Unit 3

Proportional and Linear Relationships



Big Ideas in This Unit

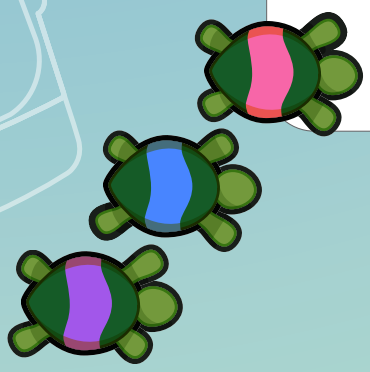
CC1 Data, Graphs, and Tables
CC2 Linear Equations
Multiple Representations of Functions
Slopes and Intercepts
CC4 Transformational Geometry



Questions for Investigation

- What can proportional relationships and slope teach you about linear relationships?
- How do equations, tables, and graphs of linear relationships connect to one another?
- What does it mean for an ordered pair to be a solution to a linear equation?

Explore: Visual Patterns
What patterns do you notice in shapes and numbers?

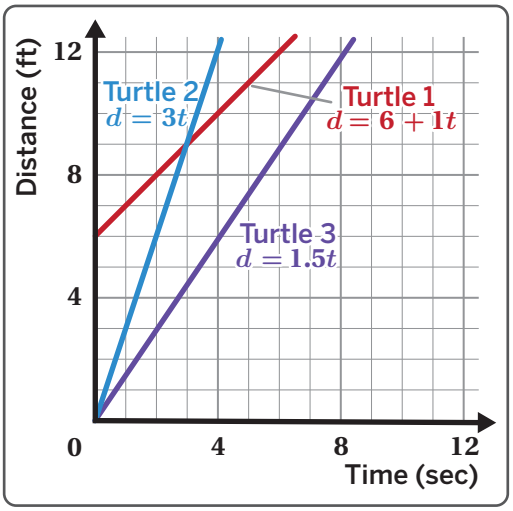


© Amplify Education, Inc and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM).

Try This

Discuss: How do you use tables, graphs, and equations to compare the turtles in a race?

Use the example if it helps with your thinking.

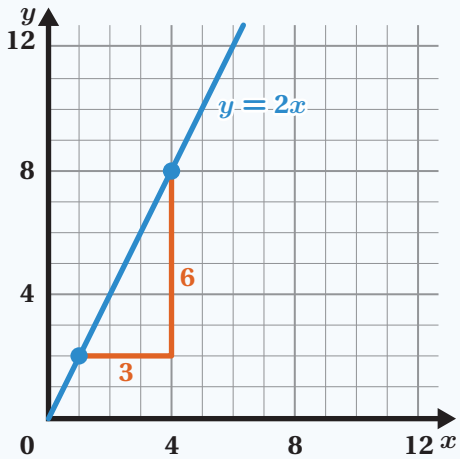


Time (sec)	Distance of Turtle 1 (ft)	Distance of Turtle 2 (ft)	Distance of Turtle 3 (ft)
0	6	0	0
1	7	3	1.5
2	8	6	3

Summary | Lesson 01

A line that passes through the origin, $(0, 0)$, represents a *proportional relationship*. The *slope* of the line represents a *unit rate* for this relationship.

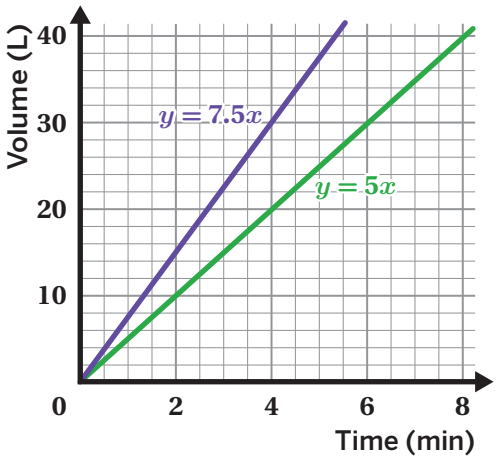
Here is a graph of the equation $y = 2x$. The slope of the line is $\frac{6}{3}$, or 2.



Try This

How are the equations of proportional relationships related to their graphs?

Use the examples if it helps with your thinking.



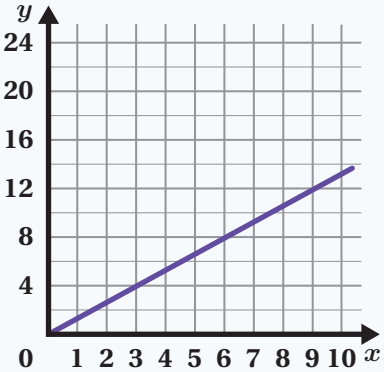
Summary | Lesson 02

We can represent a proportional relationship using the equation $y = mx$, where m represents both a unit rate and the slope of the line.

Generally, you can derive the equation $y = mx$ to represent a proportional relationship, where m is the slope of the line and also represents the unit rate for the situation. The graph of this line passes through the origin.

Here are some examples.

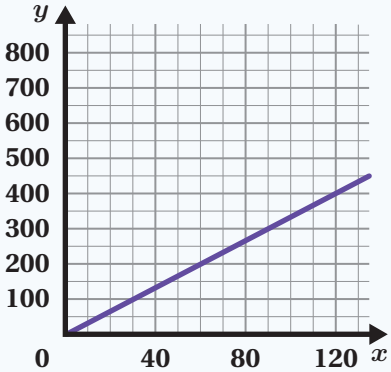
Graph A



slope: $\frac{4}{3}$

equation: $y = \frac{4}{3}x$

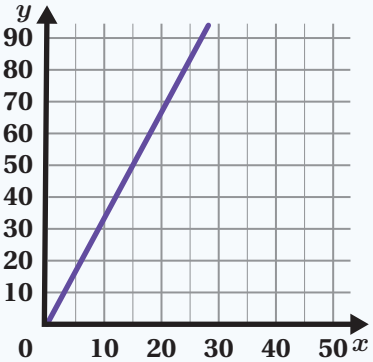
Graph B



slope: $\frac{100}{30} = \frac{10}{3}$

equation: $y = \frac{10}{3}x$

Graph C



slope: $\frac{50}{15} = \frac{10}{3}$

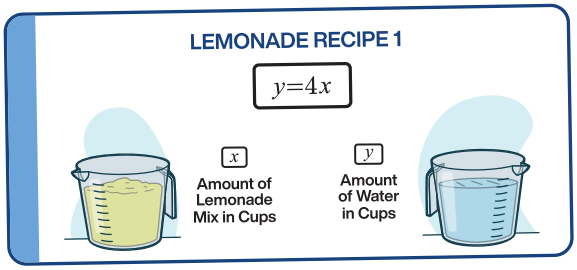
equation: $y = \frac{10}{3}x$

You can see that the slopes for Graph B and Graph C are equivalent. This means they have the same proportional relationship, even though the lines may look like they don't have the same steepness.

Try This

Discuss: How can you compare proportional relationships when they are represented differently?

Use the example if it helps with your thinking.



LEMONADE RECIPE 2

Lemonade Mix (cups)	Water (cups)
10	50
13	65
21	105

Summary | Lesson 03

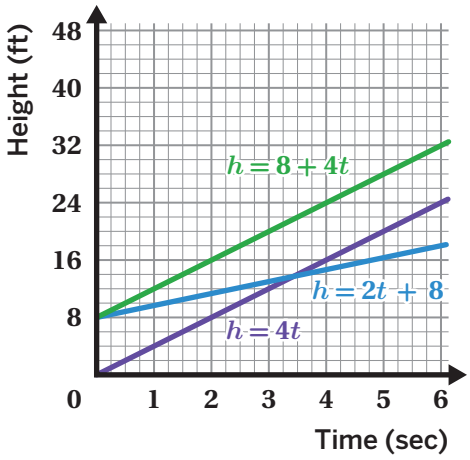
Using different representations is helpful when comparing proportional relationships, such as equations, tables, and graphs.

You can represent proportional relationships with the equation $y = mx$, where m is the slope of a line and also represents a unit rate for the situation. You can identify the slope or unit rate using all of the different representations, or by using slope triangles within a graph.

Try This

Discuss: How can you identify a non-proportional linear relationship on a graph and in an equation?

Use the example if it helps with your thinking.



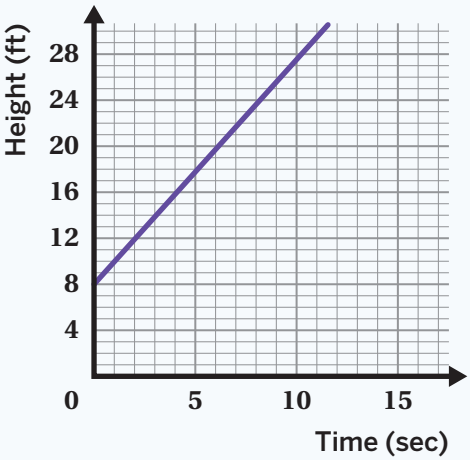
Summary | Lesson 04

Linear relationships are represented by graphs that are lines. Some linear relationships are proportional relationships and some are not. Proportional relationships have an **initial value** of zero, while non-proportional linear relationships have an initial value that is not zero.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.



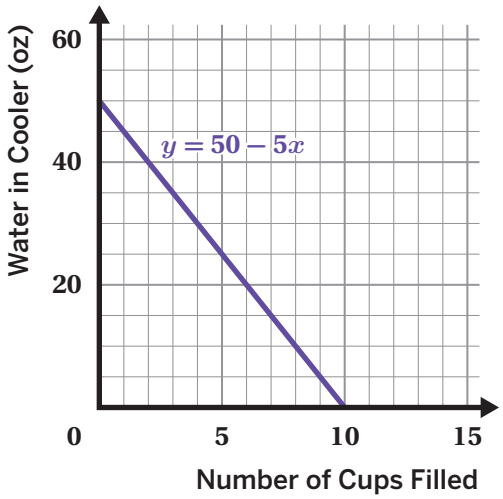
initial value The starting amount in a context.

linear A relationship is called linear when its graph is a line.

Try This

Discuss: What are 2–3 things you can determine about a situation from its graph?

Use the graph if it helps with your thinking.

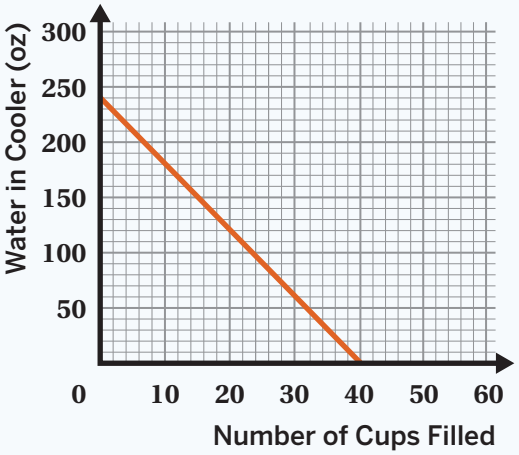


Summary | Lesson 05

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

- The **vertical intercept**, also called the *y-intercept*, is $(0, 240)$. In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the *x-intercept*, is $(40, 0)$. In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.



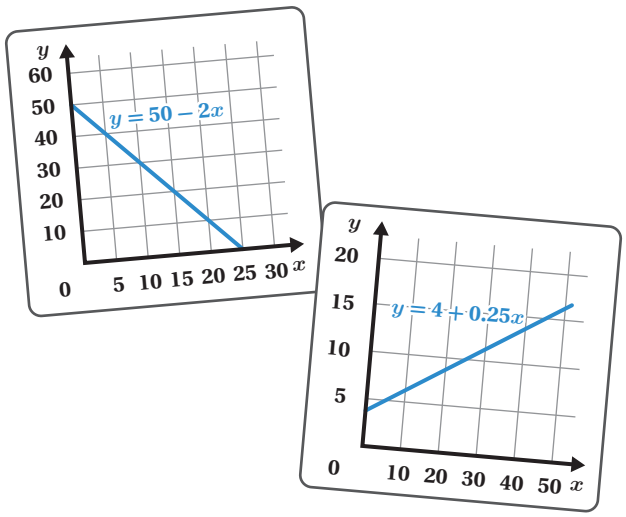
horizontal intercept (x-intercept) The point where the graph of a line crosses the horizontal axis or when $y = 0$.

vertical intercept (y-intercept) The point where the graph of a line crosses the vertical axis or when $x = 0$.

Try This

Discuss: How can you determine whether a situation has a positive or negative slope?

Use the example if it helps with your thinking.



Summary | Lesson 06

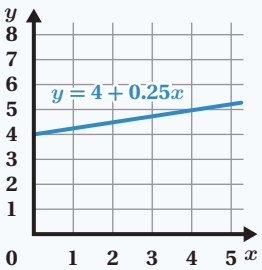
Generally, you can derive the equation $y = mx + b$ to represent any linear relationship, where m represents the slope and b represents the vertical intercept.

For linear relationships with a *positive* slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a *negative* slope, the y-values decrease at a constant rate as the x-values increase.

Here are two examples.

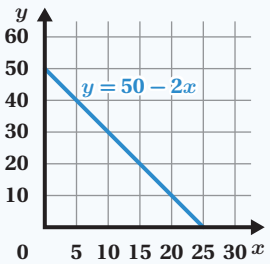
Positive slope: A medium-sized frozen yogurt costs $\$4$, plus $\$0.25$ per topping.

Let y represent the total cost of the frozen yogurt after adding x toppings.



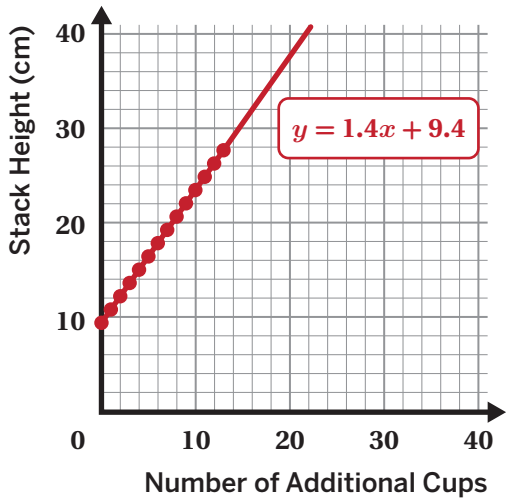
Negative slope: A student loads an arcade game card with $\$50$. Every time she plays a game, $\$2$ is subtracted from the amount available on the game card.

Let y represent the amount in dollars on the card after the student plays x games.



Try This

Discuss: How can you use a linear relationship to model a situation and make predictions?



Summary | Lesson 07

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the value of the other variable.

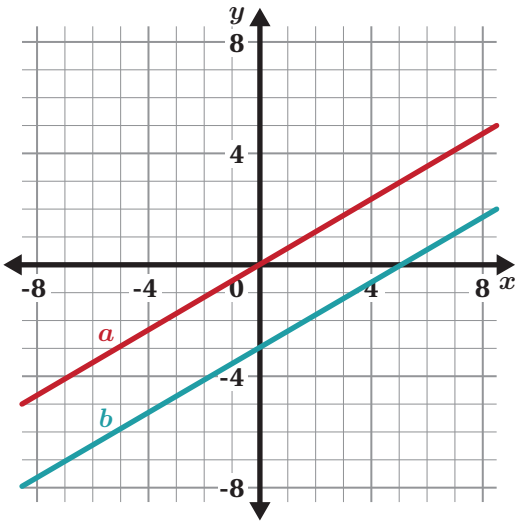
Let's say we want to know how many stacked cups will have a height of 30 centimeters. We can look at a graph and determine the number of additional cups, x, that correspond to $y = 30$. In this situation, the slope of the line tells us how many centimeters the height of the stacked cups increases by every time we add a cup to the stack.

We might think of the y-intercept as the stack height when there are 0 cups, but this doesn't make sense in this situation. Instead, the y-intercept tells us the height of the initial cup. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

Try This

Here is a graph of two equations.

How are the equations of line a and line b alike and different?



Summary | Lesson 08

A *translation* of a line that represents a proportional relationship creates a line that is parallel to the pre-image, but changes the location of the vertical intercept, also known as the y-intercept.

The equation $y = mx$ represents a line that passes through the origin. The equation $y = mx + b$ represents a vertical translation of line $y = mx$ by b units.

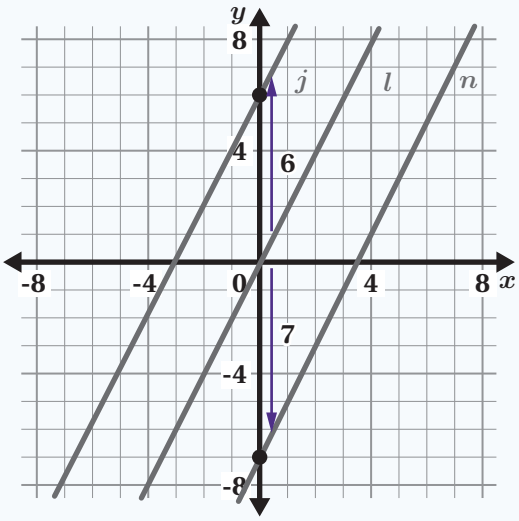
If $b > 0$, the line is translated up.

If $b < 0$, the line is translated down.

For example, the equation of line l is $y = 2x$.

If line l is translated 6 units up to produce line j, the equation of line j is $y = 2x + 6$.

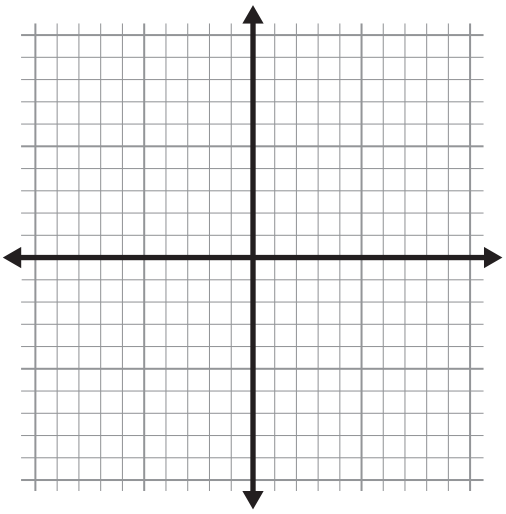
If line l is translated 7 units down to produce line n, the equation of line n is $y = 2x - 7$.



Try This

What are some strategies for finding the slope of a line that goes through two given points?

Use the graph if it helps to show your thinking.

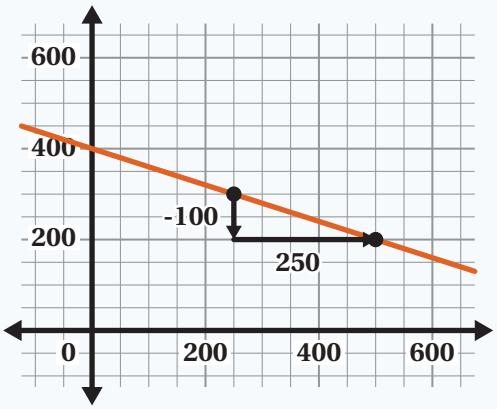


Summary | Lesson 09

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane.

Using Slope Triangles



$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = -\frac{100}{250} = -\frac{2}{5}$$

You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

Using Coordinates in a Table

x	y
250	300
500	200

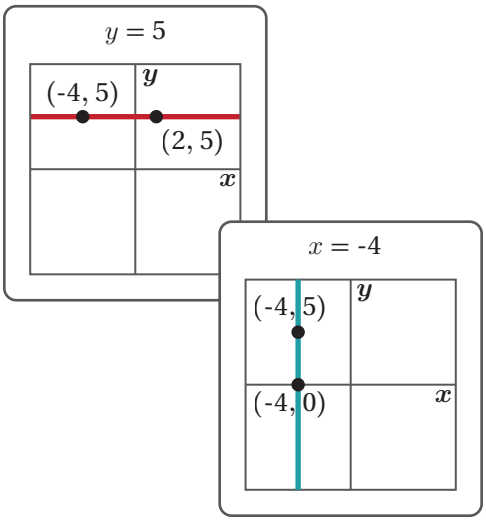
+250 (from 250 to 500) -100 (from 300 to 200)

$$\frac{\text{change in } y}{\text{change in } x} = \frac{-100}{250} = -\frac{2}{5}$$

The slope is the ratio of the vertical change to the horizontal change.

Try This

How can you tell from looking at a linear equation if its graph is a horizontal or vertical line?



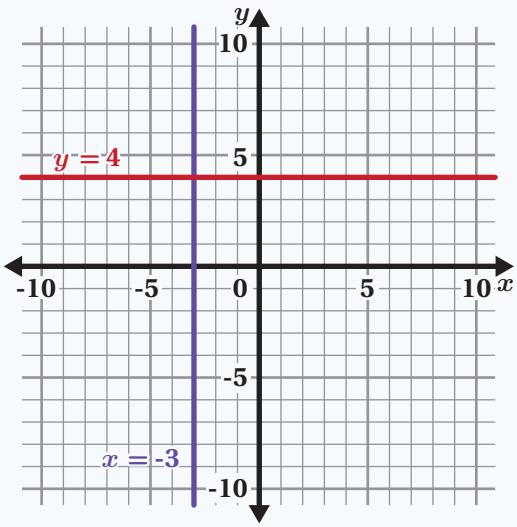
Summary | Lesson 10

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0. A horizontal equation is of the form $y = mx + b$, where the slope is zero.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

For example, the equation $y = 4$ represents the horizontal line because every point on the line has the same y-coordinate, 4.

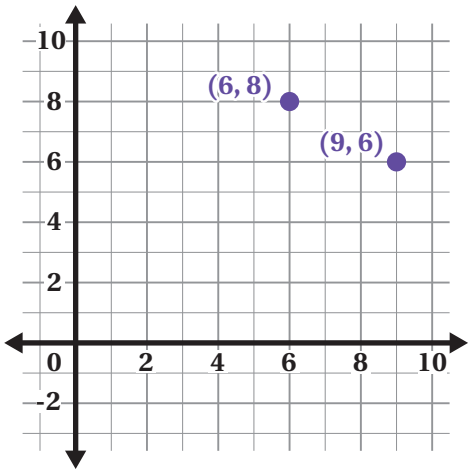
The equation $x = -3$ represents the vertical line because every point on the line has the same x-coordinate, -3.



Try This

Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.



Summary | Lesson 11

Here are two different strategies for writing the equation of a line using two given points.

Strategy Using a Table

First, calculate the slope using a table. Next, work backwards until the x-value is 0 to determine the initial value or y-intercept. Then write the equation in the form $y = mx + b$.

Tariq

x	y
3	9
4	7

+1 (x), −2 (y)

x	y	
0	15	
1	13	+2
2	11	+2
3	9	+2
4	7	+2
−1		
−1		
−1		
−1		

slope: $\frac{-2}{1}$ y-intercept = (0,15)

I can place the values in a table and work backwards to find the initial value.

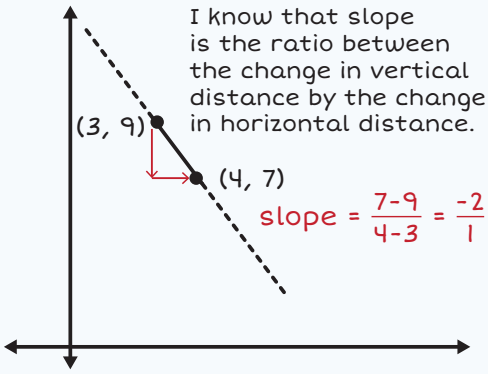
y = mx + b, where m is the slope and b is the y-intercept.

The equation is y = -2x + 15.

Strategy Using Slope Triangles

First, calculate the slope by drawing a slope triangle to determine the ratio between the change in the vertical distance by the change in the horizontal distance. Next, substitute the x- and y-value of one of the ordered pairs. Then write the equation in the form $y = mx + b$.

Nia



y = -2x + b

I'll substitute (3, 9) in for x and y

9 = -2(3) + b

9 = -6 + b

9 + 6 = b

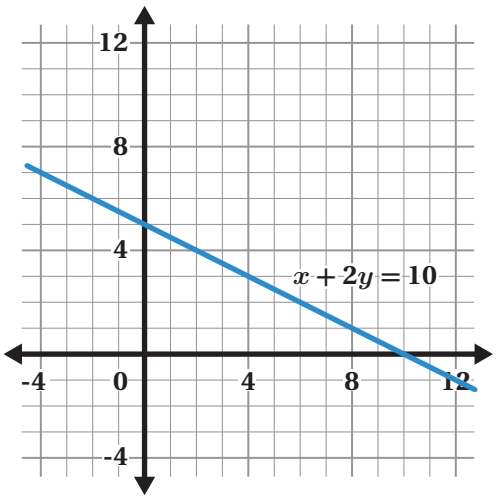
15 = b

The equation is y = -2x + 15

Try This

Discuss: How can you determine whether a point is a solution to an equation?

Use the example if it helps with your thinking.

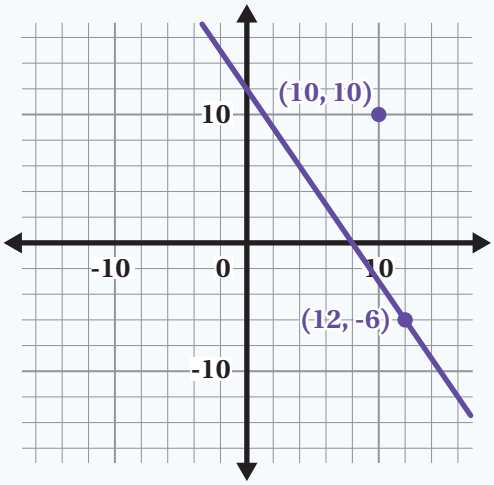


Summary | Lesson 12

A **solution to an equation** with two variables is an ordered pair (x, y) that makes the equation true.

Every point that lies on a line is a solution to that equation. Points that do not lie on the line are *not* solutions to the equation.

Here is a graph of the linear equation $3x + 2y = 24$.



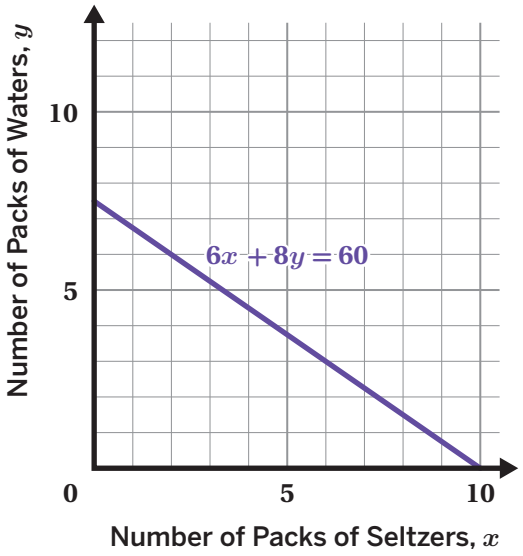
- $(12, -6)$ is a solution to the equation $3x + 2y = 24$ because the point is on the graph of the line, and because $3(12) + 2(-6) = 24$.
- $(10, 10)$ is *not* a solution because the point is not on the line, and because $3(10) + 2(10) = 50$, not 24.
- Although we can't see it on the graph, $(-10, 27)$ is also a solution because $3(-10) + 2(27) = 24$.

solution to an equation The value that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y)

Try This

Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.

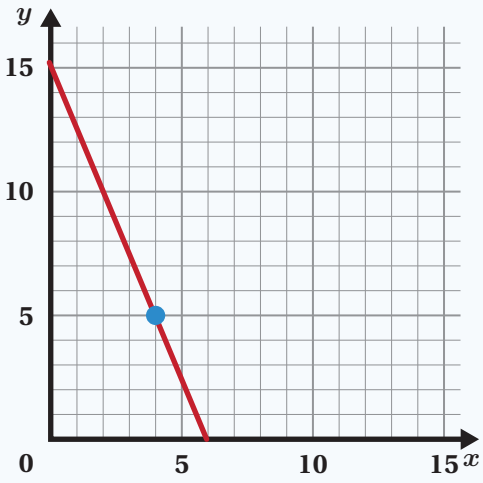


Summary | Lesson 13

The four representations of a linear relationship — table, graph, equation, and verbal description — are all useful when solving real-world problems.

Let's say a coach has a $120 budget to buy dinner for their team. Pizzas cost $20 and sandwiches cost $8. x represents the number of pizzas bought and y represents the number of sandwiches bought.

This situation can be modeled by the linear relationship $20x + 8y = 120$.



Here are two ways to show that 4 pizzas and 5 sandwiches is one solution to the equation:

- The values $x = 4$ and $y = 5$ make the equation true.

$$20(4) + 8(5) = 120$$
$$80 + 40 = 120$$
$$120 = 120$$

- The point $(4, 5)$ is on the graph of the linear relationship.

Lesson 1

Responses vary. In a table, I can calculate the speed of each turtle by seeing how much distance it gains in one second. In a graph, I can determine the speed of each turtle by examining the steepness of each turtle's graph. The steepest one, which is the one with the greatest slope, is fastest. In an equation, the coefficient (the number that is multiplied by t) determines the speed of the turtle. The greatest coefficient is the fastest turtle.

Lesson 2

Responses vary. The equations of proportional relationships always have the form $y = mx$, where m is the slope of the line.

Lesson 3

Responses vary. You can compare proportional relationships that are represented differently by looking at the unit rate. For example, Recipe 1 gives an equation where the unit rate is 4 cups of water for every 1 cup of lemonade mix. Recipe 2 gives a table where the unit rate is 5 cups of water for every 1 cup of lemonade mix.

Lesson 4

Responses vary. I can identify a non-proportional linear relationship on a graph because the line does not pass through the point $(0, 0)$. In an equation, I can look for the initial value, like the value 8 in the equations $h = 8 + 4t$ and $h = 2t + 8$.

Lesson 5

Responses vary.

- The starting point. For example, the water cooler here started with 50 ounces of water in it before any cups were filled.
- Whether there is an increase or decrease in the y-variable. In this case, after each cup was filled, the water cooler lost 5 ounces of water.
- The x-value when y is 0. On this graph, for example, the water cooler ran out of water after 10 cups.

Lesson 6

Responses vary. You can tell if a situation has a negative or positive slope by determining if the rate of change is increasing or decreasing. If it is increasing, the slope is positive. If it is decreasing, the slope is negative.

Lesson 7

Responses vary. First I can graph the relationship between the number of cups and the height of the stacked cups. Then I can look at the graph to determine the number of cups, x, that match the height of the table.

Lesson 8

Responses vary. The equations are alike because they both have the same slope, $\frac{3}{5}$. The equations are different because you have to translate vertically to get from the graph of line a, $y = \frac{3}{5}x$, to the graph of line b, $y = \frac{3}{5}x - 3$.

Lesson 9

Responses vary.

- One strategy is to draw a slope triangle using the two points on a coordinate plane, then determine the ratio of vertical change to horizontal change.
- Another strategy is to use a table to determine the horizontal change between the x-coordinates and the vertical change between the y-coordinates, then write the ratio of the change in y to the change in x.

Lesson 10

Responses vary.

- The graph of a linear equation is a horizontal line when it's in the form $y = a$.
- The graph of a linear equation is a vertical line when it's in the form $x = b$.

Lesson 11

Responses vary. First you calculate the slope by determining the change in x and change in y. Then you determine the y-intercept by setting up the equation $y = mx + b$, where m is the slope. Substitute a point in for x and y and solve for b, which represents the y-intercept. Then you can write your final equation.

Lesson 12

Responses vary.

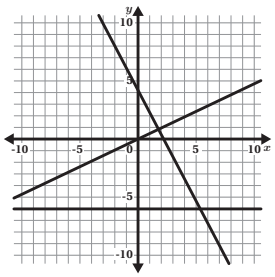
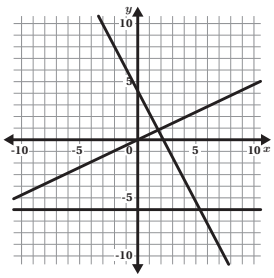
- I know the point is a solution if substituting in the x- and y-values makes the equation true.
- I know the point is a solution if it lies on the line representing the equation.

Lesson 13

Responses vary.

- I can substitute 7 for x and 2 for y and see whether these values make the equation true.
- I can check whether the point (7, 2) is on the line of the graph.

Grade 8 Unit 3 Glossary/8.º grado Unidad 3 Glosario

English	Español
C	
coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.	**coeficiente** Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.
constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity.	**constante de proporcionalidad** En una relación proporcional, el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.
E	
expression A set of numbers, variables, operations, and grouping symbols that represent a quantity. Expressions do not include an equal sign (=). For example, $5x$, $m - 4$, $3y + 0.5$, and $2(n + 8)$ are all expressions.	**expresión** Un conjunto de números, variables, operaciones y símbolos de agrupación que representan una cantidad. Las expresiones no incluyen un signo de igualdad (=). Por ejemplo, $5x$, $m - 4$, $3y + 0.5$ y $2(n + 8)$ son expresiones.
H	
horizontal intercept See x-intercept.	**intersección horizontal** Ver intersección con el eje x.
I	
initial value The starting amount in a context.	**valor inicial** La cantidad inicial en un contexto.
L	
linear A relationship is called linear when its graph is a line. 	**lineal** Una relación se llama lineal cuando su gráfica es una recta. 

© Amplify Education, Inc and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM).

English

P

proportional relationship Two variables are in a proportional relationship if the values for one variable are each multiplied by the same number to get the values for the other variable.

Carpet (sq. ft.)	Cost (dollars)
10	15.00
20	30.00
50	75.00

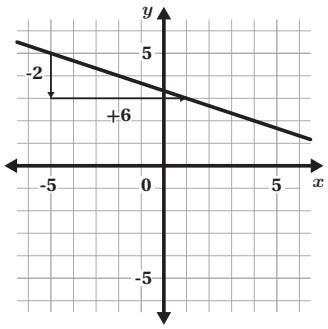
The graph of a proportional relationship is a straight line passing through the origin, $(0, 0)$.

The equation of a proportional relationship is of the form $y = kx$, where k is the constant of proportionality.

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

S

slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.



The slope of this line is $-\frac{2}{6} = -\frac{1}{3}$.

Español

P

relación proporcional Dos variables están en una relación proporcional si los valores de una cantidad se pueden multiplicar por el mismo número para obtener los valores de la otra variable.

Alfombra (pies cuadrados)	Costo (dólares)
10	15.00
20	30.00
50	75.00

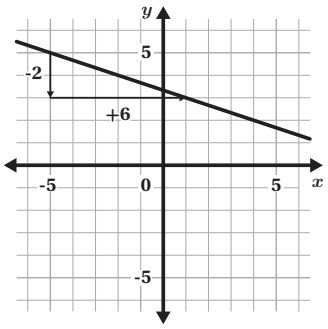
La gráfica de una relación proporcional es una recta que pasa por el origen, $(0, 0)$.

La ecuación de una relación proporcional tiene la forma $y = kx$, donde k es la constante de proporcionalidad.

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

S

pendiente Un número que describe la dirección e inclinación de una línea. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Es por eso que la pendiente de una recta a veces se denomina tasa de cambio. Para calcular la pendiente, la distancia vertical entre dos puntos cualesquiera en la recta se divide entre la distancia horizontal entre dichos puntos.

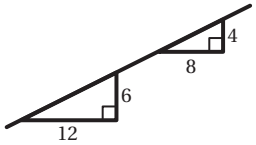


La pendiente de esta recta es $-\frac{2}{6} = -\frac{1}{3}$.

Grade 8 Unit 3 Glossary/8.º grado Unidad 3 Glosario

English

slope triangle A triangle drawn to include two points on a line in order to determine the slope of that line. The longest side lies on the line and the other two sides are vertical and horizontal. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.



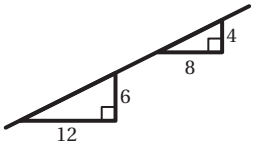
Both of these triangles are slope triangles.

solution to an equation The value or set of values that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y).

The solution to the equation $x + 15 = 8$ is $x = -7$ because $(-7) + 15 = 8$. One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.

Español

triángulo de pendiente Un triángulo que se traza para incorporar dos puntos de una recta y así determinar la pendiente de dicha recta. El lado más largo se sitúa sobre la recta y los dos lados restantes son uno vertical y otro horizontal. La altura del triángulo de pendiente representa la distancia vertical entre los puntos, mientras que la base del triángulo representa la distancia horizontal entre los puntos.



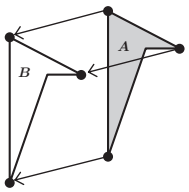
Estos dos triángulos son triángulos de pendiente.

solución de una ecuación El valor o conjunto de valores que hacen que la ecuación sea verdadera. Una solución de una ecuación con dos variables es un par de valores que hacen que la ecuación sea verdadera, y a menudo se escribe como un par ordenado, (x, y).

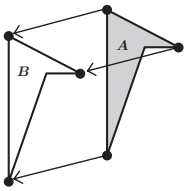
La solución de la ecuación $x + 15 = 8$ es $x = -7$ porque $(-7) + 15 = 8$. Una solución de la ecuación $4x + 3y = 24$ es $(6, 0)$ porque $4(6) + 3(0) = 24$.

T

translation A transformation that moves every point in a figure a given distance in a given direction.



traslación Una transformación que mueve cada punto de una figura una determinada distancia en una determinada dirección.



U

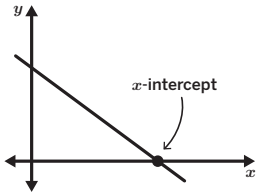
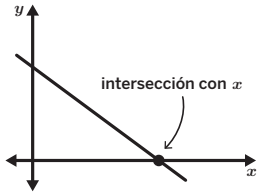
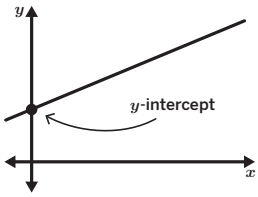
unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

© Amplify Education, Inc and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM).

English		Español
	V	
vertical intercept See y-intercept.		**intersección vertical** Ver intersección con el eje y.
	X	
x-intercept The point where the graph of a line crosses the horizontal axis or when $y = 0$. The x–intercept is sometimes called the horizontal intercept. 		**intersección con el eje x** El punto donde la gráfica de una recta se cruza con el eje horizontal o cuando $y = 0$. La intersección con el eje x a veces se denomina intersección horizontal. 
	Y	
y-intercept The point where the graph of a line crosses the vertical axis or when $x = 0$. The y-intercept is sometimes called the vertical intercept. 		**intersección con el eje y** El punto donde la gráfica de una recta se cruza con el eje vertical o cuando $x = 0$. La intersección con el eje y a veces se denomina intersección vertical. 