Amplify Desmos Math FLORIDA

Teacher Edition

GRADE 7

Volume 1



Amplify Desmos Math FLORIDA

Grade 7

Volume 1: Units 1–4

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Teacher,

Welcome to Amplify Desmos Math Florida! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

- A program designed around the **Florida B.E.S.T. Standards for Mathematics**, grounded in the strands, standards, and benchmarks, and the importance of student exploration and investigation.
- **Deep and intentional alignment** to the Florida B.E.S.T. standards, Florida English Language Arts standards for K-12 Mathematics, and Florida English Language Development Standards related to the Language of Mathematics.
- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Manipulatives, visual models, discussions, and drawings to encourage perseverance and revision.
  - » Easy-to-follow lesson plans, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency, help students review previous topics, and gain experience with FAST Assessment problem types.
- **Recommended differentiation moves** that meet the needs of all learners.
- **Diagnostic, formative, and summative assessments** along with lesson-level checks for understanding.
- **A caregiver resource** for each unit that includes explanations of key math concepts and problems to try with your student.

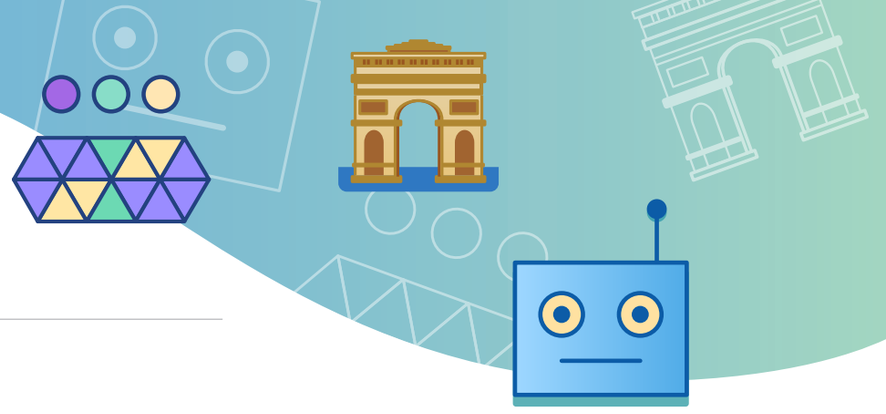
We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math Florida team



Unit 1 Scale Drawings

Students examine and create scaled copies of figures, then explore scale drawings of real-life objects.



Pre-Unit Check 1L

Sub-Unit 1 Scaled Copies 2

		B.E.S.T. Benchmark Connection
1.01	Scaling Machines | What Are Scaled Copies? 3A	1.01: MA.7.GR.1.5, MTR.4.1, MTR.5.1
1.02	Scaling Robots | Lengths and Scaled Copies 12A	1.02: MA.7.GR.1.5, MTR.4.1, MTR.5.1
1.03	Make It Scale | Drawing Scaled Copies 19A	1.03: MA.7.GR.1.5, MTR.4.1
1.04	Scale Factor Challenges | Effects of Scale Factors 26A	1.04: MA.7.GR.1.5, MTR.4.1, MTR.5.1
1.05	Tiles | Scaling and Area 34A	1.05: MA.7.GR.1.5, MTR.1.1, MTR.3.1, MTR.4.1

Sub-Unit Quiz 41A

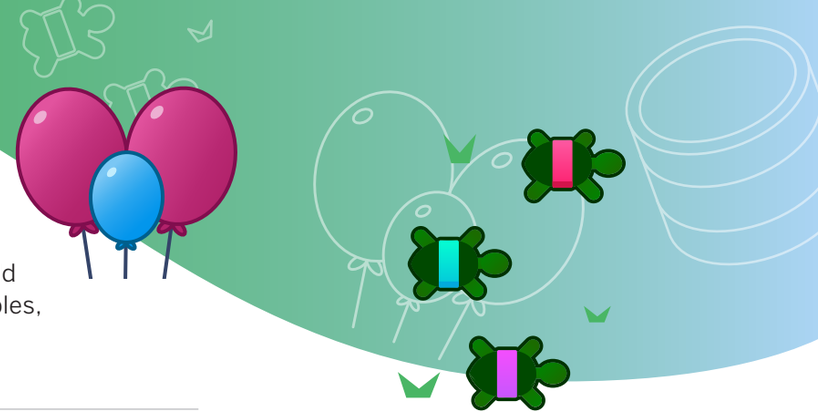
Sub-Unit 2 Scale Drawings 42

		B.E.S.T. Benchmark Connection
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1.07	Will It Fit? | Scale Drawings 49A	1.07: MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1
1.08	Scaling States, Part 1 | Creating Scale Drawings 56A	1.08: MA.7.GR.1.5, MTR.4.1, MTR.7.1
1.09	Scaling States, Part 2 | Creating Scale Drawings 62A	1.09: MA.7.GR.1.5, MTR.5.1, MTR.7.1
1.10	Scaling Buildings | Same Object, Different Scales 68A	1:10: MA.7.GR.1.5, MTR.2.1, MTR.4.1, MTR.5.1

End-of-Unit Assessment 75A

Unit 2 Introducing Proportional Relationships

Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.



Pre-Unit Check 77L

Sub-Unit 1 Proportional Relationships in Tables 78

B.E.S.T. Benchmark Connection
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2.02 Balloon Float | Introducing Proportional Relationships With Tables ..... 86A — **2.02: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2,** MTR.4.1, MTR.5.1
2.03 Sugar, Spice, and Everything Rice | Constant of Proportionality ..... 94A — **2.03: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2,** MTR.2.1, MTR.6.1

Sub-Unit 2 Proportional Relationships in Equations 100

B.E.S.T. Benchmark Connection
2.04 Robot Factory | Proportional Relationships and Equations ..... 101A — **2.04: MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4,** MTR.2.1, MTR.3.1
2.05 Snapshots | More Equations of Proportional Relationships ..... 107A — **2.05: MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.2.1, MTR.7.1
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Sub-Unit Quiz 127A

Sub-Unit 3 Proportional Relationships in Graphs 128

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2.08 DinoPops | Introducing Graphs of Proportional Relationships ..... 129A — **2.08: MA.7.AR.3.2, MA.7.AR.4.1,** MTR.2.1, MTR.5.1
2.09 Gallon Challenge | Interpreting Graphs of Proportional Relationships ..... 137A — **2.09: MA.7.AR.3.2, MA.7.AR.4.2,** MTR.3.1, MTR.7.1
2.10 Three Turtles | Proportional Relationships and Graphs ..... 144A — **2.10: MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.1.1, MTR.2.1, MTR.4.1

Sub-Unit 4 Using Proportional Relationships 152

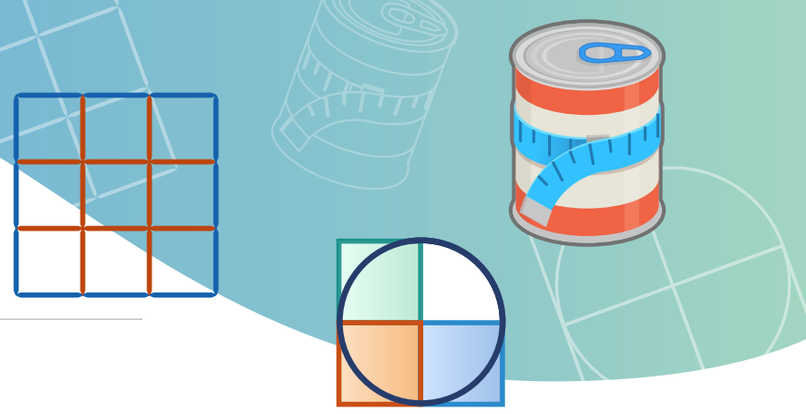
B.E.S.T. Benchmark Connection
2.11 Four Representations | Connecting Descriptions, Tables, Graphs, and Equations ..... 153A — **2.11: MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4,** MTR.2.1, MTR.7.1
2.12 Water Efficiency, Part 1 | Let's Put It to Work ..... 159A — **2.12: MA.7.AR.3.2, MA.7.AR.4.2 MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.1.1, MTR.7.1
2.13 Water Efficiency, Part 2 | Let's Put It to Work ..... 165A — **2.13: MA.7.AR.3.2, MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.2.1

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End-of-Unit Assessment 171A

Unit 3 Measuring Circles

Students explore the relationships between the radius, diameter, circumference, and area of a circle.



Pre-Unit Check 173L

Sub-Unit 1 Circumference of a Circle 174

B.E.S.T. Benchmark Connection
3.01: Building Toward MA.7.GR.1.3, MTR.5.1, MTR.6.1
3.02: MA.7.GR.1.3, MTR.4.1
3.03: MA.7.GR.1.3, MTR.1.1, MTR.4.1, MTR.6.1

3.01 Toothpicks | Perimeter and Proportional Relationships 175A

3.02 Is It a Circle? | Parts of a Circle 182A

3.03 Measuring Around | Estimating and Calculating Circumference 189A

Sub-Unit Quiz 195A

Sub-Unit 2 Area of a Circle 196

B.E.S.T. Benchmark Connection
3.04: MA.7.GR.1.2, MTR.1.1, MTR.4.1, MTR.6.1
3.05: MA.7.GR.1.4, MTR.4.1, MTR.5.1, MTR.6.1
3.06: MA.7.GR.1.4, MTR.1.1, MTR.3.1, MTR.4.1

3.04 Area Strategies | Estimating and Calculating Area 197A

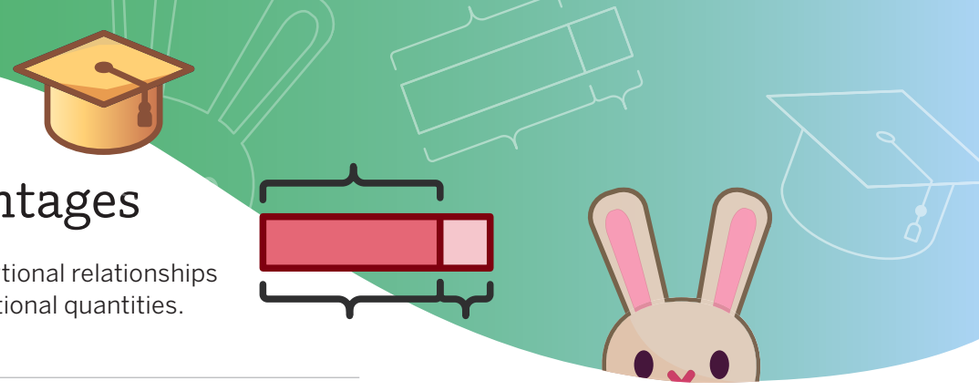
3.05 Radius Squares | Exploring Circle Area 203A

3.06 Circle Area | Calculating Circle Area 210A

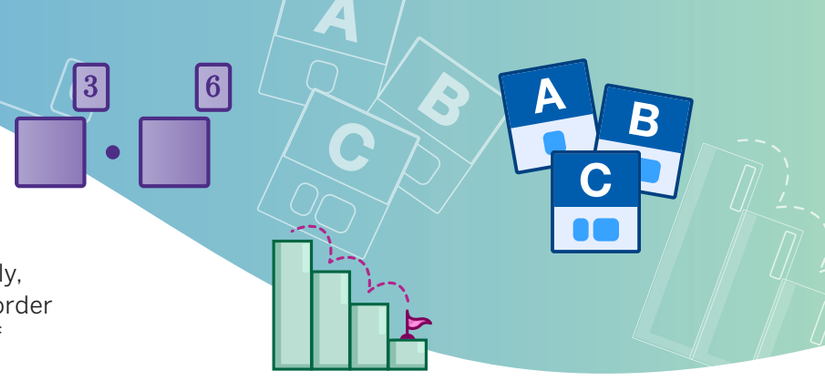
End-of-Unit Assessment 215A

Unit 4 Proportional Relationships and Percentages

Students apply what they have learned about proportional relationships to solve problems involving percent change and fractional quantities.



		B.E.S.T. Benchmark Connection
	Pre-Unit Check 217L	
Sub-Unit 1	Percentages as Proportional Relationships 218	B.E.S.T. Benchmark Connection
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4.03	100% | Percent Increase and Decrease With Double Number Lines 234A	**4.03: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.1.1, MTR.2.1
4.04	Percent Machines | Calculating Unknowns With Percentages 241A	**4.04: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.2.1, MTR.4.1
4.05	Back in My Day | Real-World Situations Involving Percent Increase 248A	**4.05: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.6.1, MTR.7.1
	Sub-Unit Quiz 254B	
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4.06	Tax, Tips, and Fees | Multistep Percent Problems 257A	**4.06: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.3.1, MTR.4.1
4.07	How Simple, How Interesting | Calculating Simple Interest Using a Formula 265A	**4.07: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.3.1, MTR.7.1
4.08	Bookcase Builder | Percent Error 271A	**4.08: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.3.1, MTR.4.1
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4.11	Pen Pals | Converting Units 295A	**4.11: MA.7.AR.3.3,** MTR.2.1, MTR.4.1
4.12	Decimal Deep Dive | Writing Fractions as Decimals 302A	**4.12: MA.7.NSO.1.2,** MTR.5.1
4.13	Population and Fast Fashion | Posing Problems Involving Percents 309A	**4.13: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.4.1, MTR.7.1
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	End-of-Unit Assessment 316A	



Unit 5 Positive and Negative Rational Numbers

Students extend what they learned in Grade 6 to add, subtract, multiply, and divide positive and negative rational numbers. Students also use order of operations to evaluate numerical expressions and apply the Laws of Exponents to generate equivalent expressions.

Pre-Unit Check 317L

Sub-Unit 1 Operations with Rational Numbers 318

5.01	Code Breaking | Equivalent Forms	319A
5.02	Bumpers | Adding and Subtracting on the Number Line	326A
5.03	Back in Time | Position, Rate, and Time	333A
5.04	Speeding Turtles | Dividing Rational Numbers	341A
5.05	Order Matters | Evaluating Numerical Expressions	348A

Sub-Unit Quiz 355A

B.E.S.T. Benchmark Connection
5.01: MA.7.NSO.1.2, MTR.3.1, MTR.4.1
5.02: MA.7.NSO.2.2, MTR.3.1, MTR.4.1, MTR.5.1
5.03: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1
5.04: MA.7.NSO.2.2, MA.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1
5.05: MA.7.NSO.2.1, MA.7.NSO.2.2, MTR.1.1, MTR.5.1

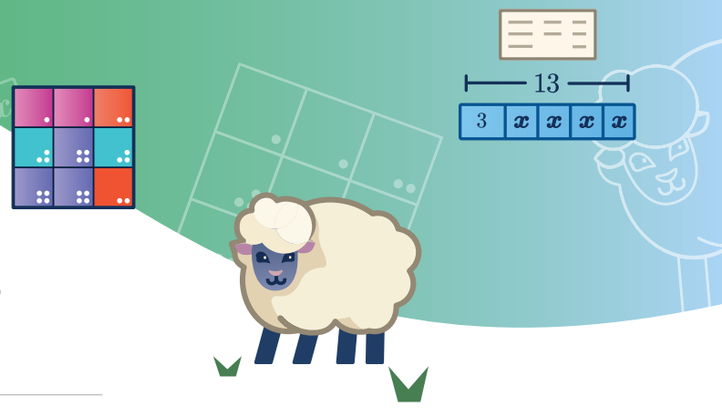
Sub-Unit 2 Laws of Exponents 356

5.06	Combining Exponents | Equivalent Expressions with Exponents	357A
5.07	Power Pairs | Multiplying Powers and Powers of Powers	364A
5.08	Rewriting Powers | Rewriting Exponential Expressions as a Single Power	370A
5.09	Exponent Zero | Using Patterns to Understand Zero Exponents	377A
5.10	Negative Bases | Powers With Negative Bases	383A

End-of-Unit Assessment 389A

B.E.S.T. Benchmark Connection
5.06: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
5.07: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
5.08: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
5.09: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1
5.10: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1

Unit 6 Expressions, Equations, and Inequalities



Students extend what they learned in Grade 6 about solving one-step equations to solve equations of the form $px + q = r$ and $p(x + q) = r$, and equations that include expanding, factoring, or adding terms. Students also solve inequalities and graph their solutions on a number line.

Pre-Unit Check 391L

Sub-Unit 1 Equations and Tape Diagrams 392

B.E.S.T. Benchmark Connection
6.01 Smudged Receipts | Connecting Contexts and Tape Diagrams ..... 393A — **6.01: MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.5.1, MTR.7.1
6.02 Equations | Representing Contexts With Tape Diagrams and Equations ..... 400A — **6.02: MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.03 Seeing Structure | Practice With Tape Diagrams and Equations ..... 408A — **6.03: MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.5.1, MTR.7.1

Sub-Unit 2 Solving Equations 414

B.E.S.T. Benchmark Connection
6.04 Balancing Moves | Introduction to Balanced Hangers ..... 415A — **6.04: MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.05 Balancing Equations | Solving Equations With Balanced Hangers ..... 422A — **6.05: MA.7.NSO.2.2, MA.7.AR.1.2, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.06 Keeping It True | Solving Equations ..... 429A — **6.06: MA.7.NSO.2.2, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.07 Factoring and Expanding | Options for Solving One Equation ..... 435A — **6.07: MA.7.NSO.2.2, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.08 Always-Equal Machines | Equivalent Expressions ..... 442A — **6.08: MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.09 The Same but Different | Determining Whether Expressions are Equivalent ..... 450A — **6.09: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1
6.10 Collect the Squares | Adding and Subtracting Expressions ..... 457A — **6.10: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1,** MTR.3.1, MTR.4.1, MTR.5.1
6.11 Pass the Equation | Solving Equations Multiple Ways ..... 465A — **6.11: MA.7.NSO.2.2, MA.7.AR.2.2,** MTR.2.1, MTR.3.1, MTR.5.1
6.12 Community Day | Using Equations to Solve Problems ..... 471A — **6.12: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.7.1

Sub-Unit Quiz 476B

Sub-Unit 3 Inequalities 477–478

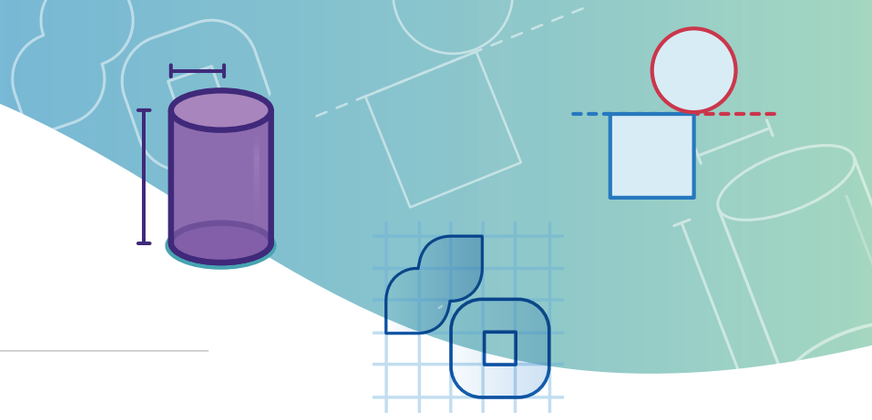
B.E.S.T. Benchmark Connection
6.13 I Saw the Signs | Inequalities on the Number Line ..... 479A — **6.13: Building Toward MA.7.AR.2.1,** MTR.2.1, MTR.4.1, MTR.5.1
6.14 Unbalanced Hangers | Solutions to Inequalities ..... 487A — **6.14: MA.7.NSO.2.2, MA.7.AR.2.1,** MTR.2.1, MTR.3.1, MTR.4.1
6.15 Shira the Sheep | Solving Inequalities With Positive and Negative Numbers ..... 495A — **6.15: MA.7.NSO.2.2, MA.7.AR.2.1,** MTR.3.1, MTR.4.1, MTR.6.1
6.16 Write Them and Solve Them | Modeling With Inequalities ..... 502A — **6.16: MA.7.NSO.2.2, MA.7.AR.2.1,** MTR.3.1, MTR.6.1, MTR.7.1

Math in Action 508

End-of-Unit Assessment 508A

Unit 7 Polygons and Cylinders

Students calculate the areas of quadrilaterals, polygons, and composite figures. Students also explore the volume and surface area of cylinders.



Pre-Unit Check 509L

Sub-Unit 1 Area 510

B.E.S.T. Benchmark Connection
7.01: MA.7.GR.1.1, MTR.1.1, MTR.2.1, MTR.5.1
7.02: MA.7.GR.1.2, MTR.1.1, MTR.4.1, MTR.5.1
7.03: MA.7.GR.1.1, MA.7.GR.1.2, MTR.2.1, MTR.4.1, MTR.7.1
7.04: MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.4.1, MTR.5.1
7.05: MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.3.1

7.01 Access to All Areas | Investigating Areas of Special Quadrilaterals 511A

7.02 Piles of Polygons | Investigating Polygons and Their Areas 518A

7.03 Shapes Inside Shapes | Area of Composite Figures 525A

7.04 Area Challenges | Calculating Areas of Complex Shapes 532A

7.05 Circle vs. Square | Calculating Area Given Perimeter 538A

Sub-Unit Quiz 543A

Sub-Unit 2 Solid Geometry 544

B.E.S.T. Benchmark Connection
7.06: MA.7.GR.2.3, MTR.2.1, MTR.3.1, MTR.4.1
7.07: MA.7.GR.2.3, MTR.1.1, MTR.5.1, MTR.7.1
7.08: MA.7.GR.2.1, MA.7.GR.2.2, MTR.1.1, MTR.2.1, MTR.7.1

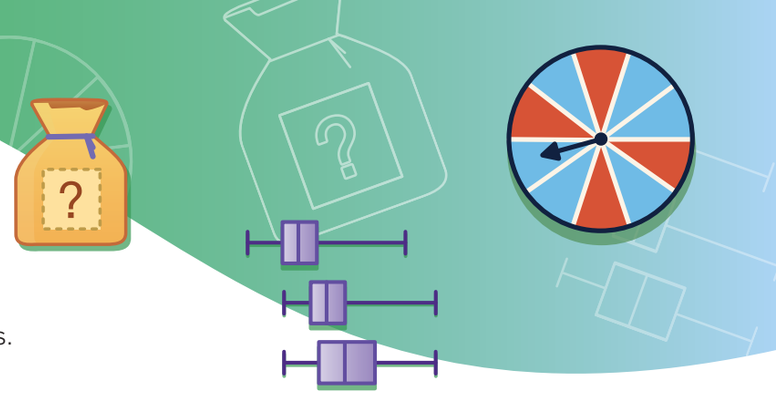
7.06 Cylinders | Volumes of Cylinders 545A

7.07 Scaling Cylinders | Applying Volumes of Cylinders 553A

7.08 From Flat to Round | Surface Area of Cylinders 560A

End-of-Unit Assessment 567A

Unit 8 Probability and Sampling



Students learn about probability as a way to describe the likelihood of unknown events and estimate the probability of real-world situations. They use samples to draw conclusions about and compare populations.

Pre-Unit Check 569L

Sub-Unit 1 Probability 570

8.01 How Likely? | Chance Experiments 571A
8.02 Prob-bear-bilities | Intro to Probability 577A
8.03 Mystery Bag | Predicting Sample Spaces 586A
8.04 Spin Class | Repeated Experiments 594A
8.05 Is It Fair? | Comparing Probabilities and Experiments 602A

Sub-Unit Quiz 609A

Sub-Unit 2 Sampling 610

8.06 The County Fair | Creating Circle Graphs 611A
8.07 Choices, Choices | Choosing an Appropriate Representation for Data 619A
8.08 On Display | Creating an Appropriate Representation for Data 626A
8.09 Car, Bike, or Train? | Using Mean, Median, and Range to Compare Groups 633A
8.10 Crab Island | Sampling From Large Populations 640A
8.11 Flower Power | Using Percentages to Predict Populations 648A
8.12 Plots and Samples | Using Statistical Measures to Make Predictions 655A
8.13 School Newspaper | Comparing Populations 663A

Math in Action 670

End-of-Unit Assessment 670A

B.E.S.T. Benchmark Connection
8.01: MA.7.DP.2.1, MTR.2.1, MTR.3.1
8.02: MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1
8.03: MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.3.1, MTR.6.1
8.04: MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.1.1, MTR.4.1
8.05: MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.5.1, MTR.6.1

B.E.S.T. Benchmark Connection
8.06: MA.7.DP.1.4, MTR.4.1, MTR.5.1
8.07: MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.4.1, MTR.7.1
8.08: MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.7.1
8.09: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1
8.10: MA.7.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1
8.11: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3, MTR.4.1, MTR.5.1, MTR.6.1
8.12: MA.7.DP.1.1, MA.7.DP.1.2, MTR.5.1, MTR.6.1
8.13: MA.7.DP.1.2, MTR.4.1, MTR.7.1

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math Florida, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

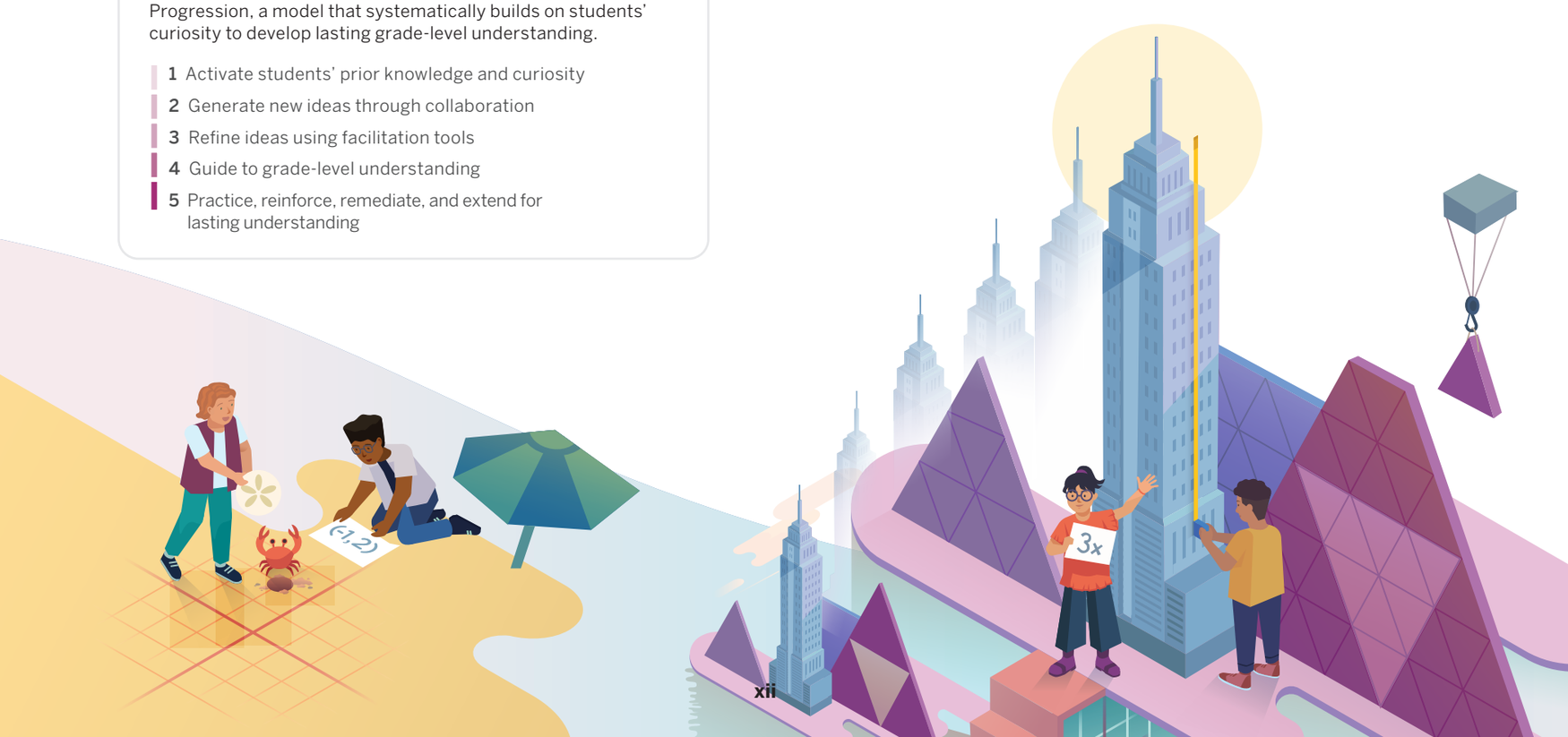
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.





Student thinking is valuable and can be made evident

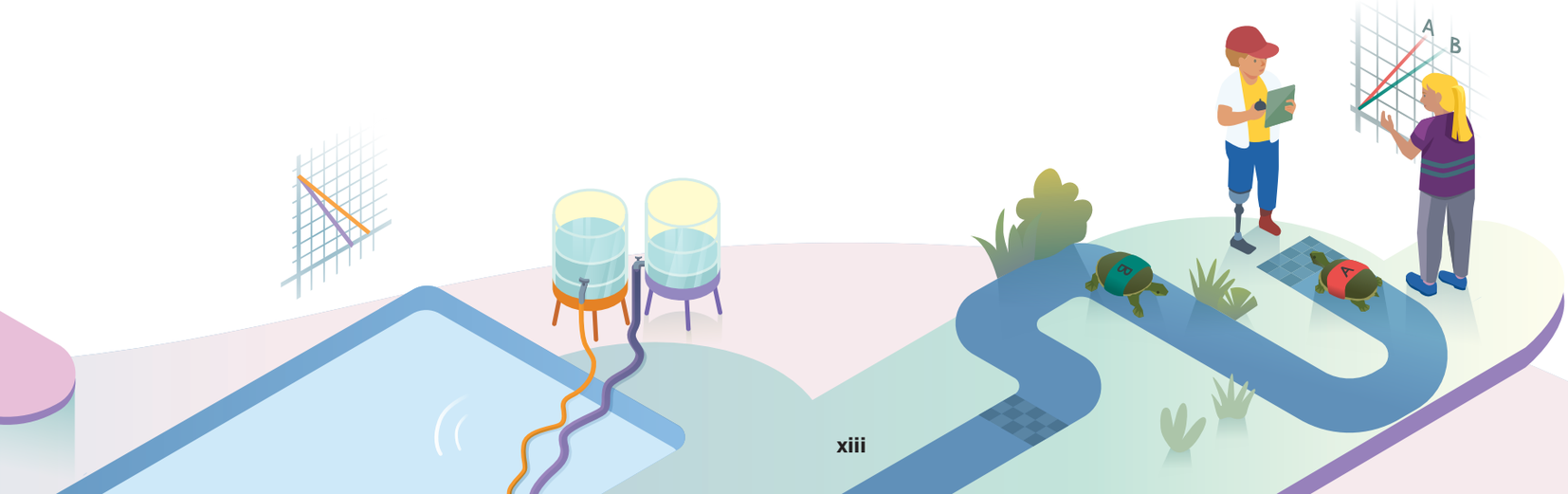
Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Math that motivates students

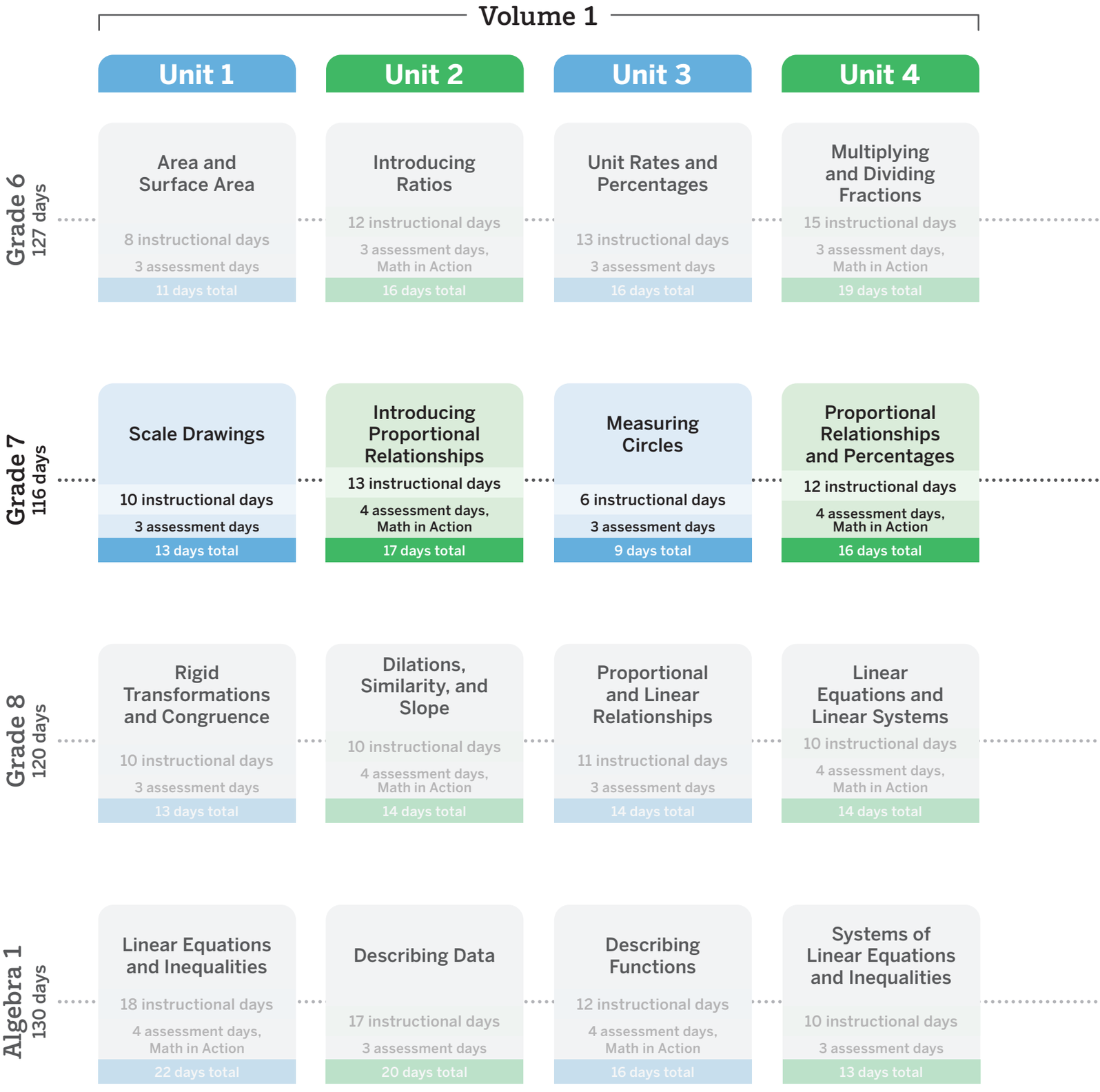
Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity.

Our curriculum supports collaborative classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.



Scope and Sequence | Grade 6–Algebra 1



Volume 2

Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Decimal Arithmetic 10 instructional days 3 assessment Days 13 days total	**Expressions and Equations** 11 instructional days 4 assessment Days, Math in Action 15 days total	**Positive and Negative Numbers** 18 instructional days 3 assessment Days 21 days total	**Describing Data** 12 instructional days 4 assessment Days, Math in Action 16 days total	
Positive and Negative Rational Numbers 10 instructional days 3 assessment days 13 days total	**Equations, Expressions, and Inequalities** 16 instructional days 4 assessment days, Math in Action 20 days total	**Polygons and Cylinders** 8 instructional days 3 assessment days 11 days total	**Probability and Sampling** 13 instructional days 4 assessment days, Math in Action 17 days total	
Functions 7 instructional days 3 assessment days 10 days total	**Associations in Data** 9 instructional days 4 assessment days, Math in Action 13 days total	**Exponents and Scientific Notation** 13 instructional days 3 assessment days 16 days total	**The Pythagorean Theorem and Irrational Numbers** 15 instructional days 4 assessment days, Math in Action 19 days total	**Probability** 5 instructional days 2 assessment days 7 days total
Exponential Functions 15 instructional days 4 assessment days, Math in Action 19 days total	**Quadratic Functions** 15 instructional days 3 assessment days 18 days total	**Quadratic Equations** 18 instructional days 4 assessment days, Math in Action 22 days total		

Program Resources

For Students


Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1

Make It Scale

Let's draw scaled copies.

Warm-Up

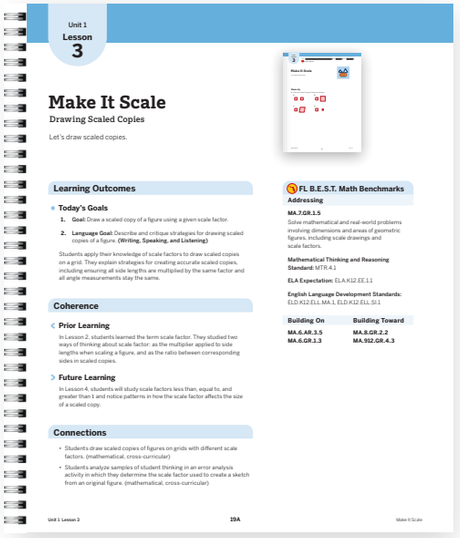
1. Which pair doesn't belong? Explain your thinking.

A. B. C. D.

Unit 1 Lesson 3 19 Warm-Up

- Student Edition (two-volume)

For Teachers


Unit 1
Lesson
3

Make It Scale

Drawing Scaled Copies

Let's draw scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Draw a scaled copy of a figure using a given scale factor.
2. **Language Goal:** Describe and critique strategies for drawing scaled copies of a figure. **(Writing, Speaking, and Listening)**

Students apply their knowledge of scale factors to draw scaled copies on a grid. They explain strategies for creating accurate scaled copies, including ensuring all side lengths are multiplied by the same factor and all angle measurements stay the same.

Coherence

Prior Learning

In Lesson 2, students learned the term scale factor. They studied two ways of thinking about scale factor: as the multiplier applied to side lengths when scaling a figure, and as the ratio between corresponding sides in scaled copies.

Future Learning

In Lesson 4, students will study scale factors less than, equal to, and greater than 1 and notice patterns in how the scale factor affects the size of a scaled copy.

Connections

- Students draw scaled copies of figures on grids with different scale factors. (mathematical, cross-curricular)
- Students analyze samples of student thinking in an error analysis activity in which they determine the scale factor used to create a sketch from an original figure. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Unit 1 Lesson 3 19A Make It Scale

- Teacher Edition (two-volume)

Additional Resources



Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior grade Mini-Lessons
- Extensions

Optional: Manipulative Kits



hand2mind

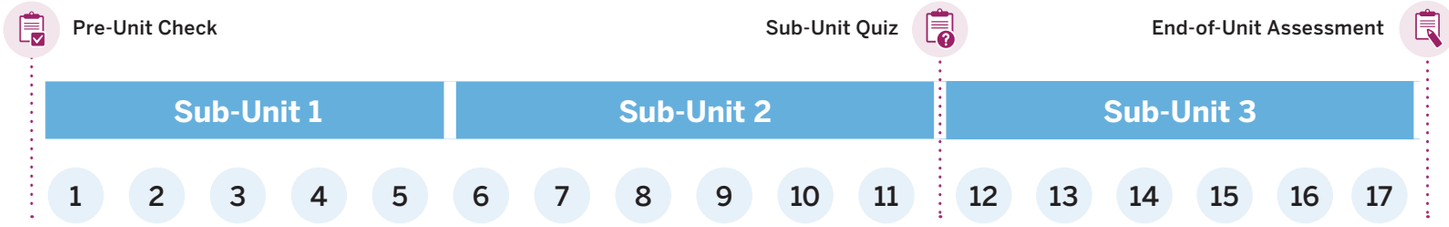
Program Architecture

Course

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
13 days	17 days	9 days	16 days	13 days	20 days	11 days	17 days

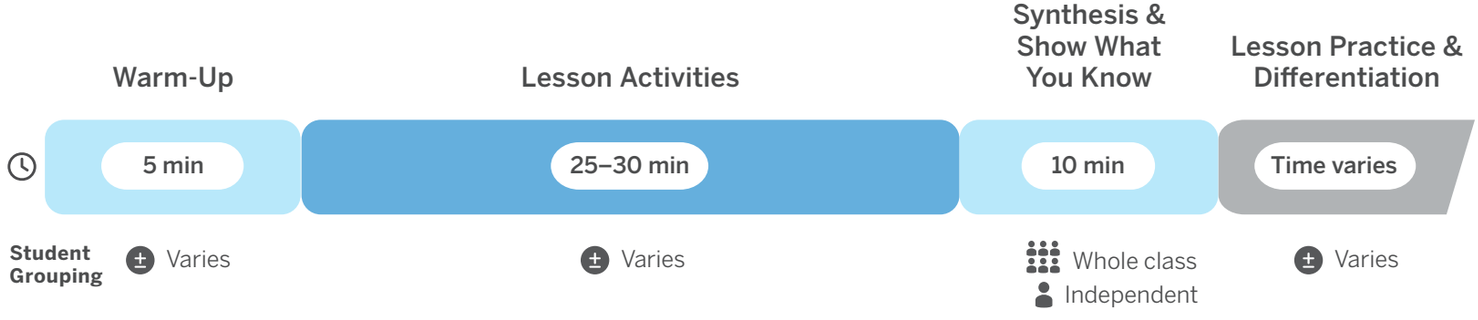
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

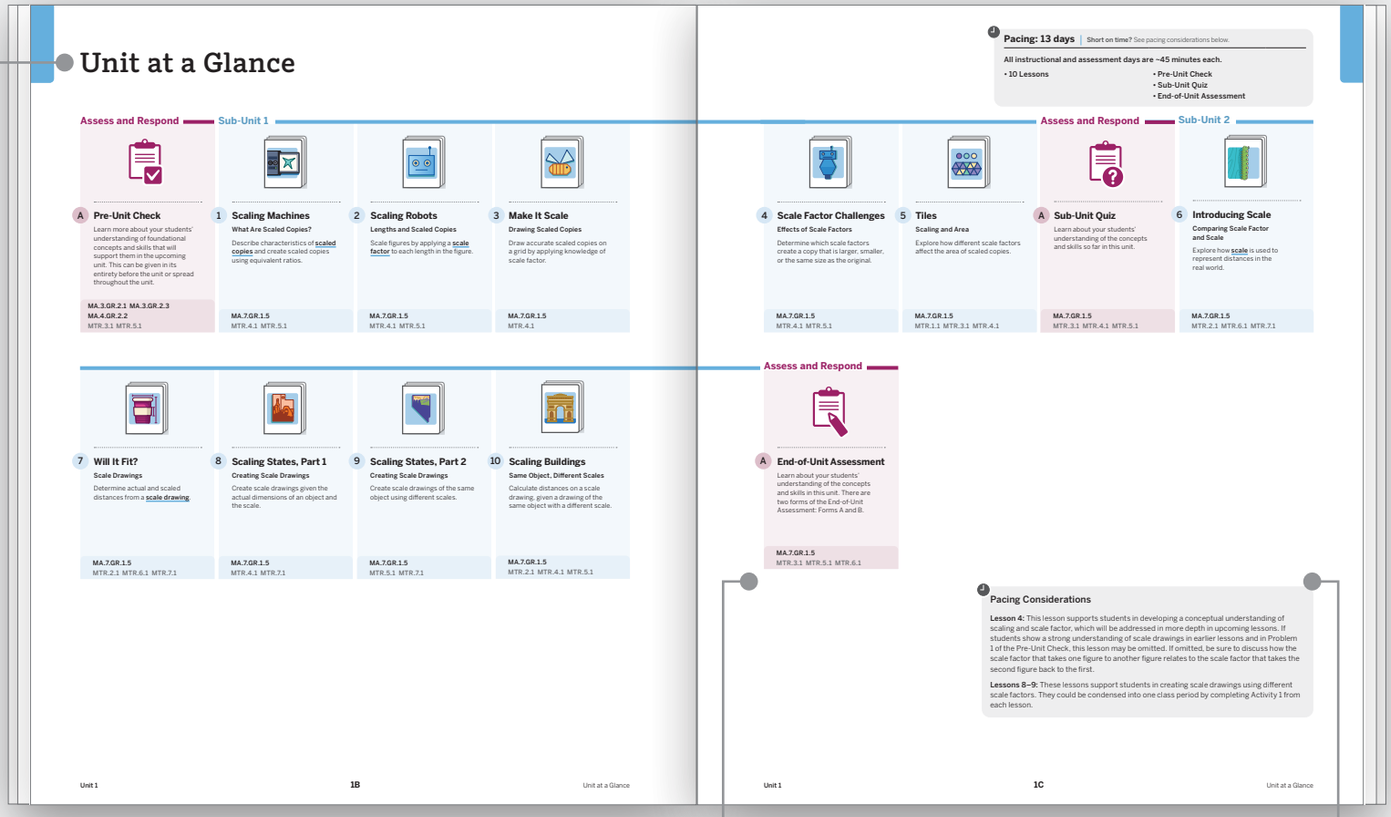
Lesson



Navigating This Program

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Unit at a Glance

Assess and Respond — Sub-Unit 1

A Pre-Unit Check
Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.
MA.3.GR.2.1 MA.3.GR.2.3 MA.4.GR.2.2 MTR.3.1 MTR.5.1

1 Scaling Machines
What Are Scaled Copies?
Describe characteristics of scaled copies and create scaled copies using equivalent ratios.
MA.7.GR.1.5 MTR.4.1 MTR.5.1

2 Scaling Robots
Lengths and Scaled Copies
Scale figures by applying a scale factor to each length in the figure.
MA.7.GR.1.5 MTR.4.1 MTR.5.1

3 Make It Scale
Drawing Scaled Copies
Draw accurate scaled copies on a grid by applying knowledge of scale factor.
MA.7.GR.1.5 MTR.4.1

7 Will It Fit?
Scale Drawings
Determine actual and scaled distances from a scale drawing.
MA.7.GR.1.5 MTR.2.1 MTR.6.1 MTR.7.1

8 Scaling States, Part 1
Creating Scale Drawings
Create scale drawings given the actual dimensions of an object and the scale.
MA.7.GR.1.5 MTR.4.1 MTR.7.1

9 Scaling States, Part 2
Creating Scale Drawings
Create scale drawings of the same object using different scales.
MA.7.GR.1.5 MTR.5.1 MTR.7.1

10 Scaling Buildings
Same Object, Different Scales
Calculate distances on a scale drawing, given a drawing of the same object with a different scale.
MA.7.GR.1.5 MTR.2.1 MTR.4.1 MTR.5.1

Unit 1 1B Unit at a Glance

Pacing: 13 days | Short on time? See pacing considerations below.
All instructional and assessment days are ~45 minutes each.
• 10 Lessons
• Pre-Unit Check
• Sub-Unit Quiz
• End-of-Unit Assessment

Assess and Respond — Sub-Unit 2

4 Scale Factor Challenges
Effects of Scale Factors
Determine which scale factors create a copy that is larger, smaller, or the same size as the original.
MA.7.GR.1.5 MTR.4.1 MTR.5.1

5 Tiles
Scaling and Area
Explore how different scale factors affect the area of scaled copies.
MA.7.GR.1.5 MTR.1.1 MTR.3.1 MTR.4.1

A Sub-Unit Quiz
Learn about your students' understanding of the concepts and skills so far in this unit.
MA.7.GR.1.5 MTR.3.1 MTR.4.1 MTR.5.1

6 Introducing Scale
Comparing Scale Factor and Scale
Explore how scale is used to represent distances in the real world.
MA.7.GR.1.5 MTR.2.1 MTR.6.1 MTR.7.1

Assess and Respond

A End-of-Unit Assessment
Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.
MA.7.GR.1.5 MTR.3.1 MTR.5.1 MTR.6.1

Pacing Considerations

Lesson 4: This lesson supports students in developing a conceptual understanding of scaling and scale factor, which will be addressed in more depth in upcoming lessons. If students show a strong understanding of scale drawings in earlier lessons and in Problem 1 of the Pre-Unit Check, this lesson may be omitted. If omitted, be sure to discuss how the scale factor that takes one figure to another figure relates to the scale factor that takes the second figure back to the first.

Lessons 8–9: These lessons support students in creating scale drawings using different scale factors. They could be condensed into one class period by completing Activity 1 from each lesson.

Unit 1 1C Unit at a Glance

Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

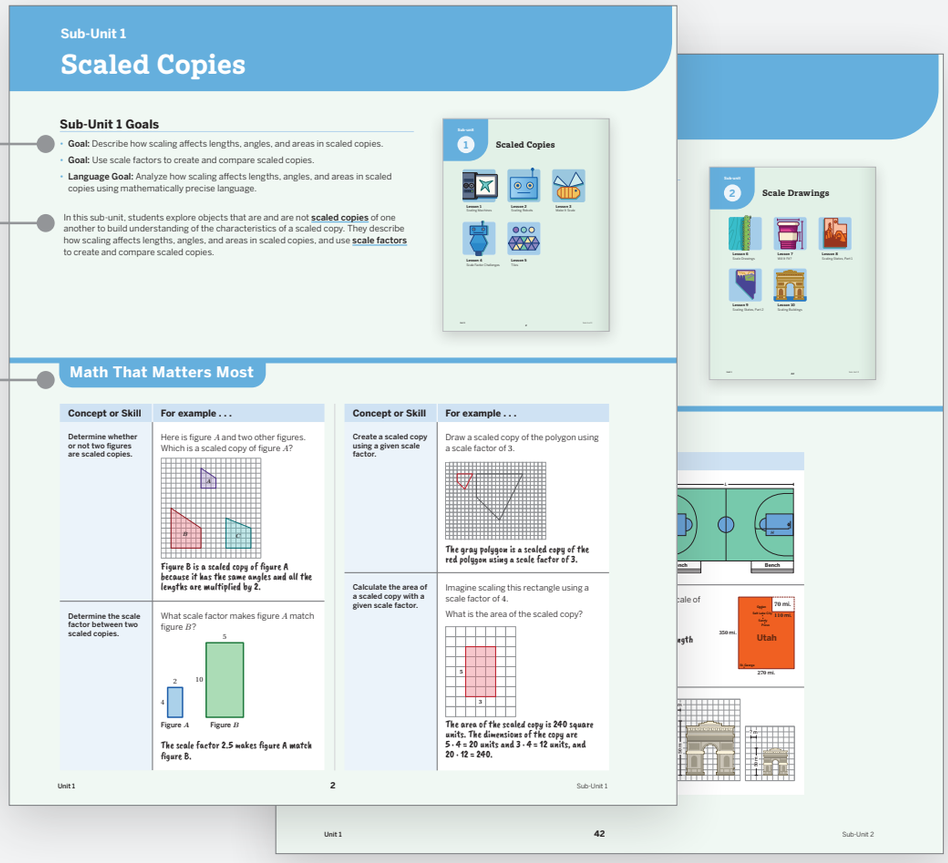
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, a preview of each of the unit assessments, and guidance about differentiation, accessibility, language development, materials, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



Sub-Unit 1

Scaled Copies

Sub-Unit 1 Goals

- **Goal:** Describe how scaling affects lengths, angles, and areas in scaled copies.
- **Goal:** Use scale factors to create and compare scaled copies.
- **Language Goal:** Analyze how scaling affects lengths, angles, and areas in scaled copies using mathematically precise language.

In this sub-unit, students explore objects that are and are not **scaled copies** of one another to build understanding of the characteristics of a scaled copy. They describe how scaling affects lengths, angles, and areas in scaled copies, and use **scale factors** to create and compare scaled copies.

Math That Matters Most

Concept or Skill	For example . . .
Determine whether or not two figures are scaled copies.	Here is figure *A* and two other figures. Which is a scaled copy of figure *A*? Figure B is a scaled copy of figure A because it has the same angles and all the lengths are multiplied by 2.
Determine the scale factor between two scaled copies.	What scale factor makes figure *A* match figure *B*? Figure *A* Figure *B* The scale factor 2.5 makes figure A match figure B.
Create a scaled copy using a given scale factor.	Draw a scaled copy of the polygon using a scale factor of 3. The gray polygon is a scaled copy of the red polygon using a scale factor of 3.
Calculate the area of a scaled copy with a given scale factor.	Imagine scaling this rectangle using a scale factor of 4. What is the area of the scaled copy? The area of the scaled copy is 240 square units. The dimensions of the copy are 5 · 4 = 20 units and 3 · 4 = 12 units, and 20 · 12 = 240.

Unit 1 2 Sub-Unit 1

Unit 1 42 Sub-Unit 2

The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

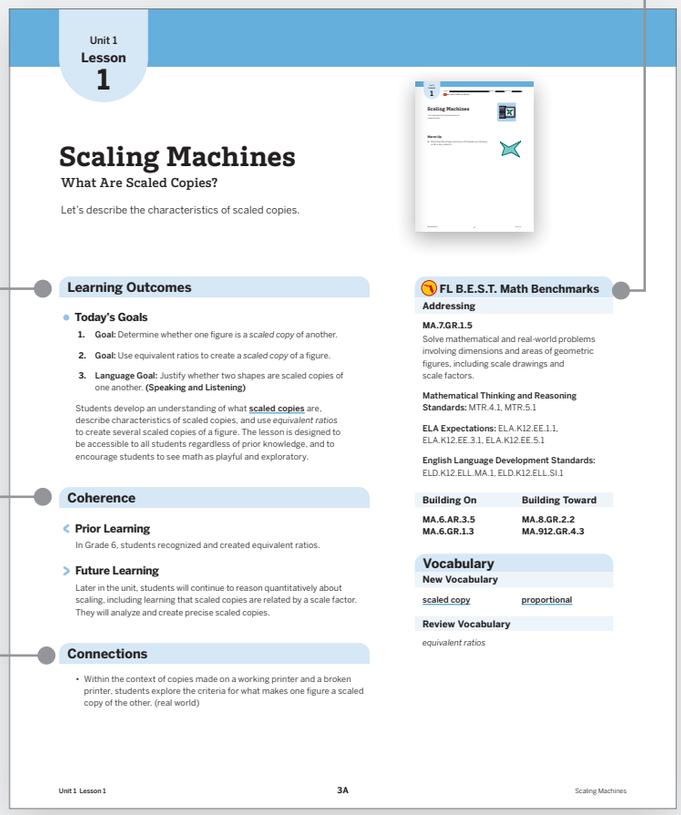
The lessons within each unit are grouped into sub-units that address a related group of concepts. All sub-units include lesson practice and Show What You Know formative assessments, and targeted sub-units include a quiz.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

The **Florida B.E.S.T. Math Benchmarks** section will list all standards addressed in this lesson, including the **Mathematical Thinking and Reasoning Standards,** the **ELA Expectations,** and the **English Language Development Standards.** This section shows the standards the lesson builds on and the standards the lesson builds toward.



The **Learning Outcomes** section is where teachers will find the goals of the lesson, including language goals

In the **Coherence** section, teachers will find information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

In the **Connections** section, teachers will find opportunities to help their students make connections cross-curricularly, to the real world, or to other mathematical learning.

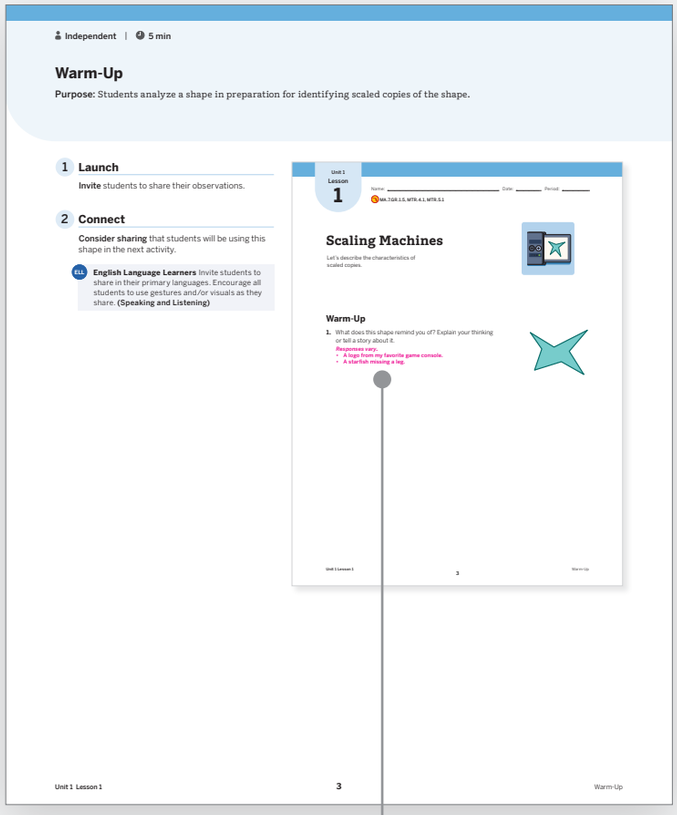
Lesson Overview

This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.





The **Prep Checklist** displays all materials students will need for the lesson.

Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class. The page also lists any hands-on materials that may be needed.

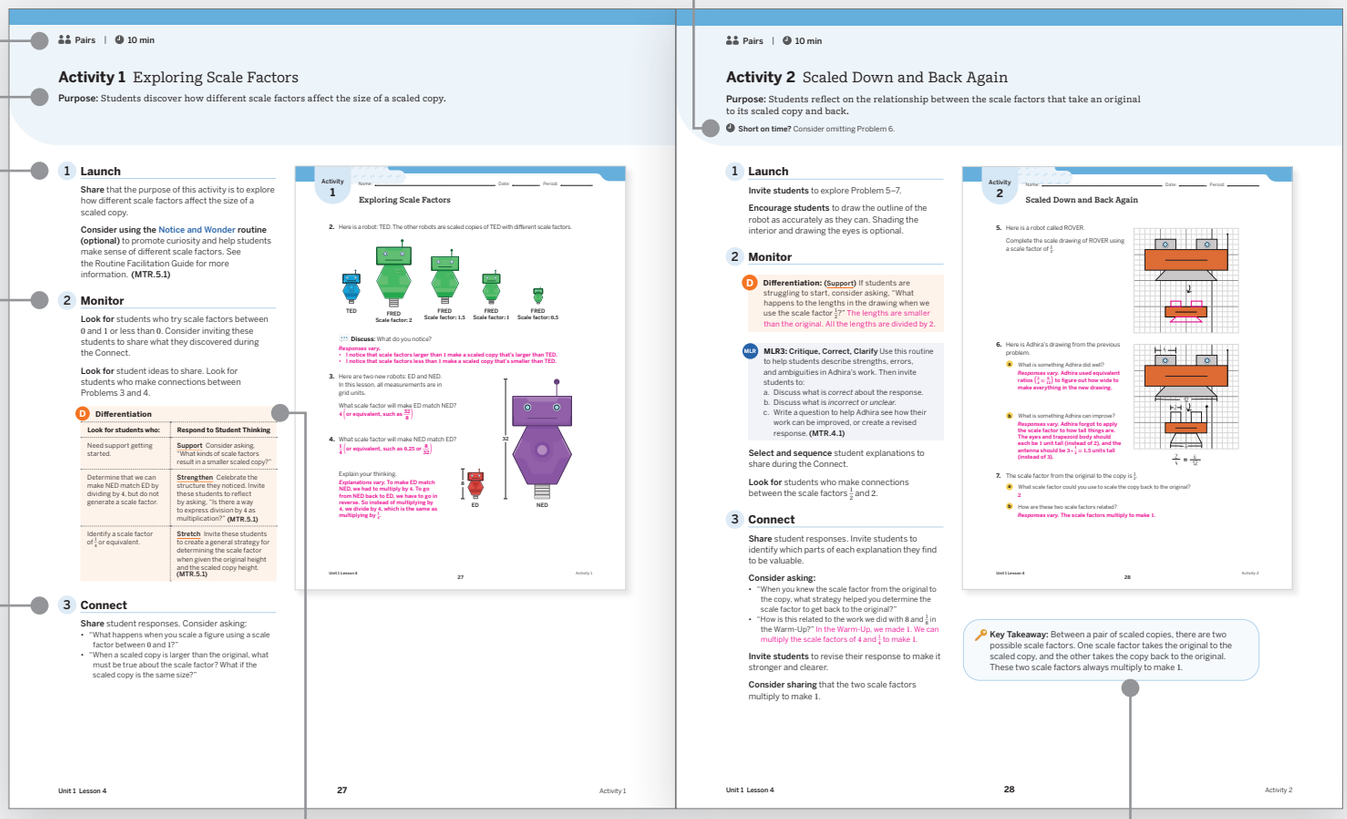
Warm-Up

Every Amplify Desmos Math Florida lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping** and **time frame** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.



Pairs | 10 min

Activity 1 Exploring Scale Factors

Purpose: Students discover how different scale factors affect the size of a scaled copy.

1 Launch

Share that the purpose of this activity is to explore how different scale factors affect the size of a scaled copy.

Consider using the Notice and Wonder routine (optional) to promote curiosity and help students make sense of different scale factors. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

2 Monitor

Look for students who try scale factors between 0 and 1 or less than 0. Consider inviting these students to share what they discovered during the Connect.

Look for student ideas to share. Look for students who make connections between Problems 3 and 4.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What kinds of scale factors result in a smaller scaled copy?"
Determine that we can make NED match ED by dividing by 4, but do not generate a scale factor.	**Strengthen** Celebrate the structure they noticed. Invite these students to reflect by asking, "Is there a way to express division by 4 as multiplication?" **(MTR.5.1)**
Identify a scale factor of $\frac{1}{4}$ or equivalent.	**Stretch** Invite these students to create a general strategy for determining the scale factor when given the original height and the scaled copy height. **(MTR.5.1)**

3 Connect

Share student responses. Consider asking:

- "What happens when you scale a figure using a scale factor between 0 and 1?"
- "When a scaled copy is larger than the original, what must be true about the scale factor? What if the scaled copy is the same size?"

Activity 1 — Exploring Scale Factors

2. Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.

TED; FRED Scale factor: 2; FRED Scale factor: 1.5; FRED Scale factor: 1; FRED Scale factor: 0.5

Discuss: What do you notice?

Responses vary.
- I notice that scale factors larger than 1 make a scaled copy that's larger than TED.
- I notice that scale factors less than 1 make a scaled copy that's smaller than TED.

3. Here are two new robots: ED and NED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED? 4 (or equivalent, such as $\frac{32}{8}$)

4. What scale factor will make NED match ED? $\frac{1}{4}$ (or equivalent, such as 0.25 or $\frac{8}{32}$)

Explain your thinking.

Explanations vary. To make ED match NED, we had to multiply by 4. To go from NED back to ED, we have to go in the opposite direction of multiplying by 4, we divide by 4, which is the same as multiplying by $\frac{1}{4}$.

ED; NED; 32

Unit 1 Lesson 4 — 27 — Activity 1

Pairs | 10 min

Activity 2 Scaled Down and Back Again

Purpose: Students reflect on the relationship between the scale factors that take an original to its scaled copy and back.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to explore Problem 5–7.

Encourage students to draw the outline of the robot as accurately as they can. Shading the interior and drawing the eyes is optional.

2 Monitor

D Differentiation: (Support) If students are struggling to start, consider asking, "What happens to the lengths in the drawing when we use the scale factor $\frac{1}{2}$?" The lengths are smaller than the original. All the lengths are divided by 2.

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Adhira's work. Then invite students to:
a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Adhira see how their work can be improved, or create a revised response. **(MTR.4.1)**

Select and sequence student explanations to share during the Connect.

Look for students who make connections between the scale factors $\frac{1}{2}$ and 2.

3 Connect

Share student responses. Invite students to identify which parts of each explanation they find to be valuable.

Consider asking:
- "When you knew the scale factor from the original to the copy, what strategy helped you determine the scale factor to get back to the original?"
- "How is this related to the work we did with 8 and $\frac{1}{8}$ in the Warm-Up?" In the Warm-Up, we made 1. We can multiply the scale factors of 4 and $\frac{1}{4}$ to make 1.

Invite students to revise their response to make it stronger and clearer.

Consider sharing that the two scale factors multiply to make 1.

Activity 2 — Scaled Down and Back Again

5. Here is a robot called ROVER.

Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.

6. Here is Adhira's drawing from the previous problem.

What is something Adhira did well?

Responses vary. Adhira used equivalent ratios ($\frac{1}{2} = \frac{2}{4}$) to figure out how wide to make everything in the new drawing.

What is something Adhira can improve?

Responses vary. Adhira forgot to apply the scale factor to how tall things are. The eyes and trapezoid body should each be 1 unit tall (instead of 2), and the antenna should be $3 \times \frac{1}{2} = 1.5$ units tall (instead of 3).

7. The scale factor from the original to the copy is $\frac{1}{2}$.

What scale factor could you use to scale the copy back to the original?

2

How are these two scale factors related?

Responses vary. The scale factors multiply to make 1.

Unit 1 Lesson 4 — 28 — Activity 2

Key Takeaway: Between a pair of scaled copies, there are two possible scale factors. One scale factor takes the original to the scaled copy, and the other takes the copy back to the original. These two scale factors always multiply to make 1.

In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a **D Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

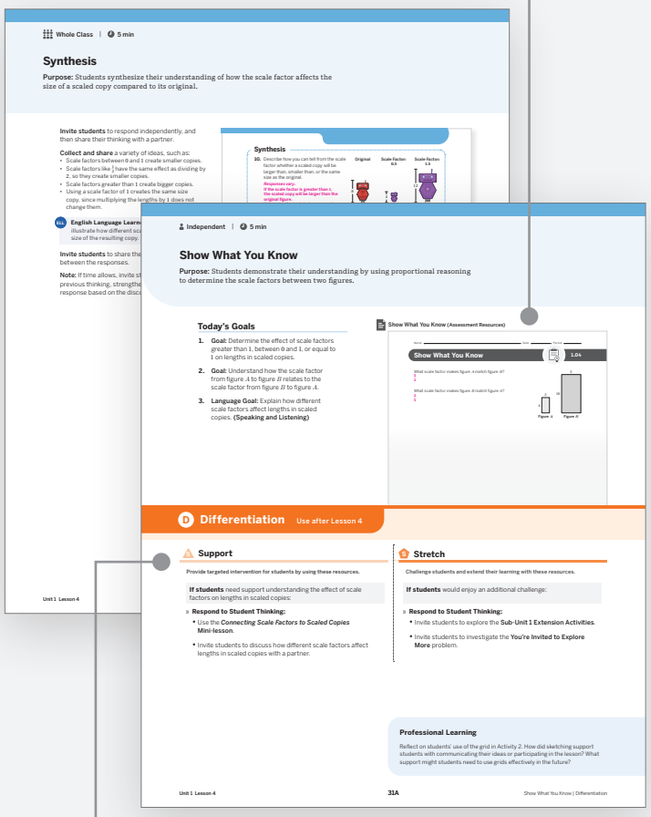
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

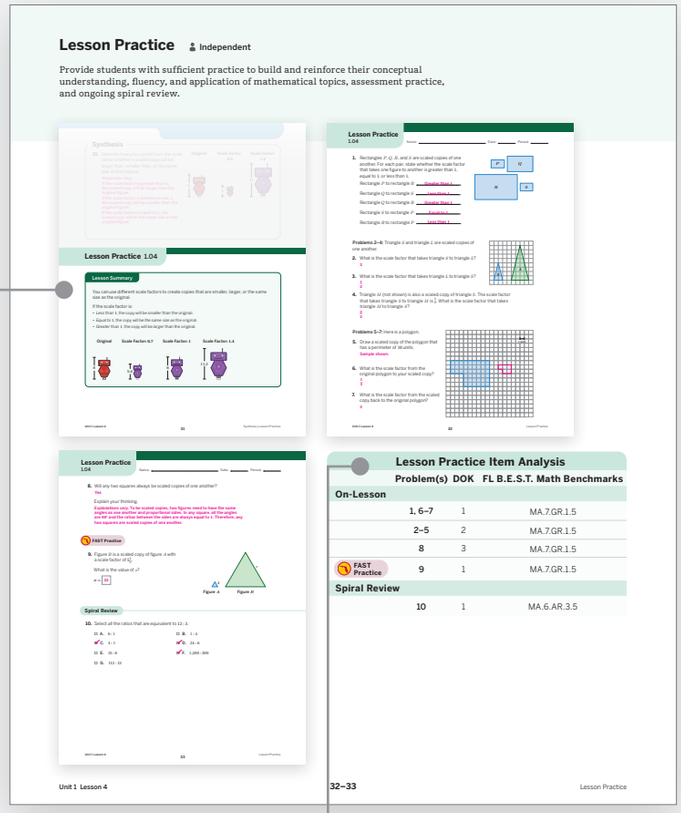
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, and ways to increase access to the language and content. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** begins with a **Summary** of the big ideas in the lesson, often including a worked example. Students can highlight parts of the summary or share it with a caregiver or classmate.



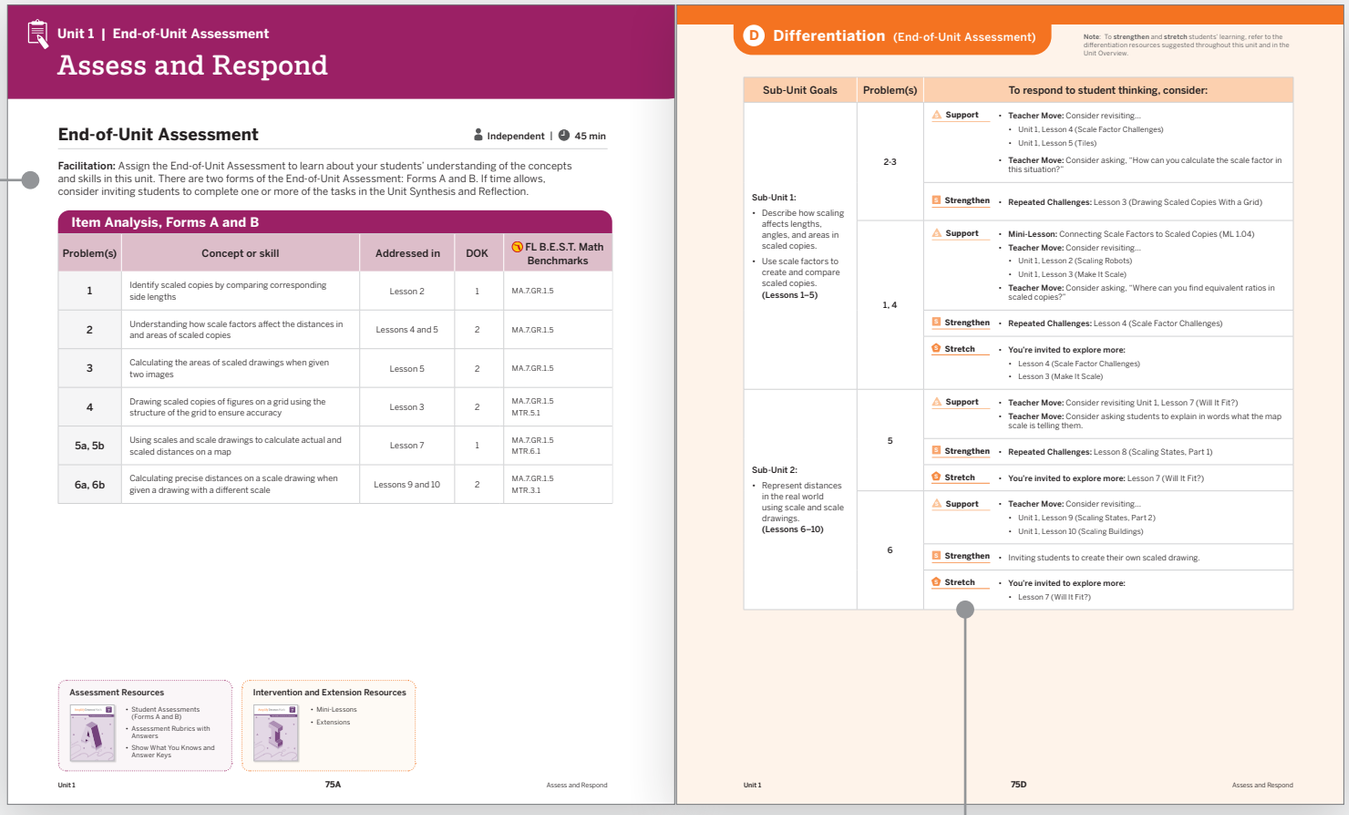
A **Practice Problem Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding B.E.S.T. Math Benchmarks.

Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

Lesson Practice

Daily practice problems for the day's lesson are included both online and in the print Student Edition, including Fluency, FAST Practice, and Spiral Review.



Unit 1 | End-of-Unit Assessment

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify scaled copies by comparing corresponding side lengths	Lesson 2	1	MA.7.GR.1.5
2	Understanding how scale factors affect the distances in and areas of scaled copies	Lessons 4 and 5	2	MA.7.GR.1.5
3	Calculating the areas of scaled drawings when given two images	Lesson 5	2	MA.7.GR.1.5
4	Drawing scaled copies of figures on a grid using the structure of the grid to ensure accuracy	Lesson 3	2	MA.7.GR.1.5 MTR.5.1
5a, 5b	Using scales and scale drawings to calculate actual and scaled distances on a map	Lesson 7	1	MA.7.GR.1.5 MTR.6.1
6a, 6b	Calculating precise distances on a scale drawing when given a drawing with a different scale	Lessons 9 and 10	2	MA.7.GR.1.5 MTR.3.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Unit 1 75A Assess and Respond

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Describe how scaling affects lengths, angles, and areas in scaled copies.
• Use scale factors to create and compare scaled copies.
(Lessons 1–5)
	2-3	**Support**
		• **Teacher Move:** Consider revisiting...
		  • Unit 1, Lesson 4 (Scale Factor Challenges)
		  • Unit 1, Lesson 5 (Tiles)
		• **Teacher Move:** Consider asking, "How can you calculate the scale factor in this situation?"

		Strengthen • **Repeated Challenges:** Lesson 3 (Drawing Scaled Copies With a Grid)

	1, 4	**Support**
		• **Mini-Lesson:** Connecting Scale Factors to Scaled Copies (ML 1.04)
		• **Teacher Move:** Consider revisiting...
		  • Unit 1, Lesson 2 (Scaling Robots)
		  • Unit 1, Lesson 3 (Make It Scale)
		• **Teacher Move:** Consider asking, "Where can you find equivalent ratios in scaled copies?"

		Strengthen • **Repeated Challenges:** Lesson 4 (Scale Factor Challenges)

		Stretch
		• **You're invited to explore more:**
		  • Lesson 4 (Scale Factor Challenges)
		  • Lesson 3 (Make It Scale)

Sub-Unit 2: • Represent distances in the real world using scale and scale drawings. **(Lessons 6–10)**	5	**Support** • **Teacher Move:** Consider revisiting Unit 1, Lesson 7 (Will It Fit?) • **Teacher Move:** Consider asking students to explain in words what the map scale is telling them.
		Strengthen • **Repeated Challenges:** Lesson 8 (Scaling States, Part 1)
		Stretch • **You're invited to explore more:** Lesson 7 (Will It Fit?)

	6	**Support**
		• **Teacher Move:** Consider revisiting...
		  • Unit 1, Lesson 9 (Scaling States, Part 2)
		  • Unit 1, Lesson 10 (Scaling Buildings)

		Strengthen • Inviting students to create their own scaled drawing.
		Stretch • **You're invited to explore more:** • Lesson 7 (Will It Fit?)

Unit 1 75D Assess and Respond

An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math Florida is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions*.

1 Launch



The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor



As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect



Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

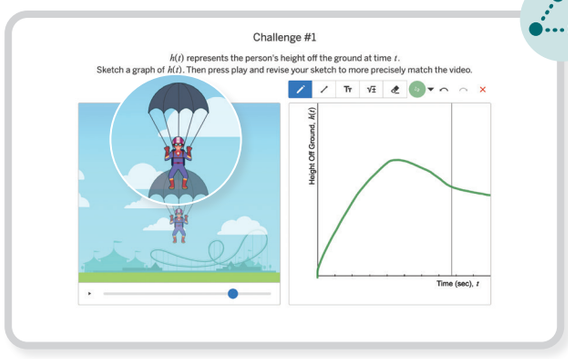
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

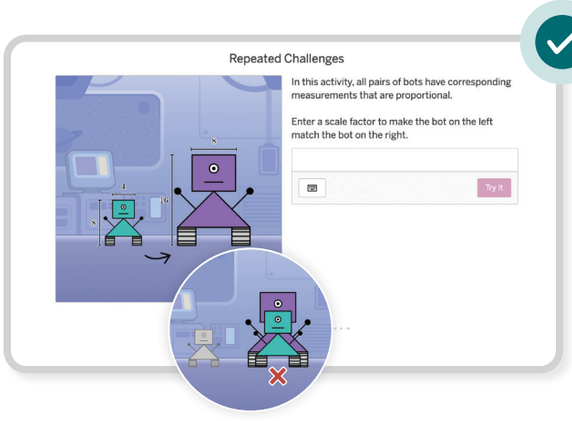
The Power of Digital

Amplify Desmos Math Florida provides the option for teachers to deliver lessons using the power of digital technology. This includes the use of visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. These interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



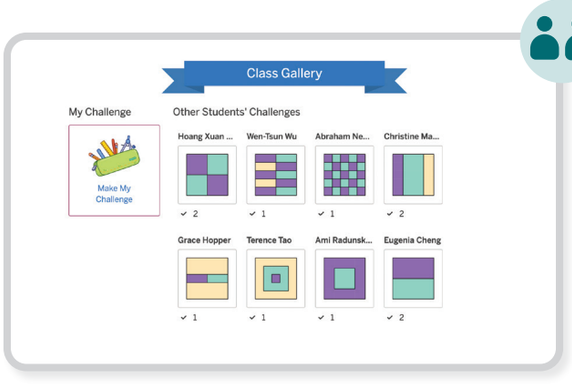
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math Florida, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

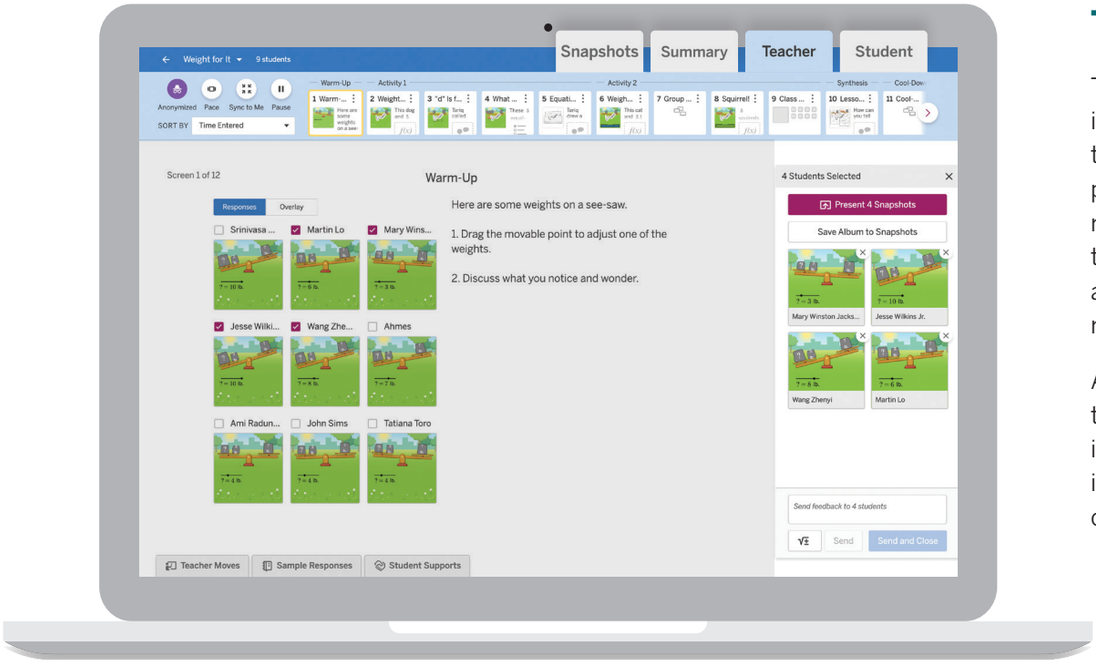


Collaborative experiences

Digital tools allow students to interact with each other's mathematical ideas in a variety of ways. Students can use the "Share With Class" button to exchange mathematical thinking with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Facilitation Tools

Amplify Desmos Math Florida includes a suite of digital facilitation tools. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

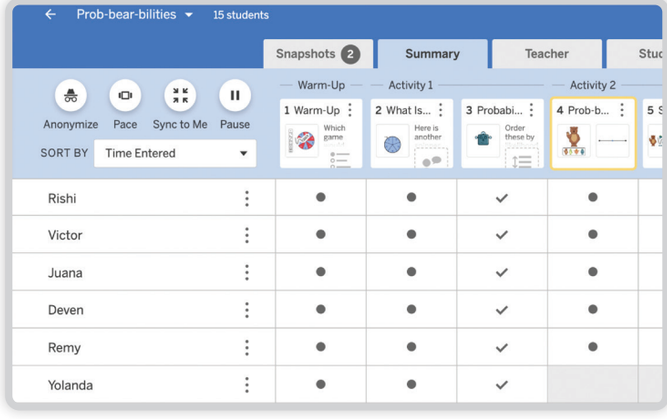
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math in Action

Each Math in Action feature highlights an individual whose work in the field of mathematics, or a connected field, have made an impact on American society. These features can be used to help make connections between the mathematical content of the grade and how math shows up in a wide variety of careers.

Career Connections

Amplify Desmos Math Florida intentionally celebrates traditional mathematics fields as well as elevates work that engages mathematics in unique ways.

Representation of Mathematicians

Students can read about the contributions and work of a variety of individuals.



Unit 6 | Math in Action

Career Connection

Did you know that there are over 400 different types of oranges in the world? And that oranges make up about 70% of all the citrus produced worldwide? The state of Florida is the second largest producer of citrus in the United States.

How many types of oranges can you name? A few are shown in the picture.

Horticulturalists are among the many careers that involve the citrus industry. They are people who study and whose work includes growing fruits, vegetables, flowers, and other plants. They use expressions and equations to determine measurements, temperatures and to describe the characteristics of the plants they are studying. Data collection and graphing will allow a horticulturalist to create new varieties of plants through cross-pollination and other techniques.

B.E.S.T. Mathematics Benchmark Connection

Horticulturalists and others involved in producing citrus use many math concepts in their work. For example, they apply inequality concepts and use number lines to track temperature and growth rates (MA.7.AR.2.1) so that plants grow in the best conditions possible. IT Specialists will use diagrams and equations (MA.7.NSO.2.3, MA.7.AR.2.2) as they use technology tools to monitor weather conditions, and control watering systems.

Mathematical Thinking and Reasoning Connection

Horticulturalists use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as diagrams or equations (MTR.2.1).

Meet Lue Gim Gong

Lue Gim Gong was a horticulturist who is credited for his work in growing the American citrus industry. He developed his interest in plants while he was working in orange groves in China and his mother taught him about cross-pollination. Even when he moved to Massachusetts to work in a shoe factory, he continued to study plants. When he moved to Florida, he focused his studies on the cold frosts that damaged orange crops and is credited for developing a cold-resistant orange!

Nature and Science/Alamy Stock Photo

Grade 7 Unit 6

508



Eugenia Cheng

Mathematics and music? Eugenia Cheng found a way to combine her interest in both, studying how patterns and ratios are applied to make unique sounds.



Ebonya Washington

Mathematics and money? Ebonya Washington found a way to combine her interest in both, studying how percents are applied to make good banking decisions.



Lue Gim Gong

Mathematics and horticulture? Lue Lim Gong found a way to combine his interest in both, studying how temperatures impacted the growth of citrus to invent a cold-resistant orange.



Franklin R. Chang-Diaz

Mathematics and space? Franklin R. Chang-Diaz found a way to combine his interest in both, studying how geometry and engineering are applied to construct space vehicles.

Facilitating Meaningful Conversations

Amplify Desmos Math Florida is designed to support interactive and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the ideas of others, and appreciate different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the real-world and mathematical experiences that students bring to the classroom.

Use the Tell a Story routine to help students make sense of the turtle race animation. Invite students to write a story about the animation using their experiences, perspectives, and creativity. **(MP1)**



In the lesson **Turtle Time Trials**, students are invited to use the **Tell a Story** routine to create their own narratives about what the turtles in the animation are doing and why. This is a chance for students to use their creativity while engaging deeply in the math.

1 Launch

Direct students' attention to the images of the rice dishes.

To surface connections, consider asking, "What is your favorite rice dish? When or where do you eat it?"



In the lesson **Rice Ratios**, students are introduced to the concept of equivalent ratios in the context of making rice. The Teacher Edition provides teachers with prompts they can use to help students connect the rice dishes in the activities to their own lives.

Supporting Student Collaboration

Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.

How will you support student collaboration?

- Discuss what supporting each other without giving answers could look or sound like.
- Invite a student from each group to read the task aloud.

The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

In the lesson **Subway Seats**, students use the **Which One Doesn't Belong?** routine to begin exploring two-variable equations. Classroom conversations about activities like this are a way for teachers to elicit different kinds of student brilliance. The Teacher Edition provides suggestions for how teachers can engage students in conversation and reinforce the idea that each student has something uniquely valuable to contribute.

Accessibility

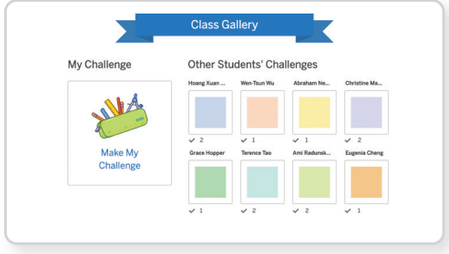
Amplify Desmos Math Florida is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

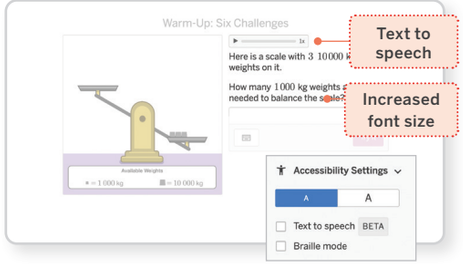
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome student interests, support sustaining effort and persistence, and develop an awareness of others.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

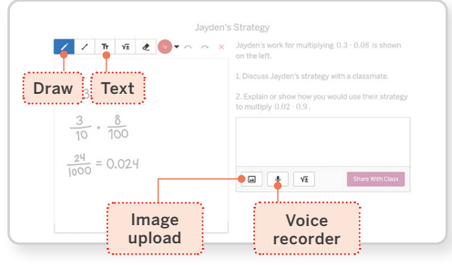
Learners make sense of information differently. Amplify Desmos Math Florida includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math Florida ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

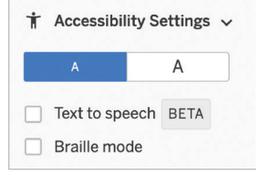
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math Florida enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.

Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Differentiation: Beyond the Lesson

In each lesson, students' understanding of the learning goals is broken down for teachers into three categories: students who need support to understand the learning goals, students who need to continue strengthening their understanding of the learning goals, and students who are ready to stretch their learning.

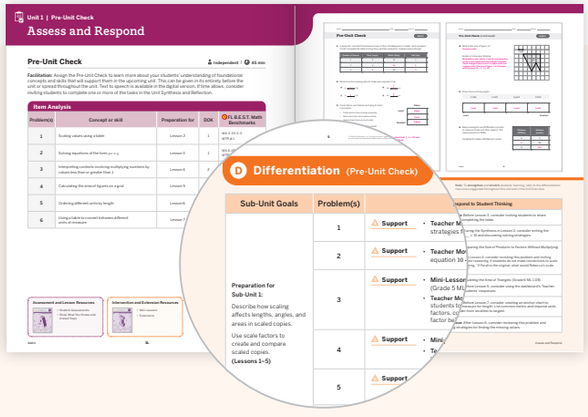
Each **Support**, **Strengthen**, and **Stretch** resource is designed to take 15 minutes:

- **Mini-Lessons:** Targeted intervention for students who need additional support or need more time.
- **Extensions:** Small group or independent challenges.



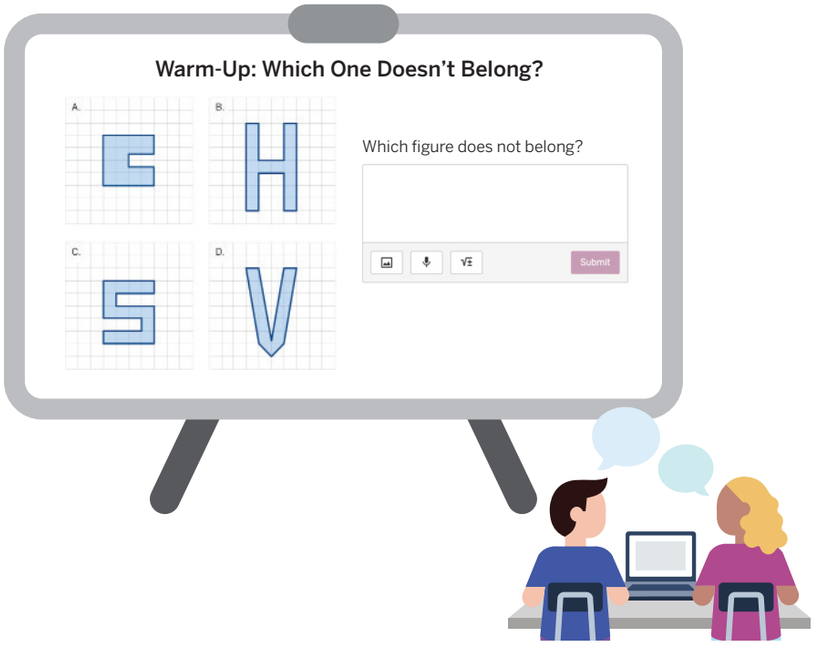
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

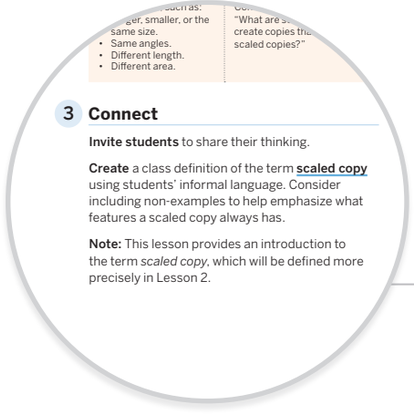
- The **Which One Doesn't Belong?** routine communicates to students that their ideas have value, that there are many ways to be correct in math, and that they can learn math by sharing their math thinking with each other.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

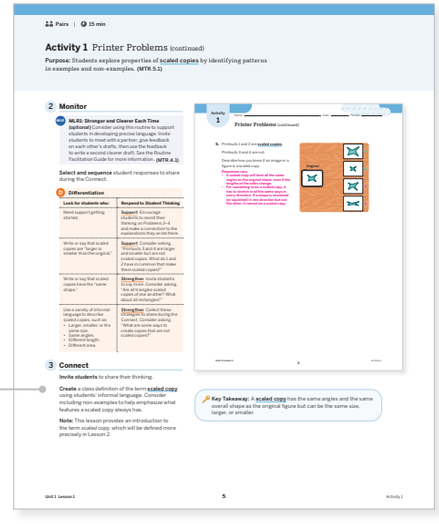
Each instructional routine included in an Amplify Desmos Math Florida lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' mathematical thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math Florida curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?





For more information about the Instructional Routines embedded in the Amplify Desmos Math Florida curriculum, see the Routine Facilitation Guide in the Professional Development Library.

Math Language Development

At Amplify Desmos Math Florida, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of English Language Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math Florida includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for English Language Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* (′). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does *pre* mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Calculate the area of a parallelogram on a grid using a variety of strategies.
  2. **Language Goal:** Compare and contrast strategies for determining the area of a parallelogram. **(Writing, Speaking, and Listening)**
  3. **Language Goal:** Describe parallelograms using precise mathematical language. **(Speaking and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

English Language Learners Support

Supports for ELL **English Language Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for English Language Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. English Language Learner supports may also be attached to MLRs.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Developing Fluency

Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another*.

In Grade 7, Florida students are expected to perform all four operations with rational numbers with procedural fluency.

Expectation

Exploration

To develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.

Procedural Reliability

To utilize skills to develop an accurate, reliable method that aligns with the student's understanding and learning style.

Procedural Fluency

To utilize skills to become fluent with an efficient, generalizable and accurate procedure, including a standard algorithm.

How Amplify Desmos Math Florida Supports the Expectation

- Manipulatives and visual models and diagrams are used extensively to support student's exploration of new concepts.
- Discussions are strongly encouraged in each lesson, including 💬 Discuss prompts in the student edition as well as suggested discussion prompts in the teacher's edition.
- Students are encouraged to estimate and compare estimations to actual answers to support reasonableness of problem solving solutions.
- Students are often asked to analyze a drawing or to make a drawing to explain their reasoning and strengthen their understanding of the math concepts being studied.

- Frequently in each course, students are asked to analyze and evaluate work presented in their student edition that represents work done by other students for accuracy, preference to learning style, and for efficiency.

- Program features such as practice, spiral reviews, mini lessons, and pre-unit assessments are all designed to assist students with achieving procedural fluency.

*National Research Council [2001, 2005, 2012]; Star [2005]

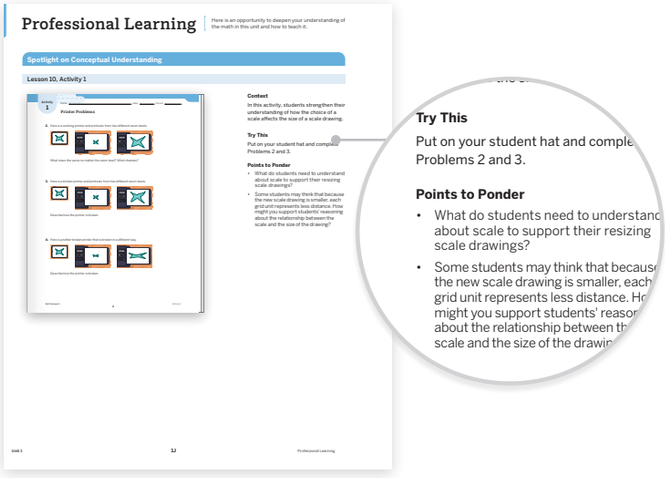
Professional Learning

The Amplify Desmos Math Florida program includes regular opportunities for teachers to deepen their understanding while they teach.

Embedded Professional Development

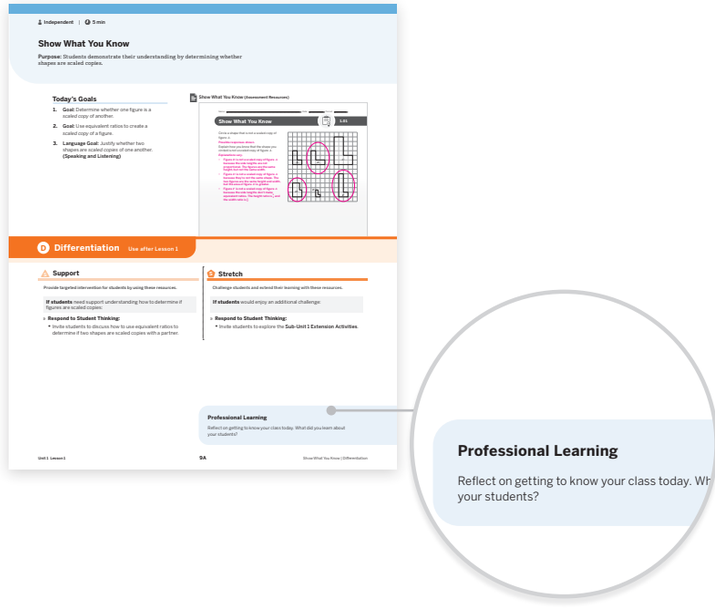
Unit-Level Professional Development

The Unit Overview for each unit includes a Professional Development page, which lists opportunities for teachers to deepen their understanding of the math in the unit and how to teach it.



Lesson-Level Professional Development

Each lesson provides a moment of professional learning as an opportunity for teachers to reflect on how well the lesson went. These professional learning moments can relate to principles, instruction, pedagogy, or program content.



Professional Development Sessions

Amplify's professional development team supports both teachers and administrators throughout their entire implementation journey. These sessions are available as on-site or virtual options.

Launch

Launch sessions introduce Amplify programs and support strong implementation. After learning about the program's foundational principles, you'll practice implementing the program in a collaborative environment.

Strengthen

Strengthen sessions deepen understanding of the program. These offerings are targeted and are meant to take your practice – and your students' learning to the next level.

Coach

These sessions are tailored to elevate instructional practices and meet the unique needs of teachers and leaders. These sessions can be customized leveraging our menu of supports.

Professional learning on your schedule

The PD Library serves as a centralized location where educators can access an ever-growing collection of professional learning resources and courses designed to support the implementation of Amplify Desmos Math Florida. Educators will find bite-sized how to videos, unit and lesson planning guides and templates, resources for leaders, and so much more!

Program Assessments

A variety of performance data in Amplify Desmos Math Florida provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math Florida, understanding is made continually visible.

Lesson-Level Assessments

Amplify Desmos Math lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.



Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.



Pre-Unit Check

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.



Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.



End-of-Unit Assessment

Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight. Two forms are included to help facilitate assessment administration in the classroom.

Florida's B.E.S.T. Standards for Mathematics

The following correlations show the alignment of Amplify Desmos Math Florida to the FL B.E.S.T. Mathematics Standards for Grade 7.

Benchmark	B.E.S.T Mathematics Benchmark	Amplify Desmos Math Lesson(s)
Number Sense and Operations		
MA.7.NSO.1.1	Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.	**Unit 5** Lessons 6, 7, 8, 9, 10
MA.7.NSO.1.2	Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.	**Unit 5** Lessons 1, 2, 6, 7, 8, 9 ,10 **Unit 8** Lessons 3, 4, 5
MA.7.NSO.2.1	Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.	**Unit 5** Lessons 5, 6, 7 **Unit 6** Lessons 9, 10
MA.7.NSO.2.2	Add, subtract, multiply and divide rational numbers with procedural fluency.	**Unit 5** Lessons 2, 3, 4, 5 **Unit 6** Lessons 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16 **Unit 8** Lessons 9, 11
MA.7.NSO.2.3	Solve real-world problems involving any of the four operations with rational numbers.	**Unit 5** Lessons 3, 4 **Unit 6** Lessons 1, 2, 3, 12 **Unit 7** Lessons 6, 7, 8 **Unit 8** Lessons 9, 11
Algebraic Reasoning		
MA.7.AR.1.1	Apply properties of operations to add and subtract linear expressions with rational coefficients.	**Unit 6** Lesson 10
MA.7.AR.1.2	Determine whether two linear expressions are equivalent.	**Unit 6** Lessons 4, 5, 8, 9
MA.7.AR.2.1	Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.	**Unit 6** Lessons 13, 14, 15, 16
MA.7.AR.2.2	Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.	**Unit 6** Lessons 1, 2, 3, 5, 6, 7, 11, 12
MA.7.AR.3.1	Apply previous understanding of percentages and ratios to solve multi-step realworld percent problems.	**Unit 4** Lessons 1, 2, 3, 4, 5, 6, 7, 8, 13
MA.7.AR.3.2	Apply previous understanding of ratios to solve real-world problems involving proportions.	**Unit 2** Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13 **Unit 4** Lessons 9, 10 **Unit 8** Lesson 11
MA.7.AR.3.3	Solve mathematical and real-world problems involving the conversion of units across different measurement systems.	**Unit 4** Lesson 11
MA.7.AR.4.1	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.	**Unit 2** Lessons 2, 3, 7, 8, 10
MA.7.AR.4.2	Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.	**Unit 2** Lessons 2, 3, 4, 5, 6, 7, 9, 10, 11, 12 **Unit 4** Lessons 9, 10 **Unit 8** Lessons 3, 4, 5

Florida's B.E.S.T. Standards for Mathematics

MA.7.AR.4.3	Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.	**Unit 2** Lessons 11, 23
MA.7.AR.4.4	Given any representation of a proportional relationship, translate the representation to a written description, table or equation.	**Unit 2** Lessons 4, 5, 6, 7, 9, 10, 11, 13 **Unit 4** Lesson 9
MA.7.AR.4.5	Solve real-world problems involving proportional relationships.	**Unit 2** Lessons 5, 6, 7, 10, 12, 13 **Unit 4** Lessons 9, 10 **Unit 8** Lesson 11
	Geometric Reasoning	
MA.7.GR.1.1	Apply formulas to find the areas of trapezoids, parallelograms and rhombi.	**Unit 7** Lessons 1, 3
MA.7.GR.1.2	Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.	**Unit 3** Lesson 4 **Unit 7** Lessons 2, 3
MA.7.GR.1.3	Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.	**Unit 3** Lessons 1, 2, 3
MA.7.GR.1.4	Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.	**Unit 3** Lessons 5, 6 **Unit 7** Lessons 4, 5
MA.7.GR.1.5	Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.	**Unit 1** Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **Unit 3** Lesson 2 **Unit 4** Lesson 9 **Unit 7** Lessons 4, 5
MA.7.GR.2.1	Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure's net.	**Unit 7** Lesson 8
MA.7.GR.2.2	Solve real-world problems involving surface area of right circular cylinders.	**Unit 7** Lesson 8
MA.7.GR.2.3	Solve mathematical and real-world problems involving volume of right circular cylinders.	**Unit 7** Lessons 6, 7
	Data Analysis and Probability	
MA.7.DP.1.1	Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.	**Unit 8** Lessons 9, 12
MA.7.DP.1.2	Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.	**Unit 8** Lessons 9, 12, 13
MA.7.DP.1.3	Given categorical data from a random sample, use proportional relationships to make predictions about a population.	**Unit 8** Lessons 10, 11
MA.7.DP.1.4	Use proportional reasoning to construct, display and interpret data in circle graphs.	**Unit 8** Lessons 6, 7, 8

Florida's B.E.S.T. Standards for Mathematics

MA.7.DP.1.5	Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.	**Unit 8** Lessons 7, 8
MA.7.DP.2.1	Determine the sample space for a simple experiment.	**Unit 8** Lesson 2
MA.7.DP.2.2	Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.	**Unit 8** Lessons 1, 2, 3, 4
MA.7.DP.2.3	Find the theoretical probability of an event related to a simple experiment.	**Unit 8** Lessons 2, 3, 4, 5
MA.7.DP.2.4	Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.	**Unit 8** Lessons 3, 4, 5

Mathematical Thinking and Reasoning Standards

The following correlations show the alignment of Amplify Desmos Math Florida to the Mathematical Thinking and Reasoning Standards.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Unit 1 Lesson 5
Unit 2 Lessons 7, 10, 12
Unit 3 Lessons 3, 4, 6
Unit 4 Lessons 2, 3
Unit 5 Lesson 5
Unit 7 Lessons 1, 2, 4, 5, 7, 8
Unit 8 Lessons 4, 10

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Unit 1 Lessons 6, 7, 10
Unit 2 Lessons 3, 4, 5, 6, 8, 10, 11, 12
Unit 4 Lessons 1, 3, 4, 9, 11
Unit 5 Lessons 6, 7, 8
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
Unit 7 Lessons 1, 3, 6, 8
Unit 8 Lessons 1, 2, 7, 8

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Unit 1 Lesson 5
Unit 2 Lessons 4, 9
Unit 3 Lesson 6
Unit 4 Lessons 6, 7, 8
Unit 5 Lessons 1, 2, 3, 4
Unit 6 Lessons 11, 14, 15, 16
Unit 7 Lessons 5, 6
Unit 8 Lessons 1, 2, 3

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Unit 1 Lessons 1, 2, 3, 4, 5, 8, 10
Unit 2 Lessons 1, 2, 7, 10
Unit 3 Lessons 2, 3, 4, 5, 6
Unit 4 Lessons 4, 6, 8, 9, 10, 11, 13
Unit 5 Lessons 1, 2, 9, 10
Unit 6 Lessons 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15
Unit 7 Lessons 2, 3, 4, 6
Unit 8 Lessons 4, 6, 7, 9, 10, 11, 13

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Unit 1 Lessons 1, 2, 4, 9, 10
Unit 2 Lessons 1, 2, 6, 7, 8
Unit 3 Lessons 1, 5
Unit 4 Lesson 12
Unit 5 Lessons 2, 3, 4, 5, 6, 7, 8, 9, 10
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13
Unit 7 Lessons 1, 2, 4, 7
Unit 8 Lessons 5, 6, 11, 12

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Unit 1 Lessons 6, 7
Unit 2 Lesson 3
Unit 3 Lessons 1, 3, 4, 5
Unit 4 Lesson 5
Unit 5 Lessons 6, 7, 8
Unit 6 Lessons 15, 16
Unit 8 Lessons 3, 5, 11, 12

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Unit 1 Lessons 6, 7, 8, 9
Unit 2 Lessons 5, 9, 11, 12
Unit 4 Lessons 5, 7, 13
Unit 5 Lessons 3, 4
Unit 6 Lessons 1, 3, 12, 16
Unit 7 Lessons 3, 7, 8
Unit 8 Lessons 7, 8, 9, 10, 13

Florida's B.E.S.T. Standards for English Language Arts

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

Unit 1 Lessons 2, 3, 5
Unit 2 Lessons 5, 6, 7, 10, 11
Unit 3 Lessons 3
Unit 4 Lessons 6, 8
Unit 5 Lessons 1, 2, 4, 5, 6, 10
Unit 6 Lessons 1, 2, 4, 5, 6, 7, 10, 14, 16
Unit 7 Lessons 3, 5, 8
Unit 8 Lessons 3, 4, 6, 7, 10, 11, 12, 13

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

Unit 1 Lessons 1, 5
Unit 2 Lesson 7
Unit 3 Lesson 3
Unit 4 Lessons
Unit 5 Lessons
Unit 6 Lesson 12
Unit 7 Lessons 3, 7
Unit 8 Lessons 2, 4, 9

ELA.K12.EE.3.1 Make inferences to support comprehension.

Unit 1 Lessons 1, 2, 4, 6, 7, 8, 10
Unit 2 Lessons 2, 3, 5, 6, 7, 9, 10, 13
Unit 3 Lessons 1, 3, 5, 6
Unit 4 Lessons 3, 4, 5, 10, 11, 12
Unit 5 Lessons 6, 7, 9
Unit 6 Lessons 1, 3, 6
Unit 7 Lesson 7
Unit 8 Lesson 11

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

Unit 1 Lessons 2, 5, 7, 10
Unit 2 Lessons 9, 12
Unit 3 Lessons 2, 4, 6
Unit 4 Lessons 1, 2, 3, 6, 7, 10, 11
Unit 5 Lessons 3, 8, 9, 10
Unit 6 Lessons 3, 6, 9, 11, 14
Unit 7 Lessons 1, 7, 8
Unit 8 Lessons 7, 8, 9, 10

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

Unit 1 Lessons 1, 4, 5, 8, 9, 10
Unit 2 Lessons 1, 2, 4, 9, 12, 13
Unit 3 Lesson 4
Unit 4 Lessons 1, 5, 6, 9, 11, 12, 13
Unit 5 Lessons 1, 3, 4, 7
Unit 6 Lessons 2, 5, 7, 8, 9, 10, 11, 13, 15
Unit 7 Lessons 1, 2, 3, 4, 6, 7
Unit 8 Lessons 1, 2, 3, 5, 8, 9, 10, 12

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

Unit 1 Lesson 6
Unit 2 Lessons 1, 5, 11
Unit 3 Lesson 2
Unit 4 Lesson 2, 4, 7, 8, 13
Unit 5 Lessons 4, 5
Unit 6 Lessons 1, 2
Unit 7 Lesson 3
Unit 8 Lessons 6, 7, 13

English Language Development Standards

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

Unit 1 All Lessons
Unit 2 All Lessons
Unit 3 All Lessons
Unit 4 All Lessons
Unit 5 All Lessons
Unit 6 All Lessons
Unit 7 All Lessons
Unit 8 All Lessons

ELD.K12.ELL.SI.1 English language learners communicate for social and instructional purposes within the school setting.

Unit 1 All Lessons
Unit 2 All Lessons
Unit 3 All Lessons
Unit 4 All Lessons
Unit 5 All Lessons
Unit 6 All Lessons
Unit 7 All Lessons
Unit 8 All Lessons



UNIT 1

Scale Drawings

Students examine and create scaled copies of figures, then explore scale drawings of real-life objects.

Essential Questions

- How can you tell whether a figure is a scaled copy of another figure?
- How are lengths, angles, and areas affected when creating scaled copies?
- How do scales and scale drawings help bring the world to a more manageable size?

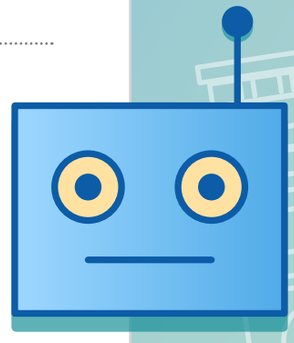
Sub-Unit Goals

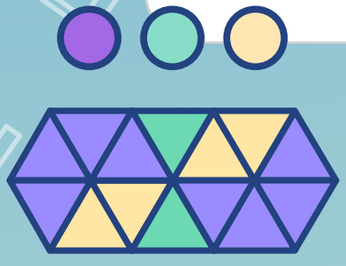
Sub-Unit 1

- Describe how scaling affects lengths, angles, and areas in scaled copies.
- Use scale factors to create and compare scaled copies.

Sub-Unit 2

- Represent distances in the real world using scale and scale drawings.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Solve problems involving two-dimensional figures, including circles. **(MA.7.GR.1)**

Benchmarks Addressed in This Unit: MA.7.GR.1.5

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.6.1**

Coherence Telling a story within and across grades

In this Unit

Students examine and create scaled copies of figures, as well as explore scale drawings of real-life objects.

Prior Learning

- **Grade 4**: Measurement units within one system **(MA.4.M.1.2)**
- **Grade 6,** Unit 4: Multiplying and dividing fractions **(MA.6.NSO.2.2)**
- **Grade 6,** Unit 1: Areas of polygons **(MA.6.GR.2.1, MA.6.GR.2.2)**
- **Grade 6,** Unit 2: Ratio reasoning **(MA.6.AR.3.1, MA.6.AR.3.2, MA.6.AR.3.3)**

Future Learning

- **Grade 7,** Units 2 and 4: Proportional relationships **(MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.5)**
- **Grade 8,** Unit 2: Similarity and dilations **(MA.8.GR.2.2, MA.8.GR.2.4)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 6–9**, students develop conceptual understanding of scaling by drawing scaled copies and studying their characteristics.

Procedural Fluency

In **Lesson 4**, students develop fluency in determining the scale factor between two figures.

Application

In **Lessons 7–8**, students apply their understanding of scale factor to determine the actual distances and areas represented by scale drawings.

Strategic Competence

In **Lesson 5**, students develop strategic competence as they analyze different methods for calculating the area of a scaled copy.

Adaptive Reasoning

In **Lesson 3**, students develop adaptive reasoning as they reflect on tools and strategies that help them draw scaled copies.

Productive Disposition

In **Lesson 2**, students have opportunities to see sense in mathematics as they explore scaling in a creative context.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.4.FR.2.4 MA.4.M.1.2 MA.6.AR.3.3 MA.6.AR.3.5 MA.6.GR.2.2
MTR.4.1, MTR.5.1, MTR.6.1

Sub-Unit 1

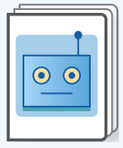


1 Scaling Machines

What Are Scaled Copies?

Describe characteristics of **scaled copies** and create scaled copies using equivalent ratios.

MA.7.GR.1.5
MTR.4.1 MTR.5.1



2 Scaling Robots

Lengths and Scaled Copies

Scale figures by applying a **scale factor** to each length in the figure.

MA.7.GR.1.5
MTR.4.1 MTR.5.1



3 Make It Scale

Drawing Scaled Copies

Draw accurate scaled copies on a grid by applying knowledge of scale factor.

MA.7.GR.1.5
MTR.4.1



7 Will It Fit?

Scale Drawings

Determine actual and scaled distances from a **scale drawing**.

MA.7.GR.1.5
MTR.2.1 MTR.6.1 MTR.7.1



8 Scaling States, Part 1

Creating Scale Drawings

Create scale drawings given the actual dimensions of an object and the scale.

MA.7.GR.1.5
MTR.4.1 MTR.7.1



9 Scaling States, Part 2

Creating Scale Drawings

Create scale drawings of the same object using different scales.

MA.7.GR.1.5
MTR.5.1 MTR.7.1



10 Scaling Buildings

Same Object, Different Scales

Calculate distances on a scale drawing, given a drawing of the same object with a different scale.

MA.7.GR.1.5
MTR.2.1 MTR.4.1 MTR.5.1

Pacing: 13 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 10 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment



4 **Scale Factor Challenges**

Effects of Scale Factors

Determine which scale factors create a copy that is larger, smaller, or the same size as the original.

MA.7.GR.1.5
MTR.4.1 MTR.5.1



5 **Tiles**

Scaling and Area

Explore how different scale factors affect the area of scaled copies.

MA.7.GR.1.5
MTR.1.1 MTR.3.1 MTR.4.1

Assess and Respond

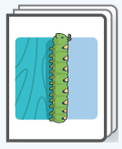


A **Sub-Unit Quiz**

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.GR.1.5
MTR.3.1 MTR.4.1 MTR.5.1

Sub-Unit 2



6 **Scale Drawings**

Comparing Scale Factor and Scale

Explore how **scale** is used to represent distances in the real world.

MA.7.GR.1.5
MTR.2.1 MTR.6.1 MTR.7.1

Assess and Respond



A **End-of-Unit Assessment**

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.GR.1.5
MTR.3.1 MTR.5.1 MTR.6.1

Pacing Considerations

Lesson 4: This lesson supports students in developing a conceptual understanding of scaling and scale factor, which will be addressed in more depth in upcoming lessons. If students show a strong understanding of scale drawings in earlier lessons and in Problem 1 of the Pre-Unit Check, this lesson may be omitted. If omitted, be sure to discuss how the scale factor that takes one figure to another figure relates to the scale factor that takes the second figure back to the first.

Lessons 8–9: These lessons support students in creating scale drawings using different scale factors. They could be condensed into one class period by completing Activity 1 from each lesson.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

Copy: Show What You Know

Lesson 2

Copy: Show What You Know

Lesson 3

Copy: Show What You Know

Materials: graph paper (*as needed*), index cards

Lesson 4

Copy: Show What You Know, Activity 3 You're Invited to Explore More Sheet (*one per student, as needed*)

Lesson 5

Copy: Show What You Know

Lesson 6

Copy: Show What You Know, Activity 2 You're Invited to Explore More Sheet *(one per student, as needed)*

Materials: calculators, measuring tools (*in centimeters*)

Lesson 7

Copy: Show What You Know

Materials: rulers

Lesson 8

Copy: Show What You Know

Materials: protractors, rulers

Lesson 9

Copy: Show What You Know

Materials: calculators, rulers

Lesson 10

Copy: Show What You Know

Materials: graph paper (*as needed*)

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

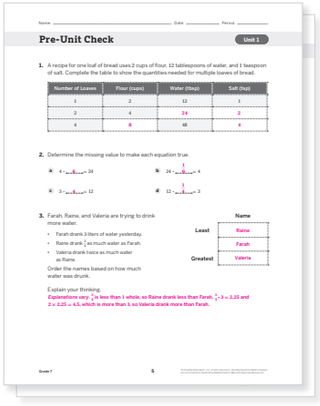
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards assessed: MA.4.FR.2.4, MA.4.M.1.2, MA.6.AR.3.3, MA.6.AR.3.5, MA.6.GR.2.2


Pre-Unit Check

Unit 1

1. A recipe for one loaf of bread uses 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. Complete the table to show the quantities needed for multiple loaves of bread.

Number of Loaves	Flour (cups)	Water (tbsp)	Salt (tsp)
1	2	12	1
2	4	24	2
4	8	48	4

2. Determine the missing value to make each equation true.

a. $4 \cdot \square = 24$

b. $24 \cdot \frac{1}{\square} = 6$

c. $3 \cdot \square = 12$

d. $12 \cdot \frac{1}{\square} = 3$

3. Farah, Raine, and Valeria are trying to drink more water.

- Farah drank 3 liters of water yesterday.
- Raine drank $\frac{3}{4}$ as much water as Farah.
- Valeria drank twice as much water as Raine.

Order the names based on how much water was drunk.

	Name
Least	Raine
	Farah
Greatest	Valeria

Explain your thinking.

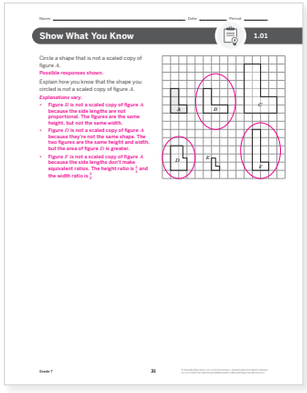
Explanations vary. $\frac{3}{4}$ is less than 1 whole, so Raine drank less than Farah. $\frac{3}{4} \cdot 3 = 2.25$ and $2 \times 2.25 = 4.5$, which is more than 3, so Valeria drank more than Farah.

Grade 7 5

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

Standards assessed on Lesson 1 Show What You Know: MA.7.GR.1.5


Show What You Know

1.01

Circle a shape that is not a scaled copy of figure *A*.

Possible responses shown.

Explain how you know that the shape you circled is not a scaled copy of figure *A*.

Explanations vary.

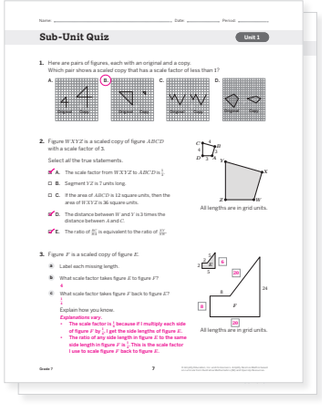
- Figure *B* is not a scaled copy of figure *A* because the side lengths are not proportional. The figures are the same height, but not the same width.
- Figure *D* is not a scaled copy of figure *A* because they're not the same shape. The figures are the same height and width, but the area of figure *D* is greater.
- Figure *F* is not a scaled copy of figure *A* because the side lengths don't make equivalent ratios. The height ratio is $\frac{1}{1}$ and the width ratio is $\frac{1}{2}$.

Grade 7 31

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

Standards assessed: MA.7.GR.1.5


Sub-Unit Quiz

Unit 1

1. Here are pairs of figures, each with an original and a copy. Which pair shows a scaled copy that has a scale factor of less than 1?

A. B. C. D.

2. Figure $WXYZ$ is a scaled copy of figure $ABCD$ with a scale factor of 3.

Select all the true statements.

- [x] A. The scale factor from $WXYZ$ to $ABCD$ is $\frac{1}{3}$.
- [ ] B. Segment YZ is 7 units long.
- [ ] C. If the area of $ABCD$ is 12 square units, then the area of $WXYZ$ is 36 square units.
- [x] D. The distance between W and Y is 3 times the distance between A and C.
- [x] E. The ratio of $\frac{WX}{YZ}$ is equivalent to the ratio of $\frac{AB}{CD}$.

All lengths are in grid units.

3. Figure F is a scaled copy of figure E.

a. Label each missing length.

b. What scale factor takes figure E to figure F?

4

c. What scale factor takes figure F back to figure E?

$\frac{1}{4}$

Explain how you know.

Explanations vary.

- The scale factor is $\frac{1}{4}$ because I multiply each side of figure F by $\frac{1}{4}$ get the side lengths of figure E.
- The ratio of any side length in figure E to the same side length in figure F is $\frac{1}{4}$. This is the scale factor I use to scale figure F back to figure E.

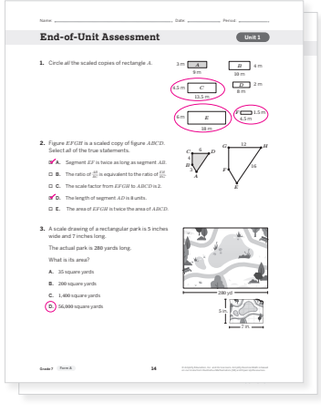
All lengths are in grid units.

Grade 7 7

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

Standards assessed: MA.7.GR.1.5


End-of-Unit Assessment

Unit 1

1. Circle all the scaled copies of rectangle *A*.

2. Figure $EFGH$ is a scaled copy of figure $ABCD$. Select all of the true statements.

- [x] A. Segment EF is twice as long as segment AB.
- [ ] B. The ratio of $\frac{EH}{FG}$ is equivalent to the ratio of $\frac{DA}{CB}$.
- [ ] C. The scale factor from $EFGH$ to $ABCD$ is 2.
- [x] D. The length of segment AD is 8 units.
- [ ] E. The area of $EFGH$ is twice the area of $ABCD$.

3. A scale drawing of a rectangular park is 5 inches wide and 7 inches long.

The actual park is 280 yards long.

What is its area?

A. 35 square yards

B. 200 square yards

C. 1,400 square yards

D. 56,000 square yards

Grade 7 Form A 14

D Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

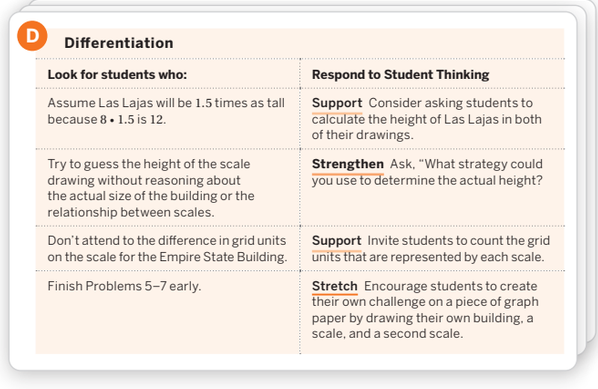
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Describe how scaling affects lengths, angles, and areas in scaled copies. • Use scale factors to create and compare scaled copies.	**Mini-Lesson** • *Connecting Scale Factors to Scaled Copies*	**Repeated Challenges** • Lesson 3 (*Drawing Scaled Copies With a Grid*) • Lesson 4 (*Scale Factor Challenges*) **Practice** • Lessons 1–5	**You're invited to explore more.** • Lesson 3 (*Make It Scale*) • Lesson 4 (*Scale Factor Challenges*) **DOK3 Practice** • Lesson 2–5
Sub-Unit 2 (Lessons 6–10) • Represent distances in the real world using scale and scale drawings.	**Mini-Lesson** • *Determining Unknowns in Scale Drawings*	**Repeated Challenges** • Lesson 8 (*Scaling States, Part 1*) **Practice** • Lessons 6–10	**You're invited to explore more.** • Lesson 6 (*Scale Drawings*) • Lesson 7 (*Will It Fit?*) **DOK3 Practice** • Lesson 7

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 10**, where students reproduce scale drawings of buildings at different scales.



Differentiation

Look for students who:	Respond to Student Thinking
Assume Las Lajas will be 1.5 times as tall because $8 \cdot 1.5$ is 12.	**Support** Consider asking students to calculate the height of Las Lajas in both of their drawings.
Try to guess the height of the scale drawing without reasoning about the actual size of the building or the relationship between scales.	**Strengthen** Ask, "What strategy could you use to determine the actual height?
Don't attend to the difference in grid units on the scale for the Empire State Building.	**Support** Invite students to count the grid units that are represented by each scale.
Finish Problems 5–7 early.	**Stretch** Encourage students to create their own challenge on a piece of graph paper by drawing their own building, a scale, and a second scale.

Accessibility

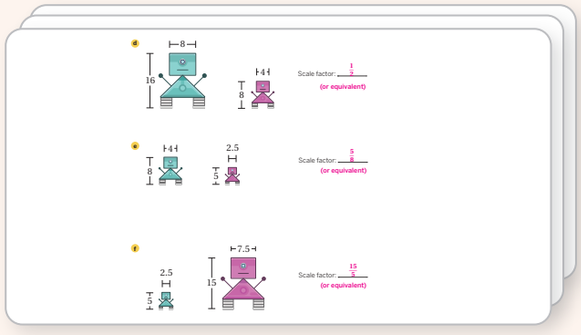
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action and Expression.

Spotlight on Expression and Communication

Lesson 1: In this lesson, students build fluency determining the scale factor between corresponding parts of two objects progressing from measurements with whole numbers to measurements with rational numbers.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Visual-Spatial Processing

Here are two suggestions in this unit for supporting students with interpreting visual representations.

Lesson 2, Activity 2

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to identify pairs of corresponding sides.

Lesson 8, Warm-Up

A **Accessibility: Visual-Spatial Processing** As students make sense of the visual representations, invite them to annotate the map of Utah by decomposing it into two rectangular sections and/or drawing dotted line segments to form a larger rectangle.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
proportional **scaled copy**	1
corresponding parts **scale factor**	2
scale	6
scale drawing	7

Review Vocabulary
area
equivalent ratios

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	1, 5–6, and 10
MLR3: Critique, Correct, Clarify	4
MLR7: Compare and Connect	8

Sub-Unit Language Goals

- Analyze how scaling affects lengths, angles, and areas in scaled copies.
- Analyze strategies for creating a scale drawing to represent actual distances and areas.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 4**, where students are using proportional reasoning to determine the scale factor from one figure to another. Students are encouraged to share in their primary language and to use gestures or visuals as they share to make sense of the mathematical strategies.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Proportional Relationships

In this unit, students connect scale factors to proportional relationships in the context of scaled copies. **In Grade 7, Units 2 and 4,** students will study proportional relationships in depth, making connections between tables, graphs, and equations.

FL B.E.S.T. Math Benchmarks:
MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.5

Example:

There is a proportional relationship between the size of the carpet and its cost. In this relationship, every square foot of carpet costs $\$1.50$. The number 1.5 is called a *constant of proportionality*. The constant of proportionality is connected to the idea of scale factor from this unit.

Carpet (sq. ft)	Cost ($)
$10 \xrightarrow{\bullet 1.5}$	15.00
$20 \xrightarrow{\bullet 1.5}$	30.00
$50 \xrightarrow{\bullet 1.5}$	75.00

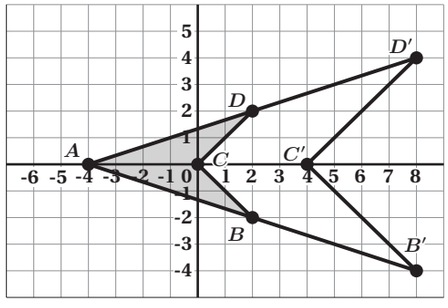
Similarity and Dilations

In this unit, students use scale factors to create and compare scaled copies. **In Grade 8, Unit 2,** students will apply this to understand similar figures, which are scaled copies of one another.

FL B.E.S.T. Math Benchmarks:
MA.8.GR.2.2, MA.8.GR.2.4

Example:

Figures are *similar* if one can fit exactly over the other after rigid transformations (translations, rotations, and reflections) and dilations. *Dilations* are a transformation in which each point on a figure moves along a line and changes its distance from a fixed point (called the center of dilations). Each distance is multiplied by the same scale factor.



In this example, each point in $AB'C'D'$ is twice as far from the center of dilation (A) as it is in $ABCD$.

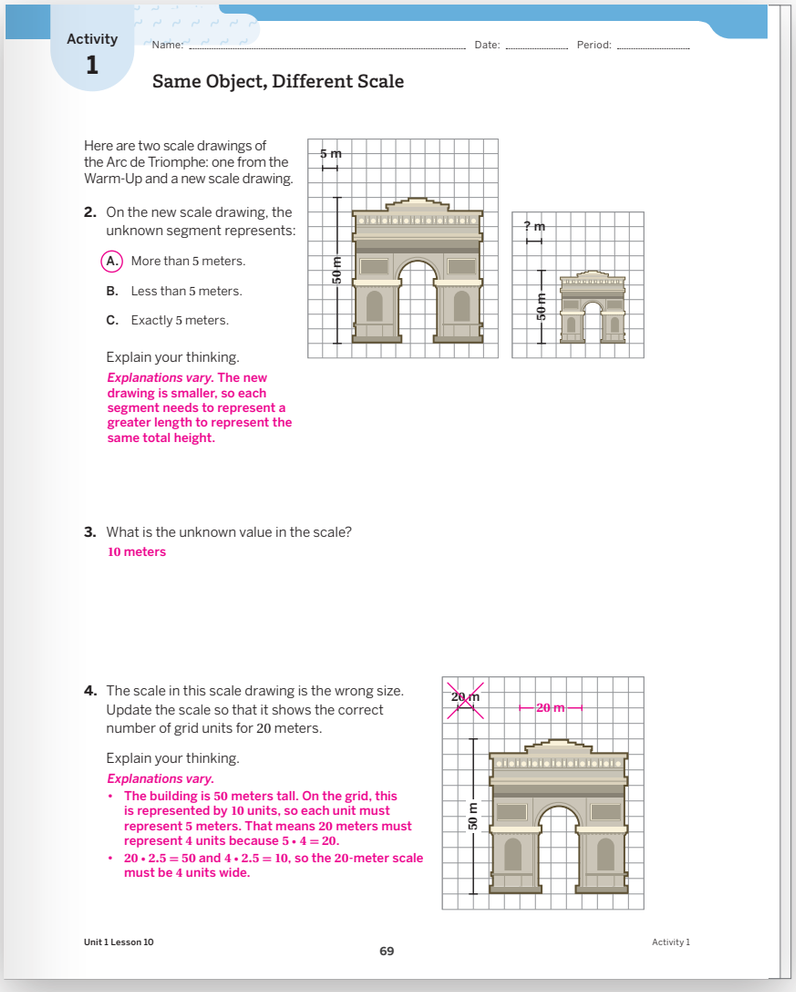
The scale factor from figure $ABCD$ to figure $AB'C'D'$ is 2, and $ABCD$ is similar to $AB'C'D'$.

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 10, Activity 1



Activity 1

Name: _____ Date: _____ Period: _____

Same Object, Different Scale

Here are two scale drawings of the Arc de Triomphe: one from the Warm-Up and a new scale drawing.

2. On the new scale drawing, the unknown segment represents:

A. More than 5 meters.

B. Less than 5 meters.

C. Exactly 5 meters.

Explain your thinking.

Explanations vary. The new drawing is smaller, so each segment needs to represent a greater length to represent the same total height.

3. What is the unknown value in the scale?

10 meters

4. The scale in this scale drawing is the wrong size. Update the scale so that it shows the correct number of grid units for 20 meters.

Explain your thinking.

Explanations vary.

- The building is 50 meters tall. On the grid, this is represented by 10 units, so each unit must represent 5 meters. That means 20 meters must represent 4 units because $5 \cdot 4 = 20$.
- $20 \cdot 2.5 = 50$ and $4 \cdot 2.5 = 10$, so the 20-meter scale must be 4 units wide.

Unit 1 Lesson 10 69 Activity 1

Context

In this activity, students strengthen their understanding of how the choice of a scale affects the size of a scale drawing.

Try This

Put on your student hat and complete Problems 2 and 3.

Points to Ponder

- What do students need to understand about scale to support their resizing scale drawings?
- Some students may think that because the new scale drawing is smaller, each grid unit represents less distance. How might you support students' reasoning about the relationship between the scale and the size of the drawing?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Scaling values using a table	Lesson 2	1	MA.6.AR.3.3
2	Multiplying fraction by whole number of whole number by fraction	Lesson 3	1	MA.4.FR.2.4
3	Interpreting contexts involving multiplying numbers by values less than or greater than 1	Lesson 4	2	MA.4.FR.2.4 MTR.6.1
4	Calculating the area of figures on a grid	Lesson 5	2	MA.6.GR.2.2 MTR.4.1, MTR.5.1
5	Ordering different units by length	Lesson 6	1	MA.4.M.1.2
6	Using a table to convert between different units of measure	Lesson 7	1	MA.6.AR.3.5

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 1

1. A recipe for one loaf of bread uses 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. Complete the table to show the quantities needed for multiple loaves of bread.

Number of Loaves	Flour (cups)	Water (tbsp)	Salt (tsp)
1	2	12	1
2	4	24	2
4	8	48	4

2. Determine the missing value to make each equation true.

a. $4 \cdot \underline{6} = 24$

b. $24 \cdot \underline{\frac{1}{6}} = 4$

c. $3 \cdot \underline{4} = 12$

d. $12 \cdot \underline{\frac{1}{4}} = 3$

3. Farah, Raine, and Valeria are trying to drink more water.

- Farah drank 3 liters of water yesterday.
- Raine drank $\frac{3}{4}$ as much water as Farah.
- Valeria drank twice as much water as Raine.

Order the names based on how much water was drunk.

	Name
Least	Raine
	Farah
Greatest	Valeria

Explain your thinking.

Explanations vary. $\frac{3}{4}$ is less than 1 whole, so Raine drank less than Farah. $\frac{3}{4} \cdot 3 = 2.25$ and $2 \times 2.25 = 4.5$, which is more than 3, so Valeria drank more than Farah.

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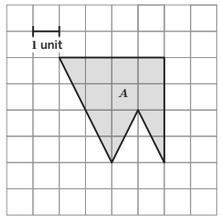
Name: Date: Period:

Pre-Unit Check (continued)

Unit 1

4. What is the area of figure A?

10 square units



Explain or show your thinking.

Explanations vary. Figure A can be surrounded by a 4-by-4-unit square with two triangles removed. Those triangles have areas of 4 square units and 2 square units. The area of figure A is 10 square units because $16 - 4 - 2 = 10$.

5. Order these units by length.

1 mile	1 inch	1 yard	1 foot

1 inch	1 foot	1 yard	1 mile

Least Greatest

6. Marine biologists use Wiffle balls in photos to measure corals and other objects. This measurement is a Wiffle.

Complete the table of Wiffles and inches.

Distance (Wiffles)	Distance (inches)
2	7
6	21
3	10.5

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

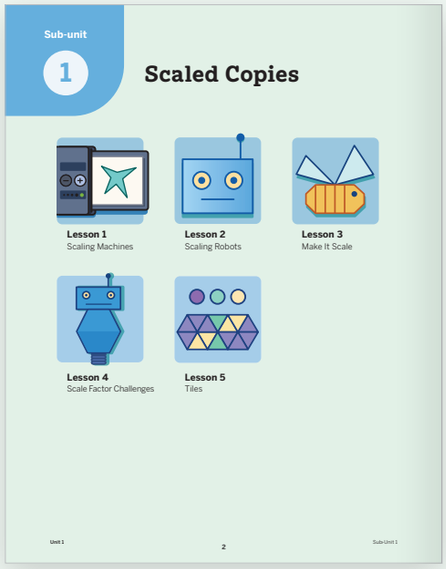
Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: Describe how scaling affects lengths, angles, and areas in scaled copies. Use scale factors to create and compare scaled copies. **(Lessons 1–5)**	1	**Support** • **Teacher Move:** Before Lesson 3, consider inviting students to share strategies for completing the table.
	2	**Support** • **Teacher Move:** During the Synthesis in Lesson 2, consider writing the equation $10 \cdot _____ = 15$ and discussing solving strategies.
	3	**Support** • **Mini-Lesson:** *Comparing Products without Multiplying* • **Teacher Move:** After Lesson 4, consider revisiting this problem and inviting students to share their reasoning. If students do not make connections to scale factors, consider asking, "If Farah is the original, what would Rebecca's scale factor be? Valeria's?"
	4	**Support** • **Mini-Lesson:** *Calculating the Area of Triangles*
	5	**Support** • **Teacher Move:** Before Lesson 7, consider creating an anchor chart to review units of measure for length. List common metric and imperial units of length in order from smallest to largest.
	6	**Support** • **Teacher Move:** After Lesson 6, consider reviewing this problem and discussing strategies for finding the missing values.

Scaled Copies

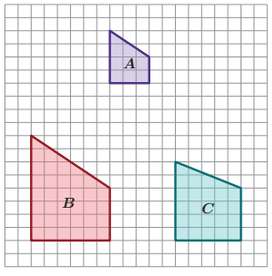
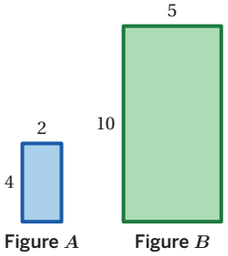
Sub-Unit 1 Goals

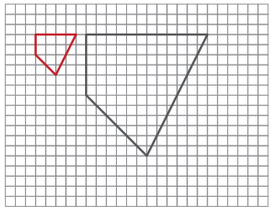
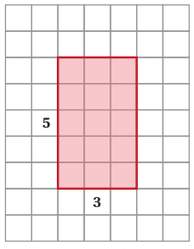
- **Goal:** Describe how scaling affects lengths, angles, and areas in scaled copies.
- **Goal:** Use scale factors to create and compare scaled copies.
- **Language Goal:** Analyze how scaling affects lengths, angles, and areas in scaled copies using mathematically precise language.

In this sub-unit, students explore objects that are and are not **scaled copies** of one another to build understanding of the characteristics of a scaled copy. They describe how scaling affects lengths, angles, and areas in scaled copies, and use **scale factors** to create and compare scaled copies.

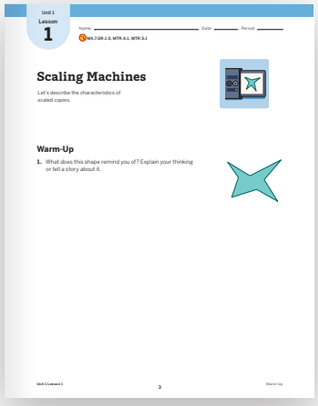


Math That Matters Most

Concept or Skill	For example . . .
Determine whether or not two figures are scaled copies.	Here is figure A and two other figures. Which is a scaled copy of figure A?  Figure B is a scaled copy of figure A because it has the same angles and all the lengths are multiplied by 2.
Determine the scale factor between two scaled copies.	What scale factor makes figure A match figure B?  Figure A Figure B The scale factor 2.5 makes figure A match figure B.

Concept or Skill	For example . . .
Create a scaled copy using a given scale factor.	Draw a scaled copy of the polygon using a scale factor of 3.  The gray polygon is a scaled copy of the red polygon using a scale factor of 3.
Calculate the area of a scaled copy with a given scale factor.	Imagine scaling this rectangle using a scale factor of 4. What is the area of the scaled copy?  The area of the scaled copy is 240 square units. The dimensions of the copy are $5 \cdot 4 = 20$ units and $3 \cdot 4 = 12$ units, and $20 \cdot 12 = 240$.

Unit 1
Lesson
1


Unit 1
Lesson
1
Name _____ Date _____ Period _____
MA.7.GR.1.5, MTR.4.1, MTR.5.1
Scaling Machines
Let's describe the characteristics of scaled copies.
Warm-Up
1. What does this shape remind you of? Explain your thinking or tell a story about it.
3
Warm-Up

Scaling Machines

What Are Scaled Copies?

Let's describe the characteristics of scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether one figure is a *scaled copy* of another.
2. **Goal:** Use equivalent ratios to create a *scaled copy* of a figure.
3. **Language Goal:** Justify whether two shapes are scaled copies of one another. **(Speaking and Listening)**

Students develop an understanding of what **scaled copies** are, describe characteristics of scaled copies, and use *equivalent ratios* to create several scaled copies of a figure. The lesson is designed to be accessible to all students regardless of prior knowledge, and to encourage students to see math as playful and exploratory.

Coherence

Prior Learning

In Grade 6, students recognized and created equivalent ratios.

Future Learning

Later in the unit, students will continue to reason quantitatively about scaling, including learning that scaled copies are related by a scale factor. They will analyze and create precise scaled copies.

Connections

- Within the context of copies made on a working printer and a broken printer, students explore the criteria for what makes one figure a scaled copy of the other. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scaled copy
proportional

Review Vocabulary

equivalent ratios

Lesson at a Glance

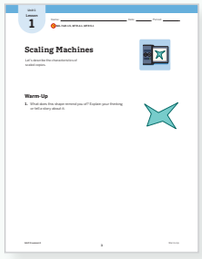
~ 45 min

MA.7.GR.1.5

Warm-Up

Independent | 5 min

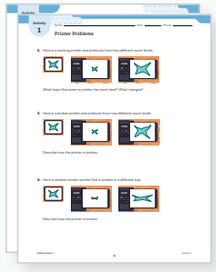
Students analyze a shape in preparation for identifying scaled copies of the shape.



Activity 1

Pairs | 15 min

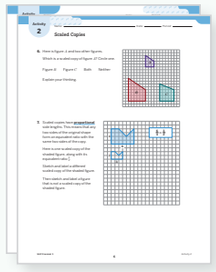
Students explore properties of **scaled copies** by identifying patterns in examples and non-examples. **(MTR.5.1)**



Activity 2

Pairs | 10 min

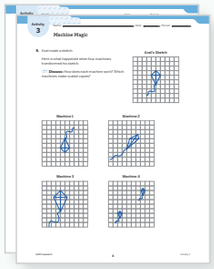
Students determine whether or not shapes are scaled copies by reasoning about *equivalent ratios* between sides.



Activity 3

Pairs | 5 min

Students discover ways that a shape can be changed to create or not create a scaled copy.

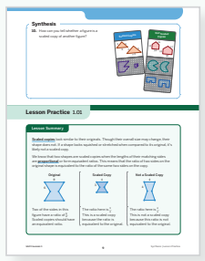


Synthesis

Whole Class | 5 min

Students synthesize their understanding of the important features of scaled copies.

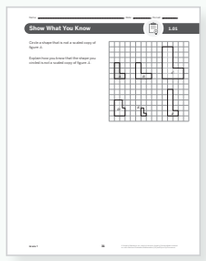
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether shapes are scaled copies.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students analyze a shape in preparation for identifying scaled copies of the shape.

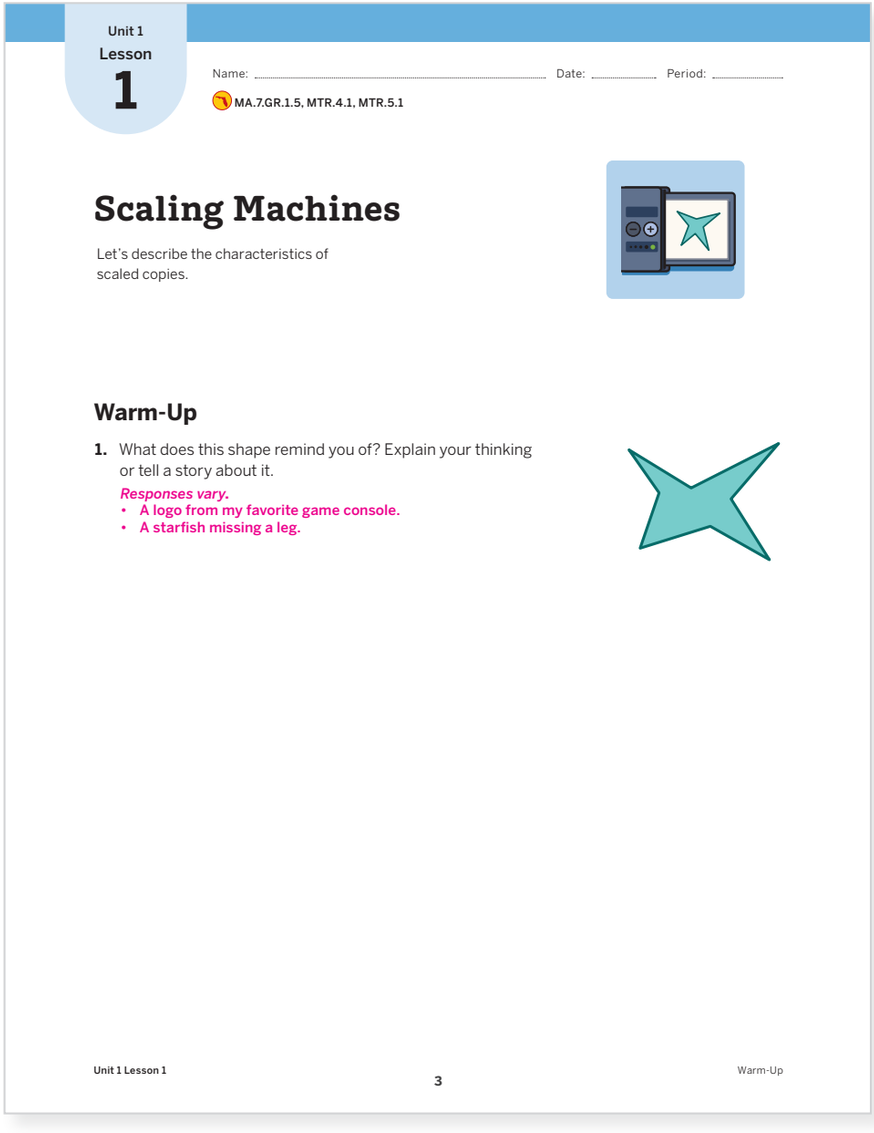
1 Launch

Invite students to share their observations.

2 Connect

Consider sharing that students will be using this shape in the next activity.

ELL **English Language Learners** Invite students to share in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**



Unit 1
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scaling Machines

Let's describe the characteristics of scaled copies.

Warm-Up

1. What does this shape remind you of? Explain your thinking or tell a story about it.

Responses vary.
- A logo from my favorite game console.
- A starfish missing a leg.

Unit 1 Lesson 1 3 Warm-Up

Pairs | 15 min

Activity 1 Printer Problems

Purpose: Students explore properties of **scaled copies** by identifying patterns in examples and non-examples. **(MTR.5.1)**

1 Launch

To surface prior knowledge, consider asking students what experiences they have had with printers or copiers.

Consider asking:

- "Have you ever used the zoom feature on a web page?"
- "What happens to the picture or text when you zoom in or zoom out?"

2 Monitor

Listen for informal language students use to describe the working printer, such as "same shape, but different size."

Note: Allow students to use their own language to describe what they see. The term *scaled copy* will be introduced later in the activity.

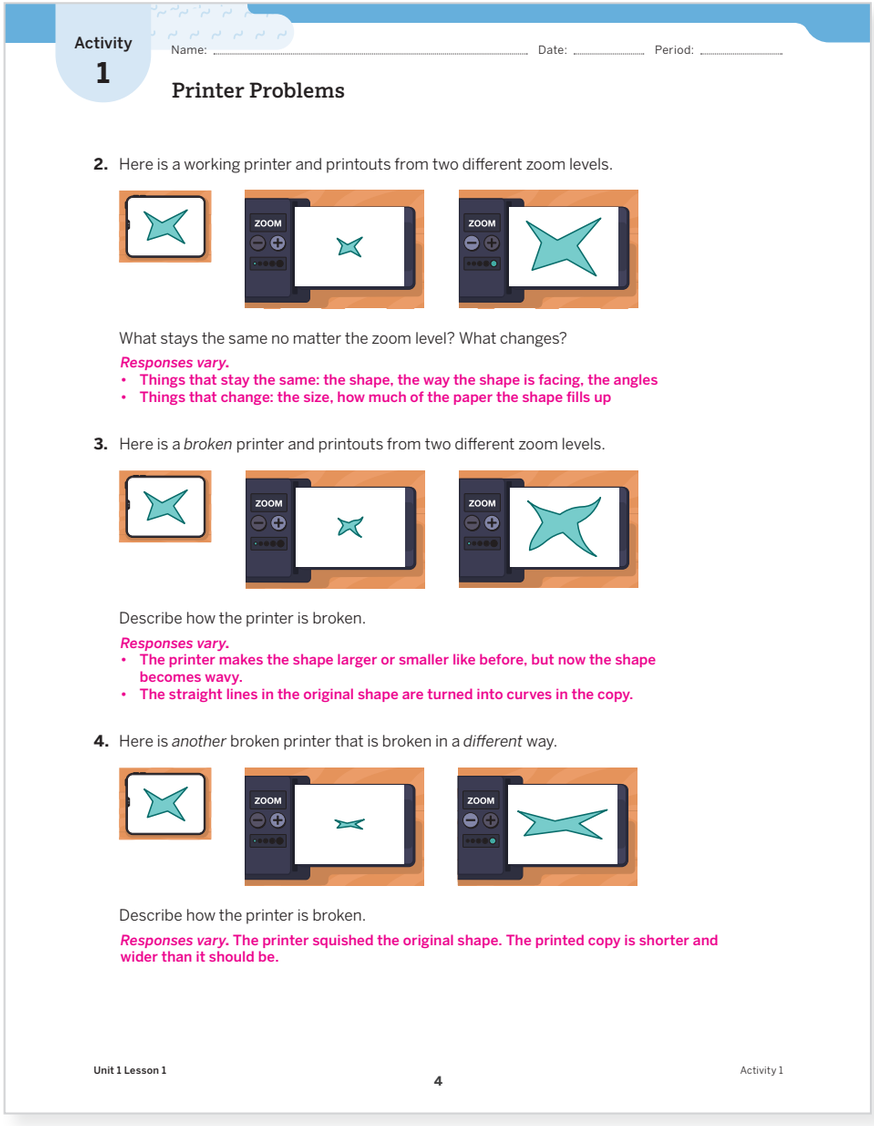
Consider asking, "How is this similar to the working printer? How is it different?" **(MTR.5.1)**

Responses vary.

- All the printers make the shape larger or smaller depending on the zoom level.
- The broken printer makes the shape wavy. *(Problem 3)*
- The broken printer squishes the shape by making it shorter or wider than it should be. *(Problem 4)*

Listen to partner discussions for student language and ideas that might be useful to share during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Printer Problems

2. Here is a working printer and printouts from two different zoom levels.

What stays the same no matter the zoom level? What changes?

Responses vary.
- **Things that stay the same: the shape, the way the shape is facing, the angles**
- **Things that change: the size, how much of the paper the shape fills up**

3. Here is a *broken* printer and printouts from two different zoom levels.

Describe how the printer is broken.

Responses vary.
- **The printer makes the shape larger or smaller like before, but now the shape becomes wavy.**
- **The straight lines in the original shape are turned into curves in the copy.**

4. Here is *another* broken printer that is broken in a *different* way.

Describe how the printer is broken.

Responses vary. **The printer squished the original shape. The printed copy is shorter and wider than it should be.**

Unit 1 Lesson 1 4 Activity 1

Activity 1 Printer Problems (continued)

Purpose: Students explore properties of **scaled copies** by identifying patterns in examples and non-examples. **(MTR.5.1)**

2 Monitor

MLR1: Stronger and Clearer Each Time (optional) Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Select and sequence student responses to share during the Connect.

Differentiation

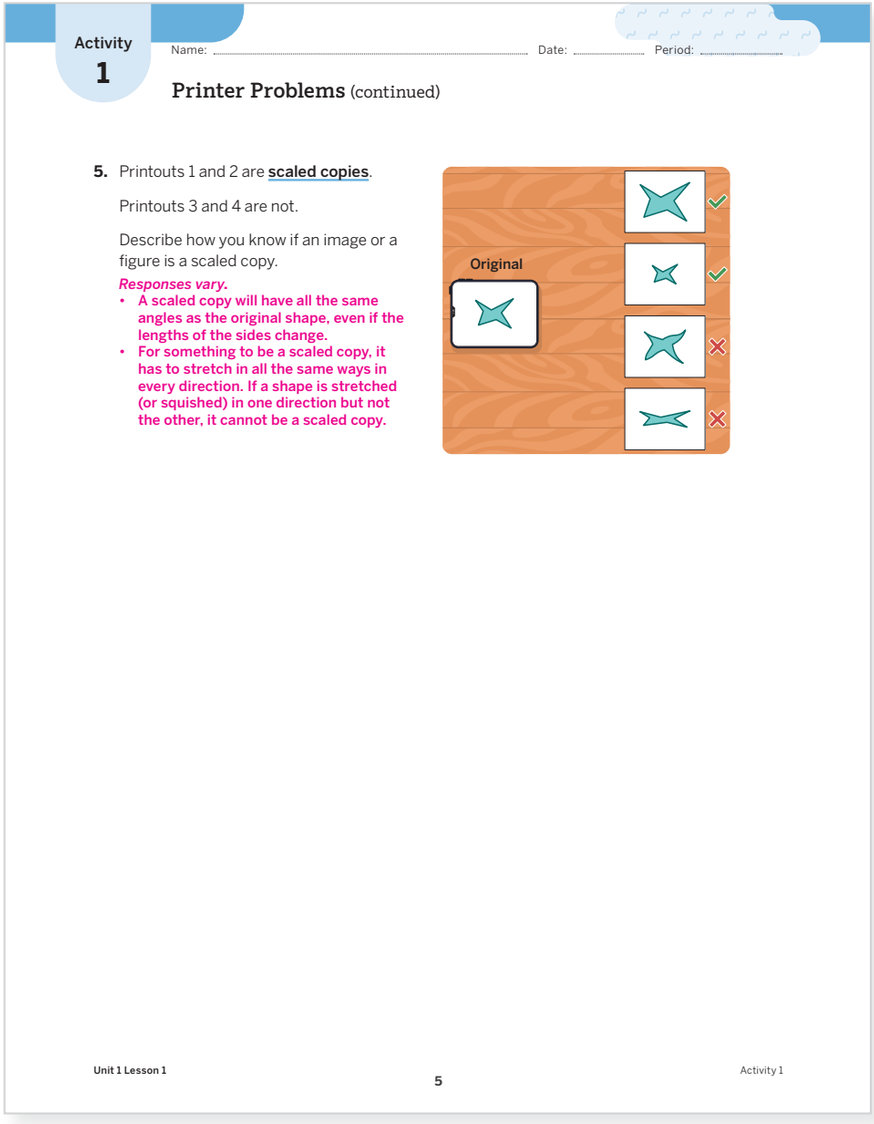
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to revisit their thinking on Problems 2–4 and make a connection to the explanations they wrote there.
Write or say that scaled copies are "larger or smaller than the original."	**Support** Consider asking, "Printouts 3 and 4 are larger and smaller but are not scaled copies. What do 1 and 2 have in common that make them scaled copies?"
Write or say that scaled copies have the "same shape."	**Strengthen** Invite students to say more. Consider asking, "Are all triangles scaled copies of one another? What about all rectangles?"
Use a variety of informal language to describe scaled copies, such as: • Larger, smaller, or the same size. • Same angles. • Different length. • Different area.	**Strengthen** Collect these strategies to share during the Connect. Consider asking, "What are some ways to create copies that are not scaled copies?"

3 Connect

Invite students to share their thinking.

Create a class definition of the term **scaled copy** using students' informal language. Consider including non-examples to help emphasize what features a scaled copy always has.

Note: This lesson provides an introduction to the term *scaled copy*, which will be defined more precisely in Lesson 2.



Activity 1

Name: Date: Period:

Printer Problems (continued)

5. Printouts 1 and 2 are **scaled copies**.

Printouts 3 and 4 are not.

Describe how you know if an image or a figure is a scaled copy.

Responses vary.
- A scaled copy will have all the same angles as the original shape, even if the lengths of the sides change.
- For something to be a scaled copy, it has to stretch in all the same ways in every direction. If a shape is stretched (or squished) in one direction but not the other, it cannot be a scaled copy.

Unit 1 Lesson 1 5 Activity 1

Key Takeaway: A **scaled copy** has the same angles and the same overall shape as the original figure but can be the same size, larger, or smaller.

Pairs | 10 min

Activity 2 Scaled Copies

Purpose: Students determine whether or not shapes are scaled copies by reasoning about *equivalent ratios* between sides.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to make and justify a choice.

Differentiation: (Support) To support students getting started, consider asking, "Which copy has the same angles as the original?"

Note: It is okay if students are unsure. They will have more opportunities to explore what makes a scaled copy.

2 Monitor

Invite students to read the text aloud to one another and to explain what they think **proportional** means in their own words.

Consider asking, "What strategies will you use to draw a scaled copy of the figure and draw a figure that is not a scaled copy?"

Accessibility: Memory and Attention To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Note: If further discussion about scaled copies is needed, consider pausing briefly to show a shape with a height of 6 and a length of 8. Consider asking, "Why might someone think this is a scaled copy?"

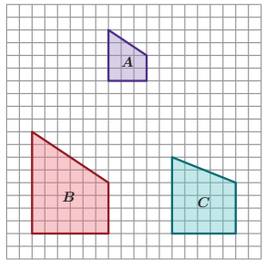
Activity 2 continued >

Activity 2

Name: Date: Period:

Scaled Copies

6. Here is figure A and two other figures.



Which is a scaled copy of figure A? Circle one.

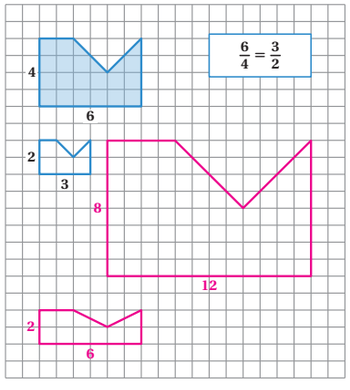
(Figure B) Figure C Both Neither

Explain your thinking.

Explanations vary.

- Figure B is the only one that has the same shape as figure A. Figure C is close to the same shape, but its top side is a little flatter.
- Figure B has the same angles as figure A, and the side lengths are all 2 times longer. Figure C doesn't have the same angles as figure A.

7. Scaled copies have **proportional** side lengths. This means that any two sides of the original shape form an *equivalent ratio* with the same two sides of the copy.



Here is one scaled copy of the shaded figure, along with its equivalent ratio $\frac{3}{2}$.

Sketch and label a *different* scaled copy of the shaded figure.

Then sketch and label a figure that is *not* a scaled copy of the shaded figure.

Responses vary.

Unit 1 Lesson 1 6 Activity 2

Pairs | 10 min

Activity 2 Scaled Copies (continued)

Purpose: Students determine whether or not shapes are scaled copies by reasoning about *equivalent ratios* between sides.

Short on time? Consider omitting Problem 6.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about whether the pairs of shapes are scaled copies. **(MTR.4.1)**

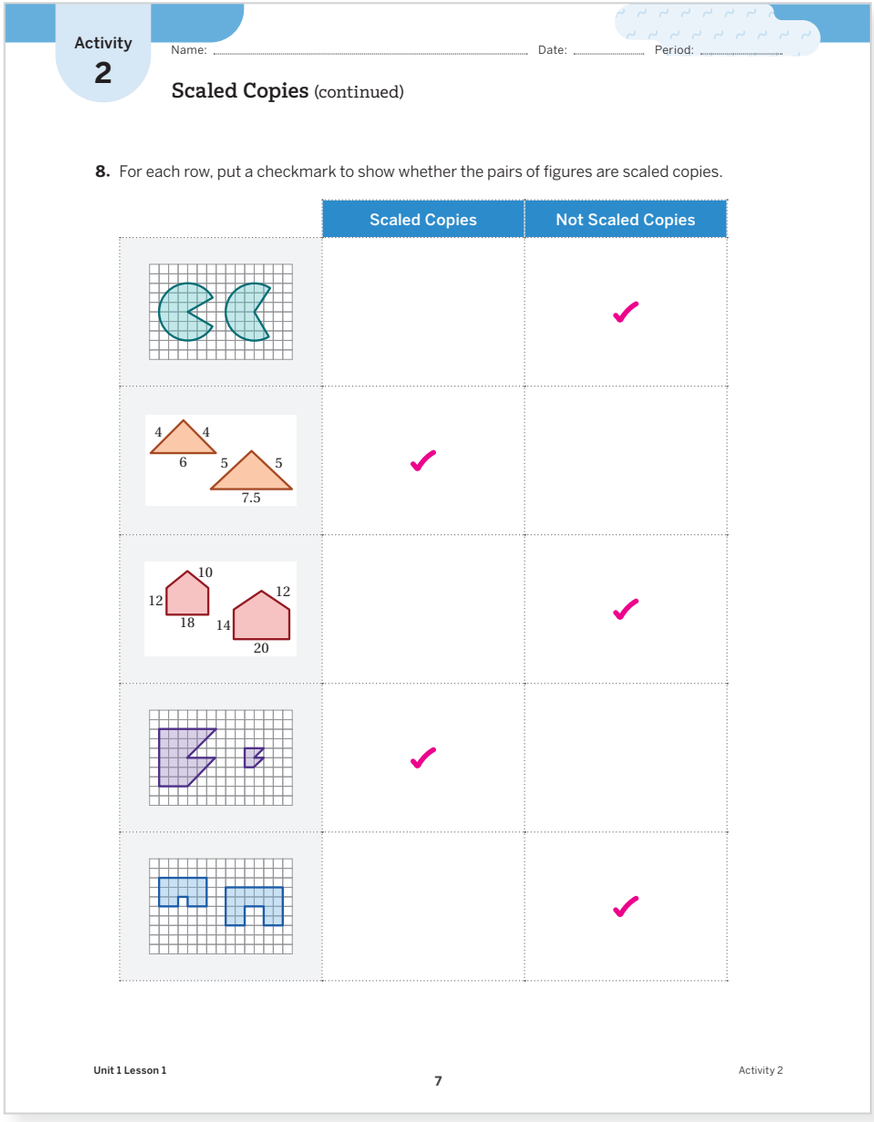
ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I can tell these are scaled copies because _____."). **(Writing and Speaking)**

Listen for a variety of student strategies for categorizing the pairs of shapes including both using *equivalent ratios* and inferring using the overall shape of each figure.

3 Connect

Invite students to share explanations and strategies for how they categorized the pairs of shapes.

To surface the Key Takeaway, consider asking, "How do the gridlines or measurements help you determine whether the shapes are scaled copies?"



Activity 2

Name: Date: Period:

Scaled Copies (continued)

8. For each row, put a checkmark to show whether the pairs of figures are scaled copies.

	Scaled Copies	Not Scaled Copies
		✓
	✓	
		✓
	✓	
		✓

Unit 1 Lesson 1 7 Activity 2

Key Takeaway: Scaled copies have **proportional** side lengths. This means that any two sides of one shape form an *equivalent ratio* with the two matching sides of the other.

Pairs | 5 min

Activity 3 Machine Magic

Purpose: Students discover ways that a shape can be changed to create or not create a scaled copy.

Short on time? Consider omitting Activity 3.

1 Launch

Direct students' attention to Axel's sketch. Invite students to share their observations about the characteristics of the kite.

To generate excitement, consider asking, "Have you ever flown a kite?"

2 Monitor

Listen for a variety of formal and informal language students use to describe the machines.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Select and sequence student ideas to share during the Connect.

D **Differentiation**

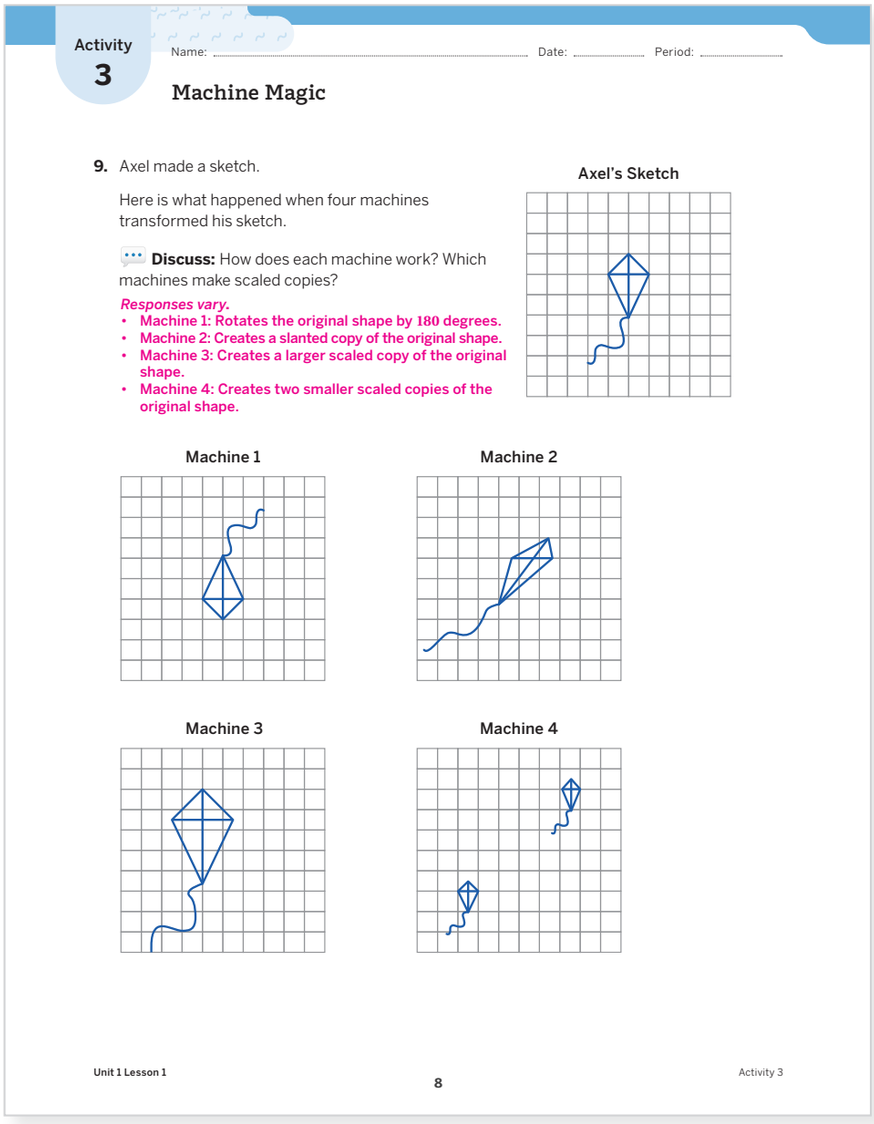
Look for students who:	Respond to Student Thinking
Think Machine 1 does not create scaled copies because it rotates the sketch.	**Support** Consider asking, "What are the important characteristics of scaled copies?"
Actively listen to their partners and build on each other's ideas or language.	**Strengthen** Celebrate these students one-on-one or publicly.
Identify that Machines 1, 3, and 4 create scaled copies.	**Stretch** Consider asking, "What do those three machines have in common?"

Listen for student strategies and invite students to share more about their explanations and ideas.

3 Connect

Discuss the characteristics students generated to describe which machines make scaled copies.

Consider asking, "What do the machines that create scaled copies have in common?"



Activity 3

Name: Date: Period:

Machine Magic

9. Axel made a sketch.

Here is what happened when four machines transformed his sketch.

Discuss: How does each machine work? Which machines make scaled copies?

Responses vary.
- Machine 1: Rotates the original shape by 180 degrees.
- Machine 2: Creates a slanted copy of the original shape.
- Machine 3: Creates a larger scaled copy of the original shape.
- Machine 4: Creates two smaller scaled copies of the original shape.

Axel's Sketch

Machine 1

Machine 2

Machine 3

Machine 4

Unit 1 Lesson 1 8 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the important features of scaled copies.

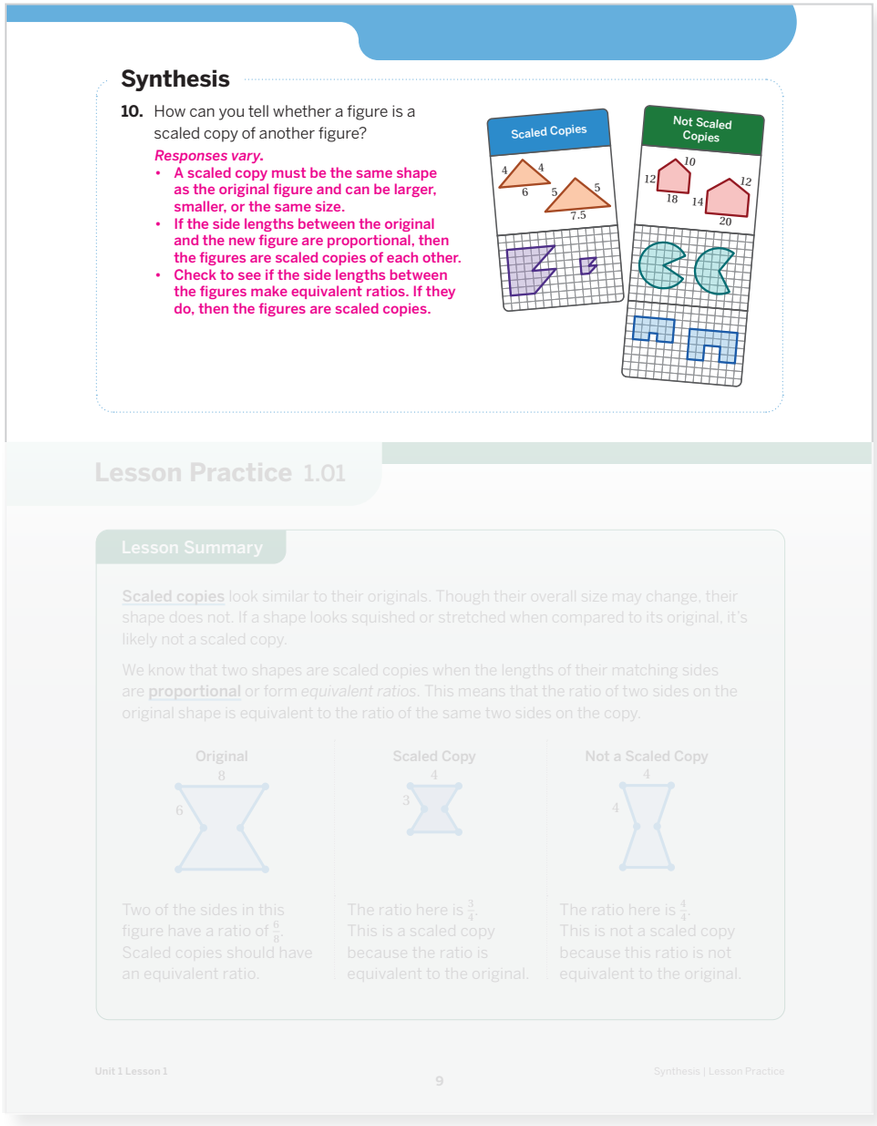
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Scaled copies are the same overall shape, but can be different sizes.
- You can check whether two shapes are scaled copies by looking for equivalent ratios between the sides (checking whether the side lengths are proportional).
- Figures that are not scaled copies of each other might be distorted. They have lines or angles that do not look like the original.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *scaled copies* and *proportional* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to restate the definitions to a partner in their own words, and share what questions they still have.



Synthesis

10. How can you tell whether a figure is a scaled copy of another figure?

Responses vary.

- A scaled copy must be the same shape as the original figure and can be larger, smaller, or the same size.
- If the side lengths between the original and the new figure are proportional, then the figures are scaled copies of each other.
- Check to see if the side lengths between the figures make equivalent ratios. If they do, then the figures are scaled copies.

Lesson Practice 1.01

Lesson Summary

Scaled copies look similar to their originals. Though their overall size may change, their shape does not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

We know that two shapes are scaled copies when the lengths of their matching sides are proportional or form *equivalent ratios*. This means that the ratio of two sides on the original shape is equivalent to the ratio of the same two sides on the copy.

Original	Scaled Copy	Not a Scaled Copy
8, 6	4, 3	4, 4
Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.	The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.	The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

Unit 1 Lesson 1 9 Synthesis | Lesson Practice

Lesson Takeaway: A **scaled copy** is a copy with the same overall shape as the original figure. It can be the same size, larger, or smaller. The two figures must have **proportional** side lengths, meaning they form *equivalent ratios*.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether shapes are scaled copies.

Today's Goals

1. **Goal:** Determine whether one figure is a *scaled copy* of another.
2. **Goal:** Use equivalent ratios to create a *scaled copy* of a figure.
3. **Language Goal:** Justify whether two shapes are *scaled copies* of one another. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.01

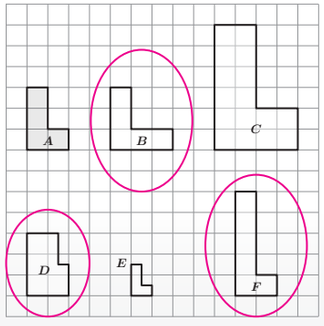
Circle a shape that is not a scaled copy of figure A.

Possible responses shown.

Explain how you know that the shape you circled is *not* a scaled copy of figure A.

Explanations vary.

- Figure B is not a scaled copy of figure A because the side lengths are not proportional. The figures are the same height, but not the same width.
- Figure D is not a scaled copy of figure A because they're not the same shape. The two figures are the same height and width, but the area of figure D is greater.
- Figure F is not a scaled copy of figure A because the side lengths don't make equivalent ratios. The height ratio is $\frac{5}{3}$ and the width ratio is $\frac{2}{2}$.



D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine if figures are scaled copies:

» **Respond to Student Thinking:**

- Invite students to discuss how to use equivalent ratios to determine if two shapes are scaled copies with a partner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on getting to know your class today. What did you learn about your students?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How can you tell whether a figure is a scaled copy of another figure?

Responses vary.

- A scaled copy must be the same shape as the original figure and can be larger, smaller, or the same size.
- If the side lengths between the original and the new figure are proportional, then the figures are scaled copies of each other.
- Check to see if the side lengths between the figures make equivalent ratios. If they do, then the figures are scaled copies.

Lesson Practice 1.01

Lesson Summary

Scaled copies look similar to their originals. Though their overall size may change, their shape does not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

We know that two shapes are scaled copies when the lengths of their matching sides are **proportional** or form *equivalent ratios*. This means that the ratio of two sides on the original shape is equivalent to the ratio of the same two sides on the copy.

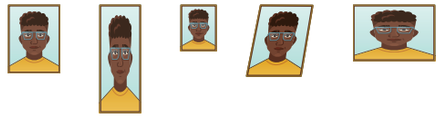
Original	Scaled Copy	Not a Scaled Copy
		
Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.	The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.	The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

Lesson Practice 1.01

Name: _____ Date: _____ Period: _____

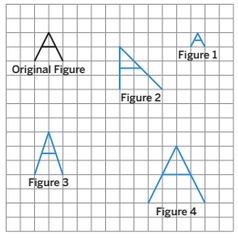
1. Andre's grandma ordered school pictures. She thought she would receive the original portrait, but instead she got the following images. How is each image different from the original portrait of Andre?

Original Portrait | Image A | Image B | Image C | Image D



Responses vary. Image A is "squished" from the sides. Image B looks like a smaller copy of the original. Image C is slanted. Image D is "squished" from the top.

Problems 2–4: Here is the original figure along with several other figures.



2. Which figures are scaled copies of the original figure?

Figures 1 and 4

3. How does each scaled copy compare to the original?

Responses vary.

- Figure 1 is a scaled copy of the original because it's the same shape, the lengths form equivalent ratios, and the angles have the same measure.
- Figure 4 is a scaled copy of the original because its dimensions are proportional to the original figure. It is twice as tall and twice as wide as the original figure.

4. How do you know a figure is *not* a scaled copy?

Responses vary.

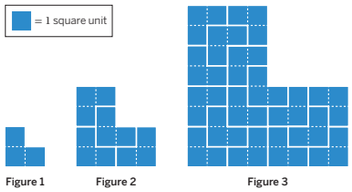
- Figure 2 is not a scaled copy of the original because it is not the same shape as the original figure. The angles at the top of the "A" are different from the original.
- Figure 3 is not a scaled copy of the original because its height and width aren't proportional to the original.

Lesson Practice 1.01

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 5–7: Figures 1, 2, and 3 are scaled copies of each other. Each small square has a side length of 1 unit and an area of 1 square unit.



5. Complete the table.

	Figure 1	Figure 2	Figure 3
Perimeter (units)	8	16	32
Area (sq. units)	3	12	48

6. Describe any patterns you see.

Responses vary. The perimeter of each figure that follows is twice the perimeter of the previous figure. The area of each figure that follows is four times greater than the area of the previous figure.

7. If the pattern continues, what will be the perimeter and area of Figure 4?

Perimeter: 64 units Area: 192 sq. units

Spiral Review

Problems 8–11: Complete each equation with a number that makes it true.

8. $5 \cdot \boxed{3} = 15$

9. $6 \cdot \boxed{1.5} = 9$

10. $\boxed{8} \cdot 4 = 32$

11. $\boxed{\frac{1}{4}} \cdot 12 = 3$

Lesson Practice Item Analysis

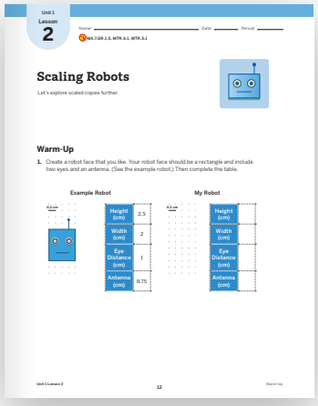
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.GR.1.5
FAST Practice	5–7	2	MA.7.GR.1.5
Spiral Review			
	8–11	1	MA.5.FR.2.2

Unit 1
Lesson
2

Scaling Robots

Lengths and Scaled Copies

Let's explore scaled copies further.


Unit 1
Lesson 2
Name: Date: Period:
MA.7.GR.1.5, MTR.4.1, MTR.5.1
Scaling Robots
Let's explore scaled copies further.
Warm-Up
1. Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.
Example Robot
My Robot
Height (cm)
Width (cm)
Eye Distance (cm)
Antenna (cm)
2.5
2
1
0.75
12

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that a *scale factor* is the number multiplied by each side length in an original figure to create a scaled copy.
2. **Goal:** Understand that there is a proportional relationship between lengths in a figure and corresponding lengths in a scaled copy.
3. **Language Goal:** Explain how scaling affects side lengths in scaled copies. **(Speaking and Listening)**

Students scale images, make connections between **scale factor** and equivalent ratios, and analyze other students' thinking about scaled copies. This lesson provides opportunities to communicate to students that incorrect, imprecise, and unfinished thinking are valuable.

Coherence

Prior Learning

In Lesson 1, students learned what scaled copies are and described their characteristics.

Future Learning

In Lesson 3, students will draw and analyze scaled copies with greater precision.

Connections

- Students extend their understanding of scaled figures to use corresponding parts to determine if a figure is a scaled copy of the other. (mathematical)
- Students critique student thinking and provide feedback for how to improve their understanding about how to make a scaled copy of an object. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scale factor
corresponding parts

Lesson at a Glance

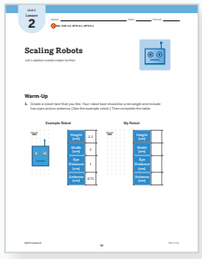
~ 45 min

MA.7.GR.1.5

Warm-Up

Independent | 5 min

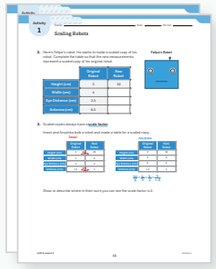
Students express their creativity as they design a robot.



Activity 1

Pairs | 15 min

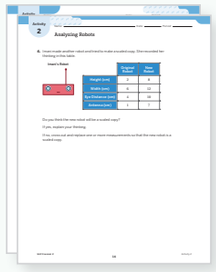
Students scale a robot and look for patterns between matching lengths, identifying two ways to conceptualize **scale factor**. **(MTR.5.1)**



Activity 2

Pairs | 15 min

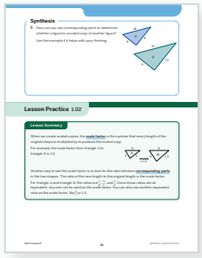
Students revise other students' work using their understanding of scale factor.



Synthesis

Whole Class | 5 min

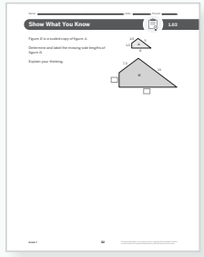
Students synthesize their understanding of scale factor and how it relates to lengths in scaled copies.



Show What You Know

Independent | 5 min

Students demonstrate their understanding of scale factor by determining the side lengths of a scaled copy.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students express their creativity as they design a robot.

1 Launch

Share that in today's lesson, students will explore scaled copies using robots.

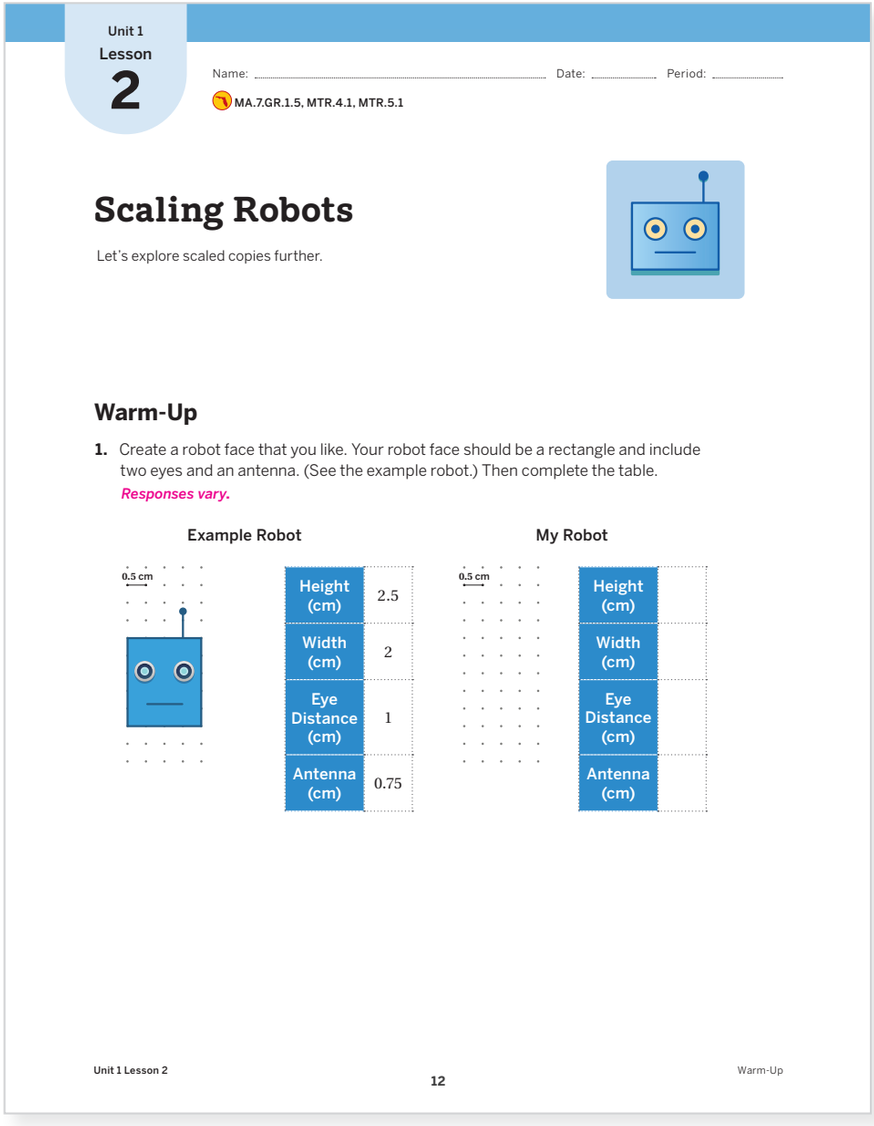
To encourage playfulness, invite students to make up a name for, or a story about, their robot.

2 Connect

Consider using the Think-Pair-Share routine (optional) to support student-to-student discourse and connection. Invite students to think independently and then take turns sharing their robot and its name or story with a partner. Invite several students to share about their or their partner's robot.

ELL **English Language Learners** Consider personally inviting students to share if they feel comfortable, including encouraging them to use their primary languages. **(Speaking and Listening)**

To surface connections, consider asking "How do you think we are going to use what we learned in the previous lesson with the robots you designed?"



Unit 1
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scaling Robots

Let's explore scaled copies further.

Warm-Up

1. Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.

Responses vary.

Example Robot

0.5 cm

Height (cm)	2.5
Width (cm)	2
Eye Distance (cm)	1
Antenna (cm)	0.75

My Robot

0.5 cm

Height (cm)	
Width (cm)	
Eye Distance (cm)	
Antenna (cm)	

Unit 1 Lesson 2 12 Warm-Up

Pairs | 15 min

Activity 1 Scaling Robots

Purpose: Students scale a robot and look for patterns between matching lengths, identifying two ways to conceptualize scale factor. **(MTR.5.1)**

1 Launch

Consider asking, "What happens if only some parts are resized and not others? Would it still look like the same robot?"

2 Monitor

Invite students to write a response independently, then discuss their ideas with a partner.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Select and sequence student explanations to share during the Connect. Consider including imprecise or unfinished explanations.

D Differentiation

Look for students who:	Respond to Student Thinking
Explain only Imani's strategy.	**Support** Encourage them to describe what Anushka wrote. Consider asking, "Why might Anushka have paired 10 and 5 together in a ratio?"
Describe Anushka's strategy without identifying the scale factor.	**Strengthen** Consider asking, "How do the ratios Anushka wrote connect to the scale factor of 2?"
Include words such as equivalent ratio and proportional in their explanation.	**Stretch** Consider inviting these students to share during the Connect or celebrating them publicly for attending to precision.

3 Connect

Invite students to share their explanations.

To surface the Key Takeaway, create a class definition of the term **scale factor** using students' language and the glossary. Record it for students to refer to later in the unit. Consider updating the class definition of *scaled copy* to reflect students' growing understanding of the term.

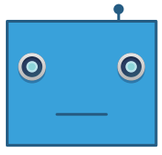
Activity 1

Name: Date: Period:

Scaling Robots

2. Here's Felipe's robot. He wants to make a scaled copy of his robot. Complete the table so that the new measurements represent a scaled copy of his original robot.

Felipe's Robot



	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	4	8
Eye Distance (cm)	2.5	5
Antenna (cm)	0.5	1

3. Scaled copies always have a **scale factor**.

Imani and Anushka built a robot and made a table for a scaled copy.

Imani

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

Anushka

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

$\frac{10}{5} = \frac{6}{3} = \frac{4}{2} = \frac{3}{1.5}$

Show or describe where in their work you can see the scale factor is 2.

Responses vary.

- Imani's work shows the scale factor as the number you need to multiply the values in the first column by to get the values in the second column. For example, $5 \cdot 2 = 10$ and $3 \cdot 2 = 6$.
- Anushka wrote all the lengths as ratios, and those ratios are equivalent. They all have the same value, 2, which must be the scale factor.

Unit 1 Lesson 2 13 Activity 1

Key Takeaway: Scaled copies are related by the **scale factor**. We can think of scale factor in two ways: it is the number all side lengths are multiplied by to create a scaled copy, and it is the ratio between matching sides of a scaled copy and an original figure.

Pairs | 15 min

Activity 2 Analyzing Robots

Purpose: Students revise other students' work using their understanding of scale factor.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to inspect Imani's new robot's dimensions in the table and continue to the next problem when they have made and justified a choice.

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means. Consider reviewing the class definition of scaled copy. **(Reading)**

Note: Some students may believe that the new robot is a scaled copy because each measurement is added by 6. It's okay — even desirable — to lack consensus at this stage.

2 Monitor

Invite students to work independently for a few minutes, then share their responses with a partner.

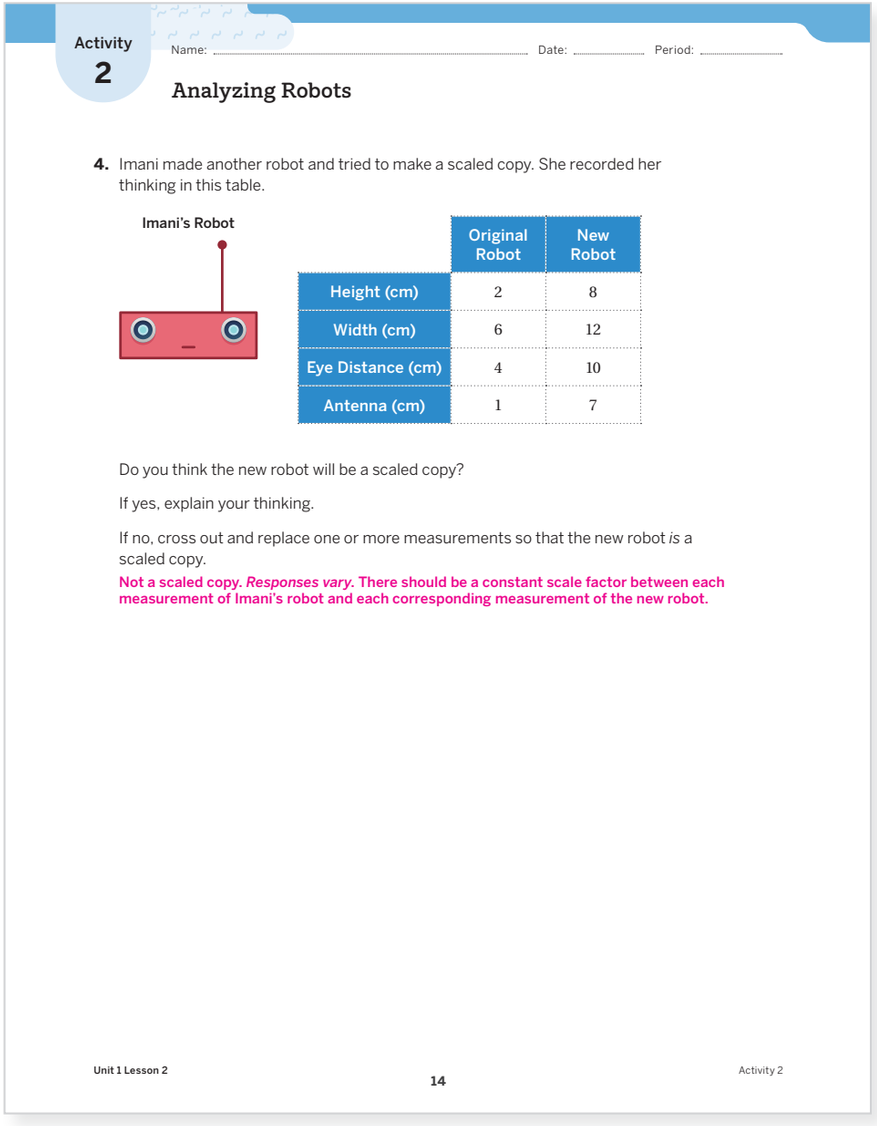
Encourage students who finish early to try creating a robot with a different scale factor.

Select and Sequence several student tables that have different scale factors. Consider including some tables that do not create scaled copies.

Pause briefly and share examples of some students' tables. Consider asking:

- "Does this table represent a scaled copy?"
- "What scale factor does this table use?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Analyzing Robots

4. Imani made another robot and tried to make a scaled copy. She recorded her thinking in this table.

Imani's Robot

	Original Robot	New Robot
Height (cm)	2	8
Width (cm)	6	12
Eye Distance (cm)	4	10
Antenna (cm)	1	7

Do you think the new robot will be a scaled copy?

If yes, explain your thinking.

If no, cross out and replace one or more measurements so that the new robot *is* a scaled copy.

Not a scaled copy. *Responses vary.* There should be a constant scale factor between each measurement of Imani's robot and each corresponding measurement of the new robot.

Unit 1 Lesson 2 14 Activity 2

Pairs | 15 min

Activity 2 Analyzing Robots (continued)

Purpose: Students revise other students' work using their understanding of scale factor.

Short on time? Consider omitting Problem 6.

2 Monitor

Invite students to read the definition of **corresponding parts** and identify them in Anushka's robots.

D **Differentiation: (Support)** Consider asking, "How can you tell the two figures are not scaled copies?"

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to identify pairs of corresponding sides.

Encourage students to read others' responses and discuss their ideas with a partner.

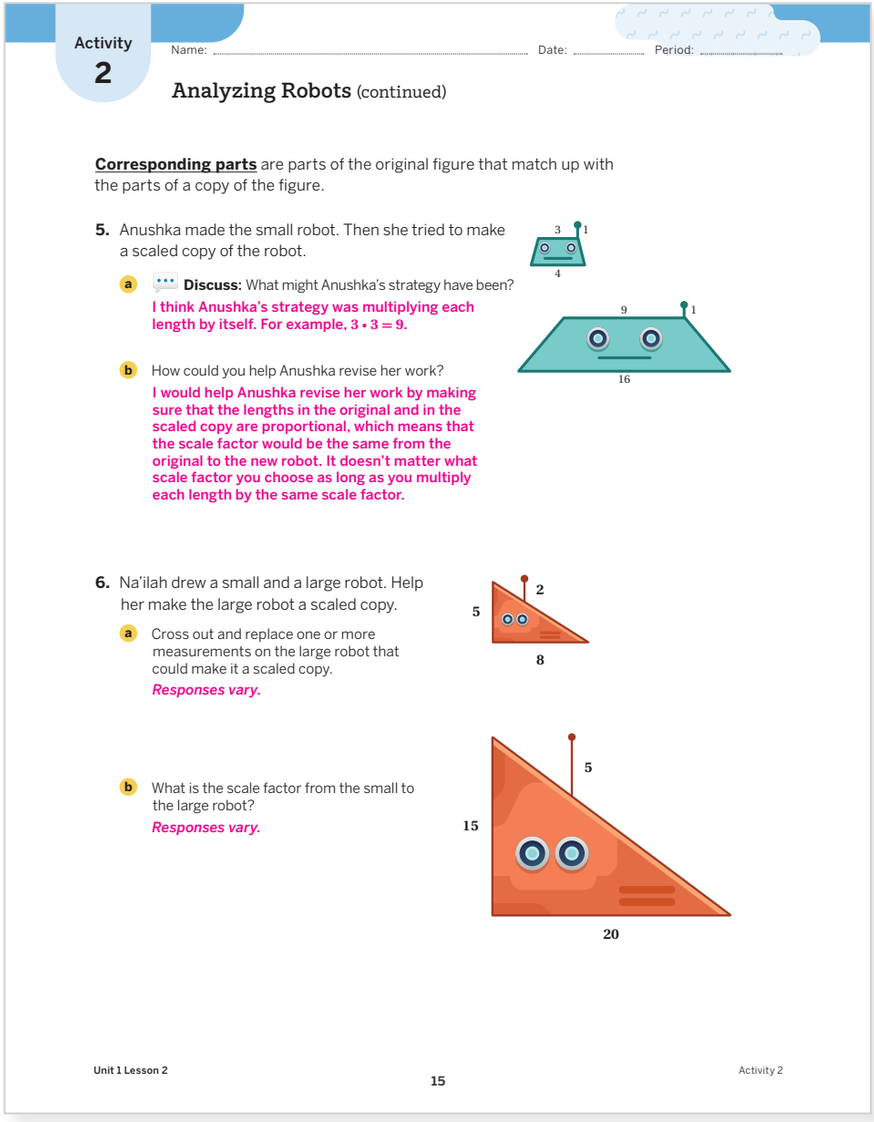
D **Differentiation: (Stretch)** To encourage exploration, consider asking, "If the antenna of the scaled copy was 5.5, what would the measurements of the height and width need to be?"

Look for strategies to share during the Connect. Look for a variety of solutions.

3 Connect

Share several solutions. As a class, consider creating a list of all the scaled copies students generated along with their accompanying scale factors.

To surface the Key Takeaway, consider asking, "Is it possible to create a scaled copy by only changing the width? Why or why not?" No, because the height and antenna would still have two different scale factors.



Activity 2

Name: Date: Period:

Analyzing Robots (continued)

Corresponding parts are parts of the original figure that match up with the parts of a copy of the figure.

5. Anushka made the small robot. Then she tried to make a scaled copy of the robot.

a. Discuss: What might Anushka's strategy have been?

I think Anushka's strategy was multiplying each length by itself. For example, 3 • 3 = 9.

b. How could you help Anushka revise her work?

I would help Anushka revise her work by making sure that the lengths in the original and in the scaled copy are proportional, which means that the scale factor would be the same from the original to the new robot. It doesn't matter what scale factor you choose as long as you multiply each length by the same scale factor.

6. Na'ilah drew a small and a large robot. Help her make the large robot a scaled copy.

a. Cross out and replace one or more measurements on the large robot that could make it a scaled copy.

Responses vary.

b. What is the scale factor from the small to the large robot?

Responses vary.

Unit 1 Lesson 2 15 Activity 2

Key Takeaway: To create a scaled copy, every length in the original figure is multiplied by the same scale factor. If lengths are multiplied by different scale factors, the result is not a scaled copy.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of scale factor and how it relates to lengths in scaled copies.

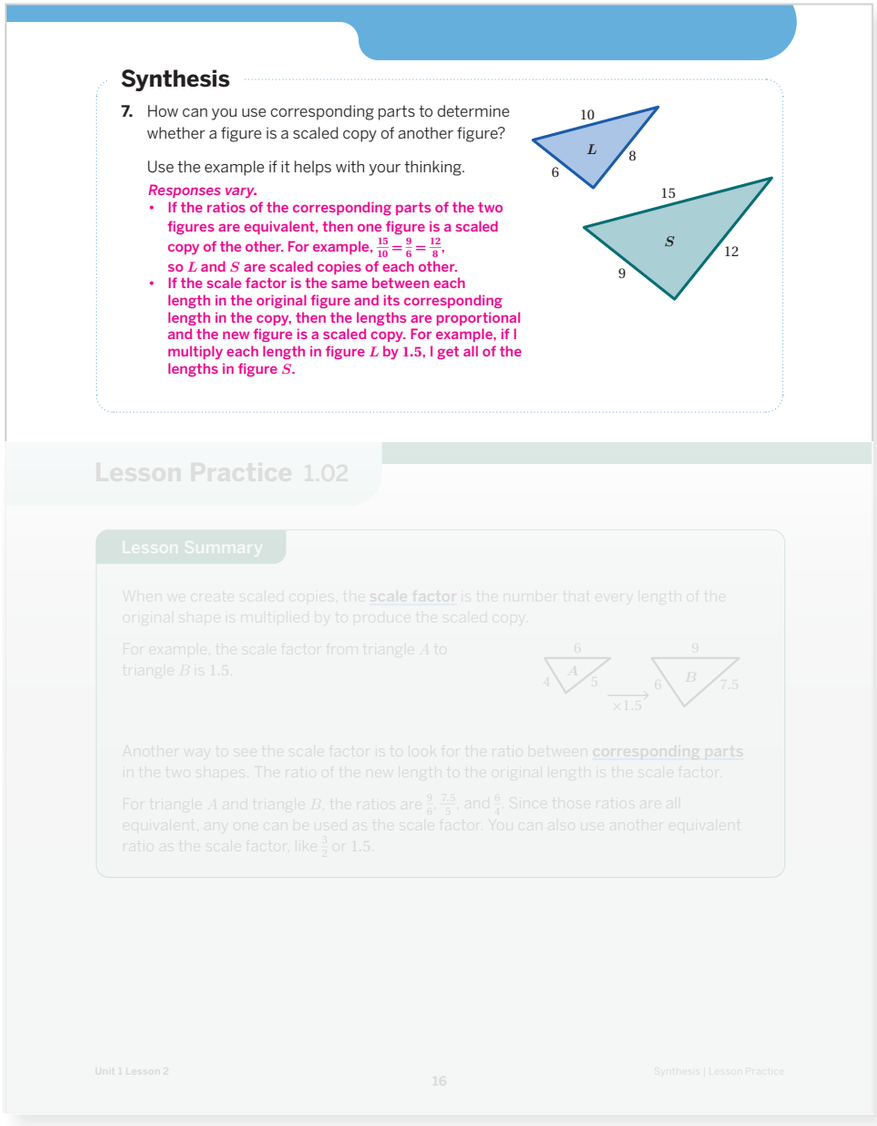
Invite students to respond independently, and then share their thinking with a partner.

Note: Encourage students to use a calculator to test ideas they have about the relationship between the side lengths.

Collect and share a variety of ideas, such as:

- Looking for a multiplicative pattern between corresponding lengths. In the example, the lengths in L are multiplied by 1.5 to create the lengths in S.
- Checking the ratios of corresponding side lengths. If they are equivalent, the figure is a scaled copy. In the example, $\frac{15}{10} = \frac{12}{8} = \frac{9}{6}$.

Note: If time allows, invite students to revise their previous response based on the discussion.



Synthesis

7. How can you use corresponding parts to determine whether a figure is a scaled copy of another figure?

Use the example if it helps with your thinking.

Responses vary.

- If the ratios of the corresponding parts of the two figures are equivalent, then one figure is a scaled copy of the other. For example, $\frac{15}{10} = \frac{9}{6} = \frac{12}{8}$, so L and S are scaled copies of each other.
- If the scale factor is the same between each length in the original figure and its corresponding length in the copy, then the lengths are proportional and the new figure is a scaled copy. For example, if I multiply each length in figure L by 1.5, I get all of the lengths in figure S.

Lesson Practice 1.02

Lesson Summary

When we create scaled copies, the scale factor is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle A to triangle B is 1.5.

Another way to see the scale factor is to look for the ratio between corresponding parts in the two shapes. The ratio of the new length to the original length is the scale factor.

For triangle A and triangle B, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Unit 1 Lesson 2 16 Synthesis | Lesson Practice

Lesson Takeaway: The lengths in an original figure and in a scaled copy are proportional and are related by the **scale factor**.

Independent | 5 min

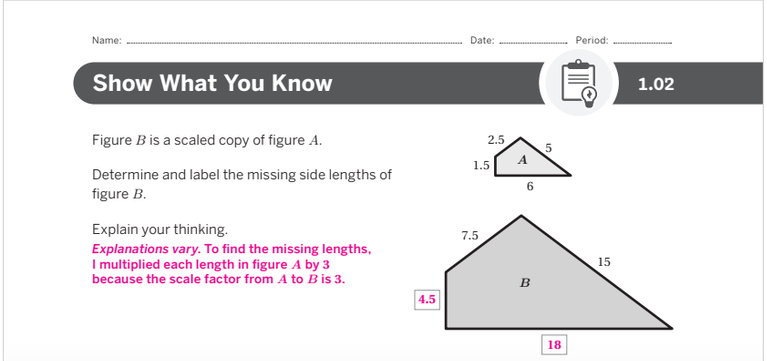
Show What You Know

Purpose: Students demonstrate their understanding of scale factor by determining the side lengths of a scaled copy.

Today's Goals

1. **Goal:** Recognize that a *scale factor* is the number multiplied by each side length in an original figure to create a scaled copy.
2. **Goal:** Understand that there is a proportional relationship between lengths in a figure and corresponding lengths in a scaled copy.
3. **Language Goal:** Explain how scaling affects side lengths in scaled copies. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: Date: Period:

Show What You Know 1.02

Figure B is a scaled copy of figure A.

Determine and label the missing side lengths of figure B.

Explain your thinking.

Explanations vary. To find the missing lengths, I multiplied each length in figure A by 3 because the scale factor from A to B is 3.

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need additional support understanding the effects of scaling on side lengths:

» **Respond to Student Thinking:**

- Review Imani and Anushka's methods from Activity 1.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on the strategies students used today. What opportunities were there to share specific students' strategies with the class and celebrate them? What facilitation techniques (e.g., naming strategies after the students) can help make this a part of your classroom culture? How might you continue to reference student strategies from today in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. How can you use corresponding parts to determine whether a figure is a scaled copy of another figure?

Use the example if it helps with your thinking.

Responses vary.

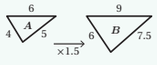
- If the ratios of the corresponding parts of the two figures are equivalent, then one figure is a scaled copy of the other. For example, $\frac{15}{10} = \frac{9}{6} = \frac{12}{8}$, so L and S are scaled copies of each other.
- If the scale factor is the same between each length in the original figure and its corresponding length in the copy, then the lengths are proportional and the new figure is a scaled copy. For example, if I multiply each length in figure L by 1.5, I get all of the lengths in figure S.

Lesson Practice 1.02

Lesson Summary

When we create scaled copies, the **scale factor** is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle A to triangle B is 1.5.



Another way to see the scale factor is to look for the ratio between **corresponding parts** in the two shapes. The ratio of the new length to the original length is the scale factor.

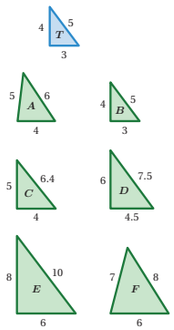
For triangle A and triangle B, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Unit 1 Lesson 2 16 Synthesis | Lesson Practice

Lesson Practice 1.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Here are seven triangles.



1. Which triangles are scaled copies of triangle T?

   Triangles B, D, and E

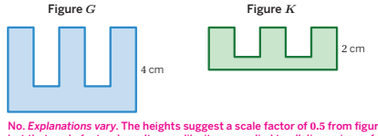
2. For each scaled copy, write the scale factor that takes triangle T to that triangle. Leave blank if it is not a scaled copy.

Triangle	Scale Factor
A	
B	1
C	
D	1.5
E	2
F	

3. List two triangles whose lengths can be represented by the equivalent ratios $\frac{4}{8} = \frac{5}{10} = \frac{3}{6}$.

   Triangles T and E or triangles B and E

4. Tyler says that figure K is a scaled copy of figure G. Is Tyler correct? Explain your thinking.



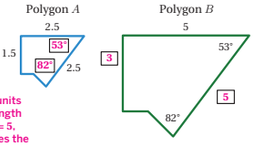
No. *Explanations vary.* The heights suggest a scale factor of 0.5 from figure G to K, but that scale factor doesn't seem like it was applied to *all* dimensions of figure G.

Unit 1 Lesson 2 17 Lesson Practice

Lesson Practice 1.02

Name: _____ Date: _____ Period: _____

Problems 5–7: Polygon B is a scaled copy of polygon A.



5. What is the scale factor that takes polygon A to polygon B? Explain your thinking.

   2. *Explanations vary.* The side length of 5 units in polygon B corresponds with the side length of 2.5 units in polygon A. Because $2.5 \cdot 2 = 5$, all the side lengths in polygon B are 2 times the corresponding side lengths in polygon A.

6. Enter the missing lengths in polygon B.

   Responses shown on diagram.

7. Enter the missing angle measurements in polygon A.

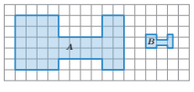
   Responses shown on diagram.

FAST Practice

8. Figures A and B are scaled copies.

   What scale factor takes A to B?

   $\frac{1}{4}$.



9. Using the digits 0 to 9 without repetition, fill in the blanks to make rectangles that are scaled copies.

   Responses vary.



Spiral Review

Problems 10–13: Evaluate each expression.

10. $\frac{1}{4} \cdot 32 = 8$
11. $\frac{1}{4} \cdot 5.6 = 1.4$
12. $7.2 \cdot \frac{1}{9} = 0.8$
13. $2 \div \frac{1}{4} = 8$

Unit 1 Lesson 2 18 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.GR.1.5
	3–7	2	MA.7.GR.1.5
	8	3	MA.7.GR.1.5
FAST Practice	9	2	MA.7.GR.1.5
Spiral Review			
	10–13	1	MA.5.FR.2.2

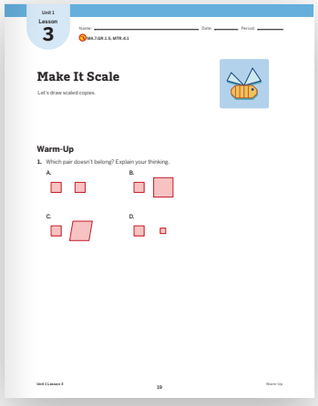
Unit 1

Lesson 3

Make It Scale

Drawing Scaled Copies

Let's draw scaled copies.


Unit 1
Lesson 3
Name: Date: Period:
MA.7.GR.1.5, MTR.4.1
Make It Scale
Let's draw scaled copies.
Warm-Up
1. Which pair doesn't belong? Explain your thinking.
A.
B.
C.
D.
19
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Draw a scaled copy of a figure using a given scale factor.
2. **Language Goal:** Describe and critique strategies for drawing scaled copies of a figure. **(Writing, Speaking, and Listening)**

Students apply their knowledge of scale factors to draw scaled copies on a grid. They explain strategies for creating accurate scaled copies, including ensuring all side lengths are multiplied by the same factor and all angle measurements stay the same.

Coherence

Prior Learning

In Lesson 2, students learned the term scale factor. They studied two ways of thinking about scale factor: as the multiplier applied to side lengths when scaling a figure, and as the ratio between corresponding sides in scaled copies.

Future Learning

In Lesson 4, students will study scale factors less than, equal to, and greater than 1 and notice patterns in how the scale factor affects the size of a scaled copy.

Connections

- Students draw scaled copies of figures on grids with different scale factors. (mathematical, cross-curricular)
- Students analyze samples of student thinking in an error analysis activity in which they determine the scale factor used to create a sketch from an original figure. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	**MA.8.GR.2.2**
MA.6.GR.1.3	**MA.912.GR.4.3**

Lesson at a Glance

~ 45 min

MA.7.GR.1.5

Warm-Up

Independent | 5 min

Students use the **Which One Doesn't Belong?** routine to surface their understanding of lengths and angles in scaled copies.



Activity 1

Pairs | 15 min

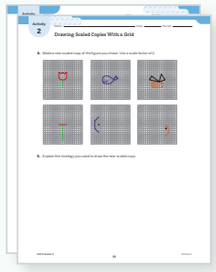
Students attempt to draw a scaled copy without measuring tools and consider what tools would be helpful in drawing a precise scaled copy.



Activity 2

Pairs | 15 min

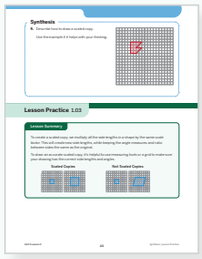
Students strategically use a grid to draw precise scaled copies and analyze the strategies of other students. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

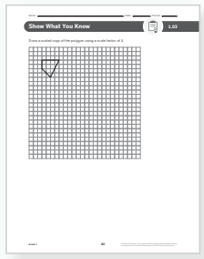
Students synthesize their understanding of how to draw a scaled copy, attending to both side lengths and angle measurements.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using a grid to create a scaled copy of a polygon.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- graph paper (as needed)
- index cards

Independent | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to surface their understanding of lengths and angles in scaled copies.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing and precisely describing properties of scaled copies.

Draw students' attention to all four pairs of shapes. Invite students to choose one they believe doesn't belong and justify their choice. **(MTR.4.1)**

Consider sharing that each pair does not belong in some way. Encourage students to look for more than one possibility.

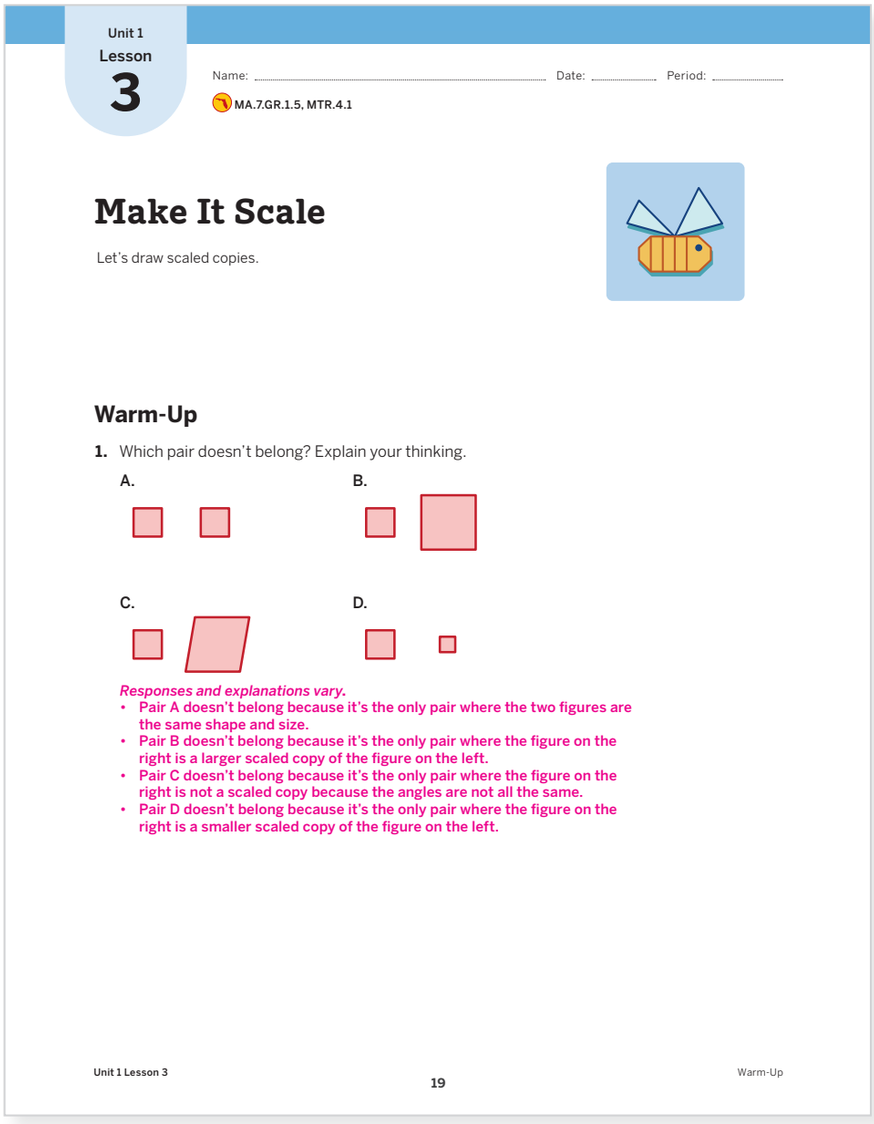
2 Connect

Invite students to share at least one reason why *each* pair doesn't belong.

ELL **English Language Learners** Provide sentence frames (e.g., "_____doesn't belong because _____." or "_____is the only one that has/ doesn't have_____."). **(Writing and Speaking)**

To surface the Key Takeaway, encourage students to explain how they know that pairs are or are not scaled copies.

Note: If it does not come up naturally, consider mentioning the relationship between the angles in the original and the copy for the bottom left pair.



Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1

Make It Scale

Let's draw scaled copies.

Warm-Up

1. Which pair doesn't belong? Explain your thinking.

A. B.

C. D.

Responses and explanations vary.
- Pair A doesn't belong because it's the only pair where the two figures are the same shape and size.
- Pair B doesn't belong because it's the only pair where the figure on the right is a larger scaled copy of the figure on the left.
- Pair C doesn't belong because it's the only pair where the figure on the right is not a scaled copy because the angles are not all the same.
- Pair D doesn't belong because it's the only pair where the figure on the right is a smaller scaled copy of the figure on the left.

Unit 1 Lesson 3 19 Warm-Up

Key Takeaway: Scale factor can change the side lengths of a figure, but scaled copies always have the same angle measurements as the original.

Pairs | 15 min

Activity 1 Drawing Scaled Copies Without a Grid

Purpose: Students attempt to draw a scaled copy without measuring tools and consider what tools would be helpful in drawing a precise scaled copy.

Short on time? Consider omitting the discussion on Problem 3 and encouraging students to read others' responses instead.

1 Launch

Invite students to choose an image that interests them.

Distribute index cards to students. Consider modeling using the index card as a straightedge to draw a segment.

2 Monitor

Encourage students to not spend too much time on this drawing. Explain that they will have the opportunity to create a more precise drawing later in the lesson.

A **Accessibility: Fine Motor Skills** To reduce anxiety about precision, share that the goal is to create a rough draft, rather than to be precise when using the sketch tool. Consider encouraging students to choose the flower, which is the figure with the fewest segments.

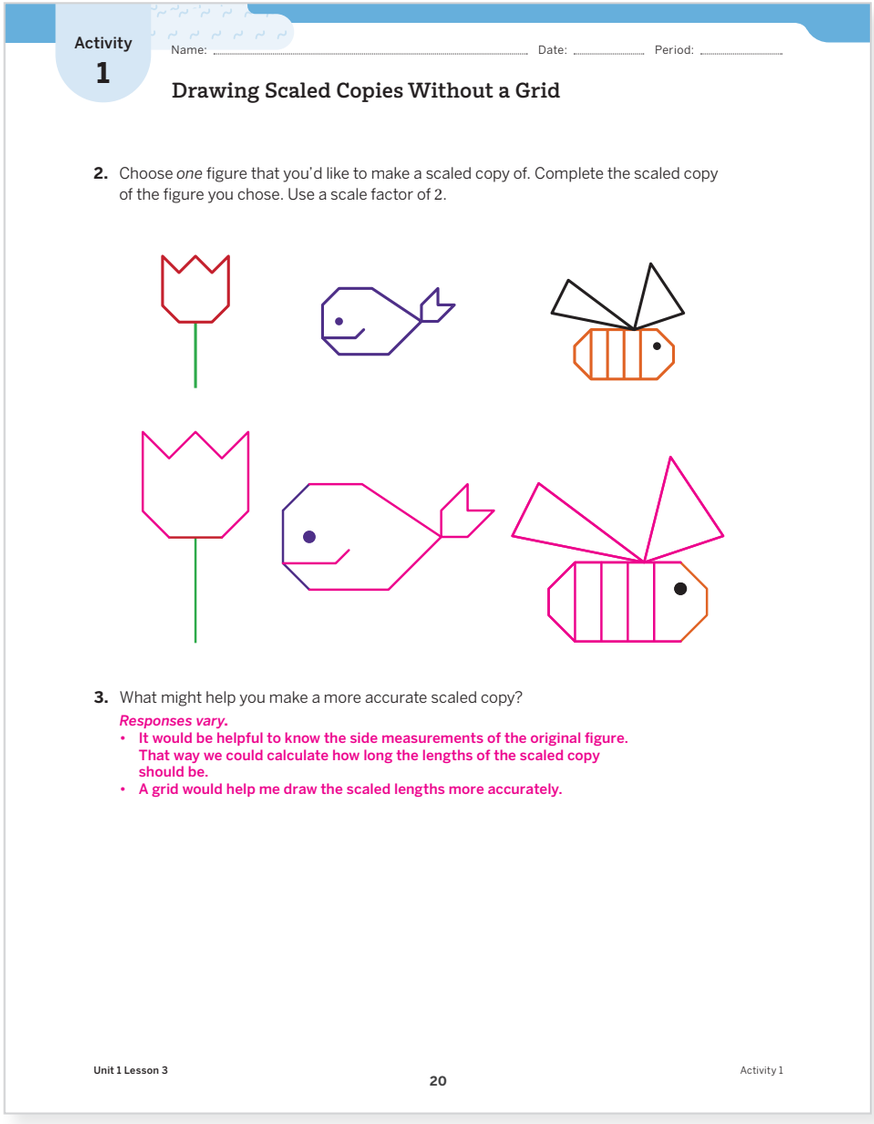
Listen for explanations to share during the Connect. Look for students whose responses mention lengths, angles, or measuring tools.

3 Connect

Share the student explanations. Consider asking, "Were your thoughts similar or different?"

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

To surface the Key Takeaway, consider asking, "How did it feel to create a scale drawing without any measurements or measuring tools?"



Activity 1

Name: Date: Period:

Drawing Scaled Copies Without a Grid

2. Choose *one* figure that you'd like to make a scaled copy of. Complete the scaled copy of the figure you chose. Use a scale factor of 2.

3. What might help you make a more accurate scaled copy?

Responses vary.

- It would be helpful to know the side measurements of the original figure. That way we could calculate how long the lengths of the scaled copy should be.
- A grid would help me draw the scaled lengths more accurately.

Unit 1 Lesson 3 20 Activity 1

Key Takeaway: Creating an accurate scale drawing requires keeping the original angle measurements and knowing precise side lengths so you can apply the scale factor.

Pairs | 15 min

Activity 2 Drawing Scaled Copies With a Grid

Purpose: Students strategically use a grid to draw precise scaled copies and analyze the strategies of other students. **(MTR.4.1)**

1 Launch

Share that students will now have the opportunity to recreate their scaled copy using a grid. Encourage students to strive for a more precise copy than their previous attempt.

2 Monitor

Look for students' progress on their scaled copies.

D Differentiation

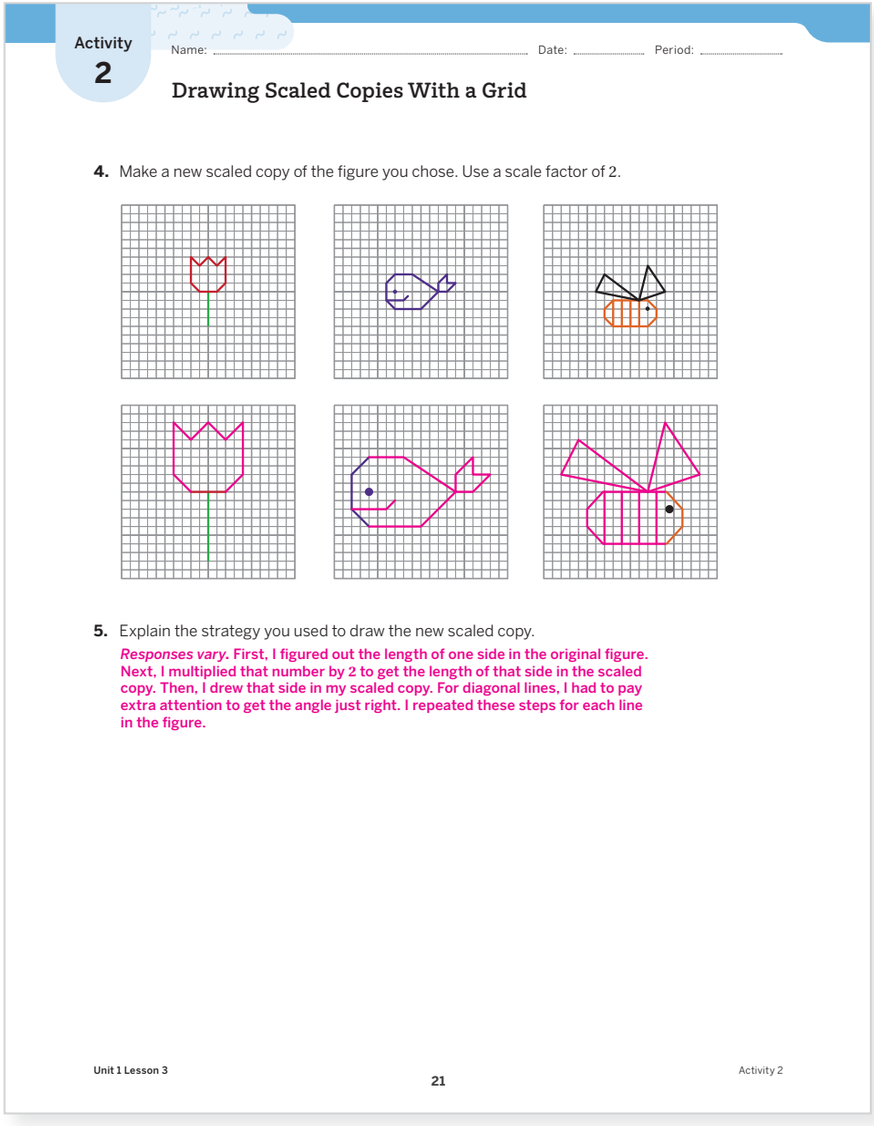
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to choose a segment in the original figure and count its length. Consider asking, "How would the length change if we applied a scale factor of 2?"
Scale side lengths accurately, but do not consider the size of corresponding angles.	**Support** Consider asking, "What strategies can you try for drawing diagonal lengths?"
Check their work after drawing each segment.	**Strengthen** Consider asking, "What pattern do you notice between the original segment and the copy? What do you predict will be the new length of the next segment?"

Look for a variety of student strategies to share. Consider including strategies that are incomplete or imprecise, then using the discussion to strengthen these explanations as a class.

Pause briefly to share examples of student's strategies. Invite students to reflect on which strategies they used or would like to try.

Note: If it does not come up naturally, consider asking, "What were your strategies for drawing diagonal lines?" *Responses vary.* I counted the number of units down and to the right in the original and multiplied those by the scale factor.

Activity 2 continued >



Activity 2

Name: Date: Period:

Drawing Scaled Copies With a Grid

4. Make a new scaled copy of the figure you chose. Use a scale factor of 2.

5. Explain the strategy you used to draw the new scaled copy.

Responses vary. First, I figured out the length of one side in the original figure. Next, I multiplied that number by 2 to get the length of that side in the scaled copy. Then, I drew that side in my scaled copy. For diagonal lines, I had to pay extra attention to get the angle just right. I repeated these steps for each line in the figure.

Unit 1 Lesson 3 21 Activity 2

Pairs | 15 min

Activity 2 Drawing Scaled Copies With a Grid (continued)

Purpose: Students strategically use a grid to draw precise scaled copies and analyze the strategies of other students. **(MTR.4.1)**

2 Monitor

Encourage students to apply the strategies they wrote about in Problem 5.

Invite students to discuss their thinking with a partner.

Listen to partner discussions for student ideas to highlight during the Connect.

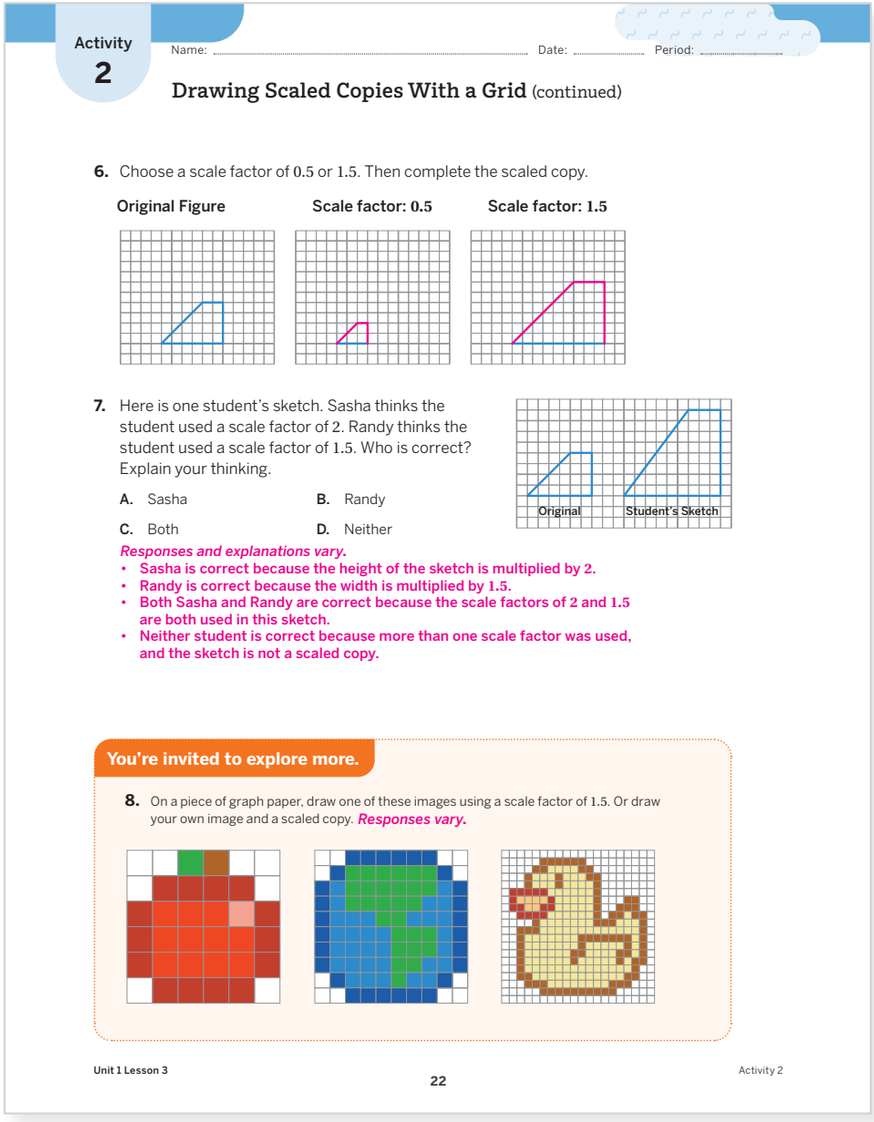
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore scale drawings further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need graph paper.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Consider using the Decide and Defend routine (optional) to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:
- "Where do you see Sasha's scale factor in the sketch?"
- "Where do you see Randy's scale factor?"
- "Did this student create a scaled copy?"



Activity 2

Name: Date: Period:

Drawing Scaled Copies With a Grid (continued)

6. Choose a scale factor of 0.5 or 1.5. Then complete the scaled copy.

Original Figure	Scale factor: 0.5	Scale factor: 1.5

7. Here is one student's sketch. Sasha thinks the student used a scale factor of 2. Randy thinks the student used a scale factor of 1.5. Who is correct? Explain your thinking.

Original Student's Sketch

A. Sasha B. Randy
C. Both D. Neither

Responses and explanations vary.
- Sasha is correct because the height of the sketch is multiplied by 2.
- Randy is correct because the width is multiplied by 1.5.
- Both Sasha and Randy are correct because the scale factors of 2 and 1.5 are both used in this sketch.
- Neither student is correct because more than one scale factor was used, and the sketch is not a scaled copy.

You're invited to explore more.

8. On a piece of graph paper, draw one of these images using a scale factor of 1.5. Or draw your own image and a scaled copy. *Responses vary.*

Unit 1 Lesson 3 22 Activity 2

Key Takeaway: A measuring tool, like a grid, is helpful to create scaled copies. It can help us precisely measure lengths and make sure that every length is multiplied by the same scale factor.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to draw a scaled copy, attending to both side lengths and angle measurements.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Multiplying all side lengths by the same scale factor.
- Drawing horizontal and vertical lines first.
- Using different strategies for checking that angle measurements remain the same from the original figure to the copy.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. Describe how to draw a scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- When making a scaled copy, make sure all the side lengths are multiplied by the same scale factor.
- When drawing the new figure, keep in mind how you draw the angles so that the angles remain the same in both figures.

Lesson Practice 1.03

Lesson Summary

To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies

Not Scaled Copies

Unit 1 Lesson 3 23 Synthesis | Lesson Practice

Lesson Takeaway: In a scaled copy, all the side lengths are multiplied by the scale factor, but the angle measures remain the same. Using a grid can help you create a precise scaled copy.

Independent | 5 min

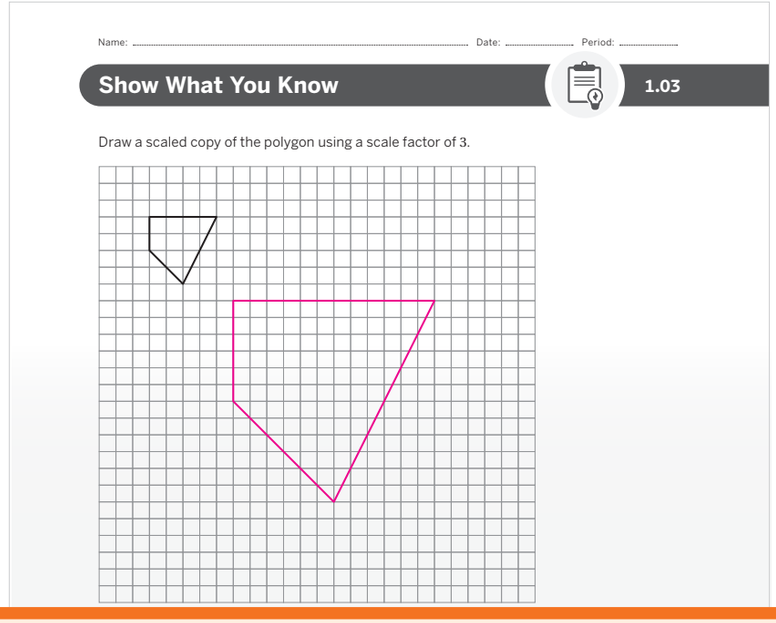
Show What You Know

Purpose: Students demonstrate their understanding by using a grid to create a scaled copy of a polygon.

Today's Goals

1. **Goal:** Draw a scaled copy of a figure using a given scale factor.
2. **Language Goal:** Describe and critique strategies for drawing scaled copies of a figure. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)


Name: Date: Period:

Show What You Know 1.03

Draw a scaled copy of the polygon using a scale factor of 3.

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to draw a scaled copy of a figure:

» **Respond to Student Thinking:**

- the Lesson Summary and provide targeted feedback on select Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to investigate the **You're Invited to Explore More** problem.

Professional Learning

Reflect on the student thinking that was shared in this lesson. Who participated and who did not participate in sharing? What trends do you see in participation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how to draw a scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- When making a scaled copy, make sure all the side lengths are multiplied by the same scale factor.
- When drawing the new figure, keep in mind how you draw the angles so that the angles remain the same in both figures.

Lesson Practice 1.03

Lesson Summary

To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies



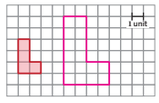
Not Scaled Copies



Lesson Practice 1.03

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a polygon.



1. Draw a scaled copy of the polygon using a scale factor of 2.

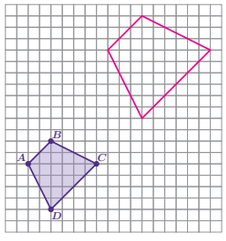
Sample shown.

2. What is the area and perimeter of your scaled copy?

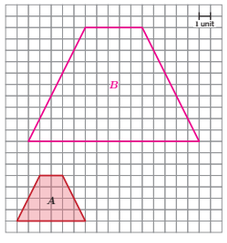
The area is 16 square units. The perimeter is 20 units.

3. Draw a scaled copy of figure $ABCD$ using a scale factor of 1.5.

Sample shown.



Problems 4–5: Imagine there is a quadrilateral B that is a scaled copy of quadrilateral A. Its shortest side is 5 units long.



4. What is the scale factor from quadrilateral A to B?

2.5

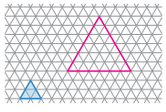
5. Draw quadrilateral B.

Sample shown.

Lesson Practice 1.03

Name: _____ Date: _____ Period: _____

Problems 6–7: Here is an equilateral triangle.



6. Draw a scaled copy of this equilateral triangle using a scale factor of 3.

Responses vary.

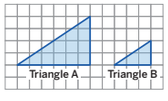
7. Equilateral triangles are always scaled copies. What other shapes are always scaled copies? **squares, circles, or any regular shape (shapes where all sides and all angles are equal)**

FAST Practice

8. Triangle B is a scaled copy of triangle A. Gabriela says the scale factor is 0.5. Is she correct?

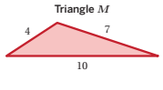
Select **ONE** correct answer in each box to complete the sentences.

Gabriela is (A.) correct **B.** incorrect. The height and base of triangle A is multiplied by (A.) 0.5 **B.** 2.



Spiral Review

9. Triangle Z is a scaled copy of triangle M. Select *all* sets of values that could be the side lengths of triangle Z.



☐ **A.** 8, 11, and 14 ☑ **B.** 10, 17.5, and 25

☐ **C.** 6, 9, and 11 ☑ **D.** 6, 10.5, and 15

☑ **E.** 8, 14, and 20

Problems 10–12: Solve each equation. Show or explain your thinking.

10. $6x = 156$

$x = 26$

Work varies.

$6x \div 6 = 156 \div 6$

$x = 26$

11. $16x = 8$

$x = \frac{1}{2}$

Work varies.

$16x \div 16 = 8 \div 16$

$x = \frac{1}{2}$

12. $\frac{1}{5}x = 1$

$x = 5$

Work varies.

$\frac{1}{5}x \div \frac{1}{5} = 1 \div \frac{1}{5}$

$x = 5$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	1	MA.7.GR.1.5
	6–7	3	MA.7.GR.1.5
FAST Practice	8	2	MA.7.GR.1.5
Spiral Review			
	9	2	MA.7.GR.1.5
	10–12	1	MA.6.AR.2.3

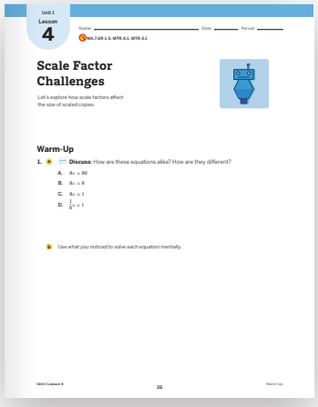
Unit 1

Lesson 4

Scale Factor Challenges

Effects of Scale Factors

Let's explore how scale factors affect the size of scaled copies.


Unit 1
Lesson 4
Name _____ Date _____ Period _____
MA.7.GR.1.5, MTR.4.1, MTR.5.1
Scale Factor Challenges
Let's explore how scale factors affect the size of scaled copies.
Warm-Up
1. Discuss: How are these equations alike? How are they different?
A. $8x = 80$
B. $8x = 8$
C. $8x = 1$
D. $\frac{1}{8}x = 1$
Use what you noticed to solve each equation mentally.
Unit 1 Lesson 4
26
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Determine the effect of scale factors greater than 1, between 0 and 1, or equal to 1 on lengths in scaled copies.
2. **Goal:** Understand how the scale factor from figure A to figure B relates to the scale factor from figure B to figure A.
3. **Language Goal:** Explain how different scale factors affect lengths in scaled copies. **(Speaking and Listening)**

Students determine which scale factors create a scaled copy that is larger than, smaller than, or the same size as the original. They discover patterns in how scale factors affect the size of a scaled copy and apply these patterns by using scale factors that take a scaled copy back to the original figure. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 3, students used grids to create scaled copies with a given scale factor.

Future Learning

In Lesson 5, students will continue to explore how scaling a figure affects its size, noticing that area does not scale in the same way as lengths.

Connections

- Students determine if a scaled copy of a robot will be larger than, smaller than, or the same size as the original using the scale factor. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Lesson at a Glance

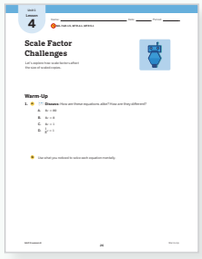
~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

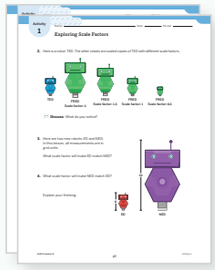
Students think about pairs of numbers that multiply to make 1.



Activity 1

Pairs | 10 min

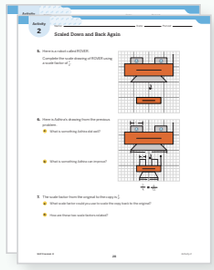
Students discover how different scale factors affect the size of a scaled copy.



Activity 2

Pairs | 10 min

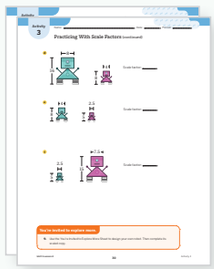
Students reflect on the relationship between the scale factors that take an original to its scaled copy and back. **Routine:** MLR3: Critique, Correct, Clarify



Activity 3

Pairs | 10 min

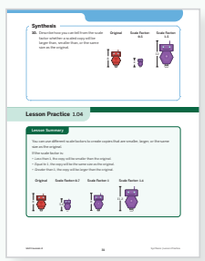
Students use proportional reasoning to determine the scale factor from one figure to another.



Synthesis

Whole Class | 5 min

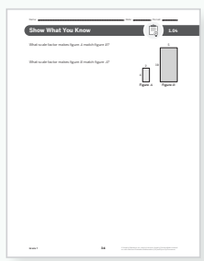
Students synthesize their understanding of how scale factor affects the size of a scaled copy compared to its original.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using proportional reasoning to determine the scale factors between two figures.



Prep Checklist

Prepare Activity 3 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More Sheet, one per student (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students think about pairs of numbers that multiply to make 1.

1 Launch

Direct students' attention to the equations and invite students to discuss how they think they are alike and different.

Listen for students who notice a connection between equations C and D. Consider inviting these students to share their ideas during the Connect.

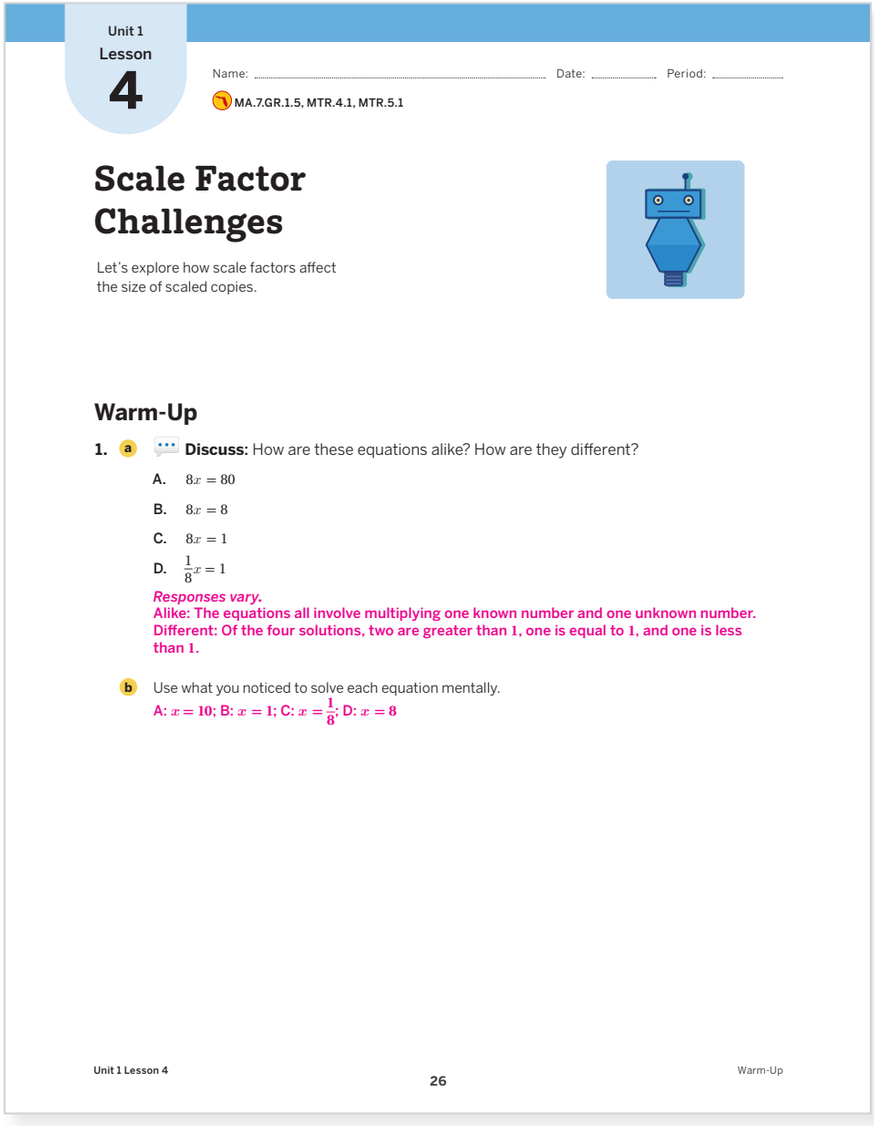
2 Connect

Invite students to share their responses.

To surface connections, consider showing how to verify the solutions to the equations using substitution. If it does not come up naturally, consider highlighting that the last two equations both show 8 and $\frac{1}{8}$ multiplying to make 1.

Consider asking, "What other pairs of numbers multiply to make 1?"

Note: The word *reciprocal* is not required in this lesson or unit, but you may choose to introduce it if it would be helpful to describe this relationship.



Unit 1
Lesson
4

Name: ..... Date: ..... Period: .....

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scale Factor Challenges

Let's explore how scale factors affect the size of scaled copies.

Warm-Up

1. a. **Discuss:** How are these equations alike? How are they different?

   A. $8x = 80$

   B. $8x = 8$

   C. $8x = 1$

   D. $\frac{1}{8}x = 1$

   Responses vary.
   Alike: The equations all involve multiplying one known number and one unknown number.
   Different: Of the four solutions, two are greater than 1, one is equal to 1, and one is less than 1.

   b. Use what you noticed to solve each equation mentally.
   A: $x = 10$; B: $x = 1$; C: $x = \frac{1}{8}$; D: $x = 8$

Unit 1 Lesson 4 26 Warm-Up

Pairs | 10 min

Activity 1 Exploring Scale Factors

Purpose: Students discover how different scale factors affect the size of a scaled copy.

1 Launch

Share that the purpose of this activity is to explore how different scale factors affect the size of a scaled copy.

Consider using the Notice and Wonder routine (optional) to promote curiosity and help students make sense of different scale factors. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

2 Monitor

Look for students who try scale factors between 0 and 1 or less than 0. Consider inviting these students to share what they discovered during the Connect.

Look for student ideas to share. Look for students who make connections between Problems 3 and 4.

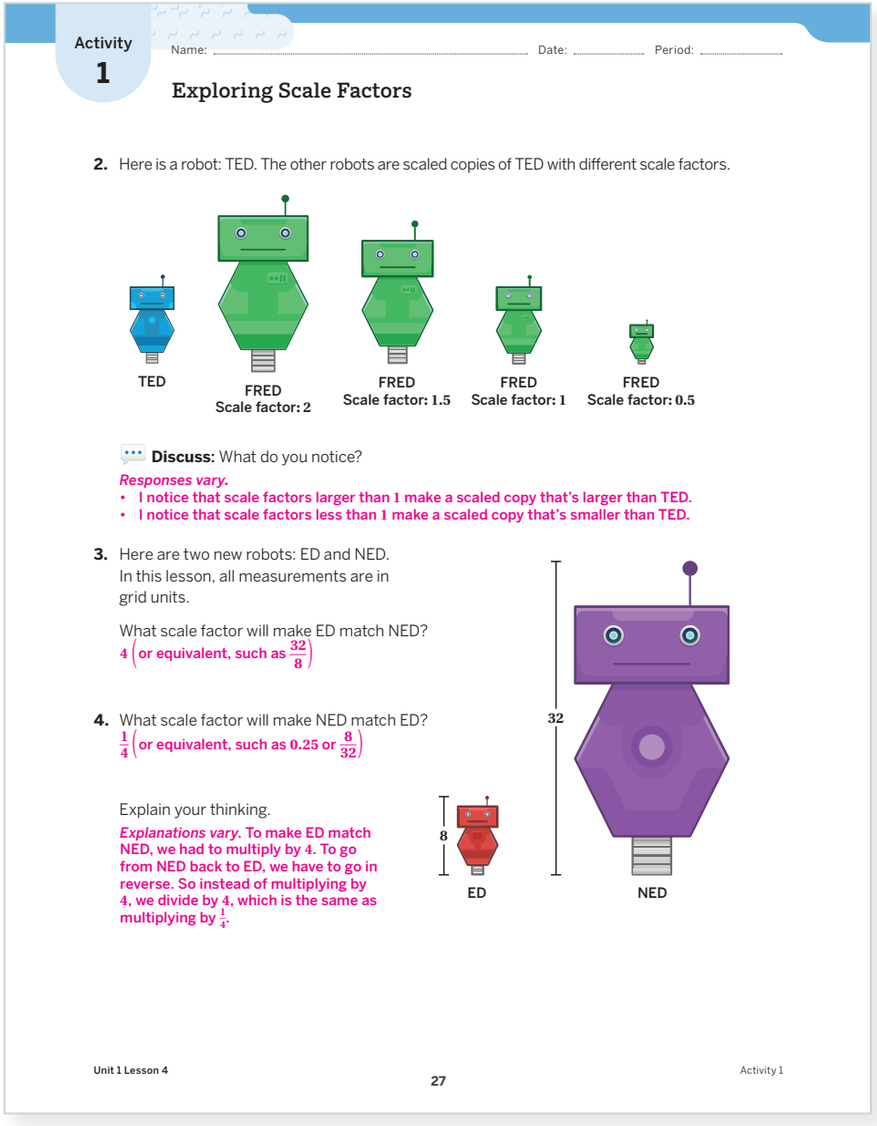
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What kinds of scale factors result in a smaller scaled copy?"
Determine that we can make NED match ED by dividing by 4, but do not generate a scale factor.	**Strengthen** Celebrate the structure they noticed. Invite these students to reflect by asking, "Is there a way to express division by 4 as multiplication?" **(MTR.5.1)**
Identify a scale factor of $\frac{1}{4}$ or equivalent.	**Stretch** Invite these students to create a general strategy for determining the scale factor when given the original height and the scaled copy height. **(MTR.5.1)**

3 Connect

Share student responses. Consider asking:

- "What happens when you scale a figure using a scale factor between 0 and 1?"
- "When a scaled copy is larger than the original, what must be true about the scale factor? What if the scaled copy is the same size?"



Activity 1

Name: Date: Period:

Exploring Scale Factors

2. Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.

TED

FRED
Scale factor: 2

FRED
Scale factor: 1.5

FRED
Scale factor: 1

FRED
Scale factor: 0.5

Discuss: What do you notice?

Responses vary.
- I notice that scale factors larger than 1 make a scaled copy that's larger than TED.
- I notice that scale factors less than 1 make a scaled copy that's smaller than TED.

3. Here are two new robots: ED and NED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED?

4 (or equivalent, such as $\frac{32}{8}$)

4. What scale factor will make NED match ED?

$\frac{1}{4}$ (or equivalent, such as 0.25 or $\frac{8}{32}$)

Explain your thinking.

Explanations vary. To make ED match NED, we had to multiply by 4. To go from NED back to ED, we have to go in reverse. So instead of multiplying by 4, we divide by 4, which is the same as multiplying by $\frac{1}{4}$.

8

ED

32

NED

Unit 1 Lesson 4 27 Activity 1

Pairs | 10 min

Activity 2 Scaled Down and Back Again

Purpose: Students reflect on the relationship between the scale factors that take an original to its scaled copy and back.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to explore Problem 5–7.

Encourage students to draw the outline of the robot as accurately as they can. Shading the interior and drawing the eyes is optional.

2 Monitor

Differentiation: (Support) If students are struggling to start, consider asking, "What happens to the lengths in the drawing when we use the scale factor $\frac{1}{2}$?" The lengths are smaller than the original. All the lengths are divided by 2.

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Adhira's work. Then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Adhira see how their work can be improved, or create a revised response. **(MTR.4.1)**

Select and sequence student explanations to share during the Connect.

Look for students who make connections between the scale factors $\frac{1}{2}$ and 2.

3 Connect

Share student responses. Invite students to identify which parts of each explanation they find to be valuable.

Consider asking:

- "When you knew the scale factor from the original to the copy, what strategy helped you determine the scale factor to get back to the original?"
- "How is this related to the work we did with 8 and $\frac{1}{8}$ in the Warm-Up?" In the Warm-Up, we made 1. We can multiply the scale factors of 4 and $\frac{1}{4}$ to make 1.

Invite students to revise their response to make it stronger and clearer.

Consider sharing that the two scale factors multiply to make 1.

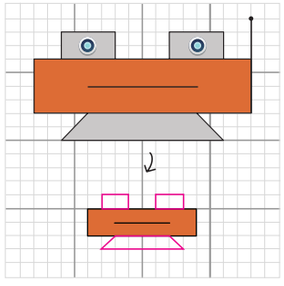
Activity 2

Name: Date: Period:

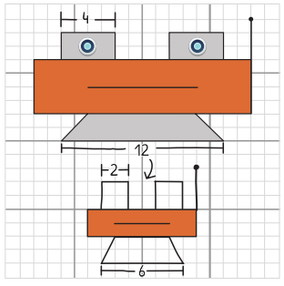
Scaled Down and Back Again

5. Here is a robot called ROVER.

Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.



6. Here is Adhira's drawing from the previous problem.



$\frac{2}{4} = \frac{6}{12}$

a. What is something Adhira did well?

Responses vary. Adhira used equivalent ratios $\left(\frac{2}{4} = \frac{6}{12}\right)$ to figure out how wide to make everything in the new drawing.

b. What is something Adhira can improve?

Responses vary. Adhira forgot to apply the scale factor to how tall things are. The eyes and trapezoid body should each be 1 unit tall (instead of 2), and the antenna should be $3 \cdot \frac{1}{2} = 1.5$ units tall (instead of 3).

7. The scale factor from the original to the copy is $\frac{1}{2}$.

a. What scale factor could you use to scale the copy back to the original?

2

b. How are these two scale factors related?

Responses vary. The scale factors multiply to make 1.

Unit 1 Lesson 4 28 Activity 2

Key Takeaway: Between a pair of scaled copies, there are two possible scale factors. One scale factor takes the original to the scaled copy, and the other takes the copy back to the original. These two scale factors always multiply to make 1.

Pairs | 10 min

Activity 3 Practicing With Scale Factors

Purpose: Students use proportional reasoning to determine the scale factor from one figure to another.

Short on time? Consider sharing with students that it is okay if they do not complete all the challenges.

1 Launch

Invite students to solve as many challenges as they have time for. Share that sense-making is more important than speed.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

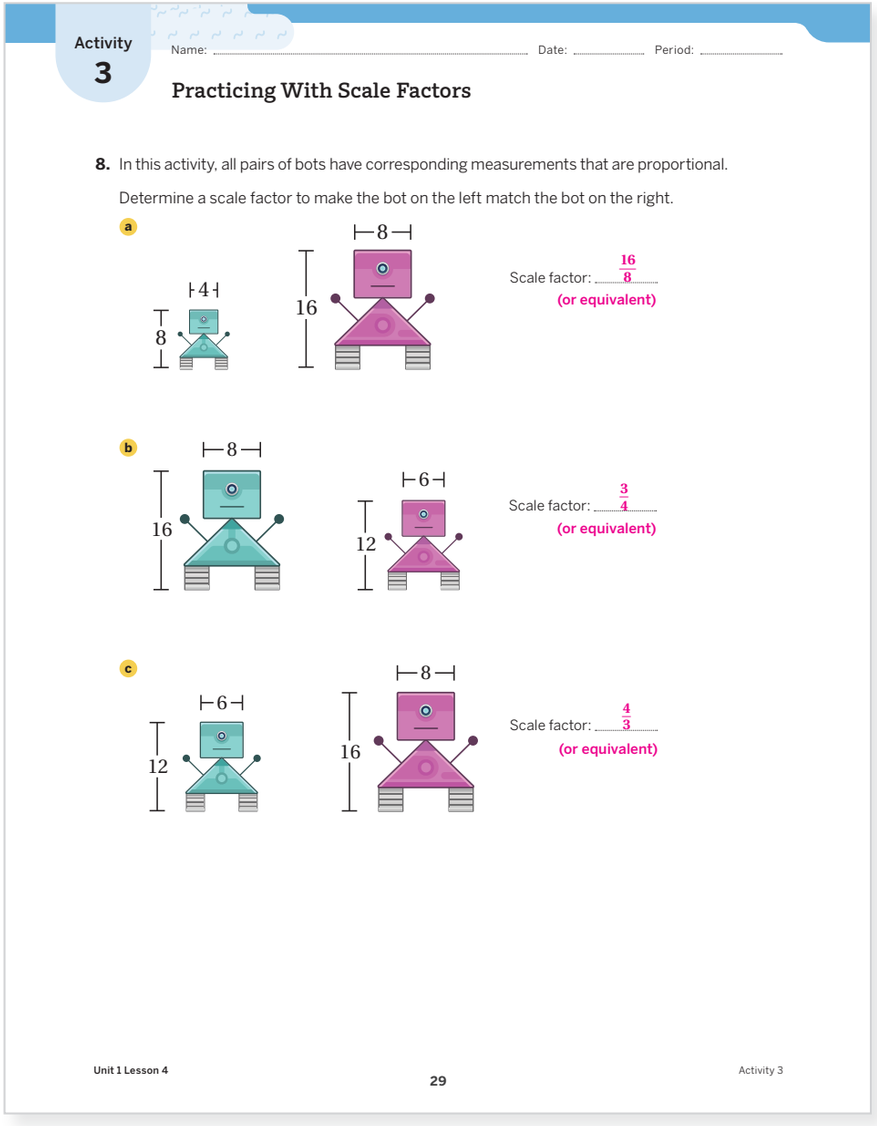
Listen for student strategies and partner discussions, offering help or encouragement where needed.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to identify pairs of matching sides and look for a relationship between them.
Choose a scale factor greater than 1 when the scaled copy is smaller than the original.	**Support** Consider asking, "What can you tell about the scale factor when the scaled copy is smaller than the original?"
Recognize that the scale factor is the ratio $\frac{\text{new height}}{\text{original height}}$.	**Strengthen** Invite students to think about how they would share their strategies during the Connect.
Use estimation to determine the scale factor.	**Stretch** Encourage these students to look for structure. Consider asking, "How could you set up an equation or expression to determine the scale factor?"

Look for a variety of strategies to share, including strategies that identify the scale factor as a ratio of new to old lengths.

Activity 3 continued >



Activity 3

Name: Date: Period:

Practicing With Scale Factors

8. In this activity, all pairs of bots have corresponding measurements that are proportional.

Determine a scale factor to make the bot on the left match the bot on the right.

a. 4, 8 and 8, 16. Scale factor: $\frac{16}{8}$ (or equivalent)

b. 8, 16 and 6, 12. Scale factor: $\frac{3}{4}$ (or equivalent)

c. 6, 12 and 8, 16. Scale factor: $\frac{4}{3}$ (or equivalent)

Unit 1 Lesson 4 29 Activity 3

Pairs | 10 min

Activity 3 Practicing With Scale Factors (continued)

Purpose: Students use proportional reasoning to determine the scale factor from one figure to another.

Short on time? Consider sharing with students that it is okay if they do not complete all the challenges.

2 Monitor

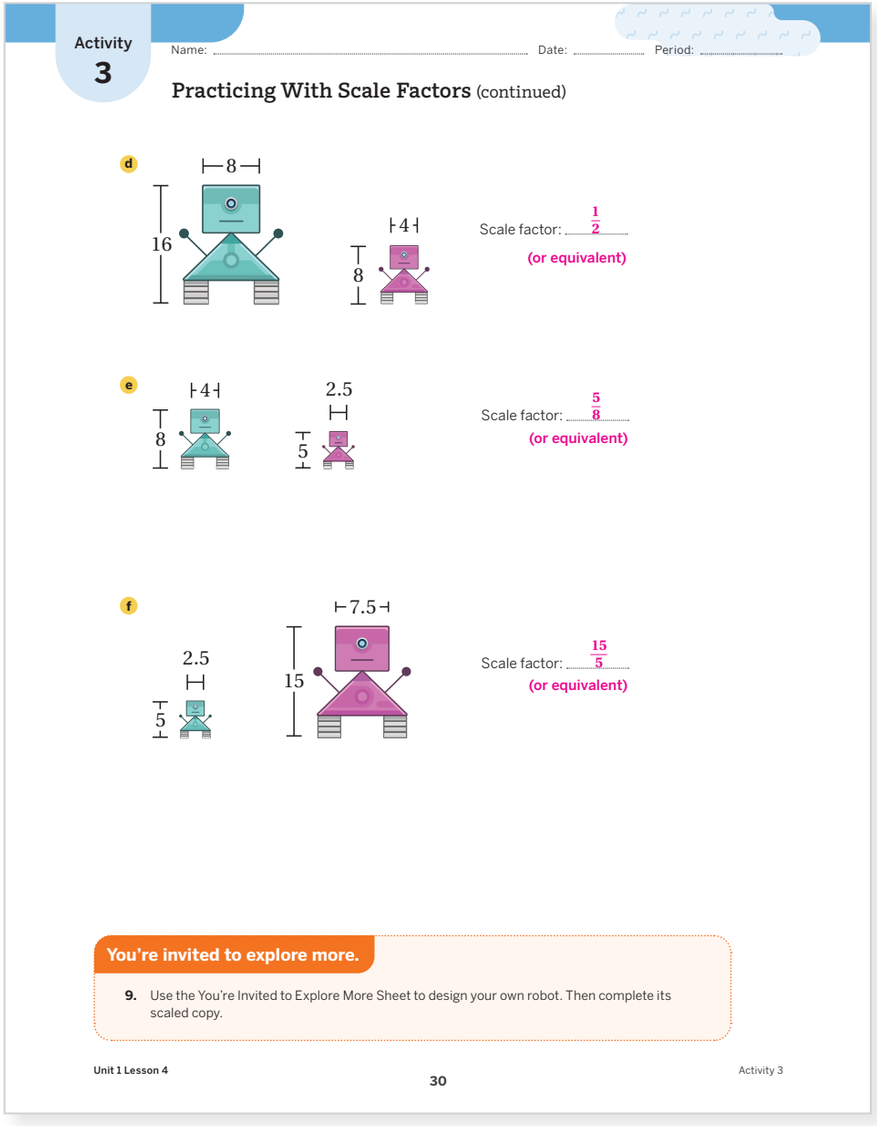
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who want to explore using scale factors further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share strategies they found helpful or what they learned from mistakes they made while solving these challenges.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Consider sharing the student-created robot designs.



Activity 3

Name: Date: Period:

Practicing With Scale Factors (continued)

d

Scale factor: $\frac{1}{2}$ (or equivalent)

e

Scale factor: $\frac{5}{8}$ (or equivalent)

f

Scale factor: $\frac{15}{5}$ (or equivalent)

You're invited to explore more.

9. Use the You're Invited to Explore More Sheet to design your own robot. Then complete its scaled copy.

Unit 1 Lesson 4 30 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how the scale factor affects the size of a scaled copy compared to its original.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Scale factors between 0 and 1 create smaller copies.
- Scale factors like $\frac{1}{2}$ have the same effect as dividing by 2, so they create smaller copies.
- Scale factors greater than 1 create bigger copies.
- Using a scale factor of 1 creates the same size copy, since multiplying the lengths by 1 does not change them.

ELL **English Language Learners** Use gestures to illustrate how different scale factors affect the size of the resulting copy.

Invite students to share the connections they see between the responses.

Note: If time allows, invite students to revise their previous thinking, strengthening or clarifying their response based on the discussion.



Synthesis

10. Describe how you can tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original.

Responses vary.
If the scale factor is greater than 1, the scaled copy will be larger than the original figure.
If the scale factor is between 0 and 1, the scaled copy will be smaller than the original figure.
If the scale factor is equal to 1, the scaled copy will be the same size as the original figure.

Lesson Practice 1.04

Lesson Summary

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:

- *Less* than 1, the copy will be *smaller* than the original.
- *Equal to* 1, the copy will be *the same size* as the original.
- *Greater* than 1, the copy will be *larger* than the original.

Unit 1 Lesson 4 31 Synthesis | Lesson Practice

Lesson Takeaway: The scale factor affects the size of the scaled copy. Scale factors greater than 1 create larger copies, while scale factors between 0 and 1 create smaller copies. A scale factor of 1 creates a copy the same size as the original.

Independent | 5 min

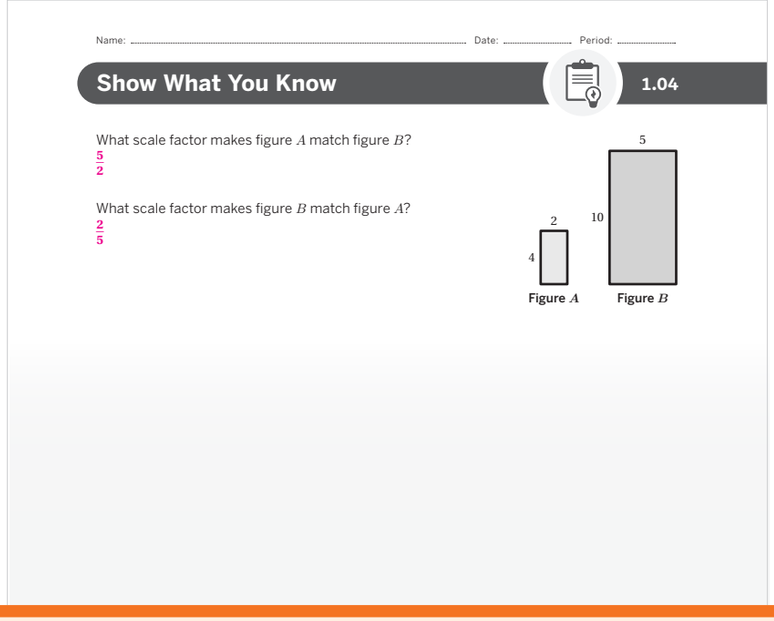
Show What You Know

Purpose: Students demonstrate their understanding by using proportional reasoning to determine the scale factors between two figures.

Today's Goals

1. **Goal:** Determine the effect of scale factors greater than 1, between 0 and 1, or equal to 1 on lengths in scaled copies.
2. **Goal:** Understand how the scale factor from figure A to figure B relates to the scale factor from figure B to figure A.
3. **Language Goal:** Explain how different scale factors affect lengths in scaled copies. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.04

What scale factor makes figure A match figure B?

$\frac{5}{2}$

What scale factor makes figure B match figure A?

$\frac{2}{5}$

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding the effect of scale factors on lengths in scaled copies:

» **Respond to Student Thinking:**

- Use the ***Connecting Scale Factors to Scaled Copies* Mini-lesson**.
- Invite students to discuss how different scale factors affect lengths in scaled copies with a partner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to investigate the **You're Invited to Explore More** problem.

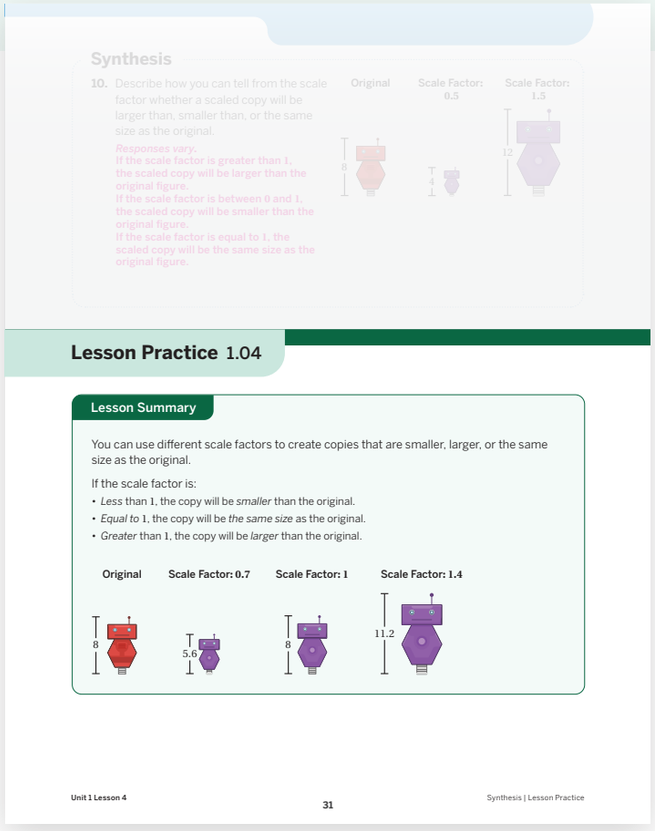
Professional Learning

Reflect on students' use of the grid in Activity 2. How did sketching support students with communicating their ideas or participating in the lesson? What support might students need to use grids effectively in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. Describe how you can tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original.

Responses vary.
If the scale factor is greater than 1, the scaled copy will be larger than the original figure.
If the scale factor is between 0 and 1, the scaled copy will be smaller than the original figure.
If the scale factor is equal to 1, the scaled copy will be the same size as the original figure.

Original | Scale Factor: 0.5 | Scale Factor: 1.5

8 | 4 | 12

Lesson Practice 1.04

Lesson Summary

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:
- *Less* than 1, the copy will be *smaller* than the original.
- *Equal to* 1, the copy will be *the same size* as the original.
- *Greater* than 1, the copy will be *larger* than the original.

Original | Scale Factor: 0.7 | Scale Factor: 1 | Scale Factor: 1.4

8 | 5.6 | 8 | 11.2

Unit 1 Lesson 4 31 Synthesis | Lesson Practice

Lesson Practice 1.04

Name: _____ Date: _____ Period: _____

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, state whether the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.

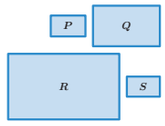
Rectangle P to rectangle R Greater than 1

Rectangle Q to rectangle S Less than 1

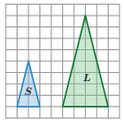
Rectangle Q to rectangle R Greater than 1

Rectangle S to rectangle P Equal to 1

Rectangle R to rectangle P Less than 1



Problems 2–4: Triangle S and triangle L are scaled copies of one another.

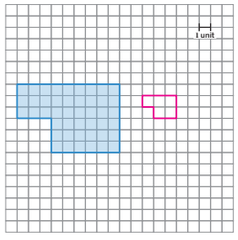


2. What is the scale factor that takes triangle S to triangle L?
2

3. What is the scale factor that takes triangle L to triangle S?
$\frac{1}{2}$

4. Triangle M (not shown) is also a scaled copy of triangle S. The scale factor that takes triangle S to triangle M is $\frac{3}{2}$. What is the scale factor that takes triangle M to triangle S?
$\frac{2}{3}$

Problems 5–7: Here is a polygon.



5. Draw a scaled copy of the polygon that has a perimeter of 10 units.
Sample shown.

6. What is the scale factor from the original polygon to your scaled copy?
$\frac{1}{3}$

7. What is the scale factor from the scaled copy back to the original polygon?
3

Unit 1 Lesson 4 32 Lesson Practice

Lesson Practice 1.04

Name: _____ Date: _____ Period: _____

8. Will any two squares always be scaled copies of one another?
Yes

Explain your thinking.
Explanations vary. To be scaled copies, two figures need to have the same angles as one another and proportional sides. In any square, all the angles are 90° and the ratios between the sides are always equal to 1. Therefore, any two squares are scaled copies of one another.

FAST Practice

9. Figure B is a scaled copy of figure A with a scale factor of $5\frac{1}{2}$.

What is the value of x?

$x =$ 22

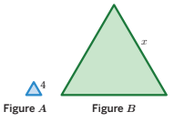


Figure A Figure B

Spiral Review

10. Select *all* the ratios that are equivalent to 12 : 3.

- ☐ A. 6 : 1
- ☐ B. 1 : 4
- ☑ C. 4 : 1
- ☑ D. 24 : 6
- ☐ E. 15 : 6
- ☑ F. 1,200 : 300
- ☐ G. 112 : 13

Unit 1 Lesson 4 33 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 6–7	1	MA.7.GR.1.5
	2–5	2	MA.7.GR.1.5
	8	3	MA.7.GR.1.5
FAST Practice	9	1	MA.7.GR.1.5
Spiral Review			
	10	1	MA.6.AR.3.5

Unit 1

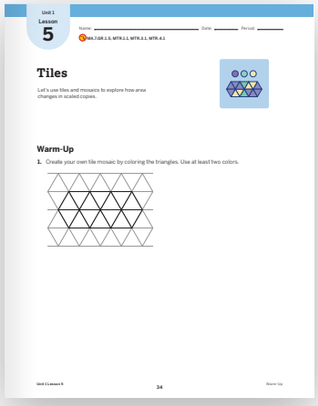
Lesson

5

Tiles

Scaling and Area

Let's use tiles and mosaics to explore how area changes in scaled copies.


Unit 1
Lesson
5
Name _____ Date _____ Period _____
MA.7.GR.1.5, MTR.1.1, MTR.3.1, MTR.4.1
Tiles
Let's use tiles and mosaics to explore how area changes in scaled copies.
Warm-Up
1. Create your own tile mosaic by coloring the triangles. Use at least two colors.
34
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the area of a scaled copy.
2. **Language Goal:** Explain how scaling affects lengths and areas in scaled copies. **(Reading, Writing, Speaking, and Listening)**

Students explore how different scale factors affect the *area* of scaled copies. They notice that when you create a scaled copy, the area is not scaled by the same scale factor as the side lengths. They then use repeated reasoning to generate strategies for determining the area of a scaled copy. **(MTR.3.1)**

Coherence

Prior Learning

In Lesson 4, students analyzed how scale factors less than, equal to, or greater than 1 affect the size of a scaled copy compared to the original figure.

Future Learning

In Lessons 7 and 8, students will begin to create scale drawings, attending to their lengths, angles, and areas.

Connections

- Students apply their understanding of scale factor to identify patterns in mosaics and create their own designs. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	**MA.8.GR.2.2**
MA.6.GR.1.3	**MA.912.GR.3.4**
MA.6.GR.2.2	**MA.912.GR.4.3**
	MA.912.GR.4.4

Vocabulary

Review Vocabulary

area

Lesson at a Glance

~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

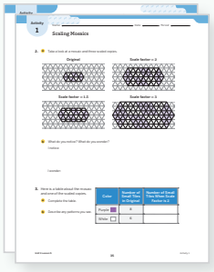
Students create a mosaic they will use to explore how scaling affects *area*.



Activity 1

Pairs | 15 min

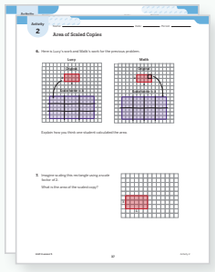
Students use a mosaic to make sense of how the area changes when a figure is scaled by a scale factor. **Routines:** Notice and Wonder, Decide and Defend **(MTR.1.1)**



Activity 2

Pairs | 15 min

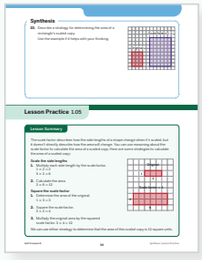
Students develop and use strategies for calculating the area of a scaled copy.
Routine: Think-Pair-Share



Synthesis

Whole Class | 5 min

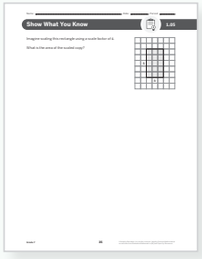
Students synthesize their understanding of the relationship between the areas of scaled copies.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the area of a scaled rectangle.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students create a mosaic they will use to explore how scaling affects *area*.

1 Launch

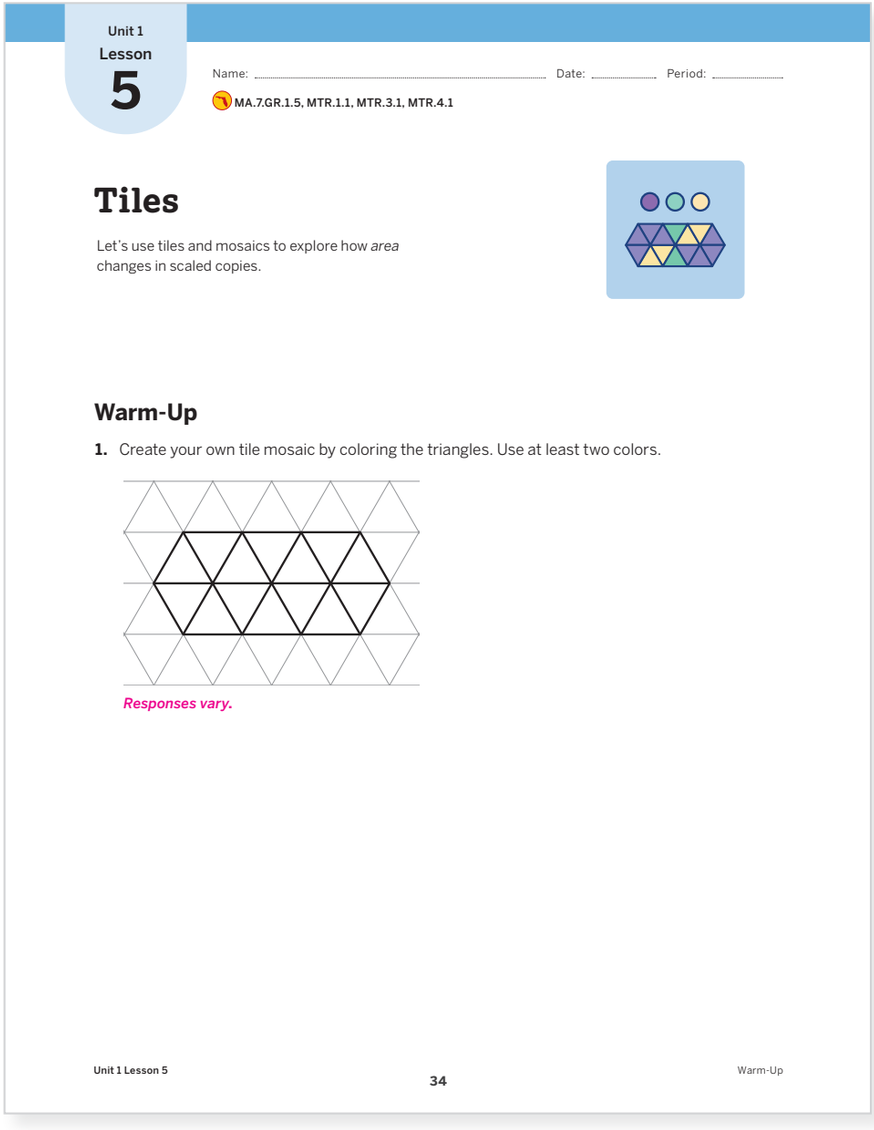
To surface connections, invite students to share with a partner all the things they could measure in their mosaic.

Consider sharing that students will use a similar mosaic in the next activity.

To encourage creativity, invite students to create a mosaic they like.

2 Connect

Invite students to share the mosaics they created.



Unit 1
Lesson
5

Name: Date: Period:

MA.7.GR.1.5, MTR.1.1, MTR.3.1, MTR.4.1

Tiles

Let's use tiles and mosaics to explore how *area* changes in scaled copies.

Warm-Up

1. Create your own tile mosaic by coloring the triangles. Use at least two colors.

Responses vary.

Unit 1 Lesson 5 34 Warm-Up

Pairs | 15 min

Activity 1 Scaling Mosaics

Purpose: Students use a mosaic to make sense of how the area changes when a figure is scaled by a scale factor. **(MTR.1.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of how scaling affects *area*.

A **Accessibility: Visual-Spatial Processing** Invite students to work with a partner to count the shaded tiles at different scale factors.

2 Monitor

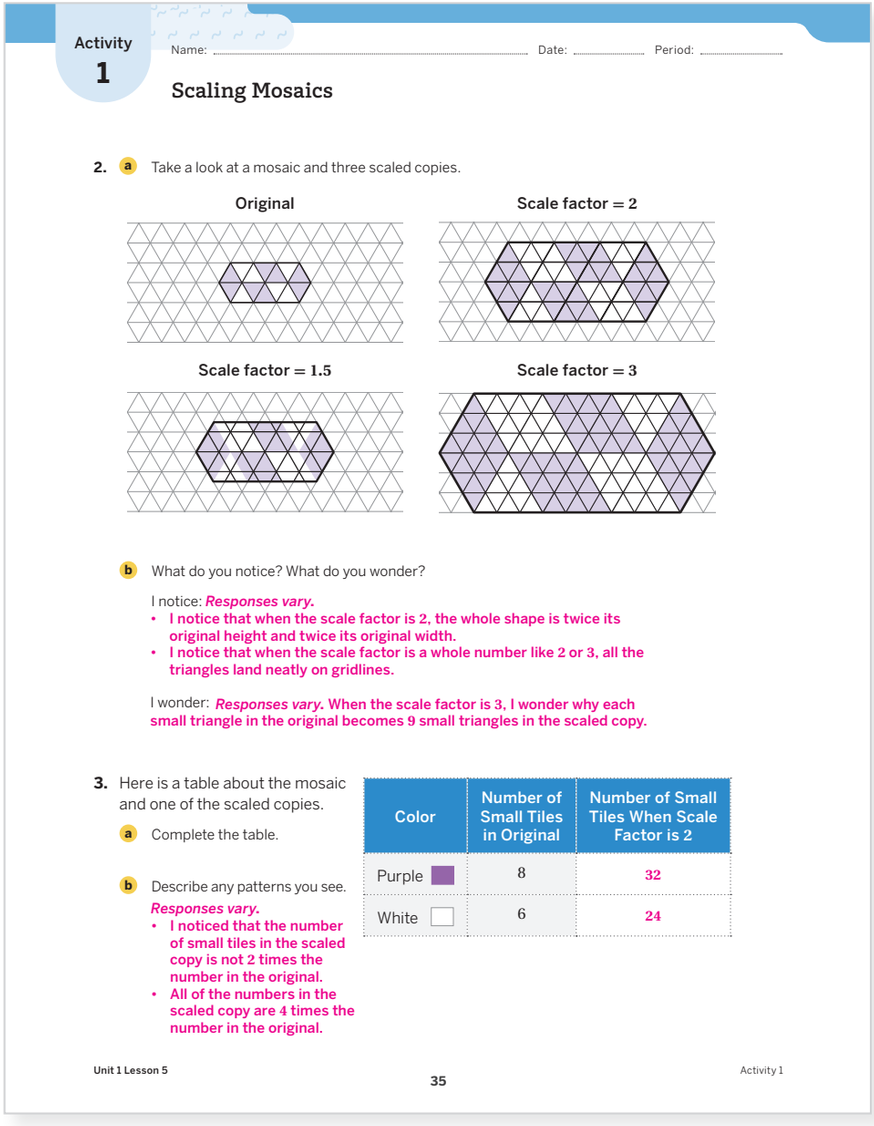
Look for students who notice a multiplicative pattern in the table. Encourage students to identify the factor by which the number of tiles is multiplied.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: area, length, scale factor, and scaled copy. **(Writing and Speaking)**

A **Accessibility: Executive Functioning** To support students who would benefit from more processing time, provide a pre-completed table and have students focus on describing patterns. Consider asking, "How did the side lengths change from the original to the scaled copy? How did the number of tiles change?"

Note: Consider waiting until most students have completed Problem 3 to discuss as a whole class. It is okay if students do not yet see the relationship between the scale factor and the change in number of tiles.

Activity 1 continued >



Activity 1

Name: Date: Period:

Scaling Mosaics

2. a Take a look at a mosaic and three scaled copies.

Original

Scale factor = 2

Scale factor = 1.5

Scale factor = 3

b What do you notice? What do you wonder?

I notice: *Responses vary.*
- I notice that when the scale factor is 2, the whole shape is twice its original height and twice its original width.
- I notice that when the scale factor is a whole number like 2 or 3, all the triangles land neatly on gridlines.

I wonder: *Responses vary.* When the scale factor is 3, I wonder why each small triangle in the original becomes 9 small triangles in the scaled copy.

3. Here is a table about the mosaic and one of the scaled copies.

a Complete the table.

Color	Number of Small Tiles in Original	Number of Small Tiles When Scale Factor is 2
Purple	8	32
White	6	24

b Describe any patterns you see.

Responses vary.
- I noticed that the number of small tiles in the scaled copy is not 2 times the number in the original.
- All of the numbers in the scaled copy are 4 times the number in the original.

Unit 1 Lesson 5 35 Activity 1

Pairs | 15 min

Activity 1 Scaling Mosaics (continued)

Purpose: Students use a mosaic to make sense of how the area changes when a figure is scaled by a scale factor. **(MTR.1.1)**

2 Monitor

Encourage students to sketch to show their thinking.

Listen for a variety of student ideas to highlight during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Believe the number of tiles will be 3 times the original, because the scale factor is 3.	**Support** Encourage students to choose a color and count the number of tiles of that color in both figures.
Notice that the number of tiles is not multiplied by the scale factor.	**Strengthen** Consider asking, "How would you describe how the number of tiles changed from the original to the scaled copy?"
Notice that the scaled copy has 9 times as many tiles as the original.	**Stretch** Consider asking, "How many tiles would there be if the scale factor were 4 instead of 3?"

Invite students to use as many different ways as they can to calculate the *area* of the large rectangle.

Note: Consider waiting to discuss Problem 5 until Activity 2. Students will see two strategies for this same problem on the following page.

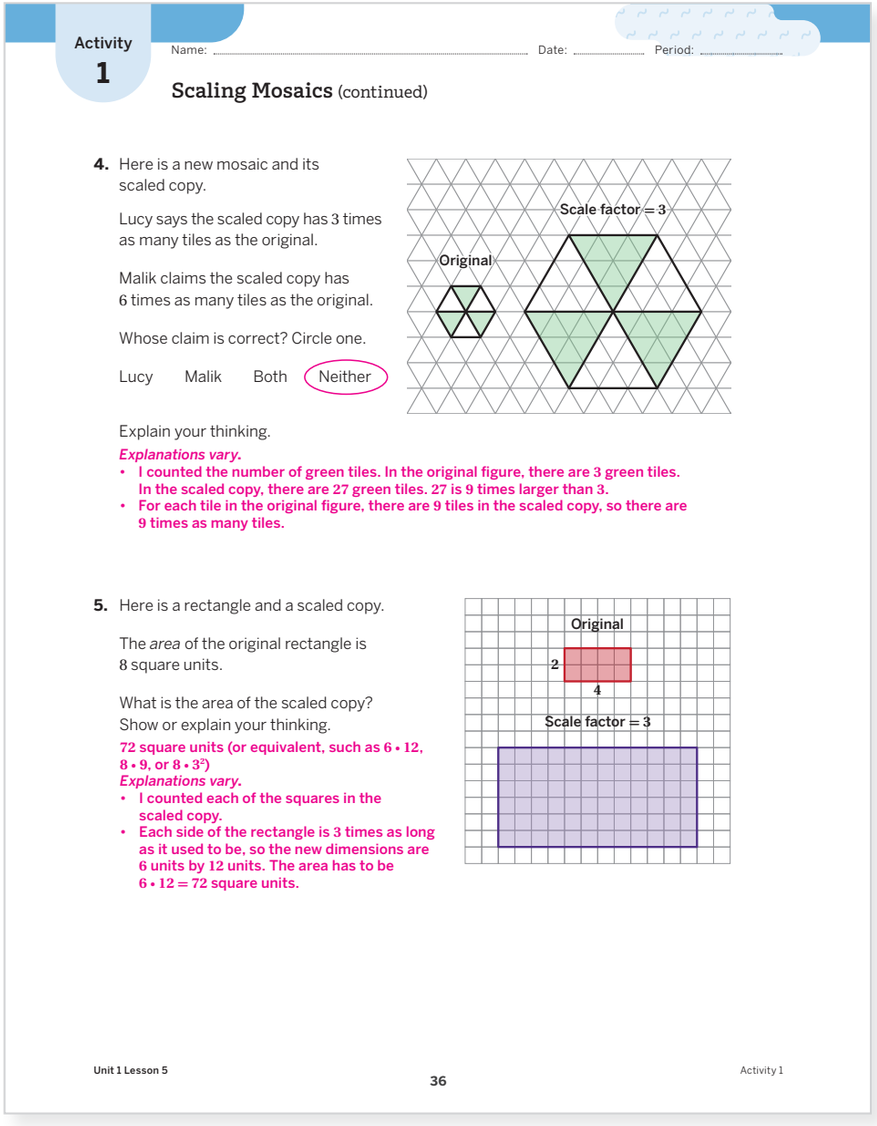
3 Connect

Invite students to share their work.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:

- "Where do you see the scale factor of 3 in the scaled copy?" Each side is 3 times as large as the original figure.
- "How does scaling affect the number of tiles?" The number of tiles increases. It is not 3 times as large like when we scale the sides.



Activity 1

Name: Date: Period:

Scaling Mosaics (continued)

4. Here is a new mosaic and its scaled copy.

Lucy says the scaled copy has 3 times as many tiles as the original.

Malik claims the scaled copy has 6 times as many tiles as the original.

Whose claim is correct? Circle one.

Lucy Malik Both Neither

Explain your thinking.

Explanations vary.
- I counted the number of green tiles. In the original figure, there are 3 green tiles. In the scaled copy, there are 27 green tiles. 27 is 9 times larger than 3.
- For each tile in the original figure, there are 9 tiles in the scaled copy, so there are 9 times as many tiles.

5. Here is a rectangle and a scaled copy.

The *area* of the original rectangle is 8 square units.

What is the area of the scaled copy? Show or explain your thinking.

72 square units (or equivalent, such as $6 \cdot 12$, $8 \cdot 9$, or $8 \cdot 3^2$)

Explanations vary.
- I counted each of the squares in the scaled copy.
- Each side of the rectangle is 3 times as long as it used to be, so the new dimensions are 6 units by 12 units. The area has to be $6 \cdot 12 = 72$ square units.

Unit 1 Lesson 5 36 Activity 1

Key Takeaway: When a figure is scaled by a scale factor, the *area* is *not* scaled by the same scale factor.

Pairs | 15 min

Activity 2 Area of Scaled Copies

Purpose: Students develop and use strategies for calculating the area of a scaled copy.

Short on time? Consider omitting Problems 8 and 9 and focusing the discussion on student strategies from Problem 7.

1 Launch

Consider sharing that both Lucy and Malik entered $8 \cdot 9$ for the area.

Use the Think-Pair-Share routine to help students shape their ideas about Lucy's and Malik's strategies. See the Routine Facilitation Guide for more information.

Encourage students to make connections between the expression $8 \cdot 9$ and the sketches Malik and Lucy created.

Discuss each strategy, including students' ideas from Problem 5. Consider polling the class by asking:

- "Who saw it the way Malik saw it? How would you describe what he did?"
- "Who saw it the way Lucy saw it? What did she do?"
- "Who saw it a different way? How would you describe your strategy?"

Consider creating a record or anchor chart of strategies so students can refer back to them during the rest of the activity.

2 Monitor

Invite students to use the strategies from Problem 6 or a new strategy to determine the area of the scaled copy.

Look for students who draw sketches or write area expressions that suggest they are noticing structure, such as:

- $15 \cdot 4$
- $3 \cdot 5 \cdot 2^2$

D **Differentiation: (Support)** Students may not yet be confident applying the strategies discussed in the launch. Encourage these students to sketch the scaled copy, then look for a pattern or connection they can apply to future problems. **(MTR.3.1)**

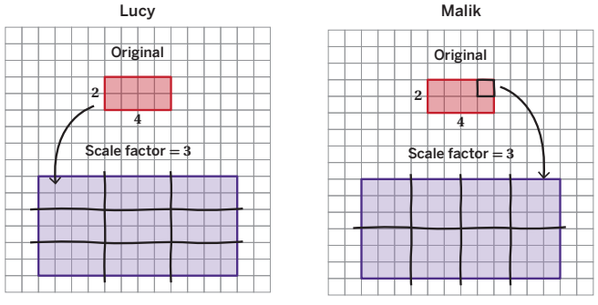
Activity 2 continued >

Activity 2

Name: Date: Period:

Area of Scaled Copies

6. Here is Lucy's work and Malik's work for the previous problem.



Explain how you think one student calculated the area.

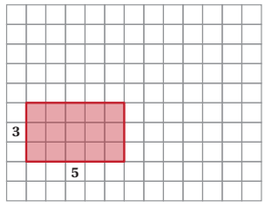
Responses vary.

- Lucy's strategy: Lucy created 9 copies of the original rectangle. Since the original rectangle had an area of 8 square units, the total area is $8 \cdot 9 = 72$ square units.
- Malik's strategy: Malik realized that each tile in the original became a 3-by-3 square in the scaled copy. There are 8 groups of 9 tiles in the scaled copy, so the area is $8 \cdot 9 = 72$ square units.

7. Imagine scaling this rectangle using a scale factor of 2.

What is the area of the scaled copy?

60 square units



Unit 1 Lesson 5 37 Activity 2

Pairs | 15 min

Activity 2 Area of Scaled Copies (continued)

Purpose: Students develop and use strategies for calculating the area of a scaled copy.

Short on time? Consider omitting Problems 8 and 9 and focusing the discussion on student strategies from Problem 7.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation

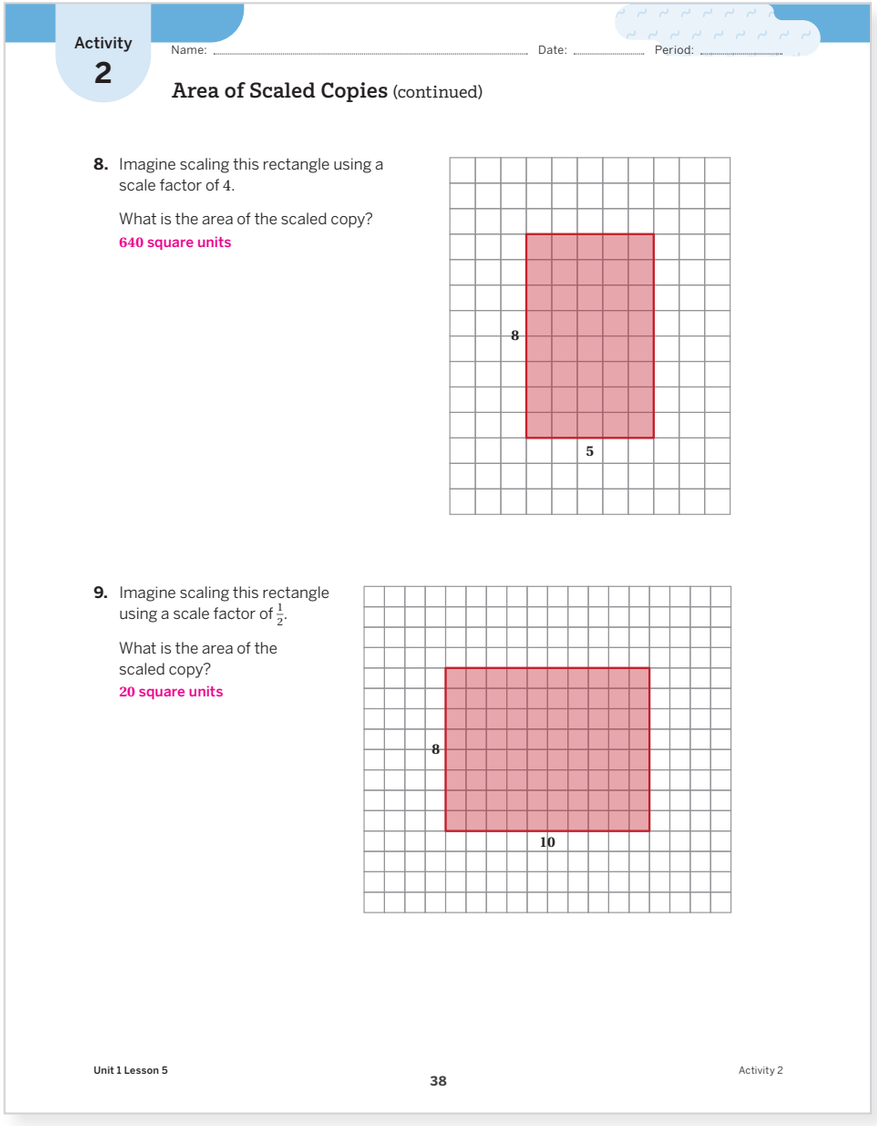
Look for students who:	Respond to Student Thinking
Create a sketch of the scaled rectangle.	**Support** Encourage students to try to determine the scaled lengths using *scale factor*.
Calculate the area of the original rectangle, then multiply by the scale factor.	**Support** Consider asking, "What are the length and width of the scaled copy? What is the area of a rectangle with those dimensions?"
Use a variety of strategies to determine the area, including: • Multiplying each length by the scale factor, then using those lengths to calculate area (e.g., $(8 \cdot 4) \cdot (5 \cdot 4)$). • Strategies similar to Lucy's or Malik's.	**Strengthen** Consider inviting these students to share their strategies during the Connect.
Apply the square of the scale factor, or multiply by the scale factor twice, to calculate the area.	**Stretch** Consider asking, "What relationship do you see between scale factor and area?"

3 Connect

Invite students to make connections between different strategies.

Consider asking, "What is similar or different about these strategies?"

If students generate expressions or sketches that represent the square of the scale factor, consider asking, "How is the 16 in this expression connected to the scale factor of 4?"



Activity 2

Name: Date: Period:

Area of Scaled Copies (continued)

8. Imagine scaling this rectangle using a scale factor of 4.

What is the area of the scaled copy?

640 square units

9. Imagine scaling this rectangle using a scale factor of $\frac{1}{2}$.

What is the area of the scaled copy?

20 square units

Unit 1 Lesson 5 38 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the relationship between the areas of scaled copies.

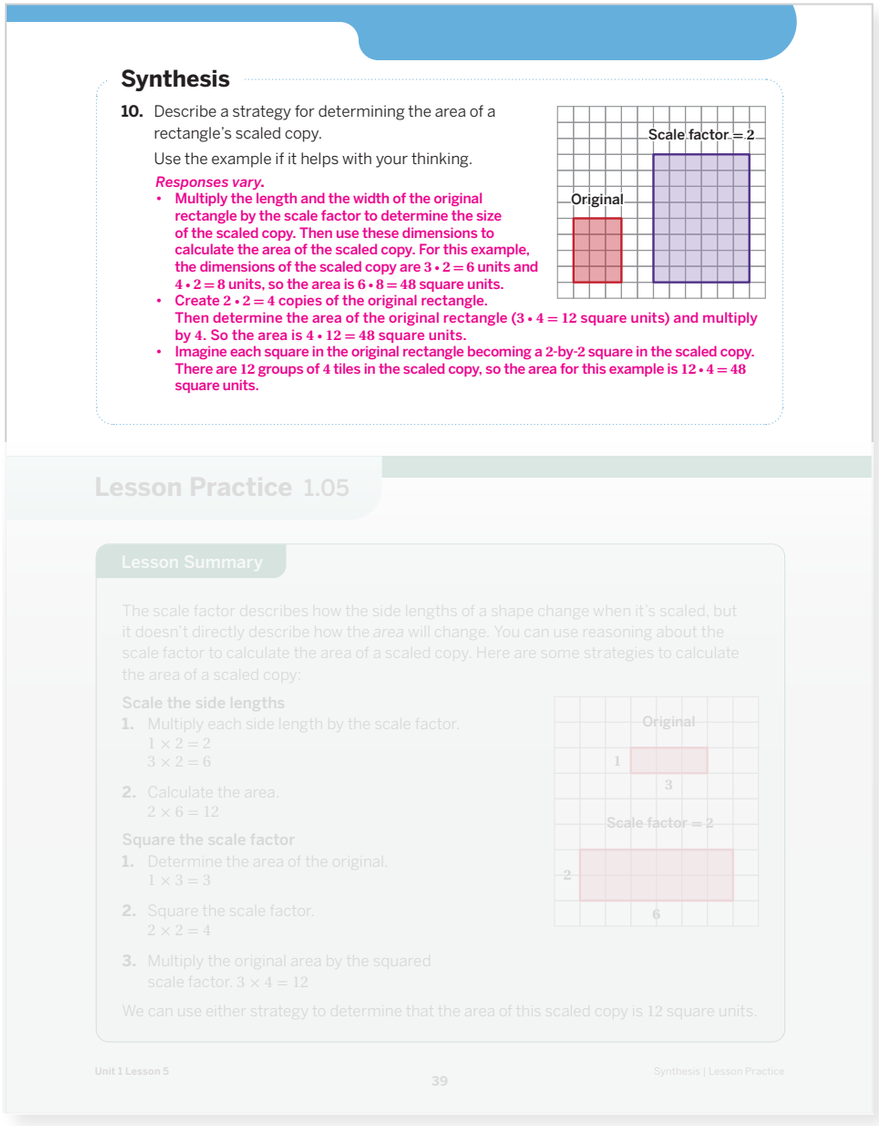
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Multiplying both length and width by the scale factor, then using these dimensions to calculate area.
- Creating 4 copies of the original rectangle, determining the original area, and multiplying by 4.
- Imagining each square in the original rectangle becoming a 2-by-2 square in the scaled copy. There would be 12 of these 2-by-2 squares, so the area is $12 \cdot 4 = 48$.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. Describe a strategy for determining the area of a rectangle's scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- Multiply the length and the width of the original rectangle by the scale factor to determine the size of the scaled copy. Then use these dimensions to calculate the area of the scaled copy. For this example, the dimensions of the scaled copy are $3 \cdot 2 = 6$ units and $4 \cdot 2 = 8$ units, so the area is $6 \cdot 8 = 48$ square units.
- Create $2 \cdot 2 = 4$ copies of the original rectangle. Then determine the area of the original rectangle ($3 \cdot 4 = 12$ square units) and multiply by 4. So the area is $4 \cdot 12 = 48$ square units.
- Imagine each square in the original rectangle becoming a 2-by-2 square in the scaled copy. There are 12 groups of 4 tiles in the scaled copy, so the area for this example is $12 \cdot 4 = 48$ square units.

Lesson Practice 1.05

Lesson Summary

The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the *area* will change. You can use reasoning about the scale factor to calculate the area of a scaled copy. Here are some strategies to calculate the area of a scaled copy:

Scale the side lengths

1. Multiply each side length by the scale factor.
   $1 \times 2 = 2$
   $3 \times 2 = 6$
2. Calculate the area.
   $2 \times 6 = 12$

Square the scale factor

1. Determine the area of the original.
   $1 \times 3 = 3$
2. Square the scale factor.
   $2 \times 2 = 4$
3. Multiply the original area by the squared scale factor. $3 \times 4 = 12$

We can use either strategy to determine that the area of this scaled copy is 12 square units.

Unit 1 Lesson 5 39 Synthesis | Lesson Practice

Lesson Takeaway: When a figure is scaled, its lengths are all scaled by the scale factor, but the *area* is not. However, you can still use the scale factor to calculate the area of a scaled copy.

Independent | 5 min

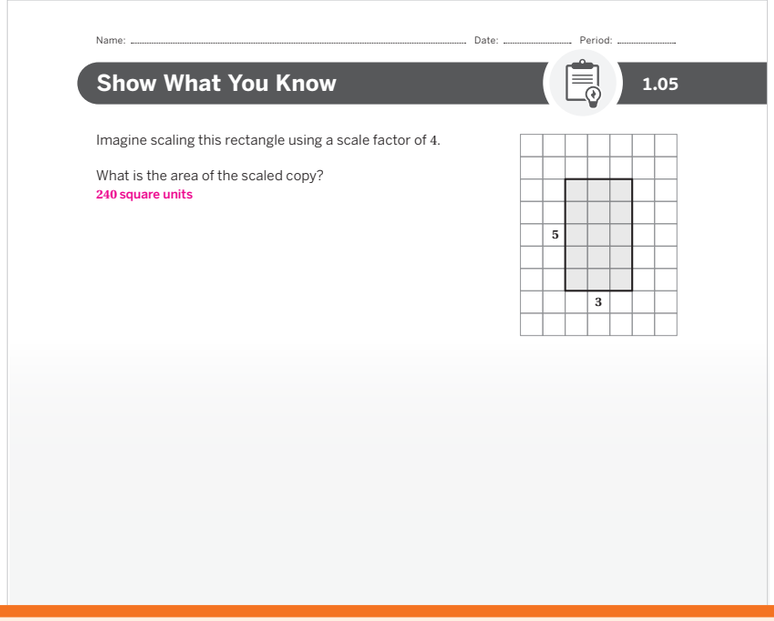
Show What You Know

Purpose: Students demonstrate their understanding by calculating the area of a scaled rectangle.

Today's Goals

1. **Goal:** Calculate the area of a scaled copy.
2. **Language Goal:** Explain how scaling affects lengths and areas in scaled copies. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.05

Imagine scaling this rectangle using a scale factor of 4.

What is the area of the scaled copy?

240 square units

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the area of a scaled copies:

» **Respond to Student Thinking:**

- Invite students to discuss how to determine the length and width of a scaled rectangle with a partner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on an instructional routine from this lesson. Students will use these routines throughout the year. What about the routine felt successful? What are areas for growth next time? Consider consulting the routine facilitation guide for more ideas.

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe a strategy for determining the area of a rectangle's scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- Multiply the length and the width of the original rectangle by the scale factor to determine the size of the scaled copy. Then use these dimensions to calculate the area of the scaled copy. For this example, the dimensions of the scaled copy are $3 \cdot 2 = 6$ units and $4 \cdot 2 = 8$ units, so the area is $6 \cdot 8 = 48$ square units.
- Create $2 \cdot 2 = 4$ copies of the original rectangle. Then determine the area of the original rectangle ($3 \cdot 4 = 12$ square units) and multiply by 4. So the area is $4 \cdot 12 = 48$ square units.
- Imagine each square in the original rectangle becoming a 2-by-2 square in the scaled copy. There are 12 groups of 4 tiles in the scaled copy, so the area for this example is $12 \cdot 4 = 48$ square units.

Lesson Practice 1.05

Lesson Summary

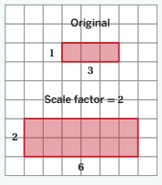
The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the *area* will change. You can use reasoning about the scale factor to calculate the area of a scaled copy. Here are some strategies to calculate the area of a scaled copy:

Scale the side lengths

1. Multiply each side length by the scale factor.
   $1 \times 2 = 2$
   $3 \times 2 = 6$
2. Calculate the area.
   $2 \times 6 = 12$

Square the scale factor

1. Determine the area of the original.
   $1 \times 3 = 3$
2. Square the scale factor.
   $2 \times 2 = 4$
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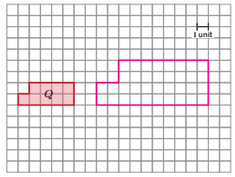


We can use either strategy to determine that the area of this scaled copy is 12 square units.

Lesson Practice 1.05

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is polygon Q.



1. Draw a scaled copy of polygon Q using a scale factor of 2.
Sample shown.

2. What is the area of the scaled copy?
36 square units

3. Complete the table for different scaled copies of polygon Q.

Scale Factor	Perimeter (units)	Area (sq. units)
1	14	9
2	28	**36**
3	**42**	**81**

4. How is the relationship between the scale factor and perimeter different from the relationship between the scale factor and area?
Responses vary. **The perimeter of a scaled copy grows by the scale factor. For example, a scaled copy of polygon Q using a scale factor of 3 has a perimeter of 42 ($14 \cdot 3 = 42$). The area doesn't grow by the scale factor because $9 \cdot 3 \neq 81$.**

5. Imagine a right triangle with an area of 36 square units.

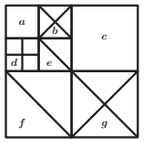
Scaled copies are made using different scale factors. Complete the table with the area of each scaled copy.

Scale Factor	Area (sq. units)
1	36
2	**144**
3	**324**
5	**900**
$\frac{1}{2}$	**9**
$\frac{2}{3}$	**16**

Lesson Practice 1.05

Name: _____ Date: _____ Period: _____

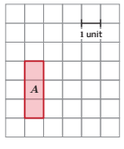
6. The square shown is made up of scaled copies of a triangle and a square. The area of region d is 1 square unit. Determine as many different areas as you can.



Region	a	b	c	d	e	f	g
Area (sq. units)	4	1	16	1	2	8	4

FAST Practice

Problems 7–8: Ivory created a scaled copy of rectangle A. The area of Ivory's scaled copy is 48 square units.



7. How many times greater is the area of Ivory's scaled copy than the area of rectangle A?

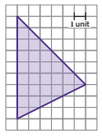
16 times greater

8. What scale factor did Ivory use to create their scaled copy?

4

Spiral Review

9. Determine the area of this triangle.
27 square units



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.GR.1.5
	6	3	MA.7.GR.1.5
FAST Practice	7–8	2	MA.7.GR.1.5
Spiral Review			
	9	2	MA.6.GR.2.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying scaled copies that have a scale factor of less than 1	Lesson 1	1	MA.7.GR.1.5 MTR.5.1
2	Comparing lengths and areas in original figures and scaled copies	Lesson 2	1	MA.7.GR.1.5
3a	Calculating missing lengths in scaled copies	Lesson 2	1	MA.7.GR.1.5
3b	Determining scale factors between original figures and scaled copies	Lesson 3	1	MA.7.GR.1.5
3c	Determining and justifying scale factors between original figures and scaled copies	Lesson 4	2	MA.7.GR.1.5
4a	Determining and justifying if the side lengths in two figures are proportional	Lesson 2	2	MA.7.GR.1.5 MTR.3.1
4b	Drawing scaled copies	Lesson 3	2	MA.7.GR.1.5
5a	Connecting scaled copies and area	Lesson 5	3	MA.7.GR.1.5
5b	Determining and justifying scale factors	Lesson 5	2	MA.7.GR.1.5 MTR.4.1

Assessment and Lesson Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



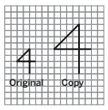
- Mini-Lessons
- Extensions

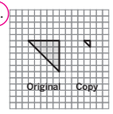
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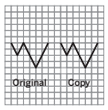
Sub-Unit Quiz

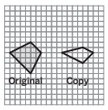
Unit 1

1. Here are pairs of figures, each with an original and a copy.
Which pair shows a *scaled* copy that has a scale factor of less than 1?

A. 

B. 

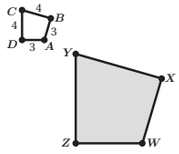
C. 

D. 

2. Figure $WXYZ$ is a scaled copy of figure $ABCD$ with a scale factor of 3.

Select *all* the true statements.

- ☑ **A.** The scale factor from $WXYZ$ to $ABCD$ is $\frac{1}{3}$.
- ☐ **B.** Segment YZ is 7 units long.
- ☐ **C.** If the area of $ABCD$ is 12 square units, then the area of $WXYZ$ is 36 square units.
- ☑ **D.** The distance between W and Y is 3 times the distance between A and C.
- ☑ **E.** The ratio of $\frac{BC}{BA}$ is equivalent to the ratio of $\frac{XY}{XW}$.



All lengths are in grid units.

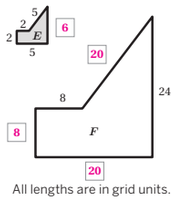
3. Figure F is a scaled copy of figure E.

a. Label each missing length.

b. What scale factor takes figure E to figure F?
4

c. What scale factor takes figure F back to figure E?
$\frac{1}{4}$

Explain how you know.
Explanations vary.
- The scale factor is $\frac{1}{4}$ because if I multiply each side of figure F by $\frac{1}{4}$, I get the side lengths of figure E.
- The ratio of any side length in figure E to the same side length in figure F is $\frac{1}{4}$. This is the scale factor I use to scale figure F back to figure E.



All lengths are in grid units.

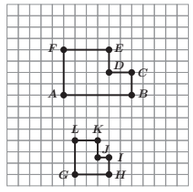
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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 1

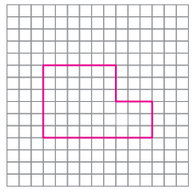
4. Here are figures $ABCDEF$ and $GHIJKL$.



a. Are the side lengths in $ABCDEF$ proportional to the side lengths in $GHIJKL$?
No

Explain how you know.
Explanations vary.
- Even though the width of $GHIJKL$ is half the width of $ABCDEF$, its height is not half. So the side lengths of the figures are not proportional.
- The side lengths of the figures are not proportional because they don't have equivalent ratios. For example, the widths form a ratio of $\frac{6}{3} = 2$, but the heights form a ratio of $\frac{4}{3}$, which is not equal to 2.

b. Draw a scaled copy of figure $ABCDEF$ using a scale factor of $\frac{3}{2}$.

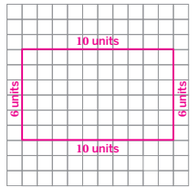


5. Rectangle S is 3 units by 5 units.

a. Draw a scaled copy of rectangle S with an area of 60 square units. Label each side length of the copy.

b. What is the scale factor between rectangle S and your copy?
2

Explain how you know.
Explanations vary. Student response depends on their response in Problem 5a.
If their response in Problem 5a is correct, here is a sample response: All of the lengths of the scaled copy are twice the lengths of the original figure. For example, the height of the original figure is 3 units, while the height of the scaled copy is $3 \cdot 2 = 6$ units.



D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

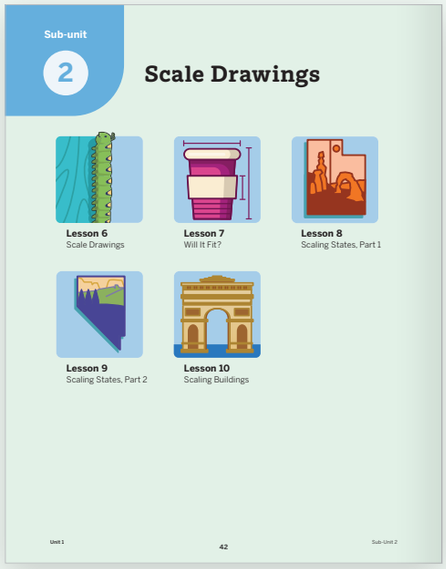
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Describe how scaling affects lengths, angles, and areas in scaled copies. • Use scale factors to create and compare scaled copies. **(Lessons 1–5)**	1, 4b	**Support**	• **Teacher Move:** Consider revisiting Unit 1, Lesson 1 (Scaling Machines) and Unit 1, Lesson 4 (Scale Factor Challenges)
		Strengthen	• **Practicing With Scale Factors:** Lesson 4 (Scale Factor Challenges)
		Stretch	• **You're invited to explore more:** Lesson 3 (Make It Scale)
	3	**Support**	• **Mini-Lesson:** Connecting Scale Factors to Scaled Copies (ML 1.04)
		Strengthen	• **Teacher Move:** Consider asking, "Which side measurements should be used to determine a scale factor between two figures?"
		Stretch	• **You're invited to explore more:** Lesson 4 (Scale Factor Challenges)
	2, 4a, 5	**Support**	• **Teacher Move:** Consider revisiting Unit 1, Lesson 3 (Make It Scale) and Unit 1, Lesson 5 (Tiles)
		Strengthen	• **Teacher Move:** Consider asking, "How does scaling impact the area of a scaled copy? How does it impact the side lengths?"
		Stretch	• **You're invited to explore more:** Lesson 5 (Back in My Day)

Scale Drawings

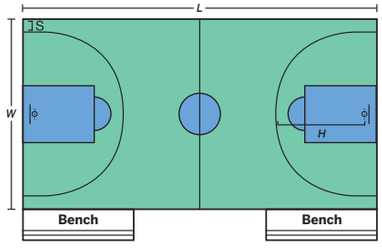
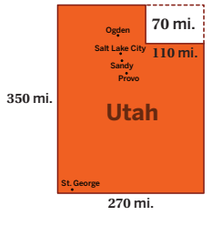
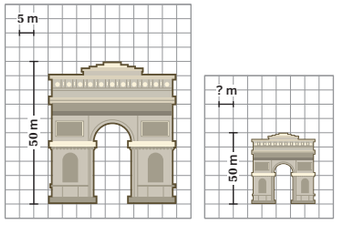
Sub-Unit 2 Goals

- **Goal:** Represent distances in the real world using scale and scale drawings.
- **Language Goal:** Analyze strategies for creating a scale drawing to represent actual distances and areas.

In this sub-unit, students use what they know about scaled copies to create and analyze **scale drawings** of real-life objects. They transition from using scale factor to using **scale**, determine scales of drawings, and justify whether two scales are equivalent.



Math That Matters Most

Concept or Skill	Example
Calculate actual and scaled distances from a scale drawing.	Determine a scale for the basketball court so that it will fit in a 20-by-20-meter park area. *I can use a scale where 2 centimeters on the scale drawing represents 10 meters on the actual court.* 
Use actual distances to create scale drawings.	How long is the western edge of Utah using a scale of 1 centimeter to 50 miles? *Since every 1 centimeter represents 50 miles I can divide 350 by 50 to get that the length on my scale drawing will be 7 centimeters.* 
Reason about multiple scale drawings of the same object using different scales.	Will the value of the unknown scale be greater or less than the scale of 5 meters? *The unknown scale will be more than 5 meters because each segment needs to represent a greater length to represent the same total height.* 

Unit 1

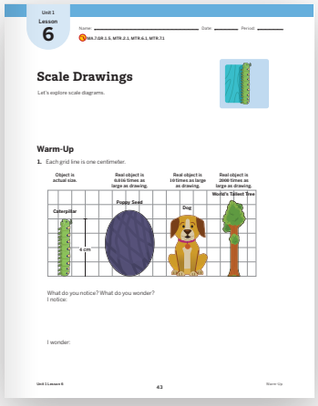
Lesson

6

Scale Drawings

Comparing Scale Factor and Scale

Let's explore scale diagrams.


Unit 1
Lesson
6
Name: Date: Period:
MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1
Scale Drawings
Let's explore scale diagrams.
Warm-Up
1. Each grid line is one centimeter.
Object is actual size.
Real object is 0.016 times as large as drawing.
Real object is 10 times as large as drawing.
Real object is 2000 times as large as drawing.
Caterpillar
Poppy Seed
Dog
World's Tallest Tree
4 cm
What do you notice? What do you wonder?
I notice:
I wonder:
43
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that a *scale* is a ratio of distance in a drawing to the corresponding distance in an object in the real world.
2. **Goal:** Interpret the *scale* of a drawing.
3. **Language Goal:** Explain how a *scale* helps to represent actual distances. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to the term **scale**, and begin to explore why scales are useful for representing very large or very small real-world objects. They describe how using a scale is similar or different to using a scale factor. They use scales to estimate actual lengths of objects.

Coherence

Prior Learning

In Lessons 1–5, students used scale factors to create and compare scaled copies.

Future Learning

In Lessons 7 and 8, students will create a scale drawing given a scale.

Connections

- Students apply their understanding of scale to estimate the measurements of real-world objects. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scale

Lesson at a Glance

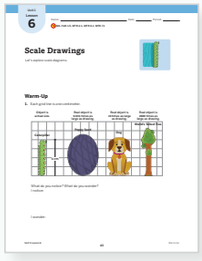
~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

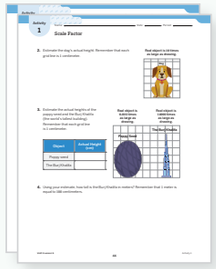
Students notice and wonder about very large or small objects represented on a grid with a scale factor. **Routine:** Notice and Wonder



Activity 1

Pairs | 15 min

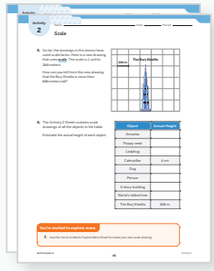
Students use scale factors to estimate the actual height of objects.



Activity 2

Pairs | 15 min

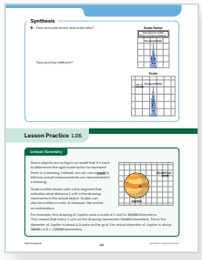
Students use **scale** to interpret drawings of real-world objects and to estimate the height of objects. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

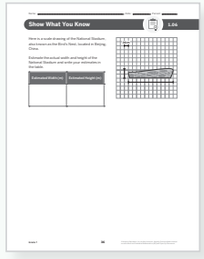
Students synthesize their understanding of scale. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of scale by estimating the actual width and height of National Stadium.



Prep Checklist

Prepare Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student
  - calculators
- Activity 2 You're Invited to Explore More Sheet, one per student (as needed)
  - measuring tools (in centimeters)

Pairs | 5 min

Warm-Up

Purpose: Students notice and wonder about very large or small objects represented on a grid with a scale factor.

1 Launch

Consider sharing that students will examine drawings of actual objects in the world in this lesson and for the rest of the unit.

2 Connect

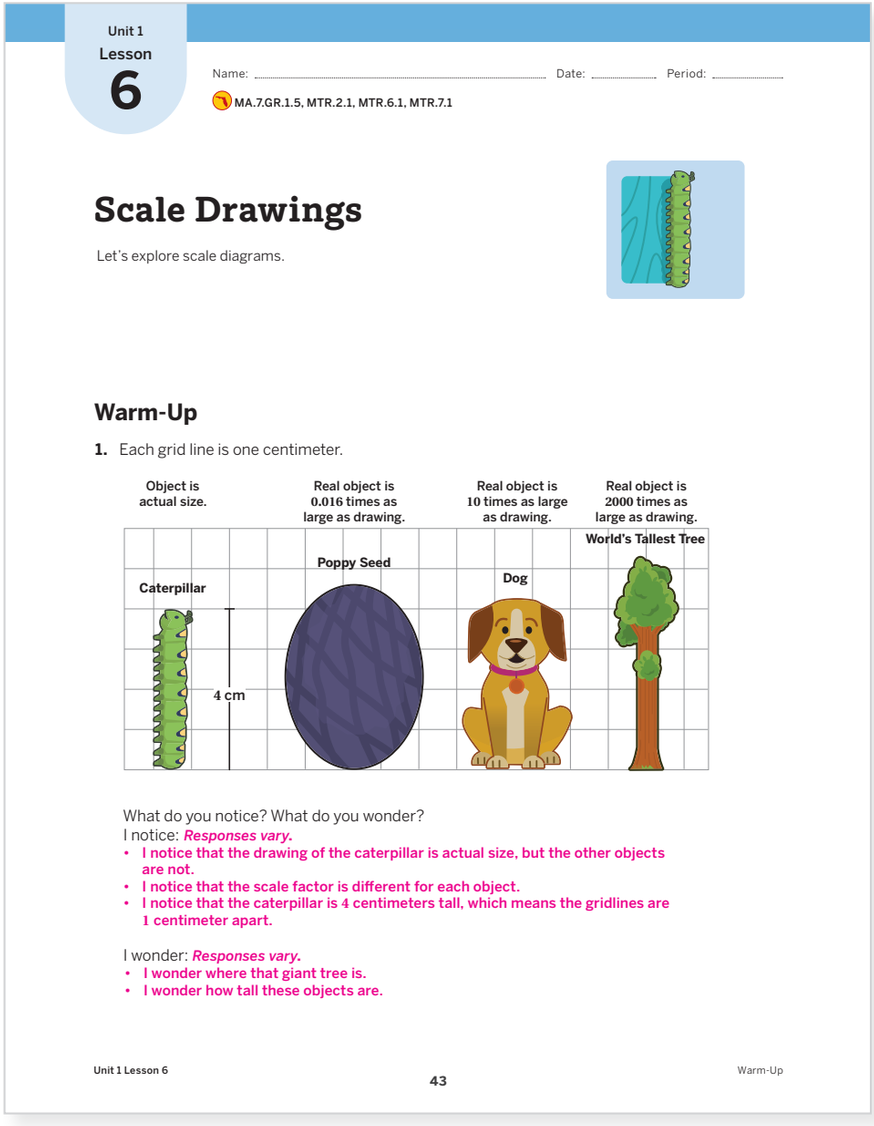
Use the Notice and Wonder routine to promote curiosity and help students make sense of the visual.

A **Accessibility: Visual-Spatial Processing**
Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does the sentence tell us about the size of the actual dog?"

Invite students to share their observations. Consider asking, "Where have you seen a poppy seed in real life? Is it bigger or smaller than a caterpillar?"

Note: If the following ideas do not arise naturally during the discussion, consider sharing them with the class:

- The number in the sentence "Real object is 10 times as large" means that the scale factor from this dog to the actual dog is 10.
- The grid is 1 cm by 1 cm based on the given information in the problem.



Unit 1
Lesson
6

Name: ..... Date: ..... Period: .....

MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1

Scale Drawings

Let's explore scale diagrams.

Warm-Up

1. Each grid line is one centimeter.

Object is actual size. | Real object is 0.016 times as large as drawing. | Real object is 10 times as large as drawing. | Real object is 2000 times as large as drawing.

Caterpillar | Poppy Seed | Dog | World's Tallest Tree

4 cm

What do you notice? What do you wonder?
I notice: *Responses vary.*
- I notice that the drawing of the caterpillar is actual size, but the other objects are not.
- I notice that the scale factor is different for each object.
- I notice that the caterpillar is 4 centimeters tall, which means the gridlines are 1 centimeter apart.

I wonder: *Responses vary.*
- I wonder where that giant tree is.
- I wonder how tall these objects are.

Unit 1 Lesson 6 43 Warm-Up

Pairs | 15 min

Activity 1 Scale Factor

Purpose: Students use scale factors to estimate the actual height of objects.

Short on time? Consider omitting Problem 4.

1 Launch

Consider asking, "What is the scale factor from the drawing to the actual dog?"

Invite students to check in with a partner before moving on.

2 Monitor

Select and sequence student strategies for using the scale factor, including those that show the operations used.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How tall does the object appear to be?"
Use the caterpillar as a reference point for the size of the poppy seed or Burj Khalifa.	**Support** Consider asking, "What does this sentence tell you about the scale factor? How does the grid help us make sense of the size?"
Count the units of each object on the grid and multiply by the scale factor.	**Stretch** Consider asking, "What would change about the scale factor if the grid were made up of smaller squares?"
Mention how small or large the numbers are for the poppy seed and the Burj Khalifa.	**Stretch** Consider asking, "How else could you measure the height of those objects?"

Encourage students who finish early to determine how tall the Burj Khalifa is in kilometers. Consider asking, "Which unit of measurement makes the most sense?" **(MTR.6.1)**

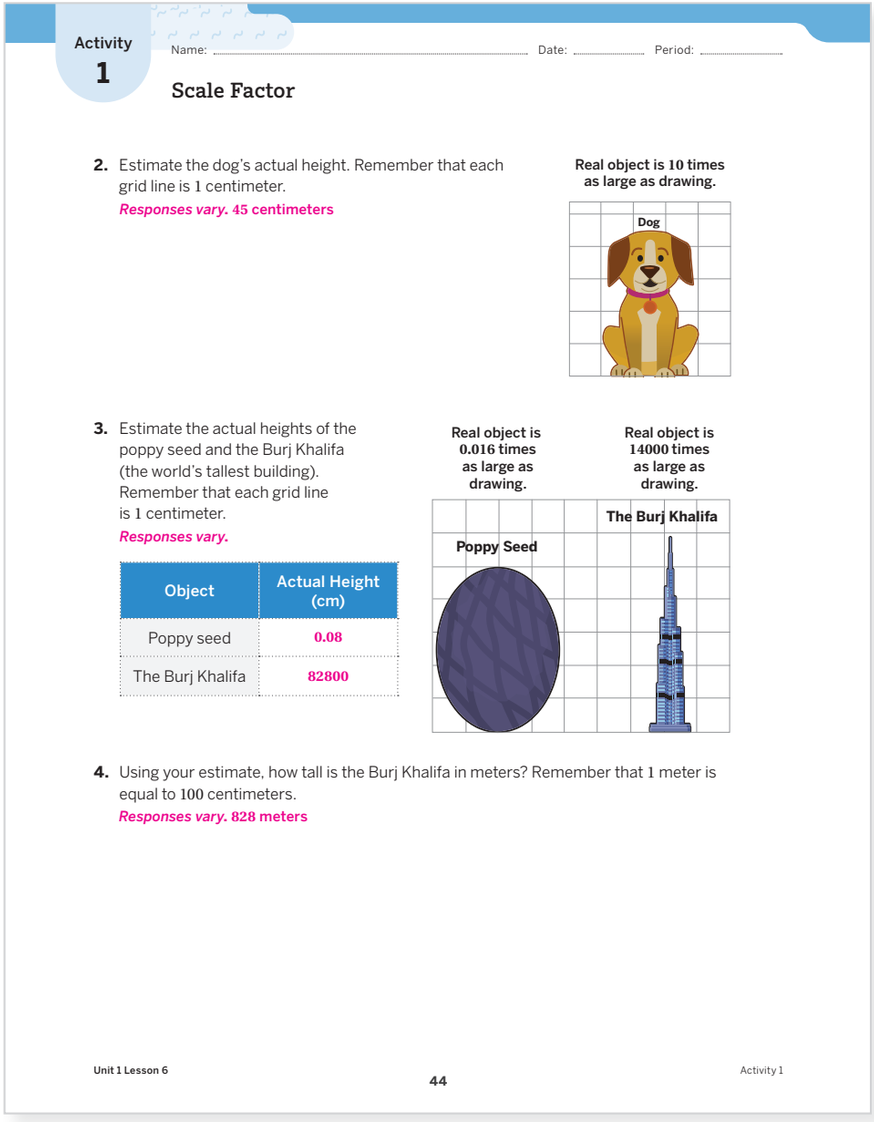
Invite students to share their estimates with their partner for the height of the Burj Khalifa before moving on to converting between centimeters and meters.

3 Connect

Invite students to discuss their strategies. Consider asking, "What might be challenging about using scale factor to estimate the size of real objects?" **(MTR.7.1)**

ELL

English Language Learners Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**



Activity 1

Name: Date: Period:

Scale Factor

2. Estimate the dog's actual height. Remember that each grid line is 1 centimeter.

Responses vary. 45 centimeters

Real object is 10 times as large as drawing.

Dog

3. Estimate the actual heights of the poppy seed and the Burj Khalifa (the world's tallest building). Remember that each grid line is 1 centimeter.

Responses vary.

Object	Actual Height (cm)
Poppy seed	0.08
The Burj Khalifa	82800

Real object is 0.016 times as large as drawing.

Real object is 14000 times as large as drawing.

Poppy Seed

The Burj Khalifa

4. Using your estimate, how tall is the Burj Khalifa in meters? Remember that 1 meter is equal to 100 centimeters.

Responses vary. 828 meters

Unit 1 Lesson 6 44 Activity 1

Pairs | 15 min

Activity 2 Scale

Purpose: Students use **scale** to interpret drawings of real-world objects and to estimate the height of objects. **(MTR.2.1)**

1 Launch

Invite students to read the problem and then pause to ask, "How is the scale on the grid similar or different from what you know about scale factor?" **(MTR.2.1)**

Create a class definition of the term **scale** using students' language and the glossary. Encourage students to note the differences between scale and scale factor.

2 Monitor

Distribute the Activity 2 Sheet to students.

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., presenting one problem at a time), which will aid students who benefit from support with organizational skills.

Look for students who count units on the grid and multiply by the scale. Look for these expressions to share during the Connect.

Note: Students may not have seen the symbol μ. Consider telling students that a meter is one million times as large as a micrometer.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore using scale to represent the height of real-world objects further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share how they determined the actual height of a few objects. Consider displaying the multiplicative expressions in addition to the height to ask about what each part of the multiplication statement represents in the situation. **(MTR.2.1)**

To surface the Key Takeaway, consider asking, "Why might it be useful for a scale to involve different units?"

Activity 2

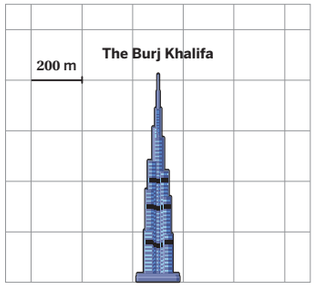
Name: Date: Period:

Scale

5. So far, the drawings in this lesson have used scale factor. Here is a new drawing that uses **scale**. The scale is 1 unit to 200 meters.

How can you tell from this new drawing that the Burj Khalifa is more than 800 meters tall?

Responses vary.
- The Burj Khalifa is a little more than 4 units tall, which means it's a little more than $4 \cdot 200 = 800$ meters tall.
- Count the gridlines up from the bottom of the drawing: 200, 400, 600, 800. This means the Burj Khalifa is a little more than 800 meters tall.



6. The Activity 2 Sheet contains scale drawings of all the objects in the table.

Estimate the actual height of each object.

Responses vary. Samples shown in table.

Object	Actual Height
Amoeba	140 μm
Poppy seed	800 μm
Ladybug	5.5 mm
Caterpillar	4 cm
Dog	45 cm
Person	180 cm
5-story building	15 m
World's tallest tree	115 m
The Burj Khalifa	828 m

You're invited to explore more.

7. Use the You're Invited to Explore More Sheet to create your own scale drawing.

Unit 1 Lesson 6 45 Activity 2

Key Takeaway: A **scale** can be used to show how lengths on a drawing relate to the lengths on the actual object. Scales can be helpful for representing distances on very large or very small objects.

Whole Class | 5 min

Synthesis

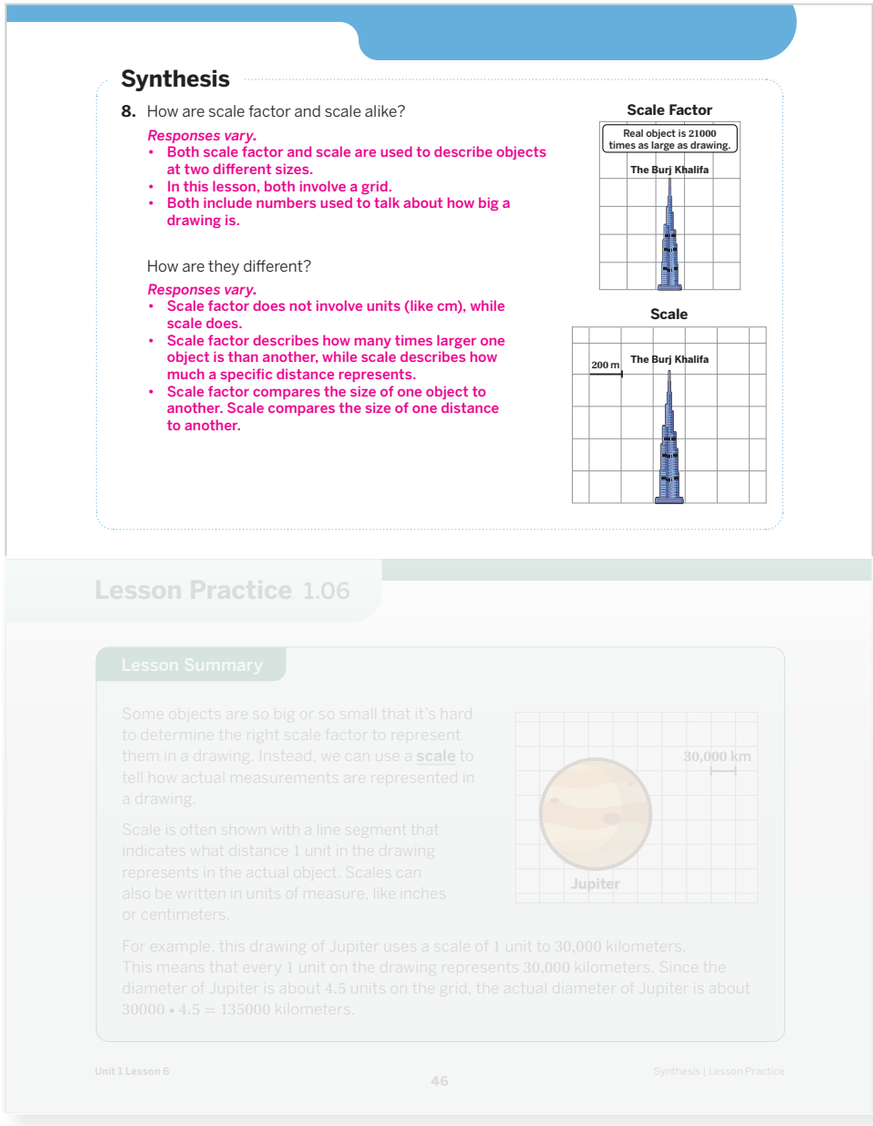
Purpose: Students synthesize their understanding of scale.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Both scale factor and scale are used to describe objects at two different sizes.
- Both include values used to talk about how big or small a drawing is in comparison to the real object.
- Scale factor does not involve units, but scale does.
- Scale factor describes how many times larger one object is than another, while scale describes how much a specific distance represents.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about similarities and differences between *scale factor* and *scale* as they share. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. How are scale factor and scale alike?

Responses vary.
- Both scale factor and scale are used to describe objects at two different sizes.
- In this lesson, both involve a grid.
- Both include numbers used to talk about how big a drawing is.

How are they different?

Responses vary.
- Scale factor does not involve units (like cm), while scale does.
- Scale factor describes how many times larger one object is than another, while scale describes how much a specific distance represents.
- Scale factor compares the size of one object to another. Scale compares the size of one distance to another.

Scale Factor

Real object is 21000 times as large as drawing.

The Burj Khalifa

Scale

200 m

The Burj Khalifa

Lesson Practice 1.06

Lesson Summary

Some objects are so big or so small that it's hard to determine the right scale factor to represent them in a drawing. Instead, we can use a scale to tell how actual measurements are represented in a drawing.

Scale is often shown with a line segment that indicates what distance 1 unit in the drawing represents in the actual object. Scales can also be written in units of measure, like inches or centimeters.

30,000 km

Jupiter

For example, this drawing of Jupiter uses a scale of 1 unit to 30,000 kilometers. This means that every 1 unit on the drawing represents 30,000 kilometers. Since the diameter of Jupiter is about 4.5 units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.

Unit 1 Lesson 6 46 Synthesis | Lesson Practice

Lesson Takeaway: A scale describes how much a specific distance in a drawing represents on the real object. Scales can be written in units of measure, e.g., 1 unit on the grid represents 2 miles.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of scale by estimating the actual width and height of National Stadium.

Today's Goals

1. **Goal:** Recognize that a *scale* is a ratio of distance in a drawing to the corresponding distance in an object in the real world.
2. **Goal:** Interpret the *scale* of a drawing.
3. **Language Goal:** Explain how a *scale* helps to represent actual distances. **(Reading, Writing, Speaking, and Listening)**

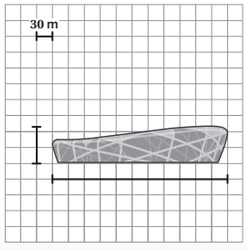
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.06

Here is a scale drawing of the National Stadium, also known as the Bird's Nest, located in Beijing, China.

Estimate the actual width and height of the National Stadium and write your estimates in the table.



Estimated Width (m)	Estimated Height (m)
Responses between 330 and 340 meters are considered correct.	Responses between 60 and 80 meters are considered correct.

Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how the scale of a drawing corresponds to real world objects:

» **Respond to Student Thinking:**

- Students will also have more opportunities to develop this concept in future lessons, so intervention is not necessary at this time.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to investigate the **You're Invited to Explore More** problem.

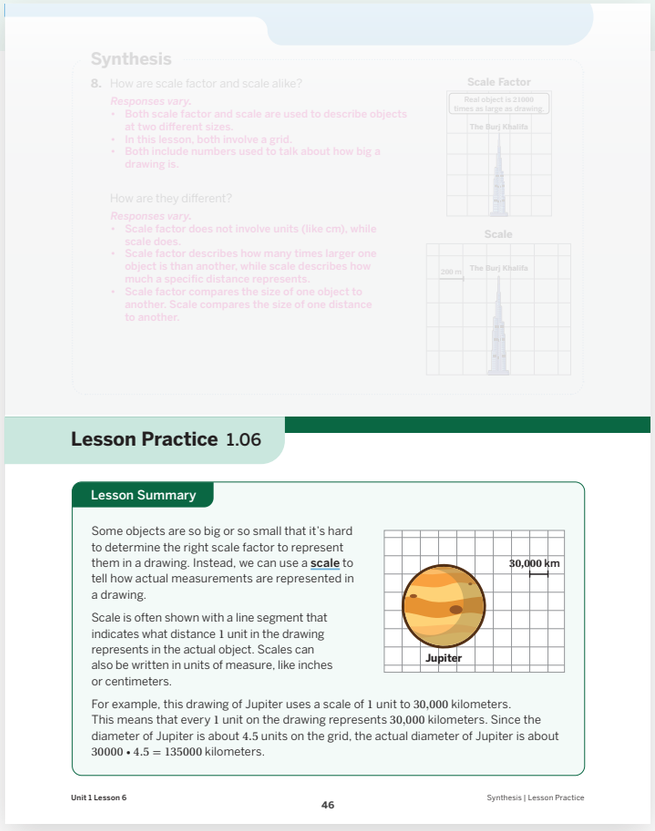
Professional Learning

Today students explored **scale** as it relates to heights in the real world. In the rest of the sub-unit, students will explore more scale drawings of actual places. How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. How are scale factor and scale alike?

Responses vary.

- Both scale factor and scale are used to describe objects at two different sizes.
- In this lesson, both involve a grid.
- Both include numbers used to talk about how big a drawing is.

How are they different?

Responses vary.

- Scale factor does not involve units (like cm), while scale does.
- Scale factor describes how many times larger one object is than another, while scale describes how much a specific distance represents.
- Scale factor compares the size of one object to another. Scale compares the size of one distance to another.

Scale Factor

Real object is 21000 times as large as drawing

The Burj Khalifa

Scale

200 m

The Burj Khalifa

Lesson Practice 1.06

Lesson Summary

Some objects are so big or so small that it's hard to determine the right scale factor to represent them in a drawing. Instead, we can use a **scale** to tell how actual measurements are represented in a drawing.

Scale is often shown with a line segment that indicates what distance 1 unit in the drawing represents in the actual object. Scales can also be written in units of measure, like inches or centimeters.

For example, this drawing of Jupiter uses a scale of 1 unit to 30,000 kilometers. This means that every 1 unit on the drawing represents 30,000 kilometers. Since the diameter of Jupiter is about 4.5 units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.

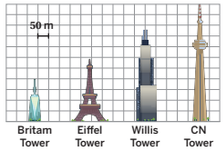
Lesson Practice 1.06

Name: _____ Date: _____ Period: _____

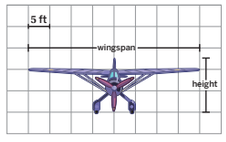
1. Here is a scale drawing of some towers in different cities. Use the scale to estimate how tall each tower is.

Responses vary.

Tower	Estimated Height (m)
Britam Tower	200
Eiffel Tower	300
Willis Tower	440
CN Tower	550



Problems 2–3: The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here is a scale drawing that shows the Lysander.

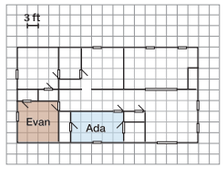


Approximate the actual lengths.

2. The wingspan of the plane:
   About 40 feet

3. The height of the plane:
   About 12 feet

Ada and Evan are siblings. Here is a scale drawing of their house that shows their rooms.



4. Complete the table with the dimensions of their rooms.

	Length (ft)	Width (ft)
Ada's Room	15	9
Evan's Room	12	12

Lesson Practice 1.06

Name: _____ Date: _____ Period: _____

FAST Practice

5. Here is a map of a town, with a scale at the bottom. Estimate how far Aki's house is from the pizza place.

About 400 feet

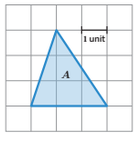


Spiral Review

1. Complete the table so that the two values in each row are related by the same ratio.

3	12
1.5	6
4	16
0.25	1

Problems 7–8: Tyrone created a scaled copy of triangle A with an area of 72 square units.



2. How many times greater is the area of the scaled copy compared to the area of triangle A?
   16 times greater

3. What scale factor did Tyrone apply to triangle A to create the scaled copy?
   4

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.GR.1.5
FAST Practice	5	2	MA.7.GR.1.5
Spiral Review			
	6	1	MA.6.AR.3.3
	7–8	2	MA.7.GR.1.5

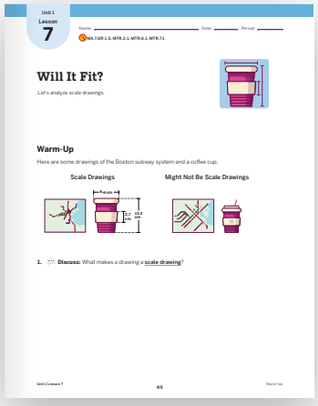
Unit 1

Lesson 7

Will It Fit?

Scale Drawings

Let's analyze scale drawings.


Unit 1
Lesson 7

Will It Fit?

Let's analyze scale drawings.

Warm-Up

Here are some drawings of the Boston subway system and a coffee cup.

Scale Drawings

Might Not Be Scale Drawings

1. Discuss: What makes a drawing a scale drawing?

49

Learning Outcomes

Today's Goals

1. **Goal:** Calculate actual and scaled distances from a *scale drawing*.
2. **Goal:** Calculate actual areas from a *scale drawing*.
3. **Language Goal:** Describe strategies for creating *scale drawings* to represent actual distances and areas. **(Reading and Writing)**

Students begin to analyze **scale drawings** (scaled two-dimensional representations of actual objects or places). They use scale drawings to determine actual distances and areas, then use these measurements and reasoning to solve real-world problems.

Coherence

Prior Learning

In Lessons 2–5, students explored scaled copies and how distances in a figure correspond to distances in a copy of the figure. In Lesson 6, they explored and interpreted the scale of a drawing.

Future Learning

In Lessons 8 and 9, students will create and reproduce scale drawings at specified scales, and will determine appropriate scales to use in different situations.

Connections

- Students determine whether a basketball court will fit in a specific location using scale drawings, measurements, and reasoning. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scale drawing

Lesson at a Glance

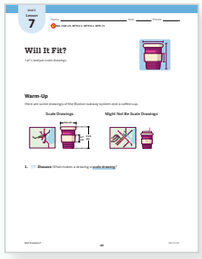
~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

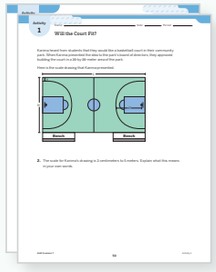
Students identify important properties of a **scale drawing**. **Routine:** Think-Pair-Share



Activity 1

Pairs | 15 min

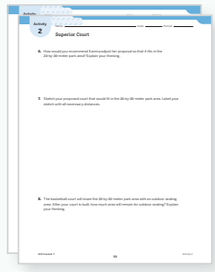
Students analyze a scale drawing to determine whether a basketball court will fit in a location. **(MTR.2.1)**



Activity 2

Pairs | 15 min

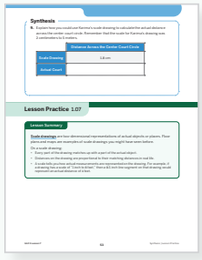
Students revise the planned dimensions and area of the court to ensure it will fit in the designated location. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of determining actual distances from a scale drawing.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using a scale to determine an actual distance.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- rulers

Pairs | 5 min

Warm-Up

Purpose: Students identify important properties of a **scale drawing**.

1 Launch

Direct students' attention to the images of the subway system and coffee cup.

To surface connections, invite students to share with their partners where they have seen images like these before.

Use the Think-Pair-Share routine to help students shape their ideas about scale drawings. See the Routine Facilitation Guide for more information.

2 Connect

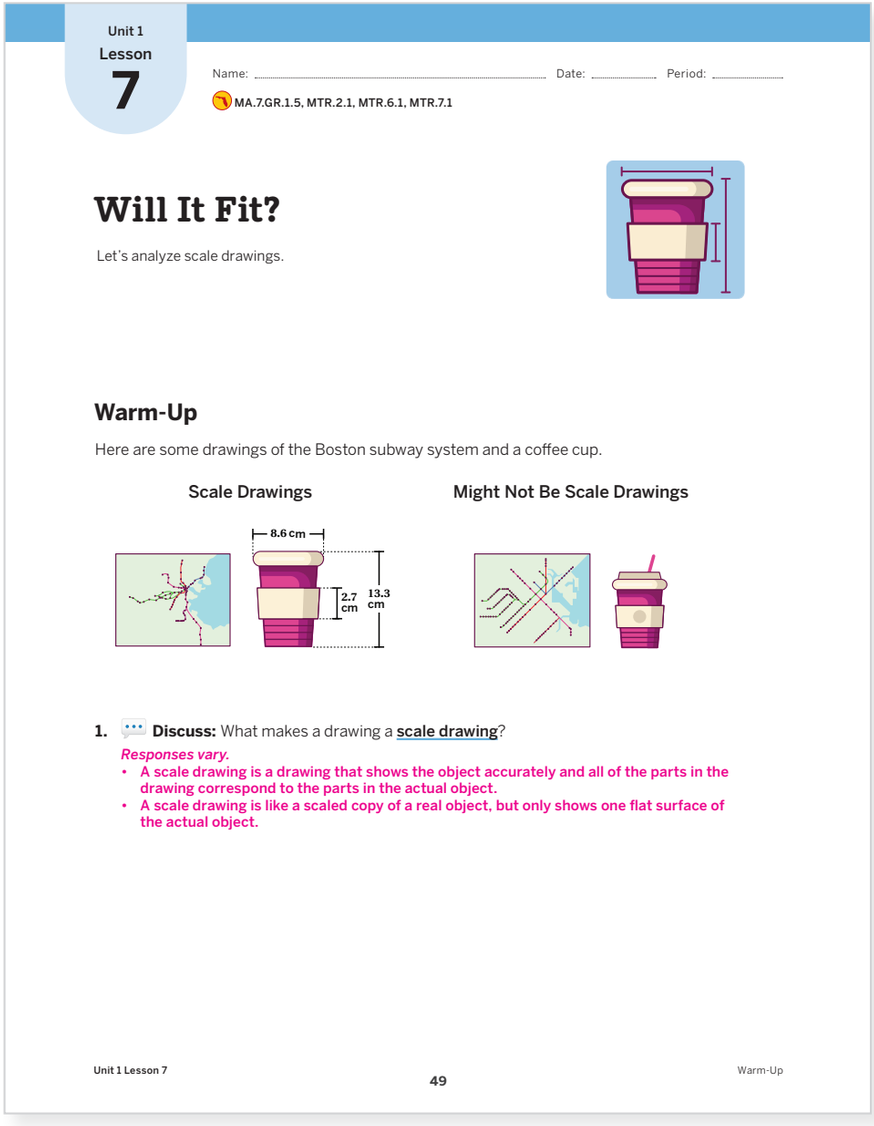
Invite students to share by asking:

- "How are distances portrayed in both types of drawings?"
- "Which set of drawings would be more useful for making decisions about building the subway? Which would be more useful for navigating the subway? What does this tell you about scale drawings?" **(MTR.7.1)**

Create a class definition for the term **scale drawing** using the students' language and the glossary.

To surface the Key Takeaway, consider asking:

- "What's similar about the two drawings of the map or cup? What's different?"
- "If you were to create a scale drawing of our school or city, what are things you would need to consider?"



Unit 1
Lesson
7

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1

Will It Fit?

Let's analyze scale drawings.

Warm-Up

Here are some drawings of the Boston subway system and a coffee cup.

Scale Drawings

Might Not Be Scale Drawings

8.6 cm
2.7 cm
13.3 cm

1. **Discuss:** What makes a drawing a **scale drawing**?

Responses vary.
- A scale drawing is a drawing that shows the object accurately and all of the parts in the drawing correspond to the parts in the actual object.
- A scale drawing is like a scaled copy of a real object, but only shows one flat surface of the actual object.

Unit 1 Lesson 7 49 Warm-Up

Key Takeaway: A **scale drawing** is a drawing that represents an actual place, object, or person. All measurements in a scale drawing correspond to the measurements of the actual object by the same scale.

Pairs | 15 min

Activity 1 Will the Court Fit?

Purpose: Students analyze a scale drawing to determine whether a basketball court will fit in a location. **(MTR.2.1)**

Short on time? Consider omitting the measurement from the 3-point line to the side line.

1 Launch

Direct students' attention to the model of the basketball court and distribute rulers for students to use as they create precise scale drawings. Read Karima's situation as a class.

Invite students to predict if they think the basketball courts will fit in the designated area. **(MTR.6.1)**

ELL **English Language Learners** Display photos of a basketball court and invite students to annotate them with words in English and in their primary language. **(Reading, Writing, Listening, and Speaking)**

2 Monitor

Look for strategies that students use to measure and organize their thinking.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What distance does 1 centimeter represent on the actual basketball court?"
Confuse the units on the scale when trying to calculate the actual distances on the court.	**Support** Consider asking, "What unit would make sense to measure a basketball court? What unit would make sense to represent a basketball court on paper?" **(MTR.7.1)**
Use their rulers to measure distances on the scale drawings.	**Stretch** Consider asking, "Which distances are helpful to measure to determine if the court will fit?"

A **Accessibility: Fine Motor Skills** Consider providing one or more of the measurements of the scale drawing so students can focus on determining the actual measurements of the court.

Activity 1 continued >

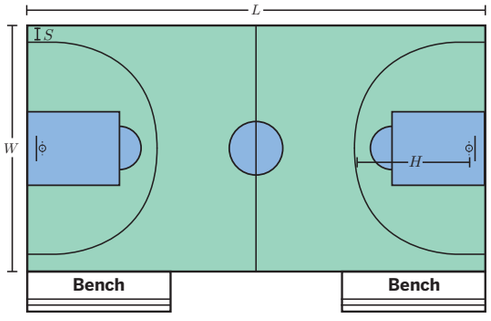
Activity 1

Name: _____ Date: _____ Period: _____

Will the Court Fit?

Karima heard from students that they would like a basketball court in their community park. When Karima presented the idea to the park's board of directors, they approved building the court in a 20-by-20-meter area of the park.

Here is the scale drawing that Karima presented.



2. The scale for Karima's drawing is 2 centimeters to 5 meters. Explain what this means in your own words.

Responses vary. It means that 2 centimeters on the scale drawing represents 5 meters on the actual court.

Unit 1 Lesson 7 50 Activity 1

Pairs | 15 min

Activity 1 Will the Court Fit? (continued)

Purpose: Students analyze a scale drawing to determine whether a basketball court will fit in a location. **(MTR.2.1)**

Short on time? Consider omitting the measurement from the 3-point line to the side line.

2 Monitor

Select and sequence student tables that clearly show how they calculated the distances on the actual court, as well as explanations for how students determined if the court would fit. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started determining the actual distances.	**Support** Consider asking, "How can you use the scale to determine the actual distance? What does 1 centimeter on the scale drawing represent in actual distance?"
See the relationship between Karima's scale and the scale 1 centimeter to 2.5 meters.	**Strengthen** Consider inviting them to share during the Connect. If they seem nervous, have them practice their response with a partner.
Determine the court won't fit because the length is greater than 20 meters.	**Strengthen** Consider asking, "How could you use the area of the actual court to show that it will not fit?"
Finish early.	**Stretch** **You're invited to explore more.** Invite students who would like to explore scale drawings further to complete this optional task. Encourage them to discuss their thinking with a partner.

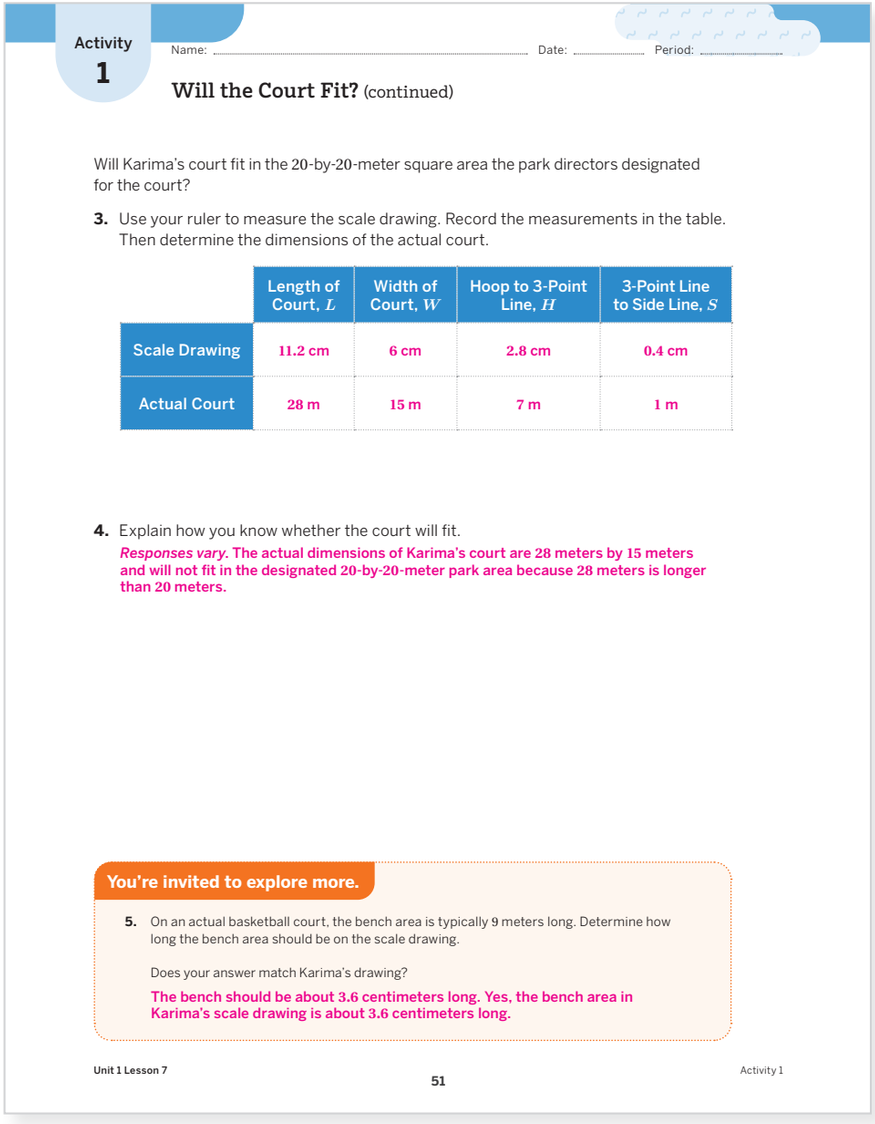
3 Connect

Direct students' attention to the table with the distances on the scale drawing. Invite students to share their strategies for determining the actual distances of the court.

Note: Spend adequate time on this discussion, making sure students understand at least one strategy for calculating actual dimensions at scale.

To surface the Key Takeaway, consider asking:

- "Why was it a good idea to make a scale drawing before going to build the basketball court?"
- "How does the scale help us to understand the distances on the basketball court?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

Will the Court Fit? (continued)

Will Karima's court fit in the 20-by-20-meter square area the park directors designated for the court?

3. Use your ruler to measure the scale drawing. Record the measurements in the table. Then determine the dimensions of the actual court.

	Length of Court, L	Width of Court, W	Hoop to 3-Point Line, H	3-Point Line to Side Line, S
Scale Drawing	11.2 cm	6 cm	2.8 cm	0.4 cm
Actual Court	28 m	15 m	7 m	1 m

4. Explain how you know whether the court will fit.

Responses vary. The actual dimensions of Karima's court are 28 meters by 15 meters and will not fit in the designated 20-by-20-meter park area because 28 meters is longer than 20 meters.

You're invited to explore more.

5. On an actual basketball court, the bench area is typically 9 meters long. Determine how long the bench area should be on the scale drawing.

Does your answer match Karima's drawing?

The bench should be about 3.6 centimeters long. Yes, the bench area in Karima's scale drawing is about 3.6 centimeters long.

Unit 1 Lesson 7 51 Activity 1

Key Takeaway: Scale drawings can be used to calculate actual distances and dimensions. They also help us to visualize large objects or places in a way that is easier for us to understand. Each distance on a scale drawing can be multiplied by the scale to determine the actual distance in the real world.

Pairs | 15 min

Activity 2 Superior Court

Purpose: Students revise the planned dimensions and area of the court to ensure it will fit in the designated location. **(MTR.7.1)**

1 Launch

To encourage creativity, invite students to think outside the box with their task of helping Karima adjust her scale drawing.

Encourage students to discuss and plan what dimensions of the basketball court they will adjust before they begin to sketch. **(MTR.2.1)**

2 Monitor

To support students getting started, ask, "What is important to include in the final version of the basketball court?"

Look for students who resize the court proportionally or who creatively meet the design constraints.

To encourage creativity, consider asking:

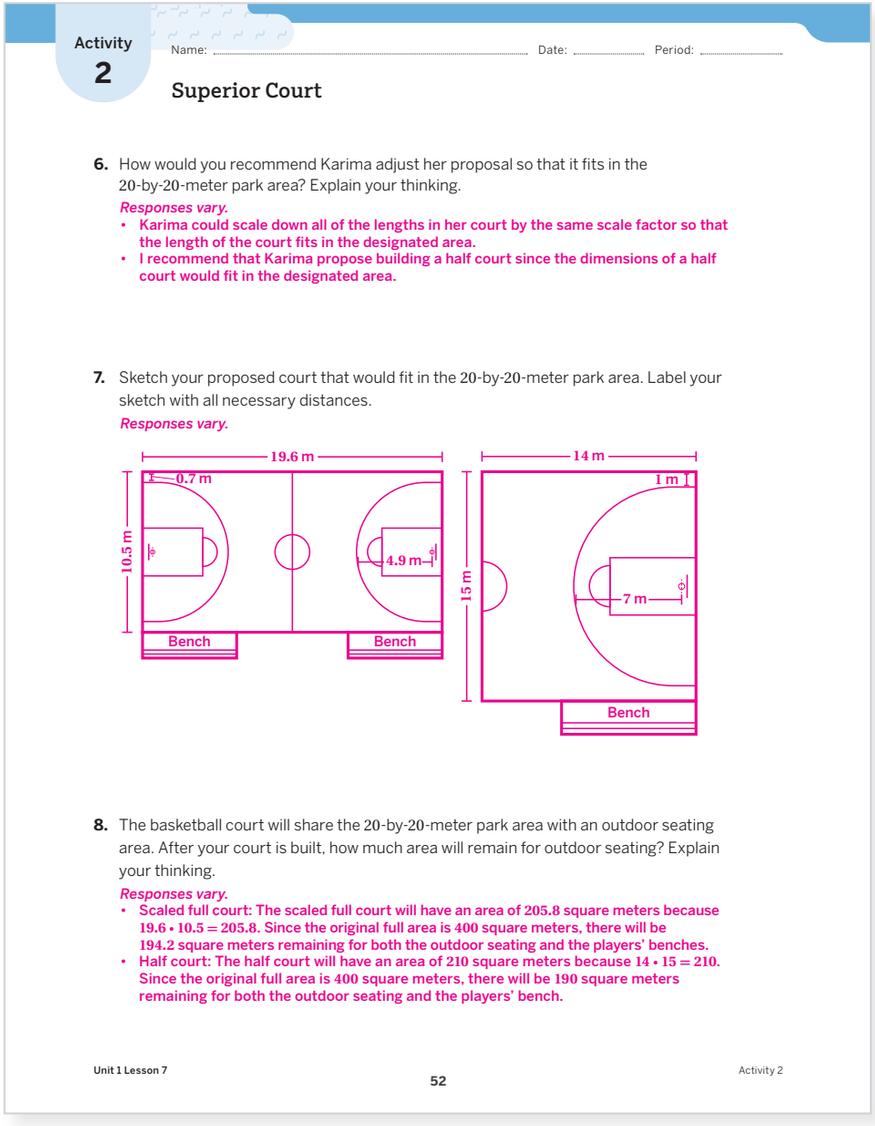
- "How do the benches impact your revised court?"
- "How would it affect basketball games if we changed the size of the court?"
- "Does your court need to be a rectangle?" **(MTR.7.1)**

3 Connect

Invite student pairs to share their strategies for designing their proposed courts and making sure they fit in the area.

Consider asking:

- "What strategies did you see in another pair's work that was different from your own?"
- "What is a creative solution or strategy from your classmates that you would want to include in your proposed court?"



Activity 2

Name: Date: Period:

Superior Court

6. How would you recommend Karima adjust her proposal so that it fits in the 20-by-20-meter park area? Explain your thinking.

Responses vary.

- Karima could scale down all of the lengths in her court by the same scale factor so that the length of the court fits in the designated area.
- I recommend that Karima propose building a half court since the dimensions of a half court would fit in the designated area.

7. Sketch your proposed court that would fit in the 20-by-20-meter park area. Label your sketch with all necessary distances.

Responses vary.

8. The basketball court will share the 20-by-20-meter park area with an outdoor seating area. After your court is built, how much area will remain for outdoor seating? Explain your thinking.

Responses vary.

- Scaled full court: The scaled full court will have an area of 205.8 square meters because $19.6 \cdot 10.5 = 205.8$. Since the original full area is 400 square meters, there will be 194.2 square meters remaining for both the outdoor seating and the players' benches.
- Half court: The half court will have an area of 210 square meters because $14 \cdot 15 = 210$. Since the original full area is 400 square meters, there will be 190 square meters remaining for both the outdoor seating and the players' bench.

Unit 1 Lesson 7 52 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of determining actual distances from a scale drawing.

Invite students to respond independently, and then share their thinking with a partner.

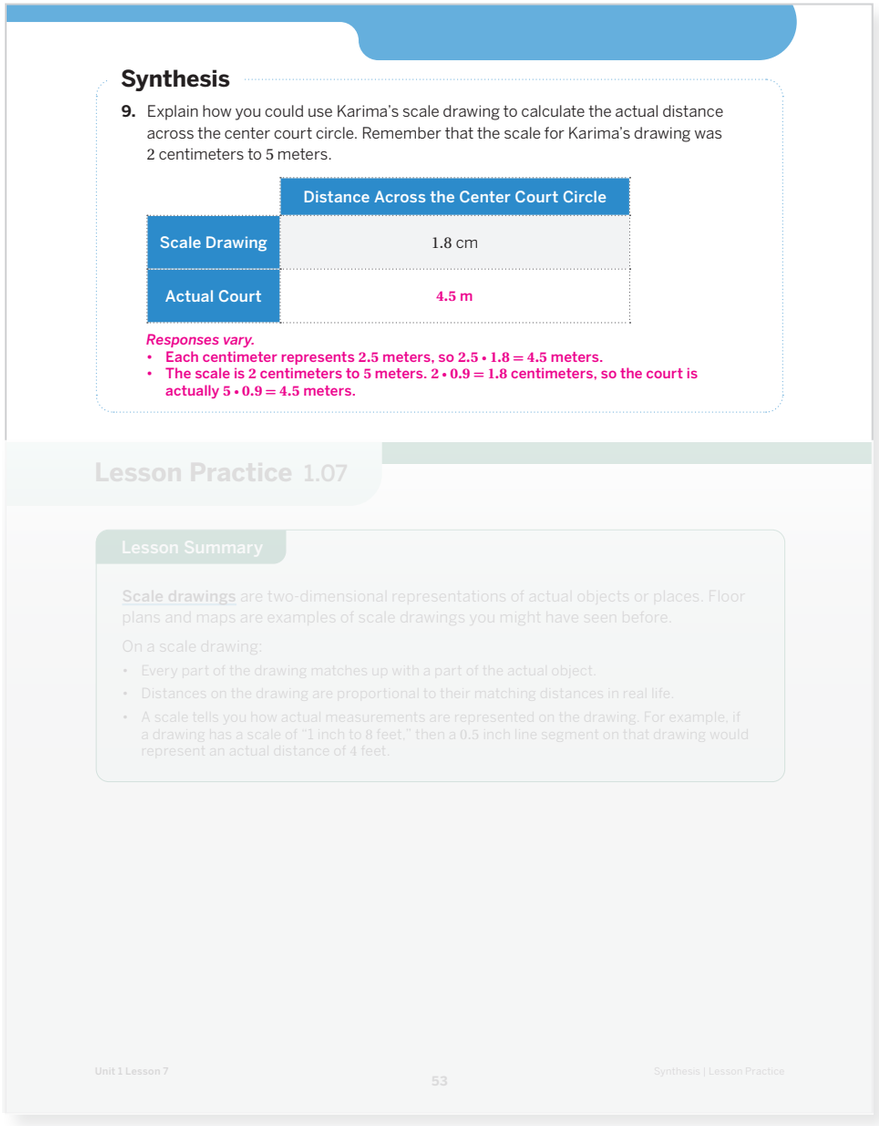
Collect and share a variety of ideas for determining the actual distance of the center court circle, such as:

- Using the scale of 2 centimeters to 5 meters to divide the 1.8 by 2 and multiply by 5 to get the actual distance.
- Using a simplified scale of 1 centimeter to 2.5 meters to multiply the 1.8 by 2.5 to get the actual distance.

Consider asking:

- "When is it important or useful to create a scale drawing?"
- "When might a scale drawing not be very useful or appropriate to create?"

Note: If time allows, invite students to revise their basketball courts based on the discussion.



Synthesis

9. Explain how you could use Karima's scale drawing to calculate the actual distance across the center court circle. Remember that the scale for Karima's drawing was 2 centimeters to 5 meters.

	Distance Across the Center Court Circle
Scale Drawing	1.8 cm
Actual Court	4.5 m

Responses vary.

- Each centimeter represents 2.5 meters, so $2.5 \cdot 1.8 = 4.5$ meters.
- The scale is 2 centimeters to 5 meters. $2 \cdot 0.9 = 1.8$ centimeters, so the court is actually $5 \cdot 0.9 = 4.5$ meters.

Lesson Practice 1.07

Lesson Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are examples of scale drawings you might have seen before.

On a scale drawing:

- Every part of the drawing matches up with a part of the actual object.
- Distances on the drawing are proportional to their matching distances in real life.
- A scale tells you how actual measurements are represented on the drawing. For example, if a drawing has a scale of "1 inch to 8 feet," then a 0.5 inch line segment on that drawing would represent an actual distance of 4 feet.

Unit 1 Lesson 7 53 Synthesis | Lesson Practice

Lesson Takeaway: Scale drawings are proportional, two-dimensional representations of actual objects or places that can be used to determine actual distances and areas. Decisions can be made about whether something will fit in a specific area using scale drawings because all the relationships between distances and area are the same, just at a different size.

Independent | 5 min

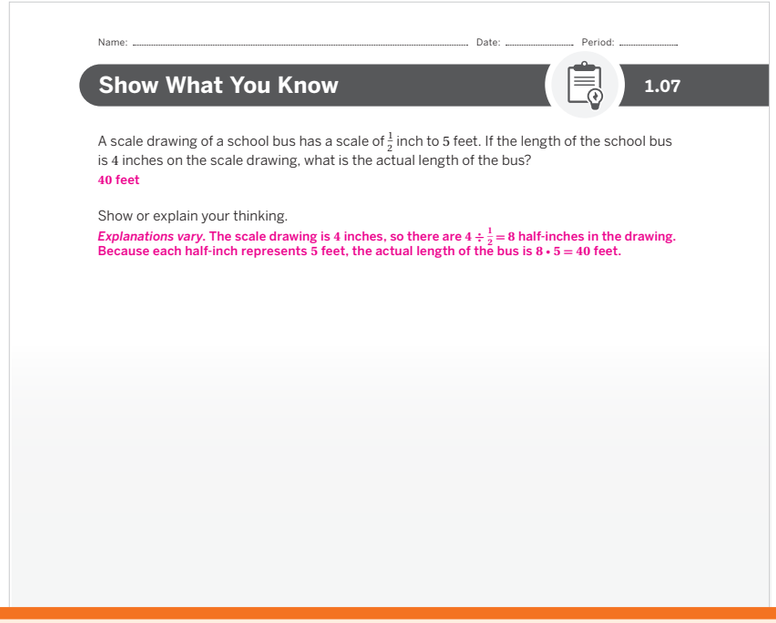
Show What You Know

Purpose: Students demonstrate their understanding by using a scale to determine an actual distance.

Today's Goals

1. **Goal:** Calculate actual and scaled distances from a *scale drawing*.
2. **Goal:** Calculate actual areas from a *scale drawing*.
3. **Language Goal:** Describe strategies for creating *scale drawings* to represent actual distances and areas. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.07

A scale drawing of a school bus has a scale of $\frac{1}{2}$ inch to 5 feet. If the length of the school bus is 4 inches on the scale drawing, what is the actual length of the bus?

40 feet

Show or explain your thinking.

Explanations vary. **The scale drawing is 4 inches, so there are $4 \div \frac{1}{2} = 8$ half-inches in the drawing. Because each half-inch represents 5 feet, the actual length of the bus is $8 \cdot 5 = 40$ feet.**

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate distances from a scaled drawing:

» **Respond to Student Thinking:**

- Students will also have more opportunities to develop this concept in future lessons, so intervention is not necessary at this time.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to investigate the **You're invited to explore more** activity.

Professional Learning

Take a moment to reflect on the strategies students used to create their basketball courts. What opportunities were there to celebrate specific students' strategies for creating a **scale drawing**? How can you connect back to these strategies as students create new scale drawings in Lessons 8 and 9?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Explain how you could use Karima's scale drawing to calculate the actual distance across the center court circle. Remember that the scale for Karima's drawing was 2 centimeters to 5 meters.

	Distance Across the Center Court Circle
Scale Drawing	1.8 cm
Actual Court	4.5 m

Responses vary.

- Each centimeter represents 2.5 meters, so $2.5 \cdot 1.8 = 4.5$ meters.
- The scale is 2 centimeters to 5 meters. $2 \cdot 0.9 = 1.8$ centimeters, so the court is actually $5 \cdot 0.9 = 4.5$ meters.

Lesson Practice 1.07

Lesson Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are examples of scale drawings you might have seen before.

On a scale drawing:

- Every part of the drawing matches up with a part of the actual object.
- Distances on the drawing are proportional to their matching distances in real life.
- A scale tells you how actual measurements are represented on the drawing. For example, if a drawing has a scale of "1 inch to 8 feet," then a 0.5 inch line segment on that drawing would represent an actual distance of 4 feet.

Unit 1 Lesson 7 53 Synthesis | Lesson Practice

Lesson Practice 1.07

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is part of a scale drawing of Zahra's office. The scale is 6 centimeters to 15 feet.



1. What are the dimensions of the actual office?
10 feet by 5 feet

2. What is the actual area of her office?
50 square feet

3. Zahra wants to put a couch in her office that is 3 feet wide. How wide would the couch be if it were drawn on the scale drawing?
1.2 centimeters

4. Fabiana's office is near Zahra's office and measures 4 centimeters by 8 centimeters in the scale drawing. Is Fabiana's office twice the area of Zahra's office? Explain your thinking.
No. *Explanations vary.*

- **The actual dimensions of Fabiana's office are 10 feet by 20 feet, so the area is 200 square feet, or 4 times the area of Zahra's office.**
- **Since each dimension of Zahra's office is doubled, the area of Fabiana's office would be more than doubled.**

Problems 5–7: Jada is looking at a map of a square park that has a scale of 5 centimeters to 200 feet. On the map, each side of the park is 10 centimeters long.



5. Jada lives 500 feet from the park. How long would this distance be on the map?
12.5 centimeters

6. If Jada ran around the perimeter of the park once, what distance would she run?
1,600 feet

7. Jada wants to run a mile (5,280 feet). About how many times would she need to run around the park in order to reach her goal?
About 3.3 times

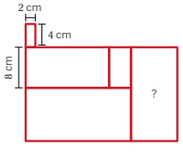
Unit 1 Lesson 7 54 Lesson Practice

Lesson Practice 1.07

Name: _____ Date: _____ Period: _____

8. Here is the whole floor that Zahra's office is on. Each room is a scaled copy of every other room. The scale for this blueprint is 6 centimeters to 15 feet.

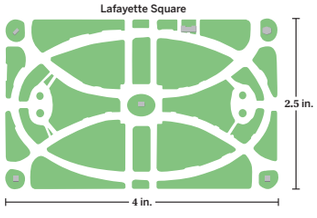
Use any strategy to determine the *actual area* of the room with the question mark.
The dimensions of the room with the question mark are 18 centimeters by 9 centimeters, so its actual area is $45 \cdot 22.5 = 1012.5$ square feet.



FAST Practice

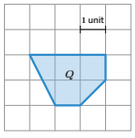
9. Here is a scale drawing of Lafayette Square in Washington, D.C. The scale is 1 inch to 200 feet. Determine the actual dimensions of Lafayette Square.

About **800** feet long and **500** feet wide



Spiral Review

Problems 10–11: Here is polygon Q.



10. What is the area of polygon Q?
4.5 square units

11. Lola drew a scaled copy of polygon Q using a scale factor of 2. What is the area of Lola's polygon? Explain your thinking.
18 square units. *Explanations vary.* When comparing areas, you use the square of the scale factor. So the area of Lola's polygon is 4 times greater than the area of polygon Q. $4.5 \cdot 4 = 18$.

Unit 1 Lesson 7 55 Lesson Practice

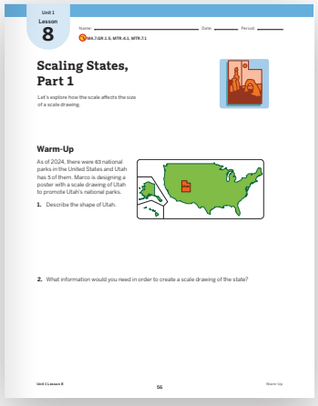
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.GR.1.5
	3–7	2	MA.7.GR.1.5
	8	3	MA.7.GR.1.5
FAST Practice	9	2	MA.7.GR.1.5
Spiral Review			
	10–11	2	MA.7.GR.1.5

Unit 1

Lesson

8


Unit 1
Lesson
8
Name: Date: Period:
MA.7.GR.1.5, MTR.4.1, MTR.7.1
Scaling States, Part 1
Let's explore how the scale affects the size of a scale drawing.
Warm-Up
As of 2024, there were 63 national parks in the United States and Utah has 5 of them. Marco is designing a poster with a scale drawing of Utah to promote Utah's national parks.
1. Describe the shape of Utah.
2. What information would you need in order to create a scale drawing of the state?
56

Scaling States, Part 1

Creating Scale Drawings

Let's explore how the scale affects the size of a scale drawing.

Learning Outcomes

Today's Goals

1. **Goal:** Create a scale drawing given the actual dimensions of the object and the scale.
2. **Goal:** Determine the scale given actual lengths and lengths on a scale drawing.
3. **Language Goal:** Compare and contrast strategies for determining lengths in scale drawings. **(Reading, Writing, Speaking, and Listening)**

Students make a scale drawing of Utah using real-world distances. They begin to explore how the choice of scale influences lengths in the drawing, which supports students in developing important understanding about distances and angles in scale drawings.

Coherence

Prior Learning

In Lesson 7, students used scales and scale drawings to calculate actual and scaled distances.

Future Learning

In Lesson 9, students will use their understanding of scale drawings to describe how scale affects lengths in a scale drawing.

Connections

- Students use scaled distances and angles to make a scale drawing of Utah using real-world distances. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	**MA.8.GR.2.2**
MA.6.GR.2.2	**MA.912.GR.4.3**

Lesson at a Glance

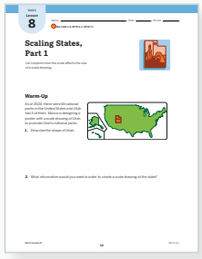
~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

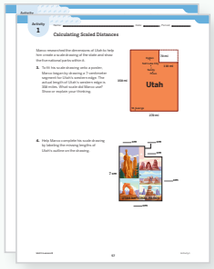
Students analyze the shape of Utah in preparation for creating scale drawings of the state.



Activity 1

Pairs | 15 min

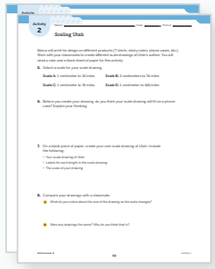
Students determine a scale using the border distances of Utah and use that information to create a scale drawing.



Activity 2

Small groups | 15 min

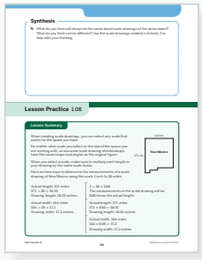
Students use different scales to create a precise scale drawing of Utah and notice how the scale used impacts their drawing. **MLR7: Compare and Connect**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how scale impacts scale drawings.



Show What You Know

Independent | 5 min

Students demonstrate their understanding of scale drawings by creating a precise scale drawing of a swimming pool.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- protractors
- rulers

Pairs | 5 min

Warm-Up

Purpose: Students analyze the shape of Utah in preparation for creating scale drawings of the state.

1 Launch

Direct students' attention to the map of the United States with Utah highlighted. Consider reading the text as a class before having students make sense of the shape of Utah.

ELL **English Language Learners** Invite students to talk through the shape of Utah with a partner before writing it down. **(Writing and Speaking)**

A **Accessibility: Visual-Spatial Processing** As students make sense of the visual representations, invite them to annotate the map of Utah by decomposing it into two rectangular sections and/or drawing dotted line segments to form a larger rectangle.

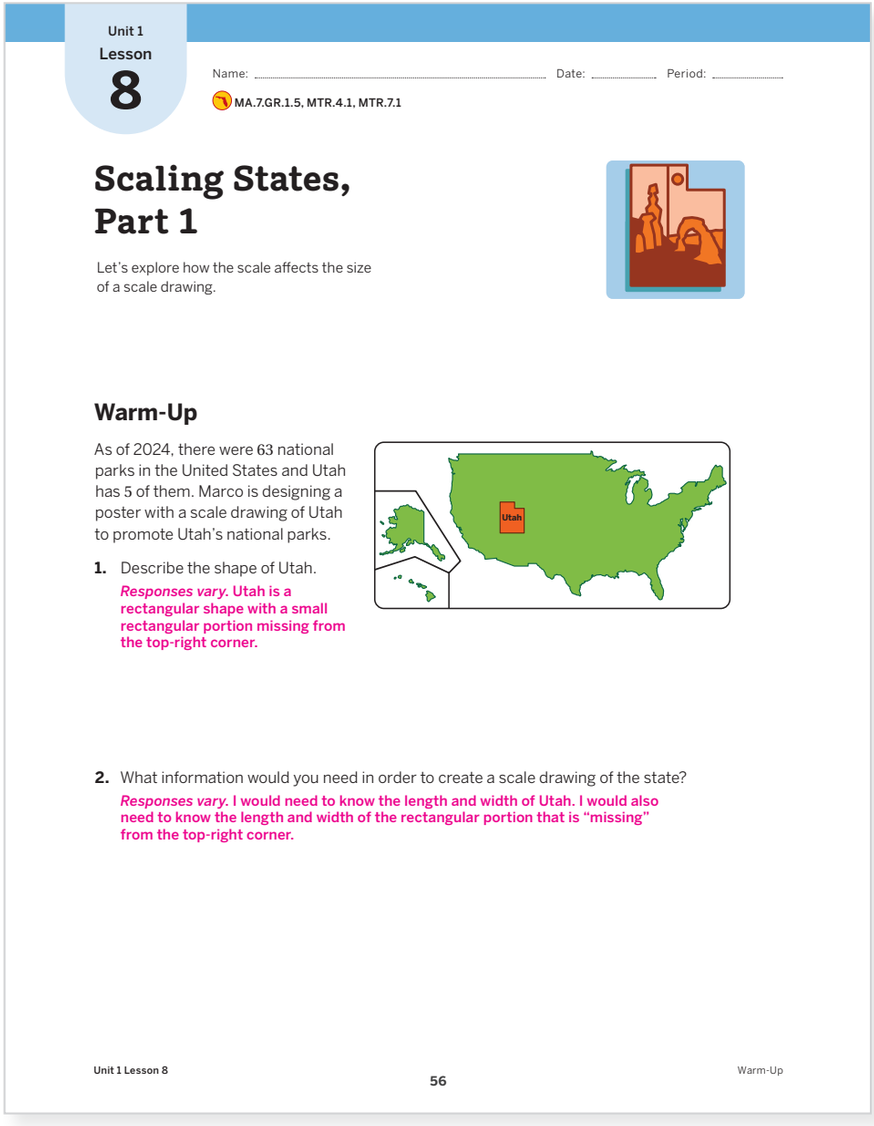
Look for responses that include needing to know the different lengths that form the border of Utah.

2 Connect

Invite students to share what details they noticed about Utah's shape and how they might be able to create a scale drawing.

Consider asking:

- "Why are scale drawings created?"
- "Which would be easier: a scale drawing of Utah or of Michigan? Florida? Hawaii?"
- "What tools do you think you need to create a scale drawing of Utah?"



Unit 1
Lesson
8

Name: ..... Date: ..... Period: .....

MA.7.GR.1.5, MTR.4.1, MTR.7.1

Scaling States, Part 1

Let's explore how the scale affects the size of a scale drawing.

Warm-Up

As of 2024, there were 63 national parks in the United States and Utah has 5 of them. Marco is designing a poster with a scale drawing of Utah to promote Utah's national parks.

1. Describe the shape of Utah.

   Responses vary. Utah is a rectangular shape with a small rectangular portion missing from the top-right corner.

2. What information would you need in order to create a scale drawing of the state?

   Responses vary. I would need to know the length and width of Utah. I would also need to know the length and width of the rectangular portion that is "missing" from the top-right corner.

Unit 1 Lesson 8 56 Warm-Up

Pairs | 15 min

Activity 1 Calculating Scaled Distances

Purpose: Students determine a scale using the border distances of Utah and use that information to create a scale drawing.

1 Launch

Direct students' attention to the map of Utah with actual distances.

To surface connections, ask students what they know about Utah. Consider asking about the geography and what they notice about the state park images in Problem 4.

2 Monitor

To support students getting started, ask, "How can you determine the distances along the northern and eastern borders of the state?" Consider displaying an image of a compass to support thinking about cardinal directions.

Select and sequence methods that use the scale factor to determine every side and those that use the relationship between opposite sides in the scale drawing.

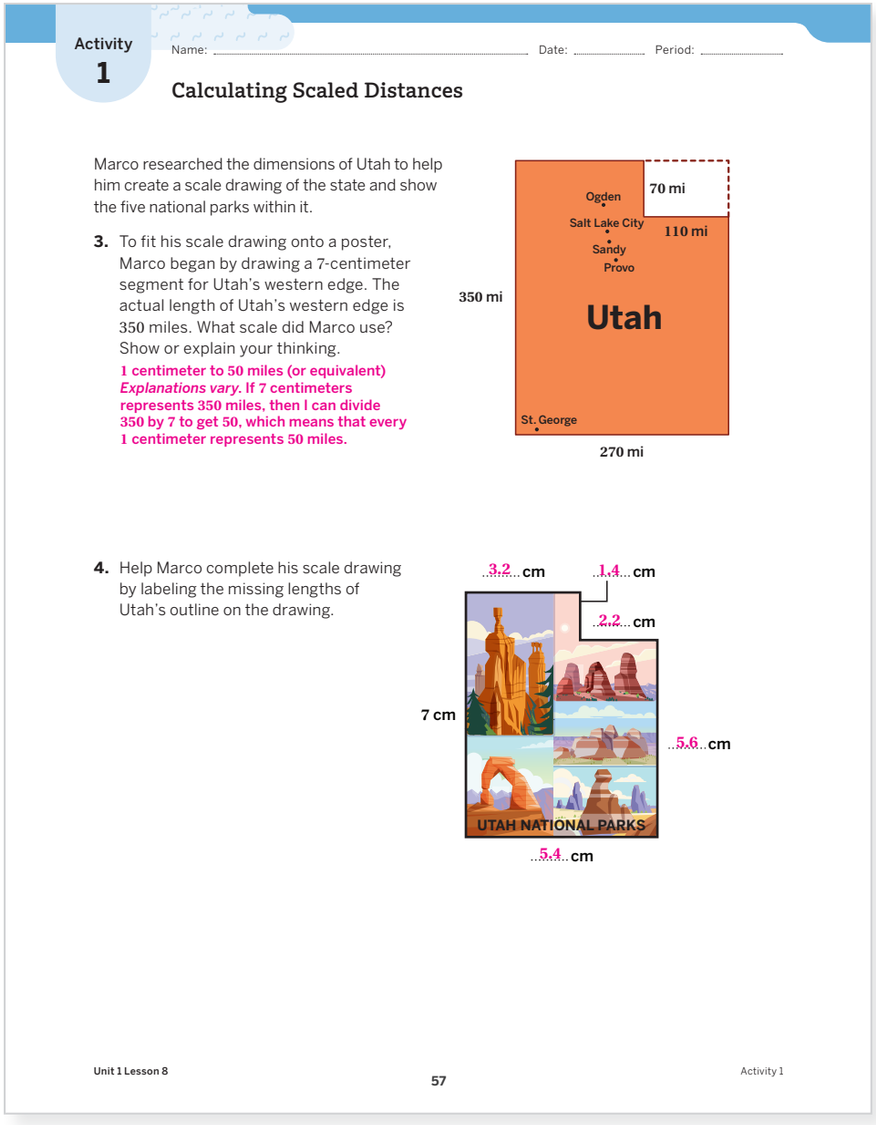
D Differentiation

Look for students who:	Respond to Student Thinking
Use 350 and 270 to represent the northern and eastern borders of the state.	**Support** Consider drawing the rectangle that shows the students what their thinking represents. Ask, "How does the state of Utah compare to this model?"
Use the scale of 7 to 350 for their calculations.	**Strengthen** Consider asking, "Can you reduce that scale factor for more efficient calculation?"
Multiply by the scale factor to determine the length of every side.	**Strengthen** Consider asking, "How can you use sides of the scale drawing that you have calculated to determine the opposite sides?"

3 Connect

Share strategies for determining the distances in the scale drawing. Invite students to share the thinking behind each strategy.

To surface the Key Takeaway, ask "What information did you use to determine the scale for Marco's scale drawing?"



Activity 1

Name: _____ Date: _____ Period: _____

Calculating Scaled Distances

Marco researched the dimensions of Utah to help him create a scale drawing of the state and show the five national parks within it.

3. To fit his scale drawing onto a poster, Marco began by drawing a 7-centimeter segment for Utah's western edge. The actual length of Utah's western edge is 350 miles. What scale did Marco use? Show or explain your thinking.

1 centimeter to 50 miles (or equivalent)
***Explanations vary.* If 7 centimeters represents 350 miles, then I can divide 350 by 7 to get 50, which means that every 1 centimeter represents 50 miles.**

4. Help Marco complete his scale drawing by labeling the missing lengths of Utah's outline on the drawing.

Unit 1 Lesson 8 57 Activity 1

Key Takeaway: To determine a scale, you must know both the actual lengths of an object and the desired length of at least one part of the scaled copy.

Small Groups | 15 min

Activity 2 Scaling Utah

Purpose: Students use different scales to create a precise scale drawing of Utah and notice how the scale used impacts their drawing.

Short on time? Consider omitting Problem 6.

1 Launch

Distribute the protractors and rulers, and ensure each student has access to blank paper.

Pause briefly to show how to use the tools to produce precise right angles.

2 Monitor

Encourage students in the same group to choose different scales.

To support students getting started, ask "What do you need to determine before beginning your scale drawing?"

Invite students to use their tools to create a precise scale drawing.

Look for students who organize their scale dimensions and devise strategies for precisely replicating angles in their drawings.

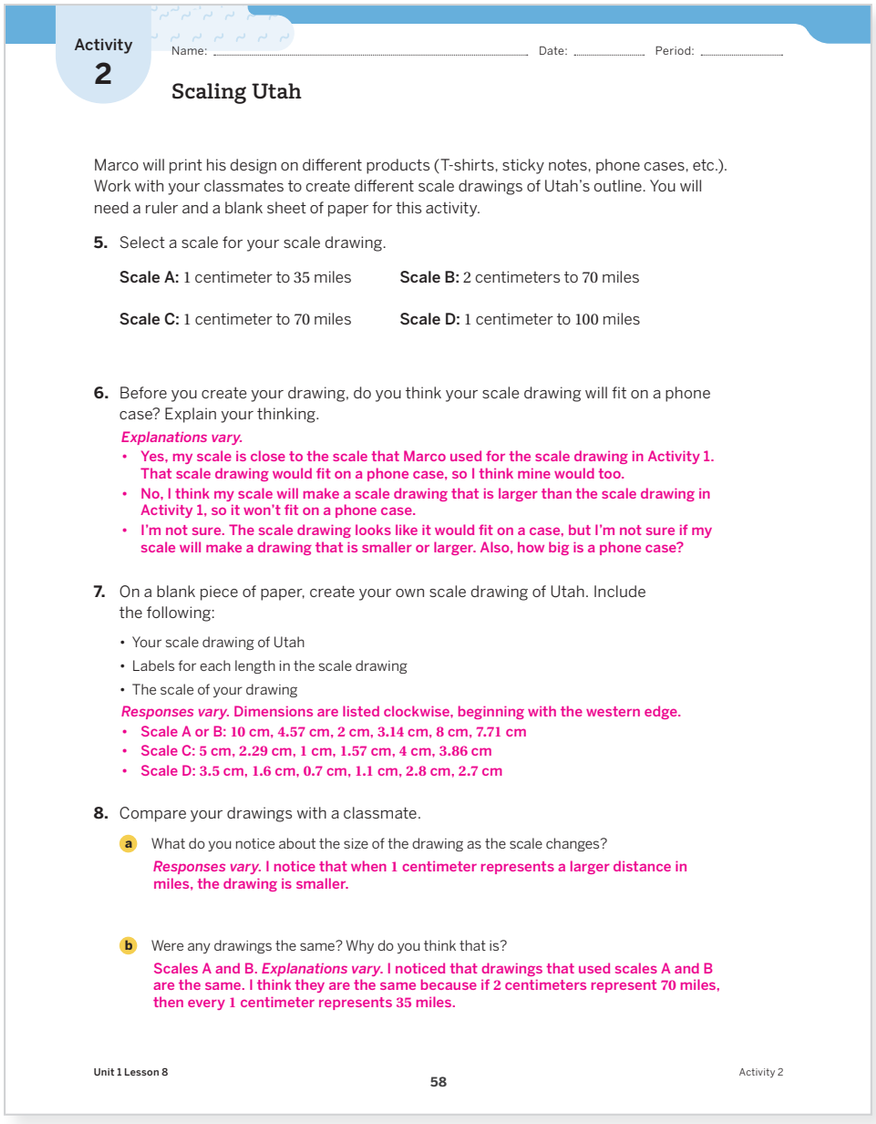
3 Connect

Invite students to share their scale drawings of Utah. Consider asking, "How do each of these scale drawings compare to the actual distances of Utah?" **(MTR.7.1)**

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each drawing, then identify ways in which the scales are different (compare) and alike (connect). **(MTR.4.1)**

Note: If it does not come up naturally, consider asking, "Why do scales A and B produce scale drawings of the same size?"

To surface the Key Takeaway, consider asking:
- "What stays the same in each scale drawing?"
- "What changes about the distances? The angles?"
- "Why is it important for some things about the scale drawings to stay the same?"



Activity 2

Name: _____ Date: _____ Period: _____

Scaling Utah

Marco will print his design on different products (T-shirts, sticky notes, phone cases, etc.). Work with your classmates to create different scale drawings of Utah's outline. You will need a ruler and a blank sheet of paper for this activity.

5. Select a scale for your scale drawing.

Scale A: 1 centimeter to 35 miles **Scale B:** 2 centimeters to 70 miles

Scale C: 1 centimeter to 70 miles **Scale D:** 1 centimeter to 100 miles

6. Before you create your drawing, do you think your scale drawing will fit on a phone case? Explain your thinking.

Explanations vary.
- Yes, my scale is close to the scale that Marco used for the scale drawing in Activity 1. That scale drawing would fit on a phone case, so I think mine would too.
- No, I think my scale will make a scale drawing that is larger than the scale drawing in Activity 1, so it won't fit on a phone case.
- I'm not sure. The scale drawing looks like it would fit on a case, but I'm not sure if my scale will make a drawing that is smaller or larger. Also, how big is a phone case?

7. On a blank piece of paper, create your own scale drawing of Utah. Include the following:
- Your scale drawing of Utah
- Labels for each length in the scale drawing
- The scale of your drawing

Responses vary. Dimensions are listed clockwise, beginning with the western edge.
- Scale A or B: 10 cm, 4.57 cm, 2 cm, 3.14 cm, 8 cm, 7.71 cm
- Scale C: 5 cm, 2.29 cm, 1 cm, 1.57 cm, 4 cm, 3.86 cm
- Scale D: 3.5 cm, 1.6 cm, 0.7 cm, 1.1 cm, 2.8 cm, 2.7 cm

8. Compare your drawings with a classmate.

a. What do you notice about the size of the drawing as the scale changes?

Responses vary. I notice that when 1 centimeter represents a larger distance in miles, the drawing is smaller.

b. Were any drawings the same? Why do you think that is?

Scales A and B. *Explanations vary.* I noticed that drawings that used scales A and B are the same. I think they are the same because if 2 centimeters represent 70 miles, then every 1 centimeter represents 35 miles.

Unit 1 Lesson 8 58 Activity 2

Key Takeaway: You can create scale drawings of the same object using different scales. When the scales are equivalent, the drawings are the same size.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how scale impacts scale drawings.

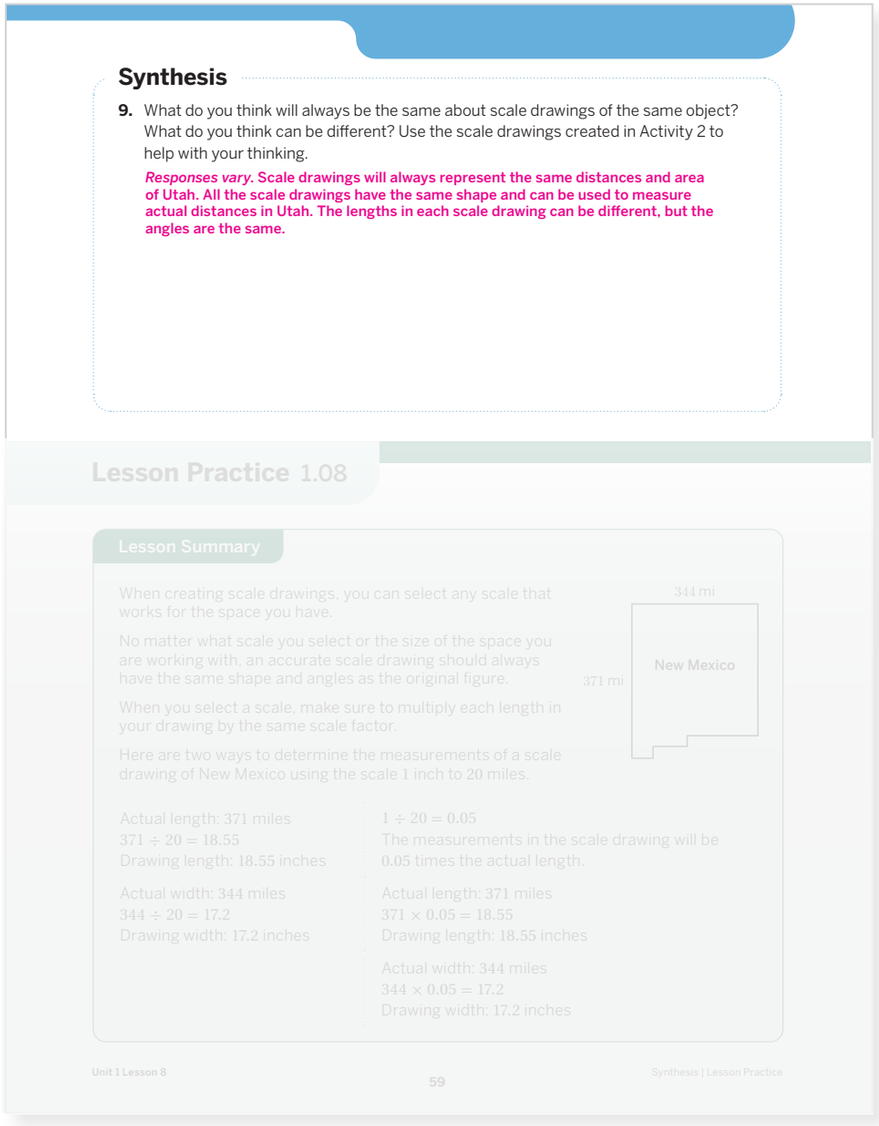
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Scale drawings represent the actual distances of an object.
- Scale drawings have the same shape and can be used to make sense of the relationship between actual distances.
- When scales are equivalent, their scale drawings will be the same size.

Invite students to make a prediction about which scale drawings will be the same and which will be the largest.

Note: If time allows, invite students to revise their scale drawings of Utah based on the discussion.



Synthesis

9. What do you think will always be the same about scale drawings of the same object? What do you think can be different? Use the scale drawings created in Activity 2 to help with your thinking.

Responses vary. Scale drawings will always represent the same distances and area of Utah. All the scale drawings have the same shape and can be used to measure actual distances in Utah. The lengths in each scale drawing can be different, but the angles are the same.

Lesson Practice 1.08

Lesson Summary

When creating scale drawings, you can select any scale that works for the space you have.

No matter what scale you select or the size of the space you are working with, an accurate scale drawing should always have the same shape and angles as the original figure.

When you select a scale, make sure to multiply each length in your drawing by the same scale factor.

Here are two ways to determine the measurements of a scale drawing of New Mexico using the scale 1 inch to 20 miles.

344 mi

371 mi

New Mexico

Actual length: 371 miles
$371 \div 20 = 18.55$
Drawing length: 18.55 inches

Actual width: 344 miles
$344 \div 20 = 17.2$
Drawing width: 17.2 inches

$1 \div 20 = 0.05$
The measurements in the scale drawing will be 0.05 times the actual length.

Actual length: 371 miles
$371 \times 0.05 = 18.55$
Drawing length: 18.55 inches

Actual width: 344 miles
$344 \times 0.05 = 17.2$
Drawing width: 17.2 inches

Unit 1 Lesson 8 59 Synthesis | Lesson Practice

Lesson Takeaway: When creating scale drawings, you multiply each side length by a scale factor. It is important to attend to precision when drawing the angles so they are the same as in the original figure. Any resulting scale drawing should have the same shape as the original.

Independent | 5 min

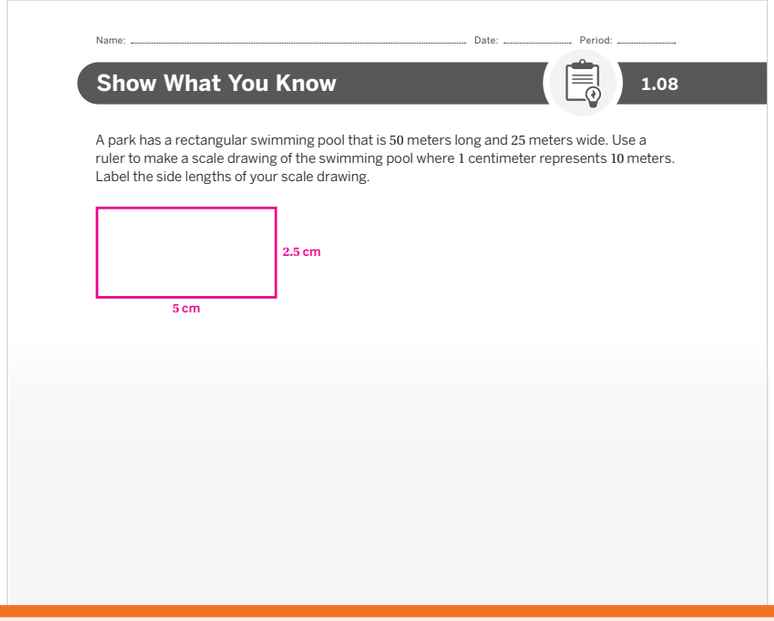
Show What You Know

Purpose: Students demonstrate their understanding of scale drawings by creating a precise scale drawing of a swimming pool.

Today's Goals

1. **Goal:** Create a scale drawing given the actual dimensions of the object and the scale.
2. **Goal:** Determine the scale given actual lengths and lengths on a scale drawing.
3. **Language Goal:** Compare and contrast strategies for determining lengths in scale drawings. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.08

A park has a rectangular swimming pool that is 50 meters long and 25 meters wide. Use a ruler to make a scale drawing of the swimming pool where 1 centimeter represents 10 meters. Label the side lengths of your scale drawing.

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to create a scale drawing given actual dimensions of an object and the scale:

» **Respond to Student Thinking:**

- Review Activity 1, Problems 3 and 4.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Think back to how you learned about scale drawings. How did that impact the way you planned for this lesson? How did it impact how you taught the lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What do you think will always be the same about scale drawings of the same object? What do you think can be different? Use the scale drawings created in Activity 2 to help with your thinking.

Responses vary. Scale drawings will always represent the same distances and area of Utah. All the scale drawings have the same shape and can be used to measure actual distances in Utah. The lengths in each scale drawing can be different, but the angles are the same.

Lesson Practice 1.08

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Actual length: 371 miles
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Actual length: 371 miles
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Drawing length: 18.55 inches

Actual width: 344 miles
$344 \times 0.05 = 17.2$
Drawing width: 17.2 inches

Lesson Practice 1.08

Name: _____ Date: _____ Period: _____

Problems 1–2: A book measures 6 inches wide and 9 inches tall. The publisher wants to display an image of the book on a billboard. The width of the book on the billboard is 36 inches.

1. How tall is the book on the billboard?

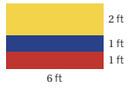
54 inches tall

2. What scale is used for the image on the billboard? Show or explain your thinking.

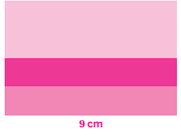
6 inches on the billboard for every 1 inch of the actual book. Sample shown in the table.

Actual	Scaled
6 in. wide	36 in. wide
1 in.	6 in.

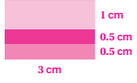
Problems 3–4: Here are the dimensions of the Colombian flag.



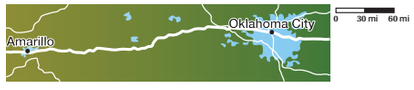
3. Create a scale drawing of the flag so that the top yellow stripe is 3 centimeters tall. Label the dimensions.



4. Create a scale drawing of the flag where 1 centimeter represents 2 feet.



5. This map shows parts of Texas and Oklahoma.



Approximately how far is it from Amarillo to Oklahoma City?

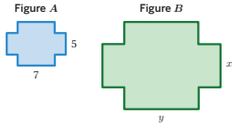
About 270 miles

Lesson Practice 1.08

Name: _____ Date: _____ Period: _____

FAST Practice

6. Figures A and B are scaled copies. The scale that takes figure A to figure B is 1 unit to $6\frac{3}{4}$ units.



What are the values of x and y?

$x =$ $33\frac{3}{4}$ (or equivalent)

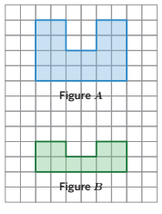
$y =$ $47\frac{1}{4}$ (or equivalent)

Spiral Review

7. Are figures A and B scaled copies? Explain your thinking.

No. *Explanations vary.*

- Even though the height of figure B is half the height of figure A, their widths are the same. That means the same scale factor wasn't used for each length.
- Not every side length is multiplied by the same scale factor. $4 \cdot \frac{1}{2} = 2$, but $6 \cdot 1 = 6$.



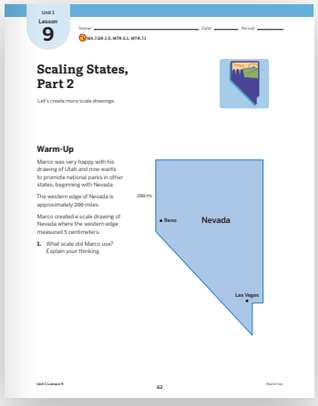
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.GR.1.5
FAST Practice	6	1	MA.7.GR.1.5
Spiral Review			
	7	2	MA.7.GR.1.5

Unit 1

Lesson

9



Scaling States, Part 2

Creating Scale Drawings

Let's create more scale drawings.

Learning Outcomes

Today's Goals

1. **Goal:** Create scale drawings of the same object using different scales.
2. **Goal:** Determine the scale given actual lengths and lengths on a scale drawing.
3. **Language Goal:** Analyze how different scales impact scale drawings of the same object. **(Reading, Writing, Speaking, and Listening)**

Students continue to use actual distances to calculate scaled distances and create scale drawings. They also further explore how choice of scale affects the size of their scale drawing.

Coherence

Prior Learning

In Lesson 8, students determined the scale of a scale drawing and created their own scale drawings.

Future Learning

In Lesson 10, students will extend their understanding of scale drawings to compare and recreate scale drawings on grids with different scales.

Connections

- Students use actual distances to calculate an appropriate scale as they create scale drawings on t-shirt designs. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.7.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.2.2	MA.912.GR.4.3

Lesson at a Glance

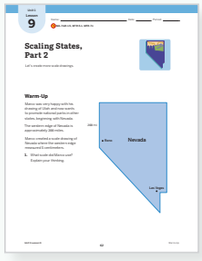
~ 45 min

MA.7.GR.1.5

Warm-Up

Independent | 5 min

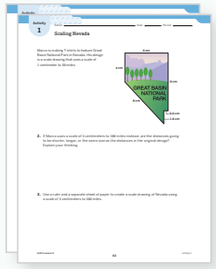
Students determine the scale used to create a scale drawing.



Activity 1

Pairs | 15 min

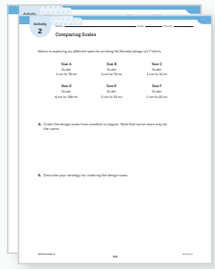
Students reproduce a scale drawing at a different scale to compare how different scales impact the size.



Activity 2

Small Groups | 15 min

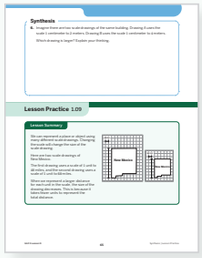
Students develop and implement strategies to compare the size of scale drawings by looking at the scale.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how scales impact the size of scale drawings.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by changing a scale and determining new lengths based on their scale.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators
- rulers

Independent | 5 min

Warm-Up

Purpose: Students determine the scale used to create a scale drawing.

1 Launch

Direct students' attention to the image of the state of Nevada. Consider reading the context with students before having them determine the scale.

Look for students who use division to determine the scale, both students who divide 200 by 5 and those who divide both of the numbers in the ratio 5 : 200 by 5.

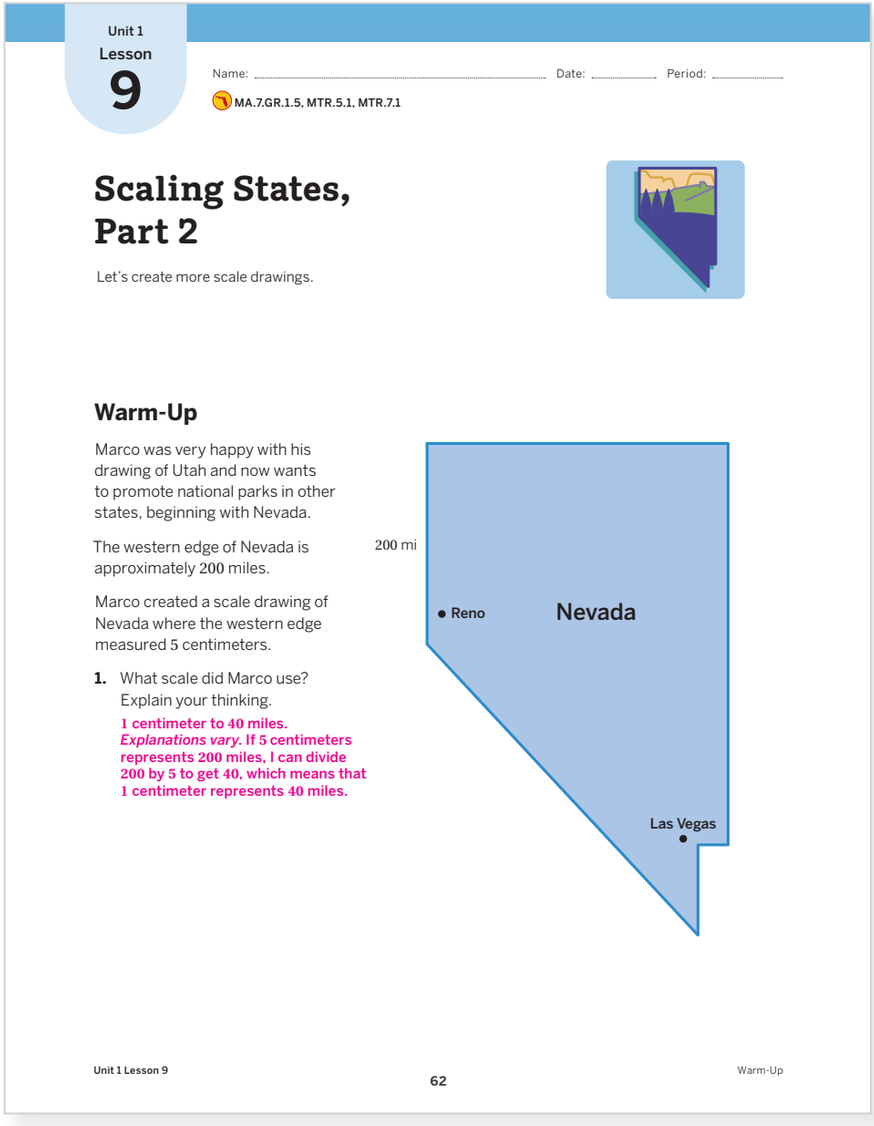
Consider asking students who finish early to determine the distance between Reno and Las Vegas using their scale.

2 Connect

Invite students to share their strategies for determining the scale.

Consider asking:

- "What information do you need to determine the scale used to create any scale drawing?" I need to know the actual distance and the distance represented in the scale drawing.
- "How do you know whether two scales are equivalent?" If two scales are equivalent ratios, then they produce equivalent scale drawings.



Unit 1
Lesson
9

Name: Date: Period:

MA.7.GR.1.5, MTR.5.1, MTR.7.1

Scaling States, Part 2

Let's create more scale drawings.

Warm-Up

Marco was very happy with his drawing of Utah and now wants to promote national parks in other states, beginning with Nevada.

The western edge of Nevada is approximately 200 miles.

Marco created a scale drawing of Nevada where the western edge measured 5 centimeters.

1. What scale did Marco use? Explain your thinking.

   1 centimeter to 40 miles. *Explanations vary.* If 5 centimeters represents 200 miles, I can divide 200 by 5 to get 40, which means that 1 centimeter represents 40 miles.

Unit 1 Lesson 9 62 Warm-Up

Pairs | 15 min

Activity 1 Scaling Nevada

Purpose: Students reproduce a scale drawing at a different scale to compare how different scales impact the size.

1 Launch

To generate excitement, consider letting students research Great Basin National Park. Consider asking, "Why would Marco want to create T-shirts for National Parks?"

Direct students' attention to Marco's design and consider having students share with their partners the distance of the northern border of Nevada to practice using the scale of his design.

Distribute the rulers and calculators. Ensure each student has access to blank paper.

2 Monitor

Select and sequence student strategies that convert one of the scales (either Marco's original or the new one in the problem) so they can be compared.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Ask, "How can the new scale be converted so you can compare it to Marco's?"
Calculate the actual distances of Nevada to multiply by the new scale.	**Stretch** Celebrate their strategy. Ask, "How could we use the relationship between the two scales to determine the new distances in our scale drawing?"
Use a scale of 1.5 to create their scale drawing of Nevada from Marco's scale.	**Stretch** Celebrate their thinking. Ask, "How would your strategy change if the new scale was 5 centimeters to 75 miles?"

Look for students who determine the actual distances of Nevada to use with the new scale and those who convert directly from Marco's scale drawing to their drawing. **(MTR.7.1)**

3 Connect

Invite students to share their strategies for determining the size of the new scale drawing.

To surface the Key Takeaway, consider asking:

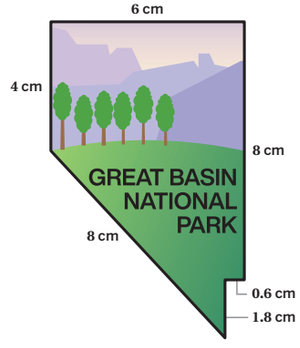
- "If we used the scale of 5 centimeters to 150 miles, would the new scale drawing be larger or smaller than Marco's original?"
- "What strategies can we use to compare how different scales make a drawing larger or smaller?"

Activity 1

Name: _____ Date: _____ Period: _____

Scaling Nevada

Marco is making T-shirts to feature Great Basin National Park in Nevada. His design is a scale drawing that uses a scale of 1 centimeter to 50 miles.



2. If Marco uses a scale of 3 centimeters to 100 miles instead, are the distances going to be shorter, longer, or the same size as the distances in the original design? Explain your thinking.

   Longer. *Explanations vary.* It will take 3 centimeters to represent 100 miles, which is more centimeters (or a longer distance) than it would take to represent 100 miles with the 1 centimeter to 50 miles scale.

3. Use a ruler and a separate sheet of paper to create a scale drawing of Nevada using a scale of 3 centimeters to 100 miles.

   Dimensions are listed clockwise, beginning with the western edge: 6 cm, 9 cm, 12 cm, 0.9 cm, 2.7 cm, 12 cm

Unit 1 Lesson 9 63 Activity 1

Key Takeaway: When we change the scale of a drawing so that one unit represents less actual distance, the new drawing is larger. This is because it takes more units to represent each actual length.

Small Groups | 15 min

Activity 2 Comparing Scales

Purpose: Students develop and implement strategies to compare the size of scale drawings by looking at the scale.

1 Launch

Direct students' attention to the six sizes. Invite students to make a prediction about which size they think will be the largest and share with their group.

Consider sharing that there are many strategies to ordering the sizes and each group should brainstorm about potential strategies before starting.

A **Accessibility: Memory and Attention** Consider providing a table for students to keep track of the value of the scales they have calculated and a way to organize them from smallest to largest.

2 Monitor

Look for students who follow the pattern of writing each scale as "1 cm to ___ miles" to easily compare. **(MTR.5.1)**

ELL **English Language Learners** Invite students to ensure that everyone in the group can explain how to convert various scales to a unit rate. Vary who is called on to represent the work of the group. **(Writing, Speaking, and Listening)**

Select and sequence explanations that include converting to a unit scale and those that make thoughtful comparisons between pairs of scales.

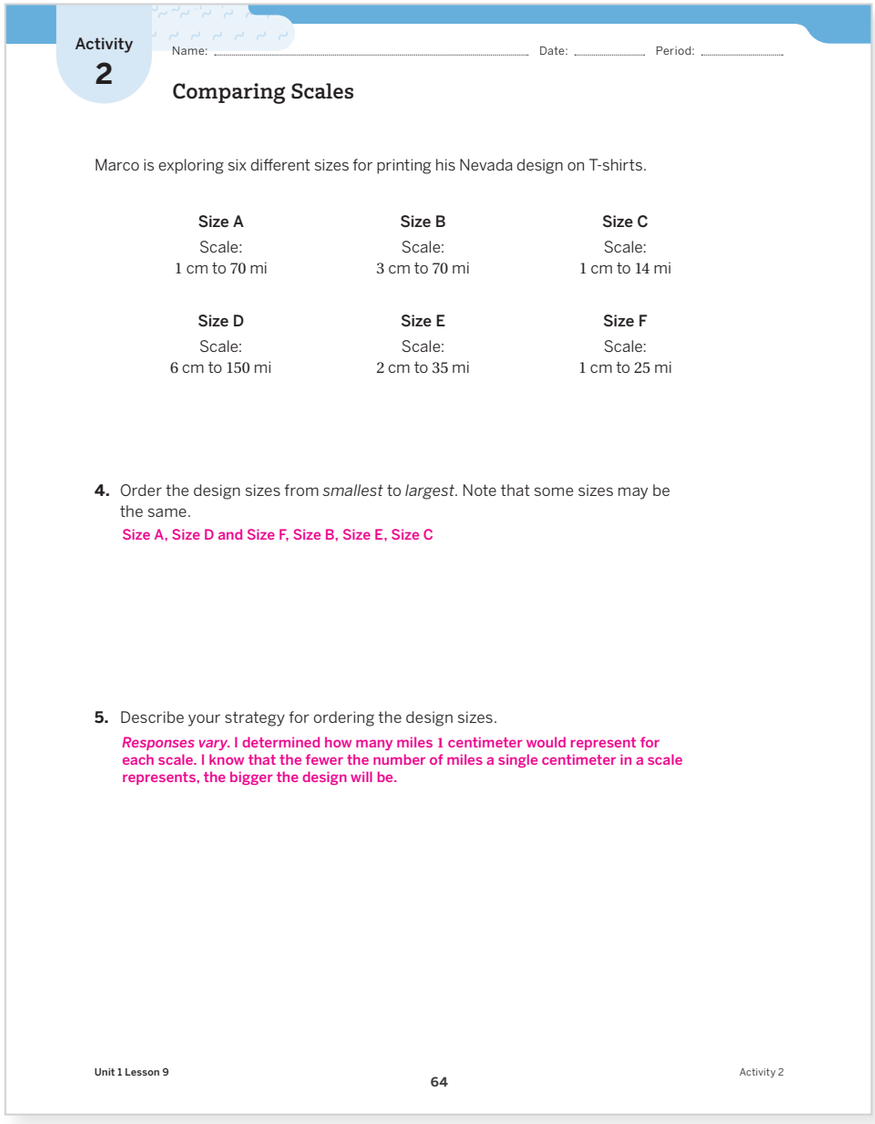
3 Connect

Share and discuss strategies. Invite students to share how the strategies show which scales are the largest.

Encourage students to reflect on their predictions by asking, "What evidence do we have now that helped solidify our prediction or convinced us to change our thinking?"

To surface the Key Takeaway, consider asking:

- "What were some things that each of our strategies had in common for the ordering?"
- "Which two scales produce the same size design? How do you know?"



Activity 2

Name: Date: Period:

Comparing Scales

Marco is exploring six different sizes for printing his Nevada design on T-shirts.

Size A	Size B	Size C
Scale: 1 cm to 70 mi	Scale: 3 cm to 70 mi	Scale: 1 cm to 14 mi
Size D	**Size E**	**Size F**
Scale: 6 cm to 150 mi	Scale: 2 cm to 35 mi	Scale: 1 cm to 25 mi

4. Order the design sizes from *smallest* to *largest*. Note that some sizes may be the same.

Size A, Size D and Size F, Size B, Size E, Size C

5. Describe your strategy for ordering the design sizes.

Responses vary. I determined how many miles 1 centimeter would represent for each scale. I know that the fewer the number of miles a single centimeter in a scale represents, the bigger the design will be.

Unit 1 Lesson 9 64 Activity 2

Key Takeaway: When comparing scales, it is helpful to convert all of them to share a common value.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how scales impact the size of scale drawings.

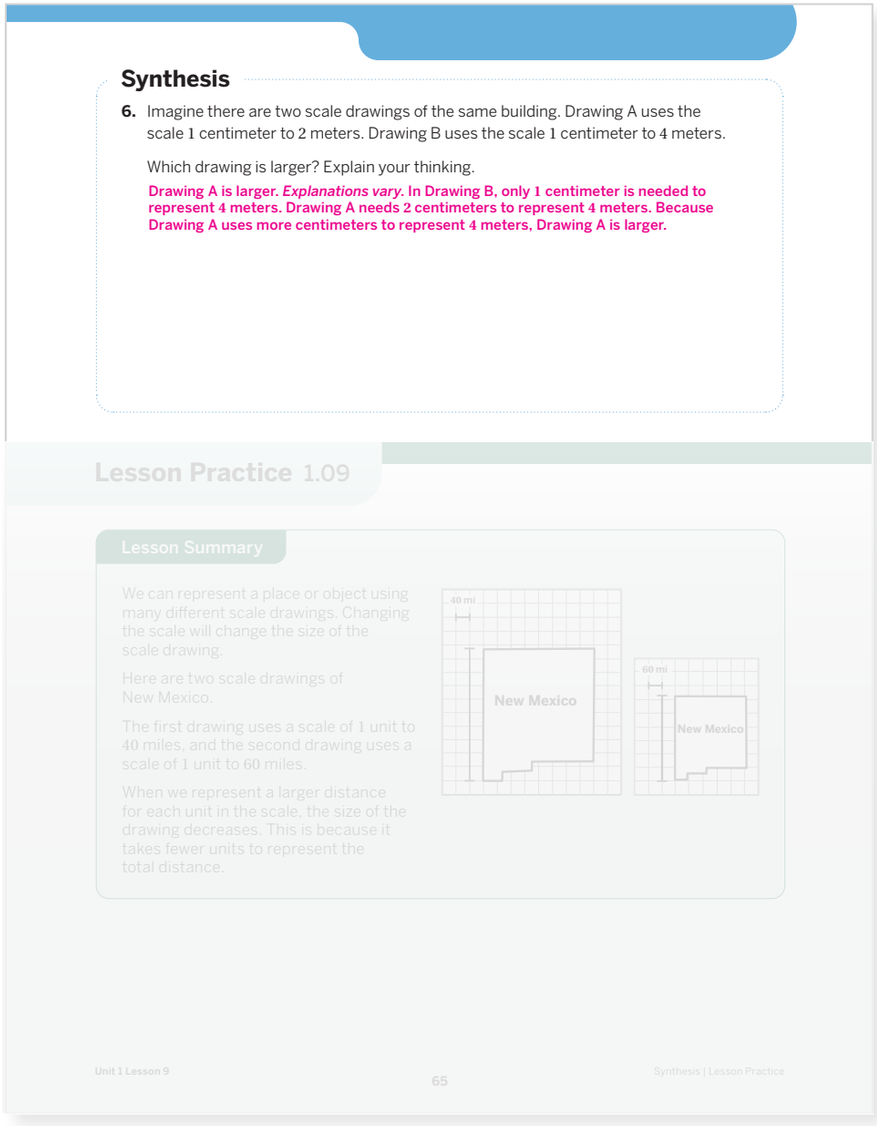
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Drawing A needs more centimeters to represent the same actual distance so that drawing would be larger.
- It would take 2 centimeters to represent 4 meters in Drawing A so it would be larger.

Invite students to share their thinking for determining which scale is larger.

Create an anchor chart that highlights one of the strategies for comparing scales. Consider naming the strategy after a student who developed it during class.



Synthesis

6. Imagine there are two scale drawings of the same building. Drawing A uses the scale 1 centimeter to 2 meters. Drawing B uses the scale 1 centimeter to 4 meters.

Which drawing is larger? Explain your thinking.

Drawing A is larger. *Explanations vary.* In Drawing B, only 1 centimeter is needed to represent 4 meters. Drawing A needs 2 centimeters to represent 4 meters. Because Drawing A uses more centimeters to represent 4 meters, Drawing A is larger.

Lesson Practice 1.09

Lesson Summary

We can represent a place or object using many different scale drawings. Changing the scale will change the size of the scale drawing.

Here are two scale drawings of New Mexico.

The first drawing uses a scale of 1 unit to 40 miles, and the second drawing uses a scale of 1 unit to 60 miles.

When we represent a larger distance for each unit in the scale, the size of the drawing decreases. This is because it takes fewer units to represent the total distance.

Unit 1 Lesson 9 65 Synthesis | Lesson Practice

Lesson Takeaway: As the distance represented by a single unit in a scale gets smaller, the scale drawing created using that scale gets larger.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by changing a scale and determining new lengths based on their scale.

Today's Goals

1. **Goal:** Create scale drawings of the same object using different scales.
2. **Goal:** Determine the scale given actual lengths and lengths on a scale drawing.
3. **Language Goal:** Analyze how different scales impact scale drawings of the same object. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.09

Aaliyah is making a scale drawing of a rectangular swimming pool. The pool is 50 meters long and 25 meters wide. She originally used a scale of 1 centimeter to 10 meters for her drawing of the pool.

a. Aaliyah needs her drawing to be larger. Suggest a new scale for Aaliyah.
Responses vary. 1 centimeter to 5 meters

b. Determine the side lengths of the scale drawing using the new scale.
Responses vary. If she uses a scale of 1 centimeter to 5 meters, the scale drawing will be 10 centimeters long and 5 centimeters wide.

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to choose a scale that would result in a larger scale drawing:

» **Respond to Student Thinking:**

- Review Activity 2, Problem 4.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how students' understanding of the words *scale*, *scale factor*, *scaled drawing*, and *scaled copy* have developed in this unit. How has the definition of scaled copy from Lesson 1 expanded throughout the unit? How are students using these words to talk about scale drawings?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. Imagine there are two scale drawings of the same building. Drawing A uses the scale 1 centimeter to 2 meters. Drawing B uses the scale 1 centimeter to 4 meters.

Which drawing is larger? Explain your thinking.

Drawing A is larger. *Explanations vary.* In Drawing B, only 1 centimeter is needed to represent 4 meters. Drawing A needs 2 centimeters to represent 4 meters. Because Drawing A uses more centimeters to represent 4 meters, Drawing A is larger.

Lesson Practice 1.09

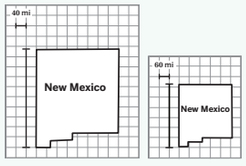
Lesson Summary

We can represent a place or object using many different scale drawings. Changing the scale will change the size of the scale drawing.

Here are two scale drawings of New Mexico.

The first drawing uses a scale of 1 unit to 40 miles, and the second drawing uses a scale of 1 unit to 60 miles.

When we represent a larger distance for each unit in the scale, the size of the drawing decreases. This is because it takes fewer units to represent the total distance.



Lesson Practice 1.09

Name: _____ Date: _____ Period: _____

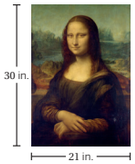
1. Tyler made a scale model of Earth in science class. The scale that maps the scale model to the actual Earth is 1 inch to 1,070 miles. If the height in the scale model is 7.4 inches, what is the actual height of Earth?

7,918 miles



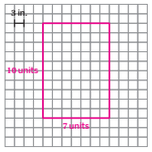
max dallocco/Sutterstock.com

Problems 2–4: Leonardo Da Vinci's famous painting, the *Mona Lisa*, measures 21 inches by 30 inches.

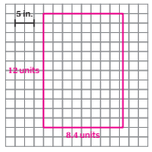


Public Domain

2. Create a scale drawing of the outline of the *Mona Lisa* where 1 unit represents 3 inches.



3. Create a scale drawing of the outline of the *Mona Lisa* where 2 units represents 5 inches.



4. Imagine a new scale drawing of the *Mona Lisa* where 3 units represent 1 inch. Is this drawing smaller, larger, or equal in size compared to your previous drawings? Explain your thinking.

Larger. *Explanations vary.*
- The new drawing would be 90 units tall because $3 \cdot 30 = 90$, which is taller than either of the drawings.
- In the first drawing, 3 inches is represented by 1 unit. In the new drawing, 3 inches would be represented by 9 units.

5. The floorplan of a restaurant shows a scale of 1 inch to 12 feet. The floorplan shows the area of the restaurant as 60 square inches. Hoang says the actual area of the restaurant is 720 square feet. Is Hoang correct? Explain your thinking.

No. *Explanations vary.* Each dimension is multiplied by 12, making the area $12^2 = 144$ times greater. The area of the actual restaurant is $60 \cdot 144 = 8640$, or 8,640 square feet.

Lesson Practice 1.09

Name: _____ Date: _____ Period: _____

FAST Practice

6. James created a blueprint for his high school basketball court. The court is 4.2 inches long and 2.5 inches wide on the blueprint. The actual dimensions of the court are 84 feet long and 50 feet wide. What scale did James use?

1 inch to [20] feet

Spiral Review

Problems 7–8: Circle the phrase that makes each statement true.

7. The value of the expression $15 \cdot 0.75$ is . . .

(Less than 15) Greater than 15

8. The value of the expression $15 \cdot 1.3$ is . . .

Less than 15 (Greater than 15)

Problems 9–12: Determine the missing value in each equation.

9. $32 \cdot \boxed{\frac{1}{8}} = 4$

10. $24 \cdot \boxed{\frac{1}{8}} = 3$

11. $16 \cdot \boxed{\frac{1}{8}} = 2$

12. $8 \cdot \boxed{\frac{1}{8}} = 1$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.GR.1.5
	2–5	2	MA.7.GR.1.5
FAST Practice	6	2	MA.7.GR.1.5
Spiral Review			
	7–8	1	MA.5.FR.2.3
	9–12	2	MA.4.FR.2.4

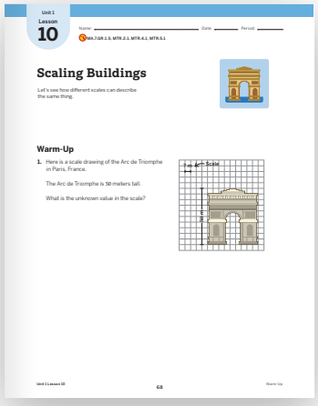
Unit 1

Lesson 10

Scaling Buildings

Same Object, Different Scales

Let's see how different scales can describe the same thing.


Unit 1
Lesson 10
Name _____ Date _____ Period _____
MA.7.GR.1.5, MTR.2.1, MTR.4.1, MTR.5.1

Scaling Buildings

Let's see how different scales can describe the same thing.

Warm-Up

1. Here is a scale drawing of the Arc de Triomphe in Paris, France.

   The Arc de Triomphe is 50 meters tall.

   What is the unknown value in the scale?

1 cm : ? m

68

Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Calculate distances on a scale drawing, given a drawing of the same object with a different scale.
2. **Goal:** Determine the scale of a scale drawing.
3. **Language Goal:** Explain how different scales produce scale drawings that are smaller, the same size, or larger than the original. **(Reading and Writing)**

Students reason about multiple scale drawings of the same object using different scales. Students use precision to determine the scale of a drawing and calculate specific distances on one drawing based on another. They compare scales, look for and recognize patterns as the scale drawings change size, and justify whether two different scales are equivalent. **(MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In Lessons 1–5, students used scale factors to create and compare scaled copies. In Lessons 6–9, students represented distances in the world using scales with units and scale drawings.

Future Learning

In Grade 8, Unit 2, students will extend their understanding of scale factor to perform dilations.

Connections

- Students apply precision to determine the scale used in a drawing of real-world landmarks and reproduce them using a different scale. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5

Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	**MA.8.GR.2.2** **MA.912.GR.4.3**

Lesson at a Glance

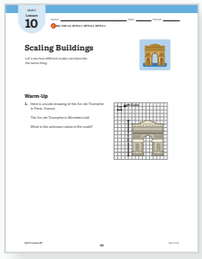
~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

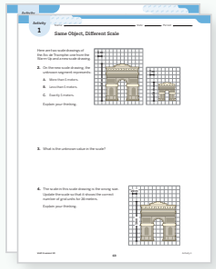
Students share their intuitions about how changing the scale affects a scale drawing.



Activity 1

Pairs | 20 min

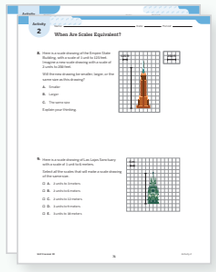
Students reproduce scale drawings at different scales and reason about how a drawing will change when the scale changes.



Activity 2

Pairs | 10 min

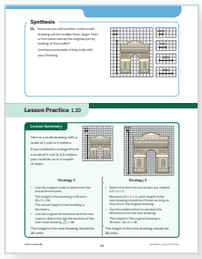
Students explore equivalent ways of representing a scale.



Synthesis

Whole Class | 5 min

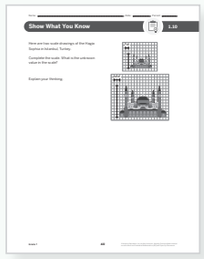
Students synthesize their understanding of how different scales change the size of a scale drawing. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by comparing two scale drawings and using the scale of one to determine the scale of the other.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- graph paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students share their intuitions about how changing the scale affects a scale drawing.

1 Launch

Invite students to share what they know about the Arc de Triomphe. Consider showing pictures of the Arc to build context for students about the actual size.

Look for students using the grid to make sense of the scale.

A **Accessibility: Visual-Spatial Processing**
Throughout this lesson, invite students to sketch their thinking on graph paper to increase access to the task.

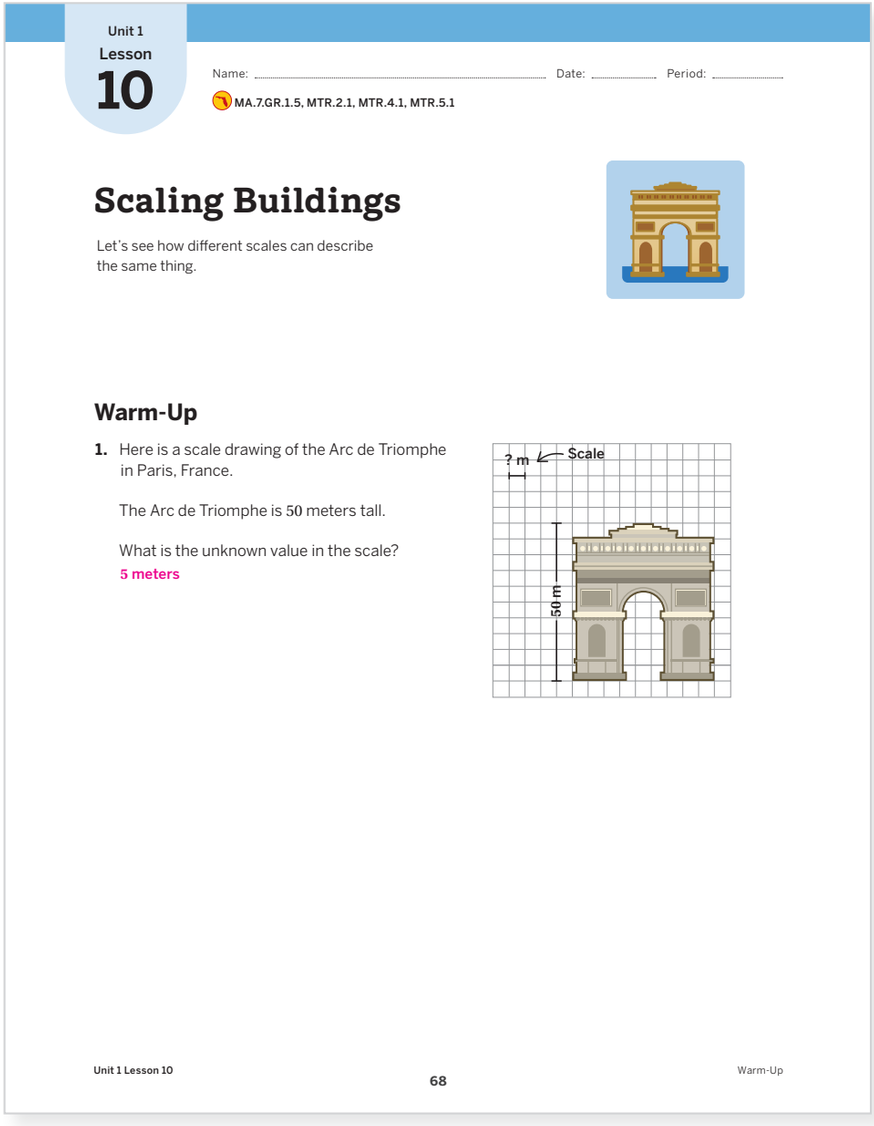
2 Connect

Invite students to share how they determined the scale. Record different strategies so they can be referenced for support.

Consider asking:

- "Why is it helpful to create scale drawings of tall buildings?"
- "What else has changed in the scale drawing of the Arc de Triomphe? What stayed the same?"

Note: Throughout the lesson, students may have questions about where the measurements for the buildings stop. Have students use the line segment that represents the height of the building in these cases.



Unit 1
Lesson
10

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.2.1, MTR.4.1, MTR.5.1

Scaling Buildings

Let's see how different scales can describe the same thing.

Warm-Up

1. Here is a scale drawing of the Arc de Triomphe in Paris, France.

   The Arc de Triomphe is 50 meters tall.

   What is the unknown value in the scale?

   5 meters

Unit 1 Lesson 10 68 Warm-Up

Pairs | 20 min

Activity 1 Same Object, Different Scale

Purpose: Students reproduce scale drawings at different scales and reason about how a drawing will change when the scale changes.

Short on time? Consider having students discuss their thinking in Problem 4 with a partner instead of writing a response.

1 Launch

Invite students to share their reasoning with their partner and work together to reach an agreement. Call attention to any conflict or consensus you see.

- If there is consensus, consider highlighting one response.
- If there is conflict, consider selecting and sequencing responses for each choice. Share the responses and invite those students to share their reasoning. **(MTR.4.1)**

Note: It's okay — even desirable — to lack consensus at this stage. The activity will build toward consensus in Problem 3 when students determine the actual scale.

2 Monitor

To support students getting started, consider asking, "What strategy from the Warm-up can you use to determine the scale?"

Look for students who notice that the new scale drawing has a 50-meter segment that is divided into fewer grid units, so each grid unit must represent a larger actual distance. **(MTR.5.1)**

Look for responses that include:

- Noticing that each grid unit represents 5 meters, so it will take 4 grid units to represent 20 meters.
- Discussing how many times larger 50 meters is than 20 meters and how that relates to the segment that represents the scale.

D **Differentiation:** (Support) Consider providing students with graph paper so they can sketch out their strategies.

Consider pausing to invite students to share their strategies for fixing the scale.

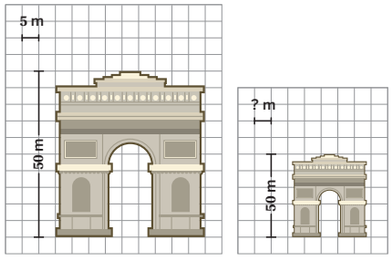
Activity 1 continued >

Activity 1

Name: Date: Period:

Same Object, Different Scale

Here are two scale drawings of the Arc de Triomphe: one from the Warm-Up and a new scale drawing.



2. On the new scale drawing, the unknown segment represents:

A. More than 5 meters.

B. Less than 5 meters.

C. Exactly 5 meters.

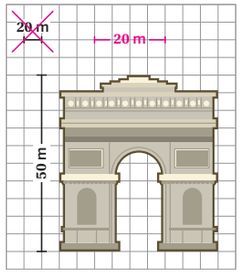
Explain your thinking.

Explanations vary. The new drawing is smaller, so each segment needs to represent a greater length to represent the same total height.

3. What is the unknown value in the scale?

10 meters

4. The scale in this scale drawing is the wrong size. Update the scale so that it shows the correct number of grid units for 20 meters.



Explain your thinking.

Explanations vary.

- The building is 50 meters tall. On the grid, this is represented by 10 units, so each unit must represent 5 meters. That means 20 meters must represent 4 units because $5 \cdot 4 = 20$.
- $20 \cdot 2.5 = 50$ and $4 \cdot 2.5 = 10$, so the 20-meter scale must be 4 units wide.

Unit 1 Lesson 10 69 Activity 1

Pairs | 20 min

Activity 1 Same Object, Different Scale (continued)

Purpose: Students reproduce scale drawings at different scales and reason about how a drawing will change when the scale changes.

Short on time? Consider having students discuss their thinking in Problem 4 with a partner instead of writing a response.

2 Monitor

To generate excitement, tell students that they will be analyzing scale drawings of buildings from around the world.

Encourage students to read the location of each building and share with their partner any personal connections they might have to the buildings or places.

Invite students to work on Problems 5–7 with a partner.

D Differentiation

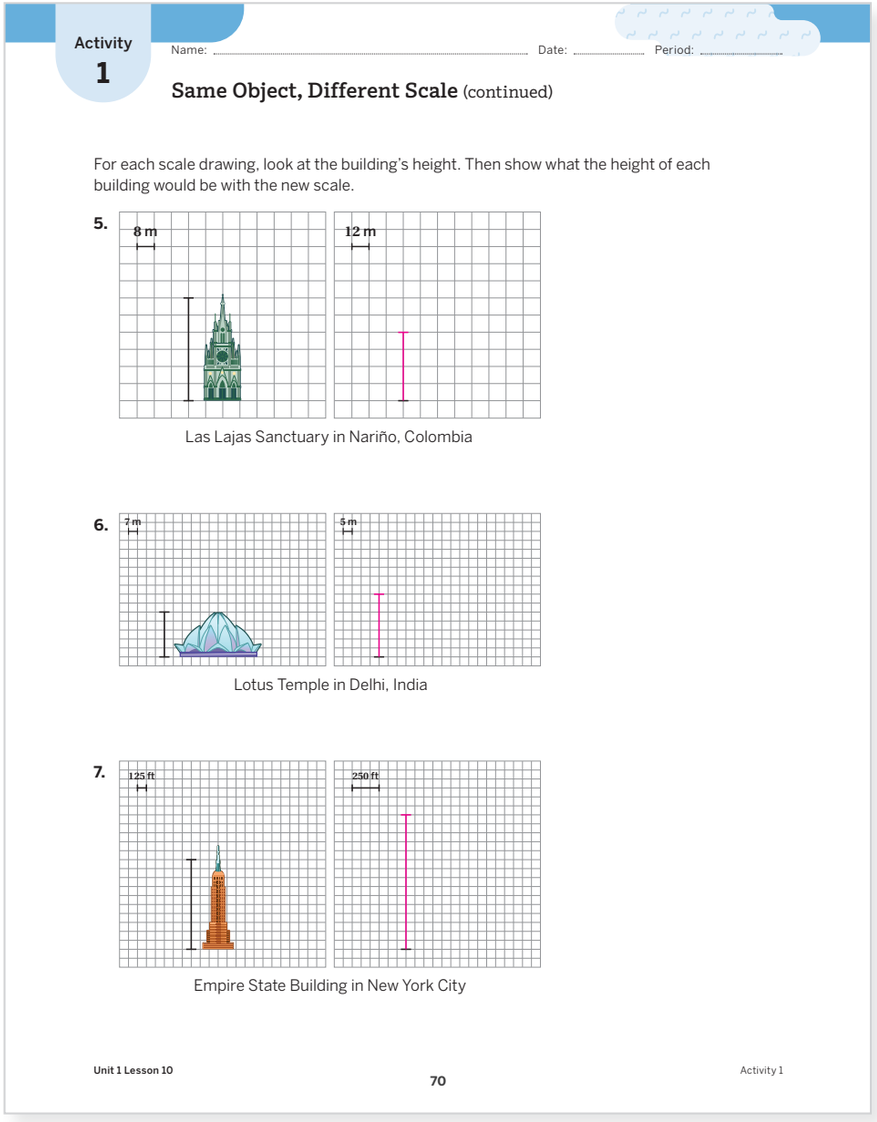
Look for students who:	Respond to Student Thinking
Assume Las Lajas will be 1.5 times as tall because $8 \cdot 1.5$ is 12.	**Support** Consider asking students to calculate the height of Las Lajas in both of their drawings.
Try to guess the height of the scale drawing without reasoning about the actual size of the building or the relationship between scales.	**Strengthen** Ask, "What strategy could you use to determine the actual height?
Don't attend to the difference in grid units on the scale for the Empire State Building.	**Support** Invite students to count the grid units that are represented by each scale.
Finish early.	**Stretch** Encourage students to create their own challenge on a piece of graph paper by drawing their own building, a scale, and a second scale.

3 Connect

Consider asking students to discuss their strategies for determining the size of the new scale models. **(MTR.2.1)**

MLR MLR1: Stronger and Clearer Each Time (optional) Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider asking, "How do we know that the new scale drawing of the Lotus Temple (on the right) will be larger without doing any calculations?" **(MTR.5.1)**



Activity 1

Name: Date: Period:

Same Object, Different Scale (continued)

For each scale drawing, look at the building's height. Then show what the height of each building would be with the new scale.

5.

Las Lajas Sanctuary in Nariño, Colombia

6.

Lotus Temple in Delhi, India

7.

Empire State Building in New York City

Unit 1 Lesson 10 70 Activity 1

Key Takeaway: Comparing scales to one another helps us determine the relative sizes of scale drawings.

Pairs | 10 min

Activity 2 When are Scales Equivalent?

Purpose: Students explore equivalent ways of representing a scale.

1 Launch

Invite students to make a prediction and discuss their thinking with a partner.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., The new drawing will be _____ because _____.).

2 Monitor

Collect and share strategies that explain the height will be the same because they are equivalent scales.

D Differentiation

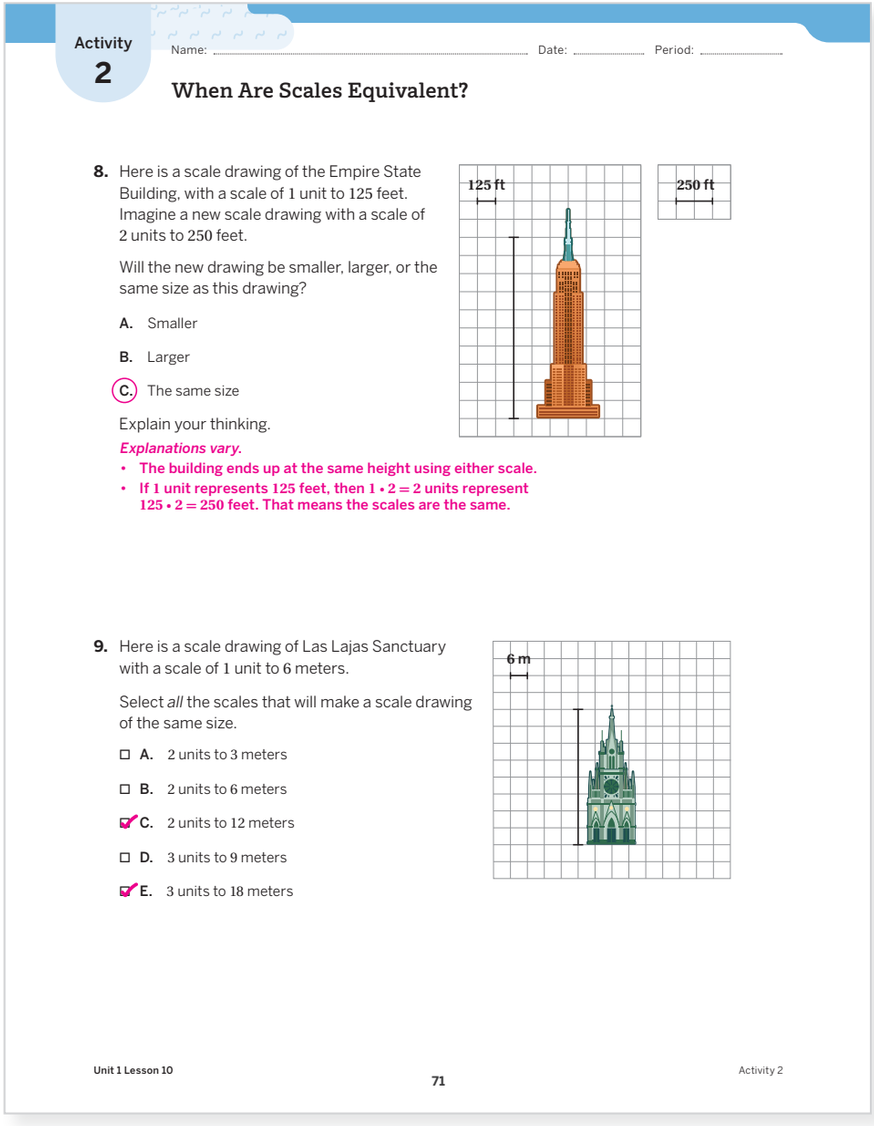
Look for students who:	Respond to Student Thinking
Select "Smaller" or "Larger."	**Support** Consider asking, "What does one unit on the grid represent in both scales?"
Select "The same size," but struggle to justify their choice.	**Support** Consider inviting them to determine the height of the building with both scales. Consider asking, "What is similar or different about the scales?"
Finish early.	**Stretch** Invite them to think of why someone might have selected a different answer and to create a convincing counterargument.

Invite students to discuss strategies for determining whether scales are equivalent.

Invite students to share their strategies for determining equivalent scales. Consider asking:

- "Why might someone say that 2 units to 3 meters is the same scale as this drawing?"
- "How would you explain how to determine equivalent scales to someone who missed class today?"

Activity 2 continued >



Activity 2

Name: Date: Period:

When Are Scales Equivalent?

8. Here is a scale drawing of the Empire State Building, with a scale of 1 unit to 125 feet. Imagine a new scale drawing with a scale of 2 units to 250 feet.

Will the new drawing be smaller, larger, or the same size as this drawing?

A. Smaller

B. Larger

C. The same size

Explain your thinking.

Explanations vary.

- The building ends up at the same height using either scale.
- If 1 unit represents 125 feet, then $1 \cdot 2 = 2$ units represent $125 \cdot 2 = 250$ feet. That means the scales are the same.

125 ft

250 ft

9. Here is a scale drawing of Las Lajas Sanctuary with a scale of 1 unit to 6 meters.

Select *all* the scales that will make a scale drawing of the same size.

- ☐ A. 2 units to 3 meters
- ☐ B. 2 units to 6 meters
- ☑ C. 2 units to 12 meters
- ☐ D. 3 units to 9 meters
- ☑ E. 3 units to 18 meters

6 m

Unit 1 Lesson 10 71 Activity 2

Pairs | 10 min

Activity 2 When are Scales Equivalent? (continued)

Purpose: Students explore equivalent ways of representing a scale.

2 Monitor

Look for matches of equivalent scales. Consider selecting some incomplete matches to finish as a class.

D Differentiation

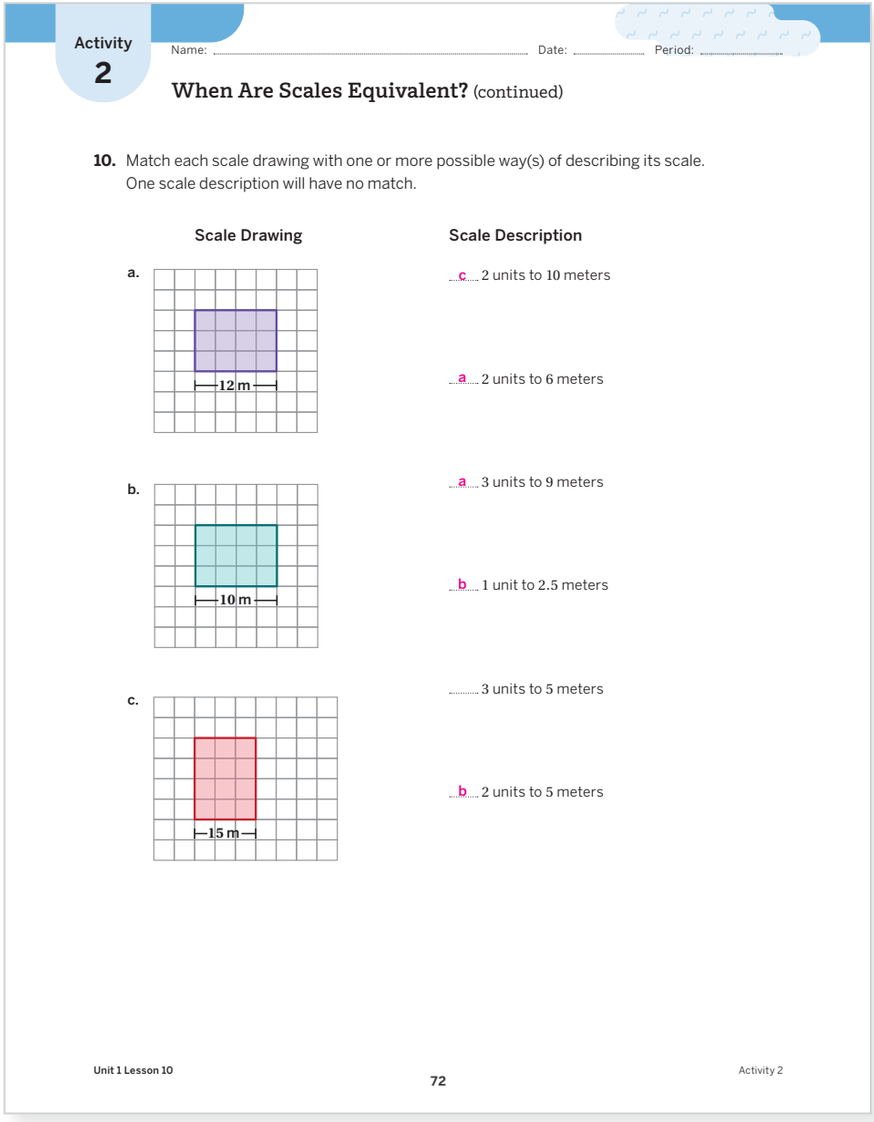
Look for students who:	Respond to Student Thinking
Need support getting started with the drawings.	**Support** Consider asking, "What would one unit represent?"
Multiply the values in the scales to match with the lengths of sides (e.g., matching the scale 3 units to 5 meters to the rectangle with side length 15 meters).	**Support** Consider asking them to explain what the phrase "3 units to 5 meters" means in their own words.
Finish early.	**Stretch** Invite them to create a drawing for the leftover scale description or think of another scale for each drawing.

3 Connect

Invite students to explain how they matched the scale drawings with the scale descriptions.

To surface the Key Takeaway, consider asking:

- "What is an equivalent scale to 3 units to 5 meters?"
- "If we made a scale drawing of a building with each of the scales we just created, what would the drawings have in common?"



Activity 2

Name: Date: Period:

When Are Scales Equivalent? (continued)

10. Match each scale drawing with one or more possible way(s) of describing its scale. One scale description will have no match.

Scale Drawing	Scale Description
a.	c 2 units to 10 meters
	a 2 units to 6 meters
b.	a 3 units to 9 meters
	b 1 unit to 2.5 meters
c.	3 units to 5 meters
	b 2 units to 5 meters

Unit 1 Lesson 10 72 Activity 2

Key Takeaway: For scale drawings to be the same size, they need to be created by equivalent scales.

Whole Class | 5 min

Synthesis

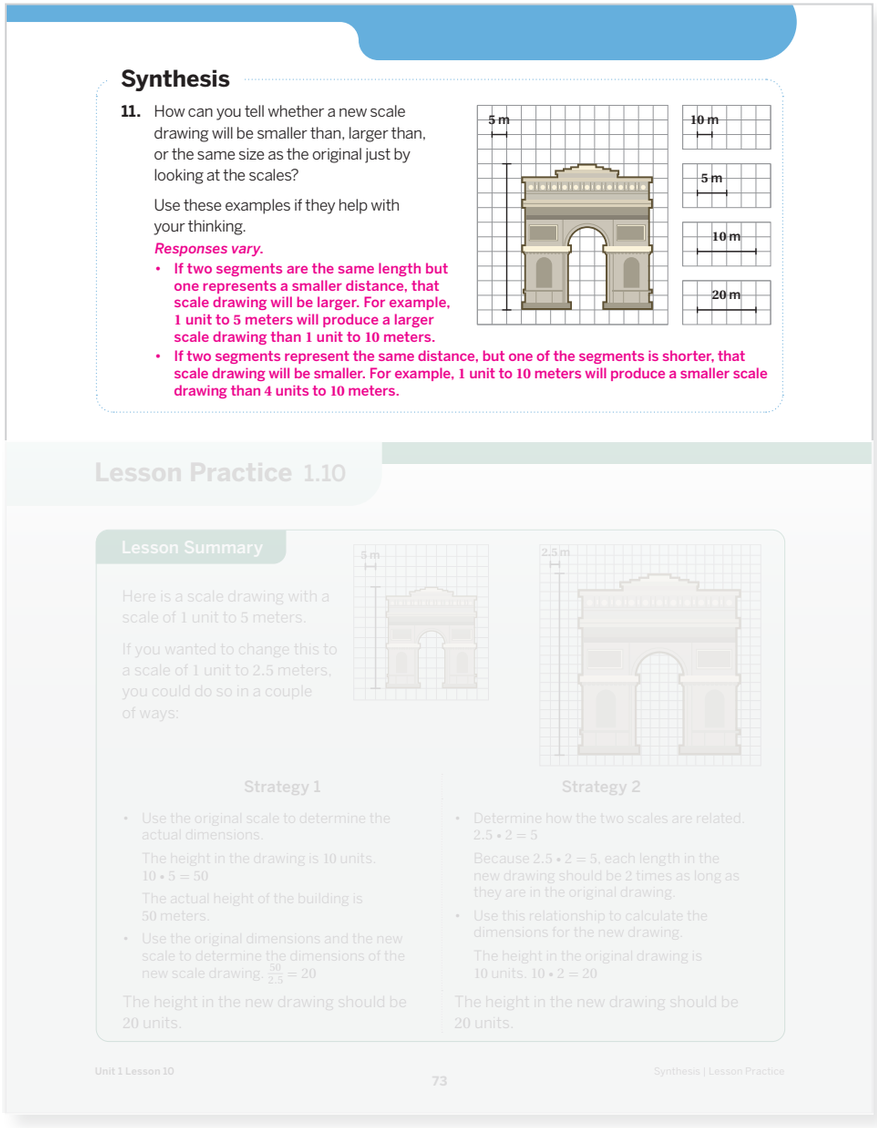
Purpose: Students synthesize their understanding of how different scales change the size of a scale drawing.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Scale drawings that use the same (or equivalent) scales will be the same size.
- If two segments are the same length but one represents a smaller distance, that scale drawing will be larger. For example, 1 unit to 5 meters will produce a larger scale drawing than 1 unit to 10 meters.
- If two segments represent the same distance, but one of the segments is shorter, that scale drawing will be smaller. For example, 1 unit to 10 meters will produce a smaller scale drawing than 4 units to 10 meters.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display for students to reference and add to for the rest of the unit.



Synthesis

11. How can you tell whether a new scale drawing will be smaller than, larger than, or the same size as the original just by looking at the scales?

Use these examples if they help with your thinking.

Responses vary.

- If two segments are the same length but one represents a smaller distance, that scale drawing will be larger. For example, 1 unit to 5 meters will produce a larger scale drawing than 1 unit to 10 meters.
- If two segments represent the same distance, but one of the segments is shorter, that scale drawing will be smaller. For example, 1 unit to 10 meters will produce a smaller scale drawing than 4 units to 10 meters.

Lesson Practice 1.10

Lesson Summary

Here is a scale drawing with a scale of 1 unit to 5 meters.

If you wanted to change this to a scale of 1 unit to 2.5 meters, you could do so in a couple of ways:

Strategy 1

- Use the original scale to determine the actual dimensions.

  The height in the drawing is 10 units. $10 \cdot 5 = 50$

  The actual height of the building is 50 meters.
- Use the original dimensions and the new scale to determine the dimensions of the new scale drawing. $\frac{50}{2.5} = 20$

The height in the new drawing should be 20 units.

Strategy 2

- Determine how the two scales are related. $2.5 \cdot 2 = 5$

  Because $2.5 \cdot 2 = 5$, each length in the new drawing should be 2 times as long as they are in the original drawing.
- Use this relationship to calculate the dimensions for the new drawing.

  The height in the original drawing is 10 units. $10 \cdot 2 = 20$

The height in the new drawing should be 20 units.

Unit 1 Lesson 10 73 Synthesis | Lesson Practice

Lesson Takeaway: Different scales can be used to create scale drawings that accurately depict the dimensions of an object. Comparing scales helps determine if a drawing will be smaller than, equivalent to, or larger than another.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by comparing two scale drawings and using the scale of one to determine the scale of the other.

Today's Goals

1. **Goal:** Calculate distances on a scale drawing, given a drawing of the same object with a different scale.
2. **Goal:** Determine the scale of a scale drawing.
3. **Language Goal:** Explain how different scales produce scale drawings that are smaller, the same size, or larger than the original. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.10

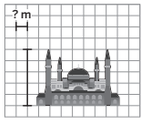
Here are two scale drawings of the Hagia Sophia in Istanbul, Turkey.

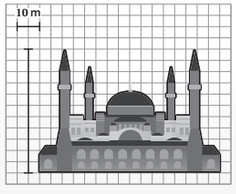
Complete the scale. What is the unknown value in the scale?

11 meters

Explain your thinking.

Explanations vary. In the bottom drawing, 2 units represent 10 meters, so 1 unit represents 5 meters. The Hagia Sophia is 11 units tall in this drawing, so the actual height is $\frac{5}{11} = 55$ meters. In the top drawing, the Hagia Sophia is 5 units tall, so each unit must represent 11 meters.





D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate distances on scale drawings:

» **Respond to Student Thinking:**

- Assign the ***Determining Unknowns in Scale Drawings*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

In this lesson, students created scale drawings of the same objects with different scales. How did students use proportional relationships to justify if a new scale drawing would be larger or smaller?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you tell whether a new scale drawing will be smaller than, larger than, or the same size as the original just by looking at the scales?

Use these examples if they help with your thinking.

Responses vary.

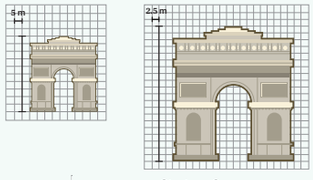
- If two segments are the same length but one represents a smaller distance, that scale drawing will be larger. For example, 1 unit to 5 meters will produce a larger scale drawing than 1 unit to 10 meters.
- If two segments represent the same distance, but one of the segments is shorter, that scale drawing will be smaller. For example, 1 unit to 10 meters will produce a smaller scale drawing than 4 units to 10 meters.

Lesson Practice 1.10

Lesson Summary

Here is a scale drawing with a scale of 1 unit to 5 meters.

If you wanted to change this to a scale of 1 unit to 2.5 meters, you could do so in a couple of ways:



Strategy 1

- Use the original scale to determine the actual dimensions.

  The height in the drawing is 10 units.
  $10 \cdot 5 = 50$

  The actual height of the building is 50 meters.
- Use the original dimensions and the new scale to determine the dimensions of the new scale drawing. $\frac{50}{2.5} = 20$

The height in the new drawing should be 20 units.

Strategy 2

- Determine how the two scales are related.
  $2.5 \cdot 2 = 5$

  Because $2.5 \cdot 2 = 5$, each length in the new drawing should be 2 times as long as they are in the original drawing.
- Use this relationship to calculate the dimensions for the new drawing.

  The height in the original drawing is 10 units. $10 \cdot 2 = 20$

The height in the new drawing should be 20 units.

Lesson Practice 1.10

Name: _____ Date: _____ Period: _____

Problems 1–4: Ali and Kiana buried a treasure together on their school's field. The field is 400 feet wide. Ali made a map that is 8 inches wide to record its location.

1. Write two possible scales Ali could have used to make her drawing.

Responses vary. 1 inch to 50 feet or 2 inches to 100 feet

2. Kiana made her own map using a scale of 1 inch to 20 feet. Whose map is larger: Kiana's or Ali's? Explain your thinking.

Kiana's. *Explanations vary.* On Ali's map, the field is 8 inches wide. On Kiana's, the field is 20 inches wide because $400 \div 20 = 20$.

3. On Kiana's map, the treasure is 2 inches from the south edge of the field. How far is the treasure from the south edge on Ali's map?

0.8 inches (or equivalent)

4. On Kiana's map, the area of the field is 16 square inches. Kiana says that the actual area of the field is 320 square feet. Is Kiana correct? Explain your thinking.

No. *Explanations vary.* When you create a scaled copy, the area is not scaled by the same scale factor, so the area would not be $16 \cdot 20 = 320$.

5. Select *all* the scales that are equivalent to 3 centimeters to 15 meters.

- ☐ A. 3 inches to 15 inches
- ☑ B. 1 centimeter to 5 meters
- ☐ C. 3 meters to 15 centimeters
- ☑ D. 4 millimeters to 2 meters
- ☐ E. 1 inch to 5 feet

Explain your thinking for the scale(s) you selected.

Explanations vary.
Scale B is equivalent because 3 centimeters with this scale would be $3 \cdot 5 = 15$ meters.
Scale D is equivalent because 4 millimeters can be rewritten as 0.4 centimeters.
If each value in the scale 0.4 centimeters to 2 meters is multiplied by 7.5, the result is 3 centimeters to 15 meters.

Lesson Practice 1.10

Name: _____ Date: _____ Period: _____

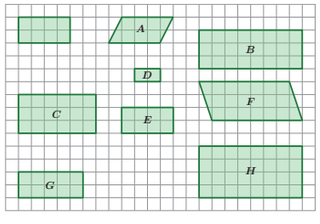
FAST Practice

6. On a blueprint, the living room is 2.1 inches wide. The blueprint has a scale of 1 inch to 10 feet. How wide would the living room be on a blueprint that has a scale of 1 inch to 15 feet?

A. 1.4 inches B. 2.1 inches C. 3.15 inches D. 21 inches

Spiral Review

7. Here is an unlabeled rectangle and several quadrilaterals that are labeled.



In the table, mark the quadrilaterals that are scaled copies of the unlabeled rectangle. Then, for all scaled copies, write the scale factor used to create it.

	A	B	C	D	E	F	G	H
Is It a Scaled Copy?			✓	✓	✓			✓
Scale Factor			1.5	0.5	1			2

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.GR.1.5
FAST Practice	6	1	MA.7.GR.1.5
Spiral Review			
	7	2	MA.7.GR.1.5

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify scaled copies by comparing corresponding side lengths	Lesson 2	1	MA.7.GR.1.5
2	Understanding how scale factors affect the distances in and areas of scaled copies	Lessons 4 and 5	2	MA.7.GR.1.5
3	Calculating the areas of scaled drawings when given two images	Lesson 5	2	MA.7.GR.1.5
4	Drawing scaled copies of figures on a grid using the structure of the grid to ensure accuracy	Lesson 3	2	MA.7.GR.1.5 MTR.5.1
5a, 5b	Using scales and scale drawings to calculate actual and scaled distances on a map	Lesson 7	1	MA.7.GR.1.5 MTR.6.1
6a, 6b	Calculating precise distances on a scale drawing when given a drawing with a different scale	Lessons 9 and 10	2	MA.7.GR.1.5 MTR.3.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



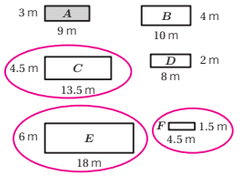
- Mini-Lessons
- Extensions

Name: Date: Period:

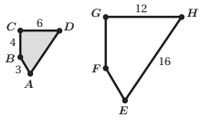
End-of-Unit Assessment

Unit 1

1. Circle *all* the scaled copies of rectangle A.



2. Figure $EFGH$ is a scaled copy of figure $ABCD$. Select *all* of the true statements.

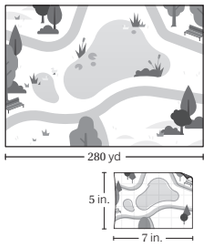


- ☑ **A.** Segment EF is twice as long as segment AB.
- ☐ **B.** The ratio of $\frac{AB}{BC}$ is equivalent to the ratio of $\frac{EH}{HG}$.
- ☐ **C.** The scale factor from $EFGH$ to $ABCD$ is 2.
- ☑ **D.** The length of segment AD is 8 units.
- ☐ **E.** The area of $EFGH$ is twice the area of $ABCD$.

3. A scale drawing of a rectangular park is 5 inches wide and 7 inches long.

The actual park is 280 yards long.

What is its area?



- **A.** 35 square yards
- **B.** 200 square yards
- **C.** 1,400 square yards
- **D.** 56,000 square yards

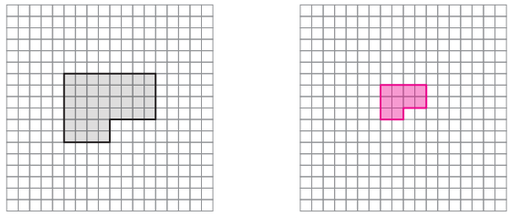
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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

4. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{2}$.



The position may vary, but size and shape must match.

5. Roberto is drawing a map of his town.

a He wants to include a school and a hospital, which are 10 kilometers apart.

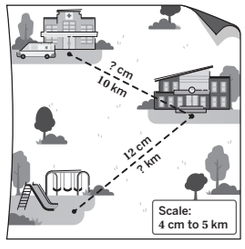
What should the distance between the school and the hospital be on the map?

8 centimeters

b On the map, the school and the playground are 12 centimeters apart.

What is the actual distance between the school and the playground?

15 kilometers



Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

6. A runner gets a new map of her favorite running trail.

- Her old map has a scale of 1 centimeter to 100 meters.
- Her new map has a scale of 1 centimeter to 500 meters.

a If the maps represent the same area, are the distances on the new map longer, shorter, or the same size as the old map?

- **A.** Longer
- **B.** Shorter
- **C.** The same size

Explain your thinking.

Explanations vary. If each centimeter on the new map represents five times the distance on the old map, the new map is much smaller. Each distances on the new map would be $\frac{1}{5}$ of the distance on the old map.

b Her favorite running trail was 40 centimeters long on her old map.

How long is this trail on her new map?

8 centimeters

Explain your thinking.

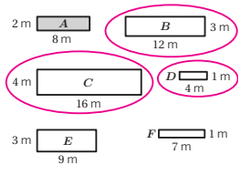
Explanations vary. Using the scales of both maps, a trail that is 40 centimeters long on her old map would be equivalent to an actual distance of 4,000 meters or 4 kilometers, since $40 \cdot 100 = 4000$. On her new map, this same distance would be 8 centimeters because $\frac{4000}{500} = 8$.

Name: Date: Period:

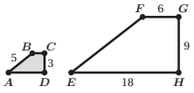
End-of-Unit Assessment

Unit 1

1. Circle *all* the scaled copies of rectangle A.



2. Figure $EFGH$ is a scaled copy of figure $ABCD$. Select *all* of the true statements.

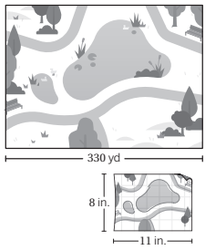


- ☐ **A.** Segment GH is three times as long as segment AB.
- ☑ **B.** The ratio of $\frac{AB}{BC}$ is equivalent to the ratio of $\frac{EF}{FG}$.
- ☑ **C.** The scale factor from $EFGH$ to $ABCD$ is $\frac{1}{3}$.
- ☑ **D.** The length of segment BC is 2 units.
- ☐ **E.** The area of $EFGH$ is three times the area of $ABCD$.

3. A scale drawing of a rectangular park is 8 inches wide and 11 inches long.

The actual park is 330 yards long.

What is its area?



A. 88 square yards

B. 240 square yards

C. 2,640 square yards

D. 79,200 square yards

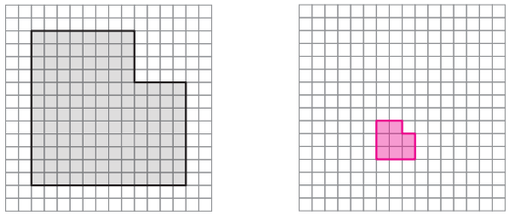
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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

4. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{4}$.



The position may vary, but size and shape must match.

5. Anya is drawing a map of her town.

a She wants to include a school and a hospital, which are 15 kilometers apart.

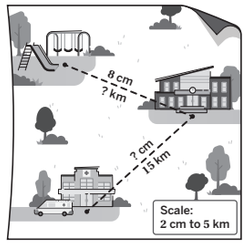
What should the distance between the school and the hospital be on the map?

6 centimeters

b On the map, the school and the playground are 8 centimeters apart.

What is the actual distance between the school and the playground?

20 kilometers



Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

6. A runner gets a new map of her favorite running trail.

- Her old map has a scale of 1 centimeter to 400 meters.
- Her new map has a scale of 1 centimeter to 100 meters.

a If the maps represent the same area, will the new map be larger, smaller, or the same size as the old map?

A. Larger

B. Smaller

C. The same size

Explain your thinking.

***Explanations vary*. Each centimeter on the old map represents four times the distance on the new map. This means the new map will need more space to represent the same distance.**

b Her favorite running trail was 20 centimeters long on her old map.

How long is this trail on her new map?

80 centimeters

Explain your thinking.

***Explanations vary*. Using the scales of both maps, a trail that is 20 centimeters long on her old map would be equivalent to an actual distance of 8,000 meters or 8 kilometers, since $20 \cdot 400 = 8000$. On her new map, this same distance would be 80 centimeters because $\frac{8000}{100} = 80$.**

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	

Sub-Unit 1:
• Describe how scaling affects lengths, angles, and areas in scaled copies.
• Use scale factors to create and compare scaled copies.
(Lessons 1–5)
	2-3	**Support**	• **Teacher Move:** Consider revisiting...
			  • Unit 1, Lesson 4 (Scale Factor Challenges)
			  • Unit 1, Lesson 5 (Tiles)
			• **Teacher Move:** Consider asking, "How can you calculate the scale factor in this situation?"

		Strengthen	• **Repeated Challenges:** Lesson 3 (Drawing Scaled Copies With a Grid)

	1, 4	**Support**	• **Mini-Lesson:** Connecting Scale Factors to Scaled Copies (ML 1.04)
			• **Teacher Move:** Consider revisiting...
			  • Unit 1, Lesson 2 (Scaling Robots)
			  • Unit 1, Lesson 3 (Make It Scale)
			• **Teacher Move:** Consider asking, "Where can you find equivalent ratios in scaled copies?"

		Strengthen	• **Repeated Challenges:** Lesson 4 (Scale Factor Challenges)

		Stretch	• **You're invited to explore more:**
			  • Lesson 4 (Scale Factor Challenges)
			  • Lesson 3 (Make It Scale)

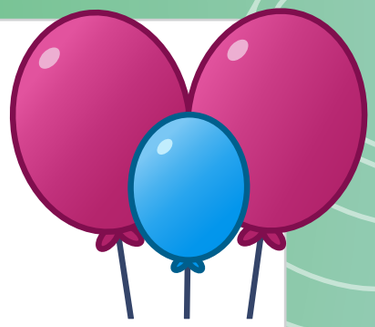
Sub-Unit 2: • Represent distances in the real world using scale and scale drawings. **(Lessons 6–10)**	5	**Support**	• **Teacher Move:** Consider revisiting Unit 1, Lesson 7 (Will It Fit?) • **Teacher Move:** Consider asking students to explain in words what the map scale is telling them.
		Strengthen	• **Repeated Challenges:** Lesson 8 (Scaling States, Part 1)
		Stretch	• **You're invited to explore more:** Lesson 7 (Will It Fit?)

	6	**Support**	• **Teacher Move:** Consider revisiting...
			  • Unit 1, Lesson 9 (Scaling States, Part 2)
			  • Unit 1, Lesson 10 (Scaling Buildings)

		Strengthen	• Inviting students to create their own scaled drawing.
		Stretch	• **You're invited to explore more:** • Lesson 7 (Will It Fit?)

UNIT 2

Introducing Proportional Relationships



Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.

Essential Questions

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are the representations related?

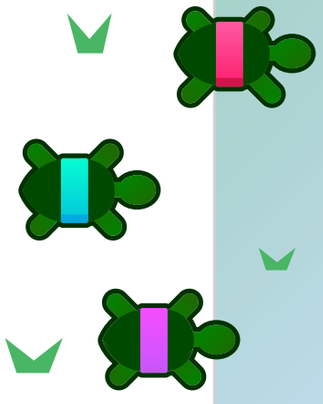
Sub-Unit Goals

Sub-Unit 1
- Use tables to recognize proportional relationships.
- Determine a constant of proportionality from a table.

Sub-Unit 2
- Write and use equations to describe proportional relationships.
- Determine a constant of proportionality from an equation.

Sub-Unit 3
- Use graphs to recognize proportional relationships.
- Determine a constant of proportionality from a graph.

Sub-Unit 4
- Use proportional relationships to solve real-world and mathematical problems.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Use percentages and proportional reasoning to solve problems. **(MA.7.AR.3)**

Analyze and represent two-variable proportional relationships. **(MA.7.AR.4)**

Benchmarks Addressed in This Unit: MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.3 MA.7.AR.4.4 MA.7.AR.4.5

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.6.1**

Coherence Telling a story within and across grades

In this Unit

Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.

Prior Learning

- **Grade 6,** Unit 2: Write and interpret ratios **(MA.6.AR.3.1)**
- **Grade 6,** Unit 2: Equivalent ratios **(MA.6.AR.3.3, MA.6.AR.3.5)**
- **Grade 6,** Unit 3: Unit rates **(MA.6.AR.3.2, MA.6.AR.3.5)**
- **Grade 7,** Unit 7: Graph points in a coordinate plane **(MA.6.GR.1.1)**

Future Learning

- **Grade 7,** Unit 4: Proportional relationships as percents **(MA.7.AR.3.1)**
- **Grade 8,** Unit 2: Slope and rate of change **(MA.8.AR.3.2)**
- **Grade 8,** Unit 3: Linear relationships **(MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–9**, students develop conceptual understanding of proportionality by analyzing multiple representations of proportional relationships.

Strategic Competence

In **Lesson 2**, students develop strategic competence as they use different strategies to determine unknown values in a table.

Procedural Fluency

In **Lessons 3–6**, students develop procedural fluency as they calculate constants of proportionality.

Adaptive Reasoning

In **Lesson 6**, students develop adaptive reasoning as they consider how one relationship can be represented in many ways.

Application

In **Lessons 12–13**, students apply their understanding of proportional relationships to determine whether a bath or shower uses more water.

Productive Disposition

In **Lesson 8**, students have opportunities to develop a productive disposition as they problem solve in a situation like manufacturing lollipops.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.5.GR.4.1 MA.6.AR.3.2 MA.6.AR.3.1 MA.6.AR.3.5 MTR.4.1 MTR.7.1

Sub-Unit 1



1 Paint

Using Equivalent Ratios

Explore equivalent ratios and have initial, informal conversations about the key ideas in proportional relationships.

MA.7.AR.3.2 MTR.4.1 MTR.5.1



2 Balloon Float

Introducing Proportional Relationships With Tables

Explore **proportional relationships** and analyze tables of equivalent ratios.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MTR.4.1 MTR.5.1



3 Sugar, Spice, and Everything Rice

Constant of Proportionality

Determine, interpret, and use **constants of proportionality** to make sense of proportional relationships.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MTR.2.1 MTR.6.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5 MTR.5.1 MTR.6.1 MTR.7.1

Sub-Unit 3



8 DinoPops

Introducing Graphs of Proportional Relationships

Explore what a proportional relationship looks like graphically.

MA.7.AR.3.2 MA.7.AR.4.1 MTR.2.1 MTR.5.1

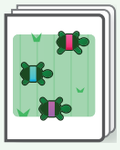


9 Gallon Challenge

Interpreting Graphs of Proportional Relationships

Use graphs to identify constants of proportionality.

MA.7.AR.3.2 MA.7.AR.4.2 MTR.3.1 MTR.7.1



10 Three Turtles

Proportional Relationships and Graphs

Use graphs and equations to compare proportional relationships.

MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5 MTR.1.1 MTR.2.1 MTR.4.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5 MTR.3.1 MTR.4.1 MTR.6.1 MTR.7.1

Pacing: 17 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 13 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2



4 Robot Factory

Proportional Relationships and Equations

Use repeated reasoning about tables to write equations for proportional relationships.

MA.7.AR.3.2 MA.7.AR.4.2 MA.7.AR.4.4
MTR.2.1 MTR.3.1



5 Snapshots

More Equations of Proportional Relationships

Write and interpret equations that represent proportional relationships.

MA.7.AR.3.2 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.2.1 MTR.7.1

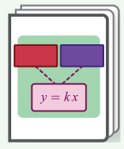


6 Two and Two

Two Equations for Each Relationship

Find the two constants of proportionality for a proportional relationship, discover they are **reciprocals**, and use them to write two equations representing the relationship.

MA.7.AR.3.2 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.2.1 MTR.5.1



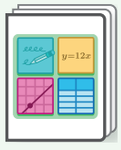
7 All Kinds of Equations

Equations of Proportional Relationships

Compare proportional and non-proportional relationships.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.1.1 MTR.4.1 MTR.5.1

Sub-Unit 4



11 Four Representations

Connecting Descriptions, Tables, Graphs, and Equations

Examine the connection between verbal descriptions, tables, equations, and graphs of proportional relationships.

MA.7.AR.4.2 MA.7.AR.4.3 MA.7.AR.4.4
MTR.2.1 MTR.7.1



12 Water Efficiency, Part 1

Let's Put It to Work

Use proportional relationships to model the water usage of baths and showers.

MA.7.AR.3.2 MA.7.AR.4.2 MA.7.AR.4.3 MA.7.AR.4.4 MA.7.AR.4.5
MTR.1.1 MTR.7.1



13 Water Efficiency, Part 2

Let's Put It to Work

Reflect on how the mathematical modeling process offers insight into a real-world context.

MA.7.AR.3.2 MA.7.AR.4.3 MA.7.AR.4.4 MA.7.AR.4.5
MTR.2.1

M Math in Action

Connecting Ratios to Music

Explore the connection between ratios and the rhythmic pattern of music, as well as the contributions of Eugenia Cheng.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2 MA.7.GR.4.4
MTR.1.1 MTR.3.1

Pacing Considerations

Lesson 1: This lesson supports students in developing their understanding of key ideas about proportional relationships, which will be addressed in more depth in upcoming lessons. If most students demonstrate a strong understanding of proportional relationships, this lesson may be omitted.

Lesson 5: This lesson supports students in developing fluency writing and using equations to make sense of proportional relationships. If students show a strong understanding working with equations of proportional relationships in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how each part of an equation of a proportional relationship relates to a situation in Lessons 6, 7, and 10.

Lesson 11: This lesson gives students an opportunity to connect descriptions, tables, equations, and graphs of proportional relationships. There is no new content introduced in this lesson.

Lessons 12–13: This lesson gives students an opportunity to use their creativity and personal experiences to apply the concepts they learned in this unit to explore whether baths or showers use more water. There is no new content introduced in these lessons.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

Copy: Show What You Know, Activity 3 Sheet *(one per student)*

Materials: calculators *(as needed)*

Lesson 2

Copy: Show What You Know

Lesson 3

Copy: Show What You Know

Materials: scientific calculators *(as needed)*

Lesson 4

Copy: Show What You Know

Lesson 5

Copy: Show What You Know, Activity 3 You're Invited to Explore More Sheet *(one per student, as needed)*

Lesson 6

Copy: Show What You Know

Lesson 7

Copy: Show What You Know, Activity 2 You're Invited to Explore More Sheet *(one per student, as needed)*

Materials: calculators *(as needed)*

Lesson 8

Copy: Show What You Know

Lesson 9

Copy: Show What You Know

Lesson 10

Copy: Show What You Know, Activity 3 You're Invited to Explore More Sheet *(one per student, as needed)*

Materials: straightedges *(as needed)*

Lesson 11

Copy: Show What You Know

Lesson 12

Copy: Show What You Know

Lesson 13

Copy: Show What You Know

Materials: graph paper, markers, poster paper, straightedges

Resource Books

Assessment and Lesson Resources



- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources



- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

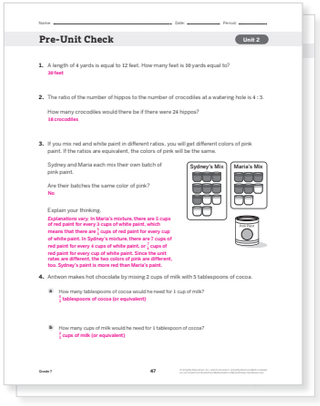
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards assessed: MA.5.GR.4.1, MA.6.AR.3.1, MA.6.AR.3.2, MA.6.AR.3.5


Name _____ Date _____ Period _____

Pre-Unit Check Unit 2

1. A length of 4 yards is equal to 12 feet. How many feet is 18 yards equal to?
54 feet

2. The ratio of the number of hippos to the number of crocodiles at a watering hole is 4 : 3.
How many crocodiles would there be if there were 24 hippos?
18 crocodiles

3. If you mix red and white paint in different ratios, you will get different colors of pink paint. If the ratios are equivalent, the colors of pink will be the same.
Sydney and Maria each mix their own batch of pink paint.
Sydney's Mix | Maria's Mix
Are their batches the same color of pink?
No
Explain your thinking.
Explanations vary. In Maria's mixture, there are 5 cups of red paint for every 3 cups of white paint, which means that there are $\frac{5}{3}$ cups of red paint for every cup of white paint. In Sydney's mixture, there are 7 cups of red paint for every 4 cups of white paint, or $\frac{7}{4}$ cups of red paint for every cup of white paint. Since the unit rates are different, the two colors of pink are different, too. Sydney's paint is more red than Maria's paint.

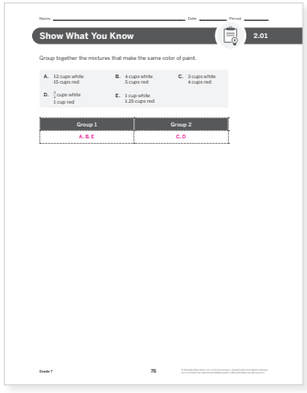
4. Antwon makes hot chocolate by mixing 2 cups of milk with 5 tablespoons of cocoa.
a. How many tablespoons of cocoa would he need for 1 cup of milk?
$\frac{5}{2}$ tablespoons of cocoa (or equivalent)
b. How many cups of milk would he need for 1 tablespoon of cocoa?
$\frac{2}{5}$ cups of milk (or equivalent)

Grade 7 47

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

Standards assessed on Lesson 1 Show What You Know: MA.7.AR.3.2


Name _____ Date _____ Period _____

Show What You Know 2.01

Group together the mixtures that make the same color of paint.

A. 12 cups white, 15 cups red
B. 4 cups white, 5 cups red
C. 3 cups white, 4 cups red
D. 1 cup white, 1 cup red
E. 1 cup white, 1.25 cups red

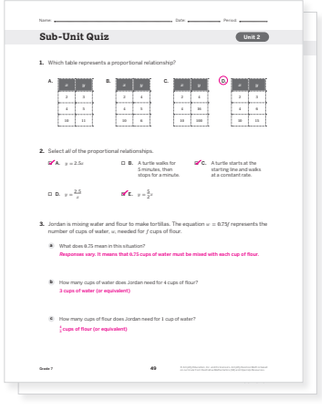
Group 1	Group 2
A, B, E	C, D

Grade 7 75

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

Standards assessed: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5


Name _____ Date _____ Period _____

Sub-Unit Quiz Unit 2

1. Which table represents a proportional relationship?
A. | x | y | 2 | 5 | 4 | 9 | 10 | 11 |
B. | x | y | 2 | 4 | 5 | 7 | 10 | 6 |
C. | x | y | 2 | 4 | 5 | 10 | 10 | 100 |
D. | x | y | 2 | 5 | 4 | 10 | 10 | 25 |

2. Select all of the proportional relationships.
☑ A. $y = 2.5x$
☐ B. A turtle walks for 5 minutes, then stops for a minute.
☑ C. A turtle starts at the starting line and walks at a constant rate.
☐ D. $y = \frac{2.5}{x}$
☑ E. $y = \frac{5}{2}x$

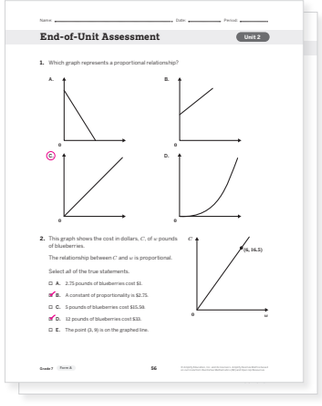
3. Jordan is mixing water and flour to make tortillas. The equation $w = 0.75f$ represents the number of cups of water, w, needed for f cups of flour.
a. What does 0.75 mean in this situation?
Responses vary. It means that 0.75 cups of water must be mixed with each cup of flour.
b. How many cups of water does Jordan need for 4 cups of flour?
3 cups of water (or equivalent)
c. How many cups of flour does Jordan need for 1 cup of water?
$\frac{4}{3}$ cups of flour (or equivalent)

Grade 7 49

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

Standards assessed: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5


Name _____ Date _____ Period _____

End-of-Unit Assessment Unit 2

1. Which graph represents a proportional relationship?
A. B. C. D.

2. This graph shows the cost in dollars, C, of w pounds of blueberries.
The relationship between C and w is proportional.
Select all of the true statements.
☐ A. 2.75 pounds of blueberries cost $1.
☑ B. A constant of proportionality is $2.75.
☐ C. 5 pounds of blueberries cost $15.50.
☑ D. 12 pounds of blueberries cost $33.
☐ E. The point (3, 9) is on the graphed line.

Grade 7 Form A 56

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

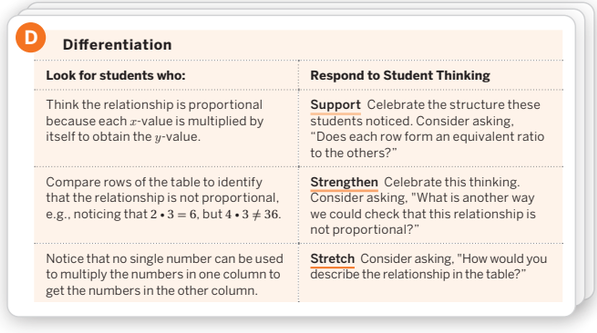
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–3) • Use tables to recognize proportional relationships. • Determine a constant of proportionality from a table.	**Mini-Lesson** • *Determining the Constant of Proportionality*	**Challenge Creator** • Lesson 1 (*Paint*) **Practice** • Lessons 1–3	**You're invited to explore more.** • Lesson 2 (*Balloon Float*)
Sub-Unit 2 (Lessons 4–7) • Write and use equations to describe proportional relationships. • Determine a constant of proportionality from an equation.	**Mini-Lesson** • *Finding More Than One Equation to Represent Proportional Relationships*	**Practice** • Lessons 4–7	**You're invited to explore more.** • Lesson 5 (*Snapshots*) • Lesson 6 (*Two and Two*) • Lesson 7 (*All Kinds of Equations*)
Sub-Unit 3 (Lessons 8–10) • Use graphs to recognize proportional relationships. • Determine a constant of proportionality from a graph.	**Mini-Lesson** • *Comparing Proportional Relationships With Graphs*	**Turtle Challenges** • Lesson 10 (*Three Turtles*) **Practice** • Lessons 8–10	**You're invited to explore more.** • Lesson 8 (*DinoPops*) • Lesson 10 (*Three Turtles*)
Sub-Unit 4 (Lessons 11–13) • Use proportional relationships to solve real-world and mathematical problems.	**Mini-Lesson** • *Representing Proportional Relationships*	**Practice** • Lessons 11–13	**You're invited to explore more.** • Lesson 13 (*Water Efficiency, Part 2*)

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 2**, where students analyze tables of equivalent ratios to build conceptual understanding of proportional relationships.



Differentiation

Look for students who:	Respond to Student Thinking
Think the relationship is proportional because each x-value is multiplied by itself to obtain the y-value.	**Support** Celebrate the structure these students noticed. Consider asking, "Does each row form an equivalent ratio to the others?"
Compare rows of the table to identify that the relationship is not proportional, e.g., noticing that $2 \cdot 3 = 6$, but $4 \cdot 3 \neq 36$.	**Strengthen** Celebrate this thinking. Consider asking, "What is another way we could check that this relationship is not proportional?"
Notice that no single number can be used to multiply the numbers in one column to get the numbers in the other column.	**Stretch** Consider asking, "How would you describe the relationship in the table?"

A Accessibility

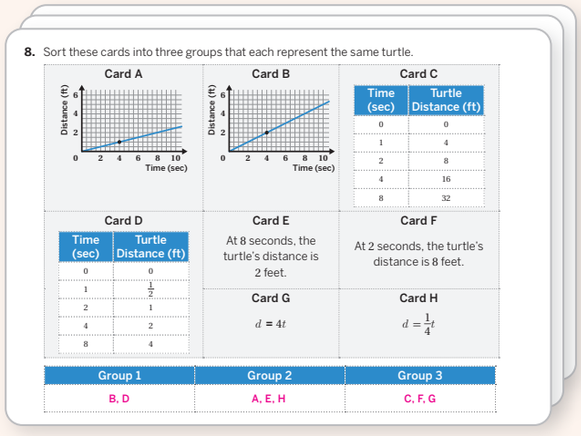
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Language and Symbols . . .

Lesson 10: In this lesson, students explore the relationship between graphs, equations, and tables, making explicit links between representations.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Executive Functioning . . .

Here are two suggestions in this unit for supporting students with initiating complex tasks.

Lesson 6, Activity 1

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one table at a time and look for an equation to match it.

Lesson 11, Activity 2

A **Accessibility: Executive Functioning** Consider pre-selecting two quantities from Activity 1 and providing a corresponding table of values for the relationship. This will allow students to focus on analyzing the connections between the table, graph, and equation.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
proportional relationship	2
constant of proportionality	3
reciprocal	6
origin	8

Review Vocabulary
coordinate plane
equivalent ratio
scaled copy

Math Language Routines

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	**1 and 11**
MLR2: Collect and Display	**1–4, 7, 9, and 12**
MLR3: Critique, Correct, Clarify	**4**
MLR7: Compare and Connect	**2, 9, and 13**

Sub-Unit Language Goals

- Justify whether a table represents a proportional relationship.
- Describe the relationship between values in a table using terms such as *proportional, non-proportional,* and *constant of proportionality.*
- Describe relationships between values in an equation using terms such as *proportional, non-proportional,* and *constant of proportionality.*
- Describe the relationship between values in a graph using terms such as *proportional, non-proportional,* and *constant of proportionality.*
- Interpret different representations of a proportional relationship to make sense of a situation in context.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 7**, where students are analyzing relationships in multiple representations to determine whether a relationship is proportional. Students are given sentence frames to support them in expressing their thinking about proportional relationships.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "___ is proportional/not proportional because___."). **(Writing and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

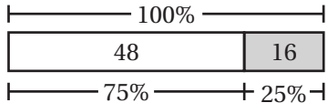
Proportional Relationships as Percents

In this unit, students use proportional relationships in tables, equations, and graphs to solve real-world problems. **In Grade 7, Unit 4,** students will study percentages as proportional relationships.

FL B.E.S.T. Math Benchmark:
MA.7.AR.3.1

Example:

Here is an example of a *percentage decrease*. The computer club had 64 students. Then, they lost 16 students. This is a 25% decrease because $\frac{16}{64} = 0.25$. The club now has 48 students, which is 75% of the starting amount: $0.75 \cdot 64 = 48$.



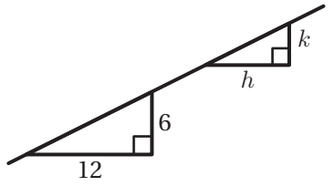
Slope and Rate of Change

In this unit, students learn about the constant of proportionality. **In Grade 8, Unit 2,** students learn the term *slope*.

FL B.E.S.T. Math Benchmark:
MA.8.AR.3.2

Example:

The slope of a line is a number we can calculate using any two points on the line. To calculate the slope, divide the vertical distance between the points by the horizontal distance. The slope of this line is $\frac{k}{h} = \frac{6}{12} = \frac{1}{2}$.



Linear Relationships

In this unit, students write equations to describe proportional relationships. **In Grade 8, Unit 3,** students revisit proportional relationships and write equations for linear relationships that are not proportional.

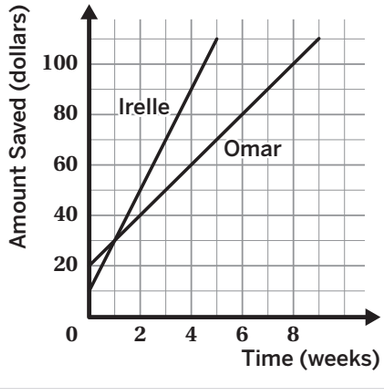
FL B.E.S.T. Math Benchmarks:
MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3

Example:

For example, Irelle and Omar save some of the money they earned. Let w represent the number of weeks passed. Let s represent the amount saved.

Equation for Irelle's savings: $s = 20w + 10$

Equation for Omar's savings: $s = 10w + 20$

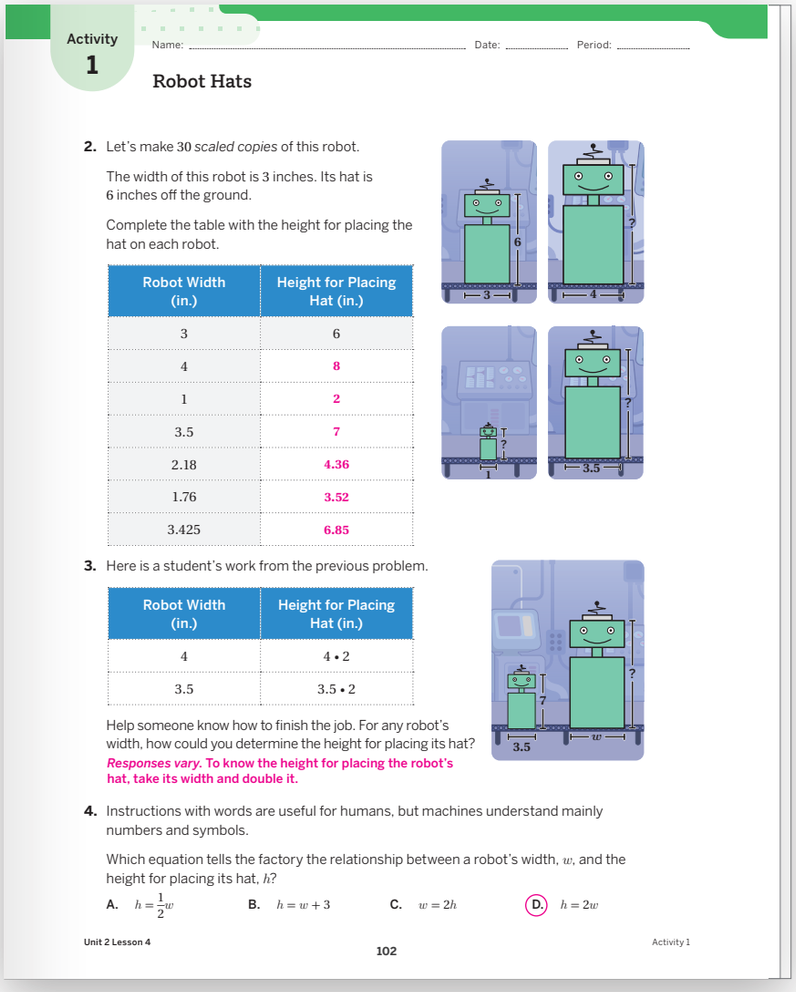


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 4, Activity 1



Activity 1

Name: _____ Date: _____ Period: _____

Robot Hats

2. Let's make 30 *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.

Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	8
1	2
3.5	7
2.18	4.36
1.76	3.52
3.425	6.85

3. Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
4	4 • 2
3.5	3.5 • 2

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?

Responses vary. To know the height for placing the robot's hat, take its width and double it.

4. Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?

A. $h = \frac{1}{2}w$ B. $h = w + 3$ C. $w = 2h$ D. $h = 2w$

Unit 2 Lesson 4 102 Activity 1

Context

In this activity, students extend their understanding of proportional relationships in tables to write equations using the constant of proportionality.

Try This

Put on your student hat and complete Activity 1.

Points to Ponder

- In what ways will students be able to build on their prior knowledge of proportional relationships in tables to write equations?
- Students will deepen their understanding of how the constant of proportionality can be represented. What will be challenging for students as they try to move between the table and equation representations? How can you support them through these challenges?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1, 2	Calculating equivalent ratios	Lesson 1	1	MA.6.AR.3.1, MA.6.AR.3.5, MTR.7.1
3	Explaining whether or not two ratios are equivalent	Lesson 1	2	MA.6.AR.3.1, MTR.4.1
4	Calculating two different unit rates for one relationship	Lesson 1	1	MA.6.AR.3.5, MTR.7.1
5	Scaling values in a table	Lesson 2	1	MA.6.AR.3.5, MTR.7.1
6	Calculating and using unit rates	Lesson 2	1	MA.6.AR.3.2, MA.6.AR.3.5 MTR.4.1
7	Graphing points in the coordinate plane	Lesson 8	1	MA.5.GR.4.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 2

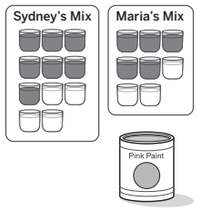
1. A length of 4 yards is equal to 12 feet. How many feet is 10 yards equal to?
   30 feet

2. The ratio of the number of hippos to the number of crocodiles at a watering hole is 4 : 3.

   How many crocodiles would there be if there were 24 hippos?
   18 crocodiles

3. If you mix red and white paint in different ratios, you will get different colors of pink paint. If the ratios are equivalent, the colors of pink will be the same.

   Sydney and Maria each mix their own batch of pink paint.

   Are their batches the same color of pink?
   No

   Explain your thinking.
   Explanations vary. **In Maria's mixture, there are 5 cups of red paint for every 3 cups of white paint, which means that there are $\frac{5}{3}$ cups of red paint for every cup of white paint. In Sydney's mixture, there are 7 cups of red paint for every 4 cups of white paint, or $\frac{7}{4}$ cups of red paint for every cup of white paint. Since the unit rates are different, the two colors of pink are different, too. Sydney's paint is more red than Maria's paint.**

4. Antwon makes hot chocolate by mixing 2 cups of milk with 5 tablespoons of cocoa.

   a. How many tablespoons of cocoa would he need for 1 cup of milk?
   $\frac{5}{2}$ tablespoons of cocoa (or equivalent)

   b. How many cups of milk would he need for 1 tablespoon of cocoa?
   $\frac{2}{5}$ cups of milk (or equivalent)

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Name: Date: Period:

Pre-Unit Check (continued)

Unit 2

5. An airplane flew across the Pacific Ocean at a constant speed. The table shows the amount of time and the distance traveled. Complete the table.

Time (hr)	Distance Traveled (mi)
2	**1,100**
3	1,650
6	**3,300**

6. Blueberries cost $4.00 per pound.

   a. How many pounds of blueberries can you buy for $1.00?
   0.25 pounds (or equivalent)

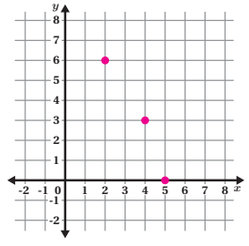
   b. How many pounds of blueberries can you buy for $13.00?
   3.25 pounds (or equivalent)

   Explain or show your thinking.
   Explanations vary. **For every $1, I can buy 0.25 pounds of blueberries. So for $13, I can buy 3.25 pounds of blueberries (because $13 \cdot 0.25 = 3.25$).**

7. The table shows some coordinate pairs.

x	y
4	3
2	6
5	0

   Plot these points in the coordinate plane.



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D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Use tables to recognize proportional relationships. • Determine a constant of proportionality from a table. **(Lessons 1–3)**	1–3	**Support** • **Mini-Lesson:** *Generating Equivalent Ratios* • **Teacher Move:** During **Lesson 1**, Activity 1, consider reviewing how to use representations such as double number lines and tables of equivalent ratios.
	5, 6	**Support** • **Teacher Move:** After the **Lesson 2** Warm-Up, invite students to add pairs of values to the table.
	4	**Support** • **Mini-Lesson:** *Calculating Unit Rates* • **Teacher Move:** During the **Lesson 6** Synthesis, invite students to share strategies for determining the unit rates in this problem.
	7	**Support** • **Mini-Lesson:** *Plotting Points on the Coordinate Plane* • **Teacher Move:** During **Lesson 8**, Activity 1, consider using the movable point to review the meaning of the x and y coordinates.

Sub-Unit 1

Proportional Relationships in Tables

Sub-Unit 1 Goals

- **Goal:** Use tables to recognize proportional relationships.
- **Goal:** Determine a constant of proportionality from a table.
- **Language Goal:** Justify whether a table represents a proportional relationship.
- **Language Goal:** Describe the relationship between values in a table using terms such as *proportional, non-proportional,* and *constant of proportionality.*

In this sub-unit, students create and compare equivalent ratios using a variety of representations. They recognize **proportional relationships** represented in tables and determine the **constant of proportionality** using tables.



Math that Matters Most

Concept or Skill	For example . . .
Generate equivalent ratios.	Darryl mixed 4 cups of white paint with 6 cups of red paint, but he didn't have enough to finish painting his wall. How much red paint would he need to add to 1 cup of white paint to match the color? **He would need to add $\frac{6}{4}$ cups of red paint.**
Determine unknown values in tables representing proportional relationships.	Brielle mixed 6 cups of blue paint with 3 cups of white paint to make the perfect color to paint her mural. Complete the table to show several other ways to make this color.

Blue Paint (cups)	White Paint (cups)
6	3
8	**4**
12	6
30	15

Determine a constant of proportionality from a table.	When Deven makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. This table shows how to make batches of different sizes. What is a constant of proportionality in this relationship? **1.5 because the cups of milk can be multiplied by 1.5 to find the tablespoons of chocolate.**

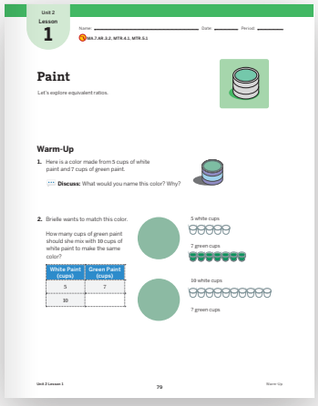
Milk (cups)	Chocolate Syrup (tbsp)
2	3
8	12
1	$\frac{3}{2}$
10	15

Unit 2
Lesson
1

Paint

Using Equivalent Ratios

Let's explore equivalent ratios.


Unit 2
Lesson
1
Name _____ Date _____ Period _____
MA.7.AR.3.2, MTR.4.1, MTR.5.1
Paint
Let's explore equivalent ratios.
Warm-Up
1. Here is a color made from 5 cups of white paint and 7 cups of green paint.
Discuss: What would you name this color? Why?
2. Brielle wants to match this color.
How many cups of green paint should she mix with 10 cups of white paint to make the same color?
White Paint (cups) | Green Paint (cups)
5 | 7
10
5 white cups
7 green cups
10 white cups
7 green cups
Unit 2 Lesson 1
79
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Understand and generate equivalent ratios.
2. **Language Goal:** Justify whether two ratios are equivalent. **(Reading, Writing, Speaking, and Listening)**

Students have initial, informal conversations about the key ideas in proportional relationships. They use paint color as a context for exploring *equivalent ratios* and use various strategies to determine whether ratios are equivalent.

Coherence

Prior Learning

In Grade 6, students used tables, tape diagrams, and double number line diagrams to solve problems with equivalent ratios. In Unit 1, students explored how scaled copies have equivalent ratios of corresponding side lengths.

Future Learning

In Lesson 2, students will identify patterns and calculate unknown quantities in tables representing proportional relationships.

Connections

- Students use equivalent ratios to mix paint colors in order to create a color match for a different volume of paint. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.3	**MA.7.AR.4.4**
MA.6.AR.3.5	**MA.7.AR.4.5**
	MA.8.AR.3.1

Vocabulary

Review Vocabulary

equivalent ratios

Lesson at a Glance

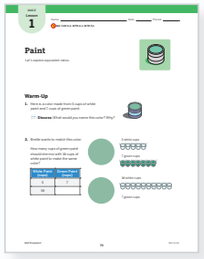
~ 45 min

MA.7.AR.3.2

Warm-Up

Pairs | 5 min

Students use a ratio table to generate a new mixture that will create the same shade of the given paint color.



Activity 1

Pairs | 10 min

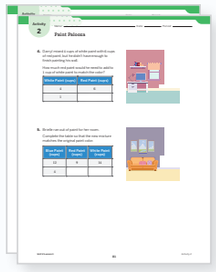
Students define the term *equivalent ratios*, using the context of paint colors to support their reasoning. MLR1: Stronger and Clearer Each Time



Activity 2

Pairs | 10 min

Students use tables to generate equivalent ratios in the context of matching a given paint color.



Activity 3

Independent | 10 min

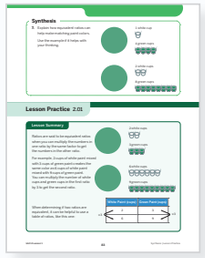
Students explore equivalent ratios by creating their own paint color matching challenge and solving classmates' challenges.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of strategies for creating equivalent ratios. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of equivalent ratios by matching different amounts of paint that result in the same color.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheet, one per student
- calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use a ratio table to generate a new mixture that will create the same shade of the given paint color.

1 Launch

To generate excitement, share that this lesson is about determining the exact right color of paint. Consider asking, "Why might someone need to mix together different colors of paint? Have you ever mixed paint?"

ELL **English Language Learners** Invite students to share what they know about paint or painting to increase access to the task. Consider creating a list of student ideas, including words such as *paint*, *lighter*, and *darker*, along with translations or visuals representing the terms. **(Reading and Listening)**

Invite students to use their creativity to name the shade of green created using 5 cups of white paint and 7 cups of green paint.

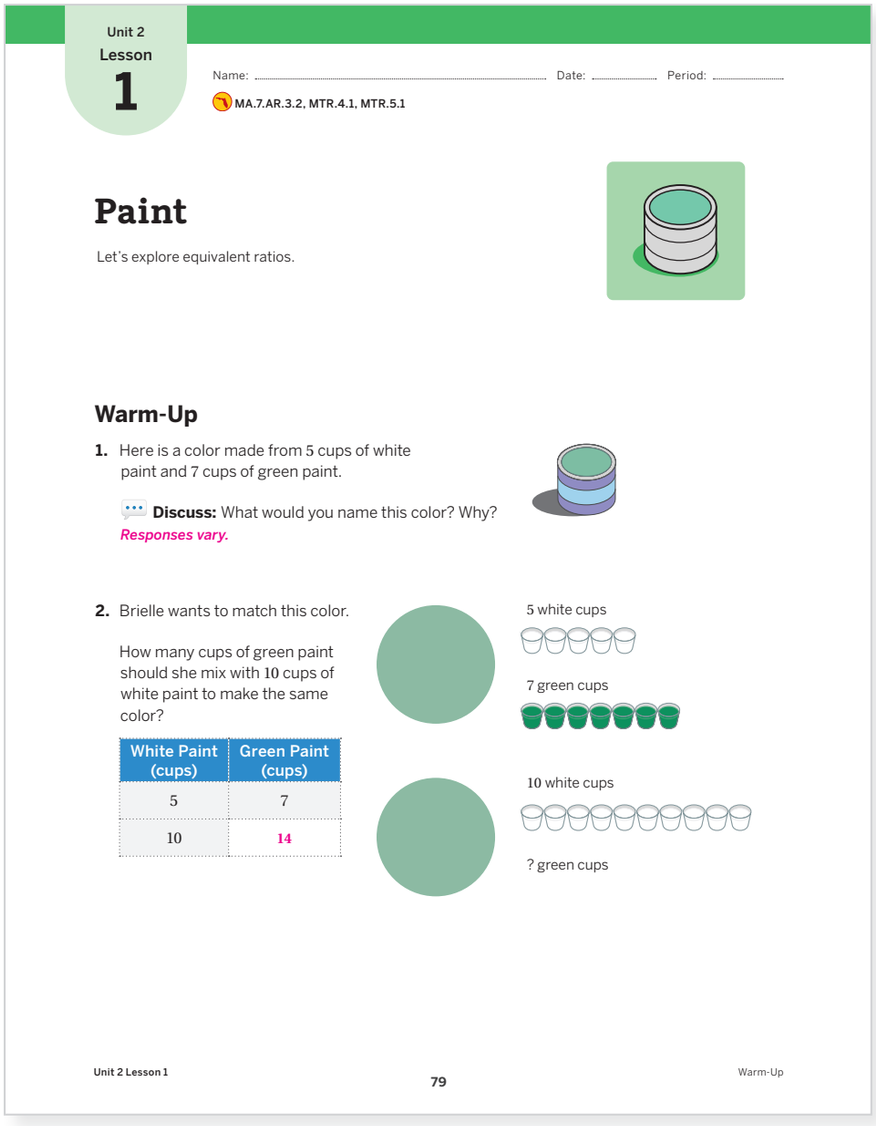
2 Connect

Invite students to share their tables.

Consider asking, "How did you determine how many cups of green paint to use?"

Note: Consider using informal language as students develop their own strategies for how to make different mixtures that produce the same color. The phrase *equivalent ratios* will be more formally introduced in the next activity.

Invite students to reflect on how the shared strategies were similar to or different from their own.



Unit 2
Lesson
1

Name: ..... Date: ..... Period: .....

MA.7.AR.3.2, MTR.4.1, MTR.5.1

Paint

Let's explore equivalent ratios.

Warm-Up

1. Here is a color made from 5 cups of white paint and 7 cups of green paint.

   Discuss: What would you name this color? Why?

   Responses vary.

2. Brielle wants to match this color.

   How many cups of green paint should she mix with 10 cups of white paint to make the same color?

White Paint (cups)	Green Paint (cups)
5	7
10	14

5 white cups

7 green cups

10 white cups

? green cups

Unit 2 Lesson 1 79 Warm-Up

Pairs | 10 min

Activity 1 Color Match

Purpose: Students define the term *equivalent ratios*, using the context of paint colors to support their reasoning.

1 Launch

Consider asking, "How can you tell that these two mixtures will create the same paint color?" The amount of each color has been doubled, so it's like mixing two batches of the same paint. The color will match, but there is twice as much.

2 Monitor

Look for ways that students are making use of structure to determine which mixture is an equivalent ratio. **(MTR.5.1)**

D Differentiation

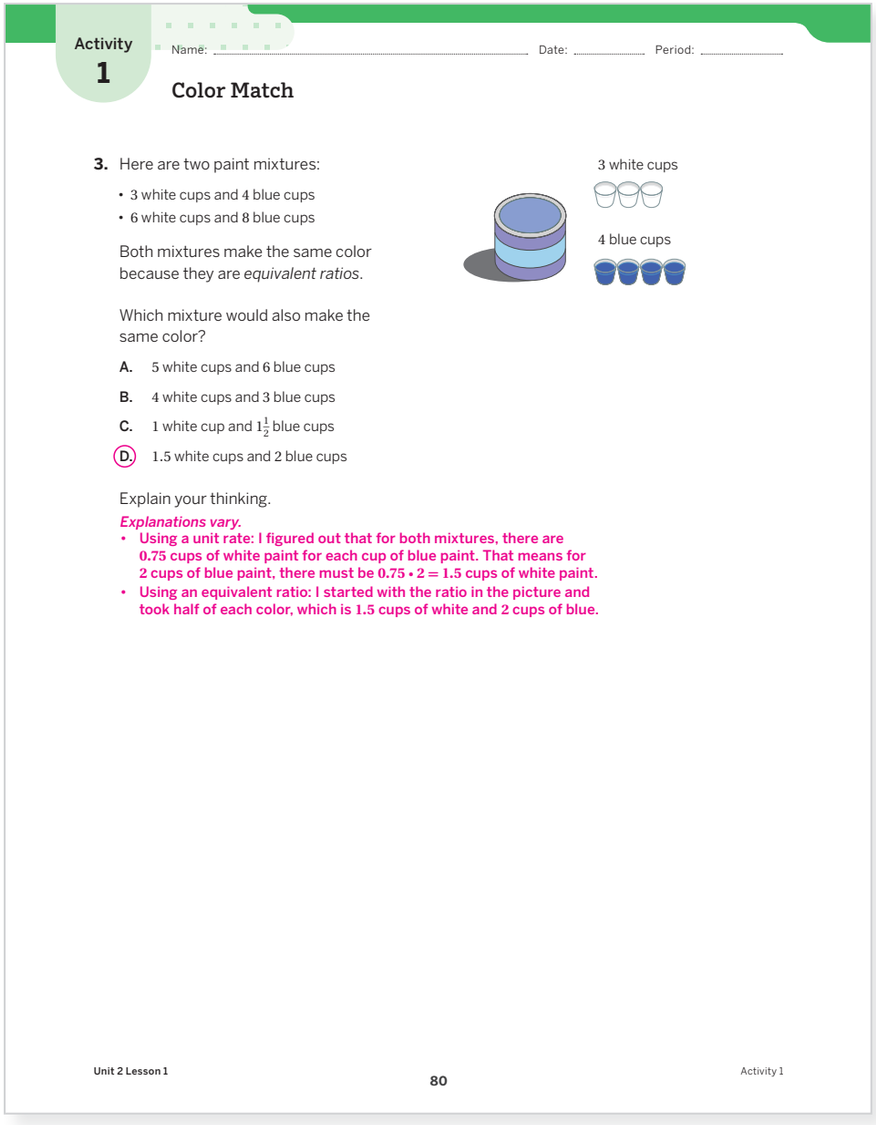
Look for students who:	Respond to Student Thinking
Select "5 white cups and 6 blue cups" because both paint colors increased by 2 cups.	**Support** Invite students to reflect on the given equivalent ratios. Consider asking, "What is the relationship between these two ratios? What operation could be used to create equivalent ratios?"
Select "4 white cups and 3 blue cups" because it uses the same numbers as the given ratio.	**Support** Encourage students to consider which quantity is larger. Consider asking, "Will a color that uses more white paint than blue look the same as one that uses more blue paint than white?"
Divide the amounts in the given ratio by 2 to generate an equivalent ratio.	**Strengthen** Consider asking, "How is this related to the strategy of multiplying by 2 to find an equivalent ratio?"
Use a unit rate to decide which mixture is equivalent.	**Stretch** Consider asking, "How could you change one value in each of the other mixtures so that they are all equivalent?"

3 Connect

Invite students to write a definition of the phrase *equivalent ratios* in their own words.

MLR1: Stronger and Clearer Each Time Use this routine to support students in developing a precise definition of *equivalent ratios*. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second, clearer draft. **(MTR.4.1)**

Create a class definition of the phrase *equivalent ratios* based on student language. Record this definition for students to refer to throughout the lesson and unit.



Activity 1

Name: _____ Date: _____ Period: _____

Color Match

3. Here are two paint mixtures:

- 3 white cups and 4 blue cups
- 6 white cups and 8 blue cups

Both mixtures make the same color because they are *equivalent ratios*.

3 white cups

4 blue cups

Which mixture would also make the same color?

A. 5 white cups and 6 blue cups

B. 4 white cups and 3 blue cups

C. 1 white cup and $1\frac{1}{2}$ blue cups

D. 1.5 white cups and 2 blue cups

Explain your thinking.

Explanations vary.

- Using a unit rate: I figured out that for both mixtures, there are 0.75 cups of white paint for each cup of blue paint. That means for 2 cups of blue paint, there must be $0.75 \cdot 2 = 1.5$ cups of white paint.
- Using an equivalent ratio: I started with the ratio in the picture and took half of each color, which is 1.5 cups of white and 2 cups of blue.

Unit 2 Lesson 1 80 Activity 1

Pairs | 10 min

Activity 2 Paint Palooza

Purpose: Students use tables to generate equivalent ratios in the context of matching a given paint color.

Short on time? Consider omitting Problem 4.

1 Launch

To surface connections, consider asking, "How is this problem different from Problems 2 and 3? How is it alike?"

Encourage students to refer to the class definition of *equivalent ratios*.

2 Monitor

Select and sequence a variety of student strategies, including those that show division expressions in the tables.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to guess and use the feedback to think of a strategy. Invite students to use a calculator to support them with performing calculations.
Use estimation and responsive feedback to complete the table.	**Strengthen** Celebrate this strategy. Consider asking, "How could you use equivalent ratios to confirm your answers?"
Divide the given paint amounts by 4 (Problem 4) or 3 (Problem 5) to determine the unknown amounts of red and white paint.	**Stretch** Encourage these students to make a connection with scale factors. Consider asking, "What scale factor are you using to go from the original paint mixture to the new one?"

3 Connect

Invite students to share their strategies.

A Accessibility: Executive Functioning
To support students in keeping track of information, create an anchor chart that describes strategies for calculating equivalent ratios for future reference.

To surface the Key Takeaway, consider asking:

- "How did you figure out how many cups of red paint you need? White paint?"
- "What are some ways to check that the mixtures will make the same color?"
- "Why is it important for us to know that the mixtures are equivalent?"

Activity 2

Name: _____ Date: _____ Period: _____

Paint Palooza

4. Darryl mixed 4 cups of white paint with 6 cups of red paint, but he didn't have enough to finish painting his wall.

How much red paint would he need to add to 1 cup of white paint to match the color?

White Paint (cups)	Red Paint (cups)
4	6
1	$\frac{6}{4}$ or 1.5 (or equivalent)



5. Brielle ran out of paint for her room.

Complete the table so that the new mixture matches the original paint color.

Blue Paint (cups)	Red Paint (cups)	White Paint (cups)
12	9	14
4	3	$\frac{14}{3}$ (or equivalent)



Unit 2 Lesson 1 81 Activity 2

Key Takeaway: Using equivalent ratios allows us to match one mixture to another, such as mixing a batch of paint that is the same color as another.

Independent | 10 min

Activity 3 Colorful Challenge

Purpose: Students explore equivalent ratios by creating their own paint color matching challenge and solving classmates' challenges.

1 Launch

To generate excitement, share that students will create their own challenge similar to the previous problem.

Demonstrate how students can create their paint color challenge.

Note: Students will need one or more partners for this activity. Distribute one Activity 3 Sheet to each student so they can create their paint color challenge.

Invite students to make a challenge they think no one in their class will be able to solve.

A **Accessibility: Memory and Attention** Invite students to use additional paper or a calculator to support them with performing calculations.

2 Monitor

Listen for strategies students use in discussing how to solve the problems, such as looking for a scale factor between columns of the table, identifying the ratio between two colors in one row and determining an equivalent ratio, or using a unit rate.

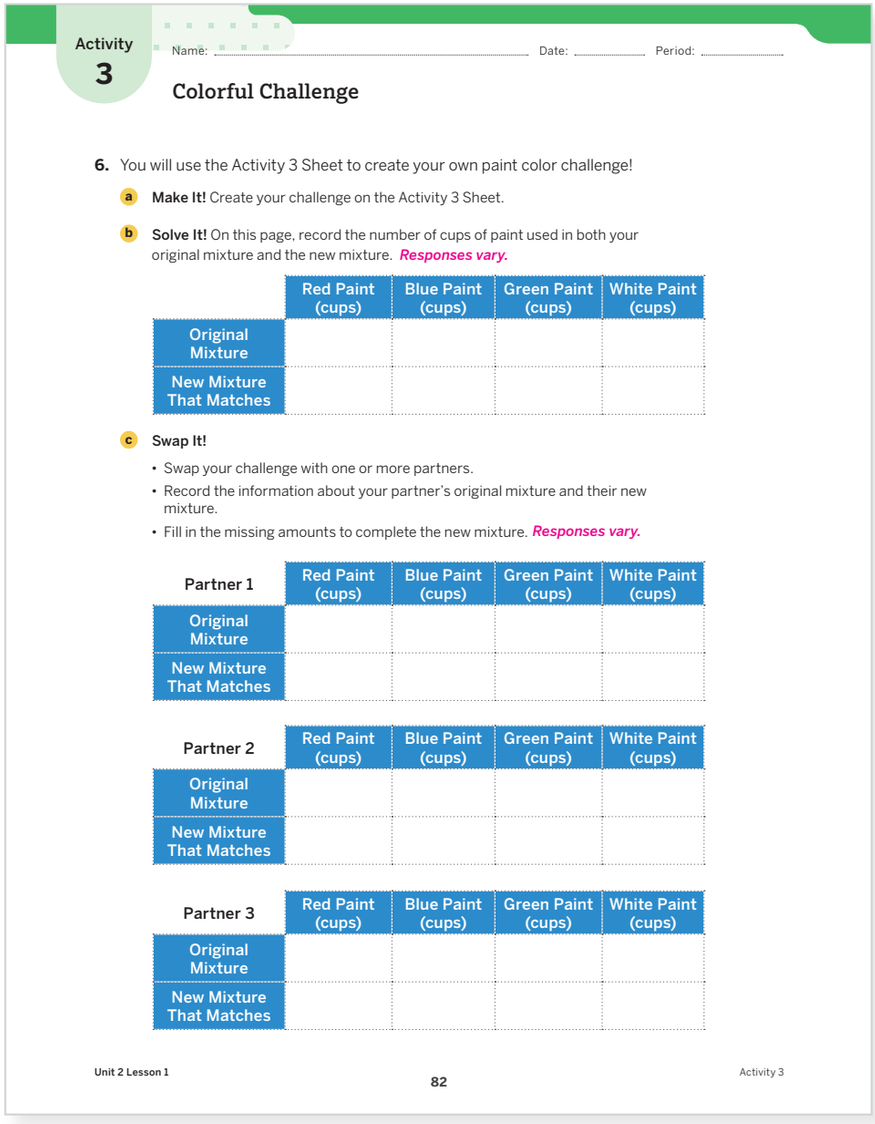
Look for several creative challenges and solutions to share during the Connect.

3 Connect

Share student paint color challenges. Invite students to share how they decided what new amount to use in the second row of their challenge table.

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "Which challenges seemed more complex than others? What strategies did you use to approach them?"
- "Was there a strategy you heard that surprised you?"



Activity 3

Name: _____ Date: _____ Period: _____

Colorful Challenge

6. You will use the Activity 3 Sheet to create your own paint color challenge!

a **Make It!** Create your challenge on the Activity 3 Sheet.

b **Solve It!** On this page, record the number of cups of paint used in both your original mixture and the new mixture. *Responses vary.*

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

c **Swap It!**

- Swap your challenge with one or more partners.
- Record the information about your partner's original mixture and their new mixture.
- Fill in the missing amounts to complete the new mixture. *Responses vary.*

Partner 1	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Partner 2	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Partner 3	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Unit 2 Lesson 1 82 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for creating equivalent ratios.

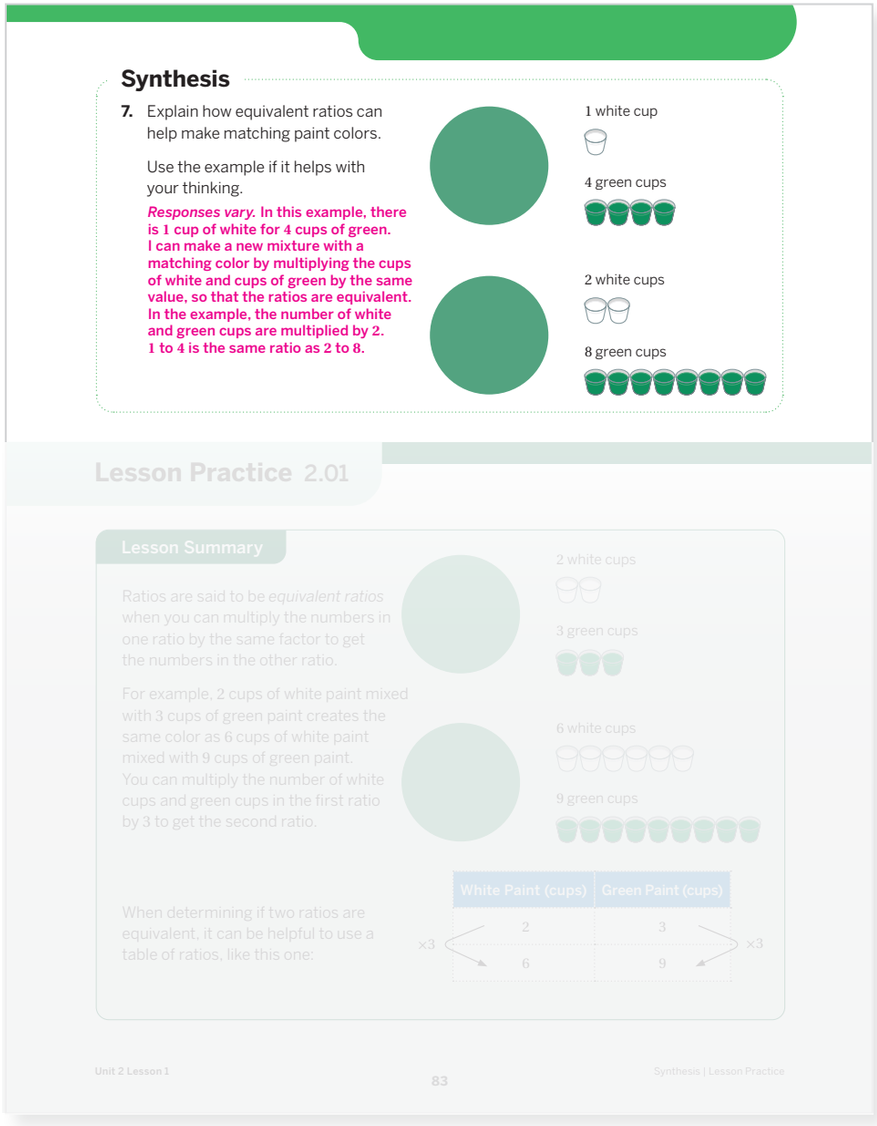
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Multiplying both values in a ratio by the same number will create an equivalent ratio.
- Dividing both values in a ratio by the same number will create an equivalent ratio.
- Finding how much of one color you need for every 1 cup of the other color can help you create a lot of new paint mixtures that are all the same color.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *equivalent ratios* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

7. Explain how equivalent ratios can help make matching paint colors.

Use the example if it helps with your thinking.

Responses vary. In this example, there is 1 cup of white for 4 cups of green. I can make a new mixture with a matching color by multiplying the cups of white and cups of green by the same value, so that the ratios are equivalent. In the example, the number of white and green cups are multiplied by 2. 1 to 4 is the same ratio as 2 to 8.

1 white cup

4 green cups

2 white cups

8 green cups

Lesson Practice 2.01

Lesson Summary

Ratios are said to be *equivalent ratios* when you can multiply the numbers in one ratio by the same factor to get the numbers in the other ratio.

For example, 2 cups of white paint mixed with 3 cups of green paint creates the same color as 6 cups of white paint mixed with 9 cups of green paint. You can multiply the number of white cups and green cups in the first ratio by 3 to get the second ratio.

2 white cups

3 green cups

6 white cups

9 green cups

When determining if two ratios are equivalent, it can be helpful to use a table of ratios, like this one:

White Paint (cups)	Green Paint (cups)
2	3
6	9

×3 ×3

Unit 2 Lesson 1 83 Synthesis | Lesson Practice

Lesson Takeaway: There are many ways to create *equivalent ratios*, including multiplying the numbers in a ratio by the same factor or dividing the numbers by the same factor.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of equivalent ratios by matching different amounts of paint that result in the same color.

Today's Goals

1. **Goal:** Understand and generate equivalent ratios.
2. **Language Goal:** Justify whether two ratios are equivalent. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.01

Group together the mixtures that make the same color of paint.

A. 12 cups white, 15 cups red

B. 4 cups white, 5 cups red

C. 3 cups white, 4 cups red

D. $\frac{3}{4}$ cups white, 1 cup red

E. 1 cup white, 1.25 cups red

Group 1	Group 2
A, B, E	C, D

D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to generate equivalent ratios:

» **Respond to Student Thinking:**

- Revisit Grade 6, Unit 2, Lesson 3, Activity 2 (Fruit Lab).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students expanded on their prior knowledge of equivalent ratios. What strengths did your students exhibit today? How can you build on those strengths to support their developing understanding of proportional relationships?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Explain how equivalent ratios can help make matching paint colors.

Use the example if it helps with your thinking.

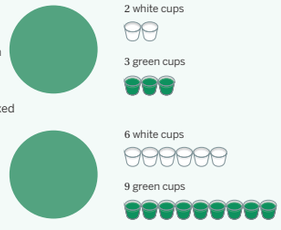
Responses vary. In this example, there is 1 cup of white for 4 cups of green. I can make a new mixture with a matching color by multiplying the cups of white and cups of green by the same value, so that the ratios are equivalent. In the example, the number of white and green cups are multiplied by 2. 1 to 4 is the same ratio as 2 to 8.

Lesson Practice 2.01

Lesson Summary

Ratios are said to be *equivalent ratios* when you can multiply the numbers in one ratio by the same factor to get the numbers in the other ratio.

For example, 2 cups of white paint mixed with 3 cups of green paint creates the same color as 6 cups of white paint mixed with 9 cups of green paint. You can multiply the number of white cups and green cups in the first ratio by 3 to get the second ratio.



When determining if two ratios are equivalent, it can be helpful to use a table of ratios, like this one:

	White Paint (cups)	Green Paint (cups)	
×3	2	3	×3
	6	9	

Lesson Practice 2.01

Name: _____ Date: _____ Period: _____

Problems 1–5: The table shows an original recipe for orange-pineapple juice, as well as four other recipes.

Recipe	Orange Juice (cups)	Pineapple Juice (cups)
Original	10	15
Recipe A	4	6
Recipe B	3	2
Recipe C	9	12
Recipe D	6	9

1. Select *all* the recipes that taste the same as the original.
   - ☑ A. Recipe A
   - ☐ B. Recipe B
   - ☐ C. Recipe C
   - ☑ D. Recipe D

2. Choose one recipe that tastes the same as the original and explain your thinking.
   Responses vary. Recipe A uses 1.5 cups of pineapple juice for every cup of orange juice, just like the original.

3. Choose a recipe that does not taste the same as the original. How do the two recipes compare?
   Responses vary. Recipe B uses 1.5 cups of orange juice for every cup of pineapple juice, so it would taste much more orangey than the original recipe.

4. How much pineapple juice would you need to mix with 1 cup of orange juice to make a mixture that tastes the same as the original?
   1.5 cups

5. Complete the table to create a different recipe that will also taste the same as the original.
   Responses vary. Sample shown in table.

Orange Juice (cups)	Pineapple Juice (cups)
12	18

Lesson Practice 2.01

Name: _____ Date: _____ Period: _____

6. Brielle mixed 6 cups of blue paint with 3 cups of white paint to make the perfect color to paint her mural.

   Complete the table to show several other ways to make this color.

Blue Paint (cups)	White Paint (cups)
6	3
8	4
12	6
30	15

7. Determine whether $\frac{6}{14}$ and $\frac{9}{21}$ are equivalent ratios.
   Yes, they are equivalent.

8. Create equivalent ratios. Fill in each blank using the digits 0 to 9 as many times as you want.
   Responses vary.

   6 : 2 = 1 8 : 6 = 3 6 : 1 2

FAST Practice

9. Select *all* the values that are *not* equivalent to $\frac{8}{10}$.
   - ☐ A. $\frac{16}{20}$
   - ☐ B. 0.8
   - ☑ C. $\frac{12}{20}$
   - ☑ D. $\frac{36}{40}$
   - ☐ E. $\frac{20}{25}$

Spiral Review

10. A grocery store has three types of fish for sale. Carp costs $5 for 3 pounds, mullet costs $4 for 2 pounds, and herring costs $3 for 2 pounds. Which fish costs the least per pound?
    Herring

    Show or explain your thinking.
    Explanations vary. $5 \div 3 = 1.66666$, so the carp costs $1.67 per pound. $4 \div 2 = 2$, so the mullet costs $2.00 per pound. $3 \div 2 = 1.5$, so the herring costs $1.50 per pound.

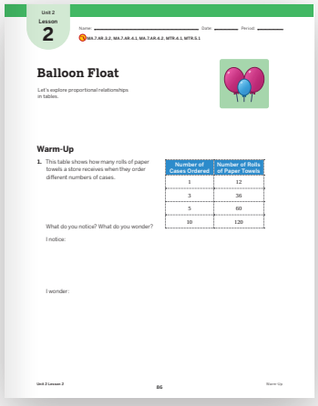
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.7.AR.3.2
	7	1	MA.7.AR.3.2
	8	3	MA.7.AR.3.2
FAST Practice	9	2	MA.7.AR.3.2
Spiral Review			
	10	2	MA.6.AR.3.5

Unit 2

Lesson

2


Unit 2
Lesson
2
Name: _____ Date: _____ Period: _____
MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MTR.4.1, MTR.5.1
Balloon Float
Let's explore proportional relationships in tables.
Warm-Up
1. This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?
I notice:
I wonder:
Unit 2 Lesson 2
86
Warm-Up

Balloon Float

Introducing Proportional Relationships With Tables

Let's explore proportional relationships in tables.

Learning Outcomes

Today's Goals

1. **Goal:** Use a table to calculate unknown quantities in a *proportional relationship*.
2. **Goal:** Determine whether two quantities are in a *proportional relationship*.
3. **Language Goal:** Describe patterns in tables that represent *proportional relationships*. **(Reading, Writing, Speaking, and Listening)**

Students explore **proportional relationships** by analyzing tables of equivalent ratios. Students learn that they can use patterns of multiplicative strategies to determine unknown values for quantities that are in a proportional relationship. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 1, students began to build their understanding of proportional relationships by exploring equivalent ratios of paint color mixtures.

Future Learning

In Lesson 3, students will learn about constants of proportionality and use them to compare relationships.

Connections

- Students apply proportional reasoning to determine if two quantities are in a proportional relationship in real world contexts such as the weight of an object and number of balloons needed to keep it afloat. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

Also Addressing: **MA.7.AR.4.2**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.3	**MA.7.AR.4.3**
MA.6.AR.3.5	**MA.7.AR.4.4**
	MA.7.AR.4.5
	MA.8.AR.3.1

Vocabulary

New Vocabulary

proportional relationship

Lesson at a Glance

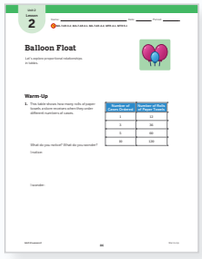
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2

Warm-Up

Pairs | 5 min

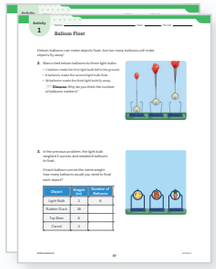
Students use the **Notice and Wonder** routine to identify patterns among related quantities in a table.



Activity 1

Pairs | 10 min

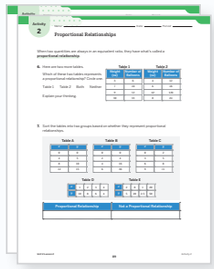
Students develop strategies for completing a table that represents a **proportional relationship**. **MLR7: Compare and Connect**



Activity 2

Pairs | 20 min

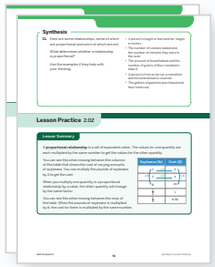
Students are introduced to the term *proportional relationship* and apply their understanding to analyze several tables. **MLR2: Collect and Display**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to determine whether a relationship is proportional.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by completing a table to show a proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to identify patterns among related quantities in a table.

Short on time? Consider omitting Problem 1 and using the discussion in Activity 1 Problem 2 as a Warm-Up for this lesson.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the table. Encourage students to look for structure and patterns in the rows and columns of the table. **(MTR.5.1)**

2 Connect

Invite students to share what they noticed and wondered.

Consider asking, "Did anyone notice that the number of cases increases as we move down the table? Does this make you wonder whether the table could be extended forever?"

Note: If it doesn't come up naturally, consider asking students what patterns they notice both *across* and *down* the table. Consider introducing one or more as noticings from a student in a previous class or year.

ELL **English Language Learners** Create a list of words used in reference to tables that students might not know, such as *row*, *column*, *horizontal*, and *vertical*. Include visuals or translations of each word in languages used by your students. **(Reading and Listening)**

To surface the Key Takeaway, consider sharing that this lesson will involve studying tables that represent a variety of contexts. Consider asking, "What are the key features you look at when you are analyzing a table?"

Unit 2
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MTR.4.1, MTR.5.1

Balloon Float



Let's explore proportional relationships in tables.

Warm-Up

1. This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?

I notice:

Responses vary.
- I notice that each row is an equivalent ratio to the other rows.
- I notice there are 12 rolls of paper towels in every case.
- I notice that if I multiply every number in the first column by 12, I get the number in the second column.

I wonder:

Responses vary.
- I wonder what's the largest number of cases that can be ordered.
- I wonder if the relationship between number of cases and number of rolls of paper towels would continue forever.

Unit 2 Lesson 2 86 Warm-Up

Key Takeaway: You can use a table to represent a relationship between two quantities. When analyzing a table, you can look for patterns between the rows and between the columns.

Pairs | 10 min

Activity 1 Balloon Float

Purpose: Students develop strategies for completing a table that represents a **proportional relationship**.

1 Launch

To surface prior knowledge, consider asking students what experiences they have had with helium balloons.

Consider asking, "What do you think it means for an object to float?" Floating means the object will stay in place, not falling to the ground or flying away.

Invite students to describe what is happening in the picture.

2 Monitor

A Accessibility: Visual-Spatial Processing
Provide additional printed copies of the table for students to draw on or highlight.

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which of the patterns we noticed in the Warm-Up might be useful here?"
Identify the relationship between the columns of the table by multiplying each weight by 3 to determine the number of balloons.	Consider capturing the work of these students as the basis for the Connect, rather than that of the sample students. **Stretch** Consider asking, "What if we saw an object that floated with 27 balloons, and we needed to determine its weight? How would you apply your strategy?"
Identify a scale factor between rows of the table, then apply that scale factor to determine unknown weights.	
Identify that an object weighing 1 ounce would need 3 balloons to float.	**Stretch** Consider asking, "Why is it helpful to determine the number of balloons needed for a 1 ounce object?"

ELL English Language Learners Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Balloon Float

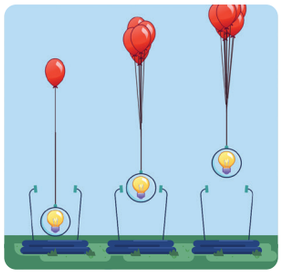
Helium balloons can make objects float, but too many balloons will make objects fly away!

2. Marco tied helium balloons to three light bulbs.

- 1 balloon made the first light bulb fall to the ground.
- 6 balloons made the second light bulb float.
- 10 balloons made the third light bulb fly away.

Discuss: Why do you think the number of balloons matters?

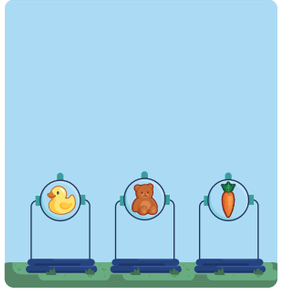
Responses vary. If there aren't enough balloons, the light bulb will fall to the ground. And if there are too many balloons, the light bulb will fly away. The number of balloons has to be just right to make the light bulb float.



3. In the previous problem, the light bulb weighed 2 ounces and needed 6 balloons to float.

If each balloon carries the same weight, how many balloons would you need to float each object?

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	18
Carrot	3	9



Unit 2 Lesson 2 87 Activity 1

Pairs | 10 min

Activity 1 Balloon Float (continued)

Purpose: Students develop strategies for completing a table that represents a proportional relationship.

2 Monitor

Listen for a variety of student thinking to highlight during the Connect, including:

- Using equivalent ratios to represent the relationships between the columns or rows.
- Using multiplication (or division) to determine the number of balloons.

Note: Students might use terms such as *scale factor* or *unit rate*, but it is not necessary at this point. Vocabulary for describing these relationships will be formalized later in the lesson and the unit.

D **Differentiation: (Support)** Support students getting started by encouraging them to look back at Ariel's and Emma's strategies. Consider asking, "Could you use these strategies to complete the table? Which do you think would be more helpful?"

Look for students who move flexibly between different strategies to complete the table, such as scaling up the light bulb row to find the weight of the cake and multiplying the weight of the banana by 3.

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their work in Problem 3. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:

- "Why does Ariel's strategy work? Emma's?"
- "How are Ariel's and Emma's strategies alike? How are they different?"
- "When might you use each strategy?"

Activity 1

Name: _____ Date: _____ Period: _____

Balloon Float (continued)

4. Here are two strategies for determining the number of balloons needed to make the rubber duck float.

Ariel

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	
Carrot	3	

Emma

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	
Carrot	3	

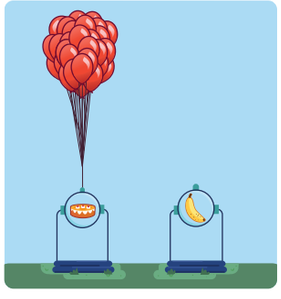
Discuss: How might Ariel and Emma use their strategies to finish their tables?

Responses vary.

- Ariel might finish the table by scaling the numbers in the first row (2 ounces and 6 balloons). To determine how many balloons would make the toy bear float, she would need to multiply by a scale factor of 3, which would give her 18 balloons. To determine the number of balloons for the carrot, she would need to multiply by a scale factor of 1.5, which would give her 9 balloons.
- Emma might finish the table by multiplying each weight by 3 to get the number of balloons. The unit rate (balloons per ounce) is 3, which tells us how many balloons are required to carry 1 ounce of weight.

5. Here are some new objects. Complete the table so that each object floats.

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Cake	20	60
Banana	$3\frac{1}{3}$	10



Unit 2 Lesson 2 88 Activity 1

Key Takeaway: To complete a table showing a proportional relationship between two quantities, we look for multiplicative patterns between the rows or columns of the table.

Pairs | 20 min

Activity 2 Proportional Relationships

Purpose: Students are introduced to the term *proportional relationship* and apply their understanding to analyze several tables.

1 Launch

Invite students to read the definition of **proportional relationship** and rephrase it in their own words. Consider sharing that the relationship between balloons and ounces in Activity 1 is proportional.

Encourage students to refer to the class definition of *equivalent ratios*.

Listen for a variety of ideas, such as:
- Equivalent ratios are created by multiplying, so we should look for a multiplicative pattern.
- Scaled copies also have equivalent ratios. A proportional relationship is like scaling.
- The relationship should stay the same. From one quantity to another, we always see the same ratio.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and language as they discuss with a partner. Display for students to reference and add to for the rest of the unit.

D **Differentiation: (Support)** Students may think Table 1 shows a proportional relationship because the number of balloons is always 3 more than the weight. Encourage these students to check for equivalent ratios in each row of the table. Consider asking, "How many balloons do you need to hold each ounce?"

Invite students to share their reasoning.

Note: Spend adequate time on this discussion to ensure that all students notice the multiplicative relationships in the proportional table, including both the constant ratio between columns and the scaling relationship between rows.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the tables.

D **Differentiation: (Support)** Encourage students to relate the tables to the balloon and weight context. Consider asking, "If the columns were labeled ounces and balloons, what would each row mean? Do the values make sense for a proportional relationship?"

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Proportional Relationships

When two quantities are always in an equivalent ratio, they have what's called a **proportional relationship**.

6. Here are two more tables.

Which of these two tables represents a proportional relationship? Circle one.

Table 1 (Table 2) Both Neither

Explain your thinking.

Explanations vary. The number of balloons is always 3 times the weight in ounces. That means they're always in a 3 to 1 ratio, which means their relationship is proportional.

Table 1

Weight (oz)	Number of Balloons
3	6
7	10
9	12
30	33

Table 2

Weight (oz)	Number of Balloons
4	12
6	18
42	126
8	24

7. Sort the tables into two groups based on whether they represent proportional relationships.

Table A

x	y
0	0
4	5
8	10
12	15

Table B

x	y
0	0
2	4
4	16
6	36

Table C

x	y
0	2
3	5
6	8
9	11

Table D

x	1	2	3	4
y	10	8	6	4

Table E

x	2	8	1	20
y	5	20	2.5	50

Proportional Relationship	Not a Proportional Relationship
Table A, Table E	Table B, Table C, Table D

Unit 2 Lesson 2 89 Activity 2

Pairs | 20 min

Activity 2 Proportional Relationships (continued)

Purpose: Students are introduced to the term *proportional relationship* and apply their understanding to analyze several tables.

2 Monitor

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Think the relationship is proportional because each x-value is multiplied by itself to obtain the y-value.	**Support** Celebrate the structure these students noticed. Consider asking, "Does each row form an equivalent ratio to the others?"
Compare rows of the table to identify that the relationship is not proportional, e.g., noticing that $2 \cdot 3 = 6$, but $4 \cdot 3 \neq 36$.	**Strengthen** Celebrate this thinking. Consider asking, "What is another way we could check that this relationship is not proportional?"
Notice that no single number can be used to multiply the numbers in one column to get the numbers in the other column.	**Stretch** Consider asking, "How would you describe the relationship in the table?"

Listen for the assumptions students make about each situation, such as that the marathon runner maintains a consistent pace during the race. Select and sequence student ideas to share during the Connect.

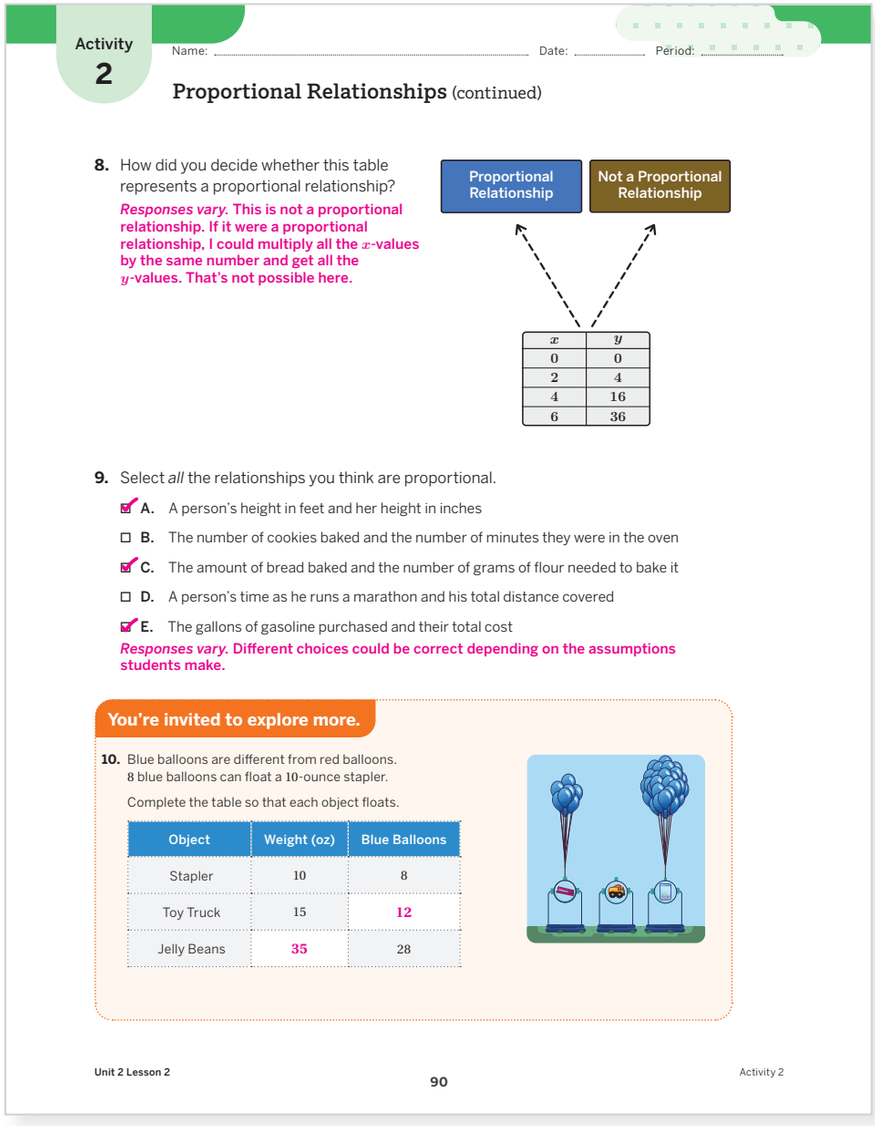
D Differentiation: You're invited to explore more. **(Stretch)** Invite students who would like to explore tables representing proportional relationships further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their responses and discuss each option.

Consider asking, "What criteria did you use to decide whether a relationship is proportional?" *Responses vary.* In a proportional relationship, there must be a consistent ratio. The relationship does not change over time.

Note: It is okay — even desirable — to lack consensus on every option. Use the discussion to surface what assumptions students made and clarify the essential features of a proportional relationship.



Activity 2

Name: Date: Period:

Proportional Relationships (continued)

8. How did you decide whether this table represents a proportional relationship?

Responses vary. This is not a proportional relationship. If it were a proportional relationship, I could multiply all the x-values by the same number and get all the y-values. That's not possible here.

Proportional Relationship | Not a Proportional Relationship

x	y
0	0
2	4
4	16
6	36

9. Select *all* the relationships you think are proportional.

- ☑ **A.** A person's height in feet and her height in inches
- ☐ **B.** The number of cookies baked and the number of minutes they were in the oven
- ☑ **C.** The amount of bread baked and the number of grams of flour needed to bake it
- ☐ **D.** A person's time as he runs a marathon and his total distance covered
- ☑ **E.** The gallons of gasoline purchased and their total cost

Responses vary. Different choices could be correct depending on the assumptions students make.

You're invited to explore more.

10. Blue balloons are different from red balloons. 8 blue balloons can float a 10-ounce stapler.

Complete the table so that each object floats.

Object	Weight (oz)	Blue Balloons
Stapler	10	8
Toy Truck	15	12
Jelly Beans	35	28

Unit 2 Lesson 2 90 Activity 2

Key Takeaway: In a proportional relationship, the two quantities always form an equivalent ratio. If the ratio changes, the relationship is not proportional.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to determine whether a relationship is proportional.

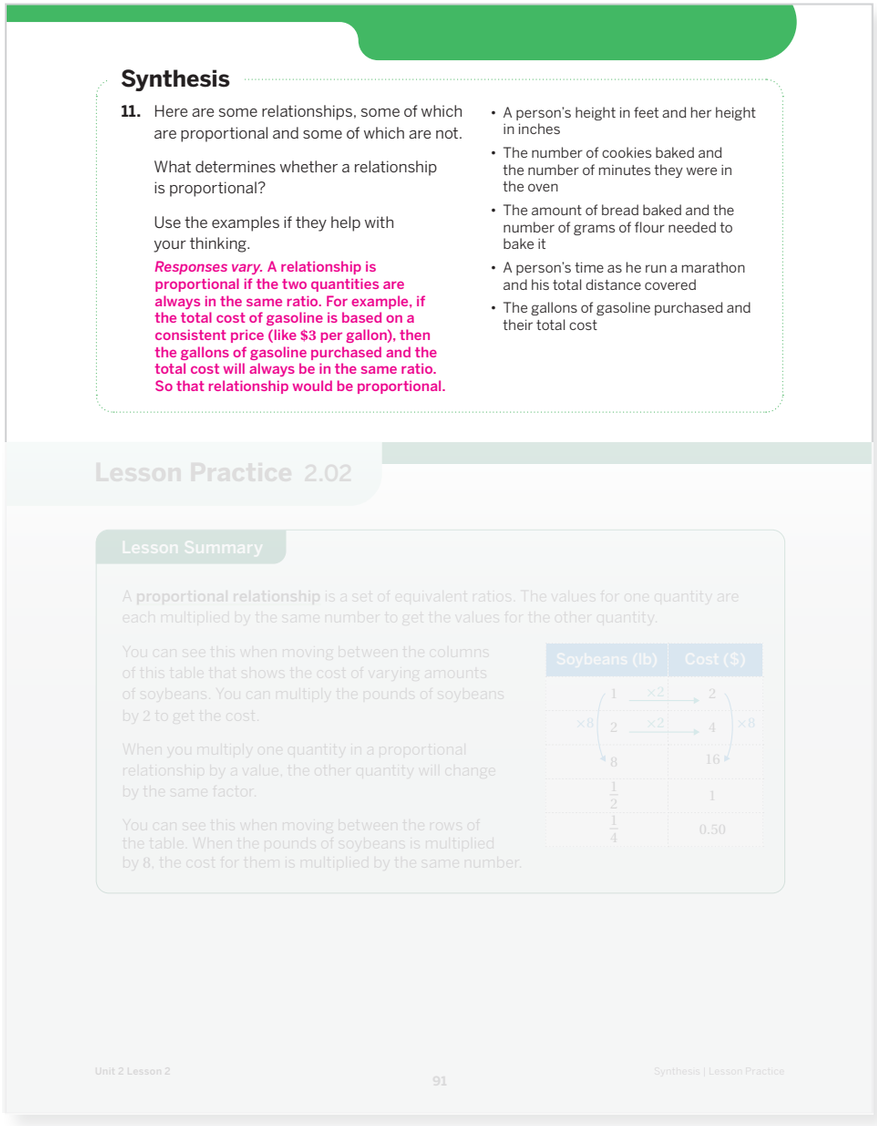
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- A relationship is proportional if the two quantities are always in the same ratio.
- If the rate in a situation is changing, like a runner who takes a break to walk, the relationship is not proportional.
- A relationship is proportional if you can double one quantity and the other quantity also doubles.
- If one quantity stays the same while the other changes, the relationship is not proportional. When baking, if you decide to make more cookies, you don't need to change the oven temperature.

Encourage students to add their ideas and language to the display about proportional relationships.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here are some relationships, some of which are proportional and some of which are not.

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary. **A relationship is proportional if the two quantities are always in the same ratio. For example, if the total cost of gasoline is based on a consistent price (like $3 per gallon), then the gallons of gasoline purchased and the total cost will always be in the same ratio. So that relationship would be proportional.**

- A person's height in feet and her height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as he run a marathon and his total distance covered
- The gallons of gasoline purchased and their total cost

Lesson Practice 2.02

Lesson Summary

A **proportional relationship** is a set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by 8, the cost for them is multiplied by the same number.

Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Unit 2 Lesson 2 91 Synthesis | Lesson Practice

Lesson Takeaway: Two quantities are in a **proportional relationship** when they change together in a multiplicative pattern. When one quantity is multiplied, the other is multiplied by the same factor. This also creates equivalent ratios.

Independent | 5 min

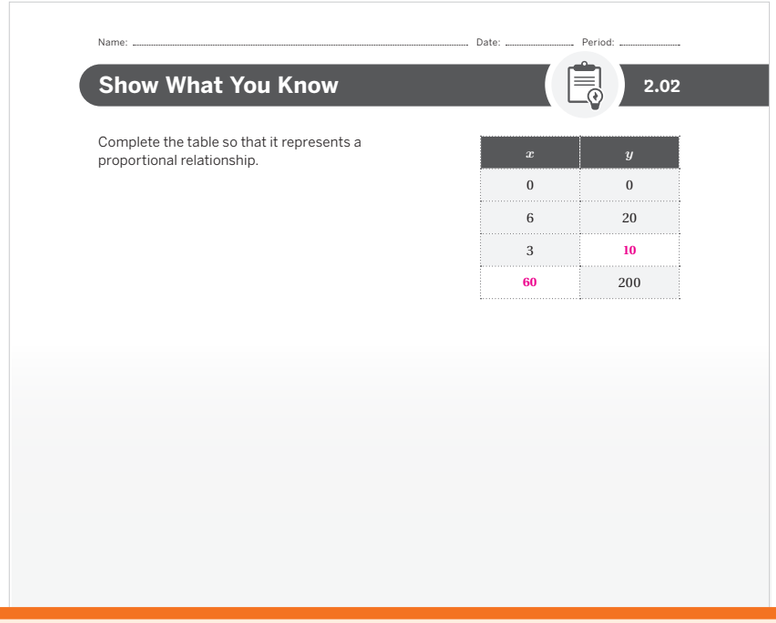
Show What You Know

Purpose: Students demonstrate their understanding by completing a table to show a proportional relationship.

Today's Goals

1. **Goal:** Use a table to calculate unknown quantities in a *proportional relationship*.
2. **Goal:** Determine whether two quantities are in a *proportional relationship*.
3. **Language Goal:** Describe patterns in tables that represent *proportional relationships*. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.02

Complete the table so that it represents a proportional relationship.

x	y
0	0
6	20
3	10
60	200

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to complete a table to show a proportional relationship:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of this concept in Lesson 3. You may choose to not intervene at this time

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Reflect on your facilitation of the **MLR2: Collect and Display** routine in this lesson. How did students engage in the routine? How did your facilitation of the routine support student understanding of the term **proportional relationship**? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are some relationships, some of which are proportional and some of which are not.

- A person's height in feet and her height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as he run a marathon and his total distance covered
- The gallons of gasoline purchased and their total cost

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary. A relationship is proportional if the two quantities are always in the same ratio. For example, if the total cost of gasoline is based on a consistent price (like $3 per gallon), then the gallons of gasoline purchased and the total cost will always be in the same ratio. So that relationship would be proportional.

Lesson Practice 2.02

Lesson Summary

A **proportional relationship** is a set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by 8, the cost for them is multiplied by the same number.

Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Lesson Practice 2.02

Name: _____ Date: _____ Period: _____

Problems 1–4: Complete the tables so that the relationship is proportional.

1.

x	y
30	3
120	12
100	10

2.

x	y
1	1.5
3	4.5
8	12

3.

x	y
15	45
1	3
0	0

4.

x	y
0.2	1
1	5
4	20

5. Entrance to a state park costs $6 per vehicle, plus $2 per person. The table shows the entry cost for several recent groups of visitors. Is the relationship between the number of people and the total entrance cost a proportional relationship?

No

Explain your thinking.

Explanations vary. When I look at the ratio of people in the vehicle to the total entry cost, I can tell these are not equivalent ratios.

Number of People in Vehicle	Total Cost ($)
2	10
3	12
4	14
10	26

6. A store charges $4.80 for 16 ounces of bubble tea. Complete the table so that it shows a proportional relationship between ounces of tea and cost.

Response shown in table.

Tea (oz)	Cost ($)
16	4.80
20	6.00
24	7.20

Lesson Practice 2.02

Name: _____ Date: _____ Period: _____

FAST Practice

7. A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Complete the table so that it shows a proportional relationship between the amount of honey and the amount of flour.

Honey (tbsp)	Flour (cups)
8	10
20	25
13	16.25
10	12.5

Spiral Review

Problems 8–10: Solve each equation. Show your thinking. *Work varies.*

8. $\frac{1}{2} + x = 2$

$\frac{1}{2} + x - \frac{1}{2} = 2 - \frac{1}{2}$

$x = 1\frac{1}{2}$

9. $\frac{2}{3}y = 6$

$\frac{2}{3}y \div \frac{2}{3} = 6 \div \frac{2}{3}$

$y = 9$

10. $3 = \frac{1}{4}b$

$3 \div \frac{1}{4} = \frac{1}{4}b \div \frac{1}{4}$

$3 \cdot 4 = b$

$12 = b$

11. A student makes a scaled copy of a rectangle. The dimensions of the original rectangle are 3.5 by 2.5 meters. Select *all* the possible dimensions of the scaled copy.

- ☑ **A.** 1.75 meters and 1.25 meters
- ☑ **B.** 7 meters and 5 meters
- ☐ **C.** 7 meters and 6 meters
- ☐ **D.** 10 meters and 2.5 meters
- ☑ **E.** 10.5 meters and 7.5 meters

Lesson Practice Item Analysis

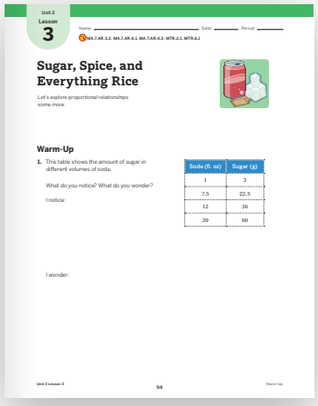
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.AR.4.2
	5	2	MA.7.AR.3.2, MA.7.AR.4.1
	6	2	MA.7.AR.3.2, MA.7.AR.4.2
FAST Practice	7	2	MA.7.AR.3.2, MA.7.AR.4.2
Spiral Review			
	8–10	1	MA.6.AR.2.2, MA.6.AR.2.3
	11	1	MA.7.GR.1.5

Unit 2
Lesson
3

Sugar, Spice, and Everything Rice

Constant of Proportionality

Let's explore proportional relationships some more.


Unit 2
Lesson
3

Sugar, Spice, and Everything Rice

Let's explore proportional relationships some more.

Warm-Up

1. This table shows the amount of sugar in different volumes of soda.

What do you notice? What do you wonder?

I notice:

Soda (fl. oz)	Sugar (g)
1	3
7.5	22.5
12	36
20	60

I wonder:

94

Learning Outcomes

Today's Goals

1. **Goal:** Determine the *constant of proportionality* for a proportional relationship represented in a table.
2. **Goal:** Use the *constant of proportionality* to determine values in a table.
3. **Language Goal:** Justify whether a table represents a proportional relationship, using terms such as *constant of proportionality*. **(Reading and Writing)**

Students determine, interpret, and use **constants of proportionality** to make sense of proportional relationships. They compare proportional relationships in context, using the constant of proportionality to support their reasoning. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 2, students observed equivalent ratios in tables and identified patterns representing proportional relationships. Students also used tables to determine unknown values for quantities in a proportional relationship.

Future Learning

In Lesson 4, students will write equations to represent proportional relationships.

Connections

- Students use the constant of proportionality to make sense of proportional relationships, such as comparing how "sugary" one drink is to another, modifying ingredients in a recipe, and more. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

Also Addressing: **MA.7.AR.3.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.7.AR.4.4**
MA.6.AR.3.3	**MA.7.AR.4.5**
MA.6.AR.3.5	**MA.8.AR.3.1**
	MA.8.AR.3.2

Vocabulary

New Vocabulary

constant of proportionality

Lesson at a Glance

~ 45 min

MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2

Warm-Up

Independent | 5 min

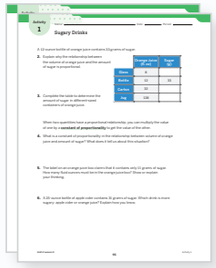
Students use the **Notice and Wonder** routine to make sense of the relationship between the volume of a soda and how many grams of sugar it contains.



Activity 1

Pairs | 15 min

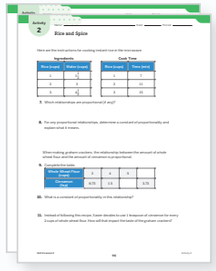
Students explore how the **constant of proportionality** can be helpful when analyzing a relationship. **MLR2: Collect and Display** **(MTR.2.1)**



Activity 2

Pairs | 15 min

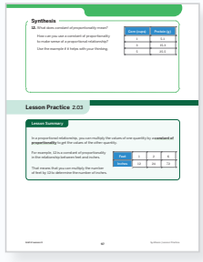
Students use constants of proportionality to compare cooking instructions and ingredients in other recipes.



Synthesis

Whole Class | 5 min

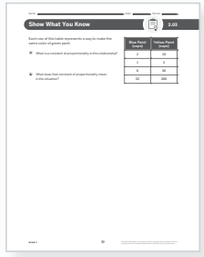
Students synthesize their understanding of using a constant of proportionality to make sense of relationships. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining and interpreting the constant of proportionality from a table.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- scientific calculators (as needed)

Independent | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of the relationship between the volume of a soda and how many grams of sugar it contains.

Short on time? Consider omitting the Warm-Up and using the Notice and Wonder routine to start Problem 2.

1 Launch

To surface connections, consider asking, "What do you know and wonder about soda?"

ELL **English Language Learners** Consider discussing the word soda and what other word(s) your students use to describe this product. In particular, invite students to respond in their primary languages. **(Writing and Speaking)**

Direct students' attention to the table.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between grams of sugar in different volumes of soda.

2 Connect

Consider asking:

- "What do you notice and wonder when observing the table?"
- "What are you noticing and wondering about the relationship between sugar and ounces of soda?"

Note: If it doesn't come up naturally, consider asking, "What happens as the amount of soda gets bigger?"

Unit 2
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MTR.2.1, MTR.6.1

Sugar, Spice, and Everything Rice

Let's explore proportional relationships some more.



Warm-Up

1. This table shows the amount of sugar in different volumes of soda.

Soda (fl. oz)	Sugar (g)
1	3
7.5	22.5
12	36
20	60

What do you notice? What do you wonder?

I notice:

Responses vary.

- As the volume increases, the amount of sugar also increases.
- You can multiply the number of fluid ounces by 3 to get the number of grams of sugar.
- One fluid ounce of soda contains 3 grams of sugar.
- The relationship between the volume of soda and the amount of sugar is proportional.

I wonder:

Responses vary.

- What is a typical serving size for soda?
- Could you figure out how much sugar is in any amount of soda?
- How does the amount of sugar in soda compare to the amount of caffeine it has?
- How does the amount of sugar in soda compare to the amount of sugar in other drinks?

Unit 2 Lesson 3 94 Warm-Up

Pairs | 15 min

Activity 1 Sugary Drinks

Purpose: Students explore how the **constant of proportionality** can be helpful when analyzing a relationship. **(MTR.2.1)**

1 Launch

To generate excitement, consider asking, "What do you think about orange juice?"

ELL **English Language Learners** Consider discussing the different containers and describing how they are connected to volume. **(Speaking, Writing, and Listening)**

2 Monitor

To support students getting started, consider asking, "How would the amount of sugar change as the bottle of orange juice gets bigger? Smaller?" **(MTR.6.1)**

A **Accessibility: Memory and Attention** Invite students to refer back to the previous lesson on proportional relationships to activate prior knowledge. Consider asking, "What do you remember from yesterday's lesson?"

Pause after most students have completed Problems 2 and 3, discuss the definition of **constant of proportionality**.

MLR **MLR2: Collect and Display** Use this routine to collect students' language as they consider the new term. Display for students to reference and add to for the rest of the unit.

ELL **English Language Learners** Invite students to share what the new term reminds them of (e.g., "When I hear *constant of proportionality*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

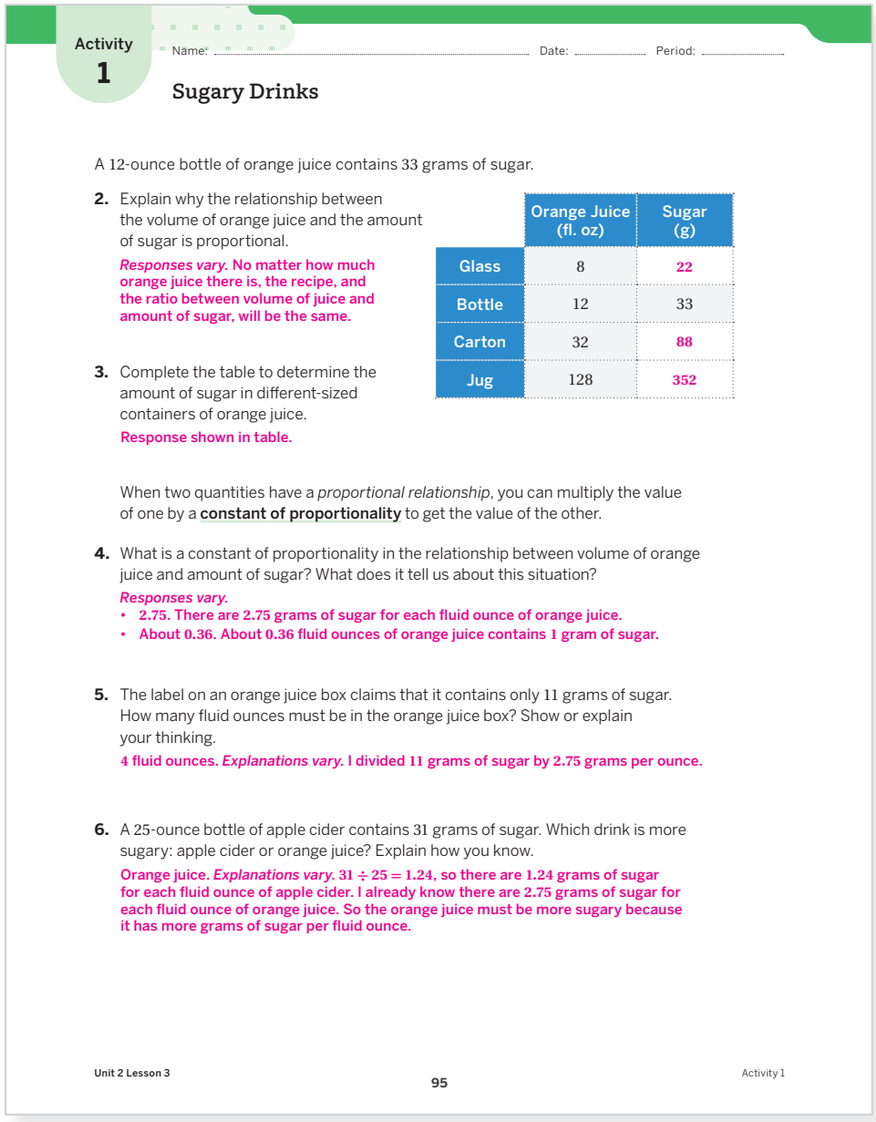
Look for students using different strategies to solve for and apply the constant of proportionality, such as:

- Utilizing the relationship across a row in a table to find a common multiplier.
- Solving for the amount of sugar for 1 ounce of orange juice and using this to scale up. **(MTR.2.1)**

3 Connect

Invite students to share their responses to Problem 6.

To surface the Key Takeaway, consider asking, "What can the constant of proportionality tell us about the taste of each beverage?"



Activity 1

Name: Date: Period:

Sugary Drinks

A 12-ounce bottle of orange juice contains 33 grams of sugar.

2. Explain why the relationship between the volume of orange juice and the amount of sugar is proportional.

Responses vary. No matter how much orange juice there is, the recipe, and the ratio between volume of juice and amount of sugar, will be the same.

3. Complete the table to determine the amount of sugar in different-sized containers of orange juice.

Response shown in table.

	Orange Juice (fl. oz)	Sugar (g)
Glass	8	22
Bottle	12	33
Carton	32	88
Jug	128	352

When two quantities have a *proportional relationship*, you can multiply the value of one by a **constant of proportionality** to get the value of the other.

4. What is a constant of proportionality in the relationship between volume of orange juice and amount of sugar? What does it tell us about this situation?

Responses vary.
- 2.75. There are 2.75 grams of sugar for each fluid ounce of orange juice.
- About 0.36. About 0.36 fluid ounces of orange juice contains 1 gram of sugar.

5. The label on an orange juice box claims that it contains only 11 grams of sugar. How many fluid ounces must be in the orange juice box? Show or explain your thinking.

4 fluid ounces. *Explanations vary.* I divided 11 grams of sugar by 2.75 grams per ounce.

6. A 25-ounce bottle of apple cider contains 31 grams of sugar. Which drink is more sugary: apple cider or orange juice? Explain how you know.

Orange juice. *Explanations vary.* $31 \div 25 = 1.24$, so there are 1.24 grams of sugar for each fluid ounce of apple cider. I already know there are 2.75 grams of sugar for each fluid ounce of orange juice. So the orange juice must be more sugary because it has more grams of sugar per fluid ounce.

Unit 2 Lesson 3 95 Activity 1

Key Takeaway: The **constant of proportionality** is the multiplier between two quantities in a proportional relationship. Given one value in the relationship, you can multiply by the constant of proportionality to find the other value.

Pairs | 15 min

Activity 2 Rice and Spice

Purpose: Students use constants of proportionality to compare cooking instructions and ingredients in other recipes.

1 Launch

Direct students' attention to the two tables.

To surface connections, consider asking:
- "What quantities do these tables tell us about?"
- "What experience do you have with rice?"

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

2 Monitor

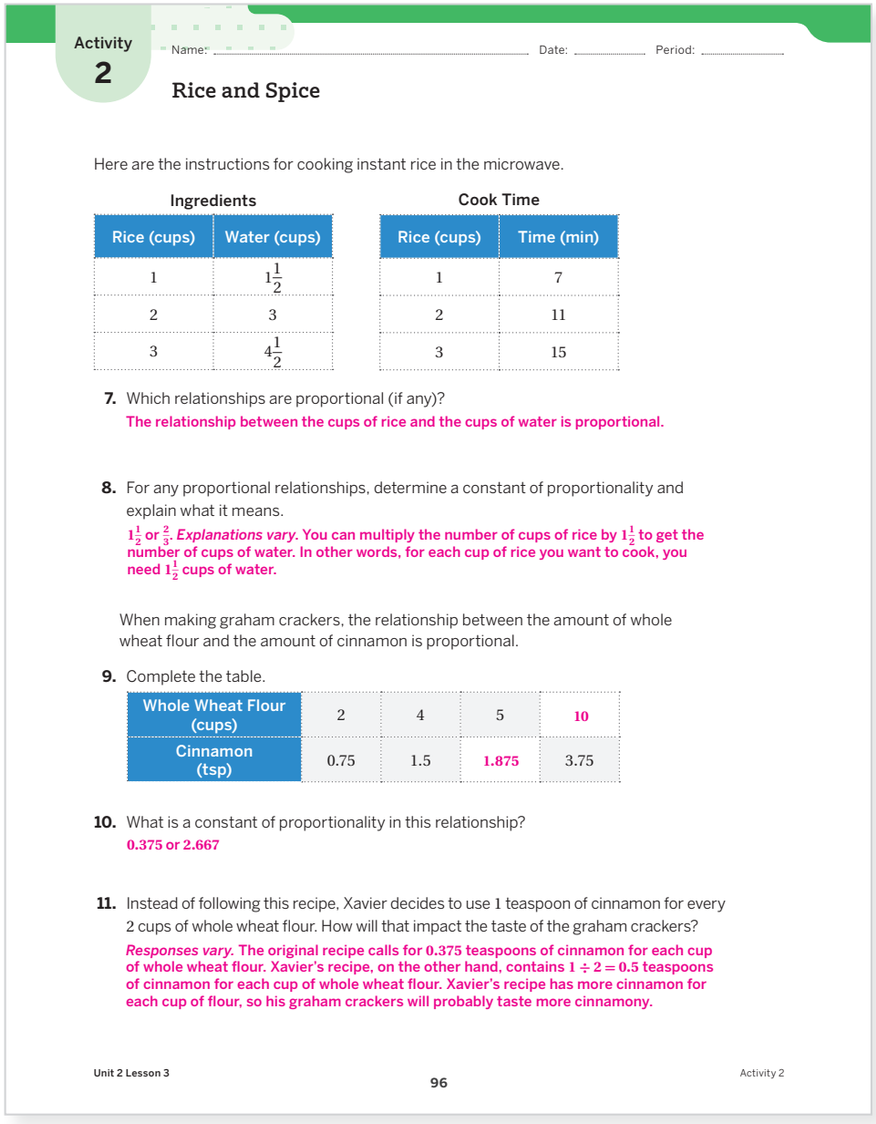
D **Differentiation** *(Problems 7–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What would *proportional* mean in each scenario?"
Think both tables are proportional. *(Problem 7)*	**Support** Invite them to revisit the class discussion on proportional relationships from the previous lesson.
Need support computing with fractions or decimals.	**Support** Consider asking, "What could you type in a calculator to determine that value?"
Use different strategies for determining and using the constant of proportionality, such as: • Utilizing the value that corresponds with 1 in the table. • Dividing values from the table to find a multiplier and confirming it is consistent.	**Strengthen** Invite these students to share during the Connect. Consider asking, "Can you think of an alternative strategy to solve for the constant of proportionality?" **(MTR.2.1)**

3 Connect

Invite students to share their solution for the constant of proportionality. Encourage students to make connections between the constants of proportionality and the taste of the graham crackers.

To surface connections, consider asking, "What is similar about our solutions? What is different?"



Activity 2

Name: _____ Date: _____ Period: _____

Rice and Spice

Here are the instructions for cooking instant rice in the microwave.

Ingredients

Rice (cups)	Water (cups)
1	$1\frac{1}{2}$
2	3
3	$4\frac{1}{2}$

Cook Time

Rice (cups)	Time (min)
1	7
2	11
3	15

7. Which relationships are proportional (if any)?

The relationship between the cups of rice and the cups of water is proportional.

8. For any proportional relationships, determine a constant of proportionality and explain what it means.

$1\frac{1}{2}$ or $\frac{2}{3}$. *Explanations vary.* You can multiply the number of cups of rice by $1\frac{1}{2}$ to get the number of cups of water. In other words, for each cup of rice you want to cook, you need $1\frac{1}{2}$ cups of water.

When making graham crackers, the relationship between the amount of whole wheat flour and the amount of cinnamon is proportional.

9. Complete the table.

Whole Wheat Flour (cups)	2	4	5	10
Cinnamon (tsp)	0.75	1.5	1.875	3.75

10. What is a constant of proportionality in this relationship?

0.375 or 2.667

11. Instead of following this recipe, Xavier decides to use 1 teaspoon of cinnamon for every 2 cups of whole wheat flour. How will that impact the taste of the graham crackers?

Responses vary. The original recipe calls for 0.375 teaspoons of cinnamon for each cup of whole wheat flour. Xavier's recipe, on the other hand, contains $1 \div 2 = 0.5$ teaspoons of cinnamon for each cup of whole wheat flour. Xavier's recipe has more cinnamon for each cup of flour, so his graham crackers will probably taste more cinnamony.

Unit 2 Lesson 3 96 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using a constant of proportionality to make sense of relationships.

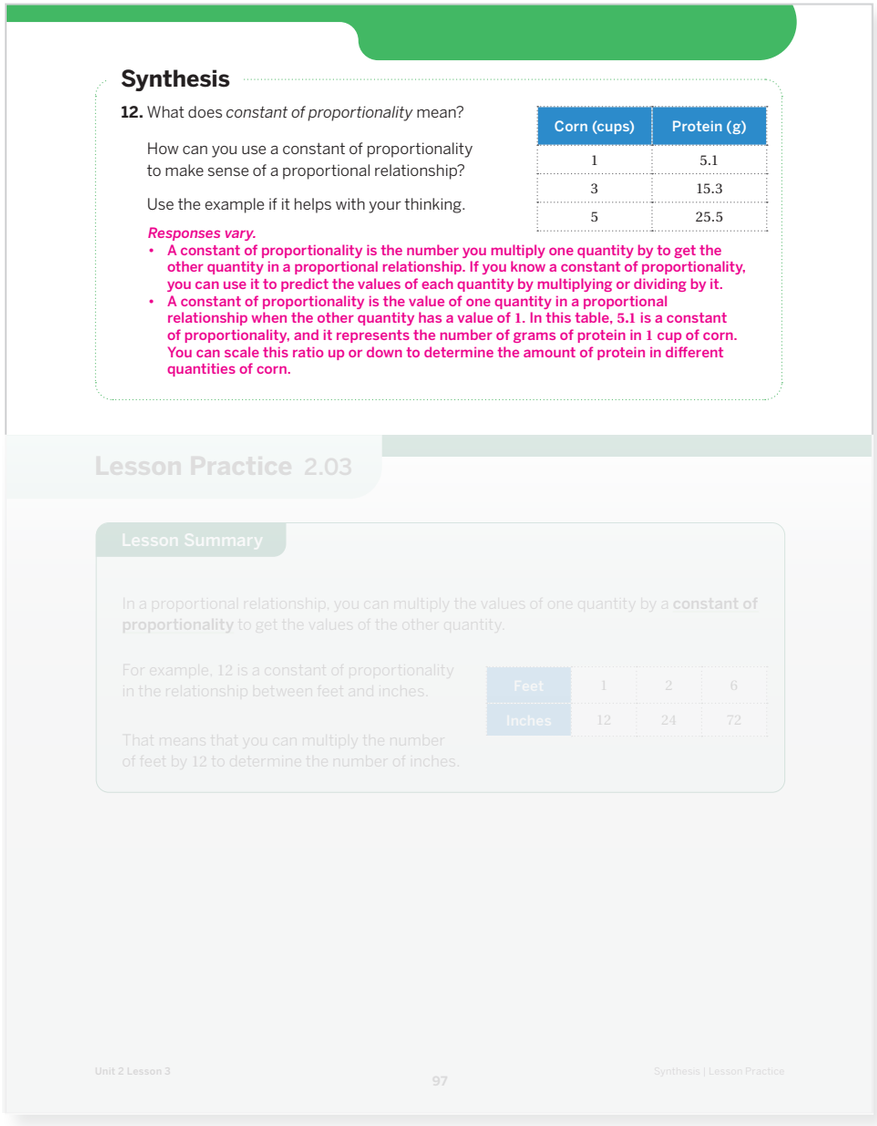
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas about constants of proportionality, such as:

- It is the number you multiply one quantity by to get the other quantity.
- It is the value of one quantity when the other quantity has a value of 1.
- It can be used to find values in a proportional relationship.
- In this table it is 5.1, and it represents the number of grams of protein in 1 cup of corn.

MLR **MLR2: Collect and Display** Use this routine to revisit the class definition of *constant of proportionality*. Add any new ideas to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their responses based on the discussion.



Synthesis

12. What does *constant of proportionality* mean?

How can you use a constant of proportionality to make sense of a proportional relationship?

Use the example if it helps with your thinking.

Corn (cups)	Protein (g)
1	5.1
3	15.3
5	25.5

Responses vary.

- A constant of proportionality is the number you multiply one quantity by to get the other quantity in a proportional relationship. If you know a constant of proportionality, you can use it to predict the values of each quantity by multiplying or dividing by it.
- A constant of proportionality is the value of one quantity in a proportional relationship when the other quantity has a value of 1. In this table, 5.1 is a constant of proportionality, and it represents the number of grams of protein in 1 cup of corn. You can scale this ratio up or down to determine the amount of protein in different quantities of corn.

Lesson Practice 2.03

Lesson Summary

In a proportional relationship, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

Feet	1	2	6
Inches	12	24	72

That means that you can multiply the number of feet by 12 to determine the number of inches.

Unit 2 Lesson 3 97 Synthesis | Lesson Practice

Lesson Takeaway: You can use a table to justify whether a relationship is proportional by using strategies such as calculating the ratio for each pair of values to determine if there is a **constant of proportionality**.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining and interpreting the constant of proportionality from a table.

Today's Goals

1. **Goal:** Determine the *constant of proportionality* for a proportional relationship represented in a table.
2. **Goal:** Use the *constant of proportionality* to determine values in a table.
3. **Language Goal:** Justify whether a table represents a proportional relationship, using terms such as *constant of proportionality*. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.03

Each row of this table represents a way to make the same color of green paint.

Blue Paint (cups)	Yellow Paint (cups)
2	10
1	5
6	30
52	260

a. What is a constant of proportionality in this relationship?
5 or $\frac{1}{5}$

b. What does that constant of proportionality mean in the situation?
Responses vary. The constant of proportionality 5 means that this particular color of paint uses 5 cups of yellow paint for every 1 cup of blue paint.

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify a constant of proportionality:

» **Respond to Student Thinking:**

- Review the Lesson Summary.
- Use the ***Determining the Constant of Proportionality*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- If students would like to explore nutrition in greater depth, invite them to research sodas, sport drinks, and juices and compare the amount of sugar or caffeine per ounce.

Professional Learning

Why was the context of sugar and cooking particularly useful in building students' understanding of **constant of proportionality** in this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What does *constant of proportionality* mean?

How can you use a constant of proportionality to make sense of a proportional relationship?

Use the example if it helps with your thinking.

Corn (cups)	Protein (g)
1	5.1
3	15.3
5	25.5

Responses vary.

- A constant of proportionality is the number you multiply one quantity by to get the other quantity in a proportional relationship. If you know a constant of proportionality, you can use it to predict the values of each quantity by multiplying or dividing by it.
- A constant of proportionality is the value of one quantity in a proportional relationship when the other quantity has a value of 1. In this table, 5.1 is a constant of proportionality, and it represents the number of grams of protein in 1 cup of corn. You can scale this ratio up or down to determine the amount of protein in different quantities of corn.

Lesson Practice 2.03

Lesson Summary

In a proportional relationship, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

Feet	1	2	6
Inches	12	24	72

That means that you can multiply the number of feet by 12 to determine the number of inches.

Lesson Practice 2.03

Name: _____ Date: _____ Period: _____

Problems 1–2: When Deven makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. This table shows how to make batches of different sizes.

Milk (cups)	Chocolate Syrup (tbsp)
2	3
8	12
1	$\frac{3}{2}$
10	15

1. Is there a proportional relationship between cups of milk and tablespoons of chocolate syrup?

Yes

Show or explain your thinking.

Explanations vary. The relationship is proportional because there are 1.5 tablespoons of chocolate syrup for every cup of milk.

2. What is a constant of proportionality for this relationship?

1.5 or $\frac{2}{3}$

Problems 3–4: When you mix two colors of paint in equivalent ratios, the color you get is always the same.

3. Complete the table so that each row makes the same color purple.

Responses that are in a 1 : 3 ratio of blue cups to red cups are considered correct.

Blue (cups)	Red (cups)
2	6
1	3

4. What is a constant of proportionality for this relationship?

3 or $\frac{1}{3}$

Problems 5–7: Here is some information about the side lengths of two scaled copies, triangle A and triangle B.

Side Length of Triangle A (in.)	Side Length of Triangle B (in.)
1	6
$\frac{1}{2}$	3
$1\frac{1}{3}$	8

5. Complete the table to determine the missing side lengths of each triangle.

6. What is a constant of proportionality in this relationship?

6 or $\frac{1}{6}$

7. What does that constant of proportionality tell you about the triangles?

Responses vary. Each side length in triangle A is $\frac{1}{6}$ the length of the corresponding side in triangle B.

Lesson Practice 2.03

Name: _____ Date: _____ Period: _____

8. Table A shows different amounts of blue and white paint that can be mixed together to create a color called "Blue Breeze." Sai wants to create a new color that is *bluer* than "Blue Breeze" and call it "Sea Glass Blue."

a. Complete the first row of Table B to create a possible mixture for "Sea Glass Blue."

Responses vary. Sample shown in table.

b. Complete the rest of Table B so that the relationship between cups of blue and cups of white is proportional.

Responses vary. Sample shown in table.

Table A Blue Breeze		Table B Sea Glass Blue	
Blue (cups)	White (cups)	Blue (cups)	White (cups)
1	3	4	8
2	6	7	14
5	15	0.5	1
8	24	12	24

FAST Practice

9. Sarah orders a small poster that is 100 square inches and costs $5.00. She then orders a large poster that is 500 square inches and costs $25.00. What does the constant of proportionality tell you about the costs of the posters?

The cost is $ 0.05 for each square inch of poster.

Spiral Review

10. Select *all* the scales that are equivalent to the scale 1 cm to 5 m.

- ☑ A. $\frac{1}{2}$ cm to $2\frac{1}{2}$ m
- ☐ B. 1 cm to 5 km
- ☐ C. 5 cm to 1 km
- ☑ D. 5 cm to 25 m
- ☑ E. 10 cm to 50 m

11. Select *all* of the ratios that are equivalent to 4 : 7.

- ☐ A. 8 : 15
- ☑ B. 16 : 28
- ☐ C. 7 : 4
- ☑ D. 20 : 35
- ☐ E. 6 : 9

Lesson Practice Item Analysis

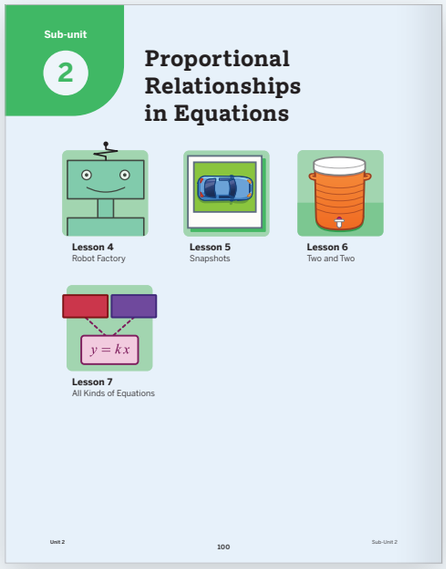
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.7.AR.4.1
	2, 4, 6	1	MA.7.AR.4.2
	3, 5	2	MA.7.AR.3.2
	7	2	MA.7.AR.4.2
	8	3	MA.7.AR.4.2
FAST Practice	9	2	7.MA.AR.4.2
Spiral Review			
	10	2	MA.7.GR.1.5
	11	1	MA.6.AR.3.1

Proportional Relationships in Equations

Sub-Unit 2 Goals

- **Goal:** Write and use equations to describe proportional relationships.
- **Goal:** Determine a constant of proportionality from an equation.
- **Language Goal:** Describe relationships represented with equations using terms such as *proportional*, *non-proportional*, and *constant of proportionality*.

In this sub-unit, students write equations describing proportional relationships and recognize proportional relationships represented with equations. They also determine the constant of proportionality using equations. Students write two equations that represent the same proportional relationship and notice that the two constants of proportionality are **reciprocals**.



Math that Matters Most

Concept or Skill	For example . . .
Write equations of the form $y = kx$ to represent proportional relationships.	The table represents a proportional relationship. Determine the constant of proportionality that completes the equation. $P =$ 4 s
Explore and interpret the concept of *reciprocals*.	Alexis thinks that the equation $y = 100x$ matches Table A. Carlos thinks that the equation $x = \frac{1}{100}y$ matches Table A. Whose claim is correct? Both. The equation y = 100x is saying that you multiply each x-value by 100 to get the y-value. The equation $x = \frac{1}{100}y$ is saying that you multiply each y-value by $\frac{1}{100}$ to get the x-value.

s	P
2	8
3	12
5	20
10	40

Table A

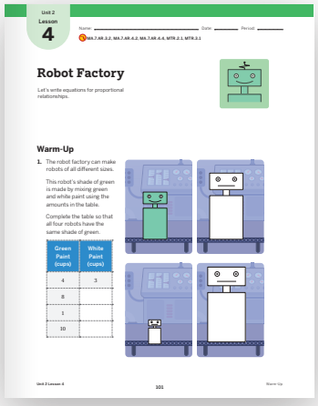
Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Unit 2
Lesson
4

Robot Factory

Proportional Relationships and Equations

Let's write equations for proportional relationships.


Unit 2
Lesson
4

Robot Factory

Let's write equations for proportional relationships.

Warm-Up

1. The robot factory can make robots of all different sizes.

This robot's shade of green is made by mixing green and white paint using the amounts in the table.

Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	
1	
10	

101

Learning Outcomes

Today's Goals

1. **Goal:** Write equations of the form $y = kx$ to represent proportional relationships.
2. **Language Goal:** Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality. **(Speaking and Listening)**

Students are given tables and use repeated reasoning to write equations for proportional relationships. This may be one of the first times that students encounter equations with two variables. This lesson is intended to expose students to the power of using equations as representations of proportional relationships. **(MTR.3.1)**

Coherence

Prior Learning

In Lessons 2 and 3, students determined the constant of proportionality from a table of values and used reasoning with the constant of proportionality to determine unknown values.

Future Learning

In Lesson 5, students will practice writing and interpreting equations for different scenarios.

Connections

- Students write equations relating proportional relationships in real world contexts such as scaling new parts of a robot to size, converting money exchanges while traveling, and distance traveled over time. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph **or written description of a proportional relationship**.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, **table or equation**.

Also Addressing: **MA.7.AR.3.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.1**
MA.6.AR.3.3	**MA.8.AR.3.2**
MA.6.AR.3.5	**MA.8.AR.3.3**
MA.7.GR.1.5	

Vocabulary

Review Vocabulary

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Lesson at a Glance

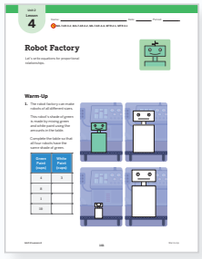
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4

Warm-Up

Pairs | 5 min

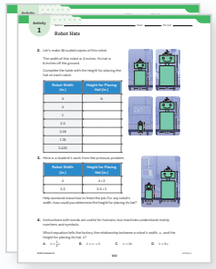
Students are introduced to the context of creating a robot and discuss what the constant of proportionality represents.



Activity 1

Pairs | 15 min

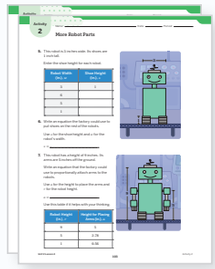
Students use repeated reasoning in tables to generate equations of the form $y = kx$ to produce robots fitting a design. MLR3: Critique, Correct, Clarify (MTR.3.1)



Activity 2

Pairs | 15 min

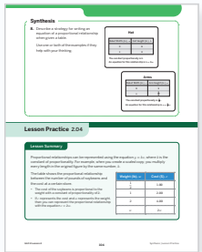
Students write equations to represent proportional relationships.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using the constant of proportionality to write an equation that represents a proportional relationship. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing the equation of a proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of creating a robot and discuss what the constant of proportionality represents.

1 Launch

Direct students' attention to the robots of different sizes.

Consider asking, "What strategies can we use to determine the amount of white paint for each robot?"

Listen for students who use equivalent ratios, multiplicative relationships, or determine a constant of proportionality. **(MTR.2.1)**

D **Differentiation:** (Support) For students who need help getting started, consider asking, "How did we use a constant of proportionality in previous lessons to determine values in tables like this?"

Invite students to check their responses with a partner.

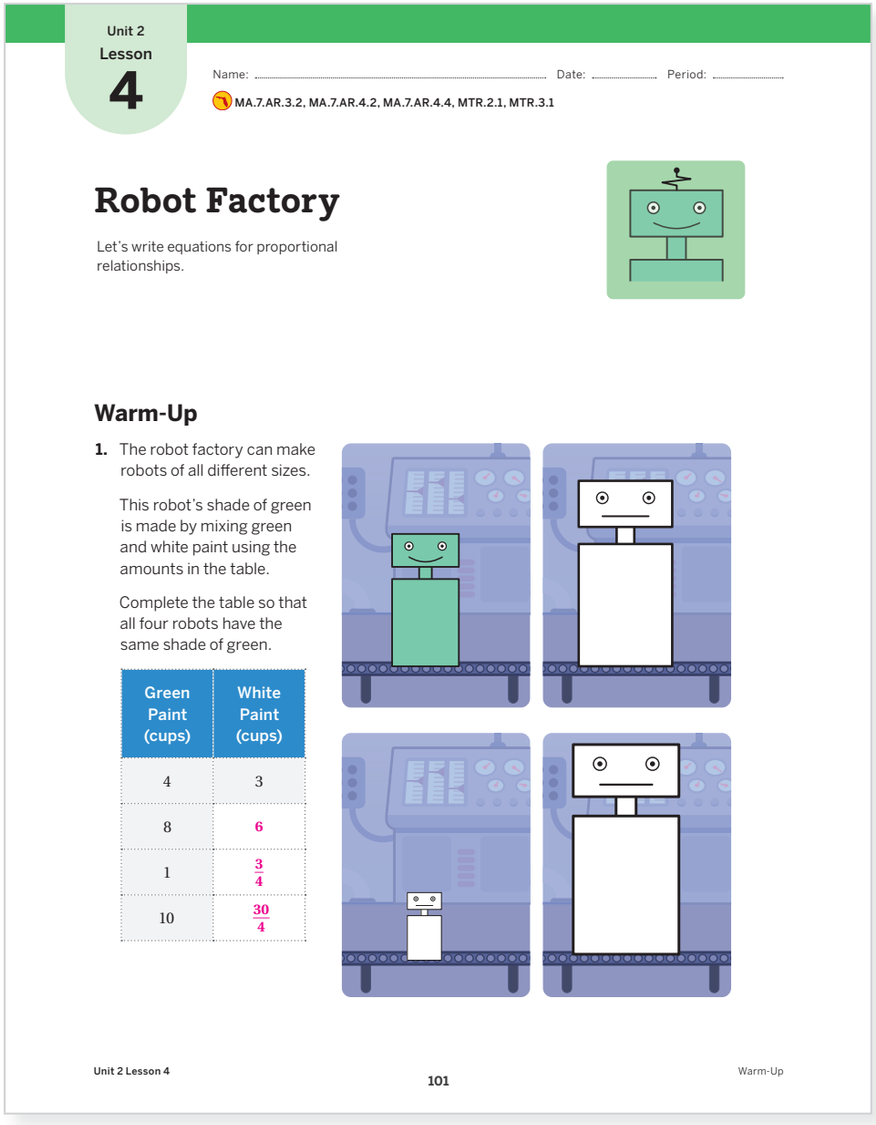
2 Connect

Invite students to share their strategies for completing the table.

Consider asking:

- "What is the constant of proportionality and where can you see it in the table?"**(MTR.2.1)**
- "What does the constant of proportionality represent in this situation?"

Note: If it does not come up naturally, consider discussing whether students believe it is a coincidence that the constant of proportionality is $\frac{3}{4}$ and the number paired with 1 is $\frac{3}{4}$.



Unit 2
Lesson
4

Name: Date: Period:

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MTR.2.1, MTR.3.1

Robot Factory

Let's write equations for proportional relationships.

Warm-Up

1. The robot factory can make robots of all different sizes.

   This robot's shade of green is made by mixing green and white paint using the amounts in the table.

   Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	6
1	$\frac{3}{4}$
10	$\frac{30}{4}$

Unit 2 Lesson 4 101 Warm-Up

Pairs | 15 min

Activity 1 Robot Hats

Purpose: Students use repeated reasoning in tables to generate equations of the form $y = kx$ to produce robots fitting a design. **(MTR.3.1)**

1 Launch

Consider sharing that students will be in charge of creating 30 *scaled copies* of their robot and they will be accessorized with hats, shoes, and arms.

2 Monitor

Select and sequence strategies for determining the height for placing the hat.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to form a strategy. Consider asking, "What relationships do you notice across or down the table?"
Attempt to use additive relationships to determine the height.	**Support** Consider asking, "What does it mean to be a scaled copy? How do the sides of scaled copies relate to each other?"
Represent the height by multiplying the width by 2.	**Stretch** Consider asking, "How could you represent your thinking in the table with an equation?"

MLR MLR3: Critique, Correct, Clarify Instead of using the discussion question in Problem 3, have students improve upon a statement, such as "Multiply by 2."
a. Discuss what is *correct* about the statement.
b. Discuss what is *incorrect* or *unclear.*
c. Write a question to help a student revise their statement.

3 Connect

Share and discuss the student strategies used to determine the height for placing the hat.

Invite students to discuss with a partner about which equation represents the relationship.

To surface the Key Takeaway, consider asking:
- "How would you explain what the equation $h = 2w$ means in a sentence?"
- "Why might someone think the equation $w = 2h$ is correct? What would you say in response?"
- "Why do we want to generate an equation?" **(MTR.3.1)**

Note: If time allows, consider using the equation $h = w + 3$ to highlight that an equation should successfully predict the height of all robots.

Activity 1

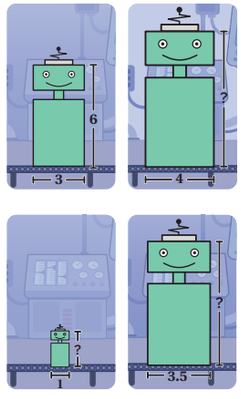
Name: _____ Date: _____ Period: _____

Robot Hats

2. Let's make 30 *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

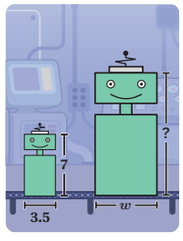
Complete the table with the height for placing the hat on each robot.



Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	8
1	2
3.5	7
2.18	4.36
1.76	3.52
3.425	6.85

3. Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
4	$4 \cdot 2$
3.5	$3.5 \cdot 2$



Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?

Responses vary. To know the height for placing the robot's hat, take its width and double it.

4. Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?

A. $h = \frac{1}{2}w$ B. $h = w + 3$ C. $w = 2h$ D. $h = 2w$

Unit 2 Lesson 4 102 Activity 1

Key Takeaway: You can use an equation to represent a proportional relationship that needs to be evaluated many times. It is helpful to determine the constant of proportionality to write the equation.

Pairs | 15 min

Activity 2 More Robot Parts

Purpose: Students write equations to represent proportional relationships.

Short on time? Consider omitting Problem 6. Students will have more opportunities with these types of equations in future lessons.

1 Launch

To generate excitement, consider asking, "What do you think your robot could be used for?"

2 Monitor

Look for students who use multiplication and division to represent the proportional relationships in the equations.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What operations did we use in Activity 1 to write the equation? Can we use similar operations here?"
Divide the width by 3 or multiply the width by $\frac{1}{3}$ to write $d = \frac{w}{3}$ or $d = \frac{1}{3}w$. *(Problem 6)*	**Stretch** Consider asking, "How does the constant of proportionality show up in your equation?"
Try to use whole number values to represent the proportional relationship. *(Problem 7)*	**Support** Invite students to use decimal or fractional values when writing their equations.

Invite students to share their strategies for how they are determining the equations for their robot.

A Accessibility: Memory and Attention To activate prior knowledge, consider asking, "What information have we used earlier that might help us now?"

3 Connect

Share the captured equations and invite students to justify whether each equation will build the robots correctly.

Consider asking, "Where do we see the height of the robot and how far the arms are off the ground in the equation?"

Note: The relationship in Problem 7 is challenging and students may need support to write an equation to represent it. Students will have several more opportunities to practice over the subsequent lessons.

Activity 2

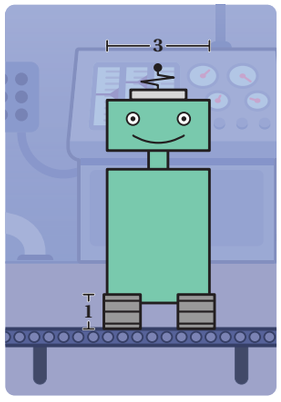
Name: _____ Date: _____ Period: _____

More Robot Parts

5. This robot is 3 inches wide. Its shoes are 1 inch tall.

Enter the shoe height for each robot.

Robot Width (in.), w	Shoe Height (in.), s
3	1
6	2
5	$\frac{5}{3}$
1	$\frac{1}{3}$



6. Write an equation the factory could use to put shoes on the rest of the robots.

Use s for the shoe height and w for the robot's width.

$s =$ $\frac{w}{3}$ or $\frac{1}{3}w$ (or equivalent)

7. This robot has a height of 9 inches. Its arms are 5 inches off the ground.

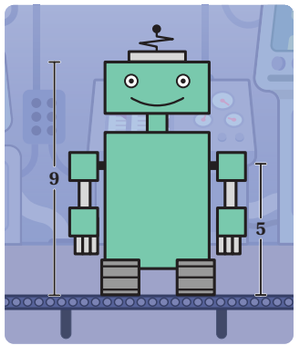
Write an equation that the factory could use to proportionally attach arms to the robots.

Use a for the height to place the arms and r for the robot height.

$a =$ $\frac{5}{9}r$ or $0.56r$ (or equivalent)

Use this table if it helps with your thinking.

Robot Height (in.), r	Height for Placing Arms (in.), a
9	5
5	2.78
1	0.56



Unit 2 Lesson 4 103 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using the constant of proportionality to write an equation that represents a proportional relationship.

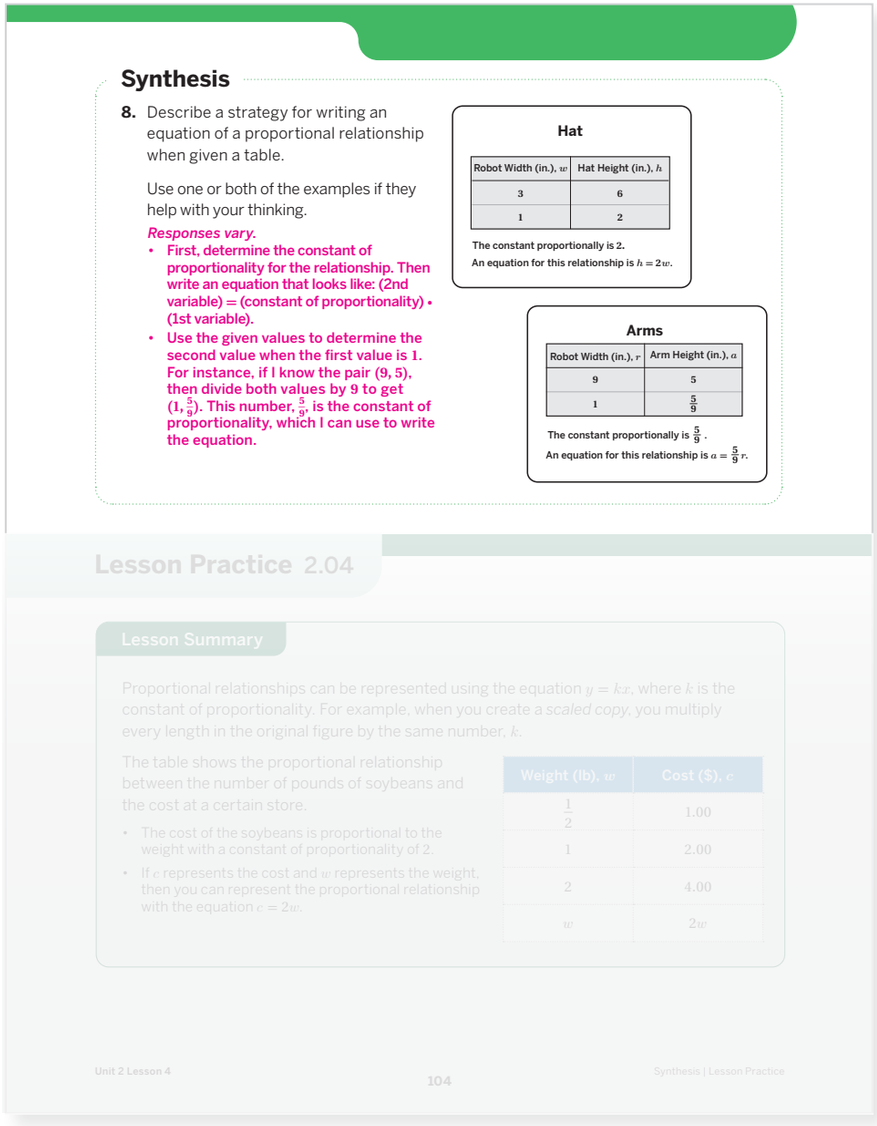
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Determining the constant of proportionality and using it to write the equation in the form $y = kx$.
- Determining the value of the second quantity when the first quantity is 1 (calculating the unit rate).
- Checking the equation and making sure that it works for all given values in the relationship.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for writing an equation as they share. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Responses vary.

- First, determine the constant of proportionality for the relationship. Then write an equation that looks like: (2nd variable) = (constant of proportionality) • (1st variable).
- Use the given values to determine the second value when the first value is 1. For instance, if I know the pair (9, 5), then divide both values by 9 to get $(1, \frac{5}{9})$. This number, $\frac{5}{9}$, is the constant of proportionality, which I can use to write the equation.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionality is 2.

An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionality is $\frac{5}{9}$.

An equation for this relationship is $a = \frac{5}{9}r$.

Lesson Practice 2.04

Lesson Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality. For example, when you create a *scaled copy*, you multiply every length in the original figure by the same number, k.

The table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

Weight (lb), w	Cost ($), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Unit 2 Lesson 4 104 Synthesis | Lesson Practice

Lesson Takeaway: It is helpful to determine the constant of proportionality when writing an equation that represents a proportional relationship. Once we have determined it, we can write the equation in the form $y = kx$, where k is the constant of proportionality.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing the equation of a proportional relationship.

Today's Goals

1. **Goal:** Write equations of the form $y = kx$ to represent proportional relationships.
2. **Language Goal:** Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.04

In Great Britain, two weeks is called a *fortnight*.

a. Complete the table.

Weeks, w	Fortnights, f
1	0.5
2	1
8	4

b. Write an equation for the number of fortnights, f, in any number of weeks, w.
$f = 0.5w$ (or equivalent)

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the equation in a proportional relationship:

» **Respond to Student Thinking:**

- Consider reviewing the strategies for completing the table in Activity 1 before Lesson 5. Consider asking, "What relationships do you see between the rows and columns of the table?

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how students' understanding of the term *constant of proportionality* has developed throughout this course. How has their language usage of *scaled copies*, *scale factor*, and *equivalent ratios* developed over the course? How do you hear students using these words to represent the multiplicative relationships that exist within proportional relationships?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Responses vary.

- First, determine the constant of proportionality for the relationship. Then write an equation that looks like: (2nd variable) = (constant of proportionality) • (1st variable).
- Use the given values to determine the second value when the first value is 1. For instance, if I know the pair $(9, 5)$, then divide both values by 9 to get $(1, \frac{5}{9})$. This number, $\frac{5}{9}$, is the constant of proportionality, which I can use to write the equation.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionality is 2.
An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionality is $\frac{5}{9}$.
An equation for this relationship is $a = \frac{5}{9}r$.

Lesson Practice 2.04

Lesson Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality. For example, when you create a *scaled copy*, you multiply every length in the original figure by the same number, k.

The table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

Weight (lb), w	Cost ($), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Lesson Practice 2.04

Name: _____ Date: _____ Period: _____

1. The ceilings in many basements are made up of rectangular tiles. For one basement, each square meter of ceiling requires 10.75 tiles. Complete the table.

Area of Ceiling (sq. m)	Number of Tiles
1	10.75
10	107.5
5	53.75
x	$10.75x$

Problems 2–3: Each table represents a proportional relationship. Determine the constant of proportionality that completes each equation.

2.

s	P
2	8
3	12
5	20
10	40

$P =$ 4 s

3.

d	C
2	6.28
3	9.42
5	15.7
10	31.4

$C =$ 3.14 d

Problems 4–5: A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.

4. Complete the table.

Time (hr)	Distance (mi)
1	610
1.5	915
2	1,220
2.5	1,525
t	$610t$

5. How far would the plane fly in 10 hours at this speed?

6,100 miles

Lesson Practice 2.04

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 6–8: While she was traveling in 2023, Mai received 342.50 Norwegian kroner in exchange for 50 Australian dollars.

6. How many kroner would Mai have received in exchange for 1 Australian dollar?

6.85 Norwegian kroner

7. Write an equation to represent the amount of kroner, k, received in exchange for a Australian dollars.

$k =$ $6.85a$

8. Determine the number of kroner Mai would receive in exchange for 120 Australian dollars.

822 Norwegian kroner

Spiral Review

Problems 9–10: A bicycle travels 21 meters in 3 seconds.

9. Complete the table.

Time (sec)	Distance (m)
3	21
$1\frac{1}{2}$	$10\frac{1}{2}$
$\frac{9}{10}$	$6\frac{3}{10}$

10. What is a constant of proportionality in this relationship?

7 or $\frac{1}{7}$ (or equivalent)

What does this represent in the situation?

Responses vary. It represents that the bicycle travels 7 meters every second.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–5	2	MA.7.AR.4.5
	2–3	1	MA.7.AR.4.2, MA.7.AR.4.4
FAST Practice	6–8	2	MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
Spiral Review			
	9	2	MA.7.AR.3.2
	10	1	MA.7.AR.4.2

Unit 2

Lesson

5


Unit 2
Lesson
5
Snapshots
Let's use equations to make sense of proportional relationships in the world.
Warm-Up
1. Write a story about this car's trip.
107

Snapshots

More Equations of Proportional Relationships

Let's use equations to make sense of proportional relationships in the world.

Learning Outcomes

Today's Goals

1. **Goal:** Connect each part of an equation of the form $y = kx$ to the situation it represents.
2. **Goal:** Create equations representing proportional relationships and use them to solve problems.
3. **Language Goal:** Interpret each part of an equation that represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Students develop fluency in writing and interpreting equations representing proportional relationships. Students explain the meaning of each part of an equation and solve problems in various contexts, moving between the abstract representation and concrete situation. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 4, students were introduced to equations with two variables and began writing equations to represent proportional relationships.

Future Learning

In Lesson 6, students will determine the two constants of proportionality for a proportional relationship and write two equations to represent the relationship.

Connections

- Students extend their understanding of linear equations to solve problems involving real world proportional relationships such as taking a trip, adjusting a recipe, and comparing different units of measure. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.3.2, MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.1**
MA.6.AR.3.3	**MA.8.AR.3.2**
MA.6.AR.3.5	**MA.8.AR.3.3**

Lesson at a Glance

~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Pairs | 5 min

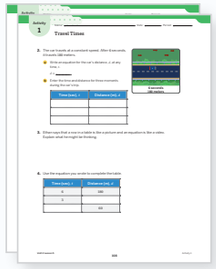
Students use the **Tell a Story** routine to explore how snapshots can provide information and invite questions about a situation.



Activity 1

Pairs | 10 min

Students make connections between tables and equations and consider how each abstract representation relates to the situation.



Activity 2

Pairs | 10 min

Students practice using equations of the form $y = kx$ to reason about proportional relationships.

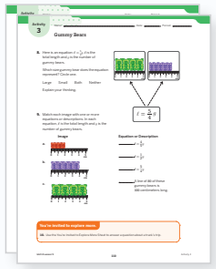


Activity 3

Pairs | 10 min

Students develop strategies to categorize descriptions, equations, and images that represent the same proportional relationship.

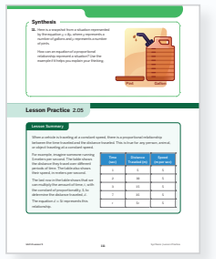
Routine: **Decide and Defend**



Synthesis

Whole Class | 5 min

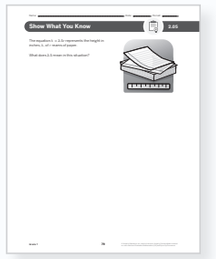
Students synthesize their understanding of equations representing proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting the meaning of one part of an equation in context.



Prep Checklist

Prepare Activity 3 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More, one per student (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Tell a Story routine to explore how snapshots can provide information and invite questions about a situation.

1 Launch

To surface connections, consider asking students about their experience with taking trips in a car.

Invite students to share what they see in the snapshots.

Use the Tell a Story routine to help students make sense of the snapshots. Invite students to write a story about the images using their unique experiences and creativity. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Invite students to write stories in their primary language. **(Writing)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

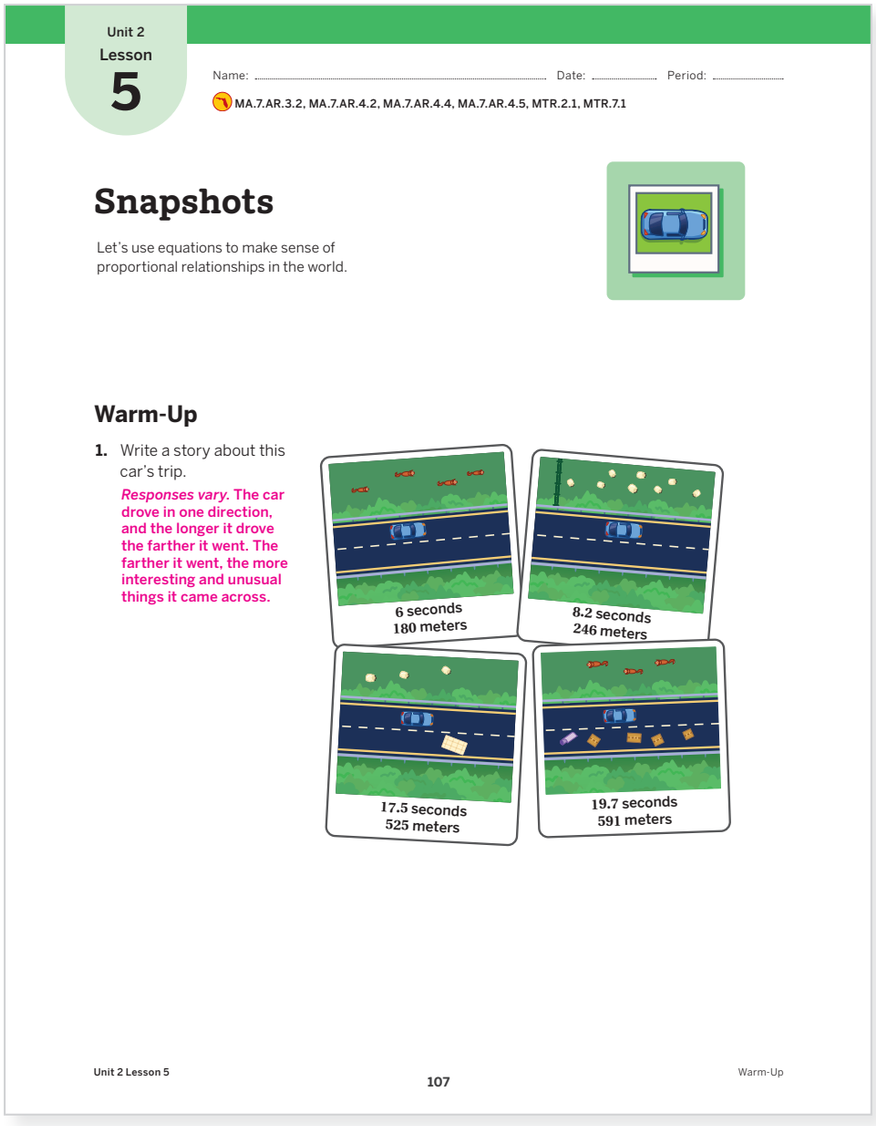
Listen for students who notice that the car is traveling at a constant speed. Consider inviting these students to share their thinking during Activity 1.

2 Connect

Invite students to share their stories.

Encourage students to ask questions about each other's stories. Consider asking:

- "What questions do you have about the car's trip after just looking at the pictures?"
- "Can we know the whole story from these snapshots? Why or why not?"



Unit 2
Lesson
5

Name: ..... Date: ..... Period: .....

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.7.1

Snapshots

Let's use equations to make sense of proportional relationships in the world.

Warm-Up

1. Write a story about this car's trip.

Responses vary. The car drove in one direction, and the longer it drove the farther it went. The farther it went, the more interesting and unusual things it came across.

Unit 2 Lesson 5 107 Warm-Up

Pairs | 10 min

Activity 1 Travel Times

Purpose: Students make connections between tables and equations and consider how each abstract representation relates to the situation. **(MTR.2.1)**

1 Launch

Invite students to read the problem aloud and restate it in their own words.

Consider asking:
- "What does the phrase *constant speed* mean to you?"
- "How does knowing that the car has a constant speed help you write an equation?" **(MTR.7.1)**

Pause briefly once most students have written a correct equation and invite students to share their equations.

Note: Make sure that every student is able to write a correct equation and then complete the table. **(MTR.2.1)**

2 Monitor

Capture a variety of explanations, including some that are incomplete or unclear.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

A **Accessibility: Conceptual Processing** Invite students to read their equation aloud with variable definitions instead of d and t (e.g., "The distance is equal to 30 times the number of seconds.").

Look for a variety of student thinking.

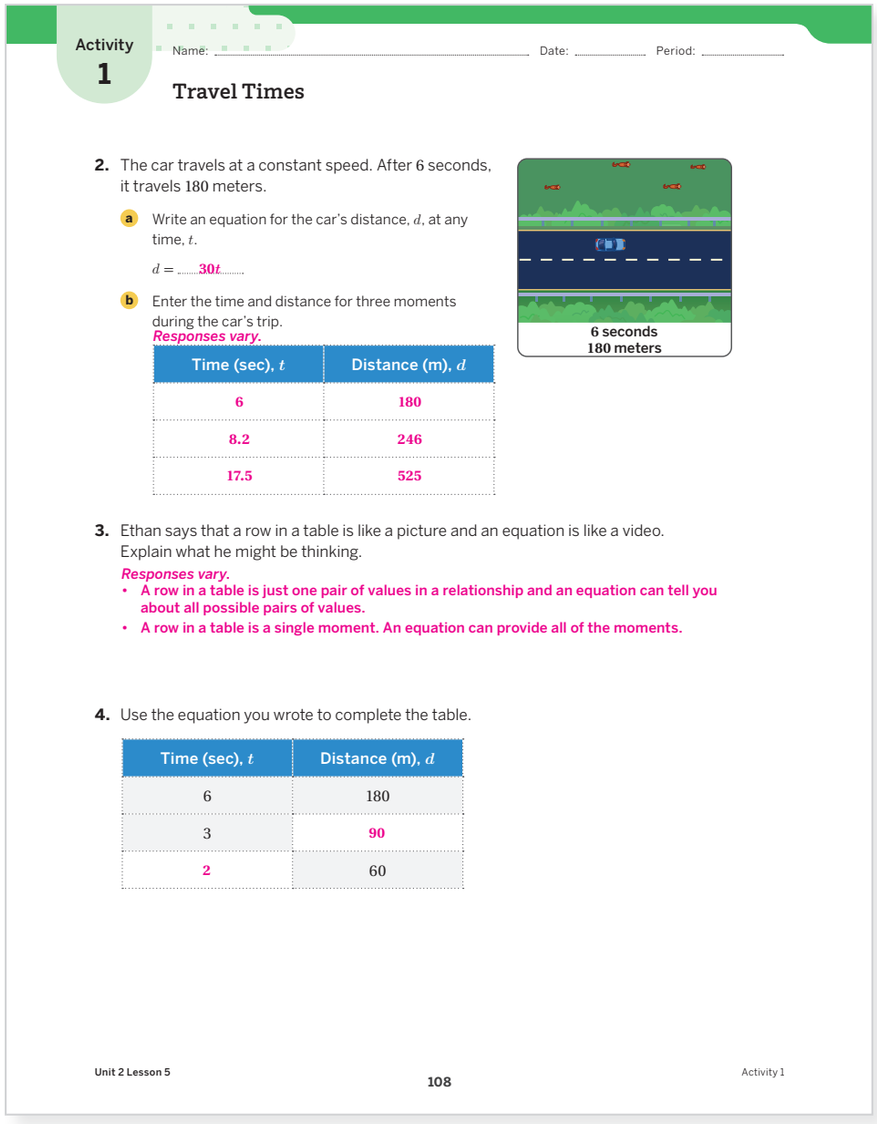
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to try substituting the given values into their equation.
Multiply a given distance by 30 to calculate a value for time.	**Support** Consider asking, "Which variable is multiplied by 30 in your equation?"
Scale rows in the table to find unknown values.	**Strengthen** Consider asking, "How can you check your work using the equation you wrote?"

3 Connect

Discuss the captured explanations.

Consider creating an anchor chart to compare and contrast when a table or equation is a more useful representation.



Activity 1

Name: _____ Date: _____ Period: _____

Travel Times

2. The car travels at a constant speed. After 6 seconds, it travels 180 meters.

a. Write an equation for the car's distance, d, at any time, t.

$d =$ 30t

b. Enter the time and distance for three moments during the car's trip.
Responses vary.

Time (sec), t	Distance (m), d
6	180
8.2	246
17.5	525

3. Ethan says that a row in a table is like a picture and an equation is like a video. Explain what he might be thinking.
Responses vary.
- A row in a table is just one pair of values in a relationship and an equation can tell you about all possible pairs of values.
- A row in a table is a single moment. An equation can provide all of the moments.

4. Use the equation you wrote to complete the table.

Time (sec), t	Distance (m), d
6	180
3	90
2	60

Unit 2 Lesson 5 108 Activity 1

Key Takeaway: You can use the equation representing a relationship to predict any pair of values in that relationship.

Pairs | 10 min

Activity 2 Cakes

Purpose: Students practice using equations of the form $y = kx$ to reason about proportional relationships.

1 Launch

To surface connections, consider asking students about their experience with baking cakes.

Discuss each part of the equation $m = 6c$.

Note: Students may interpret this equation to mean that 1 ounce of milk makes 6 cakes. Consider rewriting the equation as $m = 6 \cdot c$ to emphasize that c is a quantity being multiplied, not a label of units.

2 Monitor

Look for students who write the expression $6 \cdot 12$.

ELL **English Language Learners** Encourage students to discuss with their partners. Provide sentence frames to help students explain their strategy (e.g., "I know that _____, so I _____."). **(Speaking)**

Listen for a variety of student thinking.

D Differentiation

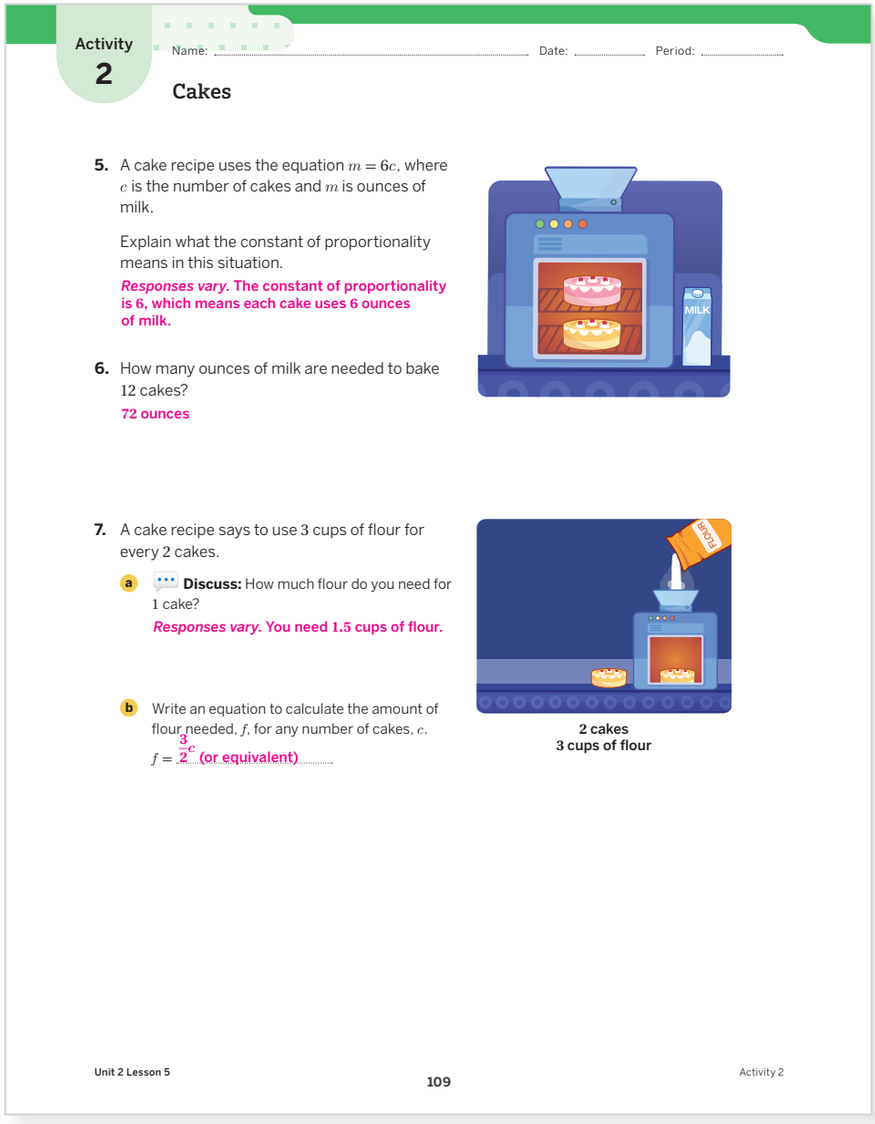
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to make a table of values to help them find the constant of proportionality.
Write $3f = 2c$.	**Support** Encourage students to make a table of values and use substitution in their equation to check their work.
Write $f = \frac{2}{3}c$.	**Support** Encourage students to write an equivalent ratio using the quantities in the original recipe to check whether their equation is correct.
Write $f = 1.5c$ or equivalent.	**Strengthen** Consider asking, "How could you use this equation to find the amount of flour for any number of cakes?" **(MTR.7.1)**

Look for student equations to share during the Connect.

3 Connect

Display the captured equations and invite students to explain what each part of the equation represents.

To surface the Key Takeaway, consider asking, "How did you find the constant of proportionality in this situation?"



Activity 2

Name: _____ Date: _____ Period: _____

Cakes

5. A cake recipe uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

Explain what the constant of proportionality means in this situation.

Responses vary. The constant of proportionality is 6, which means each cake uses 6 ounces of milk.

6. How many ounces of milk are needed to bake 12 cakes?

72 ounces

7. A cake recipe says to use 3 cups of flour for every 2 cakes.

a. **Discuss:** How much flour do you need for 1 cake?

Responses vary. You need 1.5 cups of flour.

b. Write an equation to calculate the amount of flour needed, f, for any number of cakes, c.

$f = \frac{3}{2}c$ (or equivalent)

2 cakes
3 cups of flour

Unit 2 Lesson 5 109 Activity 2

Key Takeaway: In an equation of the form $y = kx$, k is the constant of proportionality. To find the constant of proportionality, you can divide any x-value and the y-value it is paired with.

Pairs | 10 min

Activity 3 Gummy Bears

Purpose: Students develop strategies to categorize descriptions, equations, and images that represent the same proportional relationship.

Short on time? Consider omitting Problem 9 and discussing Problem 8 during the Connect.

1 Launch

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Invite students to share their responses.

Consider asking, "What does $\frac{5}{4}$ mean in this situation?"

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match the images with the equations and description.

Look for a variety of student thinking.

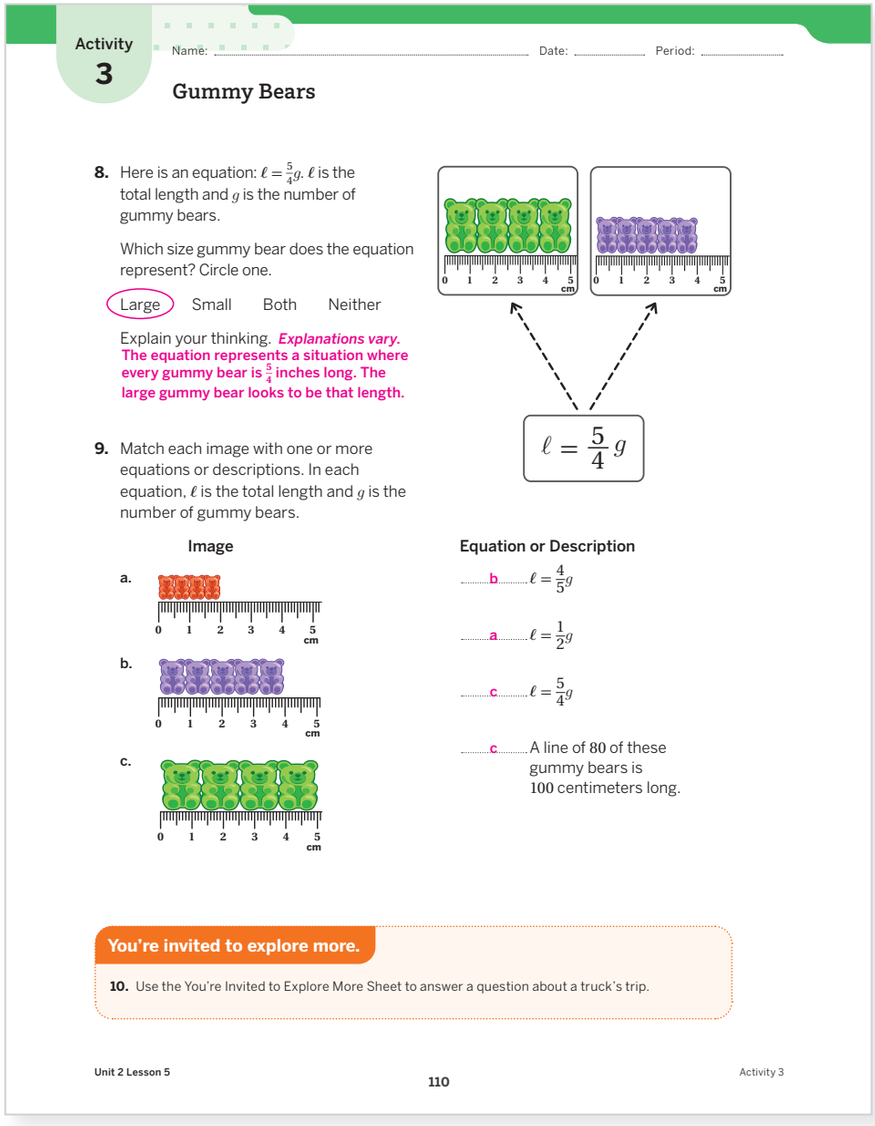
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to identify the constant of proportionality in one equation. Consider asking, "Which image has the same constant of proportionality?"
Think that the equation $l = \frac{4}{5}g$ means that 4 gummy bears will have a length of 5 centimeters.	**Support** Encourage students to check their work by substituting those values into the equation.
Interpret the constant of proportionality as the length of one gummy bear.	**Stretch** Consider asking, "How did you decide where to group the description?" Invite students to create their own descriptions for groups without one.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore proportional relationships further to complete this optional task. Encourage them to discuss their ideas with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their matches. For incorrect matches, discuss why someone might make a match in this way.



Activity 3

Name: _____ Date: _____ Period: _____

Gummy Bears

8. Here is an equation: $\ell = \frac{5}{4}g$. ℓ is the total length and g is the number of gummy bears.

Which size gummy bear does the equation represent? Circle one.

(Large) Small Both Neither

Explain your thinking. *Explanations vary.* The equation represents a situation where every gummy bear is $\frac{5}{4}$ inches long. The large gummy bear looks to be that length.

9. Match each image with one or more equations or descriptions. In each equation, ℓ is the total length and g is the number of gummy bears.

Image	Equation or Description
a.	b $\ell = \frac{4}{5}g$
b.	a $\ell = \frac{1}{2}g$
c.	c $\ell = \frac{5}{4}g$
	c A line of 80 of these gummy bears is 100 centimeters long.

You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to answer a question about a truck's trip.

Unit 2 Lesson 5 110 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of equations representing proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Variables represent the quantities in the relationship.
- Recognizing the number in the equation is the constant of proportionality and multiplying it by one variable to find the value of the other variable.
- Using the equation to find points from the relationship and creating a table.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here is a snapshot from a situation represented by the equation $g = 8p$, where g represents a number of gallons and p represents a number of pints.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Responses vary. **I think the equation $g = 8p$ represents the situation because it tells us that there are 8 pints in a gallon.**

Lesson Practice 2.05

Lesson Summary

When a vehicle is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

Time (sec)	Distance Traveled (m)	Speed (m per sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

The last row in the table shows that we can multiply the amount of time, t, with the constant of proportionality, 5, to determine the distance traveled, d.

The equation $d = 5t$ represents this relationship.

Unit 2 Lesson 5 111 Synthesis | Lesson Practice

Lesson Takeaway: Tables and equations are both tools for making sense of proportional relationships in different situations. You can use an equation of the form $y = kx$ to create a table or solve problems once you make sense of what the pieces of the equation mean.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting the meaning of one part of an equation in context.

Today's Goals

1. **Goal:** Connect each part of an equation of the form $y = kx$ to the situation it represents.
2. **Goal:** Create equations representing proportional relationships and use them to solve problems.
3. **Language Goal:** Interpret each part of an equation that represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

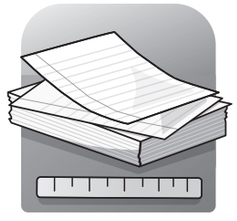
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Show What You Know 2.05

The equation $h = 2.5r$ represents the height in inches, h, of r reams of paper.

What does 2.5 mean in this situation?

Responses vary. 2.5 inches is how tall one ream of paper is.



D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to interpret the parts of an equation of the form $y = kx$:

» **Respond to Student Thinking:**

- Encourage students to practice reading each equation using variable definitions (e.g., read $m = 6c$ as "The number of ounces of milk is 6 times the number of cakes.").

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

In this lesson, students continue developing their understanding of equations with two variables. How does this model integrate with other representations of proportional relationships? How did you see students making sense of the connections between equations, tables, and situations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here is a snapshot from a situation represented by the equation $g = 8p$, where g represents a number of gallons and p represents a number of pints.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Responses vary. I think the equation $g = 8p$ represents the situation because it tells us that there are 8 pints in a gallon.

Lesson Practice 2.05

Lesson Summary

When a vehicle is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

Time (sec)	Distance Traveled (m)	Speed (m per sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

The last row in the table shows that we can multiply the amount of time, t, with the constant of proportionality, 5, to determine the distance traveled, d.

The equation $d = 5t$ represents this relationship.

Lesson Practice 2.05

Name: _____ Date: _____ Period: _____

Problems 1–3: A performer expects to sell 5,000 tickets for an upcoming concert. He plan to make $311,000 from the sales of these tickets. Assume that all tickets have the same price.

1. What is the price for one ticket? Show or explain your thinking.
$62.20. *Explanations vary.* I divided $311,000 by 5,000 to determine the price for one ticket.

2. Write an equation to represent the relationship between the number of tickets sold, x, and the total amount of money (in dollars) that he makes, y.
$y = 62.2x$

3. If he makes $379,420, how many tickets has he sold?
6,100 tickets

Problems 4–5: A car is traveling on a highway at a constant speed. The equation that represents the distance traveled in miles, d, for t hours is $d = 65t$.

4. What does the value 65 represent in this situation?
***Responses vary.* The value 65 is the speed, or constant of proportionality. The car travels 65 miles every hour.**

5. At this rate, how many miles will the car travel in 1.5 hours? Show your thinking.
97.5 miles. *Work varies.*
$d = 65t$
$d = 65(1.5)$
$d = 97.5$

Problems 6–7: On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.



6. This table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Pittsburgh to Saint Louis	1	550	**550**
Saint Louis to Albuquerque	1.7	**935**	**550**
Albuquerque to Phoenix	**0.6**	330	**550**

7. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.
$d = 550t$ (or equivalent)

Lesson Practice 2.05

Name: _____ Date: _____ Period: _____

8. Na'ilah is making a pitcher of a lemon-flavored sports drink. The drink mix container says to mix $\frac{1}{4}$ cups of powdered drink mix with 2 quarts of water. She prefers her sports drink to taste *more* lemon-y than the recipe on the container. Complete the equation to represent a mixture of cups of drink mix, c, and quarts of water, w, that would be *more* lemon-y than the original mixture.
Responses where the constant of proportionality is greater than $\frac{1}{8}$ are considered correct.
$c =$ **$\frac{1}{5}$** w

FAST Practice

9. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours. Complete the equation to represent the distance, d, that the train travels at any time, t.

$d =$ **134** t

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☑ **A.**

x	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

☐ **B.**

x	0	$\frac{1}{2}$	2	3
y	0	$\frac{1}{4}$	4	9

☑ **C.**

x	0	4	8	12
y	0	2	4	6

☑ **D.**

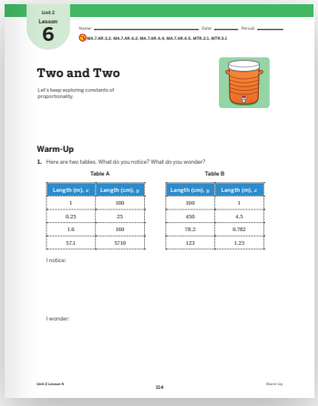
x	0	1	2	3
y	0	4	8	12

11. Is 4.5 a solution to the equation $1.5 + x = 6$?
Yes

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4	1	MA.7.AR.4.2
	2, 7	2	MA.7.AR.4.4
	3	2	MA.7.AR.4.5
	5	2	MA.7.AR.4.4, MA.7.AR.4.5
	6	2	MA.7.AR.3.2
	8	3	MA.7.AR.4.4, MA.7.AR.4.5
FAST Practice	**9**	3	MA.7.AR.4.2, MA.7.AR.4.4
Spiral Review			
	10	2	MA.7.AR.4.1
	11	1	MA.6.AR.2.1

Unit 2
Lesson
6


Unit 2
Lesson
6

Two and Two

Let's keep exploring constants of proportionality.

Warm-Up

1. Here are two tables. What do you notice? What do you wonder?

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

I notice:

I wonder:

114

Two and Two

Two Equations for Each Relationship

Let's keep exploring constants of proportionality.

Learning Outcomes

Today's Goals

1. **Goal:** Write two equations that represent the same proportional relationship, such as $y = kx$ and $x = \frac{1}{k}y$.
2. **Language Goal:** Explain the relationship between the two constants of proportionality in a proportional relationship using terms such as *reciprocal*. **(Reading and Writing)**

Students determine the two constants of proportionality for a proportional relationship and discover they are **reciprocals**. They interpret these constants of proportionality in context and use them to write two equations that represent each relationship. **(MTR.2.1)**

Coherence

Prior Learning

In Grade 6, students discovered reciprocals. In Lesson 5, students developed fluency in writing and using equations to make sense of proportional relationships in various contexts.

Future Learning

In Lesson 8, students will represent proportional relationships graphically.

Connections

- Students apply linear equations to real-world proportional relationships such as the amount of time needed to fill a cooler based on the rate of water flow. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.3.2, MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.1**
MA.6.AR.3.3	**MA.8.AR.3.2**
MA.6.AR.3.5	**MA.8.AR.3.3**

Vocabulary

New Vocabulary

reciprocal

Lesson at a Glance

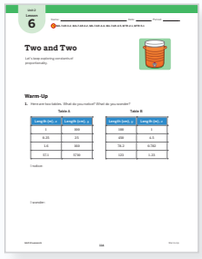
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Whole Class | 5 min

Students use the **Notice and Wonder** routine to compare two tables representing the relationship between meters and centimeters.



Activity 1

Pairs | 10 min

Students make connections between tables and equations representing the same proportional relationship. **Routine: Decide and Defend (MTR.2.1)**



Activity 2

Pairs | 20 min

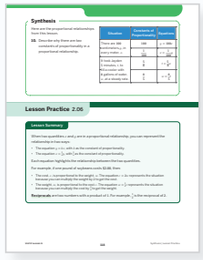
Students determine the two constants of proportionality for a relationship and discuss the concept of **reciprocals** to enrich their understanding.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of why there are two constants of proportionality in a proportional relationship.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing two equations to represent the same proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to compare two tables representing the relationship between meters and centimeters.

1 Launch

Direct students' attention to the two tables.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between the two tables.

2 Connect

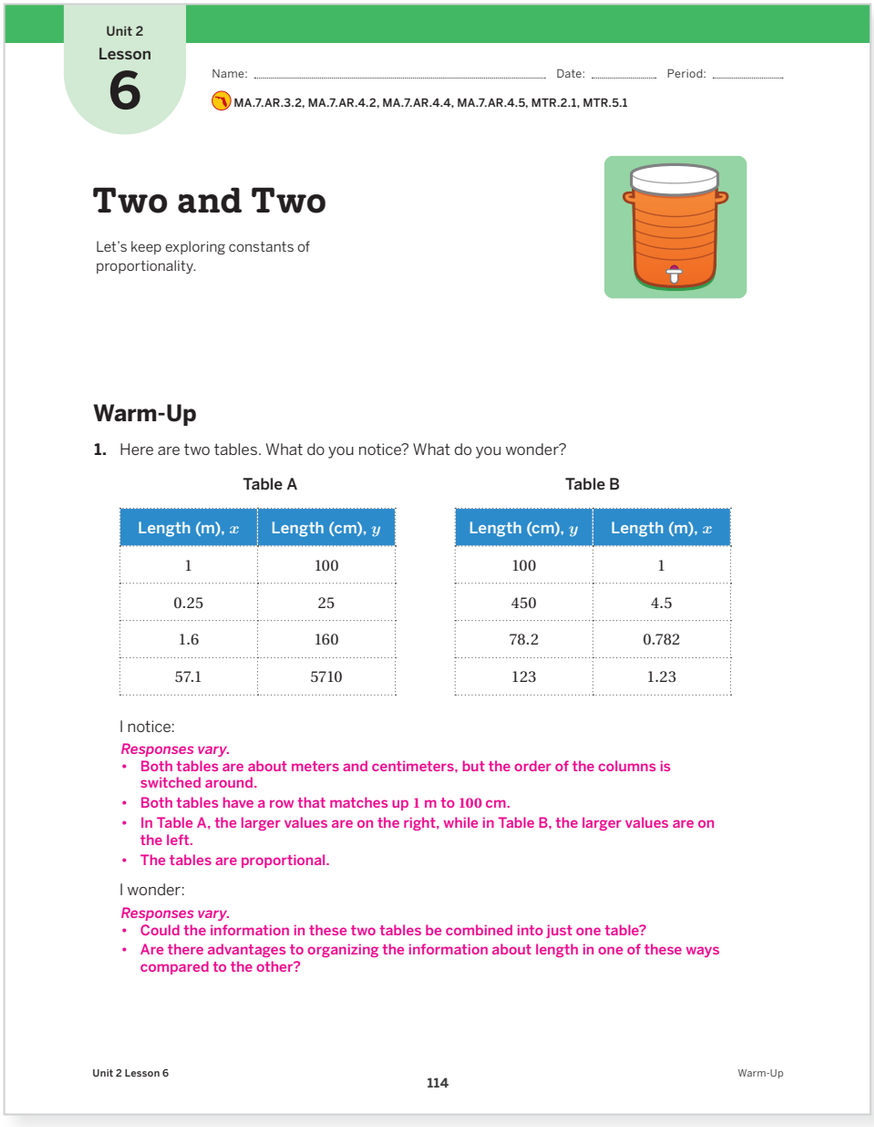
Invite students to share what they noticed and wondered.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Consider asking, "Did anyone notice that the column labels on the tables are the same, but reversed? Does this make you wonder about whether we could combine these tables?" **(MTR.5.1)**

Note: If it doesn't come up naturally, consider sharing that both tables show the relationship between meters and centimeters.

Invite students to use gestures such as holding out their arms and pinching their fingers to demonstrate the approximate length of 1 meter and 1 centimeter.



Unit 2
Lesson
6

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.5.1

Two and Two

Let's keep exploring constants of proportionality.

Warm-Up

1. Here are two tables. What do you notice? What do you wonder?

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

I notice:

Responses vary.
- Both tables are about meters and centimeters, but the order of the columns is switched around.
- Both tables have a row that matches up 1 m to 100 cm.
- In Table A, the larger values are on the right, while in Table B, the larger values are on the left.
- The tables are proportional.

I wonder:

Responses vary.
- Could the information in these two tables be combined into just one table?
- Are there advantages to organizing the information about length in one of these ways compared to the other?

Unit 2 Lesson 6 114 Warm-Up

Pairs | 10 min

Activity 1 Table Equation Match-Up

Purpose: Students make connections between tables and equations representing the same proportional relationship. **(MTR.2.1)**

Short on time? Problem 2 may be omitted.

1 Launch

Direct students' attention to the equations.

To surface connections, consider asking, "How are these equations alike? How are they different?" **(MTR.2.1)**

2 Monitor

Listen for a variety of student thinking.

D Differentiation *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look for a constant of proportionality in each table by checking for a multiplier from one column to the other.
Determine that 100 is a constant of proportionality in Table A because it is paired with the value 1.	**Strengthen** Consider asking, "How did you decide whether $y = 100x$ or $x = 100y$ best represents the relationship you noticed?"
Substitute values from the tables into each equation to determine a match.	**Strengthen** Celebrate this strategy. Consider asking, "What is the constant of proportionality in each table? Where do you see it in the equations?"

A Accessibility: Executive Functioning Consider chunking this activity by inviting students to choose one table at a time and look for an equation to match it.

3 Connect

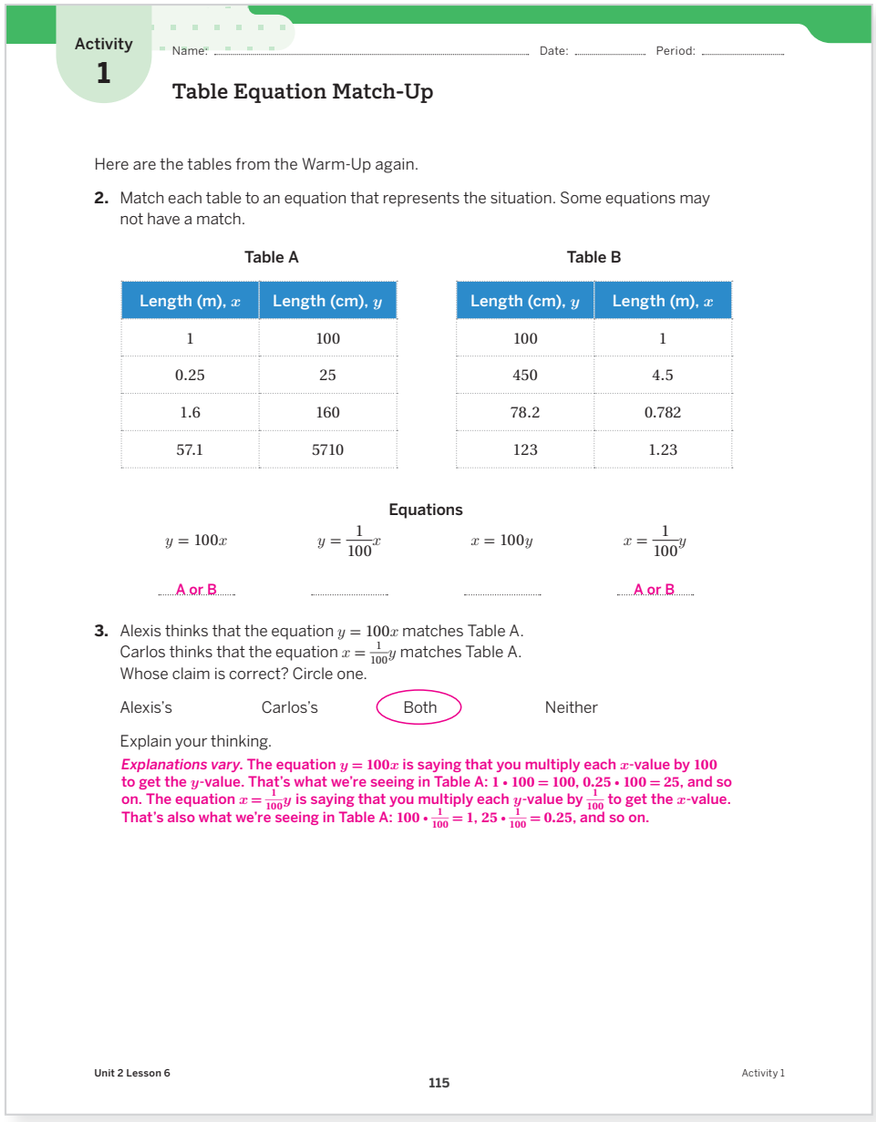
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Display the distribution of responses by polling the class.

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking, "What evidence or language was most convincing to you?"

To surface the Key Takeaway, consider asking:
- "What is the same about the two equations for Table A?"
- "What is different?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

Table Equation Match-Up

Here are the tables from the Warm-Up again.

2. Match each table to an equation that represents the situation. Some equations may not have a match.

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

Equations

$y = 100x$ — A or B

$y = \frac{1}{100}x$

$x = 100y$

$x = \frac{1}{100}y$ — A or B

3. Alexis thinks that the equation $y = 100x$ matches Table A. Carlos thinks that the equation $x = \frac{1}{100}y$ matches Table A. Whose claim is correct? Circle one.

Alexis's Carlos's Both Neither

Explain your thinking.

Explanations vary. The equation $y = 100x$ is saying that you multiply each x-value by 100 to get the y-value. That's what we're seeing in Table A: $1 \cdot 100 = 100$, $0.25 \cdot 100 = 25$, and so on. The equation $x = \frac{1}{100}y$ is saying that you multiply each y-value by $\frac{1}{100}$ to get the x-value. That's also what we're seeing in Table A: $100 \cdot \frac{1}{100} = 1$, $25 \cdot \frac{1}{100} = 0.25$, and so on.

Unit 2 Lesson 6 115 Activity 1

Key Takeaway: You can write two equations to represent the same proportional relationship. The equations use the same variables, but they have different constants of proportionality.

Pairs | 20 min

Activity 2 Jayden's Cooler

Purpose: Students determine the two constants of proportionality for a relationship and discuss the concept of **reciprocals** to enrich their understanding.

1 Launch

Consider sharing the context for the activity and inviting students to use gestures to show the approximate size of a gallon of water.

ELL **English Language Learners** Display the image of the cooler being filled to increase access to the task. **(Reading and Listening)**

Direct students' attention to the instructions and invite students to complete Part 1 or Part 2. Encourage students to discuss their thinking with a partner completing the same part.

Note: Later in this activity, students will pair up with a classmate who completed the other part. Consider ensuring that about half of the class completes each part.

2 Monitor

Look for a variety of student thinking.

D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to restate the information they know about Jayden, then look for a row in the table they can fill in.
Need support determining a constant of proportionality.	**Support** Invite students to reflect on strategies they have previously used to determine constants of proportionality.
Need support writing an equation.	**Strengthen** Consider asking, "Where do you see the constant of proportionality in this table?"
Finish Problem 4 early.	**Stretch** Invite students to complete the other part in preparation for actively listening to and comparing with a partner.

Direct students' attention to the discussion question.

Invite students to find a partner who completed the other part and complete Problems 5–8 together.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Jayden's Cooler

4. It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

Complete either Part 1 or Part 2.

Part 1

a. Complete the table.

b. What is a constant of proportionality in this relationship?

$\frac{8}{5}$

c. Complete the equation for this proportional relationship.

$w = \frac{8}{5}t$

Time (min), t	Water (gal), w
0	0
1	$\frac{8}{5}$
2.5	4
5	8
t	$\frac{8}{5}t$

Part 2

a. Complete the table.

b. What is a constant of proportionality in this relationship?

$\frac{5}{8}$

c. Complete the equation for this proportional relationship.

$t = \frac{5}{8}w$

Water (gal), w	Time (min), t
0	0
1	$\frac{5}{8}$
4	2.5
8	5
w	$\frac{5}{8}w$

5. Find a partner who completed the part you *didn't* complete.

Discuss: How are your responses alike? How are they different?

Responses vary. Our tables are both proportional and have two pairs of values that are the same: $t = 2.5$ and $w = 4$, $t = 5$ and $w = 8$. Our constants of proportionality are not identical but they are related: the numerator and denominator are just switched. The equations have the same structure but different constants of proportionality, and the variables are switched.

Unit 2 Lesson 6 116 Activity 2

Pairs | 20 min

Activity 2 Jayden's Cooler (continued)

Purpose: Students determine the two constants of proportionality for a relationship and discuss the concept of **reciprocals** to enrich their understanding.

2 Monitor

Collect a variety of language and ideas students use to describe the relationship between the two constants of proportionality, such as:

- They have the same numbers.
- The numerator and denominator are switched.
- They multiply to make 1.
- They are *reciprocals*.

Listen for and celebrate the different ways that students explain what each constant of proportionality represents in context.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore constants of proportionality further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Direct students' attention to Problem 4 and invite students to share the constants of proportionality they generated for each table.

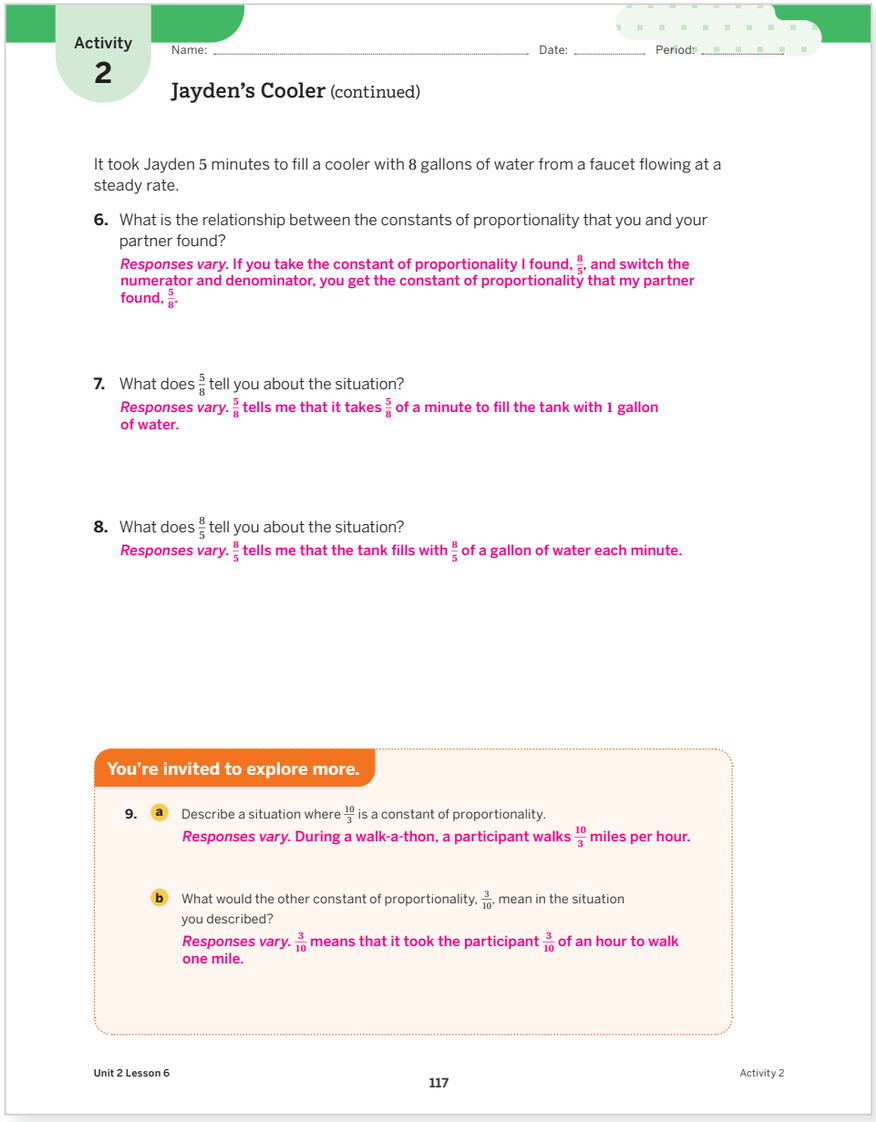
Share student examples and invite students to discuss their work further.

Consider asking:

- "How do you see the relationship between the two constants of proportionality?"
- "Where do you see the constant of proportionality in the table? In the equation?"
- "What does each constant of proportionality tell us about the situation?" **(MTR.2.1)**

Share that the relationship between the two constants of proportionality can be described using the term *reciprocals*.

Create a class definition of the term **reciprocal** using students' language, examples of equations, and the glossary. Record it for students to refer to later in the unit.



Activity 2

Name: _____ Date: _____ Period: _____

Jayden's Cooler (continued)

It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

6. What is the relationship between the constants of proportionality that you and your partner found?

Responses vary. If you take the constant of proportionality I found, $\frac{8}{5}$, and switch the numerator and denominator, you get the constant of proportionality that my partner found, $\frac{5}{8}$.

7. What does $\frac{5}{8}$ tell you about the situation?

Responses vary. $\frac{5}{8}$ tells me that it takes $\frac{5}{8}$ of a minute to fill the tank with 1 gallon of water.

8. What does $\frac{8}{5}$ tell you about the situation?

Responses vary. $\frac{8}{5}$ tells me that the tank fills with $\frac{8}{5}$ of a gallon of water each minute.

You're invited to explore more.

9. a. Describe a situation where $\frac{10}{3}$ is a constant of proportionality.

Responses vary. During a walk-a-thon, a participant walks $\frac{10}{3}$ miles per hour.

b. What would the other constant of proportionality, $\frac{3}{10}$, mean in the situation you described?

Responses vary. $\frac{3}{10}$ means that it took the participant $\frac{3}{10}$ of an hour to walk one mile.

Unit 2 Lesson 6 117 Activity 2

Key Takeaway: For every proportional relationship, there are two constants of proportionality that are **reciprocals** of each other.

Whole Class | 5 min

Synthesis

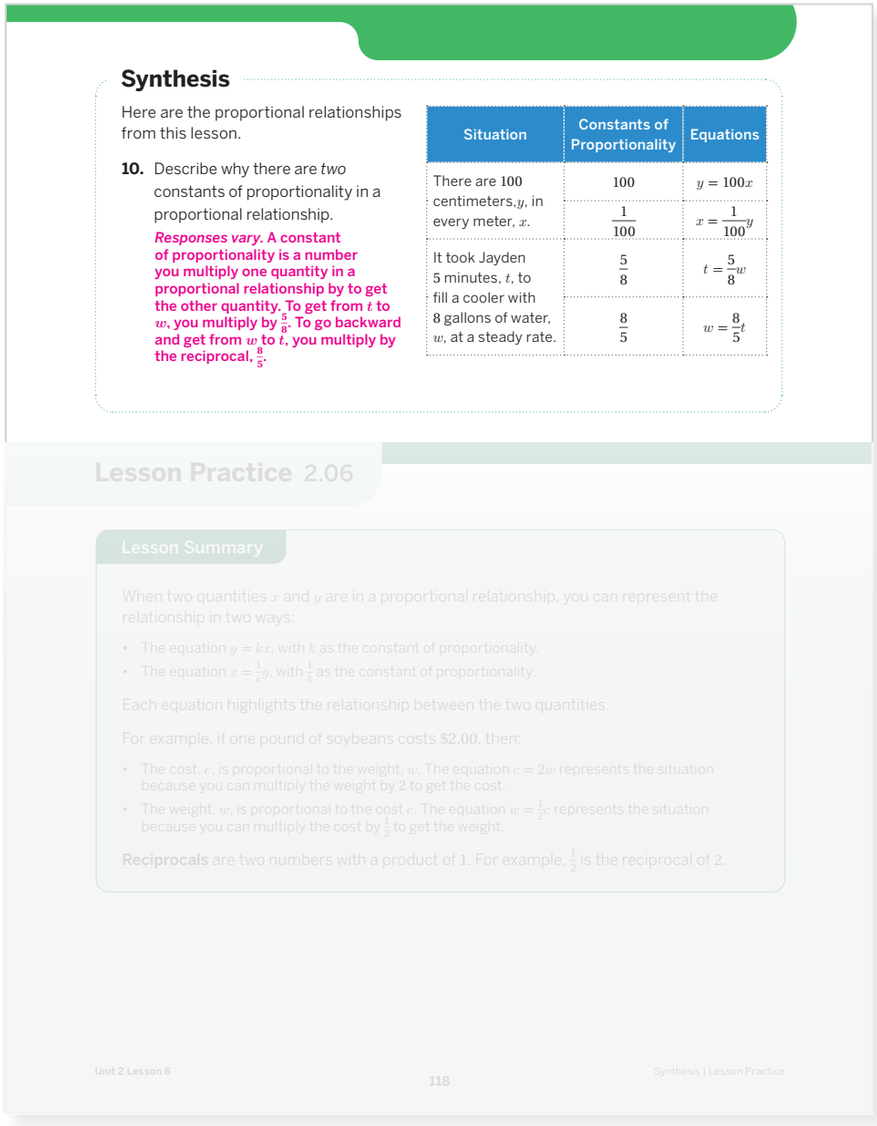
Purpose: Students synthesize their understanding of why there are two constants of proportionality in a proportional relationship.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The constant of proportionality tells you what to multiply to get from one quantity to another. There are two because you need to go back and forth between two quantities.
- The two constants of proportionality are like the two scale factors between scaled copies.
- In Jayden's example, the constants of proportionality represent how much water he used per minute or how many minutes it took to add one gallon of water to the cooler.
- In the measurement example, it is possible to convert from centimeters to meters or from meters to centimeters.

Consider asking, "If you knew one constant of proportionality, how might you figure out the other?"



Synthesis

Here are the proportional relationships from this lesson.

10. Describe why there are *two* constants of proportionality in a proportional relationship.

Responses vary. **A constant of proportionality is a number you multiply one quantity in a proportional relationship by to get the other quantity. To get from t to w, you multiply by $\frac{5}{8}$. To go backward and get from w to t, you multiply by the reciprocal, $\frac{8}{5}$.**

Situation	Constants of Proportionality	Equations
There are 100 centimeters, y, in every meter, x.	100	$y = 100x$
	$\frac{1}{100}$	$x = \frac{1}{100}y$
It took Jayden 5 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.	$\frac{5}{8}$	$t = \frac{5}{8}w$
	$\frac{8}{5}$	$w = \frac{8}{5}t$

Lesson Practice 2.06

Lesson Summary

When two quantities x and y are in a proportional relationship, you can represent the relationship in two ways:

- The equation $y = kx$, with k as the constant of proportionality.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

For example, if one pound of soybeans costs $2.00, then:

- The cost, c, is proportional to the weight, w. The equation $c = 2w$ represents the situation because you can multiply the weight by 2 to get the cost.
- The weight, w, is proportional to the cost c. The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

Reciprocals are two numbers with a product of 1. For example, $\frac{1}{2}$ is the reciprocal of 2.

Unit 2 Lesson 6 118 Synthesis | Lesson Practice

Lesson Takeaway: For every proportional relationship, there are two constants of proportionality that are **reciprocals** of each other. Two equations can be written to represent each proportional relationship.

Independent | 5 min

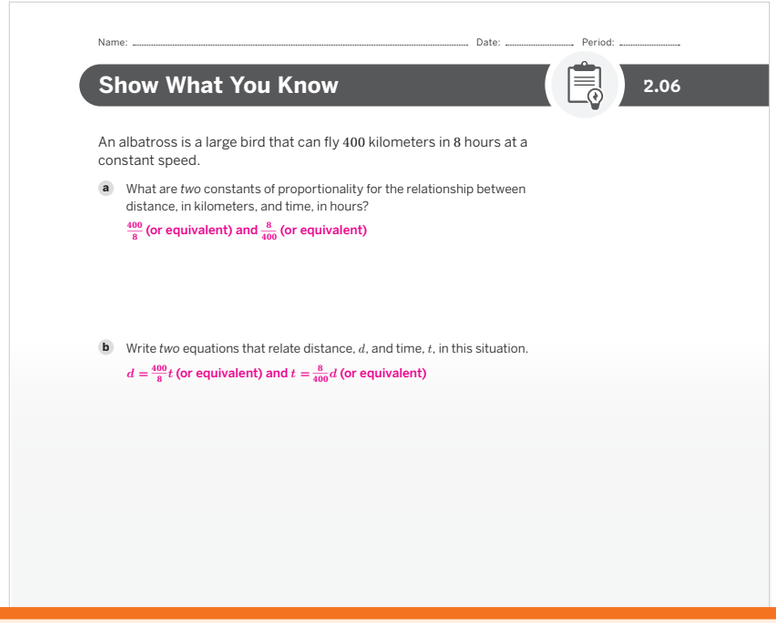
Show What You Know

Purpose: Students demonstrate their understanding by writing two equations to represent the same proportional relationship.

Today's Goals

1. **Goal:** Write two equations that represent the same proportional relationship, such as $y = kx$ and $x = \frac{1}{k}y$.
2. **Language Goal:** Explain the relationship between the two constants of proportionality in a proportional relationship using terms such as *reciprocal*. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.06

An albatross is a large bird that can fly 400 kilometers in 8 hours at a constant speed.

a. What are *two* constants of proportionality for the relationship between distance, in kilometers, and time, in hours?

$\frac{400}{8}$ (or equivalent) and $\frac{8}{400}$ (or equivalent)

b. Write *two* equations that relate distance, d, and time, t, in this situation.

$d = \frac{400}{8}t$ (or equivalent) and $t = \frac{8}{400}d$ (or equivalent)

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify both constants of proportionality in a relationship:

» **Respond to Student Thinking:**

- Use the ***Finding More Than One Equation to Represent Proportional Relationships*** **Mini-Lesson.**

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner

Professional Learning

Take a moment to reflect on students' work in pairs today. What trends do you see in participation? How might you invite students to celebrate each other's brilliance as they work with partners in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

Here are the proportional relationships from this lesson.

Situation	Constants of Proportionality	Equations
There are 100 centimeters, y, in every meter, x.	100	$y = 100x$
	$\frac{1}{100}$	$x = \frac{1}{100}y$
It took Jayden 5 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.	$\frac{5}{8}$	$t = \frac{5}{8}w$
	$\frac{8}{5}$	$w = \frac{8}{5}t$

10. Describe why there are *two* constants of proportionality in a proportional relationship.

Responses vary. A constant of proportionality is a number you multiply one quantity in a proportional relationship by to get the other quantity. To get from t to w, you multiply by $\frac{5}{8}$. To go backward and get from w to t, you multiply by the reciprocal, $\frac{8}{5}$.

Lesson Practice 2.06

Lesson Summary

When two quantities x and y are in a proportional relationship, you can represent the relationship in two ways:

- The equation $y = kx$, with k as the constant of proportionality.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

For example, if one pound of soybeans costs $2.00, then:

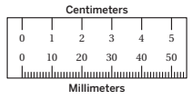
- The cost, c, is proportional to the weight, w. The equation $c = 2w$ represents the situation because you can multiply the weight by 2 to get the cost.
- The weight, w, is proportional to the cost c. The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

Reciprocals are two numbers with a product of 1. For example, $\frac{1}{2}$ is the reciprocal of 2.

Lesson Practice 2.06

Name: _____ Date: _____ Period: _____

Problems 1–3: There is a proportional relationship between a length measured in centimeters and the same length measured in millimeters.



1. Complete the table and then determine a constant of proportionality.

Length (cm)	Length (mm)
4	40
12	120
50	500
88.49	884.9

Constant of proportionality: 10 or $\frac{1}{10}$ (or equivalent)

2. Complete the table and then determine a constant of proportionality.

Length (mm)	Length (cm)
25	2.5
240	24
4	0.4
699.1	69.91

Constant of proportionality: $\frac{1}{10}$ or 10 (or equivalent)

3. How are these two constants of proportionality related to each other?

Responses vary. They are reciprocals of each other.

4. Amoli and Emika are converting measurements between inches and feet.

Amoli says that the constant of proportionality for the relationship between inches and feet is 12. Emika says it is $\frac{1}{12}$.

Whose claim is correct?

A. Amoli's B. Emika's C. Both D. Neither

Explain your thinking.

Explanations vary. Amoli is saying that there are 12 inches for every 1 foot. Emika is saying that there is $\frac{1}{12}$ of a foot for every 1 inch.

Lesson Practice 2.06

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 5–8: A recipe for granola calls for $\frac{5}{2}$ cups of rolled oats and $\frac{3}{4}$ cups of shredded coconut.

5. Complete the table to show how many cups of coconut should be used for different amounts of oats.

Oats (cups)	Coconut (cups)
1	$\frac{3}{10}$ or equivalent
5	$1\frac{1}{2}$ or equivalent
12	$3\frac{3}{5}$ or equivalent

6. What is a constant of proportionality in this relationship? What does it represent?

$\frac{3}{10}$ (or $\frac{10}{3}$). *Responses vary.* For each cup of oats, you should use $\frac{3}{10}$ cups of coconut.

7. What is the *other* constant of proportionality? What does it represent?

$\frac{10}{3}$ (or $\frac{3}{10}$). *Responses vary.* For each cup of coconut, you should use $\frac{10}{3}$ cups of oats.

8. In a recipe, t represents the number of cups of oats, and c represents the number of cups of coconut. Select *all* equations that represent the relationship between t and c.

- ☑ A. $c = \frac{3}{10}t$
- ☐ B. $t = \frac{3}{10}c$
- ☐ C. $c = \frac{10}{3}t$
- ☑ D. $t = \frac{10}{3}c$
- ☐ E. $c = \frac{5}{2}t$
- ☐ F. $t = \frac{2}{5}c$

Spiral Review

9. The Wrangell-St. Elias National Park and Preserve in Alaska has an area of 20,625 square miles. A scale drawing of the Wrangell-St. Elias National Park and Preserve has an area of 8.25 square inches. What is the scale of the drawing?

1 inch to 50 miles (or equivalent)

10. Which equation has a constant of proportionality equal to 5?

A. $5y = 5x$ B. $5y = 15x$ C. $3y = 5x$ D. $3y = 15x$

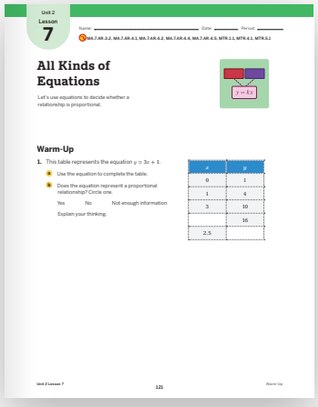
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.4.2, MA.7.AR.4.4
	4	2	MA.7.AR.4.2
FAST Practice	5–8	2	MA.7.AR.4.2 MA.7.AR.4.4
Spiral Review			
	9	2	MA.7.GR.1.5
	10	1	MA.7.AR.4.2

Unit 2

Lesson

7



All Kinds of Equations

Equations of Proportional Relationships

Let's use equations to decide whether a relationship is proportional.

Learning Outcomes

Today's Goals

1. **Goal:** Understand that equations of the form $y = kx$ represent proportional relationships.
2. **Language Goal:** Justify whether a relationship is proportional or not by looking at the structure of an equation. **(Reading, Writing, Speaking, and Listening)**
3. **Language Goal:** Compare and contrast equations that do and do not represent proportional relationships. **(Speaking and Listening)**

Students return to comparing proportional and non-proportional relationships, focusing on the connection between the structure of the equation and the type of relationship it represents. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students found the two constants of proportionality for a proportional relationship, explained how they are related to each other, and used them to write two equations representing the relationship.

Future Learning

In Lesson 8, students will explore what a proportional relationship looks like graphically.

Connections

- Students use the constant of proportionality to determine if real world relationships are proportional such as hours worked and money earned. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.1**
MA.6.AR.3.3	**MA.8.AR.3.2**
MA.6.AR.3.5	**MA.8.AR.3.3**

Lesson at a Glance

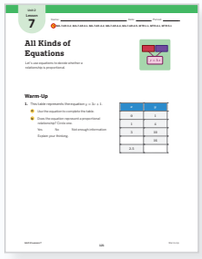
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Pairs | 5 min

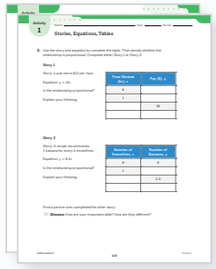
Students complete and use a table to describe whether a given relationship is proportional.



Activity 1

Pairs | 15 min

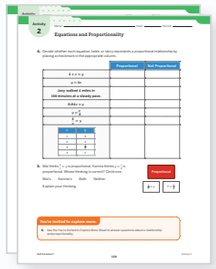
Students analyze multiple representations of relationships with a partner to surface strategies for deciding if an equation represents a proportional relationship.



Activity 2

Pairs | 15 min

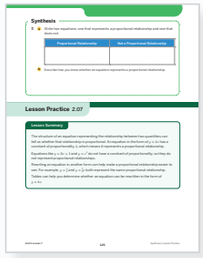
Students analyze relationships in multiple representations and use the **Decide and Defend** routine to construct viable arguments about whether a relationship is proportional.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of proportional relationships by writing an equation that represents a proportional relationship and another that does not. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting equations that represent proportional relationships.



Prep Checklist

Prepare Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 You're Invited to Explore More Sheet, one per student (as needed)
- calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students complete and use a table to describe whether a given relationship is proportional.

1 Launch

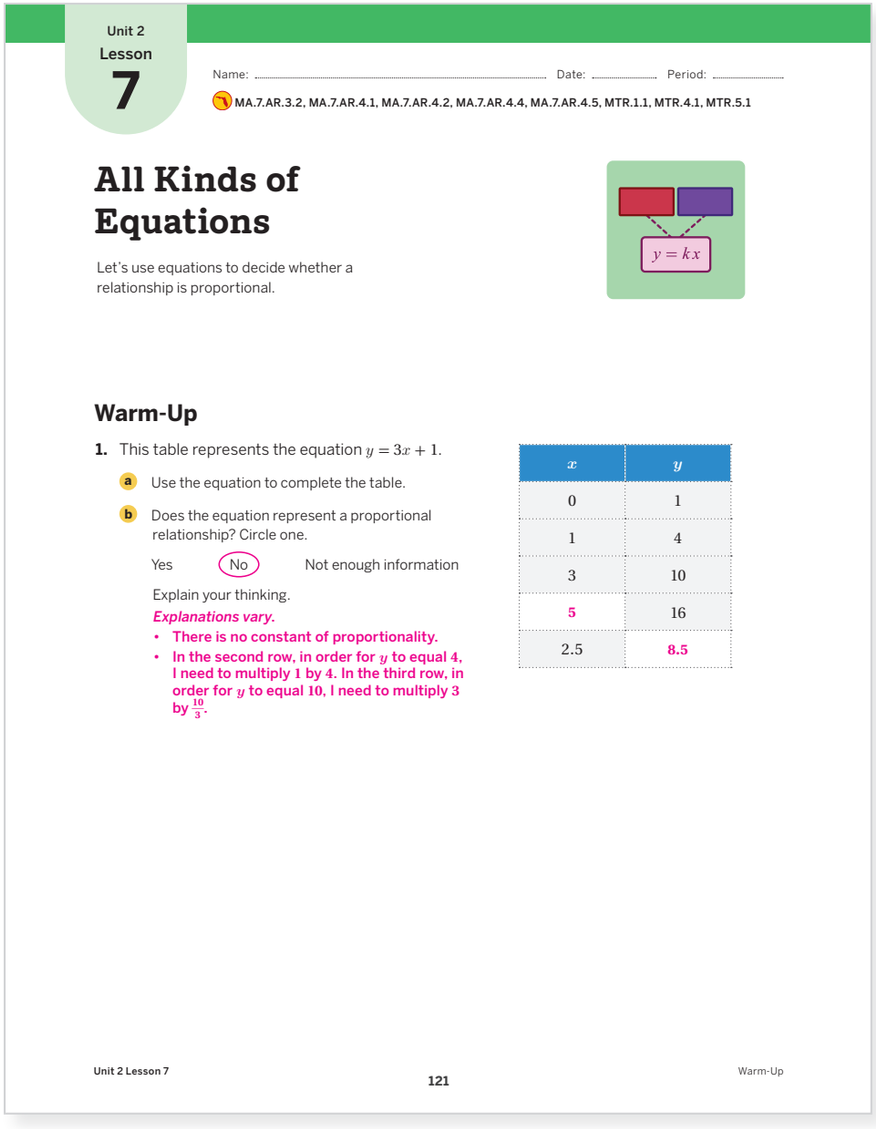
To surface prior knowledge, consider asking "What does it mean for a relationship to be proportional?"

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations throughout this lesson.

2 Connect

Collect and share several different student responses for proportional and not proportional.

Consider asking, "How did you decide if the relationship was proportional or not proportional?"



Unit 2
Lesson
7

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.1.1, MTR.4.1, MTR.5.1

All Kinds of Equations

Let's use equations to decide whether a relationship is proportional.

Warm-Up

1. This table represents the equation $y = 3x + 1$.

a. Use the equation to complete the table.

b. Does the equation represent a proportional relationship? Circle one.

Yes (No) Not enough information

Explain your thinking.

Explanations vary.

- There is no constant of proportionality.
- In the second row, in order for y to equal 4, I need to multiply 1 by 4. In the third row, in order for y to equal 10, I need to multiply 3 by $\frac{10}{3}$.

x	y
0	1
1	4
3	10
5	16
2.5	8.5

Unit 2 Lesson 7 121 Warm-Up

Pairs | 15 min

Activity 1 Stories, Equations, Tables

Purpose: Students analyze multiple representations of relationships with a partner to surface strategies for deciding if an equation represents a proportional relationship.

Short on time? Consider omitting the movement of connecting with a partner and instead hold the discussion briefly as a class.

1 Launch

To surface connections, consider asking, "What representations can we use to capture a story?"

Direct students' attention to the directions for the activity. Students will complete Story 1 or Story 2 and then compare responses.

ML/EL **MLR6: Three Reads (optional)** Consider inviting students to read the story to themselves multiple times, first focusing on what the situation is about, then what the quantities are, and finally the question, in order to support them in making sense of the relationship it represents. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

2 Monitor

Look for a variety of student strategies.

D **Differentiation** *(Stories 1–2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you use the equation to determine values in tables like this?"
Think the relationships are not proportional.	**Support** Encourage students to compare with classmates that worked on the same column to hear their strategies.
Keep increasing the x-values by 1 without analyzing the given y-value in the 3rd row.	**Support** Considering asking, "How can you use the equation to confirm values in each row of your table?"

Pause after most students have discussed Stories 1 and 2. Invite students to share whether their relationships were proportional and how they knew.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Stories, Equations, Tables

2. Use the story and equation to complete the table. Then decide whether the relationship is proportional. Complete either Story 1 or Story 2.

Story 1

Story: Lucia earns \$12 per hour.

Equation: $y = 12x$

Is the relationship proportional?
Yes
Explain your thinking.
Explanations vary. The constant of proportionality is 12.

Time Worked (hr), x	Pay (\$), y
0	0
1	12
2.5	30
3	36

Responses in the last row vary. Sample shown.

Story 2

Story: A recipe recommends 1 banana for every 2 smoothies.

Equation: $y = 0.5x$

Is the relationship proportional?
Yes
Explain your thinking.
Explanations vary. The constant of proportionality is 0.5.

Number of Smoothies, x	Number of Bananas, y
0	0
1	0.5
5	2.5
8	4

Responses in the last row vary. Sample shown.

Find a partner who completed the other story.

Discuss: How are your responses alike? How are they different?

Responses vary. The equations, tables, and stories were different, but both relationships were proportional even though they had different constants of proportionality.

Unit 2 Lesson 7 122 Activity 1

Pairs | 15 min

Activity 1 Stories, Equations, Tables (continued)

Purpose: Students analyze multiple representations of relationships with a partner to surface strategies for deciding if an equation represents a proportional relationship.

2 Monitor

Invite students to complete Story 3 or Story 4.

D Differentiation *(Stories 3–4)*

Look for students who:	Respond to Student Thinking
Need support with remembering how to work with exponents. *(Story 4)*	**Support** Remind them another way to write x^2 is $x \cdot x$. Consider asking, "How does the story about the area help us determine the missing values?"
Thinking that the situation is proportional because there is a rate of \$35 per month. *(Story 3)*	**Support** Invite them to revisit other problems identified as proportional and consider asking, "What was the relationship between the left column and right column in the table?"
Would like to explore relationships in stories further.	**Stretch** Consider asking, "Can you write another story that would be represented by the same equation and table?" **Stretch** Consider asking, "How could you describe the relationship between constants of proportionality and coefficients to a classmate?"

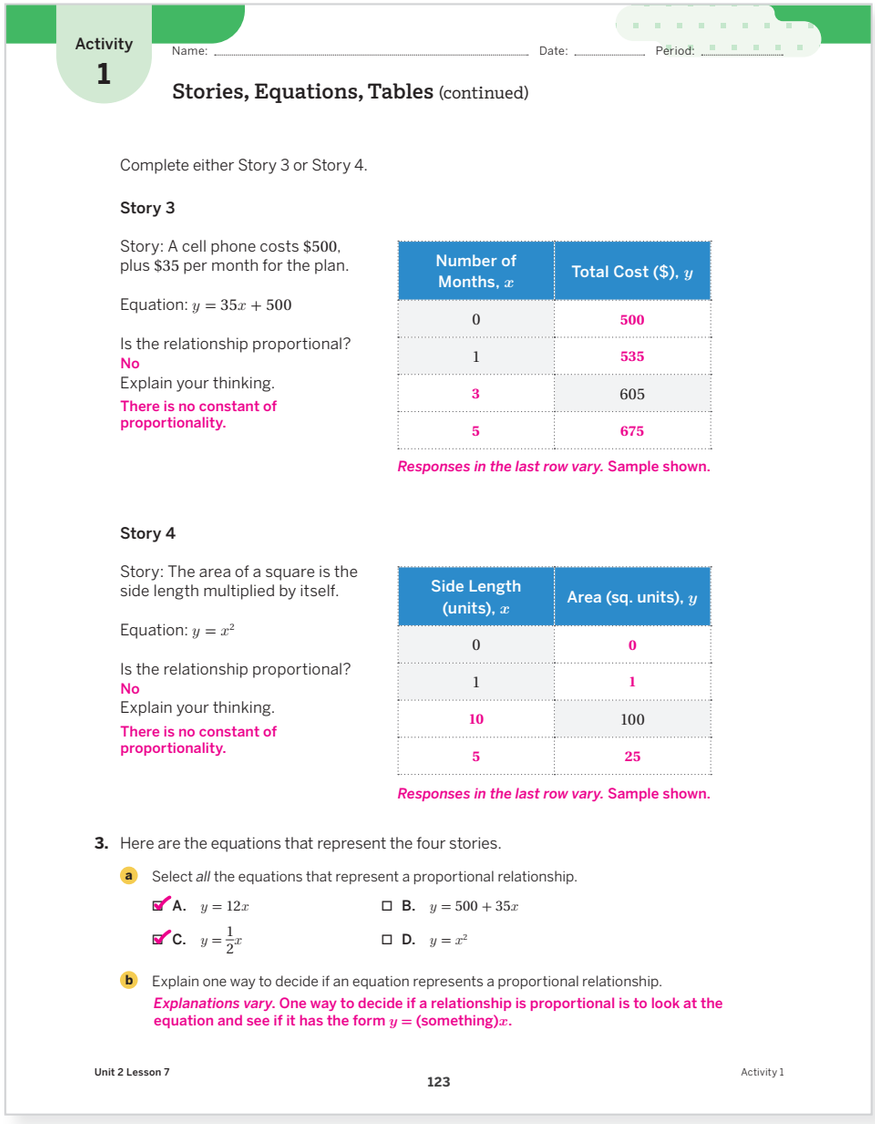
Invite students to share their thinking after comparing with a partner.

Listen for student explanations to highlight during the Connect, such as:
- Identifying the constant of proportionality.
- Looking at the equation and making generalizations about the structure, e.g., "It has the form $y =$ (something)x." **(MTR.5.1)**

3 Connect

Share the captured student explanations to highlight different strategies for deciding if an equation represents a proportional relationship.

To surface the Key Takeaway, consider asking:
- "How can you tell from the equation if a relationship is proportional?"
- "Where can you see the constant of proportionality in the equation?"
- "What does the constant of proportionality tell us about the story?"



Activity 1

Name: Date: Period:

Stories, Equations, Tables (continued)

Complete either Story 3 or Story 4.

Story 3

Story: A cell phone costs \$500, plus \$35 per month for the plan.

Equation: $y = 35x + 500$

Is the relationship proportional?
No
Explain your thinking.
There is no constant of proportionality.

Number of Months, x	Total Cost (\$), y
0	500
1	535
3	605
5	675

Responses in the last row vary. Sample shown.

Story 4

Story: The area of a square is the side length multiplied by itself.

Equation: $y = x^2$

Is the relationship proportional?
No
Explain your thinking.
There is no constant of proportionality.

Side Length (units), x	Area (sq. units), y
0	0
1	1
10	100
5	25

Responses in the last row vary. Sample shown.

3. Here are the equations that represent the four stories.

a. Select *all* the equations that represent a proportional relationship.

- ☑ A. $y = 12x$
- ☐ B. $y = 500 + 35x$
- ☑ C. $y = \frac{1}{2}x$
- ☐ D. $y = x^2$

b. Explain one way to decide if an equation represents a proportional relationship.
Explanations vary. One way to decide if a relationship is proportional is to look at the equation and see if it has the form $y =$ (something)x.

Unit 2 Lesson 7 123 Activity 1

Key Takeaway: If an equation represents a proportional relationship, then one variable is multiplied or divided by the constant of proportionality.

Pairs | 15 min

Activity 2 Equations and Proportionality

Purpose: Students analyze relationships in multiple representations and use the Decide and Defend routine to construct viable arguments about whether a relationship is proportional.

1 Launch

Invite students to make sense of the relationships independently. Encourage students to share their reasoning with a partner and work together to reach an agreement about how to determine whether each equation, table, or story represents a proportional relationship.

2 Monitor

Listen for different reasoning students give for why representations are proportional or not.

A **Accessibility: Executive Function** Invite students to focus on one representation at a time to support students in carrying out multiple steps.

D **Differentiation: (Support)** Students may struggle with equations where the y is not isolated. Invite them to work with a partner to rewrite the equation or create a table of values.

Listen and look for a variety of student responses.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "___ is proportional/not proportional because ___."). **(Writing and Speaking)**

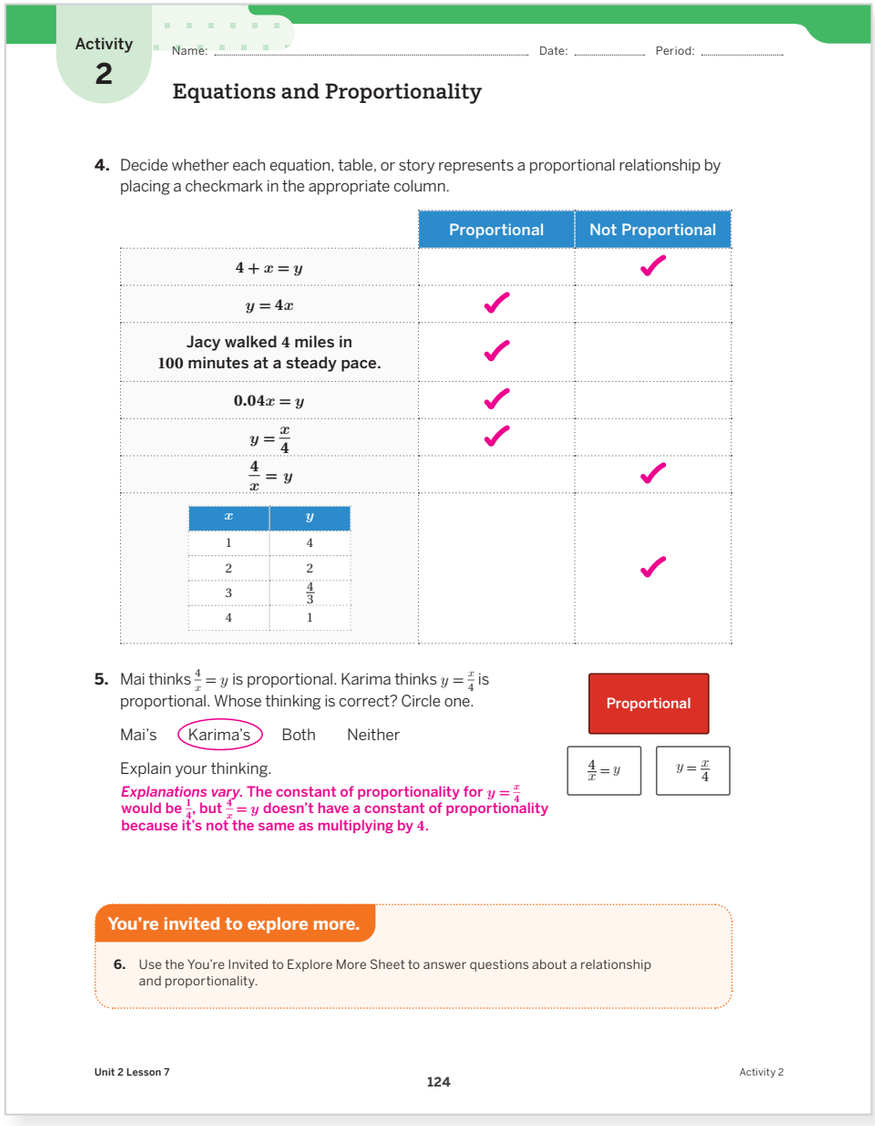
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who want to explore proportional and not proportional situations further to complete this optional task. Encourage them to discuss their thinking with a partner. **Note:** Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students to share their reasoning for each choice. Consider asking, "How are each of these like other equations we have used today? How are they different?"



Activity 2

Name: _____ Date: _____ Period: _____

Equations and Proportionality

4. Decide whether each equation, table, or story represents a proportional relationship by placing a checkmark in the appropriate column.

	Proportional	Not Proportional
$4 + x = y$		✓
$y = 4x$	✓	
Jacy walked 4 miles in 100 minutes at a steady pace.	✓	
$0.04x = y$	✓	
$y = \frac{x}{4}$	✓	
$\frac{4}{x} = y$		✓
x | y: 1 | 4; 2 | 2; 3 | $\frac{4}{3}$; 4 | 1		✓

5. Mai thinks $\frac{4}{x} = y$ is proportional. Karima thinks $y = \frac{x}{4}$ is proportional. Whose thinking is correct? Circle one.

Mai's (Karima's) Both Neither

Explain your thinking.

Explanations vary. The constant of proportionality for $y = \frac{x}{4}$ would be $\frac{1}{4}$, but $\frac{4}{x} = y$ doesn't have a constant of proportionality because it's not the same as multiplying by 4.

Proportional

$\frac{4}{x} = y$ $y = \frac{x}{4}$

You're invited to explore more.

6. Use the You're Invited to Explore More Sheet to answer questions about a relationship and proportionality.

Unit 2 Lesson 7 124 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of proportional relationships by writing an equation that represents a proportional relationship and another that does not.

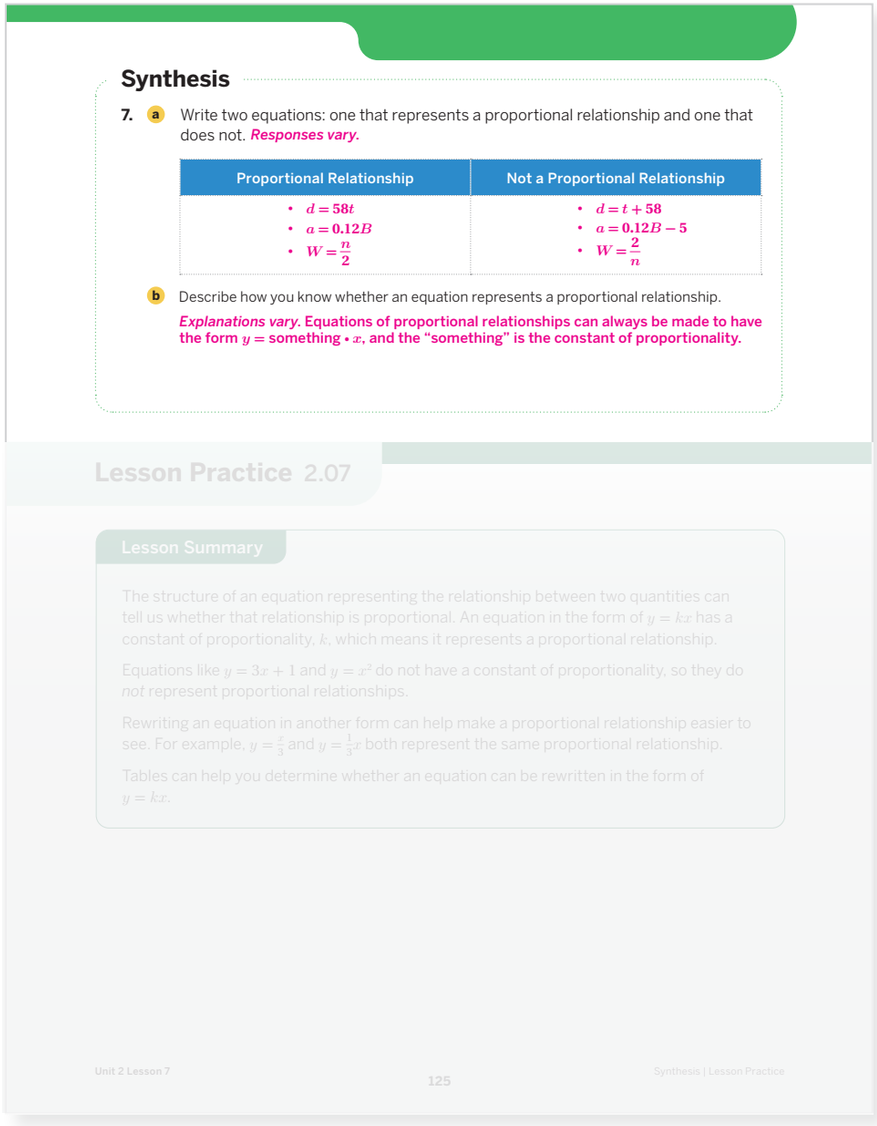
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of equations and ideas, such as:

- Proportional equations look like $y =$ (something)x.
- Non-proportional equations sometimes have a plus in them.
- Non-proportional equations sometimes have exponents.

MLR **MLR2: Collect and Display** Use this routine to collect students' equations as they generate them with a partner. Add examples of proportional and non-proportional relationships to the class anchor chart. Consider displaying the chart for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to add to their previous thinking for the term *constant of proportionality*, based on the discussion about equations.



Synthesis

7. **a** Write two equations: one that represents a proportional relationship and one that does not. ***Responses vary.***

Proportional Relationship	Not a Proportional Relationship
• $d = 58t$ • $a = 0.12B$ • $W = \frac{n}{2}$	• $d = t + 58$ • $a = 0.12B - 5$ • $W = \frac{2}{n}$

b Describe how you know whether an equation represents a proportional relationship.

Explanations vary. **Equations of proportional relationships can always be made to have the form $y =$ something $\cdot x$, and the "something" is the constant of proportionality.**

Lesson Practice 2.07

Lesson Summary

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a constant of proportionality, k, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a constant of proportionality, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Unit 2 Lesson 7 125 Synthesis | Lesson Practice

Lesson Takeaway: Proportional relationships, in equation form, can always be rewritten to have the form $y = k \cdot x$, where k is the constant of proportionality.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting equations that represent proportional relationships.

Today's Goals

1. **Goal:** Understand that equations of the form $y = kx$ represent proportional relationships.
2. **Language Goal:** Justify whether a relationship is proportional or not by looking at the structure of an equation. **(Reading, Writing, Speaking, and Listening)**
3. **Language Goal:** Compare and contrast equations that do and do not represent proportional relationships. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.07

Select *all* the equations that represent proportional relationships.

- ☐ A. $180 - x = y$
- ☑ B. $1.08x = y$
- ☐ C. $y = x^8$
- ☑ D. $y = \frac{x}{8}$
- ☑ E. $y = 8x$

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify equations that represent a proportional relationship:

» **Respond to Student Thinking:**

- Revisit Activity 2 Problem 4 as a class and invite students to share strategies for identifying equations that represent a proportional relationship.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on students' progress with proportional relationships. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. a. Write two equations: one that represents a proportional relationship and one that does not. *Responses vary.*

Proportional Relationship	Not a Proportional Relationship
• $d = 58t$ • $a = 0.12B$ • $W = \frac{n}{2}$	• $d = t + 58$ • $a = 0.12B - 5$ • $W = \frac{2}{n}$

b. Describe how you know whether an equation represents a proportional relationship.

Explanations vary. Equations of proportional relationships can always be made to have the form $y =$ something $\cdot x$, and the "something" is the constant of proportionality.

Lesson Practice 2.07

Lesson Summary

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a constant of proportionality, k, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a constant of proportionality, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Lesson Practice 2.07

Name: _____ Date: _____ Period: _____

Problems 1–2: The relationship between a distance in yards, y, and the same distance in miles, x, is represented by the equation $y = 1760x$.

1. Complete the table.

Distance (mi), x	Distance (yd), y
1	1,760
5	8,800
2	3,520
10	17,600

2. Is there a proportional relationship between a distance in yards and the same distance in miles?

Yes

Explain your thinking.

Explanations vary. The equation is of the form $y = kx$, where $k = 1760$.

Problems 3–4: Determine whether or not each relationship is proportional. Explain your thinking.

3.

x	y
2	5
3	7.5
6	15

Yes. *Explanations vary.* It could be proportional because $k = \frac{5}{2}$ for each pair of values listed in the table.

4. $y = 3.2x + 5$

No. *Explanations vary.* It's not proportional because the equation is not of the form $y = kx$.

Problems 5–6: Determine whether each relationship is proportional or not proportional. Explain your thinking.

5. The weight of a stack of standard 8.5-by-11-inch paper and the number of sheets of paper.

Proportional. *Explanations vary.* The weight of each sheet of paper is constant.

6. The weight of a stack of different-sized books (where each book weighs a different amount) and the number of books in a stack.

Not proportional. *Explanations vary.* Each book weighs a different amount, so there is no constant weight.

Lesson Practice 2.07

Name: _____ Date: _____ Period: _____

7. Determine whether or not each relationship is proportional.

	Proportional	Not Proportional
The remaining length, L, of a 120-inch rope after x inches have been cut off: $120 - x = L$.	☐	☑
The total cost, t, after 8% sales tax is added to an item's price p: $1.08p = t$.	☑	☐
The number of marbles, x, each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$.	☑	☐
The volume, V, of a rectangular prism whose height is 12 centimeters and whose base is a square with side lengths of s centimeters: $V = 12s^2$.	☐	☑

FAST Practice

8. Liam and Sadia are running a 60-meter race. Each of their distances can be represented by an equation in the form $y = kx$ where y is the distance in meters and x is the time in seconds.

Use this information to complete the table.

- Liam's distance is represented with the equation $y = 6x$.
- At 6 seconds, Liam is 12 meters ahead of Sadia.

Time (sec)	Liam's Distance (m)	Sadia's Distance (m)
0	0	0
2	12	8
4	24	16
6	36	24

Spiral Review

9. The equation $y = 3.5x$ can be used to determine the total cost, y, in dollars, of x ounces of blueberries. What does the number 3.5 represent in the equation?

A. The number of blueberries that \$1 can buy.

B. The number of blueberries in x ounces.

C. The cost of 1 ounce of blueberries.

D. The cost of x ounces of blueberries.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.7.AR.4.1, MA.7.AR.4.4
	3–4, 7	1	MA.7.AR.4.1
	5–6	2	MA.7.AR.4.1
FAST Practice	8	3	MA.7.AR.4.1, MA.7.AR.4.4
Spiral Review			
	9	2	MA.7.AR.4.2

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining whether quantities are in a proportional relationship from a table	Lesson 2	2	MA.7.AR.4.1, MTR.5.1
2	Determining whether quantities are in a proportional relationship from an equation or situation	Lesson 4	2	MA.7.AR.4.1, MA.7.AR.4.4
3a	Explaining the meaning of a constant of proportionality	Lesson 3	2	MA.7.AR.3.2, MA.7.AR.4.5, MTR.6.1
3b, 3c	Solving problems using equations of proportional relationships	Lesson 5	2	MA.7.AR.4.5
4a	Determining values in a table of a proportional relationship	Lesson 4	2	MA.7.AR.3.2
4b	Determining a constant of proportionality for a proportional relationship represented in a table	Lesson 6	2	MA.7.AR.4.2
4c	Writing an equation for a proportional relationship	Lesson 5	2	MA.7.AR.4.4, MTR.7.1
5a	Determining two constants of proportionality for a proportional relationship	Lesson 6	2	MA.7.AR.4.2
5b	Writing two equations for a proportional relationship	Lesson 7	2	MA.7.AR.4.4, MTR.7.1
5c	Solving problems using equations of proportional relationships	Lesson 7	2	MA.7.AR.4.5

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 2

1. Which table represents a proportional relationship?

A.

x	y
2	3
4	5
10	11

B.

x	y
2	4
4	5
10	6

C.

x	y
2	4
4	16
10	100

D.

x	y
2	3
4	6
10	15

2. Select *all* of the proportional relationships.

- ☑ A. $y = 2.5x$
- ☐ B. A turtle walks for 5 minutes, then stops for a minute.
- ☑ C. A turtle starts at the starting line and walks at a constant rate.
- ☐ D. $y = \frac{2.5}{x}$
- ☑ E. $y = \frac{5}{2}x$

3. Jordan is mixing water and flour to make tortillas. The equation $w = 0.75f$ represents the number of cups of water, w, needed for f cups of flour.

a. What does 0.75 mean in this situation?
Responses vary. It means that 0.75 cups of water must be mixed with each cup of flour.

b. How many cups of water does Jordan need for 4 cups of flour?
3 cups of water (or equivalent)

c. How many cups of flour does Jordan need for 1 cup of water?
$\frac{4}{3}$ cups of flour (or equivalent)

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 2

4. When you mix two colors of paint in equivalent ratios, the result is always the same color.

a. Complete the table so that there is a proportional relationship between cups of blue paint and cups of red paint.

Blue Paint (cups), b	Red Paint (cups), r
1	$\frac{3}{2}$ (or equivalent)
2	3
$\frac{14}{3}$ (or equivalent)	7
10	15 (or equivalent)

b. What is a constant of proportionality for this relationship?
$\frac{3}{2}$ (or equivalent) or $\frac{2}{3}$ (or equivalent)

What does it represent in this situation?
Explanations vary.
- The constant of proportionality $\frac{3}{2}$ represents the number of cups of red paint needed for each cup of blue paint.
- The constant of proportionality $\frac{2}{3}$ represents the number of cups of blue paint needed for each cup of red paint.

c. Write an equation for the relationship between the number of cups of blue paint, b, and the number of cups of red paint, r.
$r = \frac{3}{2}b$ (or equivalent) or $b = \frac{2}{3}r$ (or equivalent)

5. Titus took 30 minutes to walk 2 miles at a constant rate.
- d is the distance Titus walks, in miles.
- t is the time it takes Titus to walk, in minutes.

a. Write *two* different constants of proportionality for this relationship.
15 (or equivalent), $\frac{1}{15}$ (or equivalent)

b. Write *two* different equations to represent the relationship between d and t.
$t = 15d$ (or equivalent), $d = \frac{1}{15}t$ (or equivalent)

c. If Titus walked 7.5 miles at this same rate, how many minutes would it take? Explain or show your thinking.
112.5 minutes. *Explanations vary.* I multiplied 7.5 miles by the constant of proportionality, 15, to get $7.5 \cdot 15 = 112.5$ minutes.

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Use tables to recognize proportional relationships • Determine a constant of proportionality from a table **(Lessons 1–3)**	1, 4a, 4b	Support	• **Mini-Lesson: Determining the Constant of Proportionality**
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 3, Activity 2 and discuss Problems 7 and 8. Ask students to share with a partner their thinking for the constant of proportionality found.
		Stretch	• **You're invited to explore more:** Lesson 2 (Balloon Float)
Sub-Unit 2: • Write and use equations to describe proportional relationships. • Determine a constant of proportionality from an equation. **(Lessons 4–7)**	3, 4c	Support Strengthen	• **Mini-Lesson: Finding More Than One Equation to Represent Proportional Relationships**
		Stretch	• **Teacher Move:** Consider revisiting Lesson 7, Activity 2 and discuss Problem 4. Ask students to share with a partner their thinking for each given situation.
	2, 5	Support	• **Teacher Move:** Consider revisiting Lesson 5, Activity 2 and asking, "How can you use a hanger diagram to solve the equation?" • **Challenge Creator:** Lesson 4 (Balancing Moves)
		Strengthen	• **Teacher Move:** Consider asking, "How does the diagram connect with the expression and situation? What is known and unknown?" Then invite students to collaborate and revise Problem 5.
		Stretch	• **You're invited to explore more:** Lesson 7 (All Kinds of Equations)

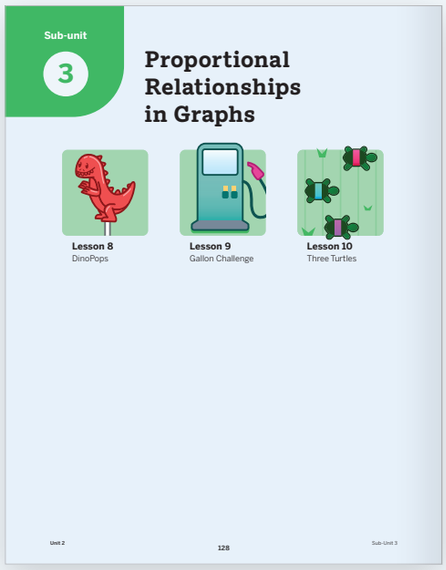
Sub-Unit 3

Proportional Relationships in Graphs

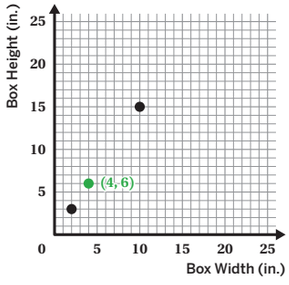
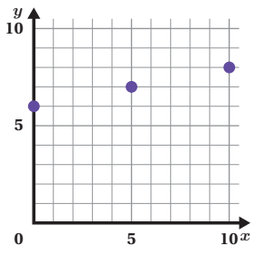
Sub-Unit 3 Goals

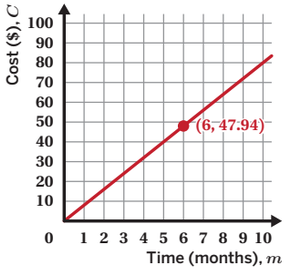
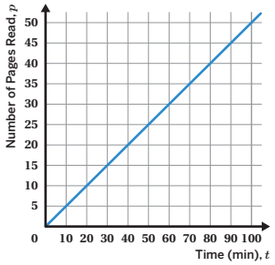
- **Goal:** Use graphs to recognize proportional relationships.
- **Goal:** Determine a constant of proportionality from a graph.
- **Language Goal:** Describe relationships represented with graphs using terms such as *proportional*, *non-proportional*, and *constant of proportionality*.

In this sub-unit, students recognize proportional relationships represented in graphs and determine the constant of proportionality using graphs. They learn that a proportional relationship can be represented by points on a line that passes through the **origin** on a coordinate plane.



Math that Matters Most

Concept or Skill	For example . . .
Use a graph of a proportional relationship to predict other points in the same relationship.	The graphed point represents a TriceraPop in a box. Add at least two more points to the graph to represent other boxes that fit a TriceraPop. 
Identify whether a relationship is proportional by studying a graph.	Determine whether the graph represents a proportional relationship. Explain your thinking.  *It is not proportional. The points are on a line, but it does not go through the origin.*

Concept or Skill	For example . . .
Determine a constant of proportionality from a graph.	What is the constant of proportionality in this relationship?  *The constant of proportionality is 7.99.*
Write an equation for a proportional relationship given a graph.	Write an equation that represents the relationship between the time in minutes, t, and the total number of pages Joel has read, p.  $p = \frac{1}{2}t$

Unit 2

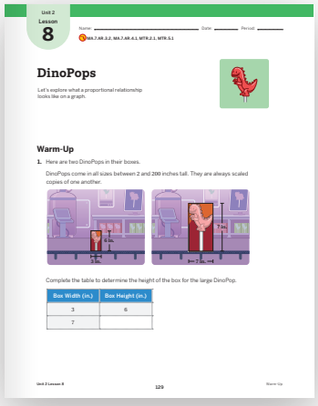
Lesson

8

DinoPops

Introducing Graphs of Proportional Relationships

Let's explore what a proportional relationship looks like on a graph.


Unit 2
Lesson
8
Name _____ Date _____ Period _____
MA.7.AR.3.2, MA.7.AR.4.1, MTR.2.1, MTR.5.1
DinoPops
Let's explore what a proportional relationship looks like on a graph.
Warm-Up
1. Here are two DinoPops in their boxes.
DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.
Complete the table to determine the height of the box for the large DinoPop.
Box Width (in.) | Box Height (in.)
3 | 6
7 |
Unit 2 Lesson 8
129
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Use a graph that represents a proportional relationship to predict other points in that relationship.
2. **Goal:** Understand that a proportional relationship can be represented by points on a line that passes through the *origin*.
3. **Language Goal:** Justify whether a graph represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Students explore what a proportional relationship looks like graphically. This is the first time in Grade 7 that students encounter a *coordinate plane*. Students reason abstractly and quantitatively about how lollipop boxes can be represented as points on a coordinate plane.

Coherence

Prior Learning

In Lesson 7, students developed strategies for deciding whether a relationship is proportional based on the structure of its equation.

Future Learning

In Lesson 9, students will work with graphs of proportional relationships and identify the constant of proportionality using a graph.

Connections

- Within the context of comparing box sizes of Dinopops, students determine if a relationship is proportional by its graph. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5 **MA.6.GR.1.1**	**MA.8.AR.3.1**

Vocabulary

New Vocabulary

origin

Review Vocabulary

coordinate plane

Lesson at a Glance

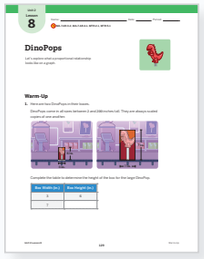
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.1

Warm-Up

Pairs | 5 min

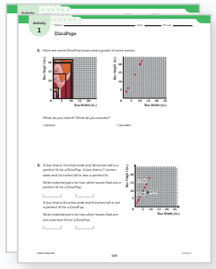
Students familiarize themselves with the lesson context and establish that the relationship between lollipop box width and box height is proportional.



Activity 1

Pairs | 8 min

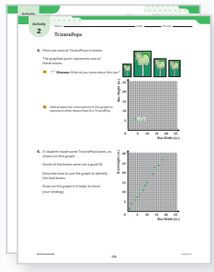
Students use the **Notice and Wonder** routine to explore graphs of proportional relationships, including their connection to the **origin**.



Activity 2

Pairs | 12 min

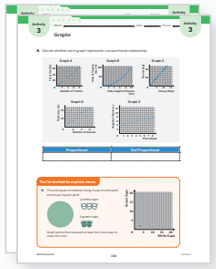
Students continue to graph proportional relationships and discuss the key features that allow them to determine proportionality from a graph.



Activity 3

Pairs | 10 min

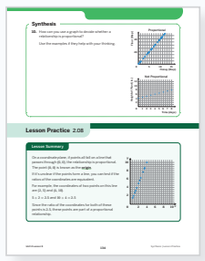
Students interpret graphs and discuss whether or not they represent a proportional relationship.



Synthesis

Whole Class | 5 min

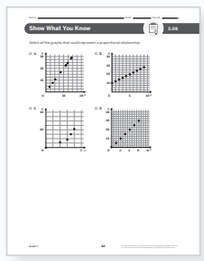
Students synthesize their understanding of representing proportional relationships using graphs.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying graphs of proportional relationships.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students familiarize themselves with the lesson context and establish that the relationship between lollipop box width and box height is proportional.

1 Launch

To generate excitement, consider asking students to guess the height of the record-holding world's tallest lollipop. (See's Candies made a lollipop that was approximately 199 inches tall in 2012.)

Invite students to determine measurements for other boxes that would fit a DinoPop.

2 Connect

Consider asking, "Is the relationship between box width and box height proportional? How can you tell?" *Responses vary.*

- Yes. The problem says they are scaled copies, and scaled copies are always proportional.
- Yes. In the table, you can always multiply the width by 2 to get the height. There is a constant of proportionality of 2.
- Yes. It could be represented by the equation $h = 2w$.

Unit 2
Lesson
8

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.1, MTR.2.1, MTR.5.1

DinoPops

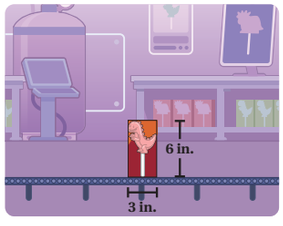
Let's explore what a proportional relationship looks like on a graph.

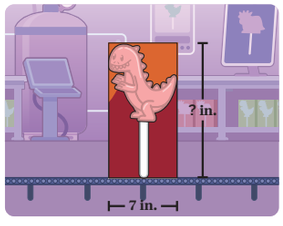


Warm-Up

1. Here are two DinoPops in their boxes.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.





Complete the table to determine the height of the box for the large DinoPop.

Box Width (in.)	Box Height (in.)
3	6
7	14

Unit 2 Lesson 8 129 Warm-Up

Pairs | 8 min

Activity 1 DinoPops

Purpose: Students use the Notice and Wonder routine to explore graphs of proportional relationships, including their connection to the **origin**.

1 Launch

Direct students' attention to the DinoPop boxes and the graph.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the graph. **(MTR.2.1)**

Create a record of what students noticed and wondered, along with the names of the students who shared.

Note: If it does not come up naturally, consider sharing that the graph shows points on a *coordinate plane*. Invite students to read the axis labels to identify the quantities represented.

2 Monitor

Invite students to write ordered pairs for boxes that are a perfect fit for a DinoPop and for boxes that are not a perfect fit for a DinoPop.

Listen for a variety of informal language students use to describe the graph.

3 Connect

To surface the Key Takeaway, consider asking, "What do you notice about the points representing a 'perfect fit' for a DinoPop?" **(MTR.5.1)**

Note: If it does not come up naturally, consider sharing that you could draw a straight line through the points, starting at $(0, 0)$. Share that this point is called the **origin**.

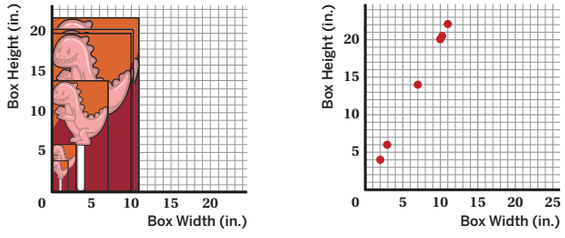
Consider asking, "Do you think a point at the origin represents a perfect DinoPops box? Why or why not?"

Activity 1

Name: _____ Date: _____ Period: _____

DinoPops

2. Here are some DinoPop boxes and a graph of some points.



What do you notice? What do you wonder?

I notice: *Responses vary.*
- I notice that each point on the graph corresponds to one DinoPop box.
- I notice that you can draw a straight line through all of the points.
- I notice that the imaginary line passes through the lower-left corner of the graph, (0, 0).

I wonder: *Responses vary.*
- I wonder if all of the DinoPop boxes will be on this imaginary line.
- I wonder if other types of boxes make points that line up nicely.
- I wonder if other types of boxes form a line through (0, 0).

3. A box that is 5 inches wide and 10 inches tall is a perfect fit for a DinoPop. A box that is 7 inches wide and 14 inches tall is also a perfect fit.

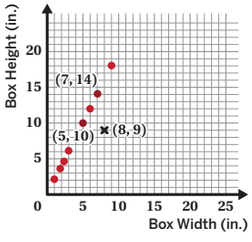
Write ordered pairs for two other boxes that are a perfect fit for a DinoPop. *Responses vary.*

(__4__, __8__) (__8__, __16__)

A box that is 8 inches wide and 9 inches tall is *not* a perfect fit for a DinoPop.

Write ordered pairs for two other boxes that are *not* a perfect fit for a DinoPop. *Responses vary.*

(__2__, __10__) (__7__, __5__)



Unit 2 Lesson 8 130 Activity 1

Key Takeaway: You can represent a proportional relationship on a graph with a straight line passing through the point $(0, 0)$, which is called the **origin**.

Pairs | 12 min

Activity 2 TriceraPops

Purpose: Students continue to graph proportional relationships and discuss the key features that allow them to determine proportionality from a graph.

Short on time? Consider omitting Problem 6 and instead using it as a differentiation support for students needing an access point for Problem 7.

1 Launch

Invite students to discuss and add points to the graph.

D **Differentiation: (Support)** To support students getting started, consider asking, "What do you think each coordinate in the point $(4, 6)$ means?" **(MTR.2.1)**

Consider asking, "What strategies did you use to predict other points that would create a perfect TriceraPops box?"

A **Accessibility: Memory and Attention** Invite students to record strategies as they are shared to support students with keeping track of information. Students can use these strategies as they complete Problem 5.

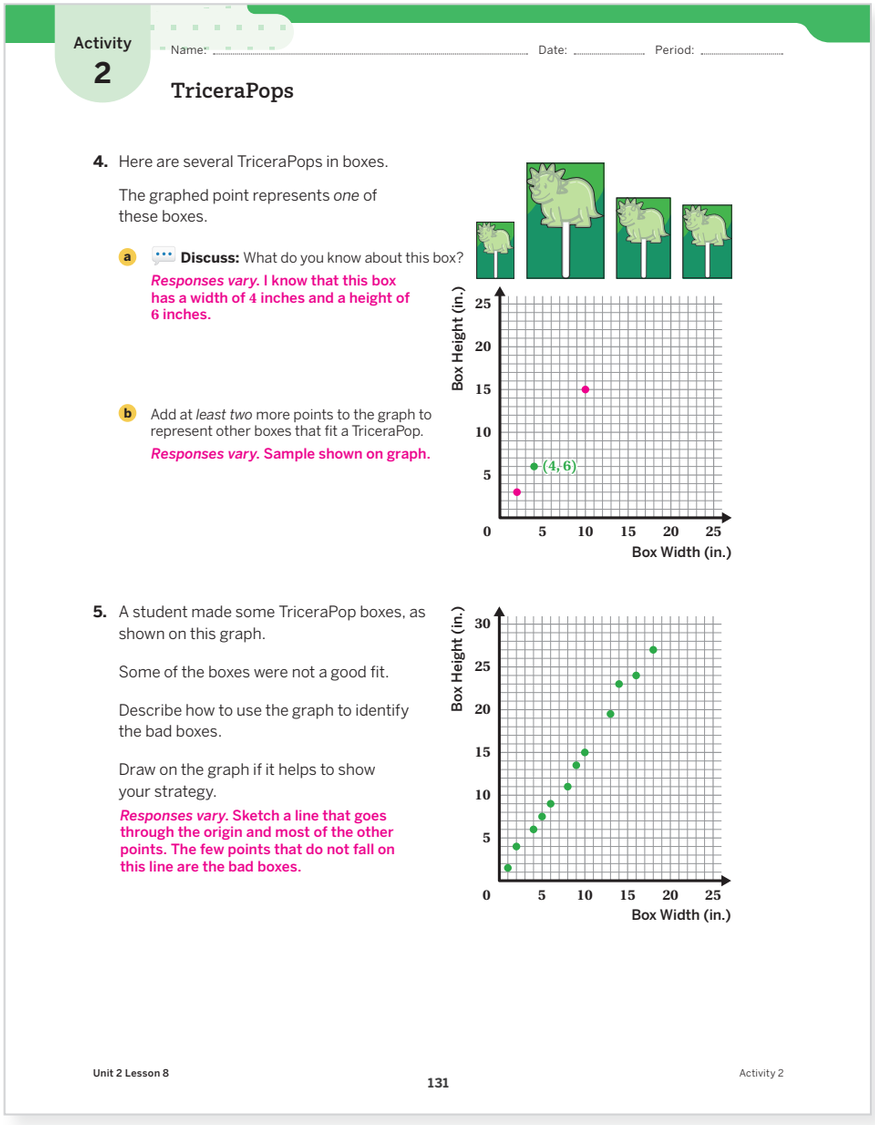
2 Monitor

Invite students to find a few boxes that fit a TriceraPop, then continue to the next problem.

Listen for explanations of how students determined which boxes were not a perfect fit, including students who drew a line through the origin.

Pause briefly to invite students to share their work. Consider asking, "How can we recognize a proportional relationship on a graph?"

Activity 2 continued >



Activity 2

Name: Date: Period:

TriceraPops

4. Here are several TriceraPops in boxes.

The graphed point represents *one* of these boxes.

a. **Discuss:** What do you know about this box?

Responses vary. I know that this box has a width of 4 inches and a height of 6 inches.

b. Add at *least two* more points to the graph to represent other boxes that fit a TriceraPop.

Responses vary. Sample shown on graph.

5. A student made some TriceraPop boxes, as shown on this graph.

Some of the boxes were not a good fit.

Describe how to use the graph to identify the bad boxes.

Draw on the graph if it helps to show your strategy.

Responses vary. Sketch a line that goes through the origin and most of the other points. The few points that do not fall on this line are the bad boxes.

Unit 2 Lesson 8 131 Activity 2

Pairs | 12 min

Activity 2 TriceraPops (continued)

Purpose: Students continue to graph proportional relationships and discuss the key features that allow them to determine proportionality from a graph.

Short on time? Consider omitting Problem 6 and instead using it as a differentiation support for students needing an access point for Problem 7.

2 Monitor

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Select "I'm not sure."	**Support** Encourage students to create a table of values and check for proportionality using their table.
Think the relationship is proportional because the points fall along a straight line.	**Support** Encourage students to test the width to height ratio for two boxes on the graph. Consider asking, "Do the two boxes form equivalent ratios?"
Think the relationship is proportional because the points are evenly spaced.	**Support** Invite students to refer to the strategies captured in Activity 1. Consider asking, "What is similar or different about this relationship?"
Determine that the relationship is not proportional because the points lie on a line that doesn't include the origin.	**Strengthen** Consider asking, "How else could you verify your answer?"

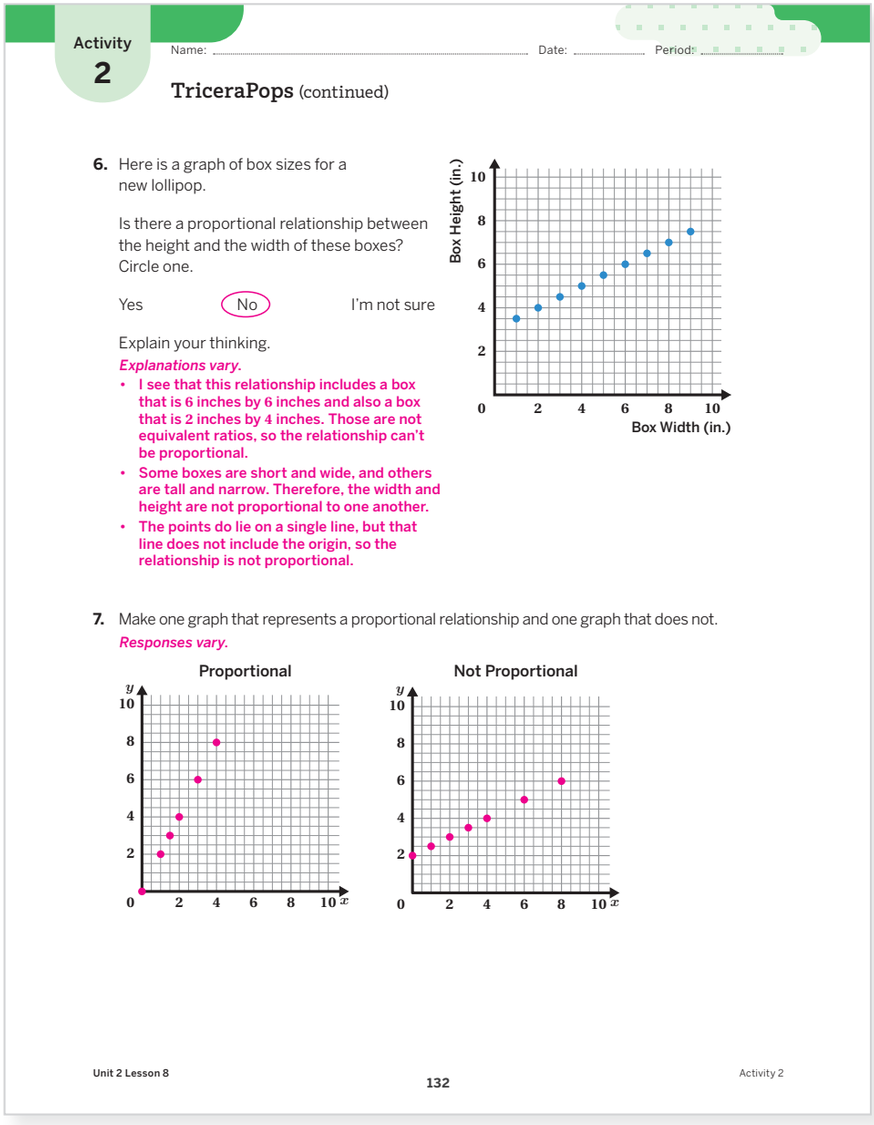
Select and sequence a variety of correct and incorrect responses to share during the Connect, such as:

- Points that lie along a curve.
- Points that are evenly or unevenly spaced.
- Points that lie along a straight line through the origin.
- Points that lie along a straight line not through the origin.

3 Connect

Consider asking, "What features do the proportional graphs have in common? What about the non-proportional graphs?" **(MTR.5.1)**

Create a class anchor chart showing the characteristics of proportional and non-proportional graphs based on the discussion.



Activity 2

Name: Date: Period:

TriceraPops (continued)

6. Here is a graph of box sizes for a new lollipop.

Is there a proportional relationship between the height and the width of these boxes? Circle one.

Yes (No) I'm not sure

Explain your thinking.

Explanations vary.

- I see that this relationship includes a box that is 6 inches by 6 inches and also a box that is 2 inches by 4 inches. Those are not equivalent ratios, so the relationship can't be proportional.
- Some boxes are short and wide, and others are tall and narrow. Therefore, the width and height are not proportional to one another.
- The points do lie on a single line, but that line does not include the origin, so the relationship is not proportional.

7. Make one graph that represents a proportional relationship and one graph that does not.

Responses vary.

Proportional

Not Proportional

Unit 2 Lesson 8 132 Activity 2

Pairs | 10 min

Activity 3 Graphs

Purpose: Students interpret graphs and discuss whether or not they represent a proportional relationship.

1 Launch

Invite students to make sense of the relationships and share their reasoning with a partner to reach an agreement about whether each graph represents a proportional relationship.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: origin, coordinate plane, graph, proportional, etc. **(Speaking)**

2 Monitor

Look for a variety of student thinking.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer to the anchor chart showing the features of proportional graphs.
Associate proportionality with even spacing of points, e.g., think Graph E is proportional and Graph C is not.	**Support** Consider asking, "Do these graphs show every possible pair of values for the relationship? How can we check for proportionality even with limited information?"
Think any graph with points that fall along a straight line, e.g., Graph E, is proportional.	**Support** Encourage students to identify the coordinates of two points and check for equivalent ratios to verify their work.
Use strategies identified earlier in the lesson.	**Strengthen** Celebrate this thinking. Consider inviting these students to share during the Connect.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore graphs of proportional relationships further to complete this optional task. Encourage them to discuss their thinking with a partner.

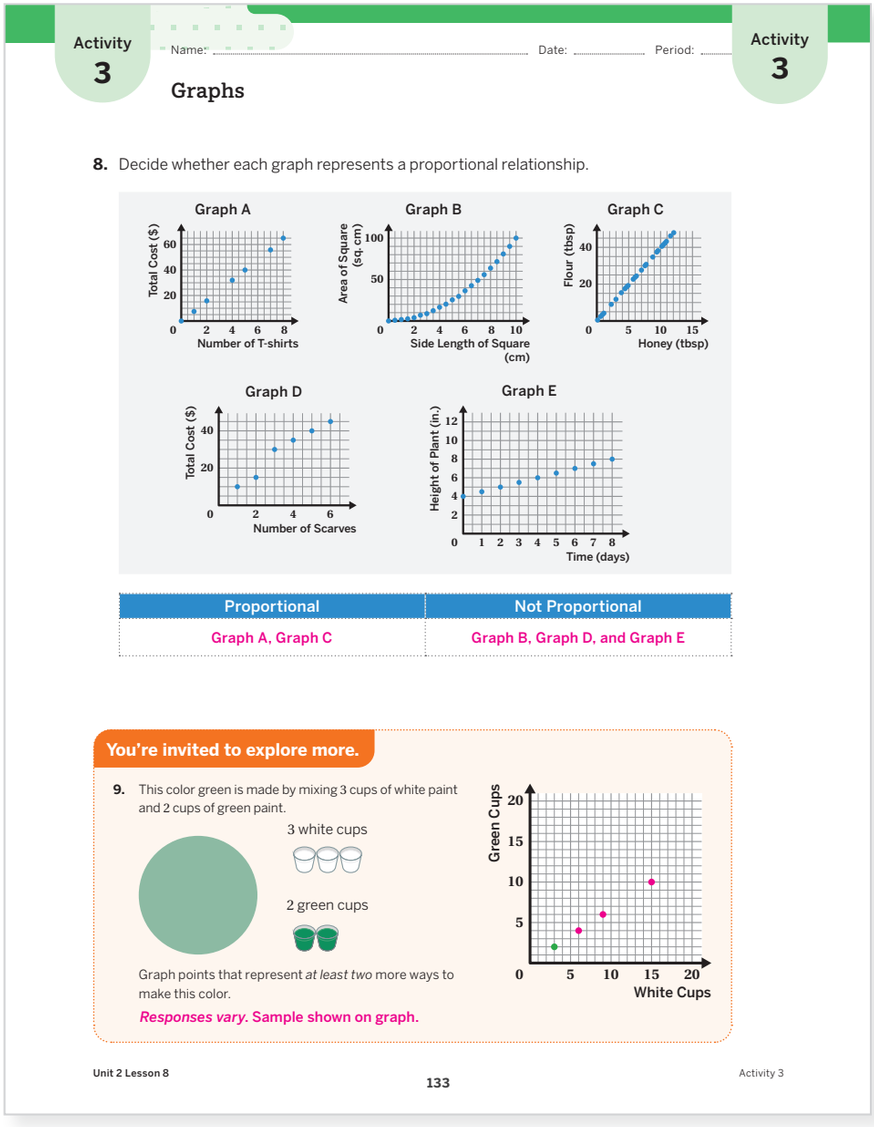
3 Connect

Invite students to share the strategies they used to determine whether each graph represents a proportional relationship.

Consider asking:

- "Which graphs were the most challenging? How did you decide whether the graph represents a proportional relationship?"
- "How would you describe the points in the graphs you sorted as proportional?" **(MTR.2.1)**

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.



Activity 3

Name: Date: Period:

Activity 3

Graphs

8. Decide whether each graph represents a proportional relationship.

Graph A

Graph B

Graph C

Graph D

Graph E

Proportional	Not Proportional
Graph A, Graph C	Graph B, Graph D, and Graph E

You're invited to explore more.

9. This color green is made by mixing 3 cups of white paint and 2 cups of green paint.

3 white cups

2 green cups

Graph points that represent *at least two* more ways to make this color.

Responses vary. Sample shown on graph.

Unit 2 Lesson 8 133 Activity 3

Whole Class | 5 min

Synthesis

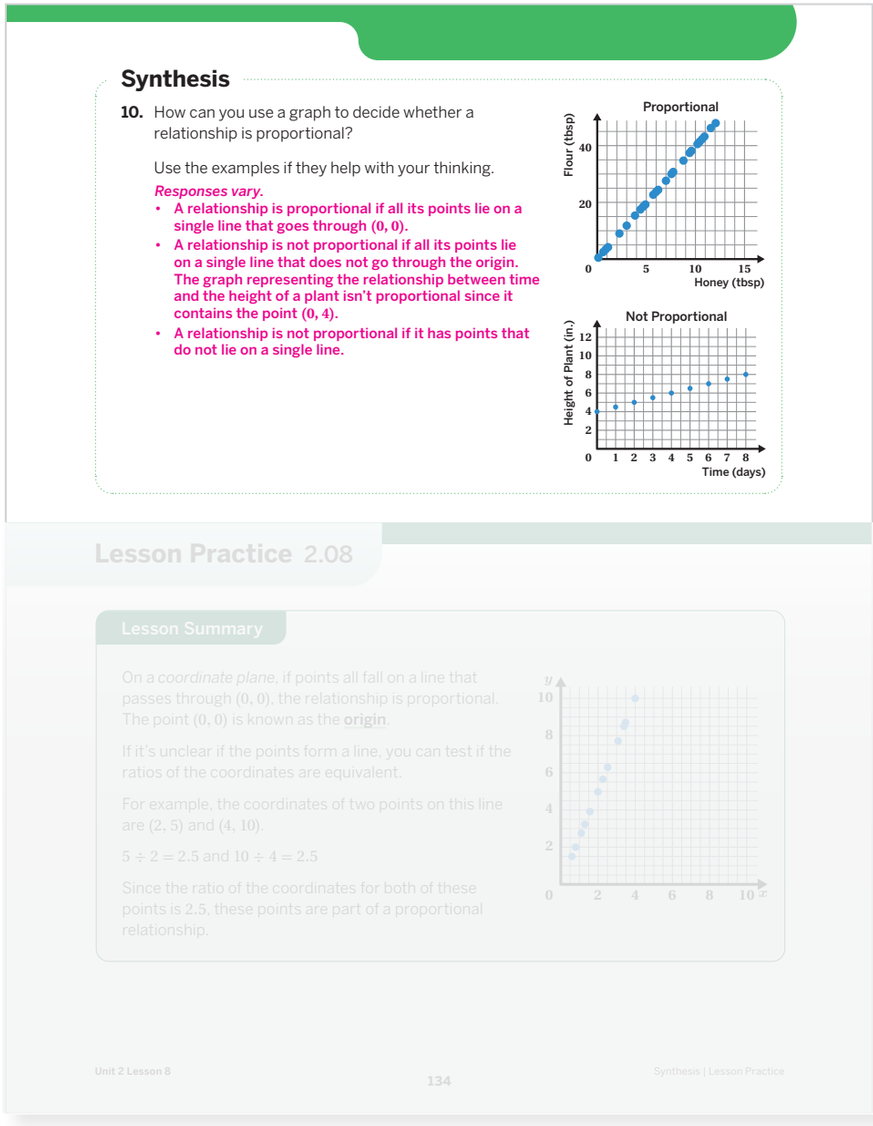
Purpose: Students synthesize their understanding of representing proportional relationships using graphs.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Proportional relationships can be represented as points that lie on the same line.
- If the line passing through the points does not include the origin, the relationship is not proportional.
- The points on a graph of a proportional relationship do not need to be evenly spaced, as long as they lie on the same line.
- In a proportional relationship, the coordinates of points on the graph form equivalent ratios.

Note: If time allows, make additions to the anchor chart for the class and invite students to revise their previous thinking based on the discussion.



Synthesis

10. How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary.

- A relationship is proportional if all its points lie on a single line that goes through (0, 0).
- A relationship is not proportional if all its points lie on a single line that does not go through the origin. The graph representing the relationship between time and the height of a plant isn't proportional since it contains the point (0, 4).
- A relationship is not proportional if it has points that do not lie on a single line.

Lesson Practice 2.08

Lesson Summary

On a *coordinate plane*, if points all fall on a line that passes through (0, 0), the relationship is proportional. The point (0, 0) is known as the **origin**.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are (2, 5) and (4, 10).

$5 \div 2 = 2.5$ and $10 \div 4 = 2.5$

Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.

Unit 2 Lesson 8 134 Synthesis | Lesson Practice

Lesson Takeaway: On a coordinate plane, if points all fall on a line that passes through the **origin**, the relationship is proportional.

Independent | 5 min

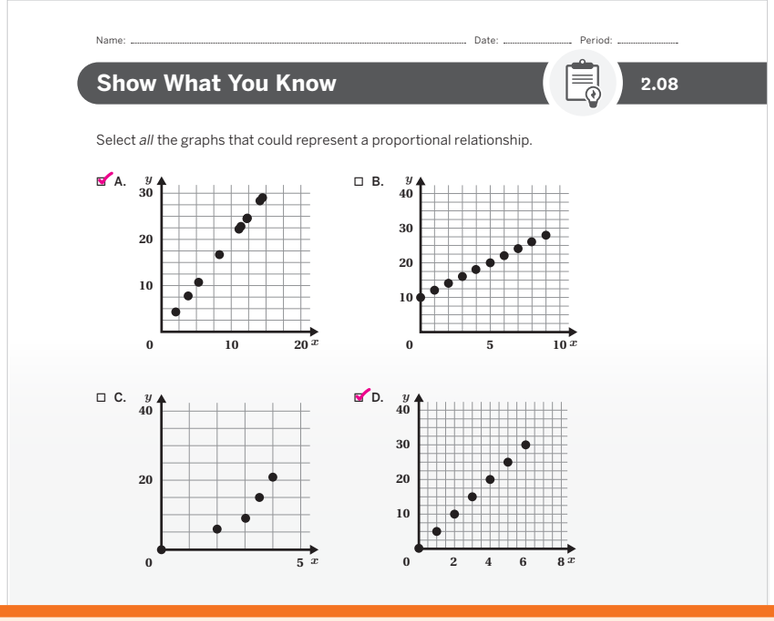
Show What You Know

Purpose: Students demonstrate their understanding by identifying graphs of proportional relationships.

Today's Goals

1. **Goal:** Use a graph that represents a proportional relationship to predict other points in that relationship.
2. **Goal:** Understand that a proportional relationship can be represented by points on a line that passes through the *origin*.
3. **Language Goal:** Justify whether a graph represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.08

Select *all* the graphs that could represent a proportional relationship.

☑ A.

☐ B.

☐ C.

☑ D.

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine proportionality from a graph:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of this concept in Lessons 9–10. You may choose to not intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on the strategies students used to sort the graphs in Activity 3. How did students use proportional relationships to justify their reasoning? How can you connect these strategies to other representations of proportional relationships?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

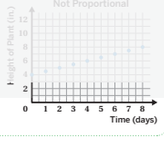
Synthesis

10. How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary.

- A relationship is proportional if all its points lie on a single line that goes through (0, 0).
- A relationship is not proportional if all its points lie on a single line that does not go through the origin. The graph representing the relationship between time and the height of a plant isn't proportional since it contains the point (0, 4).
- A relationship is not proportional if it has points that do not lie on a single line.



Lesson Practice 2.08

Lesson Summary

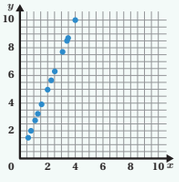
On a *coordinate plane*, if points all fall on a line that passes through (0, 0), the relationship is proportional. The point (0, 0) is known as the **origin**.

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For example, the coordinates of two points on this line are (2, 5) and (4, 10).

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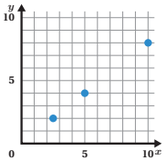
Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.



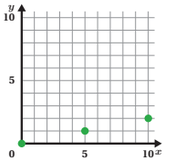
Lesson Practice 2.08

Name: _____ Date: _____ Period: _____

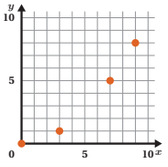
Problems 1–4: Determine if each graph represents a proportional relationship. Explain your thinking.

1. 

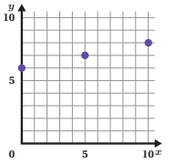
Proportional. *Explanations vary.* The line going through the points passes through the origin.

2. 

Proportional. *Explanations vary.* The line going through the points passes through the origin.

3. 

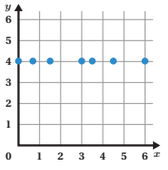
Not proportional. *Explanations vary.* The points lie along a curve.

4. 

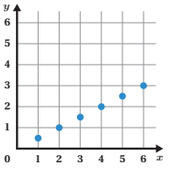
Not proportional. *Explanations vary.* The line passing through the points does not include the origin.

5. Select *all* the representations that show a proportional relationship.

☐ A.



☑ B.



☐ C.

x	0	2	4	6
y	0	3	9	27

☑ D.

x	0	2	4	6
y	0	12	24	36

Lesson Practice 2.08

Name: _____ Date: _____ Period: _____

6. Arturo earns $33.00 for babysitting for 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your thinking.

$57.75. *Explanations vary.* Arturo's rate is $8.25 per hour. 7 hours at that rate is $57.75.

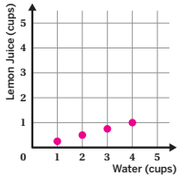
FAST Practice

7. A lemonade recipe calls for a $\frac{1}{4}$ cup of lemon juice for every cup of water. The table shows different amounts of water and lemon juice you can use to make this recipe. Graph the ordered pairs to determine whether the relationship between water and lemon juice is proportional.

Water (cups)	Lemon Juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Select **ONE** correct answer in each box to complete the sentences.

The relationship (**A.** is) **B.** is not proportional. The points (**A.** fall) **B.** do not fall on a line that passes through the origin.



Spiral Review

8. A turtle is walking away from a rock. x represents the time in minutes that the turtle is walking. y represents the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* the true statements.

☑ **A.** The equation $y = 3x$ could represent the distance that the turtle walks.

☐ **B.** The turtle walks for a bit and then stops for a minute before walking again.

☑ **C.** The turtle walks away from the rock at a constant rate.

☐ **D.** The equation $y = x + 3$ could represent the distance that the turtle walks.

☐ **E.** After 6 minutes, the turtle walks 18 meters, and after 10 minutes, the turtle walks 20 meters.

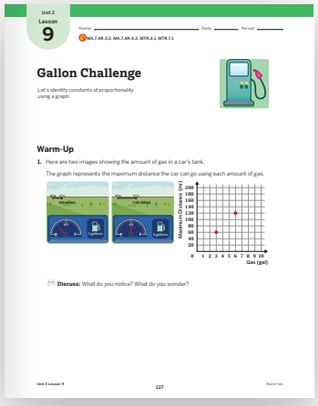
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.AR.4.1
	5	2	MA.7.AR.4.1
	6	2	MA.7.AR.3.2
FAST Practice	7	2	MA.7.AR.4.1
Spiral Review			
	8	2	MA.7.AR.4.4 MA.7.AR.4.5

Unit 2

Lesson

9


Unit 2
Lesson
9
Name: Date: Period:
MA.7.AR.3.2, MA.7.AR.4.2, MTR.3.1, MTR.7.1
Gallon Challenge
Let's identify constants of proportionality using a graph.
Warm-Up
1. Here are two images showing the amount of gas in a car's tank.
The graph represents the maximum distance the car can go using each amount of gas.
Maximum Distance (mi)
Gas (gal)
Discuss: What do you notice? What do you wonder?
137

Gallon Challenge

Interpreting Graphs of Proportional Relationships

Let's identify constants of proportionality using a graph.

Learning Outcomes

Today's Goals

1. **Goal:** Identify the constant of proportionality from the graph of a proportional relationship.
2. **Language Goal:** Interpret points on the graph of a proportional relationship in context, including $(1, k)$ where k is the constant of proportionality. **(Reading and Writing)**

Students continue to work with graphs of proportional relationships and use them to identify constants of proportionality. They analyze graphs of different vehicles' gas mileage and reason about how the concept of gas mileage is connected to the constant of proportionality.

Coherence

Prior Learning

In Lesson 8, students explored what a proportional relationship looks like graphically.

Future Learning

In Lesson 10, students will make comparisons of proportional relationships using graphs and equations. Students will also connect different representations of proportional relationships of the same situation.

Connections

- Students analyze graphs of different vehicles' gas mileage and relate gas mileage to the constant of proportionality. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.7.AR.4.3
MA.7.AR.4.1	MA.8.AR.3.1
	MA.8.AR.3.2

Lesson at a Glance

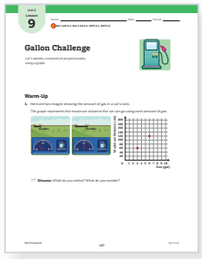
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2

Warm-Up

Pairs | 5 min

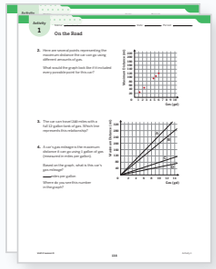
Students explore how values in a proportional relationship translate to points on a graph.
Routine: Notice and Wonder



Activity 1

Pairs | 15 min

Students develop strategies for determining the gas mileage of a vehicle using its graph.
Routines: Think-Pair-Share, MLR7: Compare and Connect



Activity 2

Pairs | 15 min

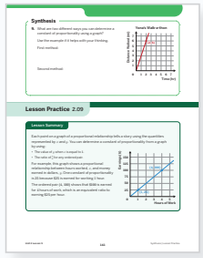
Students generalize strategies for calculating the constant of proportionality from a graph by finding the gas mileage of different vehicles.
MLR2: Collect and Display **(MTR.3.1)**



Synthesis

Whole Class | 5 min

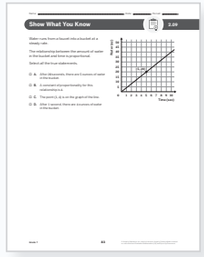
Students synthesize their understanding of determining constants of proportionality using graphs of proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting points on a graph of a proportional relationship, including the point $(1, k)$.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students explore how values in a proportional relationship translate to points on a graph.

1 Launch

To surface connections, consider asking students what they know about gas in cars.

Invite students to analyze the images and the related graph.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between gallons of gas and maximum distance.

2 Connect

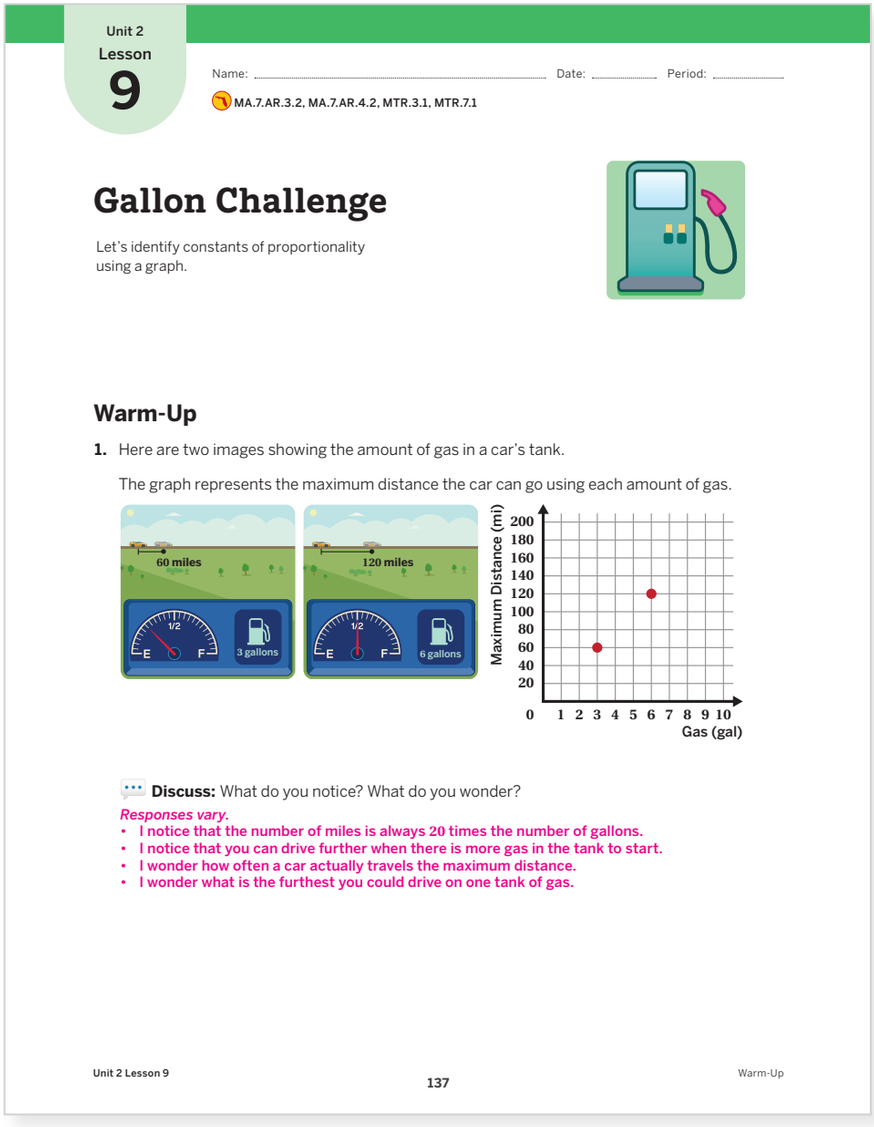
Invite students to share what they noticed and wondered.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., "I notice . . ." or "I wonder . . ."). **(Writing and Speaking)**

Note: If it doesn't come up naturally, consider asking, "What do you think the phrase *maximum distance* means?" Maximum distance is how far the car can travel before it runs out of gas, if it is running in the most efficient way possible.

To surface the Key Takeaway, consider asking:

- "How might we use the graph to predict how far the car can travel with more than 6 gallons of gas?"
- "What might be another point on the graph that fits the pattern?"



Unit 2
Lesson
9

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.2, MTR.3.1, MTR.7.1

Gallon Challenge

Let's identify constants of proportionality using a graph.

Warm-Up

1. Here are two images showing the amount of gas in a car's tank.

The graph represents the maximum distance the car can go using each amount of gas.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the number of miles is always 20 times the number of gallons.
- I notice that you can drive further when there is more gas in the tank to start.
- I wonder how often a car actually travels the maximum distance.
- I wonder what is the furthest you could drive on one tank of gas.

Unit 2 Lesson 9 137 Warm-Up

Key Takeaway: A graph is a way to collect all the possible points that make a relationship true.

Pairs | 15 min

Activity 1 On the Road

Purpose: Students develop strategies for determining the gas mileage of a vehicle using its graph.

Short on time? Consider omitting the Think-Pair-Share routine and inviting students to explore Problems 2–4 in partners.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about representing relationships with graphs.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

2 Monitor

Listen for students who mention the importance of the point $(12, 240)$.

Select and sequence explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where on the graph might you find information about 1 gallon of gas?"
Look for the maximum distance when the number of gallons is 1.	Select and sequence these to share during the Connect.
Divide the maximum distance by the number of gallons, e.g., $\frac{240}{12}$.	**Strengthen** Consider asking, "What other points on the graph could you use to find the gas mileage?"

3 Connect

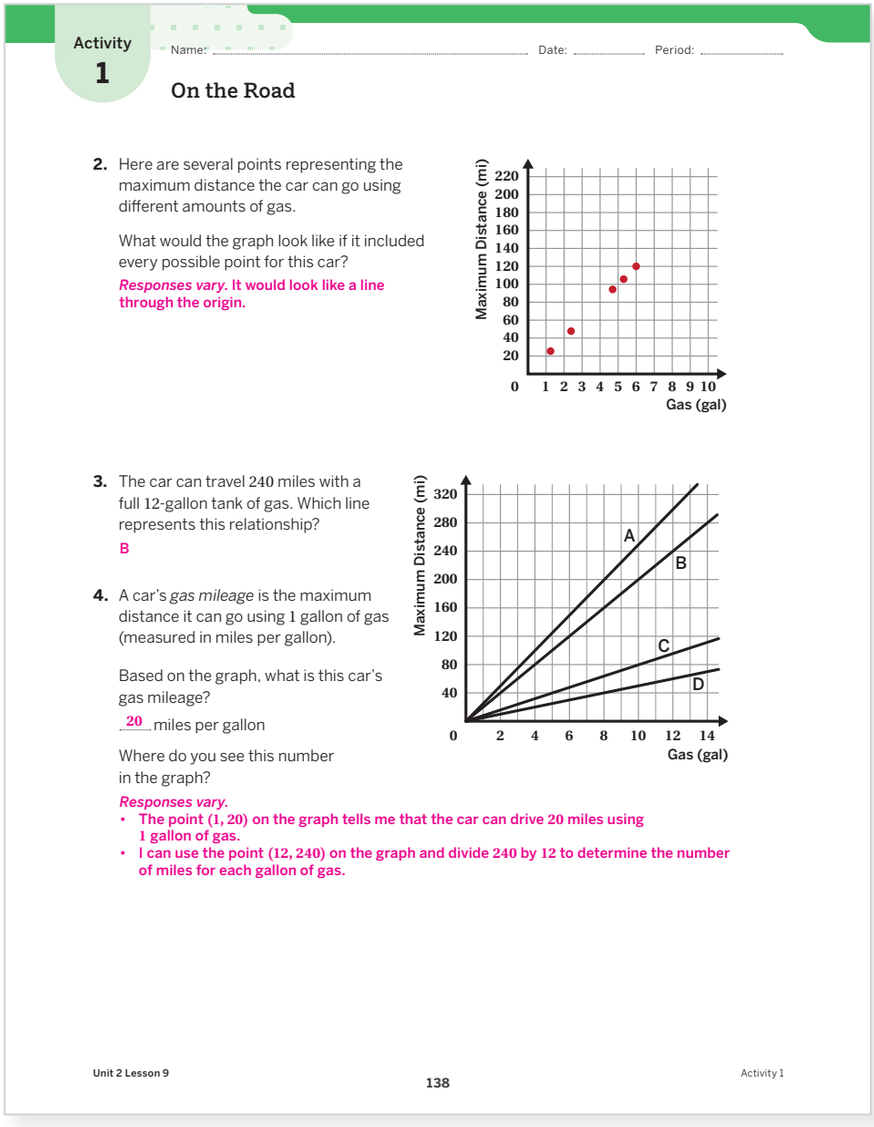
Invite students to share their strategies, ensuring at least two different strategies are showcased.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy.

ELL **English Language Learners** To support students in sharing their thinking, consider displaying sentence frames such as, "First, they _____. Next, they _____. Finally, _____." or "I see ____ in both strategies." **(Writing and Speaking)**

Consider asking:

- "How are these strategies alike? How are they different?"
- "Why did the two strategies lead to the same outcome?"



Activity 1

Name: Date: Period:

On the Road

2. Here are several points representing the maximum distance the car can go using different amounts of gas.

What would the graph look like if it included every possible point for this car?

Responses vary. It would look like a line through the origin.

3. The car can travel 240 miles with a full 12-gallon tank of gas. Which line represents this relationship?

B

4. A car's *gas mileage* is the maximum distance it can go using 1 gallon of gas (measured in miles per gallon).

Based on the graph, what is this car's gas mileage?

20 miles per gallon

Where do you see this number in the graph?

Responses vary.

- The point (1, 20) on the graph tells me that the car can drive 20 miles using 1 gallon of gas.
- I can use the point (12, 240) on the graph and divide 240 by 12 to determine the number of miles for each gallon of gas.

Unit 2 Lesson 9 138 Activity 1

Key Takeaway: You can use different strategies to find the constant of proportionality from a graph. You can look for the quantity of y when x is 1, or choose a point and divide the y-coordinate by the x-coordinate.

Pairs | 15 min

Activity 2 Gas Mileage

Purpose: Students generalize strategies for calculating the constant of proportionality from a graph by finding the gas mileage of different vehicles. **(MTR.3.1)**

1 Launch

To surface connections, share that in this activity, students will compare the gas mileage of different vehicles.

Invite students to think back to the previous activity and restate the meaning of gas mileage in their own words.

Consider asking:

- "What does it mean for one vehicle to have better gas mileage than another?"
- "What kind of vehicles would you expect to have the best gas mileage? What about the worst?"

2 Monitor

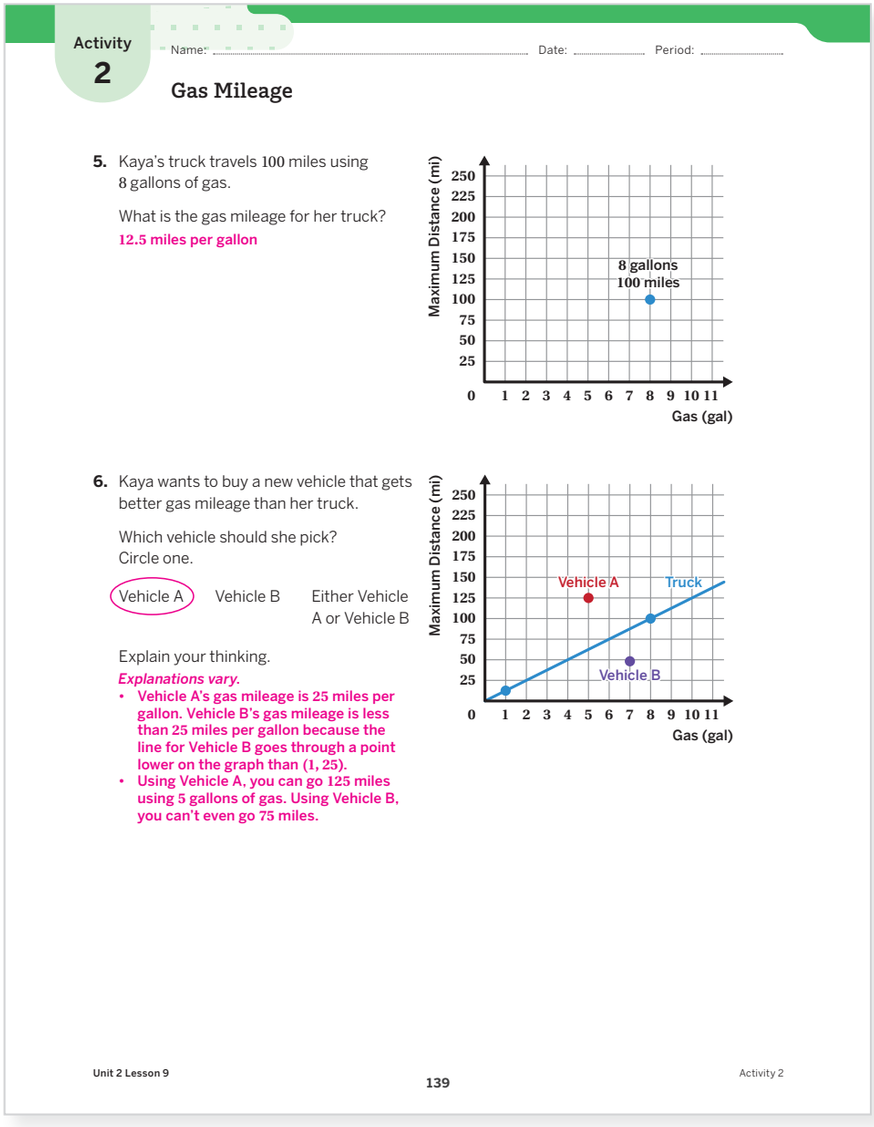
Encourage students to use a strategy from Activity 1 to calculate the gas mileage of Kaya's truck. **(MTR.7.1)**

Look for a variety of student strategies.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are the coordinates of the point for Vehicle A? What can that tell you about the gas mileage?"
Think that either vehicle will be better than the truck because the points appear to the left of (8, 100).	**Support** Celebrate the structure these students noticed in the x-coordinates of the points. Encourage them to consider how the y-coordinate of each point affects gas mileage.
Think that Vehicle A has better gas mileage because the point is above the line for the truck.	**Strengthen** Celebrate the structure these students noticed. Consider asking, "How do you know that a point higher than the line represents better gas mileage?"
Draw a line to represent each vehicle and compare the maximum distances for the same number of gallons.	**Stretch** Celebrate this strategy. Consider asking, "How might you calculate the exact gas mileage for each vehicle?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Gas Mileage

5. Kaya's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?

12.5 miles per gallon

6. Kaya wants to buy a new vehicle that gets better gas mileage than her truck.

Which vehicle should she pick? Circle one.

(Vehicle A) Vehicle B Either Vehicle A or Vehicle B

Explain your thinking.

Explanations vary.

- Vehicle A's gas mileage is 25 miles per gallon. Vehicle B's gas mileage is less than 25 miles per gallon because the line for Vehicle B goes through a point lower on the graph than (1, 25).
- Using Vehicle A, you can go 125 miles using 5 gallons of gas. Using Vehicle B, you can't even go 75 miles.

Unit 2 Lesson 9 139 Activity 2

Pairs | 15 min

Activity 2 Gas Mileage (continued)

Purpose: Students generalize strategies for calculating the constant of proportionality from a graph by finding the gas mileage of different vehicles. **(MTR.3.1)**

2 Monitor

Encourage students to try different strategies until they find one that is consistently successful, and to verify their work by finding the constant of proportionality in multiple ways. **(MTR.3.1)**

A **Accessibility: Conceptual Processing** Invite students to describe what they have tried to support self-monitoring of understanding.

Look for students who find the constant of proportionality by looking for the maximum distance corresponding to 1 gallon of gas or by dividing the y-value of a point on the line by the x-value of the same point. Consider inviting these students to share during the Connect.

D **Differentiation: (Stretch)** Invite students who would like to explore comparing proportional relationships further to compare the fuel efficiencies of the three vehicles to a car that gets 20 miles per gallon and carries 1–4 people.

3 Connect

Invite students to share their strategies for finding constants of proportionality given a graph.

ML/EL **MLR2: Collect and Display** Use this routine to collect students' ideas, language, and strategies as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Consider asking:

- "How is the constant of proportionality like the gas mileage in this situation?"
- "Who used a different strategy and arrived at the same constant of proportionality?"
- "Which strategy was easier to use for each vehicle? Why?"

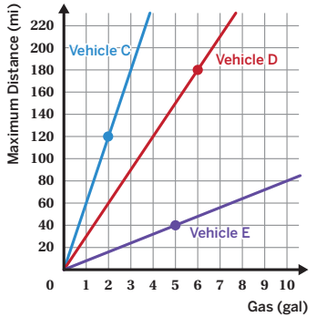
Activity 2

Name: Date: Period:

Gas Mileage (continued)

7. In the relationship between amount of gas and maximum distance, the vehicle's gas mileage is a constant of proportionality.

What is that constant of proportionality for each vehicle?



Vehicle	Constant of Proportionality (miles per gallon)
C	60
D	30
E	8

8. Here is the maximum distance traveled by three vehicles using a certain amount of gas.

Plane	Cruise Ship	Train
		
Note: The average plane has 100 passengers.	Note: The average cruise ship has 3000 passenger.	Note: The average train has 300 passengers.

Compare the fuel efficiency for each vehicle.

Responses vary. The plane gets 0.5 miles per gallon and carries 100 passengers. The cruise ship gets 0.02 miles per gallon and carries 3000 passengers. The train gets 0.2 miles per gallon and carries 300 passengers. That means a full cruise ship and train can carry passengers the same distance per gallon of gas, while the plane is less fuel efficient.

Unit 2 Lesson 9 140 Activity 2

Whole Class | 5 min

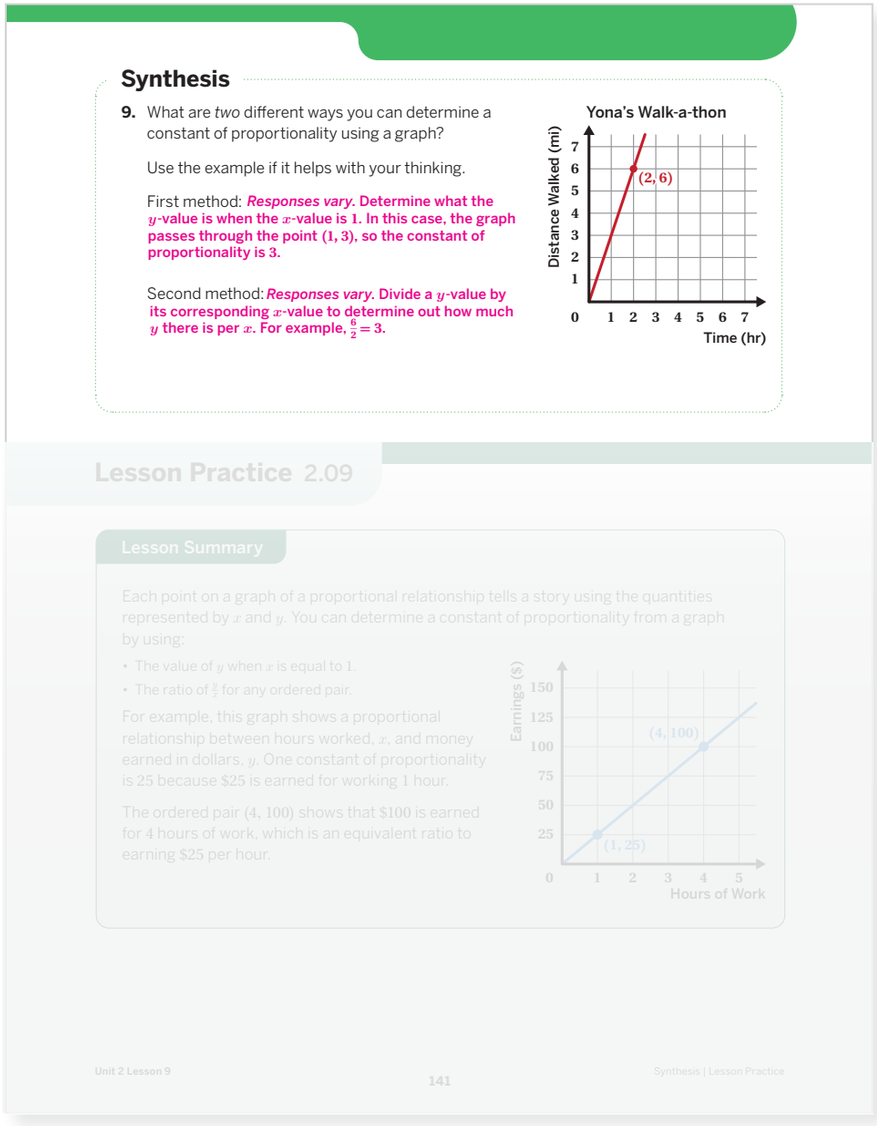
Synthesis

Purpose: Students synthesize their understanding of determining constants of proportionality using graphs of proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Looking for the y-value when the x-value is 1.
- Choosing a point and dividing the y-value by the x-value.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. What are *two* different ways you can determine a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method: ***Responses vary.*** **Determine what the y-value is when the x-value is 1. In this case, the graph passes through the point (1, 3), so the constant of proportionality is 3.**

Second method: ***Responses vary.*** **Divide a y-value by its corresponding x-value to determine out how much y there is per x. For example, $\frac{6}{2} = 3$.**

Lesson Practice 2.09

Lesson Summary

Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y. You can determine a constant of proportionality from a graph by using:

- The value of y when x is equal to 1.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, x, and money earned in dollars, y. One constant of proportionality is 25 because \$25 is earned for working 1 hour.

The ordered pair (4, 100) shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.

Unit 2 Lesson 9 141 Synthesis | Lesson Practice

Lesson Takeaway: A graph can be used to determine the constant of proportionality and make comparisons among quantities of proportional relationships.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting points on a graph of a proportional relationship, including the point $(1, k)$.

Today's Goals

1. **Goal:** Identify the constant of proportionality from the graph of a proportional relationship.
2. **Language Goal:** Interpret points on the graph of a proportional relationship in context, including $(1, k)$ where k is the constant of proportionality. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

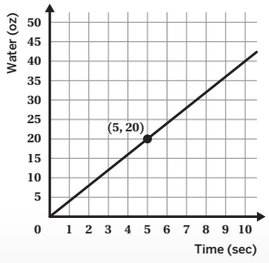
Show What You Know 2.09

Water runs from a faucet into a bucket at a steady rate.

The relationship between the amount of water in the bucket and time is proportional.

Select *all* the true statements.

- ☐ A. After 20 seconds, there are 5 ounces of water in the bucket.
- ☑ B. A constant of proportionality for this relationship is 4.
- ☑ C. The point $(1, 4)$ is on the graph of the line.
- ☑ D. After 1 second, there are 4 ounces of water in the bucket.



Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine constants of proportionality from graphs:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- If students would like to explore the context further, Invite them to research real cars and consider whether the relationship between gas and distance traveled is always proportional.

Professional Learning

In this lesson, students are introduced to graphs of proportional relationships as lines rather than collections of points. Why was the context of gas mileage particularly useful in building students' understanding of graphs? How did it support students deepening their understanding of constants of proportionality?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What are *two* different ways you can determine a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method: *Responses vary.* Determine what the y-value is when the x-value is 1. In this case, the graph passes through the point $(1, 3)$, so the constant of proportionality is 3.

Second method: *Responses vary.* Divide a y-value by its corresponding x-value to determine out how much y there is per x. For example, $\frac{3}{1} = 3$.

Lesson Practice 2.09

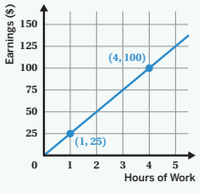
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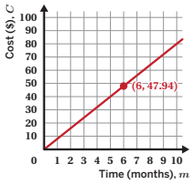
The ordered pair $(4, 100)$ shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.



Lesson Practice 2.09

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a graph that shows a proportional relationship between the number of months Tiara had a streaming service subscription and the total amount of money she paid for the subscription.



After 6 months, Tiara paid \$47.94.

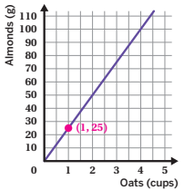
1. What is a constant of proportionality in this relationship?

7.99 (or equivalent) or $\frac{1}{7.99}$ (or equivalent)

2. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

$C = 7.99m$ or $m = \frac{1}{7.99}C$ (or equivalent)

Problems 3–4: A recipe for granola calls for a mix of almonds and oats. The graph shows the amount of almonds, in grams, for different amounts of oats, in cups.



3. Determine a constant of proportionality for this relationship, then explain its meaning.

25 or $\frac{25}{1}$. *Explanations vary.* The graph contains the point (2, 50), which gives the ratio $\frac{50}{2} = 25$. This means 25 grams of almonds are needed for every 1 cup of oats.

4. Label one place you see that constant of proportionality on the graph.

5. The graph shows the cost for two different varieties of apples. Which variety of apples has a higher cost per pound?

Gala



Explain your thinking.

Explanations vary. A constant of proportionality for Gala apples is 3 and a constant of proportionality for Red Delicious apples is 1. That means Gala apples have a higher unit price (\$3 per pound) than Red Delicious apples (\$1 per pound).

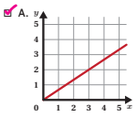
Lesson Practice 2.09

Name: _____ Date: _____ Period: _____

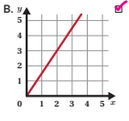
FAST Practice

6. Which of these relationships has the same constant of proportionality as the equation $y = \frac{2}{3}x$? Select *all* that apply.

☑ A.



☐ B.



☑ C.

x	y
0	0
3	2
6	4
9	6

☐ D.

x	y
0	0
2	3
4	6
6	9

Spiral Review

7. Write an expression that is equivalent to $8(n + 6)$.

Responses vary. $8n + 48$

Problems 8–10: Haru and Irene were running laps around the track. Their coach recorded their times at the end of laps 2, 4, and 6.

Haru's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	4	2
4	9	2.25
6	15	2.5

Irene's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	5	2.5
4	10	2.5
6	15	2.5

8. Complete the tables with the minutes per lap for each run.

9. Based on the table, is Haru running at a constant speed? Explain your thinking.

No. *Explanations vary.* Haru is not running at a constant speed because the number of minutes per lap is not the same for each row in his table.

10. Based on the table, is Irene running at a constant speed? Explain your thinking.

Yes. *Explanations vary.* Irene may be running at a constant speed because the number of minutes per lap is the same value for each row of her table. To be more certain, I'd also want to know her times for laps 1, 3, and 5.

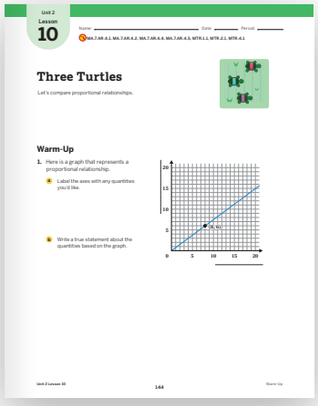
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.3.2, MA.7.AR.4.2
FAST Practice	6	2	MA.7.AR.4.2
Spiral Review			
	7	1	MA.6.AR.1.4
	8	1	MA.6.AR.3.2
	9–10	2	MA.7.AR.4.2

Unit 2

Lesson

10



Three Turtles

Proportional Relationships and Graphs

Let's compare proportional relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Write an equation of a proportional relationship given a point on a graph.
2. **Language Goal:** Compare proportional relationships using their graphs and equations. **(Reading and Writing)**

Students use graphs and equations to compare proportional relationships. They make connections between graphs, equations, and the context of turtles in a race. They reason abstractly as they use graphs and equations to make claims about the turtles based on the steepness of a line or a constant of proportionality. **(MTR.2.1)**

Coherence

Prior Learning

In Lessons 8 and 9, students identified graphs of proportional relationships and connected graphs of these relationships to constants of proportionality.

Future Learning

In Lesson 11, students will create their own representations of proportional relationships (equations, tables, graphs, and descriptions).

Connections

- Students compare and contrast different representations of proportional relationships in real world contexts such as analyzing the results of distance/time relationships of different runners. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.4

Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

MA.7.AR.4.5

Solve real-world problems involving proportional relationships.

Also Addressing: **MA.7.AR.4.1, MA.7.AR.4.2**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.1	**MA.8.AR.3.1**
MA.6.AR.3.2	**MA.8.AR.3.2**
	MA.8.AR.3.3

Lesson at a Glance

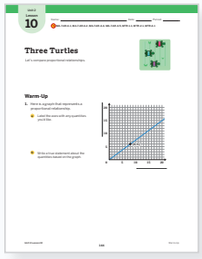
~ 45 min

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Pairs | 5 min

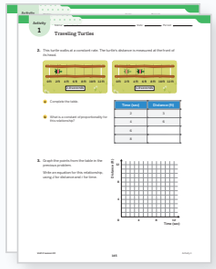
Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.



Activity 1

Pairs | 5 min

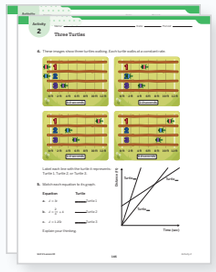
Students connect information in the graph, table, and equation of a single proportional relationship.



Activity 2

Pairs | 10 min

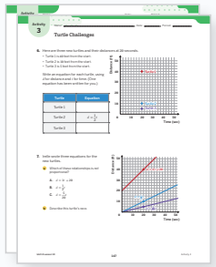
Students use graphs and equations to decontextualize a situation about three turtles in a race. **Routine:** Notice and Wonder **(MTR.2.1)**



Activity 3

Pairs | 15 min

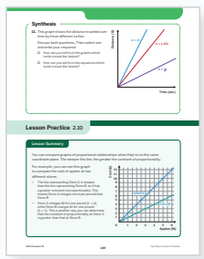
Students create and compare representations of proportional relationships. They also revisit how to differentiate between relationships that are proportional and not proportional.



Synthesis

Whole Class | 5 min

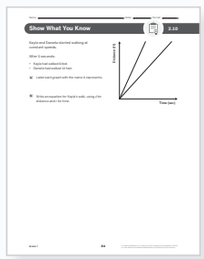
Students synthesize their ideas about using graphs and equations to compare proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by labeling two relationships on a graph and writing an equation.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

Student Edition
Show What You Know

- Activity 3 You're Invited to Explore More Sheet, one per student (as needed)
- straightedges (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.

Short on time? Consider omitting this Warm-Up or doing it as a class.

1 Launch

To surface prior knowledge, consider asking, "Is this a graph of a proportional relationship? How do you know?"

To encourage creativity, invite students to think of axes labels related to something they care about or different from what they think their classmates will choose.

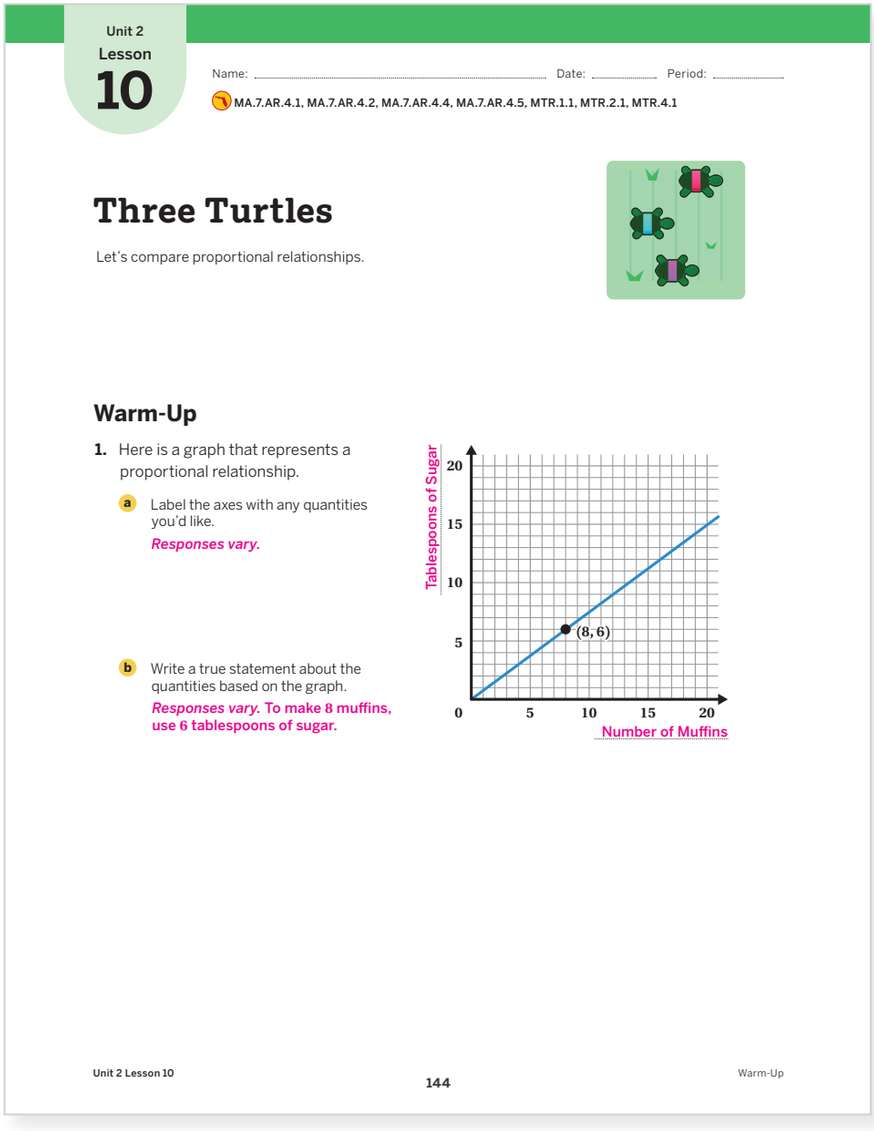
2 Connect

Look for a variety of graphs and statements, including ones that mention a constant of proportionality ($\frac{3}{4}$ or $\frac{4}{3}$).

Invite students to share their statements and to explain their reasoning.

Consider asking:

- "Where do you see the 8 and the 6 in _____'s statement? What do they mean?"
- "How were our statements alike? Different?"



Unit 2
Lesson
10

Name: _____ Date: _____ Period: _____

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.1.1, MTR.2.1, MTR.4.1

Three Turtles

Let's compare proportional relationships.

Warm-Up

1. Here is a graph that represents a proportional relationship.

a. Label the axes with any quantities you'd like.

Responses vary.

b. Write a true statement about the quantities based on the graph.

Responses vary. To make 8 muffins, use 6 tablespoons of sugar.

Unit 2 Lesson 10 144 Warm-Up

Pairs | 5 min

Activity 1 Traveling Turtles

Purpose: Students connect information in the graph, table, and equation of a single proportional relationship.

1 Launch

Direct students' attention to the turtle's position in the image.

Invite students to measure the turtle's distance at 2 and 4 seconds.

2 Monitor

Look for students who write each of the two constants of proportionality ($\frac{3}{2}$ and $\frac{2}{3}$).

Encourage students to look for connections between the table and the graph before writing their equation. **(MTR.2.1)**

D Differentiation

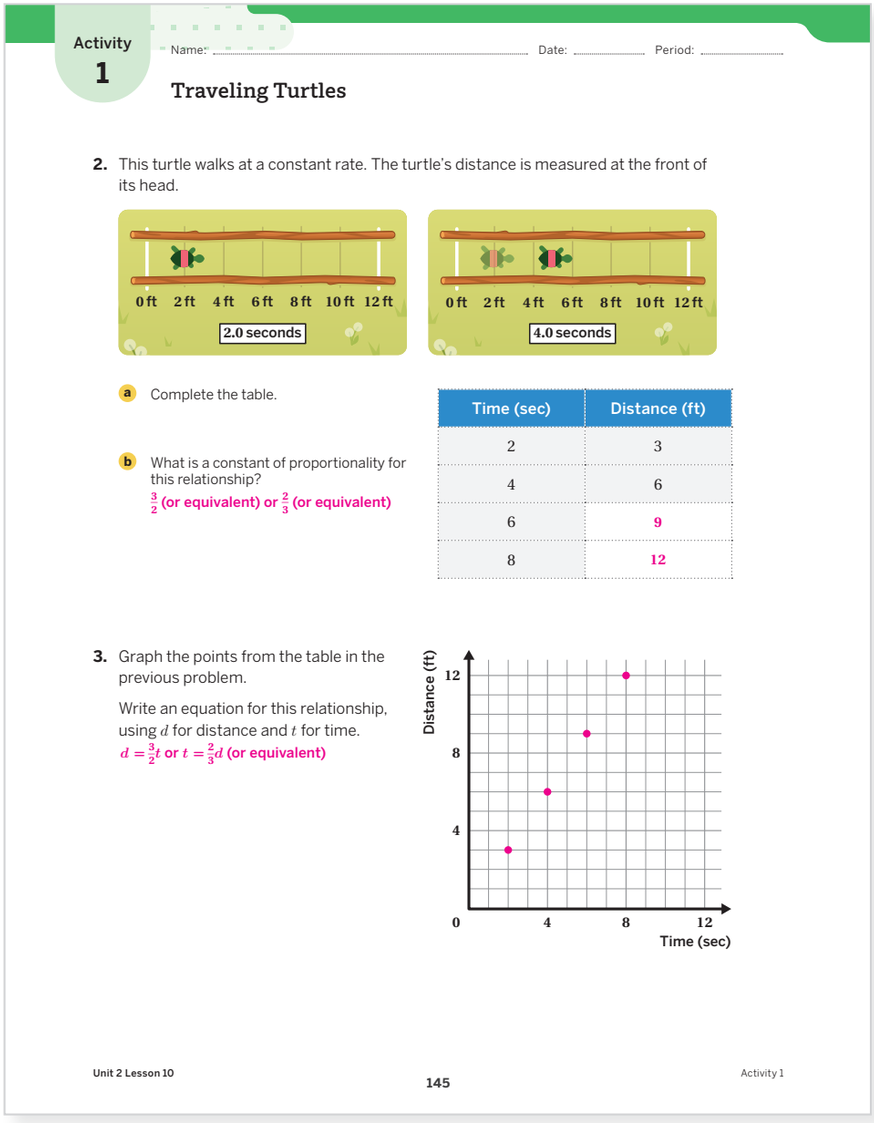
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit their work from earlier lessons. Consider asking, "What do equations of proportional relationships typically look like?"
Write $t = \frac{3}{2}d$ or a different equation with the variables swapped.	**Support** Invite them to test the points from the table or graph in the equation, then to revise their thinking.
Use a variety of strategies, such as: • Noticing that every 2 seconds, the turtle travels 3 feet. • Using a constant of proportionality in their equation. • Testing the equation using the table or graph.	**Strengthen** Invite students to think about how they would share their strategy during the Connect. **Stretch** Invite students to write a *different* equation for this proportional relationship.

3 Connect

Invite students to share connections they see between the representations of the turtle situation.

Consider asking:

- "Where do you see the $\frac{3}{2}$ in the table? In the equation? In the graph?"
- "What might the graph and equation look like for a turtle who is moving faster? What about a turtle who is moving more slowly?"



Activity 1

Name: Date: Period:

Traveling Turtles

2. This turtle walks at a constant rate. The turtle's distance is measured at the front of its head.

a. Complete the table.

b. What is a constant of proportionality for this relationship?
$\frac{3}{2}$ (or equivalent) or $\frac{2}{3}$ (or equivalent)

Time (sec)	Distance (ft)
2	3
4	6
6	9
8	12

3. Graph the points from the table in the previous problem.

Write an equation for this relationship, using d for distance and t for time.
$d = \frac{3}{2}t$ or $t = \frac{2}{3}d$ (or equivalent)

Unit 2 Lesson 10 145 Activity 1

Pairs | 10 min

Activity 2 Three Turtles

Purpose: Students use graphs and equations to decontextualize a situation about three turtles in a race. **(MTR.2.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students notice features of the turtle race that will support them as they connect the movement of the turtles with graphs and equations. **(MTR.1.1)**

Note: This may be the first time students have seen a coordinate plane without a grid.

2 Monitor

Listen for students who make connections between the steepness of the graph and the speed of the turtle or the starting point of the turtle and the value at 0 on the graph.

A **Accessibility: Visual-Spatial Processing** Invite students to trace each graph with a pencil, then discuss how they are alike and different.

Select and sequence graphs and explanations to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to test out different values of t in each equation, like $t = 0$ and $t = 1$.
Swap the two proportional equations and graphs.	**Support** Consider asking, "How could you test to see if this equation makes sense with the graphs and turtles?"
Make connections between the steepness of the graph and the constant of proportionality.	**Strengthen** Consider inviting them to revise their response to convince a friend who made different matches.
Recognize that the turtle with a head start does not represent a proportional relationship.	**Stretch** Consider asking, "What would the turtle's race whose equation is $d = \frac{2}{3}t$ look like?

3 Connect

Share student graphs and explanations. If there is conflict, consider sharing 1–2 responses for each choice, and invite those students to share their reasoning.

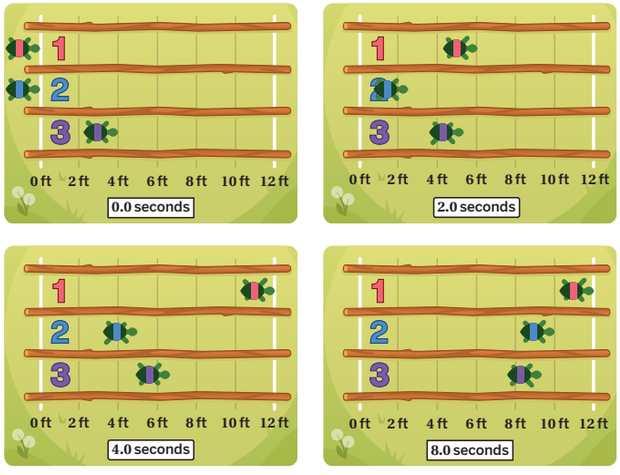
To surface the Key Takeaway, consider asking, "How do you see the information from each equation in its graph?"

Activity 2

Name: Date: Period:

Three Turtles

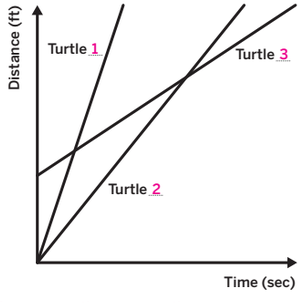
4. These images show three turtles walking. Each turtle walks at a constant rate.



Label each line with the turtle it represents: Turtle 1, Turtle 2, or Turtle 3.

5. Match each equation to its graph.

Equation	Turtle
a. $d = 3t$	a Turtle 1
b. $d = \frac{2}{3}t + 4$	c Turtle 2
c. $d = 1.25t$	b Turtle 3



Explain your thinking.

Explanations vary. The turtle with a head start has an equation representing a relationship that is not proportional. The slower proportional turtle matches the equation that has the smaller constant of proportionality.

Unit 2 Lesson 10 146 Activity 2

Key Takeaway: We can use graphs and equations to represent situations in context. Steeper graphs and equations with larger constants of proportionality represent a faster rate. Graphs that do not start at $(0, 0)$ or have equations that look like $d =$ _____ $t +$ _____ represent a head start.

Pairs | 15 min

Activity 3 Turtle Challenges

Purpose: Students create and compare representations of proportional relationships. They also revisit how to differentiate between relationships that are proportional and not proportional.

Short on time? Consider omitting Problem 7.

1 Launch

Direct students' attention to the instructions and consider clarifying them as a class.

To surface prior knowledge, consider asking, "Where can you see on the graph that Turtle 1 is 40 feet from the start after 20 seconds?" **(MTR.2.1)**

2 Monitor

Listen for a variety of strategies students use to write their equations and respond individually.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to sketch a line they think represents Turtle 1. Consider asking, "Where is the constant of proportionality in this line?"
Write an equation that does not go through the point on the graph.	**Support** Consider asking, "How could you test to see if this equation makes sense with the graphs and turtles?"
Use strategies such as: • Drawing a line and looking for $(1, k)$. • Calculating a constant of proportionality. • Writing the equation of a line not through the origin.	**Strengthen** Consider asking, "Is this the only equation of a line that could represent this turtle?"

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own.

Activity 3 continued >

Activity 3

Name: Date: Period:

Turtle Challenges

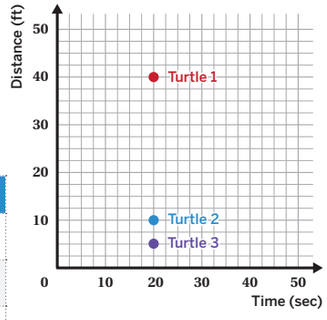
6. Here are three new turtles and their distances at 20 seconds.

- Turtle 1 is 40 feet from the start.
- Turtle 2 is 10 feet from the start.
- Turtle 3 is 5 feet from the start.

Write an equation for each turtle, using d for distance and t for time. (One equation has been written for you.)

Responses vary.

Turtle	Equation
Turtle 1	$d = 2t$ $d = 30 + \frac{t}{2}$
Turtle 2	$d = \frac{1}{2}t$
Turtle 3	$d = \frac{1}{4}t$ $d = 25 - t$



7. Irelle wrote three equations for the new turtles.

a. Which of these relationships is *not* proportional?

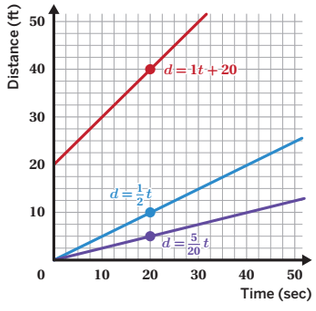
A. $d = 1t + 20$

B. $d = \frac{1}{2}t$

C. $d = \frac{5}{20}t$

b. Describe this turtle's race.

Responses vary. This turtle began the race with a 20-foot head start, and then traveled 1 foot per second for 30 seconds.



Unit 2 Lesson 10 147 Activity 3

Pairs | 15 min

Activity 3 Turtle Challenges (continued)

Purpose: Students create and compare representations of proportional relationships. They also revisit how to differentiate between relationships that are proportional and not proportional.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the representations.

A **Accessibility: Executive Function** Invite students to focus on one representation at a time to support students in carrying out multiple steps.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *table, graph, equation, constant of proportionality*, etc. **(Speaking)**

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation. **(MTR.4.1)**

Encourage students to use straightedges to sketch their own lines to meet the criteria. They can add more than 3 lines or erase and draw new lines if they would like.

To encourage creativity, consider inviting students to imagine a turtle race and then try to represent it on their graphs.

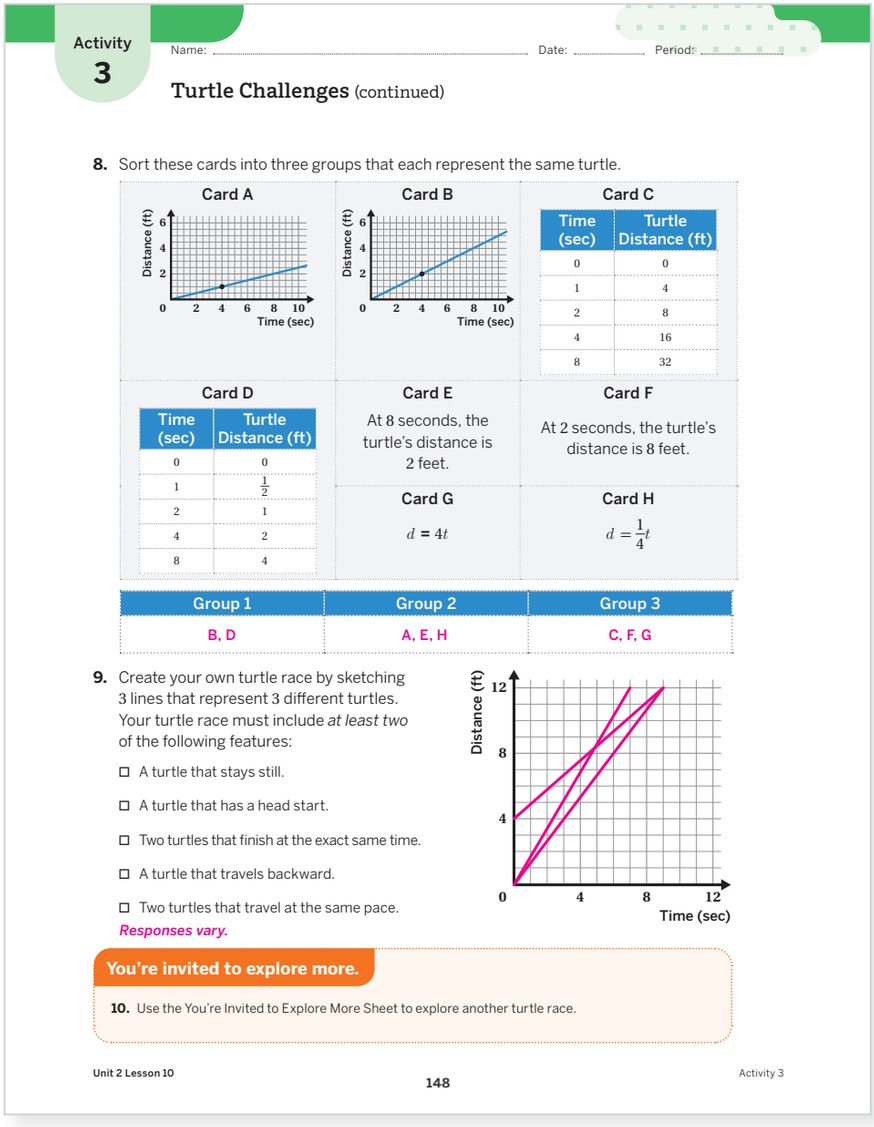
D **Differentiation: You're invited to explore more.** (Stretch) Invite students who would like to explore the relationship between turtle speed and graphs further to complete this optional task. Encourage them to discuss their thinking with a partner. **Note:** Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their groupings and consider discussing each group one at a time.

Consider asking:

- "Do _____ and _____ represent the same relationship? How do you know?"
- "Which was the last card you sorted? How did you decide where it should go?"
- "What strategies were helpful for deciding which representations were in the same group?"



Activity 3

Name: Date: Period:

Turtle Challenges (continued)

8. Sort these cards into three groups that each represent the same turtle.

Card A

Card B

Card C

Time (sec)	Turtle Distance (ft)
0	0
1	4
2	8
4	16
8	32

Card D

Time (sec)	Turtle Distance (ft)
0	0
1	$\frac{1}{2}$
2	1
4	2
8	4

Card E

At 8 seconds, the turtle's distance is 2 feet.

Card F

At 2 seconds, the turtle's distance is 8 feet.

Card G

$d = 4t$

Card H

$d = \frac{1}{4}t$

Group 1	Group 2	Group 3
B, D	A, E, H	C, F, G

9. Create your own turtle race by sketching 3 lines that represent 3 different turtles. Your turtle race must include *at least two* of the following features:

- ☐ A turtle that stays still.
- ☐ A turtle that has a head start.
- ☐ Two turtles that finish at the exact same time.
- ☐ A turtle that travels backward.
- ☐ Two turtles that travel at the same pace.

Responses vary.

You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to explore another turtle race.

Unit 2 Lesson 10 148 Activity 3

Whole Class | 5 min

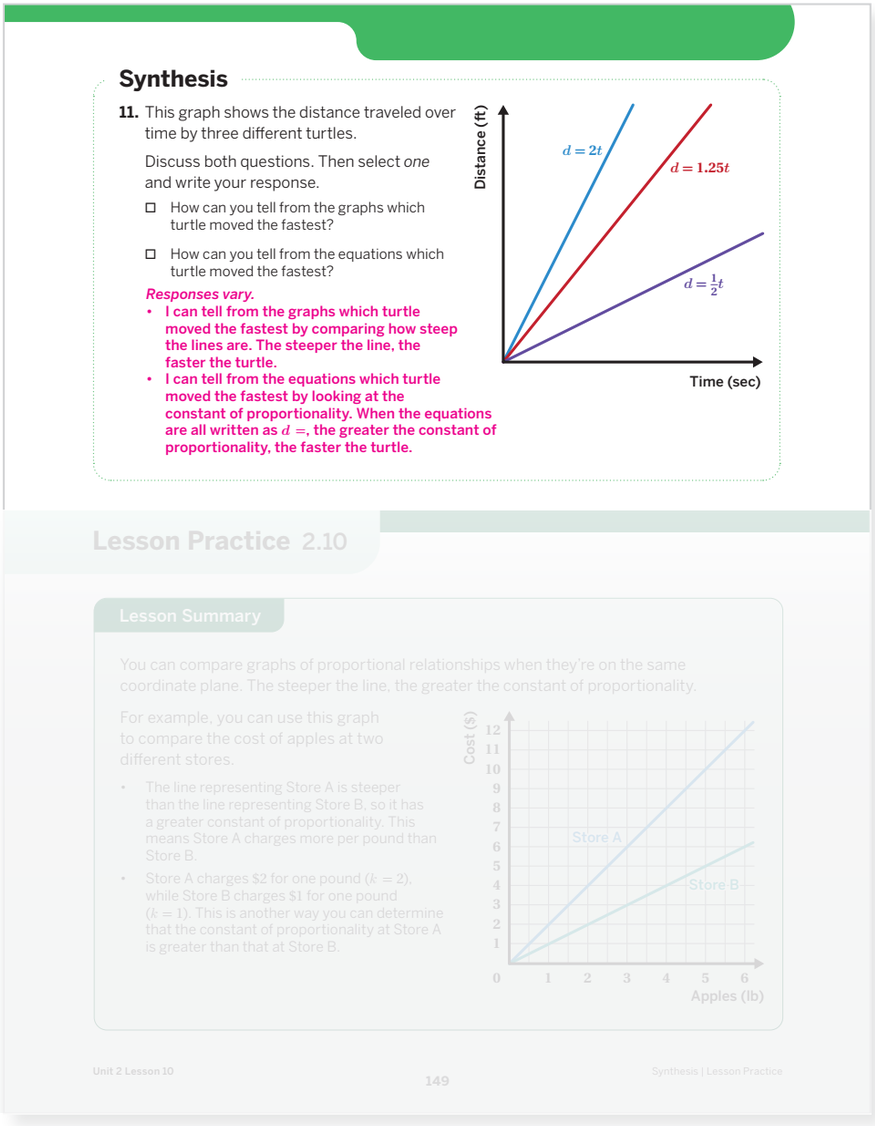
Synthesis

Purpose: Students synthesize their ideas about using graphs and equations to compare proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Steeper lines represent faster turtles.
- The coefficient in the equation is also a constant of proportionality.
- The larger the constant of proportionality, the faster the turtle.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- ☐ How can you tell from the graphs which turtle moved the fastest?
- ☐ How can you tell from the equations which turtle moved the fastest?

Responses vary.
- I can tell from the graphs which turtle moved the fastest by comparing how steep the lines are. The steeper the line, the faster the turtle.
- I can tell from the equations which turtle moved the fastest by looking at the constant of proportionality. When the equations are all written as $d =$, the greater the constant of proportionality, the faster the turtle.

Lesson Practice 2.10

Lesson Summary

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.

- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges $2 for one pound ($k = 2$), while Store B charges $1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.

Unit 2 Lesson 10 149 Synthesis | Lesson Practice

Lesson Takeaway: The constant of proportionality in an equation of a proportional relationship is connected to the steepness of the graph of its line and the rate in a situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by labeling two relationships on a graph and writing an equation.

Today's Goals

1. **Goal:** Write an equation of a proportional relationship given a point on a graph.
2. **Language Goal:** Compare proportional relationships using their graphs and equations. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.10

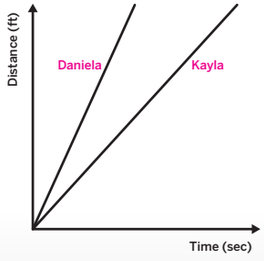
Kayla and Daniela started walking at constant speeds.

After 3 seconds:

- Kayla had walked 6 feet.
- Daniela had walked 12 feet.

a. Label each graph with the name it represents.

b. Write an equation for Kayla's walk, using d for distance and t for time.
$d = 2t$ (or equivalent)



Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to connect situations, graphs, and equations:

» **Respond to Student Thinking:**

- Use the ***Comparing Proportional Relationships with Graphs*** **Mini-Lesson**.
- Revisiting Activity 3's card sort as a class and inviting students to share strategies for making connections between representations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for connecting representations and writing equations? What opportunities might there be in future lessons to connect back to the strategies students used today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- ☐ How can you tell from the graphs which turtle moved the fastest?
- ☐ How can you tell from the equations which turtle moved the fastest?

Responses vary.

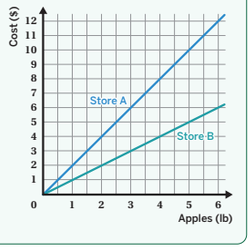
- I can tell from the graphs which turtle moved the fastest by comparing how steep the lines are. The steeper the line, the faster the turtle.
- I can tell from the equations which turtle moved the fastest by looking at the constant of proportionality. When the equations are all written as $d =$, the greater the constant of proportionality, the faster the turtle.

Lesson Practice 2.10

Lesson Summary

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.



- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges \$2 for one pound ($k = 2$), while Store B charges \$1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.

Lesson Practice 2.10

Name: _____ Date: _____ Period: _____

1. Meteoroid Perseid 245 and Asteroid X travel through the solar system. This graph shows how much distance they travel over time.



Does Asteroid X travel faster or slower than Perseid 245? **Faster**

Explain your thinking.

Explanations vary. **Asteroid X travels faster because its line is steeper. So it takes less time for Asteroid X to travel the same distance as Perseid 245.**

2. Having a crew on the International Space Station requires life support and crew supplies. The graph shows the relationship between the cost of life support and crew supplies, and the number of days spent in space.



Which costs less, life support or crew supplies?

Life support

Explain your thinking.

Explanations vary. **Life support costs less because its line is less steep than the line for crew supplies. If I draw a vertical line starting from the horizontal axis, it will intersect the life support line before the crew supplies line.**

Problems 3–6: At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for \$5.40.

3. How much does the honey cost per ounce? Show your thinking.

\$0.45 per ounce. $\frac{5.40}{12} = 0.45$

4. How much honey can be bought per dollar? Show your thinking.

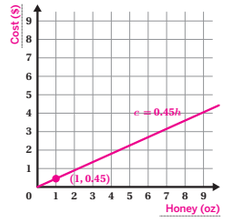
About 2.2 ounces per dollar. $\frac{12}{5.40} \approx 2.2$

5. Write *two* different equations representing this situation. Use h for ounces of honey and c for cost in dollars.

$c = 0.45h$ and $h = 2.2c$ (or equivalent)

6. Choose *one* equation and draw its graph on the coordinate plane. Be sure to label the axes.

Sample shown on graph.

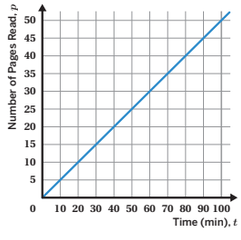


Lesson Practice 2.10

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 7–10: The graph shows the relationship between time in minutes, t, and the total number of pages Joel has read, p.



7. Is this relationship proportional?

Explain your thinking.

Yes. *Explanations vary.* The graph is a straight line that passes through the origin.

8. What is a constant of proportionality in this relationship?

$\frac{1}{2}$ (or equivalent) or 2 (or equivalent)

9. What does this number mean in this situation?

Responses vary. **Joel is reading a book at a constant rate of a $\frac{1}{2}$ page per minute.**

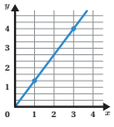
10. Write an equation that represents the relationship between the time in minutes, t, and the total number of pages Joel has read, p.

Equation: **$p = \frac{1}{2}t$ or $t = 2p$** (or equivalent)

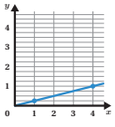
Spiral Review

Problems 11–14: Match each equation with its graph.

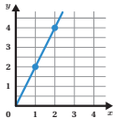
Graph A



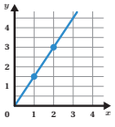
Graph B



Graph C



Graph D



11. $y = \frac{1}{4}x$ **B** **12.** $y = \frac{3}{2}x$ **D** **13.** $y = 2x$ **C** **14.** $y = \frac{4}{3}x$ **A**

Lesson Practice Item Analysis

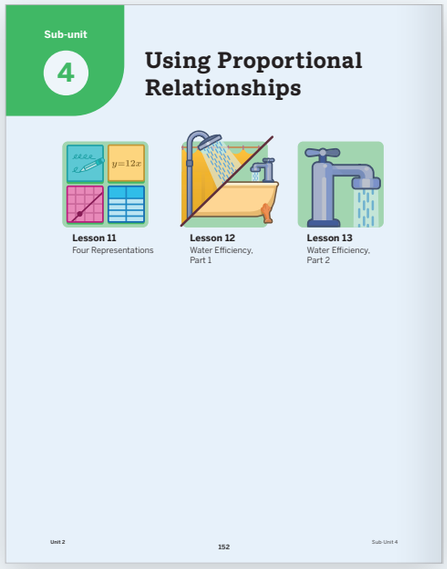
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.4.5
	5–6	2	MA.7.AR.4.4
FAST Practice	7– 10	2	MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4
Spiral Review			
	11–14	1	MA.7.AR.4.4

Using Proportional Relationships

Sub-Unit 4 Goals

- **Goal:** Use proportional relationships to solve real-world and mathematical problems.
- **Language Goal:** Interpret different representations of a proportional relationship to make sense of a situation in context.

In this sub-unit, students use proportional relationships to solve real-world and mathematical problems.



Math that Matters Most

Concept or Skill	For example . . .
Determine the constant of proportionality in multiple representations.	What is the constant of proportionality in each of these representations? Explain your thinking.

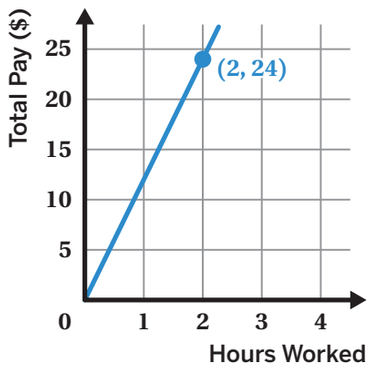
Equation

$y = 12x$

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Graph



Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96
10	120

Equation: 12 because that is the k value in the equation of the form y = kx.

Description: 12 because that is the rate in the description.

Graph: 12 because the y-value of the point (1, 12) represents the constant of proportionality.

Table: 12 because that is the value of y when x is 1.

Use proportional relationships to solve real-world problems	Elena can buy 10 ounces of popcorn for $2.50. Write two different equations that represent this situation. Use p for ounces of popcorn and c for cost in dollars.

I can write c = 0.25p because the popcorn costs $0.25 per ounce. I can also write p = 4c because Elena can buy 4 ounces of popcorn per dollar.

Unit 2

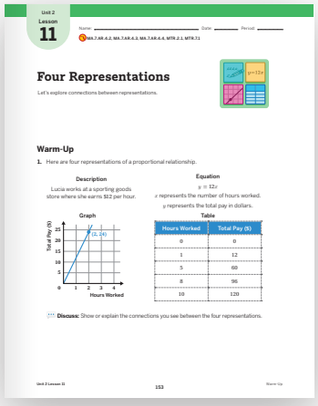
Lesson

11

Four Representations

Connecting Descriptions, Tables, Graphs, and Equations

Let's explore connections between representations.


Unit 2
Lesson
11
Four Representations
Let's explore connections between representations.
Warm-Up
1. Here are four representations of a proportional relationship.
Description
Lucia works at a sporting goods store where she earns $12 per hour.
Equation
$y = 12x$
x represents the number of hours worked.
y represents the total pay in dollars.
Graph
Table
Hours Worked | Total Pay ($)
0 | 0
1 | 12
5 | 60
8 | 96
10 | 120
Discuss: Show or explain the connections you see between the four representations.
153

Learning Outcomes

Today's Goals

1. **Goal:** Create four different representations of a proportional relationship: description, table, graph, and equation.
2. **Language Goal:** Interpret proportional relationships in context using tables, equations, graphs, and verbal descriptions. **(Writing)**

Students examine the connections between verbal descriptions, tables, equations, and graphs of proportional relationships. As they work, students pay close attention to the variables as they build contextual meaning of the constant of proportionality.

Coherence

Prior Learning

Throughout this unit, students studied various representations of proportional relationships and how to find the constant of proportionality.

Future Learning

In Lessons 12 and 13, students will apply their understanding of proportional relationships to explore a real-world problem. They will determine which quantities are important and model the relationship using different representations.

Connections

- Students continue to build their understanding of different representations of proportional relationships in real world contexts such as total pay per hours worked and converting between different units of measure. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.3
Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.4.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.AR.3.3**
MA.6.AR.3.5	**MA.8.AR.3.4**
MA.6.GR.1.1	

Lesson at a Glance

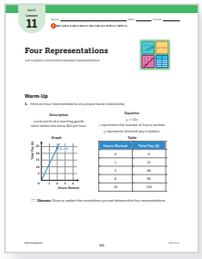
~ 45 min

7.MA.AR.4.2, 7.MA.AR.4.3, 7.MA.AR.4.4

Warm-Up

Pairs | 5 min

Students describe the similarities and differences between the four different representations of a proportional relationship.



Activity 1

Pairs | 10 min

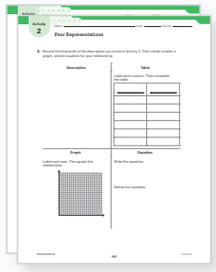
Students write and revise a description of a relationship, then justify whether it is proportional. MLR1: Stronger and Clearer Each Time



Activity 2

Pairs | 20 min

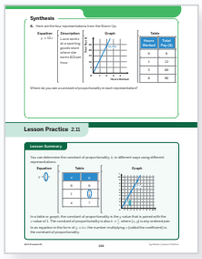
Students define, develop, describe, and represent their own proportional relationship and think about what quantities are related.



Synthesis

Whole Class | 5 min

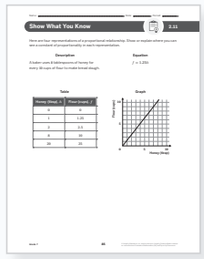
Students synthesize their understanding of the different representations and their uses for describing a proportional relationship.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the constant of proportionality in different representations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students describe the similarities and differences between the four different representations of a proportional relationship.

1 Launch

Direct student's attention to the four representations of the proportional relationship.

To surface prior knowledge, consider inviting students to try to find the same information in all four representations. **(MTR.2.1)**

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

2 Connect

Invite students to share the connections they see between the four representations.

Consider asking, "Which of these representations is the most useful?" Each representation is useful to highlight a different property of the function. For example, in the equation, it is easy to see the exact value of the constant of proportionality.

Unit 2
Lesson
11

Name: _____ Date: _____ Period: _____

MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4, MTR.2.1, MTR.7.1

Four Representations

Let's explore connections between representations.



Warm-Up

1. Here are four representations of a proportional relationship.

Description

Lucia works at a sporting goods store where she earns $12 per hour.

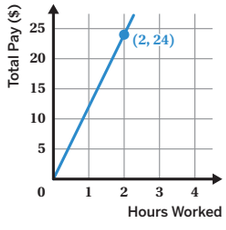
Equation

$y = 12x$

x represents the number of hours worked.

y represents the total pay in dollars.

Graph



Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96
10	120

Discuss: Show or explain the connections you see between the four representations.

Responses vary. The constant of proportionality in the equation is 12. I also see this in the second row of the table, which says 1 hour worked and 12 dollars of pay. The point (2, 24) is plotted on the graph, and $24 \div 2 = 12$. This means the graph has a constant of proportionality of 12, too. The description mentions the same constant of proportionality, noting that Lucy earns $12 per hour.

Unit 2 Lesson 11 153 Warm-Up

Pairs | 10 min

Activity 1 Stronger and Clearer Each Time

Purpose: Students write and revise a description of a relationship, then justify whether it is proportional.

Short on time? Consider limiting the rounds of feedback on the proportional relationships to two.

1 Launch

Direct student's attention to the list of items. To encourage playfulness, consider asking, "Which two options on these lists would be surprising if they were paired together?"

Invite students to choose a pair of options with their partner. Encourage students to choose a pair that no one else in class will choose.

2 Monitor

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft and use feedback to make their thinking stronger and clearer.

D Differentiation

Look for students who:	Respond to Student Thinking
Are overwhelmed with the choices from the table.	**Support** Consider showing them how the Warm-Up related two quantities.
Need support with giving useful feedback.	**Support** Invite these students to look for clear values and units in each relationship. Consider having them practice giving feedback with their partner before giving feedback to other groups.
Finish early.	**Stretch** Invite them to come up with their own categories or units.

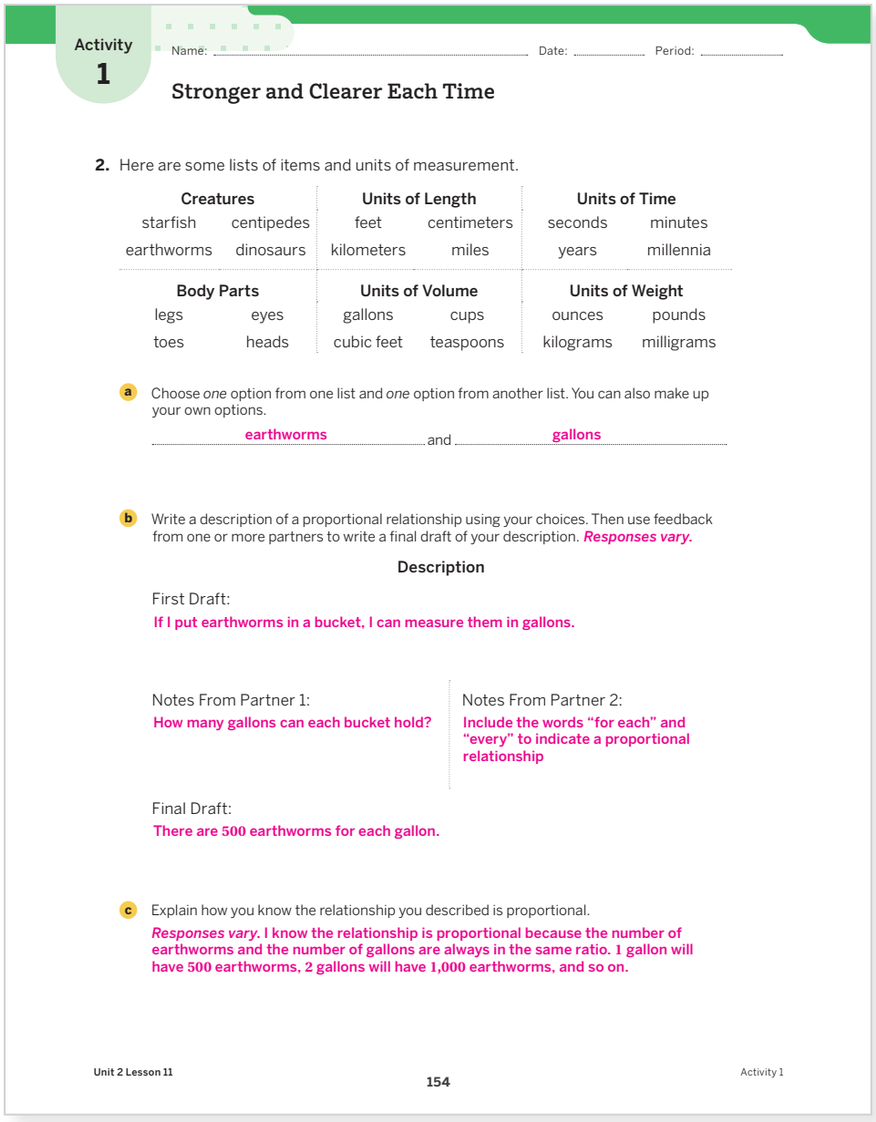
Invite students to share their drafts with two other pairs of partners and give each other feedback.

Encourage students to use the feedback they got from their partners to write a second draft that is stronger and clearer (with more precision, organization, etc.).

3 Connect

Share student drafts that were made stronger through feedback. Consider asking, "What specific pieces of feedback made the descriptions stronger and clearer?" **(MTR.7.1)**

To surface the Key Takeaway, consider asking, "How do you know that you have created a proportional relationship?"



Activity 1

Name: _____ Date: _____ Period: _____

Stronger and Clearer Each Time

2. Here are some lists of items and units of measurement.

Creatures		Units of Length		Units of Time	
starfish	centipedes	feet	centimeters	seconds	minutes
earthworms	dinosaurs	kilometers	miles	years	millennia
Body Parts		**Units of Volume**		**Units of Weight**	
legs	eyes	gallons	cups	ounces	pounds
toes	heads	cubic feet	teaspoons	kilograms	milligrams

a. Choose *one* option from one list and *one* option from another list. You can also make up your own options.

earthworms and gallons

b. Write a description of a proportional relationship using your choices. Then use feedback from one or more partners to write a final draft of your description. *Responses vary.*

Description

First Draft:

If I put earthworms in a bucket, I can measure them in gallons.

Notes From Partner 1:

How many gallons can each bucket hold?

Notes From Partner 2:

Include the words "for each" and "every" to indicate a proportional relationship

Final Draft:

There are 500 earthworms for each gallon.

c. Explain how you know the relationship you described is proportional.

Responses vary. I know the relationship is proportional because the number of earthworms and the number of gallons are always in the same ratio. 1 gallon will have 500 earthworms, 2 gallons will have 1,000 earthworms, and so on.

Unit 2 Lesson 11 154 Activity 1

Key Takeaway: A strong description of a proportional relationship includes the value and units for how the two quantities change together. This often is represented by the constant of proportionality.

Pairs | 20 min

Activity 2 Four Representations

Purpose: Students define, develop, describe, and represent their own proportional relationship and think about what quantities are related.

1 Launch

Direct student's attention to the four representations from the Warm-Up. Share with students that they will create a graph, table, and an equation for their proportional relationship. **(MTR.2.1)**

A **Accessibility: Executive Functioning** Consider pre-selecting two quantities from Activity 1 and providing a corresponding table of values for the relationship. This will allow students to focus on analyzing the connections between the table, graph, and equation.

2 Monitor

Select and sequence examples that clearly show how the four representations are related to share during the Connect.

D **Differentiation**

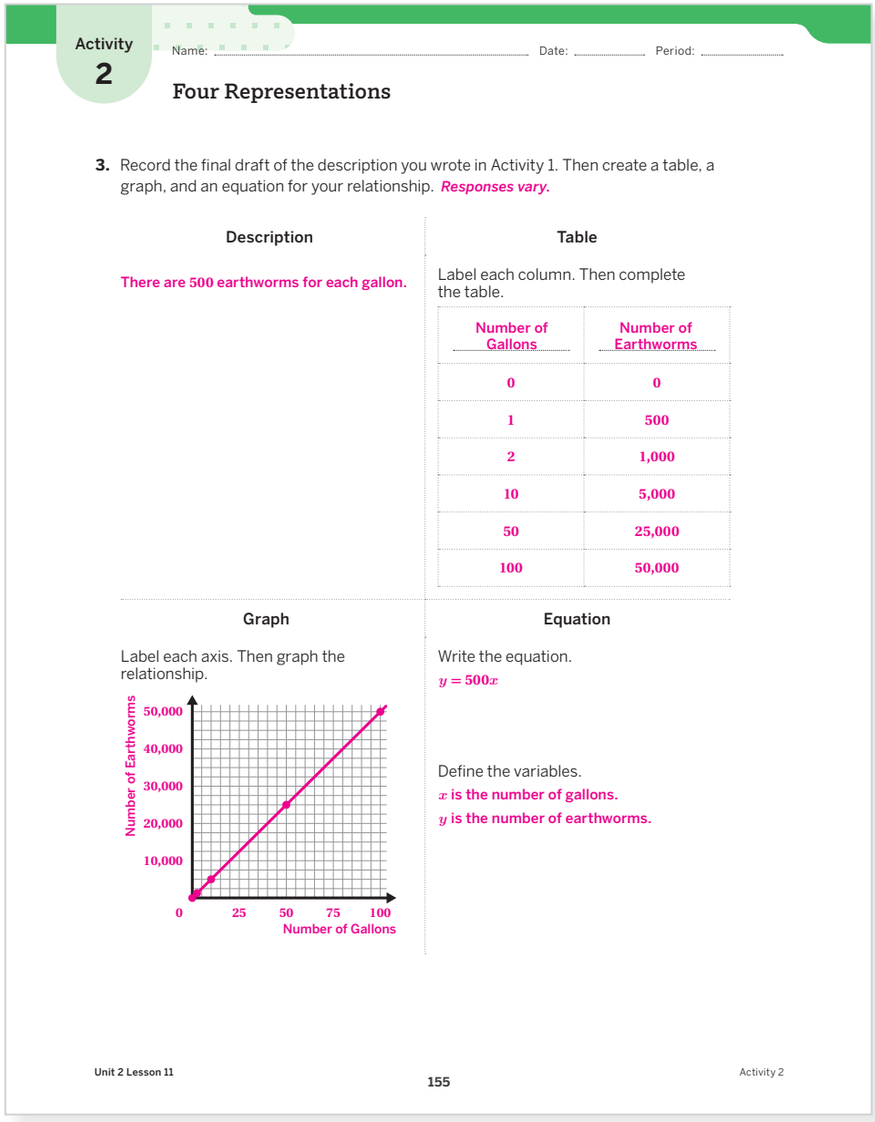
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to start by creating a table of values that represents their relationship.
Use different values in their tables and graphs.	**Support** Consider asking, "How can you clearly show the connection between the tables and the graph?"
Create tables or graphs without labels.	**Strengthen** Invite them to be precise with the values along the axes and the labels for the columns and axes.
Clearly identify the constant of proportionality in all of the representations.	**Strengthen** Consider asking, "What would change about each of your representations if you made your constant of proportionality larger?"

3 Connect

Share and discuss the examples that clearly show how the four representations are connected. Invite students to share how they can see the constant of proportionality in each representation. **(MTR.2.1)**

To surface the Key Takeaway, consider asking:

- "What do the different representations clearly show about your relationship? (For instance, what does a graph clearly show? A table? An equation?)"
- "Why would someone want to have representations about the relationship you have chosen?" **(MTR.7.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Four Representations

3. Record the final draft of the description you wrote in Activity 1. Then create a table, a graph, and an equation for your relationship. *Responses vary.*

Description

There are 500 earthworms for each gallon.

Table

Label each column. Then complete the table.

Number of Gallons	Number of Earthworms
0	0
1	500
2	1,000
10	5,000
50	25,000
100	50,000

Graph

Label each axis. Then graph the relationship.

Equation

Write the equation.

$y = 500x$

Define the variables.

x is the number of gallons.

y is the number of earthworms.

Unit 2 Lesson 11 155 Activity 2

Key Takeaway: Each representation communicates something very clearly about the proportional relationship. In each, you can determine the constant of proportionality.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the different representations and their uses for describing a proportional relationship.

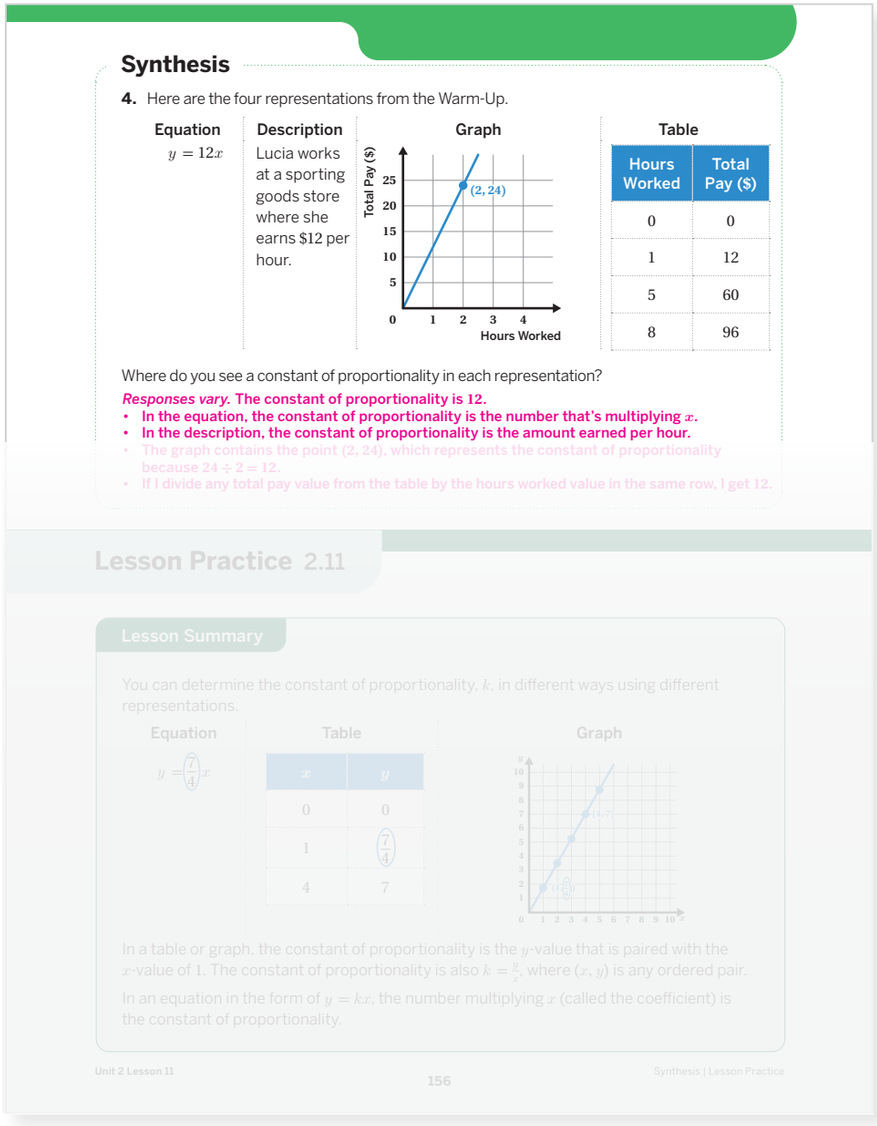
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The description often has a "per" or "for each" description that describes the constant of proportionality.
- The equation is in the form $y = kx$ and the constant of proportionality is k.
- The steepness of the graph shows the constant of proportionality.
- The value that is paired with 1 hour worked is the constant of proportionality.

Create a chart that highlights how students can identify the constant of proportionality in each representation.

Note: If time allows, invite students to share with a partner what makes sense about each representation and what questions they still have.



Synthesis

4. Here are the four representations from the Warm-Up.

Equation

$y = 12x$

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Graph

Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96

Where do you see a constant of proportionality in each representation?

Responses vary. The constant of proportionality is 12.

- In the equation, the constant of proportionality is the number that's multiplying x.
- In the description, the constant of proportionality is the amount earned per hour.
- The graph contains the point $(2, 24)$, which represents the constant of proportionality because $24 \div 2 = 12$.
- If I divide any total pay value from the table by the hours worked value in the same row, I get 12.

Lesson Practice 2.11

Lesson Summary

You can determine the constant of proportionality, k, in different ways using different representations.

Equation

$y = \frac{7}{4}x$

Table

x	y
0	0
1	$\frac{7}{4}$
4	7

Graph

In a table or graph, the constant of proportionality is the y-value that is paired with the x-value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where (x, y) is any ordered pair.

In an equation in the form of $y = kx$, the number multiplying x (called the coefficient) is the constant of proportionality.

Unit 2 Lesson 11 156 Synthesis | Lesson Practice

Lesson Takeaway: Each representation of a proportional relationship can help us to determine the constant of proportionality. It is the k value in an equation of the form $y = kx$, the y-value paired with 1 in a table or graph, or the value associated with "for each" or "per" in a description.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the constant of proportionality in different representations.

Today's Goals

1. **Goal:** Create four different representations of a proportional relationship: description, table, graph, and equation.
2. **Language Goal:** Interpret proportional relationships in context using tables, equations, graphs, and verbal descriptions. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.11

Here are four representations of a proportional relationship. Show or explain where you can see a constant of proportionality in each representation.

Description

A baker uses 8 tablespoons of honey for every 10 cups of flour to make bread dough.

Responses vary. A constant of proportionality is the number of cups of flour for each tablespoon of honey: $\frac{10}{8}$.

Equation

$f = 1.25h$

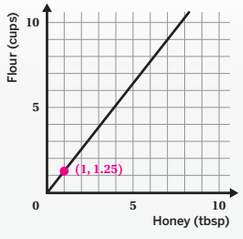
Responses vary. A constant of proportionality is k in the equation $f = kh$, so it's 1.25.

Table

Honey (tbsp), h	Flour (cups), f
0	0
1	1.25
2	2.5
8	10
20	25

Responses vary. A constant of proportionality is 1.25, which is the y-value paired with the x-value of 1.

Graph



D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify the constant of proportionality in different representations:

» **Respond to Student Thinking:**

- Use the ***Representing Proportional Relationships*** **Mini-lesson.**
- Revisit Activity 2 and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Reflect on how students gave and received feedback to each other. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of each student's descriptions? How can you support students in giving and receiving feedback to each other in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

4. Here are the four representations from the Warm-Up.

Equation

$y = 12x$

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Graph

Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96

Where do you see a constant of proportionality in each representation?

Responses vary. The constant of proportionality is 12.

- In the equation, the constant of proportionality is the number that's multiplying x.
- In the description, the constant of proportionality is the amount earned per hour.
- The graph contains the point (2, 24), which represents the constant of proportionality because $24 \div 2 = 12$.
- If I divide any total pay value from the table by the hours worked value in the same row, I get 12.

Lesson Practice 2.11

Lesson Summary

You can determine the constant of proportionality, k, in different ways using different representations.

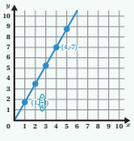
Equation

$y = \frac{7}{4}x$

Table

x	y
0	0
1	$\frac{7}{4}$
4	7

Graph



In a table or graph, the constant of proportionality is the y-value that is paired with the x-value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where (x, y) is an ordered pair.

In an equation in the form of $y = kx$, the number multiplying x (called the coefficient) is the constant of proportionality.

Lesson Practice 2.11

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine a constant of proportionality in each representation. Show your thinking.

1. David walked 15 feet in 3 seconds.

5 or $\frac{1}{5}$. Work varies. $k = 15 \div 3 = 5$ *or* $k = 3 \div 15 = \frac{1}{5}$

2. $C = 4.8n$

4.8. Explanations vary. The coefficient in the equation is 4.8.

3.

x	y
0	0
$\frac{7}{10}$	$\frac{28}{15}$

$\frac{8}{3}$ or $\frac{3}{8}$. *Work varies.*

$k = \frac{28}{15} \div \frac{7}{10} = \frac{28}{15} \cdot \frac{10}{7} = \frac{8}{3}$

or $k = \frac{7}{10} \div \frac{28}{15} = \frac{7}{10} \cdot \frac{15}{28} = \frac{3}{8}$

4.



2.75 or 0.3636 . . . Work varies.

$k = 5.5 \div 2 = 2.75$ or $k = 2 \div 5.5 = 0.3636 \ldots$

5. Here is a description, a graph, and a table.

Description

Yosef has to work 12 hours in a week to earn $180.

Graph



Table

Hours Worked	Total Pay ($)
0	0
2	25
6	75
10	125
20	250

Which one represents a different relationship than the others? Explain your thinking.

The table. Explanations vary. The description has a constant of proportionality of $180 \div 12$, which is equal to 15. The graph has a constant of proportionality of $60 \div 4$, which is also equal to 15. The table has a constant of proportionality of $25 \div 2$, which is equal to 12.5.

Lesson Practice 2.11

Name: _____ Date: _____ Period: _____

Problems 6–8: The equation $c = 2.95g$ gives the cost of gas on a particular day, where c represents the cost in dollars and g represents the number of gallons of gas purchased at a neighborhood gas station.

6. Write four ordered pairs that fit this relationship, where the x-value is the number of gallons of gas and the y-value is the cost.

Responses vary. (0, 0), (1, 2.95), (2, 5.90), (3, 8.85)

7. What does the value 2.95 represent in this situation?

Responses vary. It's the price of each gallon of gas.

8. Hailey's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? Explain your thinking.

Yes. *Explanations vary.* The reciprocal of 2.95 is $\frac{1}{2.95} \approx 0.339$, which is about a third.

FAST Practice

9. $\left(\frac{1}{2}, 3\right)$ is a point on a line that represents a proportional relationship. Select *all* the points that are also on this line.

- ☐ A. $\left(9, \frac{3}{2}\right)$
- ☑ B. $(1, 6)$
- ☐ C. $(1, 3)$
- ☐ D. $\left(\frac{3}{2}, \frac{1}{4}\right)$
- ☑ E. $\left(\frac{7}{2}, 21\right)$

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☑ A.

x	y
0	0
$\frac{1}{4}$	$\frac{1}{8}$
$\frac{2}{4}$	$\frac{2}{8}$
$\frac{3}{4}$	$\frac{3}{8}$

☐ B.

x	y
0	0
$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{3}$	$\frac{1}{5}$
$\frac{1}{4}$	$\frac{1}{6}$

☐ C.

x	y
0	0
1	1
2	4
4	16

☑ D.

x	y
0	0
1	4
3	12
4	16

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.4.2
	5	3	MA.7.AR.4.2
	6–8	2	MA.7.AR.4.2, MA.7.AR.4.4
FAST Practice	9	2	MA.7.AR.4.2
Spiral Review			
	10	2	MA.7.AR.4.1

Unit 2

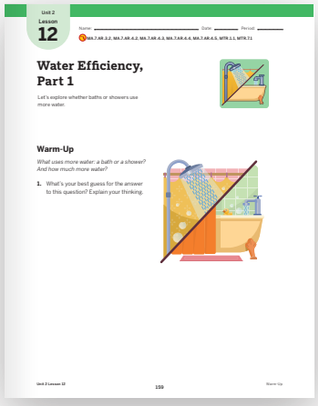
Lesson

12

Water Efficiency, Part 1

Let's Put It to Work

Let's explore whether baths or showers use more water.


Unit 2
Lesson
12

Water Efficiency, Part 1

Let's explore whether baths or showers use more water.

Warm-Up

What uses more water: a bath or a shower? And how much more water?

1. What's your best guess for the answer to this question? Explain your thinking.

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Learning Outcomes

Today's Goals

1. **Goal:** Model a real-world situation using proportional relationships.
2. **Language Goal:** Interpret a situation in context in order to model it with a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Students use their understanding of proportional relationships to explore whether baths or showers use more water. They decide which quantities are important to answering this question, make reasonable assumptions about the values for those quantities, and create a mathematical model to test and report on which uses more water. **(MTR.7.1)**

Coherence

Prior Learning

In this unit, students have explored tables, equations, graphs, and descriptions of proportional relationships. They have used each of these mathematical representations to identify proportional relationships.

Future Learning

In Lesson 13, students will participate in a **Gallery Tour** of their classmates' posters. They have the opportunity to revise and reflect on their work with their group.

Connections

- Students apply proportional relationships to represent data using multiple representations when relating the amount of water used by baths and showers. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.5
Solve real-world problems involving proportional relationships.

Also Addressing: **MA.7.AR.3.2, MA.7.AR.4.3, MA.7.AR.4.4**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Lesson at a Glance

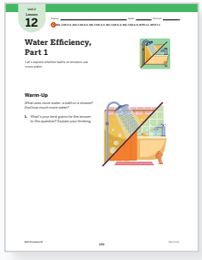
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Pairs | 5 min

Students make a prediction about whether a bath or shower uses more water.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

Students make assumptions and estimates that they will use in their models of bathtubs and showers. MLR2: Collect and Display **(MTR.7.1)**



Activity 2

Pairs | 20 min

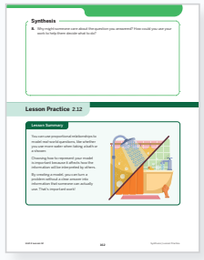
Students formulate a plan and use proportional reasoning to compute whether a bathtub or shower uses more water. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

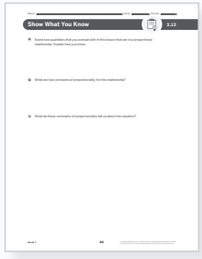
Students synthesize their understanding of how mathematical models can be used to answer questions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by reflecting on the quantities they used in the lesson and how they create a proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students make a prediction about whether a bath or shower uses more water.

1 Launch

Direct students' attention to the bath faucet and the showerhead. To surface connections, consider asking, "Why is it important to save water?"

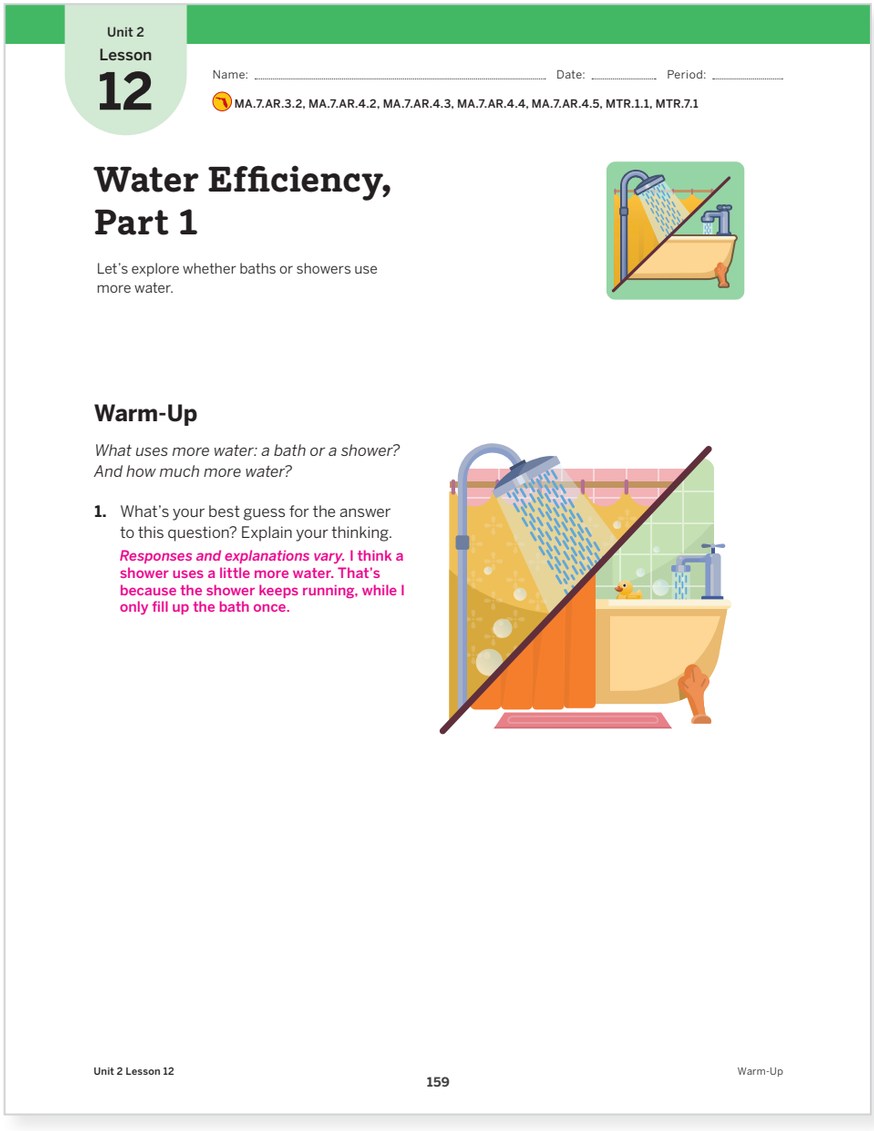
To generate excitement, consider asking students to make a prediction — does a shower or a bath use more water? Poll the class and record students' votes.

Use the Think-Pair-Share routine to help students deepen their thinking around their initial hunches. Invite students to think independently and then take turns sharing with a partner about why they think a bath or shower uses more water.

2 Connect

Invite students to share their thinking about whether a shower or a bath uses more water.

Consider asking, "Why would someone care about the answer to this question?"



Unit 2
Lesson
12

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5, MTR.1.1, MTR.7.1

Water Efficiency, Part 1

Let's explore whether baths or showers use more water.

Warm-Up

What uses more water: a bath or a shower? And how much more water?

1. What's your best guess for the answer to this question? Explain your thinking.

   Responses and explanations vary. **I think a shower uses a little more water. That's because the shower keeps running, while I only fill up the bath once.**

Unit 2 Lesson 12 159 Warm-Up

Pairs | 10 min

Activity 1 Choosing Values

Purpose: Students make assumptions and estimates that they will use in their models of bathtubs and showers. **(MTR.7.1)**

1 Launch

To generate excitement, tell students that it is their job to determine the answer to the question from the Warm-Up. Invite students to brainstorm with a partner what information they need to make a decision. **(MTR.1.1)**

Consider asking, "What do we need to know about showers and baths to answer these questions?"

A **Accessibility: Executive Functioning** To support students identifying information they need to start, consider identifying possible places to start, such as how long a shower takes.

2 Monitor

Select and sequence information that students decide to collect to share during the Connect.

Invite students to do research on shower and bath usage, such as flow rates of bathtub faucets and shower heads or capacities of bathtubs. **(MTR.7.1)**

D **Differentiation** *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are things that impact the amount of water used in a bath or shower?"
Decide on specific values for their model quickly. *(Problem 3)*	**Strengthen** Consider asking, "Are these the only values that could be used to solve this problem? What else might you consider as you create your model?"
Decide on specific values for a single bath or shower. *(Problem 3)*	**Stretch** Consider asking, "How does this information help us describe monthly or yearly water use from showers/baths?"

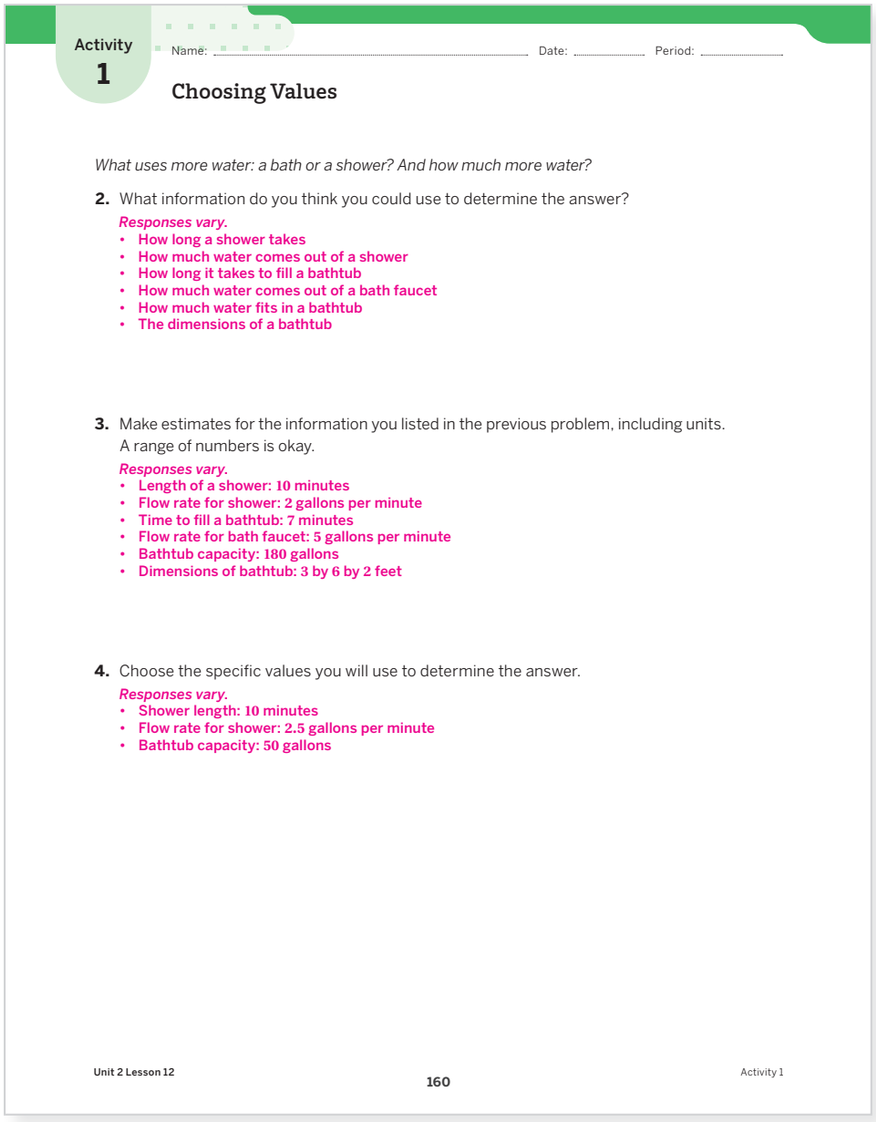
Listen for values that will be helpful for students in writing proportional relationships in the next activity.

3 Connect

Invite students to share the values that their group has decided to explore.

MLR **MLR2: Collect and Display** Use this routine to collect students' specific values as they share during the Connect. Display for students to reference and add to during the lesson.

Note: If time allows, consider giving students the opportunity to revise their values based on the discussion.



Activity 1

Name: _____ Date: _____ Period: _____

Choosing Values

What uses more water: a bath or a shower? And how much more water?

2. What information do you think you could use to determine the answer?

Responses vary.
- How long a shower takes
- How much water comes out of a shower
- How long it takes to fill a bathtub
- How much water comes out of a bath faucet
- How much water fits in a bathtub
- The dimensions of a bathtub

3. Make estimates for the information you listed in the previous problem, including units. A range of numbers is okay.

Responses vary.
- Length of a shower: 10 minutes
- Flow rate for shower: 2 gallons per minute
- Time to fill a bathtub: 7 minutes
- Flow rate for bath faucet: 5 gallons per minute
- Bathtub capacity: 180 gallons
- Dimensions of bathtub: 3 by 6 by 2 feet

4. Choose the specific values you will use to determine the answer.

Responses vary.
- Shower length: 10 minutes
- Flow rate for shower: 2.5 gallons per minute
- Bathtub capacity: 50 gallons

Unit 2 Lesson 12 160 Activity 1

Key Takeaway: Agreeing upon assumptions and values to use in a mathematical model is important. Once they have been agreed upon, then the work of using them to solve a problem can begin.

Pairs | 20 min

Activity 2 Bath vs. Shower

Purpose: Students formulate a plan and use proportional reasoning to compute whether a bathtub or shower uses more water. **(MTR.7.1)**

Short on time? Problem 6 may be omitted.

1 Launch

Consider asking, "Which representation do you want to use to check your prediction? Why?" **(MTR.7.1)**

Invite students to share with the class which representations they might use and why.

2 Monitor

Select and sequence different representations that clearly show the conditions where a bath or shower uses more water.

D Differentiation *(Problems 5–6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What relationship can you measure between the values you are exploring?"
Need support writing a proportional relationship. *(Problem 5)*	**Strengthen** Invite students to write out in words or make a table to represent how their values change together (i.e., how much water fills the bath over time).
Create a table of values, a graph, or an equation to represent their relationship. *(Problem 6)*	**Strengthen** Consider asking, "How can your representation clearly show which uses more water? How do you know that you are right?"
Finish their analysis quickly.	**Stretch** Consider asking, "What are ways that someone could misinterpret your solution? How can you make it clearer?"

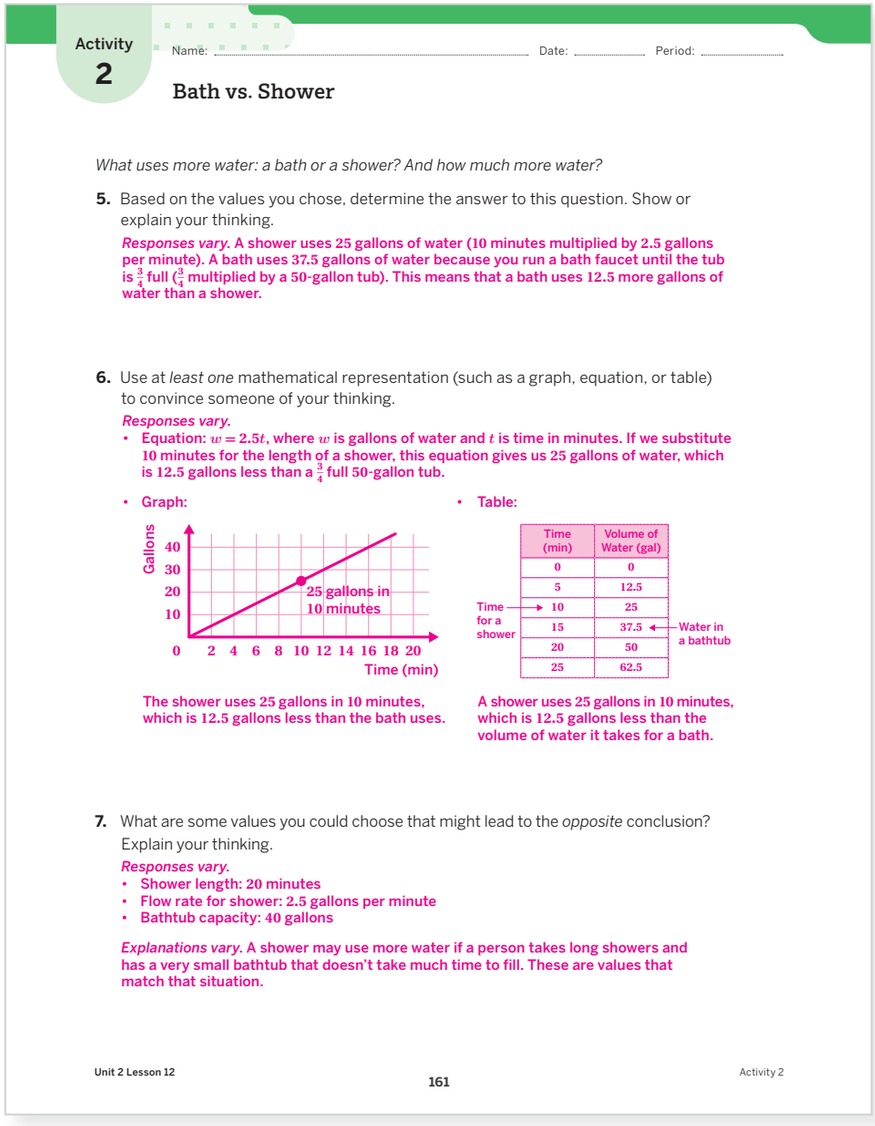
Note: It is okay if students need support with creating a convincing form of their representations. Consider highlighting particularly strong examples and give students time to revise when creating their posters in Lesson 13.

3 Connect

Invite students to share the relationships they explored and their representations with the class.

To surface the Key Takeaway, consider asking:

- "How do you know your values have proportional relationships?"
- "What makes these representations clear and convincing?"



Activity 2

Name: Date: Period:

Bath vs. Shower

What uses more water: a bath or a shower? And how much more water?

5. Based on the values you chose, determine the answer to this question. Show or explain your thinking.

Responses vary. A shower uses 25 gallons of water (10 minutes multiplied by 2.5 gallons per minute). A bath uses 37.5 gallons of water because you run a bath faucet until the tub is $\frac{3}{4}$ full ($\frac{3}{4}$ multiplied by a 50-gallon tub). This means that a bath uses 12.5 more gallons of water than a shower.

6. Use at *least one* mathematical representation (such as a graph, equation, or table) to convince someone of your thinking.

Responses vary.

- Equation: $w = 2.5t$, where w is gallons of water and t is time in minutes. If we substitute 10 minutes for the length of a shower, this equation gives us 25 gallons of water, which is 12.5 gallons less than a $\frac{3}{4}$ full 50-gallon tub.
- Graph:

The shower uses 25 gallons in 10 minutes, which is 12.5 gallons less than the bath uses.

- Table:

Time (min)	Volume of Water (gal)
0	0
5	12.5
10	25
15	37.5
20	50
25	62.5

Time for a shower → 10

15 ← Water in a bathtub

A shower uses 25 gallons in 10 minutes, which is 12.5 gallons less than the volume of water it takes for a bath.

7. What are some values you could choose that might lead to the *opposite* conclusion? Explain your thinking.

Responses vary.

- Shower length: 20 minutes
- Flow rate for shower: 2.5 gallons per minute
- Bathtub capacity: 40 gallons

Explanations vary. A shower may use more water if a person takes long showers and has a very small bathtub that doesn't take much time to fill. These are values that match that situation.

Unit 2 Lesson 12 161 Activity 2

Key Takeaway: You can use different representations of proportional relationships to show when a bath or shower uses more water. Graphs, tables, equations, and words all show information in different compelling ways.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how mathematical models can be used to answer questions.

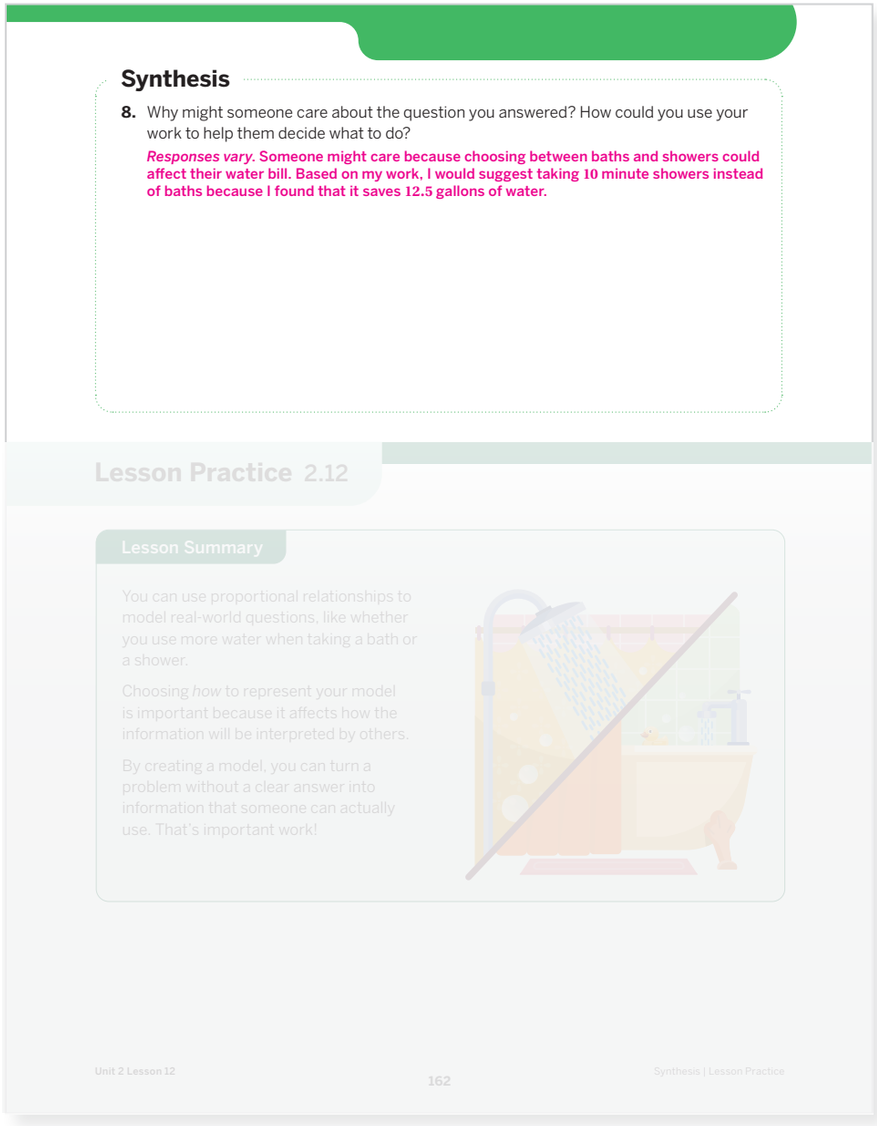
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Knowing which uses more water can help people save money on their water bill.
- Someone may want to change their habits to conserve water.
- Representatives of the water company may want to know so they can help people understand how to save money.
- Proportional relationships can show how the water usage changes over time, and a table can be used to show which uses more.
- A combination of an equation and a graph can be used to show that the shower uses less water per minute and show the point at which they use the same amount.

Consider asking, "How might math from this unit be helpful in convincing someone about which uses more water?"

Note: If time allows, invite students to revise their models of how much water a bath or shower uses based on the discussion.



Synthesis

8. Why might someone care about the question you answered? How could you use your work to help them decide what to do?

Responses vary. Someone might care because choosing between baths and showers could affect their water bill. Based on my work, I would suggest taking 10 minute showers instead of baths because I found that it saves 12.5 gallons of water.

Lesson Practice 2.12

Lesson Summary

You can use proportional relationships to model real-world questions, like whether you use more water when taking a bath or a shower.

Choosing *how* to represent your model is important because it affects how the information will be interpreted by others.

By creating a model, you can turn a problem without a clear answer into information that someone can actually use. That's important work!

Unit 2 Lesson 12 162 Synthesis | Lesson Practice

Lesson Takeaway: We can use mathematical models — such as proportional relationships — to compare how two quantities change. Different representations — such as graphs, tables, and equations — can help us determine specific properties of those relationships more easily.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by reflecting on the quantities they used in the lesson and how they create a proportional relationship.

Today's Goals

1. **Goal:** Model a real-world situation using proportional relationships.
2. **Language Goal:** Interpret a situation in context in order to model it with a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.12

a. Name two quantities that you worked with in this lesson that are in a proportional relationship. Explain how you know.

Responses vary. The length of a shower and the water used during a shower are in a proportional relationship because there is a constant amount of water coming out of the shower head. The amount of water used divided by the amount of time spent in the shower is always the same number.

b. What are two constants of proportionality for this relationship?

Responses vary. If the rate of water coming from the shower is 2.5 gallons per minute, then 2.5 and $\frac{1}{2.5}$ are constants of proportionality.

c. What do these constants of proportionality tell us about the situation?

Responses vary. These constants of proportionality tell me about the relationship between the length of a shower and how much water is used.

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to find constants of proportionality for their relationships:

» **Respond to Student Thinking:**

- Check in with students during Lesson 13, Activity 1. Consider asking, "What would the constant of proportionality represent in your situation?"

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.
- If students would like to explore water efficiency in greater depth, invite them to research water-saving showerheads and faucets and consider their benefits and challenges.

Professional Learning

Today, students applied their understanding of proportional relationships to the topic of water usage. What questions might your students have about the ways in which water is used in their community? What resources might you and the class use to help you learn more about these questions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Why might someone care about the question you answered? How could you use your work to help them decide what to do?

Responses vary. Someone might care because choosing between baths and showers could affect their water bill. Based on my work, I would suggest taking 10 minute showers instead of baths because I found that it saves 12.5 gallons of water.

Lesson Practice 2.12

Lesson Summary

You can use proportional relationships to model real-world questions, like whether you use more water when taking a bath or a shower.

Choosing *how* to represent your model is important because it affects how the information will be interpreted by others.

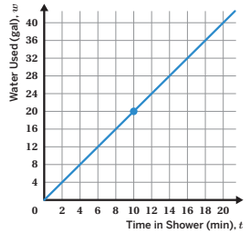
By creating a model, you can turn a problem without a clear answer into information that someone can actually use. That's important work!



Lesson Practice 2.12

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is a graph representing how many gallons of water, w, a shower used in t minutes.



1. Explain how you know that these quantities are in a proportional relationship.

Responses vary. The graph is a line that passes through the origin.

2. What are the two constants of proportionality for this relationship?

2 (or equivalent) and $\frac{1}{2}$ (or equivalent)

3. What do these constants of proportionality mean in this situation?

Responses vary. The shower uses 2 gallons of water for every minute it's running. It takes $\frac{1}{2}$ a minute for the shower to use a gallon of water.

4. Write two equations representing this relationship.

$w = 2t$ and $t = \frac{1}{2}w$ (or equivalent)

FAST Practice

5. On its way from Los Angeles to Boston, a plane traveling at a constant speed flew over Las Vegas, Denver, Cedar Rapids, and Detroit. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Las Vegas to Denver	1	575	575
Denver to Cedar Rapids	1.2	690	575
Cedar Rapids to Detroit	0.8	460	575

Lesson Practice 2.12

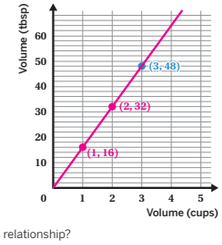
Name: _____ Date: _____ Period: _____

Spiral Review

Problems 6–8: There is a proportional relationship between cups and tablespoons.

6. Plot and label *at least two* more points that fit this relationship. Then draw a line to represent the relationship.

Sample shown on graph.



7. For which value of y is the point $(1, y)$ on the line?

16

8. What is a constant of proportionality for this relationship?

16 (or equivalent) or $\frac{1}{16}$ (or equivalent)

9. Write two equations representing the relationship shown in the table.

$y = 12x$, $x = \frac{1}{12}y$ (or equivalent)

x	y
3	36
5	60
12	144

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.7.AR.4.3
	2 –3	2	MA.7.AR.4.2
	4	2	MA.7.AR.4.4
FAST Practice	5	3	MA.7.AR.3.2
Spiral Review			
	6	2	MA.7.AR.4.3
	7–8	2	MA.7.AR.4.2
	9	2	MA.7.AR.4.4

Unit 2

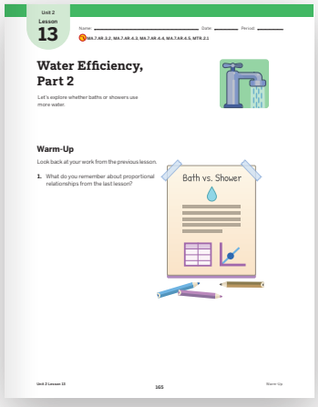
Lesson

13

Water Efficiency, Part 2

Let's Put It to Work

Let's explore whether baths or showers use more water.


Unit 2
Lesson
13

Water Efficiency, Part 2

Let's explore whether baths or showers use more water.

Warm-Up

Look back at your work from the previous lesson.

1. What do you remember about proportional relationships from the last lesson?

Bath vs. Shower

165

Learning Outcomes

Today's Goals

1. **Goal:** Use a model of a real-world situation to make recommendations.
2. **Language Goal:** Analyze a problem about water usage using proportional relationships. **(Reading, Writing, Speaking, and Listening)**

Students revisit their posters before giving and receiving feedback on their work. Then, students take some time to reflect on how the process of making assumptions, doing calculations, and applying proportional reasoning offers them insight into a real-world context.

Coherence

Prior Learning

In Lesson 12, students used their understanding of proportional relationships to explore whether baths or showers use more water.

Future Learning

In Unit 3, students will explore the relationships between the radius, diameter, circumference, and area of a circle.

Connections

- Students use evidence-based reasoning and mathematical representations to defend whether a bath or shower uses more water. (real world, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.3

Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.

MA.7.AR.4.4

Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.3.2, MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standard: MTR.2.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Lesson at a Glance

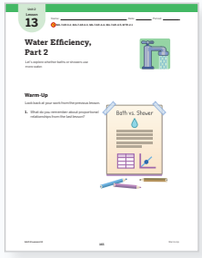
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Pairs | 5 min

Students reflect on the work they have done with proportional relationships in the previous lesson.



Activity 1

Small Groups | 15 min

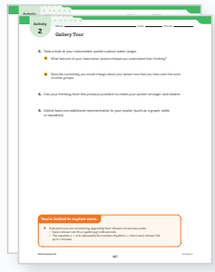
Students create a poster using multiple representations to display their thinking about whether a bath or shower uses more water. **(MTR.2.1)**



Activity 2

Small Groups | 15 min

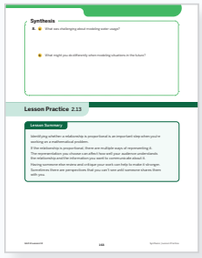
Students participate in a **Gallery Tour** to see how different representations help to make an argument stronger and clearer.
MLR7: Compare and Connect



Synthesis

Whole Class | 5 min

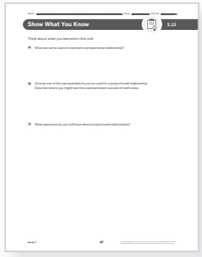
Students synthesize their understanding of modeling water usage using representations of proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by describing ways proportional relationships can be represented.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- graph paper
- markers
- poster paper
- straightedges

Pairs | 5 min

Warm-Up

Purpose: Students reflect on the work they have done with proportional relationships in the previous lesson.

1 Launch

Invite students to discuss with their partner what they are proud of in their work to determine whether a bath or shower uses more water.

Encourage students to reflect with their partner about how they determined which uses more water.

Consider asking:

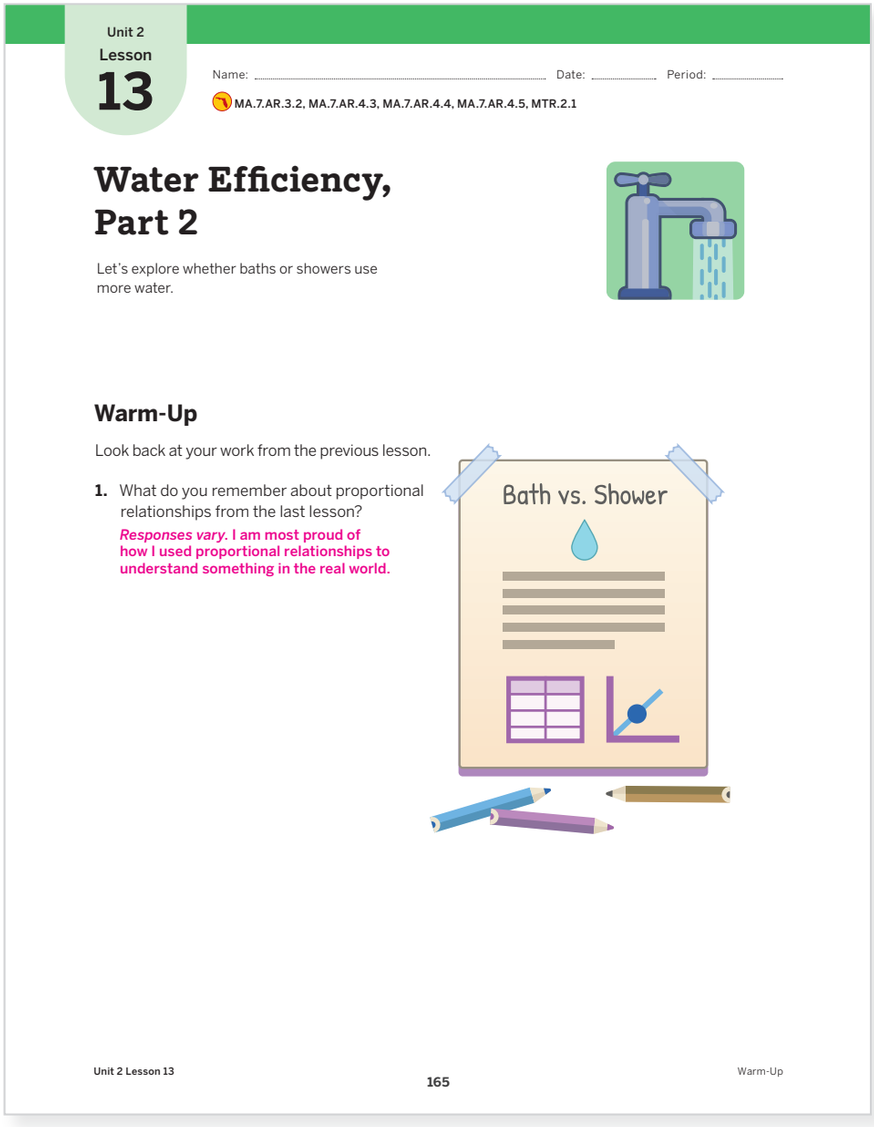
- "What values did you decide were important to consider?"
- "What mathematical representations did you use?"

2 Connect

Consider asking, "What evidence helped you convince yourself about which uses more water?"

Invite students to revise their work to make it stronger and clearer after the discussion. Consider sharing that they are about to make a poster and that having a solid draft will be helpful.

ELL **English Language Learners** Provide sentence frames to help students explain their models (e.g., "I noticed that _____, so I _____." or "First, I _____ because _____."). **(Writing and Speaking)**



Unit 2
Lesson
13

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1

Water Efficiency, Part 2

Let's explore whether baths or showers use more water.

Warm-Up

Look back at your work from the previous lesson.

1. What do you remember about proportional relationships from the last lesson?

Responses vary. I am most proud of how I used proportional relationships to understand something in the real world.

Unit 2 Lesson 13 165 Warm-Up

Small Groups | 15 min

Activity 1 Create a Poster

Purpose: Students create a poster using multiple representations to display their thinking about whether a bath or shower uses more water. **(MTR.2.1)**

1 Launch

To surface connections, consider asking, "How have you convinced yourself which uses more water? How can you show that to other people?"

Direct students' attention to the types of representations they could choose to put on their poster. **(MTR.2.1)**

Consider sharing that students will have time to look at their classmates' posters and then revise their own.

Distribute poster making supplies to students.

2 Monitor

To encourage creativity, consider inviting students to do research on their bath and shower models to convince themselves of their argument.

Select and sequence different representations that answer the questions from Problem 2 on their poster.

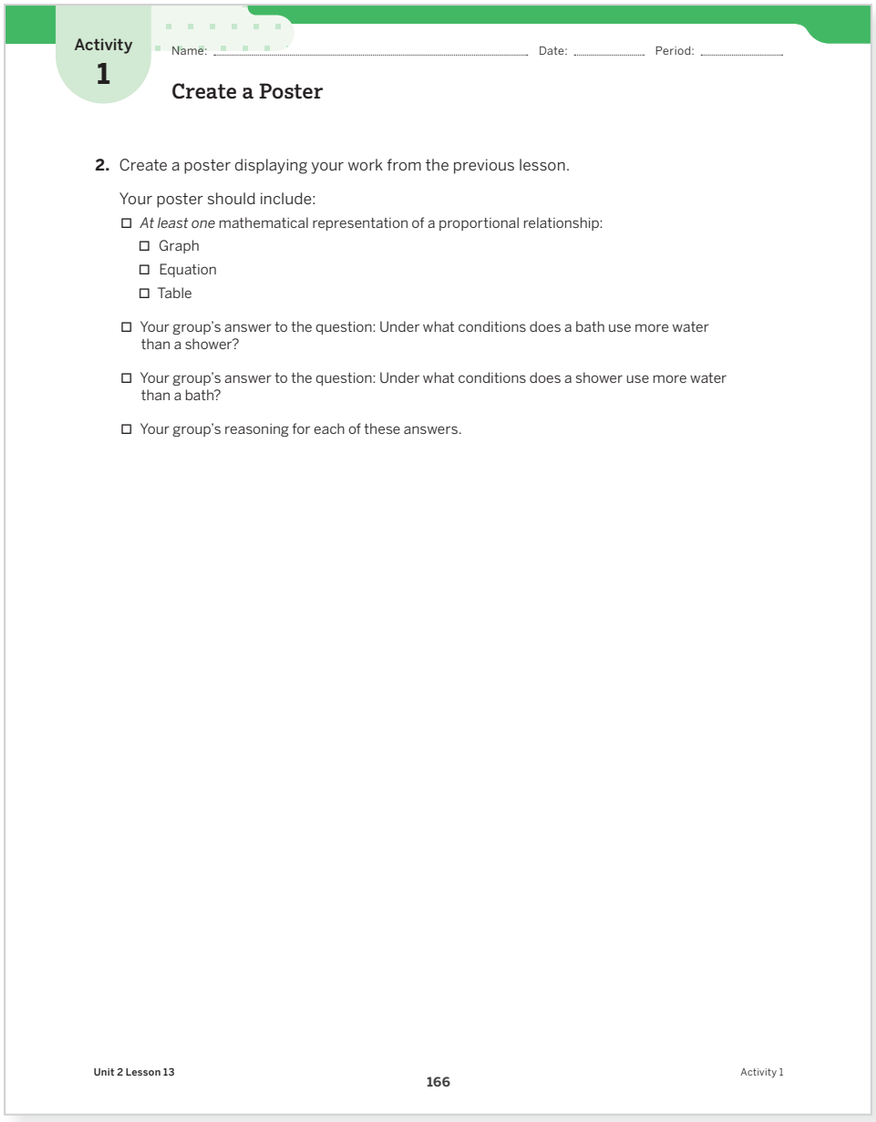
D Differentiation *(Problem 2)*

Look for students who:	Respond to Student Thinking
Determine that a bath uses less water than a very long shower.	**Strengthen** Consider asking, "How can you use mathematical representations to make your conclusion clear to others?"
Determine that a short shower uses less water than a bath.	
Have different representations, but have not shown how they all connect to form an argument.	**Strengthen** Consider asking, "How can you show others how the different representations connect to one another visually?" Encourage students to try using colors, labels, or diagrams to make connections.

3 Connect

To surface the Key Takeaway, consider asking, "What do your different representations show about when a bath or shower uses more water?" **(MTR.2.1)**

Note: After the discussion, consider giving students a few minutes to revise their posters before moving to Activity 2. Consider sharing that they will have another opportunity to revise after they have seen other groups' posters.



Activity 1

Name: _____ Date: _____ Period: _____

Create a Poster

2. Create a poster displaying your work from the previous lesson.

Your poster should include:

- ☐ *At least one* mathematical representation of a proportional relationship:
  - ☐ Graph
  - ☐ Equation
  - ☐ Table
- ☐ Your group's answer to the question: Under what conditions does a bath use more water than a shower?
- ☐ Your group's answer to the question: Under what conditions does a shower use more water than a bath?
- ☐ Your group's reasoning for each of these answers.

Unit 2 Lesson 13 166 Activity 1

Key Takeaway: Graphs, tables, and equations all highlight different aspects of a proportional relationship. It is often helpful to see a relationship shown in different ways to pick out the most important properties.

Small Groups | 15 min

Activity 2 Gallery Tour

Purpose: Students participate in a Gallery Tour to see how different representations help to make an argument stronger and clearer.

Short on time? Consider having students only look at 1–2 other group posters before having them reflect.

1 Launch

To generate engagement, consider having students share with their group what they are proud of from their poster.

Invite students to share their posters through a Gallery Tour. Encourage them to silently view their classmates' work and leave feedback about the representations and the strength of their mathematical argument. Then give each group time to review the feedback left on their work.

Encourage students to go to each poster and write one feature of each poster that helped convince them of the group's argument about baths and showers.

A **Accessibility: Memory and Attention** Provide students with a graphic organizer to support them keeping track of the information and features they see on other posters.

2 Monitor

Look for students who highlight how they might change their representations on their own posters.

Invite students to make revisions to their own posters. Consider asking, "What about other groups' posters do you want to incorporate into your own?"

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore comparing proportional relationships further to complete this optional task. Encourage them to discuss their thinking with a partner.

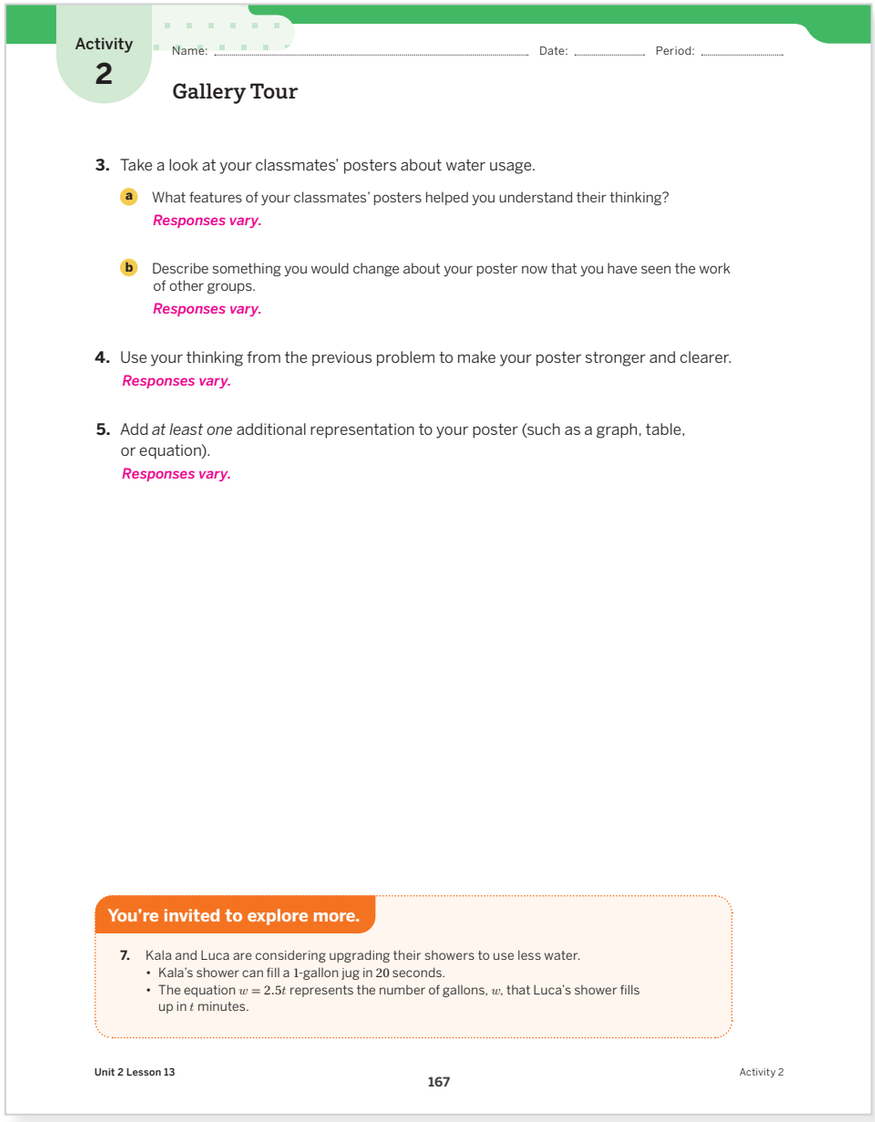
3 Connect

Invite students to display their posters.

To surface the Key Takeaway, consider asking:

- "What makes a strong visual display of a proportional relationship?"
- "Why didn't all of our groups come to the same exact conclusion? Why might we have different thinking on these topics?"

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each poster, and then connect to their group's poster. See the Routine Facilitation Guide for more information.



Activity 2

Name: _____ Date: _____ Period: _____

Gallery Tour

3. Take a look at your classmates' posters about water usage.
   a. What features of your classmates' posters helped you understand their thinking?
      Responses vary.
   b. Describe something you would change about your poster now that you have seen the work of other groups.
      Responses vary.
4. Use your thinking from the previous problem to make your poster stronger and clearer.
   Responses vary.
5. Add *at least one* additional representation to your poster (such as a graph, table, or equation).
   Responses vary.

You're invited to explore more.

7. Kala and Luca are considering upgrading their showers to use less water.
   - Kala's shower can fill a 1-gallon jug in 20 seconds.
   - The equation $w = 2.5t$ represents the number of gallons, w, that Luca's shower fills up in t minutes.

Unit 2 Lesson 13 167 Activity 2

Key Takeaway: Using different representations like graphs, tables, or equations can help highlight different properties of a situation and make a visual argument stronger and clearer.

Whole Class | 5 min

Synthesis

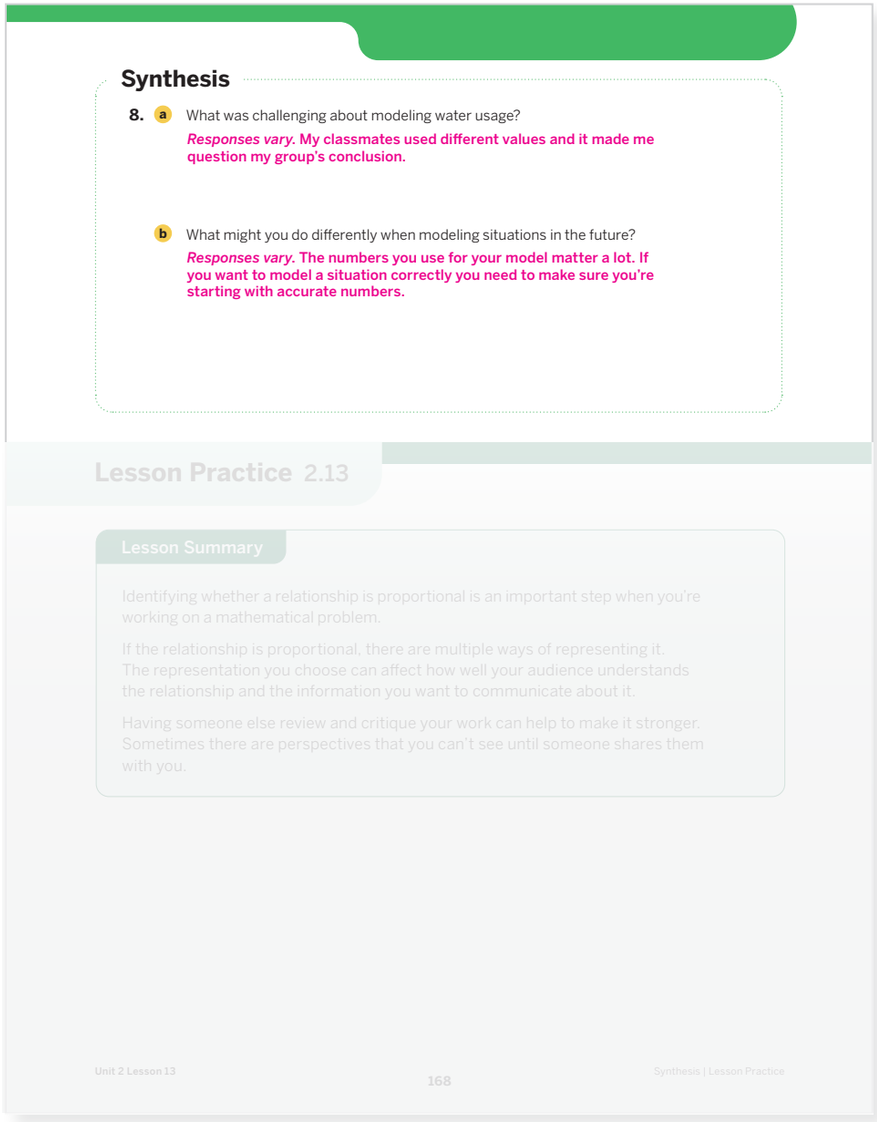
Purpose: Students synthesize their understanding of modeling water usage using representations of proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- It is helpful to check assumptions in a variety of different ways to make sure that the right information is used.
- It was challenging to come up with all of the variables and assumptions to model the bath and shower.
- It was sometimes necessary to adjust models based on feedback from calculations and classmates.

Invite students to discuss why proportional relationships are powerful tools for creating models in the real world. Consider asking, "How might models with proportional relationships be used in the world?"



Synthesis

8. a. What was challenging about modeling water usage?

Responses vary. My classmates used different values and it made me question my group's conclusion.

b. What might you do differently when modeling situations in the future?

Responses vary. The numbers you use for your model matter a lot. If you want to model a situation correctly you need to make sure you're starting with accurate numbers.

Lesson Practice 2.13

Lesson Summary

Identifying whether a relationship is proportional is an important step when you're working on a mathematical problem.

If the relationship is proportional, there are multiple ways of representing it. The representation you choose can affect how well your audience understands the relationship and the information you want to communicate about it.

Having someone else review and critique your work can help to make it stronger. Sometimes there are perspectives that you can't see until someone shares them with you.

Unit 2 Lesson 13 168 Synthesis | Lesson Practice

Lesson Takeaway: Proportional relationships can be used to model things that happen in the real world. When modeling, it is important to interpret and revise our conclusions to make the strongest and clearest response to a question.

Independent | 5 min

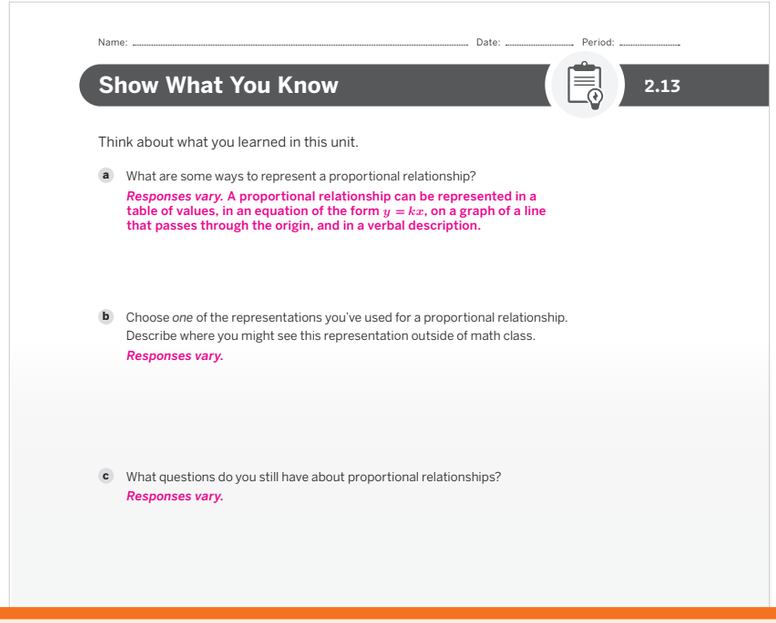
Show What You Know

Purpose: Students demonstrate their understanding by describing ways proportional relationships can be represented.

Today's Goals

1. **Goal:** Use a model of a real-world situation to make recommendations.
2. **Language Goal:** Analyze a problem about water usage using proportional relationships. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.13

Think about what you learned in this unit.

a. What are some ways to represent a proportional relationship?

Responses vary. A proportional relationship can be represented in a table of values, in an equation of the form $y = kx$, on a graph of a line that passes through the origin, and in a verbal description.

b. Choose *one* of the representations you've used for a proportional relationship. Describe where you might see this representation outside of math class.

Responses vary.

c. What questions do you still have about proportional relationships?

Responses vary.

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to describe proportional relationships outside of class:

» **Respond to Student Thinking:**

- Select a few related and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner

Professional Learning

Take a moment to reflect on how students applied what they've learned about proportional relationships to the ways water is used in baths and showers. How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. a. What was challenging about modeling water usage?

Responses vary. My classmates used different values and it made me question my group's conclusion.

b. What might you do differently when modeling situations in the future?

Responses vary. The numbers you use for your model matter a lot. If you want to model a situation correctly you need to make sure you're starting with accurate numbers.

Lesson Practice 2.13

Lesson Summary

Identifying whether a relationship is proportional is an important step when you're working on a mathematical problem.

If the relationship is proportional, there are multiple ways of representing it. The representation you choose can affect how well your audience understands the relationship and the information you want to communicate about it.

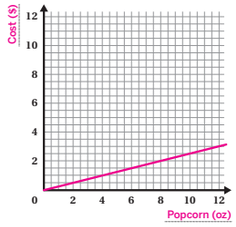
Having someone else review and critique your work can help to make it stronger. Sometimes there are perspectives that you can't see until someone shares them with you.

Lesson Practice 2.13

Name: _____ Date: _____ Period: _____

Problems 1–4: Elena goes to a store where customers can scoop their own popcorn and pay by the ounce. She buys 10 ounces of popcorn for $2.50.

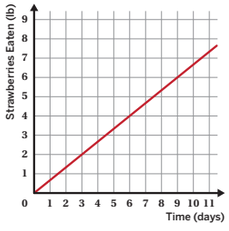
1. How much does the popcorn cost per ounce?
   $0.25
2. How much popcorn can Elena buy per dollar?
   4 ounces
3. Write two different equations that represent this situation. Use p for ounces of popcorn and c for cost in dollars.
   $c = 0.25p$ and $p = 4c$



4. Graph the line of one of the equations. Be sure to label the axes. **Sample shown on graph.**

FAST Practice

Problems 5–7: This graph shows a proportional relationship. Represent this relationship in different ways:



5. In a table:

Time (days)	Strawberries Eaten (lb)
3	2
6	4
9	6

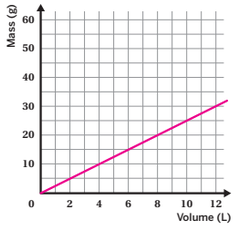
6. With a story:
   Responses vary. **Lin ate 2 pounds of strawberries every 3 days.**
7. With an equation using t for the time in days and s for the pounds of strawberries eaten:
   Equation: $s = \frac{2}{3}t$

Lesson Practice 2.13

Name: _____ Date: _____ Period: _____

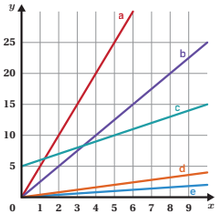
Spiral Review

8. A chemical substance has a proportional relationship between its mass and volume. One constant of proportionality is 2.5 grams per liter. Graph the proportional relationship.
   Sample shown on graph.



9. Match each equation with its graph.

d	$y = \frac{2}{5}x$
a	$y = 5x$
b	$y = \frac{5}{2}x$
e	$y = \frac{1}{5}x$
c	$y = x + 5$



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.7.AR.4.5
	3–4	2	MA.7.AR.4.4
FAST Practice	5–7	2	MA.7.AR.4.4
Spiral Review			
	8	2	MA.7.AR.4.3
	9	2	MA.7.AR.4.4

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students explored proportional relationships in a variety of real-world contexts and made comparisons using representations such as tables, equations, and graphs.

Career Connection | Meet Eugenia Cheng

Invite students to read about how mathematician and concert pianist Eugenia Cheng has helped to explain the connections between mathematical ratios and music.

B.E.S.T. Benchmark Connections

Consider asking:

- "What other math concepts have you studied in Units 1 and 2 that you could apply to music?" Sample response: Designing musical instruments that are in the same family, such as woodwinds (flutes, piccolo), involves geometry so that you use the right shapes and materials so that they create the right sounds. (MA.7.GR.1.5)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "If you visit a store that sells musical instruments, you will find many different types of each kind. How would this apply to Mathematical Thinking and Reasoning standard MTR.7.1 in the work that people who design and make musical instruments do ?" Sample response: Many musical instruments come in different shapes and sizes. These characteristics are often decided when the designer or musician wants to make an instrument create a special sound, handle in a certain way, or use new technology.



Unit 2 | Math in Action

Career Connection

How might ratios alter the mood of a song?

Musicians perform and create musical pieces for people to listen to and enjoy. They use ratios and proportional relationships to create sounds that when played together, are pleasing to the ear.

Ratio of 3 : 4

Ratio of 2 : 3

B.E.S.T. Mathematics Benchmark Connection

Believe it or not, musicians apply math extensively in their work. For example, the combination of notes that are played depend on proportional relationships (MA.7.AR.3.2, MA.7.AR.4.1) to create a certain sound or to choose how fast or slow to play a part of a musical piece. Musicians who write and play music also apply their understanding of ratios and proportions (MA.7.GR.4.2, MA.7.GR.4.4) as they determine which instruments are played during certain parts of a piece of music. How the music is played often sets the tone for a piece of music, even when there are no words being sung.

Mathematical Thinking and Reasoning Connection

People who are involved with music use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work with people they collaborate with, such as the music arranger, the conductor of the orchestra, or fellow musicians in a band (MTR.1.1). When musicians study arrangements for performances such as a concert, they learn to complete the tasks of organizing and displaying the sheet music, attachments for their instruments, and other details with fluency (MTR.3.1).

Meet Eugenia Cheng

Eugenia Cheng, a mathematician and concert pianist, has helped to explain how math and music are connected. Both can involve patterns and ratios. The duration of the notes in a sound creates a rhythmic pattern based on ratios. Different rhythmic patterns can set the mood for a song. According to Eugenia Cheng's research, a rhythmic pattern where the first note takes up $\frac{3}{4}$ of the beat tends to sound more lively, while a rhythmic pattern where the first note takes up $\frac{2}{3}$ of the beat might sound more relaxed.

Paul Crisanti

Grade 7 Unit 2

171

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying graphs that are proportional	Lesson 8	1	MA.7.AR.4.1
2	Determining a constant of proportionality and using it to answer questions	Lesson 9	2	MA.7.AR.4.2, MTR.6.1
3	Creating equations for proportional situations	Lesson 10	2	MA.7.AR.4.4, MTR.7.1
4	Comparing and interpreting graphs of proportional relationships and justifying a claim	Lesson 10	2	MA.7.AR.4.5, MTR.4.1
5	Determining values in a table of a proportional relationship	Lesson 3	2	MA.7.AR.3.2
6	Determining whether or not equations represent a proportional relationship	Lesson 7	2	MA.7.AR.3.2, MTR.3.1
7a	Determining how to label the x-axis and y-axis from a situation and a coordinate pair	Lesson 10	3	MA.7.AR.3.2
7b	Writing an equation to represent a proportional relationship when given a situation and a coordinate pair	Lesson 10	3	MA.7.AR.4.4
7c	Explaining what a point in the form (1, r) means in a proportional relationship	Lesson 10	3	MA.7.AR.4.5, MTR.6.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

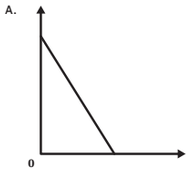
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End-of-Unit Assessment

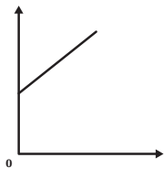
Unit 2

1. Which graph represents a proportional relationship?

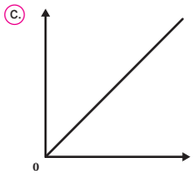
A.



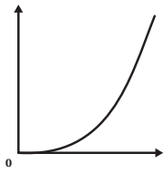
B.



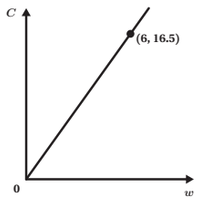
C.



D.



2. This graph shows the cost in dollars, C, of w pounds of blueberries.



The relationship between C and w is proportional.

Select *all* of the true statements.

- ☐ **A.** 2.75 pounds of blueberries cost \$1.
- ☑ **B.** A constant of proportionality is \$2.75.
- ☐ **C.** 5 pounds of blueberries cost \$15.50.
- ☑ **D.** 12 pounds of blueberries cost \$33.
- ☐ **E.** The point (3, 9) is on the graphed line.

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Name: Date: Period:

End-of-Unit Assessment (continued)

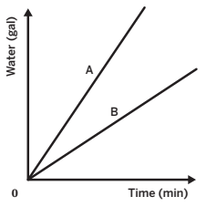
Unit 2

3. Kadeem rode his bike at a constant speed. He rode 1 mile in 5 minutes.

Which equation represents the amount of time in minutes, t, that it took for him to ride a distance of d miles?

A. $t = 5d$ **B.** $t = \frac{1}{5}d$ **C.** $t = d + 4$ **D.** $t = d - 4$

4. These two lines represent the amount of water filling two tanks of the same size over time.



Which tank is filling more quickly? Circle one.

Tank A Tank B Not enough information

Explain or show your thinking.

Explanations vary. I know Tank A is filling more quickly because it's represented by the steeper graph. I can also choose a time and see how much water is in the two tanks at that moment.

5. This table shows the weight of 100 raspberries at a market.

Complete the table so there is a proportional relationship between the number of raspberries and their weight.

Number of Raspberries	Weight (kg)
40	0.16 (or equivalent)
100	0.40
300	1.2 (or equivalent)

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 2

6. The equation $F = \frac{9}{5}C + 32$ represents the relationship between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F.

Is there a proportional relationship between C and F?

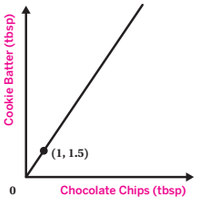
No

Explain or show your thinking.

Explanations vary. The equation $F = \frac{9}{5}C + 32$ is not in the form $y =$ (something)x.

7. A cookie recipe uses 3 tablespoons of cookie batter for every 2 tablespoons of chocolate chips.

This line represents the relationship between tablespoons of cookie batter and tablespoons of chocolate chips in the recipe. The point (1, 1.5) is on the line.



a Label the axes to represent the situation.

b Write an equation for the line.

Use b for the number of tablespoons of cookie batter and c for the number of tablespoons of chocolate chips.

$b = 1.5c$ (or equivalent)

c Explain what the point (1, 1.5) means in this situation.

Explanations vary. The point (1, 1.5) tells me that the recipe calls for 1 tablespoon of chocolate chips for every 1.5 tablespoons of cookie batter. This point gives a unit rate.

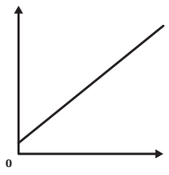
Name: Date: Period:

End-of-Unit Assessment

Unit 2

1. Which graph represents a proportional relationship?

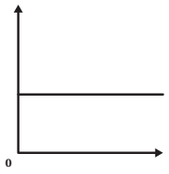
A.



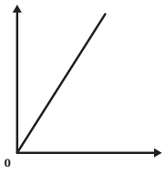
B.



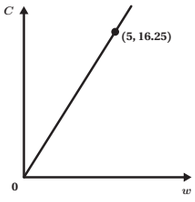
C.



D.



2. This graph shows the cost in dollars, C, of w pounds of strawberries.



The relationship between C and w is proportional.

Select *all* of the true statements.

- ☐ **A.** 3.25 pounds of strawberries cost \$1.
- ☑ **B.** A constant of proportionality is \$3.25.
- ☑ **C.** 4 pounds of strawberries cost \$13.
- ☐ **D.** 10 pounds of strawberries cost \$32.
- ☐ **E.** The point $(3, 10)$ is on the graphed line.

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Name: Date: Period:

End-of-Unit Assessment (continued)

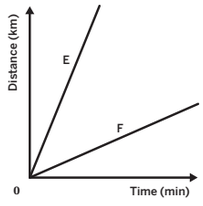
Unit 2

3. Abdullah walked to school at a constant speed. He walked 1 mile in 15 minutes.

Which equation represents the amount of time in minutes, t, that it took for him to walk a distance of d miles?

A. $t = d + 14$ **B.** $t = d - 14$ **C.** $t = 15d$ **D.** $t = \frac{1}{15}d$

4. These two lines represent the distance that two runners ran in a race.



Which runner ran more slowly? Circle one.

Runner E (Runner F) Not enough information

Explain or show your thinking.

Explanations vary. I know that Runner F ran slower because Runner F's graph is less steep. Another way to figure out which person ran slower is to choose a time and see which runner traveled the shorter distance in that time.

5. This table shows the weight of 100 blueberries at a market.

Complete the table so there is a proportional relationship between the number of blueberries and their weight.

Number of Blueberries	Weight (kg)
60	0.36 (or equivalent)
100	0.60
400	2.4 (or equivalent)

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 2

6. The equation $p = 2.2x$ represents the relationship between mass in pounds, p, and mass in kilograms, x.

Is there a proportional relationship between p and x?

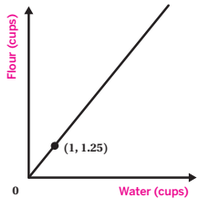
Yes

Explain or show your thinking.

Explanations vary. One way to decide if a relationship is proportional is to look at the equation and see if it has the form $y = (\text{something})x$. The "something" is the constant of proportionality. In this case, 2.22 is the constant of proportionality.

7. A recipe for tortillas uses 5 cups of flour for every 4 cups of water.

This line represents the relationship between the amount of flour and the amount of water needed for this recipe. The point $(1, 1.25)$ is on the line.



a. Label the axes to represent the situation.

b. Write an equation for the line.

Use f for the number of cups of flour and w for the number of cups of water.

$f = 1.25w$ (or equivalent)

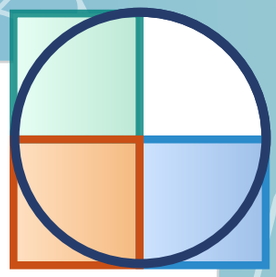
c. Explain what the point $(1, 1.25)$ means in this situation.

Explanations vary. The point $(1, 1.25)$ indicates that the recipe uses 1 cup of water for every 1.25 cups of flour.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Use table to Recognize proportional Relationships. • Determine a constant of proportionality from a table. **(Lessons 1–3)**	5	**Support**	• **Mini Lesson:** Determining the Constant of Proportionality • **Teacher Move:** Consider revisiting Lesson 3 (*Sugar, Spice, and Everything Rice*)
		Strengthen	• **Colorful Challenge:** Lesson 1 (*Paint*)
		Stretch	• **You're invited to explore more:** Lesson 2 (*Balloon Float*)
Sub-Unit 2: • Write and use equations to describe Proportional Relationships. • Determine a constant of proportionality from an equation. **(Lessons 4–7)**	6	**Support**	• **Mini Lesson:** Finding More Than One Equation to Represent Proportional Relationships
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 7 (*All Kinds of Equations*)
		Stretch	• **You're invited to explore more:** Lesson 7 (*All Kinds of Equations*)
Sub-Unit 3: • Use graphs to Recognize proportional Relationships. • Determine a constant of proportionality from a graph. **(Lessons 8–10)**	1, 2	**Support**	• **Teacher Move:** Consider revisiting Lesson 8 (DinoPops) and Unit 2, Lesson 9 (Gallon Challenge)
		Strengthen	• Invite students to create a poster using multiple representations to display their mathematical thinking.
		Stretch	• **You're invited to explore more:** Lesson 8 (*DinoPops*).
	3, 4,7	**Support**	• **Mini-Lesson:** Comparing Proportional Relationships With Graphs • **Teacher Move:** Consider revisiting Lesson 9 (*Gallon Challenge*) and Unit 2, Lesson 10 (*Three Turtles*)
		Strengthen	• **Turtle Challenges:** Lesson 10 (*Three Turtles*)
		Stretch	• **You're invited to explore more:** Lesson 10 (*Three Turtles*)



UNIT 3

Measuring Circles

Students explore the relationships between the radius, diameter, circumference, and area of a circle.

Essential Questions

- How do we measure circles when all of our tools are straight?
- What is π and what does it have to do with circles?
- How can squares help you measure the space inside circles?



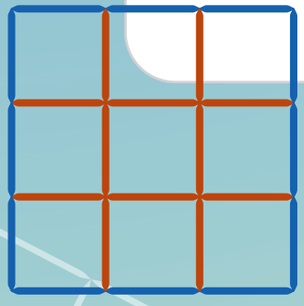
Sub-Unit Goals

Sub-Unit 1

- Use the relationships between the radius, diameter, and circumference of a circle to calculate unknown measurements.

Sub-Unit 2

- Estimate the areas of shapes with curved edges.
- Calculate the areas of circles.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Solve problems involving two-dimensional figures, including circles. **(MA.7.GR.1)**

Benchmarks Addressed in This Unit: MA.7.GR.1.2 MA.7.GR.1.3 MA.7.GR.1.4

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Assess the reasonableness of solutions. **(MTR.6.1)**

Coherence Telling a story within and across grades

In this Unit

Students explore the relationships between the radius, diameter, circumference, and area of a circle. They determine whether these relationships are proportional or not, write equations to model them, and use their equations to solve problems.

Prior Learning

- **Grade 7,** Unit 2: Equations of proportional relationships (**MA.7.AR.4.1, MA.7.AR.4.4)**
- **Grade 6,** Unit 1: Areas of triangles and rectangles **(MA.6.GR.2.1, MA.6.GR.2.2)**
- **Grade 6,** Unit 6: Evaluating expressions **(MA.6.NSO.3.3, MA.6.AR.1.3)**

Future Learning

- **Grade 7,** Unit 7: Volume and surface area of cylinders **(MA.7.GR.2.1, MA.7.GR.2.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 4–5**, students use decomposing and rearranging to find the area of complex shapes. They generate a formula for the area of circles by making sense of several visual representations.

Strategic Competence

In **Lesson 6**, students generate methods for solving multistep problems involving circumference and area.

Procedural Fluency

In **Lessons 3–6**, students build fluency in calculating the circumference of circles and the perimeter of complex shapes that include parts of circles.

Adaptive Reasoning

In **Lesson 6**, students reflect on the precision of their estimates for the area of a circle, determining whether they are greater than or less than the exact area.

Application

In **Lesson 3**, students apply their understanding of proportional relationships to the relationship between the diameter and circumference of a circle.

Productive Disposition

In **Lesson 4**, students use their creativity to create challenges for their classmates and develop their own strategies for calculating circumference.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.GR.2.1 MA.6.GR.2.2 MA.6.NSO.3.3 MA.7.AR.4.4
MTR.2.1 MTR.3.1 MTR.5.1 MTR.7.1

Sub-Unit 1



1 Toothpicks

Perimeter and Proportional Relationships

Examine the relationship between the diagonal length and perimeter of a square in preparation to explore analogous relationships in circles.

Building Toward: MA.7.GR.1.3
MTR.5.1 MTR.6.1

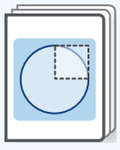


2 Is It a Circle?

Parts of a Circle

Study both examples and non-examples of circles and develop language to describe them precisely, including the terms **radius** and **diameter**.

MA.7.GR.1.3
MTR.4.1



5 Radius Squares

Exploring Circle Area

Use repeated reasoning to make sense of the relationship between radius squares and the area of a circle.

MA.7.GR.1.4
MTR.4.1 MTR.5.1 MTR.6.1



6 Circle Area

Calculating Circle Area

Discover a formula for the relationship between the radius and area of a circle.

MA.7.GR.1.4
MTR.1.1 MTR.3.1 MTR.4.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.GR.1.3 MA.7.GR.1.4 MA.7.AR.4.1
MTR.2.1 MTR.3.1

Pacing: 9 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 6 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment



3 Measuring Around

Estimating and Calculating Circumference

Explore the relationship between a circle's diameter and **circumference**.

MA.7.GR.1.3
MTR.1.1 MTR.4.1 MTR.6.1

Assess and Respond

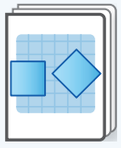


A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.AR.3.2 MA.7.AR.4.1 MA.7.AR.4.2
MA.7.GR.1.3

Sub-Unit 2



4 Area Strategies

Estimating and Calculating Area

Surface strategies to determine the area of complex shapes and estimate the areas of shapes with curved edges.

MA.7.GR.1.2
MTR.1.1 MTR.4.1 MTR.6.1

Pacing Considerations

Lesson 1: This lesson supports students in developing their understanding of key ideas about proportional relationships in geometric figures, which will be addressed in more depth in upcoming lessons.

Lesson 4: The purpose of this lesson is to surface strategies for calculating area from Grade 6. This prepares students for estimating the area of circles in the next lesson. This lesson may be omitted as it mostly addresses standards from a prior course.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know, Activity 1 You're Invited to Explore More Sheet *(one per student, as needed)*

Lesson 2

- **Copy:** Show What You Know
- **Cut:** Activity 2 Sheet *(one per student)*

Lesson 3

- **Copy:** Show What You Know
- **Materials:** a variety of circular objects *(one per pair, if possible)*, measuring tools, string

Lesson 4

- **Copy:** Show What You Know
- **Materials:** coloring tools

Lesson 5

- **Copy:** Show What You Know, Activity 3 Sheet (*one per student)*
- **Cut:** Activity 2 Cards *(one per group)*
- **Materials:** graph paper *(optional)*, scissors, tape

Lesson 6

- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

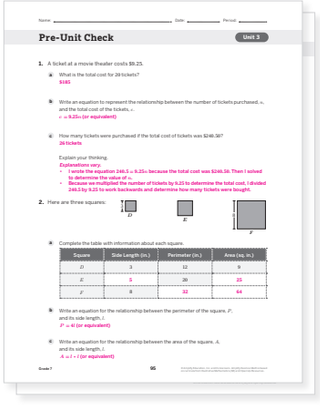
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.GR.2.1, MA.6.GR.2.2, MA.6.NSO.3.3, MA.7.AR.4.4



Pre-Unit Check

Unit 3

1. A ticket at a movie theater costs $9.25.
   a. What is the total cost for 28 tickets?
      $185
   b. Write an equation to represent the relationship between the number of tickets purchased, n, and the total cost of the tickets, c.
      $c = 9.25n$ (or equivalent)
   c. How many tickets were purchased if the total cost of tickets was $240.50?
      26 tickets
      Explain your thinking.
      Explanations vary.
      - I wrote the equation $240.5 = 9.25n$ because the total cost was $240.50. Then I solved to determine the value of n.
      - Because we multiplied the number of tickets by 9.25 to determine the total cost, I divided 240.5 by 9.25 to work backwards and determine how many tickets were bought.
2. Here are three squares.

   a. Complete the table with information about each square.

Square	Side Length (in.)	Perimeter (in.)	Area (sq. in.)
D	3	12	9
E	5	20	25
F	8	32	64

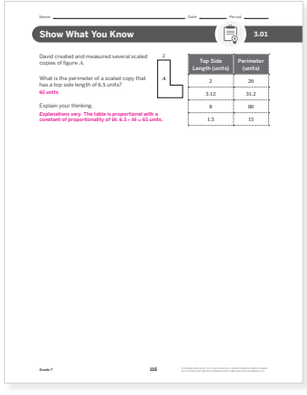
   b. Write an equation for the relationship between the perimeter of the square, P, and its side length, l.
      $P = 4l$ (or equivalent)
   c. Write an equation for the relationship between the area of the square, A, and its side length, l.
      $A = l \cdot l$ (or equivalent)

Grade 7 95

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 1 Show What You Know: MA.7.GR.1.3



Show What You Know

3.01

David created and measured several scaled copies of figure A.

Top Side Length (units)	Perimeter (units)
2	20
3.12	31.2
8	80
1.5	15

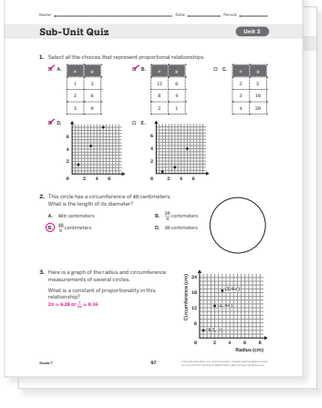
What is the perimeter of a scaled copy that has a top side length of 6.5 units?
65 units
Explain your thinking.
Explanations vary. The table is proportional with a constant of proportionality of 10. $6.5 \cdot 10 = 65$ units.

Grade 7 115

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2, MA.7.GR.1.3



Sub-Unit Quiz

Unit 3

1. Select all the choices that represent proportional relationships.

2. This circle has a circumference of 40 centimeters. What is the length of its diameter?
   A. 40π centimeters
   B. $\frac{20}{\pi}$ centimeters
   C. $\frac{40}{\pi}$ centimeters
   D. 20 centimeters

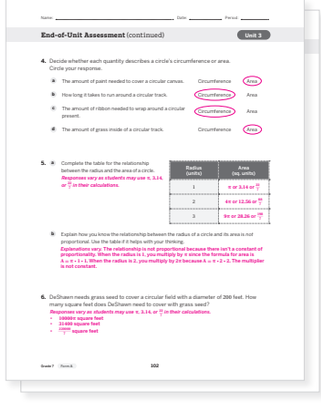
3. Here is a graph of the radius and circumference measurements of several circles. What is a constant of proportionality in this relationship?
   $2\pi \approx 6.28$ or $\frac{1}{2\pi} \approx 0.16$

Grade 7 97

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.GR.1.3, MA.7.GR.1.4, MA.7.AR.4.1



End-of-Unit Assessment (continued)

Unit 3

4. Decide whether each quantity describes a circle's circumference or area. Circle your response.
   a. The amount of paint needed to cover a circular canvas. Circumference (Area)
   b. How long it takes to run around a circular track. (Circumference) Area
   c. The amount of ribbon needed to wrap around a circular present. (Circumference) Area
   d. The amount of grass inside of a circular track. Circumference (Area)
5. a. Complete the table for the relationship between the radius and the area of a circle.
   Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Radius (units)	Area (sq. units)
1	π or 3.14 or $\frac{22}{7}$
2	4π or 12.56 or $\frac{88}{7}$
3	9π or 28.26 or $\frac{198}{7}$

   b. Explain how you know the relationship between the radius of a circle and its area is not proportional. Use the table if it helps with your thinking.
   Explanations vary. The relationship is not proportional because there isn't a constant of proportionality. When the radius is 1, you multiply by π since the formula for area is $A = \pi \cdot r \cdot r$. When the radius is 2, you multiply by 2π because $A = \pi \cdot 2 \cdot 2$. The multiplier is not constant.
6. DeShawn needs grass seed to cover a circular field with a diameter of 200 feet. How many square feet does DeShawn need to cover with grass seed?
   Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.
   - 10000π square feet
   - 31 400 square feet
   - $\frac{220000}{7}$ square feet

Grade 7 Form A 102

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

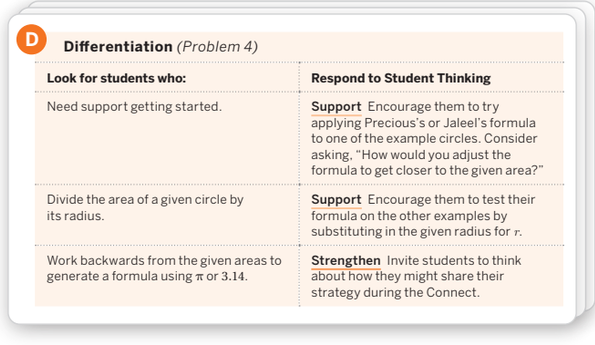
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–3) • Use the relationships between the radius, diameter, and circumference of a circle to calculate unknown measurements.	**Mini-Lesson** • *Determining the Circumference of a Circle*	**Practice** • Lessons 1–3	**You're invited to explore more.** • Lesson 1 (*Toothpicks*) • Lesson 2 (*Is It a Circle?*)
Sub-Unit 2 (Lessons 4–6) • Estimate the areas of shapes with curved edges. • Calculate the areas of circles.	**Mini-Lesson** • *Determining the Area of a Circle*	**Practice** • Lessons 4–6	**You're invited to explore more.** • Lesson 4 (*Area Strategies*)

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 6**, where students use a given area and a radius or diameter of four circles to generalize a formula for calculating the area of any circle.



Differentiation *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try applying Precious's or Jaleel's formula to one of the example circles. Consider asking, "How would you adjust the formula to get closer to the given area?"
Divide the area of a given circle by its radius.	**Support** Encourage them to test their formula on the other examples by substituting in the given radius for r.
Work backwards from the given areas to generate a formula using π or 3.14.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.

Accessibility

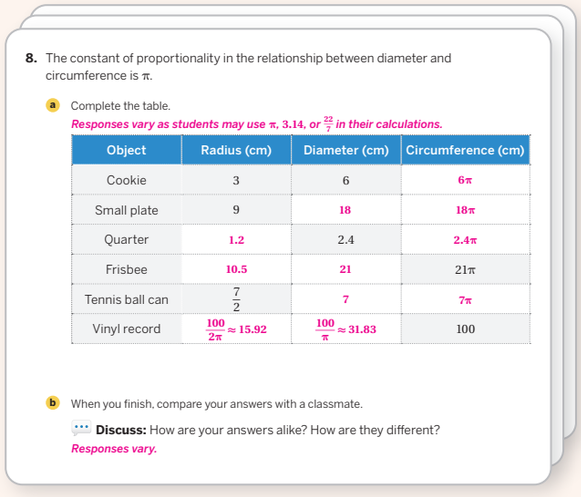
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Strategy Development

Lesson 3: In this lesson, students have opportunities for strategizing and planning when they use the circumference formula to calculate various unknown measurements and choose the best approximation for π in their calculations.



8. The constant of proportionality in the relationship between diameter and circumference is π.

a. Complete the table.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Cookie	3	6	6π
Small plate	9	18	18π
Quarter	1.2	2.4	2.4π
Frisbee	10.5	21	21π
Tennis ball can	$\frac{7}{2}$	7	7π
Vinyl record	$\frac{100}{2\pi} \approx 15.92$	$\frac{100}{\pi} \approx 31.83$	100

b. When you finish, compare your answers with a classmate.

Discuss: How are your answers alike? How are they different?

Responses vary.

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Visual-Spatial Processing

Here are two suggestions in this unit for supporting students with solving abstract problems and applying concepts in new situations.

Lesson 1, Activity 1

A **Accessibility: Visual-Spatial Processing** Invite students to draw or write in their Student Edition or on a printed copy of the visual to support interpreting visual representations.

Lesson 4, Activity 1

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to support making connections between the two shapes.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)	Review Vocabulary	Additional Vocabulary
diameter **radius**	2	circle constant of proportionality	approximation
circumference **pi**	3		

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	2
MLR2: Collect and Display	3 and 6
MLR7: Compare and Connect	4

Sub-Unit Language Goals

- Explain that the relationships between the radius, diameter, and circumference of a circle are proportional and that pi is the constant of proportionality relating circumference and diameter.
- Describe the relationship between the radius of any circle and its area.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 5**, where students estimate the area of circles using radius squares (squares whose side lengths are equal to the radius of the circle). Making sense of this term supports students in eventually generating a formula for determining the area of a circle.

ELL **English Language Learners** Invite students to share what each word in the new term *radius square* reminds them of (e.g., "When I hear the word radius, I think ___." and "When I hear the word square, I think of___."). Then invite students to share what they think it means when the two words are used together. **(Listening and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

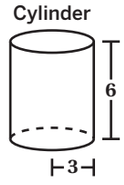
Volume and Surface Area of Cylinders

In this unit, students use the formula for the area of a circle to solve problems. **In Grade 7, Unit 7**, students will use the area of a circle to help them calculate the volume and surface area of cylinders.

FL B.E.S.T. Math Benchmarks:
MA.7.GR.2.1, MA.7.GR.2.3

Example:

The volume of the cylinder is given by the formula $V = B \cdot h$. Since the base of the cylinder is a circle, $V = \pi r^2 \cdot h$. Using the given values for radius and height, $V = \pi \cdot (3)^2 \cdot (6)$. The volume of the cylinder is 54π cubic units.

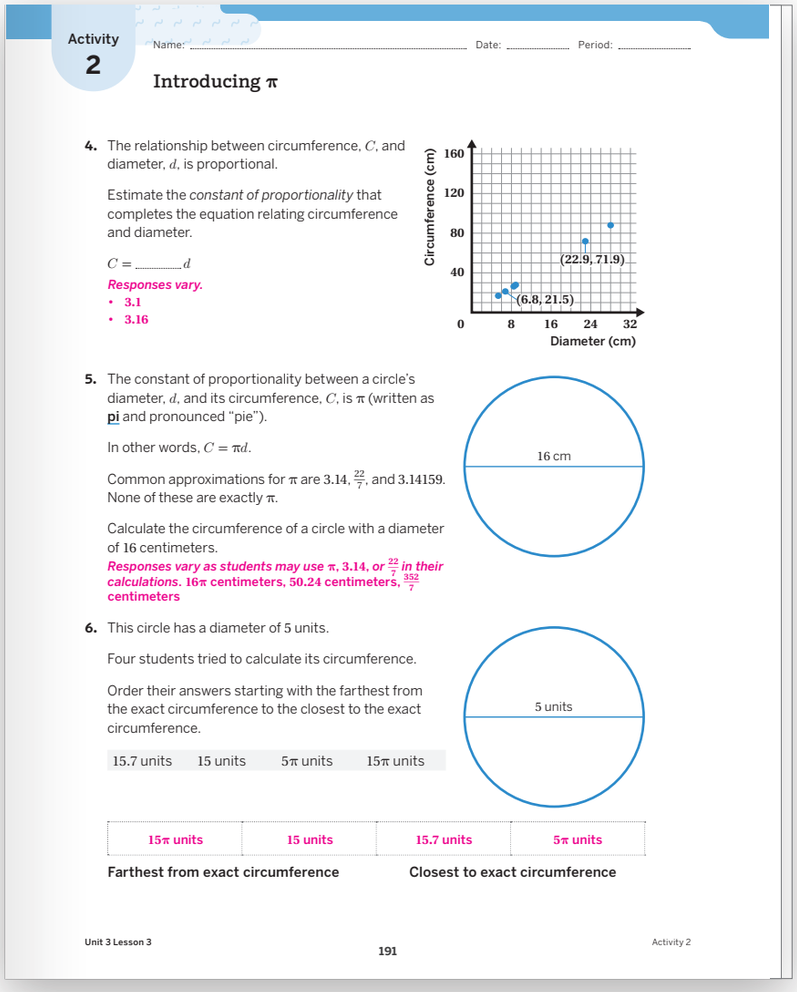


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 3, Activity 2



Activity 2

Name: _____ Date: _____ Period: _____

Introducing π

4. The relationship between circumference, C, and diameter, d, is proportional.

Estimate the *constant of proportionality* that completes the equation relating circumference and diameter.

$C =$ _____ d

Responses vary.
- **3.1**
- **3.16**

5. The constant of proportionality between a circle's diameter, d, and its circumference, C, is π (written as **pi** and pronounced "pie").

In other words, $C = \pi d$.

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. None of these are exactly π.

Calculate the circumference of a circle with a diameter of 16 centimeters.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations. 16π centimeters, 50.24 centimeters, $\frac{352}{7}$ centimeters

6. This circle has a diameter of 5 units.

Four students tried to calculate its circumference.

Order their answers starting with the farthest from the exact circumference to the closest to the exact circumference.

15.7 units	15 units	5π units	15π units

15π units	**15 units**	**15.7 units**	**5π units**

Farthest from exact circumference **Closest to exact circumference**

Unit 3 Lesson 3 191 Activity 2

Context

In this activity, students encounter the circumference and the quantity π for the first time. They make sense of these new ideas by applying their existing knowledge of proportional relationships to the relationship between diameter and circumference.

Try This

Put on your student hat and complete Problems 4 and 5.

Points to Ponder

- What do students need to understand about proportional relationships to support their sense-making around circumference?
- Some students may think that expressions such as 16π have the same value as 16. How might you support students to interpret the π symbol?
- How is the proportional relationship between diameter and circumference related to the work students did with scaled copies in Unit 1?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Writing and using equations of proportional relationships	Lesson 1	1	MA.7.AR.4.4 MTR.7.1
2	Modeling relationships between side length, perimeter, and area of a square	Lessons 1, 4	1	MA.6.GR.2.1,
3	Determining the area of triangles	Lesson 4	1	MA.6.GR.2.1
4	Determining the area of complex polygons on a grid	Lesson 4	1	MA.6.GR.2.2 MTR.3.1
5	Evaluating expressions involving exponents	Lesson 5	1	MA.6.NSO.3.3, MA.6.GR.2.2 MTR.2.1, MTR.5.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 3

1. A ticket at a movie theater costs $9.25.

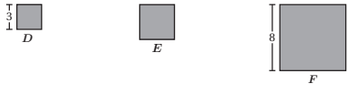
a. What is the total cost for 20 tickets?
$185

b. Write an equation to represent the relationship between the number of tickets purchased, n, and the total cost of the tickets, c.
$c = 9.25n$ (or equivalent)

c. How many tickets were purchased if the total cost of tickets was $240.50?
26 tickets

Explain your thinking.
Explanations vary.
- **I wrote the equation $240.5 = 9.25n$ because the total cost was $240.50. Then I solved to determine the value of n.**
- **Because we multiplied the number of tickets by 9.25 to determine the total cost, I divided 240.5 by 9.25 to work backwards and determine how many tickets were bought.**

2. Here are three squares:



a. Complete the table with information about each square.

Square	Side Length (in.)	Perimeter (in.)	Area (sq. in.)
D	3	12	9
E	**5**	20	**25**
F	8	**32**	**64**

b. Write an equation for the relationship between the perimeter of the square, P, and its side length, l.
$P = 4l$ (or equivalent)

c. Write an equation for the relationship between the area of the square, A, and its side length, l.
$A = l \cdot l$ (or equivalent)

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Name: ..... Date: ..... Period: .....

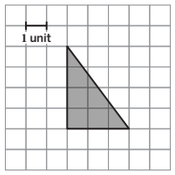
Pre-Unit Check (continued)

Unit 3

3. One formula for the area of a triangle is $A = \frac{1}{2} \cdot b \cdot h$.

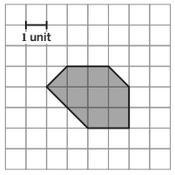
The variable b represents the length of the base of the triangle and h represents its height.

What is the area of this triangle?
6 square units



4. Use any strategy to determine the area of this figure. Draw on the figure if it helps you with your thinking.
9 square inches

Explain your thinking.
Explanations vary.
- **I counted each of the whole number tiles and combined each of the half tiles to form wholes.**
- **I found the area of the large 3-by-4 rectangle and then subtracted the parts that weren't shaded in.**



5. The area of this shape is $4 \cdot 3^2$ square units.

What is another way to write the shape's area?

A. 36 square units
B. 144 square units
C. Neither



Explain your thinking.
Explanations vary. **This is another way of saying 4 copies of a 3-by-3 square. The area of each square is 9 square units, so the total area is $4 \cdot 9 = 36$ square units.**

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

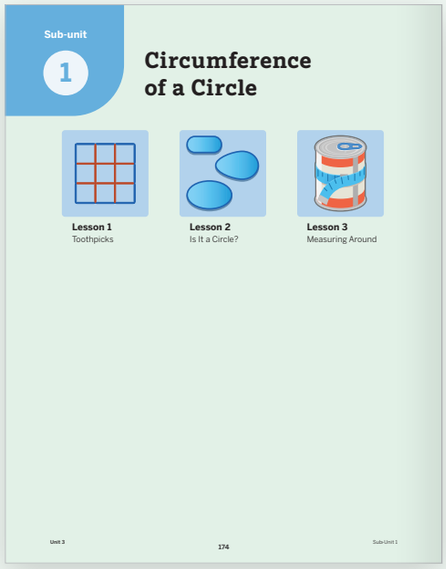
Sub-Unit Goal	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Use the relationships between the radius, diameter, and circumference of a circle to calculate unknown measurements. **(Lessons 1–3)**	**1, 2b**	**Support** • **Mini-Lesson:** *Writing Equations from Descriptions and Tape Diagrams* • **Teacher Move:** Consider revisiting Unit 2, Lesson 5 *(Snapshots)* • **Teacher Move:** Before Lesson 1, invite students to share the strategies they used to determine the total cost in Problem 1a and the number of tickets in Problem 1c.
	2a	**Support** • **Mini-Lesson:** *Solving Problems Involving Area and Perimeter* • **Teacher Move:** During Activity 1 of Lesson 1, invite students to share strategies for determining the perimeter of a shape.
	3, 4	**Support** • **Teacher Move:** Consider revisiting Grade 6, Unit 1, Lesson 4 *(Letters)* and Grade 6, Unit 1, Lesson 2 *(Off the Grid)*
	2c, 5	**Support** • **Mini-Lesson:** *Evaluating Expressions With Exponents* • **Teacher Move:** Consider revisiting Grade 6, Unit 6, Lesson 10 *(Powers)* • **Teacher Move:** During the Lesson 5 Warm-Up, invite students to share expressions for the area of the larger square.

Circumference of a Circle

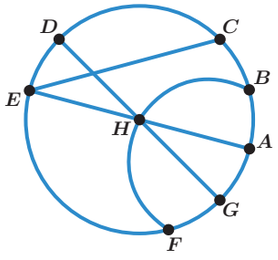
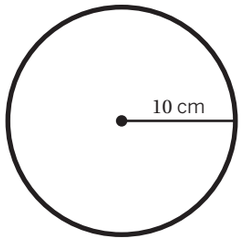
Sub-Unit 1 Goals

- **Goal:** Use the relationships between the radius, diameter, and circumference of a circle to calculate unknown measurements.
- **Language Goal:** Explain that the relationships between the radius, diameter, and circumference of a circle are proportional and that pi is the constant of proportionality relating circumference and diameter.

In this sub-unit, students build on what they know about scaled copies and proportional relationships. Students generate characteristics that define a circle and learn the terms **radius** and **diameter** to describe parts of circles. Students make sense of the relationship between the diameter of a circle and its **circumference**, allowing them to calculate unknown measurements using π.



Math That Matters Most

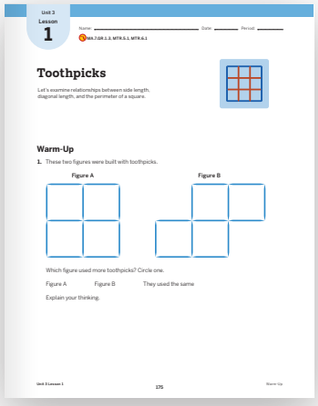
Concept or Skill	For example . . .
Identify the diameter and radius of a circle.	Here is a circle. Point H is the circle's center. Identify a segment that is a radius and a segment that is a diameter. *Segment AH is a radius because it connects the center of the circle to the edge. Segment DG is a diameter because it goes from one side of the circle to the other, through the center.* 
Given radius, diameter, or circumference of a circle, calculate the other two measurements.	What is the diameter of the circle? What is the circumference of the circle? *The diameter of the circle is $10 \cdot 2 = 20$ centimeters because a diameter is always twice the length of the radius. The circumference is 20π centimeters because you calculate circumference by multiplying the diameter by π.* 

Unit 3
Lesson
1

Toothpicks

Perimeter and Proportional Relationships

Let's examine relationships between side length, diagonal length, and the perimeter of a square.


Unit 3
Lesson
1
Name _____ Date _____ Period _____
MA.7.GR.1.3, MTR.5.1, MTR.6.1
Toothpicks
Let's examine relationships between side length, diagonal length, and the perimeter of a square.
Warm-Up
1. These two figures were built with toothpicks.
Figure A
Figure B
Which figure used more toothpicks? Circle one.
Figure A Figure B They used the same
Explain your thinking.
175
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Use proportional relationships to determine missing side lengths, diagonal lengths, and perimeters.
2. **Language Goal:** Explain whether or not the relationship between the length of a side or a diagonal of a polygon and its perimeter is proportional. **(Writing, Speaking, and Listening)**

This lesson builds on what students know about scaled copies and proportional relationships. They examine the relationship between the side length of a square and its perimeter, as well as the relationship between the diagonal length of a square and its perimeter. They discover that both relationships are proportional, which will support them in thinking about the relationship between quantities in circles later in the unit.

Coherence

Prior Learning

In Unit 2, students analyzed proportional relationships and calculated constants of proportionality using tables, graphs, descriptions, and equations. In Unit 1, students studied the characteristics of scaled copies.

Future Learning

In Lesson 3, students will explore the proportional relationship between the diameter of a circle and its circumference.

Connections

- Students use visual models to explore changing the characteristics of shapes and describe patterns. (mathematical)

FL B.E.S.T. Math Benchmarks

Building Toward

MA.7.GR.1.3
Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.GR.1.3
MA.7.AR.4.1
MA.7.AR.4.2

Lesson at a Glance

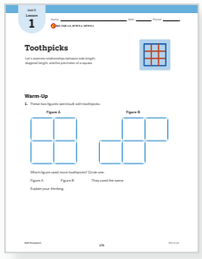
~ 45 min

MA.7.GR.1.3

Warm-Up

Independent | 5 min

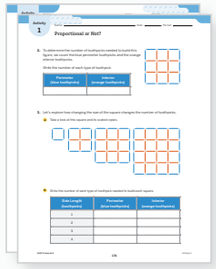
Students surface strategies for counting the toothpicks used to build a figure.



Activity 1

Pairs | 15 min

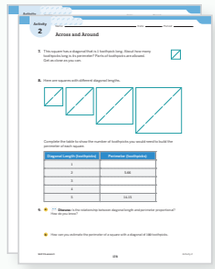
Students analyze the relationships between side length, perimeter, and number of interior toothpicks.



Activity 2

Pairs | 15 min

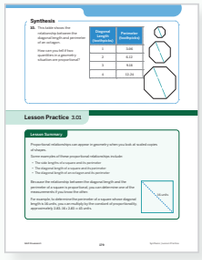
Students use what they know about proportional relationships to calculate the perimeter of a square using the length of its diagonal.



Synthesis

Whole Class | 5 min

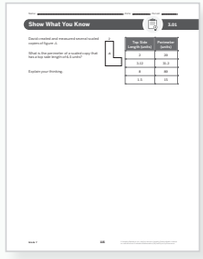
Students synthesize their understanding of the proportional relationship between the diagonal length of an octagon and its perimeter.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using a given side length to determine the perimeter of a scaled copy of a figure.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 You're Invited to Explore More Sheet, one per student (as needed)

Independent | 5 min

Warm-Up

Purpose: Students surface strategies for counting the toothpicks used to build a figure.

1 Launch

To surface connections, consider asking, "What do these figures remind you of?"

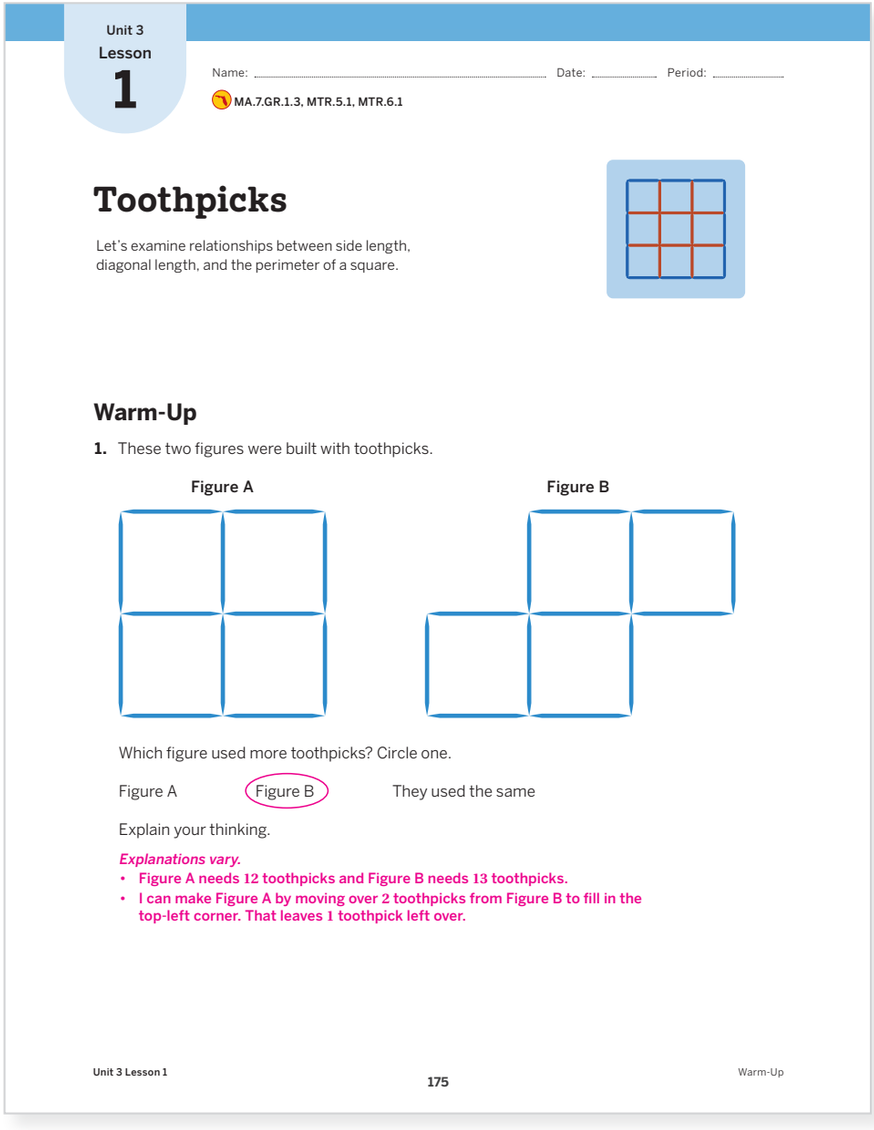
Invite students to make a guess about which figure uses more toothpicks, without counting.

2 Connect

Consider asking, "Was your initial guess correct? What helped you verify your idea or change your mind?" **(MTR.6.1)**

Discuss students' strategies for counting or comparing the number of toothpicks in each figure.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**



Unit 3
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.GR.1.3, MTR.5.1, MTR.6.1

Toothpicks

Let's examine relationships between side length, diagonal length, and the perimeter of a square.

Warm-Up

1. These two figures were built with toothpicks.

Figure A

Figure B

Which figure used more toothpicks? Circle one.

Figure A (Figure B) They used the same

Explain your thinking.

Explanations vary.

- Figure A needs 12 toothpicks and Figure B needs 13 toothpicks.
- I can make Figure A by moving over 2 toothpicks from Figure B to fill in the top-left corner. That leaves 1 toothpick left over.

Unit 3 Lesson 1 175 Warm-Up

Pairs | 15 min

Activity 1 Proportional or Not?

Purpose: Students analyze the relationships between side length, perimeter, and number of interior toothpicks.

Short on time? Consider beginning the discussion of Problem 4 once most students have completed one or two rows of the table in Problem 3.

1 Launch

Consider asking, "How might you group these toothpicks to count them more efficiently?" You can count the blue toothpicks on each side. You can group the orange toothpicks into rows and columns of the same length.

2 Monitor

Encourage students to write expressions for the number of blue and orange toothpicks, and to look for patterns in their table. **(MTR.5.1)**

A **Accessibility: Visual-Spatial Processing** Invite students to draw or write in their Student Edition or on a printed copy of the visual to support interpreting visual representations.

Note: Students do not need to write algebraic expressions or generalize a rule for determining the toothpicks in a square of any size. The focus of this problem should be developing a counting strategy and noticing patterns.

Listen for students who notice that the blue toothpicks can be counted by multiplying the side length by 4 or who use words such as *proportional* or *scale factor* in their partner discussions.

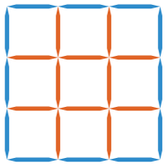
Activity 1 continued >

Activity 1

Name: Date: Period:

Proportional or Not?

2. To determine the number of toothpicks needed to build this figure, we count the blue perimeter toothpicks and the orange interior toothpicks.

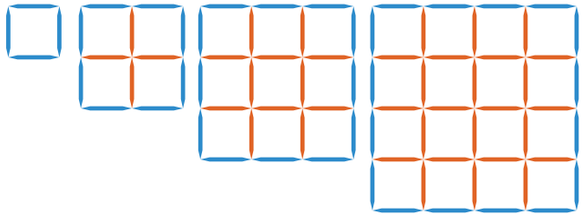


Write the number of each type of toothpick.

Perimeter (blue toothpicks)	Interior (orange toothpicks)
12	12

3. Let's explore how changing the size of the square changes the number of toothpicks.

a. Take a look at this square and its scaled copies.



b. Write the number of each type of toothpick needed to build each square.

Side Length (toothpicks)	Perimeter (blue toothpicks)	Interior (orange toothpicks)
1	4	0
2	8	4
3	12	12
4	16	24

Unit 3 Lesson 1 176 Activity 1

Pairs | 15 min

Activity 1 Proportional or Not? (continued)

Purpose: Students analyze the relationships between side length, perimeter, and number of interior toothpicks.

Short on time? Consider beginning the discussion of Problem 4 once most students have completed one or two rows of the table in Problem 3.

2 Monitor

Select and sequence a variety of explanations to highlight during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Select "Neither" because neither graph includes a point at $(0, 0)$.	**Support** Consider asking, "What would a point at $(0, 0)$ represent in these situations? Could we include that point on the graph?"
Say the graph of side length and perimeter is proportional because the points form a straight line.	**Strengthen** Invite these students to think about how they might share their thinking during the Connect.
Calculate a constant of proportionality for the relationship between side length and perimeter.	**Stretch** Consider asking, "Do you think there is a proportional relationship between side length and perimeter in other shapes, too?"

Look for student explanations that reference the number of equal sides in the square or the multiplicative relationship between side length and perimeter.

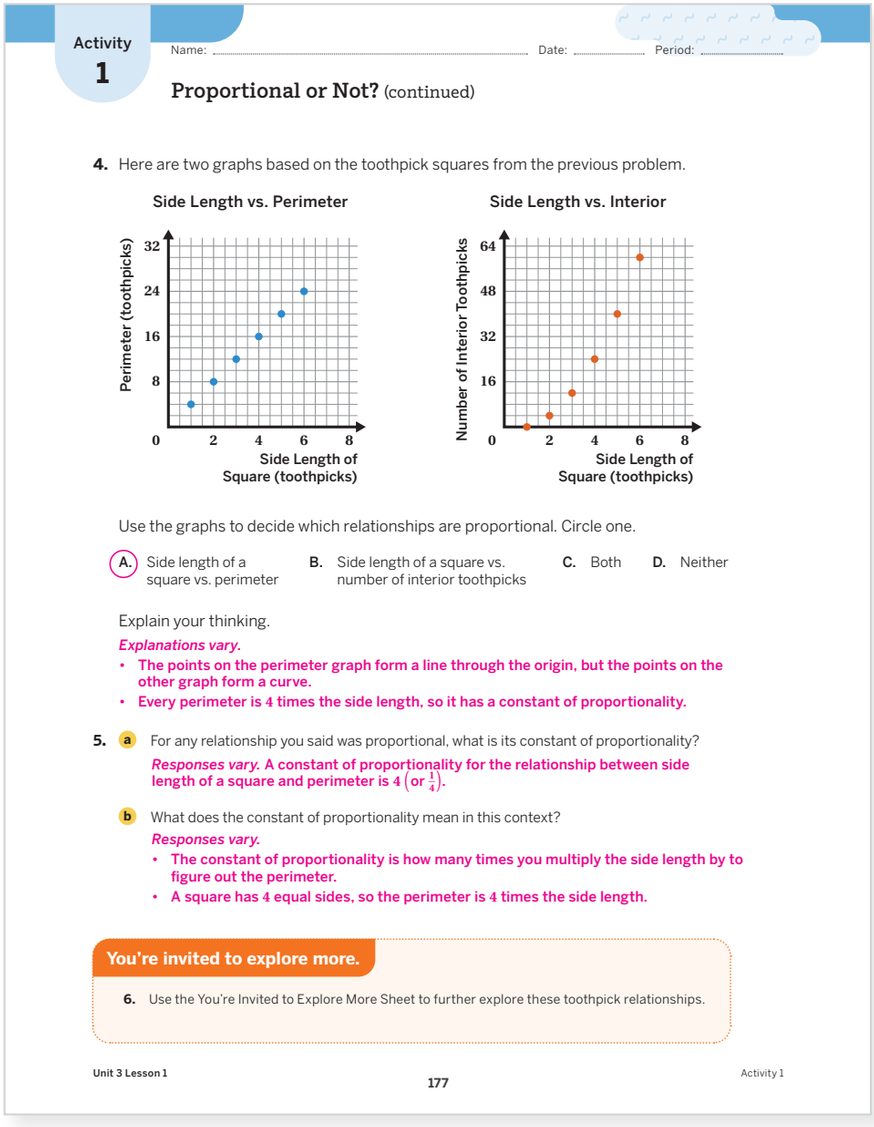
D Differentiation: You're invited to explore more. **(Stretch)** Invite students who would like to explore the relationships between side length, perimeter, and interior toothpicks further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their responses and reasoning.

Consider asking:
- "Why might someone think that both relationships are proportional?"
- "How would you determine the perimeter of a square with 100 toothpicks on each side?" **(MTR.5.1)**

Note: If time allows, consider inviting students to share the constant of proportionality they identified and what it represents in this context.



Activity 1

Name: Date: Period:

Proportional or Not? (continued)

4. Here are two graphs based on the toothpick squares from the previous problem.

Side Length vs. Perimeter

Side Length vs. Interior

Use the graphs to decide which relationships are proportional. Circle one.

A. Side length of a square vs. perimeter
B. Side length of a square vs. number of interior toothpicks
C. Both
D. Neither

Explain your thinking.

Explanations vary.
- The points on the perimeter graph form a line through the origin, but the points on the other graph form a curve.
- Every perimeter is 4 times the side length, so it has a constant of proportionality.

5. a. For any relationship you said was proportional, what is its constant of proportionality?

Responses vary. A constant of proportionality for the relationship between side length of a square and perimeter is 4 (or $\frac{1}{4}$).

b. What does the constant of proportionality mean in this context?

Responses vary.
- The constant of proportionality is how many times you multiply the side length by to figure out the perimeter.
- A square has 4 equal sides, so the perimeter is 4 times the side length.

You're invited to explore more.

6. Use the You're Invited to Explore More Sheet to further explore these toothpick relationships.

Unit 3 Lesson 1 177 Activity 1

Pairs | 15 min

Activity 2 Across and Around

Purpose: Students use what they know about proportional relationships to calculate the perimeter of a square using the length of its diagonal.

Short on time? Consider limiting the time students spend on Problem 8.

1 Launch

Direct student's attention to the square with a diagonal length of 1 and consider asking, "Do you think the perimeter will be more or less than 4 toothpicks long?"

Invite students to make an estimate and share their estimate with a peer. If estimates differ, students should justify their reasoning and adapt their answer as needed. **(MTR.6.1)**

Note: Students are not expected to calculate the exact value $2\sqrt{2}$. This will be discussed in Grade 8.

2 Monitor

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to use the perimeter for a square with a diagonal length of 1 to make an estimate.
Use guess and check to fill in the table.	**Support** Celebrate their persistence. Consider asking, "What strategies are you using to decide on your first estimate for each perimeter?"
Multiply each diagonal length by the same value to calculate the corresponding perimeter.	**Strengthen** Consider asking, "Is this a proportional relationship? How can you tell?"

Listen for students who use multiplication or division to check for proportionality.

ELL English Language Learners Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the course such as *constant of proportionality*. **(Speaking and Listening)**

Capture student explanations to share during the Connect.

3 Connect

Share the captured explanations and consider asking:

- "How did you decide whether the relationship was proportional or not?"
- "How can knowing a relationship is proportional help you estimate the perimeter with 100 toothpicks? 200 toothpicks? Any number?"

Activity 2

Name: Date: Period:

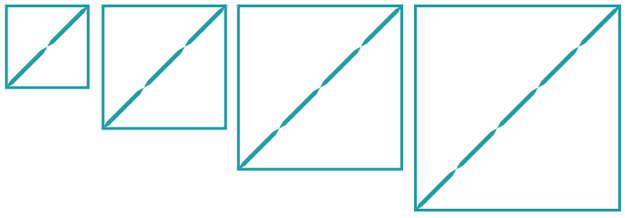
Across and Around

7. This square has a diagonal that is 1 toothpick long. About how many toothpicks long is its perimeter? Parts of toothpicks are allowed. Get as close as you can.



Responses vary. ≈ 2.83

8. Here are squares with different diagonal lengths.



Complete the table to show the number of toothpicks you would need to build the perimeter of each square.

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	2.83
2	5.66
3	8.49
4	11.32
5	14.15

9. a. **Discuss:** Is the relationship between diagonal length and perimeter proportional? How do you know?

Yes. *Responses vary.* The relationship is proportional because I can multiply each diagonal length by the same value, 2.83, to get the number of toothpicks needed for the perimeter.

b. How can you estimate the perimeter of a square with a diagonal of 100 toothpicks.

Responses vary. I can multiply 100 by the constant of proportionality, which is 2.83. This means there will be about $2.83 \cdot 100 = 283$ toothpicks along the perimeter when the diagonal is 100 toothpicks.

Unit 3 Lesson 1 178 Activity 2

Key Takeaway: Some relationships in scaled copies of geometric figures are proportional. For example, there is a proportional relationship between the length of a diagonal of a square and the perimeter of a square.

Whole Class | 5 min

Synthesis

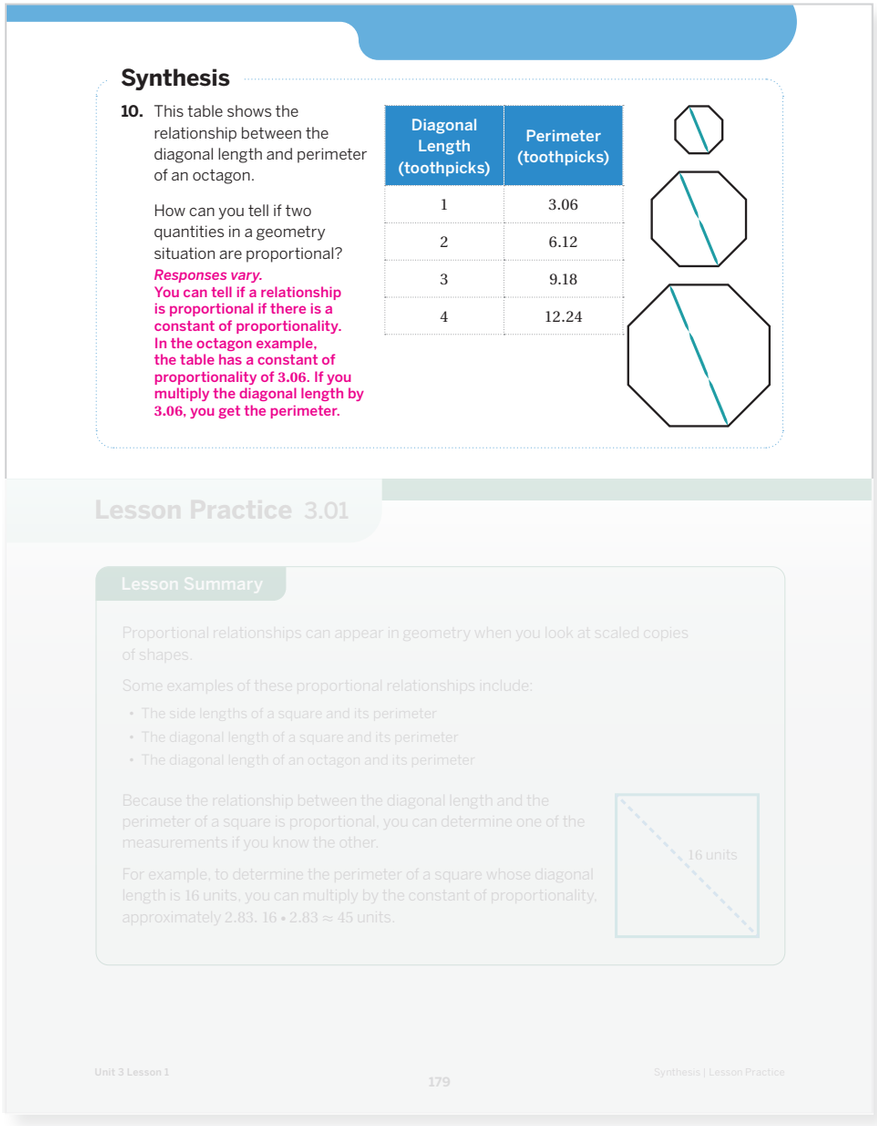
Purpose: Students synthesize their understanding of the proportional relationship between the diagonal length of an octagon and its perimeter.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Determining that the table has a constant of proportionality means the relationship between the diagonal length and the perimeter is proportional.
- Multiplying each diagonal length by the same number (3.06) to determine the perimeter.
- Lengths in scaled copies are proportional, and perimeter is a measurement of length.

Note: If time allows, invite students to share with a partner what makes sense about each explanation and what questions they still have.



Synthesis

10. This table shows the relationship between the diagonal length and perimeter of an octagon.

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	3.06
2	6.12
3	9.18
4	12.24

How can you tell if two quantities in a geometry situation are proportional?

Responses vary.
You can tell if a relationship is proportional if there is a constant of proportionality. In the octagon example, the table has a constant of proportionality of 3.06. If you multiply the diagonal length by 3.06, you get the perimeter.

Lesson Practice 3.01

Lesson Summary

Proportional relationships can appear in geometry when you look at scaled copies of shapes.

Some examples of these proportional relationships include:

- The side lengths of a square and its perimeter
- The diagonal length of a square and its perimeter
- The diagonal length of an octagon and its perimeter

Because the relationship between the diagonal length and the perimeter of a square is proportional, you can determine one of the measurements if you know the other.

For example, to determine the perimeter of a square whose diagonal length is 16 units, you can multiply by the constant of proportionality, approximately 2.83. $16 \cdot 2.83 \approx 45$ units.

16 units

Unit 3 Lesson 1 179 Synthesis | Lesson Practice

Lesson Takeaway: We can find examples of proportional relationships in geometry. In scaled copies of a figure, there is a proportional relationship between diagonal length and perimeter.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using a given side length to determine the perimeter of a scaled copy of a figure.

Today's Goals

1. **Goal:** Use proportional relationships to determine missing side lengths, diagonal lengths, and perimeters.
2. **Language Goal:** Explain whether or not the relationship between the length of a side or a diagonal of a polygon and its perimeter is proportional. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.01

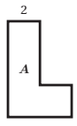
David created and measured several scaled copies of figure A.

What is the perimeter of a scaled copy that has a top side length of 6.5 units?

65 units

Explain your thinking.

Explanations vary. The table is proportional with a constant of proportionality of 10. $6.5 \cdot 10 = 65$ units.



Top Side Length (units)	Perimeter (units)
2	20
3.12	31.2
8	80
1.5	15

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify proportional relationships:

» **Respond to Student Thinking:**

- Use the ***Determining the Constant of Proportionality*** **Mini-Lesson** from Unit 2, Lesson 3.
- Revisit Unit 2, Lesson 3, Activity 1 Sugar, Spice, and Everything Rice).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

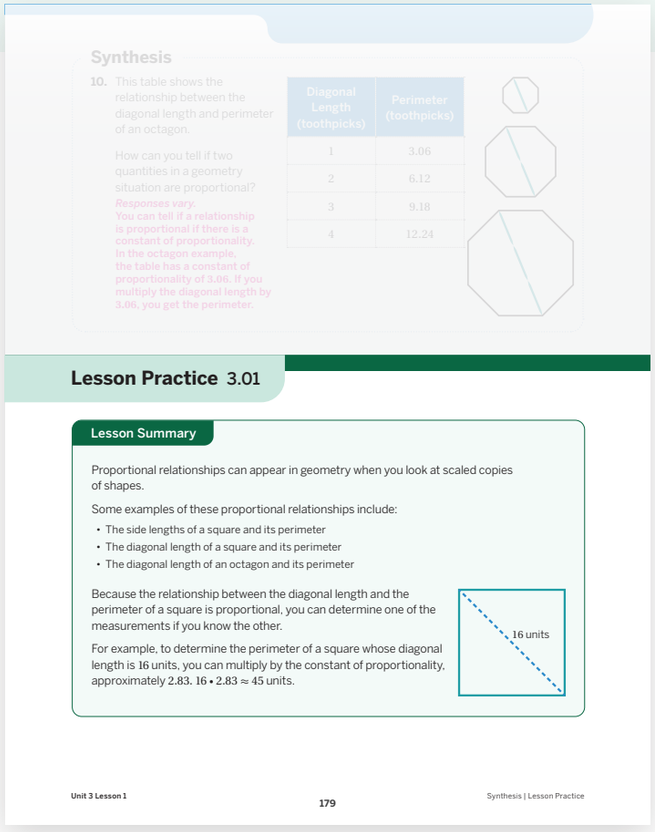
Professional Learning

Consider the next several lessons you will teach. How will students' work today in studying the relationship between diagonal length and perimeter prepare them for studying the relationship between diameter and circumference in Lesson 3? What are ways that you might incorporate students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. This table shows the relationship between the diagonal length and perimeter of an octagon.

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	3.06
2	6.12
3	9.18
4	12.24

How can you tell if two quantities in a geometry situation are proportional?

Responses vary. You can tell if a relationship is proportional if there is a constant of proportionality. In the octagon example, the table has a constant of proportionality of 3.06. If you multiply the diagonal length by 3.06, you get the perimeter.

Lesson Practice 3.01

Lesson Summary

Proportional relationships can appear in geometry when you look at scaled copies of shapes.

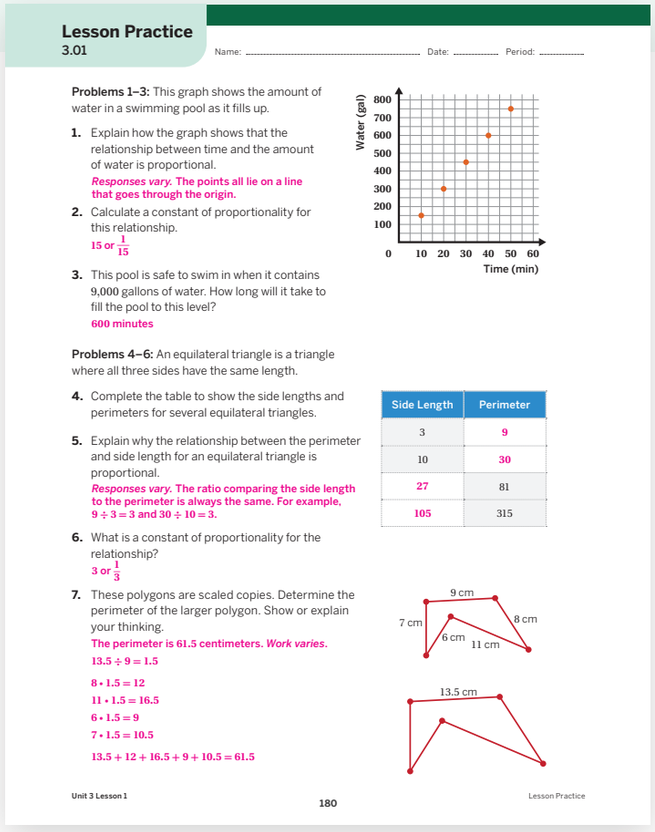
Some examples of these proportional relationships include:

- The side lengths of a square and its perimeter
- The diagonal length of a square and its perimeter
- The diagonal length of an octagon and its perimeter

Because the relationship between the diagonal length and the perimeter of a square is proportional, you can determine one of the measurements if you know the other.

For example, to determine the perimeter of a square whose diagonal length is 16 units, you can multiply by the constant of proportionality, approximately 2.83. $16 \cdot 2.83 \approx 45$ units.

16 units



Lesson Practice 3.01

Name: Date: Period:

Problems 1–3: This graph shows the amount of water in a swimming pool as it fills up.

1. Explain how the graph shows that the relationship between time and the amount of water is proportional.
   Responses vary. The points all lie on a line that goes through the origin.
2. Calculate a constant of proportionality for this relationship.
   15 or $\frac{1}{15}$
3. This pool is safe to swim in when it contains 9,000 gallons of water. How long will it take to fill the pool to this level?
   600 minutes

Problems 4–6: An equilateral triangle is a triangle where all three sides have the same length.

4. Complete the table to show the side lengths and perimeters for several equilateral triangles.

Side Length	Perimeter
3	9
10	30
27	81
105	315

5. Explain why the relationship between the perimeter and side length for an equilateral triangle is proportional.
   Responses vary. The ratio comparing the side length to the perimeter is always the same. For example, $9 \div 3 = 3$ and $30 \div 10 = 3$.
6. What is a constant of proportionality for the relationship?
   3 or $\frac{1}{3}$
7. These polygons are scaled copies. Determine the perimeter of the larger polygon. Show or explain your thinking.
   The perimeter is 61.5 centimeters. *Work varies.*
   $13.5 \div 9 = 1.5$
   $8 \cdot 1.5 = 12$
   $11 \cdot 1.5 = 16.5$
   $6 \cdot 1.5 = 9$
   $7 \cdot 1.5 = 10.5$
   $13.5 + 12 + 16.5 + 9 + 10.5 = 61.5$

Lesson Practice 3.01

Name: Date: Period:

FAST Practice

8. Select *all* of the coordinate pairs that could be part of a proportional relationship with a constant of proportionality of 4.
   - ☑ A. (3, 12)
   - ☐ B. (6, 10)
   - ☐ C. (4, 8)
   - ☑ D. $\left(\frac{1}{2}, 2\right)$
   - ☑ E. (3.5, 14)

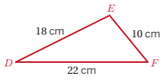
Spiral Review

Problems 9–10: Here are two recipes for making sparkling lemonade. For each recipe, determine how many tablespoons of lemonade mix are required *per cup* of sparkling water. Show or explain your thinking.

9. 4 tablespoons of lemonade mix and 12 cups of sparkling water
   $\frac{1}{3}$ tablespoons of lemonade mix per cup of sparkling water. $\frac{4}{12} = \frac{1}{3}$

10. 4 tablespoons of lemonade mix and 6 cups of sparkling water
    $\frac{2}{3}$ tablespoons of lemonade mix per cup of sparkling water. $\frac{4}{6} = \frac{2}{3}$

11. Here is triangle DEF and its dimensions. Which measurements represent the dimensions of a triangle that is a scaled copy of triangle DEF?



- A. 20 cm, 12 cm, 24 cm
- B. 9 cm, 5 cm, 10 cm
- C. 27 cm, 16 cm, 33 cm
- (D.) 13.5 cm, 7.5 cm, 16.5 cm

Lesson Practice Item Analysis

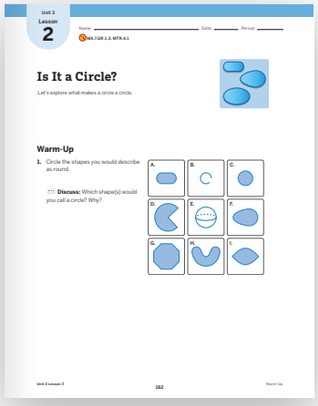
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.GR.1.3*
	6	1	MA.7.GR.1.3*
	7	2	MA.7.GR.1.3*
FAST Practice	8	2	MA.7.GR.1.3*
Spiral Review			
	9–10	1	MA.7.AR.3.2
	11	2	MA.7.GR.1.5

*This problem builds toward the standard shown.

Unit 3

Lesson

2


Unit 3
Lesson
2
Name _____ Date _____ Period _____
MA.7.GR.1.3, MTR.4.1
Is It a Circle?
Let's explore what makes a circle a circle.
Warm-Up
1. Circle the shapes you would describe as round.
Discuss: Which shape(s) would you call a circle? Why?
A.
B.
C.
D.
E.
F.
G.
H.
I.
Unit 3 Lesson 2
182
Warm-Up

Is It a Circle?

Parts of a Circle

Let's explore what makes a circle a circle.

Learning Outcomes

Today's Goals

1. **Goal:** Identify the *diameter* and *radius* of a circle and explain how they are related.
2. **Language Goal:** Describe the properties of circles using terms such as *center*, *radius*, and *diameter*. **(Reading, Writing, Speaking, and Listening)**

Students study both examples and non-examples of circles and develop language to describe them precisely. They begin by using informal words such as "round" and build towards defining the terms **radius** and **diameter**. Students also study the relationship between radius and diameter, noticing that the diameter is always twice the length of the radius. **(MTR.3.1)**

Coherence

Prior Learning

In Kindergarten, students named shapes and described objects in the real world using the term circle.

Future Learning

In Lesson 3, students will determine the circumference of a circle given the radius or diameter.

Connections

- Students recognize the importance of common vocabulary when communicating with others about parts of a circle. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.3
Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.1	MA.912.GR.7.2

Vocabulary

New Vocabulary

diameter, radius

Review Vocabulary

circle

Lesson at a Glance

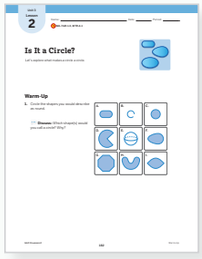
~ 45 min

MA.7.GR.1.3

Warm-Up

Pairs | 5 min

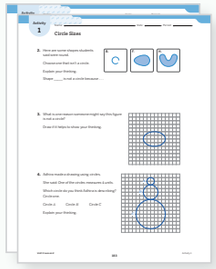
Students use informal language to describe shapes, justifying whether the shapes are or are not *circles*.



Activity 1

Pairs | 15 min

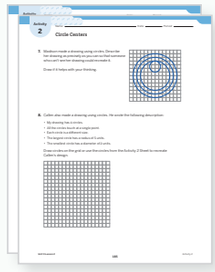
Students use the terms **radius** and **diameter** to describe the size of a circle.
Routine: Think-Pair-Share



Activity 2

Pairs | 15 min

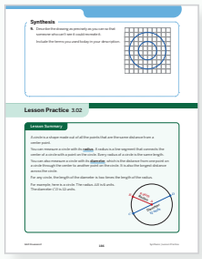
Students use precise language to describe a drawing made up of circles and create a drawing to match a description.



Synthesis

Whole Class | 5 min

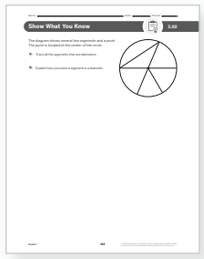
Students synthesize their understanding of measuring and describing a circle using its *radius* and diameter.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the diameter of a circle.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student, pre-cut

Pairs | 5 min

Warm-Up

Purpose: Students use informal language to describe shapes, justifying whether the shapes are or are not *circles*.

1 Launch

To encourage playfulness, consider asking:

- "What do these shapes remind you of?"
- "What would you call each of these shapes?"

Invite students to select as many or as few round shapes as they like and to prepare to justify their choices.

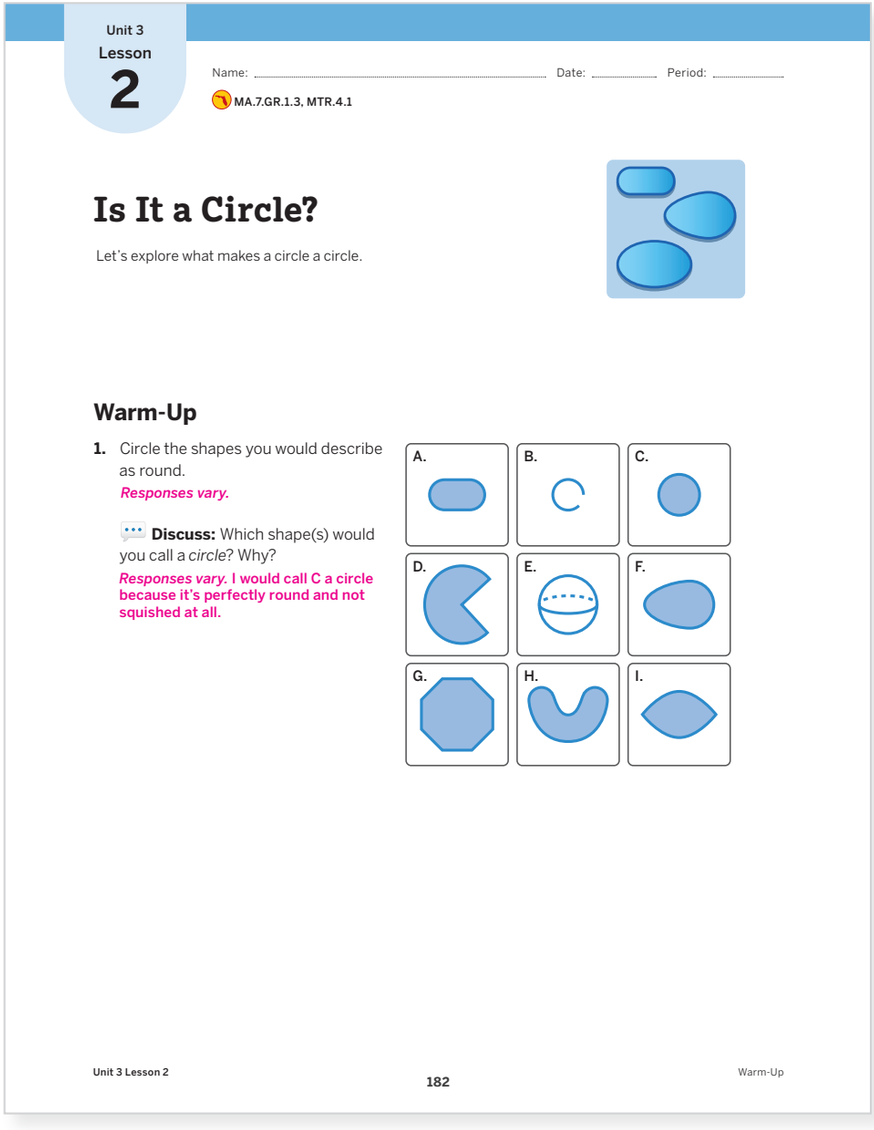
ELL **English Language Learners** Review examples of objects in real life that are round before inviting students to select round shapes. **(Reading and Listening)**

2 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Consider displaying an anonymized student response and asking:

- "How do you think this student would define the word *round*?"
- "Would you call any of these shapes a *circle*? Why or why not?"



Unit 3
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.GR.1.3, MTR.4.1

Is It a Circle?

Let's explore what makes a circle a circle.

Warm-Up

1. Circle the shapes you would describe as round.

Responses vary.

Discuss: Which shape(s) would you call a *circle*? Why?

Responses vary. I would call C a circle because it's perfectly round and not squished at all.

A. B. C. D. E. F. G. H. I.

Unit 3 Lesson 2 182 Warm-Up

Pairs | 15 min

Activity 1 Circle Sizes

Purpose: Students use the terms **radius** and **diameter** to describe the size of a circle.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about the characteristics of circles. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class. **(MTR.4.1)**

2 Monitor

Encourage students to think back to how they described circles and other round shapes during the Warm-Up, to support them in attending to precision in their language.

Listen for students who notice that the height and width of the shape are not equal.

Pause briefly to create an anchor chart showing the language students are using to describe circles and non-circles. If it does not come up naturally, consider sharing that every circle has a *center* that is the same distance from every point on the circle.

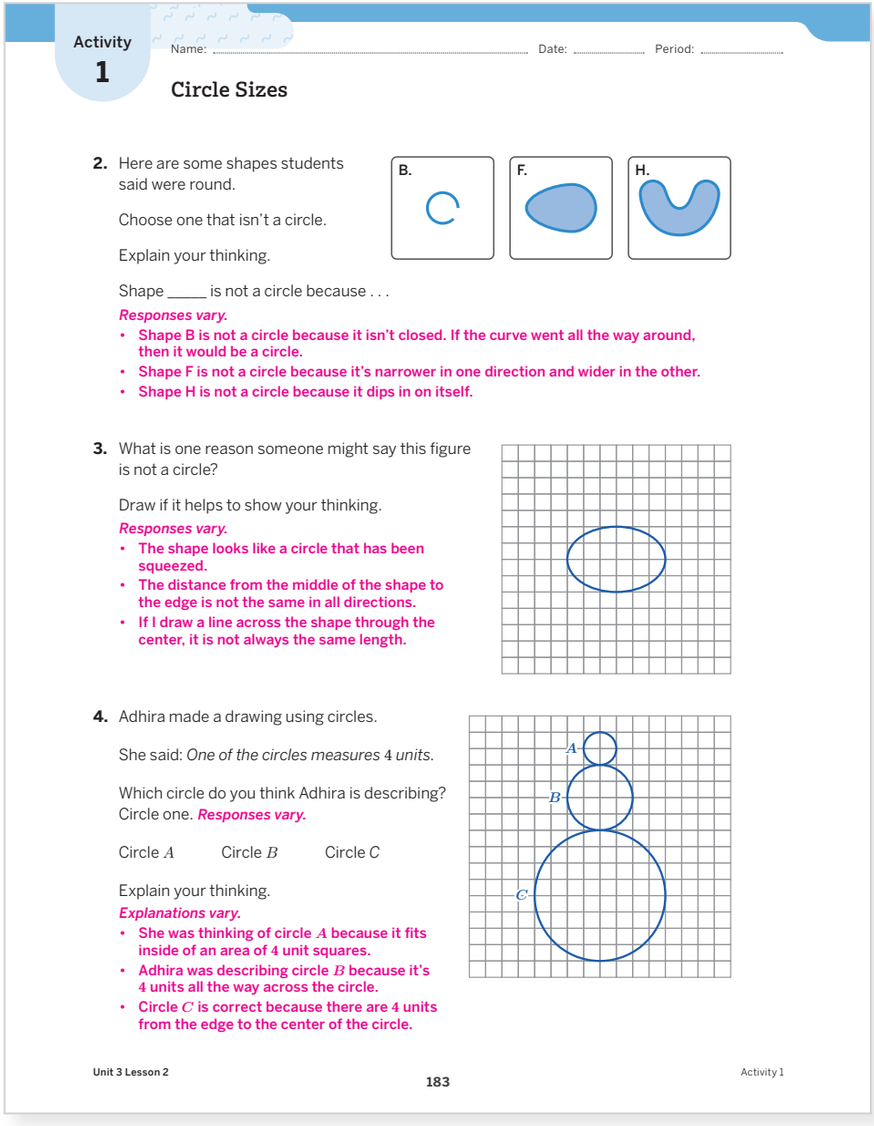
Listen for informal language students use to describe the *radius* and *diameter* of the circles, such as *halfway across, all the way across, from the middle to the edge*, etc.

D **Differentiation: (Stretch)** Encourage students who finish Problem 4 early to discuss with a partner how Adhira might have been thinking about each of the three circles.

Consider asking:

- "Do you think that Adhira was measuring across the circle, up and down, or diagonally?"
- "What could Adhira have been thinking about if she was describing Circle A?"
- "What about Circle B?"
- "What about Circle C?" **(MTR.4.1)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Circle Sizes

2. Here are some shapes students said were round.

Choose one that isn't a circle.

Explain your thinking.

B. F. H.

Shape _____ is not a circle because . . .

Responses vary.

- Shape B is not a circle because it isn't closed. If the curve went all the way around, then it would be a circle.
- Shape F is not a circle because it's narrower in one direction and wider in the other.
- Shape H is not a circle because it dips in on itself.

3. What is one reason someone might say this figure is not a circle?

Draw if it helps to show your thinking.

Responses vary.

- The shape looks like a circle that has been squeezed.
- The distance from the middle of the shape to the edge is not the same in all directions.
- If I draw a line across the shape through the center, it is not always the same length.

4. Adhira made a drawing using circles.

She said: *One of the circles measures 4 units.*

Which circle do you think Adhira is describing? Circle one. *Responses vary.*

Circle A Circle B Circle C

Explain your thinking.

Explanations vary.

- She was thinking of circle A because it fits inside of an area of 4 unit squares.
- Adhira was describing circle B because it's 4 units all the way across the circle.
- Circle C is correct because there are 4 units from the edge to the center of the circle.

Unit 3 Lesson 2 183 Activity 1

Pairs | 15 min

Activity 1 Circle Sizes (continued)

Purpose: Students use the terms **radius** and **diameter** to describe the size of a circle.

2 Monitor

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on radius first, labeling the radius of each circle and filling in that column of the table. Then, repeat with diameter.
Think the radius and diameter of Circle A are both 2 units.	**Support** Invite them to draw the radius and diameter of the circle. Consider asking, "How are these lengths related?"
Use a variety of language to describe the diameter and radius of the circles.	**Strengthen** Invite students to think about how they might share their descriptions during the Connect. **Stretch** Consider asking, "Do the radius and diameter always need to be measured horizontally?"

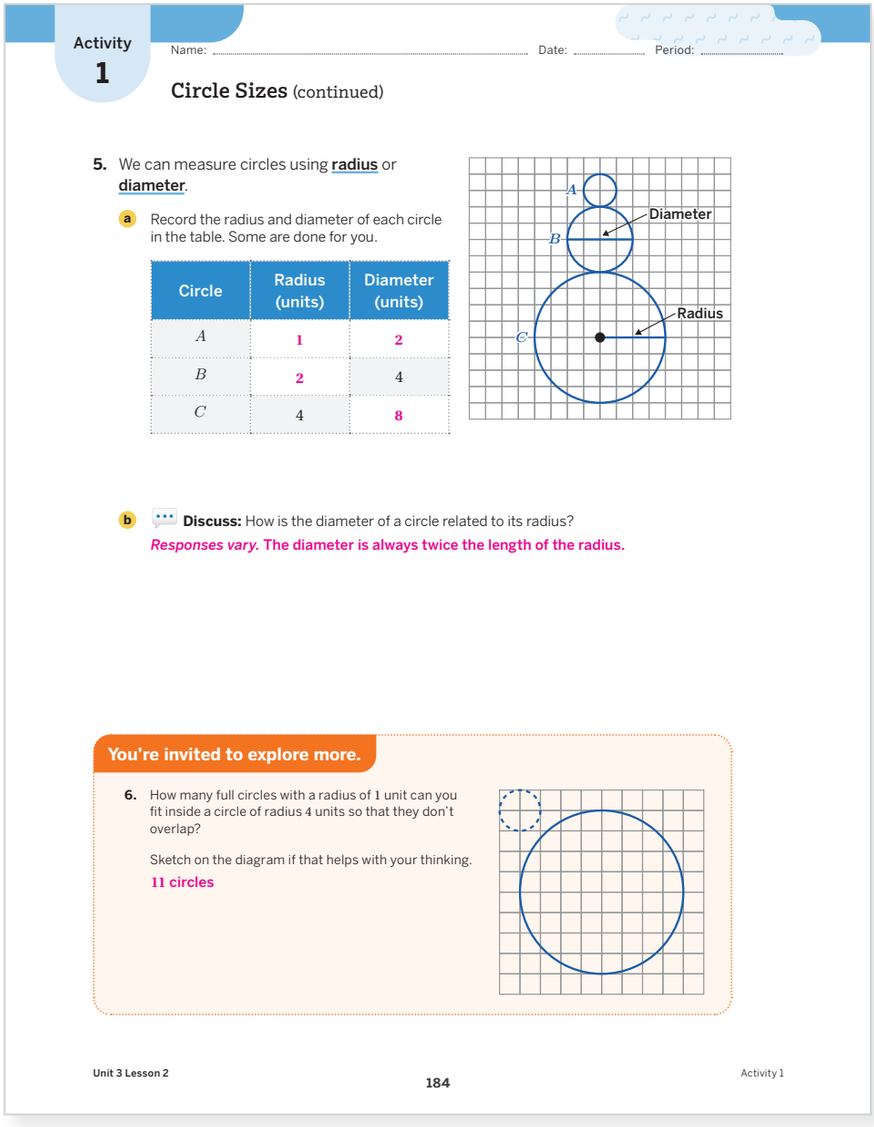
D Differentiation: You're invited to explore more. **(Stretch)** Invite students who would like to explore radius and area of circles further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Create class definitions of the words **radius** and **diameter**. To help generate student language, consider asking:

- "Where can the radius be measured on a circle?"
- "Where can the diameter be measured on a circle?"
- "What is the relationship between these two measurements?"

ELL English Language Learners Use gestures to illustrate the meaning of the terms *radius* and *diameter*. **(Reading and Listening)**



Activity 1

Name: Date: Period:

Circle Sizes (continued)

5. We can measure circles using **radius** or **diameter**.

a. Record the radius and diameter of each circle in the table. Some are done for you.

Circle	Radius (units)	Diameter (units)
A	1	2
B	2	4
C	4	8

b. **Discuss:** How is the diameter of a circle related to its radius?

Responses vary. The diameter is always twice the length of the radius.

You're invited to explore more.

6. How many full circles with a radius of 1 unit can you fit inside a circle of radius 4 units so that they don't overlap?

Sketch on the diagram if that helps with your thinking.

11 circles

Unit 3 Lesson 2 184 Activity 1

Key Takeaway: You can measure a *circle* by its **diameter** or **radius**. The radius is the distance from the center of the circle to the edge. The diameter is the distance across the circle from edge to edge, through the center. The diameter is always twice the length of the radius.

Pairs | 15 min

Activity 2 Circle Centers

Purpose: Students use precise language to describe a drawing made up of circles and create a drawing to match a description.

Short on time? Consider omitting the routine **MLR1: Stronger and Clearer Each Time** or limiting students to one round of feedback.

1 Launch

Direct students' attention to Madison's drawing and invite students to informally describe what they see.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to write a first draft, meet with 1–2 different partners, give feedback on each other's drafts, and then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

2 Monitor

Distribute the Activity 2 Sheet to students and invite them to use the circles to explore possible recreations of Callen's drawing.

A **Accessibility: Executive Functioning** Chunk this task into smaller, more manageable parts by encouraging students to read the bullet points one at a time, adjusting their drawing at each step to reflect the new information.

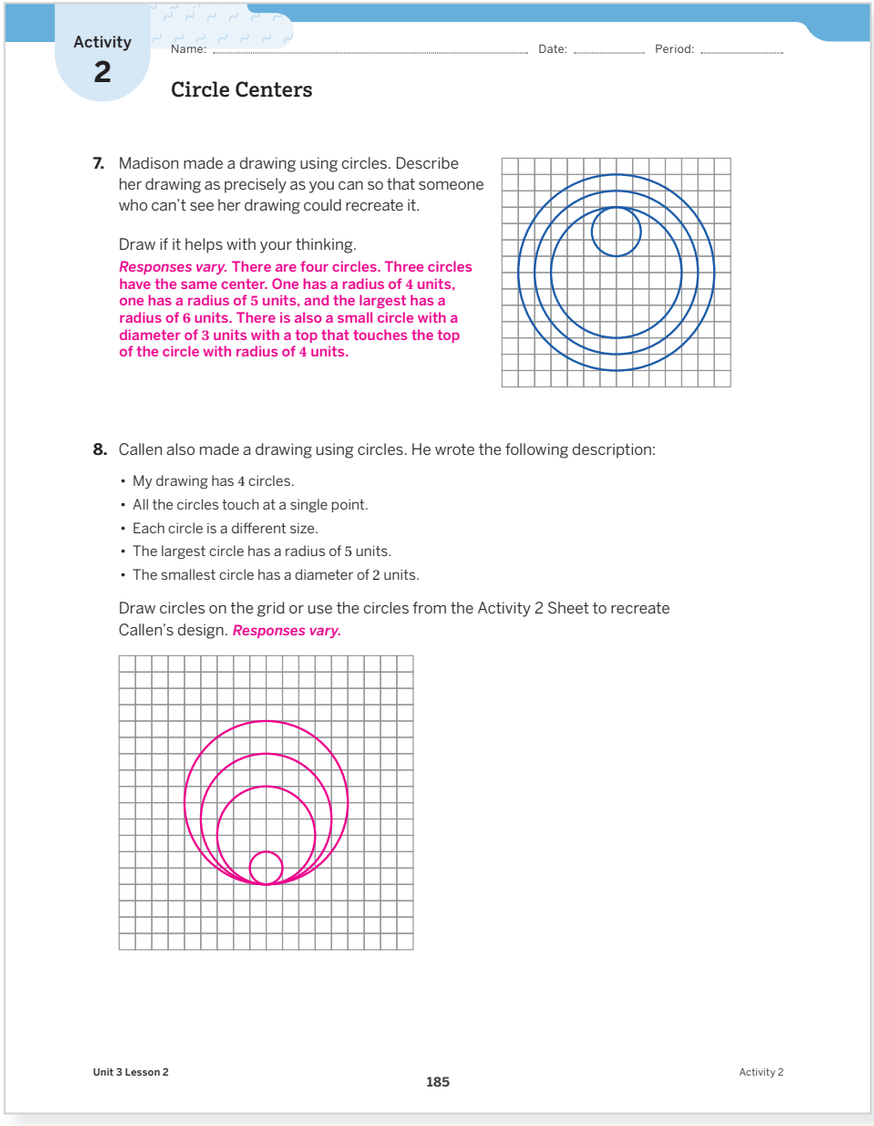
Capture several unique or creative student drawings.

3 Connect

Display the captured drawings and invite the authors to share their process or inspiration.

Consider asking:

- "Which parts of the description did you draw first? Why?"
- "Did you get stuck? How did you get unstuck?"
- "Are there any parts of Callen's description that you would change to make it clearer?"



Activity 2

Name: Date: Period:

Circle Centers

7. Madison made a drawing using circles. Describe her drawing as precisely as you can so that someone who can't see her drawing could recreate it.

Draw if it helps with your thinking.

Responses vary. There are four circles. Three circles have the same center. One has a radius of 4 units, one has a radius of 5 units, and the largest has a radius of 6 units. There is also a small circle with a diameter of 3 units with a top that touches the top of the circle with radius of 4 units.

8. Callen also made a drawing using circles. He wrote the following description:

- My drawing has 4 circles.
- All the circles touch at a single point.
- Each circle is a different size.
- The largest circle has a radius of 5 units.
- The smallest circle has a diameter of 2 units.

Draw circles on the grid or use the circles from the Activity 2 Sheet to recreate Callen's design. *Responses vary.*

Unit 3 Lesson 2 185 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of measuring and describing a circle using its radius and diameter.

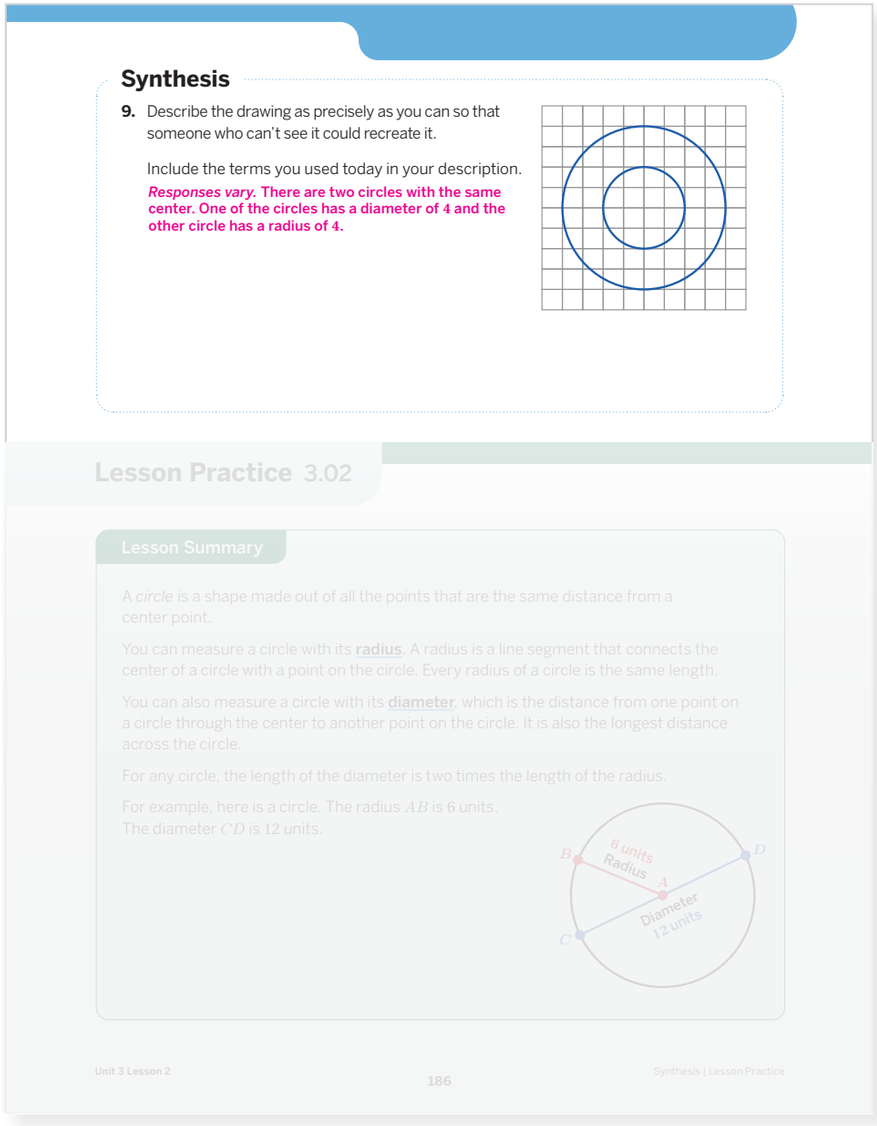
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- There are two circles with the same center.
- One circle has a radius of 4 units and the other has a radius of 2 units.
- One circle has a diameter of 8 units and the other has a diameter of 4 units.
- The smaller circle is in the middle of the larger circle.

Display any references created during the lesson and consider adding student ideas and language from the discussion to them.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Describe the drawing as precisely as you can so that someone who can't see it could recreate it.

Include the terms you used today in your description.

Responses vary. There are two circles with the same center. One of the circles has a diameter of 4 and the other circle has a radius of 4.

Lesson Practice 3.02

Lesson Summary

A *circle* is a shape made out of all the points that are the same distance from a center point.

You can measure a circle with its **radius**. A radius is a line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

You can also measure a circle with its **diameter**, which is the distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

For any circle, the length of the diameter is two times the length of the radius.

For example, here is a circle. The radius AB is 6 units.
The diameter CD is 12 units.

Unit 3 Lesson 2 186 Synthesis | Lesson Practice

Lesson Takeaway: To communicate clearly to others, it helps to use precise language. Terms like **diameter**, **radius**, and center help us communicate about *circles*. The radius is the distance from the center of the circle to the edge. The diameter is the distance across the circle from edge to edge, through the center. The diameter is always twice the length of the radius.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the diameter of a circle.

Today's Goals

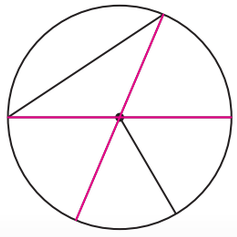
1. **Goal:** Identify the *diameter* and *radius* of a circle and explain how they are related.
2. **Language Goal:** Describe the properties of circles using terms such as *center*, *radius*, and *diameter*. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.02

The diagram shows several line segments and a point. The point is located at the center of the circle.



a. Trace all the segments that are diameters.
Response shown on diagram.

b. Explain how you know a segment is a diameter.
***Responses vary.* A segment is a diameter if it goes from one point on a circle to another point on the circle and passes through the center.**

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify of radius and diameter:

» **Respond to Student Thinking:**

- Students will have more opportunity to develop their understanding of these terms in Lesson 3. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Today, students built from informal to formal language to describe circles. Which students' ideas and language were you able to highlight? Who participated and who did not participate in the creation of the class definitions of **radius** and **diameter**? What trends do you see in participation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe the drawing as precisely as you can so that someone who can't see it could recreate it.

   Include the terms you used today in your description.

   Responses vary. There are two circles with the same center. One of the circles has a diameter of 4 and the other circle has a radius of 4.

Lesson Practice 3.02

Lesson Summary

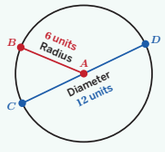
A *circle* is a shape made out of all the points that are the same distance from a center point.

You can measure a circle with its **radius**. A radius is a line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

You can also measure a circle with its **diameter**, which is the distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

For any circle, the length of the diameter is two times the length of the radius.

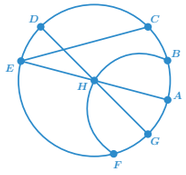
For example, here is a circle. The radius AB is 6 units. The diameter CD is 12 units.



Lesson Practice 3.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a circle. Point H is the circle's center.



1. Trace a segment that is a radius.

   Responses vary.
   - Segment AH
   - Segment DH
   - Segment EH
   - Segment GH

2. Trace a segment that is a diameter.

   Responses vary.
   - Segment DG
   - Segment AE

3. Trace a segment that is neither a radius nor a diameter.

   Segment CE

4. A circle has a radius of 4.5 units. What is its diameter?

   9 units

5. Can a circle with a radius of 5.2 inches have a diameter of 13 inches?

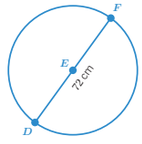
   No

   Explain your thinking.

   Explanations vary. If I double the radius of 5.2, the diameter would be 10.4, not 13.

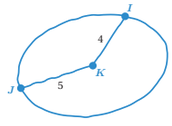
6. What is the length of the radius in the circle?

   36 centimeters



7. Kala's teacher asked her to create a circle and draw a diameter. Here is her drawing. Identify and describe her mistakes.

   Responses vary. In Kala's shape, the distance from the center to a point on the circle is not the same all the way around, so it's not a circle. The "diameter" she drew is not one line segment that passes through the center. It's two line segments that meet at point K.



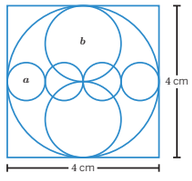
Lesson Practice 3.02

Name: _____ Date: _____ Period: _____

FAST Practice

8. This square contains several circles. Determine the radius and diameter of circles a and b.

Circle	Radius (cm)	Diameter (cm)
a	0.5	1
b	1	2



Spiral Review

9. A lemonade recipe calls for $\frac{1}{4}$ cups of lemon juice, $\frac{1}{2}$ cups of sugar, and 1 cup of water. Complete the table to show the amount of lemon juice for different-sized batches.

Sugar (cups)	Lemon Juice (cups)
2	1
10	5
5	$\frac{5}{2}$

10. The graph of a proportional relationship includes the point (3, 12).

    What is one constant of proportionality for this relationship? Show or explain your thinking.

    4 or $\frac{1}{4}$. *Explanations vary.* $\frac{12}{3} = 4$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3, 5, 7	2	MA.7.GR.1.3
	4, 6	1	MA.7.GR.1.3
FAST Practice	8	3	MA.7.GR.1.3
Spiral Review			
	9	2	MA.7.AR.3.2
	10	1	MA.7.AR.4.2

Unit 3

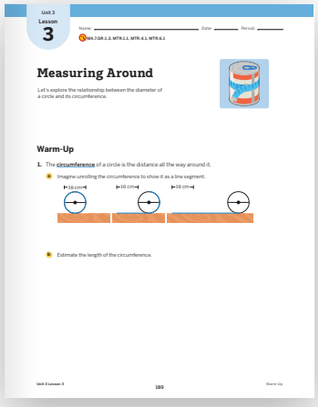
Lesson

3

Measuring Around

Estimating and Calculating Circumference

Let's explore the relationship between the diameter of a circle and its circumference.


Unit 3
Lesson
3
Name _____ Date _____ Period _____
MA.7.GR.1.3, MTR.1.1, MTR.4.1, MTR.6.1
Measuring Around
Let's explore the relationship between the diameter of a circle and its circumference.
Warm-Up
1. The circumference of a circle is the distance all the way around it.
a. Imagine unrolling the circumference to show it as a line segment.
▶ 10 cm ◀
▶ 20 cm ◀
▶ 30 cm ◀
b. Estimate the length of the circumference.
Unit 3 Lesson 3
189
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Given the radius, diameter, or *circumference* of a circle, calculate the other two measurements.
2. **Language Goal:** Explain that the relationship between the diameter and *circumference* of a circle is proportional and that the constant of proportionality is π. **(Reading, Writing, Speaking, and Listening)**

Students explore the relationship between a circle's diameter and its **circumference**. They determine that the relationship is proportional, and the *constant of proportionality* is **pi** (π). They use their knowledge of proportional relationships and the equation $C = \pi d$ to calculate unknown measurements.

Coherence

Prior Learning

In Lesson 2, students described the size of a circle using the measurements of the radius and diameter. In Units 1 and 2, students explored proportional relationships, including the relationship between lengths in scaled copies.

Future Learning

In Lesson 4, students will estimate and calculate the areas of complex shapes including shapes with curved edges.

Connections

- Students determine the measures of different circular objects, such as tires, pizzas, dishes, tennis ball cans, and more. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.3

Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2	MA.912.GR.7.2

Vocabulary

New Vocabulary

circumference **pi (π)**

Review Vocabulary

constant of proportionality

Additional Vocabulary

approximation

Lesson at a Glance

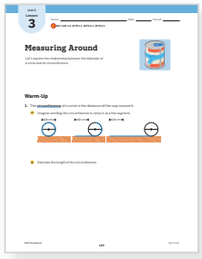
~ 45 min

MA.7.GR.1.3

Warm-Up

Independent | 5 min

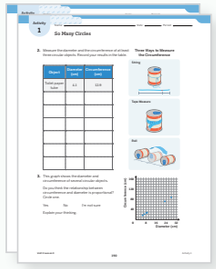
Students are introduced to the term **circumference** and estimate the circumference of a circle by unrolling it into a straight line.



Activity 1

Pairs | 10 min

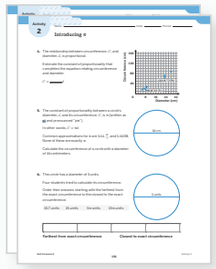
Students use various tools to measure the circumference of round objects and determine there is a proportional relationship between a circle's diameter and its circumference.



Activity 2

Pairs | 10 min

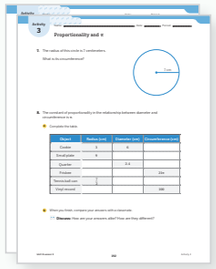
Students discover that π is the *constant of proportionality* relating the diameter and circumference of a circle and use different approximations for **pi** (π) to calculate unknown measurements. MLR2: Collect and Display



Activity 3

Pairs | 10 min

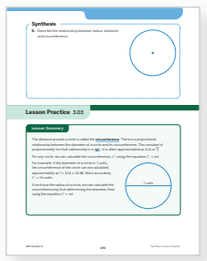
Students use their knowledge of proportional relationships and the equation $C = \pi d$ to calculate unknown measurements.



Synthesis

Whole Class | 5 min

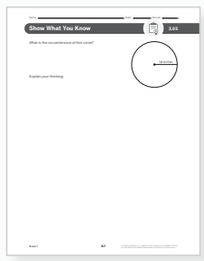
Students synthesize their understanding of the relationship between radius, diameter, and circumference in circles. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the circumference of a circle.



Prep Checklist

Prepare the circular objects, measuring tools, and string. A print option is also available.

This lesson includes:

- Student Edition
- Show What You Know
- a variety of circular objects (one per pair, if possible)
- measuring tools
- string

Independent | 5 min

Warm-Up

Purpose: Students are introduced to the term **circumference** and estimate the circumference of a circle by unrolling it into a straight line.

1 Launch

To surface connections, read the definition of **circumference** aloud. Consider asking, "What other words do you know to describe the distance around a shape?"

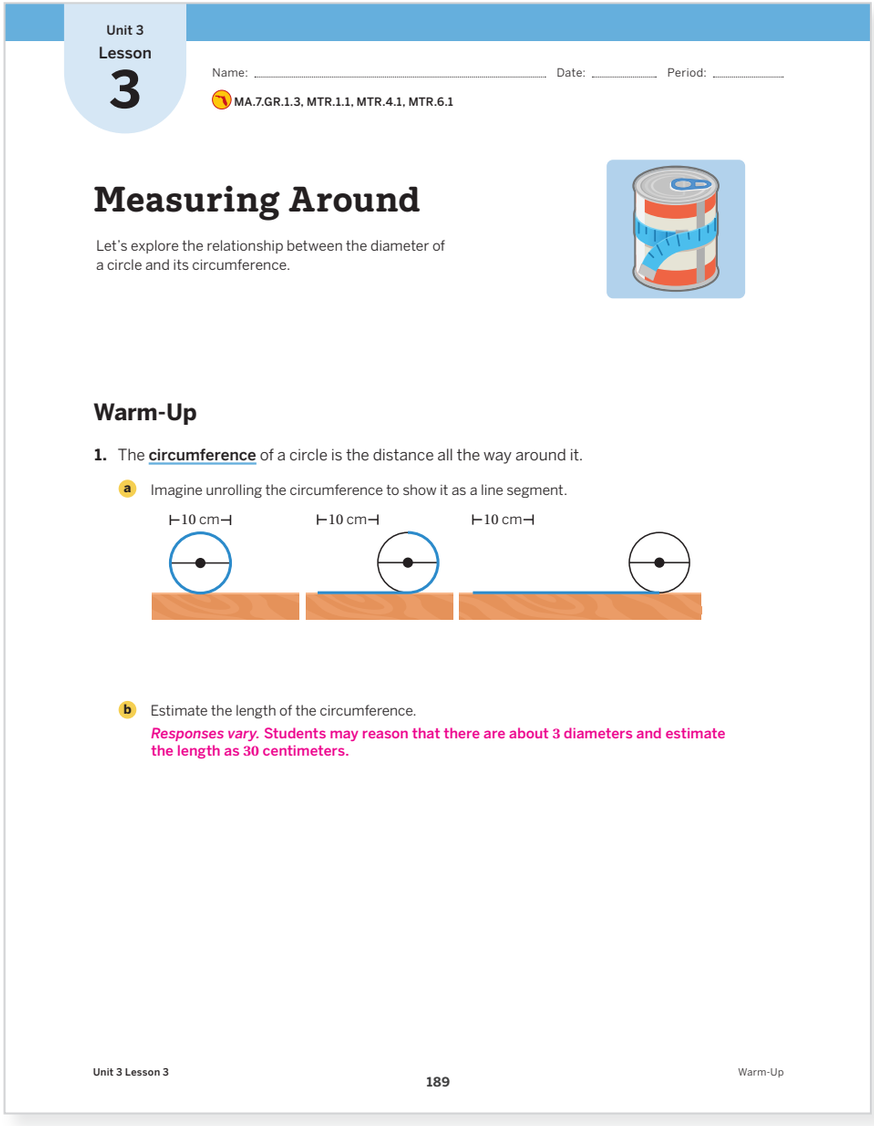
ELL **English Language Learners** Use gestures to illustrate the meaning of the terms *radius*, *diameter*, and *circumference*. **(Reading and Listening)**

2 Connect

Capture and share a variety of estimates. Emphasize the range of student responses. It's okay — even desirable — to lack consensus at this stage. Consider sharing with students that they will build toward calculating the circumference more precisely in this lesson.

Consider asking, "When might it be useful to know the distance around a circle or a circular object?"

Create an anchor chart that describes and depicts the *radius*, *diameter*, and *circumference* of a circle for students to refer to throughout the lesson.



Unit 3
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.GR.1.3, MTR.1.1, MTR.4.1, MTR.6.1

Measuring Around

Let's explore the relationship between the diameter of a circle and its circumference.

Warm-Up

1. The **circumference** of a circle is the distance all the way around it.

   a. Imagine unrolling the circumference to show it as a line segment.

   b. Estimate the length of the circumference.

   Responses vary. Students may reason that there are about 3 diameters and estimate the length as 30 centimeters.

Unit 3 Lesson 3 189 Warm-Up

Pairs | 10 min

Activity 1 So Many Circles

Purpose: Students use various tools to measure the circumference of round objects and determine there is a proportional relationship between a circle's diameter and its circumference.

Short on time? Consider limiting students' measuring time or encouraging pairs to collect data for only one or two objects.

1 Launch

Demonstrate measuring a circular object using each of the methods shown in the image.

Consider asking, "Why might you get two different answers for the diameter of the same object? What should you do if that happens?" Encourage students to consider the limitations and advantages of each method. **(MTR.6.1)**

Distribute the circular objects and measuring tools (including string).

2 Monitor

Encourage students to measure each object using two different methods and come to consensus about which measurement to record. **(MTR.1.1)**

Invite students to add their own points to the graph based on the objects they measured.

Select and sequence sketches and explanations for each of the most popular choices.

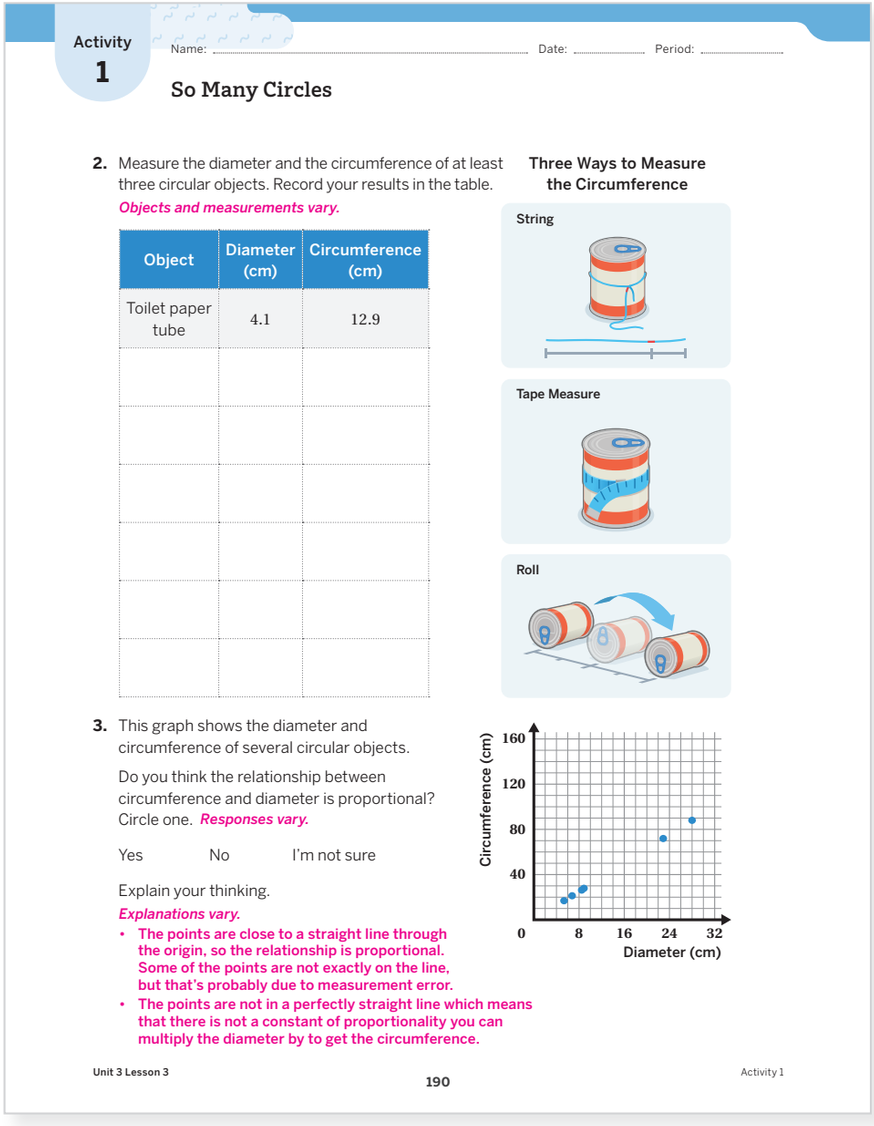
D Differentiation

Look for students who:	Respond to Student Thinking
Think the relationship is not proportional because there is at least one point that does not lie on a line through the origin.	**Support** Consider asking, "Why might it be true that the points do not lie perfectly on a line? Does this mean the relationship is not proportional?"
Use the graph to support their argument.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.
Refer to measurement error in their explanation.	**Stretch** Consider asking, "How can we be sure our measurements are as accurate as possible? Which tools do you think gave the most accurate measurements?"

3 Connect

Invite students to share their work, calling attention to any conflict or consensus.

Invite students to make arguments and justify their reasoning for each response if there is no consensus. **(MTR.4.1)**



Activity 1

Name: Date: Period:

So Many Circles

2. Measure the diameter and the circumference of at least three circular objects. Record your results in the table.

Objects and measurements vary.

Object	Diameter (cm)	Circumference (cm)
Toilet paper tube	4.1	12.9

Three Ways to Measure the Circumference

String

Tape Measure

Roll

3. This graph shows the diameter and circumference of several circular objects.

Do you think the relationship between circumference and diameter is proportional? Circle one. *Responses vary.*

Yes No I'm not sure

Explain your thinking.

Explanations vary.

- The points are close to a straight line through the origin, so the relationship is proportional. Some of the points are not exactly on the line, but that's probably due to measurement error.
- The points are not in a perfectly straight line which means that there is not a constant of proportionality you can multiply the diameter by to get the circumference.

Unit 3 Lesson 3 190 Activity 1

Key Takeaway: There is a proportional relationship between a circle's diameter and its **circumference**.

Pairs | 10 min

Activity 2 Introducing π

Purpose: Students discover that π is the *constant of proportionality* relating the diameter and circumference of a circle and use different approximations for **pi** (π) to calculate unknown measurements.

1 Launch

To surface prior knowledge, consider asking, "What do you remember about *constants of proportionality*?"

Invite students to make the closest estimate they can. **(MTR.6.1)**

D **Differentiation: (Stretch)** Encourage students to use a fraction to estimate the constant of proportionality, using the smallest numerator and denominator possible.

Invite students to share the constants they identified.

2 Monitor

Direct students' attention to Problem 5 and read each statement aloud. Consider inviting students to rephrase the definition of **pi** in their own words.

Capture a variety of correct answers that use different values for π.

Invite students to share what value they used for π.

Encourage students to compare their order with another pair of students, sharing their reasoning for each choice. **(MTR.4.1)**

3 Connect

Invite a few students to share their work

Consider asking:
- "How is the value of 15 different from the value of 15π?"
- "Which value represents the exact circumference? How do you know?"

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they discuss the relationship between diameter and circumference and the different ways to approximate pi. Display for students to reference and add to for the rest of the unit.

Activity 2

Name: ..... Date: ..... Period: .....

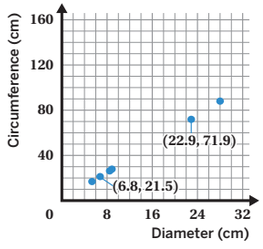
Introducing π

4. The relationship between circumference, C, and diameter, d, is proportional.

Estimate the *constant of proportionality* that completes the equation relating circumference and diameter.

$C =$ ..... d

Responses vary.
- 3.1
- 3.16



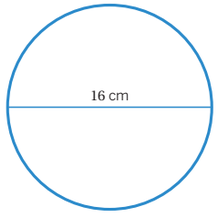
5. The constant of proportionality between a circle's diameter, d, and its circumference, C, is π (written as **pi** and pronounced "pie").

In other words, $C = \pi d$.

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. None of these are exactly π.

Calculate the circumference of a circle with a diameter of 16 centimeters.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations. 16π centimeters, 50.24 centimeters, $\frac{352}{7}$ centimeters

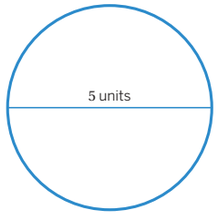


6. This circle has a diameter of 5 units.

Four students tried to calculate its circumference.

Order their answers starting with the farthest from the exact circumference to the closest to the exact circumference.

15.7 units | 15 units | 5π units | 15π units



15π units	15 units	15.7 units	5π units

Farthest from exact circumference — Closest to exact circumference

Unit 3 Lesson 3 191 Activity 2

Pairs | 10 min

Activity 3 Proportionality and π

Purpose: Students use their knowledge of proportional relationships and the equation $C = \pi d$ to calculate unknown measurements.

1 Launch

To surface connections, consider asking, "How is this question different from the previous one? How is it alike?"

2 Monitor

Encourage students to work together to determine each missing measurement.

A **Accessibility: Memory and Attention** Invite students to refer to Problem 5 or to the class display to see various approximations for pi.

Select and sequence several student tables to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Multiply a given circumference by pi to determine a diameter, e.g., write 100π for the diameter of the vinyl record.	**Support** Consider asking, "How is your strategy for determining the diameter different from your strategy for determining the circumference?"
Use decimal or fraction *approximations* of pi.	**Strengthen** Consider asking, "What is another way you could write the circumference? Which is closer to the exact value?"
Write expressions using the symbol π.	Capture these to share during the Connect.

3 Connect

Invite students to share their strategies for determining each unknown value. Consider focusing on one or two rows of the table.

To surface the Key Takeaway, consider asking:

- "What patterns did you notice that helped you complete the table?"
- "What was your strategy for figuring out the diameter if you knew the circumference?"

Activity 3

Name: Date: Period:

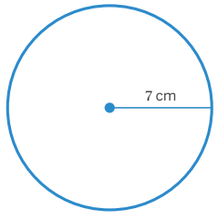
Proportionality and π

7. The radius of this circle is 7 centimeters.

What is its circumference?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- 14π centimeters
- 43.96 centimeters
- 44 centimeters



8. The constant of proportionality in the relationship between diameter and circumference is π.

a. Complete the table.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Cookie	3	6	6π
Small plate	9	18	18π
Quarter	1.2	2.4	2.4π
Frisbee	10.5	21	21π
Tennis ball can	$\frac{7}{2}$	7	7π
Vinyl record	$\frac{100}{2\pi} \approx 15.92$	$\frac{100}{\pi} \approx 31.83$	100

b. When you finish, compare your answers with a classmate.

Discuss: How are your answers alike? How are they different?

Responses vary.

Unit 3 Lesson 3 192 Activity 3

Key Takeaway: The *constant of proportionality* in the relationship between the diameter and circumference of a circle is **pi** (π). Some common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. You can use the equation $C = \pi d$ to determine unknown measurements of a circle.

Whole Class | 5 min

Synthesis

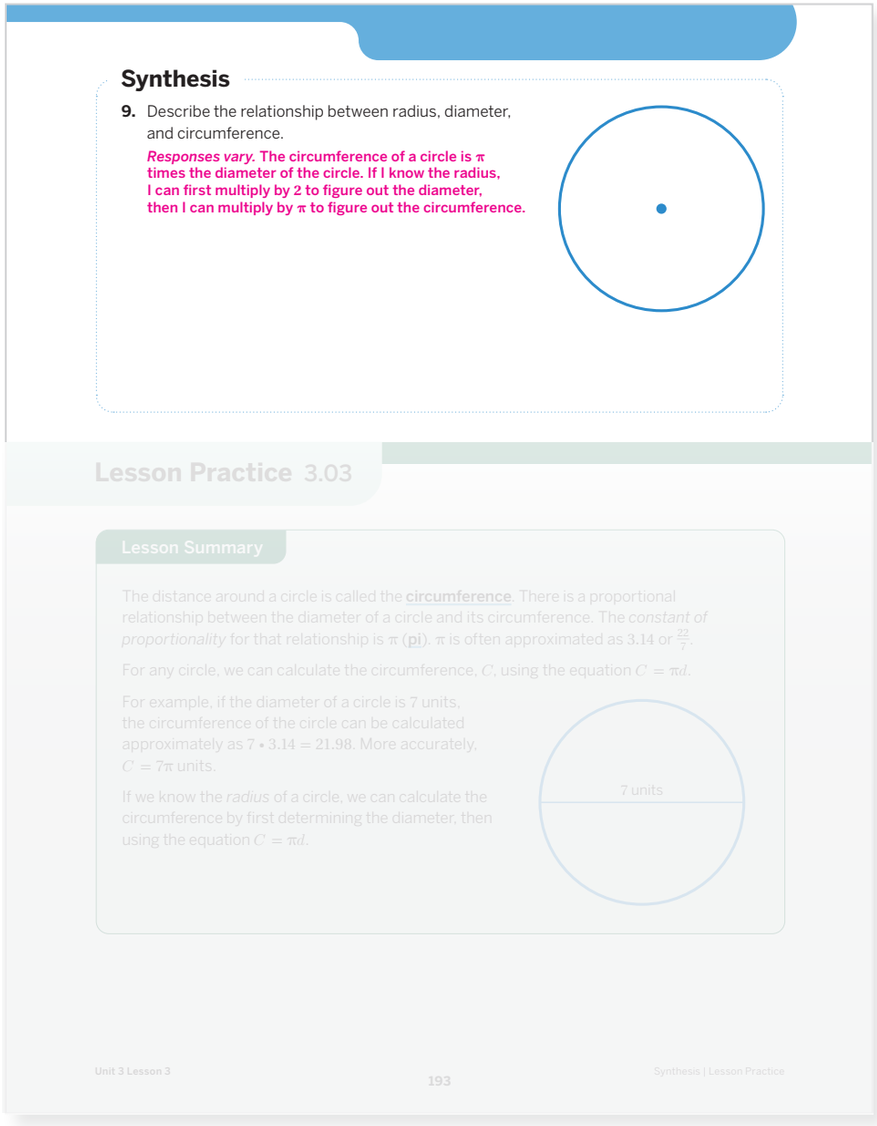
Purpose: Students synthesize their understanding of the relationship between radius, diameter, and circumference in circles.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The relationship between radius, diameter, and circumference is proportional. Between radius and diameter, there is a scale factor of 2. Between diameter and circumference, there is a scale factor of π.
- Multiplying the radius by 2 to calculate the diameter.
- Dividing the diameter by 2 to calculate the radius.
- Multiplying the diameter by π to calculate the circumference.

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas about the relationships between radius, diameter, circumference, and pi. Add any new ideas to the display for students to reference for the rest of the unit.



Synthesis

9. Describe the relationship between radius, diameter, and circumference.

Responses vary. The circumference of a circle is π times the diameter of the circle. If I know the radius, I can first multiply by 2 to figure out the diameter, then I can multiply by π to figure out the circumference.

Lesson Takeaway: The circumference of a circle is equal to the diameter times π.

Independent | 5 min

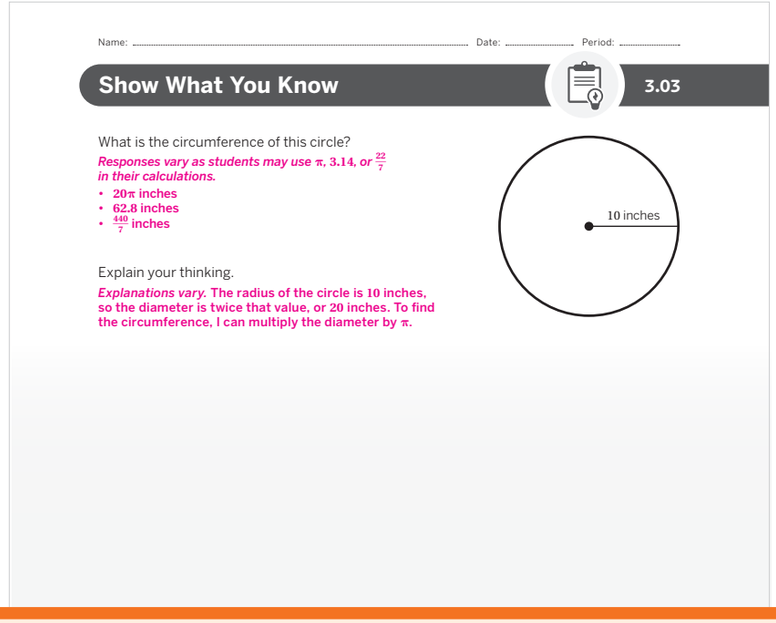
Show What You Know

Purpose: Students demonstrate their understanding by calculating the circumference of a circle.

Today's Goals

1. **Goal:** Given the radius, diameter, or *circumference* of a circle, calculate the other two measurements.
2. **Language Goal:** Explain that the relationship between the diameter and *circumference* of a circle is proportional and that the constant of proportionality is π. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.03

What is the circumference of this circle?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- 20π inches
- 62.8 inches
- $\frac{440}{7}$ inches

Explain your thinking.

Explanations vary. The radius of the circle is 10 inches, so the diameter is twice that value, or 20 inches. To find the circumference, I can multiply the diameter by π.

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding the relationship between the diameter and circumference:

» **Respond to Student Thinking:**

- Use the **Determining the Circumference of a Circle Mini-Lesson.**
- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- If students would like to explore pi in greater depth, invite them to research Pi Day.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students developed early ideas about the constant π. What values, approximate or exact, did you see students use for π? How will their understanding of π build over the next several lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe the relationship between radius, diameter, and circumference.

Responses vary. The circumference of a circle is π times the diameter of the circle. If I know the radius, I can first multiply by 2 to figure out the diameter, then I can multiply by π to figure out the circumference.

Lesson Practice 3.03

Lesson Summary

The distance around a circle is called the **circumference**. There is a proportional relationship between the diameter of a circle and its circumference. The *constant of proportionality* for that relationship is π (**pi**). π is often approximated as 3.14 or $\frac{22}{7}$.

For any circle, we can calculate the circumference, C, using the equation $C = \pi d$.

For example, if the diameter of a circle is 7 units, the circumference of the circle can be calculated approximately as $7 \cdot 3.14 = 21.98$. More accurately, $C = 7\pi$ units.

If we know the *radius* of a circle, we can calculate the circumference by first determining the diameter, then using the equation $C = \pi d$.



Lesson Practice 3.03

Name: _____ Date: _____ Period: _____

1. Lucy measured the diameter and circumference of several circular objects and recorded her measurements in a table.

Object	Diameter (cm)	Circumference (cm)
Half dollar coin	3	10
Flying disc	23	50
Jar lid	8	25
Flower pot	15	48

Which object's circumference measurement do you think is the least accurate?

Flying disc

Explain your thinking.

Explanations vary. **The constant of proportionality should be a little more than 3, but $23 \cdot 2.17$ is approximately 50, so the constant of proportionality is less than 3.**

2. Complete the table to determine the missing measurements for each object.

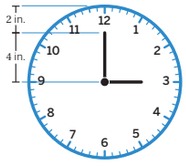
Some responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Object	Radius	Diameter	Circumference
Hula hoop	**18 inches**	36 inches	**36π inches**
Circular pond	**≈88.5 feet**	**≈177 feet**	556 feet
Magnifying glass	**2.6 centimeters**	5.2 centimeters	**5.2π centimeters**
Car tire	**≈11.4 inches**	**≈22.8 inches**	71.6 inches

3. The circumference of a circle is approximately 50.27 inches. Determine the length of the radius of the circle in inches. Round your answer to the nearest whole number.

8 inches

Problems 4–5: The minute hand of a circular clock measures 4 inches. The distance from the end of the minute hand to the outer edge of the clock is 2 inches.



4. What is the length of the radius of the clock?

6 inches

5. What is the circumference of the clock?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- **12π inches**
- **37.68 inches**
- **$\frac{264}{7}$ inches**

Lesson Practice 3.03

Name: _____ Date: _____ Period: _____

Problems 6–7: The size of a drum affects its pitch. When comparing drums, the one with a larger diameter has a lower pitch because the frequency of its sound waves is slower.

6. If a drummer wants a drum with a lower pitch, should she choose a drum with a circumference of 18π inches or 22.5π inches?

22.5π inches

7. Determine the diameter of the drum she should choose.

22.5 inches

FAST Practice

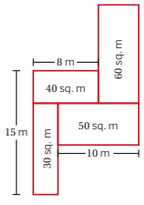
8. Determine whether the measurement described is the diameter, radius, or circumference.

	Diameter	Radius	Circumference
The tires on a tractor are 4.5 feet tall.	☑	☐	☐
The distance from the tip of a slice of pizza to the crust is 6 inches.	☐	☑	☐
The length of the metal rim around a glass lens is 190 millimeters.	☐	☐	☑

Spiral Review

9. This shape is made up of four rectangles. Determine the total perimeter of the shape.

70 meters



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	2	MA.7.GR.1.3
FAST Practice	**8**	2	MA.7.GR.1.3
Spiral Review			
	9	3	MA.4.GR.2.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining whether quantities are in a proportional relationship from tables and graphs	Lesson 1	2	MA.7.AR.4.1
2	Determining the diameter of circles when given the circumference	Lesson 3	2	MA.7.GR.1.3
3	Determining a constant of proportionality for a proportional relationship represented in a graph	Lesson 1	2	MA.7.AR.4.2
4a	Determining the circumference of circles	Lesson 3	1	MA.7.GR.1.3
4b	Determining values in a table of a proportional relationship	Lesson 3	1	MA.7.AR.3.2
4c	Determining a constant of proportionality for a proportional relationship represented in a table	Lesson 3	1	MA.7.AR.4.2

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 3

1. Select *all* the choices that represent proportional relationships.

☑ A.

x	y
1	3
2	6
3	9

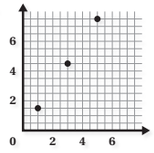
☑ B.

x	y
12	6
8	4
2	1

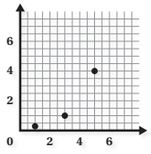
☐ C.

x	y
2	5
3	10
4	20

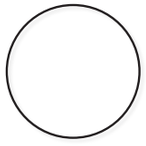
☑ D.



☐ E.



2. This circle has a circumference of 40 centimeters. What is the length of its diameter?



A. 40π centimeters

B. $\frac{20}{\pi}$ centimeters

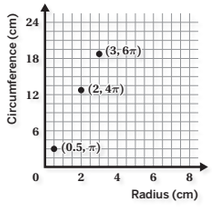
(B.) $\frac{40}{\pi}$ centimeters

D. 20 centimeters

3. Here is a graph of the radius and circumference measurements of several circles.

What is a constant of proportionality in this relationship?

$2\pi \approx 6.28$ or $\frac{1}{2\pi} \approx 0.16$



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Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 3

4. A wheel has a radius of 35 centimeters.



a. Determine its circumference.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- 70π centimeters
- 219.8 centimeters
- 220 centimeters

b. Complete the table showing how far this wheel travels for each number of rotations.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Number of Rotations	Distance (cm)
1	70π or 219.8 or 220
2	140π
5	350π
10	700π or 2198 or 2200

c. What is a constant of proportionality for this relationship?

Responses vary.

- 70π or $\frac{1}{70\pi}$
- 219.8 or 0.0045
- 220 or 0.0045

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

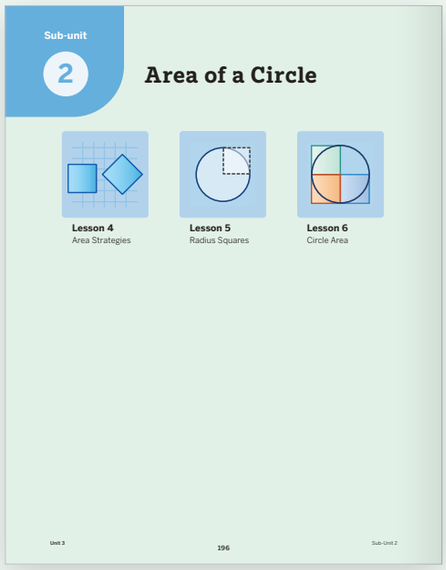
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: Use the relationships between the radius, diameter, and circumference of a circle to calculate unknown measurements. **(Lessons 1–3)**	2, 4a	Support	• **Mini-Lesson:** Determining the Circumference of a Circle (ML 3.03)
		Strengthen	• **Teacher Move:** Consider revisiting Unit 3, Lesson 3 (Measuring Around)
	1, 3	Support	• **Teacher Move:** Consider revisiting Unit 2, Lesson 3 (Sugar, Spice, and Everything Rice) and Unit 2, Lesson 8 (DinoPops)
		Stretch	• **You're invited to explore more:** Lesson 1 (Toothpicks)
	4b, 4c	Support	• **Teacher Move:** Consider revisiting Unit 3, Lesson 1 (Toothpicks) and Unit 3, Lesson 3 (Measuring Around)
		Strengthen	• **Teacher Move:** Invite students to share strategies to determine constant of proportionality
		Stretch	• **You're invited to explore more:** Lesson 1 (Toothpicks)

Area of a Circle

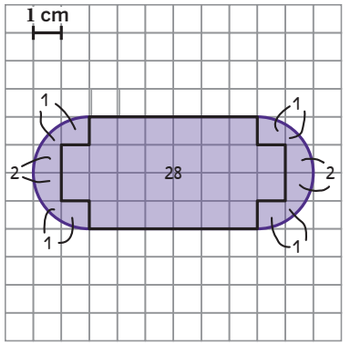
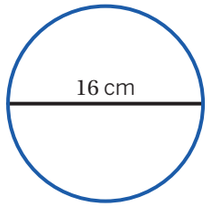
Sub-Unit 2 Goals

- **Goal:** Estimate the areas of shapes with curved edges.
- **Goal:** Calculate the areas of circles.
- **Language Goal:** Describe the relationship between the radius of any circle and its area.

In this sub-unit, students break apart and rearrange complex shapes with curved edges as a strategy to approximate area. Students discover the formula for the area of a circle and use it to solve problems. Students will use this work to determine the volumes and surface areas of cylinders in Unit 7.



Math That Matters Most

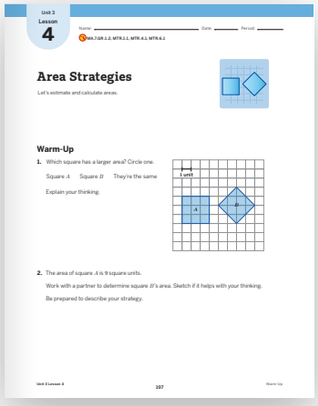
Concept or Skill	For example . . .
Estimate the area of a shape with curved edges.	Estimate the area of the shape. *The area is about 36 square units. I counted the total number of whole squares. For the curved edges, I imagined that the squares that were less than half-shaded could be cut up and filled into the spaces that were missing in the squares that were mostly shaded, and I counted each of the mostly shaded squares: 28 + 2 + 2 + 1 + 1 + 1 + 1 = 36 square units.* 
Calculate the area of a circle.	Calculate the exact area of the circle. *The radius of the circle is 8 centimeters. The area is equal to $\pi \cdot 8^2 = 64\pi$ square centimeters.* 

Unit 3
Lesson
4

Area Strategies

Estimating and Calculating Area

Let's estimate and calculate areas.


Unit 3
Lesson
4

Area Strategies

Let's estimate and calculate areas.

Warm-Up

1. Which square has a larger area? Circle one.

   Square A Square B They're the same

   Explain your thinking.

2. The area of square A is 9 square units.

   Work with a partner to determine square B's area. Sketch if it helps with your thinking.

   Be prepared to describe your strategy.

197

Learning Outcomes

Today's Goals

1. **Goal:** Determine the area of a complex shape using a variety of strategies, such as counting whole and partial tiles, decomposing and rearranging, and surrounding and subtracting.
2. **Goal:** Estimate the area of a shape that has curved edges.
3. **Language Goal:** Describe different strategies to calculate the area of a shape. **(Reading, Writing, Speaking, and Listening)**

Students surface strategies for determining the area of complex shapes and estimate the areas of shapes with curved edges. They discuss breaking up and rearranging a shape as a way of approximating its area. This lesson prepares students to estimate the area of a circle in Activity 1 of the next lesson.

Coherence

Prior Learning

In Lesson 3, students learned that the relationship between a circle's diameter and circumference is proportional with a constant of proportionality π.

Future Learning

In Lesson 5, students will apply the strategy of decomposing and rearranging to make more precise estimates of the areas of circles.

Connections

- Students communicate and apply multiple strategies to estimate the area of irregular figures. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.2
Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.

Mathematical Thinking and Reasoning Standard: MTR.1.1, MTR.4.1, MTR.6.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	**MA.912.GR.3.4** **MA.912.GR.4.4**

Lesson at a Glance

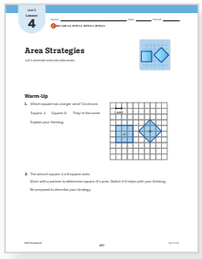
~ 45 min

MA.7.GR.1.2

Warm-Up

Pairs | 5 min

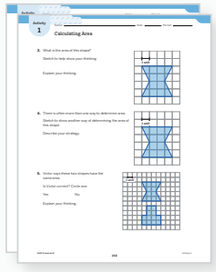
Students decide which of two shapes has a larger area and justify their choice. **(MTR.4.1)**



Activity 1

Pairs | 15 min

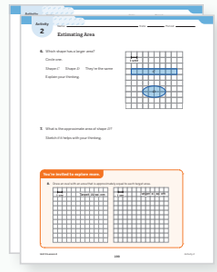
Students analyze and apply a variety of strategies for calculating the area of a complex shape, then justify why they result in the same answer.
MLR7: Compare and Connect **(MTR.4.1)**



Activity 2

Pairs | 15 min

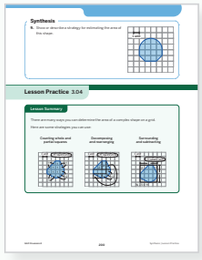
Students estimate the areas of shapes with curved edges.
Routine: Think-Pair-Share



Synthesis

Whole Class | 5 min

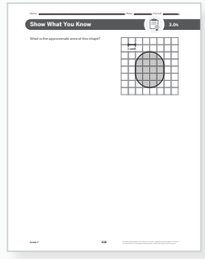
Students synthesize their strategies for estimating the area of figures with curved edges.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by estimating the area of a complex shape.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- coloring tools

Pairs | 5 min

Warm-Up

Purpose: Students decide which of two shapes has a larger area and justify their choice. **(MTR.4.1)**

1 Launch

Invite students to make a prediction about which shape has a larger area. Consider sharing that students will calculate the exact area in the next problem.

Encourage students to show their thinking about the area of each shape.

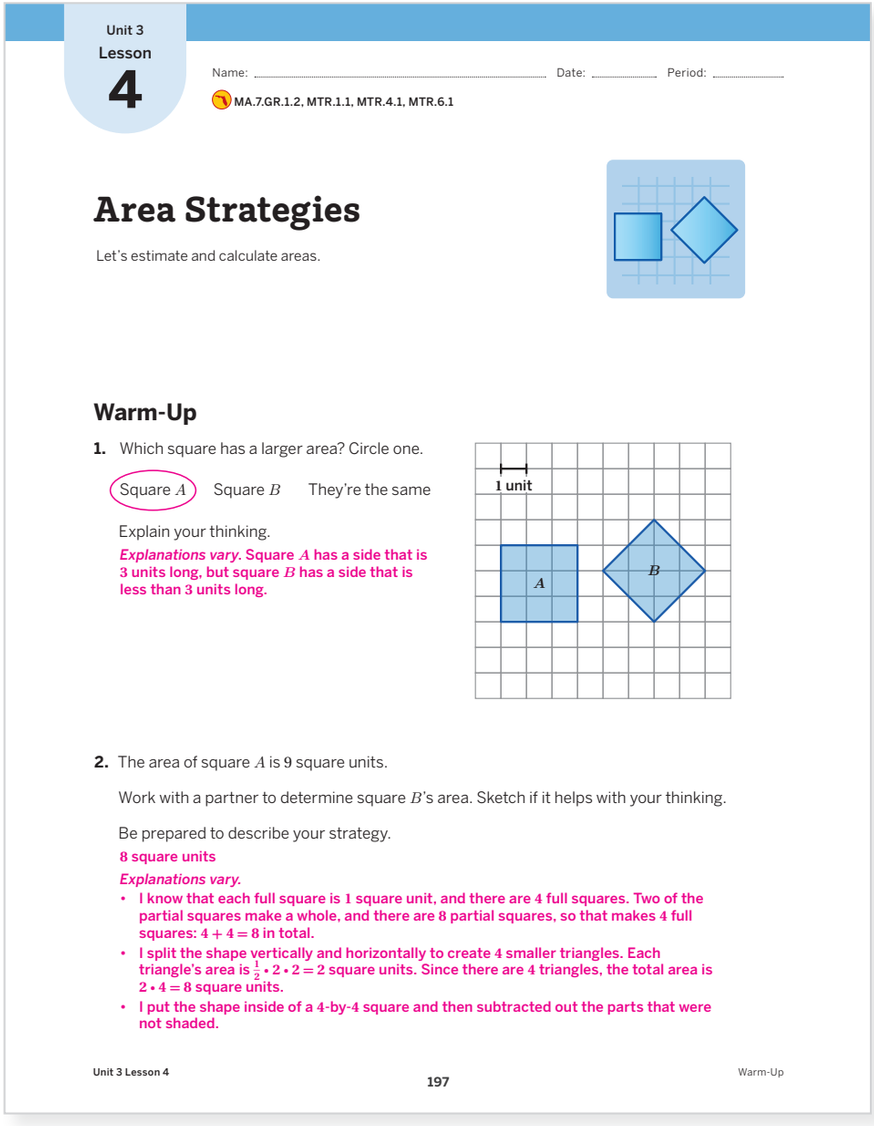
ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and inviting them to share and receive feedback in their primary language. **(Writing, Speaking, and Listening)**

2 Connect

Capture and share student sketches that show a variety of strategies for determining the area of the squares, such as counting whole and partial squares, decomposing and rearranging, or surrounding and subtracting.

Consider asking, "When might you use each of these strategies?" Invite students to make arguments and justify their reasoning for each method. **(MTR.4.1)**

Note: If students need more support, consider creating a record of strategies as they are shared for students to refer to throughout Activity 1.



Unit 3
Lesson
4

Name: _____ Date: _____ Period: _____

MA.7.GR.1.2, MTR.1.1, MTR.4.1, MTR.6.1

Area Strategies

Let's estimate and calculate areas.

Warm-Up

1. Which square has a larger area? Circle one.

Square A Square B They're the same

Explain your thinking.

Explanations vary. Square A has a side that is 3 units long, but square B has a side that is less than 3 units long.

1 unit

A B

2. The area of square A is 9 square units.

Work with a partner to determine square B's area. Sketch if it helps with your thinking.

Be prepared to describe your strategy.

8 square units

Explanations vary.

- I know that each full square is 1 square unit, and there are 4 full squares. Two of the partial squares make a whole, and there are 8 partial squares, so that makes 4 full squares: $4 + 4 = 8$ in total.
- I split the shape vertically and horizontally to create 4 smaller triangles. Each triangle's area is $\frac{1}{2} \cdot 2 \cdot 2 = 2$ square units. Since there are 4 triangles, the total area is $2 \cdot 4 = 8$ square units.
- I put the shape inside of a 4-by-4 square and then subtracted out the parts that were not shaded.

Unit 3 Lesson 4 197 Warm-Up

Pairs | 15 min

Activity 1 Calculating Area

Purpose: Students analyze and apply a variety of strategies for calculating the area of a complex shape, then justify why they result in the same answer. **(MTR.4.1)**

1 Launch

Consider asking, "Why might it be useful to have more than one strategy for determining area?"

To encourage creativity, invite students to show their thinking about the shape with a sketch.

2 Monitor

Select and sequence sketches and explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer back to the strategies used during the Warm-Up and apply one of them to the new shape.
Calculate the area in a variety of ways, such as: • Counting squares. • Decomposing. • Surrounding and subtracting.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.
Rearrange the shape so it consists only of whole squares.	

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own. **(MTR.4.1)**

A Accessibility: Visual-Spatial Processing
Invite students to use coloring tools to support making connections between the two shapes.

3 Connect

Display at least two captured strategies for determining the area of the shape. If possible, include a student who used a decomposing and rearranging strategy.

MLR MLR7: Compare and Connect Invite students to analyze and compare each strategy, and then connect to their strategy. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:
- "What did each student do?"
- "Why did the different strategies lead to the same area?"
- "Does moving the pieces change the area? Why or why not?"

Activity 1

Name: Date: Period:

Calculating Area

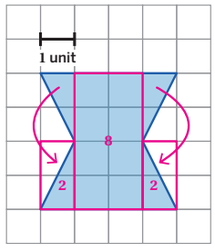
3. What is the area of this shape?

Sketch to help show your thinking.

12 square units

Explain your thinking.

Explanations vary. I put two triangles together to form a 2-by-1 rectangle, so I ended up with a 2-by-4 rectangle and two 2-by-1 rectangles. $8 + 2 + 2 = 12$ square units.

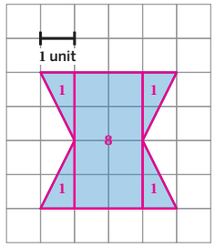


4. There is often more than one way to determine area.

Sketch to show another way of determining the area of this shape.

Describe your strategy.

Responses vary. I broke the shape up into one long rectangle and four small triangles. The area of the rectangle is $2 \cdot 4 = 8$ square units, and the area of each triangle is 1 square unit. $8 + 4 \cdot 1 = 12$ square units.



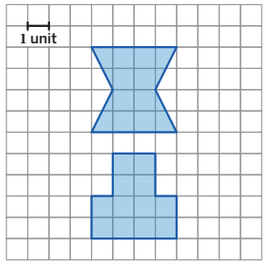
5. Victor says these two shapes have the same area.

Is Victor correct? Circle one.

(Yes) No

Explain your thinking.

Explanations vary. The shapes have the same area because you can rearrange the top shape to look like the bottom shape, which means they cover the same total number of squares.



Unit 3 Lesson 4 198 Activity 1

Key Takeaway: There are many ways to determine the area of a shape. You can count whole and partial squares, decompose and rearrange the pieces of the shape, or surround the shape and subtract the area of the empty space.

Pairs | 15 min

Activity 2 Estimating Area

Purpose: Students estimate the areas of shapes with curved edges.

Short on time? Consider omitting the Think-Pair-Share routine and inviting students to write a response, then continue.

1 Launch

Consider sharing that this activity includes shapes with curved edges. Consider asking, "What might be different about determining the area of a curved shape compared to one with only straight edges?" **(MTR.1.1)**

Note: If it does not come up naturally, share that students will be determining estimated or *approximate* areas.

Use the Think-Pair-Share routine to help students shape their ideas about areas of curved shapes. See the Routine Facilitation Guide for more information.

2 Monitor

Select and sequence student sketches to share during the Connect.

D Differentiation

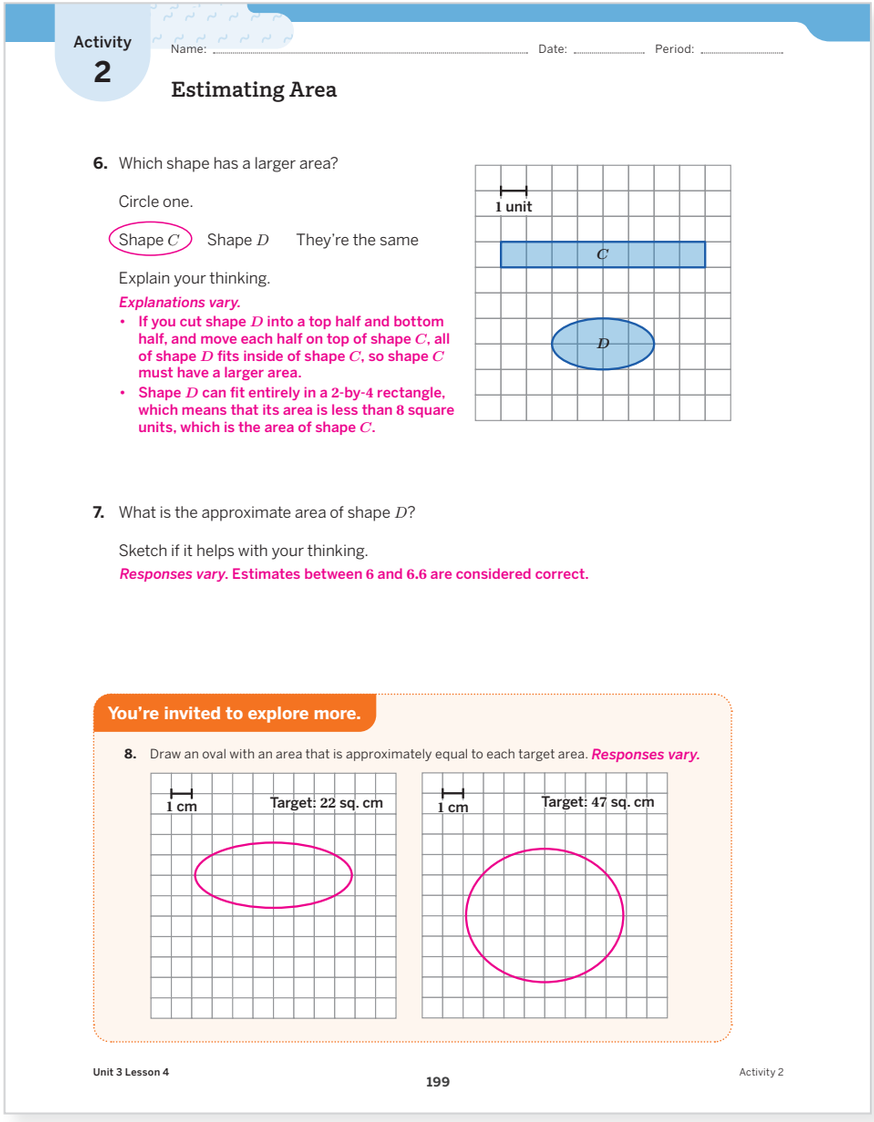
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How could you apply one of the strategies from the previous activity to this new shape?"
Surround the shape with a 2-unit by 4-unit rectangle and subtract the estimated area of the empty space.	**Strengthen** Invite students to think about how they might share their strategy during the Connect. Consider asking, "Is there anything you could do to make your estimate more precise?"
Count whole and partial squares, estimating values for the partial squares containing curved segments.	

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore estimating areas further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Share the captured student work and invite students to share the reasoning behind their estimates. **(MTR.6.1)**

Consider asking, "How was estimating the area of the curved shape like calculating the area of the shapes with only straight sides? How was it different?"



Activity 2

Name: Date: Period:

Estimating Area

6. Which shape has a larger area?

Circle one.

Shape C Shape D They're the same

Explain your thinking.

Explanations vary.

- If you cut shape D into a top half and bottom half, and move each half on top of shape C, all of shape D fits inside of shape C, so shape C must have a larger area.
- Shape D can fit entirely in a 2-by-4 rectangle, which means that its area is less than 8 square units, which is the area of shape C.

7. What is the approximate area of shape D?

Sketch if it helps with your thinking.

Responses vary. Estimates between 6 and 6.6 are considered correct.

You're invited to explore more.

8. Draw an oval with an area that is approximately equal to each target area. *Responses vary.*

Unit 3 Lesson 4 199 Activity 2

Whole Class | 5 min

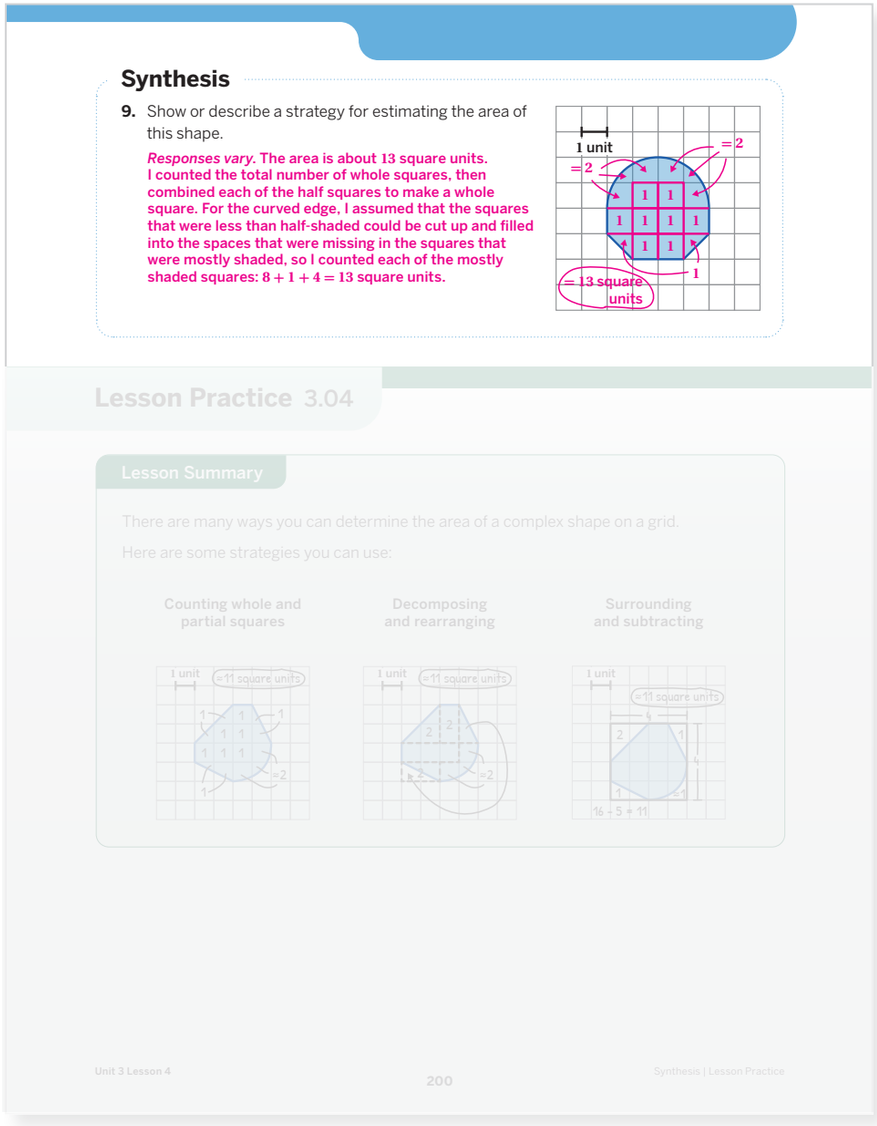
Synthesis

Purpose: Students synthesize their strategies for estimating the area of figures with curved edges.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Counting whole squares and estimating the area of partial squares.
- Surrounding the shape in a 4-by-4-unit square and subtracting the excess.



Synthesis

9. Show or describe a strategy for estimating the area of this shape.

Responses vary. The area is about 13 square units. I counted the total number of whole squares, then combined each of the half squares to make a whole square. For the curved edge, I assumed that the squares that were less than half-shaded could be cut up and filled into the spaces that were missing in the squares that were mostly shaded, so I counted each of the mostly shaded squares: $8 + 1 + 4 = 13$ square units.

Lesson Practice 3.04

Lesson Summary

There are many ways you can determine the area of a complex shape on a grid.

Here are some strategies you can use:

Counting whole and partial squares

Decomposing and rearranging

Surrounding and subtracting

Unit 3 Lesson 4 200 Synthesis | Lesson Practice

Lesson Takeaway: Counting partial and whole tiles, decomposing and rearranging, and surrounding and subtracting are a few strategies to estimate and calculate the area of shapes. You can use these strategies to estimate the area of shapes with curved edges.

Independent | 5 min

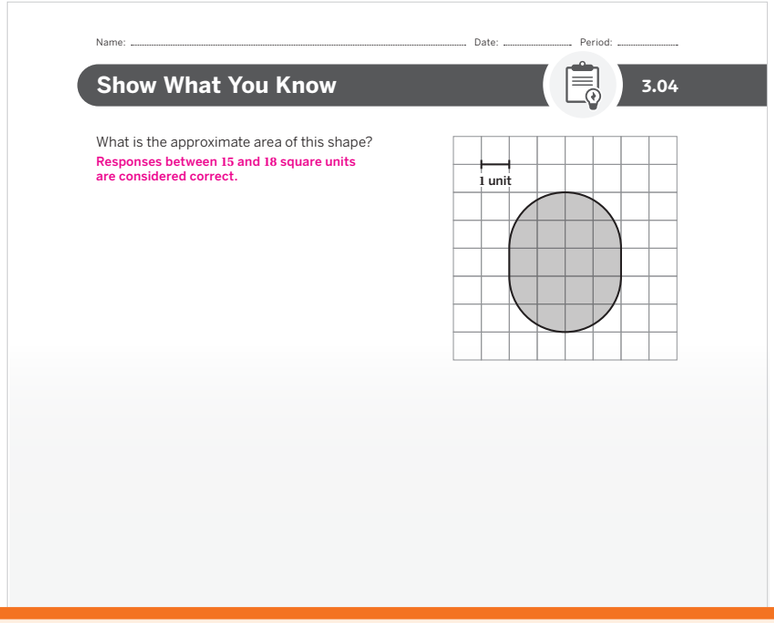
Show What You Know

Purpose: Students demonstrate their understanding by estimating the area of a complex shape.

Today's Goals

1. **Goal:** Determine the area of a complex shape using a variety of strategies, such as counting whole and partial tiles, decomposing and rearranging, and surrounding and subtracting.
2. **Goal:** Estimate the area of a shape that has curved edges.
3. **Language Goal:** Describe different strategies to calculate the area of a shape. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)


Name: _____ Date: _____ Period: _____

Show What You Know 3.04

What is the approximate area of this shape?

Responses between 15 and 18 square units are considered correct.

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to decompose shapes to determine area:

» **Respond to Student Thinking:**

- Revisit Grade 6, Unit 1, Lesson 4 (Letters).
- Revisit Activity 2 as a class and invite students to share strategies for determining area.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Reflect on your facilitation of the MLR7: Compare and Connect in this lesson. How did you prepare to facilitate the routine as you monitored student work in Problems 3 and 4? What about how you facilitated the routine promoted student learning and engagement? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Show or describe a strategy for estimating the area of this shape.

Responses vary. The area is about 13 square units. I counted the total number of whole squares, then combined each of the half squares to make a whole square. For the curved edge, I assumed that the squares that were less than half-shaded could be cut up and filled into the spaces that were mostly shaded, so I counted each of the mostly shaded squares: $8 + 1 + 4 = 13$ square units.

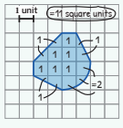
Lesson Practice 3.04

Lesson Summary

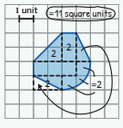
There are many ways you can determine the area of a complex shape on a grid.

Here are some strategies you can use:

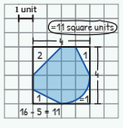
Counting whole and partial squares



Decomposing and rearranging



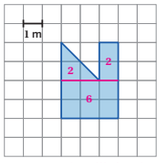
Surrounding and subtracting



Lesson Practice 3.04

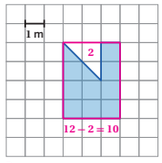
Name: _____ Date: _____ Period: _____

1. What is the area of this shape? Show your thinking.



10 square meters. *Work varies.*

2. Show or describe another way to determine the area of this shape.

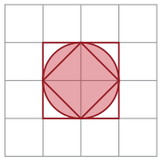


Responses vary.

3. Here is a diagram of two squares and a circle.

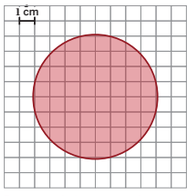
Explain why the area of the circle is more than 2 square units but less than 4 square units.

Responses vary. The unshaded square is larger than the circle, and its area is 4 square units. The shaded square is smaller than the circle, and its area is 2 square units. The area of the circle has to be between 2 and 4 square units.



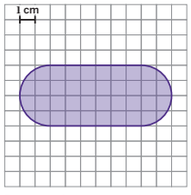
Problems 4–5: Estimate the area of each shape.

4.



Responses between 49 and 56 square centimeters are considered correct.

5.



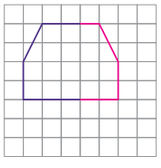
Responses between 32 and 38 square centimeters are considered correct.

Lesson Practice 3.04

Name: _____ Date: _____ Period: _____

6. Basheera started drawing a polygon. Complete the drawing so that the polygon has an area of 18 square units.

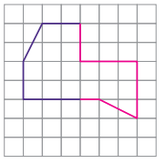
Responses vary.



FAST Practice

7. Basheera started drawing another polygon. Complete Basheera's drawing in a different way so that the polygon has an area of 18 square units.

Responses vary.



Spiral Review

8. Select *all* the expressions that are equivalent to 48.

- ☑ A. $3 \cdot 4^2$
- ☐ B. $6 + 6 \cdot 4$
- ☑ C. 50% of 96
- ☐ D. $(3 \cdot 4)^2$
- ☑ E. $4(6 + 6)$

9. What is the greatest common factor of 48 and 64?

16

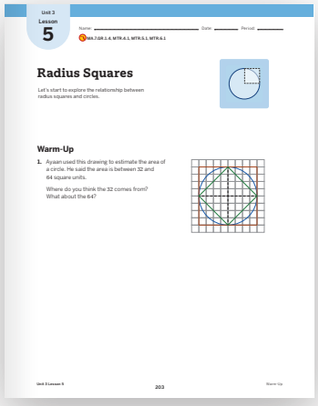
10. What is the least common multiple of 4 and 6?

12

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.GR.1.2
	3–5	2	MA.7.GR.1.2
	6	3	MA.7.GR.1.2
FAST Practice	7	3	MA.7.GR.1.2
Spiral Review			
	8	2	MA.5.AR.2.2 MA.6.NSO.3.3 MA.6.AR.3.4
	9–10	1	MA.6.NSO.3.1

Unit 3
Lesson
5


Unit 3
Lesson
5
Name: Date: Period:
MA.7.GR.1.4, MTR.4.1, MTR.5.1, MTR.6.1
Radius Squares
Let's start to explore the relationship between radius squares and circles.
Warm-Up
1. Ayaan used this drawing to estimate the area of a circle. He said the area is between 32 and 64 square units.
Where do you think the 32 comes from? What about the 64?
203

Radius Squares

Exploring Circle Area

Let's start to explore the relationship between radius squares and circles.

Learning Outcomes

Today's Goals

1. **Goal:** Estimate the area of a circle.
2. **Language Goal:** Informally describe the relationship between the radius of any circle and its area by referring to radius squares. **(Reading, Writing, Speaking, and Listening)**

Students use repeated reasoning to make sense of the relationship between radius squares (squares whose side lengths are equal to the length of the radius of a circle) and the area of the circle. Students discover the formula for the area of a circle by decomposing a circle into equal-size wedges and then rearranging the wedges to form a rectangle. This lesson prepares students to use a formula for calculating the area of a circle in Lesson 6. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 4, students developed strategies for estimating the area of a shape with curved edges.

Future Learning

In Lesson 6, students will use radius squares to generate the formula $A = \pi r^2$. They will apply the formula to calculate the area of circles.

Connections

- Students extend their understanding of calculating the area of a square to the area of a circle to determine the number of squares it takes to fill in a circle with the same radius as the side of the square. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.4
Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.1	**MA.7.GR.2.1**
MA.6.GR.2.2	**MA.7.GR.2.2**
	MA.7.GR.2.3

Lesson at a Glance

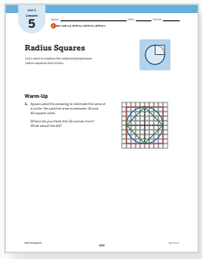
~ 45 min

MA.7.GR.1.4

Warm-Up

Pairs | 5 min

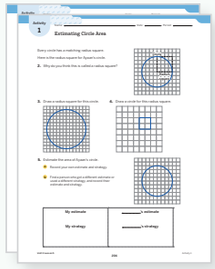
Students analyze another student's work to estimate the area of a circle.



Activity 1

Pairs | 10 min

Students estimate the area of a circle and are introduced to radius squares as a tool for estimating circle area.

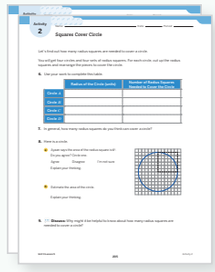


Activity 2

Small Groups | 10 min

Students visualize the relationship between the radius and area of a circle by cutting and rearranging radius squares to approximate the area of a circle.

Routine: Notice and Wonder

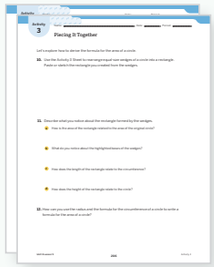


Activity 3

Pairs | 10 min

Students discover the formula for the area of a circle by rearranging the equal-sized wedges of a circle to form a rectangle.

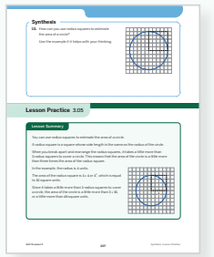
Routine: Notice and Wonder **(MTR.5.1)**



Synthesis

Whole Class | 5 min

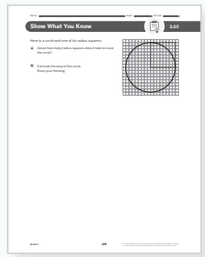
Students synthesize their understanding of using radius squares to estimate the areas of circles.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by estimating the area of a circle.



Prep Checklist

Prepare the Activity 2 Cards, scissors, and tape. Invite students to use their Student Edition.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one per student, pre-cut
  - graph paper (optional)
- Activity 3 Sheet, one per student
  - scissors
  - tape

Pairs | 5 min

Warm-Up

Purpose: Students analyze another student's work to estimate the area of a circle.

1 Launch

Direct students' attention to the drawing of the circle and invite students to analyze Ayaan's work.

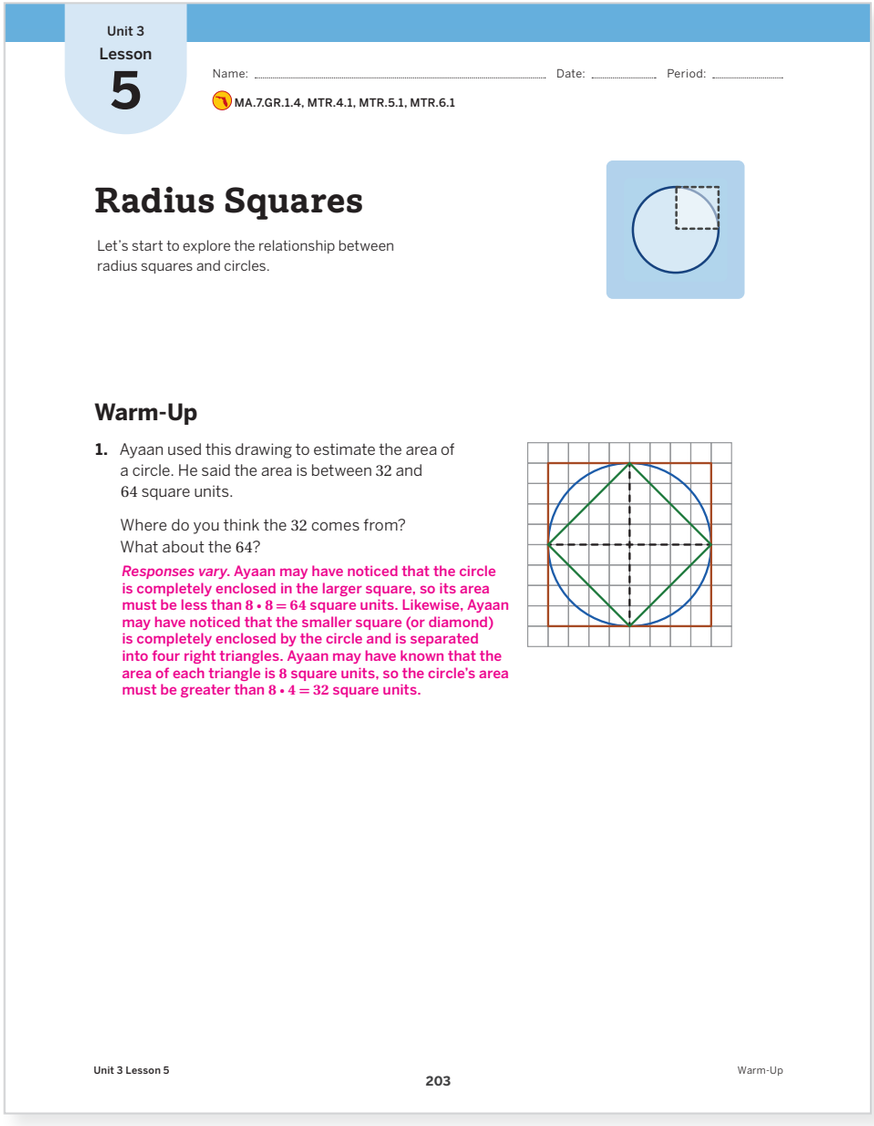
A **Accessibility: Visual-Spatial Processing** Invite students to draw on the diagram to support interpreting visual representations.

ELL **English Language Learners** Consider providing a word bank to support students in expressing their ideas with words such as *area, radius, side length, square*, and *square units*. **(Writing and Speaking)**

2 Connect

Capture and share a variety of student ideas about Ayaan's diagram, including ones similar to the sample responses.

Note: If it does not come up naturally, consider sharing that the area of the outer square can be represented as $8 \cdot 8$ or 8^2.



Unit 3
Lesson
5

Name: ..... Date: ..... Period: .....

MA.7.GR.1.4, MTR.4.1, MTR.5.1, MTR.6.1

Radius Squares

Let's start to explore the relationship between radius squares and circles.

Warm-Up

1. Ayaan used this drawing to estimate the area of a circle. He said the area is between 32 and 64 square units.

Where do you think the 32 comes from? What about the 64?

Responses vary. Ayaan may have noticed that the circle is completely enclosed in the larger square, so its area must be less than $8 \cdot 8 = 64$ square units. Likewise, Ayaan may have noticed that the smaller square (or diamond) is completely enclosed by the circle and is separated into four right triangles. Ayaan may have known that the area of each triangle is 8 square units, so the circle's area must be greater than $8 \cdot 4 = 32$ square units.

Unit 3 Lesson 5 203 Warm-Up

Pairs | 10 min

Activity 1 Estimating Circle Area

Purpose: Students estimate the area of a circle and are introduced to radius squares as a tool for estimating circle area.

1 Launch

To surface connections, consider asking, "What do you notice about the circle and the radius square?"

ELL **English Language Learners** Invite students to share what each word in the new term *radius square* reminds them of. (e.g., "When I hear the word radius, I think ____." and "When I hear the word square, I think of____."). Then invite students to share what they think it means when the two words are used together. **(Listening and Speaking)**

2 Monitor

Encourage students to share their responses to Problems 3 and 4 with a partner.

Consider asking, "Is there only one way to draw the radius square for the circle?"

Select and sequence estimation strategies for Ayaan's circle to share during the Connect.

D **Differentiation** (*Problem 5*)

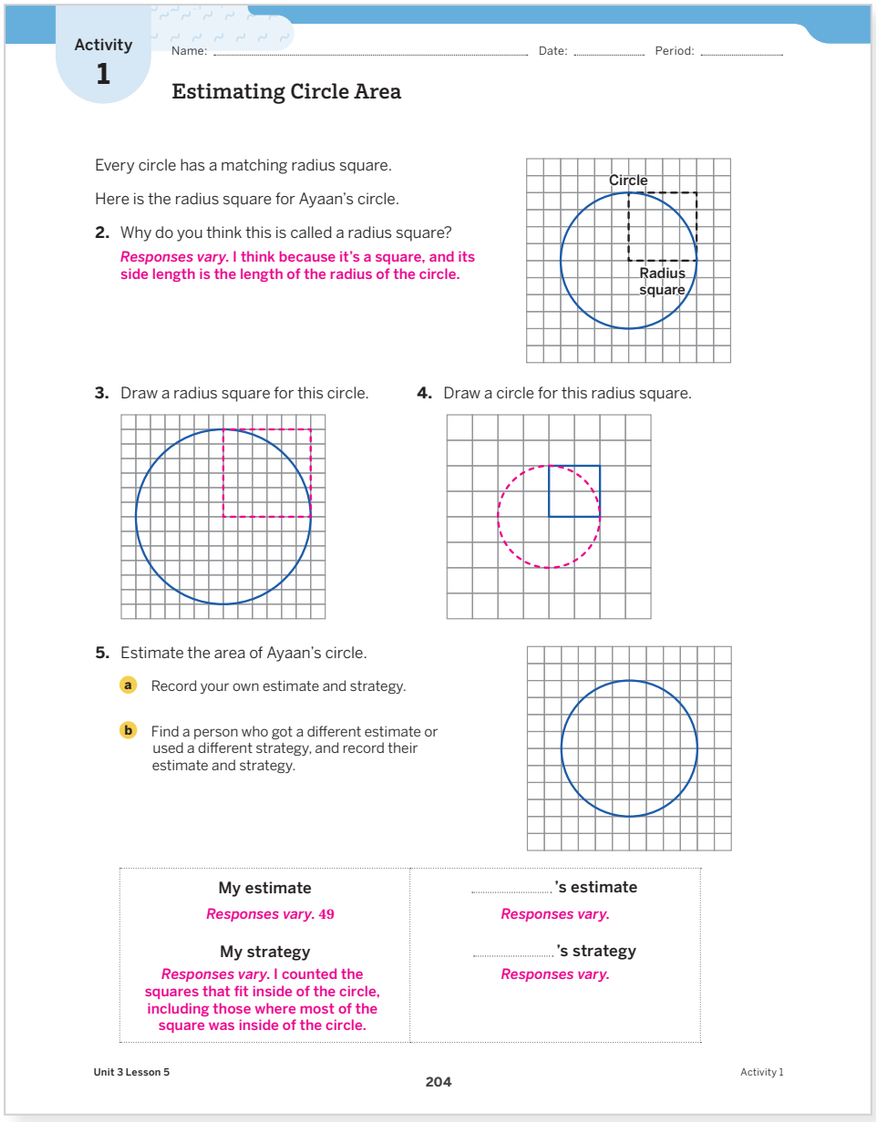
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer back to the Warm-Up. Consider asking, "What's an estimate that is too high? Too low?"
Count whole and partial squares.	**Strengthen** Consider asking, "How might you make your estimate more precise?"
Use the area of a radius square to make an estimate.	**Strengthen** Consider asking, "How did you decide how many radius squares would cover the circle?"
Make an overestimate and an underestimate.	**Strengthen** Consider asking, "Which estimate do you think is closer to the exact area? How do you know?" **(MTR.6.1)**

3 Connect

Display the captured strategies and discuss each one.

Consider asking, "Would this strategy work for any circle? How do you know?"

Discuss why there might be a range of responses to this question, emphasizing that our estimates can still be useful even if they do not represent the exact area of the circle. **(MTR.6.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Estimating Circle Area

Every circle has a matching radius square.

Here is the radius square for Ayaan's circle.

2. Why do you think this is called a radius square?

Responses vary. I think because it's a square, and its side length is the length of the radius of the circle.

3. Draw a radius square for this circle.

4. Draw a circle for this radius square.

5. Estimate the area of Ayaan's circle.

a. Record your own estimate and strategy.

b. Find a person who got a different estimate or used a different strategy, and record their estimate and strategy.

My estimate	_____'s estimate
Responses vary. 49	*Responses vary.*
My strategy	**_____'s strategy**
Responses vary. I counted the squares that fit inside of the circle, including those where most of the square was inside of the circle.	*Responses vary.*

Unit 3 Lesson 5 204 Activity 1

Small Groups | 10 min

Activity 2 Squares Cover Circle

Purpose: Students visualize the relationship between the radius and area of a circle by cutting and rearranging radius squares to approximate the area of a circle.

Short on time? Consider completing Problem 8 as a class.

1 Launch

Direct students' attention to the instructions and read them aloud, clarifying them as a class.

Distribute the Activity 2 Cards, scissors, and tape.

2 Monitor

Look for a variety of strategies students use to determine the number of radius squares needed to cover each circle.

D Differentiation (*Problem 6*)

Look for students who:	Respond to Student Thinking
Cover the circle with all four squares, rather than cutting.	**Support** Consider asking, "How could you cut the radius squares to fit inside the circle? Do you think you would need to use all four?"
Trim the excess off of all four squares and need support counting how many radius squares were used.	**Support** Encourage them to try using some of the trimmed scraps to cover part of the circle. Consider asking, "How many whole squares do you need? How many parts of squares?"
Finish early.	**Stretch** Encourage them to use graph paper to determine how many radius squares it would take to cover a circle of their own choosing.

Listen for groups who notice that it takes about 3 radius squares to cover the entire circle. Capture a variety of language students use to describe this amount.

3 Connect

Direct students' attention to the table and invite students from several groups to share how many radius squares they used to cover each circle.

Use the Notice and Wonder routine to promote curiosity and help students make connections between different parts of the table and different groups' responses. **(MTR.4.1)**

Direct students' attention to the circle. Consider sharing that this example is an opportunity to apply the patterns they noticed in the table to develop a strategy for estimating area. **(MTR.5.1)**

To surface the Key Takeaway, consider asking:

- "Why do you think Ayaan would write the area of the radius square as 6^2?"
- "Why might it be helpful to know about how many radius squares are needed to cover a circle?"

Activity 2

Name: _____ Date: _____ Period: _____

Squares Cover Circle

Let's find out how many radius squares are needed to cover a circle.

You will get four circles and four sets of radius squares. For each circle, cut up the radius squares and rearrange the pieces to cover the circle.

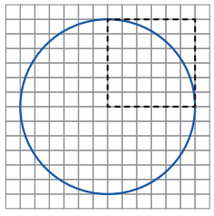
6. Use your work to complete this table.

	Radius of the Circle (units)	Number of Radius Squares Needed to Cover the Circle
Circle A	9	*Responses vary.* ~3
Circle B	2	*Responses vary.* ~3
Circle C	7	*Responses vary.* ~3
Circle D	5	*Responses vary.* ~3

7. In general, how many radius squares do you think can cover a circle?

Responses vary. It takes about 3 radius squares to cover a circle.

8. Here is a circle.



a. Ayaan says the area of the radius square is 6^2. Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Explanations vary. 6^2 is the same as $6 \cdot 6$. The side length of the square is 6, and to calculate its area you multiply the side length by itself.

b. Estimate the area of the circle.

Responses vary. A little more than 108 square units

Explain your thinking.

Explanations vary. The area of the radius square is 36 square units and there are a little more than 3 radius squares in a circle. $36 \cdot 3 = 108$, so the area of the circle would be a little more than 108 square units.

9. **Discuss:** Why might it be helpful to know about how many radius squares are needed to cover a circle?

Responses vary. Knowing this can help me estimate the area of a circle. It's easy to calculate the area of a radius square. You just multiply the radius by itself. If I know how many radius squares are needed to cover a circle, I can multiply that value by the area of the radius square to approximate the area of the circle.

Unit 3 Lesson 5 205 Activity 2

Key Takeaway: It takes a little more than 3 radius squares to cover a circle. The area of each radius square is $r \cdot r$, or r^2, where r represents the radius of the circle.

Pairs | 10 min

Activity 3 Piecing It Together

Purpose: Students discover the formula for the area of a circle by rearranging the equal-sized wedges of a circle to form a rectangle. **(MTR.5.1)**

1 Launch

Consider sharing that students will cut apart a circle and rearrange the parts to form another shape.

Distribute the Activity 3 Sheet, scissors, and tape.

A **Accessibility: Fine Motor Skills** To support students using scissors, consider pairing students, assigning one student to cut the wedges and another student to tape the wedges together.

To support prior knowledge, consider asking:

- "What is circumference?"
- "What are some real-world examples of circumference?"
- "What will you highlight on the circle in order to show the circumference?"

2 Monitor

Use the Notice and Wonder **routine** to encourage students to look for what has changed and what has not changed when rearranging the parts of the circle to form a rectangle. See the Routine Facilitation Guide for more information.

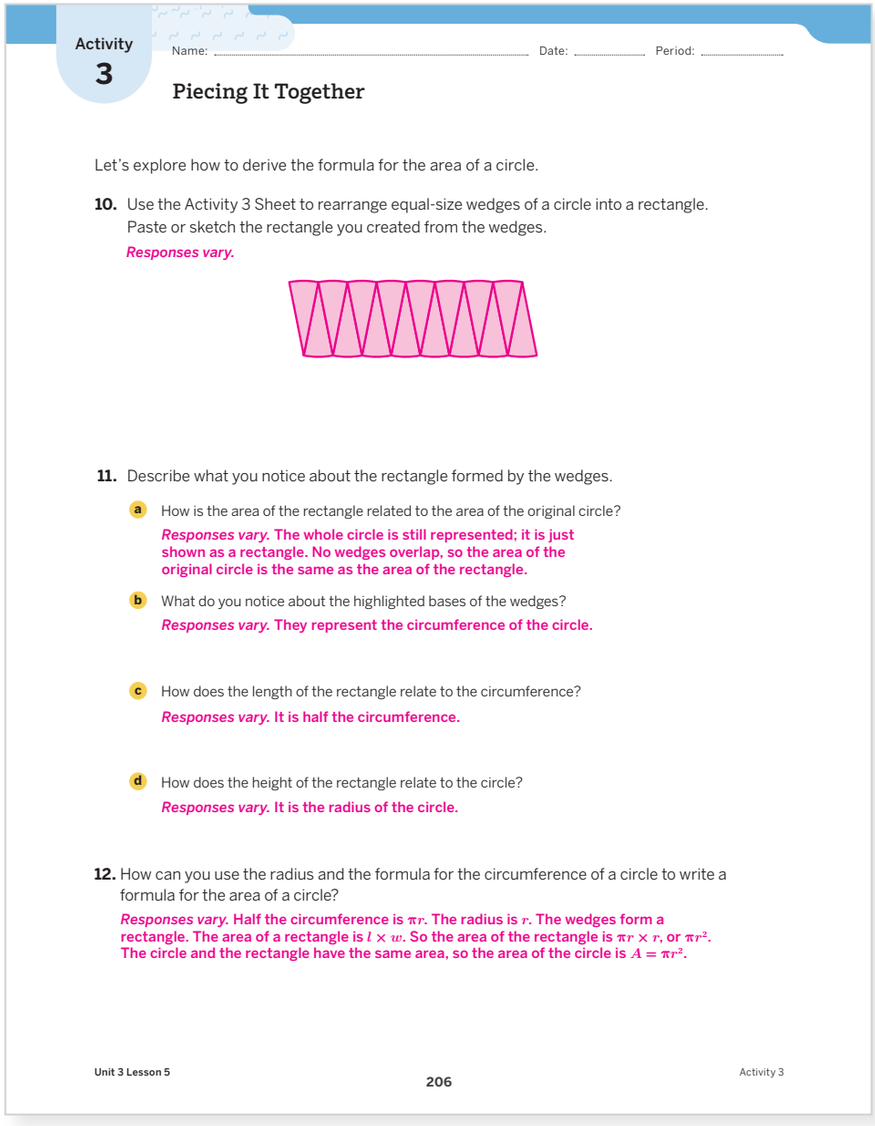
Consider asking, "Have you lost any part of the original circle?"

Look for students who write the formula for the circumference of half of the circle using r: $\frac{1}{2}\pi d = \pi r$.

3 Connect

Invite students to share their strategies for using the rectangle they created to develop the formula for the area of a circle.

Create an anchor chart that shows how a rectangle formed from a circle relates to the circumference and radius of the circle by labeling the rectangle's length as πr and its width as r. **(MTR.5.1)**



Activity 3

Name: _____ Date: _____ Period: _____

Piecing It Together

Let's explore how to derive the formula for the area of a circle.

10. Use the Activity 3 Sheet to rearrange equal-size wedges of a circle into a rectangle. Paste or sketch the rectangle you created from the wedges.

Responses vary.

11. Describe what you notice about the rectangle formed by the wedges.

a. How is the area of the rectangle related to the area of the original circle?

Responses vary. The whole circle is still represented; it is just shown as a rectangle. No wedges overlap, so the area of the original circle is the same as the area of the rectangle.

b. What do you notice about the highlighted bases of the wedges?

Responses vary. They represent the circumference of the circle.

c. How does the length of the rectangle relate to the circumference?

Responses vary. It is half the circumference.

d. How does the height of the rectangle relate to the circle?

Responses vary. It is the radius of the circle.

12. How can you use the radius and the formula for the circumference of a circle to write a formula for the area of a circle?

Responses vary. Half the circumference is πr. The radius is r. The wedges form a rectangle. The area of a rectangle is $l \times w$. So the area of the rectangle is $\pi r \times r$, or πr^2. The circle and the rectangle have the same area, so the area of the circle is $A = \pi r^2$.

Unit 3 Lesson 5 206 Activity 3

Whole Class | 5 min

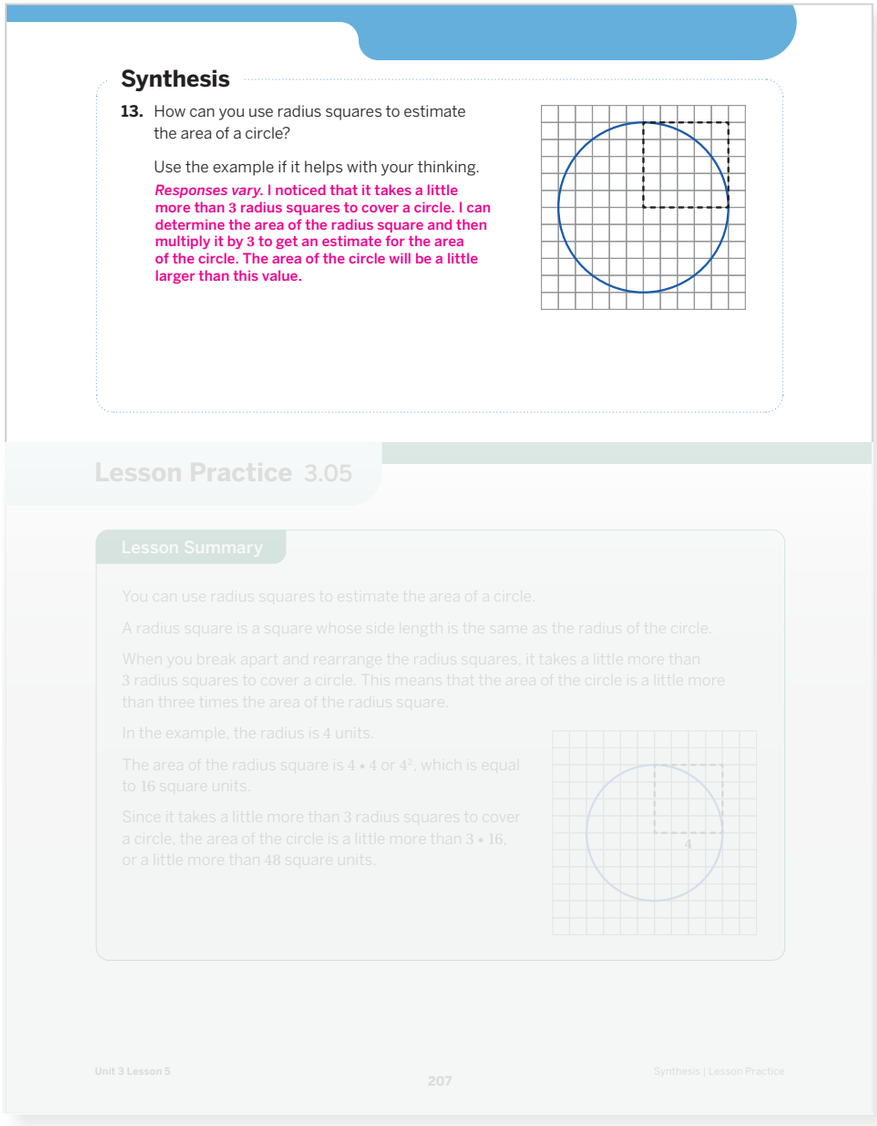
Synthesis

Purpose: Students synthesize their understanding of using radius squares to estimate the areas of circles.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- It takes a little more than 3 radius squares to cover a circle.
- Calculating the area of a radius square and multiplying it by 3.



Synthesis

13. How can you use radius squares to estimate the area of a circle?

Use the example if it helps with your thinking.

Responses vary. **I noticed that it takes a little more than 3 radius squares to cover a circle. I can determine the area of the radius square and then multiply it by 3 to get an estimate for the area of the circle. The area of the circle will be a little larger than this value.**

Lesson Practice 3.05

Lesson Summary

You can use radius squares to estimate the area of a circle.

A radius square is a square whose side length is the same as the radius of the circle.

When you break apart and rearrange the radius squares, it takes a little more than 3 radius squares to cover a circle. This means that the area of the circle is a little more than three times the area of the radius square.

In the example, the radius is 4 units.

The area of the radius square is $4 \cdot 4$ or 4^2, which is equal to 16 square units.

Since it takes a little more than 3 radius squares to cover a circle, the area of the circle is a little more than $3 \cdot 16$, or a little more than 48 square units.

4

Unit 3 Lesson 5 207 Synthesis | Lesson Practice

Lesson Takeaway: You can estimate the area of a circle by determining the area of a radius square, then multiplying it by 3. Each radius square has an area of r^2, where r represents the radius of the circle. The area of the circle is a little more than 3 times this area.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by estimating the area of a circle.

Today's Goals

1. **Goal:** Estimate the area of a circle.
2. **Language Goal:** Informally describe the relationship between the radius of any circle and its area by referring to radius squares. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.05

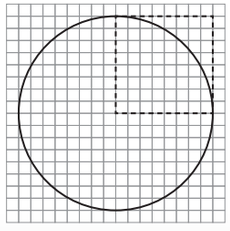
Here is a circle and one of its radius squares.

a. About how many radius squares does it take to cover the circle?

Responses vary. About 3 radius squares.

b. Estimate the area of the circle. Show your thinking.

Responses and work vary. About 192 square units.
$8 \cdot 8 = 64$
$3 \cdot 64 = 192$



Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support visualizing the meanings of r and r 2:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of this concept in Lesson 6. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 2 Extension Activities

Professional Learning

In this lesson, students are introduced to radius squares. Why was the context of radius squares particularly useful in building students' understanding of the area of circles in this lesson? In Lesson 6, students will generate a formula for calculating the exact area of a circle. What opportunities might there be to connect back to the strategies students shared today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. How can you use radius squares to estimate the area of a circle?

Use the example if it helps with your thinking.

Responses vary. I noticed that it takes a little more than 3 radius squares to cover a circle. I can determine the area of the radius square and then multiply it by 3 to get an estimate for the area of the circle. The area of the circle will be a little larger than this value.

Lesson Practice 3.05

Lesson Summary

You can use radius squares to estimate the area of a circle.

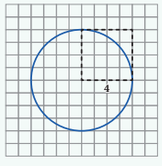
A radius square is a square whose side length is the same as the radius of the circle.

When you break apart and rearrange the radius squares, it takes a little more than 3 radius squares to cover a circle. This means that the area of the circle is a little more than three times the area of the radius square.

In the example, the radius is 4 units.

The area of the radius square is $4 \cdot 4$ or 4^2, which is equal to 16 square units.

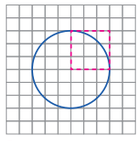
Since it takes a little more than 3 radius squares to cover a circle, the area of the circle is a little more than $3 \cdot 16$, or a little more than 48 square units.



Lesson Practice 3.05

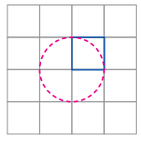
Name: _____ Date: _____ Period: _____

1. Draw a radius square for this circle.



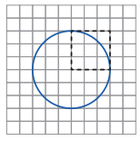
Responses vary.

2. Draw a circle for this radius square.



Responses vary.

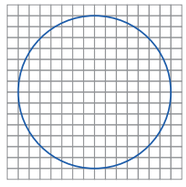
Problems 3–4: Prisha says the area of this circle is a little more than 27 square units.



3. What is one way Prisha might have determined her estimate?

Responses vary. Prisha might know that it takes a little bit more than 3 radius squares to cover a circle. The radius of this circle is 3 units, so each radius square has an area of 9 square units. 9 multiplied by 3 is 27.

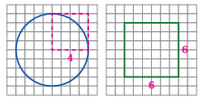
4. Estimate the area of this new circle. Show or explain your thinking.



Responses vary. A little more than 147 square units. The radius of the circle is 7 units, so the area of a radius square is 49 square units. 49 multiplied by 3 is 147.

FAST Practice

5. Compare the areas of the circle and the square.



Select **ONE** correct answer in each box to complete the sentence.

The area of the (A.) circle B. square is larger because the circle's area is a little over A. 24 B. 36 (C.) 48 square units and the square's area is A. 24 (B.) 36 C. 48 square units.

Lesson Practice 3.05

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 6–8: When the High Roller observation wheel opened in Las Vegas, NV, in 2014, it became the tallest observation wheel in the world, breaking the record set by the Singapore Flyer in Downtown Core, Singapore, in 2008.

Singapore Flyer



High Roller



6. The Singapore Flyer has a diameter of 150 meters. If you rode all the way around the observation wheel, how far would you travel?

$150\pi \approx 471$ meters

7. The High Roller has a diameter of 158.5 meters. If you rode all the way around the observation wheel, how far would you travel?

$158.5\pi \approx 498$ meters

8. If you rode once all the way around each observation wheel, how much farther would you travel on the High Roller than on the Singapore Flyer?

I would travel about 27 meters farther on the High Roller than on the Singapore Flyer.

9. The cost for 16 ounces of strawberries is $4.00.

Fill in the blank to write an equation that represents the relationship between the number of ounces of strawberries, s, and the cost, c.

$c =$ 0.25 s

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.GR.1.4
	3–4	2	MA.7.GR.1.4
FAST Practice	5	2	MA.7.GR.1.4
Spiral Review			
	6–7	2	MA.7.GR.1.3
	8	2	MA.7.GR.1.3
	9	1	MA.7.AR.4.2 MA.7.AR.4.4

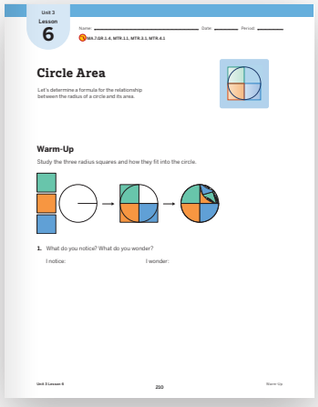
Unit 3

Lesson 6

Circle Area

Calculating Circle Area

Let's determine a formula for the relationship between the radius of a circle and its area.


Unit 3 Lesson 6

Name: Date: Period:

MA.7.GR.1.4, MTR.1.1, MTR.3.1, MTR.4.1

Circle Area

Let's determine a formula for the relationship between the radius of a circle and its area.

Warm-Up

Study the three radius squares and how they fit into the circle.

1. What do you notice? What do you wonder?

I notice: I wonder:

210

Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the area of a circle given the radius.
2. **Language Goal:** Describe the relationship between the radius of any circle and its area. **(Writing)**

Students discover a formula for the relationship between the radius of a circle and its area. This is the second of two lessons that deal with calculating the areas of circles. Students build on their noticing that the area of a circle is slightly greater than the area of 3 radius squares, identifying that it is exactly equal to the area of π radius squares. They calculate the exact areas of circles using the formula $A = \pi r^2$.

Coherence

Prior Learning

In Lesson 5, students explored how radius squares can be used to estimate the area of a circle. They determined that the area of a circle is slightly more than 3 times the area of a radius square.

Future Learning

In Unit 7, students will use the formula for the area of a circle to determine surface areas and volumes of cylinders.

Connections

- Students calculate the area of circular figures such as pizzas, bike wheels, and coins. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.4

Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	**MA.7.GR.2.1** **MA.7.GR.2.2** **MA.7.GR.2.3**

Lesson at a Glance

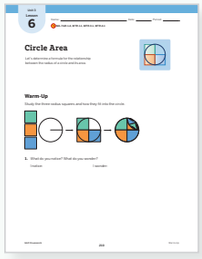
~ 45 min

MA.7.GR.1.4

Warm-Up

Independent | 5 min

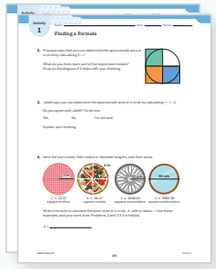
Students compare the area of three radius squares to the area of a circle.
Routine: Notice and Wonder



Activity 1

Pairs | 15 min

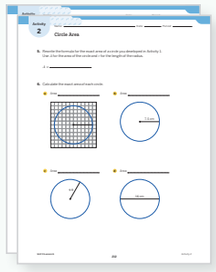
Students use repeated reasoning to write a formula to calculate the area of a circle given the radius length.
Routine: Think-Pair-Share



Activity 2

Pairs | 15 min

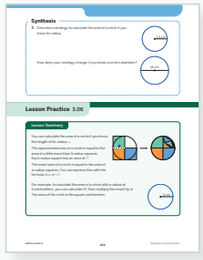
Students develop fluency with calculating the exact areas of circles. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

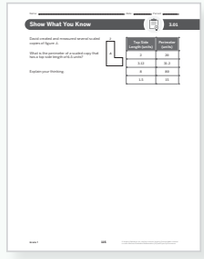
Students synthesize their understanding of how to calculate the exact area of a circle.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the area of a circle given its diameter.



Prep Checklist

Invite students to use their Student Edition.

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students compare the area of three radius squares to the area of a circle.

1 Launch

Direct students' attention to the images of the three radius squares and the three circles.

Use the Notice and Wonder routine to promote curiosity and surface prior knowledge about the relationship between the area of the radius squares and the area of the circle. See the Routine Facilitation Guide for more information.

2 Connect

Invite students to share what they noticed and wondered.

Consider asking, "Did anyone notice that there is some white space left over in the last circle? Does this make you wonder about how many radius squares would cover the circle completely?"

Unit 3
Lesson
6

Name: _____ Date: _____ Period: _____

MA.7.GR.1.4, MTR.1.1, MTR.3.1, MTR.4.1

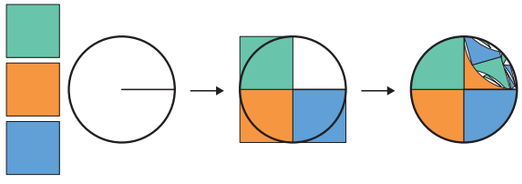
Circle Area

Let's determine a formula for the relationship between the radius of a circle and its area.



Warm-Up

Study the three radius squares and how they fit into the circle.



1. What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice that 3 radius squares do not exactly equal the area of the circle.
- I notice there is white space left over, so the exact number of radius squares is slightly more than 3.

I wonder:

Responses vary.

- I wonder how much more the area of the circle is compared to the area of the 3 radius squares.
- I wonder if there is a formula that I can use to more accurately calculate the area of a circle.

Unit 3 Lesson 6 210 Warm-Up

Pairs | 15 min

Activity 1 Finding a Formula

Purpose: Students use repeated reasoning to write a formula to calculate the area of a circle given the radius length.

1 Launch

Direct students' attention to the image of the three radius squares.

Use the Think-Pair-Share routine to help students shape their ideas about Precious's expression. See the Routine Facilitation Guide for more information.

Invite students to share where they see the 3 and the r^2 in the diagram.

2 Monitor

Encourage students to share their reasoning with a partner and reach an agreement about Jaleel's expression. **(MTR.4.1)**

ELL **English Language Learners** To support students in sharing their thinking, consider displaying sentence frames such as:

- The r represents _____.
- The 3 represents _____.
- I see _____ in both strategies.
- What is clear to me is _____. What is unclear to me is _____. **(Writing and Speaking)**

Select and sequence a variety of correct and incorrect formulas to share during the Connect.

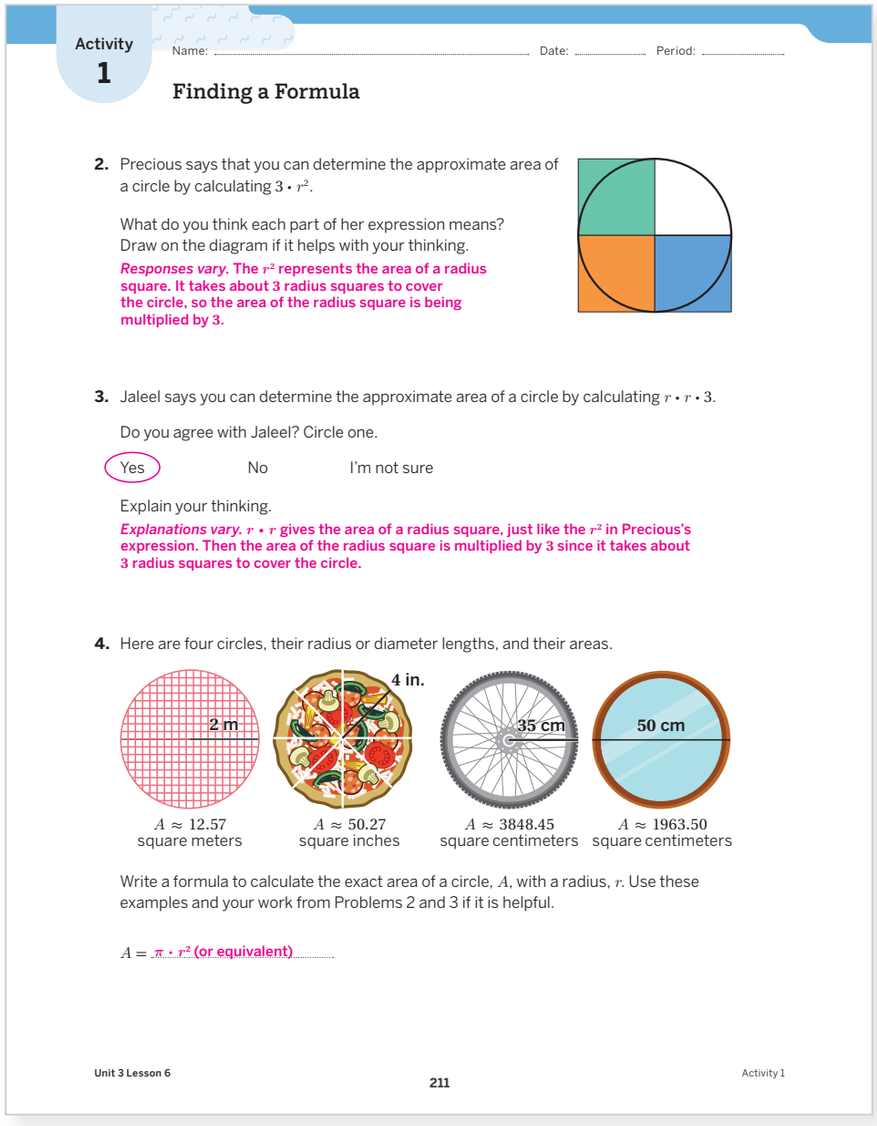
D **Differentiation** (*Problem 4*)

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try applying Precious's or Jaleel's formula to one of the example circles. Consider asking, "How would you adjust the formula to get closer to the given area?"
Divide the area of a given circle by its radius.	**Support** Encourage them to test their formula on the other examples by substituting in the given radius for r.
Work backwards from the given areas to generate a formula using π or 3.14.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.

3 Connect

Direct students' attention to the example circles and test several formulas one by one to see if they result in the correct areas. **(MTR.1.1)**

Create a record of formulas that work for all four circles.



Activity 1

Name: _____ Date: _____ Period: _____

Finding a Formula

2. Precious says that you can determine the approximate area of a circle by calculating $3 \cdot r^2$.

What do you think each part of her expression means? Draw on the diagram if it helps with your thinking.

Responses vary. The r^2 represents the area of a radius square. It takes about 3 radius squares to cover the circle, so the area of the radius square is being multiplied by 3.

3. Jaleel says you can determine the approximate area of a circle by calculating $r \cdot r \cdot 3$.

Do you agree with Jaleel? Circle one.

Yes No I'm not sure

Explain your thinking.

Explanations vary. $r \cdot r$ gives the area of a radius square, just like the r^2 in Precious's expression. Then the area of the radius square is multiplied by 3 since it takes about 3 radius squares to cover the circle.

4. Here are four circles, their radius or diameter lengths, and their areas.

2 m — $A \approx 12.57$ square meters

4 in. — $A \approx 50.27$ square inches

35 cm — $A \approx 3848.45$ square centimeters

50 cm — $A \approx 1963.50$ square centimeters

Write a formula to calculate the exact area of a circle, A, with a radius, r. Use these examples and your work from Problems 2 and 3 if it is helpful.

$A =$ $\pi \cdot r^2$ (or equivalent)

Unit 3 Lesson 6 211 Activity 1

Key Takeaway: You can calculate the exact area of a circle using the formula $A = \pi r^2$. The r^2 represents the area of a radius square, and it takes π radius squares to completely cover a circle.

Pairs | 15 min

Activity 2 Circle Area

Purpose: Students develop fluency with calculating the exact areas of circles. **(MTR.3.1)**

Short on time? Consider inviting students to choose two or three out of the four circles in Problem 6.

1 Launch

To support students getting started, invite them to rewrite the formula they developed in Activity 1.

Consider asking, "What is the difference between an exact area and an *approximate* area?"

Consider sharing that in this activity, it is more important to make sense of the area formula than it is to work quickly. **(MTR.3.1)**

2 Monitor

Encourage students to share their reasoning with a partner and reach an agreement about the area of each circle.

Select and sequence a variety of student thinking.

D Differentiation (*Problem 6*)

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to restate what each part of the area formula represents, i.e., "The r stands for radius. The radius of this circle is 5 units."
Multiply the radius by 2 rather than squaring it.	**Support** Invite students to draw a radius square for the circle. Consider asking, "What is the area of the radius square?"
Use the diameter of the circle in place of the radius. (*Problem 6d*)	**Support** Consider asking, "How is this problem like the previous ones? How is it different?"
Use an approximate value for π.	**Strengthen** Celebrate these students' thinking. Consider asking, "How could you make your answer more precise?"

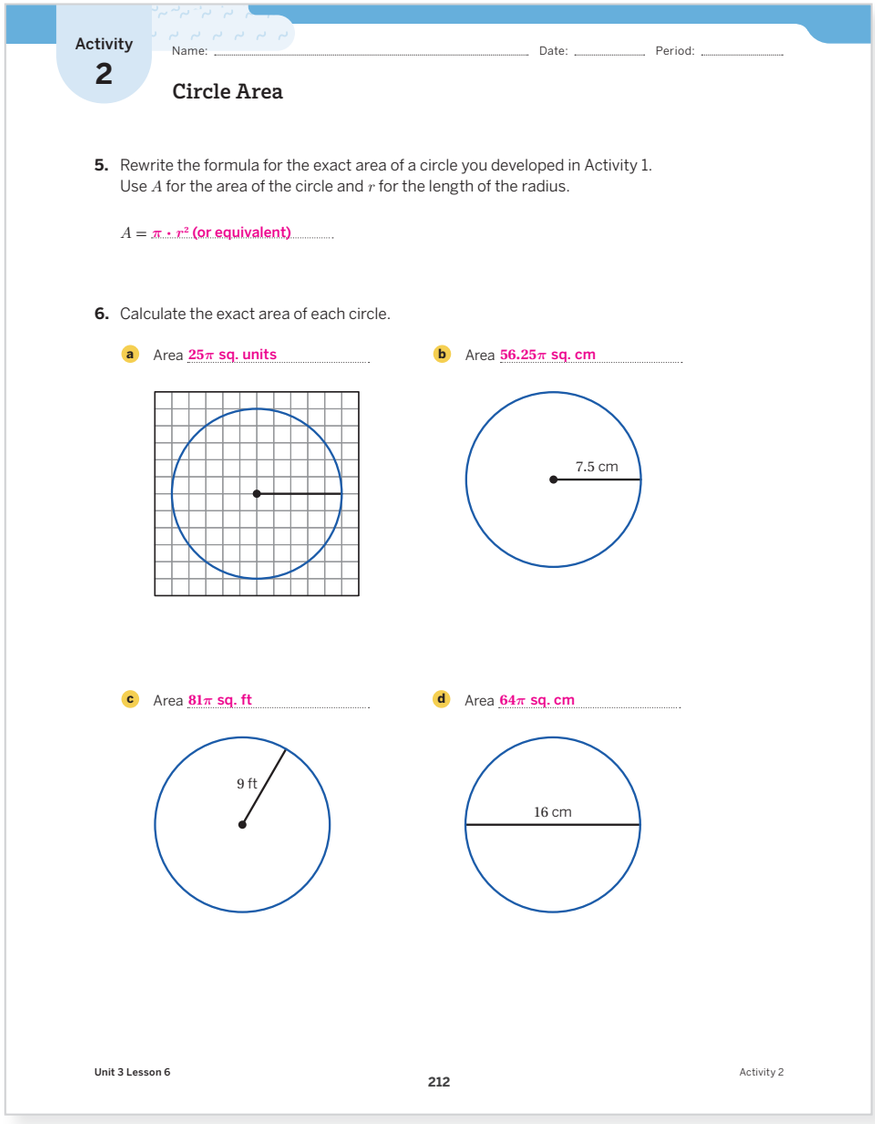
A Accessibility: Conceptual Processing Invite students to talk aloud about their thinking to support self-monitoring of understanding.

3 Connect

Direct students' attention to one or more of the circles from Problem 6.

Share the captured student work and invite the authors to discuss their thinking for the circle shown.

To surface the Key Takeaway, consider asking, "How does the process of calculating the area change when you are given the diameter instead of the radius?" **(MTR.3.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Circle Area

5. Rewrite the formula for the exact area of a circle you developed in Activity 1. Use A for the area of the circle and r for the length of the radius.

$A = \pi \cdot r^2$ (or equivalent)

6. Calculate the exact area of each circle.

a. Area 25π sq. units

b. Area 56.25π sq. cm

c. Area 81π sq. ft

d. Area 64π sq. cm

Unit 3 Lesson 6 212 Activity 2

Key Takeaway: You can calculate the exact area of a circle using the formula $A = \pi r^2$. If you know the diameter of a circle, you can divide it by 2 to determine the radius, then use the formula for area.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to calculate the exact area of a circle.

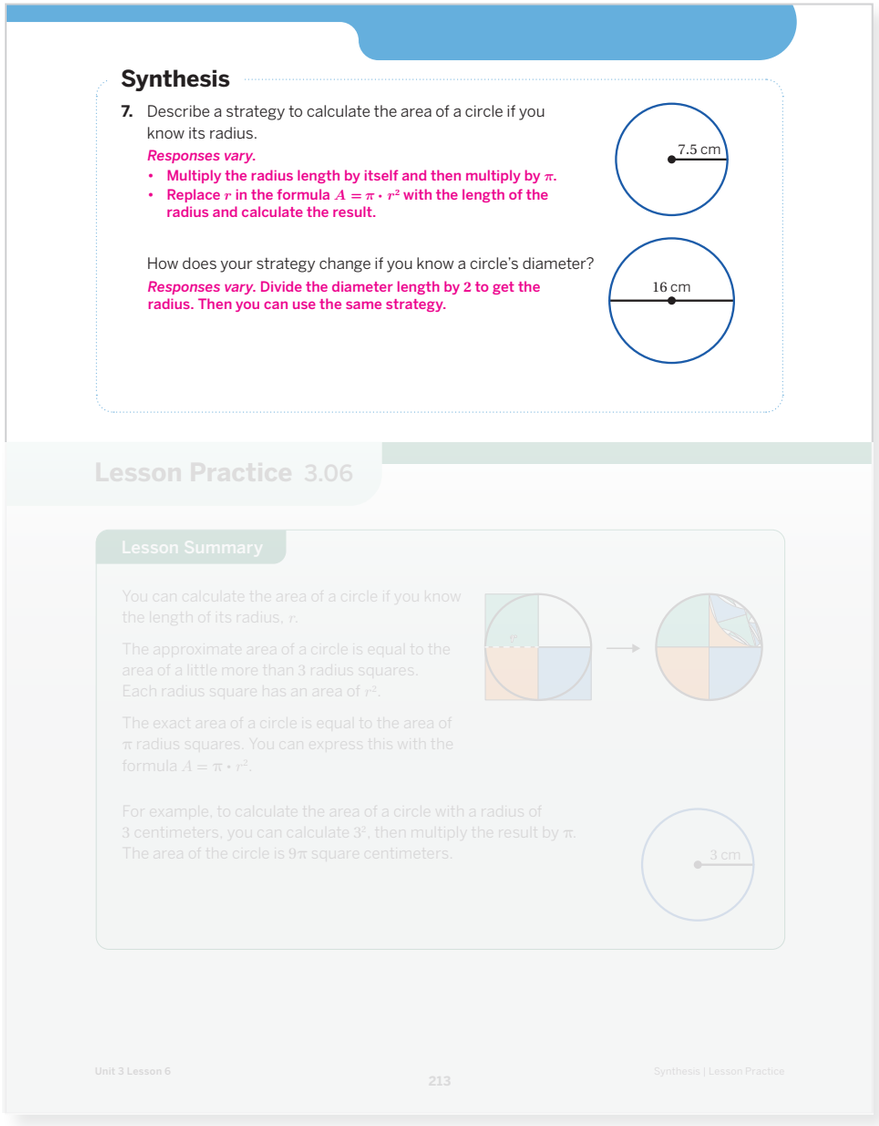
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Substituting a value for r into the formula.
- Multiplying the radius by itself, then multiplying by π.
- When given the diameter, dividing it by 2 to calculate the radius.

MLR **MLR2: Collect and Display** Use this routine to formalize a strategy for calculating the exact area of a circle using students' language. Display any new language, examples, or diagrams for students to reference later. Consider including a visual representation of r^2 and $2r$, to support students distinguishing these expressions.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

7. Describe a strategy to calculate the area of a circle if you know its radius.

Responses vary.

- Multiply the radius length by itself and then multiply by π.
- Replace r in the formula $A = \pi \cdot r^2$ with the length of the radius and calculate the result.

How does your strategy change if you know a circle's diameter?

Responses vary. Divide the diameter length by 2 to get the radius. Then you can use the same strategy.

Lesson Practice 3.06

Lesson Summary

You can calculate the area of a circle if you know the length of its radius, r.

The approximate area of a circle is equal to the area of a little more than 3 radius squares. Each radius square has an area of r^2.

The exact area of a circle is equal to the area of π radius squares. You can express this with the formula $A = \pi \cdot r^2$.

For example, to calculate the area of a circle with a radius of 3 centimeters, you can calculate 3^2, then multiply the result by π. The area of the circle is 9π square centimeters.

Unit 3 Lesson 6 213 Synthesis | Lesson Practice

Lesson Takeaway: The area of a circle is equal to the area of π radius squares. You can calculate the area of a circle using the formula $\pi \cdot r^2$, where r represents the radius of the circle.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by calculating the area of a circle given its diameter.

Today's Goals

1. **Goal:** Calculate the area of a circle given the radius.
2. **Language Goal:** Describe the relationship between the radius of any circle and its area. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.01

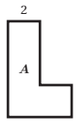
David created and measured several scaled copies of figure A.

What is the perimeter of a scaled copy that has a top side length of 6.5 units?

65 units

Explain your thinking.

Explanations vary. The table is proportional with a constant of proportionality of 10. $6.5 \cdot 10 = 65$ units.



Top Side Length (units)	Perimeter (units)
2	20
3.12	31.2
8	80
1.5	15

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the areas of circles:

» **Respond to Student Thinking:**

- Use the ***Determining the Area of a Circle*** **Mini-lesson.**
- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- If students would enjoy an additional challenge, invite them to compare and contrast the ways that we calculate the circumference and area of circles

Professional Learning

Today, students used their prior knowledge of circles and area to generate a formula that can be used to calculate the area of any circle. How did students tap into their creativity today? How did you recognize and encourage students' creativity?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Describe a strategy to calculate the area of a circle if you know its radius.

Responses vary.

- Multiply the radius length by itself and then multiply by π.
- Replace r in the formula $A = \pi \cdot r^2$ with the length of the radius and calculate the result.

How does your strategy change if you know a circle's diameter?

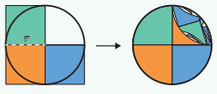
Responses vary. Divide the diameter length by 2 to get the radius. Then you can use the same strategy.

Lesson Practice 3.06

Lesson Summary

You can calculate the area of a circle if you know the length of its radius, r.

The approximate area of a circle is equal to the area of a little more than 3 radius squares. Each radius square has an area of r^2.



The exact area of a circle is equal to the area of π radius squares. You can express this with the formula $A = \pi \cdot r^2$.

For example, to calculate the area of a circle with a radius of 3 centimeters, you can calculate 3^2, then multiply the result by π. The area of the circle is 9π square centimeters.

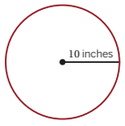


Lesson Practice 3.06

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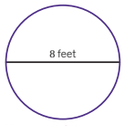
Problems 1–2: Calculate the exact area of each circle.

1.



100π square inches

2.



16π square feet

Problems 3–4: The table shows the diameters of 4 different coins.

Coin	Diameter (cm)	Area (sq. cm)
Penny	1.9	0.9025π
Nickel	2.1	1.1025π
Dime	1.8	0.81π
Quarter	2.4	1.44π

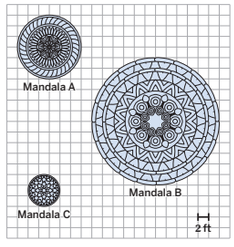
3. To determine how much metal is on one face of a coin, it is more useful to use the area rather than the circumference. Explain why this is the case.

Responses vary. The circumference measures the distance around the edge of the coin. The area measures the amount of metal needed to cover the face of the coin.

4. Complete the table by calculating the exact area of each coin.

5. A mandala is a geometric figure. The word *mandala* is Sanskrit for *circle*.

Saanvi designed three mandalas. Determine the exact circumference and area for each mandala.



	Circumference (ft)	Area (sq. ft)
A	12π	36π
B	24π	144π
C	6π	9π

Lesson Practice 3.06

Name: _____ Date: _____ Period: _____

FAST Practice

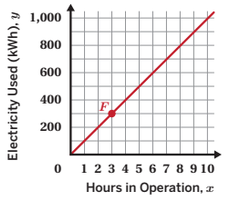
6. The radius of Earth is approximately 6,400 kilometers. The equator is the circle around Earth that divides it into the northern and southern hemispheres.

What is the length of the equator?

$12800\pi \approx 40212$ kilometers

Spiral Review

Problems 7–9: This graph shows a proportional relationship between the number of hours a manufacturing factory is in operation and the number of kilowatt-hours (kWh) of electricity used. Use the graph to determine whether each statement is true or false.



7. Point F represents the number of kilowatt-hours of electricity used when the factory is in operation for 3 hours.

True

8. The factory uses 6 kilowatt-hours of electricity when it is in operation for 600 hours.

False

9. The factory uses 700 kilowatt-hours of electricity when it is in operation for 7 hours.

True

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4	1	MA.7.GR.1.4
	3, 5	2	MA.7.GR.1.4
FAST Practice	6	2	MA.7.GR.1.4
Spiral Review			
	7–9	2	MA.7.AR.3.2

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Item Analysis Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Calculating the area of a circle given its radius	Lesson 6	1	MA.7.GR.1.3
2	Making sense of circumference/area values written in terms of π	Lesson 3, 6	2	MA7.GR.1.3, MTR.3.1
3	Determining whether quantities are in a proportional relationship from tables and graphs	Lesson 1, 3	2	MA.7.AR.4.1
4	Distinguishing area from circumference in various real-world contexts	Lesson 3, 4	2	MA.7.GR.1.3, MA.7.AR.1.4
5a	Calculating the area of circles	Lesson 4	1	MA.7.GR.1.3, MA.7.AR.1.4
5b	Determining whether the relationship between radius and area is proportional	Lesson 3	2	MA.7.AR.1.4
6	Solving a real-world problem involving circle area	Lesson 6	2	MA.7.AR.1.4, MTR.2.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



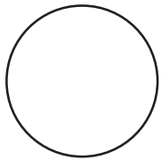
- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

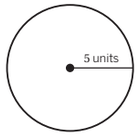
Unit 3

1. A circle has a radius of 50 centimeters. Which of these is closest to its area?



- **A.** 157 square centimeters
- **B.** 314 square centimeters
- **C.** 7,854 square centimeters
- **D.** 15,708 square centimeters

2. This circle has a radius of 5 units. Three students tried to calculate the circumference.



Order their answers from farthest from the exact circumference to closest to the exact circumference.

31.4 units	31.4π units	10π units
31.4π units	**31.4 units**	**10π units**
Farthest from exact circumference		**Closest to exact circumference**

3. Determine whether each relationship is proportional:

a The radius and circumference of a circle

Radius (cm)	Circumference (cm)
0	0
2	4π
5	10π

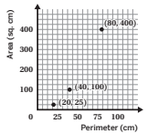
- ☑ Proportional
- ☐ Not proportional

b The radius and diameter of a circle

Radius (cm)	Diameter (cm)
5	10
10	20
15	30

- ☑ Proportional
- ☐ Not proportional

c The perimeter and area of a square



- ☐ Proportional
- ☑ Not proportional

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 3

4. Decide whether each quantity describes a circle's circumference or area. Circle your response.

a	The amount of paint needed to cover a circular canvas.	Circumference	(Area)
b	How long it takes to run around a circular track.	(Circumference)	Area
c	The amount of ribbon needed to wrap around a circular present.	(Circumference)	Area
d	The amount of grass inside of a circular track.	Circumference	(Area)

5. **a** Complete the table for the relationship between the radius and the area of a circle.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Radius (units)	Area (sq. units)
1	π or 3.14 or $\frac{22}{7}$
2	4π or 12.56 or $\frac{88}{7}$
3	9π or 28.26 or $\frac{198}{7}$

b Explain how you know the relationship between the radius of a circle and its area is *not* proportional. Use the table if it helps with your thinking.

Explanations vary. The relationship is not proportional because there isn't a constant of proportionality. When the radius is 1, you multiply by π since the formula for area is $A = \pi \cdot 1 \cdot 1$. When the radius is 2, you multiply by 2π because $A = \pi \cdot 2 \cdot 2$. The multiplier is not constant.

6. DeShawn needs grass seed to cover a circular field with a diameter of 200 feet. How many square feet does DeShawn need to cover with grass seed?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

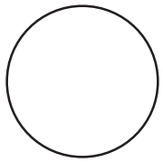
- 10000π square feet
- 31400 square feet
- $\frac{220000}{7}$ square feet

Name: Date: Period:

End-of-Unit Assessment

Unit 3

1. A circle has a radius of 40 centimeters. Which of these is closest to its circumference?



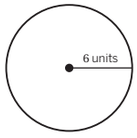
A. 126 centimeters

B. 251 centimeters

C. 1,600 centimeters

D. 5,027 centimeters

2. This circle has a radius of 6 units. Three students tried to calculate the area.



Order their answers from farthest from the exact area to closest to the exact area.

113.1 square units	36π square units	113.1π square units
113.1π sq. units	**113.1 sq. units**	**36π sq. units**

Farthest from exact area **Closest to exact area**

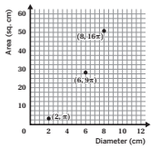
3. Determine whether each relationship is proportional:

a The diameter and radius of a circle

Diameter (cm)	Radius (cm)
8	4
16	8
24	12

☑ Proportional
☐ Not proportional

b The diameter and area of a circle



☐ Proportional
☑ Not proportional

c The radius and circumference of a circle

Radius (cm)	Circumference (cm)
1	2π
3	6π
4	8π

☑ Proportional
☐ Not proportional

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 3

4. Decide whether each quantity describes a circle's circumference or area. Circle your response.

a	The amount of glass in a circular window.	Circumference	(Area)
b	The number of tiles needed to go around a circular pool.	(Circumference)	Area
c	The amount of paint needed to paint a circular table top.	Circumference	(Area)
d	The distance a toy car travels around a circular track.	(Circumference)	Area

5. **a** Complete the table for the relationship between the radius and the circumference of a circle.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Radius (units)	Circumference (units)
1	2π or 6.28 or $\frac{44}{7}$
2	4π or 12.56 or $\frac{88}{7}$
3	6π or 18.84 or $\frac{132}{7}$

b Explain how you know the relationship between the radius of a circle and its circumference is proportional. Use the table if it helps with your thinking.

Explanations vary. The relationship is proportional because there is a constant of proportionality. In each row of the table, you multiply the radius by 2π to get the circumference.

6. Brielle needs to paint the bottom of a circular pool with a diameter of 24 feet. How many square feet does Brielle need to cover with paint?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- 144π square feet
- 452.16 square feet
- $\frac{3168}{7}$ square feet

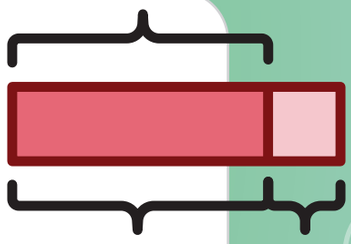
D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: Use the relationships between the radius diameter and circumference of a circle to calculate unknown measurements. **(Lessons 1–3)**	1, 2, 5a	**Support**	• **Mini-Lesson:** Determining the Circumference of a Circle (ML 3.03) • **Teacher Move:** Consider revisiting Unit 3, Lesson 3 (Measuring Around), Lesson 6 (Circle Area)
		Strengthen	• **Teacher Move:** Invite students to share aloud the formulas for area and circumference and discuss how to evaluate them when given the radius or diameter.
		Stretch	• **You're invited to explore more:** Lesson 2 (Is It A Circle?)
Sub-Unit 2: Calculate the areas of circles **(Lessons 4–6)**	3, 5b	**Support**	• **Teacher Move:** Consider revisiting Unit 3, Lesson 3 (Measuring Around)
		Strengthen	• **Teacher Move:** Invite students to share strategies for determining if relationship are proportional
	4	**Support**	• **Teacher Move:** Consider reviewing Problem 4 and then inviting students to generate additional examples of area and circumference in the real world.
		Strengthen	• **Teacher Move:** Invite students to share strategies for distinguishing the differences between area and circumference
	6	**Support**	• **Mini-Lesson:** Determining the Area of a Circle (ML 3.06)
		Strengthen	• **Teacher Move:** Consider revisiting Unit 3, Lesson 4 (Area Strategies)

UNIT 4

Proportional Relationships and Percentages



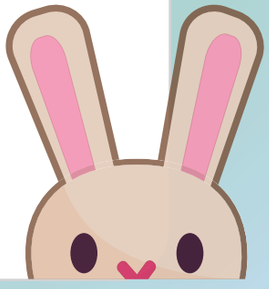
Students apply what they have learned about proportional relationships to solve problems involving percent change and fractional quantities.

Essential Questions

- How are percentages and fractions related to proportional relationships?
- How do proportional relationships and percentages represent change in the real world?

Sub-Unit Goals

Sub-Unit 1	• Solve problems involving percent change using representations, such as tape diagrams, double number lines, tables, and equations.
Sub-Unit 2	• Use proportional reasoning to solve problems in real-world situations involving percent increase, percent decrease, and percent error. • Interpret situations involving percent change and apply appropriate representations to solve problems.
Sub-Unit 3	• Determine unknown values in proportional relationships involving fractional quantities. • Convert between fraction and decimal representations.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Rewrite numbers in equivalent forms. **(MA.7.NSO.1)**

Use percentages and proportional reasoning to solve problems. **(MA.7.AR.3)**

Analyze and represent two-variable proportional relationships. **(MA.7.AR.4)**

Solve problems involving two-dimensional figures, including circles. **(MA.7.GR.1)**

Benchmarks Addressed in This Unit: MA.7.NSO.1.2 MA.7.AR.3.1 MA.7.AR.3.2 MA.7.AR.3.3 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5 MA.7.GR.1.5

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.6.1**

Coherence Telling a story within and across grades

In this Unit

Students apply proportional relationships to solve problems involving percent change and fractional quantities. They will deepen their understanding of multiple representations for percent change and apply this knowledge to solve a variety of real-world context problems.

Prior Learning

- **Grade 6,** Unit 3: Unit rate problems **(MA.6.AR.3.2, MA.6.AR.3.5)**
- **Grade 6,** Unit 3: Calculating percentages **(MA.6.AR.3.4)**
- **Grade 6,** Unit 5: Fractions, decimals, and percentages **(MA.6.NSO.3.5)**

Future Learning

- **Grade 7,** Unit 4: Solving equations **(MA.7.AR.2.2)**
- **High School:** Exponential functions **(MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–5**, students build conceptual understanding of percent increase and percent decrease as it relates to proportional relationships and answer questions about the world.

Strategic Competence

In **Lessons 4, 10, and 12**, students develop strategic competence as they compare different strategies, representations, and solution pathways to apply proportional relationships to solve problems involving percent change and fractional quantities.

Procedural Fluency

In **Lesson 3**, students gain fluency in solving percent increase or decrease problems to determine the original or new amount.

Adaptive Reasoning

In **Lesson 1**, students develop adaptive reasoning as they consider strategies using both tape diagrams and tables to make sense of percent increase and decrease.

Application

In **Lesson 6**, students apply their understanding of writing expressions and equations to solve percentage problems involving sales tax and tips.

Productive Disposition

In **Lesson 13**, students have opportunities to develop a productive disposition as they relate the mathematics they do in class to solving real issues of the world.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.NSO.3.5 MA.6.AR.1.4
MA.6.AR.3.4 MA.6.AR.3.5
MTR.2.1 MTR.5.1 MTR.7.1

Sub-Unit 1

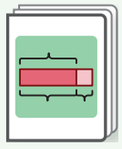


1 More and Less

Percent Increase and Decrease With Tape Diagrams

Use both tape diagrams and tables to represent **percent increase** and **percent decrease**, and then use these representations to solve problems.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.2.1

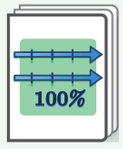


2 All the Equations

Percent Increase and Decrease With Equations

Write and use equations to describe the relationship between original and new amounts after a percent increase or decrease.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.1.1



3 100%

Percent Increase and Decrease With Double Number Lines

Expand on understanding of double number line diagrams and percentages to model and make sense of problems involving percent change.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.1.1 MTR.2.1

Sub-Unit 3

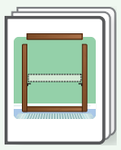


7 How Simple, How Interesting

Calculating Simple Interest Using a Formula

Use the simple interest formula to determine the interest earned on deposits and **interest** paid on loans.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.3.1 MTR.7.1



8 Bookcase Builder

Percent Error

Consider why **percent error** is useful, determine percent error, and decide whether a measurement is within an acceptable range.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.3.1 MTR.4.1



9 Sticker Sizes

Revisiting Proportional Relationships

Use tools, such as tables, constants of proportionality, and scale factors to determine unknown values in proportional relationships, some of which involve fractional quantities.

MA.7.AR.3.2 MA.7.AR.4.2
MA.7.AR.4.4 MA.7.AR.4.5
MA.7.GR.1.5
MTR.2.1 MTR.4.1



10 Peach Cobbler

Ratios and Rates With Fractions

Use constants of proportionality to compare proportional relationships involving fractional quantities.

MA.7.AR.3.2 MA.7.AR.4.2
MA.7.AR.4.5
MTR.4.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.AR.3.1 MA.7.AR.3.2
MA.7.AR.4.2 MA.7.AR.4.5
MTR.3.1 MTR.7.1

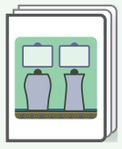
Pacing: 17 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 13 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Assess and Respond

Sub-Unit 2



4 Percent Machines

Calculating Unknowns With Percentages

Make connections between the representations of double number line diagrams, tables, and equations.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.2.1 MTR.4.1



5 Back in My Day

Real-World Situations Involving Percent Increase

Analyze and think critically about what it means for prices to increase by different percentages over time.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.6.1 MTR.7.1



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.AR.3.1
MTR.3.1 MTR.6.1 MTR.7.1



6 Tax, Tips, and Fees

Multistep Percent Problems

Solve multistep percent problems in a common context and calculate the total cost after applying sales tax, tips, and fees.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.3.1 MTR.4.1



11 Pen Pals

Converting Units

Convert measurements from one unit to another in different measurement systems using ratios.

MA.7.AR.3.3
MTR.2.1 MTR.4.1



12 Decimal Deep Dive

Writing Fractions as Decimals

Convert fractions to decimals using long division and recognize patterns in the calculations for classifying decimals as **terminating** or **repeating**.

MA.7.NSO.1.2
MTR.5.1



13 Population and Fast Fashion

Posing Problems Involving Percents

Ask and answer questions about the world by applying knowledge of increasing and decreasing percentages.

MA.7.NSO.1.2 MA.7.AR.3.1
MTR.4.1 MTR.7.1

M Math in Action

Connecting Percents to Finance

Explore the connection between percents, interest rates, and financial goals, as well as the contributions of Ebonya Washington to the field of public finance.

MA.7.NSO1.2 MA.7.AR.3.1
MA.7.AR.3.1
MTR.1.1

Pacing Considerations

Lesson 5: This lesson gives students an opportunity to use what they've learned about proportional relationships and percent change to analyze changes in costs over time. If students show a strong understanding of working with percent change in earlier lessons in the unit, this lesson can be omitted.

Lesson 13: This lesson gives students an opportunity to apply what they have learned about percent increase and decrease in this unit to make sense of a real-world situation. There is no new math content introduced in this lesson.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

Copy: Show What You Know

Lesson 2

Copy: Show What You Know

Materials: calculators *(as needed)*

Lesson 3

Copy: Show What You Know

Lesson 4

Copy: Show What You Know, Activity 3 Sheet *(one per student)*

Materials: scientific calculators *(as needed)*

Lesson 5

Copy: Show What You Know

Materials: calculators *(as needed)*

Lesson 6

Copy: Show What You Know

Materials: calculators

Lesson 7

Copy: Show What You Know

Materials: calculators

Lesson 8

Copy: Show What You Know

Lesson 9

Copy: Show What You Know

Materials: calculators

Lesson 10

Copy: Show What You Know

Materials: coloring tools *(optional)*

Lesson 11

Copy: Show What You Know

Lesson 12

Copy: Show What You Know

Materials: graph paper *(optional)*

Lesson 13

Copy: Show What You Know

Materials: calculators, coloring tools, poster paper

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

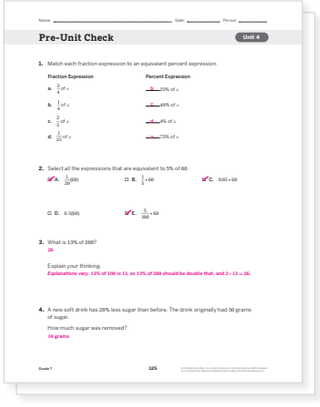
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.NSO.3.5, MA.6.AR.1.4, MA.6.AR.3.4, MA.6.AR.3.5



Name: _____ Date: _____ Period: _____

Pre-Unit Check Unit 4

1. Match each fraction expression to an equivalent percent expression.

Fraction Expression	Percent Expression
a. $\frac{1}{4}$ of x	25% of x
b. $\frac{1}{2}$ of x	50% of x
c. $\frac{2}{5}$ of x	4% of x
d. $\frac{2}{25}$ of x	75% of x

2. Select all the expressions that are equivalent to 5% of 60.
   - ✓ A. $\frac{1}{20}(60)$
   - ☐ B. $\frac{1}{5} \div 60$
   - ✓ C. 0.05×60
   - ☐ D. $0.5(60)$
   - ✓ E. $\frac{5}{60} \times 60$

3. What is 13% of 200?
   26
   Explain your thinking.
   Explanations vary. 13% of 100 is 13, so 13% of 200 should be double that, and 2 • 13 = 26.

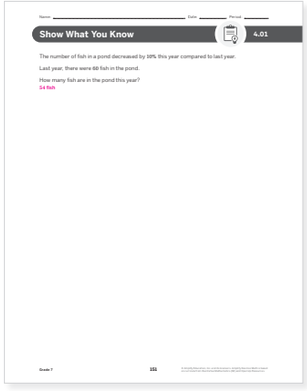
4. A new soft drink has 20% less sugar than before. The drink originally had 50 grams of sugar.
   How much sugar was removed?
   10 grams

Grade 7 125

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 1 Show What You Know: MA.7.NSO.1.2, MA.7.AR.3.1



Name: _____ Date: _____ Period: _____

Show What You Know 4.01

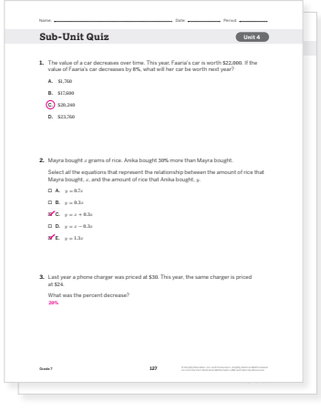
The number of fish in a pond decreased by 10% this year compared to last year.
Last year, there were 60 fish in the pond.
How many fish are in the pond this year?
54 fish

Grade 7 151

Sub-Unit Quizzes

Assign **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmark assessed: MA.7.AR.3.1



Name: _____ Date: _____ Period: _____

Sub-Unit Quiz Unit 4

1. The value of a car decreases over time. This year, Faaria's car is worth $22,000. If the value of Faaria's car decreases by 8%, what will her car be worth next year?
   - A. $1,760
   - B. $17,600
   - (C) $20,240
   - D. $23,760

2. Mayra bought x grams of rice. Anika bought 30% more than Mayra bought.
   Select all the equations that represent the relationship between the amount of rice that Mayra bought, x, and the amount of rice that Anika bought, y.
   - ☐ A. $y = 0.7x$
   - ☐ B. $y = 0.3x$
   - ✓ C. $y = x + 0.3x$
   - ☐ D. $y = x - 0.3x$
   - ✓ E. $y = 1.3x$

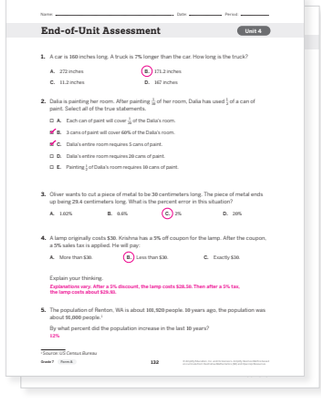
3. Last year a phone charger was priced at $30. This year, the same charger is priced at $24.
   What was the percent decrease?
   20%

Grade 7 127

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.AR.3.1, MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.5



Name: _____ Date: _____ Period: _____

End-of-Unit Assessment Unit 4

1. A car is 160 inches long. A truck is 7% longer than the car. How long is the truck?
   - A. 272 inches
   - (B) 171.2 inches
   - C. 11.2 inches
   - D. 167 inches

2. Dalia is painting her room. After painting $\frac{1}{2}$ of her room, Dalia has used $\frac{3}{4}$ of a can of paint. Select all of the true statements.
   - ☐ A. Each can of paint will cover $\frac{1}{8}$ of the Dalia's room.
   - ✓ B. 3 cans of paint will cover 66% of the Dalia's room.
   - ✓ C. Dalia's entire room requires 5 cans of paint.
   - ☐ D. Dalia's entire room requires 28 cans of paint.
   - ☐ E. Painting $\frac{1}{4}$ of Dalia's room requires 10 cans of paint.

3. Oliver wants to cut a piece of metal to be 30 centimeters long. The piece of metal ends up being 29.4 centimeters long. What is the percent error in this situation?
   - A. 1.602%
   - B. 9.4%
   - (C) 2%
   - D. 29%

4. A lamp originally costs $30. Krishna has a 5% off coupon for the lamp. After the coupon, a 5% sales tax is applied. He will pay:
   - A. More than $30.
   - (B) Less than $30.
   - C. Exactly $30.

   Explain your thinking.
   Explanations vary. After a 5% discount, the lamp costs $28.50. Then after a 5% tax, the lamp costs about $29.93.

5. The population of Renton, WA is about 100,920 people. 10 years ago, the population was about 91,000 people.*
   By what percent did the population increase in the last 10 years?
   11%

   Source: US Census Bureau

Grade 7 Form A 132

D Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

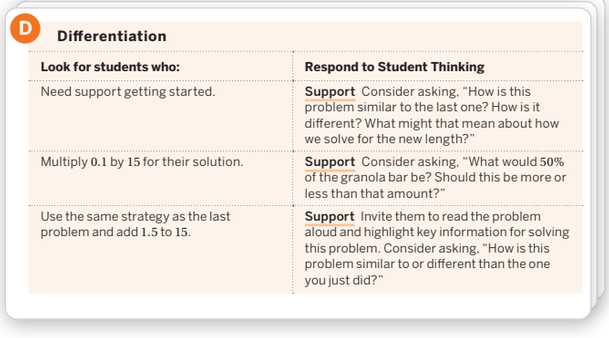
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Solve problems involving percent change using representations, such as tape diagrams, double number lines, tables, and equations.	**Mini-Lesson** • *Calculating Percentages*	**Challenge Creator** • Lesson 4 (*Percent Machines*) **Practice** • Lessons 1–5	**You're invited to explore more.** • Lesson 1 (*More and Less*) • Lesson 3 (*100%*) • Lesson 5 (*Back in My Day*)
Sub-Unit 2 (Lessons 6–8) • Use proportional reasoning to solve problems in real-world situations involving percent increase, percent decrease, and percent error. • Interpret situations involving percent change and apply appropriate representations to solve problems.	**Mini-Lesson** • *Finding Sales Tax and Tip*	**Repeated Challenges** • Lesson 8 (*Bookcase Builder*) **Practice** • Lessons 6–8	**You're invited to explore more.** • Lesson 8 (*Bookcase Builder*)
Sub-Unit 3 (Lessons 9–13) • Determine unknown values in proportional relationships involving fractional quantities. • Convert between fraction and decimal representations.	**Mini-Lesson** • *Applying Fractions in Proportional Relationships*	**Practice** • Lessons 9–13	**You're invited to explore more.** • Lesson 9 (*Sticker Sizes*) • Lesson 11 (*Pen Pals*)

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 1**, where students shape their ideas about percent decrease by solving for a new length when given the original length and percent change.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How is this problem similar to the last one? How is it different? What might that mean about how we solve for the new length?"
Multiply 0.1 by 15 for their solution.	**Support** Consider asking, "What would 50% of the granola bar be? Should this be more or less than that amount?"
Use the same strategy as the last problem and add 1.5 to 15.	**Support** Invite them to read the problem aloud and highlight key information for solving this problem. Consider asking, "How is this problem similar to or different than the one you just did?"

Accessibility

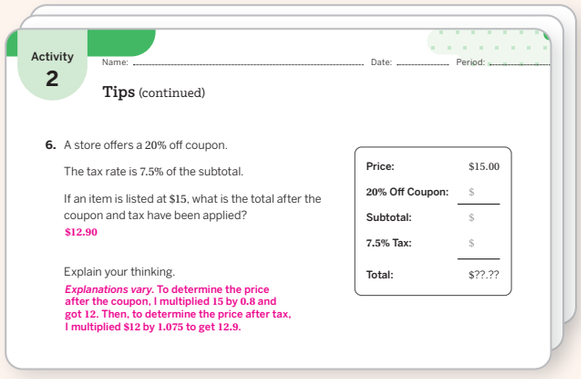
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action and Expression.

Spotlight on Welcoming Interests

Lesson 6: This lesson highlights the utility and relevance of the math related to tax, tip and discounts through meaningful activities.



Activity 2

Name: Date: Period:

Tips (continued)

6. A store offers a 20% off coupon.

The tax rate is 7.5% of the subtotal.

If an item is listed at $15, what is the total after the coupon and tax have been applied?

$12.90

Explain your thinking.

Explanations vary. **To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied $12 by 1.075 to get 12.9.**

Price:	$15.00
20% Off Coupon:	$
Subtotal:	$
7.5% Tax:	$
Total:	$??.??

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Memory and Attention

Here are two suggestions in this unit for supporting students with accessing prior knowledge.

Lesson 8, Activity 2

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

Lesson 9, Activity 1

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson
percent decrease **percent increase**	1
markdown **markup**	3
interest **principal** **simple interest**	7
percent error	8
bar notation **repeating decimal** **terminating decimal**	12

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	13
MLR2: Collect and Display	1, 2, 5–8, and 11–12
MLR3: Critique, Correct, Clarify	4 and 13
MLR7: Compare and Connect	4, 9, and 11

Sub-Unit Language Goals

- Interpret verbal descriptions and strategies to solve problems about percent change.
- Interpret scenarios depicting percent change and percent error.
- Analyze strategies for solving problems involving proportional relationships with fractional and decimal quantities.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 1**, where students are sorting a series of expressions representing percent increase or percent decrease. Students are encouraged to begin by sharing about their ordering of the values in their primary language to open access to the task.

ELL **English Language Learners** Encourage students to discuss how to order the values in their primary language, clarifying any unknown words. **(Speaking and Reading)**

Connections to Future Learning

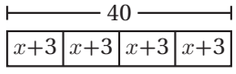
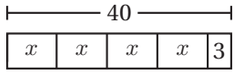
Here is how the content in this unit connects to where your students are headed on their math journeys.

Solving Equations

In this unit, students use tape diagrams, tables, double number lines, and equations to solve problems involving percentages. **In Grade 7, Unit 6,** students will use tape diagrams and other representations to solve equations.

FL B.E.S.T. Math Benchmark:
MA.7.AR.2.2

Example:

Situation	Equation	Solution	Tape Diagram
Some decks of playing cards in Italy and Spain have 40 cards. There are four suits. Each suit has 3 face cards and x non-face cards.	$40 = 4(x + 3)$	$x = 7$	
A chef at a Burmese restaurant makes 40 quarts of mohinga, a noodle and fish soup. She uses 3 quarts now and divides the rest equally into 4 containers to freeze.	$40 = 4x + 3$	$x = 9.25$	

Exponential Functions

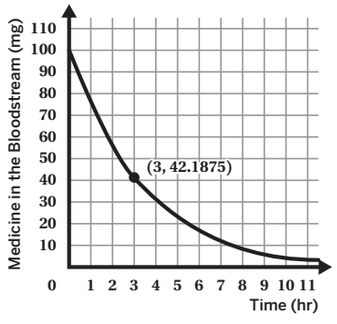
In this unit, students interpret and solve problems involving percent change. **In High School**, students will apply their understanding of percent change to study percent increase and decrease in exponential functions.

FL B.E.S.T. Math Benchmarks:
MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6

Example:

Imagine you take 100 milligrams of medicine and each hour, 25% of the medicine is used up. The equation $y = 100 \cdot 0.75$ models this relationship because 75% of the medicine is left in your body each hour.

- x is the number of hours after taking the medicine.
- y is the number of milligrams of medicine left in your body.

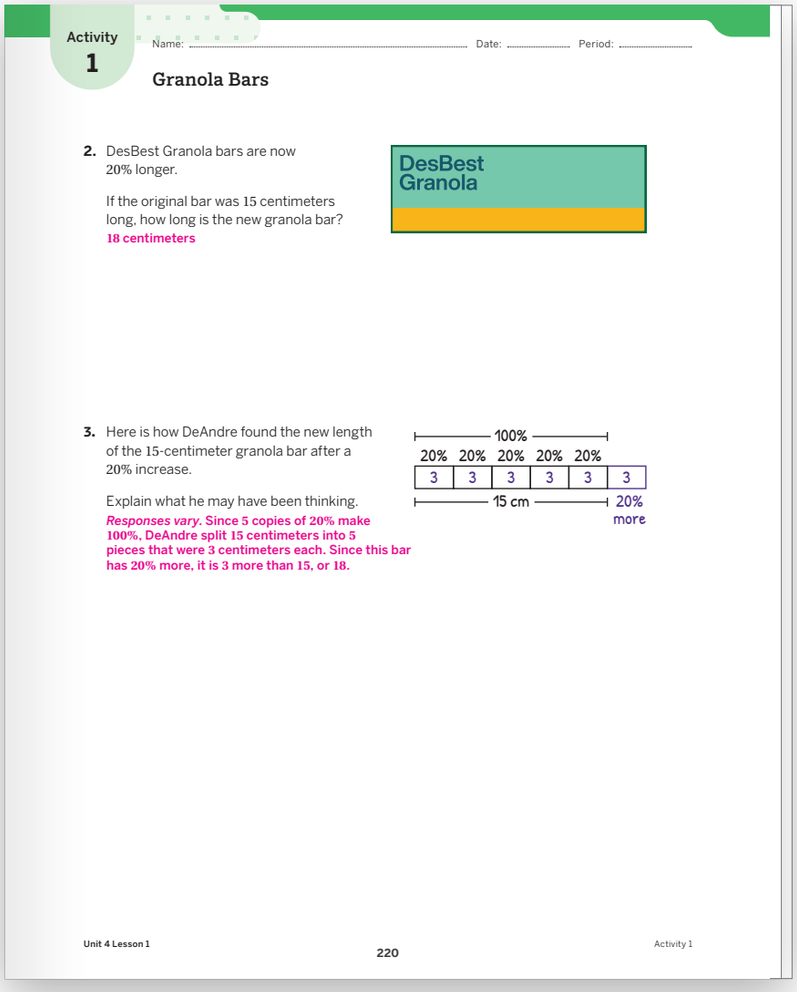


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 1, Activity 1



Activity 1

Name: Date: Period:

Granola Bars

2. DesBest Granola bars are now 20% longer.

If the original bar was 15 centimeters long, how long is the new granola bar?

18 centimeters

DesBest Granola

3. Here is how DeAndre found the new length of the 15-centimeter granola bar after a 20% increase.

Explain what he may have been thinking.

Responses vary. **Since 5 copies of 20% make 100%, DeAndre split 15 centimeters into 5 pieces that were 3 centimeters each. Since this bar has 20% more, it is 3 more than 15, or 18.**

100%

20% 20% 20% 20% 20%

3 3 3 3 3 3

15 cm 20% more

Unit 4 Lesson 1 220 Activity 1

Context

In this activity, students develop adaptive reasoning as they move fluently between tape diagrams and tables to make sense of the percent increase and percent decrease problems.

Try This

Put on your student hat and complete Problems 2–3.

Points to Ponder

- What do students need to understand about both tape diagrams and tables to make sense of these representations as useful approaches to problems with a percent increase or decrease?
- How does representing percent increase and decrease problems with tape diagrams and tables build from their understanding of multiple representations of proportional relationships?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying equivalent fractions and percentages	Lesson 1	1	MA.6.NSO.3.5, MTR.2.1
2	Identifying equivalent fractions, decimals, and percentages	Lessons 1	1	MA.6.NSO.3.5, MTR.2.1, MTR.5.1
3	Calculating the percentage of a number	Lesson 1	1	MA.6.AR.3.4
4	Reasoning about percent increase or decrease	Lesson 2	1	MA.6.AR.3.4 MTR.7.1
5	Identifying equivalent expressions	Lesson 2	1	MA.6.AR.1.4
6	Using double number lines involving percentages	Lesson 3	1	MA.6.AR.3.4 MTR.5.1
7	Using rates and ratio reasoning to solve problems	Lesson 9	1	MA.6.AR.3.5, MTR.2.1, MTR.7.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 4

1. Match each fraction expression to an equivalent percent expression.

Fraction Expression	Percent Expression
a. $\frac{3}{4}$ of x	b 25% of x
b. $\frac{1}{4}$ of x	c 40% of x
c. $\frac{2}{5}$ of x	d 4% of x
d. $\frac{1}{25}$ of x	a 75% of x

2. Select *all* the expressions that are equivalent to 5% of 60.

- ☑ A. $\frac{1}{20}(60)$
- ☐ B. $\frac{1}{5} \cdot 60$
- ☑ C. $0.05 \cdot 60$
- ☐ D. $0.5(60)$
- ☑ E. $\frac{5}{100} \cdot 60$

3. What is 13% of 200?

26

Explain your thinking.

Explanations vary. 13% of 100 is 13, so 13% of 200 should be double that, and $2 \cdot 13 = 26$.

4. A new soft drink has 20% less sugar than before. The drink originally had 50 grams of sugar.

How much sugar was removed?

10 grams

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Name: ..... Date: ..... Period: .....

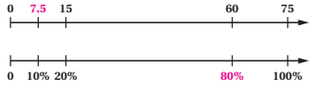
Pre-Unit Check (continued)

Unit 4

5. Select *all* the expressions that are equivalent to $0.4x$.

- ☑ A. $(1 - 0.6)x$
- ☐ B. $1 - 0.6x$
- ☐ C. $x - 0.6$
- ☑ D. $x - 0.6x$
- ☑ E. $\frac{40}{100}x$

6. Gabriel created this double number line. Fill in the value at each unlabeled tick mark.



7. Last year, Thiago hosted a spaghetti dinner for the school soccer team. He made 6 boxes of spaghetti to feed 20 people.

This year, 50 people are coming! How many boxes of spaghetti should Thiago make to feed all of his guests?

15 boxes

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking

Preparation for Sub-Unit 1:
• Solve problems involving percent change using representations, such as tape diagrams, double number lines, tables, and equations. **(Lessons 1–5)**
Preparation for Sub-Unit 2:
• Use proportional reasoning to solve problems in real-world situations involving percent increase, percent decrease, and percent error.
• Interpret situations involving percent change and apply appropriate representations to solve problems. **(Lessons 6–8)**
	1, 2	**Support**
		• **Teacher Move:** Consider revisiting...
		  • Grade 6, Unit 5, Lesson 9 (*Coining New Representations*)
		  • Grade 6, Unit 5, Lesson 10 (*Grocery Prices*)

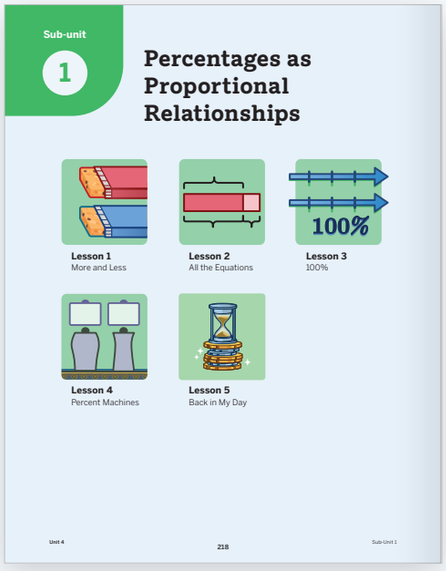
	3, 4, 6	**Support** • **Mini-Lesson:** *Calculating Unknown Percentages* • **Teacher Move:** Consider revisiting Grade 6, Unit 3, Lesson 8 (*Lucky Duckies*) • **Teacher Move:** During **Lesson 3**, Activity 2, consider helping students identify a strategy in which they feel most confident.
	5	**Support** • **Teacher Move:** During **Lesson 2**, Activity 2, consider showing students how to check if equations are equivalent through substitution.
	7	**Support** • **Teacher Move:** During **Lesson 9**, Activity 1, consider reviewing examples of ratio reasoning with whole numbers.

Percentages as Proportional Relationships

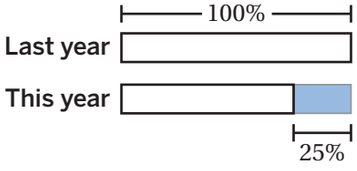
Sub-Unit 1 Goals

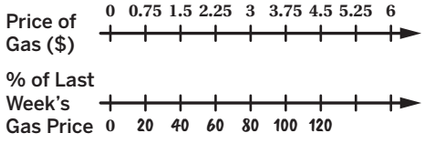
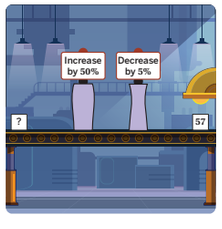
- **Goal:** Solve problems involving percent change using representations, such as tape diagrams, double number lines, tables, and equations.
- **Language Goal:** Interpret verbal descriptions and strategies to solve problems about percent change.

In this sub-unit, students use tape diagrams, tables, and double number line diagrams to represent and solve proportional relationships involving **percent increase** and **percent decrease**. They also practice writing and using equations of proportional relationships that represent percent change.



Math That Matters Most

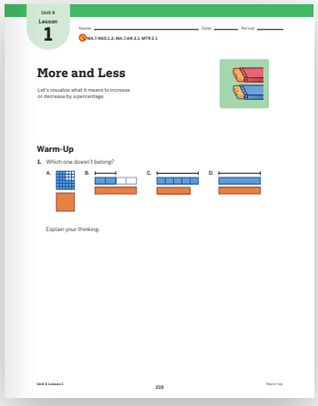
Concept or Skill	For example . . .
Use tape diagrams to represent situations involving a percent increase or decrease.	If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?  I know this year's harvest is 75% of 80, and 0.75 · 80 = 60. So 60 pounds of peaches were harvested this year.
Use equations to represent situations involving a percent increase or decrease.	Jayla's bank account increased by 7% this year. Write an equation to represent the relationship. between the amount that Jayla started with, b, and the amount she has now, c. One way to write this would be c = (1 + 0.07)b, where b is the starting amount and c is her current amount.

Concept or Skill	For example . . .
Use double number lines to represent situations involving a percent increase or decrease.	Last week, gas was \$3.75 per gallon. This week, gas was \$4.50 per gallon. Create a double number line diagram to find the percent change.  The percent change is 120%.
Calculate the original amount given the new amount and a percent change for one-step and multistep percent problems.	A number went into this machine, and 57 came out. What number went in?  First divide 57 by the percent decrease: 57 ÷ 0.95 = 60. Then divide 60 by the percent increase: 60 ÷ 1.50 = 40. The number 40 was the original number that went into the machine.

Unit 4

Lesson

1



More and Less

Percent Increase and Decrease With Tape Diagrams

Let's visualize what it means to increase or decrease by a percentage.

Learning Outcomes

Today's Goals

1. **Goal:** Use tape diagrams and tables to represent situations involving a *percent increase* or *decrease*.
2. **Goal:** Determine a new amount given an original amount and a *percent increase* or *decrease*.
3. **Language Goal:** Interpret a verbal description to solve problems about *percent increase* or *decrease*. **(Reading and Writing)**

Students visualize what it means to increase or decrease an amount by a percentage, and use that visual to help them calculate unknown values. They see both tape diagrams and tables used to represent **percent increase** and **percent decrease**, and they use these representations to solve problems. **(MTR.2.1)**

Coherence

Prior Learning

In Grade 6, students solved problems involving ratios and percents.

Future Learning

In Lesson 2, students will use equations to represent percent increase and decrease.

Connections

- Students use models to solve problems involving percent increase and percent decrease in real world contexts such as comparing the lengths of granola bars and the amount of fruit harvested over two years. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2

Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1

Apply previous understanding to percentages and ratios to solve real-world multi-step percentage problems.

Mathematical Thinking and Reasoning Standard: MTR.2.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.AR.3.4

Vocabulary

New Vocabulary

percent decrease **percent increase**

Lesson at a Glance

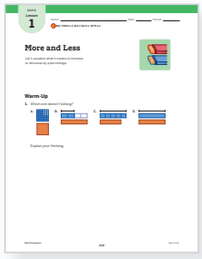
~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

Students make sense of the structure of tape diagrams to be ready to use them to represent a relationship between two variables. **Routine:** Which One Doesn't Belong?



Activity 1

Pairs | 13 min

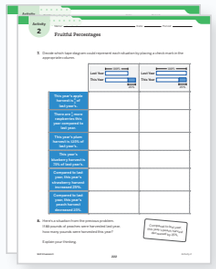
Students use the context of granola bars and representations, including tape diagrams and tables, to visualize **percent increase** and **percent decrease**.
Routine: Think-Pair-Share



Activity 2

Pairs | 10 min

Students apply the concepts from Activity 1 to new contexts and continue to make connections between representations.
MLR2: Collect and Display **(MTR.2.1)**



Activity 3

Pairs | 7 min

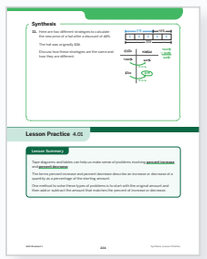
Students distinguish between a percent of a number and percent change.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of different strategies and tools to solve for percent increase and percent decrease.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting a percent decrease situation.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students make sense of the structure of tape diagrams to be ready to use them to represent a relationship between two variables.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing the structure of tape diagrams.

Consider sharing that each diagram does not belong in some way. Encourage students to look for more than one possibility.

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "How would you describe the relationship between the blue and orange figure in diagram A? B?"

ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share their ideas with others. **(Writing and Speaking)**

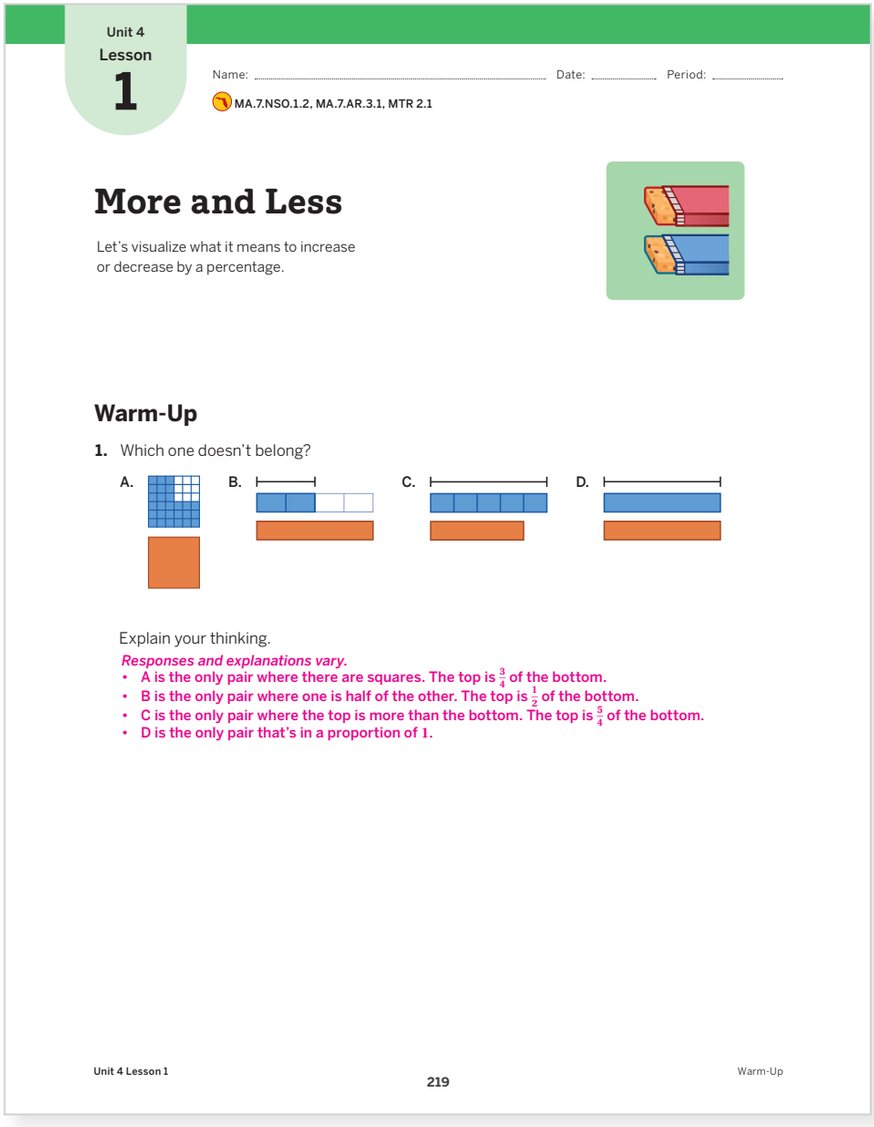
2 Connect

Invite students to share at least one reason why each diagram doesn't belong.

Consider asking:

- "What is another way to say why diagram ___ doesn't belong?"
- "Is there another reason why diagram ___ doesn't belong?"

Note: If it doesn't come up naturally, consider asking, "How could we use these diagrams to represent percentages?"



Unit 4
Lesson
1

Name: Date: Period:

MA.7.NSO.1.2, MA.7.AR.3.1, MTR 2.1

More and Less

Let's visualize what it means to increase or decrease by a percentage.

Warm-Up

1. Which one doesn't belong?

A. B. C. D.

Explain your thinking.

Responses and explanations vary.

- A is the only pair where there are squares. The top is $\frac{3}{4}$ of the bottom.
- B is the only pair where one is half of the other. The top is $\frac{1}{2}$ of the bottom.
- C is the only pair where the top is more than the bottom. The top is $\frac{5}{4}$ of the bottom.
- D is the only pair that's in a proportion of 1.

Unit 4 Lesson 1 219 Warm-Up

Pairs | 13 min

Activity 1 Granola Bars

Purpose: Students use the context of granola bars and representations, including tape diagrams and tables, to visualize **percent increase** and **percent decrease**.

Short on time? Consider omitting Problem 6 to allocate time on the strategies for percent increase and percent decrease.

1 Launch

For Problem 2, **Direct students' attention** to the granola bar. Consider asking, "Where have you seen phrases such as 20% more or 20% longer in your lives?"

ELL **English Language Learners** Invite students to read the problem aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

Note: This may be the first time students encounter a **percent increase**. Encourage students to use blank paper to help them revise their thinking.

2 Monitor

To support students getting started, consider asking, "How long is 10% of this granola bar? How can that help you find 20%?"

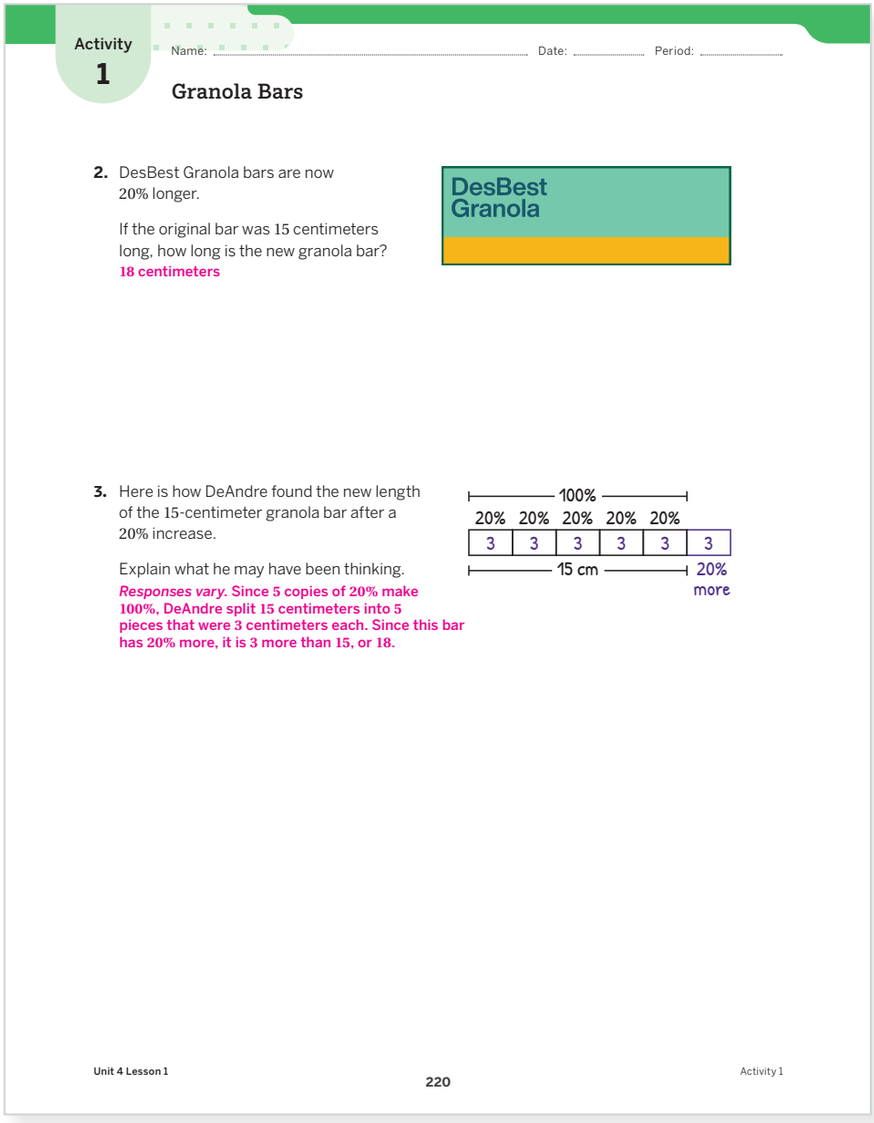
For Problem 3, **look for and share** a variety of student explanations for the steps in DeAndre's strategy, such as:

- 20% is $\frac{1}{5}$ of 100% or 5 copies of 20% make 100%.
- 3 is $\frac{1}{5}$ of 15 or 3 is 20% of 15.

Encourage students to read other student responses. Consider asking, "How are the explanations similar to or different from your own?"

Pause briefly to discuss DeAndre's strategy. Consider asking, "Why do you think DeAndre decided to divide the rectangle into five equal parts? Where did the number 3 come from?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Granola Bars

2. DesBest Granola bars are now 20% longer.

If the original bar was 15 centimeters long, how long is the new granola bar?

18 centimeters

3. Here is how DeAndre found the new length of the 15-centimeter granola bar after a 20% increase.

Explain what he may have been thinking.

Responses vary. Since 5 copies of 20% make 100%, DeAndre split 15 centimeters into 5 pieces that were 3 centimeters each. Since this bar has 20% more, it is 3 more than 15, or 18.

Unit 4 Lesson 1 220 Activity 1

Pairs | 13 min

Activity 1 Granola Bars (continued)

Purpose: Students use the context of granola bars and representations, including tape diagrams and tables, to visualize **percent increase** and **percent decrease.**

Short on time? Consider omitting Problem 6 to allocate time on the strategies for percent increase and percent decrease.

1 Monitor

For Problem 4, **Use the Think-Pair-Share routine** to help students shape their ideas about **percent decrease**. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Listen for a variety of student strategies to solve for the length of the new granola bar.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How is this problem similar to the last one? How is it different? What might that mean about how we solve for the new length?"
Multiply 0.1 by 15 for their solution.	**Support** Consider asking, "What would 50% of the granola bar be? Should this be more or less than that amount?"
Use the same strategy as the last problem and add 1.5 to 15.	**Support** Invite them to read the problem aloud and highlight key information for solving this problem. Consider asking, "How is this problem similar or different to the one you just did?"

For Problem 5, **encourage students** to read other student responses. Consider asking, "How are the explanations similar to or different from your own?"

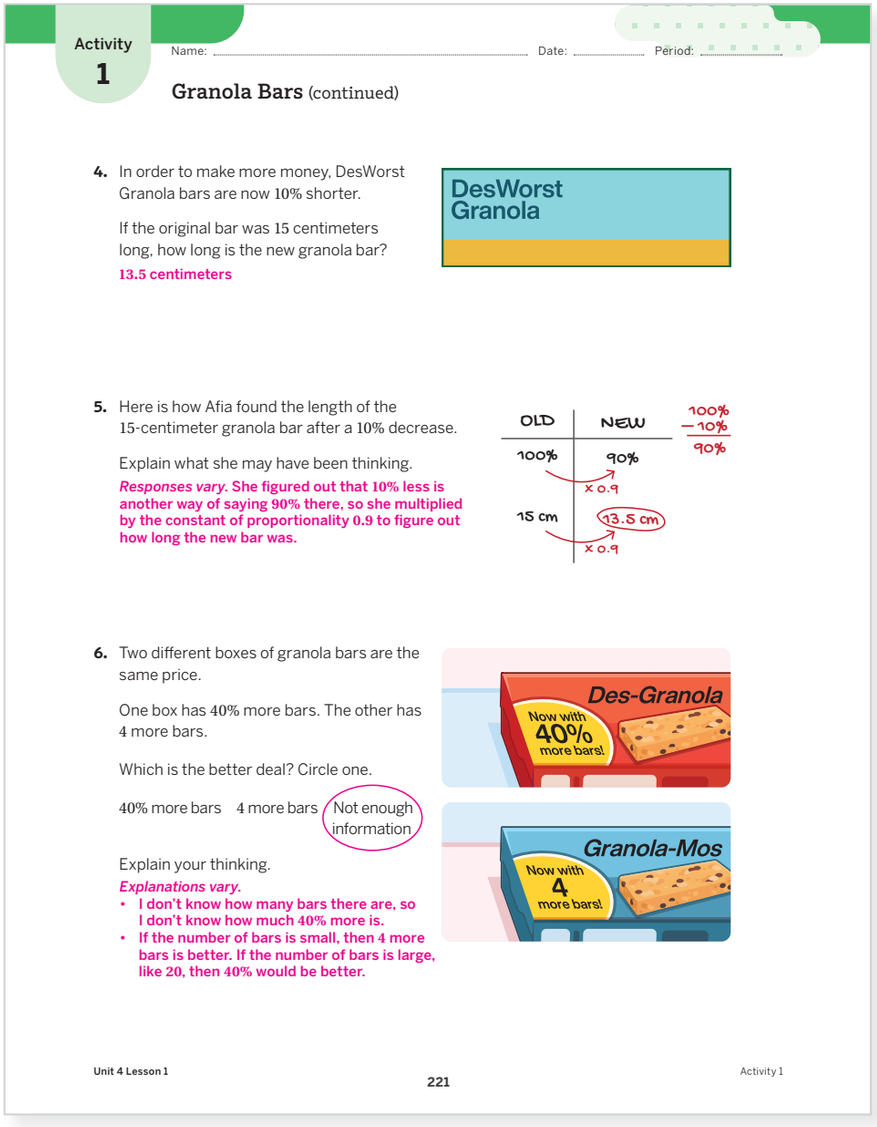
Consider asking, "How would DeAndre draw 40% more bars?"

2 Connect

For Problem 5, consider asking:

- "How do we see parts of the question in Afia's work?"
- "How is this strategy similar to DeAndre's? Different?"

Note: If time allows, consider asking, "How is this table similar to or different from tables we have used so far this year?"



Activity 1

Name: Date: Period:

Granola Bars (continued)

4. In order to make more money, DesWorst Granola bars are now 10% shorter.

If the original bar was 15 centimeters long, how long is the new granola bar?

13.5 centimeters

5. Here is how Afia found the length of the 15-centimeter granola bar after a 10% decrease.

Explain what she may have been thinking.

Responses vary. **She figured out that 10% less is another way of saying 90% there, so she multiplied by the constant of proportionality 0.9 to figure out how long the new bar was.**

6. Two different boxes of granola bars are the same price.

One box has 40% more bars. The other has 4 more bars.

Which is the better deal? Circle one.

40% more bars 4 more bars Not enough information

Explain your thinking.

Explanations vary.

- **I don't know how many bars there are, so I don't know how much 40% more is.**
- **If the number of bars is small, then 4 more bars is better. If the number of bars is large, like 20, then 40% would be better.**

Unit 4 Lesson 1 221 Activity 1

Key Takeaway: One strategy for determining the total after a **percent increase** and **percent decrease** is to find a benchmark percentage to add to or subtract from 100%. Another strategy is to multiply by a decimal that is representative of the ending desired percentage.

Pairs | 10 min

Activity 2 Fruitful Percentages

Purpose: Students apply the concepts from Activity 1 to new contexts and continue to make connections between representations. **(MTR.2.1)**

1 Launch

Direct students' attention to the diagrams in the table. Consider asking, "What do you notice about the two diagrams?"

2 Monitor

Encourage students to discuss what words or phrases help them decide whether a situation represents a percent increase or decrease and work together to reach an agreement about which diagram represents each situation. **(MTR.2.1)**

A **Accessibility: Executive Function** Invite students to focus on one situation at a time to support students in carrying out multiple steps.

Note: If time allows, invite pairs to compare their tables, justify their thinking, and make revisions based on their conversation.

Look for student responses to share during the Connect and listen for student strategies for distinguishing between situations for percent increase and decrease.

For Problem 8, **look for** a variety of student explanations for percent decrease.

D **Differentiation: (Support)** To support students getting started, consider asking, "How might you use DeAndre's or Alfi's strategies to solve for the pounds of peaches?"

3 Connect

For Problem 7, **invite students** to share their work and explain how they distinguished between situations for percent increase and decrease.

To surface the Key Takeaway, consider asking:

- "Which words in a situation connect to a percent increase diagram? A percent decrease diagram?"
- "Why might it be helpful to have more than one way to ask for the same percent increase or decrease?"

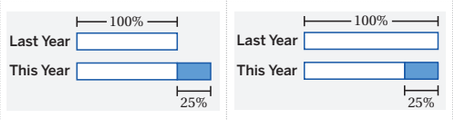
MLR **MLR2: Collect and Display** Use this routine to create a class definition of the terms *percent increase* and *percent decrease* using students' language. Display any new language, examples, or diagrams for students to reference later.

Activity 2

Name: Date: Period:

Fruitful Percentages

7. Decide which tape diagram could represent each situation by placing a check mark in the appropriate column.



This year's apple harvest is $\frac{3}{4}$ of last year's.		✓
There are $\frac{1}{4}$ more raspberries this year compared to last year.	✓	
This year's plum harvest is 125% of last year's.	✓	
This year's blueberry harvest is 75% of last year's.		✓
Compared to last year, this year's strawberry harvest increased 25%.	✓	
Compared to last year, this year's peach harvest decreased 25%.		✓

8. Here's a situation from the previous problem.

If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?

60 pounds

Explain your thinking.

Explanations vary. **I know this year's harvest is 75% of 80, and $0.75 \cdot 80 = 60$.**

Compared to last year, this year's peach harvest decreased by 25%.

Unit 4 Lesson 1 222 Activity 2

Key Takeaway: A percent increase can be expressed using words like more, percentages higher than 100, and a tape diagram with a shaded region that is more than the original length. A percent decrease can be expressed using words like less, percentages lower than 100, and a tape diagram with a shaded region that is less than the original length.

Pairs | 7 min

Activity 3 Percent Practice

Purpose: Students distinguish between a percent of a number and percent change.

1 Launch

For Problem 9, consider asking, "Where have you seen percentages? When might you want to compare percentages?"

2 Monitor

Listen for a variety of strategies students use to order the values.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What words or phrases can you use to help estimate and then compute the values?"
Are unsure how to compute 25% more of 20 or 75% less than 60.	**Support** Consider asking, "How might you use DeAndre's or Alfi's strategies to solve ____?"
Use different tools to support thinking, such as a table, a tape diagram, or a calculator.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.

A Accessibility: Executive Functioning Invite students to make a table on a separate piece of paper to organize their thinking.

ELL English Language Learners Encourage students to discuss how to order the values in their primary language, clarifying any unknown words. **(Speaking and Reading)**

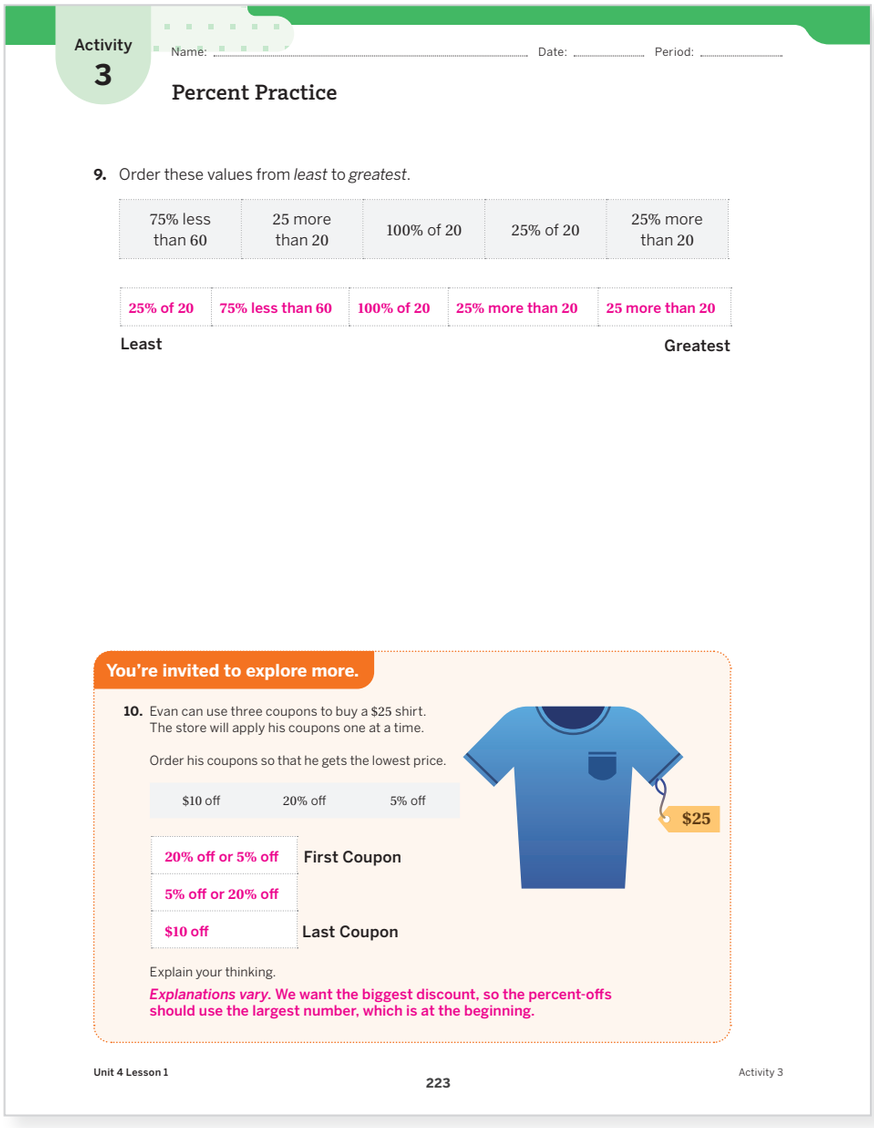
D Differentiation: You're invited to explore more. (Stretch) Invite students who want to explore percent increase and decrease further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

For Problem 9, **display** the values and invite students to share strategies for ordering the values.

Consider asking, "What was easy about this ordering task? What was challenging? What do you want to remember about determining percentages?"

Note: If time allows, try to have students that used different models share how they were helpful for this problem and add them to the anchor chart.



Activity 3

Name: Date: Period:

Percent Practice

9. Order these values from *least* to *greatest*.

75% less than 60	25 more than 20	100% of 20	25% of 20	25% more than 20

25% of 20	75% less than 60	100% of 20	25% more than 20	25 more than 20

Least Greatest

You're invited to explore more.

10. Evan can use three coupons to buy a $25 shirt. The store will apply his coupons one at a time.

Order his coupons so that he gets the lowest price.

$10 off	20% off	5% off

20% off or 5% off	First Coupon
5% off or 20% off	
$10 off	Last Coupon

Explain your thinking.

Explanations vary. We want the biggest discount, so the percent-offs should use the largest number, which is at the beginning.

Unit 4 Lesson 1 223 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of different strategies and tools to solve for percent increase and percent decrease.

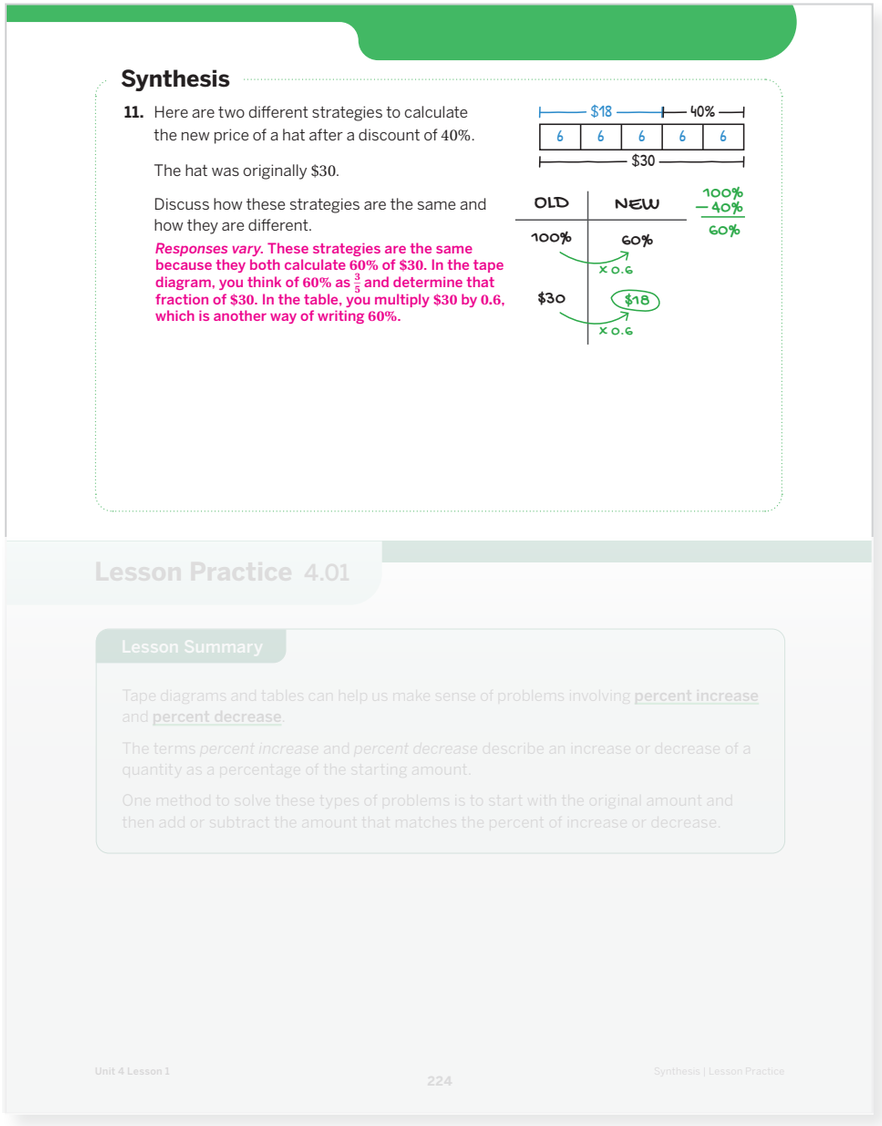
For Problem 11, **invite students** to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- A tape diagram can be a useful representation in displaying a percent discount, where the segments in a tape diagram align with the number necessary to represent the percent decrease in a given problem.
- A table can be a useful representation in explaining percent discount as the same constant of proportionality can be applied to a row displaying percentages and a row displaying price.

Consider asking, "Why might it be helpful to have multiple strategies for solving for percent decrease?"

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. Here are two different strategies to calculate the new price of a hat after a discount of 40%.

The hat was originally $30.

Discuss how these strategies are the same and how they are different.

Responses vary. These strategies are the same because they both calculate 60% of $30. In the tape diagram, you think of 60% as $\frac{3}{5}$ and determine that fraction of $30. In the table, you multiply $30 by 0.6, which is another way of writing 60%.

Lesson Practice 4.01

Lesson Summary

Tape diagrams and tables can help us make sense of problems involving percent increase and percent decrease.

The terms *percent increase* and *percent decrease* describe an increase or decrease of a quantity as a percentage of the starting amount.

One method to solve these types of problems is to start with the original amount and then add or subtract the amount that matches the percent of increase or decrease.

Unit 4 Lesson 1 224 Synthesis | Lesson Practice

Lesson Takeaway: Tape diagrams, tables, or other tools can help make sense of problems involving **percent increase** and **percent decrease**.

Independent | 5 min

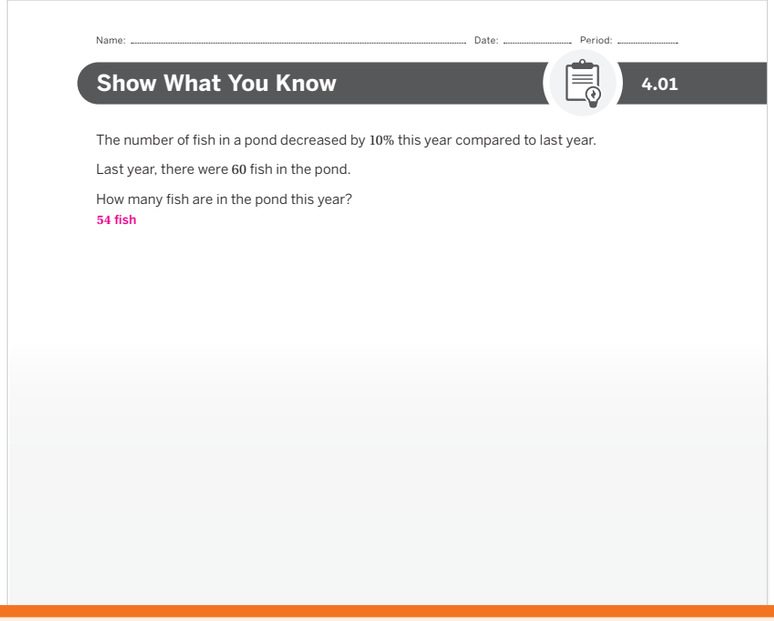
Show What You Know

Purpose: Students demonstrate their understanding by interpreting a percent decrease situation.

Today's Goals

1. **Goal:** Use tape diagrams and tables to represent situations involving a *percent increase* or *decrease*.
2. **Goal:** Determine a new amount given an original amount and a *percent increase* or *decrease*.
3. **Language Goal:** Interpret a verbal description to solve problems about *percent increase* or *decrease*. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.01

The number of fish in a pond decreased by 10% this year compared to last year.

Last year, there were 60 fish in the pond.

How many fish are in the pond this year?

54 fish

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support with understanding percent increase and percent decrease:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of these concepts in Lessons 2–4. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to create another problem similar to the **Explore More** task, then challenge a partner to solve for the final price.

Professional Learning

In this lesson, students are introduced to tape diagrams as a model to support their understanding of **percent increase** and **percent decrease**. How does this model integrate with other representations (i.e., double number lines, tables, etc.) of percent increase and percent decrease?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are two different strategies to calculate the new price of a hat after a discount of 40%.

The hat was originally $30.

Discuss how these strategies are the same and how they are different.

Responses vary. These strategies are the same because they both calculate 60% of $30. In the tape diagram, you think of 60% as $\frac{3}{5}$ and determine that fraction of $30. In the table, you multiply $30 by 0.6, which is another way of writing 60%.

Lesson Practice 4.01

Lesson Summary

Tape diagrams and tables can help us make sense of problems involving **percent increase** and **percent decrease**.

The terms *percent increase* and *percent decrease* describe an increase or decrease of a quantity as a percentage of the starting amount.

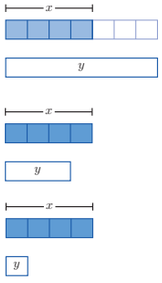
One method to solve these types of problems is to start with the original amount and then add or subtract the amount that matches the percent of increase or decrease.

Lesson Practice 4.01

Name: _____ Date: _____ Period: _____

1. Match each situation with a diagram.

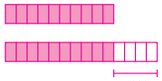
Situation	Diagram
a. Hoang drinks x ounces of juice. Nekeisha drinks $\frac{1}{4}$ less than that.	b
b. Hoang runs x miles. Nekeisha runs $\frac{3}{4}$ more than that.	a
c. Hoang buys x pounds of almonds. Nekeisha buys $\frac{1}{4}$ of that.	c



Problems 2–3: Draw a diagram to represent each situation.

2. The amount of flour that the bakery used this month increased 40% compared to last month.

Responses vary.



3. The amount of milk that the bakery used this month decreased 75% compared to last month.

Responses vary.



Lesson Practice 4.01

Name: _____ Date: _____ Period: _____

Problems 4–6: Fill in the blanks to describe each increase or decrease as a percentage of the original amount.

4. This year, there was 40% more snow than last year.

The amount of snow this year is 140 % of the amount last year.

5. This year, there were 25% fewer sunny days than last year.

The number of sunny days this year is 75 % of the number of sunny days last year.

6. A restaurant adds a 93% markup to the price of the ingredients to set the menu price.

The menu price is 193 % of the price of ingredients.

FAST Practice

7. At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, a family has eaten 30% of it. Which expression represents the ounces of peanut butter left in the pantry?

A. $0.7 \cdot 80$ B. $0.3 \cdot 80$ C. $8 - 0.30$ D. $(1 + 0.3) \cdot 80$

Spiral Review

Problems 8–9: A store sells strawberries for $1.38 per pound.

8. Write an equation relating the cost, c, and the pounds of strawberries, p.

$c = 1.38p$ (or equivalent)

9. One strawberry order costs $8.97. How many pounds did the person order?

6.5 pounds

Problems 10–11: Solve each equation.

10. $x \cdot \frac{7}{3} = 1$

$x = \frac{3}{7}$

11. $1 \div \frac{11}{2} = x$

$x = \frac{2}{11}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.7.AR.3.1
	2–6	2	MA.7.AR.3.1
FAST Practice	7	2	MA.7.AR.3.1
Spiral Review			
	8–9	2	MA.7.AR.4.4 MA.7.AR.4.5
	10–11	1	MA.6.AR.2.3

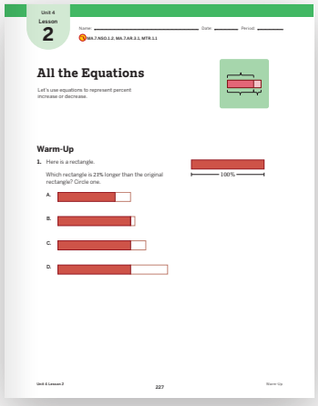
Unit 4

Lesson 2

All the Equations

Percent Increase and Decrease With Equations

Let's use equations to represent percent increase or decrease.



Learning Outcomes

Today's Goals

1. **Goal:** Use equations to represent situations involving a percent increase or decrease.
2. **Goal:** Use a tape diagram where 100% corresponds to the original value to represent adding or subtracting a percent of the original value.
3. **Language Goal:** Interpret a verbal description to write an equation involving percent increase or decrease. **(Reading)**

Students write and use equations to describe the relationship between original and new amounts after a percent increase or decrease. They also make connections between different ways to write their equations.

Coherence

Prior Learning

In Lesson 1, students used tape diagrams to visualize percent increase and decrease and to calculate unknown values.

Future Learning

In Lesson 3, students will practice calculating the original amount, new amount, or the percent change given the other two quantities using representations such as a double number line diagram, table, and equation.

Connections

- Students leverage different forms of equations to describe the relationship between the original and new amounts of an item after a percent increase or decrease in many real world situations such as the paper used in a copy shop. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standard: MTR.1.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.912.AR.1.1

Lesson at a Glance

~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

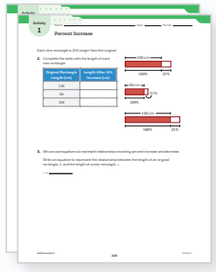
Students visualize increasing by the percentages they will represent with an equation in Activity 1.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

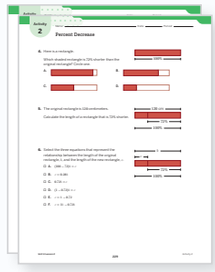
Students consider the relationship between original and new amounts after a percent increase and write an equation to represent it.
MLR2: Collect and Display **(MTR.1.1)**



Activity 2

Pairs | 12 min

Students make sense of using equations to represent percent decrease.



Activity 3

Pairs | 8 min

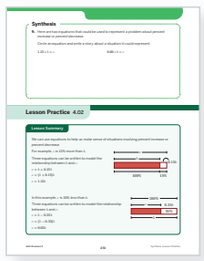
Students deepen their understanding of the connection between the equations that represent percent increase and percent decrease.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of percent increase and decrease by using the Tell a Story routine to describe a given equation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equation to represent the relationship between an original and new amount after a percent increase.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students visualize increasing by the percentages they will represent with an equation in Activity 1.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about percent increase. Invite students to think independently about how to identify a rectangle that is 21% longer than the original rectangle and then take turns sharing with a partner.

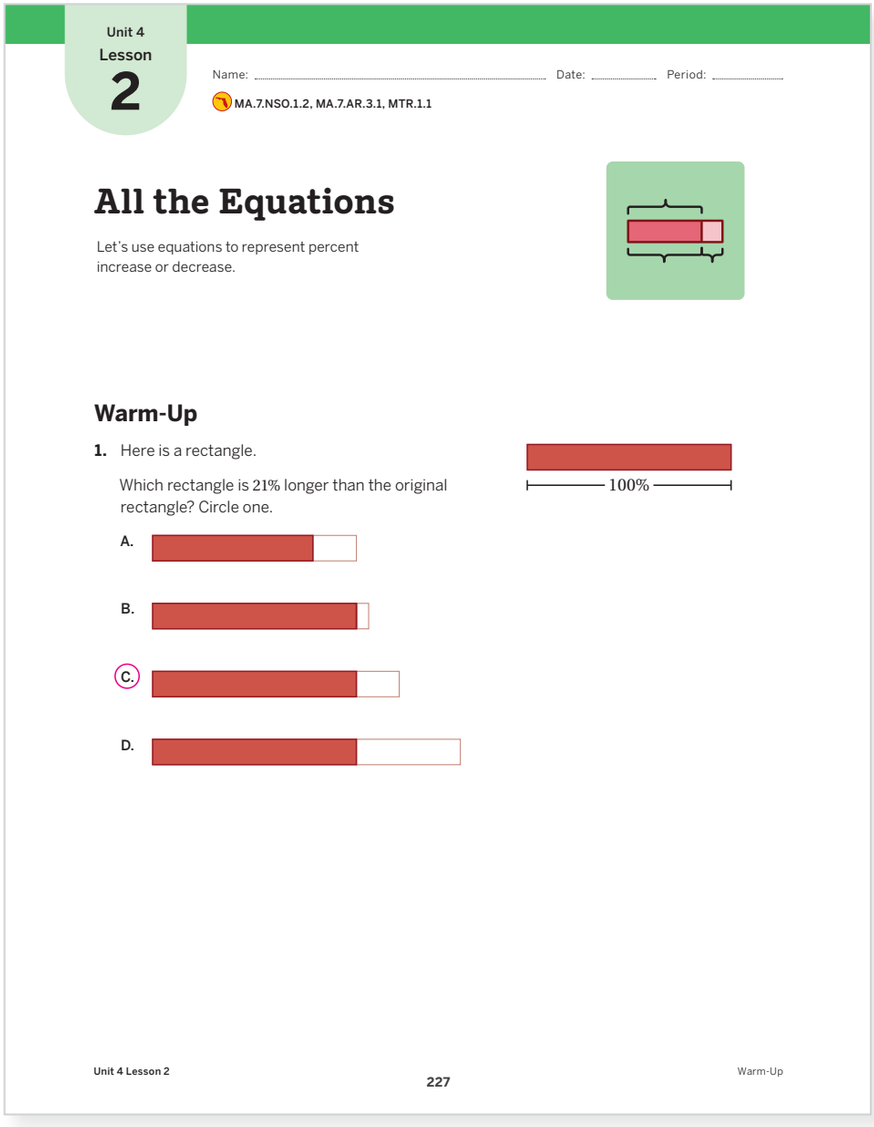
Invite students to consider the rectangle choices. Consider asking, "How are the rectangles similar to or different from the original?"

ELL **English Language Learners** Use gestures to illustrate the term longer. (**Reading and Listening**)

2 Connect

Invite students to share their or their partner's idea for identifying the rectangle that is 21% longer with the class. Encourage several students to share.

Consider asking, "How did you determine how wide the section representing 21% should be based on the model for 100%?"



Unit 4
Lesson
2

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.1.1

All the Equations

Let's use equations to represent percent increase or decrease.

Warm-Up

1. Here is a rectangle.

Which rectangle is 21% longer than the original rectangle? Circle one.

A.

B.

C.

D.

Unit 4 Lesson 2 227 Warm-Up

Pairs | 10 min

Activity 1 Percent Increase

Purpose: Students consider the relationship between original and new amounts after a percent increase and write an equation to represent it. **(MTR.1.1)**

Short on time? Consider having students complete two of the three rows in Problem 2.

1 Launch

To surface connections, consider asking, "Is 21% the same amount for each of these rectangles? How do you know?"

2 Monitor

Look for a variety of student strategies to compute the length of the new rectangles.

Differentiation: (Support) To support students getting started, consider working together with students on the first rectangle before inviting them to try one independently.

Note: This relationship is challenging and students may need support writing an equation to represent it. Encourage them to write their equation even if they are unsure; they can edit their equation after getting feedback from their partner. **(MTR.1.1)**

Select and sequence student equations to share during the Connect.

Differentiation

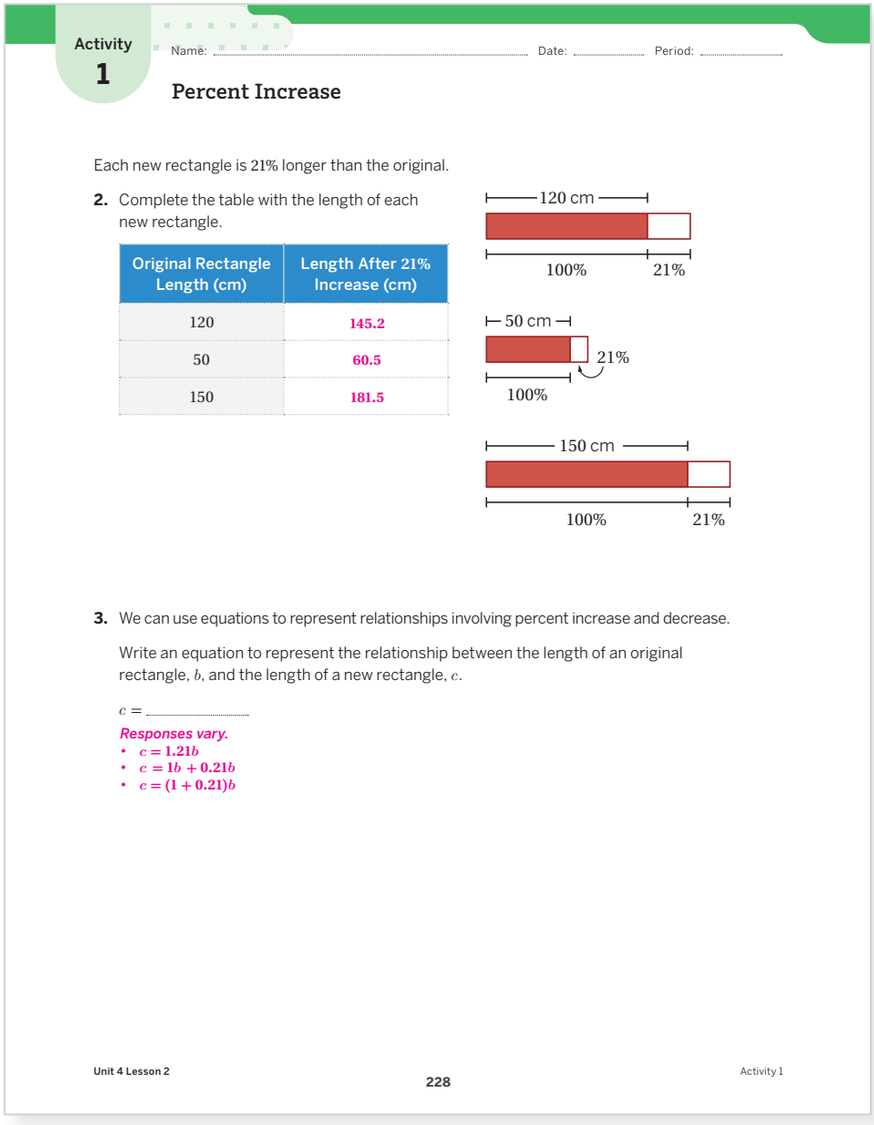
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking them to write any equation that fits one pair of values and use the feedback to revise their thinking.
Write the equation $c = 0.21b$.	**Support** Consider asking, "Why do you think the value decreased? How could we represent the 100%?"
Represent the 21% increase as the decimal 1.21.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.
Want to explore writing percent increase equations further.	**Stretch** Invite students to think of a different correct equation for this relationship.

3 Connect

Share the selected student equations and consider asking:

- "Where do we see 21% in this equation?"
- "Where do we see the value of the original rectangle?"
- "How do we know this equation will result in an increase?"

MLR2: Collect and Display Use this routine to synthesize strategies for writing an equation for percent increase using students' language. Display any new language, examples, or diagrams for students to reference later.



Activity 1

Name: Date: Period:

Percent Increase

Each new rectangle is 21% longer than the original.

2. Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	Length After 21% Increase (cm)
120	145.2
50	60.5
150	181.5

3. We can use equations to represent relationships involving percent increase and decrease.

Write an equation to represent the relationship between the length of an original rectangle, b, and the length of a new rectangle, c.

$c =$ _____

Responses vary.

- $c = 1.21b$
- $c = 1b + 0.21b$
- $c = (1 + 0.21)b$

Unit 4 Lesson 2 228 Activity 1

Pairs | 12 min

Activity 2 Percent Decrease

Purpose: Students make sense of using equations to represent percent decrease.

1 Launch

To surface connections, consider asking, "How is this like our Warm-Up? How is it different?"

2 Monitor

To support students getting started, consider asking, "Will the amount shaded be larger or smaller than half of the original rectangle?"

Listen for a variety of student strategies to compute the length of the new rectangles.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started or are using only a guess and check method.	**Support** Consider asking, "How would you approach this problem if the percentage was 75%?" Encourage students to build from this strategy.
Use strategies to calculate the length of the shorter rectangle, such as: • Calculating the remaining percentage of the rectangle's length (28%). • Calculating 72% of 120 and subtracting it from 120.	**Strengthen** Invite students to think about how their strategy might be captured as an equation.

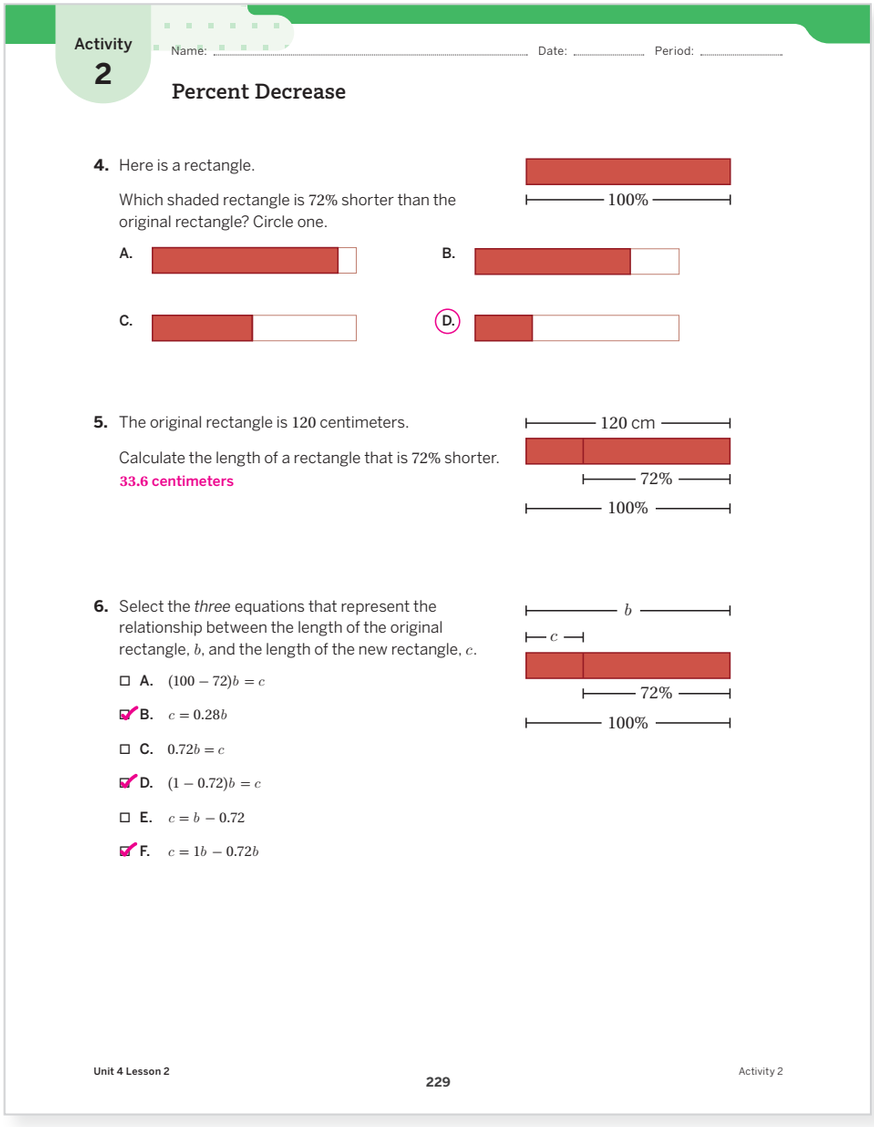
Encourage students to work together to reach an agreement about which three equations represent the relationship.

D Differentiation: (Support) For students that need support getting started, consider inviting them to test the values 120 and 33.6 from the previous problem in different equations and see what they notice.

3 Connect

Invite students to share their work. Consider adding equations to the anchor chart from Activity 1 for all students to reference throughout the unit.

To surface the Key Takeaway, consider asking, "How are each of the three correct equations related?"



Activity 2

Name: Date: Period:

Percent Decrease

4. Here is a rectangle.

Which shaded rectangle is 72% shorter than the original rectangle? Circle one.

A.

B.

C.

D.

5. The original rectangle is 120 centimeters.

Calculate the length of a rectangle that is 72% shorter.

33.6 centimeters

6. Select the *three* equations that represent the relationship between the length of the original rectangle, b, and the length of the new rectangle, c.

- ☐ A. $(100 - 72)b = c$
- ☑ B. $c = 0.28b$
- ☐ C. $0.72b = c$
- ☑ D. $(1 - 0.72)b = c$
- ☐ E. $c = b - 0.72$
- ☑ F. $c = 1b - 0.72b$

Unit 4 Lesson 2 229 Activity 2

Key Takeaway: There are multiple equivalent ways to write an equation for percent decrease. These all are different versions of writing the percentage as a decimal, subtracting it from 1 (or 100%), and then multiplying by the original value.

Pairs | 8 min

Activity 3 Percents and Equations

Purpose: Students deepen their understanding of the connection between the equations that represent percent increase and percent decrease.

1 Launch

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to identify the equations that represent percent increase and decrease.

Note: If time allows, invite pairs to compare their responses, justify their thinking, and make revisions based on their conversation.

2 Monitor

Listen for student strategies for distinguishing between equations for percent increase and decrease.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "This equation represents ____ because ____." or "This equation does not represent an increase or decrease because ____."). **(Writing and Speaking)**

Pause briefly to invite students to share strategies for distinguishing between equations for percent increase and decrease.

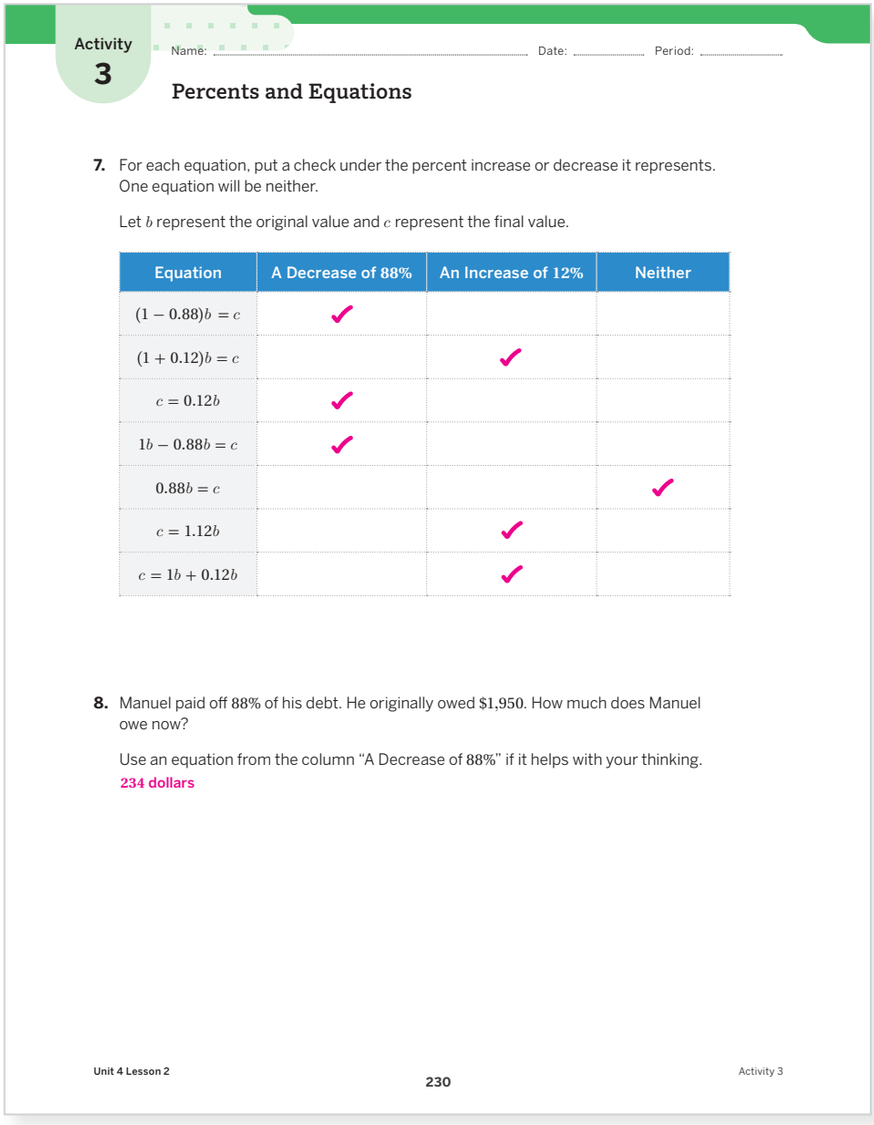
To support students getting started, consider asking, "Will 1950 represent b or c?"

Look for a variety of student strategies (e.g., tape diagrams, tables, equations, or individual strategies) to determine how much Manuel owes.

3 Connect

Discuss the three equations in the image. Consider asking:

- "Does the 1950 represent b or c in each equation?"
- "Is the amount he owes now more or less than half of what he owed originally? How do you know?"
- "What does the 0.12 represent in the equation? Where does it come from?"



Activity 3

Name: Date: Period:

Percents and Equations

7. For each equation, put a check under the percent increase or decrease it represents. One equation will be neither.

Let b represent the original value and c represent the final value.

Equation	A Decrease of 88%	An Increase of 12%	Neither
$(1 - 0.88)b = c$	✓		
$(1 + 0.12)b = c$		✓	
$c = 0.12b$	✓		
$1b - 0.88b = c$	✓		
$0.88b = c$			✓
$c = 1.12b$		✓	
$c = 1b + 0.12b$		✓	

8. Manuel paid off 88% of his debt. He originally owed $1,950. How much does Manuel owe now?

Use an equation from the column "A Decrease of 88%" if it helps with your thinking.

234 dollars

Unit 4 Lesson 2 230 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of percent increase and decrease by using the Tell a Story routine to describe a given equation.

Invite students to spend 2 minutes responding to the prompt. Ask them to share their thinking with a partner.

Use the Tell a Story routine to help students make sense of percent increase or decrease using their unique experiences, perspectives, and creativity.

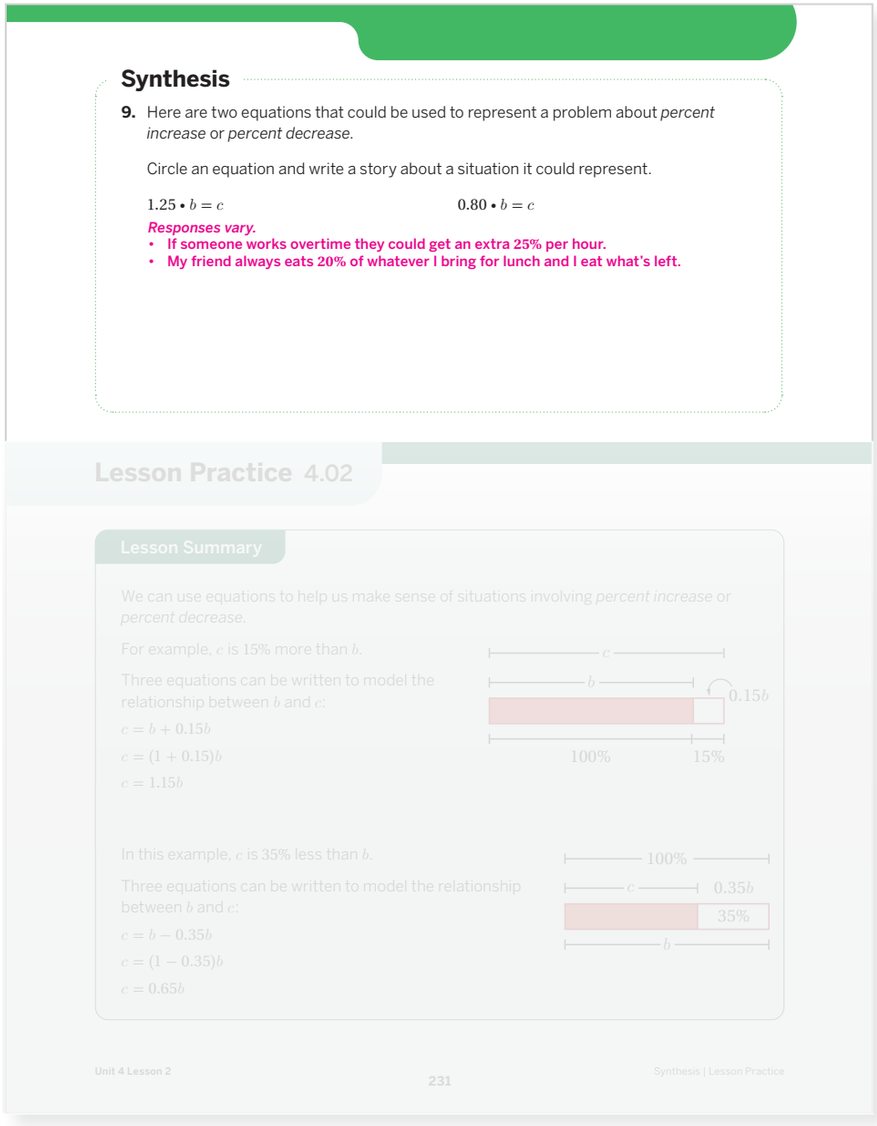
Consider sharing that stories do not have to be complete or perfect to be valuable.

ELL **English Language Learners** Invite students to write first in their primary language and then translate it to English during the second draft. **(Writing)**

Encourage students to ask questions about each other's stories. Consider asking:

- "How do you see the parts of ___'s story in the equation?"
- "What parts of each of our stories are alike? How are they different?"
- "What questions do you have about each other's stories?"

Note: If time allows, return to the anchor chart and add several of the student stories, with their names, as examples to refer back to throughout the unit.



Synthesis

9. Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$1.25 \cdot b = c$ $0.80 \cdot b = c$

Responses vary.

- If someone works overtime they could get an extra 25% per hour.
- My friend always eats 20% of whatever I bring for lunch and I eat what's left.

Lesson Practice 4.02

Lesson Summary

We can use equations to help us make sense of situations involving *percent increase* or *percent decrease*.

For example, c is 15% more than b.

Three equations can be written to model the relationship between b and c:

$c = b + 0.15b$

$c = (1 + 0.15)b$

$c = 1.15b$

In this example, c is 35% less than b.

Three equations can be written to model the relationship between b and c:

$c = b - 0.35b$

$c = (1 - 0.35)b$

$c = 0.65b$

Unit 4 Lesson 2 231 Synthesis | Lesson Practice

Lesson Takeaway: Equations are one way to represent percent increase or decrease. If you write a percentage as a decimal, then add or subtract it from 1 (or 100%), you can multiply that number by the original to get the new value.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing an equation to represent the relationship between an original and new amount after a percent increase.

Today's Goals

1. **Goal:** Use equations to represent situations involving a percent increase or decrease.
2. **Goal:** Use a tape diagram where 100% corresponds to the original value to represent adding or subtracting a percent of the original value.
3. **Language Goal:** Interpret a verbal description to write an equation involving percent increase or decrease. **(Reading)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.02

Jayla's bank account increased by 7% this year.

Write an equation to represent the relationship between the amount that Jayla started with, b, and the amount she has now, c.

Responses vary.

- $1b + 0.07b = c$
- $1.07b = c$
- $c = (1 + 0.07)b$

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use equations to represent percent change:

» **Respond to Student Thinking:**

- Revisit Activity 3 with students before providing individualized feedback on the relevant Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. Did the order in which you sequenced student ideas support the learning goals of the lesson? What might you change for the next time you facilitate a lesson like this?

Lesson Practice

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$1.25 \cdot b = c$ $0.80 \cdot b = c$

Responses vary.
- If someone works overtime they could get an extra 25% per hour.
- My friend always eats 20% of whatever I bring for lunch and I eat what's left.

Lesson Practice 4.02

Lesson Summary

We can use equations to help us make sense of situations involving *percent increase* or *percent decrease*.

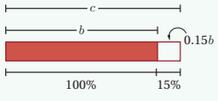
For example, c is 15% more than b.

Three equations can be written to model the relationship between b and c:

$c = b + 0.15b$

$c = (1 + 0.15)b$

$c = 1.15b$



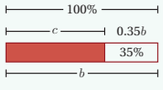
In this example, c is 35% less than b.

Three equations can be written to model the relationship between b and c:

$c = b - 0.35b$

$c = (1 - 0.35)b$

$c = 0.65b$



Lesson Practice 4.02

Name: _____ Date: _____ Period: _____

1. Draw a diagram that represents this situation:

The number of people in a town has increased by 50% in the past decade.



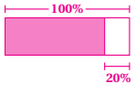
2. Write a story that matches the diagram.



Responses vary. The number of pets in my family increased by 25%.

3. Draw a diagram that represents this situation:

The amount of paper that the copy shop used this month decreased by 20% compared to what they used last month.



4. A new video game costs $60 on the day it gets released. The price of a new game generally drops by about 12% within a few weeks. What will the price of this new game be a few weeks after release? Show or explain your thinking.

$52.80. *Explanations vary.*

$100\% - 12\% = 88\%$

Let c represent the original cost of the game and s represent the price. Then $0.88 \cdot c = s$. If $c = 60$, then $0.88 \cdot 60 = s$, so $52.80 = s$.

5. Kai and Amari both wrote an equation to represent an increase of 3%, where x represents the amount before the increase and y represents the new amount.

Kai wrote the equation $y = 1x + 0.03x$. Amari wrote the equation $y = 1.03x$.

Whose equation is correct? Circle one.

Kai Amari (Both) Neither

Explain your thinking.

Explanations vary. If I substitute a number in for either equation, I get a value that is 3% more than the value I started with.

Lesson Practice 4.02

Name: _____ Date: _____ Period: _____

Problems 6–7: A sneaker store raised its prices 15% compared to last year.

6. If x is the price before the increase and y is the price after the increase, which equations are correct? Select *all* that apply.

☑ A. $y = 1.15x$ ☐ B. $y = x + 0.15$ ☑ C. $y = x + 0.15x$

☐ D. $y = 15 + x$ ☑ E. $y = (1 + 0.15)x$

7. A pair of designer sneakers was $120 last year. What is the price this year?

$138

FAST Practice

8. Two stores each advertise a discount on the same type of sweatshirt. At both stores, the original price of the sweatshirt is $25.

- Store A discounts the price of the sweatshirt by 20%.
- Store B discounts the price of the sweatshirt by 15%.

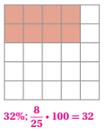
How much less is the discounted price of the sweatshirt at Store A than the discounted price of the sweatshirt at Store B?

A. $0.75 B. $1.00 (C.) $1.25 D. $1.75

Spiral Review

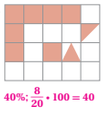
Problems 9–10: Determine the percentage of each grid that is shaded.

9.



32%; $\frac{8}{25} \cdot 100 = 32$

10.



40%; $\frac{8}{20} \cdot 100 = 40$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	2	MA.7.AR.3.1
FAST Practice	8	2	MA.7.AR.3.1
Spiral Review			
	9–10	1	MA.6.AR.3.4

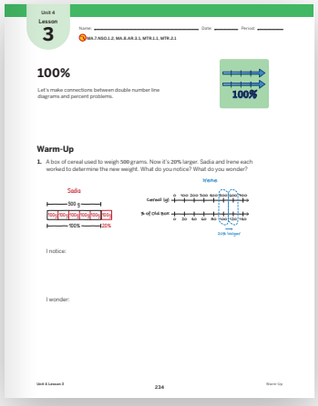
Unit 4

Lesson 3

100%

Percent Increase and Decrease With Double Number Lines

Let's make connections between double number line diagrams and percent problems.



Learning Outcomes

Today's Goals

1. **Goal:** Use double number lines to represent situations involving a percent increase or decrease.
2. **Goal:** Determine the original amount given the new amount and a percent increase or decrease.
3. **Language Goal:** Interpret a situation to solve problems about percent increase or decrease using mathematically precise language. **(Reading and Writing)**

Students expand on their understanding of double number line diagrams and percentages from Grade 6 to make sense of and model problems involving percent change. Later in the lesson, students make connections between tables, equations, and double number line diagrams to determine the new value when given the original value and the percent of increase or decrease. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 2, students represented situations involving percent increase or decrease using equations.

Future Learning

In Lesson 4, students will practice calculating the original value, new value, or the percent of change given the other two quantities.

Connections

- Students determine the new value of an item when given the original value and the percent of increase (or decrease) in real world contexts such as the decrease in the size of popcorn bags in a movie theater. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.AR.3.4

Vocabulary

New Vocabulary

markdown

markup

Lesson at a Glance

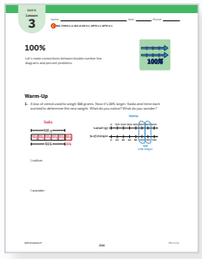
~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

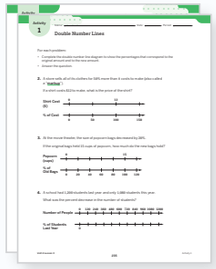
Students use the **Notice and Wonder** routine to analyze two student work samples for a percent increase problem. **(MTR.2.1)**



Activity 1

Pairs | 15 min

Students use double number line diagrams to calculate the original amount, new amount, or percent change, given the other two quantities.



Activity 2

Pairs | 15 min

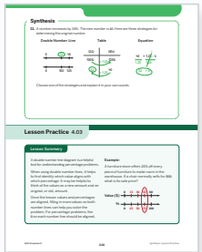
Students use multiple representations to determine how a turtle population has changed over a year. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using tables, equations, and double number line diagrams to solve problems involving percent increase and decrease.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving for the new value when given the percent increase and original value.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to analyze two student work samples for a percent increase problem. **(MTR.2.1)**

1 Launch

To generate engagement, consider asking, "What is your favorite cereal? How would you feel if the box was 20% larger?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of the math representations. **(MTR.2.1)**

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means. Clarify any words or phrases they don't know. **(Reading)**

Encourage students to think independently and then share their thinking with a partner about the two diagrams.

2 Connect

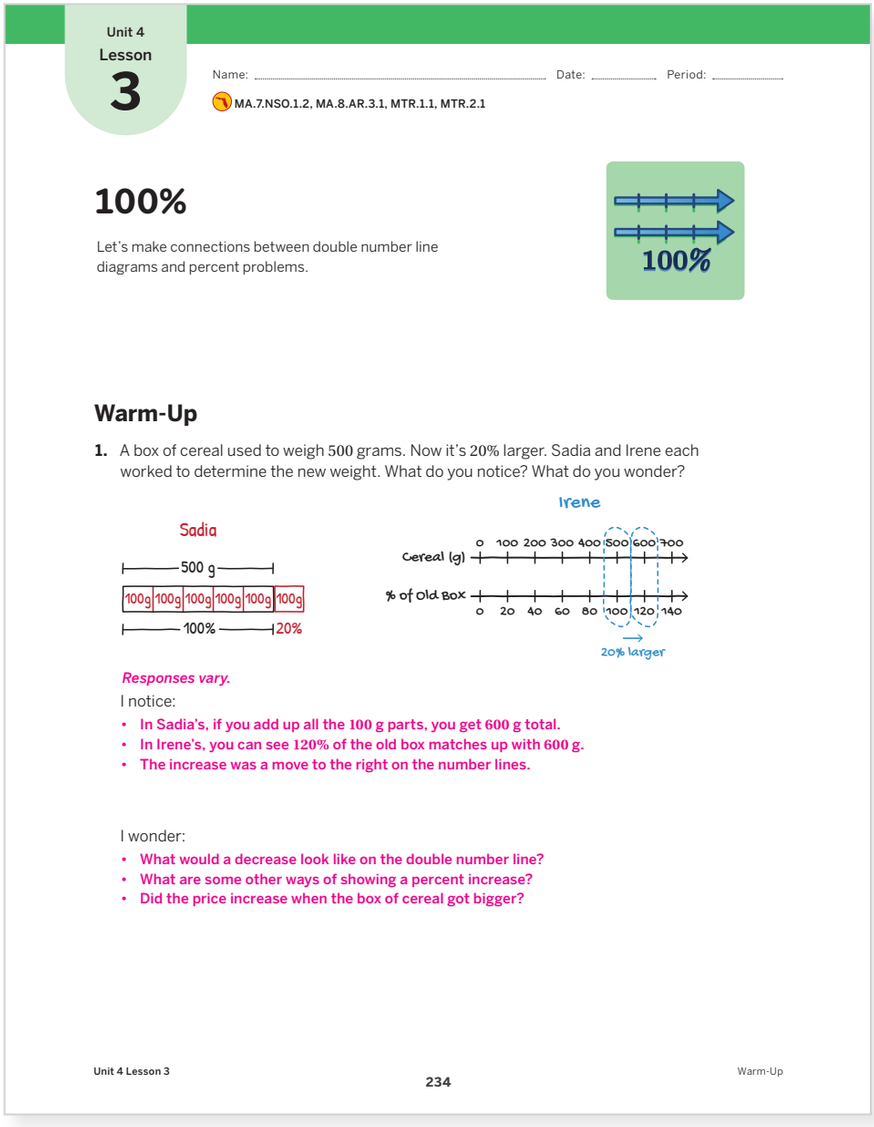
Discuss student explanations for interpreting each representation. Consider asking:

- "How are these two representations connected?"
- "Where can you see the percent increase in both representations?"
- "What would a percent decrease look like in each of these representations?"

Create a record of what students noticed and wondered, along with the name of the students who shared them.

A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific point on each representation and describe what that point means to support interpreting visual representations.

Note: If it doesn't come up naturally, consider asking, "What do you recall about double number lines?"



Unit 4 Lesson 3

Name: Date: Period:

MA.7.NSO.1.2, MA.8.AR.3.1, MTR.1.1, MTR.2.1

100%

Let's make connections between double number line diagrams and percent problems.

Warm-Up

1. A box of cereal used to weigh 500 grams. Now it's 20% larger. Sadia and Irene each worked to determine the new weight. What do you notice? What do you wonder?

Responses vary.

I notice:

- In Sadia's, if you add up all the 100 g parts, you get 600 g total.
- In Irene's, you can see 120% of the old box matches up with 600 g.
- The increase was a move to the right on the number lines.

I wonder:

- What would a decrease look like on the double number line?
- What are some other ways of showing a percent increase?
- Did the price increase when the box of cereal got bigger?

Unit 4 Lesson 3 234 Warm-Up

Pairs | 15 min

Activity 1 Double Number Lines

Purpose: Students use double number line diagrams to calculate the original amount, new amount, or percent change, given the other two quantities.

Short on time? Consider omitting Problem 3, as students will work with double number lines with 5 partitions and percent decrease in other problems.

1 Launch

Direct students' attention to Irene's work in the Warm-Up.

- "Where do you see connections between Irene's work and the representation?" **(MTR.2.1)**
- "Why did Irene choose the number of partitions for her number line?" **(MTR.2.1)**
- "What might be a problem that would benefit from using 4 partitions? 10 partitions?" **(MTR.2.1)**

Invite students to do a first read of Problems 2–7 to initially determine whether each situation shows a percent increase or decrease and consider what number of partitions will be most helpful.

Create a class definition for the terms **markup** and **markdown** using students' language and the glossary.

Encourage students to move several times throughout the activity to share their reasoning with a partner and reach an agreement on the double number line that represents the problem.

2 Monitor

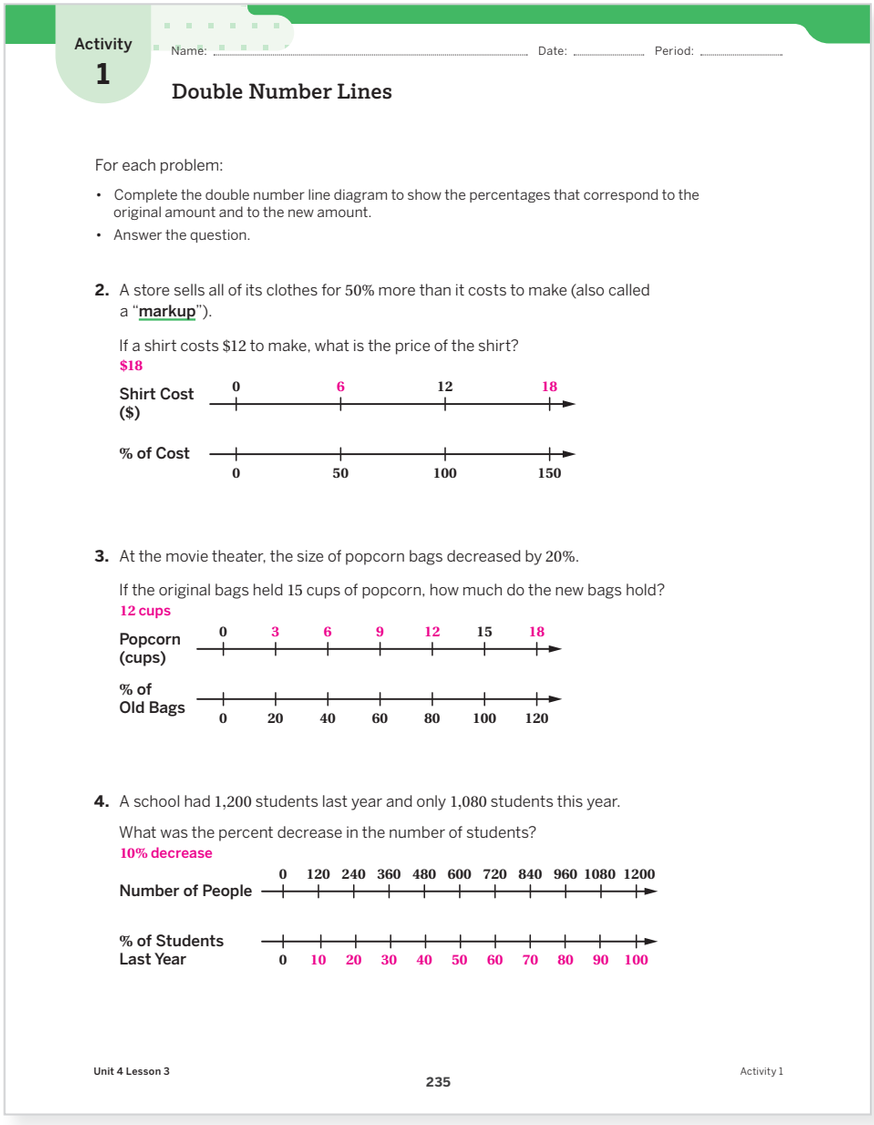
To support students getting started, invite them to determine what value represents 100% (original amount) in each problem or if that value is unknown.

ELL **English Language Learners** Invite students to use one color to highlight the original amount — even if unknown — in each problem and use the same color to locate that quantity on the double number line diagram. Have them repeat this process for the new amount, using a different color. **(Reading)**

D **Differentiation: (Support)** For students who think that the given amount always represents the original value (100%), encourage them to think of relatable examples, such as items marked on sale and how the price shown is not always the original price.

A **Accessibility: Executive Functioning** To support students in keeping track of information, consider providing or displaying a table (column suggestions: original amount, percent increase/decrease [if applicable], new amount) that students could use to record these quantities for each problem.

Activity 1 continued >



Activity 1

Name: Date: Period:

Double Number Lines

For each problem:

- Complete the double number line diagram to show the percentages that correspond to the original amount and to the new amount.
- Answer the question.

2. A store sells all of its clothes for 50% more than it costs to make (also called a "markup").

If a shirt costs $12 to make, what is the price of the shirt?

$18

Shirt Cost ($): 0, 6, 12, 18

% of Cost: 0, 50, 100, 150

3. At the movie theater, the size of popcorn bags decreased by 20%.

If the original bags held 15 cups of popcorn, how much do the new bags hold?

12 cups

Popcorn (cups): 0, 3, 6, 9, 12, 15, 18

% of Old Bags: 0, 20, 40, 60, 80, 100, 120

4. A school had 1,200 students last year and only 1,080 students this year.

What was the percent decrease in the number of students?

10% decrease

Number of People: 0, 120, 240, 360, 480, 600, 720, 840, 960, 1080, 1200

% of Students Last Year: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

Unit 4 Lesson 3 235 Activity 1

Pairs | 15 min

Activity 1 Double Number Lines (continued)

Purpose: Students use double number line diagrams to calculate the original amount, new amount, or percent change, given the other two quantities.

Short on time? Consider omitting Problem 3, as students will work with double number lines with 5 partitions and percent decrease in other problems.

2 Monitor

Look for a variety of student strategies, such as:

- Making sense of the problems to decide whether the amount given would increase or decrease based on the description of the percentage. **(MTR.1.1)**
- Realizing that the previous year's or previous week's value corresponds to 100% on the double number line diagram. *(Problems 3–4)*
- Dividing 100% in equal intervals on the double number line diagram. *(Problems 5–6)*

D **Differentiation: (Support)** For students who are unsure how many partitions to utilize in their double number line diagrams, consider asking, "How does the percent increase or decrease for this problem help us determine how many partitions are needed?"

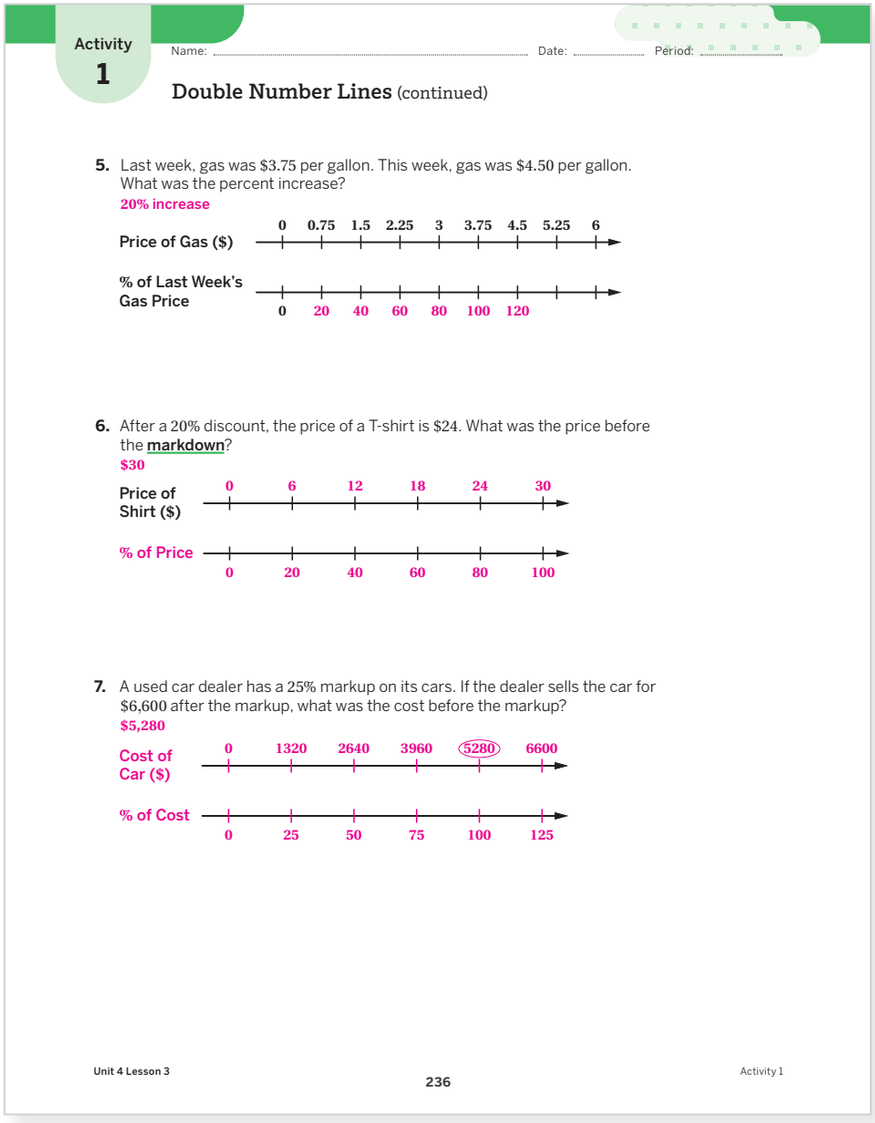
Look for good examples of students' double number lines for Problem 6 to share during the Connect.

ELL **English Language Learners** During this lesson, consider personally inviting students to share responses, asking if they would feel comfortable sharing with the class. Consider inviting students to respond in their primary languages. **(Speaking and Listening)**

3 Connect

To surface the Key Takeaway, consider asking:

- "How did you decide how many partitions were useful for this context?"
- "How does your representation help you see how much the original price of the car was before the markup?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

Double Number Lines (continued)

5. Last week, gas was $3.75 per gallon. This week, gas was $4.50 per gallon. What was the percent increase?

20% increase

Price of Gas ($): 0, 0.75, 1.5, 2.25, 3, 3.75, 4.5, 5.25, 6

% of Last Week's Gas Price: 0, 20, 40, 60, 80, 100, 120

6. After a 20% discount, the price of a T-shirt is $24. What was the price before the markdown?

$30

Price of Shirt ($): 0, 6, 12, 18, 24, 30

% of Price: 0, 20, 40, 60, 80, 100

7. A used car dealer has a 25% markup on its cars. If the dealer sells the car for $6,600 after the markup, what was the cost before the markup?

$5,280

Cost of Car ($): 0, 1320, 2640, 3960, 5280, 6600

% of Cost: 0, 25, 50, 75, 100, 125

Unit 4 Lesson 3 236 Activity 1

Key Takeaway: You can use a double number line diagram to represent a percent increase or a percent decrease. To create the diagram, fill in the information that's given and then use those clues to complete the diagram. For example, the original amount always aligns with 100%. A percent increase always aligns with that percent plus 100%. To determine how to partition the rest of the diagram, consider the difference between the percentages of quantities given. For example, for a 25% increase, make tick marks every 25%. For a decrease from 40 to 32, make tick marks every 8.

Pairs | 15 min

Activity 2 Green Sea Turtles

Purpose: Students use multiple representations to determine how a turtle population has changed over a year. **(MTR.2.1)**

1 Launch

Direct students' attention to the instructions and read them aloud, clarifying them as a class.

To surface connections, share with students that these mathematical representations can support sensemaking beyond markups and markdowns. Consider reading the prompt aloud and sharing that a sanctuary is defined as a safe place.

2 Monitor

To support students getting started, consider asking, "Does 234 represent the original amount or the new amount of nesting turtles?"

Look for a variety of student ideas, such as:

- Creating tables, double number line diagrams, or equations to showcase clear thinking.
- Realizing the equation would utilize 0.90 to represent the percent decrease for this context.
- Realizing that the number of nesting turtles would be greater than 234 in the previous year and less than 234 next year.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore percent decrease in context further to complete this optional task. Encourage them to discuss their thinking with a partner.

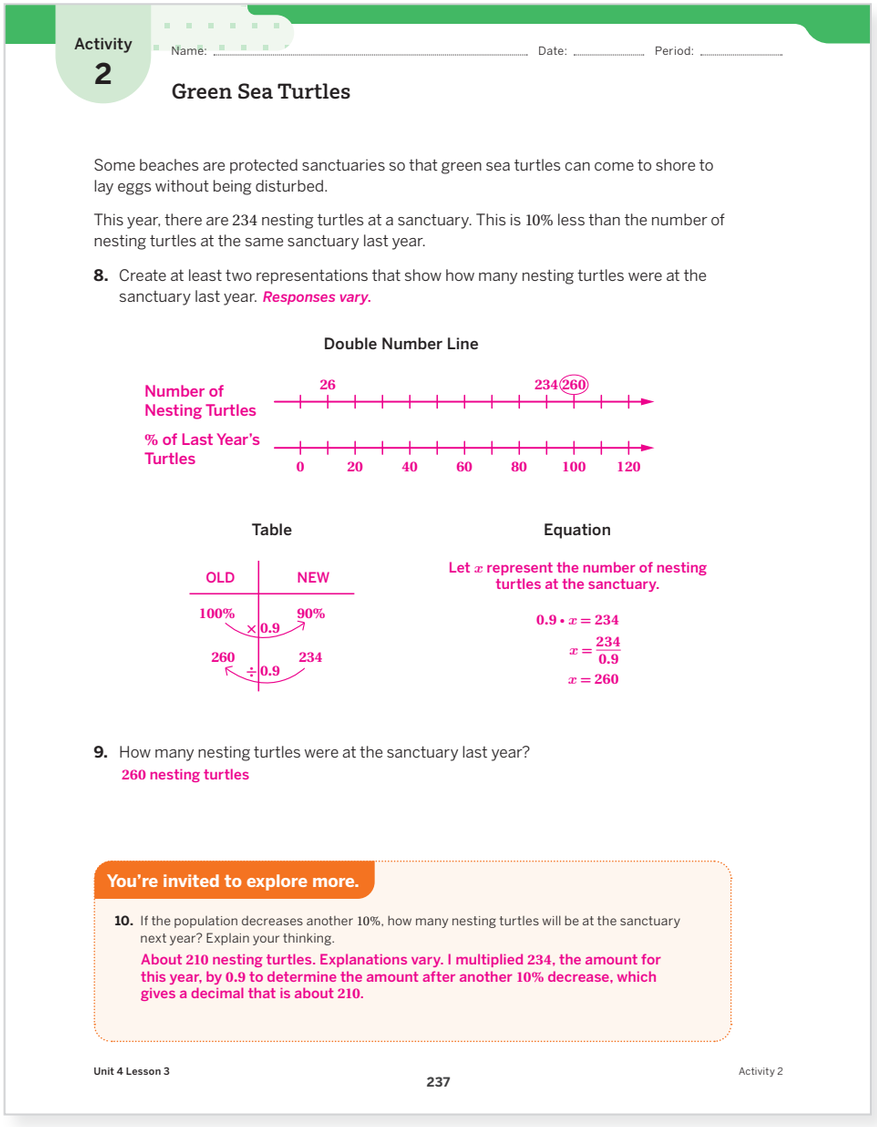
3 Connect

Invite students to share their representations for the context around sea turtles. Consider asking, "Which representation do you prefer? Why?"

ELL **English Language Learners** Provide sentence frames to support students as they share their preferred representations. For example, "I like the double number line diagram because it shows _____." or "The table helps me understand _____ because _____." **(Reading and Speaking)**

To surface the Key Takeaway, consider asking:

- "Where do you see the same information in the double number line diagram, table, and equation?"
- "How do your representations help you see how many turtles are at the sanctuary?" **(MTR.2.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Green Sea Turtles

Some beaches are protected sanctuaries so that green sea turtles can come to shore to lay eggs without being disturbed.

This year, there are 234 nesting turtles at a sanctuary. This is 10% less than the number of nesting turtles at the same sanctuary last year.

8. Create at least two representations that show how many nesting turtles were at the sanctuary last year. *Responses vary.*

Double Number Line

Table

OLD	NEW
100%	90%
260	234

×0.9 (100% → 90%); ÷0.9 (260 → 234)

Equation

Let x represent the number of nesting turtles at the sanctuary.

$0.9 \cdot x = 234$

$x = \frac{234}{0.9}$

$x = 260$

9. How many nesting turtles were at the sanctuary last year?

260 nesting turtles

You're invited to explore more.

10. If the population decreases another 10%, how many nesting turtles will be at the sanctuary next year? Explain your thinking.

About 210 nesting turtles. Explanations vary. I multiplied 234, the amount for this year, by 0.9 to determine the amount after another 10% decrease, which gives a decimal that is about 210.

Unit 4 Lesson 3 237 Activity 2

Key Takeaway: You can use multiple representations, including double number lines, tables, equations, to determine the original or new amount or to determine the percent change given the other two quantities.

Whole Class | 5 min

Synthesis

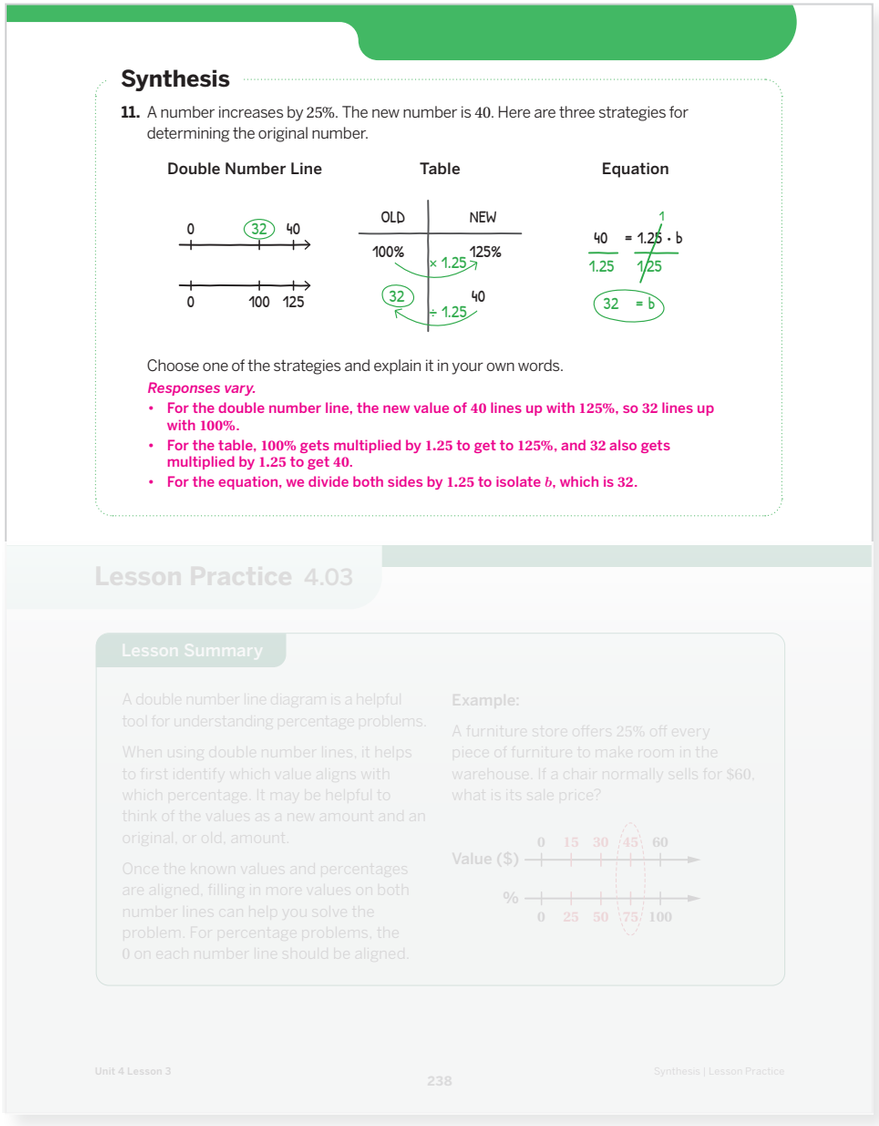
Purpose: Students synthesize their understanding of using tables, equations, and double number line diagrams to solve problems involving percent increase and decrease.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Explaining the relationship in the double number line diagram and how the percent increase aligns with the new value.
- Using a constant of proportionality in the table to multiply by 100% in the first row and divide the new value by in the second row to find the original number.
- Converting the percent increase to a decimal, writing an equation that relates the new number to the original number, and solving the equation.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for determining the original number.

Double Number Line

Table

Equation

Choose one of the strategies and explain it in your own words.

Responses vary.

- For the double number line, the new value of 40 lines up with 125%, so 32 lines up with 100%.
- For the table, 100% gets multiplied by 1.25 to get to 125%, and 32 also gets multiplied by 1.25 to get 40.
- For the equation, we divide both sides by 1.25 to isolate b, which is 32.

Lesson Practice 4.03

Lesson Summary

A double number line diagram is a helpful tool for understanding percentage problems.

When using double number lines, it helps to first identify which value aligns with which percentage. It may be helpful to think of the values as a new amount and an original, or old, amount.

Once the known values and percentages are aligned, filling in more values on both number lines can help you solve the problem. For percentage problems, the 0 on each number line should be aligned.

Example:

A furniture store offers 25% off every piece of furniture to make room in the warehouse. If a chair normally sells for $60, what is its sale price?

Unit 4 Lesson 3 238 Synthesis | Lesson Practice

Lesson Takeaway: A double number line diagram can be an appropriate tool to represent a percent increase or a percent decrease situation. The diagram can help you solve for the original amount, new amount, or percent change, when given the other two quantities. Other representations, such as a table or an equation, are more concise tools for solving the same types of problems.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving for the new value when given the percent increase and original value.

Today's Goals

1. **Goal:** Use double number lines to represent situations involving a percent increase or decrease.
2. **Goal:** Determine the original amount given the new amount and a percent increase or decrease.
3. **Language Goal:** Interpret a situation to solve problems about percent increase or decrease using mathematically precise language. **(Reading and Writing)**

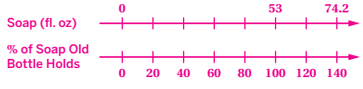
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.03

A company's new container holds 40% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold? Show or explain your thinking.

74.2 fluid ounces. *Explanations vary.* 53 fluid ounces represents the original 100%. I multiplied 53 by 1.4 to determine how much the new container holds.



Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use double number lines to solve percent increase and decrease problems:

» **Respond to Student Thinking:**

- Use the ***Calculating Percentages*** **Mini-lesson**.
- Assigning a few **Practice Problems** in class and inviting students to share their strategies for representing percent increase and percent decrease with a double number line.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students were reintroduced to double number lines as a representation for solving percent increase and percent decrease problems. How will this representation connect to other strategies in the upcoming lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for determining the original number.

Double Number Line

Table

Equation

Choose one of the strategies and explain it in your own words.

Responses vary.

- For the double number line, the new value of 40 lines up with 125%, so 32 lines up with 100%.
- For the table, 100% gets multiplied by 1.25 to get to 125%, and 32 also gets multiplied by 1.25 to get 40.
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Lesson Practice 4.03

Lesson Summary

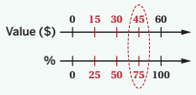
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Once the known values and percentages are aligned, filling in more values on both number lines can help you solve the problem. For percentage problems, the 0 on each number line should be aligned.

Example:

A furniture store offers 25% off every piece of furniture to make room in the warehouse. If a chair normally sells for $60, what is its sale price?



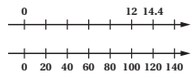
Lesson Practice 4.03

Name: _____ Date: _____ Period: _____

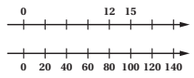
1. Abdullah and Kanna are working on this problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How many fluid ounces of juice does the new packaging hold?

Abdullah's Double Number Line



Kanna's Double Number Line



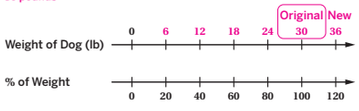
Whose double number line represents the problem? Explain your thinking.

Abdullah's. *Explanations vary.* The number 12 should be aligned with 100% because that is the original amount of fluid ounces.

Problems 2–3: Read each scenario and then answer the question. Complete the double number line diagram if it helps with your thinking.

2. A dog weighs 20% more than it did three months ago. It weighs 36 pounds now. How much did the dog weigh three months ago?

30 pounds



3. A bakery used 25% less butter this month than last month. The bakery used 240 kilograms of butter this month. How much did it use last month?

320 kilograms



Lesson Practice 4.03

Name: _____ Date: _____ Period: _____

Problems 4–5: Next week, the price of oranges at the farmer's market will decrease by 20%.

4. Select *all* the equations that represent the relationship between the price of oranges this week, x, and the price of oranges next week, y.

- ☐ A. $y = \frac{1}{5}x$
- ☑ B. $y = \frac{4}{5}x$
- ☑ C. $y = x - \frac{1}{5}x$
- ☐ D. $y = 0.2x$
- ☑ E. $y = 0.8x$

5. The price of oranges this week is $4.50. What will the price be next week?

$3.60

6. Without determining the actual percent increase, explain why it is not reasonable to say that a price increase from $4.00 to $5.00 for a bag of clementines represents a 125% increase.

***Responses vary.* A 125% increase would mean the new price is 225% of the original price, so it more than doubles. $5.00 is not more than twice $4.00.**

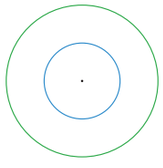
FAST Practice

7. The number of fish in a lake decreased by 15% between last year and this year. This year, there are 51 fish in the lake. What was the population last year?

60 fish

Spiral Review

Problems 8–9: Here is a circle and a scaled copy of the circle with a scale factor of 2.



8. How does the circumference of the scaled copy compare to the circumference of the original circle?

***Responses vary.* The circumference of the scaled copy is twice the circumference of the original.**

9. How does the area of the scaled copy compare to the area of the original circle?

***Responses vary.* The area of the scaled copy is 4 times the area of the original.**

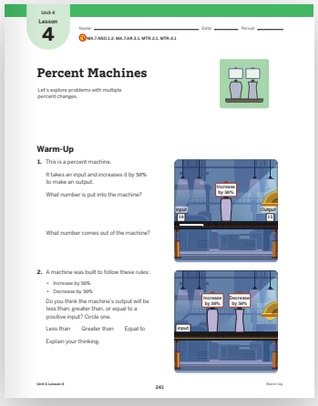
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.3.1
	6	3	MA.7.AR.3.1
FAST Practice	7	2	MA.7.AR.3.1
Spiral Review			
	8–9	3	MA.7.GR.1.5

Unit 4

Lesson

4


Unit 4
Lesson
4
Name _____ Date _____ Period _____
MA.7.NSO.1.2, MA.7.AR.3.1, MTR.2.1, MTR.4.1
Percent Machines
Let's explore problems with multiple percent changes.
Warm-Up
1. This is a percent machine.
It takes an input and increases it by 50% to make an output.
What number is put into the machine?
What number comes out of the machine?
Input
Output
2. A machine was built to follow these rules:
• Increase by 50%
• Decrease by 50%
Do you think the machine's output will be less than, greater than, or equal to a positive input? Circle one.
Less than Greater than Equal to
Explain your thinking.
241
Warm-Up

Percent Machines

Calculating Unknowns With Percentages

Let's explore problems with multiple percent changes.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the original amount given the new amount and a percent change for one-step and multistep percent problems.
2. **Language Goal:** Explain a strategy for calculating an original value, new value, or percent change given the other two quantities. **(Writing, Speaking, and Listening)**

Students practice calculating the original value, new value, or percent change given the other two quantities. This is also the first lesson in which students experience multistep percent problems, so they will spend time analyzing and explaining strategies to approach these problems. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 3, students made connections between double number line diagrams and percent problems. They learned that percent increase and percent decrease can be represented with double number line diagrams, tables, and equations.

Future Learning

In Lesson 6, students will use understanding of multistep percent problems to make sense of tax and tip.

Connections

- Students determine the percent increase or percent decrease when given the original value of an item and the new value in real world contexts, such as amount of fruit harvested. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.8.AR.2.1

Lesson at a Glance

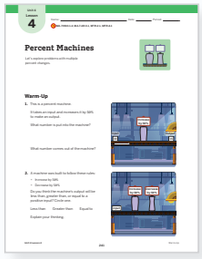
~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

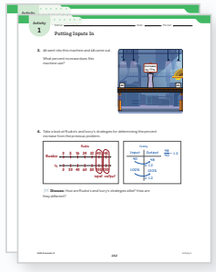
Students are introduced to percent machines, which take an input value and increase or decrease that value by a percent to produce an output.



Activity 1

Pairs | 10 min

Students develop strategies for calculating the original value, the new value, or the percent change given the other two quantities.
MLR7: Compare and Connect



Activity 2

Pairs | 10 min

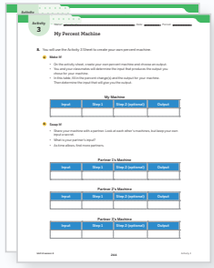
Students use the MLR3: Critique, Correct, Clarify routine to hone strategies for determining other unknown quantities in percent change problems.



Activity 3

Pairs | 10 min

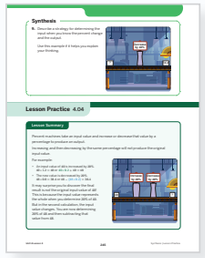
Students express their creativity by creating their own challenge and also practice calculating the original value given the new value and a percent change.



Synthesis

Whole Class | 5 min

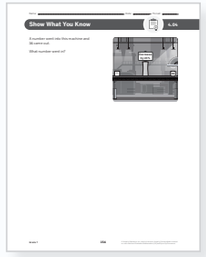
Students synthesize strategies for determining an input given the percent change and output values.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using a percent machine to calculate an input value.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheet, one per student
- scientific calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to percent machines, which take an input value and increase or decrease that value by a percent to produce an output.

1 Launch

Direct students' attention to the percent machine. Consider asking, "What number was put into the machine? What number came out of the machine?"

ELL **English Language Learners** Invite students to share what the terms *input* and *output* remind them of for this context. (e.g., "When I hear *[input/output]*, I think___."). **(Reading and Listening)**

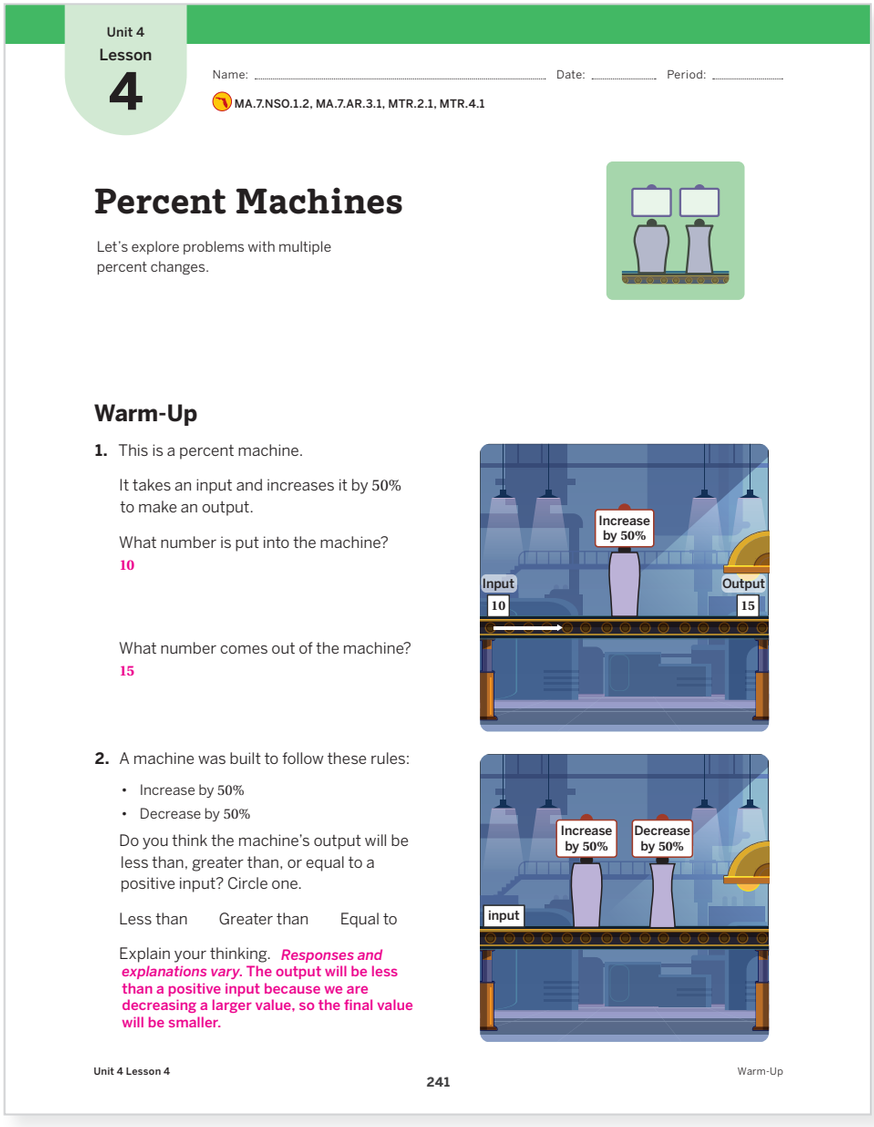
Invite students to respond individually to the question for 1–2 minutes before sharing their thinking with a partner.

2 Connect

Invite students to justify whether they believe the output will be greater than, less than, or equal to the input and to critique one another's reasoning. **(MTR.4.1)**

Consider asking, "In what ways could using specific values help to make our arguments stronger and clearer?"

Note: If it does not come up naturally, consider sharing when working with machines with multiple percent changes, students should complete the first percent change and use their solution to solve the second.



Unit 4
Lesson
4

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.2.1, MTR.4.1

Percent Machines

Let's explore problems with multiple percent changes.

Warm-Up

1. This is a percent machine.

It takes an input and increases it by 50% to make an output.

What number is put into the machine?
10

What number comes out of the machine?
15

2. A machine was built to follow these rules:

- Increase by 50%
- Decrease by 50%

Do you think the machine's output will be less than, greater than, or equal to a positive input? Circle one.

Less than Greater than Equal to

Explain your thinking. *Responses and explanations vary.* The output will be less than a positive input because we are decreasing a larger value, so the final value will be smaller.

Unit 4 Lesson 4 241 Warm-Up

Pairs | 10 min

Activity 1 Putting Inputs In

Purpose: Students develop strategies for calculating the original value, the new value, or the percent change given the other two quantities.

1 Launch

To surface connections, consider sharing that students will encounter a variety of problems with percent machines.

Consider asking, "How has this problem changed from the Warm-up?" Invite students to test their reasoning from the Warm-Up on Problem 3 and then continue through each problem at their own pace.

A **Accessibility: Executive Functioning** Provide a graphic organizer to support students in documenting their guesses/strategy.

2 Monitor

Listen for a variety of student strategies, such as tables, tape diagrams, or double number lines.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider encouraging them to try to put in an estimate and use the feedback to revise their thinking.
Calculate $\frac{48}{40} = 1.2$ and enter the percent increase as 2%.	**Support** Consider asking, "What does 0.2 represent as a percent? How does that compare to 0.02 as a percent?"
Create tables, double number lines, drawings, or use a calculator to support their thinking. **(MTR.2.1)**	**Strengthen** Invite them to share their strategies during the Connect.

3 Connect

Direct students' attention to Rudra's and Ivory's strategies for solving percent problems. Invite students to think individually about the question before sharing their thinking with a partner.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

To surface the Key Takeaway, consider asking:

- "How did ____ solve the problem?"
- "Where do you see the same information in Rudra's and Ivory's strategy?" **(MTR.2.1)**
- "How does each strategy compare to your own?" **(MTR.4.1)**

Activity 1

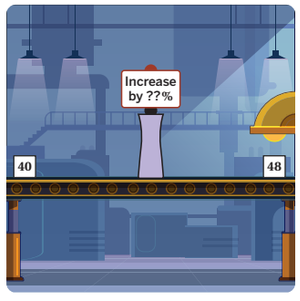
Name: Date: Period:

Putting Inputs In

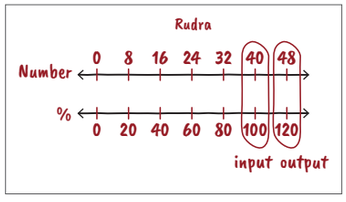
3. 40 went into this machine and 48 came out.

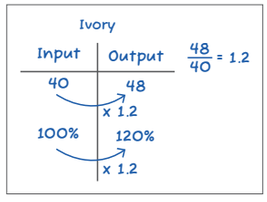
What percent increase does this machine use?

20%



4. Take a look at Rudra's and Ivory's strategies for determining the percent increase from the previous problem.





Discuss: How are Rudra's and Ivory's strategies alike? How are they different?

Responses vary. They both found an increase of 20%, but Rudra used double number lines and Ivory used a table.

Unit 4 Lesson 4 242 Activity 1

Key Takeaway: To solve for an unknown value in a percent increase problem, you can represent the relationship in an equation, table, or double number line. When a percentage is represented as a decimal, it is important to represent that value as "out of 100".

Pairs | 10 min

Activity 2 Getting Outputs Out

Purpose: Students use the MLR3: Critique, Correct, Clarify routine to hone strategies for determining other unknown quantities in percent change problems.

1 Launch

To surface connections, direct students' attention the percent machine and share with students that they will continue to find unknown values in percent machine challenges.

2 Monitor

Differentiation: (Support) To support students getting started, consider asking, "Do you think the input is greater than, less than, or equal to 36?"

Look for a variety of student strategies for computing the input for this percent machine.

Listen for a variety of responses about Kai's work, and consider highlighting ideas such as:

- Increasing by 20% and then decreasing that value by 20% do not balance each other out.
- The constant of proportionality should be the same within each relationship (in this case, 1.20).
- It can be helpful to think of the input as 100%.
- You can divide by the constant of proportionality in order to work backward to figure out the input.

Note: The purpose of this problem is to introduce students to multistep machines in which they need to determine the original value.

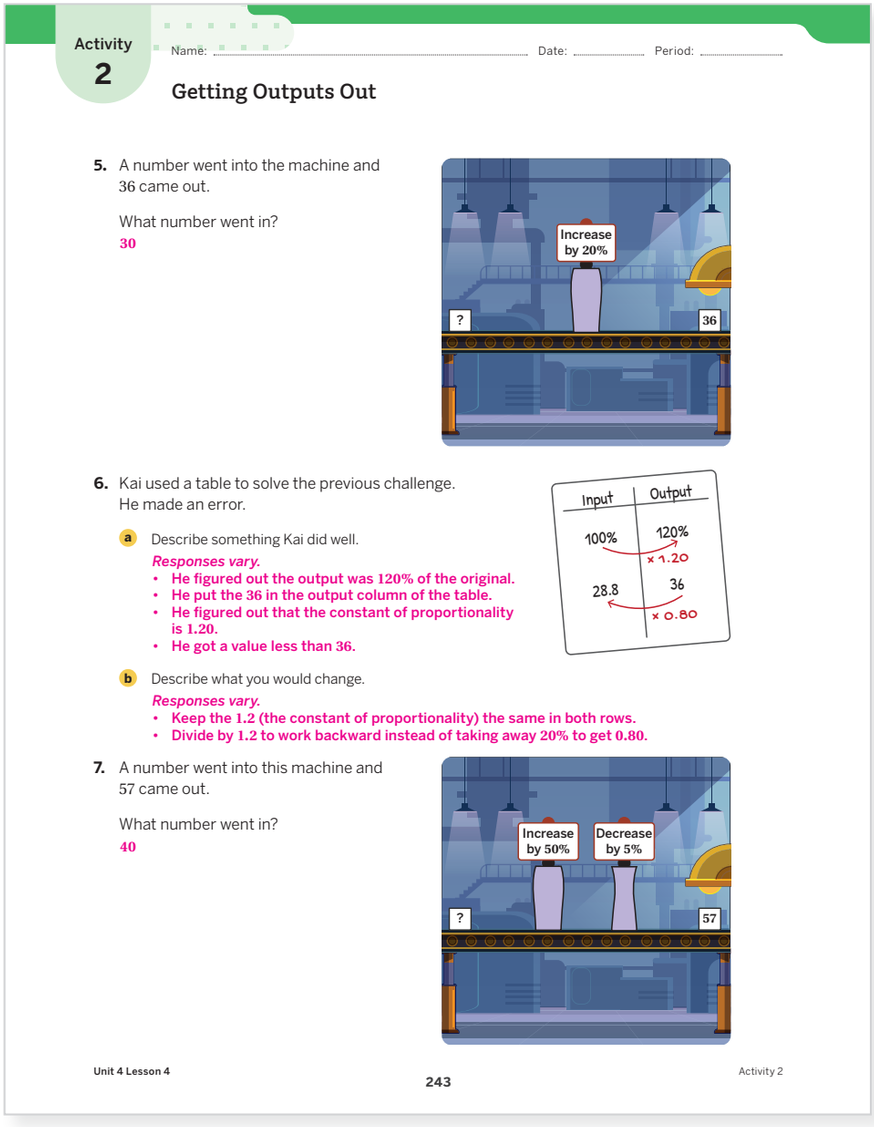
3 Connect

Invite students to share their thinking and explain how they determined the original value.

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Kai's work. Then invite students to:

- Discuss what is *correct* about the response.
- Discuss what is *incorrect* or *unclear*.
- Write a question to help the student see how their work can be improved, or create a revised response. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "How could we utilize Kai's strategy to solve the multistep problem in Problem 7?" Consider capturing the discussion for students to reference throughout the remainder of the lesson.



Activity 2

Name: Date: Period:

Getting Outputs Out

5. A number went into the machine and 36 came out.

What number went in?

30

6. Kai used a table to solve the previous challenge. He made an error.

a. Describe something Kai did well.

Responses vary.

- He figured out the output was 120% of the original.
- He put the 36 in the output column of the table.
- He figured out that the constant of proportionality is 1.20.
- He got a value less than 36.

b. Describe what you would change.

Responses vary.

- Keep the 1.2 (the constant of proportionality) the same in both rows.
- Divide by 1.2 to work backward instead of taking away 20% to get 0.80.

7. A number went into this machine and 57 came out.

What number went in?

40

Unit 4 Lesson 4 243 Activity 2

Key Takeaway: We can represent each percent change as a constant of proportionality that relates the input and outputs. For multistep problems, it is helpful to determine a constant of proportionality for both percent changes in the problem.

Pairs | 10 min

Activity 3 My Percent Machine

Purpose: Students express their creativity by creating their own challenge and also practice calculating the original value given the new value and a percent change.

Short on time? Consider inviting students to complete fewer of their classmates' challenges.

1 Launch

Consider sharing that students will create their own challenge like the percent machines we have seen throughout this lesson.

Distribute the Activity 3 Sheet to students.

2 Monitor

Listen for strategies students use in discussing how to solve the problems, such as:

- Setting up a table of the relationship and utilizing a strategy similar to Ivory's to solve for the unknown.
- Creating a double number line, similar to Rudra's, and using it to solve the percent machine challenges.
- Writing equations and working backwards to solve for the unknown value in each percent machine problem.

A **Accessibility: Executive Functioning** To support students initiating complex tasks, support students in identifying a challenge that would be a good place to start.

D **Differentiation: (Support)** For students working on multistep challenges, encourage them to refer back to Kai's strategy and support them with setting up a multistep table.

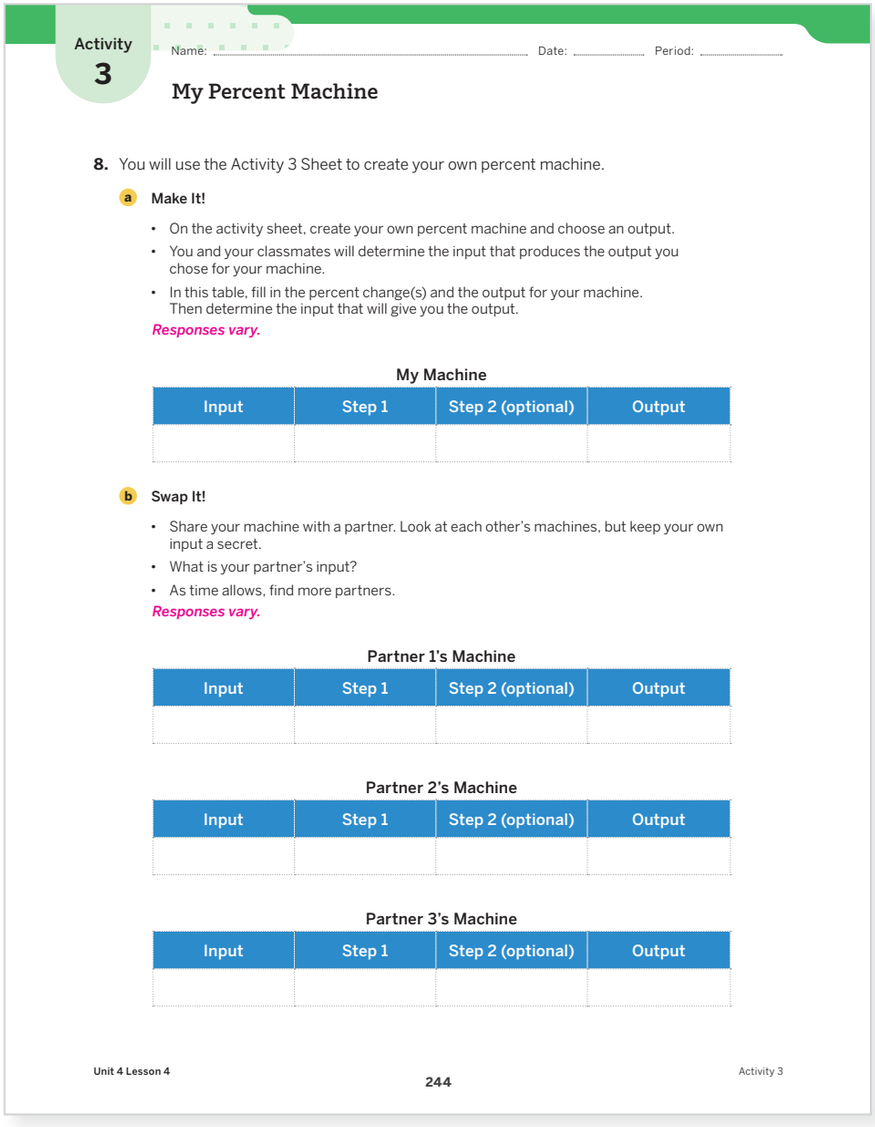
Look for several creative challenges and solutions to share during the Connect.

3 Connect

Share the student examples of percent machines. Invite students to discuss strategies that felt most useful in solving for the unknowns.

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "What made some challenges more complex than others?"
- "Was there a strategy you heard that surprised you?"



Activity 3

Name: Date: Period:

My Percent Machine

8. You will use the Activity 3 Sheet to create your own percent machine.

a **Make It!**

- On the activity sheet, create your own percent machine and choose an output.
- You and your classmates will determine the input that produces the output you chose for your machine.
- In this table, fill in the percent change(s) and the output for your machine. Then determine the input that will give you the output.

Responses vary.

My Machine

Input	Step 1	Step 2 (optional)	Output

b **Swap It!**

- Share your machine with a partner. Look at each other's machines, but keep your own input a secret.
- What is your partner's input?
- As time allows, find more partners.

Responses vary.

Partner 1's Machine

Input	Step 1	Step 2 (optional)	Output

Partner 2's Machine

Input	Step 1	Step 2 (optional)	Output

Partner 3's Machine

Input	Step 1	Step 2 (optional)	Output

Unit 4 Lesson 4 244 Activity 3

Whole Class | 5 min

Synthesis

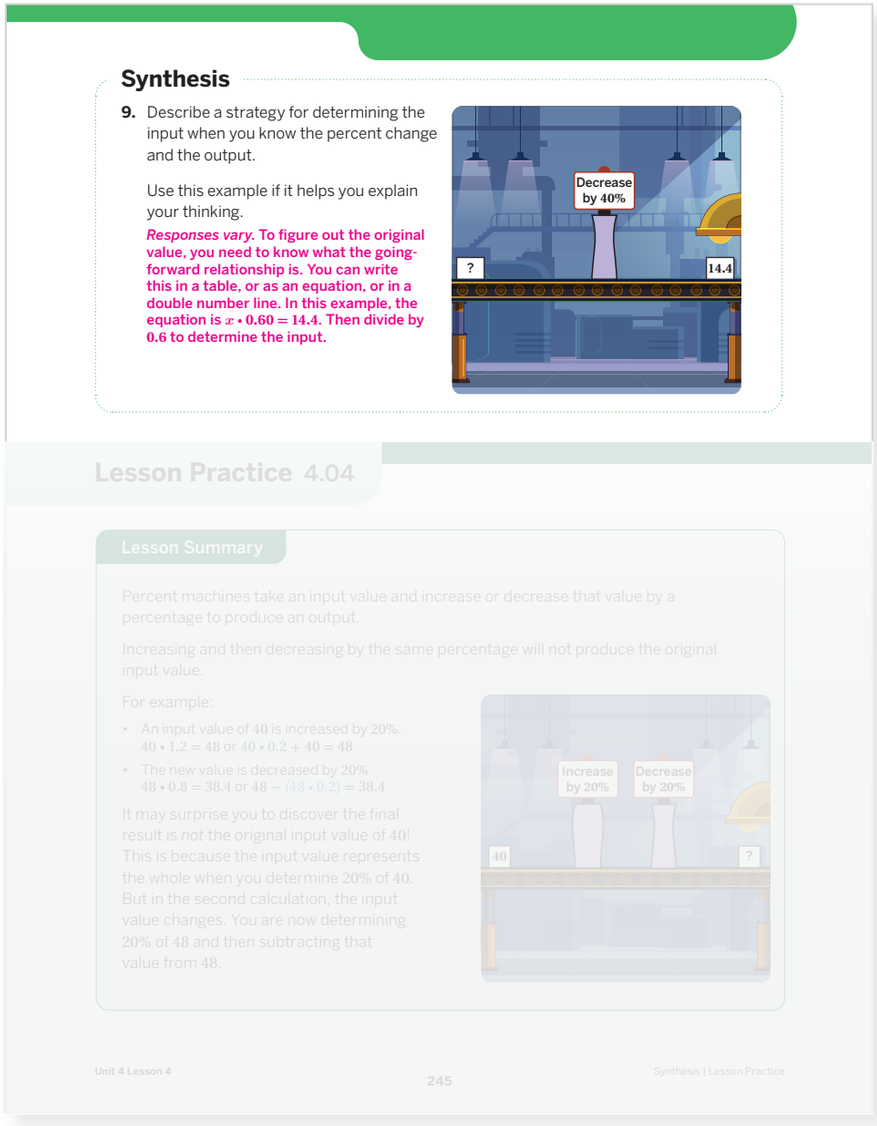
Purpose: Students synthesize strategies for determining an input given the percent change and output values.

To surface connections, encourage students to estimate whether the value should be less than, greater than, or equal to the output. Then give students 2–3 minutes to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Determining the original value by representing the relationship in a table or a double number line and utilizing the structure to reverse to solve for the unknown.
- Solving for an original value in a percent decrease problem by writing an equation with the percent as a decimal: $(1 - \text{percent decrease})x = \text{output}$ where x represents the input, and then solving the equation for the missing input.
- In this example, the equation is $x \cdot 0.60 = 14.4$. Then divide by 0.6 to determine the input.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. Describe a strategy for determining the input when you know the percent change and the output.

Use this example if it helps you explain your thinking.

Responses vary. **To figure out the original value, you need to know what the going-forward relationship is. You can write this in a table, or as an equation, or in a double number line. In this example, the equation is $x \cdot 0.60 = 14.4$. Then divide by 0.6 to determine the input.**

Lesson Practice 4.04

Lesson Summary

Percent machines take an input value and increase or decrease that value by a percentage to produce an output.

Increasing and then decreasing by the same percentage will not produce the original input value.

For example:

- An input value of 40 is increased by 20%. $40 \cdot 1.2 = 48$ or $40 \cdot 0.2 + 40 = 48$
- The new value is decreased by 20%. $48 \cdot 0.8 = 38.4$ or $48 - (48 \cdot 0.2) = 38.4$

It may surprise you to discover the final result is *not* the original input value of 40! This is because the input value represents the whole when you determine 20% of 40. But in the second calculation, the input value changes. You are now determining 20% of 48 and then subtracting that value from 48.

Unit 4 Lesson 4 245 Synthesis | Lesson Practice

Lesson Takeaway: To calculate an input value, percent change, or the output value, we can represent the percent change as the constant of proportionality between each input and output value. It is often helpful to set up equations, tables, number lines, or other representations to help us remember what value represents 100%.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using a percent machine to calculate an input value.

Today's Goals

1. **Goal:** Calculate the original amount given the new amount and a percent change for one-step and multistep percent problems.
2. **Language Goal:** Explain a strategy for calculating an original value, new value, or percent change given the other two quantities. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

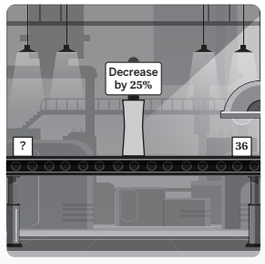
Name: _____ Date: _____ Period: _____

Show What You Know 4.04

A number went into this machine and 36 came out.

What number went in?

48



Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to [insert skill here]:

» **Respond to Student Thinking:**

- Revisit the challenges in Activities 1–2 and invite students to share strategies for determining the unknown values.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on the **MLR3: Critique, Correct, Clarify** routine in this lesson. This routine will help students when they again encounter incorrect thinking of their own or a peer. How can you support students in making a clear and concise claim, asking clarifying questions, and posing counterarguments? What about the routine felt successful? What are areas for growth next time?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe a strategy for determining the input when you know the percent change and the output.

Use this example if it helps you explain your thinking.

Responses vary. To figure out the original value, you need to know what the going-forward relationship is. You can write this in a table, or as an equation, or in a double number line. In this example, the equation is $x \cdot 0.60 = 14.4$. Then divide by 0.6 to determine the input.

Lesson Practice 4.04

Lesson Summary

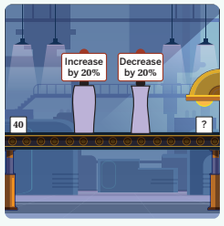
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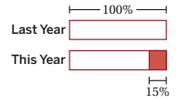


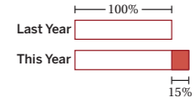
Lesson Practice 4.04

Name: _____ Date: _____ Period: _____

1. Match each situation with a diagram that represents it.

Situation	Diagram
a. The amount of apples this year decreased by 15% compared to last year's amount.	a, c
b. The amount of cherries this year increased by 15% compared to last year's amount.	b, d
c. The amount of pears this year is 85% of last year's amount.	
d. The amount of oranges this year is 115% of last year's amount.	





2. Mateo's aunt bought a trading card many years ago. The value of the card increased by 80%. Its value is now $270. What was the value of the card when Mateo's aunt bought it?

$150

Problems 3–5: A small town had a population of 4,000 in 1990.

3. By 2000, the population increased to 5,000. What was the percent change from 1990 to 2000? Show or explain your thinking.

25%. *Work varies.* $\frac{5000}{4000} \cdot 100 = 125$ and $125\% - 100\% = 25\%$

4. In 2010, the population decreased back to 4,000. What was the percent change from 2000 to 2010? Show or explain your thinking.

20%. *Work varies.* $\frac{4000}{5000} \cdot 100 = 80$ and $100\% - 80\% = 20\%$

5. Explain why the percent change in Problem 3 is not equal to the percent change in Problem 4.

Responses vary. Although in each decade the population changed by 1,000, the original population in each situation is different.

Lesson Practice 4.04

Name: _____ Date: _____ Period: _____

FAST Practice

6. Tyler is paid by the hour. Last week, he earned $136 for 8 hours of work. This week, he got a 5% raise. What is his new hourly wage?

$ 17.85

Spiral Review

Problems 7–8: Write an equation to represent each situation.

7. A worker is paid $15.00 per hour. What is their total earnings, t, for h hours worked?

$t = 15h$ (or equivalent)

8. What is the value of y if it represents 40% of a given value x?

$y = 0.40x$ (or equivalent)

9. Eliza walked 12 miles. Then she walked $\frac{1}{4}$ of that distance. How many miles did she walk altogether? Select *all* that apply.

- ☐ A. $12 + \frac{1}{4}$
- ☐ B. $12 \cdot \frac{1}{4}$
- ☑ C. $12 + 12 \cdot \frac{1}{4}$
- ☑ D. $12\left(1 + \frac{1}{4}\right)$
- ☐ E. $12 \cdot \frac{3}{4}$
- ☑ F. $12 \cdot \frac{5}{4}$

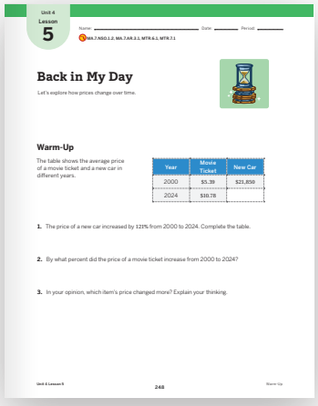
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.AR.3.1
	2–5	2	MA.7.AR.3.1
FAST Practice	6	2	MA.7.AR.3.1
Spiral Review			
Fluency	7–8	1	MA.7.AR.4.4
	9	2	MA.5.AR.2.1

Unit 4

Lesson

5


Unit 4
Lesson
5
Name: _____ Date: _____ Period: _____
MA.7.NSO.1.2, MA.7.AR.3.1, MTR.6.1, MTR.7.1
Back in My Day
Let's explore how prices change over time.
Warm-Up
The table shows the average price of a movie ticket and a new car in different years.

Year	Movie Ticket	New Car
2000	$5.39	$21,850
2024	$10.78	

1. The price of a new car increased by 121% from 2000 to 2024. Complete the table.
2. By what percent did the price of a movie ticket increase from 2000 to 2024?
3. In your opinion, which item's price changed more? Explain your thinking.

Unit 4 Lesson 5
248
Warm-Up

Back in My Day

Real-World Situations Involving Percent Increase

Let's explore how prices have changed over time.

Learning Outcomes

Today's Goals

1. **Goal:** Solve problems about price changes over time, including calculating the new price or the percentage change.
2. **Goal:** Solve problems involving percent increase of more than 100%.
3. **Language Goal:** Compare how different prices have changed using percentages. **(Reading, Writing, and Speaking)**

Students use percent increase to solve problems about the cost of items over time. They analyze how costs changed over time and think critically about what it means when prices change by different percentages. This is the first lesson where students encounter percentage changes over 100%.

Coherence

Prior Learning

In Lessons 1–4, students used a variety of representations to study percent increase and decrease. They practiced calculating new amounts, original amounts, and percent changes when given the other two values.

Future Learning

In Lesson 6, students will solve multistep percent problems as they calculate the tax, tips, and fees in various situations.

Connections

- Students apply percent increase to solve problems about the cost of items over time, such as the prices of movie tickets and cars. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	**MA.7.AR.3.1** **MA.7.AR.3.2**

Lesson at a Glance

~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

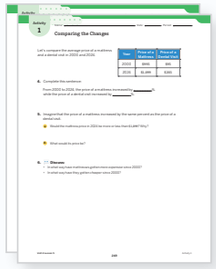
Students analyze a table showing how the price of items has changed over time, then reflect on which price has changed the most.



Activity 1

Pairs | 10 min

Students study how the price of a mattress and the price of a dental visit have changed over time as both a percent increase and a dollar amount increase.



Activity 2

Pairs | 20 min

Students use percent increase to analyze how the costs of college and other expenses have changed over time.

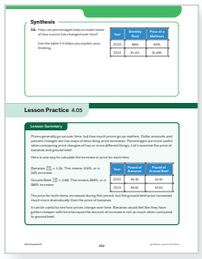
Routines: Think-Pair-Share, Notice and Wonder



Synthesis

Whole Class | 5 min

Students synthesize their understanding of percent increase as a tool for making sense of data showing a change over time.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the new price of an item when given the percent increase.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students analyze a table showing how the price of items has changed over time, then reflect on which price has changed the most.

1 Launch

To surface connections, consider sharing that the title of this lesson is *Back in My Day*. Consider asking:

- "Have you ever heard someone use that phrase? What did they mean by it?"
- "What are some ways that you think life was different in the year 2000?"

Direct students' attention to the table of prices. Consider asking, "Do any of the values in this table surprise you?"

Encourage students to use a calculator as needed throughout this lesson to support them with performing calculations.

ELL **English Language Learners** Consider grouping students with different English language proficiency levels so students can listen to and learn from their peers. **(Speaking and Listening)**

2 Connect

To surface prior knowledge, invite students to share their strategies for determining percent increase.

Discuss students' responses to Problem 3.

To surface the Key Takeaway, consider asking:

- "Why is it helpful to look at both *percent* increase and *dollar* increase?"
- "What ideas might we miss if we only looked at how the dollar amounts have changed?"

Unit 4
Lesson
5

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.6.1, MTR.7.1

Back in My Day

Let's explore how prices change over time.



Warm-Up

The table shows the average price of a movie ticket and a new car in different years.

Year	Movie Ticket	New Car
2000	$5.39	$21,850
2024	$10.78	$48,288.50

1. The price of a new car increased by 121% from 2000 to 2024. Complete the table.

2. By what percent did the price of a movie ticket increase from 2000 to 2024?
   100%

3. In your opinion, which item's price changed more? Explain your thinking.
   Responses vary. The price of the new car changed more because it had a greater percent change and a greater change in terms of dollars.

Unit 4 Lesson 5 248 Warm-Up

Key Takeaway: Prices tend to increase over time. To see how a price has changed, you can look at the increase in dollars or the percent increase. Together, these two values give us a more complete picture of what has changed.

Pairs | 10 min

Activity 1 Comparing the Changes

Purpose: Students study how the price of a mattress and the price of a dental visit have changed over time as both a percent increase and a dollar amount increase.

1 Launch

Direct students' attention to the table and read the column and row labels.

Consider sharing that prices tend to increase over time but that the percent increase differs for different goods and services.

Consider asking, "Why might we want to talk about the change in prices over time?" You can determine whether an increase in price is reasonable by comparing the percent increase for different products.

2 Monitor

Look for a variety of student strategies and offer support where needed.

D Differentiation

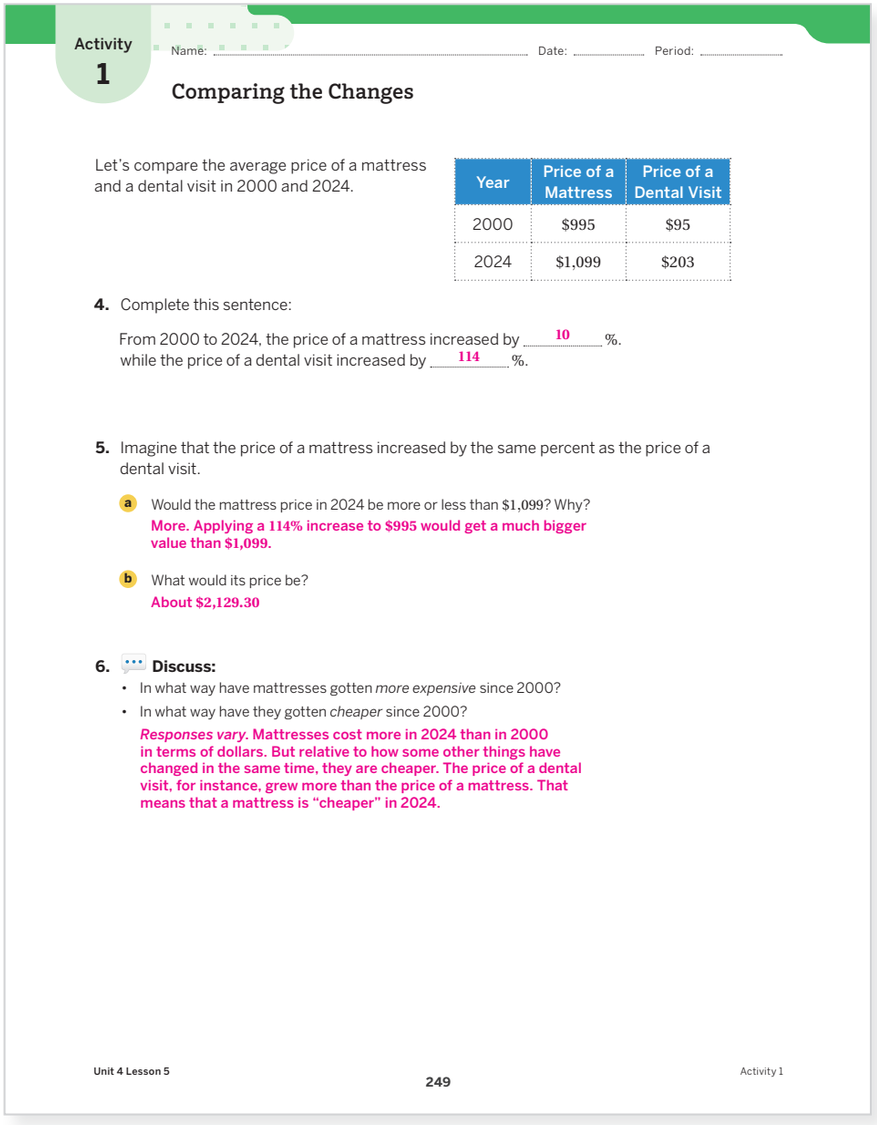
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to refer back to displays or notes that show strategies for calculating percent increase.
Calculate the change as a percent of the 2024 values rather than the 2000 values.	**Support** Consider asking, "Which row in the table represents the original values, and which shows the new values?"

Collect a variety of mathematical language students use to determine how a different percent increase affects the price of a mattress.

A Accessibility: Affective Functioning To support students in sharing their thinking, ask students in advance if they, you, or a classmate can share their thinking during the class discussion.

3 Connect

Invite students to share their thoughts about how mattresses got cheaper or more expensive between 2000 and 2024. Encourage students to use mathematical evidence to support their claims. **(MTR.7.1)**



Activity 1

Name: Date: Period:

Comparing the Changes

Let's compare the average price of a mattress and a dental visit in 2000 and 2024.

Year	Price of a Mattress	Price of a Dental Visit
2000	$995	$95
2024	$1,099	$203

4. Complete this sentence:

From 2000 to 2024, the price of a mattress increased by 10 %. while the price of a dental visit increased by 114 %.

5. Imagine that the price of a mattress increased by the same percent as the price of a dental visit.

a. Would the mattress price in 2024 be more or less than $1,099? Why?
More. Applying a 114% increase to $995 would get a much bigger value than $1,099.

b. What would its price be?
About $2,129.30

6. Discuss:
- In what way have mattresses gotten *more expensive* since 2000?
- In what way have they gotten *cheaper* since 2000?

Responses vary. Mattresses cost more in 2024 than in 2000 in terms of dollars. But relative to how some other things have changed in the same time, they are cheaper. The price of a dental visit, for instance, grew more than the price of a mattress. That means that a mattress is "cheaper" in 2024.

Unit 4 Lesson 5 249 Activity 1

Pairs | 20 min

Activity 2 Change Over Time

Purpose: Students use percent increase to analyze how the costs of college and other expenses have changed over time.

Short on time? Consider completing Problems 10–12 as a class.

1 Launch

Direct students' attention to the table. Consider sharing additional context about how public and private colleges are funded, as well as common ways students pay for college, such as scholarships or student loans.

Use the Think-Pair-Share routine to support student-to-student discourse about the cost of college.

ELL **Multilingual/English Language Learners**
Encourage students to paraphrase each other's ideas to help make connections. Consider providing sentence frames to support these connections, such as:

- "I think college should be _____."
- "I think you are saying _____."
- "I agree that _____, but I disagree about _____."

(Speaking and Listening)

2 Monitor

Encourage students to use math to justify why the values they wrote for Problem 7 are reasonable. **(MTR.6.1)**

D **Differentiation: (Support)** Students may feel unsure about what values seem reasonable. Invite them to look back at the data from the Warm-Up and Activity 1 and consider how other prices have changed since 2000.

Consider sharing that inflation reflects the rise in the price of common goods and services purchased by households over time.

Listen for a variety of language students use to describe percent increases greater than 100%.

Pause briefly and consider asking:

- "What does this tell us about the cost of college with fees in 2000? In 2024?"

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Change Over Time

Now let's look at the cost of college. In 2000, the cost of one year of college was $3,400 based on the average tuition and fees at U.S. public universities.

Year	Cost of One Year of College
2000	$3,400
2024	

7. Let's think about the cost of a year of college in 2024. What are the values that seem reasonable to you? Give at least two and explain your reasoning.
Responses and explanations vary.

Value: $7,276 Explanation: From 2000 to 2024, the price of a dental visit increased by 114%, so I applied the same percent increase to the cost of college in 2000.

Value: $3,740 Explanation: I don't think the price of college should increase any more than the price of a mattress. The mattress price increased from 2000 to 2024 by about 10%, so I applied that to the cost of college in 2000.

8. From 2000 to 2024, the cost of college increased by about 240%.

a. What does it mean to have an increase of more than 200%?
Responses vary. An increase of 100% doubles the original amount and an increase of 200% triples the original amount. So, an increase of more than 200% means the original amount more than tripled.

b. What was the cost of college in 2024? Show your thinking.
About $11,560. $3400 \bullet 3.40 = 11560$

9. The rate of inflation from 2000 and 2024 was 83%. Imagine the cost of college increased by the same percent. What would the cost of college be in 2024?
About $6,222

Unit 4 Lesson 5 250 Activity 2

Pairs | 20 min

Activity 2 Change Over Time (continued)

Purpose: Students use percent increase to analyze how the costs of college and other expenses have changed over time.

Short on time? Consider completing Problems 10–12 as a class.

2 Monitor

Use the Notice and Wonder routine to promote curiosity and help students make sense of the table.

Create a record of what students noticed and wondered, along with the names of the students who shared them.

Look for several unique student comparisons to share during the Connect. **(MTR.7.1)**

D Differentiation *(Problem 12–13)*

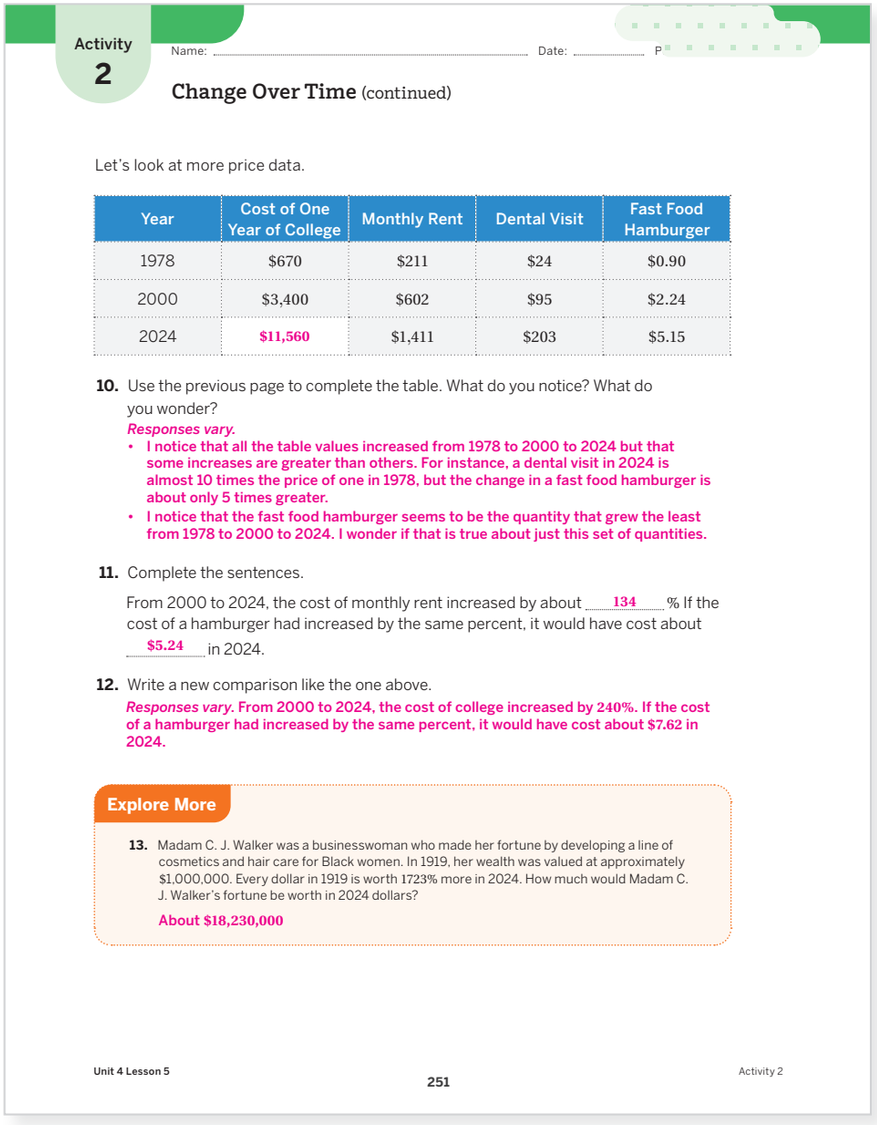
Look for students who:	Respond to Student Thinking
Need support writing their own comparison *(Problem 12)*	**Support** Invite students to create a sentence frame like the one modeled in Problem 11, then use the frame to support their problem-solving.
Make comparisons that show increases greater than 100%.	**Strengthen** Consider asking, "What is your reaction to this data? What does this tell you about how the price has changed over time?"
Would like to explore changes in the value of money over time further. *(Problem 13)*	**Stretch You're invited to explore more.** Invite students who would like to explore further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Share the captured student comparisons and invite students to share their reactions to them.

Consider asking:

- "What parts of the data surprised you?"
- "How do you think these changes affected people? How would it feel in 1978, 2000, or 2024?"
- "What other prices or dollar values would be interesting to track over time?"



Activity 2

Name: Date: P

Change Over Time (continued)

Let's look at more price data.

Year	Cost of One Year of College	Monthly Rent	Dental Visit	Fast Food Hamburger
1978	$670	$211	$24	$0.90
2000	$3,400	$602	$95	$2.24
2024	$11,560	$1,411	$203	$5.15

10. Use the previous page to complete the table. What do you notice? What do you wonder?

Responses vary.

- I notice that all the table values increased from 1978 to 2000 to 2024 but that some increases are greater than others. For instance, a dental visit in 2024 is almost 10 times the price of one in 1978, but the change in a fast food hamburger is about only 5 times greater.
- I notice that the fast food hamburger seems to be the quantity that grew the least from 1978 to 2000 to 2024. I wonder if that is true about just this set of quantities.

11. Complete the sentences.

From 2000 to 2024, the cost of monthly rent increased by about 134 % If the cost of a hamburger had increased by the same percent, it would have cost about $5.24 in 2024.

12. Write a new comparison like the one above.

Responses vary. From 2000 to 2024, the cost of college increased by 240%. If the cost of a hamburger had increased by the same percent, it would have cost about $7.62 in 2024.

Explore More

13. Madam C. J. Walker was a businesswoman who made her fortune by developing a line of cosmetics and hair care for Black women. In 1919, her wealth was valued at approximately $1,000,000. Every dollar in 1919 is worth 1723% more in 2024. How much would Madam C. J. Walker's fortune be worth in 2024 dollars?

About $18,230,000

Unit 4 Lesson 5 251 Activity 2

Key Takeaway: Prices tend to increase over time, but many other factors affect whether things feel cheaper or more expensive to the person buying them. Looking at the percent change and considering both prices and wages helps us better understand the changes.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of percent increase as a tool for making sense of data showing a change over time.

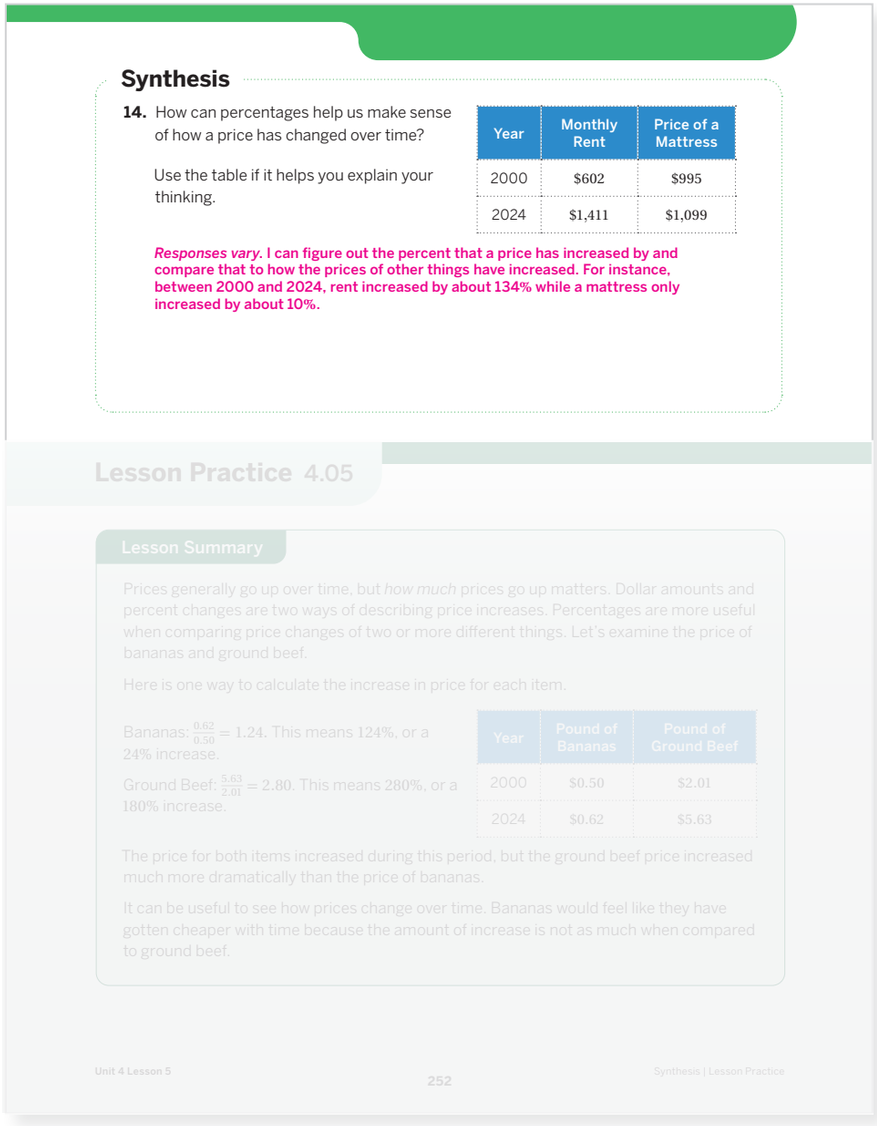
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Comparing percentages gives a common frame of reference to compare prices of items that are very different, like a movie ticket and a mattress.
- Something that increased by a higher percentage than another item will feel like it has gotten more expensive, even if the change was fewer dollars.

Consider asking, "How would our conclusions about prices be different if we only looked at dollar increases instead of percent increases?" We might think that one item is now more expensive than another if it had a larger increase in terms of dollars, but it could have a smaller percent increase, so its price went up a lesser amount compared to the percent increase of the other item.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they share during the Connect. Display for students to reference and add to for the rest of the lesson.



Synthesis

14. How can percentages help us make sense of how a price has changed over time?

Use the table if it helps you explain your thinking.

Year	Monthly Rent	Price of a Mattress
2000	$602	$995
2024	$1,411	$1,099

Responses vary. I can figure out the percent that a price has increased by and compare that to how the prices of other things have increased. For instance, between 2000 and 2024, rent increased by about 134% while a mattress only increased by about 10%.

Lesson Practice 4.05

Lesson Summary

Prices generally go up over time, but *how much* prices go up matters. Dollar amounts and percent changes are two ways of describing price increases. Percentages are more useful when comparing price changes of two or more different things. Let's examine the price of bananas and ground beef.

Here is one way to calculate the increase in price for each item.

Bananas: $\frac{0.62}{0.50} = 1.24$. This means 124%, or a 24% increase.

Ground Beef: $\frac{5.63}{2.01} = 2.80$. This means 280%, or a 180% increase.

Year	Pound of Bananas	Pound of Ground Beef
2000	$0.50	$2.01
2024	$0.62	$5.63

The price for both items increased during this period, but the ground beef price increased much more dramatically than the price of bananas.

It can be useful to see how prices change over time. Bananas would feel like they have gotten cheaper with time because the amount of increase is not as much when compared to ground beef.

Unit 4 Lesson 5 252 Synthesis | Lesson Practice

Lesson Takeaway: Calculating percent change can help you make comparisons in real-world situations, like changes in price or in wages.

Independent | 5 min

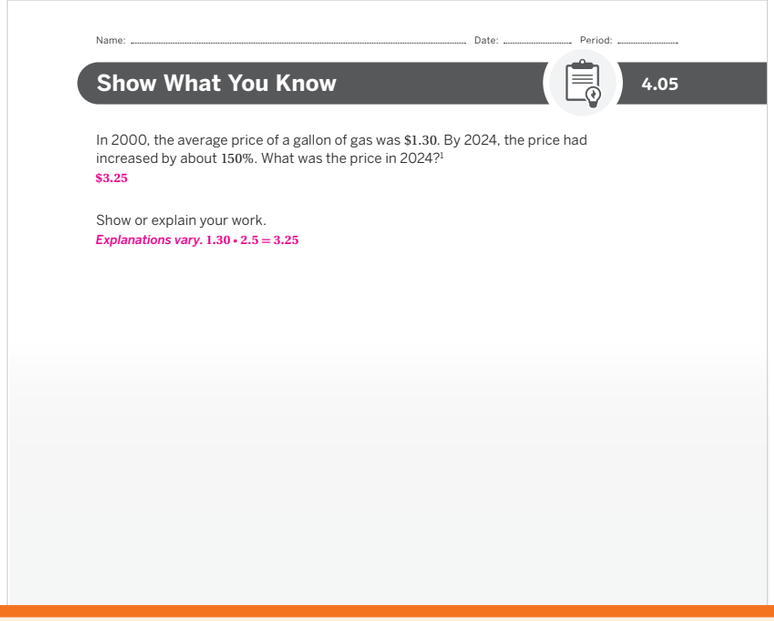
Show What You Know

Purpose: Students demonstrate their understanding by calculating the new price of an item when given the percent increase.

Today's Goals

1. **Goal:** Solve problems about price changes over time, including calculating the new price or the percentage change.
2. **Goal:** Solve problems involving percent increase of more than 100%.
3. **Language Goal:** Compare how different prices have changed using percentages. **(Reading, Writing, and Speaking)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.05

In 2000, the average price of a gallon of gas was $1.30. By 2024, the price had increased by about 150%. What was the price in 2024?[1]

$3.25

Show or explain your work.

Explanations vary. $1.30 \cdot 2.5 = 3.25$

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate percent increases, new values, or original values:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and discuss the calculations together.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task in Activity 2.

Professional Learning

Take a moment to reflect on how students applied what they've learned about percent increase to price changes. In what ways were students able to make connections between math ideas and their experiences in this lesson? How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

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Lesson Practice 4.05

Name: _____ Date: _____ Period: _____

1. Jayla purchased a brand new car for $41,500. Her car is now worth $25,730. What is the percent decrease between the car's original price and its value now?

38%

Problems 2–4: The table shows the price of a 55-inch 4K television set in 2023 and 2024.

Year	Price
2023	$400
2024	$292

2. Determine the percent decrease for the price of the television set between 2023 and 2024.

27%

3. If the percent decrease continues for another year, what will the price be in 2025?

$213.16

4. If the same percent decrease was true from 2022 to 2023, what was the price of the television set in 2021?

About $547.95

Problems 5–6: Ecuador and Peru are neighboring countries in South America. Here are their populations in 2005 and 2024.

Year	Ecuador	Peru
2005	13,770,000	28,147,000
2024	18,100,000	34,200,00

5. Circle the expression that could help you determine the percent change in Ecuador's population from 2005 to 2024.

$\frac{18100}{13770}$ $\frac{13770}{18100}$

Explain your thinking.

Explanations vary. The population increased during this time, so we are looking for an expression that is greater than 1 because those expressions represent an increase.

6. If the population of Peru had grown by the same rate percent as Ecuador from 2005 to 2024, which would be true about Peru's population? Circle one.

Greater than 34,200,000 Less than 34,200,000

Explain your thinking.

Explanations vary. Ecuador's growth rate was larger than Peru's, so if Peru had Ecuador's growth rate, its population would be greater than what it turned out to be.

Lesson Practice 4.05

Name: _____ Date: _____ Period: _____

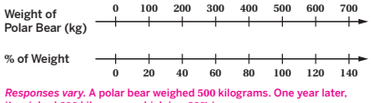
FAST Practice

7. Cassi has money in a savings account that grows 1.5% every year. Write an equation to represent the money in her account as it grows from one year to the next. Use x for this year's amount and y for next year's amount.

$y = 1.015x$

Spiral Review

8. Write a situation that matches the diagram.



Responses vary. A polar bear weighed 500 kilograms. One year later, it weighed 600 kilograms, which is a 20% increase.

9. A phone keeps track of the number of steps a person has taken and the distance traveled. Does the information in the table show a proportional relationship between the two quantities? Show or explain your thinking.

Number of Steps	Distance (km)
1,150	1
4,654	3.3
6,795	5.1

No. *Explanations vary.* There is not a proportional relationship between the two quantities because there is not a constant of proportionality for the pairs of values.

$\frac{1150}{1} = 1150$ $\frac{4654}{3.3} = 1410.3$ $\frac{6795}{5.1} = 1332.4$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.3.1
	4	2	MA.7.AR.3.1
	5–6	2	MA.7.AR.3.1
FAST Practice	7	1	MA.7.AR.3.1
Spiral Review			
	8	2	MA.7.AR.3.1
	9	2	MA.7.AR.4.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining the result of a percent change	Lesson 1	1	MA.7.AR.3.1
2	Representing a percent change using equations	Lesson 2	1	MA.7.AR.3.1, MTR.7.1
3	Calculating a percent increase or decrease from two values	Lesson 3	1	MA.7.AR.3.1, MTR.3.1
4	Calculating values from a percent increase of more than one hundred percent and explaining the result	Lesson 5	2	MA.7.AR.3.1, MTR.6.1
5a	Determining the result of percent increases or decreases	Lesson 1	1	MA.7.AR.3.1
5b	Writing an equation to represent a percent change	Lesson 2	1	MA.7.AR.3.1
5c	Determining an original price given a percent increase and the increased price and explaining the strategy used	Lesson 3	2	MA.7.AR.3.1

Assessment and Lesson Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 4

1. The value of a car decreases over time. This year, Faaria's car is worth $22,000. If the value of Faaria's car decreases by 8%, what will her car be worth next year?

A. $1,760

B. $17,600

C. $20,240

D. $23,760

2. Mayra bought x grams of rice. Anika bought 30% more than Mayra bought.

Select *all* the equations that represent the relationship between the amount of rice that Mayra bought, x, and the amount of rice that Anika bought, y.

☐ **A.** $y = 0.7x$

☐ **B.** $y = 0.3x$

☑ **C.** $y = x + 0.3x$

☐ **D.** $y = x - 0.3x$

☑ **E.** $y = 1.3x$

3. Last year a phone charger was priced at $30. This year, the same charger is priced at $24.

What was the percent decrease?

20%

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Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 4

4. In 2023, Anya spent an average of 4 hours a day looking at screens. In 2024, Anya's weekly screen time increased by 250%.

What was Anya's average weekly screen time in 2024?

14 hours

Explain your thinking.

Explanations vary. A 250% increase means 350% of the starting amount. $350\% = 3.5$ so I did $3.5 \cdot 4 = 14$, which means the 2024 screen time is 14 hours per week.

5. A storekeeper increased the price of hats by 5%.

a A hat was originally priced at $15.00. What is the new cost of the hat?

$15.75

b Write an equation to calculate the new total cost, c, when the original price of a hat is p dollars.

$c = 1.05p$

c If the price of a hat (after the increase) is $33.60, what was the original price?

$32.00

Explain your thinking.

Explanations vary. I used the equation $c = 1.05p$ and substituted $33.60 in place of c. Then I divided both sides by 1.05 to get p, the original price of the hat.

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	

Sub-Unit 1:
• Solve problems involving percent change using representations, such as tape diagrams, double number lines, tables, and equations. **(Lessons 1–5)**
	1, 4, 5a	**Support**
		Strengthen
			• **Teacher Move:** Consider revisiting:
			  • Lesson 1 (*More and Less*)
			  • Lesson 5 (*Back in My Day*)

		Stretch	• **You're invited to explore more:**
			  • Lesson 1 (*More and Less*)
			  • Lesson 5 (*Back in My Day*)

	2, 5b	**Support**	• **Teacher Move:** Consider revisiting Lesson 2 (*All the Equations*)
		Strengthen	• **Teacher Move:** Invite students to share strategies they use to determine which equation represents percent increase or decrease.

	3, 5c	**Support**
		Strengthen
			• **Mini-Lesson: Calculating Percentages**
			• **Teacher Move:** Consider revisiting:
			  • Lesson 3 (*100%*)
			  • Lesson 4 (*Percent Machines*)

		Stretch	• **You're invited to explore more:** • Lesson 3 (*100%*)

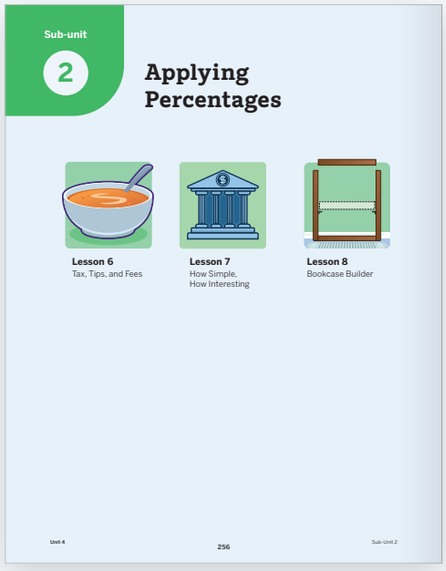
Sub-Unit 2

Applying Percentages

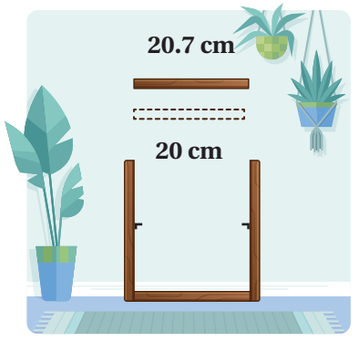
Sub-Unit 2 Goals

- **Goal:** Use proportional reasoning to solve problems in real-world situations involving percent increase, percent decrease, and percent error.
- **Goal:** Interpret situations involving percent change and apply appropriate representations to solve problems.
- **Language Goal:** Interpret scenarios depicting percent change and percent error.

In this sub-unit, students interpret and solve real-world problems involving proportional relationships, percent change, **simple interest**, and **percent error**.



Math That Matters Most

Concept or Skill	For example . . .
Calculate an original amount, new amount, or percent change in the context of sales tax and tip.	A store offers a 20% off coupon. The tax rate is 7.5% of the subtotal. If an item is listed at $15, what is the total after the coupon and tax have been applied? *To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied $12 by 1.075 to get 12.9. The new total is $12.90.*
Calculate simple interest earned on deposits or paid on loans.	You deposit $120 into an account that pays 3% simple interest. If you leave the money in the account for 10 years, how much interest will your original deposit earn? *The formula for simple interest is I = prt. The principal is $120, the interest rate is 3%, and the time is 10 years: I = 120(0.03)(10) = 36.* *The account will earn $36 in interest on the original deposit.*
Decide whether a value is within an acceptable percent error.	The acceptable percent error is 5% for a shelf to fit. Will the shelf fit? Why or why not? *The shelf will fit because the error is 0.7 centimeters and the original is 20 centimeters. 0.7 is 3.5% of 20, which is less than 5%.* 

Unit 4

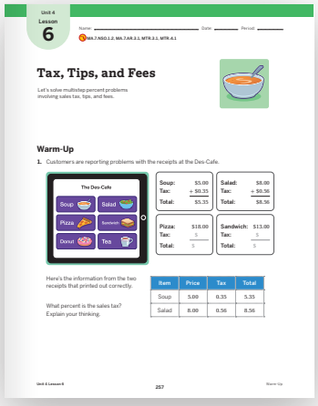
Lesson

6

Tax, Tips, and Fees

Multistep Percent Problems

Let's solve multistep percent problems involving sales tax, tips, and fees.


Unit 4
Lesson
6
Tax, Tips, and Fees
Let's solve multistep percent problems involving sales tax, tips, and fees.
Warm-Up
1. Customers are reporting problems with the receipts at the Deo Cafe.
The Deo Cafe
Soup
Salad
Pizza
Sandwich
Donut
Tea
Soup: $5.00 Tax: + $0.35 Total: $5.35
Salad: $8.00 Tax: + $0.56 Total: $8.56
Pizza: $10.00 Tax: $ Total: $
Sandwich: $13.00 Tax: $ Total: $
Here's the information from the two receipts that printed out correctly.
What percent is the sales tax? Explain your thinking.
Item | Price | Tax | Total
Soup | 5.00 | 0.35 | 5.35
Salad | 8.00 | 0.56 | 8.56
257

Learning Outcomes

Today's Goals

1. **Goal:** Calculate an original amount, new amount, or percent change in the context of sales tax and tip, including situations that involve multiple percent increases or decreases (e.g., sales tax and a discount).

2. **Language Goal:** Interpret situations involving tax and tip using mathematically precise language. **(Speaking and Listening)**

Students apply what they've learned in the first section of the unit to solve multistep percent problems. They determine the total cost after applying sales tax, tips, and fees and calculate the total after two percentages are applied (e.g., a discount followed by sales tax).

Coherence

Prior Learning

In Lesson 4, students worked with problems that involved multiple percentages.

Future Learning

Lessons 7–8 will continue to give students ample opportunity to work with percentages in new contexts.

Connections

- Students calculate the total price after two percentages in real world contexts, such as calculating the cost of an item in a store after a coupon and sales tax. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2

Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1

Apply previous understanding of percentages and ratios to solve multi-step real world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.8.AR.2.1
	MA.912.AR.1.1

Lesson at a Glance

~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Independent | 5 min

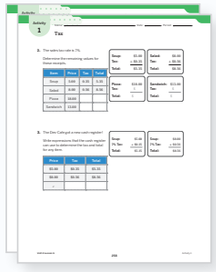
Students explore cash register receipts as an introduction to the concept of sales tax.



Activity 1

Pairs | 10 min

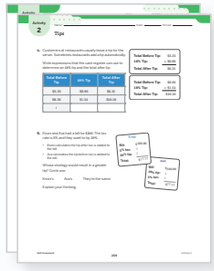
Students write expressions for the cost of a meal, including sales tax, to make sense of how tax increases the total amount.



Activity 2

Pairs | 10 min

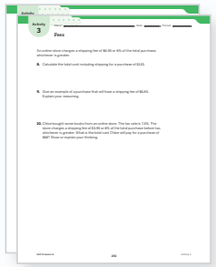
Students calculate tips to make sense of how to solve multistep percentage problems.
Routine: Decide and Defend



Activity 3

Pairs | 10 min

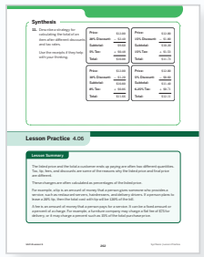
Students calculate the total cost of a purchase including shipping fees, which are calculated as the greater amount of a flat fee or a percentage of the total purchase.



Synthesis

Whole Class | 5 min

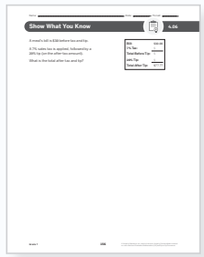
Students synthesize strategies for solving problems that involve calculating the total cost for something after multiple percentage increases or decreases. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the cost of a meal that includes tax and tip.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators

Independent | 5 min

Warm-Up

Purpose: Students explore cash register receipts as an introduction to the concept of sales tax.

1 Launch

To surface connections, consider asking, "Have you ever had something cost more than you expected because of tax?"

To support students getting started, consider asking, "How do you think the cash register is calculating the amount of the tax? Could you create an expression or equation to represent what it is doing?"

ELL **English Language Learners** Consider inviting students to share what they know about the terms *taxes* and *percentages*. Discuss how the meanings of these terms are alike and different. **(Reading and Listening)**

A **Accessibility: Memory and Attention** Invite students to use a calculator, double number line, or ratio table to support them with performing calculations.

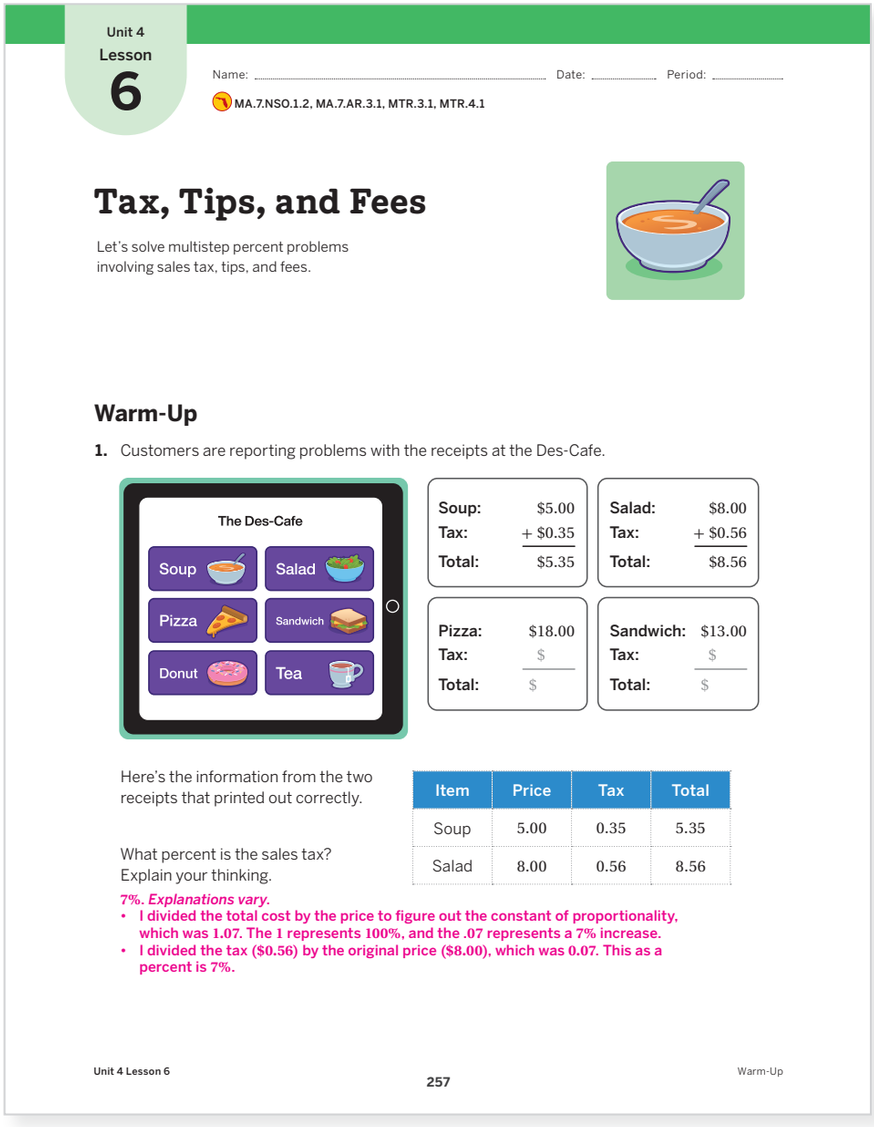
2 Connect

Encourage students to read others' responses. Consider asking, "How are others' strategies similar or different to your own?" **(MTR.4.1)**

Discuss as a whole-class what the values 0.07 and 1.07 represent in this context.

To surface the Key Takeaway, consider asking:

- "If we know the price and the tax, how can we figure out the sales tax percentage?"
- "How are the numbers 0.07 and 1.07 related to each other in this situation?"



Unit 4
Lesson
6

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.3.1, MTR.4.1

Tax, Tips, and Fees

Let's solve multistep percent problems involving sales tax, tips, and fees.

Warm-Up

1. Customers are reporting problems with the receipts at the Des-Cafe.

Soup:	$5.00	Salad:	$8.00
Tax:	+ $0.35	Tax:	+ $0.56
Total:	$5.35	Total:	$8.56
Pizza:	$18.00	Sandwich:	$13.00
Tax:	$	Tax:	$
Total:	$	Total:	$

Here's the information from the two receipts that printed out correctly.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56

What percent is the sales tax? Explain your thinking.

7%. Explanations vary.

- I divided the total cost by the price to figure out the constant of proportionality, which was 1.07. The 1 represents 100%, and the .07 represents a 7% increase.
- I divided the tax ($0.56) by the original price ($8.00), which was 0.07. This as a percent is 7%.

Unit 4 Lesson 6 257 Warm-Up

Key Takeaway: To determine the sales tax rate, you can divide the tax by the price to get the percentage as a decimal. You can divide the total by the pre-tax price to find the constant of proportionality for the percent increase.

Pairs | 10 min

Activity 1 Tax

Purpose: Students write expressions for the cost of a meal, including sales tax, to make sense of how tax increases the total amount.

1 Launch

Invite students to use the sales tax rate they discussed in the Warm-Up to figure out the values on the missing receipts.

2 Monitor

Look for strategies students use to determine the tax and total for their items, including those that show expressions in the table.

D Differentiation

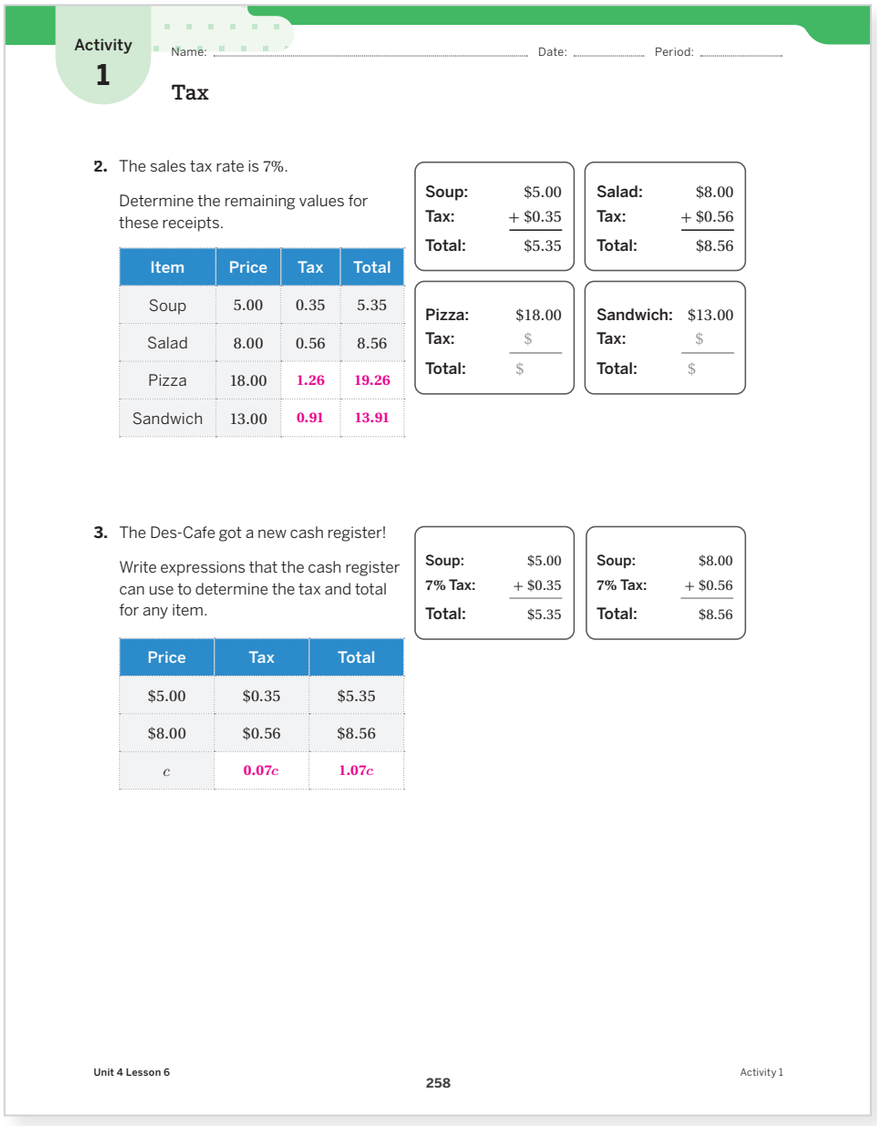
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What strategies can you use to represent 7%?" Consider referring to the strategies from Lesson 2.
Use the decimal 0.7 to represent 7%.	**Support** Consider asking, "What does 7% mean? How can we represent that as a decimal?"
Determine 1% of the price and use that to determine the tax and total.	**Strengthen** Celebrate their thinking. Consider asking, "How can we determine the tax with just one calculation?"
Type expressions that involve multiplying by 0.07 for the tax or by 1.07 for the total.	**Strengthen** Consider asking, "How does this support you writing your equations in Problem 3?"
Write the expression $0.07c + c$ to represent calculating the tax, then adding the cost.	**Strengthen** Consider asking, "Can you write this as a single multiplication expression?" **(MTR.3.1)**

A Accessibility: Memory and Attention Invite students to record their calculations for tax and total on a separate sheet of paper so they can reference their work when trying to determine an equation on Problem 4.

3 Connect

Invite students to share their strategies for determining tax and total. Invite students to share how the different pieces of their equations represent the tax and total.

To surface the Key Takeaway, consider asking, "How can we determine the sales tax and total for any item sold at the Des-Cafe?"



Activity 1

Name: Date: Period:

Tax

2. The sales tax rate is 7%.

Determine the remaining values for these receipts.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56
Pizza	18.00	1.26	19.26
Sandwich	13.00	0.91	13.91

Soup: $5.00 Tax: + $0.35 Total: $5.35

Salad: $8.00 Tax: + $0.56 Total: $8.56

Pizza: $18.00 Tax: $ Total: $

Sandwich: $13.00 Tax: $ Total: $

3. The Des-Cafe got a new cash register!

Write expressions that the cash register can use to determine the tax and total for any item.

Price	Tax	Total
$5.00	$0.35	$5.35
$8.00	$0.56	$8.56
c	$0.07c$	$1.07c$

Soup: $5.00 7% Tax: + $0.35 Total: $5.35

Soup: $8.00 7% Tax: + $0.56 Total: $8.56

Unit 4 Lesson 6 258 Activity 1

Key Takeaway: To determine a percentage of an amount, you can write the percent as a decimal or fraction, then multiply it by the amount. To increase an amount by a certain percent, you can multiply it by 100% plus the percent of the increase. For example, to determine the total after an 8% increase, you can multiply the original amount by 1.08. This is the same as finding 8% of the amount and adding it to the original.

Pairs | 10 min

Activity 2 Tips

Purpose: Students calculate tips to make sense of how to solve multistep percentage problems.

Short on time? Consider completing Problem 7 as a class.

1 Launch

To surface connections, consider asking, "What do you know about tipping at a cafe?"

Note: There are many different customs for tipping. Consider each strategy that a student offers for tipping equitably and with care, acknowledging how it can be discussed mathematically.

2 Monitor

To support students getting started, consider asking, "How can the strategies we discussed for determining tax be used here?"

Invite students to spend 1–2 minutes thinking independently about the problem before discussing it with a partner.

A **Accessibility: Executive Functioning** Consider providing a graphic organizer to support students in calculating multiple percentages for a single problem.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Consider asking, "Why does the order we calculate tax and tip matter for the total bill?" **(MTR.3.1)**

Activity 2 continued >

Activity 2

Name: Date: Period:

Tips

4. Customers at restaurants usually leave a tip for the server. Sometimes restaurants add a tip automatically.

Write expressions that the cash register can use to determine an 18% tip and the total after tip.

Total Before Tip	18% Tip	Total After Tip
$5.35	$0.96	$6.31
$8.56	$1.54	$10.10
t	$0.18t$	$1.18t$

Total Before Tip:	$5.35
18% Tip:	+ $0.96
Total After Tip:	$6.31

Total Before Tip:	$8.56
18% Tip:	+ $1.54
Total After Tip:	$10.10

5. Kiran and Ava had a bill for $100. The tax rate is 5% and they want to tip 20%.

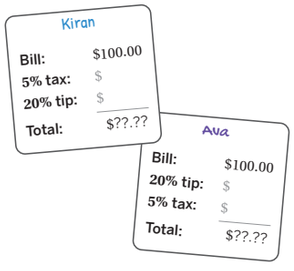
- Kiran calculates the tip after tax is added to the bill.
- Ava calculates the tip *before* tax is added to the bill.

Whose strategy would result in a greater tip? Circle one.

Kiran's Ava's They're the same

Explain your thinking.

Explanations vary. Ava is calculating 20% of $100. Kiran is calculating 20% of more than $100.



Unit 4 Lesson 6 259 Activity 2

Pairs | 10 min

Activity 2 Tips (continued)

Purpose: Students calculate tips to make sense of how to solve multistep percentage problems.

Short on time? Consider completing Problem 7 as a class.

2 Monitor

To surface connections, consider asking, "How is a discount similar to or different from a tax?"

ELL **English Language Learners** Invite students to share situations in which they have used coupons or gotten discounts on things they have purchased to increase access to the task. **(Reading and Listening)**.

Look for a variety of student strategies for determining the total price.

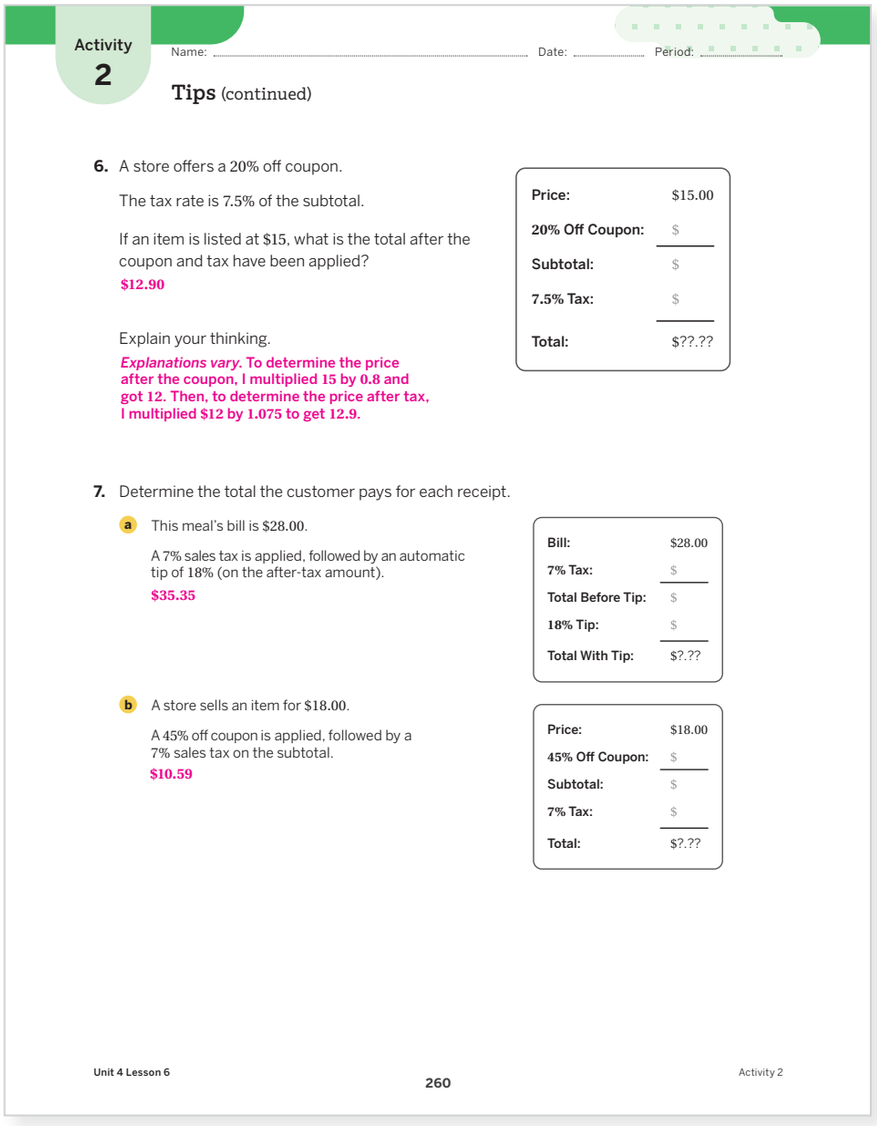
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Does a 20% coupon represent a percent increase or decrease?"
Calculate the tax before the coupon.	**Support** Encourage them to follow the order on the receipt. Consider asking, "Why does the order matter?"
Subtract the tax from the discounted price.	**Support** Consider asking, "Should the tax increase or decrease the total?"
Represent the total as an expression like $15(.8)(1 + 0.075)$.	**Stretch** Invite students to share during the Connect. Consider asking, "How do each of your values or operations represent something in the situation?"

3 Connect

Invite students to share how they determined the total price after the discount and tax.

To surface the Key Takeaway, consider asking, "Why is it important to consider the order in which taxes, discounts, or tips are applied to a total?"



Activity 2

Name: Date: Period:

Tips (continued)

6. A store offers a 20% off coupon.

The tax rate is 7.5% of the subtotal.

If an item is listed at $15, what is the total after the coupon and tax have been applied?

$12.90

Price:	$15.00
20% Off Coupon:	$
Subtotal:	$
7.5% Tax:	$
Total:	$??.??

Explain your thinking.

Explanations vary. **To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied $12 by 1.075 to get 12.9.**

7. Determine the total the customer pays for each receipt.

a This meal's bill is $28.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

$35.35

Bill:	$28.00
7% Tax:	$
Total Before Tip:	$
18% Tip:	$
Total With Tip:	$?.??

b A store sells an item for $18.00.

A 45% off coupon is applied, followed by a 7% sales tax on the subtotal.

$10.59

Price:	$18.00
45% Off Coupon:	$
Subtotal:	$
7% Tax:	$
Total:	$?.??

Unit 4 Lesson 6 260 Activity 2

Key Takeaway: It is important to consider the order in which percentages of a total are added to a tip. Taking off 20% and then adding an 8% tax is different from the opposite order because we apply the percentages to different amounts depending on the order.

Pairs | 10 min

Activity 3 Fees

Purpose: Students calculate the total cost of a purchase including shipping fees, which are calculated as the greater amount of a flat fee or a percentage of the total purchase.

1 Launch

Consider sharing that fees can be a fixed amount, a percentage of a charge, or both.

2 Monitor

To support students getting started, invite them to calculate 6% of different purchase amounts to see which ones result in a shipping fee that is less than $6.95.

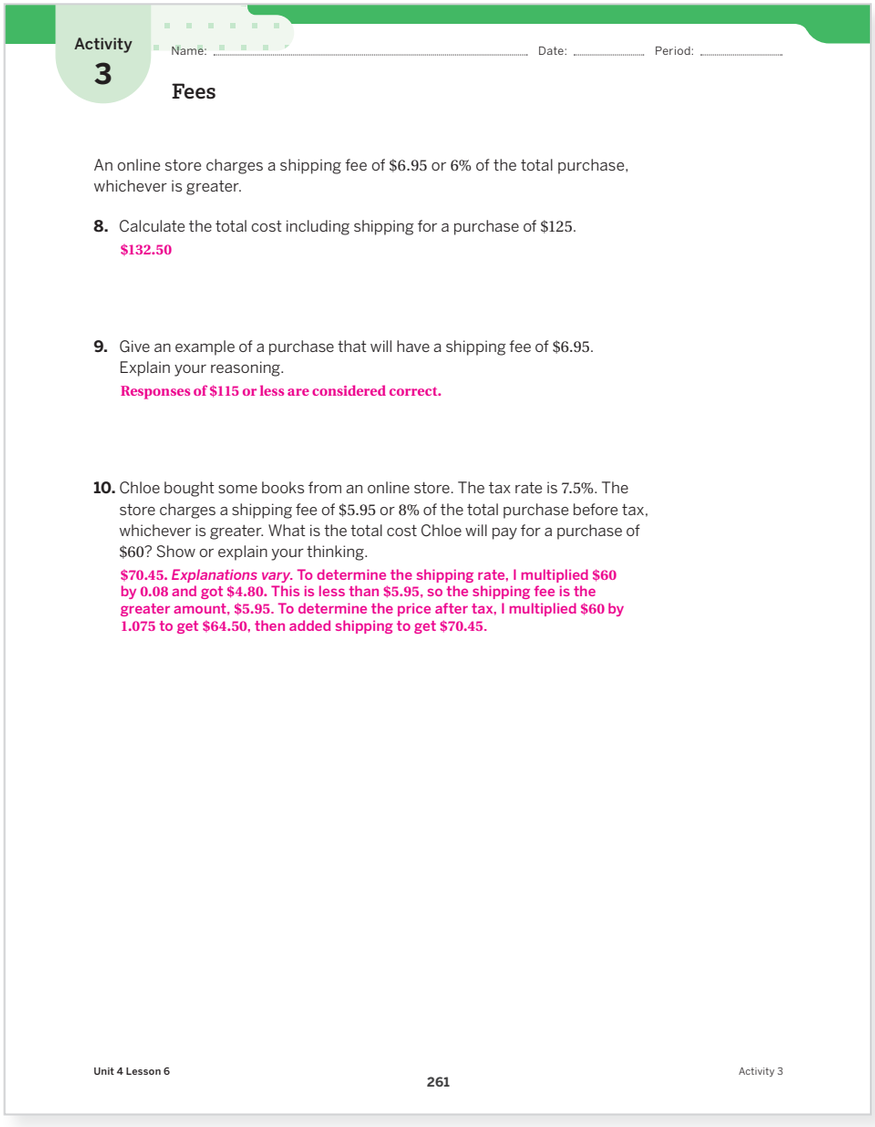
Look for students who write an equation to determine the exact purchase price that changes from a fixed shipping fee to a percentage of the purchase.

3 Connect

Invite students to share how they determined the total price after tax and shipping.

Consider asking:

- "What is the minimum shipping rate?"
- "Is there a maximum shipping rate?"
- "Why do you think some companies offer free shipping when the total purchase is above a certain amount?"



Activity 3

Name: Date: Period:

Fees

An online store charges a shipping fee of $6.95 or 6% of the total purchase, whichever is greater.

8. Calculate the total cost including shipping for a purchase of $125.

$132.50

9. Give an example of a purchase that will have a shipping fee of $6.95. Explain your reasoning.

Responses of $115 or less are considered correct.

10. Chloe bought some books from an online store. The tax rate is 7.5%. The store charges a shipping fee of $5.95 or 8% of the total purchase before tax, whichever is greater. What is the total cost Chloe will pay for a purchase of $60? Show or explain your thinking.

$70.45. *Explanations vary.* To determine the shipping rate, I multiplied $60 by 0.08 and got $4.80. This is less than $5.95, so the shipping fee is the greater amount, $5.95. To determine the price after tax, I multiplied $60 by 1.075 to get $64.50, then added shipping to get $70.45.

Unit 4 Lesson 6 261 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for solving problems that involve calculating the total cost for something after multiple percentage increases or decreases.

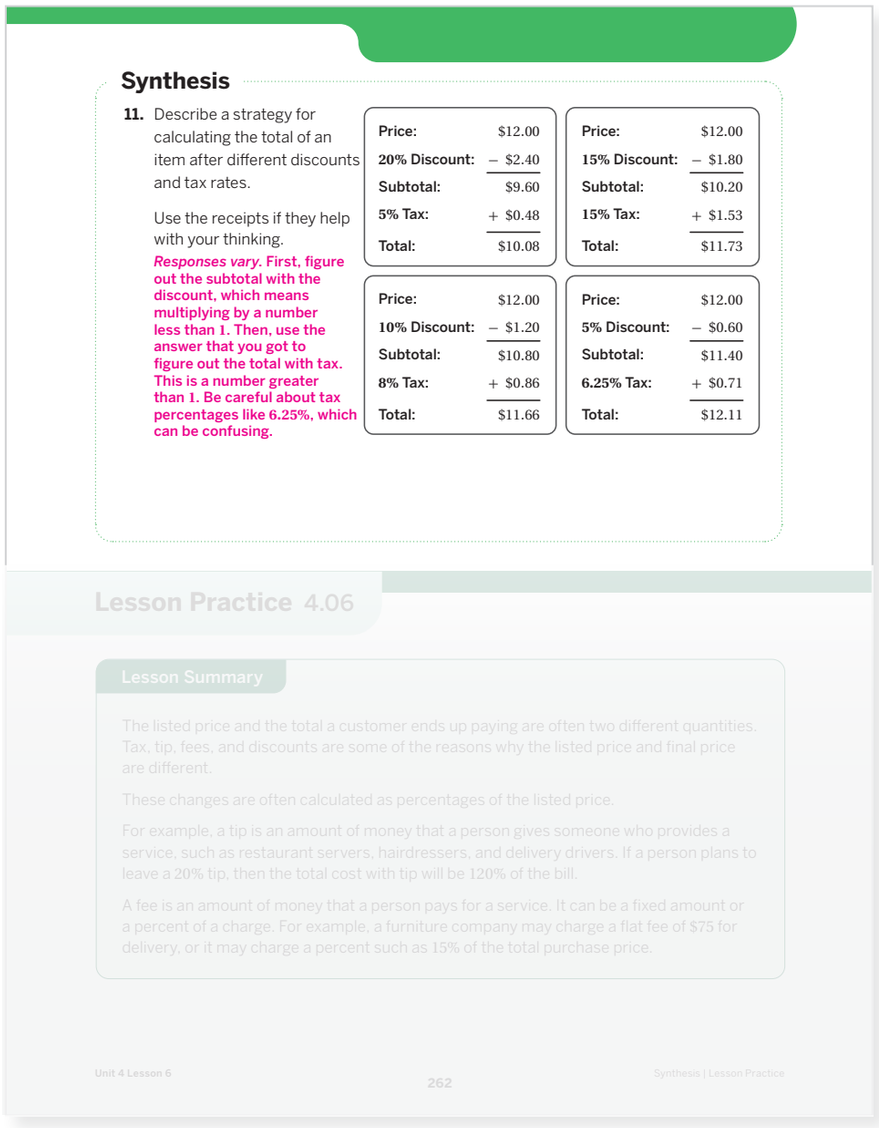
Invite students to respond independently and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using a representation to show the percent increase or decrease.
- Writing a single expression that includes multiplying by a constant of proportionality to represent the percent change.
- Dividing the total costs by tax rates to determine what percent the tax is of the total.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies around determining a percent change as they share during the Synthesis. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. Describe a strategy for calculating the total of an item after different discounts and tax rates.

Use the receipts if they help with your thinking.

Responses vary. First, figure out the subtotal with the discount, which means multiplying by a number less than 1. Then, use the answer that you got to figure out the total with tax. This is a number greater than 1. Be careful about tax percentages like 6.25%, which can be confusing.

Price:	$12.00
20% Discount:	− $2.40
Subtotal:	$9.60
5% Tax:	+ $0.48
Total:	$10.08

Price:	$12.00
15% Discount:	− $1.80
Subtotal:	$10.20
15% Tax:	+ $1.53
Total:	$11.73

Price:	$12.00
10% Discount:	− $1.20
Subtotal:	$10.80
8% Tax:	+ $0.86
Total:	$11.66

Price:	$12.00
5% Discount:	− $0.60
Subtotal:	$11.40
6.25% Tax:	+ $0.71
Total:	$12.11

Lesson Practice 4.06

Lesson Summary

The listed price and the total a customer ends up paying are often two different quantities. Tax, tip, fees, and discounts are some of the reasons why the listed price and final price are different.

These changes are often calculated as percentages of the listed price.

For example, a tip is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. If a person plans to leave a 20% tip, then the total cost with tip will be 120% of the bill.

A fee is an amount of money that a person pays for a service. It can be a fixed amount or a percent of a charge. For example, a furniture company may charge a flat fee of $75 for delivery, or it may charge a percent such as 15% of the total purchase price.

Unit 4 Lesson 6 262 Synthesis | Lesson Practice

Lesson Takeaway: Sales tax, tips, and discounts are a few examples of contexts that involve percent increases or decreases.

Independent | 5 min

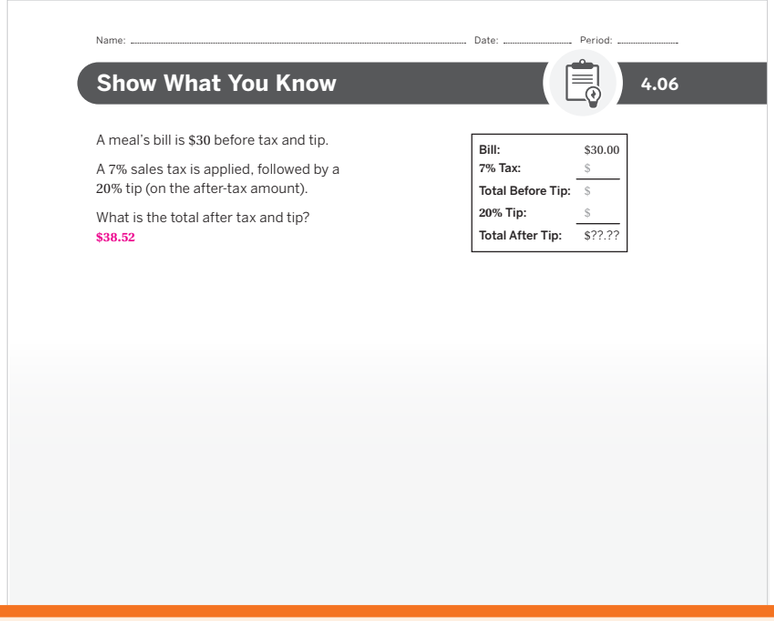
Show What You Know

Purpose: Students demonstrate their understanding by determining the cost of a meal that includes tax and tip.

Today's Goals

1. **Goal:** Calculate an original amount, new amount, or percent change in the context of sales tax and tip, including situations that involve multiple percent increases or decreases (e.g., sales tax and a discount).
2. **Language Goal:** Interpret situations involving tax and tip using mathematically precise language. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.06

A meal's bill is $30 before tax and tip.

A 7% sales tax is applied, followed by a 20% tip (on the after-tax amount).

What is the total after tax and tip?

$38.52

Bill:	$30.00
7% Tax:	$
Total Before Tip:	$
20% Tip:	$
Total After Tip:	$??.??

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding multistep percentage problems in the context of money:

» **Respond to Student Thinking:**

- Use the ***Finding Sales Tax and Tip*** **Mini-lesson**.
- Revisit the **Synthesis** and discuss each example as a class.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Today, students explored the topics of sales tax, discounts, and tipping. Were there any moments of the conversation about the amount to tip that became difficult or ambiguous? How can you prepare to acknowledge different levels of understanding that students bring to a conversation while keeping the discussion focused on mathematical topics (in this unit around percentage)?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. Describe a strategy for calculating the total of an item after different discounts and tax rates.

Use the receipts if they help with your thinking.

Responses vary. First, figure out the subtotal with the discount, which means multiplying by a number less than 1. Then, use the answer that you got to figure out the total with tax. This is a number greater than 1. Be careful about tax percentages like 6.25%, which can be confusing.

Price:	$12.00
20% Discount:	− $2.40
Subtotal:	$9.60
5% Tax:	+ $0.48
Total:	$10.08

Price:	$12.00
15% Discount:	− $1.80
Subtotal:	$10.20
15% Tax:	+ $1.53
Total:	$11.73

Price:	$12.00
10% Discount:	− $1.20
Subtotal:	$10.80
8% Tax:	+ $0.86
Total:	$11.66

Price:	$12.00
5% Discount:	− $0.60
Subtotal:	$11.40
6.25% Tax:	+ $0.71
Total:	$12.11

Lesson Practice 4.06

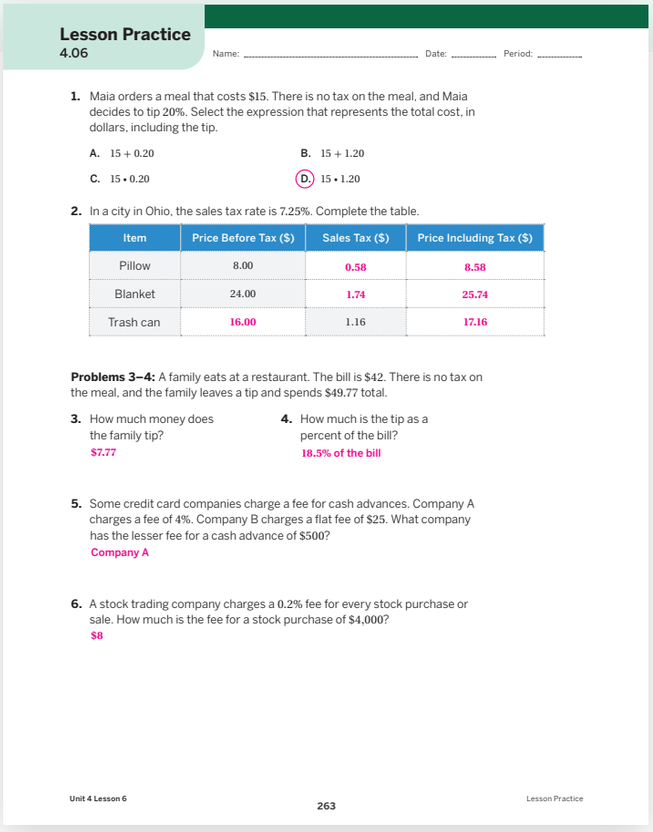
Lesson Summary

The listed price and the total a customer ends up paying are often two different quantities. Tax, tip, fees, and discounts are some of the reasons why the listed price and final price are different.

These changes are often calculated as percentages of the listed price.

For example, a tip is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. If a person plans to leave a 20% tip, then the total cost with tip will be 120% of the bill.

A fee is an amount of money that a person pays for a service. It can be a fixed amount or a percent of a charge. For example, a furniture company may charge a flat fee of $75 for delivery, or it may charge a percent such as 15% of the total purchase price.



Lesson Practice 4.06

Name: _____ Date: _____ Period: _____

1. Maia orders a meal that costs $15. There is no tax on the meal, and Maia decides to tip 20%. Select the expression that represents the total cost, in dollars, including the tip.

   A. $15 + 0.20$ B. $15 + 1.20$

   C. $15 \cdot 0.20$ D. $15 \cdot 1.20$

2. In a city in Ohio, the sales tax rate is 7.25%. Complete the table.

Item	Price Before Tax ($)	Sales Tax ($)	Price Including Tax ($)
Pillow	8.00	0.58	8.58
Blanket	24.00	1.74	25.74
Trash can	16.00	1.16	17.16

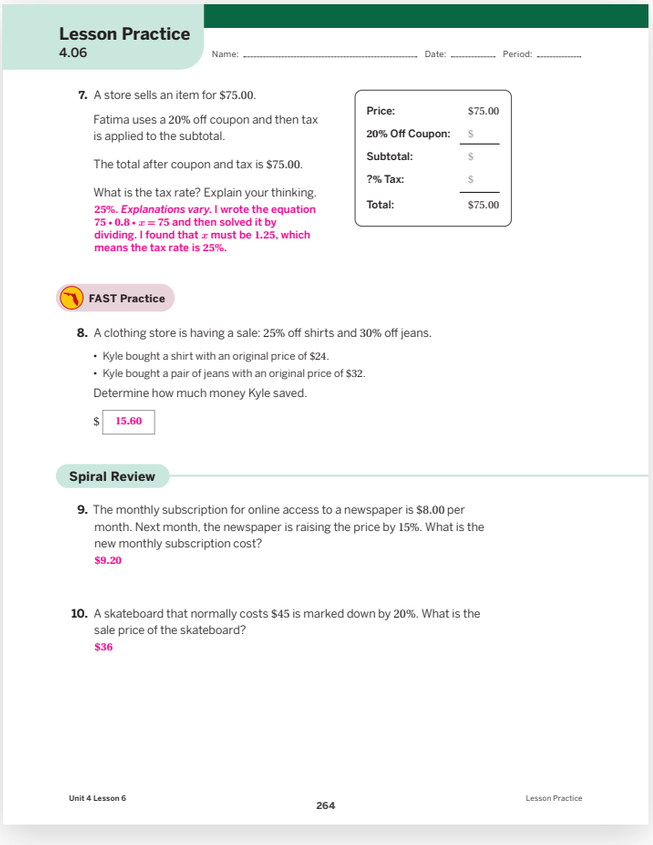
Problems 3–4: A family eats at a restaurant. The bill is $42. There is no tax on the meal, and the family leaves a tip and spends $49.77 total.

3. How much money does the family tip?
   $7.77

4. How much is the tip as a percent of the bill?
   18.5% of the bill

5. Some credit card companies charge a fee for cash advances. Company A charges a fee of 4%. Company B charges a flat fee of $25. What company has the lesser fee for a cash advance of $500?
   Company A

6. A stock trading company charges a 0.2% fee for every stock purchase or sale. How much is the fee for a stock purchase of $4,000?
   $8



Lesson Practice 4.06

Name: _____ Date: _____ Period: _____

7. A store sells an item for $75.00.

   Fatima uses a 20% off coupon and then tax is applied to the subtotal.

   The total after coupon and tax is $75.00.

   What is the tax rate? Explain your thinking.

   25%. *Explanations vary.* I wrote the equation $75 \cdot 0.8 \cdot x = 75$ and then solved it by dividing. I found that x must be 1.25, which means the tax rate is 25%.

Price:	$75.00
20% Off Coupon:	$
Subtotal:	$
?% Tax:	$
Total:	$75.00

FAST Practice

8. A clothing store is having a sale: 25% off shirts and 30% off jeans.
   - Kyle bought a shirt with an original price of $24.
   - Kyle bought a pair of jeans with an original price of $32.

   Determine how much money Kyle saved.

   $ 15.60

Spiral Review

9. The monthly subscription for online access to a newspaper is $8.00 per month. Next month, the newspaper is raising the price by 15%. What is the new monthly subscription cost?
   $9.20

10. A skateboard that normally costs $45 is marked down by 20%. What is the sale price of the skateboard?
    $36

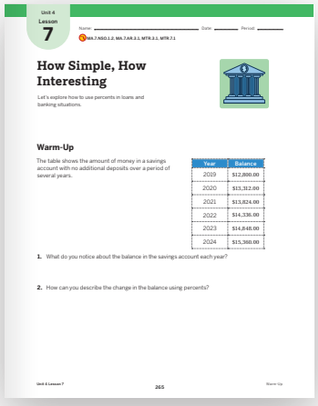
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.3.1
	6	1	MA.7.AR.3.1
	7	3	MA.7.AR.3.1
FAST Practice	8	2	MA.7.AR.3.1
Spiral Review			
	9–10	2	MA.7.AR.3.1

Unit 4

Lesson

7



How Simple, How Interesting

Calculating Simple Interest Using a Formula

Let's explore how to use percentages in loans and banking situations.

Learning Outcomes

Today's Goals

1. **Goal:** Solve real-world problems involving *simple interest* earned.
2. **Language Goal:** Describe how to use the *simple interest* formula to solve real-world problems about borrowing and investing money. **(Reading, Writing)**

Students extend their knowledge of using percents to solve real-world problems to solving problems involving **simple interest**. They use the simple interest formula to determine the **interest** earned on deposits and interest paid on loans.

Coherence

Prior Learning

Throughout Unit 4, students have solved a variety of mathematical and real-world problems using percents, such as discounts, markups, tax, tips, and fees.

Future Learning

In high school, students will solve compound interest problems, where interest accumulates on both the principal and on the previous interest.

Connections

- Students calculate the simple interest paid on a loan over a period of time using the formula $I = prt$. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.912.FL.3.1 MA.912.FL.3.2

Vocabulary

New Vocabulary

interest
principal
simple interest

Lesson at a Glance

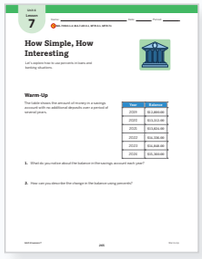
~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

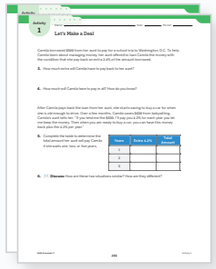
Students compare the values in a table, including using percents.



Activity 1

Pairs | 15 min

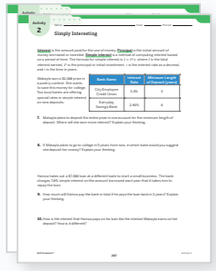
Students explore real-world situations about borrowing and lending money.
Routine: Think-Pair-Share **(MTR.7.1)**



Activity 2

Pairs | 15 min

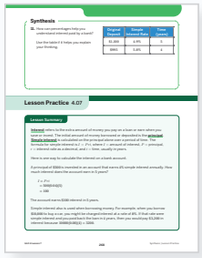
Students use the **simple interest** formula to solve real-world problems about banking.
MLR2: Collect and Display **(MTR.3.1)**



Synthesis

Whole Class | 5 min

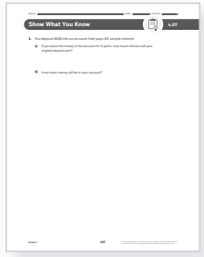
Students synthesize their understanding of simple interest.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by,..



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators

Pairs | 5 min

Warm-Up

Purpose: Students compare the values in a table, including using percents.

1 Launch

Direct students' attention to the table of values.

Invite students to select several rows from the table and compare consecutive rows.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

2 Connect

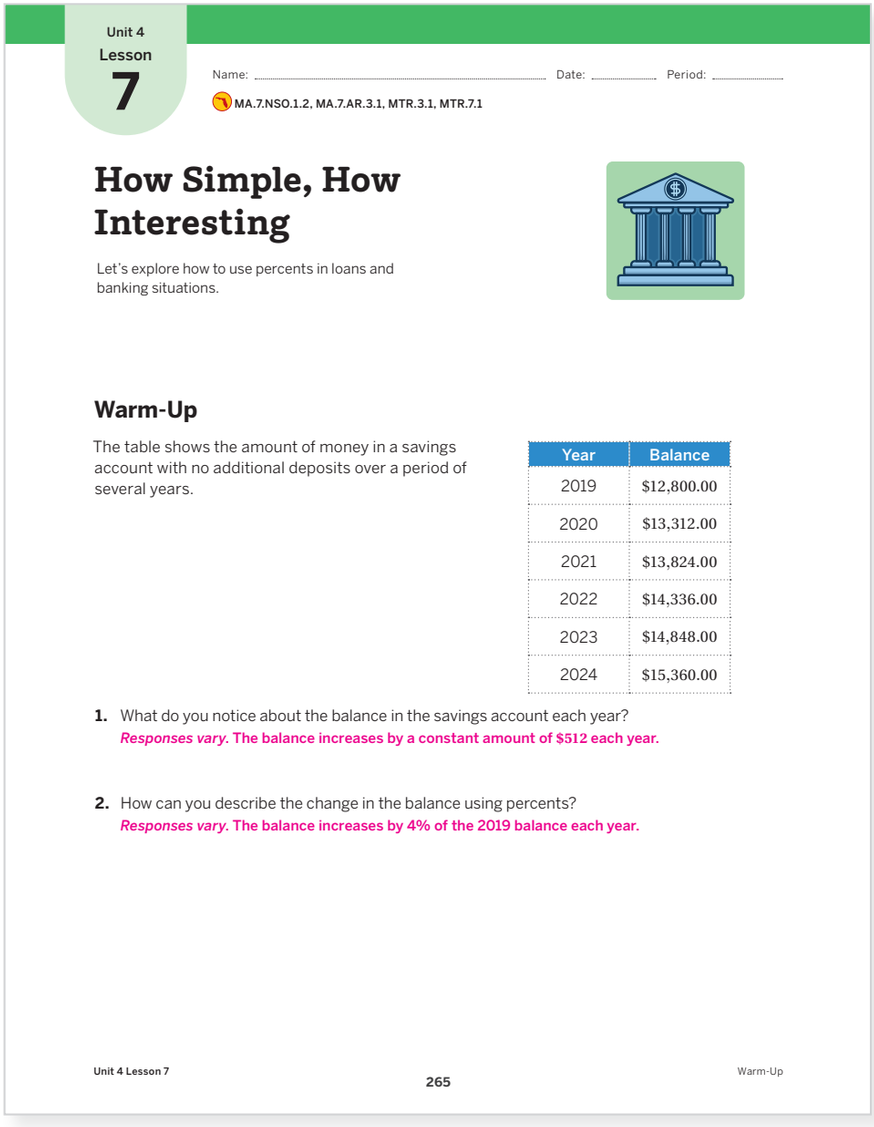
Encourage students to share their responses with a partner. Consider asking, "How are others' responses similar to or different from your own?"

Discuss students' strategies for calculating the percents they used to compare the annual balances in the table.

Look for different ways that students compare the balances:

- Constant amount of increase
- Percent of the balance in 2019
- Decreasing percent of the annual balance
- $512 per year
- About 3.5% to 4% each year

ELL **English Language Learners** Invite students to share their ideas with a partner before they share them with others. **(Writing and Speaking)**



Unit 4
Lesson
7

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.3.1, MTR.7.1

How Simple, How Interesting

Let's explore how to use percents in loans and banking situations.

Warm-Up

The table shows the amount of money in a savings account with no additional deposits over a period of several years.

Year	Balance
2019	$12,800.00
2020	$13,312.00
2021	$13,824.00
2022	$14,336.00
2023	$14,848.00
2024	$15,360.00

1. What do you notice about the balance in the savings account each year?
Responses vary. The balance increases by a constant amount of $512 each year.

2. How can you describe the change in the balance using percents?
Responses vary. The balance increases by 4% of the 2019 balance each year.

Unit 4 Lesson 7 265 Warm-Up

Pairs | 13 min

Activity 1 Let's Make a Deal

Purpose: Students explore real-world situations about borrowing and lending money. **(MTR.7.1)**

Short on time? Consider discussing Problem 6 as a class.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about the deal that Camila's aunt makes with her. See the Routine Facilitation Guide for more information.

2 Monitor

Select and sequence strategies for determining the interest and total amount for different numbers of years.

Look for connections between percents and equations.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look for patterns in the table like in the Warm-Up.
Have difficulty calculating.	**Strengthen** Invite them to solve a simpler problem using a number that is easier to calculate with, like 2%.
Use the table to calculate the total.	**Strengthen** Consider asking, "How can you show this with an equation?"
Quickly identify the interest paid and total.	**Stretch** Challenge them to write and solve a similar problem, where a greater amount of money is involved.

3 Connect

Discuss students' responses to Problem 6. Consider asking:

- "What kinds of values are involved in both problems?"
- "How does time factor into solving these problems?"

To surface the Key Takeaway, consider asking, "How did you use percents to solve these problems?" **(MTR.7.1)**

Activity 1

Name: Date: Period:

Let's Make a Deal

Camila borrowed $500 from her aunt to pay for a school trip to Washington, D.C. To help Camila learn about managing money, her aunt offered to loan Camila the money with the condition that she pay back an extra 2.4% of the amount borrowed.

3. How much extra will Camila have to pay back to her aunt?

$12

4. How much will Camila have to pay in all? How do you know?

$512. *Responses vary.* She will have to pay back $500, plus the $12 that is 2.4% of $500.

After Camila pays back the loan from her aunt, she starts saving to buy a car for when she is old enough to drive. Over a few months, Camila saves $650 from babysitting. Camila's aunt tells her, "If you lend me the $650, I'll pay you 4.2% for each year you let me keep the money. Then when you are ready to buy a car, you can have this money back plus the 4.2% per year."

5. Complete the table to determine the total amount her aunt will pay Camila if she waits one, two, or five years.

Years	Extra 4.2%	Total Amount
1	$27.30	$677.30
2	$54.60	$704.60
5	$136.50	$786.50

6. **Discuss:** How are these two situations similar? How are they different?

Responses vary. In both situations, one person is giving money to the other and is adding a percent of the loan to the total amount to be paid back. In the first situation, Camila is borrowing money, and the percent is applied to the original amount once. In the second situation, Camila is lending money, and the percent is applied multiple times, once every year.

Unit 4 Lesson 7 266 Activity 1

Key Takeaway: Percents are used to solve problems about the cost of borrowing money or the value of lending money.

Pairs | 15 min

Activity 2 Simply Interesting

Purpose: Students use the **simple interest** formula to solve real-world problems about banking. **(MTR.3.1)**

Short on time? Consider discussing Problem 10 as a class.

1 Launch

Direct students' attention to the *simple interest* formula.

To surface connections, consider asking, "What steps do you take to use a formula to solve problems?"

Consider creating an anchor chart of the simple interest formula emphasizing the connection between the variables (I,p,r, and t) and their meanings for students to refer to throughout the lesson.

2 Monitor

To support students getting started, consider asking, "How can you use a formula to calculate the interest earned at each bank?" **(MTR.3.1)**

Listen for students who notice that 5 years is less than the minimum length of deposit for the interest rate at Everyday Savings Bank.

Look for students who use the percent without changing the value to a decimal. Consider asking, "How do you write a percent as a decimal?"

A **Accessibility: Executive Function** Consider providing tables for students with columns labeled Principal (P), Interest Rate (r), Time (t), and Interest (I).

3 Connect

Invite students to share their strategies for determining the total that Hamza pays the bank. If students do not suggest it, note that they could write an expression like $7500(1 + 0.078)(3)$ instead of adding the interest to the amount borrowed.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of **interest**, **simple interest**, and **principal** using students' language. Display any new language, examples, or diagrams for students to reference later.

Invite students to share their responses and justify their thinking.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., They are similar because both are ____. They are different because Hamza____ while Makayla ____.). **(Writing and Speaking)**

Activity 2

Name: Date: Period:

Simply Interesting

Interest is the amount paid for the use of money. **Principal** is the initial amount of money borrowed or invested. **Simple interest** is a method of computing interest based on a period of time. The formula for simple interest is $I = Prt$, where I is the total interest earned, P is the principal or initial investment, r is the interest rate as a decimal, and t is the time in years.

Makayla won a $2,500 prize in a poetry contest. She wants to save this money for college. Two local banks are offering special rates in simple interest on new deposits.

Bank Name	Interest Rate	MInimum Length of Deposit (years)
City Employee Credit Union	3.5%	5
Everyday Savings Bank	2.95%	6

7. Makayla plans to deposit the entire prize in one account for the minimum length of deposit. Where will she earn more interest? Explain your thinking.

Everyday Savings Bank; *Explanations vary.* At the credit union, she will earn $2500(0.035)(5) = \$437.50$ interest over 5 years, but at the savings bank, she will earn $2500(0.0295)(6) = \$442.50$ over 6 years.

8. If Makayla plans to go to college in 5 years from now, in which bank would you suggest she deposit her money? Explain your thinking.

City Employee Credit Union; *Responses vary.* She should use the City Employee Credit Union because the minimum length of deposit at Everyday Savings Bank is 6 years to get the 2.95% interest rate and she wants to go to college before that time is up.

Hamza takes out a $7,500 loan at a different bank to start a small business. The bank charges 7.8% simple interest on the amount borrowed each year that it takes him to repay the loan.

9. How much will Hamza pay the bank in total if he pays the loan back in 3 years? Explain your thinking.

$9,255. *Explanations vary.* He will pay $7,500 plus the interest owed of $1,755.

10. How is the interest that Hamza pays on his loan like the interest Makayla earns on her deposit? How is it different?

Responses vary. Both are examples of simple interest. The interest rates are different rates, but both are determined by a percent of the original amount over a number of years. Makayla deposited money, so she will earn interest, but Hamza borrowed money, so he will pay interest.

Unit 4 Lesson 7 267 Activity 2

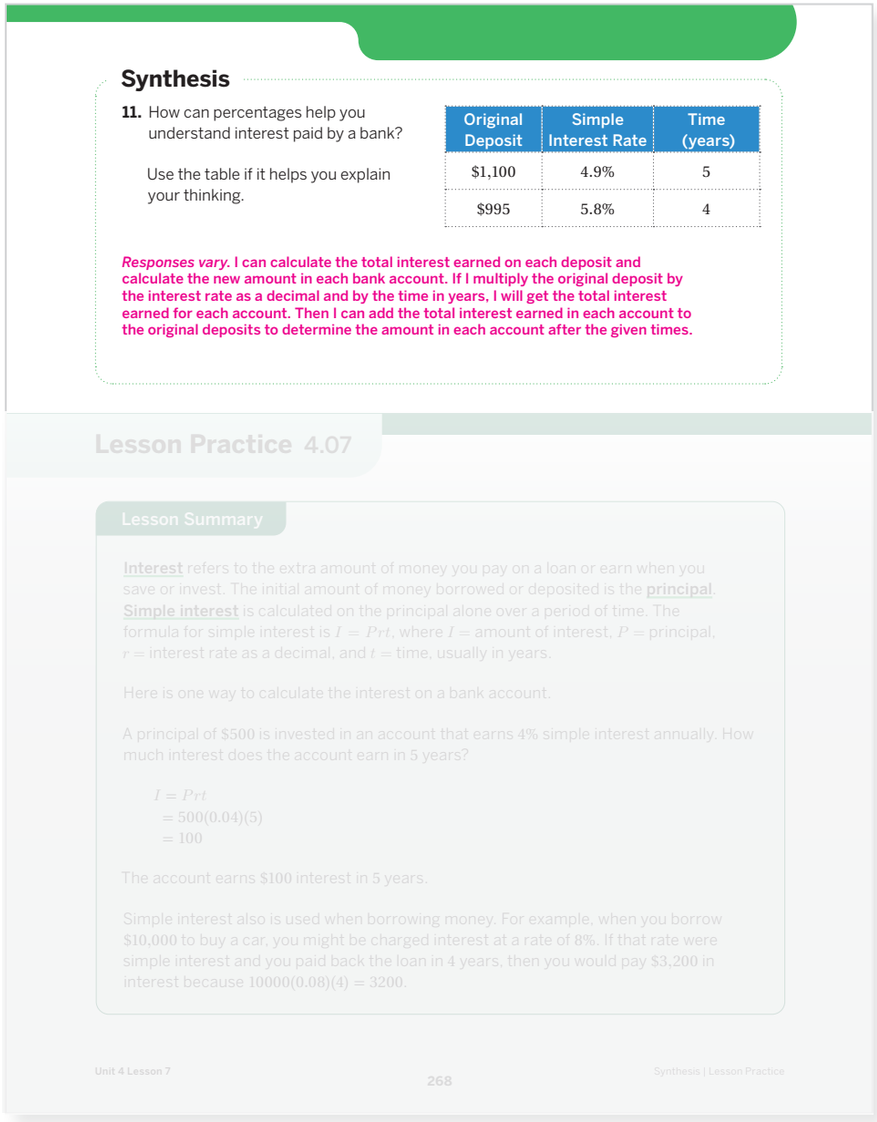
Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of simple interest.

Invite students to respond independently, and then share their thinking about simple interest with a partner.

Create a record of how students calculated simple interest or how they described how to calculate it.



Synthesis

11. How can percentages help you understand interest paid by a bank?

Use the table if it helps you explain your thinking.

Original Deposit	Simple Interest Rate	Time (years)
$1,100	4.9%	5
$995	5.8%	4

Responses vary. I can calculate the total interest earned on each deposit and calculate the new amount in each bank account. If I multiply the original deposit by the interest rate as a decimal and by the time in years, I will get the total interest earned for each account. Then I can add the total interest earned in each account to the original deposits to determine the amount in each account after the given times.

Lesson Practice 4.07

Lesson Summary

Interest refers to the extra amount of money you pay on a loan or earn when you save or invest. The initial amount of money borrowed or deposited is the principal. Simple interest is calculated on the principal alone over a period of time. The formula for simple interest is $I = Prt$, where I = amount of interest, P = principal, r = interest rate as a decimal, and t = time, usually in years.

Here is one way to calculate the interest on a bank account.

A principal of $500 is invested in an account that earns 4% simple interest annually. How much interest does the account earn in 5 years?

$$\begin{aligned} I &= Prt \\ &= 500(0.04)(5) \\ &= 100 \end{aligned}$$

The account earns $100 interest in 5 years.

Simple interest also is used when borrowing money. For example, when you borrow $10,000 to buy a car, you might be charged interest at a rate of 8%. If that rate were simple interest and you paid back the loan in 4 years, then you would pay $3,200 in interest because $10000(0.08)(4) = 3200$.

Unit 4 Lesson 7 268 Synthesis | Lesson Practice

Lesson Takeaway: Simple interest may be earned on deposits or paid on loans and is calculated using the formula $I = Prt$.

Independent | 5 min

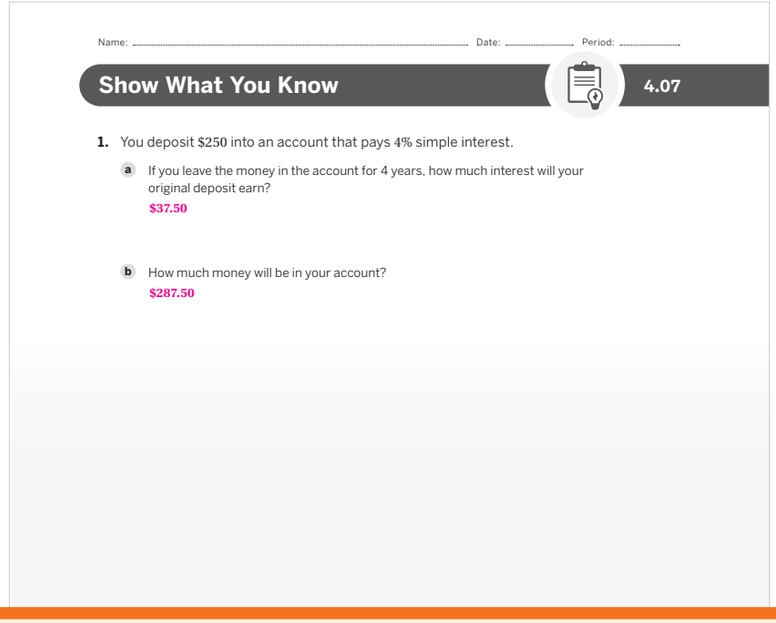
Show What You Know

Purpose: Students demonstrate their understanding by,..

Today's Goals

1. **Goal:** Solve real-world problems involving simple interest earned.
2. **Language Goal:** Describe how to use the simple interest formula to solve real-world problems about borrowing and investing money. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.07

1. You deposit $250 into an account that pays 4% simple interest.
   a. If you leave the money in the account for 4 years, how much interest will your original deposit earn?
   $37.50
   b. How much money will be in your account?
   $287.50

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate simple interest using a formula:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how students' understanding of the term percent has developed throughout this unit. How has their definition of percent expanded over the last few lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can percentages help you understand interest paid by a bank?

Use the table if it helps you explain your thinking.

Original Deposit	Simple Interest Rate	Time (years)
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Responses vary. I can calculate the total interest earned on each deposit and calculate the new amount in each bank account. If I multiply the original deposit by the interest rate as a decimal and by the time in years, I will get the total interest earned for each account. Then I can add the total interest earned in each account to the original deposits to determine the amount in each account after the given times.

Lesson Practice 4.07

Lesson Summary

Interest refers to the extra amount of money you pay on a loan or earn when you save or invest. The initial amount of money borrowed or deposited is the **principal**. **Simple interest** is calculated on the principal alone over a period of time. The formula for simple interest is $I = Prt$, where $I =$ amount of interest, $P =$ principal, $r =$ interest rate as a decimal, and $t =$ time, usually in years.

Here is one way to calculate the interest on a bank account.

A principal of $500 is invested in an account that earns 4% simple interest annually. How much interest does the account earn in 5 years?

$$\begin{aligned} I &= Prt \\ &= 500(0.04)(5) \\ &= 100 \end{aligned}$$

The account earns $100 interest in 5 years.

Simple interest also is used when borrowing money. For example, when you borrow $10,000 to buy a car, you might be charged interest at a rate of 8%. If that rate were simple interest and you paid back the loan in 4 years, then you would pay $3,200 in interest because $10000(0.08)(4) = 3200$.

Lesson Practice 4.07

Name: _____ Date: _____ Period: _____

Problems 1–2: You deposit $1,200 into an account that pays 3% simple interest.

1. Complete the table to determine the interest earned and the total amount in the account for the first five years the account is open.

Years	Interest	Total Amount
1	$36	$1,236
2	$72	$1,272
3	$108	$1,308
4	$144	$1,344
5	$180	$1,380

2. If you leave the money in the account for 10 years, what will be the total amount of money in the account? Explain your thinking.

$1,560. *Explanations vary.* The account will earn $360 in interest because $(1200)(0.03)(10) = 360$. Add this to the initial deposit: $1200 + 360 = 1560$.

3. Jin purchased a brand new car. She got a loan for $35,000, which she will pay back over 6 years. The terms of Jin's loan are that she will pay 9% simple interest on the original principal each year that she is paying off the loan. How much interest will Jin pay?

$18,900

Problems 4–5: A bank is offering a special where it will charge 6.75% simple interest annually on any loan paid back within 7 years.

4. What will the total amount paid back be if the borrower takes 7 years to pay back a $1,780 loan?

$2,621.05

5. If the total paid on a $980 loan was $1,310.75 including the simple interest, how many years did it take to pay off the loan? Explain your thinking.

5 years. *Explanations vary.* If $1,310.75 was paid on a $980 loan, then $330.75 was the amount paid in interest because $1310.75 - 980 = 330.75$. I can use the simple interest formula to calculate that $66.15 is paid in interest each year $980(0.0675)(3) = 66.15$. Finally, I will divide the amount of interest paid by the interest per year to determine how many years it took to pay off the loan: $330.75 \div 66.15 = 5$.

Lesson Practice 4.07

Name: _____ Date: _____ Period: _____

Problems 6–8: Greenville Credit Union and Parkville Bank are both offering special rates on new deposits. The simple interest rate and minimum deposit for new accounts at each bank are shown in the table.

Bank	Interest Rate	Minimum Deposit
Greenville Credit Union	5.75%	$500
Parkville Bank	6.1%	$1,500

6. How much interest will a $3,000 deposit earn at each bank in 4 years?

Greenville Credit Union: $690

Parkville Bank: $732

7. What will the value be in an account with a $850 deposit at Greenville Credit Union in 3 years? Show or explain your thinking.

About $996.63. *Explanations vary.* $850(0.0575)(3) = 146.625$, and $850 + 146.625 \approx 996.63$.

8. What will the value be in an account with a $850 deposit at Parkville Bank after 3 years? Show or explain your thinking.

Explanations vary. You cannot deposit $850 into a new account at Parkville Bank because there is a minimum deposit of $1,500.

FAST Practice

9. Your bank is offering a simple interest rate of 7.5% for a loan paid back in 3 years. Determine the total amount you would pay if you borrowed $900 at this rate.

The simple interest on a loan for $900 at a rate of 7.5% over 3 years is $ 202.50 .

So I would pay back a total of $ 1,102.50 .

Spiral Review

10. The sales tax rate is 5.5%. Determine the price after tax for an item that sells for $40 regularly and that is 20% off for this week. Explain your reasoning.

$33.76. *Explanations vary.* The discounted price is $40(0.8) = 32$. The cost including tax is $32(1.055) = 33.76$.

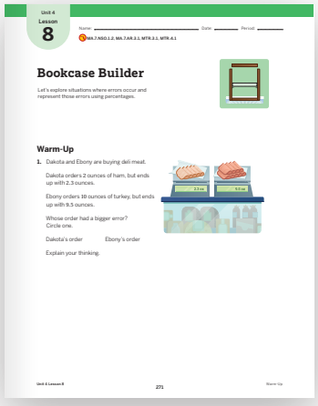
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.AR.3.1
	2–4, 6–8	2	MA.7.AR.3.1
	5	3	MA.7.AR.3.1
FAST Practice	9	2	MA.7.AR.3.1
Spiral Review			
	10	2	MA.7.AR.3.1

Unit 4

Lesson

8


Unit 4
Lesson
8
Name: Date: Period:
MA.7.NSO.1.2, MA.7.AR.3.1, MTR.3.1, MTR.4.1
Bookcase Builder
Let's explore situations where errors occur and represent those errors using percentages.
Warm-Up
1. Dakota and Ebony are buying deli meat.
Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.
Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.
Whose order had a bigger error? Circle one.
Dakota's order Ebony's order
Explain your thinking.
271

Bookcase Builder

Percent Error

Let's explore situations where errors occur and represent those errors using percentages.

Learning Outcomes

Today's Goal

1. **Goal:** Know that *percent error* expresses error as a percentage of a desired value.
2. **Goal:** Decide whether a value is within an acceptable *percent error*.
3. **Language Goal:** Explain a strategy for solving a problem involving *percent error*. **(Writing)**

Students will be introduced to real-world contexts in which error occurs. Students will consider why **percent error** is useful, practice determining percent error, and express whether a measurement is within an acceptable range.

Coherence

Prior Learning

In Lesson 6, students worked with contexts involving percent change, such as tax, tip, and discounts.

Future Learning

In Grade 8, students will continue to explore proportional relationships by comparing them using equations, tables, and graphs.

Connections

- Students calculate percent error and express whether a measurement is within an acceptable range in real-world contexts, such as the size of screws when building a bookshelf. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.8.AR.2.1

Vocabulary

New Vocabulary

percent error

Lesson at a Glance

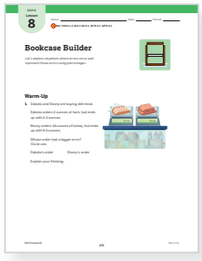
~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Independent | 5 min

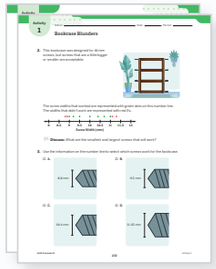
Students encounter a context involving two errors and use the Decide and Defend routine to describe which error is larger. **(MTR.4.1)**



Activity 1

Pairs | 15 min

Students come to understand that **percent error** is one way to describe how far a value is above or below a desired measurement. MLR2: Collect and Display



Activity 2

Pairs | 15 min

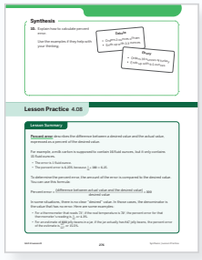
Students practice deciding whether a value is within a range of acceptable percent errors. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding by explaining how to find percent error for values in context. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the percent error for a new context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students encounter a context involving two errors and use the Decide and Defend routine to describe which error is larger. **(MTR.4.1)**

1 Launch

To surface connections, consider asking students whether they have ever had an experience at a store similar to Dakota's and Ebony's experiences.

Invite students to decide whose order had a bigger error and to explain their reasoning.

A **Accessibility: Affective Functioning** To support students in sharing their thinking, ask students in advance if they, you, or a classmate can share their thinking during the whole class discussion.

2 Connect

Invite students to share their work.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

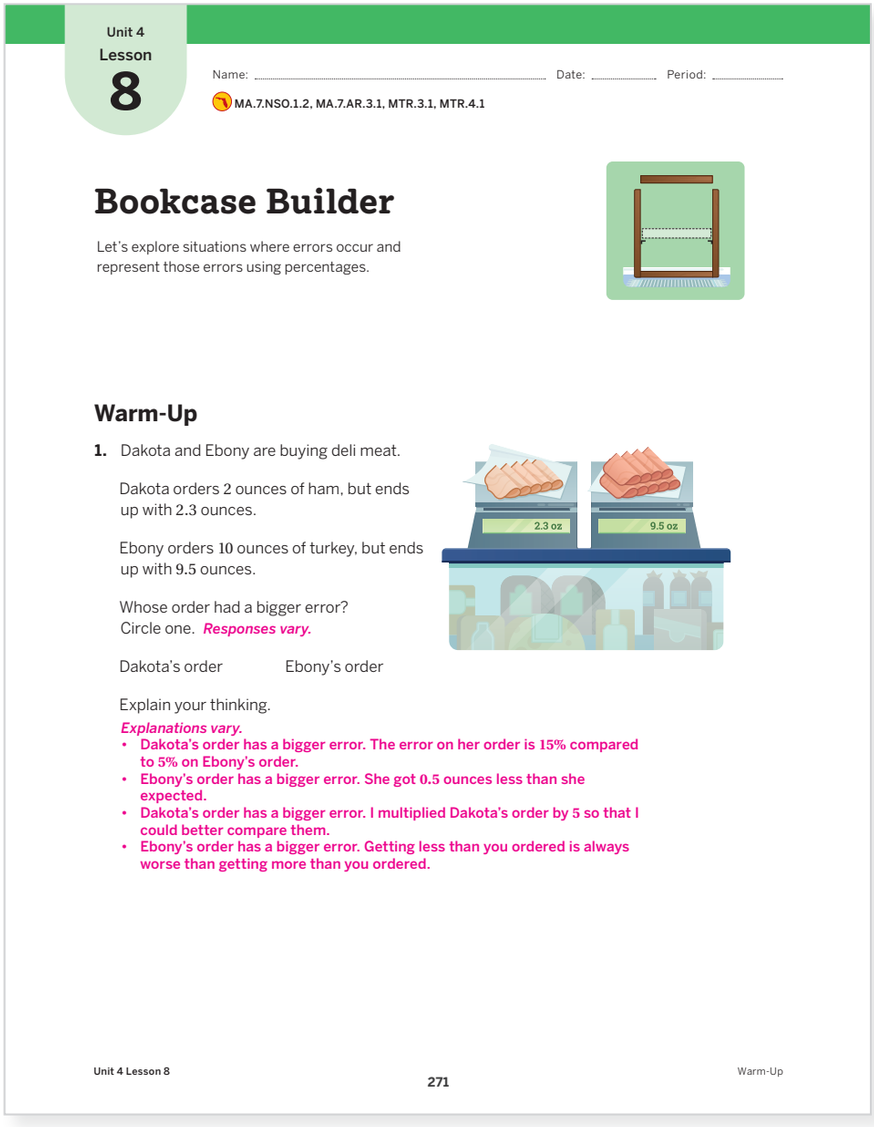
- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How do you know?

(Writing and Speaking)

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

Consider asking:

- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"
- "Where do you see mathematical thinking in selecting either order?" **(MTR.4.1)**



Unit 4
Lesson
8

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.3.1, MTR.4.1

Bookcase Builder

Let's explore situations where errors occur and represent those errors using percentages.

Warm-Up

1. Dakota and Ebony are buying deli meat.

Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.

Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.

Whose order had a bigger error? Circle one. *Responses vary.*

Dakota's order Ebony's order

Explain your thinking.

Explanations vary.

- Dakota's order has a bigger error. The error on her order is 15% compared to 5% on Ebony's order.
- Ebony's order has a bigger error. She got 0.5 ounces less than she expected.
- Dakota's order has a bigger error. I multiplied Dakota's order by 5 so that I could better compare them.
- Ebony's order has a bigger error. Getting less than you ordered is always worse than getting more than you ordered.

Unit 4 Lesson 8 271 Warm-Up

Pairs | 15 min

Activity 1 Bookcase Blunders

Purpose: Students come to understand that **percent error** is one way to describe how far a value is above or below a desired measurement.

1 Launch

To surface connections, consider asking, "Have you ever needed to assemble furniture when given a set of instructions and parts? Does this match your experience with a building project?"

Direct students' attention to the line plot. Consider asking, "What are some measurements that lie between the tick marks on the number line?"

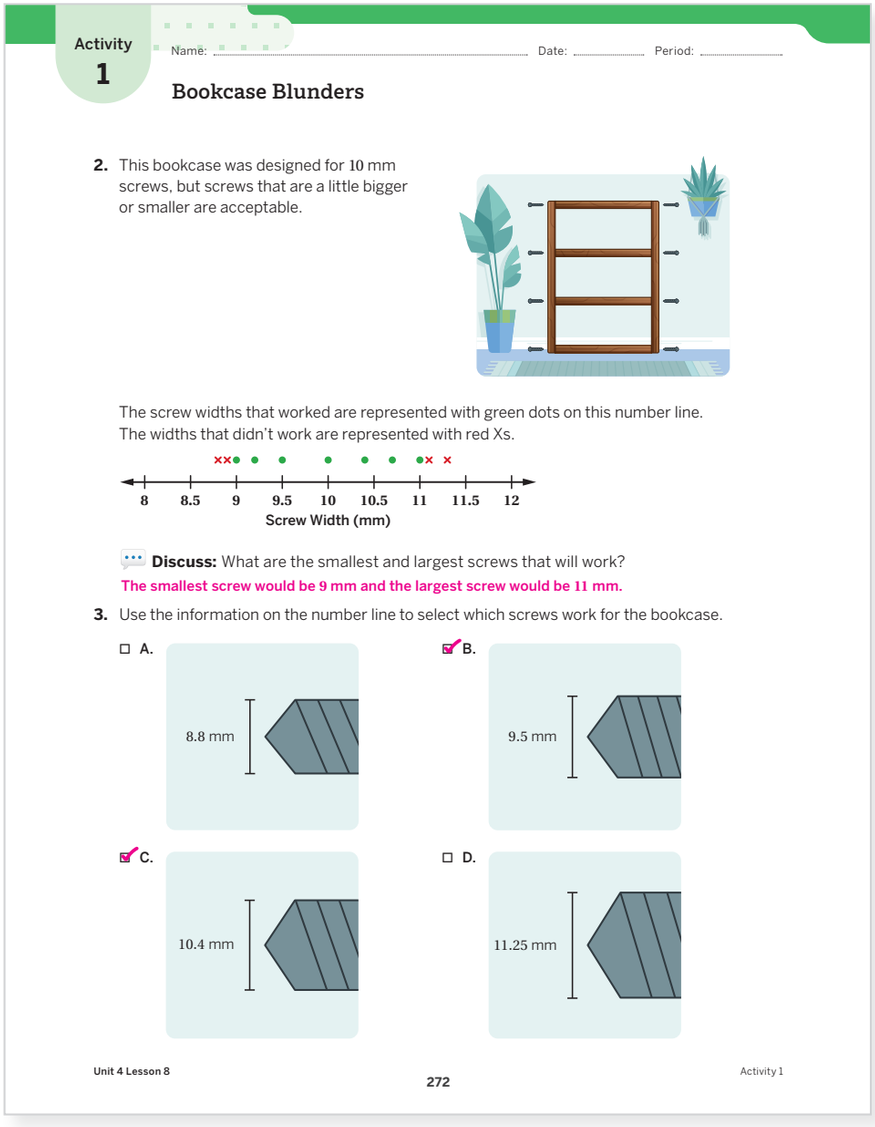
2 Monitor

Encourage students to determine the largest and smallest screw sizes that will work.

A **Accessibility: Visual-Spatial Processing**
Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does the green dot over 9.5 mean? What about the red X between 11 and 11.5?"

Listen for a variety of student strategies to determine how they estimated the location of each measurement on the line plot. Consider inviting them to share with a partner to reach agreement before continuing.

Activity 1 continued >



Activity 1

Name: Date: Period:

Bookcase Blunders

2. This bookcase was designed for 10 mm screws, but screws that are a little bigger or smaller are acceptable.

The screw widths that worked are represented with green dots on this number line. The widths that didn't work are represented with red Xs.

8 8.5 9 9.5 10 10.5 11 11.5 12

Screw Width (mm)

Discuss: What are the smallest and largest screws that will work?

The smallest screw would be 9 mm and the largest screw would be 11 mm.

3. Use the information on the number line to select which screws work for the bookcase.

☐ A. 8.8 mm

☑ B. 9.5 mm

☑ C. 10.4 mm

☐ D. 11.25 mm

Unit 4 Lesson 8 272 Activity 1

Pairs | 15 min

Activity 1 Bookcase Blunders (continued)

Purpose: Students come to understand that **percent error** is one way to describe how far a value is above or below a desired measurement.

2 Monitor

Encourage students to determine the largest and smallest screw sizes that will work.

Listen for explanations for both similarities and differences to share during the discussion.

Differentiation: (Strengthen) For students that find the acceptable range is between $\frac{1}{10}$ (or 10%) smaller and the same amount larger than the desired value for both screws, invite these students to share their thinking during the discussion.

Consider asking, "What is similar about the range of acceptable values for both the 6-millimeter and 10-millimeter screws?"

Consider sharing that **percent error** is a way to describe error, expressed as a percentage of the desired value. Invite students to calculate the percent error of each screw.

Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Work together on the provided example to determine the percent error for the 6 mm screw. Encourage them to refer to this example as they continue.
Want to explore percent error further.	**Stretch** Encourage them to think of a rule that might be used to figure out what screws work for any desired size screw.

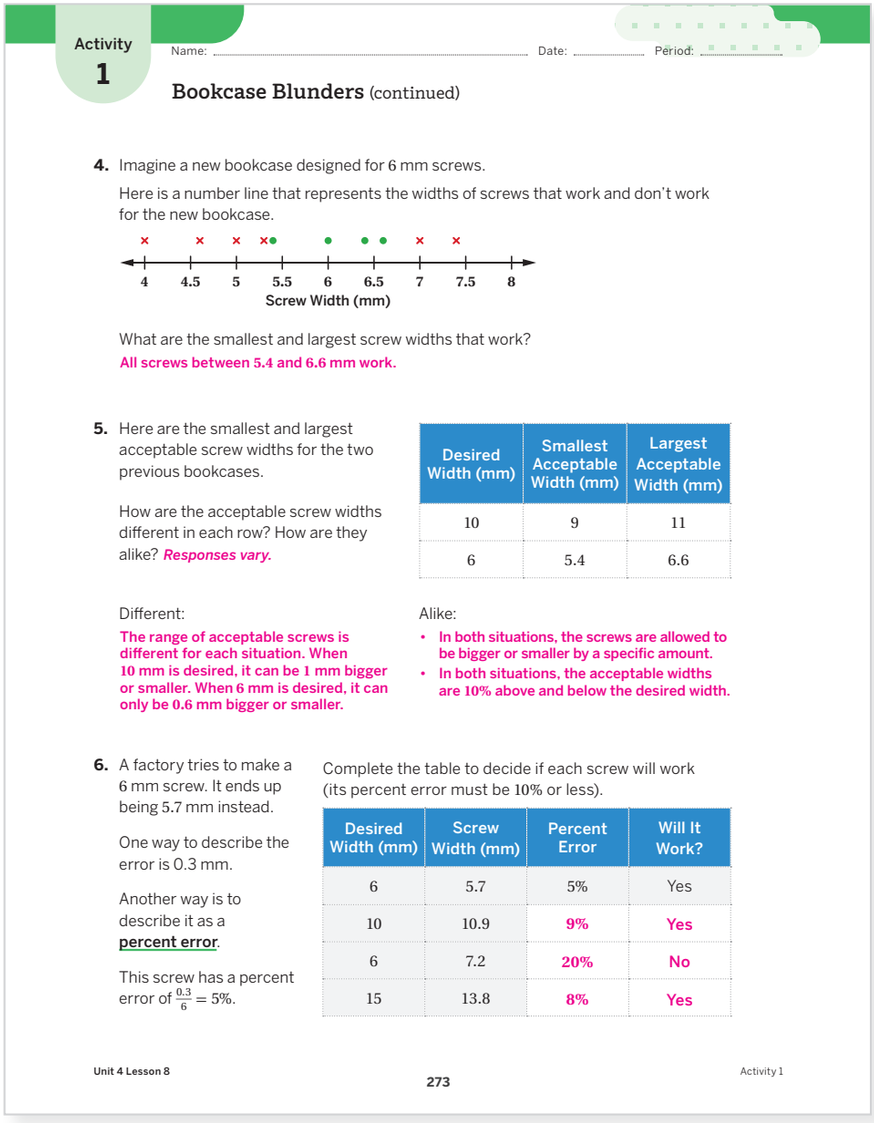
3 Connect

Share and discuss the percent error table.

MLR2: Collect and Display Use this routine to create a class definition or anchor chart for *percent error* using students' language. Display any new language, examples, or diagrams for students to reference later.

Consider asking:

- "When do you think calculating percent error might be useful in your life or in the lives of people in your community?"
- "When would it be most necessary to have a small percent error?"
- "When would it be acceptable to have a larger percent error?"



Activity 1

Name: Date: Period:

Bookcase Blunders (continued)

4. Imagine a new bookcase designed for 6 mm screws.

Here is a number line that represents the widths of screws that work and don't work for the new bookcase.

What are the smallest and largest screw widths that work?

All screws between 5.4 and 6.6 mm work.

5. Here are the smallest and largest acceptable screw widths for the two previous bookcases.

How are the acceptable screw widths different in each row? How are they alike? *Responses vary.*

Desired Width (mm)	Smallest Acceptable Width (mm)	Largest Acceptable Width (mm)
10	9	11
6	5.4	6.6

Different:

The range of acceptable screws is different for each situation. When 10 mm is desired, it can be 1 mm bigger or smaller. When 6 mm is desired, it can only be 0.6 mm bigger or smaller.

Alike:

- In both situations, the screws are allowed to be bigger or smaller by a specific amount.
- In both situations, the acceptable widths are 10% above and below the desired width.

6. A factory tries to make a 6 mm screw. It ends up being 5.7 mm instead.

One way to describe the error is 0.3 mm.

Another way is to describe it as a **percent error**.

This screw has a percent error of $\frac{0.3}{6} = 5\%$.

Complete the table to decide if each screw will work (its percent error must be 10% or less).

Desired Width (mm)	Screw Width (mm)	Percent Error	Will It Work?
6	5.7	5%	Yes
10	10.9	9%	Yes
6	7.2	20%	No
15	13.8	8%	Yes

Unit 4 Lesson 8 273 Activity 1

Pairs | 15 min

Activity 2 Challenge Your Shelf

Purpose: Students practice deciding whether a value is within a range of acceptable percent errors. **(MTR.3.1)**

Short on time? Consider inviting students to complete fewer repeated challenges in Problem 8.

1 Launch

Direct students' attention to the bookcase and consider asking, "What might be some sources of error when building a bookshelf?" *Responses vary.* You might drill holes at the wrong height, cut the shelves too short, or cut the shelves too long.

Invite students to use the strategies they tried in the previous problem to determine if the shelf will fit, and to explain their reasoning.

2 Monitor

Invite students to solve as many challenges as they have time for. Consider inviting students to set their own goal for a streak before beginning.

Note: Share that sense-making is more important than speed to encourage students to prioritize deepening their understanding of percent error.

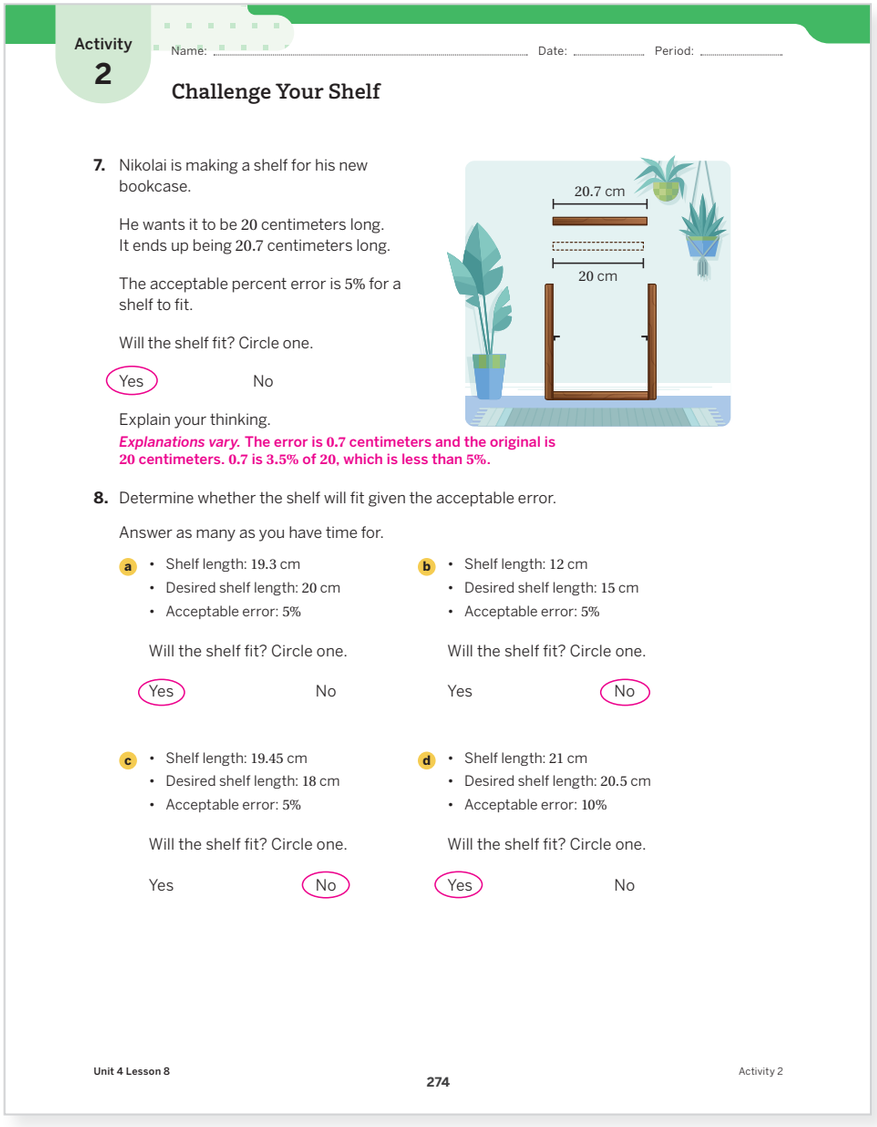
A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

Listen for a variety of student strategies and partner discussions, offering help or encouragement where needed.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to look back at the example for determining percent error for the 6 mm screw. Encourage them to refer to this example as they continue.
Repeatedly use guessing for whether the shelf will fit or not.	**Support** Consider asking, "What strategies shared from Activity 1 might help you to improve your challenge streak?"
Repeatedly calculate percent error with a reliable strategy (i.e., applying the percent error equation). **(MTR.3.1)**	**Strengthen** Invite students to think about how they would share their strategies during the Connect.

Activity 2 continued >



Activity 2

Name: Date: Period:

Challenge Your Shelf

7. Nikolai is making a shelf for his new bookcase.

He wants it to be 20 centimeters long. It ends up being 20.7 centimeters long.

The acceptable percent error is 5% for a shelf to fit.

Will the shelf fit? Circle one.

(Yes) No

Explain your thinking.

Explanations vary. The error is 0.7 centimeters and the original is 20 centimeters. 0.7 is 3.5% of 20, which is less than 5%.

8. Determine whether the shelf will fit given the acceptable error.

Answer as many as you have time for.

a
- Shelf length: 19.3 cm
- Desired shelf length: 20 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

(Yes) No

b
- Shelf length: 12 cm
- Desired shelf length: 15 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes (No)

c
- Shelf length: 19.45 cm
- Desired shelf length: 18 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes (No)

d
- Shelf length: 21 cm
- Desired shelf length: 20.5 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

(Yes) No

Unit 4 Lesson 8 274 Activity 2

Pairs | 15 min

Activity 2 Challenge Your Shelf (continued)

Purpose: Students practice deciding whether a value is within a range of acceptable percent errors. **(MTR.3.1)**

Short on time? Consider inviting students to complete fewer repeated challenges in Problem 8.

2 Monitor

Note: Refer to the previous page for guidance.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who want to explore percent error further to complete this optional task. Encourage them to discuss their thinking with a partner.

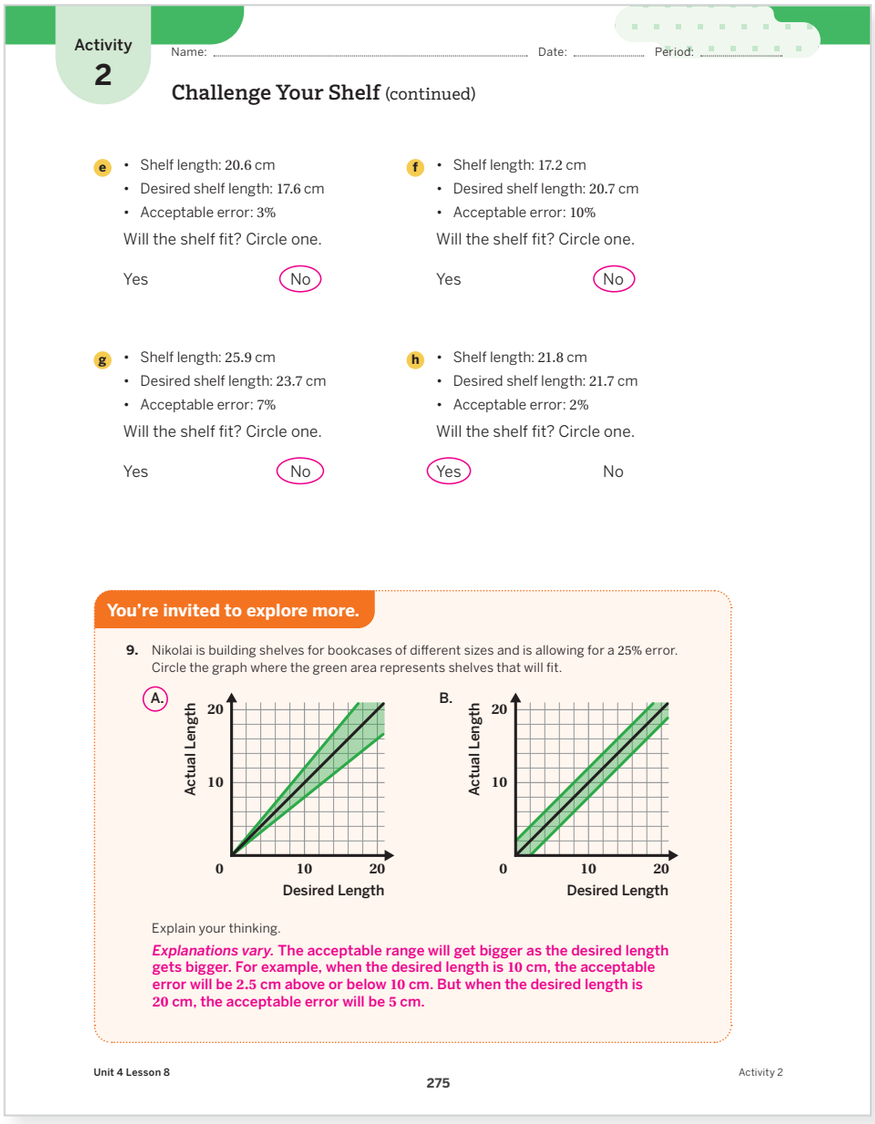
3 Connect

Invite students to share strategies they found helpful to reach their streak goal of solving multiple challenges. **(MTR.3.1)**

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

To surface the Key Takeaway, consider asking:

- "Were there any mathematical representations that helped you to solve for whether a shelf would fit or not for multiple challenges?"
- "How did you adjust the representation for each new shelf scenario?"
- "What can we add to our definition of *percent error*?"



Activity 2

Name: Date: Period:

Challenge Your Shelf (continued)

e
- Shelf length: 20.6 cm
- Desired shelf length: 17.6 cm
- Acceptable error: 3%

Will the shelf fit? Circle one.

Yes (No)

f
- Shelf length: 17.2 cm
- Desired shelf length: 20.7 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

Yes (No)

g
- Shelf length: 25.9 cm
- Desired shelf length: 23.7 cm
- Acceptable error: 7%

Will the shelf fit? Circle one.

Yes (No)

h
- Shelf length: 21.8 cm
- Desired shelf length: 21.7 cm
- Acceptable error: 2%

Will the shelf fit? Circle one.

(Yes) No

You're invited to explore more.

9. Nikolai is building shelves for bookcases of different sizes and is allowing for a 25% error. Circle the graph where the green area represents shelves that will fit.

(A.)

B.

Explain your thinking.

Explanations vary. The acceptable range will get bigger as the desired length gets bigger. For example, when the desired length is 10 cm, the acceptable error will be 2.5 cm above or below 10 cm. But when the desired length is 20 cm, the acceptable error will be 5 cm.

Unit 4 Lesson 8 275 Activity 2

Key Takeaway: Knowing the desired measurement for a situation and the **percent error** helps you calculate the highest and lowest acceptable values. For example, if the desired value is 30 and the percent error is 5%, you can calculate 5% of 30, then add that amount to or subtract it from 30 to find other values that will work.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding by explaining how to find percent error for values in context.

Invite students to respond independently, and then share their thinking with a partner.

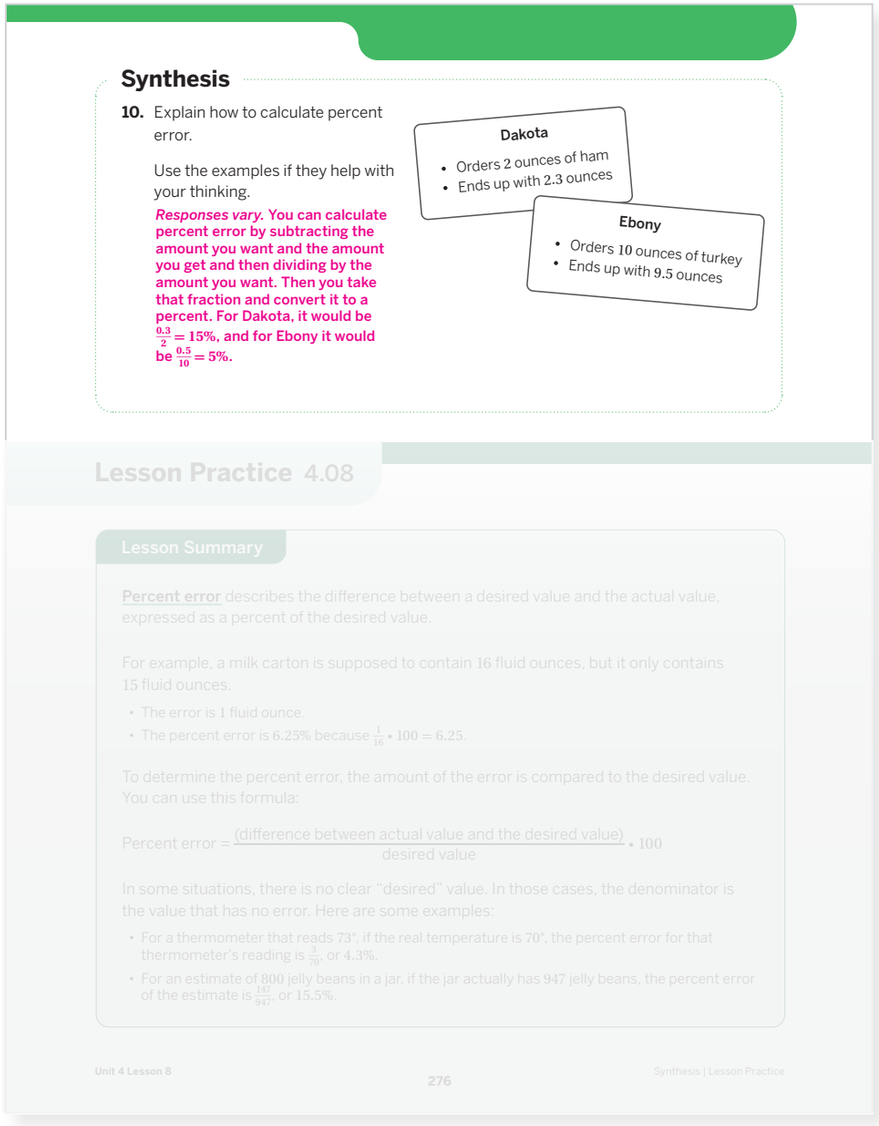
Collect and share a variety of student ideas, such as:

- Explaining calculating percent error using the relationship between actual value and desired value.
- Determining the percent error for Dakota's and Ebony's deli orders.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *percent error* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, consider asking students, "How can percent error be useful in situations like these deli meat orders?" *Responses vary.* Percent error allows us to compare situations with different amounts.

Note: If time allows, invite students to come up with a new situation where percent error might be helpful.



Synthesis

10. Explain how to calculate percent error.

Use the examples if they help with your thinking.

Responses vary. **You can calculate percent error by subtracting the amount you want and the amount you get and then dividing by the amount you want. Then you take that fraction and convert it to a percent. For Dakota, it would be $\frac{0.3}{2} = 15\%$, and for Ebony it would be $\frac{0.5}{10} = 5\%$.**

Dakota
- Orders 2 ounces of ham
- Ends up with 2.3 ounces

Ebony
- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

Lesson Practice 4.08

Lesson Summary

Percent error describes the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a milk carton is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

- The error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

To determine the percent error, the amount of the error is compared to the desired value. You can use this formula:

$$\text{Percent error} = \frac{(\text{difference between actual value and the desired value})}{\text{desired value}} \cdot 100$$

In some situations, there is no clear "desired" value. In those cases, the denominator is the value that has no error. Here are some examples:

- For a thermometer that reads 73°, if the real temperature is 70°, the percent error for that thermometer's reading is $\frac{3}{70}$, or 4.3%.
- For an estimate of 800 jelly beans in a jar, if the jar actually has 947 jelly beans, the percent error of the estimate is $\frac{147}{947}$, or 15.5%.

Unit 4 Lesson 8 276 Synthesis | Lesson Practice

Lesson Takeaway: Percent error is one way to describe how far above or below a desired measurement a value is. To compute percent error, find the difference between the desired value and actual value divided by the desired value (for example, $\frac{0.3}{2} = 15\%$ computes the percent error of 2.3 ounces of ham when 2 ounces of ham is desired).

Independent | 5 min

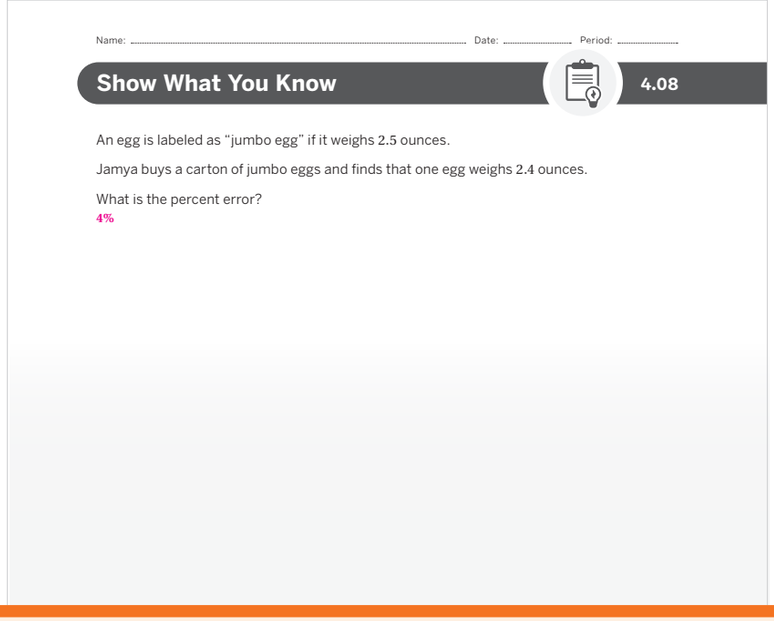
Show What You Know

Purpose: Students demonstrate their understanding by determining the percent error for a new context.

Today's Goals

1. **Goal:** Know that *percent error* expresses error as a percentage of a desired value.
2. **Goal:** Decide whether a value is within an acceptable *percent error*.
3. **Language Goal:** Explain a strategy for solving a problem involving *percent error*. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.08

An egg is labeled as "jumbo egg" if it weighs 2.5 ounces.

Jamya buys a carton of jumbo eggs and finds that one egg weighs 2.4 ounces.

What is the percent error?

4%

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine whether a value is within an acceptable percent error:

» **Respond to Student Thinking:**

- Revisit the **Synthesis** and invite students to share strategies for calculating percent error in each example.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on students' progress with percent increase, percent decrease, and percent error in Sub-Units 1 and 2. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Explain how to calculate percent error.

Use the examples if they help with your thinking.

Responses vary. You can calculate percent error by subtracting the amount you want and the amount you get and then dividing by the amount you want. Then you take that fraction and convert it to a percent. For Dakota, it would be $\frac{0.3}{2} = 15\%$, and for Ebony it would be $\frac{0.5}{10} = 5\%$.

Dakota
- Orders 2 ounces of ham
- Ends up with 2.3 ounces

Ebony
- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

Lesson Practice 4.08

Lesson Summary

Percent error describes the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a milk carton is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

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To determine the percent error, the amount of the error is compared to the desired value. You can use this formula:

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In some situations, there is no clear "desired" value. In those cases, the denominator is the value that has no error. Here are some examples:

- For a thermometer that reads 73°, if the real temperature is 70°, the percent error for that thermometer's reading is $\frac{3}{70}$, or 4.3%.
- For an estimate of 800 jelly beans in a jar, if the jar actually has 947 jelly beans, the percent error of the estimate is $\frac{147}{947}$, or 15.5%.

Lesson Practice 4.08

Name: _____ Date: _____ Period: _____

1. A baker needs 500 grams of flour for a batter, but accidentally only puts 300 grams of flour into the mixing bowl. What is the percent error?

40%

2. A speed detection device measured the speed of a baseball at 93 miles per hour. If the baseball was actually going 90 miles per hour, what was the percent error in this measurement?

About 3.3%

Problems 3–4: In Burlington, all city departments get an annual budget. They are allowed to spend 1.5% over their budget. If they spend more than that, the mayor conducts a review.

3. The Department of Health has a budget of $90,000. What is the largest acceptable amount for their spending? Show or explain your thinking.

$91,350. *Explanations vary.*
$90000 \cdot 0.015 = 1350$
$90000 + 1350 = 91350$

4. The Department of Parks and Recreation has a budget of $30,000, and they spent $31,000. Will the mayor conduct a review of their spending? Show or explain your thinking.

Yes. *Explanations vary.*
$31000 - 30000 = 1000$
$1000 \div 30000 = 0.0333\ldots$
$0.033 \cdot 100 = 3.3$
3.3% is more than 1.5%.

FAST Practice

5. Any can of seltzer should have 12 fluid ounces of liquid to be appropriately carbonated. One can on the assembly line contains 13.5 fluid ounces. What is the percent error?

12.5 %

Lesson Practice 4.08

Name: _____ Date: _____ Period: _____

Spiral Review

6. Hoang is buying 5 concert tickets that cost $95.00 each. He finds a coupon to receive a 10% discount. A sales tax of 8% is applied after the discount. How much did he pay after the discount and sales tax?

A. $92.34 B. $427.50 C. $461.70 D. $513.00

7. Ahmed bikes x kilometers. Kimaya bikes $\frac{3}{10}$ less than that. Using y for Kimaya's distance, write an equation that describes the relationship between the two quantities.

$y = \frac{7}{10}x$ (or equivalent)

Problems 8–11: Complete each blank using the symbols >, <, or =.

8. $-\frac{3}{4}$ < $\frac{4}{3}$

9. 3.24 = $-(-3.24)$

10. $-\frac{2}{3}$ > $-\frac{5}{6}$

11. $|-4|$ = $-|-4|$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.3.1
FAST Practice	5	2	MA.7.AR.3.1
Spiral Review			
	6	2	MA.7.AR.3.1
	7	2	MA.7.AR.4.4
	8–11	1	MA.6.NSO.1.1

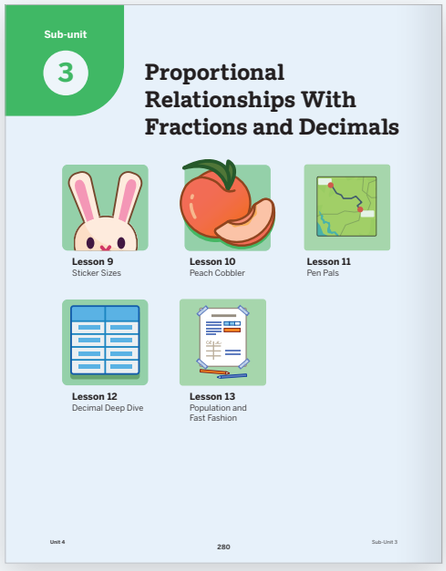
Sub-Unit 3

Proportional Relationships With Fractions and Decimals

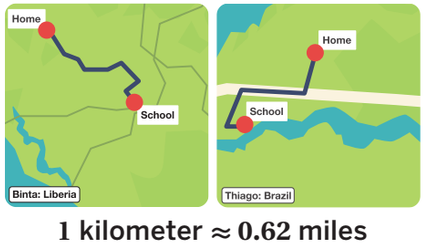
Sub-Unit 3 Goals

- **Goal:** Determine unknown values in proportional relationships involving fractional quantities.
- **Goal:** Convert between fraction and decimal representations.
- **Language Goal:** Analyze strategies for solving problems involving proportional relationships with fractional and decimal quantities.

In this sub-unit, students extend their previous knowledge of the constant of proportionality to determine unknown fractional quantities in proportional relationships. They also practice converting fractions to decimals using long division and examining patterns that result in **terminating** and **repeating decimals**.



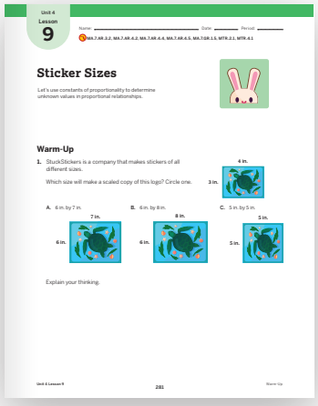
Math That Matters Most

Concept or Skill	For example . . .
Use a constant of proportionality to solve problems involving fractional quantities.	How much would $11\frac{1}{2}$ sheets cost? *It would cost \$40.25 for $11\frac{1}{2}$ sheets. I can determine the cost per sheet by dividing 14 by 4 to get 3.5 and then multiply 3.5 by $11\frac{1}{2}$ to get the total cost of 40.25.* | Number of Sheets | Total Cost (\$) | | 4 | 14 | | $11\frac{1}{2}$ | |
Convert measurements from one unit to another in different measurement systems.	Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil. Who lives closer to their school? Explain your thinking. *Binta lives 15 miles from school. Thiago lives 20 kilometers, or 12.4 miles, from his school, so Thiago lives closer.*  **1 kilometer ≈ 0.62 miles**
Convert a fraction to a decimal using long division to determine whether a fraction will be represented as a terminating or repeating decimal.	Finish writing $\frac{5}{8}$ as a decimal. *$\frac{5}{8}$ is a terminating decimal because there are a set number of digits in the decimal and it does not have a repeating pattern.* 0.625 $8)\overline{5.000}$ -0 50 -48 20 -16 40 -40 0

Unit 4

Lesson

9


Unit 4
Lesson
9
Sticker Sizes
Let's use constants of proportionality to determine unknown values in proportional relationships.
Warm-Up
1. StuckStickers is a company that makes stickers of all different sizes.
Which size will make a scaled copy of this logo? Circle one.
A. 6 in. by 7 in.
B. 6 in. by 8 in.
C. 5 in. by 5 in.
Explain your thinking.
281

Sticker Sizes

Revisiting Proportional Relationships

Let's use constants of proportionality to determine unknown values in proportional relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Determine an unknown value in a proportional relationship using a table.
2. **Language Goal:** Compare and contrast strategies for solving problems involving proportional relationships with fractional quantities, such as scale factor and constant of proportionality. **(Speaking and Listening)**

Students use tools, such as tables, constants of proportionality, and scale factors to determine unknown values in proportional relationships, some of which involve fractional quantities.

Coherence

Prior Learning

In Units 1 and 2, students learned about scaled copies and proportional relationships.

Future Learning

In Lesson 10, students will continue to develop their understanding of proportional relationships using a scale factor with fractional values.

Connections

- Students determine unknown values in proportional relationships in real-world contexts such as designing scaled sizes of stickers. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Also Addressing: **MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.GR.2.2**
MA.6.AR.3.5	**MA.8.GR.2.4**
	MA.912.GR.4.3

Lesson at a Glance

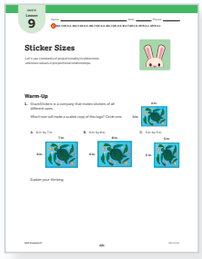
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.7.GR.1.5

Warm-Up

Pairs | 5 min

Students are introduced to the scenario of creating different-sized stickers from a logo and recall what they know about scaled copies.



Activity 1

Pairs | 20 min

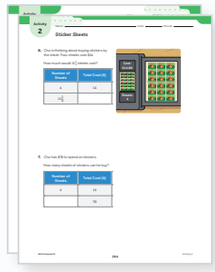
Students make connections between scale factor and constant of proportionality by discussing when each is useful for determining measurements. MLR7: Compare and Connect



Activity 2

Pairs | 10 min

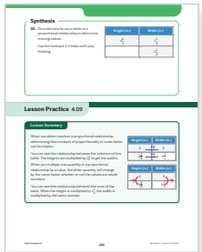
Students practice strategies for determining an unknown value in a proportional relationship that involves fractional quantities.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using tables, constants of proportionality, and scale factors to determine values in proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining a value in a proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the scenario of creating different-sized stickers from a logo and recall what they know about scaled copies.

1 Launch

To surface connections, consider asking, "Have you ever created your own logo or sticker?" Invite students to select which size would be a scaled copy and to defend their choice.

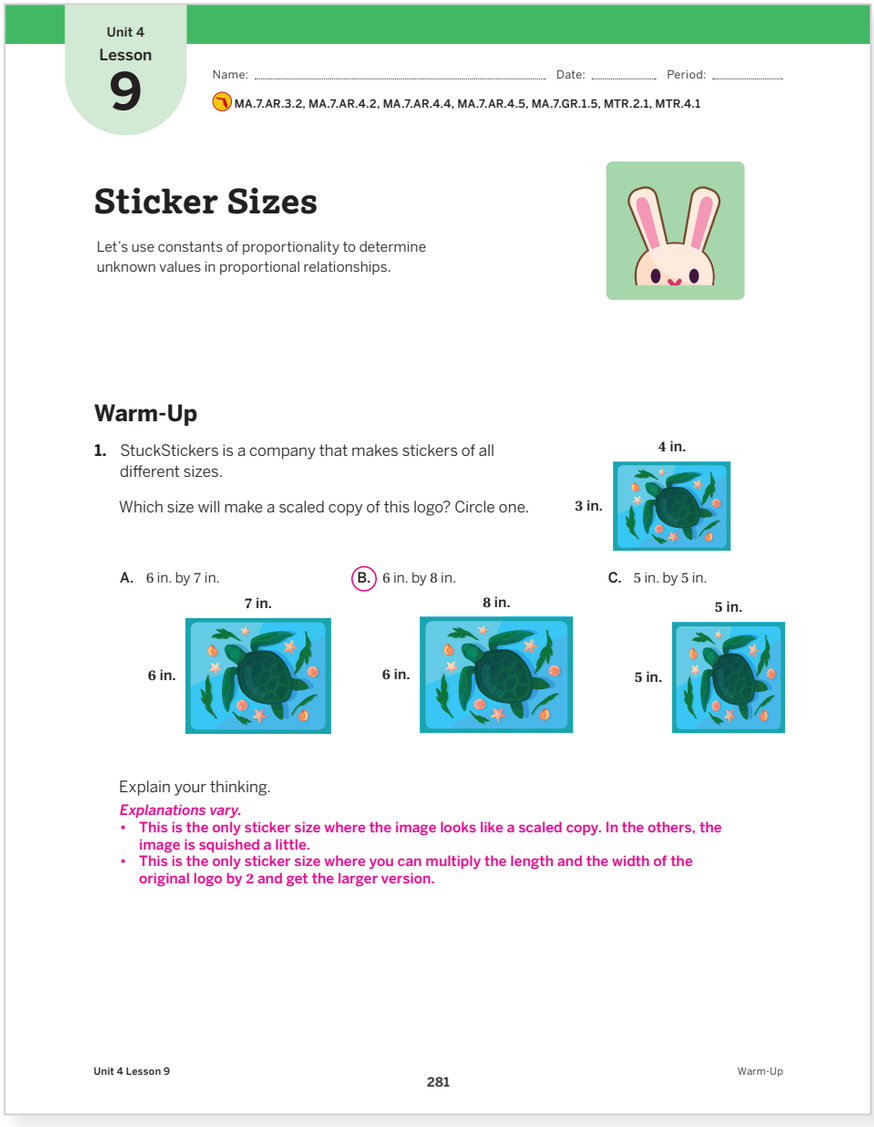
To support students getting started, consider asking, "What is the relationship between a scale copy and the original figure?"

Look for student responses that mention the properties of scaled copies, such as:
- The sides must be multiplied by the same scale factor.
- The sides need to create equivalent ratios.

2 Connect

Invite students to share their reasoning in order to resurface the properties of scaled copies for the class.

Consider asking, "Why might someone think the 6-inch by 7-inch sticker is a scaled copy of the original?"



Unit 4
Lesson
9

Name: _____ Date: _____ Period: _____

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.7.GR.1.5, MTR.2.1, MTR.4.1

Sticker Sizes

Let's use constants of proportionality to determine unknown values in proportional relationships.

Warm-Up

1. StuckStickers is a company that makes stickers of all different sizes.

   Which size will make a scaled copy of this logo? Circle one.

A. 6 in. by 7 in.

B. 6 in. by 8 in.

C. 5 in. by 5 in.

Explain your thinking.

Explanations vary.
- This is the only sticker size where the image looks like a scaled copy. In the others, the image is squished a little.
- This is the only sticker size where you can multiply the length and the width of the original logo by 2 and get the larger version.

Unit 4 Lesson 9 281 Warm-Up

Pairs | 20 min

Activity 1 Scaling Stickers

Purpose: Students make connections between scale factor and constant of proportionality by discussing when each is useful for determining measurements.

Short on time? Consider omitting Problem 3 and discussing the strategies for finding the width in Problem 4.

1 Launch

To surface connections, consider asking, "What is the relationship between the length and width of the original sticker?"

A Accessibility: Memory and Attention Invite students to use a calculator to support them with performing calculations.

2 Monitor

Encourage students to read others' responses or to discuss their response with a partner. Consider asking, "How are others' strategies similar to or different from yours?" **(MTR.4.1)**

Look for different approaches or tools that students use to determine the unknown values.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Can you draw a picture to show what we know and what we are trying to find?"
Use guess and check to find the right width.	**Strengthen** Encourage them to read the strategies in Problem 4 and then revisit their work on this problem.
Create tables, double number lines, drawings, or use a calculator to support their thinking. **(MTR.2.1)**	**Strengthen** Invite these students to share their strategy during the discussion of Problem 4. Consider having them compare how their strategy is similar to or different from the two shown.

Activity 1 continued >

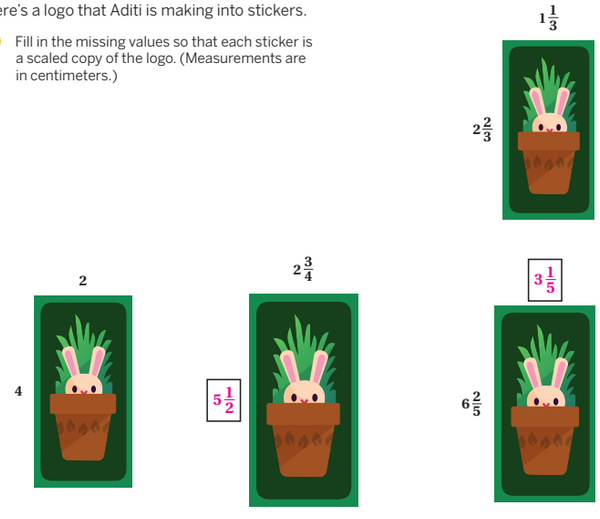
Activity 1

Name: Date: Period:

Scaling Stickers

2. Here's a logo that Aditi is making into stickers.

a. Fill in the missing values so that each sticker is a scaled copy of the logo. (Measurements are in centimeters.)



b. Describe your strategy.

Explanations vary. I noticed that the height is 2 times as long as the width. So to determine the height, I multiplied the width by 2.

3. Hamza wants to use this design to create a bumper sticker.

If the bumper sticker has a height of $3\frac{1}{2}$ inches, what will the width of this bumper sticker be?

$8\frac{3}{4}$ inches



Unit 4 Lesson 9 282 Activity 1

Pairs | 20 min

Activity 1 Scaling Stickers (continued)

Purpose: Students make connections between scale factor and constant of proportionality by discussing when each is useful for determining measurements.

Short on time? Consider omitting Problem 3 and discussing the strategies for finding the width in Problem 4.

2 Monitor

Invite students to think for 1–2 minutes about each strategy before facilitating a whole class discussion.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy for finding the width of Hazma's bumper sticker. **(MTR.4.1)**

Consider asking:
- "In what situations might Polina's strategy be more useful? What about Jamir's strategy?"
- "Where do you see the constant of proportionality in each strategy (including your own)?"

Look for student strategies or representations to share during the Connect.

D **Differentiation**

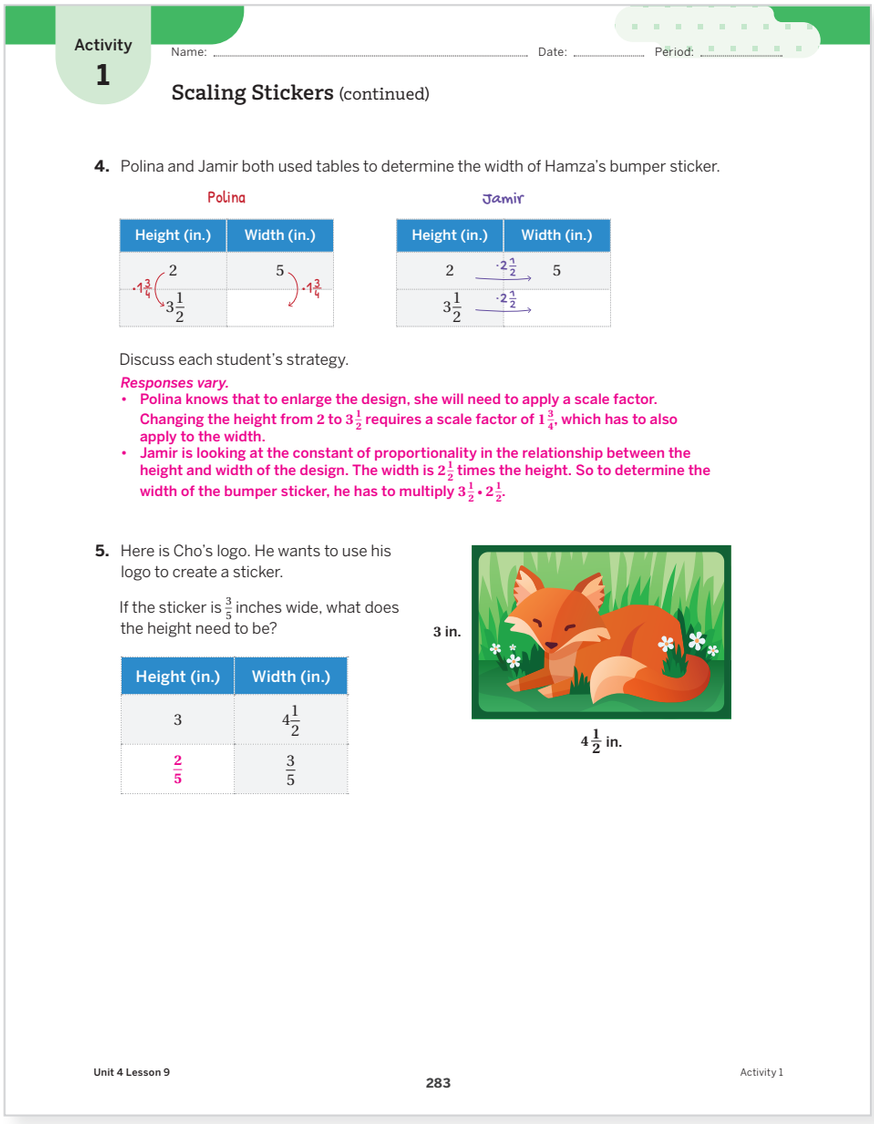
Look for students who:	Respond to Student Thinking
Reverse the width and the height.	**Support** Consider asking, "How does the table help you determine which side of the logo corresponds to the width?"
Create tables, double number lines, drawings, or use a calculator to support their thinking. **(MTR.2.1)**	**Strengthen** Invite these students to share their strategy during the Connect.
Finish early.	**Stretch** Encourage students to use a different tool and to determine which tool is more effective for this problem and why.

Note: If it does not come up naturally, consider displaying a strategy, such as a double number line, table, drawings, or using a calculator to determine the height of the sticker. **(MTR.2.1)**

3 Connect

Invite students to share their strategies with the class, encouraging them to make connections between their representations and other strategies shared.

To surface the Key Takeaway, consider asking:
- "Which of these strategies or representations do you want to remember the next time you have to determine measurements in scaled copies?"
- "Why would you choose one of these representations over another?"



Activity 1

Name: Date: Period:

Scaling Stickers (continued)

4. Polina and Jamir both used tables to determine the width of Hamza's bumper sticker.

Polina

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	

Jamir

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	

Discuss each student's strategy.

Responses vary.
- Polina knows that to enlarge the design, she will need to apply a scale factor. Changing the height from 2 to $3\frac{1}{2}$ requires a scale factor of $1\frac{3}{4}$, which has to also apply to the width.
- Jamir is looking at the constant of proportionality in the relationship between the height and width of the design. The width is $2\frac{1}{2}$ times the height. So to determine the width of the bumper sticker, he has to multiply $3\frac{1}{2} \cdot 2\frac{1}{2}$.

5. Here is Cho's logo. He wants to use his logo to create a sticker.

If the sticker is $\frac{3}{5}$ inches wide, what does the height need to be?

Height (in.)	Width (in.)
3	$4\frac{1}{2}$
$\frac{2}{5}$	$\frac{3}{5}$

3 in.

$4\frac{1}{2}$ in.

Unit 4 Lesson 9 283 Activity 1

Key Takeaway: Tables, tape diagrams, drawings, and calculators are some of the tools and representations that can help us to make sense of proportional relationships. These tools can be particularly helpful when attending to precision in relationships with fractional values.

Pairs | 10 min

Activity 2 Sticker Sheets

Purpose: Students practice strategies for determining an unknown value in a proportional relationship that involves fractional quantities.

1 Launch

Consider asking, "How is the printer figuring out how many sheets to print?"

ELL **English Language Learners** Invite students to read the situation aloud and paraphrase what it means. If needed, clarify that students are finding the price per sheet, not the price per sticker. **(Reading)**

2 Monitor

Invite students to use paper to show their thinking.

Look for a variety of student strategies, such as tables, tape diagrams, or double number lines.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to estimate the solution and confirm with a partner what they are solving for in this problem.
Determine a unit rate for 1 sheet of paper.	**Strengthen** Invite them to use a representation to show how they determined their unit rate and to share during the discussion.
Input their responses as expressions, such as $14 \cdot \left(\frac{11.5}{4}\right)$.	**Strengthen** Invite them to share their expressions during the discussion. Consider asking, "What would change about your expression if the number of sheets changed? Why is your expression a helpful representation?"

Pause briefly to have students share strategies for how they are determining the number of sheets or the cost. Consider asking, "How do these strategies relate to the ones we explored in Activity 1?"

Activity 2 continued >

Activity 2

Name: Date: Period:

Sticker Sheets

6. Cho is thinking about buying stickers by the sheet. Four sheets cost $14.

How much would $11\frac{1}{2}$ sheets cost?

Number of Sheets	Total Cost ($)
4	14
$11\frac{1}{2}$	40.25



7. Cho has $70 to spend on stickers.

How many sheets of stickers can he buy?

Number of Sheets	Total Cost ($)
4	14
20	70

Unit 4 Lesson 9 284 Activity 2

Pairs | 10 min

Activity 2 Sticker Sheets (continued)

Purpose: Students practice strategies for determining an unknown value in a proportional relationship that involves fractional quantities.

2 Monitor

Invite students to respond individually to the question for 1–2 minutes before sharing their thinking with a partner.

To support students getting started, invite them to test values in the equations and see which ones work.

Look for students who only select one option. Consider asking:

- "Does your equation relate to the rows or columns of the table?"
- "How does your equation relate to previous strategies we have talked about?"
- "What is another way we can relate the two quantities in the table?"

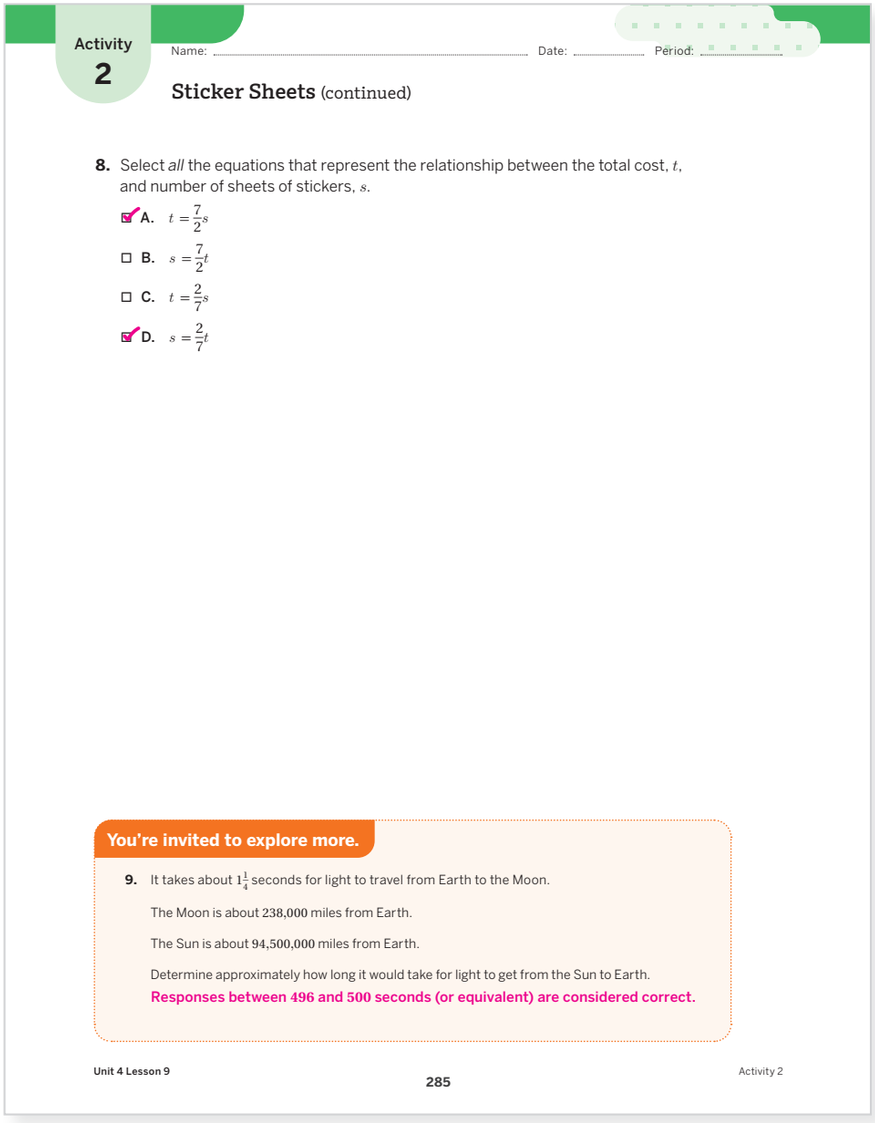
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore scaling by fractional values further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share why they think different equations represent the total cost for a specific number of sheets.

Consider asking:

- "Why is there more than one equation that represents this situation?"
- "What is the relationship between the constant of proportionality in the two equations?"
- "Does it matter which equation we choose to use?"
- "Why might an equation be helpful in this situation?"



Activity 2

Name: Date: Period:

Sticker Sheets (continued)

8. Select *all* the equations that represent the relationship between the total cost, t, and number of sheets of stickers, s.

- ☑ A. $t = \frac{7}{2}s$
- ☐ B. $s = \frac{7}{2}t$
- ☐ C. $t = \frac{2}{7}s$
- ☑ D. $s = \frac{2}{7}t$

You're invited to explore more.

9. It takes about $1\frac{1}{4}$ seconds for light to travel from Earth to the Moon.

The Moon is about 238,000 miles from Earth.

The Sun is about 94,500,000 miles from Earth.

Determine approximately how long it would take for light to get from the Sun to Earth.

Responses between 496 and 500 seconds (or equivalent) are considered correct.

Unit 4 Lesson 9 285 Activity 2

Whole Class | 5 min

Synthesis

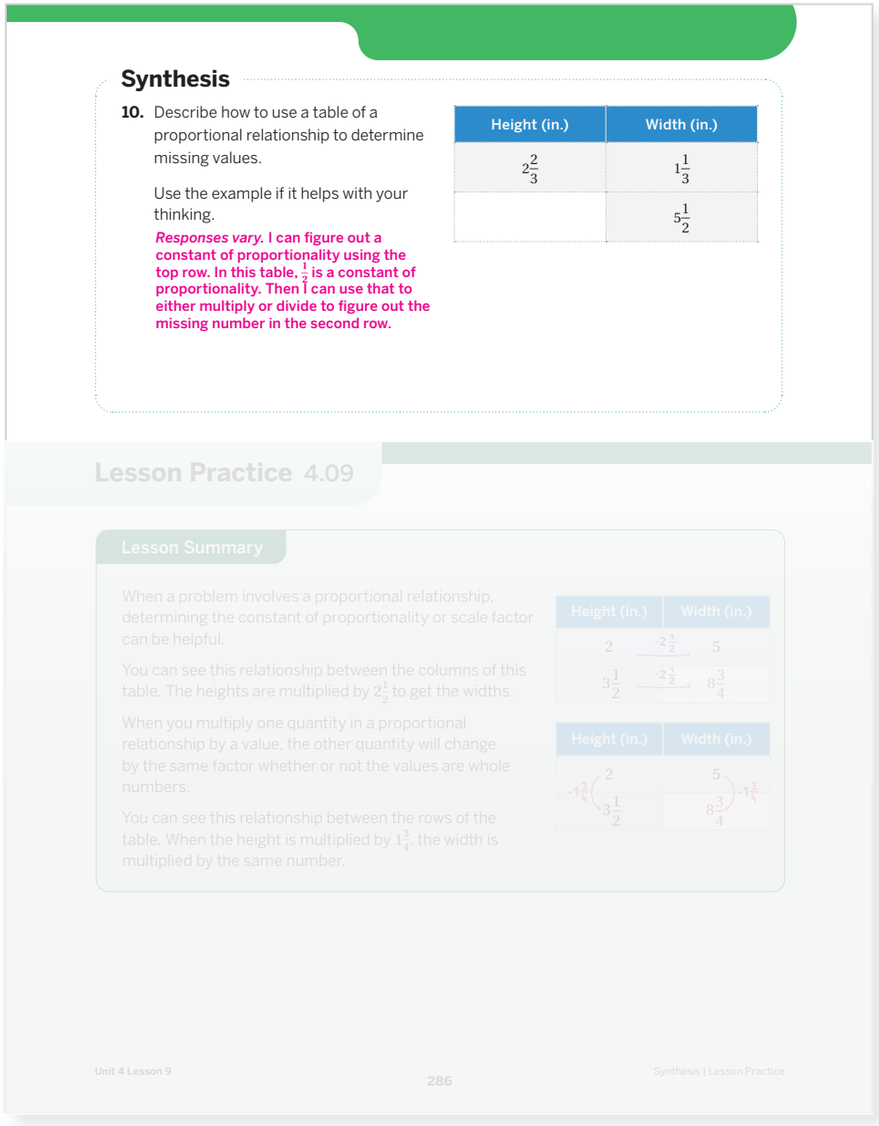
Purpose: Students synthesize their understanding of using tables, constants of proportionality, and scale factors to determine values in proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The rows and columns of the table help to determine the constant of proportionality.
- The height can be divided by the width to determine that the constant of proportionality in this table is $\frac{1}{2}$, and the width can be multiplied by 2 to get the height in the second row.
- The width in the second row can be divided by the width in the first row to determine a constant of proportionality of $\frac{33}{8}$, and $2\frac{2}{3}$ can be multiplied by the constant to get $11\frac{1}{2}$.
- Depending on the situation, it is sometimes easier to see the constant of proportionality between the rows or columns of a table.

Consider asking, "What tools or representations do you want to remember to use when working with proportional relationships with fractional values in the future?"



Synthesis

10. Describe how to use a table of a proportional relationship to determine missing values.

Use the example if it helps with your thinking.

Height (in.)	Width (in.)
$2\frac{2}{3}$	$1\frac{1}{3}$
	$5\frac{1}{2}$

Responses vary. I can figure out a constant of proportionality using the top row. In this table, $\frac{1}{2}$ is a constant of proportionality. Then I can use that to either multiply or divide to figure out the missing number in the second row.

Lesson Practice 4.09

Lesson Summary

When a problem involves a proportional relationship, determining the constant of proportionality or scale factor can be helpful.

You can see this relationship between the columns of this table. The heights are multiplied by $2\frac{1}{2}$ to get the widths.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor whether or not the values are whole numbers.

You can see this relationship between the rows of the table. When the height is multiplied by $1\frac{3}{4}$, the width is multiplied by the same number.

Unit 4 Lesson 9 286 Synthesis | Lesson Practice

Lesson Takeaway: Using a table, tape diagram, double number line, or a calculator can be helpful to determine unknown values in proportional relationships with fractional values.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining a value in a proportional relationship.

Today's Goals

1. **Goal:** Determine an unknown value in a proportional relationship using a table.
2. **Language Goal:** Compare and contrast strategies for solving problems involving proportional relationships with fractional quantities, such as scale factor and constant of proportionality. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.09

It costs \$3.75 to buy $\frac{3}{4}$ of a pound of walnuts.

How many pounds of walnuts can you buy with \$11.25?

Walnuts (lb)	Cost (\$)
$\frac{3}{4}$	3.75
$2\frac{1}{4}$ (or equivalent)	11.25

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine unknown fractional values in a table:

» **Respond to Student Thinking:**

- Students will have more opportunities to explore this in Lesson 10. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

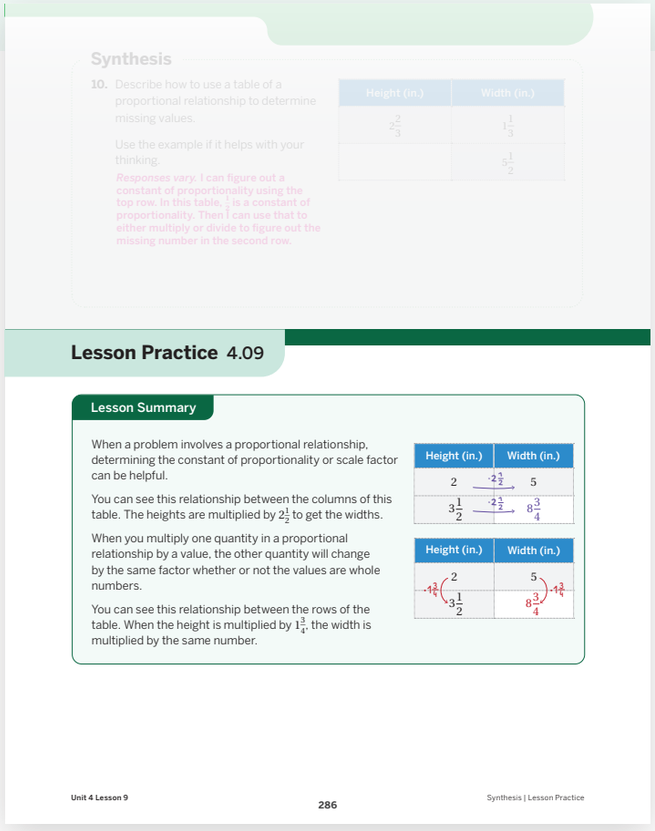
Professional Learning

Reflect on students' progress with proportional relationships in Grade 7 so far. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. Describe how to use a table of a proportional relationship to determine missing values.

Use the example if it helps with your thinking.

Responses vary. I can figure out a constant of proportionality using the top row. In this table, $\frac{1}{3}$ is a constant of proportionality. Then I can use that to either multiply or divide to figure out the missing number in the second row.

Height (in.)	Width (in.)
$2\frac{2}{3}$	$1\frac{1}{3}$
	$5\frac{1}{2}$

Lesson Practice 4.09

Lesson Summary

When a problem involves a proportional relationship, determining the constant of proportionality or scale factor can be helpful.

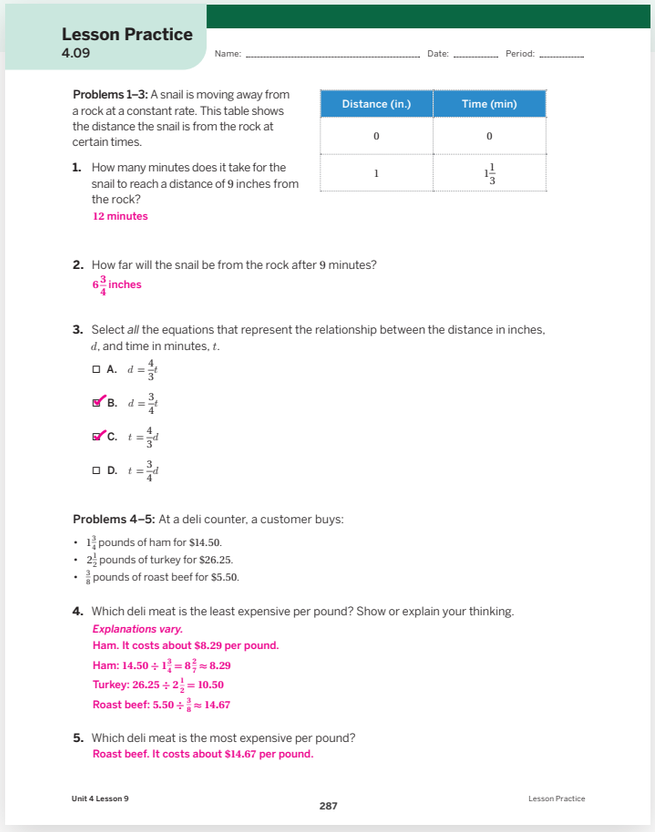
You can see this relationship between the columns of this table. The heights are multiplied by $2\frac{1}{2}$ to get the widths.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor whether or not the values are whole numbers.

You can see this relationship between the rows of the table. When the height is multiplied by $1\frac{3}{4}$, the width is multiplied by the same number.

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	$8\frac{3}{4}$

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	$8\frac{3}{4}$



Lesson Practice
4.09

Name: _____ Date: _____ Period: _____

Problems 1–3: A snail is moving away from a rock at a constant rate. This table shows the distance the snail is from the rock at certain times.

Distance (in.)	Time (min)
0	0
1	$1\frac{1}{3}$

1. How many minutes does it take for the snail to reach a distance of 9 inches from the rock?
   12 minutes

2. How far will the snail be from the rock after 9 minutes?
   $6\frac{3}{4}$ inches

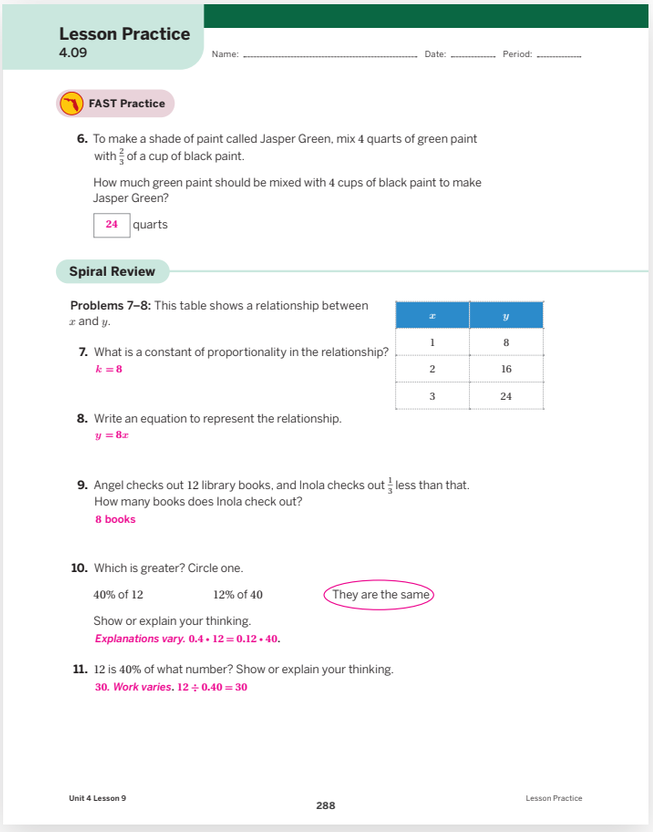
3. Select *all* the equations that represent the relationship between the distance in inches, d, and time in minutes, t.
   - ☐ A. $d = \frac{4}{3}t$
   - ☑ B. $d = \frac{3}{4}t$
   - ☑ C. $t = \frac{4}{3}d$
   - ☐ D. $t = \frac{3}{4}d$

Problems 4–5: At a deli counter, a customer buys:

- $1\frac{3}{4}$ pounds of ham for $14.50.
- $2\frac{1}{2}$ pounds of turkey for $26.25.
- $\frac{3}{8}$ pounds of roast beef for $5.50.

4. Which deli meat is the least expensive per pound? Show or explain your thinking.
   Explanations vary.
   Ham. It costs about $8.29 per pound.
   Ham: $14.50 \div 1\frac{3}{4} = 8\frac{2}{7} \approx 8.29$
   Turkey: $26.25 \div 2\frac{1}{2} = 10.50$
   Roast beef: $5.50 \div \frac{3}{8} \approx 14.67$

5. Which deli meat is the most expensive per pound?
   Roast beef. It costs about $14.67 per pound.



Lesson Practice
4.09

Name: _____ Date: _____ Period: _____

FAST Practice

6. To make a shade of paint called Jasper Green, mix 4 quarts of green paint with $\frac{2}{3}$ of a cup of black paint.

   How much green paint should be mixed with 4 cups of black paint to make Jasper Green?

   [24] quarts

Spiral Review

Problems 7–8: This table shows a relationship between x and y.

x	y
1	8
2	16
3	24

7. What is a constant of proportionality in the relationship?
   $k = 8$

8. Write an equation to represent the relationship.
   $y = 8x$

9. Angel checks out 12 library books, and Inola checks out $\frac{1}{3}$ less than that. How many books does Inola check out?
   8 books

10. Which is greater? Circle one.

    40% of 12 12% of 40 (They are the same)

    Show or explain your thinking.
    Explanations vary. **$0.4 \cdot 12 = 0.12 \cdot 40$.**

11. 12 is 40% of what number? Show or explain your thinking.
    30. *Work varies.* $12 \div 0.40 = 30$

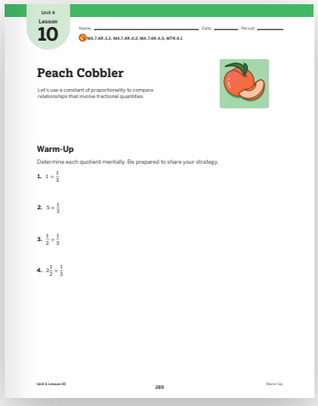
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.7.AR.4.5
	3	2	MA.7.AR.4.4
	4–5	2	MA.7.AR.3.2
FAST Practice	6	1	MA.7.AR.3.2
Spiral Review			
	7–8	2	MA.7.AR.4.2 MA.7.AR.4.4
	9	2	MA.4.FR.2.4
	10–11	2	MA.6.AR.3.4

Unit 4

Lesson

10



Peach Cobbler

Ratios and Rates With Fractions

Let's use a constant of proportionality to compare relationships that involve fractional quantities.

Learning Outcomes

Today's Goals

1. **Goal:** Use a constant of proportionality to solve problems involving fractional quantities.
2. **Language Goal:** Describe strategies for solving problems involving proportional relationships with fractional quantities. **(Writing)**

Students compare proportional relationships involving fractional quantities. They use constants of proportionality to make an argument to decide which recipe is the least sweet and scale a recipe to serve a different number of people. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 9, students used constants of proportionality to create scaled copies of stickers involving fractional values.

Future Learning

In Grade 8, students will continue to explore proportional relationships when exploring relationships among similar figures and extending their understanding to linear equations.

Connections

- Students apply constants of proportionality to make an argument to decide which recipe is the least sweet and scale a recipe to serve a different number of people. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

Also Addressing: **MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.1**
MA.6.AR.3.3	**MA.8.AR.3.2**
MA.6.AR.3.5	

Lesson at a Glance

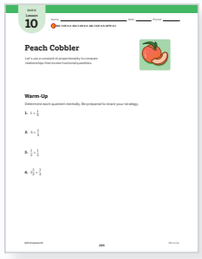
~ 45 min

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.5

Warm-Up

Independent | 10 min

Students discuss strategies for dividing whole numbers and fractions into fractional parts.
Routine: Number Talk



Activity 1

Pairs | 15 min

Students apply prior knowledge about constants of proportionality to make an argument about recipes involving fractional quantities. **(MTR.4.1)**



Activity 2

Pairs | 10 min

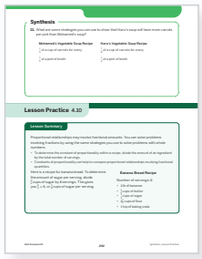
Students use a constant of proportionality to adjust a recipe involving fractional quantities.



Synthesis

Whole Class | 5 min

Students synthesize strategies for comparing two quantities that have fractional values.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by comparing two proportional relationships with fractional unit rates.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- coloring tools (optional)

Independent | 10 min

Warm-Up

Purpose: Students discuss strategies for dividing whole numbers and fractions into fractional parts.

1 Launch

Direct students' attention to each problem one at a time.

Use the Number Talk routine to surface prior knowledge about dividing fractions.

Invite students to determine the first quotient independently and then share their strategies aloud. Encourage them to consider how they can communicate their ideas in ways others can understand, and share that everyone is responsible for trying to understand each others' ideas.

Create a record of each strategy that students share, along with the name of the student who shared it.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

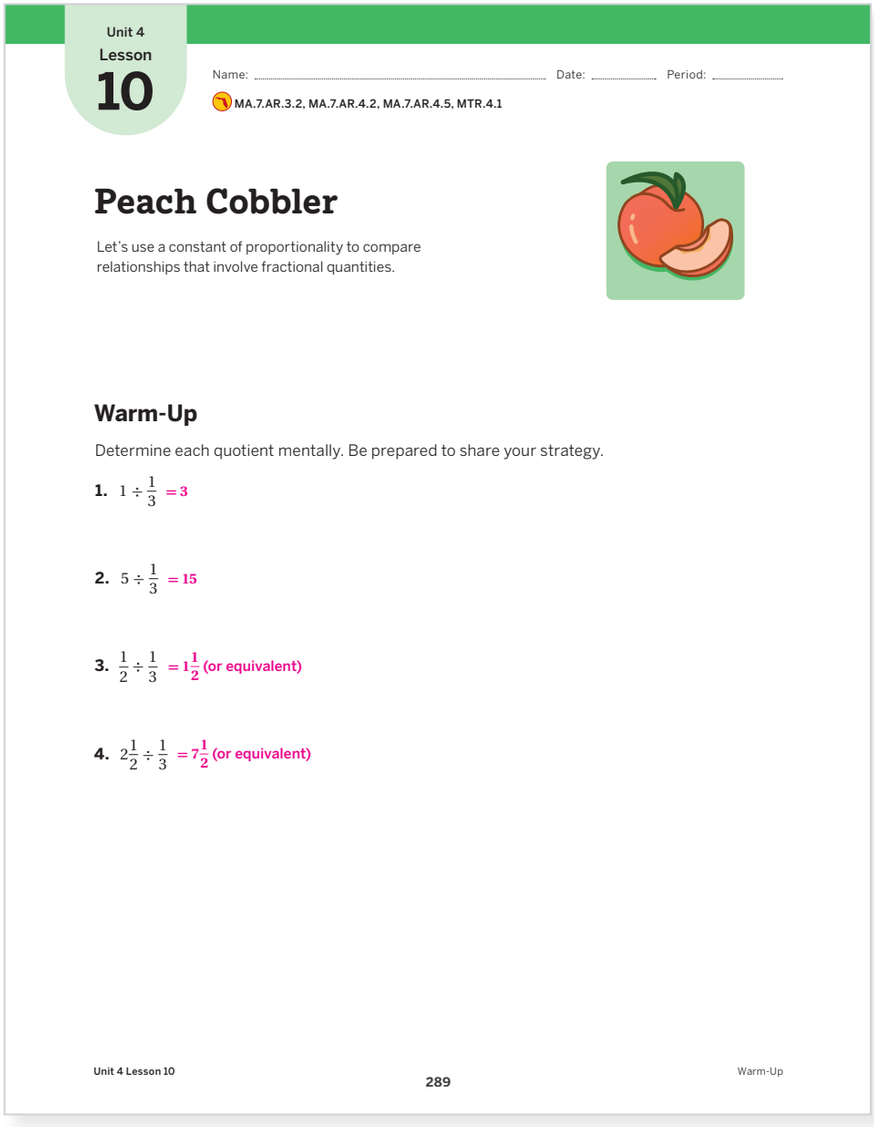
Consider asking:
- "Who thought about it in a different way?"
- "Can someone explain ___'s strategy in their own words?" **(MTR.4.1)**

Note: Repeat this structure for each quotient in the Number Talk.

2 Connect

Invite students to compare and contrast the different strategies used to determine the quotient.

Consider asking:
- "What connections do you notice among the different strategies for dividing by $\frac{1}{3}$?"
- "How is dividing 5 by $\frac{1}{3}$ similar to dividing $2\frac{1}{2}$ by $\frac{1}{3}$?"
- "What representations could be helpful to determine these quotients?"



Unit 4
Lesson
10

Name: ..... Date: ..... Period: .....

MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.5, MTR.4.1

Peach Cobbler

Let's use a constant of proportionality to compare relationships that involve fractional quantities.

Warm-Up

Determine each quotient mentally. Be prepared to share your strategy.

1. $1 \div \frac{1}{3} = 3$

2. $5 \div \frac{1}{3} = 15$

3. $\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2}$ (or equivalent)

4. $2\frac{1}{2} \div \frac{1}{3} = 7\frac{1}{2}$ (or equivalent)

Unit 4 Lesson 10 289 Warm-Up

Pairs | 15 min

Activity 1 Which Recipe?

Purpose: Students apply prior knowledge about constants of proportionality to make an argument about recipes involving fractional quantities. **(MTR.4.1)**

1 Launch

Direct students' attention to the recipe cards. To surface connections, consider asking, "Who has used measuring cups or a scale when baking something?"

Discuss what it means to be the "least sweet". Have students discuss with their partner how they will convince someone that a recipe is the least sweet.

Note: If necessary, have a discussion about why a recipe represents a proportional relationship.

A **Accessibility: Visual-Spatial Processing** Invite students to use colors or highlights to support making connections between the quantities in each recipe.

2 Monitor

Look for strategies or representations for comparing the recipes to share during the Connect.

D **Differentiation** *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is difficult about comparing the three recipes right now? How can you make the comparisons easier?"
Compare the amount of sugar in each recipe, without considering the number of servings.	**Support** Consider asking, "How could you tell how much sugar is in each serving of cobbler?"
Convert each recipe to have the same number of servings. (*Problem 6*)	**Strengthen** Celebrate their thinking. Consider asking, "How could you show the amount of sugar per serving?"
Determine the amount of sugar in a serving for each recipe. (*Problems 6–7*)	**Strengthen** Celebrate their thinking. Consider asking, "How can these values be used to craft a convincing argument?" **(MTR.4.1)**

3 Connect

Invite students to share strategies they used to determine which recipe was the least sweet. Consider recording strategies for determining the constant of proportionality for students to reference later.

To surface the Key Takeaway, consider asking, "How does the amount of sugar per serving represent a constant of proportionality? Why is that helpful?"

Activity 1

Name: Date: Period:

Which Recipe?

Amara is making peach cobbler. She has three recipes and is deciding which one to make.

Recipe A	Recipe B	Recipe C
Number of servings: 9	**Number of servings: 12**	**Number of servings: $4\frac{1}{2}$**
• $2\frac{7}{10}$ lb of peaches	• 4 lb of peaches	• $1\frac{4}{5}$ lb of peaches
• $\frac{1}{2}$ cups of butter	• $\frac{3}{4}$ cups of butter	• $\frac{1}{4}$ cups of butter
• 1 cup of flour	• $\frac{3}{4}$ cups of flour	• $\frac{2}{3}$ cups of flour
• $1\frac{1}{8}$ cups of sugar	• $1\frac{1}{3}$ cups of sugar	• $\frac{3}{4}$ cups of sugar
• $\frac{1}{2}$ tsp of lemon juice	• $\frac{1}{2}$ tsp of lemon juice	• 1 tsp of lemon juice

Amara wants to make a recipe that isn't too sweet.

5. She thinks Recipe C will be the least sweet because it has the least amount of sugar. Do you agree? Explain your thinking.

No. *Explanations vary.* Recipe A has $\frac{1}{8}$ cups of sugar per serving. Recipe B has $\frac{1}{9}$ cups of sugar per serving, and Recipe C has $\frac{1}{9}$ cups of sugar per serving. Recipe C has the least total amount of sugar, but it has the most sugar per serving. This means it will be the sweetest tasting recipe.

6. Which recipe should she make? Explain your thinking.

Recipe B. *Explanations vary.* Recipe B has the least amount of sugar per serving.

7. Is the relationship between number of servings and total amount of sugar a proportional relationship? Explain your thinking.

Yes. *Explanations vary.* The constant of proportionality between the amount of sugar and number of servings stays the same. For example, if the number of servings is doubled, the amount of sugar will need to be doubled.

Unit 4 Lesson 10 290 Activity 1

Key Takeaway: Determining the constant of proportionality for each ingredient, or the amount of that ingredient per serving, helps you compare recipes of different sizes.

Pairs | 10 min

Activity 2 Adjusting a Recipe

Purpose: Students use a constant of proportionality to adjust a recipe involving fractional quantities.

Short on time? Consider omitting Problem 10.

1 Launch

Direct students' attention to the recipe cards. To surface connections, consider asking, "Have you ever had to adjust, double, or halve a recipe before? Why might someone want to increase or decrease the number of servings?"

2 Monitor

Look for strategies for scaling the recipe down to one serving and then determining other serving sizes.

D Differentiation *(Problems 8–9)*

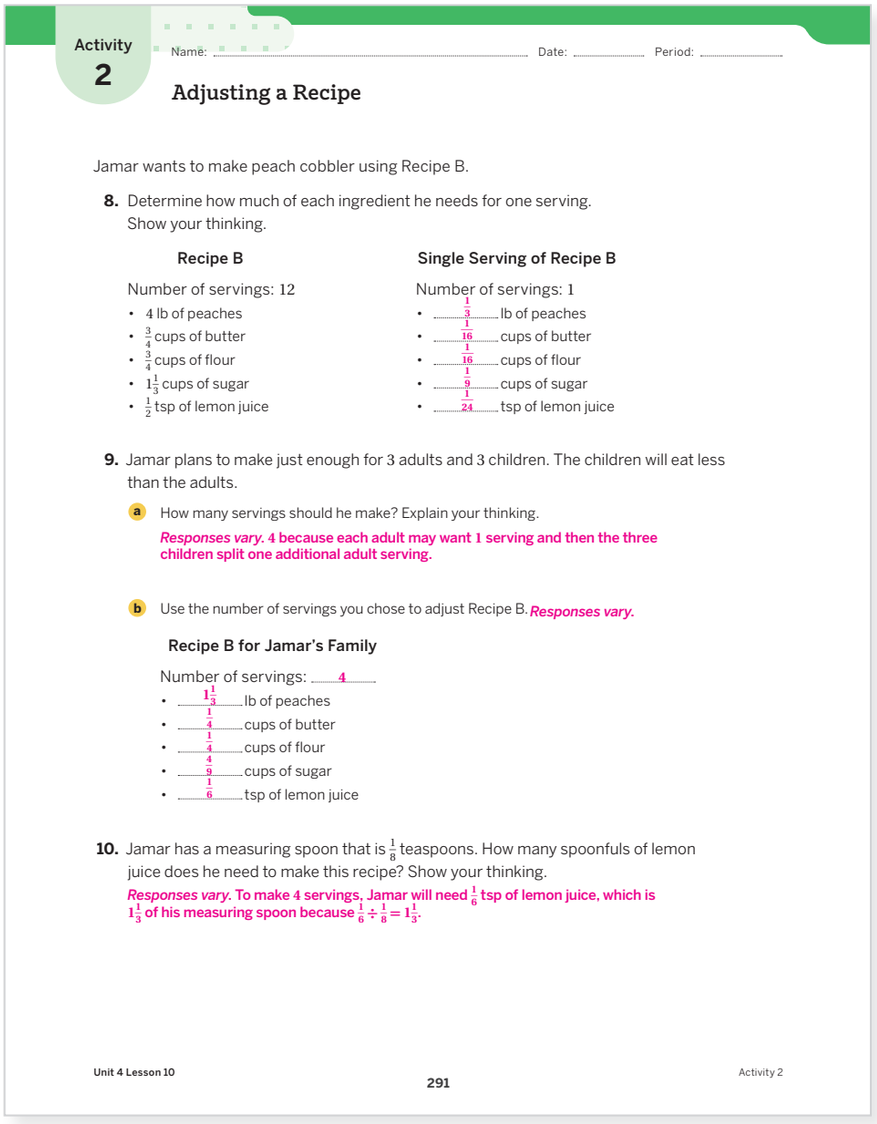
Look for students who:	Respond to Student Thinking
Need support performing the calculations. (*Problem 8*)	**Support** Consider having students use tape diagrams or double number lines to visualize the division or multiplication.
Divide each ingredient by 12 to get the single serving. (*Problem 8*)	**Strengthen** Consider asking, "What does your single serving represent in this proportional relationship?"
Use the single serving as a constant of proportionality to determine the amount for Jamar's family. (*Problem 9*)	**Strengthen** Celebrate their thinking. Consider asking, "How could you have used the original recipe to determine the amount for Jamar's family?"
Finish early.	**Stretch** Invite students to figure out how they could use standard measuring cups, tablespoons, teaspoons, and a scale to create the recipe for Jamar.

ELL English Language Learners Consider providing a reference that includes descriptions and visuals of measuring spoons and cups. **(Reading, Writing, Speaking, and Listening)**

3 Connect

Direct students' attention to the recipe card for Jamar's family. Invite students to share their thinking around how much of each ingredient is needed.

Consider asking, "Why is it helpful to know how much of each ingredient is in a single serving?" *Responses vary.* Knowing how much is in a single serving is helpful when trying to adjust a recipe for a certain number of servings. A single serving is the constant of proportionality for each ingredient.



Activity 2

Name: _____ Date: _____ Period: _____

Adjusting a Recipe

Jamar wants to make peach cobbler using Recipe B.

8. Determine how much of each ingredient he needs for one serving. Show your thinking.

Recipe B	Single Serving of Recipe B
Number of servings: 12	Number of servings: 1
• 4 lb of peaches	• $\frac{1}{3}$ lb of peaches
• $\frac{3}{4}$ cups of butter	• $\frac{1}{16}$ cups of butter
• $\frac{3}{4}$ cups of flour	• $\frac{1}{16}$ cups of flour
• $1\frac{1}{3}$ cups of sugar	• $\frac{1}{9}$ cups of sugar
• $\frac{1}{2}$ tsp of lemon juice	• $\frac{1}{24}$ tsp of lemon juice

9. Jamar plans to make just enough for 3 adults and 3 children. The children will eat less than the adults.

a. How many servings should he make? Explain your thinking.

Responses vary. 4 because each adult may want 1 serving and then the three children split one additional adult serving.

b. Use the number of servings you chose to adjust Recipe B. *Responses vary.*

Recipe B for Jamar's Family

Number of servings: 4

- $1\frac{1}{3}$ lb of peaches
- $\frac{1}{4}$ cups of butter
- $\frac{1}{4}$ cups of flour
- $\frac{4}{9}$ cups of sugar
- $\frac{1}{6}$ tsp of lemon juice

10. Jamar has a measuring spoon that is $\frac{1}{8}$ teaspoons. How many spoonfuls of lemon juice does he need to make this recipe? Show your thinking.

Responses vary. To make 4 servings, Jamar will need $\frac{1}{6}$ tsp of lemon juice, which is $1\frac{1}{3}$ of his measuring spoon because $\frac{1}{6} \div \frac{1}{8} = 1\frac{1}{3}$.

Unit 4 Lesson 10 291 Activity 2

Whole Class | 5 min

Synthesis

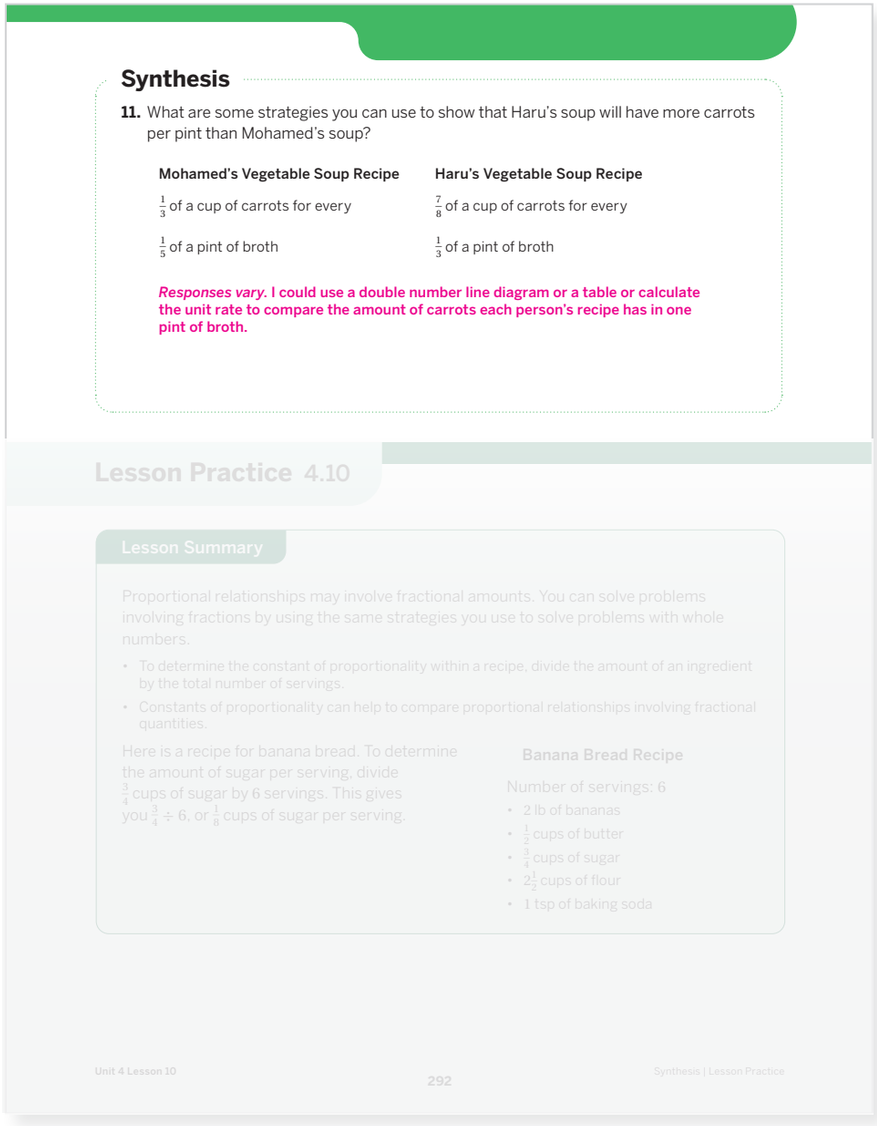
Purpose: Students synthesize strategies for comparing two quantities that have fractional values.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using a double number line or tape diagram.
- Determining a unit rate to compare in both recipes.
- Converting both recipes to similar sizes to compare.

Consider asking:
- "How does a constant of proportionality help us to compare relationships?"
- "What are proportional relationships you might want to compare in your life?"

Note: If time allows, invite students to revise their previous thinking in Activity 2 based on the discussion.



Synthesis

11. What are some strategies you can use to show that Haru's soup will have more carrots per pint than Mohamed's soup?

Mohamed's Vegetable Soup Recipe	Haru's Vegetable Soup Recipe
$\frac{1}{3}$ of a cup of carrots for every	$\frac{7}{8}$ of a cup of carrots for every
$\frac{1}{5}$ of a pint of broth	$\frac{1}{3}$ of a pint of broth

Responses vary. **I could use a double number line diagram or a table or calculate the unit rate to compare the amount of carrots each person's recipe has in one pint of broth.**

Lesson Practice 4.10

Lesson Summary

Proportional relationships may involve fractional amounts. You can solve problems involving fractions by using the same strategies you use to solve problems with whole numbers.

- To determine the constant of proportionality within a recipe, divide the amount of an ingredient by the total number of servings.
- Constants of proportionality can help to compare proportional relationships involving fractional quantities.

Here is a recipe for banana bread. To determine the amount of sugar per serving, divide $\frac{3}{4}$ cups of sugar by 6 servings. This gives you $\frac{3}{4} \div 6$, or $\frac{1}{8}$ cups of sugar per serving.

Banana Bread Recipe

Number of servings: 6
- 2 lb of bananas
- $\frac{1}{2}$ cups of butter
- $\frac{3}{4}$ cups of sugar
- $2\frac{1}{2}$ cups of flour
- 1 tsp of baking soda

Unit 4 Lesson 10 292 Synthesis | Lesson Practice

Lesson Takeaway: You can compare proportional relationships involving fractional quantities by determining the constant of proportionality for each relationship.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by comparing two proportional relationships with fractional unit rates.

Today's Goals

1. **Goal:** Use a constant of proportionality to solve problems involving fractional quantities.
2. **Language Goal:** Describe strategies for solving problems involving proportional relationships with fractional quantities. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.10

Aba and Esteban each make lemonade

Aba	Esteban
Aba mixes $2\frac{1}{2}$ cups of water with $\frac{1}{3}$ cup of lemon juice.	Esteban mixes $1\frac{2}{3}$ cups of water with $\frac{1}{4}$ cup of lemon juice.

Whose lemonade mixture has more lemon juice per cup? Show or explain your thinking.

Esteban's. *Explanations vary.* Aba uses $7\frac{1}{2}$ cups of water per 1 cup of lemon juice because $2\frac{1}{2} \div \frac{1}{3} = 7\frac{1}{2}$. Esteban uses $6\frac{2}{3}$ cups of water per 1 cup of lemon juice because $1\frac{2}{3} \div \frac{1}{4} = 6\frac{2}{3}$. Esteban's mixture has less water for the same amount of lemon juice.

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to compare proportional relationships with fractional quantities:

» **Respond to Student Thinking:**

- Use the ***Applying Fractions in Proportional Relationships*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to research a recipe and scale it up or down to feed their family, friends, or community.

Professional Learning

How did your previous experience with recipes or cooking impact how you taught this lesson? How did students' previous experiences with recipes or cooking impact how you taught the lesson? How can you bring students' experiences into the mathematics that you explore in your classroom?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. What are some strategies you can use to show that Haru's soup will have more carrots per pint than Mohamed's soup?

Mohamed's Vegetable Soup Recipe	Haru's Vegetable Soup Recipe
$\frac{1}{3}$ of a cup of carrots for every	$\frac{7}{8}$ of a cup of carrots for every
$\frac{1}{5}$ of a pint of broth	$\frac{1}{3}$ of a pint of broth

Responses vary. I could use a double number line diagram or a table or calculate the unit rate to compare the amount of carrots each person's recipe has in one pint of broth.

Lesson Practice 4.10

Lesson Summary

Proportional relationships may involve fractional amounts. You can solve problems involving fractions by using the same strategies you use to solve problems with whole numbers.

- To determine the constant of proportionality within a recipe, divide the amount of an ingredient by the total number of servings.
- Constants of proportionality can help to compare proportional relationships involving fractional quantities.

Here is a recipe for banana bread. To determine the amount of sugar per serving, divide $\frac{3}{4}$ cups of sugar by 6 servings. This gives you $\frac{3}{4} \div 6$, or $\frac{1}{8}$ cups of sugar per serving.

Banana Bread Recipe

Number of servings: 6

- 2 lb of bananas
- $\frac{1}{2}$ cups of butter
- $\frac{3}{4}$ cups of sugar
- $2\frac{1}{2}$ cups of flour
- 1 tsp of baking soda

Lesson Practice 4.10

Name: _____ Date: _____ Period: _____

1. A recipe calls for $\frac{1}{2}$ cups of sugar and 1 cup of flour. Complete the table to show how much sugar and flour is needed for different batches of the recipe.

Sugar (cups)	Flour (cups)
$\frac{1}{2}$	1
$\frac{3}{4}$	$1\frac{1}{2}$
$\frac{7}{8}$	$1\frac{3}{4}$
1	2
$1\frac{1}{4}$	$2\frac{1}{2}$

Problems 2–4: A punch recipe calls for $1\frac{1}{2}$ quarts of sparkling water and $\frac{3}{4}$ quarts of grape juice.

2. How much sparkling water would you need to mix with 9 quarts of grape juice?

18 quarts

3. How much grape juice would you need to mix with $3\frac{3}{4}$ quarts of sparkling water?

$\frac{15}{8}$ quarts (or equivalent). Note: The ratio $1\frac{1}{2}$ quarts of sparkling water to $\frac{3}{4}$ quarts of grape juice is equivalent to a 2 : 1 ratio.

4. How much of each ingredient would you need to make 75 quarts of punch?

Sparkling Water (qt): 50

Grape Juice (qt): 25

FAST Practice

Problems 5–6: To make a specific color of green paint, a painter mixes $\frac{1}{2}$ of a gallon of blue paint with $\frac{4}{5}$ of a gallon of yellow paint.

5. How many gallons of yellow paint are needed to mix with 3 gallons of blue paint?

$4\frac{4}{5}$ gallons

6. How many gallons of each color are needed to make 26 total gallons of this color?

Blue Paint: 10 gallons

Yellow Paint: 16 gallons

Lesson Practice 4.10

Name: _____ Date: _____ Period: _____

Spiral Review

7. Complete the table based on $y = \frac{2}{3}x$.

x	y
12	8
24	16

8. Select *all* the ratios that are equivalent to 4 : 5.

- ☑ A. 2 : 2.5
- ☐ B. 3 : 4
- ☑ C. 3 : 3.75
- ☑ D. 8 : 10
- ☐ E. 14 : 27.5

9. On a map, DeAndre's house is 2.5 inches from his grandma's house. The map has a scale of 1 inch to 20 miles. How far apart, in inches, would DeAndre's house be from his grandma's house on a map that has a scale of 1 inch to 80 miles?

A. 0.5 (B.) 0.625 C. 1.60 D. 1.75

10. What is 40% of 160?

64

11. What is 160% of 40?

64

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.3.2
FAST Practice	5–6	2	MA.7.AR.3.2
Spiral Review			
	7	2	MA.6.AR.2.3
	8	1	MA.6.AR.3.5
	9	2	MA.7.GR.1.5
	10–11	1	MA.7.AR.3.4

Unit 4

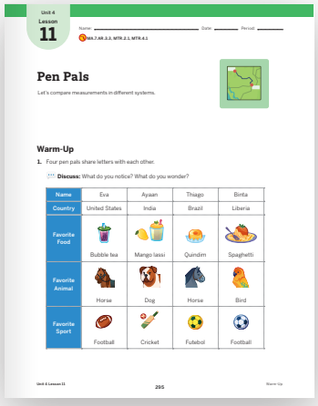
Lesson

11

Pen Pals

Converting Units

Let's compare measurements in different systems.


Unit 4
Lesson
11
Name _____ Date _____ Period _____
MA.7.AR.3.3, MTR.2.1, MTR.4.1
Pen Pals
Let's compare measurements in different systems.
Warm-Up
1. Four pen pals share letters with each other.
Discuss: What do you notice? What do you wonder?
Name | Eva | Ayaan | Thiago | Binta
Country | United States | India | Brazil | Liberia
Favorite Food | Bubble tea | Mango lassi | Quindim | Spaghetti
Favorite Animal | Horse | Dog | Horse | Bird
Favorite Sport | Football | Cricket | Futebol | Football
Unit 4 Lesson 11
295
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Convert measurements from one unit to another in different measurement systems.
2. **Language Goal:** Compare and contrast strategies for converting between units. **(Writing, Speaking, and Listening)**

Students convert measurements from one unit to another in different measurement systems. Students learn that they can represent unit conversion problems in multiple ways and that different representations can be useful in different situations. **(MTR.2.1)**

Coherence

Prior Learning

In Grade 6, students converted measurements within the same system to solve mathematical and real-world problems. In Unit 2, students created tables to help make sense of rates and proportional relationships.

Future Learning

In Grade 8, students will apply their understanding of unit conversions to solve real-world geometric problems.

Connections

- Students convert measurements from one unit to another using tables and double number-lines. (real world)
- Students describe strategies used by others to convert measurements to deepen their own understanding. (mathematical, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.3
Solve mathematical and real-world problems involving the conversion of units across different measurement systems.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.1.2

Lesson at a Glance

~ 45 min

MA.7.AR.3.3

Warm-Up

Pairs | 5 min

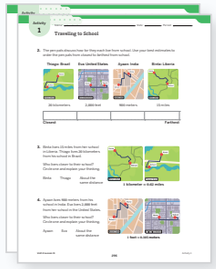
Students are introduced to the context of the lesson by comparing and contrasting some favorite things around the world.
Routine: Notice and Wonder



Activity 1

Pairs | 10 min

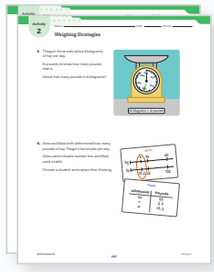
Students compare distances using conversion rates.
Routine: Think-Pair-Share



Activity 2

Pairs | 10 min

Students analyze different tools and strategies for converting measurements from one unit to another. MLR7: Compare and Connect (MTR.2.1)



Activity 3

Pairs | 10 min

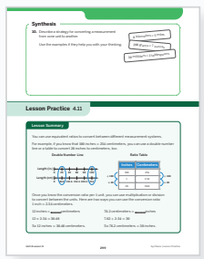
Students practice converting amounts of money from one currency to another.



Synthesis

Whole Class | 5 min

Students synthesize their understanding that ratio reasoning can be used to convert measurements from one unit to another in different measurement systems. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by converting among units of measurement in different systems using a ratio in terms other than 1.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of the lesson by comparing and contrasting some favorite things around the world.

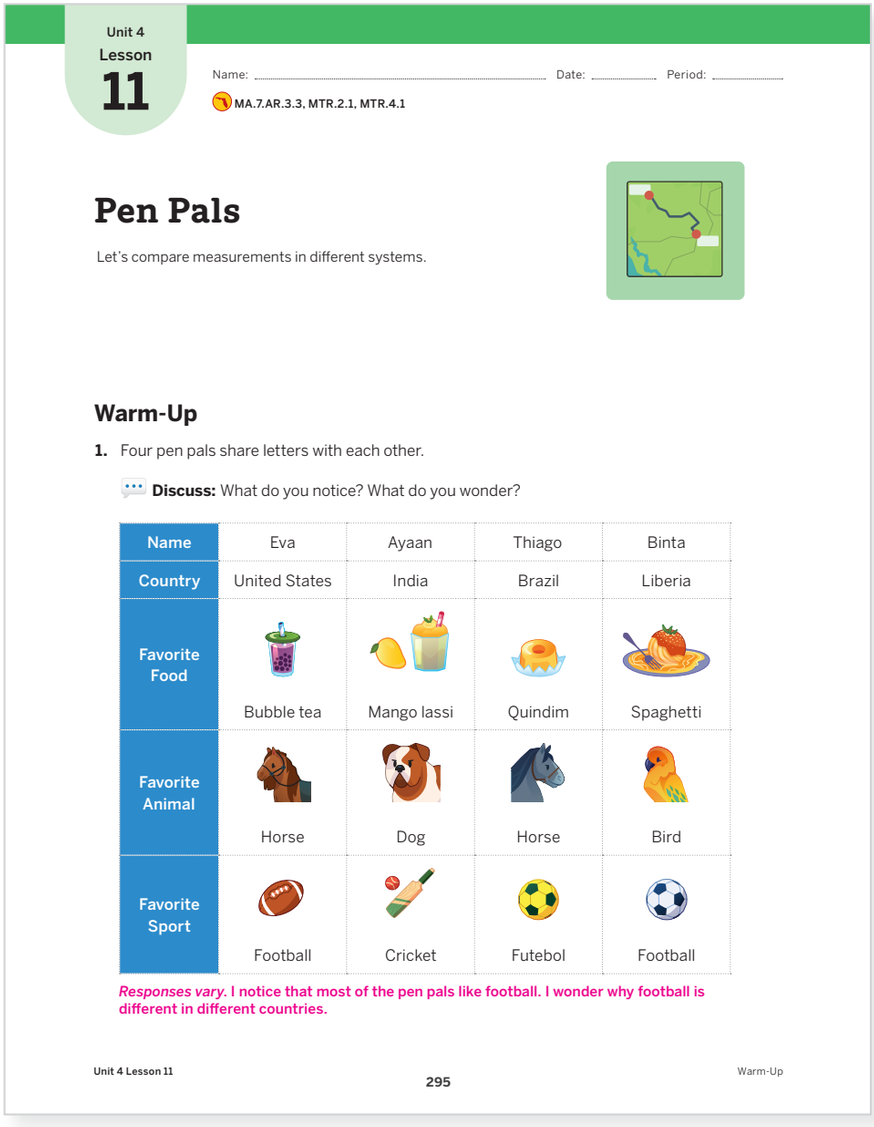
1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the context. See the Routine Facilitation Guide for more information.

To surface connections, consider asking students what they know about pen pals, and whether anyone in the class has ever had a pen pal.

2 Connect

Listen for students making personal connections to the images of the favorite things.



Unit 4
Lesson
11

Name: _____ Date: _____ Period: _____

MA.7.AR.3.3, MTR.2.1, MTR.4.1

Pen Pals

Let's compare measurements in different systems.

Warm-Up

1. Four pen pals share letters with each other.

Discuss: What do you notice? What do you wonder?

Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food	Bubble tea	Mango lassi	Quindim	Spaghetti
Favorite Animal	Horse	Dog	Horse	Bird
Favorite Sport	Football	Cricket	Futebol	Football

Responses vary. I notice that most of the pen pals like football. I wonder why football is different in different countries.

Unit 4 Lesson 11 295 Warm-Up

Pairs | 10 min

Activity 1 Traveling to School

Purpose: Students compare distances using conversion rates.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

Invite a few students to share their work. It is okay — even desirable — to lack consensus at this stage. The activity will build toward consensus when students are given a ratio to use to convert between different units.

2 Monitor

Pause briefly to share with students that we'll be helping the pen pals convert distances from school from one unit of measurement to another.

Invite students to use a calculator or paper to help with their thinking throughout this lesson.

Look for a variety of strategies and explanations.

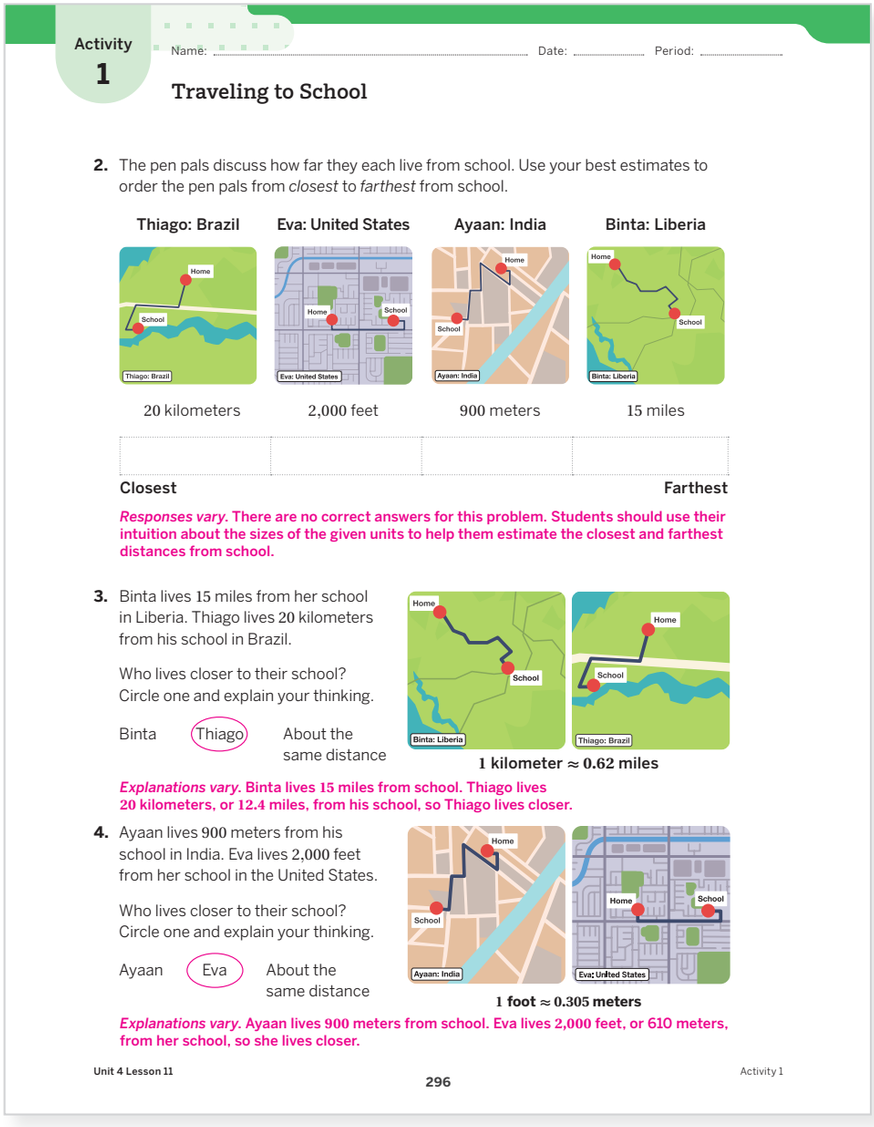
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What unit of measurement is longer? What is the relationship between the two?"
Only multiply or divide to do their conversion.	**Support** Invite them to use a double number line or table to support their ratio reasoning.
Use a table or double number line to support their ratio reasoning.	**Strengthen** Invite them to share their thinking during the Connect.

Note: This may be the first time students see the ≈ symbol used to represent an approximate equivalence.

3 Connect

Invite students to ask questions of each classmate after they share, including clarifying the strategies for making sense of the different units.



Activity 1

Name: Date: Period:

Traveling to School

2. The pen pals discuss how far they each live from school. Use your best estimates to order the pen pals from *closest* to *farthest* from school.

Thiago: Brazil	Eva: United States	Ayaan: India	Binta: Liberia
20 kilometers	2,000 feet	900 meters	15 miles

Closest Farthest

Responses vary. There are no correct answers for this problem. Students should use their intuition about the sizes of the given units to help them estimate the closest and farthest distances from school.

3. Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school? Circle one and explain your thinking.

Binta (Thiago) About the same distance

1 kilometer ≈ 0.62 miles

Explanations vary. Binta lives 15 miles from school. Thiago lives 20 kilometers, or 12.4 miles, from his school, so Thiago lives closer.

4. Ayaan lives 900 meters from his school in India. Eva lives 2,000 feet from her school in the United States.

Who lives closer to their school? Circle one and explain your thinking.

Ayaan (Eva) About the same distance

1 foot ≈ 0.305 meters

Explanations vary. Ayaan lives 900 meters from school. Eva lives 2,000 feet, or 610 meters, from her school, so she lives closer.

Unit 4 Lesson 11 296 Activity 1

Pairs | 10 min

Activity 2 Weighing Strategies

Purpose: Students analyze different tools and strategies for converting measurements from one unit to another. **(MTR2.1)**

1 Launch

Direct students' attention to the scale and consider asking:

- "Which is heavier, 1 pound or 1 kilogram?"
- "How many pounds do you think is about the same as 5 kilograms?"

Invite students to check their responses with a partner.

2 Monitor

Look and listen for a variety of tools and strategies to share during the Connect. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is a solution that is too high? Too low?"
Have an inaccurate response.	**Strengthen** Invite them to use a double number line or table to support their ratio reasoning.
Use strategies similar to Alina's or Maia's on the next problem or other creative strategies.	**Strengthen** Invite them to share their thinking during the Connect.

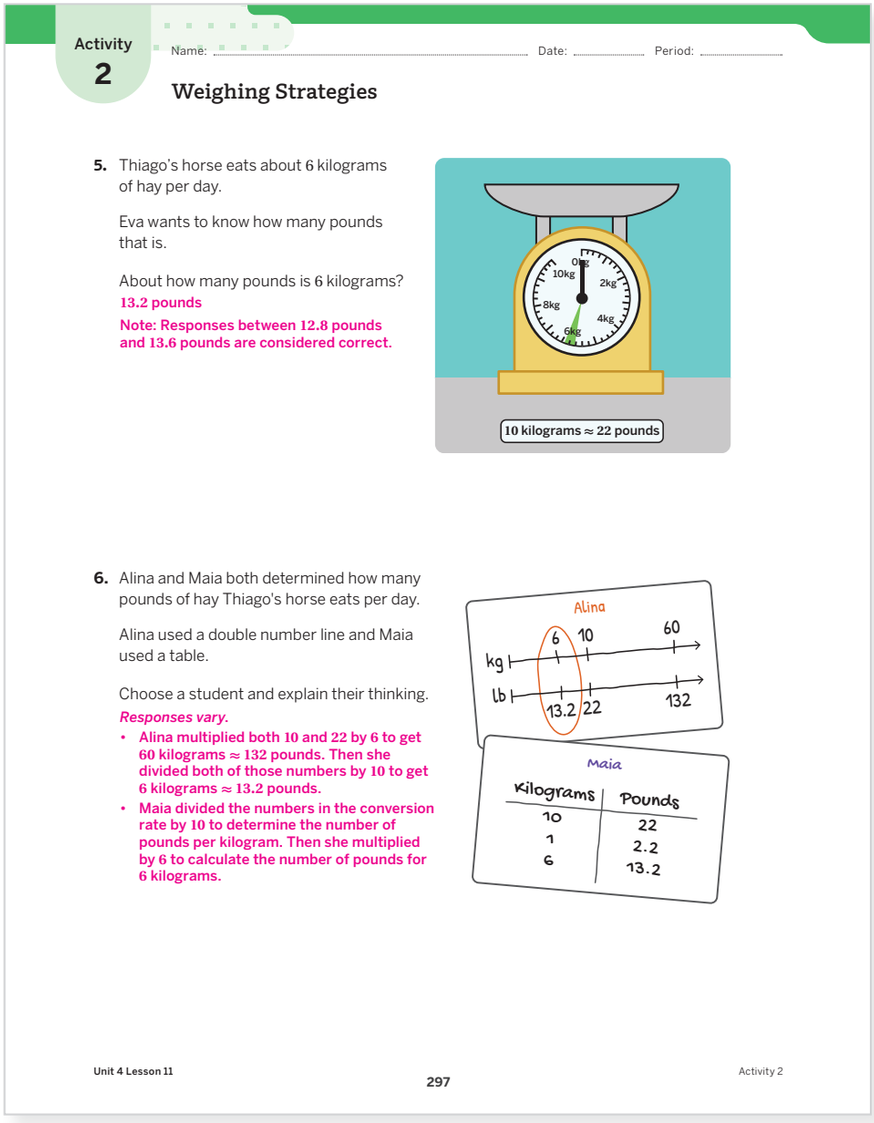
3 Connect

Invite students to select a strategy that makes the most sense to them.

MLR **MLR7: Compare and Connect** Invite students to analyze each representation, then find ways in which the representations are different (compare) and alike (connect). Consider asking:

- "How would you describe Alina's strategy in your own words? Maia's?"
- "Why do you think Maia used a 1 in her ratio table?"
- "What are the advantages of Alina's strategy? Maia's?"

Invite students that used similar or other creative strategies to share. Consider celebrating students who justify the strategy they used and compare the efficiency of their strategy to other strategies. **(MTR.4.1)**



Activity 2

Name: Date: Period:

Weighing Strategies

5. Thiago's horse eats about 6 kilograms of hay per day.

Eva wants to know how many pounds that is.

About how many pounds is 6 kilograms?

13.2 pounds

Note: Responses between 12.8 pounds and 13.6 pounds are considered correct.

6. Alina and Maia both determined how many pounds of hay Thiago's horse eats per day.

Alina used a double number line and Maia used a table.

Choose a student and explain their thinking.

Responses vary.

- **Alina multiplied both 10 and 22 by 6 to get 60 kilograms ≈ 132 pounds. Then she divided both of those numbers by 10 to get 6 kilograms ≈ 13.2 pounds.**
- **Maia divided the numbers in the conversion rate by 10 to determine the number of pounds per kilogram. Then she multiplied by 6 to calculate the number of pounds for 6 kilograms.**

Unit 4 Lesson 11 297 Activity 2

Key Takeaway: When students are given a ratio to use to convert between different units, a table or double number line can be used to organize and find equivalent ratios.

Pairs | 10 min

Activity 3 Favorite Things

Purpose: Students practice converting amounts of money from one currency to another.

Short on time? Consider omitting problem 7.

1 Launch

To surface connections, consider asking students if they have ever seen or used foreign currency, and what sorts of differences from U.S. currency they noticed.

A **Accessibility: Visual-Spatial Processing**
Provide graph paper, blank tables, or tape diagram templates for students to draw on to increase access to the task.

2 Monitor

Look and listen for students who may need individual support to be comfortable with at least one strategy.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started or use a guessing and checking strategy.	**Support** Invite students to either make a table or double number line. Consider asking, "Where would ____ go?"
Are able to find equivalent ratios that are close (e.g., $2 : 168.80$, $11 : 63.80$, and $16 : 3032.32$) to the desired ratio but are not sure what to do next.	**Strengthen** Celebrate their attempt as an estimation strategy. Consider asking, "Between what two values will the solution be?" Then encourage them to revisit Alina's and Maia's strategies.
Use a table, double number line, or other creative strategies to support their ratio reasoning.	**Strengthen** Invite them to share their thinking during the Connect. **Strengthen** Encourage students to try another strategy.

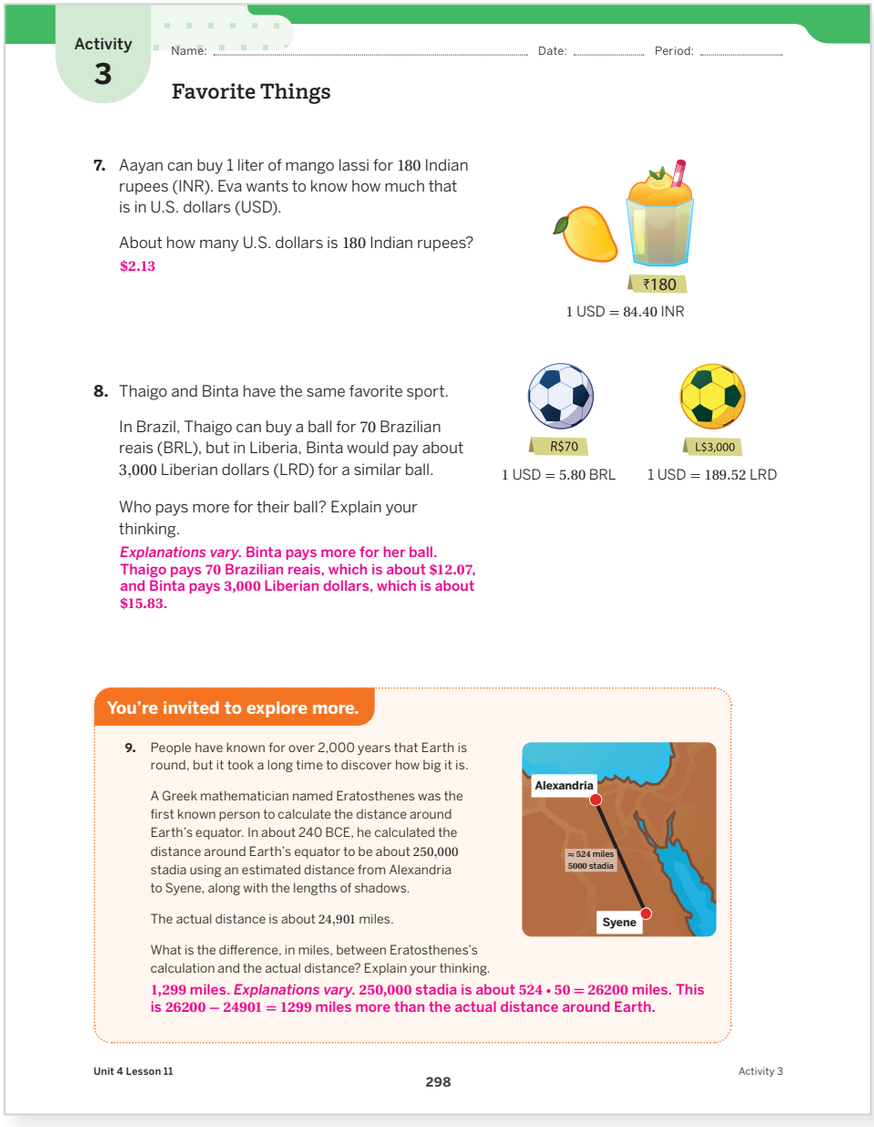
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore different units further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students that had inaccuracies in their work to share their mistakes and their new, refined work or give the students an opportunity to ask the class for feedback.

Invite students that used tables, double number lines, or other creative strategies to share.

Consider asking, "How can equivalent ratios help us to convert units?"



Activity 3

Name: Date: Period:

Favorite Things

7. Aayan can buy 1 liter of mango lassi for 180 Indian rupees (INR). Eva wants to know how much that is in U.S. dollars (USD).

About how many U.S. dollars is 180 Indian rupees?

$2.13

1 USD = 84.40 INR

8. Thaigo and Binta have the same favorite sport.

In Brazil, Thaigo can buy a ball for 70 Brazilian reais (BRL), but in Liberia, Binta would pay about 3,000 Liberian dollars (LRD) for a similar ball.

Who pays more for their ball? Explain your thinking.

Explanations vary. Binta pays more for her ball. Thaigo pays 70 Brazilian reais, which is about $12.07, and Binta pays 3,000 Liberian dollars, which is about $15.83.

1 USD = 5.80 BRL

1 USD = 189.52 LRD

You're invited to explore more.

9. People have known for over 2,000 years that Earth is round, but it took a long time to discover how big it is.

A Greek mathematician named Eratosthenes was the first known person to calculate the distance around Earth's equator. In about 240 BCE, he calculated the distance around Earth's equator to be about 250,000 stadia using an estimated distance from Alexandria to Syene, along with the lengths of shadows.

The actual distance is about 24,901 miles.

What is the difference, in miles, between Eratosthenes's calculation and the actual distance? Explain your thinking.

1,299 miles. *Explanations vary.* 250,000 stadia is about 524 • 50 = 26200 miles. This is 26200 − 24901 = 1299 miles more than the actual distance around Earth.

Unit 4 Lesson 11 298 Activity 3

Whole Class | 5 min

Synthesis

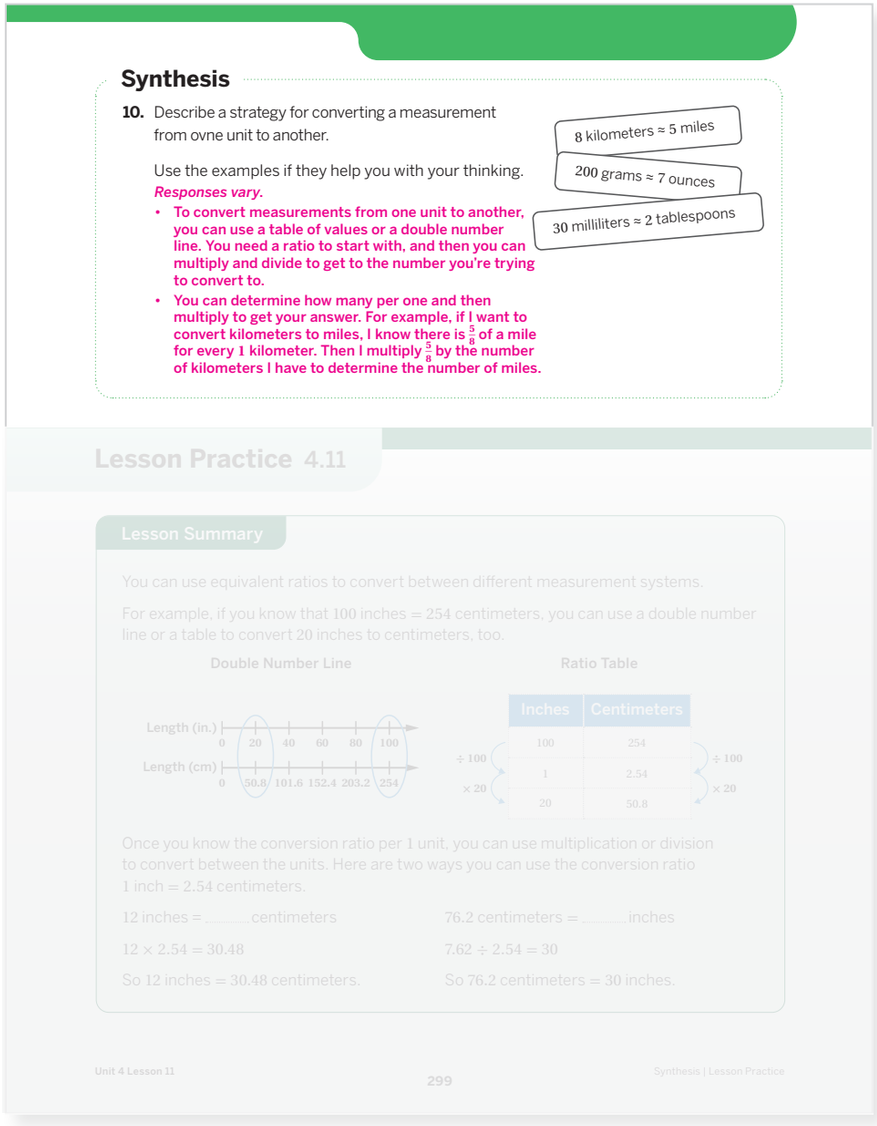
Purpose: Students synthesize their understanding that ratio reasoning can be used to convert measurements from one unit to another in different measurement systems.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Writing your conversation rate as a ratio on a table or double number line to help organize and calculate equivalent ratios.
- Making an equivalent ratio in which one of the values is a 1 can be helpful to get your desired ratio.
- Using a common multiple of the original and desired values can be helpful when finding a ratio that can be used as a middle step.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategies for converting units using students' language. Display any new language, examples, or diagrams for students to reference later.



Synthesis

10. Describe a strategy for converting a measurement from ovne unit to another.

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

30 milliliters ≈ 2 tablespoons

Responses vary.

- To convert measurements from one unit to another, you can use a table of values or a double number line. You need a ratio to start with, and then you can multiply and divide to get to the number you're trying to convert to.
- You can determine how many per one and then multiply to get your answer. For example, if I want to convert kilometers to miles, I know there is $\frac{5}{8}$ of a mile for every 1 kilometer. Then I multiply $\frac{5}{8}$ by the number of kilometers I have to determine the number of miles.

Lesson Practice 4.11

Lesson Summary

You can use equivalent ratios to convert between different measurement systems.

For example, if you know that 100 inches = 254 centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line

Length (in.): 0, 20, 40, 60, 80, 100

Length (cm): 0, 50.8, 101.6, 152.4, 203.2, 254

Ratio Table

Inches	Centimeters
100	254
1	2.54
20	50.8

÷ 100, × 20

Once you know the conversion ratio per 1 unit, you can use multiplication or division to convert between the units. Here are two ways you can use the conversion ratio 1 inch = 2.54 centimeters.

12 inches = _____ centimeters

$12 \times 2.54 = 30.48$

So 12 inches = 30.48 centimeters.

76.2 centimeters = _____ inches

$7.62 \div 2.54 = 30$

So 76.2 centimeters = 30 inches.

Unit 4 Lesson 11 299 Synthesis | Lesson Practice

Lesson Takeaway: Ratio reasoning can be used to convert measurements from one unit to another in different measurement systems. A table or double number line can be used to organize and find helpful equivalent ratios.

Independent | 5 min

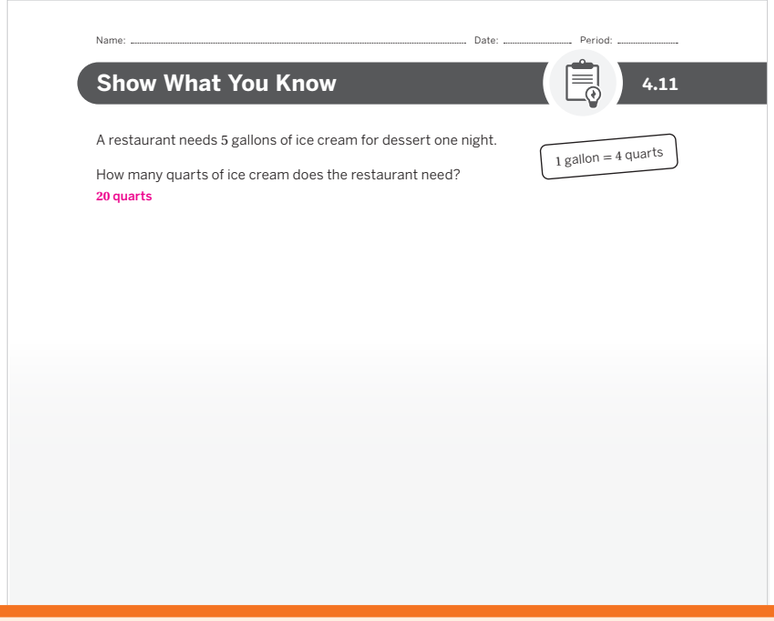
Show What You Know

Purpose: Students demonstrate their understanding by converting among units of measurement in different systems.

Today's Goals

1. **Goal:** Convert measurements from one unit to another in different measurement systems.
2. **Language Goal:** Compare and contrast strategies for converting between units. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.11

A restaurant needs 5 gallons of ice cream for dessert one night.

How many quarts of ice cream does the restaurant need?

20 quarts

1 gallon = 4 quarts

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to convert among units of measurement in different systems using a ratio in terms other than 1:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Consider the proportional relationships you will teach in Unit 8. How will students' work today in Pen Pals prepare them for using ratio reasoning to construct circle graphs and using a sample to make predictions about a larger population. What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe a strategy for converting a measurement from one unit to another.

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

30 milliliters ≈ 2 tablespoons

Responses vary.

- To convert measurements from one unit to another, you can use a table of values or a double number line. You need a ratio to start with, and then you can multiply and divide to get to the number you're trying to convert to.
- You can determine how many per one and then multiply to get your answer. For example, if I want to convert kilometers to miles, I know there is $\frac{5}{8}$ of a mile for every 1 kilometer. Then I multiply $\frac{5}{8}$ by the number of kilometers I have to determine the number of miles.

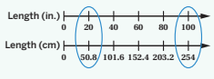
Lesson Practice 4.11

Lesson Summary

You can use equivalent ratios to convert between different measurement systems.

For example, if you know that 100 inches = 254 centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line



Ratio Table

Inches	Centimeters
100	254
1	2.54
20	50.8

÷ 100, × 20 | ÷ 100, × 20

Once you know the conversion ratio per 1 unit, you can use multiplication or division to convert between the units. Here are two ways you can use the conversion ratio 1 inch = 2.54 centimeters.

12 inches = ... centimeters

$12 \times 2.54 = 30.48$

So 12 inches = 30.48 centimeters.

76.2 centimeters = ... inches

$7.62 \div 2.54 = 30$

So 76.2 centimeters = 30 inches.

Lesson Practice 4.11

Name: ..... Date: ..... Period: .....

1. Malik is 57 inches tall. If 1 inch = 2.54 centimeters, which value is closest to his height in centimeters?

A. 22.4 centimeters

B. 57 centimeters

C. 144.8 centimeters

D. 3,551 centimeters

Problems 2–3: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show or explain your thinking.

1 kilogram = 1000 grams	3 ounces ≈ 85 grams
1 pound ≈ 0.454 kilograms	4 kilograms ≈ 141 ounces

2. 15 ounces ≈ 425 grams

Explanations vary.

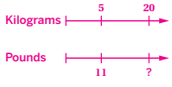
Ounces	Grams
3	85
15	?

×5 | ×5

$85 \cdot 5 = 425$

3. 20 kilograms ≈ 44 pounds

Explanations vary.



$11 \cdot 4 = 44$

4. Dhruv's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Responses shown in table.

Dollars	Pesos
250	4,250
25	425
1	17
3	51
30	510

Lesson Practice 4.11

Name: ..... Date: ..... Period: .....

FAST Practice

5. A yard is equal to 3 feet, and there are 1,760 yards in 1 mile. How many feet are there in 4 miles?

A. 3,520

B. 7,040

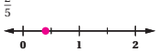
C. 5,280

D. 21,120

Spiral Review

Problems 6–9: Plot each value on the number line.

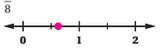
6. $\frac{2}{5}$



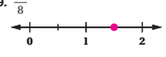
7. $1\frac{2}{5}$



8. $\frac{5}{8}$



9. $\frac{12}{8}$



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.3.3
FAST Practice	5	2	MA.7.AR.3.3
Spiral Review			
	6–9	1	MA.6.NSO.1.1

Unit 4

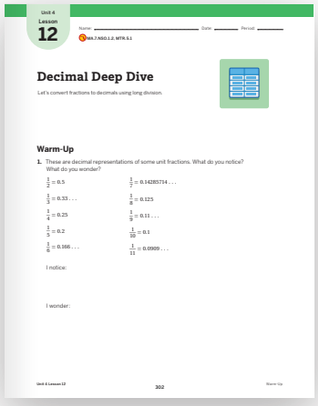
Lesson

12

Decimal Deep Dive

Writing Fractions as Decimals

Let's convert fractions to decimals using long division.


Unit 4
Lesson
12
Name: Date: Period:
MA.7.NSO.1.2, MTR.5.1
Decimal Deep Dive
Let's convert fractions to decimals using long division.
Warm-Up
1. These are decimal representations of some unit fractions. What do you notice? What do you wonder?
$\frac{1}{2} = 0.5$
$\frac{1}{3} = 0.33\ldots$
$\frac{1}{4} = 0.25$
$\frac{1}{5} = 0.2$
$\frac{1}{6} = 0.166\ldots$
$\frac{1}{7} = 0.142857142\ldots$
$\frac{1}{8} = 0.125$
$\frac{1}{9} = 0.11\ldots$
$\frac{1}{10} = 0.1$
$\frac{1}{11} = 0.0909\ldots$
I notice:
I wonder:
Unit 4 Lesson 12
302
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Explore patterns that lead to *terminating* or *repeating decimals* when converting a fraction to a decimal using long division.
2. **Language Goal:** Describe strategies for determining whether a fraction will be represented as a *terminating* or *repeating decimal*. **(Writing, Speaking, and Listening)**

Students convert fractions to decimals using long division. They then notice patterns in their calculations for classifying decimals as terminating or repeating. **(MTR.5.1)**

Coherence

Prior Learning

In Lessons 9–10, students solved problems involving proportional relationships with fractional and decimal values.

Future Learning

In Unit 5, students will represent rational numbers in equivalent forms as fractions, mixed numbers, decimals, and percents.

Connections

- Students use long division to convert fractions to repeating or terminating decimals. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standard: MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.1.1	**MA.7.NSO.2.1**
MA.6.NSO.3.5	**MA.7.NSO.2.2**
	MA.8.NSO.1.1
	MA.8.NSO.1.2

Vocabulary

New Vocabulary

bar notation
repeating decimal
terminating decimal

Lesson at a Glance

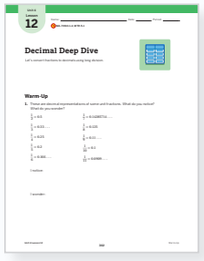
~ 45 min

MA.7.NSO.1.2

Warm-Up

Pairs | 5 min

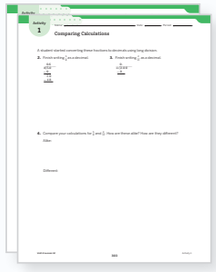
Students look at decimal expansions of unit fractions to prepare for the upcoming activities involving **terminating** and **repeating decimals**.
Routine: Notice and Wonder



Activity 1

Pairs | 10 min

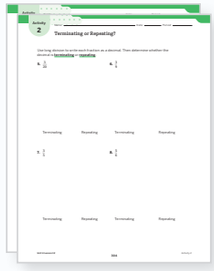
Students analyze one strategy for writing fractions as decimals and compare calculations that result in terminating and repeating decimals.



Activity 2

Pairs | 10 min

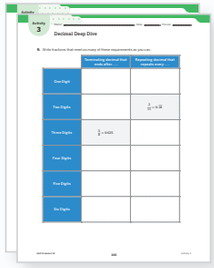
Students practice using long division to write fractions as decimals.



Activity 3

Pairs | 10 min

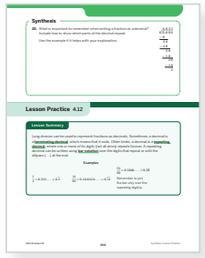
Students creatively apply the patterns they have noticed to determine fractions with specific numbers of repeating digits. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize strategies for writing a fraction as a decimal.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by converting a fraction to a decimal using long division.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- graph paper (optional)

Pairs | 5 min

Warm-Up

Purpose: Students look at decimal expansions of unit fractions to prepare for the upcoming activities involving **terminating** and **repeating decimals**.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of fractions and their equivalent decimal values.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I notice___." or "I wonder___."). **(Writing and Speaking)**

2 Connect

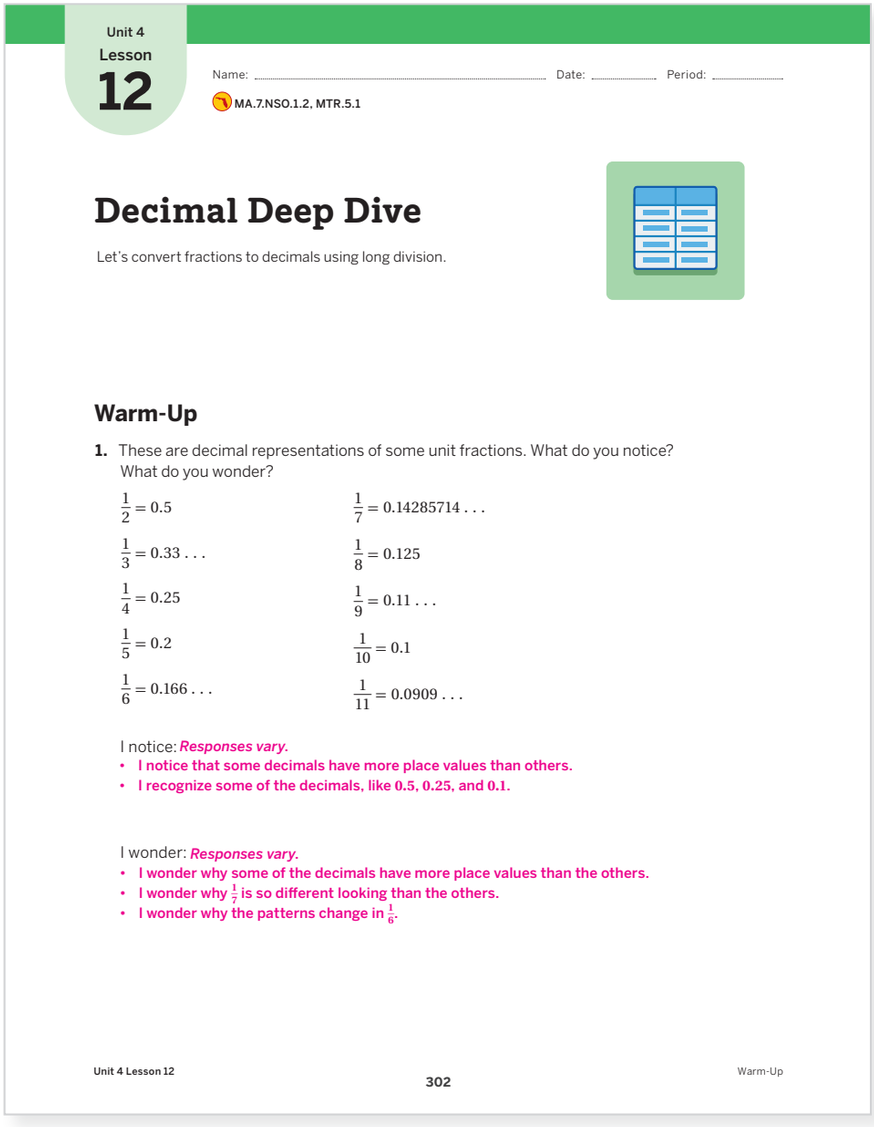
Invite students to share things they noticed and wondered.

Create a record of the different properties of fractions and decimals students noticed and wondered, along with the name of the student who shared them.

Note: If it does not come up naturally, consider asking, "Did anyone wonder about the ". . ." symbols? Does this make you wonder anything about the decimal values?"

Invite students to use a calculator to view the calculator's representation of a few of these values, including but not limited to, $\frac{1}{6}$ and $\frac{1}{7}$.

Consider asking, "How do the calculator's representations compare to the values calculated here?"



Unit 4
Lesson
12

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.2, MTR.5.1

Decimal Deep Dive

Let's convert fractions to decimals using long division.

Warm-Up

1. These are decimal representations of some unit fractions. What do you notice? What do you wonder?

$\frac{1}{2} = 0.5$

$\frac{1}{3} = 0.33\ldots$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.166\ldots$

$\frac{1}{7} = 0.14285714\ldots$

$\frac{1}{8} = 0.125$

$\frac{1}{9} = 0.11\ldots$

$\frac{1}{10} = 0.1$

$\frac{1}{11} = 0.0909\ldots$

I notice: *Responses vary.*
- I notice that some decimals have more place values than others.
- I recognize some of the decimals, like 0.5, 0.25, and 0.1.

I wonder: *Responses vary.*
- I wonder why some of the decimals have more place values than the others.
- I wonder why $\frac{1}{7}$ is so different looking than the others.
- I wonder why the patterns change in $\frac{1}{6}$.

Unit 4 Lesson 12 302 Warm-Up

Pairs | 10 min

Activity 1 Comparing Calculations

Purpose: Students analyze one strategy for writing fractions as decimals and compare calculations that result in terminating and repeating decimals.

1 Launch

Encourage students to spend 1–2 minutes trying the problems on their own before checking their work with their partner.

A **Accessibility: Memory and Attention** Provide students with an anchor chart or graphic organizer to support students with keeping track of information in their division problems.

2 Monitor

Look for a variety of organizational strategies for the division and responses to what is the same or different.

D **Differentiation** *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problems 2–3)*	**Support** Consider working through the first steps of the division algorithm together.
Misalign the columns during the long division algorithm. *(Problems 2–3)*	**Support** Invite students to use graph paper or other organizational strategies to align the columns.
Continue to divide after the division for $\frac{2}{11}$ has repeated a few times. *(Problem 3)*	**Strengthen** Consider asking, "What pattern do you notice is happening in your division?"
Notice that the decimal for $\frac{2}{11}$ repeats the digits 18. *(Problems 3–4)*	**Stretch** Consider asking, "How can you explain to someone else how you know that the decimal will continue to repeat?"

3 Connect

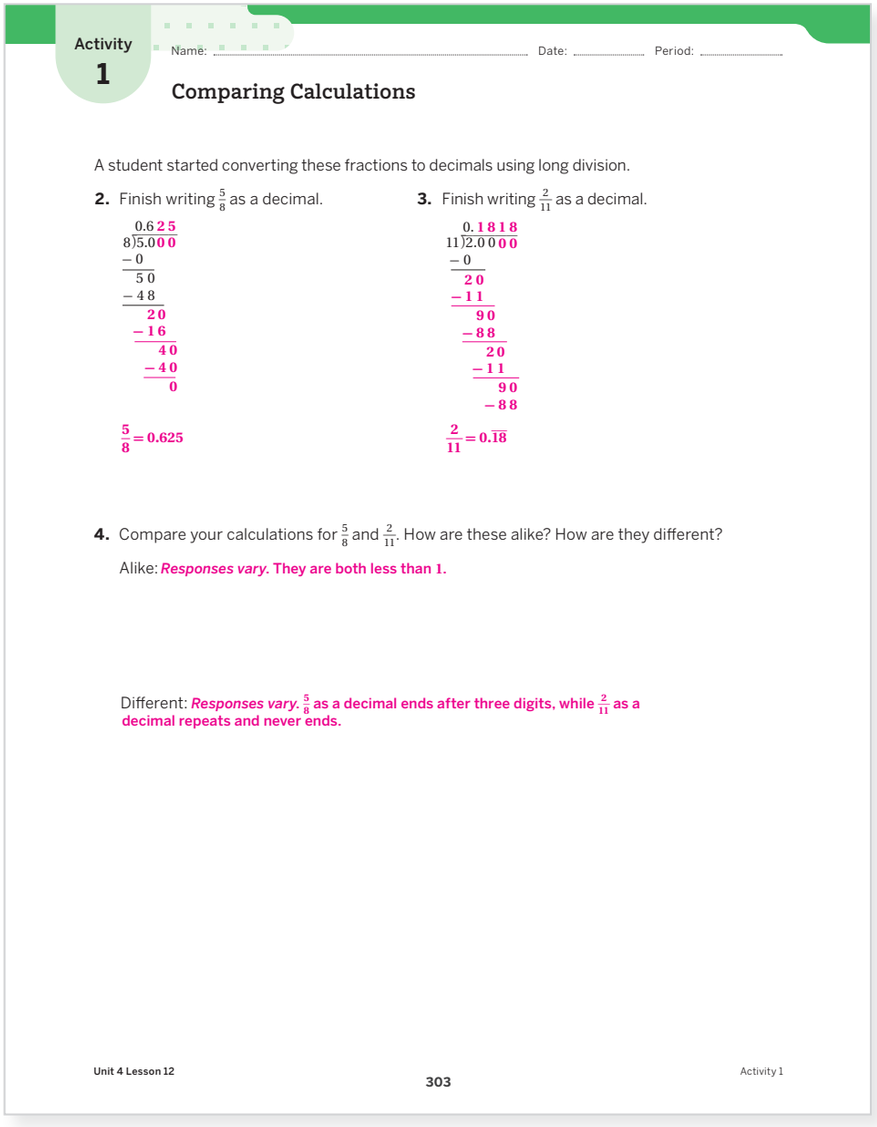
Direct students' attention to the two division problems.

Invite students to share their decimals and what they noticed is the same or different. Consider asking, "What pattern did you notice in Problem 3 that did not happen in Problem 2?" **(MTR.5.1)**

Demonstrate how to use **bar notation** as a way of representing repeating digits.

Create a class definition for the terms **terminating decimal** and **repeating decimal** using students' language and the glossary.

To surface the Key Takeaway, consider asking:
- "How do we know a decimal is terminating?"
- "How do we know a decimal is repeating?"



Activity 1

Name: Date: Period:

Comparing Calculations

A student started converting these fractions to decimals using long division.

2. Finish writing $\frac{5}{8}$ as a decimal.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -0 \\ \hline 50 \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\frac{5}{8} = 0.625$

3. Finish writing $\frac{2}{11}$ as a decimal.

$$\begin{array}{r} 0.1818 \\ 11\overline{)2.0000} \\ -0 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \end{array}$$

$\frac{2}{11} = 0.\overline{18}$

4. Compare your calculations for $\frac{5}{8}$ and $\frac{2}{11}$. How are these alike? How are they different?

Alike: *Responses vary.* They are both less than 1.

Different: *Responses vary.* $\frac{5}{8}$ as a decimal ends after three digits, while $\frac{2}{11}$ as a decimal repeats and never ends.

Unit 4 Lesson 12 303 Activity 1

Key Takeaway: A **terminating decimal** is a value that has a set number of non-zero digits, such as 0.625. A **repeating decimal** is a value where one or more of its digits eventually form a string of numbers that repeats forever, such as $0.181818\ldots$, which we represent as $0.\overline{18}$.

Pairs | 10 min

Activity 2 Terminating or Repeating?

Purpose: Students practice using long division to write fractions as decimals.

Short on time? Consider having students complete just Problems 5 and 6 instead of all 4.

1 Launch

To surface connections, consider asking, "Make a prediction: Which of these four fractions do you think will result in a repeating decimal?" Have students make a prediction independently and then discuss with a partner.

Invite students to choose a problem to begin to test whether their prediction was correct.

A **Accessibility: Memory and Attention** To support students as they activate prior information they learned in Activity 1, create and maintain a display of the terms *terminating* and *repeating*, including definitions, examples, and non-examples.

2 Monitor

Look for student long division algorithms to share during the Connect.

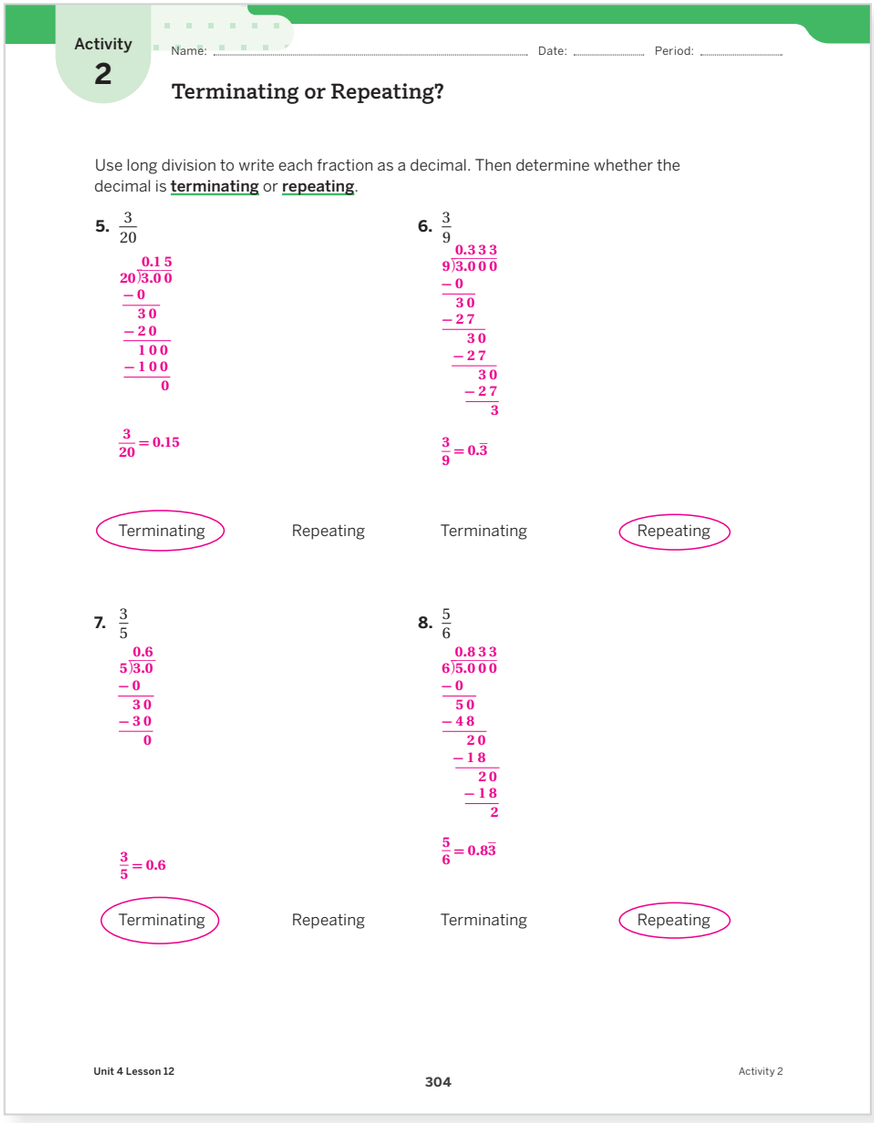
D **Differentiation** *(Problems 5–8)*

Look for students who:	Respond to Student Thinking
Need support with long division. *(Problems 5–8)*	**Support** Look back with them at Activity 1 and have them annotate the examples to make sense of how to follow the long division algorithm.
Put the overline over the entire decimal instead of starting at 3. *(Problem 8)*	**Strengthen** Consider asking, "What numbers would be repeating in your decimal? How can you adjust to make sure to not include the 8?"
Clearly show in their work the repeating decimal values. *(Problems 5–8)*	**Strengthen** Celebrate their thinking and encourage them to share during the Connect. Consider asking, "How are you clearly showing that the decimal is repeating?"
Finish early.	**Stretch** Invite these students to try to write one fraction whose decimal representation is repeating and one that is terminating.

3 Connect

Invite students to share their strategies for determining if the decimal representation was terminating or repeating.

Consider asking, "What patterns did you notice during your calculations? What was challenging?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Terminating or Repeating?

Use long division to write each fraction as a decimal. Then determine whether the decimal is **terminating** or **repeating**.

5. $\frac{3}{20}$

$20\overline{)3.00}$ → 0.15; -0, 30, -20, 100, -100, 0

$\frac{3}{20} = 0.15$

(Terminating) Repeating

6. $\frac{3}{9}$

$9\overline{)3.000}$ → 0.333; -0, 30, -27, 30, -27, 30, -27, 3

$\frac{3}{9} = 0.\overline{3}$

Terminating (Repeating)

7. $\frac{3}{5}$

$5\overline{)3.0}$ → 0.6; -0, 30, -30, 0

$\frac{3}{5} = 0.6$

(Terminating) Repeating

8. $\frac{5}{6}$

$6\overline{)5.000}$ → 0.833; -0, 50, -48, 20, -18, 20, -18, 2

$\frac{5}{6} = 0.8\overline{3}$

Terminating (Repeating)

Unit 4 Lesson 12 304 Activity 2

Pairs | 10 min

Activity 3 Decimal Deep Dive

Purpose: Students creatively apply the patterns they have noticed to determine fractions with specific numbers of repeating digits. **(MTR.5.1)**

Short on time? Consider assigning a row to a pair of students and have them work on their row to complete the class table.

1 Launch

Direct students' attention to the class table. To generate excitement, consider creating a challenge for the class to collectively determine every blank spot in the table.

ELL **English Language Learners** Invite students to share different meanings of the term digit. Clarify which meaning this word will have for this activity and how that will impact what types of fractions go in each row. **(Reading and Listening)**

2 Monitor

To support students getting started, consider asking, "What patterns did you notice about the two division problems we have done today that resulted in repeating decimals? Terminating?"

Look for students who apply patterns they notice in trying to find fractions that meet the requirement, such as: **(MTR.5.1)**

- When a fraction has a denominator of 2, 5, or 10, it results in a terminating decimal.
- When a fraction has a denominator of 3 and the numerator is not divisible by 3, then it results in a repeating decimal.
- When a decimal is divided by 2, it either adds another digit to a terminating decimal or to the number of digits that repeat in a repeating decimal.

Invite students to add fractions to the class table as they determine values that fit in each blank spot. Invite others in the class to check the division to make sure they belong in each row.

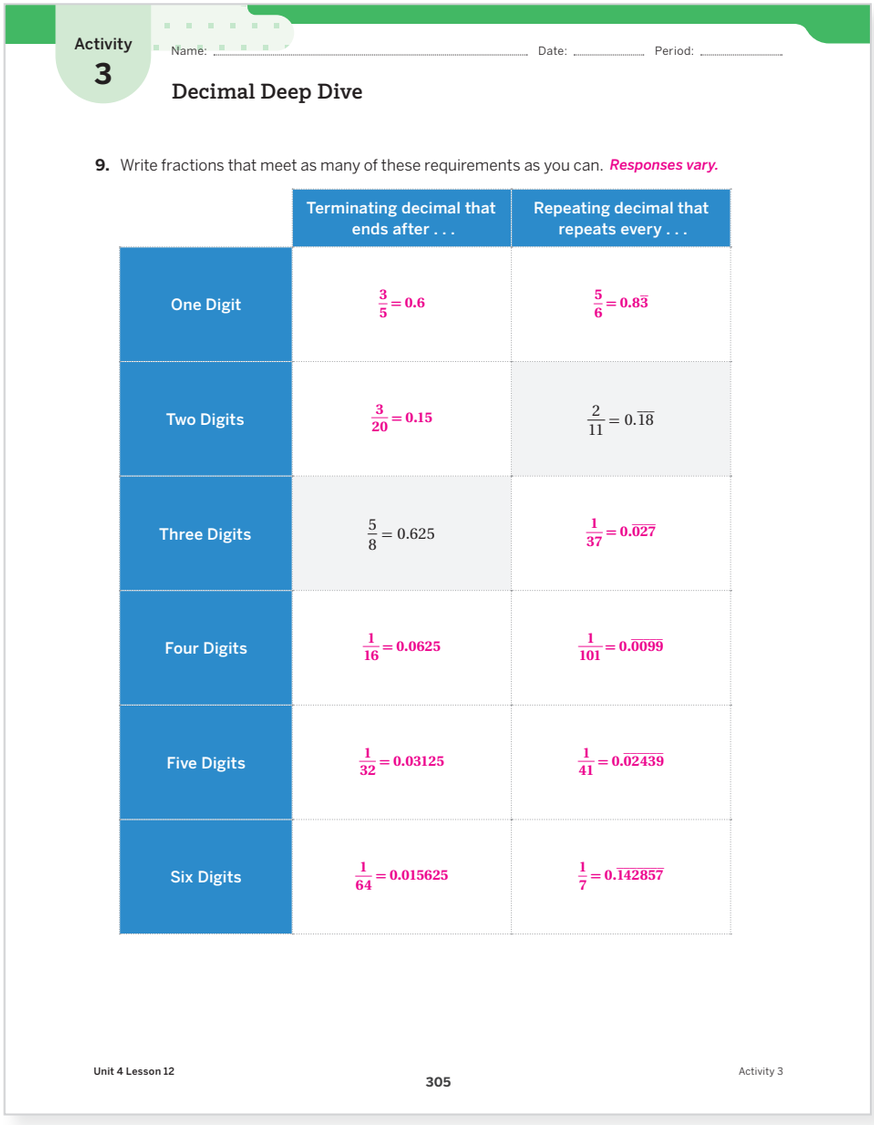
Note: Some students may benefit from access to a calculator after a time of productive struggle.

3 Connect

Invite students to share their strategies for determining fractions for each row.

Consider asking:

- "What are some patterns or strategies that you want to remember when solving problems in the future?" **(MTR.5.1)**
- "Were there particular boxes in the table that were easiest to determine a solution? Hardest to determine?"
- "What was challenging about determining decimals with a specific number of digits?"



Activity 3

Name: Date: Period:

Decimal Deep Dive

9. Write fractions that meet as many of these requirements as you can. *Responses vary.*

	Terminating decimal that ends after . . .	Repeating decimal that repeats every . . .
One Digit	$\frac{3}{5} = 0.6$	$\frac{5}{6} = 0.8\overline{3}$
Two Digits	$\frac{3}{20} = 0.15$	$\frac{2}{11} = 0.\overline{18}$
Three Digits	$\frac{5}{8} = 0.625$	$\frac{1}{37} = 0.\overline{027}$
Four Digits	$\frac{1}{16} = 0.0625$	$\frac{1}{101} = 0.\overline{0099}$
Five Digits	$\frac{1}{32} = 0.03125$	$\frac{1}{41} = 0.\overline{02439}$
Six Digits	$\frac{1}{64} = 0.015625$	$\frac{1}{7} = 0.\overline{142857}$

Unit 4 Lesson 12 305 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for writing a fraction as a decimal.

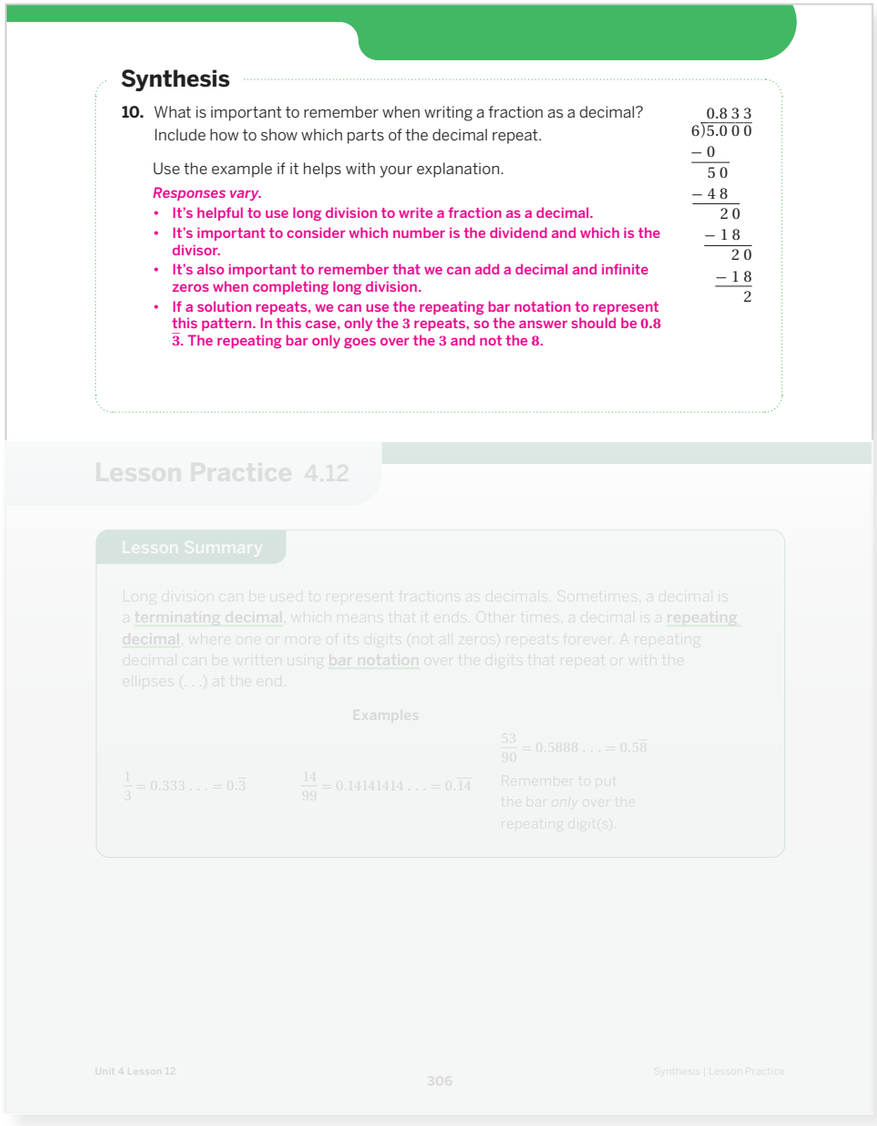
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The digits of a repeating decimal can be represented with a line over the digits that repeat.
- Terminating decimals have digits that end, but an infinite number of zeros can be added to the end of a terminating decimal.
- It is helpful to add 0 to the end of a number when doing long division.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies and examples about writing fractions as decimals as they share.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. What is important to remember when writing a fraction as a decimal? Include how to show which parts of the decimal repeat.

Use the example if it helps with your explanation.

$$\begin{array}{r} 0.833 \\ 6\overline{)5.000} \\ -\,0 \\ \hline 50 \\ -\,48 \\ \hline 20 \\ -\,18 \\ \hline 20 \\ -\,18 \\ \hline 2 \end{array}$$

Responses vary.

- It's helpful to use long division to write a fraction as a decimal.
- It's important to consider which number is the dividend and which is the divisor.
- It's also important to remember that we can add a decimal and infinite zeros when completing long division.
- If a solution repeats, we can use the repeating bar notation to represent this pattern. In this case, only the 3 repeats, so the answer should be $0.8\overline{3}$. The repeating bar only goes over the 3 and not the 8.

Lesson Practice 4.12

Lesson Summary

Long division can be used to represent fractions as decimals. Sometimes, a decimal is a terminating decimal, which means that it ends. Other times, a decimal is a repeating decimal, where one or more of its digits (not all zeros) repeats forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (. . .) at the end.

Examples

$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$

$\frac{14}{99} = 0.14141414\ldots = 0.\overline{14}$

$\frac{53}{90} = 0.5888\ldots = 0.5\overline{8}$

Remember to put the bar *only* over the repeating digit(s).

Unit 4 Lesson 12 306 Synthesis | Lesson Practice

Lesson Takeaway: Fractions can be written as decimals by using the numerator as the dividend and the denominator as the divisor. Decimals whose digits end are called **terminating decimals**. Decimals where one or more digits repeat forever are called **repeating decimals**.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by converting a fraction to a decimal using long division.

Today's Goals

1. **Goal:** Explore patterns that lead to *terminating* or *repeating decimals* when converting a fraction to a decimal using long division.
2. **Language Goal:** Describe strategies for determining whether a fraction will be represented as a *terminating* or *repeating decimal*. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.12

Use long division to write $\frac{4}{15}$ as a decimal. Determine whether the decimal is a terminating or repeating decimal.

$0.2\overline{6}$

Work varies.

$$
\begin{array}{r}
0.2666 \\
15\overline{)4.0000} \\
-0 \\
\hline
40 \\
-30 \\
\hline
100 \\
-90 \\
\hline
100 \\
-90 \\
\hline
100 \\
-90 \\
\hline
10
\end{array}
$$

This is a repeating decimal because the 6 repeats forever.

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support understanding long division:

» **Respond to Student Thinking:**

- Revisit Activity 2 and invite students to share their strategies for one or two problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In what ways did students' work in Activity 3 surprise you today? How were you able to celebrate student strategies as they explored **terminating** and **repeating decimals** with long division?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. What is important to remember when writing a fraction as a decimal? Include how to show which parts of the decimal repeat.

Use the example if it helps with your explanation.

$6\overline{)15.000}$ → 0.833, remainders 0, 50, 48, 20, 18, 20, 18, 2

Responses vary.

- It's helpful to use long division to write a fraction as a decimal.
- It's important to consider which number is the dividend and which is the divisor.
- It's also important to remember that we can add a decimal and infinite zeros when completing long division.
- If a solution repeats, we can use the repeating bar notation to represent this pattern. In this case, only the 3 repeats, so the answer should be $0.8\overline{3}$. The repeating bar only goes over the 3 and not the 8.

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Long division can be used to represent fractions as decimals. Sometimes, a decimal is a **terminating decimal**, which means that it ends. Other times, a decimal is a **repeating decimal**, where one or more of its digits (not all zeros) repeats forever. A repeating decimal can be written using **bar notation** over the digits that repeat or with the ellipses (. . .) at the end.

Examples

$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$

$\frac{14}{99} = 0.14141414\ldots = 0.\overline{14}$

$\frac{53}{90} = 0.5888\ldots = 0.5\overline{8}$

Remember to put the bar *only* over the repeating digit(s).

Lesson Practice 4.12

Name: _____ Date: _____ Period: _____

1. Finish writing $\frac{3}{8}$ as a decimal. Then determine if the decimal representation is terminating or repeating. Explain your thinking.

$8\overline{)3.000}$ → 0.375, remainders 0, 30, 24, 60, 56, 40, 40, 0

$\frac{3}{8} = 0.375$. Terminating.

Explanations vary. This decimal stops after three digits with no remainder.

Problems 2–4: Use the fractions $\frac{9}{25}$, $\frac{11}{30}$, and $\frac{4}{11}$.

2. Match each fraction to its decimal equivalent.

Fraction	Decimal
a. $\frac{9}{25}$	a. 0.36
b. $\frac{4}{11}$	c. $0.3\overline{6}$
c. $\frac{11}{30}$	b. $0.\overline{36}$

3. What is the same about the decimal representations and what is different?

Responses vary. All three fractions have the same values in the tenths and hundredths place, but $\frac{9}{25}$ terminates, while $\frac{11}{30}$ and $\frac{4}{11}$ repeat.

4. Which of these fractions has the greatest value? Explain how you know.

$\frac{11}{30}$. *Explanations vary.* The decimal representations of all three fractions have the same values in the tenths and hundredths place, and $\frac{11}{30}$ has the greatest value in the thousandths place.

5. Diego was writing fractions as repeating decimals. He used long division and determined $\frac{52}{225} = 0.23111111$. He wrote his answer as $0.2\overline{31}$.
Is Diego correct? Explain your thinking.

No. *Explanations vary.* Diego's answer of $0.2\overline{31}$ represents $0.231231231\ldots$
He should have written $0.23\overline{1}$.

FAST Practice

6. Complete the table.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
Decimal	0.5	0.25	0.125	0.0625	0.03125
Number of Digits in Decimal	1	2	3	4	5

How many digits will be in the decimal for the fraction $\frac{1}{256}$? [8] digits

Lesson Practice 4.12

Name: _____ Date: _____ Period: _____

Spiral Review

7. Select *all* the true statements.

- ☑ A. $2.25 > 2$
- ☐ B. $2.25 < 2$
- ☐ C. $-2.25 > -2$
- ☑ D. $-2.25 < -2$
- ☑ E. $2.25 > -2$

8. Irelle buys the following ingredients:

- $2\frac{1}{2}$ cups of chocolate chips for $8.00
- 2 cups of raisins for $6.50
- $\frac{2}{3}$ cups of shredded coconut for $2.00

Which ingredient is the most expensive per cup? Circle one.

Chocolate Chips (Raisins) Shredded Coconut

9. For each description or equation, decide whether it matches Diagram A, Diagram B, or neither. x represents the original value and y represents the final (new) value.

Diagram A

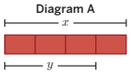
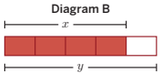


Diagram B



B An increase by 25%

A A decrease by 25%

Neither $y = 1.\overline{6}x$

Neither $y = 1.\overline{3}x$

A $y = 0.75x$

B $y = 1.25x$

10. You can make dried cucumbers through the process of dehydration, which involves removing water. A cucumber weighs 14 ounces. After dehydration, the cucumber weighs 0.56 ounces. What is the percent decrease in the weight of the cucumber before and after dehydration?

96%

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.NSO.1.2, MA.7.NSO.2.2
	3–5	2	MA.7.NSO.1.2
FAST Practice	6	3	MA.7.NSO.1.2
Spiral Review			
	7	1	MA.6.NSO.1.1
	8	2	MA.7.AR.3.2
	9–10	2	MA.6.AR.3.1

Unit 4

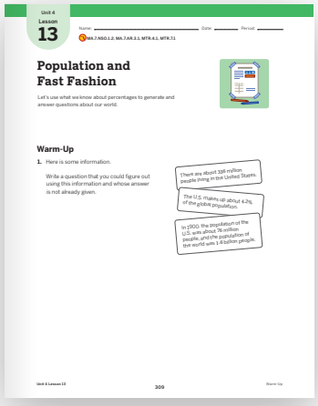
Lesson

13

Population and Fast Fashion

Posing Problems Involving Percents

Let's use what we know about percentages to generate and answer questions about our world.


Unit 4
Lesson
13
Name: Date: Period:
MA.7.NSO.1.2, MA.7.AR.3.1, MTR.4.1, MTR.7.1
Population and Fast Fashion
Let's use what we know about percentages to generate and answer questions about our world.
Warm-Up
1. Here is some information.
Write a question that you could figure out using this information and whose answer is not already given.
There are about 328 million people living in the United States.
The U.S. makes up about 4.2% of the global population.
In 1900, the population of the U.S. was about 76 million people, and the population of the world was 1.6 billion people.
Unit 4 Lesson 13
309
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Answer questions involving percent increase and decrease in the world.
2. **Language Goal:** Write questions about a situation that can be answered using proportional reasoning and percent increase or decrease. **(Reading, Writing, Speaking, and Listening)**

Students apply what they've learned about percent increase and decrease to write and answer questions about the world. Students generate interesting and answerable questions related to fast fashion based on given information. Then, they collaborate to discuss and revise their questions before working with a small group to answer their questions.

Coherence

Prior Learning

In Lessons 5–7, students solved problems involving percent changes in real-world contexts.

Future Learning

In Grade 8, students will continue to explore proportional relationships by comparing them using equations, tables, and graphs.

Connections

- Students use percentages to analyze problem situations in a real-world context of clothing manufacturing. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2

Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

MA.7.AR.3.1

Apply previous understanding of percentages and ratios to solve multi-step real world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building Toward

MA.6.AR.3.4

Lesson at a Glance

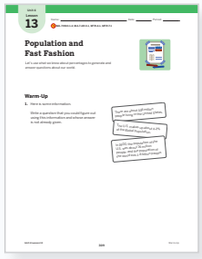
~ 45 min

MA.7.NSO.1.2, MA.7.AR.3.1

Warm-Up

Independent | 10 min

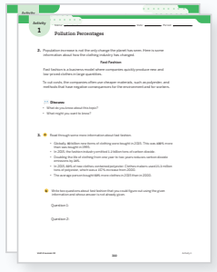
Students practice writing questions from a set of statistics and develop characteristics of a "good" question. MLR3: Critique, Correct, Clarify



Activity 1

Independent | 10 min

Students generate questions about a real-world situation involving percent increase or decrease and then work with several classmates to refine their questions.
Routines: Think-Pair-Share, MLR1: Stronger and Clearer Each Time



Activity 2

Small Groups | 15 min

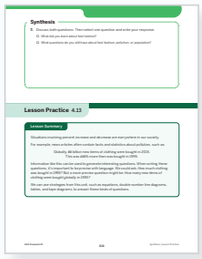
Students collaborate to choose appropriate tools and representations for determining the answers to their revised questions and create a poster to share with their classmates.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using percentages to make sense of contexts that affect the world, like the impact of the fashion industry.



Show What You Know

Independent | 5 min

Students demonstrate their understanding of percentages by writing and answering a question using two given facts.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- calculators
- coloring tools
- poster paper

Independent | 10 min

Warm-Up

Purpose: Students practice writing questions from a set of statistics and develop characteristics of a "good" question.

1 Launch

Direct students' attention to the information and invite a student to read the three statistics aloud. Consider asking, "What do these statistics make you want to know about?" **(MTR.7.1)**

Invite students to write a question that they can figure out using the information and is not already answered in the information cards.

ELL **English Language Learners** Invite a student to share an example question and display it for reference. **(Writing and Speaking)**

2 Connect

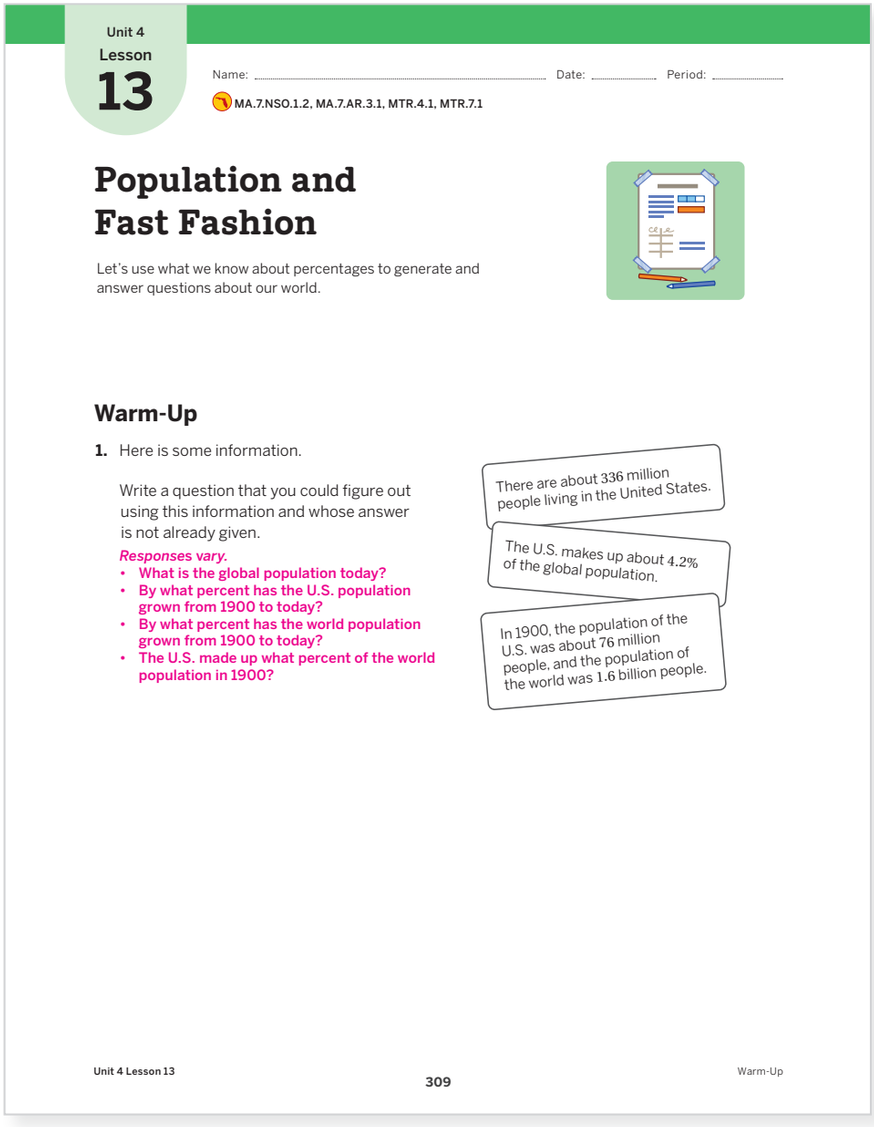
Capture and share a variety of questions, including ones that could be stronger or clearer.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in questions students generated. Invite students to discuss:

a. What is *clear* about the written question.
b. What is *unclear* or *unanswerable*. Consider asking: "Are there any words that are unclear?" and "Do you have all the information you would need to answer your question?"
c. How their question can be improved, or suggest a revised question. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "What do you think makes a good question about this information?"

Create a record of words or phrases students use to describe a "good" question for students to refer back to during the rest of the lesson.



Unit 4
Lesson
13

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.2, MA.7.AR.3.1, MTR.4.1, MTR.7.1

Population and Fast Fashion

Let's use what we know about percentages to generate and answer questions about our world.

Warm-Up

1. Here is some information.

   Write a question that you could figure out using this information and whose answer is not already given.

   Responses vary.
   - What is the global population today?
   - By what percent has the U.S. population grown from 1900 to today?
   - By what percent has the world population grown from 1900 to today?
   - The U.S. made up what percent of the world population in 1900?

There are about 336 million people living in the United States.

The U.S. makes up about 4.2% of the global population.

In 1900, the population of the U.S. was about 76 million people, and the population of the world was 1.6 billion people.

Unit 4 Lesson 13 309 Warm-Up

Key Takeaway: People who study real-world topics like population growth might ask many types of questions about the topic. Quantitative questions, as in questions dealing with quantities, should be clear, answerable, and use precise language. Additionally, well-written questions will be related to information that is known or given, but not immediately obvious from that information.

Independent | 10 min

Activity 1 Fast Fashion Percentages

Purpose: Students generate questions about a real-world situation involving percent increase or decrease and then work with several classmates to refine their questions.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse about population and fast fashion. See the Routine Facilitation Guide for more information.

Encourage students to read the information and identify words or phrases they are unsure of. Discuss these and consider creating a record for students to reference throughout the lesson.

Note: If time allows, invite students to research more about the impacts of fast fashion and about organizations working to change these impacts.

2 Monitor

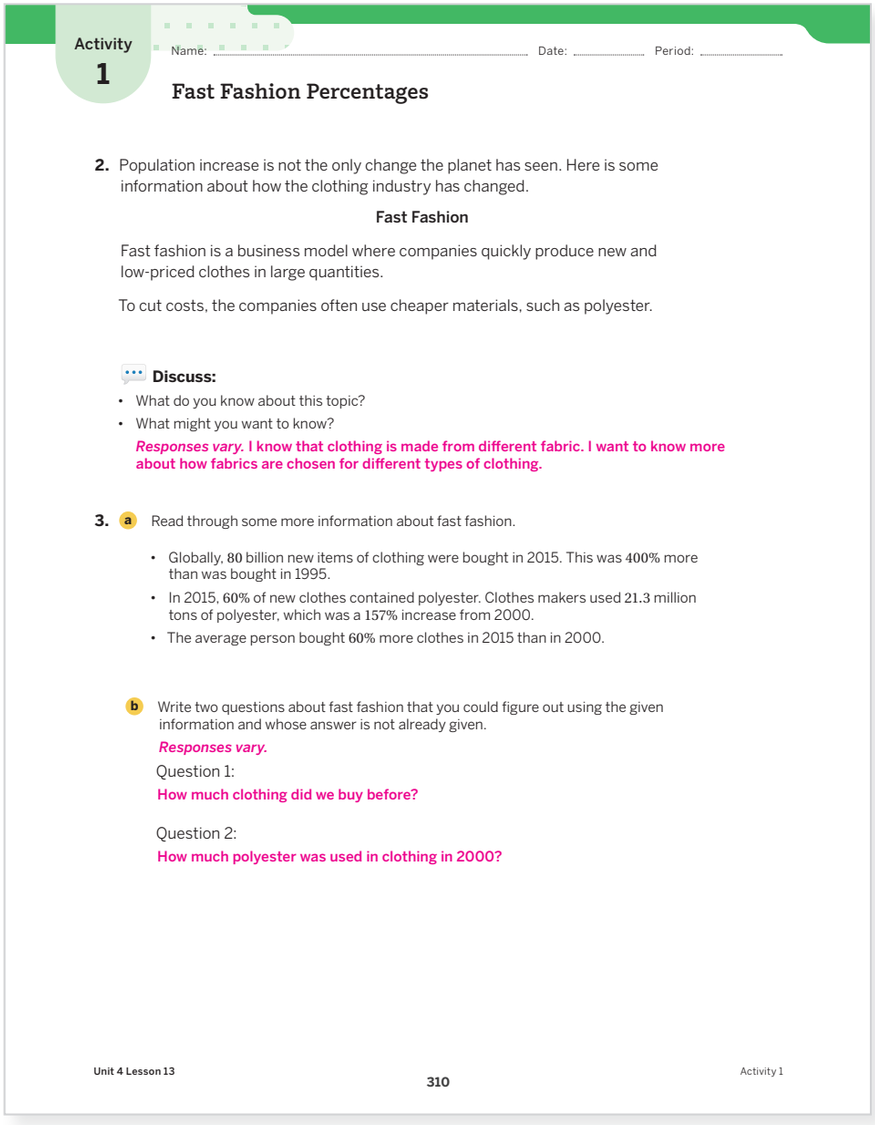
Invite students to write two questions that they could figure out using the information they have and for which the answers are not already given. **(MTR.7.1)**

Listen for students who reference different parts of the information as they write their question.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is something you would like to know but is not included in the information?"
Ask a question that can be answered without performing a calculation, such as, "Has there been an increase in new clothing items produced every year?"	**Support** Invite students to read the information to themselves again. Consider asking, "Can your question be answered without performing any calculations?"
Identify something is unknown from the information, such as: • The amount of polyester used in 2000. • The amount of new clothing items purchased in 1995.	**Strengthen** Invite students to pose their unknown as a question and to start thinking about what information they would need to answer their question.

Activity 1 continued >



Activity 1

Name: Date: Period:

Fast Fashion Percentages

2. Population increase is not the only change the planet has seen. Here is some information about how the clothing industry has changed.

Fast Fashion

Fast fashion is a business model where companies quickly produce new and low-priced clothes in large quantities.

To cut costs, the companies often use cheaper materials, such as polyester.

Discuss:

- What do you know about this topic?
- What might you want to know?

Responses vary. I know that clothing is made from different fabric. I want to know more about how fabrics are chosen for different types of clothing.

3. a Read through some more information about fast fashion.

- Globally, 80 billion new items of clothing were bought in 2015. This was 400% more than was bought in 1995.
- In 2015, 60% of new clothes contained polyester. Clothes makers used 21.3 million tons of polyester, which was a 157% increase from 2000.
- The average person bought 60% more clothes in 2015 than in 2000.

b Write two questions about fast fashion that you could figure out using the given information and whose answer is not already given.

Responses vary.

Question 1:

How much clothing did we buy before?

Question 2:

How much polyester was used in clothing in 2000?

Unit 4 Lesson 13 310 Activity 1

Activity 1 Fast Fashion Percentages (continued)

Purpose: Students generate questions about a real-world situation involving percent increase or decrease and then work with several classmates to refine their questions.

2 Monitor

Consider sharing that first drafts do not have to be complete or perfect to be valuable.

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft and use feedback to make their questions stronger and clearer.

Invite students to find partners in different locations to share their drafts, to encourage movement.

Encourage students to use the discussion questions from the problem and the record of good questions from the Warm-Up to provide support for their partners in writing new drafts that are stronger and clearer.

A **Accessibility: Affective Functioning** To support students in sharing and receiving feedback, model giving and receiving constructive feedback as a class.

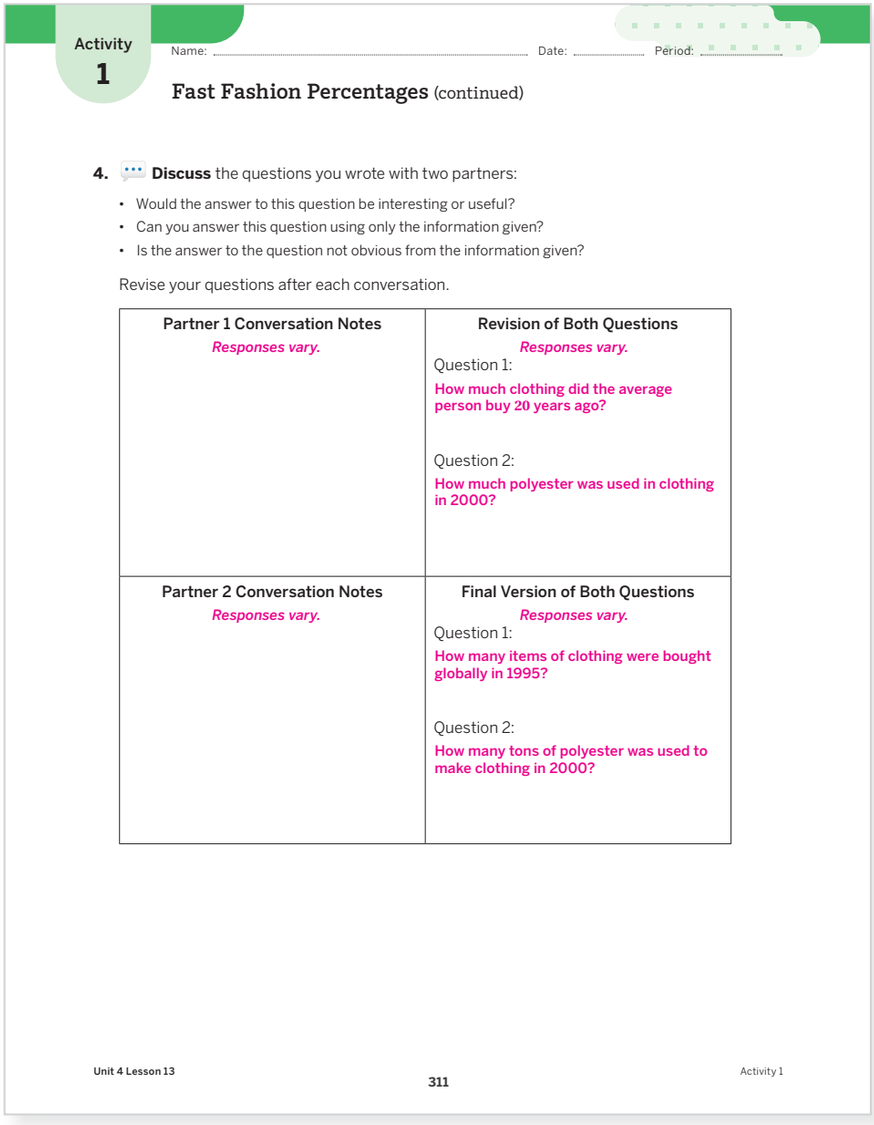
ELL **English Language Learners** Encourage students to rephrase their partners' questions before providing any feedback. Provide sentence frames, such as, "I think you are asking _____. Is that correct?" or "You are wondering _____ about _____. Is that correct?" **(Speaking and Listening)**

Invite students to use the feedback they got from their partners to write a second draft that is stronger (with more or better evidence) or clearer (with more precision, organization, etc.).

3 Connect

Invite students to share their final version of the two questions and how the conversations with their partners were helpful in revising their questions to be stronger and clearer. **(MTR.4.1)**

Consider asking, "How are the questions in your final version more precise than those from your first draft?"



Activity 1

Name: Date: Period:

Fast Fashion Percentages (continued)

4. **Discuss** the questions you wrote with two partners:
   - Would the answer to this question be interesting or useful?
   - Can you answer this question using only the information given?
   - Is the answer to the question not obvious from the information given?

   Revise your questions after each conversation.

Partner 1 Conversation Notes	Revision of Both Questions
Responses vary.	*Responses vary.* Question 1: How much clothing did the average person buy 20 years ago? Question 2: How much polyester was used in clothing in 2000?
Partner 2 Conversation Notes	**Final Version of Both Questions**
Responses vary.	*Responses vary.* Question 1: How many items of clothing were bought globally in 1995? Question 2: How many tons of polyester was used to make clothing in 2000?

Unit 4 Lesson 13 311 Activity 1

Small Groups | 15 min

Activity 2 Make a Poster

Purpose: Students collaborate to choose appropriate tools and representations for determining the answers to their revised questions and create a poster to share with their classmates.

Short on time? Consider omitting Problem 6 to maximize time for question-posing and problem-solving.

1 Launch

Invite students to select two questions from the ones they have written and answer them.

Distribute the poster making supplies to students, as needed.

2 Monitor

Look for student strategies for choosing representations or tools that will help them to answer their questions.

D Differentiation

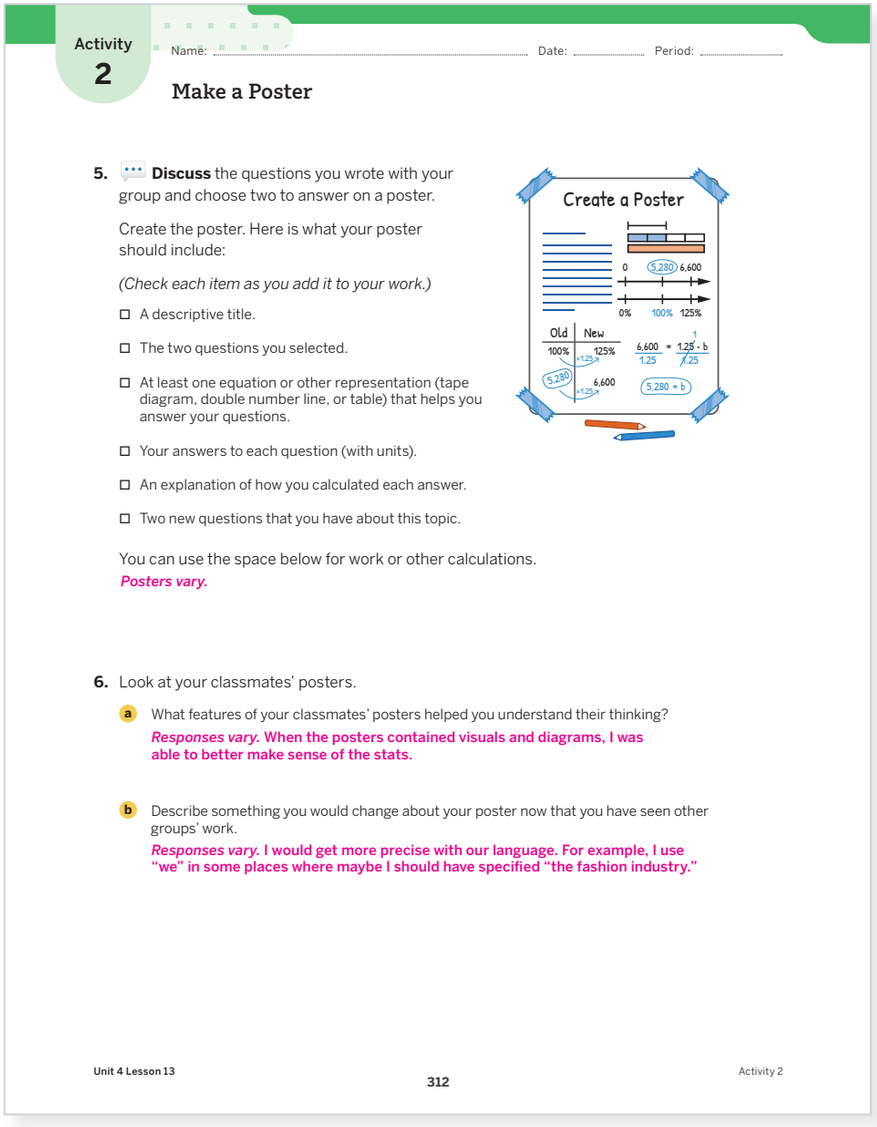
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What piece of information is missing (e.g., the original amount, new amount, or percent change)?"
Think that one representation can work to answer both questions.	**Support** Consider sharing this may happen with a double number line, diagram, or table, but in an equation, the unknown value will change according to the situation, so a second equation is necessary.
Need support creating representations.	**Support** Consider providing access to blank templates of double number line diagrams, tables, and equation structures for them to use to complete and add to their posters.

Note: If time allows, invite students to do a Gallery Tour and observe other groups' work. Encourage them to consider strengths in the representations other groups chose and the questions they asked.

3 Connect

Invite students to share what surprised them after determining the answers to their questions.

Note: If it does not come up naturally, consider highlighting the potential to reduce carbon emissions by extending the life of clothes. This may help to showcase how positive change is happening.



Activity 2

Name: Date: Period:

Make a Poster

5. **Discuss** the questions you wrote with your group and choose two to answer on a poster.

Create the poster. Here is what your poster should include:

(Check each item as you add it to your work.)

- ☐ A descriptive title.
- ☐ The two questions you selected.
- ☐ At least one equation or other representation (tape diagram, double number line, or table) that helps you answer your questions.
- ☐ Your answers to each question (with units).
- ☐ An explanation of how you calculated each answer.
- ☐ Two new questions that you have about this topic.

You can use the space below for work or other calculations.

Posters vary.

6. Look at your classmates' posters.

a. What features of your classmates' posters helped you understand their thinking?

Responses vary. When the posters contained visuals and diagrams, I was able to better make sense of the stats.

b. Describe something you would change about your poster now that you have seen other groups' work.

Responses vary. I would get more precise with our language. For example, I use "we" in some places where maybe I should have specified "the fashion industry."

Unit 4 Lesson 13 312 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using percentages to make sense of contexts that affect the world, like the impact of the fashion industry.

Invite students to respond independently, and then share their thinking with a partner.

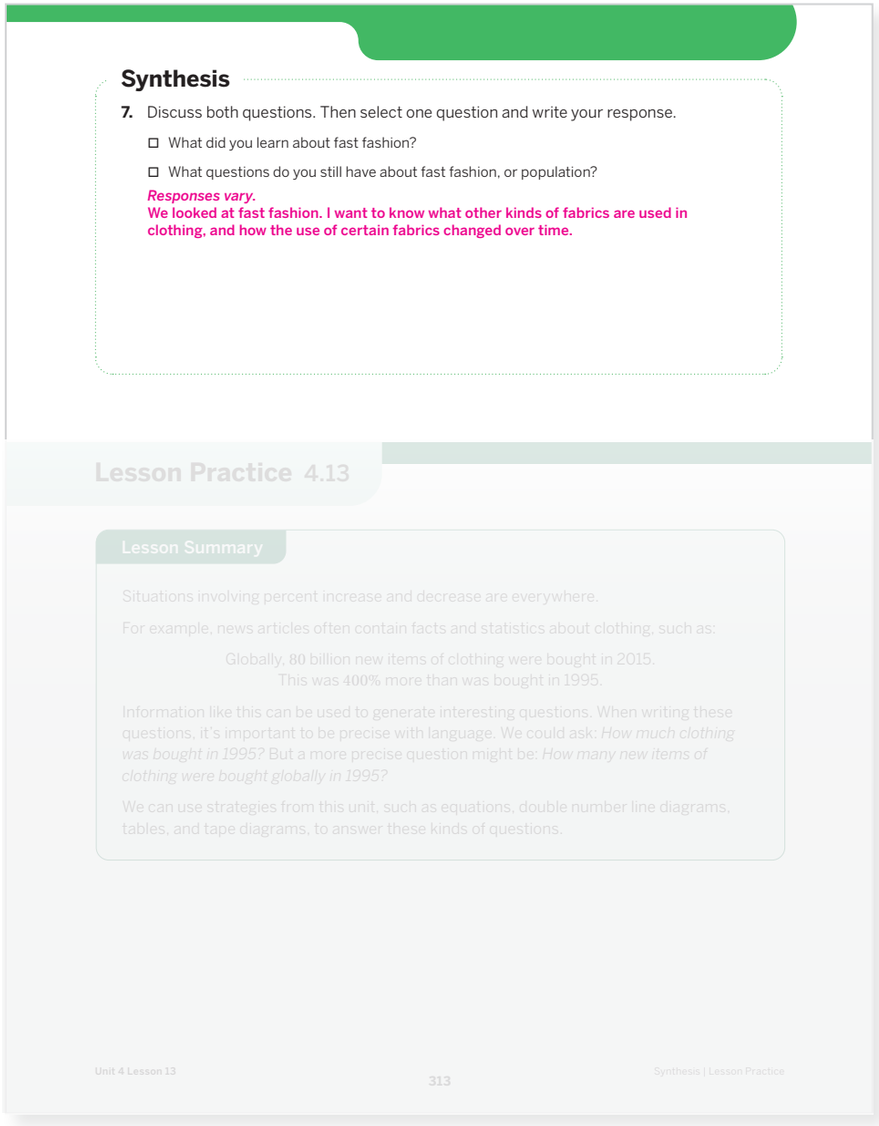
Consider sharing, "Mathematics is a powerful tool used to make sense of our world and it can help us find ways to advocate for change that will improve issues like fast fashion." **(MTR.7.1)**

Invite students to share the new questions they have about fast fashion. Consider asking:

- "What did you learn about fast fashion that was surprising or new?"
- "Where might you go to learn more about these topics?"
- "What actions might you suggest for yourself or others around these topics?"

Note: If time allows, discuss both the population information and fast fashion information. Invite students to discuss the percent increases in each of these topics and any possible connections between these topics.

Note: If time allows, invite students to share other topics they are interested in knowing more about that can be analyzed using percent change.



Synthesis

7. Discuss both questions. Then select one question and write your response.

- ☐ What did you learn about fast fashion?
- ☐ What questions do you still have about fast fashion, or population?

Responses vary.
We looked at fast fashion. I want to know what other kinds of fabrics are used in clothing, and how the use of certain fabrics changed over time.

Lesson Practice 4.13

Lesson Summary

Situations involving percent increase and decrease are everywhere.

For example, news articles often contain facts and statistics about clothing, such as:

Globally, 80 billion new items of clothing were bought in 2015.
This was 400% more than was bought in 1995.

Information like this can be used to generate interesting questions. When writing these questions, it's important to be precise with language. We could ask: *How much clothing was bought in 1995?* But a more precise question might be: *How many new items of clothing were bought globally in 1995?*

We can use strategies from this unit, such as equations, double number line diagrams, tables, and tape diagrams, to answer these kinds of questions.

Unit 4 Lesson 13 313 Synthesis | Lesson Practice

Lesson Takeaway: Mathematics is a powerful lens that supports us with answering questions about the world in which we live. Tape diagrams, double number lines, tables, and equations are all representations that support answering questions about proportional relationships in our world that involve a percent increase or percent decrease.

Independent | 5 min

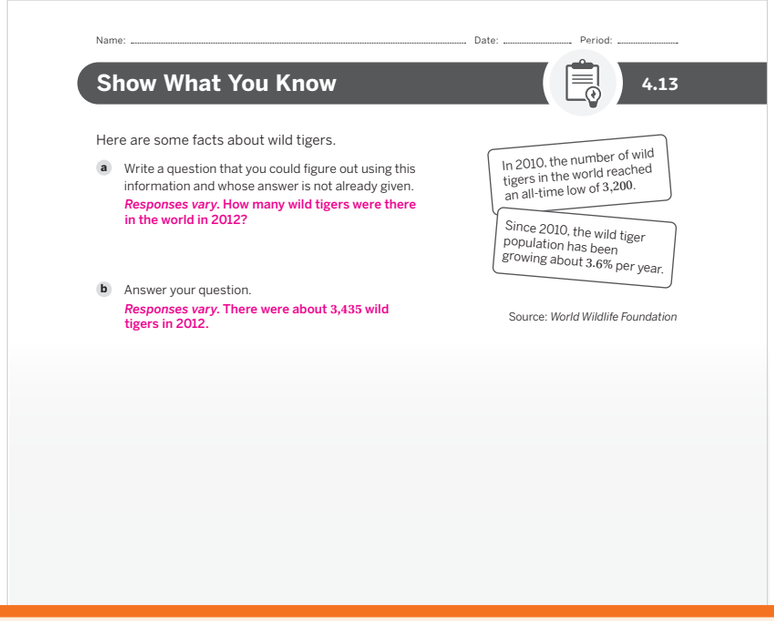
Show What You Know

Purpose: Students demonstrate their understanding of percentages by writing and answering a question using two given facts.

Today's Goals

1. **Goal:** Answer questions involving percent increase and decrease in the world.
2. **Language Goal:** Write questions about a situation that can be answered using proportional reasoning and percent increase or decrease. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.13

Here are some facts about wild tigers.

a. Write a question that you could figure out using this information and whose answer is not already given.
Responses vary. How many wild tigers were there in the world in 2012?

b. Answer your question.
Responses vary. There were about 3,435 wild tigers in 2012.

In 2010, the number of wild tigers in the world reached an all-time low of 3,200.

Since 2010, the wild tiger population has been growing about 3.6% per year.

Source: *World Wildlife Foundation*

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of write questions about percent change based on data:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to research initiatives related to fast fashion and then share what they learn.

Professional Learning

Consider the important issues raised in this lesson. What aspects of fast fashion are you more knowledgeable about? Less knowledgeable? In what ways were students able to make connections between math ideas and the real-world situations in this lesson? How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Discuss both questions. Then select one question and write your response.

   ☐ What did you learn about fast fashion?

   ☐ What questions do you still have about fast fashion, or population?

Responses vary.
We looked at fast fashion. I want to know what other kinds of fabrics are used in clothing, and how the use of certain fabrics changed over time.

Lesson Practice 4.13

Lesson Summary

Situations involving percent increase and decrease are everywhere.

For example, news articles often contain facts and statistics about clothing, such as:

Globally, 80 billion new items of clothing were bought in 2015.
This was 400% more than was bought in 1995.

Information like this can be used to generate interesting questions. When writing these questions, it's important to be precise with language. We could ask: *How much clothing was bought in 1995?* But a more precise question might be: *How many new items of clothing were bought globally in 1995?*

We can use strategies from this unit, such as equations, double number line diagrams, tables, and tape diagrams, to answer these kinds of questions.

Lesson Practice 4.13

Name: _____ Date: _____ Period: _____

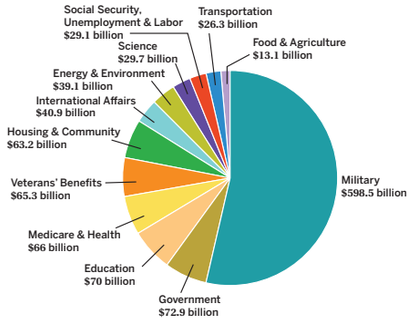
Problems 1–2: A city has a 5% sales tax.

1. A toothbrush costs $3.40 before tax. How much does it cost including tax?
   $3.57

2. A book costs $32.55 after tax. How much did it cost before tax?
   $31

Problems 3–5: In the federal government's budget, there is mandatory spending and discretionary spending. Discretionary spending includes all spending where lawmakers choose how much to spend. The pie chart shows the discretionary spending in 2015. That year, discretionary spending represented about 29% of the total budget.

Total U.S. federal government discretionary spending in 2015: $1.11 trillion



3. Complete each sentence: *In 2015, military spending made up about* ___54___*% of the discretionary budget. This was about* ___16___*% of the federal government's total budget.*

4. Write a question that you could figure out using the given information and whose answer is not already given.
   Responses vary. **What percent of the government's discretionary spending went to education in 2015?**

5. Answer the question you wrote.
   Responses vary. **About 6.3%.**

Lesson Practice 4.13

Name: _____ Date: _____ Period: _____

6. The price of gold is often reported per ounce. At the end of 2005, gold was $513 per ounce. At the end of 2015, it was $1,060 per ounce. By what percent did the price increase?
   By about 107%

7. A grocery store allows customers to use multiple coupons when checking out. Suppose you have a $5 off coupon and a 10% off coupon. The register will calculate the new price after each coupon is used. Does the order you use the coupons make a difference? Explain your thinking.
   Yes, the order makes a difference. For buying an item that costs $10, you save an extra $0.50 by using the 10% off coupon first.
   Explanations vary. **I will test what will happen if I buy an item that costs $10 and check both possible orders for the coupons.**

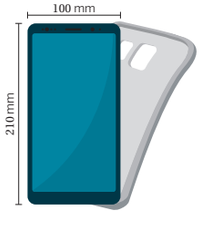
Using $5 off first:	Using 10% off first:
$10 - 5 = 5$	$10 \cdot 0.9 = 9$
$5 \cdot 0.9 = 4.50$	$9 - 5 = 4$
The item will cost $4.50.	The item will cost $4.00.

FAST Practice

8. A greeting card costs $6 before tax. A customer has a coupon for a 10% discount. Then a 5% sales tax is added. How much will the customer pay for the greeting card?
   $ 5.67

Spiral Review

9. Here are the dimensions of a phone case designed by the Soft Shield company. The size of a case can be up to 1% off and still fit a phone. Determine the largest and smallest width and height of the case that will fit a phone.
   The phone case's width can be as small as 99 millimeters and as wide as 101 millimeters. Its height can be as small as 207.9 millimeters and as large as 212.1 millimeters.



10. What is the decimal equivalent of $\frac{13}{100}$?
    0.13

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.AR.3.1
	3–6	2	MA.7.AR.3.1
	7	3	MA.7.AR.3.1
FAST Practice	8	2	MA.7.AR.3.1
Spiral Review			
	8–9	2	MA.7.AR.3.1
	10–11	1	MA.7.NSO.1.2

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students applied what they have learned about proportional relationships to solve problems involving percent change and fractional quantities.

Career Connection | Meet Ebonya Washington

Invite students to read about how banks have different kinds of accounts that people can use to help manage their money. Some banks pay interest as a percentage of the money in the account. Then invite students to read about economics professor Ebonya Washington.

Consider asking:

- "Why might it be important to research the types of accounts different banks might offer before opening an account with a particular bank?"

B.E.S.T. Benchmark Connections

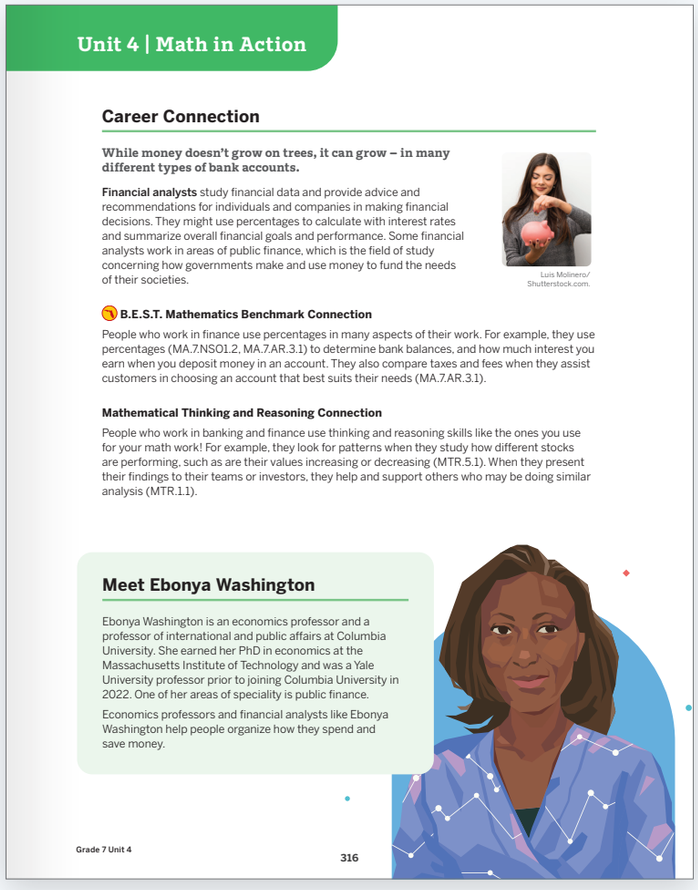
Consider asking:

- "What other math concepts have you studied in Units 1–4 that you could apply to banking and finance?" Sample response: Explaining information about different bank accounts or expenses can be done using tables, graphs, and equations to look for relationships (MA.7.AR.4.2, MA.7.AR.4.4).

Mathematical Thinking and Reasoning Connections

Consider asking:

- Ebonya Washington is a financial analyst. She specializes in public finance, which involves collecting taxes. How could she and others who work in public finance apply the strategies of Mathematical Thinking and Reasoning standard MTR.2.1 to help her teams and people who want to understand taxes better ?" Sample response: She can use models and drawings to help build understanding of how taxes are calculated.



Unit 4 | Math in Action

Career Connection

While money doesn't grow on trees, it can grow – in many different types of bank accounts.

Financial analysts study financial data and provide advice and recommendations for individuals and companies in making financial decisions. They might use percentages to calculate with interest rates and summarize overall financial goals and performance. Some financial analysts work in areas of public finance, which is the field of study concerning how governments make and use money to fund the needs of their societies.

Luis Molinero/ Shutterstock.com.

B.E.S.T. Mathematics Benchmark Connection

People who work in finance use percentages in many aspects of their work. For example, they use percentages (MA.7.NSO1.2, MA.7.AR.3.1) to determine bank balances, and how much interest you earn when you deposit money in an account. They also compare taxes and fees when they assist customers in choosing an account that best suits their needs (MA.7.AR.3.1).

Mathematical Thinking and Reasoning Connection

People who work in banking and finance use thinking and reasoning skills like the ones you use for your math work! For example, they look for patterns when they study how different stocks are performing, such as are their values increasing or decreasing (MTR.5.1). When they present their findings to their teams or investors, they help and support others who may be doing similar analysis (MTR.1.1).

Meet Ebonya Washington

Ebonya Washington is an economics professor and a professor of international and public affairs at Columbia University. She earned her PhD in economics at the Massachusetts Institute of Technology and was a Yale University professor prior to joining Columbia University in 2022. One of her areas of speciality is public finance.

Economics professors and financial analysts like Ebonya Washington help people organize how they spend and save money.

Grade 7 Unit 4

316

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining the result of a percent increase	Lesson 1	1	MA.7.AR.3.1
2	Selecting true statements associated with ratios of fractions	Lesson 10	2	MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.5
3	Calculating the percent error given a desired measurement and an actual measurement	Lesson 8	1	MA.7.AR.3.1
4	Reasoning about multistep percent problems	Lesson 1	2	MA.7.AR.3.1
5	Determining the percent change given an original value and a new value	Lesson 3	1	MA.7.AR.3.1
6a	Writing equations that model a percent increase or decrease	Lesson 6	1	MA.7.AR.3.1
6b	Calculating the original amount given a percent change and new amount	Lesson 4	1	MA.7.AR.3.1
7a	Calculate the result of percent increases or decreases	Lesson 13	1	MA.7.AR.3.1
7b	Writing equations that represent a percent increase or decrease	Lesson 6	1	MA.7.AR.3.1, MTR.7.1
7c	Determining the result of a percent increase	Lesson 6	1	MA.7.AR.3.1, MTR.3.1
8	Calculating the interest paid over a period of time	Lesson 7	1	MA.7.AR.3.1, MTR.7.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 4

1. A car is 160 inches long. A truck is 7% longer than the car. How long is the truck?

A. 272 inches **B.** 171.2 inches

C. 11.2 inches **D.** 167 inches

2. Dalia is painting her room. After painting $\frac{1}{10}$ of her room, Dalia has used $\frac{1}{2}$ of a can of paint. Select *all* of the true statements.

- ☐ **A.** Each can of paint will cover $\frac{1}{20}$ of the Dalia's room.
- ☑ **B.** 3 cans of paint will cover 60% of the Dalia's room.
- ☑ **C.** Dalia's entire room requires 5 cans of paint.
- ☐ **D.** Dalia's entire room requires 20 cans of paint.
- ☐ **E.** Painting $\frac{1}{2}$ of Dalia's room requires 10 cans of paint.

3. Oliver wants to cut a piece of metal to be 30 centimeters long. The piece of metal ends up being 29.4 centimeters long. What is the percent error in this situation?

A. 1.02% **B.** 0.6% **C.** 2% **D.** 20%

4. A lamp originally costs $30. Krishna has a 5% off coupon for the lamp. After the coupon, a 5% sales tax is applied. He will pay:

A. More than $30. **B.** Less than $30. **C.** Exactly $30.

Explain your thinking.

Explanations vary. **After a 5% discount, the lamp costs $28.50. Then after a 5% tax, the lamp costs about $29.93.**

5. The population of Renton, WA is about 101,920 people. 10 years ago, the population was about 91,000 people.[1]

By what percent did the population increase in the last 10 years?

12%

[1] Source: *US Census Bureau*

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

6. A store is offering a 20% discount on all items.

a Write an equation for the relationship between the discount price of an item, d, and its original price, p.

Responses vary.

- $d = 0.8p$
- $d = p - 0.2p$
- $d = (1 - 0.2)p$
- $d = \frac{4}{5}p$

b The price of a hat after the discount is $18. What was the original price?

$22.50

7. The cost of college is expected to increase by 3.5% next year.

a The cost to attend Westish College is currently $18,000 per year. If the cost to attend Westish increases by 3.5%, what would the cost be next year?

$18,630

b Write an equation to represent the cost of *any* college as it increases by 3.5%. Use t to represent this year's cost and n to represent next year's cost.

Responses vary.

- $n = 1.035t$
- $n = t + 0.035t$
- $n = (1 + 0.035)t$

c The cost to attend Faber College increased by 3.5% since last year. The new cost is $23,805. What was the cost last year? Explain or show your thinking.

$23,000. ***Explanations vary.*** **Since $23,805 is the amount after an increase, I divided. $\frac{23805}{1.035}$ is $23,000.**

8. A loan is taken out to cover the cost of a home renovation. The bank offers a simple interest rate of 6.2% per year for a 4-year term. If the loan amount is $1,200, what is the total amount that must be repaid at the end of 4 years?

$1,297.60

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 4

1. A pencil is 120 millimeters long. A marker is 9% longer than the pencil. How long is the marker?

A. 130.8 millimeters ✓

B. 10.8 millimeters

C. 228 millimeters

D. 129 millimeters

2. Kayleen is mowing a field. After mowing $\frac{1}{8}$ of the field, she used $\frac{1}{2}$ of a gallon of gas. Select *all* of the true statements.

☑ **A.** Mowing the entire field requires 4 gallons of gas.

☐ **B.** Mowing the entire field requires 16 gallons of gas.

☑ **C.** Each gallon of gas will allow Kayleen to mow $\frac{1}{4}$ of the field.

☑ **D.** Mowing $\frac{1}{2}$ of the field requires 2 gallons of gas.

☐ **E.** 3 gallons of gas will allow Kayleen to mow 50% of the field.

3. The temperature in a room is 20.0°C. A thermometer measures it as 20.8°C. What is the percent error in this situation?

A. 40%

B. 4% ✓

C. 1.04%

D. 0.8%

4. Na'ilah purchases a rug that costs $50. A 10% sales tax is applied. After the tax, Na'ilah uses a 10% off coupon. She will pay:

A. Exactly $50.

B. More than $50.

C. Less than $50. ✓

Explain your thinking.

Explanations vary. **After a 10% tax, the rug costs $55. Then after a 10% coupon, the rug costs $49.50.**

5. The population of Bristol, PA is about 53,110 people. Thirty years ago, the population was about 56,500 people.[1]

By what percent did the population decrease in the last 30 years?

6%

[1] Source: *US Census Bureau*

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Name: ..... Date: ..... Period: .....

6. Nicolas has a coupon for a 60% discount at his favorite store.

a. Write an equation for the relationship between the discount price of an item, d, and its original price, p.

Responses vary.
- $d = 0.4p$
- $d = p - 0.6p$
- $d = (1 - 0.6)p$
- $d = \frac{2}{5}p$

b. The price of a coat after the discount is $21. What was the original price?

$52.50

7. The price of new cars and trucks is expected to increase by 2.9% next year.

a. The price of a specific car is $17,000. If the price of the car increases by 2.9%, what would the price be next year?

$17,493

b. Write an equation to represent the price of *any* car as it increases by 2.9%. Use t to represent this year's price and n to represent next year's price.

Responses vary.
- $n = 1.029t$
- $n = t + 0.029t$
- $n = (1 + 0.029)t$

c. The price of a truck increased by 2.9% since last year. The new price is $22,638. What was the price last year? Explain or show your thinking.

$22,000. *Explanations vary.* **Since $22,638 is the amount after an increase, I divided. $\frac{22638}{1.029}$ is $22,000.**

8. A loan is taken out to cover the cost of purchasing new furniture. The bank offers a simple interest rate of 5.8% per year for a 3-year term. If the loan amount is $1,500, what is the total amount that must be repaid at the end of 3 years?

$1,761

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Solve problems involving percent change using representations, such as tape diagrams, double number lines, tables, and equations. **(Lessons 1–5)**	1, 5, 6b	Support	• **Mini-Lesson: Calculating Percentages**
		Strengthen	• **Teacher Move:** Consider revisiting: • Lesson 1 (*More or Less*) • Lesson 3 (*All the Equations*) • Lesson 4 (*Percent Machines*)
		Stretch	• **You're invited to explore more:** • Lesson 1 (*More or Less*) • Lesson 2 (*All the Equations*) • Lesson 3 (*100%*)
	6a, 7b	Support	• **Teacher Move:** Consider revisiting Lesson 2 (*All the Equations*)
		Strengthen	• **Teacher Move:** Invite students to share strategies with a partner.
		Stretch	• **You're invited to explore more:** Lesson 2 (*All the Equations*)
Sub-Unit 2: • Use proportional reasoning to solve problems in real-world situations involving percent increase, percent decrease, and percent error. • Interpret situations involving percent change and apply appropriate representations to solve problems. **(Lessons 6–8)**	3	Support	• **Teacher Move:** Consider revisiting Lesson 8 (*Bookcase Builder*)
		Strengthen	• **Challenge Your Shelf:** Lesson 8 (*Bookcase Builder*)
		Stretch	• **You're invited to explore more:** Lesson 8 (*Bookcase Builder*)
	4	Support	• **Mini-Lesson: Finding Sales Tax and Tip**
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 6 (*Tax,Tips, and Fees*)
	7a, 7c	Support	• **Teacher Move:** Consider revisiting: • Lesson 6 (*Tax, Tips, and Fees*) • Lesson 8 (*Bookcase Builder*)
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 6 (*Tax,Tips, and Fees*)
Sub-Unit 3: • Determine unknown values in proportional relationships involving fractional quantities. • Convert between fraction and decimal representations. **(Lessons 9–13)**	2	Support	• **Mini-Lesson**: **Applying Fractions in Proportional Relationships**
		Strengthen	• **Teacher Move:** Invite students to share strategies with a partner.
		Stretch	• **You're invited to explore more:** Lesson 9 (*Sticker Sizes*)

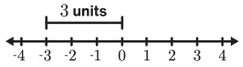
Glossary/Glosario

English

Español

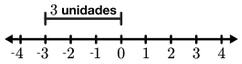
A

absolute value The distance from 0 to a number on a number line is its absolute value.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

valor absoluto La distancia del 0 a un número en una recta numérica es su valor absoluto.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

approximation A rounded value that you can use to represent a number that may be difficult to work with, such as a never-ending decimal.

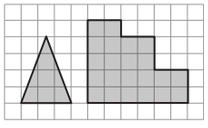
For example, the exact value of pi (π) is a never-ending decimal, so we often use the approximate value of 3.14 in calculations involving pi.

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un decimal infinito

Por ejemplo, el valor exacto de pi (π) es un decimal infinito, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

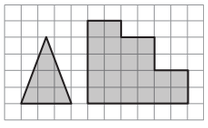
area The space inside a two-dimensional figure. It is expressed in square units.

The area of the triangle is 6 square units. The area of the other shape is 22 square units.



área El espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.

El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.



B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.

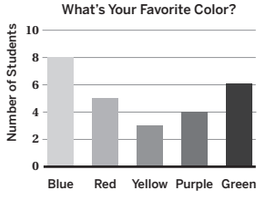
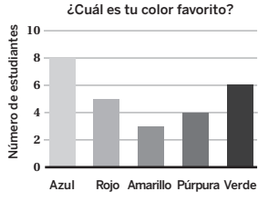


gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.



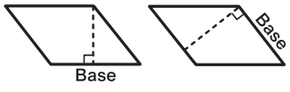
bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

For example, the repeating decimal $0.090909...$ can be written in bar notation as $0.\overline{09}$.

raya indicadora de decimales periódicos (vinculum) Una forma de representar los dígitos que se repiten en un número decimal. Sobre los dígitos que se repiten se traza una pequeña línea.

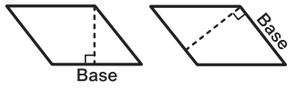
Por ejemplo, el decimal periódico $0.090909...$ puede escribirse con la raya indicadora de decimales periódicos (vinculum) como $0.\overline{09}$.

base (of a parallelogram) One side of a parallelogram. We can choose any side to be the base.



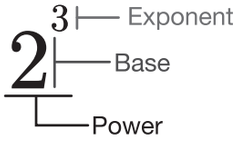
The base can also refer to the length of this side. The height of a shape is perpendicular to the base.

base (de un paralelogramo) Un lado de un paralelogramo. Podemos elegir cualquier lado como base.



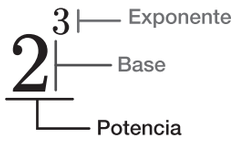
La base también puede referirse a la longitud de este lado. La altura de una figura es perpendicular a la base.

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.



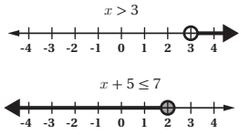
In the expression 2^3, 2 is the base.

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.



En la expresión 2^3, 2 es la base.

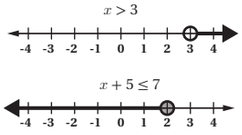
boundary point The point on a number line or graph that separates solutions of an inequality from non-solutions.



If the boundary point is a solution to the inequality (i.e., $\geq$ or $\leq$), it is represented with a closed circle on the graph. If it's not a solution (i.e., $>$ or $<$), it is represented with an open circle on the graph.

The boundary point of $x > 3$ is 3. The boundary point of $x + 5 \leq 7$ is 2.

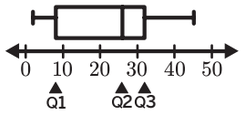
punto límite El punto en una recta numérica o una gráfica que separa las soluciones de una desigualdad de los valores que no son soluciones.
Si el punto límite es una solución de la desigualdad (es decir, $\geq$ o $\leq$), se representa con un círculo cerrado en la gráfica. Si no es una solución (es decir, $>$ o $<$), se representa con un círculo abierto en la gráfica.



El punto límite de $x > 3$ es 3. El punto límite de $x + 5 \leq 7$ es 2.

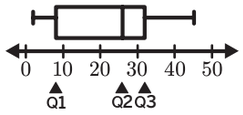
English

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



Español

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.

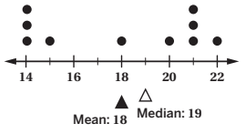


C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

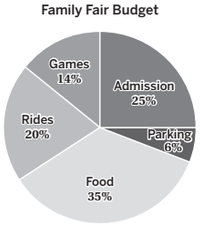
"What kind of pet do you have?" is a question that would result in categorical data.

center A single value that can be used to summarize the typical value in a data set. Mean and median are measures of center.



circle A shape made out of all the points that are the same distance from a center point.

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a pie chart.



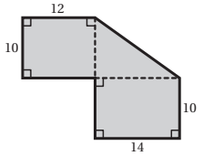
circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string. The circumference of a circle, C, can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression $5x + 8$, 5 is the coefficient of x.

composite figure A two- or three-dimensional figure that can be decomposed into smaller figures.

The composite figure can be decomposed into two rectangles and a triangle.



constant A value that does not change. In an algebraic expression, the constant term does not contain a variable.

In the expression $7x + 9$, 9 is the constant term.

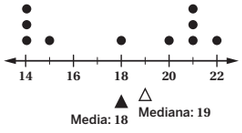
In the expression $5x - 8$, -8 is the constant term.

In the expression $12 - 4x$, 12 is the constant term.

datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

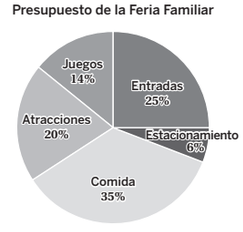
"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

centro Un solo valor que puede utilizarse para resumir el valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central, o sea, medidas de centro.



círculo Una figura formada por todos los puntos que están a la misma distancia de un punto central.

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado gráfico de torta.



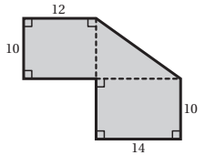
circunferencia La distancia alrededor de un círculo. Si imaginamos el círculo como un trozo de cuerda, es la longitud de la cuerda. La circunferencia de un círculo, C, puede calcularse mediante la fórmula $C = \pi d$, donde d es el diámetro del círculo, o $C = 2\pi r$, donde r es el radio.

coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

En la expresión $5x + 8$, el coeficiente de x es 5.

figura compuesta Figura bidimensional o tridimensional que puede descomponerse en figuras más pequeñas.

La figura compuesta se puede descomponer en dos rectángulos y un triángulo.



constante Un valor que no cambia. En una expresión algebraica, el término constante no contiene una variable.

En la expresión $7x + 9$, 9 es el término constante.

En la expresión $5x - 8$, -8 es el término constante.

En la expresión $12 - 4x$, 12 es el término constante.

Glossary/Glosario

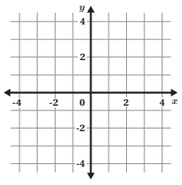
English

constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

Carpet (sq. ft.)		Cost (dollars)
10	$\times\frac{2}{3}$	15.00
20	$\times\frac{2}{3}$	30.00
50	$\times\frac{2}{3}$	75.00

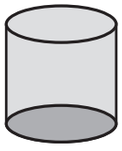
In this table, one constant of proportionality is $\frac{2}{3}$.

coordinate plane The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



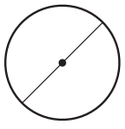
correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

cylinder A 3-D figure that has two parallel circular bases connected by a curved surface.

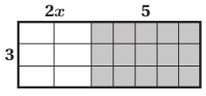


D

diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.



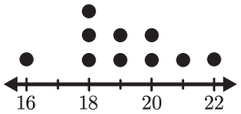
distributive property The property that says $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.



Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dot plot A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other. A dot plot is sometimes called a line plot.



Number of Jelly Beans

For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

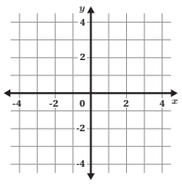
Español

constante de proporcionalidad En una relación proporcional, una constante de proporcionalidad es el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)		Costo (dólares)
10	$\times\frac{2}{3}$	15.00
20	$\times\frac{2}{3}$	30.00
50	$\times\frac{2}{3}$	75.00

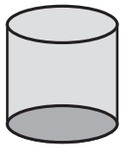
En esta tabla, una constante de proporcionalidad es $\frac{2}{3}$.

plano de coordenadas, plano cartesiano El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).

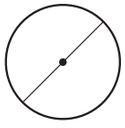


corresponder (partes correspondientes) Corresponder es coincidir. Cuando una parte de una figura original corresponde con una parte de una copia, se llaman partes correspondientes. Pueden ser puntos, segmentos, ángulos o distancias.

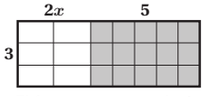
cilindro Una figura tridimensional que tiene dos bases circulares paralelas conectadas por una superficie curva.



diámetro La distancia entre un punto y otro en un círculo, pasando por el centro. También es la distancia mayor entre un punto y otro en un círculo.



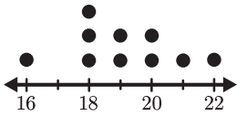
propiedad distributiva La propiedad que indica que $a(b + c) = ab + ac$. Significa que multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente y luego sumar los productos.



El área como un producto
$3(2x + 5)$

Por ejemplo, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

diagrama de puntos Una forma de visualizar conjuntos de datos numéricos, en la que cada punto de datos se representa mediante un punto en una recta numérica. Los puntos de datos con el mismo valor se apilan unos sobre otros. Un diagrama de puntos a veces se conoce como gráfica de puntos.



Cantidad de frijolitos

Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

English

Español

E

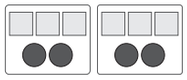
equivalent expressions Expressions that are equal for every value of a variable.

For example, $6x + 2x$ is equivalent to $5x + 3x$. No matter what the value of x is, the two expressions will be equal.

expresiones equivalentes Expresiones que son iguales para cualquier valor de una variable.

Por ejemplo, $6x + 2x$ es equivalente a $5x + 3x$. Independientemente del valor de x, las dos expresiones serán iguales.

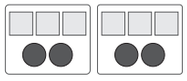
equivalent ratios Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



$3 : 2$ is equivalent to $6 : 4$ because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. Both recipes will taste the same.

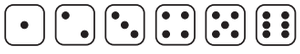
razónes equivalentes Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



$3 : 2$ es equivalente a $6 : 4$ porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

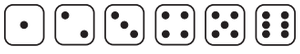
Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. Ambas recetas tendrán el mismo sabor.

event A set of one or more outcomes in a chance experiment.



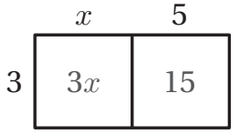
For example, if we roll a number cube, there are six possible outcomes.

evento Un conjunto de uno o más resultados en un experimento de azar.



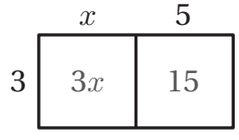
Por ejemplo, si tiramos un dado, hay seis resultados posibles.

expand To expand is to use the distributive property to multiply the factors in an expression and rewrite the expression as a sum. The new expression is equivalent to the original expression.



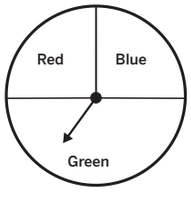
For example, we can expand $3(x + 5)$ to get the equivalent expression $3x + 15$.
The sum of the two boxes in the diagram show the expression in expanded form.

expandir Expandir significa usar la propiedad distributiva para multiplicar los factores en una expresión y reescribir la expresión como una suma. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos expandir $3(x + 5)$ para obtener la expresión equivalente $3x + 15$.
La suma de las dos casillas en el diagrama muestra la expresión en forma expandida.

experiment An action or activity that you can do over and over again without knowing what the outcome will be each time.



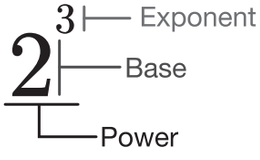
For example, each time you spin the spinner, it could land on red, blue, or green.

experimento Una acción o actividad que se puede llevar a cabo una y otra vez sin saber cuál será el resultado en cada ocasión.



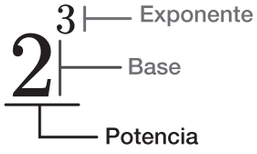
Por ejemplo, cada vez que se gira la ruleta, podría caer en rojo, azul o verde.

exponent A number used to describe repeated multiplication.



In this example, 3 is the exponent, which means $2^3 = 2 \cdot 2 \cdot 2$.

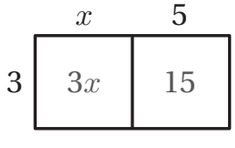
exponente Un número que se usa para describir multiplicaciones repetidas.



En este ejemplo, 3 es el exponente, lo cual significa que $2^3 = 2 \cdot 2 \cdot 2$.

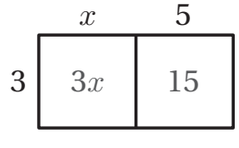
F

factor To factor is to use the distributive property to rewrite an expression as the product of two or more factors. The new expression is equivalent to the original expression.



For example, we can factor $3x + 15$ to get the equivalent expression $3(x + 5)$.
The product of the two sides in the diagram show the expression in factored form.

factorizar Factorizar significa usar la propiedad distributiva para reescribir una expresión como el producto de dos o más factores. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos factorizar $3x + 15$ para obtener la expresión equivalente $3(x + 5)$.
El producto de los dos lados del diagrama muestra la expresión en forma factorizada.

Glossary/Glosario

English

Español

H

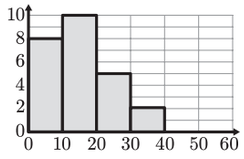
height (of a parallelogram) The shortest distance between a base and its opposite side. The height is always perpendicular to the base.



altura (de un paralelogramo) La distancia más corta entre una base y su lado opuesto. La altura siempre es perpendicular a la base.

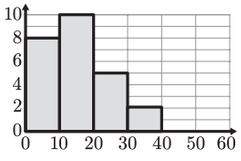


histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.



For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.

histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.



Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

I

integer All whole numbers and their opposites.

For example, 35, -15, and 1 are integers. 0.3 and $\frac{1}{3}$ are not.

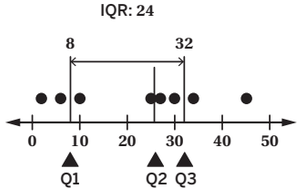
enteros Todos los números enteros y sus opuestos.

Por ejemplo, 35, -15 y 1 son números enteros. 0.3 y $\frac{1}{3}$ no lo son.

interest The extra money that you pay on a loan or the money that you earn when you save or invest.

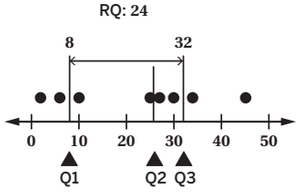
interés El dinero extra que pagas por un préstamo o el dinero que ganas cuando ahorras o inviertes.

interquartile range (IQR) One wayto measure how spread out a data set is. We also call this the IQR. The IQR is calculated as the distance from the first quartile (Q1) to the third quartile (Q3).



For example, the IQR of this data set is 24 because $32 - 8 = 24$.

rango intercuartílico (RQ) Una formade medir cuán disperso está un conjunto de datos. También lo denominamos RQ. El RQ se calcula como la distancia entre el primer cuartil (Q1) y el tercer cuartil (Q3).



Por ejemplo, el RQ de este conjunto de datos es 24 porque $32 - 8 = 24$.

inverse operations Two operations that undo each other are called inverse operations.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

L

like terms Terms with variables and exponents that are the same.

For example, $8x$ and $12x$ are like terms because both terms have a variable of x. $3x$ and $3x^2$ are not like terms because they have different exponents.

términos semejantes Términos con variables y exponentes iguales.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable que incluye x. $3x$ y $3x^2$ no son términos semejantes porque tienen exponentes diferentes.

M

markdown The amount of decrease between the original price of an item and its sale price.

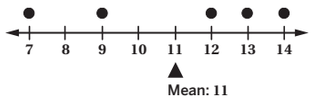
markup The amount of increase between the store's cost of an item and the item's selling price.

rebaja El monto de descuento entre el precio original de un artículo y su precio de venta.

sobreprecio La cantidad de aumento entre el costo de un artículo en la tienda y el precio de venta del artículo.

English

mean A measure of center. If you equally distribute a set of items into different groups, the mean is the number of items in each group.



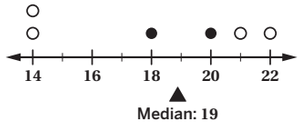
It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.

For example, for the data set $7, 9, 12, 13, 14$, the mean is 11 because $7 + 9 + 12 + 13 + 14 = 55$ and $\frac{55}{5} = 11$.

measure of center A single number that summarizes all of the values in a data set. It is usually a typical value for a data set. Mean and median are measures of center.

measure of variability A single number that describes the spread, or distribution, of a data set. Range and interquartile range are measures of variability. Also called a measure of variation.

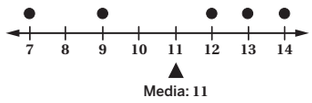
median A measure of center. The median is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values.



The median of this data set is 19 because it is the average of the two middle values: 18 and 20.

Español

media Una medida de tendencia central. Si distribuyes equitativamente un conjunto de elementos en diferentes grupos, la media es el número de artículos en cada grupo.



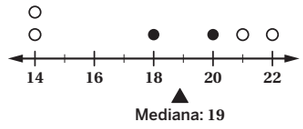
Para calcular la media, se pueden sumar los valores de todos los puntos de datos y luego dividir por la cantidad de puntos de datos.

Por ejemplo, en el conjunto de datos $7, 9, 12, 13, 14$, la media es 11 porque $7 + 9 + 12 + 13 + 14 = 55$ y $\frac{55}{5} = 11$.

medida de tendencia central, medida de centro Un solo número que resume todos los valores de un conjunto de datos. Suele ser un valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central.

medida de variabilidad Un solo número que describe la dispersión, o la distribución, de un conjunto de datos. El rango y el rango intercuartil son medidas de variabilidad. También llamada medida de variación.

mediana Una medida de tendenciacentral. La mediana es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay un número par de puntos de datos, la mediana es el promedio de los dos números centrales.

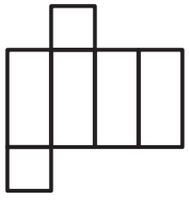


La mediana de este conjunto de datos es 19 porque es el promedio de los dos valores centrales: 18 y 20.

N

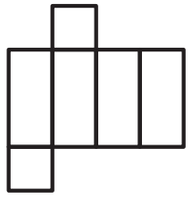
net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

Here is a net for a rectangular prism.



red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.

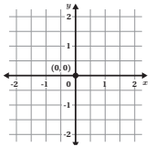
Esta es una red de un prisma rectangular.



O

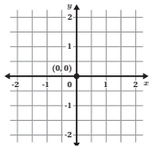
order of operations A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone. The standard order of operations is parentheses/grouping symbols, exponents/roots, multiplication/division, and then addition/subtraction.

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



orden de las operaciones Un orden coherente aplicado a una expresión con múltiples operaciones para que cualquiera pueda evaluar la expresión de la misma manera. El orden estándar de las operaciones es paréntesis/símbolos de agrupación, exponentes/raíces, multiplicación/división y, luego, suma/resta.

origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan el eje x y el eje y.



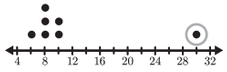
Glossary/Glosario

English

outcome One of the possible results of an experiment.

For example, the possible outcomes of tossing a coin are heads and tails.

outlier A value that is much higher or much lower than the other values in a set of data.



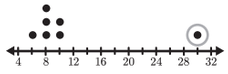
The circled point is an outlier.

Español

resultado Uno de los posibles resultados de un experimento.

Por ejemplo, los posibles resultados de lanzar una moneda son cara y sello.

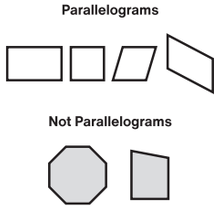
valor atípico Un valor que es mucho mayor o mucho menor que los demás valores de un conjunto de datos.



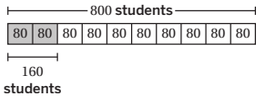
El punto encerrado en un círculo es un valor atípico.

P

parallelogram A quadrilateral that has two pairs of parallel sides. Opposite, or parallel, sides of a parallelogram are the same length.

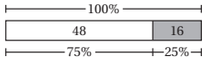


percent (percentage) Percent means for every 100. It is represented by the percent symbol: %. We use percentages to represent ratios and fractions.



For example, 20% means $20 : 100$. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number.
Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

percent decrease How much a quantity goes down, expressed as a percentage of the starting amount.

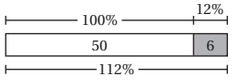


For example, a store had 64 hats in stock on Friday.
They had 48 hats left on Saturday. The amount of hats went down by 16.
This was a 25% decrease because 16 is 25% of 64.

percent error A way to describe the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a box is supposed to have 150 folders in it. Clare counts only 147 folders in the box. This is an error of 3 folders. The percent error is 2% because 3 is 2% of 150. $\frac{3}{150} = 0.02 = 2\%$

percent increase How much a quantity goes up, expressed as a percentage of the starting amount.



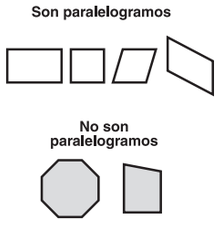
For example, Elena had $\$50$ on Monday. She helped a neighbor, so she had $\$56$ on Tuesday. The amount went up by $\$6$.
This was a 12% increase because 6 is 12% of 50. $\frac{6}{50} = 0.12 = 12\%$

perpendicular Describes a line that crosses or meets another line at a $90°$ angle.

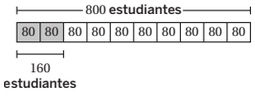
pi A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π.

Some common approximations for π are 3.14 and $\frac{22}{7}$.

paralelogramo Un cuadrilátero que tiene dos pares de lados paralelos. Los lados opuestos, o paralelos, de un paralelogramo tienen la misma longitud.

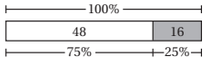


por ciento (porcentaje) Por ciento significa por cada 100. Se representa con el símbolo de porcentaje: %. Usamos porcentajes para representar razones y fracciones.



Por ejemplo, 20% significa $20 : 100$. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número.
Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos están en una excursión, entonces eso es 160 estudiantes porque 20 están de viaje por cada 100 estudiantes en total.

disminución porcentual Describe cuánto disminuye una cantidad, expresada como un porcentaje de la cantidad inicial.

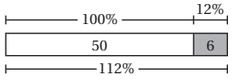


Por ejemplo, una tienda tenía 64 sombreros en inventario el viernes. El sábado le quedaban 48 sombreros.
La cantidad de sombreros se redujo en 16.
Fue una disminución del 25% porque 16 es el 25% de 64.

porcentaje de error, error porcentual Una forma de describir la diferencia entre un valor deseado y el valor real, expresada como un porcentaje del valor deseado.

Por ejemplo, se supone que una caja contiene 150 carpetas. Clare cuenta solo 147 carpetas en la caja. Ese es un error de 3 carpetas. El porcentaje de error es del 2% porque 3 es el 2% de 150. $\frac{3}{150} = 0.02 = 2\%$

aumento porcentual Cuánto aumenta una cantidad, expresada como un porcentaje de la cantidad inicial.



Por ejemplo, Elena tenía $\$50$ el lunes. Ayudó a un vecino, así que el martes tenía $\$56$. La cantidad aumentó en $\$6$.
Fue un aumento del 12% porque 6 es el 12% de 50. $\frac{6}{50} = 0.12 = 12\%$

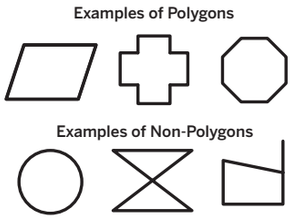
perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de $90°$.

pi Un número que representa la constante de proporcionalidad entre el diámetro y la circunferencia de cualquier círculo. El símbolo de pi es π.

Algunas aproximaciones comunes de π son 3.14 y $\frac{22}{7}$.

English

polygon A closed 2-D shape with straight sides that do not cross each other.



population A set of people or things that are being studied.

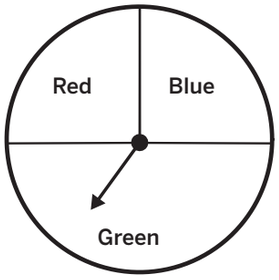
For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

power of ten A number written in the form 10^n, where n represents the number of times 10 is multiplied.

principal The total amount (initial value) of money borrowed or invested, not including any interest.

probability A number that tells us howlikely an event is to happen. A probability of 1 means the event is certain. A probability of 0 means the event is impossible.

For example, the probability of spinning red is $\frac{1}{4}$.



proportional Quantities are proportional if they form equivalent ratios.

For example, the heights and bases of these triangles are proportional because the ratio $6:12$ is equivalent to $4:8$.

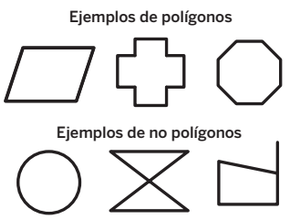
proportional relationship A collection of pairs of numbers that are in equivalent ratios. Two variables are in a proportional relationship if you can multiply the values for one variable by the same number to get the values for the other variable. The graph of a proportional relationship is a straight line passing through the origin, $(0, 0)$.

Carpet (sq. ft)	Cost (dollars)
10	15.00
20	30.00
50	75.00

For example, if $y = kx$, then y is said to be directly proportional to x and the constant of proportionality is k. Every cost in the table is equal to 1.5 times the number of square feet of carpet.

Español

polígono Una figura bidimensionalcerrada con lados rectos que no se cruzan.



población Conjunto de personas o cosas que se estudian.

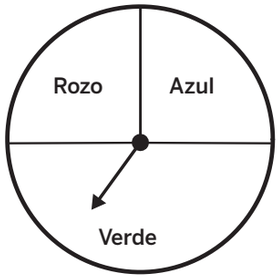
Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

potencia de diez Un número escrito de la forma 10^n, donde n representa el número de veces que se multiplica el 10.

principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, excluyendo los intereses.

probabilidad Número que indica qué tan probable es que ocurra un evento.Una probabilidad de 1 significa que el evento es seguro. Una probabilidad de 0 significa que el evento es imposible.

Por ejemplo, la probabilidad de girar y que caiga rojo es $\frac{1}{4}$.



proporcional Diferentes cantidades son proporcionales si forman razones equivalentes.

Por ejemplo, las alturas y las bases de estos triángulos son proporcionales porque la relación $6:12$ es equivalente a $4:8$.

relación proporcional Un conjunto de pares de números que están en proporciones equivalentes. Dos variables están en una relación proporcional si puedes multiplicar los valores de una variable por el mismo número para obtener los valores para la otra variable. La gráfica de una relación proporcional es una línea recta que pasa por el origen, $(0, 0)$.

Alfombra (pies cuadrados)	Costo (dólares)
10	15.00
20	30.00
50	75.00

Por ejemplo, si $y = kx$, entonces se dice que y es directamente proporcional a x y la constante de proporcionalidad es k. Cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

Q

quadrilateral A polygon that has four sides.

Squares, rectangles, and parallelograms are examples of quadrilaterals.

cuadrilátero Un polígono que tiene cuatro lados.

Los cuadrados, rectángulos y paralelogramos son ejemplos de cuadriláteros.

Glossary/Glosario

English

Español

R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.



radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.

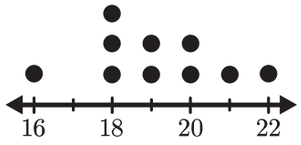


random Outcomes of a chance experiment are random if they are all equally likely to happen.

aleatorio Los resultados de un experimento de azar son aleatorios si todos tienen la misma probabilidad de suceder.

range A measure of spread. It is the difference between the maximum and minimum values in a data set.

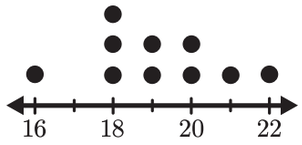
For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.



Number of Jelly Beans

rango Una medida de dispersión. Es la diferencia entre los valores máximo y mínimo de un conjunto de datos.

Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.



Cantidad de frijolitos

rational number Any number that can be written as a fraction, including whole numbers and negative numbers.

For example, -10, 2.5, $\frac{3}{7}$, and 82 are all rational numbers.

número racional Cualquier número que pueda escribirse como una fracción, incluidos los números enteros y los números negativos.

Por ejemplo, -10, 2.5, $\frac{3}{7}$ y 82 son números racionales.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \times \frac{2}{3} = 1$.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \times \frac{2}{3} = 1$.

relative frequency The number of times an outcome happens divided by the number of repeated experiments. After several experiments, the relative frequency of an outcome gets close to its probability.

For example, if you spin a spinner 12 times and it lands on red 5 times, that means the relative frequency of landing on red is $\frac{5}{12}$.

frecuencia relativa El número de veces que se produce un resultado dividido por el número de experimentos repetidos. Al cabo de varios experimentos, la frecuencia relativa de un resultado se aproxima a su probabilidad.

Por ejemplo, si se hace girar una ruleta 12 veces y cae en rojo 5 veces, la frecuencia relativa de que caiga en rojo es $\frac{5}{12}$.

repeating decimal A decimal with one or more digits (not all zeros) that repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (...) at the end.

If the repeating digits are all zeros, it would be called a terminating decimal.

For example, the decimal representation of $\frac{1}{3}$ is $0.\overline{3}$, which means $0.33333...$

The decimal representation of $\frac{25}{22}$ is $1.1\overline{36}$, which means $1.1363636...$

decimal periódico Un decimal con uno o más dígitos (no todos son ceros) que se repiten infinitamente. Un decimal periódico puede escribirse usando la raya indicadora de decimales periódicos (vinculum) encima de los dígitos que se repiten o con puntos suspensivos (...) al final.

Si todos los dígitos periódicos son ceros, entonces se denomina decimal exacto.

Por ejemplo, la representación decimal de $\frac{1}{3}$ es $0.\overline{3}$, lo que significa $0.33333...$

La representación decimal de $\frac{25}{22}$ es $1.1\overline{36}$, o sea $1.1363636...$

rhombus A quadrilateral containing four equal-length sides.



rombo Un cuadrilátero que contiene cuatro lados de igual longitud.



S

sample A part of a population.

For example, a population could be all the seventh grade students at one school. One sample of that population is all the seventh grade students who are in band.

muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

English

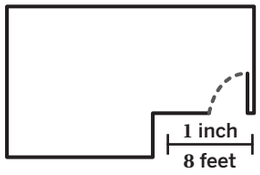
sample space The list of every possible outcome for a chance experiment.

For example, the sample space for tossing two coins is: heads-heads, tails-heads, heads-tails, tails-tails.



scale A scale tells us how the actual measurements of an object are represented in a drawing.

The scale of this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room.



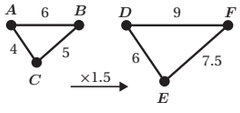
scale drawing A two-dimensional representation of an actual object or place. All the measurements in the drawing correspond to the measurements of the actual object by the same scale. Floor plans and maps are examples of scale drawings.



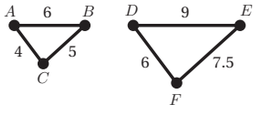
A scale may show that 1 centimeter on a map represents 30 miles on land. The distance marked on this map is 8 centimeters, or 240 miles.

scale factor The number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from ABC to DEF is 1.5.



scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a shape looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.



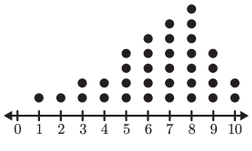
For example, triangle DEF is a scaled copy of triangle ABC.

shape A description of the distribution (or pattern) of the data within a data set. Descriptions of the shape of a distribution can include: symmetry, skew, clusters, and outliers.

simple interest Interest that is calculated solely based on the initial amount.

$I = prt$, where I = interest, p = principal, r = rate, and t = time

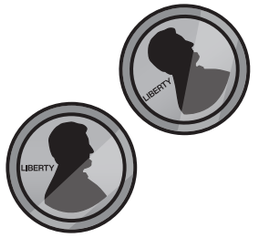
skewed A data distribution where there are more values concentrated on one end of the data, and few values on the other end.



Español

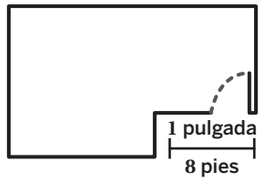
espacio muestral La lista de todos los posibles resultados de un experimento de azar.

Por ejemplo, el espacio muestral del lanzamiento de dos monedas es: cara-cara, sello-cara, cara-sello, sello-sello.



escala Una escala nos indica cómo están representadas en un dibujo o diagrama las medidas reales de un objeto.

La escala de este plano nos indica que 1 pulgada en el dibujo representa 8 pies en la habitación real.



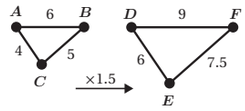
dibujo a escala Una representación bidimensional de un objeto o lugar reales. Todas las medidas del dibujo corresponden a las medidas del objeto real usando la misma escala. Los planos y los mapas son ejemplos de dibujos a escala.



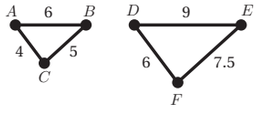
Una escala puede mostrar que 1 centímetro en un mapa representa 30 millas en tierra. La distancia mostrada en este mapa es 8 centímetros o 240 millas.

factor de escala El valor por el que se multiplica cada longitud de la figura original para producir la copia a escala.

Por ejemplo, el factor de escala de ABC a DEF es 1.5.



copia a escala Una copia de una imagen que puede cambiar de tamaño, pero que siempre conserva la forma y las medidas de los ángulos de la imagen original. Si una figura luce aplastada o estirada en comparación con la original, no es una copia a escala. Para crear una copia a escala, multiplicamos todas las longitudes de la figura original por el mismo número.



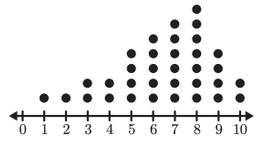
Por ejemplo, el triángulo DEF es una copia a escala del triángulo ABC.

figura Una descripción de la distribución (o patrón) de los datos en un conjunto de datos. Las descripciones de la forma de una distribución pueden incluir: simetría, asimetría, agrupaciones y valores atípicos.

interés simple Interés que se calcula solo según la cantidad inicial.

$I = prt$, donde I = interés, p = principal, r = tasa y t = tiempo

asimétrico Una distribución de datos en la que hay valores concentrados en un extremo de los datos y pocos valores en el otro extremo.



Glossary/Glosario

English

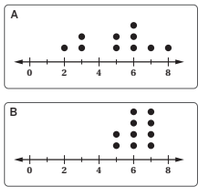
solution to an equation A value of a variable that makes the equation true.

For example, 5 is a solution to the equation $3x = 15$ because $3(5) = 15$. 6 is not a solution to the equation $3x = 15$ because $3(6) = 15$ is not true.

solutions to an inequality All of the values of a variable that make that inequality true.

For example, $x < 5$ represents the solutions to the inequality $3x < 15$ because any value that is less than 5 will make the inequality true.

spread A measure of how close together or far apart the values in a data set are from each other and from the center. The spread also describes the variability of a data set. Range and interquartile range measure the spread of data.



For example, Dot Plot A has a larger spread than Dot Plot B.

stem-and-leaf plot A table that organizes data by place value to compare data frequencies.

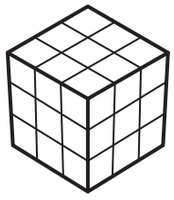
The stem-and-leaf plot shows the data set (5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64).

Stem	Leaf
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

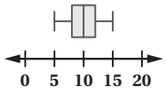
Key: 1 | 0 = 10

surface area The sum of the areas of the surfaces of a solid.

For example, the six faces of this cube each have an area of 9 square centimeters, so the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



symmetrical (data) A data display where the left side of the distribution is a mirror image of the right side.



Español

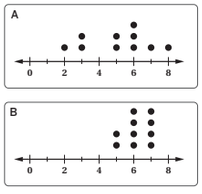
solución de una ecuación Un valor de una variable que hace que la ecuación sea verdadera.

Por ejemplo, 5 es una solución de la ecuación $3x = 15$ porque $3(5) = 15$. 6 no es una solución de la ecuación $3x = 15$ porque $3(6) = 15$ no es verdadero.

soluciones de una desigualdad Todos los valores de una variable que hacen que la desigualdad sea verdadera.

Por ejemplo, $x < 5$ representa las soluciones de la desigualdad $3x < 15$ porque cualquier valor que sea menor que 5 hará que la desigualdad sea verdadera.

dispersión Una medida que indica cuán agrupados o dispersos están los valores de un conjunto de datos entre sí y con respecto al centro. La dispersión también describe la variabilidad de un conjunto de datos. El rango y el rango intercuartil miden la dispersión de datos.



Por ejemplo, el diagrama de puntos A tiene una dispersión mayor que el diagrama de puntos B.

diagrama de tallo y hojas Una tabla que organiza los datos por valor posicional para comparar frecuencias de datos.

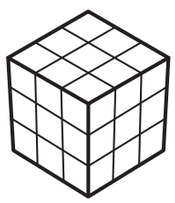
El diagrama de tallo y hojas muestra el conjunto de datos (5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64).

Tallo	Hoja
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

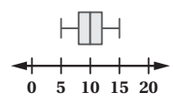
Leyenda: 1 | 0 = 10

área de superficie La suma de las áreas de las superficies de un sólido.

Por ejemplo, cada una de las seis caras de este cubo tiene un área de 9 centímetros cuadrados, por lo tanto, el área de superficie del cubo mide $6 \cdot 9$ o 54 centímetros cuadrados.

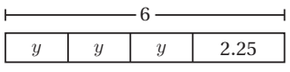


simétricos (datos) Una representación de datos donde el lado izquierdo de la distribución es una imagen reflejada del lado derecho.



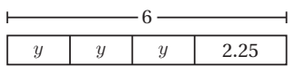
T

tape diagram A way to represent relationships between quantities (like ratios) as lengths of a tape. The entire tape diagram represents the whole and is divided to represent the parts.



This tape diagram shows a relationship that can be represented with the equation $3y + 2.25 = 6$.

diagrama de cinta Una forma de representar las relaciones entre cantidades (como razones) en forma de longitudes de una cinta. El diagrama de cinta completo representa el todo y se divide para representar las partes.

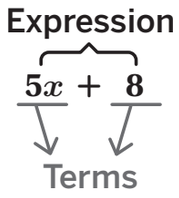


Este diagrama de cinta muestra una relación que puede representarse con la ecuación $3y + 2.25 = 6$.

English

term A part of an expression. A term can be a single number, a variable, or a number and a variable multiplied together.

For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.



terminating decimal A decimal with a finite number of non-zero digits after the decimal point.

For example, 0.08, 1.5, and 0.2563 are all terminating decimals.

trapezoid A quadrilateral with *at least* one pair of parallel sides.



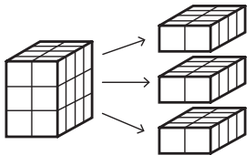
V

variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

volume The number of cubic units that fill a 3-D region without any gaps or overlaps.

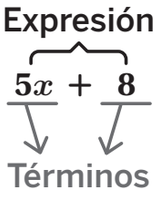
For example, the volume of this rectangular prism is 24 cubic units because it is made of 3 layers that are each 8 cubic units.



Español

término Una parte de una expresión. Un término puede ser un número individual, una variable, o una variable y un número multiplicados.

Por ejemplo, la expresión $5x + 8$ tiene dos términos. El primer término es $5x$ y el segundo término es 8.



decimal exacto Un decimal con un número finito de dígitos distintos de cero después del punto decimal.

Por ejemplo: 0.08, 1.5 y 0.2563 son decimales exactos.

trapezoide Un cuadrilátero con al menos un par de lados paralelos.

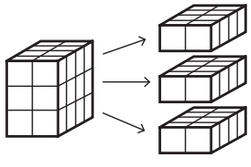


variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

volumen La cantidad de unidades cúbicas que llenan un espacio tridimensional, sin vacíos ni superposiciones.

Por ejemplo, el volumen de este prisma rectangular mide 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.



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Unit 1: Scale Drawings
Unit 2: Introducing Proportional Relationships
Unit 3: Measuring Circles
Unit 4: Proportional Relationships and Percentages

VOL. 2

Unit 5: Positive and Negative Rational Numbers
Unit 6: Expressions, Equations, and Inequalities
Unit 7: Polygons and Cylinders
Unit 8: Probability and Sampling



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