Amplify Desmos Math FLORIDA

Algebra 1

A1

Volume 1

Teacher Edition



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Algebra 1

Volume 1: Units 1–4

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Teacher,

Welcome to Amplify Desmos Math Florida! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

- A program designed around the **Florida B.E.S.T. Standards for Mathematics**, grounded in the strands, standards, and benchmarks, and the importance of student exploration and investigation.
- **Deep and intentional alignment** to the Florida B.E.S.T. standards, Florida English Language Arts standards for K-12 Mathematics, and Florida English Language Development Standards related to the Language of Mathematics.
- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Manipulatives, visual models, discussions, and drawings to encourage perseverance and revision.
  - » Easy-to-follow lesson plans, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency, help students review previous topics, and gain experience with FAST Assessment problem types.
- **Recommended differentiation moves** that meet the needs of all learners.
- **Diagnostic, formative, and summative assessments** along with lesson-level checks for understanding.
- **A caregiver resource** for each unit that includes explanations of key math concepts and problems to try with your student.

We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math Florida team



Unit 1 Linear Equations and Inequalities

Students solve one-variable equations and inequalities to make sense of multi-variable equations and two-variable inequalities.



Pre-Unit Check 1L

Sub-Unit 1 One-Variable Equations 2

1.01	Working Backwards	Solving Equations with Inverse Operations	3A
1.02	Solving Strategies	More Solving with One-Variable Equations	10A
1.03	Same Position	No Solution and Infinitely Many Solutions	17A

B.E.S.T. Benchmark Connection

1.01: MA.912.AR.2.1, MTR.1.1, MTR.2.1, MTR.5.1

1.02: MA.912.AR.2.1, MTR.4.1, MTR.5.1

1.03 MA.8.AR.2.1, MA.912.AR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Sub-Unit 2 Multi-Variable Equations 26

1.04	Subway Seats	Representing Situations with Two-Variable Equations	27A
1.05	Various Variables	Solving Multi-Variable Equations	34A
1.06	Shelley the Snail	Connecting Graphs and Linear Equations	42A
1.07	Five Representations	Linear Relationships in Equations, Tables, and Graphs	49A
1.08	Whale Growth Rate	Finding Equations of Parallel Lines	56A
1.09	House Design	Finding Equations of Perpendicular Lines	64A

B.E.S.T. Benchmark Connection

1.04: MA.912.AR.1.2, MA.912.AR.2.2 MA.912.AR.2.5, MA.912.AR.2.6, MTR.2.1, MTR.3.1, MTR.4.1

1.05: MA.912.AR.1.2, MA.912.AR.2.2 MA.912.AR.2.5, MA.912.AR.2.6, MTR.3.1, MTR.4.1

1.06: MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MA.912.AR.2.5, MA.912.AR.2.6, MTR.1.1, MTR.2.1

1.07: MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.3, MA.912.AR.2.4, MA.912.AR.2.5, MTR.4.1, MTR.6.1, MTR.7.1

1.08: MA.912.AR.2.3, MTR.4.1, MTR.7.1

1.09: MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.3, MA.912.AR.2.4, MA.912.AR.2.5, MTR.4.1, MTR.6.1, MTR.7.1

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Sub-Unit 3 One-Variable and Two-Variable Inequalities 72

1.10	Pizza Delivery	Representing Situations with One-Variable Inequalities	73A
1.11	Graphing Inequalities	Inequalities on the Number Line	81A
1.12	Solutions and Sheep	Solving One-Variable Inequalities	88A
1.13	Tick Tock	One-Variable Compound Linear Inequalities	96A
1.14	Absolute Value Solutions	Solving Absolute Value Equations and Inequalities	106A
1.15	Getting Absolute	Creating and Solving Absolute Value Equations	114A
1.16	Bracelet Budgets	Introduction to Two-Variable Inequalities	123A
1.17	All of the Solutions	Graphing Solutions to Two-Variable Inequalities	132A
1.18	Concert Planning	Graphing Two-Variable Inequalities in Context	140A

B.E.S.T. Benchmark Connection

1.10: MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1

1.11: MA.K12.MTR.1.1, MA.912.AR.2.6, MA.912.AR.2.8, MTR.1.1, MTR.2.1, MTR.4.1

1.12: MA.912.AR.2.6, MTR.1.1, MTR.4.1, MTR.5.1

1.13: MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1

1.14: MA.912.AR.4.1, MTR.2.1, MTR.5.1

1.15: MA.912.AR.4.1, MTR.1.1, MTR.4.1, MTR.5.1

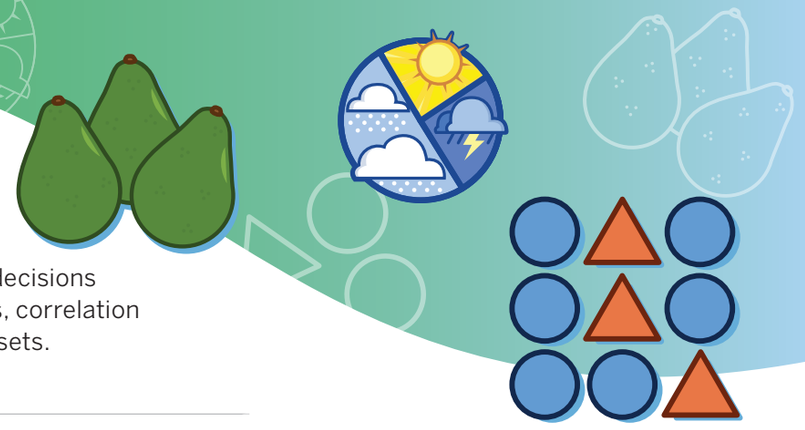
1.16: MA.912.AR.2.8, MTR.1.1, MTR.2.1

1.17: MA.912.AR.2.8, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

1.18: MA.912.AR.2.7, MA.912.AR.2.8, MTR.1.1, MTR.5.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 148A

Unit 2 Describing Data



Students use dot plots, histograms, box plots, line graphs, and measures of center and spread to analyze and describe one-variable data sets. Students use two-way tables to make decisions and predictions based on data. Students also use scatter plots, correlation coefficients, and lines of best fit to describe two-variable data sets.

Pre-Unit Check	149L	
Sub-Unit 1 Representing Categorical Data	150	B.E.S.T. Benchmark Connection
2.01 Survey Says | What Kinds of Data Can I Collect?	151A	**2.01: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.4.1
2.02 Data Dimensions | Univariate and Bivariate Vocabulary	159A	**2.02: MA.912.DP.1.2,** MTR.3.1, MTR.4.1, MTR.5.1
Sub-Unit 2 Summarizing One-Variable Data	166	B.E.S.T. Benchmark Connection
2.03 Data Driven | Revisiting Dot Plots and Histograms	167A	**2.03: MA.912.DP.1.1,** MTR.4.1, MTR.5.1, MTR.7.1
2.04 Better Weather? | Revisiting Box Plots	176A	**2.04: MA.912.DP.1.1,** MTR.2.1, MTR.4.1, MTR.7.1
2.05 Quick Pick | Revisiting Measures of Center	184A	**2.05: MA.912.DP.1.2,** MTR.3.1, MTR.4.1
2.06 Far Out | Identifying Outliers	192A	**2.06: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.2.1, MTR.4.1, MTR.6.1
2.07 Dynamic Decades | Comparing Data Using Measures of Center and Spread	200A	**2.07: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.1.1, MTR.4.1, MTR.7.1
2.08 How Big? | Estimate a Population Using Sample Survey Data	206A	**2.08: MA.912.DP.1.4,** MTR.4.1, MTR.5.1, MTR.7.1
2.09 How Many? | Estimate Mean or Percentage Using Survey Data	212A	**2.09: MA.912.DP.1.4,** MTR.1.1, MTR.4.1, MTR.7.1
2.10 Dribble, Draw, and Decide | Margin of Error through Simulation	218A	**2.10: MA.912.DP.1.4,** MTR.1.1, MTR.4.1, MTR.7.1
Sub-Unit Quiz 1	223A	
Sub-Unit 3 Summarizing Two-Variable Data	224	B.E.S.T. Benchmark Connection
2.11 Trains and Traffic | Two-Way Tables and Relative Frequency Tables	225A	**2.11: MA.912.DP.3.1,** MTR.4.1, MTR.7.1
2.12 Remodel Choices | Making Decisions with Frequency Tables	233A	**2.12: MA.912.DP.3.1,** MTR.2.1, MTR.4.1, MTR.7.1
2.13 Connecting the Dots | Numerical Bivariate Data	241A	**2.13: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.1.1, MTR.3.1, MTR.6.1
2.14 City Slopes | Interpreting Slope and y-intercept in Context	250A	**2.14: MA.912.DP.2.4, MA.912.DP.2.6,** MTR.1.1, MTR.4.1, MTR.6.1, MTR.7.1
2.15 Residual Fruit | Residuals and Residual Plots	257A	**2.15: MA.912.DP.2.4, MA.912.DP.2.6,** MTR.1.1, MTR.4.1, MTR.7.1
2.16 Penguin Populations | Using Technology to Generate the Line of Best Fit	265A	**2.16: MA.912.DP.2.4, MA.912.DP.2.6,** MTR.1.1, MTR.4.1, MTR.7.1
2.17 Behind the Headlines | Causation vs. Correlation	274A	**2.17: MA.912.DP.1.3,** MTR.1.1, MTR.4.1, MTR.7.1
End-of-Unit Assessment	280A	

Unit 3 Describing Functions



Students explore what makes a relationship a function and how functions can model situations and tell stories. Students represent functions using tables, graphs, equations, and words. They use function notation to describe key features of functions, compare different functions, and define functions. Finally, students explore new types of functions that model situations with different rules for different inputs.

Pre-Unit Check 281L

Sub-Unit 1 Function Notation 282

				B.E.S.T. Benchmark Connection
3.01	Mystery Rule	What Is a Function?	283A	3.01: MA.912.F.1.1, MTR.3.1, MTR.4.1
3.02	Pricing Pizzas	Introducing Function Notation	291A	3.02: MA.912.F.1.2, MA.912.AR.2.2, MTR.2.1, MTR.4.1
3.03	Rule Breakers	Domain and Range Notation	299A	3.03: MA.912.F.1.1, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.4.1, MTR.5.1
3.04	Toy Factory	Function Notation and Equations	306A	3.04: MA.912.F.1.2, MA.912.AR.2.2, MTR.3.1, MTR.4.1, MTR.5.1

Sub-Unit Quiz 1 ..... 313A

Sub-Unit 2 Key Features of Functions 314

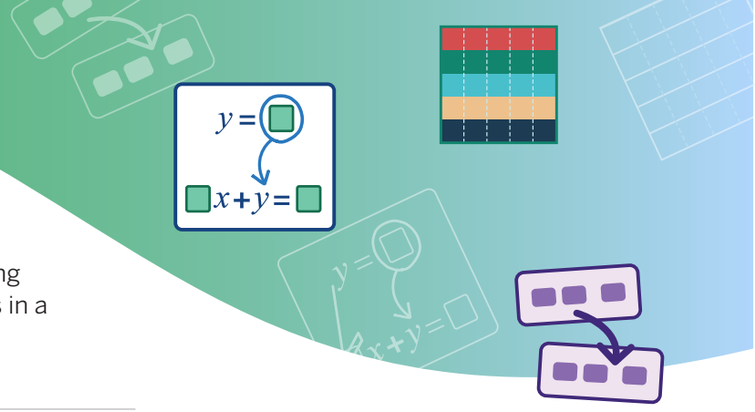
				B.E.S.T. Benchmark Connection
3.05	Function Carnival	Creating and Interpreting Graphs of Functions	315A	3.05: MA.912.F.1.2, MA.912.F.1.6, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1
3.06	Craft-a-Graph	Key Features of Graphs	323A	3.06: MA.912.F.1.6, MTR.4.1
3.07	Plane, Train, and Automobile	Average Rate of Change	331A	3.07: MA.912.F.1.3, MA.912.F.1.5, MTR.1.1, MTR.2.1, MTR.4.1
3.08	Space Race	Comparing Graphs	340A	3.08: MA.912.F.1.5, MA.912.F.1.6, MTR.4.1
3.09	Ins and Outs	Introducing Domain and Range	348A	3.09: MA.8.F.1.1, MA.912.F.1.2, MA.912.F.1.5, MTR.4.1
3.10	Elevator Stories	Describing Domain and Range with Inequalities	356A	3.10: MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.2.1, MTR.4.1

Sub-Unit 3 Special Types of Functions 366

				B.E.S.T. Benchmark Connection
3.11	What's Your Score?	Absolute Value Functions, Part 1	367A	3.11: MA.912.AR.4.3, MTR.2.1, MTR.5.1
3.12	Absolute Value Machines	Absolute Value Functions, Part 2	375A	3.12: MA.912.AR.4.3, MTR.4.1, MTR.5.1

End-of-Unit Assessment 384B

Unit 4 Systems of Linear Equations and Inequalities



Students explore how to solve systems of equations and inequalities using different strategies. Students also analyze the structure of the equations in a system to strategize about which solving method to choose.

Pre-Unit Check 385L

Sub-Unit 1 Systems of Equations 386

				B.E.S.T. Benchmark Connection
4.01	Eliminating Shapes	Introduction to Elimination	387A	**4.01: MA.912.AR.9.1,** MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
4.02	Process of Elimination	Elimination Using Equivalent Equations	396A	**4.02: MA.912.AR.9.1,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.03	Solution by Substitution	Solving Systems by Substitution	404A	**4.03: MA.912.AR.9.1,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.04	Lizard Lines	Graphing Systems of Linear Equations	411A	**4.04: MA.912.AR.9.1,** MTR.2.1, MTR.4.1, MTR.5.1
4.05	City Development	Solving Graphically and Symbolically	419A	**4.05: MA.912.AR.9.1, MA.912.AR.9.6,** MTR.1.1, MTR.6.1
4.06	Bus Systems	Writing and Solving Systems of Equations	427A	**4.06: MA.912.AR.9.1, MA.912.AR.9.6,** MTR.2.1, MTR.4.1, MTR.5.1

Sub-Unit Quiz 1 ..... 433A

Sub-Unit 2 Systems of Inequalities 434

				B.E.S.T. Benchmark Connection
4.07	Quilts	Introduction to Systems of Inequalities	435A	**4.07: MA.912.AR.9.4,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.08	Seeking Solutions	Solutions to Systems of Inequalities	442A	**4.08: MA.912.AR.9.4, MA.912.AR.9.6,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.09	Boundaries and Shading	Graphing Systems of Inequalities	451A	**4.09: MA.912.AR.9.4, MA.912.AR.9.6,** MTR.4.1, MTR.5.1, MTR.6.1
4.10	Restaurant Meals	Using Systems of Inequalities to Make Decisions	458A	**4.10: MA.912.AR.9.4, MA.912.AR.9.6,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 464A

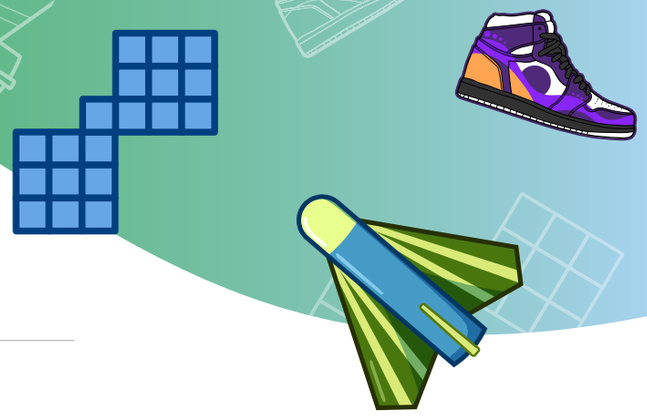
Unit 5 Exponential Functions

Students learn about relationships that grow quickly over time. Students learn how to distinguish between linear and exponential relationships, and create equations to represent them. Students model situations that increase or decrease by a percentage. Students also model compound interest with exponential functions.



		B.E.S.T. Benchmark Connection
Pre-Unit Check 465L		
Sub-Unit 1 Comparing Linear and Exponential Functions 466		B.E.S.T. Benchmark Connection
5.01 Growing Globs | Patterns of Growth 467A		**5.01: MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8,** MTR.4.1, MTR.5.1, MTR.7.1
5.02 Going Viral | Graphs of Exponential Relationships 475A		**5.02: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.F.1.6, MA.912.F.1.8,** MTR.4.1, MTR.5.1, MTR.7.1
5.03 Return of the Globs | Connecting Representations of Linear and Exponential Functions 483A		**5.03: MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1
5.04 Carlos and Corals | Evaluating Exponential Functions 489A		**5.04: MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.F.1.2, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1
5.05 Differences and Factors | Changes Over Equal Intervals 497A		**5.05: MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.F.1.2, MA.912.F.1.8, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.5.1, MTR.7.1
Sub-Unit 2 Exponential Growth and Decay 504		B.E.S.T. Benchmark Connection
5.06 Growing Mold | Percent Increase and Decrease, Part 1 ..... 505A		**5.06: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.NSO.1.1,** MTR.2.1, MTR.6.1, MTR.7.1
5.07 At a Loss | Percent Increase and Decrease, Part 2 ..... 513A		**5.07: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.NSO.1.1,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1
Sub-Unit Quiz 1 ..... 521A		
Sub-Unit 3 Modeling with Exponentials 522		B.E.S.T. Benchmark Connection
5.08 Thinking Rationally | Writing Expressions Using Radicals and Rational Exponents 523A		**5.08: MA.912.NSO.1.1, MA.912.NSO.1.4,** MTR.2.1, MTR.4.1, MTR.5.1
5.09 Writing Radicals | Writing Equivalent Expressions Using Rational Exponents and Radicals 532A		**5.09: MA.912.NSO.1.1, MA.912.NSO.1.4,** MTR.2.1, MTR.4.1, MTR.5.1
5.10 Rule the Roots | Operations with Radicals 538A		**5.10: MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4,** MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
5.11 Tame the Terms | Adding and Subtracting Radicals 547A		**5.11: MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4,** MTR.1.1, MTR.2.1, MTR.4.1, MTR.5.1
5.12 Bank Accounts | Introducing Simple and Compound Interest 555A		**5.12: MA.912.FL.3.2, MA.912.FL.3.4, MA.912.F.1.6, MA.912.F.1.8,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1
5.13 Payday Loan | Revisiting Compound Interest 563A		**5.13: MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1
5.14 Credit Card Compounding | Different Compound Intervals 571A		**5.14: MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1
5.15 Exploring Interest | Relating Interest to Growth 580A		**5:15: MA.912.FL.3.4,** MTR.2.1, MTR.4.1, MTR.7.1
End-of-Unit Assessment 587A		

Unit 6 Quadratic Functions

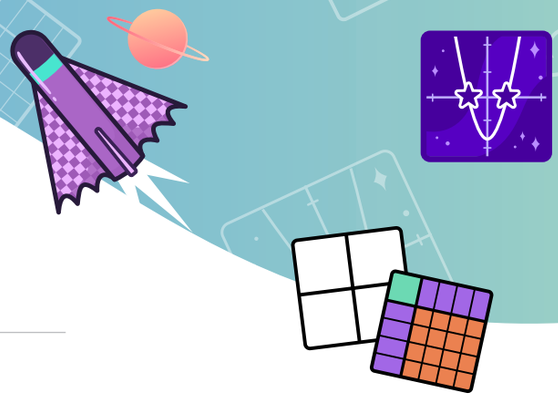


Students learn about how quadratic relationships are different from linear and exponential relationships, and graph them. Students model situations with quadratic functions and make predictions and decisions based on your models.

Pre-Unit Check 589L		
Sub-Unit 1 Introduction to Quadratic Functions 590	B.E.S.T. Benchmark Connection	
6.01 Revisiting Visual Patterns | A New Type of Pattern 591A	**6.01: MA.912.F.1.8,** MTR.2.1, MTR.5.1	
6.02 Quadratic Visual Patterns | Expressions for Quadratic Patterns 599A	**6.02: MA.912.F.1.8,** MTR.2.1, MTR.5.1	
6.03 Sorting Relationships | Comparing Linear, Exponential, and Quadratic Relationships 608A	**6.03: MA.912.F.1.1, MA.912.F.1.8,** MTR.2.1, MTR.5.1	
6.04 On the Fence | Quadratics in Context 615A	**6.04: MA.912.F.1.1, MA.912.F.1.8, MA.912.AR.3.7,** MTR.2.1, MTR.5.1, MTR.7.1	
6.05 Stomp Rockets | Projectiles and Predictions 623A	**6.05: MA.912.AR.3.7, MA.912.AR.3.8,** MTR.1.1, MTR.2.1, MTR.3.1, MTR.5.1	
6.06 Plenty of Parabolas | Key Features of Parabolas 631A	**6.06: MA.912.AR.3.7,** MTR.2.1, MTR.4.1, MTR.5.1	
6.07 Robot Launch | Key Features of Graphs in Context 639A	**6.07: MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.4.1, MTR.5.1	
Sub-Unit 2 Standard Form and Factored Form 646	B.E.S.T. Benchmark Connection	
6.08 What's My Graph? | Creating Graphs of Quadratics 647A	**6.08: MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.4.1	
6.09 Two for One | Standard Form and Factored Form 653A	**6.09: MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.5.1	
6.10 Interesting Intercepts | Intercepts in Factored and Standard Forms 659A	**6.10: MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.4.1, MTR.5.1	
6.11 Break Through: Parabolas | Building Quadratics in Factored Form 667A	**6.11: MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1,** MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1	
6.12 Sneaker Drop | Exploring Revenue 675A	**6.12: MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2,** MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1	
Sub-Unit Quiz 1 ..... 681A		
Sub-Unit 3 Vertex Form 682	B.E.S.T. Benchmark Connection	
6.13 Vertex Form | Translating Quadratic Functions 683A	**6.13: MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1,** MTR.2.1, MTR.4.1, MTR.5.1	
6.14 Stretch It Out | Vertical Stretches and Vertex Form 692A	**6.14: MA.912.AR.3.4, MA.912.AR.3.7, MA.912.F.2.1,** MTR.2.1, MTR.5.1	
6.15 Predicting Sales | Linear, Exponential, and Quadratic Modeling 701A	**6.15: MA.912.F.1.2, MA.912.F.1.8, MA.912.AR.3.8,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1	
End-of-Unit Assessment 708B		

Unit 7 Quadratic Equations

Students learn different methods for solving quadratic equations and develop strategies for choosing your solving method strategically. Students also connect your solving process to the graphs and key features of quadratic functions.



Pre-Unit Check 709L

Sub-Unit 1 Multiplying and Factoring 710

		B.E.S.T. Benchmark Connection
7.01	Sums and Differences | Adding and Subtracting Linear and Quadratic Expressions 711A	**7.01: MA.912.AR.1.3, MA.912.NSO.1.1,** MTR.1.1, MTR.4.1, MTR.5.1
7.02	Two-Factor Multiplication | Rewriting Factored-Form Expressions in Standard Form 719A	**7.02: MA.912.AR.1.3, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.1.1, MTR.3.1, MTR.4.1, MTR.5.1
7.03	Standard Feature | Patterns in Factored-Form and Standard-Form Expressions 727A	**7.03: MA.912.AR.1.3, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.1.1, MTR.3.1, MTR.5.1
7.04	X-Factor | Factoring Quadratic Expressions 735A	**7.04: MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.3.1, MTR.5.1, MTR.6.1
7.05	Form Up | More Factoring Quadratic Expressions 743A	**7.05: MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
7.06	Divide and Conquer | Dividing Polynomial Expressions 750A	**7.06: MA.912.AR.1.4, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.3.1, MTR.4.1, MTR.5.1
7.07	Consider the Factors | Factoring Polynomial Expressions 757A	**7.07: MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.5.1
7.08	Shooting Stars | Determining the x-intercepts of Quadratic Functions 766A	**7.08: MA.912.AR.1.7, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
7.09	Make It Zero | Solving Quadratic Equations Using the Zero-Product Property 773A	**7.09: MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1

Sub-Unit Quiz 1 ..... 780A

Sub-Unit 2 Solving Equations and Completing the Square 782

		B.E.S.T. Benchmark Connection
7.10	Zero, One, or Two? | Solving Equations by Reasoning 783A	**7.10: Building toward: MA.912.AR.3.1,** MTR.4.1, MTR.5.1, MTR.6.1
7.11	Graph to Solve | Solving Quadratic Equations by Graphing 791A	**7.11: MA.912.AR.3.1, MA.912.AR.3.6,** MTR.2.1, MTR.4.1, MTR.5.1
7.12	Couldn't Square Less | Solving by Taking the Square Root 799A	**7.12: MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1
7.13	Square Dance | Perfect Square Expressions 807A	**7.13: MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1,** MTR.2.1, MTR.4.1, MTR.5.1
7.14	Square Tactic | Solving by Completing the Square 815A	**7.14: MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.5.1, MTR.6.1
7.15	Back and Forth | Rewriting Quadratic Expressions in Vertex Form 823A	**7.15: MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

Sub-Unit 3 The Quadratic Formula and More 830

		B.E.S.T. Benchmark Connection
7.16	Formula Foundations | Introducing the Quadratic Formula 831A	**7.16: MA.912.AR.1.2, MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.3.1, MTR.4.1, MTR.5.1
7.17	Formula Fluency | Solving Quadratic Equations Using the Quadratic Formula 839A	**7.17: MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.3.1, MTR.4.1, MTR.5.1
7.18	Stomp Rockets in Space | Solving Quadratic Equations in Context 846A	**7.18: MA.912.AR.3.1, MA.912.AR.3.8, MA.912.NSO.1.4,** MTR.3.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 854A

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math Florida, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

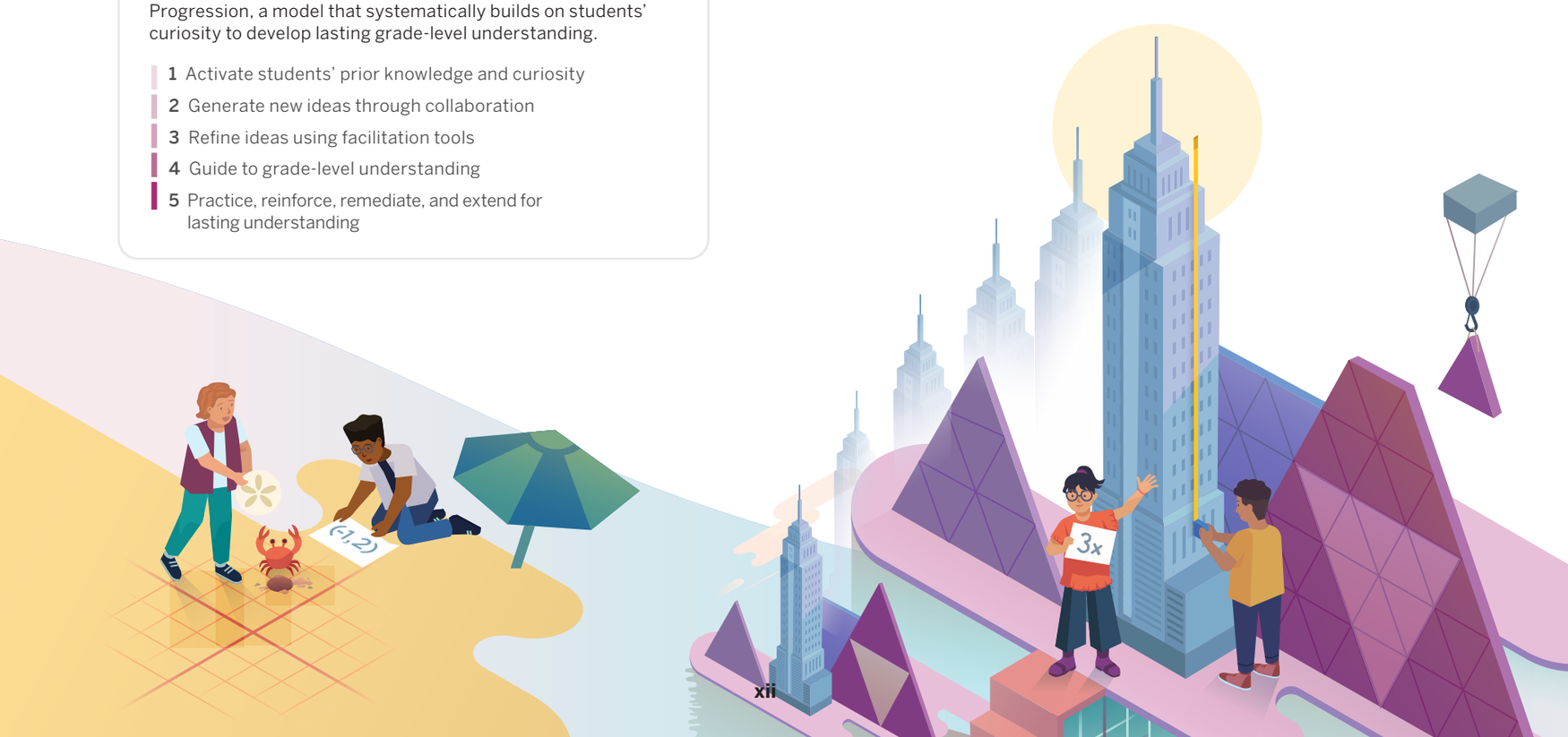
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.





Student thinking is valuable and can be made evident

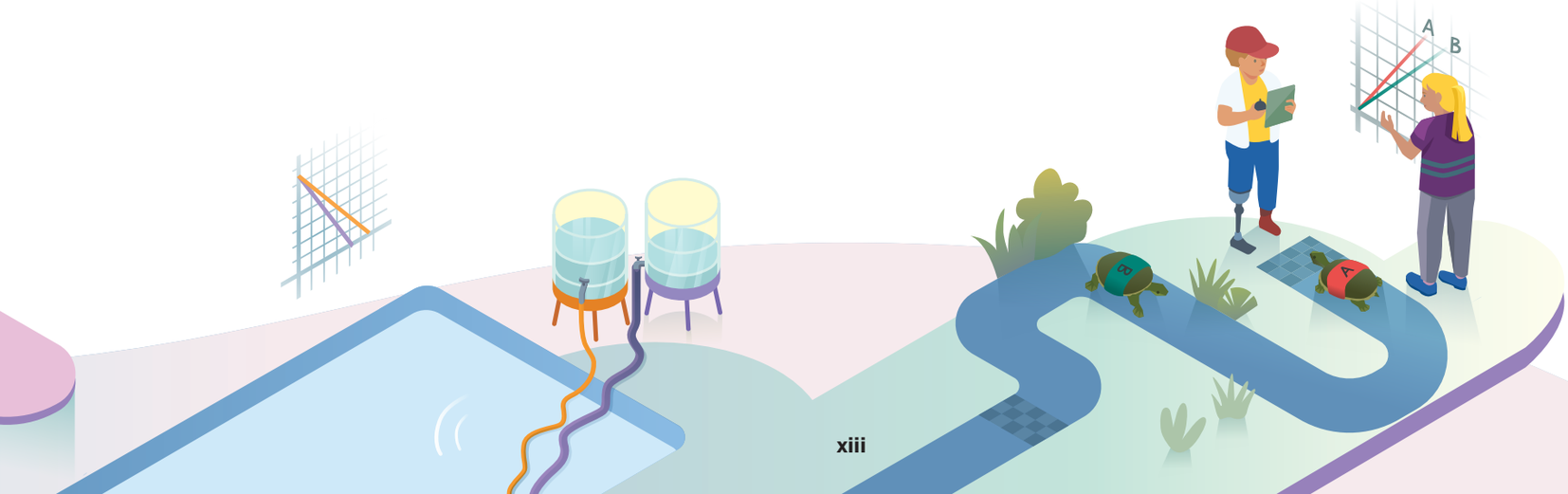
Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Math that motivates students

Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity.

Our curriculum supports collaborative classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.



Scope and Sequence | Grade 6–Algebra 1

Volume 1

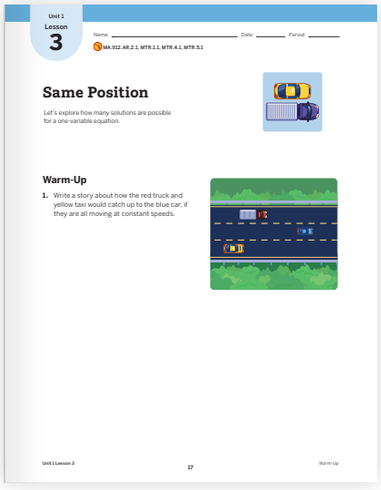
	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 127 days	**Area and Surface Area** 8 instructional days 3 assessment days 11 days total	**Introducing Ratios** 12 instructional days 3 assessment days, Math in Action 16 days total	**Unit Rates and Percentages** 13 instructional days 3 assessment days 16 days total	**Multiplying and Dividing Fractions** 15 instructional days 3 assessment days, Math in Action 19 days total
Grade 7 116 days	**Scale Drawings** 10 instructional days 3 assessment days 13 days total	**Introducing Proportional Relationships** 13 instructional days 4 assessment days, Math in Action 17 days total	**Measuring Circles** 6 instructional days 3 assessment days 9 days total	**Proportional Relationships and Percentages** 12 instructional days 4 assessment days, Math in Action 16 days total
Grade 8 120 days	**Rigid Transformations and Congruence** 10 instructional days 3 assessment days 13 days total	**Dilations, Similarity, and Slope** 10 instructional days 4 assessment days, Math in Action 14 days total	**Proportional and Linear Relationships** 11 instructional days 3 assessment days 14 days total	**Linear Equations and Linear Systems** 10 instructional days 4 assessment days, Math in Action 14 days total
Algebra 1 130 days	**Linear Equations and Inequalities** 18 instructional days 4 assessment days, Math in Action 22 days total	**Describing Data** 17 instructional days 3 assessment days 20 days total	**Describing Functions** 12 instructional days 4 assessment days, Math in Action 16 days total	**Describing Functions** 10 instructional days 3 assessment days 13 days total

Volume 2

Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Decimal Arithmetic 10 instructional days 3 assessment Days 13 days total	**Expressions and Equations** 11 instructional days 4 assessment Days, Math in Action 15 days total	**Positive and Negative Numbers** 18 instructional days 3 assessment Days 21 days total	**Describing Data** 12 instructional days 4 assessment Days, Math in Action 16 days total	
Positive and Negative Rational Numbers 10 instructional days 3 assessment days 13 days total	**Equations, Expressions, and Inequalities** 16 instructional days 4 assessment days, Math in Action 20 days total	**Polygons and Cylinders** 8 instructional days 3 assessment days 11 days total	**Probability and Sampling** 13 instructional days 4 assessment days, Math in Action 17 days total	
Functions 7 instructional days 3 assessment days 10 days total	**Associations in Data** 9 instructional days 4 assessment days, Math in Action 13 days total	**Exponents and Scientific Notation** 13 instructional days 3 assessment days 16 days total	**The Pythagorean Theorem and Irrational Numbers** 15 instructional days 4 assessment days, Math in Action 19 days total	**Probability** 5 instructional days 2 assessment days 7 days total
Exponential Functions 15 instructional days 4 assessment days, Math in Action 17 days total	**Quadratic Functions** 15 instructional days 3 assessment days 18 days total	**Quadratic Equations** 18 instructional days 4 assessment days, Math in Action 22 days total		

Program Resources

For Students


Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.AR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Same Position

Let's explore how many solutions are possible for a one-variable equation.

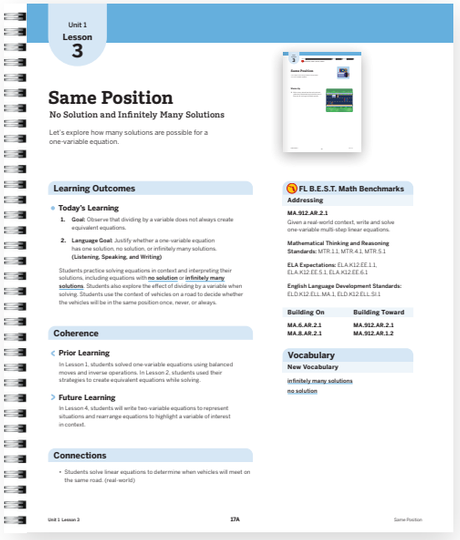
Warm-Up

1. Write a story about how the red truck and yellow taxi would catch up to the blue car, if they are all moving at constant speeds.

Unit 1 Lesson 3 17 Warm-Up

- Student Edition (two-volume)

For Teachers


Unit 1
Lesson
3

Same Position

No Solution and Infinitely Many Solutions

Let's explore how many solutions are possible for a one-variable equation.

Learning Outcomes

Today's Learning

1. **Goal:** Observe that dividing by a variable does not always create equivalent equations.
2. **Language Goal:** Justify whether a one-variable equation has one solution, no solution, or infinitely many solutions. **(Listening, Speaking, and Writing)**

Students practice solving equations in context and interpreting their solutions, including equations with **no solution** or **infinitely many solutions**. Students also explore the effect of dividing by a variable when solving. Students use the context of vehicles on a road to decide whether the vehicles will be in the same position once, never, or always.

Coherence

Prior Learning

In Lesson 1, students solved one-variable equations using balanced moves and inverse operations. In Lesson 2, students used their strategies to create equivalent equations while solving.

Future Learning

In Lesson 4, students will write two-variable equations to represent situations and rearrange equations to highlight a variable of interest in context.

Connections

- Students solve linear equations to determine when vehicles will meet on the same road. (real-world)

FL B.E.S.T. Math Benchmarks Addressing

MA.912.AR.2.1

Given a real-world context, write and solve one-variable multi-step linear equations.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.1	MA.912.AR.2.1
MA.8.AR.2.1	MA.912.AR.1.2

Vocabulary

New Vocabulary

infinitely many solutions
no solution

Unit 1 Lesson 3 27A Same Position

- Teacher Edition (two-volume)

Additional Resources



Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior grade Mini-Lessons
- Extensions

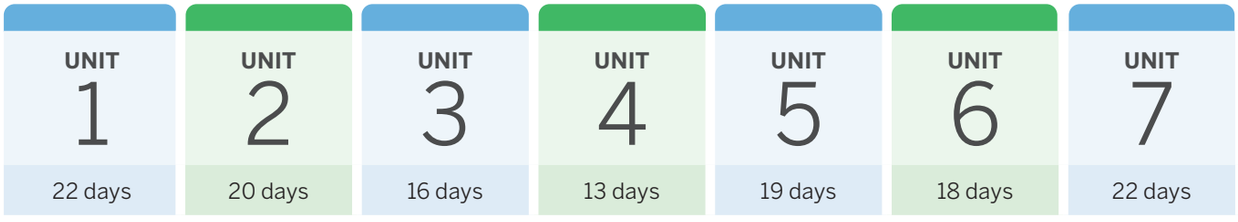
Optional: Manipulative Kits



hand2mind

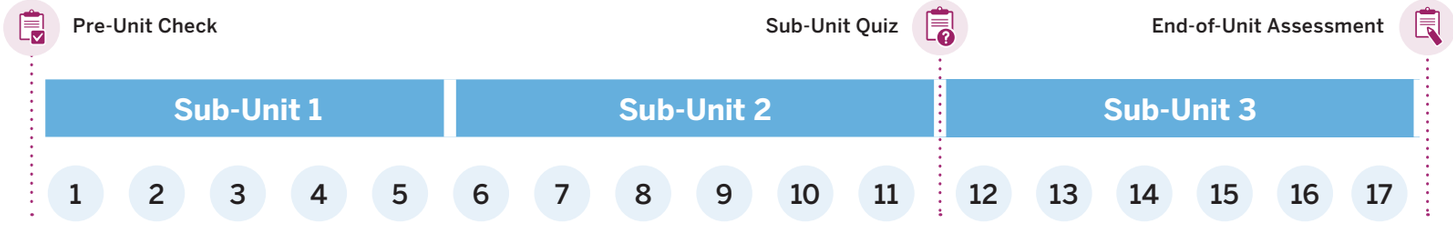
Program Architecture

Course



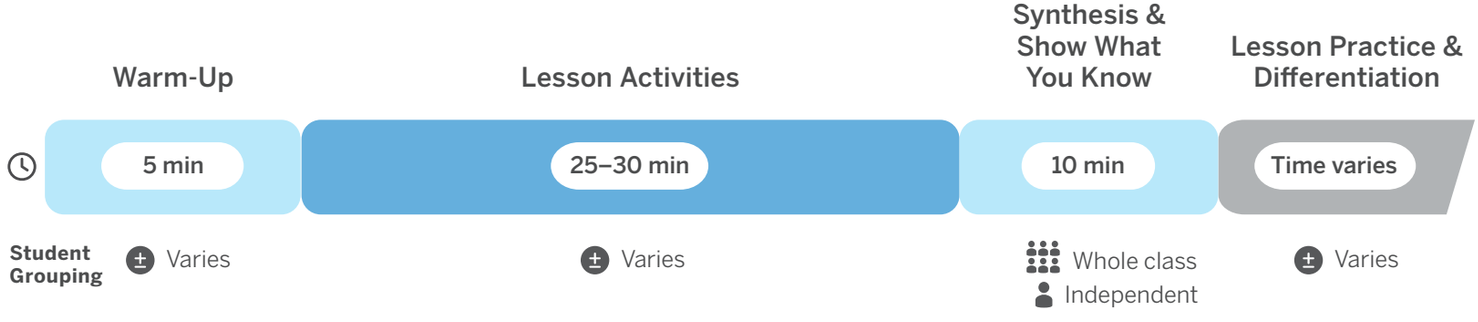
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

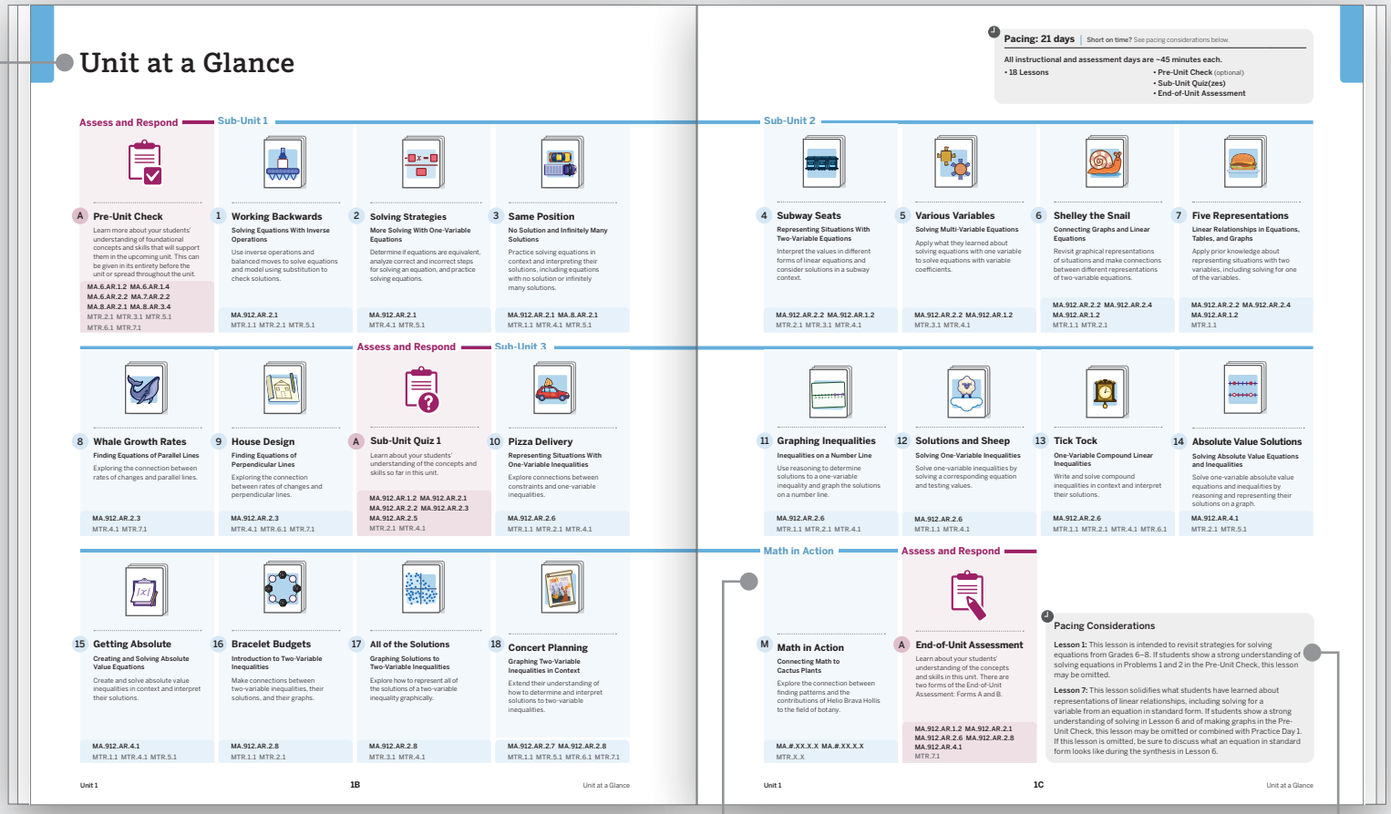
Lesson



Navigating This Program

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

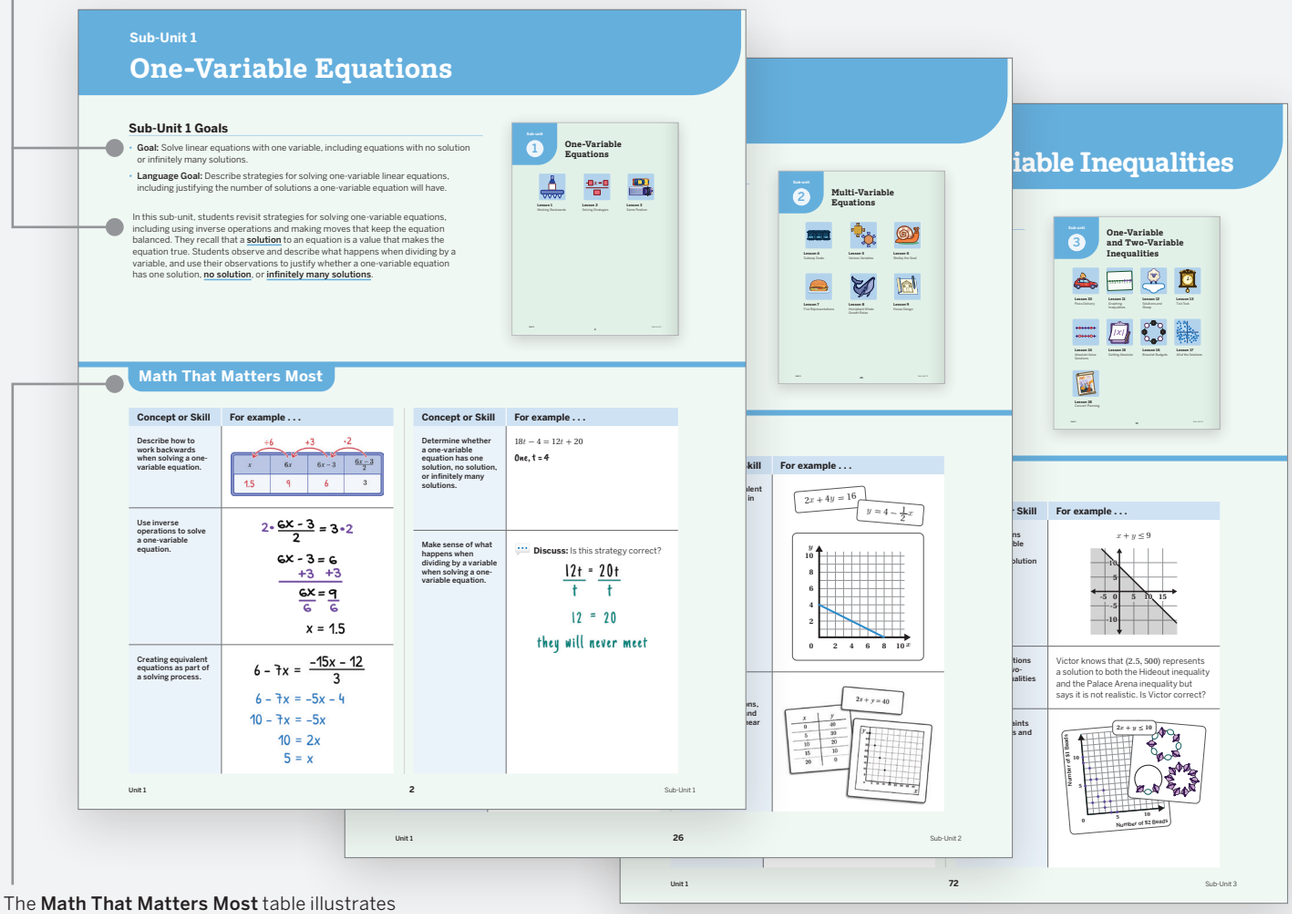
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, a preview of each of the unit assessments, and guidance about differentiation, accessibility, language development, materials, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

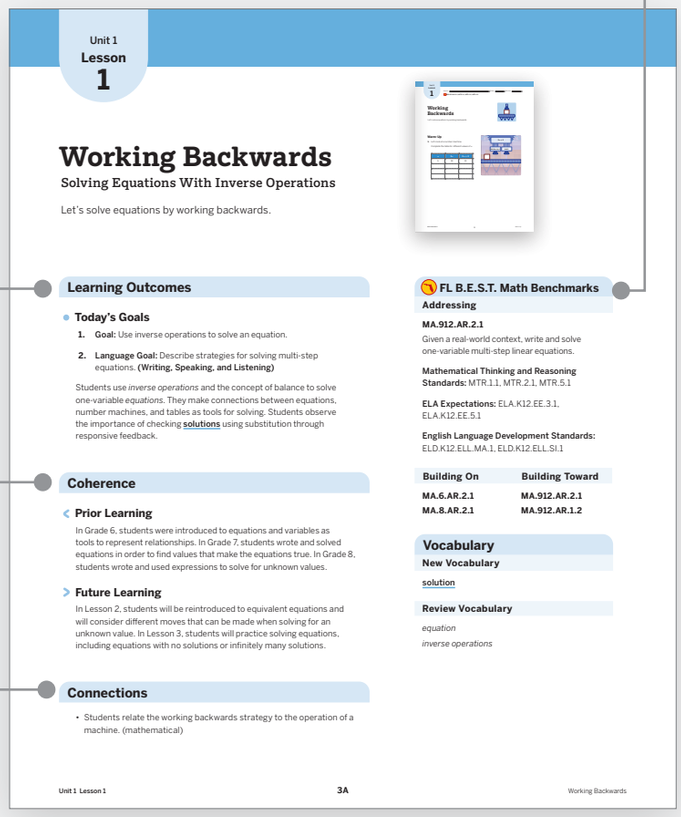
The lessons within each unit are grouped into sub-units that address a related group of concepts. All sub-units include lesson practice and Show What You Know formative assessments, and targeted sub-units include a quiz.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

The **Florida B.E.S.T. Math Benchmarks** section will list all standards addressed in this lesson, including the **Mathematical Thinking and Reasoning Standards,** the **ELA Expectations,** and the **English Language Development Standards.** This section shows the standards the lesson builds on and the standards the lesson builds toward.



Unit 1
Lesson
1

Working Backwards

Solving Equations With Inverse Operations

Let's solve equations by working backwards.

Learning Outcomes

- **Today's Goals**
  1. **Goal:** Use inverse operations to solve an equation.
  2. **Language Goal:** Describe strategies for solving multi-step equations. **(Writing, Speaking, and Listening)**

Students use *inverse operations* and the concept of balance to solve one-variable *equations*. They make connections between equations, number machines, and tables as tools for solving. Students observe the importance of checking **solutions** using substitution through responsive feedback.

Coherence

Prior Learning

In Grade 6, students were introduced to equations and variables as tools to represent relationships. In Grade 7, students wrote and solved equations in order to find values that make the equations true. In Grade 8, students wrote and used expressions to solve for unknown values.

Future Learning

In Lesson 2, students will be reintroduced to equivalent equations and will consider different moves that can be made when solving for an unknown value. In Lesson 3, students will practice solving equations, including equations with no solutions or infinitely many solutions.

Connections

- Students relate the working backwards strategy to the operation of a machine. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.1
Given a real-world context, write and solve one-variable multi-step linear equations.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.1	MA.912.AR.2.1
MA.8.AR.2.1	MA.912.AR.1.2

Vocabulary

New Vocabulary

solution

Review Vocabulary

equation
inverse operations

Unit 1 Lesson 1 3A Working Backwards

The **Learning Outcomes** section is where teachers will find the goals of the lesson, including language goals

In the **Coherence** section, teachers will find information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

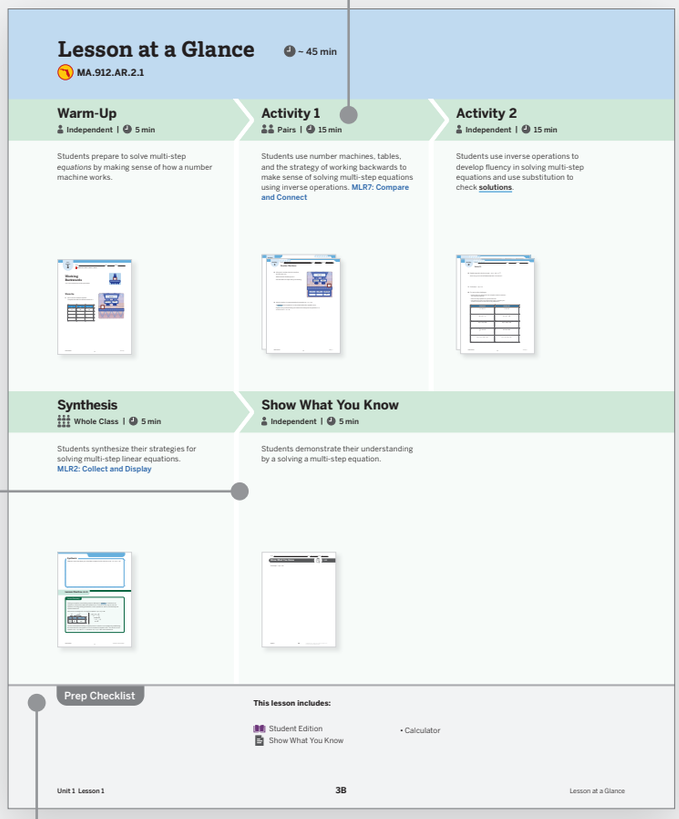
In the **Connections** section, teachers will find opportunities to help their students make connections cross-curricularly, to the real world, or to other mathematical learning.

Lesson Overview

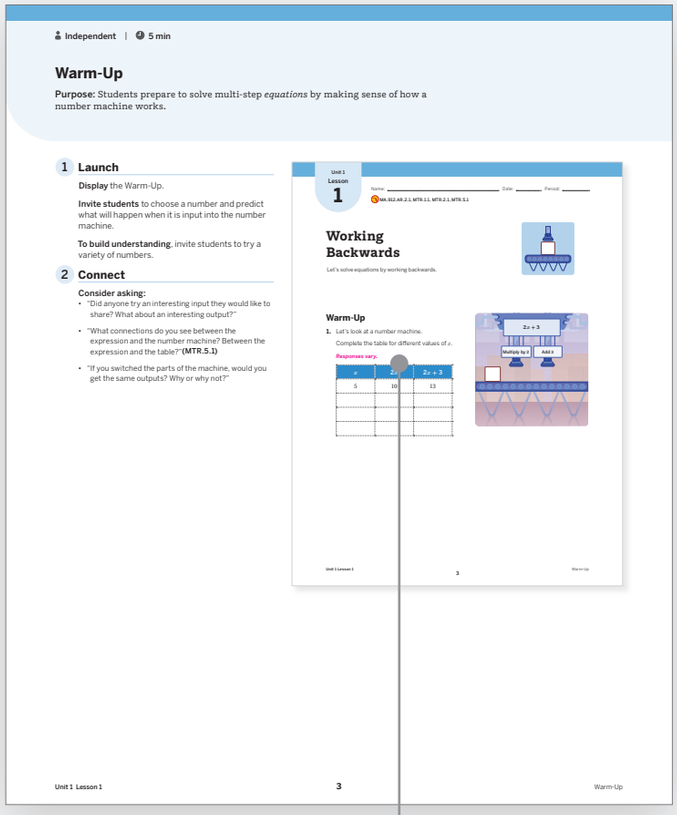
This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.



The **Prep Checklist** displays all materials students will need for the lesson.



Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class. The page also lists any hands-on materials that may be needed.

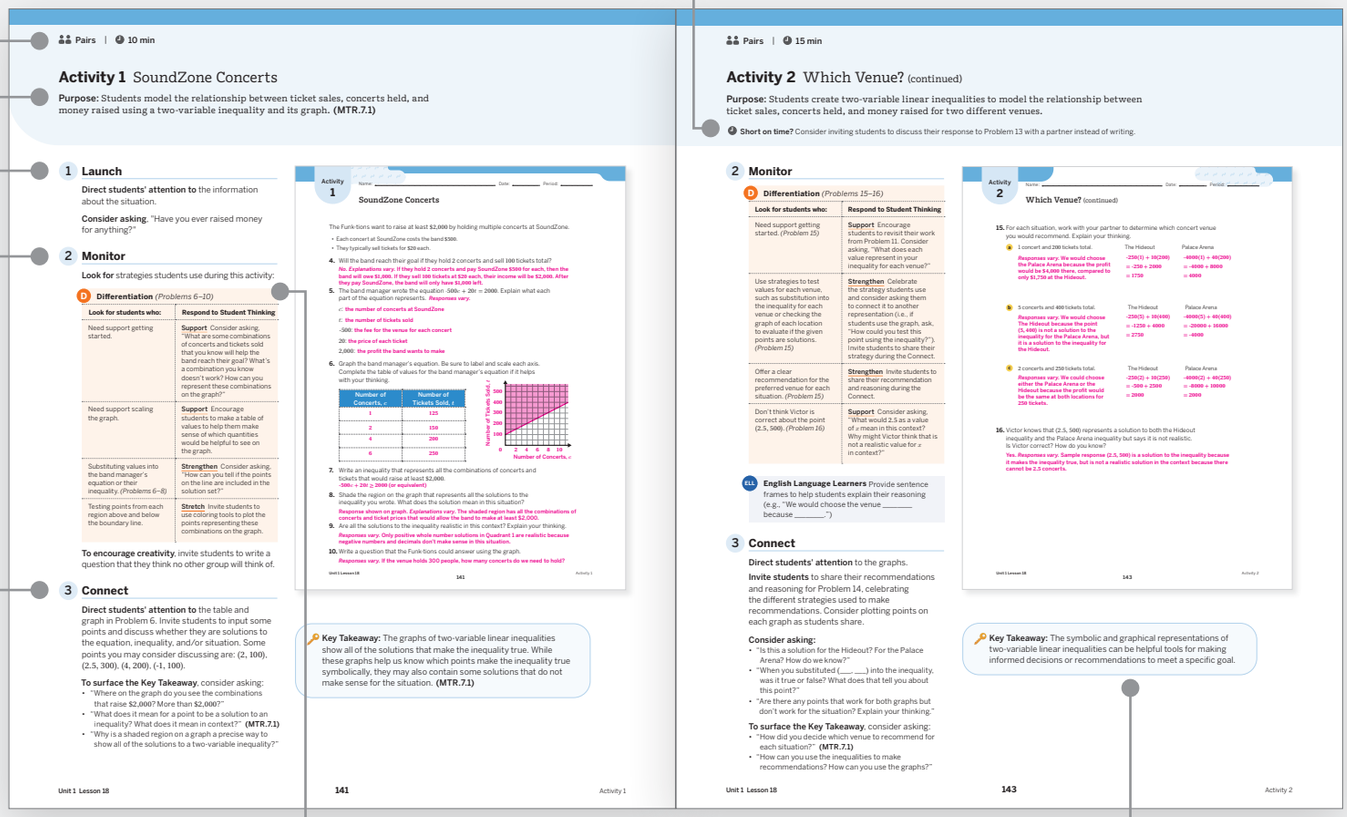
Warm-Up

Every Amplify Desmos Math Florida lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping** and **time frame** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.



Pairs | 10 min

Activity 1 SoundZone Concerts

Purpose: Students model the relationship between ticket sales, concerts held, and money raised using a two-variable inequality and its graph. **(MTR.7.1)**

1 Launch

Direct students' attention to the information about the situation.

Consider asking, "Have you ever raised money for anything?"

2 Monitor

Look for strategies students use during this activity:

D Differentiation *(Problems 6–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are some combinations of concerts and tickets sold that you know will help the band reach their goal? What's a combination you know doesn't work? How can you represent these combinations on the graph?"
Need support scaling the graph.	**Support** Encourage students to make a table of values to help them make sense of which quantities would be helpful to see on the graph.
Substituting values into the band manager's equation or their inequality. *(Problems 6–8)*	**Strengthen** Consider asking, "How can you tell if the points on the line are included in the solution set?"
Testing points from each region above and below the boundary line.	**Stretch** Invite students to use coloring tools to plot the points representing these combinations on the graph.

To encourage creativity, invite students to write a question that they think no other group will think of.

3 Connect

Direct students' attention to the table and graph in Problem 6. Invite students to input some points and discuss whether they are solutions to the equation, inequality, and/or situation. Some points you may consider discussing are: (2, 100), (2.5, 300), (4, 200), (-1, 100).

To surface the Key Takeaway, consider asking:

- "Where on the graph do you see the combinations that raise $2,000? More than $2,000?"
- "What does it mean for a point to be a solution to an inequality? What does it mean in context?" **(MTR.7.1)**
- "Why is a shaded region on a graph a precise way to show all of the solutions to a two-variable inequality?"

Activity 1

Name _____ Date _____ Period _____

SoundZone Concerts

The Funk-tions want to raise at least $2,000 by holding multiple concerts at SoundZone.

- Each concert at SoundZone costs the band $300.
- They typically sell tickets for $20 each.

4. Will the band reach their goal if they hold 2 concerts and sell 100 tickets total?
No. Explanations vary. If they hold 2 concerts and pay SoundZone $300 for each, then the band will owe $1,000. If they sell 100 tickets at $20 each, their income will be $2,000. After they pay SoundZone, the band will only have $1,000 left.
5. The band manager wrote the equation $-500c + 20t \geq 2000$. Explain what each part of the equation represents. *Responses vary.*
c: the number of concerts at SoundZone
t: the number of tickets sold
-500: the fee for the venue for each concert
20: the price of each ticket
2,000: the profit the band wants to make
6. Graph the band manager's equation. Be sure to label and scale each axis. Complete the table of values for the band manager's equation if it helps with your thinking.

Number of Concerts, c	Number of Tickets Sold, t
1	125
2	150
4	200
6	250

7. Write an inequality that represents all the combinations of concerts and tickets that would raise at least $2,000.
$-500c + 20t \geq 2000$ (or equivalent)
8. Shade the region on the graph that represents all the solutions to the inequality you wrote. What does the solution mean in this situation?
Response shown on graph. Explanations vary. The shaded region has all the combinations of concerts and ticket prices that would allow the band to make at least $2,000.
9. Are all the solutions to the inequality realistic in this context? Explain your thinking.
Responses vary. Only positive whole number solutions in Quadrant I are realistic because negative numbers and decimals don't make sense in this situation.
10. Write a question that the Funk-tions could answer using the graph.
Responses vary. If the venue holds 300 people, how many concerts do we need to hold?

Number of Tickets Sold, t — Number of Concerts, c

Unit 1 Lesson 18 141 Activity 1

Key Takeaway: The graphs of two-variable linear inequalities show all of the solutions that make the inequality true. While these graphs help us know which points make the inequality true symbolically, they may also contain some solutions that do not make sense for the situation. **(MTR.7.1)**

Pairs | 15 min

Activity 2 Which Venue? (continued)

Purpose: Students create two-variable linear inequalities to model the relationship between ticket sales, concerts held, and money raised for two different venues.

Short on time? Consider inviting students to discuss their response to Problem 13 with a partner instead of writing.

2 Monitor

D Differentiation *(Problems 15–16)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 15)*	**Support** Encourage students to revisit their work from Problem 11. Consider asking, "What does each value represent in your inequality for each venue?"
Use strategies to test values for each venue, such as substitution into the inequality for each venue or checking the graph of each location to evaluate if the given points are solutions. *(Problem 15)*	**Strengthen** Celebrate the strategy students use and consider asking them to connect it to another representation (i.e., if students use the graph, ask, "How could you test this point using the inequality?"). Invite students to share their strategy during the Connect.
Offer a clear recommendation for the preferred venue for each situation. *(Problem 15)*	**Strengthen** Invite students to share their recommendation and reasoning during the Connect.
Don't think Victor is correct about the point (2.5, 500). *(Problem 16)*	**Support** Consider asking, "What would 2.5 as a value of x mean in this context? Why might Victor think that is not a realistic value for x in context?"

ELL English Language Learners Provide sentence frames to help students explain their reasoning (e.g., "We would choose the venue _____ because _____.")

3 Connect

Direct students' attention to the graphs.

Invite students to share their recommendations and reasoning for Problem 14, celebrating the different strategies used to make recommendations. Consider plotting points on each graph as students share.

Consider asking:

- "Is this a solution for the Hideout? For the Palace Arena? How do we know?"
- "When you substituted (___, ___) into the inequality, was it true or false? What does that tell you about this point?"
- "Are there any points that work for both graphs but don't work for the situation? Explain your thinking."

To surface the Key Takeaway, consider asking:

- "How did you decide which venue to recommend for each situation?" **(MTR.7.1)**
- "How can you use the inequalities to make recommendations? How can you use the graphs?"

Activity 2

Name _____ Date _____ Period _____

Which Venue? (continued)

15. For each situation, work with your partner to determine which concert venue you would recommend. Explain your thinking.

a. 1 concert and 200 tickets total.
The Hideout: $-250(1) + 18(200)$ $= -250 + 2900$ $= 1750$
Palace Arena: $-400(1) + 40(290)$ $= -4000 + 8000$ $= 4000$
Responses vary. We would choose the Palace Arena because the profit would be $4,000 there, compared to only $1,750 at the Hideout.

b. 5 concerts and 400 tickets total.
The Hideout: $-250(5) + 18(400)$ $= -1250 + 4000$ $= 2750$
Palace Arena: $-400(5) + 40(400)$ $= -20000 + 16000$ $= -4000$
Responses vary. We would choose The Hideout because the point (5, 400) is not a solution to the inequality for the Palace Arena, but it is a solution to the inequality for the Hideout.

c. 2 concerts and 250 tickets total.
The Hideout: $-250(2) + 18(250)$ $= -500 + 2500$ $= 2000$
Palace Arena: $-400(2) + 40(250)$ $= -8000 + 10000$ $= 2000$
Responses vary. We could choose either the Palace Arena or the Hideout because the profit would be the same at both locations for 250 tickets.

16. Victor knows that (2.5, 500) represents a solution to both the Hideout inequality and the Palace Arena inequality but says it is not realistic. Is Victor correct? How do you know?
Yes. Responses vary. Sample response (2.5, 500) is a solution to the inequality because it makes the inequality true, but is not a realistic solution in the context because there cannot be 2.5 concerts.

Unit 1 Lesson 18 143 Activity 2

Key Takeaway: The symbolic and graphical representations of two-variable linear inequalities can be helpful tools for making informed decisions or recommendations to meet a specific goal.

In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a **D Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

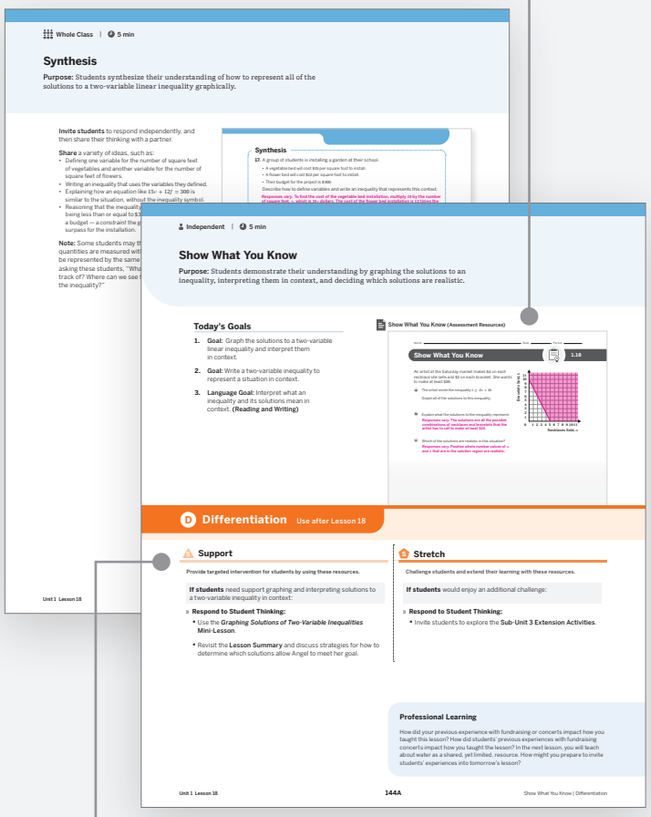
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

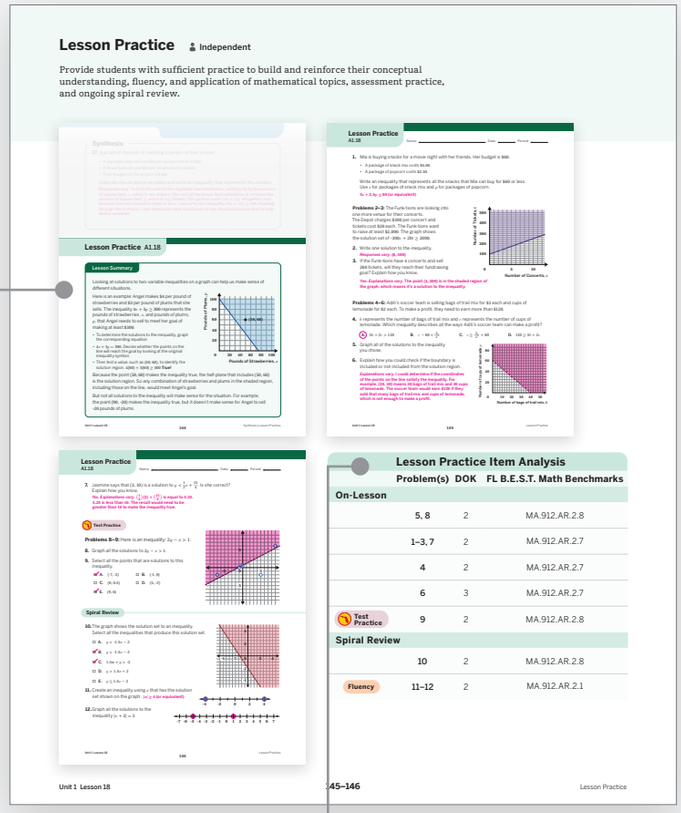
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, and ways to increase access to the language and content. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** begins with a **Summary** of the big ideas in the lesson, often including a worked example. Students can highlight parts of the summary or share it with a caregiver or classmate.



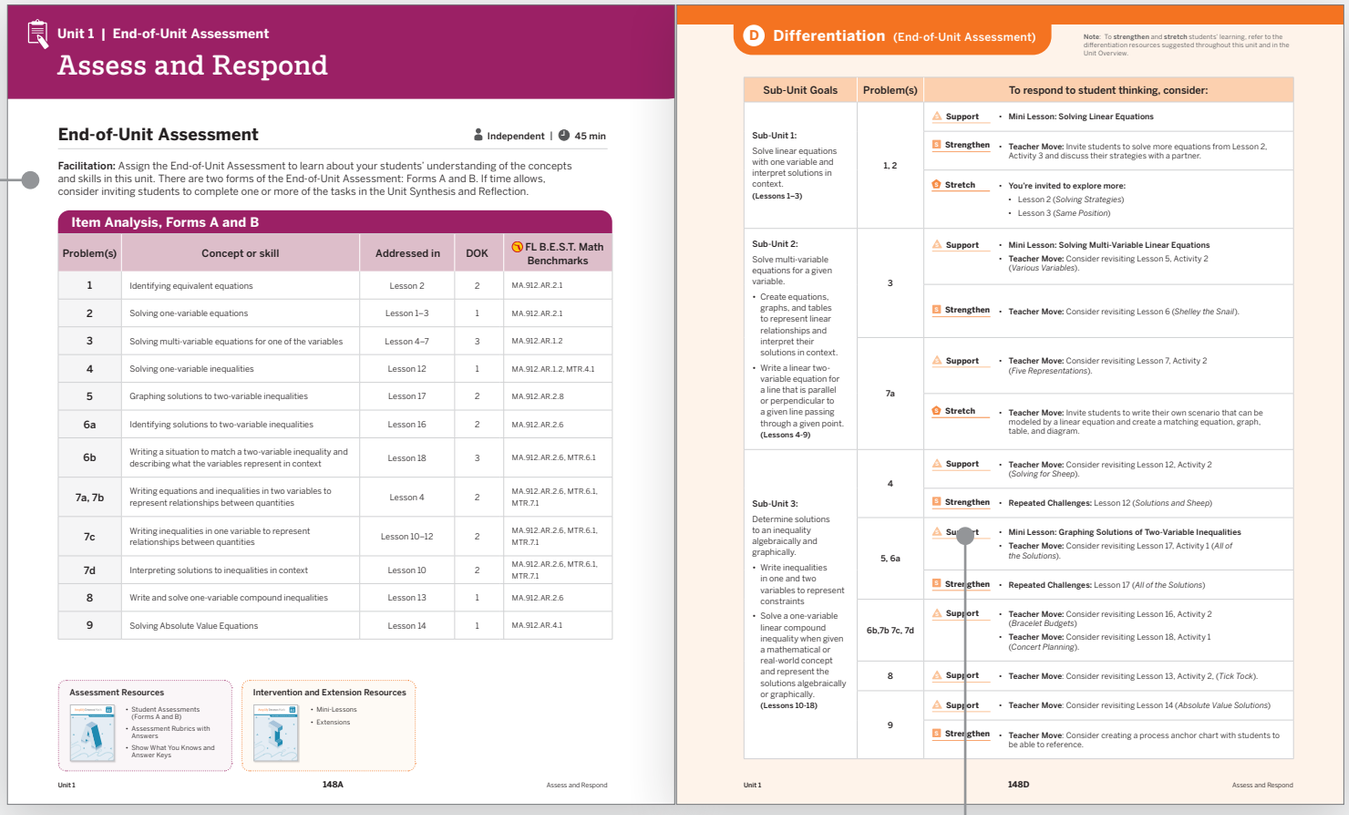
A **Practice Problem Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding B.E.S.T. Math Benchmarks.

Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

Lesson Practice

Daily practice problems for the day's lesson are included both online and in the print Student Edition, including Fluency, FAST Practice, and Spiral Review.



Unit 1 | End-of-Unit Assessment

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying equivalent equations	Lesson 2	2	MA.912.AR.2.1
2	Solving one-variable equations	Lesson 1–3	1	MA.912.AR.2.1
3	Solving multi-variable equations for one of the variables	Lesson 4–7	3	MA.912.AR.1.2
4	Solving one-variable inequalities	Lesson 12	1	MA.912.AR.1.2, MTR.4.1
5	Graphing solutions to two-variable inequalities	Lesson 17	2	MA.912.AR.2.8
6a	Identifying solutions to two-variable inequalities	Lesson 16	2	MA.912.AR.2.6
6b	Writing a situation to match a two-variable inequality and describing what the variables represent in context	Lesson 18	3	MA.912.AR.2.6, MTR.6.1
7a, 7b	Writing equations and inequalities in two variables to represent relationships between quantities	Lesson 4	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
7c	Writing inequalities in one variable to represent relationships between quantities	Lesson 10–12	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
7d	Interpreting solutions to inequalities in context	Lesson 10	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
8	Write and solve one-variable compound inequalities	Lesson 13	1	MA.912.AR.2.6
9	Solving Absolute Value Equations	Lesson 14	1	MA.912.AR.4.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Unit 1 148A Assess and Respond

Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: Solve linear equations with one variable and interpret solutions in context. **(Lessons 1–3)**	1, 2	**Support** • **Mini Lesson: Solving Linear Equations** **Strengthen** • **Teacher Move:** Invite students to solve more equations from Lesson 2, Activity 3 and discuss their strategies with a partner. **Stretch** • **You're invited to explore more:** Lesson 2 (*Solving Strategies*); Lesson 3 (*Same Position*)
Sub-Unit 2: Solve multi-variable equations for a given variable. • Create equations, graphs, and tables to represent linear relationships and interpret their solutions in context. • Write a linear two-variable equation for a line that is parallel or perpendicular to a given line passing through a given point. **(Lessons 4-9)**	3	**Support** • **Mini Lesson: Solving Multi-Variable Linear Equations** • **Teacher Move:** Consider revisiting Lesson 5, Activity 2 (*Various Variables*). **Strengthen** • **Teacher Move:** Consider revisiting Lesson 6 (*Shelley the Snail*).
	7a	**Support** • **Teacher Move:** Consider revisiting Lesson 7, Activity 2 (*Five Representations*). **Stretch** • **Teacher Move:** Invite students to write their own scenario that can be modeled by a linear equation and create a matching equation, graph, table, and diagram.
Sub-Unit 3: Determine solutions to an inequality algebraically and graphically. • Write inequalities in one and two variables to represent constraints • Solve a one-variable linear compound inequality when given a mathematical or real-world concept and represent the solutions algebraically or graphically. **(Lessons 10-18)**	4	**Support** • **Teacher Move:** Consider revisiting Lesson 12, Activity 2 (*Solving for Sheep*). **Strengthen** • **Repeated Challenges:** Lesson 12 (*Solutions and Sheep*)
	5, 6a	**Support** • **Mini Lesson: Graphing Solutions of Two-Variable Inequalities** • **Teacher Move:** Consider revisiting Lesson 17, Activity 1 (*All of the Solutions*). **Strengthen** • **Repeated Challenges:** Lesson 17 (*All of the Solutions*)
	6b,7b 7c, 7d	**Support** • **Teacher Move:** Consider revisiting Lesson 16, Activity 2 (*Bracelet Budgets*) • **Teacher Move:** Consider revisiting Lesson 18, Activity 1 (*Concert Planning*).
	8	**Support** • **Teacher Move**: Consider revisiting Lesson 13, Activity 2, (*Tick Tock*).
	9	**Support** • **Teacher Move**: Consider revisiting Lesson 14 (*Absolute Value Solutions*) **Strengthen** • **Teacher Move**: Consider creating a process anchor chart with students to be able to reference.

Unit 1 148D Assess and Respond

An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math Florida is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions*.

1 Launch



The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor



As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect



Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

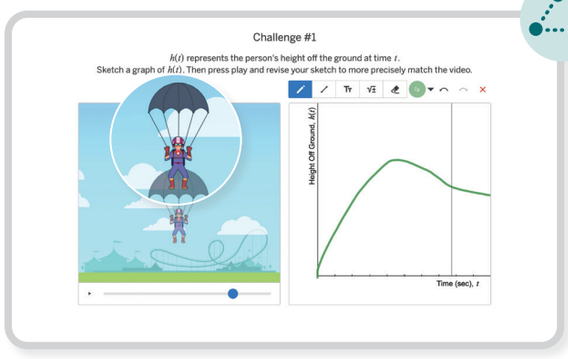
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

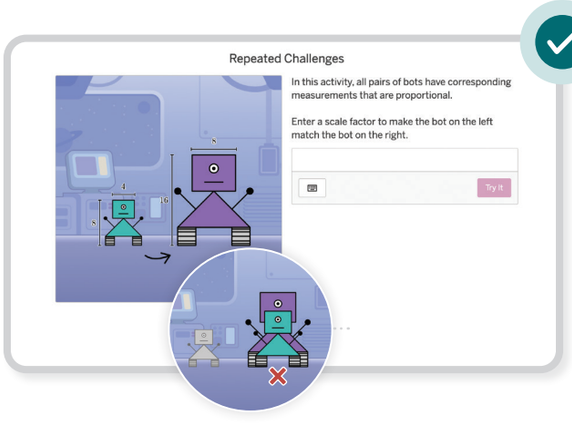
The Power of Digital

Amplify Desmos Math Florida provides the option for teachers to deliver lessons using the power of digital technology. This includes the use of visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. These interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



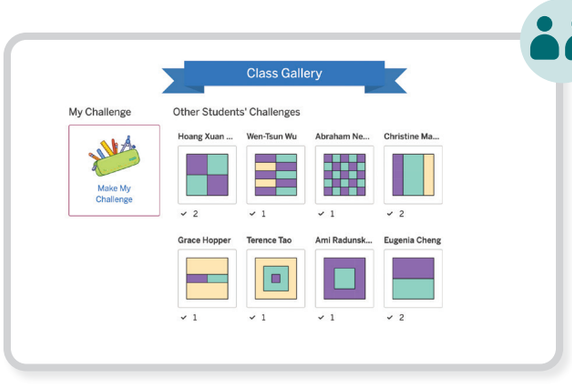
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math Florida, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

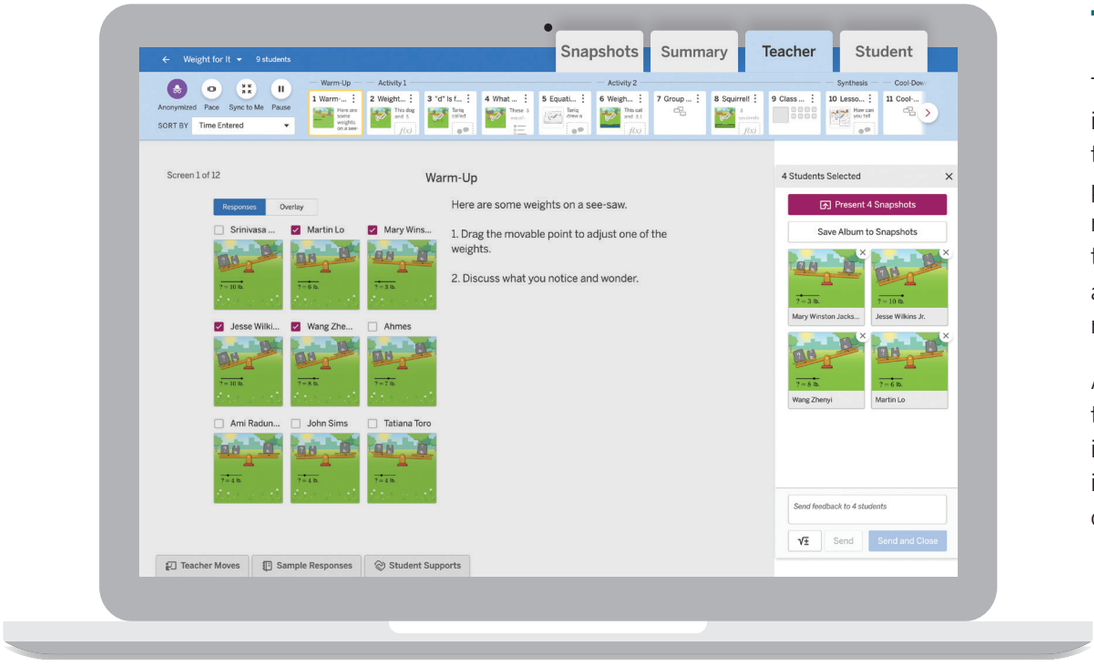


Collaborative experiences

Digital tools allow students to interact with each other's mathematical ideas in a variety of ways. Students can use the "Share With Class" button to exchange mathematical thinking with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Facilitation Tools

Amplify Desmos Math Florida includes a suite of digital facilitation tools. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

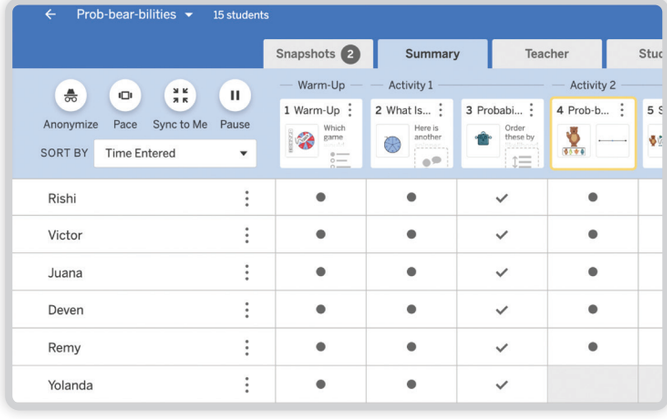
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math in Action

Each Math in Action feature highlights an individual whose work in the field of mathematics, or a connected field, have made an impact on American society. These features can be used to help make connections between the mathematical content of the grade and how math shows up in a wide variety of careers.

Career Connections

Amplify Desmos Math Florida intentionally celebrates traditional mathematics fields as well as elevates work that engages mathematics in unique ways.

Representation of Mathematicians

Students can read about the contributions and work of a diverse range of individuals with varying backgrounds, ages, careers, and ethnicities.



Unit 5 | Math in Action

Career Connection

How is city planning related to solving a puzzle?

Solving a puzzle can involve thinking about different strategies, trying out ideas, and thinking about constraints. One famous historical puzzle, the "Hundred Fowls Problem," was created by 5th century Chinese mathematician Zhang Quijian.

KieferPix/Shutterstock.com.

City planners think about constraints as they design cities to be attractive and functional. Some constraints might be geographic location, access to natural water sources, or available budget to spend. City planners might create systems of equations to help decide where to place buildings, public transit systems, and parks.

B.E.S.T. Mathematics Benchmark Connection

Many city planners rely on algebra concepts in their work. For example, they solve systems of linear equations (MA.912.AR.9.1) when they analyze various costs for materials and labor. They may also represent constraints as systems of equations or inequalities (MA.912.F.1.5) to determine what combination of supplies to use when they want to stay within budget.

Mathematical Thinking and Reasoning Connection

People who work on city planning use thinking and reasoning skills like the ones you use for your math work! For example, teams have different areas of expertise, so when they start a new project, they have to learn about the location, building materials, community requirements, and other key elements as individuals and as a group (MTR.1.1). They use multiple representations to explain budgets, schedules, supplies, and equipment needs so that the other teams involved in the planning understand their work (MTR.2.1).

Meet Zhang Qiujian

While little is known about Chinese mathematician Zhang Quijian's biography, he wrote a famous puzzle called the "Hundred Fowls" problem in his 5th century book, *Zhang Qiujian Suanjing*. Can you solve it? How can a system of equations help you get started?

"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens and chicks are there?"

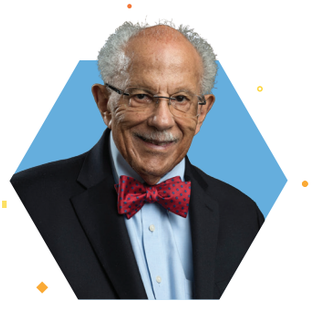
Mathematicians and others often study problems as if they are puzzles. They use equations and graphs to analyze the information they are given, and test values to find succesful solutions.

Algebra 1 Unit 5 587



Helia Bravo Hollis

Mathematics and botany? Helia Bravo Hollis found a way to combine her interest in both, studying documenting and writing books about different types of cacti.



Warren Washington

Mathematics and weather? Warren Washington found a way to combine her interest in both, developing atmospheric computer models to better understand weather patterns.



Zhang Qiujian

Mathematics and puzzles? Zhang Qiujian found a way to combine his interest in both by creating a famous puzzle called the "Hundred Fowls" that can be solved using systems of equations.



Po-Shen Loh

Mathematics and history? Po-Shen Loh found a way to combine his interest in both, researching ways mathematicians solved problems hundreds and thousands of years ago.

Facilitating Meaningful Conversations

Amplify Desmos Math Florida is designed to support interactive and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the ideas of others, and appreciate different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the real-world and mathematical experiences that students bring to the classroom.

Use the Tell a Story routine to help students make sense of the turtle race animation. Invite students to write a story about the animation using their unique experiences, perspectives, identities, and creativity. **(MP1)**



In the lesson **Turtle Time Trials**, students are invited to use the **Tell a Story** routine to create their own narratives about what the turtles in the animation are doing and why. This is a chance for students to use their creativity while engaging deeply in the math.

1 Launch

Direct students' attention to the images of the rice dishes.

To surface connections, consider asking, "What is your favorite rice dish? When or where do you eat it?"



In the lesson **Rice Ratios**, students are introduced to the concept of equivalent ratios in the context of making rice. The Teacher Edition provides teachers with prompts they can use to help students connect the rice dishes in the activities to their own lives.

Supporting Student Collaboration

Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.

How will you support student collaboration?

- Discuss what supporting each other without giving answers could look or sound like.
- Invite a student from each group to read the task aloud.

The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

In the lesson **Subway Seats**, students use the **Which One Doesn't Belong?** routine to begin exploring two-variable equations. Classroom conversations about activities like this are a way for teachers to elicit different kinds of student brilliance. The Teacher Edition provides suggestions for how teachers can engage students in conversation and reinforce the idea that each student has something uniquely valuable to contribute.

Accessibility

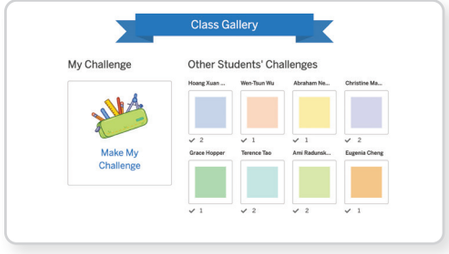
Amplify Desmos Math Florida is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

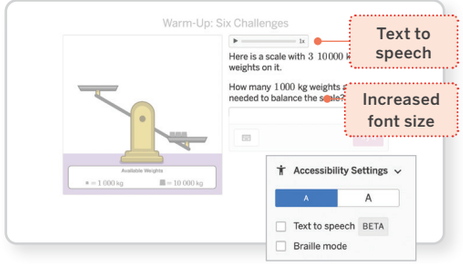
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome interests and identities, support sustaining effort and persistence, and develop emotional capacity.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

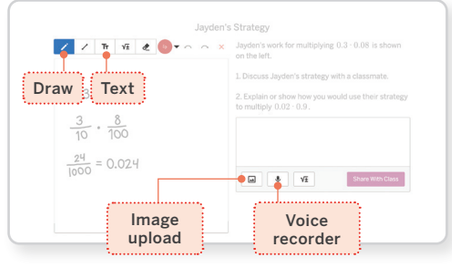
Learners make sense of information differently. Amplify Desmos Math Florida includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math Florida ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

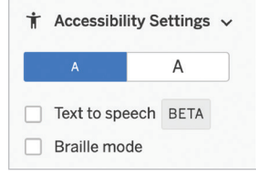
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math Florida enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

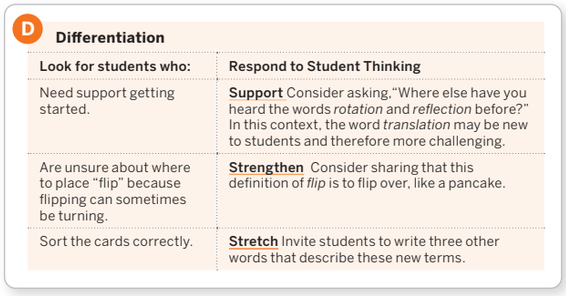
Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.



Differentiation

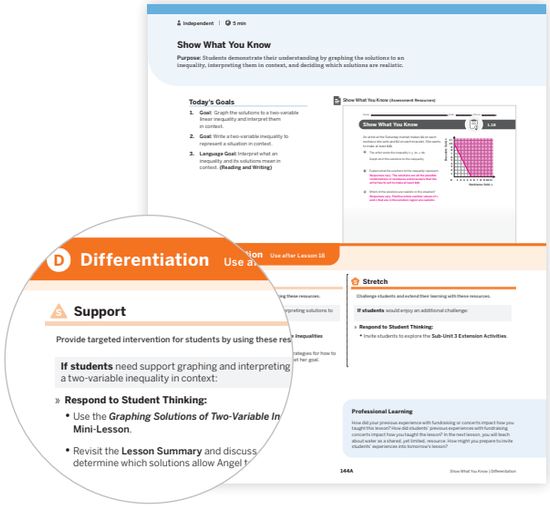
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Differentiation: Beyond the Lesson

In each lesson, students' understanding of the learning goals is broken down for teachers into three categories: students who need support to understand the learning goals, students who need to continue strengthening their understanding of the learning goals, and students who are ready to stretch their learning.

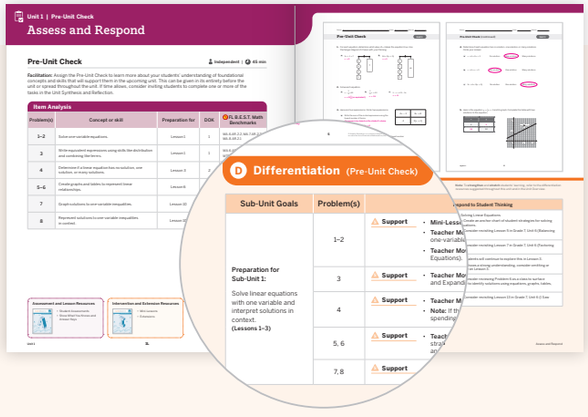
Each **Support**, **Strengthen**, and **Stretch** resource is designed to take 15 minutes:

- **Mini-Lessons:** Targeted intervention for students who need additional support or need more time.
- **Extensions:** Small group or independent challenges.



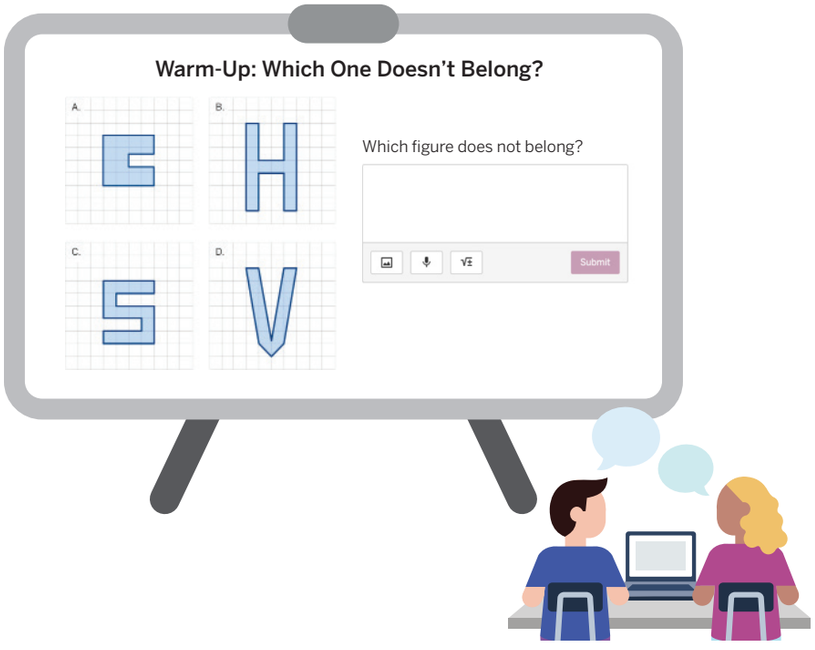
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

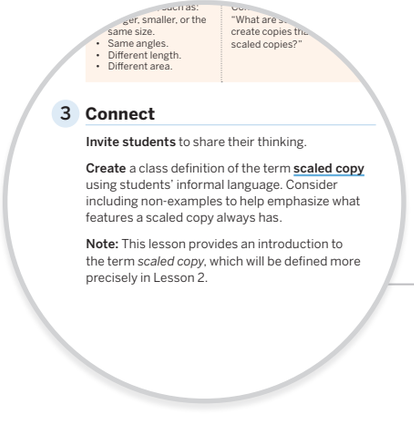
- The **Which One Doesn't Belong?** routine communicates to students that their ideas have value, that there are many ways to be correct in math, and that they can learn math by sharing their math thinking with each other.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

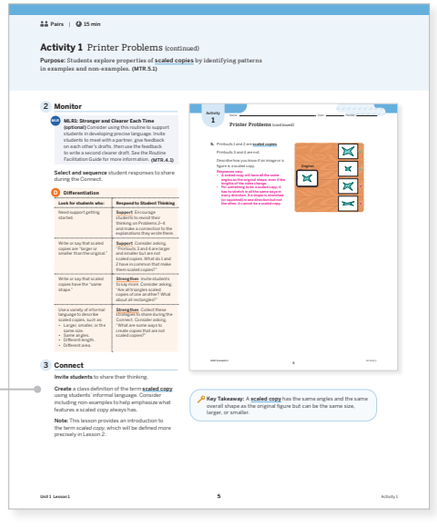
Each instructional routine included in an Amplify Desmos Math Florida lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' mathematical thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math Florida curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?





For more information about the Instructional Routines embedded in the Amplify Desmos Math Florida curriculum, see the Routine Facilitation Guide in the Professional Development Library.

Math Language Development

At Amplify Desmos Math Florida, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of English Language Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math Florida includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for English Language Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* ($'$). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does *pre* mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Calculate the area of a parallelogram on a grid using a variety of strategies.
  2. **Language Goal:** Compare and contrast strategies for determining the area of a parallelogram. **(Writing, Speaking, and Listening)**
  3. **Language Goal:** Describe parallelograms using precise mathematical language. **(Speaking and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

English Language Learners Support

Supports for ELL **English Language Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for English Language Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. English Language Learner supports may also be attached to MLRs.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Developing Fluency

Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another*.

In Algebra 1, Florida students are expected to perform all four operations with integers, positive decimals and positive fractions with procedural fluency.

Expectation

Exploration

To develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.

Procedural Reliability

To utilize skills to develop an accurate, reliable method that aligns with the student's understanding and learning style.

Procedural Fluency

To utilize skills to become fluent with an efficient, generalizable and accurate procedure, including a standard algorithm.

How Amplify Desmos Math Florida Supports the Expectation

- Manipulatives and visual models and diagrams are used extensively to support student's exploration of new concepts.
- Discussions are strongly encouraged in each lesson, including 💬 Discuss prompts in the student edition as well as suggested discussion prompts in the teacher's edition.
- Students are encouraged to estimate and compare estimations to actual answers to support reasonableness of problem solving solutions.
- Students are often asked to analyze a drawing or to make a drawing to explain their reasoning and strengthen their understanding of the math concepts being studied.

- Frequently in each course, students are asked to analyze and evaluate work presented in their student edition that represents work done by other students for accuracy, preference to learning style, and for efficiency.

- Program features such as practice, spiral reviews, mini lessons, and pre-unit assessments are all designed to assist students with achieving procedural fluency.

*National Research Council [2001, 2005, 2012]; Star [2005]

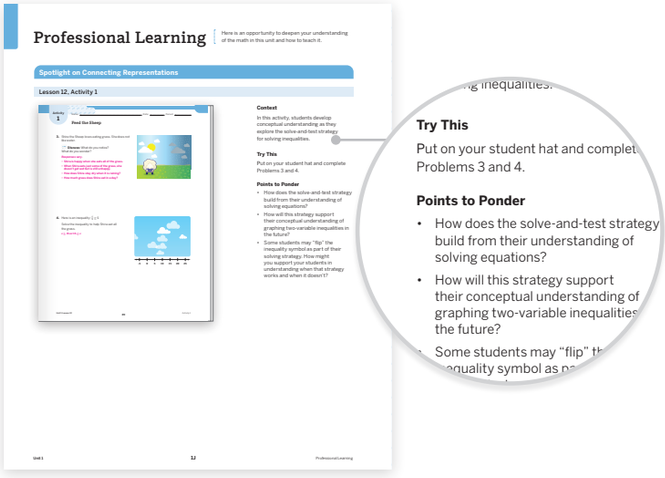
Professional Learning

The Amplify Desmos Math Florida program includes regular opportunities for teachers to deepen their understanding while they teach.

Embedded Professional Development

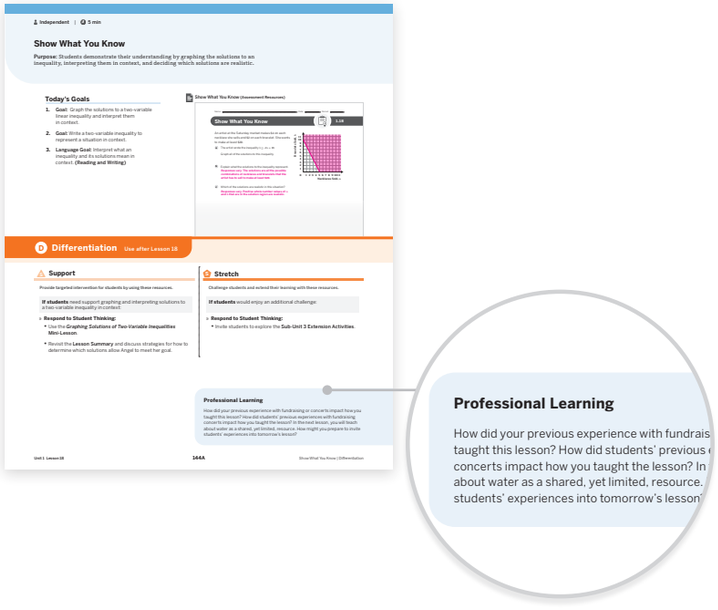
Unit-Level Professional Development

The Unit Overview for each unit includes a Professional Development page, which lists opportunities for teachers to deepen their understanding of the math in the unit and how to teach it.



Lesson-Level Professional Development

Each lesson provides a moment of professional learning as an opportunity for teachers to reflect on how well the lesson went. These professional learning moments can relate to principles, instruction, pedagogy, or program content.



Professional Development Sessions

Amplify's professional development team supports both teachers and administrators throughout their entire implementation journey. These sessions are available as on-site or virtual options.

Launch

Launch sessions introduce Amplify programs and support strong implementation. After learning about the program's foundational principles, you'll practice implementing the program in a collaborative environment.

Strengthen

Strengthen sessions deepen understanding of the program. These offerings are targeted and are meant to take your practice – and your students' learning to the next level.

Coach

These sessions are tailored to elevate instructional practices and meet the unique needs of teachers and leaders. These sessions can be customized leveraging our menu of supports.

Professional learning on your schedule

The PD Library serves as a centralized location where educators can access an ever-growing collection of professional learning resources and courses designed to support the implementation of Amplify Desmos Math Florida. Educators will find bite-sized how to videos, unit and lesson planning guides and templates, resources for leaders, and so much more!

Program Assessments

A variety of performance data in Amplify Desmos Math Florida provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math Florida, understanding is made continually visible.

Lesson-Level Assessments

Amplify Desmos Math lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.



Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.



Pre-Unit Check

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.



Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.



End-of-Unit Assessment

Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight. Two forms are included to help facilitate assessment administration in the classroom.

Florida's B.E.S.T. Standards for Mathematics

The following correlations show the alignment of Amplify Desmos Math Florida to the FL B.E.S.T. Mathematics Standards for Algebra 1.

Benchmark	B.E.S.T Mathematics Benchmark	Amplify Desmos Math Lesson(s)
	Number Sense and Operations	
MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.	**Unit 5** Lessons 5, 6, 10, 11, 12, 13, 15 **Unit 7** Lessons 3, 4, 5, 6, 7, 8, 9, 10
MA.912.NSO.1.2	Generate equivalent algebraic expressions using the properties of exponents.	**Unit 5** Lessons 5, 6, 12, 13, 15 **Unit 7** Lessons 4, 5, 6, 7, 8, 9, 10
MA.912.NSO.1.4	Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.	**Unit 5** Lessons 10, 11, 12, 13 **Unit 7** Lessons 14, 18
	Algebraic Reasoning	
MA.912.AR.1.1	Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.	**Unit 7** Lessons 3, 4, 5, 6
MA.912.AR.1.2	Rearrange equations or formulas to isolate a quantity of interest.	**Unit 1** Lessons 5, 6, 7, 8, 9 **Unit 7** Lessons 10, 14, 15, 16, 18
MA.912.AR.1.3	Add, subtract and multiply polynomial expressions with rational number coefficients.	**Unit 7** Lessons 3, 4, 5, 15, 16
MA.912.AR.1.4	Divide a polynomial expression by a monomial expression with rational number coefficients.	**Unit 7** Lessons 7, 8, 9
MA.912.AR.1.7	Rewrite a polynomial expression as a product of polynomials over the real number system.	**Unit 7** Lessons 6, 7, 9, 10, 14, 15, 16
MA.912.AR.2.1	Given a real-world context, write and solve one-variable multi-step linear equations.	**Unit 1** Lessons 2, 3
MA.912.AR.2.2	Write a linear two-variable equation to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.	**Unit 1** Lessons 5, 6, 7, 8, 9 **Unit 4** Lesson 6
MA.912.AR.2.3	Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.	**Unit 1** Lesson 9
MA.912.AR.2.4	Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.	**Unit 1** Lessons 7, 8
MA.912.AR.2.5	Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.	**Unit 1** Lessons 5, 6, 7, 8, 9
MA.912.AR.2.6	Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.	**Unit 1** Lessons 5, 6, 7, 8, 11, 12, 13, 14
MA.912.AR.2.7	Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.	**Unit 1** Lesson 18

Florida's B.E.S.T. Standards for Mathematics

MA.912.AR.2.8	Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.	**Unit 1** Lessons 12, 17, 18
MA.912.AR.3.1	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.	**Unit 7** Lessons 10, 13, 14, 15, 16, 18
MA.912.AR.3.4	Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.	**Unit 6** Lesson 15
MA.912.AR.3.5	Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function.	**Unit 6** Lesson 11
MA.912.AR.3.6	Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.	**Unit 6** Lessons 12, 13, 15 **Unit 7** Lessons 10, 13, 16
MA.912.AR.3.7	Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.	**Unit 6** Lessons 6, 7, 8, 10, 11, 12, 13
MA.912.AR.3.8	Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.	**Unit 6** Lessons 7, 13
MA.912.AR.4.1	Given a mathematical or real-world context, write and solve one-variable absolute value equations.	**Unit 1** Lessons 15, 16
MA.912.AR.4.3	Given a table, equation or written description of an absolute value function, graph that function and determine its key features.	**Unit 3** Lesson 12
MA.912.AR.5.3	Given a mathematical or real-world context, classify an exponential function as representing growth or decay.	**Unit 5** Lessons 5, 6
MA.912.AR.5.4	Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.	**Unit 5** Lesson 4
MA.912.AR.5.6	Given a table, equation or written description of an exponential function, graph that function and determine its key features.	**Unit 5** Lessons 4, 7
MA.912.AR.9.1	Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.	**Unit 4** Lessons 3, 4, 5, 6, 7
MA.912.AR.9.4	Graph the solution set of a system of two-variable linear inequalities.	**Unit 4** Lessons 9, 10
MA.912.AR.9.6	Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.	**Unit 4** Lessons 7, 10
	Functions	
MA.912.F.1.1	Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.	**Unit 3** Lesson 4 **Unit 6** Lessons 5, 6
MA.912.F.1.2	Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.	**Unit 3** Lessons 3, 6, 10 **Unit 5** Lesson 6 **Unit 6** Lessons 8, 10, 11, 12, 13

Florida's B.E.S.T. Standards for Mathematics

MA.912.F.1.3	Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.	**Unit 3** Lesson 8
MA.912.F.1.5	Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.	**Unit 3** Lessons 4, 8, 9, 10
MA.912.F.1.6	Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.	**Unit 3** Lessons 4, 6, 7, 9 **Unit 5** Lessons 3, 4, 14
MA.912.F.1.8	Determine whether a linear, quadratic or exponential function best models a given real-world situation.	**Unit 5** Lessons 3, 4, 14 **Unit 6** Lessons 3, 4, 5, 6
MA.912.F.2.1	Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.	**Unit 6** Lesson 15
	Financial Literacy	
MA.912.FL.3.2	Solve real-world problems involving simple, compound and continuously compounded interest.	**Unit 5** Lessons 14, 15
MA.912.FL.3.4	Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.	**Unit 5** Lessons 14, 15
	Data Analysis & Probability	
MA.912.DP.1.1	Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.	**Unit 2** Lessons 2, 4, 5, 6, 7, 8, 14
MA.912.DP.1.2	Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.	**Unit 2** Lessons 2, 6, 7, 8, 14
MA.912.DP.1.3	Explain the difference between correlation and causation in the contexts of both numerical and categorical data.	**Unit 2** Lesson 17
MA.912.DP.1.4	Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.	**Unit 2** Lessons 9, 10
MA.912.DP.2.4	Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.	**Unit 2** Lessons 15, 16, 17
MA.912.DP.2.6	Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.	**Unit 2** Lessons 15, 16, 17
MA.912.DP.3.1	Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.	**Unit 2** Lessons 12, 13

Mathematical Thinking and Reasoning Standards

The following correlations show the alignment of Amplify Desmos Math Florida to the Mathematical Thinking and Reasoning Standards.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Unit 1 Lessons 1, 3, 6, 7, 10, 11, 12, 13, 15, 16, 18
Unit 2 Lessons 7, 9, 10, 13, 14, 15, 16, 17
Unit 3 Lessons 3, 7, 10
Unit 4 Lesson 5
Unit 5 Lesson 11
Unit 6 Lessons 5, 11
Unit 7 Lessons 1, 2, 3

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Unit 1 Lessons 1, 4, 6, 7, 10, 11, 13, 14, 16, 17
Unit 2 Lessons 4, 6, 12
Unit 3 Lessons 2, 5, 7, 10, 11
Unit 4 Lessons 1, 2, 3, 4, 6, 7, 8, 10
Unit 5 Lessons 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15
Unit 7 Lessons 4, 5, 7, 8, 9, 11, 12, 13, 15

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Unit 1 Lessons 4, 5
Unit 2 Lessons 2, 5, 13
Unit 3 Lessons 1, 4
Unit 4 Lessons 1, 2, 3, 7, 8, 10
Unit 5 Lessons 3, 10
Unit 6 Lessons 5, 11
Unit 7 Lessons 2, 3, 4, 6, 16, 17, 18

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Unit 1 Lessons 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15, 17
Unit 2 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17
Unit 3 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12
Unit 4 Lessons 1, 4, 6, 9
Unit 5 Lessons 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15
Unit 6 Lessons 6, 7, 8, 10, 13, 15
Unit 7 Lessons 1, 2, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Unit 1 Lessons 1, 2, 3, 12, 14, 15, 17, 18
Unit 2 Lessons 2, 3, 8
Unit 3 Lessons 3, 4, 11, 12
Unit 4 Lessons 1, 4, 6, 9
Unit 5 Lessons 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15
Unit 7 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Unit 1 Lessons 9, 10, 13, 18
Unit 2 Lessons 6, 13, 14
Unit 3 Lesson 5
Unit 4 Lessons 2, 3, 5, 7, 8, 9, 10
Unit 5 Lessons 4, 6, 13, 14
Unit 6 Lessons 11, 12, 15
Unit 7 Lessons 4, 10, 12, 14, 15, 18

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Unit 1 Lessons 8, 9, 18
Unit 2 Lessons 3, 4, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17
Unit 3 Lesson 5
Unit 4 Lessons 2, 3, 7, 8, 10
Unit 5 Lessons 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 15
Unit 6 Lessons 4, 12, 15
Unit 7 Lesson 18

English Language Arts B.E.S.T. Standards

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

Unit 1 Lessons 3, 8, 11, 12, 13
Unit 2 Lessons 1, 5, 6, 11
Unit 3 Lessons 1, 2, 4, 10
Unit 4 Lessons 1, 4, 7, 10
Unit 5 Lessons 1, 6, 8, 13, 15
Unit 6 Lessons 1, 2, 10
Unit 7 Lessons 2, 3, 13, 15, 18

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

Unit 1 Lesson 18
Unit 2 Lesson 13
Unit 3 Lessons 2, 3, 5, 6, 9, 10, 11
Unit 4 Lesson 6
Unit 5 Lessons 7, 12, 13, 14, 15
Unit 6 Lesson 15
Unit 7 Lesson 16, 18

ELA.K12.EE.3.1 Make inferences to support comprehension.

Unit 1 Lessons 1, 4, 8, 12, 13, 14, 15, 16, 17
Unit 2 Lessons 1, 2, 4, 5, 7, 9, 10, 11, 13, 15, 17
Unit 3 Lessons 4, 5, 7, 8, 11
Unit 4 Lessons 1, 3, 4, 5, 7
Unit 5 Lessons 2, 3, 4, 6, 9
Unit 6 Lessons 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 14, 15
Unit 7 Lessons 3, 5, 8, 16, 18

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

Unit 1 Lessons 2, 4, 5, 8, 9, 13, 14, 15, 16, 17
Unit 2 Lessons 3, 4, 9, 10, 11, 12, 14, 16, 17
Unit 3 Lessons 1, 6, 7, 9, 12
Unit 4 Lessons 2, 3, 5, 6, 10
Unit 5 Lessons 3, 5, 7, 8, 10, 11, 12, 15
Unit 6 Lessons 2, 4, 5, 6, 13
Unit 7 Lessons 1, 2, 5, 7, 8, 9, 11, 13, 14

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

Unit 1 Lessons 1, 2, 3, 5, 6, 7, 10, 11, 14, 15, 16, 17
Unit 2 Lessons 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17
Unit 3 Lessons 1, 11
Unit 4 Lessons 1, 2, 3, 4, 5, 6, 8, 9
Unit 5 Lessons 1, 2, 3, 9
Unit 6 Lessons 3, 5, 6, 7, 8, 9, 10, 11, 13
Unit 7 Lessons 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 15, 16, 17

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

Unit 1 Lessons 3, 4, 8, 9, 10, 13
Unit 2 Lessons 1, 4, 5, 8, 12, 14, 15, 16
Unit 3 Lesson 1
Unit 4 Lessons 8, 9
Unit 5 Lessons 1, 2, 14
Unit 6 Lesson 12
Unit 7 Lessons 9, 11

English Language Development Standards

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

Unit 1 Lessons - All Lessons
Unit 2 Lessons - All Lessons
Unit 3 Lessons - All Lessons
Unit 4 Lessons - All Lessons
Unit 5 Lessons - All Lessons
Unit 6 Lessons - All Lessons
Unit 7 Lessons - All Lessons

ELD.K12.ELL.SI.1 English language learners communicate for social and instructional purposes within the school setting.

Unit 1 Lessons - All Lessons
Unit 2 Lessons - All Lessons
Unit 3 Lessons - All Lessons
Unit 4 Lessons - All Lessons
Unit 5 Lessons - All Lessons
Unit 6 Lessons - All Lessons
Unit 7 Lessons - All Lessons

UNIT 1

Linear Equations and Inequalities

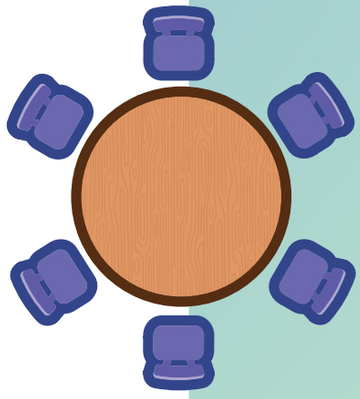


Students revisit strategies for solving one-variable equations and inequalities and extend their knowledge to make sense of multi-variable equations and two-variable inequalities.

Essential Questions

- What does it mean to be a solution to an equation or inequality?
- How are the different representations of equations or inequalities connected?
- How do you determine and graph the solution of an inequality?

Sub-Unit Goals



Sub-Unit 1

- Solve linear equations with one variable and interpret solutions in context.

Sub-Unit 2

- Solve multi-variable equations for a given variable.
- Create equations, graphs, and tables to represent linear relationships and interpret their solutions in context.
- Write a linear two-variable equation for a line that is parallel or perpendicular to a given line passing through a given point.

Sub-Unit 3

- Determine solutions to an inequality algebraically and graphically.
- Write inequalities in one and two variables to represent constraints
- Solve a one-variable linear compound inequality when given a mathematical or real-world concept and represent the solutions algebraically or graphically.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Rearrange equations or formulas to isolate a quantity of interest. **(MA.912.AR.1.2)**

Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context. **(MA.912.AR.2.2)**

Other Standards Addressed in This Unit: MA.912.AR.2.1 MA.8.AR.2.1 MA.912.AR.2.3 MA.912.AR.2.4 MA.912.AR.2.6 MA.912.AR.2.7 MA.912.AR.2.8 MA.912.AR.4.1

Mathematical Thinking and Reasoning Standards

- Students connect representations of situations to make sense of an equation or inequality. **(MTR.1.1)**
- Students write equations to represent contexts and interpret their solutions. **(MTR.2.1)**
- Students use the structure of an equation or inequality to reason about their solving strategy. **(MTR.5.1)**
- Students model constraints with equations and inequalities to represent different situations. **(MTR.7.1)**

Also Addressed: **MTR.3.1, MTR.4.1, MTR.6.1**

Coherence Telling a story within and across grades

In This Unit

Students solve linear equations and inequalities with one variable, including equations with no solution or infinitely many solutions. They write equations representing linear situations and solve multi-variable equations for a given variable. Students represent constraints by writing inequalities and determine their solutions algebraically and graphically.

Prior Learning

- **Grade 6,** Unit 6: Write expressions involving variables to represent situations **(MA.6.A.R.2.2)**
- **Grade 7,** Unit 6: Expand and factor expressions with positive and negative coefficients **(MA.7.NSO.2.2)**
- **Grade 7,** Unit 6: Represent inequalities symbolically and graphically **(MA.7.AR.2.1)**
- **Grade 8,** Unit 3: Represent solutions of an equation or inequality on a graph **(MA.8.AR.3.4)**
- **Grade 8,** Unit 4: Solve linear equations with variables on both sides **(MA.8.AR.2.1)**

Future Learning

- **Algebra 1,** Unit 4: Solve systems of equations and inequalities **(MA.912.AR.9.1)**
- **Algebra 1,** Unit 7: Determine the number of solutions for quadratic equations **(MA.912.AR.3.1)**
- **Algebra 1,** Unit 7: Solve quadratic equations graphically and algebraically **(MA.912.AR.3.6, MA.912.AR.3.1)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 6**, students develop conceptual understanding by creating equivalent forms of a linear equation and recognizing that both equations represent the same situation, yet reveal different properties.

Strategic Competence

In **Lesson 17**, students develop strategic competence as they consider how many values to test to determine the solution region.

Procedural Fluency

In **Lesson 3**, students develop procedural fluency as they practice solving one-variable equations and interpret their solutions in context.

Adaptive Reasoning

In **Lesson 2**, students critically analyze others' process for solving equations, identifying correct and incorrect steps.

Application

In **Lesson 5**, students apply their solving strategies for one-variable equations to rearrange multi-variable equations to highlight a variable of interest.

Productive Disposition

In **Lesson 18**, students see mathematics as a powerful tool to analyze real-life data and help make decisions.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.AR.1.2 MA.6.AR.1.4 MA.6.AR.2.2 MA.7.AR.2.2 MA.8.AR.2.1 MA.8.AR.3.4
MTR.2.1 MTR.3.1 MTR.5.1 MTR.6.1 MTR.7.1

Sub-Unit 1

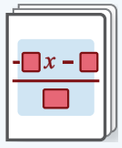


1 Working Backwards

Solving Equations With Inverse Operations

Use inverse operations and balanced moves to solve equations and model using substitution to check solutions.

MA.912.AR.2.1
MTR.1.1 MTR.2.1 MTR.5.1



2 Solving Strategies

More Solving With One-Variable Equations

Determine if equations are equivalent, analyze correct and incorrect steps for solving an equation, and practice solving equations.

MA.912.AR.2.1
MTR.4.1 MTR.5.1



3 Same Position

No Solution and Infinitely Many Solutions

Practice solving equations in context and interpreting their solutions, including equations with no solution or infinitely many solutions.

MA.912.AR.2.1 MA.8.AR.2.1
MTR.1.1 MTR.4.1 MTR.5.1

Assess and Respond



A Sub-Unit Quiz 1

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.912.AR.1.2 MA.912.AR.2.1 MA.912.AR.2.2 MA.912.AR.2.3 MA.912.AR.2.5
MTR.2.1 MTR.4.1

Sub-Unit 3



8 Whale Growth Rates

Finding Equations of Parallel Lines

Exploring the connection between rates of changes and parallel lines.

MA.912.AR.2.3
MTR.4.1 MTR.7.1



9 House Design

Finding Equations of Perpendicular Lines

Exploring the connection between rates of changes and perpendicular lines.

MA.912.AR.2.3
MTR.4.1 MTR.6.1 MTR.7.1



10 Pizza Delivery

Representing Situations With One-Variable Inequalities

Explore connections between constraints and one-variable inequalities.

MA.912.AR.2.6
MTR.1.1 MTR.2.1 MTR.4.1

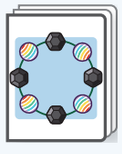


15 Getting Absolute

Creating and Solving Absolute Value Equations

Create and solve absolute value inequalities in context and interpret their solutions.

MA.912.AR.4.1
MTR.1.1 MTR.4.1 MTR.5.1



16 Bracelet Budgets

Introduction to Two-Variable Inequalities

Make connections between two-variable inequalities, their solutions, and their graphs.

MA.912.AR.2.8
MTR.1.1 MTR.2.1



17 All of the Solutions

Graphing Solutions to Two-Variable Inequalities

Explore how to represent all of the solutions of a two-variable inequality graphically.

MA.912.AR.2.8
MTR.3.1 MTR.4.1



18 Concert Planning

Graphing Two-Variable Inequalities in Context

Extend their understanding of how to determine and interpret solutions to two-variable inequalities.

MA.912.AR.2.7 MA.912.AR.2.8
MTR.1.1 MTR.5.1 MTR.6.1 MTR.7.1

Pacing: 21 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 18 Lessons
- Pre-Unit Check
- Sub-Unit Quiz(zes)
- End-of-Unit Assessment

Sub-Unit 2

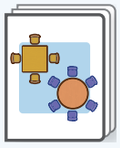


4 Subway Seats

Representing Situations With Two-Variable Equations

Interpret the values in different forms of linear equations and consider solutions in a subway context.

MA.912.AR.2.2 MA.912.AR.1.2
MTR.2.1 MTR.3.1 MTR.4.1



5 Various Variables

Solving Multi-Variable Equations

Apply what they learned about solving equations with one variable to solve equations with variable coefficients.

MA.912.AR.2.2 MA.912.AR.1.2
MTR.3.1 MTR.4.1



6 Shelley the Snail

Connecting Graphs and Linear Equations

Revisit graphical representations of situations and make connections between different representations of two-variable equations.

MA.912.AR.2.2 MA.912.AR.2.4
MA.912.AR.1.2
MTR.1.1 MTR.2.1



7 Five Representations

Linear Relationships in Equations, Tables, and Graphs

Apply prior knowledge about representing situations with two variables, including solving for one of the variables.

MA.912.AR.2.2 MA.912.AR.2.4
MA.912.AR.1.2
MTR.1.1



11 Graphing Inequalities

Inequalities on a Number Line

Use reasoning to determine solutions to a one-variable inequality and graph the solutions on a number line.

MA.912.AR.2.6
MTR.1.1 MTR.2.1 MTR.4.1



12 Solutions and Sheep

Solving One-Variable Inequalities

Solve one-variable inequalities by solving a corresponding equation and testing values.

MA.912.AR.2.6
MTR.1.1 MTR.4.1

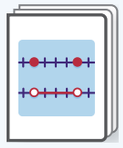


13 Tick Tock

One-Variable Compound Linear Inequalities

Write and solve compound inequalities in context and interpret their solutions.

MA.912.AR.2.6
MTR.1.1 MTR.2.1 MTR.4.1 MTR.6.1



14 Absolute Value Solutions

Solving Absolute Value Equations and Inequalities

Solve one-variable absolute value equations and inequalities by reasoning and representing their solutions on a graph.

MA.912.AR.4.1
MTR.2.1 MTR.5.1

Math in Action

M Math in Action

Connecting Math to Cactus Plants

Explore the connection between finding patterns and the contributions of Helio Brava Hollis to the field of botany.

MA.#.XX.X.X MA.#.XX.X.X
MTR.X.X

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.AR.1.2 MA.912.AR.2.1
MA.912.AR.2.6 MA.912.AR.2.8
MA.912.AR.4.1
MTR.7.1

Pacing Considerations

Lesson 1: This lesson is intended to revisit strategies for solving equations from Grades 6–8. If students show a strong understanding of solving equations in Problems 1 and 2 in the Pre-Unit Check, this lesson may be omitted.

Lesson 7: This lesson solidifies what students have learned about representations of linear relationships, including solving for a variable from an equation in standard form. If students show a strong understanding of solving in Lesson 6 and of making graphs in the Pre-Unit Check, this lesson may be omitted or combined with Practice Day 1. If this lesson is omitted, be sure to discuss what an equation in standard form looks like during the synthesis in Lesson 6.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1
Copy: Show What You Know
Materials: scientific calculator *(as needed)*

Lesson 2
Copy: Show What You Know
Materials: no required materials

Lesson 3
Copy: Show What You Know
Materials: no required materials

Lesson 4
Copy: Show What You Know
Materials: scientific calculator *(as needed)*

Lesson 5
Copy: Show What You Know
Materials: chips or coins *(as needed)*, highlighters *(as needed)*

Lesson 6
Copy: Show What You Know
Materials: no required materials

Lesson 7
Copy: Show What You Know
Materials: Activity 1 Cards, one per pair, pre-cut; coloring tools *(as needed)*; group-sized whiteboards or chart paper *(optional)*

Lesson 8
Copy: Show What You Know
Materials: graph paper, straightedges, scientific calculators

Lesson 9
Copy: Show What You Know
Materials: graph paper, straightedges, scientific calculators

Lesson 10
Copy: Show What You Know
Materials: no required materials

Lesson 11
Copy: Show What You Know
Materials: no required materials

Lesson 12
Copy: Show What You Know
Materials: scientific calculator *(optional)*

Lesson 13
Copy: Show What You Know
Materials: no required materials

Lesson 14
Copy: Show What You Know
Materials: no required materials

Lesson 15
Copy: Show What You Know
Materials: no required materials

Lesson 16
Copy: Show What You Know
Materials: coloring tools

Lesson 17
Copy: Show What You Know
Materials: scientific calculator *(as needed)*

Lesson 18
Copy: Show What You Know
Materials: scientific calculators *(optional)* coloring tools *(optional)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

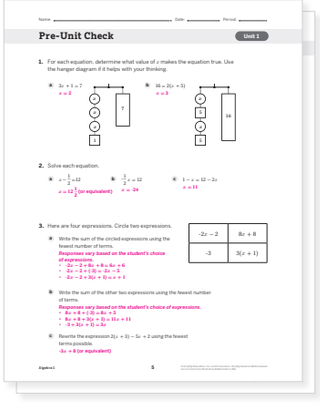
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.AR.1.2, MA.6.AR.1.4, MA.6.AR.2.2, MA.7.AR.2.2, MA.8.AR.2.1, MA.8.AR.3.4


Pre-Unit Check

Unit 1

1. For each equation, determine what value of x makes the equation true. Use the hanger diagram if it helps with your thinking.

a. $3x + 1 = 7$
$x = 2$

b. $16 = 2(x + 5)$
$x = 3$

2. Solve each equation.

a. $x - \frac{1}{2} = 12$
$x = 12\frac{1}{2}$ (or equivalent)

b. $\frac{1}{2}x = 12$
$x = 24$

c. $1 - x = 12 - 2x$
$x = 11$

3. Here are four expressions. Circle two expressions.

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

a. Write the sum of the circled expressions using the fewest number of terms.
Responses vary based on the student's choice of expressions.
- $8x + 8 + (-2x - 2) = 6x + 6$
- $-2x - 2 + 3(x + 1) = 1x - 1$
- $8x - 2 + 3(x + 1) = x + 1$

b. Write the sum of the other two expressions using the fewest number of terms.
Responses vary based on the student's choice of expressions.
- $8x + 8 + 3(x + 1) = 11x + 11$
- $-3 + 3(x + 1) = 3x$

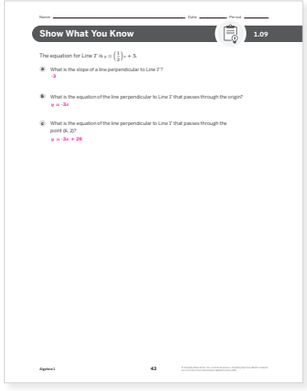
c. Rewrite the expression $2(x + 3) - 5x + 2$ using the fewest terms possible.
$-3x + 8$ (or equivalent)

Algebra 1 5

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed: on Lesson 9 Show What You Know: MA.912.AR.2.3


Show What You Know

1.09

The equation for Line 2 is $y = \left(\frac{1}{3}\right)x + 5$.

a. What is the slope of a line perpendicular to Line 2?
-3

b. What is the equation of the line perpendicular to Line 2 that passes through the origin?
$y = -3x$

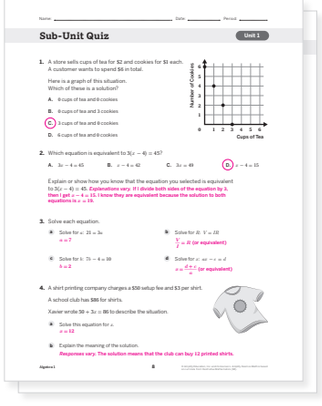
c. What is the equation of the line perpendicular to Line 2 that passes through the point (6, 2)?
$y = -3x + 20$

Algebra 1 43

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.912.AR.1.2, MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.3, MA.912.AR.2.5


Sub-Unit Quiz

Unit 1

1. A store sells cups of tea for $2 and cookies for $1 each. A customer wants to spend $6 in total.
Here is a graph of this situation.
Which of these is a solution?

A. 6 cups of tea and 6 cookies
B. 6 cups of tea and 3 cookies
C. 3 cups of tea and 6 cookies
D. 4 cups of tea and 6 cookies

2. Which equation is equivalent to $3(x - 4) = 45$?
A. $3x - 4 = 45$ B. $x - 4 = 42$ C. $3x = 49$ D. $x - 4 = 15$
Explain or show how you know that the equation you selected is equivalent to $3(x - 4) = 45$. Explanations vary. If I divide both sides of the equation by 3, then I get $x - 4 = 12$. I know they are equivalent because the solution to both equations is $x = 16$.

3. Solve each equation.
a. Solve for x: $25 = 3x$
$x = 7$
b. Solve for R: $\frac{V}{I} = R$
$V = IR$ (or equivalent)
c. Solve for t: $2t - 4 = 10$
$t = 2$
d. Solve for x: $ax - c = d$
$x = \frac{d + c}{a}$ (or equivalent)

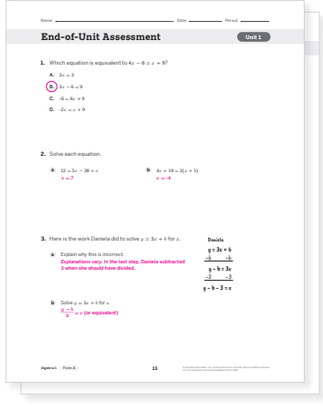
4. A shirt printing company charges a $50 setup fee and $3 per shirt.
A school club has $86 for shirts.
Xavier wrote $50 + 3x = 86$ to describe the situation.
a. Solve this equation for x.
$x = 12$
b. Explain the meaning of the solution.
Responses vary. The solution means that the club can buy 12 printed shirts.

Algebra 1 8

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.912.AR.1.2, MA.912.AR.2.1, MA.912.AR.2.6, MA.912.AR.2.8, MA.912.AR.4.1


End-of-Unit Assessment

Unit 1

1. Which equation is equivalent to $4x - 6 = x + 9$?
A. $3x = 3$
B. $3x - 6 = 9$
C. $-6 = 6x + 9$
D. $-2x = x + 9$

2. Solve each equation.
a. $12 = 5x - 38 + x$
$x = 7$
b. $4x + 10 = 2(x + 1)$
$x = -4$

3. Here is the work Daniela did to solve $y = 3x + b$ for x.

Daniela

$y = 3x + b$
$-b \quad -b$
$y - b = 3x$
$\frac{-3}{} \quad \frac{-3}{}$
$y - b - 3 = x$

a. Explain why this is incorrect.
Explanations vary. In the last step, Daniela subtracted 3 when she should have divided.
b. Solve $y = 3x + b$ for x.
$\frac{y - b}{3} = x$ (or equivalent)

Algebra 1 Form B 13

D Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

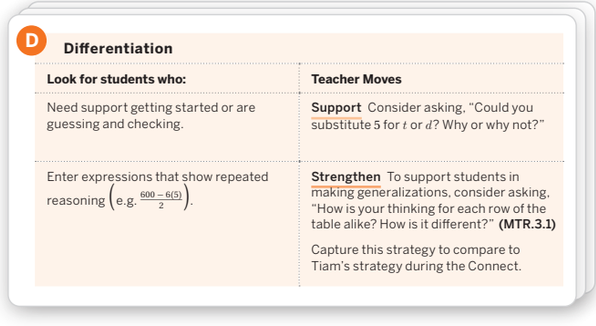
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–3) Solve linear equations with one variable, including equations with no solution or infinitely many solutions.	**Mini-Lessons** • Solving Linear Equations **Previous Lessons** • Grade 7, Unit 6, Lesson 10 (*Collect the Squares*) • Grade 8, Unit 4, Lesson 2 (*All, Some, or None?*)	**Repeated Challenges** • Lesson 1 (*Working Backwards*)	**You're Invited to Explore More.** • Lesson 2 (*Solving Strategies*) • Lesson 3 (*Same Position*) • Lesson 3, Problems 6–7
Sub-Unit 2 (Lessons 4–9) • Solve multi-variable equations for a given variable. • Write equations to represent linear situations.	**Mini-Lessons** • Solving Multi-Variable Linear Equations **Previous Lessons** • Grade 6, Unit 6, Lesson 6 (*Vari-apples*)	**Teacher Moves** • In Lesson 5, Activity 2, invite students to solve for a different variable in the second equation of each problem.	**Teacher Moves** • In Lesson 9 (*House Design*), invite students to create their own house design, like in Activity 1, and write the equations of their roof lines.
Sub-Unit 3 (Lessons 10–18) • Determine solutions to an inequality algebraically and graphically. • Write inequalities in one and two variables to represent constraints	**Mini-Lessons** • Graphing Solutions of Two-Variable Inequalities **Previous Lesson** • Grade 7, Unit 6, Lesson 15 (*Shira the Sheep*)	**Repeated Challenges** • Lesson 12 (*Solutions and Sheep*) • In Lesson 15, Activity 1, Problem 3, invite students to solve the sorted equations to verify if they have two, one, or no solution(s).	**Teacher Moves** • In Lesson 18, Activity 2, invite students to research the costs of reserving two different local concert venues and complete the activity with this information.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 4**, where students make sense of what combinations of seating and standing capacity will work for a given equation. As we develop students' ability to create and interpret these combinations in context, some students may need support with getting started. Other students may need scaffolding to see how their expressions demonstrate how to rearrange an equation to solve for a variable of interest.



Differentiation

Look for students who:	Teacher Moves
Need support getting started or are guessing and checking.	**Support** Consider asking, "Could you substitute 5 for t or d? Why or why not?"
Enter expressions that show repeated reasoning (e.g. $\frac{600-6(5)}{2}$).	**Strengthen** To support students in making generalizations, consider asking, "How is your thinking for each row of the table alike? How is it different?" **(MTR.3.1)** Capture this strategy to compare to Tiam's strategy during the Connect.

Accessibility

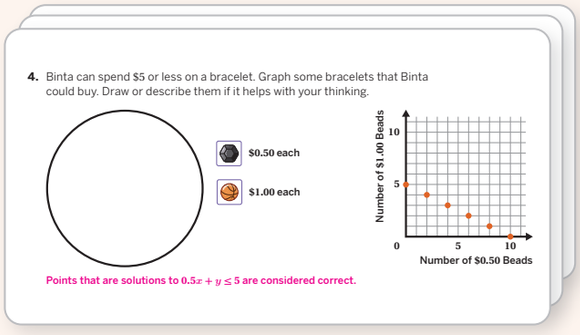
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support engagement.

Spotlight on Recruiting Interest . . .

In **Lesson 16:** Students optimize individual choice and autonomy as they create their own unique combination of beads to craft a bracelet that is within a given budget.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing . . .

Here are two suggestions in this unit for supporting students with conceptual processing.

Lesson 1, Activity 1

Accessibility: Conceptual Processing Invite students to revisit their work to help them think about working backwards. Consider asking:

- "To get 13, what number do we need to put in before adding 3?" 10 **(MTR.2.1)**
- "To get 10, what number do we need to put in before that number is multiplied by 2?" 5 **(MTR.2.1)**

Lesson 11, Activity 2

Accessibility: Conceptual Processing Invite students to read each inequality aloud (e.g., "Five times a number plus four is greater than or equal to seven times the same number").

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
solution	1
equivalent equations	2
infinitely many solutions	3
no solution	3
x-intercept	6
y-intercept	6
standard form (of a linear equation)	7
slope-intercept form	7
constraint	10
absolute value	14
compound inequality	13
boundary line	17

Review Vocabulary

boundary point
distributive property
equation
inverse operations
linear relationship
region
solution set
substitute
variable

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	1–3, 7, 14, 17
MLR3: Critique, Correct, Clarify	2, 6, 11, 17
MLR6: Three Reads	18
MLR7: Compare and Connect	1, 3, 5, 10, 14, 16

Sub-Unit Language Goals

Students engage in the following language functions:

- Justify that each step in solving a linear equation creates an equivalent equation with the same solution **(Reading and Writing)**
- Describe connections between equations, tables, descriptions, and graphs of linear situations. **(Writing and Speaking)**
- Interpret two-variable inequalities that represent constraints in a situation. **(Writing and Speaking)**
- Analyze two-variable linear inequalities in order to make sense of an issue in society. **(Reading, Writing, Speaking, and Listening)**

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 16**, where students describe what the numbers and variables in an inequality represent.

ELL **English Language Learners** Consider providing sentence frames such as: "The ___ represents ___." **(Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Solving Systems of Equations

In this unit, students solve one- and two-variable equations. **In Unit 4,** students will solve systems of equations algebraically using elimination.

FL B.E.S.T. Math Benchmarks:
MA.912.AR.9.1, MA.912.AR.9.6

Example:

Elimination is a method that generalizes how to solve systems of two or more linear equations. When solving a system of equations using elimination, we add or subtract equations (or multiples of equations) in order to eliminate one of the variables, which determines an equation in one variable.

$$x + 2y = 10$$
$$x + y = 7$$

In this example, subtracting the two equations leaves $y = 3$. Substituting this value into either equation results in $x = 4$, so the solution to this system of equations is the ordered pair $(4, 3)$.

Solving Systems of Inequalities

In this unit, students solve one- and two-variable inequalities. **In Unit 4,** students will solve systems of inequalities.

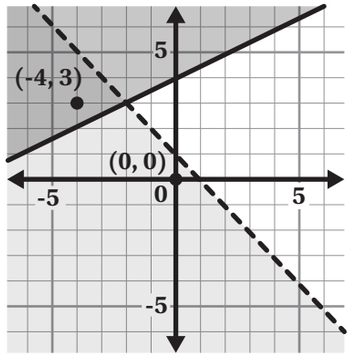
FL B.E.S.T. Math Benchmarks:
MA.912.AR.9.4, MA.912.AR.9.6

Example:

Solving a system of inequalities means finding an ordered pair (x, y) that makes both inequalities true. Testing different (x, y) ordered pairs is one way to determine which region contains the solutions to a system of inequalities.

For example, in this graph, the point $(0, 0)$ was tested in the system of inequalities shown. When substituted into both inequalities, $(0, 0)$ makes one inequality true, but not both. When the point $(-4, 3)$ is substituted, both inequalities are true. This indicates that region that contains $(-4, 3)$ is the solution region.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$



Solving Quadratic Equations Graphically and Algebraically

In this unit, students determine the number of solutions of a one-variable linear equation. **In Unit 7,** students will determine the number of solutions of a quadratic equation.

FL B.E.S.T. Math Benchmarks:
MA.912.AR.3.1

Example:

The structure of a quadratic equation can help determine the number of solutions.

For example, quadratics can have no solution, one solution, or two solutions. By looking at the structure of the equations, students can reason about whether the vertex of the quadratic is greater than, less than, or equal to a value to help determine the number of solutions of a quadratic equation.

No Solution

$$x^2 + 4 = 0$$

One Solution

$$(x + 4)^2 = 0$$

Two Solutions

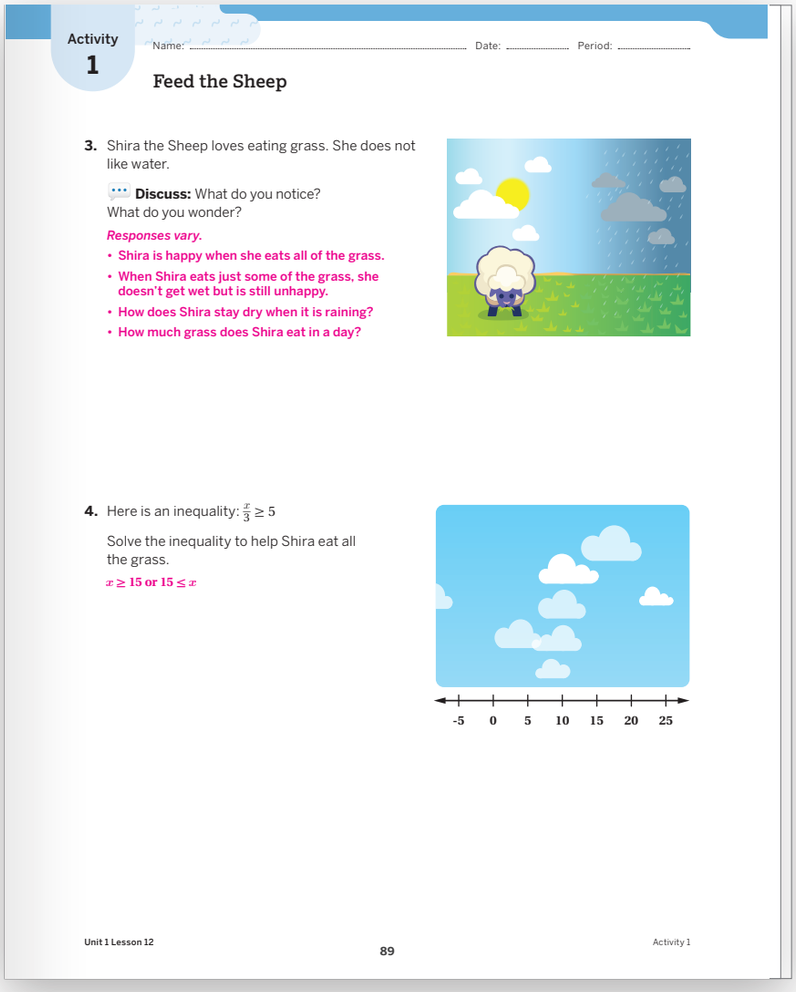
$$(x + 4)^2 = 1$$

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Connecting Representations

Lesson 12, Activity 1



Activity 1

Name: Date: Period:

Feed the Sheep

3. Shira the Sheep loves eating grass. She does not like water.

Discuss: What do you notice? What do you wonder?

Responses vary.

- Shira is happy when she eats all of the grass.
- When Shira eats just some of the grass, she doesn't get wet but is still unhappy.
- How does Shira stay dry when it is raining?
- How much grass does Shira eat in a day?

4. Here is an inequality: $\frac{x}{3} \geq 5$

Solve the inequality to help Shira eat all the grass.

$x \geq 15$ or $15 \leq x$

-5 0 5 10 15 20 25

Unit 1 Lesson 12 89 Activity 1

Context

In this activity, students develop conceptual understanding as they explore the solve-and-test strategy for solving inequalities.

Try This

Put on your student hat and complete Problems 3 and 4.

Points to Ponder

- How does the solve-and-test strategy build from their understanding of solving equations?
- How will this strategy support their conceptual understanding of graphing two-variable inequalities in the future?
- Some students may "flip" the inequality symbol as part of their solving strategy. How might you support your students in understanding when that strategy works and when it doesn't?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1–2	Solve one-variable equations.	Lesson 1	1	MA.6.AR.2.2, MA.7.AR.2.2, MA.8.AR.2.1
3	Write equivalent expressions using skills like distribution and combining like terms.	Lesson 1	1	MA.6.AR.1.4 MTR.3.1
4	Determine if a linear equation has no solution, one solution, or many solutions.	Lesson 3	2	MA.8.AR.2.1 MTR.5.1
5–6	Create graphs and tables to represent linear relationships.	Lesson 6	2	MA.8.AR.3.4
7	Graph solutions to one-variable inequalities.	Lesson 10	2	MA.6.AR.1.2
8	Represent solutions to one-variable inequalities in context.	Lesson 10	2	MA.6.AR.1.2 MTR.6.1, MTR.7.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

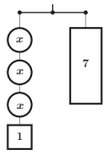
Pre-Unit Check

Unit 1

1. For each equation, determine what value of x makes the equation true. Use the hanger diagram if it helps with your thinking.

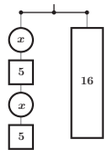
a $3x + 1 = 7$

$x = 2$



b $16 = 2(x + 5)$

$x = 3$



2. Solve each equation.

a $x - \frac{1}{2} = 12$

$x = 12\frac{1}{2}$ (or equivalent)

b $-\frac{1}{2}x = 12$

$x = -24$

c $1 - x = 12 - 2x$

$x = 11$

3. Here are four expressions. Circle two expressions.

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

a Write the sum of the circled expressions using the fewest number of terms.

Responses vary based on the student's choice of expressions.

- $-2x - 2 + 8x + 8 = 6x + 6$
- $-2x - 2 + (-3) = -2x - 5$
- $-2x - 2 + 3(x + 1) = x + 1$

b Write the sum of the other two expressions using the fewest number of terms.

Responses vary based on the student's choice of expressions.

- $8x + 8 + (-3) = 8x + 5$
- $8x + 8 + 3(x + 1) = 11x + 11$
- $-3 + 3(x + 1) = 3x$

c Rewrite the expression $2(x + 3) - 5x + 2$ using the fewest terms possible.

$-3x + 8$ (or equivalent)

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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 1

4. Determine if each equation has no solution, one solution, or many solutions. Circle your answer.

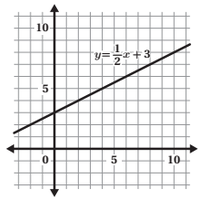
a $x + 6 = 2x + 3$ — No solution — **One solution** — Many solutions

b $x + 6 = x + 3$ — **No solution** — One solution — Many solutions

c $2x + 6 = 2(x + 3)$ — No solution — One solution — **Many solutions**

5. Here is the equation $y = \frac{1}{2}x + 3$ and its graph. Complete the table with two solutions to the equation.

x	y
4	5
18	12



D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

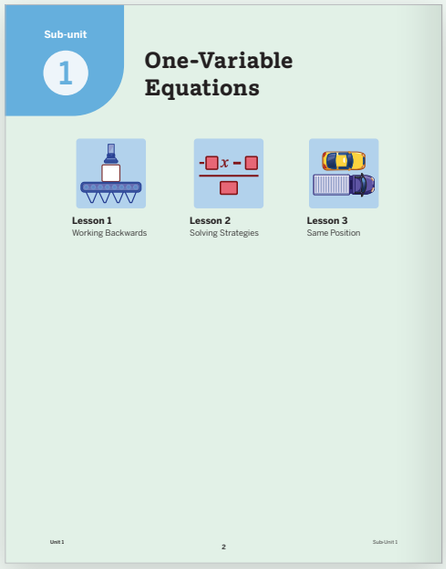
Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: Solve linear equations with one variable and interpret solutions in context. **(Lessons 1–3)**	1–2	**Support** • **Mini-Lesson:** Solving Linear Equations • **Teacher Move:** Create an anchor chart of student strategies for solving one-variable equations. • **Teacher Move:** Consider revisiting Lesson 5 in Grade 7, Unit 6 (Balancing Equations).
	3	**Support** • **Teacher Move:** Consider revisiting Lesson 7 in Grade 7, Unit 6 (Factoring and Expanding).
	4	**Support** • **Teacher Move:** Students will continue to explore this in Lesson 3. • **Note:** If the class shows a strong understanding, consider omitting or spending less time on Lesson 3.
	5, 6	**Support** • **Teacher Move:** Consider reviewing Problem 6 as a class to surface strategies for how to identify solutions using equations, graphs, tables, and situations.
	7, 8	**Support** • **Teacher Move:** Consider revisiting Lesson 13 in Grade 7, Unit 6 (I Saw the Signs).

One-Variable Equations

Sub-Unit 1 Goals

- **Goal:** Solve linear equations with one variable, including equations with no solution or infinitely many solutions.
- **Language Goal:** Describe strategies for solving one-variable linear equations, including justifying the number of solutions a one-variable equation will have.

In this sub-unit, students revisit strategies for solving one-variable equations, including using inverse operations and making moves that keep the equation balanced. They recall that a **solution** to an equation is a value that makes the equation true. Students observe and describe what happens when dividing by a variable, and use their observations to justify whether a one-variable equation has one solution, **no solution**, or **infinitely many solutions**.



Math That Matters Most

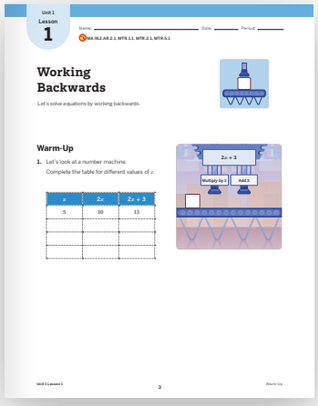
Concept or Skill	For example . . .
Describe how to work backwards when solving a one-variable equation.	÷6 +3 ·2 x | $6x$ | $6x-3$ | $\frac{6x-3}{2}$ 1.5 | 9 | 6 | 3
Use inverse operations to solve a one-variable equation.	$2 \cdot \frac{6x-3}{2} = 3 \cdot 2$ $6x - 3 = 6$ $+3 \quad +3$ $\frac{6x}{6} = \frac{9}{6}$ $x = 1.5$
Creating equivalent equations as part of a solving process.	$6 - 7x = \frac{-15x - 12}{3}$ $6 - 7x = -5x - 4$ $10 - 7x = -5x$ $10 = 2x$ $5 = x$

Concept or Skill	For example . . .
Determine whether a one-variable equation has one solution, no solution, or infinitely many solutions.	$18t - 4 = 12t + 20$ One, t = 4
Make sense of what happens when dividing by a variable when solving a one-variable equation.	**Discuss:** Is this strategy correct? $\frac{12t}{t} = \frac{20t}{t}$ $12 = 20$ they will never meet

Unit 1

Lesson

1



Working Backwards

Solving Equations With Inverse Operations

Let's solve equations by working backwards.

Learning Outcomes

Today's Goals

1. **Goal:** Use inverse operations to solve an equation.
2. **Language Goal:** Describe strategies for solving multi-step equations. **(Writing, Speaking, and Listening)**

Students use *inverse operations* and the concept of balance to solve one-variable *equations*. They make connections between equations, number machines, and tables as tools for solving. Students observe the importance of checking **solutions** using substitution through responsive feedback.

Coherence

Prior Learning

In Grade 6, students were introduced to equations and variables as tools to represent relationships. In Grade 7, students wrote and solved equations in order to find values that make the equations true. In Grade 8, students wrote and used expressions to solve for unknown values.

Future Learning

In Lesson 2, students will be reintroduced to equivalent equations and will consider different moves that can be made when solving for an unknown value. In Lesson 3, students will practice solving equations, including equations with no solutions or infinitely many solutions.

Connections

- Students relate the working backwards strategy to the operation of a machine. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.1

Given a real-world context, write and solve one-variable multi-step linear equations.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.1	MA.912.AR.2.1
MA.8.AR.2.1	MA.912.AR.1.2

Vocabulary

New Vocabulary

solution

Review Vocabulary

equation

inverse operations

Lesson at a Glance

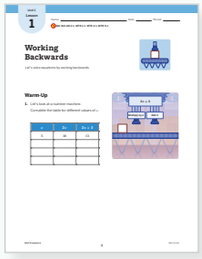
~ 45 min

MA.912.AR.2.1

Warm-Up

Independent | 5 min

Students prepare to solve multi-step *equations* by making sense of how a number machine works.



Activity 1

Pairs | 15 min

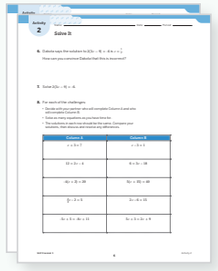
Students use number machines, tables, and the strategy of working backwards to make sense of solving multi-step equations using inverse operations. MLR7: Compare and Connect



Activity 2

Independent | 15 min

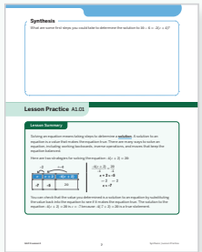
Students use inverse operations to develop fluency in solving multi-step equations and use substitution to check **solutions**.



Synthesis

Whole Class | 5 min

Students synthesize their strategies for solving multi-step linear equations.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by a solving a multi-step equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students prepare to solve multi-step *equations* by making sense of how a number machine works.

1 Launch

Display the Warm-Up.

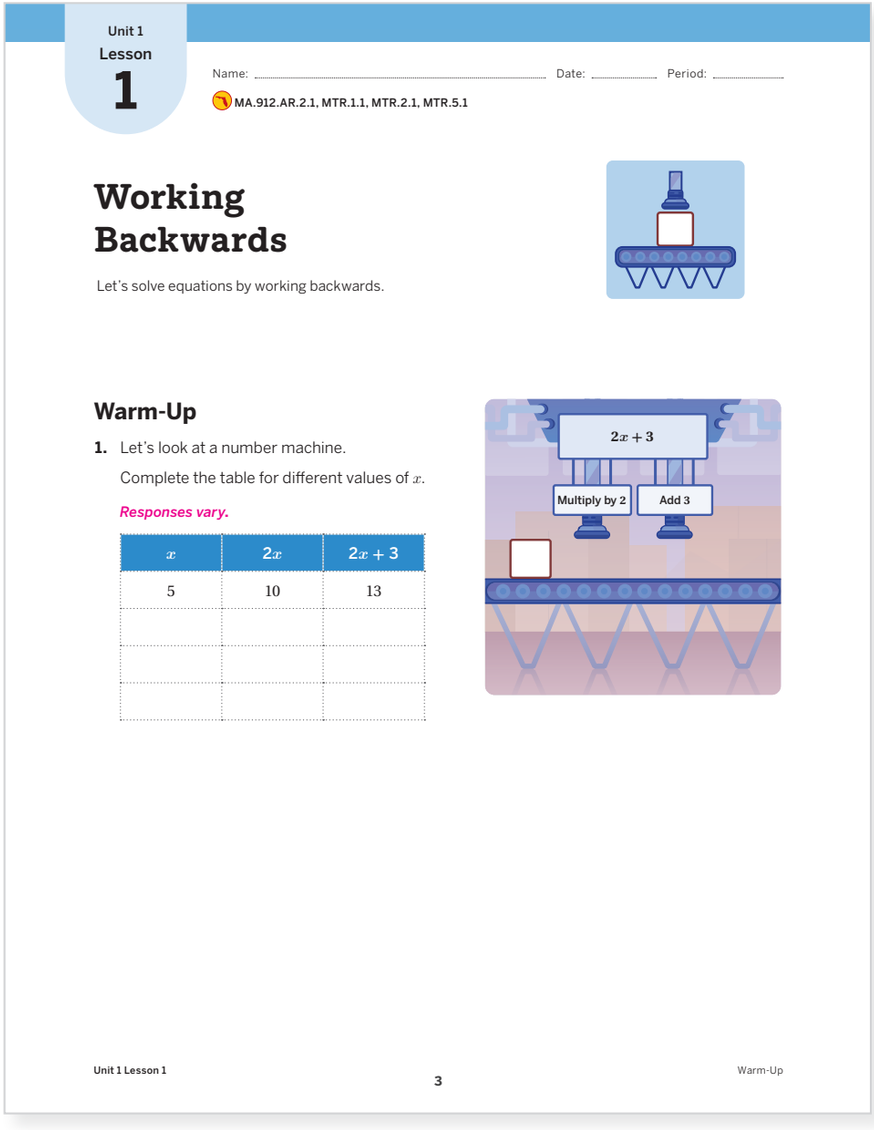
Invite students to choose a number and predict what will happen when it is input into the number machine.

To build understanding, invite students to try a variety of numbers.

2 Connect

Consider asking:

- "Did anyone try an interesting input they would like to share? What about an interesting output?"
- "What connections do you see between the expression and the number machine? Between the expression and the table?" **(MTR.5.1)**
- "If you switched the parts of the machine, would you get the same outputs? Why or why not?"



Unit 1
Lesson
1

Name: _____ Date: _____ Period: _____

MA.912.AR.2.1, MTR.1.1, MTR.2.1, MTR.5.1

Working Backwards

Let's solve equations by working backwards.

Warm-Up

1. Let's look at a number machine.

Complete the table for different values of x.

Responses vary.

x	$2x$	$2x + 3$
5	10	13

Unit 1 Lesson 1 3 Warm-Up

Pairs | 15 min

Activity 1 Number Machines

Purpose: Students use number machines, tables, and the strategy of working backwards to make sense of solving multi-step equations using inverse operations.

1 Launch

Invite students to make suggestions for what number they think Alina put into the number machine. Choose several student suggestions to input into the machine.

2 Monitor

ELL **English Language Learners** Invite students to share different meanings of the term *solution* (e.g., the solution to a problem, a solution in chemistry class, the solution to an equation). Clarify which meaning this word will have for this unit. **(Reading and Listening)**

Look for strategies students use on the problems:

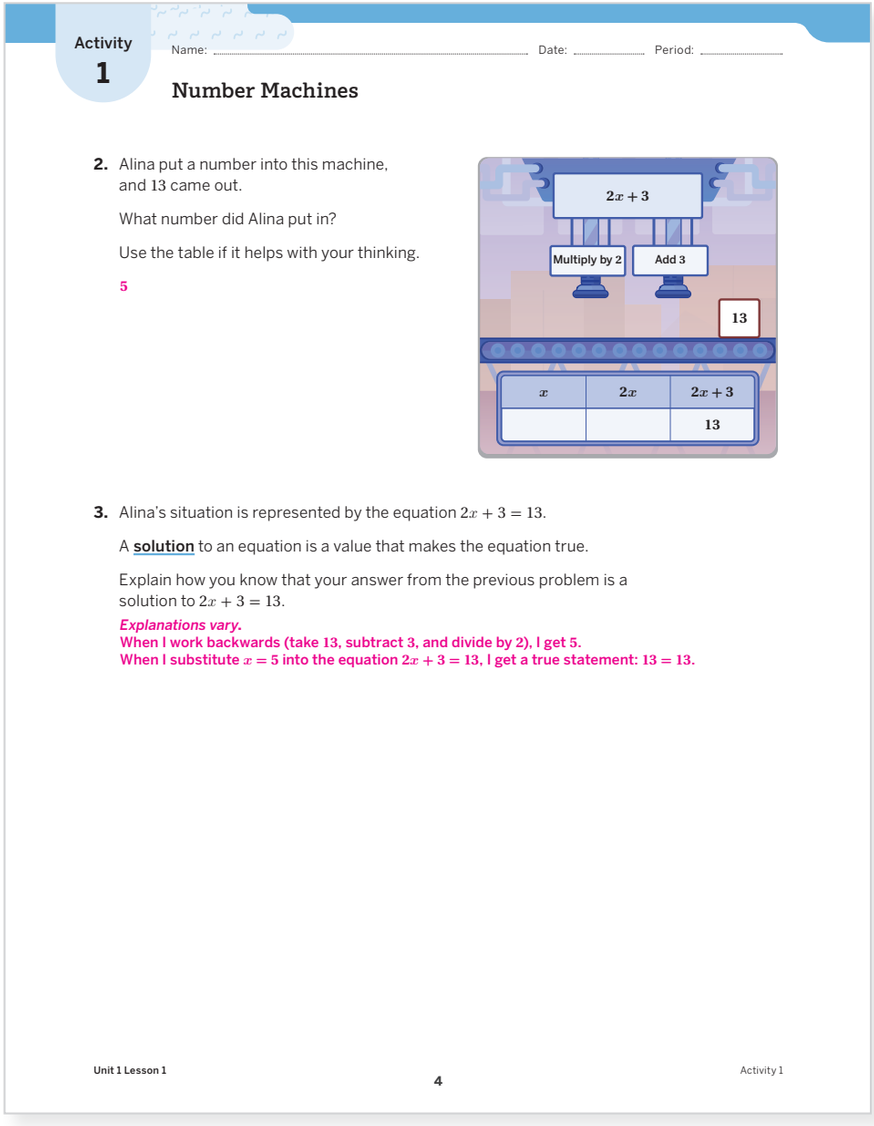
D **Differentiation**

Look for students who:	Teacher Moves
Need support getting started.	**Support** Encourage students to make a reasonable estimate for a number to try in the machine and use the feedback to revise their thinking.
Use guess and check.	**Strengthen** Invite them to create a number machine or table to complete the problem.
Use inverse operations to work backwards.	**Strengthen** Celebrate this student's strategy. Capture this strategy to share during the Connect.
Use reasoning or a unique strategy.	**Stretch** Consider asking, "Are there any output numbers that the machine cannot produce? How do you know?"

A **Accessibility: Conceptual Processing** Invite students to revisit their work to help them think about working backwards. Consider asking:

- "To get 13, what number do we need to put in before adding 3?" 10 **(MTR.2.1)**
- "To get 10, what number do we need to put in before that number is multiplied by 2?" 5 **(MTR.2.1)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Number Machines

2. Alina put a number into this machine, and 13 came out.

What number did Alina put in?

Use the table if it helps with your thinking.

5

3. Alina's situation is represented by the equation $2x + 3 = 13$.

A **solution** to an equation is a value that makes the equation true.

Explain how you know that your answer from the previous problem is a solution to $2x + 3 = 13$.

Explanations vary.
When I work backwards (take 13, subtract 3, and divide by 2), I get 5.
When I substitute $x = 5$ into the equation $2x + 3 = 13$, I get a true statement: $13 = 13$.

Unit 1 Lesson 1 4 Activity 1

Pairs | 15 min

Activity 1 Number Machines (continued)

Purpose: Students use number machines, tables, and the strategy of working backwards to make sense of solving multi-step equations using inverse operations.

2 Monitor

ELL **English Language Learners** Provide sentence frames for students to use as they explain their thinking. For example, "The original equation uses the operations _____ and _____. To work backwards, I first _____. Then I _____. The solution is _____." **(Reading and Writing)**

Look for students who use strategies similar to Jin's and Nasir's.

Invite students to share their strategies.

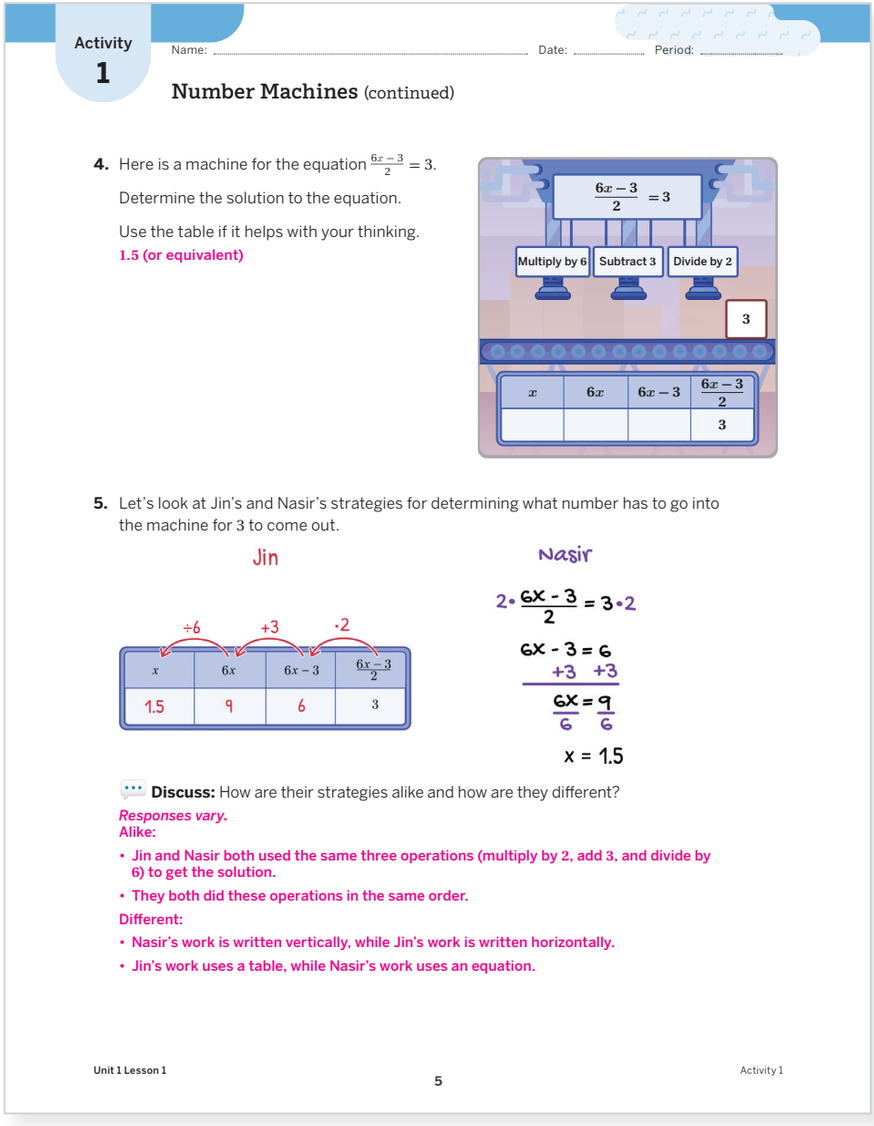
3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own solving strategy.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one strategy at a time before comparing them.

Consider asking:

- "What did each student do?"
- "Why did the two strategies both lead to the same solution?" **(MTR.2.1)**
- "How did Nasir use the idea of balance to solve the equation?" **(MTR.5.1)**
- "Do solving moves need to balance both sides of the equation? Why or why not?"
- "How does each strategy compare to your own?"
- "Where do you see the solution in each strategy?"



Activity 1

Name: Date: Period:

Number Machines (continued)

4. Here is a machine for the equation $\frac{6x-3}{2} = 3$.

Determine the solution to the equation.

Use the table if it helps with your thinking.

1.5 (or equivalent)

x	$6x$	$6x-3$	$\frac{6x-3}{2}$
			3

5. Let's look at Jin's and Nasir's strategies for determining what number has to go into the machine for 3 to come out.

Jin

÷6	+3	·2	
x	$6x$	$6x-3$	$\frac{6x-3}{2}$
1.5	9	6	3

Nasir

$2 \cdot \frac{6x-3}{2} = 3 \cdot 2$

$6x - 3 = 6$

$+3 \quad +3$

$\frac{6x}{6} = \frac{9}{6}$

$x = 1.5$

Discuss: How are their strategies alike and how are they different?

Responses vary.

Alike:

- Jin and Nasir both used the same three operations (multiply by 2, add 3, and divide by 6) to get the solution.
- They both did these operations in the same order.

Different:

- Nasir's work is written vertically, while Jin's work is written horizontally.
- Jin's work uses a table, while Nasir's work uses an equation.

Unit 1 Lesson 1 5 Activity 1

Key Takeaway: There is more than one way to solve an equation. One way to solve an equation is to work backwards using *inverse operations*.

Independent | 15 min

Activity 2 Solve It

Purpose: Students use inverse operations to develop fluency in solving multi-step equations and use substitution to check **solutions**.

Short on time? Consider inviting students to complete fewer Repeated Challenges on Problem 8.

1 Launch

Consider asking, "How would you know whether you've correctly solved an equation?"

Invite students to show how they would check to see if $x = \frac{1}{2}$ is a solution.

Capture and share several student sketches that show a variety of ways to check to see if $x = \frac{1}{2}$ is a solution.

2 Monitor

Encourage students to record their thinking.

Invite students to set their own goal for how many problems they would like to complete, sharing that sense-making is more important than speed.

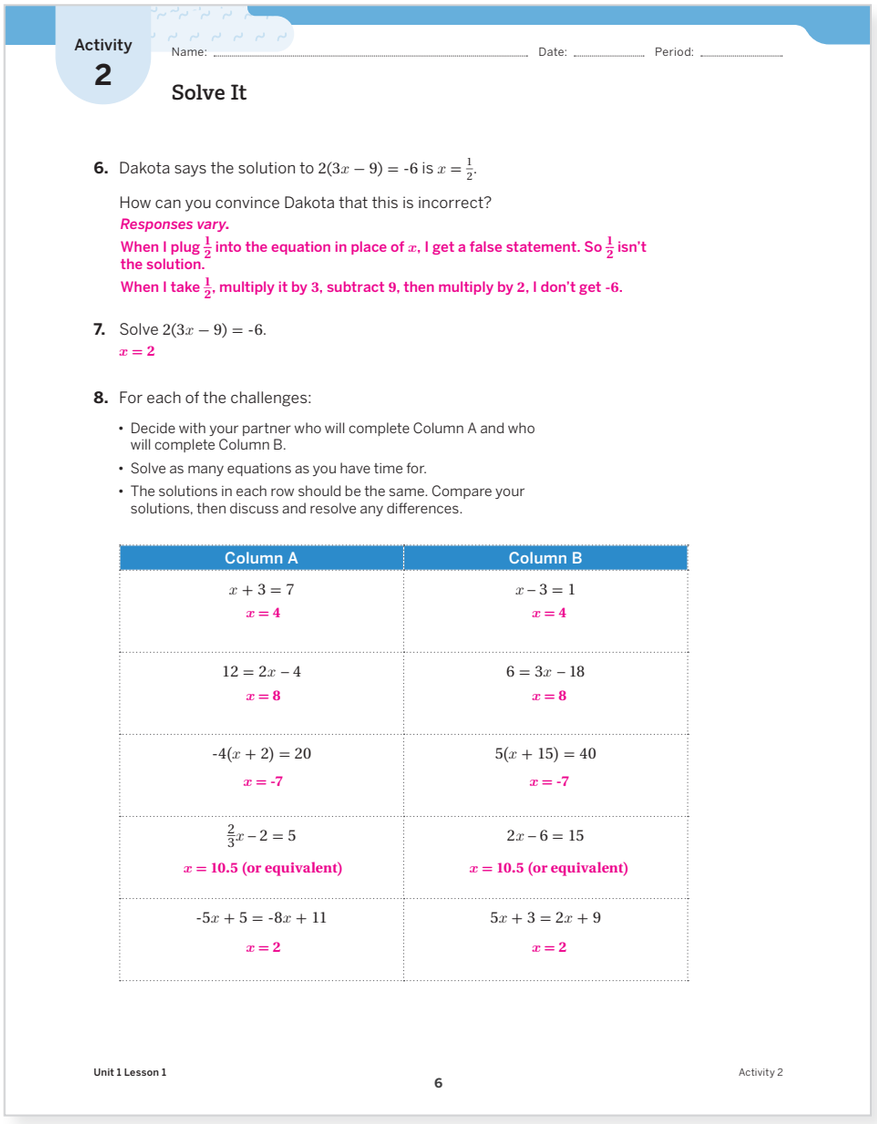
Students will need a partner for this activity. They will solve and compare solutions to pairs of problems that have the same solutions instead of receiving responsive feedback.

D **Differentiation: (Strengthen)** Look for strategies students use to solve the challenge with a fractional coefficient $\left(\frac{2}{3}x - 2 = 5\right)$. Encourage students to use a calculator to help them evaluate. Consider asking, "How could you work backwards to move the $\frac{2}{3}$? The 2? The 3?"

Students will have more opportunities to develop strategies for solving equations with fractional coefficients in the next few lessons.

3 Connect

Invite students to share strategies they found helpful or what they learned from mistakes they made while solving these challenges.



Activity 2

Name: Date: Period:

Solve It

6. Dakota says the solution to $2(3x - 9) = -6$ is $x = \frac{1}{2}$.

How can you convince Dakota that this is incorrect?

Responses vary.

When I plug $\frac{1}{2}$ into the equation in place of x, I get a false statement. So $\frac{1}{2}$ isn't the solution.

When I take $\frac{1}{2}$, multiply it by 3, subtract 9, then multiply by 2, I don't get -6.

7. Solve $2(3x - 9) = -6$.

$x = 2$

8. For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many equations as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$x + 3 = 7$ $x = 4$	$x - 3 = 1$ $x = 4$
$12 = 2x - 4$ $x = 8$	$6 = 3x - 18$ $x = 8$
$-4(x + 2) = 20$ $x = -7$	$5(x + 15) = 40$ $x = -7$
$\frac{2}{3}x - 2 = 5$ $x = 10.5$ (or equivalent)	$2x - 6 = 15$ $x = 10.5$ (or equivalent)
$-5x + 5 = -8x + 11$ $x = 2$	$5x + 3 = 2x + 9$ $x = 2$

Unit 1 Lesson 1 6 Activity 2

Key Takeaway: To check to see if a value is the **solution** to an equation, substitute the value into the equation and see if it makes the equation true.

Whole Class | 5 min

Synthesis

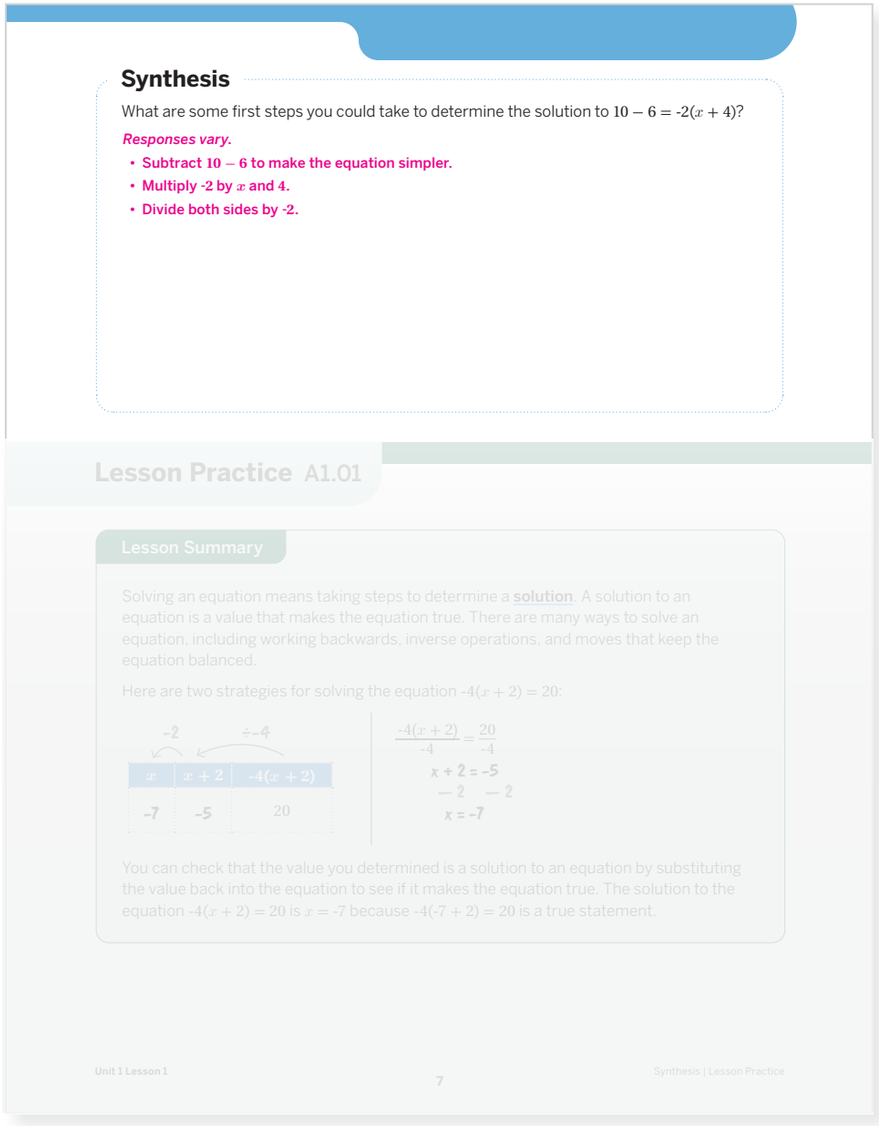
Purpose: Students synthesize their strategies for solving multi-step linear equations.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, including:
- Using inverse operations.
- Working backwards.
- Using moves that keep the equation balanced.
- Using substitution to check solutions.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for finding **solutions** to multi-step linear equations using students' language. Consider providing examples of inverse operations and moves that keep the equation balanced.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

What are some first steps you could take to determine the solution to $10 - 6 = -2(x + 4)$?

Responses vary.

- Subtract $10 - 6$ to make the equation simpler.
- Multiply -2 by x and 4.
- Divide both sides by -2.

Lesson Practice A1.01

Lesson Summary

Solving an equation means taking steps to determine a **solution**. A solution to an equation is a value that makes the equation true. There are many ways to solve an equation, including working backwards, inverse operations, and moves that keep the equation balanced.

Here are two strategies for solving the equation $-4(x + 2) = 20$:

You can check that the value you determined is a solution to an equation by substituting the value back into the equation to see if it makes the equation true. The solution to the equation $-4(x + 2) = 20$ is $x = -7$ because $-4(-7 + 2) = 20$ is a true statement.

Unit 1 Lesson 1 7 Synthesis | Lesson Practice

Lesson Takeaway: There are many ways to solve an *equation*. One strategy is to use *inverse operations* and moves that keep the equation balanced. A **solution** to an equation can be checked by substituting the value back into the equation to see if it makes the equation true.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by a solving a multi-step equation.

Today's Goals

1. **Goal:** Use inverse operations to solve an equation.
2. **Language Goal:** Describe strategies for solving multi-step equations. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.01

Solve $3(2x - 6) = -15$.

$x = \frac{1}{2}$ (or equivalent)

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support when solving linear equations with parentheses:

» **Respond to Student Thinking:**

- Use the ***Solving Linear Equations with Parentheses*** **Mini-Lesson**.
- Revisiting **Grade 8, Unit 4, Lesson 1** *(Equation Roundtable)*

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to list as many mathematical operations as they can think of and determine their inverse operation, if possible. Challenge students to research operations beyond what they have used in this lesson or even seen before (like % or the factorial operation !), and determine if these operations have inverse operations (or if not, why not).

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for solving equations? What opportunities might there be in future lessons to connect back to the strategies students used today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

What are some first steps you could take to determine the solution to $10 - 6 = -2(x + 4)$?

Responses vary.

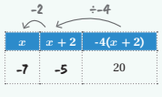
- Subtract 10 − 6 to make the equation simpler.
- Multiply -2 by x and 4.
- Divide both sides by -2.

Lesson Practice A1.01

Lesson Summary

Solving an equation means taking steps to determine a **solution**. A solution to an equation is a value that makes the equation true. There are many ways to solve an equation, including working backwards, inverse operations, and moves that keep the equation balanced.

Here are two strategies for solving the equation $-4(x + 2) = 20$:



$$\frac{-4(x+2)}{-4} = \frac{20}{-4}$$
$$x + 2 = -5$$
$$-2 \quad -2$$
$$x = -7$$

You can check that the value you determined is a solution to an equation by substituting the value back into the equation to see if it makes the equation true. The solution to the equation $-4(x + 2) = 20$ is $x = -7$ because $-4(-7 + 2) = 20$ is a true statement.

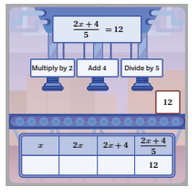
Lesson Practice A1.01

Name: ..... Date: ..... Period: .....

Problems 1–2: Solve each equation for x. Use the tables if they help with your thinking.

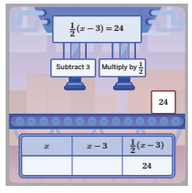
1. Here is a machine for the equation $\frac{2x+4}{5} = 12$.

$x = 28$



2. Here is a machine for the equation $\frac{1}{2}(x - 3) = 24$.

$x = 51$



Problems 3–4: Solve each equation for x.

3. $5(3x - 2) = -55$

$x = -3$

4. $\frac{6x-2}{2} = x + 9$

$x = 5$

Problems 5–6: Zwena made a mistake when solving $3(x - 4) = 5x$ for x.

5. Show or explain her mistake.

Responses vary. After Step 2, Zwena added $5x$ to both sides of the equation, but $5x + 5x$ does not equal 0.

6. What is the correct solution to this equation?

$x = -6$

Zwena

Step 1: $3(x - 4) = 5x$
Step 2: $3x - 12 = 5x$
Step 3: $8x - 12 = 0$
Step 4: $8x = 12$
Step 5: $x = 1.5$

Lesson Practice A1.01

Name: ..... Date: ..... Period: .....

Problems 7–10: Solve each equation for x.

7. $3x + 19 = 40$

$x = 7$

8. $5 = x + 1 + 3x$

$x = 1$

9. $3x + 5 = 4x + 1$

$x = 4$

10. Determine the value of x that makes the given equation true: $2(5x + 6) = 17$.

$x = 0.5$ (or equivalent)

Test Practice

11. $4x + 18 = 6x$

A. $x = 16$ B. $x = -9$ C. $x = -16$ D. $x = 9$

Spiral Review

12. Select *all* the equations where $x = 0.5$ is a solution.

- ☐ A. $8 = 4x$
- ☑ B. $13x = 6.5$
- ☑ C. $3x + 8 = 9.5$
- ☐ D. $3 = 2x + 1$
- ☐ E. $5 - x = 5.5$

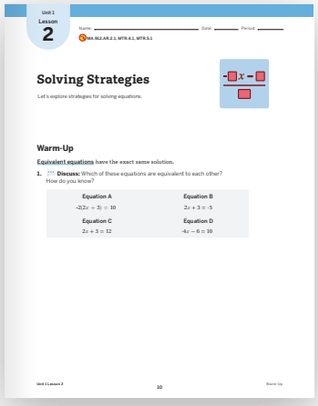
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.912.AR.2.1
	7–8	1	MA.912.AR.2.1
	9–10	2	MA.912.AR.2.1
Test Practice	11	2	MA.912.AR.2.1
Spiral Review			
Fluency	12	2	MA.8.AR.2.1

Unit 1

Lesson

2



Solving Strategies

More Solving With One-Variable Equations

Let's explore strategies for solving equations.

Learning Outcomes

Today's Goals

1. **Goal:** Solve one-variable linear equations.
2. **Goal:** Analyze others' reasoning when solving equations.
3. **Language Goal:** Justify that each step in solving a linear equation creates an equivalent equation with the same solution. **(Reading and Writing)**

Students practice solving one-variable equations using **equivalent equations**, the concept of balance, and inverse operations. Students decide whether pairs of equations are equivalent, analyze both correct and incorrect steps for solving an equation, and then practice solving a variety of equations.

Coherence

Prior Learning

In Lesson 1, students used inverse operations and the concept of balance to solve equations, building on prior learning on solving equations from Grade 8.

Future Learning

In Lesson 3, students will analyze equations with one solution, no solutions, or infinitely many solutions. In Lessons 4–7, students will generalize their solving strategies by extending them to equations with variable coefficients.

Connections

- Students make choices on solving strategies based on analysis of the given conditions and solving preferences. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.1

Given a real-world context, write and solve one-variable multi-step linear equations.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.1	MA.912.AR.2.1
MA.8.AR.2.1	MA.912.AR.1.2

Vocabulary

New Vocabulary

equivalent equations

Review Vocabulary

distributive property

Lesson at a Glance

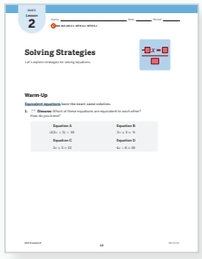
~ 45 min

MA.912.AR.2.1

Warm-Up

Pairs | 5 min

Students are introduced to the term **equivalent equations** and surface strategies for determining whether two equations are equivalent to one another. **Routine:** Think-Pair-Share



Activity 1

Pairs | 10 min

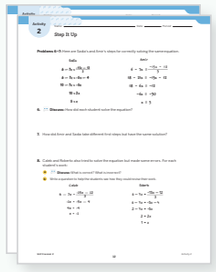
Students decide whether pairs of equations are equivalent by either justifying their reasoning or writing a new equivalent equation. **(MTR4.1)** MLR2: Collect and Display



Activity 2

Pairs | 10 min

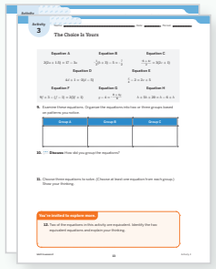
Students use the definition of equivalent equations to analyze both correct and incorrect strategies for solving an equation. **Routines:** Notice and Wonder; MLR3: Critique, Correct, Clarify



Activity 3

Small Groups | 10 min

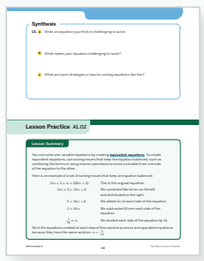
Students develop fluency by solving a variety of equations with different structures. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their strategies for solving one-variable equations with challenging structures.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving two equations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the term **equivalent equations** and surface strategies for determining whether two equations are equivalent to one another.

1 Launch

Display the equations.

Use the Think-Pair-Share routine to support collaborative learning. Invite students to think independently, then to take turns sharing with a partner.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to compare two equations at a time.

ELL **English Language Learners** Invite students to share what each word in the new term *equivalent equations* reminds them of. (E.g., When I hear the word *equivalent*, I think _____. When I hear the word *equations*, I think of _____.) Then invite students to share what they think it means when the two words are used together. **(Speaking and Listening)**

2 Connect

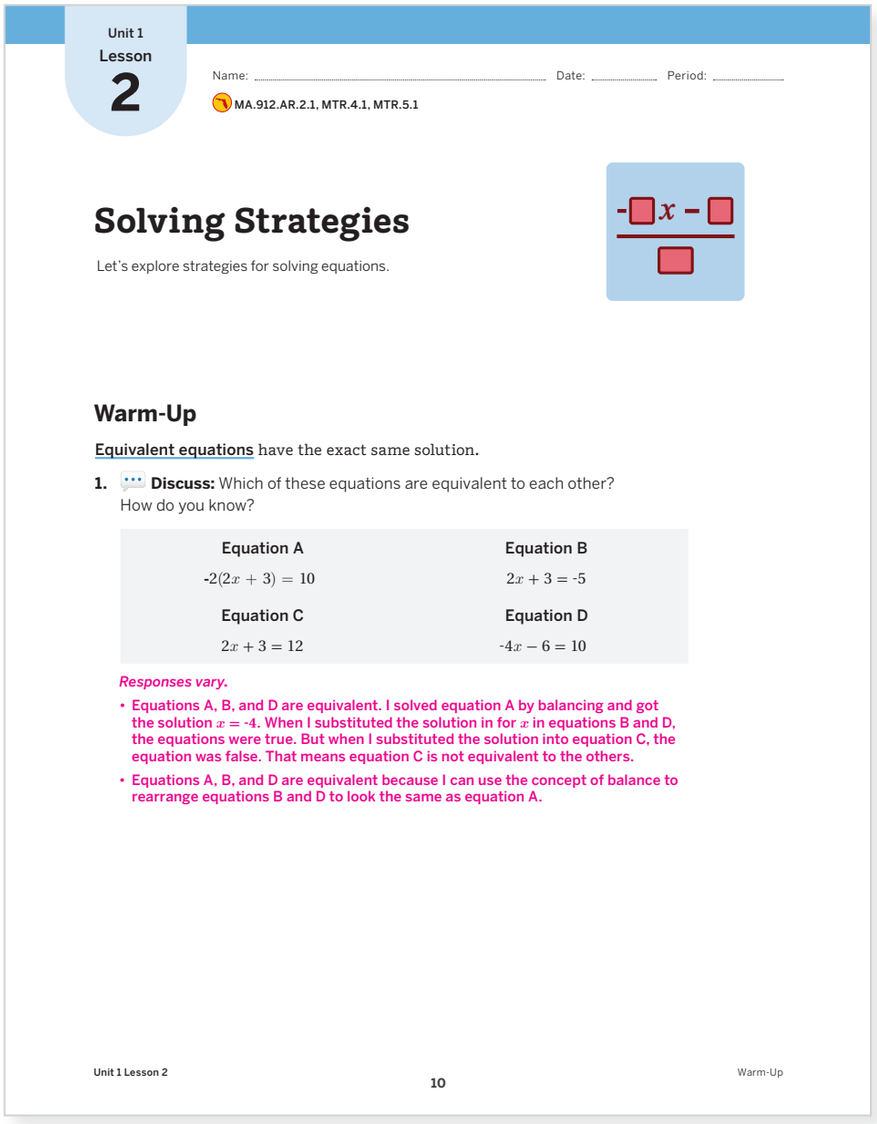
Invite students to share which equations they and their partner decided were equivalent and explain why.

D **Differentiation: (Support)** Students may say that Equations A and C are equivalent because you can add -2 to both sides of Equation A to result in Equation C. Consider asking these students, "What operation is -2 doing in $-2x + 3 = 10$?"

Discuss students' strategies for deciding whether pairs of equations are equivalent. Some ideas to highlight might include:

- Using the concept of balance or inverse operations to transform one equation to look like another (e.g., Distributing -2 in Equation A and simplifying so it looks like Equation D).
- Solving to see if the solutions are the same.
- Using the transitive property to justify equivalence (e.g., If Equations A and B are equivalent and Equations B and D are equivalent, then Equations A and D must also be equivalent).
- Solving one equation and substituting the solution into other equations to see if it makes the equation true (e.g., Solving Equation A for $x = -4$, then substituting -4 in for x in Equations B, C, and D).

Consider sharing that the goal of today's lesson is to use the concept of equivalent equations having the same solution to help solve one-variable equations.



Unit 1
Lesson
2

Name: _____ Date: _____ Period: _____

MA.912.AR.2.1, MTR.4.1, MTR.5.1

Solving Strategies

Let's explore strategies for solving equations.

$-\square x - \square$
$\square$

Warm-Up

Equivalent equations have the exact same solution.

1. **Discuss:** Which of these equations are equivalent to each other? How do you know?

Equation A	Equation B
$-2(2x + 3) = 10$	$2x + 3 = -5$
Equation C	**Equation D**
$2x + 3 = 12$	$-4x - 6 = 10$

Responses vary.

- Equations A, B, and D are equivalent. I solved equation A by balancing and got the solution $x = -4$. When I substituted the solution in for x in equations B and D, the equations were true. But when I substituted the solution into equation C, the equation was false. That means equation C is not equivalent to the others.
- Equations A, B, and D are equivalent because I can use the concept of balance to rearrange equations B and D to look the same as equation A.

Unit 1 Lesson 2 10 Warm-Up

Key Takeaway: **Equivalent equations** have the same exact solutions. There are several ways to determine if two equations are equivalent, including rearranging them using inverse operations, solving each equation to see if the solutions are the same, or solving one equation and substituting the solution in the other equation to see if the solution makes the equation true.

Pairs | 10 min

Activity 1 Equivalent or Not?

Purpose: Students decide whether pairs of equations are equivalent by either justifying their reasoning or writing a new equivalent equation. **(MTR.4.1)**

1 Launch

Draw students attention to the equations and the prompt.

Consider asking, "How can we create two equations that are not equivalent?"
Responses vary. We can create equations that aren't equivalent by using moves that unbalance the equation.

ELL **English Language Learners** Consider providing a word bank to support students in explaining their ideas with words such as: variable, both sides, add, subtract, multiply, divide, first, next, equivalent, solution, etc. **(Reading and Writing)**

2 Monitor

A **Accessibility: Memory and Attention** Invite students to use paper or a calculator to support students with performing calculations.

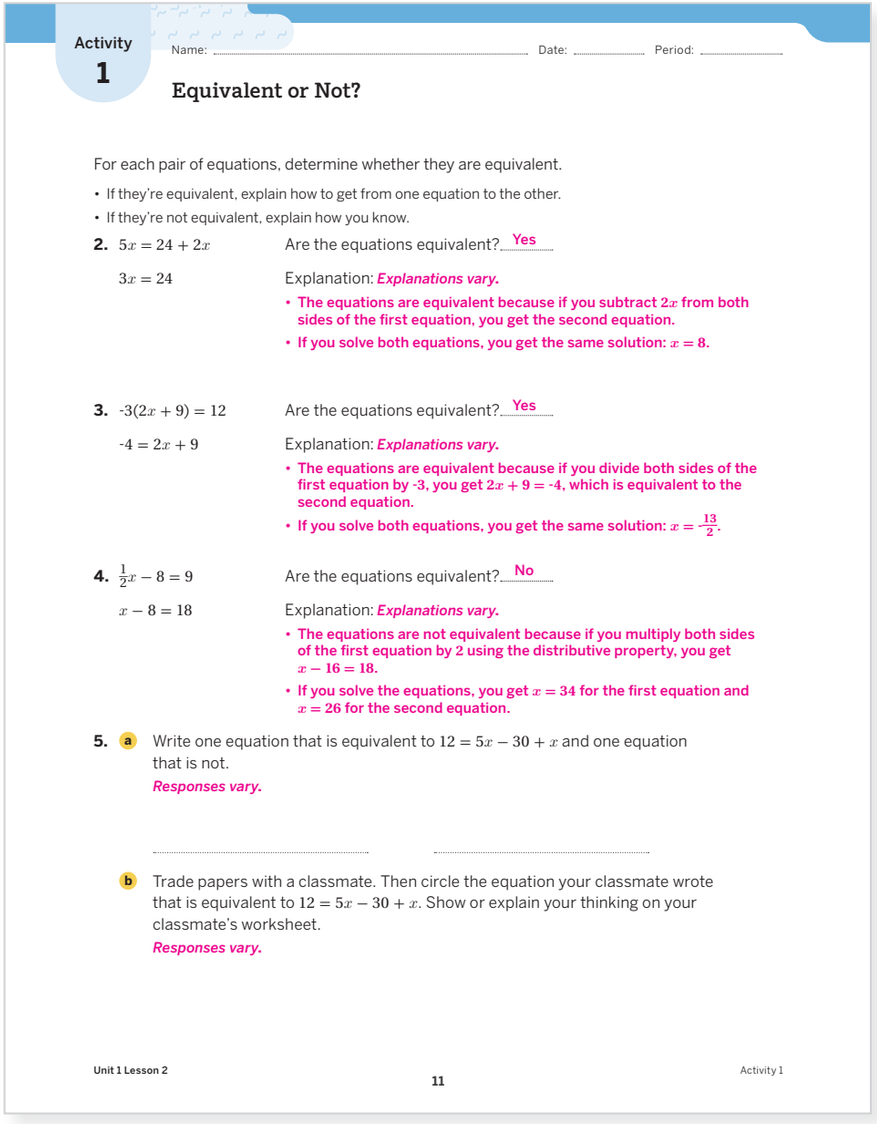
D **Differentiation**

Look for students who:	Teacher Moves
Need support getting started. *(Problems 2–4)*	**Support** Encourage students to use the strategies they learned in Lesson 1 to help them determine whether or not the equations are equivalent. Consider asking, "What inverse operation might we use with this equation?"
Determine which moves keep an equation balanced and equivalent. *(Problems 2–4)*	**Strengthen** Consider asking, "How did you know that the -3 in Problem 3 was multiplying?" Invite them to share their strategy during the Connect.
Would like a challenge. *(Problem 5)*	**Stretch** Invite students to create an equivalent equation to $12 = 5x - 30 + x$ that they think no one else will write!

3 Connect

MLR **MLR2: Collect and Display** Use this routine to gather strategies and ideas and create an anchor chart with moves that create **equivalent equations**.

Consider asking:
- "How can you tell if two equations are not equivalent?"
- "What are some moves that create equivalent equations?"



Activity 1

Name: _____ Date: _____ Period: _____

Equivalent or Not?

For each pair of equations, determine whether they are equivalent.

- If they're equivalent, explain how to get from one equation to the other.
- If they're not equivalent, explain how you know.

2. $5x = 24 + 2x$ Are the equations equivalent? Yes

$3x = 24$ Explanation: *Explanations vary.*

- The equations are equivalent because if you subtract $2x$ from both sides of the first equation, you get the second equation.
- If you solve both equations, you get the same solution: $x = 8$.

3. $-3(2x + 9) = 12$ Are the equations equivalent? Yes

$-4 = 2x + 9$ Explanation: *Explanations vary.*

- The equations are equivalent because if you divide both sides of the first equation by -3, you get $2x + 9 = -4$, which is equivalent to the second equation.
- If you solve both equations, you get the same solution: $x = -\frac{13}{2}$.

4. $\frac{1}{2}x - 8 = 9$ Are the equations equivalent? No

$x - 8 = 18$ Explanation: *Explanations vary.*

- The equations are not equivalent because if you multiply both sides of the first equation by 2 using the distributive property, you get $x - 16 = 18$.
- If you solve the equations, you get $x = 34$ for the first equation and $x = 26$ for the second equation.

5. a Write one equation that is equivalent to $12 = 5x - 30 + x$ and one equation that is not.
Responses vary.

b Trade papers with a classmate. Then circle the equation your classmate wrote that is equivalent to $12 = 5x - 30 + x$. Show or explain your thinking on your classmate's worksheet.
Responses vary.

Unit 1 Lesson 2 11 Activity 1

Pairs | 10 min

Activity 2 Step It Up

Purpose: Students use the definition of equivalent equations to analyze both correct and incorrect strategies for solving an equation.

1 Launch

Draw students attention to Sadia's and Amir's work.

Use the Notice and Wonder routine to support students in making sense of the equation they will analyze in this activity.

Consider asking:

- "What move did Sadia use to make the first equation equivalent to the second equation? The third equation to the fourth equation?"
- "How do we know that each equation in Sadia's and Amir's work is equivalent to the others?"
- "Are all of Sadia's equations equivalent to Amir's? How do you know?"

2 Monitor

Encourage students to use the Student Edition to record their thinking.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one student's work at a time.

Pause briefly once most students have completed Problem 7 to ensure students understand why each step creates equivalent equations.

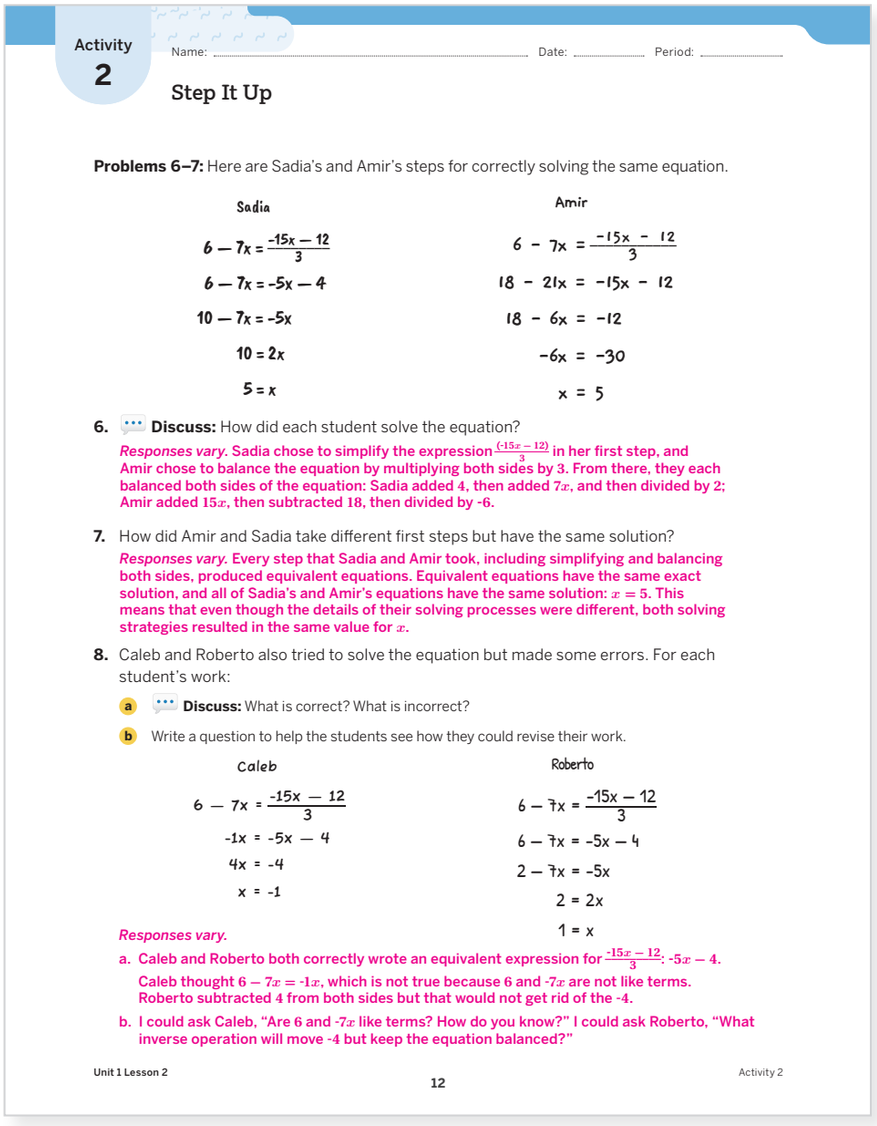
3 Connect

Draw students attention to Caleb's and Roberto's work.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Caleb's and Roberto's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1, MTR.5.1)**

To surface the Key Takeaway, consider asking:

- "What is the same about each set of equations? What changed?"
- "What move did Caleb make? Are the two equations equivalent after that move? How do you know?"
- "What inverse operation did Roberto use to move the -4?"
- "Are all of Caleb's and Roberto's equations equivalent to one another? How do you know?"
- "How do equivalent equations help with solving?"



Activity 2

Name: Date: Period:

Step It Up

Problems 6–7: Here are Sadia's and Amir's steps for correctly solving the same equation.

Sadia	Amir
$6 - 7x = \frac{-15x - 12}{3}$	$6 - 7x = \frac{-15x - 12}{3}$
$6 - 7x = -5x - 4$	$18 - 21x = -15x - 12$
$10 - 7x = -5x$	$18 - 6x = -12$
$10 = 2x$	$-6x = -30$
$5 = x$	$x = 5$

6. **Discuss:** How did each student solve the equation?

Responses vary. Sadia chose to simplify the expression $\frac{(-15x - 12)}{3}$ in her first step, and Amir chose to balance the equation by multiplying both sides by 3. From there, they each balanced both sides of the equation: Sadia added 4, then added $7x$, and then divided by 2; Amir added $15x$, then subtracted 18, then divided by -6.

7. How did Amir and Sadia take different first steps but have the same solution?

Responses vary. Every step that Sadia and Amir took, including simplifying and balancing both sides, produced equivalent equations. Equivalent equations have the same exact solution, and all of Sadia's and Amir's equations have the same solution: $x = 5$. This means that even though the details of their solving processes were different, both solving strategies resulted in the same value for x.

8. Caleb and Roberto also tried to solve the equation but made some errors. For each student's work:

   a. **Discuss:** What is correct? What is incorrect?

   b. Write a question to help the students see how they could revise their work.

Caleb	Roberto
$6 - 7x = \frac{-15x - 12}{3}$	$6 - 7x = \frac{-15x - 12}{3}$
$-1x = -5x - 4$	$6 - 7x = -5x - 4$
$4x = -4$	$2 - 7x = -5x$
$x = -1$	$2 = 2x$
	$1 = x$

Responses vary.

a. Caleb and Roberto both correctly wrote an equivalent expression for $\frac{-15x - 12}{3}$: $-5x - 4$. Caleb thought $6 - 7x = -1x$, which is not true because 6 and $-7x$ are not like terms. Roberto subtracted 4 from both sides but that would not get rid of the -4.

b. I could ask Caleb, "Are 6 and $-7x$ like terms? How do you know?" I could ask Roberto, "What inverse operation will move -4 but keep the equation balanced?"

Unit 1 Lesson 2 12 Activity 2

Key Takeaway: Creating **equivalent equations** is a useful tool for ensuring that an equation is solved correctly. When solving, equations will remain equivalent if the operations used keep both sides of the equation balanced. A move that unbalances the equation will create an equation that is not equivalent.

Small Groups | 10 min

Activity 3 The Choice Is Yours

Purpose: Students develop fluency by solving a variety of equations with different structures. **(MTR.5.1)**

Short on time? Invite students to solve fewer than three equations in Problem 11.

1 Launch

Display the equations.

Consider sharing that there are many different ways to sort the equations and that the purpose of this sort is to analyze the structure of the equations, not to solve them yet. **(MTR.5.1)**

2 Monitor

Select and sequence strategies for solving the equations in each group to share during the Connect.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started. *(Problem 9)*	**Support** Encourage them to compare the equations in the first row (A–D). Consider asking, "Which equations would you group together? Why?"
Create groups of equations but are only able to solve some of their chosen equations correctly. *(Problem 11)*	**Strengthen** Celebrate the reasoning behind how they grouped the equations. Invite them to look back at their work from Activity 2. Consider asking, "Is there an equation where Sadia's or Amir's first step would be helpful?"
Solve an equation from each group successfully.	**Stretch You're invited to explore more.** Celebrate their success. Share with students that there are two equations in the set that are equivalent. Challenge them to find them by completing this optional task.

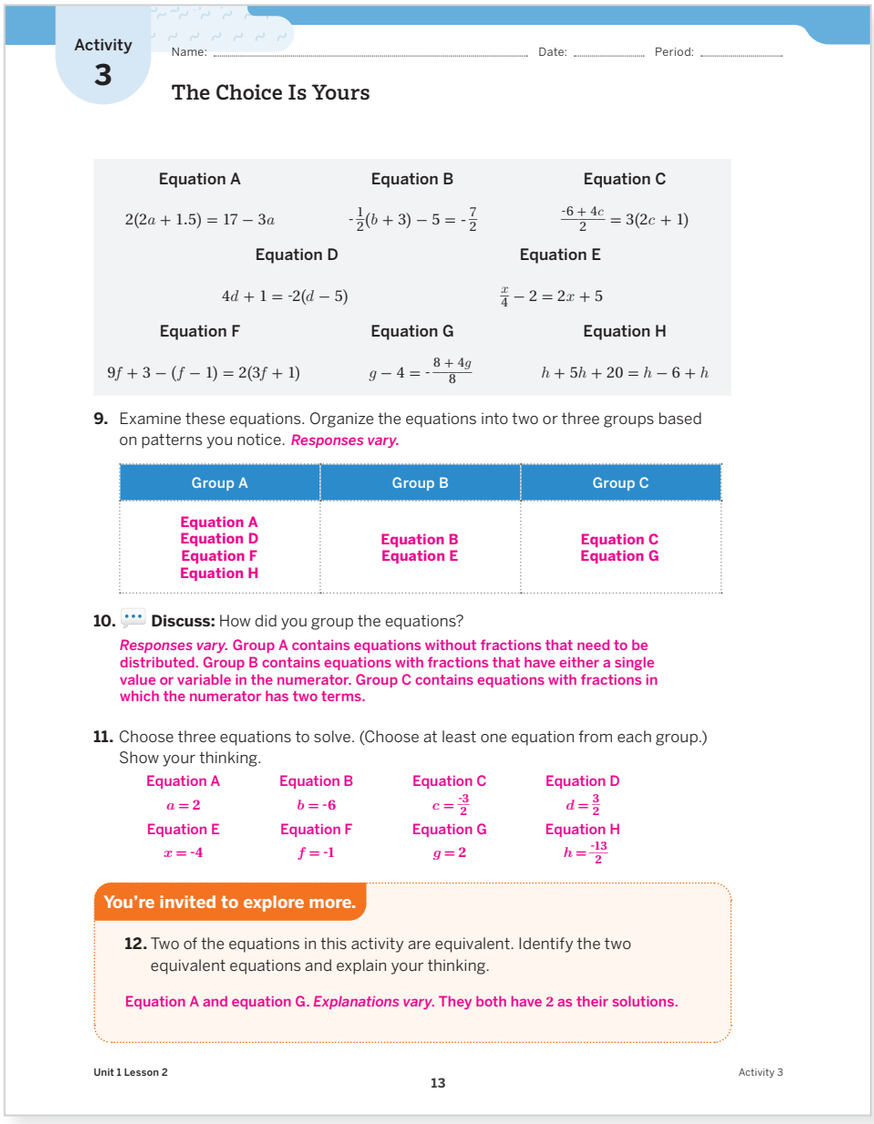
3 Connect

Invite students to share how they sorted the cards, highlighting the structure that students noticed in each group. (E.g. "I heard you say that each equation in this group has a fractional coefficient.") **(MTR.5.1)**

Consider asking:

- "What strategies were most helpful for solving these equations?"
- "Which equations were less difficult to solve than you expected? More difficult?"

Consider sharing an answer key after most students have solved their equations.



Activity 3

Name: Date: Period:

The Choice Is Yours

Equation A	Equation B	Equation C
$2(2a + 1.5) = 17 - 3a$	$-\frac{1}{2}(b + 3) - 5 = -\frac{7}{2}$	$\frac{-6 + 4c}{2} = 3(2c + 1)$
Equation D		**Equation E**
$4d + 1 = -2(d - 5)$		$\frac{x}{4} - 2 = 2x + 5$
Equation F	**Equation G**	**Equation H**
$9f + 3 - (f - 1) = 2(3f + 1)$	$g - 4 = -\frac{8 + 4g}{8}$	$h + 5h + 20 = h - 6 + h$

9. Examine these equations. Organize the equations into two or three groups based on patterns you notice. *Responses vary.*

Group A	Group B	Group C
Equation A Equation D Equation F Equation H	Equation B Equation E	Equation C Equation G

10. **Discuss:** How did you group the equations?

Responses vary. Group A contains equations without fractions that need to be distributed. Group B contains equations with fractions that have either a single value or variable in the numerator. Group C contains equations with fractions in which the numerator has two terms.

11. Choose three equations to solve. (Choose at least one equation from each group.) Show your thinking.

Equation A	Equation B	Equation C	Equation D
$a = 2$	$b = -6$	$c = \frac{-3}{2}$	$d = \frac{3}{2}$
Equation E	**Equation F**	**Equation G**	**Equation H**
$x = -4$	$f = -1$	$g = 2$	$h = \frac{-13}{2}$

You're invited to explore more.

12. Two of the equations in this activity are equivalent. Identify the two equivalent equations and explain your thinking.

Equation A and equation G. *Explanations vary.* They both have 2 as their solutions.

Unit 1 Lesson 2 13 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for solving one-variable equations with challenging structures.

Invite students to respond independently, and then share their thinking with a partner.

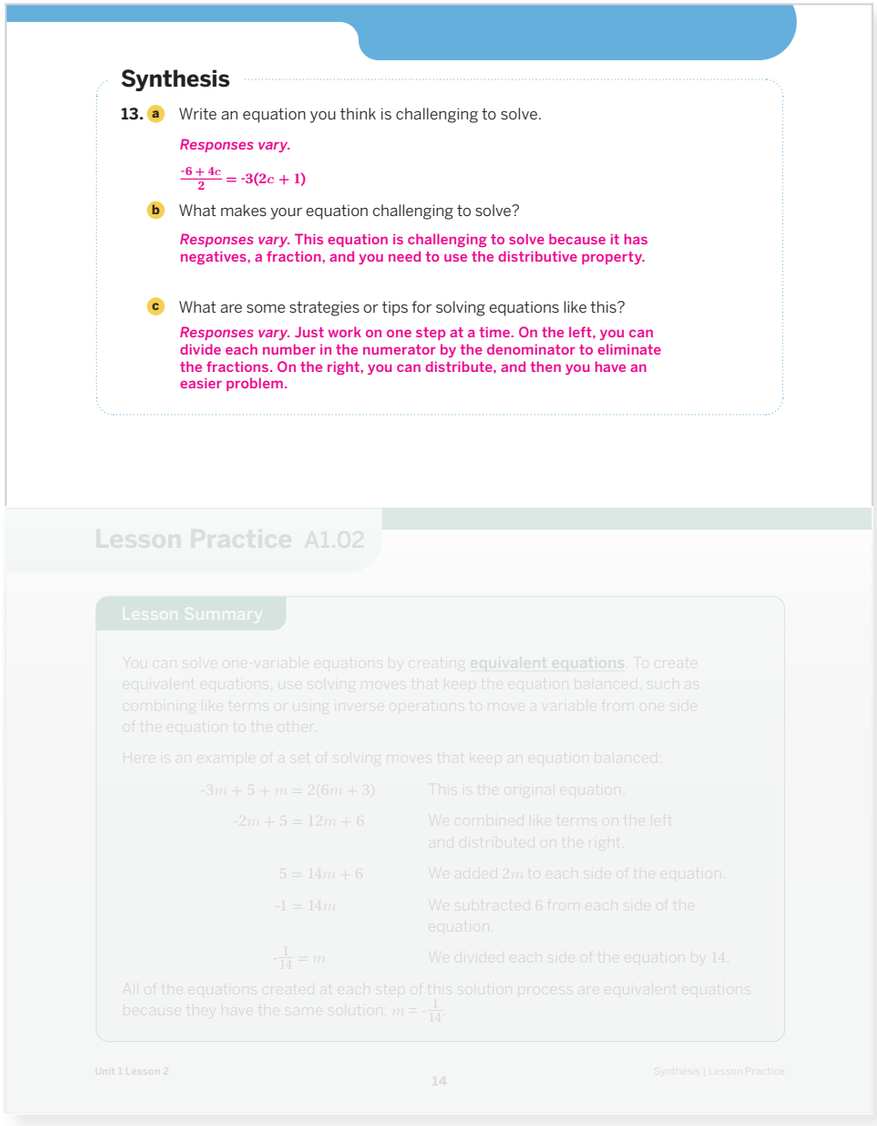
Invite students to share the equations they wrote and their strategies for solving them.

Consider discussing:

- "What makes _____'s equation challenging to solve?"
- "What strategies might help someone solve _____'s equation?"

ELL **English Language Learners** Consider providing sentence frames to help students explain their reasoning (e.g., This equation is difficult to solve because _____.). **(Speaking)**

MLR **MLR2: Collect and Display (optional)** If time allows, consider adding student strategies for solving equations to the anchor chart created in Activity 1.



Synthesis

13. a. Write an equation you think is challenging to solve.

Responses vary.

$\frac{-6+4c}{2} = -3(2c+1)$

b. What makes your equation challenging to solve?

Responses vary. This equation is challenging to solve because it has negatives, a fraction, and you need to use the distributive property.

c. What are some strategies or tips for solving equations like this?

Responses vary. Just work on one step at a time. On the left, you can divide each number in the numerator by the denominator to eliminate the fractions. On the right, you can distribute, and then you have an easier problem.

Lesson Practice A1.02

Lesson Summary

You can solve one-variable equations by creating **equivalent equations**. To create equivalent equations, use solving moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of solving moves that keep an equation balanced:

$-3m+5+m=2(6m+3)$	This is the original equation.
$-2m+5=12m+6$	We combined like terms on the left and distributed on the right.
$5=14m+6$	We added $2m$ to each side of the equation.
$-1=14m$	We subtracted 6 from each side of the equation.
$-\frac{1}{14}=m$	We divided each side of the equation by 14.

All of the equations created at each step of this solution process are equivalent equations because they have the same solution: $m=-\frac{1}{14}$.

Unit 1 Lesson 2 14 Synthesis | Lesson Practice

Lesson Takeaway: One-variable equations can be solved using a variety of different strategies. In all of the strategies, the moves made in each line of the solving process should create **equivalent equations**. This will ensure that the original equation, every equation in the solving process, and the last equation all have exactly the same solution.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving two equations.

Today's Goals

1. **Goal:** Solve one-variable linear equations.
2. **Goal:** Analyze others' reasoning when solving equations.
3. **Language Goal:** Justify that each step in solving a linear equation creates an equivalent equation with the same solution. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.02

Solve each equation. Show your thinking. *Work varies.*

a $-2(x+5)=3x+5$

$x=-3$

$-2(x+5)=3x+5$

$-2x-10=3x+5$

$-10=5x+5$

$-15=5x$

$-3=x$

b $x-2x+10=\frac{2x-28}{2}$

$x=12$

$x-2x+10=\frac{2x-28}{2}$

$-x+10=x-14$

$10=2x-14$

$24=2x$

$12=x$

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support when solving multistep equations:

» **Respond to Student Thinking:**

- Use the ***Solving Linear Equations*** **Mini-Lesson**.
- Revisit **Grade 8, Unit 4, Lesson 3** (Strategic Solving).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write their own equations, trade them with a partner, and solve them.
- Invite them to complete the **You're invited to explore more** task and discuss their thinking with a partner.

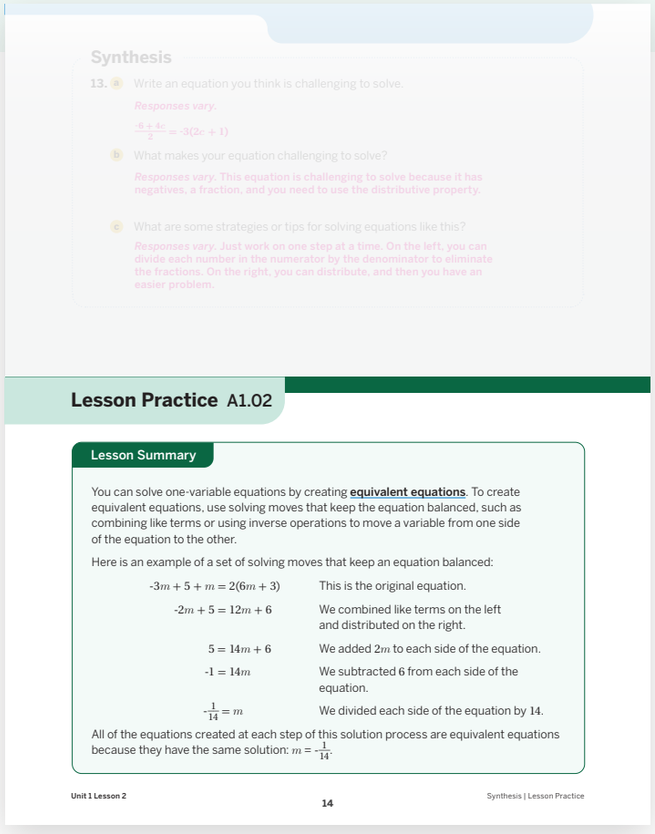
Professional Learning

Reflect on how students analyzed student work and solved equations today. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of each student's work? How can you support students in valuing learning from both correct and incorrect thinking?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

13. a. Write an equation you think is challenging to solve.

Responses vary.

$\frac{-6+4x}{2} = -3(2x+1)$

b. What makes your equation challenging to solve?

Responses vary. This equation is challenging to solve because it has negatives, a fraction, and you need to use the distributive property.

c. What are some strategies or tips for solving equations like this?

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Lesson Practice A1.02

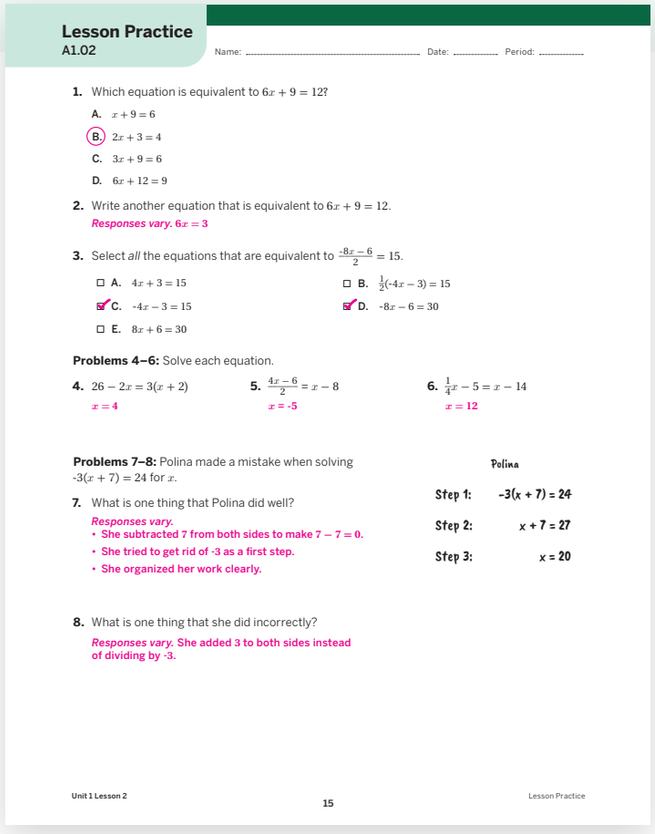
Lesson Summary

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Here is an example of a set of solving moves that keep an equation balanced:

$-3m + 5 + m = 2(6m + 3)$	This is the original equation.
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$-1 = 14m$	We subtracted 6 from each side of the equation.
$-\frac{1}{14} = m$	We divided each side of the equation by 14.

All of the equations created at each step of this solution process are equivalent equations because they have the same solution: $m = -\frac{1}{14}$.



Lesson Practice A1.02

Name: Date: Period:

1. Which equation is equivalent to $6x + 9 = 12$?

A. $x + 9 = 6$
B. $2x + 3 = 4$
C. $3x + 9 = 6$
D. $6x + 12 = 9$

2. Write another equation that is equivalent to $6x + 9 = 12$.

Responses vary. $6x = 3$

3. Select *all* the equations that are equivalent to $\frac{-8x-6}{2} = 15$.

☐ A. $4x + 3 = 15$
☐ B. $\frac{1}{2}(-4x - 3) = 15$
☑ C. $-4x - 3 = 15$
☑ D. $-8x - 6 = 30$
☐ E. $8x + 6 = 30$

Problems 4–6: Solve each equation.

4. $26 - 2x = 3(x + 2)$
$x = 4$

5. $\frac{4x-6}{2} = x - 8$
$x = -5$

6. $\frac{1}{4}x - 5 = x - 14$
$x = 12$

Problems 7–8: Polina made a mistake when solving $-3(x + 7) = 24$ for x.

Polina

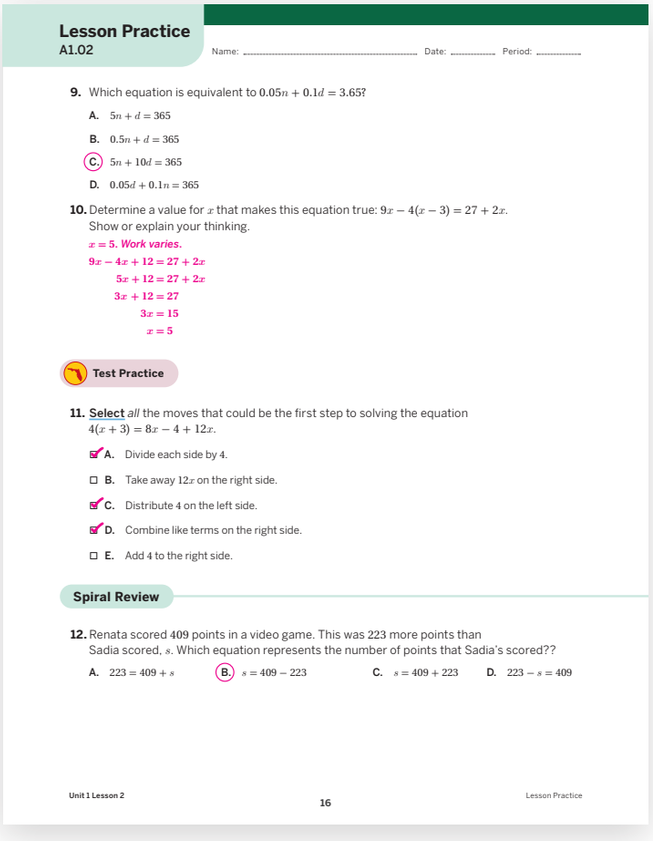
Step 1: $-3(x + 7) = 24$
Step 2: $x + 7 = 27$
Step 3: $x = 20$

7. What is one thing that Polina did well?

Responses vary.
- She subtracted 7 from both sides to make $7 - 7 = 0$.
- She tried to get rid of -3 as a first step.
- She organized her work clearly.

8. What is one thing that she did incorrectly?

Responses vary. She added 3 to both sides instead of dividing by -3.



Lesson Practice A1.02

Name: Date: Period:

9. Which equation is equivalent to $0.05n + 0.1d = 3.65$?

A. $5n + d = 365$
B. $0.5n + d = 365$
C. $5n + 10d = 365$
D. $0.05d + 0.1n = 365$

10. Determine a value for x that makes this equation true: $9x - 4(x - 3) = 27 + 2x$. Show or explain your thinking.

$x = 5$. *Work varies.*

$9x - 4x + 12 = 27 + 2x$
$5x + 12 = 27 + 2x$
$3x + 12 = 27$
$3x = 15$
$x = 5$

Test Practice

11. **Select** *all* the moves that could be the first step to solving the equation $4(x + 3) = 8x - 4 + 12x$.

☑ A. Divide each side by 4.
☐ B. Take away $12x$ on the right side.
☑ C. Distribute 4 on the left side.
☑ D. Combine like terms on the right side.
☐ E. Add 4 to the right side.

Spiral Review

12. Renata scored 409 points in a video game. This was 223 more points than Sadia scored, s. Which equation represents the number of points that Sadia's scored??

A. $223 = 409 + s$ B. $s = 409 - 223$ C. $s = 409 + 223$ D. $223 - s = 409$

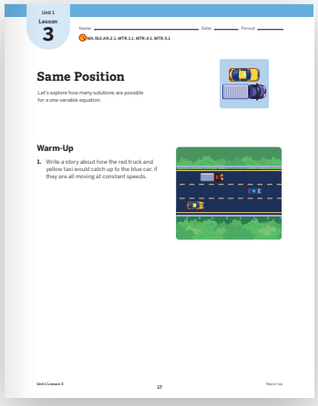
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.2.1
	2–10	2	MA.912.AR.2.1
Test Practice	11	2	MA.912.AR.2.1
Spiral Review			
	12	1	MA.6.AR.2.3

Unit 1

Lesson

3



Same Position

No Solution and Infinitely Many Solutions

Let's explore how many solutions are possible for a one-variable equation.

Learning Outcomes

Today's Learning

1. **Goal:** Observe that dividing by a variable does not always create equivalent equations.
2. **Language Goal:** Justify whether a one-variable equation has one solution, no solution, or infinitely many solutions. **(Listening, Speaking, and Writing)**

Students practice solving equations in context and interpreting their solutions, including equations with **no solution** or **infinitely many solutions**. Students also explore the effect of dividing by a variable when solving. Students use the context of vehicles on a road to decide whether the vehicles will be in the same position once, never, or always.

Coherence

Prior Learning

In Lesson 1, students solved one-variable equations using balanced moves and inverse operations. In Lesson 2, students used their strategies to create equivalent equations while solving.

Future Learning

In Lesson 4, students will write two-variable equations to represent situations and rearrange equations to highlight a variable of interest in context.

Connections

- Students solve linear equations to determine when vehicles will meet on the same road. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.1

Given a real-world context, write and solve one-variable multi-step linear equations.

Also Addressing: **MA.8.AR.2.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.1	**MA.912.AR.2.1**
MA.8.AR.2.1	**MA.912.AR.1.2**

Vocabulary

New Vocabulary

infinitely many solutions

no solution

Lesson at a Glance

~ 45 min

MA.8.AR.2.1, MA.912.AR.2.1

Warm-Up

Independent | 5 min

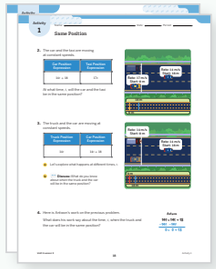
Students express their creativity by using the Tell a Story routine to make sense of the context of passing vehicles used throughout this lesson. **(MTR.1.1)**



Activity 1

Pairs | 20 min

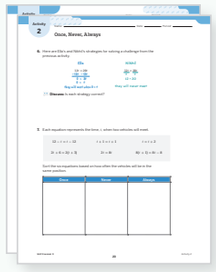
Students solve equations that represent the positions of vehicles on a road and connect the vehicle's position to the structure of equations with different types of solutions. **(MTR.5.1)**



Activity 2

Pairs | 10 min

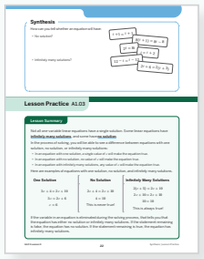
Students critique the reasoning of others as they consider the effect of dividing by a variable when solving an equation. **Routines:** Compare and Connect; MLR7: Decide and Defend **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their strategies for deciding how many solutions there are for an equation. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the number of possible solutions to a one-variable equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students express their creativity by using the Tell a Story routine to make sense of the context of passing vehicles used throughout this lesson. **(MTR.1.1)**

1 Launch

Use the Tell a Story routine to help students make sense of the context. Invite students to write a story about the image using their unique experiences, perspectives, and creativity. **(MTR.1.1)**

A **Accessibility: Expressive Language** Invite students to talk through what they notice and wonder about the image with a partner before writing their story. Provide sentence frames to help students write their story (e.g., First, _____. Then, _____. Finally, _____.).

ELL **English Language Learners** Invite students to annotate the images of trucks, cars, and taxis in their Student Editions with the words in English and/or in their primary language. Consider asking student volunteers to act out each scenario. For example, invite two students to walk at different speeds and start in different locations. Ask each student to describe their movement in their own words, using the quantities from each expression. **(Reading, Writing, Speaking, and Listening)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

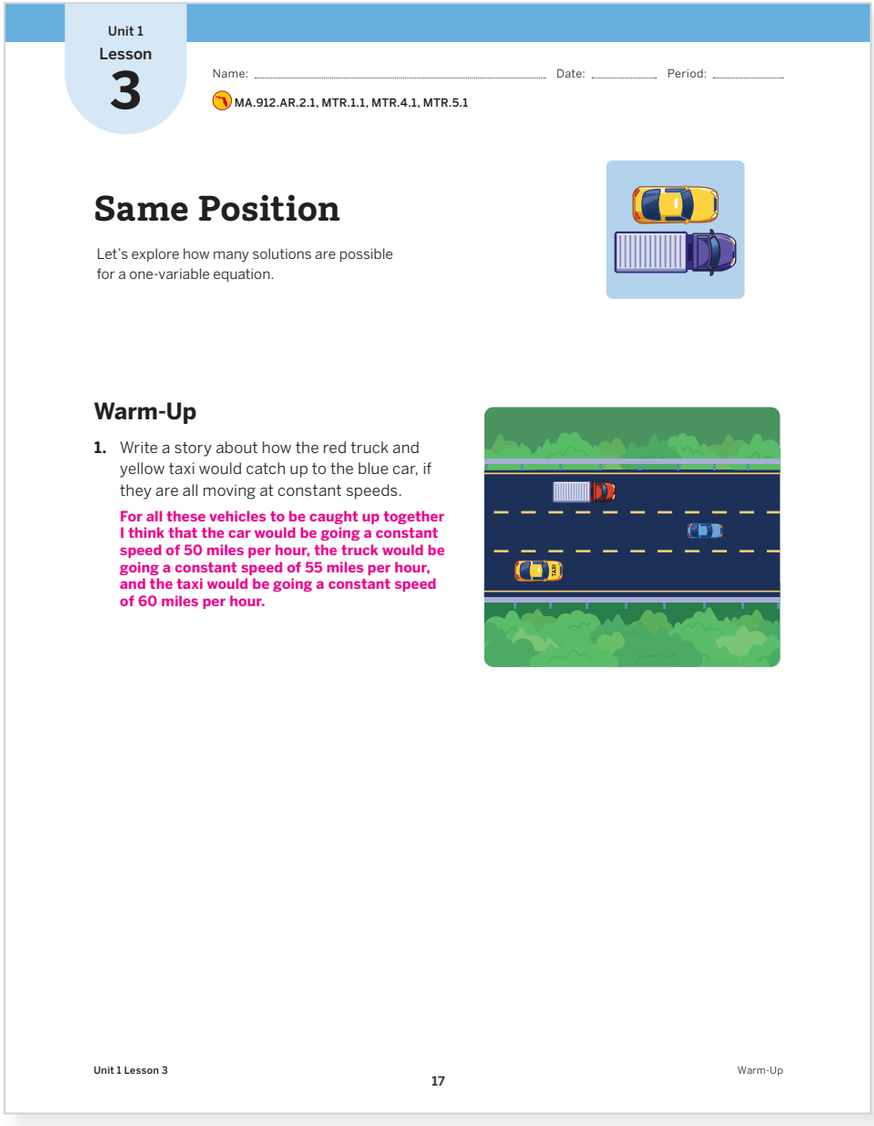
2 Connect

Invite several students to share their stories, encouraging them to make connections to the vehicle speed scenarios as they share.

Encourage students to ask questions about each other's stories.

Consider asking:

- "How does your story connect to the vehicle speed scenarios?"
- "When ___ said, "___," what do you think they meant?" (E.g., "When Madison said, "The two cars were even," what do you think she meant?")



Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.AR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Same Position

Let's explore how many solutions are possible for a one-variable equation.

Warm-Up

1. Write a story about how the red truck and yellow taxi would catch up to the blue car, if they are all moving at constant speeds.

   For all these vehicles to be caught up together I think that the car would be going a constant speed of 50 miles per hour, the truck would be going a constant speed of 55 miles per hour, and the taxi would be going a constant speed of 60 miles per hour.

Unit 1 Lesson 3 17 Warm-Up

Pairs | 20 min

Activity 1 Same Position

Purpose: Students solve equations that represent the positions of vehicles on a road and connect the vehicle's position to the structure of equations with different types of solutions. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges on Problem 5.

1 Launch

Direct students' attention to the scenario in Problem 2. Consider asking students to share what "m/s" means. Meters per second

Invite students to make connections between the rates and starting positions in the image and the expressions. **(MTR.5.1)**

Consider asking, "What does 14 in $14t + 18$ tell us about the car? What about 18?" **(MTR.5.1)**

D **Differentiation: (Support)** Look for students who think the car is moving faster than the taxi because 18 is greater than 17. Consider asking these students, "Where will each car be after 1 second? After 2 seconds?"

2 Monitor

Look for these strategies for solving equations.

D **Differentiation**

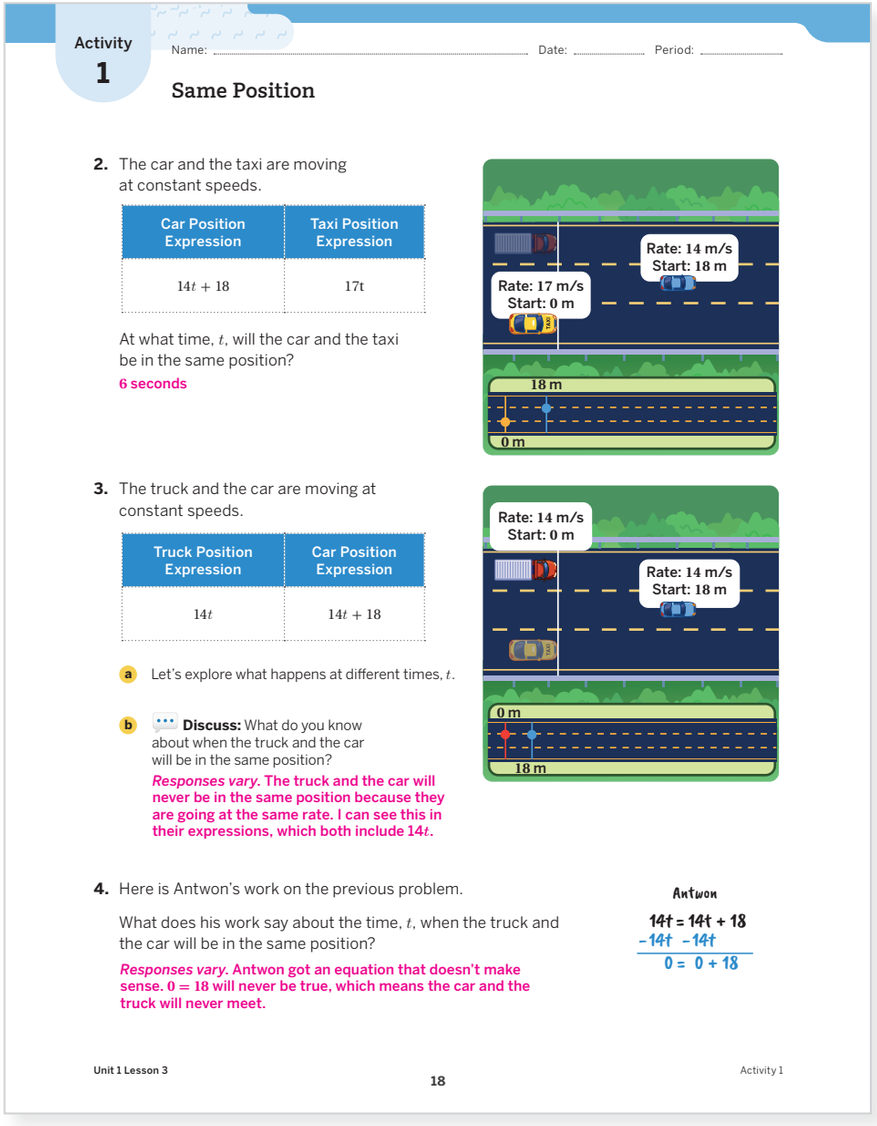
Look for students who:	Teacher Moves
Need support getting started.	**Support** Encourage them to talk through the problem aloud with their partner.
Use guess and check.	**Strengthen** Consider asking, "How can an equation help us figure out when the two vehicles will be in the same position?"
Make a table of values.	**Strengthen** Capture this strategy to share during the Connect.
Write and solve an equation like Antwon's on Problem 4.	**Strengthen** Consider renaming Antwon's strategy after a student in your class who used it. Capture this strategy and ask them to share during the Connect.

Invite students to suggest times to enter and discuss their reasoning with a partner, revising their thinking.

Pause to discuss Antwon's work. Consider asking:

- "Why does it make sense that Antwon ended up with an untrue equation?"
- "What does the untrue equation ($0 = 0 + 18$) tell us about when the vehicles will meet?" *Responses vary.* An untrue statement means that there is never a time when the car and truck will be in the same position. **(MTR.5.1)**
- "How can we tell from Antwon's work that this equation has no solutions?" *Responses vary.* There is no value for t that makes this equation true, so that means this equation has no solution.

Activity 1 continued ›



Activity 1

Name: _____ Date: _____ Period: _____

Same Position

2. The car and the taxi are moving at constant speeds.

Car Position Expression	Taxi Position Expression
$14t + 18$	$17t$

At what time, t, will the car and the taxi be in the same position?

6 seconds

3. The truck and the car are moving at constant speeds.

Truck Position Expression	Car Position Expression
$14t$	$14t + 18$

a. Let's explore what happens at different times, t.

b. **Discuss:** What do you know about when the truck and the car will be in the same position?

Responses vary. The truck and the car will never be in the same position because they are going at the same rate. I can see this in their expressions, which both include $14t$.

4. Here is Antwon's work on the previous problem.

What does his work say about the time, t, when the truck and the car will be in the same position?

Responses vary. Antwon got an equation that doesn't make sense. $0 = 18$ will never be true, which means the car and the truck will never meet.

Antwon

$14t = 14t + 18$
$-14t \quad -14t$
$0 = 0 + 18$

Unit 1 Lesson 3 18 Activity 1

Pairs | 20 min

Activity 1 Same Position (continued)

Purpose: Students solve equations that represent the positions of vehicles on a road and connect the vehicle's position to the structure of equations with different types of solutions. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges on Problem 5.

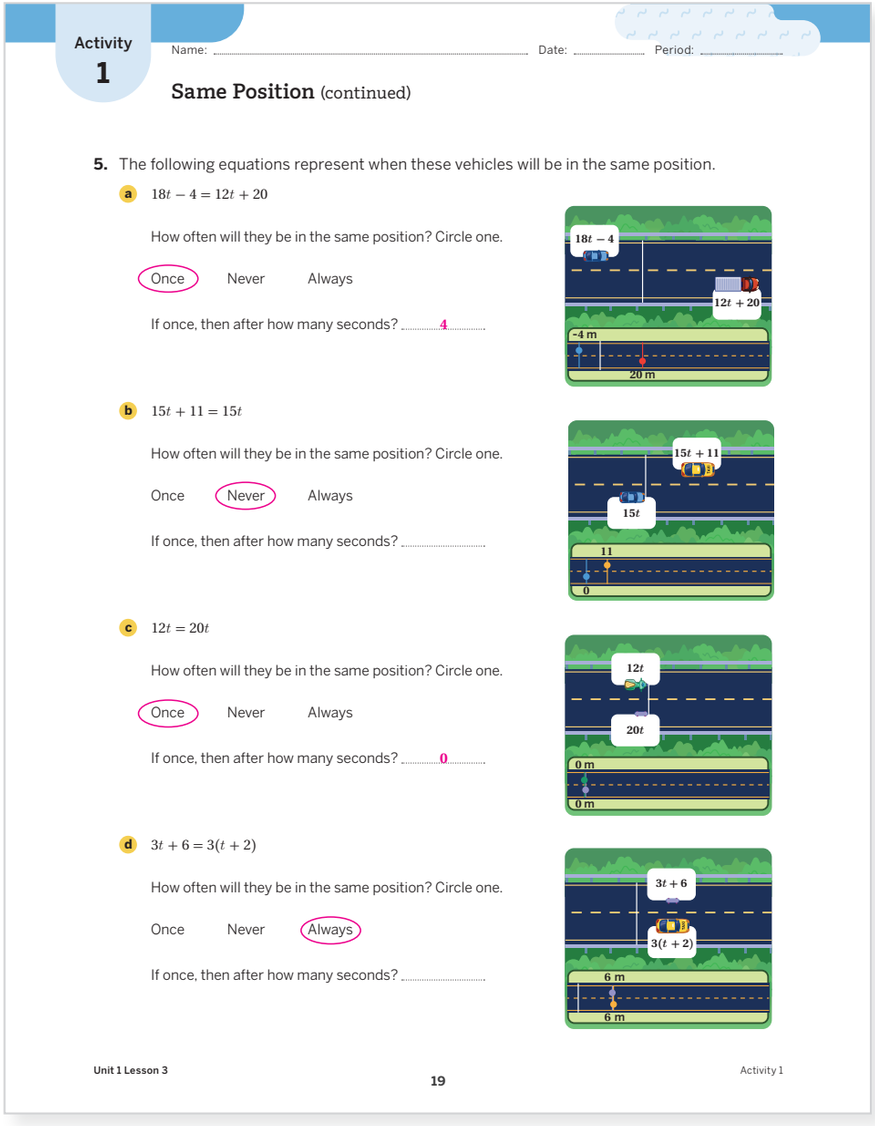
2 Monitor

Listen for student strategies and partner discussions using structure of the equations or reasoning about the situation. Offer help or encouragement where needed. **(MTR.5.1)**

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.

A **Accessibility: Memory and Attention** For students who would benefit from having more time to think deeply on fewer problems, invite them to complete fewer challenges.



Activity 1

Name: Date: Period:

Same Position (continued)

5. The following equations represent when these vehicles will be in the same position.

a. $18t - 4 = 12t + 20$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? 4

b. $15t + 11 = 15t$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds?

c. $12t = 20t$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? 0

d. $3t + 6 = 3(t + 2)$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds?

Unit 1 Lesson 3 19 Activity 1

Key Takeaway: Linear equations can have the same value on both sides of the equation once, never, or always. This value is a solution that makes the equation true.

Pairs | 10 min

Activity 2 Once, Never, Always

Purpose: Students critique the reasoning of others as they consider the effect of dividing by a variable when solving an equation. **(MTR.4.1)**

1 Launch

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy.

Consider asking:

- "What did each student do?"
- "How are Ella's and Nikhil's strategies alike? How are they different?"
- "Why did the different strategies lead to different outcomes?"
- "Will the vehicles be in the same position at $t = 0$ or never? How did you decide?"
- "What happened when Nikhil divided by t?"

D **Differentiation:** (Stretch) Encourage students to create a different equation that also has $0 = t$ as the solution.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the equations.

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation. Students will sort equations based on if the vehicles they represent would be in the same position once, never, or always.

ELL **English Language Learners** Invite students to annotate the words "always," "never," and "once" in their Student Editions with words from their primary language to support their access to the language used in this activity. **(Reading, Writing, Listening, and Speaking)**

A **Accessibility: Conceptual Processing** Invite students to read each equation aloud and discuss how they are similar and different before sorting the cards.

Activity 2 continued >

Activity 2

Name: Date: Period:

Once, Never, Always

6. Here are Ella's and Nikhil's strategies for solving a challenge from the previous activity.

Ella

$12t = 20t$
$-12t \quad -12t$
$0 = 8t$
$0 = t$
they will meet when $0 = t$

Nikhil

$\frac{12t}{t} = \frac{20t}{t}$
$12 = 20$
they will never meet

Discuss: Is each strategy correct?

Ella's strategy is correct. Nikhil's strategy is not correct. *Explanations vary.* Ella's strategy is correct because when I plugged 0 back in for t, the equation was true. Nikhil's strategy is not correct because 12 is not equal to 20. Something went wrong when he divided both sides by t.

7. Each equation represents the time, t, when two vehicles will meet.

$12 - t = t - 12$	$t + 1 = t + 1$	$t = t + 2$
$2t + 6 = 2(t + 3)$	$2t = 8t$	$8(t + 1) = 8t - 8$

Sort the six equations based on how often the vehicles will be in the same position.

Once	Never	Always
$12 - t = t - 12$ $2t = 8t$	$t = t + 2$ $8(t + 1) = 8t - 8$	$t + 1 = t + 1$ $2t + 6 = 2(t + 3)$

Unit 1 Lesson 3 20 Activity 2

Pairs | 10 min

Activity 2 Once, Never, Always (continued)

Purpose: Students critique the reasoning of others as they consider the effect of dividing by a variable when solving an equation. **(MTR.4.1)**

2 Monitor

Collect student responses by polling the class.

ELL **English Language Learners** Invite students to read each student's idea aloud and paraphrase what it means, clarifying any unknown words or concepts before responding.

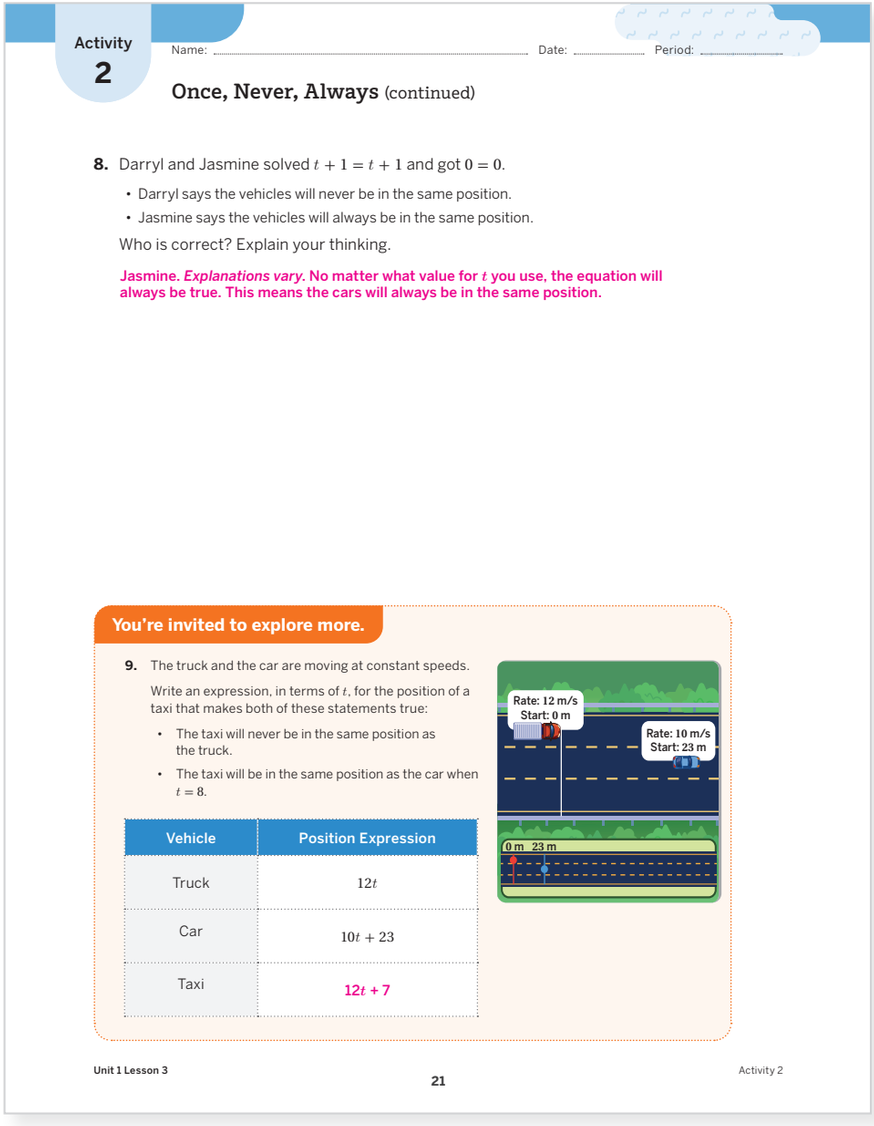
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore this concept further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"



Activity 2

Name: Date: Period:

Once, Never, Always (continued)

8. Darryl and Jasmine solved $t + 1 = t + 1$ and got $0 = 0$.

- Darryl says the vehicles will never be in the same position.
- Jasmine says the vehicles will always be in the same position.

Who is correct? Explain your thinking.

Jasmine. *Explanations vary.* No matter what value for t you use, the equation will always be true. This means the cars will always be in the same position.

You're invited to explore more.

9. The truck and the car are moving at constant speeds.

Write an expression, in terms of t, for the position of a taxi that makes both of these statements true:

- The taxi will never be in the same position as the truck.
- The taxi will be in the same position as the car when $t = 8$.

Vehicle	Position Expression
Truck	$12t$
Car	$10t + 23$
Taxi	$12t + 7$

Unit 1 Lesson 3 21 Activity 2

Key Takeaway: When solving an equation, the variable is sometimes eliminated. If the statement that remains is true, then there are **infinitely many solutions** to the equation. If the statement that remains is false, then there is **no solution** to the equation.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for deciding how many solutions there are for an equation.

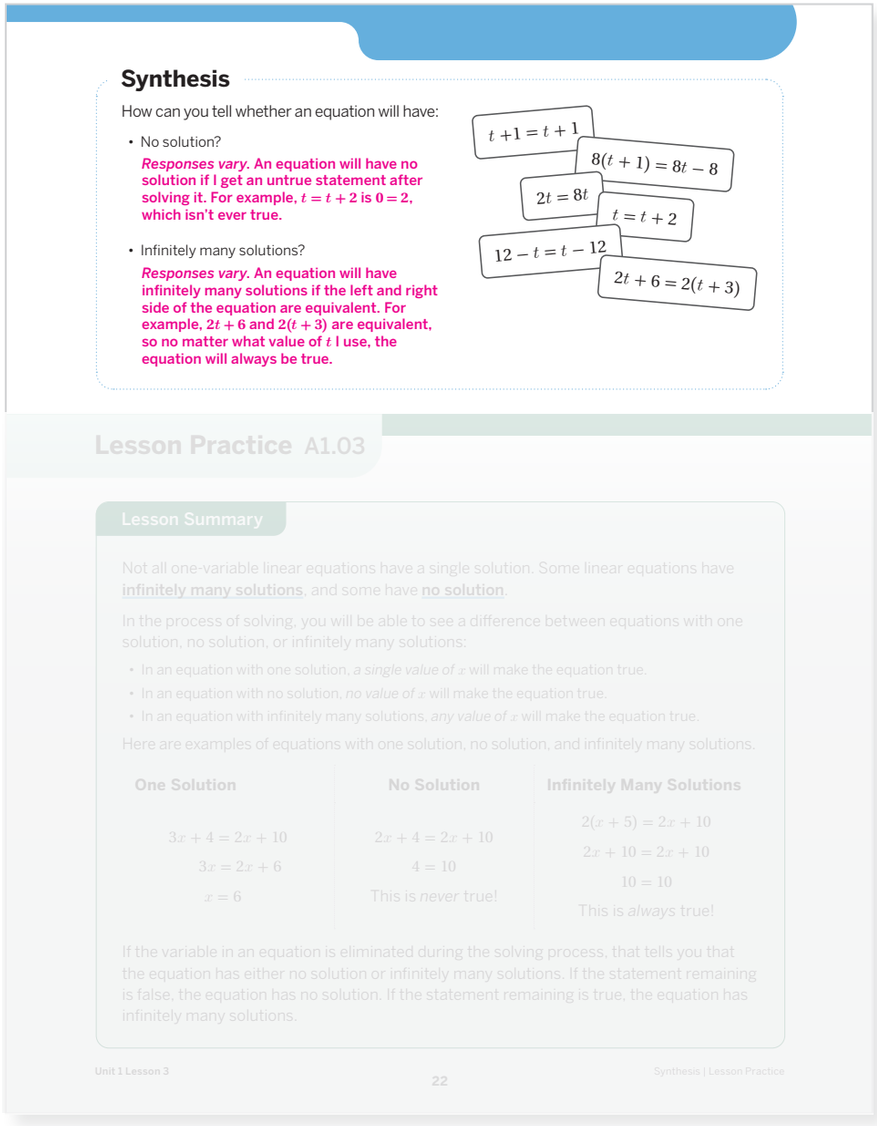
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- A linear equation can have one solution, no solution, or infinitely many solutions.
- When eliminating a variable in the solving process results in a true statement, the equation has infinitely many solutions.
- When eliminating a variable in the solving process results in a false statement, the equation has no solution.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of **infinitely many solutions**, **no solution**, and one solution using students' language, drawing connections between these words and the language used in this lesson (once, never, always). Include an example equation for each.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

How can you tell whether an equation will have:

- No solution?

  Responses vary. An equation will have no solution if I get an untrue statement after solving it. For example, $t = t + 2$ is $0 = 2$, which isn't ever true.

- Infinitely many solutions?

  Responses vary. An equation will have infinitely many solutions if the left and right side of the equation are equivalent. For example, $2t + 6$ and $2(t + 3)$ are equivalent, so no matter what value of t I use, the equation will always be true.

$t + 1 = t + 1$

$8(t + 1) = 8t - 8$

$2t = 8t$

$t = t + 2$

$12 - t = t - 12$

$2t + 6 = 2(t + 3)$

Lesson Practice A1.03

Lesson Summary

Not all one-variable linear equations have a single solution. Some linear equations have infinitely many solutions, and some have no solution.

In the process of solving, you will be able to see a difference between equations with one solution, no solution, or infinitely many solutions:

- In an equation with one solution, *a single value of* x will make the equation true.
- In an equation with no solution, *no value of* x will make the equation true.
- In an equation with infinitely many solutions, *any value of* x will make the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 4 = 2x + 10$	$2x + 4 = 2x + 10$	$2(x + 5) = 2x + 10$
$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	This is *never* true!	$10 = 10$
		This is *always* true!

If the variable in an equation is eliminated during the solving process, that tells you that the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

Unit 1 Lesson 3 22 Synthesis | Lesson Practice

Lesson Takeaway: Linear equations can have one solution, **no solution**, or **infinitely many solutions**. Equations that result in a true statement have infinitely many solutions, while equations that result in a false statement have no solution.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the number of possible solutions to a one-variable equation. **(MTR.4.1)**

Today's Goals

1. **Goal:** Observe that dividing by a variable does not always create equivalent equations.
2. **Language Goal:** Justify whether a one-variable equation has one solution, no solution, or infinitely many solutions. **(Listening, Speaking, and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.03

How many solutions does the equation $10x = 2.5x$ have? Circle one.

(One solution) No solution Infinitely many solutions

If there is one solution, what is the solution? $x = 0$

If there are no solutions or infinitely many solutions, show or explain how you know.

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support identifying the number of possible solutions of a linear equation:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problem 6** to review Nikhil's work.
- Revisit **Grade 8, Unit 4, Lesson 2** *(All, Some, or None, Part 1)*

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite them to complete the **You're invited to explore more task** and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on how students' understanding of the meaning of *solution* has developed in Lessons 1–3. In Lesson 1, a solution to an equation is defined as a value that makes that equation true. How has this definition expanded today? How did students use the structure of equations to support their growing understanding of the term *solution*? **(MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

How can you tell whether an equation will have:

- No solution?

Responses vary. An equation will have no solution if I get an untrue statement after solving it. For example, $t = t + 2$ is $0 = 2$, which isn't ever true.

- Infinitely many solutions?

Responses vary. An equation will have infinitely many solutions if the left and right side of the equation are equivalent. For example, $2t + 6$ and $2(t + 3)$ are equivalent, so no matter what value of t I use, the equation will always be true.

$t + 1 = t + 1$ $\quad 8(t + 1) = 8t - 8$ $\quad 2t = 8t$ $\quad t = t + 2$ $\quad 12 - t = t - 12$ $\quad 2t + 6 = 2(t + 3)$

Lesson Practice A1.03

Lesson Summary

Not all one-variable linear equations have a single solution. Some linear equations have **infinitely many solutions**, and some have **no solution**.

In the process of solving, you will be able to see a difference between equations with one solution, no solution, or infinitely many solutions:

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One Solution	No Solution	Infinitely Many Solutions
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$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	This is *never* true!	$10 = 10$
		This is *always* true!

If the variable in an equation is eliminated during the solving process, that tells you that the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

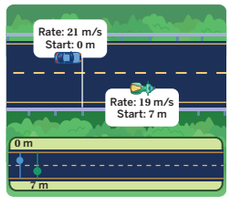
Lesson Practice A1.03

Name: ..... Date: ..... Period: .....

1. The car and scooter are moving at constant speeds. The equation $21t = 19t + 7$ represents the time, t, when they will be in the same position.

When will the car and scooter be in the same position?

3.5 seconds



Problems 2–3: The equation $10t = 2.5t$ represents the time, t, when two vehicles will be in the same position.

2. When will these two vehicles be in the same position? Circle one.

(Once) Never Always

3. Explain how you know.

Explanations vary. $t = 0$ **makes the equation true because** $10(0) = 2.5(0)$**. No other values make the equation true, so the vehicles will be in the same position once.**

4. Here is Kiandra's work to solve $16x = 10x$. She says there is no solution.

Is this correct? Show or explain your thinking.

No. Explanations vary. **If Kiandra subtracted** $10x$ **from each side of the equation, she would get** $6x = 0$**. If she then divided by 6, she would get a solution:** $x = 0$**.**

Kiandra

$\frac{16x}{x} = \frac{10x}{x}$

$16 = 10$

There is no solution.

Problems 5 and 6: Create two different equations that each have a solution of $x = 1$.

5. Fill in each blank using the digits 0 to 9 only once each.

Responses vary.

- $1x + 8 = 2x + 7$ **and** $3x + 6 = 4x + 5$
- $7x + 1 = 6x + 2$ **and** $5x + 3 = 8x + 0$

$\square x + \square = \square x + \square$

$\square x + \square = \square x + \square$

6. Explain what you notice about your equations.

Explanations vary. **The digits used on each side of each equation sum to the same total.**

Lesson Practice A1.03

Name: ..... Date: ..... Period: .....

Test Practice

7. Group the equations based on their number of solutions.

A. $5t = 3t$ B. $2t = 10 - 2t$ C. $15 - 3(t + 5) = -3t$

D. $4t + 7 = 4(t + 2)$ E. $6t + 2 = -3 + 6t$

One Solution	No Solution	Infinitely Many Solutions
A. $5t = 3t$ B. $2t = 10 - 2t$	D. $4t + 7 = 4(t + 2)$ E. $6t + 2 = -3 + 6t$	C. $15 - 3(t + 5) = -3t$

Spiral Review

8. Select *all* the equations where $x = 2$ is a solution.

☐ A. $\frac{x}{4} = 8$ ☑ B. $19 = 2(x + 6) + 3$ ☐ C. $2x + 10 = 2x + 8$

☑ D. $5 - 3x = -1$ ☑ E. $4 - x = x$

9. Select all the expressions that are equivalent to $2(x + 3)$.

☑ A. $(x + 3) \cdot 2$ ☐ B. $2x + 5$ ☑ C. $2x + 3 \cdot 2$

☐ D. $2x + 3$ ☑ E. $2x + 6$

10. Select the expression that is equivalent to $6 - 2(x + 1)$.

A. $4(x + 1)$

B. $7 - 2x$

(C.) $4 - 2x$

D. $4x + 4$

Lesson Practice Item Analysis

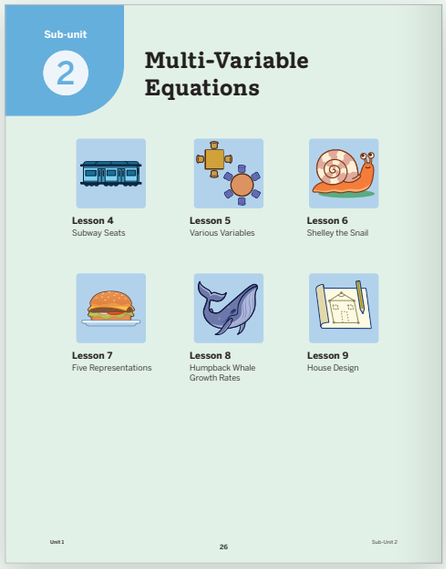
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.2.1
	2–4	2	MA.912.AR.2.1
	5–6	3	MA.912.AR.2.1
Test Practice	7	2	MA.912.AR.2.1
Spiral Review			
	8	2	MA.7.AR.1.2
	9–10	1	MA.7.AR.1.2

Multi-Variable Equations

Sub-Unit 2 Goals

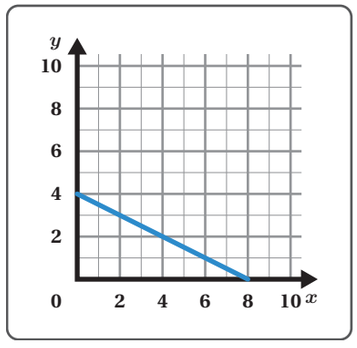
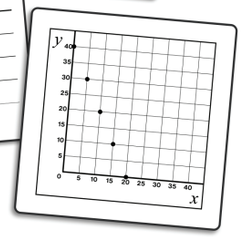
- **Goal:** Solve multi-variable equations for a given variable.
- **Goal:** Create equations, graphs, and tables to represent linear relationships and interpret their solutions in context.
- **Goal:** Write a linear two-variable equation for a line that is parallel or perpendicular to a given line passing through a given point.
- **Language Goal:** Describe connections between equations, tables, descriptions, and graphs of linear situations. **(Writing and Speaking)**

In this sub-unit, students apply their strategies for solving equations from Sub-Unit 1 to rearrange equations to reveal information about a situation, including the **x-intercept** and **y-intercept**. They represent situations using two-variable equations and rewrite those equations to highlight a variable of interest. Students develop flexibility and fluency in translating between different representations of linear situations.



Math That Matters Most

Concept or Skill	For example . . .
Represent constraints with two-variable linear equations.	The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.
Interpret the solutions to two-variable equations in context.	Select a point on your graph and explain what it means in this situation. Point: Responses vary. (0, 12) Explanation: *Explanations vary.* The point means that Cameron might have sold 0 paintings and 12 frames.
Rearrange a linear equation to highlight a variable of interest.	$6r + 4s = 240$ $-6r \quad -6r$ $\frac{4s}{4} = \frac{240 - 6r}{4}$ $s = 60 - 1.5r$

Concept or Skill	For example . . .
Write two equivalent linear equations in different forms.	$2x + 4y = 16$ $y = 4 - \frac{1}{2}x$ 
Describe connections between equations, tables, graphs, and description of linear equations.	$2x + y = 40$ x | y 0 | 40 5 | 30 10 | 20 15 | 10 20 | 0 

Unit 1

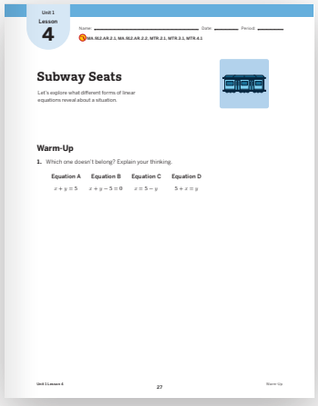
Lesson

4

Subway Seats

Representing Situations With Two-Variable Equations

Let's explore what different forms of linear equations reveal about a situation.


Unit 1
Lesson
4
Name: Date: Period:
MA.912.AR.2.1, MA.912.AR.2.2, MTR.2.1, MTR.3.1, MTR.4.1
Subway Seats
Let's explore what different forms of linear equations reveal about a situation.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
Equation A Equation B Equation C Equation D
$x + y = 5$ $x + y - 5 = 0$ $x = 5 - y$ $5 = x + y$
Unit 1 Lesson 4 27 Warm-Up

Learning Outcomes

Today's Learning

1. **Goal:** Identify equivalent forms of linear equations and understand that different forms of a linear equation can be useful for different purposes.
2. **Goal:** Represent constraints with two-variable equations and interpret their solutions.
3. **Language Goal:** Interpret different forms of a two-variable linear equation in context. **(Writing and Speaking)**

Students are introduced to two-variable equations in the context of sitting and standing capacities in a subway car. They consider the meaning of solutions to two-variable equations by solving an equation for one of the variables and considering how the new form reveals different insights about the situation and the relationship between the variables. **(MTR.3.1)**

Coherence

Prior Learning

In Lesson 3, students practiced solving equations in context and interpreted their solutions, including equations with no solution and infinitely many solutions.

Future Learning

In Lesson 5, students will apply what they learned about solving equations with one-variable to solve equations with coefficients represented by letters.
Note: The term *constraint* will be formally defined in Lesson 10.

Connections

- Students write equivalent forms of two-variable linear equations to redesign subway cars to accommodate more people. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.2
Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

MA.912.AR.1.2
Rearrange equations or formulas to isolate a quantity of interest.

Also Addressing: **MA.912.AR.2.5, MA.912.AR.2.6**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.1	**MA.912.AR.5.5**
MA.8.AR.2.2	**MA.912.AR.8.2**
	MA.912.AR.9.8

Vocabulary

Additional Vocabulary

constraint

Lesson at a Glance

~ 45 min

MA.912.AR.2.2, MA.912.AR.1.2, MA.912.AR.2.5, MA.912.AR.2.6

Warm-Up

Independent | 5 min

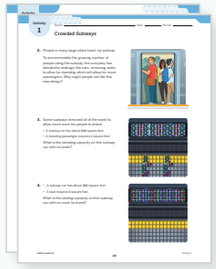
Students use the **Which One Doesn't Belong?** routine to compare the different structures of two-variable equations that they will explore in this lesson.



Activity 1

Pairs | 20 min

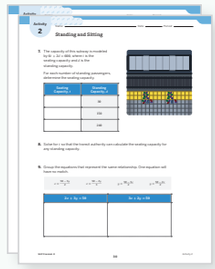
Students explore solutions that represent capacities in a subway car to make sense of how writing equations in different forms reveals new information about a situation.
Routine: Think-Pair-Share **(MTR.3.1)**



Activity 2

Pairs | 10 min

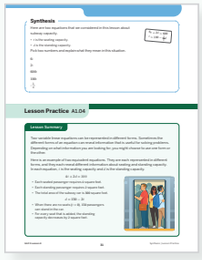
Students rearrange two-variable equations to gain new insights about the relationships between the variables when the equations are written in different forms. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

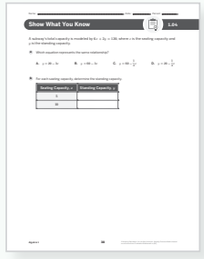
Students synthesize their understanding of how different forms of the same equation can provide new insights into a situation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by rewriting an equation to highlight a variable of interest and use the equations to answer questions in context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to compare the different structures of two-variable equations that they will explore in this lesson.

Short on time? Consider omitting the Warm-Up.

1 Launch

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

ELL **English Language Learners** Provide sentence frames. (E.g., "Equation ___ doesn't belong because ___." or "Equation ___ is the only one that has/doesn't have___."). **(Writing and Speaking)**

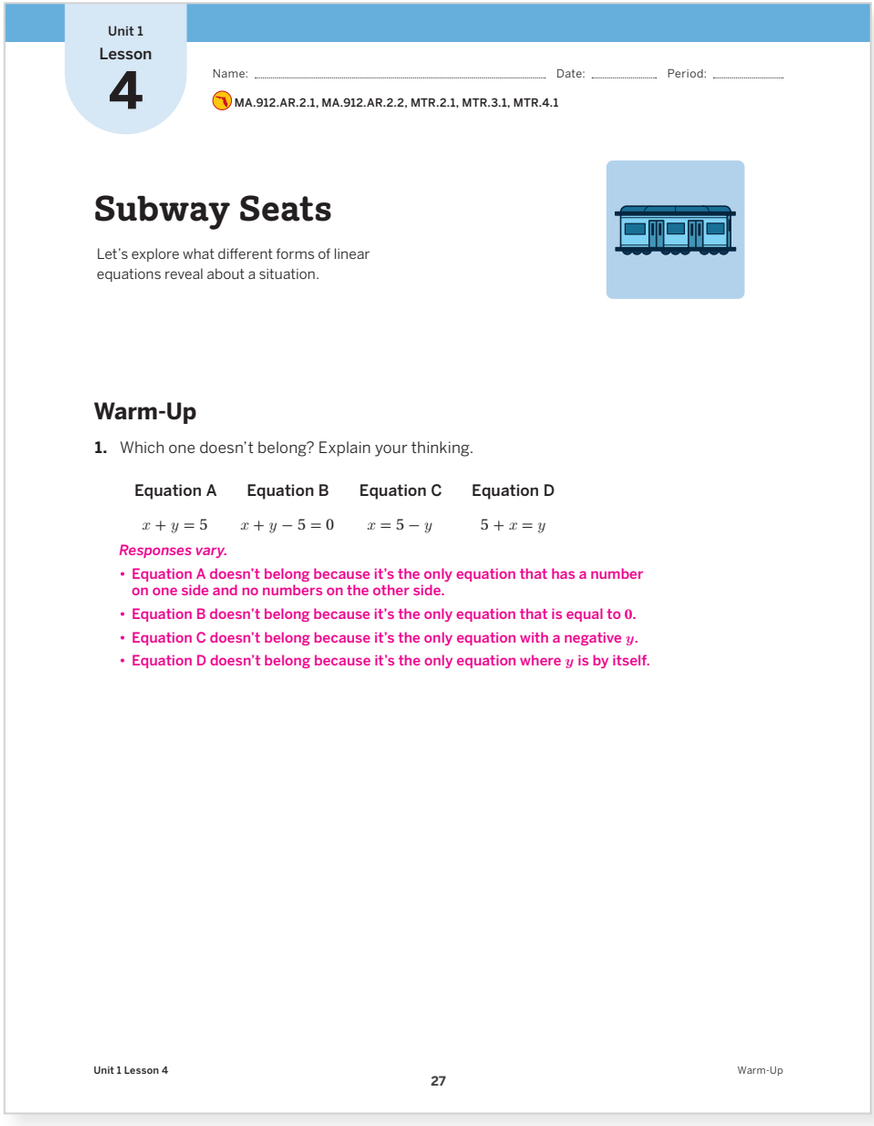
2 Connect

Discuss why each representation doesn't belong, asking students to justify their choice.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

To surface the Key Takeaway, consider asking:

- "Which equations are equivalent? How do you know?" Equations A, B, and C are all equivalent because they can be rearranged to show the same equation.
- "What is a possible value for x and y for equation ___?" *Responses vary.* One possible set of values could be $x = 2$ and $y = 3$ for Equations A, B, or C.
- "Do these values work for all of these equations?" No. *Explanations vary.* If $x = 2$ and $y = 3$, the remaining statement for Equation D is not true (i.e., $5 + 2 \neq 3$).



Unit 1
Lesson
4

Name: _____ Date: _____ Period: _____

MA.912.AR.2.1, MA.912.AR.2.2, MTR.2.1, MTR.3.1, MTR.4.1

Subway Seats

Let's explore what different forms of linear equations reveal about a situation.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

Equation A	Equation B	Equation C	Equation D
$x + y = 5$	$x + y - 5 = 0$	$x = 5 - y$	$5 + x = y$

Responses vary.

- Equation A doesn't belong because it's the only equation that has a number on one side and no numbers on the other side.
- Equation B doesn't belong because it's the only equation that is equal to 0.
- Equation C doesn't belong because it's the only equation with a negative y.
- Equation D doesn't belong because it's the only equation where y is by itself.

Unit 1 Lesson 4 27 Warm-Up

Key Takeaway: Two-variable equations can be written in different, but equivalent, forms.

Pairs | 20 min

Activity 1 Crowded Subways

Purpose: Students explore solutions that represent capacities in a subway car to make sense of how writing equations in different forms reveals new information about a situation. **(MTR.3.1)**

1 Launch

To surface prior knowledge, consider asking:

- "Have you ever been on a subway? What was it like?"
- "Have you ever stood on a bus or train when it was moving?"

ELL **English Language Learners** Invite students to act out scenarios, show videos/images, use gestures, or talk about their experiences on subways or other public transportation before starting the task. **(Reading and Listening)**

Use the Think-Pair-Share routine to help students shape their ideas. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's ideas with the class. **(MTR.4.1)**

Consider asking:

- "What do you think the transit authority should do about the crowded subways?"
- "How do you think people would feel about having seats removed?"

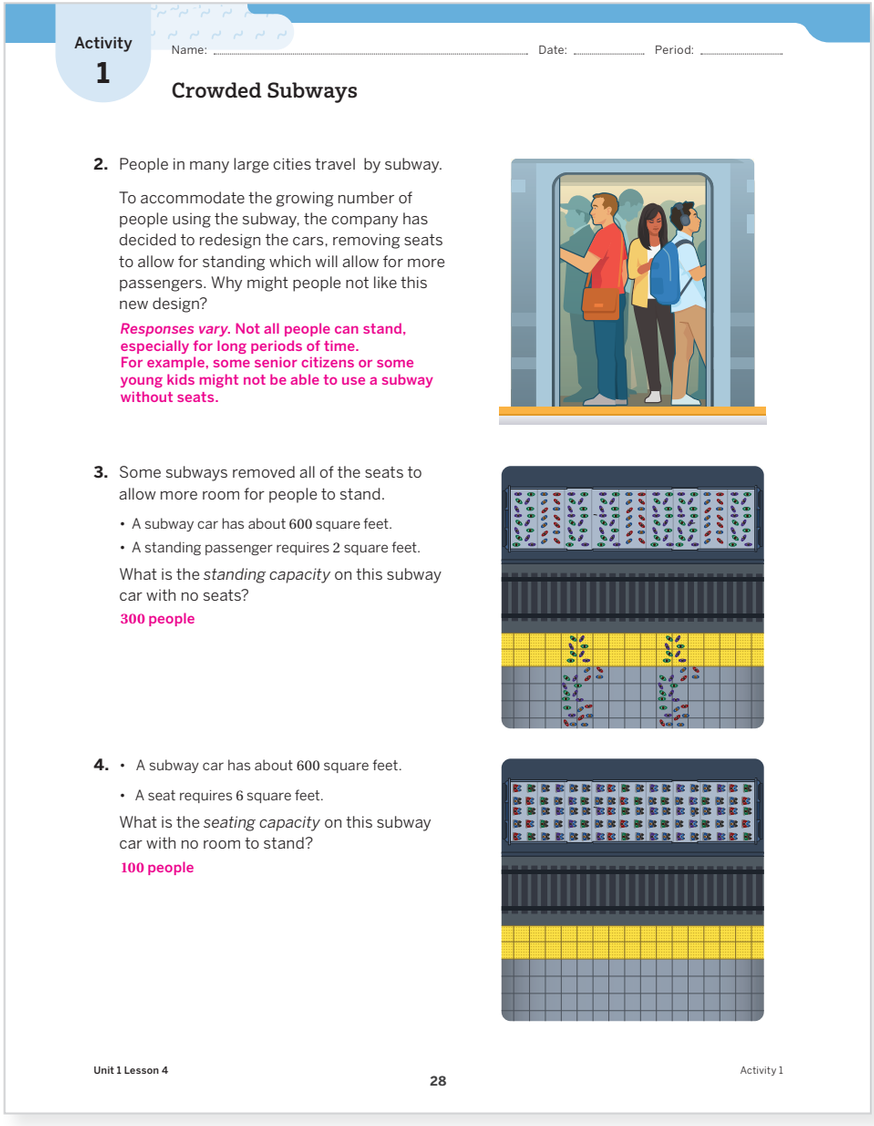
2 Monitor

ELL **English Language Learners** Invite students to describe the meaning of *capacity* in their own words to increase access to the task. **(Reading and Listening)**

Encourage students to use the feedback from their partners to help them revise their thinking. Invite students to use a calculator as needed.

A **Accessibility: Conceptual Processing** Consider asking a similar question with more friendly numbers (e.g., "If the subway car were 8 square feet and each person standing required 2 square feet, how many people could stand in the car?"). Invite students to describe their thought process and encourage them to apply this same strategy to the problem.

Activity 1 continued >



Activity 1

Name: Date: Period:

Crowded Subways

2. People in many large cities travel by subway.

To accommodate the growing number of people using the subway, the company has decided to redesign the cars, removing seats to allow for standing which will allow for more passengers. Why might people not like this new design?

Responses vary. Not all people can stand, especially for long periods of time. For example, some senior citizens or some young kids might not be able to use a subway without seats.

3. Some subways removed all of the seats to allow more room for people to stand.

- A subway car has about 600 square feet.
- A standing passenger requires 2 square feet.

What is the *standing capacity* on this subway car with no seats?

300 people

4.

- A subway car has about 600 square feet.
- A seat requires 6 square feet.

What is the *seating capacity* on this subway car with no room to stand?

100 people

Unit 1 Lesson 4 28 Activity 1

Pairs | 20 min

Activity 1 Crowded Subways (continued)

Purpose: Students explore solutions that represent capacities in a subway car to make sense of how writing equations in different forms reveals new information about a situation. **(MTR.3.1)**

2 Monitor

Encourage students to use paper to help with their thinking.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started or are guessing and checking.	**Support** Consider asking, "Could you substitute 5 for t or d? Why or why not?"
Enter expressions that show repeated reasoning (e.g., $\frac{600-6(5)}{2}$).	**Strengthen** To support students in making generalizations, consider asking, "How is your thinking for each row of the table alike? How is it different?" **(MTR.2.1)** Capture this strategy to compare to Tiam's strategy during the Connect.

A Accessibility: Conceptual Processing Invite students to say what each equation means replacing t and d with variable definitions (e.g., "6 times the seating capacity plus 2 times the standing capacity is equal to 600" or "The standing capacity equals 300 people and decreases by 3 for each seated passenger."). **(MTR.3.1)**

Capture a variety of interpretations to share during the Connect.

ELL English Language Learners Provide a word bank to support students in expressing their ideas with words such as: people, feet, seat, standing, more, less, subway car, etc. **(Writing and Speaking)**

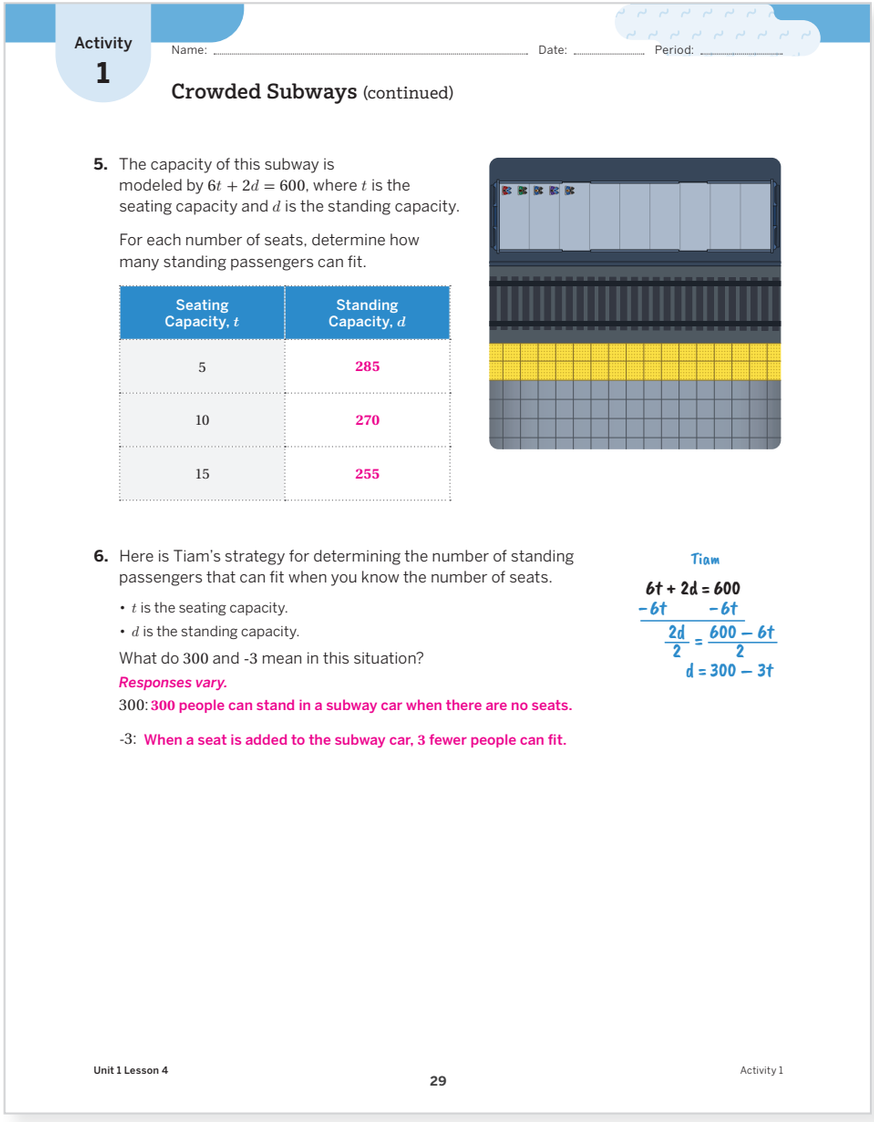
3 Connect

Consider asking:

- "What steps did Tiam take? Why?"
- "What connections can you describe between your strategy on Problem 5 and Tiam's strategy?"

Share the captured interpretations. For each interpretation, invite students to highlight strengths and ideas to make each stronger and clearer. **(MTR.3.1)**

To surface the Key Takeaway, consider asking, "What insights did $6t + 2d = 600$ reveal about the situation that Tiam's final equation did not?"



Activity 1

Name: Date: Period:

Crowded Subways (continued)

5. The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of seats, determine how many standing passengers can fit.

Seating Capacity, t	Standing Capacity, d
5	285
10	270
15	255

6. Here is Tiam's strategy for determining the number of standing passengers that can fit when you know the number of seats.

- t is the seating capacity.
- d is the standing capacity.

What do 300 and -3 mean in this situation?

Responses vary.

300: 300 people can stand in a subway car when there are no seats.

-3: When a seat is added to the subway car, 3 fewer people can fit.

Tiam

$6t + 2d = 600$

$-6t \quad -6t$

$\frac{2d}{2} = \frac{600-6t}{2}$

$d = 300 - 3t$

Unit 1 Lesson 4 29 Activity 1

Key Takeaway: Different forms of equations reveal different insights about the situation.

Pairs | 10 min

Activity 2 Standing and Sitting

Purpose: Students rearrange two-variable equations to gain new insights about the relationships between the variables when the equations are written in different forms. **(MTR.2.1)**

1 Launch

Consider asking, "How is this problem like what you did for problems 5 and 6? How is it different?"

2 Monitor

Encourage students to look for patterns in their repeated reasoning to support them in generalizing on the next problem. **(MTR.5.1)**

To support students getting started, invite students to review Tiam's process on problem 6.

Encourage students to use paper to help with their thinking.

A **Accessibility: Visual-Spatial Processing** To support students in distinguishing between similar equations, invite pairs to discuss how they are similar and different before grouping the equations.

Invite students to share their reasoning with a partner and work together to reach an agreement about how to sort the equations. **(MTR.4.1, MTR.2.1)**

D **Differentiation: (Stretch)** Encourage students to describe a situation that each group of equations could represent. **(MTR.2.1)**

3 Connect

Invite students to share different strategies they used to match equivalent equations.

Discuss additional equivalent equations to add to each group (e.g., $y = \frac{50 - 3(x)}{2}$ could be written as $y = 25 - \frac{3}{2}x$).

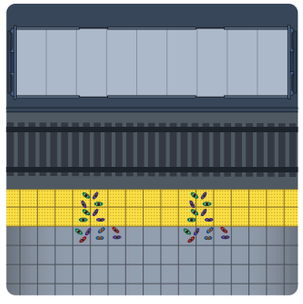
Activity 2

Name: Date: Period:

Standing and Sitting

7. The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of standing passengers, determine the seating capacity.



Seating Capacity, t	Standing Capacity, d
90	30
50	150
20	240

8. Solve for t so that the transit authority can calculate the seating capacity for any standing capacity.

$t = \frac{600 - 2d}{6}$ (or equivalent)

9. Group the equations that represent the same relationship. One equation will have no match.

$x = \frac{50 - 3y}{2}$ $x = \frac{50 - 2y}{3}$ $y = \frac{50 - 3x}{2}$ $y = \frac{50 - 2x}{2}$

$2x + 3y = 50$	$3x + 2y = 50$
$x = \frac{50 - 3y}{2}$	$x = \frac{50 - 2y}{3}$ $y = \frac{50 - 3x}{2}$

Equation with no match: $y = \frac{50 - 2x}{2}$

Unit 1 Lesson 4 30 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how different forms of the same equation can provide new insights into a situation.

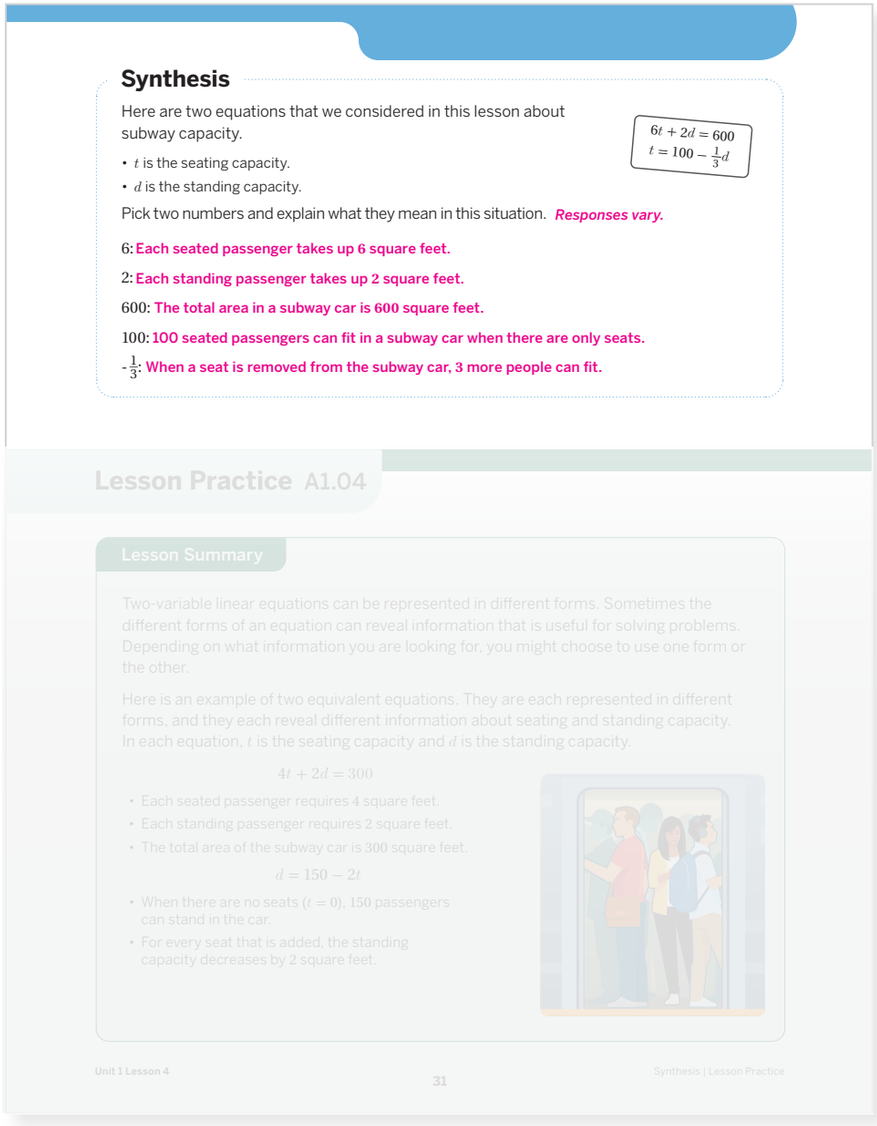
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of student interpretations. See sample responses for examples.

Consider asking:

- "Which numbers are the easiest for you to interpret? Which do you find more challenging?"
- "When might each equation be helpful for the transportation authority?"
- "When in general might it be helpful to solve an equation for a specific variable?"

MLR **MLR1: Stronger and Clearer Each Time (Optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**



Synthesis

Here are two equations that we considered in this lesson about subway capacity.

$6t + 2d = 600$
$t = 100 - \frac{1}{3}d$

- t is the seating capacity.
- d is the standing capacity.

Pick two numbers and explain what they mean in this situation. *Responses vary.*

6: Each seated passenger takes up 6 square feet.

2: Each standing passenger takes up 2 square feet.

600: The total area in a subway car is 600 square feet.

100: 100 seated passengers can fit in a subway car when there are only seats.

$-\frac{1}{3}$: When a seat is removed from the subway car, 3 more people can fit.

Lesson Practice A1.04

Lesson Summary

Two-variable linear equations can be represented in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations. They are each represented in different forms, and they each reveal different information about seating and standing capacity. In each equation, t is the seating capacity and d is the standing capacity.

$4t + 2d = 300$

- Each seated passenger requires 4 square feet.
- Each standing passenger requires 2 square feet.
- The total area of the subway car is 300 square feet.

$d = 150 - 2t$

- When there are no seats ($t = 0$), 150 passengers can stand in the car.
- For every seat that is added, the standing capacity decreases by 2 square feet.

Unit 1 Lesson 4 31 Synthesis | Lesson Practice

Lesson Takeaway: Equations can be rearranged to highlight different variables of interest, revealing different insights into the situations they represent.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by rewriting an equation to highlight a variable of interest and use the equations to answer questions in context.

Today's Goals

1. **Goal:** Identify equivalent forms of linear equations and understand that different forms of a linear equation can be useful for different purposes.
2. **Goal:** Represent constraints with two-variable equations and interpret their solutions.
3. **Language Goal:** Interpret different forms of a two-variable linear equation in context. **(Writing and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.04

A subway's total capacity is modeled by $6x + 2y = 120$, where x is the seating capacity and y is the standing capacity.

a. Which equation represents the same relationship?

A. $y = 20 - 3x$ B. $y = 60 - 3x$ C. $y = 60 - \frac{1}{3}x$ D. $y = 20 - \frac{1}{3}x$

b. For each seating capacity, determine the standing capacity.

Seating Capacity, x	Standing Capacity, y
5	45
10	30

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support rearranging two-variable equations to highlight a variable of interest:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problem 9** and invite students to share their strategies for identifying matching equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how the class discussed fairness within the subway context. How did your and students' previous experiences with public transportation impact how you taught the lesson? In what ways were students able to make connections between math ideas and their or others' communities in this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

Here are two equations that we considered in this lesson about subway capacity.

$6t + 2d = 600$

$t = 100 - \frac{1}{3}d$

- t is the seating capacity.
- d is the standing capacity.

Pick two numbers and explain what they mean in this situation. *Responses vary.*

6: Each seated passenger takes up 6 square feet.

2: Each standing passenger takes up 2 square feet.

600: The total area in a subway car is 600 square feet.

100: 100 seated passengers can fit in a subway car when there are only seats.

$-\frac{1}{3}$: When a seat is removed from the subway car, 3 more people can fit.

Lesson Practice A1.04

Lesson Summary

Two-variable linear equations can be represented in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

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- The total area of the subway car is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats ($t = 0$), 150 passengers can stand in the car.
- For every seat that is added, the standing capacity decreases by 2 square feet.



Lesson Practice A1.04

Name: Date: Period:

Problems 1–4: Adriana spent \$24 on fruit punch and apple juice. Fruit punch costs \$3 per bottle. Apple juice costs \$2 per bottle.

1. How many bottles of *fruit punch* could Adriana buy if she didn't get any apple juice?
   8 bottles
2. How many bottles of *apple juice* could Adriana buy if she did not get any fruit punch?
   12 bottles

Adriana wrote the equation $3f + 2a = 24$ to represent the situation.

3. Use the equation to help you complete the table.

f	a
2	9
6	3

4. Which equation represents the same relationship?

A. $a = 8 - \frac{2}{3}f$ B. $a = 8 - \frac{3}{2}f$
C. $a = 12 - \frac{2}{3}f$ (D.) $a = 12 - \frac{3}{2}f$

Problems 5–6: Here is an equation: $2x + 4y = 80$.

5. Use the equation to help you complete the table.

x	y
6	17
12	14

6. Which equation represents the same relationship?

A. $y = 20 - 2x$ B. $y = 40 - 2x$
(C.) $y = 20 - \frac{1}{2}x$ D. $y = 40 - \frac{1}{2}x$

Test Practice

Problems 7–8: Nia is buying bananas and apples for her family. Bananas cost \$0.50 each. Apples cost \$1.00 each.

7. The equation $0.5b + 1a = 3.50$ represents the number of bananas and apples that Nia can buy for \$3.50. Solve this equation for a.
   $a = 3.50 - 0.5b$
8. Select *all* the combinations of bananas and apples that Nia could buy for exactly \$3.50.

☑ A. 1 banana and 3 apples ☑ B. 5 bananas and 1 apple
☐ C. 1 banana and 2 apples ☑ D. 3 bananas and 2 apples
☐ E. 5 bananas and 2 apples

Lesson Practice A1.04

Name: Date: Period:

Spiral Review

9. Select *all* the expressions that are equivalent to $8 - 12 - (6 + 4)$.

☐ A. $(6 + 4) - 8 - 12$
☐ B. $8 - 6 - 12 + 4$
☑ C. $8 - 12 - 6 - 4$
☑ D. $8 - (6 + 4) - 12$
☐ E. $(8 - 12) - 6 + 4$

10. Which equation is equivalent to $\frac{1}{3}m + \frac{1}{2}n = 9$?

A. $3m + 2n = 9$
(B.) $2m + 3n = 54$
C. $m + 3n = 27$
D. $2m + n = 18$

11. Explain how you know that equation A and equation B are equivalent.

Equation A	Equation B
$48 - 5x = 13$	$5x = 35$

Responses vary.
- The solution to each equation is $x = 7$.
- If you subtract 48 from both sides of equation A, the equation would be $-5x = -35$. Then if you divide or multiply both sides by -1, equation A would be equivalent to equation B.

12. Create two equivalent equations by filling in the blanks using the digits 0 to 9 only once.

Responses vary.
- $6x + 2y = 8$ and $y = 4 - 3x$
- $8x + 2y = 6$ and $y = 3 - 4x$

$\square x + \square y = \square$

$y = \square - \square x$

Lesson Practice Item Analysis

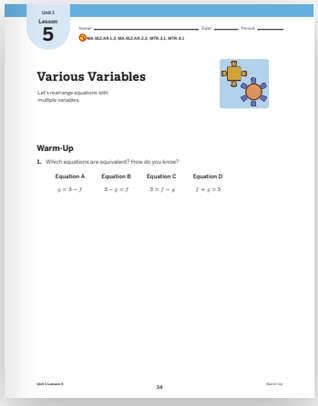
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 5	1	MA.912.AR.2.2*
	3	1	MA.912.AR.1.2*
	4	2	MA.912.AR.1.2
	5–6	2	MA.912.AR.2.1
Test Practice	7–8	2	MA.912.AR.2.2
Spiral Review			
	9	1	MA.6.AR.1.4
	10–11	2	MA.7.AR.1.2
	12	3	MA.912.AR.1.2

*These problems build toward the standard shown.

Unit 1

Lesson

5



Various Variables

Solving Multi-Variable Equations

Let's rearrange equations with multiple variables.

Learning Outcomes

Today's Goals

1. **Goal:** Rearrange multi-variable equations to highlight a variable of interest.
2. **Goal:** Solve multi-variable equations for a variable of interest.
3. **Language Goal:** Explain how solving for a variable is similar to rearranging an equation to isolate a variable of interest. **(Speaking and Writing)**

Students solve multi-variable equations for a variable of interest. They use strategies for solving, including keeping the equation balanced, using inverse operations, and working backwards. Students generalize how solving a one-variable equation compares to solving an equation with multiple variables. **(MTR.3.1)**

Coherence

Prior Learning

In Lessons 1–4, students solved equations using strategies such as working backwards and using inverse operations. They considered how rearranging an equation can reveal different relationships between variables.

Future Learning

In Lesson 6, students will rearrange linear equations in standard form to highlight a variable of interest and make connections to the graph. Students will be introduced to the terms *standard form* and *slope-intercept form* in Lesson 7. In Lessons 9–10, students will solve one-variable inequalities.

Connections

- Students write equivalent forms of two-variable linear equations to determine different seating arrangements at a party. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.2

Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

MA.912.AR.1.2

Rearrange equations or formulas to isolate a quantity of interest.

Also Addressing: **MA.912.AR.2.5, MA.912.AR.2.6**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.1	**MA.912.AR.5.5**
MA.8.AR.2.2	**MA.912.AR.8.2**
	MA.912.AR.9.8

Vocabulary

Review Vocabulary

substitute

variable

Lesson at a Glance

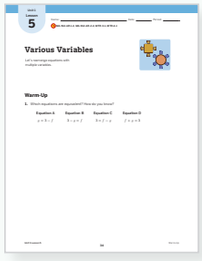
~ 45 min

MA.912.AR.2.2, MA.912.AR.1.2, MA.912.AR.2.5, MA.912.AR.2.6

Warm-Up

Pairs | 5 min

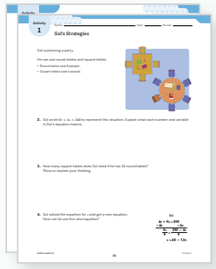
Students determine which of four equations are equivalent to surface strategies for rearranging two-variable equations. **Routine:** Think-Pair-Share



Activity 1

Pairs | 15 min

Students rearrange a two-variable equation to highlight each variable as they solve problems about different-sized tables at a party. MLR7: Compare and Connect



Activity 2

Pairs | 15 min

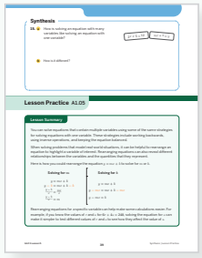
Students develop fluency in solving multi-variable equations by using repeated reasoning and the same strategies they used to solve one-variable equations. MLR7: Compare and Connect **(MTR3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the strategies used for solving equations and consider how solving one-variable and multi-variable equations are similar and different.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving for a variable of interest in a one-variable equation and in a multi-variable equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students determine which of four equations are equivalent to surface strategies for rearranging two-variable equations.

1 Launch

Display the equations.

To surface prior knowledge, consider asking, "What does it mean for equations to be equivalent?" *Responses vary.* Equivalent equations have exactly the same solution. If two equations are equivalent, they can be rearranged to show that one is exactly like the other.

Use the Think-Pair-Share routine to help students shape their ideas about equivalent equations. See the Routine Facilitation Guide for more information.

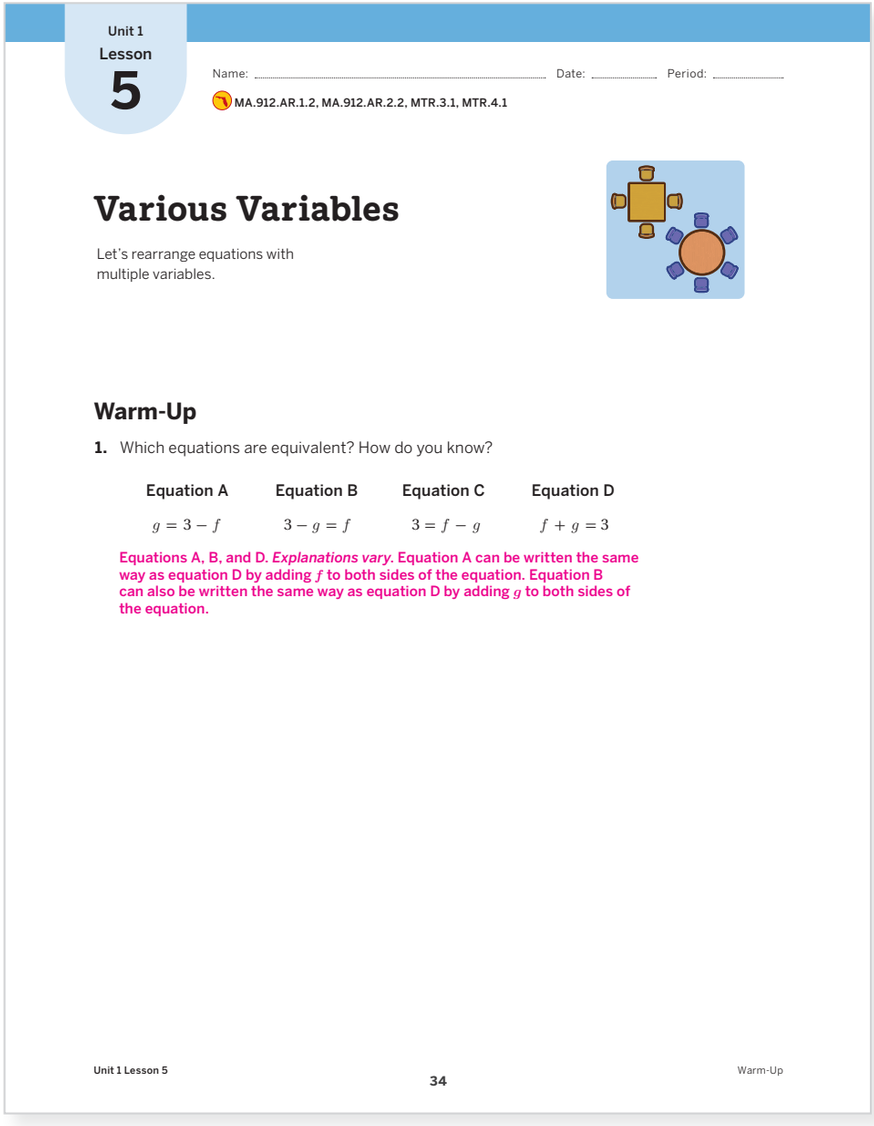
ELL **English Language Learners** Consider providing students with a word bank with words to support them in describing their solution strategies, such as: rearrange, *substitute*, balanced moves, add, subtract, inverse operations, and *variable*. **(Reading and Writing)**

Consider asking, "What would it mean if you could rearrange one equation to look exactly like another one?"

A **Accessibility: Conceptual Processing** To support students as they solve abstract problems, invite them to try concrete examples for the variables f and g (e.g., "If $f = 1$ and $g = 2$, which equations would be equivalent? Would the equations be equivalent for any value of f and g you chose?").

2 Connect

Invite students to share strategies for how they decided which equations were equivalent, such as substituting values in for f and g or rearranging equations.



Unit 1
Lesson
5

Name: _____ Date: _____ Period: _____

MA.912.AR.1.2, MA.912.AR.2.2, MTR.3.1, MTR.4.1

Various Variables

Let's rearrange equations with multiple variables.

Warm-Up

1. Which equations are equivalent? How do you know?

Equation A	Equation B	Equation C	Equation D
$g = 3 - f$	$3 - g = f$	$3 = f - g$	$f + g = 3$

Equations A, B, and D. *Explanations vary.* Equation A can be written the same way as equation D by adding f to both sides of the equation. Equation B can also be written the same way as equation D by adding g to both sides of the equation.

Unit 1 Lesson 5 34 Warm-Up

Pairs | 15 min

Activity 1 Sol's Strategies

Purpose: Students rearrange a two-variable equation to highlight each variable as they solve problems about different-sized tables at a party.

1 Launch

Draw students' attention to the image of tables.

Consider asking:

- "What is the largest party you've ever attended? What was it like?"
- "Have you ever seen a party with many tables?"
- "If Sol had 5 square tables and 10 round tables, how many seats would he have at his party?"

2 Monitor

A **Accessibility: Conceptual Processing** To support students in solving abstract problems, consider using objects such as chips or coins to represent the different types of tables.

To support students getting started with explaining the meaning of numbers and variables, consider asking, "What connections do you see between the numbers in the equation and the number of people that can sit at each type of table?"

D **Differentiation:** (Strengthen) *(Problem 2)* Students may describe that r represents "round tables" and s represents "square tables." Celebrate this connection and encourage them to specify. Consider asking, "What about the round tables? The number of people sitting at them? The amount of them?"

ELL **English Language Learners** Invite students to highlight key phrases such as "round table" and "square table" to help decide which information is known and what variable needs to be solved for. **(Reading)**

Look for and celebrate the different strategies that students use to determine the number of square tables in Problem 3, such as:

- Substituting 16 for r in the original equation.
- Using guess-and-check to see which number of square tables plus 16 round tables will seat 240 people.

D **Differentiation:** (Support) *(Problem 4)* Encourage students to walk through each step of Sol's work to make sure they understand their thinking before moving forward. If enough students are struggling, consider leading a class discussion on Problem 4. During the discussion, consider asking, "What problems might Sol's new equation help them solve?"

Activity 1 continued >

Activity 1

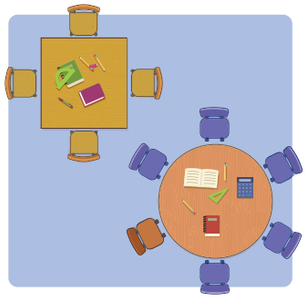
Name: Date: Period:

Sol's Strategies

Sol is planning a party.

He can use round tables and square tables.

- Round tables seat 6 people.
- Square tables seat 4 people.



2. Sol wrote $6r + 4s = 240$ to represent this situation. Explain what each number and variable in Sol's equation means.

Responses vary.

- 6 represents the number of people that can be seated at a round table and r represents the number of round tables at the party.
- 4 represents the number of people that can be seated at a square table and s represents the number of square tables at the party.
- 240 represents the total number of people who will be seated at the party.

3. How many square tables does Sol need if he has 16 round tables? Show or explain your thinking.

36 square tables. *Work varies.*

$$\begin{aligned} 6r + 4s &= 240 \\ 6(16) + 4s &= 240 \\ 96 + 4s &= 240 \\ 96 + 4s - 96 &= 240 - 96 \\ 4s &= 144 \\ \frac{4s}{4} &= \frac{144}{4} \\ s &= 36 \end{aligned}$$

4. Sol solved the equation for s and got a new equation. How can he use this new equation?

Responses vary. He can use this new equation to more easily solve for the number of square tables when he knows the number of round tables being used.

Sol

$$\begin{aligned} 6r + 4s &= 240 \\ -6r \qquad &\quad -6r \\ \frac{4s}{4} &= \frac{240 - 6r}{4} \\ s &= 60 - 1.5r \end{aligned}$$

Unit 1 Lesson 5 35 Activity 1

Pairs | 15 min

Activity 1 Sol's Strategies (continued)

Purpose: Students rearrange a two-variable equation to highlight each variable as they solve problems about different-sized tables at a party.

2 Monitor

D Differentiation *(Problems 5–7)*

Look for students who:	Teacher Moves
Substitute the value given for round tables, r, in to s, the variable for square tables. *(Problem 5)*	**Support** Consider asking, "What does the r represent in this situation?" Consider inviting students to use a highlighter to annotate each problem to show what information they were given (e.g., writing $r = 10$ for Problem 5).
Divide only some terms by 6 when solving to isolate r. *(Problem 7)*	**Support** Invite students to revisit Sol's work in Problem 4. Consider asking, "Why did Sol divide both 240 and $-6r$ by 4?" *Responses vary. Sol divided $4s$ by 4, so they should also divide every term on the other side by 4 to keep the equation balanced.*
Solve the equation using a strategy similar to Sol's. *(Problem 7)*	Consider renaming the strategy Sol uses after one or more students in your class and inviting them to explain their thinking during the Connect.
Correctly solve Sol's equation for r. *(Problem 7)*	**Stretch** Consider asking, "Is it possible for Sol to buy the same number of round and square tables? If so, how many of each would he need?"

3 Connect

Draw students' attention to Sol's reasoning and solution.

MLR **MLR7: Compare and Connect** Invite students to make sense of Sol's work, compare Sol's strategies for Problems 6 and 7, and share any connections they notice to their own strategy. See the Routine Facilitation Guide for more information. **(MTR.3.1)**

ELL **English Language Learners** Consider providing a word bank to support students in expressing their ideas with words such as: variable, both sides, add, subtract, multiply, divide, first, next, etc. **(Reading and Writing)**

To surface the Key Takeaway, consider asking:

- "Sol divided by 6 in both problems. How did this move work differently in each problem?" **(MTR.3.1)**
- "For what value of s would we get the solution Sol found in Problem 6? Why?"
- "Why might we want to solve a multi-variable equation for a specific variable?"

Activity 1

Name: Date: Period:

Sol's Strategies (continued)

5. Use Sol's new equation, $s = 60 - 1.5r$, to decide how many square tables he needs if he uses 10 round tables. Show or explain your thinking.

45 square tables. *Work varies.*

$s = 60 - 1.5r$

$s = 60 - 1.5(10)$

$s = 60 - 15$

$s = 45$

6. How many round tables does Sol need if he will use 18 square tables? Show or explain your thinking.

28 round tables. *Work varies.*

$6r + 4(18) = 240$

$6r + 72 = 240$

$6r = 168$

$r = 28$

7. Solve Sol's original equation $6r + 4s = 240$ for r. Show or explain your thinking.

$r = 40 - \frac{2}{3}s$. ***Work varies.***

$6r + 4s = 240$

$6r = 240 - 4s$

$\frac{6r}{6} = \frac{240}{6} - \frac{4s}{6}$

$r = 40 - \frac{2}{3}s$

Unit 1 Lesson 5 36 Activity 1

Key Takeaway: Rearranging multi-variable equations to highlight a variable of interest can be a helpful strategy for solving problems.

Pairs | 15 min

Activity 2 Equations and Formulas

Purpose: Students develop fluency in solving multi-variable equations by using repeated reasoning and the same strategies they used to solve one-variable equations. **(MTR.3.1)**

Short on time? Consider omitting Problems 9 or 13.

1 Launch

Display the equations and invite students to share how they are alike and different. **(MTR.3.1)**

Invite students to solve Problem 8 independently, then consider modeling how to solve each equation for t.

Encourage students to work independently on Problems 9–10 before checking in with a partner.

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, encourage them to write each solving step on its own line, or provide a graphic organizer that has space for each step of each problem.

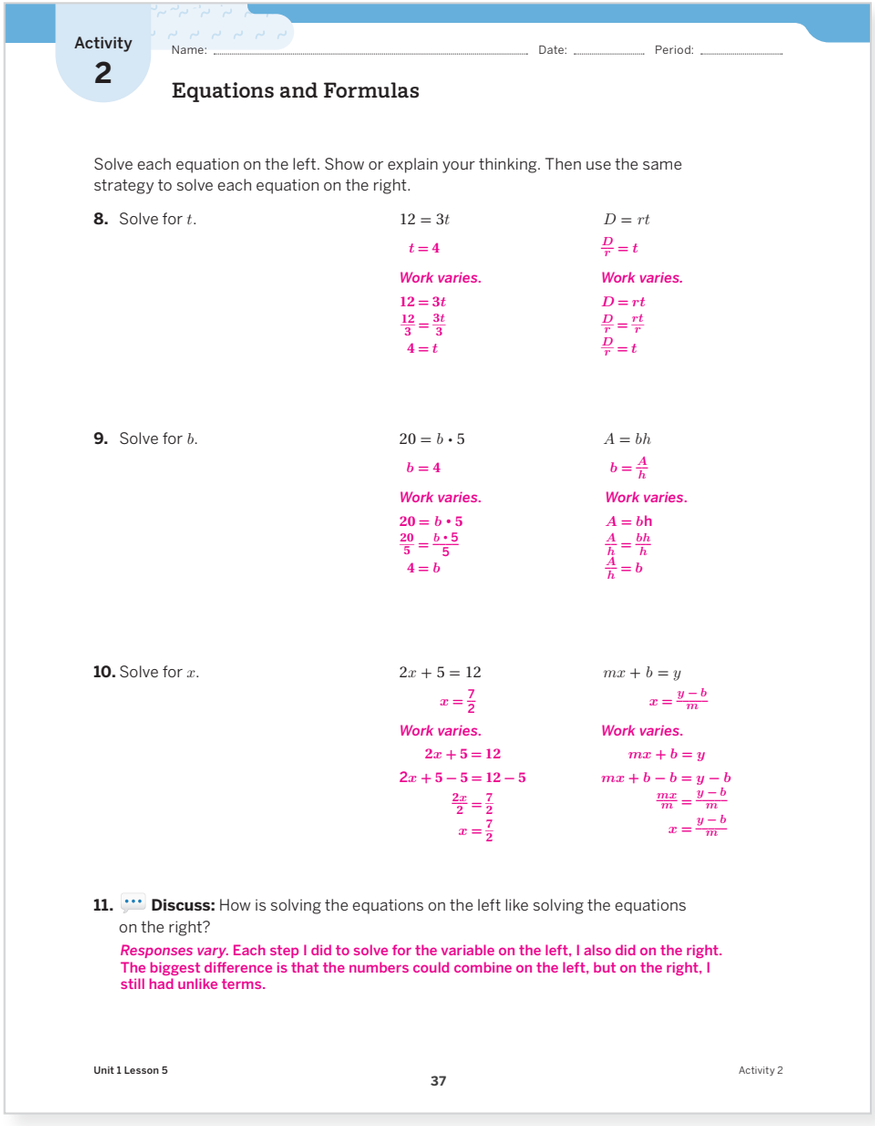
2 Monitor

To support students getting started, consider asking, "What strategies do you remember using to solve a one-variable equation?"

Encourage students to discuss with their partners whenever they get different answers and make revisions based on their conversation. **(MTR.4.1)**

ELL **English Language Learners** Pause briefly after most students have solved the first three pairs of equations to invite students to compare and connect how solving one-variable equations is similar to solving multi-variable equations. **(MTR.3.1)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Equations and Formulas

Solve each equation on the left. Show or explain your thinking. Then use the same strategy to solve each equation on the right.

8. Solve for t.

$12 = 3t$

$t = 4$

Work varies.

$12 = 3t$

$\frac{12}{3} = \frac{3t}{3}$

$4 = t$

$D = rt$

$\frac{D}{r} = t$

Work varies.

$D = rt$

$\frac{D}{r} = \frac{rt}{r}$

$\frac{D}{r} = t$

9. Solve for b.

$20 = b \cdot 5$

$b = 4$

Work varies.

$20 = b \cdot 5$

$\frac{20}{5} = \frac{b \cdot 5}{5}$

$4 = b$

$A = bh$

$b = \frac{A}{h}$

Work varies.

$A = bh$

$\frac{A}{h} = \frac{bh}{h}$

$\frac{A}{h} = b$

10. Solve for x.

$2x + 5 = 12$

$x = \frac{7}{2}$

Work varies.

$2x + 5 = 12$

$2x + 5 - 5 = 12 - 5$

$\frac{2x}{2} = \frac{7}{2}$

$x = \frac{7}{2}$

$mx + b = y$

$x = \frac{y - b}{m}$

Work varies.

$mx + b = y$

$mx + b - b = y - b$

$\frac{mx}{m} = \frac{y - b}{m}$

$x = \frac{y - b}{m}$

11. **Discuss:** How is solving the equations on the left like solving the equations on the right?

Responses vary. Each step I did to solve for the variable on the left, I also did on the right. The biggest difference is that the numbers could combine on the left, but on the right, I still had unlike terms.

Unit 1 Lesson 5 37 Activity 2

Pairs | 15 min

Activity 2 Equations and Formulas (continued)

Purpose: Students develop fluency in solving multi-variable equations by using repeated reasoning and the same strategies they used to solve one-variable equations. **(MTR.3.1)**

Short on time? Consider omitting Problems 9 or 13.

2 Monitor

D Differentiation *(Problems 12–14)*

Look for students who:	Teacher Moves
Correctly solve the equation on the left, but need support getting started solving the equation on the right.	**Support** Celebrate how students solved the left equation.Consider asking, "What was your first step? What is a similar step you could apply to the equation on the right?"
Use a move that is not an inverse operation to one in the equation (e.g., adding k as the inverse of $-k$ in Problem 12).	**Support** Consider asking, "What operation is happening between $-k$ and x?" Invite them to share what they learned in the Connect.
Use an inverse operation, but do not use the move in a way that keeps the equation balanced (e.g., multiplying both sides by g, but not distributing g to every term in Problem 13).	**Support** Celebrate the inverse operation they chose. Consider asking, "What are you multiplying by g? How can you keep the equations equivalent?" Invite them to share what they learned in the Connect.
Solve each equation correctly.	**Strengthen** Invite students to think of other multi-variable equations they have encountered in math or science and solve them for a variable of interest.

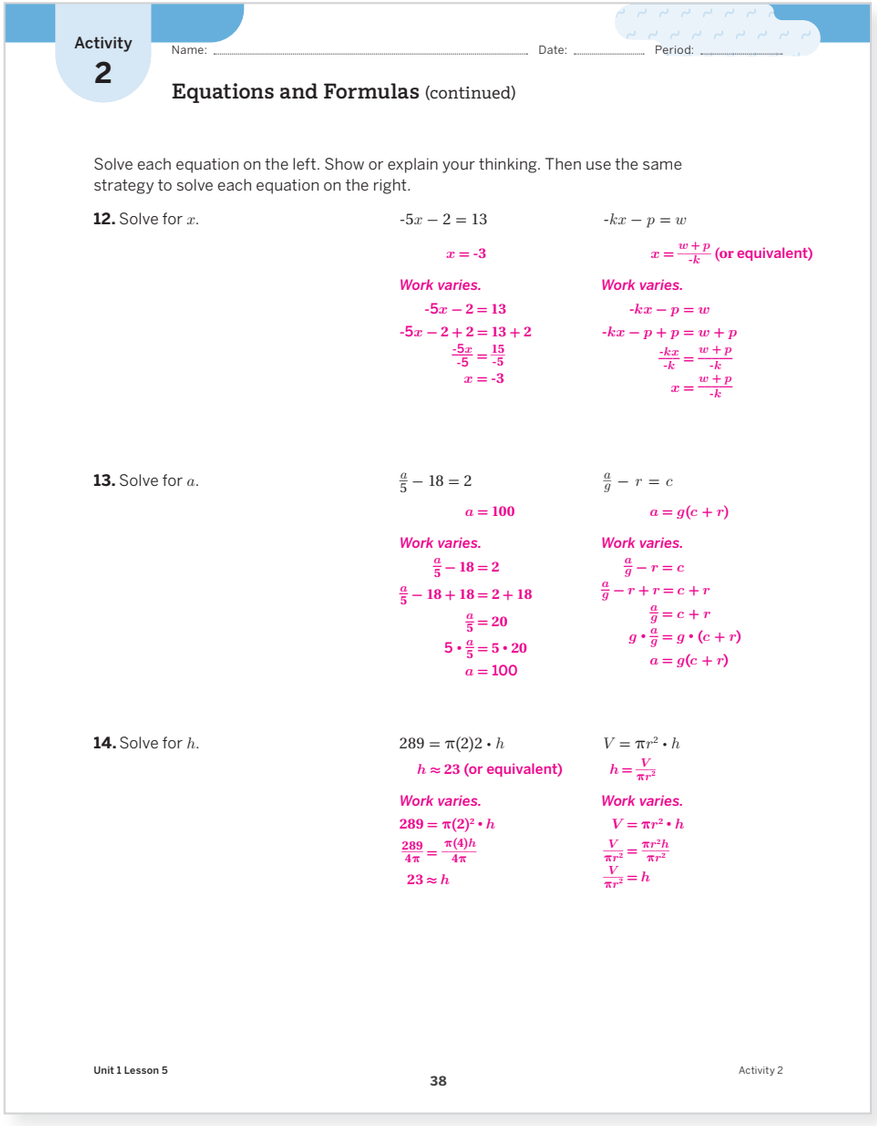
A Accessibility: Affective Functioning To support students in sharing their thinking, check in with students in advance to let them know you will ask for their thoughts.

3 Connect

Consider asking:

- "Which multi-variable equation was a challenge for you to solve? Which ones were easier? Why?"
- "Why do mathematicians and scientists often use multi-variable equations to communicate relationships?"
- "When is it helpful to use a multi-variable equation to communicate a relationship? When is it helpful to use a one-variable equation?"

ELL English Language Learners Encourage students to paraphrase each other's ideas to help make connections and use vocabulary from the unit. **(Speaking and Listening)**



Activity 2

Name: Date: Period:

Equations and Formulas (continued)

Solve each equation on the left. Show or explain your thinking. Then use the same strategy to solve each equation on the right.

12. Solve for x.

$-5x - 2 = 13$

$x = -3$

Work varies.

$-5x - 2 = 13$

$-5x - 2 + 2 = 13 + 2$

$\frac{-5x}{-5} = \frac{15}{-5}$

$x = -3$

$-kx - p = w$

$x = \frac{w+p}{-k}$ (or equivalent)

Work varies.

$-kx - p = w$

$-kx - p + p = w + p$

$\frac{-kx}{-k} = \frac{w+p}{-k}$

$x = \frac{w+p}{-k}$

13. Solve for a.

$\frac{a}{5} - 18 = 2$

$a = 100$

Work varies.

$\frac{a}{5} - 18 = 2$

$\frac{a}{5} - 18 + 18 = 2 + 18$

$\frac{a}{5} = 20$

$5 \cdot \frac{a}{5} = 5 \cdot 20$

$a = 100$

$\frac{a}{g} - r = c$

$a = g(c + r)$

Work varies.

$\frac{a}{g} - r = c$

$\frac{a}{g} - r + r = c + r$

$\frac{a}{g} = c + r$

$g \cdot \frac{a}{g} = g \cdot (c + r)$

$a = g(c + r)$

14. Solve for h.

$289 = \pi(2)2 \cdot h$

$h \approx 23$ (or equivalent)

Work varies.

$289 = \pi(2)^2 \cdot h$

$\frac{289}{4\pi} = \frac{\pi(4)h}{4\pi}$

$23 \approx h$

$V = \pi r^2 \cdot h$

$h = \frac{V}{\pi r^2}$

Work varies.

$V = \pi r^2 \cdot h$

$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$

$\frac{V}{\pi r^2} = h$

Unit 1 Lesson 5 38 Activity 2

Whole Class | 5 min

Synthesis

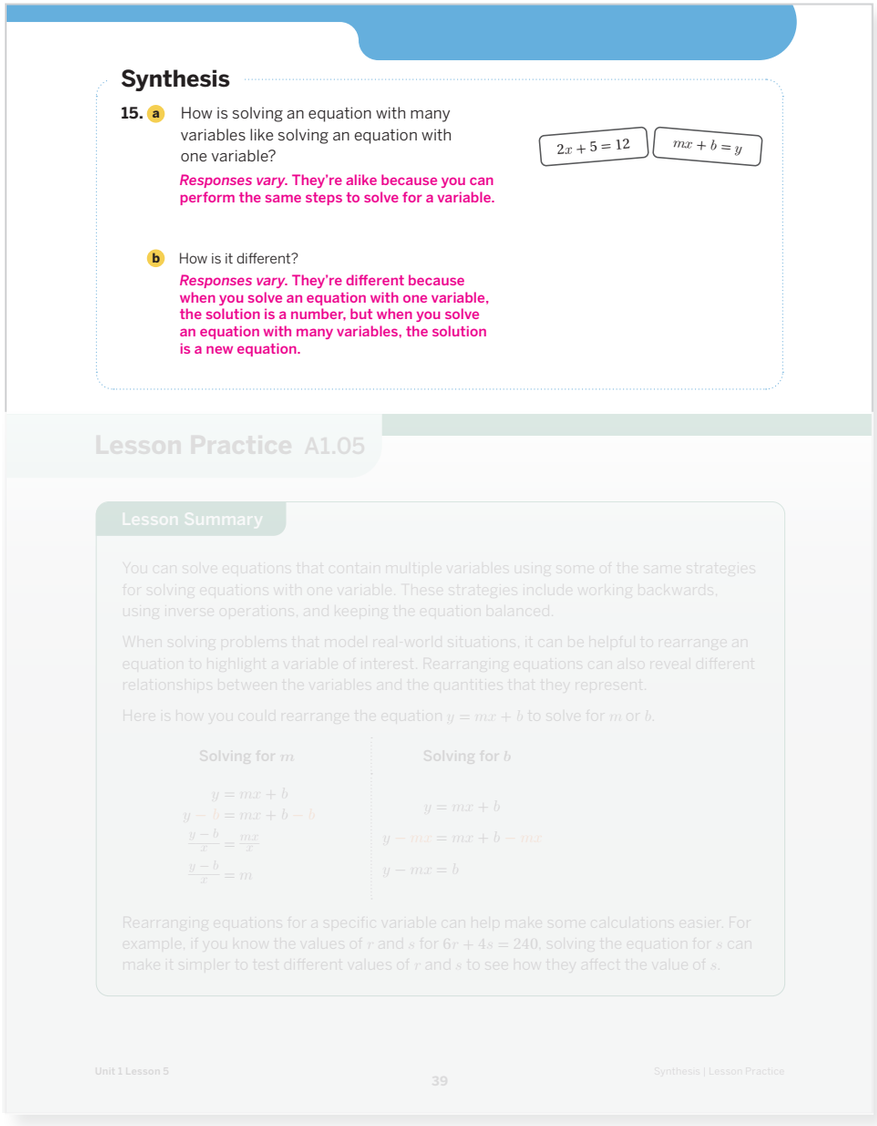
Purpose: Students synthesize their understanding of the strategies used for solving equations and consider how solving one-variable and multi-variable equations are similar and different.

Encourage students to respond independently, and then share their thinking with a partner. **(MTR.4.1)**

Invite students to share similarities and differences between solving one-variable and multi-variable equations, such as:

- Using moves that keep an equation balanced.
- Applying inverse operations.
- Working backwards.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

15. a How is solving an equation with many variables like solving an equation with one variable?

$2x + 5 = 12$ $mx + b = y$

Responses vary. **They're alike because you can perform the same steps to solve for a variable.**

b How is it different?

Responses vary. **They're different because when you solve an equation with one variable, the solution is a number, but when you solve an equation with many variables, the solution is a new equation.**

Lesson Practice A1.05

Lesson Summary

You can solve equations that contain multiple variables using some of the same strategies for solving equations with one variable. These strategies include working backwards, using inverse operations, and keeping the equation balanced.

When solving problems that model real-world situations, it can be helpful to rearrange an equation to highlight a variable of interest. Rearranging equations can also reveal different relationships between the variables and the quantities that they represent.

Here is how you could rearrange the equation $y = mx + b$ to solve for m or b.

Solving for m

$y = mx + b$

$y - b = mx + b - b$

$\frac{y-b}{x} = \frac{mx}{x}$

$\frac{y-b}{x} = m$

Solving for b

$y = mx + b$

$y - mx = mx + b - mx$

$y - mx = b$

Rearranging equations for a specific variable can help make some calculations easier. For example, if you know the values of r and s for $6r + 4s = 240$, solving the equation for s can make it simpler to test different values of r and s to see how they affect the value of s.

Unit 1 Lesson 5 39 Synthesis | Lesson Practice

Lesson Takeaway: The strategies used for solving a one-variable equation can be used to solve for a variable of interest in a multi-variable equation. Isolating different variables in multi-variable equations can help to surface connections for solving problems in context.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving for a variable of interest in a one-variable equation and in a multi-variable equation.

Today's Goals

1. **Goal:** Rearrange multi-variable equations to highlight a variable of interest.
2. **Goal:** Solve multi-variable equations for a variable of interest.
3. **Language Goal:** Explain how solving for a variable is similar to rearranging an equation to isolate a variable of interest. **(Speaking and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.05

Solve each equation for x. Show or explain your thinking.

a $2x + 8(3) = 16$

$x = -4$

Work varies.

$2x + 8(3) = 16$

$2x + 24 = 16$

$2x + 24 - 24 = 16 - 24$

$\frac{2x}{2} = \frac{8}{2}$

$x = -4$

b $2x + 8y = 16$

$x = 8 - 4y$ **(or equivalent)**

Work varies.

$2x + 8y = 16$

$2x + 8y - 8y = 16 - 8y$

$\frac{2x}{2} = \frac{16 - 8y}{2}$

$x = 8 - 4y$

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support solving multi-variable equations:

» **Respond to Student Thinking:**

- Use the ***Solving Multi-Variable Linear Equations*** **Mini-Lesson**
- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

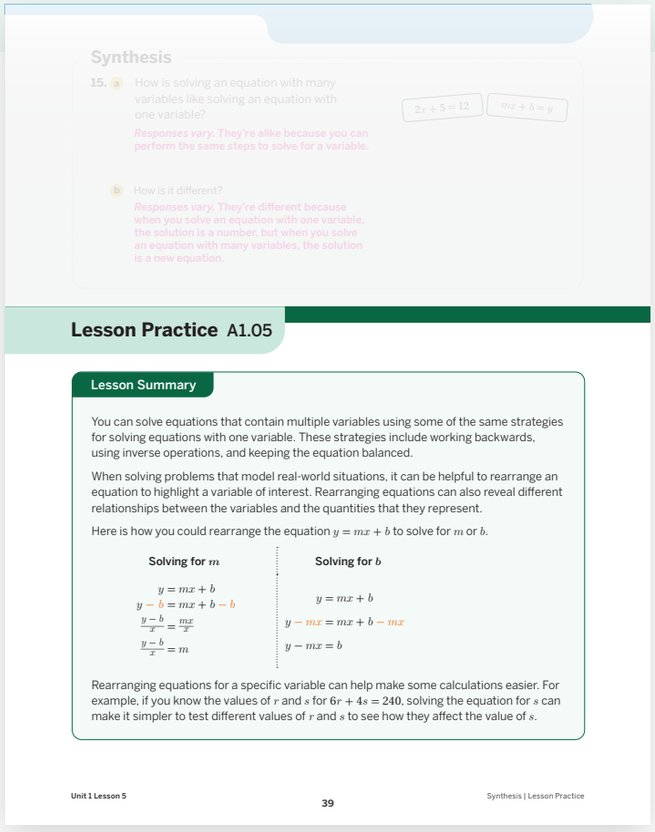
Professional Learning

Reflect on the facilitation of the **Think-Pair-Share** routine used in the Warm-Up of this lesson. Students will use this routine again in Sub-Unit 3. Who shared in the whole class discussion? Who did not? What about the routine felt successful? What are some areas for growth next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

15. a. How is solving an equation with many variables like solving an equation with one variable?

$2x + 5 = 12$ $mx + b = y$

Responses vary. They're alike because you can perform the same steps to solve for a variable.

b. How is it different?

Responses vary. They're different because when you solve an equation with one variable, the solution is a number, but when you solve an equation with many variables, the solution is a new equation.

Lesson Practice A1.05

Lesson Summary

You can solve equations that contain multiple variables using some of the same strategies for solving equations with one variable. These strategies include working backwards, using inverse operations, and keeping the equation balanced.

When solving problems that model real-world situations, it can be helpful to rearrange an equation to highlight a variable of interest. Rearranging equations can also reveal different relationships between the variables and the quantities that they represent.

Here is how you could rearrange the equation $y = mx + b$ to solve for m or b.

Solving for m

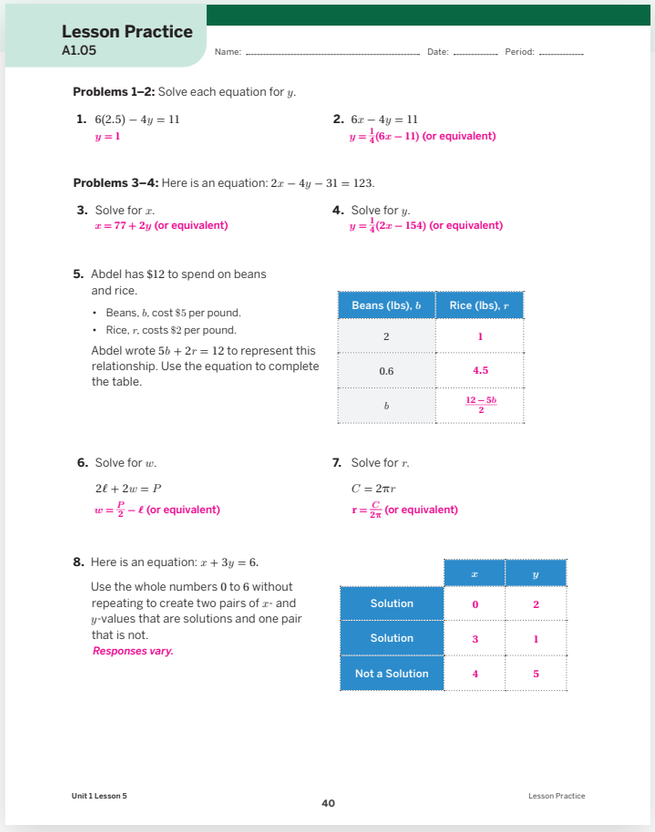
$$y = mx + b$$
$$y - b = mx + b - b$$
$$\frac{y-b}{x} = \frac{mx}{x}$$
$$\frac{y-b}{x} = m$$

Solving for b

$$y = mx + b$$
$$y - mx = mx + b - mx$$
$$y - mx = b$$

Rearranging equations for a specific variable can help make some calculations easier. For example, if you know the values of r and s for $6r + 4s = 240$, solving the equation for s can make it simpler to test different values of r and s to see how they affect the value of s.

Unit 1 Lesson 5 39 Synthesis | Lesson Practice



Lesson Practice
A1.05

Name: Date: Period:

Problems 1–2: Solve each equation for y.

1. $6(2.5) - 4y = 11$
   $y = 1$

2. $6x - 4y = 11$
   $y = \frac{1}{4}(6x - 11)$ (or equivalent)

Problems 3–4: Here is an equation: $2x - 4y - 31 = 123$.

3. Solve for x.
   $x = 77 + 2y$ (or equivalent)

4. Solve for y.
   $y = \frac{1}{4}(2x - 154)$ (or equivalent)

5. Abdel has $12 to spend on beans and rice.
   - Beans, b, cost $5 per pound.
   - Rice, r, costs $2 per pound.

   Abdel wrote $5b + 2r = 12$ to represent this relationship. Use the equation to complete the table.

Beans (lbs), b	Rice (lbs), r
2	1
0.6	4.5
b	$\frac{12-5b}{2}$

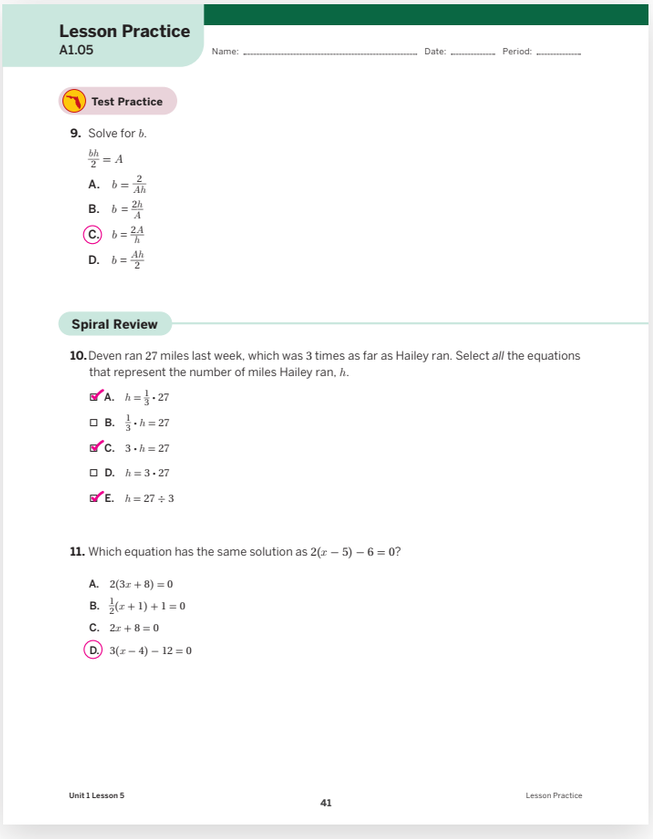
6. Solve for w.
   $2\ell + 2w = P$
   $w = \frac{P}{2} - \ell$ (or equivalent)

7. Solve for r.
   $C = 2\pi r$
   $r = \frac{C}{2\pi}$ (or equivalent)

8. Here is an equation: $x + 3y = 6$.
   Use the whole numbers 0 to 6 without repeating to create two pairs of x- and y-values that are solutions and one pair that is not.
   Responses vary.

	x	y
Solution	0	2
Solution	3	1
Not a Solution	4	5

Unit 1 Lesson 5 40 Lesson Practice



Lesson Practice
A1.05

Name: Date: Period:

Test Practice

9. Solve for b.
   $\frac{bh}{2} = A$
   A. $b = \frac{2}{Ah}$
   B. $b = \frac{2h}{A}$
   C. $b = \frac{2A}{h}$
   D. $b = \frac{Ah}{2}$

Spiral Review

10. Deven ran 27 miles last week, which was 3 times as far as Hailey ran. Select *all* the equations that represent the number of miles Hailey ran, h.
   ☑ A. $h = \frac{1}{3} \cdot 27$
   ☐ B. $\frac{1}{3} \cdot h = 27$
   ☑ C. $3 \cdot h = 27$
   ☐ D. $h = 3 \cdot 27$
   ☑ E. $h = 27 \div 3$

11. Which equation has the same solution as $2(x - 5) - 6 = 0$?
   A. $2(3x + 8) = 0$
   B. $\frac{1}{2}(x + 1) + 1 = 0$
   C. $2x + 8 = 0$
   D. $3(x - 4) - 12 = 0$

Unit 1 Lesson 5 41 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.AR.2.2
	3–5	2	MA.912.AR.2.2
	6, 7	2	MA.912.AR.1.2
	8	3	MA.912.AR.2.2
Test Practice	9	2	MA.912.AR.1.2
Spiral Review			
	10	2	MA.7.AR.2.1
	11	2	MA.8.AR.2.1

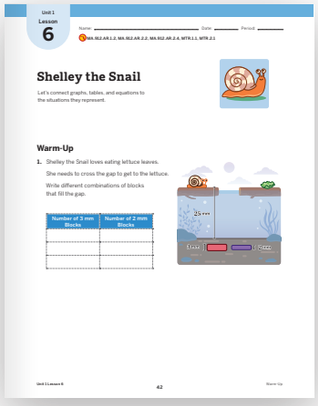
Unit 1

Lesson 6

Shelley the Snail

Connecting Graphs and Linear Equations

Let's connect graphs, tables, and equations to the situations they represent.


Unit 1
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Shelley the Snail
Let's connect graphs, tables, and equations to the situations they represent.
Warm-Up
1. Shelley the Snail loves eating lettuce leaves.
She needs to cross the gap to get to the lettuce.
Write different combinations of blocks that fill the gap.
Number of 3 mm Blocks
Number of 2 mm Blocks
25 mm
42

Learning Outcomes

Today's Goals

1. **Goal:** Understand that the graph of a linear equation represents all the pairs of values that are solutions to the equation.
2. **Goal:** Solve an equation in standard form for one of its variables and connect the new equation to its graph.
3. **Language Goal:** Discuss how the graph of a linear equation can be written in different but equivalent forms. **(Speaking and Listening)**

Students revisit the concept that situations can be represented graphically and that each point on a graph is a solution to an equation the graph represents. Students make sense of problems when explaining connections between equations, situations, and graphs. **(MTR.1.1)**

Coherence

Prior Learning

In Grade 8, students were introduced to situations that can be modeled by linear equations and made connections to their graphs. In Lesson 5, students were introduced to the idea that equations modeling the same situation can be written in different and equivalent forms.

Future Learning

In Lesson 7, students will be introduced to the terms *standard form* and *slope-intercept form* as two forms of a linear equation, and put together everything they have learned about representing situations with two variables.

Connections

- Students use linear relationships to determine the number of blocks needed to close a gap between a starting point and a destination, as well as the number of school supplies that can be purchased with a set amount of money. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.2

Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

MA.912.AR.2.4

Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.

Also Addressing: **MA.912.AR.1.2, MA.912.AR.2.5, MA.912.AR.2.6**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1

ELA Expectations: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
8.EE.B	MA.912.AR.9.8
MA.8.AR.2.1	MA.912.AR.9.9
MA.8.AR.3.3	MA.912.AR.9.10
MA.8.AR.3.4	

Vocabulary

New Vocabulary

x-intercept

y-intercept

Review Vocabulary

linear relationship

Lesson at a Glance

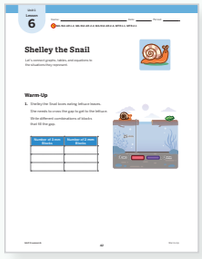
~ 45 min

MA.912.AR.2.2, MA.912.AR.2.4, MA.912.AR.1.2, MA.912.AR.2.5, MA.912.AR.2.6

Warm-Up

Pairs | 5 min

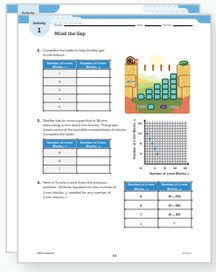
Students make sense of the situation by identifying multiple combinations of blocks that can help Shelley the Snail cross a gap. **(MTR.1.1)**



Activity 1

Pairs | 15 min

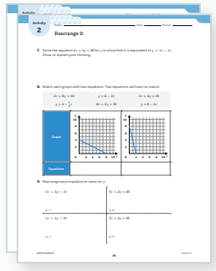
Students represent the solutions of a situation using a table, a graph, and multiple forms of an equation, and describe the connections between these representations. **(MTR.2.1)**



Activity 2

Pairs | 15 min

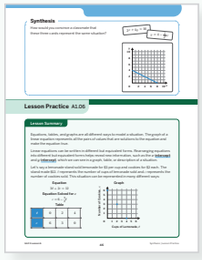
Students match equivalent representations and create equivalent equations by solving for a variable. MLR3: Critique, Correct, Clarify



Synthesis

Whole Class | 5 min

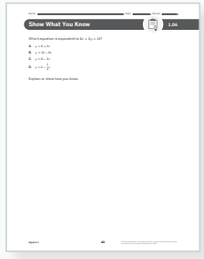
Students synthesize their understanding of the connection between a situation, a graph, and two different forms of an equation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying a different but equivalent form of an equation and justifying their thinking.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students identify multiple combinations of blocks that can help Shelley the Snail cross a gap. **(MTR.1.1)**

1 Launch

Consider asking:

- "How can Shelley get to the lettuce?"
- "What might happen if we do not fill the gap?"

ELL **English Language Learners** Encourage students to label the diagram in their Student Edition with words that help them connect it to the table, such as *snail*, *gap*, and *blocks*.

2 Connect

Draw students attention to the Warm-Up problem. Invite students to share a combination of blocks that they know works to help Shelley the Snail cross the gap.

Consider asking:

- "Is there more than one combination of blocks that will help Shelley get to the lettuce?"
- "Is it possible to fill the gap with only 4-millimeter blocks? Only 2-millimeter blocks?"
- "What's a combination of blocks that you know won't work? How do you know?"

D **Differentiation:** (Stretch) Invite students to find as many combinations as they can and to explain how they know when they've found all of the possible combinations.

Unit 1
Lesson
6

Name: _____ Date: _____ Period: _____

MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MTR.1.1, MTR.2.1

Shelley the Snail



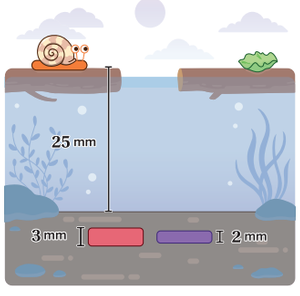
Let's connect graphs, tables, and equations to the situations they represent.

Warm-Up

1. Shelley the Snail loves eating lettuce leaves.

She needs to cross the gap to get to the lettuce.

Write different combinations of blocks that fill the gap.



Responses vary.

Number of 3 mm Blocks	Number of 2 mm Blocks
5	5
7	2
1	11

Unit 1 Lesson 6 42 Warm-Up

Pairs | 15 min

Activity 1 Mind the Gap

Purpose: Students represent the solutions of a situation using a table, a graph, and multiple forms of an equation, and describe the connections between these representations. **(MTR.2.1)**

1 Launch

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to complete one row of the table at a time and confirm their entries with a partner, revising as needed. **(MTR.3.1)**

2 Monitor

Look for different strategies that students use such as guessing and checking, comparing the sizes of the two types of blocks, using repeated addition, or working backwards.

Encourage students to use the feedback from sharing their reasoning with a partner, and revise as needed. **(MTR.3.1)**

Consider asking, "What patterns do you see in the table? What does that mean for this situation?" **(MTR.2.1)**

To support students getting started, consider selecting a point on the graph and asking, "What does this point on the graph represent in Shelley the Snail's situation?" **(MTR.2.1)**

A **Accessibility: Visual-Spatial Processing** Invite students to use the diagram on the previous Problem to help them make sense of the problem.

Pause briefly to discuss:

- "Why are there only integer values plotted?"
- "Does it make sense to connect the points in this situation? Why or why not?"
- "Is it possible to have points with decimal values for this situation? Why or why not?"

To support students getting started, consider asking, "What connections do you see between the two table columns?"

Consider asking:

- "How do the parts of the equation you wrote match the situation?"
- "How can writing an equation to represent this situation be helpful?"

Activity 1 continued >

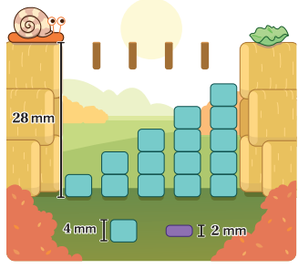
Activity 1

Name: Date: Period:

Mind the Gap

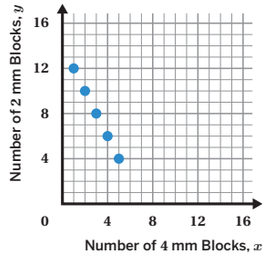
2. Complete the table to help Shelley get to the lettuce.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
1	12
2	10
3	8
4	6
5	4



3. Shelley has to cross a gap that is 28 mm deep using 4 mm and 2 mm blocks. This graph shows some of the possible combinations of blocks. Complete the table.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14
6	2
7	0



4. Here is Tyrone's work from the previous problem. Write an equation for the number of 2 mm blocks, y, needed for any number of 4 mm blocks, x.

$y = 14 - 2x$ **(or equivalent)**

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14 − 2(0)
6	14 − 2(6)
7	14 − 2(7)
x	?

Unit 1 Lesson 6 43 Activity 1

Pairs | 15 min

Activity 1 Mind the Gap (continued)

Purpose: Students represent solutions to a situation using a table, a graph, and multiple forms of an equation and describe connections between the representations. **(MTR.2.1)**

2 Monitor

Invite students to talk with a partner who chose a different representation and compare where they see intercepts.

Encourage students to show their thinking.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started.	**Support** Consider chunking this Problem into more manageable parts by asking, "Where do you see the 14 in the graph? In the diagram?"
Describe how the combinations of blocks are solutions to the equations.	**Strengthen** Capture a variety of student sketches that show connections to share during the Connect. Include things that surprise you or that other students may not have noticed.
Connect the stack of 14 blocks in the diagram to the 14 in the equation.	
Notice that there is a y-intercept of $(0, 14)$ on the graph and in the equation $y = 14 - 2x$.	
See that as the number of 4-millimeter blocks increases by 1, the number of 2-millimeter blocks decreases by 2.	

3 Connect

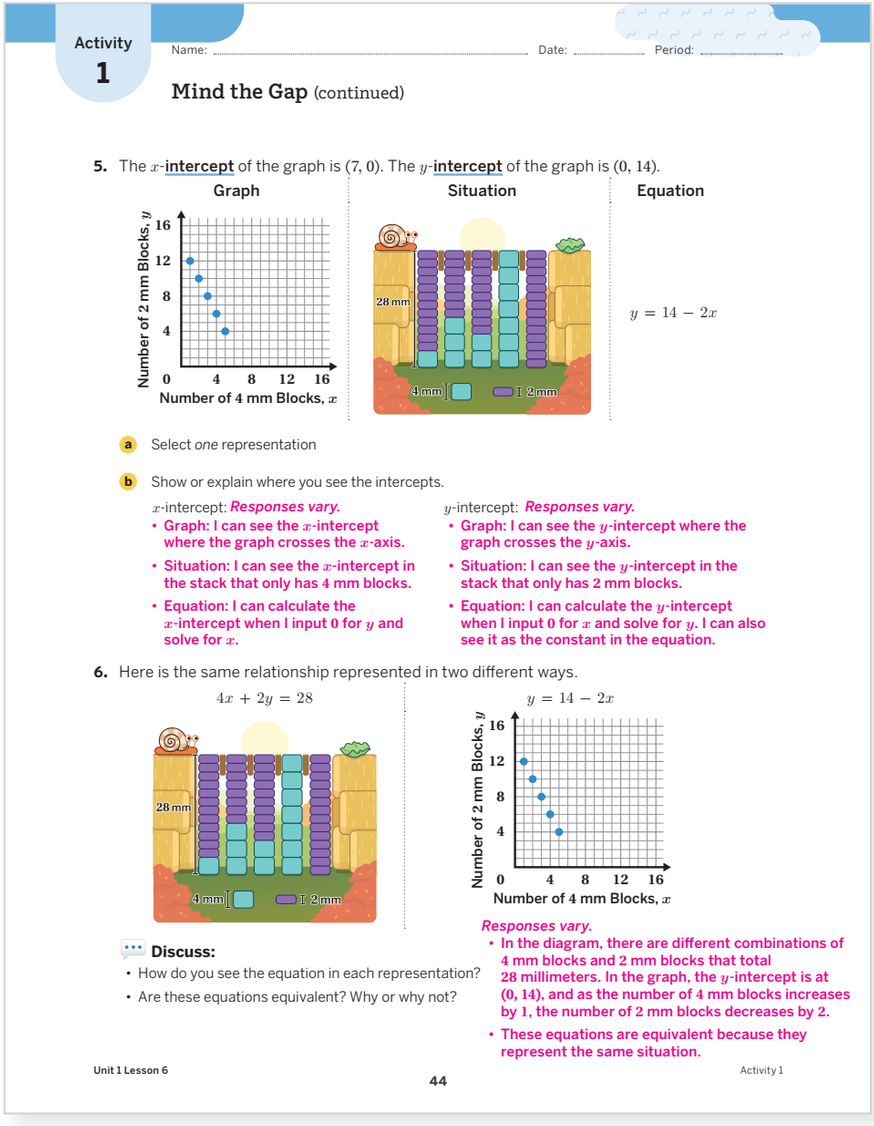
Create a class definition for the terms **x-intercept** and **y-intercept**.

- x-intercept: the point where the graph crosses the x-axis or when $y = 0$.
- y-intercept: the point where the graph crosses the y-axis or when $x = 0$.

Share and discuss the captured student sketches.

To surface the Key Takeaway, consider asking:

- "How can you tell these equations match the situation? The graph? The diagram?" **(MTR.2.1)**
- "Where can you see the $4x$, $2y$, 28, 14, and $-2x$ in the diagram or graph? **(MTR.2.1)**
- "How can you tell whether these two equations are equivalent?" *Responses vary.* The equations are equivalent because they represent the same situation.



Activity 1

Name: Date: Period:

Mind the Gap (continued)

5. The x-**intercept** of the graph is $(7, 0)$. The y-**intercept** of the graph is $(0, 14)$.

Graph | Situation | Equation

Number of 2 mm Blocks, y (16, 12, 8, 4; 0, 4, 8, 12, 16)

Number of 4 mm Blocks, x

28 mm; 4 mm; 2 mm

$y = 14 - 2x$

a. Select *one* representation

b. Show or explain where you see the intercepts.

x-intercept: *Responses vary.*

- Graph: I can see the x-intercept where the graph crosses the x-axis.
- Situation: I can see the x-intercept in the stack that only has 4 mm blocks.
- Equation: I can calculate the x-intercept when I input 0 for y and solve for x.

y-intercept: *Responses vary.*

- Graph: I can see the y-intercept where the graph crosses the y-axis.
- Situation: I can see the y-intercept in the stack that only has 2 mm blocks.
- Equation: I can calculate the y-intercept when I input 0 for x and solve for y. I can also see it as the constant in the equation.

6. Here is the same relationship represented in two different ways.

$4x + 2y = 28$

$y = 14 - 2x$

Discuss:

- How do you see the equation in each representation?
- Are these equations equivalent? Why or why not?

Responses vary.

- In the diagram, there are different combinations of 4 mm blocks and 2 mm blocks that total 28 millimeters. In the graph, the y-intercept is at (0, 14), and as the number of 4 mm blocks increases by 1, the number of 2 mm blocks decreases by 2.
- These equations are equivalent because they represent the same situation.

Unit 1 Lesson 6 44 Activity 1

Key Takeaway: The graph of a linear equation represents all the pairs of values that are solutions to the equation and make the equation true. There are different, equivalent forms of linear equations, and each form helps us understand different parts of the graph, table, and situation.

Pairs | 15 min

Activity 2 Rearrange It

Purpose: Students match equivalent representations and create equivalent equations by solving for a variable.

Short on time? Consider limiting the number of challenges on Problem 9.

1 Launch

Consider sharing that because we know both equations represent the same situation, our new goal is to show they are equivalent algebraically.

Consider asking, "What could be a possible first step?"

2 Monitor

Encourage students to start with representations that they feel the most confident with.

Invite students to compare their matches, justify their decisions, and make revisions based on their conversation.

Note: monitor student pair discussions to gain insight as to how they match equations with graphs, offering support as needed.

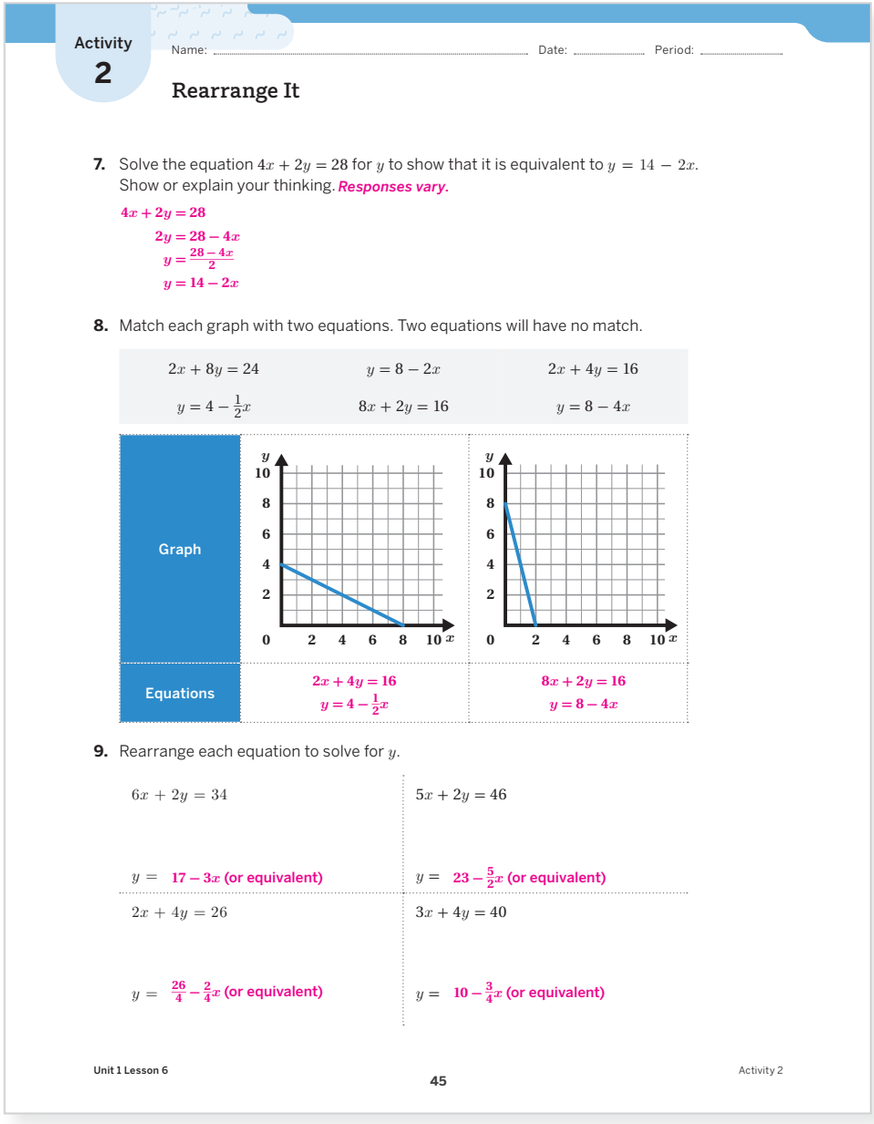
A **Accessibility: Executive Functioning** To support initiating complex tasks, identify one or two matches to help students have a successful start.

Invite students to set their own goal for how many challenges they would like to complete, sharing that sense-making is more important than speed.

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Invite students to analyze each other's work by asking, "What has this student done well? What would you recommend as a next step to this student?" **(MTR.3.1)**

Invite students to share strategies that they found helpful or mistakes they made while solving these challenges.



Activity 2

Name: Date: Period:

Rearrange It

7. Solve the equation $4x + 2y = 28$ for y to show that it is equivalent to $y = 14 - 2x$. Show or explain your thinking. *Responses vary.*

$4x + 2y = 28$
$2y = 28 - 4x$
$y = \frac{28 - 4x}{2}$
$y = 14 - 2x$

8. Match each graph with two equations. Two equations will have no match.

$2x + 8y = 24$	$y = 8 - 2x$	$2x + 4y = 16$
$y = 4 - \frac{1}{2}x$	$8x + 2y = 16$	$y = 8 - 4x$

Graph		
Equations	$2x + 4y = 16$ $y = 4 - \frac{1}{2}x$	$8x + 2y = 16$ $y = 8 - 4x$

9. Rearrange each equation to solve for y.

$6x + 2y = 34$	$5x + 2y = 46$
$y =$ $17 - 3x$ (or equivalent)	$y =$ $23 - \frac{5}{2}x$ (or equivalent)
$2x + 4y = 26$	$3x + 4y = 40$
$y =$ $\frac{26}{4} - \frac{2}{4}x$ (or equivalent)	$y =$ $10 - \frac{3}{4}x$ (or equivalent)

Unit 1 Lesson 6 45 Activity 2

Key Takeaway: Rearranging equations into different equivalent forms helps reveal new information that we can see in a graph, situation, or table.

Whole Class | 5 min

Synthesis

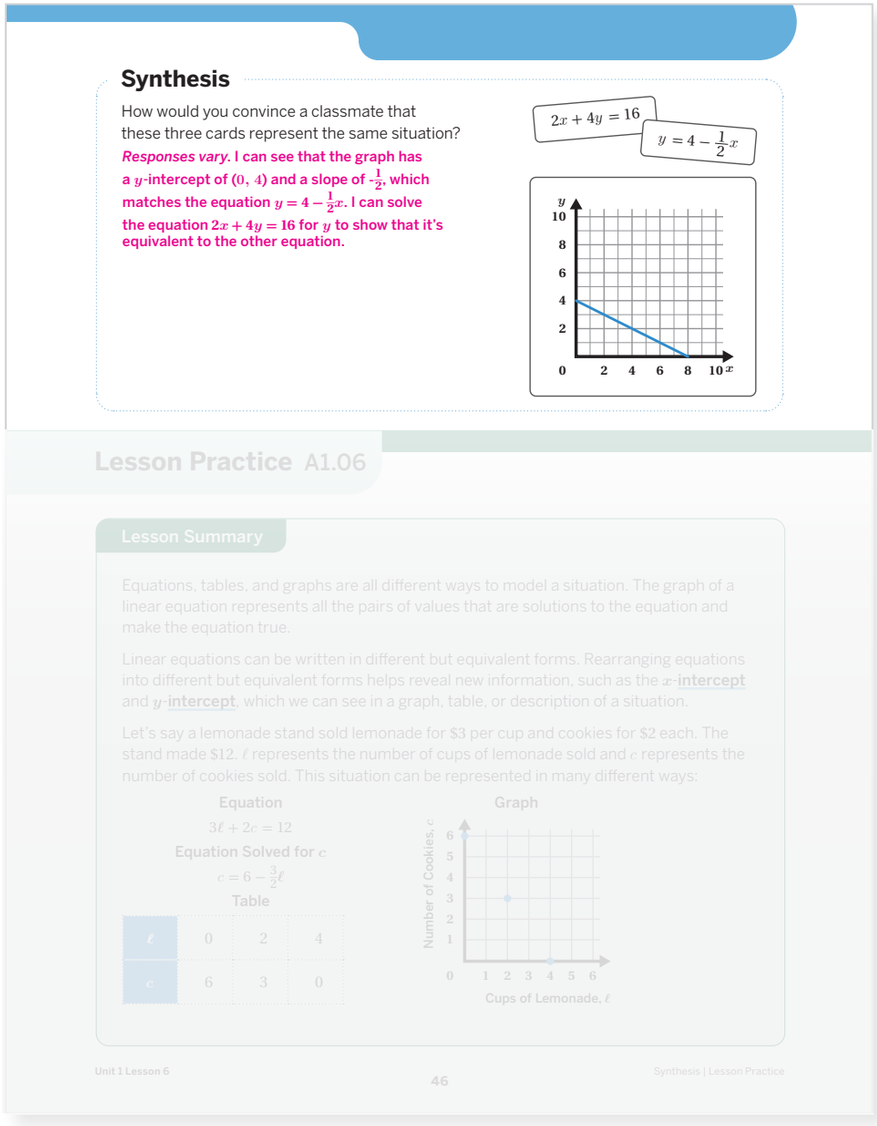
Purpose: Students synthesize their understanding of the connection between a situation, a graph, and two different forms of an equation.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, including:

- How each equation matches the graph.
- How to see the x- or y-intercept in the graph and equations.
- How to rearrange one equation to create the other.
- How the points on the graph are solutions that make both equations true.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

How would you convince a classmate that these three cards represent the same situation?

Responses vary. I can see that the graph has a y-intercept of $(0, 4)$ and a slope of $-\frac{1}{2}$, which matches the equation $y = 4 - \frac{1}{2}x$. I can solve the equation $2x + 4y = 16$ for y to show that it's equivalent to the other equation.

$2x + 4y = 16$

$y = 4 - \frac{1}{2}x$

Lesson Practice A1.06

Lesson Summary

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation and make the equation true.

Linear equations can be written in different but equivalent forms. Rearranging equations into different but equivalent forms helps reveal new information, such as the x-intercept and y-intercept, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for $3 per cup and cookies for $2 each. The stand made $12. ℓ represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in many different ways:

Equation

$3\ell + 2c = 12$

Equation Solved for c

$c = 6 - \frac{3}{2}\ell$

Table

ℓ	0	2	4
c	6	3	0

Graph

Unit 1 Lesson 6 46 Synthesis | Lesson Practice

Lesson Takeaway: Linear equations can be written in different, but equivalent forms. Each form reveals different properties in the graph, table, and situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying a different but equivalent form of an equation and justifying their thinking.

Today's Goals

1. **Goal:** Understand that the graph of a linear equation represents all the pairs of values that are solutions to the equation.
2. **Goal:** Solve an equation in standard form for one of its variables and connect the new equation to its graph.
3. **Language Goal:** Discuss how the graph of a linear equation can be written in different but equivalent forms. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.06

Which equation is equivalent to $6x + 2y = 12$?

A. $y = 6 + 3x$

B. $y = 12 - 6x$

C. $y = 6 - 3x$

D. $y = 2 - \frac{1}{3}x$

Explain or show how you know.

Explanations vary. I subtracted $6x$ from both sides and got $2y = 12 - 6x$. Then I divided everything by 2.

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support identifying an equivalent but different form of a mult-variable equation:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problems 7 and 9** and invite students to share their strategies.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write their own equations to rearrange for a specific variable, then trade with a partner to rearrange. Consider challenging students to write an equation that they think would be challenging to rearrange for a specific variable and to explain why.

Professional Learning

Reflect on the connections students made between different representations of two-variable linear relationships. Students will continue making connections between tables, graphs, equations, and situations in Lesson 7. Which representations did your students describe clear connections between? Which representations did students struggle to connect? How did students show and describe connections?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

How would you convince a classmate that these three cards represent the same situation?

Responses vary. I can see that the graph has a y-intercept of $(0, 4)$ and a slope of $-\frac{1}{2}$, which matches the equation $y = 4 - \frac{1}{2}x$. I can solve the equation $2x + 4y = 16$ for y to show that it's equivalent to the other equation.

$2x + 4y = 16$

$y = 4 - \frac{1}{2}x$

Lesson Practice A1.06

Lesson Summary

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation and make the equation true.

Linear equations can be written in different but equivalent forms. Rearranging equations into different but equivalent forms helps reveal new information, such as the x-**intercept** and y-**intercept**, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for $3 per cup and cookies for $2 each. The stand made $12. ℓ represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in many different ways:

Equation

$3\ell + 2c = 12$

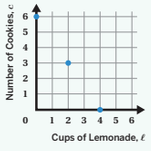
Equation Solved for c

$c = 6 - \frac{3}{2}\ell$

Table

ℓ	0	2	4
c	6	3	0

Graph



Lesson Practice A1.06

Name: ..... Date: ..... Period: .....

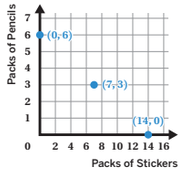
Problems 1–2: Here is a graph about school supplies.

1. A teacher spent $21 on packs of stickers and packs of pencils for her class.

- Stickers cost $1.50 per pack.
- Pencils cost $3.50 per pack.

Show or explain how you know that this graph represents this situation.

Responses vary. I know this graph represents this situation because each coordinate pair corresponds to a total of $21. When I multiply the x-value by 1.50 and the y-value by 3.50, the total is always 21.



2. Circle a coordinate pair and explain what it means in this situation.

(0, 6) (7, 3) (14, 0)

Responses vary.

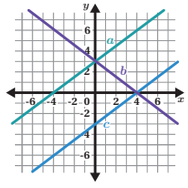
- (0, 6) means that the teacher bought 0 packs of stickers and 6 packs of pencils for $21.
- (7, 3) means that the teacher bought 7 packs of stickers and 3 packs of pencils for $21.
- (14, 0) means that the teacher bought 14 packs of stickers and 0 packs of pencils for $21.

3. Circle the line that represents $12 = 3x + 4y$.

Line a (Line b) Line c

Show or explain how you know.

Explanations vary. When I solve $12 = 3x + 4y$ for y, I get $y = 3 - \frac{3}{4}x$. Line b is the only choice that starts at 3 on the y-axis and has a negative slope.



4. Which equation is equivalent to $15x + 3y = 2$?

A. $y = \frac{2}{3} + 5x$ (B.) $y = \frac{2}{3} - 5x$ C. $y = 2 - 15x$ D. $y = 2 - 5x$

5. Match each equation with its equivalent equation.

a $4x + 6y = 20$ b $3x - 6y = 16$ c $2x - 3y = 10$ d $-3x + 6y = 16$

d $y = \frac{8}{3} + \frac{1}{2}x$ c $y = -\frac{10}{3} + \frac{2}{3}x$ a $y = \frac{10}{3} - \frac{2}{3}x$ b $y = \frac{8}{3} + \frac{1}{2}x$

Lesson Practice A1.06

Name: ..... Date: ..... Period: .....

6. Makayla is trying to predict the price of her next taxi ride.

She recorded her last three rides in the table.

How much can she expect to pay to travel 10 miles?

$38

Distance (mi)	Price ($)
2	10
4	17
7	27.50

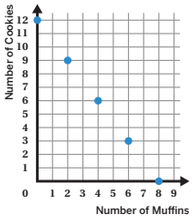
Test Practice

7. Jordan had a bake sale. Muffins were $3 each and cookies were $2 each. Jordan earned $24.

Here is a graph of Jordan's situation.

Select *all* the combinations of muffins and cookies Jordan could have sold.

- ☐ A. 0 muffins and 8 cookies
- ☐ B. 9 muffins and 2 cookies
- ☑ C. 2 muffins and 9 cookies
- ☐ D. 6 muffins and 4 cookies
- ☑ E. 4 muffins and 6 cookies



Spiral Review

8. Select *all* the equations where $x = -2$ is a solution.

- ☐ A. $4x = 4 + 2x$
- ☑ B. $2(x + 5) = x + 8$
- ☐ C. $3x - 5 = 1$
- ☐ D. $19 = 2(x - 6) + 3$
- ☑ E. $5 + 3x = -1$

9. Solve $-3x + 4y = 28$ for y.

$y = \frac{3}{4}x + 7$ (or equivalent)

10. Solve $6x - 3y = 36$ for y.

$y = -12 + 2x$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 6	2	MA.912.AR.2.4
	3	1	MA.912.AR.2.2
	4–5	2	MA.912.AR.1.2
Test Practice	7	2	MA.912.AR.2.4
Spiral Review			
	8	2	MA.7.AR.2.2
	9–10	2	MA.912.AR.1.2

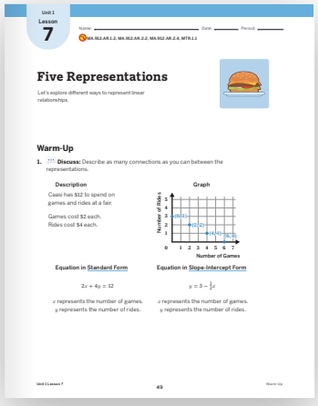
Unit 1

Lesson 7

Five Representations

Linear Relationships in Equations, Tables, and Graphs

Let's explore different ways to represent linear relationships.


Unit 1
Lesson 7
Five Representations
Let's explore different ways to represent linear relationships.
Warm-Up
1. Discuss: Describe as many connections as you can between the representations.
Description
Graph
Equation in Standard Form
Equation in Slope-Intercept Form
Number of Rides
Number of Games
49

Learning Outcomes

Today's Goals

1. **Goal:** Write two linear equations to represent the same situation: one in standard form and one in slope-intercept form.
2. **Language Goal:** Describe connections between equations, tables, descriptions, and graphs of linear situations. **(Writing and Speaking)**

Students make sense of situations with two variables by representing them in graphs, tables, and equations in multiple forms. Students reason abstractly and quantitatively about linear relationships and make connections between the different representations. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 6, students solved linear equations in standard form for one of their variables and connected the new equations to their graphs. Students also described how the graphs of linear equations can be written in different but equivalent forms.

Future Learning

In Lessons 10-12, students will continue investigating the solutions of one-variable linear inequalities using multiple representations. Students will deepen this investigation in Lessons 13–14 when they represent the solutions of two-variable linear inequalities.

Connections

- Students use linear relationships to determine combinations of items that can be purchased for a certain price. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.2

Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

MA.912.AR.2.4

Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.

Also Addressing: **MA.912.AR.1.2, MA.912.AR.2.5, MA.912.AR.2.6**

Mathematical Thinking and Reasoning Standard: MTR.1.1

ELA Expectations: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.1	MA.912.AR.9.8
MA.8.AR.3.3	MA.912.AR.9.9
MA.8.AR.3.4	MA.912.AR.9.10

Vocabulary

New Vocabulary

standard form (of a linear equation)

slope-intercept form

Lesson at a Glance

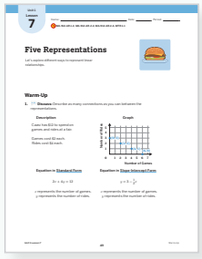
~ 45 min

MA.912.AR.2.2, MA.912.AR.2.4, MA.912.AR.1.2, MA.912.AR.2.5, MA.912.AR.2.6

Warm-Up

Pairs | 5 min

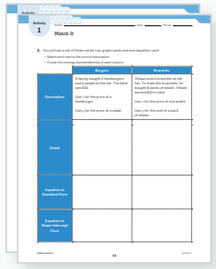
Students surface connections between different representations of a linear situation and highlight the information that each one reveals about the situation.



Activity 1

Pairs | 10 min

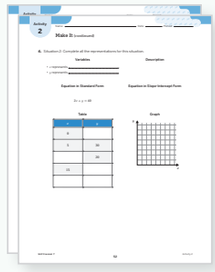
Students match graphs and equations to descriptions about a situation and use the given information to create missing representations.



Activity 2

Small Groups | 20 min

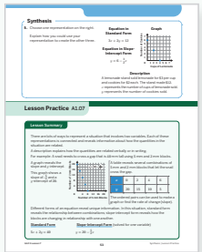
Students use one representation to complete all the missing representations for two linear situations.



Synthesis

Whole Class | 5 min

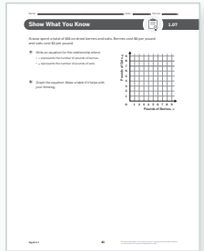
Students synthesize their understanding of the different ways to represent a linear situation and how to use the information in one representation to create another one. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing and graphing a linear equation.



Prep Checklist

Prepare the Activity 1 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 1 Sheet, one per 3 pairs of students, precut.

Pairs | 5 min

Warm-Up

Purpose: Students surface connections between different representations of a linear situation and highlight the information that each one reveals about the situation.

1 Launch

Draw students' attention to the representations and consider asking, "What experience do you have with fairs or carnivals?"

A **Accessibility: Conceptual Processing** Invite students to label or color-code features from each representation and how they are shown on the equation, description, or graph.

D **Differentiation: (Support)** If students need help getting started, consider asking them to choose one representation and explain how it is connected to each of the others.

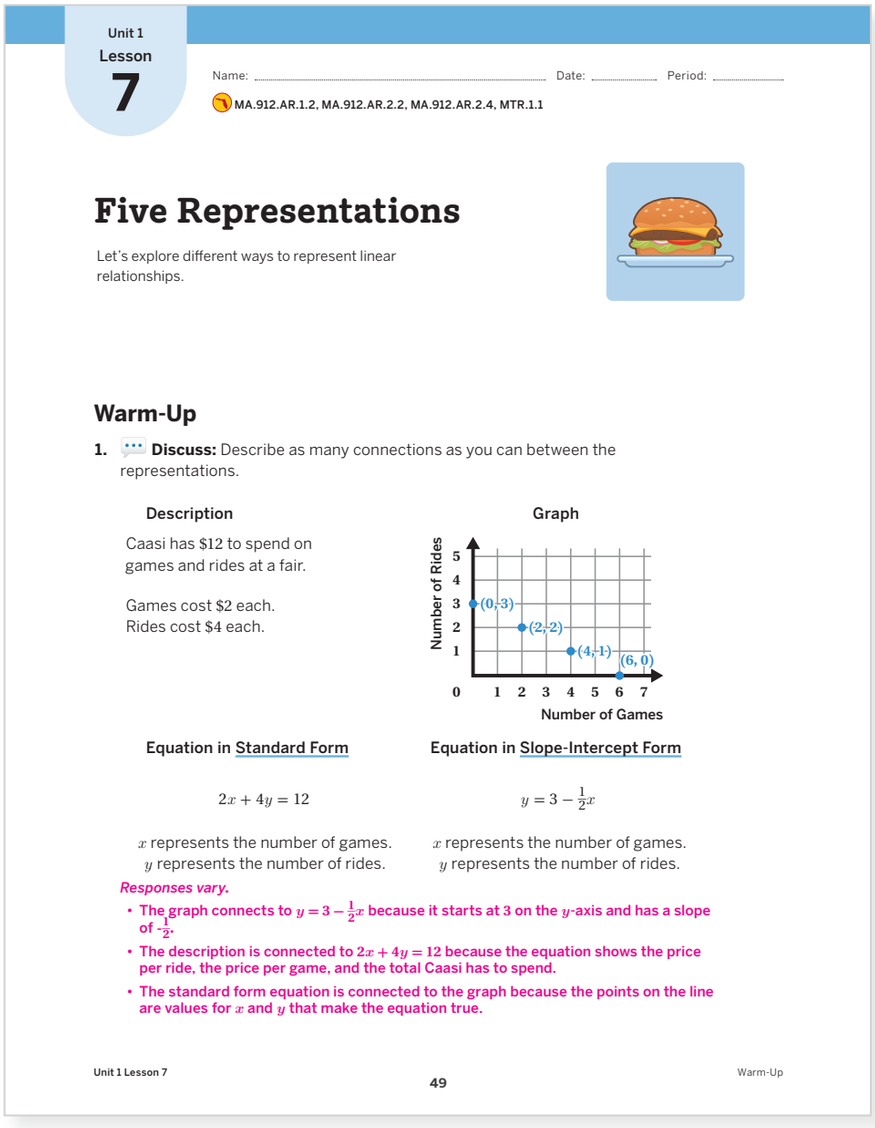
2 Connect

To surface the Key Takeaway, consider asking:

- "What connection(s) do you see between the equation in standard form and the equation in slope-intercept form?" *Responses vary.* They are equivalent and represent the same situation. I can rearrange the equation in standard form to get the equation in slope-intercept form by solving for y.
- "What connection(s) do you see between the description and the graph?"
- "Where do you see \$12, \$2, and \$4 from the description in the equations?"
- "How could you use one representation to create the others?"

Create an anchor chart for **standard form** and **slope-intercept form** with features students notice about equations in each form.

Note: The terms *standard form* and *slope-intercept form* are included here to help students communicate more clearly about different forms of the equation and make connections between different representations. The goal of introducing these terms is for students to have language to discuss structures of equations. If students find these terms helpful for discussing structures of equations, encourage their use. Students may use other language to refer to these forms.



Unit 1
Lesson 7

Name: _____ Date: _____ Period: _____

MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MTR.1.1

Five Representations

Let's explore different ways to represent linear relationships.

Warm-Up

1. **Discuss:** Describe as many connections as you can between the representations.

Description

Caasi has \$12 to spend on games and rides at a fair.

Games cost \$2 each.
Rides cost \$4 each.

Graph

Equation in Standard Form

$2x + 4y = 12$

x represents the number of games.
y represents the number of rides.

Equation in Slope-Intercept Form

$y = 3 - \frac{1}{2}x$

x represents the number of games.
y represents the number of rides.

Responses vary.

- The graph connects to $y = 3 - \frac{1}{2}x$ because it starts at 3 on the y-axis and has a slope of $-\frac{1}{2}$.
- The description is connected to $2x + 4y = 12$ because the equation shows the price per ride, the price per game, and the total Caasi has to spend.
- The standard form equation is connected to the graph because the points on the line are values for x and y that make the equation true.

Unit 1 Lesson 7 49 Warm-Up

Key Takeaway: Linear situations with two variables can be represented by a description, an equation in **slope-intercept form** or **standard form**, and a graph, each of which is connected to the other representations and reveals specific information about the situation.

Pairs | 10 min

Activity 1 Match It

Purpose: Students match graphs and equations to descriptions about a situation and use the given information to create missing representations.

1 Launch

Distribute the Activity 1 Cards.

Draw students' attention to the two context descriptions

Invite students to read the prompts and paraphrase them to a partner. **(MTR.1.1)**

ELL **English Language Learners** To help students access background knowledge, act out scenarios (or invite students to), show videos or images, or use gestures to describe both situations before students begin the task. **(Listening and Reading)**

Consider asking, "How are the two situations alike? How are they different?" *Responses vary.* Both situations involve two different variables with different prices. In the first situation, both variables contribute to the total cost. In the second situation, one variable represents the seller's revenue and the other variable represe.nts cost. **(MTR.1.1)**

2 Monitor

Encourage students to annotate the graphs to make connections to the equation and the descriptions.

Look for strategies for matching the cards that students can share during the Connect, such as:

- Connecting the information in the standard-form equation to the numbers in the description.
- Calculating the slope of each line and connecting it to the equation in slope-intercept form.
- Identifying the x- and y-intercepts of each line and connecting them to the equation or situation.

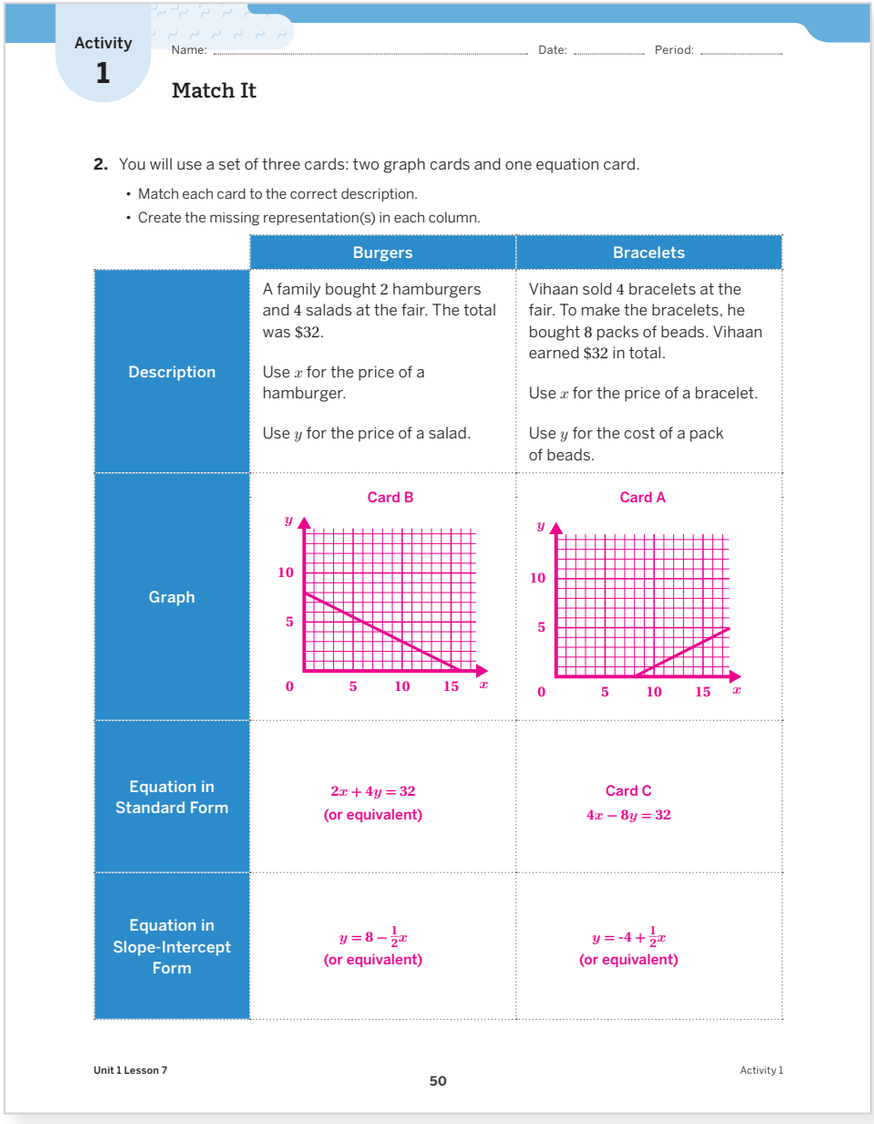
D **Differentiation: (Support)** If students need support with scaling the axis on the graphs, consider pausing to briefly discuss student strategies for selecting a scale.

3 Connect

Discuss students' strategies for matching the cards to the two descriptions.

Invite students to share their strategies for creating missing representations, such as:

- Rearranging equations by solving for y.
- Writing an equation in slope-intercept form using the graph.



Activity 1

Name: Date: Period:

Match It

2. You will use a set of three cards: two graph cards and one equation card.

- Match each card to the correct description.
- Create the missing representation(s) in each column.

	Burgers	Bracelets
Description	A family bought 2 hamburgers and 4 salads at the fair. The total was \$32. Use x for the price of a hamburger. Use y for the price of a salad.	Vihaan sold 4 bracelets at the fair. To make the bracelets, he bought 8 packs of beads. Vihaan earned \$32 in total. Use x for the price of a bracelet. Use y for the cost of a pack of beads.
Graph	Card B	Card A
Equation in Standard Form	$2x + 4y = 32$ (or equivalent)	Card C $4x - 8y = 32$
Equation in Slope-Intercept Form	$y = 8 - \frac{1}{2}x$ (or equivalent)	$y = -4 + \frac{1}{2}x$ (or equivalent)

Unit 1 Lesson 7 50 Activity 1

Small Groups | 20 min

Activity 2 Make It

Purpose: Students use one representation to complete all the missing representations for two linear situations.

Short on time? Consider having students prioritize completing all the representations for Situation 1.

1 Launch

Draw students' attention to the variables and description for Situation 1.

Consider asking, "What is a situation where you may need to buy more than one type of a similar thing, like Kiana buying bus and train rides?"

Note: Students may appreciate a shared thinking space, such as a whiteboard or chart paper.

To encourage creativity, consider sharing that representations can be completed in any order.

2 Monitor

A **Accessibility: Executive Functioning** To support students initiating complex tasks, identify possible places to start. Consider inviting students to fill in the missing values in the table first.

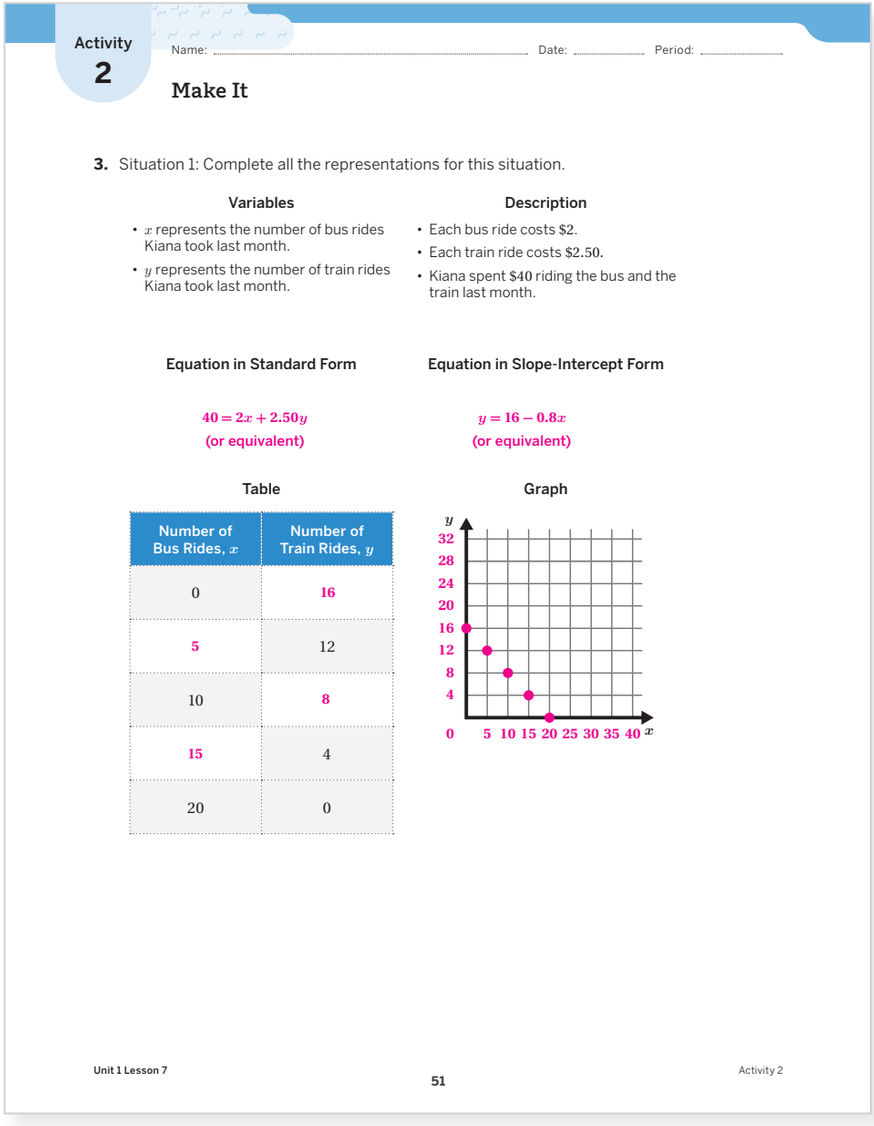
Look for student strategies for making connections across representations and building missing ones, such as:
- Using patterns in the table to write an equation.
- Using the table or an equation to create a graph.

To surface connections, consider asking small groups, "What do the values in each equation reveal about the situation? Where do you see those values on the graph?" *Responses vary.* 16 is the number of train rides Kiana took last month, if she took 0 bus rides. It is also the y-intercept of the graph, and can be located where the line $y = 16 - 0.8x$ crosses the y-axis.

D **Differentiation: (Support)** If students make a graph that is scaled such that it is challenging to read, consider supporting them in using the table by asking, "What is the difference between the first two x-values in the table? The first two y-values? How could this help you scale your graph?"

Note: If many groups are struggling with Situation 1, consider pausing briefly and inviting students to share productive strategies before students move to Situation 2.

Activity 2 continued >



Activity 2

Name: Date: Period:

Make It

3. Situation 1: Complete all the representations for this situation.

Variables
- x represents the number of bus rides Kiana took last month.
- y represents the number of train rides Kiana took last month.

Description
- Each bus ride costs $2.
- Each train ride costs $2.50.
- Kiana spent $40 riding the bus and the train last month.

Equation in Standard Form

$40 = 2x + 2.50y$ (or equivalent)

Equation in Slope-Intercept Form

$y = 16 - 0.8x$ (or equivalent)

Table

Number of Bus Rides, x	Number of Train Rides, y
0	16
5	12
10	8
15	4
20	0

Graph

Unit 1 Lesson 7 51 Activity 2

Activity 2 Make It (continued)

Purpose: Students use one representation to complete all the missing representations for two linear situations.

2 Monitor

Select and sequence a variety of strategies students use to create missing representations to share during the Connect.

D Differentiation *(Problem 4)*

Look for students who:	Teacher Moves
Need support getting started writing a description.	**Support** Encourage students to brainstorm things that cost $1 and things that cost $2. Consider asking, "How could you write a description with those items?"
Create at least one representation that does not match the others.	**Support** Invite students to check their representation by comparing it to the given information and any correct representations they have made (e.g., checking table entries against the graph, substituting table entries or points into the equations, etc.).
Create multiple representations that match.	**Strengthen** Celebrate their work and invite students to describe connections they see between the representations.
Create an accurate graph using a scale that is helpful to view the relationship.	**Strengthen** Celebrate students' attention to precision in constructing the graph and invite them to share during the Connect.

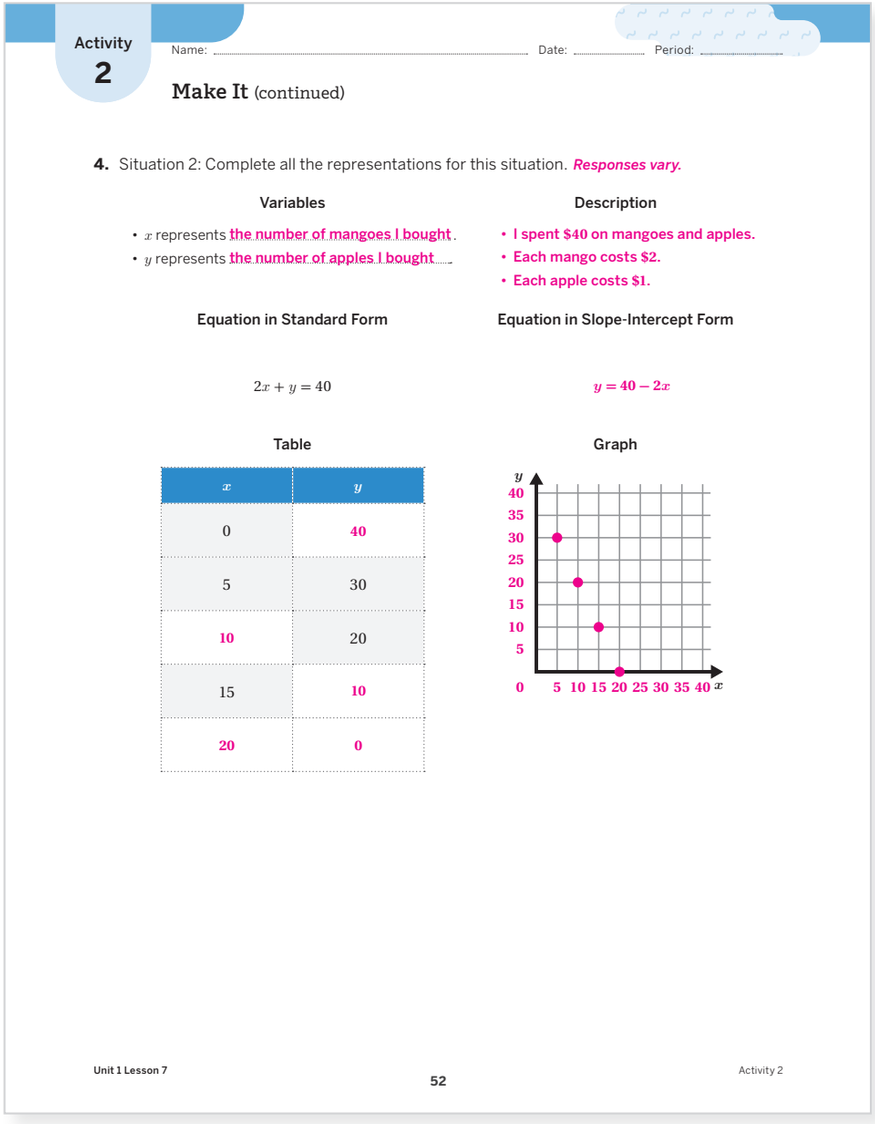
3 Connect

Share the table, graph, and equation for Situation 2 to discuss how students created the missing representations.

To surface the Key Takeaway, consider asking:

- "Which representations were easier to create? Which ones were more challenging?"
- "Was there ever a time when you didn't have enough information to create a missing representation?"

Note: If time allows, consider inviting students to share what their preferred order of making representations would be and why. Celebrate that we all have different preferences and strengths and our preferences might change based on the situation. *Responses vary.* I would prefer to start with the standard form equation, so I could rearrange it to find the slope-intercept form equation, and then make a graph.



Activity 2

Name: Date: Period:

Make It (continued)

4. Situation 2: Complete all the representations for this situation. *Responses vary.*

Variables

- x represents the number of mangoes I bought.
- y represents the number of apples I bought.

Description

- I spent $40 on mangoes and apples.
- Each mango costs $2.
- Each apple costs $1.

Equation in Standard Form

$2x + y = 40$

Equation in Slope-Intercept Form

$y = 40 - 2x$

Table

x	y
0	40
5	30
10	20
15	10
20	0

Graph

Unit 1 Lesson 7 52 Activity 2

Key Takeaway: Each representation of a linear situation (e.g., a graph, a description, or an equation) provides enough information to create the other representations.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the different ways to represent a linear situation and how to use the information in one representation to create another one.

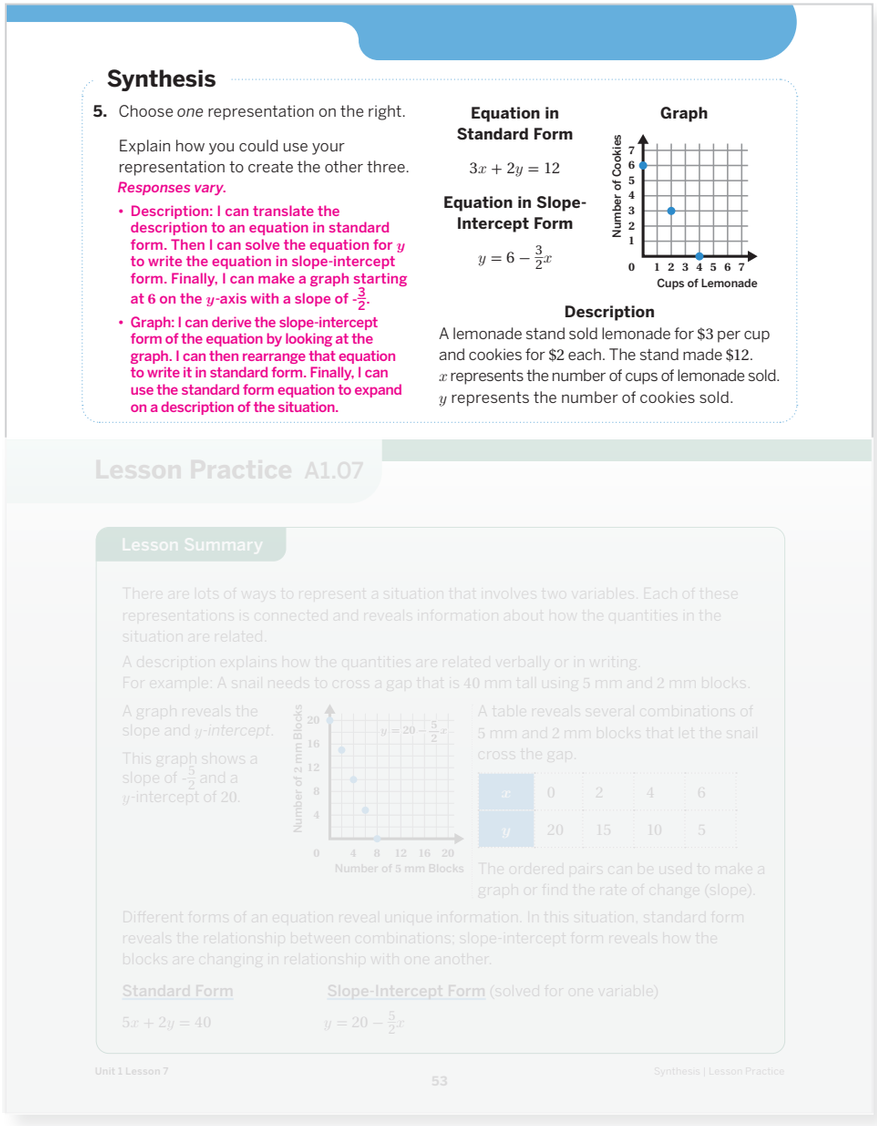
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- How to create an equation from the description.
- How to rearrange an equation to make another equation.
- How to write a slope-intercept form equation from a graph.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for translating between representations. Display for students to reference and add to throughout the rest of this unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

5. Choose *one* representation on the right.

Explain how you could use your representation to create the other three.

Responses vary.

- Description: I can translate the description to an equation in standard form. Then I can solve the equation for y to write the equation in slope-intercept form. Finally, I can make a graph starting at 6 on the y-axis with a slope of $-\frac{3}{2}$.
- Graph: I can derive the slope-intercept form of the equation by looking at the graph. I can then rearrange that equation to write it in standard form. Finally, I can use the standard form equation to expand on a description of the situation.

Equation in Standard Form

$3x + 2y = 12$

Equation in Slope-Intercept Form

$y = 6 - \frac{3}{2}x$

Graph

Number of Cookies

Cups of Lemonade

Description

A lemonade stand sold lemonade for \$3 per cup and cookies for \$2 each. The stand made \$12.
x represents the number of cups of lemonade sold.
y represents the number of cookies sold.

Lesson Practice A1.07

Lesson Summary

There are lots of ways to represent a situation that involves two variables. Each of these representations is connected and reveals information about how the quantities in the situation are related.

A description explains how the quantities are related verbally or in writing.
For example: A snail needs to cross a gap that is 40 mm tall using 5 mm and 2 mm blocks.

A graph reveals the slope and y-intercept.

This graph shows a slope of $-\frac{5}{2}$ and a y-intercept of 20.

Number of 2 mm Blocks

Number of 5 mm Blocks

$y = 20 - \frac{5}{2}x$

A table reveals several combinations of 5 mm and 2 mm blocks that let the snail cross the gap.

x	0	2	4	6
y	20	15	10	5

The ordered pairs can be used to make a graph or find the rate of change (slope).

Different forms of an equation reveal unique information. In this situation, standard form reveals the relationship between combinations; slope-intercept form reveals how the blocks are changing in relationship with one another.

Standard Form: $5x + 2y = 40$

Slope-Intercept Form (solved for one variable): $y = 20 - \frac{5}{2}x$

Unit 1 Lesson 7 53 Synthesis | Lesson Practice

Lesson Takeaway: There are many strategies for representing linear situations in two variables, including writing a description, creating an equation (which can be in **slope-intercept form** or **standard form**), and making a graph. Each of these representations has enough information about the situation to make the others.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing and graphing a linear equation.

Today's Goals

1. **Goal:** Write two linear equations to represent the same situation: one in standard form and one in slope-intercept form.
2. **Language Goal:** Describe connections between equations, tables, descriptions, and graphs of linear situations. **(Writing and Speaking)**

Show What You Know (Assessment Resources)

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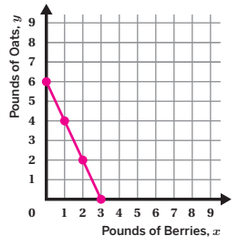
Show What You Know 1.07

Ariana spent a total of $18 on dried berries and oats. Berries cost $6 per pound and oats cost $3 per pound.

a. Write an equation for this relationship where:
   - x represents the number of pounds of berries.
   - y represents the number of pounds of oats.

   $6x + 3y = 18$

b. Graph the equation. Make a table if it helps with your thinking.



Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support writing an equation to match a scenario or graphing a linear equation:

» **Respond to Student Thinking:**

- Use the ***Interpreting Slope and Intercepts of Linear Relationships*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write their own scenario that can be modeled by a linear equation and create as many of the representations as they can.

Professional Learning

Reflect on student progress with solving one- and two-variable equations throughout Sub-Units 1 and 2. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you utilize student strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

5. Choose *one* representation on the right.

Explain how you could use your representation to create the other three.

Responses vary.

- Description: I can translate the description to an equation in standard form. Then I can solve the equation for y to write the equation in slope-intercept form. Finally, I can make a graph starting at 6 on the y-axis with a slope of $-\frac{3}{2}$.
- Graph: I can derive the slope-intercept form of the equation by looking at the graph. I can then rearrange that equation to write it in standard form. Finally, I can use the standard form equation to expand on a description of the situation.

Equation in Standard Form

$3x + 2y = 12$

Equation in Slope-Intercept Form

$y = 6 - \frac{3}{2}x$

Graph

Description

A lemonade stand sold lemonade for $3 per cup and cookies for $2 each. The stand made $12.

x represents the number of cups of lemonade sold.

y represents the number of cookies sold.

Lesson Practice A1.07

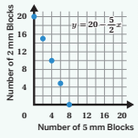
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There are lots of ways to represent a situation that involves two variables. Each of these representations is connected and reveals information about how the quantities in the situation are related.

A description explains how the quantities are related verbally or in writing.
For example: A snail needs to cross a gap that is 40 mm tall using 5 mm and 2 mm blocks.

A graph reveals the slope and y-*intercept*.

This graph shows a slope of $-\frac{5}{2}$ and a y-intercept of 20.



A table reveals several combinations of 5 mm and 2 mm blocks that let the snail cross the gap.

x	0	2	4	6
y	20	15	10	5

The ordered pairs can be used to make a graph or find the rate of change (slope).

Different forms of an equation reveal unique information. In this situation, standard form reveals the relationship between combinations; slope-intercept form reveals how the blocks are changing in relationship with one another.

Standard Form

$5x + 2y = 40$

Slope-Intercept Form (solved for one variable)

$y = 20 - \frac{5}{2}x$

Lesson Practice A1.07

Name: Date: Period:

Problems 1–3: A school is putting on a play. Adult tickets cost $8 each and student tickets cost $4 each. The school collected $320 total.

- a represents the number of adult tickets sold.
- s represents the number of student tickets sold.

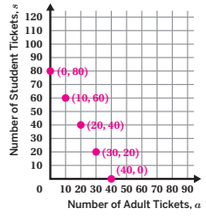
1. Which equation represents this relationship?

A. $a + s = 320$
B. $8a + 4s = 320$
C. $8a - 4s = 320$
D. $8s + 4a = 320$

2. Complete the table representing this relationship. *Responses in the final row vary.*

Number of Adult Tickets, a	Number of Student Tickets, s
0	80
20	40
40	0

3. Make a graph showing this relationship.



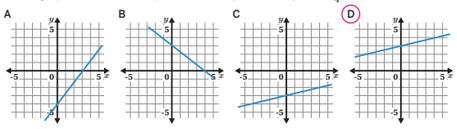
Problems 4–5: Here is the equation $-x + 4y = 12$.

4. Explain how you can determine that $-x + 4y = 12$ is equivalent to $y = 3 + \frac{1}{4}x$.

Responses vary. When I add x to each side and then divide by 4, I get $y = 3 + \frac{1}{4}x$.

5. Which graph matches the equations $-x + 4y = 12$ and $y = 3 + \frac{1}{4}x$?

A B C D



Lesson Practice A1.07

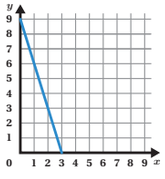
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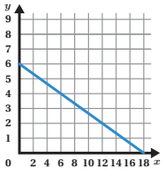
Test Practice

Problems 6–7: Match each graph to two equations. One equation will have no match.

Equation A	Equation B	Equation C	Equation D	Equation E
$y = 6 - \frac{1}{3}x$	$2y + 6x = 18$	$y = 9 - 3x$	$6y + 2x = 36$	$3y + 6x = 18$

6. Equation B Equation C **7.** Equation D Equation A





Spiral Review

8. Write at least *three* different equations that are equivalent to $\frac{1}{2}(-10x + 2) = 6$. *Responses vary.*

Equation 1	Equation 2	Equation 3
$-10x + 2 = 12$	$-10x = 10$	$x = -1$

9. Select *all* the equations that have no solution.

☐ A. $2t + 1 = 1 + 2t$ ☑ B. $2t + 1 = 2t$ ☐ C. $2t + t = 2t + 1$
☐ D. $2t = 1$ ☑ E. $2(t + 1) = 2t + 1$

10. Write an equation equivalent to $x = \frac{10 - 5y}{3}$ in the form $Ax + By = C$.

$3x + 5y = 10$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 2	2	MA.912.AR.2.2
	3	2	MA.912.AR.2.4
	4	2	MA.912.AR.1.2
	5	2	MA.912.AR.2.2 MA.912.AR.2.4 MA.912.AR.1.2
Test Practice	6, 7	2	MA.912.AR.2.2 MA.912.AR.2.4 MA.912.AR.1.2
Spiral Review			
	8	2	MA.7.AR.1.2
	9	2	MA.8.AR.2.2
	10	2	MA.912.AR.1.2

Unit 1

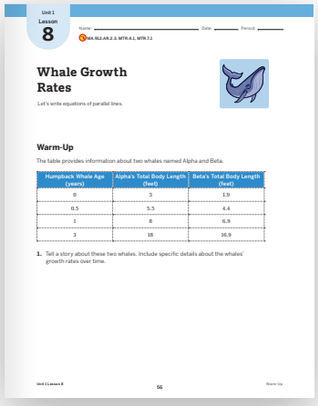
Lesson

8

Whale Growth Rates

Finding Equations of Parallel Lines

Let's write equations of parallel lines.


Unit 1
Lesson
8
Name _____ Date _____ Period _____
MA.912.AR.2.3, MTR.4.1, MTR.7.1

Whale Growth Rates

Let's write equations of parallel lines.

Warm-Up

The table provides information about two whales named Alpha and Beta.

Humpback Whale Age (years)	Alpha's Total Body Length (feet)	Beta's Total Body Length (feet)
0	3	1.9
0.5	5.3	4.4
1	8	6.9
3	18	16.9

1. Tell a story about these two whales. Include specific details about the whales' growth rates over time.

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Learning Outcomes

Today's Goal

1. **Goal:** Find the equation of a line through a given point that is parallel to a given line.
2. **Goal:** Interpret and apply equations of parallel line using real-world context.
3. **Language Goal:** Explain the relationship between two lines that are parallel, describing how they are alike and how they are different.

Students find equations of lines that are parallel to a given line when provided information about the given line. Students apply what they have learned about writing linear equations in slope-intercept form and demonstrate their understanding of how parallel lines can model real-world contexts. **(MTR.4.1)**

Coherence

Prior Learning

In Lessons 4–7, students became fluent in writing multivariable equations and represented real-world contexts with two variables using descriptions, tables, graphs, or algebraic equations.

Future Learning

In Lesson 9, students will extend their understanding of linear relationships to analyze perpendicular line relationships in context.

Connections

- Students use equations in a real-world context as they compare the growth rates of various whales species. (real-world, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.3
Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.2	**MA.912.AR.9.1**
MA.912.AR.2.4	**MA.912.AR.9.6**

Vocabulary

New Vocabulary

parallel lines

Review Vocabulary

slope	*rate of change*
y-intercept	*slope-intercept form*

Lesson at a Glance

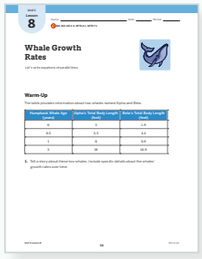
~ 45 min

MA.912.AR.2.3

Warm-Up

Pairs | 5 min

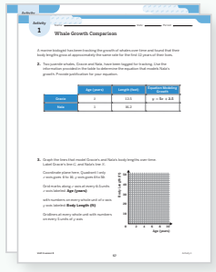
Students apply their knowledge of proportions and slope to interpret the values in a table and use the Tell a Story routine to describe the equation using the given data about whale growth. **Routines:** Tell a Story, Think-Pair-Share (optional)



Activity 1

Pairs | 10 min

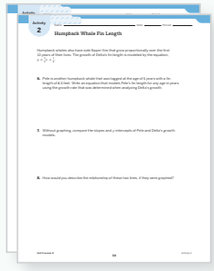
Students use the Think-Pair-Share routine to create parallel line equations and compare two lines in terms of the given context. **Routine:** Think-Pair-Share **(MTR.7.1)**



Activity 2

Pairs | 10 min

Students use a given equation to determine the constant rate of change and create a parallel equation to then interpret the two lines in terms of the context. **Routine:** Notice and Wonder (optional) **(MTR.4.1)**



Activity 3

Small Groups | 10 min

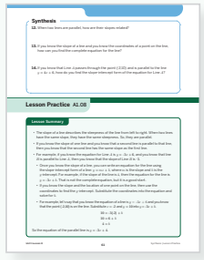
Students create real-world stories using information about a given line to find a parallel line that meets defined specifications. **Routine:** Tell a Story **(MTR.4.1, MTR.7.1)**



Synthesis

Whole Class | 5 min

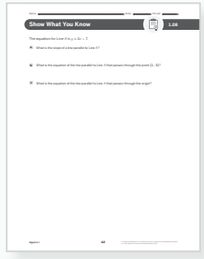
Students synthesize their understanding of how to find the equation of a parallel line that passes through a given point. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by finding parallel line equations using the information provided about the given lines and describe their meaning in context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students apply their knowledge of proportions and slope to interpret the values in a table and use the Tell a Story routine to describe the equation using the given data about whale growth.

1 Launch

Direct students' attention to the table that showcases two whales' growth in feet over time in years.

Use the Tell a Story routine to support students making sense of the data table. Encourage students to justify their story using mathematical facts and computations to support what they know about the two whales.

MLR **MLR5: Co-craft Questions and Problems** Encourage students to justify their story using mathematical facts and computations to support what they know about the two whales.

Consider using the Think-Pair-Share routine to help students shape their ideas about how they can determine the constant rate of growth in whale length, and how this is related to the slope of the line that defines growth. See the Routine Facilitation Guide for more information.

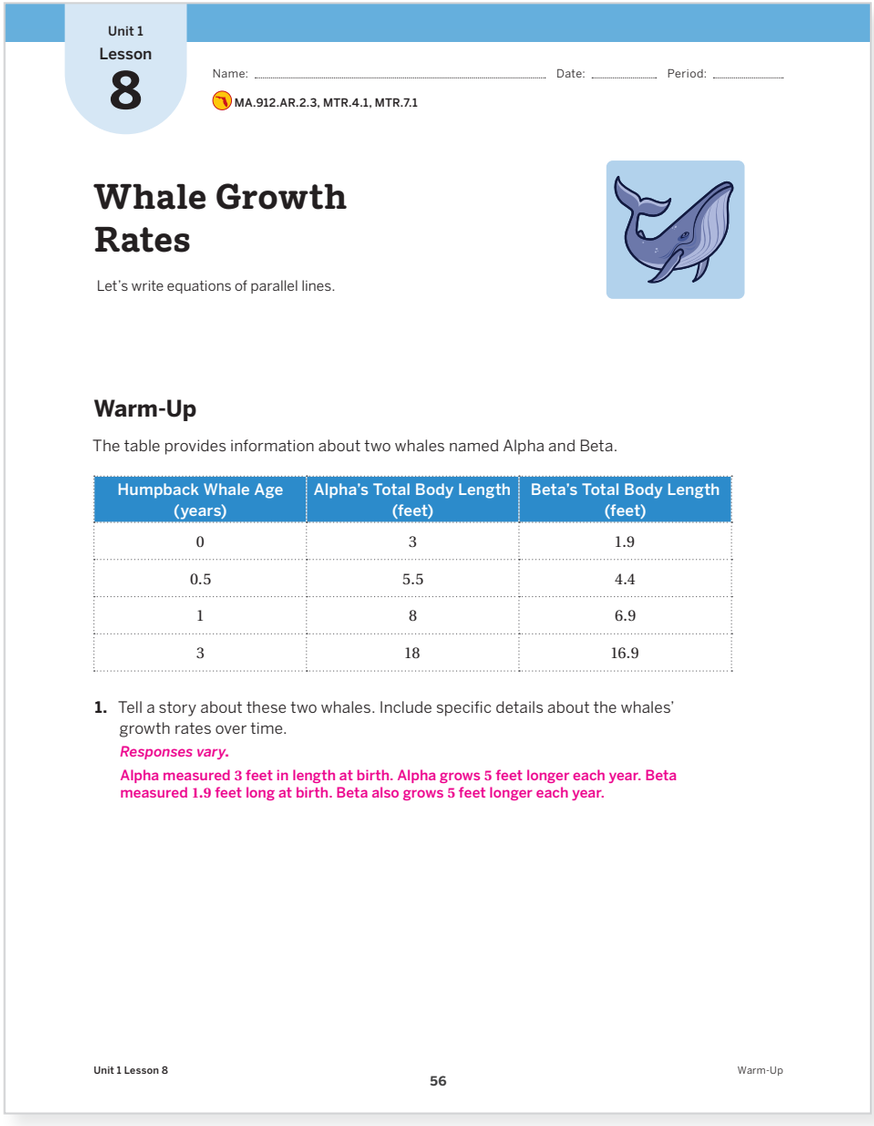
2 Connect

Invite students to share their stories about the 2 whales and how the data supports their story.

Consider using the Think-Pair-Share routine to develop students' interpretation of the data as they compare Alpha's growth to Beta's growth.

ELL **English Language Learners** Consider grouping students with different English language proficiency levels, so they can listen to and learn from their peers as they express their interpretation of context with the terms in the linear equation, such as how the slope is the constant rate of growth and how the y-intercept is the length of the whale at birth.

Discuss as a large group how students found the rate of change that represents Alpha's and Beta's growth.



Unit 1
Lesson
8

Name: _____ Date: _____ Period: _____

MA.912.AR.2.3, MTR.4.1, MTR.7.1

Whale Growth Rates

Let's write equations of parallel lines.

Warm-Up

The table provides information about two whales named Alpha and Beta.

Humpback Whale Age (years)	Alpha's Total Body Length (feet)	Beta's Total Body Length (feet)
0	3	1.9
0.5	5.5	4.4
1	8	6.9
3	18	16.9

1. Tell a story about these two whales. Include specific details about the whales' growth rates over time.

Responses vary.

Alpha measured 3 feet in length at birth. Alpha grows 5 feet longer each year. Beta measured 1.9 feet long at birth. Beta also grows 5 feet longer each year.

Unit 1 Lesson 8 56 Warm-Up

Pairs | 10 min

Activity 1 Whale Growth Comparison

Purpose: Students use the Think-Pair-Share routine to create parallel line equations and compare two lines in terms of the given context. **(MTR.4.1)**

Short on time? Consider assigning a specific whale to each student. Then, regroup into pairs for the comparison tasks.

1 Launch

Invite students to share what they had determined as a growth rate for humpback whales from the Warm-up.

Consider asking:

- What do you know about the growth of Gracie?
- How can you use Gracie's growth information with the information on Nala to create Nala's growth model?

Encourage students to define their numeric and algebraic answers in terms of the context to assist them in the comparison portion of the activity.

2 Monitor

Use the Think-Pair-Share routine to support collaborative learning. Invite students to think independently, then to take turns sharing with a partner as they complete problems 3 and 4.

Capture a few of students' explanations that showcase reflection and revision of reasoning to share during the Connect.

Activity 1 continued >

Activity 1

Name: Date: Period:

Whale Growth Comparison

A marine biologist has been tracking the growth of whales over time and found that their body lengths grow at approximately the same rate for the first 12 years of their lives.

2. Two juvenile whales, Gracie and Nala, have been tagged for tracking. Use the information provided in the table to determine the equation that models Nala's growth. Provide justification for your equation.

	Age (years)	Length (feet)	Equation Modeling Growth
Gracie	2	12.5	$y = 5x + 2.5$
Nala	3	16.2	

Responses vary. Since whales grow at the same rate, their growth rates will all be modeled by the equation $y = 5x + b$. Using the point (3,16.2), substitute the values in for the variables.

$16.2 = 5(3) + b$

$16.2 = 15 + b$ therefore $b = 1.2$.

So Nala's growth is modeled by the equation $y = 5x + 1.2$.

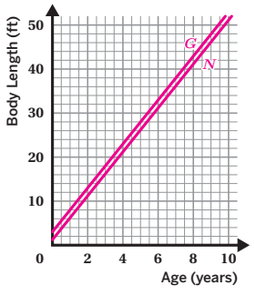
3. Graph the lines that model Gracie's and Nala's body lengths over time. Label Gracie's line G, and Nala's line N.

Coordinate plane here, Quadrant I only
x-axis goes 0 to 10, y-axis goes 0 to 50.

Grid marks along x-axis at every 0.5 units
x-axis labeled: **Age (years)**

with numbers on every whole unit of x-axis
y-axis labeled: **Body Length (ft)**

Gridlines at every whole unit with numbers on every 5 units of y-axis



Unit 1 Lesson 8 57 Activity 1

Key Takeaway: When interpreting equations of **parallel lines**, the slopes are the same because the growth rate of whales is a constant rate of change. The y-intercepts are different since whales' lengths at birth vary.

Pairs | 10 min

Activity 1 Whale Growth Comparison (continued)

Purpose: Students use the Think-Pair-Share routine to create parallel line equations and compare two lines in terms of the given context. **(MTR.4.1)**

Short on time? Consider assigning a specific whale to each student. Then, regroup into pairs for the comparison tasks.

D Differentiation *(Problems 4–5)*

Look for students who:	Respond to Student Thinking
Need support getting started. (Problem 4)	**Support** Encourage students to reflect on the whale growth facts they know from the equations and graphs to aid them in analyzing whale lengths.
Have substitution problems with the given point to find the y-intercept. (Problem 5)	**Strengthen** Consider asking, "What does the given point on the line mean in terms of whale growth?" or "How can you use substitution to find the length of the whale when $x = 0$?"
Would like a challenge. (Problem 5)	**Stretch** Invite students to create a new whale equation for a whale that would always have a length that's between the sizes of Nala and Omega.

Look for students' explanations to share about the comparison and invite them to share their reasoning. **(MTR.7.1)**

2 Monitor

To support students' reasoning, ask:

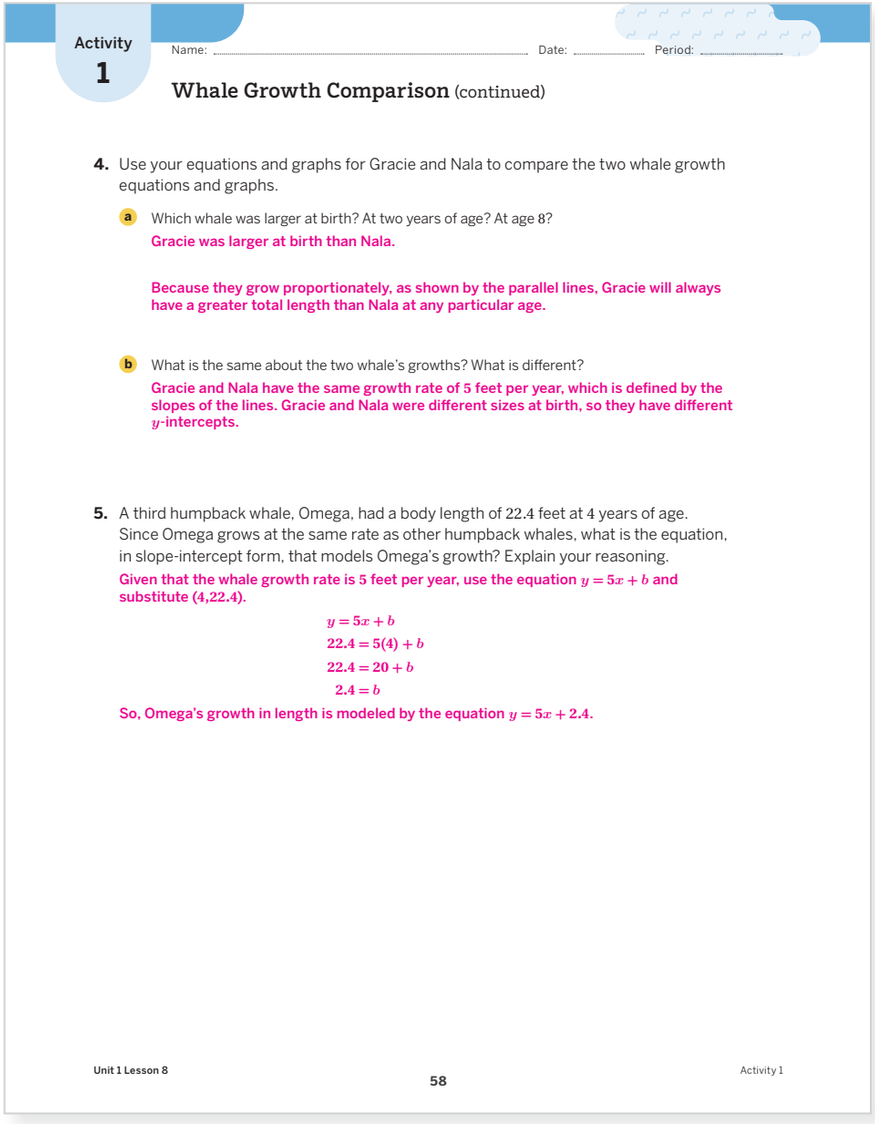
- "How are Gracie's and Nala's equations for growth the same?"
- "What does this likeness mean in terms of their growth?"
- "How are Gracie's and Nala's equations for growth different?"
- "What does this difference mean in terms of their growth?"

3 Connect

Invite students to share a mistake they made and what reasoning or strategy they used in resolving it.

To surface the Key Takeaway, ask: (MTR.7.1)

- "What would be true about the key features of the lines that model the growth of six humpback whales?"



Activity 1

Name: Date: Period:

Whale Growth Comparison (continued)

4. Use your equations and graphs for Gracie and Nala to compare the two whale growth equations and graphs.

a. Which whale was larger at birth? At two years of age? At age 8?

Gracie was larger at birth than Nala.

Because they grow proportionately, as shown by the parallel lines, Gracie will always have a greater total length than Nala at any particular age.

b. What is the same about the two whale's growths? What is different?

Gracie and Nala have the same growth rate of 5 feet per year, which is defined by the slopes of the lines. Gracie and Nala were different sizes at birth, so they have different y-intercepts.

5. A third humpback whale, Omega, had a body length of 22.4 feet at 4 years of age. Since Omega grows at the same rate as other humpback whales, what is the equation, in slope-intercept form, that models Omega's growth? Explain your reasoning.

Given that the whale growth rate is 5 feet per year, use the equation $y = 5x + b$ and substitute $(4, 22.4)$.

$$y = 5x + b$$
$$22.4 = 5(4) + b$$
$$22.4 = 20 + b$$
$$2.4 = b$$

So, Omega's growth in length is modeled by the equation $y = 5x + 2.4$.

Unit 1 Lesson 8 58 Activity 1

Pairs | 10 min

Activity 2 Humpback Whale Fin Length

Purpose: Students use a given equation to determine the constant rate of change and create a parallel equation to then interpret the two lines in terms of the context. **(MTR.3.1)**

Short on time? Consider using a whole group discussion for completing problems 7– 8.

1 Launch

Discuss the initial text that describes the growth of fin length of whales over time.

Consider using the Notice and Wonder routine to promote discussion and reasoning as students find the equation that models the growth of Pele's fin length.

Discuss: What do you notice? What do you wonder?

Consider asking:

- "How do we use what we know about calculating slope to determine the constant rate of fin growth?"
- "What are two ways that the slope can be represented?"
- "Which representation of slope do you prefer? Why?"

2 Monitor

A **Accessibility: Executive Function** Invite students to use paper or a calculator to support students with performing calculations.

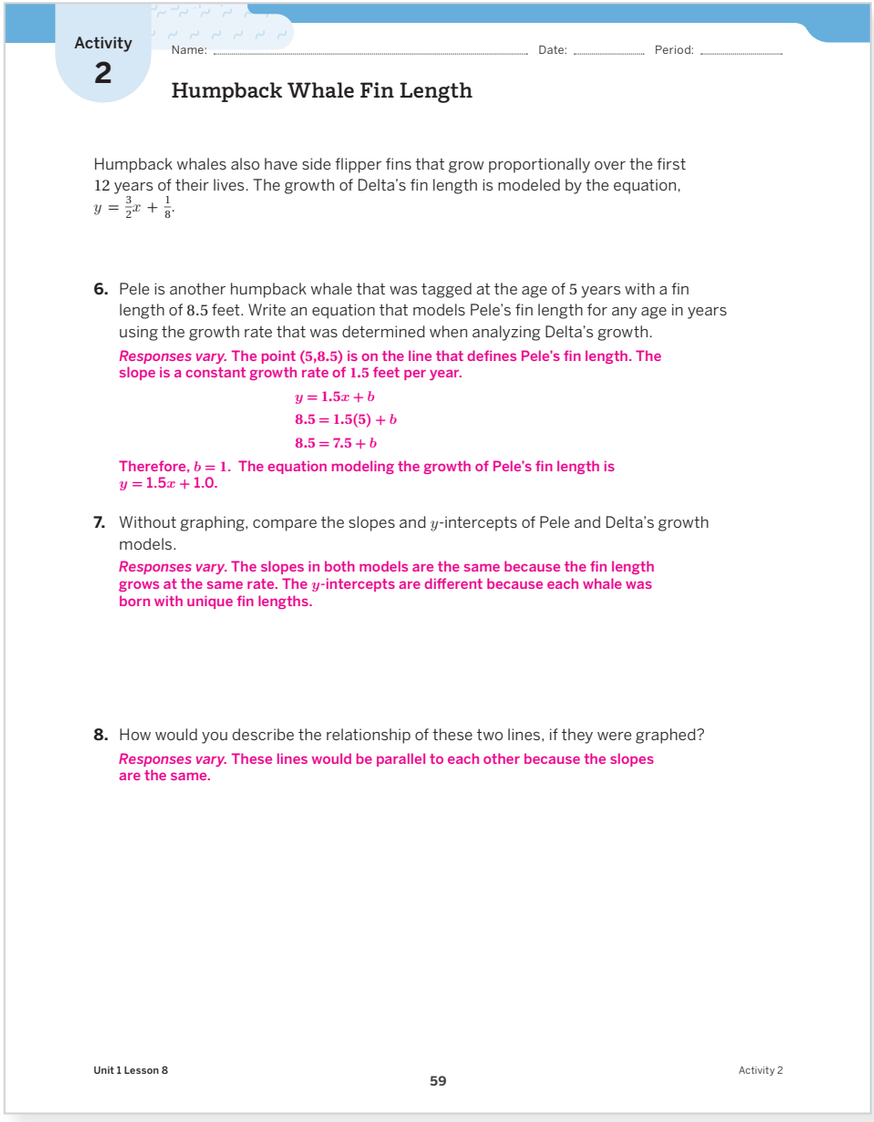
Select and sequence students' reasoning throughout the activity to share during the Connect.

D **Differentiation** *(Problems 6–8)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to consider what values are needed to write the equation of a line.
Student response example 2. (Problem 6)	**Strengthen** Invite them to share what points they used to find the equation.
Student response example 3. (Problem 8)	**Strengthen** Consider asking, "What is the point on the line that defines Pele's growth?"
Student response. (Problem 8)	**Stretch** Encourage fluency in representing slope by asking, "What is another way you can write this equation?"

3 Connect

Invite students to share and discuss the relationships between Delta and Pele's growth equations.



Activity 2

Name: Date: Period:

Humpback Whale Fin Length

Humpback whales also have side flipper fins that grow proportionally over the first 12 years of their lives. The growth of Delta's fin length is modeled by the equation, $y = \frac{3}{2}x + \frac{1}{8}$.

6. Pele is another humpback whale that was tagged at the age of 5 years with a fin length of 8.5 feet. Write an equation that models Pele's fin length for any age in years using the growth rate that was determined when analyzing Delta's growth.

Responses vary. The point (5,8.5) is on the line that defines Pele's fin length. The slope is a constant growth rate of 1.5 feet per year.

$$y = 1.5x + b$$
$$8.5 = 1.5(5) + b$$
$$8.5 = 7.5 + b$$

Therefore, $b = 1$. The equation modeling the growth of Pele's fin length is $y = 1.5x + 1.0$.

7. Without graphing, compare the slopes and y-intercepts of Pele and Delta's growth models.

Responses vary. The slopes in both models are the same because the fin length grows at the same rate. The y-intercepts are different because each whale was born with unique fin lengths.

8. How would you describe the relationship of these two lines, if they were graphed?

Responses vary. These lines would be parallel to each other because the slopes are the same.

Unit 1 Lesson 8 59 Activity 2

Small Groups | 20 min

Activity 3 Write a Story

Purpose: Students create real-world stories using information about a given line to find a parallel line that meets defined specifications. **(MTR.3.1, MTR.4.1)**

1 Launch

Use the Think-Pair-Share routine as you prompt students to brainstorm scenarios that might model the given equation and resulting parallel line.

Encourage students to think independently as they create scenario ideas. Group students into pairs to share their ideas and then work together to find the equation, and interpret the equations using at least one context.

Invite several student pairs to share their interpretation of the slope and y-intercept in context.

ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and inviting them to share in that language. (Writing, Speaking, and Listening)

2 Monitor

Encourage students while proceeding through problems 10 and 11, to try to reason algebraically.

D **Differentiation:** (*Problems 10–11*) If students are struggling, offer the option to create tables and graphs to help support their algebraic reasoning.

Pause briefly once most students have completed each problem to ensure students have the correct equation before describing the parallel lines in context.

Invite students to share the real-world scenario that each pair of parallel lines from problem 10 and 11 might model.

Consider asking: "What is different about the slope of these lines compared to the two previous activities?"

3 Connect

Invite students to share the contexts that they made for the horizontal and vertical parallel lines, justifying their scenario using the algebraic equation, table or graph.

To surface the Key Takeaway, Consider asking:

- "What is true about the slopes and y-intercepts of parallel lines?"
- "What kind of linear equation cannot be written in slope-intercept form?"

Activity 3

Name: Date: Period:

Write a Story

9. Write a story that would define a line parallel to $y = -6x + 80$ and passes through the point $(7,26)$.

Responses vary. I can join a local gym, at a cost of $6.00 per month. I have a gym savings account where I start with a balance of $80. The slope is -6 for the monthly change in account balance.

The amount of money in my savings account is modeled by $y = -6x + 80$.

My friend Liam joins the same gym and pays the same monthly fee from his gym savings account. On month 7, Liam's account had a balance of $26. So, a point on Liam's linear model is (7,26).

Since we are both paying the same monthly cost, the linear models will have the same slope and be parallel. The equation for Liam's account balance can be found by substituting the point and slope:

$$y = -6x + a$$
$$26 = -6(7) + a$$
$26 = -42 + a$ So, Liam's original account balance is $68.

Liam's account balance is modeled by $y = -6x + 68$.

10. Show how to algebraically find the equation of the line, in slope intercept form, that is parallel to $y = -4$ and goes through the point $(8,-1)$. Describe a real-world scenario that these lines would model. Be sure to label all variables.

$$y = 0x - 4$$
$$-1 = (0)(8) + b$$
$$-1 = 0 + b$$
$$-1 = b$$

Equation of parallel line: $y = 0x - 1$ or $y = -1$

Responses vary. x: days of the year, y: money owed; My brother has owed me $4 every day since the beginning of the year. I have owed my sister $1 every day since the beginning of the year.

11. What is the equation of a line through $(3,9)$ and $(3,-9)$? What is the equation of the parallel line through $(-1,4)$? Describe a scenario that these two lines model.

Responses vary. $x = 3$; $x = -1$; A city map has the capitol at the origin. There are two main avenues that run north south on each side of the capitol square defined as $x = 3$; $x = -1$.

Unit 1 Lesson 8 60 Activity 3

Key Takeaway: Slopes of lines can vary, being positive, negative, fraction, decimal, zero, or undefined and no matter what the slope, the equations of parallel lines will share the same slope, but different intercepts.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to find the equation of a parallel line that passes through a given point.

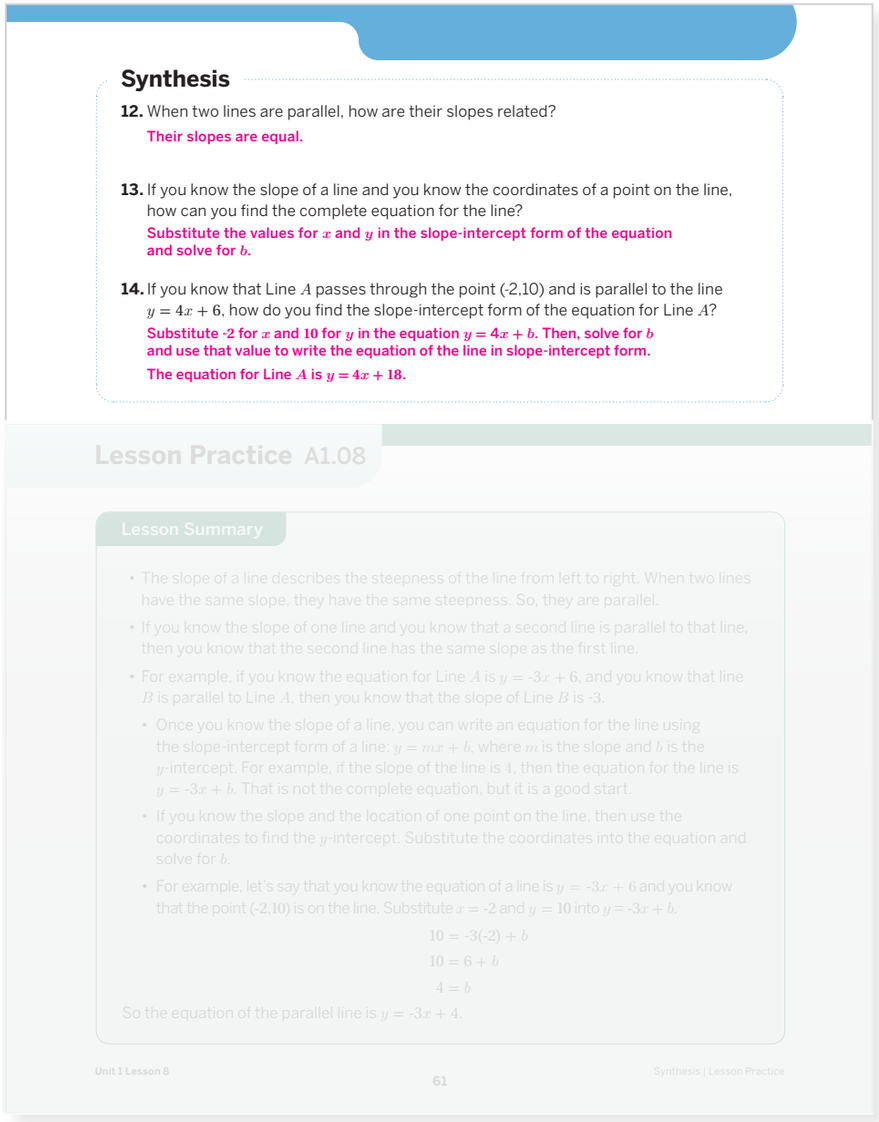
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Parallel lines have different intercepts but same slopes.
- Parallel lines can have slopes that are positive, negative, zero, or undefined.

MLR **MLR2: Collect and Display** Use this routine to formalize their strategies for finding equations of parallel lines using students' language. Display any new language, tables, or graphs publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion



Synthesis

12. When two lines are parallel, how are their slopes related?

Their slopes are equal.

13. If you know the slope of a line and you know the coordinates of a point on the line, how can you find the complete equation for the line?

Substitute the values for x and y in the slope-intercept form of the equation and solve for b.

14. If you know that Line A passes through the point (-2,10) and is parallel to the line $y = 4x + 6$, how do you find the slope-intercept form of the equation for Line A?

Substitute -2 for x and 10 for y in the equation $y = 4x + b$. Then, solve for b and use that value to write the equation of the line in slope-intercept form.

The equation for Line A is $y = 4x + 18$.

Lesson Practice A1.08

Lesson Summary

- The slope of a line describes the steepness of the line from left to right. When two lines have the same slope, they have the same steepness. So, they are parallel.
- If you know the slope of one line and you know that a second line is parallel to that line, then you know that the second line has the same slope as the first line.
- For example, if you know the equation for Line A is $y = -3x + 6$, and you know that line B is parallel to Line A, then you know that the slope of Line B is -3.
  - Once you know the slope of a line, you can write an equation for the line using the slope-intercept form of a line: $y = mx + b$, where m is the slope and b is the y-intercept. For example, if the slope of the line is 4, then the equation for the line is $y = -3x + b$. That is not the complete equation, but it is a good start.
  - If you know the slope and the location of one point on the line, then use the coordinates to find the y-intercept. Substitute the coordinates into the equation and solve for b.
  - For example, let's say that you know the equation of a line is $y = -3x + 6$ and you know that the point (-2,10) is on the line. Substitute $x = -2$ and $y = 10$ into $y = -3x + b$.

$$10 = -3(-2) + b$$
$$10 = 6 + b$$
$$4 = b$$

So the equation of the parallel line is $y = -3x + 4$.

Unit 1 Lesson 8 61 Synthesis | Lesson Practice

Lesson Takeaway: The equations of parallel lines share the same slope but have different intercepts. The parallel line equation can be found by using the slope of the given line.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by finding the equations of parallel lines by using the information provided about the given lines and describe their meaning in context. **(MTR.4.1)**

Today's Goals

1. **Goal:** Find the equation of a line through a given point that is parallel to a given line.
2. **Goal:** Interpret and apply equations of parallel lines using real-world context.
3. **Language Goal:** Explain the relationship between two lines that are parallel, describing how they are alike and how they are different. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.08

The equation for Line S is $y = 2x - 7$.

a. What is the slope of a line parallel to Line S?
2

b. What is the equation of the line parallel to Line S that passes through the point (3, -5)?
$y = 2x - 11$

c. What is the equation of the line parallel to Line S that passes through the origin?
$y = 2x$

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support writing the equation of a line parallel to another through a given point:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create a table of values for 3 different lines, 2 of which are parallel and one that is not, using a rational slope with a denominator other than 1 or -1. Then trade their creations with a partner to determine which lines are the parallel ones.

Professional Learning

Reflect on students' responses in Activity 3 where they found equations of parallel lines and wrote contexts for the parallel lines. How did student pairs support each other in creating and revising contexts? Which students struggled to create contexts? In what ways did you offer effective, scaffolded support?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. When two lines are parallel, how are their slopes related?

Their slopes are equal.

13. If you know the slope of a line and you know the coordinates of a point on the line, how can you find the complete equation for the line?

Substitute the values for x and y in the slope-intercept form of the equation and solve for b.

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The equation for Line A is $y = 4x + 18$.

Lesson Practice A1.08

Lesson Summary

- The slope of a line describes the steepness of the line from left to right. When two lines have the same slope, they have the same steepness. So, they are parallel.
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$$10 = -3(-2) + b$$
$$10 = 6 + b$$
$$4 = b$$

So the equation of the parallel line is $y = -3x + 4$.

Lesson Practice A1.08

Name: ..... Date: ..... Period: .....

Problems 1–2: Determine which line is parallel to the given line.

1. Show how to algebraically find the slope intercept form for the equation of the line that is parallel to $3x + 3y = -18$ and goes through the point (20,-13).

Given line:
$3x + 3y = -18$
$x + y = -6$
$y = -x - 6$
Slope: $m = -1$

Parallel line through (20,-13):
$y = -1x + b$
$-13 = -1(20) + b$
$-13 = -20 + b$
$7 = b$
Equation of parallel line: $y = -1x + 7$

2. Which line is parallel to $3x - 4y = 24$?

A. $y = 3x + 4$ **B.** $y = 6 - \frac{4}{3}x$

C. $y = \frac{4}{3}x - 2$ **D.** $y = 2 + \frac{3}{4}x$

Problems 3–5: Find the equation of the line in slope-intercept form that meets the given conditions.

3. A line that passes through (2,6) and is parallel to $y = 3x + 15$.

$y = 3x$

4. A line that passes through (-10,-8) and is parallel to $x - 2y = 1$.

$y = 0.5x - 3$ or $y = \frac{1}{2}x - 3$

5. A line that passes through (-4,0) and is parallel to the line that passes through (2,12) and (-1,3).

$y = 3x + 12$

6. Road H passes through an intersection defined on a grid as (7,9). Road H is parallel to road M. Road M passes through (2,3) and (9,3). What are the slopes and equations for these two roads?

Slope:	Equation for road	Equation for road
$m =$ 0	$H:$ $y = 9$ or $y = 0x + 9$	$M:$ $y = 3$ or $y = 0x + 3$

Lesson Practice A1.08

Name: ..... Date: ..... Period: .....

Problems 7–8: The populations of three small towns have been growing at consistent rates over the last 6 years. One of the towns in the table is growing at the same rate as Town C. Town C recorded a population of 532 in year 2 and a population of 556 in year 5. Use the information in the table provided.

	Town A	Town B
Year 1	1,215	893
Year 3	1,237	909
Year 6	1,259	925

7. What is the rate of change in population and the equation, in slope-intercept form, that represents the population growth, G, of Town C?

Rate of change: 8 people per year Equation: $G = 8x + 516$

8. What is the equation that defines population, P, for any year, x, for the town with a population growth like Town C?

Town B is parallel to Town C and its population is modeled by the equation $P = 8x + 885$.

Test Practice

9. Which line passes through (-6,5) and is parallel to $2x + y = 9$?

A. $y = -2x + 7$ **B.** $y = 2x - 7$ **C.** $y = 2x + 7$ **D.** $y = -2x - 7$

Spiral Review

10. Select *all* the equations that are equivalent to $\frac{2}{3}x - \frac{1}{6}y = \frac{1}{9}$.

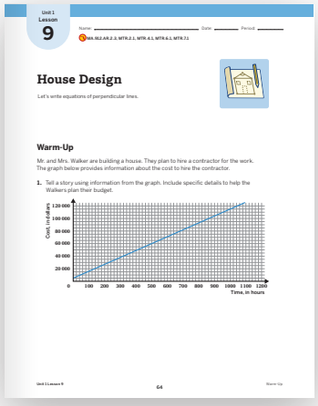
☐ **A.** $-6x + \frac{2}{3}y = 1$ ☐ **B.** $3y = -12x - 2$ ☑ **C.** $-3x + \frac{3}{4}y - \frac{1}{2} = 0$

☑ **D.** $6y - 24x = 4$ ☐ **E.** $y = -4x - \frac{2}{3}$ ☑ **F.** $y = 4x + \frac{2}{3}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	2	1	MA.912.AR.2.3
	1, 3, 4–6	2	MA.912.AR.2.3
	7, 8	3	MA.912.AR.2.3
Test Practice	9	2	MA.912.AR.2.3
Spiral Review			
	10	2	MA.912.AR.1.2

Unit 1
Lesson
9


Unit 1
Lesson
9

House Design

Let's write equations of perpendicular lines.

Warm-Up

Mr. and Mrs. Walker are building a house. They plan to hire a contractor for the work. The graph below provides information about the cost to hire the contractor.

1. Tell a story using information from the graph. Include specific details to help the Walkers plan their budget.

Time, in hours

64

House Design

Finding Equations of Perpendicular Lines

Let's write equations of perpendicular lines.

Learning Outcomes

Today's Goal

1. **Goal:** Find the equation of a line perpendicular to a given line through a specified point.
2. **Goal:** Interpret and apply perpendicular line equations in a real-world context.
3. **Language Goal:** Describe the relationship between two perpendicular lines, including the angle at which they intersect. **(Reading, Writing, Speaking, and Listening)**

Students find equations of lines perpendicular to a given line and passing through a given point. Students apply what they have learned about writing equations of lines in slope-intercept form and develop their understanding of how perpendicular lines can model real-world contexts.

Coherence

Prior Learning

In Lesson 8, students found equations of lines parallel to a given line and passing through a given point. Students applied what they have learned about writing equations in slope-intercept form and developed their understanding of how parallel lines can model real-world contexts.

Future Learning

In Lesson 10, students will expand their knowledge by exploring situations that are better represented by inequalities than equations and interpret their solutions in context.

Connections

- Students use equations in a real-world context as they examine different views of a house and evaluate the costs to hire a contractor. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.3
Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.

Also Addressing: **MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MA.912.AR.2.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.2	**MA.912.AR.9.1**
MA.912.AR.2.4	**MA.912.AR.9.6**

Vocabulary

New Vocabulary

perpendicular lines

Review Vocabulary

slope	*slope-intercept form*
y-intercept	*parallel lines*

Lesson at a Glance

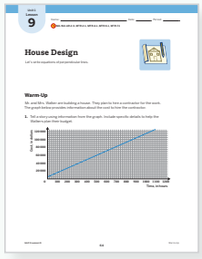
~ 45 min

MA.912.AR.2.3, MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MA.912.AR.2.5

Warm-Up

Small Groups | 5 min

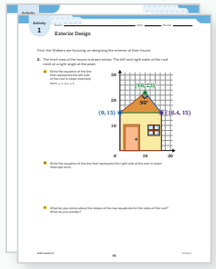
Students use the Tell a Story routine to interpret a graph in the context of cost to hire a contractor.
Routine: Tell a Story



Activity 1

Pairs | 15 min

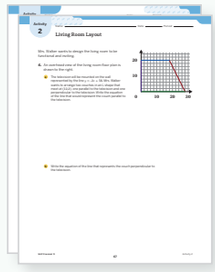
Students write equations of perpendicular lines and identify their relationships. **(MTR4.1)**



Activity 2

Pairs | 15 min

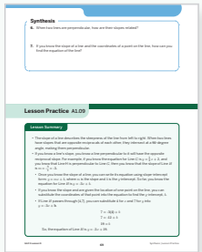
Students use the Think-Pair-Share routine to visualize the context and write equations of parallel and perpendicular lines.
Routine: Think-Pair-Share



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to find the equation of a perpendicular line that passes through a given point. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equation of a line perpendicular to another through a given point.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Small Groups | 5 min

Warm-Up

Purpose: Students use the Tell a Story routine to interpret a graph in the context of cost to hire a contractor. **(MTR.7.1)**

1 Launch

Consider asking:

- "What do you think contractors do? What types of jobs might they specialize in?"
- "What skills do you think are important for someone working as a contractor?"
- "Can you think of some ways math is used when designing and building a house?"

Invite students to share stories about their experiences with contractors, construction, or design.

Use the Tell a Story routine to help students make sense of the graph in the context. **(MTR.7.1)**

Discuss how the graph would change if the contractor charged a different base price or hourly rate.

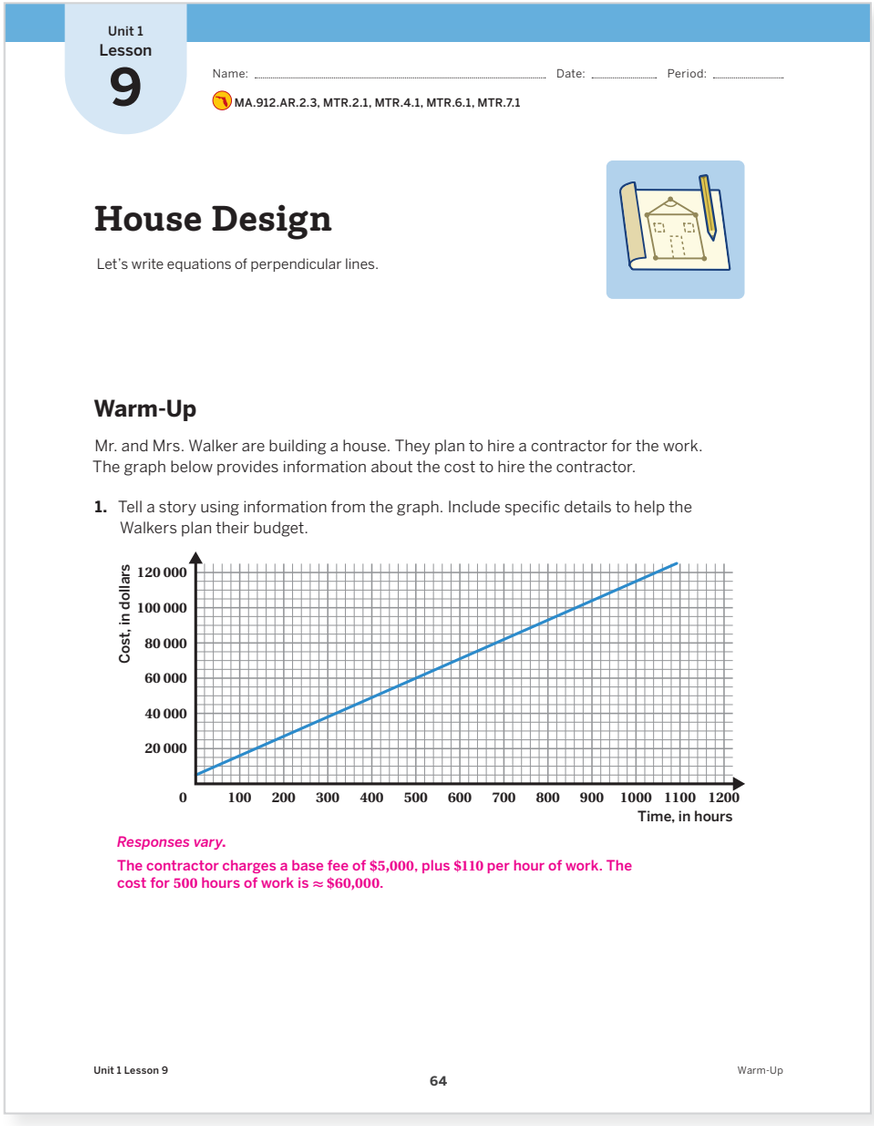
A **Accessibility: Memory and Attention** To activate prior knowledge, consider asking, "What information have we used earlier that might help us now?"

2 Connect

To generate excitement, consider asking students to imagine they owned their own business.

Consider asking:

- "What type of business would you start and why?"
- "Can you come up with 3 ways you would use math in your business?"
- "Can you create an equation and graph in the context of your business?" **(MTR.7.1)**
- "What strategy did you use to choose realistic numbers for your equation in the context of your business?" **(MTR.6.1)**



Unit 1
Lesson
9

Name: _____ Date: _____ Period: _____

MA.912.AR.2.3, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

House Design

Let's write equations of perpendicular lines.

Warm-Up

Mr. and Mrs. Walker are building a house. They plan to hire a contractor for the work. The graph below provides information about the cost to hire the contractor.

1. Tell a story using information from the graph. Include specific details to help the Walkers plan their budget.

Responses vary.

The contractor charges a base fee of $5,000, plus $110 per hour of work. The cost for 500 hours of work is ≈ $60,000.

Unit 1 Lesson 9 64 Warm-Up

Pairs | 15 min

Activity 1 Exterior Design

Purpose: Students write equations of perpendicular lines and identify their relationships. **(MTR.4.1)**

Short on time? Consider omitting Problem 2c.

1 Launch

Consider asking, "Why is it important to make thorough plans before starting a building project?"

2 Monitor

To support students getting started, share the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, and discuss which points they should substitute in.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Look for students completing the arithmetic by hand by converting mixed numbers to improper fractions.

D **Differentiation** *(Problem 2)*

Look for students who:	Respond to Student Thinking
Find the slope by using the slope formula. (Problem 2b)	**Strengthen** Invite students to find the slope another way, by using what they know about the relationship between the slopes of perpendicular lines.
Need support getting started. (Problem 2c)	**Support** Consider asking, "How might drawing or graphing the situation help you?"
Are unsure of how to visualize the problem. (Problem 2c)	**Support** Invite students to graph the point and extend the left side of the garage to visualize where the new peak of the roof would be.
Actively listen to each other.	**Strengthen** Celebrate these students one-on-one or publicly.

Activity 1 continued >

Activity 1

Name: Date: Period:

Exterior Design

First, the Walkers are focusing on designing the exterior of their house.

2. The front view of the house is drawn below. The left and right sides of the roof meet at a right angle at the peak.

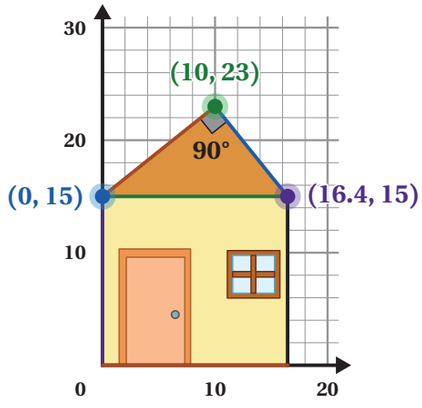
a Write the equation of the line that represents the left side of the roof in slope-intercept form, $y = mx + b$.

Use the points to find the slope of the line:

$m = \frac{23 - 15}{10 - 0} = \frac{4}{5}$

Write the equation using the known y-intercept:

$y = \frac{4}{5}x + 15$



b Write the equation of the line that represents the right side of the roof in slope-intercept form.

Use the points to find the slope of the line:

$m = \frac{15 - 23}{16.4 - 10} = \frac{-8}{6.4} = \frac{-8}{6\frac{2}{5}} = \frac{-8}{\frac{32}{5}} = -8 \cdot \frac{5}{32} = \frac{-40}{32} = -\frac{5}{4}$

Find the y-intercept by substituting in a point: $15 = -\frac{5}{4}(16.4) + b \rightarrow b = 35.5$

Write the equation: $y = -\frac{5}{4}x + 35.5$

c What do you notice about the slopes of the two equations for the sides of the roof? What do you wonder?

Responses vary. The slopes are opposite reciprocals of each other. I wonder what else I can learn about a line based on its slope.

Unit 1 Lesson 9 65 Activity 1

Pairs | 15 min

Activity 1 Exterior Design (continued)

Purpose: Students write equations of perpendicular lines and identify their relationships. **(MTR.4.1)**

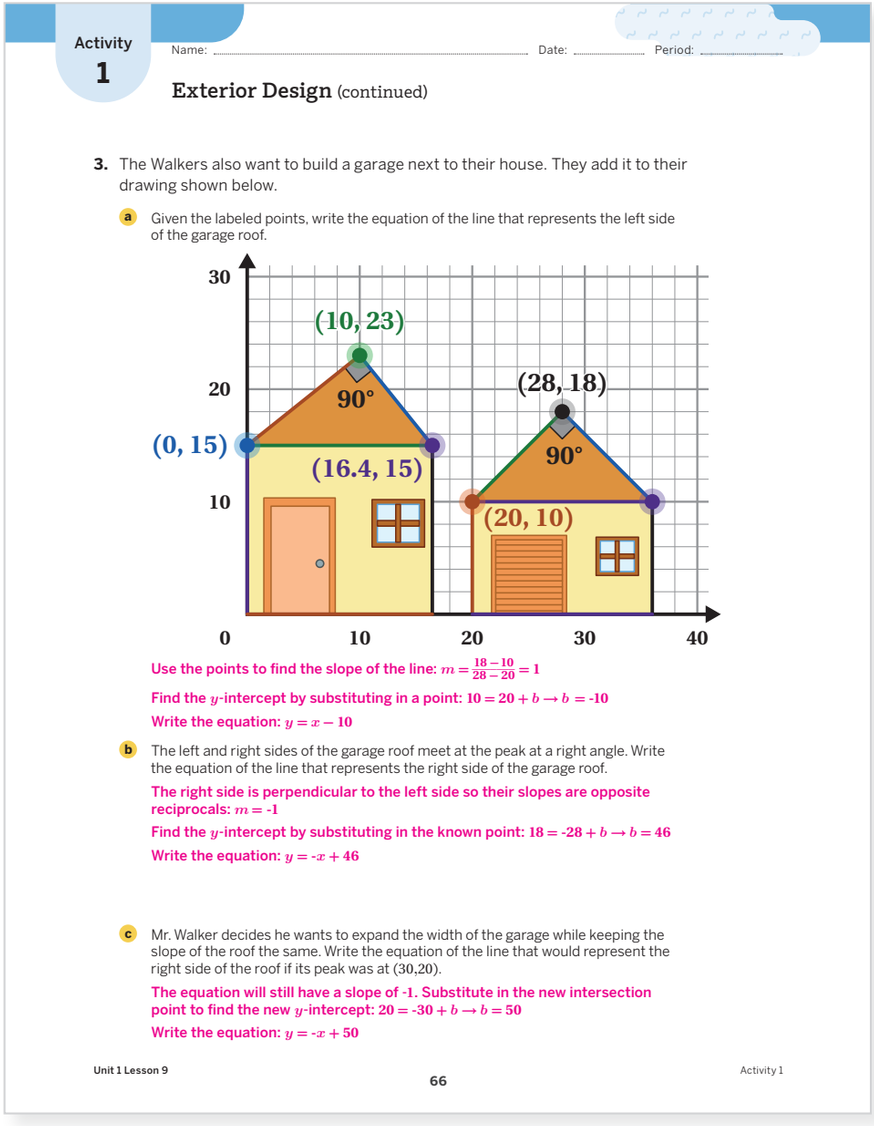
3 Connect

Share the strategy of converting mixed numbers to improper fractions for when students don't have access to a calculator.

Discuss the benefit of practicing mental arithmetic, and only using a calculator for more complex calculations.

To surface the Key Takeaway, consider asking: **(MTR.4.1)**

- "Can you explain why perpendicular lines will always have opposite reciprocal slopes?"
- "Do you have any tricks for remembering the difference between parallel and perpendicular lines and the relationships between their slopes?"



Activity 1

Name: Date: Period:

Exterior Design (continued)

3. The Walkers also want to build a garage next to their house. They add it to their drawing shown below.

a. Given the labeled points, write the equation of the line that represents the left side of the garage roof.

Use the points to find the slope of the line: $m = \frac{18-10}{28-20} = 1$

Find the y-intercept by substituting in a point: $10 = 20 + b \rightarrow b = -10$

Write the equation: $y = x - 10$

b. The left and right sides of the garage roof meet at the peak at a right angle. Write the equation of the line that represents the right side of the garage roof.

The right side is perpendicular to the left side so their slopes are opposite reciprocals: $m = -1$

Find the y-intercept by substituting in the known point: $18 = -28 + b \rightarrow b = 46$

Write the equation: $y = -x + 46$

c. Mr. Walker decides he wants to expand the width of the garage while keeping the slope of the roof the same. Write the equation of the line that would represent the right side of the roof if its peak was at (30,20).

The equation will still have a slope of -1. Substitute in the new intersection point to find the new y-intercept: $20 = -30 + b \rightarrow b = 50$

Write the equation: $y = -x + 50$

Unit 1 Lesson 9 66 Activity 1

Key Takeaway: Perpendicular lines intersect at a 90-degree angle, and their slopes are opposite reciprocals.

Pairs | 15 min

Activity 2 Living Room Layout

Purpose: Students use the Think-Pair-Share routine to visualize the context and write equations of parallel and perpendicular lines.

1 Launch

Invite students to share their experiences arranging furniture with a partner.

2 Monitor

Use the Think-Pair-Share routine to support collaboration and ensure students are on the right track. Invite students to spend time independently mapping out where each piece of furniture would lie in the room, then comparing with a partner.

A **Accessibility: Visual-Spatial Processing** Provide graph paper for students to draw on to increase access to the task.

ELL **English Language Learners** Provide a reference that includes descriptions and visuals of each piece of furniture. **(Reading, Writing, Speaking, and Listening)**

Encourage students to show their thinking.

Listen for students discussing characterizations of different shapes.

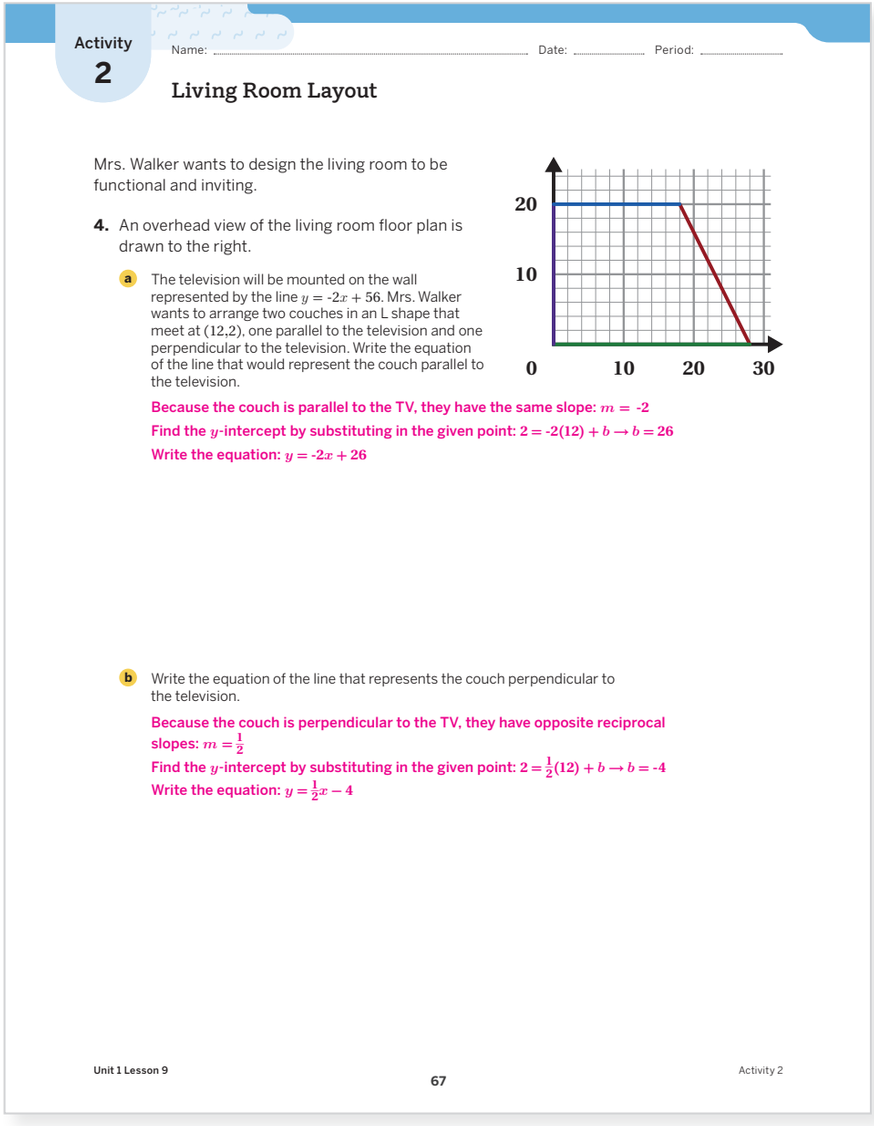
Pause to review the definitions of quadrilateral, trapezoid, kite, parallelogram, rectangle, rhombus, and square, and discuss how they are related to each other.

3 Connect

Invite students to share how sketching helps them solve the problems.

Encourage students to share which problem was the most challenging and how they solved it.

Activity 2 continued >



Activity 2

Name: Date: Period:

Living Room Layout

Mrs. Walker wants to design the living room to be functional and inviting.

4. An overhead view of the living room floor plan is drawn to the right.

a. The television will be mounted on the wall represented by the line $y = -2x + 56$. Mrs. Walker wants to arrange two couches in an L shape that meet at (12,2), one parallel to the television and one perpendicular to the television. Write the equation of the line that would represent the couch parallel to the television.

Because the couch is parallel to the TV, they have the same slope: $m = -2$
Find the y-intercept by substituting in the given point: $2 = -2(12) + b \rightarrow b = 26$
Write the equation: $y = -2x + 26$

b. Write the equation of the line that represents the couch perpendicular to the television.

Because the couch is perpendicular to the TV, they have opposite reciprocal slopes: $m = \frac{1}{2}$
Find the y-intercept by substituting in the given point: $2 = \frac{1}{2}(12) + b \rightarrow b = -4$
Write the equation: $y = \frac{1}{2}x - 4$

Unit 1 Lesson 9 67 Activity 2

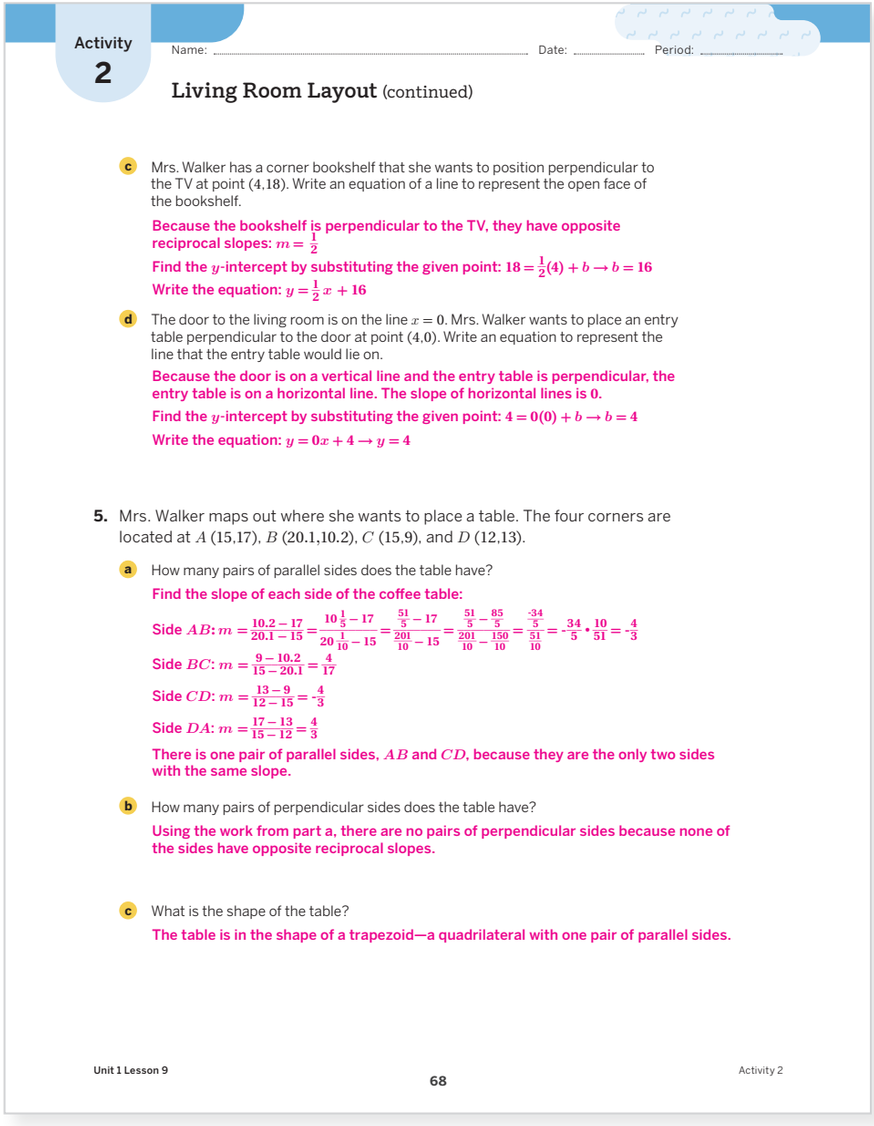
Pairs | 15 min

Activity 2 Living Room Layout (continued)

Purpose: Students use the Think-Pair-Share routine to visualize the context and write equations of parallel and perpendicular lines.

Consider asking:

- How can you apply the concept of parallel and perpendicular lines in real-world situations?
- How do you identify parallel and perpendicular lines from equations and graphs?
- Why are parallel and perpendicular lines essential in the design and construction shown in blueprints?
- How can parallel and perpendicular lines be identified in everyday objects and structures?



Activity 2

Name: Date: Period:

Living Room Layout (continued)

c. Mrs. Walker has a corner bookshelf that she wants to position perpendicular to the TV at point $(4,18)$. Write an equation of a line to represent the open face of the bookshelf.

Because the bookshelf is perpendicular to the TV, they have opposite reciprocal slopes: $m = \frac{1}{2}$

Find the y-intercept by substituting the given point: $18 = \frac{1}{2}(4) + b \rightarrow b = 16$

Write the equation: $y = \frac{1}{2}x + 16$

d. The door to the living room is on the line $x = 0$. Mrs. Walker wants to place an entry table perpendicular to the door at point $(4,0)$. Write an equation to represent the line that the entry table would lie on.

Because the door is on a vertical line and the entry table is perpendicular, the entry table is on a horizontal line. The slope of horizontal lines is 0.

Find the y-intercept by substituting the given point: $4 = 0(0) + b \rightarrow b = 4$

Write the equation: $y = 0x + 4 \rightarrow y = 4$

5. Mrs. Walker maps out where she wants to place a table. The four corners are located at A (15,17), B (20.1,10.2), C (15,9), and D (12,13).

a. How many pairs of parallel sides does the table have?

Find the slope of each side of the coffee table:

Side AB: $m = \frac{10.2 - 17}{20.1 - 15} = \frac{10\frac{1}{5} - 17}{20\frac{1}{10} - 15} = \frac{\frac{51}{5} - 17}{\frac{201}{10} - 15} = \frac{\frac{51}{5} - \frac{85}{5}}{\frac{201}{10} - \frac{150}{10}} = \frac{\frac{-34}{5}}{\frac{51}{10}} = -\frac{34}{5} \cdot \frac{10}{51} = -\frac{4}{3}$

Side BC: $m = \frac{9 - 10.2}{15 - 20.1} = \frac{4}{17}$

Side CD: $m = \frac{13 - 9}{12 - 15} = -\frac{4}{3}$

Side DA: $m = \frac{17 - 13}{15 - 12} = \frac{4}{3}$

There is one pair of parallel sides, AB and CD, because they are the only two sides with the same slope.

b. How many pairs of perpendicular sides does the table have?

Using the work from part a, there are no pairs of perpendicular sides because none of the sides have opposite reciprocal slopes.

c. What is the shape of the table?

The table is in the shape of a trapezoid—a quadrilateral with one pair of parallel sides.

Unit 1 Lesson 9 68 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to find the equation of a perpendicular line that passes through a given point.

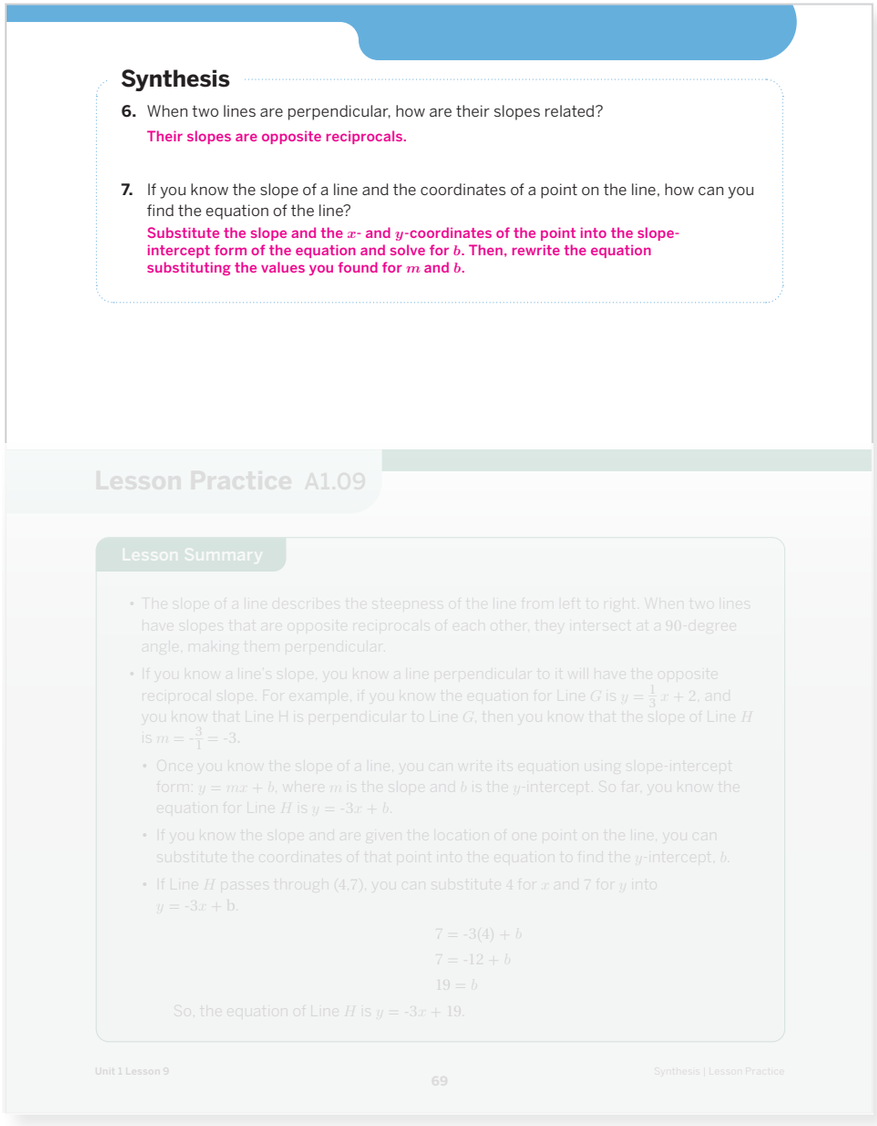
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Perpendicular lines have different y-intercepts and opposite reciprocal slopes.
- Perpendicular lines can have slopes that are positive, negative, zero, or undefined.

MLR **MLR2: Collect and Display** Use this routine to formalize strategies for finding equations of perpendicular lines using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

6. When two lines are perpendicular, how are their slopes related?

Their slopes are opposite reciprocals.

7. If you know the slope of a line and the coordinates of a point on the line, how can you find the equation of the line?

Substitute the slope and the x- and y-coordinates of the point into the slope-intercept form of the equation and solve for b. Then, rewrite the equation substituting the values you found for m and b.

Lesson Practice A1.09

Lesson Summary

- The slope of a line describes the steepness of the line from left to right. When two lines have slopes that are opposite reciprocals of each other, they intersect at a 90-degree angle, making them perpendicular.
- If you know a line's slope, you know a line perpendicular to it will have the opposite reciprocal slope. For example, if you know the equation for Line G is $y = \frac{1}{3}x + 2$, and you know that Line H is perpendicular to Line G, then you know that the slope of Line H is $m = -\frac{3}{1} = -3$.
- Once you know the slope of a line, you can write its equation using slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept. So far, you know the equation for Line H is $y = -3x + b$.
- If you know the slope and are given the location of one point on the line, you can substitute the coordinates of that point into the equation to find the y-intercept, b.
- If Line H passes through (4,7), you can substitute 4 for x and 7 for y into $y = -3x + b$.

$$7 = -3(4) + b$$
$$7 = -12 + b$$
$$19 = b$$

So, the equation of Line H is $y = -3x + 19$.

Unit 1 Lesson 9 69 Synthesis | Lesson Practice

Lesson Takeaway: Perpendicular lines have opposite reciprocal slopes. The equation of a line can be found using a point on the line and the slope of a perpendicular line.

Independent | 5 min

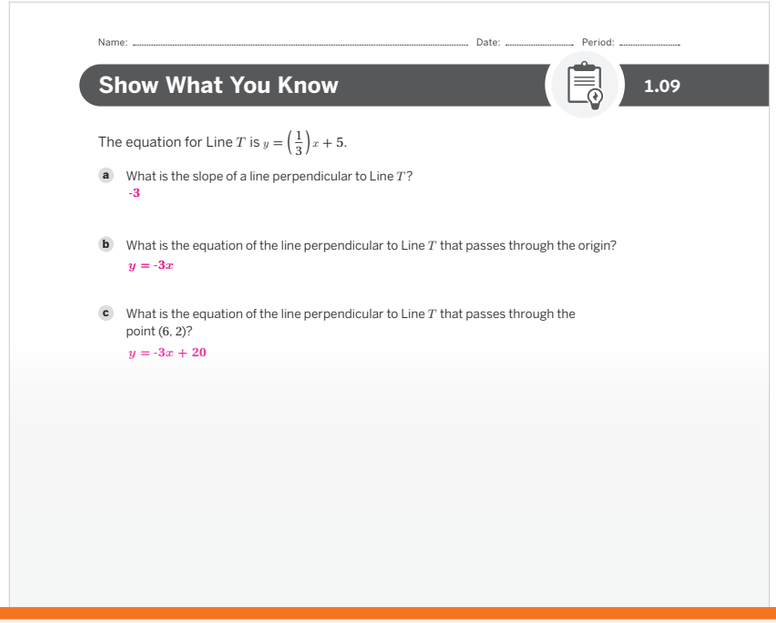
Show What You Know

Purpose: Students demonstrate their understanding by writing an equation of a line perpendicular to another through a given point.

Today's Goals

1. **Goal:** Find the equation of a line perpendicular to a given line through a specified point.
2. **Goal:** Interpret and apply perpendicular line equations in a real-world context.
3. **Language Goal:** Describe the relationship between two perpendicular lines, including the angle at which they intersect. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.09

The equation for Line T is $y = \left(\frac{1}{3}\right)x + 5$.

a. What is the slope of a line perpendicular to Line T?
-3

b. What is the equation of the line perpendicular to Line T that passes through the origin?
$y = -3x$

c. What is the equation of the line perpendicular to Line T that passes through the point $(6, 2)$?
$y = -3x + 20$

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support writing the equation of a line perpendicular to another through a given point:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create a table of values for 3 different lines, 2 of which are perpendicular and one that is not, using a rational slope with a denominator other than 1 or -1. Then trade their creations with a partner to determine which lines are the perpendicular ones.

Professional Learning

Reflect on the students' strategies in Activity 1 where they visualized a context to create equations of perpendicular lines. How did student pairs support each other in visualizing the context correctly? In what ways did you offer effective, scaffolded support?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. When two lines are perpendicular, how are their slopes related?

Their slopes are opposite reciprocals.

7. If you know the slope of a line and the coordinates of a point on the line, how can you find the equation of the line?

Substitute the slope and the x- and y-coordinates of the point into the slope-intercept form of the equation and solve for b. Then, rewrite the equation substituting the values you found for m and b.

Lesson Practice A1.09

Lesson Summary

- The slope of a line describes the steepness of the line from left to right. When two lines have slopes that are opposite reciprocals of each other, they intersect at a 90-degree angle, making them perpendicular.
- If you know a line's slope, you know a line perpendicular to it will have the opposite reciprocal slope. For example, if you know the equation for Line G is $y = \frac{1}{3}x + 2$, and you know that Line H is perpendicular to Line G, then you know that the slope of Line H is $m = -\frac{3}{1} = -3$.
  - Once you know the slope of a line, you can write its equation using slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept. So far, you know the equation for Line H is $y = -3x + b$.
  - If you know the slope and are given the location of one point on the line, you can substitute the coordinates of that point into the equation to find the y-intercept, b.
  - If Line H passes through (4,7), you can substitute 4 for x and 7 for y into $y = -3x + b$.

$$7 = -3(4) + b$$
$$7 = -12 + b$$
$$19 = b$$

So, the equation of Line H is $y = -3x + 19$.

Lesson Practice A1.09

Name: ..... Date: ..... Period: .....

1. Without graphing, find the slope-intercept equation of the line perpendicular to $4x - 12y = 32$ and passing through (-6,19).

Given line:
$4x - 12y = 32$
$4(x - 3y) = 32$
$x - 3y = 8$
$3y = x - 8$
$y = \frac{1}{3}x - \frac{8}{3}$

Slope: $m = \frac{1}{3}$

Perpendicular line through (-6,19):
Opposite reciprocal slope: $m = -3$
Substitute point:
$19 = -3(-6) + b$
$19 = 18 + b$
$b = 1$

Equation of perpendicular line:
$y = -3x + 1$

2. Which line is perpendicular to $y = 8x + \frac{1}{2}$?

A. $y = \frac{1}{2}x + 8$
B. $y = 8x - \frac{1}{2}$
C. $y = -\frac{1}{8}x - 2$
D. $y = -8x + 2$

Problems 3–4: Triangle JKL has vertices J (0,4), K (1,2), and L (4,6).

3. Determine whether triangle JKL is a right triangle. Explain.

Draw or visualize the triangle. It looks like vertex J might be at a right angle.
Determine whether sides JL and JK are perpendicular to each other:
Find the slope of JL: $m = \frac{6-4}{4-0} = \frac{2}{4} = \frac{1}{2}$ Find the slope of JK: $m = \frac{4-2}{0-1} = \frac{2}{-1} = -2$
Because JL and JK have opposite reciprocal slopes, those lines are perpendicular. Therefore, vertex J is at a 90-degree angle and triangle JKL is a right triangle.

4. Imagine you moved vertex J so that triangle JKL now formed a right angle at vertex K. Find the new equation for line JK.

For vertex K to be at a 90-degree angle, JK must be perpendicular to KL.
Find the slope of KL: $m = \frac{6-2}{4-1} = \frac{4}{3}$
The new slope of JK must be the opposite reciprocal: $m = -\frac{3}{4}$
Because you know K is on the line JK, you can substitute the point and solve for b: $2 = -\frac{3}{4}(1) + b \rightarrow b = \frac{11}{4}$
The new equation for line JK would be: $y = -\frac{3}{4}x + \frac{11}{4}$

Lesson Practice A1.09

Name: ..... Date: ..... Period: .....

5. Find the equation of Line M that passes through (-3,2) and (-3,7). Then, find the equation of Line N which is perpendicular to Line M and passes through (6,-1).

Equation for Line M: $x = -3$ Equation for Line N $y = -1$

Line M is vertical with an undefined slope. Line N is horizontal (perpendicular to Line M) with a slope of 0.

Test Practice

6. Consider line L_1 with the equation $5x + 2y = 14$. A second line, L_2, is perpendicular to L_1 and passes through the point (-5,3).

Select *all* of the true statements.

- ☑ A. The slope of L_1 is $-\frac{5}{2}$.
- ☑ B. The slope of L_2 is $\frac{2}{5}$.
- ☐ C. The y-intercept of L_1 is 14.
- ☑ D. The equation for L_2 is $y = \frac{2}{5}x + 5$.
- ☑ E. Lines L_1 and L_2 intersect at a 90-degree angle.

Spiral Review

7. Select *all* the equations that are equivalent to $\frac{1}{8}x + \frac{3}{4}y = \frac{1}{12}$.

- ☑ A. $4 = 6x + 36y$
- ☑ B. $3y = \frac{1}{3} - \frac{1}{2}x$
- ☐ C. $\frac{4}{3}y - \frac{1}{12} = 8x$
- ☑ D. $6y = \frac{2}{3} - x$
- ☐ E. $x = 3y + \frac{2}{3}$
- ☑ F. $x + 6y - \frac{2}{3} = 0$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	2	1	MA.912.AR.2.3
	1, 5	2	MA.912.AR.2.3
	3–4	3	MA.912.AR.2.3
Test Practice	6	2	MA.912.AR.2.3
Spiral Review			
Fluency	7	2	MA.912.AR.2.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying solutions using a situation or a graph	Lesson 6–7	2	MA.912.AR.2.5
2	Identifying equivalent equations and justifying the reasoning	Lesson 3	2	MA.912.AR.2.1
3a, 3c	Using the structure of one-variable equations to solve them	Lesson 1–2	1	MA.912.AR.2.1
3b, 3d	Using the structure of multi-variable equations to solve for one variable	Lesson 4–5	2	MA.912.AR.1.2
4	Solving one-variable equations and interpret solutions in context	Lesson 3	2	MA.912.AR.2.1, MTR.6.1
5a	Using an equation in two variables to represent a relationship in context	Lesson 7	3	MA.912.AR.2.2, MTR.6.1
5b	Representing a two-variable situation graphically	Lesson 6–7	2	MA.912.AR.2.5
5c	Determining whether or not a situation represents a solution and justifying the reasoning	Lesson 6	3	MA.912.AR.2.5, MTR.4.1
6a	Write a linear equation parallel to a given linear equation	Lesson 8	1	MA.912.AR.2.3
6b	Write a linear equation perpendicular to a given linear equation	Lesson 9	1	MA.912.AR.2.3

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

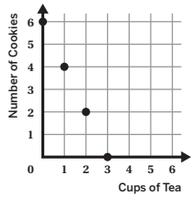
Name: Date: Period:

Sub-Unit Quiz

Unit 1

1. A store sells cups of tea for $2 and cookies for $1 each. A customer wants to spend $6 in total.

Here is a graph of this situation. Which of these is a solution?



A. 0 cups of tea and 0 cookies

B. 0 cups of tea and 3 cookies

C. 3 cups of tea and 0 cookies

D. 6 cups of tea and 0 cookies

2. Which equation is equivalent to $3(x - 4) = 45$?

A. $3x - 4 = 45$ B. $x - 4 = 42$ C. $3x = 49$ D. $x - 4 = 15$

Explain or show how you know that the equation you selected is equivalent to $3(x - 4) = 45$. *Explanations vary.* If I divide both sides of the equation by 3, then I get $x - 4 = 15$. I know they are equivalent because the solution to both equations is $x = 19$.

3. Solve each equation.

a. Solve for a: $21 = 3a$
$a = 7$

b. Solve for R: $V = IR$
$\frac{V}{I} = R$ (or equivalent)

c. Solve for b: $7b - 4 = 10$
$b = 2$

d. Solve for x: $ax - c = d$
$x = \frac{d + c}{a}$ (or equivalent)

4. A shirt printing company charges a $50 setup fee and $3 per shirt.



A school club has $86 for shirts.

Xavier wrote $50 + 3x = 86$ to describe the situation.

a. Solve this equation for x.
$x = 12$

b. Explain the meaning of the solution.
Responses vary. The solution means that the club can buy 12 printed shirts.

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 1

5. Tickets to a zoo cost $12 for each adult and $8 for each child. A family has $72 for the zoo trip.

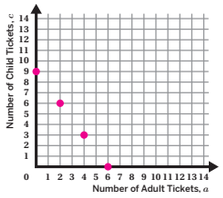


a. Write an equation for this relationship.
- Use a for the number of adult tickets.
- Use c for the number of child tickets.

$12a + 8c = 72$

b. Graph this relationship. Use the table if it helps with your thinking.

Number of Adult Tickets, a	Number of Child Tickets, c



c. Elena says that if 2 adults go to the zoo, then 7 children can also go for $72. Explain or show how you know this is incorrect.
Responses vary. I know that this is incorrect because the point is not on the graph representing this relationship.

6. The equation of a line is given: $x + 2y = 8$.

a. Which of these linear equations are parallel to the given linear equation?

A. $y = \frac{1}{2}x + 4$

B. $y = -\frac{1}{2}x - 3$

C. $y = 2x - 4$

D. $y = -2x + 3$

b. Which of these lines are perpendicular to the given linear equation?

A. $y = \frac{1}{2}x + 4$

B. $y = -\frac{1}{2}x - 3$

C. $y = 2x - 4$

D. $y = -2x + 3$

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem (s)	To respond to student thinking, consider:	
• Convert quadratic expressions between factored and standard form by multiplying and factoring. **(Lessons 1–5)**	1	**Support**	• **Teacher Move:** Consider revisiting Lesson 3 (*Standard Feature*)
		Stretch	• **You're invited to explore more:** • Lesson 3 (*Standard Feature*)
	2	**Support**	• **Teacher Move:** Consider revisiting Lesson 2 (*Two Factor Multiplication*)
	4	**Support**	• **Teacher Move:** Consider revisiting Problems 6–9 in Lesson 4 (*X-Factor*) to discuss how to use an area model to factor
• Use factoring and the zero-product property to solve equations and determine the zeros of quadratic functions. **(Lessons 6–9)**	3, 5	**Support**	• **Teacher Move:** Consider revisiting Aba's Strategy in Lesson 8 (*Shooting Stars*) to discuss how to use the zero-product property to determine zeros and solve quadratic equations.
		Stretch	• **You're invited to explore more:** • Lesson 8 (*Shooting Stars*) • Lesson 9 (*Make It Zero*)

One-Variable and Two-Variable Inequalities

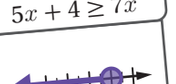
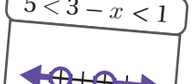
Sub-Unit 3 Goals

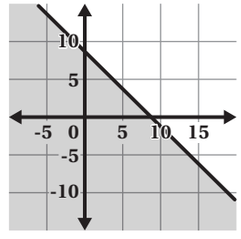
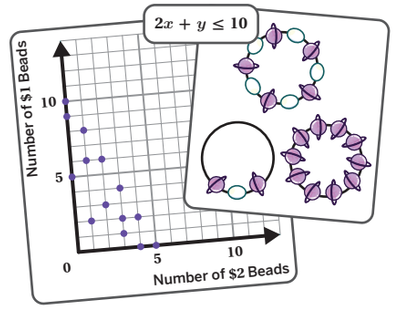
- **Goal:** Determine solutions to an inequality algebraically and graphically.
- **Goal:** Write inequalities in one and two variables to represent constraints.
- **Goal:** Solve a one-variable linear compound inequality when given a mathematical or real-world concept and represent the solutions algebraically or graphically.
- **Language Goal:** Interpret the solutions to one- and two-variable inequalities in context. **(Reading, Writing, and Speaking)**

In this sub-unit, students write inequalities in one- and two-variables to represent constraints. They model constraints algebraically and graphically and interpret the solutions to inequalities in context, including **absolute value equations** and inequalities. Students consider how inequalities that represent constraints can be used as a tool to analyze issues in society and make decisions.



Math That Matters Most

Concept or Skill	For example . . .
Understand that the solutions to an inequality are the values that make the inequality true.	$9 > 2x - 4$ $9 = 2x - 4$ $13 = 2x$ $6.5 = x$ Test $x = 6$: $9 > 2(6) - 4$, $9 > 8$ ✓ $x = 7$: $9 > 2(7) - 4$, $9 > 10$ False!
Write one- and two-variable inequalities to represent constraints.	Amoli can spend as much as \$4 on oranges. Each orange, x, costs \$0.50 Which inequality represents this situation? **A.** $0.5x \leq 4$ **C.** $4x \leq 0.5$ **B.** $0.5x \geq 4$ **D.** $4x \geq 0.5$
Graph solutions to one-variable inequalities on a number line.	$5x + 4 \geq 7x$  $5 < 3 - x < 1$ 

Concept or Skill	For example . . .
Graph solutions to a two-variable inequality by shading the solution region.	$x + y \leq 9$ 
Interpret solutions to one- and two-variable inequalities in context.	Victor knows that $(2.5, 500)$ represents a solution to both the Hideout inequality and the Palace Arena inequality but says it is not realistic. Is Victor correct?
Model constraints with equations and inequalities.	

Unit 1

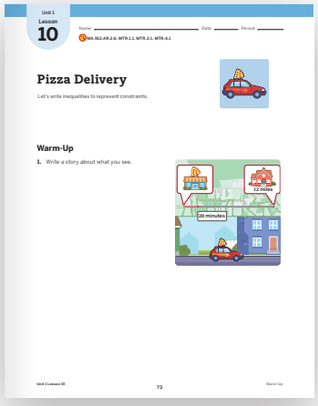
Lesson

10

Pizza Delivery

Representing Situations with One-Variable Inequalities

Let's write inequalities to represent constraints.


Unit 1
Lesson
10
Name _____ Date _____ Period _____
MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1
Pizza Delivery
Let's write inequalities to represent constraints.
Warm-Up
1. Write a story about what you see.
20 minutes
12 minutes
73

Learning Outcomes

Today's Goals

1. **Goal:** Understand that solutions to an inequality are values that make the inequality true.
2. **Goal:** Write one-variable inequalities that represent constraints.
3. **Language Goal:** Describe and interpret solutions to one-variable inequalities in context. **(Reading and Writing)**

Students explore connections between **constraints** and inequalities by analyzing several situations that involve constraints. They understand that solutions to inequalities are values that make the inequality true. In previous lessons in this unit, students worked with situations that were represented by equations. Students now explore situations that are best represented by inequalities and interpret their solutions in context. **(MTR.2.1)**

Coherence

Prior Learning

In Grade 7, students learned about one-variable inequalities, how to graph them on a number line, and how to interpret solutions to problems in context. In Lesson 6, students represented situations using equations and made connections between equations, graphs, tables, and situations.

Future Learning

In Lesson 11, students will use reasoning to determine solutions to a one-variable inequality and will graph solutions on a number line. In Lesson 14, students will graph solutions to two-variable inequalities.

Connections

- Students apply linear inequalities to contexts involving distance, time, and budget constraints. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.6

Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.2	MA.912.AR.3.3
MA.8.AR.3.4	MA.912.AR.9.8
MA.8.AR.3.5	MA.912.AR.9.9
	MA.912.AR.9.10

Vocabulary

New Vocabulary

constraint

Lesson at a Glance

~ 45 min

MA.912.AR.2.6

Warm-Up

Independent | 5 min

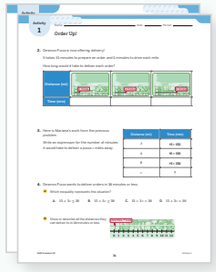
Students express their creativity using the **Tell a Story** routine to make sense of a pizza delivery context used throughout this lesson. **(MTR.1.1)**



Activity 1

Pairs | 20 min

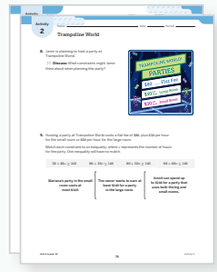
Students explore the relationship between time and distance in a pizza delivery context, representing constraints symbolically and interpreting solutions in context. **MLR7: Compare and Connect.** **(MTR.2.1)**



Activity 2

Pairs | 10 min

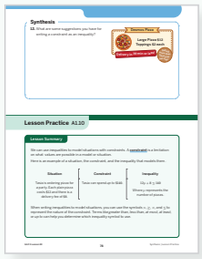
Students write inequalities to represent the constraints of a situation and use the inequalities to solve problems.



Synthesis

Whole Class | 5 min

Students synthesize their strategies for writing an inequality to represent a constraint.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by choosing an inequality that will represent the given constraints.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students express their creativity using the Tell a Story routine to make sense of a pizza delivery context used throughout this lesson. **(MTR.1.1)**

1 Launch

Consider asking:

- "What is your favorite pizza topping?"
- "Do you ever order delivery? From where?"
- "What's the longest you've ever had to wait for a delivery?"

Use the Tell a Story routine to help students make sense of the context. Invite students to write a story about the image using their unique experiences. **(MTR.1.1)**

ELL **English Language Learners** Invite students to talk through what they notice and wonder about the picture with a partner before writing their story in their primary language. Provide sentence frames to help students write their story (e.g., First, _____. Then, _____. Finally, _____.).

Consider sharing that stories do not have to be complete or perfect to be valuable.

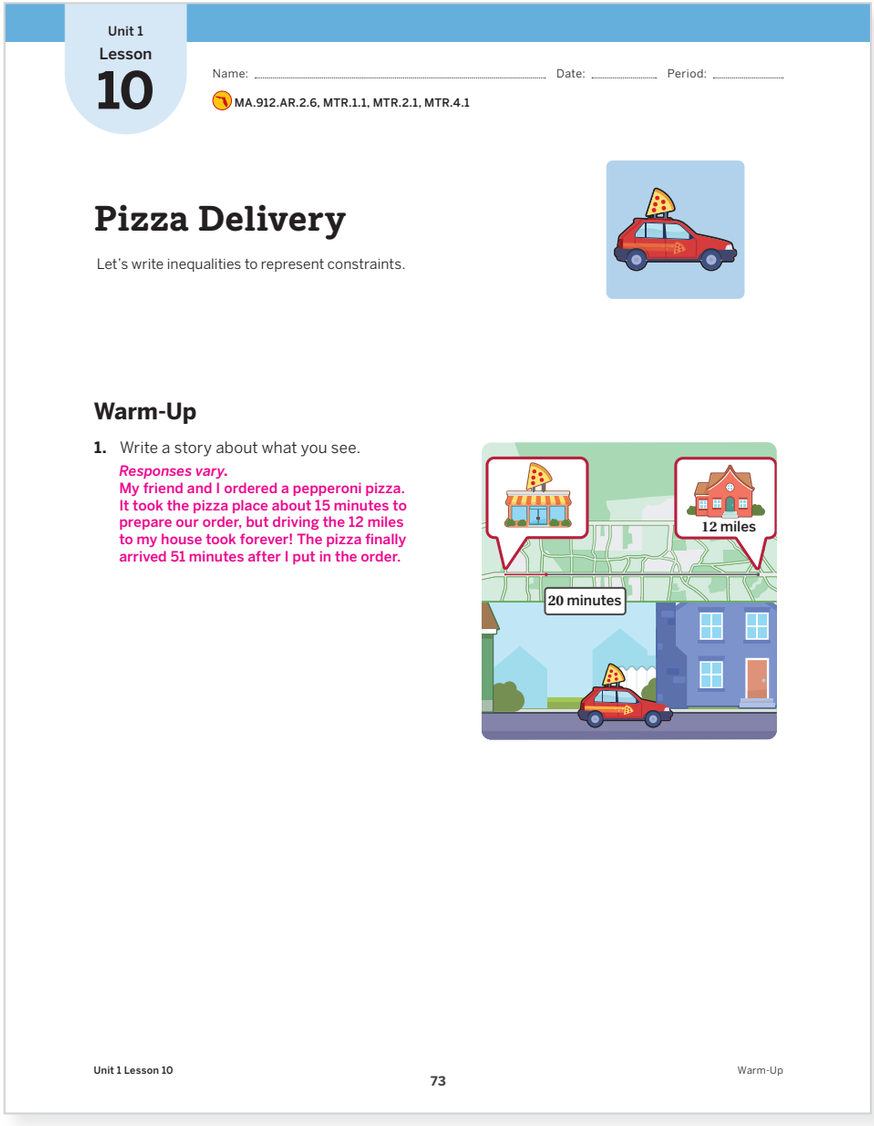
2 Connect

Invite several students to share their stories, encouraging them to make connections to the image information as they share.

Encourage students to ask questions about each other's stories.

Consider asking:

- "How does your story connect to the picture?"
- "Where do you hear times and distances in your classmates' stories?"
- "When ___ said, '___,' what do you think they meant?" (e.g., "When Omar said, 'The 51 minutes represents both the time to cook the pizza and deliver the pizza,' what do you think they meant?")



Unit 1
Lesson
10

Name: _____ Date: _____ Period: _____

MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1

Pizza Delivery

Let's write inequalities to represent constraints.

Warm-Up

1. Write a story about what you see.

Responses vary.
My friend and I ordered a pepperoni pizza. It took the pizza place about 15 minutes to prepare our order, but driving the 12 miles to my house took forever! The pizza finally arrived 51 minutes after I put in the order.

Unit 1 Lesson 10 73 Warm-Up

Pairs | 20 min

Activity 1 Order Up!

Purpose: Students explore the relationship between time and distance in a pizza delivery context, representing **constraints** symbolically and interpreting solutions in context. **(MTR.2.1)**

1 Launch

Invite students to read the situation aloud and ask a partner to describe the situation in their own words, clarifying any unknown words before answering.

Consider asking, "Why might someone want to know the number of minutes it would take to deliver a pizza?"

2 Monitor

To support students in getting started, consider asking, "How do you think Mariana got +6? +12?"

Consider asking pairs of students, "How did you know where to put x in your expression?" **(MTR.2.1)**

A **Accessibility: Memory and Attention** Take a moment to review inequality symbols and their meanings. Consider making an anchor chart for students to refer to throughout the unit.

Look for a variety of student sketches that describe all the distances Desmos Pizza can deliver to in 30 minutes or less to share during the Connect.

D **Differentiation**

Look for students who:	Teacher Moves
Need support getting started.	**Support** Consider asking, "Could Desmos Pizza deliver a pizza 2 miles away in 30 minutes or less? How about 10 miles away?"
Use words. (E.g., "5 miles or less would mean I need the symbol ≤ 30 in my expression.")	**Strengthen** Consider asking, "How could you represent your thinking using symbols, a graph, or a table?"
Plot distances on the number line.	**Strengthen** Consider asking, "How did you decide where the solutions were on your number line?"
Make a table like Mariana's on Problem 5.	**Strengthen** Consider renaming Mariana's or Camila's strategies after students in your class who used a similar strategy. Ask these students to share during the Connect.
Write and solve an equation like Camila's on Problem 5.	

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Order Up!

2. Desmos Pizza is now offering delivery!

It takes 15 minutes to prepare an order and 3 minutes to drive each mile.

How long would it take to deliver each order?

Distance (mi)			
Time (min)	21	27	39

3. Here is Mariana's work from the previous problem.

Write an expression for the number of minutes it would take to deliver a pizza x miles away.

$15 + 3x$ (or equivalent)

Distance (mi)	Time (min)
2	15 + 3(2)
4	15 + 3(4)
8	15 + 3(8)
x	?

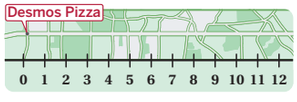
4. Desmos Pizza wants to deliver orders in 30 minutes or less.

a Which inequality represents this situation?

A. $15 + 3x \leq 30$ **B.** $15 + 3x \geq 30$ **C.** $15 + 3x > 30$ **D.** $15 + 3x < 30$

b Show or describe all the distances they can deliver to in 30 minutes or less.

Responses showing or describing 5 miles or less are considered correct.



Unit 1 Lesson 10 74 Activity 1

Pairs | 20 min

Activity 1 Order Up! (continued)

Purpose: Students explore the relationship between time and distance in a pizza delivery context, representing **constraints** symbolically and interpreting solutions in context. **(MTR.2.1)**

2 Monitor

ELL **English Language Learners** Invite students to read the situation aloud and ask a partner to paraphrase the situation in their own words, clarifying any unknown words (e.g., "toppings") before answering.

A **Accessibility: Conceptual Processing** To support students with abstract thinking, invite them to choose an inequality they know does not represent the situation and explain why. **(MTR.2.1)**

Pause briefly to ask:

- "What is a number of toppings that Mariana could not order?"
- "Can there be negative numbers of toppings? Decimal numbers? Why or why not?"

D **Differentiation: (Stretch)** Invite students to choose one of the other inequalities on this problem and write a situation that it could represent.

3 Connect

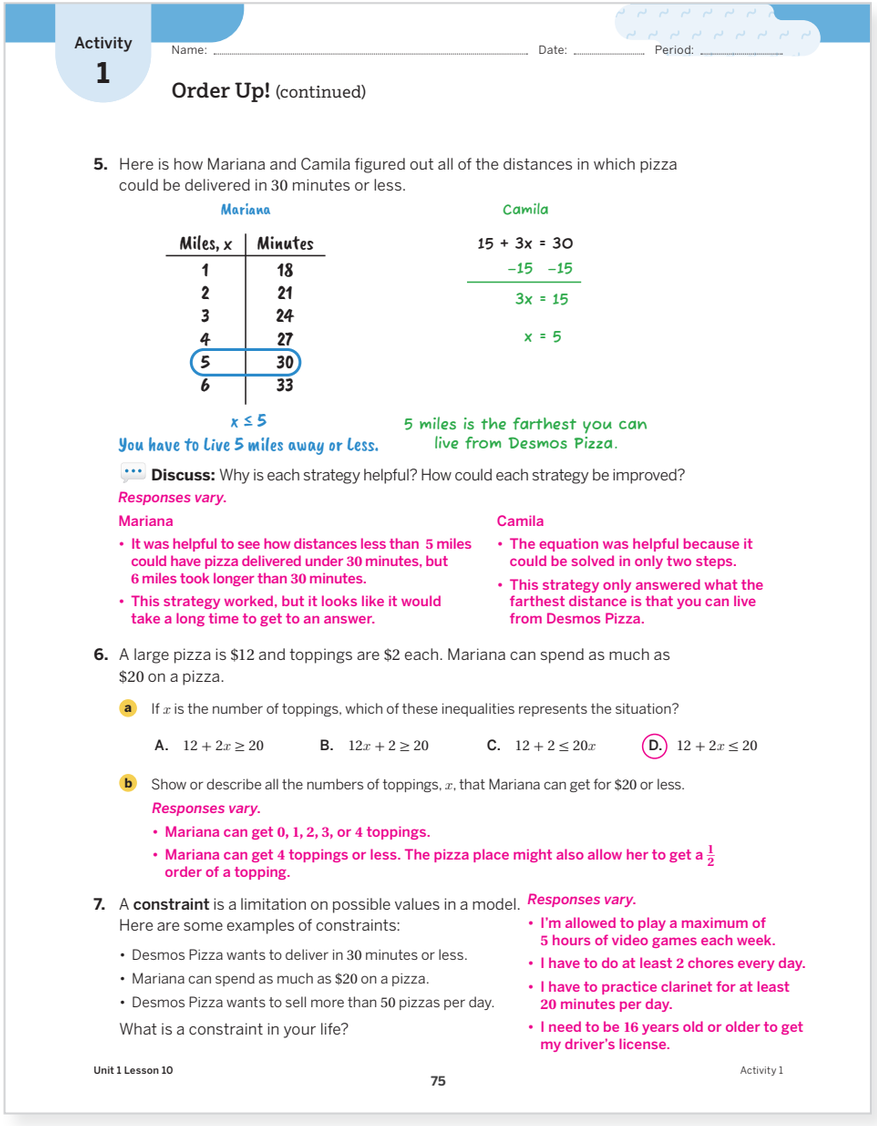
MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy from the previous problem.

Consider asking:

- "What did each student do?"
- "Where do you see the same information in Mariana's and Camila's strategies?"
- "Is 5 miles the only distance Desmos Pizza can deliver?" **(MTR.2.1)**
- "Could Desmos Pizza deliver to distances that are not whole miles? Why or why not? "
- "How do you know whether an inequality or an equation is more helpful in representing the constraints in this situation?"
- "How does each strategy compare to your own?"

Invite students to define **constraint** in their own words.

Consider sharing a constraint in your life (e.g., "My commute to work is at least 15 minutes in each direction.")



Activity 1

Name: Date: Period:

Order Up! (continued)

5. Here is how Mariana and Camila figured out all of the distances in which pizza could be delivered in 30 minutes or less.

Mariana

Miles, x	Minutes
1	18
2	21
3	24
4	27
5	30
6	33

$x \leq 5$

You have to live 5 miles away or less.

Camila

$15 + 3x = 30$

$-15 \quad -15$

$3x = 15$

$x = 5$

5 miles is the farthest you can live from Desmos Pizza.

Discuss: Why is each strategy helpful? How could each strategy be improved?

Responses vary.

Mariana

- It was helpful to see how distances less than 5 miles could have pizza delivered under 30 minutes, but 6 miles took longer than 30 minutes.
- This strategy worked, but it looks like it would take a long time to get to an answer.

Camila

- The equation was helpful because it could be solved in only two steps.
- This strategy only answered what the farthest distance is that you can live from Desmos Pizza.

6. A large pizza is \$12 and toppings are \$2 each. Mariana can spend as much as \$20 on a pizza.

a. If x is the number of toppings, which of these inequalities represents the situation?

A. $12 + 2x \geq 20$ B. $12x + 2 \geq 20$ C. $12 + 2 \leq 20x$ D. $12 + 2x \leq 20$

b. Show or describe all the numbers of toppings, x, that Mariana can get for \$20 or less.

Responses vary.

- Mariana can get 0, 1, 2, 3, or 4 toppings.
- Mariana can get 4 toppings or less. The pizza place might also allow her to get a $\frac{1}{2}$ order of a topping.

7. A **constraint** is a limitation on possible values in a model. Here are some examples of constraints:

- Desmos Pizza wants to deliver in 30 minutes or less.
- Mariana can spend as much as \$20 on a pizza.
- Desmos Pizza wants to sell more than 50 pizzas per day.

What is a constraint in your life?

Responses vary.

- I'm allowed to play a maximum of 5 hours of video games each week.
- I have to do at least 2 chores every day.
- I have to practice clarinet for at least 20 minutes per day.
- I need to be 16 years old or older to get my driver's license.

Unit 1 Lesson 10 75 Activity 1

Key Takeaway: Inequalities are a helpful way to write situations symbolically that involve **constraints**. There are many different ways to solve an inequality, including tables, graphs, and corresponding equations.

Pairs | 10 min

Activity 2 Trampoline World

Purpose: Students write inequalities to represent the constraints of a situation and use the inequalities to solve problems.

Short on time? Consider completing Problems 8 and 9 as a whole class instead of in pairs.

1 Launch

Consider asking:

- "Have you ever been to a trampoline park or planned a party?"
- "What are some things you think about when planning a party?"

Invite students to read the situation aloud and describe what the situation is about in their own words, clarifying any unknown words (e.g., "flat fee") before answering the question.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match the constraints and inequalities. **(MTR.4.1)**

Note: If time allows, invite pairs to compare their matches, justify their thinking, and make revisions based on their conversation. **(MTR.4.1)**

Consider asking pairs of students, "Where do you see the hourly fee in each inequality? The flat fee?" **(MTR.2.1)**

A **Accessibility: Executive Function** To support students in initiating complex tasks, invite students to focus on one constraint or inequality at a time.

Activity 2 continued >

Activity 2

Name: Date: Period:

Trampoline World

8. Jamir is planning to host a party at Trampoline World.

Discuss: What constraints might Jamir think about when planning this party?

Responses vary.

- How many people are invited.
- How many people fit in each room.
- How long the party will last.
- How much the party will cost.



9. Hosting a party at Trampoline World costs a flat fee of $80, plus $30 per hour for the small room or $50 per hour for the large room.

Match each constraint to an inequality, where x represents the number of hours for the party. One inequality will have no match.

$30 + 80x \geq 140$	$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Mariana's party in the small room costs at most $140.	The owner wants to earn at least $140 for a party in the large room.	Amoli can spend up to $140 for a party that uses both the big and small rooms.
$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Unit 1 Lesson 10 76 Activity 2

Pairs | 10 min

Activity 2 Trampoline World (continued)

Purpose: Students write inequalities to represent the constraints of a situation and use the inequalities to solve problems.

Short on time? Consider completing Problems 10 and 11 as a whole class instead of individually.

2 Monitor

Look for students who write the inequality $80 + 50x \geq 155$ by default because the situation says "as much as" instead of making sense of the situation. Encourage these students to read their inequality aloud, replacing the symbol $\geq$ with the words "greater than or equal to".

Select and sequence different strategies that students use to solve this problem.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started.	**Support** Consider asking, "How can the inequality be used to determine how many hours are possible?"
Solve the inequality from the previous problem and found $x \leq 1.5$.	**Strengthen** Capture these strategies and invite these students to share during the Connect.
Substitute different values into the inequality to see which make the inequality true.	**Note:** It's okay if students are still developing fluency. These strategies will be covered in more depth in Lessons 11 and 12.
Use reasoning to think about Jamir's constraints to determine how long he can have a party.	

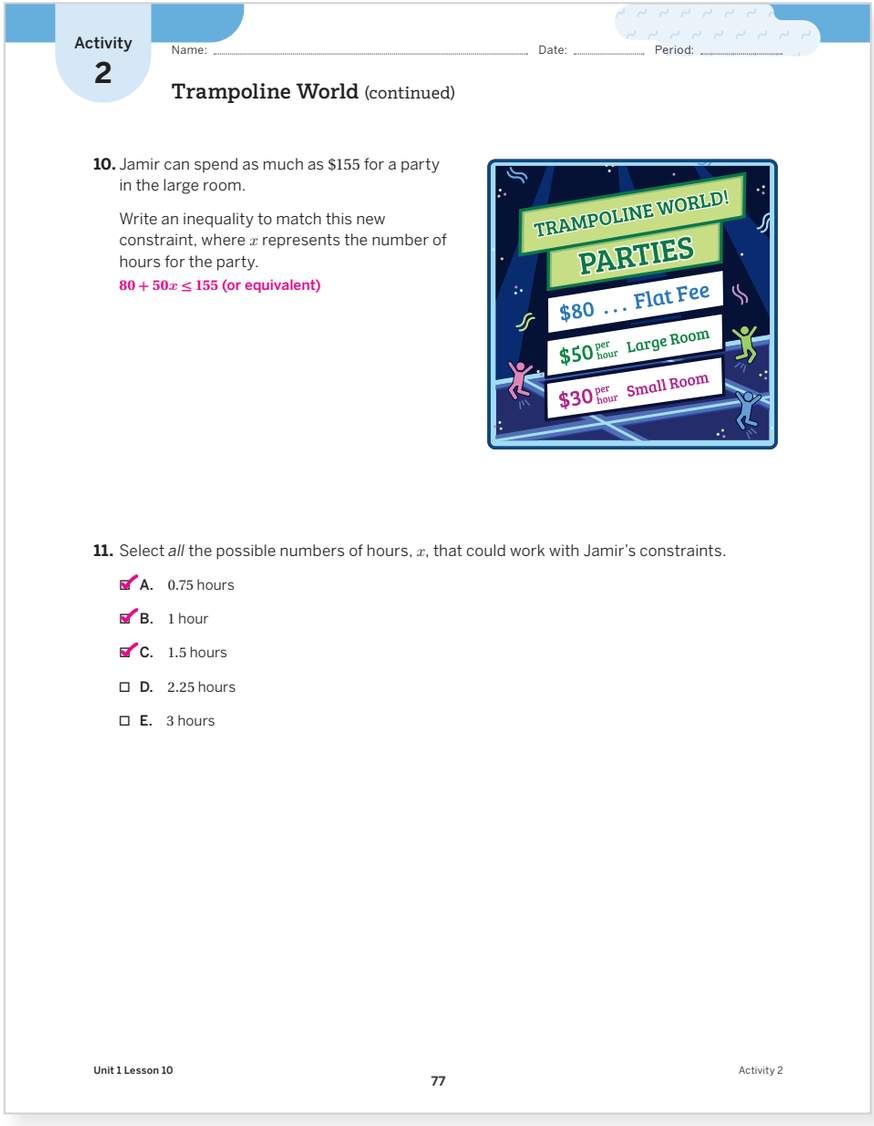
3 Connect

Invite students to share their strategies for determining all the viable solutions for Jamir's constraints.

Discuss the difference between viable and non-viable solutions. **(MTR.4.1)**

Consider asking:

- "Which constraint makes a 3-hour party a non-viable solution for Jamir's situation?"
- "Would a solution that is a decimal be viable or non-viable for this situation?"



Activity 2

Name: Date: Period:

Trampoline World (continued)

10. Jamir can spend as much as \$155 for a party in the large room.

Write an inequality to match this new constraint, where x represents the number of hours for the party.

$80 + 50x \leq 155$ (or equivalent)

11. Select *all* the possible numbers of hours, x, that could work with Jamir's constraints.

- ☑ **A.** 0.75 hours
- ☑ **B.** 1 hour
- ☑ **C.** 1.5 hours
- ☐ **D.** 2.25 hours
- ☐ **E.** 3 hours

Unit 1 Lesson 10 77 Activity 2

Key Takeaway: Inequalities can be helpful tools to represent **constraints** and solve real-world problems. When solving real-world problems, it is important to check that the solutions make the inequality true and make sense for the situation.

Whole Class | 5 min

Synthesis

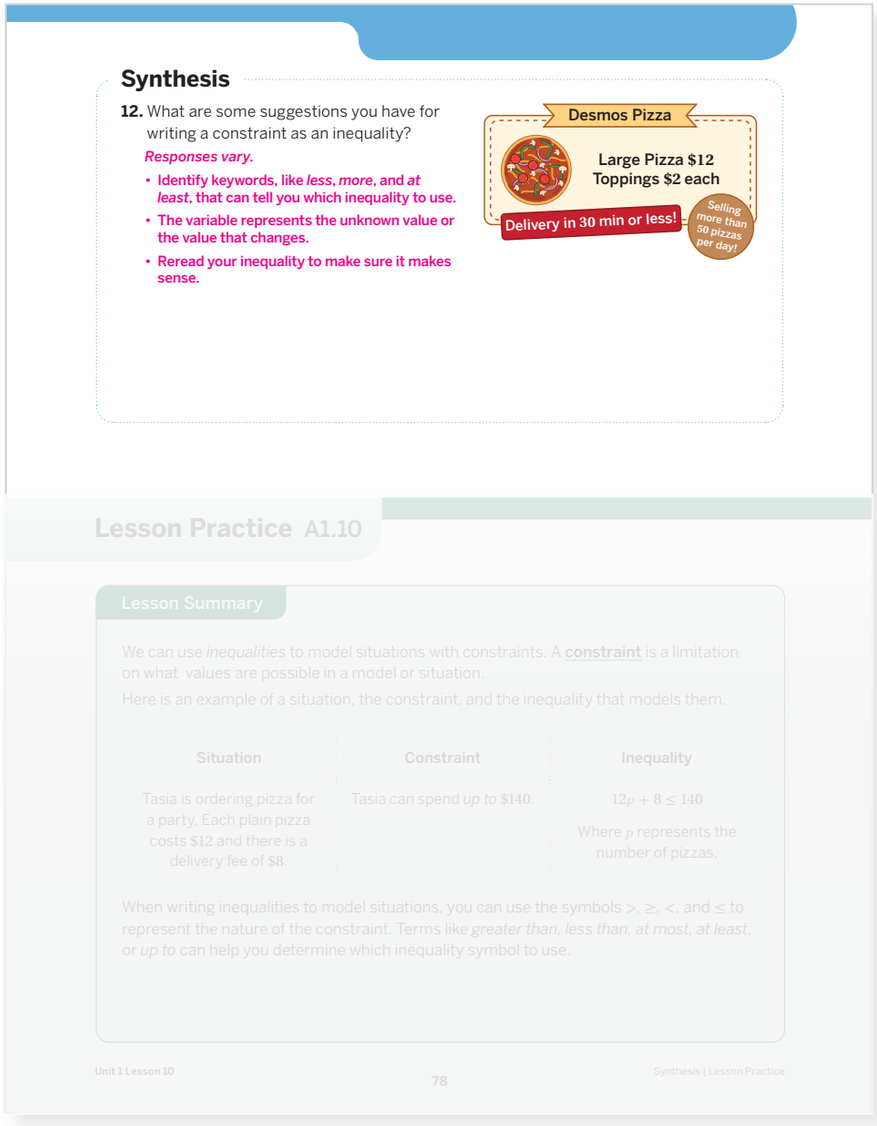
Purpose: Students synthesize their strategies for writing an inequality to represent a constraint.

Invite students to respond independently, and then share their thinking with a partner.

Gather and share a variety of ideas, such as:

- Deciding if an equation or inequality would best represent the situation.
- Deciding which inequality symbol to use.
- Deciding if a solution is viable or non-viable based on the inequality or the context of the situation.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. What are some suggestions you have for writing a constraint as an inequality?

Responses vary.

- Identify keywords, like *less*, *more*, and *at least*, that can tell you which inequality to use.
- The variable represents the unknown value or the value that changes.
- Reread your inequality to make sure it makes sense.

Lesson Practice A1.10

Lesson Summary

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs $12 and there is a delivery fee of $8.	Tasia can spend *up to* $140.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols $>$, $\geq$, $<$, and $\leq$ to represent the nature of the constraint. Terms like *greater than*, *less than*, *at most*, *at least*, or *up to* can help you determine which inequality symbol to use.

Unit 1 Lesson 10 78 Synthesis | Lesson Practice

Lesson Takeaway: Inequalities can be used to represent **constraints** and solve real-world problems. Not all solutions to inequalities are viable solutions or make sense for the situation.

Independent | 5 min

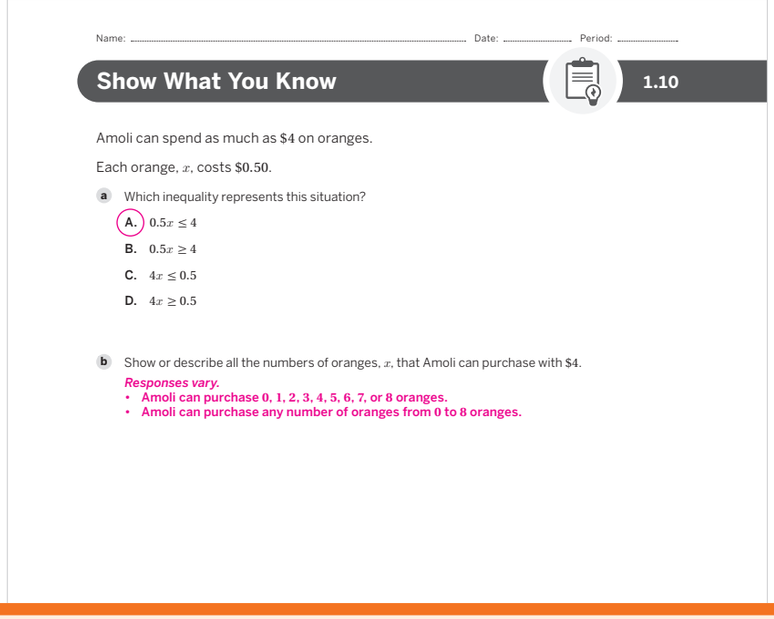
Show What You Know

Purpose: Students demonstrate their understanding by choosing an inequality that will represent the given constraints.

Today's Goals

1. **Goal:** Understand that solutions to an inequality are values that make the inequality true.
2. **Goal:** Write one-variable inequalities that represent constraints.
3. **Language Goal:** Describe and interpret solutions to one-variable inequalities in context. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.10

Amoli can spend as much as $4 on oranges.

Each orange, x, costs $0.50.

a. Which inequality represents this situation?

A. $0.5x \leq 4$

B. $0.5x \geq 4$

C. $4x \leq 0.5$

D. $4x \geq 0.5$

b. Show or describe all the numbers of oranges, x, that Amoli can purchase with $4.

Responses vary.

- Amoli can purchase 0, 1, 2, 3, 4, 5, 6, 7, or 8 oranges.
- Amoli can purchase any number of oranges from 0 to 8 oranges.

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support writing and solving one-variable inequalities:

» **Respond to Student Thinking:**

- Use the ***Solving Inequalities*** **Mini-Lesson**.
- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Reflect on the Tell a Story routine used in this lesson. Students used this routine for the first time in Lesson 3 and again today to help make sense of a situation. What about the routine felt successful? What are some areas for growth? Consider celebrating when students engage in sensemaking, naming this a mathematical strength. **(MTR.1.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What are some suggestions you have for writing a constraint as an inequality?

Responses vary.

- Identify keywords, like *less, more,* and *at least,* that can tell you which inequality to use.
- The variable represents the unknown value or the value that changes.
- Reread your inequality to make sure it makes sense.

Lesson Practice A1.10

Lesson Summary

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs $12 and there is a delivery fee of $8.	Tasia can spend *up to* $140.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols $>$, $\geq$, $<$, and $\leq$ to represent the nature of the constraint. Terms like *greater than, less than, at most, at least,* or *up to* can help you determine which inequality symbol to use.

Lesson Practice A1.10

Name: ..... Date: ..... Period: .....

1. For each constraint, write the letter of the matching inequality.

A. $x \leq 10$ B. $x > 10$ C. $x \geq 10$

A x is less than or equal to 10.

A x is at most 10.

C x is greater than or equal to 10.

B x is greater than 10.

C x is at least 10.

2. Demetrius can spend as much as $50 on shirts. Shirts, s, cost $16 each at a nearby store. Which inequality represents this situation?

A. $16s \geq 50$ B. $16s \leq 50$ C. $50s \geq 16$ D. $50s \leq 16$

Explain your thinking.

Explanations vary. $16s$ represents $16 for each shirt and 50 represents $50, which is the most that Demetrius can spend. He can spend $50 or anything less than that, so I used the symbol $\leq$.

3. List *three* values for x that would make $8 + 2x \leq 20$ true.

Any value of $x \leq 6$ is considered correct.

$x = 0$ $x = 1$ $x = \frac{5}{3}$

Problems 4–6: Write an inequality for each constraint. Use t for time (in hours).

4. Trevon practices his clarinet at least 1 hour each day.

$t \geq 1$

5. At some colleges, students must work 20 hours or less per week.

$t \leq 20$

6. The American Academy of Pediatrics recommends teenagers play video games for no more than 2 hours each day.

$t \leq 2$

Lesson Practice A1.10

Name: ..... Date: ..... Period: .....

Problems 7–8: Tell a story about each inequality. Specify the constraint and what the variable represents.

7. $x \geq 3$

Responses vary.

- Sahana needs to practice playing the clarinet at least 3 hours per week.
- x represents the number of hours that she practices each week.

8. $5 > y$

Responses vary.

- Your height must be less than 5 feet to ride the kids' roller-coaster at the fair.
- y represents your height in feet.

9. Fatima makes $9.25 per hour, h, plus $150 in commissions. She created an inequality that could be used to find the number of hours she needs to work to make at least $510 on her next paycheck.

Her inequality is $9.25 + 150h \leq 510$. Find the mistakes in Fatima's inequality and correct it.

Responses vary. There are two mistakes in Fatima's inequality:

- She misidentified what number to multiply variable h by. She multiplied it by 150 when it should be multiplied by 9.25.
- She used the wrong inequality symbol for the phrase "at least". It should be $\geq$ instead of $\leq$.
- The correct inequality is $9.25h + 150 \geq 510$.

Test Practice

10. Marquis wants to work at least 20 hours a week to earn enough money to go to a concert. Which inequality represents x, the number of hours Marquis wants to work?

A. $x > 20$ B. $x < 20$ C. $x \leq 20$ D. $x \geq 20$

Spiral Review

Problems 11–13: Solve each equation.

11. $4x - 6 = 12 - 2x$

$x = 3$

12. $\frac{1}{3}x - 8 = 12 - 3x$

$x = 6$

13. $2x + 7 - 3x = \frac{5}{2}$

$x = \frac{9}{2}$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–6	1	MA.912.AR.2.6
	2–3	2	MA.912.AR.2.6
	7–8	2	MA.912.AR.2.6
	9	3	MA.912.AR.2.6
Test Practice	10	2	MA.912.AR.2.6
Spiral Review			
	11–13	2	MA.8.AR.2.1

Unit 1

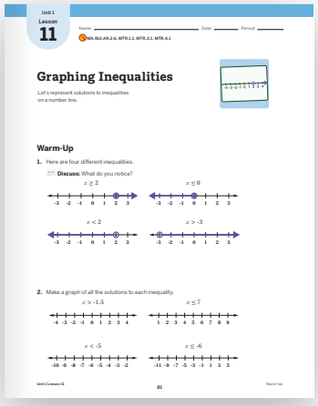
Lesson

11

Graphing Inequalities

Inequalities on the Number Line

Let's represent solutions to inequalities on a number line.


Unit 1
Lesson
11
Name: Date: Period:
MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1
Graphing Inequalities
Let's represent solutions to inequalities on a number line.
Warm-Up
1. Here are four different inequalities.
Discuss: What do you notice?
$x \geq 2$
$x \leq 0$
$x < 2$
$x > -3$
2. Make a graph of all the solutions to each inequality.
$x > -1.5$
$x \leq 7$
$x < -5$
$x \leq -6$
81
Warm-Up

Learning Outcomes

Today's Goal

1. **Goal:** Solve one-variable inequalities by reasoning.
2. **Goal:** Graph solutions to a one-variable inequality on a number line.
3. **Language Goal:** Describe how to represent the solutions to a one-variable inequality on a number line. **(Speaking and Listening)**

Students use reasoning to determine the solutions to a one-variable inequality and graph those solutions on a number line. They start by identifying individual solutions to inequalities to develop strategies for solving by reasoning. They also focus on the structure of the inequality and its graph, using number sense to develop conceptual understanding of solutions to one-variable inequalities. This builds a strong conceptual foundation for solving inequalities before students begin practicing solving corresponding equations. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 8, students explored connections between constraints given in context and the inequalities that represent them, and how solutions to inequalities are values that make the inequality true.

Future Learning

In Lesson 10, students will solve one-variable inequalities by solving their corresponding equations and testing values to determine if solutions are viable or non-viable.

Connections

- Students connect various representations to describe an inequality using symbols and graphs. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.6

Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.

Also Addressing: **MA.912.AR.2.8**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.2	**MA.912.AR.2.7**
MA.8.AR.3.4	**MA.912.AR.3.3**
MA.8.AR.3.5	

Vocabulary

Review Vocabulary

boundary point

region

solution set

Lesson at a Glance

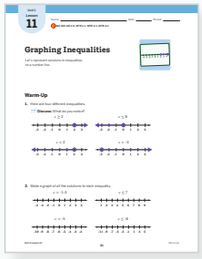
~ 45 min

MA.912.AR.2.6, MA.912.AR.2.8

Warm-Up

Pairs | 5 min

Students surface connections between one-variable inequalities and their corresponding graphs.



Activity 1

Pairs | 10 min

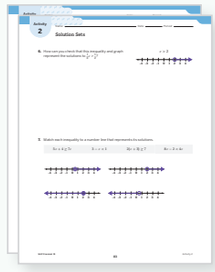
Students identify individual solutions to inequalities and reason about how to graph all the solutions of a one-variable inequality on a number line. **Routine:** Decide and Defend **(MTR.2.1)**



Activity 2

Pairs | 20 min

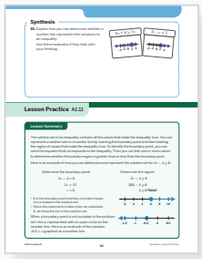
Students use reasoning to determine all the solutions to a one-variable inequality and then represent those solutions by graphing them on a number line. MLR3: Critique, Correct, Clarify



Synthesis

Whole Class | 5 min

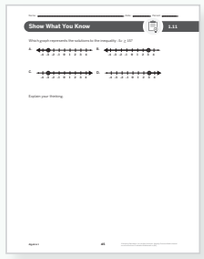
Students synthesize different creative strategies for solving and visualizing solutions to inequalities graphically.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by matching an inequality to the graph that represents its solutions.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students surface connections between one-variable inequalities and their corresponding graphs.

Short on time? Consider inviting students to complete fewer Repeated Challenges on Problem 2.

1 Launch

Draw students' attention to the Warm-Up problem.

Encourage students to identify as many connections between the inequality and the graph as possible, then continue to the next problem when they feel ready to practice graphing inequalities. **(MTR.1.1)**

Students will discuss what they notice about the graphs of inequalities with a partner before creating graphs of all of the solutions to an inequality independently.

A **Accessibility: Fine Motor Skills** Consider strategically pairing students with different strengths to support the use of virtual manipulatives throughout this lesson.

Invite students to set their own goal for how many problems they would like to complete, sharing that sense-making is more important than speed.

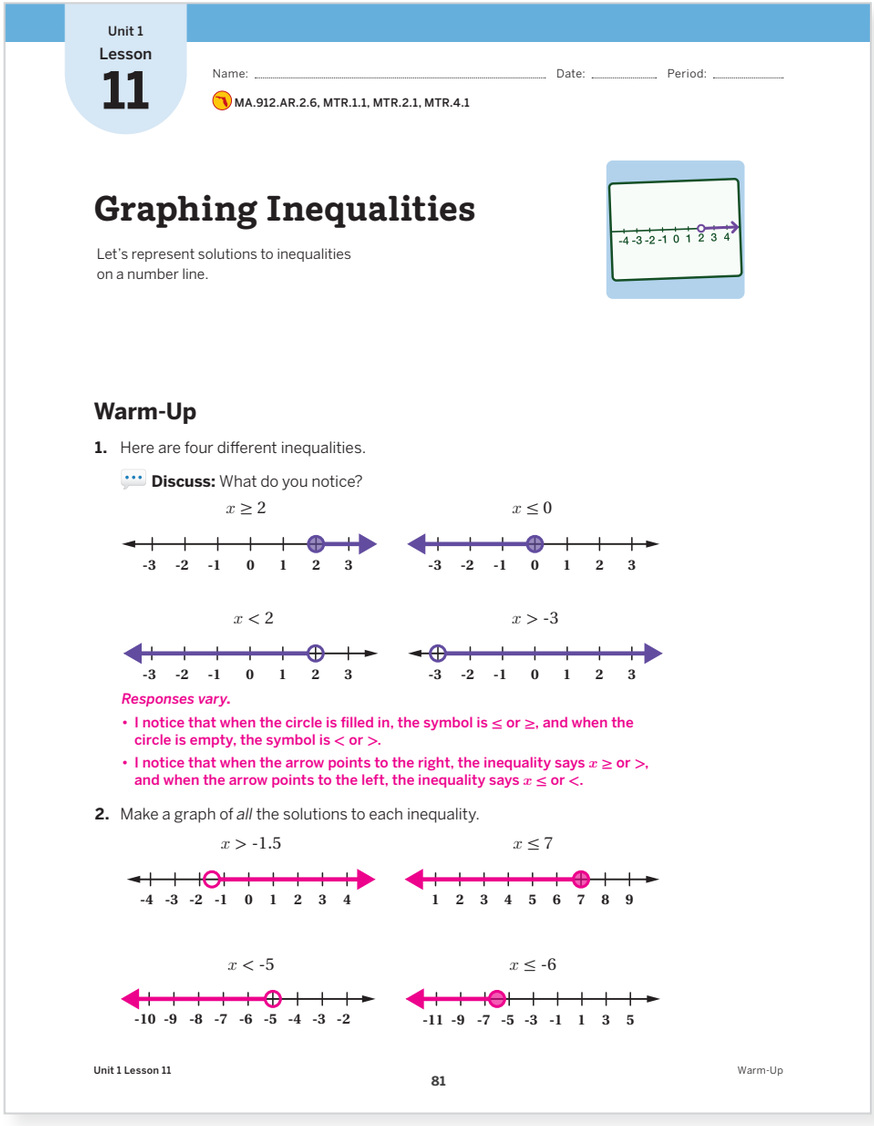
Encourage students to review and revise their challenge problems with a peer. Then discuss the answers as a class to ensure that students are ready for Activity 1. **(MTR.4.1)**

2 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.

Note: If it does not come up naturally, consider highlighting that the *boundary point* separates the number line into two *regions*, with one region representing all the solutions to the inequality. The inequality symbol determines whether the boundary point should be filled in or empty. A filled-in boundary point indicates that the value at that point is included in the *solution set*, while an empty boundary point is not included in the solution set. **(MA.7.AR.1.2)**

A **Accessibility: Memory and Attention** To support students who would benefit from having more time to think deeply on fewer problems, invite them to complete fewer challenges.



Unit 1
Lesson
11

Name: _____ Date: _____ Period: _____

MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1

Graphing Inequalities

Let's represent solutions to inequalities on a number line.

Warm-Up

1. Here are four different inequalities.

Discuss: What do you notice?

$x \geq 2$

$x \leq 0$

$x < 2$

$x > -3$

Responses vary.

- I notice that when the circle is filled in, the symbol is $\leq$ or $\geq$, and when the circle is empty, the symbol is $<$ or $>$.
- I notice that when the arrow points to the right, the inequality says $x \geq$ or $>$, and when the arrow points to the left, the inequality says $x \leq$ or $<$.

2. Make a graph of *all* the solutions to each inequality.

$x > -1.5$

$x \leq 7$

$x < -5$

$x \leq -6$

Unit 1 Lesson 11 81 Warm-Up

Pairs | 10 min

Activity 1 Show a Solution

Purpose: Students identify individual solutions to inequalities and reason about how to graph all the solutions of a one-variable inequality on a number line. **(MTR.2.1)**

1 Launch

Draw students' attention to Problem 3.

Consider asking, "How is this problem similar to the ones we completed in the Warm-Up? How is it different?"

A **Accessibility: Conceptual Processing** Invite students to read the inequality aloud (e.g., "Two times a number minus eight is greater than zero.").

2 Monitor

To support students in getting started, consider asking, "Because $2x - 8$ is greater than 0, what values of x will make this inequality true?" *Responses vary. $x > 4$ will make $2x - 8$ positive.* **(MTR.2.1)**

Invite students to share solutions with multiple partners to generate a graph or add their solution to a class number line.

D **Differentiation: (Support)** Students may think that any number greater than 0 is a solution to $2x - 8 > 0$ instead of numbers that make $2x - 8$ positive. Consider asking, "When you substitute $x = 2$, does it make the inequality true? What about $x = 3$? $x = 4$?"

Look for students who have similar thinking to Lan's on Problem 5 that the inequality has no solution. Consider asking these students, "What values have you tested to see if the inequality is true?"

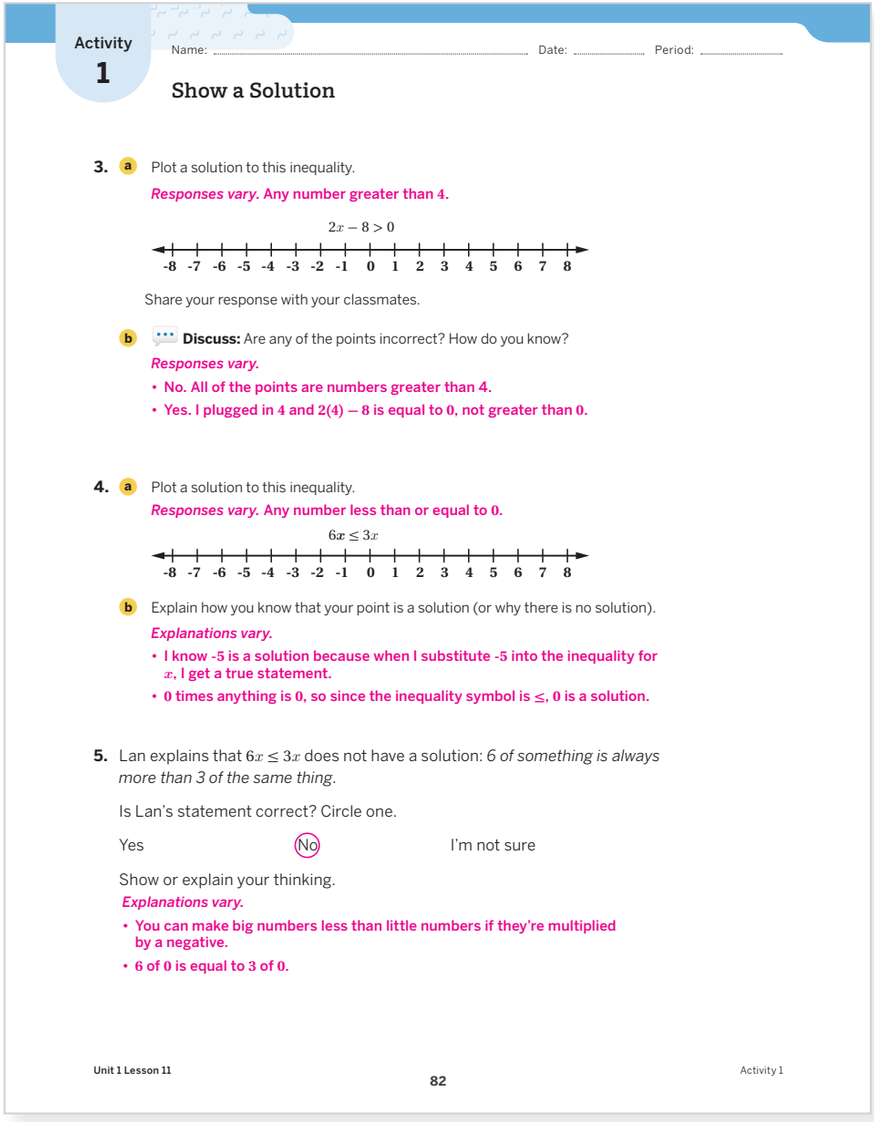
Note: It is ok — even desirable — to lack consensus at this stage. The activity will build toward consensus on Problem 5.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Consider asking:

- "What does Lan mean when he says that the inequality does not have a solution?"
- "What reasoning or evidence supports your claim?"



Activity 1

Name: Date: Period:

Show a Solution

3. a Plot a solution to this inequality.

Responses vary. Any number greater than 4.

$2x - 8 > 0$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

Responses vary.

- No. All of the points are numbers greater than 4.
- Yes. I plugged in 4 and 2(4) − 8 is equal to 0, not greater than 0.

4. a Plot a solution to this inequality.

Responses vary. Any number less than or equal to 0.

$6x \leq 3x$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

b Explain how you know that your point is a solution (or why there is no solution).

Explanations vary.

- I know -5 is a solution because when I substitute -5 into the inequality for x, I get a true statement.
- 0 times anything is 0, so since the inequality symbol is $\leq$, 0 is a solution.

5. Lan explains that $6x \leq 3x$ does not have a solution: *6 of something is always more than 3 of the same thing.*

Is Lan's statement correct? Circle one.

Yes (No) I'm not sure

Show or explain your thinking.

Explanations vary.

- You can make big numbers less than little numbers if they're multiplied by a negative.
- 6 of 0 is equal to 3 of 0.

Unit 1 Lesson 11 82 Activity 1

Key Takeaway: Inequalities have multiple solutions, with each solution making the inequality true. To represent all the solutions to an inequality on a graph, shade a *region* of a number line.

Pairs | 20 min

Activity 2 Solution Sets

Purpose: Students use reasoning to determine all the solutions to a one-variable inequality and then represent those solutions by graphing them on a number line.

Short on time? Consider asking students to discuss the question on Problem 6 instead of writing a response.

1 Launch

Invite students to share a variety of strategies for how they use reasoning to know that the graph represents all the solutions to the given inequality, such as:

- Substituting a value within (e.g., $x = 3$) or outside of the shaded region (e.g., $x = 0$) to check if the inequality is true or false.
- Using the inequality symbol to determine whether the boundary point at $x = 2$ should be filled in or empty.
- Reasoning using number sense to describe the relationship between both sides of the inequality (e.g., $\frac{1}{4}$ of a number is greater than $\frac{1}{2}$). **(MTR.2.1)**

Consider asking:

- "Where do you see the boundary point in each inequality? In the graph?" **(MTR.2.1)**
- "How can you tell if the correct region is shaded on the graph?" **(MTR.2.1)**

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match the inequalities and number lines.

Consider asking:

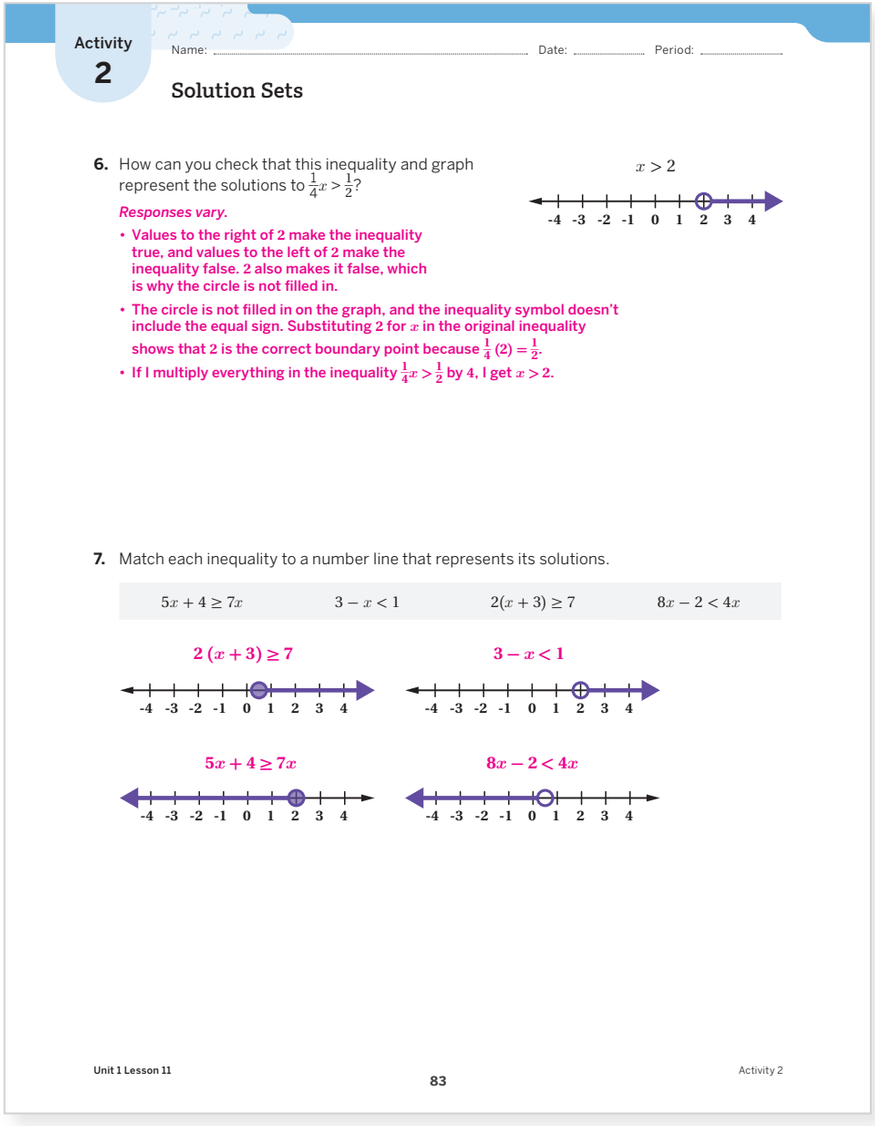
- "How did you determine the location of the boundary point?"
- "How do you know which region to shade?"

Note: If time allows, invite pairs to compare their matches, justify their thinking, and make revisions based on their conversation.

A **Accessibility: Conceptual Processing** Invite students to read each inequality aloud (e.g., "Five times a number plus four is greater than or equal to seven times the same number").

A **Accessibility: Executive Function** Invite students to look for similarities and differences among the inequalities and number lines or select ones they think might match before they start solving the inequalities.

Activity 2 continued >



Activity 2

Name: Date: Period:

Solution Sets

6. How can you check that this inequality and graph represent the solutions to $\frac{1}{4}x > \frac{1}{2}$?

$x > 2$

-4 -3 -2 -1 0 1 2 3 4

Responses vary.

- Values to the right of 2 make the inequality true, and values to the left of 2 make the inequality false. 2 also makes it false, which is why the circle is not filled in.
- The circle is not filled in on the graph, and the inequality symbol doesn't include the equal sign. Substituting 2 for x in the original inequality shows that 2 is the correct boundary point because $\frac{1}{4}(2) = \frac{1}{2}$.
- If I multiply everything in the inequality $\frac{1}{4}x > \frac{1}{2}$ by 4, I get $x > 2$.

7. Match each inequality to a number line that represents its solutions.

$5x + 4 \geq 7x$ $3 - x < 1$ $2(x + 3) \geq 7$ $8x - 2 < 4x$

$2(x + 3) \geq 7$

-4 -3 -2 -1 0 1 2 3 4

$3 - x < 1$

-4 -3 -2 -1 0 1 2 3 4

$5x + 4 \geq 7x$

-4 -3 -2 -1 0 1 2 3 4

$8x - 2 < 4x$

-4 -3 -2 -1 0 1 2 3 4

Unit 1 Lesson 11 83 Activity 2

Pairs | 20 min

Activity 2 Solution Sets (continued)

Purpose: Students use reasoning to determine all the solutions to a one-variable inequality and then represent those solutions by graphing them on a number line.

2 Monitor

Encourage students to show their thinking and listen to the responses from their classmates.

D Differentiation

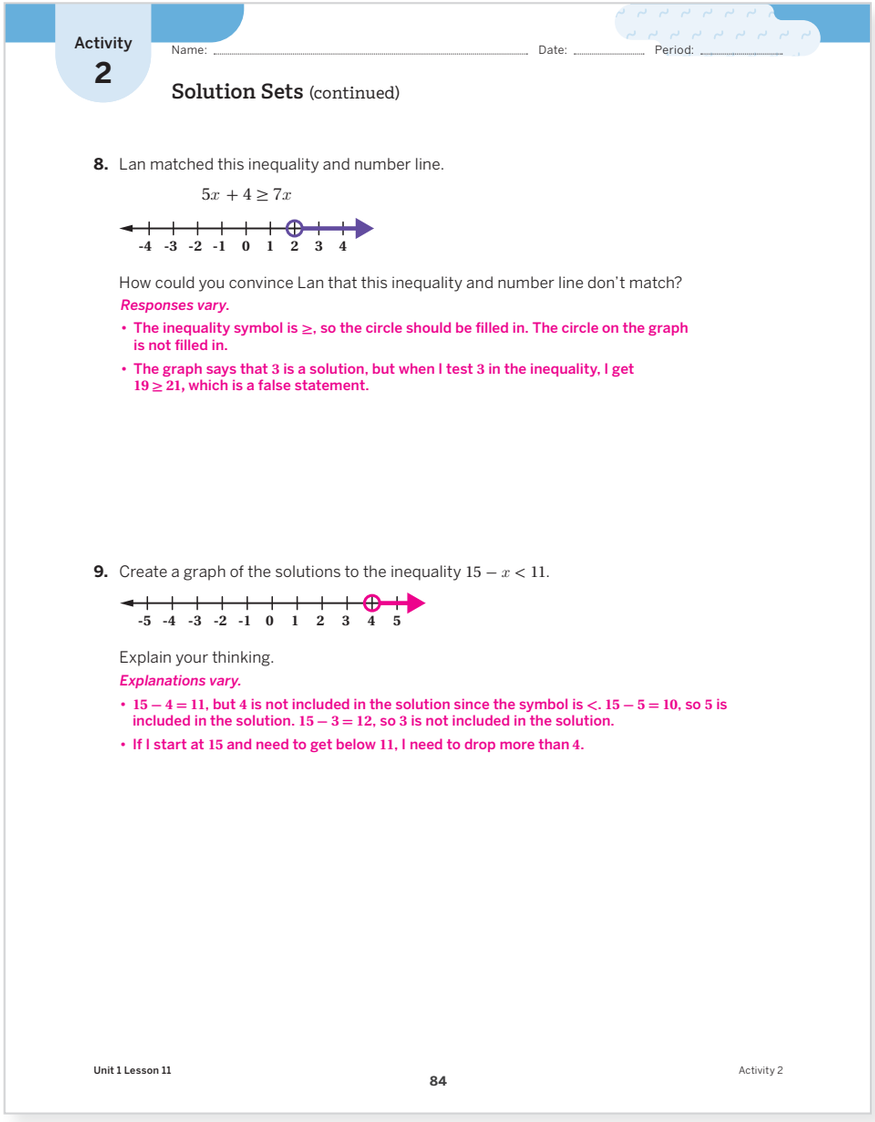
Look for students who:	Teacher Moves
Need support getting started.	**Support** Encourage them to identify individual solutions that they know make the inequality true to help them in their thinking.
Shade the number line to the left of the boundary point because of the symbol $<$.	**Support** Invite these students to check a solution from the graph by substituting the values into the inequality.
Think that the solutions are bounded by the image (e.g., between -5 and 5).	**Support** Consider asking, "What is the biggest possible value that x could be? The smallest? How do you know?"
Finish this problem early.	**Stretch** Invite students to write their own inequality and graph its solution. Encourage them to be as creative as they would like to be.

Note: This lesson intentionally does not introduce a specific method for solving inequalities with negative coefficients for x (e.g., reversing the inequality sign when multiplying or dividing by a negative number).

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe the errors in Lan's work. Invite students to share their responses from the activity. Then invite students to:

a. Discuss what is *correct* about Lan's thinking.
b. Discuss what is *incorrect* about Lan's thinking.
c. Write a question to help Lan see how their thinking can be improved, or create a revised response. **(MTR.3.1)**



Activity 2

Name: Date: Period:

Solution Sets (continued)

8. Lan matched this inequality and number line.

$5x + 4 \geq 7x$

How could you convince Lan that this inequality and number line don't match?

Responses vary.

- The inequality symbol is $\geq$, so the circle should be filled in. The circle on the graph is not filled in.
- The graph says that 3 is a solution, but when I test 3 in the inequality, I get $19 \geq 21$, which is a false statement.

9. Create a graph of the solutions to the inequality $15 - x < 11$.

Explain your thinking.

Explanations vary.

- $15 - 4 = 11$, but 4 is not included in the solution since the symbol is $<$. $15 - 5 = 10$, so 5 is included in the solution. $15 - 3 = 12$, so 3 is not included in the solution.
- If I start at 15 and need to get below 11, I need to drop more than 4.

Unit 1 Lesson 11 84 Activity 2

Key Takeaway: The *solution* set of an inequality is all the values that make an inequality true and can be represented on a number line. The inequality symbol can quickly reveal whether the *boundary point* is filled in or empty, but does not always reveal which direction to shade.

Whole Class | 5 min

Synthesis

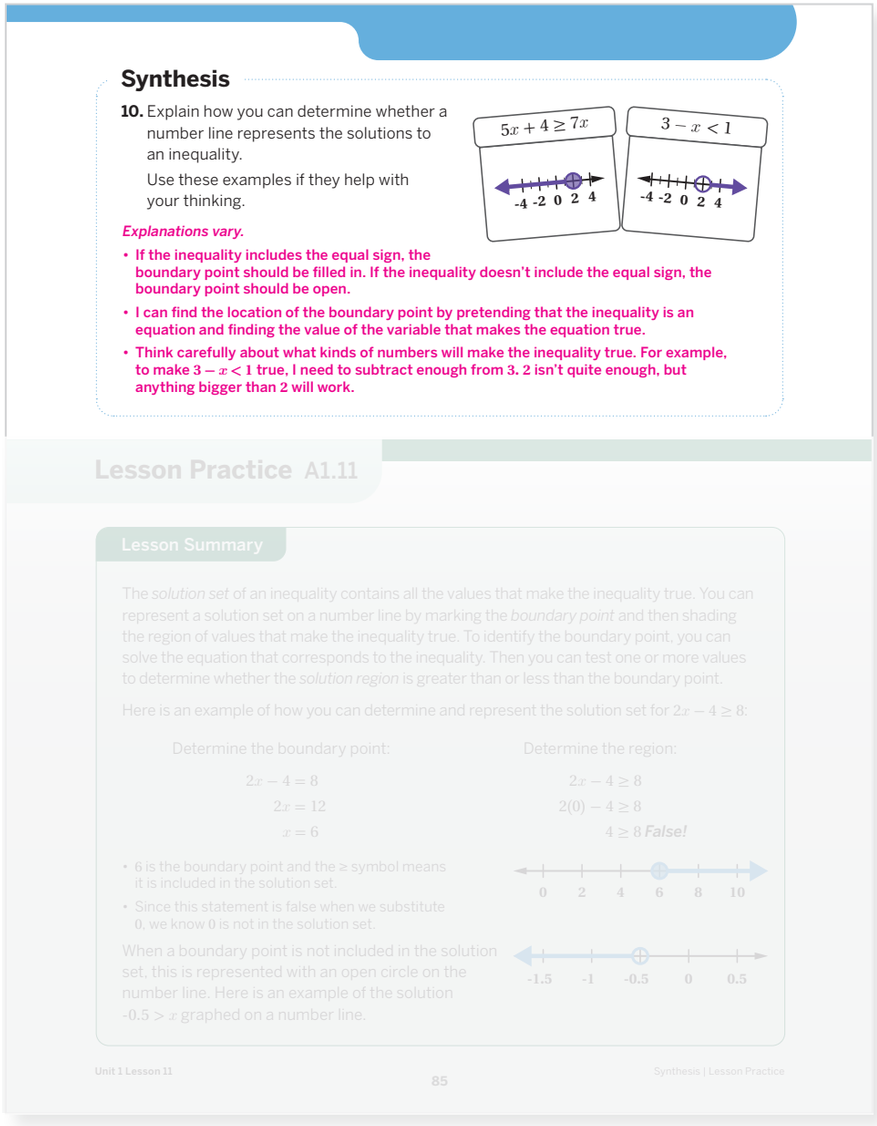
Purpose: Students synthesize different creative strategies for solving and visualizing solutions to inequalities graphically.

Invite students to respond independently, and then share their thinking with a partner.

Gather and share a variety of ideas, such as:

- If the inequality symbol includes the equal sign, the boundary point should be filled in.
- Solve the inequality like an equation. Then, test values to find which region makes the inequality true.
- The inequality symbol doesn't always reveal which direction to shade.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. Explain how you can determine whether a number line represents the solutions to an inequality.

Use these examples if they help with your thinking.

Explanations vary.

- If the inequality includes the equal sign, the boundary point should be filled in. If the inequality doesn't include the equal sign, the boundary point should be open.
- I can find the location of the boundary point by pretending that the inequality is an equation and finding the value of the variable that makes the equation true.
- Think carefully about what kinds of numbers will make the inequality true. For example, to make $3 - x < 1$ true, I need to subtract enough from 3. 2 isn't quite enough, but anything bigger than 2 will work.

Lesson Practice A1.11

Lesson Summary

The *solution set* of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution region* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for $2x - 4 \geq 8$:

Determine the boundary point:

$2x - 4 = 8$
$2x = 12$
$x = 6$

Determine the region:

$2x - 4 \geq 8$
$2(0) - 4 \geq 8$
$4 \geq 8$ *False!*

- 6 is the boundary point and the $\geq$ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.

0 2 4 6 8 10

When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution $-0.5 > x$ graphed on a number line.

-1.5 -1 -0.5 0 0.5

Unit 1 Lesson 11 85 Synthesis | Lesson Practice

Lesson Takeaway: The *solution set* to a one-variable inequality can be represented on a number line by determining the *boundary point*, examining the inequality symbol to determine whether the boundary point is included in the solution set or not, testing values on either side of the boundary point, and shading the *region* where the values make the inequality true.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by matching an inequality to the graph that represents its solutions.

Today's Goals

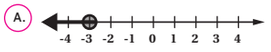
1. **Goal:** Solve one-variable inequalities by reasoning.
2. **Goal:** Graph solutions to a one-variable inequality on a number line.
3. **Language Goal:** Describe how to represent the solutions to a one-variable inequality on a number line. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

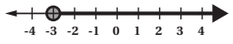
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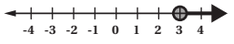
Show What You Know 1.11

Which graph represents the solutions to the inequality $-5x \geq 15$?

A. 

B. 

C. 

D. 

Explain your thinking.

Explanations vary. The boundary point is at -3 because -5(-3) = 15. I need to shade to the left because -4 makes the inequality true, but 0 does not.

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support solving and graphing one-variable inequalities:

» **Respond to Student Thinking:**

- Use the ***Solving Inequalities*** **Mini-Lesson**.
- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students learned how to graph the solutions to an inequality on a number line.

- In what ways are students taking risks when trying out new strategies for graphing inequalities?
- What types of questions might support students in exploring new strategies?
- Who did and did not participate in sharing strategies today? What trends do you see in participation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Explain how you can determine whether a number line represents the solutions to an inequality.

Use these examples if they help with your thinking.

Explanations vary.

- If the inequality includes the equal sign, the boundary point should be filled in. If the inequality doesn't include the equal sign, the boundary point should be open.
- I can find the location of the boundary point by pretending that the inequality is an equation and finding the value of the variable that makes the equation true.
- Think carefully about what kinds of numbers will make the inequality true. For example, to make $3 - x < 1$ true, I need to subtract enough from 3. 2 isn't quite enough, but anything bigger than 2 will work.

Lesson Practice A1.11

Lesson Summary

The *solution set* of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution region* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for $2x - 4 \geq 8$:

Determine the boundary point:

$$2x - 4 = 8$$
$$2x = 12$$
$$x = 6$$

Determine the region:

$$2x - 4 \geq 8$$
$$2(0) - 4 \geq 8$$
$$4 \geq 8 \textit{ False!}$$

- 6 is the boundary point and the ≥ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.



When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution $-0.5 > x$ graphed on a number line.

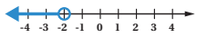


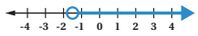
Lesson Practice A1.11

Name: ..... Date: ..... Period: .....

1. Which graph represents the solutions to $2x < -4$?

A.


B.


C.


D.


2. Create a graph of the solutions to $2x \leq 10$.



3. Leo is trying to solve $-15 + x < -14$. He knows the boundary point is at $x = 1$. How can he determine whether the solutions are $x < 1$ or $x > 1$?

Explanations vary. **Leo could test a point that is less than 1, like 0. If that point is a solution to $-15 + x < -14$, then $x < 1$ are all the solutions. If it's not, then $x > 1$ must be all the solutions.**

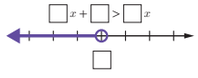
4. Diego says that $x = 5$ is a solution to $-3x > 9$ because when you divide both sides by -3, you get $x > -3$. Is this correct? Explain your thinking.

No. *Explanations vary.* I know $x = 5$ is not a solution because when you substitute 5 for x in $-3x > 9$, you get a false statement.

5. Create an inequality and the graph of its solutions by filling in each blank using the numbers 0 to 9 only once.

Responses vary.

- $3x + 8 > 5x$ and 4
- $4x + 6 > 7x$ and 2



6. Write an inequality where all the solutions are $x > 2$.

Responses vary. **$-4x < -8$**

Lesson Practice A1.11

Name: ..... Date: ..... Period: .....

Test Practice

7. What are the solutions to $-2(2 + c) + 4 < 6$?

A. $c > -3$
B. $c < -3$
C. $c > -2$
D. $c < -2$

Spiral Review

Problems 8–10: A community pool offers two different membership plans.

Plan A	Plan B
$4 per visit	An initial $12 fee, then $2 per visit

Brielle wants to spend no more than $48 at the community pool this month.

8. How many times could she visit the pool with Plan A?

12 times

9. How many times could she visit the pool with Plan B?

18 times

10. After how many visits will the cost of both plans be the same?

6 visits

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4, 6	2	MA.912.AR.2.6
	5	3	MA.912.AR.2.6
Test Practice	7	2	MA.912.AR.2.6
Spiral Review			
	8–10	2	MA.912.AR.2.1

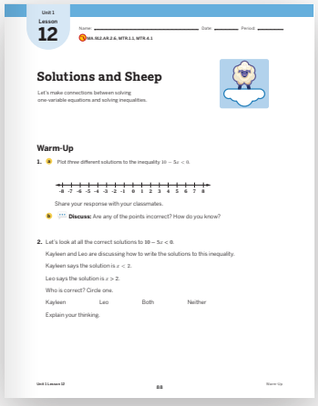
Unit 1

Lesson 12

Solutions and Sheep

Solving One-Variable Inequalities

Let's make connections between solving one-variable equations and solving inequalities.


Unit 1
Lesson 12
Name _____ Date _____ Period _____
MA.912.AR.2.6, MTR.1.1, MTR.4.1
Solutions and Sheep
Let's make connections between solving one-variable equations and solving inequalities.
Warm-Up
1. Plot three different solutions to the inequality $10 - 3x < 6$.
Share your response with your classmates.
Discuss: Are any of the points incorrect? How do you know?
2. Let's look at all the correct solutions to $10 - 3x < 6$.
Kayleen and Leo are discussing how to write the solutions to this inequality.
Kayleen says the solution is $x < 2$.
Leo says the solution is $x > 2$.
Who is correct? Circle one.
Kayleen Leo Both Neither
Explain your thinking.
88
Warm-Up

Learning Outcomes

Today's Goal

1. **Goal:** Solve a one-variable linear inequality by solving its corresponding equation and testing values.
2. **Language Goal:** Describe a strategy for solving and graphing the solutions to an inequality. **(Listening, Speaking, and Writing)**

Students solve one-variable inequalities by solving their corresponding equations. They then test values greater than and less than the boundary point of the inequality to determine which way to shade on a number line to represent the solution set.

Note: In this lesson, students develop their understanding of how to solve inequalities conceptually rather than procedurally. Resist the temptation to offer students a procedure like "flip the inequality symbol when you multiply or divide by a negative." Instead, encourage students to test points to determine the direction of the inequality.

Coherence

Prior Learning

In Lesson 11, students identified individual solutions to inequalities in order to develop strategies for solving by reasoning and to represent all solutions on a number line.

Future Learning

In Lessons 16–18, students will explore two-variable inequalities by making connections between their graphs, solutions, and corresponding equations.

Connections

- Students apply their understanding of inequalities to describe the behavior of sheep eating grass. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.6
Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.2.1 **MA.8.AR.2.1**	**MA.912.AR.2.8**

Lesson at a Glance

~ 45 min

MA.912.AR.2.6

Warm-Up

Independent | 5 min

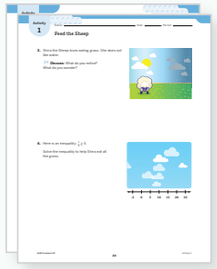
Students graph multiple solutions to an inequality on a number line and surface strategies for determining whether a value is a solution to an inequality. **Routine:** Decide and Defend



Activity 1

Pairs | 15 min

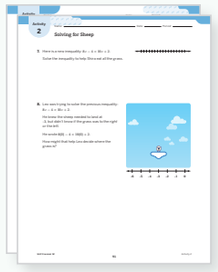
Students write and solve inequalities that represent how Shira the Sheep moves, and then practice a new strategy that can effectively solve any inequality. **Routine:** Notice and Wonder



Activity 2

Pairs | 15 min

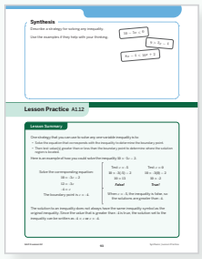
Students develop fluency as they apply the solve-and-test strategy to solve and graph the solutions to multiple inequalities.



Synthesis

Whole Class | 5 min

Students synthesize strategies for solving any inequality.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a one-variable inequality.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students graph multiple solutions to an inequality on a number line to surface strategies for determining whether a value is a solution to an inequality.

1 Launch

Consider asking, "How many solutions does an inequality have?"

A **Accessibility: Memory and Attention** To activate prior knowledge, consider asking, "What strategies have we used earlier that might help us now?"

Invite students to add their three solutions to a class number line.

D **Differentiation: (Support)** Some students may think that the numbers to the left of the boundary point are in the solution set because of the < symbol in the inequality. Consider inviting students to test one of their values in the inequality to determine if it makes the inequality true.

2 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR4.1)**

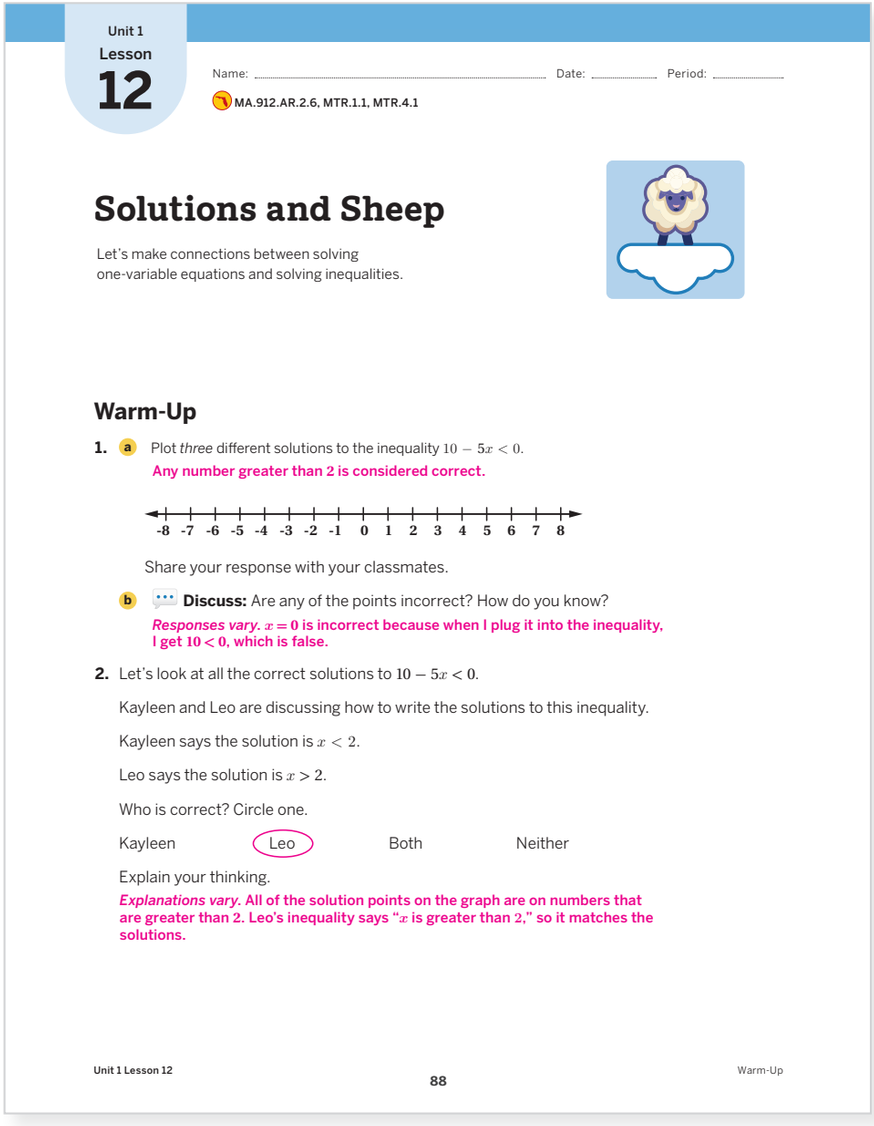
Invite students to go to different locations of the classroom based on their choice. As students discuss their reasoning, encourage students to move if they change their mind.

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

D **Differentiation: (Support)** Some students may think that the solution must have the same inequality symbol as the original inequality. Invite them to test values like $x = 1$ or $x = 3$ to see which values make the inequality true. Then ask, "How would you describe where the solutions are in relation to the boundary point $x = 2$?"

Consider asking:

- "Why do you think some students might think Kayleen's solution is correct?"
- "What strategies can help us determine which inequality symbol to use? What about which region of the number line to shade?"
- "Why can we not rely on the inequality symbol to indicate the location of the solutions?" *Responses vary.* An inequality may have a greater-than symbol but say something like $2 > x$, which could be written as $x < 2$. I have to make sense of the statement, not just read the symbol.



Unit 1
Lesson
12

Name: Date: Period:

MA.912.AR.2.6, MTR.1.1, MTR.4.1

Solutions and Sheep

Let's make connections between solving one-variable equations and solving inequalities.

Warm-Up

1. a Plot *three* different solutions to the inequality $10 - 5x < 0$.

Any number greater than 2 is considered correct.

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

Responses vary. **$x = 0$ is incorrect because when I plug it into the inequality, I get $10 < 0$, which is false.**

2. Let's look at all the correct solutions to $10 - 5x < 0$.

Kayleen and Leo are discussing how to write the solutions to this inequality.

Kayleen says the solution is $x < 2$.

Leo says the solution is $x > 2$.

Who is correct? Circle one.

Kayleen Leo Both Neither

Explain your thinking.

Explanations vary. **All of the solution points on the graph are on numbers that are greater than 2. Leo's inequality says "x is greater than 2," so it matches the solutions.**

Unit 1 Lesson 12 88 Warm-Up

Key Takeaway: Every inequality has multiple solutions. The solution to an inequality does not always have the same inequality symbol as the original inequality. Testing values in the original inequality can help determine which inequality symbol to use to represent all the solutions.

Pairs | 15 min

Activity 1 Feed the Sheep

Purpose: Students write and solve inequalities that represent how Shira the Sheep moves, and then practice a new strategy that can effectively solve any inequality.

1 Launch

Display the image from Problem 3.

ELL **English Language Learners** Create a list of words about the context, such as *grass*, *water*, *start*, *left*, *right*, etc., along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

Use the Notice and Wonder routine to get students thinking about the image from Problem 3.

2 Monitor

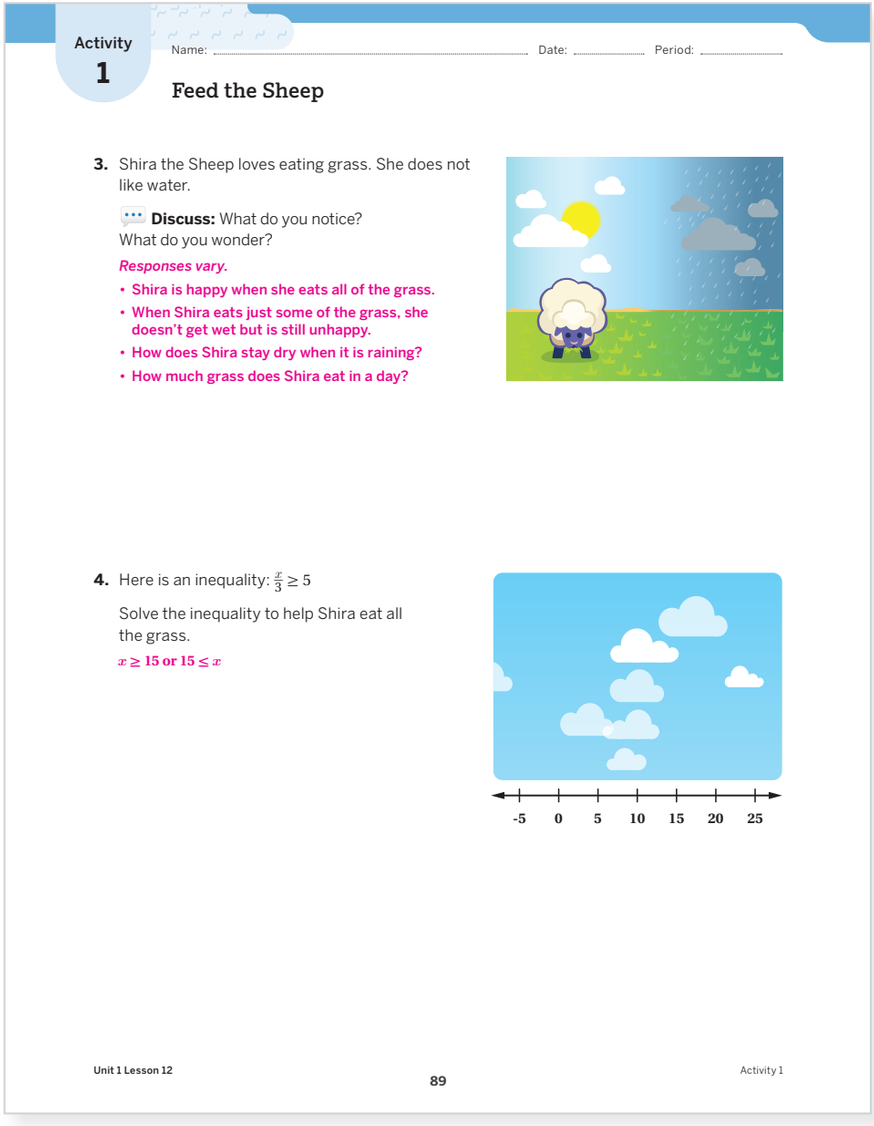
To support students in getting started, consider asking:

- "What is a number that you know is a solution?"
- "What is a number that you know is *not* a solution?"
- "Where is the boundary point?"

Encourage students to draw one or more of their solutions on the number line.

Invite students to compare their solutions with a partner.

Activity 1 continued >



Activity 1

Name: Date: Period:

Feed the Sheep

3. Shira the Sheep loves eating grass. She does not like water.

Discuss: What do you notice? What do you wonder?

Responses vary.

- Shira is happy when she eats all of the grass.
- When Shira eats just some of the grass, she doesn't get wet but is still unhappy.
- How does Shira stay dry when it is raining?
- How much grass does Shira eat in a day?

4. Here is an inequality: $\frac{x}{3} \geq 5$

Solve the inequality to help Shira eat all the grass.

$x \geq 15$ or $15 \leq x$

-5 0 5 10 15 20 25

Unit 1 Lesson 12 89 Activity 1

Pairs | 15 min

Activity 1 Feed the Sheep (continued)

Purpose: Students write and solve inequalities that represent how Shira the Sheep moves, and then practice a new strategy that can effectively solve any inequality.

2 Monitor

Look for students who used a strategy like Kayleen's in Problem 6. Consider renaming this strategy after a student in your class, or skipping Problem 6 and inviting the student in your class to share their solving strategy.

Pause to highlight the different ways that a solution to an inequality can be written (e.g., $x \geq 15$ and $15 \leq x$). Emphasize the need to read the inequality and not use the symbol to determine the location of solutions.

D **Differentiation: (Strengthen)** Consider asking:

- "Was it necessary to test both sides? Why or why not?"
- "What are the advantages and disadvantages of testing a value on each side?" *Responses vary.* One advantage of testing a value on each side is that I can check that I solved the matching equation correctly. If both tests come back with the same result, I know the equation is not solved correctly. One disadvantage of testing a value on each side is that it takes more time before I can graph or write my answer.

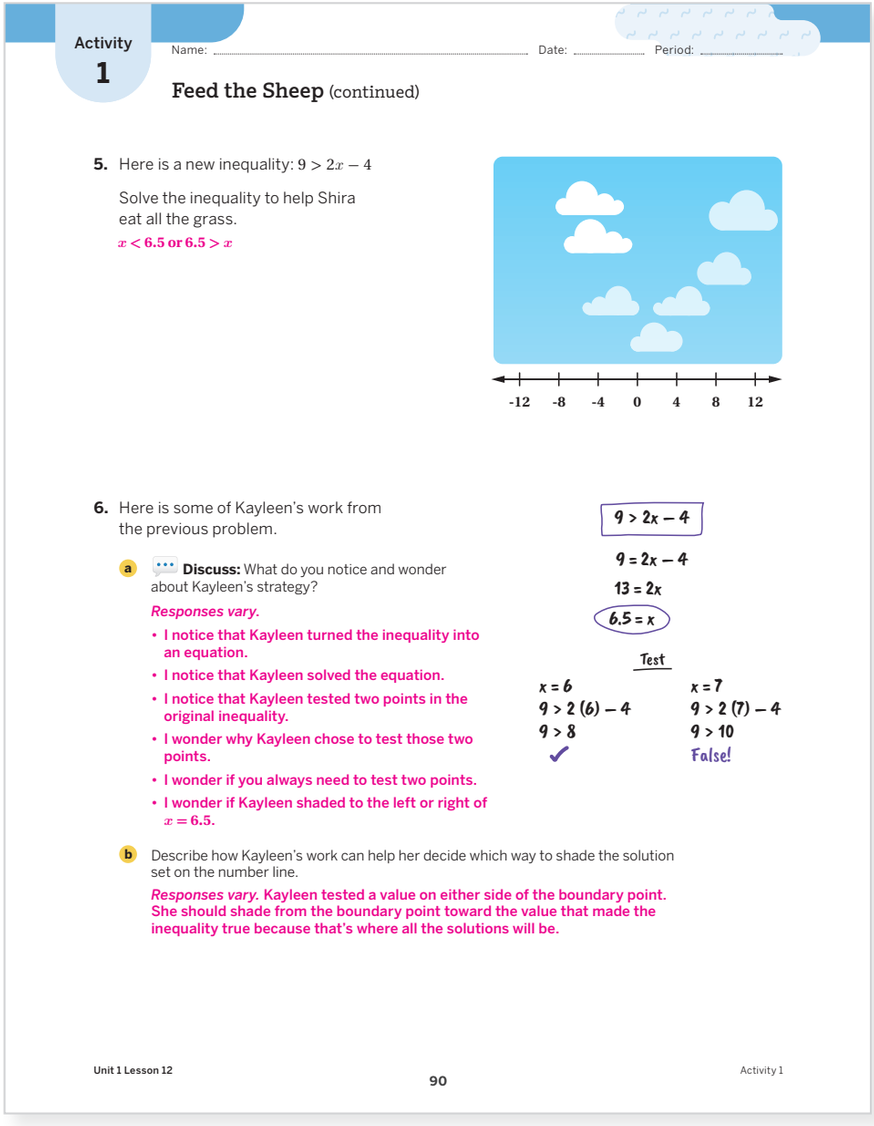
3 Connect

Use the Notice and Wonder routine to promote curiosity and help students make sense of Kayleen's strategy. **(MTR.1.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I notice . . ." or "I wonder . . .") **(Writing and Speaking)**

Consider asking:

- "What steps did Kayleen take? Why?"
- "What does the test tell us about the solutions? The graph?"
- "How could you write the solution as an inequality?"



Activity 1

Name: Date: Period:

Feed the Sheep (continued)

5. Here is a new inequality: $9 > 2x - 4$

Solve the inequality to help Shira eat all the grass.

$x < 6.5$ or $6.5 > x$

-12 -8 -4 0 4 8 12

6. Here is some of Kayleen's work from the previous problem.

$9 > 2x - 4$

$9 = 2x - 4$

$13 = 2x$

$6.5 = x$

Test

$x = 6$: $9 > 2(6) - 4$, $9 > 8$ ✓

$x = 7$: $9 > 2(7) - 4$, $9 > 10$ False!

a. **Discuss:** What do you notice and wonder about Kayleen's strategy?

Responses vary.

- I notice that Kayleen turned the inequality into an equation.
- I notice that Kayleen solved the equation.
- I notice that Kayleen tested two points in the original inequality.
- I wonder why Kayleen chose to test those two points.
- I wonder if you always need to test two points.
- I wonder if Kayleen shaded to the left or right of $x = 6.5$.

b. Describe how Kayleen's work can help her decide which way to shade the solution set on the number line.

Responses vary. Kayleen tested a value on either side of the boundary point. She should shade from the boundary point toward the value that made the inequality true because that's where all the solutions will be.

Unit 1 Lesson 12 90 Activity 1

Key Takeaway: The boundary point of an inequality can be determined by solving the corresponding equation. Testing values greater than or less than the boundary point is a strategy for determining whether the solutions are greater than or less than the boundary point.

Pairs | 15 min

Activity 2 Solving for Sheep

Purpose: Students develop fluency as they apply the solve-and-test strategy to solve and graph the solutions to multiple inequalities.

Short on time? Consider inviting students to complete fewer Repeated Challenges on in Problem 10.

1 Launch

Consider sharing with students that Kayleen's solve-and-test strategy can be used to solve any one-variable inequality and invite them to try her strategy in this activity.

Consider asking, "Can someone summarize Kayleen's strategy?" *Responses vary.* Write the inequality as an equation. Solve the equation. Test one value that is greater than and one value that is less than the solution to the equation to determine which inequality symbol to use.

Encourage students to write in their Student Edition and use the calculator if it helps with their thinking throughout the activity.

2 Monitor

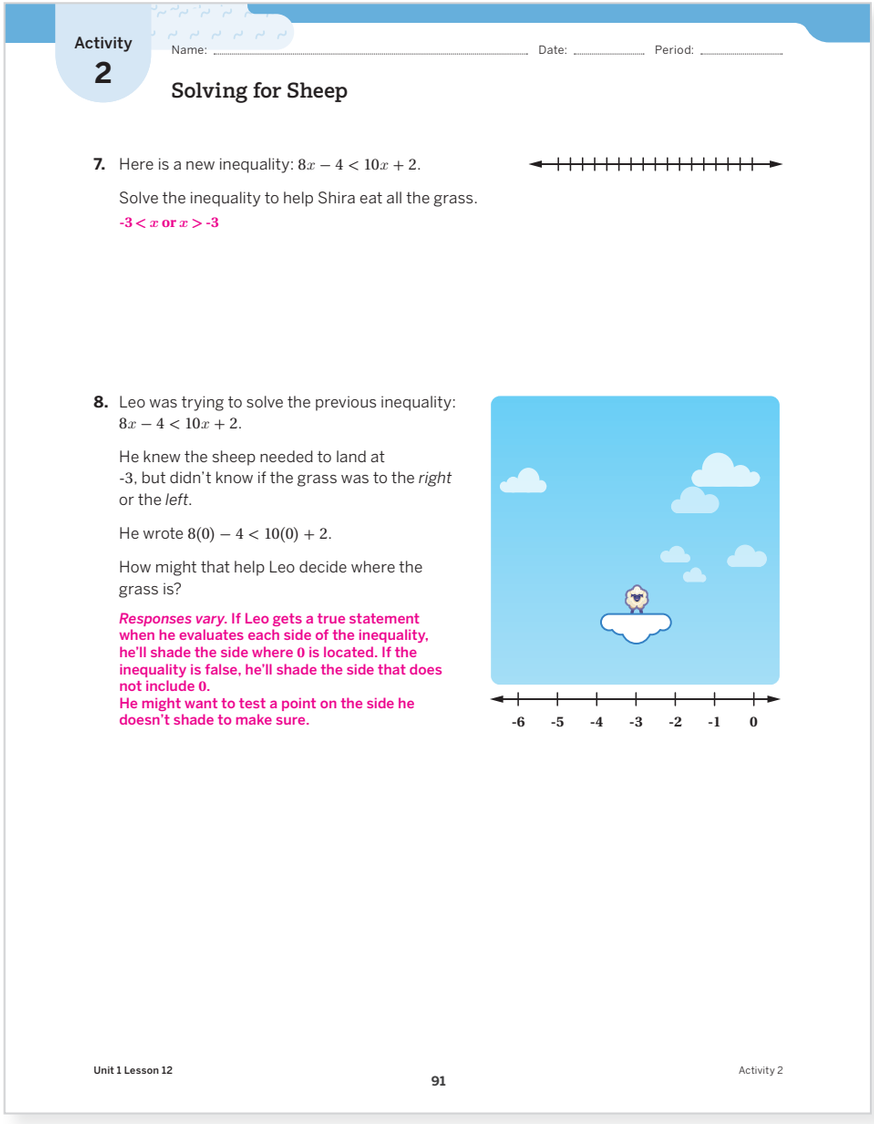
To support students in getting started, invite students to return to Kayleen's work in Problem 6 and describe her first step.

D Differentiation

Look for students who:	Teacher Moves
Are unsure why Leo chose 0 as a test value.	**Support** Consider asking, "What is helpful about the value 0?" Invite students to revisit Kayleen's strategy from the previous Problem 6.
Use the original inequality symbol by default to write the solution to the inequality.	**Support** Invite students to choose a value within the solution they've written to see if the value makes the inequality true.
Solve the inequality without using the corresponding equation.	**Support** Consider asking, "When will this strategy work? When will it *not* work?"
Are making sense of Leo's work.	**Strengthen** Consider asking: • "What point did Leo test?" • "Did the test create an inequality that was true or false?" • "What does the test say about where to shade?"

Note: If enough students need support, consider modeling how to solve the inequality using Kayleen's method.

Activity 2 continued >



Activity 2

Name: Date: Period:

Solving for Sheep

7. Here is a new inequality: $8x - 4 < 10x + 2$.

Solve the inequality to help Shira eat all the grass.

$-3 < x$ or $x > -3$

8. Leo was trying to solve the previous inequality: $8x - 4 < 10x + 2$.

He knew the sheep needed to land at -3, but didn't know if the grass was to the *right* or the *left*.

He wrote $8(0) - 4 < 10(0) + 2$.

How might that help Leo decide where the grass is?

Responses vary. If Leo gets a true statement when he evaluates each side of the inequality, he'll shade the side where 0 is located. If the inequality is false, he'll shade the side that does not include 0.
He might want to test a point on the side he doesn't shade to make sure.

-6 -5 -4 -3 -2 -1 0

Unit 1 Lesson 12 91 Activity 2

Pairs | 15 min

Activity 2 Solving for Sheep (continued)

Purpose: Students develop fluency as they apply the solve-and-test strategy to solve and graph the solutions to multiple inequalities.

Short on time? Consider inviting students to complete fewer Repeated Challenges on in Problem 10.

2 Monitor

Note: Resist the temptation to offer students a procedure like "whenever you multiply or divide by a negative, flip the inequality symbol." Instead, encourage students to test one or more points to determine the direction of the inequality.

Pause to discuss the different strategies that students used to solve the inequality. Consider asking, "Did any one choose to divide by -2 on both sides first? What about distributing the -2 as a first step?"

3 Connect

Invite students to set their own goal for how many problems they would like to complete, sharing that sense-making is more important than speed.

Students will need a partner for this activity. They will solve and compare their solutions to pairs of problems with the same solutions.

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.

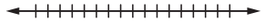
A **Accessibility: Memory and Attention** For students who would benefit from having more time to think deeply on fewer problems, invite them to complete fewer challenges.

Activity 2

Name: Date: Period:

Solving for Sheep (continued)

9. Here is a new inequality: $-2(x + 2) \leq -23$



Solve the inequality to help Shira eat all the grass.

$x \geq 9.5$ or $9.5 \leq x$

10. For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many inequalities as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$2x + 10 > 20$ $x > 5$	$\frac{x}{5} + 9 > 10$ $x > 5$
$\frac{x}{5} + 10 \leq 12$ $x \leq 10$	$5x - 10 \leq 40$ $x \leq 10$
$-10 \geq 4x + 4$ $3.5 \geq x$ (or equivalent)	$14 \geq 2x + 7$ $3.5 \geq x$ (or equivalent)
$3 - 2x > 3$ $x < 0$	$8 > 4x + 8$ $x < 0$
$\frac{x+3}{5} < 2$ $x < 7$	$\frac{x+8}{5} < 3$ $x < 7$
$5x + 10 \geq 3x + 12$ $x \geq 1$	$2x - 4 \geq x - 3$ $x \geq 1$

Unit 1 Lesson 12 92 Activity 2

Whole Class | 5 min

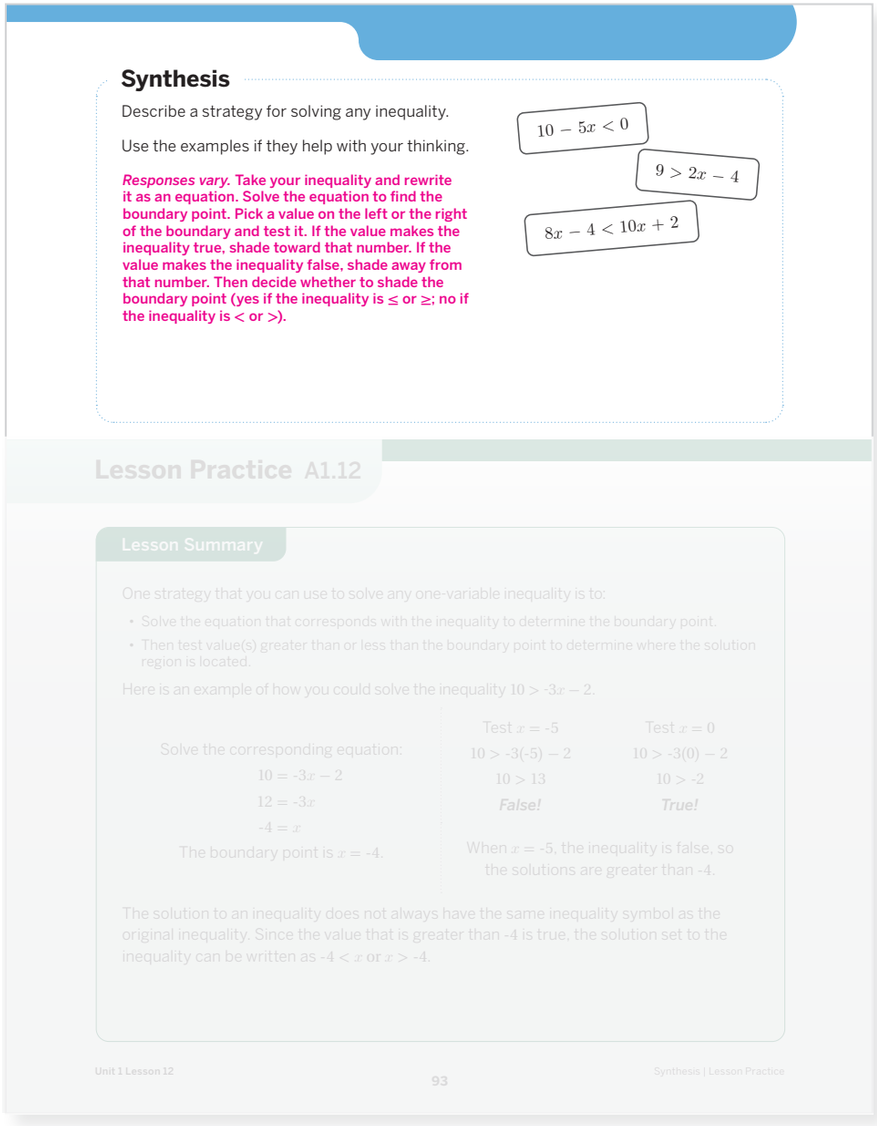
Synthesis

Purpose: Students synthesize strategies for solving any inequality.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Writing the inequality as an equation.
- Solving the equation to determine the boundary point.
- Testing a value greater than or less than the boundary point.
- Shading the region where the tested value makes the original inequality true.



Synthesis

Describe a strategy for solving any inequality.

Use the examples if they help with your thinking.

$10 - 5x < 0$

$9 > 2x - 4$

$8x - 4 < 10x + 2$

Responses vary. **Take your inequality and rewrite it as an equation. Solve the equation to find the boundary point. Pick a value on the left or the right of the boundary and test it. If the value makes the inequality true, shade toward that number. If the value makes the inequality false, shade away from that number. Then decide whether to shade the boundary point (yes if the inequality is ≤ or ≥; no if the inequality is < or >).**

Lesson Practice A1.12

Lesson Summary

One strategy that you can use to solve any one-variable inequality is to:

- Solve the equation that corresponds with the inequality to determine the boundary point.
- Then test value(s) greater than or less than the boundary point to determine where the solution region is located.

Here is an example of how you could solve the inequality $10 > -3x - 2$.

Solve the corresponding equation:

$10 = -3x - 2$

$12 = -3x$

$-4 = x$

The boundary point is $x = -4$.

Test $x = -5$	Test $x = 0$
$10 > -3(-5) - 2$	$10 > -3(0) - 2$
$10 > 13$	$10 > -2$
False!	*True!*

When $x = -5$, the inequality is false, so the solutions are greater than -4.

The solution to an inequality does not always have the same inequality symbol as the original inequality. Since the value that is greater than -4 is true, the solution set to the inequality can be written as $-4 < x$ or $x > -4$.

Unit 1 Lesson 12 93 Synthesis | Lesson Practice

Lesson Takeaway: One strategy that can be used to solve any one-variable inequality is to solve the corresponding equation to determine the boundary point. Another strategy is to test value(s) greater than and/or less than the boundary point to determine where the solutions are located.

Independent | 5 min

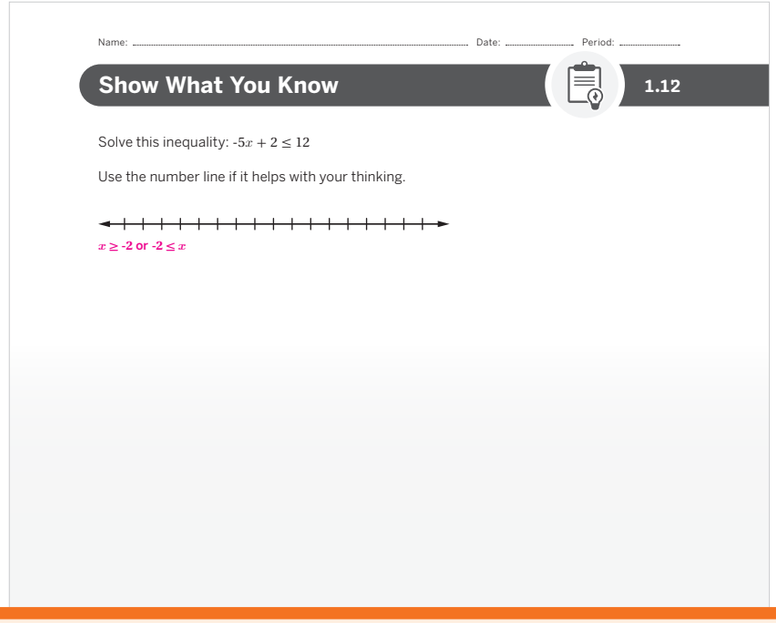
Show What You Know

Purpose: Students demonstrate their understanding by solving a one-variable inequality.

Today's Goals

1. **Goal:** Solve a one-variable linear inequality by solving its corresponding equation and testing values.
2. **Language Goal:** Describe a strategy for solving and graphing the solutions to an inequality. **(Listening, Speaking, and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.12

Solve this inequality: $-5x + 2 \leq 12$

Use the number line if it helps with your thinking.

$x \geq -2$ or $-2 \leq x$

Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support solving one-variable inequalities:

» **Respond to Student Thinking:**

- Use the ***Solving Inequalities*** **Mini-Lesson**.
- Revisit **Grade 7, Unit 6, Lesson 15** *(Shira the Sheep).*

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to write their own inequalities and trade them with a partner to solve and graph.

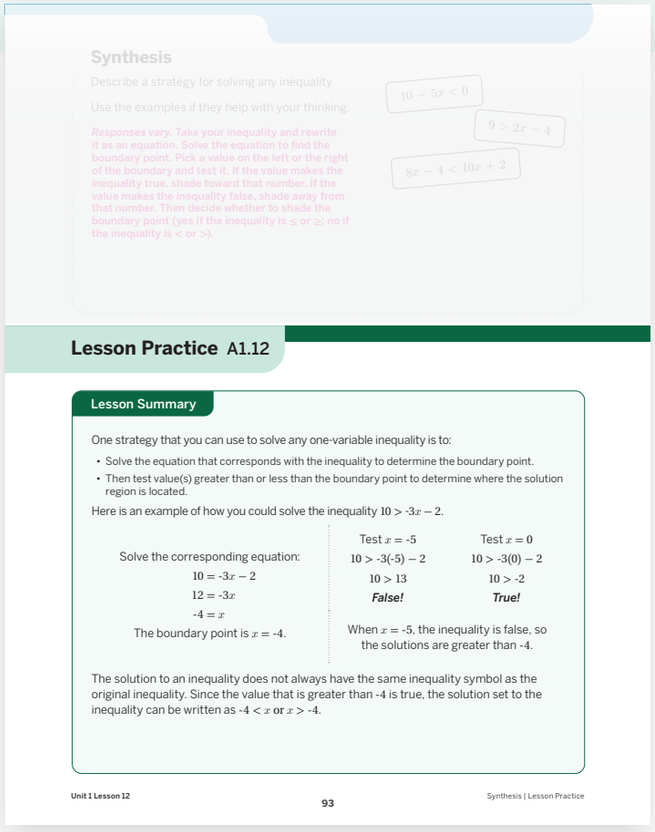
Professional Learning

In this lesson, students were introduced to a solve-and-test strategy for solving inequalities. How will this strategy support their graphing of two-variable inequalities in the upcoming lessons? How did the responsive feedback in this lesson support student learning?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

Describe a strategy for solving any inequality.

Use the examples if they help with your thinking.

$10 - 5x < 0$

$9 > 2x - 4$

$8x - 4 < 10x + 2$

Responses vary. Take your inequality and rewrite it as an equation. Solve the equation to find the boundary point. Pick a value on the left or the right of the boundary and test it. If the value makes the inequality true, shade toward that number. If the value makes the inequality false, shade away from that number. Then decide whether to shade the boundary point (yes if the inequality is $\leq$ or $\geq$; no if the inequality is $<$ or $>$).

Lesson Practice A1.12

Lesson Summary

One strategy that you can use to solve any one-variable inequality is to:

- Solve the equation that corresponds with the inequality to determine the boundary point.
- Then test value(s) greater than or less than the boundary point to determine where the solution region is located.

Here is an example of how you could solve the inequality $10 > -3x - 2$.

Solve the corresponding equation:

$10 = -3x - 2$

$12 = -3x$

$-4 = x$

The boundary point is $x = -4$.

Test $x = -5$	Test $x = 0$
$10 > -3(-5) - 2$	$10 > -3(0) - 2$
$10 > 13$	$10 > -2$
False!	***True!***

When $x = -5$, the inequality is false, so the solutions are greater than -4.

The solution to an inequality does not always have the same inequality symbol as the original inequality. Since the value that is greater than -4 is true, the solution set to the inequality can be written as $-4 < x$ or $x > -4$.

Lesson Practice
A1.12

Name: ..... Date: ..... Period: .....

Problems 1–4: Solve each inequality.

1. $4x + 5 \geq 37$

$x \geq 8$

2. $-3x + 4 \geq 12$

$x \leq -\frac{8}{3}$ (or equivalent)

3. $-6 + \frac{x}{2} < 7$

$x < 26$

4. $8x - 6 > 2x - 26$

$x > -\frac{10}{3}$

5. Here is an inequality: $7x + 6 < 3x + 2$. Select all values that are solutions.

- ☐ A. $x = 1$
- ☐ B. $x = 0$
- ☐ C. $x = -1$
- ☑ D. $x = -2$
- ☑ E. $x = -8$

6. Write three values of n that make this inequality true: $-5(n + 1) > 10$.

Responses vary. -4, -5, -6

7. Make the two inequalities equivalent by filling in each blank using the numbers 0 to 9 only once.

$\square x + \square < \square x + \square$

$x > \square$

Responses vary.

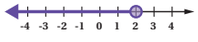
- $2x + 7 < 4x + 1$ and $x > 3$
- $5x + 6 < 8x + 0$ and $x > 2$

Lesson Practice
A1.12

Name: ..... Date: ..... Period: .....

Test Practice

8. Which inequality is represented by the graph?



- A. $2x + 6 < 10$
- B. $2x + 6 \geq 10$
- C. $-2x + 6 \geq 10$
- (D.) $-2x - 6 \geq -10$

Spiral Review

Problems 9–10: Rewrite each expression as a single power of 10.

9. $\frac{10^2 \cdot 10^5}{10^4}$

10^3

10. $(10^4)^2 \cdot (10^2)^2$

10^{12}

11. Imani is going shopping with a budget of $125. Which inequality represents the amount of dollars, x, that Imani can spend while shopping?

(A.) $x \leq 125$ B. $x \geq 125$ C. $x > 125$ D. $x < 125$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4, 6	2	MA.912.AR.2.6
	5	2	MA.912.AR.2.6
	7	3	MA.912.AR.2.6
Test Practice	8	2	MA.912.AR.2.6
Spiral Review			
Fluency	9–10	2	MA.8.AR.1.1
	11	2	MA.7.AR.2.1

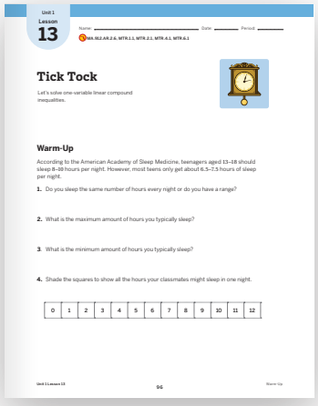
Unit 1

Lesson 13

Tick Tock

One-Variable Compound Linear Inequalities

Let's solve one-variable linear compound inequalities.


Unit 1
Lesson 13

Name _____ Date _____ Period _____

MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1

Tick Tock

Let's solve one-variable linear compound inequalities.

Warm-Up

According to the American Academy of Sleep Medicine, teenagers aged 13–18 should sleep 8–10 hours per night. However, most teens only get about 6.5–7.5 hours of sleep per night.

1. Do you sleep the same number of hours every night or do you have a range?
2. What is the maximum amount of hours you typically sleep?
3. What is the minimum amount of hours you typically sleep?
4. Shade the squares to show all the hours your classmates might sleep in one night.

0 1 2 3 4 5 6 7 8 9 10 11 12

96

Learning Outcomes

Today's Goal

1. **Goal:** Understand that a compound inequality provides a range of values for the solution set.
2. **Goal:** Write and solve compound inequalities.
3. **Language Goal:** Write stories representing compound inequalities and graphs. **(Listening, Speaking, and Writing)**

Students explore how compound inequalities provide a range of values for a solution set, including the minimum and maximum. They use mathematical and real-world contexts to write, solve, and graph one-variable compound inequalities.

Coherence

Prior Learning

In Lesson 12, students solved one-variable inequalities.

Future Learning

In Lesson 16, students will write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.

Connections

- Students use inequality statements and graphs to describe real-world behavior of sleep. (real world, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.6

Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.2	**MA.912.AR.2.7** **MA.912.AR.2.8**

Vocabulary

New Vocabulary

compound inequality

Review Vocabulary

maximum
minimum
range
inequality

Lesson at a Glance

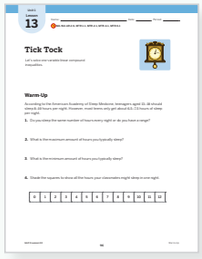
~ 45 min

MA.912.AR.2.6

Warm-Up

Small Groups | 5 min

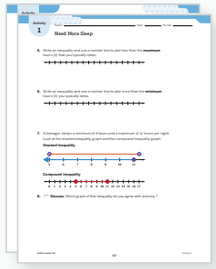
Students use the Think, Pair, and Share routine to collect data and create graphical representations of data ranges, minimums, and maximums.
Routine: Think, Pair, and Share **(MTR.1.1)**



Activity 1

Pairs | 10 min

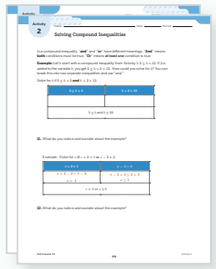
Students create data representations of compound inequalities.
Routines: Decide and Defend, Tell a Story **(MTR.2.1)**



Activity 2

Pairs | 10 min

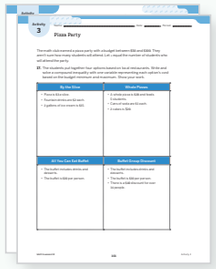
Students solve compound inequalities with and/or statements.
Routines: Co-Craft Questions, Notice and Wonder, Decide and Defend **(MTR.6.1)**



Activity 3

Small Groups | 10 min

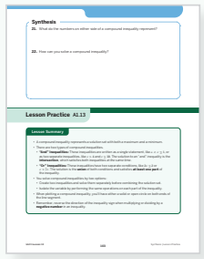
Students discuss different options based on a budget to plan for a class party.
Routine: Number Talk **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of compound inequalities.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of interpreting, writing, and solving compound inequalities.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Small Groups | 5 min

Warm-Up

Purpose: Students use the Think, Pair, and Share routine to collect data and create graphical representations of data ranges, minimums, and maximums. **(MTR.1.1)**

Short on time? Consider having students work independently and graph only their data.

1 Launch

Use the Think-Pair-Share routine to support and help students shape their ideas about sleep patterns. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's data with the class.

Consider asking:

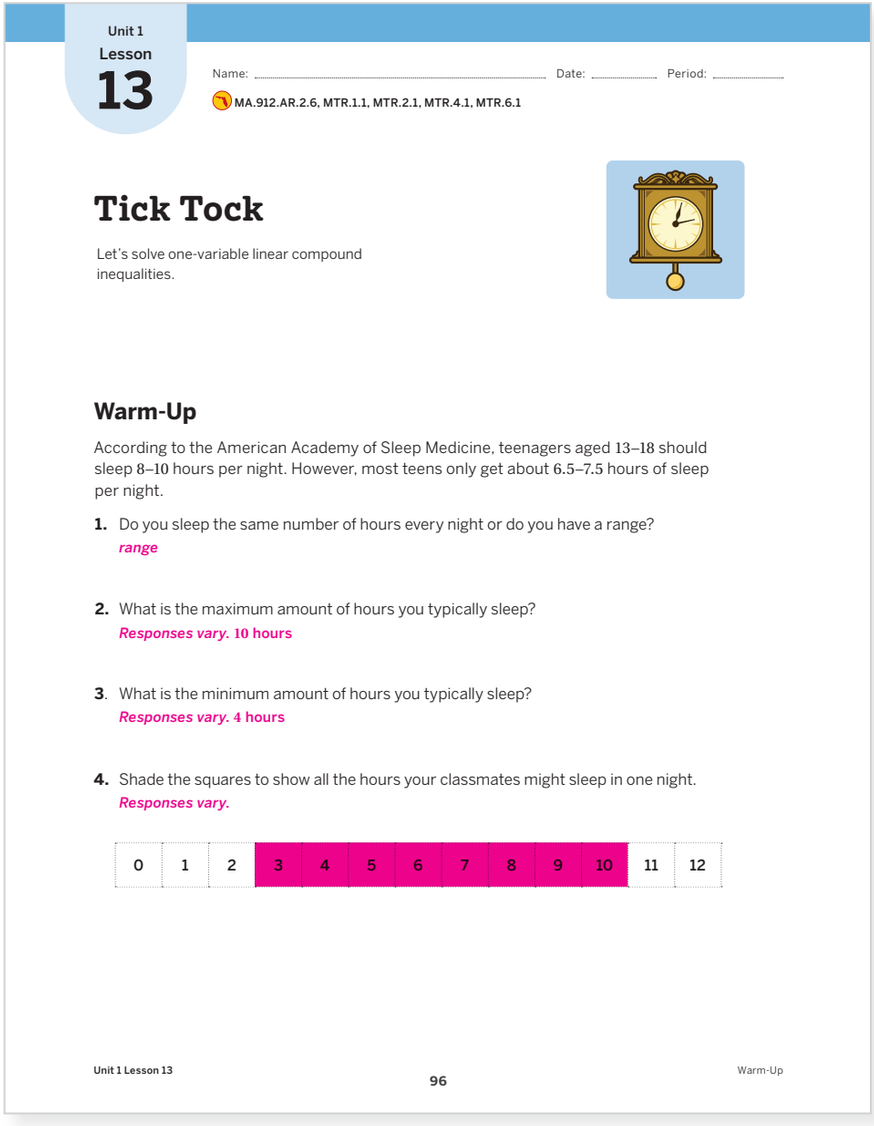
- How much sleep do you usually get each night?
- What's the least amount of sleep you may get in a week?
- What's the most amount of sleep you may get in a week?

ELL **English Language Learners** Provide a word bank to support students in expressing their strategies, with words such as range, minimum, maximum, least, most, etc. (**Speaking**)

Invite students to compare their sleep patterns: "What similarities do you see in your partner's sleep patterns?"

2 Connect

Invite students to share their sleep stories about when they may typically get more or less sleep. For example, studying for finals, nights of school activities, or family and community events that start early.



Unit 1
Lesson
13

Name: _____ Date: _____ Period: _____

MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1

Tick Tock

Let's solve one-variable linear compound inequalities.

Warm-Up

According to the American Academy of Sleep Medicine, teenagers aged 13–18 should sleep 8–10 hours per night. However, most teens only get about 6.5–7.5 hours of sleep per night.

1. Do you sleep the same number of hours every night or do you have a range?
   range

2. What is the maximum amount of hours you typically sleep?
   Responses vary. 10 hours

3. What is the minimum amount of hours you typically sleep?
   Responses vary. 4 hours

4. Shade the squares to show all the hours your classmates might sleep in one night.
   Responses vary.

0	1	2	3	4	5	6	7	8	9	10	11	12

Unit 1 Lesson 13 96 Warm-Up

Pairs | 10 min

Activity 1 Need More Sleep

Purpose: Students create data representations of compound inequalities. **(MTR.2.1)**

1 Launch

Use the Decide and Defend routine to support students in making arguments and critiquing others' reasoning. See the Routine Facilitation Guide for more information.

2 Monitor

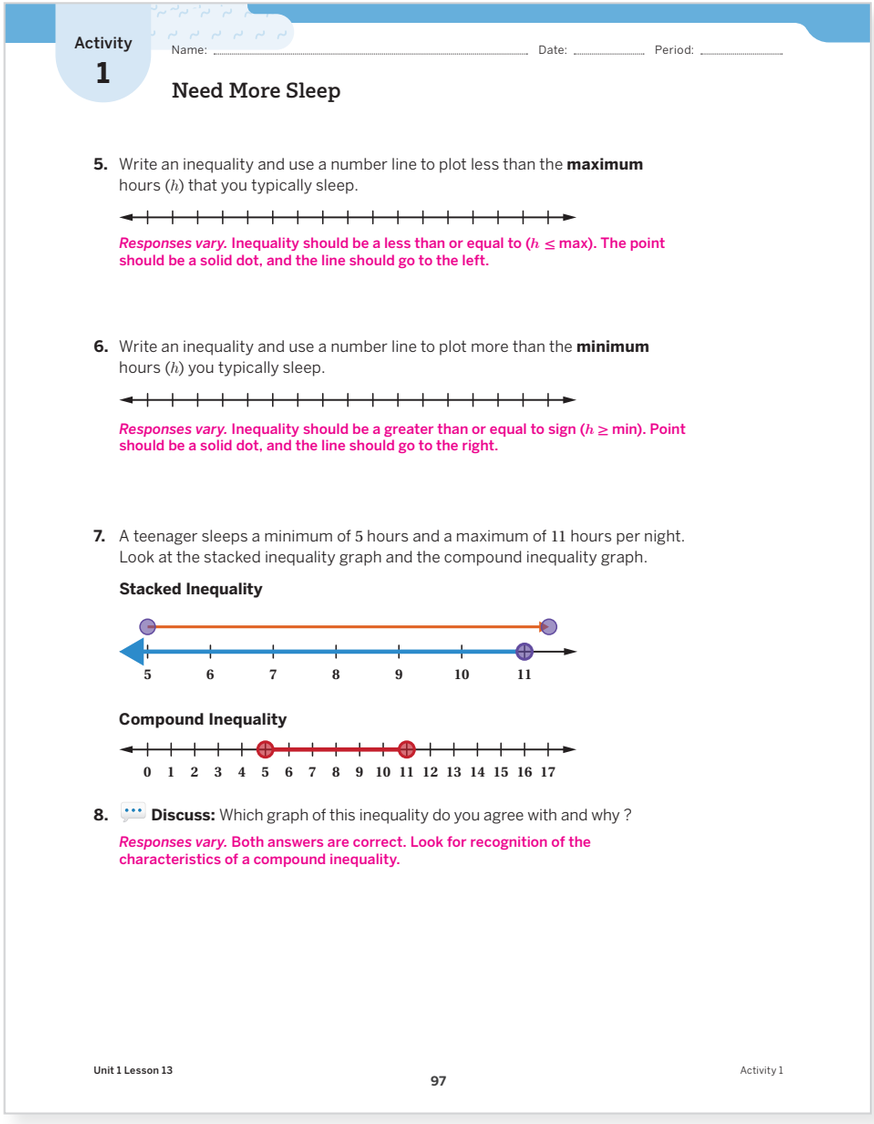
Watch for patterns in sleep data to share during the Connect.

D Differentiation *(Problems 4-6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to sketch their data to help make a line graph.
Need help understanding how to graph inequalities. (Problem 1, Problem 2)	**Strengthen** Invite them to share their minimum and maximum data points with partners to help create the graphs collaboratively.
Struggle to pick a "right" answer. (Problem 3)	**Strengthen** Consider asking, "How are the two graphs different?"
Think one answer is better than the other.	**Stretch** Show how the two graphs are showing the same information.

Look for strategies for graphing student data and invite students to share with the group.

Activity 1 continued >



Activity 1

Name: Date: Period:

Need More Sleep

5. Write an inequality and use a number line to plot less than the **maximum** hours (h) that you typically sleep.

Responses vary. Inequality should be a less than or equal to ($h \leq$ max). The point should be a solid dot, and the line should go to the left.

6. Write an inequality and use a number line to plot more than the **minimum** hours (h) you typically sleep.

Responses vary. Inequality should be a greater than or equal to sign ($h \geq$ min). Point should be a solid dot, and the line should go to the right.

7. A teenager sleeps a minimum of 5 hours and a maximum of 11 hours per night. Look at the stacked inequality graph and the compound inequality graph.

Stacked Inequality

5 6 7 8 9 10 11

Compound Inequality

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

8. **Discuss:** Which graph of this inequality do you agree with and why ?

Responses vary. Both answers are correct. Look for recognition of the characteristics of a compound inequality.

Unit 1 Lesson 13 97 Activity 1

Pairs | 10 min

Activity 1 Need More Sleep (continued)

Purpose: Students create data representations of compound inequalities. **(MTR.2.1)**

2 Monitor

Watch for patterns in sleep data to share during the Connect.

D Differentiation *(Problems 4–6)*

Look for students who:	Respond to Student Thinking
Describe the endpoints incorrectly.	**Stretch** Show how a solid endpoint differs from an open endpoint.
Have difficulty matching words, inequality signs, and graphs.	**Support** Use **number talks** to help students say and hear the common phrases between each.

Look for strategies for graphing student data and invite students to share with the group.

3 Connect

Use the Tell a Story routine to create a sleep story representing a compound inequality in number 6. Depending on time, it is possible to shorten the routine by giving one minute of individual thinking time and then inviting students to share.

To surface the Key Takeaway, ask these questions:

- What is the number on the far left of the graph?
- What is the number on the far right of the graph?
- What do these numbers represent?

Activity 1

Name: Date: Period:

Need More Sleep (continued)

9. Describe the solution for the given graph.

Graph A:



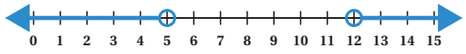
Responses vary. 5 or more hours and less than 12 hours

Graph B:



Responses vary. 5 or more hours and not more than 12 hours

Graph C:



Responses vary. Less than 5 hours or greater than 12 hours

Complete the table to show the range of hours slept, the compound inequality, and the matching graph from Problem 9.

Range of Hours Slept	Compound Inequality	Graph
5 or more hours and less than 12 hours	$5 \le h < 12$	Graph A
5 or more hours and not more than 12 hours	$5 \le h \le 12$	Graph B
less than 5 hours or greater than 12 hours	$h < 5$ or $12 < h$	Graph C

10. a. Tell a story about sleep that represents this compound inequality graph.



Responses vary. The class's range of hours of sleep is more than 3 hours and not more than 10.

b. Naoki wrote the compound inequality $3 < s \le 10$ to represent the graph. Do you agree or disagree with the inequality she wrote and why?

Responses vary. I agree because the inequality signs match more than and not more than.

Unit 1 Lesson 13 98 Activity 1

Key Takeaway: A compound inequality compares the minimum and maximum values for the range of a solution set.

Pairs | 10 min

Activity 2 Solving Compound Inequalities

Purpose: Students solve compound inequalities with and/or statements. **(MTR.6.1)**

1 Launch

MLR **MLR5: Co-Craft Questions** Use this routine to support students in making sense of compound inequalities by asking questions that include "and" and "or." For example: **(MTR.1.1, MTR.4.1)**

- Do you want to read a book and listen to music?
- Do you want to read a book or listen to music?
- Do you want peanut butter and jelly?
- Do you want peanut butter or jelly?

Use the Notice and Wonder routine to find ways to isolate the variable in a compound inequality.

2 Monitor

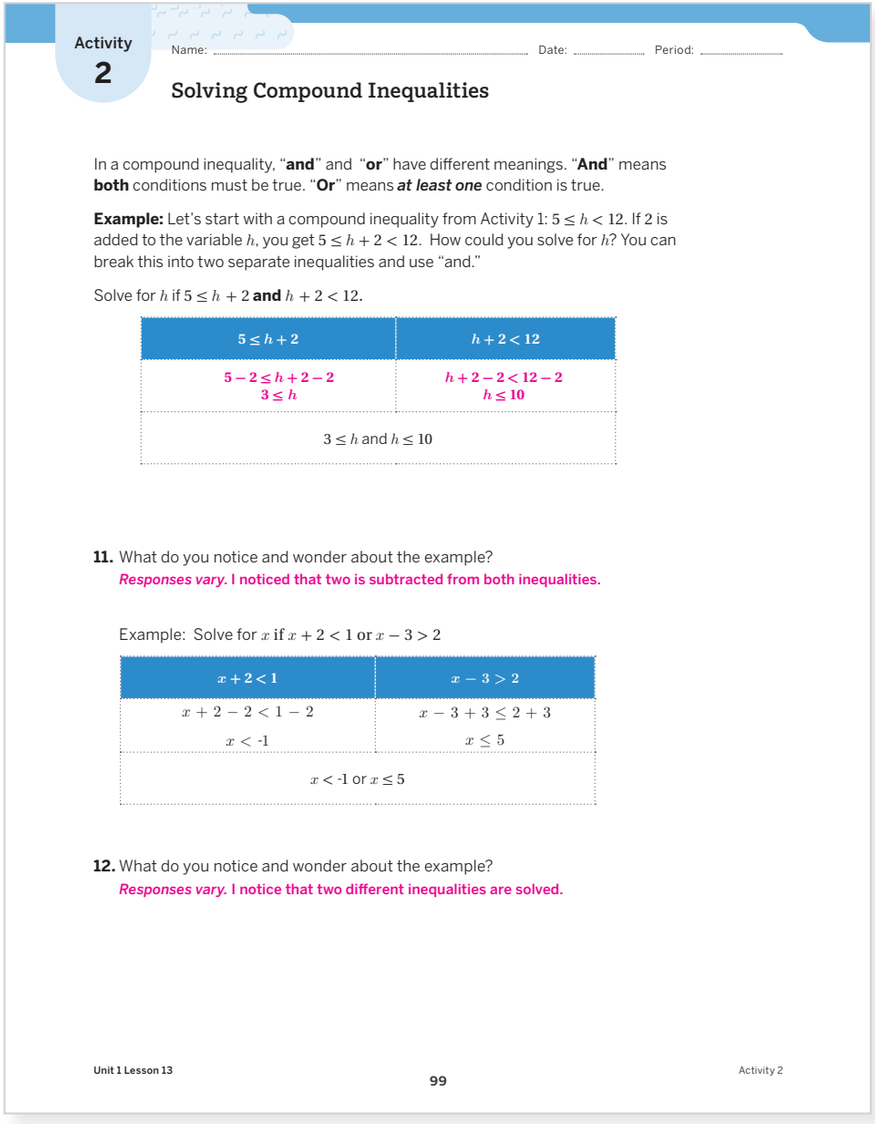
Look for students who aren't changing the sign when isolating the variable in the compound inequality.

D **Differentiation** *(Problems 1–2)*

Look for students who:	Respond to Student Thinking
Don't notice anything in the examples.	**Strengthen** Invite them to talk through the steps they did for each inequality to find that they did the same step multiple times.
Don't change the sign when isolating the variable.	**Support** Give students a visual of how to balance an equation.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Activity 2 continued >



Activity 2

Name: Date: Period:

Solving Compound Inequalities

In a compound inequality, "**and**" and "**or**" have different meanings. "**And**" means **both** conditions must be true. "**Or**" means ***at least one*** condition is true.

Example: Let's start with a compound inequality from Activity 1: $5 \leq h < 12$. If 2 is added to the variable h, you get $5 \leq h + 2 < 12$. How could you solve for h? You can break this into two separate inequalities and use "and."

Solve for h if $5 \leq h + 2$ **and** $h + 2 < 12$.

$5 \leq h + 2$	$h + 2 < 12$
$5 - 2 \leq h + 2 - 2$ $3 \leq h$	$h + 2 - 2 < 12 - 2$ $h \leq 10$
$3 \leq h$ and $h \leq 10$	

11. What do you notice and wonder about the example?

Responses vary. I noticed that two is subtracted from both inequalities.

Example: Solve for x if $x + 2 < 1$ or $x - 3 > 2$

$x + 2 < 1$	$x - 3 > 2$
$x + 2 - 2 < 1 - 2$ $x < -1$	$x - 3 + 3 \leq 2 + 3$ $x \leq 5$
$x < -1$ or $x \leq 5$	

12. What do you notice and wonder about the example?

Responses vary. I notice that two different inequalities are solved.

Unit 1 Lesson 13 99 Activity 2

Pairs | 10 min

Activity 2 Solving Compound Inequalities (continued)

Purpose: Students solve compound inequalities with and/or statements. **(MTR.6.1)**

2 Monitor

Look for students who aren't changing the sign when isolating the variable in the compound inequality.

D Differentiation *(Problems 3–5)*

Look for students who:	Respond to Student Thinking
Struggle to isolate the variable y.	**Strengthen** Remind them to look back at the examples to find what they did first.
Don't find the error with dividing by a negative number. (Problem 5)	**Stretch** Encourage students to substitute answers to test the solution to see if the answers work.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information. **(MTR.6.1)**

ELL English Language Learners Give students time to think independently before recording or verbally sharing their ideas. **(Writing, Speaking, and Listening)**

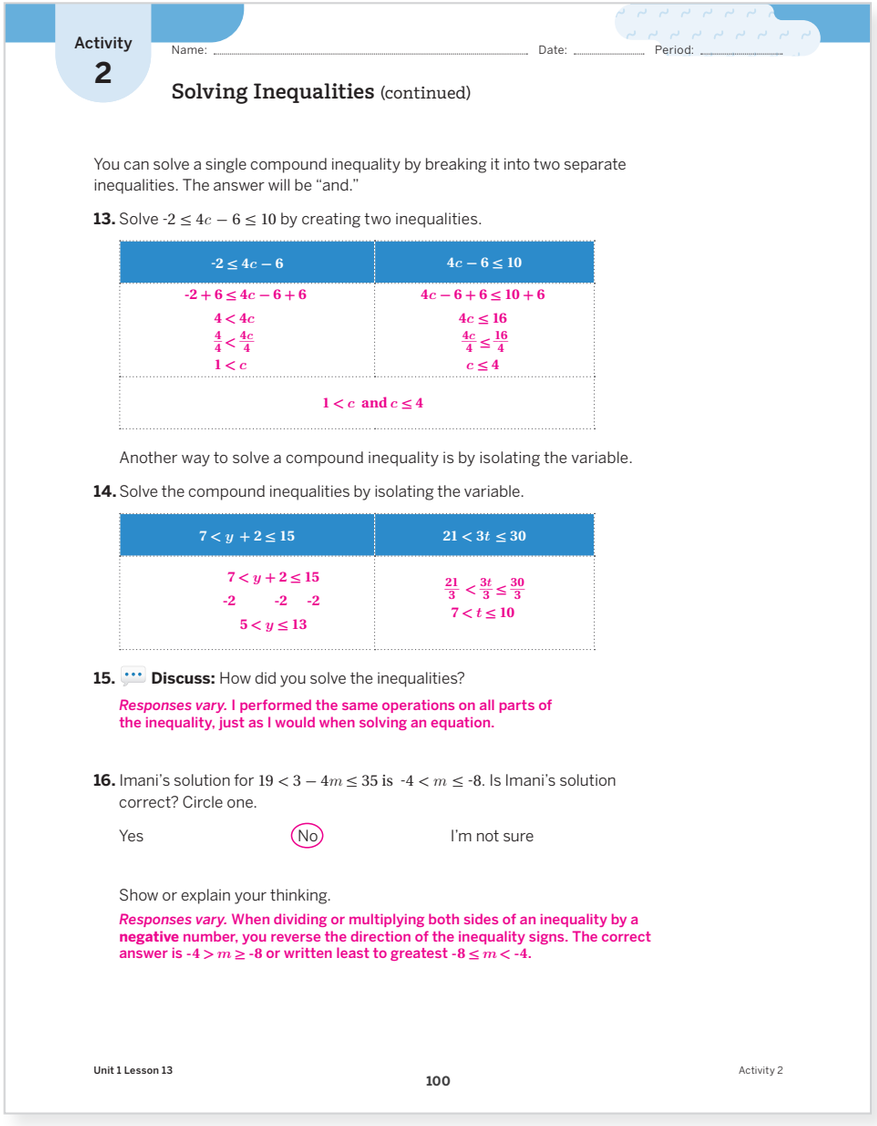
3 Connect

Share that there are two ways to solve inequalities with "and." They can solve for two separate inequalities or one compound inequality with the variable in the middle and the minimum and maximum on either side.

To surface the Key Takeaway, allow students to choose between two common classroom items or food items for lunch.

- They can have a pencil **and** a marker.
- They can have a pencil **or** a marker.
- They can have a sandwich **and** a salad.
- They can have a sandwich **or** a salad.

Ask them what is the difference between these statements. How many things do you have for each option?



Activity 2

Name: Date: Period:

Solving Inequalities (continued)

You can solve a single compound inequality by breaking it into two separate inequalities. The answer will be "and."

13. Solve $-2 \le 4c - 6 \le 10$ by creating two inequalities.

$-2 \le 4c - 6$	$4c - 6 \le 10$
$-2 + 6 \le 4c - 6 + 6$ $4 < 4c$ $\frac{4}{4} < \frac{4c}{4}$ $1 < c$	$4c - 6 + 6 \le 10 + 6$ $4c \le 16$ $\frac{4c}{4} \le \frac{16}{4}$ $c \le 4$
$1 < c$ and $c \le 4$	

Another way to solve a compound inequality is by isolating the variable.

14. Solve the compound inequalities by isolating the variable.

$7 < y + 2 \le 15$	$21 < 3t \le 30$
$7 < y + 2 \le 15$ $-2 \quad -2 \quad -2$ $5 < y \le 13$	$\frac{21}{3} < \frac{3t}{3} \le \frac{30}{3}$ $7 < t \le 10$

15. **Discuss:** How did you solve the inequalities?

Responses vary. I performed the same operations on all parts of the inequality, just as I would when solving an equation.

16. Imani's solution for $19 < 3 - 4m \le 35$ is $-4 < m \le -8$. Is Imani's solution correct? Circle one.

Yes (No) I'm not sure

Show or explain your thinking.

Responses vary. When dividing or multiplying both sides of an inequality by a negative number, you reverse the direction of the inequality signs. The correct answer is $-4 > m \ge -8$ or written least to greatest $-8 \le m < -4$.

Unit 1 Lesson 13 100 Activity 2

Key Takeaway: Compound inequalities with "and" means both conditions must be true at the same time. Inequalities with "or" means at least one condition is true.

Small Groups | 15 min

Activity 3 Pizza Party

Purpose: Students discuss different options based on a budget to plan for a class party. **(MTR.4.1)**

Short on time? Consider removing the two buffet options.

1 Launch

Use the Number Talk routine to support students' ability to think flexibly and surface strategies that students will need later in the lesson.

Consider having the students draw out the different options to help visualize the inequalities.

Invite students to share stories about their experiences with budgets.

2 Monitor

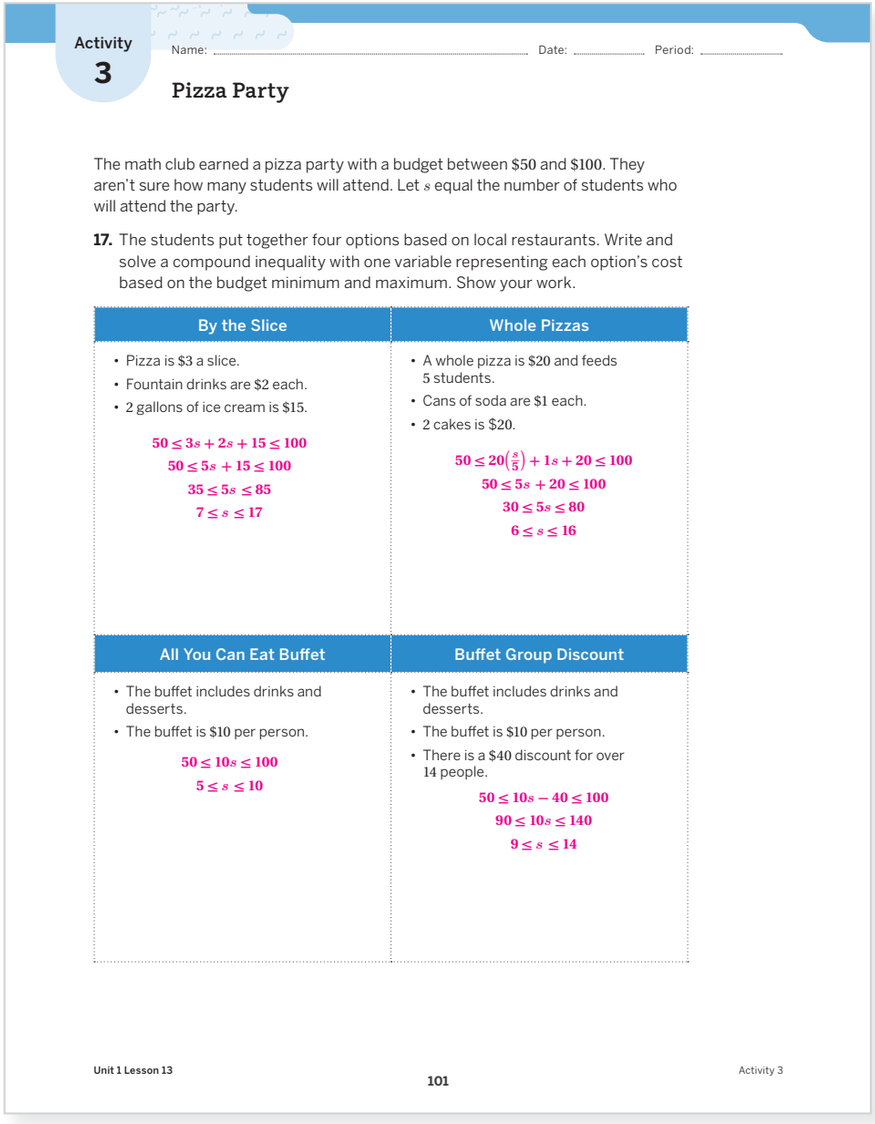
Look for times when students don't know what to do next.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to use highlighters in several colors to color code the prices for similar items.
Struggle to find common costs.	**Strengthen** Invite them to create a table to organize the various costs.
Have errors in calculations when isolating the variable.	**Support** Show students how to break a compound inequality into two statements.

Watch for options that won't work with the budget and guide students through problem-solving (Buffet Group Discount).

Activity 3 continued >



Activity 3

Name: Date: Period:

Pizza Party

The math club earned a pizza party with a budget between $50 and $100. They aren't sure how many students will attend. Let s equal the number of students who will attend the party.

17. The students put together four options based on local restaurants. Write and solve a compound inequality with one variable representing each option's cost based on the budget minimum and maximum. Show your work.

By the Slice

- Pizza is $3 a slice.
- Fountain drinks are $2 each.
- 2 gallons of ice cream is $15.

$50 \le 3s + 2s + 15 \le 100$
$50 \le 5s + 15 \le 100$
$35 \le 5s \le 85$
$7 \le s \le 17$

Whole Pizzas

- A whole pizza is $20 and feeds 5 students.
- Cans of soda are $1 each.
- 2 cakes is $20.

$50 \le 20\left(\frac{s}{5}\right) + 1s + 20 \le 100$
$50 \le 5s + 20 \le 100$
$30 \le 5s \le 80$
$6 \le s \le 16$

All You Can Eat Buffet

- The buffet includes drinks and desserts.
- The buffet is $10 per person.

$50 \le 10s \le 100$
$5 \le s \le 10$

Buffet Group Discount

- The buffet includes drinks and desserts.
- The buffet is $10 per person.
- There is a $40 discount for over 14 people.

$50 \le 10s - 40 \le 100$
$90 \le 10s \le 140$
$9 \le s \le 14$

Unit 1 Lesson 13 101 Activity 3

Small Groups | 15 min

Activity 3 Pizza Party (continued)

Purpose: Students discuss different options based on a budget to plan for a class party. **(MTR.4.1)**

Short on time? Consider removing the two buffet options.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Struggle to create compound inequalities.	**Strengthen** Consider providing math frames to help them move from data to math sentences to compound inequalities.
Have calculation errors.	**Support** Invite students to use a calculator to support them with performing calculations.

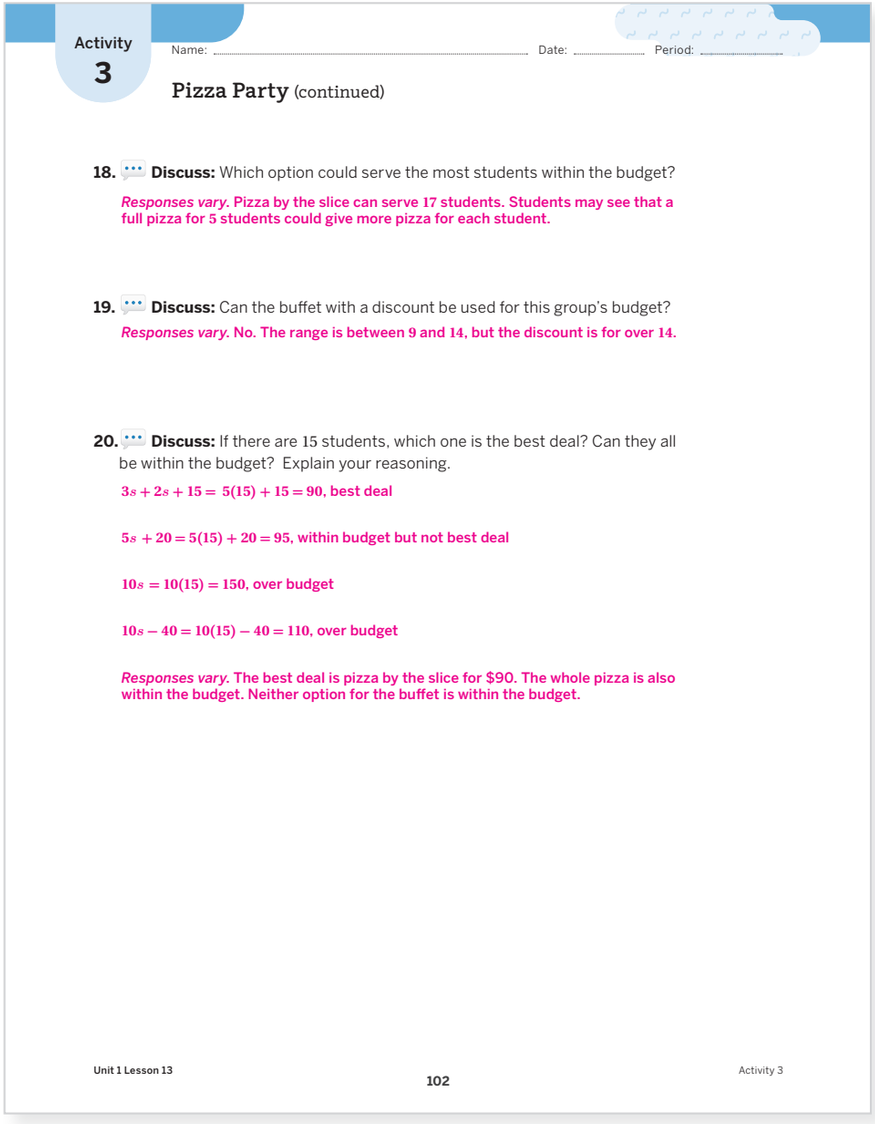
A Accessibility: Executive Functioning
Consider chunking this activity by inviting students to analyze one scenario at a time. To promote student focus, consider covering up certain quadrants until students are ready to address them.

3 Connect

Discuss as a large group how students found which option could serve the most students.

Invite students to share which option they would choose and if other considerations besides money influenced their decision.

To surface the Key Takeaway, consider role-playing by giving partners paper money to try buying food for the party for different numbers of students.



Activity 3

Name: Date: Period:

Pizza Party (continued)

18. **Discuss:** Which option could serve the most students within the budget?

Responses vary. Pizza by the slice can serve 17 students. Students may see that a full pizza for 5 students could give more pizza for each student.

19. **Discuss:** Can the buffet with a discount be used for this group's budget?

Responses vary. No. The range is between 9 and 14, but the discount is for over 14.

20. **Discuss:** If there are 15 students, which one is the best deal? Can they all be within the budget? Explain your reasoning.

$3s + 2s + 15 = 5(15) + 15 = 90$, best deal

$5s + 20 = 5(15) + 20 = 95$, within budget but not best deal

$10s = 10(15) = 150$, over budget

$10s - 40 = 10(15) - 40 = 110$, over budget

Responses vary. The best deal is pizza by the slice for $90. The whole pizza is also within the budget. Neither option for the buffet is within the budget.

Unit 1 Lesson 13 102 Activity 3

Key Takeaway: Solve problems by interpreting real-world situations using compound inequalities with budgets.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of compound inequalities.

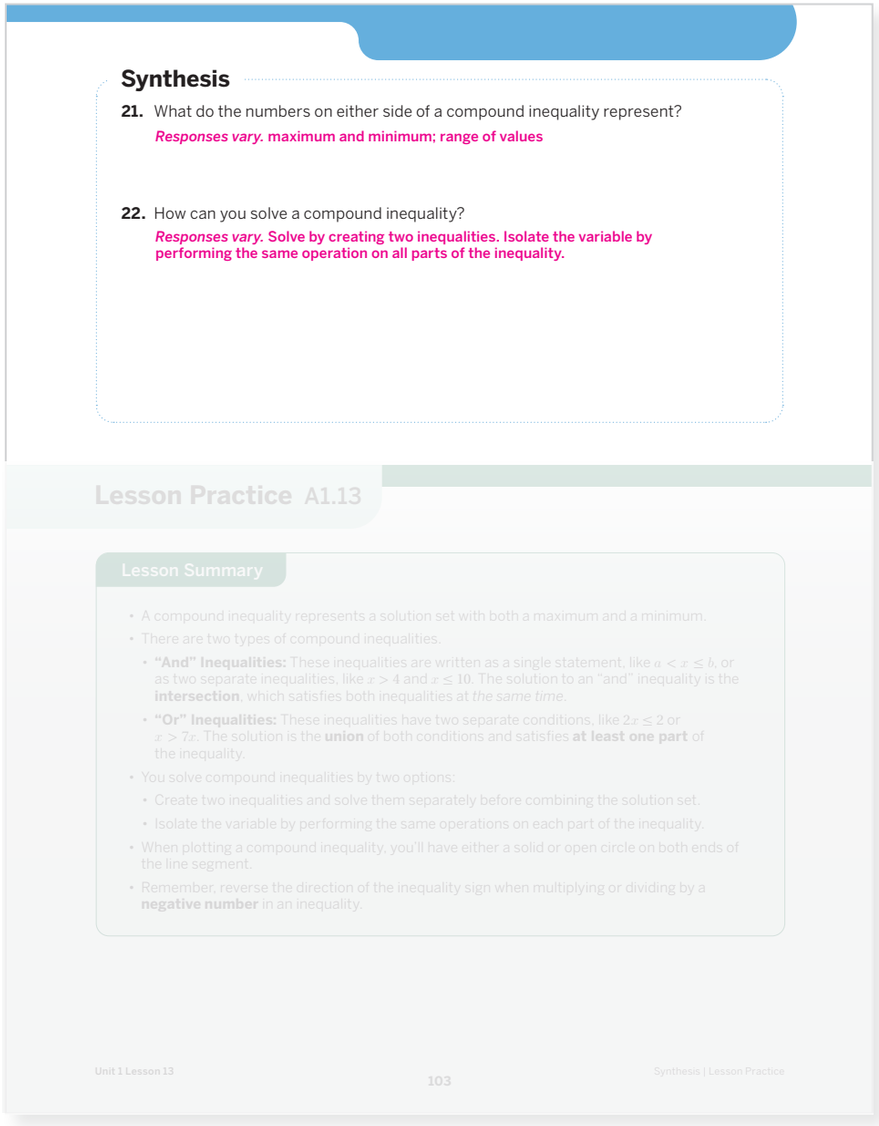
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The maximum is the largest number in the range.
- The minimum is the smallest number in the range.
- A compound inequality with "and" means both statements have to be true at the same time.
- A compound inequality with "or" means only one statement needs to be true.

MLR **MLR.2: Collect and Display** Use this routine to formalize the understanding of compound inequalities using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

21. What do the numbers on either side of a compound inequality represent?

Responses vary. maximum and minimum; range of values

22. How can you solve a compound inequality?

Responses vary. Solve by creating two inequalities. Isolate the variable by performing the same operation on all parts of the inequality.

Lesson Practice A1.13

Lesson Summary

- A compound inequality represents a solution set with both a maximum and a minimum.
- There are two types of compound inequalities.
  - **"And" Inequalities:** These inequalities are written as a single statement, like $a < x \leq b$, or as two separate inequalities, like $x > 4$ and $x \leq 10$. The solution to an "and" inequality is the **intersection**, which satisfies both inequalities at *the same time*.
  - **"Or" Inequalities:** These inequalities have two separate conditions, like $2x \leq 2$ or $x > 7x$. The solution is the **union** of both conditions and satisfies **at least one part** of the inequality.
- You solve compound inequalities by two options:
  - Create two inequalities and solve them separately before combining the solution set.
  - Isolate the variable by performing the same operations on each part of the inequality.
- When plotting a compound inequality, you'll have either a solid or open circle on both ends of the line segment.
- Remember, reverse the direction of the inequality sign when multiplying or dividing by a **negative number** in an inequality.

Unit 1 Lesson 13 103 Synthesis | Lesson Practice

Lesson Takeaway: Compound inequalities help solve problems with more than one correct answer and a range of solutions can work.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of interpreting, writing, and solving compound inequalities.

Today's Goals

1. **Goal:** Understand that a compound inequality provides a range of values for the solution set.
2. **Goal:** Write and solve compound inequalities.
3. **Language Goal:** Write stories representing compound inequalities and graphs.

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.13

Kayla is going to the bookstore. She wants to buy 4 books plus a $20 gift certificate. She wants to spend between $60 and $100.

a. Let x represent the price Kayla pays for each book. Write an inequality that can be used to determine the minimum and maximum price of the books Kayla can buy and stay in her budget.
$60 \leq 4x + 20 \leq 100$ or $40 \leq 4x \leq 80$

b. Plot the solution set on a line graph.



c. How do you interpret the maximum and minimum?
Responses vary. The maximum is the greatest price she can pay for each book and stay in her budget, and the minimum is the least amount she can pay for each book and stay in her budget.

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support writing, solving, or graphing the solution set of a compound inequality in context:

» **Respond to Student Thinking:**

- Revisit **Activity 1** to review the minimum and maximum values of a range of possible solutions to an inequality.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Revisit **Activity 3** and Invite students to create additional options for a pizza party and swapping with partners to compare the best deals.

Professional Learning

Reflect on the facilitation of the Think-Pair-Share routine in this lesson. Students will use this routine again. Who shared in the whole class discussion? Who did not? What about the routine felt successful? What are some areas for growth next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

21. What do the numbers on either side of a compound inequality represent?

Responses vary. maximum and minimum; range of values

22. How can you solve a compound inequality?

Responses vary. Solve by creating two inequalities. Isolate the variable by performing the same operation on all parts of the inequality.

Lesson Practice A1.13

Lesson Summary

- A compound inequality represents a solution set with both a maximum and a minimum.
- There are two types of compound inequalities.
  - **"And" inequalities:** These inequalities are written as a single statement, like $a < x \leq b$, or as two separate inequalities, like $x > 4$ and $x \leq 10$. The solution to an "and" inequality is the **intersection**, which satisfies both inequalities at *the same time*.
  - **"Or" inequalities:** These inequalities have two separate conditions, like $2x \leq 2$ or $x > 7x$. The solution is the **union** of both conditions and satisfies **at least one part** of the inequality.
- You solve compound inequalities by two options:
  - Create two inequalities and solve them separately before combining the solution set.
  - Isolate the variable by performing the same operations on each part of the inequality.
- When plotting a compound inequality, you'll have either a solid or open circle on both ends of the line segment.
- Remember, reverse the direction of the inequality sign when multiplying or dividing by a **negative number** in an inequality.

Lesson Practice A1.13

Name: ..... Date: ..... Period: .....

1. Choose the words to complete the sentence:

range	minimum	variable	maximum

A compound inequality represents the ___minimum___ and the ___maximum___ for the ___variable___ of the solution set.

Problems 2 and 3: Select the compound variable that represents the scenario.

2. Part-time employees can work no fewer than 20 hours and no more than 30 hours per week

(A.) $20 \leq h \leq 30$ B. $20 < h < 30$ C. $20 < h \leq 30$ D. $20 \leq h < 30$

3. The temperature for the day will be more than 70 degrees but not more than 90 degrees Fahrenheit.

A. $70 \leq t \leq 90$ B. $70 < t < 90$ (C.) $70 < t \leq 90$ D. $70 \leq t < 90$

Problems 4–6: You are a part of a work-study program. The guidelines state you must work between 5 and 10 hours each week. You always work for 3 hours on Saturday. During the weekdays, you can work for 2 hours each day. How many days a week must you work to meet the program requirements?

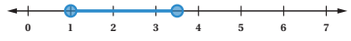
4. Write and solve a compound inequality to determine the acceptable range of weekday (d) work shifts to meet the program requirements.

$5 \leq 2d + 3 \leq 10$

$2 \leq 2d \leq 7$

$1 \leq d \leq 3.5$

5. Plot the solution set on a line graph.



Lesson Practice A1.13

Name: ..... Date: ..... Period: .....

6. How do you interpret the maximum?

Responses vary. Since the answer is looking for weekdays to work two hours, the solution would be interpreted with a maximum of a whole number.

Problems 7 and 8: Your monthly budget includes spending between $200 and $400 on groceries and dining out. You know you will spend $160 on groceries, and you want to budget to go out to eat once a week.

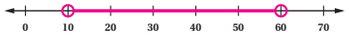
7. Let x represent the amount you spend dining out each week. Write and solve a compound inequality to determine the range of spending on dining out to stay within your monthly budget.

$200 < 160 + 4x < 400$

$40 < 4x < 240$

$10 < x < 60$

8. Plot the solution set on a line graph.



Test Practice

9. A recipe requires the oven temperature, measured in Farenheit, to be between 350 and 375 degrees, but it includes a 5-degree buffer for cooking effectiveness.

Which compound inequality represents this situation?

(A.) $350 \leq c + 5 \leq 375$ B. $350 < c - 5 < 375$ C. $350 \leq c + 5 < 375$ D. $350 < c - 5 \leq 375$

Spiral Review

Problems 10 and 11: Simplify the expression.

10. $\frac{x^{10}}{x^2}$

x^8

11. $(x^3 \cdot y^{-2})^2$

$x^6 y^{-4}$ or $\frac{x^6}{y^4}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.2.6
	2, 3	2	MA.912.AR.2.6 MA.912.AR.2.6
	4–8	3	MA.912.AR.2.6
Test Practice	9	3	MA.912.AR.2.6
Spiral Review			
Fluency	10–11	1	MA.912.NSO.1.2

Unit 1

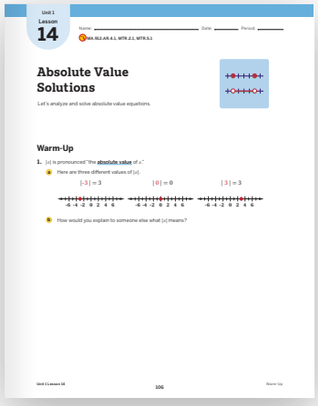
Lesson

14

Absolute Value Solutions

Solving Absolute Value Equations

Let's analyze and solve absolute value equations.


Unit 1
Lesson
14
Name _____ Date _____ Period _____
MA.912.AR.4.1, MTR.2.1, MTR.5.1
Absolute Value Solutions
Let's analyze and solve absolute value equations.
Warm-Up
1. $|x|$ is pronounced "the absolute value of x."
a. There are three different values of $|x|$.
$|-3| = 3$
$|0| = 0$
$|3| = 3$
b. How would you explain to someone else what $|x|$ means?
106

Learning Outcomes

Today's Goal

1. **Goal:** Solve one-variable absolute value equations.
2. **Goal:** Interpret solutions of absolute value equations and number line representations.
3. **Language Goal:** Explain the meaning of absolute value equations. **(Listening, Speaking, and Writing)**

Students surface prior knowledge of solving one-variable equations and understanding of absolute value as the distance of a number from 0 to develop conceptual understanding of how to solve absolute value equations. Students make use of the structure of absolute value equations and their associated graphs on number lines to develop solving strategies. **(MTR.5.1)**

Coherence

Prior Learning

In Grade 6, students learned that the absolute value of a number is that number's distance from 0 on a number line and solved real-world problems involving the comparison of absolute values. In Lessons 1–3, students developed their understanding of solving multi-step linear equations.

Future Learning

In Lesson 15, students will extend their understanding to include absolute value equations that result in two solutions, one solution, or no solution.

Connections

- Students apply their fluency with equations and inequalities as they explore absolute value relationships. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.4.1

Given a mathematical or real-world context, write and solve one-variable absolute value equations.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.1.3	**MA.912.AR.4.2**
MA.6.NSO.1.4	**MA.912.AR.4.3**

Vocabulary

Review Vocabulary

absolute value

solution

Lesson at a Glance

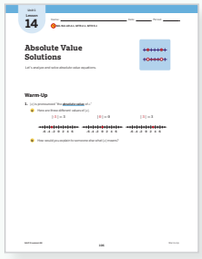
~ 45 min

MA.912.AR.4.1

Warm-Up

Independent | 5 min

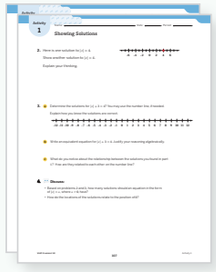
Students analyze the absolute value of given numbers using a graph to define absolute value in their own words.



Activity 1

Pairs | 10 min

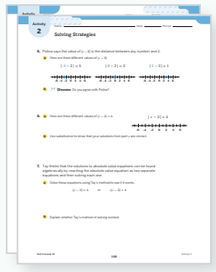
Students use the structure of absolute value equations to represent their solutions on a number line and build reasoning between the solutions and equations.
Routine: Think-Pair-Share **(MTR.2.1, MTR.5.1)**



Activity 2

Pairs | 10 min

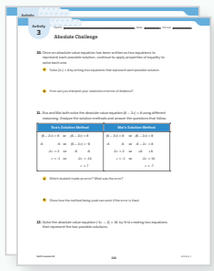
Students connect the understanding of absolute value as distance with solving equations and use structure to solve absolute value equations.
Routine: Think-Pair-Share, MLR3: Critique, Correct, and Clarify **(MTR.5.1)**



Activity 3

Small Groups | 10 min

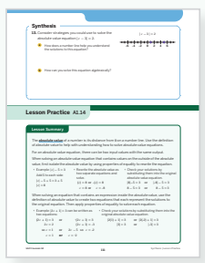
Students analyze the structure of more complex absolute value equations and find solutions.
Routine: Think-Pair-Share, MLR7: Compare and Connect



Synthesis

Whole Class | 5 min

Students synthesize their strategies for solving absolute value equations.
Routine: MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving an absolute value equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students analyze the absolute value of given numbers using a graph to define absolute value in their own words.

1 Launch

Invite students to read the phrase "the absolute value of x" aloud together as a class.

2 Connect

Direct students' attention to the three number lines and their related values after students have had time to explore on their own.

Invite students to share what they think it means to be "the absolute value" of something or what it reminds them of (e.g., "When I hear absolute value, I think of . . .").

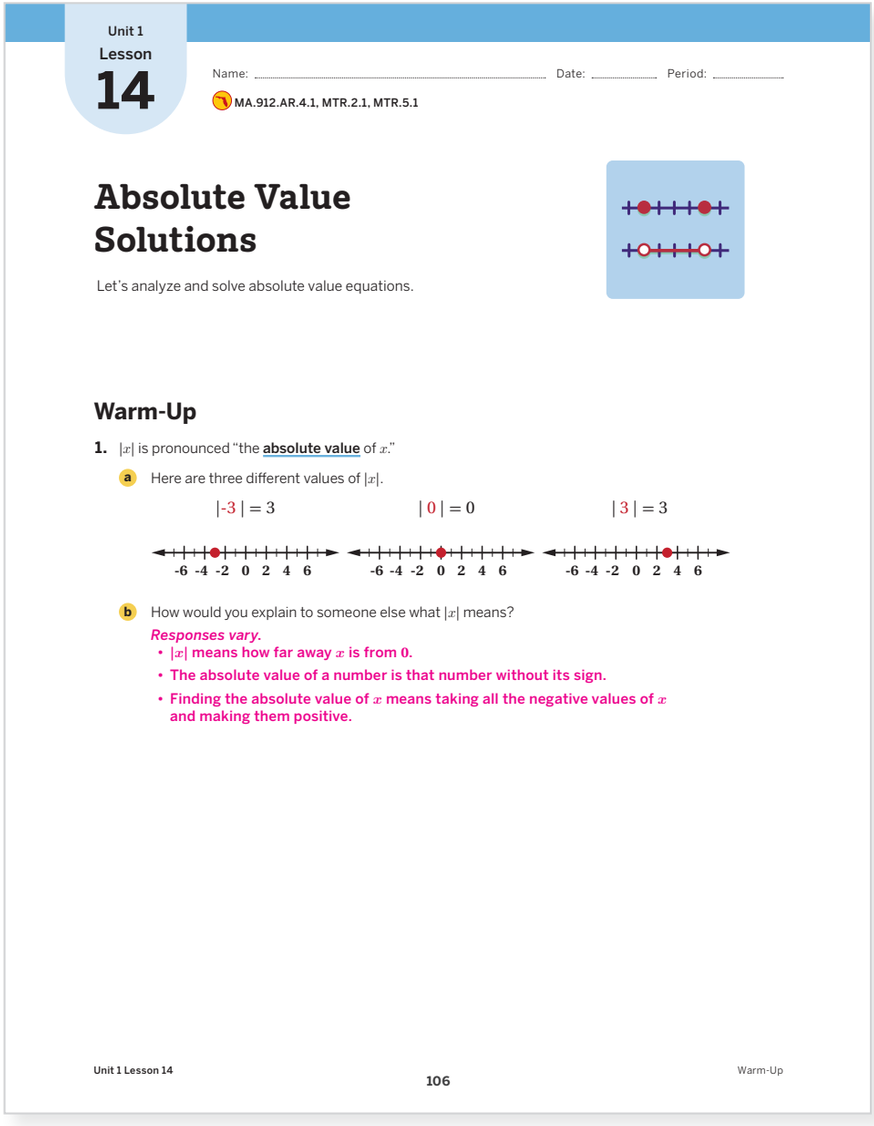
Consider asking:

- "Was there a value that you found interesting or that surprised you?"
- "Are there points on the number line that have the same absolute value?"

Consider sharing the definition of absolute value with students.

To surface the Key Takeaway, invite students to share their reasoning about traveling between home and school along the same route in a vehicle. Discuss how mileage on a vehicle measures distance, no matter which direction is traveled.

ELL **English Language Learners** Invite students to act out what it means to think about a number's value without regard to its sign or direction. Have two students move three steps away from a centrally located object in opposite directions. Consider sharing that what we are focusing on in this lesson is that they moved the same distance away, not their directions.



Unit 1
Lesson
14

Name: _____ Date: _____ Period: _____

MA.912.AR.4.1, MTR.2.1, MTR.5.1

Absolute Value Solutions

Let's analyze and solve absolute value equations.

Warm-Up

1. $|x|$ is pronounced "the **absolute value** of x."

a Here are three different values of $|x|$.

$|-3| = 3$ $\quad |0| = 0$ $\quad |3| = 3$

-6 -4 -2 0 2 4 6 $\quad$ -6 -4 -2 0 2 4 6 $\quad$ -6 -4 -2 0 2 4 6

b How would you explain to someone else what $|x|$ means?

Responses vary.

- $|x|$ means how far away x is from 0.
- The absolute value of a number is that number without its sign.
- Finding the absolute value of x means taking all the negative values of x and making them positive.

Unit 1 Lesson 14 106 Warm-Up

Lesson Takeaway: The absolute value of a number is the quantity of the number without regard to its direction or sign and is the distance a number is away from 0.

Pairs | 10 min

Activity 1 Showing Solutions

Purpose: Students use the structure of absolute value equations to represent their solutions on a number line and build reasoning between the solutions and equations. **(MTR.2.1, MTR.5.1)**

1 Launch

Consider asking, "What does it mean for 4 to be a solution to this equation?" in Problem 2.

Consider sharing with students that a solution is a value that makes the equation true.

2 Monitor

To support students in getting started, consider asking, "Is there another number that is 4 units away from 0?"

Listen to partner discussions for student ideas to highlight, such as using the idea of symmetry to explain why $|-4| = 4$. **(MTR.2.1)**

MLR **MLR: Think-Pair-Share** Use this routine in Problem 3 to help students shape their reasoning when determining the solutions using a number line. The routine will allow for discovery of how they might solve the absolute value equation algebraically.

Invite students to use what they know about absolute value and the number line to determine solutions independently. Students can explore how they might use what they know about solving equations to support their solution methods. Then, allow time to discuss with a partner. Invite several students to share their thoughts with the class.

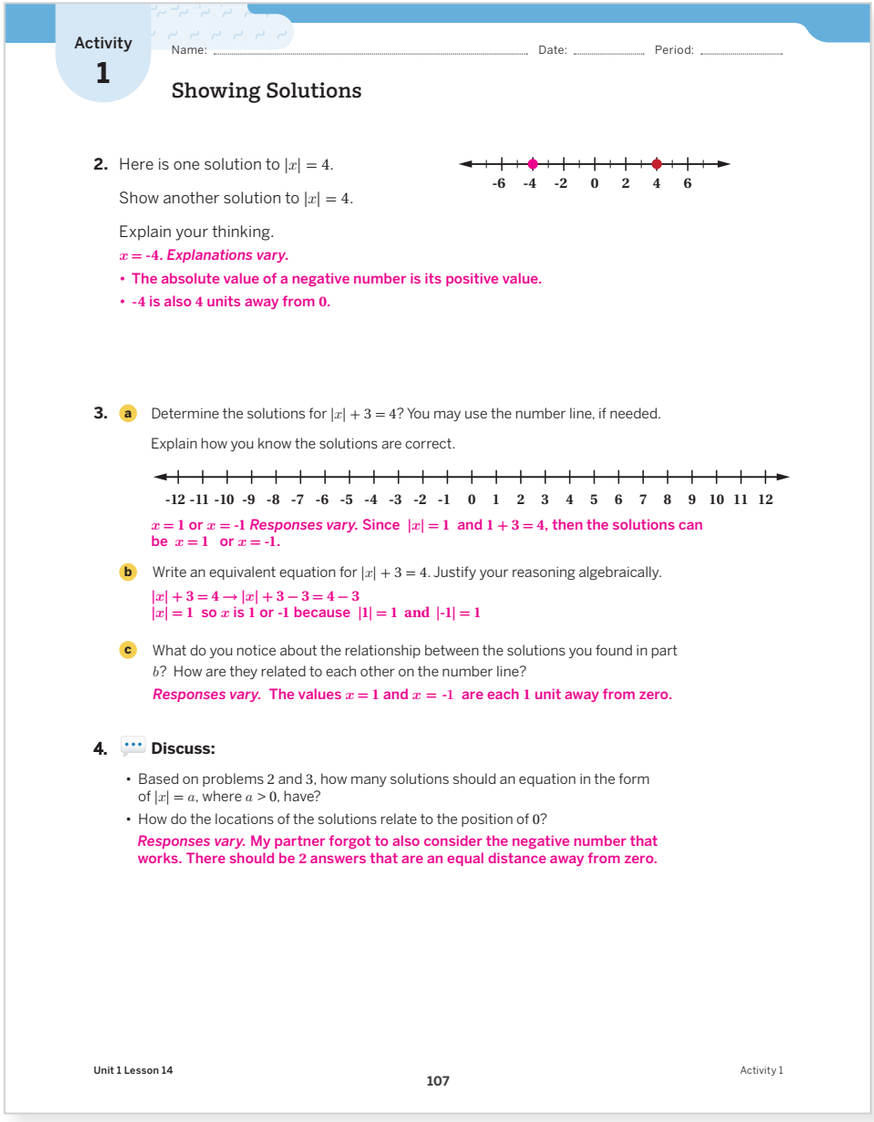
Discuss as a class how the number line for the original equation, $|x| + 3 = 4$ is related to the number line that would represent the solutions for the rewritten equation, $|x| = 1$. Invite students to share what they notice about the structure of this algebraic equation. **(MTR.5.1)**

Consider asking:

- "How are the solutions to this absolute value equation like the solutions you found for one-variable equations that you have solved in the past?"
- "What is different about the solutions for the absolute value equation when compared to past equations you have solved?

3 Connect

Listen for student reasoning that showcases the importance of attempting to find two solutions that are equidistant from a fixed value to share with the class.



Activity 1

Name: Date: Period:

Showing Solutions

2. Here is one solution to $|x| = 4$.

Show another solution to $|x| = 4$.

Explain your thinking.

$x = -4$. *Explanations vary.*

- The absolute value of a negative number is its positive value.
- -4 is also 4 units away from 0.

3. a. Determine the solutions for $|x| + 3 = 4$? You may use the number line, if needed.

Explain how you know the solutions are correct.

$x = 1$ or $x = -1$ *Responses vary.* Since $|x| = 1$ and $1 + 3 = 4$, then the solutions can be $x = 1$ or $x = -1$.

b. Write an equivalent equation for $|x| + 3 = 4$. Justify your reasoning algebraically.

$|x| + 3 = 4 \rightarrow |x| + 3 - 3 = 4 - 3$
$|x| = 1$ so x is 1 or -1 because $|1| = 1$ and $|-1| = 1$

c. What do you notice about the relationship between the solutions you found in part b? How are they related to each other on the number line?

Responses vary. The values $x = 1$ and $x = -1$ are each 1 unit away from zero.

4. **Discuss:**

- Based on problems 2 and 3, how many solutions should an equation in the form of $|x| = a$, where $a > 0$, have?
- How do the locations of the solutions relate to the position of 0?

Responses vary. My partner forgot to also consider the negative number that works. There should be 2 answers that are an equal distance away from zero.

Unit 1 Lesson 14 107 Activity 1

Pairs | 10 min

Activity 2 Solving Strategies

Purpose: Students connect the understanding of absolute value as distance with solving equations and use structure to solve absolute value equations. **(MTR.5.1)**

Short on time? Consider completing Problem 6 as a class, or grouping students in pairs to complete Problem 7.

1 Launch

Consider sharing with students that they will be looking at another strategy for solving absolute value equations in this activity.

Invite students to read $|x - 2|$ aloud (e.g., "The absolute value of the quantity x minus 2").

ELL **English Language Learners** Invite students to draw out Polina's statement on a number line using two numbers as an example (e.g., 4 and 0 are both 2 units away from 2). Consider sharing that numbers on a number line are oriented around the number 0 as a point of symmetry. This example uses 2 as its point of symmetry.

Use the Think-Pair-Share routine to help students shape their ideas about Polina's statement. Invite students to evaluate the expressions and analyze each relationship to the plotted point independently, then share with a partner. Invite several students to share their thoughts with the class.

2 Monitor

Invite students to share their substitution and calculations with a partner, reflecting and revising as needed as they complete Problem 6.

Listen for students' strategies or explanations for Problem 7 to share during the Connect.

D **Differentiation** *(Problem 7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to solve each equation separately, starting with the one on the left. Carefully apply the properties of equality.
Attempt to distribute the negative, but apply it only to the variable.	**Strengthen** Consider activating prior learning by asking, "When using the distributive property to rewrite $a(b + c)$, what is the equivalent expression?" Consider offering an approach to compare to their own, multiplying both sides by the negative to move it to the right side where it is applied to a single value. Consider asking, "How can you check your solutions to validate them?"
Would like an additional challenge.	**Stretch** Invite students to share other ways that the two equations could be written to represent the absolute value equation.

Activity 2 continued >

Activity 2

Name: Date: Period:

Solving Strategies

5. Polina says the value of $|x - 2|$ is the distance between any number and 2.

a. Here are three different values of $|x - 2|$.

$|-3 - 2| = 5$ $|0 - 2| = 2$ $|3 - 2| = 1$



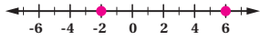
b. **Discuss:** Do you agree with Polina?

Responses vary. I agree with Polina because when you subtract, you're finding the difference between x and 2, which is the same as the distance.

6. a. Here are three different values of $|x - 2| = 4$.

Responses vary. The solutions are all the points that are 4 units away from 2.

$|x - 2| = 4$



b. Use substitution to show that your solutions from part a are correct.

Responses vary. $|-2 - 2| = 4$ or $|6 - 2| = 4$

$|-4| = 4$ or $|4| = 4$

$4 = 4$ or $4 = 4$

7. Tay thinks that the solutions to absolute value equations can be found algebraically by rewriting the absolute value equation as two separate equations and then solving each one.

a. Solve these equations using Tay's method to see if it works.

$(x - 2) = 4$ or $-(x - 2) = 4$

Responses vary. $(x - 2) = 4$ **or** $-x + 2 = 4$

$x - 2 + 2 = 4 + 2$ **or** $-x + 2 - 2 = 4 - 2$

$x = 6$ **or** $-x = 2$

$x = -2$

b. Explain whether Tay's method of solving worked.

Responses vary. Yes, Tay's method works. When I substitute the solutions back into the original absolute value equation, both values make true statements.

Unit 1 Lesson 14 108 Activity 2

Pairs | 10 min

Activity 2 Solving Strategies (continued)

Purpose: Students connect the understanding of absolute value as distance with solving equations and use structure to solve absolute value equations. **(MTR.5.1)**

Short on time? Consider completing Problem 6 as a class, or grouping students in pairs to complete Problem 7.

2 Monitor

Consider asking, "How can this absolute value equation's solutions be explained using the definition of absolute value?"
I need to find values that are 5 units away from the quantity inside the absolute value. Since $x - 3 = 0$, the solutions are 5 units from -3.

To support students getting started, invite them to describe how each student might be interpreting the equation in terms of distance.

MLR **MLR3: Critique, Correct, and Clarify** Use this routine to help students describe strengths, errors, or ambiguities in Tyler's work. Invite students to share their responses from the activity and further justify their reasoning by substituting each student's solutions into the original absolute value equation. See the Routine Facilitation Guide for more information.

Note: When creating the opposite equation that reflects the second solution, it may seem to many students that it is more direct to immediately place the negative on the "value side" of the equation; this will create a misconception and common error in Grade 7, when students are to solve absolute value inequalities. Encourage students to make the first step more representative of the definition of absolute value by placing the negative to define "the opposite of the quantity inside the absolute value."

A **Accessibility: Executive Function** Invite students to discuss the meaning of the absolute value equation with a partner. Assist their understanding by using a hand, a finger tip, or a sheet of paper to cover the quantity inside the absolute value, which makes the absolute value equation appear more like the one in Problem 2, with the "hand" symbolizing the quantity inside the absolute value, which relates to an x:

$$|(\text{✋})| = 5$$

Then articulate: "Either the expression inside the absolute value is equal to 5 or the opposite of the expression inside the absolute value is equal to 5."

$$(\text{✋}) = 5 \text{ or } -(\text{✋}) = 5$$

3 Connect

Share captured student strategies and explanations, using them to reinforce the definition of absolute value.

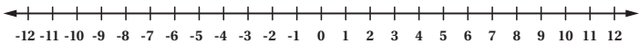
To surface the Key Takeaway, consider asking, "Why are there two equations created, where one has a negative applied to the quantity that was in the original absolute value expression?"

Activity 2

Name: Date: Period:

Solving Strategies (continued)

8. a. Show or describe all the solutions for $|x + 3| = 5$.



$x = 2$ and $x = -8$

b. What do you notice about the relationship between the equation and the solutions you found on the number line?

Responses vary. The solutions are $x = 2$ and $x = -8$, which are each exactly 5 units away from -3.

9. Ramon and Tyler each solve $|x + 3| = 5$ algebraically.

Ramon says that two related equations should be written and solved to find the solutions. Tyler says that only one equation should be written and solved, obtaining two solutions by using the positive and negative values of the solution. Analyze the reasoning shown below.

Ramon's Solution Method	Tyler's Solution Method
$(x + 3) = 5$ or $(x + 3) = -5$ $x + 3 - 3 = 5 - 3$ or $x + 3 - 3 = -5 - 3$ $x = 2$ or $x = -8$ The solutions are $x = 2$ and $x = -8$.	$(x + 3) = 5$ $x + 3 - 3 = 5 - 3$ $x = 2$ The solutions are $x = 2$ and $x = -2$.
Ramon states that the two equations reflect the outcomes. "Either what is in the parentheses is equal to 5 or the opposite of what is in the parentheses is equal to 5."	Tyler states that the absolute value equation means there is a positive and negative solution, and the solutions are opposites.

a. Whose method of solving is correct? Circle one.

(Ramon) Tyler Both Neither

b. Explain your thinking.

Explanations vary. Ramon is using the reasoning that any expression inside the absolute value will be equal either to the value on the other side or its opposite, in other words, 5 or -5. When 2 and -8 are substituted into $|x + 3| = 5$, you get $5 = 5$, which is a true statement. Graphically, 2 and -8 are each 5 units away from -3.

Unit 1 Lesson 14 109 Activity 2

Key Takeaway: Absolute value equations can have two possible solutions. The equations that will reveal the two solutions represent the definition of absolute value.

Pairs | 10 min

Activity 3 Absolute Challenge

Purpose: Students analyze the structure of more complex absolute value equations and find solutions.

Short on time? Consider using a large group discussion to complete Problem 10 together.

1 Launch

Invite students to work with a partner to discuss, solve, and explain their solutions and interpretations.

Listen for students who are using the relationship between distance and absolute value equations.

2 Monitor

Draw students' attention to Eva's and Mai's methods for solving the absolute value equation. Consider using the Think-Pair-Share routine as students analyze the solutions methods presented in Eva's and Mai's work.

MLR **MLR7: Compare and Connect** Invite students to analyze each representation, then find ways in which the equation representations, solution methods, and solutions are similar and different.

Consider asking:

- "How are Eva's and Mai's solution strategies alike? How are they different?"
- "Which solutions do Eva and Mai agree on? How do you know that this solution is correct?"
- "Where do you see an error in the algebraic reasoning?"
- "If Mai had distributed the negative through the binomial, would her method have worked?"

Invite students to first write the two algebraic equations that can be used to represent the absolute value equation. Recall the definition of absolute value, where students now can place a finger tip over the quantity inside the absolute value to help reason the two related equations. Pause to have students confirm their equations with a partner.

Invite students to check their solutions by using substitution.

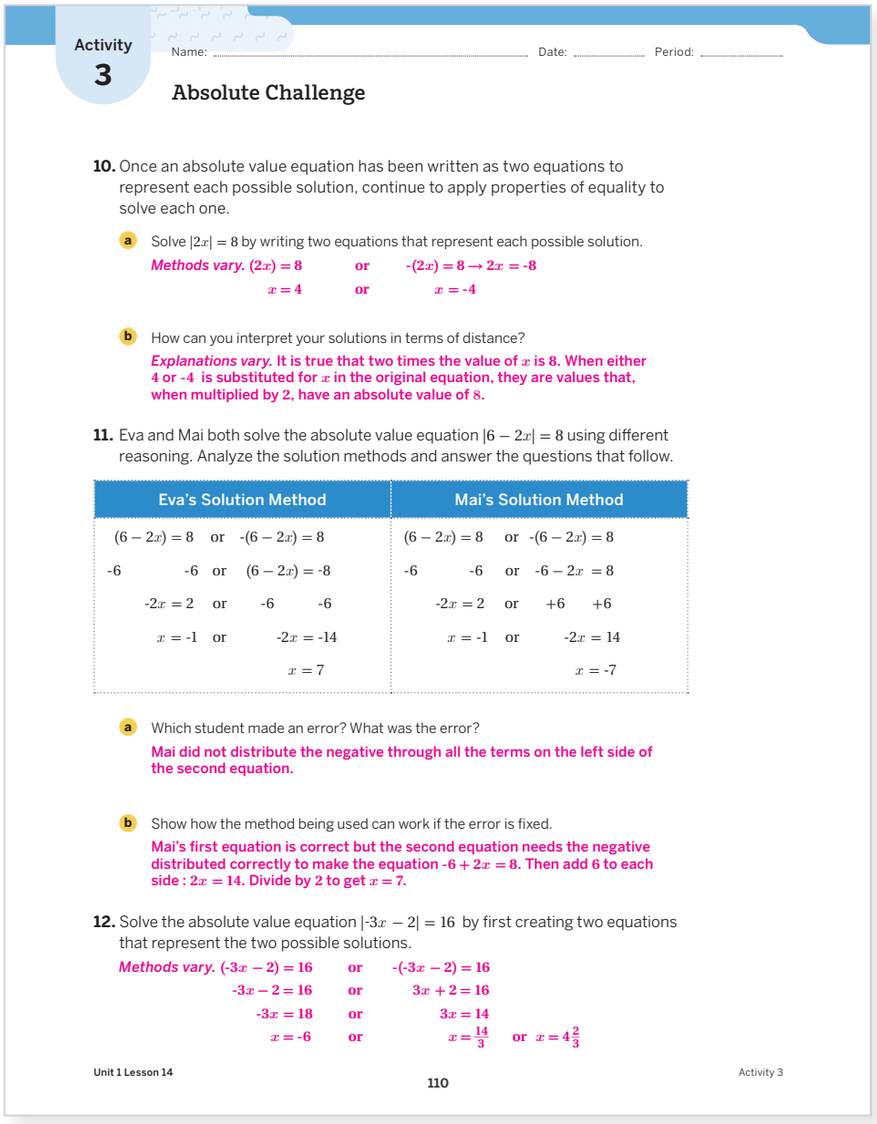
3 Connect

Discuss students' solution methods, showcasing the students who had different ways to proceed in solving the second equation (which includes the application of a negative sign to address the opposite scenario) using properties of equality.

To surface the Key Takeaway, ask:
"How can you ensure that when applying the negative sign on the second equation, the properties of equality will more likely be applied correctly?"

"How does substitution into the original absolute value equation ensure validity of your solutions?"

"Why might it not be wise to substitute your answers into the representative equation to confirm they are correct?"



Activity 3

Name: Date: Period:

Absolute Challenge

10. Once an absolute value equation has been written as two equations to represent each possible solution, continue to apply properties of equality to solve each one.

a. Solve $|2x| = 8$ by writing two equations that represent each possible solution.

Methods vary. $(2x) = 8$ or $-(2x) = 8 \rightarrow 2x = -8$

$x = 4$ or $x = -4$

b. How can you interpret your solutions in terms of distance?

Explanations vary. It is true that two times the value of x is 8. When either 4 or -4 is substituted for x in the original equation, they are values that, when multiplied by 2, have an absolute value of 8.

11. Eva and Mai both solve the absolute value equation $|6 - 2x| = 8$ using different reasoning. Analyze the solution methods and answer the questions that follow.

Eva's Solution Method	Mai's Solution Method
$(6 - 2x) = 8$ or $-(6 - 2x) = 8$	$(6 - 2x) = 8$ or $-(6 - 2x) = 8$
$-6 \quad -6$ or $(6 - 2x) = -8$	$-6 \quad -6$ or $-6 - 2x = 8$
$-2x = 2$ or $-6 \quad -6$	$-2x = 2$ or $+6 \quad +6$
$x = -1$ or $-2x = -14$	$x = -1$ or $-2x = 14$
$x = 7$	$x = -7$

a. Which student made an error? What was the error?

Mai did not distribute the negative through all the terms on the left side of the second equation.

b. Show how the method being used can work if the error is fixed.

Mai's first equation is correct but the second equation needs the negative distributed correctly to make the equation $-6 + 2x = 8$. Then add 6 to each side : $2x = 14$. Divide by 2 to get $x = 7$.

12. Solve the absolute value equation $|-3x - 2| = 16$ by first creating two equations that represent the two possible solutions.

Methods vary. $(-3x - 2) = 16$ or $-(-3x - 2) = 16$

$-3x - 2 = 16$ or $3x + 2 = 16$

$-3x = 18$ or $3x = 14$

$x = -6$ or $x = \frac{14}{3}$ or $x = 4\frac{2}{3}$

Unit 1 Lesson 14 110 Activity 3

Key Takeaway: When first creating the equations, the quantity inside the absolute value symbol stays grouped using parentheses. Then, apply the properties of equality to solve.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for solving absolute value equations.

Invite students to respond independently, then share their thinking with a partner.

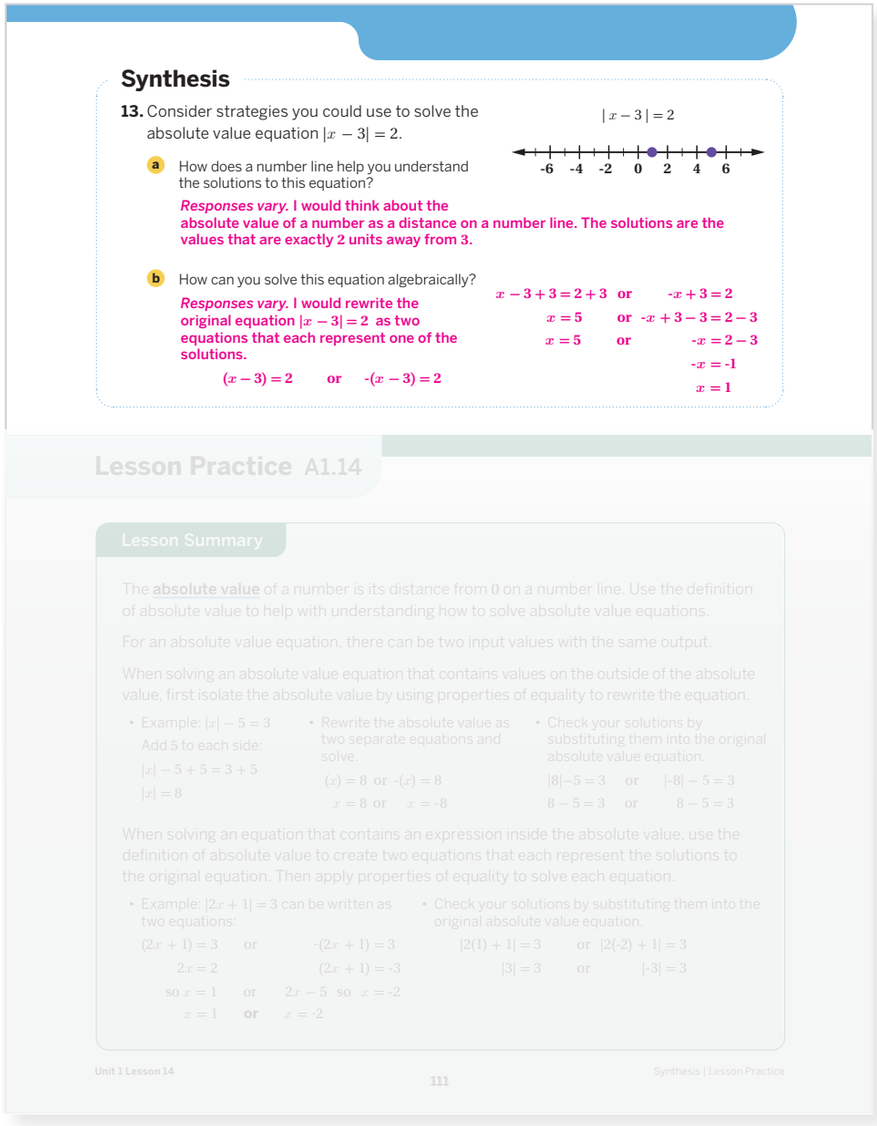
Share a variety of ideas, such as:

- Using the definition of absolute value to understand the relationship of the solutions with the given equation.
- Writing two equations to represent the absolute value equation.
- Checking solutions to confirm validity using the original absolute value equation.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for solving absolute value equations by using number line reasoning and algebraic equations, both of which are based on the definition of absolute value as a distance.

Add examples with algebraic solutions like Eva's and Mai's revised approaches, including the final validation of solutions using substitution into the original absolute value equation.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

13. Consider strategies you could use to solve the absolute value equation $|x - 3| = 2$.

a. How does a number line help you understand the solutions to this equation?

Responses vary. I would think about the absolute value of a number as a distance on a number line. The solutions are the values that are exactly 2 units away from 3.

b. How can you solve this equation algebraically?

Responses vary. I would rewrite the original equation $|x - 3| = 2$ as two equations that each represent one of the solutions.

$(x - 3) = 2$ or $-(x - 3) = 2$

$x - 3 + 3 = 2 + 3$ or $-x + 3 = 2$

$x = 5$ or $-x + 3 - 3 = 2 - 3$

$x = 5$ or $-x = 2 - 3$

$-x = -1$

$x = 1$

Lesson Practice A1.14

Lesson Summary

The **absolute value** of a number is its distance from 0 on a number line. Use the definition of absolute value to help with understanding how to solve absolute value equations.

For an absolute value equation, there can be two input values with the same output.

When solving an absolute value equation that contains values on the outside of the absolute value, first isolate the absolute value by using properties of equality to rewrite the equation.

- Example: $|x| - 5 = 3$
  Add 5 to each side:
  $|x| - 5 + 5 = 3 + 5$
  $|x| = 8$
- Rewrite the absolute value as two separate equations and solve.
  $(x) = 8$ or $-(x) = 8$
  $x = 8$ or $x = -8$
- Check your solutions by substituting them into the original absolute value equation.
  $|8| - 5 = 3$ or $|-8| - 5 = 3$
  $8 - 5 = 3$ or $8 - 5 = 3$

When solving an equation that contains an expression inside the absolute value, use the definition of absolute value to create two equations that each represent the solutions to the original equation. Then apply properties of equality to solve each equation.

- Example: $|2x + 1| = 3$ can be written as two equations:
  $(2x + 1) = 3$ or $-(2x + 1) = 3$
  $2x = 2$ $(2x + 1) = -3$
  so $x = 1$ or $2x - 5$ so $x = -2$
  $x = 1$ or $x = -2$
- Check your solutions by substituting them into the original absolute value equation.
  $|2(1) + 1| = 3$ or $|2(-2) + 1| = 3$
  $|3| = 3$ or $|-3| = 3$

Unit 1 Lesson 14 111 Synthesis | Lesson Practice

Lesson Takeaway: Absolute value equations can be solved by writing two related equations and applying the definition of absolute value and the properties of equality.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving an absolute value equation.

Today's Goals

1. **Goal:** Solve one-variable absolute value equations.
2. **Goal:** Interpret solutions of absolute value equations and number line representations.
3. **Language Goal:** Explain the meaning of absolute value equations. **(Listening, Speaking, and Writing)**

Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 1.14

Solve the absolute value equation $|x + 1| = 2$.

$x =$ -3 or $x =$ 1

D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support solving an absolute value equation:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Where in the lesson was a routine most effective in supporting the learning of a large number of students?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. Consider strategies you could use to solve the absolute value equation $|x-3|=2$.

a. How does a number line help you understand the solutions to this equation?

Responses vary. I would think about the absolute value of a number as a distance on a number line. The solutions are the values that are exactly 2 units away from 3.

b. How can you solve this equation algebraically?

Responses vary. I would rewrite the original equation $|x-3|=2$ as two equations that each represent one of the solutions.

$(x-3)=2$ or $-(x-3)=2$

$x-3+3=2+3$ or $-x+3=2$

$x=5$ or $-x+3-3=2-3$

$x=5$ or $-x=2-3$

$-x=-1$

$x=1$

Lesson Practice A1.14

Lesson Summary

The **absolute value** of a number is its distance from 0 on a number line. Use the definition of absolute value to help with understanding how to solve absolute value equations.

For an absolute value equation, there can be two input values with the same output.

When solving an absolute value equation that contains values on the outside of the absolute value, first isolate the absolute value by using properties of equality to rewrite the equation.

- Example: $|x|-5=3$
  Add 5 to each side:
  $|x|-5+5=3+5$
  $|x|=8$
- Rewrite the absolute value as two separate equations and solve.
  $(x)=8$ or $-(x)=8$
  $x=8$ or $x=-8$
- Check your solutions by substituting them into the original absolute value equation.
  $|8|-5=3$ or $|-8|-5=3$
  $8-5=3$ or $8-5=3$

When solving an equation that contains an expression inside the absolute value, use the definition of absolute value to create two equations that each represent the solutions to the original equation. Then apply properties of equality to solve each equation.

- Example: $|2x+1|=3$ can be written as two equations:
  $(2x+1)=3$ or $-(2x+1)=3$
  $2x=2$ $(2x+1)=-3$
  so $x=1$ or $2x-5$ so $x=-2$
  $x=1$ **or** $x=-2$
- Check your solutions by substituting them into the original absolute value equation.
  $|2(1)+1|=3$ or $|2(-2)+1|=3$
  $|3|=3$ or $|-3|=3$

Lesson Practice A1.14

Name: ..... Date: ..... Period: .....

1. Match each absolute value equation to its meaning in words.

A. $|x|+4=6$ b The distance of four times x from 0 is 6.

B. $|4x|=6$ c The distance of x from 4 is 6.

C. $|x-4|=6$ d Four times the distance of x from 0 is 6.

D. $4|x|=6$ a Four more than the distance of x from 0 is 6.

Problems 2–5: Solve the absolute value equations algebraically.

2. $|x-5|=8$

$x=$ 13 or $x=$ -3

3. $4|x+3|=24$

$x=$ 3 or $x=$ -9

4. $|1-2x|=7$

$x=$ -3 or $x=$ 4

5. $|\frac{1}{2}x|-3=11$

$x=$ 28 or $x=$ -28

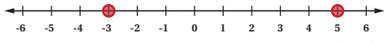
6. Which equations show the cases that you must consider when solving $|6x-12|=18$?

- ☑ A. $6x-12=18$
- ☐ B. $6x+12=18$
- ☐ C. $-6x-12=18$
- ☑ D. $-6x+12=18$

Lesson Practice A1.14

Name: ..... Date: ..... Period: .....

7. The graph shows two solutions to an equation that contains an absolute value. Explain how you can find an equation with these solutions. Include your equation in your response.



First, I need to find the point that is the same distance from both -3 and 5.
This point between -3 and 5 is $x=1$. The distance from each given point to 1 is 4 units.
An equation that has solutions of $x=-3$ and $x=5$ would be $|x-1|=4$.

8. Solve the absolute value equation: $8-|\frac{2}{3}x+3|=-13$.

$8-|\frac{2}{3}x+3|-8=-13-8$

$-|\frac{2}{3}x+3|=-21 \rightarrow |\frac{2}{3}x+3|=21$ so

$(\frac{2}{3}x+3)=21$ or $-(\frac{2}{3}x+3)=21$

$\frac{2}{3}x+3-3=21-3$ $\frac{2}{3}x+3=-21$

$(\frac{3}{2})\frac{2}{3}x=18(\frac{3}{2})$ $(\frac{3}{2})\frac{2}{3}x=-24(\frac{3}{2})$

$x=27$ $x=-36$

Test Practice

9. Solve the absolute value equation: $2+|3x-7|=6$. Choose all solutions.

A. $x=\frac{4}{3}$ B. $x=\frac{11}{3}$ (C.) $x=\frac{11}{3}$ D. $x=-\frac{4}{3}$

E. $x=-3$ (F.) $x=1$ G. $x=3$ H. $x=-1$

Spiral Review

Problems 10–11: Use the expression: $-\frac{5}{6}x(\frac{3}{4}x-\frac{2}{5})$.

10. What is the product of the expression?

$-\frac{5}{8}x^2+\frac{1}{3}x$

11. What is the value of the expression when $x=\frac{4}{3}$?

$-\frac{2}{3}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 6	1	MA.912.AR.4.1*
	2–5	2	MA.912.AR.4.1
	7, 8	3	MA.912.AR.4.1
Test Practice	9	2	MA.912.AR.4.1
Spiral Review			
	10	2	MA.8.AR.1.1
	11	2	MA.8.AR.1.1

*This problem builds toward the standard shown.

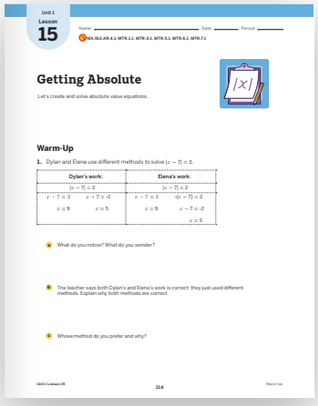
Unit 1

Lesson 15

Getting Absolute

Creating and Solving Absolute Value Equations

Let's create and solve absolute value equations.


Unit 1
Lesson 15

Getting Absolute

Let's create and solve absolute value equations.

Warm-Up

1. Dylan and Elena use different methods to solve $|x-7|=2$.

Dylan's work:		Elena's work:	
$\|x-7\|=2$		$\|x-7\|=2$	
$x-7=2$	$x-7=-2$	$x-7=2$	$-(x-7)=2$
$x=9$	$x=5$	$x=9$	$x-7=-2$
			$x=5$

a. What do you notice? What do you wonder?

b. The teacher says both Dylan's and Elena's work is correct, they just used different methods. Explain why both methods are correct.

c. Whose method do you prefer and why?

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Learning Outcomes

Today's Goal

1. **Goal:** Know how to identify whether an absolute value equation has two solutions, one solution, or no solution.
2. **Goal:** Write and solve one-variable absolute value equations given a mathematical or real-world context.
3. **Language Goal:** Describe the meaning of each term of an absolute value equation in context. **(Reading, Writing, Speaking, and Listening)**

Students learn how to identify whether an absolute value equation has two solutions, one solution, or no solution. Students practice writing and solving absolute value equations given a mathematical or real world context. **(MTR.7.1)**

Coherence

Prior Learning

In Lesson 14, students developed their conceptual understanding that an absolute value represents a distance and practiced solving simple absolute value equations.

Future Learning

In Lesson 16, students will be introduced to two-variable inequalities and graph their solution sets given mathematical or real-world contexts.

Connections

- Students connect new notation for absolute value to what they have learned about expressions and equations that involve one variable. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.4.1

Given a mathematical or real-world context, write and solve one-variable absolute value equations.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.1.3	**MA.912.AR.4.2**
MA.6.NSO.1.4	**MA.912.AR.4.3**
MA.7.NSO.2.1	

Vocabulary

Review Vocabulary

absolute value

solution

Lesson at a Glance

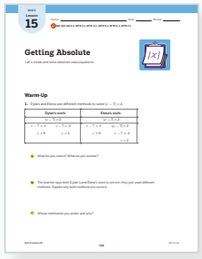
~ 45 min

MA.912.AR.4.1

Warm-Up

Whole Class | 5 min

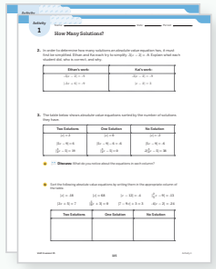
Students compare two different algebraic methods for solving absolute value equations.
Routine: Notice and Wonder



Activity 1

Small Groups | 10 min

Students practice simplifying absolute value equations and sort them by having two solutions, one solution, or no solution.
(MTR.5.1)



Activity 2

Pairs | 10 min

Students analyze real-world situations to create and solve absolute value equations.
Routine: Tell a Story
(MTR.6.1, MTR.7.1)



Activity 3

Pairs | 10 min

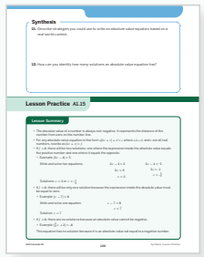
Students write absolute value equations in the context of a road trip and identify their solutions.
MLR3: Critique, Correct, and Clarify



Synthesis

Whole Class | 5 min

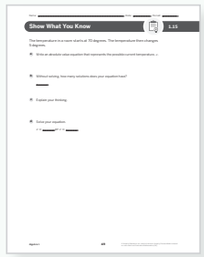
Students synthesize their strategies for creating, solving, and identifying the number of solutions of absolute value equations.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an absolute value equation for a context with restraints, identifying and justifying the number of solutions, and solving for all solutions.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students compare two different algebraic methods for solving absolute value equations.

1 Launch

As a class, discuss the meaning of absolute value equations and the role each term plays in representing a given context.

Consider asking:

- "What strategies can you use to interpret the meaning of an absolute value equation in context?"
- "What strategies can you use to solve an absolute value equation?"
- "Why is it important to pay attention to signs and order of operations when solving an absolute value equation?"

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took during Lesson 14 to activate prior knowledge.

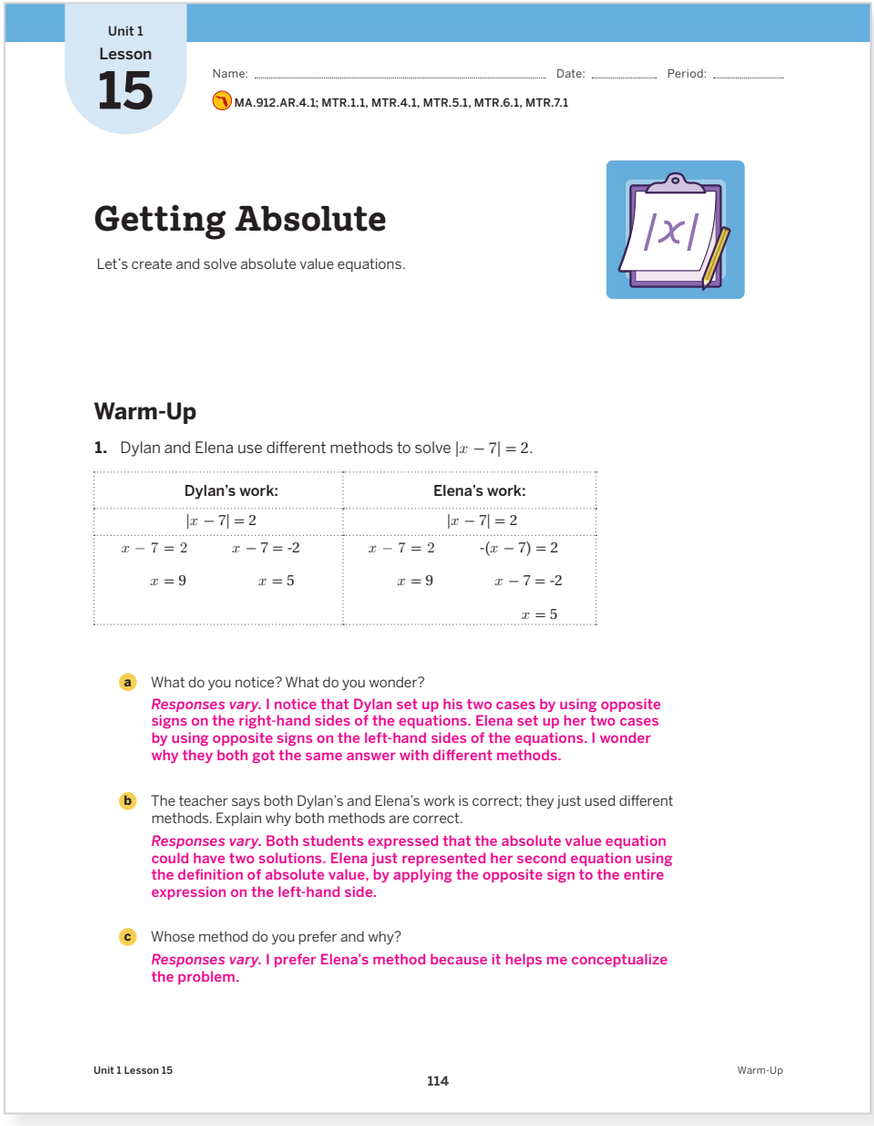
Use the Notice and Wonder routine in Problem 1a to bolster students' understanding of different methods for solving absolute value equations.

D **Differentiation: (Support)** Share student observations about the differences between Dylan's method and Elena's method.

2 Connect

Invite students to share the method they prefer and why.

Discuss why both methods work and what they each mean conceptually. Remind students that the definition of absolute value is how the two equations are first identified. Because we are only exploring absolute value equations, this initial step as used in Lesson 14 should remain in their thoughts but they can now proceed with creating the set of equations by making the constant value positive on the first equation and negative for the second equation.



Unit 1
Lesson
15

Name: _____ Date: _____ Period: _____

MA.912.AR.4.1; MTR.1.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Getting Absolute

Let's create and solve absolute value equations.

Warm-Up

1. Dylan and Elena use different methods to solve $|x - 7| = 2$.

Dylan's work:		Elena's work:	
$\|x - 7\| = 2$		$\|x - 7\| = 2$	
$x - 7 = 2$	$x - 7 = -2$	$x - 7 = 2$	$-(x - 7) = 2$
$x = 9$	$x = 5$	$x = 9$	$x - 7 = -2$
			$x = 5$

a. What do you notice? What do you wonder?

Responses vary. I notice that Dylan set up his two cases by using opposite signs on the right-hand sides of the equations. Elena set up her two cases by using opposite signs on the left-hand sides of the equations. I wonder why they both got the same answer with different methods.

b. The teacher says both Dylan's and Elena's work is correct; they just used different methods. Explain why both methods are correct.

Responses vary. Both students expressed that the absolute value equation could have two solutions. Elena just represented her second equation using the definition of absolute value, by applying the opposite sign to the entire expression on the left-hand side.

c. Whose method do you prefer and why?

Responses vary. I prefer Elena's method because it helps me conceptualize the problem.

Unit 1 Lesson 15 114 Warm-Up

Small Groups | 10 min

Activity 1 How Many Solutions?

Purpose: Students practice simplifying absolute value equations and sort them by having two solutions, one solution, or no solution. **(MTR.5.1)**

1 Launch

Consider asking, "What happens if you try to distribute a negative coefficient into an absolute value expression?"

D **Differentiation: (Support)** Invite students to share their reasoning with peers and consider the explanations of others.

2 Monitor

Direct students' attention to the table under question 3. Encourage small groups to discuss part a, then invite students to share their observations with the class.

Discuss what it means for an absolute value equation to have two solutions, one solution, or no solution, respectively.

A **Accessibility: Visual-Spatial Processing** Provide graph paper or blank number lines for students to draw on and explore substitution of values to increase access to the task.

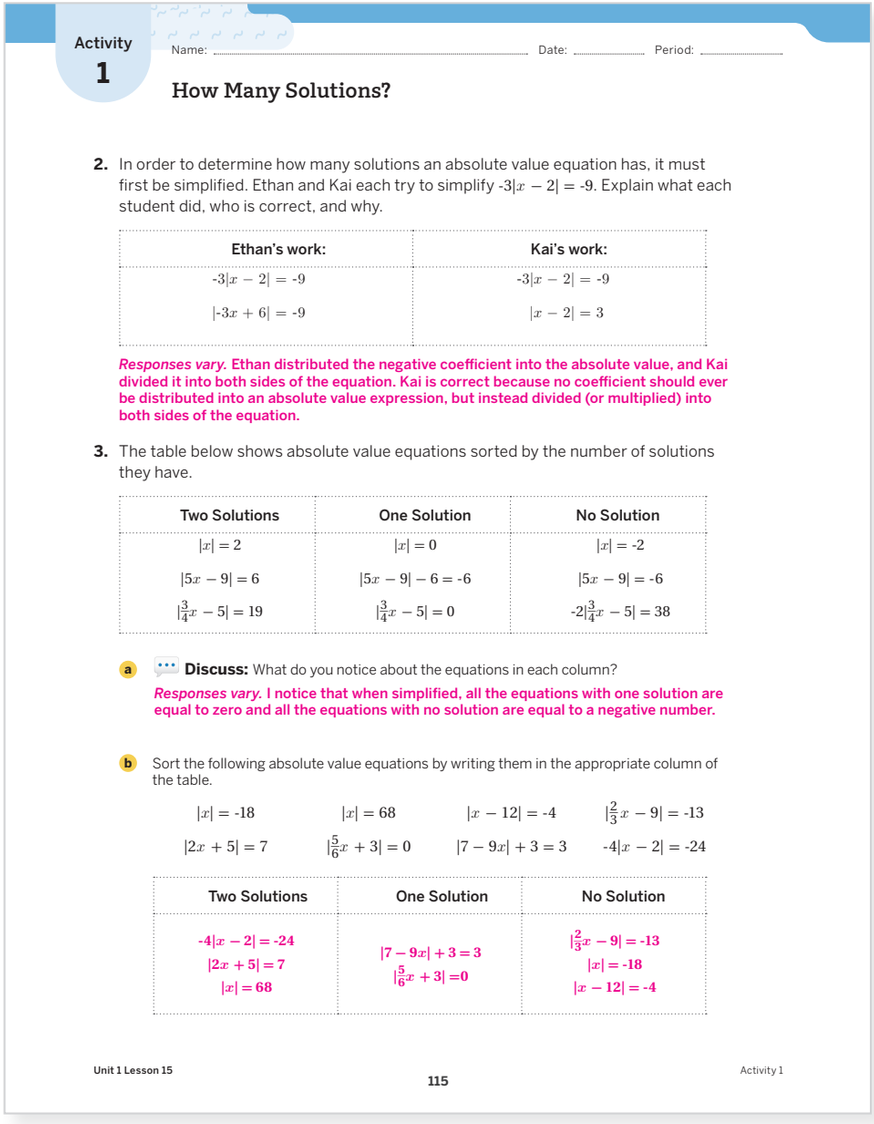
Pause briefly to remind students to simplify absolute value equations before determining how many solutions they have.

3 Connect

To surface the Key Takeaway, consider asking, "What patterns did you notice in each column of the table?" **(MTR.5.1)**

ELL **English Language Learners** Invite students to talk through their thinking with their group before writing it down. **(Writing and Speaking)**

Demonstrate *why* these patterns occur and what they mean in the context of an absolute value equation.



Activity 1

Name: Date: Period:

How Many Solutions?

2. In order to determine how many solutions an absolute value equation has, it must first be simplified. Ethan and Kai each try to simplify $-3|x-2|=-9$. Explain what each student did, who is correct, and why.

Ethan's work:	Kai's work:
$-3\|x-2\|=-9$ $\|-3x+6\|=-9$	$-3\|x-2\|=-9$ $\|x-2\|=3$

Responses vary. Ethan distributed the negative coefficient into the absolute value, and Kai divided it into both sides of the equation. Kai is correct because no coefficient should ever be distributed into an absolute value expression, but instead divided (or multiplied) into both sides of the equation.

3. The table below shows absolute value equations sorted by the number of solutions they have.

Two Solutions	One Solution	No Solution
$\|x\|=2$	$\|x\|=0$	$\|x\|=-2$
$\|5x-9\|=6$	$\|5x-9\|-6=-6$	$\|5x-9\|=-6$
$\|\frac{3}{4}x-5\|=19$	$\|\frac{3}{4}x-5\|=0$	$-2\|\frac{3}{4}x-5\|=38$

a **Discuss:** What do you notice about the equations in each column?

Responses vary. I notice that when simplified, all the equations with one solution are equal to zero and all the equations with no solution are equal to a negative number.

b Sort the following absolute value equations by writing them in the appropriate column of the table.

$|x|=-18$ $|x|=68$ $|x-12|=-4$ $|\frac{2}{3}x-9|=-13$

$|2x+5|=7$ $|\frac{5}{6}x+3|=0$ $|7-9x|+3=3$ $-4|x-2|=-24$

Two Solutions	One Solution	No Solution
$-4\|x-2\|=-24$ $\|2x+5\|=7$ $\|x\|=68$	$\|7-9x\|+3=3$ $\|\frac{5}{6}x+3\|=0$	$\|\frac{2}{3}x-9\|=-13$ $\|x\|=-18$ $\|x-12\|=-4$

Unit 1 Lesson 15 115 Activity 1

Key Takeaway: Absolute value equations in the form $|ax+b|=c$ have two solutions when $c>0$, one solution when $c=0$, and no solution when $c<0$.

Pairs | 10 min

Activity 2 Analyze and Solve

Purpose: Students analyze real-world situations to create and solve absolute value equations. **(MTR.6.1, MTR.7.1)**

Short on time? Consider omitting Problem 6 or completing Problem 7 as a class.

1 Launch

Consider sharing the absolute value equation $|x - 3| = 1$.

Use the Tell a Story routine by inviting students to come up with a real-world context that could be represented by the absolute value equation.

Discuss different student stories and why they do or do not match the absolute value equation.

2 Monitor

Look for student strategies to share during the Connect.

D **Differentiation: (Support)** Look for students who don't agree on the answer to Problem 4a. Encourage them to restate Arjun and Eliza's equations verbally, using the equation and stories from the Launch as examples.

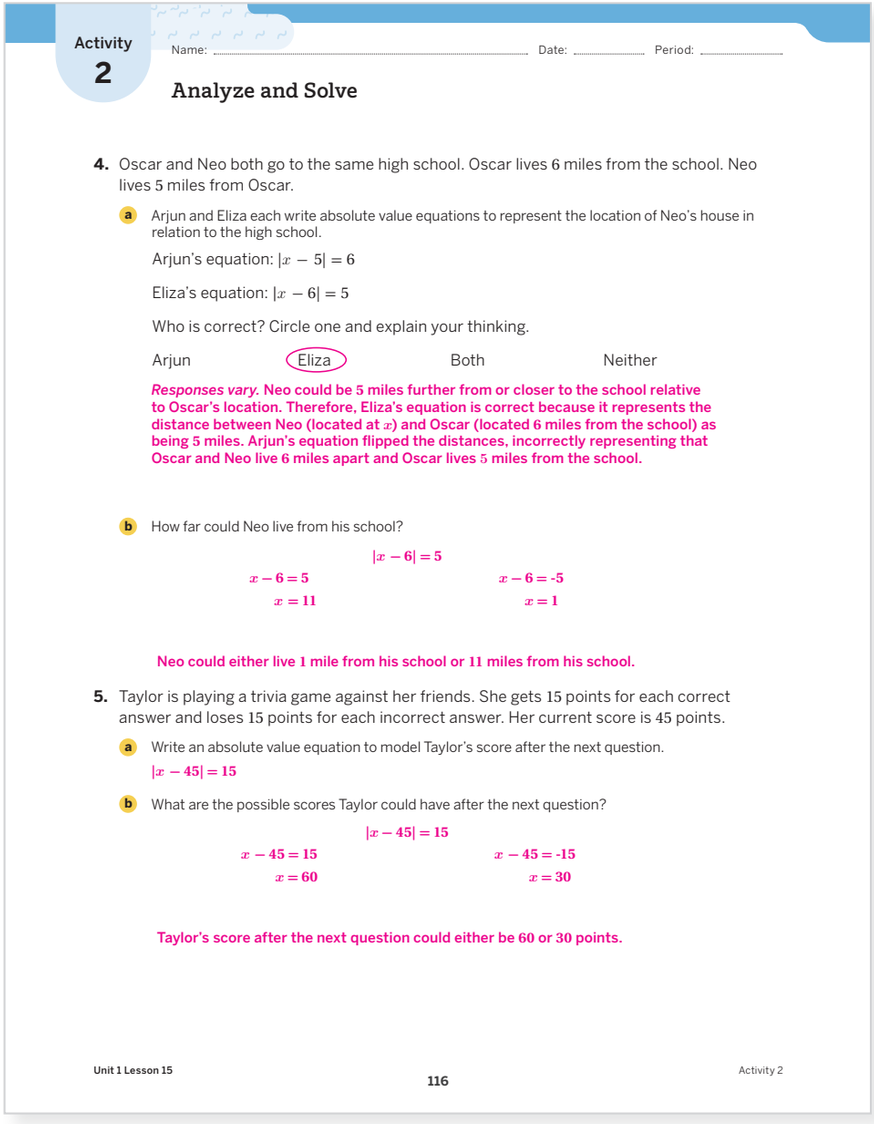
To encourage participation, pause briefly to ask students to share their favorite trivia facts. Discuss as a class and fact-check, if necessary.

Look for students who need support setting up their two equations to solve the absolute value. Direct their attention back to the Warm-Up, and consider asking, "How can you apply the same strategies in this activity?"

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate the definition of absolute value. **(Speaking and Listening)**

A **Accessibility: Conceptual Processing** To support abstract thinking, invite students to underline key information from each question before writing their equations. Consider asking, "How do you know which number is the reference point and which number is the change in distance?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Analyze and Solve

4. Oscar and Neo both go to the same high school. Oscar lives 6 miles from the school. Neo lives 5 miles from Oscar.

a. Arjun and Eliza each write absolute value equations to represent the location of Neo's house in relation to the high school.

Arjun's equation: $|x - 5| = 6$

Eliza's equation: $|x - 6| = 5$

Who is correct? Circle one and explain your thinking.

Arjun (Eliza) Both Neither

Responses vary. Neo could be 5 miles further from or closer to the school relative to Oscar's location. Therefore, Eliza's equation is correct because it represents the distance between Neo (located at x) and Oscar (located 6 miles from the school) as being 5 miles. Arjun's equation flipped the distances, incorrectly representing that Oscar and Neo live 6 miles apart and Oscar lives 5 miles from the school.

b. How far could Neo live from his school?

$|x - 6| = 5$

$x - 6 = 5$ $\quad$ $x - 6 = -5$

$x = 11$ $\quad$ $x = 1$

Neo could either live 1 mile from his school or 11 miles from his school.

5. Taylor is playing a trivia game against her friends. She gets 15 points for each correct answer and loses 15 points for each incorrect answer. Her current score is 45 points.

a. Write an absolute value equation to model Taylor's score after the next question.

$|x - 45| = 15$

b. What are the possible scores Taylor could have after the next question?

$|x - 45| = 15$

$x - 45 = 15$ $\quad$ $x - 45 = -15$

$x = 60$ $\quad$ $x = 30$

Taylor's score after the next question could either be 60 or 30 points.

Unit 1 Lesson 15 116 Activity 2

Pairs | 10 min

Activity 2 Analyze and Solve (continued)

Purpose: Students analyze real-world situations to create and solve absolute value equations. **(MTR.6.1, MTR.7.1)**

Short on time? Consider omitting Problem 6 or completing Problem 7 as a class.

2 Monitor

To generate excitement, consider filling a jar with a known number of objects (for example, unit cubes) and invite students to guess the answer.

Invite students to write their own absolute value equation to represent how sure they are about their guesses. Encourage students to state their absolute value equation verbally to surface the connection between the situation and the equation.
Ex. $|x - 26| = 5$; "I guess that there are 26 unit cubes in the jar, plus or minus 5."

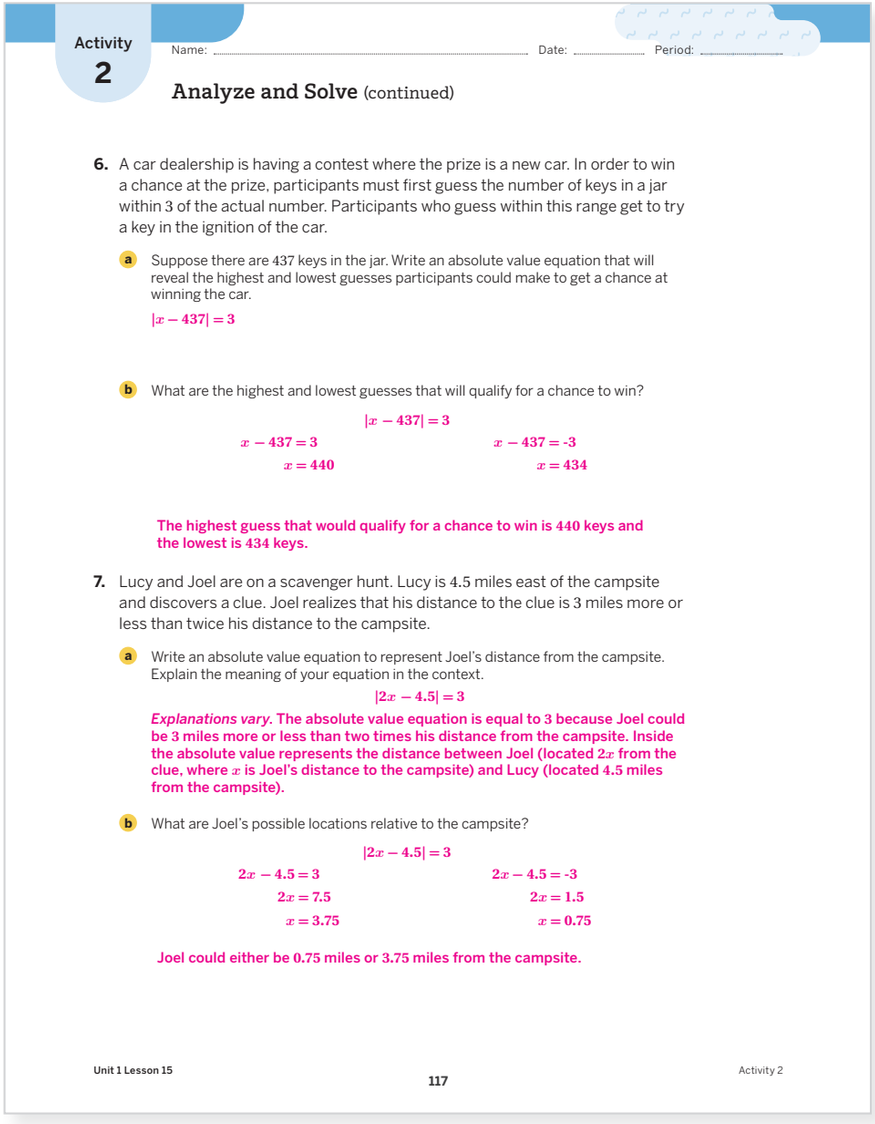
D Differentiation *(Problem 7)*

Look for students who:	Respond to Student Thinking
Are unsure how to interpret the context in Problem 7.	**Support** Work with students to break down the problem by identifying the reference point, variable, and relationship between the two. Encourage discussion about how this absolute value equation is different from the ones in previous problems.
Are using drawings or number lines to support their thinking.	**Strengthen** Consider asking, "How can you translate compass directions (e.g., east) when making sense of a context?"
Finish the problem set early.	**Stretch** Invite students to come up with another real-world context that can be represented with an absolute value equation, then have their partner write the equation based on the context.

3 Connect

Share student strategies for identifying different parts of the absolute value equation from the given context.

To surface the Key Takeaway, work as a class to list the steps for creating an absolute value equation based on a written context.



Activity 2

Name: Date: Period:

Analyze and Solve (continued)

6. A car dealership is having a contest where the prize is a new car. In order to win a chance at the prize, participants must first guess the number of keys in a jar within 3 of the actual number. Participants who guess within this range get to try a key in the ignition of the car.

a. Suppose there are 437 keys in the jar. Write an absolute value equation that will reveal the highest and lowest guesses participants could make to get a chance at winning the car.

$|x - 437| = 3$

b. What are the highest and lowest guesses that will qualify for a chance to win?

$|x - 437| = 3$

$x - 437 = 3$ $\quad$ $x - 437 = -3$

$x = 440$ $\quad$ $x = 434$

The highest guess that would qualify for a chance to win is 440 keys and the lowest is 434 keys.

7. Lucy and Joel are on a scavenger hunt. Lucy is 4.5 miles east of the campsite and discovers a clue. Joel realizes that his distance to the clue is 3 miles more or less than twice his distance to the campsite.

a. Write an absolute value equation to represent Joel's distance from the campsite. Explain the meaning of your equation in the context.

$|2x - 4.5| = 3$

Explanations vary. **The absolute value equation is equal to 3 because Joel could be 3 miles more or less than two times his distance from the campsite. Inside the absolute value represents the distance between Joel (located $2x$ from the clue, where x is Joel's distance to the campsite) and Lucy (located 4.5 miles from the campsite).**

b. What are Joel's possible locations relative to the campsite?

$|2x - 4.5| = 3$

$2x - 4.5 = 3$ $\quad$ $2x - 4.5 = -3$

$2x = 7.5$ $\quad$ $2x = 1.5$

$x = 3.75$ $\quad$ $x = 0.75$

Joel could either be 0.75 miles or 3.75 miles from the campsite.

Unit 1 Lesson 15 117 Activity 2

Key Takeaway: To write an absolute value equation based on a real-world context, identify the reference point, relate it to the variable, and express their relationship as the absolute value.

Pairs | 10 min

Activity 3 Road Trip

Purpose: Students write absolute value equations in the context of a road trip and identify their solutions. **(MTR.7.1)**

1 Launch

Consider asking:

- "Have you ever been on a road trip?"
- "Can you think of ways math is used when driving?"
- "How might an absolute value equation model the context of a road trip?"

Invite students to share their experiences with driving and road trips, and identify ways math is involved.

2 Monitor

A **Accessibility: Executive Functioning** Invite students to reflect on the strategies they used for Problem 7a before responding to Problem 8a.

Pause to discuss the similarities and differences between the contexts and information given in Problems 7 and 8.

Look for students discussing which sign to use when writing the absolute value equation for Problem 8a.

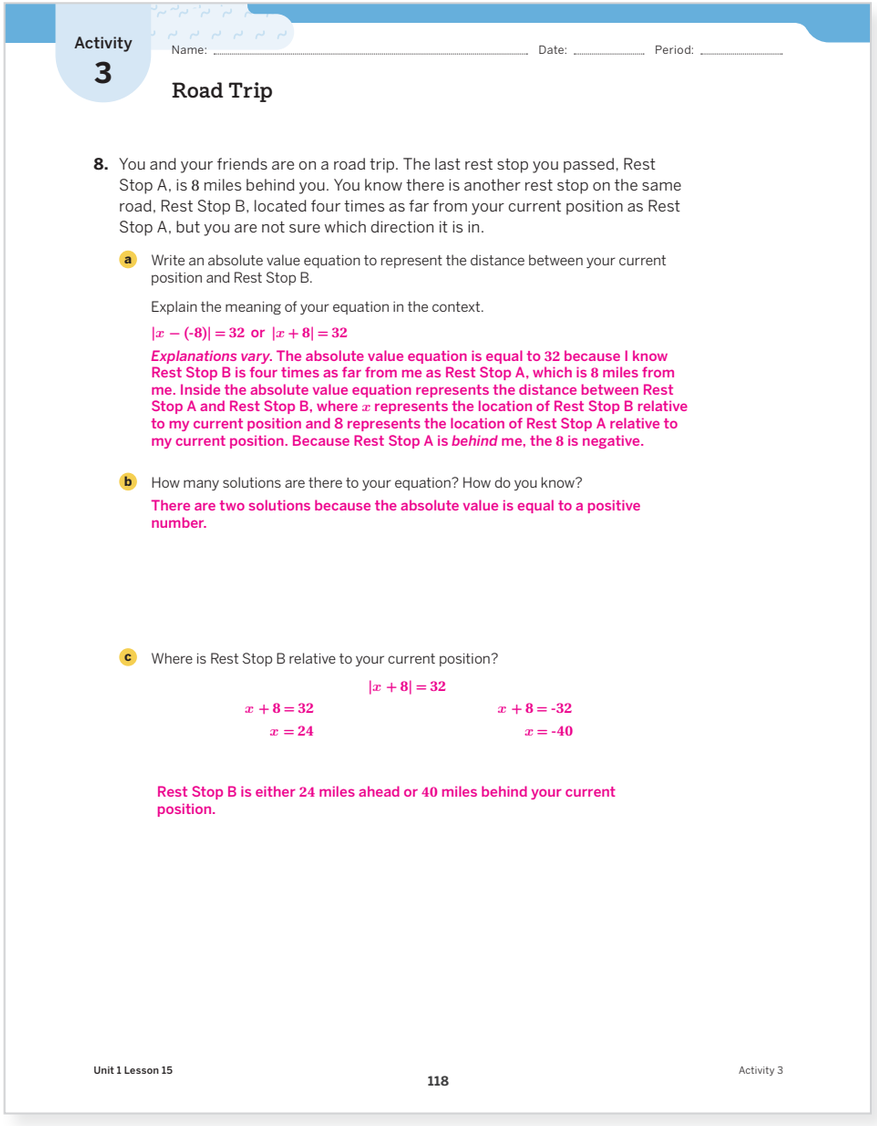
Pause to discuss the meaning of using 8 versus -8 in the equation.

A **Accessibility: Visual-Spatial Processing** To support students in interpreting spatial representations, provide blank paper or blank number lines for students to draw on to model the problems.

Capture and share strategies for understanding which sign to use when writing absolute value equations based on a given context.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Activity 3 continued >



Activity 3

Name: Date: Period:

Road Trip

8. You and your friends are on a road trip. The last rest stop you passed, Rest Stop A, is 8 miles behind you. You know there is another rest stop on the same road, Rest Stop B, located four times as far from your current position as Rest Stop A, but you are not sure which direction it is in.

a. Write an absolute value equation to represent the distance between your current position and Rest Stop B.

Explain the meaning of your equation in the context.

$|x - (-8)| = 32$ or $|x + 8| = 32$

Explanations vary. The absolute value equation is equal to 32 because I know Rest Stop B is four times as far from me as Rest Stop A, which is 8 miles from me. Inside the absolute value equation represents the distance between Rest Stop A and Rest Stop B, where x represents the location of Rest Stop B relative to my current position and 8 represents the location of Rest Stop A relative to my current position. Because Rest Stop A is *behind* me, the 8 is negative.

b. How many solutions are there to your equation? How do you know?

There are two solutions because the absolute value is equal to a positive number.

c. Where is Rest Stop B relative to your current position?

$|x + 8| = 32$

$x + 8 = 32$ $\quad$ $x + 8 = -32$

$x = 24$ $\quad$ $x = -40$

Rest Stop B is either 24 miles ahead or 40 miles behind your current position.

Unit 1 Lesson 15 118 Activity 3

Pairs | 10 min

Activity 3 Road Trip (continued)

Purpose: Students write absolute value equations in the context of a road trip and identify their solutions. **(MTR.7.1)**

2 Monitor

Invite students to discuss what it means for an absolute value equation to be equal to zero.

Consider asking, "What factors affect the gas mileage of a vehicle?"

MLR **MLR3: Critique, Correct, and Clarify** Consider using this routine during Problem 10 to help students rewrite Mariana's equation so that it makes sense in terms of the vehicle's gas mileage.

D **Differentiation: (Support)** Look for students who rewrite the equation in Problem 10b as $|x-3|=25$. Invite them to read their absolute value equation verbally (for example, "The gas mileage of the vehicle is 3 mpg, plus or minus 25 mpg."). Discuss the average gas mileage of different vehicles to help students assess the reasonableness of their numbers in the real-world context.

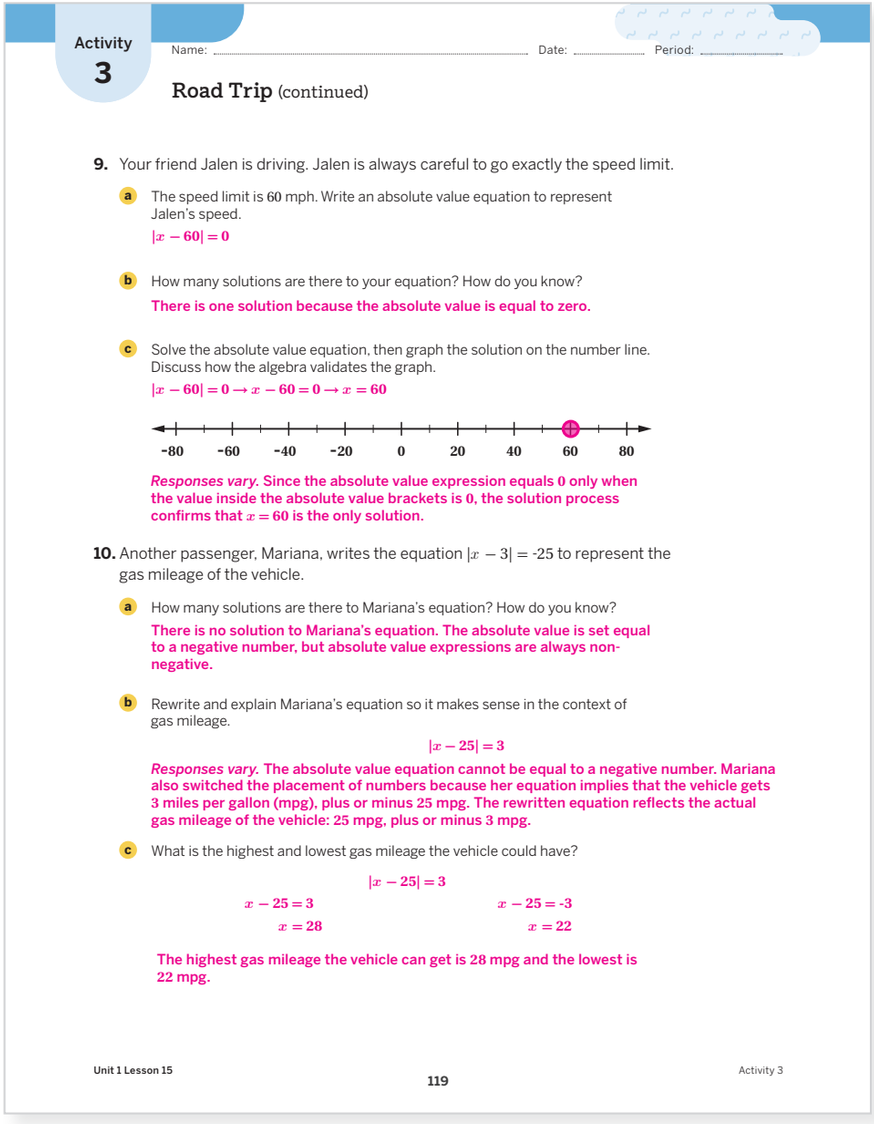
3 Connect

Encourage students to share something new they learned in this activity.

Invite students to share how using visual aids helps them solve problems.

Share captured drawings and discuss as a class how different ways of visualizing the problem can lead to the same answer.

Encourage students to share their strategies for choosing the correct sign and placement of terms for the context.



Activity 3

Name: Date: Period:

Road Trip (continued)

9. Your friend Jalen is driving. Jalen is always careful to go exactly the speed limit.

a. The speed limit is 60 mph. Write an absolute value equation to represent Jalen's speed.

$|x-60|=0$

b. How many solutions are there to your equation? How do you know?

There is one solution because the absolute value is equal to zero.

c. Solve the absolute value equation, then graph the solution on the number line. Discuss how the algebra validates the graph.

$|x-60|=0 \rightarrow x-60=0 \rightarrow x=60$

Responses vary. Since the absolute value expression equals 0 only when the value inside the absolute value brackets is 0, the solution process confirms that $x=60$ is the only solution.

10. Another passenger, Mariana, writes the equation $|x-3|=-25$ to represent the gas mileage of the vehicle.

a. How many solutions are there to Mariana's equation? How do you know?

There is no solution to Mariana's equation. The absolute value is set equal to a negative number, but absolute value expressions are always non-negative.

b. Rewrite and explain Mariana's equation so it makes sense in the context of gas mileage.

$|x-25|=3$

Responses vary. The absolute value equation cannot be equal to a negative number. Mariana also switched the placement of numbers because her equation implies that the vehicle gets 3 miles per gallon (mpg), plus or minus 25 mpg. The rewritten equation reflects the actual gas mileage of the vehicle: 25 mpg, plus or minus 3 mpg.

c. What is the highest and lowest gas mileage the vehicle could have?

$|x-25|=3$

$x-25=3$ $\quad$ $x-25=-3$

$x=28$ $\quad$ $x=22$

The highest gas mileage the vehicle can get is 28 mpg and the lowest is 22 mpg.

Unit 1 Lesson 15 119 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for creating, solving, and identifying the number of solutions of absolute value equations.

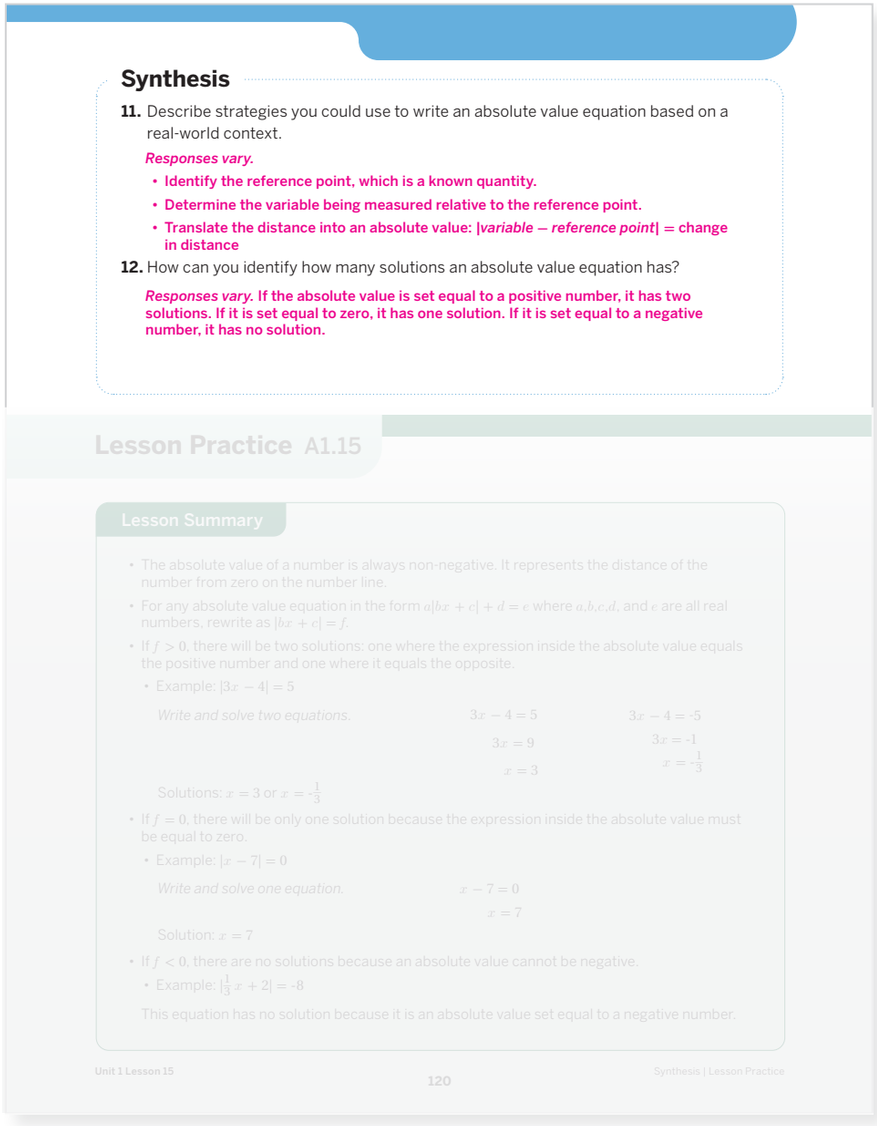
Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:

- To translate a real-world context into an absolute value equation, you must first identify the reference point and the variable.
- The formula for translating a real-world context into an absolute value equation can be written as, $|variable - reference\ point| = change\ in\ distance$.

MLR **MLR.2: Collect and Display** Use this routine to formalize the criteria for and meaning of absolute value equations having two solutions, one solution, or no solution using students' language.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. Describe strategies you could use to write an absolute value equation based on a real-world context.

Responses vary.

- Identify the reference point, which is a known quantity.
- Determine the variable being measured relative to the reference point.
- Translate the distance into an absolute value: |*variable* − *reference point*| = change in distance

12. How can you identify how many solutions an absolute value equation has?

Responses vary. If the absolute value is set equal to a positive number, it has two solutions. If it is set equal to zero, it has one solution. If it is set equal to a negative number, it has no solution.

Lesson Practice A1.15

Lesson Summary

- The absolute value of a number is always non-negative. It represents the distance of the number from zero on the number line.
- For any absolute value equation in the form $a|bx + c| + d = e$ where a,b,c,d, and e are all real numbers, rewrite as $|bx + c| = f$.
- If $f > 0$, there will be two solutions: one where the expression inside the absolute value equals the positive number and one where it equals the opposite.
  - Example: $|3x - 4| = 5$

  Write and solve two equations.

  $3x - 4 = 5$ → $3x = 9$ → $x = 3$

  $3x - 4 = -5$ → $3x = -1$ → $x = -\frac{1}{3}$

  Solutions: $x = 3$ or $x = -\frac{1}{3}$
- If $f = 0$, there will be only one solution because the expression inside the absolute value must be equal to zero.
  - Example: $|x - 7| = 0$

  Write and solve one equation.

  $x - 7 = 0$ → $x = 7$

  Solution: $x = 7$
- If $f < 0$, there are no solutions because an absolute value cannot be negative.
  - Example: $|\frac{1}{3}x + 2| = -8$

  This equation has no solution because it is an absolute value set equal to a negative number.

Unit 1 Lesson 15 120 Synthesis | Lesson Practice

Lesson Takeaway: Absolute value equations can be used to model real-world contexts and can have two solutions, one solution, or no solution.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing an absolute value equation for a context with restraints, identifying and justifying the number of solutions, and solving for all solutions.

Today's Goals

1. **Goal:** Know how to identify whether an absolute value equation has two solutions, one solution, or no solution.
2. **Goal:** Write and solve one-variable absolute value equations given a mathematical or real-world context.
3. **Language Goal:** Describe the meaning of each term of an absolute value equation in context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.15

The temperature in a room starts at 70 degrees. The temperature then changes 5 degrees.

a. Write an absolute value equation that represents the possible current temperature, x.
$|x - 70| = 5$

b. Without solving, how many solutions does your equation have?
2

c. Explain your thinking.
Responses vary. There are two solutions because the absolute value is equal to a positive number.

d. Solve your equation.
$x =$ 65 or $x =$ 75

D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support writing and solving an absolute value equation:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to create a real-world scenario where a given absolute value equation applies. Ask students to swap scenarios with a peer to verify that the provided equation accurately models the context.

Professional Learning

In this lesson, students worked with only one-variable equations. How can you help everyone feel prepared for the introduction of two-variable equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Describe strategies you could use to write an absolute value equation based on a real-world context.

Responses vary.

- Identify the reference point, which is a known quantity.
- Determine the variable being measured relative to the reference point.
- Translate the distance into an absolute value: $|variable - reference\ point| = change\ in\ distance$

12. How can you identify how many solutions an absolute value equation has?

Responses vary. If the absolute value is set equal to a positive number, it has two solutions. If it is set equal to zero, it has one solution. If it is set equal to a negative number, it has no solution.

Lesson Practice A1.15

Lesson Summary

- The absolute value of a number is always non-negative. It represents the distance of the number from zero on the number line.
- For any absolute value equation in the form $a|bx + c| + d = e$ where a,b,c,d, and e are all real numbers, rewrite as $|bx + c| = f$.
- If $f > 0$, there will be two solutions: one where the expression inside the absolute value equals the positive number and one where it equals the opposite.
  - Example: $|3x - 4| = 5$

    Write and solve two equations.

    $3x - 4 = 5$ $\quad$ $3x - 4 = -5$

    $3x = 9$ $\quad$ $3x = -1$

    $x = 3$ $\quad$ $x = -\frac{1}{3}$

    Solutions: $x = 3$ or $x = -\frac{1}{3}$
- If $f = 0$, there will be only one solution because the expression inside the absolute value must be equal to zero.
  - Example: $|x - 7| = 0$

    Write and solve one equation.

    $x - 7 = 0$

    $x = 7$

    Solution: $x = 7$
- If $f < 0$, there are no solutions because an absolute value cannot be negative.
  - Example: $|\frac{1}{3}x + 2| = -8$

    This equation has no solution because it is an absolute value set equal to a negative number.

Lesson Practice A1.15

Name: ..... Date: ..... Period: .....

1. Circle the equations that show the cases that must be considered when solving $\frac{7}{2} = |3x - 6| + \frac{1}{2}$.

A. $3x - 6 = -3$ $\quad$ **B.** $-4 = 3x - 6$ $\quad$ **C.** $4 = 3x - 6$ $\quad$ **D.** $3x - 6 = 3$

2. The NBA™ regulates the air pressure of basketballs used during games. The highest acceptable air pressure is 8.5 pounds per square inch (psi), and the lowest acceptable air pressure is 7.5 psi. Write an absolute value equation to define the highest and lowest possible pressures, p, for NBA™ basketballs.

$|p - 8| = 0.5$

NBA™ basketballs must have a pressure, p, within 0.5 psi of 8 psi.

3. Alejandro and Zoe are making corrections to practice problems they missed. They each have one incorrect problem left, but neither can figure out why their answers are wrong. Describe and correct each of their errors.

Alejandro's incorrect problem:	Zoe's incorrect problem:
$\|2x - 1\| = -9$	$\|5x + 8\| = x$
$2x - 1 = -9$ $\quad$ $-(2x - 1) = -9$	$5x + 8 = x$ $\quad$ $-(5x + 8) = x$
$2x = -8$ $\quad$ $2x - 1 = 9$	$4x + 8 = 0$ $\quad$ $5x + 8 = -x$
$x = -4$ $\quad$ $2x = 10$	$4x = -8$ $\quad$ $6x + 8 = 0$
$x = 5$	$x = -2$ $\quad$ $6x = -8$
	$x = -\frac{4}{3}$
The solutions are $x = -4$ and $x = 5$.	The solutions are $x = -2$ and $x = -\frac{4}{3}$.
Alejandro's problem is incorrect because absolute value expressions can never be equal to a negative number. Absolute values represent distances on a number line, which are always non-negative. The equation $\|2x - 1\| = -9$ has no solution.	Zoe's problem is incorrect because her solutions do not satisfy the original expression. It is crucial to check that solutions make sense in the context of the absolute value expression. For the equation $\|5x + 8\| = x$, x must be non-negative ($x \geq 0$) because the absolute value expression cannot equal a negative number. Therefore, neither of Zoe's solutions satisfy the equation. The equation $\|5x + 8\| = x$ has no solution because no valid values of x satisfy the equation.

Lesson Practice A1.15

Name: ..... Date: ..... Period: .....

4. Without solving completely, sort each equation into one of the three categories.

$|x + 13| - 2 = 7$ $\quad$ $1 + |x - 8| = -2$ $\quad$ $|\frac{3}{4}x - 5| + 4 = 4$

$|5x - 2| + 9 = 0$ $\quad$ $-3 + |\frac{1}{2}x - 8| = \frac{1}{3}$ $\quad$ $|x - 2| - 6 = 0$

No Solution	One Solution	Two Solutions
$1 + \|x - 8\| = -2$ $\|5x - 2\| + 9 = 0$	$\|\frac{3}{4}x - 5\| + 4 = 4$	$\|x - 2\| - 6 = 0$ $\|x + 13\| - 2 = 7$ $-3 + \|\frac{1}{2}x - 8\| = \frac{1}{3}$

Test Practice

5. Consider the absolute value equation $7 - |x + 5| = 3$. Select *all* of the true statements.

- **A.** $x = 1$ is a solution.
- **B.** $x = -9$ is a solution.
- **C.** The equation has no solutions.
- **D.** The solutions are numbers that are 4 units away from -5.
- **E.** $x = -1$ is a solution.
- **F.** The solutions are numbers that are 5 units away from -4.

Spiral Review

6. Solve the absolute value equation: $|2x - 5| + 8 = 22$. Choose *all* solutions.

☑ **A.** $x = -4.5$ $\quad$ ☑ **B.** $x = \frac{19}{2}$ $\quad$ ☐ **C.** $x = -9.5$ $\quad$ ☑ **D.** $x = -\frac{9}{2}$

☐ **E.** $x = -\frac{2}{19}$ $\quad$ ☐ **F.** $x = \frac{2}{9}$ $\quad$ ☑ **G.** $x = 9.5$ $\quad$ ☐ **H.** $x = 4.5$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.4.1
	2, 4	2	MA.912.AR.4.1
	3	3	MA.912.AR.4.1
Test Practice	5	2	MA.912.AR.4.1
Spiral Review			
	6	2	MA.912.AR.4.1

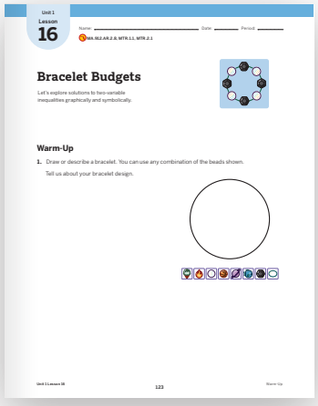
Unit 1

Lesson 16

Bracelet Budgets

Introduction to Two-Variable Inequalities

Let's explore solutions to two-variable inequalities graphically and symbolically.


Unit 1
Lesson 16
Name: Date: Period:
MA.912.AR.2.8, MTR.1.1, MTR.2.1
Bracelet Budgets
Let's explore solutions to two-variable inequalities graphically and symbolically.
Warm-Up
1. Draw or describe a bracelet. You can use any combination of the beads shown.
Tell us about your bracelet design.
123

Learning Outcomes

Today's Goal

1. **Goal:** Understand that the solutions to a two-variable inequality are points that make the inequality true.
2. **Language Goal:** Interpret two-variable inequalities that represent constraints in context. **(Writing, Speaking, and Listening)**

Students are introduced to two-variable inequalities by modeling the cost constraints of designing a beaded bracelet. They determine solutions to cost constraints and view them on a graph. Students surface multiple strategies for determining if a point is a solution to a two-variable inequality. **(MTR.2.1, MTR.3.1)**

Coherence

Prior Learning

In Lessons 4–7, students explored two-variable linear equations in multiple representations. In Lessons 12, students solved and graphed one-variable inequalities.

Future Learning

In Lesson 17, students will graph solutions to two-variable inequalities given the graph of its corresponding equations. They will also consider whether the points on the boundary line are included in the solution set or not. In Lessons 18 and 19, students will model real-word constraints with two-variable inequalities.

Connections

- Students use two-variable linear inequalities to design a bracelet within constraints of inventory and price. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.8
Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.2 **MA.8.AR.3.1**	**MA.912.AR.2.8**

Lesson at a Glance

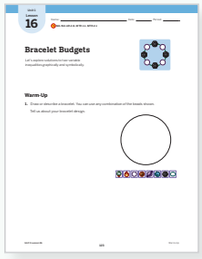
~ 45 min

MA.912.AR.2.8

Warm-Up

Independent | 5 min

Students express their creativity as they are introduced to the context of making bracelets.

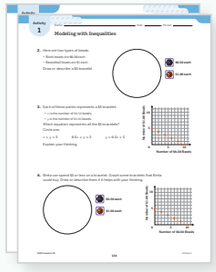


Activity 1

Pairs | 10 min

Students explore a situation that can be modeled by a two-variable inequality and what solutions look like in multiple representations.

Routines: Notice and Wonder, Decide and Defend **(MTR.2.1, MTR.7.1)**

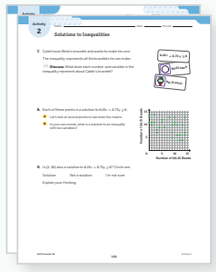


Activity 2

Pairs | 10 min

Students use multiple representations to construct viable arguments about whether or not points are solutions to two-variable inequalities.

Routines: Think-Pair-Share, MLR7: Compare and Connect **(MTR.1.1)**



Activity 3

Pairs | 10 min

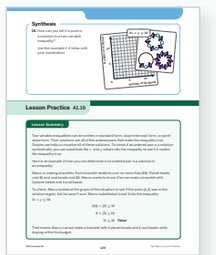
Students recognize whether an inequality is in standard, slope-intercept, or point-slope form, and convert it to another form. **Routine: Think-Pair-Share (MTR.4.1)**



Synthesis

Whole Class | 5 min

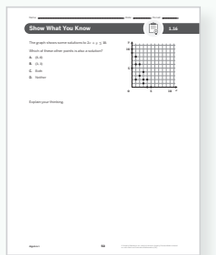
Students synthesize their understanding of what it means for a point to be a solution to a two-variable inequality.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining if points are solutions to a two-variable inequality and explaining their reasoning.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students express their creativity as they are introduced to the context of making bracelets.

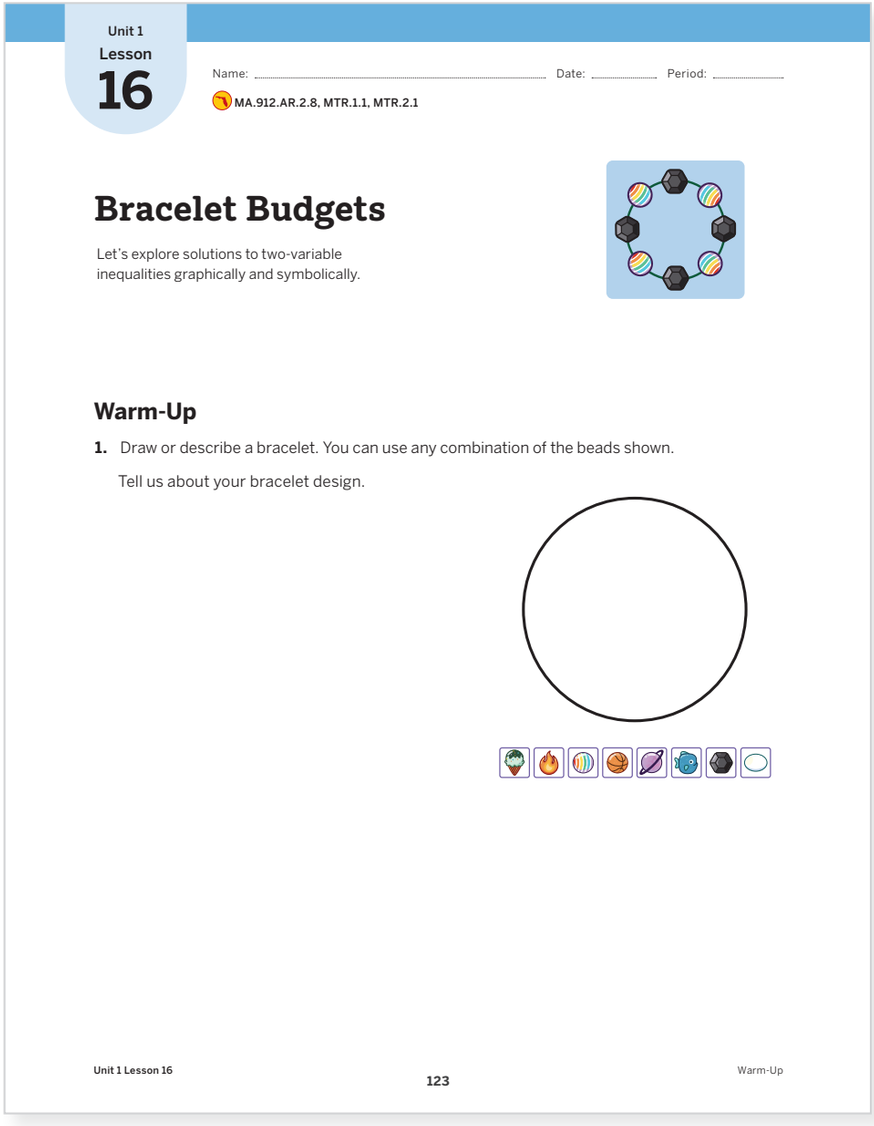
Short on time? Consider omitting the Warm-up.

1 Launch

Consider asking:

- "Is anyone wearing a bracelet now? If so, does it hold a special meaning to you?"
- "If you were going to design a bead, what would it look like?"

Invite students to draw or describe a bracelet. Encourage students to use coloring tools (e.g., coloring pencils, markers, etc.) to draw in this lesson.



Unit 1
Lesson
16

Name: Date: Period:

MA.912.AR.2.8, MTR.1.1, MTR.2.1

Bracelet Budgets

Let's explore solutions to two-variable inequalities graphically and symbolically.

Warm-Up

1. Draw or describe a bracelet. You can use any combination of the beads shown.

   Tell us about your bracelet design.

Unit 1 Lesson 16 123 Warm-Up

Pairs | 10 min

Activity 1 Modeling With Inequalities

Purpose: Students explore a situation that can be modeled by a two-variable inequality and what solutions look like in multiple representations. **(MTR.2.1, MTR.7.1)**

1 Launch

Consider asking, "Have you ever bought a gift for yourself or someone else with a specific cost in mind?"

2 Monitor

Encourage students to strategize with a partner before creating a bracelet.

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, invite them to choose how many basketball beads they want first, then to figure out how many black beads they can add to make a $5 bracelet.

A **Accessibility: Conceptual Processing** As students solve abstract problems, invite them to say out loud what each equation means with the variable definitions in place of x and y (e.g., The number of $0.50 beads plus the number of $1.00 beads is equal to $5).

Pause briefly after most students have completed Problem 3 to discuss why each equation does or does not represent the constraint. Consider asking, "How can you tell if the equation matches the situation? That the equation matches the graph?" **(MTR.2.1)**

ELL **English Language Learners** Invite students to read Binta's constraint aloud and paraphrase the meaning of "$5 or less." **(Reading)**

Consider asking pairs of students, "What do you think a graph of all the bracelets that cost $5 or less would look like?" **(MTR.2.1)**

D **Differentiation: (Stretch)** Consider asking students, "Can you make a bracelet that doesn't have whole number coordinates on the graph? Why or why not?"

Activity 1 continued >

Activity 1

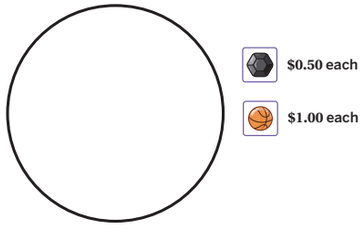
Name: Date: Period:

Modeling with Inequalities

2. Here are two types of beads:

- Black beads are $0.50 each.
- Basketball beads are $1 each.

Draw or describe a $5 bracelet.



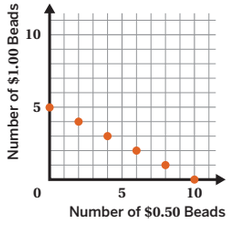
3. Each of these points represents a $5 bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

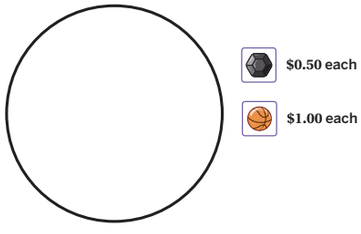
Which equation represents all the $5 bracelets? Circle one.

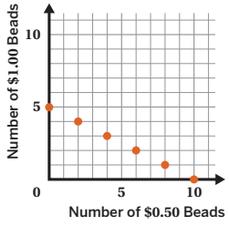
$x + y = 5$ $0.5x + y = 5$ $y = 0.5x + 5$

Explain your thinking.



4. Binta can spend $5 or less on a bracelet. Graph some bracelets that Binta could buy. Draw or describe them if it helps with your thinking.





Unit 1 Lesson 16 124 Activity 1

Pairs | 10 min

Activity 1 Modeling With Inequalities (continued)

Purpose: Students explore a situation that can be modeled by a two-variable inequality and what solutions look like in multiple representations. **(MTR.2.1, MTR.7.1)**

2 Monitor

Use the Notice and Wonder routine to look for what students connect to $0.5x + y \leq 5$ to share during the Connect. **(MTR.1.1)**

Encourage students to read others' responses and decide if others' noticings and wonderings were similar to or different from their own instead of having a whole class discussion.

A **Accessibility: Visual-Spatial Processing** As students interpret visual representations, encourage them to use a finger to trace along gridlines to the axes to help them interpret the meaning of a point. It may be helpful to review what one point tells us about the bracelet before responding.

A **Accessibility: Memory and Attention** Invite students to reference previous notes or an anchor chart about inequality symbols and their meanings.

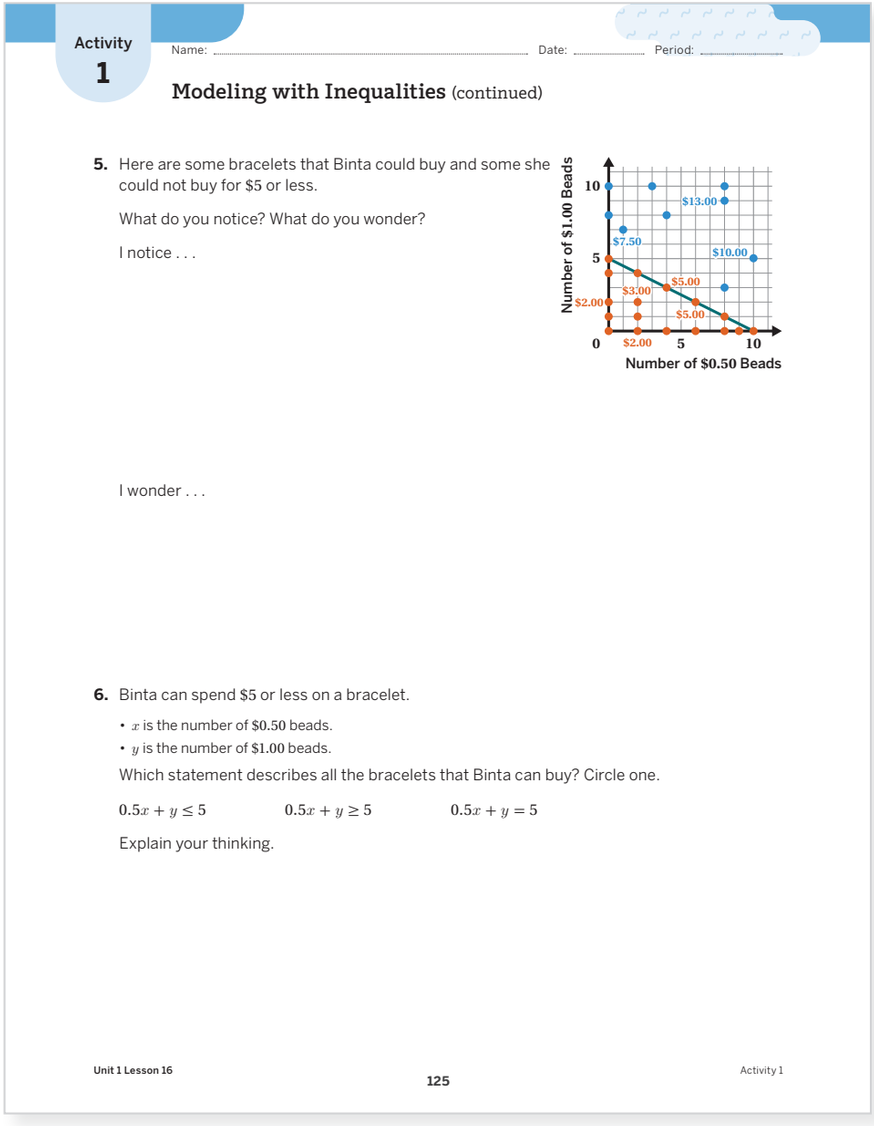
D **Differentiation**

Look for students who:	Teacher Moves
Need support getting started.	**Support** Consider asking, "What is different about each of the responses?"
Select $0.5x + y \geq 5$.	**Support** Invite them to read the inequality aloud or plug in a solution from the graph.
Select $0.5x + y \leq 5$ and have interesting explanations.	Select and sequence a variety of explanations to share during the Connect. **Stretch** Encourage students to write their own situations that match each of the other answer choices.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Invite students who made different choices to use reasoning to convince their classmates. **(optional) (MTR.4.1)**

To surface the Key Takeaway, consider asking:
- "What do each of the numbers in the inequality tell us about the situation?" **(MTR.7.1)**
- "Why does an inequality model this situation better than an equation does?" **(MTR.3.1)**



Activity 1

Name: Date: Period:

Modeling with Inequalities (continued)

5. Here are some bracelets that Binta could buy and some she could not buy for $5 or less.

What do you notice? What do you wonder?

I notice . . .

I wonder . . .

6. Binta can spend $5 or less on a bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

Which statement describes all the bracelets that Binta can buy? Circle one.

$0.5x + y \leq 5$ $0.5x + y \geq 5$ $0.5x + y = 5$

Explain your thinking.

Unit 1 Lesson 16 125 Activity 1

Key Takeaway: Two-variable linear inequalities are one way to model real-world constraints that have more than one variable to consider and more than one possible solution.

Pairs | 10 min

Activity 2 Solutions to Inequalities

Purpose: Students use multiple representations to construct viable arguments about whether or not points are solutions to two-variable inequalities. **(MTR.1.1, MTR.4.1)**

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse. To encourage movement, consider inviting students to find a partner to talk with in a different location. **(MTR.4.1)**

ELL **English Language Learners** Consider providing sentence frames such as: "The ___ represents ___." **(Speaking)**

Invite students to share their or their partner's thoughts about what each part of the inequality represents.

D **Differentiation: (Strengthen)** Students may describe that x represents "rainbow beads" and y represents "ice cream beads." Celebrate this connection and encourage them to specify. Consider asking, "What about the rainbow beads? Their cost? The number of them?"

Consider asking, "Why might someone want to buy a bracelet that costs greater than a certain amount?" **(MTR.7.1)**

2 Monitor

To support students getting started, encourage them to choose one point and describe what it means for that bracelet.

ELL **English Language Learners** Invite students to describe solutions in their primary languages. Encourage all students to use gestures and/or visuals as they discuss. **(Reading and Writing)**

D **Differentiation**

Look for students who:	Teacher Moves
Need support getting started.	**Support** Encourage them to plot the point on the graph.
Circle "I'm not sure."	**Support** Invite them to explain why they are unsure and then to explore the two strategies in Problem 10.
Use strategies similar to Marco's or Jada's or other creative strategies.	**Strengthen** Celebrate their strategies. Select and sequence these strategies to share their thinking during the Connect.

Activity 2 continued >

Activity 2

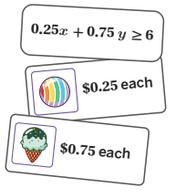
Name: Date: Period:

Solutions to Inequalities

7. Caleb loves Binta's bracelet and wants to make his own.

The inequality represents all the bracelets he can make.

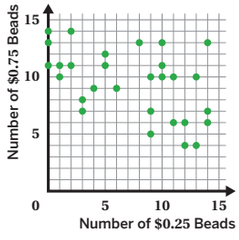
Discuss: What does each number and variable in the inequality represent about Caleb's bracelet?



8. Each of these points is a solution to $0.25x + 0.75y \geq 6$.

a. Let's look at several points to see what this means.

b. In your own words, what is a solution to an inequality with two variables?



9. Is $(2, 10)$ also a solution to $0.25x + 0.75y \geq 6$? Circle one.

Solution Not a solution I'm not sure

Explain your thinking.

Unit 1 Lesson 16 126 Activity 2

Pairs | 10 min

Activity 2 Solutions to Inequalities (continued)

Purpose: Students use multiple representations to construct viable arguments about whether or not points are solutions to two-variable inequalities. **(MTR.1.1, MTR.4.1)**

2 Monitor

A **Accessibility: Executive Functioning** Consider chunking Problem 10 by inviting students to describe one strategy at a time.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as *graph*, *line*, *point*, *plot*, *substitute*, *evaluate*, *notice*, etc. **(Writing and Speaking)**

Encourage students to confirm the location of each point from the list on the graph before responding.

D **Differentiation: (Stretch)** Invite students to determine a point that makes the inequality true, but is not a solution for this situation because of the bracelet context.

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

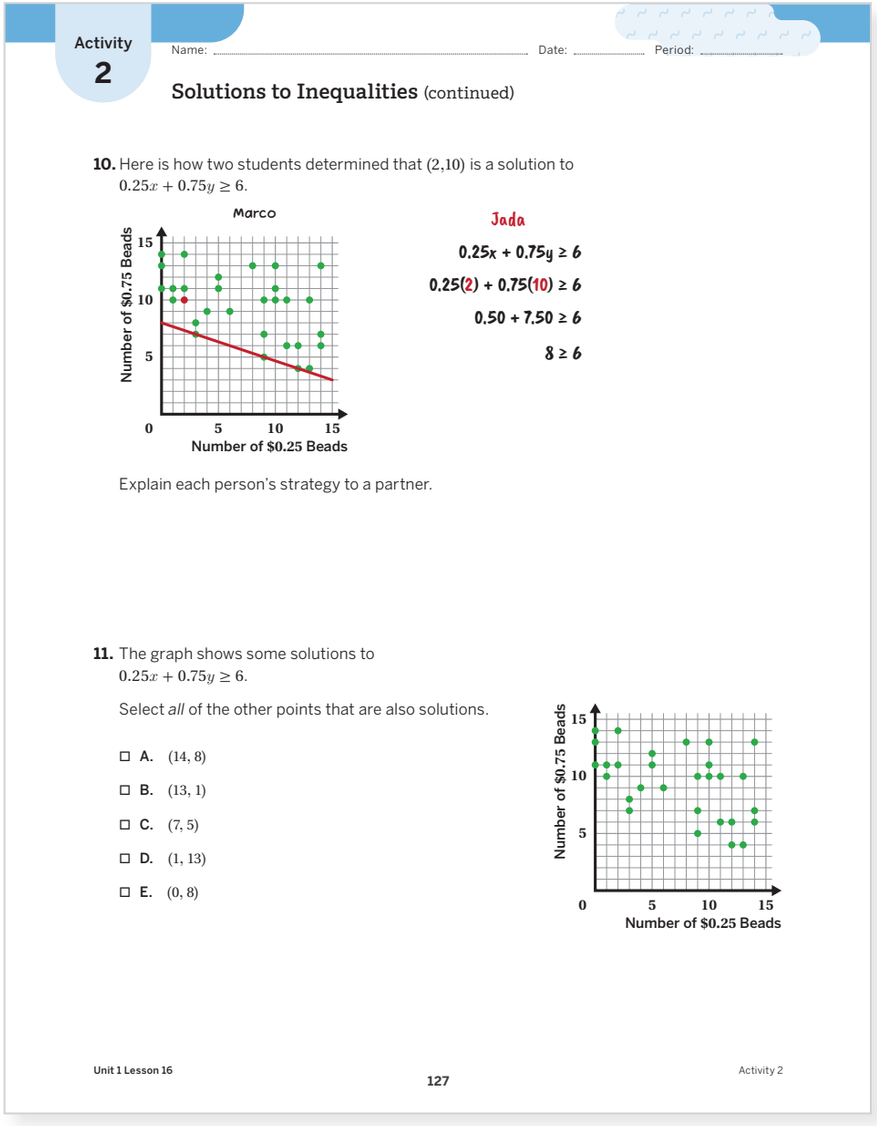
Consider asking:

- "How can you use each strategy to tell that the point is a solution?"
- "Which strategy do you prefer?"
- "When might Marco's strategy be difficult to use? Jada's?" *Responses vary.* If a point is very close to the boundary line, Marco's strategy might be hard to use. If a point has fractions or decimals and I don't have a calculator, Jada's strategy might be difficult. It's helpful to have two strategies.

Note: It is okay —even desirable— to use student language throughout this lesson to define terms such as *boundary line* and *shaded region*. Students will be formally introduced to the term boundary line in the next lesson.

Poll the class on their responses.

Invite students to make arguments and justify their reasoning for each point if there is not consensus. **(MTR.4.1)**



Activity 2

Name: Date: Period:

Solutions to Inequalities (continued)

10. Here is how two students determined that (2,10) is a solution to $0.25x + 0.75y \geq 6$.

Marco

Jada

$0.25x + 0.75y \geq 6$

$0.25(2) + 0.75(10) \geq 6$

$0.50 + 7.50 \geq 6$

$8 \geq 6$

Explain each person's strategy to a partner.

11. The graph shows some solutions to $0.25x + 0.75y \geq 6$.

Select *all* of the other points that are also solutions.

- ☐ A. (14, 8)
- ☐ B. (13, 1)
- ☐ C. (7, 5)
- ☐ D. (1, 13)
- ☐ E. (0, 8)

Unit 1 Lesson 16 127 Activity 2

Key Takeaway: The solutions to a two-variable inequality are points that make the inequality true. Substituting a point into the inequality or graphing it can be helpful strategies to determine if a point is a solution.

Pairs | 10 min

Activity 3 All About Form

Purpose: Students recognize whether an inequality is in standard, slope-intercept, or point-slope form, and convert it to another form.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse throughout this lesson. After students have initially answered each question, invite them to share with a partner, encouraging them to reflect and revise answers as needed. **(MTR.4.1)**

To surface prior knowledge, consider asking,

- "Can you recall the different forms you can write an equation in?"
- "What is the difference between writing an equation and writing an inequality?"
- "How can you apply what you know about writing equations in different forms to writing inequalities in different forms?"

Discuss the differences between standard form, slope-intercept form, and point-slope form.

Encourage students to share with their partners if they have a favorite or least favorite form, and why.

2 Monitor

D **Differentiation: (Support)** Look for students having trouble differentiating between forms. Consider sharing the formulas for each:

- Standard Form: $Ax + By = C$
- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Where m = slope, b = y-intercept, (x_1, y_1) = coordinate of a point in the solution, and A, B, and C are integers.

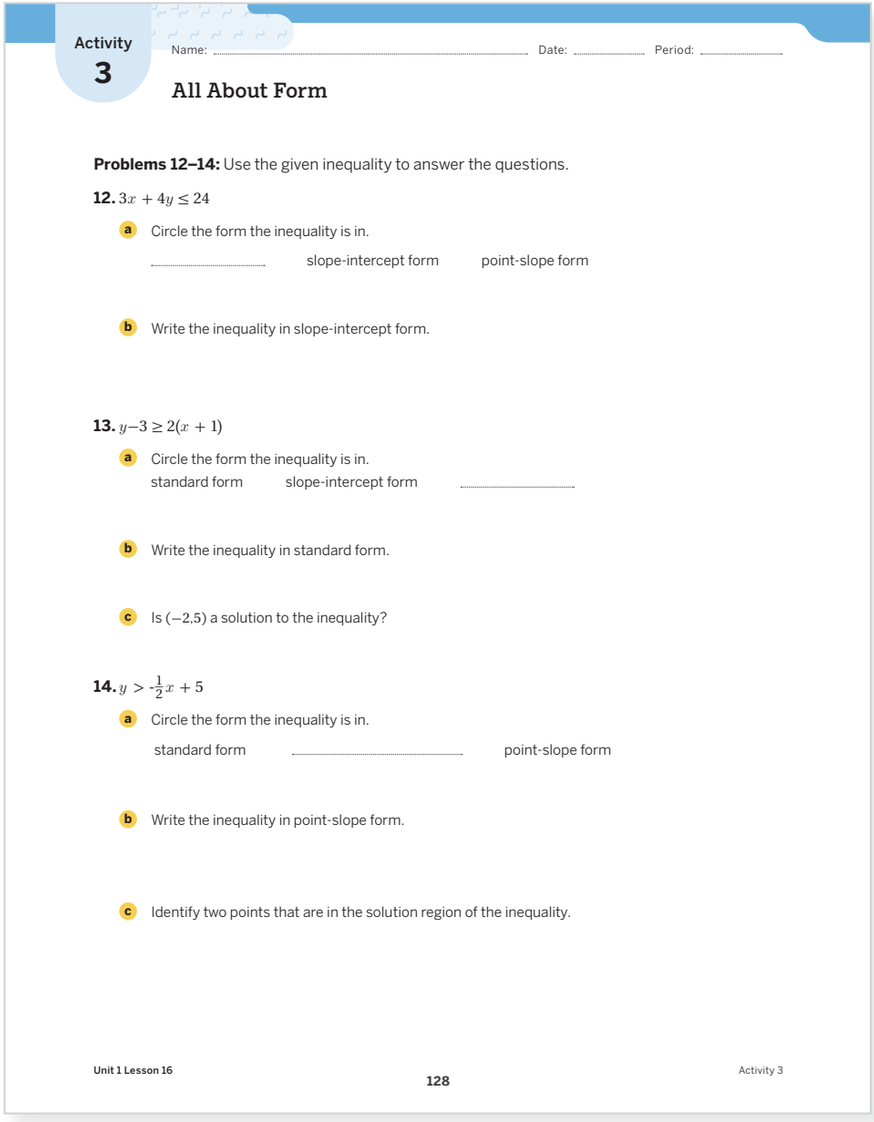
Listen for students discussing different strategies to share during the Connect. **(MTR.1.1, MTR.4.1)**

3 Connect

Share and discuss different students' strategies for identifying the form of an inequality and rewriting it in another form.

Consider asking,

- "Do you feel more confident in differentiating between different forms than you did at the beginning of the activity?"
- "Are there any questions you still have?"



Activity 3

Name: ..... Date: ..... Period: .....

All About Form

Problems 12–14: Use the given inequality to answer the questions.

12. $3x + 4y \leq 24$

a. Circle the form the inequality is in.

..... slope-intercept form point-slope form

b. Write the inequality in slope-intercept form.

13. $y - 3 \geq 2(x + 1)$

a. Circle the form the inequality is in.

standard form slope-intercept form .....

b. Write the inequality in standard form.

c. Is $(-2,5)$ a solution to the inequality?

14. $y > -\frac{1}{2}x + 5$

a. Circle the form the inequality is in.

standard form ..... point-slope form

b. Write the inequality in point-slope form.

c. Identify two points that are in the solution region of the inequality.

Unit 1 Lesson 16 128 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what it means for a point to be a solution to a two-variable inequality.

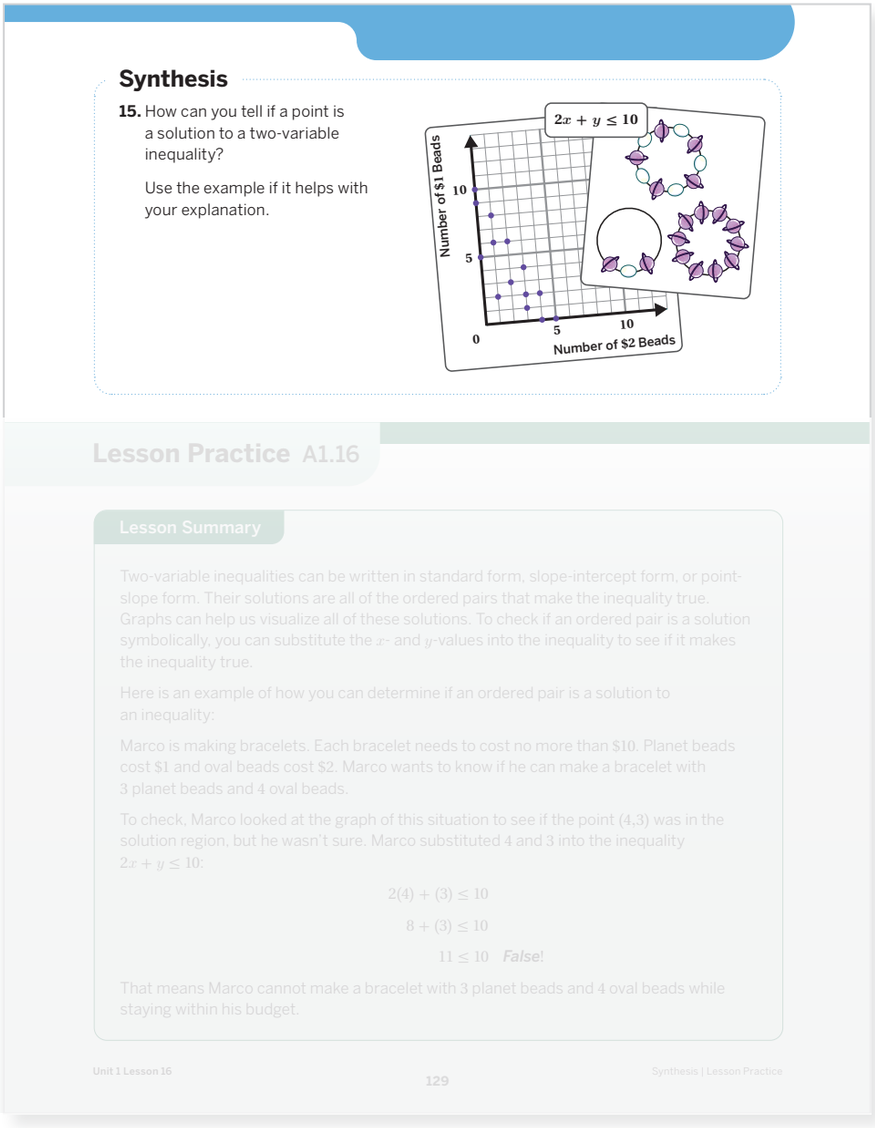
Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as *graph*, *line*, *point*, *plot*, *substitute*, *evaluate*, *notice*, etc. **(Writing and Speaking)**

Share a variety of strategies, such as:

- Substituting the point into the inequality to see if it makes a true statement.
- Plotting the point on a graph to see if it is surrounded by other points that are also solutions.
- Reasoning about if the point makes sense in the situation.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

15. How can you tell if a point is a solution to a two-variable inequality?

Use the example if it helps with your explanation.

Lesson Practice A1.16

Lesson Summary

Two-variable inequalities can be written in standard form, slope-intercept form, or point-slope form. Their solutions are all of the ordered pairs that make the inequality true. Graphs can help us visualize all of these solutions. To check if an ordered pair is a solution symbolically, you can substitute the x- and y-values into the inequality to see if it makes the inequality true.

Here is an example of how you can determine if an ordered pair is a solution to an inequality:

Marco is making bracelets. Each bracelet needs to cost no more than $10. Planet beads cost $1 and oval beads cost $2. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

To check, Marco looked at the graph of this situation to see if the point (4,3) was in the solution region, but he wasn't sure. Marco substituted 4 and 3 into the inequality $2x + y \leq 10$:

$$2(4) + (3) \leq 10$$
$$8 + (3) \leq 10$$
$$11 \leq 10 \quad \textit{False!}$$

That means Marco cannot make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

Unit 1 Lesson 16 129 Synthesis | Lesson Practice

Lesson Takeaway: The solutions to a two-variable inequality are points that make the inequality true. To check solutions graphically, look for points that are in the shaded region or on the correct side of the boundary line. To check solutions symbolically, substitute the x- and y-values into the inequality to see if they make the inequality true.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining if points are solutions to a two-variable inequality and explaining their reasoning.

Today's Goals

1. **Goal:** Understand that the solutions to a two-variable inequality are points that make the inequality true.
2. **Language Goal:** Interpret two-variable inequalities that represent constraints in context. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

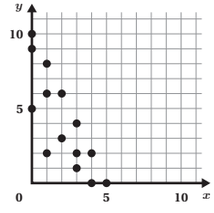
Name: _____ Date: _____ Period: _____

Show What You Know 1.16

The graph shows some solutions to $2x + y \leq 10$.

Which of these other points is also a solution?

A. $(6, 0)$
B. $(3, 3)$
C. Both
D. Neither



Explain your thinking.

Explanations vary.
- (3, 3) is between two solutions, so I think it must be a solution too.
- Substituting $x = 3$ and $y = 3$ into the inequality makes a true statement: $2(3) + (3) = 9$, and $9 \leq 10$.

D Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support determining if a point is a solution to a two-variable inequality:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problem 10** and have the students discuss the pros and cons of Marco's and Jada's strategies.
- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to determine points that make the inequality in **Activity 2** true, but which are not actually solutions because of the bracelet context.

Professional Learning

Take a moment to reflect on the **Decide and Defend** routine in this lesson and Lesson 15. Students will use this routine again in Lesson 17. How can you support students in making a clear and concise claim, asking clarifying questions, and posing counterarguments? What about the routine felt successful? What are areas for growth next time?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

15. How can you tell if a point is a solution to a two-variable inequality?

Use the example if it helps with your explanation.

Lesson Practice A1.16

Lesson Summary

Two-variable inequalities can be written in standard form, slope-intercept form, or point-slope form. Their solutions are all of the ordered pairs that make the inequality true. Graphs can help us visualize all of these solutions. To check if an ordered pair is a solution symbolically, you can substitute the x- and y-values into the inequality to see if it makes the inequality true.

Here is an example of how you can determine if an ordered pair is a solution to an inequality:

Marco is making bracelets. Each bracelet needs to cost no more than $10. Planet beads cost $1 and oval beads cost $2. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

To check, Marco looked at the graph of this situation to see if the point (4,3) was in the solution region, but he wasn't sure. Marco substituted 4 and 3 into the inequality $2x + y \leq 10$:

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$$8 + (3) \leq 10$$
$$11 \leq 10 \quad \textit{False!}$$

That means Marco cannot make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

Lesson Practice
A1.16

Name: ..... Date: ..... Period: .....

1. Write at least three coordinate pairs that are solutions to the inequality $x \leq y$.

Problems 2–3: Tyler can spend up to $45 on hats and socks. A hat costs $10 and a pair of socks costs $2.50.

- h is the number of hats.
- s is the number of pairs of socks.

2. Which inequality represents this situation?

A. $10h + 2.50s > 45$
B. $10h + 2.50s < 45$
C. $10h + 2.50s \geq 45$
D. $10h + 2.50s \leq 45$

3. Explain how you know that $h = 2$ and $s = 1$ are solutions to this situation.

4. Marco is making a bracelet for his sister using beads that cost $0.50 and $0.75. He cannot spend more than $8.00 on the bracelet.

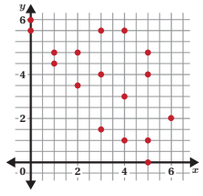
- x is the number of $0.50 beads.
- y is the number of $0.75 beads.

Marco says that the inequality $0.75y \geq 8 - 0.5x$ represents all the bracelets he can make. Do you agree with him? Explain your thinking.

5. Write an ordered pair for a point that is not a solution to $x + 3y \geq 24$.

6. This graph shows some solutions to $12 - 3x \leq 2y$.

Write an ordered pair for another point that is a solution to this inequality.



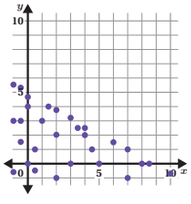
Lesson Practice
A1.16

Name: ..... Date: ..... Period: .....

Test Practice

7. This graph shows some solutions to $5x + 9y < 45$. Select *all* of the points that are also solutions.

- ☐ **A.** (1, 1)
- ☐ **B.** (4, 0)
- ☐ **C.** (10, 4)
- ☐ **D.** (0, 10)
- ☐ **E.** (6, -1)



Spiral Review

Problems 8–9: Write an inequality for each situation.

8. Duri will stay warm in her sleeping bag when the temperature is at least 30°F. Use t to represent temperatures at which Duri will stay warm.

9. Duri wants her backpack to weigh less than 45 pounds. Use w to represent the weight of the backpack.

Problems 10–11: Here is an equation: $6x + 2y = 36$.

10. For each value of x, determine the value of y.

x	y
2	
4	

11. Which equation represents the same relationship?

A. $y = 6 - 3x$
B. $y = 18 - 3x$
C. $y = 18 - \frac{1}{3}x$
D. $y = 6 - \frac{1}{3}x$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	2	1	MA.912.AR.2.8
	1, 3, 5–6	2	MA.912.AR.2.8
	4	3	MA.912.AR.2.8
Test Practice	7	1	MA.912.AR.2.8
Spiral Review			
	8	1	MA.912.AR.2.6
	9	2	MA.912.AR.2.6
Fluency	10	2	MA.912.AR.2.1
	11	2	MA.912.AR.1.2

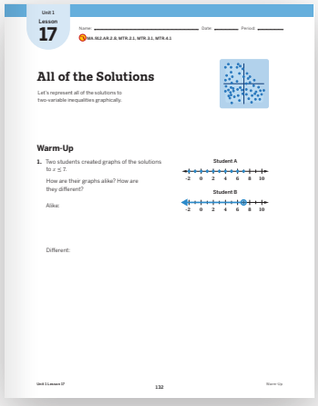
Unit 1

Lesson 17

All of the Solutions

Graphing Solutions to Two-Variable Inequalities

Let's represent all of the solutions to two-variable inequalities graphically.



Learning Outcomes

Today's Goal

1. **Goal:** Understand that the solutions to a two-variable linear inequality are represented on a graph as a half-plane (excluding the boundary in the case of a strict inequality).
2. **Goal:** Graph the solutions to a two-variable linear inequality, given the graph of its corresponding equation.
3. **Language Goal:** Describe a strategy for graphing the solutions to a two-variable inequality. **(Speaking and Listening)**

Students graph all of the solutions to a two-variable inequality, given the graph of its corresponding equation. They are introduced to the convention of using a dashed or solid **boundary line** and shading a *half-plane* to represent the solution set of a two-variable inequality. Students explore how to test points to determine which region is included in the solution set.

Coherence

Prior Learning

In Lessons 6–7, students graphed two-variable equations. In Lessons 10–12, students solved and graphed one-variable inequalities. In Lesson 16, students made sense of what it means for a point to be a solution to a two-variable inequality graphically and symbolically.

Future Learning

In Lessons 18–19, students will write and graph inequalities that represent real-world constraints and interpret their solutions in context.

Connections

- Student extend their understanding of two variable relationships to more complex situations involving graphing. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.8

Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.2	MA.912.AR.2.8

Vocabulary

New Vocabulary

boundary line

Review Vocabulary

half-plane

Lesson at a Glance

~ 45 min

MA.912.AR.2.8

Warm-Up

Pairs | 5 min

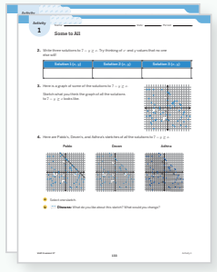
Students surface similarities and differences between some of the solutions and all of the solutions to one-variable inequalities, in preparation for representing all of the solutions to two-variable inequalities.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

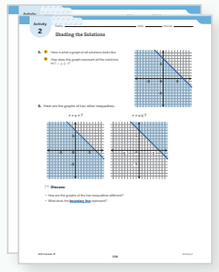
Students create and critique graphical representations of all the solutions of a two-variable inequality. MLR3: Critique, Correct, Clarify **(MTR.4.1, MTR.5.1)**



Activity 2

Pairs | 5 min

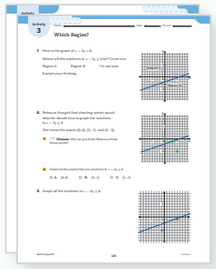
Students describe why using a dashed or solid **boundary line** and shading a *half-plane* is a precise way to graphically represent the solution set of a two-variable inequality. **(MTR.5.1)**



Activity 3

Pairs | 15 min

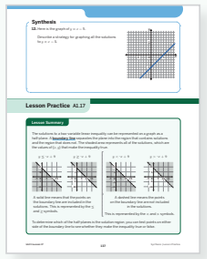
Students practice determining the solution set for a two-variable inequality, including which region to shade and whether the boundary line is included. **Routine:** Decide and Defend **(MTR.5.1)**



Synthesis

Whole Class | 5 min

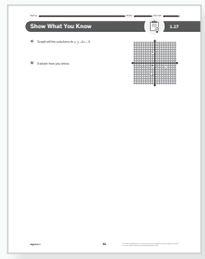
Students synthesize their understanding of how to graph all the solutions to a two-variable inequality. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating a graph of a two-variable inequality when given the graph of its corresponding equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students surface similarities and differences between some of the solutions and all of the solutions to one-variable inequalities, in preparation for representing all of the solutions to two-variable inequalities.

1 Launch

To surface prior knowledge, invite students to share where they have seen number lines like this before.

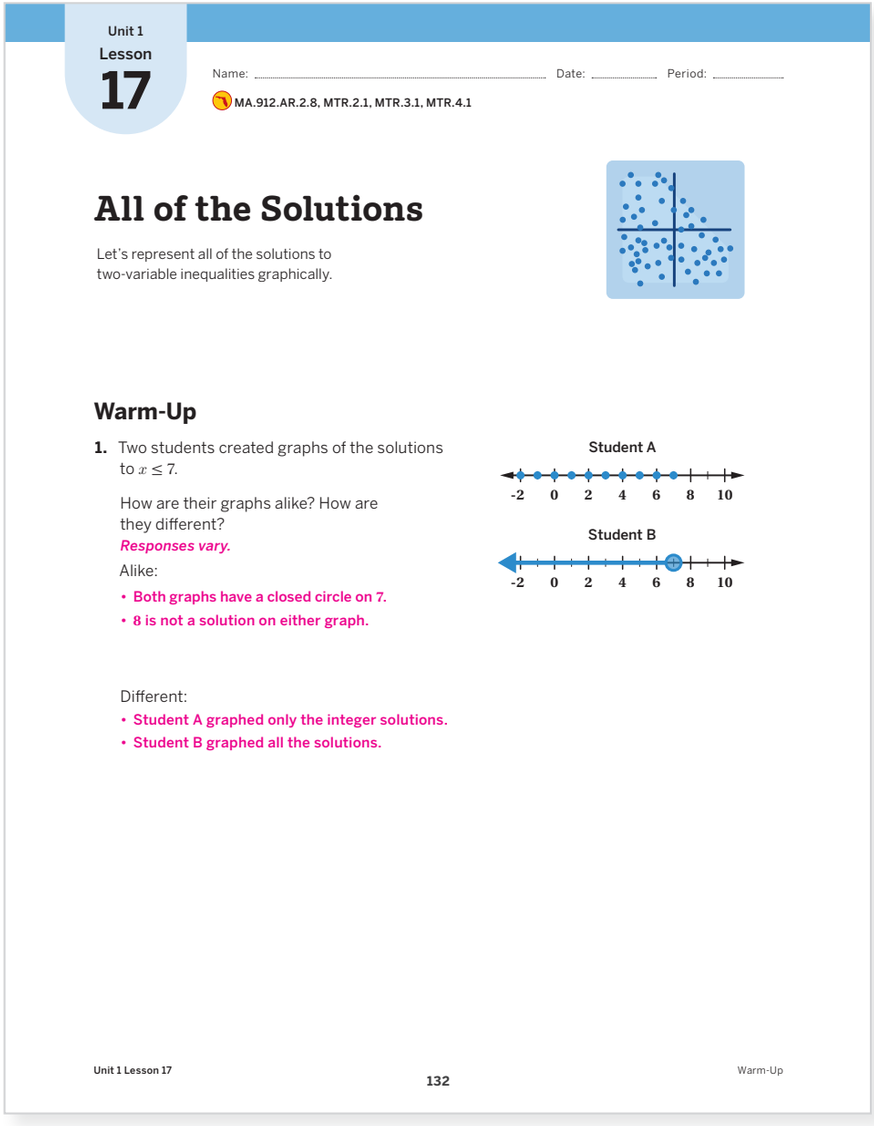
A **Accessibility: Memory and Attention** Consider inviting a student to read the inequality aloud or displaying an anchor chart about inequality symbols for students to reference throughout this lesson.

Use the Think-Pair-Share routine to help students shape their ideas about graphs of solutions of one-variable inequalities.

2 Connect

Invite students to share how the graphs are alike and different.

Consider asking, "Which graph shows some solutions? Which graph shows all the solutions?"



Unit 1
Lesson
17

Name: _____ Date: _____ Period: _____

MA.912.AR.2.8, MTR.2.1, MTR.3.1, MTR.4.1

All of the Solutions

Let's represent all of the solutions to two-variable inequalities graphically.

Warm-Up

1. Two students created graphs of the solutions to $x \leq 7$.

How are their graphs alike? How are they different?

Responses vary.

Alike:

- Both graphs have a closed circle on 7.
- 8 is not a solution on either graph.

Different:

- Student A graphed only the integer solutions.
- Student B graphed all the solutions.

Student A

Student B

Unit 1 Lesson 17 132 Warm-Up

Pairs | 10 min

Activity 1 Some to All

Purpose: Students create and critique graphical representations of all the solutions of a two-variable inequality. **(MTR.4.1, MTR.5.1)**

1 Launch

Invite students to read the inequality aloud and paraphrase it.

2 Monitor

D Differentiation

Look for students who:	Teacher Moves
Write points that aren't solutions.	**Support** Encourage students to rearrange the equation to help them revise their thinking.
Write points that include negative numbers, decimals, or fractions.	**Strengthen** Celebrate their interesting solutions. Invite them to share during the Connect.

A Accessibility: Conceptual Processing Invite them to say what each solution means in terms of two numbers summing to 7 or less (e.g., (4, 1): "4 plus 1 is equal to 5, which is less than or equal to 7").

To support students getting started, consider asking:

- "Where is the boundary between points that *are* solutions and points that *are not*?"
- "What solutions are missing? How could you show them on the graph?"

Look for creative strategies that are similar to Pablo's, Deven's, and Adhira's in Problem 4.

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in each graph. Discuss what is precise and imprecise about each graph. **(MTR.4.1)**

Consider asking:

- "Whose sketch was most like yours?"
- "What do you think should be included in the graph of all the solutions to a two-variable inequality?" **(MTR.5.1)**
- "Should only integers be included? Only points on the coordinate lines? Should fractional points be included?"

Activity 1

Name: Date: Period:

Some to All

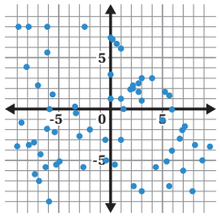
2. Write three solutions to $7 - y \geq x$. Try thinking of x- and y-values that no one else will! *Responses vary.*

Solution 1 (x, y)	Solution 2 (x, y)	Solution 3 (x, y)
(1, 6)	(5.2, -1.5)	(0, 3.4)

3. Here is a graph of some of the solutions to $7 - y \geq x$.

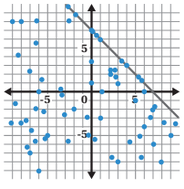
Sketch what you think the graph of all the solutions to $7 - y \geq x$ looks like.

Responses vary. See Problem 4 for three examples.

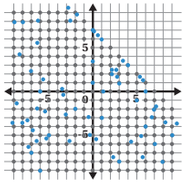


4. Here are Pablo's, Deven's, and Adhira's sketches of *all* the solutions to $7 - y \geq x$.

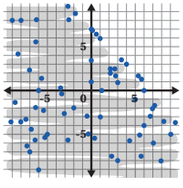
Pablo



Deven



Adhira



a. Select *one* sketch.

b. **Discuss:** What do you like about this sketch? What would you change?

Responses vary.

- I like that Pablo included a line that creates a boundary between the points that are a solution and the points that are not a solution.
- I like that Deven added many more integer solutions.
- I would add solutions with decimals to Deven's sketch.
- I would add a boundary line to Adhira's sketch, like Pablo did. That would help make her shading more precise.

Unit 1 Lesson 17 133 Activity 1

Pairs | 5 min

Activity 2 Shading the Solutions

Purpose: Students describe why using a dashed or solid **boundary line** and shading a *half-plane* is a precise way to graphically represent the solution set of a two-variable inequality.

1 Launch

Consider sharing that students will view a method that some mathematicians have agreed is a precise way of representing all of the solutions to a two-variable inequality.

Consider asking:
- "What is precise about this graph?"
- "What surprised you?"
- "How is this graph like your graph? How is it different?"

2 Monitor

D **Differentiation: (Stretch)** Invite students to discuss what they think the graph of $x + y \neq 7$ would look like with a partner.

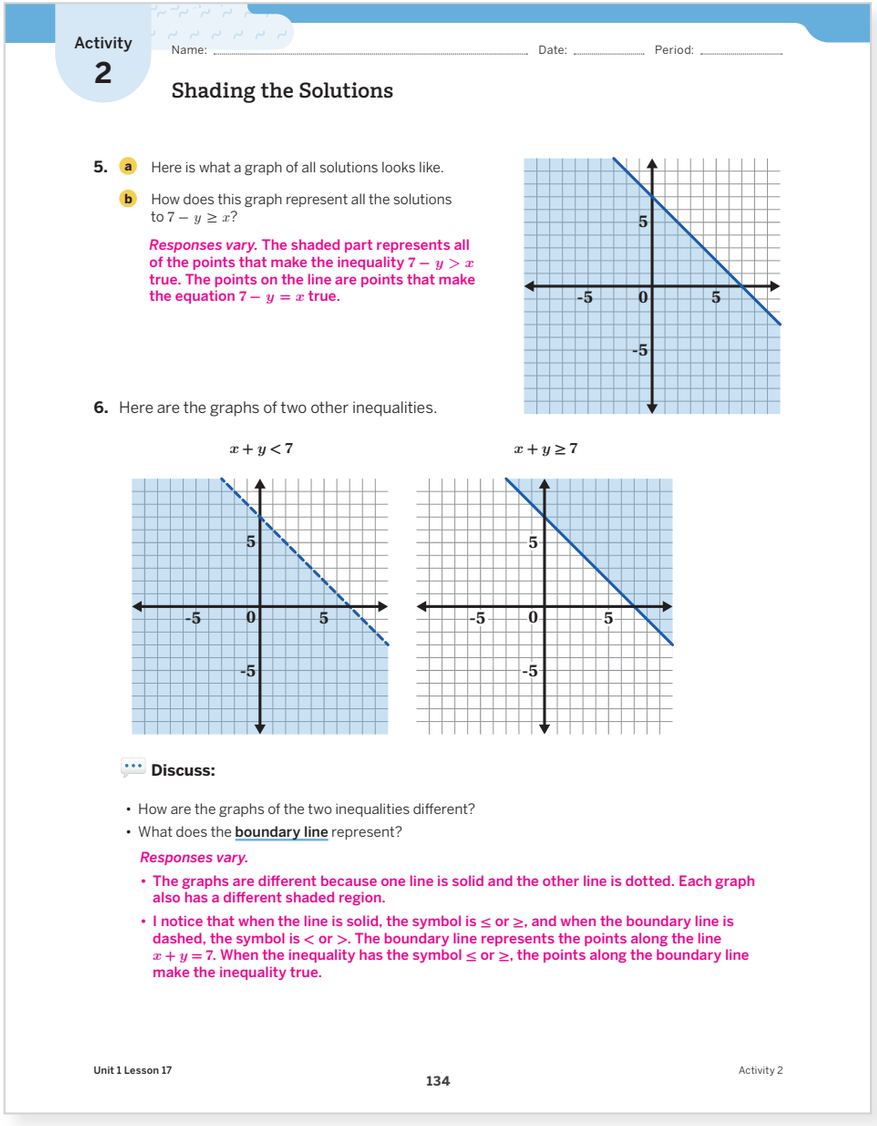
3 Connect

Invite students to share connections between the graph and the inequalities after exploring repeated cases. **(MTR.5.1)**

To surface the Key Takeaway, consider asking:
- "Why is there a dashed line when $x + y > 7$ or $x + y < 7$? What might it mean when the line is dashed?"
- "How does changing the shaded region change the inequality?"
- "How does changing the graph from a solid line to a dashed line change the inequality?"

A **Accessibility: Conceptual Processing** As students reason abstractly, consider grounding the conversation in the context of two numbers that sum to more than, less than, or equal to 7.

Note: The term *half-plane* (a set of points in the coordinate plane on one side of a boundary line) may help students understand that any boundary line splits the coordinate plane into two equal halves. We use *region* instead of *half-plane* in this lesson to be consistent with solving systems of inequalities in Unit 4. The solutions to systems of inequalities exist in a region that is rarely a half-plane.



Activity 2

Name: Date: Period:

Shading the Solutions

5. a Here is what a graph of all solutions looks like.

b How does this graph represent all the solutions to $7 - y \geq x$?

Responses vary. The shaded part represents all of the points that make the inequality $7 - y > x$ true. The points on the line are points that make the equation $7 - y = x$ true.

6. Here are the graphs of two other inequalities.

$x + y < 7$

$x + y \geq 7$

Discuss:
- How are the graphs of the two inequalities different?
- What does the **boundary line** represent?

Responses vary.
- The graphs are different because one line is solid and the other line is dotted. Each graph also has a different shaded region.
- I notice that when the line is solid, the symbol is $\leq$ or $\geq$, and when the boundary line is dashed, the symbol is $<$ or $>$. The boundary line represents the points along the line $x + y = 7$. When the inequality has the symbol $\leq$ or $\geq$, the points along the boundary line make the inequality true.

Unit 1 Lesson 17 134 Activity 2

Key Takeaway: One way mathematicians often represent all of the solutions to two-variable inequalities is by:
- Using a solid or dashed **boundary line** to indicate when the line is included in ($\geq$ or $\leq$) or excluded from ($>$ or $<$) the solution set.
- Shading the region where the solutions are located.

Pairs | 15 min

Activity 3 Which Region?

Purpose: Students practice determining the solution set for a two-variable inequality, including which region to shade and whether the boundary line is included.

Short on time? Consider inviting students to complete fewer graphs in Problem 11.

1 Launch

Invite students to think independently and then respond with a partner.

ELL **English Language Learners** Invite students to share what the term *region* reminds them of. (e.g., "When I hear *region*, I think . . .") **(Reading and Listening)**

Consider sharing that the goal of this activity is for students to surface strategies for deciding which region to shade.

2 Monitor

Listen for student ideas about why Rebecca chose those points to share during the Connect.

D **Differentiation**

Look for students who:	Teacher Moves
Need support getting started.	**Support** Invite a group of students to test the first point together. Encourage students to describe the process in their own words before completing more.
Substitute, but need support evaluating.	**Support** Celebrate their correct substitution. Invite students to use a calculator to help evaluate the expression.
Substitute a point into the inequality and evaluate, but aren't sure how to interpret the statement (e.g., $0 \geq 6$).	**Support** Invite students to read the inequality statement aloud (e.g., "Zero is greater than or equal to six.") to help them determine if the statement is true or false.
Use mathematically precise terms to describe their thinking.	**Stretch** Celebrate the mathematical brilliance of students attending to precision in their explanation by using the term *boundary line*.

Invite students to fill in the points on the graph that are solutions to the inequality.

D **Differentiation: (Support)** If students wonder why only two points are filled in, consider asking: "How can we show if a point is included or not included in the solution set? What about a line?"

Activity 3 continued >

Activity 3

Name: Date: Period:

Which Region?

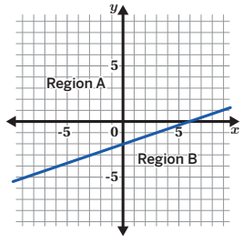
7. Here is the graph of $x - 3y = 6$.

Where will the solutions to $x - 3y \geq 6$ be? Circle one.

Region A (Region B) I'm not sure

Explain your thinking.

Explanations vary. When I plugged (0, 0) into the inequality, I got a false statement. Since (0, 0) is in Region A, I think Region B is where the solutions will be.

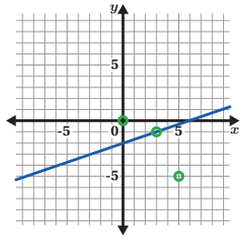


8. Rebecca thought that checking points would help her decide how to graph the solutions to $x - 3y \geq 6$.

She chose the points (0, 0), (3, -1), and (5, -5).

a. **Discuss:** Why do you think Rebecca chose these points?

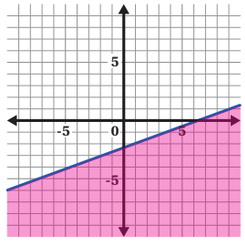
Responses vary. She chose one point above the line, one below the line, and one on the line.



b. Select *all* the points that are solutions to $x - 3y \geq 6$.

☐ A. (0, 0) ☑ B. (3, -1) ☑ C. (5, -5)

9. Graph *all* the solutions to $x - 3y \geq 6$.



Unit 1 Lesson 17 135 Activity 3

Pairs | 15 min

Activity 3 Which Region? (continued)

Purpose: Students practice determining the solution set for a two-variable inequality, including which region to shade and whether the boundary line is included.

Short on time? Consider inviting students to complete fewer graphs in Problem 11.

2 Monitor

Select and sequence a variety of student ideas to share during the Connect, such as:

- Relating the inequality symbol to the dashed line and understanding what that means about whether the points on the line are included as solutions.
- Inferring the solutions must be below the boundary line given that a point above is not a solution.
- Testing a point below the boundary line to be sure the solutions in that region make the inequality true.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

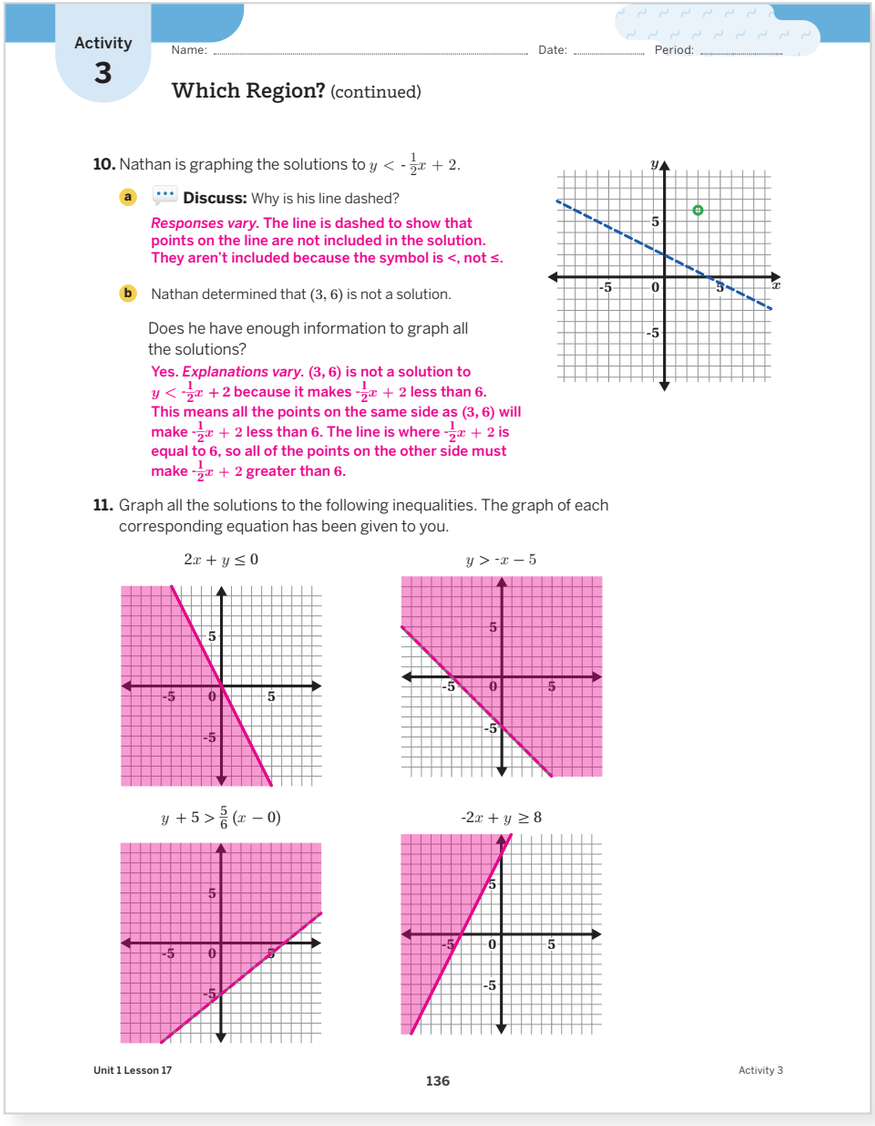
ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with you because . . .
- _____'s idea makes sense to me because ____.
- Another strategy would be _____ because _____.

(Writing and Speaking)

To surface the Key Takeaway, consider asking:

- "How can testing points help us know where to shade?"
- "How many points do we need to test to know where the solutions are located?"
- "What are the pros and cons of testing just one point versus testing a point on each side of the boundary line?"



Activity 3

Name: Date: Period:

Which Region? (continued)

10. Nathan is graphing the solutions to $y < -\frac{1}{2}x + 2$.

a. **Discuss:** Why is his line dashed?

Responses vary. The line is dashed to show that points on the line are not included in the solution. They aren't included because the symbol is <, not ≤.

b. Nathan determined that $(3, 6)$ is not a solution.

Does he have enough information to graph all the solutions?

Yes. *Explanations vary.* $(3, 6)$ is not a solution to $y < -\frac{1}{2}x + 2$ because it makes $-\frac{1}{2}x + 2$ less than 6. This means all the points on the same side as $(3, 6)$ will make $-\frac{1}{2}x + 2$ less than 6. The line is where $-\frac{1}{2}x + 2$ is equal to 6, so all of the points on the other side must make $-\frac{1}{2}x + 2$ greater than 6.

11. Graph all the solutions to the following inequalities. The graph of each corresponding equation has been given to you.

$2x + y \leq 0$

$y > -x - 5$

$y + 5 > \frac{5}{6}(x - 0)$

$-2x + y \geq 8$

Unit 1 Lesson 17 136 Activity 3

Key Takeaway: Testing a point in a region can determine if all of the points in that region are included in the solution set to a two-variable inequality. To determine if the line is included in the solution set, students can examine the inequality symbol or test a point on the line.

Whole Class | 5 min

Synthesis

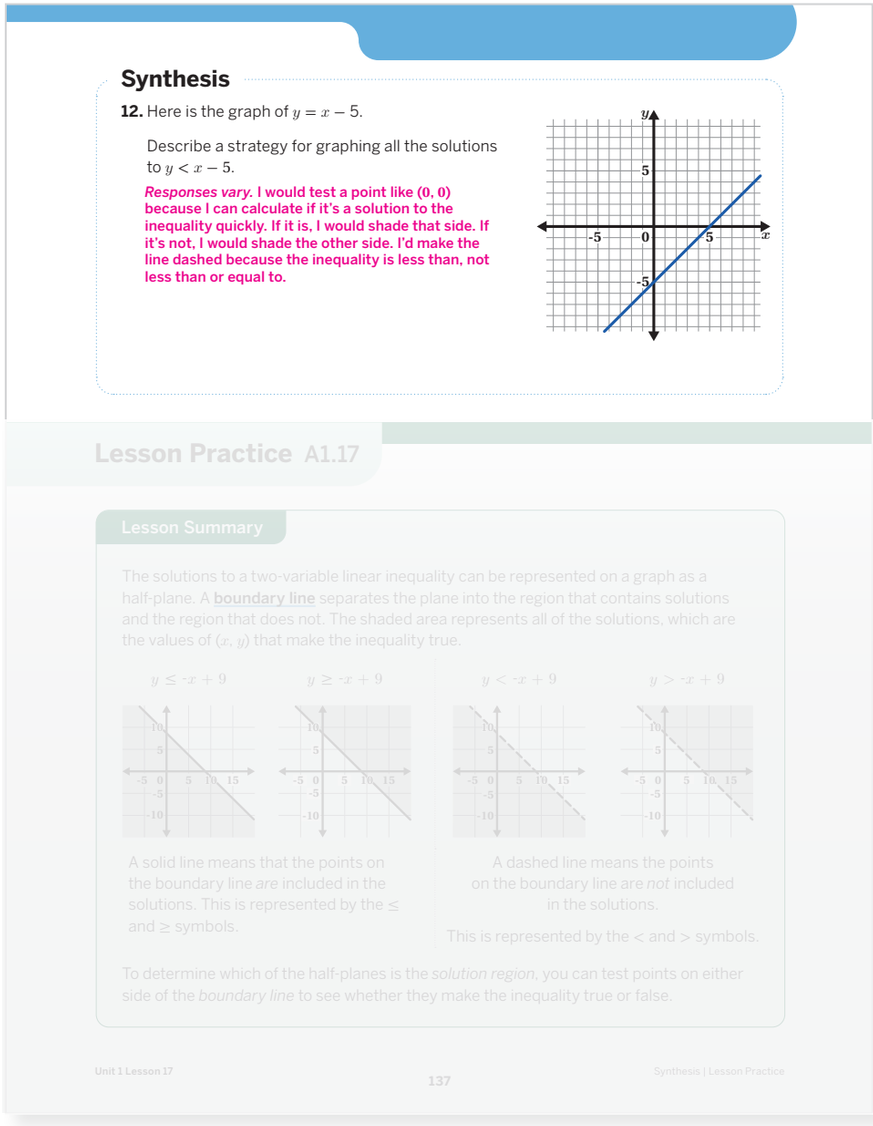
Purpose: Students synthesize their understanding of how to graph all the solutions to a two-variable inequality.

Invite students to respond independently and then share their thinking with a partner.

Share a variety of strategies for determining:
- Whether to use a dashed or solid boundary line.
- How many points to test.
- Which points to test.
- How to read an evaluation statement (e.g., $0 > 6$ is false, so the solutions are not in that region)
- Which region to shade.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for graphing two-variable inequalities. Display for students to reference and add to throughout the rest of this unit and Unit 5 when students learn about graphing systems of inequalities.

Consider asking:
- "What makes sense to you about each strategy? What does *not* make sense?"
- "What connections do you see between your classmates' strategies?"
- "What questions do you still have?"



Synthesis

12. Here is the graph of $y = x - 5$.

Describe a strategy for graphing all the solutions to $y < x - 5$.

Responses vary. I would test a point like (0, 0) because I can calculate if it's a solution to the inequality quickly. If it is, I would shade that side. If it's not, I would shade the other side. I'd make the line dashed because the inequality is less than, not less than or equal to.

Lesson Practice A1.17

Lesson Summary

The solutions to a two-variable linear inequality can be represented on a graph as a half-plane. A boundary line separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.

$y \leq -x + 9$ $y \geq -x + 9$ $y < -x + 9$ $y > -x + 9$

A solid line means that the points on the boundary line *are* included in the solutions. This is represented by the ≤ and ≥ symbols.

A dashed line means the points on the boundary line are *not* included in the solutions. This is represented by the < and > symbols.

To determine which of the half-planes is the *solution region*, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

Unit 1 Lesson 17 137 Synthesis | Lesson Practice

Lesson Takeaway: The boundary line splits the coordinate plane into two regions. Substituting a point from one region or the other can help determine which region contains the solutions to the inequality and where to shade. To determine if the boundary line is included, examine the inequality symbol or test a point on the line.

Independent | 5 min

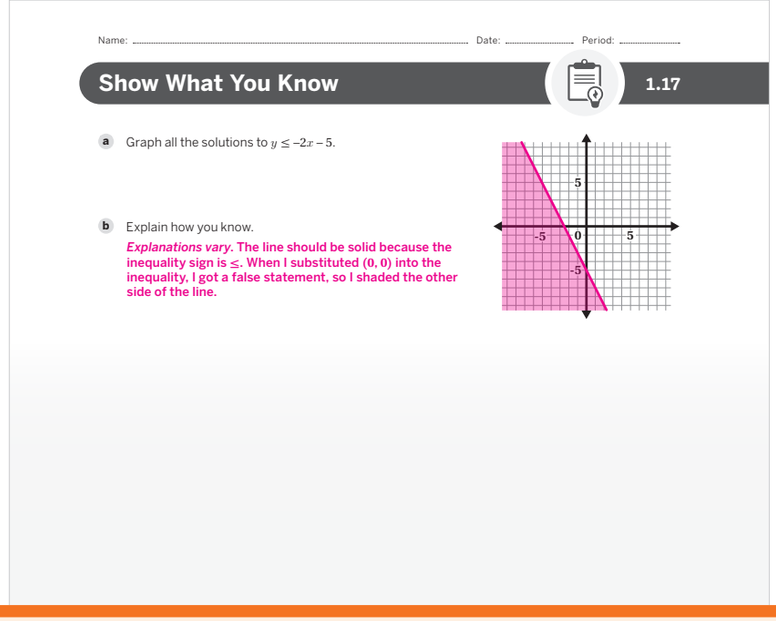
Show What You Know

Purpose: Students demonstrate their understanding by creating a graph of a two-variable inequality when given the graph of its corresponding equation.

Today's Goals

1. **Goal:** Understand that the solutions to a two-variable linear inequality are represented on a graph as a half-plane (excluding the boundary in the case of a strict inequality).
2. **Goal:** Graph the solutions to a two-variable linear inequality given the graph of its corresponding equation.
3. **Language Goal:** Describe a strategy for graphing the solutions to a two-variable inequality. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.17

a. Graph all the solutions to $y \leq -2x - 5$.

b. Explain how you know.

Explanations vary. The line should be solid because the inequality sign is ≤. When I substituted (0, 0) into the inequality, I got a false statement, so I shaded the other side of the line.

D Differentiation Use after Lesson 17

Support

Provide targeted intervention for students by using these resources.

If students need support graphing solutions to a two-variable inequality:

» **Respond to Student Thinking:**

- Use the ***Graphing Solutions of Two-Variable Inequalities*** **Mini-Lesson**.
- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to play a game with a partner: the first student graphs a line, then the second student writes the equation for that line. After each round, the partners can switch roles.

Professional Learning

Take a moment to reflect on how students' understanding of the meaning of *solution* has developed across this unit. In the first three lessons, students developed early concepts of *solution* as one value that makes an equation true. How has their definition of *solution* expanded over the last few lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Here is the graph of $y = x - 5$.

Describe a strategy for graphing all the solutions to $y < x - 5$.

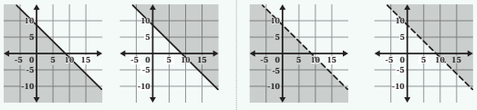
Responses vary. I would test a point like (0, 0) because I can calculate if it's a solution to the inequality quickly. If it is, I would shade that side. If it's not, I would shade the other side. I'd make the line dashed because the inequality is less than, not less than or equal to.

Lesson Practice A1.17

Lesson Summary

The solutions to a two-variable linear inequality can be represented on a graph as a half-plane. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.

$y \leq -x + 9$ $y \geq -x + 9$ $y < -x + 9$ $y > -x + 9$



A solid line means that the points on the boundary line *are* included in the solutions. This is represented by the ≤ and ≥ symbols.

A dashed line means the points on the boundary line are *not* included in the solutions.

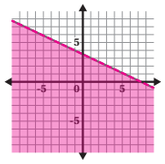
This is represented by the < and > symbols.

To determine which of the half-planes is the *solution region*, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

Lesson Practice A1.17

Name: ..... Date: ..... Period: .....

1. Graph all the solutions to $x + 2y < 7$. Explain your thinking.

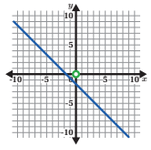


Explanations vary. The line is dashed because the inequality sign is <. When I plugged (0, 0) into the inequality, I got a true statement, so I shaded that side of the line.

2. Here is an inequality: $y \leq -x - 2$.

- Ada graphed the equation $y = -x - 2$.
- Ada noticed that (0, 0) is not a solution to $y \leq -x - 2$.

How can Ada use this information to graph the solutions to this inequality?

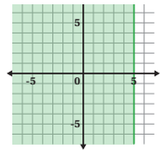


Explanations vary. Since the inequality symbol in this problem is ≤, the line should be solid. Ada knows that (0, 0) is not a solution, so Ada should shade the side of the line that does not include that point.

3. Brianna is creating a graph for the inequality $y + 2 < \frac{2}{5}(x - 0)$. She says that since the inequality has a greater-than symbol, she should shade the region above the line $y + 2 = \frac{2}{5}(x - 0)$. Is Brianna correct? Explain your thinking.

No. Explanations vary. If I plug in a point that's above the line $y + 2 = \frac{2}{5}(x - 0)$, such as (2, 0), it makes the inequality false.

4. Which inequality is shown on the graph?



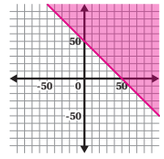
A. $y \leq 5$

B. $y \geq 5$

C. $x \geq 5$

(D.) $x \leq 5$

Problems 5–6: A food truck only sells hot dogs and hamburgers. They want to sell 50 items or more each day.



- x represents the number of hot dogs sold.
- y represents the number of hamburgers sold.

5. Which inequality represents this situation?

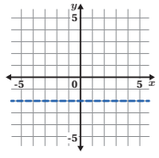
A. $x + y > 50$ B. $x + y < 50$

(C.) $x + y \geq 50$ D. $x + y \leq 50$

Lesson Practice A1.17

Name: ..... Date: ..... Period: .....

6. Complete the graph so that it represents all the solutions to the inequality for this situation.

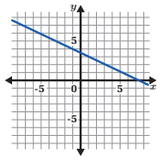


7. Lucy started to graph the inequality $y > -2$ by graphing a dashed line at $y = -2$. How might Lucy decide where to shade?

Responses vary. Pick a point above or below the line. Plug that point into the inequality and see if it makes the inequality true. If it does, shade the region that point is in. If it doesn't, shade the other region.

Test Practice

8. Here is a graph of the equation $x + 2y = 7$.



Which of these points is a solution to the inequality $x + 2y < 7$?

(A.) (0, 0)

B. (10, 0)

C. (7, 0)

D. (0, 7)

Spiral Review

9. The equation of line a is $y = -2x - 1$. Select *all* the points that are on line a.

☑ A. (-2, 3) ☐ B. (0, 4) ☑ C. (1, -3)

☐ D. (-1, -3) ☐ E. (2, 5)

10. Lola can spend up to $15 on pens and notebooks. A pen costs $2 and a notebook costs $1.50. Using p for the number of pens and n for the number of notebooks, write an inequality that represents this situation. $2p + 1.5n \leq 15$ (or equivalent)

11. A golf ball weighs 1.6 ounces and an empty metal bucket weighs 12 ounces. Neel adds golf balls one at a time to the empty metal bucket. How many golf balls will be in the bucket when the total weight is 20 ounces? 5 golf balls

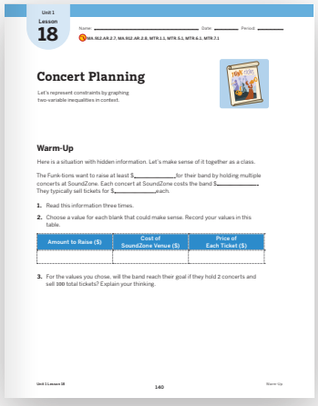
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4	1	MA.912.AR.2.8
	2–3	2	MA.912.AR.2.8
	5–7	3	MA.912.AR.2.8
Test Practice	8	2	MA.912.AR.2.8
Spiral Review			
Fluency	9	2	MA.912.AR.2.4
	10	1	MA.912.AR.2.6
	11	2	MA.912.AR.2.5

Unit 1

Lesson

18


Unit 1
Lesson
18
Name _____ Date _____ Period _____
MA.912.AR.2.7, MA.912.AR.2.8, MTR.1.1, MTR.5.1, MTR.6.2, MTR.7.1
Concert Planning
Let's represent constraints by graphing two-variable inequalities in context.
Warm-Up
Here is a situation with hidden information. Let's make sense of it together as a class.
The Funk-tions want to raise at least $_____ for their band by holding multiple concerts at SoundZone. Each concert at SoundZone costs the band $_____. They typically sell tickets for $_____ each.
1. Read this information three times.
2. Choose a value for each blank that could make sense. Record your values in this table.
Amount to Raise ($) | Cost of SoundZone Venue ($) | Price of Each Ticket ($)
3. For the values you chose, will the band reach their goal if they hold 2 concerts and sell 300 total tickets? Explain your thinking.
Unit 1 Lesson 18
140
Warm-Up

Concert Planning

Graphing Two-Variable Inequalities in Context

Let's represent constraints by graphing two-variable inequalities in context.

Learning Outcomes

Today's Goal

1. **Goal:** Graph the solutions to a two-variable linear inequality and interpret them in context.
2. **Goal:** Write a two-variable inequality to represent a situation in context.
3. **Language Goal:** Interpret what an inequality and its solutions mean in context. **(Reading and Writing)**

Students extend their understanding of how to determine and interpret the solutions to two-variable linear inequalities. Students persevere in making sense of two-variable inequalities in context and graph their solutions, considering questions that could be answered using the graphs. Students also write and graph their own inequalities and use the graphs to make decisions about concert venues. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 16, students modeled constraints of a situation using two-variable linear inequalities. In Lesson 17, students graphed the solutions of two-variable linear inequalities given the graph of the boundary line.

Future Learning

In Lesson 19, students will use two-variable inequalities and their solutions to model real-world constraints. In Unit 4, students will extend their study of two-variable inequalities to systems of linear equations and inequalities.

Connections

- Students apply the solution set of two-variable linear inequalities to solve problems involving the planning for a concert within financial constraints. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.2.7
Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.

MA.912.AR.2.8
Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.2.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.1 **MA.912.AR.2.2**	

Lesson at a Glance

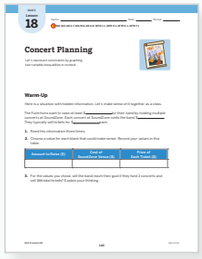
~ 45 min

MA.912.AR.2.7, MA.912.AR.2.8

Warm-Up

Whole Class | 10 min

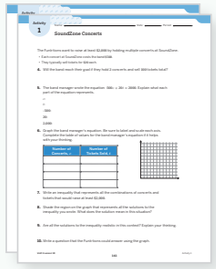
Students use MLR6: Three Reads to make sense of a context about fundraising concerts that will be used throughout this lesson. **(MTR.1.1)**



Activity 1

Pairs | 10 min

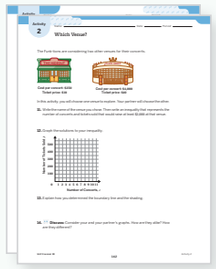
Students model the relationship between ticket sales, concerts held, and money raised using a two-variable inequality and its graph. **(MTR.7.1)**



Activity 2

Pairs | 15 min

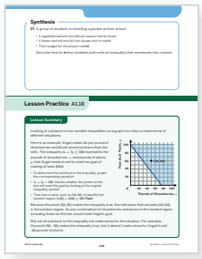
Students create two-variable linear inequalities to model the relationship between ticket sales, concerts held, and money raised for two different venues. **(MTR.4.1, MTR.7.1)**



Synthesis

Whole Class | 5 min

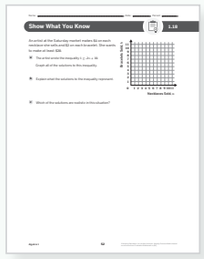
Students synthesize their understanding of how to represent all of the solutions to a two-variable linear inequality graphically.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by graphing the solutions to an inequality, interpreting them in context, and deciding which solutions are realistic.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 10 min

Warm-Up

Purpose: Students use MLR6: Three Reads to make sense of a context about fundraising concerts that will be used throughout this lesson. **(MTR.1.1)**

1 Launch

To surface connections, consider asking, "Have you or someone you know ever been to a concert? What concert was it?"

Direct students' attention to the situation with the hidden information and share with students that they will be using the Three Reads routine to make sense of the situation.

MLR **MLR6: Three Reads** Invite students to read the text three times, each with a particular focus, to support students in making sense of the situation. **(MTR.1.1)**

ELL **English Language Learners** After the read-aloud, invite students to paraphrase what the situation means. Clarify any words they don't know. **(Reading)**

Invite students to read the situation aloud, this time focusing on the quantities that could fill the blanks. Then, discuss the prompt.

Invite students to share their ideas about what would be reasonable quantities for each of these variables. Consider asking questions like:

- "What could be a reasonable price for a concert ticket?"
- "How much money do you think it costs to host a concert?"
- "What's a reasonable goal for raising money with a concert?"

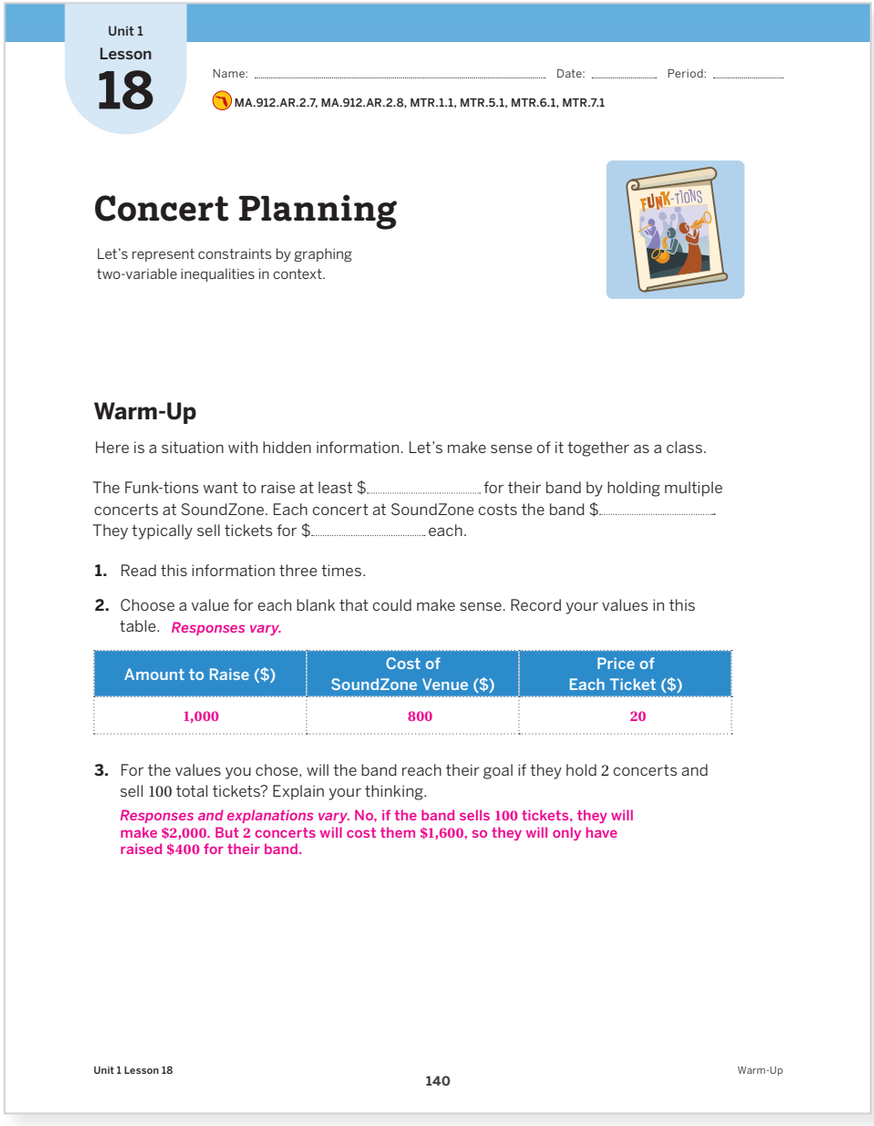
Direct students' attention to the situation again with a focus on the Funk-tions' goal. After the third read, invite students to discuss the question with a partner, then as a class.

2 Connect

Read the prompt. Consider asking:

- "How can you determine how much money the band will make?"
- "How can you determine how much money the band will owe?"
- "How did you decide whether the band reached their goal?"

Note: If it doesn't come up naturally, consider asking, "For which values will the Funk-tions meet their goal? For which values will the Funk-tions not reach their goal?" If the Funk-tions want to raise $1,000, they can meet their goal when the venue costs $200 and the tickets cost $15 each. If the Funk-tions want to raise $2,000, they cannot meet their goal when the venue costs $100 and the tickets cost $20 each.



Unit 1
Lesson
18

Name: _____ Date: _____ Period: _____

MA.912.AR.2.7, MA.912.AR.2.8, MTR.1.1, MTR.5.1, MTR.6.1, MTR.7.1

Concert Planning

Let's represent constraints by graphing two-variable inequalities in context.

Warm-Up

Here is a situation with hidden information. Let's make sense of it together as a class.

The Funk-tions want to raise at least $_____ for their band by holding multiple concerts at SoundZone. Each concert at SoundZone costs the band $_____. They typically sell tickets for $_____ each.

1. Read this information three times.
2. Choose a value for each blank that could make sense. Record your values in this table. *Responses vary.*

Amount to Raise ($)	Cost of SoundZone Venue ($)	Price of Each Ticket ($)
1,000	800	20

3. For the values you chose, will the band reach their goal if they hold 2 concerts and sell 100 total tickets? Explain your thinking.

Responses and explanations vary. No, if the band sells 100 tickets, they will make $2,000. But 2 concerts will cost them $1,600, so they will only have raised $400 for their band.

Unit 1 Lesson 18 140 Warm-Up

Pairs | 10 min

Activity 1 SoundZone Concerts

Purpose: Students model the relationship between ticket sales, concerts held, and money raised using a two-variable inequality and its graph. **(MTR.7.1)**

1 Launch

Direct students' attention to the information about the situation.

Consider asking, "Have you ever raised money for anything?"

2 Monitor

Look for strategies students use during this activity:

D Differentiation *(Problems 6–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are some combinations of concerts and tickets sold that you know will help the band reach their goal? What's a combination you know doesn't work? How can you represent these combinations on the graph?"
Need support scaling the graph.	**Support** Encourage students to make a table of values to help them make sense of which quantities would be helpful to see on the graph.
Substituting values into the band manager's equation or their inequality. *(Problems 6–8)*	**Strengthen** Consider asking, "How can you tell if the points on the line are included in the solution set?"
Testing points from each region above and below the boundary line.	**Stretch** Invite students to use coloring tools to plot the points representing these combinations on the graph.

To encourage creativity, invite students to write a question that they think no other group will think of.

3 Connect

Direct students' attention to the table and graph in Problem 6. Invite students to input some points and discuss whether they are solutions to the equation, inequality, and/or situation. Some points you may consider discussing are: $(2, 100)$, $(2.5, 300)$, $(4, 200)$, $(-1, 100)$.

To surface the Key Takeaway, consider asking:

- "Where on the graph do you see the combinations that raise $2,000? More than $2,000?"
- "What does it mean for a point to be a solution to an inequality? What does it mean in context?" **(MTR.7.1)**
- "Why is a shaded region on a graph a precise way to show all of the solutions to a two-variable inequality?"

Activity 1

Name: Date: Period:

SoundZone Concerts

The Funk-tions want to raise at least $2,000 by holding multiple concerts at SoundZone.

- Each concert at SoundZone costs the band $500.
- They typically sell tickets for $20 each.

4. Will the band reach their goal if they hold 2 concerts and sell 100 tickets total?

No. Explanations vary. If they hold 2 concerts and pay SoundZone $500 for each, then the band will owe $1,000. If they sell 100 tickets at $20 each, their income will be $2,000. After they pay SoundZone, the band will only have $1,000 left.

5. The band manager wrote the equation $-500c + 20t = 2000$. Explain what each part of the equation represents. *Responses vary.*

c: the number of concerts at SoundZone

t: the number of tickets sold

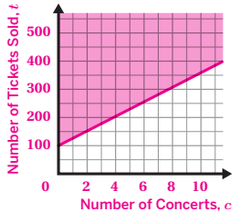
-500: the fee for the venue for each concert

20: the price of each ticket

$2{,}000$: the profit the band wants to make

6. Graph the band manager's equation. Be sure to label and scale each axis. Complete the table of values for the band manager's equation if it helps with your thinking.

Number of Concerts, c	Number of Tickets Sold, t
1	125
2	150
4	200
6	250



7. Write an inequality that represents all the combinations of concerts and tickets that would raise at least $2,000.

$-500c + 20t \geq 2000$ (or equivalent)

8. Shade the region on the graph that represents all the solutions to the inequality you wrote. What does the solution mean in this situation?

Response shown on graph. *Explanations vary.* The shaded region has all the combinations of concerts and ticket prices that would allow the band to make at least $2,000.

9. Are all the solutions to the inequality realistic in this context? Explain your thinking.

Responses vary. Only positive whole number solutions in Quadrant 1 are realistic because negative numbers and decimals don't make sense in this situation.

10. Write a question that the Funk-tions could answer using the graph.

Responses vary. If the venue holds 300 people, how many concerts do we need to hold?

Unit 1 Lesson 18 141 Activity 1

Key Takeaway: The graphs of two-variable linear inequalities show all of the solutions that make the inequality true. While these graphs help us know which points make the inequality true symbolically, they may also contain some solutions that do not make sense for the situation. **(MTR.7.1)**

Pairs | 15 min

Activity 2 Which Venue?

Purpose: Students create two-variable linear inequalities to model the relationship between ticket sales, concerts held, and money raised for two different venues.

Short on time? Consider inviting students to discuss their response to Problem 13 with a partner instead of writing.

1 Launch

Direct students' attention to the two venues and the prompt.

Invite students to think about the prompt independently, then share their thinking with a partner.

Consider asking, "Why might a band choose to play at the Hideout? Why might a band choose to play at the Palace Arena?"

Note: If it doesn't come up naturally, consider inviting students to discuss how each venue helps the Funk-tions meet their goal in different ways.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

2 Monitor

D **Differentiation** *(Problem 11–13)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit the band manager's equation from Problem 5. Consider asking, "For the venue you chose, what values change? What values stay the same? How do you know?"
Write an inequality with the symbol reversed or strict. *(Problem 11)*	**Strengthen** Celebrate the inequality students have written. Invite students to test points in their inequality and revise their inequality symbol accordingly.
Write an inequality that doesn't match the situation (e.g., switches the values and the variables that count them). *(Problem 11)*	**Strengthen** Celebrate the inequality students have written. Invite students to test possible solution values in their inequality using substitution, and consider asking, "Which value should be substituted into which variable? Why?" Invite students to revise their inequality.
Reason clearly about how they determined the boundary line and the shading. *(Problem 13)*	Celebrate the clarity of this student thinking and invite these students to share during the Connect.

Listen to group conversations for how students compare the structure of each partner's inequality in Problem 14. **(MTR.4.1)**

Activity 2 continued >

Activity 2

Name: Date: Period:

Which Venue?

The Funk-tions are considering two other venues for their concerts.



Cost per concert: $250
Ticket price: $10



Cost per concert: $4,000
Ticket price: $40

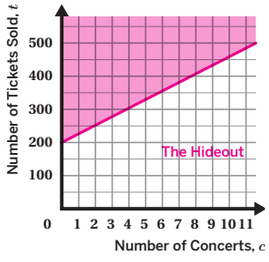
In this activity, you will choose one venue to explore. Your partner will choose the other.

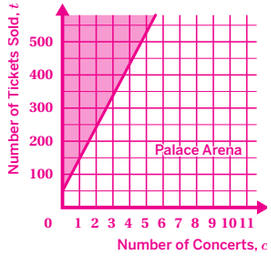
11. Write the name of the venue you chose. Then write an inequality that represents the number of concerts and tickets sold that would raise at least $2,000 at that venue.

The Hideout: $-250c + 10t \geq 2000$

Palace Arena: $-4000c + 40t \geq 2000$

12. Graph the solutions to your inequality.





13. Explain how you determined the boundary line and the shading.

Explanations vary.

- After I graphed the line, I tested a point above the line and a point below the line to decide where to shade. When I tested (4, 100), I got a false statement. When I tested (4, 500), I got a true statement.

14. **Discuss:** Consider your and your partner's graphs. How are they alike? How are they different?

Responses vary.

- The graphs are alike because both boundary lines have a positive slope. Both solution regions are also above the boundary line.
- The Palace Arena graph has a smaller y-intercept and a steeper slope than the Hideout.

Unit 1 Lesson 18 142 Activity 2

Pairs | 15 min

Activity 2 Which Venue? (continued)

Purpose: Students create two-variable linear inequalities to model the relationship between ticket sales, concerts held, and money raised for two different venues.

Short on time? Consider inviting students to discuss their response to Problem 13 with a partner instead of writing.

2 Monitor

D Differentiation *(Problems 15–16)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 15)*	**Support** Encourage students to revisit their work from Problem 11. Consider asking, "What does each value represent in your inequality for each venue?"
Use strategies to test values for each venue, such as substitution into the inequality for each venue or checking the graph of each location to evaluate if the given points are solutions. *(Problem 15)*	**Strengthen** Celebrate the strategy students use and consider asking them to connect it to another representation (i.e., if students use the graph, ask, "How could you test this point using the inequality?"). Invite students to share their strategy during the Connect.
Offer a clear recommendation for the preferred venue for each situation. *(Problem 15)*	**Strengthen** Invite students to share their recommendation and reasoning during the Connect.
Don't think Victor is correct about the point $(2.5, 500)$. *(Problem 16)*	**Support** Consider asking, "What would 2.5 as a value of x mean in this context? Why might Victor think that is not a realistic value for x in context?"

ELL English Language Learners Provide sentence frames to help students explain their reasoning (e.g., "We would choose the venue _____ because _____.")

3 Connect

Direct students' attention to the graphs.

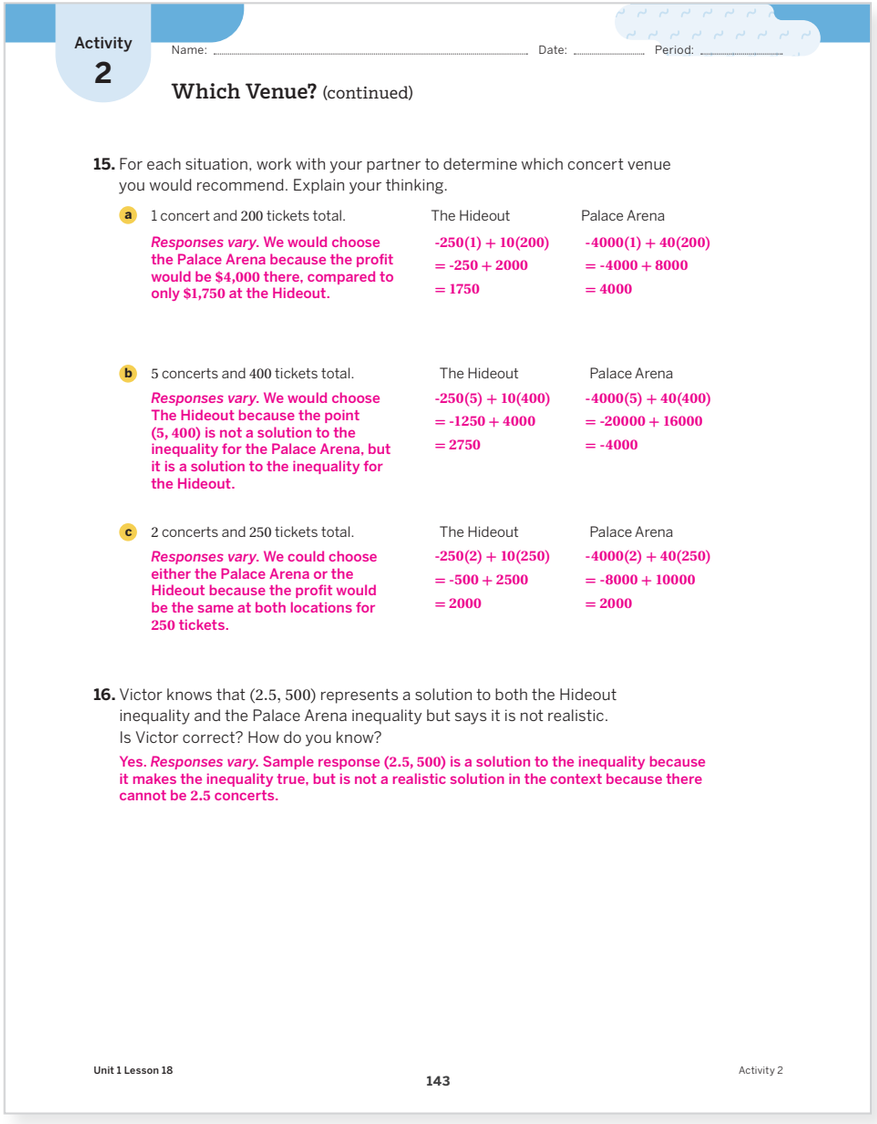
Invite students to share their recommendations and reasoning for Problem 14, celebrating the different strategies used to make recommendations. Consider plotting points on each graph as students share.

Consider asking:

- "Is this a solution for the Hideout? For the Palace Arena? How do we know?"
- "When you substituted (___, ___) into the inequality, was it true or false? What does that tell you about this point?"
- "Are there any points that work for both graphs but don't work for the situation? Explain your thinking."

To surface the Key Takeaway, consider asking:

- "How did you decide which venue to recommend for each situation?" **(MTR.7.1)**
- "How can you use the inequalities to make recommendations? How can you use the graphs?"



Activity 2

Name: Date: Period:

Which Venue? (continued)

15. For each situation, work with your partner to determine which concert venue you would recommend. Explain your thinking.

a 1 concert and 200 tickets total.

Responses vary. We would choose the Palace Arena because the profit would be $4,000 there, compared to only $1,750 at the Hideout.

The Hideout	Palace Arena
$-250(1) + 10(200)$	$-4000(1) + 40(200)$
$= -250 + 2000$	$= -4000 + 8000$
$= 1750$	$= 4000$

b 5 concerts and 400 tickets total.

Responses vary. We would choose The Hideout because the point (5, 400) is not a solution to the inequality for the Palace Arena, but it is a solution to the inequality for the Hideout.

The Hideout	Palace Arena
$-250(5) + 10(400)$	$-4000(5) + 40(400)$
$= -1250 + 4000$	$= -20000 + 16000$
$= 2750$	$= -4000$

c 2 concerts and 250 tickets total.

Responses vary. We could choose either the Palace Arena or the Hideout because the profit would be the same at both locations for 250 tickets.

The Hideout	Palace Arena
$-250(2) + 10(250)$	$-4000(2) + 40(250)$
$= -500 + 2500$	$= -8000 + 10000$
$= 2000$	$= 2000$

16. Victor knows that $(2.5, 500)$ represents a solution to both the Hideout inequality and the Palace Arena inequality but says it is not realistic. Is Victor correct? How do you know?

Yes. *Responses vary.* Sample response (2.5, 500) is a solution to the inequality because it makes the inequality true, but is not a realistic solution in the context because there cannot be 2.5 concerts.

Unit 1 Lesson 18 143 Activity 2

Key Takeaway: The symbolic and graphical representations of two-variable linear inequalities can be helpful tools for making informed decisions or recommendations to meet a specific goal.

Whole Class | 5 min

Synthesis

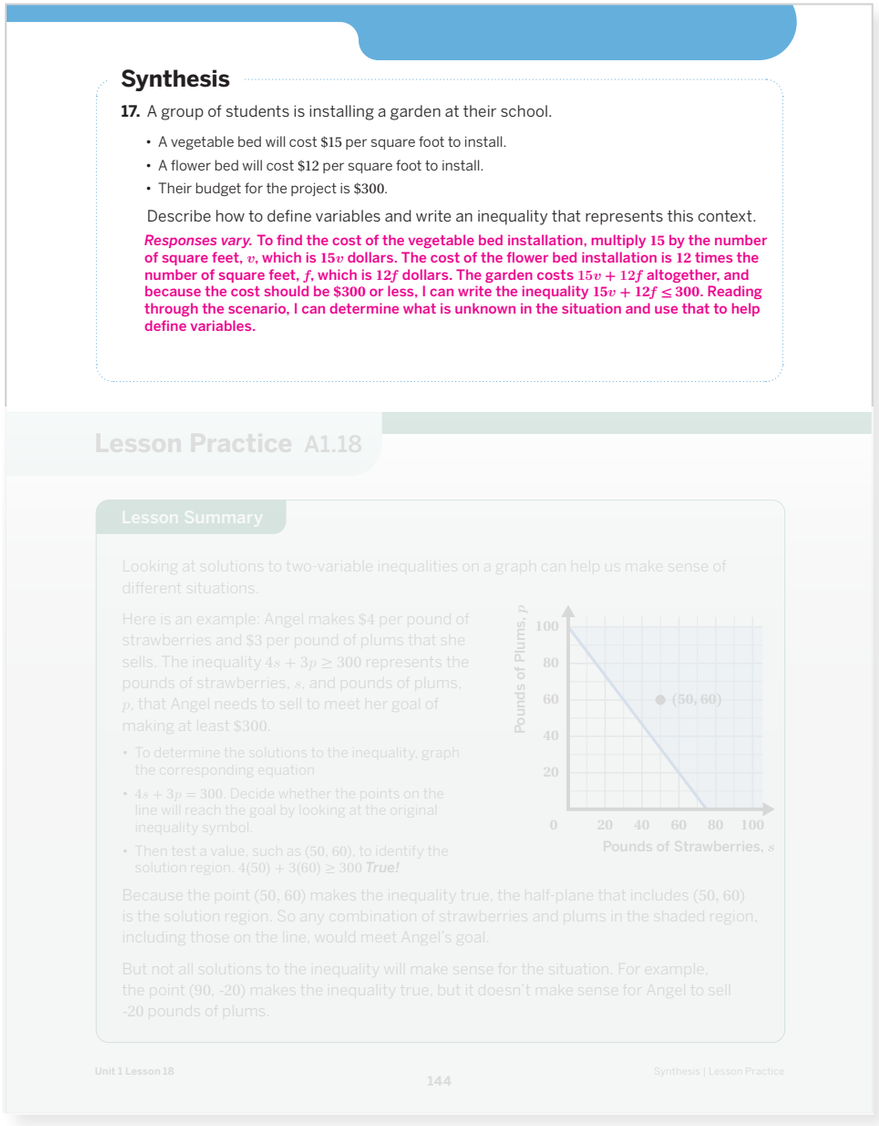
Purpose: Students synthesize their understanding of how to represent all of the solutions to a two-variable linear inequality graphically.

Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:

- Defining one variable for the number of square feet of vegetables and another variable for the number of square feet of flowers.
- Writing an inequality that uses the variables they defined.
- Explaining how an equation like $15v + 12f = 300$ is similar to the situation, without the inequality symbol.
- Reasoning that the inequality symbol should reflect being less than or equal to $300 because it represents a budget — a *constraint* the group does not want to surpass for the installation.

Note: Some students may think that because both quantities are measured with square feet, they can be represented by the same variable. Consider asking these students, "What do we need to keep track of? Where can we see that information in the inequality?"



Synthesis

17. A group of students is installing a garden at their school.

- A vegetable bed will cost $15 per square foot to install.
- A flower bed will cost $12 per square foot to install.
- Their budget for the project is $300.

Describe how to define variables and write an inequality that represents this context.

Responses vary. **To find the cost of the vegetable bed installation, multiply 15 by the number of square feet, v, which is $15v$ dollars. The cost of the flower bed installation is 12 times the number of square feet, f, which is $12f$ dollars. The garden costs $15v + 12f$ altogether, and because the cost should be $300 or less, I can write the inequality $15v + 12f \leq 300$. Reading through the scenario, I can determine what is unknown in the situation and use that to help define variables.**

Lesson Practice A1.18

Lesson Summary

Looking at solutions to two-variable inequalities on a graph can help us make sense of different situations.

Here is an example: Angel makes $4 per pound of strawberries and $3 per pound of plums that she sells. The inequality $4s + 3p \geq 300$ represents the pounds of strawberries, s, and pounds of plums, p, that Angel needs to sell to meet her goal of making at least $300.

- To determine the solutions to the inequality, graph the corresponding equation $4s + 3p = 300$. Decide whether the points on the line will reach the goal by looking at the original inequality symbol.
- Then test a value, such as $(50, 60)$, to identify the solution region. $4(50) + 3(60) \geq 300$ *True!*

Because the point $(50, 60)$ makes the inequality true, the half-plane that includes $(50, 60)$ is the solution region. So any combination of strawberries and plums in the shaded region, including those on the line, would meet Angel's goal.

But not all solutions to the inequality will make sense for the situation. For example, the point $(90, -20)$ makes the inequality true, but it doesn't make sense for Angel to sell -20 pounds of plums.

Unit 1 Lesson 18 144 Synthesis | Lesson Practice

Lesson Takeaway: Two-variable linear inequalities can be used to model relationships. Observing their solutions on a graph can be used to determine solutions that meet given constraints and support meaningful decision making.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by graphing the solutions to an inequality, interpreting them in context, and deciding which solutions are realistic.

Today's Goals

1. **Goal:** Graph the solutions to a two-variable linear inequality and interpret them in context.
2. **Goal:** Write a two-variable inequality to represent a situation in context.
3. **Language Goal:** Interpret what an inequality and its solutions mean in context. **(Reading and Writing)**

Show What You Know (Assessment Resources)

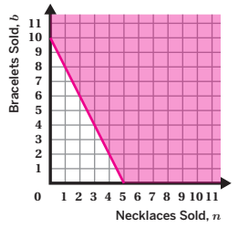
Name: _____ Date: _____ Period: _____

Show What You Know 1.18

An artist at the Saturday market makes $4 on each necklace she sells and $2 on each bracelet. She wants to make at least $20.

a. The artist wrote the inequality $b \geq -2n + 10$.

Graph all of the solutions to this inequality.



b. Explain what the solutions to the inequality represent.

Responses vary. The solutions are all the possible combinations of necklaces and bracelets that the artist has to sell to make at least $20.

c. Which of the solutions are realistic in this situation?

Responses vary. Positive whole number values of n and b that are in the solution region are realistic.

D Differentiation Use after Lesson 18

Support

Provide targeted intervention for students by using these resources.

If students need support graphing and interpreting solutions to a two-variable inequality in context:

» **Respond to Student Thinking:**

- Use the ***Graphing Solutions of Two-Variable Inequalities*** **Mini-Lesson**.
- Revisit the **Lesson Summary** and discuss strategies for how to determine which solutions allow Angel to meet her goal.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

How did your previous experience with fundraising or concerts impact how you taught this lesson? How did students' previous experiences with fundraising concerts impact how you taught the lesson? In the next lesson, you will teach about water as a shared, yet limited, resource. How might you prepare to invite students' experiences into tomorrow's lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

17. A group of students is installing a garden at their school.

- A vegetable bed will cost $15 per square foot to install.
- A flower bed will cost $12 per square foot to install.
- Their budget for the project is $300.

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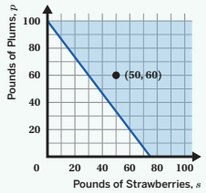
Responses vary. To find the cost of the vegetable bed installation, multiply 15 by the number of square feet, v, which is $15v$ dollars. The cost of the flower bed installation is 12 times the number of square feet, f, which is $12f$ dollars. The garden costs $15v + 12f$ altogether, and because the cost should be $300 or less, I can write the inequality $15v + 12f \leq 300$. Reading through the scenario, I can determine what is unknown in the situation and use that to help define variables.

Lesson Practice A1.18

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Here is an example: Angel makes $4 per pound of strawberries and $3 per pound of plums that she sells. The inequality $4s + 3p \geq 300$ represents the pounds of strawberries, s, and pounds of plums, p, that Angel needs to sell to meet her goal of making at least $300.



- To determine the solutions to the inequality, graph the corresponding equation $4s + 3p = 300$. Decide whether the points on the line will reach the goal by looking at the original inequality symbol.
- Then test a value, such as $(50, 60)$, to identify the solution region. $4(50) + 3(60) \geq 300$ ***True!***

Because the point $(50, 60)$ makes the inequality true, the half-plane that includes $(50, 60)$ is the solution region. So any combination of strawberries and plums in the shaded region, including those on the line, would meet Angel's goal.

But not all solutions to the inequality will make sense for the situation. For example, the point $(90, -20)$ makes the inequality true, but it doesn't make sense for Angel to sell -20 pounds of plums.

Lesson Practice
A1.18

Name: ..... Date: ..... Period: .....

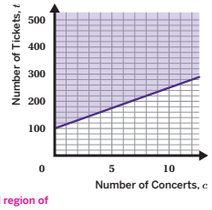
1. Mia is buying snacks for a movie night with her friends. Her budget is $60.

- A package of snack mix costs $5.00.
- A package of popcorn costs $2.50.

Write an inequality that represents all the snacks that Mia can buy for $60 or less. Use s for packages of snack mix and p for packages of popcorn.

$5s + 2.5p \leq 60$ **(or equivalent)**

Problems 2–3: The Funk-tions are looking into one more venue for their concerts. The Depot charges $300 per concert and tickets cost $20 each. The Funk-tions want to raise at least $2,000. The graph shows the solution set of $-300c + 20t \geq 2000$.



2. Write one solution to the inequality.

Responses vary. **(6, 300)**

3. If the Funk-tions have 4 concerts and sell 200 tickets, will they reach their fundraising goal? Explain how you know.

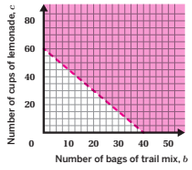
Yes. *Explanations vary.* **The point (4, 200) is in the shaded region of the graph, which means it's a solution to the inequality.**

Problems 4–6: Aditi's soccer team is selling bags of trail mix for $3 each and cups of lemonade for $2 each. To make a profit, they need to earn more than $120.

4. b represents the number of bags of trail mix and c represents the number of cups of lemonade. Which inequality describes all the ways Aditi's soccer team can make a profit?

(A.) $3b + 2c > 120$ B. $c - 60 < -\frac{3}{2}b$ C. $c \geq -\frac{3}{2}b + 60$ D. $120 \geq 3b + 2c$

5. Graph all of the solutions to the inequality you chose.



6. Explain how you could check if the boundary is included or not included from the solution region.

Explanations vary. **I could determine if the coordinates of the points on the line satisfy the inequality. For example, (20, 30) means 20 bags of trail mix and 30 cups of lemonade. The soccer team would earn $120 if they sold that many bags of trail mix and cups of lemonade, which is not enough to make a profit.**

Lesson Practice
A1.18

Name: ..... Date: ..... Period: .....

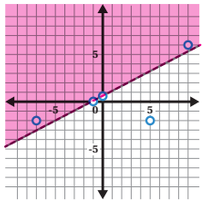
7. Jasmine says that $(3, 10)$ is a solution to $y < \frac{1}{2}x + \frac{15}{4}$. Is she correct? Explain how you know.

No. *Explanations vary.* $(\frac{1}{2})(3) + (\frac{15}{4})$ **is equal to 5.25. 5.25 is less than 10. The result would need to be greater than 10 to make the inequality true.**

Test Practice

Problems 8–9: Here is an inequality: $2y - x > 1$.

8. Graph *all* the solutions to $2y - x > 1$.

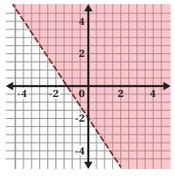


9. Select *all* the points that are solutions to this inequality.

- ☑ A. $(-7, -2)$
- ☐ B. $(-1, 0)$
- ☐ C. $(0, 0.5)$
- ☐ D. $(5, -2)$
- ☑ E. $(9, 6)$

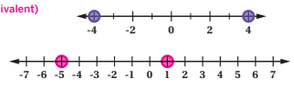
Spiral Review

10. The graph shows the solution set to an inequality. Select *all* the inequalities that produce this solution set.



- ☐ A. $y < -1.5x - 2$
- ☑ B. $y > -1.5x - 2$
- ☑ C. $1.5x + y > -2$
- ☐ D. $y > 1.5x + 2$
- ☐ E. $y \leq 1.5x - 2$

11. Create an inequality using x that has the solution set shown on the graph. $|x| \geq 4$ **(or equivalent)**



12. Graph all the solutions to the inequality $|x + 2| = 3$.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	5, 8	2	MA.912.AR.2.8
	1–3, 7	2	MA.912.AR.2.7
	4	2	MA.912.AR.2.7
	6	3	MA.912.AR.2.7
Test Practice	9	2	MA.912.AR.2.8
Spiral Review			
	10	2	MA.912.AR.2.8
Fluency	11–12	2	MA.912.AR.2.1

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students solved one- and two-variable equations and inequalities, including equations involving absolute value.

Career Connection | Meet Helia Bravo Hollis

Invite students to read about botanist Helia Bravo Hollis and how her work preserved many types of cactus plants.

Consider asking:

- "What are some characteristics of cactus plants?" Sample response: Cactus plants have a thick stem and sharp points. They can store water, which allows them to grow in very hot and dry places.

B.E.S.T. Benchmark Connections

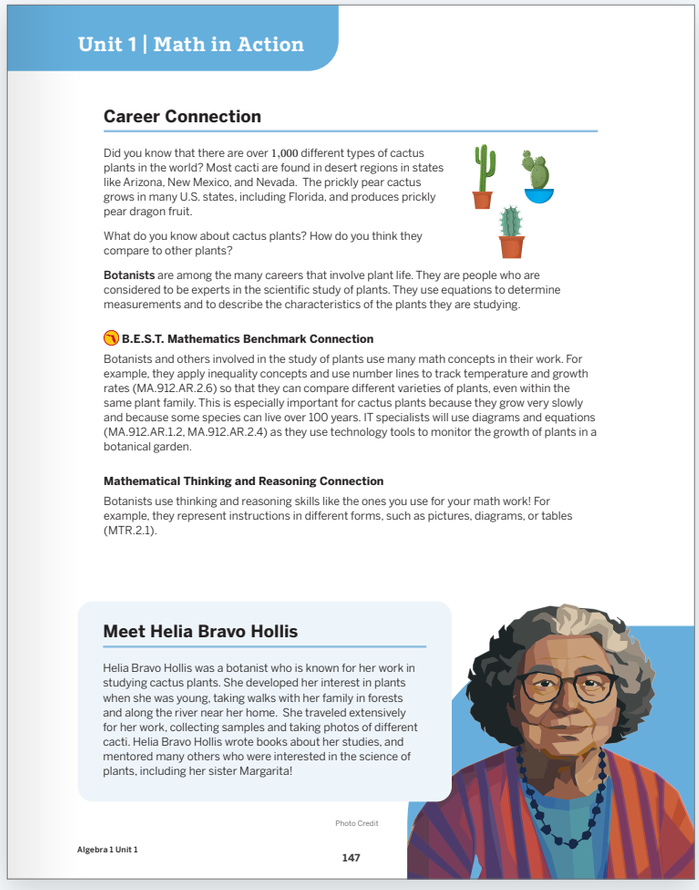
Consider asking:

- "What other math concepts have you studied in this unit that you could you apply to the study of plants?" Sample response: You can calculate and compare the growth rates of several different kinds of cactus plants. (MA.912.AR.2.5)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "Scientists often engage in discussions that reflect on their thinking. How are these actions similar to how you engage with math through Mathematical Thinking and Reasoning?" Sample response: Communicate ideas and methods. (MTR.4.1) Working on activities in this unit can help you better understand concepts like equations and inequalities, especially when using equations to solve real-world problems (like whale growth rates).



Unit 1 | Math in Action

Career Connection

Did you know that there are over 1,000 different types of cactus plants in the world? Most cacti are found in desert regions in states like Arizona, New Mexico, and Nevada. The prickly pear cactus grows in many U.S. states, including Florida, and produces prickly pear dragon fruit.

What do you know about cactus plants? How do you think they compare to other plants?

Botanists are among the many careers that involve plant life. They are people who are considered to be experts in the scientific study of plants. They use equations to determine measurements and to describe the characteristics of the plants they are studying.

B.E.S.T. Mathematics Benchmark Connection

Botanists and others involved in the study of plants use many math concepts in their work. For example, they apply inequality concepts and use number lines to track temperature and growth rates (MA.912.AR.2.6) so that they can compare different varieties of plants, even within the same plant family. This is especially important for cactus plants because they grow very slowly and because some species can live over 100 years. IT specialists will use diagrams and equations (MA.912.AR.1.2, MA.912.AR.2.4) as they use technology tools to monitor the growth of plants in a botanical garden.

Mathematical Thinking and Reasoning Connection

Botanists use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as pictures, diagrams, or tables (MTR.2.1).

Meet Helia Bravo Hollis

Helia Bravo Hollis was a botanist who is known for her work in studying cactus plants. She developed her interest in plants when she was young, taking walks with her family in forests and along the river near her home. She traveled extensively for her work, collecting samples and taking photos of different cacti. Helia Bravo Hollis wrote books about her studies, and mentored many others who were interested in the science of plants, including her sister Margarita!

Photo Credit

Algebra 1 Unit 1 147

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying equivalent equations	Lesson 2	2	MA.912.AR.2.1
2	Solving one-variable equations	Lesson 1–3	1	MA.912.AR.2.1
3	Solving multi-variable equations for one of the variables	Lesson 4–7	3	MA.912.AR.1.2
4	Solving one-variable inequalities	Lesson 12	1	MA.912.AR.1.2, MTR.4.1
5	Graphing solutions to two-variable inequalities	Lesson 17	2	MA.912.AR.2.8
6a	Identifying solutions to two-variable inequalities	Lesson 16	2	MA.912.AR.2.6
6b	Writing a situation to match a two-variable inequality and describing what the variables represent in context	Lesson 18	3	MA.912.AR.2.6, MTR.6.1
7a, 7b	Writing equations and inequalities in two variables to represent relationships between quantities	Lesson 4	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
7c	Writing inequalities in one variable to represent relationships between quantities	Lesson 10–12	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
7d	Interpreting solutions to inequalities in context	Lesson 10	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
8	Write and solve one-variable compound inequalities	Lesson 13	1	MA.912.AR.2.6
9	Solving Absolute Value Equations	Lesson 14	1	MA.912.AR.4.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 1

1. Which equation is equivalent to $4x - 6 = x + 9$?

- **A.** $3x = 3$
- **B.** $3x - 6 = 9$
- **C.** $-6 = 6x + 9$
- **D.** $-2x = x + 9$

2. Solve each equation.

a $12 = 5x - 30 + x$

$x = 7$

b $4x + 10 = 2(x + 1)$

$x = -4$

3. Here is the work Daniela did to solve $y = 3x + b$ for x.

Daniela

$y = 3x + b$

$-b \quad -b$

$y - b = 3x$

$-3 \quad -3$

$y - b - 3 = x$

a Explain why this is incorrect.

Explanations vary. In the last step, Daniela subtracted 3 when she should have divided.

b Solve $y = 3x + b$ for x.

$\frac{y - b}{3} = x$ (or equivalent)

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Name: Date: Period:

End-of-Unit Assessment (continued)

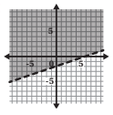
Unit 1

4. Select *all* the solutions to the inequality $2x + 6 > 3x - 12$.

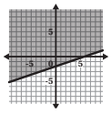
- ☐ **A.** 30
- ☑ **B.** -5
- ☐ **C.** 18
- ☑ **D.** 0
- ☑ **E.** 6

5. Which graph represents all the solutions to $x - 3y < 6$?

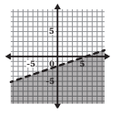
A.



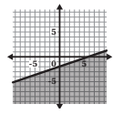
B.



C.



D.



6. Here is an inequality: $4x + 5y \leq 20$.

a Write the coordinates of three solutions to this inequality.

Responses vary.

(0, 0) (1, 0) (5, 0)

b Write a situation that this inequality could represent, including the meaning of x and y.

Responses vary. Breadsticks cost $4 and pizza costs $5. The most I can spend is $20. x is the number of breadsticks and y is the number of pizzas.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

7. A gardening club has 300 square feet to plant cucumbers and tomatoes.



Each cucumber plant uses 6 square feet of space, and each tomato plant uses 4 square feet.

- Let c represent the number of cucumber plants.
- Let t represent the number of tomato plants.

a Write an *equation* that represents the number of cucumber and tomato plants that can fill the garden.

$6c + 4t = 300$ (or equivalent)

b Write an *inequality* that represents all of the combinations of cucumber and tomato plants that can fit in 300 square feet or less.

$6c + 4t \leq 300$ (or equivalent)

The garden club decides to grow 20 cucumber plants.

c Write and solve an inequality to represent the number of tomato plants that can fit in this garden.

$6(20) + 4t \leq 300$ (or equivalent)

$t \leq 45$ (or equivalent)

d Explain what your solution says about the number of tomato plants in the garden.

Explanations vary. The solution means that I can fit 45 or fewer tomato plants in the 300-square-foot garden along with the 20 cucumber plants.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

8. The community service guidelines for Student Council state that you must complete 10 to 20 hours of community service during the summer. You have already completed 4 hours. You can work up to 3 hours on any day. How many days must you work to meet the program requirements? Write and solve a compound inequality to determine the acceptable range of day, d, work shifts to meet the program requirements.

a Choose two options that define the compound inequality for the number of days you need to work.

- ☑ **A.** $10 \leq 4 + 3d \leq 20$
- ☐ **B.** $10 \leq 3d - 4 \leq 20$
- ☐ **C.** $6 \leq 4 + 3d \leq 16$
- ☑ **D.** $6 \leq 3d \leq 16$
- ☐ **E.** $14 \leq 3d \leq 24$

b Which compound inequality defines the number of days you need to work?

- **A.** $3.67 \leq d \leq 5.33$
- **B.** $2 \leq d \leq 5.33$
- **C.** $3 \leq d \leq 5.67$
- **D.** $2.67 \leq d \leq 5.67$

9. Consider the absolute value equation $8 - |x - 3| = 2$. Select *all* of the true statements.

- ☐ **A.** $x = 3$ is a solution.
- ☑ **B.** $x = 9$ is a solution.
- ☐ **C.** The equation has no solutions.
- ☐ **D.** The solutions are numbers that are 3 units away from 6.
- ☐ **E.** $x = -1$ is a solution.
- ☑ **F.** The solutions are numbers that are 6 units away from 3.

Name: Date: Period:

End-of-Unit Assessment

Unit 1

1. Which equation is equivalent to $3x - 4 = 5x + 7$?

(A.) $-4 = 2x + 7$

B. $6x = 3$

C. $8x - 4 = 7$

D. $3 = 2x$

2. Solve each equation.

a. $4(x + 3) = 2x + 2$
$x = -5$

b. $2x - 7 - 4x = 13$
$x = -10$

3. Here is the work Dyani did to solve $y = mx + 7$ for m.

Dyani

$\frac{y}{x} = \frac{mx}{x} + 7$

$\frac{y}{x} = m + 7$

$-7 \quad -7$

$\frac{y}{x} - 7 = m$

a. Explain why this is incorrect.
Explanations vary. In the first step, Dyani divided only parts of the equation by x instead of the whole equation. Dyani could have subtracted 7 from both sides first.

b. Solve $y = mx + 7$ for m.
$\frac{y - 7}{x} = m$ (or equivalent)

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

4. Select *all* the solutions to the inequality $3x + 1 \leq 5x - 3$.

☑ A. 20

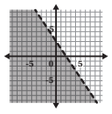
☑ B. 15

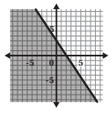
☑ C. 2

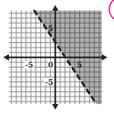
☐ D. 0

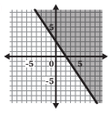
☐ E. -3

5. Which graph represents all the solutions to $2y + 3x \geq 6$?

A. 

B. 

C. 

(D.) 

6. Here is an inequality: $3x + 7y < 30$.

a. Write the coordinates of three solutions to this inequality.
Responses vary.
(0, 0) (1, 2) (9, 0)

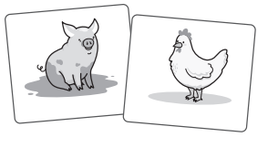
b. Write a situation that this inequality could represent, including the meaning of x and y
Responses vary. Bagels cost $3 and loaves of bread cost $7. I must spend less than $30. x is the number of bagels and y is the number of bread loaves.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

7. A farmer has a 210-square-foot barn to house chickens and pigs.



Each pig requires 50 square feet of space, and each chicken requires 4 square feet.

- Let c represent the number of chickens.
- Let p represent the number of pigs.

a. Write an *equation* that represents the number of chickens and pigs that can fill the barn.
$4c + 50p = 210$ (or equivalent)

b. Write an inequality that represents all of the combinations of chickens and pigs that can fit in 210 square feet or less.
$4c + 50p \leq 210$ (or equivalent)

The farmer decides to house 3 pigs.

c. Write and solve an *inequality* to represent the number of chickens that can fit in this barn.
$4c + 50(3) \leq 210$ (or equivalent)
$c \leq 15$ (or equivalent)

d. Explain what your solution says about the number of chickens in the barn.
Explanations vary. The solution means that the farmer can fit 15 or fewer chickens in the 210-square-foot barn along with the 3 pigs.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

8. The part-time job guidelines for the local library state that you must complete 15 to 25 hours of work each week. You always work 5 hours on Saturday. For the remaining weekdays, you can work 4 hours on any day. How many days a week must you work to meet the program requirements? Write and solve a compound inequality to determine the acceptable range of weekday, d, work shifts to meet the program requirements.

a. Choose two options that define the compound inequality for the days you can work each week.

☐ A. $15 \leq 4d - 5 \leq 25$

☑ B. $15 \leq 5 + 4d \leq 25$

☐ C. $15 \leq 4 + 5d \leq 25$

☐ D. $11 \leq 4d \leq 21$

☑ E. $10 \leq 4d \leq 20$

☐ F. $20 \leq 4d \leq 30$

b. Write the compound inequality that defines the days you can work each week.

A. $2.75 \leq d \leq 5.25$

B. $5 \leq d \leq 7.5$

(C.) $2.5 \leq d \leq 5$

D. $2 \leq d \leq 5.5$

9. Consider the absolute value equation $10 - |x + 4| = 5$. Select *all* of the true statements.

☑ A. $x = 1$ is a solution.

☐ B. $x = 4$ is a solution.

☐ C. The equation has no solutions.

☑ D. The solutions are numbers that are 5 units away from -4.

☐ E. $x = -1$ is a solution.

☐ F. The solutions are numbers that are 4 units away from 5.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

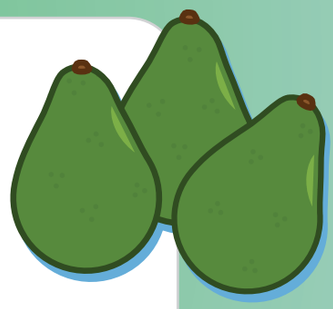
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: Solve linear equations with one variable and interpret solutions in context. **(Lessons 1–3)**	1, 2	**Support**	• **Mini Lesson: Solving Linear Equations**
		Strengthen	• **Teacher Move:** Invite students to solve more equations from Lesson 2, Activity 3 and discuss their strategies with a partner.

		Stretch	• **You're invited to explore more:**
			  • Lesson 2 (*Solving Strategies*)
			  • Lesson 3 (*Same Position*)

Sub-Unit 2: Solve multi-variable equations for a given variable. • Create equations, graphs, and tables to represent linear relationships and interpret their solutions in context. • Write a linear two-variable equation for a line that is parallel or perpendicular to a given line passing through a given point. **(Lessons 4-9)**	3	**Support**	• **Mini Lesson: Solving Multi-Variable Linear Equations** • **Teacher Move:** Consider revisiting Lesson 5, Activity 2 (*Various Variables*).
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 6 (*Shelley the Snail*).
	7a	**Support**	• **Teacher Move:** Consider revisiting Lesson 7, Activity 2 (*Five Representations*).
		Stretch	• **Teacher Move:** Invite students to write their own scenario that can be modeled by a linear equation and create a matching equation, graph, table, and diagram.
Sub-Unit 3: Determine solutions to an inequality algebraically and graphically. • Write inequalities in one and two variables to represent constraints • Solve a one-variable linear compound inequality when given a mathematical or real-world concept and represent the solutions algebraically or graphically. **(Lessons 10-18)**	4	**Support**	• **Teacher Move:** Consider revisiting Lesson 12, Activity 2 (*Solving for Sheep*).
		Strengthen	• **Repeated Challenges:** Lesson 12 (*Solutions and Sheep*)
	5, 6a	**Support**	• **Mini Lesson: Graphing Solutions of Two-Variable Inequalities** • **Teacher Move:** Consider revisiting Lesson 17, Activity 1 (*All of the Solutions*).
		Strengthen	• **Repeated Challenges:** Lesson 17 (*All of the Solutions*)
	6b,7b 7c, 7d	**Support**	• **Teacher Move:** Consider revisiting Lesson 16, Activity 2 (*Bracelet Budgets*) • **Teacher Move:** Consider revisiting Lesson 18, Activity 1 (*Concert Planning*).
	8	**Support**	• **Teacher Move**: Consider revisiting Lesson 13, Activity 2, (*Tick Tock*).
	9	**Support**	• **Teacher Move**: Consider revisiting Lesson 14 (*Absolute Value Solutions*)
		Strengthen	• **Teacher Move**: Consider creating a process anchor chart with students to be able to reference.

UNIT 2

Describing Data



Students analyze one-variable and two-variable data sets. They use dot plots, histograms, box plots, and measures of center and spread to compare one-variable data sets. They use scatter plots, correlation coefficients, and lines of best fit to describe two-variable data sets.

Essential Questions

- What tools can you use to help determine if there is an association in a set of data?
- How can you compare data using measures of center and spread?
- How can you use the correlation coefficient and line of best fit to describe the relationship between two variables?

Sub-Unit Goals

Sub-Unit 1

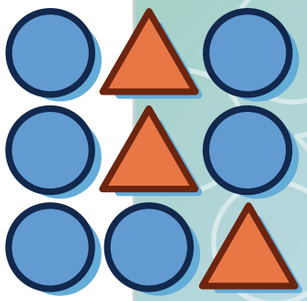
- Determine whether data is univariate or bivariate.

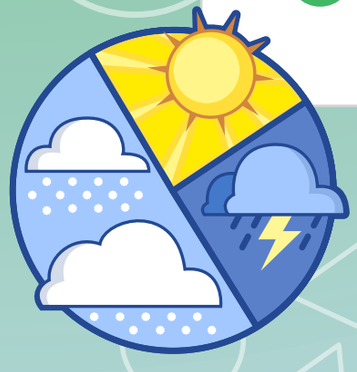
Sub-Unit 2

- Represent data with dot plots, histograms, and box plots.
- Describe the shape, center, and spread of data.
- Compare and contrast data sets using measures of center and spread appropriate to the shape of the data set and interpret the differences in context.

Sub-Unit 3

- Represent categorical data in two-way tables and relative frequency tables.
- Fit a line to data, assess the fit using residual plots, and use the linear model to make predictions.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate. **(MA.912.DP.1.1)**

Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate, and interpret the different components and quantities in the display. **(MA.912.DP.1.2)**

Other Standards Addressed in This Unit: MA.912.DP.1.3 MA.912.DP.1.4 MA.912.DP.2.4 MA.912.DP.2.6 MA.912.DP.3.1

Mathematical Thinking and Reasoning Standards

- Students construct and evaluate arguments when comparing data sets. **(MTR.4.1)**
- Students create and use lines of fit to model relationships between two variables. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.2.1, MTR.3.1, MTR.5.1**

Coherence Telling a story within and across grades

In this Unit

Students analyze one-variable and two-variable data sets. They use appropriate statistics to compare data, including the measures of center and spread for one-variable data, scatter plots, lines of best fit, and correlation coefficients for two-variable data.

Prior Learning

- **Grade 6:** Describing Data
- **Grade 8:** Associations in Data

Future Learning

- **Algebra 1,** Unit 6: Exponential Functions
- **Algebra 1,** Unit 7: Quadratic Functions

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In Lesson 15, students make connections between residuals and residual plots to determine how well a line fits data.

Procedural Fluency

In **Lesson 17**, students develop procedural fluency as they analyze news headlines and learn to be critical consumers of media.

Application

In **Lesson 12**, students apply their understanding of two-way frequency tables to support them in making informed decisions.

Strategic Competence

In **Lesson 6**, student develop strategic competence for when it is appropriate to consider the mean or the median as a typical value for describing a data set based on when outliers are present.

Adaptive Reasoning

In **Lesson 5**, students develop adaptive reasoning as they compare data provided in a box-and-whisker plot and a dot plot. Students will create a box-and-whisker plot with a specific median and range, and, if necessary, reflect on what went wrong and fix their data.

Productive Disposition

In **Lesson 9**, students have opportunities to develop a productive disposition as they use statistics to estimate how many cars are at each address in a specific town.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.4.DP.1.1 MA.5.DP.1.2 MA.6.DP.1.4
MA.8.DP.1.1 MA.8.DP.1.3 MA.8.AR.3.5
MTR.2.1, MTR.4.1, MTR.7.1

Sub-Unit 1



1 Survey Says

What Kinds of Data Can I Collect?

Make sense of the kinds of data that can be collected and use the questions to get to know each other better.

MA.912.DP.1.1 MA.912.DP.1.2
MTR.4.1



2 Data Dimensions

Univariate and Bivariate Data

Determine whether data is univariate or bivariate.

MA.912.DP.1.2
MTR.2.1 MTR.4.1 MTR.5.1

Sub-Unit 2



3 Data Driven

Revisiting Dot Plots and Histograms

Represent univariate numerical data and analyze their patterns.

MA.912.DP.1.1
MTR.4.1



8 How Big?

Estimate a Population Using Sample Survey Data

Estimating populations using data from sample surveys.

MA.912.DP.1.4
MTR.4.1 MTR.5.1 MTR.7.1



9 How Many?

Estimate a Mean or Percentage Using Sample Survey Data

Use data to estimate means and percentages.

MA.912.DP.1.4
MTR.1.1 MTR.4.1 MTR.7.1



10 Dribble, Draw, and Decide

Develop a Margin of Error Through Simulation

Analyzing the margin of error by using real world simulations.

MA.912.DP.1.4
MTR.1.1 MTR.4.1 MTR.7.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understandings of the concepts and skills so far in this unit.

MA.912.DP.1.1 MA.912.DP.1.2
MA.912.DP.1.4
MTR.4.1

Assess and Respond



15 Residual Fruit

Residuals and Residual Plots

Use **residual plots** to determine how well a line fits data.

MA.912.DP.2.4 MA.912.DP.2.6
MTR.1.1 MTR.4.1 MTR.7.1



16 Penguin Populations

Using Technology to Generate the Line of Best Fit

Explore how penguin populations have changed over time by generating and analyzing **lines of best fit**.

MA.912.DP.2.4 MA.912.DP.2.6
MTR.1.1 MTR.4.1 MTR.7.1



17 Behind the Headlines

Causation vs. Correlation

Consider the difference between **causation** and **correlation** and use that to analyze headlines.

MA.912.DP.1.3
MTR.1.1 MTR.4.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.DP.1.1 MA.912.DP.1.2
MA.912.DP.1.3 MA.912.DP.1.4
MA.912.DP.2.4 MA.912.DP.2.6
MA.912.DP.3.1
MTR.4.1 MTR.7.1

Pacing: 20 days (+1 optional days) | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 17 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

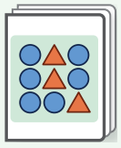


4 Better Weather

Revisiting Box Plots

Analyze categorical univariate data from different types of graphs and tables.

MA.912.DP.1.1
MTR.2.1 MTR.4.1, MTR.7.1



5 Quick Pick

Revisiting Measures of Center

Revisit mean and median and explore how extreme values impact each **measure of center**.

MA.912.DP.1.1 MA.912.DP.1.2
MTR.3.1 MTR.4.1



6 Far Out

Identifying Outliers

Learn what **outliers** are and how they affect the statistics of a data set.

MA.912.DP.1.1 MA.912.DP.1.2
MTR.2.1 MTR.4.1 MTR.6.1



7 Dynamic Decades

Comparing Data Using Measures of Center and Spread

Explore socioeconomic data from various states in the U.S. and use statistics to describe the data.

MA.912.DP.1.1 MA.912.DP.1.2
MTR.1.1 MTR.4.1 MTR.7.1

Sub-Unit 3



11 Trains and Traffic

Two-Way Tables and Relative Frequency Tables

Using categorical data to make an argument using **two-way** and **relative frequency tables**.

MA.912.DP.3.1
MTR.4.1 MTR.7.1



12 Remodel Choices

Making Decisions With Frequency Tables

Develop fluency in calculating values in relative frequency tables.

MA.912.DP.3.1
MTR.2.1 MTR.4.1 MTR.7.1



13 Connecting the Dots

Numerical Bivariate Data

Represent and analyze data sets from line graphs.

MA.912.DP.1.1 MA.912.DP.1.2
MTR.1.1 MTR.3.1 MTR.6.1



14 City Slopes

Interpreting Slope and y-Intercept in Context

Use a line of fit to describe relationships between two variables and interpret the slope and y-intercept in context.

MA.912.DP.2.4 MA.912.DP.2.6
MTR.1.1 MTR.4.1 MTR.6.1

Pacing Considerations

Lesson 5: This lesson is intended to revisit mean and median and the impact of extreme values on each measure of center from Grades 6–8. If students show a strong understanding of Problems 4–6 on the Pre-Unit Check, consider omitting this lesson. Consider spending adequate time in Lessons 10 and 12 discussing the connection between the shape of the data and its impact on each measure of center.

Lesson 7: This lesson gives students an opportunity to apply what they have learned to analyze real-world data and consider the power and challenges of mathematical modeling as they make sense of population change. There is no new math content introduced in this lesson.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1
Copy: Show What You Know
Materials: no required materials

Lesson 2
Copy: Show What You Know
Materials: no required materials

Lesson 3
Copy: Show What You Know
Materials: no required materials

Lesson 4
Copy: Show What You Know
Materials: no required materials

Lesson 5
Copy: Show What You Know
Materials: scientific calculator *(optional)*

Lesson 6
Copy: Show What You Know
Materials: scientific calculator *(optional)*/ blank number cubes

Lesson 7
Copy: Show What You Know
Materials: no required materials

Lesson 8
Copy: Show What You Know
Materials: no required materials

Lesson 9
Copy: Show What You Know
Materials: no required materials

Lesson 10
Copy: Show What You Know
Materials: no required materials

Lesson 11
Copy: Show What You Know
Materials: scientific calculator

Lesson 12
Copy: Show What You Know
Materials: scientific calculator *(as needed)*

Lesson 13
Copy: Show What You Know
Materials: no required materials

Lesson 14
Copy: Show What You Know
Materials: coloring tools (as needed)/straight-edges

Lesson 15
Copy: Show What You Know
Materials: no materials required

Lesson 16
Copy: Show What You Know
Materials: scientific calculator (as needed)

Lesson 17
Copy: Show What You Know
Materials: Activity 2 Cards, one set per pair, pre-cut Article Summary Sheet, one per student

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

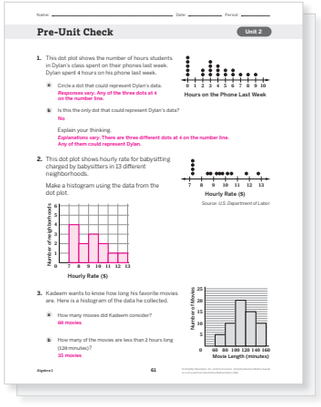
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.4.DP.1.1, MA.5.DP.1.2, MA.6.DP.1.4, MA.8.DP.1.1, MA.8.DP.1.3, MA.8.AR.3.5


Pre-Unit Check

Unit 2

1. This dot plot shows the number of hours students in Dylan's class spent on their phones last week. Dylan spent 4 hours on his phone last week.

a. Circle a dot that could represent Dylan's data.

Responses vary. Any of the three dots at 4 on the number line.

Hours on the Phone Last Week

b. Is this the only dot that could represent Dylan's data?

No

Explain your thinking.

Explanations vary. There are three different dots at 4 on the number line. Any of them could represent Dylan.

2. This dot plot shows hourly rate for babysitting charged by babysitters in 13 different neighborhoods.

Make a histogram using the data from the dot plot.

Hourly Rate ($)

Source: U.S. Department of Labor

Number of Neighborhoods

Hourly Rate ($)

3. Kadeem wants to know how long his favorite movies are. Here is a histogram of the data he collected.

a. How many movies did Kadeem consider?

60 movies

b. How many of the movies are less than 2 hours long (120 minutes)?

30 movies

Number of Movies

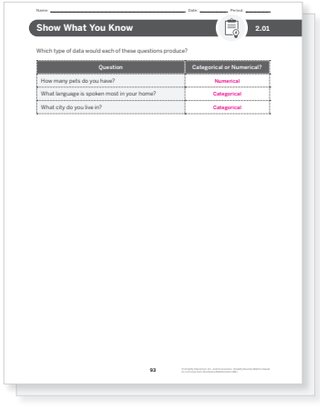
Movie Length (minutes)

Algebra 1 63

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed: on Lesson 1 Show What You Know: MA.912.DP.1.1


Show What You Know

2.01

Which type of data would each of these questions produce?

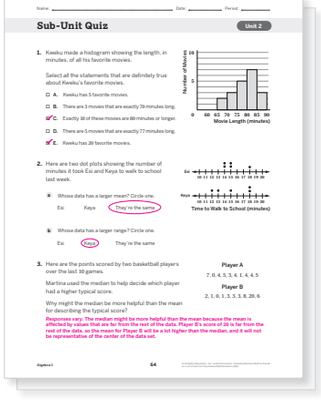
Question	Categorical or Numerical?
How many pets do you have?	Numerical
What language is spoken most in your home?	Categorical
What city do you live in?	Categorical

93

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.912.DP.1.1, MA.912.DP.1.2, MA.912.DP.1.4


Sub-Unit Quiz

Unit 2

1. Kweku made a histogram showing the length, in minutes, of all his favorite movies.

Select all the statements that are definitely true about Kweku's favorite movies.

- ☐ A. Kweku has 5 favorite movies.
- ☐ B. There are 3 movies that are exactly 76 minutes long.
- ☑ C. Exactly 30 of these movies are 80 minutes or longer.
- ☐ D. There are 5 movies that are exactly 77 minutes long.
- ☑ E. Kweku has 28 favorite movies.

Number of Movies

Movie Length (minutes)

2. Here are two dot plots showing the number of minutes it took Esi and Keya to walk to school last week.

a. Whose data has a larger mean? Circle one.

Esi Keya They're the same

Esi

Keya

Time to Walk to School (minutes)

b. Whose data has a larger range? Circle one.

Esi Keya They're the same

3. Here are the points scored by two basketball players over the last 10 games.

Player A

7, 0, 4, 5, 3, 4, 1, 4, 4, 5

Player B

2, 1, 0, 1, 3, 3, 3, 8, 20, 6

Martina used the median to help decide which player had a higher typical score.

Why might the median be more helpful than the mean for describing the typical score?

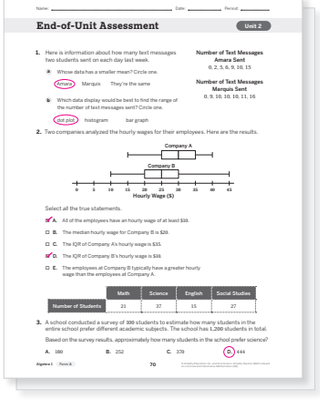
Responses vary. The median might be more helpful than the mean because the mean is affected by values that are far from the rest of the data. Player B's score of 20 is far from the rest of the data, so the mean for Player B will be a lot higher than the median, and it will not be representative of the center of the data set.

Algebra 1 64

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.912.DP.1.1, MA.912.DP.1.2, MA.912.DP.1.3, MA.912.DP.1.4, MA.912.DP.2.4, MA.912.DP.2.6, MA.912.DP.3.1


End-of-Unit Assessment

Unit 2

1. Here is information about how many text messages two students sent on each day last week.

Number of Text Messages Amara Sent

0, 2, 5, 6, 9, 10, 15

Number of Text Messages Marquis Sent

0, 9, 10, 10, 10, 11, 16

a. Whose data has a smaller mean? Circle one.

Amara Marquis They're the same

b. Which data display would be best to find the range of the number of text messages sent? Circle one.

dot plot histogram bar graph

2. Two companies analyzed the hourly wages for their employees. Here are the results.

Company A

Company B

Hourly Wage ($)

Select all the true statements.

- ☑ A. All of the employees have an hourly wage of at least $10.
- ☐ B. The median hourly wage for Company B is $20.
- ☐ C. The IQR of Company A's hourly wage is $35.
- ☑ D. The IQR of Company B's hourly wage is $10.
- ☐ E. The employees at Company B typically have a greater hourly wage than the employees at Company A.

	Math	Science	English	Social Studies
Number of Students	21	37	15	27

3. A school conducted a survey of 100 students to estimate how many students in the entire school prefer different academic subjects. The school has 1,200 students in total.

Based on the survey results, approximately how many students in the school prefer science?

A. 180 B. 252 C. 378 D. 444

Algebra 1 Form A 70

Differentiation

Every unit includes resources to support, strengthen, and stretch student learning.

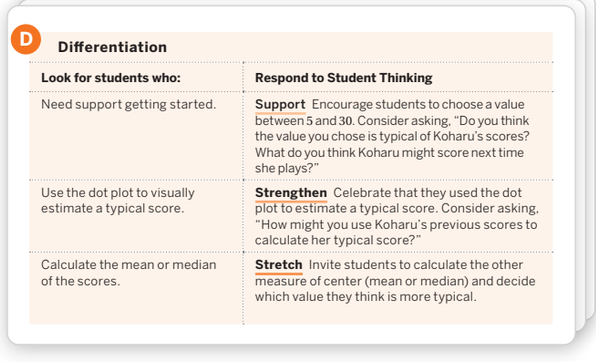
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–2) • Determine whether a given set of data is univariate or bivariate.	**Mini-Lesson** • Classifying Data	**Practice** • Lessons 1–2	**Teacher Moves** • Invite students to create a survey, gather categorical data, represent it in frequency tables, and share their findings with a partner.
Sub-Unit 2 (Lessons 3–10) • Represent data with dot plots, histograms, and box plots. • Describe the shape, center, and spread of data. • Compare and contrast data sets using measures of center and spread appropriate to the shape of the data set and interpret the differences in context.	**Mini-Lessons** • Representing Data on a Line Plot and Solving Problems • Using Line Plots to Solve Problems • Interpreting Box Plots • Estimating Population from a Sample Survey	**Practice** • Lessons 4–12	**Teacher Moves** • Invite students to research a new topic, find data from two different years or groups, and compare the data sets using appropriate statistics.
Sub-Unit 3 (Lessons 11–17) • Represent categorical data in two-way tables and relative frequency tables. • Fit a line to data, assess the fit using residual plots, and use the linear model to make predictions.	**Mini-Lessons** • Using Lines of Fit to Make Predictions • Interpreting Slope and Intercepts of Linear Relationships • Calculating Values in Two-Way and Relative Frequency Tables	**Practice** • Lessons 13–19	**DOK3 Practice** • Lesson 12 (*School Choice*) **Teacher Moves** • Invite students to select and plot data for two variables on a scatter plot. Encourage predicting the equation for the line of fit and r-value, then use technology to calculate and reflect on the reasonableness of the predictions.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 6**, where students make sense of which measure of center could be used to determine the typical value of a data set.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to choose a value between 5 and 30. Consider asking, "Do you think the value you chose is typical of Koharu's scores? What do you think Koharu might score next time she plays?"
Use the dot plot to visually estimate a typical score.	**Strengthen** Celebrate that they used the dot plot to estimate a typical score. Consider asking, "How might you use Koharu's previous scores to calculate her typical score?"
Calculate the mean or median of the scores.	**Stretch** Invite students to calculate the other measure of center (mean or median) and decide which value they think is more typical.

Accessibility

Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Comprehension . . .

Lesson 15: Students use models and visuals to make sense of how well a line fits a set of data, which guides information processing and visualization.

Accessibility: Visual-Spatial Processing
As students interpret visual representations, discuss how the point on the graph is connected to the avocado order before making a prediction.

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Visual-Spatial processing . . .

Here are two suggestions in this unit for supporting students with interpreting visual representations.

Lesson 5, Activity 2

Accessibility: Executive Functioning Consider chunking this activity by having students list what they can easily see in one dot plot, then in one box plot, then compare their notes.

Lesson 14, Activity 1

Accessibility: Visual-Spatial Processing To support interpreting visual representations, invite students to use their finger to locate 75% on the x-axis then trace up to the line of fit and estimate the y-value by tracing over to the y-axis.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
categorical data **numerical data**	1
bivariate **univariate**	2
box plot **numerical univariate data** **dot plot** **stem-and-leaf plot** **line plot** **histogram**	3
bar chart **categorical univariate data** **circle graph** **segmented bar graph**	4
measure of center	5
outlier	6
sample mean **population mean**	9

New Vocabulary	Lesson(s)
confidence interval **margin of error**	10
association **relative frequency table** **two-way table**	11
residual **residual plot**	15
line of best fit	16
causation **correlation**	17

Review Vocabulary

- bin (of a histogram)
- interquartile range (IQR)
- linear association
- line of fit
- maximum (of a data set)
- mean
- mean absolute deviation (MAD)
- median
- minimum (of a data set)
- negative association
- numerical data
- positive association
- quartile
- range
- slope
- slope-intercept form
- y-intercept

Math Language Routines and Learning Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	**4, 14**
MLR2: Collect and Display	**1–3, 5, 8–10, 12, 15–17**
MLR3: Critique, Correct, Clarify	**3 and 13**
MLR5: Co-craft Questions	**11 and 7**
MLR7: Compare and Connect	**6**

Sub-Unit Language Goals

Students engage in the following language functions:

- Compare and contrast the types of data that can be gathered from questions on a survey.
- Compare and contrast data sets using measures of center and spread appropriate to the shape of the data set and interpret differences in context.
- Describe linear associations in data using the line of fit and correlation coefficient.

English Lagnguage Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from Lesson 1, where students are provided sentence frames to support in explaining their strategy.

ML/EL **Multilingual/English Language Learners** Provide sentence frames to help students write their questions (e.g., "What percent of people _____?" or "How do people feel about _____?") **(Writing and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Exponential and Linear Functions

In this unit, students create linear models to represent situations. **In Unit 5**, students observe how exponential functions will always eventually surpass linear functions.

FL B.E.S.T. Math Benchmark: MA.912.AR.1.1

Example:

Simple and compound interest are examples of real-world situations that can be modeled by linear and exponential functions.

Which type of account is a better investment? When?

Accounts	After 5 Years ($)	After 30 Years ($)
$s(t) = 500 + 50t$	$s(5) = 750.00$	$s(30) = 2000.00$
$c(t) = 500(1.05)^t$	$c(5) = 638.14$	$c(30) = 2160.97$

Exponential Models

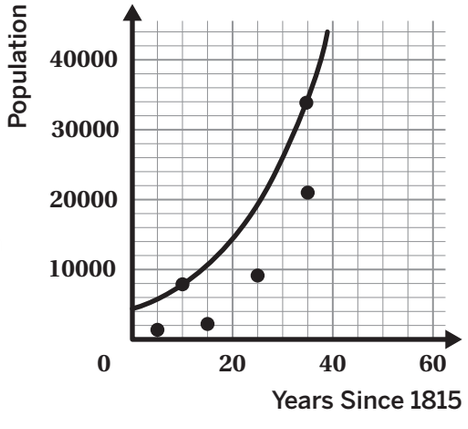
In this unit, students create linear models and assess how well a line fits data. **In Unit 5**, students will fit an exponential function to data and assess the fit.

FL B.E.S.T. Math Benchmark: MA.912.DP.2.4

Example:

As students are introduced to more functions, it is important to determine which type of model is a better fit for the situation and the data.

For example, both a linear and an exponential model could be used to fit the data in this scatter plot. A linear model would create a negative y-intercept, which may not make sense for a population of people. An exponential model may fit the data better, but using that model to make predictions will not be realistic for large values of time.



Quadratic Modeling

In this unit, students model linear situations to analyze relationships between two variables. **In Unit 6**, students will use quadratic functions to analyze nonlinear relationships in two variables. **FL B.E.S.T. Math Benchmark:** MA.912.F.1.8

Example:

If $f(x) = x$ represents the price of rent of each unit and $g(x) = 13000 - \frac{50}{100}x$ represents the number of rented units, based on price x, what is the amount of revenue that City Roots Collective would make at any price, x?

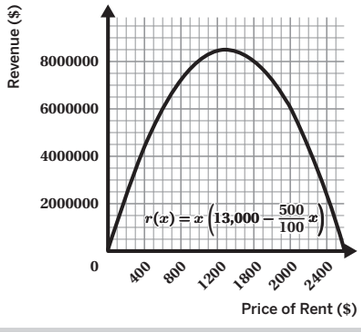
Creating a quadratic model:

Price of Rent • Number of Rented Units = Revenue

$$f(x) \cdot g(x) = r(x)$$

$$r(x) = x(13000 - \tfrac{50}{100}x)$$

It costs City Roots Collective $8,000,000 to build this project. What price should they charge for rent?

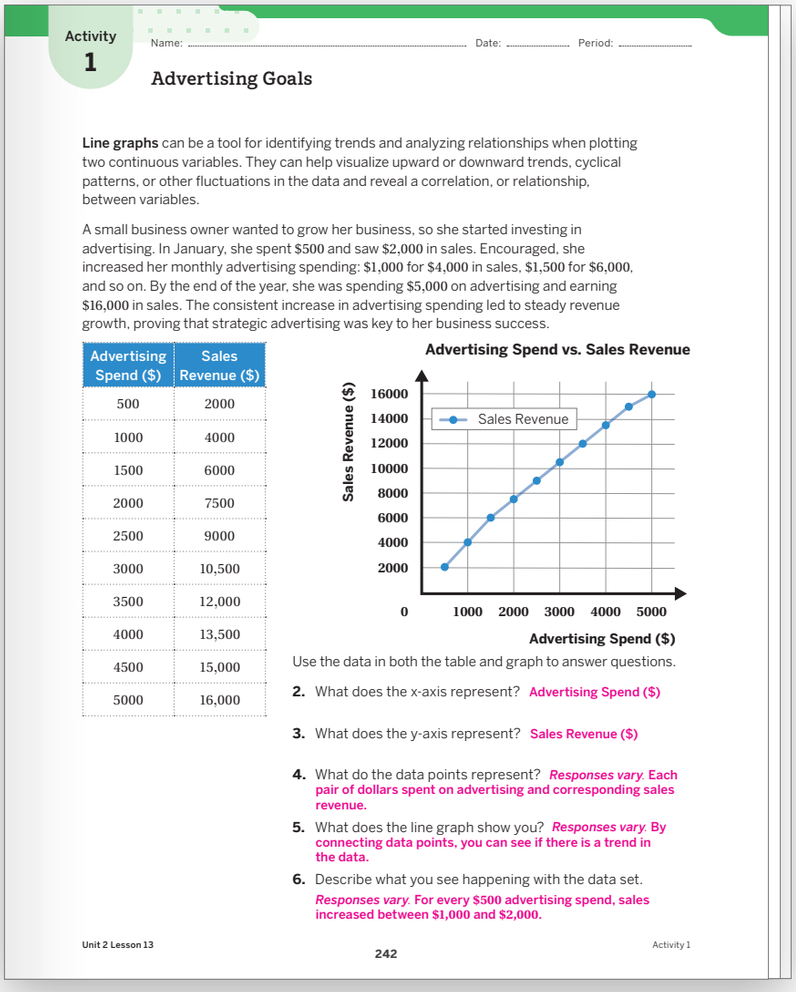


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Connecting Representations

Lesson 13, Activity 1



Activity 1

Name: Date: Period:

Advertising Goals

Line graphs can be a tool for identifying trends and analyzing relationships when plotting two continuous variables. They can help visualize upward or downward trends, cyclical patterns, or other fluctuations in the data and reveal a correlation, or relationship, between variables.

A small business owner wanted to grow her business, so she started investing in advertising. In January, she spent $500 and saw $2,000 in sales. Encouraged, she increased her monthly advertising spending: $1,000 for $4,000 in sales, $1,500 for $6,000, and so on. By the end of the year, she was spending $5,000 on advertising and earning $16,000 in sales. The consistent increase in advertising spending led to steady revenue growth, proving that strategic advertising was key to her business success.

Advertising Spend ($)	Sales Revenue ($)
500	2000
1000	4000
1500	6000
2000	7500
2500	9000
3000	10,500
3500	12,000
4000	13,500
4500	15,000
5000	16,000

Use the data in both the table and graph to answer questions.

2. What does the x-axis represent? Advertising Spend ($)
3. What does the y-axis represent? Sales Revenue ($)
4. What do the data points represent? *Responses vary.* Each pair of dollars spent on advertising and corresponding sales revenue.
5. What does the line graph show you? *Responses vary.* By connecting data points, you can see if there is a trend in the data.
6. Describe what you see happening with the data set.
   Responses vary. For every $500 advertising spend, sales increased between $1,000 and $2,000.

Unit 2 Lesson 13 242 Activity 1

Context

In this activity, students develop adaptive reasoning as they connect the data from the table of values to the data in the line graph and interpreting what the data means.

Try This

Put on your student hat and complete Problems 2–6 for Activity 1.

Points to Ponder

- What do students need to understand about the values in the table and what the x-and y-variables represent to support their decision making about what they see happening with the data set.
- Some students may only describe the direction of the line graph. How might you support your students in understanding how to describe what they see happening with the data set?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1–3	Visualize one-variable data in dot plots and histograms.	Lesson 3	2	MA.4.DP.1.1 MTR.4.1
4–5	Calculate means and medians.	Lesson 5	1	MA.5.DP.1.2
6	Use center and spread to compare data sets.	Lesson 7	2	MA.5.DP.1.2 MA.6.DP.1.4 MTR.7.1
7a–b	Interpret scatter plots.	Lesson 14	2	MA.8.DP.1.1 MTR.7.1
7c–d	Construct and use a line of fit to solve problems in context.	Lesson 16	2	MA.8.DP.1.3 MA.8.AR.3.5 MTR.7.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 2

1. This dot plot shows the number of hours students in Dylan's class spent on their phones last week. Dylan spent 4 hours on his phone last week.

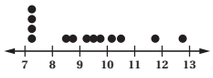


a. Circle a dot that could represent Dylan's data.
Responses vary. Any of the three dots at 4 on the number line.

b. Is this the only dot that could represent Dylan's data?
No

Explain your thinking.
Explanations vary. There are three different dots at 4 on the number line. Any of them could represent Dylan.

2. This dot plot shows hourly rate for babysitting charged by babysitters in 13 different neighborhoods.

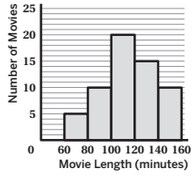


Source: *U.S. Department of Labor*

Make a histogram using the data from the dot plot.



3. Kadeem wants to know how long his favorite movies are. Here is a histogram of the data he collected.



a. How many movies did Kadeem consider?
60 movies

b. How many of the movies are less than 2 hours long (120 minutes)?
35 movies

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Name: Date: Period:

Pre-Unit Check (continued)

Unit 2

4. The table shows how many brownies Omari ate.

Day	Number of Brownies
Monday	4
Tuesday	0
Wednesday	3
Thursday	5
Friday	4

Calculate the mean (average) of this data.
3.2 brownies

5. The middle value of an ordered data set is called the *median*.

Determine the median of each data set.

a. 14, 19, 15, 20, 17
17

b. 25, 22, 19, 21, 14, 14
20

c. 18, 12, 24, 12, 18, 22, 22
18

D Differentiation (Pre-Unit Check)

Note: To **Strengthen** and **Stretch** students' learning, refer to the differentiation resources suggested throughout this unit.

Prior Knowledge	Problem(s)	Respond to Student Thinking
Visualize one-variable data in dot plots and histograms. **(Preparation for Lessons 3–4)**	1–3	**Support** • **Teacher Move:** Before Lesson 3, consider creating an anchor chart with an example of how to represent a data set using a dot plot and histogram. • **Lesson:** Consider revisiting Grade 6, Unit 8, Lesson 3 *(The Plot Thickens)* **Note:** If the class shows a strong understanding, consider omitting or spending less time on Lesson 3.
Calculate the mean and median for a data set. **(Preparation for Lessons 5–6)**	4–5	**Support** • **Teacher Move:** During Lesson 5, consider collecting common strategies and errors when calculating mean and median and creating an anchor chart for students' reference. • **Lesson:** Grade 6, Unit 8, Lesson 6 *(Snack Time)* • **Lesson:** Grade 6, Unit 8, Lesson 8 *(Toy Cars)* **Note:** If the class shows a strong understanding, consider omitting or spending less time on Lesson 5.
Use center and spread to compare data sets. **(Preparation for Lessons 7–13)**	6	**Support** • **Teacher Move:** During the Lesson 7 Warm-Up, look for students' responses related to the center and spread of the data sets and invite those students to explain their thinking. • **Lesson:** Grade 7, Unit 8, Lesson 9 (Car, Bike, or Train?)
Use scatter plots and a line of fit to analyze bivariate data. **(Preparation for Lessons 14–17)**	7	**Support** • **Teacher Move:** During Lesson 14, Activity 1, consider spending extra time gathering student strategies for using a line of fit to make a prediction. • **Lesson:** Grade 8, Unit 6, Lesson 3 *(Robots)* • **Lesson:** Grade 8, Unit 6, Lesson 6 *(Find the Fit)* • **Lesson:** Grade 8, Unit 6, Lesson 9 *(Animal Brains)*

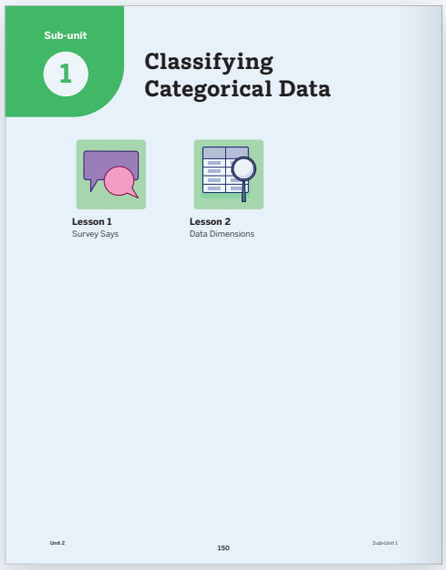
Sub-Unit 1

Classifying Data

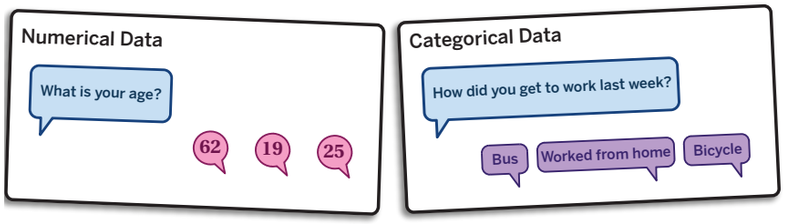
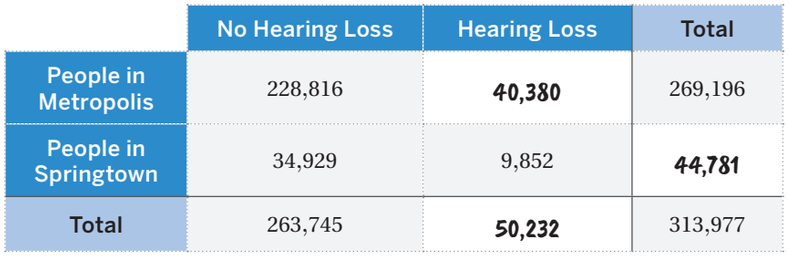
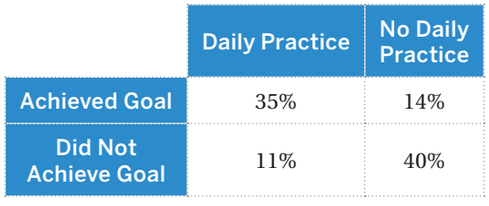
Sub-Unit 1 Goals

- **Goal:** Determine whether data is univariate or bivariate.
- **Language Goal:** Compare and contrast the types of data that can be gathered from questions on a survey.

In this sub-unit, students distinguish between the types of questions that generate **categorical data** and **quantitative data**. Students learn how to distinguish between **univariate data** and **bivariate data**.



Math that Matters Most

Concept or Skill	For example . . .
Distinguish between questions that generate categorical or numerical data.	
Calculate values in a two-way table.	
Interpret values in a relative frequency table for a situation.	A group of student athletes set personal goals for a big race. After the race, they were asked whether they achieved their goal and whether they practiced daily. This table shows the results. What does 35% represent? **35% represents the students who practiced daily and achieved their personal goal.** 

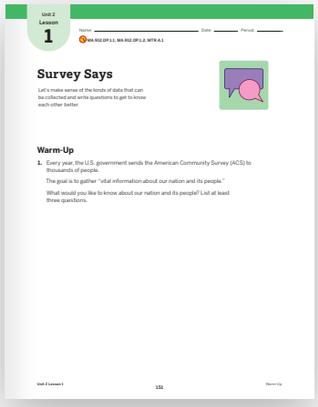
	No Hearing Loss	Hearing Loss	Total
People in Metropolis	228,816	**40,380**	269,196
People in Springtown	34,929	9,852	**44,781**
Total	263,745	**50,232**	313,977

	Daily Practice	No Daily Practice
Achieved Goal	35%	14%
Did Not Achieve Goal	11%	40%

Unit 2

Lesson

1


Unit 2
Lesson
1
Survey Says
Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.
Warm-Up
1. Every year, the U.S. government sends the American Community Survey (ACS) to thousands of people.
The goal is to gather "vital information about our nation and its people."
What would you like to know about our nation and its people? List at least three questions.

Survey Says

What Kinds of Data Can I Collect?

Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.

Learning Outcomes

Today's Goal

1. **Language Goal:** Explain the difference between *numerical* and *categorical data*. **(Writing, Speaking, and Listening)**

Students make sense of the kinds of data that can be collected and use questions to get to know each other better. They consider questions from the American Community Survey, sorting them by whether they generate **numerical data** or **categorical data**. They critique different versions of questions and think critically about the kind of data the questions would generate. They write their own questions to gather data from their classmates and summarize that data. **(MTR.4.1)**

Coherence

Prior Learning

In Grade 6, students made connections between questions, claims, and data. They also explained the difference between categorical and numerical data.

Future Learning

In Lessons 11–12, students will use two-way frequency tables to represent categorical data and determine whether there is an association in the data. In Lessons 5–17, students will investigate and represent the relationships in quantitative data.

Connections

- Students explore different kinds of data that can be collected and how to write good survey questions to produce qualitative data to learn more about their classmates. (cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Building Toward

MA.912.DP.1.1

Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.

MA.912.DP.1.2

Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.DP.1.1

MA.6.DP.1.3

Vocabulary

New Vocabulary

categorical data

quantitative data

Review Vocabulary

numerical data

Additional Vocabulary

survey

Lesson at a Glance

~ 45 min

MA.912.DP.1.1, MA.912.DP.1.2

Warm-Up

Pairs | 5 min

Students are introduced to the American Community Survey and brainstorm questions they would like to ask in a large scale survey of Americans.



Activity 1

Pairs | 10 min

Students make sense of two types of data, **categorical** and **numerical**, and determine which type of data different questions would produce.

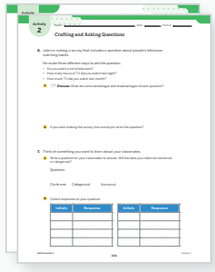
Routines: Notice and Wonder, Decide and Defend (MTR.4.1)



Activity 2

Small Groups | 20 min

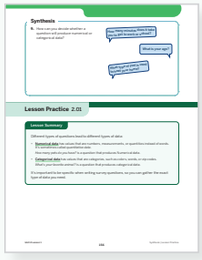
Students critique survey questions, attend to precision in writing their own questions to produce data from their classmates, then summarize and visualize their data. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

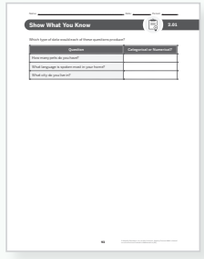
Students synthesize their understanding of the difference between categorical and numerical data and how to decide which kind a question generates. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by sorting questions according to which kind of data they will produce.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the American Community Survey and brainstorm questions they would like to ask in a large scale survey of Americans.

1 Launch

Display the information about the American Community Survey (ACS) and the prompt.

Consider asking, "Have you ever created or responded to a *survey* before? When? What was that experience like?"

Invite students to generate an example together as a whole class (e.g., "What percent of people own pets?").

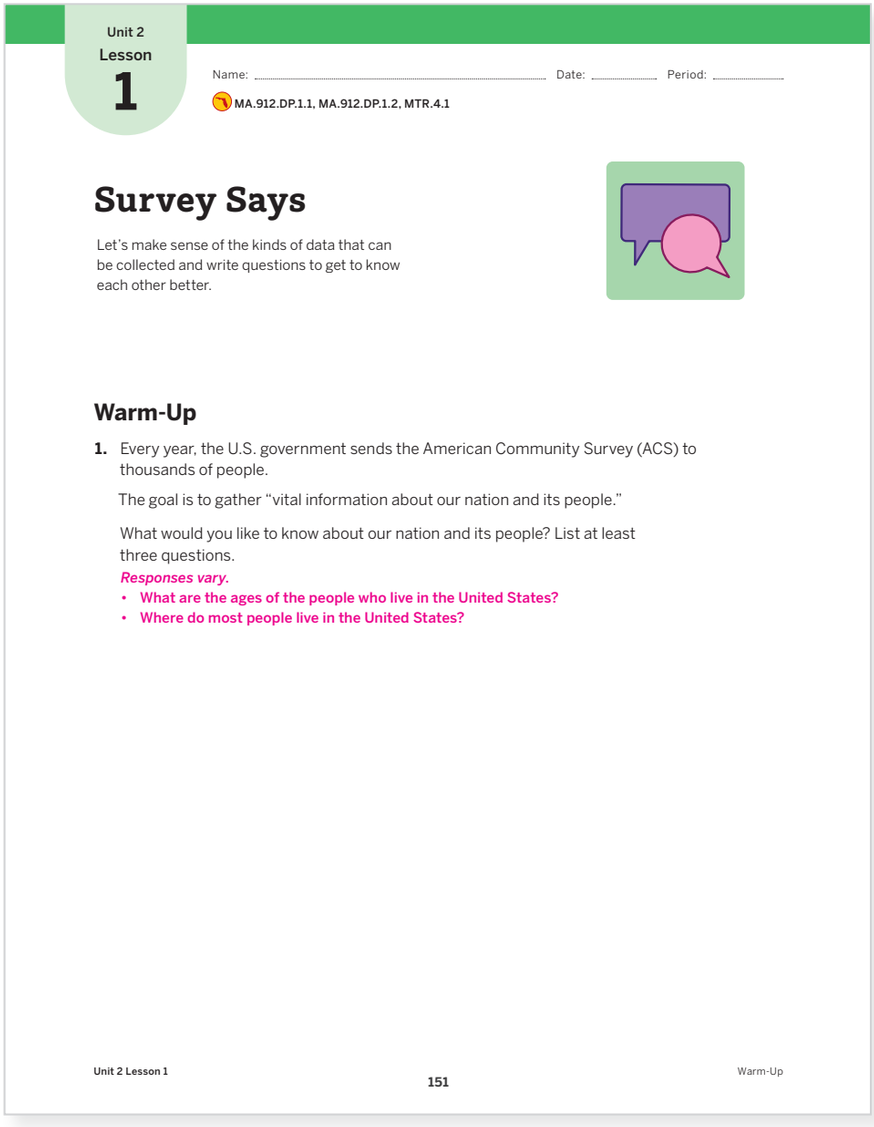
ELL **English Language Learners** Provide sentence frames to help students write their questions (e.g., "What percent of people _____?" or "How do people feel about _____?").

Consider sharing that the American Community Survey helps local officials, community leaders, and businesses understand the changes taking place in their communities.

2 Connect

Invite students to share their questions and why they are curious to know that information.

Note: When students have responded, consider making a large list of what students would like to know about, then revisiting to see if any of these questions emerge later in Activity 1.



Unit 2
Lesson
1

Name: _____ Date: _____ Period: _____

MA.912.DP.1.1, MA.912.DP.1.2, MTR.4.1

Survey Says

Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.

Warm-Up

1. Every year, the U.S. government sends the American Community Survey (ACS) to thousands of people.

   The goal is to gather "vital information about our nation and its people."

   What would you like to know about our nation and its people? List at least three questions.

   Responses vary.
   - What are the ages of the people who live in the United States?
   - Where do most people live in the United States?

Unit 2 Lesson 1 151 Warm-Up

Pairs | 10 min

Activity 1 Types of Data

Purpose: Students make sense of two types of data, **categorical** and **numerical**, and determine which type of data different questions would produce. **(MTR.4.1)**

1 Launch

Display the anonymous people, the ACS questions, and the prompt.

Consider asking:
- "Why might the ACS be interested in the answers to these questions?"
- "Which of these are you most curious about?"

ELL **English Language Learners** To help students access background knowledge, clarify each question as a class or explain phrases like "main occupation" or "monthly rent."

Use the Notice and Wonder routine to promote curiosity and help students make sense of the survey responses. **(MTR.4.1)**

Look for students who notice different things about the responses, including that sometimes the responses are words and sometimes they are numbers.

Invite students to share what they notice and wonder.

A **Accessibility: Memory and Attention** Provide students with a graphic organizer to keep track of what they notice about the responses to each question.

2 Monitor

Display the questions and the names of the two different types of data.

Listen for students who describe their thinking about categorical and numerical data in different ways.

Pause to discuss and come to consensus about the difference between categorical and numerical data. Consider asking, "How would you describe the difference between these two types of data to a student who was absent?"

MLR **MLR2: Collect and Display** Create an anchor chart with a class definition for the terms **categorical data** and **numerical data** using students' language and the glossary.

ELL **English Language Learners** Invite students to research how to say the new terms in their home language(s). Add the new term to the anchor chart in those languages, along with examples.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Types of Data

2. Here are the initials of seven people who took the American Community Survey and their responses.

a. Here are the responses to each question.

Anonymous People	What is your age?	How many people live with you, including you?	What is your main occupation?	How did you get to work or school last week?	How many miles away do you live from the closest grocery store?
M.A.	19	1	Cook	Bicycle	2
P.Y.	25	3	Construction worker	Car	10
Z.M.	53	2	Nurse	Walked	0.5
B.H.	62	5	I don't work	–	–
J.S.	44	7	Accountant	Worked from home	15
O.L.	12	3	Student	Car	–
A.B.	29	4	Carpenter	Bus	0.25

b. What do you notice about the responses? What do you wonder?

I notice:
- I notice that some responses are words, some are numbers, and some have blanks.
- I notice that not every person responded to every question.
- I notice that the youngest age is 12 and the oldest age is 62.

I wonder:
- I wonder why people gave their initials.
- I wonder how people answer the transportation question if they use multiple modes of transport.

Unit 2 Lesson 1 152 Activity 1

Pairs | 10 min

Activity 1 Types of Data (continued)

Purpose: Students make sense of two types of data, **categorical** and **numerical**, and determine which type of data different questions would produce. **(MTR.4.1)**

2 Monitor

Display the questions, and consider sharing that most of these questions come from the ACS.

A **Accessibility: Executive Functioning** Consider chunking this activity into more manageable parts by inviting students to choose one card at a time and decide what kind of data it produces.

Invite students to sort the questions by the type of data they produce.

Listen for student reasoning to share during the Connect.

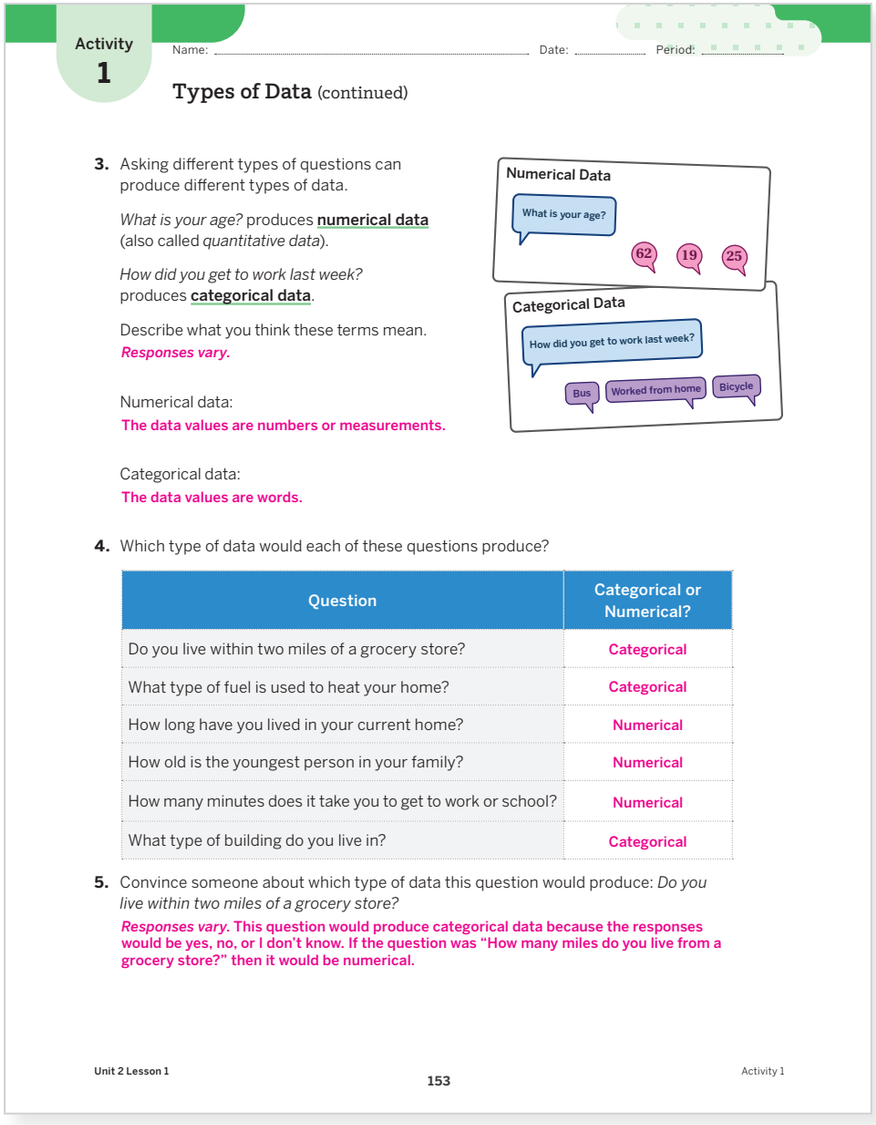
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 2)*	**Support** Encourage them to choose one question and write down different ways a person could answer it. Then, consider asking, "Do these responses fit our definition of categorical or numerical data better? Why do you think so?"
Think the question "Do you live within 2 miles of a grocery store?" produces numerical data because of the 2. *(Problems 4-5)*	**Support** Celebrate their noticing of the number as relevant information. Consider asking, "What would your answer be to this question? Is that answer numerical or categorical?"
Offer an argument that makes reference to the class definitions of numerical data and categorical data.	**Strengthen** Invite them to share their thinking during the Connect.
Would like a challenge.	**Stretch** Invite them to change the question so that it would produce numerical data instead of categorical.

3 Connect

Consider using the Decide and Defend routine to support students in making arguments. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, invite students to make arguments and justify their reasoning about the question.



Activity 1

Name: _____ Date: _____ Period: _____

Types of Data (continued)

3. Asking different types of questions can produce different types of data.

What is your age? produces **numerical data** (also called *quantitative data*).

How did you get to work last week? produces **categorical data**.

Describe what you think these terms mean.

Responses vary.

Numerical data:

The data values are numbers or measurements.

Categorical data:

The data values are words.

4. Which type of data would each of these questions produce?

Question	Categorical or Numerical?
Do you live within two miles of a grocery store?	Categorical
What type of fuel is used to heat your home?	Categorical
How long have you lived in your current home?	Numerical
How old is the youngest person in your family?	Numerical
How many minutes does it take you to get to work or school?	Numerical
What type of building do you live in?	Categorical

5. Convince someone about which type of data this question would produce: *Do you live within two miles of a grocery store?*

Responses vary. **This question would produce categorical data because the responses would be yes, no, or I don't know. If the question was "How many miles do you live from a grocery store?" then it would be numerical.**

Unit 2 Lesson 1 153 Activity 1

Key Takeaway: **Numerical data** is generated by questions that can be answered with a number, like "How many pets do you have?" **Categorical data** is generated by questions that can be answered with words, like "What kind(s) of pets do you have?"

Small Groups | 20 min

Activity 2 Crafting and Asking Questions

Purpose: Students critique survey questions, attend to precision in writing their own questions to produce data from their classmates, then summarize and visualize their data. **(MTR.4.1)**

Short on time? Consider inviting students to answer fewer of their classmates on Problem 7.

1 Launch

Display Jalen's questions and the prompts.

Invite students to read each question aloud.

Consider asking:
- "How are these questions different?"
- "Why will they produce different kinds of data?"
- "Why might these questions not produce the data Jalen is looking for?

A **Accessibility: Conceptual Processing** Invite students to answer Jalen's question first to make the comparison more concrete. Then consider asking, "How would someone else answer the question? Why might it be challenging to answer each question accurately?"

Capture and share several students' questions, discussing the advantages of each question and how to make them more precise. **(MTR.4.1)**

2 Monitor

Display the prompt.

Invite students to write their own question using similar strategies for how they improved Jalen's questions.

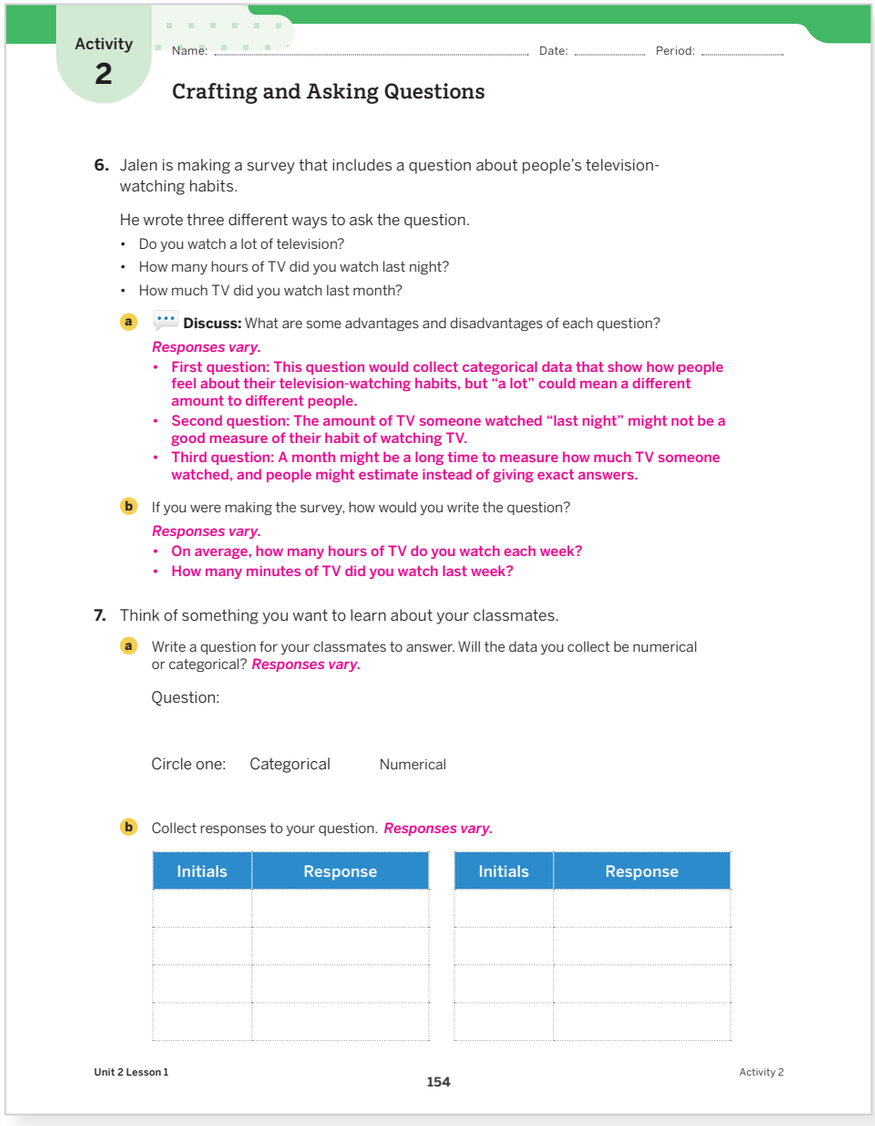
ELL **English Language Learners** Invite students to talk through their questions with their group. It may be helpful for students to alter questions from earlier in the lesson.

Encourage students to take time to answer at least ten of their classmates' questions.

A **Accessibility: Memory and Attention** Display the class definitions from earlier in the lesson for students to reference to activate prior knowledge.

D **Differentiation: (Support)** If students are unsure which kind of data their or a classmate's question will produce, encourage them to write down as many possible answers to the question as they can think of and use the class definition to help them decide.

Activity 2 continued >



Activity 2

Name: Date: Period:

Crafting and Asking Questions

6. Jalen is making a survey that includes a question about people's television-watching habits.

He wrote three different ways to ask the question.
- Do you watch a lot of television?
- How many hours of TV did you watch last night?
- How much TV did you watch last month?

a. **Discuss:** What are some advantages and disadvantages of each question?

Responses vary.
- First question: This question would collect categorical data that show how people feel about their television-watching habits, but "a lot" could mean a different amount to different people.
- Second question: The amount of TV someone watched "last night" might not be a good measure of their habit of watching TV.
- Third question: A month might be a long time to measure how much TV someone watched, and people might estimate instead of giving exact answers.

b. If you were making the survey, how would you write the question?

Responses vary.
- On average, how many hours of TV do you watch each week?
- How many minutes of TV did you watch last week?

7. Think of something you want to learn about your classmates.

a. Write a question for your classmates to answer. Will the data you collect be numerical or categorical? *Responses vary.*

Question:

Circle one: Categorical Numerical

b. Collect responses to your question. *Responses vary.*

Initials	Response	Initials	Response

Unit 2 Lesson 1 154 Activity 2

Activity 2 Crafting and Asking Questions (continued)

Purpose: Students critique survey questions, attend to precision in writing their own questions to produce data from their classmates, then summarize and visualize their data. **(MTR.4.1)**

Short on time? Consider inviting students to answer fewer of their classmates on Problem 7.

2 Monitor

Display the prompt.

Look for students who summarize their data in ways that will be revisited later in the unit (e.g., by taking an average or making a dot plot). Invite them to share during the Connect.

Note: There is no single correct way to summarize the data at this point in the unit. It is okay if students are unsure how best to describe the data generated by their classmates.

D **Differentiation:** Strengthen If students don't think their question can be improved, invite them to review the responses they received and see if any responses are surprising or were in a form they didn't expect. Then consider asking, "Is there a way you could make the question clearer, or open to a wider range of responses?" *Responses vary.* Revising the categorical question "Do you like pizza?" to "On a scale from 1 to 10, rate how much you like pizza."

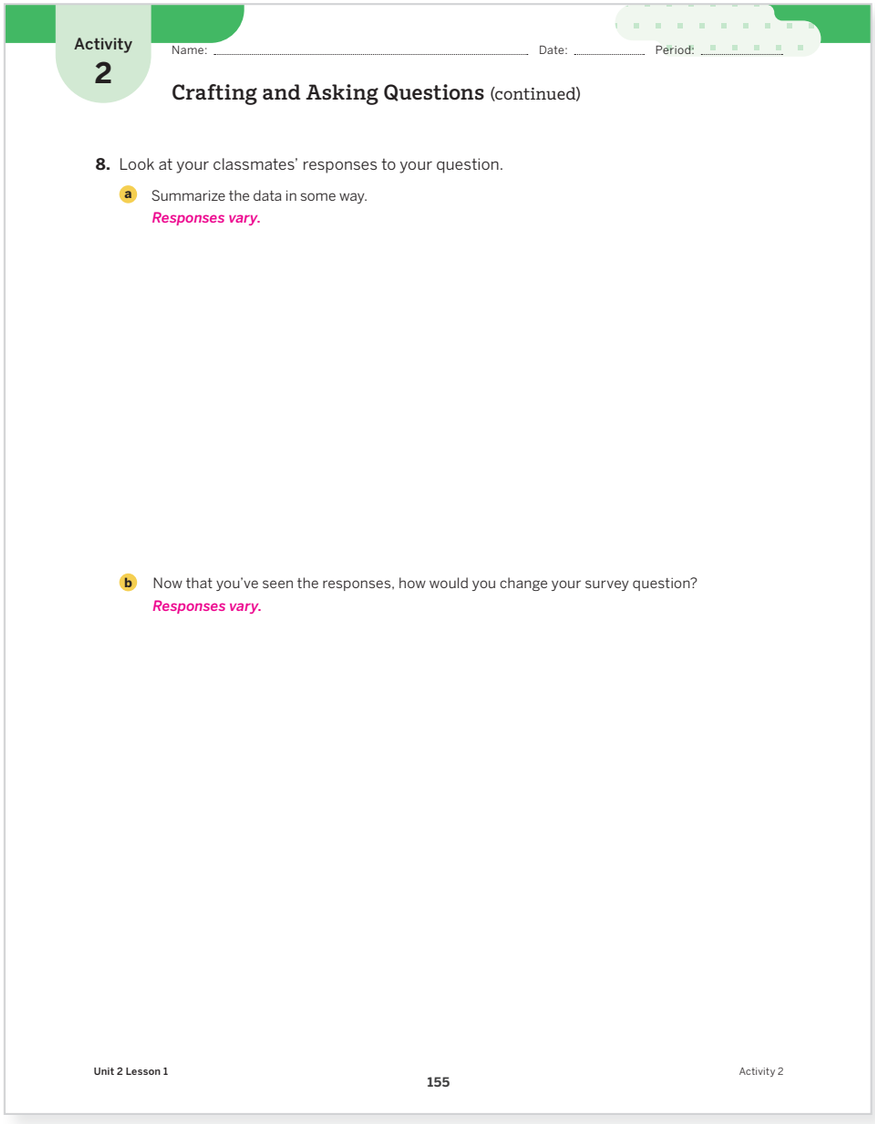
3 Connect

Encourage students to check in with their group about their summaries to see how other students described their data.

Invite students to share how they chose to summarize their data.

To surface the Key Takeaway, consider asking:

- "How did _____ summarize their data? How is this helpful for making sense of their data?"
- "What are some other ways that you could summarize this data?"



Activity 2

Name: _____ Date: _____ Period: _____

Crafting and Asking Questions (continued)

8. Look at your classmates' responses to your question.

a. Summarize the data in some way.
Responses vary.

b. Now that you've seen the responses, how would you change your survey question?
Responses vary.

Unit 2 Lesson 1 155 Activity 2

Key Takeaway: There are many ways to write survey questions so they produce useful data. Once collected, there are also many ways to summarize data visually and numerically.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the difference between categorical and quantitative data and how to decide which kind a question generates.

Invite students to respond independently, and then share their thinking with a partner.

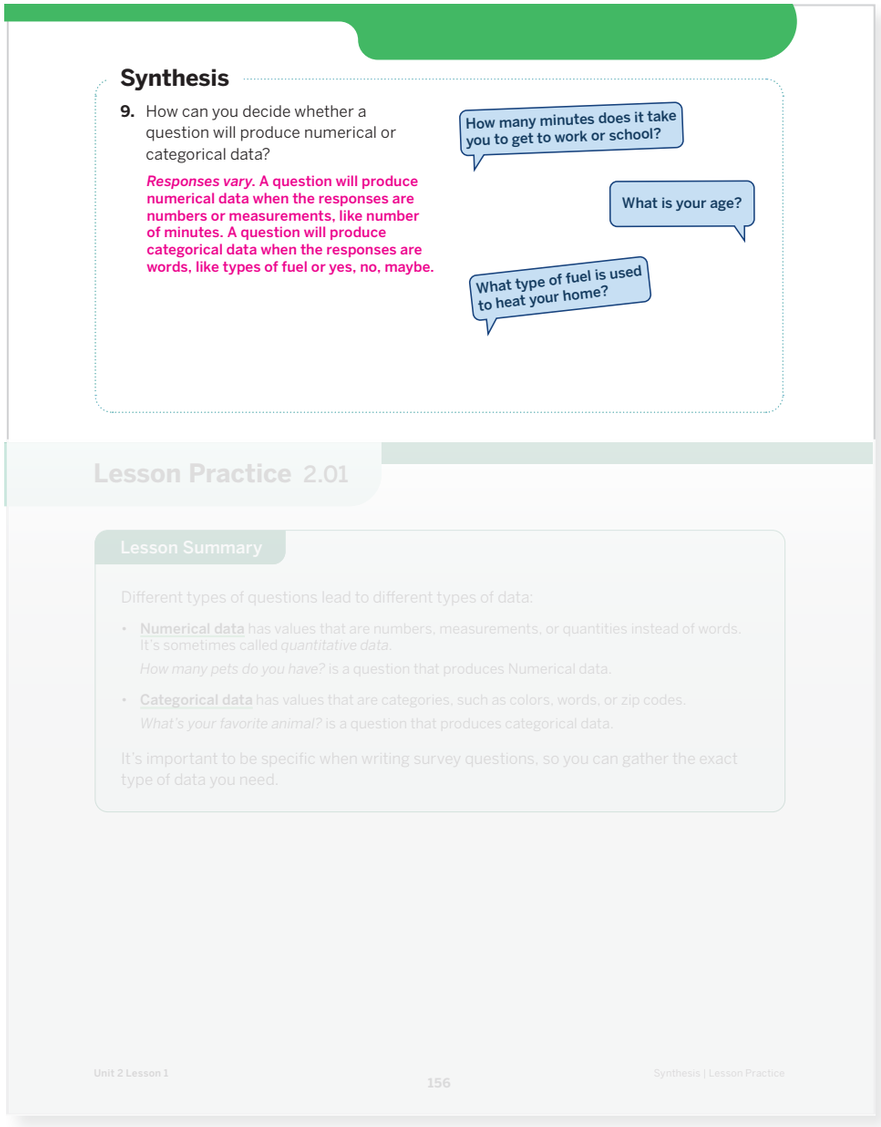
ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "A question will produce numerical data when _____. A question will produce categorical data when _____.").

Capture and share a variety of ideas, such as:

- Examples of questions that will produce numerical data.
- Examples of questions that will produce categorical data.
- Interesting or ambiguous cases, like zip codes.

MLR **MLR2: Collect and Display** Use this outine to formalize the definitions of **numerical** and **categorical data** using students' language. Use students' responses to add to the class definitions and display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. How can you decide whether a question will produce numerical or categorical data?

Responses vary. A question will produce numerical data when the responses are numbers or measurements, like number of minutes. A question will produce categorical data when the responses are words, like types of fuel or yes, no, maybe.

How many minutes does it take you to get to work or school?

What is your age?

What type of fuel is used to heat your home?

Lesson Practice 2.01

Lesson Summary

Different types of questions lead to different types of data:

- Numerical data has values that are numbers, measurements, or quantities instead of words. It's sometimes called *quantitative data*.
  How many pets do you have? is a question that produces Numerical data.
- Categorical data has values that are categories, such as colors, words, or zip codes.
  What's your favorite animal? is a question that produces categorical data.

It's important to be specific when writing survey questions, so you can gather the exact type of data you need.

Unit 2 Lesson 1 156 Synthesis | Lesson Practice

Lesson Takeaway: A statistical question can be answered in words or numbers. **Categorical data** is described using words, and distributes responses into categories. **Numerical data** is described using numbers and the responses can be used to measure and compare data sets. Categorical and numerical data can be summarized in a variety of ways to understand trends in a data set.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by sorting questions according to which kind of data they will produce.

Today's Goal

1. **Language Goal:** Explain the difference between *numerical* and *categorical data*. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.01

Which type of data would each of these questions produce?

Question	Categorical or Numerical?
How many pets do you have?	Numerical
What language is spoken most in your home?	Categorical
What city do you live in?	Categorical

D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support differentiating between categorical and numerical data:

» **Respond to Student Thinking:**

- Revisit this distinction when students look at categorical data again in **Lessons 11–12.**

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to create their own survey to share with their class or school, to determine what type of data they will collect, and to consider how they'll display the results in a meaningful way.

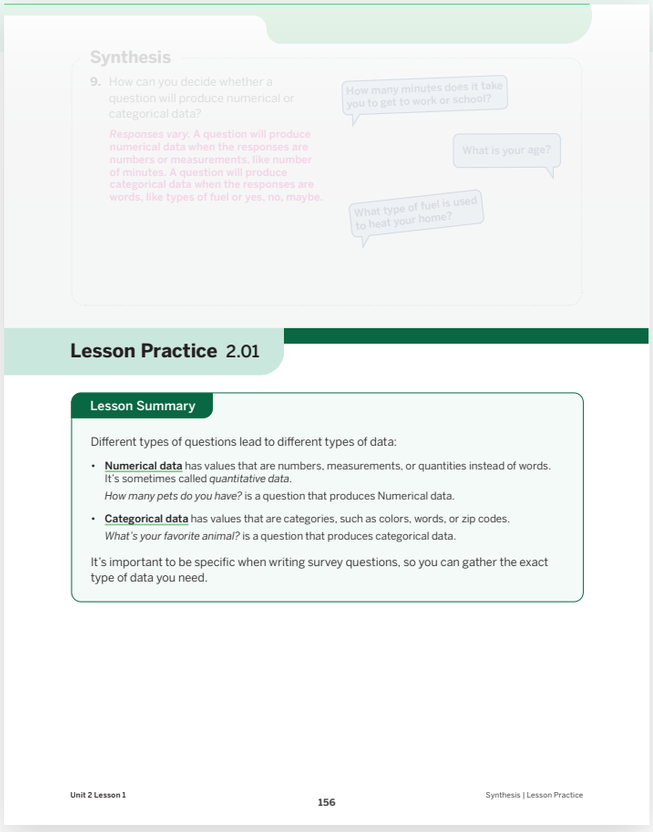
Professional Learning

In this lesson, students ask their own questions to gather data and distinguish between **categorical** and **numerical data**. How will that work support their work in the rest of this unit, which will emphasize numerical data after Lesson 12? How can you create more opportunities for students to see the ongoing importance of categorical data as they continue their statistical journey?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

9. How can you decide whether a question will produce numerical or categorical data?

Responses vary. A question will produce numerical data when the responses are numbers or measurements, like number of minutes. A question will produce categorical data when the responses are words, like types of fuel or yes, no, maybe.

How many minutes does it take you to get to work or school?

What is your age?

What type of fuel is used to heat your home?

Lesson Practice 2.01

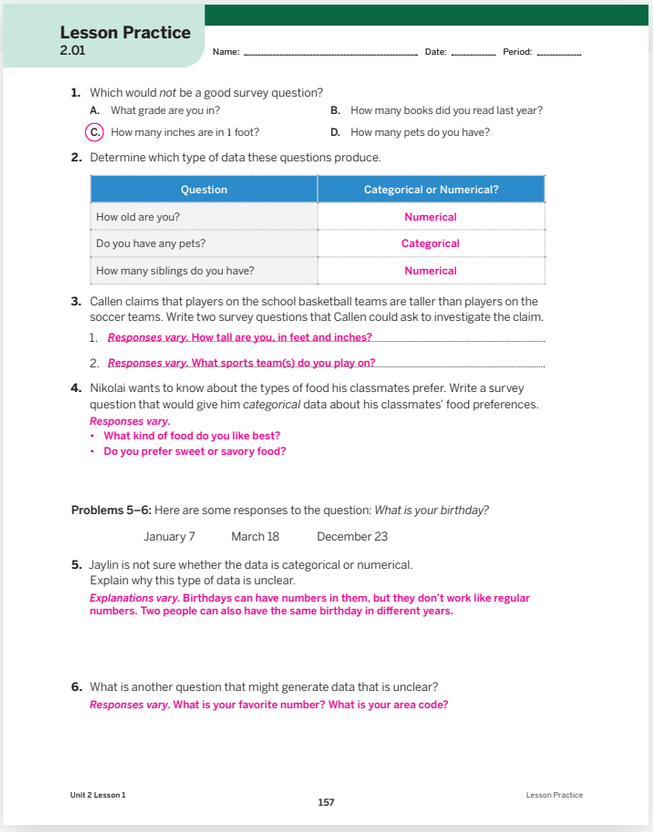
Lesson Summary

Different types of questions lead to different types of data:

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It's important to be specific when writing survey questions, so you can gather the exact type of data you need.

Unit 2 Lesson 1 156 Synthesis | Lesson Practice



Lesson Practice
2.01

Name: _____ Date: _____ Period: _____

1. Which would *not* be a good survey question?
   A. What grade are you in?
   B. How many books did you read last year?
   C. How many inches are in 1 foot?
   D. How many pets do you have?

2. Determine which type of data these questions produce.

Question	Categorical or Numerical?
How old are you?	Numerical
Do you have any pets?	Categorical
How many siblings do you have?	Numerical

3. Callen claims that players on the school basketball teams are taller than players on the soccer teams. Write two survey questions that Callen could ask to investigate the claim.
   1. *Responses vary.* How tall are you, in feet and inches?
   2. *Responses vary.* What sports team(s) do you play on?

4. Nikolai wants to know about the types of food his classmates prefer. Write a survey question that would give him *categorical* data about his classmates' food preferences.
   Responses vary.
   - What kind of food do you like best?
   - Do you prefer sweet or savory food?

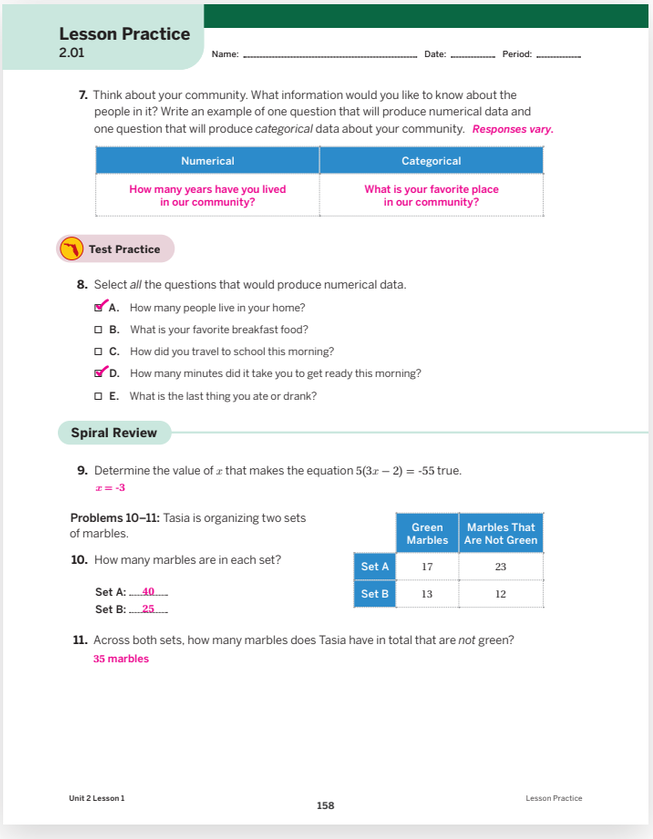
Problems 5–6: Here are some responses to the question: *What is your birthday?*

January 7 March 18 December 23

5. Jaylin is not sure whether the data is categorical or numerical. Explain why this type of data is unclear.
   Explanations vary. Birthdays can have numbers in them, but they don't work like regular numbers. Two people can also have the same birthday in different years.

6. What is another question that might generate data that is unclear?
   Responses vary. What is your favorite number? What is your area code?

Unit 2 Lesson 1 157 Lesson Practice



Lesson Practice
2.01

Name: _____ Date: _____ Period: _____

7. Think about your community. What information would you like to know about the people in it? Write an example of one question that will produce numerical data and one question that will produce *categorical* data about your community. *Responses vary.*

Numerical	Categorical
How many years have you lived in our community?	What is your favorite place in our community?

Test Practice

8. Select *all* the questions that would produce numerical data.
   ☑ A. How many people live in your home?
   ☐ B. What is your favorite breakfast food?
   ☐ C. How did you travel to school this morning?
   ☑ D. How many minutes did it take you to get ready this morning?
   ☐ E. What is the last thing you ate or drank?

Spiral Review

9. Determine the value of x that makes the equation $5(3x - 2) = -55$ true.
   $x = -3$

Problems 10–11: Tasia is organizing two sets of marbles.

	Green Marbles	Marbles That Are Not Green
Set A	17	23
Set B	13	12

10. How many marbles are in each set?
    Set A: 40
    Set B: 25

11. Across both sets, how many marbles does Tasia have in total that are *not* green?
    35 marbles

Unit 2 Lesson 1 158 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.DP.1.2
	3–7	2	MA.912.DP.1.2
Test Practice	8	1	MA.912.DP.1.2
Spiral Review			
Fluency	9	1	MA.8.AR.2.1
	10–11	1	MA.912.DP.3.1*

*These problems build toward the standard shown.

Unit 2

Lesson

2


Unit 2
Lesson
2
Name _____ Date _____ Period _____
MA.912.DP.1.1, MA.912.DP.1.2, MTR.3.1, MTR.4.1
Data Dimensions
Let's look at different data sets and determine if they are univariate or bivariate.
Warm-Up
1. Which one doesn't belong?
Unit 2 Lesson 2
159
Warm-Up

Data Dimensions

Univariate and Bivariate Data

Let's look at different data sets and determine if they are univariate or bivariate.

Learning Outcomes

Today's Goals

1. **Goal:** Describe and differentiate between univariate data and bivariate data.
2. **Language Goal:** Organize and create data sets representing univariate and bivariate data. **(Writing, Speaking, and Listening)**

Students are introduced to two types of data: univariate and bivariate. Students will sort through data sets and analyze them. They also spend time creating their own sets of data.

Coherence

Prior Learning

In Lesson 1, students learned about different types of data they could collect.

Future Learning

In Lesson 7, students will learn about describing data sets.

Connections

- Students make sense of new vocabulary using words and visual representations. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.2
Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.

Mathematical Thinking and Reasoning Standard: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.DP.1.1, MA.912.DP.1.2	**MA.912.DP.1.2**

Vocabulary

New Vocabulary

univariate

bivariate

Review Vocabulary

factor

data sets

relationship

Lesson at a Glance

~ 45 min

MA.912.DP.1.2

Warm-Up

Pairs | 5 min

Students use the Which One Doesn't Belong? routine to support their noticing and precisely describing, comparing, and contrasting different food items.



Activity 1

Whole Class | 15 min

Students learn about univariate and bivariate data.



Activity 2

Small Groups | 15 min

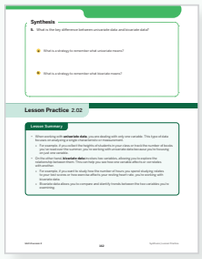
Students organize sets of data and **use the Think-Pair-Share routine** to shape their ideas about categorizing data. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

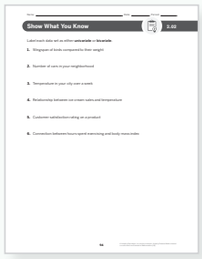
Students synthesize their understanding of univariate and bivariate data.
Routine: MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students will demonstrate their understanding of the terms univariate and bivariate by identifying data sets as one or the other.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students determine "Which One Doesn't Belong." **(MTR.4.1)**

Short on time? Consider calling on a few students instead of many.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing and precisely describing, comparing, and contrasting the different food items. This routine supports students in looking for and making use of structure. Students use their existing ideas and language to decide which of the four foods differs. Encourage students that there are no wrong answers and that creativity is important.

Consider sharing that each food does not belong in some way. Encourage students to look for more than one possibility. **(MTR.5.1)**

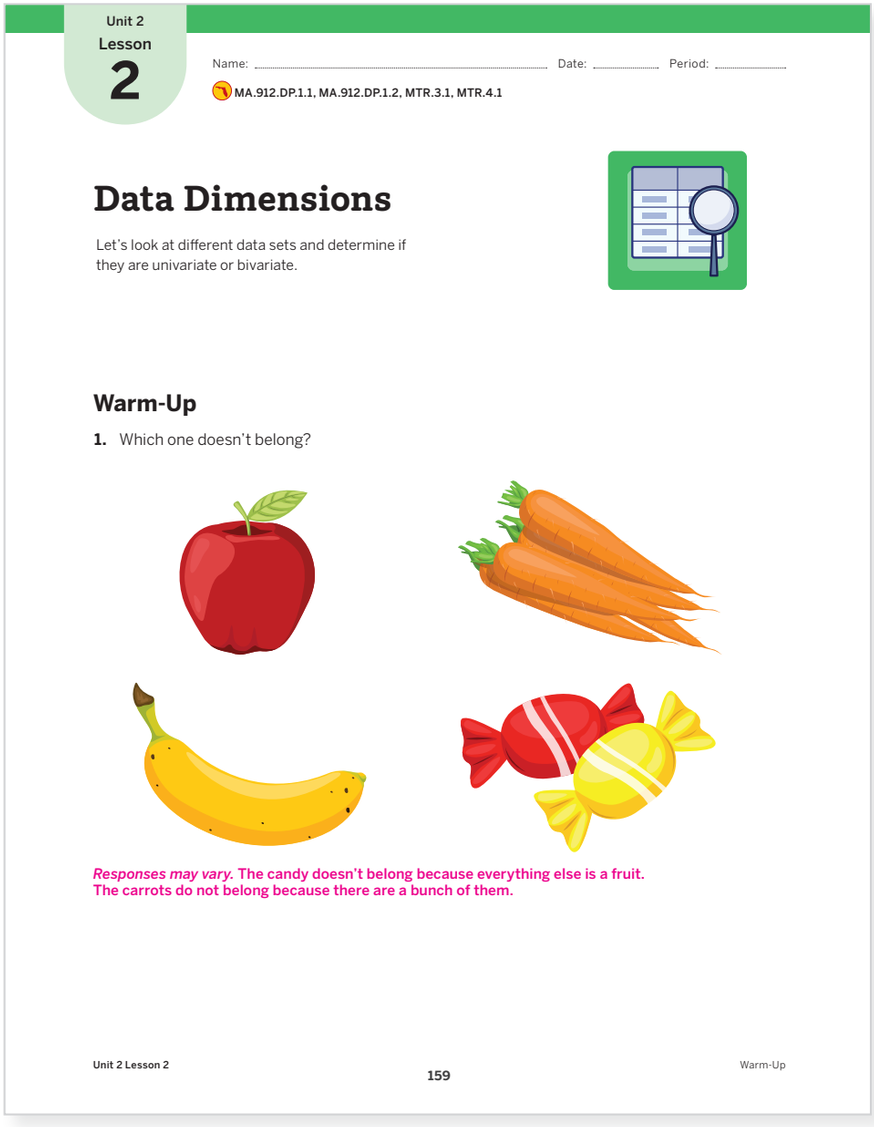
2 Connect

Invite students to share at least one reason why each food doesn't belong.

Create a list for each object and why it doesn't belong. This helps students understand what has already been said.

Consider asking:

- "How is the banana similar/different from the other foods?"
- "What is another way to say why the candy doesn't belong?"
- "Is there another reason why the carrot doesn't belong?"



Unit 2
Lesson
2

Name: _____ Date: _____ Period: _____

MA.912.DP.1.1, MA.912.DP.1.2, MTR.3.1, MTR.4.1

Data Dimensions

Let's look at different data sets and determine if they are univariate or bivariate.

Warm-Up

1. Which one doesn't belong?

Responses may vary. The candy doesn't belong because everything else is a fruit. The carrots do not belong because there are a bunch of them.

Unit 2 Lesson 2 159 Warm-Up

Whole Class | 15 min

Activity 1 Identifying Data

Purpose: Students will be introduced to data sets and create their examples. **(MTR.4.1)**

1 Launch

Discuss the time it takes for students to travel to school.

Consider asking, "Why would someone want to record this data?"

2 Monitor

Select/Capture strategies/explanations/ideas to share on a board where everyone can see.

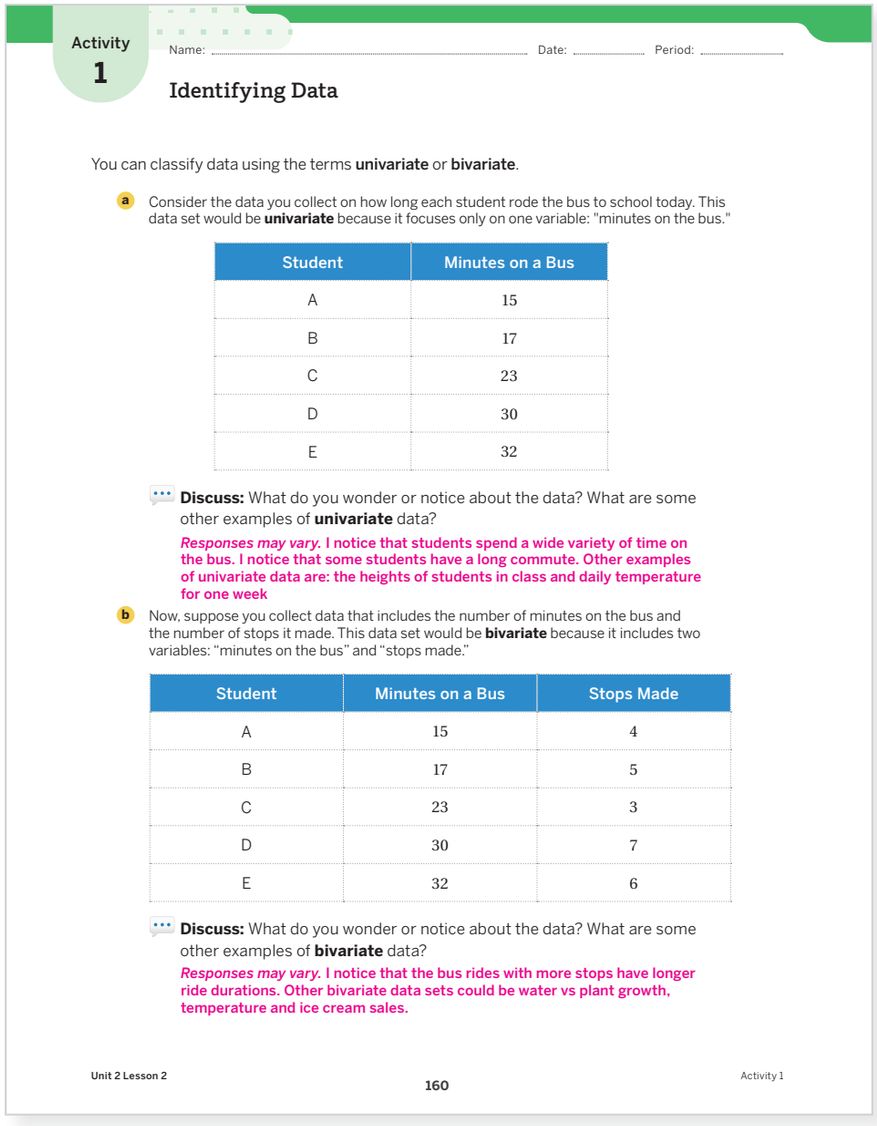
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to engage with the person next to them.
"I walk to school and do not know how long it takes."	**Strengthen** Guide them to make an accurate guess. Ask "Would it take more or less time than walking?"
"I wonder if it takes anyone longer to get to school?"	**Strengthen** Consider asking, "How long do you think it takes during morning traffic?"
"Another example of bivariate data is car speed and fuel use."	**Stretch**, Ask, "What are some other examples of bivariate data related to cars?"

Look for/listen to students creating unique sets of data and ideas.

3 Connect

Discuss: Where did each student get their idea for their data sets?



Activity 1

Name: Date: Period:

Identifying Data

You can classify data using the terms **univariate** or **bivariate**.

a Consider the data you collect on how long each student rode the bus to school today. This data set would be **univariate** because it focuses only on one variable: "minutes on the bus."

Student	Minutes on a Bus
A	15
B	17
C	23
D	30
E	32

Discuss: What do you wonder or notice about the data? What are some other examples of **univariate** data?

Responses may vary. I notice that students spend a wide variety of time on the bus. I notice that some students have a long commute. Other examples of univariate data are: the heights of students in class and daily temperature for one week

b Now, suppose you collect data that includes the number of minutes on the bus and the number of stops it made. This data set would be **bivariate** because it includes two variables: "minutes on the bus" and "stops made."

Student	Minutes on a Bus	Stops Made
A	15	4
B	17	5
C	23	3
D	30	7
E	32	6

Discuss: What do you wonder or notice about the data? What are some other examples of **bivariate** data?

Responses may vary. I notice that the bus rides with more stops have longer ride durations. Other bivariate data sets could be water vs plant growth, temperature and ice cream sales.

Unit 2 Lesson 2 160 Activity 1

Key Takeaway: Students can create examples and understand the differences between univariate and bivariate data.

Small Groups | 15 min

Activity 2 Organizing Data

Purpose: Students organize data sets and use the Think-Pair-Share routine to shape their ideas about categorizing data. **(MTR.5.1)**

Short on time? Consider asking for a few suggestions instead of many.

1 Launch

To surface connections, consider sharing a story about a field trip you recall or a trip you planned.

A **Accessibility: Memory and Attention** To activate prior knowledge, consider asking, "What information have we used earlier that might help us now?"

2 Monitor

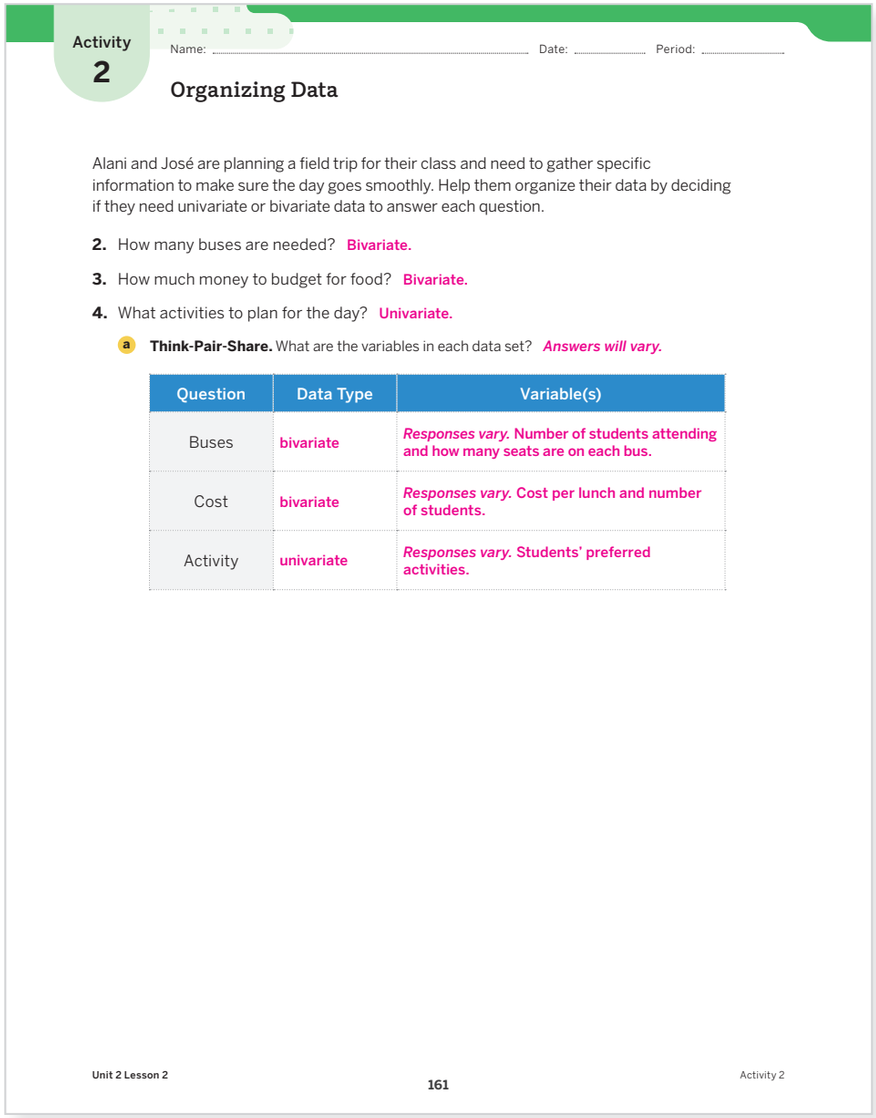
To support students in getting started, invite students to share what they recall and other ways they can categorize data.

D **Differentiation** *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Respond with "Univariate" *(Problem 2)*	**Strengthen** Invite them to list what variables are used when organizing a bus. It's more than just the number of buses.
"I think bivariate" (Problem 4a, cost)	**Strengthen** Consider asking how they know it's bivariate. What factors would they consider?
"How can I tell which is which?" (Problem 4)	**Stretch** Try listing the factors that are involved with each topic.

3 Connect

To surface the Key Takeaway, discuss how there are potentially more than one or two factors when exploring topics.



Activity 2

Name: Date: Period:

Organizing Data

Alani and José are planning a field trip for their class and need to gather specific information to make sure the day goes smoothly. Help them organize their data by deciding if they need univariate or bivariate data to answer each question.

2. How many buses are needed? Bivariate.
3. How much money to budget for food? Bivariate.
4. What activities to plan for the day? Univariate.

a. **Think-Pair-Share.** What are the variables in each data set? *Answers will vary.*

Question	Data Type	Variable(s)
Buses	bivariate	*Responses vary.* Number of students attending and how many seats are on each bus.
Cost	bivariate	*Responses vary.* Cost per lunch and number of students.
Activity	univariate	*Responses vary.* Students' preferred activities.

Unit 2 Lesson 2 161 Activity 2

Key Takeaway: Univariate and bivariate data look different based on the topic.

Whole Class | 5 min

Synthesis

Purpose: Wrap up everything the students have learned today with basic definitions and examples. This is a good time to discuss lingering questions as a class.

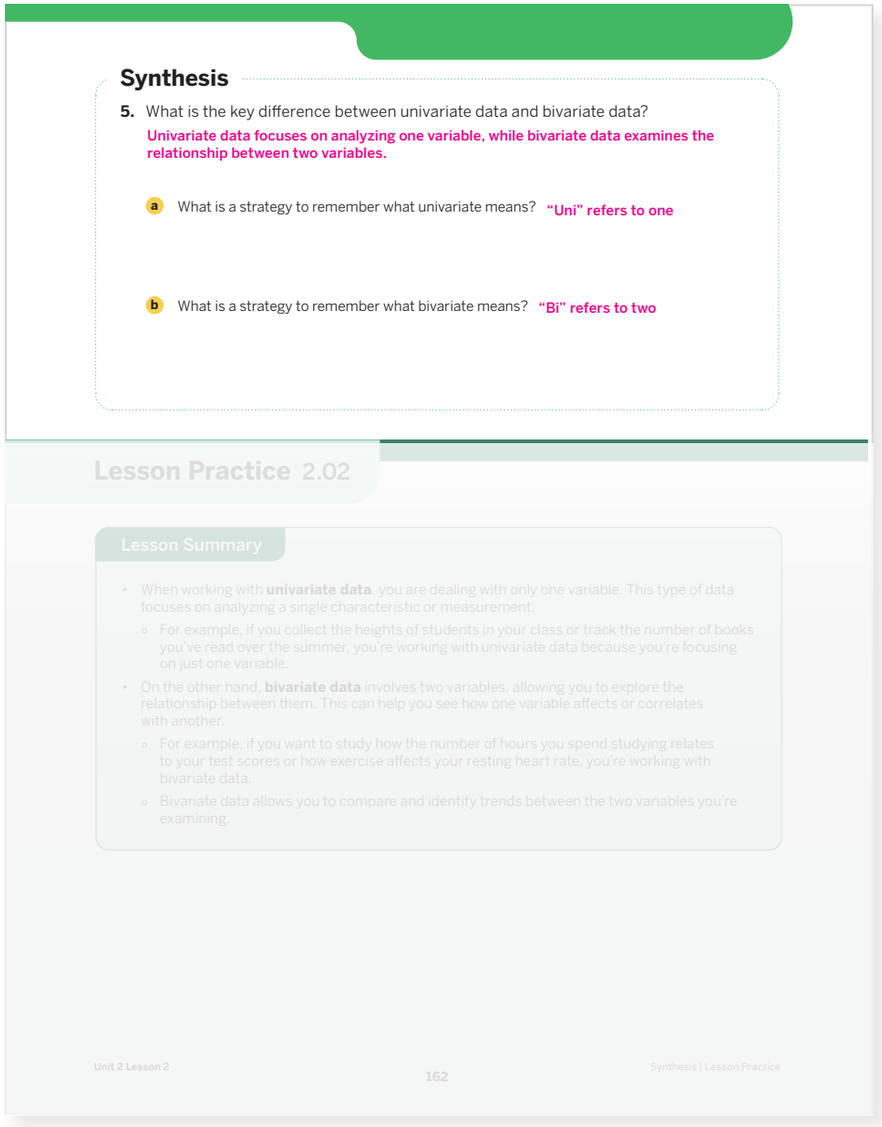
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- "Uni" refers to one variable. It's a helpful trick!
- "Bi" refers to two variables.

MLR **MLR2.1: Collect and Display** Use this routine to collect students' ideas on their answers to the questions. Use an area of the class they can all see.

Note: If time allows, invite students to create more of their own data sets.



Synthesis

5. What is the key difference between univariate data and bivariate data?

Univariate data focuses on analyzing one variable, while bivariate data examines the relationship between two variables.

a. What is a strategy to remember what univariate means? **"Uni" refers to one**

b. What is a strategy to remember what bivariate means? **"Bi" refers to two**

Lesson Practice 2.02

Lesson Summary

- When working with **univariate data**, you are dealing with only one variable. This type of data focuses on analyzing a single characteristic or measurement.
  - For example, if you collect the heights of students in your class or track the number of books you've read over the summer, you're working with univariate data because you're focusing on just one variable.
- On the other hand, **bivariate data** involves two variables, allowing you to explore the relationship between them. This can help you see how one variable affects or correlates with another.
  - For example, if you want to study how the number of hours you spend studying relates to your test scores or how exercise affects your resting heart rate, you're working with bivariate data.
  - Bivariate data allows you to compare and identify trends between the two variables you're examining.

Unit 2 Lesson 2 162 Synthesis | Lesson Practice

🔑 **Lesson Takeaway:** Univariate data focuses on analyzing one variable, while bivariate data examines the relationship between two variables.

Independent | 5 min

Show What You Know

Purpose: Students will demonstrate their understanding of the terms univariate and bivariate by identifying data sets as one or the other.

Today's Goals

1. **Goal:** Describe and differentiate between univariate data and bivariate data.
2. **Language Goal:** Organize and create data sets representing univariate and bivariate data. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.02

Label each data set as either **univariate** or **bivariate**.

1. Wingspan of birds compared to their weight
   bivariate
2. Number of cars in your neighborhood
   univariate
3. Temperature in your city over a week
   univariate
4. Relationship between ice cream sales and temperature
   bivariate
5. Customer satisfaction rating on a product
   univariate
6. Connection between hours spent exercising and body mass index
   bivariate

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support identifying if a data set is represented by univariate or bivariate data:

» **Respond to Student Thinking:**

- Use the *Classifying Data* **Mini-Lesson.**

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Have them plan an imaginary trip and list all the types of univariate and bivariate data they will need.

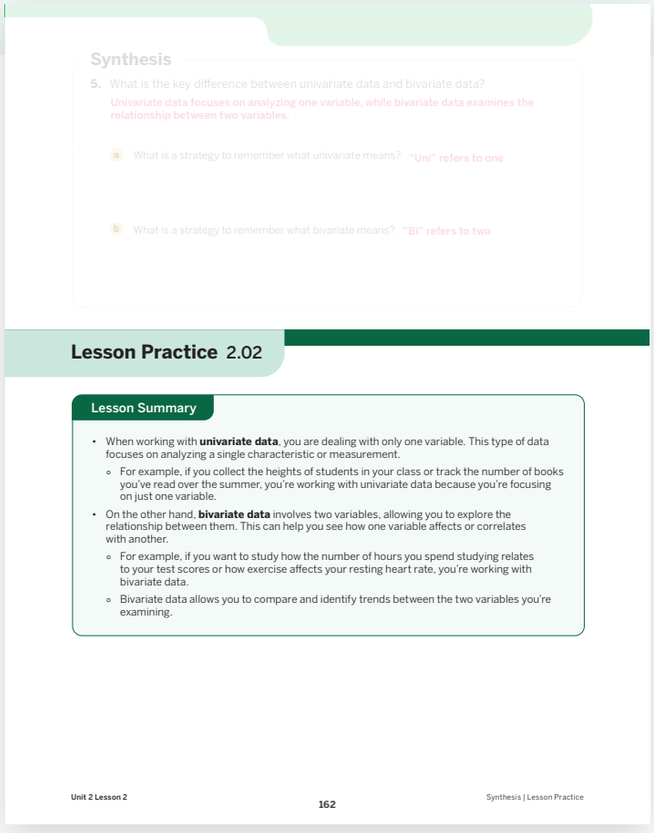
Professional Learning

Reflect on the students' strategies in Activity 2 where students organized data. How did students engage with each other? In what ways did you offer effective, scaffolded support?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

5. What is the key difference between univariate data and bivariate data?

Univariate data focuses on analyzing one variable, while bivariate data examines the relationship between two variables.

a. What is a strategy to remember what univariate means? "Uni" refers to one

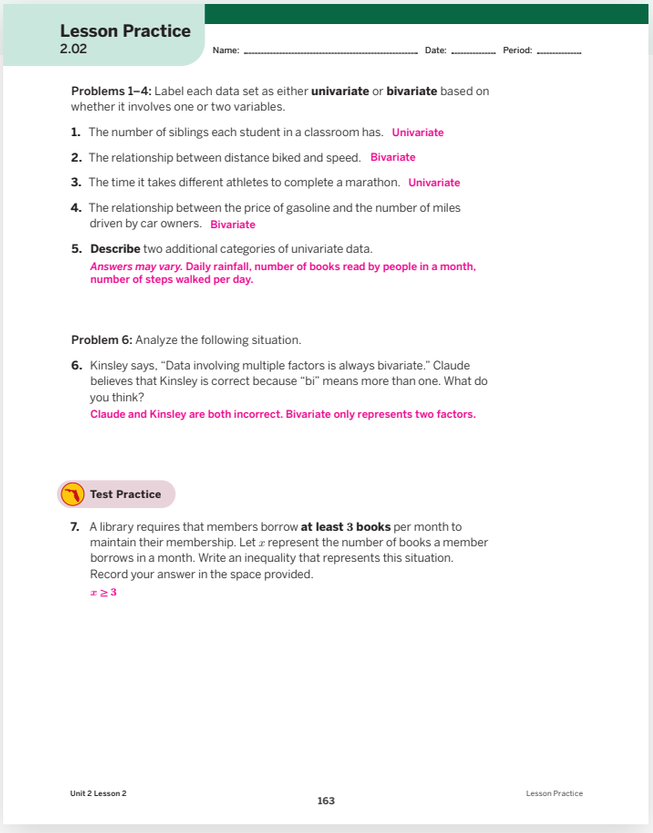
b. What is a strategy to remember what bivariate means? "Bi" refers to two

Lesson Practice 2.02

Lesson Summary

- When working with **univariate data**, you are dealing with only one variable. This type of data focuses on analyzing a single characteristic or measurement.
  - For example, if you collect the heights of students in your class or track the number of books you've read over the summer, you're working with univariate data because you're focusing on just one variable.
- On the other hand, **bivariate data** involves two variables, allowing you to explore the relationship between them. This can help you see how one variable affects or correlates with another.
  - For example, if you want to study how the number of hours you spend studying relates to your test scores or how exercise affects your resting heart rate, you're working with bivariate data.
  - Bivariate data allows you to compare and identify trends between the two variables you're examining.

Unit 2 Lesson 2 162 Synthesis | Lesson Practice



Lesson Practice
2.02

Name: ..... Date: ..... Period: .....

Problems 1–4: Label each data set as either **univariate** or **bivariate** based on whether it involves one or two variables.

1. The number of siblings each student in a classroom has. Univariate
2. The relationship between distance biked and speed. Bivariate
3. The time it takes different athletes to complete a marathon. Univariate
4. The relationship between the price of gasoline and the number of miles driven by car owners. Bivariate
5. **Describe** two additional categories of univariate data.

   Answers may vary. Daily rainfall, number of books read by people in a month, number of steps walked per day.

Problem 6: Analyze the following situation.

6. Kinsley says, "Data involving multiple factors is always bivariate." Claude believes that Kinsley is correct because "bi" means more than one. What do you think?

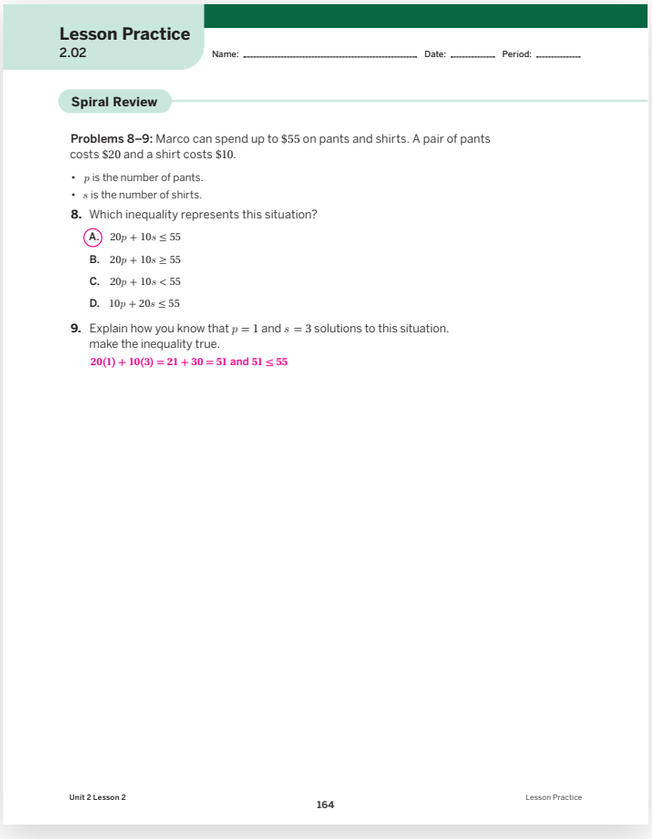
   Claude and Kinsley are both incorrect. Bivariate only represents two factors.

Test Practice

7. A library requires that members borrow **at least 3 books** per month to maintain their membership. Let x represent the number of books a member borrows in a month. Write an inequality that represents this situation. Record your answer in the space provided.

   $x \geq 3$

Unit 2 Lesson 2 163 Lesson Practice



Lesson Practice
2.02

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 8–9: Marco can spend up to \$55 on pants and shirts. A pair of pants costs \$20 and a shirt costs \$10.

- p is the number of pants.
- s is the number of shirts.

8. Which inequality represents this situation?

   A. $20p + 10s \leq 55$
   B. $20p + 10s \geq 55$
   C. $20p + 10s < 55$
   D. $10p + 20s \leq 55$

9. Explain how you know that $p = 1$ and $s = 3$ solutions to this situation. make the inequality true.

   $20(1) + 10(3) = 21 + 30 = 51$ and $51 \leq 55$

Unit 2 Lesson 2 164 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.912.DP.1.2
	5	2	MA.912.DP.1.2
Spiral Review			
Test Practice	6	2	MA.912.AR.1.2
Fluency	8–9	2	MA.912.AR.2.7

Sub-Unit 2

Summarizing One-Variable Data

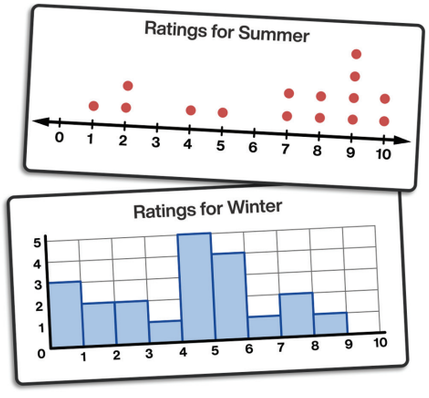
Sub-Unit 2 Goals

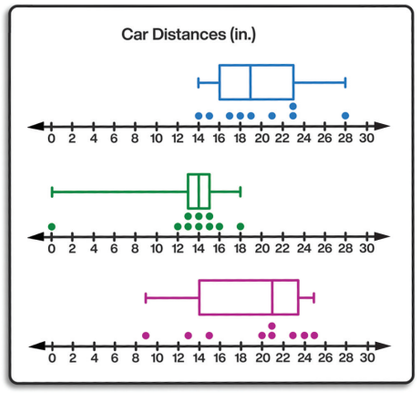
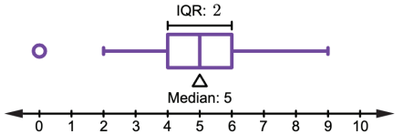
- **Goal:** Represent data with dot plots, histograms, and box plots.
- **Goal:** Describe the shape, center, and spread of data.
- **Language Goal:** Compare and contrast data sets using measures of center and spread appropriate to the shape of the data set and interpret the differences in context.

In this sub-unit, students represent **numerical univariate data** with *line plots*, *histograms*, *box plots*, and *stem-and-leaf plots* and choose the most appropriate display. Students visualize **categorical univariate data** with *bar graphs*, *circle graphs*, *frequency tables*, and *segmented bar graphs* and consider the benefits of each representation. Students use statistics appropriate to the **shape** of the data to compare data sets (*mean* and **standard deviation** or *median* and *IQR*). They use box plots to identify **outliers**.



Math that Matters Most

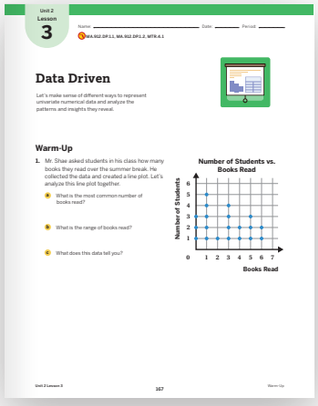
Concept or Skill	For example . . .
Represent data with dot plots, histograms, or box plots.	
Compare data sets using the mean and standard deviation.	<table><tr><th></th><th>Mean (°F)</th><th>Standard Deviation (°F)</th></tr><tr><td>Phoenix, AZ</td><td>88.22</td><td>15.11</td></tr><tr><td>Orlando, FL</td><td>84.41</td><td>8.65</td></tr></table> How do the temperatures in Phoenix compare to the temperatures in Orlando? Use statistics to support your argument.

Concept or Skill	For example . . .
Compare data using the median and IQR.	Which car would you give the Best Overall Car prize to? 
Identify outliers using box plots.	
Determine which measures of center and spread are appropriate when comparing data.	Which measures of center and spread will best help you answer your question? Explain your thinking.

Unit 2

Lesson

3


Unit 2
Lesson
3
Name _____ Date _____ Period _____
MA.912.DP.1.1, MA.912.DP.1.2, MTR.4.1
Data Driven
Let's make sense of different ways to represent univariate numerical data and analyze the patterns and insights they reveal.
Warm-Up
1. Mr. Shaw asked students in his class how many books they read over the summer break. He collected the data and created a line plot. Let's analyze this line plot together.
a. What is the most common number of books read?
b. What is the range of books read?
c. What does this data tell you?
Number of Students vs. Books Read
Number of Students
Books Read
0 1 2 3 4 5 6 7
1 2 3 4 5 6
Unit 2 Lesson 3
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Warm-Up

Data Driven

Revisiting Dot Plots and Histograms

Let's make sense of different ways to represent univariate numerical data and analyze the patterns and insights they reveal.

Learning Outcomes

Today's Goals

1. **Goal:** Analyze histograms, dot plots, stem-and-leaf plots, and box plots and compare them.
2. **Language Goal:** Describe the advantages of using histograms, dot plots, stem-and-leaf plots, and box plots. **(Writing and Speaking)**

Students make sense of **line plots, histograms, stem-and-leaf plots, and box plots** as ways to visualize one-variable data and create graphs. They examine displays that represent different real-world scenarios. **(MTR.4.1)**

Coherence

Prior Learning

In middle school grades, students represented data with line plots, histograms, box plots, and stem-and-leaf plots.

Future Learning

In Lesson 4, students will learn about categorical univariate data.

Connections

- Students analyze data representations as they relate to various real-world contexts such as time spent practicing music. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.1
Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.

Mathematical Thinking and Reasoning Standard: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.5	MA.912.DP.2.2
MA.7.DP1.5	MA.912.DP6.6
MA.8.DP.1.1	

Vocabulary

New Vocabulary

line plot, histogram

Review Vocabulary

bin (of a histogram)

Lesson at a Glance

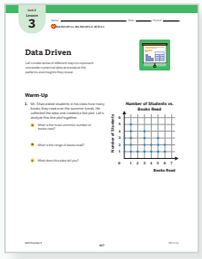
~ 45 min

MA.912.DP.1.1

Warm-Up

Pairs | 5 min

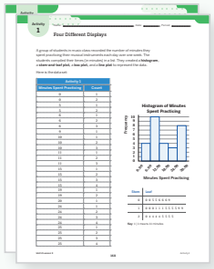
Students use the Think-Pair-Share routine to share analyze a line plot and answer questions.
Routine(s): Think-Pair-Share



Activity 1

Individual | 10 min

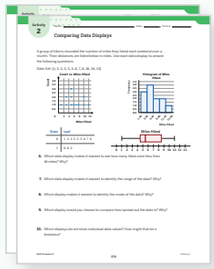
Students analyze the four data displays and answer questions. **(MTR.4.1)**
Routine(s): Collect and Display



Activity 2

Whole Groups | 10 min

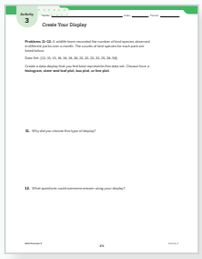
Students create their display with a new set of data and compare it to other displays.
Routine(s): Critique, Correct, Clarify **(MTR.4.1)**



Activity 3

Pairs | 10 min

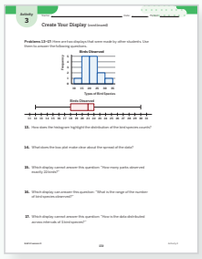
Students create their display with a new set of data and compare it to other displays. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the key features of the four different kinds of displays.



Show What You Know

Independent | 5 min

Students will demonstrate their understanding by determining the type of data display that would best represent a data set and justify their choice.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students analyze a line plot and answer questions.

Short on time? Consider omitting the warm-up.

1 Launch

To surface prior knowledge, consider asking students "Do you read books over the summer?"

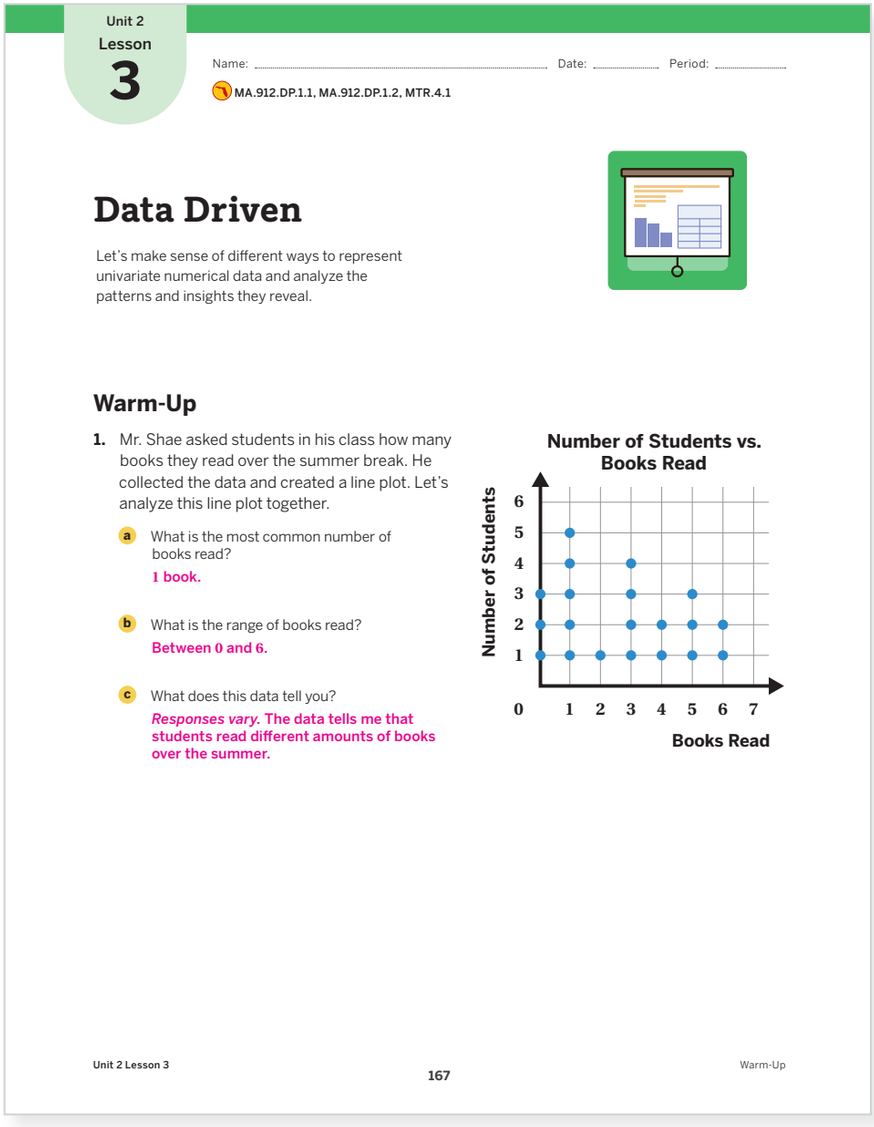
A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific point on the graph and describe what that point means to support interpreting visual representations.

2 Connect

Use the Think-Pair-Share routine to help students discuss how to interpret the line plot. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class. To encourage movement, invite students to find a partner in a different location.

Invite students to share their own thoughts on summer reading. How many books did they read?

Listen for students who understand that the line plot is a graph for showing frequency.



Unit 2
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.DP.1.1, MA.912.DP.1.2, MTR.4.1

Data Driven

Let's make sense of different ways to represent univariate numerical data and analyze the patterns and insights they reveal.

Warm-Up

1. Mr. Shae asked students in his class how many books they read over the summer break. He collected the data and created a line plot. Let's analyze this line plot together.

   a. What is the most common number of books read?
   1 book.

   b. What is the range of books read?
   Between 0 and 6.

   c. What does this data tell you?
   Responses vary. The data tells me that students read different amounts of books over the summer.

Individual | 10 min

Activity 1 Four Different Displays

Purpose: Students analyze the four data displays and answer questions. **(MTR.4.1)**

1 Launch

To surface prior knowledge, consider asking
- "What does each point represent?"
- "What does the line plot tell us about the class?"
- "What is similar about the displays?"
- "What is different about the displays?"

MLR **MLR2: Collect and Display** Use this routine to create a class line plot. Display different students' work around the room for everyone to see and compare.

2 Monitor

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts by inviting students to choose one display at a time and record what surprised them about the data.

Look for/Listen for students who can identify differences and similarities between the displays. Look for students who are struggling.

3 Connect

Invite students to summarize how different displays highlight different elements of the data. It may be helpful to ask:
- "What is the difference between stem-and-leaf plots and box plots?"
- "What is a similarity between line plots and histograms?"
- "Can you see exact values with histograms?"

Activity 1 continued >

Activity 1

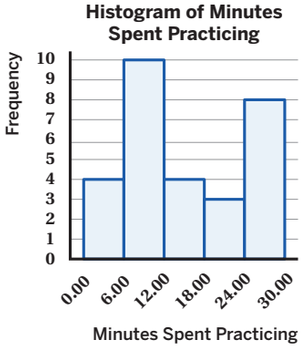
Name: Date: Period:

Four Different Displays

A group of students in music class recorded the number of minutes they spent practicing their musical instruments each day over one week. The students compiled their times (in minutes) in a list. They created a **histogram,** a **stem-and-leaf plot,** a **box plot,** and a **line plot** to represent the data.

Here is the data set:

Activity 1	
Minutes Spent Practicing	Count
0	1
0	2
5	1
5	2
6	1
6	2
6	3
9	1
10	1
10	2
10	3
11	1
11	2
11	3
15	1
15	2
15	3
15	4
19	1
19	2
20	1
24	1
24	2
24	3
24	4
25	1
25	2
25	3
25	4



Stem	Leaf
0	0 0 5 5 6 6 6 9
1	0 0 0 1 1 1 5 5 5 5 5 9 9
2	0 4 4 4 4 5 5 5 5

Key: 1 | 1 means 11 minutes

Unit 2 Lesson 3 168 Activity 1

Key Takeaway: There are four ways to represent univariate data. They all have certain advantages and disadvantages.

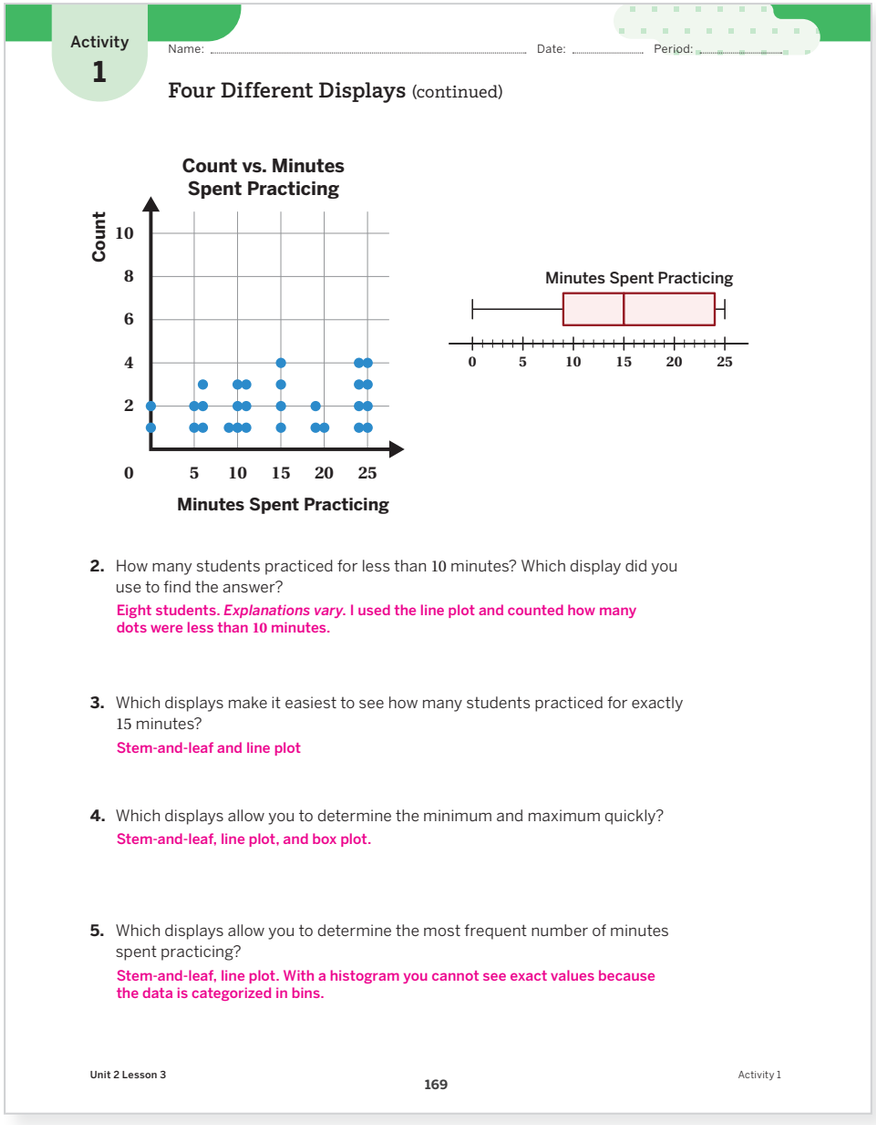
Individual | 10 min

Activity 1 Four Different Displays (continued)

Purpose: Students analyze the four data displays and answer questions. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, **(MTR.4.1)**

- "Are line plots the best visual for all data?"
- "What kind of data is best visualized by a line plot?"
- "How can a line plot help answer questions about the data it represents?
- "When might a line plot be a good way to display data, and when might another type of graph be better?
- "Imagine more data points were added to this line plot. How might that change the conclusions?



Activity 1

Name: Date: Period:

Four Different Displays (continued)

2. How many students practiced for less than 10 minutes? Which display did you use to find the answer?

   Eight students. *Explanations vary.* I used the line plot and counted how many dots were less than 10 minutes.

3. Which displays make it easiest to see how many students practiced for exactly 15 minutes?

   Stem-and-leaf and line plot

4. Which displays allow you to determine the minimum and maximum quickly?

   Stem-and-leaf, line plot, and box plot.

5. Which displays allow you to determine the most frequent number of minutes spent practicing?

   Stem-and-leaf, line plot. With a histogram you cannot see exact values because the data is categorized in bins.

Unit 2 Lesson 3 169 Activity 1

Whole Group | 10 min

Activity 2 Comparing Data Displays

Purpose: Students create their display with a new set of data and compare it to other displays. **(MTR.4.1)**

1 Launch

To generate excitement, consider asking, "Have you ever been on a hike? Where is somewhere interesting you have explored?"

2 Monitor

Invite students to share their reasoning. Discuss each possible response.

Listen for student ideas to highlight during the Connect.

D Differentiation *(Problems 6–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which display do you feel is easiest to understand? Why?"
Show confusion reading stem-and-leaf plots.	**Strengthen** "Let's understand the stem-and-leaf-plot together and make a list of the numbers it represents."
Are unable to find the mode on the line plot.	**Strengthen** Consider asking, "Which display shows the mode with ease?"
Interpret the histogram correctly.	**Stretch** Consider asking, "How would changing the bin intervals change the data?"

3 Connect

To surface the Key Takeaway, consider asking:

- "What is the difference between line plots and histograms?"
- "What is similar about line plots and histograms?"
- "What story does the histogram tell about practice times?"

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in the work.

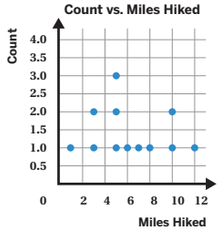
Activity 2

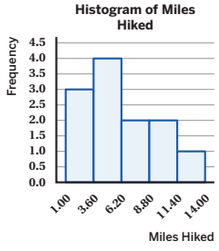
Name: Date: Period:

Comparing Data Displays

A group of hikers recorded the number of miles they hiked each weekend over a month. Their distances are listed below in miles. Use each data display to answer the following questions.

Data Set: {1, 3, 3, 5, 5, 5, 6, 7, 8, 10, 10, 12}





Stem	Leaf
0	1 3 3 5 5 5 6 7 8
1	0 0 2



6. Which data display makes it easiest to see how many hikes were less than 10 miles? Why?

   The stem-and-leaf plot makes it easiest because the "stem" of single digits (0–9) clearly shows how many values fall in this range. In this case, there are 9 hikes (1, 3, 3, 5, 5, 5, 6, 7, 8) below 10 miles.

7. Which data display makes it easiest to identify the range of the data? Why?

   ***Responses vary.* The box plot makes the range easiest to identify because the whiskers show the minimum (1 mile) and maximum (12 miles).**

8. Which display makes it easiest to identify the mode of the data? Why?

   The line plot makes the mode easiest to identify, as it visually shows the number 5 miles appearing most frequently with three dots above it.

9. Which display would you choose to compare how spread out the data is? Why?

   ***Responses vary.* The histogram is ideal for showing the spread of data because it groups distances into intervals, making it clear where most hikes fall. Box plots are a good choice as well because they show the spread of data.**

10. Which displays do *not* show individual data values? How might that be a limitation?

    The histogram and box plots do not show individual values. This limits its ability to answer specific questions, such as finding the exact distances hiked.

Unit 2 Lesson 3 170 Activity 2

Key Takeaway: Certain displays are better for certain questions. Some displays show information that others do not.

Whole Group | 10 min

Activity 3 Create Your Display

Purpose: Students create their display with a new set of data and compare it to other displays. **(MTR.4.1)**

1 Launch

To generate excitement, consider asking, "Have you ever been bird-watching?"

2 Monitor

Invite students to share their reasoning for picking certain displays. Try to understand their viewpoints.

Listen for student ideas to highlight during the Connect.

D Differentiation *(Problems 11–12)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which display do you like the most? Let's try making that together."
Are struggling with details of the display.	**Strengthen** "Make sure you have titles and labels on your displays to make them easier to read."
A display is missing key features.	**Strengthen** Consider asking, "I noticed that you are missing _____, do you think that is important for the display?"
Finished their display early.	**Stretch** Consider having them make a second display with their extra time.

Activity 3 continued >

Activity 3

Name: _____ Date: _____ Period: _____

Create Your Display

Problems 11–12: A wildlife team recorded the number of bird species observed in different parks over a month. The counts of bird species for each park are listed below:

Data Set: {12, 15, 15, 18, 18, 18, 20, 22, 22, 22, 22, 25, 28, 30}

Create a data display that you feel best represents this data set. Choose from a **histogram, stem-and-leaf plot, box plot, or line plot**. *Answers will vary.*

11. Why did you choose this type of display? *Answers will vary.*

- I chose a stem-and-leaf plot because it shows all the individual data points while also organizing the values in a way that makes it easy to see the distribution.
- I chose a histogram because it groups the data into intervals, making it easier to see the overall shape of the distribution and identify patterns like clusters or gaps.
- I chose a box plot because it summarizes the data with key features like the median, quartiles, and outliers, making it easy to compare this data set to others or analyze its spread.
- I chose a line plot because it shows how often each number occurs while keeping the individual values visible, making it easy to identify any clusters or gaps.

12. What questions could someone answer using your display? *Answers will vary.*

- "Which count of bird species is most common?" 18
- "Are there any gaps in the data?" There are a few gaps that are greater than 1.

Unit 2 Lesson 3 171 Activity 3

Pairs | 10 min

Activity 3 Create Your Display (continued)

Purpose: Students create their display with a new set of data and compare it to other displays. **(MTR.4.1)**

1 Launch

Display the two images to students for everyone to see.

2 Monitor

Select and sequence student responses about the key features of each display.

Create an anchor chart that highlights all the key features of each display. Have one student fill the chart in as students answer the questions and share with the class.

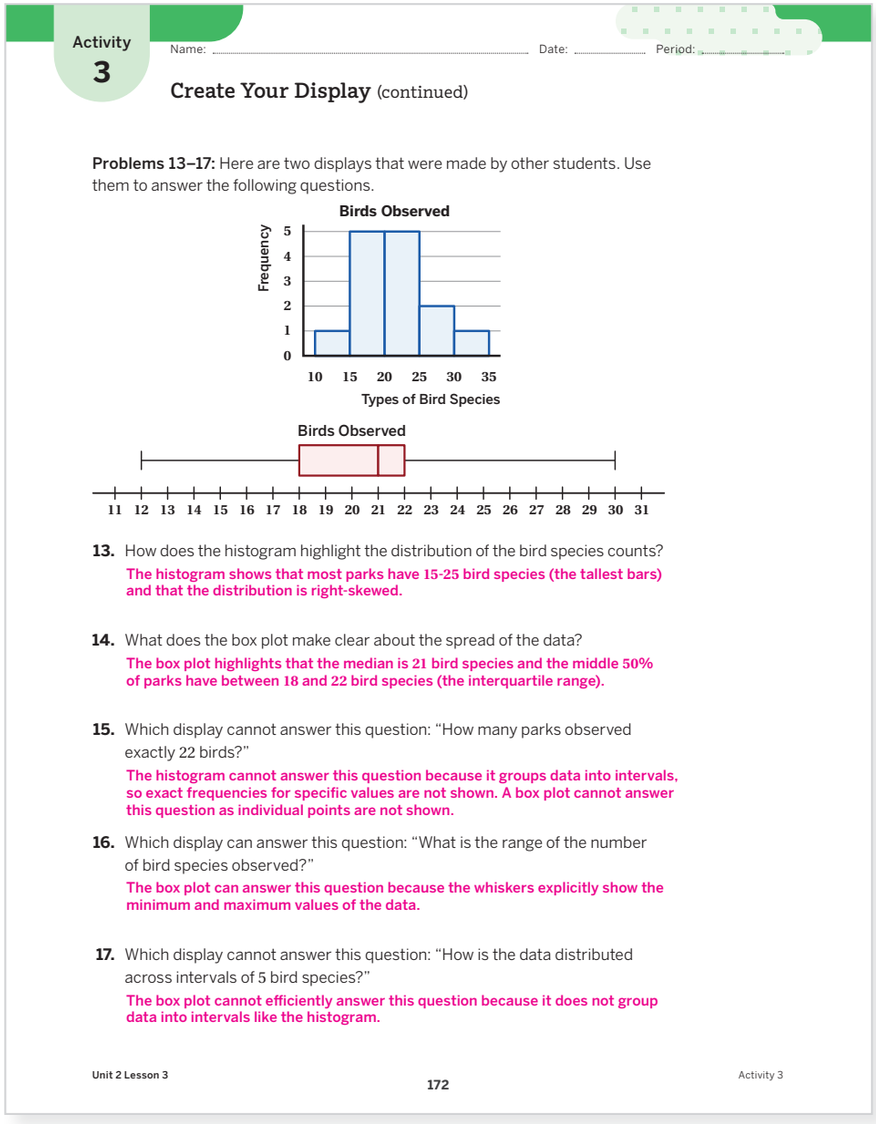
3 Connect

Display the two displays for everyone to see.

- Answer questions students have.
- Allow time for deeper thinking and extensions.

Consider asking:

- "What does the histogram reveal that the stem-and-leaf plot does not?"
- "What does the stem-and-leaf-plot reveal that the histogram does not?
- "What other questions can we ask using these displays?"
- "What other displays could be beneficial?"



Activity 3

Name: Date: Period:

Create Your Display (continued)

Problems 13–17: Here are two displays that were made by other students. Use them to answer the following questions.

13. How does the histogram highlight the distribution of the bird species counts?

The histogram shows that most parks have 15-25 bird species (the tallest bars) and that the distribution is right-skewed.

14. What does the box plot make clear about the spread of the data?

The box plot highlights that the median is 21 bird species and the middle 50% of parks have between 18 and 22 bird species (the interquartile range).

15. Which display cannot answer this question: "How many parks observed exactly 22 birds?"

The histogram cannot answer this question because it groups data into intervals, so exact frequencies for specific values are not shown. A box plot cannot answer this question as individual points are not shown.

16. Which display can answer this question: "What is the range of the number of bird species observed?"

The box plot can answer this question because the whiskers explicitly show the minimum and maximum values of the data.

17. Which display cannot answer this question: "How is the data distributed across intervals of 5 bird species?"

The box plot cannot efficiently answer this question because it does not group data into intervals like the histogram.

Unit 2 Lesson 3 172 Activity 3

Whole Class | 5 min

Synthesis

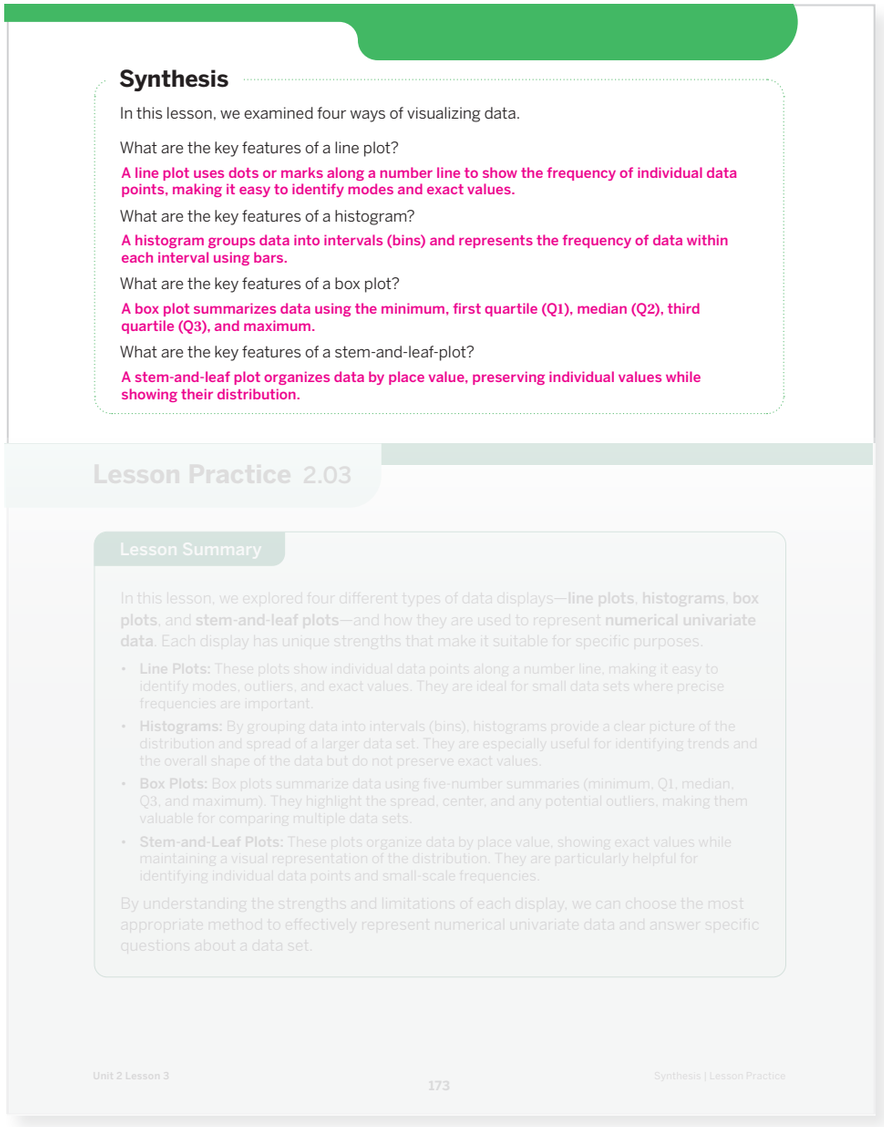
Purpose: Students synthesize their understanding of the key features of the four different kinds of displays.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Line plots show every single data point.
- Histograms are helpful when there is a lot of data or there is data that includes decimal values.
- Stem-and-leaf-plots show every single data point.
- Box plots are helpful for five number summaries.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

In this lesson, we examined four ways of visualizing data.

What are the key features of a line plot?

A line plot uses dots or marks along a number line to show the frequency of individual data points, making it easy to identify modes and exact values.

What are the key features of a histogram?

A histogram groups data into intervals (bins) and represents the frequency of data within each interval using bars.

What are the key features of a box plot?

A box plot summarizes data using the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and maximum.

What are the key features of a stem-and-leaf-plot?

A stem-and-leaf plot organizes data by place value, preserving individual values while showing their distribution.

Lesson Practice 2.03

Lesson Summary

In this lesson, we explored four different types of data displays—**line plots**, **histograms**, **box plots**, and **stem-and-leaf plots**—and how they are used to represent **numerical univariate data**. Each display has unique strengths that make it suitable for specific purposes.

- **Line Plots:** These plots show individual data points along a number line, making it easy to identify modes, outliers, and exact values. They are ideal for small data sets where precise frequencies are important.
- **Histograms:** By grouping data into intervals (bins), histograms provide a clear picture of the distribution and spread of a larger data set. They are especially useful for identifying trends and the overall shape of the data but do not preserve exact values.
- **Box Plots:** Box plots summarize data using five-number summaries (minimum, Q1, median, Q3, and maximum). They highlight the spread, center, and any potential outliers, making them valuable for comparing multiple data sets.
- **Stem-and-Leaf Plots:** These plots organize data by place value, showing exact values while maintaining a visual representation of the distribution. They are particularly helpful for identifying individual data points and small-scale frequencies.

By understanding the strengths and limitations of each display, we can choose the most appropriate method to effectively represent numerical univariate data and answer specific questions about a data set.

Unit 2 Lesson 3 173 Synthesis | Lesson Practice

Lesson Takeaway: Numerical univariate data can be represented in multiple ways, and each data display—line plots, histograms, box plots, and stem-and-leaf plots—has unique strengths and limitations.

Independent | 5 min

Show What You Know

Purpose: Students will demonstrate their understanding by determining the type of data display that would best represent a data set and justify their choice.

Today's Goals

1. **Goal:** Analyze histograms, dot plots, stem-and-leaf plots, and box plots and compare them.
2. **Language Goal:** Describe the advantages of using histograms, dot plots, stem-and-leaf plots, and box plots. **(Writing and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.03

A teacher surveyed 20 students to determine how many hours they spend on homework each week.

The data collected is as follows:

1, 2, 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 8, 9, 10, 10, 10, 12

What type of data display would best represent the distribution of homework hours, and why?

Responses vary. A box plot would be the best way to display this data because it provides a clear summary of the data distribution, showing the median, quartiles, and potential outliers. It helps visualize the spread and median of homework hours, especially when dealing with a range of values.

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support choosing the best way to display a data set:

» **Respond to Student Thinking:**

Create an anchor chart of the key features of the four display types.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on how you and your students used their graphing skills today. How did you support student learning? How did you support your facilitation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

In this lesson, we examined four ways of visualizing data.

What are the key features of a line plot?

A line plot uses dots or marks along a number line to show the frequency of individual data points, making it easy to identify modes and exact values.

What are the key features of a histogram?

A histogram groups data into intervals (bins) and represents the frequency of data within each interval using bars.

What are the key features of a box plot?

A box plot summarizes data using the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and maximum.

What are the key features of a stem-and-leaf plot?

A stem-and-leaf plot organizes data by place value, preserving individual values while showing their distribution.

Lesson Practice 2.03

Lesson Summary

In this lesson, we explored four different types of data displays—**line plots**, **histograms**, **box plots**, and **stem-and-leaf plots**—and how they are used to represent **numerical univariate data**. Each display has unique strengths that make it suitable for specific purposes.

- **Line Plots:** These plots show individual data points along a number line, making it easy to identify modes, outliers, and exact values. They are ideal for small data sets where precise frequencies are important.
- **Histograms:** By grouping data into intervals (bins), histograms provide a clear picture of the distribution and spread of a larger data set. They are especially useful for identifying trends and the overall shape of the data but do not preserve exact values.
- **Box Plots:** Box plots summarize data using five-number summaries (minimum, Q1, median, Q3, and maximum). They highlight the spread, center, and any potential outliers, making them valuable for comparing multiple data sets.
- **Stem-and-Leaf Plots:** These plots organize data by place value, showing exact values while maintaining a visual representation of the distribution. They are particularly helpful for identifying individual data points and small-scale frequencies.

By understanding the strengths and limitations of each display, we can choose the most appropriate method to effectively represent numerical univariate data and answer specific questions about a data set.

Lesson Practice 2.03

Name: _____ Date: _____ Period: _____

Problems 1–4: Arjun and Lorenzo were given a data set about the average daily rainfall (in inches) in different cities during May. They were asked to create a data display using the data. Arjun created a histogram and Lorenzo created a box plot. Below is the data set:

Data: 0.8, 1.2, 1.3, 0.7, 1.9, 2.0, 0.6, 1.5, 1.4, 2.1, 1.8, 2.2, 0.9, 1.1, 1.7

1. Why do you think Arjun chose to create a histogram?

   Arjun chose a histogram because it groups data into intervals, making it easy to see the frequency of rainfall within specific ranges and identify overall patterns in the distribution.

2. What are some limitations of using a histogram to display this data?

   A histogram does not show exact data values and depends on interval choices, which can affect how the data is interpreted.

3. Why do you think Lorenzo chose to create a box plot?

   Lorenzo chose a box plot to summarize the spread and center of the data, highlighting the median, quartiles, and range, while also identifying potential outliers.

4. Which display would you use if you wanted to explain the spread of the data? Why?

   I would use Lorenzo's box plot because it clearly shows the range, interquartile range, and median, making it ideal for explaining the spread of the data.

Problems 5–7: A biology class is studying the average growth (in centimeters) of a certain plant species under different light conditions over 10 weeks. Below is the data set:

Data: 2.1, 2.3, 2.5, 3.0, 3.2, 3.5, 3.5, 3.8, 4.0, 4.2, 4.5, 4.5, 4.8, 5.0

5. Create your own data display to represent this data. Choose from a **histogram, stem-and-leaf plot, box plot, or line plot**. Then, answer the following questions. *Answers will vary.*

Stem	Leaf
2	1 3 5
3	0 2 5 5 8
4	0 2 5 5 8
5	0

6. What are the strengths of your chosen display in representing the data? *Answers will vary.*

   Stem-and-leaf plot: It shows the exact data values and their frequencies, making it easy to see patterns like clusters or repeated values. You can clearly identify the growth each week.

7. What are the weaknesses or limitations of your chosen display for this data? *Answers will vary.*

   Stem-and-leaf: It's not as effective for summarizing large data sets or comparing ranges at a glance.

Lesson Practice 2.03

Name: _____ Date: _____ Period: _____

Test Practice

8. Which of the following data displays can be used to determine the exact values of a data set? Select all that apply.

   - [x] A. *Line Plot*
   - [ ] B. Box and Whisker Plot
   - [ ] C. Histogram
   - [x] D. Stem-and-Leaf Plot

Spiral Review

Problems 9–10: Vishant is investigating whether there is an association between wearing sneakers and participating on an athletic team. This table summarizes the data he collected from a survey of students.

	On Athletic Team	Not on Athletic Team	Total
Wear Sneakers	16%	17%	33%
Don't Wear Sneakers	32%	35%	67%
Total	48%	52%	100%

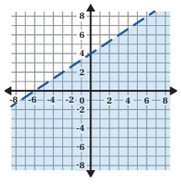
9. Interpret what 32% means in this situation.

   32% of students surveyed are on an athletic team and don't wear sneakers.

10. Is there evidence of an association between wearing sneakers and participating on an athletic team?

    No. Explanations vary. The percentage of students who are on an athletic team and wear sneakers, 16%, is very similar to the percentage of students who are not on an athletic team and wear sneakers, 17%.

11. Which inequality is shown on the graph?



    A. $y > -\frac{2}{3}x + 4$

    B. $y < -\frac{2}{3}x + 4$

    C. $y > \frac{2}{3}x + 4$

    D. $y < \frac{2}{3}x + 4$

Lesson Practice Item Analysis

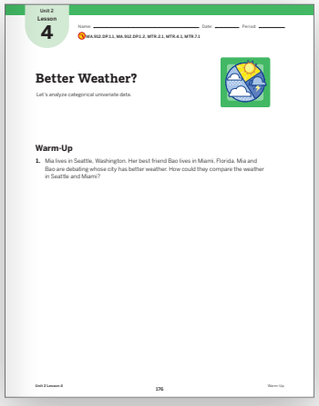
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4, 6–7	2	MA.912.DP.1.1
	5	3	MA.912.DP.1.1
Test Practice	8	1	MA.912.DP.1.1
Spiral Review			
Fluency	9–10	2	MA.912.DP.3.1*
	11	1	MA.912.AR.2.7

*These problems build toward the standard shown.

Unit 2

Lesson

4


Unit 2
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4

Name _____ Date _____ Period _____

MA.912.DP.1.1, MA.912.DP.1.2, MTR.2.1, MTR.4.1, MTR.7.1

Better Weather?

Let's analyze categorical univariate data.

Warm-Up

1. Mia lives in Seattle, Washington. Her best friend Bao lives in Miami, Florida. Mia and Bao are debating whose city has better weather. How could they compare the weather in Seattle and Miami?

Unit 2 Lesson 4 176 Warm-Up

Better Weather?

Revisiting Box Plots

Let's analyze categorical univariate data.

Learning Outcomes

Today's Goals

1. **Goal:** Describe and construct different data displays for categorical univariate data.
2. **Goal:** Understand differences in data displays for categorical univariate data and use these differences to select the most appropriate data display.
3. **Language Goal:** Write to justify the selection of a data display for categorical univariate data. **(Writing)**

Students make sense of using bar charts, circle graphs, frequency tables, and segmented bar graphs to represent categorical univariate data. They consider the information revealed in each type of data display and justify the selection of a particular data display to represent data.

Coherence

Prior Learning

In Lesson 3, students analyzed numerical univariate data.

Future Learning

In Lesson 5, students use statistics to measure the center of a dataset.

Connections

- Students analyze data using box plots within the real-world context of weather. (real world, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.1
Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.DP.1.1	**MA.912.DP.1.2**

Vocabulary

New Vocabulary

categorical univariate data

Review Vocabulary

bar chart
circle graph
frequency table
segmented bar graph

Lesson at a Glance

~ 45 min

MA.912.DP.1.1

Warm-Up Fluency

Pairs | 5 min

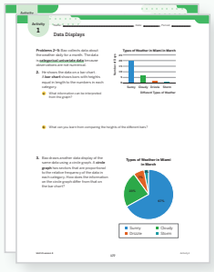
Students use the Think-Pair-Share routine to share ideas about how to compare weather.
Routine(s): Think-Pair-Share



Activity 1

Pairs | 14 min

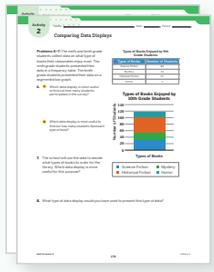
Students use the Collect and Display routine to understand how to represent categorical univariate data in multiple ways.
Routine(s): MLR2: Collect and Display **(MTR.2.1)**



Activity 2

Pairs | 8 min

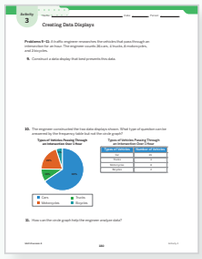
Students compare different data displays to represent real-world data. **(MTR.7.1)**



Activity 3

Pairs | 8 min

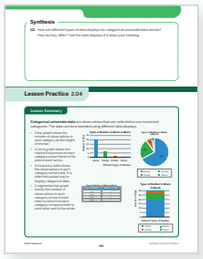
Students use the Think-Pair-Share routine to decide what data display is best for representing real-world data.
Routine(s): Think-Pair-Share **(MTR.7.1)**



Synthesis

Whole Class | 5 min

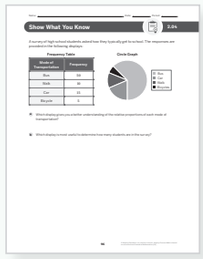
Students synthesize their understanding of using data displays to represent categorical univariate data.
Routine(s): MLR2: Collect and Display **(MTR.2.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which data display is better for gathering specific information about data gathered from a survey.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Think-Pair-Share routine to share ideas about how to compare weather.

1 Launch

To surface prior knowledge, consider asking "What do you know about Seattle and Miami?"

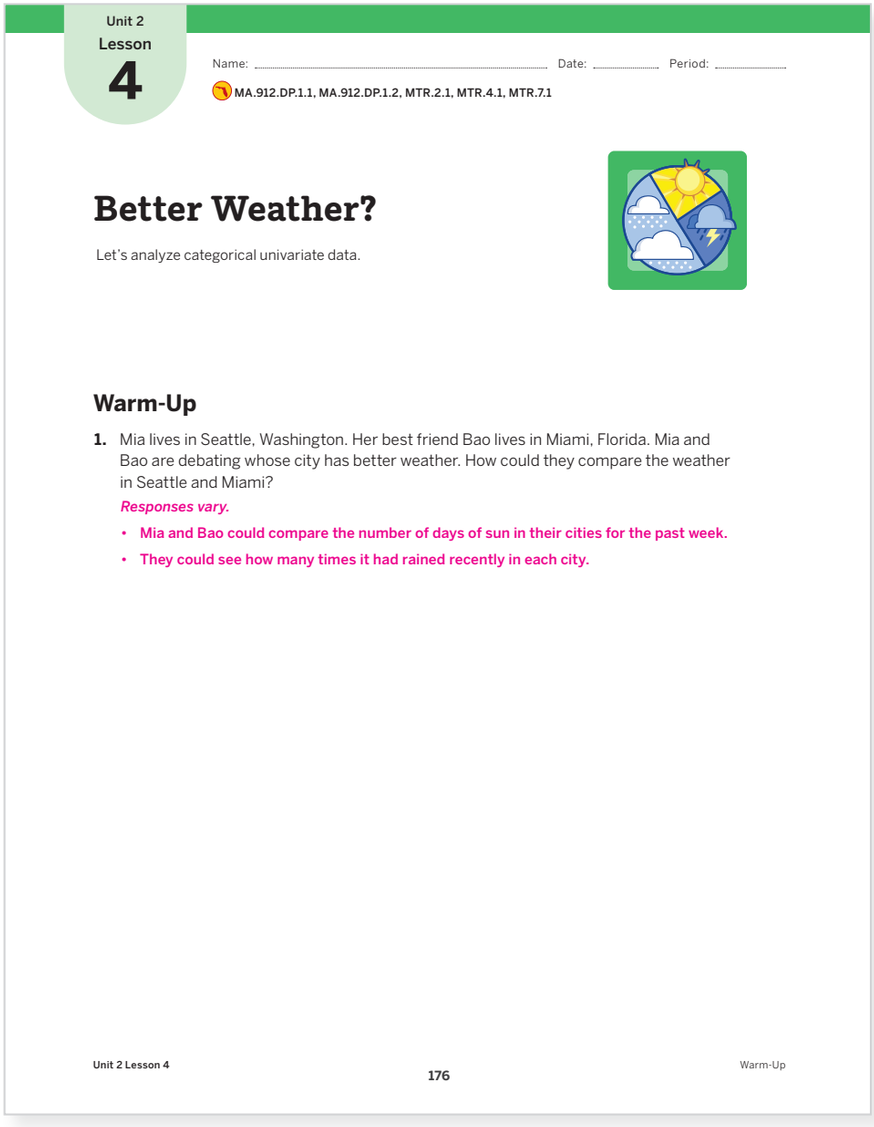
ELL **English Language Learners** Consider showing videos or images of different weather-related concepts (e.g., sunny, cloudy, drizzle, thunderstorm) to increase access to the task. **(Speaking, Writing, and Listening)**

2 Connect

Use the Think-Pair-Share routine to help students discuss what information they need to compare the weather. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class. To encourage movement, invite students to find a partner in a different location.

Invite students to share their or their partner's thoughts about Mia and Bao's debate with the class.

Listen for students who understand that Mia and Bao can collect non-numerical data to compare weather.



Unit 2
Lesson
4

Name: _____ Date: _____ Period: _____

MA.912.DP.1.1, MA.912.DP.1.2, MTR.2.1, MTR.4.1, MTR.7.1

Better Weather?

Let's analyze categorical univariate data.

Warm-Up

1. Mia lives in Seattle, Washington. Her best friend Bao lives in Miami, Florida. Mia and Bao are debating whose city has better weather. How could they compare the weather in Seattle and Miami?

   Responses vary.

   - Mia and Bao could compare the number of days of sun in their cities for the past week.
   - They could see how many times it had rained recently in each city.

Unit 2 Lesson 4 176 Warm-Up

Pairs | 14 min

Activity 1 Data Displays

Purpose: Students use the Collect and Display routine to understand how to represent categorical univariate data in multiple ways. **(MTR.2.1)**

1 Launch

Invite students to discuss with a partner what **categorical univariate data** means.

MLR **MLR2: Collect and Display** Use this routine to create a class definition of **categorical univariate data** using students' language. Display any new language, examples, or diagrams for students to reference later.

2 Monitor

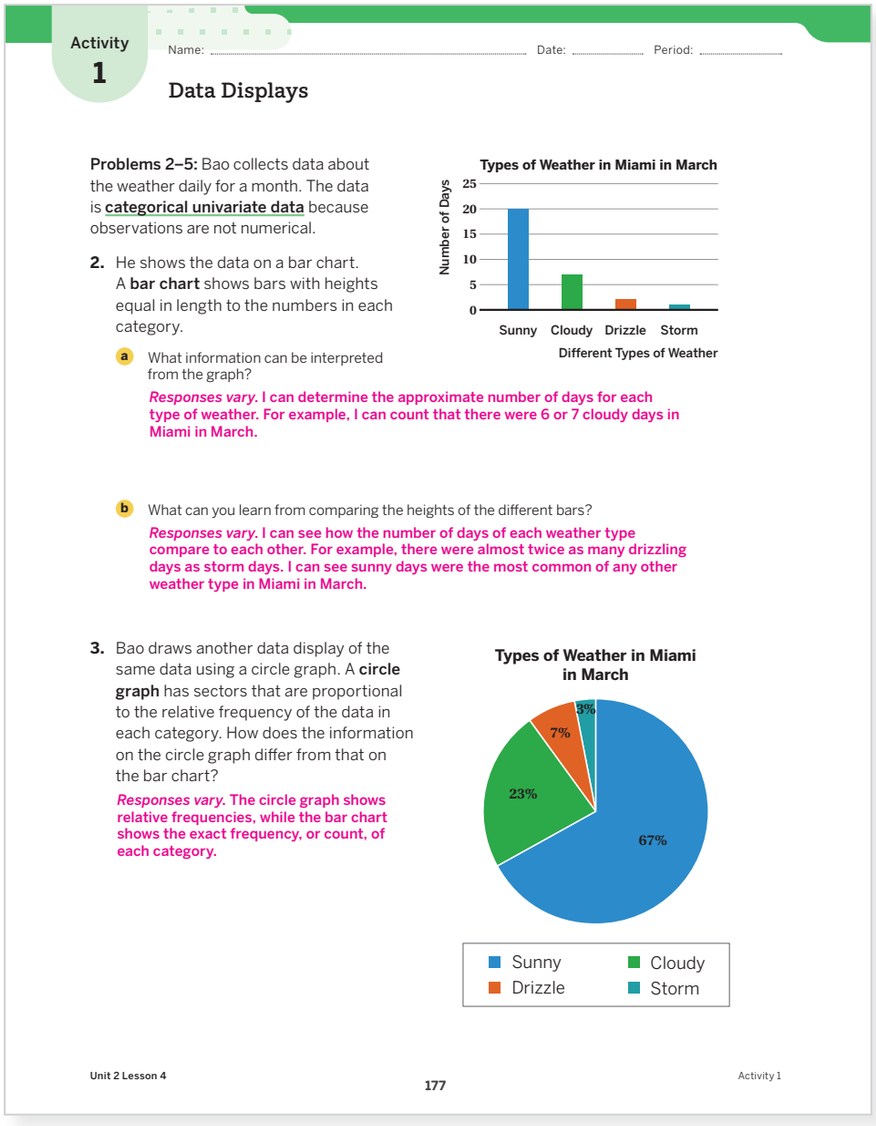
Support students getting started by asking, "How can you interpret the bar chart?"

A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific point on the graph and describe what that point means to support interpreting visual representations.

Listen for students that identify different information found on the different data displays.

Invite students to think about how they might share their observations during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Data Displays

Problems 2–5: Bao collects data about the weather daily for a month. The data is **categorical univariate data** because observations are not numerical.

2. He shows the data on a bar chart. A **bar chart** shows bars with heights equal in length to the numbers in each category.

a. What information can be interpreted from the graph?

Responses vary. I can determine the approximate number of days for each type of weather. For example, I can count that there were 6 or 7 cloudy days in Miami in March.

b. What can you learn from comparing the heights of the different bars?

Responses vary. I can see how the number of days of each weather type compare to each other. For example, there were almost twice as many drizzling days as storm days. I can see sunny days were the most common of any other weather type in Miami in March.

3. Bao draws another data display of the same data using a circle graph. A **circle graph** has sectors that are proportional to the relative frequency of the data in each category. How does the information on the circle graph differ from that on the bar chart?

Responses vary. The circle graph shows relative frequencies, while the bar chart shows the exact frequency, or count, of each category.

Unit 2 Lesson 4 177 Activity 1

Key Takeaway: **Categorical univariate data** can be represented in bar charts, circle graphs, frequency tables, and segmented bar graphs.

Pairs | 14 min

Activity 1 Data Displays (continued)

Purpose: Students use the Collect and Display routine to understand how to represent categorical univariate data in multiple ways. **(MTR.2.1)**

3 Connect

Invite students to share their reasoning. Discuss each possible response.

To surface the Key Takeaway, consider asking: **(MTR.2.1)**

- "What information is most prominent in each data representation?"
- "Is it possible to convert one data representation to another data representation?"
- "How might a business use categorical data to make decisions?
- What are some common mistakes or misunderstandings people might have when analyzing categorical data?

Activity 1

Name: Date: Period:

Data Displays (continued)

4. Bao also presents the data in a frequency table. A frequency table shows the number of observations in each category as a number.

Types of Weather in Miami in March

Types of Weather	Number of Days
Sunny	20
Cloudy	7
Drizzle	2
Storm	1

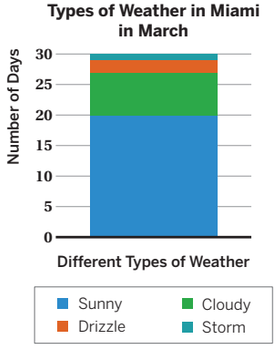
a. When would a frequency table be most useful for looking at data?

Responses vary. When the most important information is the exact number in each category.

b. When would a bar chart be more useful than a frequency table?

Responses vary. When you want to quickly compare how the numbers in each category relate to each other.

5. Lastly, Bao presents the data on a **segmented bar graph.** Here, the number of observations in each category is stacked on top of each other.



a. How is the segmented bar graph similar to a bar chart?

Responses vary. The lengths of each bar segment correspond to the number of observations in each category.

b. How is the segmented bar graph different from a bar chart?

Responses vary. On a segmented bar graph, the bars are stacked on top of each other.

c. How is the segmented bar graph similar to a circle graph?

Responses vary. The segmented bar graph shows how each category compares to the other over the whole month, like the relative size of the sectors in a circle graph.

d. How is the segmented bar graph different from a circle graph?

Responses vary. In a circle graph, all the sectors must combine to form one whole circle. On a segmented bar graph, the bars can be as tall as they need to be.

Unit 2 Lesson 4 178 Activity 1

Pairs | 8 min

Activity 2 Comparing Data Displays

Purpose: Students compare different data displays to represent real-world data. **(MTR.7.1)**

1 Launch

To generate excitement, consider asking, "What is your favorite genre of book?"

2 Monitor

Invite students to share their reasoning. Discuss each possible response.

Listen for student ideas to highlight during the Connect.

D Differentiation *(Problems 6–7)*

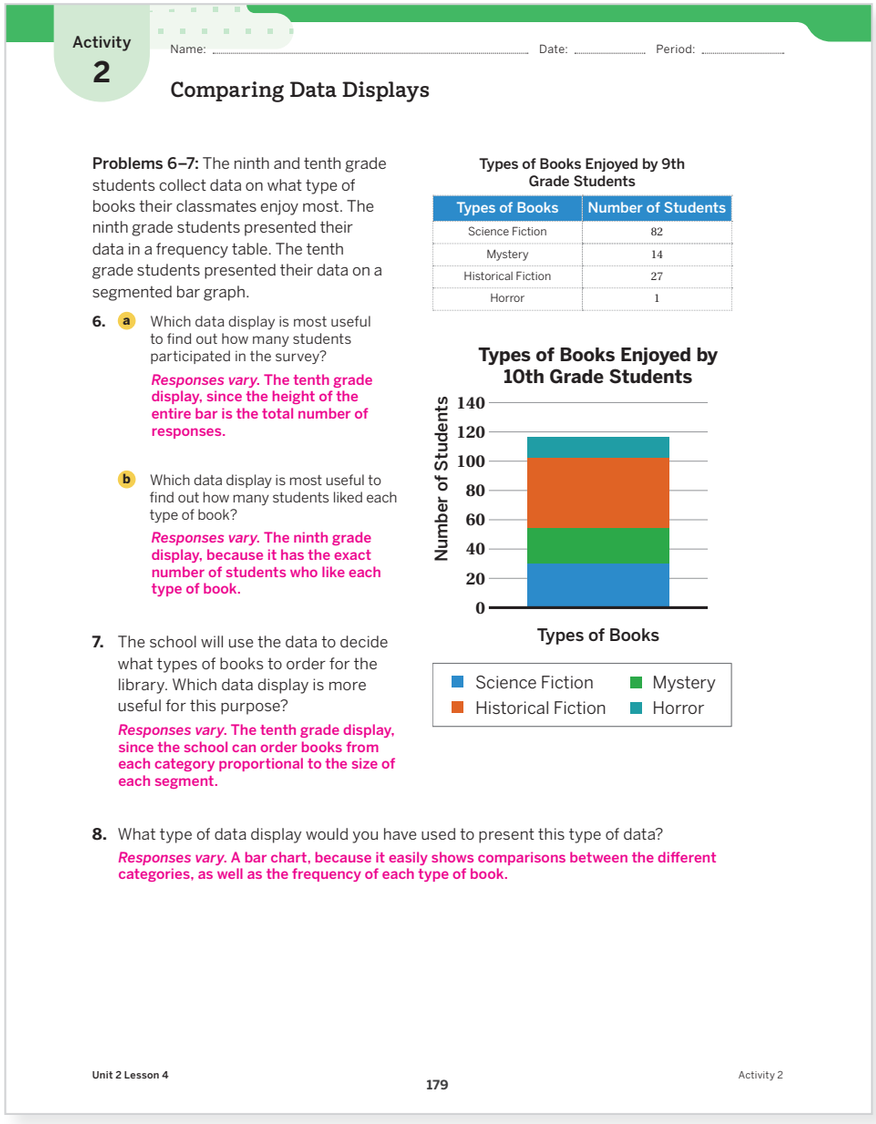
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to visit peers and hear their strategies.
Choose the frequency table. *(Problem 6)*	**Strengthen** Consider asking, "What calculations would you have to use with the numbers in the frequency table?"
Choose the segmented bar graph. *(Problem 7)*	**Strengthen** Invite them to share their reasoning with the class.
Use differences in the data displays to explain their reasoning.	**Stretch** Invite students who would like to explore further to discuss whether a circle graph would be a better data representation.

Invite students to read others' responses and decide if others' ideas were similar to or different from their own.

3 Connect

Display and celebrate a variety of students' thinking.

Discuss students' strategies for selecting the strengths and weaknesses of each data display.



Activity 2

Name: Date: Period:

Comparing Data Displays

Problems 6–7: The ninth and tenth grade students collect data on what type of books their classmates enjoy most. The ninth grade students presented their data in a frequency table. The tenth grade students presented their data on a segmented bar graph.

Types of Books Enjoyed by 9th Grade Students

Types of Books	Number of Students
Science Fiction	82
Mystery	14
Historical Fiction	27
Horror	1

6. a. Which data display is most useful to find out how many students participated in the survey?

Responses vary. The tenth grade display, since the height of the entire bar is the total number of responses.

b. Which data display is most useful to find out how many students liked each type of book?

Responses vary. The ninth grade display, because it has the exact number of students who like each type of book.

Types of Books Enjoyed by 10th Grade Students

7. The school will use the data to decide what types of books to order for the library. Which data display is more useful for this purpose?

Responses vary. The tenth grade display, since the school can order books from each category proportional to the size of each segment.

8. What type of data display would you have used to present this type of data?

Responses vary. A bar chart, because it easily shows comparisons between the different categories, as well as the frequency of each type of book.

Unit 2 Lesson 4 179 Activity 2

Pairs | 8 min

Activity 3 Creating Data Displays

Purpose: Students use the Think-Pair-Share routine to decide what data display is best for representing real-world data. **(MTR.7.1)**

Short on time? Consider omitting Problem 9 or constructing a bar chart of the data as a class.

1 Launch

To surface connections, consider discussing how this problem is similar to the work that Bao did to research weather in Miami.

2 Monitor

Use the Think-Pair-Share routine to help students shape their ideas about the best data display to use to represent the data. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

Encourage students to record their thinking as they work.

Look for opportunities to recognize the brilliance of students who have not been heard from recently.

Snapshot student strategies to share during the Connect.

3 Connect

Display and celebrate a variety of students' data displays.

To surface the Key Takeaway, consider asking: **(MTR.7.1)**

- "What information is most prominent in each data representation?"
- "Is it possible to convert one data representation to another data representation?"

Activity 3

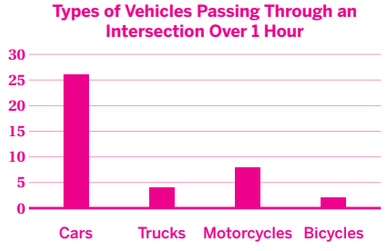
Name: Date: Period:

Creating Data Displays

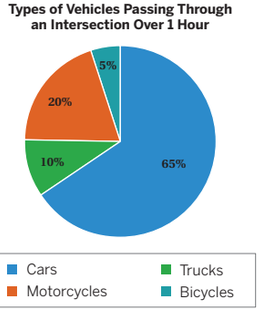
Problems 9–11: A traffic engineer researches the vehicles that pass through an intersection for an hour. The engineer counts 26 cars, 4 trucks, 8 motorcycles, and 2 bicycles.

9. Construct a data display that best presents this data.

Responses vary. Students might use a bar chart, a circle graph, a frequency table, or a segmented bar graph to display the data.



10. The engineer constructed the two data displays shown. What type of question can be answered by the frequency table but not the circle graph?



Types of Vehicles Passing Through an Intersection Over 1 Hour

Types of Vehicles	Number of Vehicles
Car	26
Trucks	4
Motorcycles	8
Bicycles	2

Responses vary. Questions that require exact numbers, such as: "How many motorcycles passed through the intersection?"

11. How can the circle graph help the engineer analyze data?

Responses vary. The circle graph shows relative frequencies, so it is easy to tell which type of vehicle is most common when compared to the others.

Unit 2 Lesson 4 180 Activity 3

Lesson Takeaway: Different types of data and different end uses determine the best data display to use when representing categorical univariate data.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using data displays to represent categorical univariate data.

Invite students to respond independently, and then share their thinking with a partner.

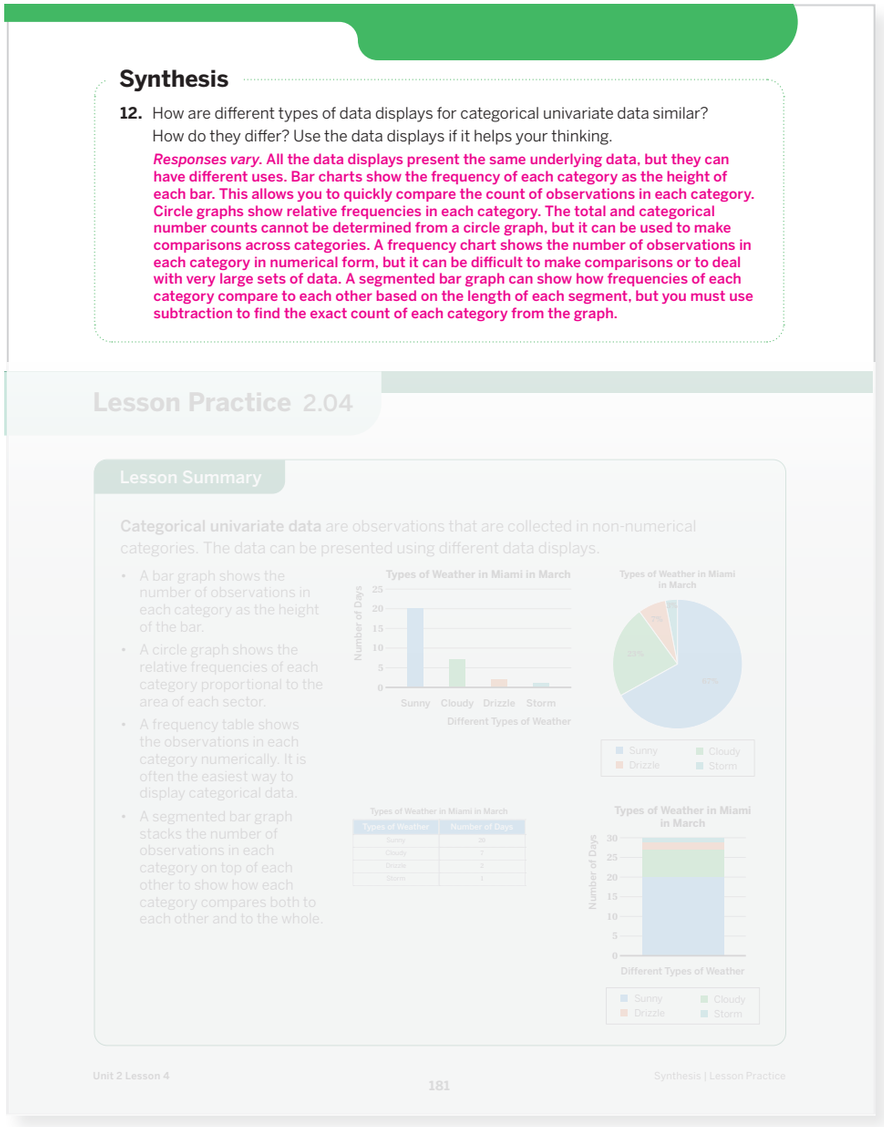
Capture and share a variety of ideas, such as:

- Bar charts show bars whose height equals the number of observations in each category.
- Circle graphs show wedges whose areas are proportional to the relative frequencies of the data in each category.
- Frequency tables show numerical values that correspond to the number of observations in each category.
- Segmented bar graphs show bars whose length equals the number of observations in each category and are stacked vertically on top of each other.

Create an anchor chart of the information shown on each data display.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. How are different types of data displays for categorical univariate data similar? How do they differ? Use the data displays if it helps your thinking.

Responses vary. All the data displays present the same underlying data, but they can have different uses. Bar charts show the frequency of each category as the height of each bar. This allows you to quickly compare the count of observations in each category. Circle graphs show relative frequencies in each category. The total and categorical number counts cannot be determined from a circle graph, but it can be used to make comparisons across categories. A frequency chart shows the number of observations in each category in numerical form, but it can be difficult to make comparisons or to deal with very large sets of data. A segmented bar graph can show how frequencies of each category compare to each other based on the length of each segment, but you must use subtraction to find the exact count of each category from the graph.

Lesson Practice 2.04

Lesson Summary

Categorical univariate data are observations that are collected in non-numerical categories. The data can be presented using different data displays.

- A bar graph shows the number of observations in each category as the height of the bar.
- A circle graph shows the relative frequencies of each category proportional to the area of each sector.
- A frequency table shows the observations in each category numerically. It is often the easiest way to display categorical data.
- A segmented bar graph stacks the number of observations in each category on top of each other to show how each category compares both to each other and to the whole.

Types of Weather in Miami in March

Number of Days

Sunny Cloudy Drizzle Storm

Different Types of Weather

Types of Weather in Miami in March

Sunny Cloudy Drizzle Storm

Types of Weather in Miami in March

Types of Weather	Number of Days
Sunny	26
Cloudy	2
Drizzle	2
Storm	1

Types of Weather in Miami in March

Number of Days

Different Types of Weather

Sunny Cloudy Drizzle Storm

Unit 2 Lesson 4 181 Synthesis | Lesson Practice

Lesson Takeaway: Different data displays for categorical univariate data can be used in different circumstances depending on the purpose.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which data display is better for gathering specific information about data gathered from a survey.

Today's Goals

1. **Goal:** Describe and construct different data displays for categorical univariate data.
2. **Goal:** Understand differences in data displays for categorical univariate data and use these differences to select the most appropriate data display.
3. **Language Goal:** Write to justify the selection of a data display for categorical univariate data. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

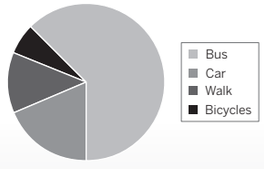
Show What You Know 2.04

A survey of high school students asked how they typically get to school. The responses are provided in the following displays:

Frequency Table

Mode of Transportation	Frequency
Bus	50
Walk	10
Car	15
Bicycle	5

Circle Graph



a. Which display gives you a better understanding of the relative proportions of each mode of transportation?
circle graph

b. Which display is most useful to determine how many students are in the survey?
frequency table

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support determining the best data display for gathering information from a survey:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** and have students make a list of pros and cons of each data display.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to find a data set that they are interested in and create two data representations to display the same data.

Professional Learning

Take a moment to reflect on how students used different data displays to represent problems today. **(MTR.2.1)**

- How did they differentiate between the different methods?
- Did students use their problem-solving strategy to guide their choice?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How are different types of data displays for categorical univariate data similar? How do they differ? Use the data displays if it helps your thinking.

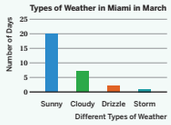
Responses vary. All the data displays present the same underlying data, but they can have different uses. Bar charts show the frequency of each category as the height of each bar. This allows you to quickly compare the count of observations in each category. Circle graphs show relative frequencies in each category. The total and categorical number counts cannot be determined from a circle graph, but it can be used to make comparisons across categories. A frequency chart shows the number of observations in each category in numerical form, but it can be difficult to make comparisons or to deal with very large sets of data. A segmented bar graph can show how frequencies of each category compare to each other based on the length of each segment, but you must use subtraction to find the exact count of each category from the graph.

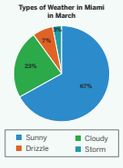
Lesson Practice 2.04

Lesson Summary

Categorical univariate data are observations that are collected in non-numerical categories. The data can be presented using different data displays.

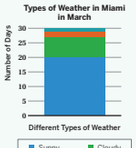
- A bar graph shows the number of observations in each category as the height of the bar.
- A circle graph shows the relative frequencies of each category proportional to the area of each sector.
- A frequency table shows the observations in each category numerically. It is often the easiest way to display categorical data.
- A segmented bar graph stacks the number of observations in each category on top of each other to show how each category compares both to each other and to the whole.





Types of Weather in Miami in March

Types of Weather	Number of Days
Sunny	20
Cloudy	7
Drizzle	2
Storm	1



Lesson Practice 2.04

Name: ..... Date: ..... Period: .....

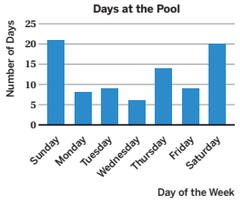
Problems 1–4: Ama records the number of times she has gone to the pool each day of the week this year in the bar chart.

1. Construct a frequency table with the data.

Responses vary.

Days at the Pool

Day of the Week	Number of Days
Sunday	21
Monday	8
Tuesday	9
Wednesday	6
Thursday	14
Friday	9
Saturday	20



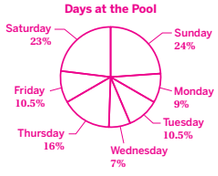
2. How can the frequency table be easier to use than the bar chart? How can the bar chart be easier to use than the frequency table?

Responses vary. It is easier to read the exact numbers from the frequency table. However, it is easy to compare data visually with the bar graph.

3. How would a segmented bar graph appear different from what Ama has drawn?

Responses vary. It would appear similar, except the bars would be stacked on top of each other instead of appearing side by side.

4. Use Ama's data to construct a circle graph.



Lesson Practice 2.04

Name: ..... Date: ..... Period: .....

5. Mateo receives survey data presented in a circle graph. What information does he need to construct a frequency table with the graph? Explain how he would use the information to construct the frequency table.

He needs to know the total number of survey respondents. Using this information, he can multiply the percentage of each category shown on the circle graph by the total number of respondents to complete each cell of the frequency table.

Test Practice

6. A survey collects data about students' favorite subjects in school. Which of these data displays can be used to present this data? Select all that apply.

- ☐ **A.** Histogram
- ☑ **B.** Bar chart
- ☐ **C.** Scatter plot
- ☐ **D.** Box plot
- ☑ **E.** Circle graph

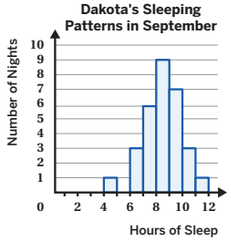
Spiral Review

7. This histogram shows how many hours of sleep Dakota got each night in September. How many nights did Dakota sleep less than 7 hours?

4 nights

8. Calculate the mean of the data set:
1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15

8



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.912.DP.1.1
	5	3	MA.912.DP.1.1
Test Practice	6	1	MA.912.DP.1.1
Spiral Review			
Fluency	7	1	MA.6.DP.1.4
	8	1	MA.5.DP.1.2

Unit 2

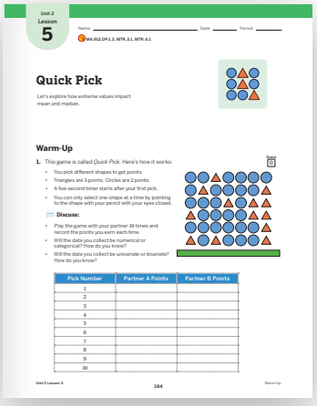
Lesson

5

Quick Pick

Revisiting Measures of Center

Let's explore how extreme values impact mean and median.


Unit 2
Lesson
5
Name: Date: Period:
MA.912.DP.1.2, MTR.3.1, MTR.4.1
Quick Pick
Let's explore how extreme values impact mean and median.
Warm-Up
1. This game is called Quick Pick. Here's how it works:
• You pick different shapes to get points.
• Triangles are 3 points. Circles are 2 points.
• A five-second timer starts after your first pick.
• You can only select one shape at a time by pointing to the shape with your pencil with your eyes closed.
Discuss:
• Play the game with your partner 10 times and record the points you earn each time.
• Will the data you collect be numerical or categorical? How do you know?
• Will the data you collect be univariate or bivariate? How do you know?
Pick Number | Partner A Points | Partner B Points
1
2
3
4
5
6
7
8
9
10
Unit 2 Lesson 5
184
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Use technology to calculate the *measures of center* (mean and median) for a data set.
2. **Language Goal:** Explain how to calculate the mean and median and what these tell us about a data set. **(Speaking and Listening)**
3. **Language Goal:** Explain the effect of extreme values on the mean and median. **(Writing, Speaking, and Listening)**

Students revisit *mean* and *median* and explore how extreme values impact each **measure of center**. They also use a calculator to create lists and calculate means and medians. This is the first instance in this unit where students use statistics to summarize data sets.

Coherence

Prior Learning

In Lessons 3–4, students revisited dot plots, histograms, and box plots and developed language for describing the shape of a data set. In Grade 6, Unit 8 students calculated measures of center and used them to describe data sets.

Future Learning

In Lesson 6, students will explore how outliers affect measures of center.

Connections

- Students explore how extreme values impact the mean and median of a data set in real-world contexts such as test scores and number of points scored in a game. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.2

Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.DP.1.1

MA.6.DP.1.3

MA.6.DP.1.5

Vocabulary

New Vocabulary

measure of center

Review Vocabulary

mean *median*

Lesson at a Glance

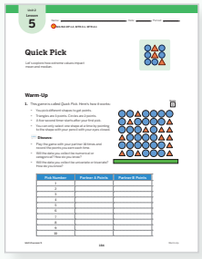
~ 45 min

MA.912.DP.1.2

Warm-Up

Independent | 5 min

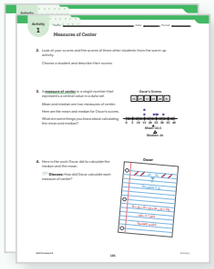
Students play a game to make sense of a context used throughout Activity 1.



Activity 1

Pairs | 10 min

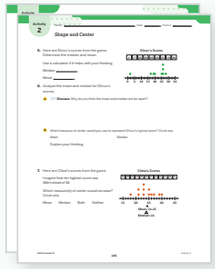
Students revisit *mean* and *median* as **measures of center** and describe how to calculate them. **Routines:** Notice and Wonder, MLR2: Collect and Display



Activity 2

Pairs | 10 min

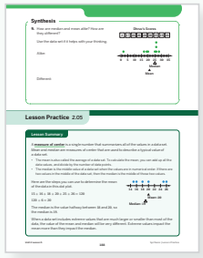
Students explore how the shape of a data set affects the mean and median. **Routine:** Decide and Defend



Synthesis

Whole Class | 5 min

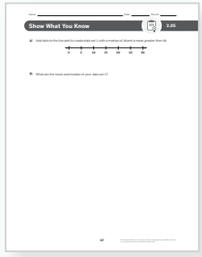
Students synthesize their understanding of mean and median by comparing the two measures of center. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating a data set that meets specific criteria and calculate the mean and median.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students play a game to make sense of a context used throughout Activity 1.

1 Launch

Demonstrate how to play one round of the game.

A **Accessibility: Fine Motor Skills** To support students in this lesson, invite them to work in pairs and strategize together about which shapes to pick before the game starts.

2 Connect

Consider asking:

- "Did you get the same score each time you played?"
- "If you had to summarize your scores with one number, what would it be? Why?"

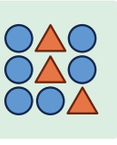
Unit 2
Lesson
5

Name: _____ Date: _____ Period: _____

MA.912.DP.1.2, MTR.3.1, MTR.4.1

Quick Pick

Let's explore how extreme values impact mean and median.

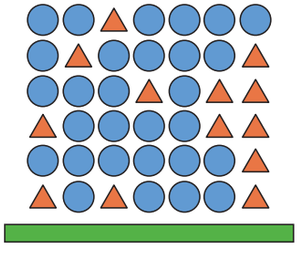


Warm-Up

1. This game is called *Quick Pick*. Here's how it works:

Score
0

- You pick different shapes to get points.
- Triangles are 3 points. Circles are 2 points.
- A five-second timer starts after your first pick.
- You can only select one shape at a time by pointing to the shape with your pencil with your eyes closed.



Discuss:

- Play the game with your partner 10 times and record the points you earn each time.
- Will the data you collect be numerical or categorical? How do you know?
- Will the data you collect be univariate or bivariate? How do you know?

Answers will vary. *Scores vary.*

Pick Number	Partner A Points	Partner B Points
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Unit 2 Lesson 5 184 Warm-Up

Pairs | 10 min

Activity 1 Measures of Center

Purpose: Students revisit *mean* and *median* as **measures of center** and describe how to calculate them.

Short on time? Consider omitting or shortening the Connect if students demonstrate a strong understanding of measures of center in the activity or the Pre-Unit Check.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of each student's scores before describing one.

2 Monitor

Look for students who use language about the shape of the data such as "skewed" or "bell-shaped" and for students who pay attention to individual data points in their descriptions.

Encourage students to read others' responses and compare them to their own.

Consider asking pairs of students, "Why might only reporting the high score not represent all of the data?"

Select and sequence students' prior knowledge about measures of center to share during the Connect.

ELL **English Language Learners** Consider discussing other contexts inside and outside of math class where the terms *mean* and *median* are used (e.g., being mean to someone, the median in a highway) in order to distinguish those from the use of mean and median in this context. **(Reading and Writing)**

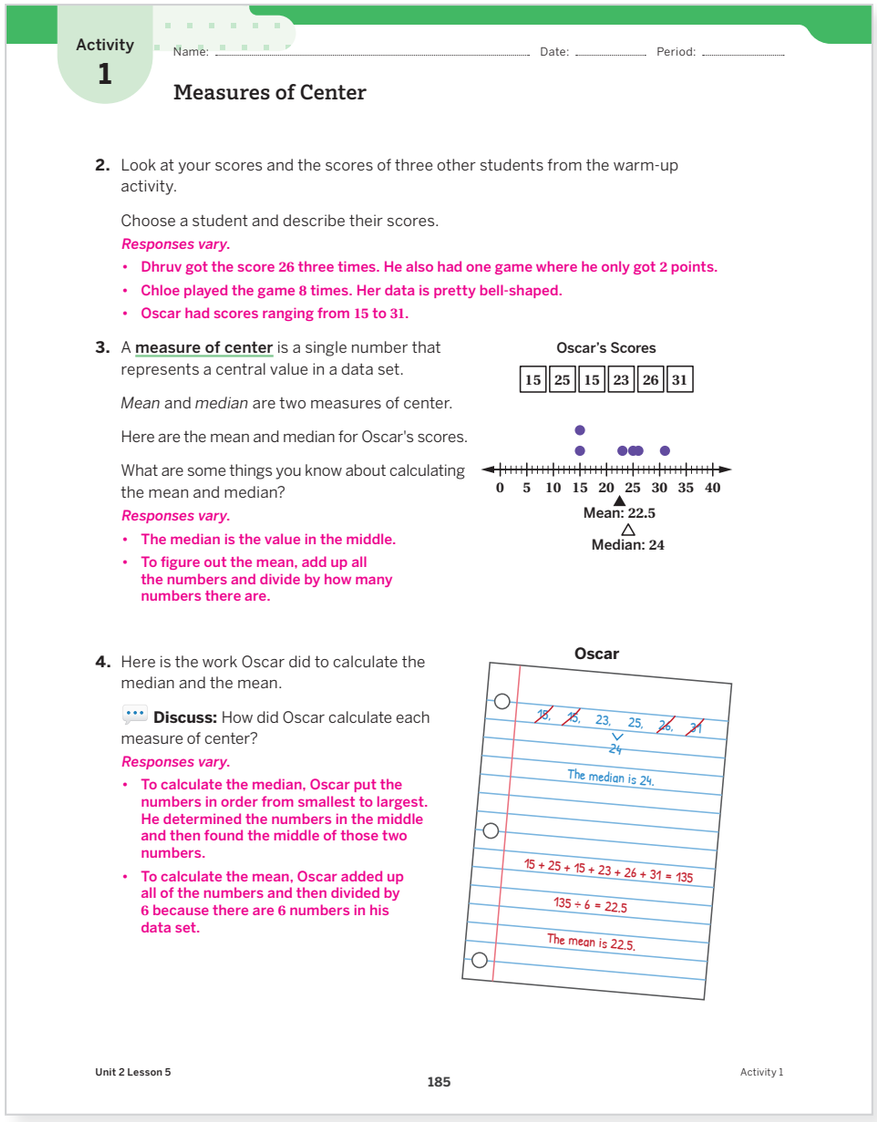
Listen for student descriptions of how to calculate each measure of center. Invite them to share in the Connect.

3 Connect

MLR **MLR2: Collect and Display** Use this routine to create class definitions of *mean*, *median*, and **measure of center** and describe what they mean in context using students' language. Display language, examples, or diagrams for students to reference later.

Share student responses and invite students to summarize strategies for determining the mean and median.

Note: If time allows, consider asking, "How would the mean and median change if Oscar's next score was 27? 3?"



Activity 1

Name: Date: Period:

Measures of Center

2. Look at your scores and the scores of three other students from the warm-up activity.

Choose a student and describe their scores.

Responses vary.

- Dhruv got the score 26 three times. He also had one game where he only got 2 points.
- Chloe played the game 8 times. Her data is pretty bell-shaped.
- Oscar had scores ranging from 15 to 31.

3. A **measure of center** is a single number that represents a central value in a data set.

Mean and *median* are two measures of center.

Here are the mean and median for Oscar's scores.

What are some things you know about calculating the mean and median?

Responses vary.

- The median is the value in the middle.
- To figure out the mean, add up all the numbers and divide by how many numbers there are.

Oscar's Scores

15 25 15 23 26 31

0 5 10 15 20 25 30 35 40

Mean: 22.5

Median: 24

4. Here is the work Oscar did to calculate the median and the mean.

Discuss: How did Oscar calculate each measure of center?

Responses vary.

- To calculate the median, Oscar put the numbers in order from smallest to largest. He determined the numbers in the middle and then found the middle of those two numbers.
- To calculate the mean, Oscar added up all of the numbers and then divided by 6 because there are 6 numbers in his data set.

Oscar

15, 15, 23, 25, 26, 31

24

The median is 24.

15 + 25 + 15 + 23 + 26 + 31 = 135

135 ÷ 6 = 22.5

The mean is 22.5.

Unit 2 Lesson 5 185 Activity 1

Key Takeaway: Two examples of **measures of center** that can be used to represent a typical value of a data set are *means* and *medians*. The mean is the average and the median is the middle data value.

Pairs | 10 min

Activity 2 Shape and Center

Purpose: Students explore how the shape of a data set affects the mean and median.

1 Launch

Encourage students to use a calculator as needed.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to revisit the strategies on the previous page and the class definitions, or consider asking, "How could the dot plot help you to determine the median?"
Think that 20 is the median because it is the middle data value in the original list. (*Problem 5*)	**Support** Consider asking, "Are those data values in order from least to greatest?"

Listen for student reasoning about why the mean and median are far apart to share in the Connect.

Share explanations of which measure of center would best represent Dhruv's typical score to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Think that both values would increase because 300 is so much larger than 30.	**Support** Consider creating a simpler situation, either by using only three data points or by using physical objects such as blocks to model the situation.
Would like to explore measures of center further.	**Support** Consider asking, "Is it possible to adjust Chloe's highest score and not change the mean nor the median? If so, what is that value? If not, why not?"

3 Connect

Discuss why the mean and median are far apart.

Display the distribution of responses, calling attention to any conflict or consensus.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

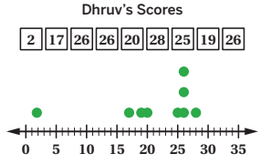
Shape and Center

5. Here are Dhruv's scores from the game. Determine the median and mean.

Use a calculator if it helps with your thinking.

Median: 25

Mean: 21



6. Analyze the mean and median for Dhruv's scores.

a. **Discuss:** Why do you think the mean and median are far apart?

Responses vary. The mean is much less than the median because of the one game where Dhruv scored only 2 points.

b. Which measure of center would you use to represent Dhruv's typical score? Circle one.

Mean Median

Explain your thinking.

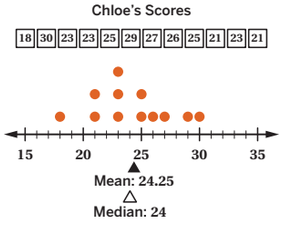
Responses and explanations vary. I would use the median because the time Dhruv scored 2 impacts the mean but not the median. That score was probably a mistake and not really typical.

7. Here are Chloe's scores from the game.

Imagine that her highest score was 300 instead of 30.

Which measure(s) of center would increase? Circle one.

(Mean) Median Both Neither



Unit 2 Lesson 5 186 Activity 2

Pairs | 10 min

Activity 2 Shape and Center (continued)

Purpose: Students explore how the shape of a data set affects the mean and median.

1 Launch

Display the dot plot and box plot for Dhruv next to each other along with the data values.

Encourage students to determine which representation would be easier to find each of the different measures of center and spread.

Display the dot plot and box plot for Chloe next to each other along with the data values.

Ask students if they agree with their statements from before about which representation better displays different measures of center and spread.

2 Monitor

Encourage students to provide examples for each opinion they make.

D **Differentiation: (Stretch)** Have students display each data set from Problem 8 in a stem and leaf plot and a histogram. Then, discuss if these representations are better for different measures of center and spread. **(MTR.3.1)**

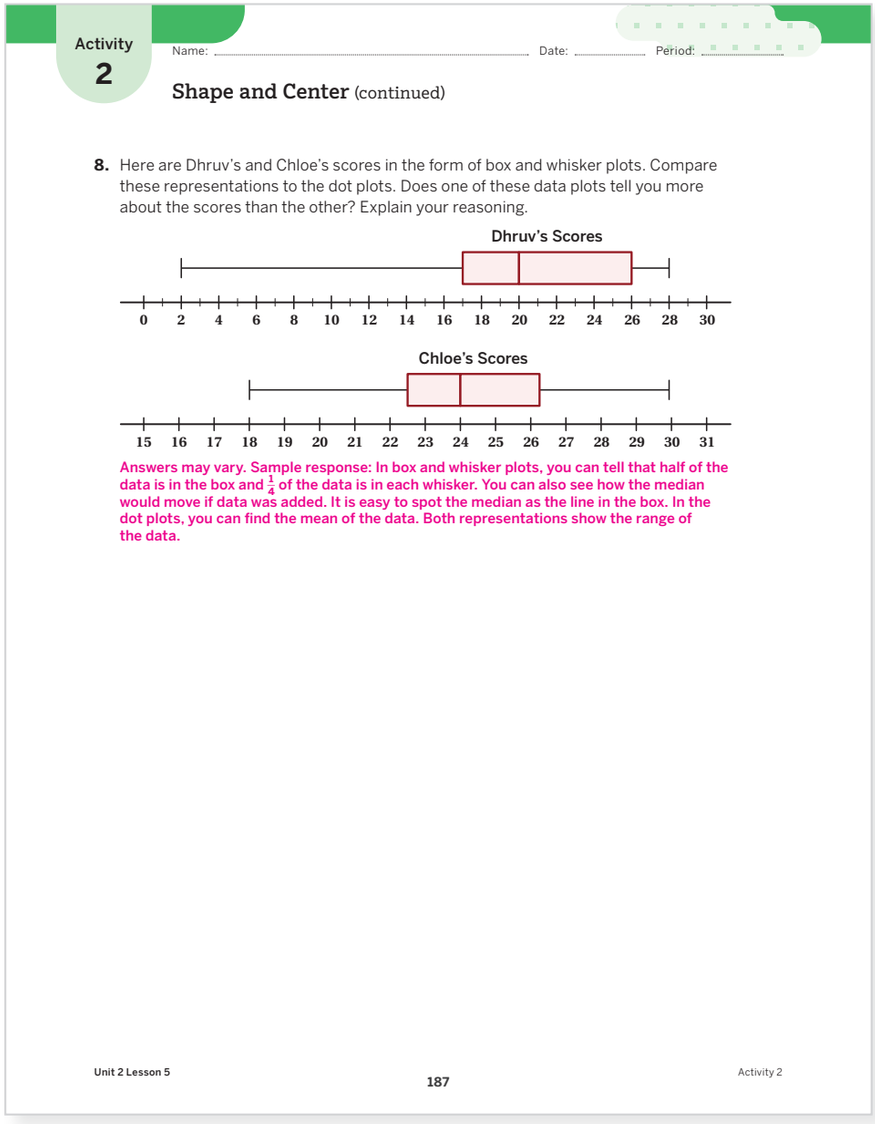
A **Accessibility: Executive Functioning** Consider chunking this activity by having students list what they can easily see in one dot plot, then in one box plot, then compare their notes.

3 Connect

Invite students to share their opinions with a partner. After listening to a partner's reasoning, do they still feel the same way about each representation and what it displays well?

To surface the Key Takeaway, consider asking:

- What types of measure does a dot plot represent well? What types of measure does a box plot represent well?
- Which representation do you prefer and why?



Activity 2

Name: Date: Period:

Shape and Center (continued)

8. Here are Dhruv's and Chloe's scores in the form of box and whisker plots. Compare these representations to the dot plots. Does one of these data plots tell you more about the scores than the other? Explain your reasoning.

Dhruv's Scores

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

Chloe's Scores

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Answers may vary. Sample response: In box and whisker plots, you can tell that half of the data is in the box and $\frac{1}{4}$ of the data is in each whisker. You can also see how the median would move if data was added. It is easy to spot the median as the line in the box. In the dot plots, you can find the mean of the data. Both representations show the range of the data.

Unit 2 Lesson 5 187 Activity 2

Key Takeaway: Different representations of data can display the same data set. However, each display is better at displaying certain measures of center and spread.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of mean and median by comparing the two measures of center.

Invite students to respond independently, and then share their thinking with a partner.

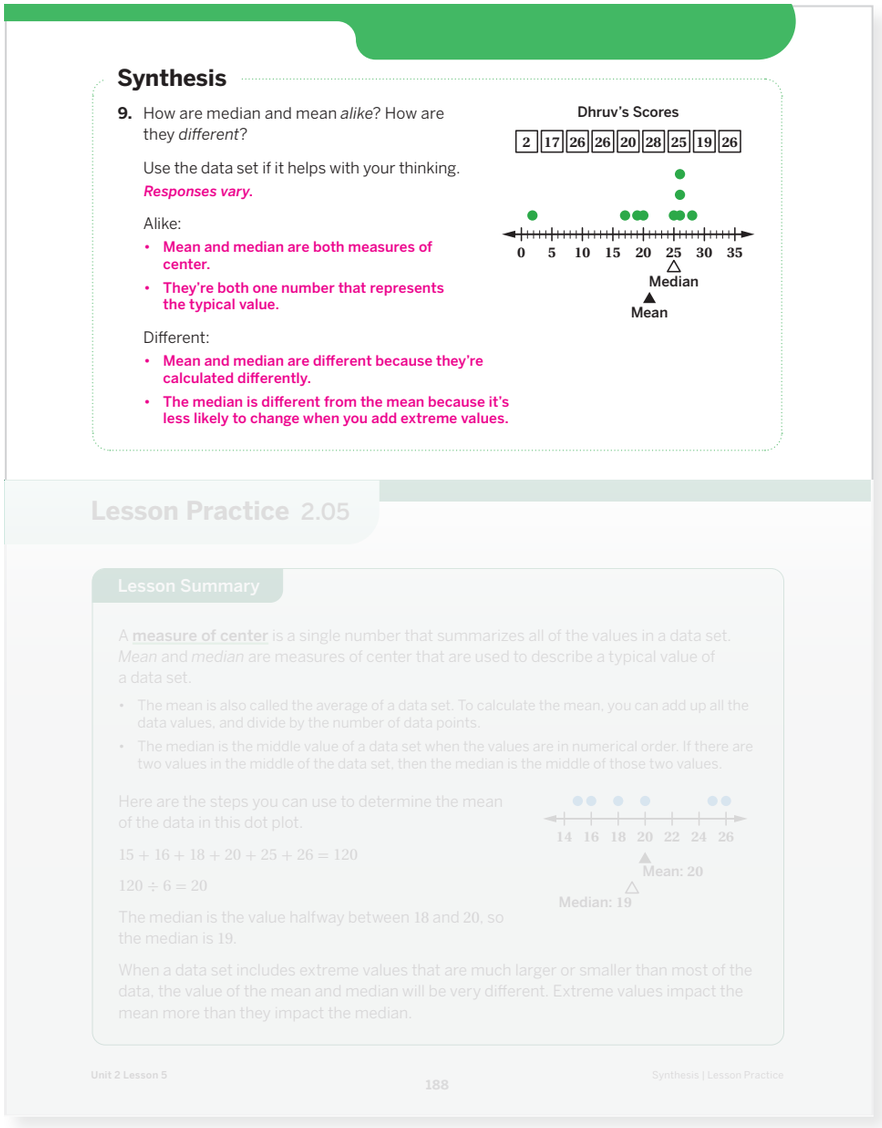
Share a variety of ideas.

Consider asking:

- "What do mean and median tell us about a data set?"
- "How is calculating mean different from calculating median?"
- "Which shapes of data are likely to have a mean and median that are close together? Far apart?"

MLR **MLR2: Collect and Display** Use this routine to add any new student ideas or language about measures of center to the class anchor chart. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. How are median and mean *alike*? How are they *different*?

Use the data set if it helps with your thinking.

Responses vary.

Alike:

- **Mean and median are both measures of center.**
- **They're both one number that represents the typical value.**

Different:

- **Mean and median are different because they're calculated differently.**
- **The median is different from the mean because it's less likely to change when you add extreme values.**

Dhruv's Scores

2 | 17 | 26 | 26 | 20 | 28 | 25 | 19 | 26

Lesson Practice 2.05

Lesson Summary

A **measure of center** is a single number that summarizes all of the values in a data set. *Mean* and *median* are measures of center that are used to describe a typical value of a data set.

- The mean is also called the average of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.
- The median is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the middle of those two values.

Here are the steps you can use to determine the mean of the data in this dot plot.

$15 + 16 + 18 + 20 + 25 + 26 = 120$

$120 \div 6 = 20$

The median is the value halfway between 18 and 20, so the median is 19.

When a data set includes extreme values that are much larger or smaller than most of the data, the value of the mean and median will be very different. Extreme values impact the mean more than they impact the median.

Unit 2 Lesson 5 188 Synthesis | Lesson Practice

Lesson Takeaway: The *mean* and *median* are both **measures of center** that can be used to represent a typical value of that data set. Means are affected more by extreme values than medians are.

Independent | 5 min

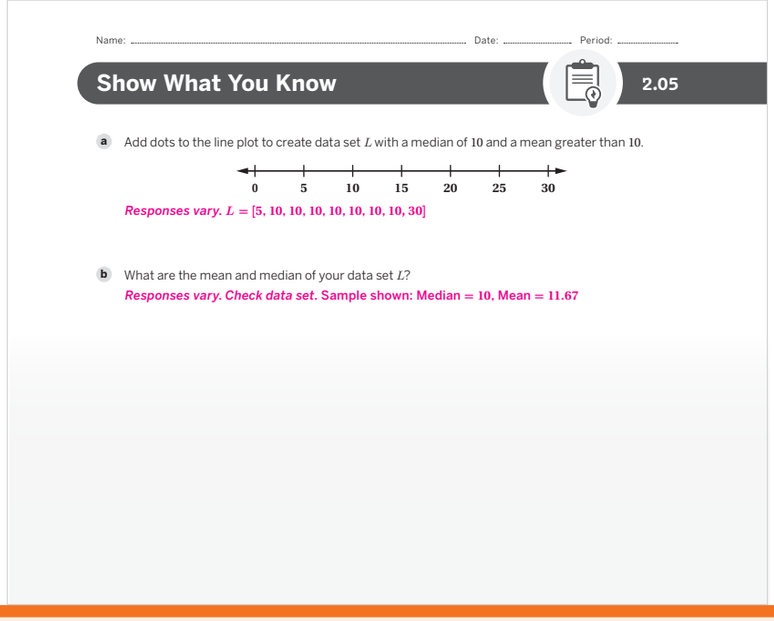
Show What You Know

Purpose: Students demonstrate their understanding by creating a data set that meets specific criteria and calculate the mean and median.

Today's Goals

1. **Goal:** Use technology to calculate the *measures of center* (mean and median) for a data set.
2. **Language Goal:** Explain how to calculate the mean and median and what these tell us about a data set. **(Speaking and Listening)**
3. **Language Goal:** Explain the effect of extreme values on the mean and median. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.05

a. Add dots to the line plot to create data set L with a median of 10 and a mean greater than 10.

0 5 10 15 20 25 30

Responses vary. $L = [5, 10, 10, 10, 10, 10, 10, 10, 30]$

b. What are the mean and median of your data set L?

Responses vary. Check data set. Sample shown: Median = 10, Mean = 11.67

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support creating a data set that meets specific criteria and/or calculating the mean and median:

» **Respond to Student Thinking:**

- Revisit **Grade 6, Unit 8, Lesson 6** (*Snack Time*) or **Lesson 8** (*Toy Cars*).
- Use the *Calculating Measures of Center and Spread* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

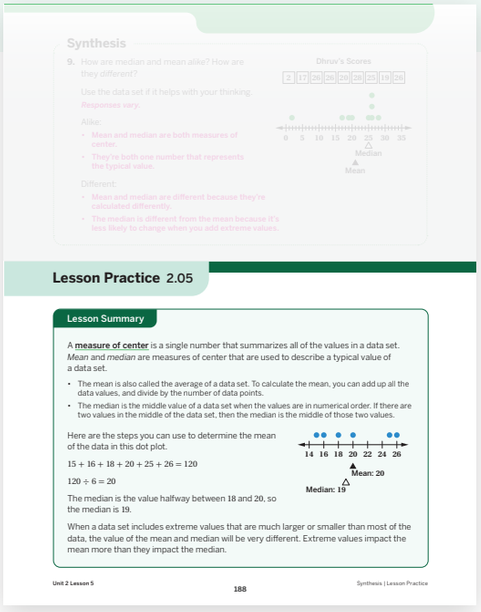
Professional Learning

Reflect on how you and your students used the calculator in today's lesson. How did using the calculator support student understanding of how shape affects the mean and median? How did it support students in looking for and expressing regularity as they repeatedly observed how changes to the data set affected means and medians? How can you continue to support students in using this tool strategically? **(MTR.3.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

9. How are median and mean alike? How are they different?

Use the data set if it helps with your thinking.

Responses vary.

Alike:

- Mean and median are both measures of center.
- They're both one number that represents the typical value.

Different:

- Mean and median are different because they're calculated differently.
- The median is different from the mean because it's less likely to change when you add extreme values.

Dhruv's Scores

2 17 26 26 28 28 28 28 28 35

Median

Mean

Lesson Practice 2.05

Lesson Summary

A **measure of center** is a single number that summarizes all of the values in a data set. *Mean* and *median* are measures of center that are used to describe a typical value of a data set.

- The mean is also called the average of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.
- The median is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the middle of those two values.

Here are the steps you can use to determine the mean of the data in this dot plot.

14 16 18 20 22 24 26

Mean: 20

Median: 19

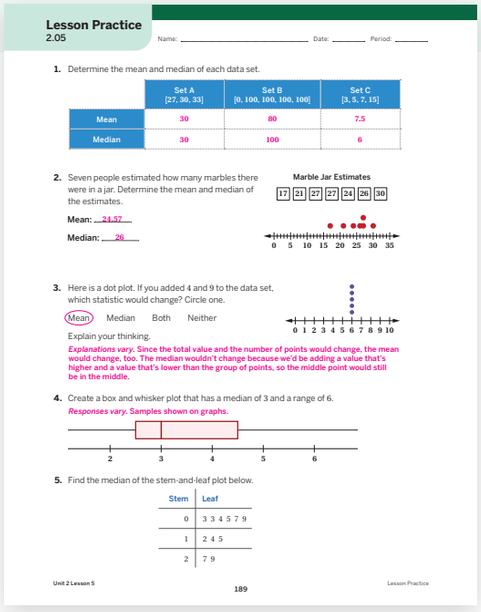
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$120 \div 6 = 20$

The median is the value halfway between 18 and 20, so the median is 19.

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Unit 2 Lesson 5 188 Synthesis | Lesson Practice



Lesson Practice 2.05

Name: _____ Date: _____ Period: _____

1. Determine the mean and median of each data set.

	Set A [27, 30, 33]	Set B [0, 100, 100, 100, 100]	Set C [3, 5, 7, 15]
Mean	30	80	7.5
Median	30	100	6

2. Seven people estimated how many marbles there were in a jar. Determine the mean and median of the estimates.

Marble Jar Estimates

17 21 27 27 24 26 30

0 5 10 15 20 25 30 35

Mean: 24.57

Median: 26

3. Here is a dot plot. If you added 4 and 9 to the data set, which statistic would change? Circle one.

Mean Median Both Neither

0 1 2 3 4 5 6 7 8 9 10

Explain your thinking.

Explanations vary. Since the total value and the number of points would change, the mean would change, too. The median wouldn't change because we'd be adding a value that's higher and a value that's lower than the group of points, so the middle point would still be in the middle.

4. Create a box and whisker plot that has a median of 3 and a range of 6.

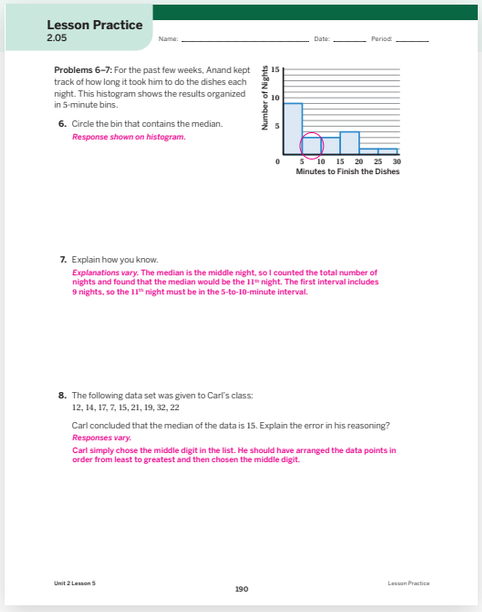
Responses vary. Samples shown on graphs.

2 3 4 5 6

5. Find the median of the stem-and-leaf plot below.

Stem	Leaf
0	3 3 4 5 7 9
1	2 4 5
2	7 9

Unit 2 Lesson 5 189 Lesson Practice



Lesson Practice 2.05

Name: _____ Date: _____ Period: _____

Problems 6–7: For the past few weeks, Anand kept track of how long it took him to do the dishes each night. This histogram shows the results organized in 5-minute bins.

6. Circle the bin that contains the median.

Response shown on histogram.

Number of Nights

0 5 10 15 20 25 30

Minutes to Finish the Dishes

7. Explain how you know.

Explanations vary. The median is the middle night, so I counted the total number of nights and found that the median would be the 11th night. The first interval includes 9 nights, so the 11th night must be in the 5-to-10-minute interval.

8. The following data set was given to Carl's class:

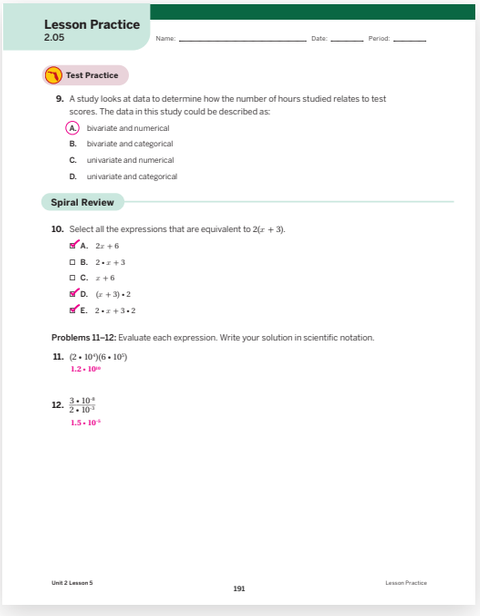
12, 14, 17, 7, 15, 21, 19, 32, 22

Carl concluded that the median of the data is 15. Explain the error in his reasoning?

Responses vary.

Carl simply chose the middle digit in the list. He should have arranged the data points in order from least to greatest and then chosen the middle digit.

Unit 2 Lesson 5 190 Lesson Practice



Lesson Practice 2.05

Name: _____ Date: _____ Period: _____

Test Practice

9. A study looks at data to determine how the number of hours studied relates to test scores. The data in this study could be described as:

A. bivariate and numerical

B. bivariate and categorical

C. univariate and numerical

D. univariate and categorical

Spiral Review

10. Select all the expressions that are equivalent to $2(x + 3)$.

☑ A. $2x + 6$

☐ B. $2 \cdot x + 3$

☐ C. $x + 6$

☑ D. $(x + 3) \cdot 2$

☑ E. $2 \cdot x + 3 \cdot 2$

Problems 11–12: Evaluate each expression. Write your solution in scientific notation.

11. $(2 \cdot 10^4)(6 \cdot 10^5)$

$1.2 \cdot 10^{10}$

12. $\frac{3 \cdot 10^4}{2 \cdot 10^3}$

$1.5 \cdot 10^1$

Unit 2 Lesson 5 191 Lesson Practice

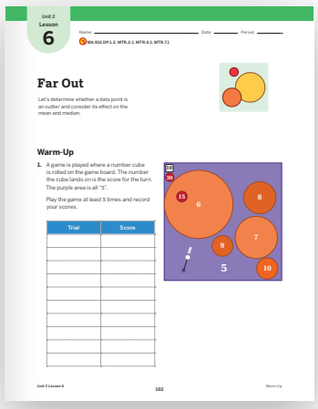
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.DP.1.1
	3	2	MA.912.DP.1.1
	4–8	2	MA.912.DP.1.1
Test Practice	9	3	MA.912.DP.1.2
Spiral Review			
	10	2	MA.6.AR.1.4
Fluency	11–12	2	MA.8.NSO.1.5

Unit 2

Lesson

6


Unit 2
Lesson
6
Far Out
Let's determine whether a data point is an outlier and consider its effect on the mean and median.
Warm-Up
1. A game is played where a number cube is rolled on the game board. The number the cube lands on is the score for the turn. The purple area is all "5".
Play the game at least 5 times and record your scores.
Trial
Score
15
8
8
7
9
5
10
Unit 2 Lesson 6
192
Warm-Up

Far Out

Identifying Outliers

Let's determine whether a data point is an outlier and consider its effect on the mean and median.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether or not a data point is an *outlier*.
2. **Language Goal:** Explain how outliers impact the mean or median of a data set. **(Speaking and Listening)**

Students learn what **outliers** are and how they affect the statistics of a data set. This lesson uses a game context to consider why an outlier might impact a typical score. Students are introduced to the rule that outliers are values further than 1.5 times the *IQR* from either *quartile* 1 or quartile 3.

Coherence

Prior Learning

In Lesson 5, students revisited measures of center and spread. They compared data using box plots and dot plots.

Future Learning

In Lesson 7, students will apply their understanding of outliers as they compare data sets in real world contexts.

Connections

- Students explore outliers in a data set and their impact on deciding which measure of central tendency to apply to a situation such as typical test score, number of texts received, and ages of people in a bike shop. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.2

Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.

Also addressing: **MA.912.DP.1.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.DP.1.1

MA.6.DP.1.3

Vocabulary

New Vocabulary

outlier

Review Vocabulary

interquartile range (IQR)

mean

median

quartile

Lesson at a Glance

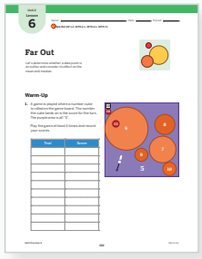
~ 45 min

MA.912.DP.1.1, MA.912.DP.1.2

Warm-Up

Independent | 5 min

Students are introduced to the game context that tey will use throughout Activity 1.

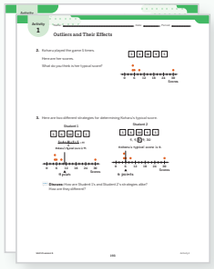


Activity 1

Pairs | 15 min

Students make sense of what an **outlier** is and its effects on the measures of center in a data set.

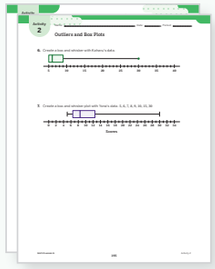
Routines: Decide and Defend, MLR7: Compare and Connect, **(MTR.2.1, MTR.4.1)**



Activity 2

Pairs | 15 min

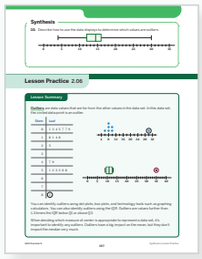
Students identify outliers by making box plots and look for points marked as outliers. **(MTR.6.1)**



Synthesis

Whole Class | 5 min

Students determine how to use different data displays to identify outliers.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which data points in a set are outliers.



Prep Checklist

Prepare copies of the Warm-Up Activity Game Board

This lesson includes:

- Student Edition
- Show What You Know
- Copy the Warm-Up Activity Game Board, one per pair
  - blank number cubes, one per pair

Independent | 5 min

Warm-Up

Purpose: Students are introduced to the game context that they will use throughout Activity 1.

1 Launch

Demonstrate how to play the game.

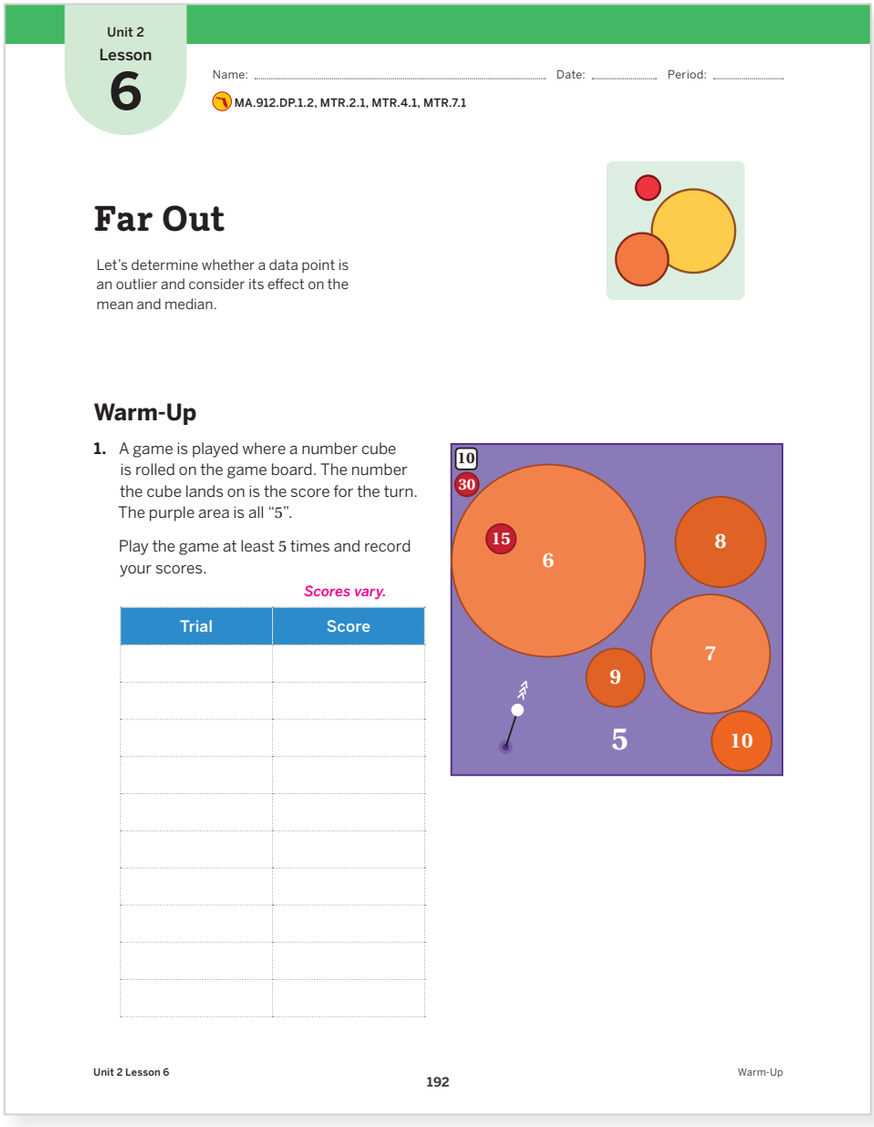
Distribute print-outs of the Game Board and number cubes, one set per pair of students.

Consider asking, "Have you ever played a game like this before? If so, where?"

2 Connect

Invite students to share what they noticed about the game and how they might compare who was a better player of this game.

Consider asking, "How would you decide which of two people is better at this game?"



Unit 2
Lesson
6

Name: _____ Date: _____ Period: _____

MA.912.DP.1.2, MTR.2.1, MTR.4.1, MTR.7.1

Far Out

Let's determine whether a data point is an outlier and consider its effect on the mean and median.

Warm-Up

1. A game is played where a number cube is rolled on the game board. The number the cube lands on is the score for the turn. The purple area is all "5".

   Play the game at least 5 times and record your scores.

Scores vary.

Trial	Score

Unit 2 Lesson 6 192 Warm-Up

Pairs | 15 min

Activity 1 Outliers and Their Effects

Purpose: Students make sense of what an **outlier** is and its effects on the measures of center in a data set.

1 Launch

Invite students to use their experience playing the game to decide how to respond. Consider sharing that there is not a single correct answer for this question.

ELL **English Language Learners** To help students access background knowledge, clarify the question as a class or explain phrases like "typical score."

2 Monitor

Look for students who use different strategies to determine the typical score (e.g., *mean*, *median*, *mode*).

D **Differentiation** (*Problems 2–3*)

Look for students who:	Teacher Moves
Need support getting started.	**Support** Encourage them to choose a value between 5 and 30. Consider asking, "Do you think this value is typical of Koharu's scores? What might Koharu score next time she plays?"
Use the dot plot to visually estimate a typical score. (*Problem 2*)	**Strengthen** Celebrate that they used the dot plot. Consider asking, "How might you use previous scores to calculate a typical score?"
Use the most frequent score (i.e., the mode = 5) to calculate the typical score. (*Problem 2*)	**Strengthen** Celebrate that they calculated a value. Consider asking, "How do the scores of 6, 9, and 30 affect her typical score?"
Calculate the mean or median of the scores.	**Stretch** Invite them to calculate the other measure of center (mean or median) and decide which value they think is more typical.

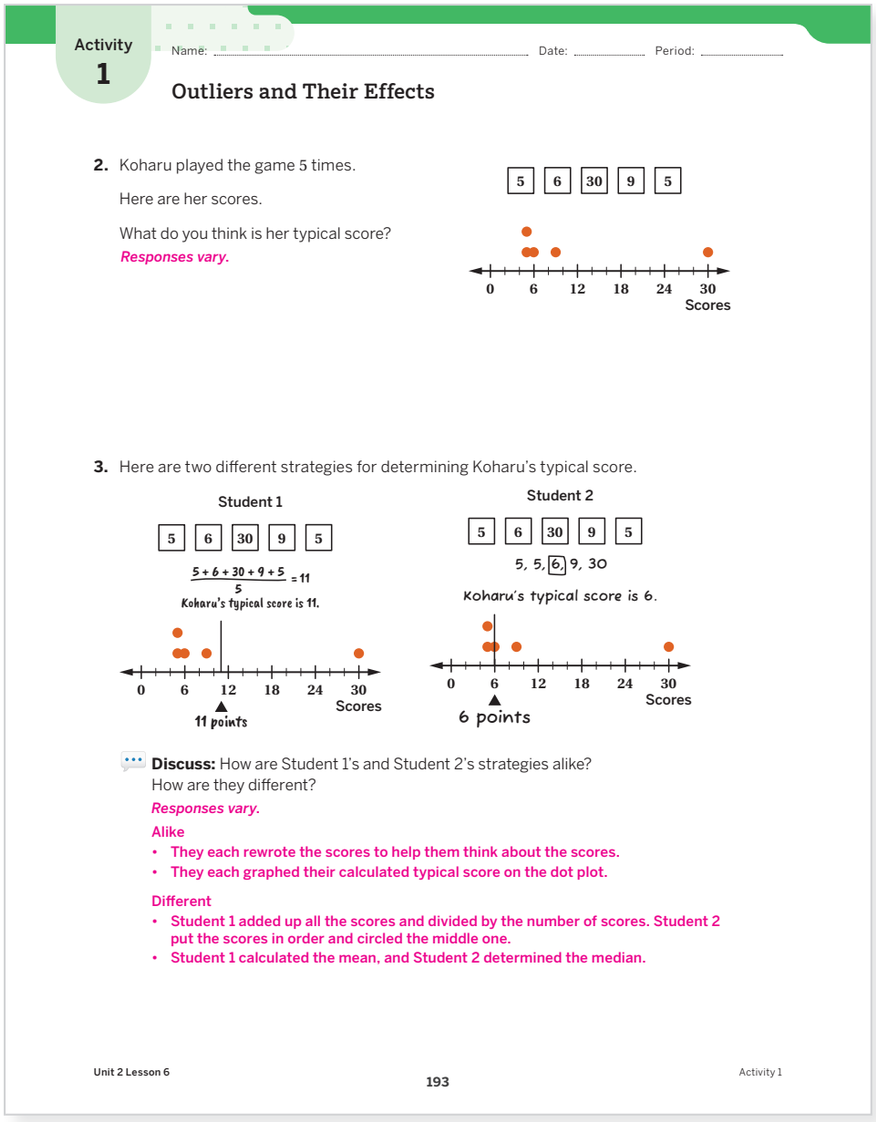
MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and connect to their own strategy on the previous page.

Look for students who notice that Student 1 calculated the mean and Student 2 calculated the median of Koharu's data set, and for students who notice that the strategies result in different values. **(MTR.2.1)**

Consider asking:

- "What did each student do?" **(MTR.2.1)**
- "How does each strategy compare to your own?"
- "When might you use each strategy?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Outliers and Their Effects

2. Koharu played the game 5 times.

Here are her scores.

What do you think is her typical score?

Responses vary.

5 6 30 9 5

3. Here are two different strategies for determining Koharu's typical score.

Student 1

5 6 30 9 5

$\frac{5+6+30+9+5}{5} = 11$

Koharu's typical score is 11.

Student 2

5 6 30 9 5

5, 5, 6, 9, 30

Koharu's typical score is 6.

Discuss: How are Student 1's and Student 2's strategies alike? How are they different?

Responses vary.

Alike

- They each rewrote the scores to help them think about the scores.
- They each graphed their calculated typical score on the dot plot.

Different

- Student 1 added up all the scores and divided by the number of scores. Student 2 put the scores in order and circled the middle one.
- Student 1 calculated the mean, and Student 2 determined the median.

Unit 2 Lesson 6 193 Activity 1

Activity 1 Outliers and Their Effects (continued)

Purpose: Students make sense of what an **outlier** is and its effects on the measures of center in a data set.

2 Monitor

Display the distribution of responses.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

Pause to discuss the shape of Koharu's scores.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

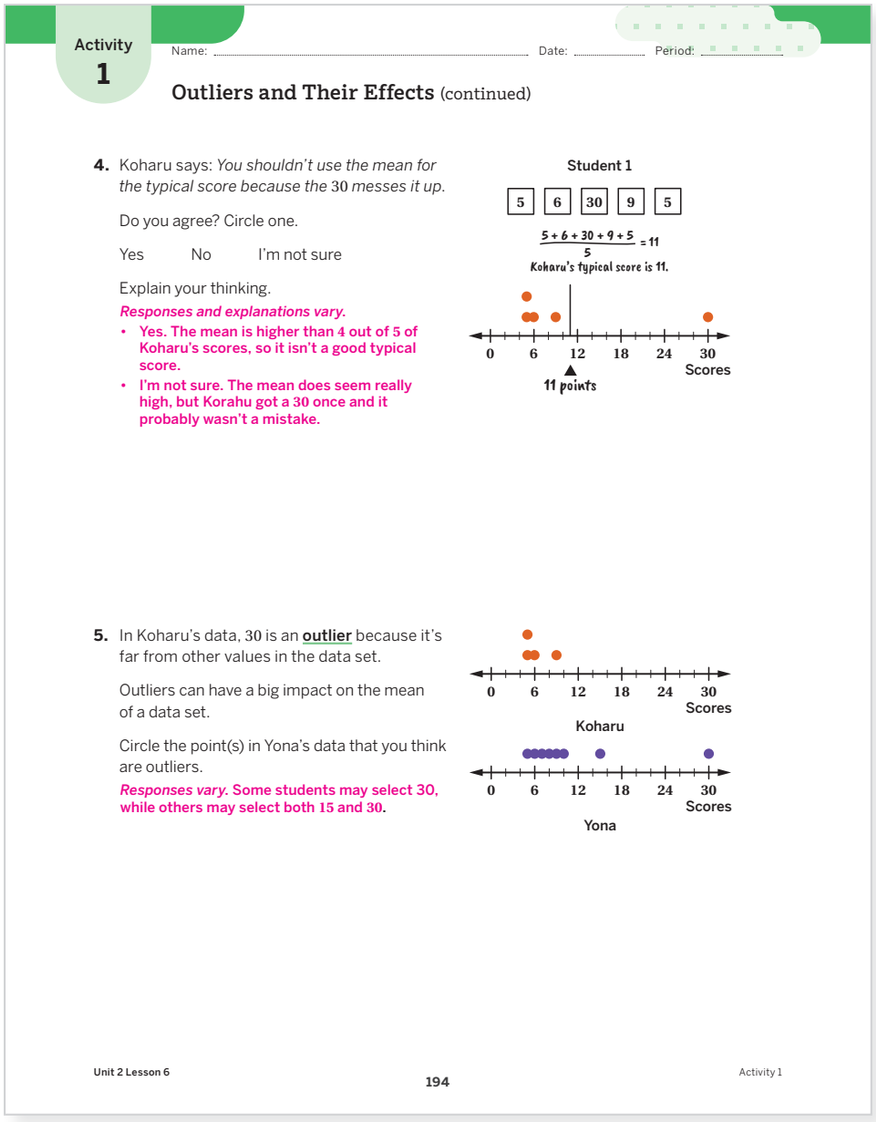
Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"
- "What is the shape of Koharu's data? How do you know?"
- "Why might Koharu choose to use the median as her typical score?"

Look for students who identify different points as outliers to share their thinking during the Connect.

3 Connect

Invite students to share what they think the term **outlier** means in their own words with a partner.

To surface the Key Takeaway, consider asking:
- "How would you describe an outlier to someone who is absent?"
- "How did you decide which points are outliers?"
- "How do the outliers affect the median of the data?"



Activity 1

Name: _____ Date: _____ Period: _____

Outliers and Their Effects (continued)

4. Koharu says: *You shouldn't use the mean for the typical score because the 30 messes it up.*

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

Responses and explanations vary.
- Yes. The mean is higher than 4 out of 5 of Koharu's scores, so it isn't a good typical score.
- I'm not sure. The mean does seem really high, but Korahu got a 30 once and it probably wasn't a mistake.

Student 1

5, 6, 30, 9, 5

$\frac{5+6+30+9+5}{5} = 11$

Koharu's typical score is 11.

5. In Koharu's data, 30 is an **outlier** because it's far from other values in the data set.

Outliers can have a big impact on the mean of a data set.

Circle the point(s) in Yona's data that you think are outliers.

Responses vary. Some students may select 30, while others may select both 15 and 30.

Unit 2 Lesson 6 194 Activity 1

Key Takeaway: An **outlier** is a data value that is far from the other values in the data set.

Pairs | 15 min

Activity 2 Outliers and Box Plots

Purpose: Students identify outliers by making box plots and look for points marked as outliers.

Short on time? Consider completing problem 7 as a whole class instead of in pairs.

1 Launch

Consider asking, "What measures make a box plot? What information does a box plot display well?"

Demonstrate how to calculate an outlier from the IQR.

Note: Students will not be assessed on their ability to use the 1.5 times IQR rule to determine outliers. Using this rule is beyond the scope of the standards for this course.

2 Monitor

To support students getting started, consider asking, "How could we create a box plot for Kaharu's data?"

Display the Problem.

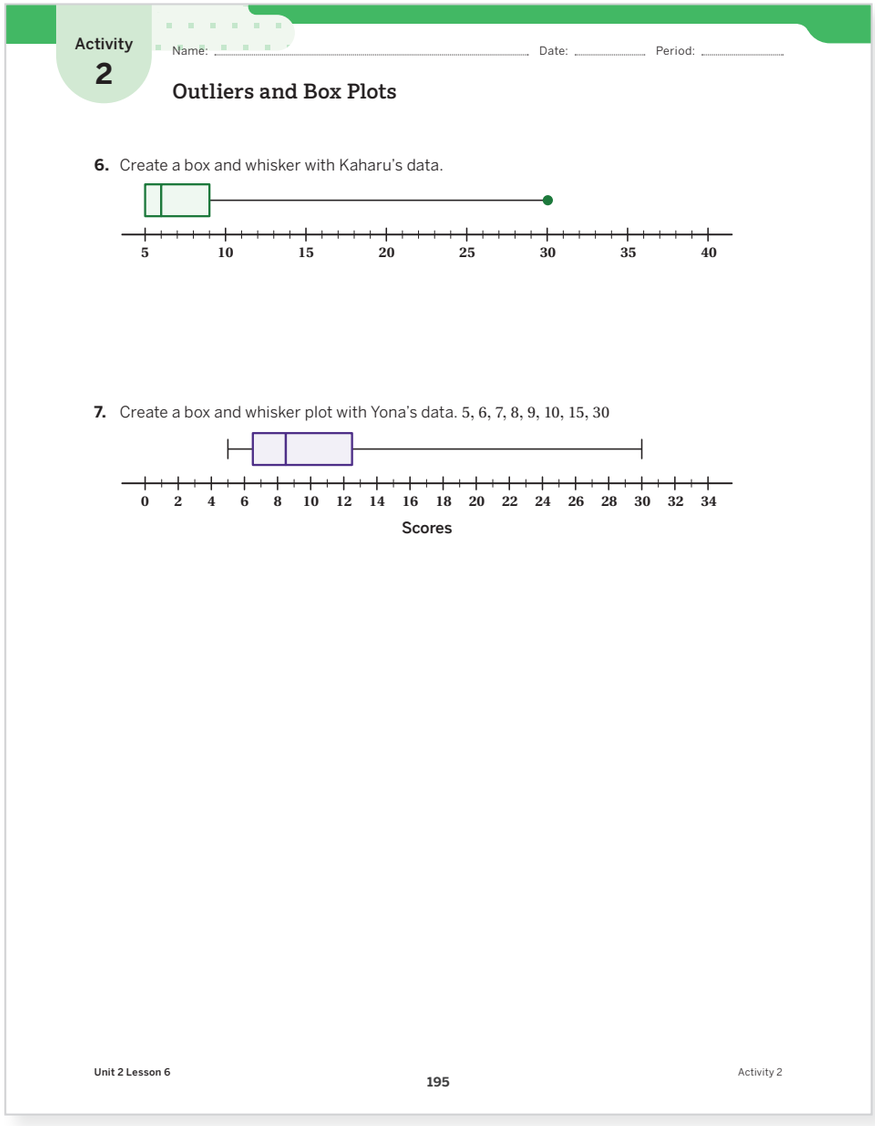
A **Accessibility: Memory and Attention:** Provide a model for students to use when creating their box plot.

Invite students to determine which value is an outlier in the given data set.

D **Differentiation: (Stretch)** Encourage students to write a rule for how to determine if a value is an outlier. **(MTR.6.1)**

Demonstrate how to create Kaharu's box plot with student input before having them create the box plot for Yona's data.

Activity 2 continued >



Activity 2

Name: Date: Period:

Outliers and Box Plots

6. Create a box and whisker with Kaharu's data.

5 10 15 20 25 30 35 40

7. Create a box and whisker plot with Yona's data. 5, 6, 7, 8, 9, 10, 15, 30

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34

Scores

Unit 2 Lesson 6 195 Activity 2

Pairs | 15 min

Activity 2 Outliers and Box Plots (continued)

Purpose: Students identify outliers by making box plots and look for points marked as outliers.

Short on time? Consider completing Problem 7 as a whole class instead of in pairs.

2 Monitor

Consider sharing that the purpose of this problem is for students to explore when data points become outliers, but not necessarily to generate the rule on their own.

Select and sequence student responses to share during the Connect.

D Differentiation *(Problem 8)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Discuss with students how to choose the intervals for the histogram from the data set.
Struggle labeling the axes.	**Strengthen** Discuss "frequency" as the value that is on the y-axis.
Notice that the outlier seems to disapper in the histogram.	**Stretch** Encourage them to begin to discuss which data points are better represented in box plots versus histograms.

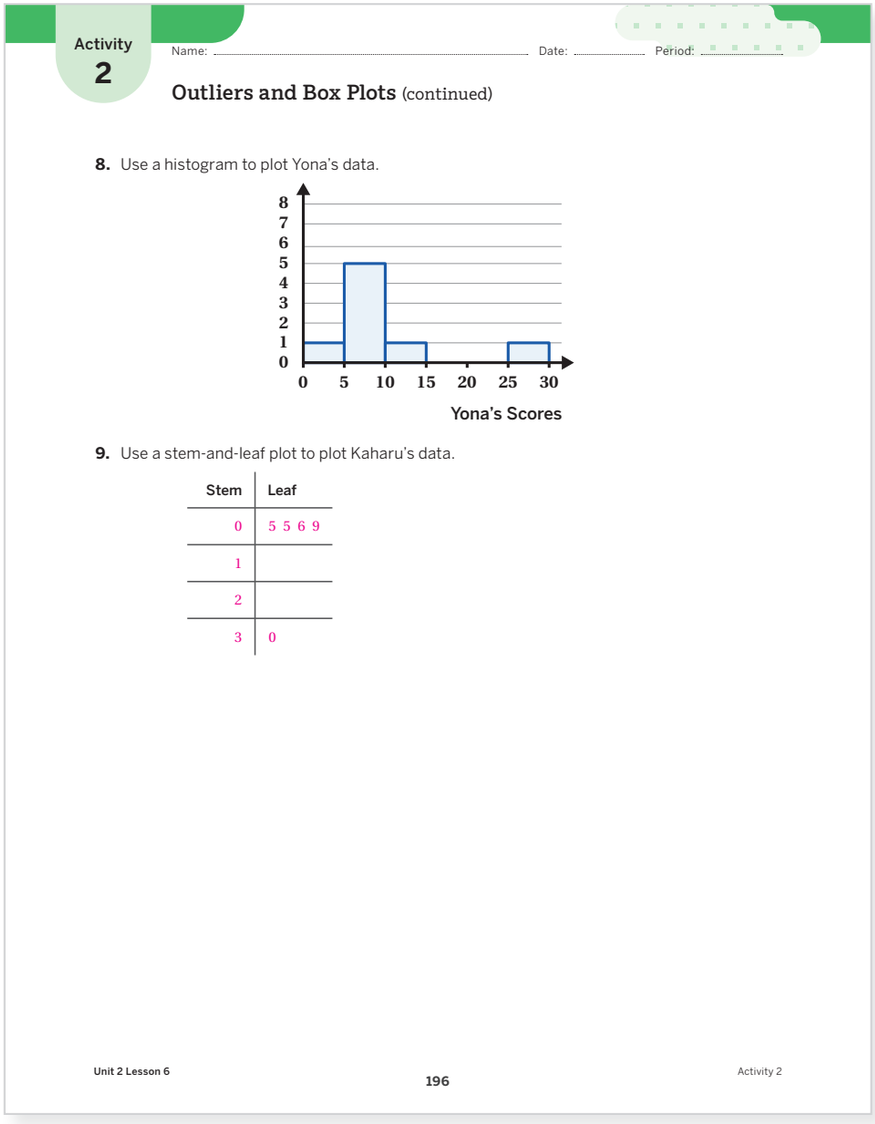
3 Connect

Share student work: compare the different representations for Kaharu and Yona.

Discuss the strengths and weaknesses for each representation. Consider asking, "Which representations can show outliers?"

To surface the Key Takeaway, consider asking:

- "When might you determine if a value is an outlier?" **(MTR.6.1)**
- "Why is it helpful to know whether a value is an outlier?"



Activity 2

Name: Date: Period:

Outliers and Box Plots (continued)

8. Use a histogram to plot Yona's data.

9. Use a stem-and-leaf plot to plot Kaharu's data.

Stem	Leaf
0	5 5 6 9
1	
2	
3	0

Unit 2 Lesson 6 196 Activity 2

Key Takeaway: Outliers can be determined by identifying values further than 1.5 times the IQR from either quartile 1 or quartile 3.

Whole Class | 5 min

Synthesis

Purpose: Students determine how to use different data displays to identify outliers.

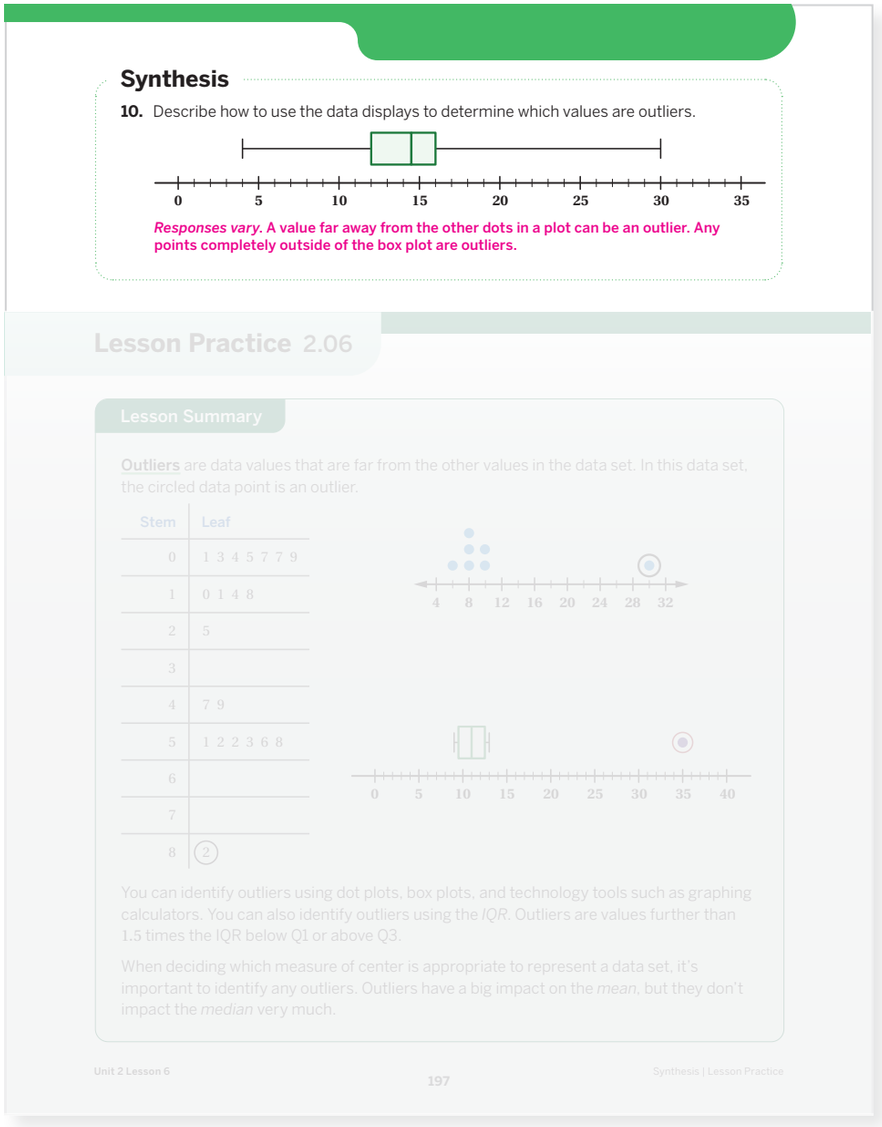
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:
- Which value(s) appear to be outliers in the dot plot and box plot?
- How can you tell they may be outliers?
- How can you check your conclusions?

Consider asking:
- "What about identifying outliers do you find challenging? What makes sense to you?"
- "What are errors someone might make in trying to identify outliers?"

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: box plot, IQR, Q1, Q3, outlier(s), data points, list, etc. **(Writing and Speaking)**

Note: If time allows, invite students to share with a partner what makes sense about determining outliers and what questions they still have.



Synthesis

10. Describe how to use the data displays to determine which values are outliers.

Responses vary. A value far away from the other dots in a plot can be an outlier. Any points completely outside of the box plot are outliers.

Lesson Practice 2.06

Lesson Summary

Outliers are data values that are far from the other values in the data set. In this data set, the circled data point is an outlier.

Stem	Leaf
0	1 3 4 5 7 7 9
1	0 1 4 8
2	5
3	
4	7 9
5	1 2 2 3 6 8
6	
7	
8	(2)

You can identify outliers using dot plots, box plots, and technology tools such as graphing calculators. You can also identify outliers using the *IQR*. Outliers are values further than 1.5 times the IQR below Q1 or above Q3.

When deciding which measure of center is appropriate to represent a data set, it's important to identify any outliers. Outliers have a big impact on the *mean*, but they don't impact the *median* very much.

Unit 2 Lesson 6 197 Synthesis | Lesson Practice

Lesson Takeaway: When a data set contains outliers, the *mean* is less representative of the data. Outliers have minimal impact on the *median* of a data set.

Independent | 5 min

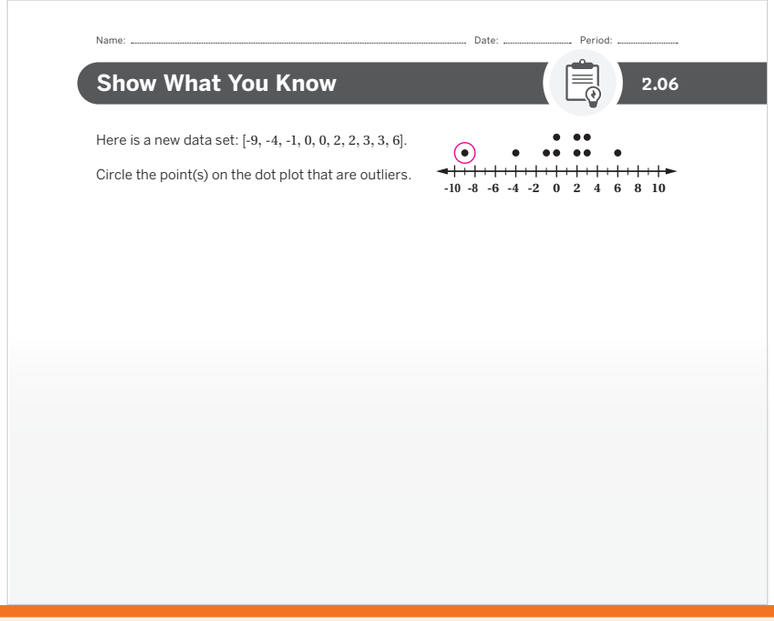
Show What You Know

Purpose: Students demonstrate their understanding by determining which data points in a set are outliers.

Today's Goals

1. **Goal:** Determine whether or not a data point is an *outlier*.
2. **Language Goal:** Explain how outliers impact the mean or median of a data set. **(Speaking and Listening)**

Show What You Know (Assessment Resources)


Name: _____ Date: _____ Period: _____

Show What You Know 2.06

Here is a new data set: [-9, -4, -1, 0, 0, 2, 2, 3, 3, 6].

Circle the point(s) on the dot plot that are outliers.

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support identifying outliers from data representations:

» **Respond to Student Thinking:**

- Revisit **Grade 6, Unit 8, Lesson 4** (*What Does It All Mean?*)
- Revisit the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on the lesson you just taught. How were you able to use questioning to encourage students to compare data displays? What about the facilitation felt successful? What are some areas for growth next time?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how to use the data displays to determine which values are outliers.

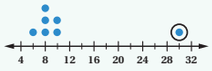
Responses vary. A value far away from the other dots in a plot can be an outlier. Any points completely outside of the box plot are outliers.

Lesson Practice 2.06

Lesson Summary

Outliers are data values that are far from the other values in the data set. In this data set, the circled data point is an outlier.

Stem	Leaf
0	1 3 4 5 7 7 9
1	0 1 4 8
2	5
3	
4	7 9
5	1 2 2 3 6 8
6	
7	
8	(2)





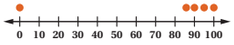
You can identify outliers using dot plots, box plots, and technology tools such as graphing calculators. You can also identify outliers using the *IQR*. Outliers are values further than 1.5 times the IQR below Q1 or above Q3.

When deciding which measure of center is appropriate to represent a data set, it's important to identify any outliers. Outliers have a big impact on the *mean*, but they don't impact the *median* very much.

Lesson Practice 2.06

Name: ..... Date: ..... Period: .....

Problems 1–2: Ricardo got the following scores on five class assignments: 87, 90, 0, 95, and 100. His teacher lets each student decide whether the student's final score will be the mean or the median of those five scores.



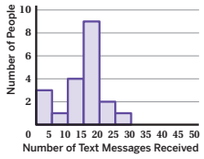
1. Would using the mean or the median give Ricardo the higher final score?

Median

2. Explain your thinking.

Explanations vary. Ricardo did really well on 4 of his 5 assignments, so reporting the median would give him the higher score.

Problems 3–4: This histogram represents the number of text messages that 20 people received in one day.



3. If you removed one person who received 3 text messages from the data set, would the median increase, decrease, or stay the same?

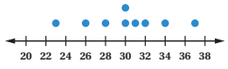
Stay the same

4. If you added a person who received 52 text messages to the data set, would the mean or median change more? Circle one.

(Mean) Median

Explain your thinking. *Explanations vary.* The mean will change more because 52 is much bigger than the rest of the numbers, so it will increase the total number of text messages by a lot but only increase the number of data points by 1.

5. Here is a data set:
26, 30, 31, 32, 28, 30, 34, 37, 23.



a. Julian says that 37 is an outlier because it is the maximum value. What could you say to Julian to help him understand his mistake?

Responses vary. I would tell Julian that the maximum is not always an outlier and that he can use a graphing calculator to determine whether or not any point is an outlier.

b. Would this data be considered numerical or categorical? Explain.

Responses vary. The data would be numerial since all data points are numbers.

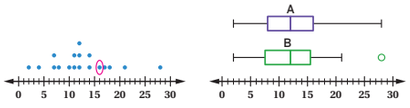
c. Would this data be considered univariate or bivariate? Explain.

Responses vary. The data is univariate because only one data point is collected.

Lesson Practice 2.06

Name: ..... Date: ..... Period: .....

6. The dot plot represents the data in box plot A. Circle one point to remove from the dot plot so that it represents box plot B instead. *Responses vary.*



Test Practice

7. This dot plot represents the ages of different people in a bike shop. Which data point would be an outlier if it were added to the data set?



(A.) 44 B. 8 C. Both D. Neither

Spiral Review

Problems 8–10: Evaluate each expression. Write your answer as a fraction.

8. $\frac{2}{3}+\frac{1}{5}$

$=\frac{13}{15}$

9. $\frac{2}{5}-\frac{3}{8}$

$=\frac{1}{40}$

10. $\frac{6}{15}+\frac{8}{20}$

$=\frac{4}{5}$

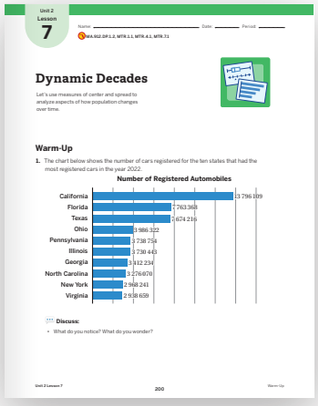
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3	1	MA.912.DP.1.2
	2, 4–5	2	MA.912.DP.1.2
	6	3	MA.912.DP.1.2
Test Practice	7	2	MA.912.DP.1.2
Spiral Review			
Fluency	8–10	1	MA.5.FR.2.1

Unit 2

Lesson

7



Dynamic Decades

Comparing Data Using Measures of Center and Spread

Let's use measures of center and spread to analyze aspects of how population changes over time.

Learning Outcomes

Today's Goals

1. **Goal:** Students explore data for various states in the U.S. and use statistics to describe the data.
2. **Goal:** Students use technology to make models and representations and find statistics from the data, including describing its shape. Based on the shape, students decide which statistics to use to make an argument about how the data has changed over time
3. **Language Goal:** Compare and contrast two data sets using measures of center and spread that are appropriate for the shape of the data. **(Reading, Writing, Speaking, and Listening)**

Students explore data on the number of car registrations in different states. They look at five number summary data to compare data and make arguments about how the data has changed over time.

Coherence

Prior Learning

In Lessons 1-6 students explored survey data and how to calculate and interpret measures of center and spread. In lesson 6, students explored outliers and explained how they affect measures of the center.

Future Learning

In Lesson 8, students will explore how to use a survey to estimate results in a population.

Connections

- Students use measures of center and spread to describe how aspects of the United States have changed over time such as postal rates. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.2

Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.

Also addressing: **MA.912.DP.1.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
6.SP.A.3	**HSS.ID.A.4**
6.SP.B.4	
6.SP.B.5.C	
6.SP.B.5.D	

Vocabulary

Review Vocabulary

interquartile range (IQR)	*minimum*
maximum	*outlier*

Lesson at a Glance

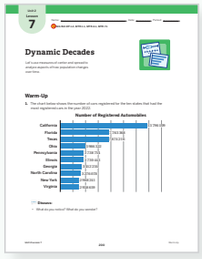
~ 45 min

MA.912.DP.1.1, MA.9.12.DP.1.2

Warm-Up

Pairs | 5 min

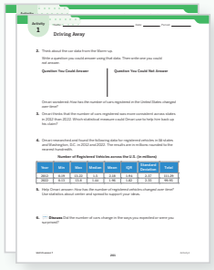
Students use the Notice and Wonder routine to reflect on state car registration data individually and collectively. **Routine(s):** Notice and Wonder **(MTR.1.1)**



Activity 1

Act 1 Small Groups | 15 min

Students use measures of center and spread to develop questions about data and engage in discussions that reflect on their thinking and the thinking of others. **Routine(s):** MLR5: Co-Craft Questions **(MTR.4.1)**



Activity 2

Act 2 Small Groups | 15 min

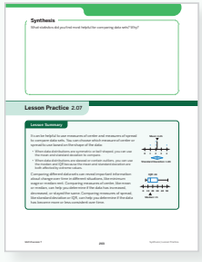
Students compare data sets in real-world contexts using the shape of the data to decide which measures of center support their reasoning.



Synthesis

Whole Class | 5 min

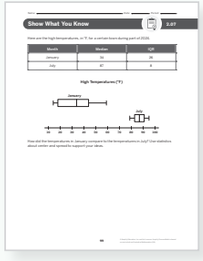
Students synthesize their understanding of how different statistics can help compare data sets and why it might be important to study how things change over time.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using statistics to compare temperature data over time in a real-world context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Warm-Up

Purpose: Students use the Notice and Wonder routine to reflect on state car registration data individually and collectively. **(MTR.1.1)**

1 Launch

Display the bar graph of car registration data.

ELL **English Language Learners** Invite students to share what they know about rules around driving cars. How old does someone have to be to drive in their state? Discuss registration and insurance. **(Speaking, Reading, and Listening)**

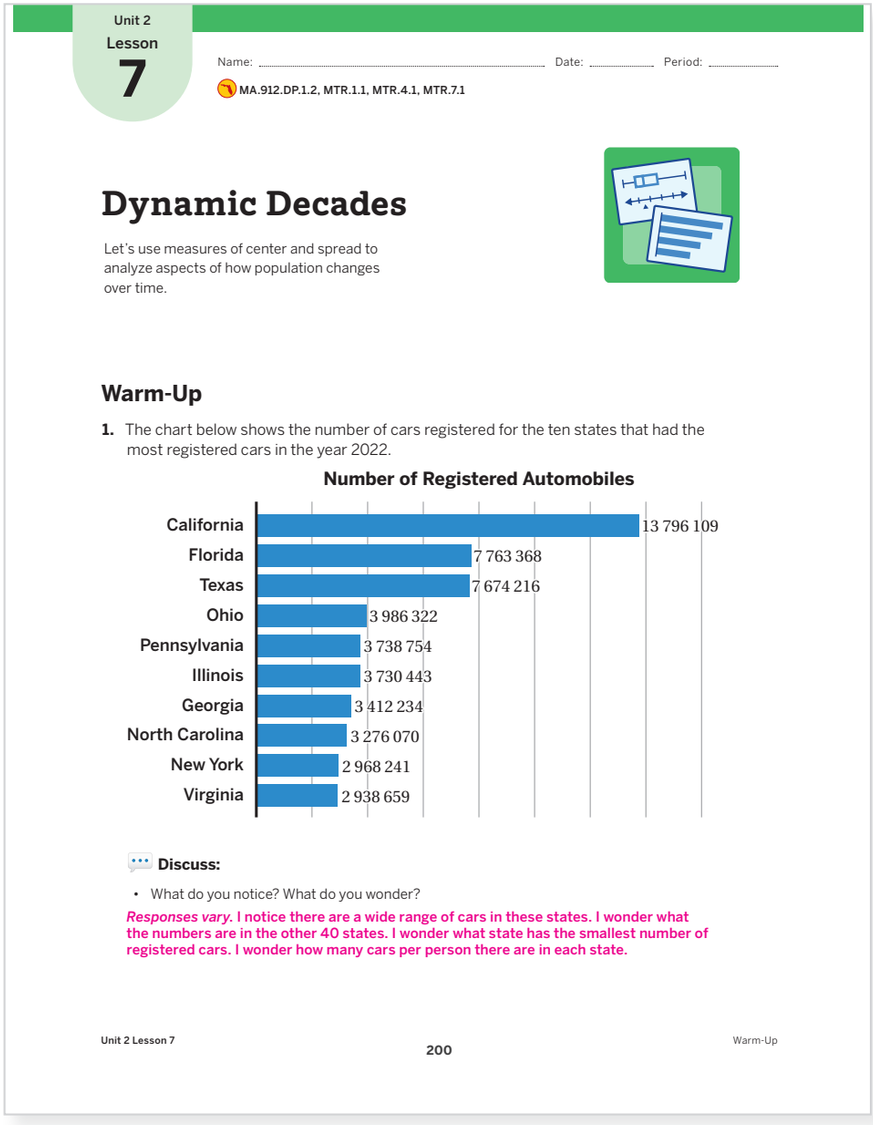
Use the **Notice and Wonder** routine to promote curiosity and help students make sense of the data in the graph. Have students work independently, then share their notice and wonders with a partner, then as a class.

A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific state from the graph and create a sentence stem for students to read and fill in the blank with the data. Example, "In _____ the number of registered cars in 2022 was _____ million."

2 Connect

Invite students to share their thinking about why someone might compare data on the number of car registrations in each state.

Consider asking,
"What does it mean for a car to be "registered? Why might someone compare the number of cars that are registered in each state?"; "Think about what you know about each state and where they are located on the map. Why might some states have so many more car registrations than others?"
Responses vary. A car is registered to the person who bought it and can legally drive it. I notice that states that are more populated have more cars registered. I wonder how many times people have more than one car registered to them. Someone might compare the number of cars per state to figure out if there are more cars per person in each state or compare the data to how the environment is affected by emissions.



Unit 2
Lesson 7

Name: _____ Date: _____ Period: _____

MA.912.DP.1.2, MTR.1.1, MTR.4.1, MTR.7.1

Dynamic Decades

Let's use measures of center and spread to analyze aspects of how population changes over time.

Warm-Up

1. The chart below shows the number of cars registered for the ten states that had the most registered cars in the year 2022.

Discuss:

- What do you notice? What do you wonder?

Responses vary. I notice there are a wide range of cars in these states. I wonder what the numbers are in the other 40 states. I wonder what state has the smallest number of registered cars. I wonder how many cars per person there are in each state.

Unit 2 Lesson 7 200 Warm-Up

Small Groups | 15 min

Activity 1 Driving Away

Purpose: Students use measures of center and spread to develop questions about data and engage in discussions that reflect on their thinking and the thinking of others. **(MTR.4.1)**

Short on time? Consider providing a list of questions for students to determine if they can or cannot answer them by using the given data.

1 Launch

MLR **MLR5: Co-Craft Questions** Use this routine to support students in making sense of the context in Problem 2, by asking questions without the pressure to produce answers.

2 Monitor

Select and sequence student reading about how the car registration data changed from 2012–2022.

D **Differentiation** *(Problems 2–6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to consider the populations in the top few states to make a notice or wonder question.
Describe the change as center or spread, but not both. *(Problem 5)*	**Strengthen** Consider asking, "How could another statistic in the table strengthen your argument?"
Were surprised by how something in the data changed from 2012 to 2022. *(Problem 6)*	**Stretch** Invite them to share why they were surprised and share why that change might have occurred.

Look for/Listen for strategies/explanations/ideas of student thinking and share/invite students to share more.

3 Connect

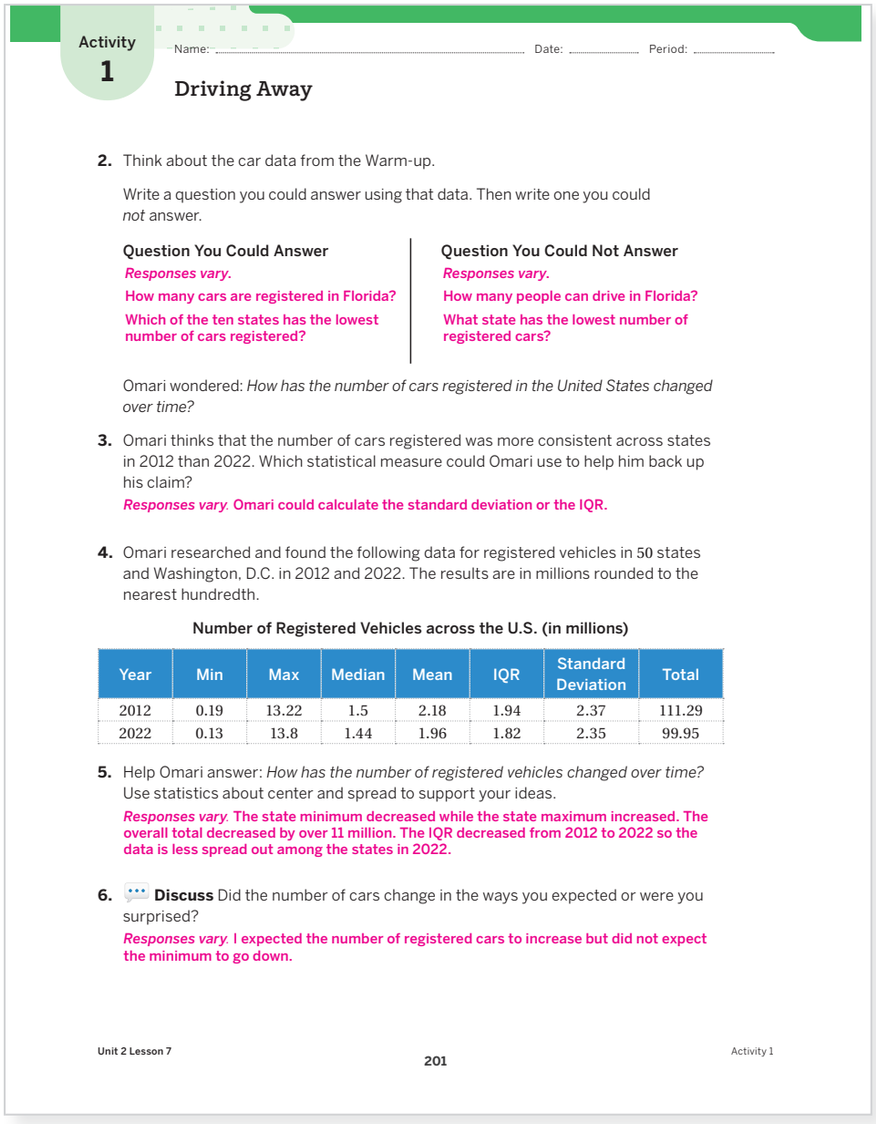
Invite students to share responses to Problem 6.

To surface the Key Takeaway, (MTR.4.1)

- How do the center and spread support your claims differently?

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., "One statistic that changed was _____, which means _____.") **(Writing and Speaking)**

Consider asking: "What did your statistical analysis on the center and spread of data tell you about car registrations over time?"



Activity 1

Name: _____ Date: _____ Period: _____

Driving Away

2. Think about the car data from the Warm-up.

Write a question you could answer using that data. Then write one you could *not* answer.

Question You Could Answer	Question You Could Not Answer
Responses vary.	*Responses vary.*
How many cars are registered in Florida?	How many people can drive in Florida?
Which of the ten states has the lowest number of cars registered?	What state has the lowest number of registered cars?

Omari wondered: *How has the number of cars registered in the United States changed over time?*

3. Omari thinks that the number of cars registered was more consistent across states in 2012 than 2022. Which statistical measure could Omari use to help him back up his claim?

Responses vary. Omari could calculate the standard deviation or the IQR.

4. Omari researched and found the following data for registered vehicles in 50 states and Washington, D.C. in 2012 and 2022. The results are in millions rounded to the nearest hundredth.

Number of Registered Vehicles across the U.S. (in millions)

Year	Min	Max	Median	Mean	IQR	Standard Deviation	Total
2012	0.19	13.22	1.5	2.18	1.94	2.37	111.29
2022	0.13	13.8	1.44	1.96	1.82	2.35	99.95

5. Help Omari answer: *How has the number of registered vehicles changed over time?* Use statistics about center and spread to support your ideas.

Responses vary. The state minimum decreased while the state maximum increased. The overall total decreased by over 11 million. The IQR decreased from 2012 to 2022 so the data is less spread out among the states in 2022.

6. **Discuss** Did the number of cars change in the ways you expected or were you surprised?

Responses vary. I expected the number of registered cars to increase but did not expect the minimum to go down.

Unit 2 Lesson 7 201 Activity 1

Key Takeaway: It is important to use data to identify changes, support claims, and understand changes over time.

Small Groups | 15 min

Activity 2 Shape of the Data

Purpose: Students compare data sets in real-world contexts using the shape of the data to decide which measures of center support their reasoning. **(MTR.7.1)**

Short on time? Consider creating one box plot and having students create the other.

1 Launch

Display the table about car registrations with the quartile 1 and quartile 3 added.

Consider asking, "How do you think aspects of life have changed from 2012 to 2022? Could these affect car registrations in the U.S. or different states?"

2 Monitor

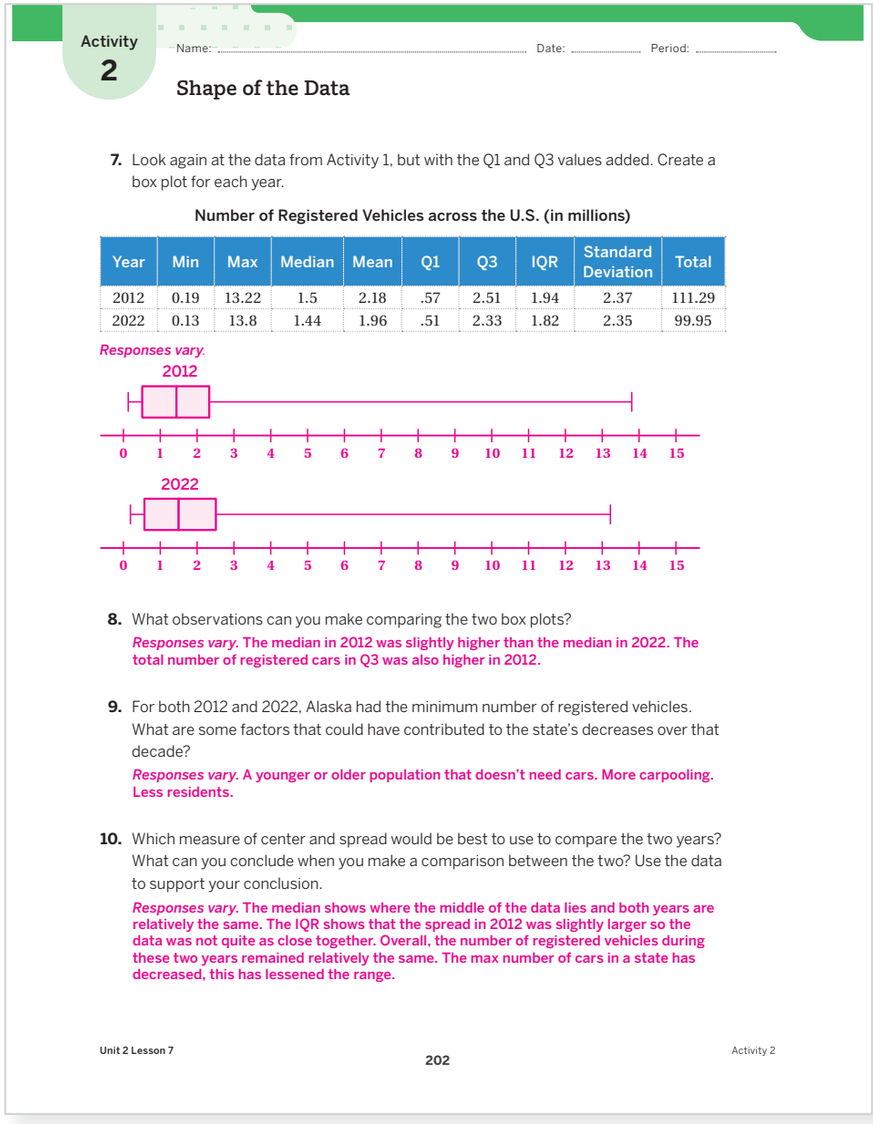
Listen for groups that talk about the different parts of the five number summary. Check that they plot those points correctly with correct markings.

D Differentiation *(Problems 7–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to use technology to graph their box plots. Or provide one of the box plots and have them graph the other.
Use mean or standard deviation to describe a data set. *(Problem 10)*	**Strengthen** Consider asking, "How might the shape of the box plots affect the mean or standard deviation?"
Would like to explore further how data can change over time. *(Problems 9-10)*	**Stretch** Invite them to research a new topic, find data from two different years to compare, and share their findings with a partner.

3 Connect

Invite students to share their box plots and discuss how they plotted "millions" and the scales they chose to use.



Activity 2

Name: Date: Period:

Shape of the Data

7. Look again at the data from Activity 1, but with the Q1 and Q3 values added. Create a box plot for each year.

Number of Registered Vehicles across the U.S. (in millions)

Year	Min	Max	Median	Mean	Q1	Q3	IQR	Standard Deviation	Total
2012	0.19	13.22	1.5	2.18	.57	2.51	1.94	2.37	111.29
2022	0.13	13.8	1.44	1.96	.51	2.33	1.82	2.35	99.95

Responses vary.

8. What observations can you make comparing the two box plots?

Responses vary. The median in 2012 was slightly higher than the median in 2022. The total number of registered cars in Q3 was also higher in 2012.

9. For both 2012 and 2022, Alaska had the minimum number of registered vehicles. What are some factors that could have contributed to the state's decreases over that decade?

Responses vary. A younger or older population that doesn't need cars. More carpooling. Less residents.

10. Which measure of center and spread would be best to use to compare the two years? What can you conclude when you make a comparison between the two? Use the data to support your conclusion.

Responses vary. The median shows where the middle of the data lies and both years are relatively the same. The IQR shows that the spread in 2012 was slightly larger so the data was not quite as close together. Overall, the number of registered vehicles during these two years remained relatively the same. The max number of cars in a state has decreased, this has lessened the range.

Unit 2 Lesson 7 202 Activity 2

Whole Class | 5 min

Synthesis

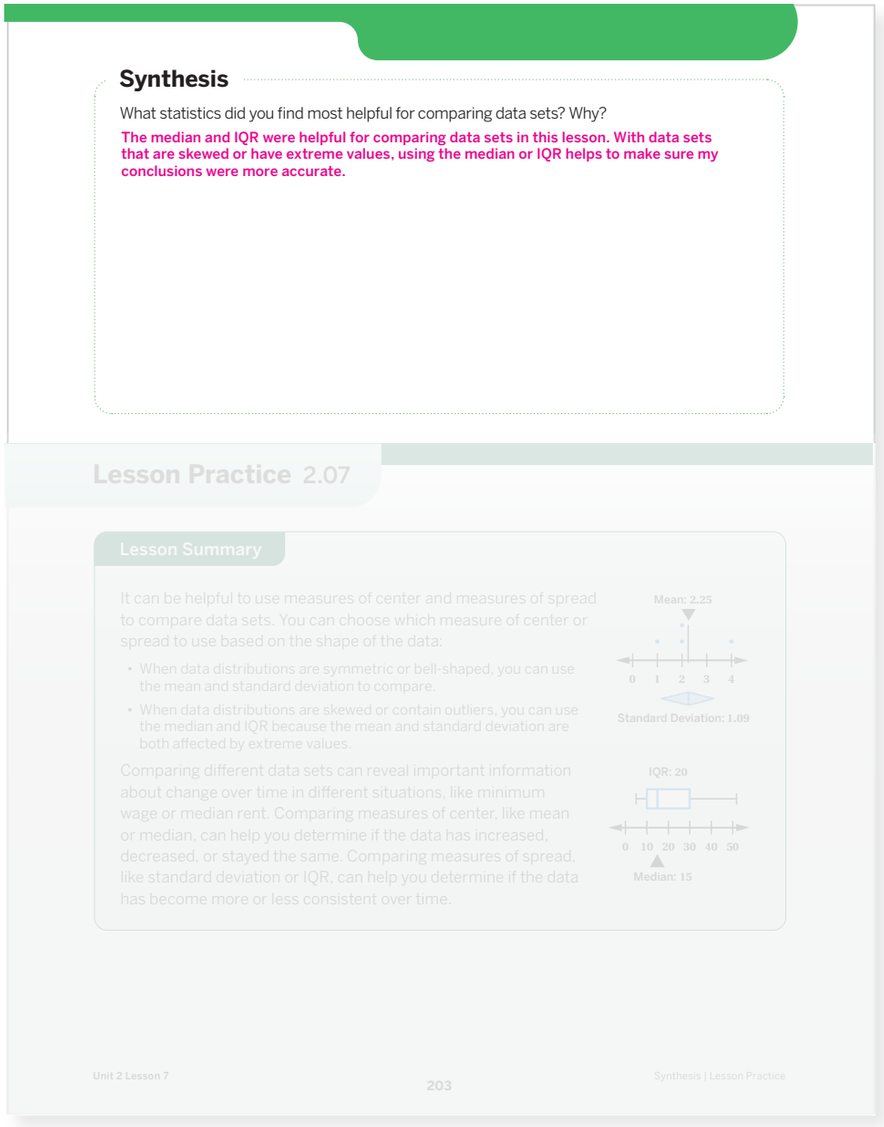
Purpose: Students synthesize their understanding of how different statistics can help compare data sets and why it might be important to study how things change over time.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The median and IQR are helpful for comparing data sets that are skewed or have extreme values.
- It is important to study how these aspects change over time because they can give information about the needs of society and what can be done to make it better.

Note: If time allows, invite students to revise their previous thinking based on the discussion and discuss any questions they still have.



Synthesis

What statistics did you find most helpful for comparing data sets? Why?

The median and IQR were helpful for comparing data sets in this lesson. With data sets that are skewed or have extreme values, using the median or IQR helps to make sure my conclusions were more accurate.

Lesson Practice 2.07

Lesson Summary

It can be helpful to use measures of center and measures of spread to compare data sets. You can choose which measure of center or spread to use based on the shape of the data:

- When data distributions are symmetric or bell-shaped, you can use the mean and standard deviation to compare.
- When data distributions are skewed or contain outliers, you can use the median and IQR because the mean and standard deviation are both affected by extreme values.

Comparing different data sets can reveal important information about change over time in different situations, like minimum wage or median rent. Comparing measures of center, like mean or median, can help you determine if the data has increased, decreased, or stayed the same. Comparing measures of spread, like standard deviation or IQR, can help you determine if the data has become more or less consistent over time.

Unit 2 Lesson 7 203 Synthesis | Lesson Practice

Lesson Takeaway: Statistics can be helpful for summarizing and comparing data sets. When choosing which statistics to use, it is important to keep in mind the shape of the data and if there are any outliers.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using statistics to compare temperature data over time in a real-world context. **(MTR.7.1)**

Today's Goals

1. **Goal:** Students explore data for various states in the U.S. and use statistics to describe the data.
2. **Goal:** Students use technology to make models and representations and find statistics from the data, including describing its shape. Based on the shape, students decide which statistics to use to make an argument about how the data has changed over time.
3. **Language Goal:** Compare and contrast two data sets using measures of center and spread that are appropriate for the shape of the data. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

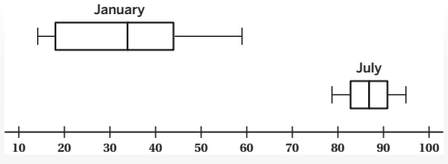
Name: _____ Date: _____ Period: _____

Show What You Know 2.07

Here are the high temperatures, in °F, for a certain town during part of 2024.

Month	Median	IQR
January	34	26
July	87	8

High Temperatures (°F)



How did the temperatures in January compare to the temperatures in July? Use statistics about center and spread to support your ideas.

Responses vary. The temperatures in January are a lot lower than the temperatures in July since the median is lower in January. The temperatures in January vary a lot more than the temperatures in July because the IQR is higher in January.

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support comparing data sets:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problems 8 and 10** as a class and invite students to share strategies for determining which measures of center and spread are appropriate for comparing data sets with different shapes.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how students engaged in constructing viable arguments and critiquing the reasoning of others in today's lesson. How did students use appropriate statistics to construct their arguments? How did they use statistics to justify their conclusions and communicate with others? How can you continue to use appropriate statistics, strategies, and tools to support their reasoning?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

What statistics did you find most helpful for comparing data sets? Why?

The median and IQR were helpful for comparing data sets in this lesson. With data sets that are skewed or have extreme values, using the median or IQR helps to make sure my conclusions were more accurate.

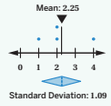
Lesson Practice 2.07

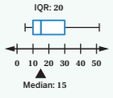
Lesson Summary

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Comparing different data sets can reveal important information about change over time in different situations, like minimum wage or median rent. Comparing measures of center, like mean or median, can help you determine if the data has increased, decreased, or stayed the same. Comparing measures of spread, like standard deviation or IQR, can help you determine if the data has become more or less consistent over time.





Lesson Practice 2.07

Name: _____ Date: _____ Period: _____

Problems 1–3: Yona is interested in the lengths of online videos. She found this data about the lengths of thousands of videos, organized by category.

	Mean Length (min)	Standard Deviation (min)
Music	4.15	0.6
Gaming	6.38	2.1

1. Write a question you could answer using this data.

Responses vary. Which category tends to have longer videos?

2. Write a question you could *not* answer using this data.

Responses vary. How many minutes long is the shortest gaming video?

3. How do the lengths of the music videos and the gaming videos compare? Use statistics to support your answer.

Responses vary. Gaming videos tend to be longer because the mean length of a gaming video is more than two minutes longer than the mean length of a music video. The gaming videos also have more variability in their length, since the standard deviation for the gaming videos is more than three times the standard deviation for the music videos.

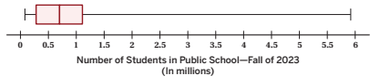
4. The table shows some statistics about the points per game scored by the Chicago Bulls in 1978 and 2020.

	Median (pts)	Mean (pts)	Standard Deviation (pts)	IQR (pts)
1978	102	104.9	10.99	15
2020	110	110.70	11.36	17.5

How has the number of points per game changed from 1978 to 2020? Use statistics to support your answer.

Responses vary. The number of points typically scored per game has increased from 1978 to 2020 because the median and mean have both increased. The variability has also increased because the standard deviation and IQR have both increased as well.

Problems 5–6: The box plot represents the number of students in public school in all 50 states and Washington, D.C. in the fall of 2023.



Number of Students in Public School—Fall of 2023
(In millions)

5. Describe what you notice about the data using the center and spread.

Responses vary. The median is almost halfway between Q1 and Q2 so the data is not skewed, however the right whisker is longer than the left whisker and shows that there is a wide range of data. The population of students between states can vary by a large amount.

Lesson Practice 2.07

Name: _____ Date: _____ Period: _____

Test Practice

6. Is the data numerical or categorical? Bivariate or Univariate?

A. The data is numerical and bivariate. (B.) The data is numerical and univariate.
C. The data is categorical and bivariate. D. The data is categorical and univariate.

Spiral Review

Problems 7–9: Use mental math to determine each value.

7. 50% of 120
60

8. 10% of 120
12

9. 90% of 120
108

Problems 10–11: How many solutions does each equation have? Circle one.

10. $6t + 2 = -3 + 6t$

A. One solution
(B.) No solution
C. Infinitely many solutions

11. $15 - 3t - 15 = -3t$

A. One solution
B. No solution
(C.) Infinitely many solutions

Problems 12–13: Here is a two-way table with data about people who live on two different islands and whether they prefer to wear a hat outside.

12. Complete the two-way table and relative frequency tables about the islands.

	Hat	No Hat	Total
Island A	2,394	6,128	8,522
Island B	7,911	33,318	41,229
Total	10,305	39,446	49,751

	Hat	No Hat	Total
Island A	~28.1%	~71.9%	100.0%
Island B	~19.2%	~80.8%	100.0%

13. Is there an association between which island a person lives on and whether they prefer to wear a hat? Explain your thinking.

Yes. *Explanations vary.* There is an association, because the percentage of people on Island B who prefer to wear a hat is almost 10% less than the percentage of people on Island A who prefer to wear a hat. Maybe Island B receives less direct sunlight, so the islanders have less of a reason to wear a hat.

Lesson Practice Item Analysis

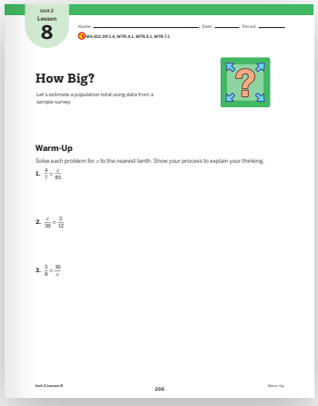
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.912.DP.1.2
	5	2	MA.912.DP.1.2
Test Practice	6	1	MA.912.DP.1.2
Spiral Review			
	7–9	1	MA.6.AR.3.4
Fluency	10–11	2	MA.8.AR.2.1
	12–13	2	MA.912.DP.3.1*

*These problems build toward the standard shown.

Unit 2

Lesson

8



How Big?

Estimate a Population Using Sample Survey Data

Let's estimate a population total using data from a sample survey.

Learning Outcomes

Today's Goals

1. **Goal:** Use a survey to find a total population.
2. **Goal:** Set up and solve proportions to estimate a population.
3. **Language Goal:** Verbally describe how to use a survey sample to estimate a total population.

Students use survey data collected from sampling a population to estimate the total number in the entire population. They will understand that using proportions is a way to determine this estimate and that the solution is a prediction.

Coherence

Prior Learning

In Lesson 4 of Unit 2 in Grade 7, students set up and solved equations with proportional relationships.

Future Learning

In Lesson 10, students will use the sample mean to find the population mean.

Connections

- Students use results from a survey and proportional reasoning to estimate populations in real-world contexts such as the total number of students in a school. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.4
Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3.2	MA.912.DP.1.2

Vocabulary

New Vocabulary

population
survey
sample

Review Vocabulary

cross multiplication
variable

Lesson at a Glance

~ 45 min

MA.912.DP.1.4

Warm-Up

Pairs | 10 min

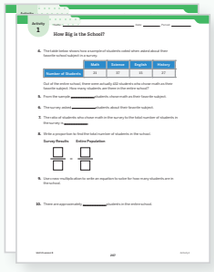
Students use the **Stronger and Clearer Each Time routine** to first write a response to the problem, then share to get feedback from a partner and revise if necessary
Routine(s): **MLR1: Stronger and Clearer Each Time** **(MTR.4.1, MTR.5.1)**



Activity 1

Partners | 10 min

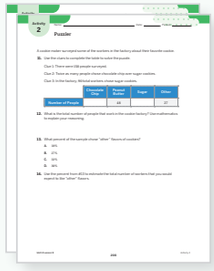
Students use the **Compare and Connect** routine to compare and contrast their approach to the problem with others.
Routine(s): **MLR7: Compare and Connect**
(MTR.5.1, MTR.7.1)



Activity 2

Partners | 15 min

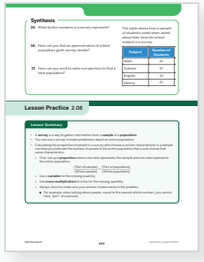
Students complete a two-way table and find a total population. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how survey results can be used to predict an estimate of a total population using proportions.
Routine(s): **MLR2: Collect and Display**
(MTR.4.1, MTR.5.1)



Show What You Know

Independent | 5 min

Students demonstrate their understanding by. using proportional reasoning to estimate a population from a survey.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 10 min

Warm-Up

Purpose: Students use the Stronger and Clearer Each Time routine to first write a response to the problem, then share to get feedback from a partner and revise if necessary. **(MTR.4.1, MTR.5.1)**

Short on time? Consider having partners each complete one problem. They will then check each other's work and provide suggestions.

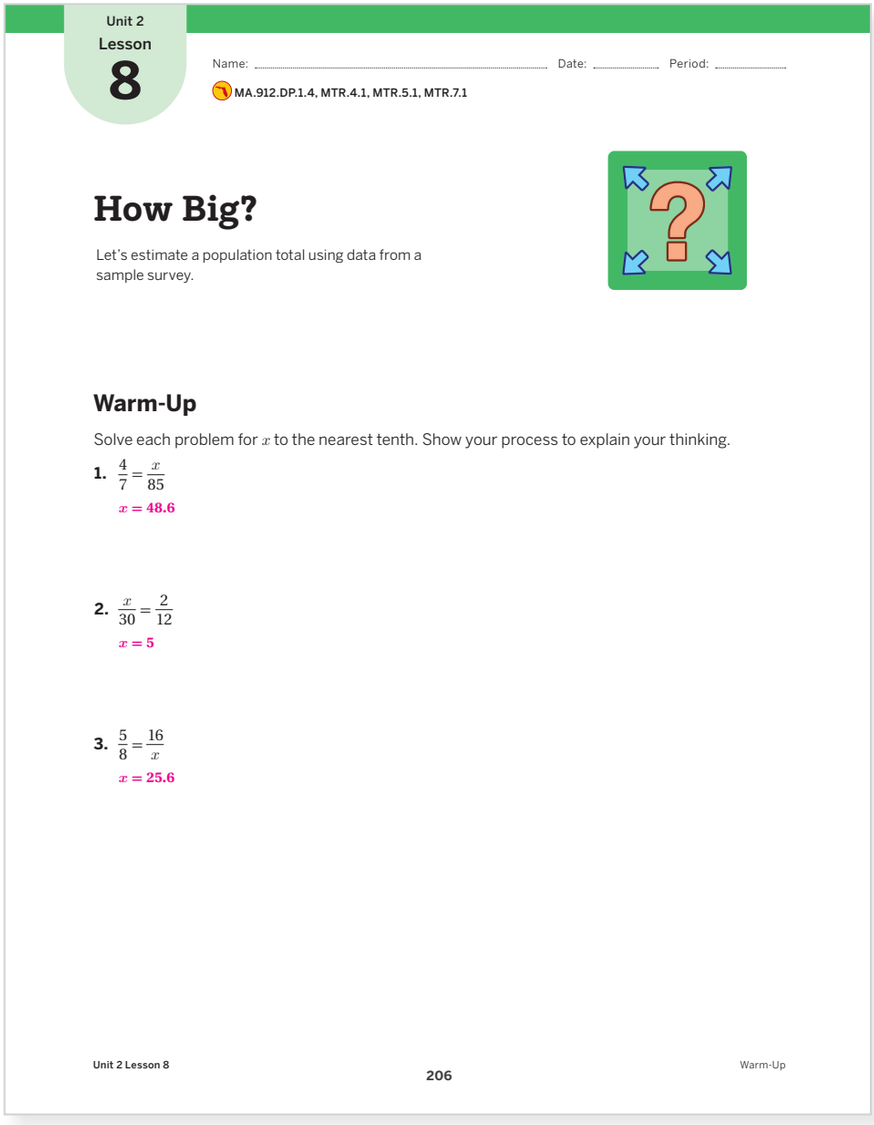
1 Launch

Use the Stronger and Clearer Each Time Routine to help students review solving proportions to prepare for the lesson.

Consider asking, "Can anyone explain the strategies they used to solve their problem? Did any partners solve their problems differently? What types of problems could you solve using proportions?"

2 Connect

To surface the Key Takeaway, invite students to share how they would define a proportion.



Unit 2
Lesson
8

Name: _____ Date: _____ Period: _____

MA.912.DP.1.4, MTR.4.1, MTR.5.1, MTR.7.1

How Big?

Let's estimate a population total using data from a sample survey.

Warm-Up

Solve each problem for x to the nearest tenth. Show your process to explain your thinking.

1. $\frac{4}{7} = \frac{x}{85}$

   $x = 48.6$

2. $\frac{x}{30} = \frac{2}{12}$

   $x = 5$

3. $\frac{5}{8} = \frac{16}{x}$

   $x = 25.6$

Unit 2 Lesson 8 206 Warm-Up

Key Takeaway: Students can solve a proportion for a variable using cross multiplication.

Partners | 10 min

Activity 1 How Big Is the School?

Purpose: Students use the Compare and Connect routine to compare and contrast their approach to the problem with others. **(MLR1) (MTR.5.1, MTR.7.1)**

1 Launch

Display the survey results to students.

Discuss how the survey could be conducted and what questions were asked. Students may connect the data to survey methods they have encountered in the past through emails or internet surveys.

2 Monitor

Create a class definition for the terms **sample** and **population** using students' language and the glossary.

Invite students to share their strategies for finding the total population.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

D **Differentiation** *(Problems 8–10)*

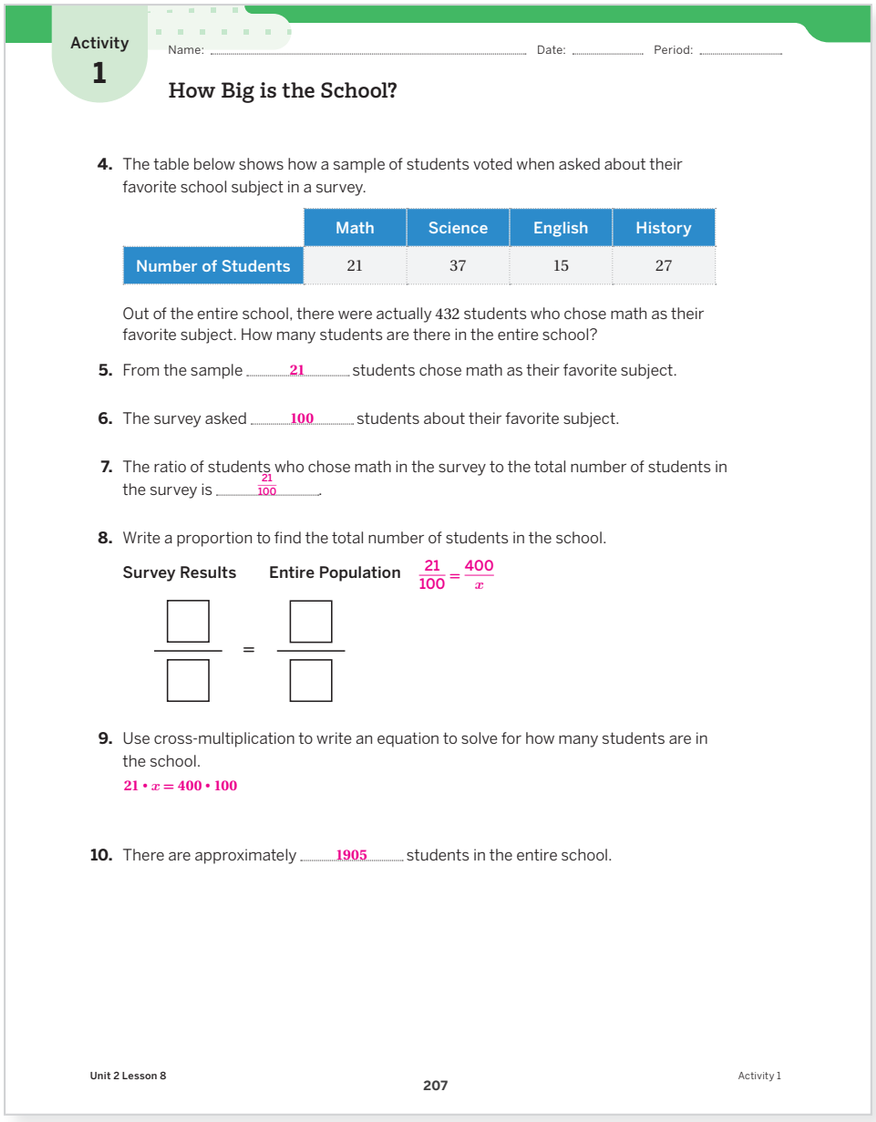
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw an X in between the ratios in Problem 8 to show what will be multiplied.
Created the correct equation. *(Problem 9)*	**Strengthen** Invite them to share how they created their equation.
Correctly calculates the total number of students in the school.	**Stretch** Have students justify their answer using substitution into the proportion from Problem 8 and the equation from Problem 9.

Look for/Listen for explanations of strategies to share that reflect student thinking and invite students to share more.

3 Connect

To surface the Key Takeaway, consider asking:

- "How does the ratio representing a survey compare to that of a total population?"



Activity 1

Name: Date: Period:

How Big is the School?

4. The table below shows how a sample of students voted when asked about their favorite school subject in a survey.

	Math	Science	English	History
Number of Students	21	37	15	27

Out of the entire school, there were actually 432 students who chose math as their favorite subject. How many students are there in the entire school?

5. From the sample 21 students chose math as their favorite subject.

6. The survey asked 100 students about their favorite subject.

7. The ratio of students who chose math in the survey to the total number of students in the survey is $\frac{21}{100}$.

8. Write a proportion to find the total number of students in the school.

Survey Results Entire Population $\frac{21}{100} = \frac{400}{x}$

$\frac{\square}{\square} = \frac{\square}{\square}$

9. Use cross-multiplication to write an equation to solve for how many students are in the school.

$21 \cdot x = 400 \cdot 100$

10. There are approximately 1905 students in the entire school.

Unit 2 Lesson 8 207 Activity 1

Key Takeaway: The ratio of the survey sample is equal to the ratio of the population.

Partners | 15 min

Activity 2 Puzzler

Purpose: Students use the Information Gap routine to complete a two-way table and find a total population. **(MTR.5.1, MTR.7.1)**

MLR **MLR4: Information Gap**

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

1 Launch

Display the clues and table to students.

Discuss how the clues can help students fill out the table and what information the table provides.

2 Monitor

Look for a variety of ways students completed the table.

Ask students to share and compare their methods.

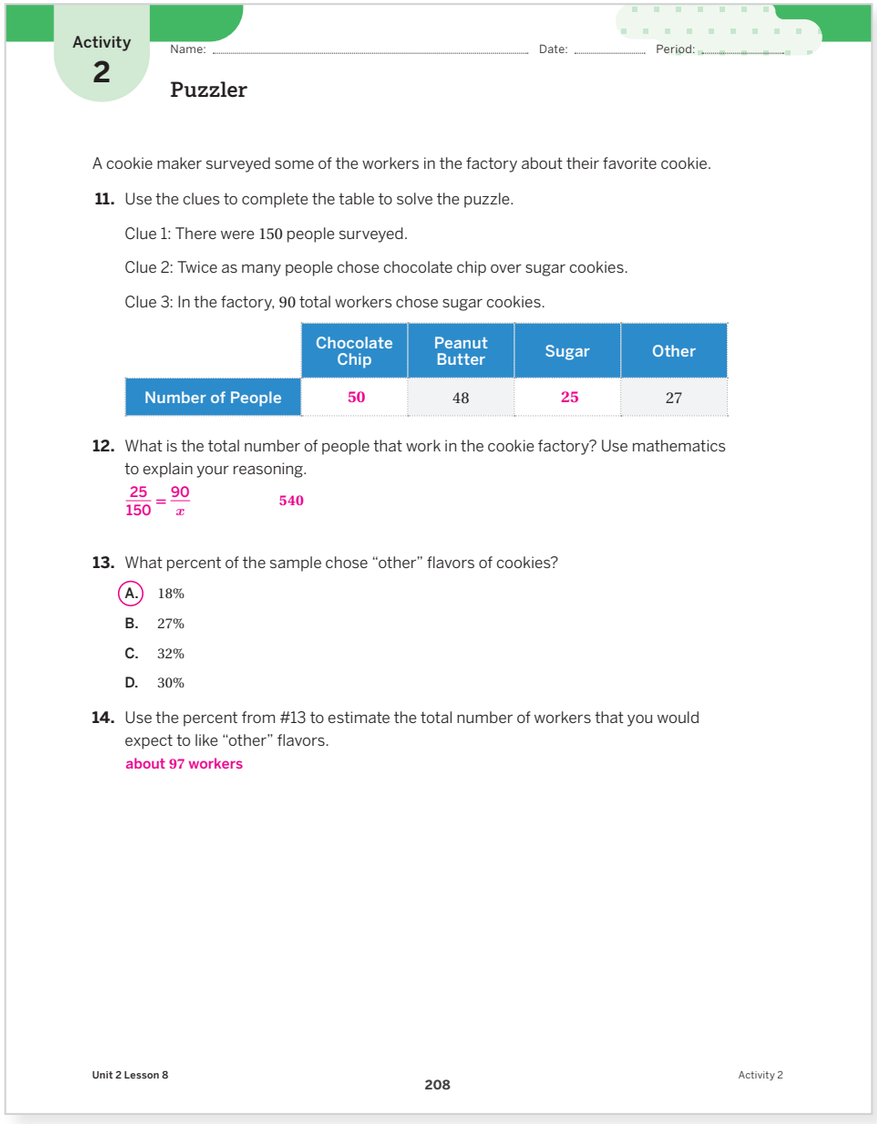
ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and inviting them to share and receive feedback in their primary language. **(Writing, Speaking, and Listening)**

D **Differentiation** *(Problems 11 and 14)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Fill in one of the open boxes and help students discuss how to fill in the last box.
Understand how to change the percent to a decimal before multiplying.	**Strengthen** Invite them to share how they used the percent to find the answer.
Correctly find the total number of workers who chose "other".	**Stretch** Have students prove or justify their answer by showing the class how the answer works in a proportion.

3 Connect

To surface the Key Takeaway, consider asking, "When setting up a proportion, how do you set up the ratios with the parts and wholes?"



Activity 2

Name: Date: Period:

Puzzler

A cookie maker surveyed some of the workers in the factory about their favorite cookie.

11. Use the clues to complete the table to solve the puzzle.

Clue 1: There were 150 people surveyed.

Clue 2: Twice as many people chose chocolate chip over sugar cookies.

Clue 3: In the factory, 90 total workers chose sugar cookies.

	Chocolate Chip	Peanut Butter	Sugar	Other
Number of People	50	48	25	27

12. What is the total number of people that work in the cookie factory? Use mathematics to explain your reasoning.

$\frac{25}{150} = \frac{90}{x}$ 540

13. What percent of the sample chose "other" flavors of cookies?

A. 18%

B. 27%

C. 32%

D. 30%

14. Use the percent from #13 to estimate the total number of workers that you would expect to like "other" flavors.

about 97 workers

Unit 2 Lesson 8 208 Activity 2

Key Takeaway: Students use clues to fill in a two-way table of the results of a survey, then use the results to make an estimate about the total population.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how survey results can be used to predict an estimate of a total population using proportions. **(MTR.4.1, MTR.5.1)**

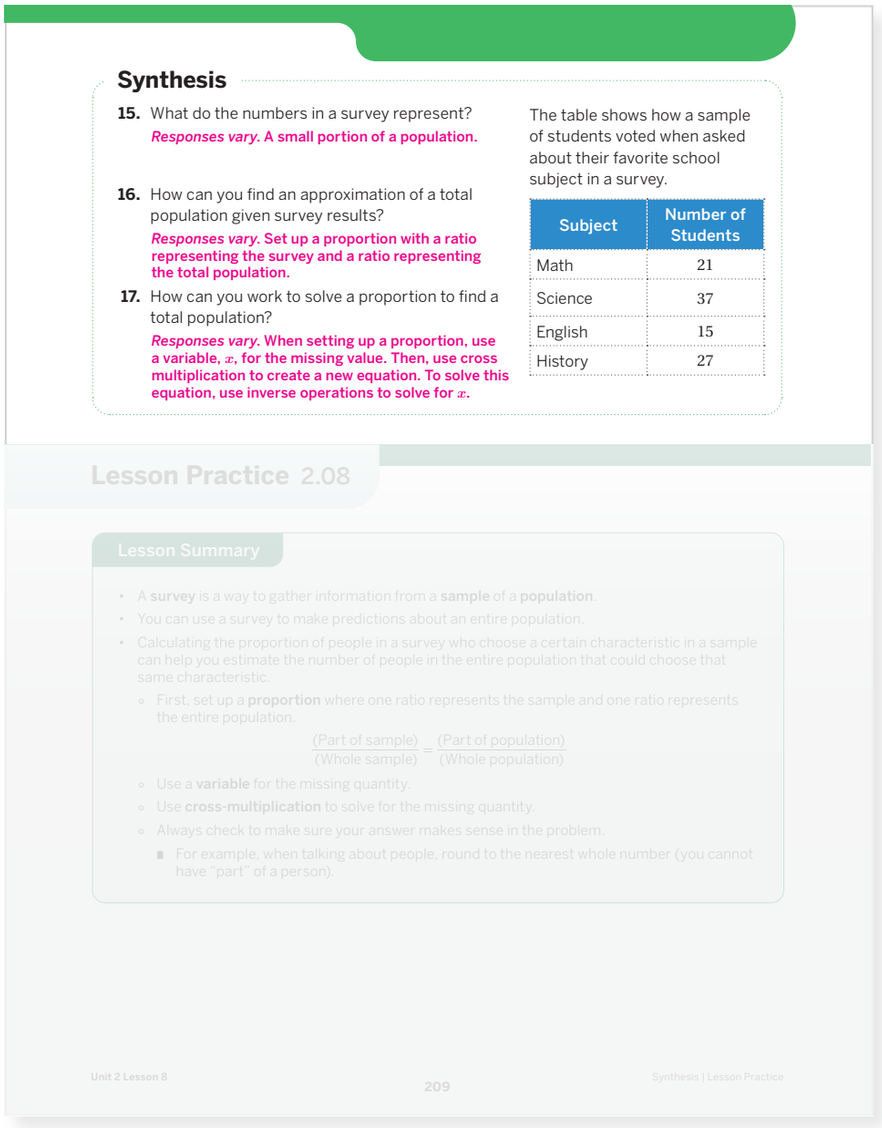
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:
- How to set up a proportion from a survey.
- How a sample can estimate a population.
- How to solve a proportion.

Create a list of vocabulary words used in the discussion.

MLR **MLR2: Collect and Display** Use the strategy for writing proportions from surveys to estimate populations using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

15. What do the numbers in a survey represent?

Responses vary. A small portion of a population.

16. How can you find an approximation of a total population given survey results?

Responses vary. Set up a proportion with a ratio representing the survey and a ratio representing the total population.

17. How can you work to solve a proportion to find a total population?

Responses vary. When setting up a proportion, use a variable, x, for the missing value. Then, use cross multiplication to create a new equation. To solve this equation, use inverse operations to solve for x.

The table shows how a sample of students voted when asked about their favorite school subject in a survey.

Subject	Number of Students
Math	21
Science	37
English	15
History	27

Lesson Practice 2.08

Lesson Summary

- A **survey** is a way to gather information from a **sample** of a **population**.
- You can use a survey to make predictions about an entire population.
- Calculating the proportion of people in a survey who choose a certain characteristic in a sample can help you estimate the number of people in the entire population that could choose that same characteristic.
  - First, set up a **proportion** where one ratio represents the sample and one ratio represents the entire population.

$$\frac{\text{(Part of sample)}}{\text{(Whole sample)}} = \frac{\text{(Part of population)}}{\text{(Whole population)}}$$

  - Use a **variable** for the missing quantity.
  - Use **cross-multiplication** to solve for the missing quantity.
  - Always check to make sure your answer makes sense in the problem.
    - For example, when talking about people, round to the nearest whole number (you cannot have "part" of a person).

Unit 2 Lesson 8 209 Synthesis | Lesson Practice

Lesson Takeaway: Students explain the meaning of the numbers in a proportion from a survey and how to solve the proportion to find an estimate of the total population.

Independent | 5 min

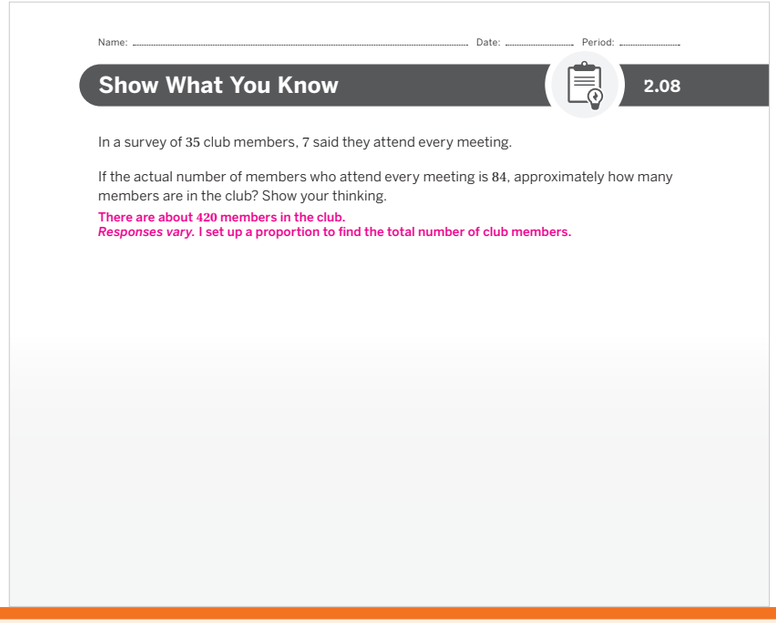
Show What You Know

Purpose: Students demonstrate their understanding by using proportional reasoning to estimate a population from a survey.

Today's Goals

1. **Goal:** Use a survey to find a total population.
2. **Goal:** Set up and solve proportions to estimate a population.
3. **Language Goal:** Verbally describe how to use a survey sample to estimate a total population.

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.08

In a survey of 35 club members, 7 said they attend every meeting.

If the actual number of members who attend every meeting is 84, approximately how many members are in the club? Show your thinking.

There are about 420 members in the club.
Responses vary. I set up a proportion to find the total number of club members.

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support using proportional reasoning to estimate a population from a survey:

» **Respond to Student Thinking:**

- Use the *Estimating Population from a Sample Survey* **Mini-Lesson**.
- Revisit the **Lesson Summary** as a class and estimate a total population using data from survey results about your students.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create their own problem with survey results and to pose a question about the total population.

Professional Learning

Reflecting on the ***facilitation of routines/activity structures***

- Reflect on your facilitation of the **Information Gap** routine in this lesson.
- How did students engage in the routine?
- What about your facilitation of the routine promoted student learning?
- How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

15. What do the numbers in a survey represent?

Responses vary. A small portion of a population.

16. How can you find an approximation of a total population given survey results?

Responses vary. Set up a proportion with a ratio representing the survey and a ratio representing the total population.

17. How can you work to solve a proportion to find a total population?

Responses vary. When setting up a proportion, use a variable, x, for the missing value. Then, use cross multiplication to create a new equation. To solve this equation, use inverse operations to solve for x.

The table shows how a sample of students voted when asked about their favorite school subject in a survey.

Subject	Number of Students
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Lesson Practice 2.08

Lesson Summary

- A **survey** is a way to gather information from a **sample** of a **population**.
- You can use a survey to make predictions about an entire population.
- Calculating the proportion of people in a survey who choose a certain characteristic in a sample can help you estimate the number of people in the entire population that could choose that same characteristic.
  - First, set up a **proportion** where one ratio represents the sample and one ratio represents the entire population.

$$\frac{\text{(Part of sample)}}{\text{(Whole sample)}} = \frac{\text{(Part of population)}}{\text{(Whole population)}}$$

  - Use a **variable** for the missing quantity.
  - Use **cross-multiplication** to solve for the missing quantity.
  - Always check to make sure your answer makes sense in the problem.
    - For example, when talking about people, round to the nearest whole number (you cannot have "part" of a person).

Lesson Practice 2.08

Name: ..... Date: ..... Period: .....

1. Choose the words to fill in each blank.

Word Bank

predictions sample population survey

A ..survey.. is given to find a ..sample.. to be used to make ..predictions.. about a ..population..

Problems 2–4:

2. A group of 40 ninth graders was surveyed about how many times a week they eat lunch from the cafeteria. Of these, 9 buy three times a week. If the actual number of ninth graders that buys three times a week is 62, about how many students are in the ninth grade?

$\frac{9}{40} = \frac{62}{x}, x \approx 276$

3. A movie theater surveyed 35 people as they entered on a Friday. Of these, 19 said they were going to see a scary movie. That same day, 67 people actually saw a scary movie. About how many people came to the movie theater on that day?

$\frac{19}{35} = \frac{67}{x}, x \approx 123$

4. A zoo surveyed 140 people on a Saturday about their favorite animal in the zoo. Of these, 84 said they were going to see giraffes during their visit. That day, 410 people actually visited the giraffes. About how many total people visited the zoo that day?

$\frac{84}{140} = \frac{410}{x}, x \approx 683$

Lesson Practice 2.08

Name: ..... Date: ..... Period: .....

Problems 5–6: Solve each problem. Show all work.

5. A zoo surveyed one day's worth of visitors about their favorite animal. On that day, there were 215 visitors and 48 of them said that the giraffe is their favorite. If in a week, 730 visitors actually said the giraffe is their favorite, how many people visited the zoo that week?

$\frac{48}{215} = \frac{730}{x}$

$48x = 730 \times 215$

$x \approx 3269.79$

About 3270 people.

Survey

Which animal is your favorite?

Giraffe	
Llama	
Two-toed sloth	

6. A teacher surveys one class about how many hours they sleep a night. Out of 24 students, 7 said that they get 8 hours of sleep a night. Out of four classes, 36 said they get 8 hours of sleep. What is the total number of students the teacher has in all four classes?

$\frac{7}{24} = \frac{36}{x}$

$36 \times 24 = 7x$

$x \approx 123.4$

123 Students

Test Practice

7. A group of 40 people were surveyed about their favorite restaurant in town. Of these, 15 chose The Variety Cafe. If 630 people in town actually like that restaurant, how many people are in the entire town? Which equation correctly describes this situation?

A. $\frac{15}{40} = \frac{630}{x}$ B. $\frac{40}{630} = \frac{15}{x}$

C. $\frac{40}{x} = \frac{630}{15}$ D. $\frac{15}{40} = \frac{x}{630}$

Spiral Review

Problems 8–10: Simplify using the laws of exponents.

8. $x^2 \cdot x^6$

x^8

9. $\frac{x^5y^4}{x^2y^8}$

$\frac{x^3}{y^4}$

10. $(x^6y^3)^3$

$x^{18}y^9$

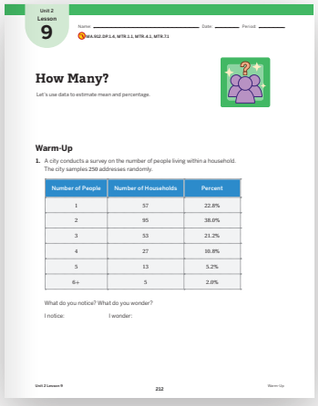
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.DP.1.4
	2–6	2	MA.912.DP.1.4
Test Practice	7	2	MA.912.DP.1.4
Spiral Review			
Fluency	8–10	2	MA.8.AR.1.1

Unit 2

Lesson

9


Unit 2
Lesson
9
Name _____ Date _____ Period _____
MA.912.DP.1.4, MTR.1.1, MTR.4.1, MTR.7.1
How Many?
Let's use data to estimate mean and percentage.
Warm-Up
1. A city conducts a survey on the number of people living within a household. The city samples 258 addresses randomly.

Number of People	Number of Households	Percent
1	57	22.8%
2	95	38.0%
3	53	21.2%
4	27	10.8%
5	13	5.2%
6+	5	2.0%

What do you notice? What do you wonder?
I notice: I wonder:
Unit 2 Lesson 9
212
Warm-Up

How Many?

Estimate Mean or Percentage Using Survey Data

Let's use data to estimate mean and percentage.

Learning Outcomes

Today's Goals

1. **Goal:** Find the population mean based on the sample mean.
2. **Goal:** Find the percentage of a population that has a certain characteristic based on a sample survey.
3. **Language Goal:** Verbally explain the relationship between a survey sample and the population.

Students use survey data collected from sampling a population to draw conclusions about that population. They understand that the characteristics of a representative random sample will be the same as the characteristics of the population as a whole. In particular, this applies to the mean and the percentages found in the sample.

Coherence

Prior Learning

In Lesson 9, students used survey data to describe a population.

Future Learning

In Lesson 11, students will estimate the true value of a population statistic based on statistics estimated from a sample and the margin of error in the sampling.

Connections

- Students use representative samples to better understand the characteristics of a population in real-world simulations such as customer satisfaction. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.4
Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.DP.1.1	MA.912.DP.5.2

Vocabulary

New Vocabulary

population mean

sample mean

Review Vocabulary

mean

population

percentage

sample

Lesson at a Glance

~ 45 min

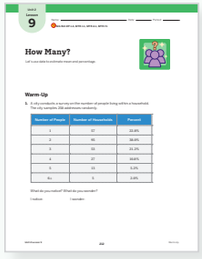
MA.912.DP.1.4

Warm-Up

WU Grouping | 5 min

Students use the Notice and Wonder routine to think about how the data in a sample survey relate to the population.

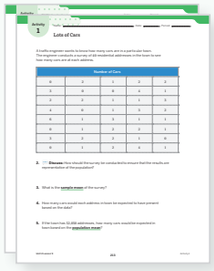
Routine(s): Notice and Wonder



Activity 1

Act 1 Grouping | 10 min

Students analyze survey data to estimate a population mean.

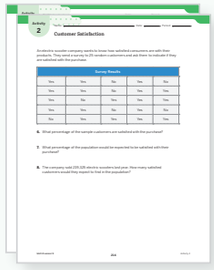


Activity 2

Act 2 Grouping | 10 min

Students analyze survey data to estimate the percentage of the population that has an opinion.

Routine(s): Think-Pair-Share **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how the mean and the percentages found in a sample are connected to the population.

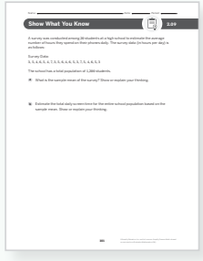
Routine(s): MLR2: Collect and Display **(MTR.4.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using proportional reasoning to calculate a sample mean based on a population mean.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to think about how the data in a sample survey relate to the population. **(MTR.1.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the data in a sample.

Create a record of the relationships students noticed and wondered, along with the name of the students who shared them.

Consider asking, "Did anyone notice the percentage column? Does this make you wonder about how the percentages were calculated?"

2 Connect

To surface the Key Takeaway, consider inviting students to share how they would define a percentage.

Unit 2
Lesson
9

Name: _____ Date: _____ Period: _____

MA.912.DP.1.4, MTR.1.1, MTR.4.1, MTR.7.1

How Many?



Let's use data to estimate mean and percentage.

Warm-Up

1. A city conducts a survey on the number of people living within a household. The city samples 250 addresses randomly.

Number of People	Number of Households	Percent
1	57	22.8%
2	95	38.0%
3	53	21.2%
4	27	10.8%
5	13	5.2%
6+	5	2.0%

What do you notice? What do you wonder?

I notice: I wonder:

Responses vary. I noticed that the table includes data that is presented as both numbers and as percentages. I wonder how the percentages were found.

Unit 2 Lesson 9 212 Warm-Up

Key Takeaway: Percentages can be used to represent the ratios in a sample.

Partners | 10 min

Activity 1 Lots of Cars

Purpose: Students analyze survey data to estimate a population mean. **(MTR.7.1)**

1 Launch

Display the survey results to students.

Discuss how such a survey could be conducted. Students may connect the data to survey methods they have encountered in the past, such as phone calls or mailers.

2 Monitor

Create a class definition for the terms **sample mean** and **population mean** using students' language and the glossary.

Invite students to share their strategies for finding the sample mean.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Look for students who respond with the same answer to Problems 3 and 4. They understand that the sample mean estimates the population mean.

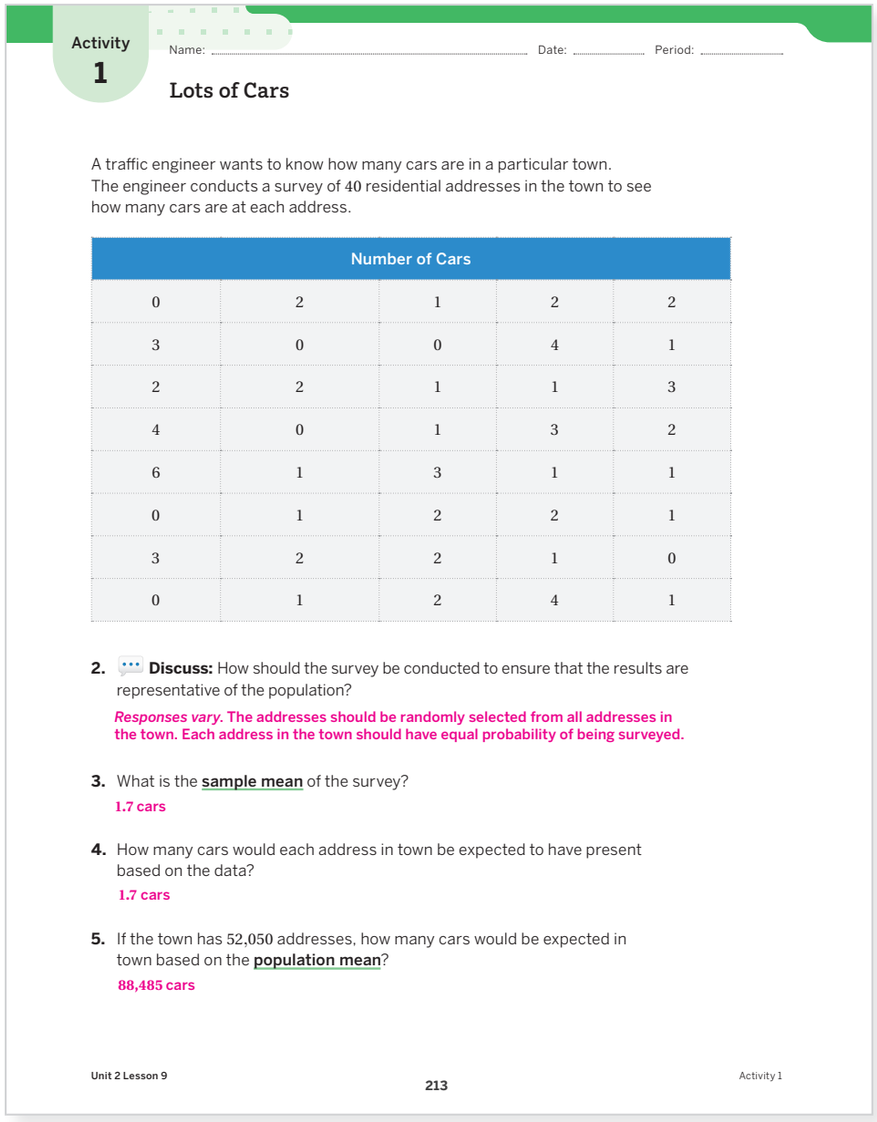
D **Differentiation** *(Problems 3–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to recall the formula for the mean of a data set.
Calculate the mean of the same by summing all the values and then dividing the sum by the number of values. *(Problem 3)*	**Strengthen** Invite them to share what they recall about measurements of center in a distribution.
Correctly calculates the expected number of cars in town. *(Problem 5)*	**Stretch** Invite students to justify their conclusion using the structure of how averages are calculated.

3 Connect

To surface the Key Takeaway, consider asking:

- "How is a sample connected to a population?"
- "What properties of a sample and a population are similar?"



Activity 1

Name: Date: Period:

Lots of Cars

A traffic engineer wants to know how many cars are in a particular town. The engineer conducts a survey of 40 residential addresses in the town to see how many cars are at each address.

Number of Cars				
0	2	1	2	2
3	0	0	4	1
2	2	1	1	3
4	0	1	3	2
6	1	3	1	1
0	1	2	2	1
3	2	2	1	0
0	1	2	4	1

2. **Discuss:** How should the survey be conducted to ensure that the results are representative of the population?

   Responses vary. The addresses should be randomly selected from all addresses in the town. Each address in the town should have equal probability of being surveyed.

3. What is the **sample mean** of the survey?

   1.7 cars

4. How many cars would each address in town be expected to have present based on the data?

   1.7 cars

5. If the town has 52,050 addresses, how many cars would be expected in town based on the **population mean**?

   88,485 cars

Unit 2 Lesson 9 213 Activity 1

Key Takeaway: The mean of a sample is equal to the mean of the population from which the sample is drawn.

Small Groups | 10 min

Activity 2 Consumer Satisfaction

Purpose: Students analyze survey data to estimate the percentage of the population that has an opinion. **(MTR.7.1)**

Short on time? Consider telling students that there were 18 yes responses and 7 no responses.

1 Launch

Display the survey results to students.

Discuss how the data in this survey is different from the one they saw in Activity 1. This survey collected categorical data while the previous survey collected quantitative data.

2 Monitor

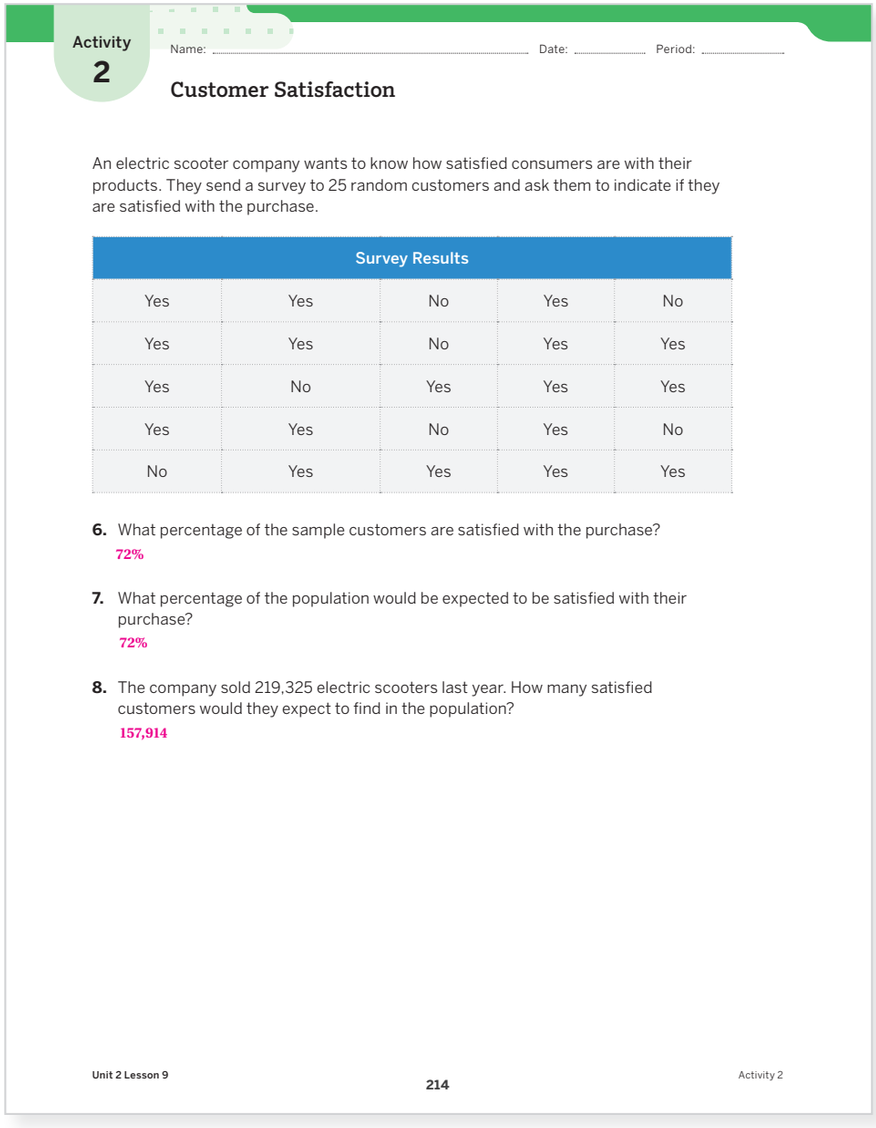
Consider asking, "How can you use percentage to describe ratios in data?"

D **Differentiation: (Support)** Students may think they cannot find the percentage using categorical data. Consider inviting them to count the number of instances of each category to find quantitative values.

Encourage students to record their thinking.

3 Connect

Use the Think-Pair-Share routine to support student-to-student discourse. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.



Activity 2

Name: Date: Period:

Customer Satisfaction

An electric scooter company wants to know how satisfied consumers are with their products. They send a survey to 25 random customers and ask them to indicate if they are satisfied with the purchase.

Survey Results				
Yes	Yes	No	Yes	No
Yes	Yes	No	Yes	Yes
Yes	No	Yes	Yes	Yes
Yes	Yes	No	Yes	No
No	Yes	Yes	Yes	Yes

6. What percentage of the sample customers are satisfied with the purchase?

72%

7. What percentage of the population would be expected to be satisfied with their purchase?

72%

8. The company sold 219,325 electric scooters last year. How many satisfied customers would they expect to find in the population?

157,914

Unit 2 Lesson 9 214 Activity 2

Whole Class | 5 min

Synthesis

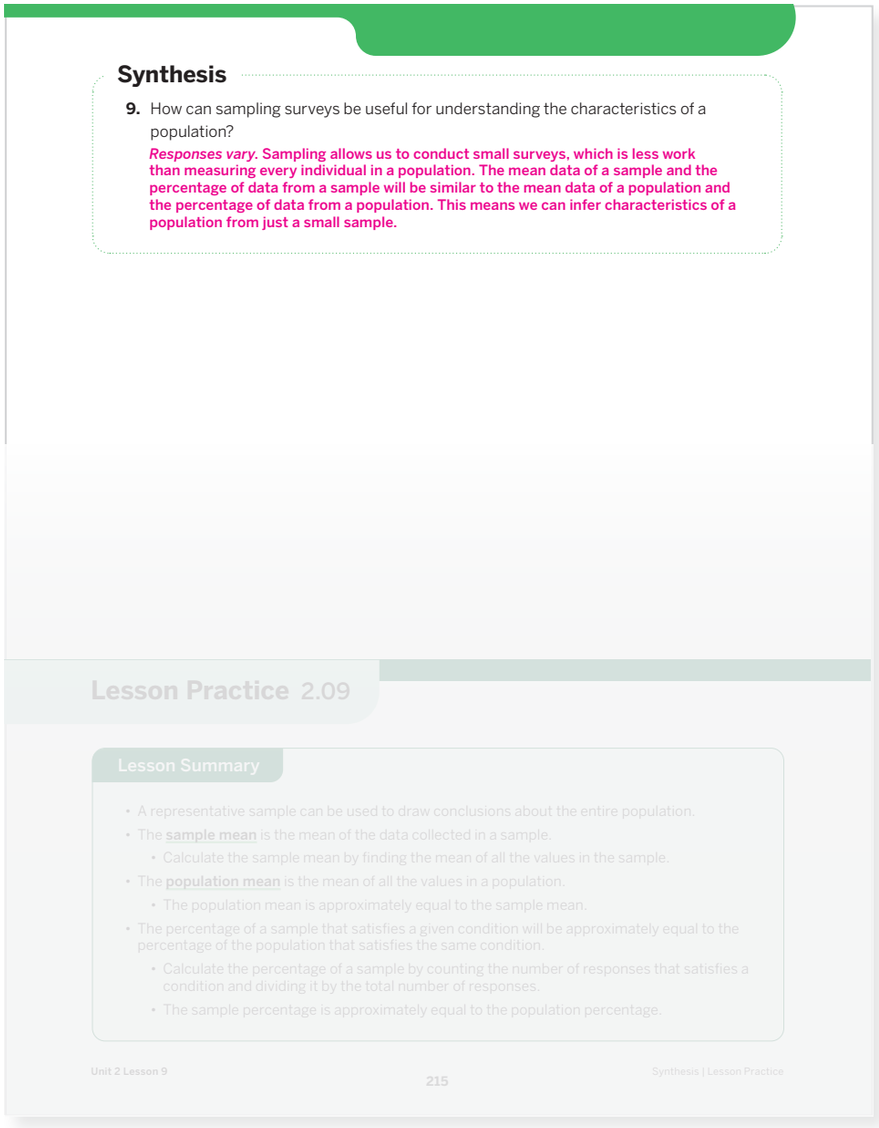
Purpose: Students synthesize their understanding of how the mean and the percentages found in a sample are connected to the population.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:
- The mean of a sample is equal to the mean of the population.
- A percentage represented in a sample is equal to the percentage found in the population.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they discuss how a sample is related to the population with a partner. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. How can sampling surveys be useful for understanding the characteristics of a population?

Responses vary. Sampling allows us to conduct small surveys, which is less work than measuring every individual in a population. The mean data of a sample and the percentage of data from a sample will be similar to the mean data of a population and the percentage of data from a population. This means we can infer characteristics of a population from just a small sample.

Lesson Practice 2.09

Lesson Summary

- A representative sample can be used to draw conclusions about the entire population.
- The sample mean is the mean of the data collected in a sample.
  - Calculate the sample mean by finding the mean of all the values in the sample.
- The population mean is the mean of all the values in a population.
  - The population mean is approximately equal to the sample mean.
- The percentage of a sample that satisfies a given condition will be approximately equal to the percentage of the population that satisfies the same condition.
  - Calculate the percentage of a sample by counting the number of responses that satisfies a condition and dividing it by the total number of responses.
  - The sample percentage is approximately equal to the population percentage.

Unit 2 Lesson 9 215 Synthesis | Lesson Practice

Lesson Takeaway: The mean of a sample will equal the mean of the population from which the sample is taken. Ratios represented as percentages in a sample will equal those same ratios in the population from which the sample is taken.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using proportional reasoning to calculate a sample mean based on a population mean.

Today's Goals

1. **Goal:** Find the population mean based on the sample mean.
2. **Goal:** Find the percentage of a population that has a certain characteristic based on a sample survey.
3. **Language Goal:** Verbally explain the relationship between a survey sample and the population.

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.09

A survey was conducted among 20 students at a high school to estimate the average number of hours they spend on their phones daily. The survey data (in hours per day) is as follows:

Survey Data:

3, 5, 4, 6, 5, 4, 7, 3, 5, 6, 4, 6, 5, 3, 7, 5, 4, 6, 5, 3

The school has a total population of 1,200 students.

a. What is the sample mean of the survey? Show or explain your thinking.

Sample Mean = 4.8 hours/day Explanations vary. I added up the number of hours to get the total for the 20 students of the sample group and then divided that total by 20.

b. Estimate the total daily screen time for the entire school population based on the sample mean. Show or explain your thinking.

5760 hours. Explanations vary. I multiplied the sample mean of 4.8 hours of screen time a day for each student by 1200 to get the total daily screen time of 1200 students.

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support using proportional reasoning to calculate a sample mean based on a population mean:

» **Respond to Student Thinking:**

- Revisit **Activity 1, Problems 3–5** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflecting on the facilitation of routines/activity structures

- Reflect on your facilitation of the Collect and Display routine in this lesson.
- How did students engage in the routine?
- What about your facilitation of the routine promoted student learning?
- How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. How can sampling surveys be useful for understanding the characteristics of a population?

Responses vary. Sampling allows us to conduct small surveys, which is less work than measuring every individual in a population. The mean data of a sample and the percentage of data from a sample will be similar to the mean data of a population and the percentage of data from a population. This means we can infer characteristics of a population from just a small sample.

Lesson Practice 2.09

Lesson Summary

- A representative sample can be used to draw conclusions about the entire population.
- The **sample mean** is the mean of the data collected in a sample.
  - Calculate the sample mean by finding the mean of all the values in the sample.
- The **population mean** is the mean of all the values in a population.
  - The population mean is approximately equal to the sample mean.
- The percentage of a sample that satisfies a given condition will be approximately equal to the percentage of the population that satisfies the same condition.
  - Calculate the percentage of a sample by counting the number of responses that satisfies a condition and dividing it by the total number of responses.
  - The sample percentage is approximately equal to the population percentage.

Lesson Practice 2.09

Name: _____ Date: _____ Period: _____

Problems 1–2: A local transportation organization conducted a survey to understand how many miles drivers commute to work each day. The survey results are shown.

Miles Driven					
12.5	15.9	12.8	12.9	18.0	10.6
7.2	23.5	17.5	27.3	14.2	18.3
15.5	1.65	19.9	19.1	16.8	11.0
38.9	22.2	11.3	45.2	13.1	13.4
26.4	31.2	48.1	13.9	25.0	16.3

1. What is the average distance that drivers in the town commute each day?

   19.3 miles

2. What information would you need to know in order to calculate the total distance driven for commuting in this town?

   The number of drivers who live in town.

Problems 3–4: The student government at a school conducts a survey for the theme of the fall dance.

Survey Results	
Fall Festival	12
Pumpkin Spice & Everything Nice	26
Starry Night	18
Time Traveler's Ball	4

3. What percentage of students at the school prefer to have a Fall Festival dance?

   20%

4. The school has 380 students. How many students would be expected to prefer a Starry Night theme?

   114

Lesson Practice 2.09

Name: _____ Date: _____ Period: _____

Test Practice

5. In a survey of 75 students, 12 students indicated that their favorite class in school was math. What percentage of the student population is expected to prefer math over all other subjects?

   A. 12%
   B. 16%
   C. 75%
   D. 625%

Spiral Review

Problems 6–7: Select the equivalent number.

6. $4.05 \times 10^2 + 5.03 \times 10^3$

   A. 9.08×10^2
   B. 9.08×10^3
   C. 5.435×10^2
   D. 5.435×10^3

7. $(7.1 \times 10^2 + 2.0 \times 10^{-1}) \times (6.4 \times 10^1)$

   A. 467.2
   B. 45,452.8
   C. 45,568
   D. 58,240

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.DP.1.2
	3–4	2	MA.912.DP.1.4
Test Practice	5	2	MA.912.DP.1.4
Spiral Review			
Fluency	6–7	2	MA.8.NSO.1.5

Unit 2

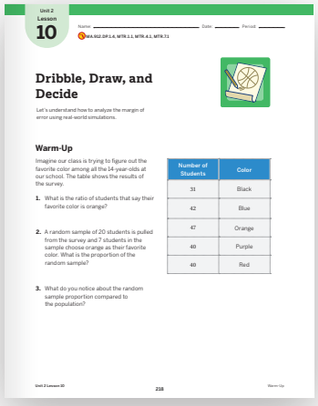
Lesson

10

Dribble, Draw, and Decide

Margin of Error through Simulation

Let's understand how to analyze the margin of error using real-world simulations.


Unit 2
Lesson
10

Name _____ Date _____ Period _____

MA.912.DP.1.4, MTR.1.1, MTR.4.1, MTR.7.1

Dribble, Draw, and Decide

Let's understand how to analyze the margin of error using real-world simulations.

Warm-Up

Imagine our class is trying to figure out the favorite color among all the 14-year-olds at our school. The table shows the results of the survey.

Number of Students	Color
31	Black
42	Blue
47	Orange
40	Purple
40	Red

1. What is the ratio of students that say their favorite color is orange?
2. A random sample of 20 students is pulled from the survey and 7 students in the sample choose orange as their favorite color. What is the proportion of the random sample?
3. What do you notice about the random sample proportion compared to the population?

Unit 2 Lesson 10 218 Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Understand how sample size and margin of error affect prediction accuracy.
2. **Goal:** Analyze data, and calculate proportions and confidence intervals.
3. **Language Goal:** Verbally explain the relationship between sample size and accurate data predictions.

Students explore how sample sizes impact prediction accuracy. Using real-life examples, they learn why larger samples improve accuracy and discover how confidence intervals can be used to make informed decisions about a population. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 10, students used data and drew conclusions about a population. They understood that the characteristics of a representative random sample are the same as the characteristics of the population as a whole. In particular, this applies to the mean and the percentages found in the sample.

Future Learning

In Lesson 12, students will use statistics to describe given data, including its shape. Based on the shape, students will decide which statistics to use to make an argument about how the data has changed over time.

Connections

- Students evaluate how sample size affects the margin of error when estimating a population proportion in real-world contexts such as basketball shooting accuracy and customer satisfaction. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.4
Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3	**MA.912.DP.1.5**
MA.7.DP.1	**MA.912.DP.2.3**
MA.7.DP.2	
MA.8.DP.2	

Vocabulary

New Vocabulary

confidence interval

margin of error

Review Vocabulary

proportional	*sample*
random	*simulation*

Lesson at a Glance

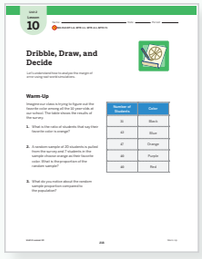
~ 45 min

MA.912.DP.1.4

Warm-Up

Independent | 5 min

Students activate prior knowledge to calculate and compare sample proportions to the total proportion.
Routine(s): Number Talk



Activity 1

Pairs | 10 min

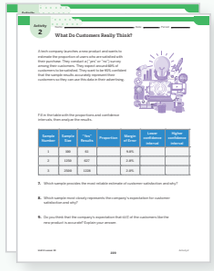
Students use a sample population to estimate a confidence level range to understand margin of error.
Routine(s): Think-Pair-Share (MTR.4.1)



Activity 2

Pairs | 10 min

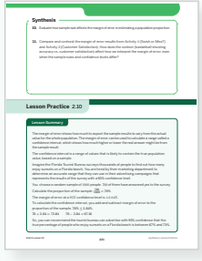
Students calculate proportions and confidence intervals to determine how margin of error provides the most reliable estimates.



Synthesis

Whole Class | 5 min

Students compare and contrast the margin of error results from Activity 1 and Activity 2 to see how to improve the accuracy of their predictions.
MLR2: Collect and Display (MTR.4.1)



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using confidence intervals, sample size, and margin of error to analyze data.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students activate prior knowledge to calculate and compare sample proportions to the total proportion. **(MTR.1.1)**

1 Launch

Use the Number Talk routine to support students' ability to surface prior knowledge about statistical ratios, proportions, populations, and samples that students will need later in the lesson. **(MTR.1.1)**

Invite students to solve the first ratio independently and then share their strategies aloud. Encourage them to consider how they can communicate their ideas in ways others can understand, and share that everyone is responsible for trying to understand each others' ideas.

ELL **English Language Learners** Provide a word bank to support students in expressing their strategies, with words such as: ratio, proportion, sample, population, etc. **(Speaking)**

Look for students who are unsure how to calculate proportions/ratios. Consider asking them if they know the sample proportion P is given by $P = \frac{X}{N}$, where X denotes the number of successes and N denotes the size of the sample in question.

Consider asking, "What connections do you notice between the different proportions?"

Note: This structure is repeated for each problem in the Number Talk.

2 Connect

To surface the Key Takeaway, consider inviting students to share how they would define the connection between sample and total proportions.

Unit 2
Lesson
10

Name: _____ Date: _____ Period: _____

MA.912.DP.1.4, MTR.1.1, MTR.4.1, MTR.7.1



Dribble, Draw, and Decide

Let's understand how to analyze the margin of error using real-world simulations.

Warm-Up

Imagine our class is trying to figure out the favorite color among all the 14-year-olds at our school. The table shows the results of the survey.

Number of Students	Color
31	Black
42	Blue
47	Orange
40	Purple
40	Red

1. What is the ratio of students that say their favorite color is orange?
   23.5%

2. A random sample of 20 students is pulled from the survey and 7 students in the sample choose orange as their favorite color. What is the proportion of the random sample?
   35%

3. What do you notice about the random sample proportion compared to the population?
   Responses vary. The proportion from the random sample was greater than the ratio for population.

Unit 2 Lesson 10 218 Warm-Up

Key Takeaway: The larger sample size was a more accurate representation of the actual proportion.

Pairs | 10 min

Activity 1 Swish or Miss?

Purpose: Students use a sample population to estimate a confidence level range to understand margin of error. **(MTR.7.1)**

1 Launch

Use the Think-Pair-Share routine for Problems 4, 5, and 6. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's solving strategy with the class.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support students with performing calculations.

2 Monitor

Create a class definition for the terms **margin of error** and **confidence interval** using students' language and the glossary.

Discuss student answers to Problems 4 and 6, using the class definitions to justify their answers.

D **Differentiation** *(Problem 4–6)*

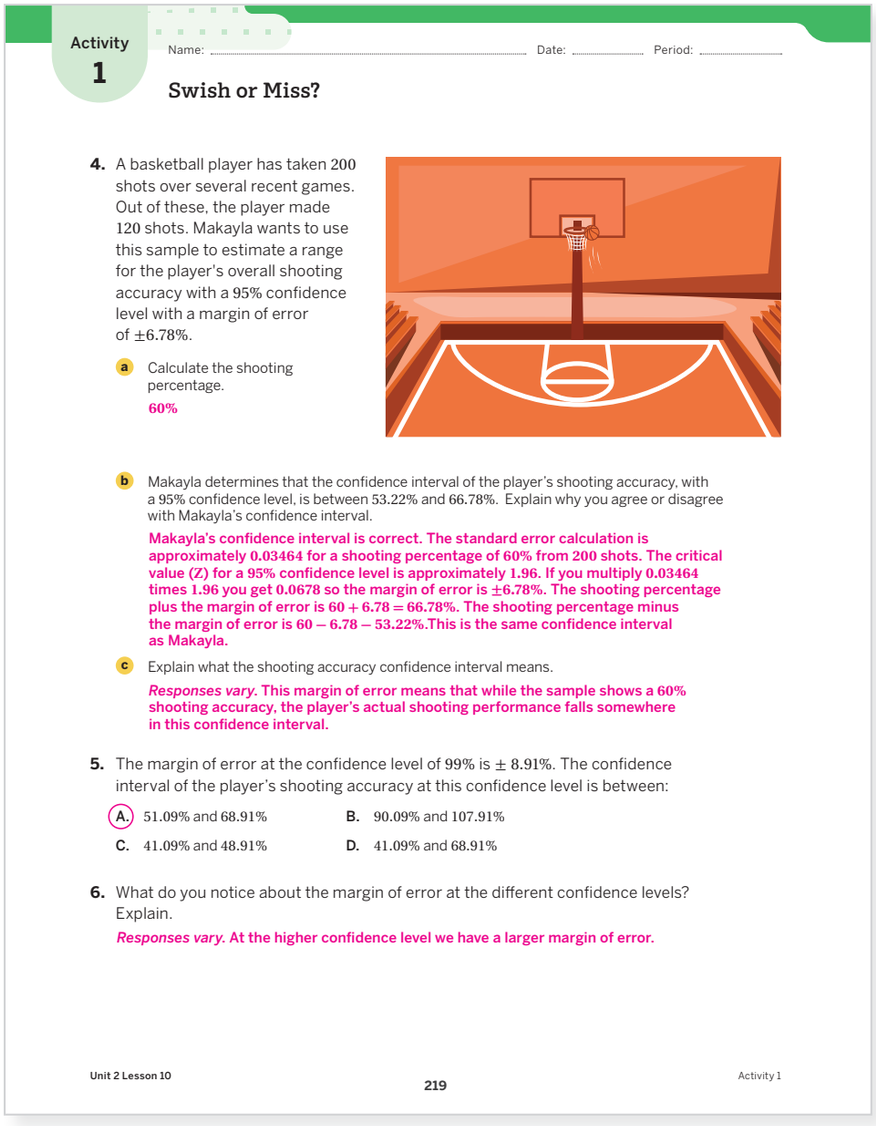
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to reread the problem, then list 1 thing they understand and 1 thing that has them stuck.
Struggle to calculate shooting percentage. *(Problem 5)*	**Strengthen** Invite students to review the formula $P = \frac{X}{N}$.
Struggle to calculate the confidence interval. *(Problem 6)*	**Strengthen** Invite students to consider that the margin of error is both positive and negative and how that can be used with the shooting percentage.
Student response *(Problem 7)*.	**Stretch.** Invite students to write their own problem to demonstrate what a confidence interval means.

Look for students who do not understand confidence intervals and the relationship to margin of error.

3 Connect

To reveal the Key Takeaway, consider asking:

- What happens when the confidence level is increased to 99%?
- What would happen to the margin of error at a confidence level below 94%?



Activity 1

Name: Date: Period:

Swish or Miss?

4. A basketball player has taken 200 shots over several recent games. Out of these, the player made 120 shots. Makayla wants to use this sample to estimate a range for the player's overall shooting accuracy with a 95% confidence level with a margin of error of ±6.78%.

a. Calculate the shooting percentage.

60%

b. Makayla determines that the confidence interval of the player's shooting accuracy, with a 95% confidence level, is between 53.22% and 66.78%. Explain why you agree or disagree with Makayla's confidence interval.

Makayla's confidence interval is correct. The standard error calculation is approximately 0.03464 for a shooting percentage of 60% from 200 shots. The critical value (Z) for a 95% confidence level is approximately 1.96. If you multiply 0.03464 times 1.96 you get 0.0678 so the margin of error is ±6.78%. The shooting percentage plus the margin of error is 60 + 6.78 = 66.78%. The shooting percentage minus the margin of error is 60 − 6.78 − 53.22%.This is the same confidence interval as Makayla.

c. Explain what the shooting accuracy confidence interval means.

Responses vary. This margin of error means that while the sample shows a 60% shooting accuracy, the player's actual shooting performance falls somewhere in this confidence interval.

5. The margin of error at the confidence level of 99% is ± 8.91%. The confidence interval of the player's shooting accuracy at this confidence level is between:

A. 51.09% and 68.91%
B. 90.09% and 107.91%
C. 41.09% and 48.91%
D. 41.09% and 68.91%

6. What do you notice about the margin of error at the different confidence levels? Explain.

Responses vary. At the higher confidence level we have a larger margin of error.

Unit 2 Lesson 10 219 Activity 1

Key Takeaway: As the confidence level increases, the margin of error becomes larger, giving us greater confidence that the true value lies within the interval, but its exact location within that range is less accurate.

Pairs | 10 min

Activity 2 What Do Customers Really Think?

Purpose: Students calculate proportions and confidence intervals to determine how margin of error provides the most reliable estimates. **(MTR.7.1)**

Short on time? Consider omitting student calculations for Sample Number 2.

1 Launch

Discuss why the reliability of accurate results would be more important in this real-world activity than in Activity 1.

2 Monitor

Consider asking, "How do sample size and margin of error affect the accuracy and reliability of survey results?"

Consider asking, "Why do larger sample sizes lead to smaller margins of error?"

D **Differentiation: (Support)** Students may think Sample 1 is most reliable because it matches the company's expectations. Consider asking them to compare the margins of error to see why Sample 3 is more reliable despite the lower proportion.

Encourage students to share other real-world survey scenarios that would use sample sizes, margin of error, and accurate, reliable survey results.

3 Connect

Use the Think-Pair-Share routine to support student-to-student interaction. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

To surface the Key Takeaway, consider asking:

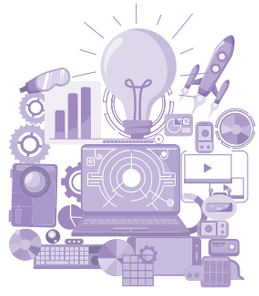
- "If you do not consider margin of error when interpreting survey data, what does that do to the reliability of your results?"
- "If you do not consider reasonable expected results when interpreting survey data, what does that do to the reliability of your results?"

Activity 2

Name: Date: Period:

What Do Customers Really Think?

A tech company launches a new product and wants to estimate the proportion of users who are satisfied with their purchase. They conduct a ("yes" or "no") survey among their customers. They expect around 60% of customers to be satisfied. They want to be 95% confident that the sample results accurately represent their customers so they can use this data in their advertising.



Fill in the table with the proportions and confidence intervals, then analyze the results.

Sample Number	Sample Size	"Yes" Results	Proportion	Margin of Error	Lower confidence interval	Higher confidence interval
1	100	61	61%	9.8%	51.20%	70.80%
2	1250	627	50%	2.8%	47.20%	52.80%
3	2500	1228	49%	2.0%	47%	51.00%

7. Which sample provides the most reliable estimate of customer satisfaction and why?

   Sample 3. Responses vary. The sample with the smallest margin of error provides the most reliable estimate of customer satisfaction.

8. Which sample most closely represents the company's expectation for customer satisfaction and why?

   Sample 1. *Responses vary.* With a proportion of 61% it is the closest to the company expectation that 60% of the customers would like the new product.

9. Do you think that the company's expectation that 60% of the customers like the new product is accurate? Explain your answer.

   No. *Responses vary.* Sample 1 is closer to their expectations but has a margin of error of 9.8%. Sample 3 has a lower margin of error at 2.0%, but the satisfaction is only at 49%. That is 11% lower than the original expectation.

Unit 2 Lesson 10 220 Activity 2

Key Takeaway: Always consider both the margin of error and how close the results are to expectations when interpreting survey data.

Whole Class | 5 min

Synthesis

Purpose: Students compare and contrast the margin of error results from Activity 1 and Activity 2 to see how to improve the accuracy of their predictions.

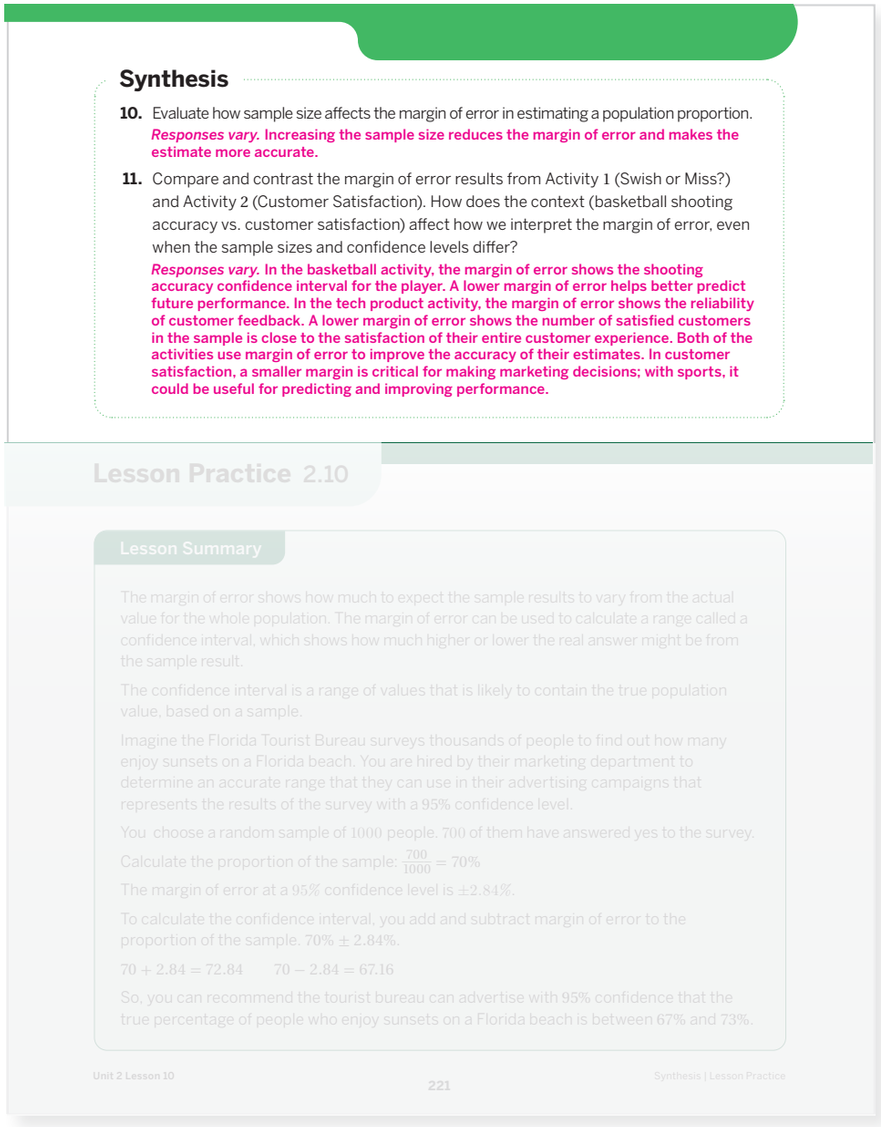
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- As the confidence level increases, the margin of error becomes larger, giving us greater confidence that the true value lies within the interval, but its exact location within that range is less accurate.
- Always consider both the margin of error and how close the results are to expectations when interpreting survey data.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they discuss how a margin of error is related to accurate predictions about the population with a partner. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Evaluate how sample size affects the margin of error in estimating a population proportion.

Responses vary. Increasing the sample size reduces the margin of error and makes the estimate more accurate.

11. Compare and contrast the margin of error results from Activity 1 (Swish or Miss?) and Activity 2 (Customer Satisfaction). How does the context (basketball shooting accuracy vs. customer satisfaction) affect how we interpret the margin of error, even when the sample sizes and confidence levels differ?

Responses vary. In the basketball activity, the margin of error shows the shooting accuracy confidence interval for the player. A lower margin of error helps better predict future performance. In the tech product activity, the margin of error shows the reliability of customer feedback. A lower margin of error shows the number of satisfied customers in the sample is close to the satisfaction of their entire customer experience. Both of the activities use margin of error to improve the accuracy of their estimates. In customer satisfaction, a smaller margin is critical for making marketing decisions; with sports, it could be useful for predicting and improving performance.

Lesson Practice 2.10

Lesson Summary

The margin of error shows how much to expect the sample results to vary from the actual value for the whole population. The margin of error can be used to calculate a range called a confidence interval, which shows how much higher or lower the real answer might be from the sample result.

The confidence interval is a range of values that is likely to contain the true population value, based on a sample.

Imagine the Florida Tourist Bureau surveys thousands of people to find out how many enjoy sunsets on a Florida beach. You are hired by their marketing department to determine an accurate range that they can use in their advertising campaigns that represents the results of the survey with a 95% confidence level.

You choose a random sample of 1000 people. 700 of them have answered yes to the survey.

Calculate the proportion of the sample: $\frac{700}{1000} = 70\%$

The margin of error at a 95% confidence level is $\pm 2.84\%$.

To calculate the confidence interval, you add and subtract margin of error to the proportion of the sample. $70\% \pm 2.84\%$.

$70 + 2.84 = 72.84$ $\quad$ $70 - 2.84 = 67.16$

So, you can recommend the tourist bureau can advertise with 95% confidence that the true percentage of people who enjoy sunsets on a Florida beach is between 67% and 73%.

Unit 2 Lesson 10 221 Synthesis | Lesson Practice

Lesson Takeaway: The margin of error in a simulated sample population improves the accuracy of predictions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using confidence intervals, sample size, and margin of error to analyze data.

Today's Goals

1. **Goal:** Understand how sample size and margin of error affect prediction accuracy.
2. **Goal:** Analyze data, and calculate proportions and confidence intervals.
3. **Language Goal:** Verbally explain the relationship between sample size and accurate data predictions.

Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 2.10

A company surveyed 300 people and found that 70 use public transportation regularly. They want to be 95% confident that the sample results accurately represent the local population within a margin of error of ±5%.

a. What is the 95% confidence interval for the proportion of people who use public transportation regularly? Show or explain your thinking.
0.18333 to 0.28333 Explanations vary.

b. Estimate the number of people in a city of 100,000 residents who regularly use public transportation based on this confidence interval. Show or explain your thinking.
18,333 to 28,333 people are estimated to regularly use public transportation.

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support using confidence intervals, sample size, and margin of error to analyze data:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write their own margin of error simulation similar to the one in **Activity 2**.

Professional Learning

Reflect on the facilitation of the Think-Pair-Share routine in this lesson. Who shared in the whole class discussion? Who did not? What about the routine felt successful? What are some areas for growth next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Evaluate how sample size affects the margin of error in estimating a population proportion.

Responses vary. Increasing the sample size reduces the margin of error and makes the estimate more accurate.

11. Compare and contrast the margin of error results from Activity 1 (Swish or Miss?) and Activity 2 (Customer Satisfaction). How does the context (basketball shooting accuracy vs. customer satisfaction) affect how we interpret the margin of error, even when the sample sizes and confidence levels differ?

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Lesson Practice 2.10

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$70 + 2.84 = 72.84 \qquad 70 - 2.84 = 67.16$

So, you can recommend the tourist bureau can advertise with 95% confidence that the true percentage of people who enjoy sunsets on a Florida beach is between 67% and 73%.

Lesson Practice 2.10

Name: ..... Date: ..... Period: .....

Problems 1–2: Determine the confidence interval of possible values.

1. A survey found that 60% of 200 participants prefer product A over product B. The survey reports a margin of error of $\pm 4\%$. What is the confidence interval of possible values for the true proportion of people who prefer product A? 56% to 64%

2. At a local dog park, 45% of dog owners, when asked, supported the new policy to leave the park open an extra hour each day. With a margin of error of $\pm 2\%$, what is the confidence interval of possible values for the true proportion of supporters? 43% to 47%

Problems 3–4: Determine the confidence interval or margin of error.

3. A company claims that 32% to 38% of their product is made from recycled materials. The actual amount of recycled materials is 35%. What is the margin of error? ±3%

4. A national poll on sock preferences sampled 1000 people. The margin of error, for brand X, at a 95% confidence level was $\pm 3.08\%$. The confidence interval is between 51.92% and 58.08%. What proportion of people surveyed preferred brand X? 55%

Problems 5–7: Explain your answer.

5. A survey finds that 48% of people prefer pens over pencils. The margin of error is $\pm 5\%$. If the sample size increases significantly, what is likely to happen to the margin of error?

Responses vary. The margin of error is likely to decrease because the larger sample size provides more data and reduces uncertainty.

6. After collecting location data from 250 visitors, a history center decides to advertise that 40% of their visitors come from out of state. The calculated margin of error for the claim is $\pm 6\%$ at a 90% confidence level. Explain how the margin of error would change if the confidence level increased to 95%, without recalculating.

Responses vary. The margin of error would increase if the confidence level increased to 95% because the higher confidence level would have a wider interval to make sure that their claim of 40% falls within that confidence interval.

7. A research company surveyed 1200 people to estimate the percentage of city residents who regularly bike on local trails. They found 68% ride regularly, with a margin of error of $\pm 2.5\%$ at a 95% confidence level. The city government plans to increase the budget for trail maintenance if at least 65% of residents bike regularly. Based on this survey, is it likely the true percentage meets this target? Explain your reasoning using the margin of error.

Responses vary. Yes, it is likely that the true percentage of residents who bike meets the city's goal of 65%. Based on the margin of error, it is reasonable to conclude that the true proportion of residents who bike is likely to meet or exceed the city's goal of 65%, as the entire confidence interval (65.5% to 70.5%) is above 65%.

Lesson Practice 2.10

Name: ..... Date: ..... Period: .....

Test Practice

8. Select all three true statements about margin of error.

- [x] **A.** The margin of error decreases when the sample size increases, making the estimate more precise.
- [ ] **B.** The margin of error increases as the sample size gets larger.
- [ ] **C.** A larger margin of error means the sample is more accurate.
- [x] **D.** Larger sample sizes generally result in a smaller margin of error.
- [x] **E.** The margin of error helps estimate how close the sample results are to the actual population value.

Spiral Review

Problems 9–11: Simplify each expression.

9. $(x^5)(x^3)$

x^8

10. $\frac{y^7}{y^4}$

y^3

11. $(3a^2b^4)^3$

$27a^6b^{12}$

Problem 12: Rewrite the expression with positive exponents.

12. $\frac{5x^{-3}}{y^2}$

$\frac{5}{(x^3y^2)}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.912.DP.1.4
	5–7	3	MA.912.DP.1.4
Test Practice	8	2	MA.912.DP.1.4
Spiral Review			
Fluency	9–12	2	MA.8.AR.1.1

Assess and Respond

Sub-Unit Quiz

Independent | 45 min

Facilitation: Assign Sub-Unit Quizzes to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Interpret a histogram in context.	Lesson 3	2	MA.912.DP.1.2 MTR.3.1
2	Compare the center and spread of two data sets.	Lesson 7	2	MA.912.DP.1.1 MTR.4.1
3	Use statistics appropriate to the shape of the data to compare the center of two data sets.	Lesson 5	2	MA.912.DP.1.2 MTR.4.1
4	Determine the appropriate data display for the given data. Estimate a population using survey data.	Lesson 8	3	MA.912.DP.1.1 MA.912.DP.1.4 MTR.7.1
5	Classify data as numerical or categorical, and univariate or bivariate. Estimate a percentage using survey data.	Lesson 9	2	MA.912.DP.1.1 MA.912.DP.1.4 MTR.7.1

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



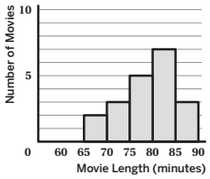
- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 2

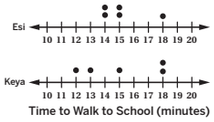
1. Kweku made a histogram showing the length, in minutes, of all his favorite movies.



Select *all* the statements that are definitely true about Kweku's favorite movies.

- ☐ **A.** Kweku has 5 favorite movies.
- ☐ **B.** There are 3 movies that are exactly 70 minutes long.
- ☑ **C.** Exactly 10 of these movies are 80 minutes or longer.
- ☐ **D.** There are 5 movies that are exactly 77 minutes long.
- ☑ **E.** Kweku has 20 favorite movies.

2. Here are two dot plots showing the number of minutes it took Esi and Keya to walk to school last week.



a Whose data has a larger mean? Circle one.

Esi Keya (They're the same)

b Whose data has a larger range? Circle one.

Esi (Keya) They're the same

3. Here are the points scored by two basketball players over the last 10 games.

Player A

7, 0, 4, 5, 3, 4, 1, 4, 4, 5

Player B

2, 1, 0, 1, 3, 3, 3, 8, 20, 6

Martina used the median to help decide which player had a higher typical score.

Why might the median be more helpful than the mean for describing the typical score?

Responses vary. The median might be more helpful than the mean because the mean is affected by values that are far from the rest of the data. Player B's score of 20 is far from the rest of the data, so the mean for Player B will be a lot higher than the median, and it will not be representative of the center of the data set.

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Name: Date: Period:

Sub-Unit Quiz (continued)

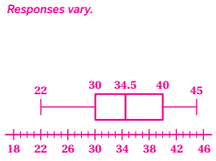
Unit 2

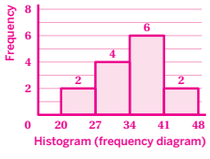
4. A company is analyzing the number of customer support calls received per day over a two-week period. The data is shown.

22, 25, 27, 30, 30, 32, 34, 35, 37, 38, 40, 40, 42, 45

a Make a data display to represent this data. Choose from a histogram, stem-and-leaf plot, box plot, and line plot.

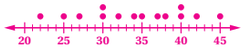
Responses vary.





Stem	Leaf
2	2 5 7
3	0 0 2 4 5 7 8
4	0 0 2 5

Key: 2 | 2 means 22 calls



b Estimate the total number of calls the company would receive in a year. Show your work.

$22 + 25 + 27 + 30 + 30 + 32 + 34 + 35 + 37 + 38 + 40 + 40 + 42 + 45 = 477$

$\frac{365}{14} \approx 26.07 \quad 477 \cdot 26.07 \approx 12{,}436$

The estimated total number of calls in a year is about 12,436.

D Differentiation (Sub-Unit Quiz)

Note: Consider inviting students to revise the corresponding assessment items after responding to student thinking.

Sub-Unit Goals	Problem (s)	To respond to student thinking, consider:
• Determine whether data is univariate or bivariate. **(Lessons 1–2)**	5a	**Support** • **Mini-Lesson:** Classifying Data • **Teacher Move:** Consider revisiting Lesson 1 (*Survey Says*)

• Represent data with dot plots, histograms, and box plots.
• Describe the shape, center, and spread of data. **(Lessons 3–6)**
	1, 2, 3, 4	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 3 (*Data Driven*)
		  • Lesson 4 (*Better Weather?*)
		  • Lesson 5 (*Quick Pick*)
		Stretch
		• **Teacher Move:** Invite students to research a new topic and find data from two different years or groups to compare using statistics appropriate to the shape of their data.

• Compare and contrast data sets using measures of center and spread appropriate to the shape of the data set and interpret the differences in context. **(Lessons 7–10)**	5b	**Support**
		• **Mini-Lesson:** Estimating Population from a Sample Survey
		• **Teacher Move:** Consider revisiting:
		  • Lesson 8 (*How Many?*)
		  • Lesson 9 (*Dribble, Draw, and Decide*)

Summarizing Two-Variable Data

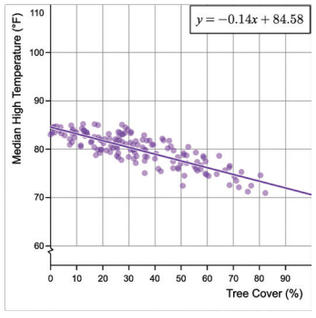
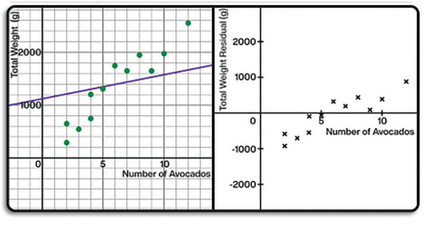
Sub-Unit 3 Goals

- **Goal:** Fit a line to data, assess the fit using residual plots, and use the linear model to make predictions.
- **Goal:** Represent categorical data in two-way tables and relative frequency tables.
- **Language Goal:** Describe linear associations in data using the line of fit and correlation coefficient.

In this sub-unit, students use the **correlation coefficient** and **lines of best fit** to describe the relationship between two variables. Students assess the fit of a linear model using **residual plots**. They also consider whether the relationship represents **causation**, or **correlation** but not causation. Students learn how to organize categorical data in **two-way tables** and **relative frequency tables**, interpreting the values they calculate in context, and using the information to analyze a situation and make a decision.



Math that Matters Most

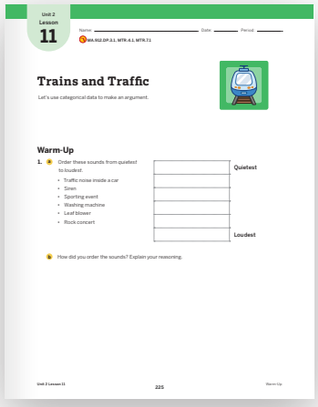
Concept or Skill	For example . . .
Interpret the correlation coefficient.	The r-value is also known as the **correlation coefficient**. What do you think *correlation* means? *I think correlation means that two things are related to each other.*
Interpret the slope and y-intercept of a linear model in context.	 What do the -0.14 and 84.58 mean about the relationship between temperature and tree cover?
Assess the fit of a linear model using a residual plot.	

Concept or Skill	For example . . .
Generate a line of best fit using technology.	In the activity, take a look at the table of data showing the Adélie penguin population over time. Then use the calculator to generate the line of best fit for this data **a** **Discuss** What does each part of $y_1 \sim mx_1 + b$ represent? **b** **Discuss** How are the parts of the equation in row 9 related to the information in row 8?
Use the line of best fit to make predictions.	Use the line of best fit to predict the cost of a hamburger at a stadium where a soda costs \$7.
Distinguish between causation and correlation.	Sydney says: *Increasing the price of a soda in a stadium causes the price of a hamburger to increase.* Do you agree with her claim? Explain your thinking.

Unit 2

Lesson

11


Unit 2
Lesson
11
Trains and Traffic
Let's use categorical data to make an argument.
Warm-Up
1. Order these sounds from quietest to loudest.
Traffic noise inside a car
Siren
Sporting event
Washing machine
Lead blower
Rock concert
Quietest
Loudest
How did you order the sounds? Explain your reasoning.
225

Trains and Traffic

Two-Way Tables and Relative Frequency Tables

Let's use categorical data to make an argument.

Learning Outcomes

Today's Goals

1. **Goal:** Represent categorical data in *two-way tables* and *relative frequency tables*.
2. **Goal:** Calculate values in two-way tables and relative frequency tables.
3. **Language Goal:** Analyze two-way tables for *associations* in data. **(Reading, Writing, and Speaking)**

Students model traffic delays data for two cities using **two-way** and **relative frequency tables**. They interpret **joint frequencies** and **marginal frequencies** and look for possible **associations** in the data. **(MTR.4.1, MTR.7.1)**

Coherence

Prior Learning

In Lesson 1, students explained the difference between quantitative and categorical data.

Future Learning

In Lesson 12, students will continue to use relative frequency tables to make decisions about a context and identify associations in data. They will also discuss the limitations of data for supporting their decision.

Connections

- Students use categorical data to share arguments regarding topics such as traffic delays and preference to when they have gym class. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.3.1
Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.8.DP.1.3

Vocabulary

New Vocabulary

association
joint frequencies
marginal frequencies
relative frequency table
two-way table

Lesson at a Glance

~ 45 min

MA.912.DP.3.1

Warm-Up

Pairs | 5 min

Students order sounds from quietest to loudest and determine if any of them could have an effect on hearing.

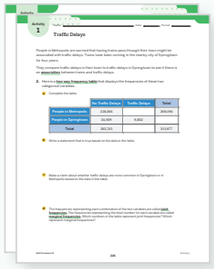


Activity 1

Pairs | 15 min

Students fill in a **two-way table** and make a **relative frequency table** to critique claims about traffic delays data for a city and a town. They interpret **joint frequencies** and **marginal frequencies** and look for possible associations in the data.

Routines: Think-Pair-Share, MLR5: Co-Craft Questions, Decide and Defend (MTR.4.1)

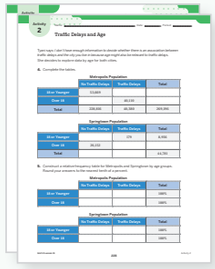


Activity 2

Small Groups | 15 min

Students use tables for age data to critique others' reasoning about the possible **association** between traffic delays and where people live.

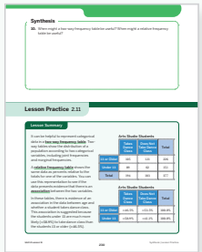
Routine: Decide and Defend (MTR.4.1)



Synthesis

Whole Class | 5 min

Students synthesize their understanding of two-way tables and relative frequency tables by describing when each is useful. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students reflect on their learning by describing something happening in their community, how it affects the residents, and what data they could gather about it.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students order sounds from quietest to loudest and determine if any of them could have an effect on hearing.

1 Launch

Display the sound sort.

ELL **English Language Learners** Invite students to read the description of each sound aloud and paraphrase what it means. Clarify any words that are unfamiliar. **(Reading)**

Invite students to order the sounds and then share their ordering and reasoning with their partner.

2 Connect

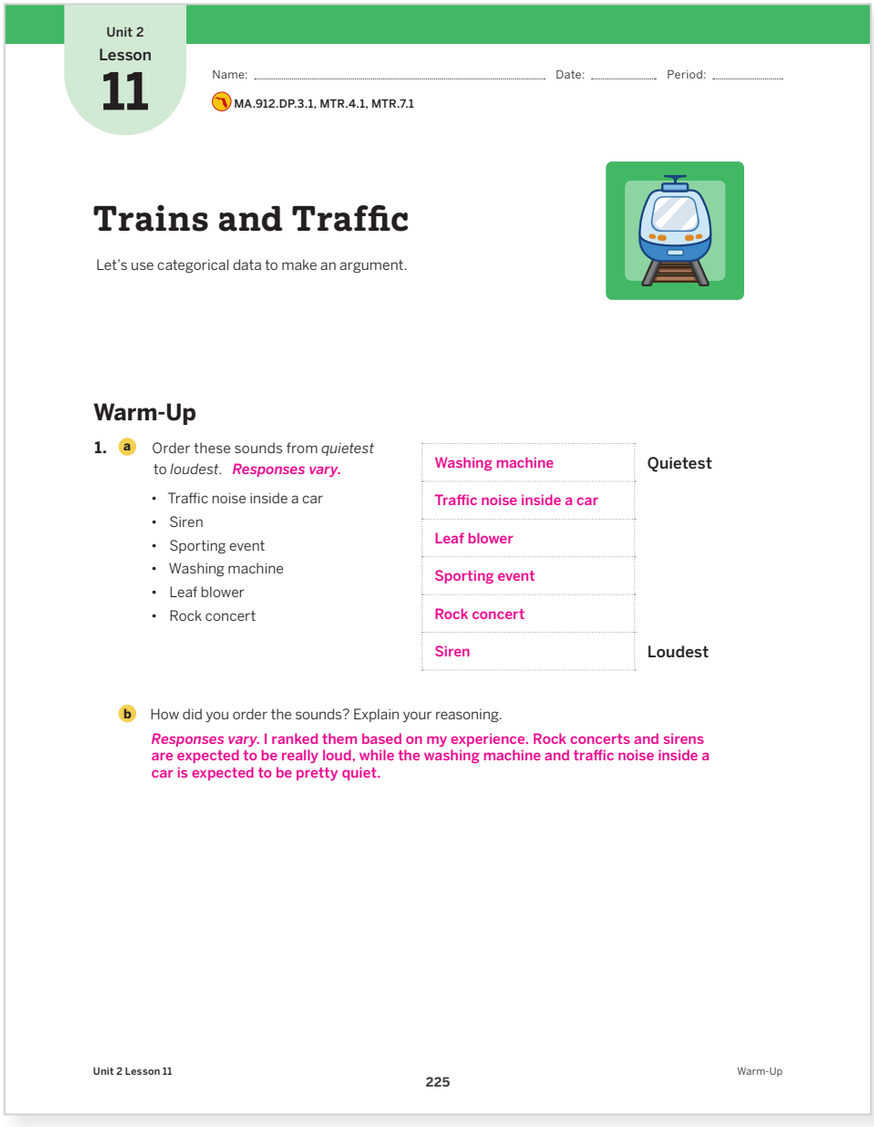
Consider asking, "Which of these noises do you predict might affect your hearing after extended exposure?"

Display the infographic and the prompt.

Invite students to share what they notice about the decibel infographic.

Consider asking:

- "Did any of the information in the graphic surprise you? How so?"
- "Do you know anyone who is deaf or who has experienced hearing loss?"



Unit 2
Lesson
11

Name: _____ Date: _____ Period: _____

MA.912.DP.3.1, MTR.4.1, MTR.7.1

Trains and Traffic

Let's use categorical data to make an argument.

Warm-Up

1. a Order these sounds from *quietest* to *loudest*. ***Responses vary.***

- Traffic noise inside a car
- Siren
- Sporting event
- Washing machine
- Leaf blower
- Rock concert

Washing machine	Quietest
Traffic noise inside a car	
Leaf blower	
Sporting event	
Rock concert	
Siren	Loudest

b How did you order the sounds? Explain your reasoning.

Responses vary. **I ranked them based on my experience. Rock concerts and sirens are expected to be really loud, while the washing machine and traffic noise inside a car is expected to be pretty quiet.**

Unit 2 Lesson 11 225 Warm-Up

Pairs | 15 min

Activity 1 Traffic Delays

Purpose: Students fill in a **two-way table** and make a **relative frequency table** to critique claims about traffic delays data for a city and a town. They interpret joint frequencies and marginal frequencies and look for possible associations in the data. **(MTR.4.1)**

1 Launch

Display the image and the prompts.

ELL **English Language Learners** Invite students to talk about their experiences with trains and traffic delays before starting the task. **(Reading and Listening)**

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class. *Responses vary.* Trains can make travel easy, but people may worry that trains can cause traffic delays.

MLR **MLR5: Co-Craft Questions** Consider inviting students to create and share questions that could be asked and answered about the context before they look at the traffic delays data. See the Routine Facilitation Guide for more information. **(MTR.7.1)**

2 Monitor

A **Accessibility: Memory and Attention** Invite students to use their Student Edition, additional paper, or a calculator to support them with performing calculations.

D **Differentiation** *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose one blank space in the table to start. Consider asking, "What does this blank represent? How is this value related to the other values in this row or column?"
Write a number in the "Traffic Delays"/"People in Metropolis" entry of the table different from 40,380.	**Support** Invite them to check their work by adding the two entries in the "People in Metropolis" row to see if the total is 269,196. Consider asking, "How could you use the two entries already in this row to find the missing entry?"
Write a number in either blank "Total" entry that is different from the true total.	**Support** Consider asking students to share in their own words what a "total" is and how they would calculate one.
Make a claim for Problem 2c that is supported by evidence in the table.	**Strengthen** Celebrate their claim and invite them to share it during the pause.

Pause to invite students to share their claims about which city traffic delays is more common.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Traffic Delays

People in Metropolis are worried that having trains pass through their town might be associated with traffic delays. Trains have been running in the nearby city of Springtown for four years.

They compare traffic delays in their town to traffic delays in Springtown to see if there is an **association** between trains and traffic delays.

2. Here is a **two-way frequency table** that displays the frequencies of these two categorical variables.

a. Complete the table.

	No Traffic Delays	Traffic Delays	Total
People in Metropolis	228,816	40,380	269,196
People in Springtown	34,929	9,852	44,781
Total	263,745	50,232	313,977

b. Write a statement that is true based on the data in the table.

Responses vary. The population of Metropolis is 269,196 people.

c. Make a claim about whether traffic delays are more common in Springtown or in Metropolis based on the data in the table.

Responses vary. Experiencing traffic delays is more common in Springtown because about 25% of people in Springtown have experienced traffic delays, while far less than 25% of people in Metropolis have experienced traffic delays.

d. The frequencies representing each combination of the two variables are called **joint frequencies**. The frequencies representing the total number for each variable are called **marginal frequencies**. Which numbers in the table represent joint frequencies? Which represent marginal frequencies?

The joint frequencies are 228,818; 40,380; 34,929; and 9,852. The marginal frequences are 264,745; 50,232; 269,196; and 44,781.

Unit 2 Lesson 11 226 Activity 1

Pairs | 15 min

Activity 1 Traffic Delays (continued)

Purpose: Students fill in a **two-way table** and make a **relative frequency table** to critique claims about traffic delays data for a city and a town. They interpret joint frequencies and marginal frequencies and look for possible associations in the data. **(MTR.4.1)**

2 Monitor

Select and sequence students' explanations of Rishi's work and their Problem 3c responses to share during the Connect.

D Differentiation *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where did the numbers in Rishi's work come from? How could you use a similar strategy for the next blank in the same row?"
Calculate the relative frequencies in different ways, e.g., dividing to find all the values or subtracting percents from 100.	**Strengthen** Celebrate their strategy and invite them to share during the Connect.
Would like to explore this possible association further.	**Stretch** Encourage students to research other factors that may impact traffic delays. Invite them to share their findings with a partner.

Look for student strategies and reasoning about whether it matters if you live in Metropolis or Springtown to share during the Connect, including:

- Comparing data from both the frequency and relative frequency tables. **(MTR.7.1)**
- Arguments about which table makes sense to use.

3 Connect

Display the tables.

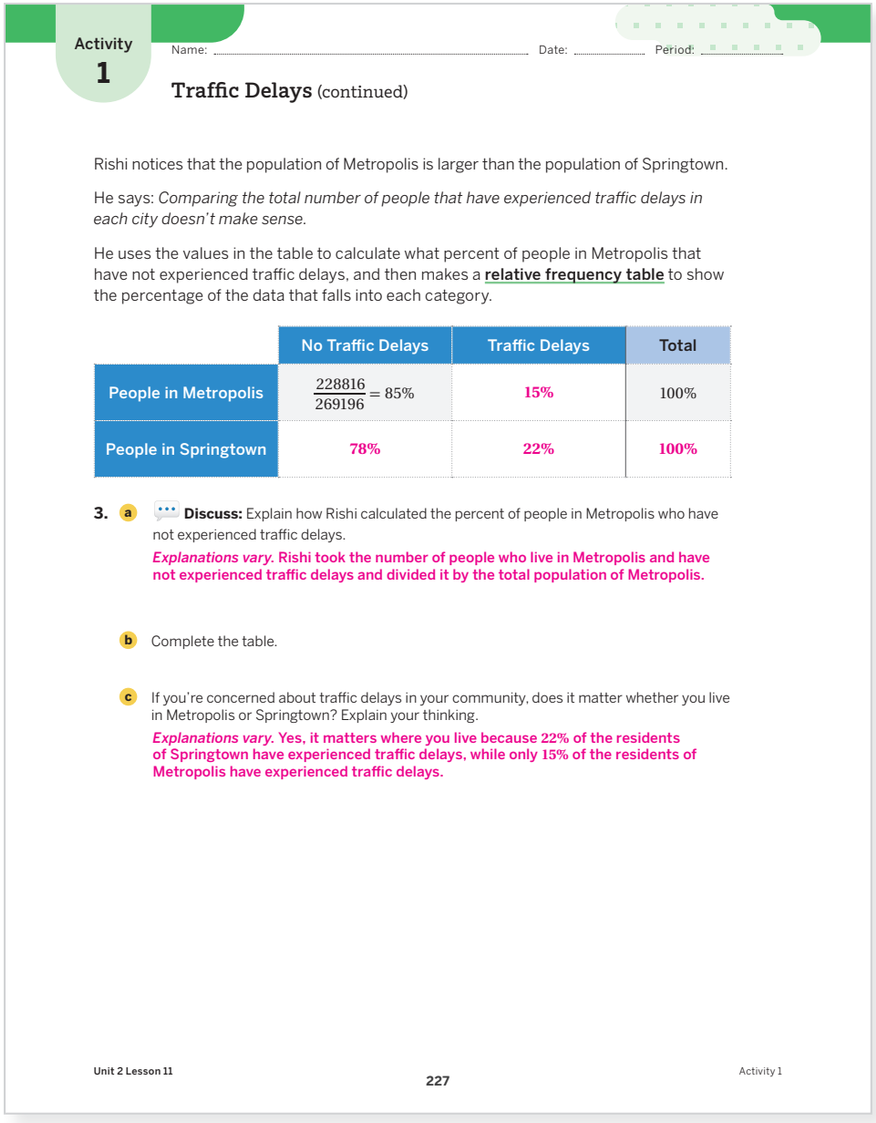
Create a class definition for **relative frequency table**. To note the differences between frequency and relative frequency, consider asking:

- "How were the two tables alike? Different?"
- "When might each type of table be helpful?"

Encourage students to refer to the tables to explain their reasoning.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Invite students to share their thinking about whether it matters where you live if you're concerned about traffic delays. If it doesn't come up naturally, consider asking, "Do you think there is an association between where you live and whether you experience traffic delays?" **(MTR.4.1)**

Note: Consider sharing that there are many reasons why there might be an association between traffic delays and where people live.



Activity 1

Name: _____ Date: _____ Period: _____

Traffic Delays (continued)

Rishi notices that the population of Metropolis is larger than the population of Springtown.

He says: *Comparing the total number of people that have experienced traffic delays in each city doesn't make sense.*

He uses the values in the table to calculate what percent of people in Metropolis that have not experienced traffic delays, and then makes a **relative frequency table** to show the percentage of the data that falls into each category.

	No Traffic Delays	Traffic Delays	Total
People in Metropolis	$\frac{228816}{269196} = 85\%$	15%	100%
People in Springtown	78%	22%	100%

3. a. **Discuss:** Explain how Rishi calculated the percent of people in Metropolis who have not experienced traffic delays.

Explanations vary. Rishi took the number of people who live in Metropolis and have not experienced traffic delays and divided it by the total population of Metropolis.

b. Complete the table.

c. If you're concerned about traffic delays in your community, does it matter whether you live in Metropolis or Springtown? Explain your thinking.

Explanations vary. Yes, it matters where you live because 22% of the residents of Springtown have experienced traffic delays, while only 15% of the residents of Metropolis have experienced traffic delays.

Unit 2 Lesson 11 227 Activity 1

Key Takeaway: A two-way frequency table shows counts and totals for categorical data split across two or more groups. A **relative frequency table** shows this data as percents of the total number of individuals in each group, which can be helpful for making comparisons especially when the groups are different sizes. In a two-way table, the frequencies representing each combination of the two variables are called **joint frequencies**. The frequencies representing the total number for each variable are called **marginal frequencies**.

Small Groups | 15 min

Activity 2 Traffic Delays and Age

Purpose: Students use tables for age data to critique others' reasoning about the possible **association** between traffic delays and where people live. **(MTR.4.1)**

Short on time? Consider omitting Problem 8 and/or Problem 9.

1 Launch

Display Tyani's statement and and use the **Decide and Defend** routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

ELL **English Language Learners** Invite students to read each claim aloud and paraphrase its meaning. Clarify any unfamiliar words. **(Reading)**

Consider asking:

- "Why might age have an effect on traffic delays?"
- "What other factors might influence the rate of traffic delays in an area?" *Responses vary.* Areas with a weaker infrastructure and worse road conditions might have higher rates of traffic delays than comparable areas.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with Tyani because . . .
- I think that makes sense because . . .
- How do you know? **(Writing and Speaking)**

2 Monitor

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one table at a time. To support focus, consider inviting students to cover up tables, columns, or rows they are not focused on at the moment.

D **Differentiation** *(Problems 4–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to revisit their work from Activity 1. Consider asking, "Which of these tables will be like the first table in Activity 1? The second?"
Want to check their work.	**Support** Invite students to check their work by verifying their entries sum to the given totals. Consider asking, "How could you use the entries already in this row/column to find the missing entry?"
Need support calculating relative frequencies.	**Support** Encourage students to refer back to Rishi's work in Activity 1. Consider asking, "How could you use a similar strategy in these tables?"
Attend to precision by calculating correct relative frequencies that are rounded to one decimal place.	**Strengthen** Celebrate these students' work and invite them to share their strategies with other students in their group or with other groups.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Traffic Delays and Age

Tyani says: *I don't have enough information to decide whether there is an association between traffic delays and the city you live in because age might also be relevant to traffic delays.* She decides to explore data by age for both cities.

4. Complete the tables.

Metropolis Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger	53,669	270	53,939
Over 18	175,147	40,110	215,257
Total	228,816	40,380	269,196

Springtown Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger	8,777	179	8,956
Over 18	26,152	9,673	35,825
Total	34,929	9,852	44,781

5. Construct a relative frequency table for Metropolis and Springtown by age groups. Round your answers to the nearest tenth of a percent.

Metropolis Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger	99.5%	0.5%	100%
Over 18	81.4%	18.6%	100%

Springtown Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger	98.0%	2.0%	100%
Over 18	73.0%	27.0%	100%

Unit 2 Lesson 11 228 Activity 2

Small Groups | 15 min

Activity 2 Traffic Delays and Age (continued)

Purpose: Students use tables for age data to critique others' reasoning about the possible **association** between traffic delays and where people live. **(MTR.4.1)**

Short on time? Consider omitting Problem 8 and/or Problem 9.

2 Monitor

Differentiation: (Support) Students may think that there is no association, or (following Tyani) think that there is not enough information to decide. Consider asking, "If there was *no* association, what would you expect the data to look like? Does this data look like that?" *Responses vary.* If there was no association, the percents would be approximately the same for each city. This data has significant differences between the percents.

Listen for student explanations about whether or not there is an association between the city people live in and traffic delays, and invite students to share during the Connect.

3 Connect

Consider asking, "Which tables would be helpful to look at for our discussion? Two-way? Relative frequency? Both?"

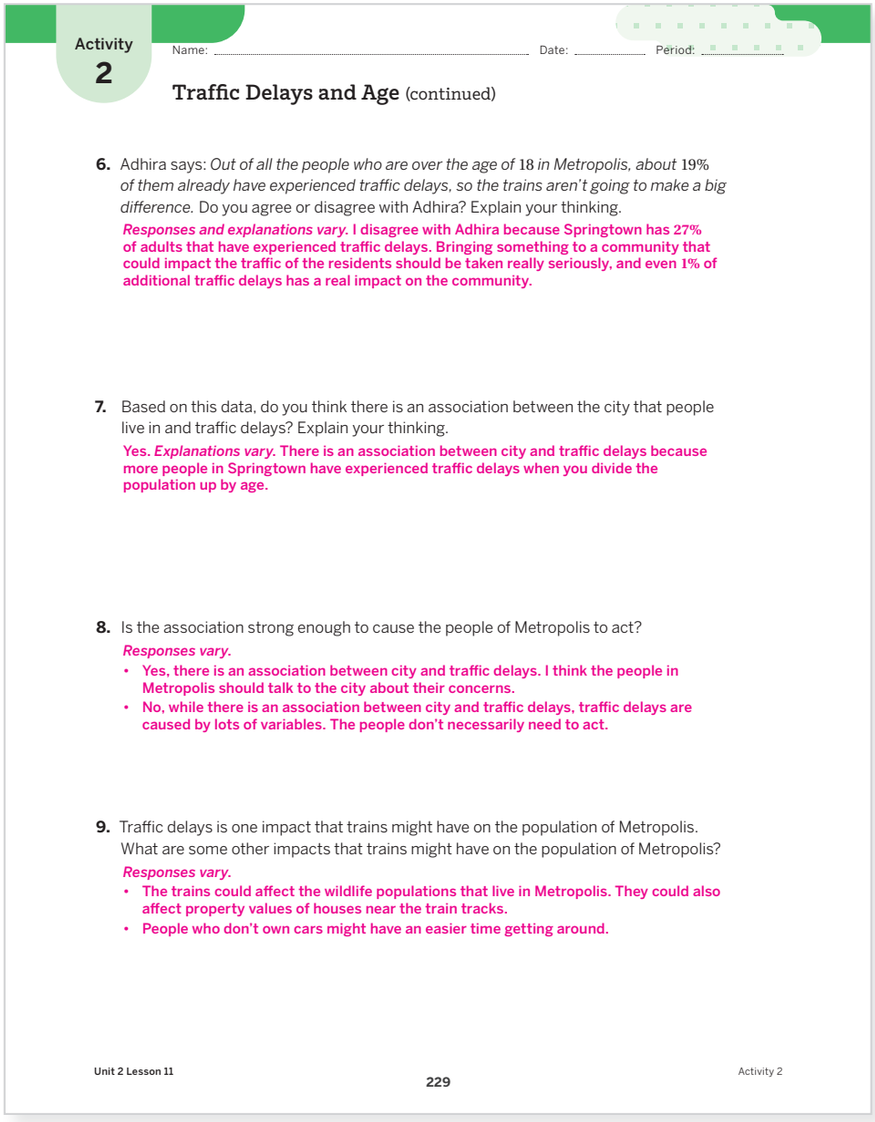
Display the tables to discuss whether or not there is an association between the city people live in and traffic delays.

Invite students to share what the new term *association* reminds them of (e.g., "When I hear association, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge.

Note: At this point we are using the words "relationship" and "association" interchangeably to describe a meaningful connection between two variables. Later in the unit students will attend to precision in their use of these terms to distinguish between the statistical treatment of links between categorical and quantitative variables.

To surface the Key Takeaway, consider asking:

- "How did representing data in two-way tables and relative frequency tables help you decide if there was an association?"
- "How did you decide whether the association was strong enough to cause the people of Metropolis to act?" *Responses vary.* The difference in the relative frequency table is almost 10% between the "Over 18" groups in the different cities; 10% of any group feels like a significant percent, which is why our group decided the people of Metropolis should act.



Activity 2

Name: _____ Date: _____ Period: _____

Traffic Delays and Age (continued)

6. Adhira says: *Out of all the people who are over the age of 18 in Metropolis, about 19% of them already have experienced traffic delays, so the trains aren't going to make a big difference.* Do you agree or disagree with Adhira? Explain your thinking.

Responses and explanations vary. **I disagree with Adhira because Springtown has 27% of adults that have experienced traffic delays. Bringing something to a community that could impact the traffic of the residents should be taken really seriously, and even 1% of additional traffic delays has a real impact on the community.**

7. Based on this data, do you think there is an association between the city that people live in and traffic delays? Explain your thinking.

Yes. *Explanations vary.* **There is an association between city and traffic delays because more people in Springtown have experienced traffic delays when you divide the population up by age.**

8. Is the association strong enough to cause the people of Metropolis to act?

Responses vary.

- **Yes, there is an association between city and traffic delays. I think the people in Metropolis should talk to the city about their concerns.**
- **No, while there is an association between city and traffic delays, traffic delays are caused by lots of variables. The people don't necessarily need to act.**

9. Traffic delays is one impact that trains might have on the population of Metropolis. What are some other impacts that trains might have on the population of Metropolis?

Responses vary.

- **The trains could affect the wildlife populations that live in Metropolis. They could also affect property values of houses near the train tracks.**
- **People who don't own cars might have an easier time getting around.**

Unit 2 Lesson 11 229 Activity 2

Key Takeaway: If there is a significant difference in the relative frequencies of one variable when grouped according to another variable, that may suggest that there is an **association** between the two variables. There may, however, be other lurking variables involved.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of two-way tables and relative frequency tables by describing when each is useful.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- A two-way table can be useful to understand exactly how many individuals are being counted.
- A relative frequency table can help make it easier to see significant differences across groups of different sizes.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about **two-way tables**, **relative frequency tables**, **joint frequences**, **marginal frequencies**, and **associations** as they share with the class. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner one thing they learned about each kind of table and any questions they might still have.



Synthesis

10. When might a two-way frequency table be useful? When might a relative frequency table be useful?

Responses vary. A two-way frequency table might be useful when you're trying to organize categorical data. A relative frequency table might be useful when you're trying to compare populations that are different sizes.

Lesson Practice 2.11

Lesson Summary

It can be helpful to represent categorical data in a **two-way frequency table**. Two-way tables show the distribution of a population according to two categorical variables, including joint frequencies and marginal frequencies.

A **relative frequency table** shows the same data as percents relative to the totals for one of the variables. You can use this representation to see if the data presents evidence that there is an **association** between the two variables.

In these tables, there is evidence of an association in the data between age and whether a student takes dance class. This association is suggested because the students under 13 are much more likely (≈58.9%) to take dance class than the students 13 or older (≈46.5%).

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	105	121	226
Under 13	89	62	151
Total	194	183	377

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	≈46.5%	≈53.5%	100.0%
Under 13	≈58.9%	≈41.1%	100.0%

Unit 2 Lesson 11 230 Synthesis | Lesson Practice

Lesson Takeaway: Categorical data can be represented in **two-way tables** that show exact counts and in **relative frequency tables** that show percents. Both kinds of tables can be useful in determining whether or not there is an **association** between categorical variables as they are distributed in two different groups.

Independent | 5 min

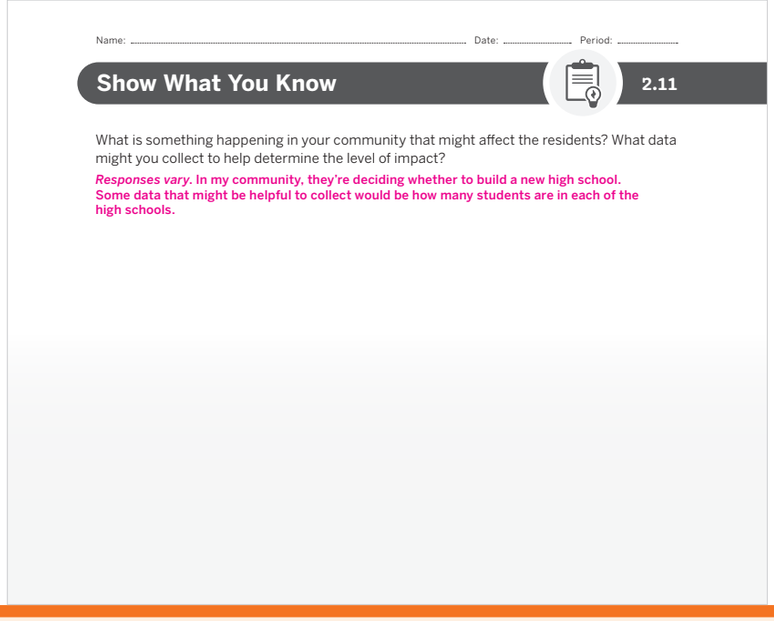
Show What You Know

Purpose: Students demonstrate their understanding by describing something happening in their community, how it affects the residents, and what data they could gather about it.

Today's Goals

1. **Goal:** Represent categorical data in *two-way tables* and *relative frequency tables*.
2. **Goal:** Calculate values in two-way tables and relative frequency tables.
3. **Language Goal:** Analyze two-way tables for *associations* in data. **(Reading, Writing, and Speaking)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.11

What is something happening in your community that might affect the residents? What data might you collect to help determine the level of impact?

Responses vary. In my community, they're deciding whether to build a new high school. Some data that might be helpful to collect would be how many students are in each of the high schools.

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support reflecting on what data can be useful when determining the impact of an event on your community:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students developed early ideas about the term **association**. How will their understanding of this term build over the next several lessons? How can you support students building on their reasoning about whether or not an association is present in ways that connect with future work students will do in this unit on bivariate data?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. When might a two-way frequency table be useful? When might a relative frequency table be useful?

Responses vary. A two-way frequency table might be useful when you're trying to organize categorical data. A relative frequency table might be useful when you're trying to compare populations that are different sizes.

Lesson Practice 2.11

Lesson Summary

It can be helpful to represent categorical data in a **two-way frequency table**. Two-way tables show the distribution of a population according to two categorical variables, including joint frequencies and marginal frequencies.

A **relative frequency table** shows the same data as percents relative to the totals for one of the variables. You can use this representation to see if the data presents evidence that there is an **association** between the two variables.

In these tables, there is evidence of an association in the data between age and whether a student takes dance class. This association is suggested because the students under 13 are much more likely (≈58.9%) to take dance class than the students 13 or older (≈46.5%).

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	105	121	226
Under 13	89	62	151
Total	194	183	377

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	≈46.5%	≈53.5%	100.0%
Under 13	≈58.9%	≈41.1%	100.0%

Lesson Practice 2.11

Name: _____ Date: _____ Period: _____

1. A random sample of teenagers were asked about their preferences when organizing a birthday party for a friend. Use this information to complete the two-way frequency table:

- 122 people responded to the survey.
- 50 of the people who said they prefer surprise parties also prefer that guests bring individual gifts.
- 68 people prefer organizing one group gift.
- 56 people prefer not to have a surprise party.

	Surprise Party	No Surprise	Total
Group Gift	16	52	68
Individual Gifts	50	4	54
Total	66	56	122

Problems 2–3: Students in the 7th, 8th, and 9th grade were asked whether they prefer to write in pen or pencil. 40 students prefer to write in pen and 60 students prefer to write in pencil.

2. Create values that could represent the number of students in the 7th, 8th, and 9th grade that responded to the survey.

Responses in the table vary, but the total number who prefer pens should be 40 and the total number who prefer pencils should be 60.

	Pen	Pencil
7th Grade	5	25
8th Grade	10	20
9th Grade	25	15
Total	40	60

3. Write a statement about the data in your table.

Responses vary. Students who are older prefer writing in pen more than younger students do.

4. This relative frequency table shows data on whether students listen to music while they do their math and Spanish homework. Based on this data, is there an association between the homework subject and whether students listen to music?

Circle one: (Yes) No

Explain your thinking.

Explanations vary. Students are more likely to listen to music when they are doing their math homework.

	Music	No Music
Math Homework	61%	39%
Spanish Homework	28%	72%

Lesson Practice 2.11

Name: _____ Date: _____ Period: _____

5. 1,430 people are surveyed about whether they take daily vitamins and whether they eat breakfast. The table shows the results. Create a relative frequency table that shows the percentage of the entire group that is in each cell.

	Take Daily Vitamins	Don't Take Daily Vitamins
Eat Breakfast	384	476
Don't Eat Breakfast	268	302

	Take Daily Vitamins	Don't Take Daily Vitamins
Eat Breakfast	~26.9%	~33.3%
Don't Eat Breakfast	~18.7%	~21.1%

Problems 6–9: This two-way frequency table shows student responses to the question: *Do you prefer to have gym class in the morning or in the afternoon?*

	Morning	Afternoon	Total
6th Grade	15	8	23
8th Grade	18	21	39
10th Grade	12	26	38
Total	45	55	100

6. How many students participated in the survey? 100

7. How many 8th grade students prefer to have gym class in the morning? 18

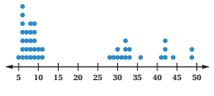
8. How many 10th grade students participated in the survey? 38

Test Practice

9. How many students prefer to have gym class in the afternoon? 55

Spiral Review

Problems 10–11: This dot plot represents the ages of people who watched a movie at a movie theater on a Saturday.



10. What kind of movie do you think this is? Explain your thinking.

Responses and explanations vary. It could be a family movie because there are lots of kids and some older family members who probably took them to the movies.

11. How many people over 40 years old watched the movie on Saturday? 7 people

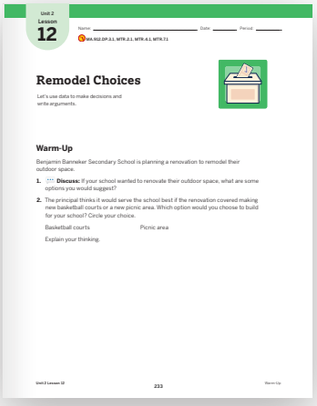
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–8	2	MA.912.DP.3.1
	2–3	3	MA.912.DP.3.1
Test Practice	9	2	MA.912.DP.3.1
Spiral Review			
Fluency	10–11	2	MA.6.DP.1.4

Unit 2

Lesson

12


Unit 2
Lesson
12
Name _____ Date _____ Period _____
MA.912.DP.3.1, MTR.2.1, MTR.4.1, MTR.7.1
Remodel Choices
Let's use data to make decisions and write arguments.
Warm-Up
Benjamin Banneker Secondary School is planning a renovation to remodel their outdoor space.
1. Discuss: If your school wanted to renovate their outdoor space, what are some options you would suggest?
2. The principal thinks it would serve the school best if the renovation covered making new basketball courts or a new picnic area. Which option would you choose to build for your school? Circle your choice.
Basketball courts
Picnic area
Explain your thinking.
Unit 2 Lesson 12
233
Warm-Up

Remodel Choices

Making Decisions With Frequency Tables

Let's use data to make decisions and write arguments.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate values in relative frequency tables by row, column, or total.
2. **Goal:** Create relative frequency tables and two-way tables.
3. **Language Goal:** Analyze relative frequency tables to determine if there is an association between two variables. **(Reading, Writing, Speaking, and Listening)**

Students use two-way tables and relative frequency tables to write an argument about whether a school should build new basketball courts or a new picnic area. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 1, students explained the difference between quantitative and categorical data. In Lesson 11, students modeled data in two-way and relative frequency tables and looked for possible associations in the data. In Grade 8, Unit 6, Lessons 8–9, students represented data in two-way frequency tables and identified possible associations between variables.

Future Learning

In Lesson 13, students will make sense of dot plots and histograms as ways to visualize one-variable data.

Connections

- Students apply their understanding of associations between categories in two-way relative frequency tables to make decisions and write arguments regarding the best way to use money to renovate an outdoor space at their school. (real world, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.3.1
Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.8.DP.1.3

Vocabulary

Additional Vocabulary

total relative frequency table

Lesson at a Glance

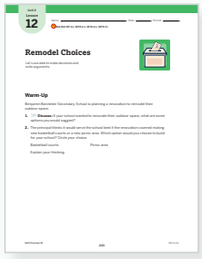
~ 45 min

MA.912.DP.3.1

Warm-Up

Pairs | 5 min

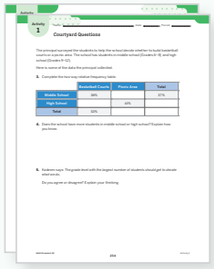
Students discuss options to improve a school's outdoor space and decide if they would prefer to build basketball courts or a picnic area. **Routine:** Think-Pair-Share



Activity 1

Pairs | 10 min

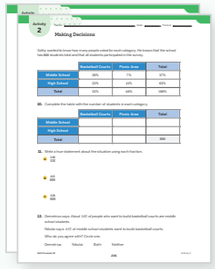
Students work with representations of survey data to decide if there is an association between grade level and preferring basketball courts or a picnic area. **(MTR.2.1, MTR.4.1)**



Activity 2

Pairs | 10 min

Students use a two-way frequency table to write and interpret statements about the situation. **(MTR.2.1, MTR.7.1)**



Activity 3

Pairs | 10 min

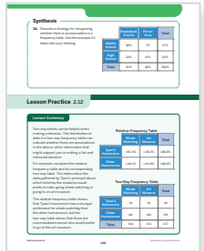
Students make an argument about which choice the school should make and apply that thinking to their own school context. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

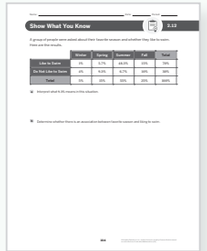
Students synthesize their strategies for recognizing whether there is an association in a frequency table. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting an entry in a two-way frequency table and determining if there is an association in the data.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students discuss options to improve a school's outdoor space and decide if they would prefer to build basketball courts or a picnic area.

1 Launch

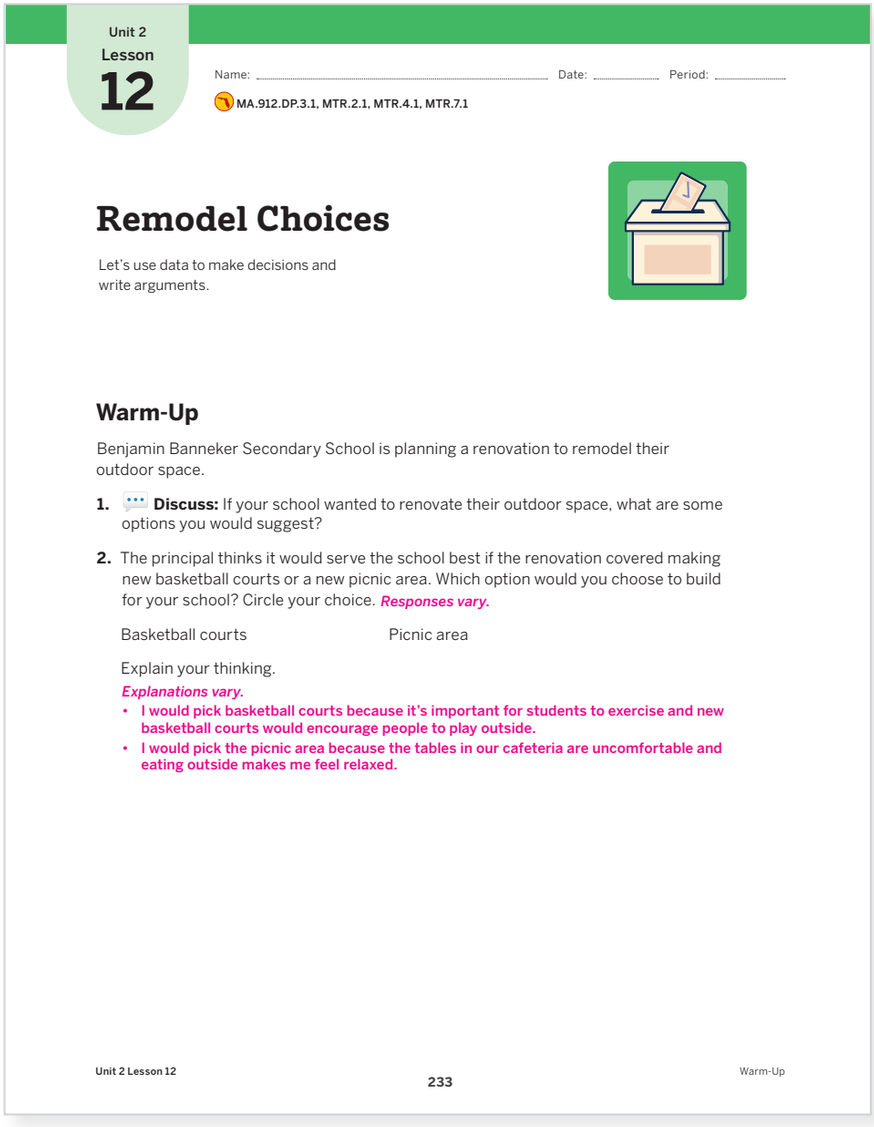
Display the prompt.

Consider asking, Why would they want to remodel their outdoor space? *Responses vary.* To be a more engaging, safe, and functional environment for students.

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's ideas with the class.

2 Connect

Invite students to raise their hands or physically move to different sides of the room to show how many would prefer each option. Consider asking one or two students with each preference to share their thinking.



Unit 2
Lesson
12

Name: _____ Date: _____ Period: _____

MA.912.DP.3.1, MTR.2.1, MTR.4.1, MTR.7.1

Remodel Choices

Let's use data to make decisions and write arguments.

Warm-Up

Benjamin Banneker Secondary School is planning a renovation to remodel their outdoor space.

1. **Discuss:** If your school wanted to renovate their outdoor space, what are some options you would suggest?
2. The principal thinks it would serve the school best if the renovation covered making new basketball courts or a new picnic area. Which option would you choose to build for your school? Circle your choice. ***Responses vary.***

   Basketball courts Picnic area

   Explain your thinking.

   Explanations vary.
   - **I would pick basketball courts because it's important for students to exercise and new basketball courts would encourage people to play outside.**
   - **I would pick the picnic area because the tables in our cafeteria are uncomfortable and eating outside makes me feel relaxed.**

Unit 2 Lesson 12 233 Warm-Up

Pairs | 10 min

Activity 1 Courtyard Questions

Purpose: Students work with representations of survey data to decide if there is an association between grade level and preferring basketball courts or a picnic area. **(MTR.2.1, MTR.4.1)**

1 Launch

Consider asking, "How might the principal collect data in order to figure out which to build?"

Display the data and the prompt.

2 Monitor

A **Accessibility: Memory and Attention** Invite students to use paper or a calculator to support them with performing calculations.

To support students getting started, consider demonstrating using a finger to trace down and across the table to help with interpreting each entry in terms of grade level and preference.

Look for students who:
- Identify the table as a relative frequency table.
- Share useful ideas and reminders about relative frequency tables (e.g., that the totals should sum to 100%).

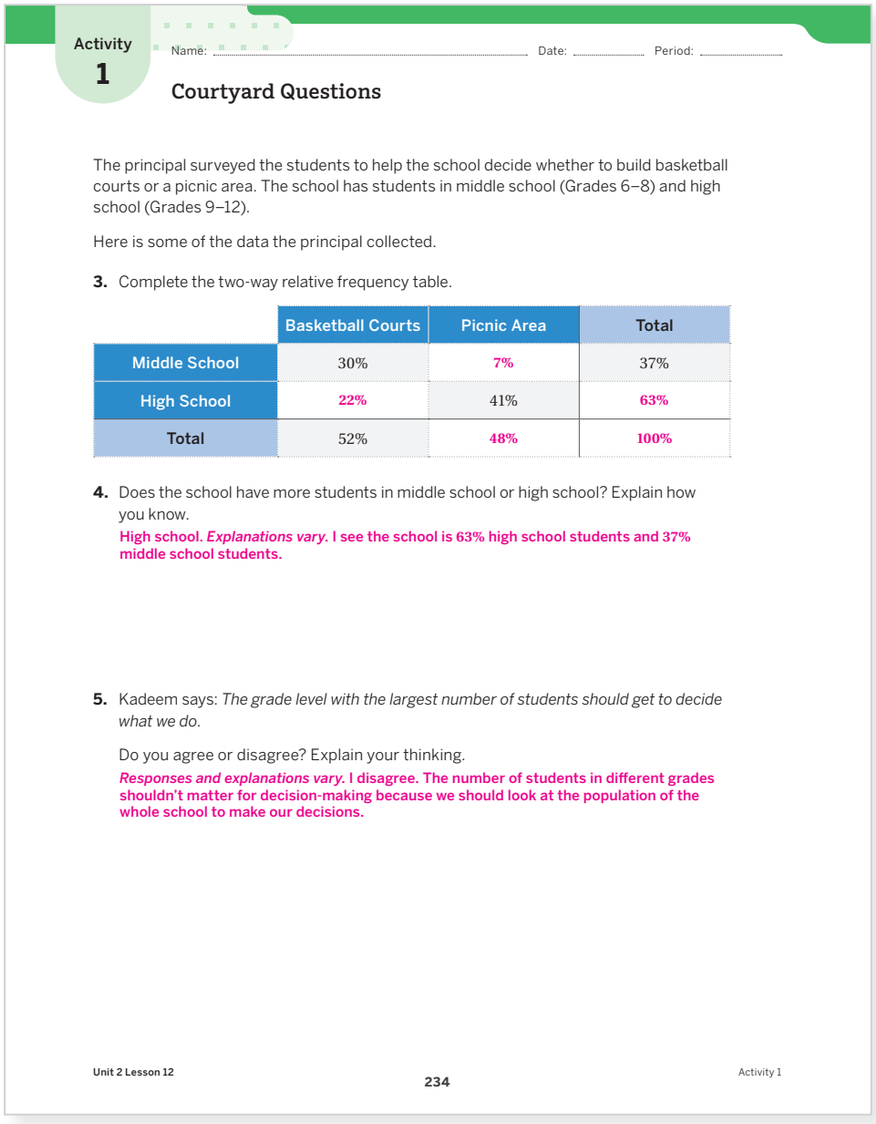
To surface connections, invite students to interpret what different numbers in the table mean about the students.

Note: A relative frequency table that includes information about totals (by row, column, or overall) is called a *total relative frequency table*. The total relative frequency tables in this lesson represent totals differently than the tables in Lesson 2. With only one "Total" column, the table could be represented like this:

	Basketball Courts	Picnic Area	Total
Middle School	~81.1%	~18.9%	100.0%
High School	~34.9%	~65.1%	100.0%

In this table, ~34.9% represents the percentage of high school students who prefer basketball courts. This corresponds to the 22% entry in the table in the Student Edition, since ~34.9% of high school students represent 22% of all students. Students will see another example of a table like this in the Lesson Summary.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Courtyard Questions

The principal surveyed the students to help the school decide whether to build basketball courts or a picnic area. The school has students in middle school (Grades 6–8) and high school (Grades 9–12).

Here is some of the data the principal collected.

3. Complete the two-way relative frequency table.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

4. Does the school have more students in middle school or high school? Explain how you know.

High school. *Explanations vary.* I see the school is 63% high school students and 37% middle school students.

5. Kadeem says: *The grade level with the largest number of students should get to decide what we do.*

Do you agree or disagree? Explain your thinking.

Responses and explanations vary. I disagree. The number of students in different grades shouldn't matter for decision-making because we should look at the population of the whole school to make our decisions.

Unit 2 Lesson 12 234 Activity 1

Pairs | 10 min

Activity 1 Courtyard Questions (continued)

Purpose: Students work with representations of survey data to decide if there is an association between grade level and preferring basketball courts or a picnic area. **(MTR.2.1, MTR.4.1)**

2 Monitor

Select and sequence strategies for comparing frequencies and deciding if there is an association to share during the Connect.

D Differentiation *(Problems 6–9)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problems 6–7)*	**Support** Encourage them to choose one entry in the table that they think might be helpful to start. Consider asking, "What does this entry represent? How do you know?"
Think that there is no association, or think that there is not enough information to decide.	**Support** Consider asking, "If there was *no* association, what would you expect the data to look like? Does this data look like that?" *Responses vary.* If there was no association, the percents would be approximately the same for each grade level. This data has significant differences between the percents.
Make a claim for Problem 9 that is supported by evidence in the table.	**Strengthen** Celebrate their claim and invite them to share it during the Connect.
Would like to explore this topic further.	**Stretch** Consider asking, "How could you represent this data in a different way?"

3 Connect

Invite students to share strategies for comparing frequencies that they used for Problems 8–9.

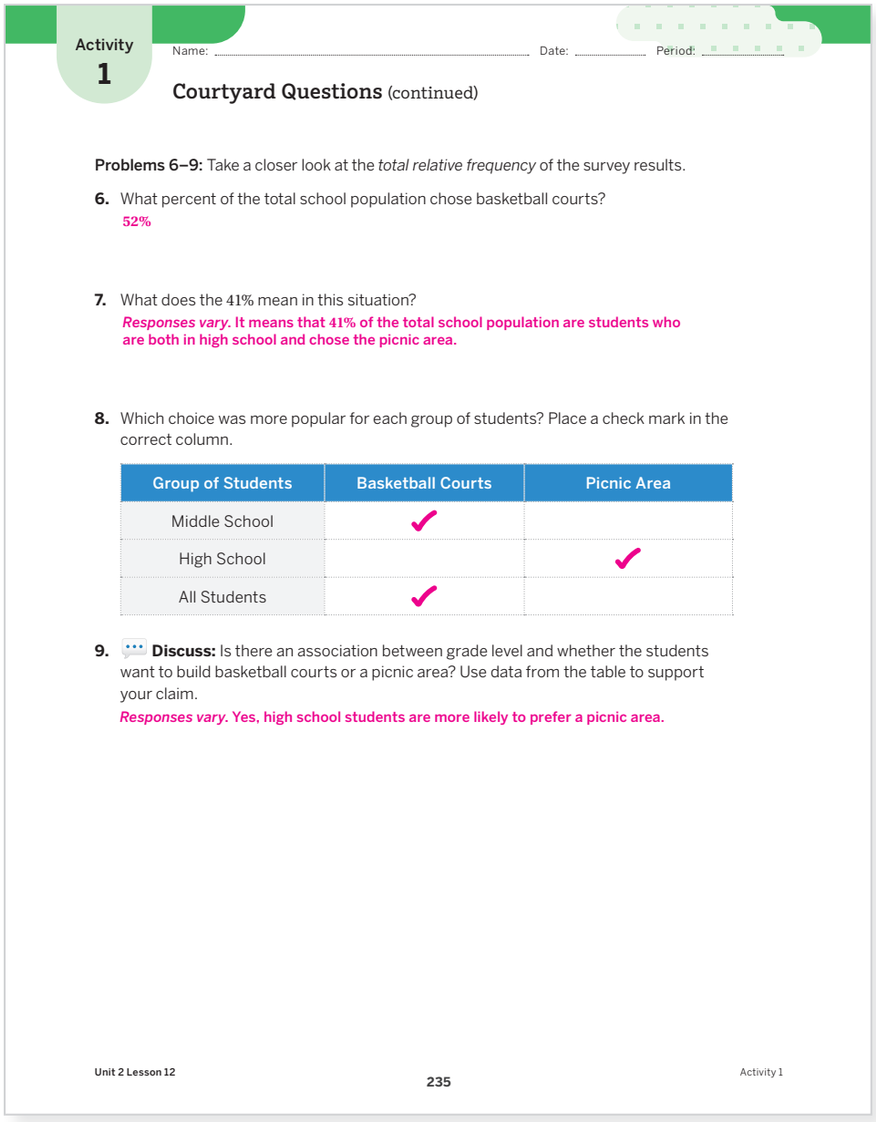
ELL English Language Learners As students share in the whole-class discussion, provide sentence frames such as:

- I know this strategy works because . . .
- _____'s idea makes sense to me because . . .
- Another strategy would be _____ because . . .

Encourage students to refer to specific entries in the table to explain their strategies. **(Writing and Speaking)**

To surface the Key Takeaway, consider asking, "How did you decide if there was an association between grade level and whether the students want to build basketball courts or a picnic area?"

Note: If time allows, consider asking how students would respond differently to Kadeem from Problem 5 after hearing other students' thinking about the association in Problem 9.



Activity 1

Name: _____ Date: _____ Period: _____

Courtyard Questions (continued)

Problems 6–9: Take a closer look at the *total relative frequency* of the survey results.

6. What percent of the total school population chose basketball courts?

52%

7. What does the 41% mean in this situation?

Responses vary. It means that 41% of the total school population are students who are both in high school and chose the picnic area.

8. Which choice was more popular for each group of students? Place a check mark in the correct column.

Group of Students	Basketball Courts	Picnic Area
Middle School	✓	
High School		✓
All Students	✓	

9. **Discuss:** Is there an association between grade level and whether the students want to build basketball courts or a picnic area? Use data from the table to support your claim.

Responses vary. Yes, high school students are more likely to prefer a picnic area.

Unit 2 Lesson 12 235 Activity 1

Key Takeaway: Sometimes making a decision that affects a whole population isn't as simple as counting how many people prefer each option. There can be associations between subgroups of a population and preferences for the decision that could be important to consider.

Pairs | 10 min

Activity 2 Making Decisions

Purpose: Students use a two-way frequency table to write and interpret statements about the situation. **(MTR.2.1, MTR.7.1)**

Short on time? Consider inviting students to interpret two of the three fractions in Problem 11.

1 Launch

Display the prompt and the accompanying image.

Invite students to share why they think it might be helpful to see how many students voted for each choice.

2 Monitor

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one table entry at a time. Encourage students to use paper or their hands to cover up extra information so it is easier to focus on one calculation at a time.

Select and sequence strategies for writing true statements from the table to share during the Connect.

ELL **English Language Learners** As students write their statements, provide sentence frames such as:

- The numerator represents _____.
- The denominator represents _____.

(Writing and Speaking)

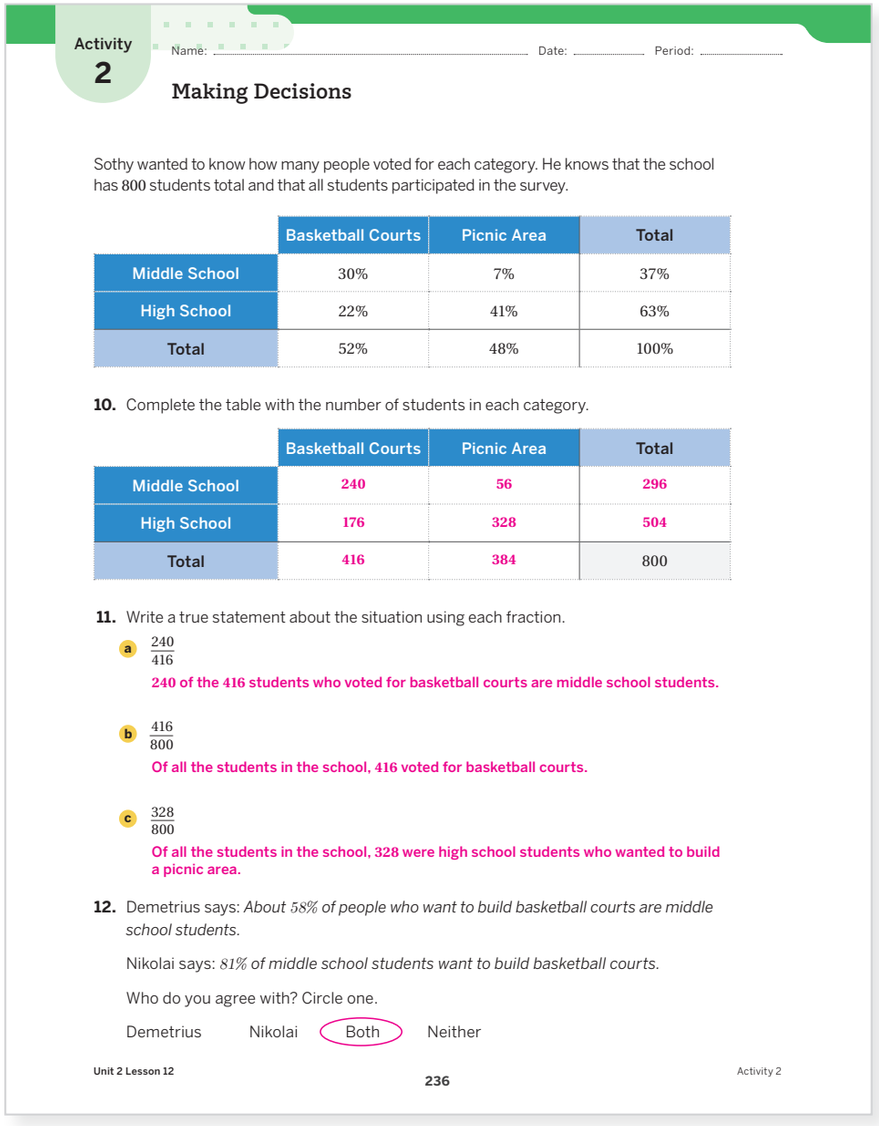
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to focus on part of the fraction. Before students write, ask them, "What does the numerator represent? The denominator?"
Disagree with Demetrius, Nikolai, or both.	**Support** Consider asking, "Are the wholes in Demetrius's and Nikolai's percentages the same, or different? How could you write them as fractions?"
Write true statements in Problem 11 that describe the numerator and denominator in terms of the table.	**Strengthen** Celebrate their statements and encourage them to think of another true statement about the same fraction. Invite them to share during the Connect.

3 Connect

Invite students to share their true statements and strategies from Problem 11.

Consider asking, "Could you write Demetrius's and Nikolai's percentages as fractions? How could that help you decide if you agree with them or not?"



Activity 2

Name: _____ Date: _____ Period: _____

Making Decisions

Sothy wanted to know how many people voted for each category. He knows that the school has 800 students total and that all students participated in the survey.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

10. Complete the table with the number of students in each category.

	Basketball Courts	Picnic Area	Total
Middle School	240	56	296
High School	176	328	504
Total	416	384	800

11. Write a true statement about the situation using each fraction.

a. $\frac{240}{416}$

240 of the 416 students who voted for basketball courts are middle school students.

b. $\frac{416}{800}$

Of all the students in the school, 416 voted for basketball courts.

c. $\frac{328}{800}$

Of all the students in the school, 328 were high school students who wanted to build a picnic area.

12. Demetrius says: *About 58% of people who want to build basketball courts are middle school students.*

Nikolai says: *81% of middle school students want to build basketball courts.*

Who do you agree with? Circle one.

Demetrius Nikolai Both Neither

Unit 2 Lesson 12 236 Activity 2

Pairs | 10 min

Activity 3 Closing Arguments

Purpose: Students make an argument about which choice the school should make and apply that thinking to their own school context. **(MTR.4.1)**

1 Launch

Display the prompt and the accompanying image.

Invite students to brainstorm and share other reasons besides data that might influence someone's choice.

2 Monitor

D **Differentiation: (Support)** To support students getting started, encourage them to choose one piece of data from the table they will use in their argument. Consider asking, "Which choice is supported by this piece of data? Why?"

Look for students with creative or unexpected decisions and explanations in their response to Problem 13 and invite them to share during the Connect.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

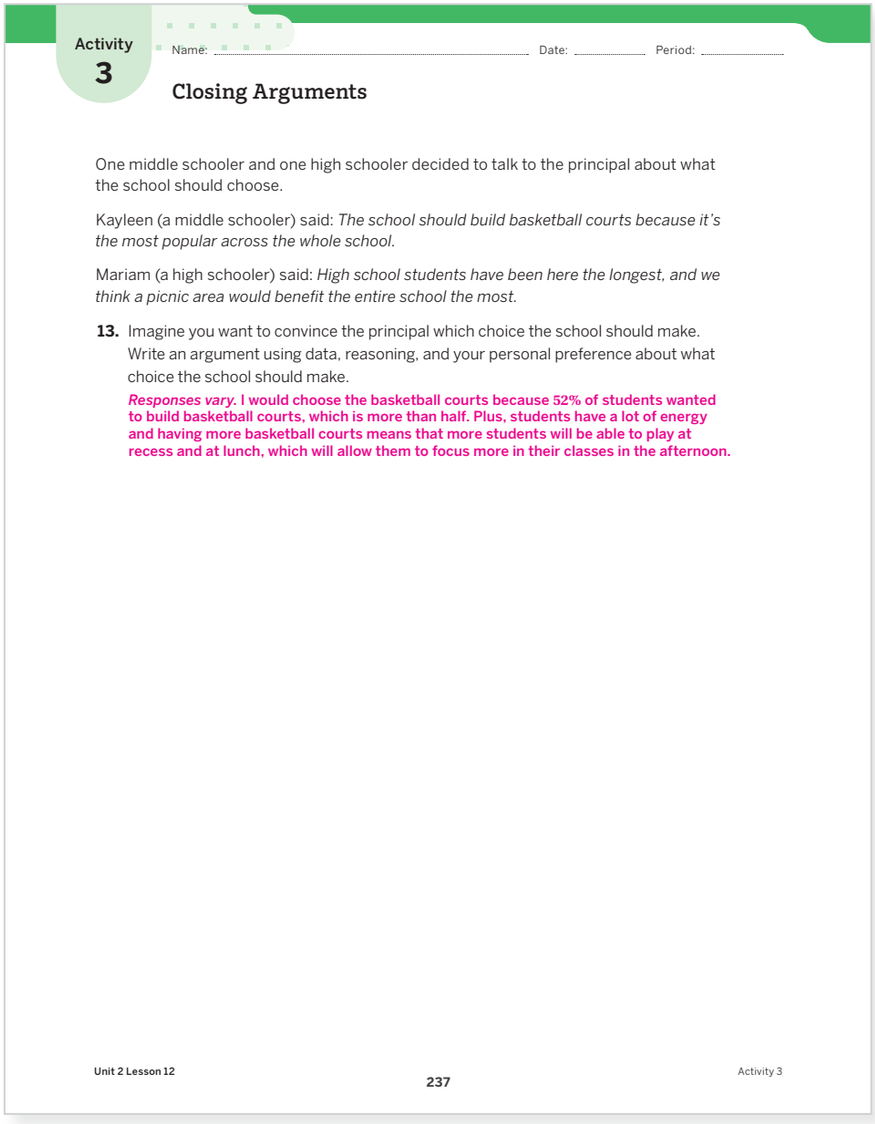
3 Connect

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Invite students to share their arguments about which choice the school should make.

To surface the Key Takeaway, consider asking:

- "How might your recommendation about the decision seem fair or unfair to different people involved?"
- "What decision at our school could a two-way frequency table help you to make?" **(MTR.4.1)**



Activity 3

Name: _____ Date: _____ Period: _____

Closing Arguments

One middle schooler and one high schooler decided to talk to the principal about what the school should choose.

Kayleen (a middle schooler) said: *The school should build basketball courts because it's the most popular across the whole school.*

Mariam (a high schooler) said: *High school students have been here the longest, and we think a picnic area would benefit the entire school the most.*

13. Imagine you want to convince the principal which choice the school should make. Write an argument using data, reasoning, and your personal preference about what choice the school should make.

Responses vary. **I would choose the basketball courts because 52% of students wanted to build basketball courts, which is more than half. Plus, students have a lot of energy and having more basketball courts means that more students will be able to play at recess and at lunch, which will allow them to focus more in their classes in the afternoon.**

Unit 2 Lesson 12 237 Activity 3

Key Takeaway: Two-way frequency tables can help to organize preference data to support decision makers in fairly representing the interests of the different groups of people affected by the decision.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for recognizing whether there is an association in a frequency table.

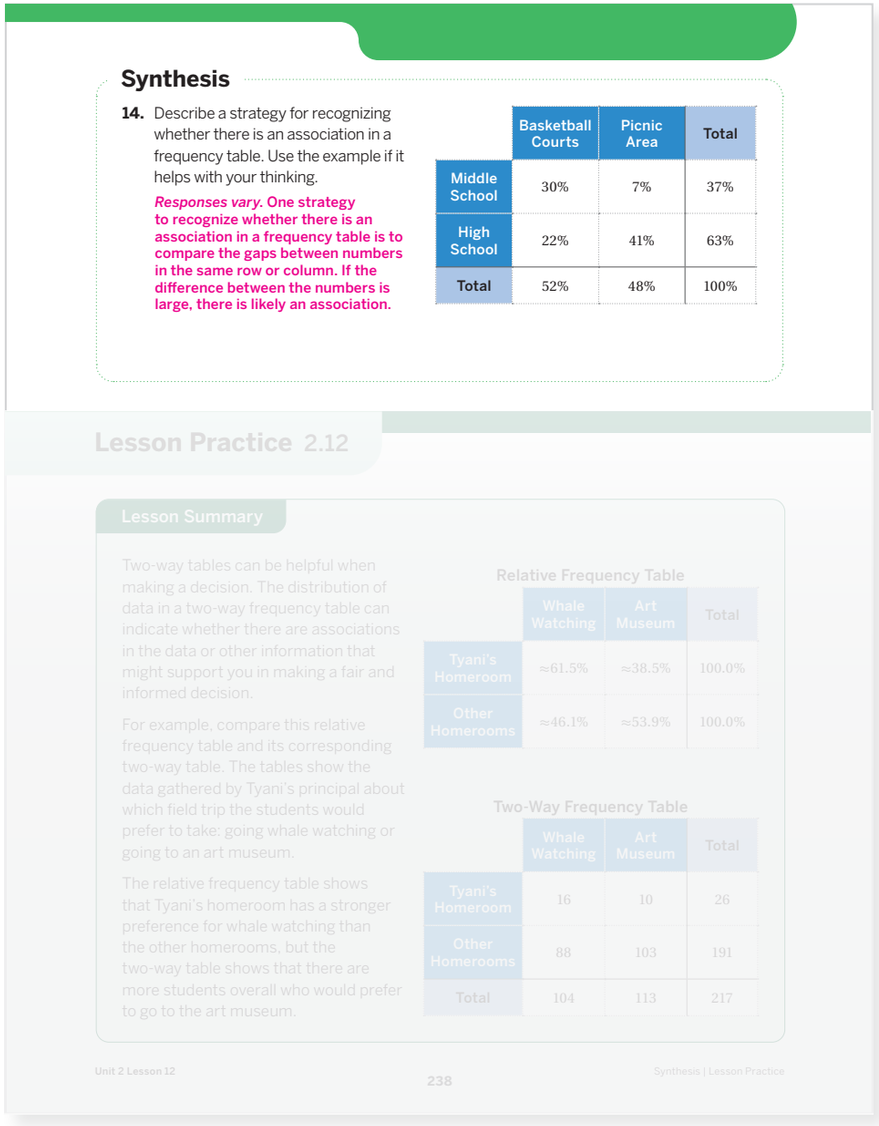
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Comparing the gaps between percents in the same row or column and seeing if there is a large difference.
- Converting a table with numbers into percents to make it easier to compare across different-sized groups.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for determining if there is an association as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

14. Describe a strategy for recognizing whether there is an association in a frequency table. Use the example if it helps with your thinking.

Responses vary. One strategy to recognize whether there is an association in a frequency table is to compare the gaps between numbers in the same row or column. If the difference between the numbers is large, there is likely an association.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

Lesson Practice 2.12

Lesson Summary

Two-way tables can be helpful when making a decision. The distribution of data in a two-way frequency table can indicate whether there are associations in the data or other information that might support you in making a fair and informed decision.

For example, compare this relative frequency table and its corresponding two-way table. The tables show the data gathered by Tyani's principal about which field trip the students would prefer to take: going whale watching or going to an art museum.

The relative frequency table shows that Tyani's homeroom has a stronger preference for whale watching than the other homerooms, but the two-way table shows that there are more students overall who would prefer to go to the art museum.

Relative Frequency Table

	Whale Watching	Art Museum	Total
Tyani's Homeroom	≈61.5%	≈38.5%	100.0%
Other Homerooms	≈46.1%	≈53.9%	100.0%

Two-Way Frequency Table

	Whale Watching	Art Museum	Total
Tyani's Homeroom	16	10	26
Other Homerooms	88	103	191
Total	104	113	217

Unit 2 Lesson 12 238 Synthesis | Lesson Practice

Lesson Takeaway: Decision making is a complex process informed by some considerations that appear in data, like what percent of people prefer a specific outcome, and some that can't be reduced to data, like ideas of fairness.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting an entry in a two-way frequency table and determining if there is an association in the data.

Today's Goals

1. **Goal:** Calculate values in relative frequency tables by row, column, or total.
2. **Goal:** Create relative frequency tables and two-way tables.
3. **Language Goal:** Analyze relative frequency tables to determine if there is an association between two variables. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.12

A group of people were asked about their favorite season and whether they like to swim. Here are the results.

	Winter	Spring	Summer	Fall	Total
Like to Swim	1%	5.7%	48.3%	15%	70%
Do Not Like to Swim	4%	9.3%	6.7%	10%	30%
Total	5%	15%	55%	25%	100%

a. Interpret what 9.3% means in this situation.

Responses vary. 9.3% is the percentage of respondents whose favorite season is spring and who do not like to swim.

b. Determine whether there is an association between favorite season and liking to swim.

Responses vary. There is an association between favorite season and liking to swim because 48.3% of respondents like to swim and picked summer as their favorite season.

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support interpreting two-way/relative frequency tables and using them to determine if there is an association:

» **Respond to Student Thinking:**

- Use the *Calculating Values in Two-Way and Relative Frequency Tables* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

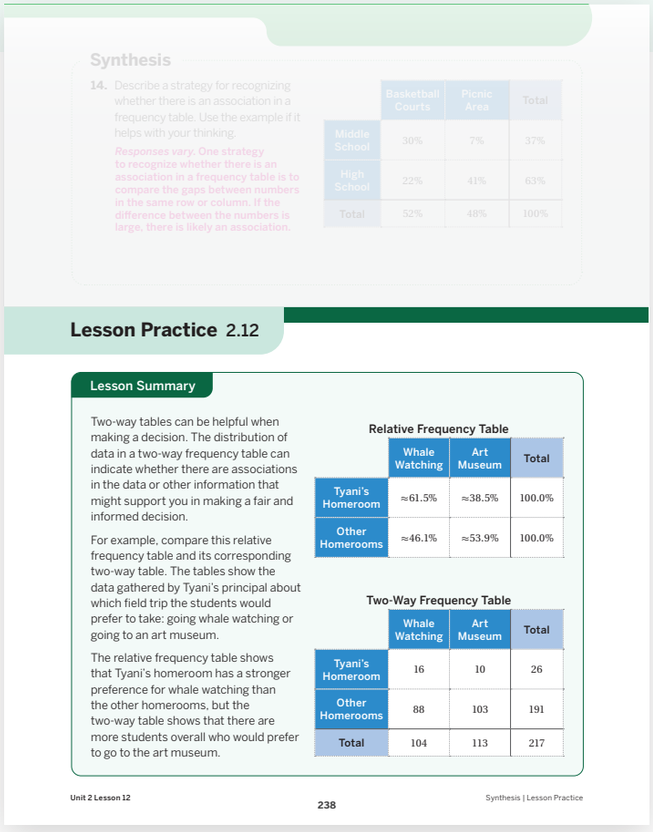
Professional Learning

In this lesson students explored the topic of decision making supported by data. What aspects of decision making in your school or local context might be connected to this work? How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

14. Describe a strategy for recognizing whether there is an association in a frequency table. Use the example if it helps with your thinking.

Responses vary. One strategy to recognize whether there is an association in a frequency table is to compare the gaps between numbers in the same row or column. If the difference between the numbers is large, there is likely an association.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
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Total	52%	48%	100%

Lesson Practice 2.12

Lesson Summary

Two-way tables can be helpful when making a decision. The distribution of data in a two-way frequency table can indicate whether there are associations in the data or other information that might support you in making a fair and informed decision.

For example, compare this relative frequency table and its corresponding two-way table. The tables show the data gathered by Tyani's principal about which field trip the students would prefer to take: going whale watching or going to an art museum.

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Relative Frequency Table

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Two-Way Frequency Table

	Whale Watching	Art Museum	Total
Tyani's Homeroom	16	10	26
Other Homerooms	88	103	191
Total	104	113	217

Lesson Practice 2.12

Name: _____ Date: _____ Period: _____

1. A group of student athletes set personal goals for a big race. After the race, they were asked whether they achieved their goal and whether they practiced daily.

   This table shows the results. What does 35% represent?

   Responses vary. 35% represents the students who practiced daily and achieved their personal goal.

	Daily Practice	No Daily Practice
Achieved Goal	35%	14%
Did Not Achieve Goal	11%	40%

2. A fashion magazine asked 250 people whether they wear a belt and a watch each day. Use the data in the relative frequency table to complete the frequency table.

Relative Frequency

	Watch	No Watch
Belt	44%	22%
No Belt	20%	14%

Frequency

	Watch	No Watch
Belt	110	55
No Belt	50	35

Problems 3–4: A group of people were asked whether they like oranges and whether they like granola bars. This two-way frequency table shows the results.

	Like Oranges	Dislike Oranges	Total
Like Granola Bars	28	30	58
Dislike Granola Bars	23	38	61
Total	51	68	119

3. What percent of people who like oranges also like granola bars?
   54.9%

4. What percent of people who dislike oranges also dislike granola bars?
   44.1%

Lesson Practice 2.12

Name: _____ Date: _____ Period: _____

Problems 5–6: A random sample of people were asked about their preferences in home decoration and their interest in fashion.

5. Use this information to complete the two-way frequency table.
   - 150 people completed the survey.
   - 70% of the people do not enjoy fashion.
   - 33 of the people who prefer neutral decorations also do not enjoy fashion.
   - 20 of the people who enjoy fashion also prefer colorful decorations in their homes.

	Prefer Colorful Decorations	Prefer Neutral Decorations	Total
Enjoy Fashion	20	25	45
Don't Enjoy Fashion	72	33	105
Total	92	58	150

6. Create a total relative frequency table for this data.

	Prefer Colorful Decorations	Prefer Neutral Decorations	Total
Enjoy Fashion	13.3%	16.7%	30%
Don't Enjoy Fashion	48%	22%	70%
Total	61.3%	38.7%	100%

Test Practice

7. A group of people were asked whether they have any siblings and whether they have any pets.

   What percent of people have no siblings and no pets? 18%

	Have Siblings	Have No Siblings
Have Pets	20.5%	26%
Have No Pets	35.5%	?

Spiral Review

Problems 8–9: Which has a different solution? Circle your choice.

8. $4(x-3)+2=2(x+5)+10$ $2(3x-5)-8=4x+12$ $2(x+5)-3=3(x-1)+9$
9. $3(2y-5)+4 <= 2y+11$ $4(y-3)+6 \le 2(y+2)+1$ $2(y+4)-5 \ge 3y+7$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.DP.3.1
	2–4	2	MA.912.DP.3.1
	5–6	2	MA.912.DP.3.1
Test Practice	7	2	MA.912.DP.3.1
Spiral Review			
Fluency	8–9	1	MA.8.AR.2.1, MA.8.AR.2.2

Unit 2

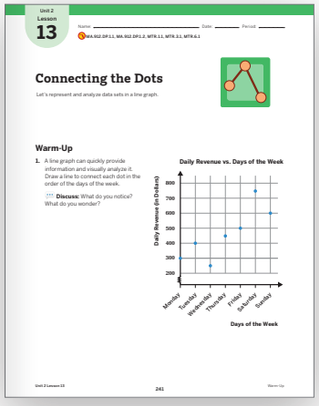
Lesson

13

Connecting the Dots

Numerical Bivariate Data

Let's represent and analyze data sets in a line graph.


Unit 2
Lesson
13

Connecting the Dots

Let's represent and analyze data sets in a line graph.

Warm-Up

1. A line graph can quickly provide information and visually analyze it. Draw a line to connect each dot in the order of the days of the week.

   Discuss: What do you notice? What do you wonder?

Daily Revenue vs. Days of the Week

Daily Revenue (in Dollars)

800
700
600
500
400
300
200

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday

Days of the Week

Unit 2 Lesson 13

241

Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Identify components and quantities displayed in line graphs.
2. **Goal:** Understand that line graphs show trends and patterns in numerical bivariate data sets.
3. **Language Goal:** Summarize and analyze data displayed in a line graph for trends. **(Reading and Speaking)**

Students use components of line graphs to answer questions about trends and patterns in different types of datasets. They will determine when line graphs are the best type of visualization and then create the representation.

Coherence

Prior Learning

In Lessons 11 and 12, students work with two-way tables and frequency tables to assist in making decisions.

Future Learning

In Lesson 14, students extend their learning of line graphs to include interpreting the slope and y-intercept.

Connections

- Students identify trends and analyze relationships between speed and reaction time of different animals using line graphs, as well as other real-world contexts. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.1
Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.

MA.912.DP.1.2
Interpret data distributions represented in various ways. State whether the data is numerical or categorical and whether it is univariate or bivariate, and interpret the different components and quantities in the display.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.6.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.5	**MA.912.DP.2.2**
MA.7.DP.1.5	**MA.912.DP.2.4**
MA.8.DP.1.1	**MA.912.DP.2.6**
	MA.912.DP.6.6

Vocabulary

Review Vocabulary

bivariate	*numerical data*
categorical data	*univariate*

Lesson at a Glance

~ 45 min

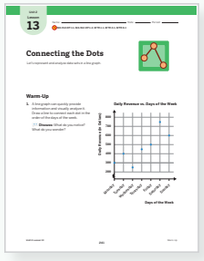
MA.912.DP.1.1, MA.912.DP.1.2

Warm-Up

Whole Class | 5 min

Students explore different types of information that can be presented visually in a line graph.

Routine(s): Notice and Wonder

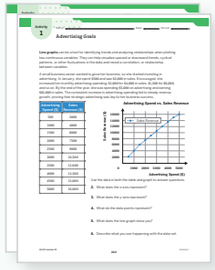


Activity 1

Small Group | 10 min

Students use data from a scenario and a table to understand how the components of line graphs are derived from the data.

Routine(s): MLR6: Three Reads (MTR.1.1)

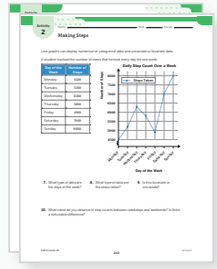


Activity 2

Small Group | 10 min

Students explore how different types of data can be presented in graphs.

Routine(s): MLR5: Co-Craft Questions (MTR.1.1, MTR.4.1)

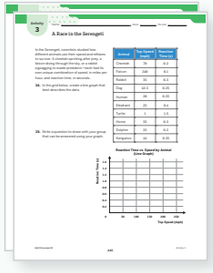


Activity 3

Pairs | 10 min

Students graph numerical bivariate data and critique similar graphs.

Routine(s): MLR3: Critique, Correct, Clarify (MTR.3.1, MTR.6.1)

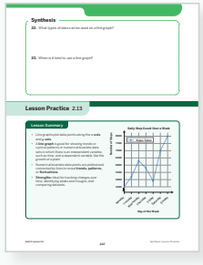


Synthesis

Whole Class | 5 min

Students synthesize their understanding of strategies for creating and analyzing line graphs from different types of data sets.

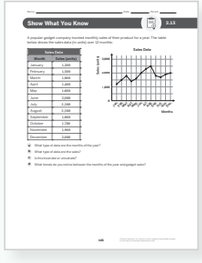
Routine(s): MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying type of data represented in a line graph, as well as trends noticed.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students explore different types of information that can be presented visually in a line graph.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of finding meaning about data on a line graph. **(MTR.1.1)**

Consider asking, "Did anyone notice a low point in the graph? Does this make you wonder why sales are high over the weekend?"

A **Accessibility: Fine Motor Skills** Consider varying the task demands by providing straightedges.

2 Connect

Ask questions about what students think about the graph and its data:

- What would you do differently?
- Is there anything confusing to you?
- What makes the most sense?
- How does a visualization of data help you see trends and patterns?

Unit 2
Lesson
13

Name: _____ Date: _____ Period: _____

MA.912.DP.1.1, MA.912.DP.1.2, MTR.1.1, MTR.3.1, MTR.6.1

Connecting the Dots

Let's represent and analyze data sets in a line graph.



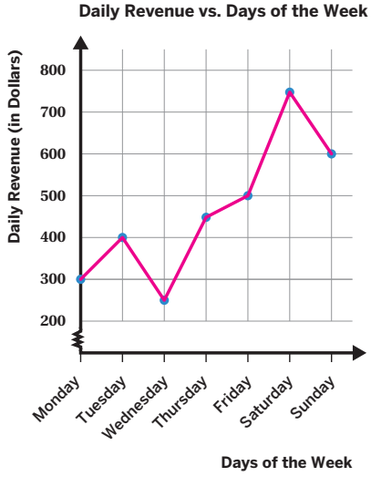
Warm-Up

1. A line graph can quickly provide information and visually analyze it. Draw a line to connect each dot in the order of the days of the week.

Discuss: What do you notice? What do you wonder?

Responses vary.

- This is one week of revenue data, Monday through Sunday.
- Sales range from $300 to about $750.
- Monday and Wednesday are the lowest days.
- There is an increase from Wednesday to Saturday.
- I wonder what happens on the weekend.
- I wonder why Wednesday is so low.



Unit 2 Lesson 13 241 Warm-Up

Small Group | 10 min

Activity 1 Advertising Goals

Purpose: Students use data from a scenario and a table to understand how the components of line graphs are derived from the data. **(MTR.1.1)**

Short on time? Consider providing the graph with dots already connected.

1 Launch

MLR **MLR6: Three Reads** Consider inviting students to read the scenario to themselves multiple times, first focusing on what the situation is about, then what the quantities are, and finally the question, to support them in making sense of the data and how it connects to the graph and table. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

Ask students what they know about business advertising, sales, and revenue to build an understanding of the context.

ELL **English Language Learners** Create a list of words about the context that students might not know, along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

2 Monitor

Consider inviting students to use different-color pens to highlight or circle data points and terms that relate to advertising costs versus revenue to draw connections between story, table, and line graph.

Capture ideas to share during the Connect.

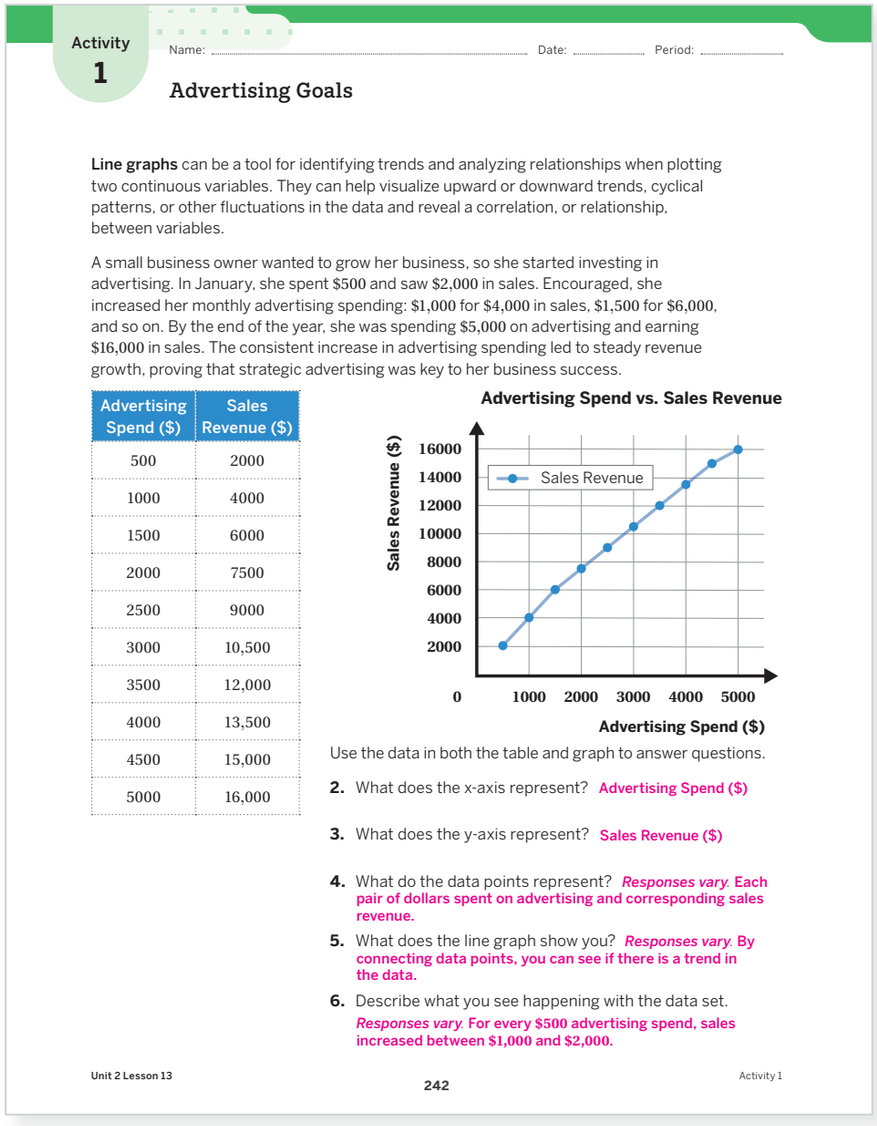
D **Differentiation** *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Struggle to label and identify the axis. *(Problems 2 & 3)*	**Strengthen** Invite them to remember working with a coordinate plane and draw attention to the similarities.
Don't understand where the data comes from for points. *(Problem 4)*	**Strengthen** Demonstrate labeling one of the coordinates and then have students complete other points. Encourage them to see the connection between table rows and points on the graph.

Look for explanations to share about the analysis of the data graph.

3 Connect

To surface the Key Takeaway, ask students to draw lines between the data in the scenario, the data in the table, and the content in the graph.



Activity 1

Name: Date: Period:

Advertising Goals

Line graphs can be a tool for identifying trends and analyzing relationships when plotting two continuous variables. They can help visualize upward or downward trends, cyclical patterns, or other fluctuations in the data and reveal a correlation, or relationship, between variables.

A small business owner wanted to grow her business, so she started investing in advertising. In January, she spent $500 and saw $2,000 in sales. Encouraged, she increased her monthly advertising spending: $1,000 for $4,000 in sales, $1,500 for $6,000, and so on. By the end of the year, she was spending $5,000 on advertising and earning $16,000 in sales. The consistent increase in advertising spending led to steady revenue growth, proving that strategic advertising was key to her business success.

Advertising Spend ($)	Sales Revenue ($)
500	2000
1000	4000
1500	6000
2000	7500
2500	9000
3000	10,500
3500	12,000
4000	13,500
4500	15,000
5000	16,000

Use the data in both the table and graph to answer questions.

2. What does the x-axis represent? Advertising Spend ($)
3. What does the y-axis represent? Sales Revenue ($)
4. What do the data points represent? *Responses vary.* Each pair of dollars spent on advertising and corresponding sales revenue.
5. What does the line graph show you? *Responses vary.* By connecting data points, you can see if there is a trend in the data.
6. Describe what you see happening with the data set. *Responses vary.* For every $500 advertising spend, sales increased between $1,000 and $2,000.

Unit 2 Lesson 13 242 Activity 1

Key Takeaway: A scenario or table provides the components of a line graph, including axis labels, intervals, and plotted points.

Small Group | 10 min

Activity 2 Making Steps

Purpose: Students explore how different types of data can be presented in graphs. **(MTR.1.1, MTR.4.1)**

Short on time? Consider eliminating Problems 10 through 13 for the second scenario.

1 Launch

ELL **MLR5: Co-Craft Questions:** Use this routine to help students make sense of a context by asking questions without the pressure to produce answers. **(MTR.1.1, MTR.4.1)**

Consider asking, "What is the difference between data types: numerical, categorical, bivariate, and univariate?"

2 Monitor

Capture different explanations of data trends and patterns to share during the Connect.

D **Differentiation:** Give students flash cards of the different data types with examples and graphical representations.

Consider giving students straightedges and different colors to annotate the graphs, label points, and draw arrows between the content and where it is on the graph.

Activity 2 continued >

Activity 2

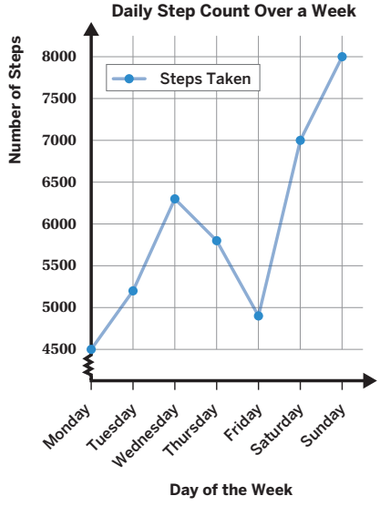
Name: Date: Period:

Making Steps

Line graphs can display numerical or categorical data and univariate or bivariate data.

A student tracked the number of steps that he took every day for one week.

Day of the Week	Number of Steps
Monday	4500
Tuesday	5200
Wednesday	6300
Thursday	5800
Friday	4900
Saturday	7000
Sunday	8000



7. What type of data are the days of the week?
Categorical

8. What type of data are the steps taken?
Numerical

9. Is this bivariate or univariate?
Univariate

10. What trend do you observe in step counts between weekdays and weekends? Is there a noticeable difference?
Responses vary. Weekend steps are higher. One possibility is that there is more time to be active.

Unit 2 Lesson 13 243 Activity 2

Activity 2 Making Steps (continued)

Purpose: Students explore how different types of data can be presented in graphs. **(MTR.1.1, MTR.4.1)**

Short on time? Consider eliminating Problems 10 through 13 for the second scenario.

3 Connect

A **Accessibility: Executive Functioning** Invite students to try different scenarios and record what they notice.

To surface the Key Takeaway, create Venn diagrams to show three types of graphs and things that make them similar, make them different, and their strengths.



Activity 2

Name: Date: Period:

Making Steps (continued)

Problems 11–13. A college surveyed students about their degree of study and career path. Use this graph to answer these questions.

11. What type of data is represented in this segmented bar chart?
Bivariate Categorical

12. What does this representation help you to visualize regarding the relationship between the data?
Responses vary. How career distributions vary by degree of study.

13. Would this data be better represented as a line graph? Why or why not?
Responses vary. No, line graphs show trends between numerical bivariate data over time.

Unit 2 Lesson 13 244 Activity 2

Key Takeaway: Different data types need different graphs to present a visualization and support analysis.

Pairs | 10 min

Activity 3 A Race in the Serengeti

Purpose: Students graph numerical bivariate data and critique similar graphs. **(MTR.3.1, MTR.6.1)**

Short on time? Consider providing a graph with some data already on it.

1 Launch

A **Accessibility: Fine Motor Skills** Consider varying the task demands by providing straightedges and multiple copies of grid paper.

2 Monitor

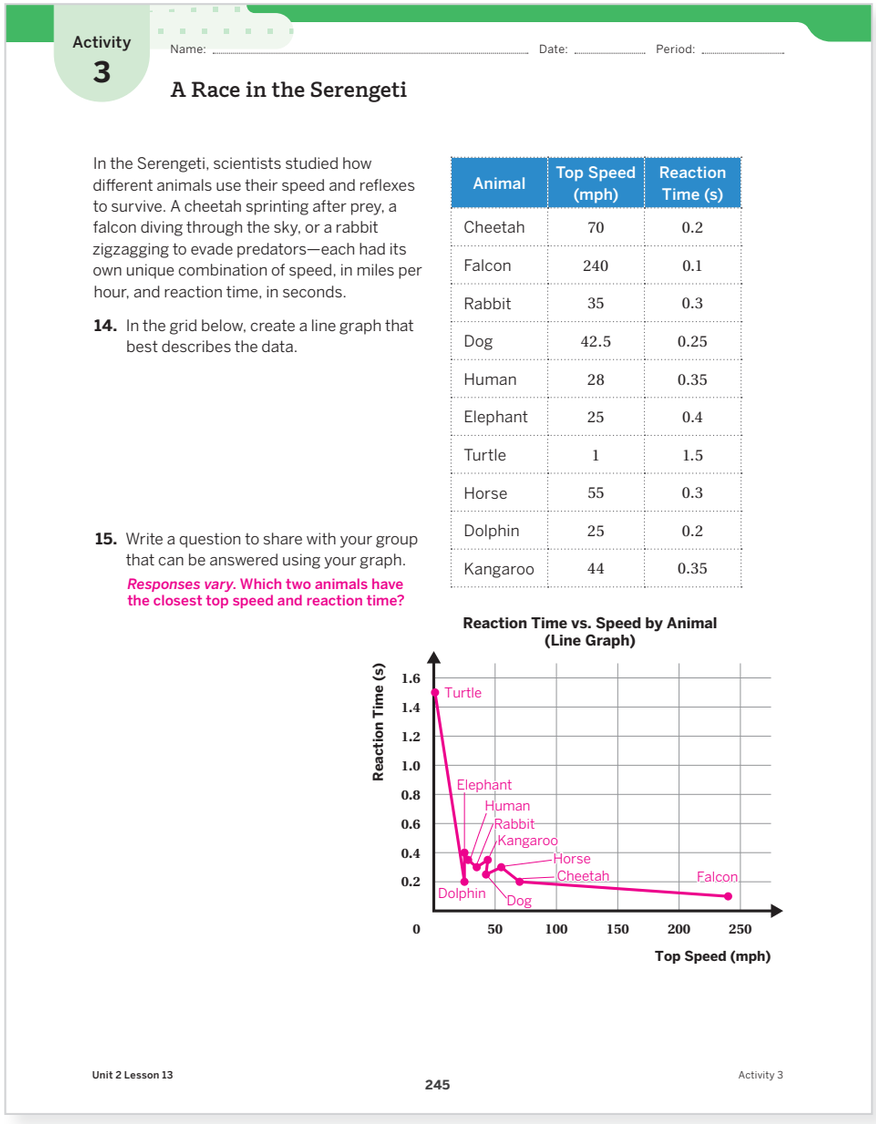
Select and sequence strategies to create a graph to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Don't see the intervals for the x- and y-axis.	**Strengthen** Invite to start with the range of the data set $(0 - 2)$ and $(0 - 240)$. Then look at the data points in the middle to see what fits best. (tenths, tens)
Have difficulties plotting the numbers on a graph.	**Strengthen** Give them a cover sheet with a rectangle cut in it for a window to see one data set at a time. Draw an X over one column and a Y over the other.
Struggle to ask a question about the data.	**Stretch** Consider asking, "What are two of your favorite animals? What do you notice about their speed and reactions? How do they compare to the whole?"

Note: If time allows, allow students to answer the questions about each other's graphs.

Activity 3 continued >



Activity 3

Name: Date: Period:

A Race in the Serengeti

In the Serengeti, scientists studied how different animals use their speed and reflexes to survive. A cheetah sprinting after prey, a falcon diving through the sky, or a rabbit zigzagging to evade predators—each had its own unique combination of speed, in miles per hour, and reaction time, in seconds.

Animal	Top Speed (mph)	Reaction Time (s)
Cheetah	70	0.2
Falcon	240	0.1
Rabbit	35	0.3
Dog	42.5	0.25
Human	28	0.35
Elephant	25	0.4
Turtle	1	1.5
Horse	55	0.3
Dolphin	25	0.2
Kangaroo	44	0.35

14. In the grid below, create a line graph that best describes the data.

15. Write a question to share with your group that can be answered using your graph.

Responses vary. Which two animals have the closest top speed and reaction time?

Unit 2 Lesson 13 245 Activity 3

Pairs | 10 min

Activity 3 A Race in the Serengeti (continued)

Purpose: Students graph numerical bivariate data and critique similar graphs. **(MTR.3.1, MTR.6.1)**

2 Monitor

ELL **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in both Daniella and Makayla's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.3.1, MTR.6.1)**

Select and sequence how other students created graphs differently.

D **Differentiation: (Support)** Consider asking, "How does a table or graph help you interpret and analyze a dataset?"

Note: If time allows, ask students to answer the questions about Daniella's and Makayla's graphs.

3 Connect

Invite the students to:

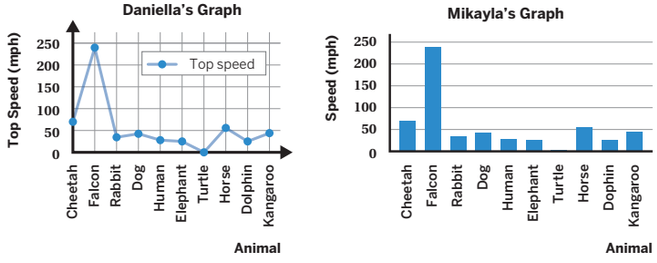
- Discuss what is *correct* about their responses.
- Discuss what is *incorrect* or *unclear*.
- Propose *correct and clear* solutions.

Activity 3

Name: Date: Period:

A Race in the Serengeti (continued)

For Problems 16–19, use the different graphs made by Daniella and Makayla representing some of the same data set.



16. What could be the title of each graph? Top Speed of Animals

17. Which animal is the second fastest? Cheetah

18. Which animals are outliers for speed? Turtle and Falcon

19. Which graph is a better choice for this data set? Why or why not? *Responses vary.*
The bar graph is a better representation because this is univariate categorical data. A line graph of univariate categorical data does not show a meaningful trend or relationship

20. When is a line graph an appropriate choice to represent a data set? *Responses vary.*
The data is continuous or progressive and I want to show:

- trends, patterns, or progressions.
- the flow or connection between data points.

21. Give an example for each type of data that a line graph could represent. *Responses vary.*

Data Type	Examples	Data Type	Examples
Continuous Data	Monthly average temperatures Patient's heart rate over time	Rate of Change	Speed vs. time in physics Energy usage vs. temperature in a building
Time Series Data	Sales revenue over months Population growth over decades	Sequential Data	Team scores across successive games.

Unit 2 Lesson 13 246 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for creating and analyzing line graphs from different types of data sets.

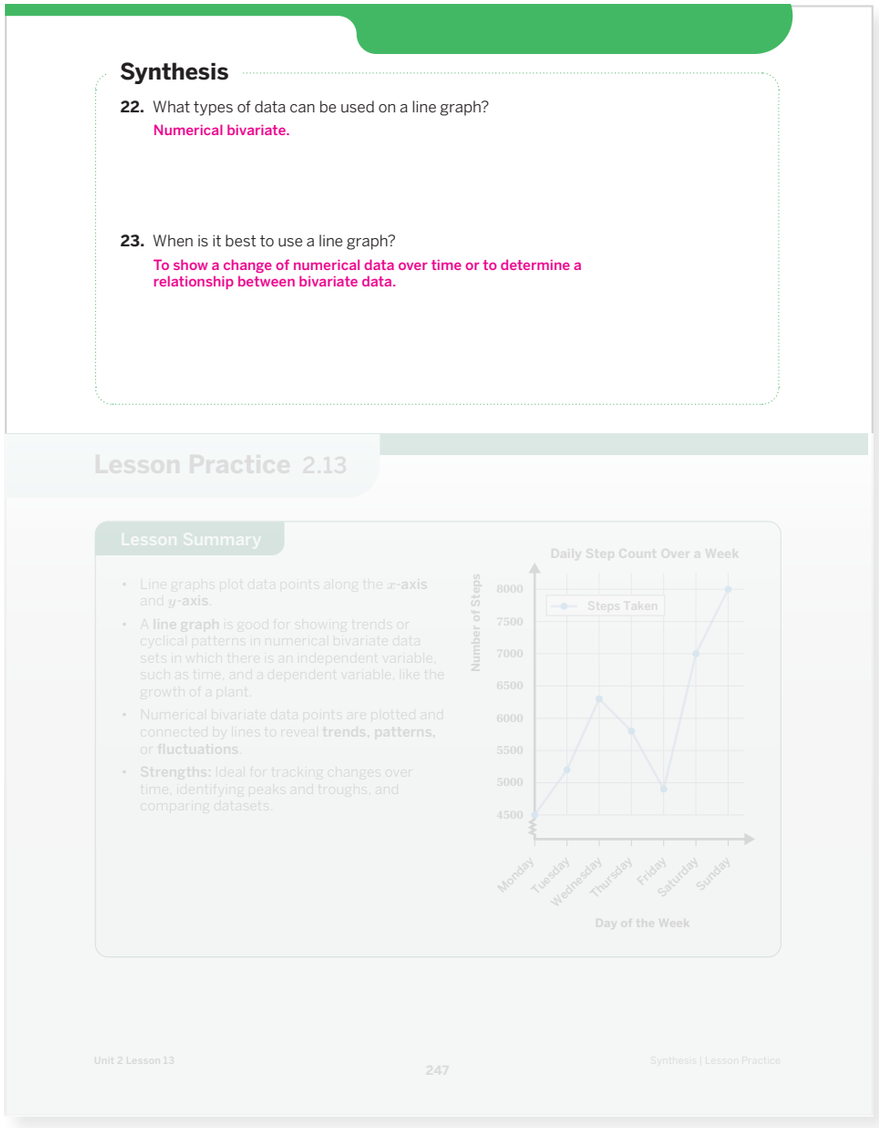
Invite students to respond independently and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Data from the scenario or table is represented on a line graph.
- Line graphs best show changes over time or find trends, patterns, and relationships.

MLR **MLR2: Collect and Display** Use this routine to collect students' examples as they discuss with a partner during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

22. What types of data can be used on a line graph?

Numerical bivariate.

23. When is it best to use a line graph?

To show a change of numerical data over time or to determine a relationship between bivariate data.

Lesson Practice 2.13

Lesson Summary

- Line graphs plot data points along the x-axis and y-axis.
- A **line graph** is good for showing trends or cyclical patterns in numerical bivariate data sets in which there is an independent variable, such as time, and a dependent variable, like the growth of a plant.
- Numerical bivariate data points are plotted and connected by lines to reveal **trends, patterns,** or **fluctuations**.
- **Strengths:** Ideal for tracking changes over time, identifying peaks and troughs, and comparing datasets.

Lesson Takeaway: Numerical bivariate data can best be represented by a line graph to determine trends and patterns.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying type of data represented in a line graph, as well as trends noticed.

Today's Goals

1. **Goal:** Identify components and quantities displayed in line graphs.
2. **Goal:** Understand that line graphs show trends and patterns in numerical bivariate data sets.
3. **Language Goal:** Summarize and analyze data displayed in a line graph for trends. **(Reading and Speaking)**

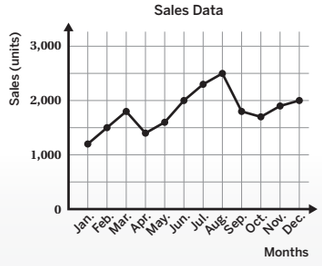
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.13

A popular gadget company tracked monthly sales of their product for a year. The table below shows the sales data (in units) over 12 months:

Sales Data	
Month	**Sales (units)**
January	1,200
February	1,500
March	1,800
April	1,400
May	1,600
June	2,000
July	2,300
August	2,500
September	1,800
October	1,700
November	1,900
December	2,000



a What type of data are the months of the year? Categorical

b What type of data are the sales? Numerical

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support identifying the type of data represented in a line graph, as well as trends noticed.

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Consider your student's responses during the Synthesis? How did their contributions demonstrate their understanding? Did all students participate? How can you ensure all students have an opportunity to share in this routine in upcoming lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

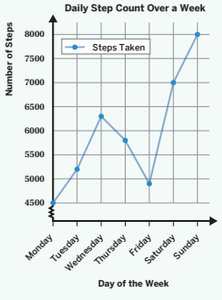
22. What types of data can be used on a line graph?
Numerical bivariate.

23. When is it best to use a line graph?
To show a change of numerical data over time or to determine a relationship between bivariate data.

Lesson Practice 2.13

Lesson Summary

- Line graphs plot data points along the x-**axis** and y-**axis**.
- A **line graph** is good for showing trends or cyclical patterns in numerical bivariate data sets in which there is an independent variable, such as time, and a dependent variable, like the growth of a plant.
- Numerical bivariate data points are plotted and connected by lines to reveal **trends, patterns,** or **fluctuations**.
- **Strengths:** Ideal for tracking changes over time, identifying peaks and troughs, and comparing datasets.



Lesson Practice 2.13

Name: _____ Date: _____ Period: _____

Problems 1–5: Use the scenario and graph provided to answer the questions.

A retail store tracks the data on the hours it stays open and the total daily sales. For example, when the store is open for 4 hours, it earns $500 in daily sales, and when it is open for 12 hours, it earns $2000. This helps the management assess whether extending store hours increases revenue proportionally.

Add the following elements to the line graph.

1. Give the graph a title. *Response varies.*
2. Label the y-axis and intervals.
3. Label the x-axis and intervals.
4. Fill in the table with the data from the graph. Estimate appropriately for any numbers not on a grid line. *Response varies.*

Time Store is Open (hrs) (x)	Daily Sales ($) ($y$)
4	500
6	~800
8	~1200
10	~1600
12	2000
14	~2600



5. What does the line graph of the data show you? *Response varies.* It shows how daily sales increase with longer operating hours.

Problems 6–8: Describe each data set (Numerical or Categorical/univariate or bivariate). Then, decide if a line graph would be a good way to represent the information. Explain your reasoning.

6.

Type of Transportation	Number of Wheels
Car	4
Unicycle	1
Motorcycle	2
Semi Truck	18

Response varies. Numerical univariate.
No, because it's not data over time.

7.

Distance Ran (Miles)	Calories Burned
1	100
2.5	250
3	300
5.5	550

Response varies. Numerical bivariate.
Yes, this is a linear relationship.

8.

Time (Min)	Number of People
0–50	5
51–100	7
101–150	8
151–200	3

Response varies. Numerical bivariate.
No, this is a distribution, and a histogram would be best.

Lesson Practice 2.13

Name: _____ Date: _____ Period: _____

Test Practice

9. Select all the true statements about using line graphs to represent data sets.
   - [x] A. Line graphs are best for showing relationships between two numerical variables over time.
   - [ ] B. Line graphs cannot display variability in data trends.
   - [x] C. Line graphs can help identify trends, peaks, and valleys in data.
   - [ ] D. Line graphs are suitable for categorical data comparisons.
   - [x] E. The x-axis typically represents the independent variable, such as time or another numerical measurement.

Spiral Review

Fill in the table for each law of exponents and solve the sample problem.

Laws of Exponents	Rule	Sample
10. Product of Powers Rule	$a^m \cdot a^n = a^{m+n}$	$j^2 \cdot j^3 = j^5$
11. Quotient of Powers Rule	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^8}{x^2} = x^6$
12. Power of a Power Rule	$(a^m)^n = a^{m \cdot n}$	$(y^3)^4 = y^{12}$
13. Power of a Product Rule	$(ab)^m = a^m \cdot b^m$	$(xy)^5 = x^5 \cdot y^5$
14. Power of a Quotient Rule	$\left(\frac{a}{b}\right)^m = \left(\frac{a^m}{b^m}\right)$	$\left(\frac{k}{n}\right)^2 = \left(\frac{k^2}{n^2}\right)$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.912.DP.1.2
	5	2	MA.912.DP.1.2
	6–8	2	MA.912.DP.1.1, MA.912.DP.1.2
Test Practice	9	2	MA.912.DP.1.1, MA.912.DP.1.2
Spiral Review			
Fluency	10–14	2	MA.8.AR.1.1

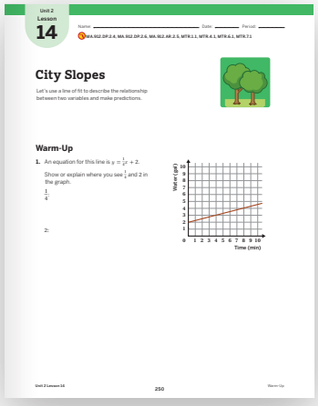
Unit 2

Lesson 14

City Slopes

Interpreting Slope and y-intercept in Context

Let's use a line of fit to describe the relationship between two variables and make predictions.



Learning Outcomes

Today's Goals

1. **Goal:** Estimate unknown values using a linear model for data on a scatter plot.
2. **Language Goal:** Interpret the slope and y-intercept for a linear model in context. **(Writing, Speaking, and Listening)**

Students use a *line of fit* to describe relationships between two variables, and interpret the *slope* and *y-intercept* in context. The line of fit is also used to make predictions for possible values in the data set.

Note: In lessons 14–15, students are introduced to the concept of a line of fit as a statistical tool to analyze relationships between two variables on a scatter plot. Across these lessons, we use the terms linear model and line of fit interchangeably to describe a line that models the general direction or trend of a group of points in a data set.

Coherence

Prior Learning

In Lesson 13, students used the correlation coefficient to describe the strength and direction of the relationship between two variables.

Future Learning

In Lesson 15, students will use residual plots to analyze how well a line fits a set of data.

Connections

- Students extend their understanding of interpreting the slope and y-intercept of a linear function to make predictions about tree cover and temperature in different cities. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.2.4
Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.

MA.912.DP.2.6
Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.DP.1	**MA.912.DP.2.7**
MA.912.DP.1.1	**MA.912.DP.2.8**
MA.912.DP.1.3	**MA.912.DP.2.9**
MA.912.DP.3.1	

Vocabulary

Review Vocabulary

line of fit	*numerical data*
scatter plot	*slope*
slope-intercept form	*y-intercept*

Lesson at a Glance

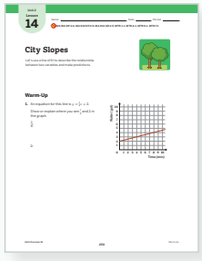
~ 45 min

MA.912.DP.2.4, MA.912.DP.2.6

Warm-Up

Pairs | 5 min

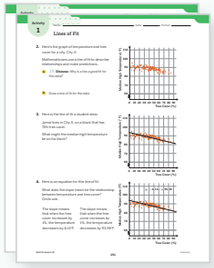
Students revisit linear equations in *slope-intercept form* ($y = mx + b$) and describe how each parameter m and b can be seen on the graph. **Routine:** Think-Pair-Share



Activity 1

Pairs | 10 min

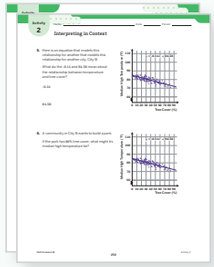
Students explore how a *line of fit* can be helpful for describing relationships between two variables and making predictions.



Activity 2

Pairs | 20 min

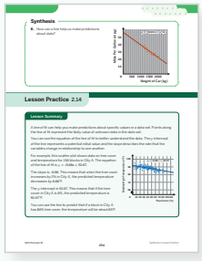
Students interpret the *slope* and y-*intercept* of linear models in context in order to compare the relationship between tree cover and temperature for different cities. **Routines:** Notice and Wonder, MLR1: Stronger and Clearer Each Time **(MTR.1.1, MTR.6.1)**



Synthesis

Whole Class | 5 min

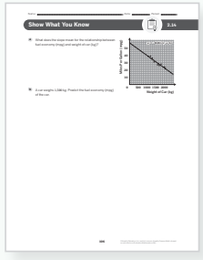
Students synthesize their understanding of how a line of fit can be used to make predictions based on data.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting the slope of a linear model in context and using the line of fit to make a prediction.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- coloring tools (as needed)
- straightedges

Pairs | 5 min

Warm-Up

Purpose: Students revisit linear equations in *slope-intercept form* ($y = mx + b$) and describe how each parameter m and b can be seen on the graph.

1 Launch

Use the Think-Pair-Share routine to support students in describing how they see the *slope* and y-*intercept* in the graph. Invite students to think independently and then take turns sharing with a partner.

Encourage students to sketch to show their thinking.

A **Accessibility: Visual-Spatial Processing**
To support students in interpreting visual representations and making connections to algebraic expressions, invite students to use coloring tools to annotate the y-intercept and slope triangle on the graph in their Student Edition before discussing with their partner.

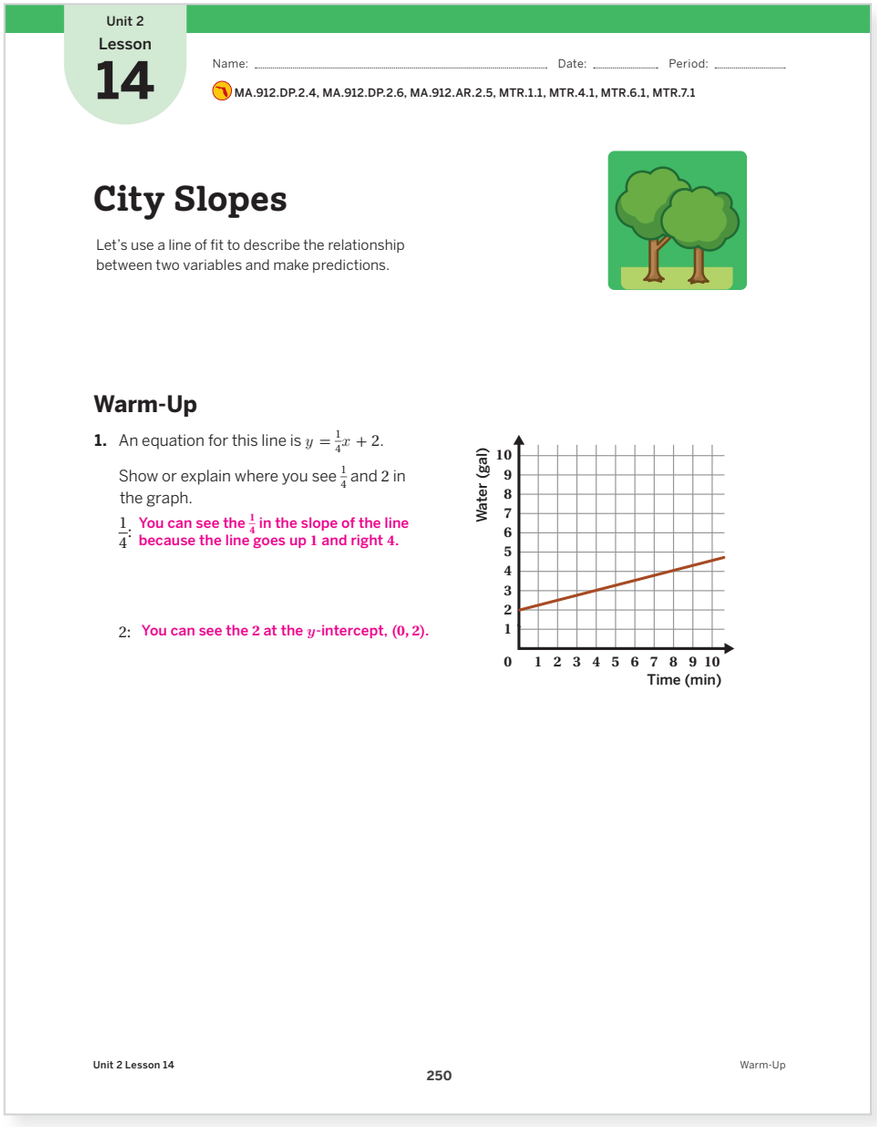
2 Connect

Invite students to share their or their partner's thinking with the class.

Demonstrate how to label the y-intercept and the slope.

Consider asking, "What does 2 mean in terms of time in minutes and gallons of water? $\frac{1}{4}$?"
Responses vary. The water tank initially contained 2 gallons of water. The number of gallons of water in the tank increased by 1 gallon every 4 minutes or by $\frac{1}{4}$ of a gallon every minute.

ELL **Multilingual/English Language Learners**
Provide sentence frames to help students interpret these values such as:
- "The slope means that, as the minutes [increase/decrease] by ___, the gallons [increase/decrease] by ___."
- "The y-intercept means that at 0___, there are ___."
(Writing and Speaking)



Unit 2
Lesson
14

Name: _____ Date: _____ Period: _____

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.AR.2.5, MTR.1.1, MTR.4.1, MTR.6.1, MTR.7.1

City Slopes

Let's use a line of fit to describe the relationship between two variables and make predictions.

Warm-Up

1. An equation for this line is $y = \frac{1}{4}x + 2$.

Show or explain where you see $\frac{1}{4}$ and 2 in the graph.

$\frac{1}{4}$: You can see the $\frac{1}{4}$ in the slope of the line because the line goes up 1 and right 4.

2: You can see the 2 at the y-intercept, (0, 2).

Unit 2 Lesson 14 250 Warm-Up

Pairs | 10 min

Activity 1 Lines of Fit

Purpose: Students explore how a *line of fit* can be helpful for describing relationships between two variables and making predictions. **(MTR.4.1)**

1 Launch

Consider sharing with students that today's topic is an extension from the previous lesson. Invite them to share what they remember about the relationship between tree cover and temperature.

Invite students to use a straightedge to draw their line of fit.

Consider asking these questions after students create their lines of fit:

- "Should we be modeling a line for this data? Why or why not?" *Responses vary.* The correlation coefficient shows that the relationship is strong and could be well modeled by a line.
- "Why isn't the correlation coefficient changing as we drag the line?" *Responses vary.* The correlation coefficient is a property of the data, not the line of fit.
- "How are all the lines of fit alike? What's different?"
- "How can we know if a line fits the data well or not?"
- "How is a linear model like a line of fit? How is it different?"

Note: All linear models provided for students in this lesson are lines of best fit. We use the term *line of fit* to support students in learning how to assess the fit of a line in Lessons 15–16 before introducing the term *line of best fit* in lesson 17.

2 Monitor

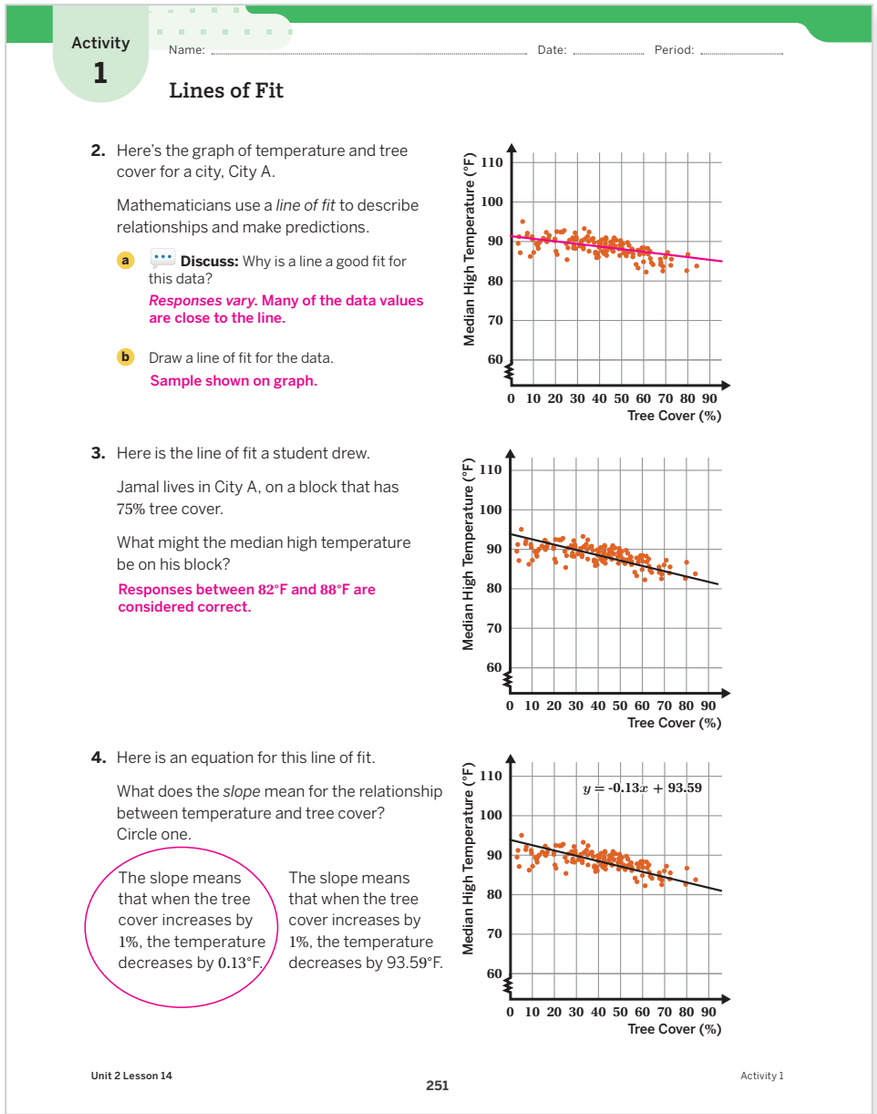
A **Accessibility: Visual-Spatial Processing** To support interpreting visual representations, invite students to use their finger to locate 75% on the x-axis then trace up to the line of fit and estimate the y-value by tracing over to the y-axis.

ELL **English Language Learners** Invite students to read the interpretation aloud and paraphrase its meaning. Clarify any unfamiliar terms. **(Reading)**

3 Connect

To surface the Key Takeaway, consider asking:

- "What information does the line of fit reveal?" It reveals the slope and y-intercept for the data.
- "What does the y-intercept mean in this situation?"
- "How can linear models be useful?"



Activity 1

Name: Date: Period:

Lines of Fit

2. Here's the graph of temperature and tree cover for a city, City A.

Mathematicians use a *line of fit* to describe relationships and make predictions.

a. **Discuss:** Why is a line a good fit for this data?

***Responses vary.* Many of the data values are close to the line.**

b. Draw a line of fit for the data.

Sample shown on graph.

3. Here is the line of fit a student drew.

Jamal lives in City A, on a block that has 75% tree cover.

What might the median high temperature be on his block?

Responses between 82°F and 88°F are considered correct.

4. Here is an equation for this line of fit.

What does the *slope* mean for the relationship between temperature and tree cover? Circle one.

The slope means that when the tree cover increases by 1%, the temperature decreases by 0.13°F.

The slope means that when the tree cover increases by 1%, the temperature decreases by 93.59°F.

Unit 2 Lesson 14 251 Activity 1

Pairs | 20 min

Activity 2 Interpreting in Context

Purpose: Students interpret the *slope* and y-*intercept* of linear models in context in order to compare the relationship between tree cover and temperature for different cities. **(MTR.1.1, MTR.6.1)**

1 Launch

Demonstrate how to click points to turn on and off the data for each city using the dashboard's Student View.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the task. **(MP1)**

A **Accessibility: Executive Functioning** To support carrying out multiple steps, consider chunking this activity by inviting students to compare two graphs at a time.

Consider asking, "If tree cover has a great impact on median high temperature, what would you predict would happen as tree cover increases?" **(MTR.4.1)**

Note: If it doesn't come up naturally, consider asking, "What do you notice about the y-intercepts of the different lines of fit? The slopes? What does that mean about tree cover and temperature?"

2 Monitor

Capture several students' interpretations of the slope and y-intercept to share before students do several rounds of MLR1: Stronger and Clearer Each Time during the Connect.

Consider sharing that students will have an opportunity to revise their first drafts, and that first drafts do not have to be complete or perfect to be valuable for learning.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: increasing, decreasing, tree cover, median high temperature, more, less, etc. **(Writing and Speaking)**

Activity 2 continued >

Activity 2

Name: Date: Period:

Interpreting in Context

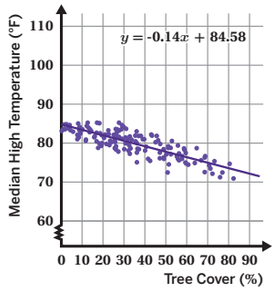
5. Here is an equation that models this relationship for another that models this relationship for another city, City B.

What do the -0.14 and 84.58 mean about the relationship between temperature and tree cover?

Responses vary.

-0.14: **-0.14 means that as the tree cover increases by 1%, the median temperature tends to decrease by about 0.14°F.**

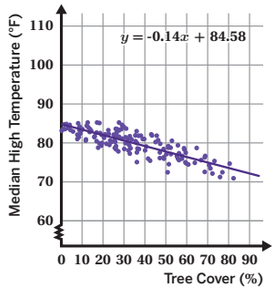
84.58: **84.58 would be the median temperature in this city if there was no tree cover.**



6. A community in City B wants to build a park.

If the park has 80% tree cover, what might its median high temperature be?

Responses between 72°F and 75°F are considered correct.



Unit 2 Lesson 14 252 Activity 2

Pairs | 20 min

Activity 2 Interpreting in Context (continued)

Purpose: Students interpret the *slope* and y-*intercept* of linear models in context in order to compare the relationship between tree cover and temperature for different cities. **(MTR.1.1, MTR.6.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit their work on Problem 1. Consider asking, "How is this like what we have done before? How is it different?"
Use either the graph or the equation of the line of fit to make a prediction. Annotate the graph to estimate the median high temperature.	**Strengthen** Celebrate this student's strategy and capture it to share during the Connect. If they used the graph, invite them to describe how they could use the equation or vice versa.
Use the line of fit model to predict and successfully interpret the value.	**Stretch** Celebrate their strategy. Consider asking, "What would be the percent of tree cover with a median temperature of 65°?"

To support students getting started, consider asking, "What does having a 'great impact' look like on the graph?"

Listen to partner discussions about how they decided to order the cities. Consider asking pairs, "How can the equation help you sort the cities?"

3 Connect

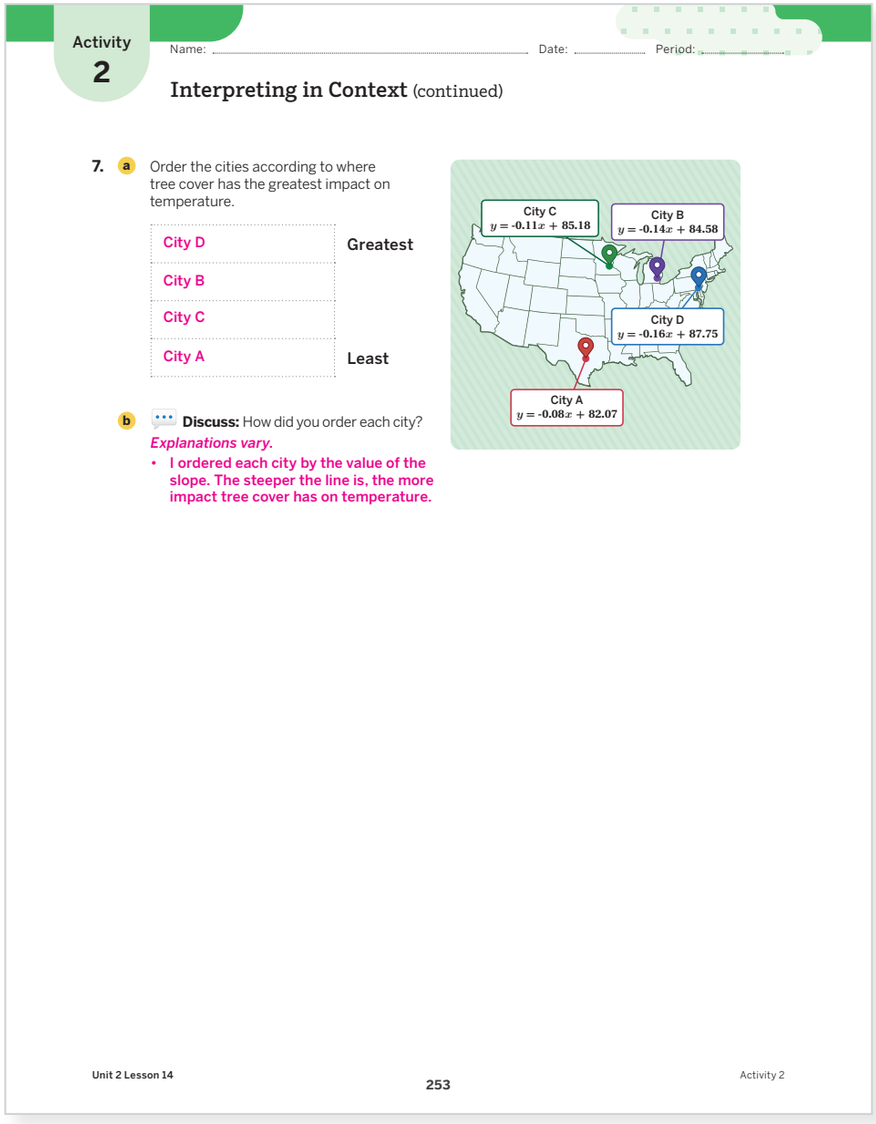
MLR1: Stronger and Clearer Each Time Use this routine to support students in developing precise language. Share and discuss the captured student interpretations of slope and y-intercept. Invite students to meet with 1–2 different partners, give feedback on each other's drafts, then use the feedback to write a second draft. **(MTR.6.1)**

Display several students' revised interpretations. Consider creating an anchor chart with sentence frames that can be helpful for interpreting slope and y-intercept, using their responses. Examples:

- On average, when there is ___% tree cover, the median high temperature is _____.
- On average, as the tree cover increases/decreases by _____, the median high temperature increases/decreases by _____.

Consider asking:

- "How did you define 'greatest impact'?" **(MTR.6.1)**
- "Why might someone say that tree cover has the greatest impact on Austin? On Philadelphia?"
- "What values did you use to support your claim?"



Activity 2

Name: _____ Date: _____ Period: _____

Interpreting in Context (continued)

7. a. Order the cities according to where tree cover has the greatest impact on temperature.

City D	Greatest
City B	
City C	
City A	Least

b. **Discuss:** How did you order each city?

Explanations vary.

- I ordered each city by the value of the slope. The steeper the line is, the more impact tree cover has on temperature.

Unit 2 Lesson 14 253 Activity 2

Lesson Takeaway: A *line of fit* is a useful tool for predicting unknown values and describing relationships.The *slope* and y-*intercept* of the line reveal the initial value and the rate at which the variables change in relationship with one another. By comparing the slope and y-intercept of each equation, we can use the information to determine in which location tree cover has the greatest impact.

Whole Class | 5 min

Synthesis

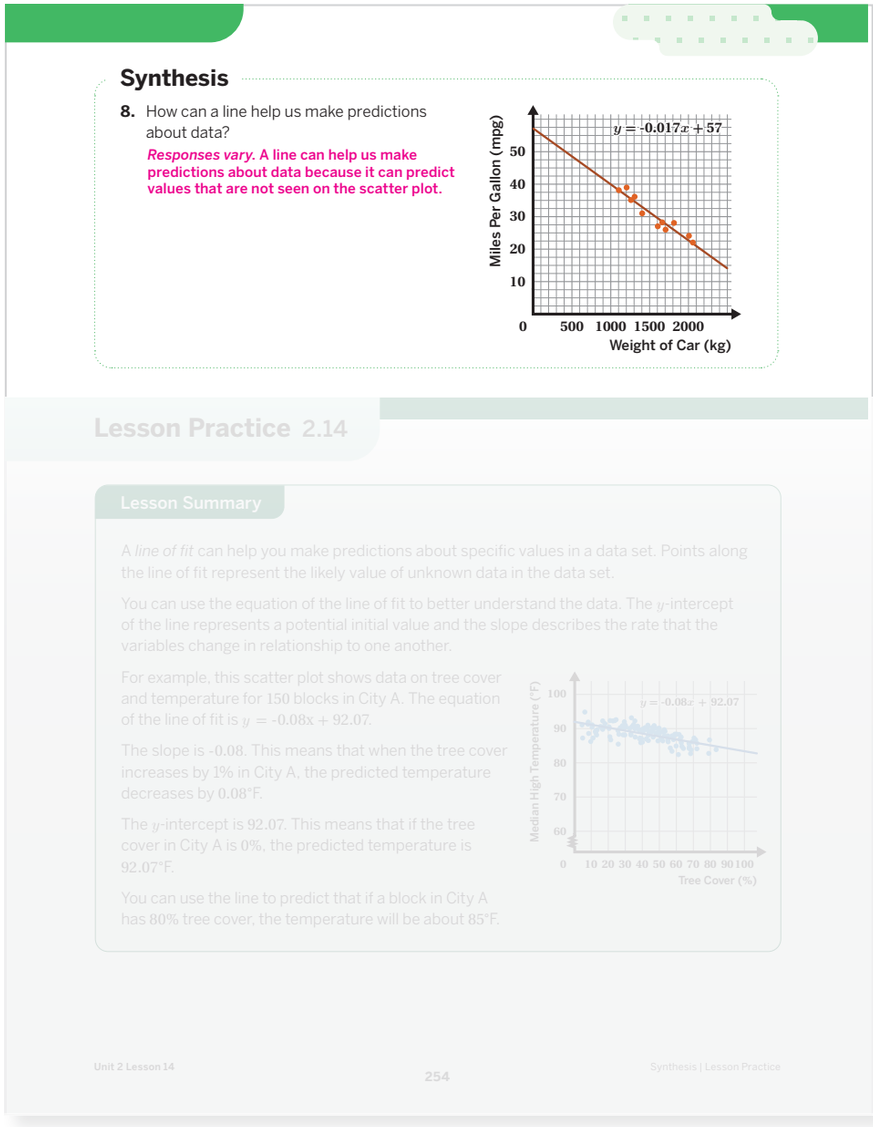
Purpose: Students synthesize their understanding of how a line of fit can be used to make predictions based on data.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The line of fit is a tool that can be used to predict x- or y-values we don't have in the scatter plot.
- The slope helps us know how the two variables are changing in relation to one another.
- The y-intercept predicts the initial value.
- The line of fit takes real data and predicts values that reveal new information.

Note: If time allows, invite students to share with a partner what questions they still have.



Lesson Takeaway: A *line of fit* with a strong correlation coefficient can be used to understand and predict the relationship between two variables such as tree cover and temperature.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting the slope of a linear model in context and using the line of fit to make a prediction.

Today's Goals

1. **Goal:** Estimate unknown values using a linear model for data on a scatter plot.
2. **Language Goal:** Interpret the slope and y-intercept for a linear model in context. **(Writing, Speaking, and Listening)**

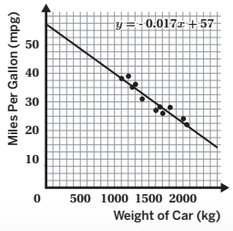
Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 2.14

a. What does the slope mean for the relationship between fuel economy (mpg) and weight of car (kg)?

Responses vary. In this situation, the slope means that as the mass of a car increases by 1 kilogram, the fuel economy tends to decrease by about 0.017 miles per gallon.



b. A car weighs 1,500 kg. Predict the fuel economy (mpg) of the car.

Responses between 30 miles per gallon and 32 miles per gallon are considered correct.

D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support interpreting slope and y-intercept in context and making predictions using a linear model:

» **Respond to Student Thinking:**

- Use the *Using Lines of Fit to Make Predictions* **Mini-Lesson**.
- Revisit **Activity 2 Problems 5 and 6** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students explored the relationship between tree cover and temperature. What other factors in the community could influence temperature? Consider building heights or how close buildings are to one another to brainstorm other ideas.

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How can a line help us make predictions about data?

Responses vary. A line can help us make predictions about data because it can predict values that are not seen on the scatter plot.

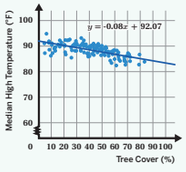
Lesson Practice 2.14

Lesson Summary

A *line of fit* can help you make predictions about specific values in a data set. Points along the line of fit represent the likely value of unknown data in the data set.

You can use the equation of the line of fit to better understand the data. The y-intercept of the line represents a potential initial value and the slope describes the rate that the variables change in relationship to one another.

For example, this scatter plot shows data on tree cover and temperature for 150 blocks in City A. The equation of the line of fit is $y = -0.08x + 92.07$.



The slope is -0.08. This means that when the tree cover increases by 1% in City A, the predicted temperature decreases by 0.08°F.

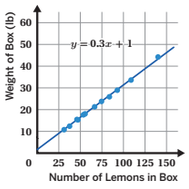
The y-intercept is 92.07. This means that if the tree cover in City A is 0%, the predicted temperature is 92.07°F.

You can use the line to predict that if a block in City A has 80% tree cover, the temperature will be about 85°F.

Lesson Practice 2.14

Name: _____ Date: _____ Period: _____

Problems 1–2: A store receives 12 boxes that contain many different numbers of lemons. They weigh each box and then count the lemons. Here is a scatter plot of that data.



1. A new box weighs 30 pounds. Approximately how many lemons are in the box?

95 lemons. Responses between 90 and 97 are considered correct.

2. The equation for the line that best fits this data is $y = 0.3x + 1$. What do the numbers 0.3 and 1 mean in this context? *Responses vary.*

The number 0.3 means that as you add each new lemon, the weight of the box tends to increase by 0.3 pounds. The number 1 means that the weight of the box is 1 pound before any lemons are added.

Problems 3–4: The store collects data about the weight of other types of fruit boxes. The table shows equations for the line of best fit for each type of fruit box, where y represents the weight of a box and x represents the number of fruits in a box.

Fruit	Line of Best Fit
Orange	$y = 0.45x + 0.95$
Lemon	$y = 0.3x + 1$
Pomegranate	$y = 1.05x + 1.15$
Mango	$y = 0.85x + 1.25$

3. Which fruit is heavier? Circle your choice.

Lemons (Pomegranates) They are the same

Explain your thinking.

***Explanations vary.* I can tell that pomegranates are heavier because the slope of the line of fit for pomegranates is higher, so the weight of a box of pomegranates increases more each time one is added.**

4. Order these fruits by their weight.

Lemon	Lightest
Orange	
Mango	
Pomegranate	Heaviest

5. Sora collects data about the number of bananas she buys at the store, x, and the total weight of the bananas in pounds, y. If Sora graphed her data, which value would be closest to the slope of the line of best fit?

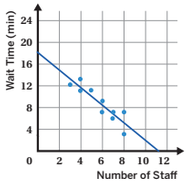
A. -4 B. -0.4

(C.) 0.4 D. 4

Lesson Practice 2.14

Name: _____ Date: _____ Period: _____

Problems 6–7: A restaurant gathered data about how long customers had to wait and how many staff members were working. The slope of the line of fit is -1.62.



6. What does the slope of -1.62 mean in this situation?

***Responses vary.* -1.62 means that for each additional staff member working, the customer wait time tends to decrease by 1.62 minutes.**

7. What equation might reasonably represent the line of fit? Explain your thinking.

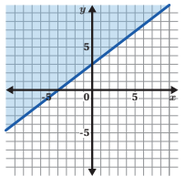
***Responses vary.* The initial value could be 18. An equation that represents the line of fit could be $y = 18 - 1.62x$, where x represents the number of staff and y represents the wait time in minutes.**

Test Practice

8. Select *all* the coordinate pairs that are solutions to $y = \frac{1}{2}x + 10$.

☐ A. $\left(0, \frac{2}{3}\right)$ ☑ B. (-10, 15) ☐ C. (4, 12) ☑ D. (2, 9) ☐ E. (5, 8)

9. Here is the graph of the inequality $y \geq 0.75 + 3$. Select *all* the coordinate pairs that are solutions to this inequality.



☐ A. (-9, -5)
☑ B. (-3, 2)
☑ C. (0, 3)
☐ D. (2, -3)
☑ E. (6, 8)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.DP.2.4
	3–4	2	MA.912.DP.2.4
	5	2	MA.912.DP.2.4
	6–7	2	MA.912.DP.2.4
Test Practice	8–9	2	MA.8.AR.3.4, MA.912.AR.2.8

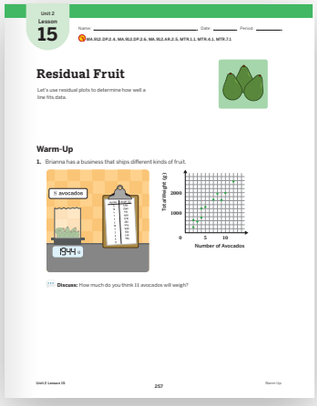
Unit 2

Lesson 15

Residual Fruit

Residuals and Residual Plots

Let's use residual plots to determine how well a line fits data.



Learning Outcomes

Today's Goals

1. **Goal:** Make connections between *residual plots* and *residuals* on a graph.
2. **Goal:** Use residuals and residual plots to understand what it means for a line to fit data well.
3. **Language Goal:** Identify when a residual plot indicates a better or worse fit. **(Writing, Speaking, and Listening)**

Students use **residual plots** to determine how well a line fits data. Students recall how lines can be used to make predictions and discuss how the actual data does not always match our predictions. Students learn that the differences between the data and our predictions are called **residuals** and that these can be used to determine how well a line fits data. Students create lines of fit in this lesson and will be introduced to lines of *best* fit in Lesson 16.

Coherence

Prior Learning

In Lesson 14, students used a line of fit to describe a relationship and make predictions about data.

Future Learning

In Lesson 16, students will explore how technology can generate the line of *best* fit for data on a scatter plot. They will use the equation of the line of best fit to make predictions.

Connections

- Students interpret residual plots to determine how well a line fits data regarding different weights of shipped fruit and the number of floors of a building and its height. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.2.4

Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.

MA.912.DP.2.6

Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.8.DP.1.2

MA.8.DP.2.2

Vocabulary

New Vocabulary

residual

residual plot

Lesson at a Glance

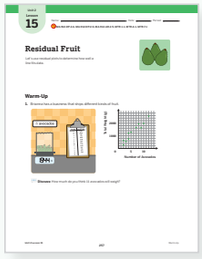
~ 45 min

MA.912.DP.2.4, MA.912.DP.2.6

Warm-Up

Pairs | 5 min

Students make sense of the context they will be using during this lesson as they analyze data about weights of different amounts of avocados and use that data to make a prediction.



Activity 1

Pairs | 10 min

Students use the **MLR2: Collect and Display** routine to describe how they decide how well a particular line fits the data.

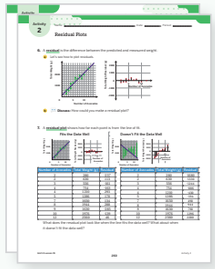


Activity 2

Pairs | 20 min

Students explore dynamic **residual plots** and use them as a tool for determining how well a line fits data.

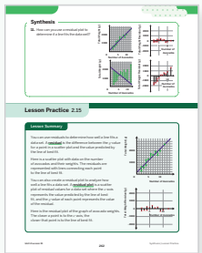
Routines: **Notice and Wonder, MLR2: Collect and Display**



Synthesis

Whole Class | 5 min

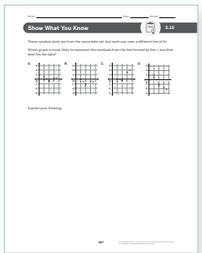
Students synthesize their understanding of how residual plots can help assess the fit of a line to data.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which residual plot represents the line that best fits a data set.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students make sense of the context they will be using during this lesson as they analyze data about weights of different amounts of avocados and use that data to make a prediction.

1 Launch

To surface connections, consider asking:

- "Have you ever had to weigh fruit? When?"
- "What types of fruit do you weigh? What types of fruit do you not weigh?"
- "Why does the weight of fruit matter?"

A **Accessibility: Visual-Spatial Processing**
As students interpret visual representations, discuss how the point on the graph is connected to the avocado order before making a prediction.

Select and sequence several student responses.

D **Differentiation**

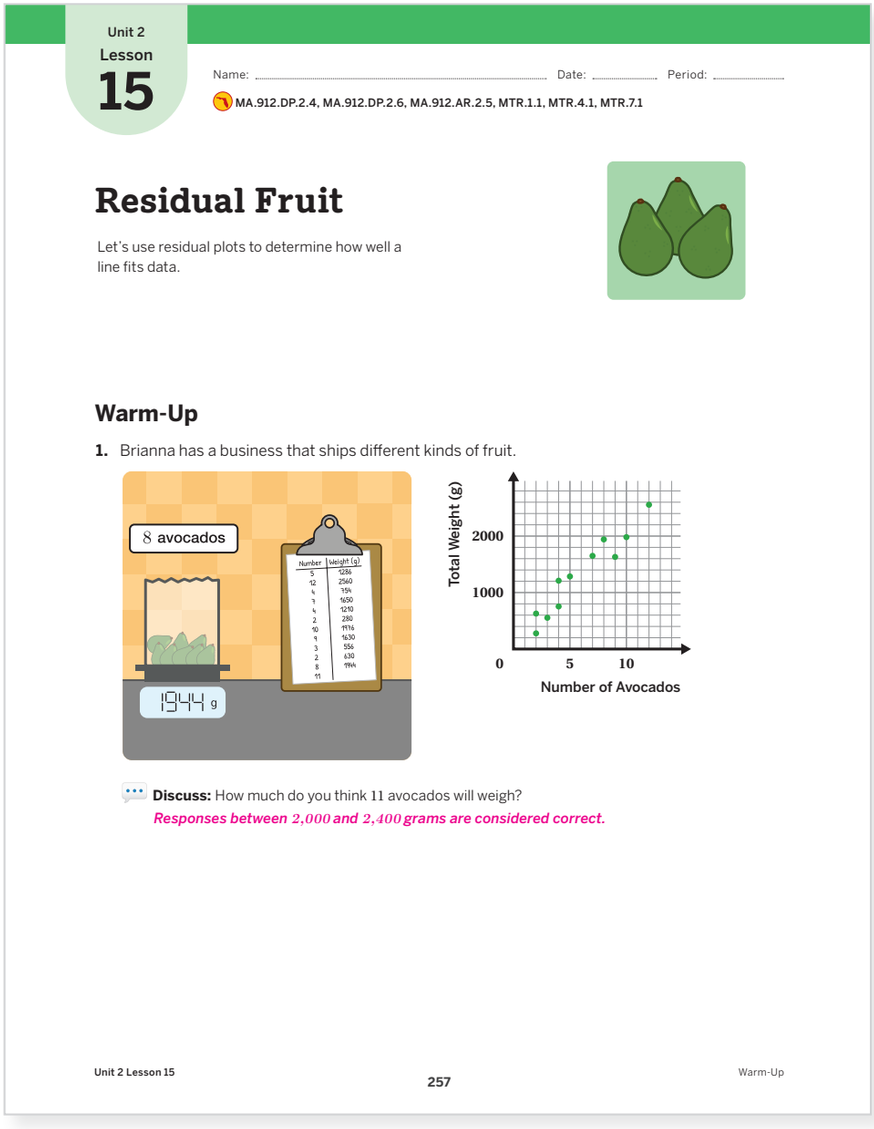
Look for students who:	Respond to Student Thinking
Find the median of the weights between 10 and 12 avocados.	
Visually estimate the weight of 11 avocados using the graph.	**Strengthen** Celebrate the strategy these students used. Invite them to share their strategy during the Connect.
Find a proportional weight for 11 avocados using weights given in the list.	
Use a line of fit to predict the weight of 11 avocados	**Strengthen** Consider inviting these students to share their strategy last to transition into Activity 1.

2 Connect

Invite students to share strategies used to make a prediction for the weight of 11 avocados.

Consider asking:

- "How did you come up with your prediction?"
- "How confident are you in your prediction? Why?" **(MTR.4.1)**
- "What could help you increase your confidence in your prediction?" **(MTR.4.1)**



Unit 2
Lesson
15

Name: Date: Period:

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.AR.2.5, MTR.1.1, MTR.4.1, MTR.7.1

Residual Fruit

Let's use residual plots to determine how well a line fits data.

Warm-Up

1. Brianna has a business that ships different kinds of fruit.

Discuss: How much do you think 11 avocados will weigh?

Responses between 2,000 and 2,400 grams are considered correct.

Unit 2 Lesson 15 257 Warm-Up

Pairs | 10 min

Activity 1 Predicting With Lines

Purpose: Students use the MLR2: Collect and Display routine to describe how they decide how well a particular line fits the data.

1 Launch

Consider asking:

- "How can a line of fit help us make predictions?"
- "If a line does not fit the data well, how might that affect the prediction?"
- "Which parts of each line (slope, y-intercept, etc.) fit the data well? Which do not?"
- "How would you adjust each line to better fit the data?"

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "The data _____, but the line _____."). **(Writing and Speaking)**

2 Monitor

MLR **MLR2: Collect and Display** Use this routine to begin collecting students' ideas about how to assess the fit of a line to data. Display for students to reference for the rest of the lesson.

Show several students' lines.

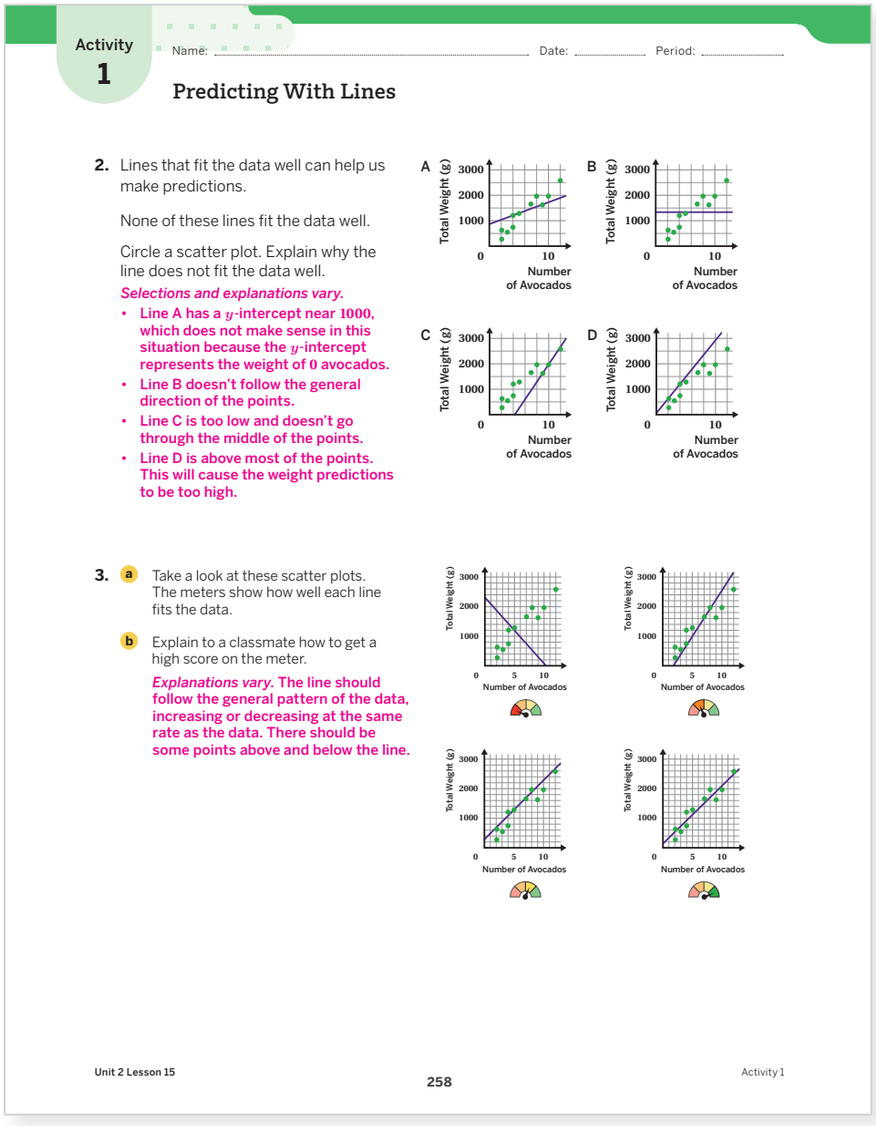
Pause briefly to discuss:

- "What does the reading on the meter tell us about the fit of the line?"
- "What criteria can help us determine if a line fits data well or not?" **(MTR.7.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Think the best lines touch the most data points.	**Support** Invite these students to create lines that touch many data points and analyze the meter. Consider asking, "What does the meter tell you about the line you created?"
Think the line of fit must go through the origin since if there are 0 avocados, then the weight must be 0.	**Stretch** Celebrate the sensemaking they made for the situation. Consider asking, "What does it mean for the line of fit to predict that 0 avocados does not weigh 0 pounds?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Predicting With Lines

2. Lines that fit the data well can help us make predictions.

None of these lines fit the data well.

Circle a scatter plot. Explain why the line does not fit the data well.

Selections and explanations vary.

- Line A has a y-intercept near 1000, which does not make sense in this situation because the y-intercept represents the weight of 0 avocados.
- Line B doesn't follow the general direction of the points.
- Line C is too low and doesn't go through the middle of the points.
- Line D is above most of the points. This will cause the weight predictions to be too high.

A B C D

Total Weight (g) 3000 2000 1000 0 10

Number of Avocados

3. a. Take a look at these scatter plots. The meters show how well each line fits the data.

b. Explain to a classmate how to get a high score on the meter.

Explanations vary. The line should follow the general pattern of the data, increasing or decreasing at the same rate as the data. There should be some points above and below the line.

Total Weight (g) 3000 2000 1000 0 5 10

Number of Avocados

Unit 2 Lesson 15 258 Activity 1

Pairs | 10 min

Activity 1 Predicting With Lines (continued)

Purpose: Students use the MLR2: Collect and Display routine to describe how they decide how well a particular line fits the data.

2 Monitor

Encourage students to read each others' responses and decide if others' strategies were similar to or different from their own.

Note: The lines of fit are all lines of best fit. To support students in developing their understanding of how to informally fit a line to data, the concept of the line of best fit will not be introduced until the next lesson.

Select and sequence student strategies for predicting the weight of 6 avocados to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to use the line of fit to predict the weight of 6 avocados. Consider asking, "Where does the line predict the weight of 6 avocados?"
Use the line of fit to estimate the weight of 6 avocados.	**Strengthen** Invite them to share during the Connect.
Would like to explore another strategy for predicting using the line of fit.	**Stretch** Provide these students with the equation for the line of fit $(y - 516.03 = 199.74(x - 2))$ and ask them how they could use it to predict the weight of 6 avocados.

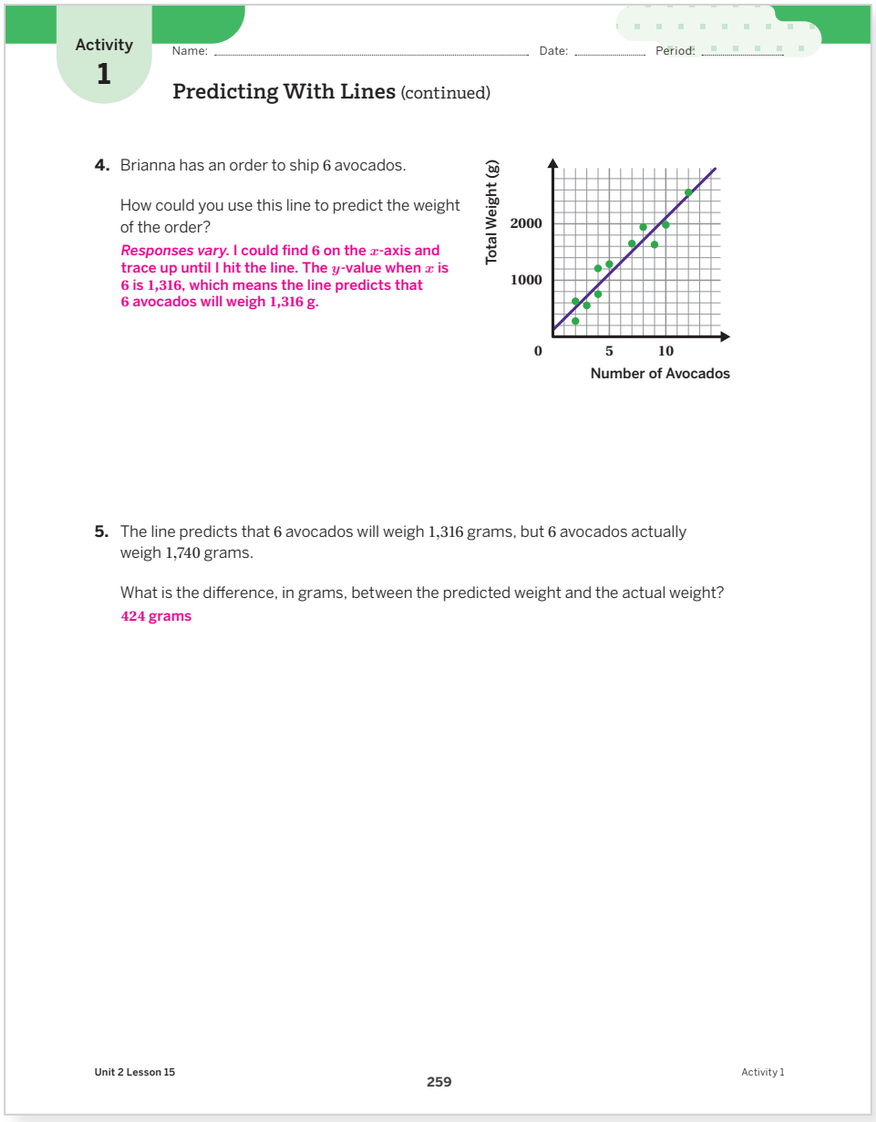
Consider sharing with students that they can input an expression in the math input and the calculator will compute the difference for them.

3 Connect

Invite students to share their strategies for predicting the weight of 6 avocados.

To surface the Key Takeaway, Consider asking:

- "What data point(s) represent orders of small avocados? Large avocados? How do you know?"
- "Will 6 avocados always weigh the same amount?"
- "How does the line of fit help increase confidence in your prediction?"
- "How can making predictions be helpful even though they are not always perfect?"



Activity 1

Name: _____ Date: _____ Period: _____

Predicting With Lines (continued)

4. Brianna has an order to ship 6 avocados.

How could you use this line to predict the weight of the order?

Responses vary. I could find 6 on the x-axis and trace up until I hit the line. The y-value when x is 6 is 1,316, which means the line predicts that 6 avocados will weigh 1,316 g.

5. The line predicts that 6 avocados will weigh 1,316 grams, but 6 avocados actually weigh 1,740 grams.

What is the difference, in grams, between the predicted weight and the actual weight?

424 grams

Unit 2 Lesson 15 259 Activity 1

Key Takeaway: Lines of fit are one tool that can help increase our confidence in making predictions for missing data points within a data set. Some lines fit the data better than others and make better predictions.

Pairs | 20 min

Activity 2 Residual Plots

Purpose: Students explore dynamic **residual plots** and use them as a tool for determining how well a line fits data.

Short on time? Consider facilitating Problem 6 as a whole class instead of individually.

1 Launch

Consider sharing that the purpose of this activity is to learn about another tool, **residuals**, that can help us assess how well a line fits the data.

Use the Notice and Wonder routine to promote curiosity and help students make sense of residuals. **(MTR.1.1)**

Consider asking:

- "What is the predicted weight? What is the measured weight? What is the difference? How do you see each of these values on the graph?"
- "What does a positive residual mean about the avocados? A negative residual?"

2 Monitor

To support students getting started, encourage them to create lines that do not fit the data well and analyze their **residual plots**. Then, have them create lines that do fit well and ask, "What changed about the residual plot?"

Encourage students to discuss with their partner properties of residual plots that do not fit the data well. Listen for:

- Residual plots that go from very negative to very positive seem to be an opposite relationship.
- If all the points in the residual plot are positive, the line of fit is below the data.

Activity 2 continued >

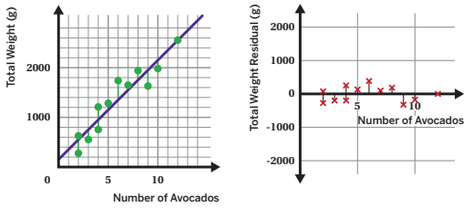
Activity 2

Name: Date: Period:

Residual Plots

6. A **residual** is the difference between the predicted and measured weight.

a. Let's see how to plot residuals.

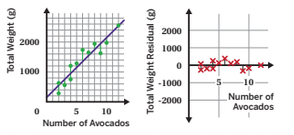


b. **Discuss:** How could you make a residual plot?

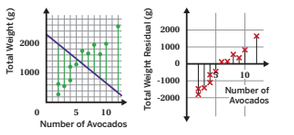
Responses vary. I could make a residual plot by measuring the distance between each point and the line. Then I would make a new graph with points that represent the distance between each number of avocados and the line.

7. A **residual plot** shows how far each point is from the line of fit.

Fits the Data Well



Doesn't Fit the Data Well



Number of Avocados	Total Weight (g)	Residual
2	280	-237
2	630	113
3	556	-161
4	754	-163
4	1210	293
5	1286	170
7	1650	134
8	1944	288
9	1630	-285
10	1976	-139
12	2560	46

Number of Avocados	Total Weight (g)	Residual
2	280	-1680
2	630	-1330
3	556	-1244
4	754	-886
4	1210	-430
5	1286	-194
7	1650	490
8	1944	944
9	1630	790
10	1976	1296
12	2560	2200

What does the residual plot look like when the line fits the data well? What about when it doesn't fit the data well?

Responses vary.

- When the line fits the data well, the points on the residual plot are close to the x-axis, and they're randomly distributed both above and below the x-axis.
- When the line doesn't fit the data well, the points on the residual plot are far from the x-axis, or there are more points either above or below the x-axis.

Unit 2 Lesson 15 260 Activity 2

Pairs | 20 min

Activity 2 Residual Plots (continued)

Purpose: Students explore dynamic **residual plots** and use them as a tool for determining how well a line fits data.

Short on time? Consider facilitating Problem 6 as a whole class instead of individually.

2 Monitor

Look for the development of student thinking about how to use residual plots to assess the fit of a line.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problems 8–10)*	**Support** Encourage them to choose one point on the residual plot to consider. Consider asking, "What does this point tell us about the location of the line in the comparison to the data?"
Draw their own line of fit and not Brianna's line. *(Problem 8)*	**Strengthen** Consider asking, "Does the line you drew match the residual plot?"
Need support creating a line of fit that gets a green meter reading. *(Problem 9)*	**Strengthen** Consider asking, "What part of your line fits the data well? The slope? The y-intercept? How can you adjust both points to make the line a better fit?"
Think Aditi's line fits the data well because it looks linear. *(Problem 10)*	**Strengthen** Invite these students to return to Problem 7. Consider asking, "What does the residual plot look like when the line fits the data well?"

3 Connect

To surface the Key Takeaway, consider asking:

- "Earlier, _____ said that the points are close to the line. How do you see that in the residual plot?"
- "How can a residual plot help us decide if a line fits data well?"

MLR2: Collect and Display Revisit students' ideas about how to assess the fit of a line to data. Collect student language used to describe what they notice about each residual plot, such as:

- "The residuals are mostly negative because the actual data is below the predicted data."
- "The line is a good fit because the residuals are balanced above and below the x-axis."

Create a class definition for **residual** and **residual plot** using student language and the glossary.

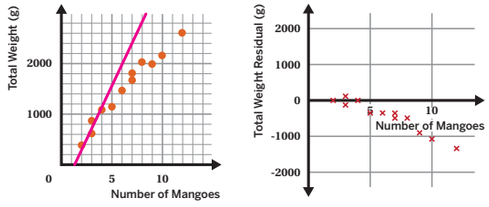
Activity 2

Name: _____ Date: _____ Period: _____

Residual Plots (continued)

8. Here is the residual plot for a line of fit Brianna created.

Sketch the line of fit you think Brianna made. *Lines vary.*

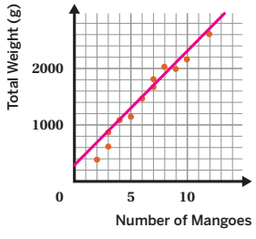


9. Let's look at Brianna's line.

a. Draw another line that is a better fit for the data. *Lines vary.*

b. **Discuss:** How will the residual plot change once the line is a better fit for the data?

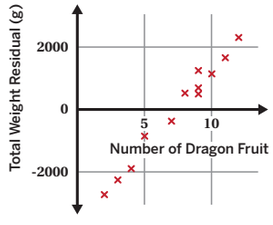
Responses vary. The points on the residual plot will move closer to the x-axis and become randomly dispersed both above and below the x-axis.



10. Aditi made a line of fit for this data showing the total weight of different numbers of dragon fruit.

Here is the graph of the residuals from Aditi's line. Based on the graph of the residuals, interpret the strength and direction of the correlation of the line of fit as it relates to number of dragon fruit and total weight in grams.

Explanations vary. Aditi's line does not model the data very well because the points on the residual plot are very far from the x-axis. So, the correlation is weak. The direction of the residuals is showing a positive slope, so this indicates a positive correlation between the number of dragon fruit and their total weight in grams.



Unit 2 Lesson 15 261 Activity 2

Key Takeaway: A **residual** is the difference between the predicted y-value on a line of fit and the actual y-value in the data. A **residual plot** is a scatter plot of all the residual values for a data set. Residuals and residual plots are helpful tools for assessing how well a line fits a set of data.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how residual plots can help assess the fit of a line to data.

Invite students to respond independently, and then share their thinking with a partner.

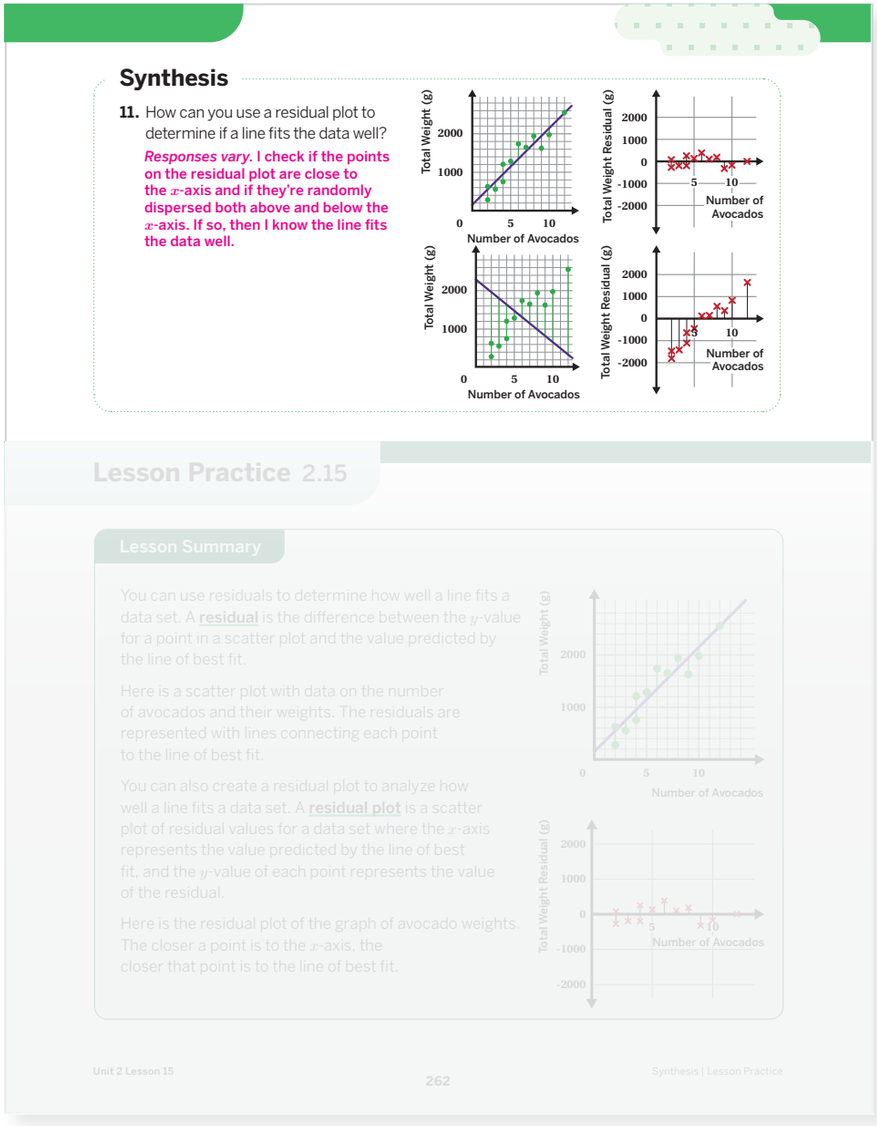
Consider sharing a variety of ideas, such as:

- Determining informally if a line visually looks like it would fit the data well.
- Looking for points on the residual plots to be close to the x-axis.
- Looking for the points on the residual plots to be balanced with some above and some below the x-axis.
- Most of the points on the residual plots are close to 0, which means the line fits the data tightly.

Consider asking:

- "What does a residual plot look like when it fits the data well?"
- "What does a residual plot look like when it does not fit data well?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. How can you use a residual plot to determine if a line fits the data well?

Responses vary. I check if the points on the residual plot are close to the x-axis and if they're randomly dispersed both above and below the x-axis. If so, then I know the line fits the data well.

Lesson Practice 2.15

Lesson Summary

You can use residuals to determine how well a line fits a data set. A residual is the difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.

Here is a scatter plot with data on the number of avocados and their weights. The residuals are represented with lines connecting each point to the line of best fit.

You can also create a residual plot to analyze how well a line fits a data set. A residual plot is a scatter plot of residual values for a data set where the x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.

Here is the residual plot of the graph of avocado weights. The closer a point is to the x-axis, the closer that point is to the line of best fit.

Unit 2 Lesson 15 262 Synthesis | Lesson Practice

Lesson Takeaway: **Residuals** are the difference between the actual and predicted value for a line of fit. Residuals can be plotted to assess if a line fits a set of data well. When a line fits data well, the points on the **residual plots** will be balanced above and below the x-axis and be tightly clustered close to 0.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which residual plot represents the line that best fits a data set.

Today's Goals

1. **Goal:** Make connections between *residual plots* and *residuals* on a graph.
2. **Goal:** Use residuals and residual plots to understand what it means for a line to fit data well.
3. **Language Goal:** Identify when a residual plot indicates a better or worse fit. **(Writing, Speaking, and Listening)**

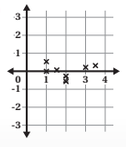
Show What You Know (Assessment Resources)

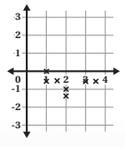
Name: _____ Date: _____ Period: _____

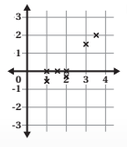
Show What You Know 2.15

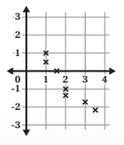
These residual plots are from the same data set, but each one uses a different line of fit.

Which graph is most likely to represent the residuals from the line formed by the x-axis that best fits the data?

A. 

B. 

C. 

D. 

Explain your thinking.

Explanations vary. The points on the residual plot are close to the x-axis and randomly dispersed both above and below the x-axis, which means it probably represents the best line of fit for the data.

D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support with how to use residuals to assess the fit of a line:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary**.
- Revisit **Activity 2 Problems 9 and 10** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Reflect on your facilitation of MLR2: Collect and Display routine in this lesson. How did students engage in the routine? What about how you facilitated the routine promoted student learning? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you use a residual plot to determine if a line fits the data well?

Responses vary. I check if the points on the residual plot are close to the x-axis and if they're randomly dispersed both above and below the x-axis. If so, then I know the line fits the data well.

Lesson Practice 2.15

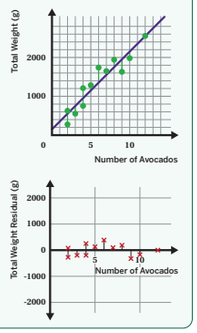
Lesson Summary

You can use residuals to determine how well a line fits a data set. A **residual** is the difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.

Here is a scatter plot with data on the number of avocados and their weights. The residuals are represented with lines connecting each point to the line of best fit.

You can also create a residual plot to analyze how well a line fits a data set. A **residual plot** is a scatter plot of residual values for a data set where the x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.

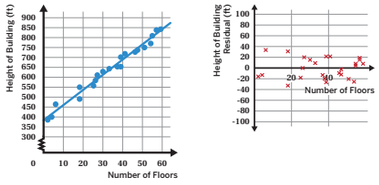
Here is the residual plot of the graph of avocado weights. The closer a point is to the x-axis, the closer that point is to the line of best fit.



Lesson Practice 2.15

Name: _____ Date: _____ Period: _____

Problems 1–2: The scatter plot shows the heights of a group of buildings, the number of floors in each building, and a line that best fits the data. The residual plot is also shown.



1. Predict the height of a building that has 50 floors.

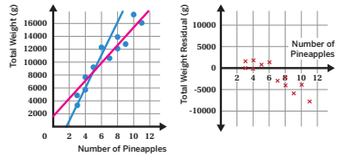
Responses between 750 and 770 feet are considered correct.

2. How can you tell that the graphed line is a good fit for the data? Use the residual plot if it helps with your thinking.

Explanations vary. The line is a good fit because the points on the residual plot are close to the x-axis, and they are both above and below the x-axis.

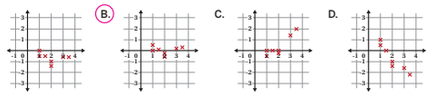
3. Here is a scatter plot and its corresponding residual plot.

Draw a better line of fit on the scatter plot.



4. These residual plots are from the same set of data, but each one represents a different line of fit. Which residual plot shows the best line of fit?

A. B. C. D.



Explain your thinking.

Explanations vary. All of the values in the graph I chose are close to the x-axis. The other three graphs have some points that are much higher or much lower.

Lesson Practice 2.15

Name: _____ Date: _____ Period: _____

Problems 5–6: This scatter plot shows the number of traffic tickets and the cost of car insurance for 16 people.

5. Which equation could represent the line of best fit?

A. $y = -62x + 220$ B. $y = 62x + 220$ C. $y = -220x + 62$ D. $y = 220x + 62$

Explain your thinking.

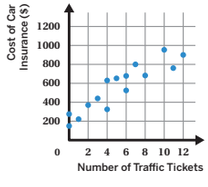
Explanations vary.

- I drew a line to fit the data. My line started near 250 and had a positive slope, so I picked $y = 62x + 220$.
- I can see that the trend has a positive slope but is not steep enough to be increasing by $220 for each ticket, so I picked $y = 62x + 220$.

Test Practice

6. Which of the following could be a reasonable prediction for the cost of car insurance for an individual with 15 traffic tickets?

A. $750 B. $900 C. $1150 D. $2000



Spiral Review

Problems 7–9: Solve each equation for x.

7. $y = 3x + 5$ → $\frac{y-5}{3} = x$

8. $4y - 2x = 10$ → $x = -5 + 2y$

9. $y = \frac{1}{6}x$ → $6y = x$

10. Are there any outliers in this data set? Use a calculator if it helps with your thinking.

[1, 1, 1, 1, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 6, 20]

Circle one. Yes No

Explain your thinking.

Explanations vary.

- I used the Desmos Graphing Calculator to create a box plot. When I clicked "Exclude outliers," I saw that 20 was an outlier.
- 20 is an outlier because it's 14 units away from the next highest number, and 14 units is bigger than the spread of the rest of the data.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.DP.2.4
	2	2	MA.912.DP.2.6
	3	2	MA.912.DP.2.4, MA.912.DP.2.6
	4	2	MA.912.DP.2.6
	5	2	MA.912.DP.2.4
Test Practice	6	2	MA.912.DP.2.4
Spiral Review			
Fluency	7–9	2	MA.912.AR.1.2
	10	1	MA.7.DP.1.2

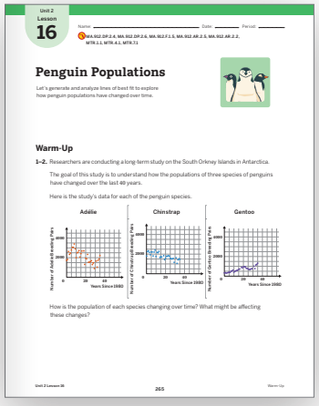
Unit 2

Lesson 16

Penguin Populations

Using Technology to Generate the Line of Best Fit

Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.


Unit 2 Lesson 16

Penguin Populations

Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.

Warm-Up

1–2. Researchers are conducting a long-term study on the South Orkney Islands in Antarctica. The goal of this study is to understand how the populations of three species of penguins have changed over the last 40 years.

Here is the study's data for each of the penguin species.

Adélie | Chinstrap | Gentoo

How is the population of each species changing over time? What might be affecting these changes?

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Learning Outcomes

Today's Goals

1. **Goal:** Use technology to generate the *line of best fit* for data on a scatter plot.
2. **Goal:** Use the equation of a line of best fit to make predictions.
3. **Language Goal:** Interpret the slope and y-intercept of the equation of a line of best fit in context. **(Speaking and Listening)**

Students explore how penguin populations have changed over time by generating and analyzing lines of best fit using a graphing calculator or a graphing tool. They learn that there is only one **line of best fit** for each set of data on a scatter plot, and use the generated line of best fit to make predictions about the penguin populations.

Coherence

Prior Learning

In Lesson 14, students used the line to estimate unknown values on a graph. In Lesson 15, students estimated lines of fit and assessed the goodness of fit by analyzing residual plots.

Future Learning

In Unit 6, students will generate a model for nonlinear data on a scatter plot and use the model to make predictions.

Connections

- Students generate and analyze best fit lines to explore how penguin populations have changed over time. (cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.2.4
Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.

MA.912.DP.2.6
Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.

Mathematical Thinking and Reasoning Standards: MTR1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.8.DP.1.2

MA.8.DP.2.2

Vocabulary

New Vocabulary

line of best fit

Lesson at a Glance

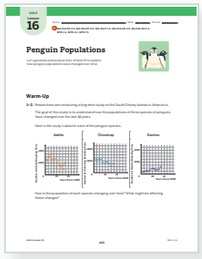
~ 45 min

MA.912.DP.2.4, MA.912.DP.2.6

Warm-Up

Pairs | 5 min

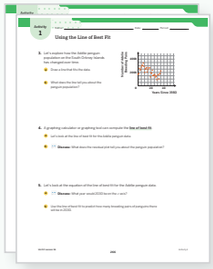
Students share their ideas about changes in penguin populations over time and what might be affecting those changes to introduce the context for the lesson.



Activity 1

Pairs | 10 min

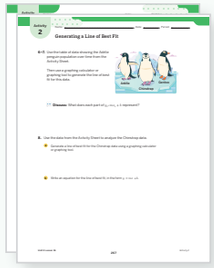
Students develop strategies to use the **line of best** fit as a tool for making precise predictions about one penguin population. **(MTR.4.1)**



Activity 2

Pairs | 10 min

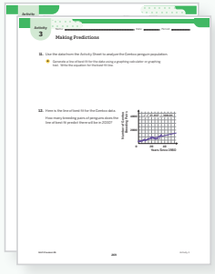
Students use the Desmos Graphing Calculator to generate a line of best fit for a new population of penguins and use the lines of best fit to compare the two populations.
Routine: Think-Pair-Share



Activity 3

Pairs | 10 min

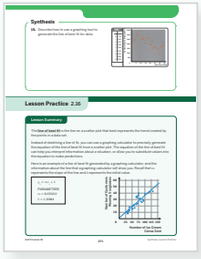
Students use lines of best fit to compare the changes with a third population of penguins and reflect on the purpose of models in making predictions.



Synthesis

Whole Class | 5 min

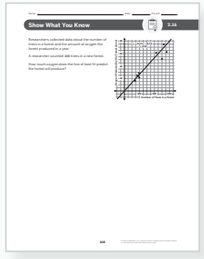
Students synthesize their understanding of how to use technology to generate the line of best fit for data on a scatter plot.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using the line of best fit to make a prediction.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Scientific calculator (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students share their ideas about changes in penguin populations over time and what might be affecting those changes to introduce the context for the lesson.

1 Launch

To surface connections, consider asking, "What do you know about Antarctica? About penguins?"

Consider sharing a map of the world to show students where the tip of South America is in relation to Antarctica.

2 Connect

Encourage students to work together in pairs to describe the general trend of the graph and then consider what that means about the penguin populations.

Select and sequence several student responses to share during the Connect.

Consider asking:

- "How can models be used to help predict what will happen to populations, like these penguin populations, in the future?"
- "Why do you think researchers might be studying how these penguin populations have changed over time?"
- "What about the data was surprising to you? What was not surprising?"
- "What kinds of questions might we use this data to answer?"

D **Differentiation: (Stretch)** Encourage students to research each of the species of penguins to learn more about them, then share their findings with the class.

Unit 2
Lesson
16

Name: _____ Date: _____ Period: _____

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.F.1.5, MA.912.AR.2.5, MA.912.AR.2.2, MTR.1.1, MTR.4.1, MTR.7.1

Penguin Populations

Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.

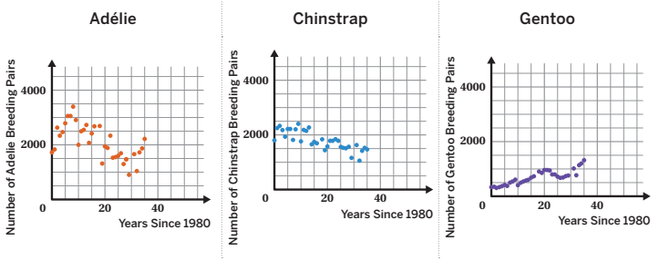


Warm-Up

1–2. Researchers are conducting a long-term study on the South Orkney Islands in Antarctica.

The goal of this study is to understand how the populations of three species of penguins have changed over the last 40 years.

Here is the study's data for each of the penguin species.



How is the population of each species changing over time? What might be affecting these changes?

Responses vary.

- The Adélie penguin population seems to be going down. But it was pretty high recently, so I'm not sure. Maybe there are different amounts of fish each year, so the penguin population keeps jumping up and down.
- The population of Chinstrap penguins seems like it's decreasing pretty steadily. Maybe there isn't enough food for them to eat, or maybe they're being eaten by predators.
- The Gentoo population is increasing! Maybe whatever changes are happening in Antarctica are actually better for them.

Unit 2 Lesson 16 265 Warm-Up

Pairs | 10 min

Activity 1 Using the Line of Best Fit

Purpose: Students develop strategies to use the line of best fit as a tool for making precise predictions about one penguin population.

1 Launch

Demonstrate how to use one point to see the values for an ordered pair (x, y).

Encourage students to estimate the y-intercept and slope for their line of fit, highlighting how each student's estimate is slightly different.

Consider asking:
- "How might a line be useful for thinking about the Adélie penguin population over time?"
- "How can we determine which line is the best for estimating how the population has changed over time?"

2 Monitor

Consider sharing with students that their line is their line of fit and the calculated line is the line of best fit.

Pause to discuss what students notice about the residuals and residual plot, highlighting that the line of best fit has the smallest possible residuals. Consider asking, "How does your line compare with the line of best fit?"

Select and sequence several strategies for using the line of best fit to make predictions to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Use the line of best fit to estimate the y-value for when $x = 50$.	**Strengthen** Celebrate the strategy they used. Consider asking, "How close do you think your prediction is to the number of penguins predicted by the line of best fit? How could we be more precise in our prediction?"
Input an expression such as $-36.13(50) + 2733.52$.	**Strengthen** Invite these students to share their strategy during the Connect.
Round the computed value to make sense for the situation.	**Strengthen** Celebrate their mathematical brilliance as they attended to precision. Consider inviting this student to share their reasoning for why it makes sense to round the answer for this situation.

3 Connect

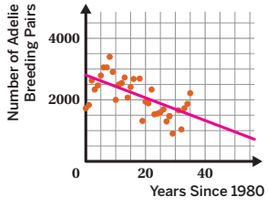
To surface the Key Takeaway, consider asking:
- "Why do you think this line is called the line of best fit?"
- "What do you think _____'s strategy was for predicting?"
- "What does the 50 mean in _____'s expression?"
- "If we use the equation, we get 927.02 breeding pairs. Is this possible? Why or why not?"

Activity 1

Name: Date: Period:

Using the Line of Best Fit

3. Let's explore how the Adélie penguin population on the South Orkney Islands has changed over time.



a. Draw a line that fits the data. *Lines vary. Sample shown on the graph.*

b. What does the line tell you about the penguin population?
Responses vary.
- The line tells me that there are about 42 fewer breeding pairs of penguins every year.
- The slope of the line is negative, so the population is going down over time.
- My line has an intercept of 2,873.7, which means there were around 2,873 breeding pairs in 1980.

4. A graphing calculator or graphing tool can compute the **line of best fit**.

a. Let's look at the line of best fit for the Adélie penguin data.

b. **Discuss:** What does the residual plot tell you about the penguin population?
Responses vary. The points on the residual plot are spread out along the x-axis.

5. Let's look at the equation of the line of best fit for the Adélie penguin data.

a. **Discuss:** What year would 2030 be on the x-axis? 50

b. Use the line of best fit to predict how many breeding pairs of penguins there will be in 2030. $-36.13(50) + 2733.52$ (or equivalent)
Responses between 926 and 928 are considered correct.

Unit 2 Lesson 16 266 Activity 1

Key Takeaway: The line of best fit is the line on a scatter plot that best represents the overall trend of the points in a data set. Out of many possible lines of fit, the line of best fit is the one that minimizes the residuals.

Pairs | 10 min

Activity 2 Generating a Line of Best Fit

Purpose: Students use a graphing calculator or a graphing tool to generate a line of best fit for a new population of penguins and use the lines of best fit to compare the two populations.

1 Launch

Consider sharing that in previous activities, students created lines of fit. In this lesson, students will generate a line of best fit using technology instead of dragging a line and visualizing if it is a good fit.

Consider asking, "How have we created a line of fit in the past? How precise was our process?"

2 Monitor

Encourage students to read the instructions and discuss each question. It may be helpful to remind students where to find the row number (in the top left corner of each row).

Pause to discuss what the values mean in the equation of the line of best fit. Consider asking:

- "What does the y_1 represent? The m? The x_1? The b?

D **Differentiation: (Support)** Students may write $y_1 \sim mx_1 + b$ and get the same results as they did for the Adélie data. Encourage them to look at the table with the Chinstrap data and consider asking, "What variables does this table use?"

Activity 2 continued >

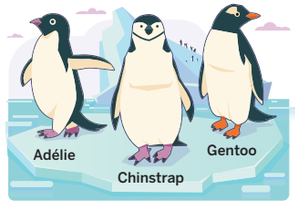
Activity 2

Name: Date: Period:

Generating a Line of Best Fit

6–7. Use the table of data showing the Adélie penguin population over time from the Activity Sheet.

Then use a graphing calculator or graphing tool to generate the line of best fit for this data.



Discuss: What does each part of $y_1 \sim mx_1 + b$ represent?

Responses vary.

- The y_1 are the values on the right side of the table.
- The m is the slope of the line.
- The x_1 are the values on the left side of the table.
- The b is the intercept of the line.

8. Use the data from the Activity Sheet to analyze the Chinstrap data.

a Generate a line of best fit for the Chinstrap data using a graphing calculator or graphing tool.

- $m = -25.4514$
- $b = 2239.59$

b Write an equation for the line of best fit, in the form $y = mx + b$.

$y = -25.45x + 2239.59$ (or equivalent)

Unit 2 Lesson 16 267 Activity 2

Pairs | 10 min

Activity 2 Generating a Line of Best Fit (continued)

Purpose: Students use a graphing calculator or a graphing tool to generate a line of best fit for a new population of penguins and use the lines of best fit to compare the two populations.

2 Monitor

Listen for students who discuss how the slope and y-intercept are powerful statistics that can be used to compare the populations of penguins (e.g., "The slope of the line of best fit for the Adélie population is -36.13 while the slope of the line of best fit for Chinstrap population is -25.45. That means the number of breeding pairs for the Adélie population is decreasing faster.").

3 Connect

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *slope*, y-*intercept*, *positive*, *negative*, *weak*, *strong*, etc. **(Writing and Speaking)**

To surface the Key Takeaway, consider asking:

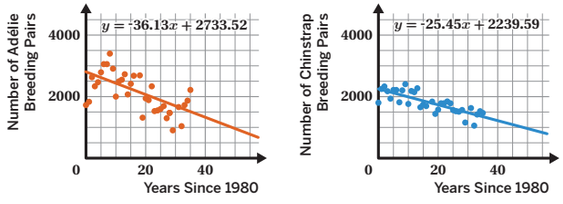
- "Which population is declining faster? What evidence do you have to support your thinking?" **(MTR.4.1)**
- "Which relationship is stronger? What evidence do you have to support your thinking?" **(MTR.4.1)**
- "How can you use the line of best fit for each population to make a prediction about the number of breeding pairs in 2030?" **(MTR.7.1)**
- "How do the lines of best fit help us compare the two populations of penguins in ways that the scatter plots do not?" **(MTR.7.1)**
- "If you were a researcher, what might your next steps be?"

Activity 2

Name: Date: Period:

Generating a Line of Best Fit (continued)

9. Compare the data of these two penguin populations.



Discuss: How are they alike? How are they different?

Responses vary.

- Both populations are decreasing over time.
- They both started with more than 2,000 breeding pairs and have had years with around 1,000 breeding pairs.
- Both of the slopes are negative.
- The Adélie started off with many more breeding pairs on the island (2,733 instead of 2,239).
- The relationship between population and time is stronger for the Chinstrap penguins. This might mean that their relationship has been more predictable so far.
- The Adélie penguin population is declining at a faster rate according to the line of best fit.

10. Recall that a linear relationship has a constant rate of change and thus, its graph is a straight line. A linear equation can be written in the form $y = mx + b$.

a. Are the equations shown on the graphs in Problem 9 linear? How do you know?

Explanations vary. Yes, these equations are linear because they show a constant rate of change and are written in the form $y = mx + b$. Their graphs are straight lines.

b. Compare these two equations and their graphs. What do you notice?

Explanations vary. The linear equation for the Adélie population has a more negative slope, which means it is decreasing faster than the linear equation for the Chinstrap population. The starting population (y-intercept) is greater for the Adélie population also.

Unit 2 Lesson 16 268 Activity 2

Key Takeaway: The line of best fit is a powerful tool that can be used to make predictions. Using substitution, it is possible to input a known value into the equation of the line of best fit and calculate a predicted value based on the generated line.

Pairs | 10 min

Activity 3 Making Predictions

Purpose: Students use lines of best fit to compare the changes with a third population of penguins and reflect on the purpose of models in making predictions.

Short on time? Consider completing Problem 10 as a whole class instead of in pairs.

1 Launch

Consider sharing that students will now compare a third population of penguins.

Encourage students to use the Adélie and Chinstrap data as examples to support them in their work.

A **Accessibility: Conceptual Processing** To support students in applying concepts to new situations, invite them to share how this situation is similar to their work on Problem 7 with a partner before responding.

2 Monitor

D **Differentiation**

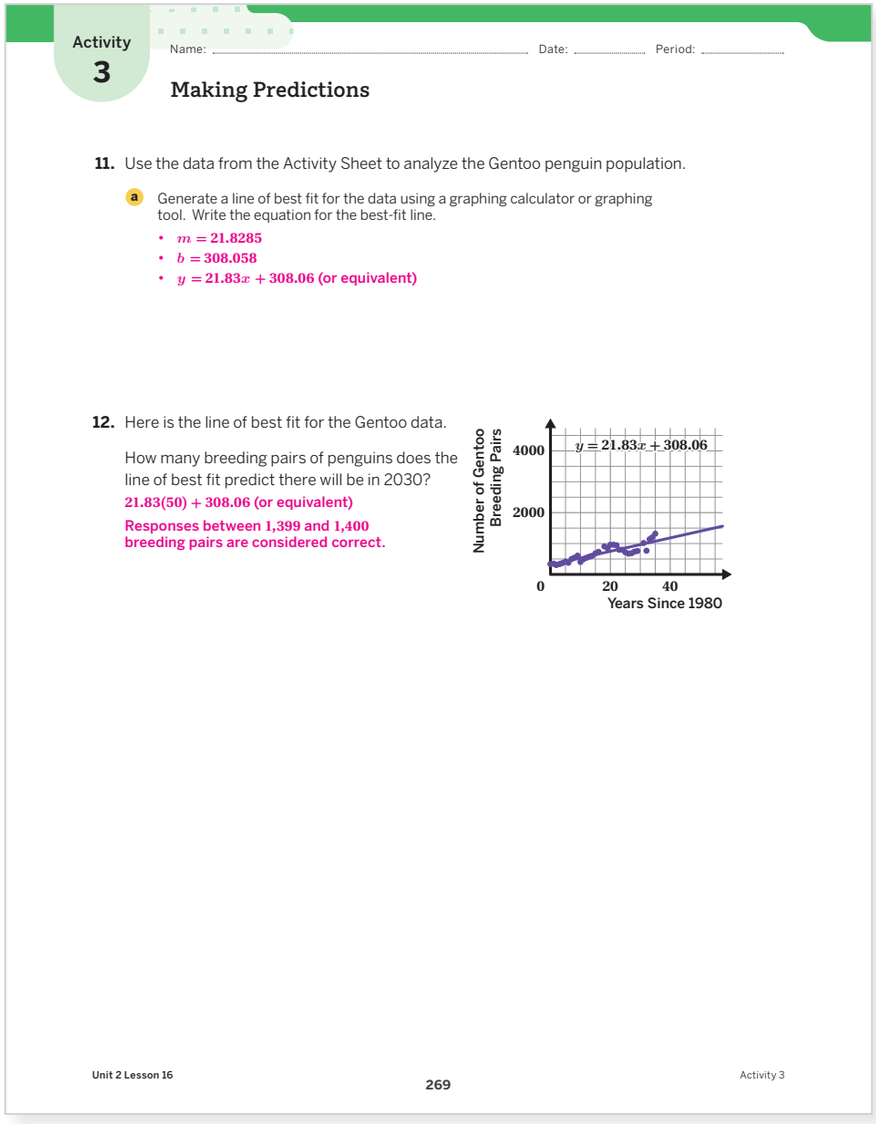
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite these students to share how this situation is similar to the work they did on Problem 7.
Write $y = mx + b$ and get sliders for m and b.	Encourage them to adjust the sliders. Consider asking them, "Is the graphing calculator or graphing tool generating the line of best fit or are you fitting the line manually? What could you type in rows 5 and 6 to see more information about the line of best fit, such as the slope and y-intercept?"
Write $y_1 \sim mx_1 + b$ and get the line of best fit for the Adélie population.	**Strengthen** Celebrate that they were able to generate a line of best fit. Consider asking, "What variables does the Gentoo data table use?"

Encourage students to enter expressions, using the input as a calculator (e.g., $21.83\ (1) + 308.06$).

Listen for students who discuss substituting the value 50 into the equation for the line of best fit as a strategy for predicting the number of breeding pairs for the Gentoo population.

D **Differentiation: (Support)** Some students may guess and check, or estimate the value by tracing along the x-axis and up to the line of best fit to find a matching y-value. Encourage these students by asking, "How can the equation of the line of best fit help us make these predictions more precise?" **(MTR.4.1)**

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Making Predictions

11. Use the data from the Activity Sheet to analyze the Gentoo penguin population.

a. Generate a line of best fit for the data using a graphing calculator or graphing tool. Write the equation for the best-fit line.
- $m = 21.8285$
- $b = 308.058$
- $y = 21.83x + 308.06$ (or equivalent)

12. Here is the line of best fit for the Gentoo data.

How many breeding pairs of penguins does the line of best fit predict there will be in 2030?

$21.83(50) + 308.06$ (or equivalent)

Responses between 1,399 and 1,400 breeding pairs are considered correct.

Unit 2 Lesson 16 269 Activity 3

Pairs | 10 min

Activity 3 Making Predictions (continued)

Purpose: Students use lines of best fit to compare the changes with a third population of penguins and reflect on the purpose of models in making predictions.

Short on time? Consider completing Problem 10 as a whole class instead of in pairs.

2 Monitor

Look for students who suggest that researchers may have questions about the rate of decline (i.e., slope) of the breeding pairs or whether the relationship is strong enough to indicate a linear relationship (i.e., how close do the data values lie related to the line). Invite these students to share during the Connect.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

Capture ideas to share and invite students to share more.

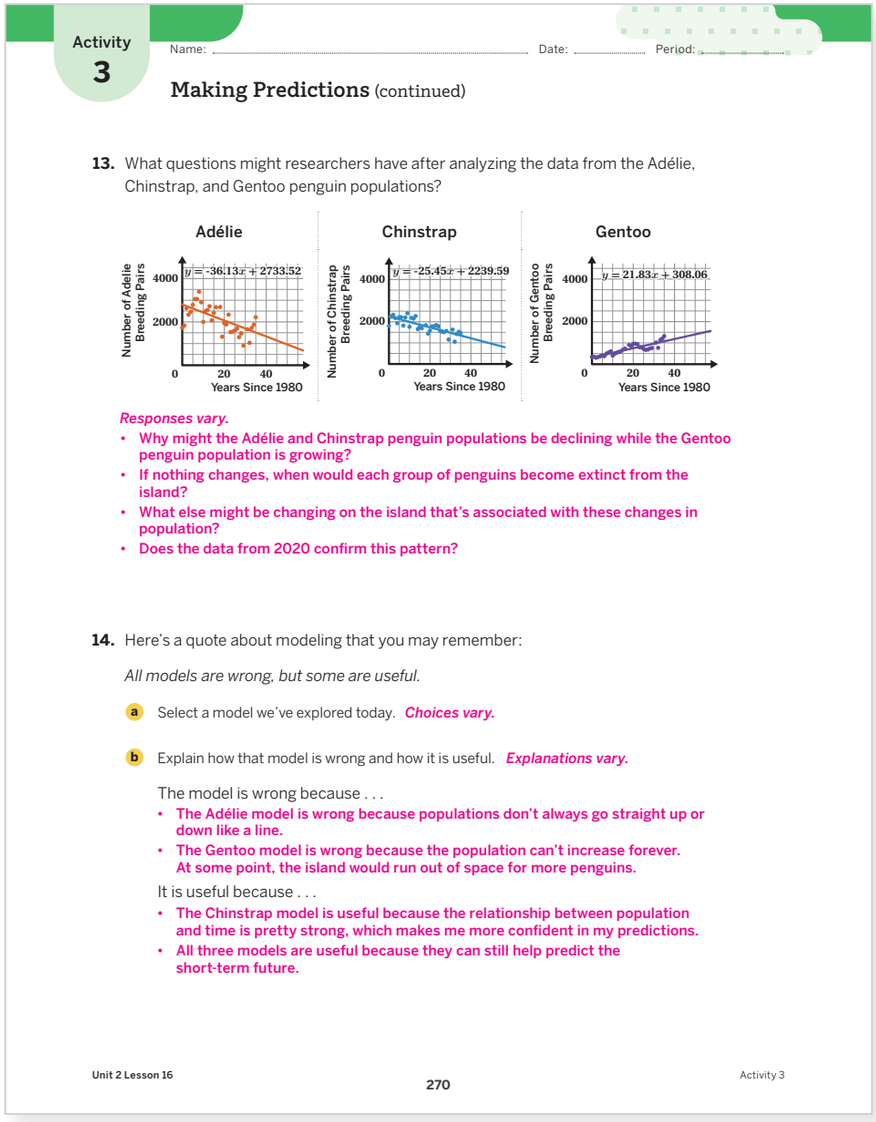
3 Connect

Consider asking:

- "What do you wonder about the populations of these penguins?"
- "What questions did the data answer about these penguin populations?"
- "What questions did the data raise about these penguin populations?"
- "How are the lines of best fit useful for making comparisons across the three populations?" **(MTR.7.1)**

To surface the Key Takeaway, consider asking:

- "Why are models wrong and useful?"
- "How were these models here useful? How were they wrong?" *Responses vary.* All models, including lines of best fit, have limitations in their ability to make accurate predictions about the future. They are useful for helping researchers know when populations are in danger, and how that danger is relative to other populations.



Activity 3

Name: _____ Date: _____ Period: _____

Making Predictions (continued)

13. What questions might researchers have after analyzing the data from the Adélie, Chinstrap, and Gentoo penguin populations?

Responses vary.

- Why might the Adélie and Chinstrap penguin populations be declining while the Gentoo penguin population is growing?
- If nothing changes, when would each group of penguins become extinct from the island?
- What else might be changing on the island that's associated with these changes in population?
- Does the data from 2020 confirm this pattern?

14. Here's a quote about modeling that you may remember:

All models are wrong, but some are useful.

a. Select a model we've explored today. *Choices vary.*

b. Explain how that model is wrong and how it is useful. *Explanations vary.*

The model is wrong because . . .

- The Adélie model is wrong because populations don't always go straight up or down like a line.
- The Gentoo model is wrong because the population can't increase forever. At some point, the island would run out of space for more penguins.

It is useful because . . .

- The Chinstrap model is useful because the relationship between population and time is pretty strong, which makes me more confident in my predictions.
- All three models are useful because they can still help predict the short-term future.

Unit 2 Lesson 16 270 Activity 3

Key Takeaway: Lines of best fit are models that can be used to make predictions. These models have limitations in their ability to make accurate predictions about the future.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to use technology to generate the line of best fit for data on a scatter plot.

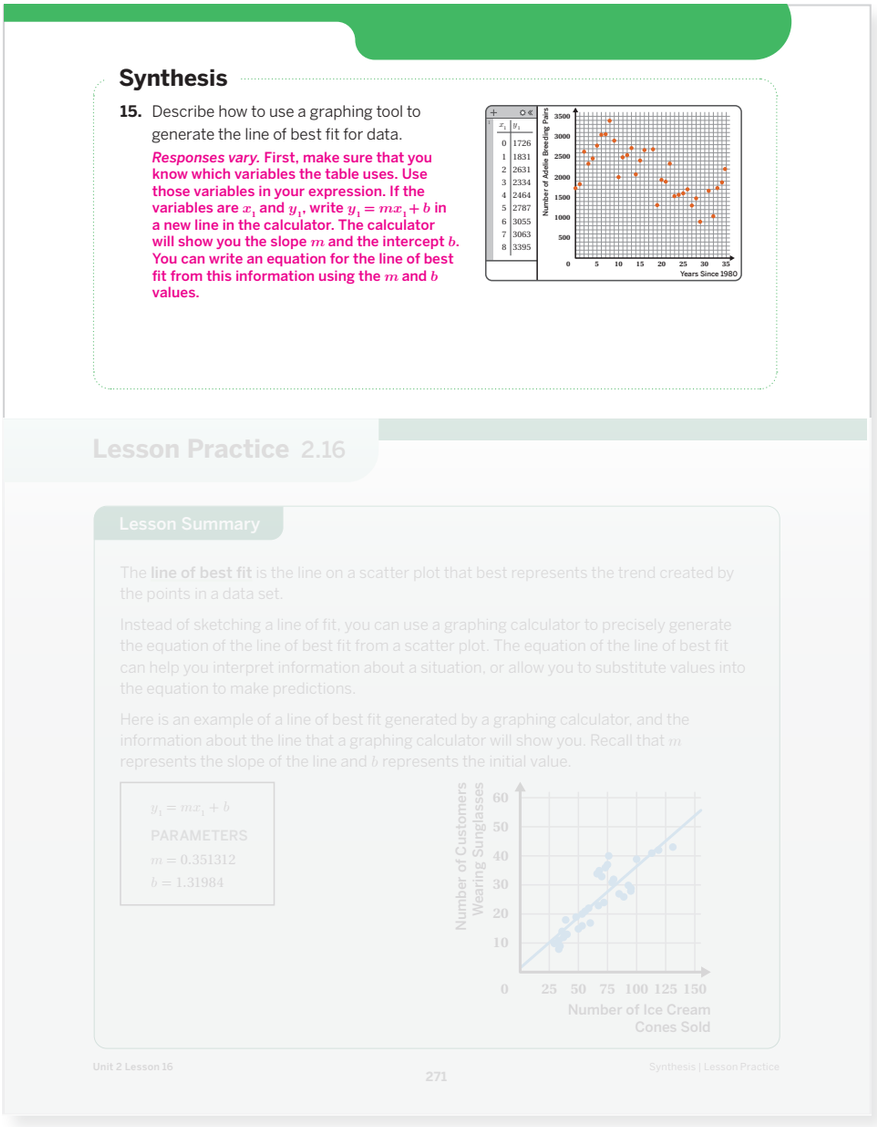
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Knowing that the variables in the table must match the equation in the expression.
- Writing $y_1 \sim mx_1 + b$ in a new line on a graphing calculator or graphing tool.
- Interpreting the data the calculator generates, such as the slope, m, and the y-intercept, b.
- Substituting the values for m and b into the equation $y = mx + b$ to write the equation for the line of best fit.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and create an anchor chart of how to generate the line of best fit. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

15. Describe how to use a graphing tool to generate the line of best fit for data.

Responses vary. First, make sure that you know which variables the table uses. Use those variables in your expression. If the variables are x_1 and y_1, write $y_1 = mx_1 + b$ in a new line in the calculator. The calculator will show you the slope m and the intercept b. You can write an equation for the line of best fit from this information using the m and b values.

x_1	y_1
0	1726
1	1831
2	2631
3	2334
4	2464
5	2787
6	3055
7	3063
8	3395

Lesson Practice 2.16

Lesson Summary

The **line of best fit** is the line on a scatter plot that best represents the trend created by the points in a data set.

Instead of sketching a line of fit, you can use a graphing calculator to precisely generate the equation of the line of best fit from a scatter plot. The equation of the line of best fit can help you interpret information about a situation, or allow you to substitute values into the equation to make predictions.

Here is an example of a line of best fit generated by a graphing calculator, and the information about the line that a graphing calculator will show you. Recall that m represents the slope of the line and b represents the initial value.

$y_1 = mx_1 + b$

PARAMETERS

$m = 0.351312$

$b = 1.31984$

Unit 2 Lesson 16 271 Synthesis | Lesson Practice

Lesson Takeaway: The **line of best fit** for a data set is a line of fit that follows the overall trend of the data as closely as possible, making the residuals as small as possible. There is only one line of best fit for any data set and not all data sets can be modeled by a line precisely. It can be used to make informed, but imperfect predictions about unknown values.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using the line of best fit to make a prediction.

Today's Goals

1. **Goal:** Use technology to generate the *line of best fit* for data on a scatter plot.
2. **Goal:** Use the equation of a *line of best fit* to make predictions.
3. **Language Goal:** Interpret the slope and y-intercept of the equation of a line of best fit in context. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.16

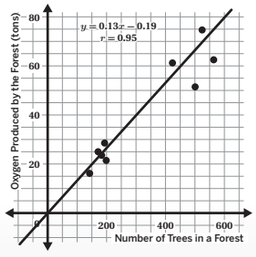
Researchers collected data about the number of trees in a forest and the amount of oxygen the forest produced in a year.

A researcher counted 400 trees in a new forest.

How much oxygen does the line of best fit predict the forest will produce?

0.13(400) − 0.19 (or equivalent)

Responses between 51 and 52 tons are considered correct.



Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support using the equation of a line of best fit to make predictions:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Take a moment to reflect on students' progress with fitting a line to a data set in this sub-unit. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for the upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

15. Describe how to use a graphing tool to generate the line of best fit for data.

Responses vary. First, make sure that you know which variables the table uses. Use those variables in your expression. If the variables are x and y, write $y_1 = mx_1 + b$ in a new line in the calculator. The calculator will show you the slope m and the intercept b. You can write an equation for the line of best fit from this information using the m and b values.

Lesson Practice 2.16

Lesson Summary

The **line of best fit** is the line on a scatter plot that best represents the trend created by the points in a data set.

Instead of sketching a line of fit, you can use a graphing calculator to precisely generate the equation of the line of best fit from a scatter plot. The equation of the line of best fit can help you interpret information about a situation, or allow you to substitute values into the equation to make predictions.

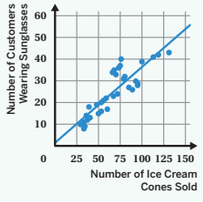
Here is an example of a line of best fit generated by a graphing calculator, and the information about the line that a graphing calculator will show you. Recall that m represents the slope of the line and b represents the initial value.

$y_1 = mx_1 + b$

PARAMETERS

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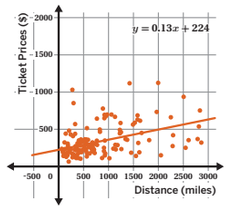
$b = 1.31984$



Lesson Practice 2.16

Name: Date: Period:

Problems 1–3: This scatter plot shows the distances of one way flights and their ticket prices.



1. The equation for the line of best fit is $y = 0.13x + 224$.

   What does 0.13 mean in this situation?

   0.13 means that for every mile that the distance increases, the price of a ticket tends to increase by $0.13.

2. The distance from Phoenix, Arizona, to Jacksonville, Florida, is $1{,}795$ miles.

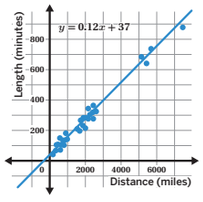
   Use the equation of line of best fit to predict the cost of a plane ticket from Phoenix to Jacksonville.

   $457.35

3. Do you think the prediction is accurate? Explain your thinking.

   Responses and explanations vary. **I don't think that the prediction is accurate because the data does not show a very strong linear correlation. Many points are not near the line.**

Problems 4–6: This scatter plot shows the distances of non-stop flights and their lengths in minutes.



4. The equation for the line of best fit is $y = 0.12x + 37$.

   What does 0.12 mean in this situation?

   0.12 means that for every mile that the distance increases, the length of a flight tends to increase by 0.12 minutes.

5. Use the equation of line of best fit to predict the length, in minutes, of a direct flight from Phoenix to Jacksonville (1,795 miles).

   252.4 minutes (or 4.2 hours)

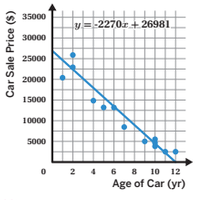
6. Do you think the prediction is accurate? Explain your thinking.

   Responses and explanations vary. **Yes, I think that the prediction is accurate because the data show a very strong linear association.**

Lesson Practice 2.16

Name: Date: Period:

Problems 7–9: Kwasi wanted to know the relationship between the ages of cars and their values. He found data on the ages of several cars (in years) and their sale prices (in dollars).



7. Describe the relationship between the age of a car and its sale price.

   Responses vary. **As the age of the car increases, the sales price decreases.**

8. Do you think one of the variables causes the other? Explain your thinking.

   Responses and explanations vary. **Yes. As cars get older, it makes sense that they would sell for a lower price.**

Test Practice

9. What else might affect this relationship? Select your answer.

   A. Option 1: How many owners have owned the car.

   B. Option 2: The mileage on the car and its maintenance.

   C. Option 3: The color of the car.

   D. Option 4: The car's brand logo placement.

Spiral Review

Problems 10–11: Here is a data set: $[1, 1, 2, 2, 3, 3, 7, 8, 9, 10, 11, 35]$

10. Complete the table. Use a graphing calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.
1	2	5	9.5	35

11. Are there any outliers in this data set? Circle one. (Yes) No

Problems 12–14: Solve each equation for y.

12. $7 = 6x - y$
    $y = 6x - 7$

13. $3y + 15x = 24$
    $y = 8 - 5x$

14. $4y - x = 44$
    $y = \frac{1}{4}x + 11$

Lesson Practice Item Analysis

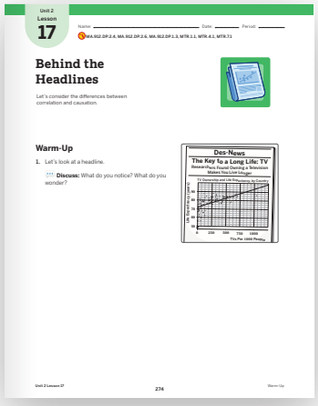
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.DP.2.4
	2–3	2	MA.912.DP.2.4, MA.912.DP.2.6
	4	1	MA.912.DP.2.4
	5–6	2	MA.912.DP.2.4, MA.912.DP.2.6
	7–8	2	MA.912.DP.2.4, MA.912.DP.1.3*
Test Practice	9	2	MA.912.DP.1.3*
Spiral Review			
	10–11	1	MA.7.DP.1.5, MA.6.DP.1.4
Fluency	12–14	1	MA.912.AR.1.2

*This problem builds toward the standard shown.

Unit 2

Lesson

17



Behind the Headlines

Causation vs. Correlation

Let's consider the differences between correlation and causation.

Learning Outcomes

Today's Goals

1. **Goal:** Determine if the relationship between two variables represents *correlation* or *causation*.
2. **Language Goal:** Analyze claims to determine if there is enough information to support causation. **(Reading, Writing, and Listening)**

Students consider the difference between **causation** and **correlation**, and use that knowledge to analyze headlines. Students apply what they've learned in earlier lessons to describe the relationship between two variables on a scatter plot, then ask themselves whether or not one variable likely causes the other. This supports students in attending to precision as they communicate how variables are related to one another. They also learn to be critical consumers of headlines, as they ask themselves questions about whether the data supports the claims that the headlines are making, or if further information is needed. **(MTR.4.1)**

Coherence

Prior Learning

In Lessons 14 and 15, students interpreted linear relationships between two variables using lines of best fit and residuals.

Future Learning

In future courses, students use causation and correlation to predict future events.

Connections

- Students explore the differences between correlation and causation to determine whether a relationship exists between two variables. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.DP.1.3
Explain the difference between correlation and causation in the contexts of both numerical and categorical data.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.DP.1.1	**MA.912.DP.5**
MA.912.DP.2.4	

Vocabulary

New Vocabulary

causation
correlation

Review Vocabulary

lurking variables
scatter plot

Lesson at a Glance

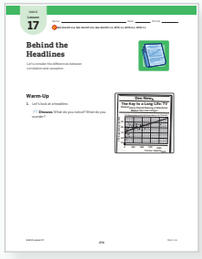
~ 45 min

MA.912.DP.1.3

Warm-Up Fluency

Whole Class | 5 min

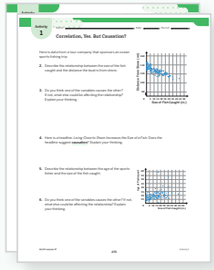
Students use the Notice and Wonder routine to make sense of data.
Routine(s): Notice and Wonder



Activity 1

Pairs | 20 min

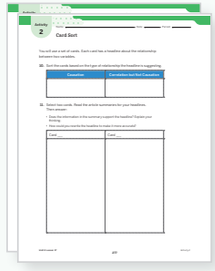
Students analyze data sets to determine whether the relationships represent causation or correlation.
Routine(s): Think-Pair-Share



Activity 2

Pairs | 10 min

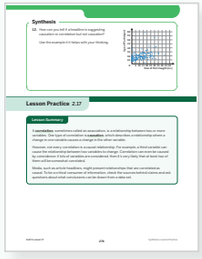
Students develop their skills as critical consumers of information as they consider whether there is enough information in the article to support each headline. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

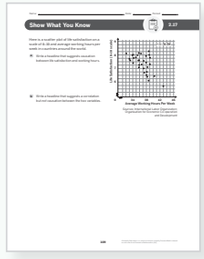
Students synthesize their understanding of the difference between a relationship that represents causation and one that represents correlation but not causation.
Routine(s): MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing a scatter plot and writing a headline that supports causation and one that supports correlation but not causation.



Prep Checklist

Prepare Activity 2 Cards and Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 2 Cards, one set per pair, pre-cut; Prepare Activity 2 Sheet, one per student

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of data.

1 Launch

Display the headline.

Consider sharing that each data point represents a different country.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the headline about the relationship between television ownership and life expectancy.

Create a record of the things students noticed and wondered, along with the names of the students who shared them.

Consider asking, "Did anyone notice any outliers for this data? Do you wonder about which country that might be?"

2 Connect

Consider asking:

- "Where have you seen headlines like this?" Responses vary. Newspapers, late-night television commercials, advertisements for buying televisions.
- "What about this headline do you find surprising, if anything?" Responses vary. I think it is interesting that the data for TV ownership is correlated with life expectancy.
- "Do you think that TV ownership is related to life expectancy? What evidence do you have to justify your thinking?"

Unit 2
Lesson
17

Name: _____ Date: _____ Period: _____

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1

Behind the Headlines

Let's consider the differences between correlation and causation.

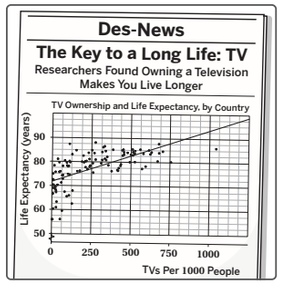


Warm-Up

1. Let's look at a headline.

   Discuss: What do you notice? What do you wonder?

   Responses vary.
   - I notice there is a positive relationship between TV ownership and life expectancy.
   - I notice that the headline sounds very convincing.
   - I notice that the data between TV ownership and life expectancy seems to be correlated.
   - I wonder if it's actually true that owning TVs causes you to live longer.
   - I wonder how the researchers collected the data.



Unit 2 Lesson 17 274 Warm-Up

Pairs | 20 min

Activity 1 Correlation, Yes. But Causation?

Purpose: Students analyze data sets to determine whether the relationships represent causation or correlation.

1 Launch

Consider asking, "How would you describe the difference between causation and correlation?"

Use the Think-Pair-Share routine, to help students shape their ideas about the difference between causation and correlation. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and inviting them to share in that language. **(Writing, Speaking, and Listening)**

Consider sharing that when we're curious about why there is a strong relationship between two variables, a *lurking variable* may explain the reason for the correlation.

Consider asking:

- "How would you describe what a causal relationship is to someone who is absent?"
- "What *lurking variables* may explain the correlation between the size of fish caught and the distance from shore?"

2 Monitor

To support students in getting started, consider asking, "Where have you seen this data before?"

Pause briefly once students have completed Problem 4 to discuss how students decided whether or not a headline suggested causation. Consider asking, "What words were clues to whether the headline suggested causation or not?"

I noticed the word "increases", which sounds like one definitely causes the other to change. If it had said "may increase" or "can increase," that would imply that one variable is related to but does not necessarily cause the other to change.

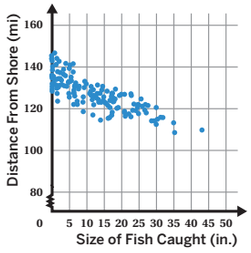
Activity 1 continued >

Activity 1

Name: Date: Period:

Correlation, Yes. But Causation?

Here is data from a tour company that sponsors an ocean sports fishing trip.



2. Describe the relationship between the size of the fish caught and the distance the boat is from shore.

   Responses vary. The relationship between the size of the fish caught and the distance the boat is from shore is strong and negative.

3. Do you think one of the variables causes the other? If not, what else could be affecting the relationship? Explain your thinking.

   Responses and explanations vary.

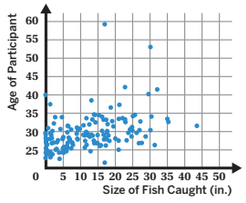
   - No, I don't think the distance from shore would cause the size of fish to decrease. I think what matters would be the type of fishing line and the type of lure used to catch fish in waters farther from shore.
   - I'm not sure if different-sized fish live in different parts of the ocean. It is possible that larger fish are found closer to shore.

4. Here is a headline: *Living Close to Shore Increases the Size of a Fish.* Does the headline suggest **causation**? Explain your thinking.

   Yes. *Explanations vary.* The headline says that living close to shore *increases* a fish's size, which suggests that the location of the fish affects the growth of the fish.

5. Describe the relationship between the age of the sports fisher and the size of the fish caught.

   Responses vary. The relationship is positive and not weak or strong.



6. Do you think one of the variables *causes* the other? If not, what else could be affecting the relationship? Explain your thinking.

   Explanations vary. It is not true that the size of the fish would change the age of the participant. It is possible that older participants have more experience and can catch larger fish, but young people can also be very skilled at fishing. The relationship might just be affected by random noise.

Unit 2 Lesson 17 275 Activity 1

Pairs | 20 min

Activity 1 Correlation, Yes. But Causation? (continued)

Purpose: Students analyze data sets to determine whether the relationships represent causation or correlation.

2 Monitor

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, with words such as positive, negative, strong, weak, relationship, correlation, causation, etc. **(Writing and Speaking)**

D **Differentiation** (*Problems 5–9*)

Look for students who:	Respond to Student Thinking
Need support describing the relationship between two variables. (*Problem 5*)	**Support** Encourage them to use the word bank. Consider asking, "What word in the word bank matches the relationship you see?"
Need support deciding if a relationship is strong or weak. (*Problem 5*)	**Support** Consider asking, "How can the correlation coefficient help us know if this relationship is strong or weak?"
Need practice justifying their thinking for why a relationship is causal or correlated. (*Problem 6*)	**Strengthen** Invite them to share their thinking with a partner. Encourage students to revise their thinking after the discussion in the Connect.
Enjoy writing and analyzing headlines as it relates to correlation versus causation.	**Stretch** Invite students to write their own headlines for any data sets in this activity. Challenge them to write one headline that implies causation and one that implies correlation. Encourage them to share their thinking with a partner.

3 Connect

To surface the Key Takeaway, consider asking:

- "How did you rewrite the headline about the number of pool users and the number of mall visitors?"
- "How did you decide whether or not a headline suggested causation?" **(MTR.4.1)**
- "Is the number of pool users in a causal relationship with the number of mall visitors? What reasoning or evidence supports your claim?"

Activity 1

Name: Date: Period:

Correlation, Yes. But Causation? (continued)

7. Here is a headline: *Older Fishers Lure Larger Fish.* How could you rewrite the headline to suggest **correlation** but *not* causation?

Headlines vary.

- Age of Fishers Are Related to Fish Size
- Positive Association Found Between Age of Fishers and Fish Size

8. Here are two headlines for the same data.

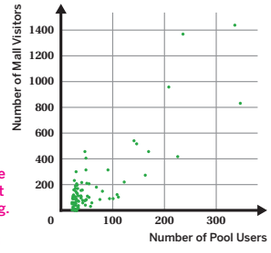
A: *Swimming Causes an Urge to Shop*

B: *Mall Shopping Correlated with Pool Swimming*

Which headline do you think is more accurate?

Explain your thinking.

Responses and explanations vary. I think headline B is more accurate because it only says that there is a relationship. It doesn't imply that swimming causes people to go shopping. It might be true that people want to go shopping after having a good workout at the pool, but it could just be that people go to the mall and go to the pool on nice days.



9. The two-way frequency table shows the number of families living in a certain town who have at least one pet and whether those families recycle. Here are two headlines.

A: *Pet Ownership Associated with Recycling*

B: *More Pets in Your Home Leads to Greater Recycling*

Which headline do you think is more accurate? Explain your thinking.

	Have at least one pet	Do not have at least one pet	Total
Recycles	244	68	312
Does not recycle	49	139	188
Total	293	207	500

Responses and explanations vary. I think headline A is more accurate because it only says that there is a relationship. Headline B implies that a family having a pet will cause the family to recycle more, which is not necessarily true.

Unit 2 Lesson 17 276 Activity 1

Key Takeaway: **Causation** is one type of correlation where a change in one variable causes a change in another. While many variables have **correlations**, more evidence is needed to justify a causal relationship.

Pairs | 10 min

Activity 2 Card Sort

Purpose: Students develop their skills as critical consumers of information as they consider whether there is enough information in the article to support each headline. **(MTR.4.1)**

Short on time? Consider inviting students to analyze the article summary for one card instead of two.

1 Launch

Distribute the Activity 2 Cards to each pair of students, and an Article Summary Sheet to each student.

Consider asking, "What can you look for to decide if there is enough information to support a headline?"

2 Monitor

Encourage students to share their reasoning with a partner and agree about sorting the headlines.

Invite pairs to compare their sortings, justify their thinking, and make revisions.

A **Accessibility: Conceptual Processing** As students look for supporting information within the articles, invite them to answer a more concrete question first (e.g., "Where in the article do you see information connected to this headline?")

3 Connect

Invite students to discuss whether they thought an article supported a headline and to justify their thinking. **(MTR.4.1)**

Note: It is okay to lack consensus about whether there is enough information to support the headline. The purpose of this activity is to develop skills as critical consumers of information.

Consider asking:

- "How did you decide whether the article supported the headline?"
- "What might the impact be of a headline that inaccurately suggests causation?"
- "What *lurking variables* could be affecting this relationship?"

Activity 2

Name: _____ Date: _____ Period: _____

Card Sort

You will use a set of cards. Each card has a headline about the relationship between two variables.

10. Sort the cards based on the type of relationship the headline is suggesting.

Causation	Correlation but Not Causation
A, B	C, D

11. Select two cards. Read the article summaries for your headlines. Then answer:

- Does the information in the summary support the headline? Explain your thinking.
- How could you rewrite the headline to make it more accurate?

Card ___	Card ___
Responses vary. **Card A** • Yes, the article provides data about the correlation between noise and honey production. It further speculates a mechanism by which the correlation might be causative. • Stress Caused By Loud Noise May Decrease Honey Production **Card B** • No, the article talks about a correlation between the number of pages assigned in a course and a student's SAT score, but the headline suggests that students would have a better chance of getting into college. • More Reading in Class Improves SAT Scores	*Responses vary.* **Card C** • No, the headline makes it seem that the purpose of the article is to describe the correlation between coffee and exercise. Instead, the article discusses the new coffee products offered at the gym. • Gym to Offer New Coffee Products **Card D** • Yes, the article describes the correlation between dog ownership and the owner's age. It hypothesizes a possible explanation for the correlation but does not draw stronger conclusions than that. • Owner's Age is Correlated to Dog Ownership

Unit 2 Lesson 17 277 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the difference between a relationship that represents causation and one that represents correlation but not causation.

IInvite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Words that are definitive such as "lowers" or "causes" might suggest causation.
- Words that are more vague or general such as "associated with," "related to," and "may lower" may suggest a correlation.
- If the relationship between two variables is strongly correlated, it doesn't mean there is enough reasoning to support causation.

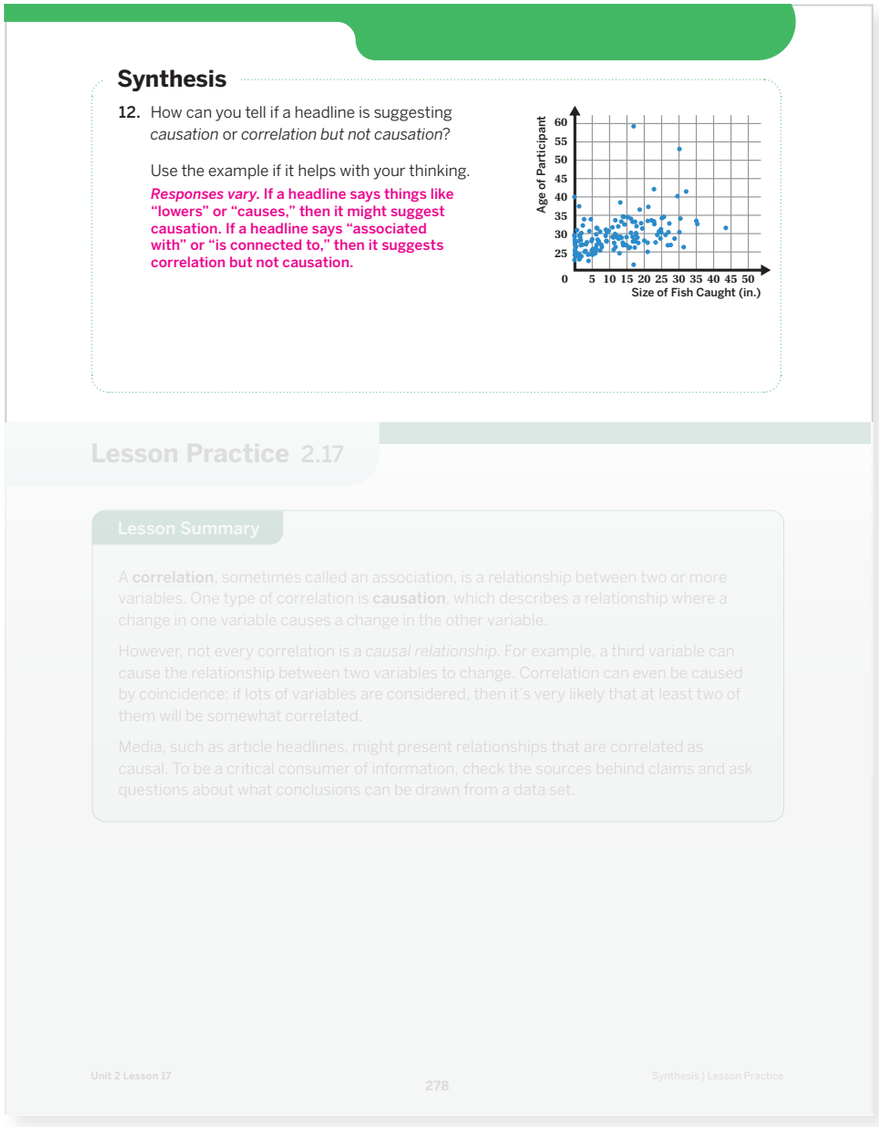
Encourage students to write one headline that suggests causation and one that suggests correlation that could match this data and share it with a partner.

Consider asking:

- "What questions might help decide if a headline represents causation or not?"
- "Why do you think it is important to be able to distinguish between causation and correlation?"
- "Why is it important to be a critical consumer of information and media?"

ELL **MLR2: Collect and Display** Use this routine to collect students' ideas to create a definition of causation and correlation but not causation, using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. How can you tell if a headline is suggesting *causation* or *correlation but not causation*?

    Use the example if it helps with your thinking.

    Responses vary. **If a headline says things like "lowers" or "causes," then it might suggest causation. If a headline says "associated with" or "is connected to," then it suggests correlation but not causation.**

Lesson Practice 2.17

Lesson Summary

A **correlation**, sometimes called an association, is a relationship between two or more variables. One type of correlation is **causation**, which describes a relationship where a change in one variable causes a change in the other variable.

However, not every correlation is a *causal relationship*. For example, a third variable can cause the relationship between two variables to change. Correlation can even be caused by coincidence: if lots of variables are considered, then it's very likely that at least two of them will be somewhat correlated.

Media, such as article headlines, might present relationships that are correlated as causal. To be a critical consumer of information, check the sources behind claims and ask questions about what conclusions can be drawn from a data set.

Unit 2 Lesson 17 278 Synthesis | Lesson Practice

Lesson Takeaway: Headlines in newspapers can be deceiving. Critical consumers make sure that the information they consume is supported by data and evidence.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing a scatter plot and writing a headline that supports causation and one that supports correlation but not causation.

Today's Goals

1. **Goal:** Determine if the relationship between two variables represents correlation or causation.
2. **Language Goal:** Analyze claims to determine if there is enough information to support causation. **(Reading, Writing, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.17

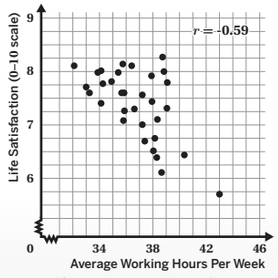
Here is a scatter plot of life satisfaction on a scale of 0–10 and average working hours per week in countries around the world.

a. Write a headline that suggests causation between life satisfaction and working hours.

Responses vary. Want to Be More Satisfied With Your Life? Work Less!

b. Write a headline that suggests a correlation but *not* causation between the two variables.

Responses vary. World Data Shows Relationship Between Work Hours and Life Satisfaction



Sources: International Labor Organization; Organisation for Economic Co-operation and Development

Differentiation Use after Lesson 17

Support

Provide targeted intervention for students by using these resources.

If students need support analyzing a scatter plot and writing a headline that supports causation and one that supports correlation but not causation.

» **Respond to Student Thinking:**

- Consider inviting groups to share their thoughts about whether a relationship is causative with the class.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to bring in news headlines they find that suggest causation or correlation throughout the year.

Professional Learning

Today, students explored the topic of **correlation** versus **causation**, and considered what it means to be a critical consumer of information. What evidence did students show of their ability to be critical consumers of information? How can you support students in developing a healthy skepticism of information and media? How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can you tell if a headline is suggesting *causation* or *correlation but not causation*?

Use the example if it helps with your thinking.

Responses vary. If a headline says things like "lowers" or "causes," then it might suggest causation. If a headline says "associated with" or "is connected to," then it suggests correlation but not causation.

Lesson Practice 2.17

Lesson Summary

A **correlation**, sometimes called an association, is a relationship between two or more variables. One type of correlation is **causation**, which describes a relationship where a change in one variable causes a change in the other variable.

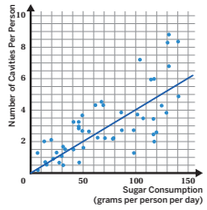
However, not every correlation is a *causal relationship*. For example, a third variable can cause the relationship between two variables to change. Correlation can even be caused by coincidence: if lots of variables are considered, then it's very likely that at least two of them will be somewhat correlated.

Media, such as article headlines, might present relationships that are correlated as causal. To be a critical consumer of information, check the sources behind claims and ask questions about what conclusions can be drawn from a data set.

Lesson Practice 2.17

Name: ..... Date: ..... Period: .....

Problems 1–2: This scatter plot shows data from different countries about average sugar consumption and tooth cavities.



1. Which statement best describes the data in the graph?

   A. The rate of tooth cavities has increased in many countries.

   B. The rate of sugar consumption has increased in many countries.

   (C.) People who eat more sugar are likely to have more cavities.

   D. People brush their teeth more frequently in some countries than others.

2. Describe the relationship between sugar consumption and teeth cavities.

   Responses vary. As sugar consumption increases, the number of cavities per person tends to increase. There is a positive correlation between sugar consumption and the number of cavities.

3. A farmer notices that the apple trees in the orchard produce more fruit when more fertilizer is applied. Can one conclude that this positive correlation shows a causal relationship?

   A. Yes, because it is not a negative correlation.

   B. Yes, because fertilizers cause trees to grow larger.

   C. No, because fertilizers only work on flowers and grass.

   (D.) No, because it is possible that the trees produce more fruit due to another factor.

Problems 4–5: Circle the type of relationship each headline suggests.

4. Brushing Twice a Day Can Prevent Gum Disease

   (Causation) Correlation Only

5. Sitting Too Much Decreases Life Expectancy

   (Causation) Correlation Only

Lesson Practice 2.17

Name: ..... Date: ..... Period: .....

Test Practice

6. Which statements are true regarding correlation and causation?

   ☐ A. Correlation is when an increase in one variable causes another variable to increase.

   ☑ B. Correlation is when a change in one variable happens at the same time as another variable.

   ☐ C. Correlation is when a change in one variable causes a change in another variable.

   ☐ D. Causation is when an increase in one variable causes another variable to increase.

   ☐ E. Causation is when a change in one variable happens at the same time as another variable.

   ☑ F. Causation is when a change in one variable causes a change in another variable.

Spiral Review

7. Select all of the equations that are equivalent to $20 + 2m - 8 = m + 17$.

   ☐ A. $20 - 8 + 17 = -m$

   ☑ B. $2m + 12 = m + 17$

   ☐ C. $3m = 17 - 20 + 8$

   ☑ D. $m = 5$

   ☑ E. $2m = m + 5$

8. Write an equation to match the table.

   $y = 5(3)^x$

x	0	1	2	3	4
y	5	15	45	135	405

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–5	1	MA.912.DP.1.3
	2–3	2	MA.912.DP.1.3
Test Practice	6	1	MA.912.DP.1.3
Spiral Review			
Fluency	7	2	MA.8.AR.2.1
	8	2	MA.912.AR.5.4

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Compare the center of two data sets and choose the appppropriate data display.	Lesson 5	2	MA.912.DP.1.2 MTR.4.1
2	Interpret data represented in box plots.	Lesson 4	2	MA.912.DP.1.1 MTR.4.1
3	Estimate a population using survey data.	Lesson 8	2	MA.912.DP.1.4 MTR.5.1
4	Describe the correlation coefficient that matches the scatter plot.	Lesson 14	1	MA.912.DP.2.6 MTR.4.1
5a	Use a line of fit to solve problems in the context of the data.	Lesson 14	2	MA.912.DP.2.4 MTR.7.1
5b	Distinguish between correlation and causation.	Lesson 17	3	MA.912.DP.2.6 MTR.7.1
6a, 6c	Assess a line of fit by analyzing a residual plot.	Lesson 15	2	MA.912.DP.2.6 MTR.4.1
6b	Fit a linear function to a scatter plot.	Lesson 15	2	MA.912.DP.2.6 MTR.7.1
7	Compare and interpret the spread of data sets in context.	Lesson 7	3	MA.912.DP.1.2 MTR.4.1
8	Estimate a percent using survey data.	Lesson 9	2	MA.912.DP.3.1 MTR.4.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 2

1. Here is information about how many text messages two students sent on each day last week.

Number of Text Messages Amara Sent
0, 2, 5, 6, 9, 10, 15

Number of Text Messages Marquis Sent
0, 9, 10, 10, 10, 11, 16

a. Whose data has a smaller mean? Circle one.

(Amara) Marquis They're the same

b. Which data display would be best to find the range of the number of text messages sent? Circle one.

(dot plot) histogram bar graph

2. Two companies analyzed the hourly wages for their employees. Here are the results.



Select *all* the true statements.

- [x] **A.** All of the employees have an hourly wage of at least $10.
- [ ] **B.** The median hourly wage for Company B is $20.
- [ ] **C.** The IQR of Company A's hourly wage is $35.
- [x] **D.** The IQR of Company B's hourly wage is $10.
- [ ] **E.** The employees at Company B typically have a greater hourly wage than the employees at Company A.

	Math	Science	English	Social Studies
Number of Students	21	37	15	27

3. A school conducted a survey of 100 students to estimate how many students in the entire school prefer different academic subjects. The school has 1,200 students in total.

Based on the survey results, approximately how many students in the school prefer science?

A. 180 **B.** 252 **C.** 370 (**D.** 444)

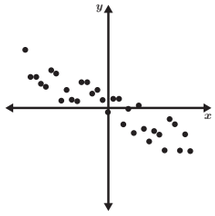
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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 2

4. The scatter plot shows the relationship between two variables. Which statement best describes the correlation in the scatter plot?



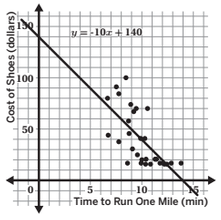
A. There is a weak positive correlation.

B. There is a strong positive correlation.

C. There is a weak negative correlation.

(**D.**) There is a strong negative correlation.

5. The scatter plot shows the number of minutes it took different people on a track team to run one mile, along with the cost of their shoes.



a. According to the model, how much would a person who runs a mile in 10 minutes pay for their shoes?

40 dollars

b. Makayla says: *Based on this data, you'll run faster if you spend more money on your shoes.*

Is Makayla's claim accurate?

No

Explain your thinking.

Explanations vary. While the data shows a strong negative association, that doesn't mean there is a causal relationship. You won't necessarily run faster if you spend more money on shoes, and even if it were true, you would need to test this theory in order to prove a causal relationship.

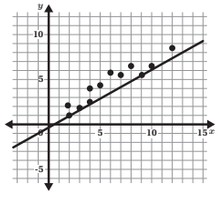
Name: Date: Period:

End-of-Unit Assessment (continued)

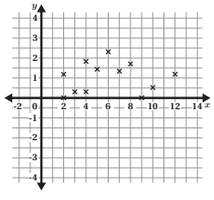
Unit 2

6. Here is the line of fit Rafael made for this scatter plot as well as the residual plot.

Scatter Plot



Residual Plot

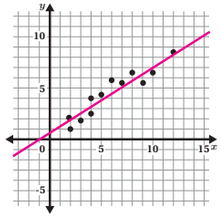


a. List one thing Rafael did well when making his line of fit and one thing he could improve.

Did Well	Could Improve
Responses vary. One thing Rafael did well was draw his line of fit through some of the points and also in the general direction of the points.	*Responses vary*. One thing Rafael could improve is to make the line go more through the middle of the points so that about half the points are above the line and half are below.

b. Draw a line that is a better fit for the data.

Lines vary. Sample shown on graph.



c. Describe what the residual plot for your line might look like.

Responses vary. The residual plot for my line of fit would have residuals that are scattered both above and below the x-axis, and they are all close to the x-axis.

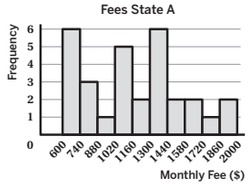
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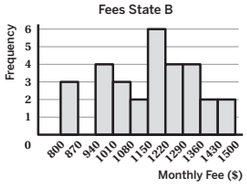
End-of-Unit Assessment (continued)

Unit 2

7. The Department of Transportation is analyzing vehicle registration fees in two different states.

The histograms below show the monthly registration fees for vehicles in two states, along with some statistics about the data.





	Mean	Median	Minimum	Q1	Q3	Maximum
State A	$1,192	$1,204	$620	$808	$1,424	$1959
State B	$1,164	$1,159	$821	$1,024	$1,332	$1,481

a. State A has a larger interquartile range (IQR) than State B. What does this tell you about the monthly registration fees in these states?

Responses vary. It tells me that there is more variability in registration fees in State A, meaning fees are more spread out. In State B, fees are more consistent within the middle 50% of the data because its IQR is smaller.

b. Why might someone think the registration fees in State A are lower?

Responses vary. Someone might think the registration fees in State A are lower because the minimum fee is lower at $620, while State B's minimum is $821. Also, State A has some fees below $800, while State B has no fees below $800.

c. Why might someone think the registration fees in State B are lower?

Responses vary. Someone might think the registration fees in State B are lower because the mean and median are both lower than in State A. Also, State A has maximum fees up to $1,959, while State B's fees do not go above $1,481.

Name: Date: Period:

End-of-Unit Assessment

Unit 2

1. Here is information about how many minutes two students spent on phone calls per day last week.

Number of Minutes Yolanda Spent
10, 15, 6, 8, 5, 7, 12

Number of Minutes Lucia Spent
25, 10, 15, 5, 0, 2, 6

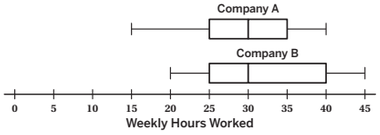
a. Whose data has a larger mean? Circle one.

Yolanda Lucia (They're the same)

b. Which data display would be best to find the median number of minutes spent on phone calls per day last week? Circle one.

dot plot histogram (box plot)

2. Two companies analyzed the weekly hours worked by their employees. Here are the results.



Select *all* the true statements.

- ☐ A. All employees worked less than 40 hours per week.
- ☑ B. Company A's and B's median weekly hours worked is 30 hours.
- ☐ C. The IQR of Company A's weekly hours worked is 5 hours.
- ☑ D. The IQR of Company B's weekly hours worked is 15 hours.
- ☑ E. The minimum number of weekly hours worked for Company B is greater than Company A.

	Math	Science	English	Social Studies
Number of Students	21	37	15	27

3. A school conducted a survey of 100 students to estimate student subject preferences. The school has 1,200 students in total.

Based on the survey results, what is the best estimate for the percentage of students in the school who prefer social studies?

A. 15 B. 21 (C. 27) D. 32

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Name: Date: Period:

End-of-Unit Assessment (continued)

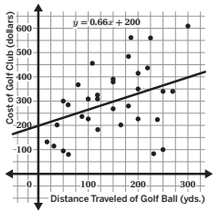
Unit 2

4. The scatter plot shows the relationship between two variables. Which statement best describes the correlation in the scatter plot?



A. There is no correlation.
B. There is a strong positive correlation.
C. There is a weak positive correlation.
D. There is a strong negative correlation.

5. This scatter plot shows the distance many golf balls traveled, along with the cost of the golf club used to hit each ball.



a. According to the model, how much would a person who hits a golf ball 300 yards pay for the golf club?

400 dollars

b. Malik says: *Based on this data, you'll hit a golf ball farther if you spend more money on your club.*

Is Malik's claim accurate?

No

Explain your thinking.

Explanations vary. **While the data shows a strong positive association, that doesn't mean there is a causal relationship. You won't necessarily hit the golf ball farther if you spend more money on the club, and even if it were true, you would need to test this theory in order to prove a causal relationship.**

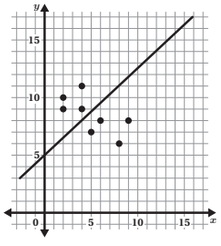
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End-of-Unit Assessment (continued)

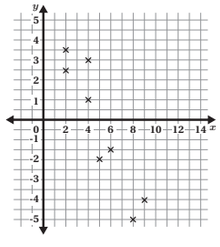
Unit 2

6. Here is the line of fit Vicente made for this scatter plot as well as the residual plot.

Scatter Plot



Residual Plot



a. List one thing Vicente did well when making his line of fit and one thing he could improve.

Did Well

Responses vary. **One thing Vicente did well was he drew his line of fit through some of the points so that about half the points are above the line and half are below.**

Could Improve

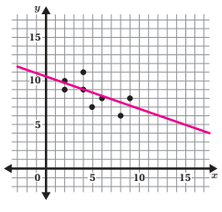
Responses vary. **One thing Vicente could improve is to make the line trend downwards instead of upwards since the points show a negative association.**

b. Draw a line that is a better fit for the data.

Lines vary. **Sample shown on graph.**

c. Describe what the residual plot for your line might look like.

Responses vary. **The residual plot for my line of fit would have residuals that are scattered both above and below the x-axis, and they are all close to the x-axis.**



Name: Date: Period:

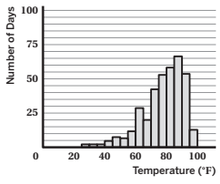
End-of-Unit Assessment (continued)

Unit 2

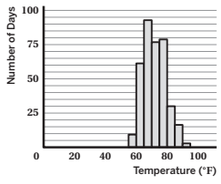
7. A family is trying to decide which of two cities has warmer weather.

Here are histograms that show the daily high temperatures in degrees Fahrenheit on each day in 2021, along with some statistics about the data.

Hattiesburg, Mississippi



San Diego, California



	Mean (°F)	Median (°F)	Minimum (°F)	Q1 (°F)	Q3 (°F)	Maximum (°F)
Hattiesburg, MS	77.7	80	25	71	88	96
San Diego, CA	71.5	71	56	66	77	92

a. Hattiesburg has a large interquartile range (IQR) than San Diego. What does that tell you about the daily high temperatures in the two cities?

Responses vary. **It tells me that the temperatures in Hattiesburg vary more than in San Diego, meaning the middle 50% of temperatures are more spread out. In San Diego, temperatures are more consistent within the middle 50% of the data.**

b. Why might someone think the temperatures in San Diego are lower?

Responses vary. **Someone might think the temperatures in San Diego are lower because the mean and median are both lower than Hattiesburg's. Also, there aren't any days where the temperature is over 92° like there are in Hattiesburg.**

c. Why might someone think the temperatures in Hattiesburg are lower?

Responses vary. **Someone might think the temperatures in Hattiesburg are lower because the minimum temperature is lower. Also, Hattiesburg has days where the temperature is less than 56°, while San Diego doesn't have days like that.**

D Differentiation (End-of-Unit Assessment)

Note: Consider inviting students to revise the corresponding assessment items after responding to student thinking.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 2
• Represent data with dot plots, histograms, and box plots.
• Describe the shape, center, and spread of data.
(Lessons 3–10)
	1, 2, 7	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 4 (*Better Weather?*)
		  • Lesson 5 (*Quick Pick*)
		  • Lesson 7 (*Dynamic Decades*)

		Stretch • **Teacher Move:** Invite students to research a new topic and find data from two different groups or years to compare using statistics appropriate to the shape of their data.
	3	**Support** • **Mini-Lesson:** Estimating Population from a Sample Survey • **Teacher Move:** Consider revisiting Lesson 8 (*How Big?*)

Sub-Unit 3
• Represent categorical data in two-way tables and relative frequency tables.
(Lessons 11–12)
• Describe linear associations in data using the line of fit and correlation coefficient.
	8	**Support**
		• **Mini-Lesson:** Calculating Values in Two-Way and Relative Frequency Tables
		• **Teacher Move:** Consider revisiting:
		  • Lesson 11 (*Trains and Traffic*)
		  • Lesson 12 (*Remodel Choices*)

	4, 5b	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 14 (*City Slopes*)
		  • Lesson 17 (*Behind the Headlines*)

• Fit a line to data, assess the fit using residual plots, and use the linear model to make predictions.
(Lessons 13–17)
	5a, 7	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 15 (*Residual Fruit*)
		  • Lesson 16 (*Penguin Populations*)

		Stretch • **Teacher Move:** Invite students to plot data for two variables of interest on a scatter plot. Encourage them to predict the equation for the line of fit and r-value before they use technology to calculate them. Then invite them to reflect on whether their predictions were reasonable.

UNIT 3

Describing Functions



Students make sense of functions represented as graphs, tables, equations, and situations. They interpret statements in function notation and describe functions using their key features, including average rate of change, domain, and range. They also explore absolute value functions.



Essential Questions

- What are the characteristics of a function and how can a function tell a story?
- What are key features of functions and how can you describe them?
- How can you use function notation as a tool to communicate precisely?

Sub-Unit Goals

Sub-Unit 1

- Determine whether a relationship is a function.
- Interpret statements in function notation.

Sub-Unit 2

- Describe functions using their key features, including domain, range, and average rate of change.
- Compare functions using function notation and their key features.

Sub-Unit 3

- Interpret, evaluate, graph, and write equations of absolute value functions.
- Analyze and model situations using function tools, including situations that model sequences and inverses of functions.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Understand the concept of a function and use function notation. **(MA.912.F.1.1)**

Interpret functions that arise in applications in terms of the context. **(MA.912.F.1.2)**

Analyze functions using different representations. **(MA.912.F.1.5)**

Other Standards Addressed in This Unit: MA.912.F.1.3 MA.912.F.1.6 MA.912.AR.2.2 MA.912.AR.4.3

Mathematical Thinking and Reasoning Standards

- Students reason abstractly and quantitatively as they interpret statements written in function notation about a context. **(MTR.2.1)**
- Students communicate precisely to describe key features of functions. **(MTR.4.1)**
- Students make use of structure to determine key features of piecewise-defined and absolute value functions. **(MTR.5.1)**

Also addressed: **MTR.1.1, MTR.3.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students describe and compare functions, making connections between their multiple representations. They create graphs to match equations and scenarios and interpret statements in function notation.

Prior Learning

- **Grade 6:** Positive and negative numbers
- **Grade 8:** Proportional and linear relationships
- **Grade 8:** Functions and volume
- **Algebra 1:** Linear equations and Inequalities

Future Learning

- **Algebra 1,** Unit 5: Exponential functions **(MA.912.AR.5.4, MA.912.F.1.6)**
- **Algebra 1,** Unit 6: Quadratic functions **(MA.912.F.1.8, MA.912.AR.3.7, MA.912.F.1.2)**
- **Algebra 1,** Units 5–6 **(MA.912.F.2.1)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 4**, students develop conceptual understanding of function notation as they write and interpret statements in function notation in context.

Strategic Competence

In **Lesson 12**, students develop strategic competence as they make connections for graphing absolute value functions by choosing appropriate x-values in the table of values.

Procedural Fluency

In **Lesson 10**, students develop fluency as they practice restricting the domain or range of a function using inequalities.

Adaptive Reasoning

In **Lesson 12**, students develop adaptive reasoning as they move fluently between equations, tables, and graphs to make sense of functions.

Application

In **Lesson 9**, students apply their understanding of the domain and range of functions to the cost of renting bikes through different companies.

Productive Disposition

In **Lesson 10**, students have opportunities to develop a productive disposition as they use their knowledge of functions to tell a story about their own (or a fictional student's) life.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.NSO.1.3 MA.7.AR.4.2 MA.8.F.1.1 MA.8.F.1.3 MA.8.AR.3.2 MA.8.AR.3.3
MTR.2.1 MTR.7.1

Sub-Unit 1



1 Mystery Rule

What Is a Function?

Consider whether or not rules are **functions** and develop a concept of a function that they will use throughout the unit.

MA.912.F.1.1
MTR.3.1 MTR.4.1

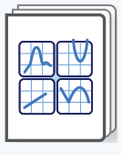


2 Pricing Pizzas

Introducing Function Notation

Learn what **function notation** is and interpret statements written in function notation in context.

MA.912.F.1.2
MTR.2.1 MTR.4.1



6 Craft-a-Graph

Key Features of Graphs

Describe and create graphs of functions using key features.

MA.912.F.1.6
MTR.4.1



7 Plane, Train, and Automobile

Average Rate of Change

Learn about **average rate of change** and how to calculate the average rate of change over specified **intervals**.

MA.912.F.1.3 MA.912.F.1.5
MTR.1.1 MTR.2.1 MTR.4.1



8 Space Race

Comparing Graphs

Use function notation and key features to compare graphs in the context of a space race.

MA.912.F.1.5 MA.912.F.1.6
MTR.4.1

Sub-Unit 3



11 What's Your Score?

Absolute Value Functions, Part 1

Make sense of the absolute value relationship as a way to measure the distance from a number.

MA.912.AR.4.3
MTR.2.1 MTR.5.1



12 Absolute Value Machines

Absolute Value Functions, Part 2

Graph an absolute value function given in function notation and analyze its key features.

MA.912.AR.4.3
MTR.4.1 MTR.5.1

M Math in Action

Connecting Statistics to Temperature

Explore the connection between statistics and temperatures as you explore the contributions of Warren Washington to meteorology.

TK

Pacing: 16 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 12 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2



3 Rule Breakers

Domain and Range Notation

Writing function rules using set-builder notation.

MA.912.F.1.1 MA.912.F.1.5 MA.912.F.1.6
MTR.1 MTR.4 MTR.5



4 Toy Factory

Function Notation and Equations

Explore functions represented as equations written in function notation.

MA.912.F.1.2 MA.912.AR.2.2
MTR.3.1 MTR.4.1 MTR.5.1



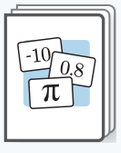
5 Function Carnival

Creating and Interpreting Graphs of Functions

Create and analyze graphs that represent stories.

MA.912.F.1.2 MA.912.F.1.6
MTR.2.1 MTR.4.1 MTR.6.1 MTR.7.1

Assess and Respond



9 Ins and Outs

Introducing Domain and Range

Define **domain** and **range** as a set of all possible inputs and outputs for a function.

MA.912.F.1.2 MA.912.F.1.5
MTR.4.1



10 Elevator Stories

Describing Domain and Range With Inequalities

Use **compound inequalities** to describe the domain and range of a function given its graph.

MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6
MTR.1.1 MTR.2.1 MTR.4.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.912.F.1.1 MA.912.F.1.2 MA.912.F.1.3
MA.912.F.1.5, MA.912.F.1.6 MA.912.AR.2.2
MTR.3.1 MTR.6.1 MTR.7.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.F.1.2 MA.912.F.1.3 MA.912.F.1.5
MA.912.F.1.6 MA.912.AR.4.3 MA.912.AR.9.1
MTR.3.1 MTR.6.1

Pacing Considerations

Lesson 1: This lesson revisits how to determine whether a rule is a function from Grade 8. If most students demonstrate a strong understanding of the definition of a function, this lesson may be omitted.
Lesson 5: This lesson invites students to make connections between graphs and the situations they represent, which supports students in defining key features of graphs later in the unit. If time runs short, this lesson may be omitted or spread out throughout Sub-Unit 2 as additional warm-up activities.
Lessons 11–12: These lessons support students in learning how to graph, evaluate, and describe key features of absolute value functions. If time runs short, these lessons can be consolidated into one by omitting Problems 8–10 of Lesson 11.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1	Lesson 2	Lesson 3	Lesson 4
Copy: Show What You Know **Materials:** no required materials	**Copy:** Show What You Know **Materials:** scientific calculators *(optional)*	**Copy:** Show What You Know **Materials:** no required materials	**Copy:** Show What You Know **Materials:** no required materials
Lesson 5	**Lesson 6**	**Lesson 7**	**Lesson 8**
Copy: Show What You Know **Materials:** no required materials	**Copy:** Show What You Know, Warm-Up Sheet **Materials:** no required materials	**Copy:** Show What You Know **Materials:** scientific calculators (optional), straightedges *(optional)*	**Copy:** Show What You Know **Materials:** no required materials
Lesson 9	**Lesson 10**	**Lesson 11**	**Lesson 12**
Copy: Show What You Know, Activity 1 Cards, one set per pair, pre-cut **Materials:** no required materials	**Copy:** Show What You Know **Materials:** no required materials	**Copy:** Show What You Know **Materials:** no required materials	**Copy:** Show What You Know **Materials:** no required materials

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Forms A and B
- Assessment Rubrics
- Show What You Knows



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

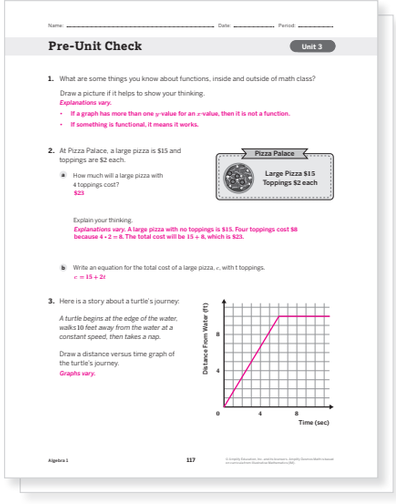
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.NSO.1.3, MA.7.AR.4.2, MA.8.F.1.1, MA.8.F.1.3, MA.8.AR.3.2, MA.8.AR.3.3



Name: _____ Date: _____ Period: _____

Pre-Unit Check Unit 3

1. What are some things you know about functions, inside and outside of math class?

   Draw a picture if it helps to show your thinking.

   Explanations vary.

   - *If a graph has more than one y-value for an x-value, then it is not a function.*
   - *If something is functional, it means it works.*

2. At Pizza Palace, a large pizza is $15 and toppings are $2 each.

   a. How much will a large pizza with 4 toppings cost?

   $23

   Explain your thinking.

   Explanations vary. A large pizza with no toppings is $15. Four toppings cost $8 because 4 • 2 = 8. The total cost will be 15 + 8, which is $23.

   Pizza Palace — Large Pizza $15, Toppings $2 each

   b. Write an equation for the total cost of a large pizza, c, with t toppings.

   $c = 15 + 2t$

3. Here is a story about a turtle's journey:

   A turtle begins at the edge of the water, walks 10 feet away from the water at a constant speed, then takes a nap.

   Draw a distance versus time graph of the turtle's journey.

   Graphs vary.

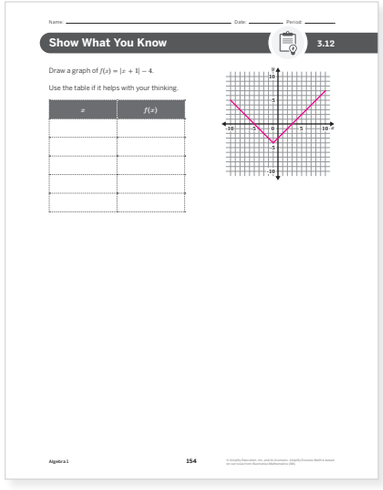
   Distance From Water (ft) / Time (sec)

Algebra 1 117

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed: on Lesson 12 Show What You Know: MA.912.AR.4.3



Name: _____ Date: _____ Period: _____

Show What You Know 3.12

Draw a graph of $f(x) = |x + 1| - 4$.

Use the table if it helps with your thinking.

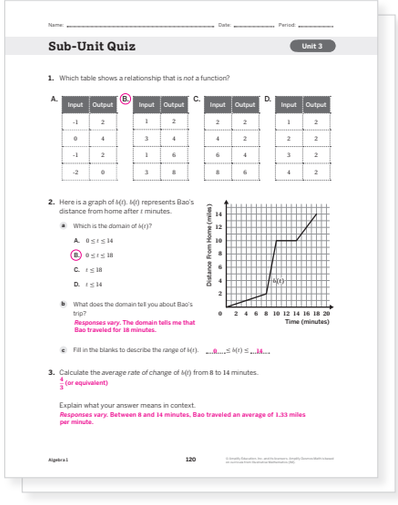
x	$f(x)$

Algebra 1 154

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.5, MA.912.F.1.6, MA.912.AR.2.2



Name: _____ Date: _____ Period: _____

Sub-Unit Quiz Unit 3

1. Which table shows a relationship that is *not* a function?

   A.

Input	Output
-1	2
0	4
-1	2
-2	0

   B.

Input	Output
1	2
3	4
1	6
3	8

   C.

Input	Output
2	2
4	2
6	4
8	6

   D.

Input	Output
1	2
2	2
3	2
4	2

2. Here is a graph of $h(t)$. $h(t)$ represents Bao's distance from home after t minutes.

   a. Which is the domain of $h(t)$?

   A. $0 \leq t \leq 14$

   B. $0 \leq t \leq 18$

   C. $t \leq 18$

   D. $t \leq 14$

   b. What does the domain tell you about Bao's trip?

   Responses vary. The domain tells me that Bao traveled for 18 minutes.

   c. Fill in the blanks to describe the range of $h(t)$. *...0...* $\leq h(t) \leq$ *...14...*

   Distance From Home (miles) / Time (minutes)

3. Calculate the average rate of change of $h(t)$ from 8 to 14 minutes.

   $\frac{4}{3}$ (or equivalent)

   Explain what your answer means in context.

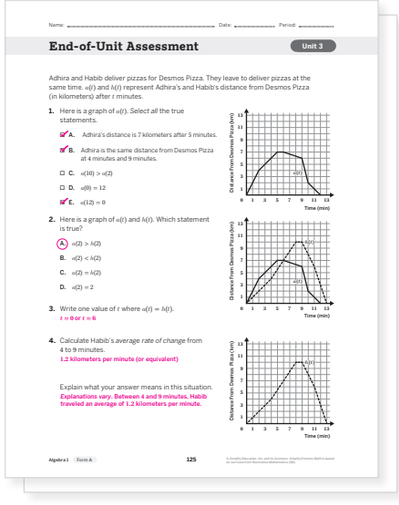
   Responses vary. Between 8 and 14 minutes, Bao traveled an average of 1.33 miles per minute.

Algebra 1 120

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.5, MA.912.F.1.6, MA.912.AR.4.3, MA.912.AR.9.1



Name: _____ Date: _____ Period: _____

End-of-Unit Assessment Unit 3

Adhira and Habib deliver pizzas for Desmos Pizza. They leave to deliver pizzas at the same time. $a(t)$ and $h(t)$ represent Adhira's and Habib's distance from Desmos Pizza (in kilometers) after t minutes.

1. Here is a graph of $a(t)$. Select all the true statements.

   ☑ A. Adhira's distance is 7 kilometers after 5 minutes.

   ☑ B. Adhira is the same distance from Desmos Pizza at 4 minutes and 9 minutes.

   ☐ C. $a(10) > a(2)$

   ☐ D. $a(9) = 12$

   ☑ E. $a(12) = 0$

   Distance From Desmos Pizza (km) / Time (min)

2. Here is a graph of $a(t)$ and $h(t)$. Which statement is true?

   A. $a(2) > h(2)$

   B. $a(2) < h(2)$

   C. $a(2) = h(2)$

   D. $a(2) = 2$

   Distance From Desmos Pizza (km) / Time (min)

3. Write one value of t where $a(t) = h(t)$.

   $t = 0$ or $t = 6$

4. Calculate Habib's average rate of change from 4 to 9 minutes.

   1.2 kilometers per minute (or equivalent)

   Explain what your answer means in this situation.

   Explanations vary. Between 4 and 9 minutes, Habib traveled an average of 1.2 kilometers per minute.

   Distance From Desmos Pizza (km) / Time (min)

Algebra 1 Form A 125

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

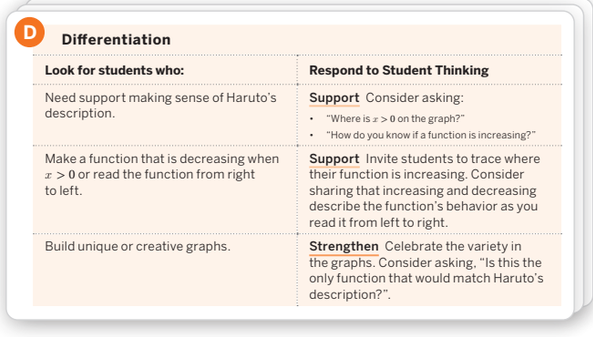
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–4) • Determine whether a relationship is a function. • Interpret statements in function notation.	**Mini-Lessons** • Evaluating Function Notation • Interpreting Slope and y-Intercept in Context • Interpreting Slope and Intercepts of Linear Relationships	**Practice** • Lessons 1–4 **Note:** Students will have additional opportunities to revisit these goals throughout Sub-Units 2–3.	**You're invited to explore more.** • Lesson 4 (*Toy Factory*)
Sub-Unit 2 (Lessons 5–10) • Describe functions using their key features, including domain, range, and average rate of change. • Compare functions using function notation and their key features.	**Mini-Lessons** • Writing Domain and Range with Inequalities • Calculating Slope Given Two Points • Interpreting Graphs	**Repeated Challenges** • Lesson 6 (*Craft-a-Graph*) **Practice** • Lessons 5–10	**You're invited to explore more.** • Lesson 5 (*Function Carnival*) • Lesson 7 (*Plane, Train, and Automobile*) • Lesson 10 (*Elevator Stories*)
Sub-Unit 3 (Lessons 11–12) • Interpret, evaluate, graph, and write equations of absolute value functions. • Analyze and model situations using function tools, including situations that model sequences and inverses of functions.	**Mini-Lessons** • Absolute Value Functions	**Repeated Challenges** • Lesson 12 (*Absolute Value Machines*) **Practice** • Lessons 11–12	**You're invited to explore more.** • Lesson 11 (*What's Your Score?*)

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 6**, where students are building graphs to match descriptions involving key features.



Differentiation

Look for students who:	Respond to Student Thinking
Need support making sense of Haruto's description.	**Support** Consider asking: • "Where is $x > 0$ on the graph?" • "How do you know if a function is increasing?"
Make a function that is decreasing when $x > 0$ or read the function from right to left.	**Support** Invite students to trace where their function is increasing. Consider sharing that increasing and decreasing describe the function's behavior as you read it from left to right.
Build unique or creative graphs.	**Strengthen** Celebrate the variety in the graphs. Consider asking, "Is this the only function that would match Haruto's description?".

Accessibility

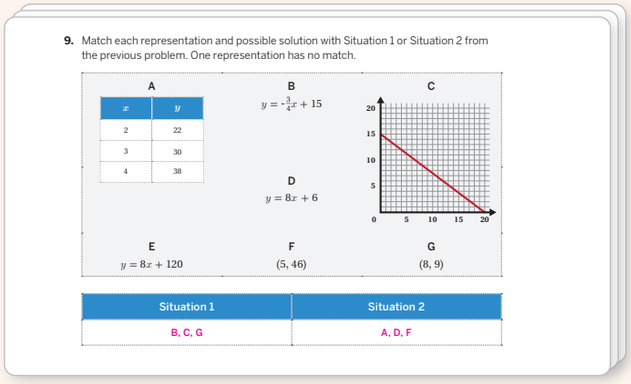
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from this unit designed to support Action & Expression.

Spotlight on Physical Action . . .

Lesson 7: Students are encouraged to use graph paper and straight-edges to support them in explaining or showing their thinking about calculating the average rate of change.



9. Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

A

x	y
2	22
3	30
4	38

B

$y = -\frac{3}{4}x + 15$

C

D

$y = 8x + 6$

E

$y = 8x + 120$

F

$(5, 46)$

G

$(8, 9)$

Situation 1	Situation 2
B, C, G	A, D, F

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Visual-Spatial Processing . . .

Here are two suggestions in this unit for supporting students with interpreting visual representations.

Lesson 8, Activity 1

A **Accessibility: Visual-Spatial Processing** Invite students to chunk the graph into smaller parts and describe what is happening at each step to support interpreting visual representations.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
function	1
function notation	2
maximum (of a function) **minimum (of a function)** **negative (interval or function)** **positive (interval or function)**	6
average rate of change **interval**	7
domain **range**	9
compound inequality	10
absolute value function	11

Review Vocabulary
absolute value
boundary point
constant difference
constant ratio
decreasing
dependent variable
increasing
independent variable
inequality
input
output
rate of change
slope
slope-intercept form
translation
x-intercept
y-intercept

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	1
MLR2: Collect and Display	1, 4, 6–7, and 9–10
MLR3: Critique, Correct, Clarify	2 and 5
MLR7: Compare and Connect	7–8 and 12

Sub-Unit Language Goals

- Interpret statements in function notation using tables, equations, and graphs.
- Describe functions using their key features, including domain, range, and average rate of change.
- Analyze and model situations using function tools, including situations that model sequences and inverses of functions.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 7**, where students are introduced to the context of the task: different ways of traveling to attend a wedding in New York City.

ML/EL **Multilingual/English Language Learners**
Invite students to act out scenarios, show images of planes, trains, or automobiles, use gestures, or talk about modes of transportation to increase access to the context used in the lesson. **(Reading and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Exponential Functions

In this unit, students graph functions, analyze their key features, and analyze the equations of piecewise-defined and absolute value functions. In **Unit 5**, students will create equations and graphs of exponential functions and interpret their key features in context.

FL B.E.S.T. Math Benchmark:
MA.912.AR.5.4, MA.912.F.1.6

Example:

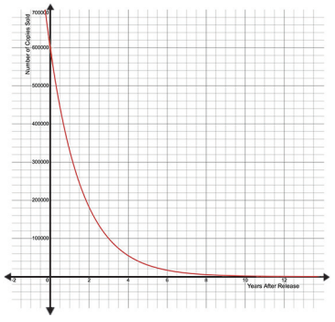
Function Equation:
$f(x) = 600000(0.55)^x$

$f(x)$ represents the number of copies sold x years after its release.

The y-intercept says that the book sold $600{,}000$ copies the year it was released. The constant ratio says that the number of copies sold decreases by 45% each year.

This function can also be used to make predictions. For example, in the third year after the release, the book sold $99{,}825$ copies because $f(3) = 99825$.

Graph:



Quadratic Functions

In this unit, students graph functions, analyze their key features, and analyze the equations of absolute value functions. **In Unit 6**, students will create equations and graphs of quadratic functions and interpret their key features in context.

FL B.E.S.T. Math Benchmarks:
MA.912.F.1.8, MA.912.AR.3.7, MA.912.F.1.2

Example:

Revenue model for Des-Kicks sneakers:

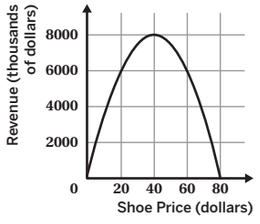
- There are $400{,}000$ people who want to buy the sneakers.
- For every $\$50$ price increase, Des-Kicks will sell 100 thousand fewer pairs.

Function Equation:

$r(x) = x(400 - 5x)$

x represents the sales price in dollars. $r(x)$ represents the revenue in thousands of dollars

Graph:



The graph and equation can be used to make decisions about what price to sell shoes for and predict the revenue for Des-Kicks. For example, selling the shoes for a price of $\$40$ (at the maximum of $r(x)$) would maximize Des-Kicks' revenue.

Transformations of Functions

In this unit, students describe how the minimum of an absolute value function could be determined from the structure of equations in the form $f(x) = |x - h| + k$. **In Units 5–6, and Algebra 2**, students will describe how key features of functions change when $f(x)$ is replaced by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k.

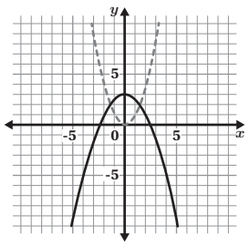
FL B.E.S.T. Math Benchmarks:
MA.912.F.2.1

Example:

The dashed line represents $f(x) = x^2$, and the solid line represents $-\frac{1}{2}f(x) + 3$.

$f(x)$ was vertically compressed by a factor of 2, reflected over the x-axis, and translated up 3 units. This affects the concavity, end behavior, and vertex of the parabola.

Graph:

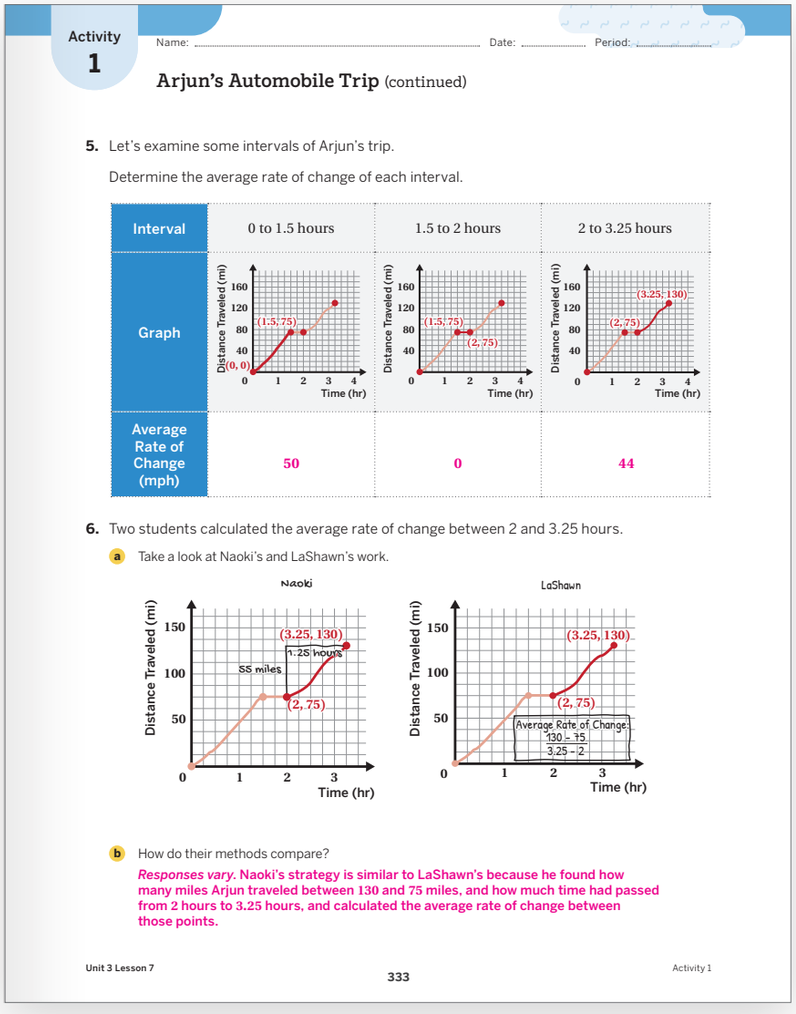


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Application

Lesson 7, Problems 4–5



Activity 1

Name: Date: Period:

Arjun's Automobile Trip (continued)

5. Let's examine some intervals of Arjun's trip.

Determine the average rate of change of each interval.

Interval	0 to 1.5 hours	1.5 to 2 hours	2 to 3.25 hours
Graph			
Average Rate of Change (mph)	50	0	44

6. Two students calculated the average rate of change between 2 and 3.25 hours.

a. Take a look at Naoki's and LaShawn's work.

b. How do their methods compare?

Responses vary. Naoki's strategy is similar to LaShawn's because he found how many miles Arjun traveled between 130 and 75 miles, and how much time had passed from 2 hours to 3.25 hours, and calculated the average rate of change between those points.

Unit 3 Lesson 7 333 Activity 1

Context

In this activity, students apply their prior knowledge of how to calculate the slope between two points to calculate the average rate of change over an interval.

Try This

Put on your student hat and complete problems 5–6.

Points to Ponder

- What do students need to understand about calculating slope between two points to support their application to average rate of change?
- Some students may think they can divide the total miles by the total time to calculate the average rate of change. How might you support them in calculating the *change* in miles divided by the change in time?
- How does calculating the average rate of change from a graph build from their understanding of calculating the rise and run of a line?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Define a function	Lesson 1	2	MA.8.F.1.1
2	Model relationships	Lesson 2	2	MA.8.AR.3.3 MTR.2.1, MTR.7.1
3	Create a graph to model a situation	Lesson 5	2	MA.8.F.1.3 MTR.7.1
4	Interpret a graph in context	Lesson 5	2	MA.7.AR.4.2
5	Calculate the slope of a line	Lesson 7	1	MA.8.AR.3.2
6–7	Reason with absolute value	Lesson 11	2	MA.6.NSO.1.3 MTR.4.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 3

1. What are some things you know about functions, inside and outside of math class?

Draw a picture if it helps to show your thinking.

Explanations vary.

- If a graph has more than one y-value for an x-value, then it is not a function.
- If something is functional, it means it works.

2. At Pizza Palace, a large pizza is $15 and toppings are $2 each.



a. How much will a large pizza with 4 toppings cost?

$23

Explain your thinking.

Explanations vary. A large pizza with no toppings is $15. Four toppings cost $8 because $4 \cdot 2 = 8$. The total cost will be $15 + 8$, which is $23.

b. Write an equation for the total cost of a large pizza, c, with t toppings.

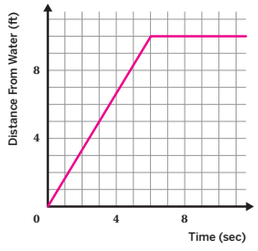
$c = 15 + 2t$

3. Here is a story about a turtle's journey:

A turtle begins at the edge of the water, walks 10 feet away from the water at a constant speed, then takes a nap.

Draw a distance versus time graph of the turtle's journey.

Graphs vary.



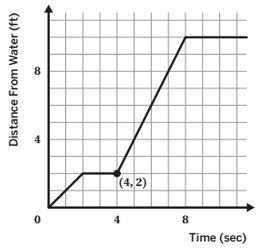
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Name: Date: Period:

Pre-Unit Check (continued)

Unit 3

4. Luca drew this graph to represent a new turtle.



a. What does the point (4, 2) tell you about the turtle's journey?

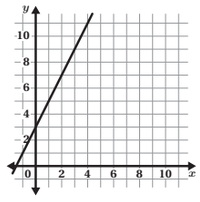
Explanations vary.
At 4 seconds, the turtle was 2 feet away from the water.

b. What was the speed of the turtle from 4 to 8 seconds?

2 feet per second

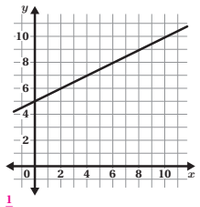
5. Determine the slope of each line.

a.



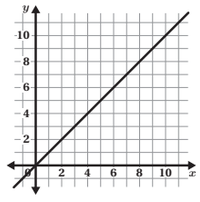
2

b.



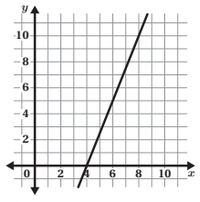
$\frac{1}{2}$

c.



1

d.



2.5

D Differentiation (Pre-Unit Check)

Note: To **Strengthen** and **Stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking

Preparation for Lesson 1: Define a function.	1	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Grade 8, Unit 5, Lesson 1 (*Guess My Rule*)
		  • Grade 8, Unit 5, Lesson 3 (*Function or Not?*)

Preparation for Lesson 2: Model relationships.	2	**Support** • **Teacher Move:** After the Warm-Up in Lesson 2, invite students to write an expression that models the price of each size pizza based on the number of toppings.
Preparation for Lesson 5: Create graphs and interpret them in context.	3–4	**Support** • **Teacher Move:** Consider revisiting: • Grade 8, Unit 5, Lesson 5 (*The Tortoise and the Hare*) • **Teacher Move:** Consider spending more time making sense of the Cannon Person and Ferris Wheel graphs in Lesson 5.

Preparation for Lesson 7: Calculate the slope of a line.	5	**Support**
		• **Mini-Lessons:**
		  • Calculating Slope by Drawing Triangles on a Coordinate Plan
		  • Calculating Slope Given Two Points
		• **Teacher Move:** Consider revisiting:
		  • Grade 8, Unit 3, Lesson 8 (*Landing Planes*)
		• **Teacher Move:** Consider inviting students to share their strategies for calculating slope and interpreting it in context before Lesson 7.

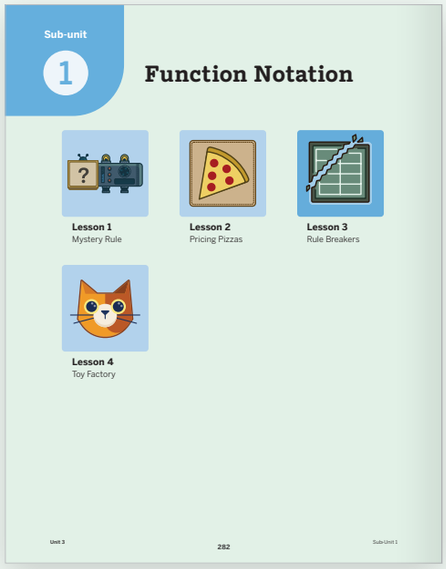
Preparation for Lesson 11: Reason with absolute value.	6	**Support**
		• **Teacher Move: Consider revisiting:**
		  • Grade 6, Unit 7, Lesson 5 (*Distance on the Number Line*)
		  • Grade 6, Unit 7, Lesson 6 (*We've Got Game(s)*)

Function Notation

Sub-Unit 1 Goals

- **Goal:** Determine whether a relationship is a function.
- **Language Goal:** Interpret statements in function notation.

In this sub-unit, students determine whether or not a relationship is a **function**. They are introduced to **function notation** as a way of writing about the inputs and outputs of a function. They interpret statements written in function notation using tables and equations.



Math That Matters Most

Concept or Skill	For example . . .
Determine whether or not a relationship is a function.	Decide if each rule is a function. Explain your thinking. **Rule H** takes any measurement in meters and converts it to centimeters. **Rule J** takes whole numbers from 1 to 15 and outputs a word of that length.

Input	Output
3 m	300 cm
2.6 m	260 cm
5.5 m	550 cm
3 m	300 cm

Input	Output
5	watch
9	vegetable
9	classroom
1	a

Rule H: Each input has exactly one output, so it's a function. For instance, when inputting 3 meters, the output will always be 300 centimeters.

Rule J: Inputs can have multiple possible outputs, so it's not a function. For instance, the table shows that the input 9 has two different outputs ("vegetable" and "classroom").

Evaluate a function for a specific input value.

If $m(t) = 15.5 + 1.5t$, what is the value of $m(4)$?

$m(4) = 15.5 + 1.5(4)$ $\quad$ $m(4) = 21.5$

Interpret statements in function notation.

$w(x) = 1.25x + 2.5$ represents the cost of a waffle cone order, where x represents the number of scoops of ice cream.

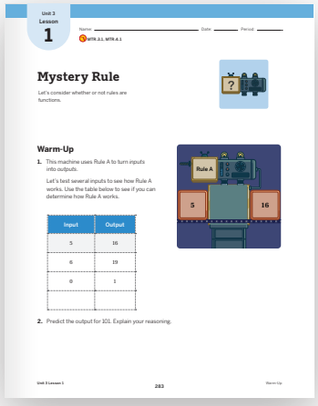
What does $w(4) = 7.5$ mean in the context of the ice cream shop?

Four scoops of ice cream in a waffle cone costs $7.50.

Unit 3

Lesson

1



Mystery Rule

What Is a Function?

Let's consider whether or not rules are functions.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether or not a rule is a *function*.
2. **Language Goal:** Explain that a function has only one output for every input. **(Writing, Speaking, and Listening)**

Students consider whether or not rules are functions and develop a definition of **function** that they will use throughout the unit. They reason repeatedly by testing different *inputs* to make sense of rules and make predictions using these rules. These rules provide the examples that students use to explain what a function is: a rule that only produces one possible *output* for a given input. **(MTR.3.1)**

Coherence

Prior Learning

In Grade 8, students learned that a function only has one output for each allowable input and justified whether a table could represent a function. In Grade 8, Unit 5, Lesson 3, students justified whether a graph could represent a function and described a situation using terms such as *function*, *input*, and *output*. In Algebra 1, students studied linear relationships.

Future Learning

In Lessons 2-4, students will continue to develop their understanding of functions as they study function notation in tables, equations, and graphs.

Connections

- Students use a machine model to explore input and output behavior of functions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.1

Given an equation or graph that defines a function, classify the function type. Given an input-output table, determine a function type that could represent it.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1.1	**MA.912.F.1.2**

Vocabulary

New Vocabulary

function

Review Vocabulary

input *output*

Additional Vocabulary

relation

Lesson at a Glance

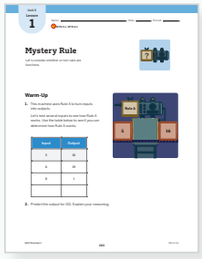
~ 45 min

MA.912.F.1.1

Warm-Up

Pairs | 5 min

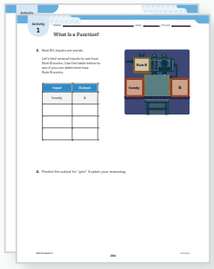
Students test *inputs* to make sense of their first rule and make predictions about the *output* value based on the input value.



Activity 1

Pairs | 20 min

Students explore rules that have only one output for each input and others that could have multiple outputs for an input in order to create a class definition of **function**. MLR2: Collect and Display (MTR.3.1)



Activity 2

Pairs | 10 min

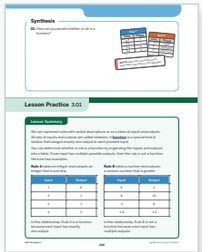
Students practice applying their class definition to determine if different rules are functions or not functions and justify their reasoning. (MTR.4.1)



Synthesis

Whole Class | 5 min

Students synthesize their understanding of what makes a rule a function. MLR1: Stronger and Clearer Each Time (MTR.4.1)



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining if rules are functions and explaining their thinking.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students test *inputs* to make sense of their first rule and make predictions about the *output* value based on the input value.

1 Launch

Display the Warm-Up.

Invite students to choose numbers and predict what will happen to the number machine.

Invite students to share their thinking about what the rule could be with a partner.

A **Accessibility: Visual-Spatial Processing** As students interpret visual representations, invite them to describe what is happening to the input to a partner.

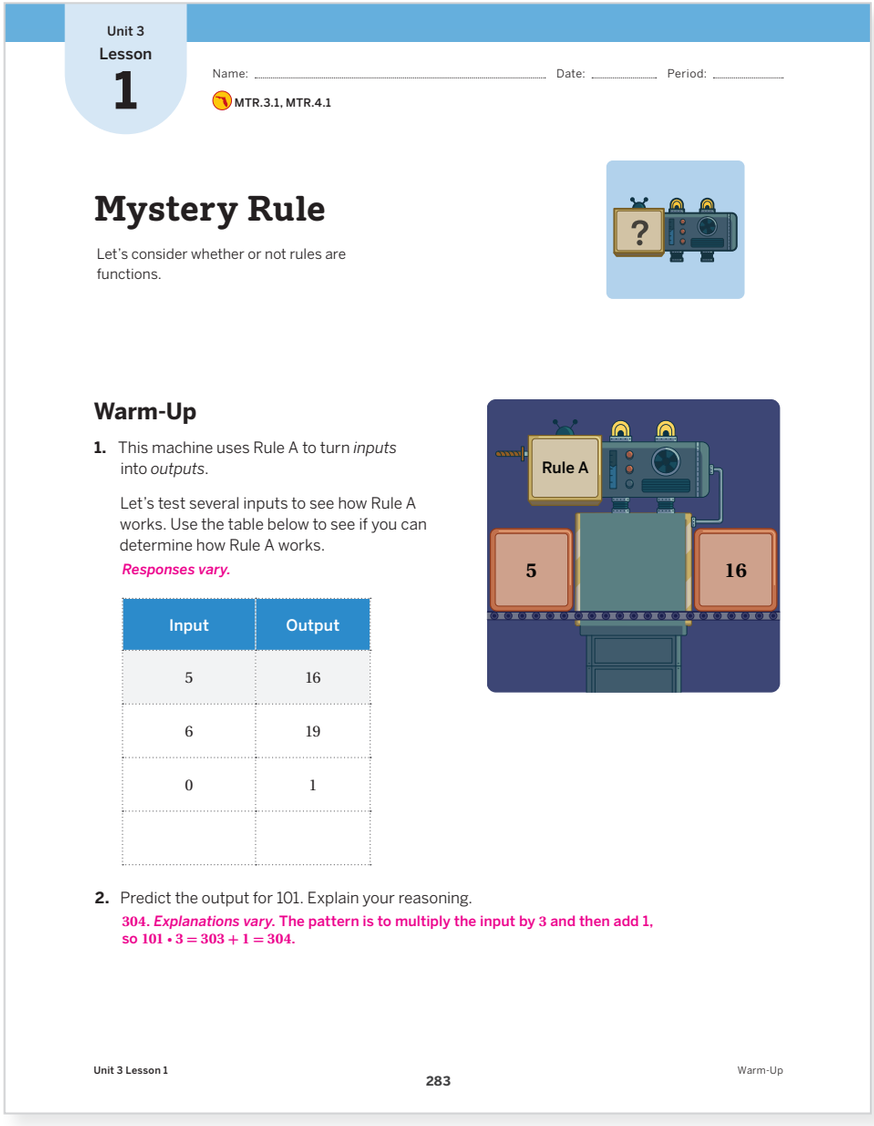
ELL **English Language Learners** Invite students to share what the terms *input* and *output* remind them of (e.g., "When I hear input/output I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

Discuss the table of inputs and outputs for Rule A. Give students enough time to complete the table.

2 Connect

Consider asking:

- "How did you determine what the output for 101 would be?"
- "Do you think there could be more than one possible output for 101? Why or why not?"



Unit 3
Lesson
1

Name: _____ Date: _____ Period: _____

MTR.3.1, MTR.4.1

Mystery Rule

Let's consider whether or not rules are functions.

Warm-Up

1. This machine uses Rule A to turn *inputs* into *outputs*.

   Let's test several inputs to see how Rule A works. Use the table below to see if you can determine how Rule A works.

   Responses vary.

Input	Output
5	16
6	19
0	1

2. Predict the output for 101. Explain your reasoning.

   304. *Explanations vary.* The pattern is to multiply the input by 3 and then add 1, so $101 \cdot 3 = 303 + 1 = 304$.

Unit 3 Lesson 1 283 Warm-Up

Pairs | 20 min

Activity 1 What Is a Function?

Purpose: Students explore rules that have only one output for each input and others that could have multiple outputs for an input in order to create a class definition of **function**. **(MTR.3.1)**

1 Launch

Invite students to share how Rule B and Rule A are alike and different.

Encourage students to give an input to a partner to predict a possible output.

ELL **English Language Learners** Encourage students to try simple words, words that have similarities (e.g., same number of letters, same starting letter, etc.), or even random strings of letters to increase access to the task. **(Reading, Writing)**

2 Monitor

A **Accessibility: Conceptual Processing** Invite students to create a hypothesis to support students in identifying relationships. Consider asking:

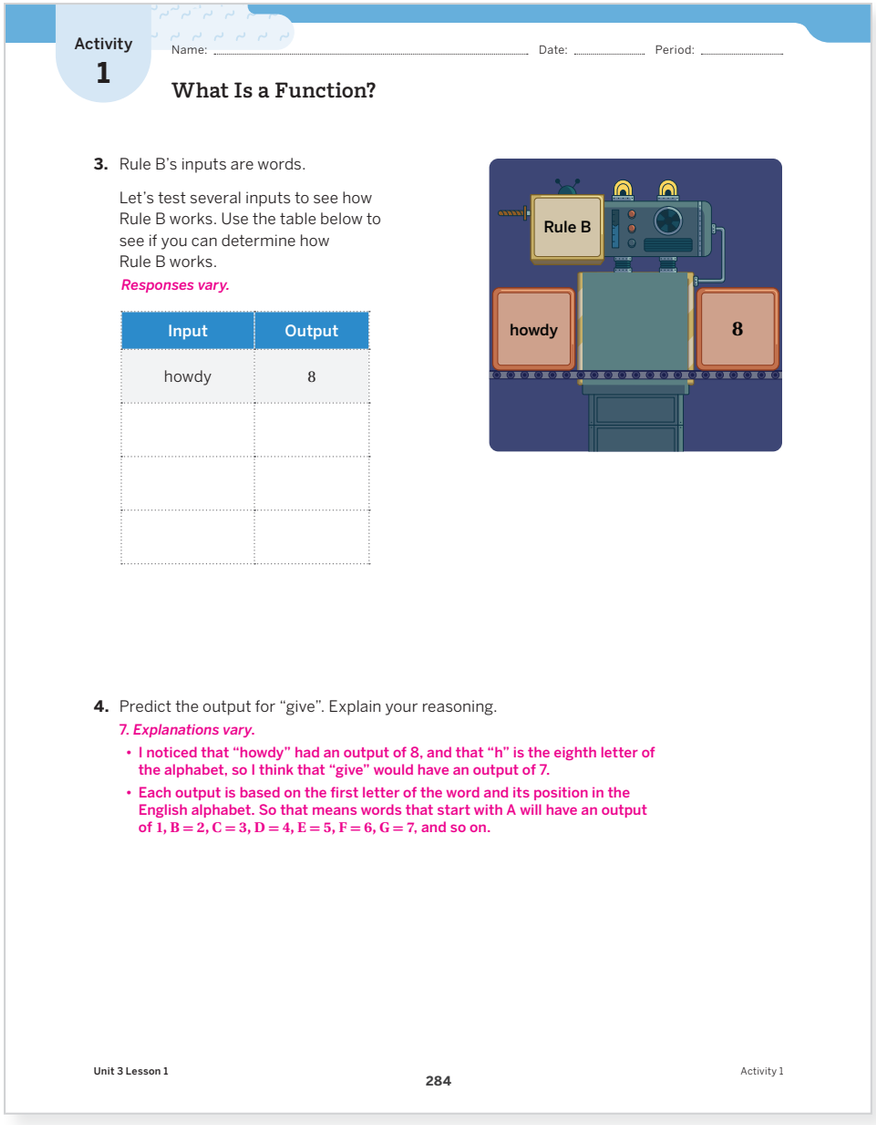
- "What patterns do you see between the input/output pairs?"
- "How does the word in the input relate to the number in the output?"
- "How can we check to see if the pattern you see matches the other input/output pairs in the table?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Describe a pattern that is different from the rule.	**Support** Consider asking, "How would your rule work for the word 'joyful?' How can you tell that your rule will work for all input/output pairs in the table?"
Have tested inputs with all different first letters.	**Strengthen** Consider asking, "Can you create two inputs that have the same output? How are these inputs alike?"
Make a prediction for "give" and explain their reasoning.	**Strengthen** Invite students to read others' responses and decide if others' reasoning is like their own or different.

Look for students who use the terms *input* and *output* in their explanations. Consider highlighting these responses during the Connect.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

What Is a Function?

3. Rule B's inputs are words.

Let's test several inputs to see how Rule B works. Use the table below to see if you can determine how Rule B works.

Responses vary.

Input	Output
howdy	8

4. Predict the output for "give". Explain your reasoning.

7. Explanations vary.

- I noticed that "howdy" had an output of 8, and that "h" is the eighth letter of the alphabet, so I think that "give" would have an output of 7.
- Each output is based on the first letter of the word and its position in the English alphabet. So that means words that start with A will have an output of 1, B = 2, C = 3, D = 4, E = 5, F = 6, G = 7, and so on.

Unit 3 Lesson 1 284 Activity 1

Pairs | 20 min

Activity 1 What Is a Function? (continued)

Purpose: Students explore rules that have only one output for each input and others that could have multiple outputs for an input in order to create a class definition of **function**. **(MTR.3.1)**

2 Monitor

Discuss with students how the machine for Rule C works by asking pairs of students to share the inputs and outputs.

Invite students to work in pairs to predict possible inputs and outputs.

D Differentiation

Look for students who:	Respond to Student Thinking
Have different responses that are all 6 letters long, and/or who notice that there are different possible outputs for 6.	**Strengthen** Invite students to share their different possible outputs and explanations during the Connect.
Would like to explore this topic further.	**Stretch** Invite students to create their own rule and have a partner guess what the rule is by trying different input/output pairs.

Listen for pairs of students who discuss getting different outputs for 6 and use that in their justification for why Rule C is not a function and invite them to share during the Connect.

3 Connect

Display the rules and the prompt.

MLR MLR2: Collect and Display Use this routine to create a class definition of **function** based on the language students used in their responses. Record language, examples, or diagrams for students to reference throughout the unit.

Note: It is okay if students are unsure about what makes a rule a function at this stage. Students will practice with input/output pairs in the next activity and other representations in future lessons.

D Differentiation: (Strengthen) Students may say that Rule C is not a function because the inputs and outputs repeat. Consider asking these students, "Are there repetitions of inputs or outputs in Rule A or B?"

To surface the Key Takeaway, consider asking:
- "What makes Rule C different from Rule A and B?"
- "In general, what makes a rule not a function? What makes a rule a function?"

Activity 1

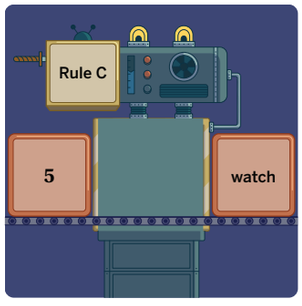
Name: _____ Date: _____ Period: _____

What Is a Function? (continued)

5. Rule C's inputs are whole numbers from 1 to 15.

Let's test several inputs to see how Rule C works. Use the table below and the machine to the right to see if you can determine how Rule C works.

Responses vary.



Input	Output
5	watch
9	vegetable

6. a. Predict the output for 6.

Responses vary. Any six-letter word.

b. Compare your response with a partner's. How are your responses alike and different?

Responses vary. My partner and I both wrote six-letter words, but they were different words.

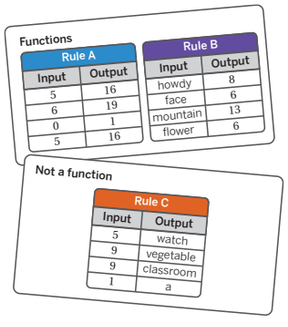
7. Rules A and B are examples of a **function**.

Rule C is *not* a function.

What do you think makes Rule C not a function?

Responses vary.

- Rule C is not a function because the input 9 has two different outputs.
- Rule C is not a function because every input does not have only one output. 9 has outputs of both "vegetable" and "classroom."



Unit 3 Lesson 1 285 Activity 1

Key Takeaway: A rule is a **function** if every *input* only has one possible *output*.

Pairs | 10 min

Activity 2 Which Rules Are Functions?

Purpose: Students practice applying their class definition to determine if different rules are functions or not functions and justify their reasoning. **(MTR.4.1)**

Short on time? Consider inviting pairs to discuss two or three of the rules instead of all four.

1 Launch

Invite students to read each rule aloud and discuss any terms that may be unclear.

A **Accessibility: Memory and Attention** To support students with keeping track of information, invite them to record their thinking for each rule as they are working.

ELL **English Language Learners** Provide sentence frames. (e.g., "_____ is a function because _____." or " _____ is not a function because _____.") **(Writing and Speaking)**

2 Monitor

Listen for different ways students explain their thinking about whether or not each rule is a function. **(MTR.4.1)**

Encourage students to refer to the class definition and incorporate words like input and output into their explanations.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to choose any one rule to focus on. Consider asking, "Is this rule more like Rules A and B from Activity 1 (a function) or like Rule C (not a function)? Why?"
Think Rule F is not a function because the output values repeat.	**Strengthen** Consider asking, "What happens when you enter the same input twice?"
Mention the repetition of inputs and outputs in their explanation. E.g., "Rule G is a function because every input word makes one string of letters as output." "Rule E is not a function because you could pick different factors as outputs."	**Stretch** Celebrate their attention to precision in using mathematical vocabulary in their explanations and invite them to share during the pause.

Pause briefly to invite groups of students to check in with their partners and share their thinking on which of the four rules are functions and why. If any pair of students has not reached a consensus, invite students to make arguments and justify their reasoning for each rule. **(MTR.4.1)**

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Which Rules Are Functions?

8. Here are four rules.

Discuss: Is each rule a function? Why or why not?

Rule D takes temperatures in Fahrenheit and outputs temperatures in Celsius.

Input	Output
50	10
77	25
100	37.8
212	100

Rule D is a function. *Explanations vary.* Every temperature in Fahrenheit has exactly one matching temperature in Celsius.

Rule E takes any integer and outputs one of its factors.

Input	Output
24	6
13	1
13	13
9	3

Rule E is not a function. *Explanations vary.* If 13 is the input, there are two possible outputs: 1 or 13.

Rule F takes any number and rounds it to a whole number.

Input	Output
32.5	33
$\frac{4}{3}$	1
0.1	0
23	23

Rule F is a function. *Explanations vary.* Rounding any number always gets you one whole number as an output.

Rule G takes any word and shifts each letter one place in the alphabet.

Input	Output
bird	cjse
house	ipvtf
hello	ifmmp
world	xpsme

Rule G is a function. *Explanations vary.* You can follow a rule to make this shift the same way every time.

Unit 3 Lesson 1 286 Activity 2

Pairs | 10 min

Activity 2 Which Rules Are Functions? (continued)

Purpose: Students practice applying their class definition to determine if different rules are functions or not functions and justify their reasoning. **(MTR.4.1)**

Short on time? Consider inviting pairs to discuss two or three of the rules instead of all four.

2 Monitor

Invite students to share their strategies out loud to one another before moving on to the next Rule.

D **Differentiation: (Strengthen)** Some students may think Rule E is a function because it is predictable. Consider asking, "What happens when you use the same input twice? Does this match our class definition? Why or why not?"

Listen for explanations of why Rule E is not a function to share during the Connect.

ELL **English Language Learners** During this lesson, consider personally inviting students to share responses, asking if they would feel comfortable sharing with the class. Consider inviting students to respond in their primary languages. **(Speaking and Listening)**

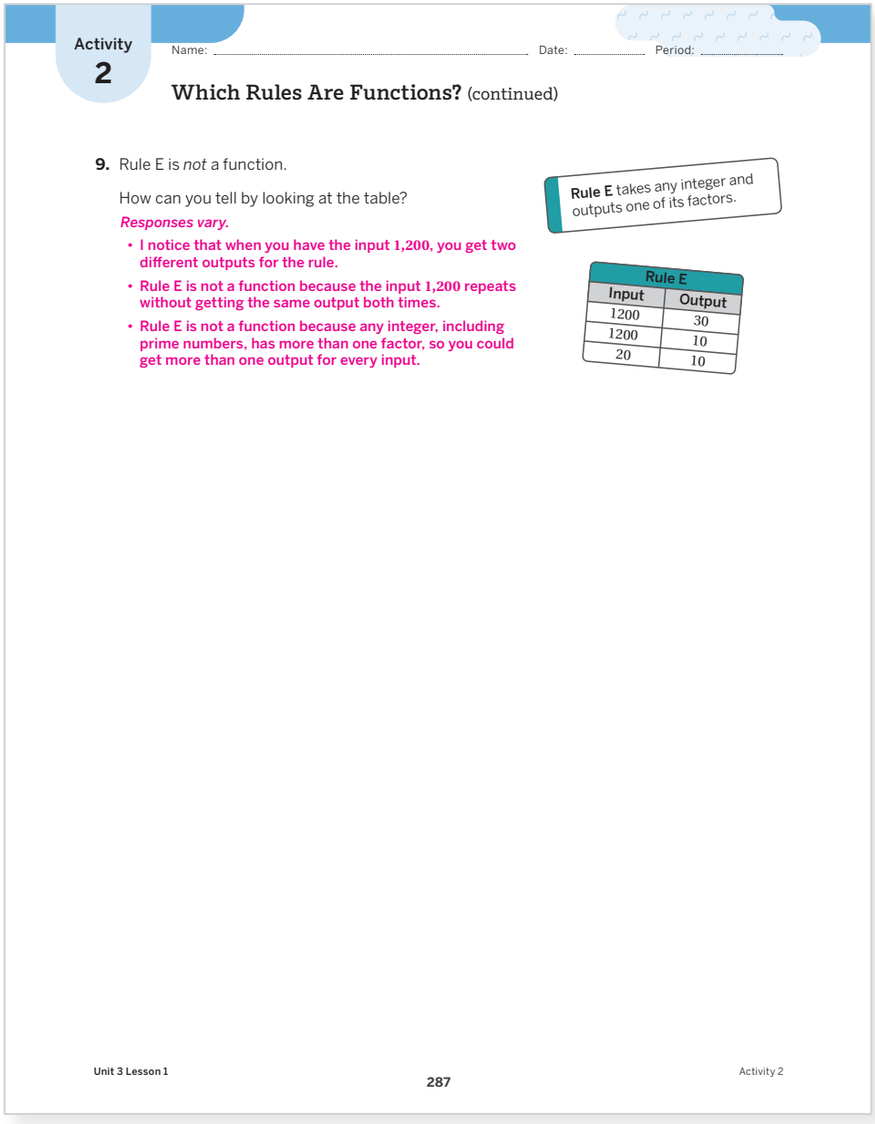
3 Connect

Invite students to discuss why Rule E is not a function.

Note: If time allows, consider revising the class definition of a function based on any new language or ideas students share. **(MTR.4.1)**

Consider sharing that most of the inputs to Rule E have multiple possible outputs, since most integers have more than one factor. Students may think that, in order for a rule not to be a function, there must be more than one input with multiple possible outputs. Share that a rule is not a function even when only *one* input can produce different outputs.

Note: Students will not be assessed on their use of the term *relation*, but you may consider sharing with students that a relation is any pairing of inputs and outputs. If it would support your students to have a name for rules that are not functions, consider sharing they can still be called relations.



Activity 2

Name: _____ Date: _____ Period: _____

Which Rules Are Functions? (continued)

9. Rule E is *not* a function.

How can you tell by looking at the table?

Responses vary.

- **I notice that when you have the input 1,200, you get two different outputs for the rule.**
- **Rule E is not a function because the input 1,200 repeats without getting the same output both times.**
- **Rule E is not a function because any integer, including prime numbers, has more than one factor, so you could get more than one output for every input.**

Rule E takes any integer and outputs one of its factors.

Rule E	
Input	Output
1200	30
1200	10
20	10

Unit 3 Lesson 1 287 Activity 2

Key Takeaway: A rule is *not* a function when the same input can produce different outputs.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what makes a rule a function. **(MTR.4.1)**

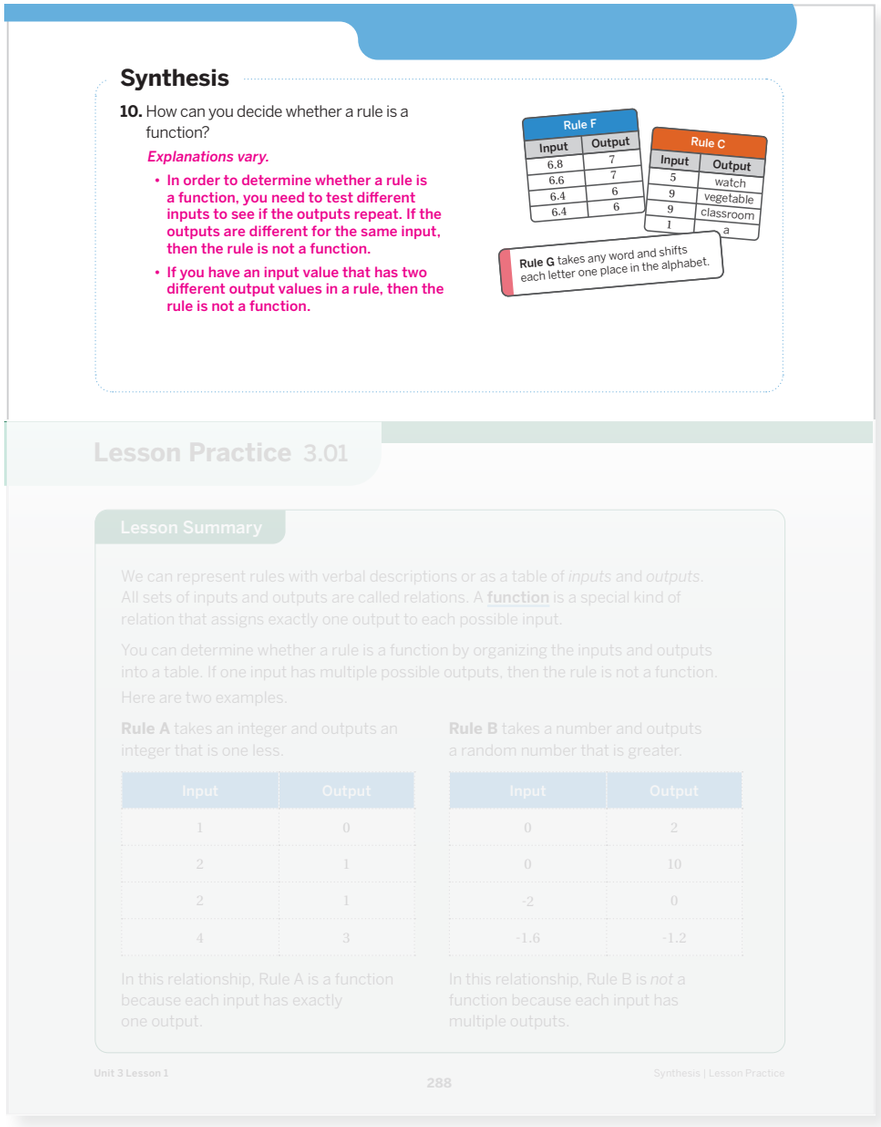
Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:

- The difference between inputs repeating versus outputs repeating. Rule F has the input 6.4 twice, but the output is the same both times so it can still be a function. Rule C has the input 9 twice but its output is different each time so it can't be a function.
- How Rule G is a function if every letter shifts one place. Every input to Rule G will have exactly one possible output, so it is a function.

Consider sharing that tables only show some of the many input/output pairs for a rule. Consider asking, "How do we know if this rule will always assign one output to every input, even for the pairs not in the table?"

MLR **MLR1: Stronger and Clearer Each Time**
Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**



Synthesis

10. How can you decide whether a rule is a function?

Explanations vary.

- **In order to determine whether a rule is a function, you need to test different inputs to see if the outputs repeat. If the outputs are different for the same input, then the rule is not a function.**
- **If you have an input value that has two different output values in a rule, then the rule is not a function.**

Rule F

Input	Output
6.8	7
6.6	7
6.4	6
6.4	6

Rule C

Input	Output
5	watch
9	vegetable
9	classroom
1	a

Rule G takes any word and shifts each letter one place in the alphabet.

Lesson Practice 3.01

Lesson Summary

We can represent rules with verbal descriptions or as a table of *inputs* and *outputs*. All sets of inputs and outputs are called relations. A **function** is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function. Here are two examples.

Rule A takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

Rule B takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not* a function because each input has multiple outputs.

Unit 3 Lesson 1 288 Synthesis | Lesson Practice

Lesson Takeaway: A rule can be a **function** or not a function. If an *input* value has two different *output* values, then the rule is not a function.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining if rules are functions and explaining their thinking.

Today's Goals

1. **Goal:** Determine whether or not a rule is a *function*.
2. **Language Goal:** Explain that a function has only one output for every input. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.01

In the Northern Hemisphere, a year is divided into four seasons.

Rule A takes a season and outputs a date.

Rule B takes a date and outputs a season.

Which rule is a function? Circle one.

Rule A (Rule B) Both Neither

Explain your thinking.

Explanations vary. **Rule B is a function because each date has exactly one season that it could be matched up with. Rule A is not a function because "Spring" is an input that has many different outputs.**

Rule A

Input	Output
Spring	May 1
Spring	May 8
Summer	August 5
Fall	October 23
Winter	December 25

Rule B

Input	Output
May 1	Spring
May 8	Spring
August 5	Summer
October 23	Fall
December 25	Winter

D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support deciding whether a rule is a function:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to look for rules that pair inputs to outputs in their life and explain whether those rules are functions.

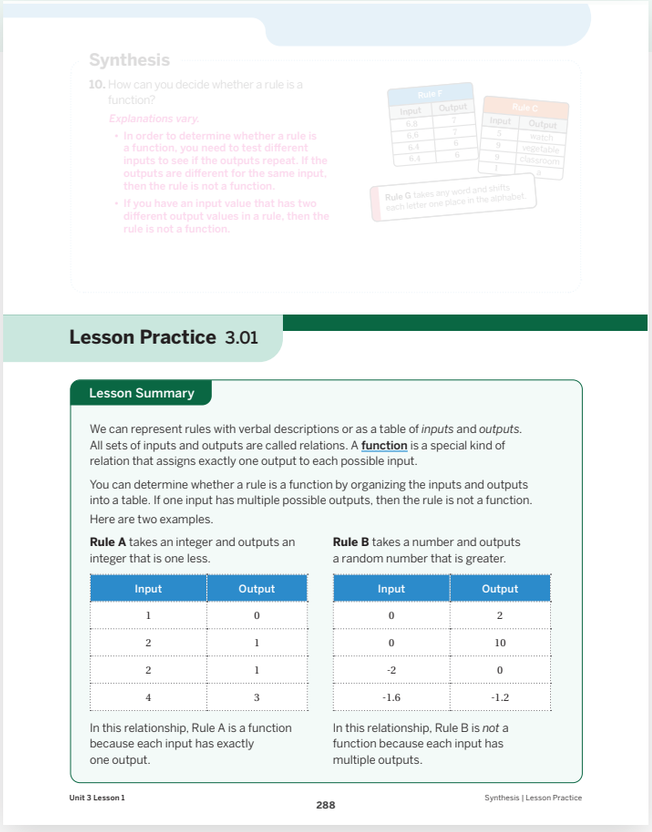
Professional Learning

In this lesson, students determined whether or not a rule is a **function**. How will that work support the thinking they will do about exponential and quadratic functions in Units 5 and 6, and the thinking they will do about other functions in future courses?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. How can you decide whether a rule is a function?

Explanations vary.

- In order to determine whether a rule is a function, you need to test different inputs to see if the outputs repeat. If the outputs are different for the same input, then the rule is not a function.
- If you have an input value that has two different output values in a rule, then the rule is not a function.

Rule F

Input	Output
6.8	7
6.6	7
6.4	6
6.4	6

Rule C

Input	Output
5	watch
9	vegetable
9	classroom
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Rule G takes any word and shifts each letter one place in the alphabet.

Lesson Practice 3.01

Lesson Summary

We can represent rules with verbal descriptions or as a table of *inputs* and *outputs*. All sets of inputs and outputs are called relations. A **function** is a special kind of relation that assigns exactly one output to each possible input.

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Input	Output
0	2
0	10
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-1.6	-1.2

In this relationship, Rule B is *not* a function because each input has multiple outputs.

Lesson Practice 3.01

Name: _____ Date: _____ Period: _____

1. Rule A takes any word as an input and writes the word backwards as an output.

Is Rule A a function? Explain your thinking.

Yes. *Explanations vary.* Rule A is a function because any word written backwards results in only one sequence of letters.

Rule A

Input	hat	sock	racecar
Output	tah	kcos	racecar

Problems 2–3: Here is Rule B.

2. Is Rule B a function? Explain your thinking.

No. *Explanations vary.* Rule B is not a function because there are two different outputs for the input 6.

Rule B

Input	4	6	6	5
Output	blue	purple	yellow	white

3. Predict what the output could be when the input is 3.

Responses vary. It can be any three-letter color, such as red.

Problems 4–5: Here is Rule C.

4. Is Rule C a function? Explain your thinking.

Yes. *Explanations vary.* Rule C is a function because every input only has one output.

Rule C

Input	6	8	12	14
Output	4	6	10	?

5. Predict the missing output for Rule C. 12

6. This table shows the total number of days in each month of a given year.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
28		×										
29		×										
30				×		×			×		×	
31	×		×		×		×	×		×		×

Imagine a rule where the input is a month and the output is the number of days in that month. Does this rule represent a function? Explain your thinking.

No. *Explanations vary.* This rule does not represent a function because the input February has two possible outputs of either 28 or 29 days.

Lesson Practice 3.01

Name: _____ Date: _____ Period: _____

7. A machine uses Rule D to turn inputs into outputs. The table shows two inputs and their outputs.

Hoang tried the input 4 again and the output was not 27. He claims that this is enough information to determine whether or not Rule D is a function.

Is Hoang correct? Explain how you know.

Yes. *Explanations vary.* If the input 4 results in any output value other than 27, then Rule D is not a function. This is because a function only has one output for every input.

Rule D

Input	Output
3	21
4	27

Test Practice

8. Which output for an input of 4 could make Rule D a function?

- ☐ A. 7
- ☐ B. 21
- ☑ C. 28
- ☐ D. 40

Spiral Review

9. Complete the table using this rule: Add 2 to the input, then multiply by 3 to get the output.

Input	-5	0	4
Output	-9	6	18

Problems 10–11: A school sells two types of tickets for a play: adult tickets and student tickets. Adult tickets are $5 each and student tickets are $2 each. The school collects $400 total.

10. Write an equation where x represents the number of adult tickets sold and y represents the number of student tickets sold.

$5x + 2y = 400$

11. How many of each ticket type could the school sell to collect $400?

Responses vary.

Adult tickets: 40 Student tickets: 100

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.F.1.1*
	2–5	2	MA.912.F.1.1*
	6	2	MA.912.F.1.1*
	7	2	MA.912.F.1.1*
TEST Practice	8	2	MA.912.F.1.1*
Spiral Review			
Fluency	9	1	MA.8.F.1.1
	10–11	2	MA.912.AR.9.1

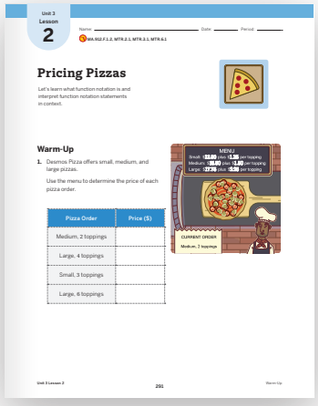
*These problems build toward the standard shown.

Unit 3
Lesson
2

Pricing Pizzas

Introducing Function Notation

Let's learn what function notation is and interpret function notation statements in context.


Unit 3
Lesson
2
Name _____ Date _____ Period _____
MA.912.F.1.2, MTR.2.1, MTR.3.1, MTR.4.1
Pricing Pizzas
Let's learn what function notation is and interpret function notation statements in context.
Warm-Up
1. Desmos Pizza offers small, medium, and large pizzas.
Use the menu to determine the price of each pizza order.
MENU
Small: $8.00 plus $1.50 per topping
Medium: $10.00 plus $1.50 per topping
Large: $12.00 plus $1.50 per topping
CURRENT ORDER
Medium, 2 toppings
Pizza Order | Price ($)
Medium, 2 toppings
Large, 4 toppings
Small, 3 toppings
Large, 6 toppings
Unit 3 Lesson 2
291
Warm-Up

Learning Outcomes

Today's Goal

1. **Language Goal:** Interpret a statement written in *function notation* in context. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to **function notation** and interpret statements written in function notation in context. They also use inputs and outputs to make sense of function notation in the context of a pizza shop.

Coherence

In Lesson 1, students tested different inputs as a strategy for deciding if a rule was a function or not. In Grade 8, students learned that a function has only one output for each possible input.

Future Learning

In Lessons 3–4, students will continue to develop their understanding of functions as they study function notation in tables, equations, and graphs. Also, in Lesson 9–10, students will consider what inputs and outputs are possible for a function (domain and range).

Connections

- Students use function notation to calculate prices of different pizza orders, as well as how long it takes to deliver a pizza to different distances. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Also addressing: **MA.912.AR.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1, ELA.K12.EE.7.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.F.1.1	**MA.912.AR.2.2** **MA.912.F.1.6**

Vocabulary

New Vocabulary

function notation

Review Vocabulary

dependent variable *independent variable*

Lesson at a Glance

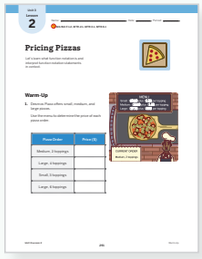
~ 45 min

MA.912.AR.2.2, MA.912.F.1.2

Warm-Up

Whole Class | 5 min

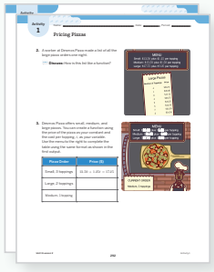
Students use information to determine the prices of pizzas of different sizes with different amounts of toppings to introduce them to the context of pizza costs. **(MTR.2.1)**



Activity 1

Pairs | 10 min

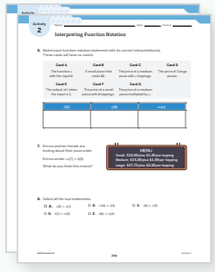
Students attend to precision in using **function notation** to write and interpret statements describing the price of a pizza based on the size and number of toppings.
Routines: Think-Pair-Share, MLR3: Critique, Correct, Clarify **(MTR.4.1)**



Activity 2

Pairs | 10 min

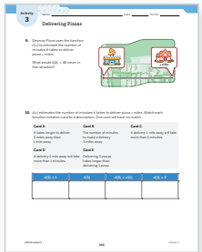
Students interpret function notation by matching function notation cards to their interpretations and interpret new statements.



Activity 3

Pairs | 10 min

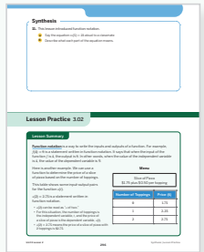
Students consider a new function that describes the number of minutes it takes to deliver a pizza and interpret statements about this new situation in function notation.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to read statements in function notation and how to interpret them for a situation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting a statement in function notation.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use information to determine the prices of pizzas of different sizes with different amounts of toppings to introduce them to the context of pizza costs.

1 Launch

Display the menu, the prompt, and the table.

Invite students to share their favorite pizza toppings and how many toppings they typically get when they order pizza.

To generate excitement, consider asking, "What's your favorite combination of pizza toppings?"

Encourage students to use additional paper to help with their thinking.

Invite students to use scientific calculators to support them in making calculations.

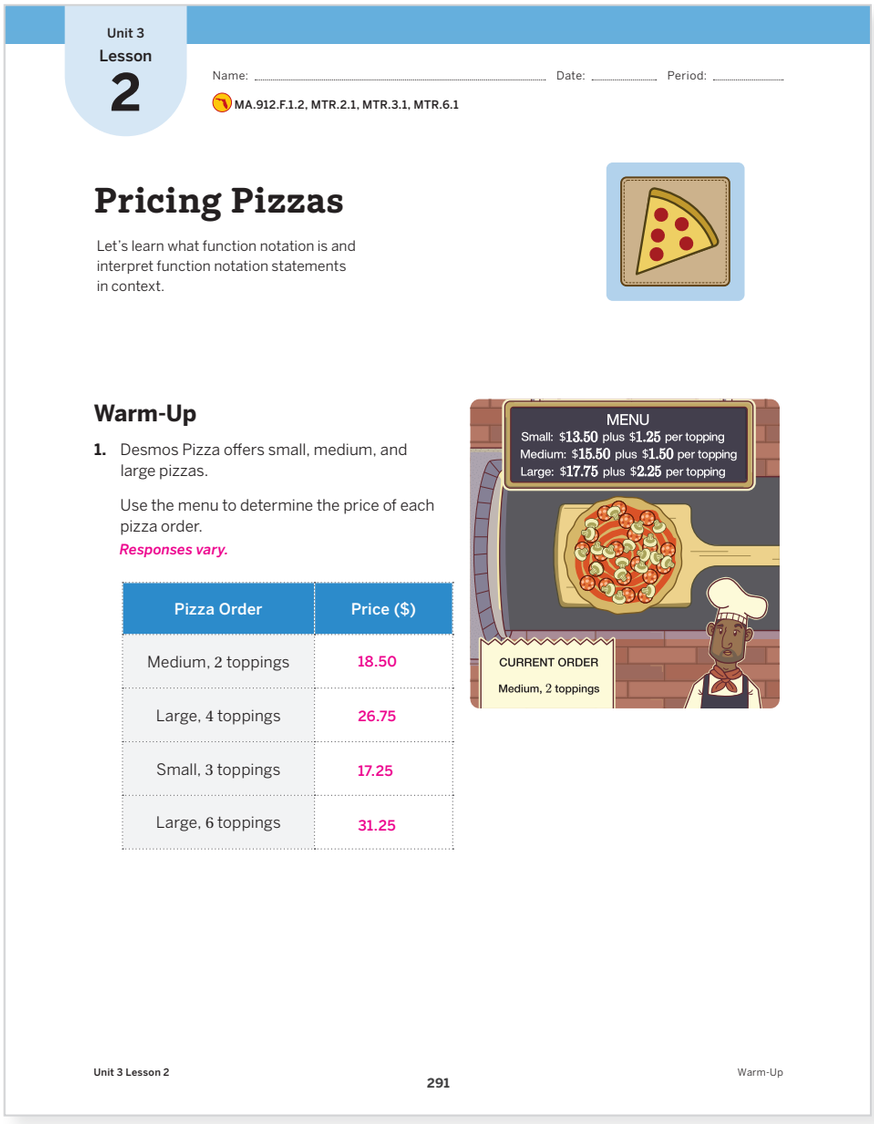
A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, invite them to make sense of the pizza menu by chunking the thinking into more manageable parts. Consider asking:

- "Where would I look to know how much a medium pizza costs?"
- "What would be the price of a medium pizza with 0 toppings?"
- "How much should we add for each topping for a medium pizza?" **(MTR.2.1)**

2 Connect

Invite students to share their strategies for finding the price of each order.

To surface connections, consider asking students when they have done calculations like this or looked at relationships like this before. *Responses vary.* This is like using a linear model: the price of each size of pizza goes up by the same amount for each topping we add.



Unit 3
Lesson
2

Name: Date: Period:

MA.912.F.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Pricing Pizzas

Let's learn what function notation is and interpret function notation statements in context.

Warm-Up

1. Desmos Pizza offers small, medium, and large pizzas.

   Use the menu to determine the price of each pizza order.

   Responses vary.

Pizza Order	Price ($)
Medium, 2 toppings	18.50
Large, 4 toppings	26.75
Small, 3 toppings	17.25
Large, 6 toppings	31.25

Unit 3 Lesson 2 291 Warm-Up

Pairs | 10 min

Activity 1 Pricing Pizzas

Purpose: Students attend to precision in using **function notation** to write and interpret statements describing the price of a pizza based on the size and number of toppings. **(MTR.4.1)**

1 Launch

Consider asking:

- "What is a function?"
- "What does it mean for one variable to be dependent on another? Which variable is the *dependent variable* in this situation? Why is number of toppings *independent*?"

ELL **English Language Learners** Review an anchor chart that displays the class definition of the term *function* to support student reasoning. **(Reading, Speaking)**

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

Note: If it doesn't come up naturally, consider asking:

- "How can we decide if the relationship between number of toppings and pizza price for a large pizza is a function?" *Responses vary.* We can check and see if every pizza size and topping combination has only one output (price).
- "Does the size of the pizza matter for deciding if this is a function? Why or why not?" *Responses vary.* It shouldn't matter because no matter what size pizza you get, you pay one price for that size with however many toppings you want.

2 Monitor

Display the menu.

To encourage participation, invite students to try out multiple pizza orders.

A **Accessibility: Visual-Spatial Processing** As students interpret visual representations, invite them to describe to a partner how the pizza order is represented by an equation. **(MTR.4.1)**

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Pricing Pizzas

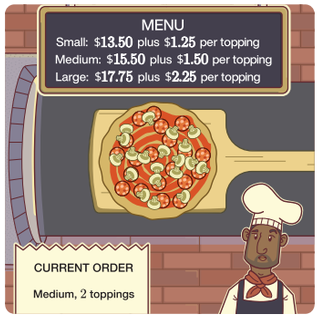
2. A worker at Desmos Pizza made a list of all the large pizza orders one night.



Discuss: How is this list like a function?

Explanations vary. This list is like a function because each number of toppings only matches up with one price. But, if this list had prices for medium and small pizzas too, it would not be a function.

3. Desmos Pizza offers small, medium, and large pizzas. You can create a function using the price of the pizza as your constant and the cost per topping, t, as your variable. Use the menu to the right to complete the table using the same format as shown in the first output.



Pizza Order	Price ($)
Small, 3 toppings	$13.50 + 1.25t = 17.25$
Large, 2 toppings	$17.75 + 2.25t = 22.25$
Medium, 1 topping	$15.50 + 1.50t = 17.00$

Unit 3 Lesson 2 292 Activity 1

Pairs | 10 min

Activity 1 Pricing Pizzas (continued)

Purpose: Students attend to precision in using **function notation** to write and interpret statements describing the price of a pizza based on the size and number of toppings. **(MTR.4.1)**

2 Monitor

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Luca's work. Then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Luca see how his work can be improved, or create a revised response. **(MTR.4.1)**

D **Differentiation: (Support)** Students may agree with Luca or think his argument makes sense if you interpret s as a variable. Celebrate this connection to thinking about variables and expressions and consider sharing that naming $s(x)$ as a function means it might not behave like a variable in all cases anymore.

Record student explanations for how to help Luca understand his mistake to share during the Connect.

3 Connect

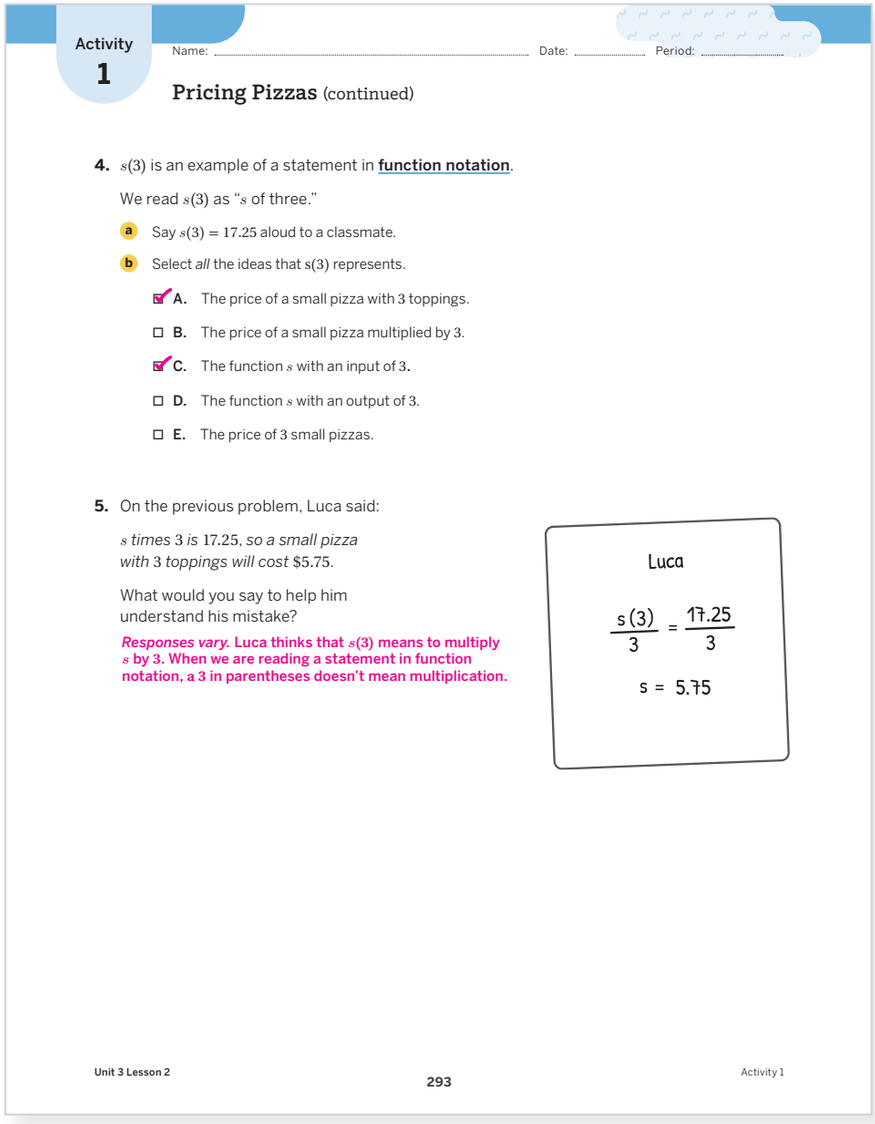
Display the statement written in function notation and its pronunciation.

Pause to model the pronunciation of $s(3)$ and invite students to share their thinking about what it means. Consider asking:

- "How would you change the statement to represent a small pizza with 4 toppings?"
- "How can we use clues to tell the difference between a statement in function notation and an expression with multiplication?" **(MTR.4.1)**
- "Where do you find the input and the output in a statement in function notation?"

Invite students to share their explanations for how to help Luca understand his mistake.

Note: If it doesn't come up naturally, consider sharing that the names of functions are arbitrary and are often chosen for convenience. It is equally possible to name a function "s," "m," or even "A&*" — any symbol (or string of symbols) will do! Although students will not need to come up with names for any functions in this lesson, they should feel empowered to name functions in a way that makes sense to them in future lessons. Consider inviting students to name functions in ways that make sense to them when the opportunity arises in lessons in this unit and throughout the year.



Activity 1

Name: Date: Period:

Pricing Pizzas (continued)

4. $s(3)$ is an example of a statement in **function notation**.

We read $s(3)$ as "s of three."

a. Say $s(3) = 17.25$ aloud to a classmate.

b. Select *all* the ideas that s(3) represents.

- ☑ **A.** The price of a small pizza with 3 toppings.
- ☐ **B.** The price of a small pizza multiplied by 3.
- ☑ **C.** The function s with an input of 3.
- ☐ **D.** The function s with an output of 3.
- ☐ **E.** The price of 3 small pizzas.

5. On the previous problem, Luca said:

s times 3 is 17.25, so a small pizza with 3 toppings will cost $5.75.

What would you say to help him understand his mistake?

Responses vary. Luca thinks that $s(3)$ means to multiply s by 3. When we are reading a statement in function notation, a 3 in parentheses doesn't mean multiplication.

Luca

$\frac{s(3)}{3} = \frac{17.25}{3}$

$s = 5.75$

Unit 3 Lesson 2 293 Activity 1

Pairs | 10 min

Activity 2 Interpreting Function Notation

Purpose: Students interpret function notation by matching function notation cards to their interpretations and interpret new statements.

Short on time? Consider giving each pair one of the statements and have them determine if it is true or false.

1 Launch

Display the card sort.

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, chunk this activity into manageable parts by inviting them to focus on one function card at a time.

2 Monitor

Look for students who use a variety of strategies and justify their thinking. **(MTR.4.1)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Pair the card $s(0)$ with the card "A small pizza that costs \$0."	**Support** Consider asking, "What is the lowest price possible for a small pizza? What else about the pizza could the 0 in $s(0)$ represent?"
Think that $m(7)$ means m multiplied by 7, or pair the card $m(x)$ with "The price of a medium pizza multiplied by x."	**Support** Invite these students to go back and refer to the statements about $s(3)$ and review Luca's mistake. Consider asking, "How can we tell the difference between a statement in function notation and an expression with multiplication?"
Would like to explore this topic further.	**Stretch** Invite students to decide if it is possible to write statements for the remaining cards and to do so if possible.

Invite students to practice speaking the statements in function notation in Problem 7. Consider asking, "What size pizza is Emma texting about when she writes $m(7)$? What about $l(5)$?"

Record explanations to share during the Connect.

3 Connect

Invite students to share their thinking to help build consensus as a class calling attention to any conflict or consensus you see.

To surface the Key Takeaway, consider asking:

- "How did you decide if each statement was true?"
- "How do the comparison statements help you to know what size pizza you would buy?"
- "Why is function notation a helpful tool when you want to describe or compare?"

Activity 2

Name: _____ Date: _____ Period: _____

Interpreting Function Notation

6. Match each function notation statement with its correct interpretation(s). Three cards will have no match.

Card A	Card B	Card C	Card D
The function s with the input 0.	A small pizza that costs \$0.	The price of a medium pizza with x toppings.	The price of 5 large pizzas.
Card E	**Card F**	**Card G**	
The output of l when the input is 5.	The price of a small pizza with 0 toppings.	The price of a medium pizza multiplied by x.	

$l(5)$	$s(0)$	$m(x)$
Card E	Card A Card F	Card C

Cards with no match: Card B, Card D, and Card G

7. Emma and her friends are texting about their pizza order.

Emma writes: $m(7) < l(5)$.

What do you think this means?

Responses vary. A medium pizza with 7 toppings costs less than a large pizza with 5 toppings.

MENU
Small: \$13.50 plus \$1.25 per topping
Medium: \$15.50 plus \$1.50 per topping
Large: \$17.75 plus \$2.25 per topping

8. Select *all* the true statements.

- ☑ **A.** $s(2) > s(1)$
- ☐ **B.** $m(4) < s(4)$
- ☑ **C.** $l(4) > s(3)$
- ☑ **D.** $l(1) = m(3)$
- ☐ **E.** $l(0) > m(2)$

Unit 3 Lesson 2 294 Activity 2

Key Takeaway: **Function notation** is an efficient way to write statements about functions and their inputs and outputs. For example, $f(2) = 3$ means that the function f at the input 2 gives the output 3.

Pairs | 10 min

Activity 3 Delivering Pizzas

Purpose: Students consider a new function that describes the number of minutes it takes to deliver a pizza and interpret statements about this new situation in function notation.

1 Launch

Display the image and the definition of $d(x)$.

Consider asking:

- "Why would a pizza company want to know how long it takes to deliver pizzas?"
- "Explain how you know $d(x)$ is a function." *Responses vary.* I think $d(x)$ is a function because if you travel 2 miles, it should take you the same amount of time. So every input should have exactly one output.

2 Monitor

Select and sequence strategies to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How do you say $d(x)$? What does $d(x)$ mean in the context of delivery times?" Invite students to go back to previous problems and refer to statements about $s(x)$.
Would like to explore this topic further.	**Stretch** Invite students to brainstorm a different situation that can be estimated with a function and share with a partner.

A Accessibility: Executive Functioning To support students in carrying out multiple steps, chunk this activity into more manageable parts by inviting them to focus on one card at a time.

3 Connect

Display the card sort.

Invite students to share their strategies and thinking for how to sort the cards.

To surface the Key Takeaway, consider asking:

- "What is the difference between $d(5) > 1$ and $d(1) > 5$?"
- "Why can't we write a statement in function notation for the remaining card?"

Activity 3

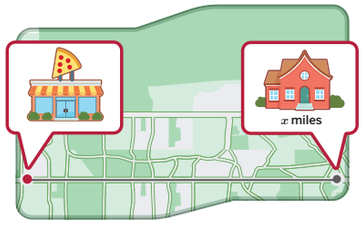
Name: _____ Date: _____ Period: _____

Delivering Pizzas

9. Desmos Pizza uses the function $d(x)$ to estimate the number of minutes it takes to deliver pizza x miles.

What would $d(2) = 30$ mean in this situation?

Responses vary. $d(2) = 30$ means that it takes 30 minutes to deliver a pizza 2 miles.



10. $d(x)$ estimates the number of minutes it takes to deliver pizza x miles. Match each function notation card to a description. One card will have no match.

Card A
It takes longer to deliver 5 miles away than 1 mile away.

Card B
The number of minutes to make a delivery 5 miles away.

Card C
A delivery 1 mile away will take more than 5 minutes.

Card D
A delivery 5 mile away will take more than 1 minutes.

Card E
Delivering 5 pizzas takes longer than delivering 1 pizza.

$d(5) > 1$	$d(5)$	$d(5) > d(1)$	$d(1) > 5$
Card D	Card B	Card A	Card C

Card with no match: Card E

Unit 3 Lesson 2 295 Activity 3

Key Takeaway: Function notation can be interpreted in context. For example, in this activity, $d(2) = 30$ means a 2-mile pizza delivery is estimated to take 30 minutes.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to read statements in function notation and how to interpret them for a situation.

Invite students to respond independently, and then share their thinking with a partner.

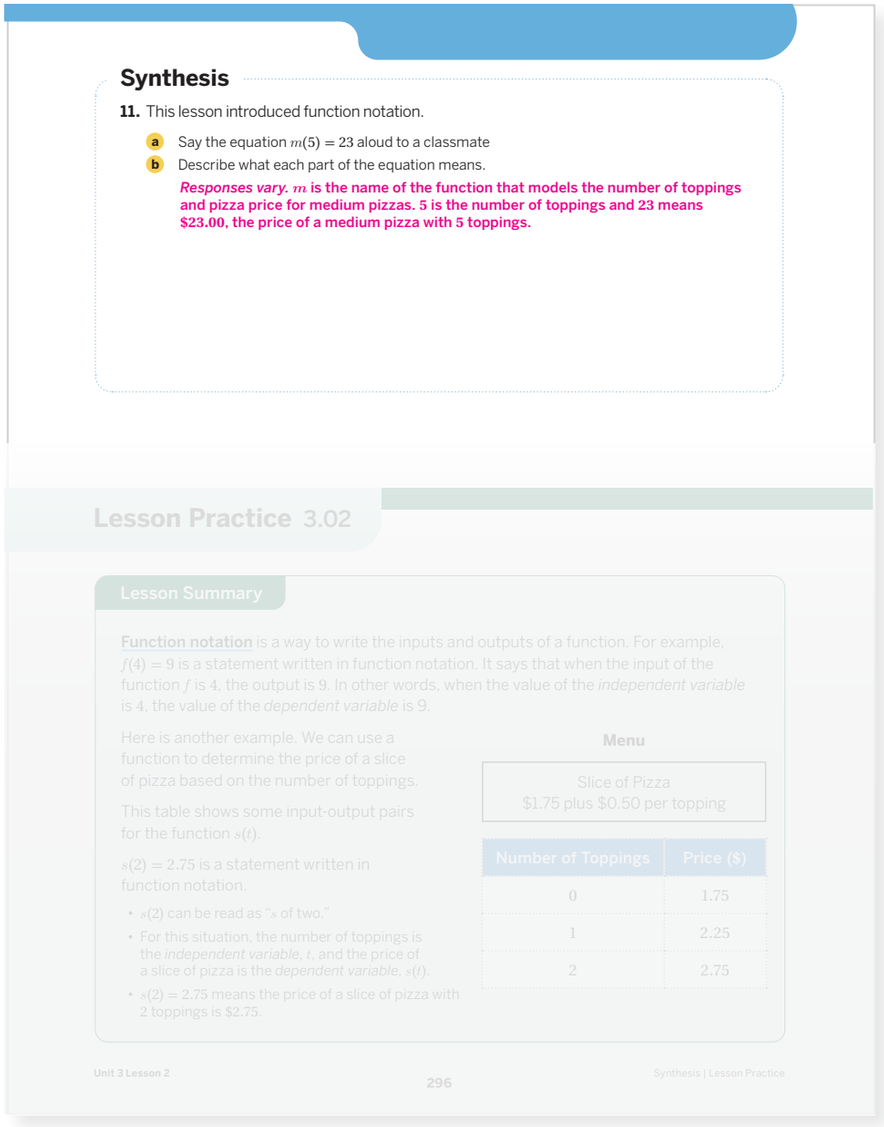
Record and share a variety of ideas, such as:

- 5 is the input and 23 is the output.
- m is the name of the function that describes medium pizzas.
- $m(5)$ is the same as 23.
- Using the function m to determine the price for a medium pizza with 5 toppings.
- A medium pizza with 5 toppings costs $23.

ELL **English Language Learners** As students process their ideas in words, invite them to talk through their thinking with a partner before writing it down.

Consider asking, "What connections do you see between your classmates' responses?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. This lesson introduced function notation.

a. Say the equation $m(5) = 23$ aloud to a classmate

b. Describe what each part of the equation means.

Responses vary. m is the name of the function that models the number of toppings and pizza price for medium pizzas. 5 is the number of toppings and 23 means $23.00, the price of a medium pizza with 5 toppings.

Lesson Practice 3.02

Lesson Summary

Function notation is a way to write the inputs and outputs of a function. For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

Here is another example. We can use a function to determine the price of a slice of pizza based on the number of toppings.

This table shows some input-output pairs for the function $s(t)$.

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two."
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is $2.75.

Menu

Slice of Pizza
$1.75 plus $0.50 per topping

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Unit 3 Lesson 2 296 Synthesis | Lesson Practice

Lesson Takeaway: Function notation is an efficient way to write statements about functions that can be interpreted in context. For example, using the functions in this lesson, the statement, "A small pizza with 4 toppings costs more than a large pizza with no toppings" can be written in function notation as $s(4) > l(0)$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting a statement in function notation.

Today's Goal

1. **Language Goal:** Interpret a statement written in *function notation* in context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.02

A veterinarian uses the function $p(x)$ to predict the weight, in pounds, of a poodle that is x months old.

What does $p(8) = 40$ mean in this situation?

Responses vary. **The veterinarian predicts that an 8-month-old poodle will weigh 40 pounds.**

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support interpreting a statement written in function notation:

» **Respond to Student Thinking:**

- Use the *Evaluating Function Notation* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students developed early ideas about the term **function notation** and how to write and interpret statements in function notation. How will their understanding of the term *function notation* build over the next several lessons? How can you support students as they continue to write, interpret, and later manipulate and solve statements in function notation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. This lesson introduced function notation.

a. Say the equation $m(5) = 23$ aloud to a classmate

b. Describe what each part of the equation means.

Responses vary. m is the name of the function that models the number of toppings and pizza price for medium pizzas, 5 is the number of toppings and 23 means $23.00, the price of a medium pizza with 5 toppings.

Lesson Practice 3.02

Lesson Summary

Function notation is a way to write the inputs and outputs of a function. For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

Here is another example. We can use a function to determine the price of a slice of pizza based on the number of toppings.

This table shows some input-output pairs for the function $s(t)$.

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two."
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is $2.75.

Menu

Slice of Pizza
$1.75 plus $0.50 per topping

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Lesson Practice 3.02

Name: _____ Date: _____ Period: _____

Problems 1–2: The function $f(t)$ models the temperature, in degrees Celsius, t hours after midnight.

1. Select the equation that represents the statement: *At 1 AM, the temperature was 20° C.*

A. $f(100) = 20$ B. $f(20) = 100$ C. $f(1) = 20$ D. $f(20) = 1$

2. Use function notation to represent each statement.

The temperature at 2 AM. $f(2)$

The temperature was the same at 9 AM and at 11 AM. $f(9) = f(11)$

The temperature was higher at 9 AM than at 2 AM. $f(9) > f(2)$

t hours after midnight, the temperature was 24°C. $f(t) = 24$

3. A restaurant sells three different salads. They use the functions $c(x)$, $g(x)$, and $p(x)$ to represent the cost of their caesar, garden, and pasta salads in dollars, with x additional ingredients added. Explain the meaning of each statement. *Explanations vary.*

$g(0) = 10$	A garden salad with no additional ingredients costs $10.
$g(3) > c(1)$	A garden salad with 3 additional ingredients costs more than a caesar salad with 1 additional ingredient.
$p(2) < g(3)$	A pasta salad with 2 additional ingredients costs less than a garden salad with 3 additional ingredients.

Problems 4–6: Use the table to determine the missing values in the function statements.

4. $f(\underline{5}) = 23$

5. $f(\text{-}5) = \underline{17}$

6. $f(\underline{\text{-}2}) = \text{-}5$

x	$f(x)$
-5	17
-2	-5
5	23

Lesson Practice 3.02

Name: _____ Date: _____ Period: _____

Test Practice

7. Which orders can be made from this menu?

A. Small with 3 toppings for $15

B. Medium with 3 toppings for $22

C. Large with 4 toppings for $30

D. Medium with 1 topping for $17

MENU
Small: $12 plus $1 per topping
Medium: $15 plus $2 per topping
Large: $18 plus $3 per topping

Spiral Review

8. Here are Rules A and B.

Which rule is a function? Circle one.

Rule A (Rule B) Both Neither

Rule A		Rule B	
Input	Output	Input	Output
4	2	1	5
9	-3	2	9
9	3	2	9

9. Solve for x: $3(2x - 4) + 5 = 2(x + 6) + x$

$6x - 12 + 5 = 2x + 12 + x$

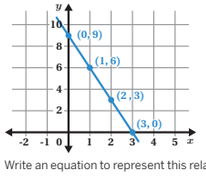
$6x - 7 = 3x + 12$

$6x - 3x = 12 + 7$

$3x = 19$

$x = \frac{19}{3}$

10. Here's a graph of a relationship and its table of values.



x	y
0	9
1	6
2	3
3	0

Write an equation to represent this relationship. $y = \text{-}3x + 9$ (or equivalent)

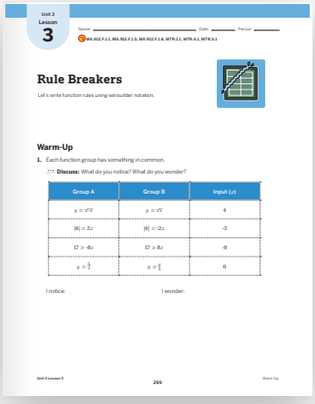
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.F.1.2
	2–3	2	MA.912.F.1.2
	4–6	1	MA.912.F.1.2
TEST Practice	7	2	MA.912.F.1.2
Spiral Review			
Fluency	8	1	MA.912.F.1.1
	9	1	MA.8.AR.2.1
	10	1	MA.8.AR.3.3

Unit 3

Lesson

3



Rule Breakers

Domain and Range Notation

Let's write function rules using set-builder and inequality notations.

Learning Outcomes

Today's Goal

1. **Goal:** Review what makes a function undefined.
2. **Goal:** Write function notation in set-builder and inequality notations.
3. **Language Goal:** Use symbols and natural language to describe the domain and range of functions. **(Reading, Writing, Speaking, and Listening)**

Students describe the domain and range for functions that limit values that would be undefined. **(MTR.1.1, MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In Lesson 2, students were introduced to function notation with different inputs and outputs.

Future Learning

In Lesson 4, students create graphs to match function stories.

Connections

- Students collaborate and use evidence-based reasoning to define a rule to determine if a set of input and output values is a function. (mathematical, cross-curricular).

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.1
Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

Also addressing: **MA.912.F.1.5, MA.912.F.1.6**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.7.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1	**MA.912.F.3**
MA.6.AR.1.2	

Vocabulary

New Vocabulary

Set-builder notation

Review Vocabulary

Interval notation
Inequality notation
Function notation
domain

Lesson at a Glance

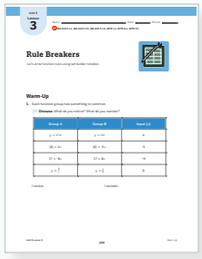
~ 45 min

MA.912.F.1.1, MA.912.F.1.5, MA.912.F.1.6

Warm-Up

Whole Class | 5 min

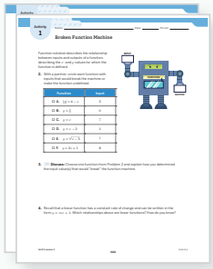
Students use the Notice and Wonder routine to identify undefined functions.
Routine(s): Notice and Wonder **(MTR.1.1)**



Activity 1

Partners | 10 min

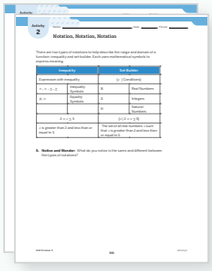
Students determine if a given value would make a function undefined.
Routine(s): Think, Pair, Share **(MTR.4.1)**



Activity 2

Small Group | 10 min

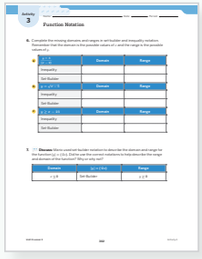
Students explore the similarities and differences between set-builder notation and inequality notation.
Routine(s): Notice and Wonder **(MTR.5.1)**



Activity 3

Whole Class | 10 min

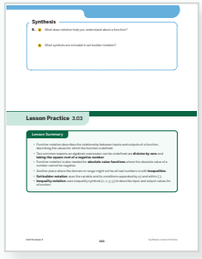
Students describe the domain and range of a function using natural language and symbols.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using set-builder and inequality notations to describe the domain and range of the function. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of set-builder and inequality notations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to identify undefined functions. **(MTR.1.1)**

Short on time? Consider reducing the number of pairs.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of defined and undefined functions. **(MTR.1.1)**

Note: If it doesn't come up naturally, consider asking, "What do you see in common with Group 1? What does it mean if there is a square root of a negative number? What happens if you try to divide by zero? Can the absolute value of a quantity be a negative number?"

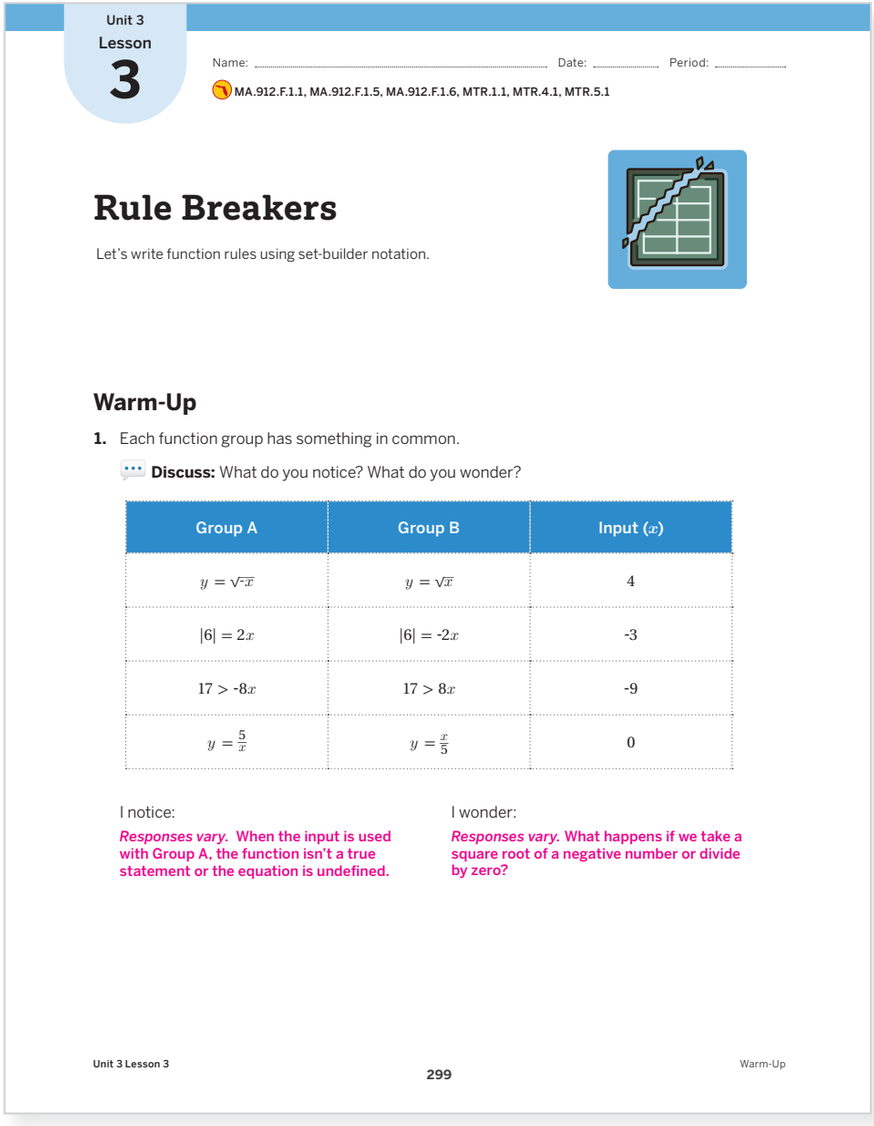
A **Accessibility: Memory and Attention** Invite students to record their ideas in their Student Edition or on paper.

2 Connect

ELL **English Language Learners** Invite students to simplify each function orally to hear the negative number with square roots and division by zero. **(Speaking and Listening)**

Invite students to compare what's different between each pair that is in different groups by asking questions like:

- What do you see different under the radical?
- What happens when you take the absolute value of a number?
- What happens when you multiply two negatives?



Unit 3
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.F.1.1, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.4.1, MTR.5.1

Rule Breakers

Let's write function rules using set-builder notation.

Warm-Up

1. Each function group has something in common.

Discuss: What do you notice? What do you wonder?

Group A	Group B	Input (x)
$y = \sqrt{-x}$	$y = \sqrt{x}$	4
$\lvert 6 \rvert = 2x$	$\lvert 6 \rvert = -2x$	-3
$17 > -8x$	$17 > 8x$	-9
$y = \frac{5}{x}$	$y = \frac{x}{5}$	0

I notice:

Responses vary. When the input is used with Group A, the function isn't a true statement or the equation is undefined.

I wonder:

Responses vary. What happens if we take a square root of a negative number or divide by zero?

Unit 3 Lesson 3 299 Warm-Up

Partners | 10 min

Activity 1 Broken Function Machine

Purpose: Students determine if a given value would make a function undefined. **(MTR.4.1)**

Short on time? Consider removing problems 2c and 2d.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse about which input values make a function undefined. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

2 Monitor

Remind students to simplify after substituting the input value for x.

Look for strategies to share during connect about how students made their matches.

D Differentiation

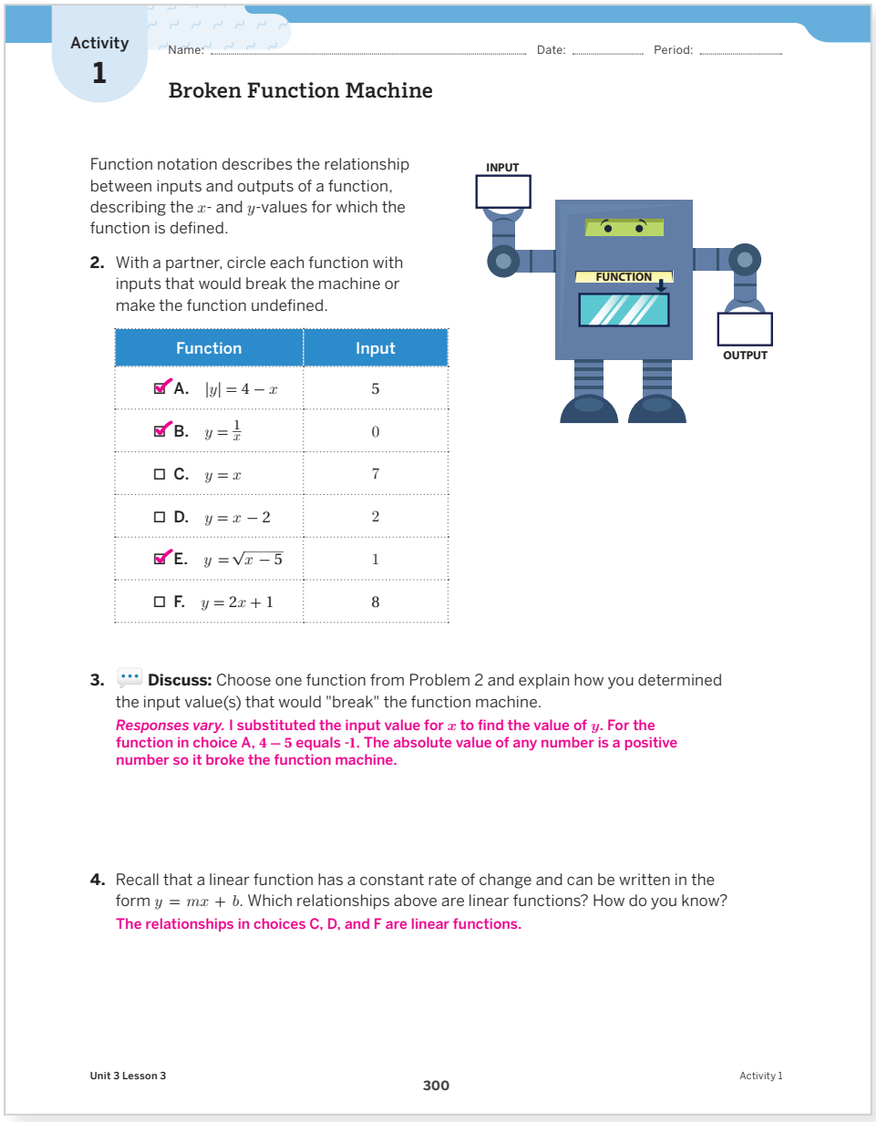
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to start solving a problem for each input value.
Don't identify the functions with a zero in the denominator.	**Strengthen** Consider asking, "What happens if you divide by zero?"
Don't identify the negative numbers under the radical.	**Strengthen** Consider asking, "What happens if you take the square root of a negative number?"
Don't identify the absolute value.	**Strengthen** Consider asking, "If the absolute value is the distance from zero, can it be a negative number?"

3 Connect

Encourage partners to compare the efficiency of their strategies to make their matches. **(MTR.4.1)**

To surface the Key Takeaway:

- What number can be squared (multiplied by itself) and have a negative number as an answer?
- How many groups of nothing can a number be broken into?
- Which functions can be written in the form $y = mx + b$? What does this tell you about the function?



Activity 1

Name: Date: Period:

Broken Function Machine

Function notation describes the relationship between inputs and outputs of a function, describing the x- and y-values for which the function is defined.

2. With a partner, circle each function with inputs that would break the machine or make the function undefined.

Function	Input
☑ A. $\lvert y\rvert = 4 - x$	5
☑ B. $y = \frac{1}{x}$	0
☐ C. $y = x$	7
☐ D. $y = x - 2$	2
☑ E. $y = \sqrt{x - 5}$	1
☐ F. $y = 2x + 1$	8

3. **Discuss:** Choose one function from Problem 2 and explain how you determined the input value(s) that would "break" the function machine.

Responses vary. I substituted the input value for x to find the value of y. For the function in choice A, $4 - 5$ equals -1. The absolute value of any number is a positive number so it broke the function machine.

4. Recall that a linear function has a constant rate of change and can be written in the form $y = mx + b$. Which relationships above are linear functions? How do you know?

The relationships in choices C, D, and F are linear functions.

Unit 3 Lesson 3 300 Activity 1

Key Takeaway: Functions have inputs that could create an undefined function with division by zero, the square root of a negative number, or the absolute value needing to result in a negative number. Linear functions have a constant rate of change and can be written in the form $y = mx + b$.

Small Group | 15 min

Activity 2 Notation, Notation, Notation

Purpose: Students explore the similarities and differences between set-builder notation and inequality notation. **(MTR.5.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of set-builder and inequality notations. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Create a list of words about the context that students might not know, along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

2 Monitor

Invite students to take turns reading each part of the function notation table aloud.

A **Accessibility: Visual-Spatial Processing** Invite students to use their Student Edition to highlight in different colors to support interpreting visual representations.

Listen for ideas to share during the Notice and Wonder about what students see as alike and different.

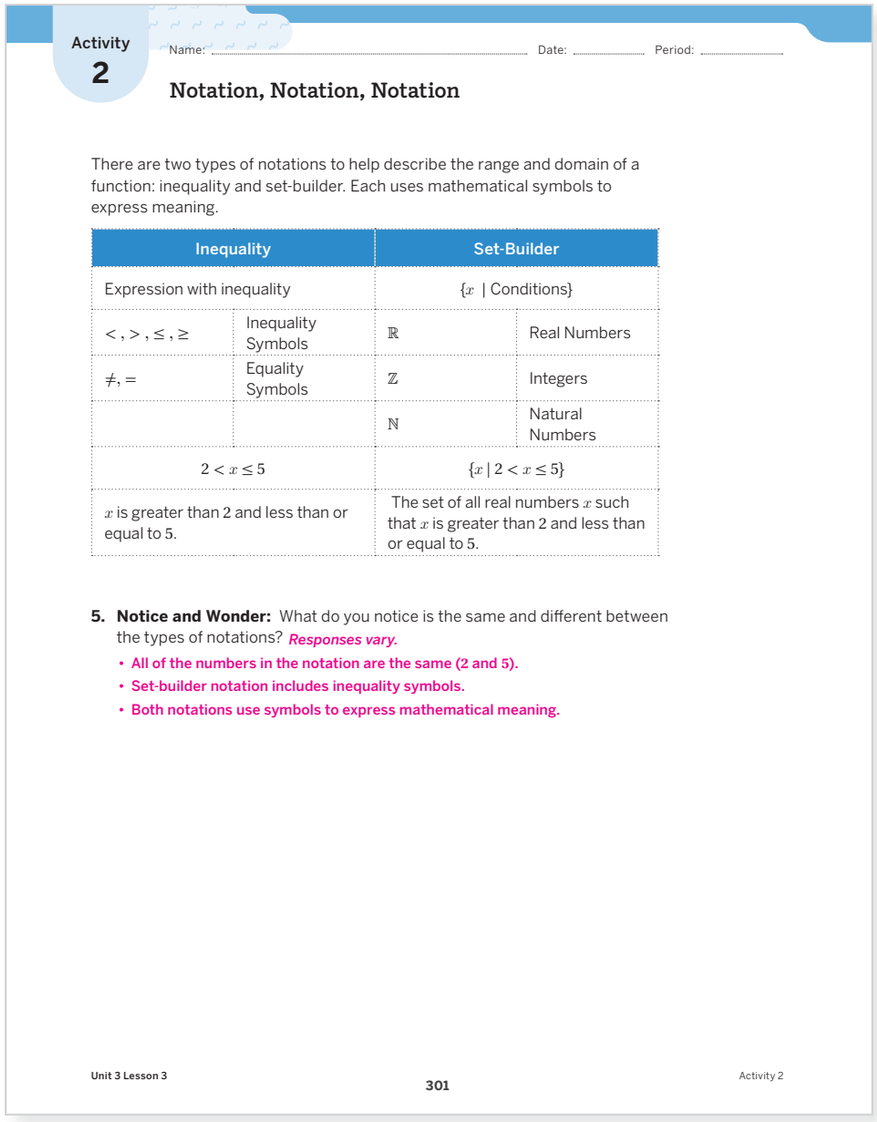
D **Differentiation: (Support)** Provide flashcards for the different types of numbers represented by $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{Z}$.

3 Connect

Consider asking students to look for the similarities between each type of function notation. **(MTR.5.1)**

To surface the Key Takeaway, ask:

- What do you see that is the same across the two types of notation?
- What similar words describe the sets of range and domain?



Activity 2

Name: Date: Period:

Notation, Notation, Notation

There are two types of notations to help describe the range and domain of a function: inequality and set-builder. Each uses mathematical symbols to express meaning.

Inequality		Set-Builder	
Expression with inequality		$\{x \mid \text{Conditions}\}$	
$<, >, \leq, \geq$	Inequality Symbols	$\mathbb{R}$	Real Numbers
$\neq, =$	Equality Symbols	$\mathbb{Z}$	Integers
		$\mathbb{N}$	Natural Numbers
$2 < x \leq 5$		$\{x \mid 2 < x \leq 5\}$	
x is greater than 2 and less than or equal to 5.		The set of all real numbers x such that x is greater than 2 and less than or equal to 5.	

5. Notice and Wonder: What do you notice is the same and different between the types of notations? *Responses vary.*

- All of the numbers in the notation are the same (2 and 5).
- Set-builder notation includes inequality symbols.
- Both notations use symbols to express mathematical meaning.

Unit 3 Lesson 3 301 Activity 2

Key Takeaway: Set-builder notation uses a structure similar to inequality notation.

Whole Class | 10 min

Activity 3 Function Notation

Purpose: Students describe the domain and range of a function using natural language and symbols.

1 Launch

Provide flashcards with inequality and set-builder notation or the chart from Activity 2 for each student to independently answer questions.

Consider giving students individual wipe boards to answer individually and share with the class.

2 Monitor

Look for strategies used to create different notations for the same function.

A **Accessibility: Memory and Attention** Invite students to refer back to the chart in Activity 2 to activate prior knowledge.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Struggle to differentiate between types of notation.	**Strengthen** Draw the connection to each type of notation and how it is written: • **Set-builder:** All of the *building* blocks to describe the set. • **Inequality:** Just the *inequality* statement.

3 Connect

Share examples of student work showing unique or creative strategies to write different forms of function notation.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Activity 3

Name: Date: Period:

Function Notation

6. Complete the missing domains and ranges in set-builder and inequality notation. Remember that the domain is the possible values of x and the range is the possible values of y.

a	$y = \frac{3}{(x-2)}$	Domain	Range
	Inequality	$x \neq 2$	$y \neq 0$
	Set-Builder	$\{x \mid x \neq 2\}$	$\{y \mid y \neq 0\}$
b	$y = \sqrt{x-5}$	Domain	Range
	Inequality	$x \geq 5$	$y \geq 0$
	Set-Builder	$\{x \mid x \geq 5\}$	$\{y \mid y \geq 0\}$
c	$y \geq x - 23$	Domain	Range
	Inequality	$(-\infty < x < \infty)$	$(-\infty < y < \infty)$
	Set-Builder	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$

7. **Discuss:** Mario used set-builder notation to describe the domain and range for the function $|y| = (-2x)$. Did he use the correct notations to help describe the range and domain of the function? Why or why not?

Domain	$\lvert y\rvert = (-2x)$	Range
$x \leq 0$	Set-Builder	$y \geq 0$

Answers may vary. Mario used inequality notation, not set builder notation.

Unit 3 Lesson 3 302 Activity 3

Whole Class | 5 min

Synthesis

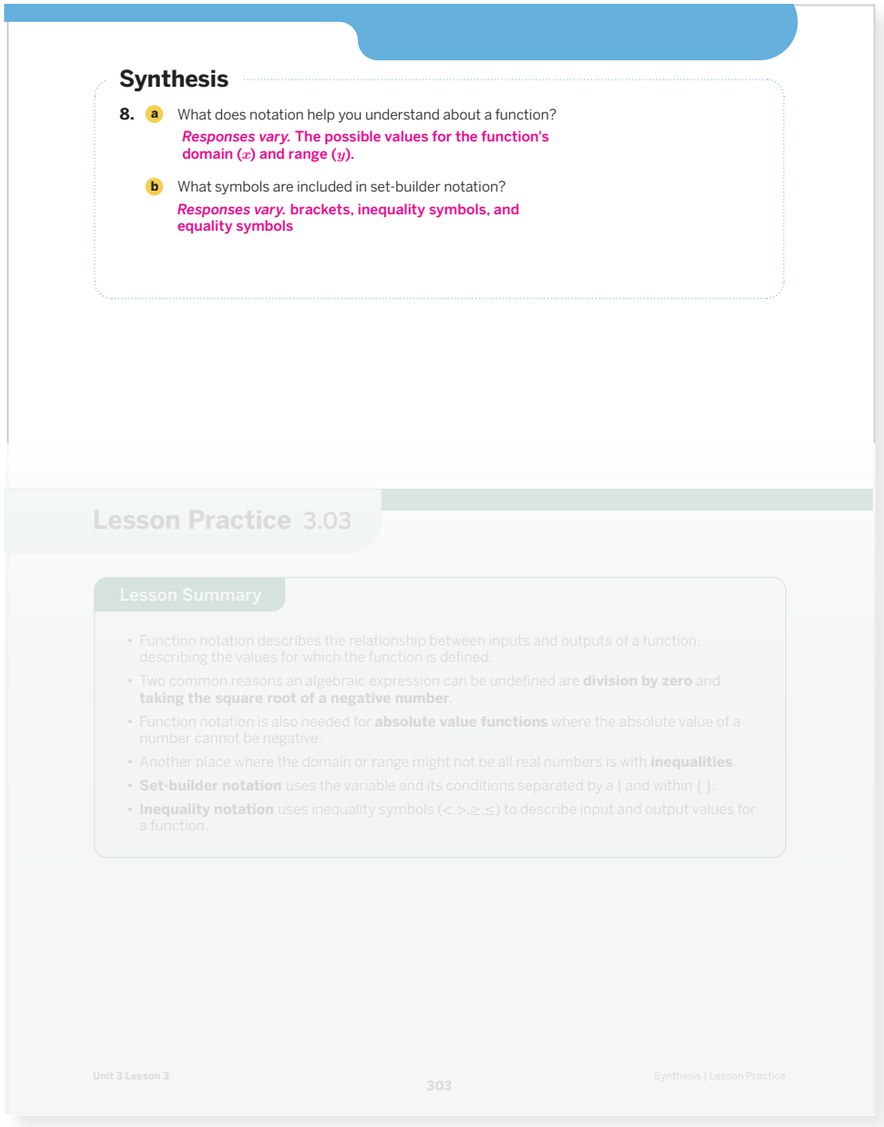
Purpose: Students synthesize their understanding of using set-builder and inequality notations to describe the domain and range of the function.

Invite students to respond independently and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Similarities and differences between set builder and inequality notations.
- Strategies for determining domain and range of functions.

MLR **MLR2: Collect and Display** Use this routine to collect students' examples as they share their ideas on notations. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each notation and what questions they still have.



Synthesis

8. a What does notation help you understand about a function?

Responses vary. The possible values for the function's domain (x) and range (y).

b What symbols are included in set-builder notation?

Responses vary. brackets, inequality symbols, and equality symbols

Lesson Practice 3.03

Lesson Summary

- Function notation describes the relationship between inputs and outputs of a function, describing the values for which the function is defined.
- Two common reasons an algebraic expression can be undefined are **division by zero** and **taking the square root of a negative number**.
- Function notation is also needed for **absolute value functions** where the absolute value of a number cannot be negative.
- Another place where the domain or range might not be all real numbers is with **inequalities**.
- **Set-builder notation** uses the variable and its conditions separated by a | and within { }.
- **Inequality notation** uses inequality symbols ($<, >, \geq, \leq$) to describe input and output values for a function.

Unit 3 Lesson 3 303 Synthesis | Lesson Practice

Lesson Takeaway: Set-builder and inequality notations describe the domain and range of functions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of set-builder and inequality notations.

Today's Goals

1. **Goal:** Review what makes a function undefined.
2. **Goal:** Write function notation in set-builder and inequality notations.
3. **Language Goal:** Use symbols and natural language to describe the domain and range of functions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.03

Determine the domain and range of $y = \sqrt{x - 9}$. Express the domain and range using set-builder notation.

Domain: $\{x \mid x \geq 9\}$

Range: $\{y \mid y \geq 0\}$

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support writing the domain and range of a function in set-builder notation:

» **Respond to Student Thinking:**

- Revisit **Activity 3, Problem 6** as a class and invite students to share how they used inequality notation to convert to set-builder notation.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Expand to problems with more complex functions, such as $f(x) = \frac{1}{x^2 - 16}$

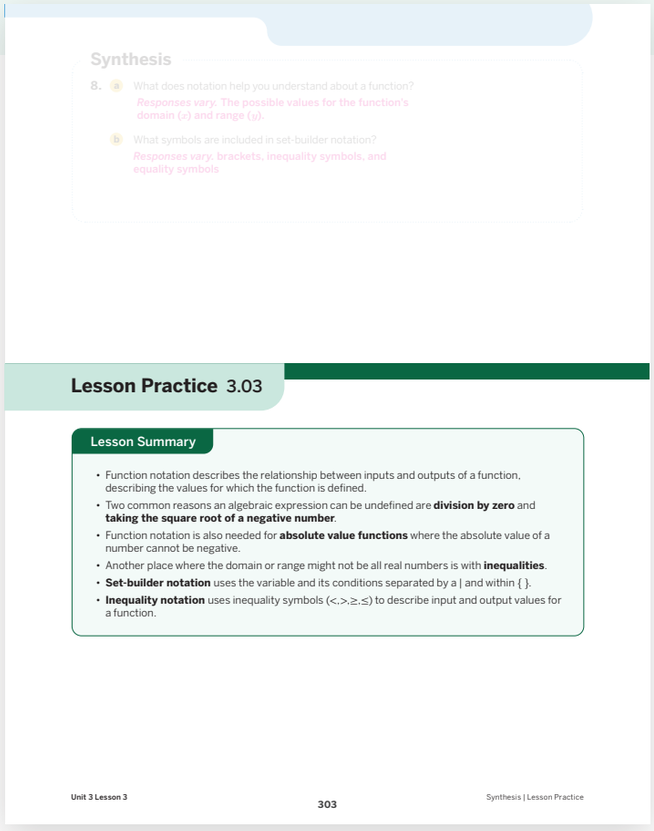
Professional Learning

Reflect on the facilitation of the Notice and Wonder routine in this lesson. Students will use this routine again. What about the routine felt successful? What are some areas for growth next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. a. What does notation help you understand about a function?

Responses vary. The possible values for the function's domain (x) and range (y).

b. What symbols are included in set-builder notation?

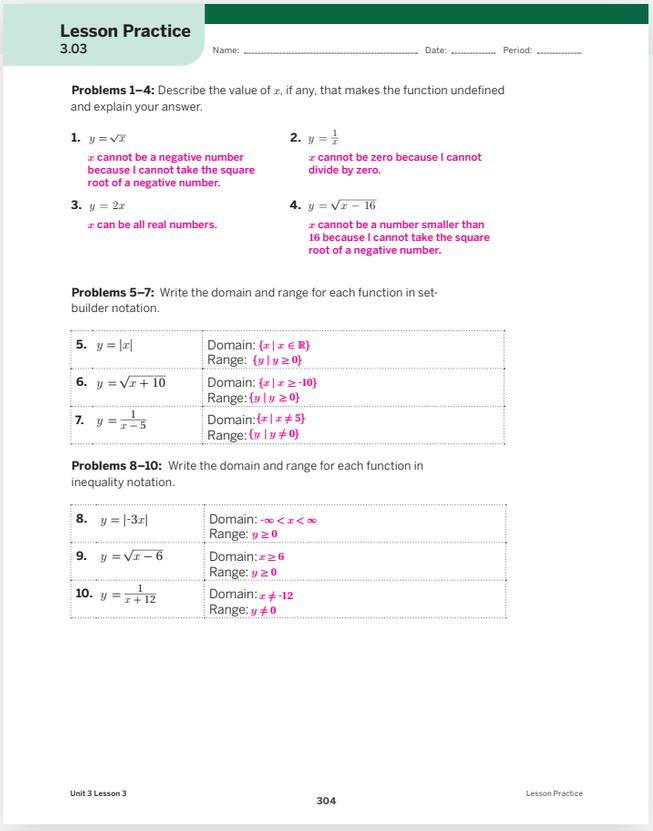
Responses vary. brackets, inequality symbols, and equality symbols

Lesson Practice 3.03

Lesson Summary

- Function notation describes the relationship between inputs and outputs of a function, describing the values for which the function is defined.
- Two common reasons an algebraic expression can be undefined are **division by zero** and **taking the square root of a negative number**.
- Function notation is also needed for **absolute value functions** where the absolute value of a number cannot be negative.
- Another place where the domain or range might not be all real numbers is with **inequalities**.
- **Set-builder notation** uses the variable and its conditions separated by a | and within { }.
- **Inequality notation** uses inequality symbols (<,>,≥,≤) to describe input and output values for a function.

Unit 3 Lesson 3 303 Synthesis | Lesson Practice



Lesson Practice 3.03

Name: Date: Period:

Problems 1–4: Describe the value of x, if any, that makes the function undefined and explain your answer.

1. $y = \sqrt{x}$

   x cannot be a negative number because I cannot take the square root of a negative number.

2. $y = \frac{1}{x}$

   x cannot be zero because I cannot divide by zero.

3. $y = 2x$

   x can be all real numbers.

4. $y = \sqrt{x - 16}$

   x cannot be a number smaller than 16 because I cannot take the square root of a negative number.

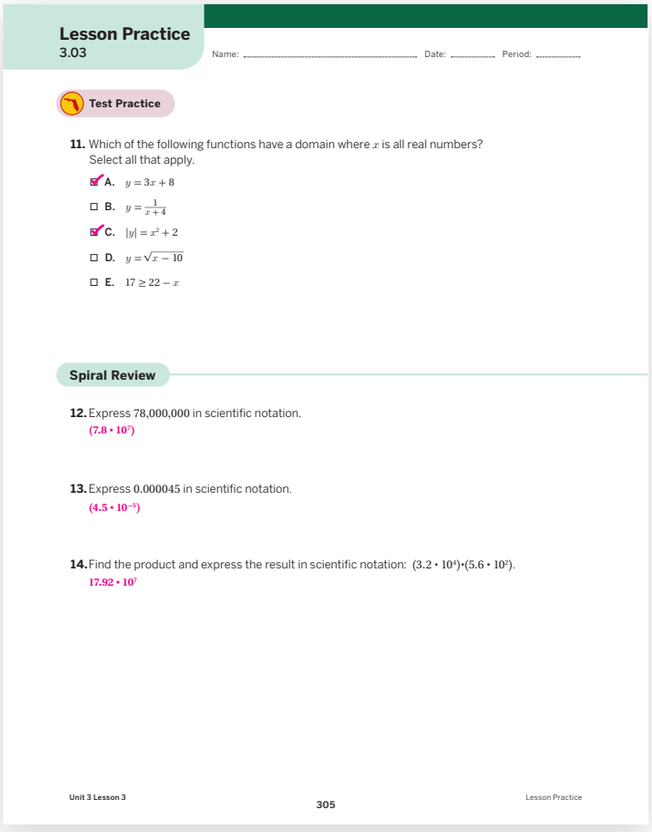
Problems 5–7: Write the domain and range for each function in set-builder notation.

5. $y = \lvert x \rvert$	Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \geq 0\}$
6. $y = \sqrt{x + 10}$	Domain: $\{x \mid x \geq -10\}$ Range: $\{y \mid y \geq 0\}$
7. $y = \frac{1}{x - 5}$	Domain: $\{x \mid x \neq 5\}$ Range: $\{y \mid y \neq 0\}$

Problems 8–10: Write the domain and range for each function in inequality notation.

8. $y = \lvert -3x \rvert$	Domain: $-\infty < x < \infty$ Range: $y \geq 0$
9. $y = \sqrt{x - 6}$	Domain: $x \geq 6$ Range: $y \geq 0$
10. $y = \frac{1}{x + 12}$	Domain: $x \neq -12$ Range: $y \neq 0$

Unit 3 Lesson 3 304 Lesson Practice



Lesson Practice 3.03

Name: Date: Period:

Test Practice

11. Which of the following functions have a domain where x is all real numbers? Select all that apply.

- ☑ A. $y = 3x + 8$
- ☐ B. $y = \frac{1}{x+4}$
- ☑ C. $|y| = x^2 + 2$
- ☐ D. $y = \sqrt{x - 10}$
- ☐ E. $17 \geq 22 - x$

Spiral Review

12. Express 78,000,000 in scientific notation.

$(7.8 \cdot 10^7)$

13. Express 0.000045 in scientific notation.

$(4.5 \cdot 10^{-5})$

14. Find the product and express the result in scientific notation: $(3.2 \cdot 10^4) \cdot (5.6 \cdot 10^2)$.

$17.92 \cdot 10^7$

Unit 3 Lesson 3 305 Lesson Practice

Lesson Practice Item Analysis

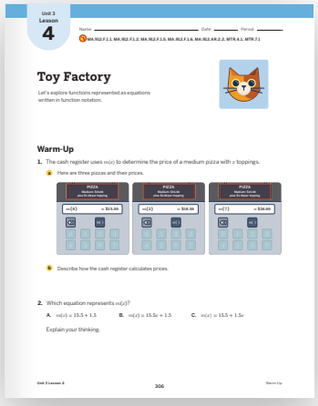
	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-Lesson			
	1–4	2	MA.912.F.1.6
	5–10	2	MA.912.F.1.6
Test Practice	11	1	MA.912.F.1.6
Spiral Review			
Fluency	12–13	1	MA.8.NSO.1.4
	14	2	MA.8.NSO.1.5

Unit 3
Lesson
4

Toy Factory

Function Notation and Equations

Let's explore functions represented as equations written in function notation.


Unit 3
Lesson
4

Toy Factory

Let's explore functions represented as equations written in function notation.

Warm-Up

1. The cash register uses $m(p)$ to determine the price of a medium pizza with p toppings.
   - Here are three pizzas and their prices.
   - Describe how the cash register calculates prices.
2. Which equation represents $m(p)$?
   A. $m(p) = 15.5 + 1.5$ B. $m(p) = 15.5p + 1.5$ C. $m(p) = 15.5 + 1.5p$
   Explain your thinking.

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Learning Outcomes

Today's Goals

1. **Goal:** Evaluate functions written in function notation.
2. **Goal:** Write equations of functions using function notation.
3. **Language Goal:** Describe equations written in function notation in context. **(Speaking and Writing)**

Students explore functions represented as equations written in function notation. They interpret, evaluate, and write equations using function notation. Through the contexts of heights and bow tie widths for toys, they also explore the power of an equation to determine the output for any input.

Coherence

Prior Learning

In Lesson 2, students were introduced to function notation to represent input-output pairs. In Unit 1, students wrote equations to represent relationships between two variables.

Future Learning

In Lesson 5, students will make connections between function notation and graphs. In Units 5 and 6, students will analyze exponential and quadratic functions defined using function notation.

Connections

- Students use linear functions to determine the price of a pizza as a function of the number of toppings ordered and other real world contexts. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

MA.912.AR.2.2

Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

Mathematical Thinking and Reasoning Standard: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Lesson at a Glance

~ 45 min

MA.912.F.1.1, MA.912.AR.2.2

Warm-Up

Pairs | 5 min

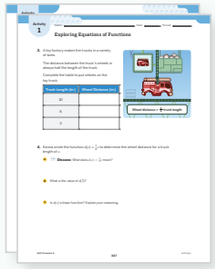
Students explore equations defined using function notation in the context of the pizza cash register from the previous lesson.



Activity 1

Pairs | 20 min

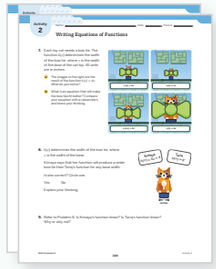
Students interpret and evaluate functions using function notation in the context of a toy factory by connecting the structure of equations to the relationship between toy parts. **(MTR.5.1)**



Activity 2

Pairs | 10 min

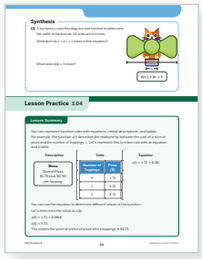
Students reason repeatedly to explore the effects of functions in context on different inputs, then write equations of functions using function notation. **Routines:** Decide and Defend **(MTR.4.1)**



Synthesis

Whole Class | 5 min

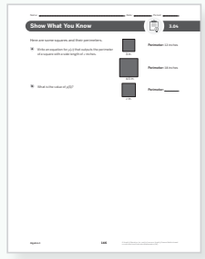
Students synthesize their understanding of equations in function notation by interpreting one equation with variables and one equation with a single input and output. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equation in function notation and evaluating it at a given input.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students explore equations defined using function notation in the context of the pizza cash register from the previous lesson.

1 Launch

Display the cash register.

Consider asking, "Where have we seen a cash register like this before?"

A **Accessibility: Conceptual Processing** As students think abstractly, invite them to answer a more concrete question first (e.g., "Why do you think that $m(3) = 20$?").

Invite students to make a prediction about the value of $m(5)$ and give it to a partner to check their prediction.

Listen for students who discuss multiplying 1.50 by the number of toppings or adding 15.50, and consider inviting them to share their thinking during the Connect.

2 Connect

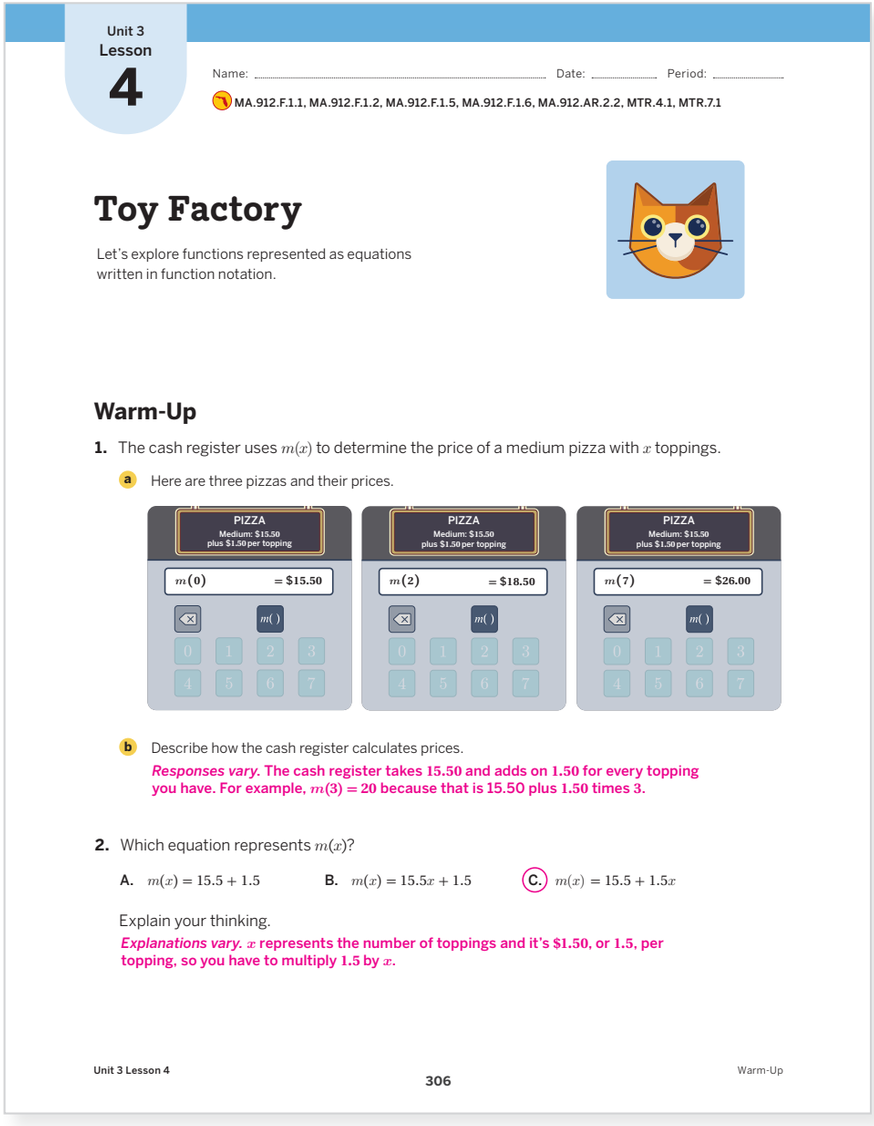
Display the equations for $m(x)$. After students have had the opportunity to respond, consider asking students to share their work, calling attention to any conflict or consensus you see.

ELL **English Language Learners** To support students in distinguishing between similar equations, invite pairs to read each aloud and discuss how they are similar and different before responding. **(Reading and Speaking)**

Invite students to share their thinking about which equation represents $m(x)$.

Consider asking:

- "How are these equations similar to ones you've seen before? How are they different?"
- "How did you decide which equation represented the cash register?"
- "How would we say $m(x) = 15.5 + 1.5x$ aloud?"



Unit 3
Lesson
4

Name: _____ Date: _____ Period: _____

MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MA.912.AR.2.2, MTR.4.1, MTR.7.1

Toy Factory

Let's explore functions represented as equations written in function notation.

Warm-Up

1. The cash register uses $m(x)$ to determine the price of a medium pizza with x toppings.

a. Here are three pizzas and their prices.

PIZZA
Medium: $15.50
plus $1.50 per topping
$m(0) = \$15.50$

PIZZA
Medium: $15.50
plus $1.50 per topping
$m(2) = \$18.50$

PIZZA
Medium: $15.50
plus $1.50 per topping
$m(7) = \$26.00$

b. Describe how the cash register calculates prices.

Responses vary. The cash register takes 15.50 and adds on 1.50 for every topping you have. For example, $m(3) = 20$ because that is 15.50 plus 1.50 times 3.

2. Which equation represents $m(x)$?

A. $m(x) = 15.5 + 1.5$ B. $m(x) = 15.5x + 1.5$ C. $m(x) = 15.5 + 1.5x$

Explain your thinking.

Explanations vary. x represents the number of toppings and it's $1.50, or 1.5, per topping, so you have to multiply 1.5 by x.

Unit 3 Lesson 4 306 Warm-Up

Pairs | 20 min

Activity 1 Exploring Equations of Functions

Purpose: Students interpret and evaluate functions using function notation in the context of a toy factory by connecting the structure of equations to the relationship between toy parts. **(MTR.5.1)**

Short on time? Consider having students evaluate 1–2 expressions instead of all three.

1 Launch

Display the image of the fire truck.

To encourage participation, consider asking, "If you had a toy factory, what kinds of toys would you make?"

2 Monitor

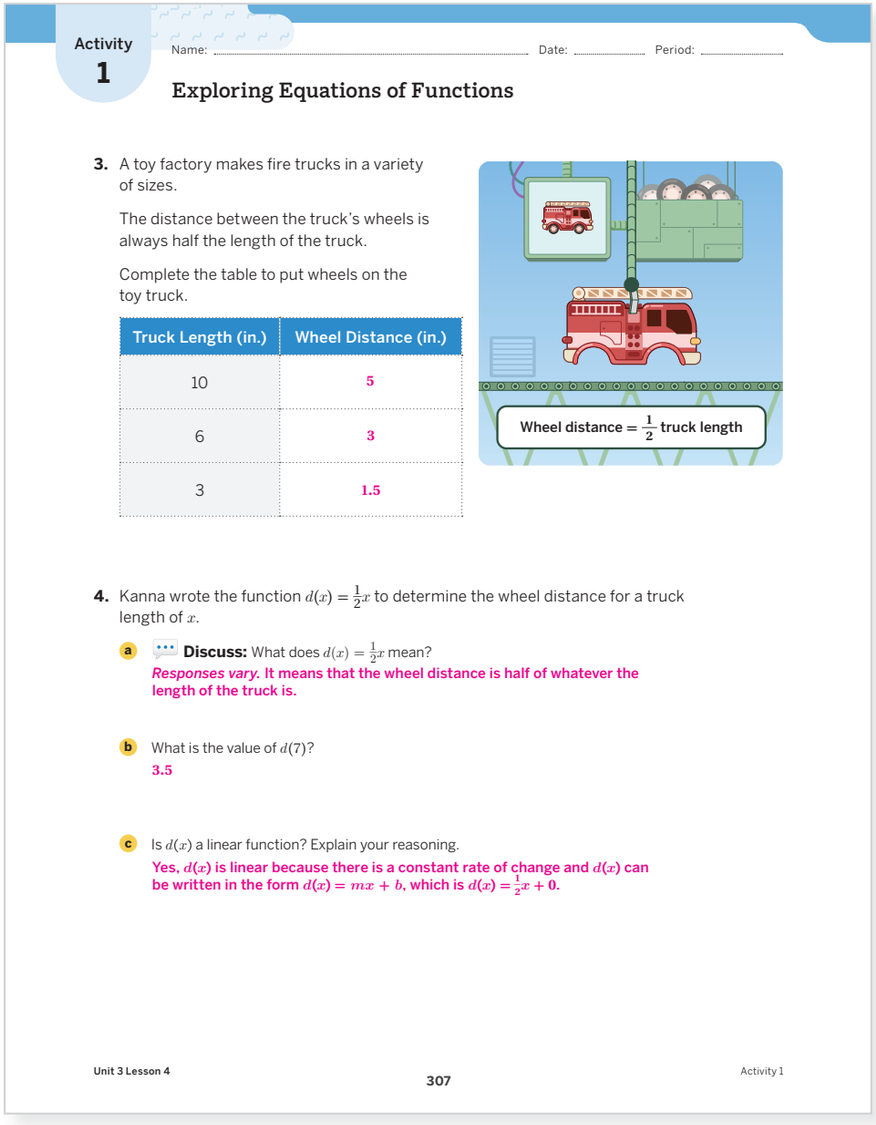
Display the equation for Kanna's function.

Listen for students who use language about inputs and outputs as they discuss what the equation means, or who make connections between the structure of the equation and the description of the context. **(MTR.5.1)**

Pause if time allows to invite students to share their thinking about what the equation for Kanna's function means.

D **Differentiation: (Support)** Some students may think that $d(7) = 14$ because 7 is half of 14. Invite these students to write out Kanna's equation and substitute 7 for x: $d(7) = \frac{1}{2}(7)$. Then consider asking, "How could you evaluate the expression on the right side to see what $d(7)$ is equal to?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Exploring Equations of Functions

3. A toy factory makes fire trucks in a variety of sizes.

The distance between the truck's wheels is always half the length of the truck.

Complete the table to put wheels on the toy truck.

Truck Length (in.)	Wheel Distance (in.)
10	5
6	3
3	1.5

4. Kanna wrote the function $d(x) = \frac{1}{2}x$ to determine the wheel distance for a truck length of x.

a. **Discuss:** What does $d(x) = \frac{1}{2}x$ mean?
Responses vary. It means that the wheel distance is half of whatever the length of the truck is.

b. What is the value of $d(7)$?
3.5

c. Is $d(x)$ a linear function? Explain your reasoning.
Yes, $d(x)$ is linear because there is a constant rate of change and $d(x)$ can be written in the form $d(x) = mx + b$, which is $d(x) = \frac{1}{2}x + 0$.

Unit 3 Lesson 4 307 Activity 1

Pairs | 20 min

Activity 1 Exploring Equations of Functions (continued)

Purpose: Students interpret and evaluate functions using function notation in the context of a toy factory by connecting the structure of equations to the relationship between toy parts. **(MTR.5.1)**

Short on time? Consider having students evaluate 1–2 expressions instead of all three.

2 Monitor

ELL **English Language Learners** To give students more processing time, invite them to read the prompt aloud and paraphrase what it means, clarifying any unknown words or concepts before responding. **(Reading, Speaking, and Listening)**

Pause and invite students to share their thinking about what $h(x) = 2x + 3$ means in this situation.

Select and sequence students' explanations for what $h(x)$ means in the situation and their strategies for evaluating $h(x)$ to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What does x represent? What does $h(x)$ represent?"
May be unsure how to interpret expressions like $h(5)$.	**Support** Consider asking, "What did expressions like $m(5)$ mean in the pizza context?"
Describe what the function does to a number instead of what it tells you about the cat (e.g., "Multiply x by 2 and then add 3").	**Strengthen** Consider asking, "What does the equation tell us about the cat?"
Complete the table on Problem 3 correctly.	**Stretch** Invite students to come up with an expression in function notation that *would not* make sense in context and explain to a partner why it doesn't make sense. *Responses vary.* $h(0)$ wouldn't make sense because you can't have a base that is 0 inches wide.

3 Connect

Invite students to share their explanations.

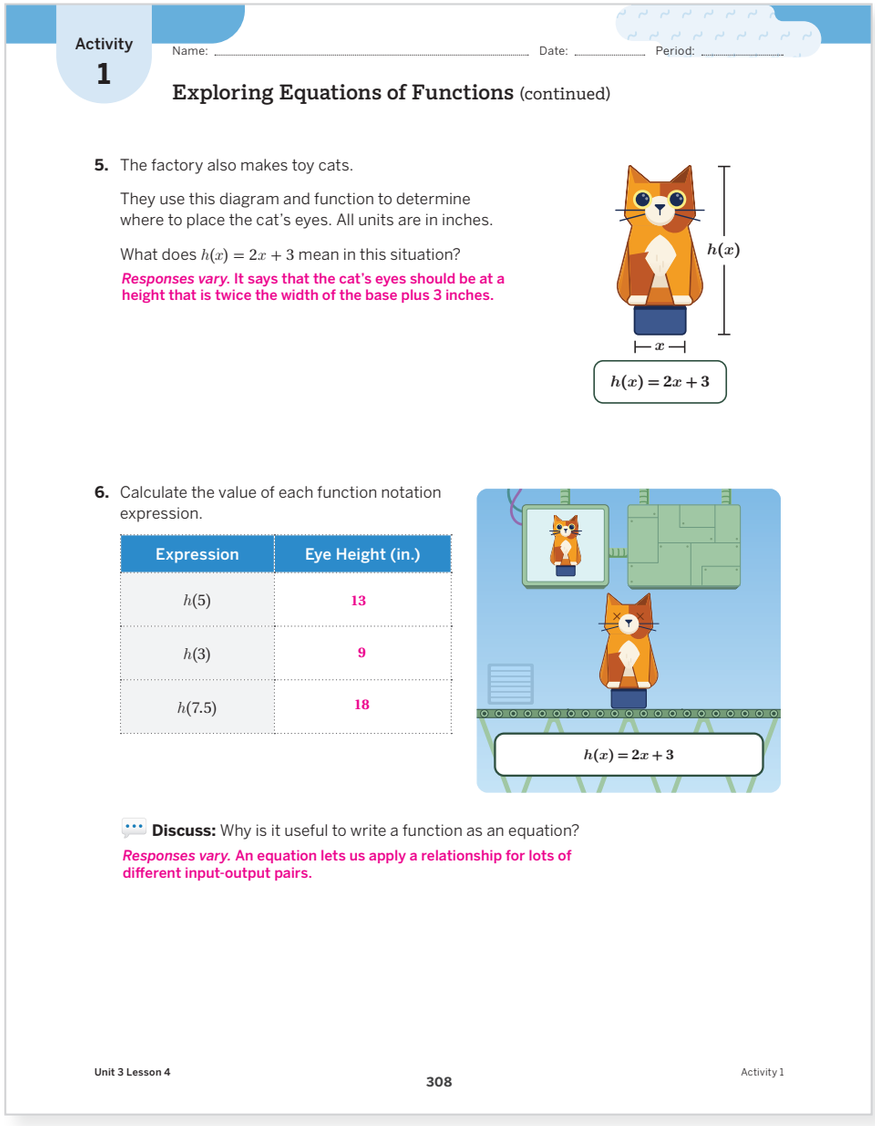
MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Display the equation for $h(x)$ and the table.

Invite students to share their strategies for evaluating $h(x)$.

To surface the Key Takeaway, consider asking:

- "What does the input represent about the toy cat? The output?" **(MTR.5.1)**
- "How can the equation help us make many different cats?"



Activity 1

Name: Date: Period:

Exploring Equations of Functions (continued)

5. The factory also makes toy cats.

They use this diagram and function to determine where to place the cat's eyes. All units are in inches.

What does $h(x) = 2x + 3$ mean in this situation?

Responses vary. It says that the cat's eyes should be at a height that is twice the width of the base plus 3 inches.

6. Calculate the value of each function notation expression.

Expression	Eye Height (in.)
$h(5)$	13
$h(3)$	9
$h(7.5)$	18

Discuss: Why is it useful to write a function as an equation?

Responses vary. An equation lets us apply a relationship for lots of different input-output pairs.

Unit 3 Lesson 4 308 Activity 1

Key Takeaway: Writing an equation for a function makes it possible to determine the output for any input and interpret that input-output pairing in context. Linear functions can be written in the form $f(x) = mx + b$.

Pairs | 10 min

Activity 2 Writing Equations of Functions

Purpose: Students reason repeatedly to explore the effects of functions in context on different inputs, then write equations of functions using function notation. **(MTR.3.1)**

1 Launch

Invite students to suggest different ways of changing the equation.

Consider asking:

- "Can you make it so that all the bow ties are the same size?" *Responses vary.* $b(x) = 10$
- "Can you make it so that bigger cats have smaller bow ties?" *Responses vary.* $b(x) = 5 \div x$

2 Monitor

A **Accessibility: Conceptual Processing** As students solve abstract problems, invite them to think of a concrete case first (e.g., If $x = 2$, whose bow tie would be wider? What if $x = 5$? $x = 1$?).

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Collect the distribution of responses for Problem 8.

Note: If everyone is in agreement, consider pausing to make a class argument and invite students to contribute their reasoning to support the collective consensus.

D **Differentiation: (Stretch)** Encourage students to change Kimaya's equation so that her statement is true.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Writing Equations of Functions

7. Each toy cat needs a bow tie. The function $b(x)$ determines the width of the bow tie, where x is the width of the base of the cat toy. All units are in inches.



a. The images to the right are the result of the function $b(x) = 4x$. What do you notice?

b. What is an equation that will make the bow ties fit better? Compare your equation with a classmate's and share your thinking.

Equations vary.

- $b(x) = 3x$
- $b(x) = 6$
- $b(x) = \frac{10}{x} + 2$

8. $b(x)$ determines the width of the bow tie, where x is the width of the base.

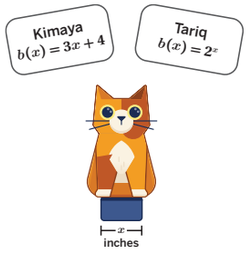
Kimaya says that her function will produce a wider bow tie than Tariq's function for any base width.

Is she correct? Circle one.

Yes (No)

Explain your thinking.

Explanations vary. Kimaya's statement is true sometimes but not for every base width. For example, if the base width is 5 inches, then Tariq's function makes a wider bow tie.



9. Refer to Problem 8. Is Kimaya's function linear? Is Tariq's function linear? Why or why not?

Explanations vary. Kimaya's function is linear because it can be written in the form $y = mx + b$. Tariq's function is not linear.

Unit 3 Lesson 4 309 Activity 2

Pairs | 10 min

Activity 2 Writing Equations of Functions (continued)

Purpose: Students reason repeatedly to explore the effects of functions in context on different inputs, then write equations of functions using function notation. **(MTR.3.1)**

2 Monitor

Display Flamingo Frank and the table.

Consider using the Notice and Wonder routine to promote curiosity and help students make sense of the table. See the Routine Facilitation Guide for more information. **(optional)**

ELL **English Language Learners** To support students with processing their ideas in words, consider pausing to share words or phrases that pairs are using in their descriptions. **(Speaking and Listening)**

Select and sequence student thinking about $f(x)$ to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on just the first two rows of the table. Consider asking, "What operations could you perform on the input 4 to result in an output of 8.5? Do those same operations work to go from 3 to 6.5? How do you know?"
Discuss multiplying 2 by the width and adding 0.5.	**Strengthen** Celebrate these observations and invite these students to share their thinking during the Connect. Consider asking, "What do the 2 and the 0.5 mean about Flamingo Frank?"
Write a correct equation for $f(x)$ in Problem 11.	**Stretch** **You're invited to explore more.** Invite students who would like to explore writing equations for functions further to complete this optional task. Encourage them to discuss their thinking with a partner.

Record student equations for $f(x)$ to share during the Connect.

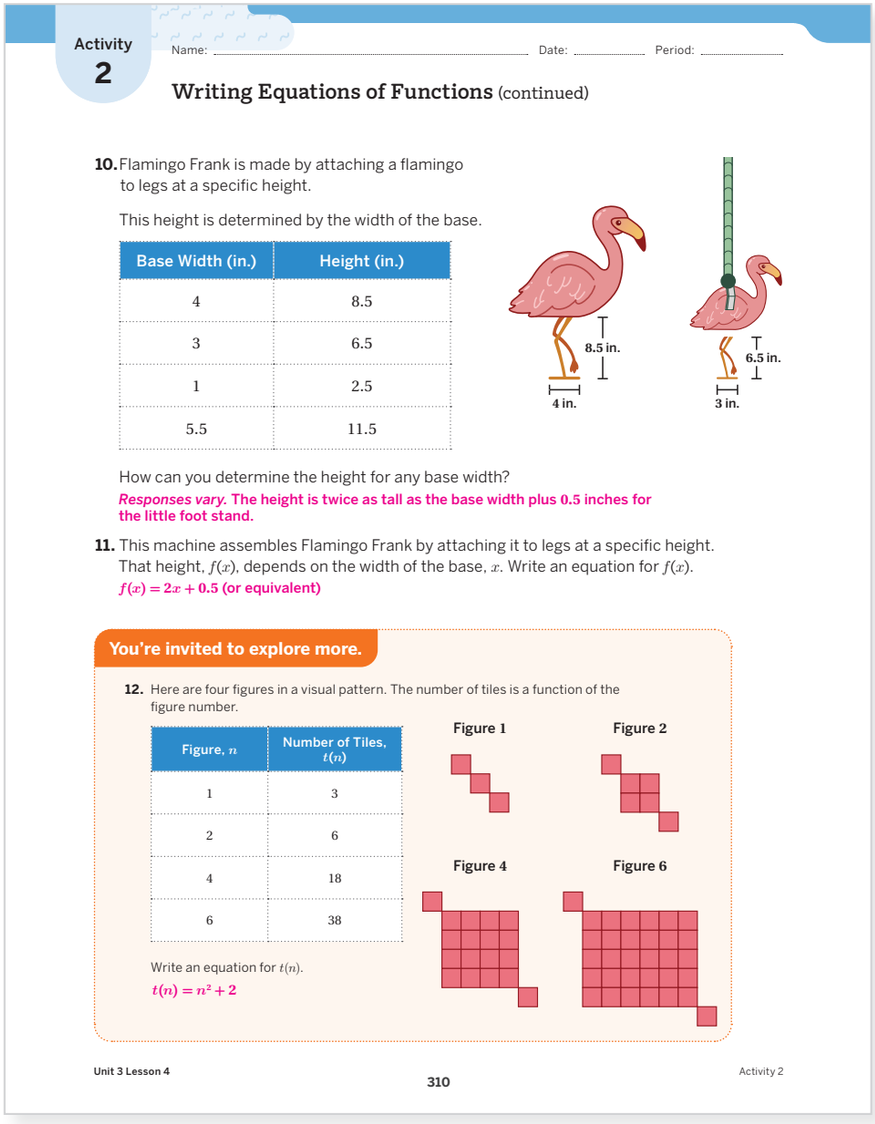
3 Connect

Invite students to share their equations for $f(x)$ and how they came up with those equations from the table.

Consider asking:

- "How did you decide what equation to write?"
- "How would we say $f(x) = 0.5 + 2x$ aloud?"
- "Why might equations be helpful in this situation?"

Note: If it does not come up naturally, consider discussing how equations can help generalize patterns of repeated reasoning. **(MTR.3.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Writing Equations of Functions (continued)

10. Flamingo Frank is made by attaching a flamingo to legs at a specific height.

This height is determined by the width of the base.

Base Width (in.)	Height (in.)
4	8.5
3	6.5
1	2.5
5.5	11.5

How can you determine the height for any base width?

Responses vary. The height is twice as tall as the base width plus 0.5 inches for the little foot stand.

11. This machine assembles Flamingo Frank by attaching it to legs at a specific height. That height, $f(x)$, depends on the width of the base, x. Write an equation for $f(x)$.

$f(x) = 2x + 0.5$ (or equivalent)

You're invited to explore more.

12. Here are four figures in a visual pattern. The number of tiles is a function of the figure number.

Figure, n	Number of Tiles, $t(n)$
1	3
2	6
4	18
6	38

Write an equation for $t(n)$.

$t(n) = n^2 + 2$

Unit 3 Lesson 4 310 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of equations in function notation by interpreting one equation with variables and one equation with a single input and output.

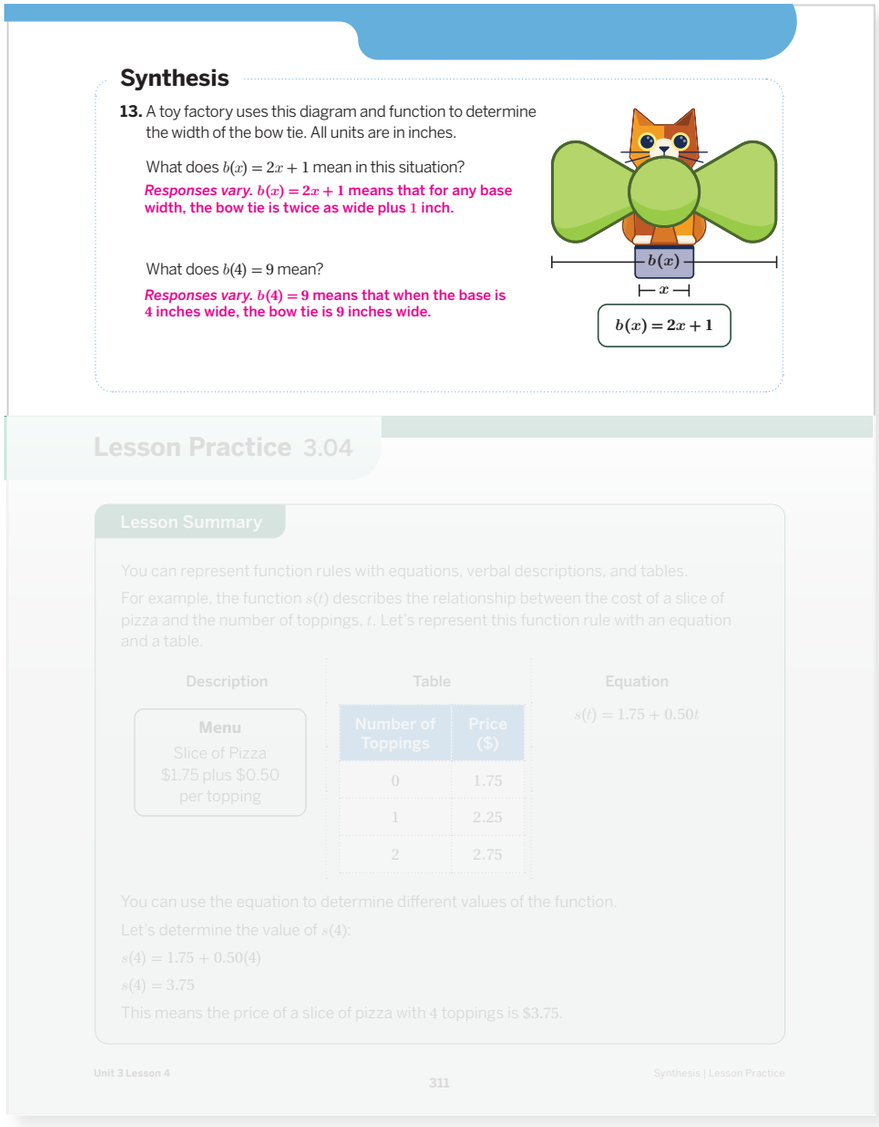
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Both statements have the same input (a number representing base width) and output (a number representing bow tie width).
- Both use function notation, which might be different from equations students have seen before.
- $b(x) = 2x + 1$ tells you the bow tie for any size cat with a base length of x inches, and $b(4) = 9$ is for a specific cat with a base length of 4 inches.

MLR **MLR2: Collect and Display** Use this routine to formalize the difference between function equations written with variables and function equations written with single inputs and outputs using students' language. Display any new language, examples, or diagrams for students to reference later.

Consider asking:
- "Where do we see the input in equations written in function notation? The output?"
- "When are equations in function notation useful?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. A toy factory uses this diagram and function to determine the width of the bow tie. All units are in inches.

What does $b(x) = 2x + 1$ mean in this situation?

Responses vary. $b(x) = 2x + 1$ means that for any base width, the bow tie is twice as wide plus 1 inch.

What does $b(4) = 9$ mean?

Responses vary. $b(4) = 9$ means that when the base is 4 inches wide, the bow tie is 9 inches wide.

Lesson Practice 3.04

Lesson Summary

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t. Let's represent this function rule with an equation and a table.

Description

Menu
Slice of Pizza
$1.75 plus $0.50 per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.

Let's determine the value of $s(4)$:

$s(4) = 1.75 + 0.50(4)$

$s(4) = 3.75$

This means the price of a slice of pizza with 4 toppings is $3.75.

Unit 3 Lesson 4 311 Synthesis | Lesson Practice

Lesson Takeaway: Equations for functions can be used to describe relationships in context. They can describe general relationships using variables, like $f(x) = 3x + 1$, or they can make statements about specific input-output pairings for a function, like $f(0) = 1$.

Independent | 5 min

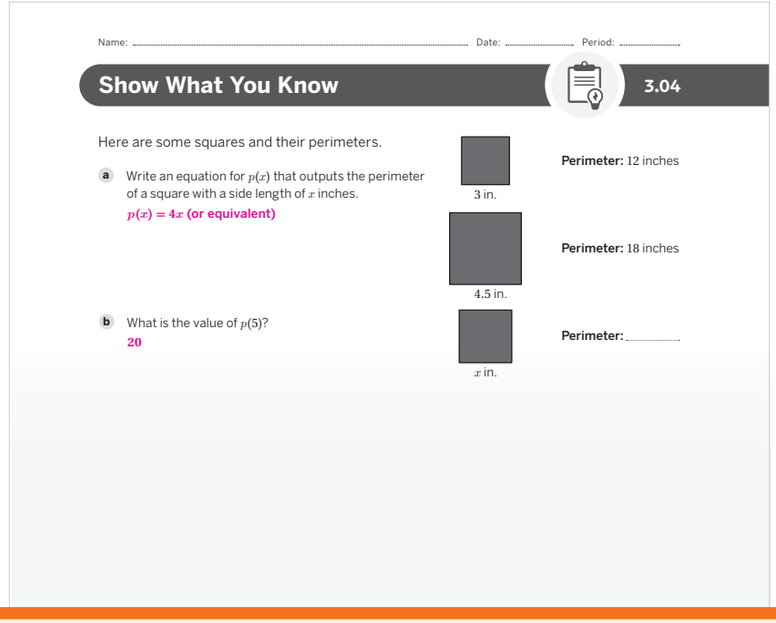
Show What You Know

Purpose: Students demonstrate their understanding by writing an equation in function notation and evaluating it at a given input.

Today's Goals

1. **Goal:** Evaluate functions written in function notation.
2. **Goal:** Write equations of functions using function notation.
3. **Language Goal:** Describe equations written in function notation in context. **(Speaking and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.04

Here are some squares and their perimeters.

a. Write an equation for $p(x)$ that outputs the perimeter of a square with a side length of x inches.
$p(x) = 4x$ (or equivalent)

3 in. **Perimeter:** 12 inches

4.5 in. **Perimeter:** 18 inches

b. What is the value of $p(5)$?
20

x in. **Perimeter:** _____

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support writing and evaluating a rule in function notation:

» **Respond to Student Thinking:**

- Use the *Interpreting Slope and y-Interept in Context* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write their own equation in function notation and evaluate it using an input of their choosing.

Professional Learning

Take a moment to reflect on how students engaged in today's lesson. How did students' repeated reasoning about functions and statements written in function notation support them in making generalizations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. A toy factory uses this diagram and function to determine the width of the bow tie. All units are in inches.

What does $b(x) = 2x + 1$ mean in this situation?

Responses vary. $b(x) = 2x + 1$ *means that for any base width, the bow tie is twice as wide plus 1 inch.*

What does $b(4) = 9$ mean?

Responses vary. $b(4) = 9$ *means that when the base is 4 inches wide, the bow tie is 9 inches wide.*

Lesson Practice 3.04

Lesson Summary

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t. Let's represent this function rule with an equation and a table.

Description

Menu
Slice of Pizza
$1.75 plus $0.50 per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.

Let's determine the value of $s(4)$:

$s(4) = 1.75 + 0.50(4)$

$s(4) = 3.75$

This means the price of a slice of pizza with 4 toppings is $3.75.

Lesson Practice 3.04

Name: _____ Date: _____ Period: _____

1. Let $f(x) = 2x + 5$.

Calculate the value of each function notation expression.

The first value is already completed.

Expression	Value
$f(0)$	5
$f(4)$	13
$f(6)$	17
$f(-3)$	-1

Problems 2–3: A toy factory makes toy bunnies. Each toy bunny holds a carrot. A bunny's height, $h(x)$, is three times the length of the carrot, x.

2. Complete the table.

x	1	2	3	4	5	6
$h(x)$	3	6	9	12	15	18

3. Write an equation for the function $h(x)$.

$h(x) = 3x$

4. The functions $f(x)$ and $g(x)$ are defined by these equations:

- $f(x) = -15x + 80$
- $g(x) = 10x + 25$

Circle which is greater: $f(2)$ or $g(2)$. Explain your thinking.

Explanations vary. $f(2) = 50$ *and* $g(2) = 45$, *so* $f(2)$ *is greater.*

Problems 5–6: The function $p(s)$ models the perimeter of a square of side length s. The perimeter is represented by the equation $p(s) = 4s$.

5. What is the value of $p(20)$?

$p(20) = 80$

6. What does your answer mean in this situation?

Responses vary. When the side length of a square is 20, the perimeter of the square is 80.

Problems 7–8: The function $w(t)$ models the weight of a pumpkin, in pounds, as a function of how many months, t, it has been growing. Explain the meaning of each statement.

7. $w(2) = 5$ *Responses vary. At 2 months, the pumpkin weighed 5 pounds.*

8. $w(6) > w(4)$ *Responses vary. The pumpkin weighed more at 6 months than it did at 4 months.*

Lesson Practice 3.04

Name: _____ Date: _____ Period: _____

Test Practice

9. Model rockets are created in various sizes. The height of a rocket in inches, $h(x)$, depends on the radius of the base of the rocket in inches, x.

Use the table to select an equation for $h(x)$ that outputs the height of the rocket with a base radius of x.

Radius (in.), x	Height (in.), $h(x)$
1	5
3	13
5	21
10	41

A. $h(x) = 4x - 1$

B. $h(x) = 3x + 2$

C. $h(x) = 4x + 1$

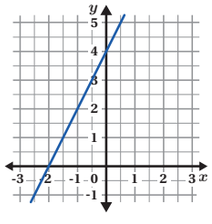
D. $h(x) = 5x - 3$

Spiral Review

Problems 10–11: Select all functions represented by the graph.

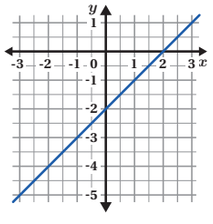
10.

- [x] a. $y = 2x + 4$
- [x] b. $2y = 4x - 8$
- [x] c. $y = 2x - 4$
- [x] d. $2y = 4x + 8$
- [x] e. $y = -2x + 4$



11.

- [x] a. $y = x + 2$
- [x] b. $y = x - 2$
- [ ] c. $2y = 2x + 4$
- [x] d. $3y = 3x - 6$
- [ ] e. $y = 2 - x$



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 5	1	MA.912.F.1.2
	4, 6, 7–8	2	MA.912.F.1.2
	3	2	MA.912.AR.2.2
TEST Practice	9	2	MA.912.AR.2.2
Spiral Review			
Fluency	10	2	MA.912.AR.2.2
	11	2	MA.8.AR.3.3

Assess and Respond

Sub-Unit Quiz

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determine if a relationship is a function from a table	Lesson 1	1	MA.912.F.1.1
2a	Determine the domain of a function from a graph	Lesson 10	2	MA.912.F.1.5
2b	Interpret the domain of a function in context	Lesson 10	2	MA.912.F.1.5 MTR.6.1
2c	Determine the range of a function from a graph	Lesson 10	2	MA.912.F.1.5
3	Calculate and interpret average rate of change	Lesson 7	2	MA.912.F.1.3
4a	Interpret a function statement in context	Lesson 2	2	MA.912.F.1.2 MTR.6.1
4b	Write an equation in function notation to represent a given context	Lesson 4	2	MA.912.AR.2.2 MTR.7.1
5	Sketch graph with given key features	Lesson 6	3	MA.912.F.1.6 MTR.3.1

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 3

1. Which table shows a relationship that is *not* a function?

A.

Input	Output
-1	2
0	4
-1	2
-2	0

B.

Input	Output
1	2
3	4
1	6
3	8

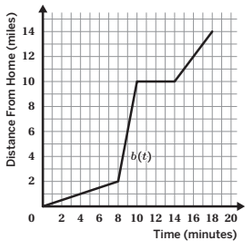
C.

Input	Output
2	2
4	2
6	4
8	6

D.

Input	Output
1	2
2	2
3	2
4	2

2. Here is a graph of $b(t)$. $b(t)$ represents Bao's distance from home after t minutes.



a. Which is the *domain* of $b(t)$?

A. $0 \le t \le 14$

B. $0 \le t \le 18$

C. $t \le 18$

D. $t \le 14$

b. What does the domain tell you about Bao's trip?

Responses vary. The domain tells me that Bao traveled for 18 minutes.

c. Fill in the blanks to describe the *range* of $b(t)$. ....0.... $\le b(t) \le$ ....14....

3. Calculate the *average rate of change* of $b(t)$ from 8 to 14 minutes.

$\frac{4}{3}$ (or equivalent)

Explain what your answer means in context.

Responses vary. Between 8 and 14 minutes, Bao traveled an average of 1.33 miles per minute.

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Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 3

4. At a toy store, customers can buy a stuffed animal and add accessories, like hats and shoes.

$g(x)$ represents the cost of a stuffed giraffe with accessories. The giraffe costs \$15 and accessories cost \$3.75 each.

a. What does $g(5) = 33.75$ say about this situation?

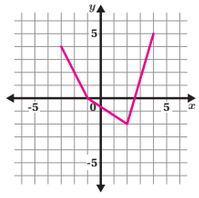
Responses vary. A giraffe with 5 accessories costs \$33.75.

b. Write an equation for $g(x)$, where x represents the number of accessories.

$g(x) =$ $15 + 3.75x$ (or equivalent)

5. Here are features of a function, $f(x)$.

- The domain of $f(x)$ is $-3 \le x \le 4$.
- $f(x)$ is decreasing when $x < 2$.
- The minimum of $f(x)$ occurs at (2, -2).



Sketch a graph that could be $f(x)$.

Graphs vary.

D Differentiation (Sub-Unit Quiz)

Note: Consider inviting students to revise the corresponding assessment items after responding to student thinking.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
• Determine whether a relationship is a function. **(Lesson 1)**	1	Support	• **Teacher Move:** Consider revisiting Lesson 1 (*Mystery Rule*) • **Teacher Move:** Invite students to look for relationships in their life and explain whether or not they are functions.

• Interpret statements in function notation. **(Lessons 2–4)**	4	Support	• **Mini-Lessons:**
			  • Function Notation
			  • Interpreting Slope and y-Intercept in Context
			• **Teacher Move:** Consider revisiting Lesson 4 (*Toy Factory*)

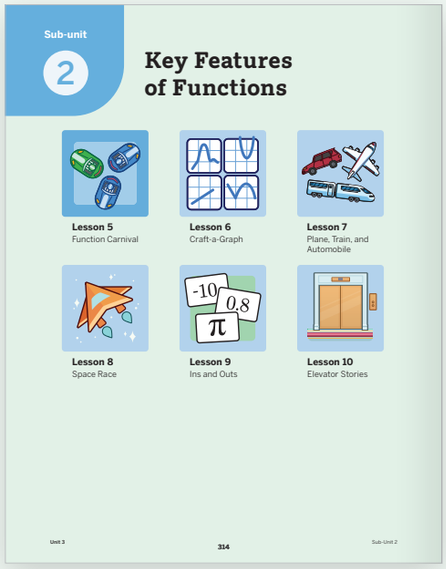
		Stretch	• **You're invited to explore more:** • Lesson 4 (*Toy Factory*)
• Describe functions using their key features, including domain, range, and average rate of change. **(Lessons 5–10)**	2	Support	• **Mini-Lesson:** • Writing Domain and Range with Inequalities • **Teacher Move:** Consider revisiting Lesson 10 (*Elevator Stories*)
		Stretch	• **You're invited to explore more:** • Lesson 10 (*Elevator Stories*)
	3	Support	• **Teacher Move:** Consider revisiting Lesson 7 (*Planes, Trains, and Automobiles*)
		Stretch	• **You're invited to explore more:** • Lesson 7 (*Plains, Trains, and Automobiles*)
	5	Support	• **Mini-Lesson:** • Writing Domain and Range with Inequalities
		Strengthen	• **Repeated Challenges:** • Lesson 6 (*Craft-a-Graph*)
		Stretch	• **You're invited to explore more:** • Lesson 5 (*Function Carnival*) • **Teacher Move:** Invite students to revisit the Polygraph from Lesson 6 (*Craft-a-Graph*), this time using new vocabulary.

Key Features of Functions

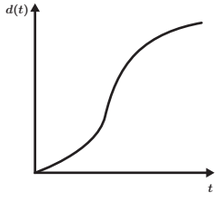
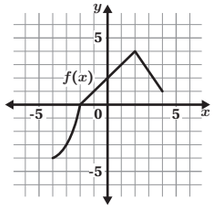
Sub-Unit 2 Goals

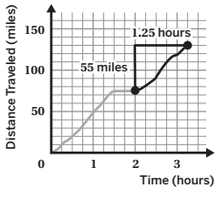
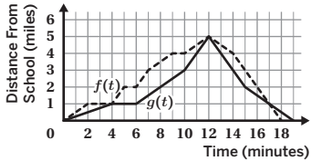
- **Language Goal:** Describe functions using their key features, including domain, range, and average rate of change.
- **Goal:** Compare graphs using function notation and their key features.

In this sub-unit, students describe functions using their key features, including **average rate of change**, **domain**, and **range**. They use those key features to compare functions. Students also restrict the domain and range of a function using **compound inequalities**.



Math That Matters Most

Concept or Skill	For example . . .
Sketch a graph to match a situation.	Jasmine races around an oval track. $d(t)$ represents how many meters were run at time t. Sketch a possible graph that could represent Jasmine's race. 
Describe and interpret key features of functions.	Identify the following key features of $f(x)$:  **a.** Increasing $-4 < x < 2$ **b.** Positive $x > -2$ **c.** Minimum $(-4, -4)$ **d.** Range $-4 \leq h(t) \leq 4$

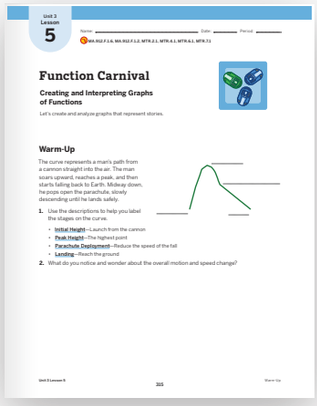
Concept or Skill	For example . . .
Calculate the average rate of change over a specific interval.	Determine Arjun's average rate of change between 2 and 3.25 hours. What does it mean about Arjun's trip?  $55 \div 1.25$ is 44, which means Arjun's average speed was 44 miles per hour in that interval.
Compare two functions using their key features.	A school has two buses. $f(t)$ and $g(t)$ represent the distance of each bus from school (in miles) after t minutes.  Complete the following so that they are true statements. **Note:** Responses vary. **a.** $f(18) = g($ 19 $)$ **b.** $g(8)$ < $f(8)$ **c.** $f(t) = g(t)$ when $t =$ 17 **d.** Both functions are decreasing after 12 minutes.

Unit 3
Lesson
5

Function Carnival

Creating and Interpreting Graphs of Functions

Let's create and analyze graphs that represent stories.


Unit 3
Lesson
5

Function Carnival

Creating and Interpreting Graphs of Functions

Let's create and analyze graphs that represent stories.

Warm-Up

The curve represents a man's path from a cannon straight into the air. The man soars upward, reaches a peak, and then starts falling back to Earth. Midway down, he pops open the parachute, slowly descending until he lands safely.

1. Use the descriptions to help you label the stages on the curve.
   - Initial Height—Launch from the cannon
   - Peak Height—The highest point
   - Parachute Deployment—Reduce the speed of the fall
   - Landing—Reach the ground
2. What do you notice and wonder about the overall motion and speed change?

Learning Outcomes

Today's Goals

1. **Goal:** Sketch a graph of a function to match a situation.
2. **Language Goal:** Describe connections between situations and graphs. **(Writing, Speaking, and Listening)**

Students create graphs of functions to match a story. They translate between stories and graphs and begin to distinguish which parts of the story graphs are most useful for telling. **(MTR.2.1)**

Coherence

Prior Learning

In this unit, students have revisited the concept of a function by evaluating expressions in function notation using equations and tables.

Future Learning

In Lessons 5–9, students will use terms such as *positive, negative, increasing, decreasing, maximum, minimum, domain, range,* and *average rate of change* to describe and compare functions represented as graphs.

Connections

- Students create and analyze graphs that represent distance and time relationships of different carnival rides. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

MA.912.F.1.6
Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1.2	**MA.912.F.1.6**
MA.8.F.1.3	

Vocabulary

Review Vocabulary

decreasing	*increasing*
rate of change	*linear function*

Lesson at a Glance

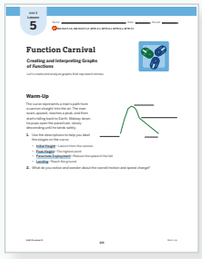
~ 45 min

MA.912.F.1.2, MA.912.F.1.6

Warm-Up

Pairs | 5 min

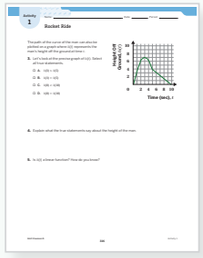
Students label a graph representing a person shot out of a cannon. They use the Notice and Wonder routine to surface key aspects before they analyze the graph of a man's height off the ground in Activity 1. **Routine(s):** Notice and Wonder



Activity 1

Pairs | 10 min

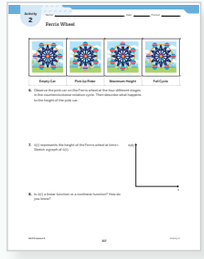
Students use a graph to analyze the journey of a man's height off the ground after they are launched out of a cannon. **(MTR.4.1, MTR.6.1)**



Activity 2

Pairs | 10 min

Students create a graph to model a Ferris wheel ride and then make sense of a graph that is not a function, describing what it would mean in context. **(MTR.2.1, MTR.4.1)**



Activity 3

Pairs | 10 min

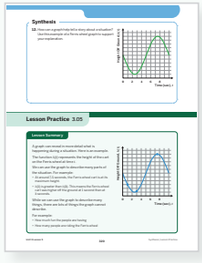
Students consider how to draw and interpret graphs that represent distance traveled, and make connections between graphs and the story they tell. MLR1: Stronger and Clearer Each Time **(MTR.6.1)**



Synthesis

Whole Class | 5 min

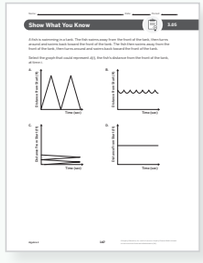
Students synthesize their understanding of how graphs can tell stories. Routine: Tell a Story **(MTR.2.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting a graph that matches a story.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students label a graph representing a person shot out of a cannon. They use the Notice and Wonder routine to surface key aspects before they analyze the graph of the person's height off the ground in Activity 1.

1 Launch

Use the Notice and Wonder routine to support students in describing how they interpret the graph. Invite students to think independently and then take turns sharing with a partner.

Consider sharing that interpretations do not have to be complete or perfect to be valuable.

ELL **English Language Learners** Invite students to write stories in their primary language and/or create a list of words about the context (e.g., cannon, parachute, launch, etc.), along with visuals or translations of each word in languages used by your students. **(Writing, Reading, and Speaking)**

2 Connect

Share and discuss some student interpretations to highlight key parts of the scenario on the graph.

Encourage students to ask questions about each others' stories. Consider asking:

- "Do you understand the parts of _____'s interpretation of the graph?"
- "What parts of each of our interpretations are alike? How are they different?"
- "When _____ said, '_____,' what do you think they meant?" (e.g., "When Madison said, 'the person switched directions,' what do you think she meant?")

Unit 3
Lesson
5

Name: _____ Date: _____ Period: _____

MA.912.F.1.6, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

Function Carnival

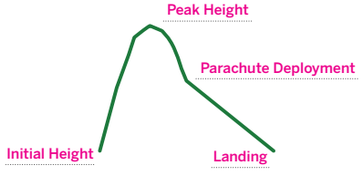


Creating and Interpreting Graphs of Functions

Let's create and analyze graphs that represent stories.

Warm-Up

The curve represents a man's path from a cannon straight into the air. The man soars upward, reaches a peak, and then starts falling back to Earth. Midway down, he pops open the parachute, slowly descending until he lands safely.



1. Use the descriptions to help you label the stages on the curve.
   - **Initial Height**—Launch from the cannon
   - **Peak Height**—The highest point
   - **Parachute Deployment**—Reduce the speed of the fall
   - **Landing**—Reach the ground
2. What do you notice and wonder about the overall motion and speed change?

Responses vary. I noticed that he goes up quickly, slows down at the top, and then starts coming back down. The speed is fastest at the beginning and slows down at the peak. The parachute slows him down. I wondered: What was the highest height? How long was he in the air? What would happen if the parachute didn't open?

Unit 3 Lesson 5 315 Warm-Up

Pairs | 10 min

Activity 1 Rocket Ride

Purpose: Students use a graph to analyze the journey of a man's height off the ground after he is launched out of a cannon. **(MTR.4.1, MTR.6.1)**

1 Launch

Consider sharing: "You are analyzing the performance of a human cannonball from the Warm Up activity. Your task is to study the height of the performer over time to understand their journey. You'll use a graph that shows their height off the ground during the launch, flight, and landing."

2 Monitor

Select and sequence a variety of student graphs, including functions and non-functions, to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to make any sketch and use the feedback to revise it.
Create a graph of the cannon person's path instead of their height, which is not a function.	**Support** Draw a graph of the cannon person's path instead of their height, which is not a function, to share during the Connect and consider asking, "What is surprising when you compare the graphs?"
Utilize different revision strategies to make their graph more precise, such as: • Plotting points for specific key features. • Using a ruler to graph a section with a constant rate of change. **(MTR.6.1)**	**Strengthen** Invite these students to share and consider naming the powerful strategies you hear after the students who use them.

3 Connect

Encourage students to discuss what each statement says about the cannon person. **(MTR.2.1)**

Display the distribution of responses, discussing any conflict or consensus you see.

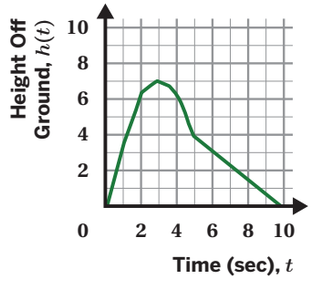
To surface the Key Takeaway, consider displaying Problem 5 and asking:
- Is $h(t)$ a linear function? Why or why not?
- What must be true about a linear function? What does the graph of $h(t)$ show?

Activity 1

Name: Date: Period:

Rocket Ride

The path of the curve of the man can also be plotted on a graph where $h(t)$ represents the man's height off the ground at time t.



3. Let's look at the precise graph of $h(t)$. Select *all* true statements.

- ☑ A. $h(3) > h(5)$
- ☐ B. $h(3) = h(5)$
- ☐ C. $h(0) < h(10)$
- ☑ D. $h(0) = h(10)$

4. Explain what the true statements say about the height of the man.

Explanations vary.
- $h(3) > h(5)$ says the man was higher off the ground at 3 seconds than at 5 seconds.
- $h(0) = h(10)$ says that the height of the man is the same at 0 seconds as at 10 seconds.

5. Is $h(t)$ a linear function? How do you know?

Explanations vary. $h(t)$ is not linear because it's graph is not a straight line which means the rate of change is not constant.

Unit 3 Lesson 5 316 Activity 1

Key Takeaway: A linear function has a constant rate of change. A nonlinear function does not have a constant rate of change. The graph of a linear function is a straight line, while the graph of a nonlinear function is not a straight line.

Pairs | 10 min

Activity 2 Ferris Wheel

Purpose: Students create a graph to model a Ferris wheel ride and then make sense of a graph that is not a function, describing what it would mean in context. **(MTR.2.1, MTR.4.1)**

Short on time? Provide students with the first part of the graph.

1 Launch

To surface connections, consider asking, "Have you ever been on a Ferris wheel? What was it like?"

ELL **English Language Learners** Create a list of words about the context (e.g., Ferris wheel, cart, ride, down, up, around, etc.), along with visuals or translations of each word in languages used by your students. **(Writing, Reading, and Speaking)**

A **Accessibility: Executive Functioning** Consider displaying transitional words to use such as: first, then, next, finally, etc.

2 Monitor

Encourage students to read others' responses and revise theirs to make them stronger and clearer.

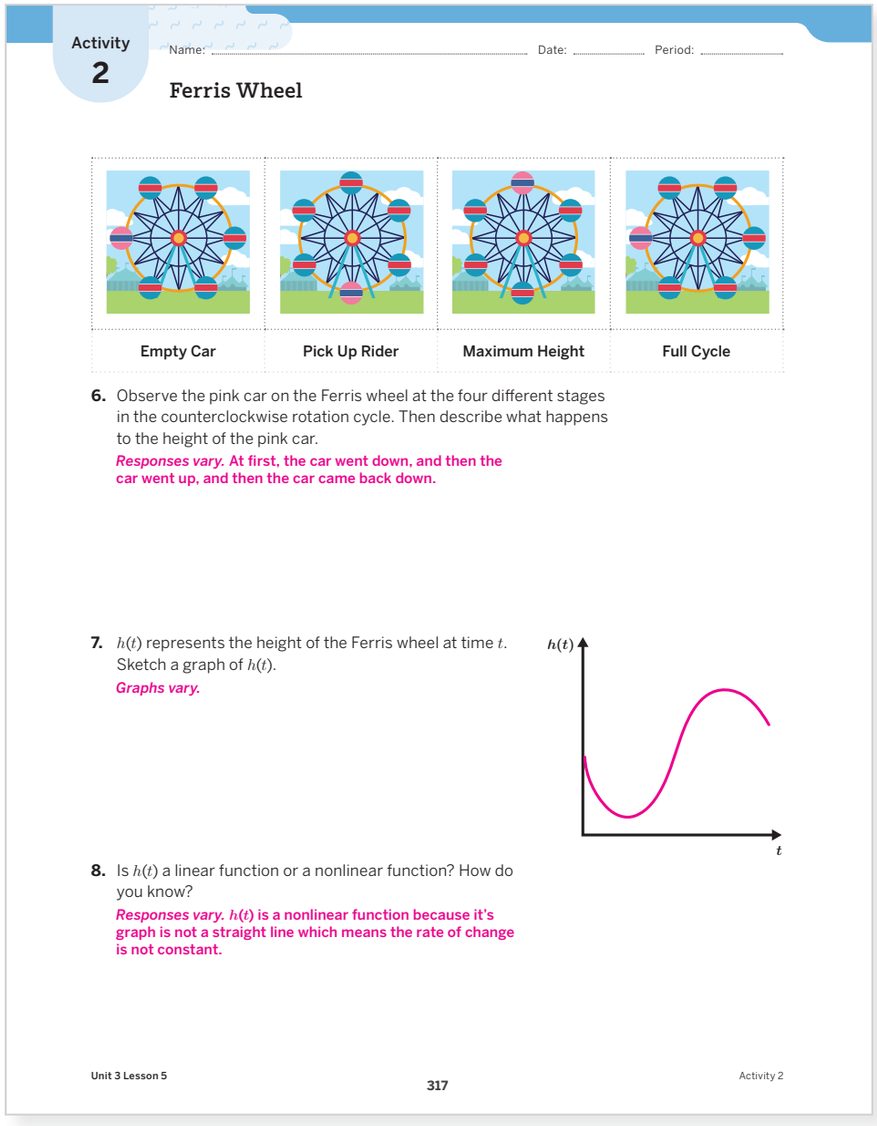
To support students getting started, consider asking. "At what height does the cart start? Does it move up or down after that?"

D **Differentiation: (Support)** Students may draw the path of the cart rather than the height of the cart over time. Consider asking, "Is it possible for one cart to be at two different heights at the same time?"

3 Connect

Display the provided sample sketch and consider asking:
- "What is precise about this sketch?"
- "What story does the sketch tell?"

To surface the Key Takeaway, consider asking:
- "Is the Ferris wheel's height over time a function?"
- "Is Liam's graph a function? Why or why not?"



Activity 2

Name: _____ Date: _____ Period: _____

Ferris Wheel

Empty Car | Pick Up Rider | Maximum Height | Full Cycle

6. Observe the pink car on the Ferris wheel at the four different stages in the counterclockwise rotation cycle. Then describe what happens to the height of the pink car.

Responses vary. At first, the car went down, and then the car went up, and then the car came back down.

7. $h(t)$ represents the height of the Ferris wheel at time t. Sketch a graph of $h(t)$.

Graphs vary.

8. Is $h(t)$ a linear function or a nonlinear function? How do you know?

Responses vary. $h(t)$ is a nonlinear function because it's graph is not a straight line which means the rate of change is not constant.

Unit 3 Lesson 5 317 Activity 2

Key Takeaway: A graph representing a situation is a function if every input has only one output. A graph is not a function if there is at least one input that has multiple outputs, which is possible in some contexts but not in others.

Pairs | 10 min

Activity 3 The Daring Dash

Purpose: Students consider how to draw and interpret graphs that represent distance traveled, and make connections between graphs and the story they tell. **(MTR.6.1)**

Short on time? Skip question 8.

1 Launch

Use the **Stronger and Clearer Each Time** routine to promote curiosity and help students make sense of what is happening with the "Daring Dash" before they draw the graph. **(MTR.6.1)**

A **Accessibility: Executive Functioning** To support students in sharing and receiving feedback, model giving and receiving constructive feedback as a class.

To encourage creativity, challenge students to try to come up with ideas that no one in their class will come up with.

Note: The goal here is to surface a variety of responses about the different things we can measure quantitatively in this story, not just to focus on distance versus time. **(MTR.2.1)**

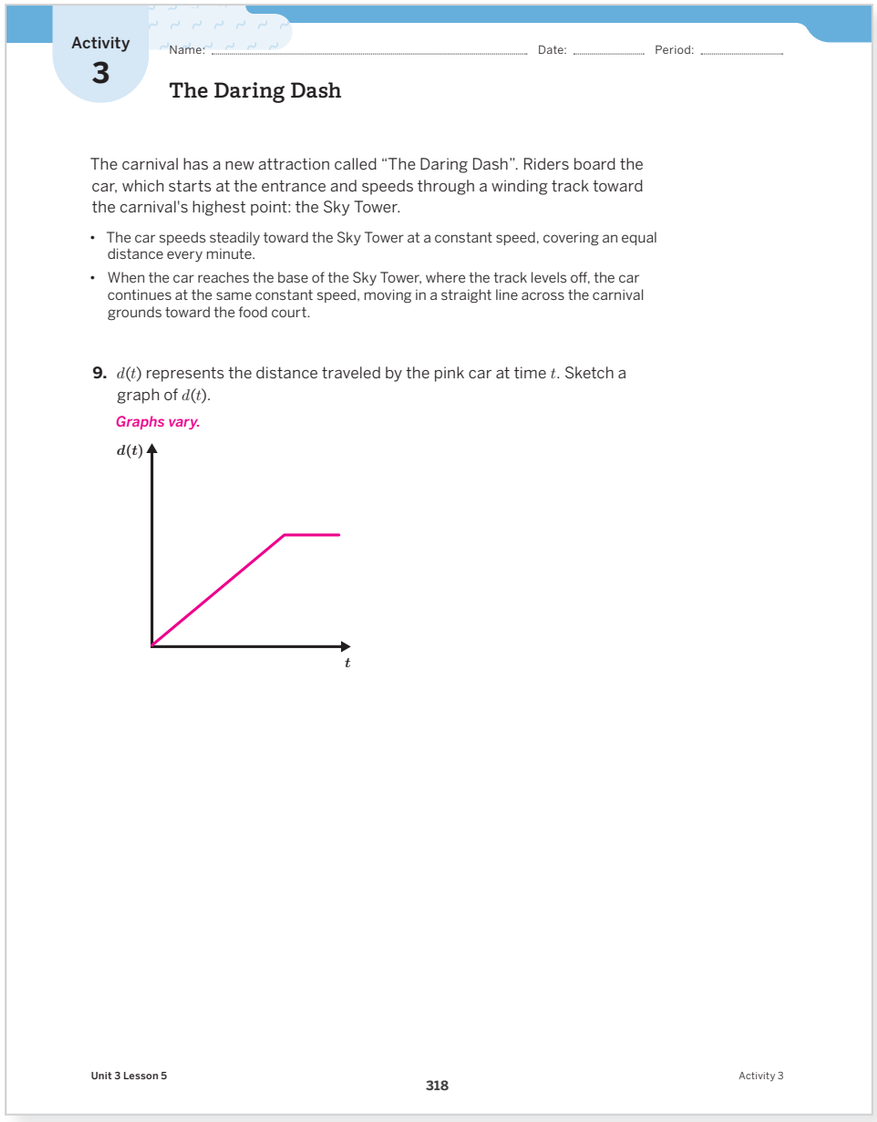
2 Monitor

Look for students who share a variety of interesting measures (e.g., the speed of the "Daring Dash," changing the location of the tower). If time allows, invite students to share.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to make a point on the graph that describes the distance the coaster has traveled at $t = 1$.
Draw the path of the coaster instead of the distance traveled.	**Support** Consider asking, "Is the graph you made a function? Explain what a function is in your own words."
Persist through struggle and continually revise their graph to make it more precise. **(MTR.6.1)**	**Strengthen** Celebrate these students and invite them to share their strategies with other students.
Want to explore function stories further.	**Stretch** Invite students to create a graph of a different variable they said could be measured on the previous problem.

Activity 3 continued >



Activity 3

Name: Date: Period:

The Daring Dash

The carnival has a new attraction called "The Daring Dash". Riders board the car, which starts at the entrance and speeds through a winding track toward the carnival's highest point: the Sky Tower.

- The car speeds steadily toward the Sky Tower at a constant speed, covering an equal distance every minute.
- When the car reaches the base of the Sky Tower, where the track levels off, the car continues at the same constant speed, moving in a straight line across the carnival grounds toward the food court.

9. $d(t)$ represents the distance traveled by the pink car at time t. Sketch a graph of $d(t)$.

Graphs vary.

Unit 3 Lesson 5 318 Activity 3

Pairs | 10 min

Activity 3 The Daring Dash (continued)

Purpose: Students consider how to draw and interpret graphs that represent distance traveled, and make connections between graphs and the story they tell. **(MTR.6.1)**

Short on time? Skip question 8.

2 Monitor

Listen to partners' discussions about how they decided which graphs matched the descriptions. Consider asking pairs, "What keywords in the descriptions help to decide what the graph should look like?"

3 Connect

ELL **MLR1: Stronger and Clearer Each Time**
Invite students to draw their own graph of the description draft and use feedback to make their thinking stronger and clearer to choose the given graphs. **(MTR.6.1)**

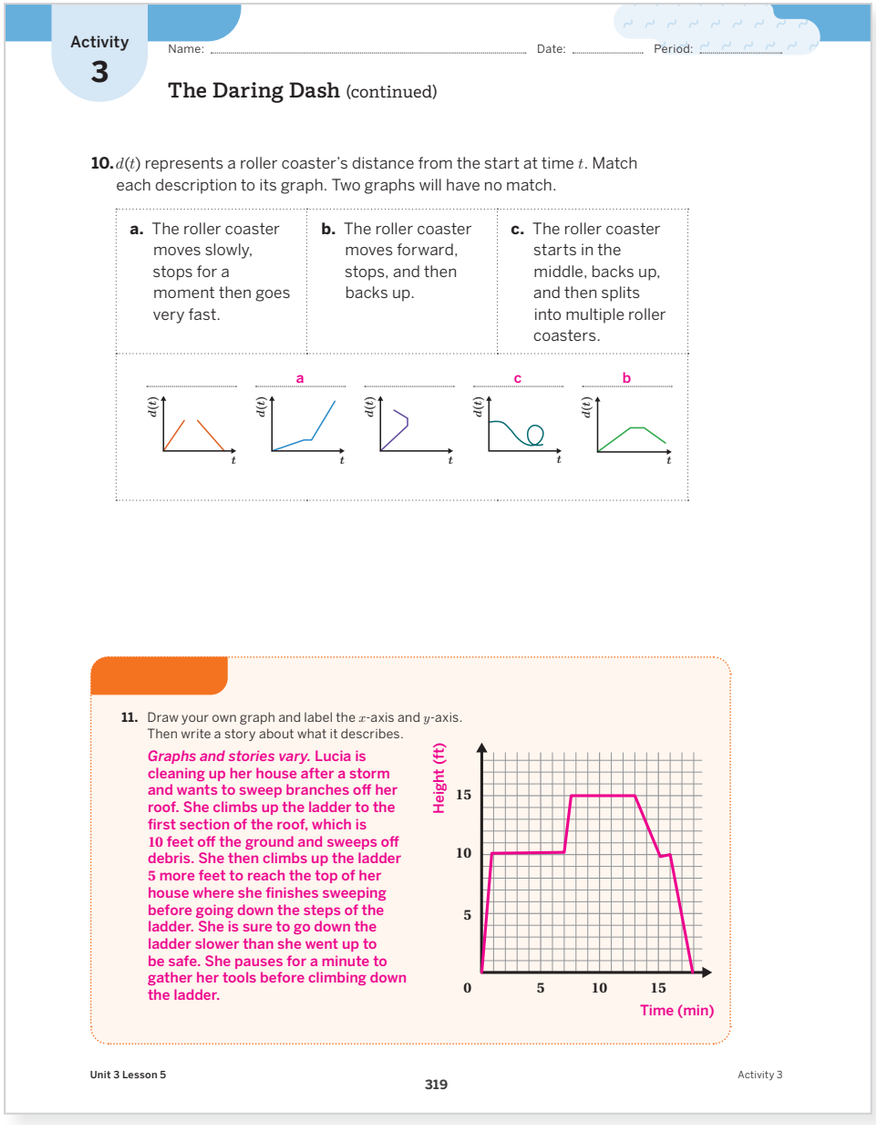
Invite students to share their graphs and keywords. Consider creating an anchor chart with sentence frames to help draw graphs to match the description.

Examples:

- When you see the word (**climb/descend**), the line direction is going (**up/down**).
- When you see the word (**faster/slower**), the slope of the line is (**wider/steeper**).

Consider asking:

- "Which graphs are functions? How can you tell?"
- "Which graphs are not functions? How can you tell?"



Activity 3

Name: Date: Period:

The Daring Dash (continued)

10. $d(t)$ represents a roller coaster's distance from the start at time t. Match each description to its graph. Two graphs will have no match.

a. The roller coaster moves slowly, stops for a moment then goes very fast.	**b.** The roller coaster moves forward, stops, and then backs up.	**c.** The roller coaster starts in the middle, backs up, and then splits into multiple roller coasters.

a c b

11. Draw your own graph and label the x-axis and y-axis. Then write a story about what it describes.

Graphs and stories vary. Lucia is cleaning up her house after a storm and wants to sweep branches off her roof. She climbs up the ladder to the first section of the roof, which is 10 feet off the ground and sweeps off debris. She then climbs up the ladder 5 more feet to reach the top of her house where she finishes sweeping before going down the steps of the ladder. She is sure to go down the ladder slower than she went up to be safe. She pauses for a minute to gather her tools before climbing down the ladder.

Height (ft) 15 10 5 0 5 10 15 Time (min)

Unit 3 Lesson 5 319 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students use the Tell a Story routine to synthesize their understanding of how graphs can tell stories. **(MTR.2.1)**

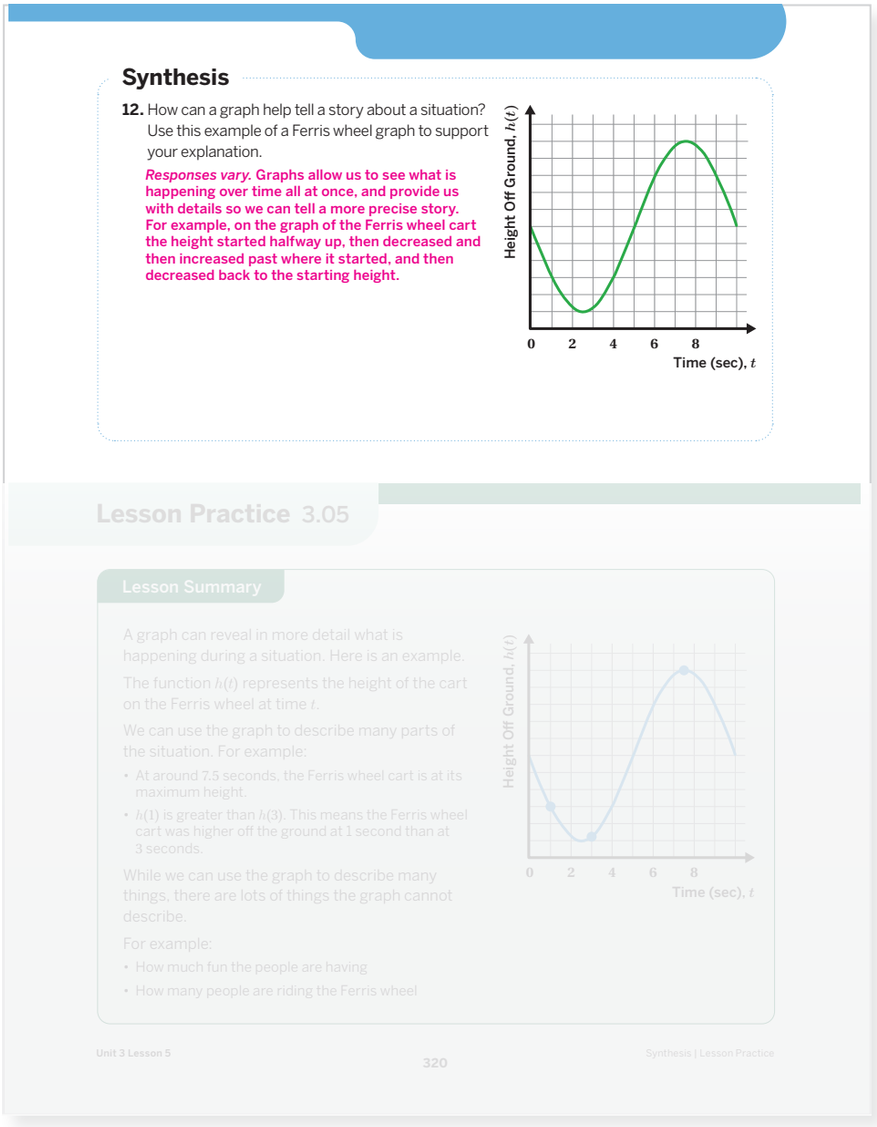
Invite students to respond independently and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- How features of graphs such as maximum or minimum can reveal important parts of stories. **(MTR.2.1)**
- How representing a story in multiple ways (e.g., with graphs, and/or words) can make a story more meaningful.
- How graphs can give more clear and precise information about a story. **(MTR.6.1)**

Consider asking:
- "What parts of a story can a graph tell? What parts of a story can a graph not tell?"
- "What is a situation you would like to see a graph of?"
- Why might a graph be helpful?"

Consider sharing with students that the sketch of $h(t)$ is what we call the graph of a function, and represents all the y-values that follow the rule, $h(t)$. In this graph, the x-values are values of the variable t and the y-values are values of the function $h(t)$. This means that $y = h(t)$.

Note: If time allows, invite students to share with a partner what makes sense about each of the ideas shared and what questions they still have.



Synthesis

12. How can a graph help tell a story about a situation? Use this example of a Ferris wheel graph to support your explanation.

Responses vary. Graphs allow us to see what is happening over time all at once, and provide us with details so we can tell a more precise story. For example, on the graph of the Ferris wheel cart the height started halfway up, then decreased and then increased past where it started, and then decreased back to the starting height.

Lesson Practice 3.05

Lesson Summary

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

We can use the graph to describe many parts of the situation. For example:
- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.

While we can use the graph to describe many things, there are lots of things the graph cannot describe.

For example:
- How much fun the people are having
- How many people are riding the Ferris wheel

Unit 3 Lesson 5 320 Synthesis | Lesson Practice

Key Takeaway: Graphs have the power to tell stories and parts of stories can be represented using graphs. Representing situations using graphs can be helpful to analyze the situation from a different perspective.

Independent | 5 min

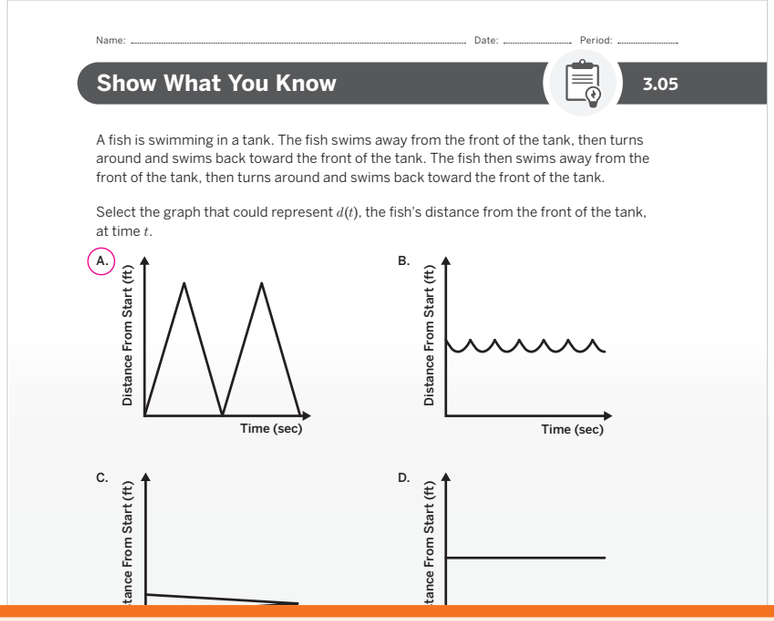
Show What You Know

Purpose: Students demonstrate their understanding by selecting a graph that matches a story.

Today's Goals

1. **Goal:** Sketch a graph of a function to match a situation.
2. **Language Goal:** Describe connections between situations and graphs. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: Date: Period:

Show What You Know 3.05

A fish is swimming in a tank. The fish swims away from the front of the tank, then turns around and swims back toward the front of the tank. The fish then swims away from the front of the tank, then turns around and swims back toward the front of the tank.

Select the graph that could represent $d(t)$, the fish's distance from the front of the tank, at time t.

A.

B.

C.

D.

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If Students need support selecting a graph that matches a story:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related Practice Problems.
- Revisit **Activity 3, Problem 10** as a class and invite students to share how they used the key features of the graphs to match them to a story.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create a graph to match the story of their journey to school.

Professional Learning

Reflect on your facilitation of the Tell a Story routine in this lesson. How did students use their knowledge and creativity to help them make sense of the story? How did you facilitate the routine and support students in reasoning abstractly and quantitatively as they made connections between the graph and the story?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can a graph help tell a story about a situation? Use this example of a Ferris wheel graph to support your explanation.

Responses vary. Graphs allow us to see what is happening over time all at once, and provide us with details so we can tell a more precise story. For example, on the graph of the Ferris wheel cart the height started halfway up, then decreased and then increased past where it started, and then decreased back to the starting height.

Lesson Practice 3.05

Lesson Summary

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

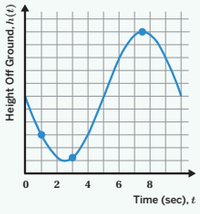
We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.

While we can use the graph to describe many things, there are lots of things the graph cannot describe.

For example:

- How much fun the people are having
- How many people are riding the Ferris wheel

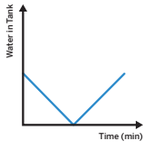


Lesson Practice 3.05

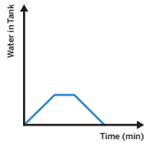
Name: Date: Period:

1. An empty water tank is filled until it is half full. Two minutes later, it drains until it is empty again. Which graph could represent this situation? Circle your choice.

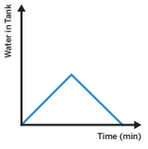
Graph A



Graph B



Graph C



Problems 2–4: Prisha rode her bike around town. Her fitness tracker made a graph to represent the distance she was from her home at any given time during her ride.

2. How many minutes was Prisha's bike ride?

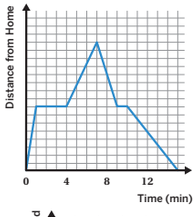
15 minutes

3. At what time was Prisha the farthest distance from her home?

7 minutes

4. How long did Prisha rest during her ride?

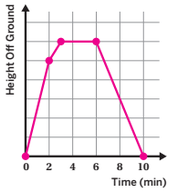
She rested for 4 minutes in total.



5. Here is some information about a hot air balloon ride.

- Ascends (goes up) quickly for 2 minutes.
- Ascends slowly for another minute until it reaches its maximum height.
- Maintains its maximum height for 3 minutes.
- Descends (goes down) for the next 4 minutes until it lands on the ground.

Make a graph that could represent the height of this hot air balloon over time.



Lesson Practice 3.05

Name: Date: Period:

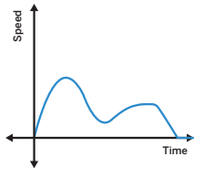
Problems 6–7 Here is a graph of speed and time.

6. Which sport could this graph represent?

Responses vary.

A. Fishing B. Skydiving C. 100-yard sprint

D. Golf E. Soccer



Test Practice

7. Describe how you think that sport fits the graph.

Explanations vary.

- Soccer. The player runs the ball down the field, then slows down to change direction, then speeds up again before stopping to kick the ball.
- 100-yard sprint. The runner gets faster until reaching top speed. The player trips and slows down before speeding up again.

Spiral Review

8. $p(t)$ represents the height of water in a bathtub, in inches, after t minutes. Match each sentence to its equation.

A.	After 20 minutes, the bathtub is empty.	C $p(10) = 4$
B.	The bathtub starts with no water.	E $p(t) = w$
C.	After 10 minutes, the height of the water is 4 inches.	A $p(20) = 0$
D.	The height of the water is 10 inches after 4 minutes.	B $p(0) = 0$
E.	The height of the water is w inches after t minutes.	D $p(4) = 10$

Lesson Practice Item Analysis

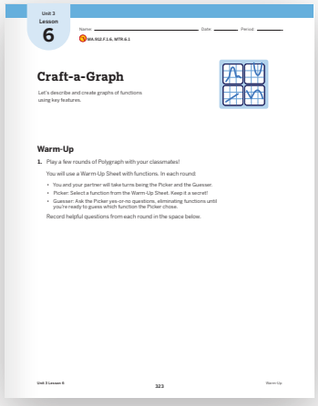
	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-Lesson			
	1	1	MA.912.F.1.6
	2–4	2	MA.912.F.1.6
	5, 6	2	MA.912.F.1.6
Test Practice	7	3	MA.912.F.1.6
Spiral Review			
Fluency	8	2	MA.912.F.1.2

Unit 3
Lesson
6

Craft-a-Graph

Key Features of Graphs

Let's describe and create graphs of functions using key features.


Unit 3
Lesson
6

Craft-a-Graph

Let's describe and create graphs of functions using key features.

Warm-Up

1. Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with functions. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a function from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating functions until you're ready to guess which function the Picker chose.

Record helpful questions from each round in the space below.

323

Learning Outcomes

Today's Goals

1. **Goal:** Use the key features of a function to build a graph of that function.
2. **Language Goal:** Describe key features of a graph using terms like *positive, negative, maximum, minimum, increasing, and decreasing*. **(Reading and Writing)**

Students describe and create graphs of functions using key features. Students make sense of the terms **positive**, **negative**, **maximum**, **minimum**, *increasing*, and *decreasing* by playing a game and analyzing examples. They match graphs to descriptions and create graphs to fit descriptions. **(MP6)**

Coherence

Prior Learning

In Lesson 5, students created graphs to help them conceptually understand terms like *positive*, *negative*, *maximum*, *minimum*, *increasing*, and *decreasing* before being introduced to them in this lesson. In Grade 8, Unit 5 students also described graphs of functions.

Future Learning

Students will use this new vocabulary as they describe and compare new functions they encounter: exponential functions in Unit 5, and quadratic functions in Unit 6. In Algebra 2, students will learn about different types of extrema.

Connections

- Students explore key features of functions (graphs) to interpret what is occurring at different points on distance-time graphs. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.6
Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Vocabulary

New Vocabulary

maximum (of a function)
minimum (of a function)
negative (interval or function)
positive (interval or function)

Review Vocabulary

decreasing (interval or function)
increasing (interval or function)
input
output

Lesson at a Glance

~ 45 min

MA.912.F.1.6

Warm-Up

Independent | 10 min

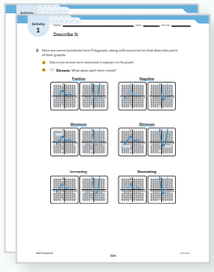
Students develop language to describe key features of functions by playing a game where they use yes-or-no questions to try to guess which function their classmate has chosen.



Activity 1

Pairs | 5 min

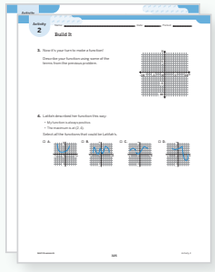
Students create class definitions of terms that describe key features of functions: *increasing*, *decreasing*, **maximum**, **minimum**, **positive**, and **negative**. MLR2: Collect and Display (MTR.4.1)



Activity 2

Pairs | 10 min

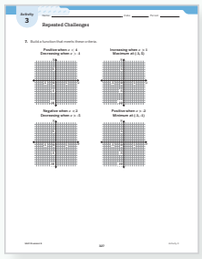
Students match graphs to descriptions and create graphs to fit descriptions involving key features.



Activity 3

Independent | 10 min

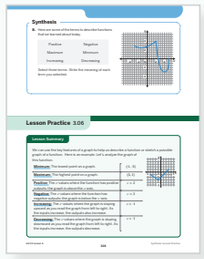
Students use their creativity and build fluency in key feature language by making graphs to match given criteria.



Synthesis

Whole Class | 5 min

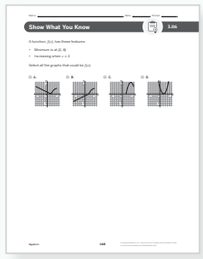
Students synthesize their understanding of the definitions of positive, negative, maximum, minimum, increasing, and decreasing. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting graphs that have specific key features.



Prep Checklist

Copy Warm-up Sheets, Set A and Set B per pair

This lesson includes:

- Student Edition
- Show What You Know

Independent | 10 min

Warm-Up

Purpose: Students develop language to describe key features of functions by playing a game where they use yes-or-no questions to try to guess which function their classmate has chosen.

1 Launch

Students will need one or more partners. Distribute the Warm-Up Sheet so that one partner has Set A and one has Set B. Invite one partner to be the Picker and the other to be the Guesser in the first round, and then switch roles for the next round.

A **Accessibility: Executive Functioning** To support multi-step activities, invite students to model an example round of the activity.

Encourage students to try and guess the function using as few questions as possible.

Note: Students will learn the terms *increasing, decreasing, maximum, minimum, positive,* and *negative* to describe functions in the next Activity.

Pause briefly to discuss common questions and terms that students are using. Some student language to highlight might include: going up/down, highest/lowest point, U-shape, wiggly, above/below the x-axis, etc.

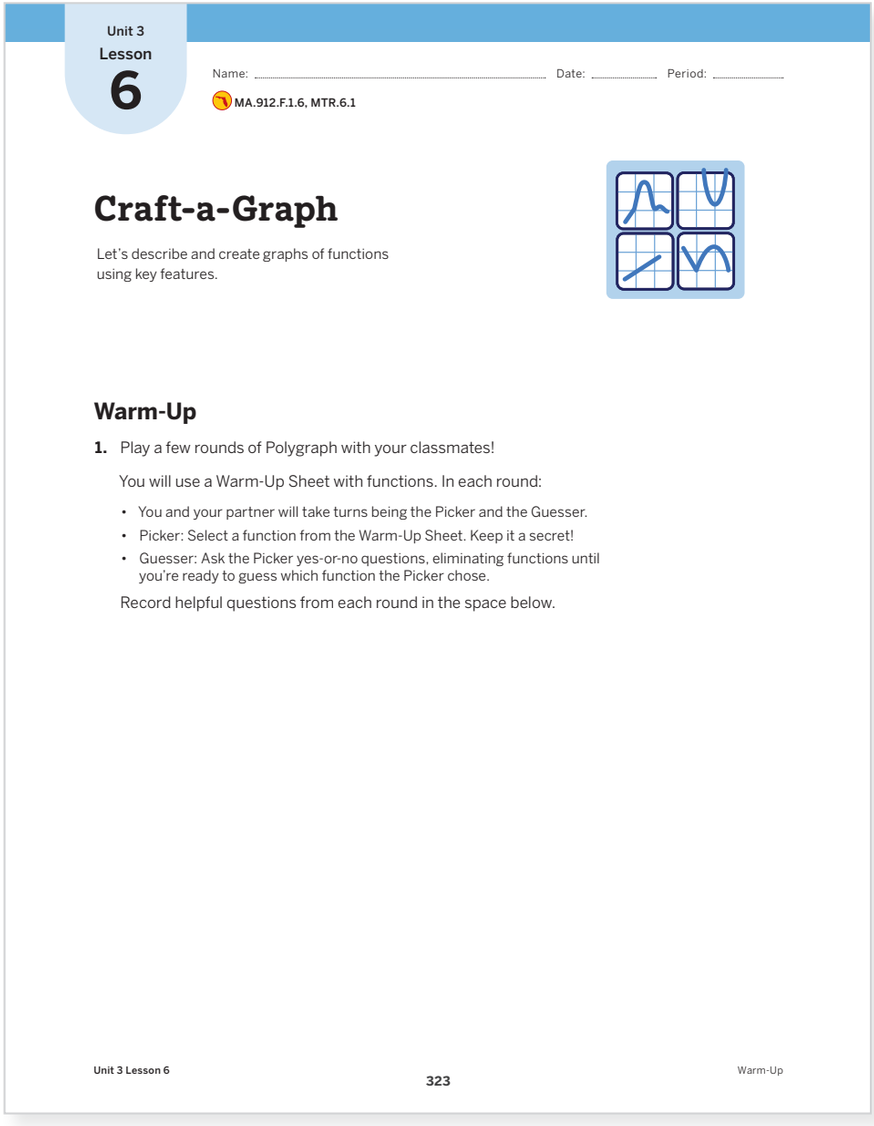
Encourage students to continue playing using language from the discussion if it is helpful for them.

2 Connect

Consider asking:

- "What were you paying attention to as you wrote your questions?"
- "What words or phrases did you find particularly helpful for distinguishing between graphs?" **(MTR.4.1)**

Listen for interesting or effective questions, including ones that describe key features of functions. Consider inviting students to add this language to the anchor chart during the **MLR2: Collect and Display** in Activity 1.



Unit 3
Lesson
6

Name: _____ Date: _____ Period: _____

MA.912.F.1.6, MTR.6.1

Craft-a-Graph

Let's describe and create graphs of functions using key features.

Warm-Up

1. Play a few rounds of Polygraph with your classmates!

   You will use a Warm-Up Sheet with functions. In each round:

   - You and your partner will take turns being the Picker and the Guesser.
   - Picker: Select a function from the Warm-Up Sheet. Keep it a secret!
   - Guesser: Ask the Picker yes-or-no questions, eliminating functions until you're ready to guess which function the Picker chose.

   Record helpful questions from each round in the space below.

Unit 3 Lesson 6 323 Warm-Up

Key Takeaway: Listening to and connecting different ways of describing functions can help us better understand the concept of a function and strengthen our mathematical community.

Pairs | 5 min

Activity 1 Describe It

Purpose: Students create class definitions of terms that describe key features of functions: *increasing*, *decreasing*, **maximum**, **minimum**, **positive**, and **negative**. **(MP6)**

1 Launch

Consider asking, "Are any of these terms like the ones you used while playing Polygraph in the Warm-Up?"

Encourage students to discuss what they think each term means.

2 Monitor

Listen for how pairs of students describe each of the new vocabulary terms. **(MTR.4.1)**

A **Accessibility: Executive Functioning** Chunk this task into smaller, more manageable parts by having students focus on one word at a time, analyzing the graphs, and recording key ideas from their discussion with their partner.

ELL **English Language Learners** Invite students to share descriptions in their primary languages. Encourage all students to use gestures and visuals as they share. **(Speaking and Listening)**

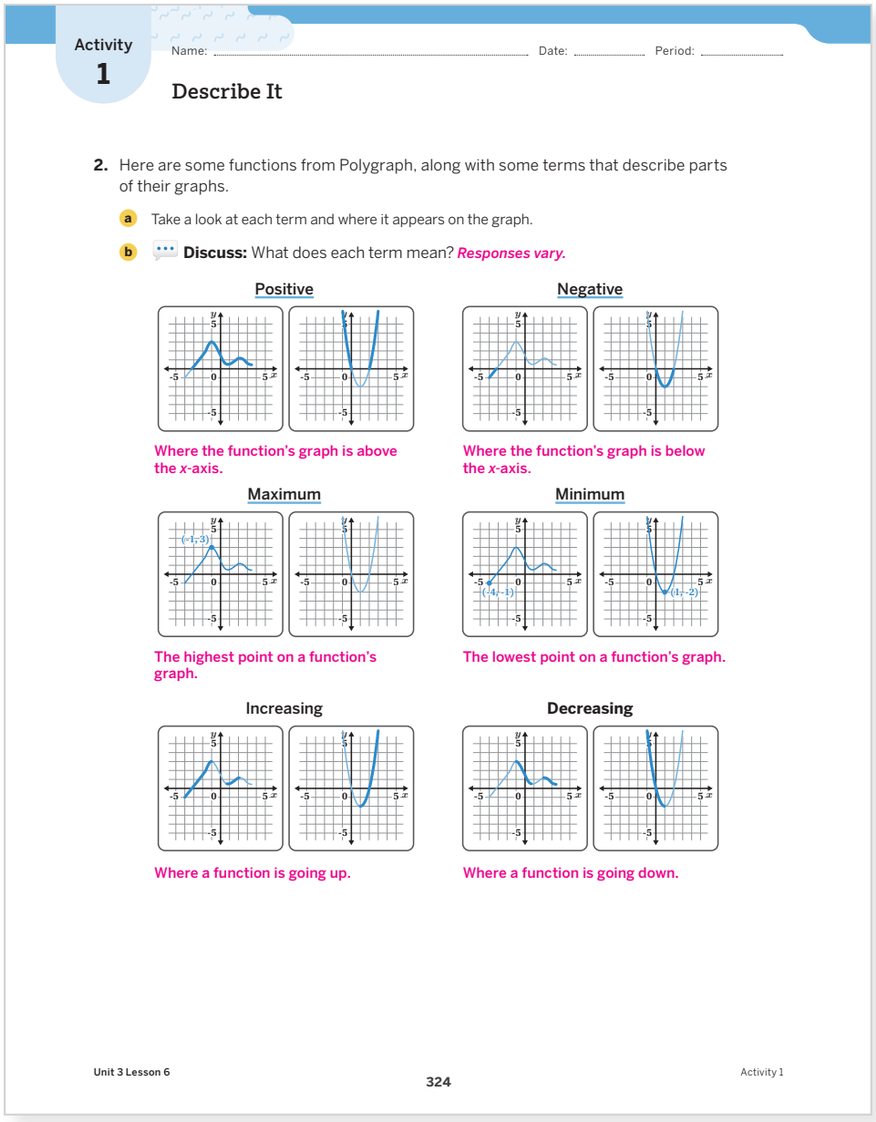
D **Differentiation: (Stretch)** Consider asking, "How could you describe the area where a function is positive, negative, increasing, or decreasing?"

3 Connect

MLR **MLR2: Collect and Display** Use this routine to create class definitions of the six terms using students' language. Display the language, examples, and diagrams for students to reference throughout the unit.

To surface the Key Takeaway, consider asking:

- "Which of these terms are related? How are they related?"
- "Which of these terms refer to points? Which of these terms refer to intervals?"
- "Are these terms describing the *inputs* or the *outputs*? How do you know?"
- "Which term means the same thing as having a positive slope? Negative slope?"



Activity 1

Name: Date: Period:

Describe It

2. Here are some functions from Polygraph, along with some terms that describe parts of their graphs.

a. Take a look at each term and where it appears on the graph.

b. **Discuss:** What does each term mean? ***Responses vary.***

Positive

Where the function's graph is above the *x*-axis.

Negative

Where the function's graph is below the *x*-axis.

Maximum

The highest point on a function's graph.

Minimum

The lowest point on a function's graph.

Increasing

Where a function is going up.

Decreasing

Where a function is going down.

Unit 3 Lesson 6 324 Activity 1

Key Takeaway: The terms *increasing*, *decreasing*, **maximum**, **minimum**, **positive**, and **negative** can be used to describe key features of functions. See the glossary for full definitions.

Pairs | 10 min

Activity 2 Build It

Purpose: Students match graphs to descriptions and create graphs to fit descriptions involving key features.

Short on time? Consider omitting Problem 6.

1 Launch

To encourage creativity, invite students to build a function they think no one else in their class will create.

2 Monitor

Look for a variety of student graphs and descriptions to share during the Connect.

Encourage students to refer to the class definitions as they describe their functions.

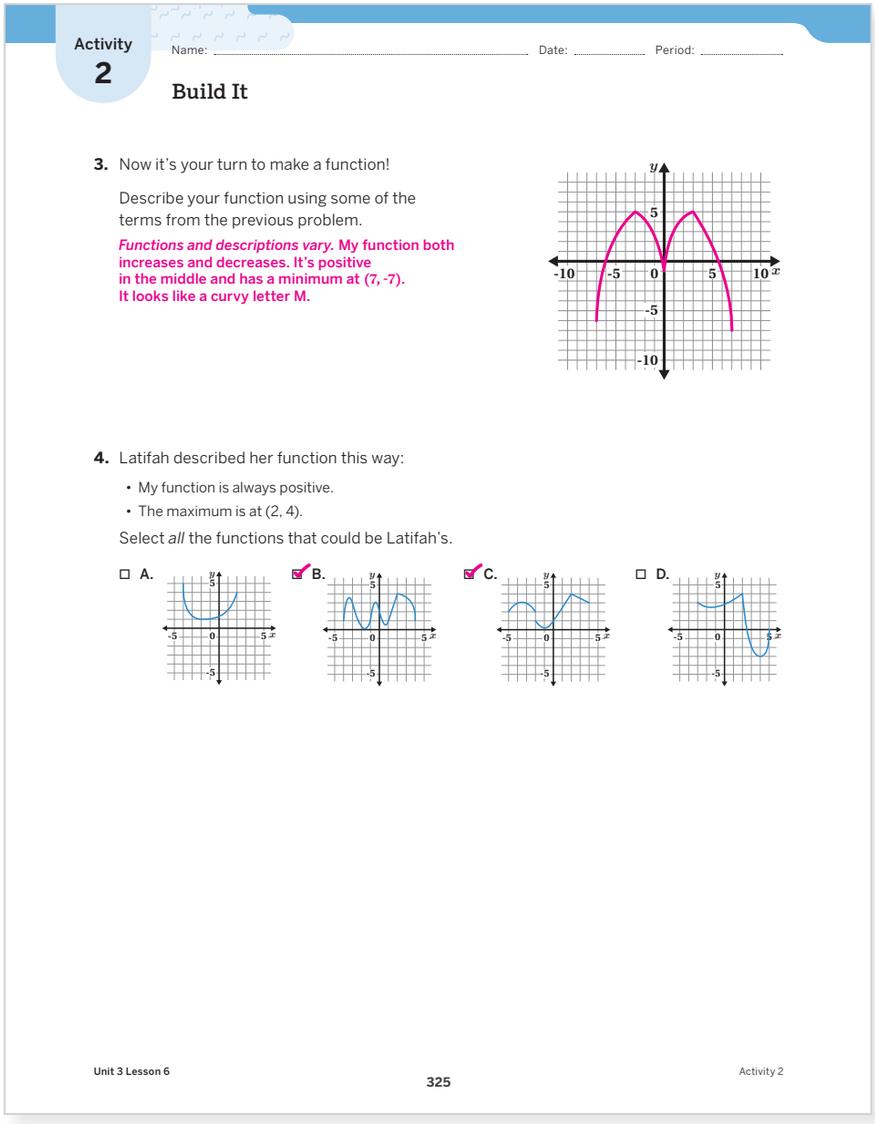
D **Differentiation: (Strengthen)** Consider asking pairs of students:
- "Where is the maximum/minimum of your function?"
- "Where is it positive/negative/increasing/decreasing?"
- "What is special about your function?"

ELL **English Language Learners** Invite students to read the function description aloud and paraphrase what it means, clarifying any unknown words or concepts before responding. **(Reading)**

D **Differentiation: (Support)** Students may select the top left graph. Consider inviting these students to trace the graph and asking: "Is (2, 4) the highest point?"

To surface connections, consider asking pairs of students:
- "How can you tell which functions are always positive?"
- "How can you determine the maximum of a function?"

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Build It

3. Now it's your turn to make a function!

Describe your function using some of the terms from the previous problem.

Functions and descriptions vary. My function both increases and decreases. It's positive in the middle and has a minimum at (7, -7). It looks like a curvy letter M.

4. Latifah described her function this way:
- My function is always positive.
- The maximum is at (2, 4).

Select *all* the functions that could be Latifah's.

☐ A.

☑ B.

☑ C.

☐ D.

Unit 3 Lesson 6 325 Activity 2

Pairs | 10 min

Activity 2 Build It (continued)

Purpose: Students match graphs to descriptions and create graphs to fit descriptions involving key features.

Short on time? Consider omitting Problem 6.

2 Monitor

To support students getting started, encourage them to create a function that meets a part of the description that they understand and then use the feedback to help them revise their response.

Note: These criteria are not exclusive. For instance, other intervals outside of $x > 0$ can be increasing, and there can be additional minimum points besides (-3, -4).

D Differentiation

Look for students who:	Respond to Student Thinking
Need support making sense of Haruto's description.	**Support** Consider asking: • "Where is $x > 0$ on the graph?" • "How do you know if a function is increasing?"
Make a function that is decreasing when $x > 0$ or read the function from right to left.	**Support** Invite students to trace where their function is increasing. Consider sharing that increasing and decreasing describe the function's behavior as you read it from left to right.
Build unique or creative graphs.	**Strengthen** Celebrate the variety in the graphs. Consider asking, "Is this the only function that would match Haruto's description?"

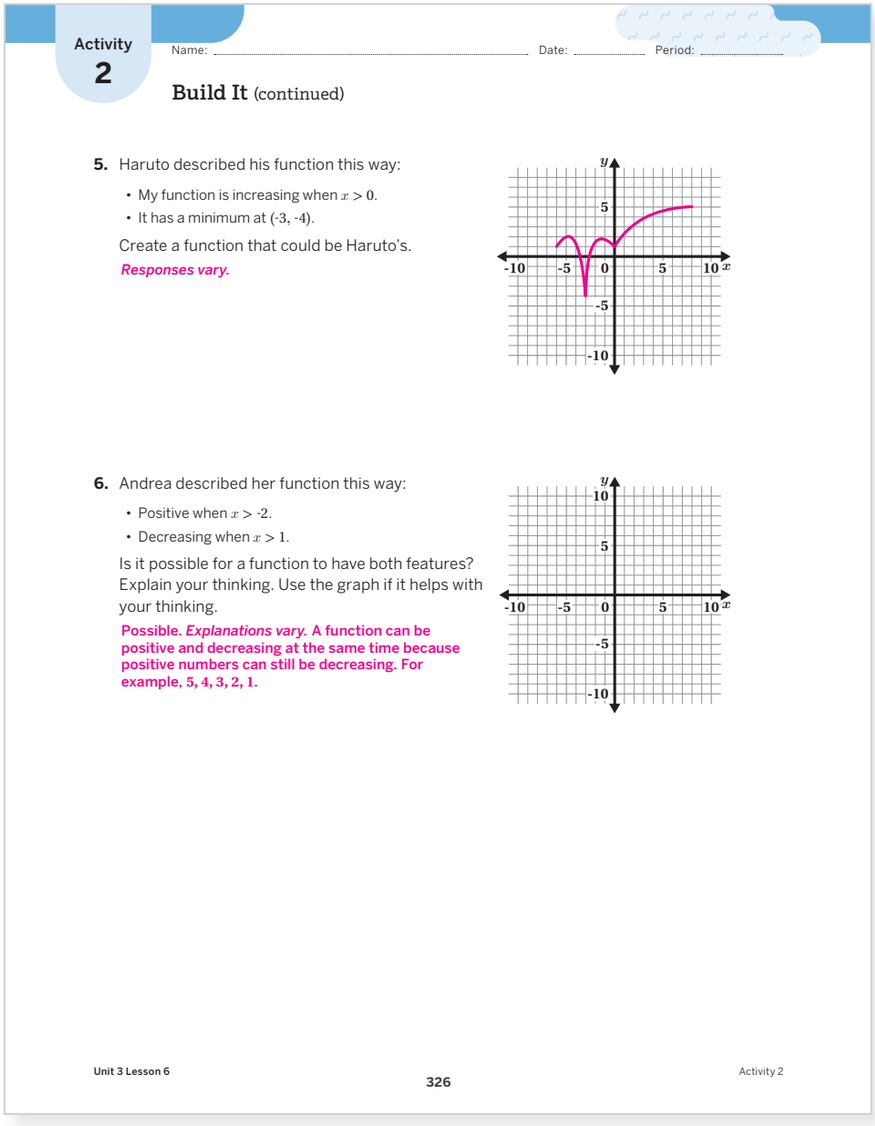
D Differentiation

Look for students who:	Respond to Student Thinking
Think this is not possible because they connect positive with a positive slope, which means increasing.	**Support** Celebrate that increasing does mean the same thing as positive slope. Consider asking, "What is the difference between positive and negative slope?"
Want to think deeper about key features of functions.	**Stretch** Invite students to make a description for a function that is not possible.

3 Connect

Encourage students to use key feature vocabulary and refer to the anchor chart as needed. Celebrate that there are multiple correct ways to describe the same function.

Note: If time allows, invite students to share their graphs. Celebrate the variety of responses and discuss why many different functions can all meet the same criteria.



Activity 2

Name: _____ Date: _____ Period: _____

Build It (continued)

5. Haruto described his function this way:

- My function is increasing when $x > 0$.
- It has a minimum at (-3, -4).

Create a function that could be Haruto's.

Responses vary.

6. Andrea described her function this way:

- Positive when $x > -2$.
- Decreasing when $x > 1$.

Is it possible for a function to have both features? Explain your thinking. Use the graph if it helps with your thinking.

Possible. *Explanations vary.* A function can be positive and decreasing at the same time because positive numbers can still be decreasing. For example, 5, 4, 3, 2, 1.

Unit 3 Lesson 6 326 Activity 2

Independent | 10 min

Activity 3 Repeated Challenges

Purpose: Students use their creativity and build fluency in key feature language by making graphs to match given criteria.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Invite students to set a goal for how many challenges they want to solve. Share that sense-making is more important than speed and that there are many correct answers for every challenge.

Students will need a partner for this activity. Encourage them to compare their solutions with their partner instead of receiving responsive feedback.

2 Monitor

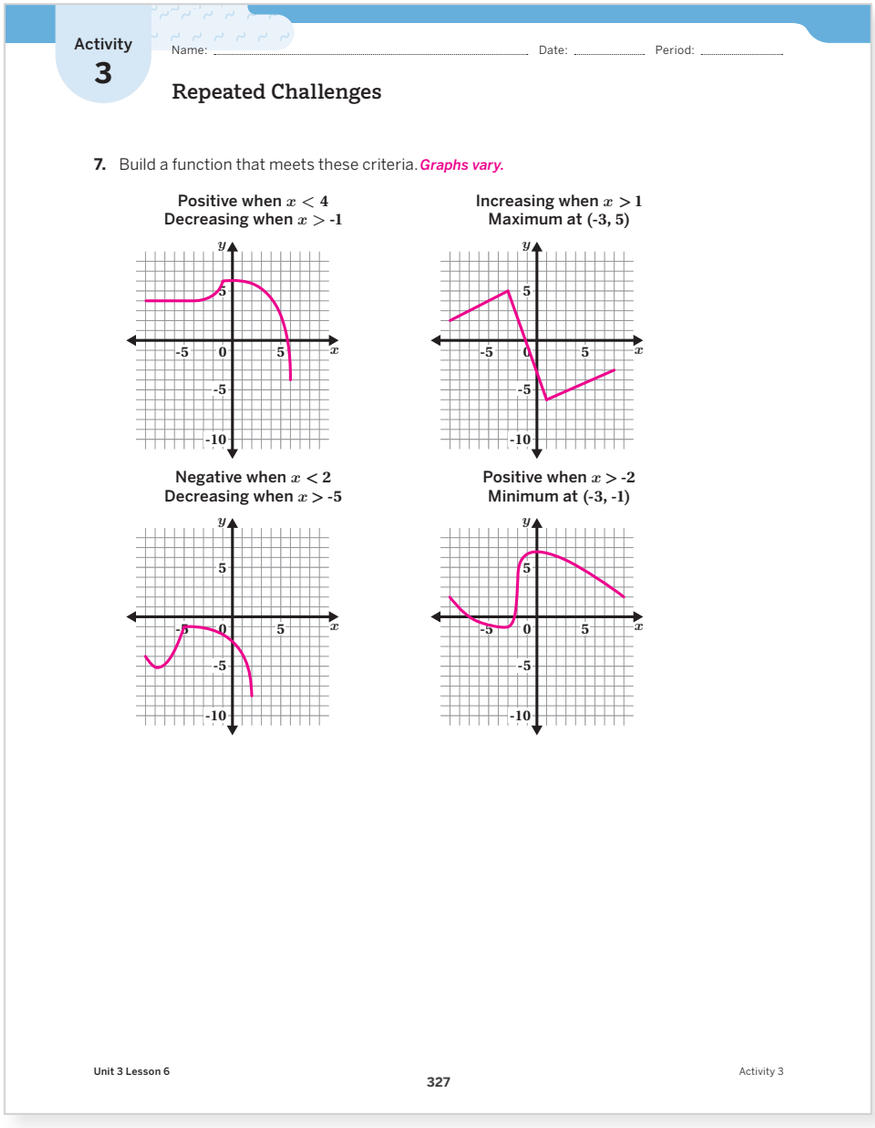
To support students getting started, encourage them to create a function that meets a part of the description that they understand and then use the feedback to help them revise their response.

Listen for mathematical language and strategies students use while discussing how to solve the problems.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one criterion at a time and build part of a function that meets that criterion.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 3

Name: Date: Period:

Repeated Challenges

7. Build a function that meets these criteria. *Graphs vary.*

Positive when $x < 4$
Decreasing when $x > -1$

Increasing when $x > 1$
Maximum at (-3, 5)

Negative when $x < 2$
Decreasing when $x > -5$

Positive when $x > -2$
Minimum at (-3, -1)

Unit 3 Lesson 6 327 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the definitions of positive, negative, maximum, minimum, increasing, and decreasing. **(MTR.4.1)**

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

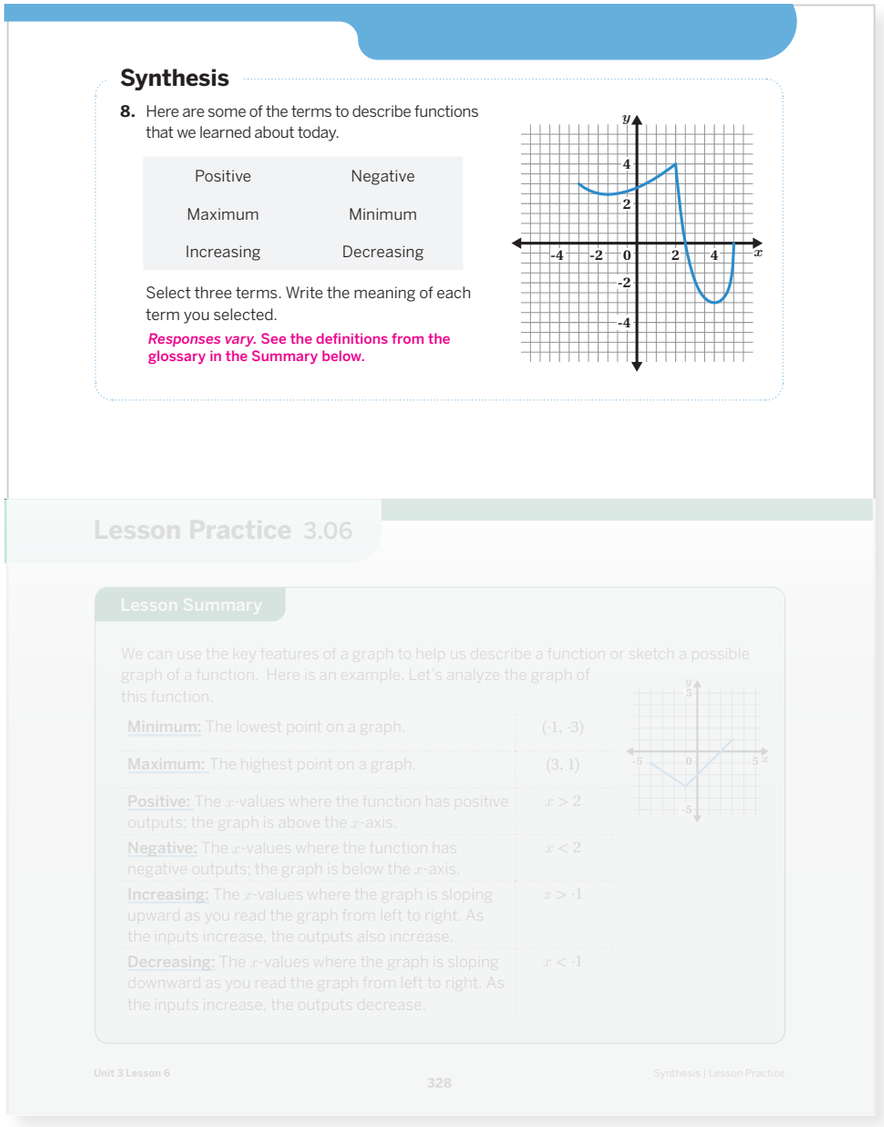
- *Positive*, *negative*, *increasing*, and *decreasing* describe intervals or sections of the graph while *maximum* and *minimum* describe a point.
- *Positive* and *negative* describe a function's relation to the x-axis.
- *Increasing* and *decreasing* describe how a function is moving when read from left to right.

Consider asking:

- "Which terms are related? How are they related?"
- "Why can having common vocabulary be helpful?"
- "Which term(s) do you understand well? Which are still unclear to you?"

MLR **MLR2: Collect and Display** Use this routine to revisit the class definitions and examples of each of the key features. Add any new ideas to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their responses based on the discussion.



Synthesis

8. Here are some of the terms to describe functions that we learned about today.

Positive	Negative
Maximum	Minimum
Increasing	Decreasing

Select three terms. Write the meaning of each term you selected.

Responses vary. **See the definitions from the glossary in the Summary below.**

Lesson Practice 3.06

Lesson Summary

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.

Minimum: The lowest point on a graph.	(-1, -3)
Maximum: The highest point on a graph.	(3, 1)
Positive: The x-values where the function has positive outputs; the graph is above the x-axis.	$x > 2$
Negative: The x-values where the function has negative outputs; the graph is below the x-axis.	$x < 2$
Increasing: The x-values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.	$x > -1$
Decreasing: The x-values where the graph is sloping downward as you read the graph from left to right. As the inputs increase, the outputs decrease.	$x < -1$

Unit 3 Lesson 6 328 Synthesis | Lesson Practice

Lesson Takeaway: There are often many functions that fit a description and many different ways to describe a single function. The terms *increasing*, *decreasing*, **maximum**, **minimum**, **positive**, and **negative** can be used to describe key features of functions precisely. See the glossary for full definitions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting graphs that have specific key features.

Today's Goals

1. **Goal:** Use the key features of a function to build a graph of that function.
2. **Language Goal:** Describe key features of a graph using terms like *positive*, *negative*, *maximum*, *minimum*, *increasing*, and *decreasing*. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

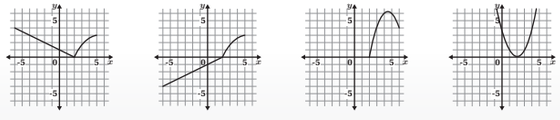
Show What You Know 3.06

A function, $f(x)$, has these features:

- Minimum is at $(2, 0)$
- Increasing when $x > 2$

Select *all* the graphs that could be $f(x)$.

☑ A. ☐ B. ☐ C. ☑ D.



D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support identifying key features of functions:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related **Practice Problems**.
- Invite students to continue using the new terms to describe functions in **Lesson 7** and throughout the unit.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Revisit the **Warm-Up** and invite students to play *Polygraph*, this time using the key feature vocabulary from this lesson.

Professional Learning

Reflect on how student language for describing functions developed throughout this lesson.

- How did your use of Polygraph and MLR2: Collect and Display support students in making sense of the new key feature vocabulary?
- How did you honor student language for describing functions today?
- How can you continue to support students in attending to precision with function feature language throughout this unit? **(MTR.4.1)**
- How do you imagine your class definitions of the new terms will grow as you explore exponential and quadratic functions later in the course and in Algebra 2?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Here are some of the terms to describe functions that we learned about today.

Positive	Negative
Maximum	Minimum
Increasing	Decreasing

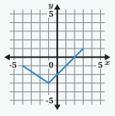
Select three terms. Write the meaning of each term you selected.

Responses vary. See the definitions from the glossary in the Summary below.

Lesson Practice 3.06

Lesson Summary

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.

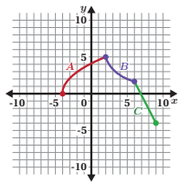


Minimum: The lowest point on a graph.	(-1, -3)
Maximum: The highest point on a graph.	(3, 1)
Positive: The x-values where the function has positive outputs; the graph is above the x-axis.	$x > 2$
Negative: The x-values where the function has negative outputs; the graph is below the x-axis.	$x < 2$
Increasing: The x-values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.	$x > -1$
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Lesson Practice 3.06

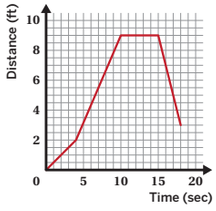
Name: Date: Period:

1. Select *all* the true statements about this graph.



- ☑ **A.** This graph is a function.
- ☐ **B.** Part A is decreasing.
- ☑ **C.** Part B is decreasing.
- ☑ **D.** The maximum is at (2, 5).
- ☐ **E.** The minimum is at (-4, 0).

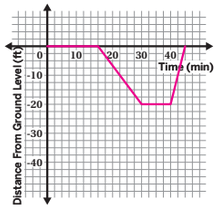
2. Manuel is watching his little brother at the park. The graph represents the distance Manuel is from his brother as a function of time.



Describe Manuel's distance from his brother over time. Use terms that you learned in this lesson.

Responses vary. Manuel starts out right next to his brother. Manuel's distance from his brother increases for about 4 seconds to about 2 feet. His distance from his brother continues to increase to a maximum distance of 9 feet, where he stays for 5 seconds. Then Manuel's distance from his brother decreases to 3 feet in about 3 seconds.

3. Ivory goes on a tour of a cave. The tour starts at ground level.



- The tour stays at ground level for 15 minutes.
- Then the tour descends (goes down) for 15 minutes to a depth of 20 feet below ground level.
- The tour stays at this level for 10 minutes.
- The tour spends the last 5 minutes ascending (going up) to ground level.

Sketch a graph describing Ivory's elevation as a function of time.

Lesson Practice 3.06

Name: Date: Period:

Problems 4–6: Here is a table that lists Seattle's temperatures for one day.

Hours After Midnight	Temperature (°F)
1	32
2	33
3	35
4	36
5	35

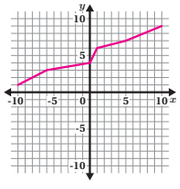
4. How many hours after midnight is the minimum temperature? 1 hour

5. How many hours after midnight is the maximum temperature? 4 hours

6. Between what hours is the temperature decreasing? Between 4 and 5 hours

Test Practice

7. Which two statements are true about the graph of a function?



- ☑ **A.** The function is always positive.
- ☑ **B.** The function is always increasing.
- ☐ **C.** The function is always decreasing.
- ☐ **D.** The function goes through (5, 0).
- ☑ **E.** The function goes through (0,5).

Spiral Review

Problems 8–10: Determine each quotient.

8. $\frac{5}{6} \div \frac{2}{3}$ $= \frac{5}{4}$ (or equivalent)

9. $\frac{8}{6} \div \frac{4}{3}$ $= 1$ (or equivalent)

10. $\frac{12}{20} \div \frac{18}{16}$ $= \frac{8}{15}$ (or equivalent)

Lesson Practice Item Analysis

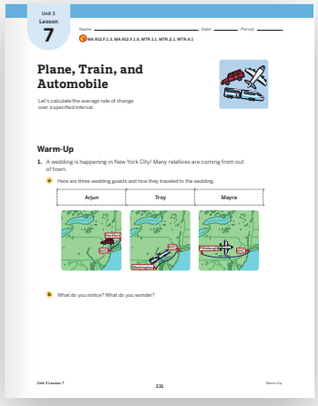
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.912.F.1.6
	2–3	2	MA.912.F.1.6
	4–6	1	MA.912.F.1.6
TEST Practice	7	2	MA.912.F.1.6
Spiral Review			
Fluency	8–10	1	MA.6.NSO.2.2

Unit 3
Lesson
7

Plane, Train, and Automobile

Average Rate of Change

Let's calculate the rate of change over a specified interval.


Unit 3
Lesson
7

Plane, Train, and Automobile

Let's calculate the average rate of change over a specified interval.

Warm-Up

1. A wedding is happening in New York City! Many relatives are coming from out of town.

Here are three wedding guests and how they traveled to the wedding.

Arjun | Troy | Mayra

What do you notice? What do you wonder?

331

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the *average rate of change* over an *interval* on a graph.
2. **Language Goal:** Interpret the average rate of change in context. **(Reading, Writing, Speaking, and Listening)**

Students use different modes of travel to the same location to learn about the term **average rate of change** and calculate the average rate of change over specified **intervals**. They use information from a graph to compare average rates of change over various intervals, and construct an argument about which mode of transportation is best based on their analysis. **(MTR.3.1)**

Note: In this lesson, students compare different modes of travel. Some students may not have experience traveling by plane or train for various reasons. Consider focusing the discussions on the fictional students' choices rather than on whether students in your class have had this experience.

Coherence

Prior Learning

In Grade 8, students calculated the slope of a line and the slope between two ordered pairs. In Lesson 5, students sketched the graph of a function to match a situation, and in Lesson 6, they described key features of graphs, including the rate of change, with terms such as increasing and decreasing.

Future Learning

In Lesson 8, students will compare key features of graphs, including their rates of change.

Connections

- Students calculate the average rate of change over a specified intervals of time to compare different modes of transportation in distance-time graphs. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.3
Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.

MA.912.F.1.5
Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Vocabulary

New Vocabulary

average rate of change

interval

Review Vocabulary

slope

Lesson at a Glance

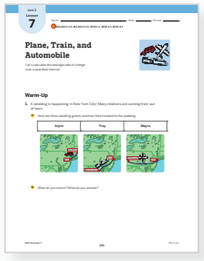
~ 45 min

MA.912.F.1.3, MA.912.F.1.5

Warm-Up

Pairs | 5 min

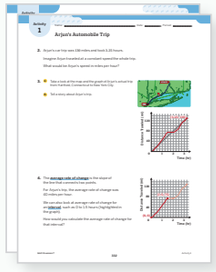
Students use the **Notice and Wonder** routine to make sense of the context of the lesson: different ways of traveling to attend a wedding in New York City.



Activity 1

Pairs | 15 min

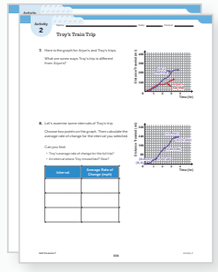
Students are introduced to the term **average rate of change** and use their prior knowledge of *slope* to calculate the average rate of change over specified **intervals**. **Routines:** Tell a Story, MLR2: Collect and Display, MLR7: Compare and Connect



Activity 2

Pairs | 7 min

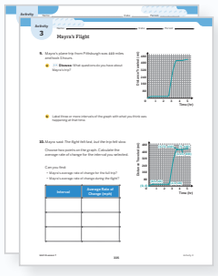
Students compare the average rate of change for two different modes of travel and interpret the average rate of change in context. **(MTR.2.1)**



Activity 3

Pairs | 8 min

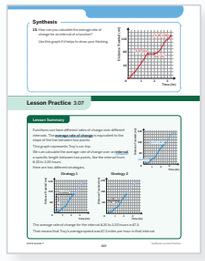
Students develop fluency in calculating the average rate of change and construct an argument about which mode of travel they prefer. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

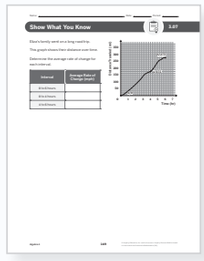
Students synthesize their strategies for how to calculate the average rate of change over a specified interval.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the average rate of change over a specified interval.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators
- Straight-edges

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of the context of the lesson: different ways of traveling to attend a wedding in New York City.

1 Launch

Consider asking:

- "Have you ever had to travel for an event that was far away?"
- "If you have to travel to an event far away, what are the different modes of transportation you might use to get there?"

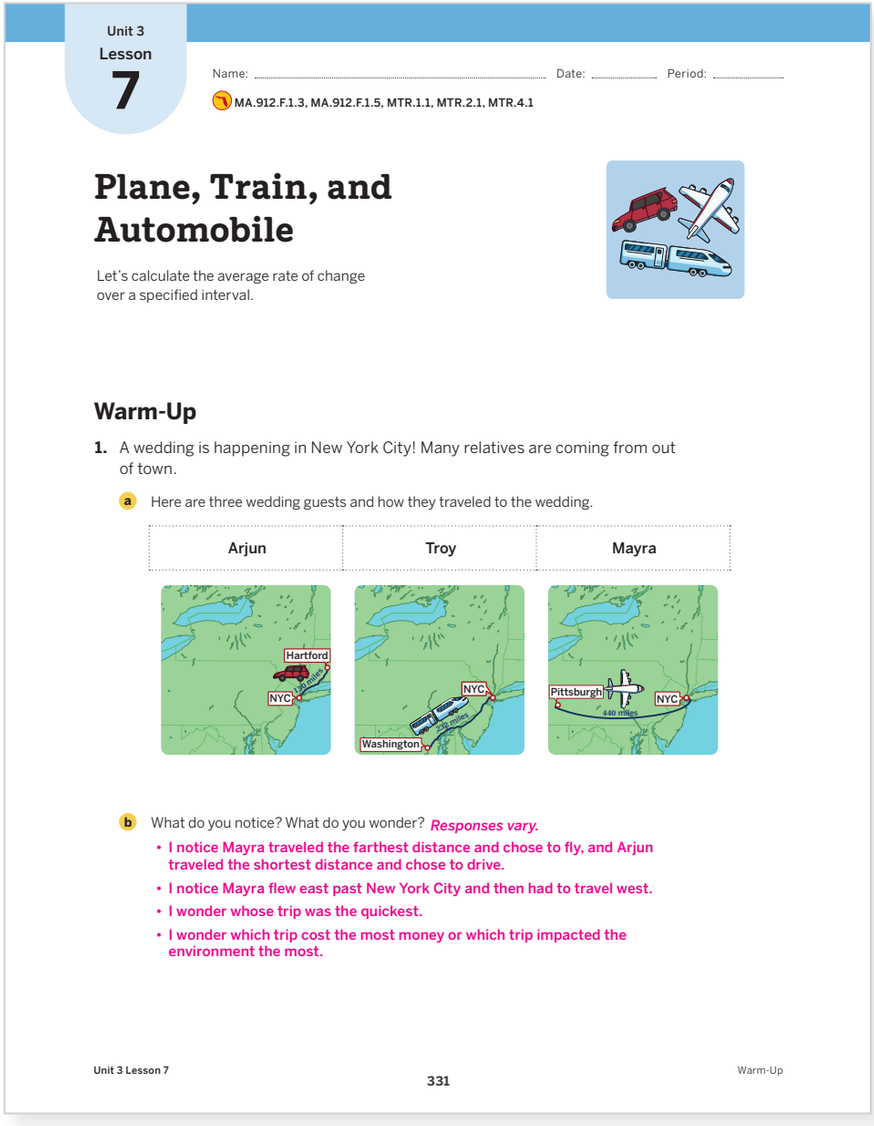
Use the Notice and Wonder routine to promote curiosity and help students make sense of the map and different modes of transportation. **(MTR.1.1)**

ELL **English Language Learners** Invite students to act out scenarios, show images of planes, trains, or automobiles, use gestures, or talk about modes of transportation to increase access to the context used in the lesson. **(Reading and Listening)**

2 Connect

Consider asking:

- "Why might someone choose to take a plane? A train? Drive a car?"
- "What are different ways to compare these different ways of traveling to the same location?" *Responses vary.* We might compare their speed, travel time, cost, sustainability, impact on environment, proximity to people, or comfort of the trip.



Unit 3
Lesson
7

Name: _____ Date: _____ Period: _____

MA.912.F.1.3, MA.912.F.1.5, MTR.1.1, MTR.2.1, MTR.4.1

Plane, Train, and Automobile

Let's calculate the average rate of change over a specified interval.

Warm-Up

1. A wedding is happening in New York City! Many relatives are coming from out of town.

   a. Here are three wedding guests and how they traveled to the wedding.

Arjun	Troy	Mayra
Hartford, NYC	NYC, Washington	Pittsburgh, NYC

   b. What do you notice? What do you wonder? *Responses vary.*

   - I notice Mayra traveled the farthest distance and chose to fly, and Arjun traveled the shortest distance and chose to drive.
   - I notice Mayra flew east past New York City and then had to travel west.
   - I wonder whose trip was the quickest.
   - I wonder which trip cost the most money or which trip impacted the environment the most.

Unit 3 Lesson 7 331 Warm-Up

Pairs | 15 min

Activity 1 Arjun's Automobile Trip

Purpose: Students are introduced to the term **average rate of change** and use their prior knowledge of *slope* to calculate the average rate of change over specified **intervals**.

1 Launch

ELL **English Language Learners** Invite students to share what the term *constant speed* reminds them of (e.g., "When I hear *constant speed*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading, Writing, Speaking, and Listening)**

2 Monitor

Look for the different strategies students use to solve Problems 2–3, including estimating the rate of change on the graph.

Encourage students to use a calculator to support their thinking.

A **Accessibility: Conceptual Processing** Invite students to answer a more concrete question first to support abstract thinking, like: "What would Arjun's speed be if he went 130 miles in 1 hour?"

Use the Tell a Story routine to make sense of the image and graph. Invite students to write a story about Arjun's trip on additional paper. Consider sharing that the gray line is representing a car that is traveling at Arjun's average speed for the whole trip. **(MTR.1.1)**

Invite students to use a straightedge to support them in calculating the average rate of change over an interval.

Listen for explanations to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Suggest using a guess and check strategy.	**Support** Celebrate their sensemaking. Consider asking, "What does the phrase 'miles per hour' tell us about how to calculate the speed?"
Divide the total miles by the total hours (e.g., $\frac{75}{1.5}$).	**Strengthen** Consider asking, "Would this strategy work if Arjun's starting point was not $(0, 0)$?" Invite these students to share during the Connect.
Calculate the slope between $(0, 0)$ and $(1.5, 75)$ algebraically or using the graph.	**Stretch** Celebrate their strategy. Consider asking, "How can you use the graph/calculate the slope to show the same information?"Invite these students to share during the Connect.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Arjun's Automobile Trip

2. Arjun's car trip was 130 miles and took 3.25 hours.

Imagine Arjun traveled at a constant speed the whole trip.

What would be Arjun's speed in miles per hour?

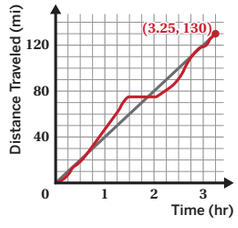
40 miles per hour

3. a Take a look at the map and the graph of Arjun's actual trip from Hartford, Connecticut to New York City.



b Tell a story about Arjun's trip.

Responses vary. At first, Arjun was traveling at about 40 miles per hour, but he needed to stop and get gas. He stayed there for 30 minutes, maybe to get food and use the bathroom. Then Arjun got back in the car and finished the trip, traveling a bit faster than he did before the stop.

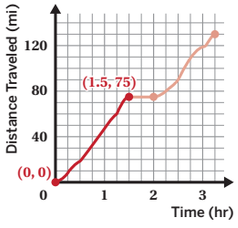


4. The **average rate of change** is the *slope* of the line that connects two points.

For Arjun's trip, the average rate of change was 40 miles per hour.

We can also look at average rate of change for an **interval**, such as 0 to 1.5 hours (highlighted in the graph).

How would you calculate the average rate of change for that interval?



Responses vary. I can use the same strategies I used for calculating the slope of a line. I would divide 75 by 1.5 to find the average rate of change for the first interval because the other endpoint is (0, 0).

Unit 3 Lesson 7 332 Activity 1

Pairs | 15 min

Activity 1 Arjun's Automobile Trip (continued)

Purpose: Students are introduced to the term **average rate of change** and use their prior knowledge of *slope* to calculate the average rate of change over specified **intervals**.

2 Monitor

Encourage students to use what they know about calculating slope to calculate the average rate of change for each interval.

D Differentiation

Look for students who:	Respond to Student Thinking
Divide the y-value by the x-value for the second interval (e.g., $\frac{130}{3.25}$).	**Support** Celebrate that they knew to use division as the operation. Consider asking, "How long was Arjun driving during that interval? How far did he travel?"
Use a strategy similar to Naoki's or LaShawn's.	**Strengthen** Consider renaming these strategies after the students in your class who used them. Invite these students to share during the Connect.
Calculate all three average rates of change correctly and are ready for more.	**Stretch** Consider asking, "When was Arjun traveling the fastest? How do you know?"

3 Connect

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about how to calculate the average rate of change for an interval as they share during the Connect. Display for students to reference and add to for the rest of the lesson.

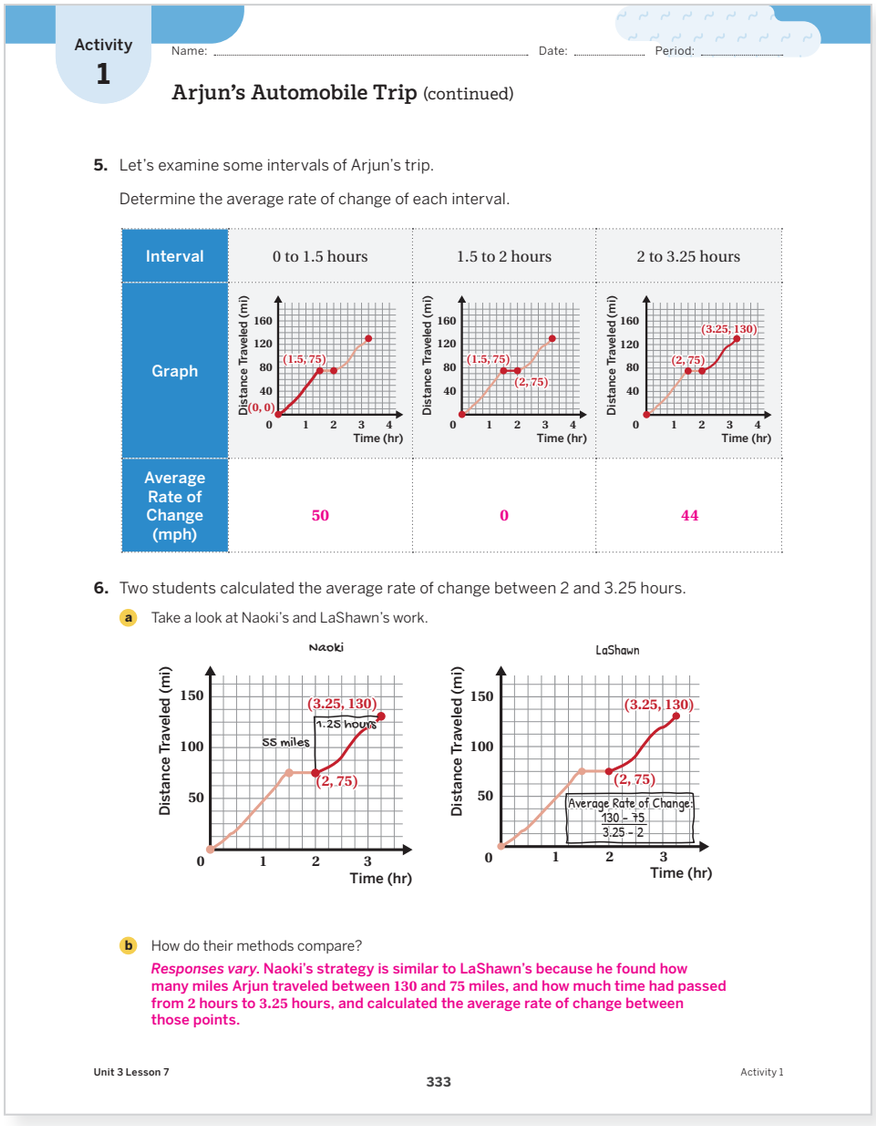
To reveal the Key Takeaway, consider asking:

- "What units are we using to measure the average rate of change? How do the units help us know what operation to use?" *Responses vary.* Miles per hour means we are using division to calculate the average rate of change.
- "Arjun's average speed is 40 miles per hour. Does that mean he traveled 40 miles per hour the entire trip?"
- "How is calculating the average rate of change for an interval similar to what you did in Problem 2?" *Responses vary.* The average rate of change is equal to the slope between two points, but the interval in between doesn't have to be a line.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

Consider asking:

- "Where do you see the same information in Naoki and LaShawn's work?"
- "How does each strategy compare to your own?"
- "How are these strategies like strategies you've used before?"



Activity 1

Name: _____ Date: _____ Period: _____

Arjun's Automobile Trip (continued)

5. Let's examine some intervals of Arjun's trip.

Determine the average rate of change of each interval.

Interval	0 to 1.5 hours	1.5 to 2 hours	2 to 3.25 hours
Graph			
Average Rate of Change (mph)	50	0	44

6. Two students calculated the average rate of change between 2 and 3.25 hours.

a. Take a look at Naoki's and LaShawn's work.

b. How do their methods compare?

Responses vary. Naoki's strategy is similar to LaShawn's because he found how many miles Arjun traveled between 130 and 75 miles, and how much time had passed from 2 hours to 3.25 hours, and calculated the average rate of change between those points.

Unit 3 Lesson 7 333 Activity 1

Key Takeaway: The **average rate of change** between two points is equal to the slope of the line that connects those two points. It can be found by dividing the change in y-values by the change in x-values for two ordered pairs. For a function $f(x)$, the average rate of change between points $(a, f(a))$ and $(b, f(b))$ is found by using the formula $\frac{f(b) - f(a)}{b - a}$.

Pairs | 7 min

Activity 2 Troy's Train Trip

Purpose: Students compare the average rate of change for two different modes of travel and interpret the average rate of change in context. **(MTR.2.1)**

1 Launch

Consider asking, "Have you ever traveled by train before? What was it like?"

2 Monitor

Encourage students to give feedback to support their reasoning.

Look for students who populate their table with multiple, different intervals to share during the Connect.

Invite students to share an expression or calculated average rate of change.

3 Connect

Display one student's table with multiple intervals and average rates of change calculated.

Consider asking:

- "During which interval is Troy moving the fastest? The slowest? How can you tell from the graph?" **(MTR.2.1)**
- "What evidence do you have to support your thinking?" **(MTR.4.1)**

Activity 2

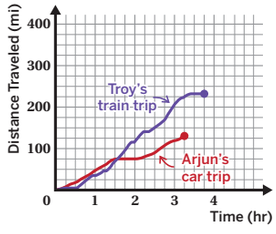
Name: _____ Date: _____ Period: _____

Troy's Train Trip

7. Here is the graph for Arjun's and Troy's trips.

What are some ways Troy's trip is different from Arjun's?

Responses vary. Troy traveled more miles, and it took more time than Arjun's trip. He also wasn't moving at the beginning of his trip because he was waiting for the train but then traveled steadily, while Arjun took a break in the middle.



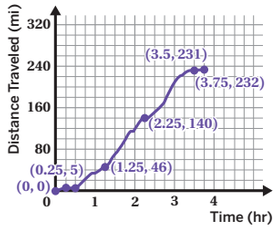
8. Let's examine some intervals of Troy's trip.

Choose two points on the graph. Then calculate the average rate of change for the interval you selected.

Responses vary based on the points students select.

Can you find:

- Troy's average rate of change for the full trip?
- An interval where Troy moved fast? Slow?



Interval	Average Rate of Change (mph)
Full trip: 0 to 3.75 hours	≈61.87
An interval where Troy moved fast: 1.25 to 2.25 hours	94
An interval where Troy moved slow: 0 to 0.25 hours	20

Unit 3 Lesson 7 334 Activity 2

Pairs | 8 min

Activity 3 Mayra's Flight

Purpose: Students develop fluency in calculating the average rate of change and construct an argument about which mode of travel they prefer. **(MTR.4.1)**

1 Launch

Consider asking, "Why might Mayra choose to travel by plane?"

Note: Some students may not have experienced traveling by plane for various reasons. Consider focusing the discussion on why Mayra chose to travel by plane, rather than on whether students in your class have had the experience.

2 Monitor

To support students getting started on Problem 10, consider asking, "Where on the graph do you think Mayra was actually in the air? How can you tell?"

Pause briefly to discuss each interval. Model curiosity by celebrating the difference in students' interpretations, inviting them to share their thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to select two points. Consider asking, "How can we use the ordered pairs to calculate Mayra's average rate of change over this interval?"
Divide the change in x-values over the change in y-values.	**Support** Consider asking, "What does the x-axis measure? The y-axis?" What would the ratio of $\frac{x}{y}$ represent for Mayra's average rate of change in this situation?" *Change in x divided by the change in y would represent the number of hours per mile.*
Think the order of subtraction for the x- and y-values does not matter.	**Strengthen** Invite these students to calculate the average rate of change for the same interval twice, switching the order of the x- or y-values, and use the result to adjust their thinking.

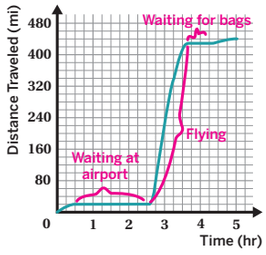
Activity 3 continued >

Activity 3

Name: _____ Date: _____ Period: _____

Mayra's Flight

9. Mayra's plane trip from Pittsburgh was 440 miles and took 5 hours.



a. **Discuss:** What questions do you have about Mayra's trip?

Responses vary.

- How long did it take her to get to the wedding from the airport?
- Why was Mayra's average rate of change 0 for such a long interval?
- At what points in the trip did Mayra get on and off the plane?
- Why did Mayra choose to fly instead of driving or taking the train?

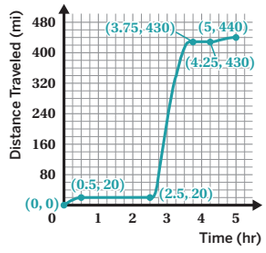
b. Label three or more intervals of the graph with what you think was happening at that time.

Responses vary.

10. Mayra said: *The flight felt fast, but the trip felt slow.*

Choose two points on the graph. Calculate the average rate of change for the interval you selected.

Responses vary based on the points students select.



Can you find:

- Mayra's average rate of change for the full trip?
- Mayra's average rate of change *during the flight*?

Interval	Average Rate of Change (mph)
Full trip: 0 to 5 hours	88
During the flight: 2.5 to 3.75 hours	328
Waiting at airport: 0.5 to 2.5 hours	0

Unit 3 Lesson 7 335 Activity 3

Pairs | 8 min

Activity 3 Mayra's Flight (continued)

Purpose: Students develop fluency in calculating the average rate of change and construct an argument about which mode of travel they prefer.

2 Monitor

Listen for student responses for which mode of travel they prefer, selecting at least one for each mode of travel.

Note: Students may define "best" for choosing a mode of travel in different ways based on their unique experiences, including sustainability, financial cost, time, and speed. Support students in seeing that "best" can mean different things to different people.

D **Differentiation: (Stretch) You're invited to explore more.** Invite students who would like to explore the average rate of change further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their responses.

To reveal the Key Takeaway, consider asking:

- "What factors might someone consider when choosing how to travel?"
- "How can the average rate of change help us compare different modes of travel?"
- "What are some other ways to define the 'best' way to travel other than speed?"

Activity 3

Name: _____ Date: _____ Period: _____

Mayra's Flight (continued)

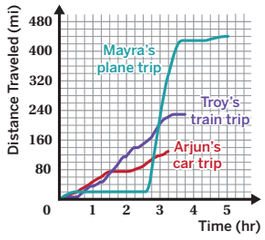
11. There are many reasons why someone may choose to travel by car, train, or plane.

Here are the graphs for all three people's trips.

Circle the trip you think is best.

Arjun (car) Troy (train) Mayra (plane)

Explain why you chose that trip.



Selections and explanations vary.

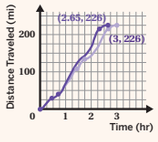
- I think traveling by car is best, especially if you are carpooling with a group of people because I love to find new and interesting places to stop along the way.
- I think traveling by train is best because it's better for the environment due to lower carbon dioxide emissions and the ability to transport more people at a time.
- I think traveling by plane is best because it gets you there fast and you don't have to be responsible for driving.

You're invited to explore more.

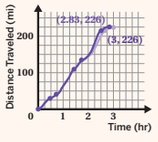
12. Public transit agencies often look at average speed (rate of change) when they look for ways to improve service.

a What is the train's average speed in each Proposal?

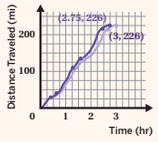
Proposal A: Skip stops along the way.



Proposal B: Upgrade one section of the tracks.



Proposal C: Buy trains with a higher top speed.



b **Discuss:**

- What is the train's average speed?
- What are some benefits and challenges for each proposal?

Responses vary. See Teacher Edition for sample responses.

Unit 3 Lesson 7 336 Activity 3

Key Takeaway: There are many different ways to compare the graphs of functions, just like there are different ways to compare modes of travel. Different people will define the "best" way to travel based on their unique experiences and perspectives.

Whole Class | 5 min

Synthesis

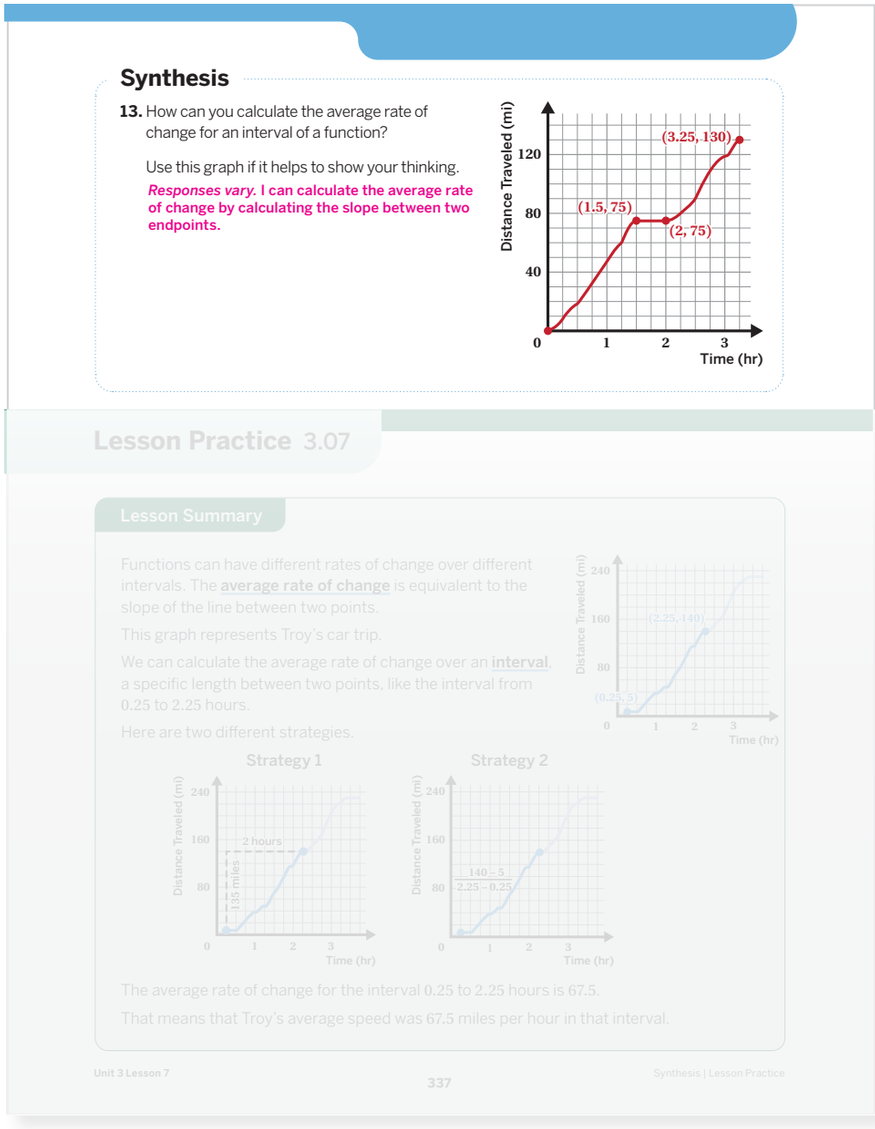
Purpose: Students synthesize their strategies for how to calculate the average rate of change over a specified interval.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Calculating the slope between two points.
- Using the graph to calculate the slope.
- Dividing the change in y-values by the change in x-values.
- Subtracting the miles traveled and the time it took to travel and dividing the differences.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

13. How can you calculate the average rate of change for an interval of a function?

Use this graph if it helps to show your thinking.

Responses vary. I can calculate the average rate of change by calculating the slope between two endpoints.

Lesson Practice 3.07

Lesson Summary

Functions can have different rates of change over different intervals. The average rate of change is equivalent to the slope of the line between two points.

This graph represents Troy's car trip.

We can calculate the average rate of change over an interval, a specific length between two points, like the interval from 0.25 to 2.25 hours.

Here are two different strategies.

Strategy 1

Strategy 2

The average rate of change for the interval 0.25 to 2.25 hours is 67.5.

That means that Troy's average speed was 67.5 miles per hour in that interval.

Unit 3 Lesson 7 337 Synthesis | Lesson Practice

Lesson Takeaway: The **average rate of change** calculates the *slope* between two points over a specified **interval** by dividing the change in y-values by the change in x-values. The average rate of change is one key feature that can be used to compare graphs.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by calculating the average rate of change over a specified interval.

Today's Goals

1. **Goal:** Calculate the *average rate of change* over an *interval* on a graph.
2. **Language Goal:** Interpret the average rate of change in context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

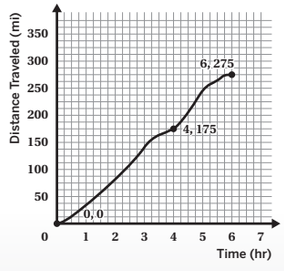
Show What You Know 3.07

Eliza's family went on a long road trip.

This graph shows their distance over time.

Determine the average rate of change for each interval.

Interval	Average Rate of Change (mph)
0 to 6 hours	45.83
0 to 4 hours	43.75
4 to 6 hours	50



Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support calculating slope:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related **Practice Problems**.
- Use the *Calculating Slope Given Two Points* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to sketch their own function and calculate its average rate of change on various intervals.

Professional Learning

In this lesson, students used their prior knowledge of how to calculate the slope between two points to support their understanding of how to calculate the **average rate of change** over a specified **interval**. How were students able to apply their thinking to this new concept? How can you support students in recognizing the connection between these two key math concepts?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. How can you calculate the average rate of change for an interval of a function?

Use this graph if it helps to show your thinking.

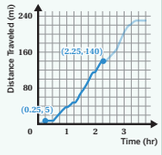
Responses vary. I can calculate the average rate of change by calculating the slope between two endpoints.

Lesson Practice 3.07

Lesson Summary

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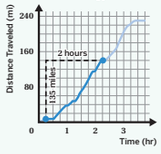
This graph represents Troy's car trip.



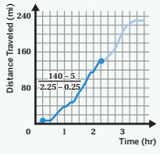
We can calculate the average rate of change over an **interval**, a specific length between two points, like the interval from 0.25 to 2.25 hours.

Here are two different strategies.

Strategy 1



Strategy 2



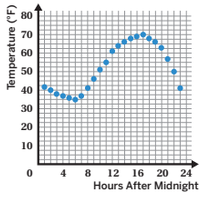
The average rate of change for the interval 0.25 to 2.25 hours is 67.5.

That means that Troy's average speed was 67.5 miles per hour in that interval.

Lesson Practice 3.07

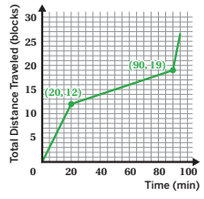
Name: _____ Date: _____ Period: _____

1. The temperature was recorded at several times in a 24-hour period. Function $t(n)$ gives the temperature in degrees Fahrenheit n hours after midnight. Use the graph to determine if the average rate of change for each interval is *positive* (+), *negative* (-), *or zero*. Place a check in the appropriate column.



	+	-	0
$n = 1$ to $n = 5$		✓	
$n = 5$ to $n = 7$	✓		
$n = 10$ to $n = 20$	✓		

Problems 2–3: This graph shows the total distance in city blocks, $d(t)$, that Pilar walked as a function of time in minutes, t.



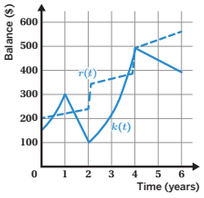
2. Determine the average rate of change between $t = 20$ and $t = 90$.

0.1

3. What do you think the average rate of change you calculated means in this situation?

Explanations vary. **The average rate of change means that Pilar walked 0.1 blocks per minute or $\frac{1}{10}$ of a block per minute, from 20 minutes to 90 minutes into her walk.**

4. $r(t)$ and $k(t)$ model the savings account balances of Rafael and Katie after t years. Select *all* the statements that are true.



- ☑ A. Katie has a lower average rate of change in the last two years.
- ☐ B. Katie's balance is always less than Rafael's.
- ☐ C. $r(2) = 100$
- ☑ D. Rafael's balance is increasing from year 0 to year 6.
- ☐ E. Rafael has a higher average rate of change in the first four years.

Lesson Practice 3.07

Name: _____ Date: _____ Period: _____

5. Match each interval to its average rate of change.

Interval	Average Rate of Change
A. X to Y	C. $\frac{1}{5}$
B. Y to Z	B. $\frac{1}{4}$
C. X to Z	A. $\frac{1}{6}$



Test Practice

Problems 6–8: This table shows the population of a city from 1988 to 2016.

Year, t	Population, $p(t)$
1988	35,700
1992	42,700
1996	33,100
2000	33,700
2004	45,000
2008	48,400
2012	40,900
2016	43,000

6. Choose the average rate of change for $p(t)$ between 1992 and 2000.

- A. **-1,125 people per year**
- B. -9,000 people per year
- C. 1,125 people per year
- D. 9,000 people per year

7. State two values of t that create an interval with a *negative* rate of change.

Responses vary. **2008 and 2012.**

8. State two values of t that create an interval with a *positive* rate of change.

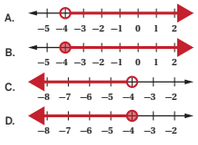
Responses vary. **1988 and 2016.**

Spiral Review

9. Jada is walking to school. The function $d(t)$ gives her distance from school, in meters, t minutes since she left home. Which equation represents this statement? *Jada is 600 meters from school after 5 minutes.*

- (A.) $d(5) = 600$
- B. $d(600) = 5$
- C. $t(5) = 600$
- D. $t(600) = 5$

10. Select the graphed solution set of $-2x + 4 < 12$.



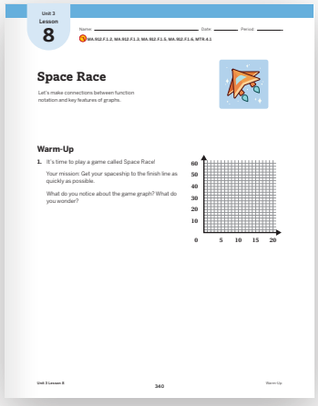
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.F.1.3
	2–3	2	MA.912.F.1.3
	4–5	2	MA.912.F.1.5 MA.912.F.1.3
TEST Practice	6–8	2	MA.912.F.1.3
Spiral Review			
Fluency	9	1	MA.912.F.1.2
	10	1	MA.8.AR.2.2

Unit 3

Lesson

8



Space Race

Comparing Graphs

Let's make connections between function notation and key features of graphs.

Learning Outcomes

Today's Goals

1. **Goal:** Interpret statements written in function notation and use them to compare key features of graphs.
2. **Language Goal:** Compare and contrast key features of functions using statements written in function notation. **(Reading, Writing, Speaking, and Listening)**

Students use function notation to compare key features of graphs in the context of a space race game. This lesson also introduces the idea that the point of intersection on a graph is where two functions are equal, e.g., $f(x) = g(x)$.

Coherence

Prior Learning

In Lesson 2–3, students used function notation to describe key features of graphs.

Future Learning

In Unit 5, students will use inequalities to compare and describe the domain and range of functions.

Connections

- Students apply their understanding of function notation and key features on graphs when playing a video game. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.5

Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.

MA.912.F.1.6

Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.8.F.1.3	**MA.912.AR.3.4**

Lesson at a Glance

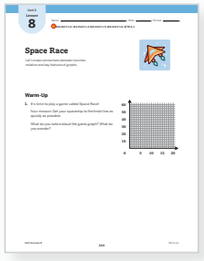
~ 45 min

MA.912.1.5, MA.912.F.1.6

Warm-Up

Independent | 5 min

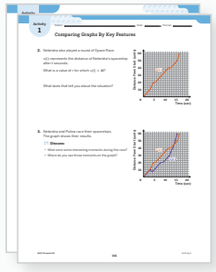
Students race their spaceship through an obstacle course to introduce them to the context of the lesson.



Activity 1

Pairs | 15 min

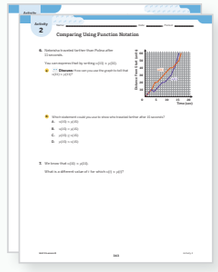
Students compare the key features of two graphs that represent space races, including their average rates of change. **Routines:** Think-Pair-Share, MLR7: Compare and Connect



Activity 2

Pairs | 15 min

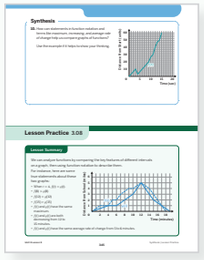
Students use statements written in function notation to compare the graphs of two functions. MLR7: Compare and Connect



Synthesis

Whole Class | 5 min

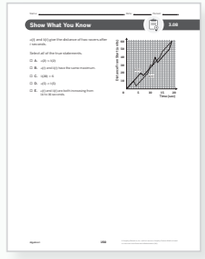
Students synthesize their understanding of how to use function notation to compare the key features of graphs.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using a graph to determine which statements written in function notation are true.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students race their spaceship through an obstacle course to introduce them to the context of the lesson.

Short on time? Consider having students play the game only once and discussing the game as a class.

1 Launch

To generate excitement, consider asking, "What's your favorite kind of video game? What do you like about it?"

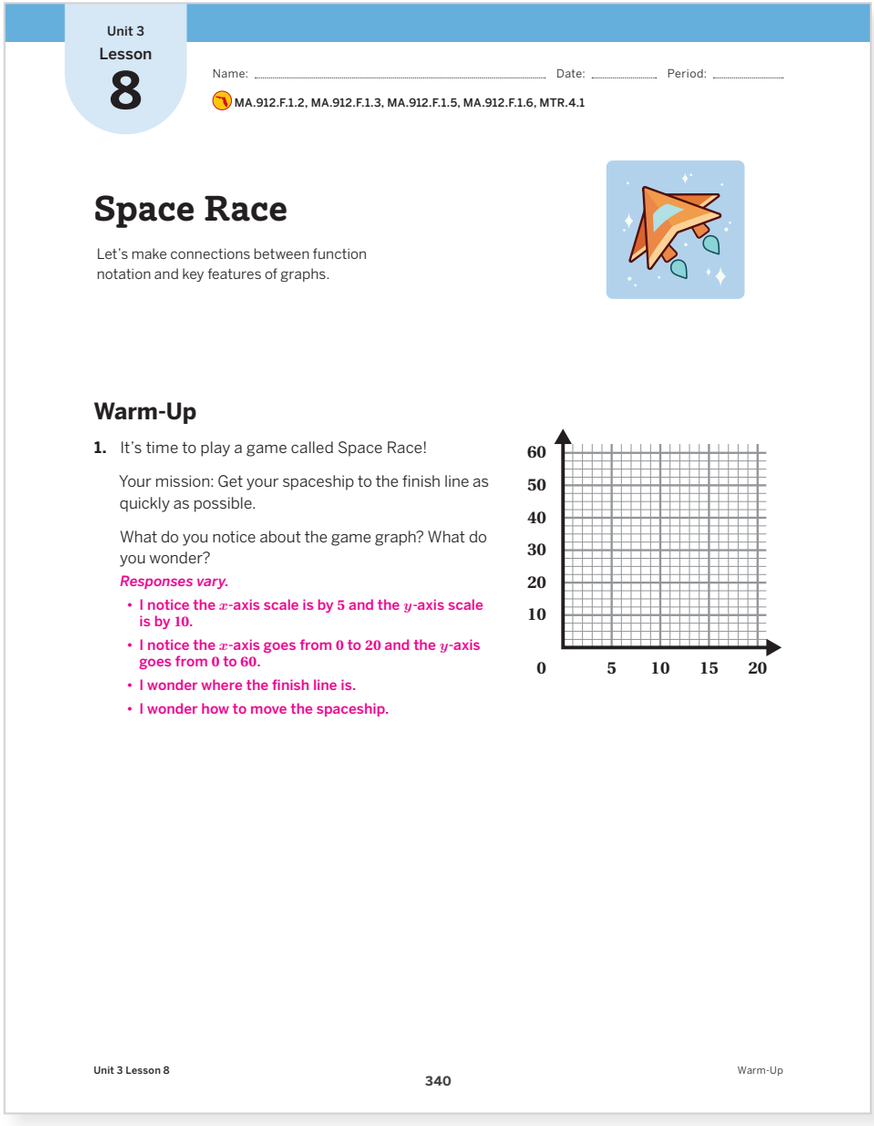
ELL **English Language Learners** Invite students to share what they know about space or video games to increase access to the task. **(Reading and Listening)**

2 Connect

Invite students to share how a graph can be used to represent the movement of the spaceship.

Consider asking:

- "In which race did you have the largest average rate of change? How can you tell?"
- "Are there any intervals where your function is decreasing?"
- "What is the maximum of your first race? Your second race? How do they compare?"



Unit 3
Lesson
8

Name: _____ Date: _____ Period: _____

MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.5, MA.912.F.1.6, MTR.4.1

Space Race

Let's make connections between function notation and key features of graphs.

Warm-Up

1. It's time to play a game called Space Race!

Your mission: Get your spaceship to the finish line as quickly as possible.

What do you notice about the game graph? What do you wonder?

Responses vary.

- I notice the x-axis scale is by 5 and the y-axis scale is by 10.
- I notice the x-axis goes from 0 to 20 and the y-axis goes from 0 to 60.
- I wonder where the finish line is.
- I wonder how to move the spaceship.

Unit 3 Lesson 8 340 Warm-Up

Pairs | 15 min

Activity 1 Comparing Graphs By Key Features

Purpose: Students compare the key features of two graphs that represent space races, including their average rates of change.

1 Launch

To access connections, consider asking:

- "Describe what happened in Nekeisha's space race."
- "What variable is being measured on the horizontal axis? How is it measured?"
- "What variable is being measured on the vertical axis? How is it measured?"

2 Monitor

Listen for strategies students use to determine a value for t.

D Differentiation

Look for students who:	Respond to Student Thinking
Think $n(35) = 10$.	**Support** Consider asking, "How can we tell from the equation if we are looking for an input value t or an output value $n(t)$?"
Estimate using the graph.	**Strengthen** Celebrate this strategy. Consider asking them, "How can we make the estimate more precise using the graph?"
Draw a horizontal line at $n(t) = 10$ to see where it intersects the graph.	**Stretch** Celebrate this student's strategy. Invite them to share their strategy during the class discussion. Consider asking, "What does the value you found for t tell you about Nekeisha's race?"
Trace up the y-axis to where $n(t) = 10$ and over to the graph to find a value of t.	

A Accessibility: Visual-Spatial Processing Invite students to chunk the graph into smaller parts and describe what is happening at each step to support interpreting visual representations.

Listen for opportunities to connect interesting moments that students notice during the space race to specific key features of the graphs. Students may discuss:

- The average rate of change by describing one ship going faster or slower than the other.
- Points of intersection as one spaceship passing the other.
- The maximum or minimum as the start or end of the race.
- Increasing, decreasing, or constant intervals as moments that describe the spaceship's movement over time. **(MTR.4.1)**

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Comparing Graphs By Key Features

2. Nekeisha also played a round of Space Race.

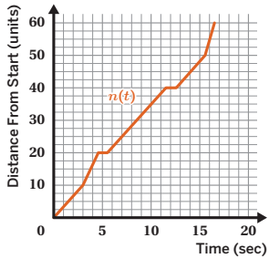
$n(t)$ represents the distance of Nekeisha's spaceship after t seconds.

What is a value of t for which $n(t) = 10$?

t = 3

What does that tell you about the situation?

Responses vary. Nekeisha's spaceship went 10 units in 3 seconds.



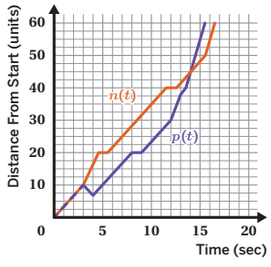
3. Nekeisha and Polina race their spaceships. The graph shows their results.

Discuss:

- What were some interesting moments during this race?
- Where do you see those moments on the graph?

Responses vary.

- There was a moment when Polina's triangle spun around and then went backwards. This is the part of the graph where Polina's function is decreasing.
- At the end of the race, Polina started zooming forward. This is the part of the graph where Polina's graph is very steep.



Unit 3 Lesson 8 341 Activity 1

Pairs | 15 min

Activity 1 Comparing Graphs By Key Features (continued)

Purpose: Students compare the key features of two graphs that represent space races, including their average rates of change.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Write an interval instead of a value into the math input.	**Support** Consider asking, "What's the difference between an interval and a value?"
Think Polina's function is decreasing at 4 seconds.	**Strengthen** Consider asking, "How can the label on the y-axis ("Distance from Start") help us describe what is happening at 4 seconds? What does it mean for the Distance from Start to decrease?"

3 Connect

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

Consider asking:

- "How can we use key features to describe key moments of the space race clearly?" **(MTR.4.1)**

MLR MLR7: Compare and Connect Invite students to analyze and compare different strategies for determining the average rate of change and connecting those strategies to their own. Students may draw a slope triangle, calculate the average rate of change using two ordered pairs, use spatial reasoning to compare how far the graphs were apart at $t = 12$ and $t = 14$, or use intuition to reason about how Polina was behind Nekeisha but ended up at the same place at the point of intersection. Listen for and share the multiple strategies that students use, helping them make connections between the different strategies. See the Routine Facilitation Guide for more information.

Activity 1

Name: _____ Date: _____ Period: _____

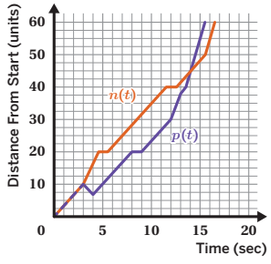
Comparing Graphs By Key Features (continued)

4. $n(t)$ and $p(t)$ represent the distances of Nekeisha's and Polina's spaceships after t seconds.

One student described part of the graph like this: *Polina's function was decreasing, and Nekeisha's function was increasing.*

Write a value of t that is in this interval of time.

Responses vary. **t-values greater than 3 seconds and less than 4 seconds are considered correct.**



Describe what was happening in the race at that moment.

Responses vary. **After 3 seconds of racing, Polina hit a star that made her spaceship go backward, while Nekeisha hit a star that zoomed her spaceship forward.**

5. Who had a greater average rate of change from 12 to 14 seconds?

A. Polina **B.** Nekeisha **C.** Their average rates of change were the same.

Explain your thinking.

Explanations vary. **Polina's graph was lower than Nekeisha's at $t = 12$ but at the same point as Nekeisha's at $t = 14$. This means Polina's spaceship needed a faster average rate of change to meet up with Nekeisha's ship at $t = 14$.**

Unit 3 Lesson 8 342 Activity 1

Key Takeaway: Using key features to compare two graphs supports us in communicating clearly with others as we describe a situation precisely.

Pairs | 15 min

Activity 2 Comparing Using Function Notation

Purpose: Students use statements written in function notation to compare the graphs of two functions.

1 Launch

Invite students to practice reading the statement $n(11) > p(11)$ aloud and describe what they think it means for Nekeisha and Polina's space race.

ELL **English Language Learners** Consider using a think-aloud to model how to use the graph to see that $n(11) > p(11)$. **(Reading, Speaking, and Listening)**

Consider asking, "How can function notation help us describe key features of graphs precisely, without using words?"

2 Monitor

Encourage students to show their thinking.

D **Differentiation: (Support)** Look for students who switch inputs and outputs on the graph. Consider asking, "Where do we find 15 seconds on the graph? How can the labels on the axes help us know whether 15 is an input or an output for this graph?"

To support students getting started, encourage students to enter a random value for t and consider what location it represents on the graph.

Look for students who use interesting values (e.g., 13.9999999 or $\frac{27}{2}$) that satisfy $n(t) > p(t)$.

Pause briefly to discuss how students might describe all the numbers that would make the statement $n(t) > p(t)$ true. Consider asking, "How can we tell from the graph that any value between 3 and 14 makes $n(t) > p(t)$ true? "

Note: Students will be introduced to using compound inequalities to write intervals to describe the domain and range in Lessons 9–10. Consider using this as an opportunity to preview student thinking rather than engage in a deep conceptual learning moment.

Activity 2 continued >

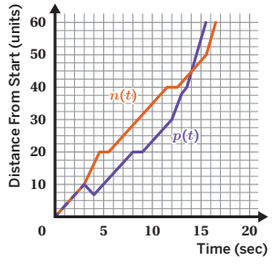
Activity 2

Name: _____ Date: _____ Period: _____

Comparing Using Function Notation

6. Nekeisha traveled farther than Polina after 11 seconds.

You can express that by writing $n(11) > p(11)$.



a. **Discuss:** How can you use the graph to tell that $n(11) > p(11)$?

Responses vary.

- At 11 seconds, Nekeisha is around 37 or 38 units from the start, while Polina is only 25 units from the start.
- If you draw a vertical line at $t = 11$, the graph of $n(t)$ intersects the line at a higher y-value than the graph of $p(t)$.

b. Which statement could you use to show who traveled farther after 15 seconds?

A. $n(15) > p(15)$

B. $n(15) = p(15)$

C. $p(15) \geq n(15)$

D. $p(15) > n(15)$

7. We know that $n(11) > p(11)$.

What is a different value of t for which $n(t) > p(t)$?

Responses vary. Any value of t greater than 3 seconds and less than 14 seconds is considered correct.

Unit 3 Lesson 8 343 Activity 2

Pairs | 15 min

Activity 2 Comparing Using Function Notation (continued)

Purpose: Students use statements written in function notation to compare the graphs of two functions.

2 Monitor

Pause briefly to discuss how to compare two graphs using the points of intersection by asking:

- "How can we see on the graph that $n(3) = p(3)$ and $n(14) = p(14)$ are true statements?"
- "When two functions are equal, what will their graphs look like?"
- "If the graph of two functions intersect when they are equal, what does it mean for $n(t) < p(t)$ and how will it look on the graph?"

3 Connect

Invite students to share their graphs and describe the story of Valeria and Zion's space race using key features.

MLR **MLR7: Compare and Connect** Invite students to compare different graphs, and then connect to their own. Discuss each of the function notation statements, supporting students in connecting each statement to a graph. See the Routine Facilitation Guide for more information.

Consider asking:

- "Where do you see the same information in ____ and ___'s graph?"
- "How can we see the moment $v(13) = z(13)$ represented on the graph? What is happening in the space race at that moment?" At 13 seconds, Valeria and Zion's spaceships are at the exact same distance from start.
- "Which of these statements will look the same on every graph? Which statements will look different?" $v(10) = 20$ is the only statement that will look the same on every graph.

Activity 2

Name: _____ Date: _____ Period: _____

Comparing Using Function Notation (continued)

8. What is a value of t for which $n(t) = p(t)$?

Responses vary. Answers in the interval $0 \le x \le 3$ and 14 are considered correct.

Describe what was happening in the race at that moment.

Descriptions vary. At that moment, Nekeisha and Polina were the same distance from the start.

9. Valeria and Zion also raced their spaceships.

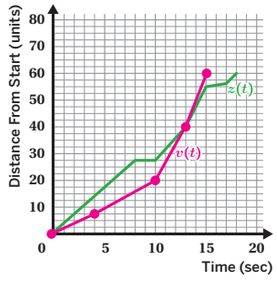
$v(t)$ and $z(t)$ represents the distances of Valeria's and Zion's spaceships after t seconds.

Here is some information about their races:

- $v(0) = z(0)$
- $v(4) < z(4)$
- $v(10) = 20$
- $v(13) = z(13)$
- $v(15) > z(15)$

Make a graph that could represent Valeria's distance traveled after t seconds.

Graphs vary. Sample shown.



Unit 3 Lesson 8 344 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to use function notation to compare the key features of graphs.

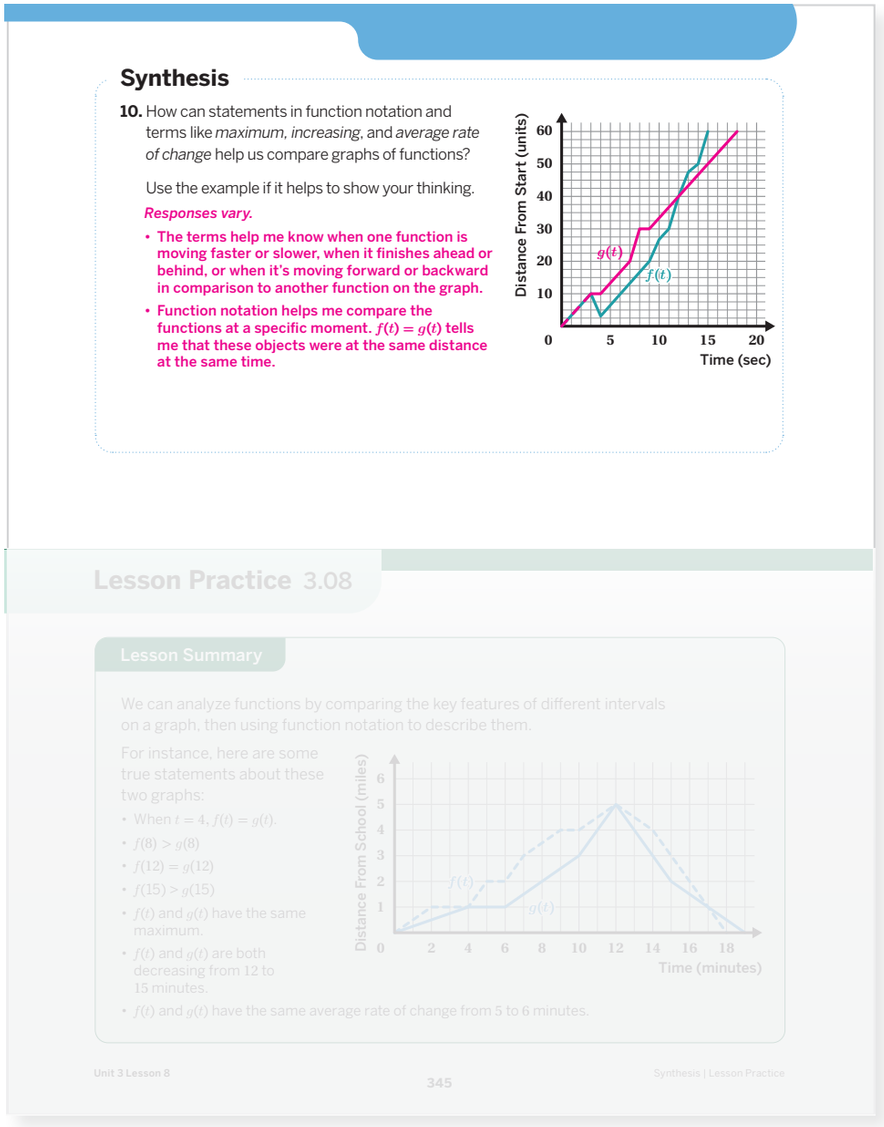
Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:

- Key features help us describe what is happening in the situation as we compare two graphs.
- The maximum or minimum describes the beginning or end.
- The average rate of change describes when one function is changing faster or slower compared to the other.
- Points of intersection describe when the functions are at the same value at the same time.
- Statements in function notation help us describe how one graph is moving in relation to the other.

Consider asking, "What does each of these terms tell you about the moment it describes on the graph?"

Note: If time allows, invite students to work with a partner to describe the graphs of the two functions as precisely as they can. Invite them to make their response stronger and clearer using statements written in function notation. **(MTR.4.1)**



Synthesis

10. How can statements in function notation and terms like *maximum*, *increasing*, and *average rate of change* help us compare graphs of functions?

Use the example if it helps to show your thinking.

Responses vary.

- The terms help me know when one function is moving faster or slower, when it finishes ahead or behind, or when it's moving forward or backward in comparison to another function on the graph.
- Function notation helps me compare the functions at a specific moment. $f(t) = g(t)$ tells me that these objects were at the same distance at the same time.

Lesson Practice 3.08

Lesson Summary

We can analyze functions by comparing the key features of different intervals on a graph, then using function notation to describe them.

For instance, here are some true statements about these two graphs:

- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

Unit 3 Lesson 8 345 Synthesis | Lesson Practice

Lesson Takeaway: Key features of graphs can be used to describe what is happening in a situation over intervals or at specific moments. Function notation helps us describe functions quantitatively at specific values or intervals.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using a graph to determine which statements written in function notation are true.

Today's Goals

1. **Goal:** Interpret statements written in function notation and use them to compare key features of graphs.
2. **Language Goal:** Compare and contrast key features of functions using statements written in function notation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

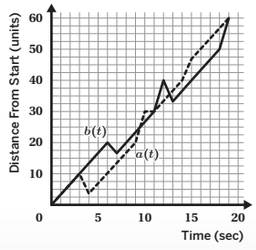
Name: _____ Date: _____ Period: _____

Show What You Know 3.08

$a(t)$ and $b(t)$ give the distance of two racers after t seconds.

Select *all* of the true statements.

- ☐ A. $a(2) > b(2)$
- ☑ B. $a(t)$ and $b(t)$ have the same maximum.
- ☐ C. $b(20) = 6$
- ☑ D. $a(5) < b(5)$
- ☑ E. $a(t)$ and $b(t)$ are both increasing from 14 to 16 seconds.



D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support interpreting statements written in function notation:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related **Practice Problems**.
- Use the *Calculating Slope Given Two Points* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Revisit **Activity 2, Problem 9** and invite students to create a new graph to compare Valeria's and Zion's graphs to their first design. Encourage them to discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on how students engaged in attending to precision in today's lesson. How did students use function notation as a tool for making their communication about comparing functions and their graphs more precise? How can you continue to support them in viewing function notation as a communication tool in this unit?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How can statements in function notation and terms like *maximum*, *increasing*, and *average rate of change* help us compare graphs of functions?

Use the example if it helps to show your thinking.

Responses vary.

- The terms help me know when one function is moving faster or slower, when it finishes ahead or behind, or when it's moving forward or backward in comparison to another function on the graph.
- Function notation helps me compare the functions at a specific moment. $f(t) = g(t)$ tells me that these objects were at the same distance at the same time.

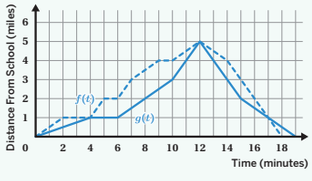
Lesson Practice 3.08

Lesson Summary

We can analyze functions by comparing the key features of different intervals on a graph, then using function notation to describe them.

For instance, here are some true statements about these two graphs:

- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

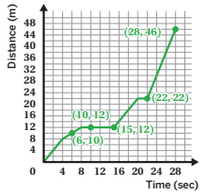


Lesson Practice 3.08

Name: ..... Date: ..... Period: .....

Problems 1–2: Mai built a model race car for a school competition.

$m(t)$ represents the distance of Mai's car, in meters, after t seconds.



1. Use the graph to determine the missing value in each function statement.

$m(\underline{6}) = 10$

$m(10) = \underline{12}$

$m(22) = \underline{22}$

$m(\underline{28}) = 46$

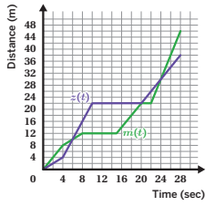
2. Over what interval did Mai's car travel the slowest?

A. 0 to 4 sec B. 4 to 8 sec (C.) 8 to 15 sec D. 15 to 20 sec

Problems 3–6: Zion also built a model race car for the school competition.

$z(t)$ represents the distance of Zion's car, in meters, after t seconds.

Did Zion or Mai have the greater average rate of change over the following intervals? Explain your thinking.



3. $t = 4$ to $t = 8$

Zion. *Explanations vary.* The average rate of change is 3 meters per second for Zion's car and 1 meter per second for Mai's car.

4. $t = 10$ to $t = 20$

Mai. *Explanations vary.* The average rate of change is 1 meter per second for Mai's car and 0 meters per second for Zion's car.

5. Name a time when Zion's and Mai's cars had traveled the same distance.

0 seconds, 6 seconds, 20 seconds, or 24 seconds

6. Select *all* the true statements.

☑ A. $m(t)$ has a greater maximum than $z(t)$.
☐ B. $z(t)$ and $m(t)$ have the same minimum at (6, 10).
☑ C. $z(20) = m(20)$
☐ D. $m(15) > z(15)$
☑ E. $m(t)$ and $z(t)$ both increase from 22 to 28 sec.

Lesson Practice 3.08

Name: ..... Date: ..... Period: .....

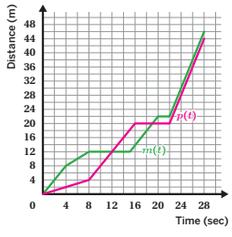
Test Practice

Problems 7–8: Parv built a race car to race against Mai.

7. Use this information to make a graph that could represent the distance of Parv's race car, $p(t)$, after t seconds:

- $p(8) < m(8)$
- $p(12) = m(12)$
- The average rate of change of $p(t)$ and $m(t)$ is the same from $t = 22$ to $t = 28$.
- $m(t)$ has a greater maximum than $p(t)$.

Graphs vary. Sample shown.



8. Which of the following equations is true?

A. $m(4) = 2$
(B.) $m(8) = 12$
C. $m(12) = 14$
D. $m(22) = 20$

Spiral Review

Problems 9–11: Nekeisha goes for a bike ride. $d(t)$ represents Nekeisha's distance from home, in miles, t minutes after she leaves.

Explain the meaning of each statement in context:

$d(0) = 0$	Nekeisha starts her bike ride at home.
$d(30) = d(60)$	Nekeisha is the same distance from home 30 minutes and 60 minutes into her bike ride.
$d(90) = 0$	Nekeisha is back home 90 minutes into her bike ride.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 6	2	MA.912.F.1.2
	2–4	2	MA.912.F.1.3
	5	2	MA.912.F.1.5
TEST Practice	7–8	2	MA.912.F.1.5, MA.912.F.1.2
Spiral Review			
Fluency	9–11	2	MA.912.F.1.2

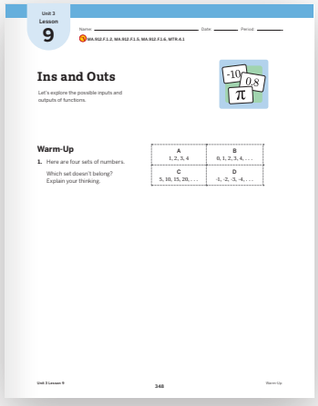
Unit 3

Lesson 9

Ins and Outs

Introducing Domain and Range

Let's explore the possible inputs and outputs of functions.


Unit 3
Lesson
9
Name _____ Date _____ Period _____
MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.4.1
Ins and Outs
Let's explore the possible inputs and outputs of functions.
-10
0.8
π
Warm-Up
1. Here are four sets of numbers.
Which set doesn't belong? Explain your thinking.
A
1, 2, 3, 4
B
0, 1, 2, 3, 4, . . .
C
5, 10, 15, 20, . . .
D
-1, -2, -3, -4, . . .
Unit 3 Lesson 9
348
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Determine if a value is a possible input or output for a function given in context.
2. **Language Goal:** Describe the *domain* and *range* of a function as the set of all possible inputs and outputs. **(Writing and Speaking)**

Students are introduced to the concept of **domain** and **range** as the set of all possible inputs and outputs. They use words to describe the domain and range for situations and graphs, considering what values are reasonable or unreasonable based on the context of a situation. Students use their understanding of domain and range to read, describe, and create graphs.

Coherence

Prior Learning

In Grade 8, students were introduced to the concept of functions when they used the terms inputs and outputs to refer to values within the domain and range. In Lesson 1, students revisited the idea of functions as they investigated the relationship between inputs and outputs to make sense of rules.

Future Learning

In Unit 5, students will use inequalities to describe the domain and range of a graph. In Lesson 10, students will use inequalities to restrict the domain and range of a given function. In Lesson 11, students will apply their knowledge of domain and range to new function types, including absolute value.

Connections

- Students explore the domain and range of real world situations such as number of hours you can rent a bike and the number of toppings that can be ordered for a pizza. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

MA.912.F.1.5

Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.3	MA.912.F.1.6

Vocabulary

New Vocabulary

domain	range

Review Vocabulary

dependent variable	*independent variable*

Additional Vocabulary

discrete

Lesson at a Glance

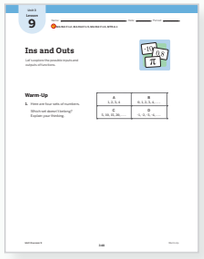
~ 45 min

MA.912.F.1.2, MA.912.F.1.5

Warm-Up

Whole Class | 5 min

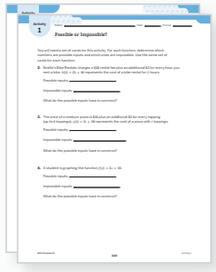
Students use the **Which One Doesn't Belong?** routine to make sense of sets of numbers.



Activity 1

Pairs | 15 min

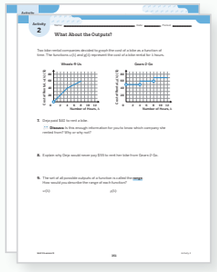
Students are introduced to the term **domain** by considering whether inputs are possible or impossible for three different situations.
Routine: Think-Pair-Share



Activity 2

Pairs | 15 min

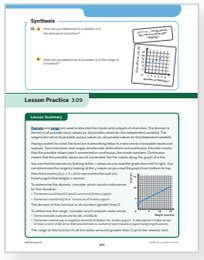
Students are introduced to the term **range** as they consider graphs of functions that model the price to rent a bike for two bike rental companies.



Synthesis

Whole Class | 5 min

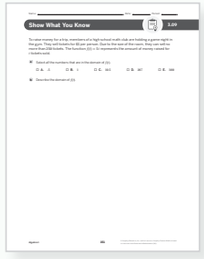
Students describe how to decide if a number is in the domain or range of a function.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by deciding if several numbers are in the domain of a function and precisely describing the domain of that function with words. **(MTR.4.1)**



Prep Checklist

Copy Activity 1 Cards, one set per pair, pre-cut.

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to make sense of sets of numbers.

1 Launch

Write out the sets of numbers and the prompt.

Invite students to think of one set of numbers that does not belong and why.

Use the Which One Doesn't Belong? routine to support students in making sense of which set of numbers does not belong.

Consider sharing that each set of numbers does not belong in some way. Encourage students to look for more than one possibility.

ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____ is the only one that has/doesn't have_____."). **(Writing and Speaking)**

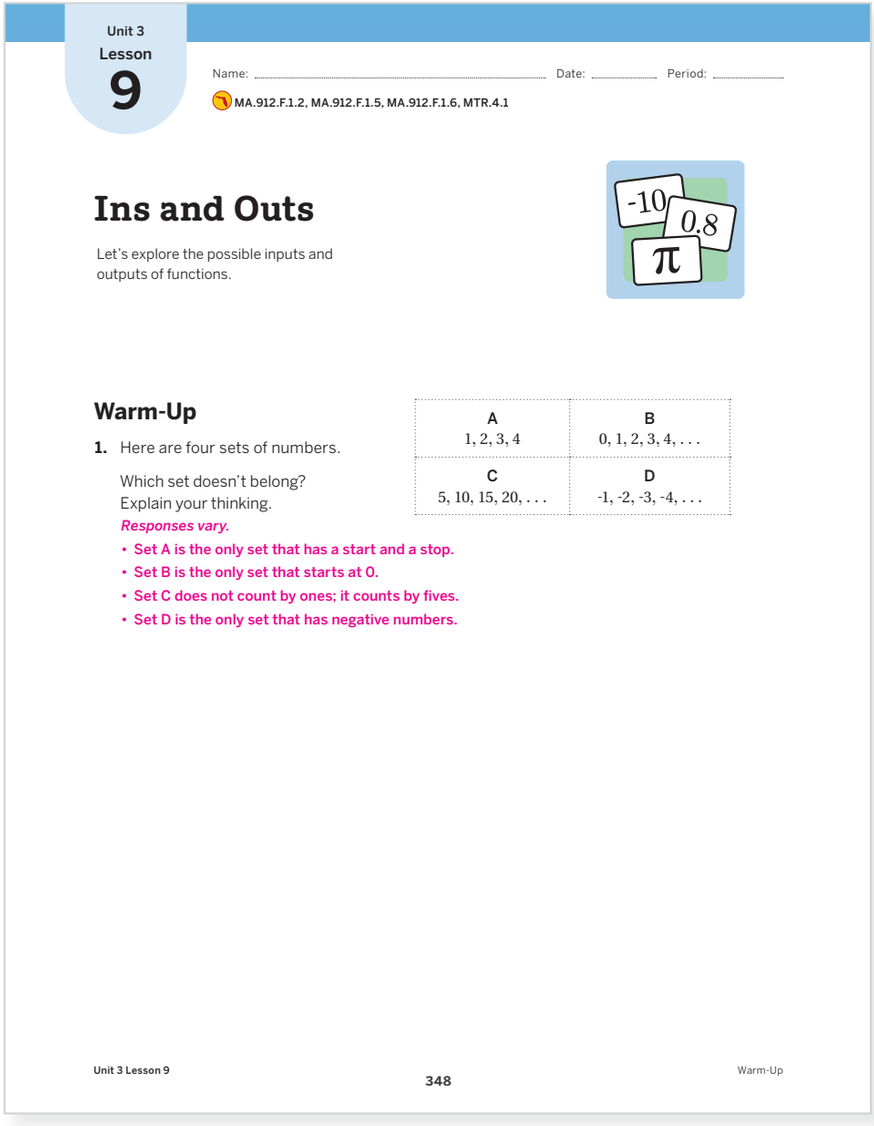
2 Connect

Invite students to share at least one reason why each set of numbers doesn't belong.

Note: At this point in the lesson, the focus is on the language that students use to describe sets of numbers, rather than terms used to describe sets of numbers (e.g., *rational*, *imaginary*, *real*).

Consider asking:

- "How would you describe this set?"
- "How is this set like the other sets? How is it different?"
- "What is another way to say why this set doesn't belong?"
- "Is there another reason why this set doesn't belong?"



Unit 3
Lesson
9

Name: _____ Date: _____ Period: _____

MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.4.1

Ins and Outs

Let's explore the possible inputs and outputs of functions.

Warm-Up

1. Here are four sets of numbers.

A	B
1, 2, 3, 4	0, 1, 2, 3, 4, . . .
C	**D**
5, 10, 15, 20, . . .	-1, -2, -3, -4, . . .

Which set doesn't belong?
Explain your thinking.

Responses vary.

- Set A is the only set that has a start and a stop.
- Set B is the only set that starts at 0.
- Set C does not count by ones; it counts by fives.
- Set D is the only set that has negative numbers.

Unit 3 Lesson 9 348 Warm-Up

Pairs | 15 min

Activity 1 Possible or Impossible?

Purpose: Students are introduced to the term **domain** by considering whether inputs are possible or impossible for three different situations.

1 Launch

Write out the image and the prompt.

Consider asking, "Which variable is *independent* in a bike rental situation? Which is *dependent*? How do you know?"

ELL **English Language Learners** Invite students to read each scenario aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

Distribute one set of cards to each pair of students.

Note: Students will use the same set of cards for all three situations.

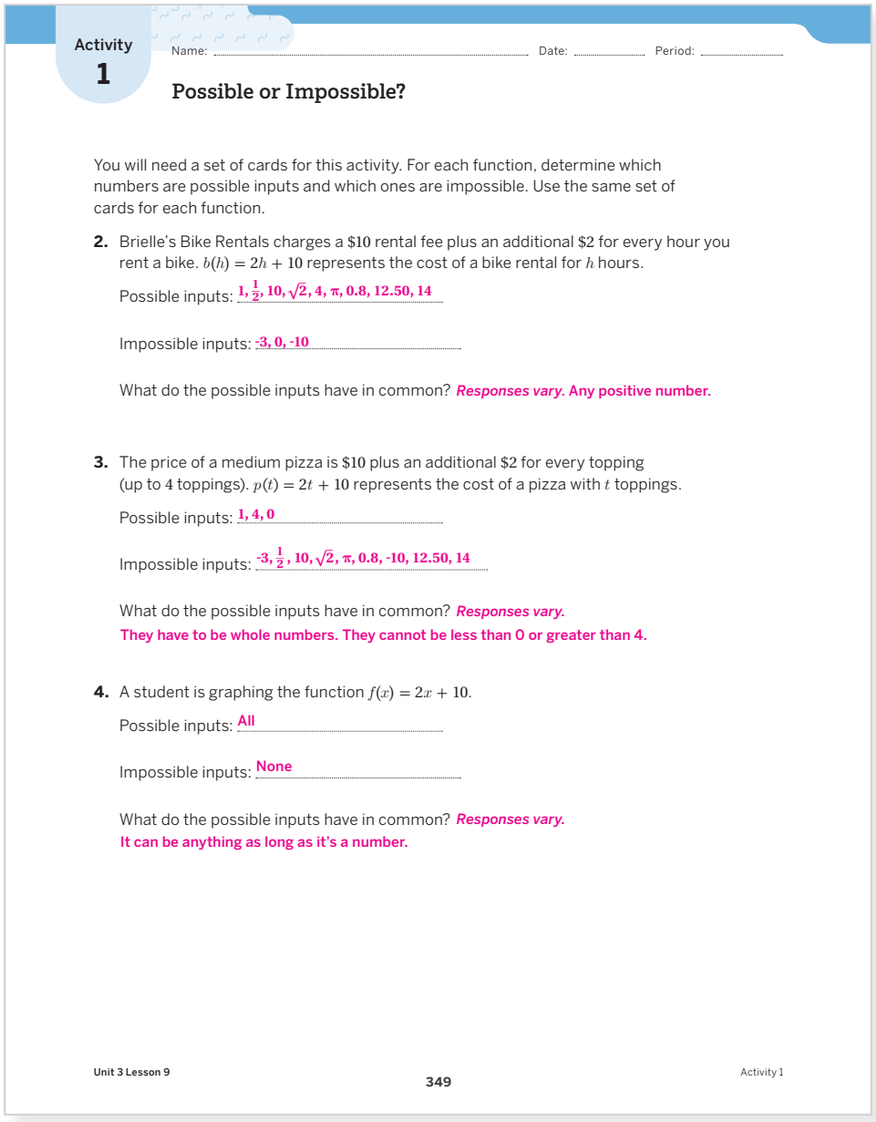
2 Monitor

Use the Think-Pair-Share routine to support student-to-student discourse about the situations. Invite students to think independently and then take turns sharing with a partner.

ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and invite them to share in that language. **(Writing, Speaking, and Listening)**

Note: Students will not be assessed on the difference between *discrete* and continuous situations in this unit. Consider sharing, however, that situations like pizza toppings where there is separation between the possible values that make sense (i.e., having a domain of whole numbers or a finite set of possible values) are discrete.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Possible or Impossible?

You will need a set of cards for this activity. For each function, determine which numbers are possible inputs and which ones are impossible. Use the same set of cards for each function.

2. Brielle's Bike Rentals charges a $10 rental fee plus an additional $2 for every hour you rent a bike. $b(h) = 2h + 10$ represents the cost of a bike rental for h hours.

Possible inputs: $1, \frac{1}{2}, 10, \sqrt{2}, 4, \pi, 0.8, 12.50, 14$

Impossible inputs: $-3, 0, -10$

What do the possible inputs have in common? *Responses vary.* Any positive number.

3. The price of a medium pizza is $10 plus an additional $2 for every topping (up to 4 toppings). $p(t) = 2t + 10$ represents the cost of a pizza with t toppings.

Possible inputs: $1, 4, 0$

Impossible inputs: $-3, \frac{1}{2}, 10, \sqrt{2}, \pi, 0.8, -10, 12.50, 14$

What do the possible inputs have in common? *Responses vary.*
They have to be whole numbers. They cannot be less than 0 or greater than 4.

4. A student is graphing the function $f(x) = 2x + 10$.

Possible inputs: All

Impossible inputs: None

What do the possible inputs have in common? *Responses vary.*
It can be anything as long as it's a number.

Unit 3 Lesson 9 349 Activity 1

Pairs | 15 min

Activity 1 Possible or Impossible? (continued)

Purpose: Students are introduced to the term **domain** by considering whether inputs are possible or impossible for three different situations.

2 Monitor

Select and sequence descriptions of the domains of the functions to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "If 10 is a possible input, what does that tell you about the set of all possible inputs?"
Construct a viable argument for why they think the descriptions of domains should be different based on the situations.	**Strengthen** Celebrate the way this student thinking exemplifies mathematical practices. Invite students to think about how they would communicate their argument or description to the class and invite them to share during the Connect. **(MTR.4.1)**
Begin to use more precise language when describing all the possible inputs, such as all positive whole numbers or $x > 0$.	
Would like to explore this topic further.	**Stretch** Invite students to think of other descriptions of domains and try to think of functions that could make sense for those domains. *Responses vary.* For the domain "All numbers less than 0," you could have a function that takes any negative number and makes it positive.

3 Connect

Invite students to share their or their partner's thinking with the class.

To reveal the Key Takeaway, consider asking:

- "Is 0 hours of a bike rental possible? Why or why not?"
- "How did you and your partner decide whether an input was possible or impossible for a situation? What evidence convinced you?" **(MTR.4.1)**
- "What's the difference between a possible input and a realistic input?"
- "How did you decide which domain matched which situation?"
- "If the three situations are all modeled by the same equation, why do you think they have different domains?"

ELL **English Language Learners** Invite students to research how to say the new term *domain* in other languages. Create an anchor chart with the term *domain* in multiple languages, along with examples. **(Reading and Listening)**

Activity 1

Name: _____ Date: _____ Period: _____

Possible or Impossible? (continued)

5. The set of all possible inputs of a function is called the **domain**.

Match each domain description with $p(t)$, $b(h)$, or $f(x)$. Be prepared to share your thinking.

Function	Domain
$f(x)$	All Numbers
$b(h)$	All Numbers Greater Than 0
$p(t)$	Whole Numbers From 0 to 4

6. Each of the scenarios $p(t)$, $b(h)$, and $f(x)$ can be modeled by the same linear relationship.

Discuss: Why are the domains of these three functions different from each other?

Responses vary. **The situations change the domain of each function. For example, you can plot $f(-3)$ on a graph, but it doesn't make sense to order a pizza with -3 toppings, or rent a bike for -3 hours.**

Unit 3 Lesson 9 350 Activity 1

Key Takeaway: The set of all possible inputs of a function is called the **domain**. The domain of a function changes based on the situation (for example, when whole or negative numbers do not make sense).

Pairs | 15 min

Activity 2 What About the Outputs?

Purpose: Students are introduced to the term **range** as they consider graphs of functions that model the price to rent a bike for two bike rental companies.

Short on time? Consider omitting Problem 11 and inviting students to discuss their graphs as a whole class instead.

1 Launch

Invite students to discuss what Deja should consider before renting a bike.

Invite students to randomly select a number from the set of cards used in Activity 1. Label one side of the room as "Possible" and the other "Impossible." Then invite students to move to either side of the room to indicate whether their number is a possible or impossible output for $w(h)$. Once they've moved, invite them to discuss with a partner and share their ideas with the class. **(MTR.4.1)**

A **Accessibility: Affective Functioning** Consider supporting students by pairing them with the card 10, which is a possible output for $w(h)$ (after 1 hour) but not for $g(h)$.

ELL **English Language Learners** As students process their ideas in words, invite them to talk through their thinking with a partner before sharing it with the class. **(Speaking)**

Note: As students discuss, they may change their minds about whether their number is possible and switch places.

2 Monitor

Listen for the ways students describe the set of all possible outputs for the situation.

D **Differentiation** *(Problems 7–9)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How are the ranges for Wheels-R-Us and Gears-2-Go the same? How are they different? Is it possible to have a bike rental that costs $50? For which company?"
Think there is not enough information to decide which company Deja rented from. (Problem 7)	**Support** Consider inviting students to trace up to 40 on the y-axis of each graph, then trace their finger horizontally towards the graph.
Begin to use more precise language.	**Strengthen** Celebrate their attention to precision and invite them to share during the Connect. **(MTR.4.1)**

Pause briefly and invite students to share their descriptions of the set of all possible outputs for $w(h)$ and $g(h)$. If there is no consensus, invite students to make arguments for what they think the ranges are for $w(h)$ and $g(h)$. **(MTR.4.1)**

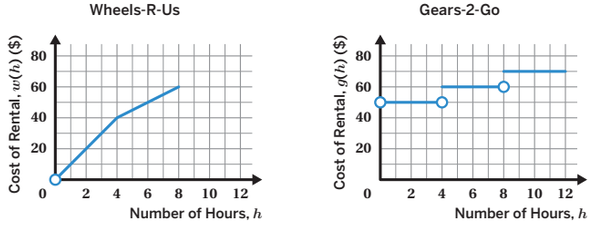
Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

What About the Outputs?

Two bike rental companies decided to graph the cost of a bike as a function of time. The functions $w(h)$ and g(h) represent the cost of a bike rental for h hours.



7. Deja paid $40 to rent a bike.

Discuss: Is this enough information for you to know which company she rented from? Why or why not?

Yes. Explanations vary. The graph of $w(h)$ shows that a 4-hour rental will cost $40. The graph of $g(h)$ does not show any outputs that are $40.

8. Explain why Deja would never pay $55 to rent her bike from Gears-2-Go.

Responses vary. She will never pay $55 because $55 is not a possible output for $g(h)$.

9. The set of all possible outputs of a function is called the **range**. How would you describe the range of each function?

$w(h)$: *Responses vary.* The range of $w(h)$ is all the numbers from 0 to 60.

$g(h)$: *Responses vary.* The range of $g(h)$ is 50, 60, and 70.

Unit 3 Lesson 9 351 Activity 2

Pairs | 15 min

Activity 2 What About the Outputs? (continued)

Purpose: Students are introduced to the term **range** as they consider graphs of functions that model the price to rent a bike for two bike rental companies.

Short on time? Consider omitting Problem 11 and inviting students to discuss their graphs as a whole class instead.

2 Monitor

Select and sequence interesting graphs that students make for Problem 10 to share during the Connect.

D Differentiation *(Problems 10–11)*

Look for students who:	Respond to Student Thinking
Create a graph that does not match the domain and range in the descriptions.	**Support** Consider asking, "What is the domain and range of your sketch?"
Create a step graph like $g(h)$ or another discrete graph that omits values.	**Strengthen** Celebrate their inclusion of whole numbers from the description and consider sharing that 0.5 fits the description for both the domain and the range. Consider asking, "How could you make sure to include *all* numbers in the descriptions of your graph?"
Would like an additional challenge.	**Stretch** Invite students to write their own description of a domain and range and swap with a partner to sketch a function that matches their descriptions.

3 Connect

Invite students to sketch their graph and share how they know it matches the descriptions.

To surface the Key Takeaway, consider asking:

- "What can we learn about a graph from the domain and range? Is there anything we cannot determine about a graph if we only know the domain and range?" *Responses vary.* They provide all the possible inputs and outputs but they do not say anything about the shape of the graph, where it is increasing or decreasing, etc.
- "How are the range and domain of a function alike? How are they different?" *Responses vary.* They are both sets of numbers, but the domain is all possible inputs and the range is all possible outputs.

ELL English Language Learners Invite students to research how to say the new term *range* in other languages. Create an anchor chart with the term *range* in multiple languages, along with examples. **(Reading and Listening)**

Activity 2

Name: _____ Date: _____ Period: _____

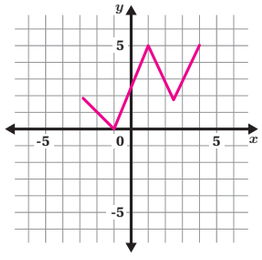
What About the Outputs? (continued)

Here is the domain and range of a function:

- Domain: All numbers from -3 to 4.
- Range: All numbers from 0 to 5.

10. Sketch a graph that matches the domain and range.

Graphs vary.



11. Compare your graph with a partner.

Discuss:

- What's the same about your graphs?
- What's different?

Responses vary.

- Both of our graphs have inputs from -3 to 4 and outputs from 0 to 5.
- My graph has increasing and decreasing segments. My partner's graph only has increasing segments.

Unit 3 Lesson 9 352 Activity 2

Lesson Takeaway: The set of all possible outputs of a function is called the **range**. Like the domain, the range can depend on the situation and sometimes has restricted values, such as the range of the Gears-2-Go cost function $g(h)$ only having three numbers in it.

Whole Class | 5 min

Synthesis

Purpose: Students describe how to decide if a number is in the domain or range of a function.

Invite students to respond independently, and then share their thinking with a partner.

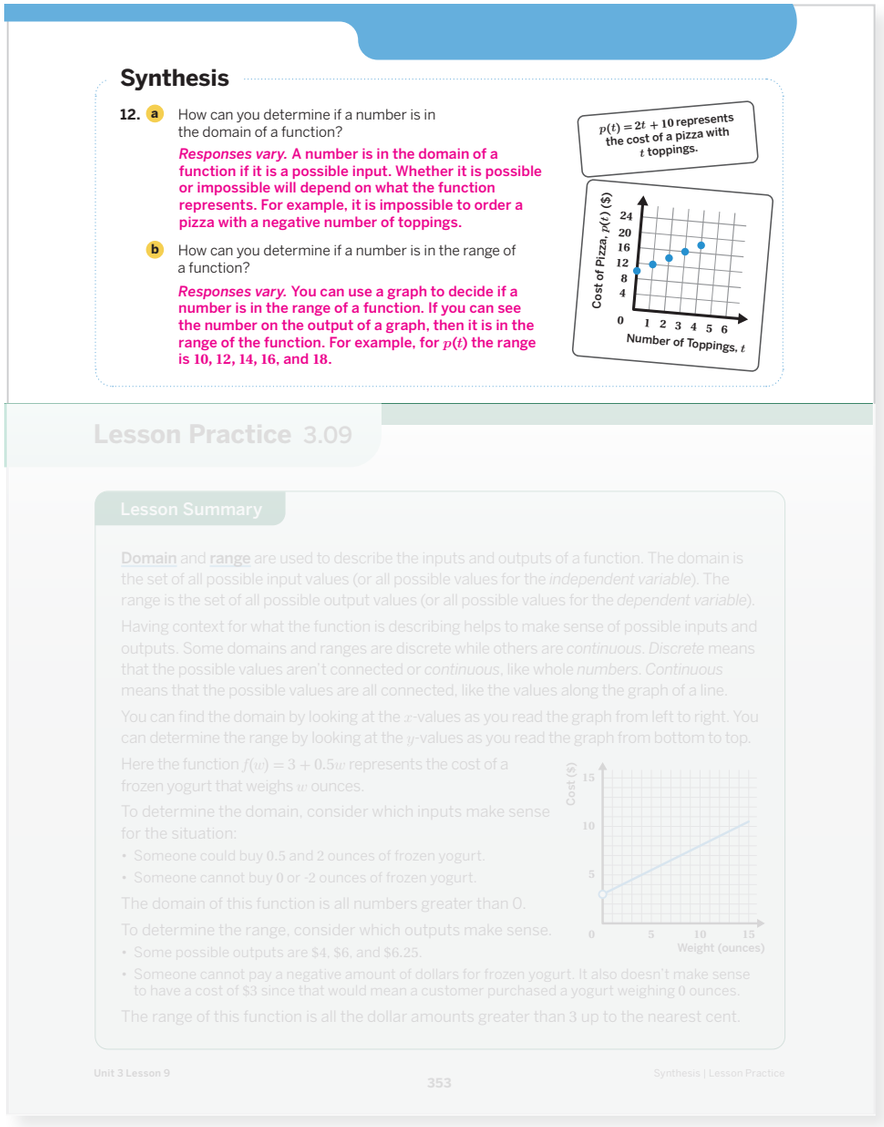
ELL **English Language Learners** As students process their ideas in words, invite them to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

Collect and share a variety of ideas, such as:

- Using the context of the situation to describe the domain or range (e.g., number of toppings can be described by positive integers).
- Using the graph to describe the domain or range (e.g., points on the graph tell me that the number of toppings and the cost must both be whole numbers).

MLR **MLR2: Collect and Display** Use this routine to create the class definitions of **domain** and **range** using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. a How can you determine if a number is in the domain of a function?

Responses vary. A number is in the domain of a function if it is a possible input. Whether it is possible or impossible will depend on what the function represents. For example, it is impossible to order a pizza with a negative number of toppings.

b How can you determine if a number is in the range of a function?

Responses vary. You can use a graph to decide if a number is in the range of a function. If you can see the number on the output of a graph, then it is in the range of the function. For example, for $p(t)$ the range is 10, 12, 14, 16, and 18.

$p(t) = 2t + 10$ represents the cost of a pizza with t toppings.

Lesson Practice 3.09

Lesson Summary

Domain and range are used to describe the inputs and outputs of a function. The domain is the set of all possible input values (or all possible values for the *independent variable*). The range is the set of all possible output values (or all possible values for the *dependent variable*).

Having context for what the function is describing helps to make sense of possible inputs and outputs. Some domains and ranges are discrete while others are *continuous*. *Discrete* means that the possible values aren't connected or *continuous*, like whole *numbers*. *Continuous* means that the possible values are all connected, like the values along the graph of a line.

You can find the domain by looking at the x-values as you read the graph from left to right. You can determine the range by looking at the y-values as you read the graph from bottom to top.

Here the function $f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.

To determine the domain, consider which inputs make sense for the situation:

- Someone could buy 0.5 and 2 ounces of frozen yogurt.
- Someone cannot buy 0 or -2 ounces of frozen yogurt.

The domain of this function is all numbers greater than 0.

To determine the range, consider which outputs make sense.

- Some possible outputs are $4, $6, and $6.25.
- Someone cannot pay a negative amount of dollars for frozen yogurt. It also doesn't make sense to have a cost of $3 since that would mean a customer purchased a yogurt weighing 0 ounces.

The range of this function is all the dollar amounts greater than 3 up to the nearest cent.

Unit 3 Lesson 9 353 Synthesis | Lesson Practice

Lesson Takeaway: The set of all possible inputs of a function is called the **domain**. The set of all possible outputs of a function is called the **range**. When describing the domain and range of a function, it is important to consider how the situation changes what inputs and outputs are possible.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by deciding if several numbers are in the domain of a function and precisely describing the domain of that function with words. **(MTR.4.1)**

Today's Goals

1. **Goal:** Determine if a value is a possible input or output for a function given in context.
2. **Language Goal:** Describe the *domain* and *range* of a function as the set of all possible inputs and outputs. **(Writing and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.09

To raise money for a trip, members of a high school math club are holding a game night in the gym. They sell tickets for $5 per person. Due to the size of the room, they can sell no more than 250 tickets. The function $f(t) = 5t$ represents the amount of money raised for t tickets sold.

a. Select *all* the numbers that are in the domain of $f(t)$.

☐ A. -5 ☑ B. 1 ☐ C. 10.5 ☑ D. 207 ☐ E. 300

b. Describe the domain of $f(t)$.

Responses vary. The domain is all whole numbers from 0 to 250.

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support describing the domain and range of functions:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related **Practice Problems**.
- Consider spending adequate time discussing strategies for describing domain and range in **Lesson 10, Activity 1**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. Students engage in modeling as they define quantities and consider how to measure them. What evidence did you see of students engaging in the modeling process in this lesson? How did considering possible and impossible inputs and outputs support students in making sense of quantities set in a context?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. a. How can you determine if a number is in the domain of a function?

Responses vary. A number is in the domain of a function if it is a possible input. Whether it is possible or impossible will depend on what the function represents. For example, it is impossible to order a pizza with a negative number of toppings.

b. How can you determine if a number is in the range of a function?

Responses vary. You can use a graph to decide if a number is in the range of a function. If you can see the number on the output of a graph, then it is in the range of the function. For example, for $p(t)$ the range is 10, 12, 14, 16, and 18.

Lesson Practice 3.09

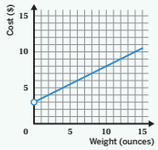
Lesson Summary

Domain and **range** are used to describe the inputs and outputs of a function. The domain is the set of all possible input values (or all possible values for the *independent variable*). The range is the set of all possible output values (or all possible values for the *dependent variable*).

Having context for what the function is describing helps to make sense of possible inputs and outputs. Some domains and ranges are discrete while others are *continuous*. *Discrete* means that the possible values aren't connected or *continuous*, like whole *numbers*. *Continuous* means that the possible values are all connected, like the values along the graph of a line.

You can find the domain by looking at the x-values as you read the graph from left to right. You can determine the range by looking at the y-values as you read the graph from bottom to top.

Here the function $f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.



To determine the domain, consider which inputs make sense for the situation:

- Someone could buy 0.5 and 2 ounces of frozen yogurt.
- Someone cannot buy 0 or -2 ounces of frozen yogurt.

The domain of this function is all numbers greater than 0.

To determine the range, consider which outputs make sense.

- Some possible outputs are $4, $6, and $6.25.
- Someone cannot pay a negative amount of dollars for frozen yogurt. It also doesn't make sense to have a cost of $3 since that would mean a customer purchased a yogurt weighing 0 ounces.

The range of this function is all the dollar amounts greater than 3 up to the nearest cent.

Lesson Practice 3.09

Name: _____ Date: _____ Period: _____

Problems 1–2: Aniyah is working at the mall wrapping presents during the holidays. She earns $2 for each present she wraps. The function $f(x) = 2x$ represents Aniyah's daily earnings for x number of presents wrapped.

1. Are the values 2.3, -10, and 0.5 possible inputs for this situation? Explain your thinking.

No. *Explanations vary.* They are not possible inputs for this situation because the input represents the number of presents wrapped. 2.3 and 0.5 represent partial presents wrapped, and in this scenario, you can't have partial presents. -10 does not work because Aniyah can't wrap negative presents.

2. Why is $24.50 an impossible daily earning amount for Aniyah?

An output of $24.50 is impossible in this scenario because all outputs are multiples of 2 and $24.50 is not a multiple of 2.

Problems 3–6: A concert is being held at a local venue. Tickets are $25 per person. The concert venue has a maximum capacity of 500 people. The function $f(t) = 25t$ represents the amount of money raised for t tickets sold.

3. Select *all* the possible values in the domain of $f(t)$.

☐ A. -0.1 ☑ B. 25 ☐ C. 3.7 ☐ D. 525 ☑ E. 173

4. Describe the domain of $f(t)$.

Responses vary. The domain is the number of tickets sold. Since the venue has a maximum capacity of 500, the domain is all whole numbers from 0 up to a maximum of 500.

5. Select all the possible values in the range of $f(t)$.

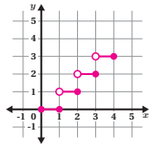
☐ A. -25 ☑ B. 0 ☑ C. 25 ☐ D. 860 ☑ E. 12,500

6. Describe the range of $f(t)$.

Responses vary. The range is the amount of money raised for t tickets sold. The range will be in multiples of 25, starting with 0 and ending with 12,500.

7. Sketch a graph of $f(x)$ that meets the following conditions:

- Domain: All numbers from 0 to 4.
- Range: All whole numbers from 0 to 3.



Lesson Practice 3.09

Name: _____ Date: _____ Period: _____

Test Practice

8. Choose the correct domain for the graph of the linear function: $p(t) = 10t + 1$.

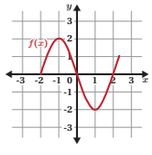
(A.) All numbers
B. Whole numbers from 0 to 5
C. All numbers from 0 to 24
D. All numbers from 1 to 10

Spiral Review

9. Use the equations for $f(x)$ and $g(x)$ to determine the outputs for each input.

	$x = -5$	$x = 0$	$x = 7$
$f(x) = 2x + 5$	-5	5	19
$g(x) = -x + 2$	7	2	-5

10. Emmanuel is describing the graph of $f(x)$. Select *all* the true statements.

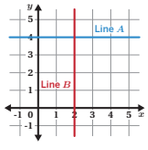


☑ A. The minimum of $f(x)$ is at $(1, -2)$.
☐ B. $f(0) = -2$
☐ C. The graph is decreasing for x, where $x \geq 1$.
☑ D. $f(0) > f(1)$
☑ E. The maximum of $f(x)$ is at $(-1, 2)$.

11. Write the equation for each line.

Line *A*: $y = 4$

Line *B*: $x = 2$



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.F.1.2
	3, 5	2	MA.912.F.1.5 MA.912.F.1.2
	4, 6	2	MA.912.F.1.5
	7	3	MA.912.AR.5.6
TEST Practice	8	1	MA.912.F.1.5
Spiral Review			
	9	1	MA.912.F.1.2
Fluency	10	1	MA.912.F.1.2, MA.912.F.1.6
	11	1	MA.8.AR.3.3

Unit 3

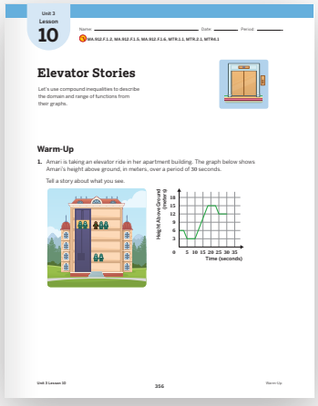
Lesson

10

Elevator Stories

Describing Domain and Range With Inequalities

Let's use compound inequalities to describe the domain and range of functions from their graphs.


Unit 3
Lesson
10
Name _____ Date _____ Period _____
MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.2.1, MTR.4.1
Elevator Stories
Let's use compound inequalities to describe the domain and range of functions from their graphs.
Warm-Up
1. Amari is taking an elevator ride in her apartment building. The graph below shows Amari's height above ground, in meters, over a period of 30 seconds.
Tell a story about what you see.
Height Above Ground (meters)
Time (seconds)
356
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Write the domain and range of a function using inequalities.
2. **Language Goal:** Interpret the meaning of the domain and range in context. **(Writing and Speaking)**

Students are introduced to **compound inequalities** as a way to describe the domain or range of a function from its graph. They also interpret the domain and range in the context of a situation. **(MTR.2.1)**

Coherence

Prior Learning

In Unit 1 Lesson 13, students were introduced to inequalities as a way to represent real-world situations. In the previous lesson, students were introduced to domain and range as the set of all possible inputs and outputs for a function, and described the domain and range using words.

Future Learning

In Lesson 11, students will evaluate and interpret outputs for absolute value functions.

Connections

- Students use compound inequalities to represent intervals of the domain and range of an elevator's height above ground as a function of time. (cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

MA.912.F.1.5
Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.

MA.912.F.1.6
Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.912.F.1.2	MA.912.AR.7.3
MA.912.AR.2.5	MA.912.AR.1.1

Vocabulary

New Vocabulary

compound inequality

Lesson at a Glance

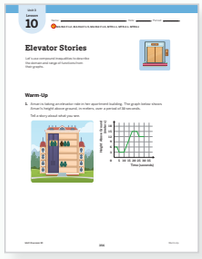
~ 45 min

MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6

Warm-Up

Independent | 5 min

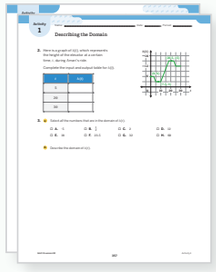
Students express their creativity by using the Tell a Story routine to make sense of the context of Amari's elevator ride used throughout this lesson. **(MTR.1.1)**



Activity 1

Pairs | 15 min

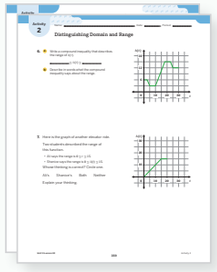
Students use words and compound inequalities to precisely describe the domain of a function from its graph. **(MTR.2.1)**



Activity 2

Pairs | 15 min

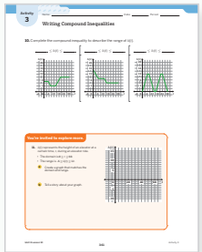
Students distinguish between describing the domain and the range with compound inequalities and interpret compound inequalities in context. **Routine:** Decide and Defend



Activity 3

Pairs | 10 min

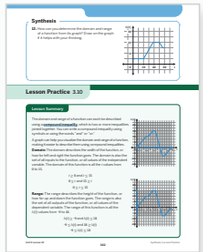
Students develop fluency writing compound inequalities to describe the range of a graph.



Synthesis

Whole Class | 5 min

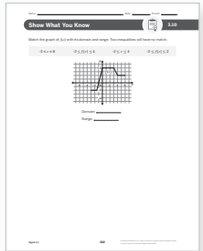
Students synthesize strategies for determining the domain and range of a function from its graph. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the compound inequalities for the domain and range of a given graph.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students express their creativity by using the Tell a Story routine to make sense of the context of Amari's elevator ride used throughout this lesson. **(MTR.1.1)**

1 Launch

Use the Tell a Story routine to help students make sense of the context. Invite students to tell a story about an elevator ride using their unique experiences, perspectives, and creativity. **(MTR.1.1)**

A **Accessibility: Executive Functioning** Consider displaying transitional words to use (e.g., first, then, next, finally, etc.).

ELL **English Language Learners** Invite students to tell the story in their primary language and/or create a list of words about the context (e.g., elevator, car, enter, exit, person, floor, up, down, etc.), along with visuals or translations of each word in languages used by your students. **(Writing, Reading, Speaking)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

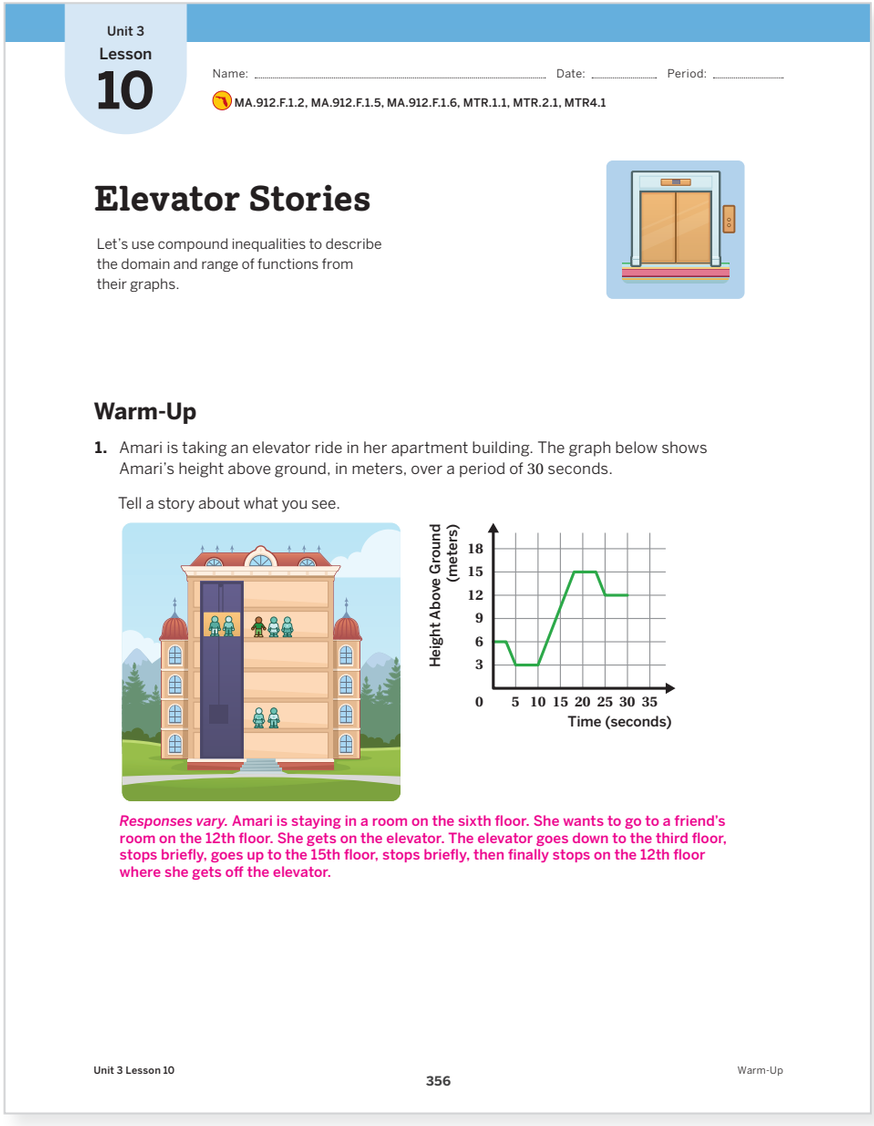
2 Connect

Invite several students to share their stories, encouraging them to make connections to the graph.

Encourage students to ask questions about each other's stories.

Consider asking:

- "How does your story connect to the graph?"
- "Where do you see different times and heights in the graph?"
- "When ___ said, '___,' what do you think they meant?" (e.g., "When Madison said, 'the elevator moved again,' what do you think she meant?")
- "How might the graph look different if we modeled the number of people on the elevator? The floor the car was on?"



Unit 3
Lesson
10

Name: _____ Date: _____ Period: _____

MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.2.1, MTR4.1

Elevator Stories

Let's use compound inequalities to describe the domain and range of functions from their graphs.

Warm-Up

1. Amari is taking an elevator ride in her apartment building. The graph below shows Amari's height above ground, in meters, over a period of 30 seconds.

Tell a story about what you see.

Responses vary. Amari is staying in a room on the sixth floor. She wants to go to a friend's room on the 12th floor. She gets on the elevator. The elevator goes down to the third floor, stops briefly, goes up to the 15th floor, stops briefly, then finally stops on the 12th floor where she gets off the elevator.

Unit 3 Lesson 10 356 Warm-Up

Pairs | 15 min

Activity 1 Describing the Domain

Purpose: Students use words and **compound inequalities** to precisely describe the domain of a function from its graph. **(MTR.2.1)**

1 Launch

Invite students to view the graph of Amari's elevator ride.

- "What does the ordered pair $(7.5, 3)$ tell us about the elevator? What about $(0, 6)$?"
- "What do the inputs to $h(t)$ tell us about Amari's elevator ride? What about the outputs?" **(MTR.2.1)**

Consider asking, "Which variable is *independent* in this situation? Which is *dependent*? How do you know?"

D **Differentiation: (Support)** Look for students who swap the values for the input and outputs. Consider asking these students, "How can the axes labels help you to make sense of the values in the ordered pair?"

Listen for strategies students use to determine whether a value is included in the domain.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider reviewing the definition of domain: The domain is the set of all possible inputs. Consider asking, "What are the inputs in this situation?"
Precisely describe the domain (e.g., all the numbers from 0 to 30).	**Strengthen** Consider celebrating these students on Problem 3b for using a precise description. **(MTR.4.1)**
Do not select $\frac{1}{2}$ or 23.5 as possible inputs for the domain.	**Support** Consider asking, "How can you use the graph to check to see if there is an output value at that input value of t?"
Select 32 and/or 60 because the elevator still exists at these times.	**Support** Consider sharing that the graph represents Amari's ride only, not the height of the elevator at any time t.
Correctly identify all of the numbers in the domain of $h(t)$.	**Stretch** Invite these students to describe how they might change the function to include their unselected values in the domain.

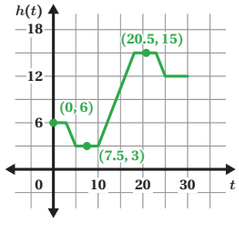
Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Describing the Domain

2. Here is a graph of $h(t)$, which represents the height of the elevator at a certain time, t, during Amari's ride.



Complete the input and output table for $h(t)$.

t	$h(t)$
5	3
20	15
30	12

3. a. Select *all* the numbers that are in the domain of $h(t)$.

- ☐ A. -5
- ☑ B. $\frac{1}{2}$
- ☑ C. 2
- ☑ D. 12
- ☑ E. 18
- ☑ F. 23.5
- ☐ G. 32
- ☐ H. 60

b. Describe the domain of $h(t)$.

Responses vary. All the numbers from 0 to 30.

Unit 3 Lesson 10 357 Activity 1

Pairs | 15 min

Activity 1 Describing the Domain (continued)

Purpose: Students use words and **compound inequalities** to precisely describe the domain of a function from its graph. **(MTR.2.1)**

Short on time? Consider facilitating a whole class discussion on Problem 5 instead of having students write their response.

2 Monitor

Invite students to identify what is different and what is the same about each graph in Problem 4a.

Listen for students who discuss ideas similar to those in the sample responses. Consider asking these students to share their reasoning during the Connect.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Listen for student ideas to highlight during the Connect:

- The direction of the inequality symbols.
- How the inequality $0 \leq t \leq 30$ compares to the others.
- How $t \geq 0$ and $0 \leq t$ are written in reverse order in the first two options but describe the same domain.

A **Accessibility: Conceptual Processing** Invite students to read each inequality aloud saying what it means (e.g., $t \geq 0$ means any number that is 0 or greater).

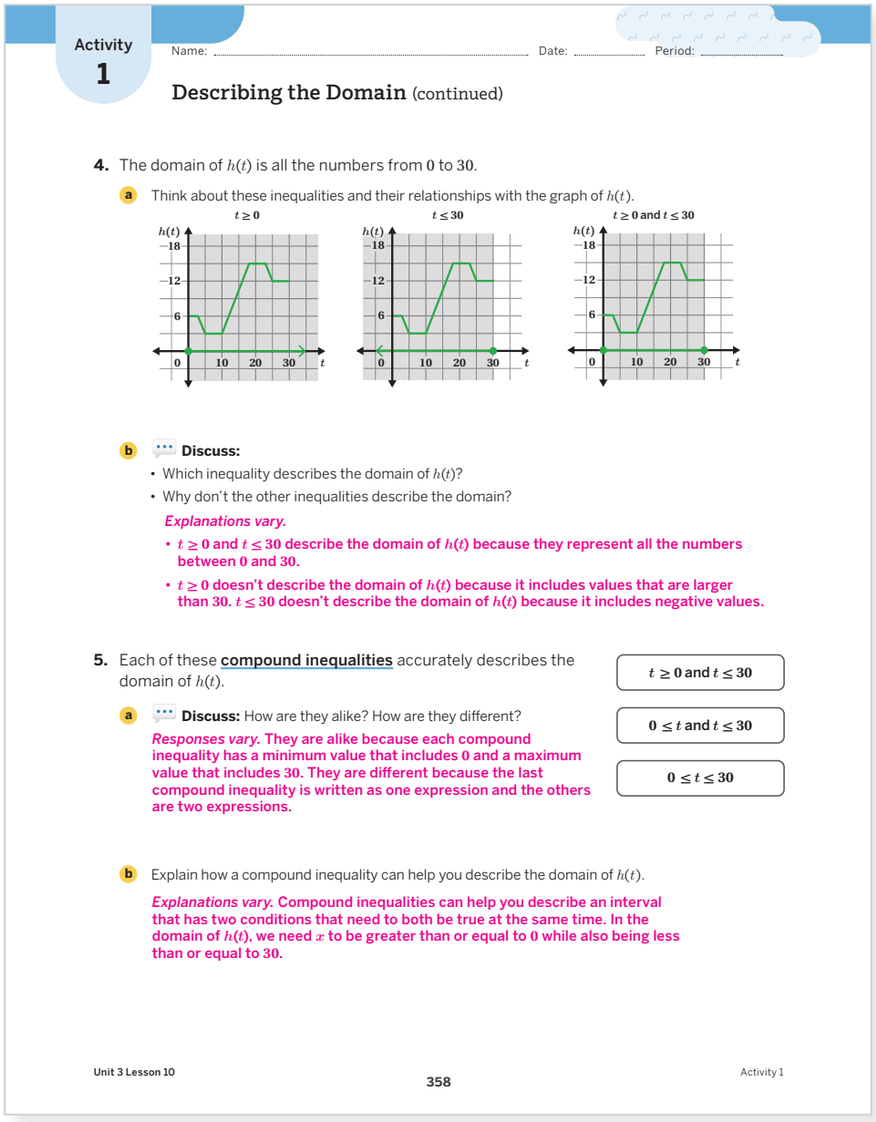
3 Connect

ELL **English Language Learners** Invite students to share what each word in the new term *compound inequality* reminds them of (e.g., "When I hear the word *compound*, I think..." or "When I hear the word *inequality*, I think ..."). Then invite students to share what they think it means when the two words are used together. **(Listening and Speaking)**

Invite students identified during the Monitor to share their thinking of what is the same and what is different about each of the compound inequalities.

To reveal the Key Takeaway, consider asking:

- "How do inequalities help to describe the domain of $h(t)$ symbolically?"
- "Why doesn't $30 \leq t \leq 0$ describe the domain of $h(t)$?"



Activity 1

Name: Date: Period:

Describing the Domain (continued)

4. The domain of $h(t)$ is all the numbers from 0 to 30.

a. Think about these inequalities and their relationships with the graph of $h(t)$.

$t \geq 0$ $t \leq 30$ $t \geq 0$ and $t \leq 30$

b. **Discuss:**

- Which inequality describes the domain of $h(t)$?
- Why don't the other inequalities describe the domain?

Explanations vary.

- $t \geq 0$ and $t \leq 30$ describe the domain of $h(t)$ because they represent all the numbers between 0 and 30.
- $t \geq 0$ doesn't describe the domain of $h(t)$ because it includes values that are larger than 30. $t \leq 30$ doesn't describe the domain of $h(t)$ because it includes negative values.

5. Each of these **compound inequalities** accurately describes the domain of $h(t)$.

$t \geq 0$ and $t \leq 30$

$0 \leq t$ and $t \leq 30$

$0 \leq t \leq 30$

a. **Discuss:** How are they alike? How are they different?

Responses vary. They are alike because each compound inequality has a minimum value that includes 0 and a maximum value that includes 30. They are different because the last compound inequality is written as one expression and the others are two expressions.

b. Explain how a compound inequality can help you describe the domain of $h(t)$.

Explanations vary. Compound inequalities can help you describe an interval that has two conditions that need to both be true at the same time. In the domain of $h(t)$, we need x to be greater than or equal to 0 while also being less than or equal to 30.

Unit 3 Lesson 10 358 Activity 1

Key Takeaway: A **compound inequality** can be a useful tool to precisely describe the domain of a function. There are multiple structures a compound inequality can take.

Pairs | 15 min

Activity 2 Distinguishing Domain and Range

Purpose: Students distinguish between describing the domain and the range with compound inequalities and interpret compound inequalities in context.

Short on time? Consider omitting the **Decide and Defend** discussion if students' responses are in consensus and their explanations show a strong understanding.

1 Launch

Invite students to share what they recall about the range.

Consider asking, "How is the range of a function like the domain? How is it different?"

2 Monitor

D **Differentiation: (Support)** To support students getting started, listen for students who recognize that the range includes values from 0 to 15 but do not focus on the variable it describes. Consider asking, "What's the same and different between these two compound inequalities? What does that difference mean?"

ELL **English Language Learners** Consider personally inviting students to share their responses if they feel comfortable, including encouraging them to use their primary languages. **(Speaking and Listening)**

Pause and list student reponses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

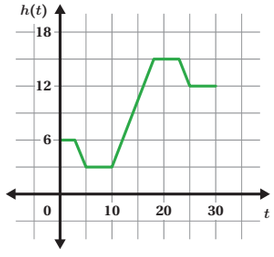
Distinguishing Domain and Range

6. **a** Write a compound inequality that describes the range of $h(t)$.

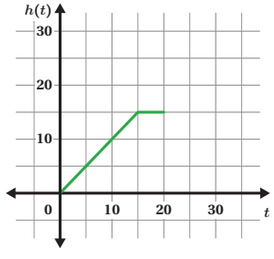
$\underline{3} \leq h(t) \leq \underline{15}$

b Describe in words what the compound inequality says about the range.

Responses vary. The compound inequality says that the range of $h(t)$ is all the numbers between 3 and 15.



7. Here is the graph of another elevator ride.



Two students described the range of this function.

- Ali says the range is $0 \leq t \leq 15$.
- Shanice says the range is $0 \leq h(t) \leq 15$.

Whose thinking is correct? Circle one.

Ali's (Shanice's) Both Neither

Explain your thinking.

Explanations vary. $h(t)$ represents the outputs, so Shanice's answer describes the range. Ali's answer uses t, which represents the inputs and describes the domain.

Unit 3 Lesson 10 359 Activity 2

Pairs | 15 min

Activity 2 Distinguishing Domain and Range (continued)

Purpose: Students distinguish between describing the domain and the range with compound inequalities and interpret compound inequalities in context.

Short on time? Consider omitting the **Decide and Defend** discussion if students' responses are in consensus and their explanations show a strong understanding.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match the graph of each function with its domain and range.

Consider asking, "How can you tell if a compound inequality is describing the domain or the range?"

A **Accessibility: Conceptual Processing** Invite students to read each compound inequality aloud (e.g., "The values of t from 0 to 30").

A **Accessibility: Executive Functioning** Invite students to look for similarities and differences among the compound inequalities and possibly sort them into a group of domains and a group of ranges before matching them with a function.

Note: If time allows, invite pairs to compare their matches, justify their thinking, and make revisions based on their conversation.

D **Differentiation: Stretch** Invite students to sketch a function that satisfies the two leftover cards.

D **Differentiation: Support** To support students getting started, consider asking, "What might the -30 mean about the elevator?"

Share and discuss student responses that show the elevator at different heights at different times. For example, elevators that start at -30 and travel up to 0 as well as elevators that start at 0 and travel down to -30.

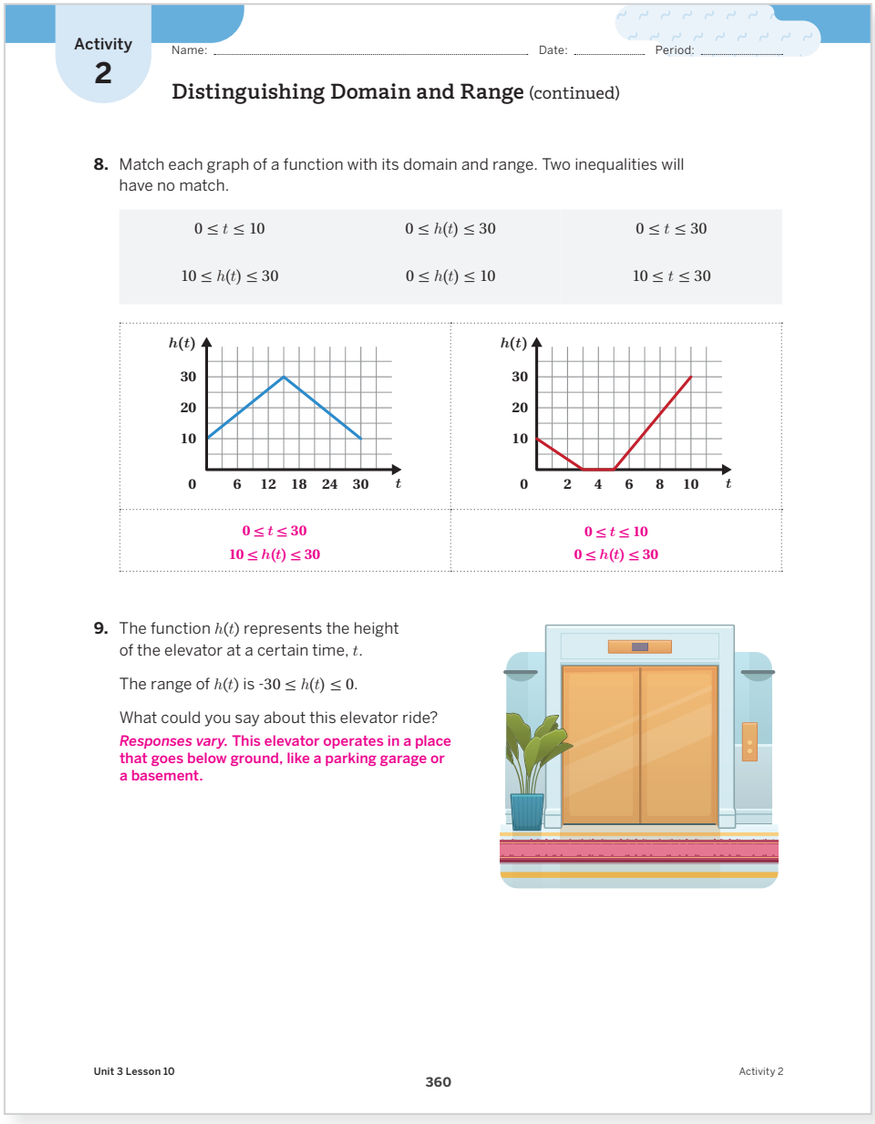
3 Connect

Share and discuss the responses making connections to the compound inequality.

Encourage students to ask questions about each other's responses.

Consider asking:

- "What key features are the range connected to?"
- "Where do you see different heights in the range?"
- "When ___ said, '___,' what do you think they meant?" (e.g., When Madison said, "the elevator moved again.")
- "How does your story connect to the compound inequality?"



Activity 2

Name: _____ Date: _____ Period: _____

Distinguishing Domain and Range (continued)

8. Match each graph of a function with its domain and range. Two inequalities will have no match.

$0 \le t \le 10$	$0 \le h(t) \le 30$	$0 \le t \le 30$
$10 \le h(t) \le 30$	$0 \le h(t) \le 10$	$10 \le t \le 30$

$0 \le t \le 30$
$10 \le h(t) \le 30$

$0 \le t \le 10$
$0 \le h(t) \le 30$

9. The function $h(t)$ represents the height of the elevator at a certain time, t.

The range of $h(t)$ is $-30 \le h(t) \le 0$.

What could you say about this elevator ride?

Responses vary. This elevator operates in a place that goes below ground, like a parking garage or a basement.

Unit 3 Lesson 10 360 Activity 2

Pairs | 10 min

Activity 3 Writing Compound Inequalities

Purpose: Students develop fluency writing compound inequalities to describe the range of a graph.

Short on time? Consider omitting Activity 3.

1 Launch

Students will need a partner for this activity. They will solve and compare their solutions to problems.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

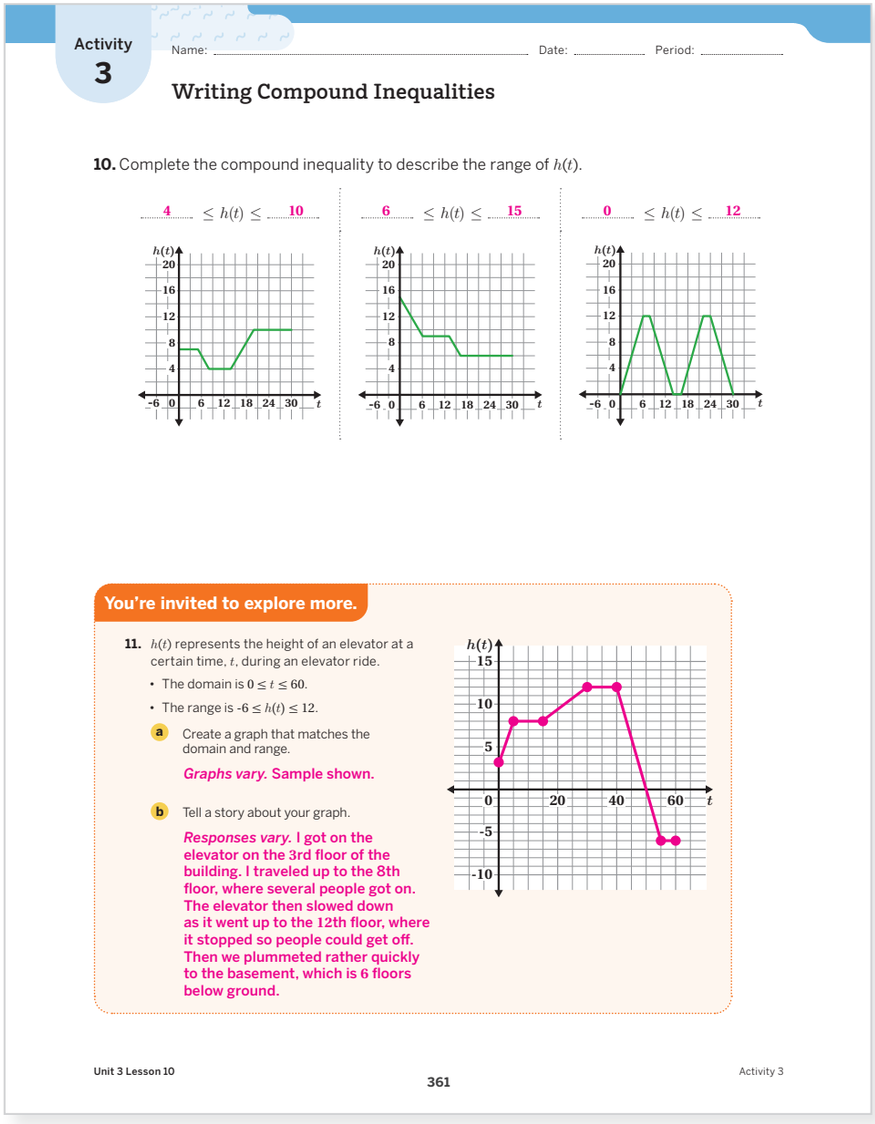
2 Monitor

Listen for strategies students use in discussing how to write the compound inequalities, such as identifying the maximum and minimum values of the range of each function.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore creating and writing a story about a function given a domain and range further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 3

Name: _____ Date: _____ Period: _____

Writing Compound Inequalities

10. Complete the compound inequality to describe the range of $h(t)$.

4 $\leq h(t) \leq$ 10

6 $\leq h(t) \leq$ 15

0 $\leq h(t) \leq$ 12

You're invited to explore more.

11. $h(t)$ represents the height of an elevator at a certain time, t, during an elevator ride.

- The domain is $0 \leq t \leq 60$.
- The range is $-6 \leq h(t) \leq 12$.

a. Create a graph that matches the domain and range.

Graphs vary. Sample shown.

b. Tell a story about your graph.

Responses vary. I got on the elevator on the 3rd floor of the building. I traveled up to the 8th floor, where several people got on. The elevator then slowed down as it went up to the 12th floor, where it stopped so people could get off. Then we plummeted rather quickly to the basement, which is 6 floors below ground.

Unit 3 Lesson 10 361 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for determining the domain and range of a function from its graph.

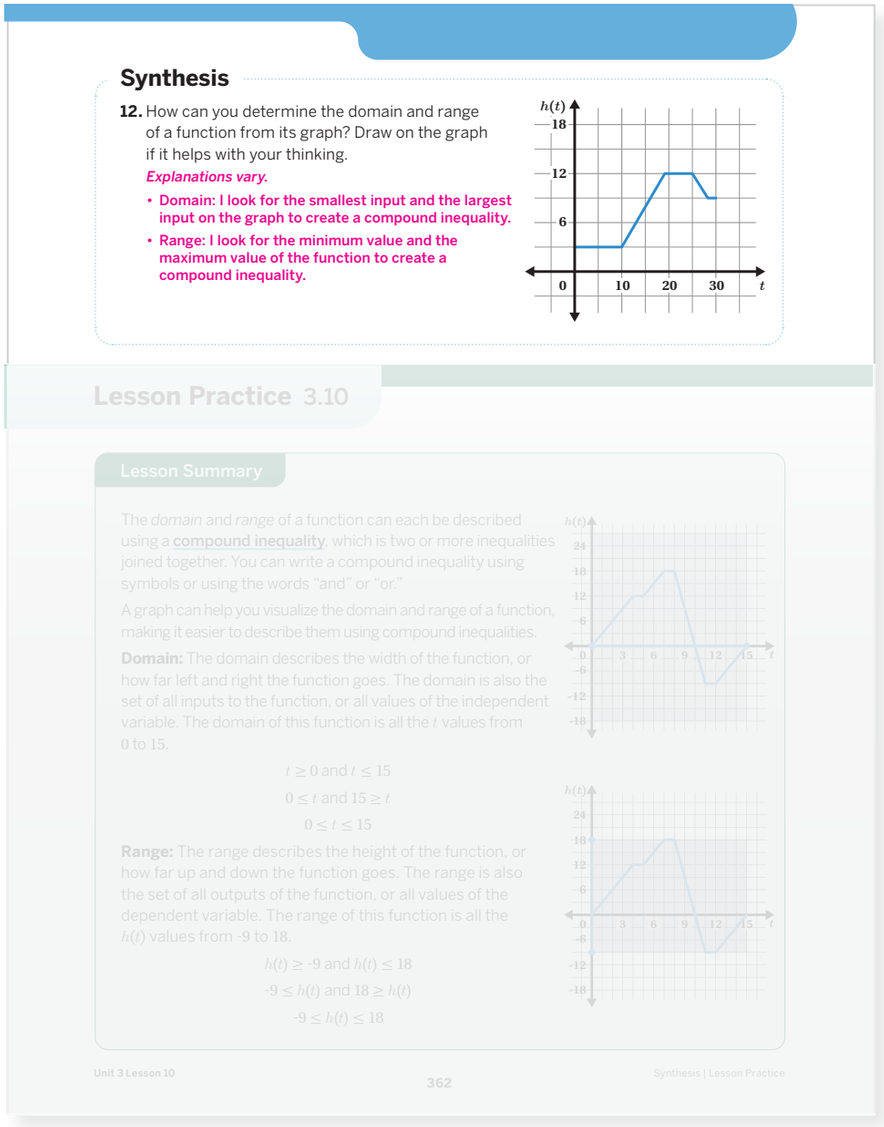
Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:

- To determine the domain, look at the values from left to right on the graph.
- To determine the range, look at the values from top to bottom on the graph.
- To determine the range, look for the minimum and maximum values on the graph.

MLR **MLR2: Collect and Display** Use this routine to collect students' language and strategies for determining the domain and range. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. How can you determine the domain and range of a function from its graph? Draw on the graph if it helps with your thinking.

Explanations vary.

- Domain: I look for the smallest input and the largest input on the graph to create a compound inequality.
- Range: I look for the minimum value and the maximum value of the function to create a compound inequality.

Lesson Practice 3.10

Lesson Summary

The *domain* and *range* of a function can each be described using a compound inequality, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words "and" or "or."

A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.

Domain: The domain describes the width of the function, or how far left and right the function goes. The domain is also the set of all inputs to the function, or all values of the independent variable. The domain of this function is all the t values from 0 to 15.

$t \geq 0$ and $t \leq 15$

$0 \leq t$ and $15 \geq t$

$0 \leq t \leq 15$

Range: The range describes the height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the $h(t)$ values from -9 to 18.

$h(t) \geq -9$ and $h(t) \leq 18$

$-9 \leq h(t)$ and $18 \geq h(t)$

$-9 \leq h(t) \leq 18$

Unit 3 Lesson 10 362 Synthesis | Lesson Practice

Lesson Takeaway: A graph can be a helpful representation to visualize the domain and range of a function when using a **compound inequality** to describe them.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the compound inequalities for the domain and range of a given graph.

Today's Goals

1. **Goal:** Write the domain and range of a function using inequalities.
2. **Language Goal:** Interpret the meaning of the domain and range in context. **(Writing and Speaking)**

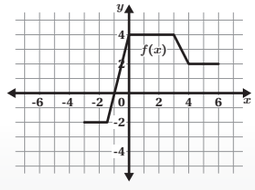
Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 3.10

Match the graph of $f(x)$ with its domain and range. Two inequalities will have no match.

$-3 \le x \le 6$ $-2 \le f(x) \le 4$ $-2 \le x \le 4$ $-2 \le f(x) \le 2$



Domain: $-3 \le x \le 6$

Range: $-2 \le f(x) \le 4$

Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support relating the domain and range of a function to its graph using compound inequalities:

» **Respond to Student Thinking:**

- Use the *Writing Domain and Range with Inequalities* **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Consider the next several lessons you will teach. How will students' work today with **compound inequalities** prepare them to understand functions in Lessons 11–12? What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can you determine the domain and range of a function from its graph? Draw on the graph if it helps with your thinking.

Explanations vary.

- Domain: I look for the smallest input and the largest input on the graph to create a compound inequality.
- Range: I look for the minimum value and the maximum value of the function to create a compound inequality.

Lesson Practice 3.10

Lesson Summary

The *domain* and *range* of a function can each be described using a **compound inequality**, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words "and" or "or."

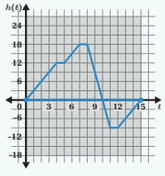
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$0 \leq t$ and $15 \geq t$

$0 \leq t \leq 15$

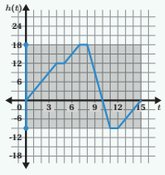


Range: The range describes the height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the $h(t)$ values from -9 to 18.

$h(t) \geq -9$ and $h(t) \leq 18$

$-9 \leq h(t)$ and $18 \geq h(t)$

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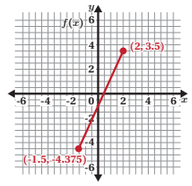


Lesson Practice 3.10

Name: _____ Date: _____ Period: _____

Problems 1–2: Valeria and Thiago disagree about the domain of $f(x)$.

- Valeria says the domain is $-1.5 \leq x \leq 2$
- Thiago says the domain is $-4.375 \leq x \leq 3.5$



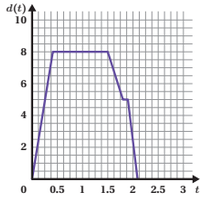
1. Whose answer is correct? Circle one.

(Valeria's) Thiago's

2. Explain why the other person's answer is incorrect.

***Explanations vary.* Thiago gave the output values for the function. That's the range.**

Problems 3–5: Haru bikes to his friend's house. After a while, he heads home. On the way, he stops at the store to buy a bottle of water. $d(t)$ represents Haru's distance from his house, in kilometers, after t hours. This graph shows Haru's distance over time.



3. Which inequality describes the domain of $d(t)$?

A. $0 \leq d(t) \leq 2.1$ B. $0 \leq d(t) \leq 8$

(C.) $0 \leq t \leq 2.1$ D. $0 \leq t \leq 8$

4. Which inequality describes the range of $d(t)$?

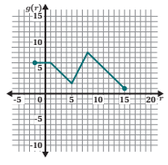
A. $0 \leq d(t) \leq 2.1$ (B.) $0 \leq d(t) \leq 8$

C. $0 \leq t \leq 2.1$ D. $0 \leq t \leq 8$

5. If Haru had not stopped at the store, would that change the domain or the range? Circle one.

(Domain) Range Both Neither

Problems 6–7: Here is the graph of $g(r)$.



6. Write a compound inequality to describe the domain.

$-2 \leq r \leq 15$

7. Write a compound inequality to describe the range.

$1 \leq g(r) \leq 8$

Lesson Practice 3.10

Name: _____ Date: _____ Period: _____

Test Practice

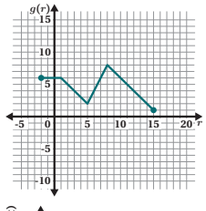
8. Which of the following compound inequalities correctly describes the domain of the function $f(x) = 2x + 5$ if the graph of $f(x)$ is only shown between $x = -3$ and $x = 4$?

☐ A. $-3 < x < 4$ ☑ B. $-3 \leq x \leq 4$

☐ C. $-3 \leq x < 4$ ☐ D. $-3 < x \leq 4$

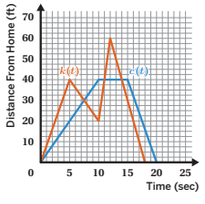
Spiral Review

9. Determine the average rate of change for each interval in the graph of $g(r)$.



Interval	Average Rate of Change
$r = -2$ to $r = 1$	0 (or equivalent)
$r = -2$ to $r = 5$	$-\frac{4}{7}$ (or equivalent)
$r = -2$ to $r = 8$	$\frac{1}{5}$ (or equivalent)

10. Functions $c(t)$ and $k(t)$ represent the distance of two cats from home after t seconds. Select *all* of the true statements.



☑ A. $k(5) > c(5)$

☐ B. $k(t)$ and $c(t)$ have the same domain and range.

☐ C. $k(t)$ keeps increasing from 0 to 13 seconds.

☑ D. $k(11) = c(11)$

☑ E. Both cats return home.

11. Tickets to the state fair cost \$10 each. The function $c(t) = 10t$ gives the total cost in dollars, $c(t)$, for the number of tickets purchased, t. Select *all* the values that are possible outputs for $c(t)$.

☑ A. 0 ☑ B. 70 ☐ C. 105 ☑ D. 880 ☐ E. 963

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
Fluency	1–2	2	MA.912.F.1.2, MA.912.F.1.5
	3–5	2	MA.912.F.1.6
	6–7	2	MA.912.F.1.6
TEST Practice	8	2	MA.912.F.1.5
Spiral Review			
	9	2	MA.912.F.1.3
	10–11	2	MA.912.F.1.2, MA.912.F.1.5

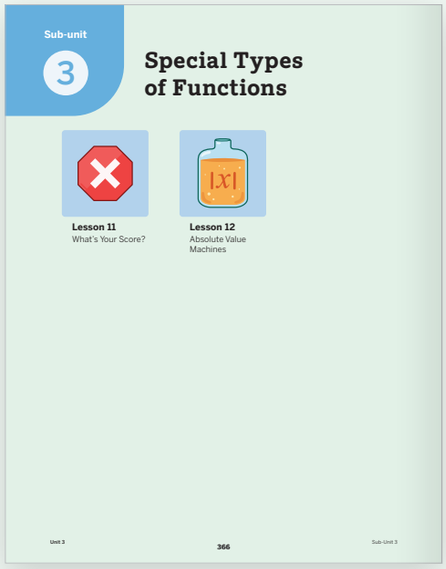
Sub-Unit 3

Special Types of Functions

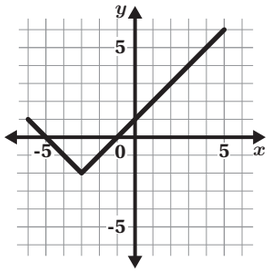
Sub-Unit 3 Goals

- **Goal:** Interpret, evaluate, graph, and write equations of absolute value functions.
- **Language Goal:** Analyze and model situations using function tools, including situations that model sequences and inverses of functions.

In this sub-unit, students interpret, evaluate, graph, and write equations of **absolute value functions**.



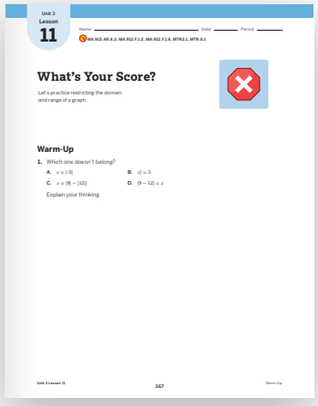
Math That Matters Most

Concept or Skill	For example . . .
Graph absolute value functions in the form $f(x) = \|x - h\| + k$ and determine its minimum.	Create the graph of $c(x) = \|x + 3\| - 2$ and determine the minimum.  Minimum: **(-3, -2)**

Unit 3

Lesson

11



What's Your Score?

Absolute Value Functions, Part 1

Let's make sense of absolute value functions.

Learning Outcomes

Today's Goals

1. **Goal:** Evaluate and interpret outputs for *absolute value functions*.
2. **Language Goal:** Describe how the absolute value function $f(x) = |x - h|$ is related to the distance from a number. **(Writing and Speaking)**

Students make sense of the **absolute value function** as the distance from a number by playing a game where one's score is the distance from a target number. **Note:** The equations for the absolute value functions in this lesson are written with *absolute value* symbols, but the equations can also be written using piecewise notation. **(MTR.2.1)**

Coherence

Prior Learning

In Grade 6, students interpreted the absolute value of a number as its distance from 0 on the number line. In Lesson 10, students described domain and range with inequalities.

Future Learning

In Lesson 12, students will graph absolute value functions and analyze their key features. In Algebra 2, students will apply transformations to absolute value functions.

Connections

- Students apply their understanding of number games as they are introduced to the concept of absolute value functions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.4.3
Given a table, equation or written description of an absolute value function, graph that function and determine its key features.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.6.NSO.3.1	**MA.912.AR.2.8**
MA.912.F.1.2	

Vocabulary

New Vocabulary

absolute value function

Review Vocabulary

absolute value

Lesson at a Glance

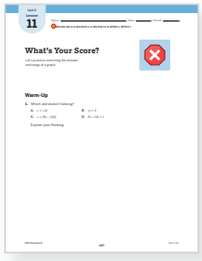
~ 45 min

MA.912.AR.4.3

Warm-Up

Pairs | 5 min

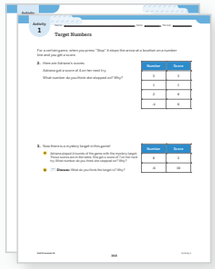
Students use the **Which One Doesn't Belong?** routine to recall the meaning of *absolute value*.



Activity 1

Pairs | 15 min

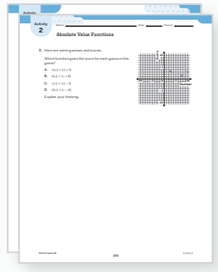
Studentsanalyze data from a game where the scores are a distance from a target number to help them make sense of the **absolute value function**.



Activity 2

Pairs | 15 min

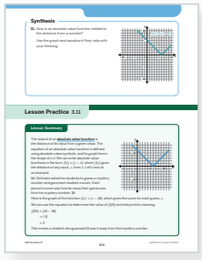
Students use the **Decide and Defend** routine to construct arguments around which absolute value equation structures match graphs and situations. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

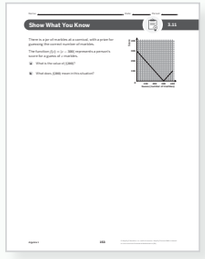
Students synthesize their understanding of how the absolute value function is related to the distance from a number. **(MTR.2.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by evaluating an absolute value function.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to recall the meaning of *absolute value*.

1 Launch

Consider asking, "What do you remember about *absolute value*?"

Use the Which One Doesn't Belong? routine to support students in noticing the features and structure of each equation. **(MTR.5.1)**

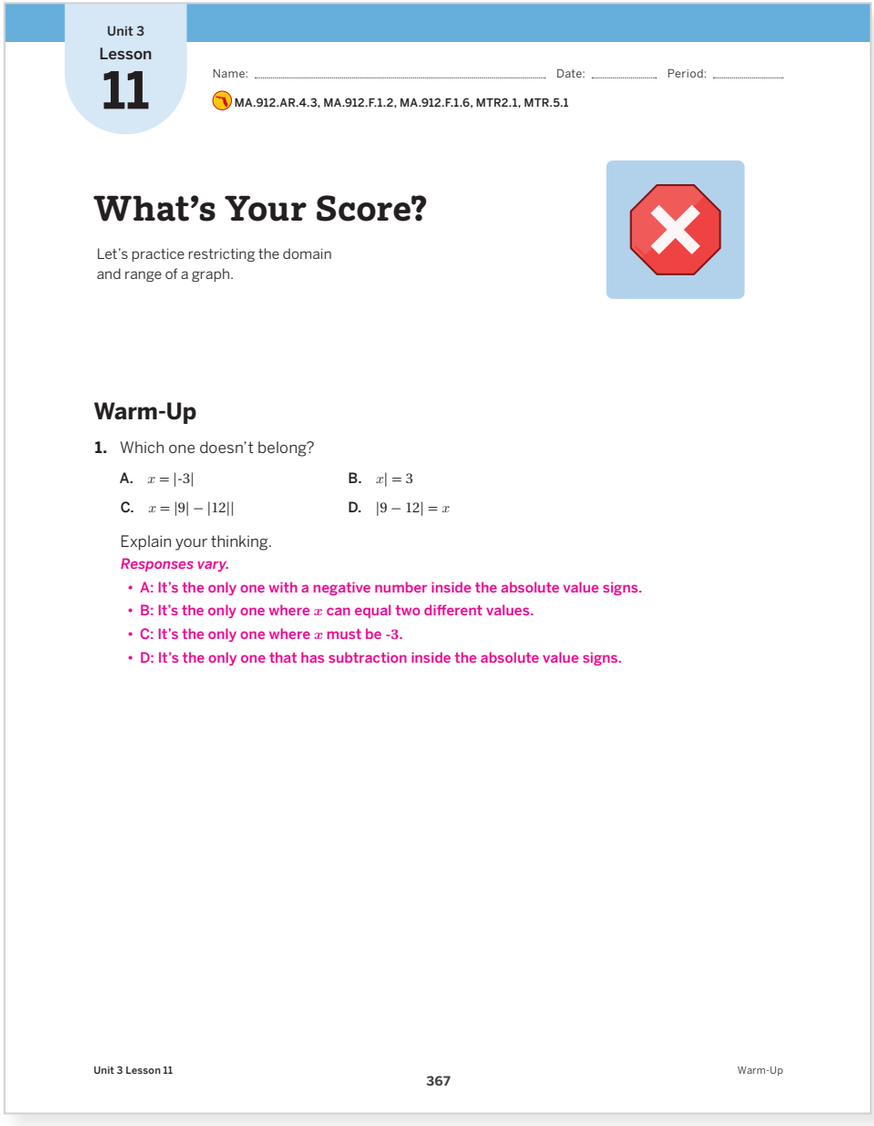
Consider sharing that each equation does not belong in some way. Encourage students to look for more than one possibility.

ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or "_____ is the only one that has/doesn't have _____."). **(Writing and Speaking)**

2 Connect

Invite students to share at least one reason why each equation doesn't belong.

To reveal the Key Takeaway, consider asking, "How is the absolute value of a number calculated, or related to, distance?"



Unit 3
Lesson
11

Name: _____ Date: _____ Period: _____

MA.912.AR.4.3, MA.912.F.1.2, MA.912.F.1.6, MTR2.1, MTR.5.1

What's Your Score?

Let's practice restricting the domain and range of a graph.

Warm-Up

1. Which one doesn't belong?

   A. $x = |-3|$
   B. $x| = 3$
   C. $x = |9| - |12||$
   D. $|9 - 12| = x$

   Explain your thinking.
   Responses vary.
   - A: It's the only one with a negative number inside the absolute value signs.
   - B: It's the only one where x can equal two different values.
   - C: It's the only one where x must be -3.
   - D: It's the only one that has subtraction inside the absolute value signs.

Unit 3 Lesson 11 367 Warm-Up

Key Takeaway: The *absolute value* of a number is its distance from 0 on the number line.

Pairs | 15 min

Activity 1 Target Numbers

Purpose: Students analyze data from a game where the scores are a distance from a target number to help them make sense of the **absolute value function**.

1 Launch

Invite students to work in pairs and discuss the game with their partner.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *stop, land on, target, distance, from,* etc. **(Writing and Speaking)**

2 Monitor

To access connections, consider asking pairs of students:

- "What patterns do you notice?"
- "How is the score related to the target?"
- "How is this game similar to or different from a time when you tried to hit a specific target in your life?"

Invite students to responses and encourage students to revise their responses if they find their classmates' thinking to be convincing.

Pause briefly and invite students to share why there are two possible numbers that Adriana could have stopped on.

To surface connections, consider asking pairs of students:

- "How is the target related to the number landed on and the score?"
- "What patterns do you see in the table?"
- "How many rounds do you need to play to determine the mystery target?"

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Target Numbers

For a certain game, when you press "Stop" it stops the arrow at a location on a number line and you get a score.

2. Here are Adriana's scores.

Number	Score
5	3
1	1
2	0
-4	6

Adriana got a score of 4 on her next try.

What number do you think she stopped on? Why?

Responses vary.

- -2 or 6 because they both are 4 units away from 2.
- -2 because it is 4 units to the left of the target, 2.
- 6 because it is 4 bigger than the target, 2.

3. Now there is a mystery target in this game!

Number	Score
6	2
-6	10

a. Adriana played 2 rounds of the game with the mystery target. Those scores are in the table. She got a score of 7 on her next try. What number do you think she stopped on? Why?

b. **Discuss:** What do you think the target is? Why?

Responses vary. She could have stopped on 3 or 11. Using the table, the number 6, with a score of 2 would be 4 numbers away. -6 with a score of 10 would be 14 numbers away. But since the magic number has to be the same for both, if we ignore the sign in front of the 6, then 10 is 4 numbers away, too. $7 - 4 = 3$ or $7 + 4 = 11$

Unit 3 Lesson 11 368 Activity 1

Pairs | 15 min

Activity 1 Target Numbers (continued)

Purpose: Students analyze data from a game where the scores are a distance from a target number to help them make sense of the **absolute value function**.

2 Monitor

A **Accessibility: Visual-Spatial Processing** Invite them to pick a point on the graph and describe what that point means in terms of the game (e.g., The point (-1, 5) means that landing on -1 gets a score of 5.).

D **Differentiation**

Look for students who:	Respond to Student Thinking
Switch the x and y in a coordinate pair or whose points do not make a V shape.	**Support** Encourage them to check a point matching the table headers to the axis labels.
Aren't sure what the graph of all possible scores will look like.	**Support** Consider asking, "What patterns do you see in the graph? What shape does it look like? Why?"
Describe the graph in helpful or creative ways such as: • It will look like a V or two lines that meet at the mystery target. • The mystery target is the minimum point.	**Stretch** Invite them to share during the Connect and consider asking, "What might the graph look like for a game with a target number of -3?"

3 Connect

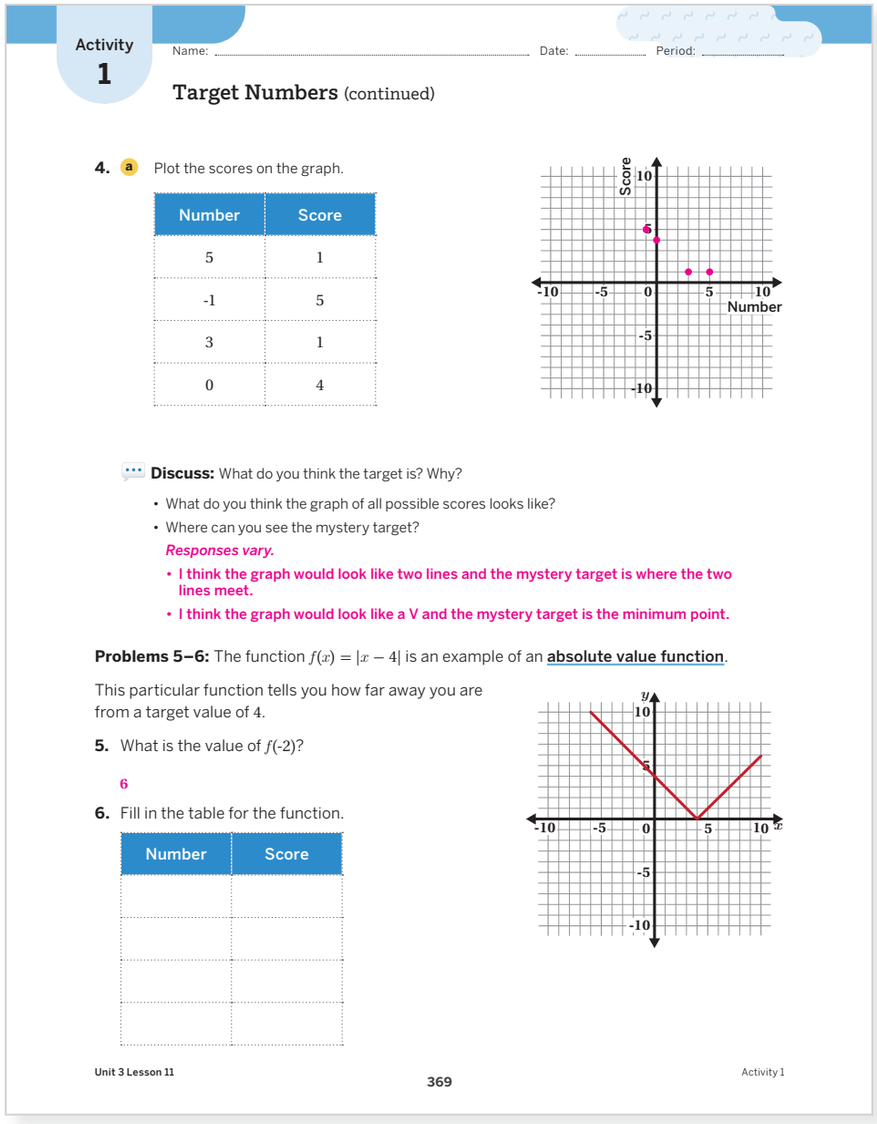
Invite students to share their responses to the discussion questions.

Invite students to discuss with their partner how this graph compares with their predictions.

A **Accessibility: Memory and Attention** To support students in activating prior knowledge, consider asking, "What do you remember about function notation? Absolute value?"

To reveal the Key Takeaway, consider asking:

- "What does $f(-2)$ tell us about the game?"
- "How do you see the mystery target in the graph? In the equation? What key feature is it revealing?"
- "Why doesn't this function have negative outputs?"
- "How is this game connected to absolute value?" *Responses vary.* Your score in the game is your distance from the target and absolute value measures distances. You can't have negative scores or distances.
- "Why is the target number subtracted from x in the equation?" Distance is determined by subtracting.
- "What would the graph and function equation look like for a target number of -3? **(MTR.2.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Target Numbers (continued)

4. a Plot the scores on the graph.

Number	Score
5	1
-1	5
3	1
0	4

Discuss: What do you think the target is? Why?

- What do you think the graph of all possible scores looks like?
- Where can you see the mystery target?

Responses vary.

- I think the graph would look like two lines and the mystery target is where the two lines meet.
- I think the graph would look like a V and the mystery target is the minimum point.

Problems 5–6: The function $f(x) = |x - 4|$ is an example of an **absolute value function**.

This particular function tells you how far away you are from a target value of 4.

5. What is the value of $f(-2)$?

6

6. Fill in the table for the function.

Number	Score

Unit 3 Lesson 11 369 Activity 1

Key Takeaway: The **absolute value function** is a function whose output is the distance of its input from a given value.

Pairs | 15 min

Activity 2 Absolute Value Functions

Purpose: Students use the **Decide and Defend** routine to construct arguments around which absolute value functions equation structures match graphs and situations. **(MTR.5.1)**

Short on time? Consider limiting or omitting the discussion about Which Function?

1 Launch

Invite students to read the instructions for the new game aloud and consider asking, "How is this game similar to and different from the previous game?"

2 Monitor

To access connections, consider asking pairs of students, "How can you see the mystery number in the graph? In the table?" *Responses vary.* It's the minimum or x-intercept on the graph. The guess that has a score of 0.

D Differentiation

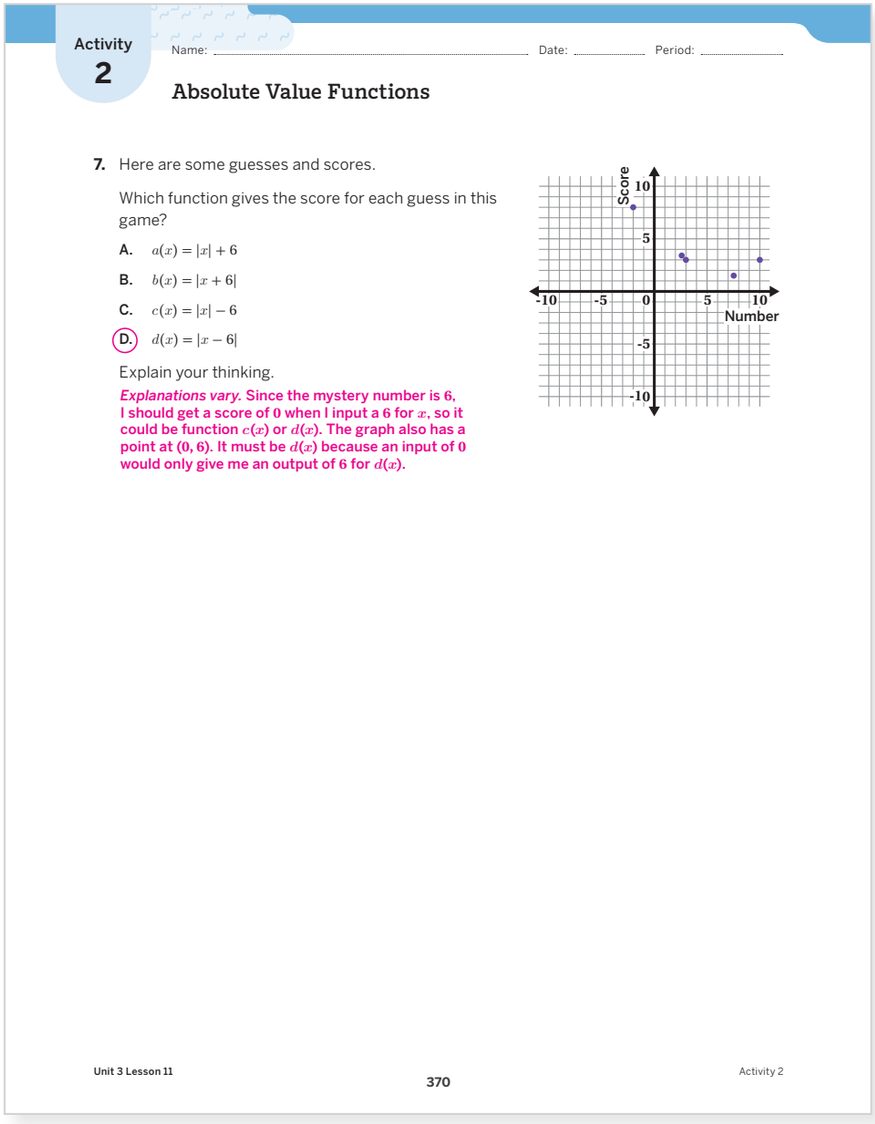
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What score should I get when I input the mystery number into the correct function? How can that help you select a function?"
Select $b(x)$ because 6 is the mystery number.	**Support** Encourage students to input points from the graph into $b(x)$ and see what happens.
Describe connections between the structure of the equation, the graph, and the situation.	**Strengthen** Invite them to share during the Connect. Consider asking, "What key features do you see in the equation? Where do you see that in the graph? In the situation?" **(MTR.5.1)**

Pause and ask students to share their observations about the mystery number for this game.

Use the Decide and Defend routine to support students in making arguments, critiquing the reasoning of others, and understanding the structure of absolute value equations.

Consider asking:
- "How can you use the equation to determine a point on the graph?"
- "How does the equation connect to the situation and the graph?" **(MTR.2.1)**

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Absolute Value Functions

7. Here are some guesses and scores.

Which function gives the score for each guess in this game?

A. $a(x) = |x| + 6$

B. $b(x) = |x + 6|$

C. $c(x) = |x| - 6$

D. $d(x) = |x - 6|$

Explain your thinking.

Explanations vary. Since the mystery number is 6, I should get a score of 0 when I input a 6 for x, so it could be function $c(x)$ or $d(x)$. The graph also has a point at (0, 6). It must be $d(x)$ because an input of 0 would only give me an output of 6 for $d(x)$.

Unit 3 Lesson 11 370 Activity 2

Pairs | 15 min

Activity 2 Absolute Value Functions (continued)

Purpose: Students use the **Decide and Defend** routine to construct arguments around which absolute value functions equation structures match graphs and situations. **(MTR.5.1)**

2 Monitor

A **Accessibility: Memory and Attention** Invite students to record their calculations.

To access connections, consider asking pairs of students:

- "What is the mystery number for this function?"
- "How do you see the 3 from the equation in the graph?"

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore absolute value functions further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share connections between the situation, graph, equation, and table.

Consider asking,

- "What is the mystery number for this function? How do you know?"
- "How do you see the 3 from the equation in the graph?"
- "What would the graph of $f(x) = |x + 5|$ look like?"
- "What key feature is revealed by an equation in the form $f(x) = |x - h|$?"
- Draw a graph of an absolute value function with the vertex somewhere along the x-axis, then ask, "What would the equation for this graph look like?"

D **Differentiation: (Stretch)** If time allows, consider inviting students to return to the functions and describe with a partner what each of those graphs would look like. Encourage them to describe how they are similar and different.

Activity 2

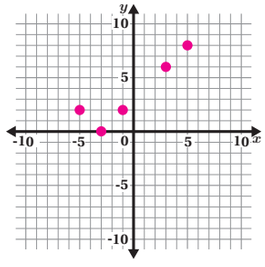
Name: _____ Date: _____ Period: _____

Absolute Value Functions (continued)

8. There is a new mystery number. The function $f(x) = |x + 3|$ gives the score for each guess, x.

Complete the table and plot the ordered pairs.

x	$f(x)$
5	8
-1	2
-5	2
-3	0
3	6



9. Compare the function $f(x) = |x + 3|$ to the function $g(x) = x + 3$ by analyzing their equations, graphs, or a table of values. What do you notice?

Responses vary. $f(x)$ is a non-linear function even though its graph is made up of two linear pieces. It does not have a constant rate of change across the entire function. $g(x)$ is a linear function because it has a constant rate of change, its graph is a straight line, and it can be written in the form $y = mx + b$. Both functions have the same vertical intercept at 3.

Explore More

10. Here are some guesses and scores for a new mystery number. Can these be scores for the same mystery number? Circle one.

Yes (No)

Explain your thinking.

Guess	Score
1	4
6	2

Explanations vary. If a guess of 1 gives a score of 4, the mystery number could be 5 or -3. If a guess of 6 gives a score of 2, the mystery number could be 4 or 8. Since there is no mystery number that works for both of these guesses, these can't be scores for the same mystery number.

Unit 3 Lesson 11 371 Activity 2

Whole Class | 5 min

Synthesis

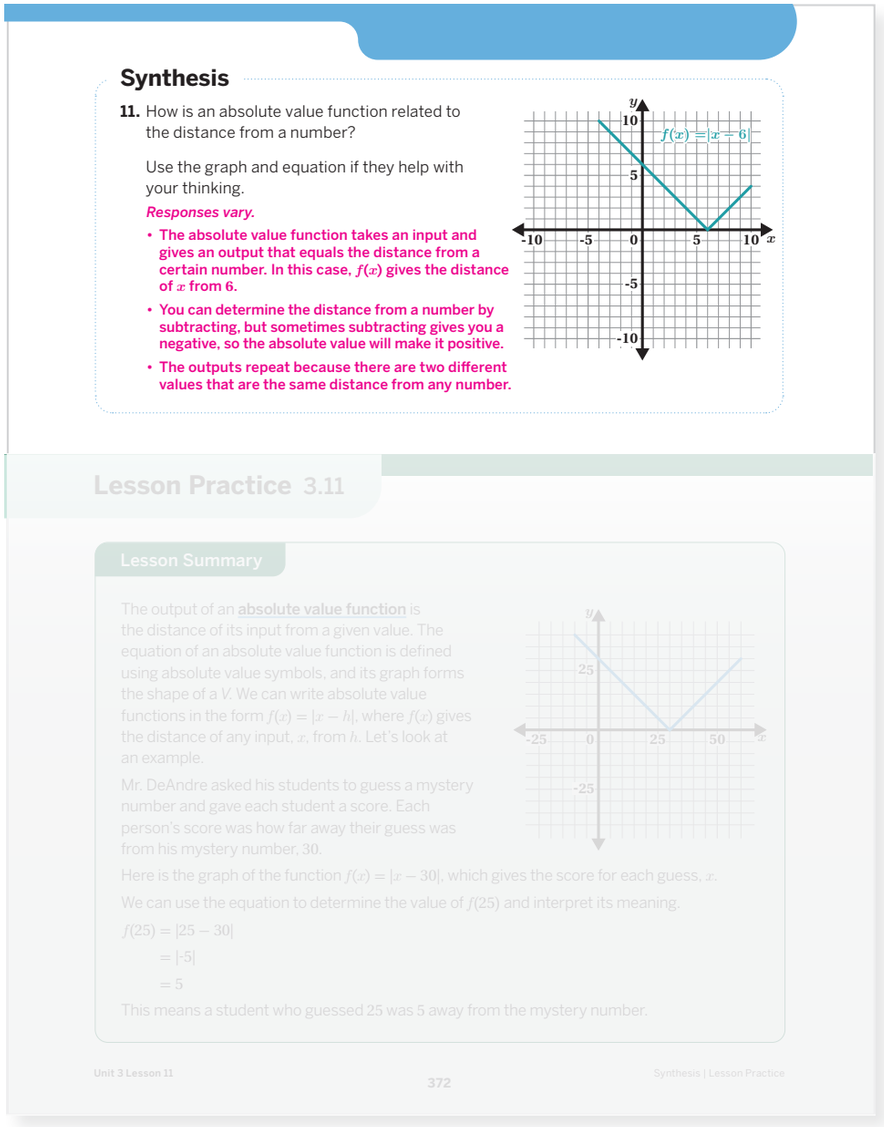
Purpose: Students synthesize their understanding of how the absolute value function is related to the distance from a number. **(MTR.2.1)**

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- You can determine distance by subtracting, which is why the equation subtracts the target number.
- The absolute value function makes all of the differences positive since distances are positive.
- Outputs can repeat because there are multiple numbers that are the same distance from a target value.

Note: If these ideas do not come up naturally, consider accessing the Lesson Takeaway by asking:
- "How is absolute value related to distance?"
- "How can you see the number you are measuring the distance from in the equation? In the graph?"
- "Why can multiple x-values have the same y-value in an absolute value function?"

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Synthesis

11. How is an absolute value function related to the distance from a number?

Use the graph and equation if they help with your thinking.

Responses vary.

- The absolute value function takes an input and gives an output that equals the distance from a certain number. In this case, $f(x)$ gives the distance of x from 6.
- You can determine the distance from a number by subtracting, but sometimes subtracting gives you a negative, so the absolute value will make it positive.
- The outputs repeat because there are two different values that are the same distance from any number.

Lesson Practice 3.11

Lesson Summary

The output of an absolute value function is the distance of its input from a given value. The equation of an absolute value function is defined using absolute value symbols, and its graph forms the shape of a *V*. We can write absolute value functions in the form $f(x) = |x - h|$, where $f(x)$ gives the distance of any input, x, from h. Let's look at an example.

Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Each person's score was how far away their guess was from his mystery number, 30.

Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$f(25) = |25 - 30|$
$= |-5|$
$= 5$

This means a student who guessed 25 was 5 away from the mystery number.

Unit 3 Lesson 11 372 Synthesis | Lesson Practice

Lesson Takeaway: The **absolute value function** can be used to model the distance of all numbers from a target value ($x = h$). The equation $f(x) = |x - h|$ and the graph looks like a V-shape with a minimum at $x = h$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by evaluating an absolute value function.

Today's Goals

1. **Goal:** Evaluate and interpret outputs for *absolute value functions*.
2. **Language Goal:** Describe how the absolute value function $f(x) = |x - h|$ is related to the distance from a number. **(Writing and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

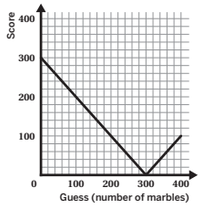
Show What You Know 3.11

There is a jar of marbles at a carnival, with a prize for guessing the correct number of marbles.

The function $f(x) = |x - 300|$ represents a person's score for a guess of x marbles.

a. What is the value of $f(200)$?
100

b. What does $f(200)$ mean in this situation?
Responses vary. A guess of 200 marbles is 100 off from the correct answer.



D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support evaluating an absolute value function:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on a few related **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Take a moment to reflect on how students looked for and made use of structure in today's lesson. **(MTR.5.1)**

- How did the **Which One Doesn't Belong?** routine support students in noticing and describing differences in structures?
- How did students connect the structure of an **absolute value function** in the form $f(x) = |x - h|$ to distance and the Mystery Target game?
- How can you continue to support students in looking for and making use of structure as they explore equations in the form $f(x) = |x - h| + k$ in the next lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How is an absolute value function related to the distance from a number?

Use the graph and equation if they help with your thinking.

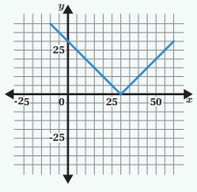
Responses vary.

- The absolute value function takes an input and gives an output that equals the distance from a certain number. In this case, $f(x)$ gives the distance of x from 6.
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Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$f(25) = |25 - 30|$

$= |-5|$

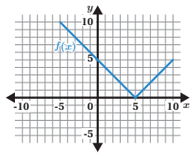
$= 5$

This means a student who guessed 25 was 5 away from the mystery number.

Lesson Practice 3.11

Name: ..... Date: ..... Period: .....

Problems 1–3: Use the graph of $f(x)$ to determine each value.

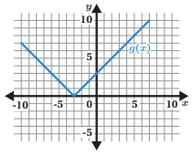


1. $f(0) =$ 5

2. $f(8) =$ 3

3. $f(5) =$ 0

4. Which equation represents the graph of $g(x)$? Circle your choice.



A. $g(x) = |x| - 3$

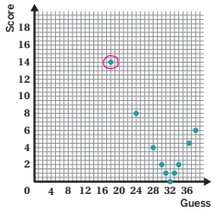
B. $g(x) = |x - 3|$

C. $g(x) = |x| + 3$

D. $g(x) = |x + 3|$

Problems 5–7: Ricardo's teacher challenged his class to guess how many marbles were in a jar. The graph shows each guess, x, and score, $m(x)$.

Each student was given a score equal to how far away their guess was from the actual number of marbles in the jar.



5. How many marbles are in the jar?
32

6. Circle the point that represents the furthest guess from the actual number of marbles.

Lesson Practice 3.11

Name: ..... Date: ..... Period: .....

Test Practice

7. Ricardo writes the equation $m(25) = 7$.

What does his equation mean?

A. Ricardo's equation shows that 25 is the correct number of marbles in the jar.

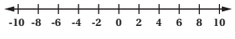
B. Ricardo's equation means a guess of 25 would give a score of 7 because it is 7 numbers away from the correct number of marbles in the jar.

C. Ricardo's equation means the correct number of marbles in the jar is 7.

D. Ricardo's equation shows that 25 and 7 are unrelated numbers.

Spiral Review

8. Select all of the true statements. Use the number line if it helps with your thinking.



☐ A. $-4 > -2$

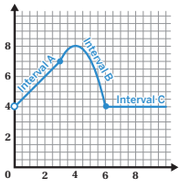
☑ B. The distance from -2 to 2 is equal to the distance from 6 to 10.

☑ C. $-4 \geq -6$

☐ D. 8 is the only number 2 units away from 6.

☑ E. $|3| = |-3|$

Problems 9–11: Match each domain to an interval of the piecewise function on the graph.



9. $3 \leq x \leq 6$ Interval B

10. $x \geq 6$ Interval C

11. $0 < x \leq 3$ Interval A

12. The California Department of Fish and Wildlife estimated there were 460,420 deer in the state in 2021. They estimated that the deer population in 2018 was 470,000.

Calculate the average rate of change during this time interval and explain what it tells us about the deer population.

About -3,193.3 deer per year. *Explanations vary.* The population of deer in California is decreasing at an average of approximately 3,193 deer per year.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
Fluency	1–3	1	MA.912.F.1.2
	4	2	MA.912.AR.4.3*
	5–6	2	MA.912.AR.4.3*
TEST Practice	7	2	MA.912.F.1.2
Spiral Review			
	8	1	MA.6.NSO.1.3
	9–11	1	MA.912.F.1.6
	12	2	MA.912.F.1.3

*This problem builds toward the standard shown.

Unit 3

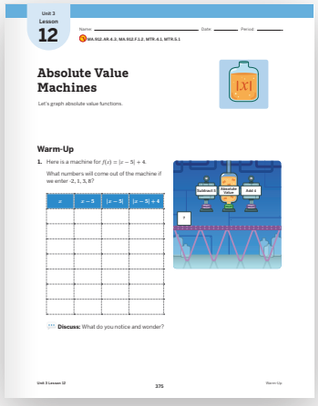
Lesson

12

Absolute Value Machines

Absolute Value Functions, Part 2

Let's graph absolute value functions.


Unit 3
Lesson
12
Absolute Value Machines
Let's graph absolute value functions.
Warm-Up
1. Here is a machine for $f(x) = |x - 5| + 4$.
What numbers will come out of the machine if we enter -2, 1, 3, 8?
Discuss: What do you notice and wonder?
375

Learning Outcomes

Today's Goals

1. **Goal:** Graph an absolute value function.
2. **Language Goal:** Analyze the key features (e.g., minimum) of an absolute value function. **(Reading, Writing, Speaking, and Listening)**

Students graph an absolute value function of the form $f(x) = |x - h| + k$ and analyze its key features. They graph an absolute value function by creating a table of values as well as by identifying the minimum from the structure of the equation. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 11, students were introduced to the absolute value function as the "distance from a number." They evaluated absolute value functions in function notation and interpreted their outputs in context.

Future Learning

In Unit 5, students will apply related reasoning to graph transformations of quadratic functions in vertex form.

Connections

- Students revisit input/output machines as they continue to develop their understanding of absolute value functions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.4.3
Given a table, equation or written description of an absolute value function, graph that function and determine its key features.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.912.F.1.2	**MA.912.AR.7.3**
	MA.912.F.2

Vocabulary

Review Vocabulary

translation

Lesson at a Glance

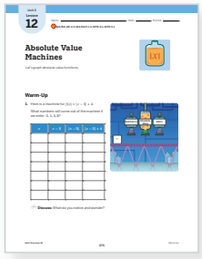
~ 45 min

MA.912.AR.4.3

Warm-Up

Whole Class | 5 min

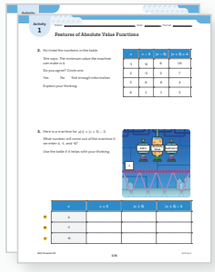
Students use the **Notice and Wonder** routine to make sense of a new machine that evaluates an absolute value function.



Activity 1

Pairs | 10 min

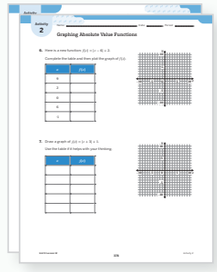
Students make sense of the minimum value for an absolute value graph, and evaluate an absolute value function to create and analyze the key features of a graph.



Activity 2

Pairs | 10 min

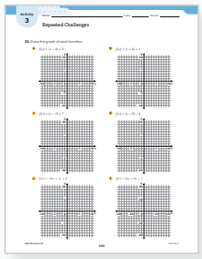
Students graph absolute value functions, and use the structure of the equation to identify its minimum. **MLR7: Compare and Connect** **(MTR.5.1)**



Activity 3

Independent | 10 min

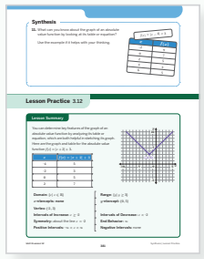
Students develop fluency creating graphs of absolute value functions.



Synthesis

Whole Class | 5 min

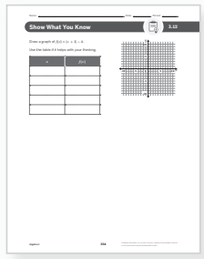
Students synthesize their understanding of connections between the structure of an absolute value equation and its graph. **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating a graph of an absolute value function.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of a new machine that evaluates an absolute value function.

1 Launch

Consider asking, "How is this machine the same as or different from other machines we have seen?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between the function, machine, and the table.

A **Accessibility: Memory and Attention** Invite students to record their ideas on paper.

Invite students to suggest values to input into the machine.

2 Connect

Invite students to share their noticings and wonderings.

Note: If it doesn't come up naturally, consider asking,

- "What happens to a value when it goes through the 'absolute value' portion of the machine?"
- "What kinds of values can we input into this machine? What kinds of values will be output by this machine?"
- "What is the smallest output you can find for this machine? What does that tell us about the graph?"

Unit 3
Lesson
12

Name: _____ Date: _____ Period: _____

MA.912.AR.4.3, MA.912.F.1.2, MTR.4.1, MTR.5.1

Absolute Value Machines

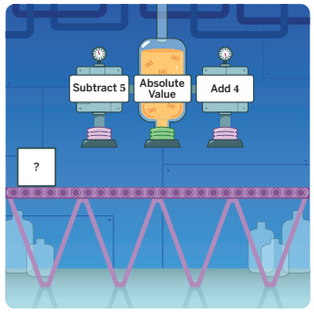
Let's graph absolute value functions.



Warm-Up

1. Here is a machine for $f(x) = |x - 5| + 4$.

What numbers will come out of the machine if we enter -2, 1, 3, 8?



x	$x - 5$	$\lvert x - 5 \rvert$	$\lvert x - 5 \rvert + 4$
-6	-11	11	15
-2			
-1	-6	6	10
1			
3			
4	-1	1	5
8			

Discuss: What do you notice and wonder?

Responses vary.

- I notice that each part of the machine updates the value in the box by performing a step in the operation. First you subtract 5, then you take the absolute value of the number, and finally you add 4.
- I wonder if it's possible to have a final answer of 0 using this machine.

Unit 3 Lesson 12 375 Warm-Up

Pairs | 10 min

Activity 1 Features of Absolute Value Functions

Purpose: Students make sense of the minimum value for an absolute value graph, and evaluate an absolute value function to create and analyze the key features of a graph.

1 Launch

Consider sharing that the function here is the same function students saw in the Warm-Up.

Invite students to think individually before sharing their ideas with a partner.

A **Accessibility: Conceptual Processing** Invite students to revisit the machine from the Warm-Up to support them in making generalizations.

2 Monitor

To support students getting started, consider asking, "When you take the absolute value of a number, what's the minimum value you can get as an output? Where is that value in the table?"

Record responses that include:

- Noticing that 4 is added in the equation and connecting this to the minimum value in the table.
- Noticing that the smallest number the absolute value of anything can be is 0.

Pause to share and celebrate a variety of observed student strategies. It is okay — even desirable — to lack consensus at this stage.

Look for pairs who may need additional support. Offer pairs support, or lead a whole-class discussion if needed.

A **Accessibility: Executive Functioning** Consider chunking the evaluation process by inviting students to use the table in Activity 1.

A **Accessibility: Conceptual Processing** Consider asking, "How would a table for the new function be similar to this table? How would it be different?"

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Features of Absolute Value Functions

2. Kiri tried the numbers in the table.

She says: *The minimum value the machine can make is 4.*

Do you agree? Circle one.

Yes No Not enough information

Explain your thinking.

Responses and explanations vary.

Yes. The smallest absolute value you can get is 0, which occurs when you input 5. Then you add 4, which is the minimum value.

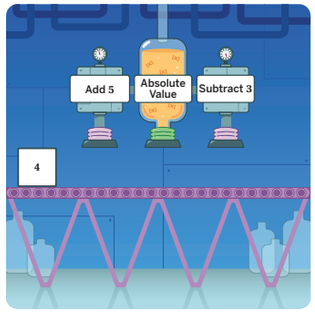
Not enough information. You can enter any number into the machine, but since you can never try them all, you can't be sure that the minimum value is 4.

x	$x-5$	$\lvert x-5\rvert$	$\lvert x+5\rvert+4$
-1	-6	6	10
2	-3	3	7
5	0	0	4
6	1	1	5

3. Here is a machine for $g(x) = |x + 5| - 3$.

What number will come out of the machine if we enter 4, -1, and -6?

Use the table if it helps with your thinking.



	x	$x+5$	$\lvert x+5\rvert$	$\lvert x+5\rvert-3$
a	4	9	9	6
b	-1	4	4	1
c	-6	-1	1	-2

Unit 3 Lesson 12 376 Activity 1

Pairs | 10 min

Activity 1 Features of Absolute Value Functions (continued)

Purpose: Students make sense of the minimum value for an absolute value graph, and evaluate an absolute value function to create and analyze the key features of a graph.

2 Monitor

Consider sharing that this is the same function from the Problem 3.

ELL **English Language Learners** Consider reviewing an anchor chart that displays the class definitions of vocabulary from this unit. **(Reading and Writing)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., The minimum of the function is at _____.). **(Writing and Speaking)**

Invite student responses that use a variety of the given terms and include different levels of precision in their response to share during the Connect.

ELL **English Language Learners** Invite students to read each option and paraphrase it. Clarify unknown words. **(Reading)**

D **Differentiation: (Stretch)** Consider sharing that equations for absolute value functions can also be written using piecewise notation. Invite students to write the equation for $g(x)$ using piecewise notation.

3 Connect

Invite students to share their responses.

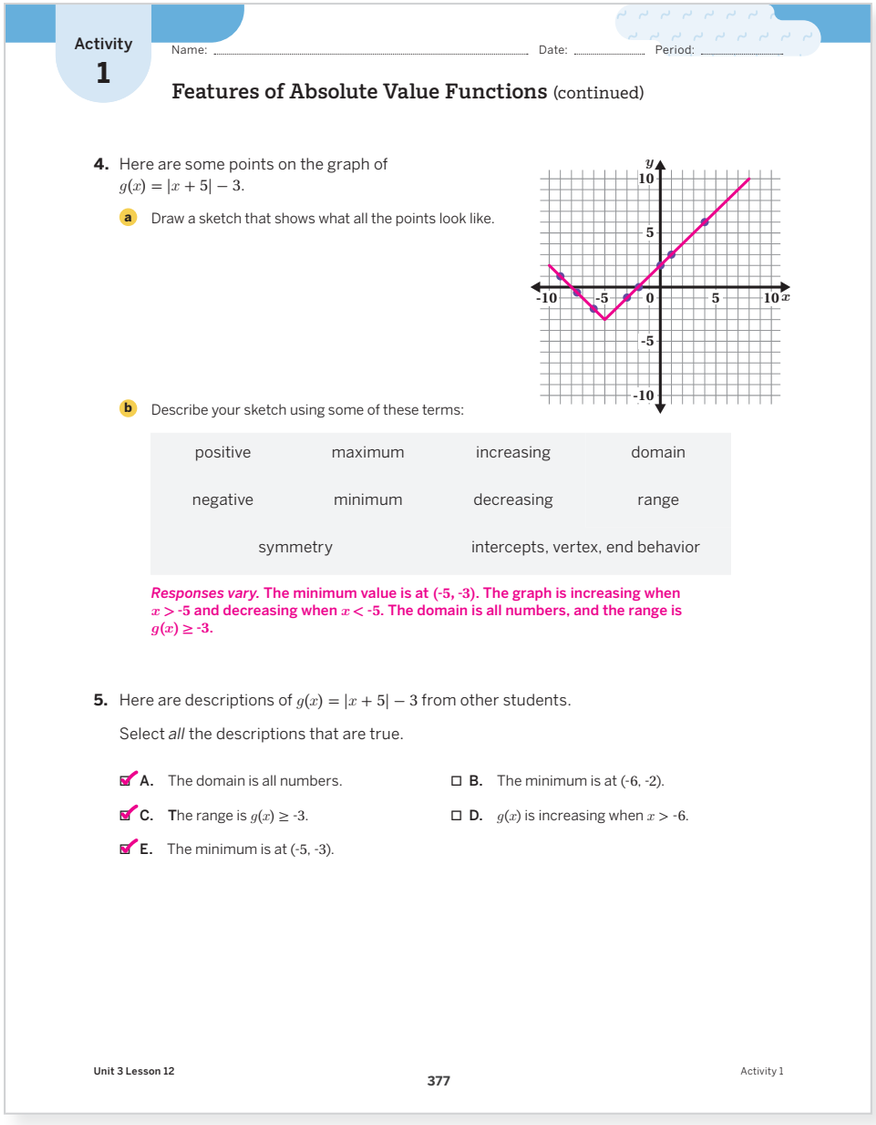
Consider asking,

- "What does ___'s description tell you about the graph?"
- "Is there another way we could say what ___ said?"

Invite students to revise their responses to help them describe the sketch of the absolute value function more precisely. **(MTR.4.1)**

Note: If time allows, consider comparing the $g(x)$ to $f(x) = |x|$ by asking:

- "How does +5 affect the graph?"
- "The -3?"
- "What changes about the graph? What does not change?"



Activity 1

Name: _____ Date: _____ Period: _____

Features of Absolute Value Functions (continued)

4. Here are some points on the graph of $g(x) = |x + 5| - 3$.

a. Draw a sketch that shows what all the points look like.

b. Describe your sketch using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry		intercepts, vertex, end behavior	

Responses vary. The minimum value is at (-5, -3). The graph is increasing when $x > -5$ and decreasing when $x < -5$. The domain is all numbers, and the range is $g(x) \geq -3$.

5. Here are descriptions of $g(x) = |x + 5| - 3$ from other students.

Select *all* the descriptions that are true.

- ☑ **A.** The domain is all numbers.
- ☐ **B.** The minimum is at (-6, -2).
- ☑ **C.** The range is $g(x) \geq -3$.
- ☐ **D.** $g(x)$ is increasing when $x > -6$.
- ☑ **E.** The minimum is at (-5, -3).

Unit 3 Lesson 12 377 Activity 1

Pairs | 10 min

Activity 2 Graphing Absolute Value Functions

Purpose: Students graph absolute value functions, and use the structure of the equation to identify its minimum. **(MTR.5.1)**

1 Launch

Consider asking, "What is the same and different about this function and $g(x) = |x + 5| - 3$ from Activity 1?"

2 Monitor

To support students getting started, consider asking,

- "What is the first thing that happens to 9 when you input that value into the function?"
- "What could go in each column of the table to help you calculate the outputs?"

A **Accessibility: Conceptual Processing** Invite students to revisit their work on the previous table to help apply concepts to new situations. Consider asking, "How would a table for the new function be similar to the other table? How would it be different?"

Record a variety of strategies.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking: • "What would go in each column header to help you complete the table?" • "What kind of numbers might be helpful to input?"
Make a graph that appears to be a line.	**Support** Celebrate their persistence. Consider asking: • "Can you check the output of a point on the other side of the graph to see if it matches your prediction?" • "What number could we input to find the minimum value?" **(MTR.5.1)**
Use effective strategies to graph the function like: • Use the table to determine points on $j(x)$. • Look for the minimum value first before making a table. • Use the symmetry of the graph to find additional points.	Select and sequence these to share during the Connect. **Strengthen** Invite students to compare their strategy with a classmate's. **Stretch** Invite students to generalize how to graph any absolute value function of the form $f(x) = |x + a| + b$.

Activity 2 continued >

Activity 2

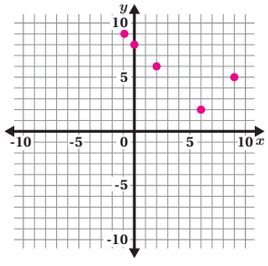
Name: _____ Date: _____ Period: _____

Graphing Absolute Value Functions

6. Here is a new function: $f(x) = |x - 6| + 2$.

Complete the table and then plot the graph of $f(x)$.

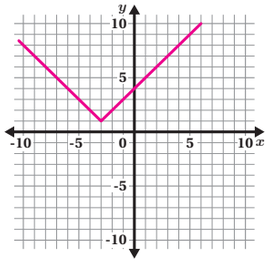
x	$f(x)$
9	5
2	6
0	8
6	2
-1	9



7. Draw a graph of $j(x) = |x + 3| + 1$.

Use the table if it helps with your thinking.

x	$j(x)$



Unit 3 Lesson 12 378 Activity 2

Pairs | 10 min

Activity 2 Graphing Absolute Value Functions (continued)

Purpose: Students graph absolute value functions, and use the structure of the equation to identify its minimum. **(MTR.5.1)**

2 Monitor

Look for students who notice that Tiana plotted the points correctly, and included the minimum value of the function.

D **Differentiation: (Stretch)** Consider asking, "What points would you plot first on the graph of $p(x) = |x - 4| + 3$? Why?"

Look for a variety of student strategies. Some students may:

- Notice the connection between the minimum and the equation.
- Reference symmetry or talk about making 0 inside the absolute value signs.

3 Connect

Share observed student graphing strategies.

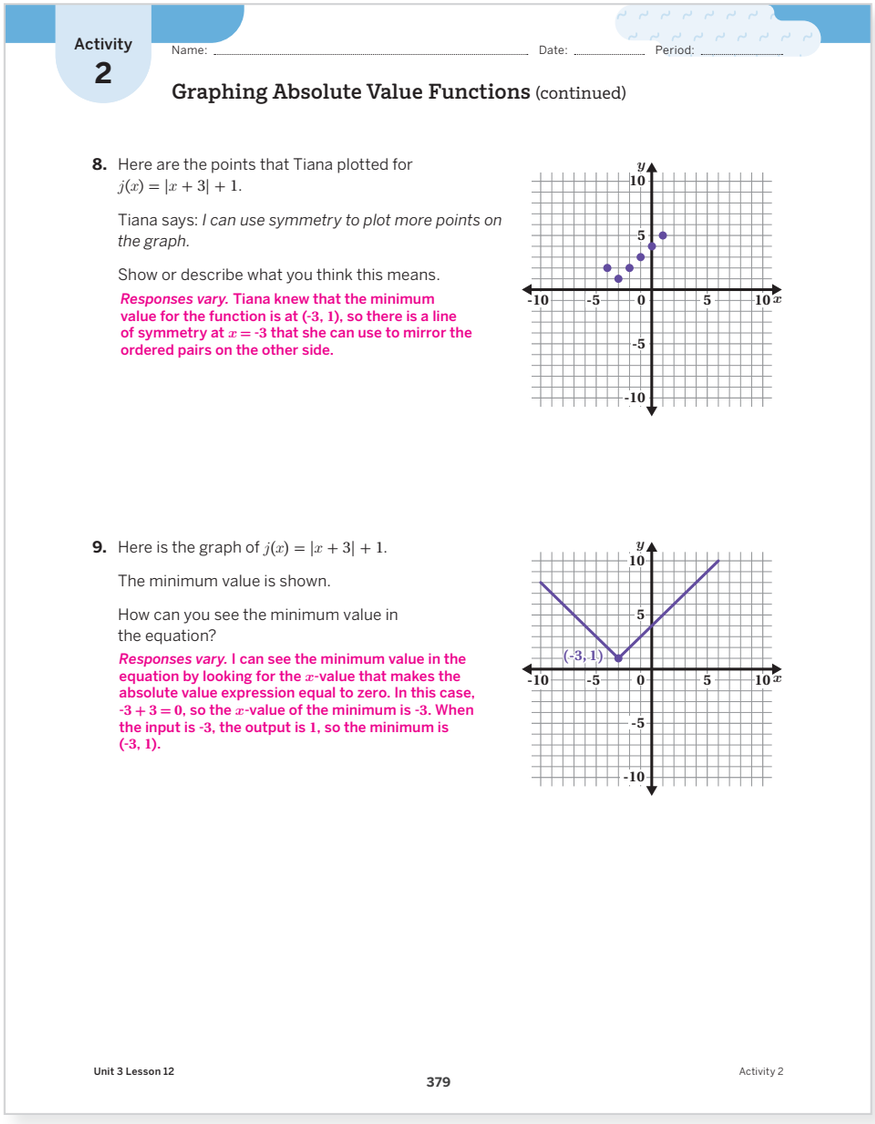
MLR **MLR7: Compare and Connect** Invite students to analyze and compare different strategies for graphing the absolute value function. Students may use a table of values, the structure of the equation, or symmetry. Highlight multiple strategies that students use, helping them make connections between the different strategies. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

To reveal the Key Takeaway, consider asking:

- "Why does it make sense that the minimum is at $(-3, 1)$?"
- "How can you know the minimum based on the equation?" **(MTR.5.1)**

Note: If time allows, consider comparing the $j(x) = |x + 3| + 1$ to $f(x) = |x|$ by asking:

- "How does $+3$ affect the graph?"
- "How does $+1$ affect the graph?"
- "What changes about the graph? What does not change?"



Activity 2

Name: _____ Date: _____ Period: _____

Graphing Absolute Value Functions (continued)

8. Here are the points that Tiana plotted for $j(x) = |x + 3| + 1$.

Tiana says: *I can use symmetry to plot more points on the graph.*

Show or describe what you think this means.

Responses vary. Tiana knew that the minimum value for the function is at (-3, 1), so there is a line of symmetry at $x = -3$ that she can use to mirror the ordered pairs on the other side.

9. Here is the graph of $j(x) = |x + 3| + 1$.

The minimum value is shown.

How can you see the minimum value in the equation?

Responses vary. I can see the minimum value in the equation by looking for the x-value that makes the absolute value expression equal to zero. In this case, $-3 + 3 = 0$, so the x-value of the minimum is -3. When the input is -3, the output is 1, so the minimum is (-3, 1).

Unit 3 Lesson 12 379 Activity 2

Key Takeaway: The minimum of an absolute value function can be identified using the structure of the equation. The x-value of the minimum is the number that makes the expression inside the absolute value equal to 0. The y-value is the corresponding output. For example, a function with the equation $f(x) = |x - h| + k$ would have a minimum at (h, k).

Independent | 10 min

Activity 3 Repeated Challenges

Purpose: Students develop fluency creating graphs of absolute value functions.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Students will need a partner for this activity. They will solve and compare their graphs.

Accessibility: Memory and Attention Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

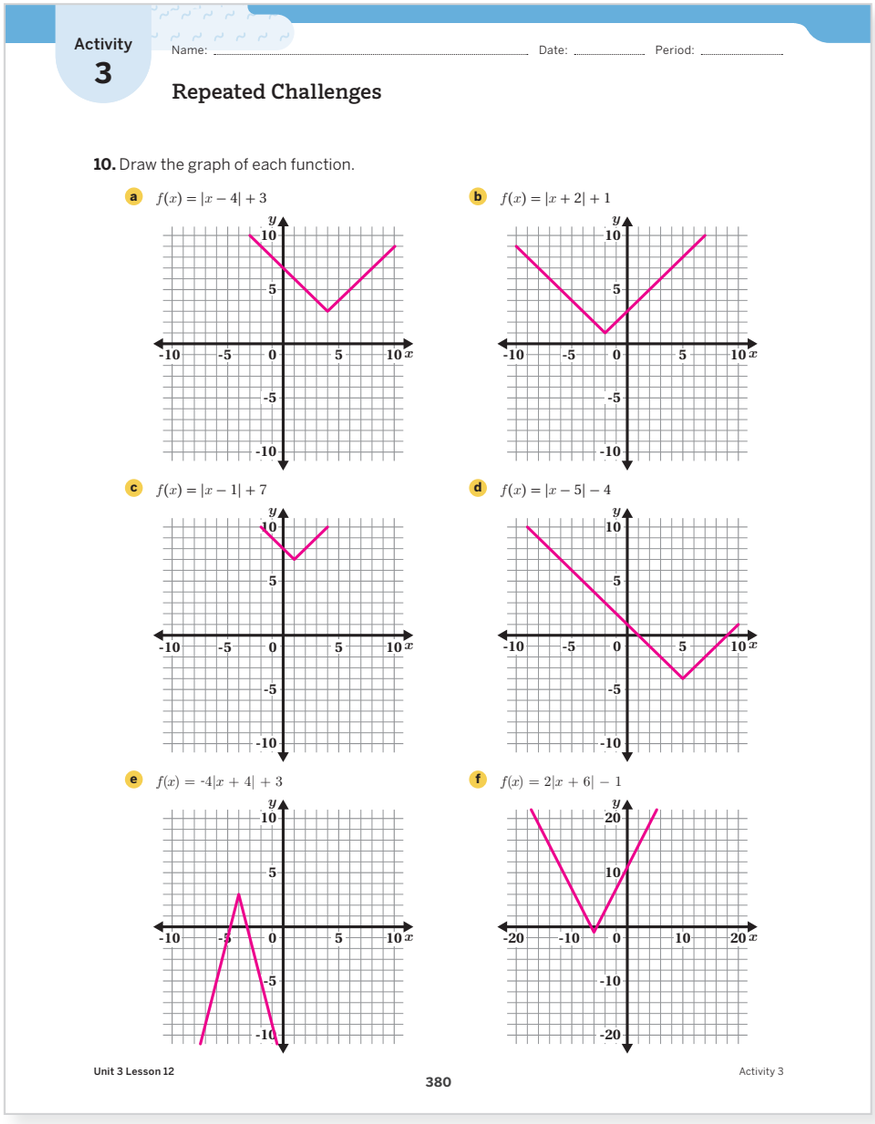
Listen for strategies students use in discussing how to graph the functions, such as:

- Making a table.
- Determining the minimum value.
- Using symmetry.
- Identifying connections between the structure of the equation and the key function features. **(MTR.5.1)**
- Recognizing patterns in how the added constants result in translations of the given function (e.g., students notice to translate the given function 4 units right and 3 units up from the structure of this equation $f(x) = |x - 4| + 3$).

Accessibility: Conceptual Processing Invite students to share how each problem is different from the previous one to help apply concepts to new situations.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 3

Name: _____ Date: _____ Period: _____

Repeated Challenges

10. Draw the graph of each function.

a. $f(x) = |x - 4| + 3$

b. $f(x) = |x + 2| + 1$

c. $f(x) = |x - 1| + 7$

d. $f(x) = |x - 5| - 4$

e. $f(x) = -4|x + 4| + 3$

f. $f(x) = 2|x + 6| - 1$

Unit 3 Lesson 12 380 Activity 3

Whole Class | 5 min

Synthesis

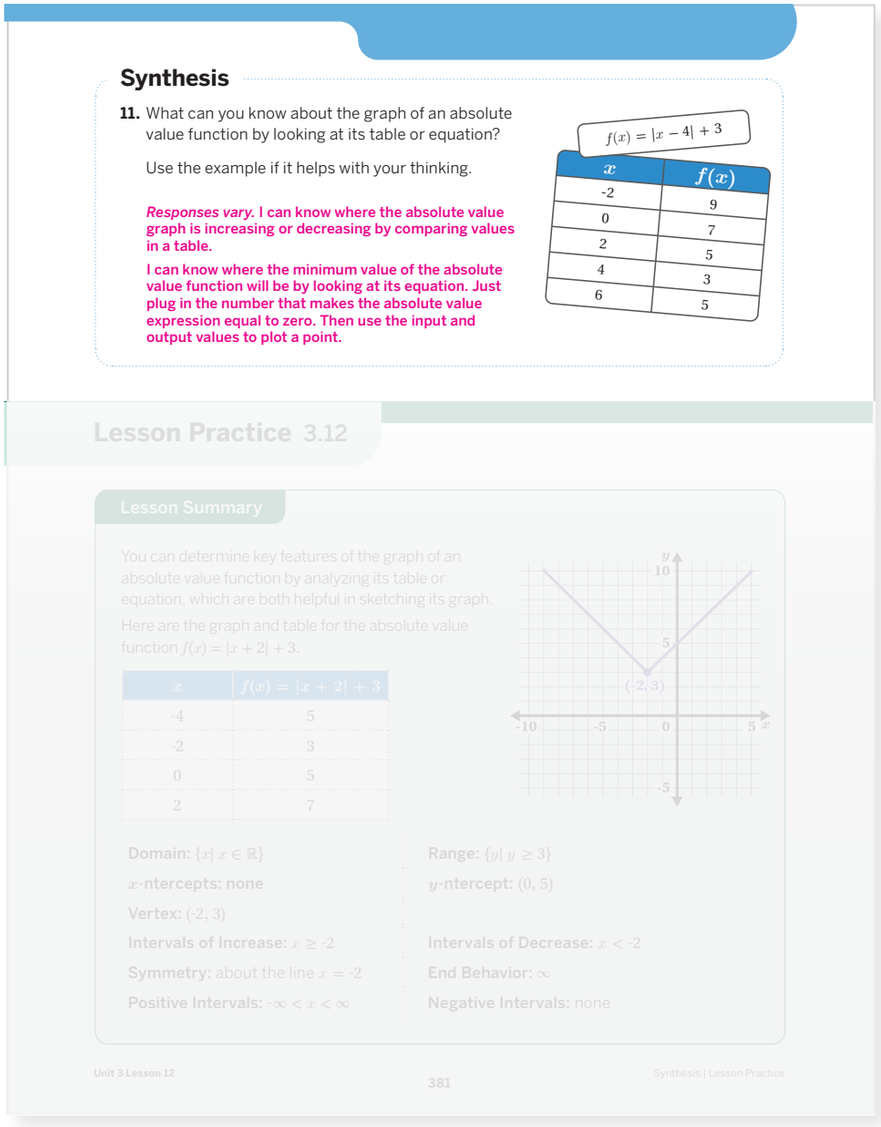
Purpose: Students synthesize their understanding of connections between the structure of an absolute value equation and its graph. **(MTR.5.1)**

Invite students to respond independently, and then share their thinking with a partner.

Record and share a variety of ideas, such as:
- Comparing values in a table to know where the absolute value graph is increasing or decreasing.
- Identifying the minimum value by using the structure of the equation. **(MTR.5.1)**
- Finding the input that makes the absolute value expression equal to zero.

Consider asking, "Can you see the minimum value in the equation, the table, and the graph?"

Note: If time allows, invite students to make their response stronger and clearer. **(MTR.4.1)**



Synthesis

11. What can you know about the graph of an absolute value function by looking at its table or equation?

Use the example if it helps with your thinking.

Responses vary. I can know where the absolute value graph is increasing or decreasing by comparing values in a table.

I can know where the minimum value of the absolute value function will be by looking at its equation. Just plug in the number that makes the absolute value expression equal to zero. Then use the input and output values to plot a point.

$f(x) = |x - 4| + 3$

x	$f(x)$
-2	9
0	7
2	5
4	3
6	5

Lesson Practice 3.12

Lesson Summary

You can determine key features of the graph of an absolute value function by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = |x + 2| + 3$.

x	$f(x) = \|x + 2\| + 3$
-4	5
-2	3
0	5
2	7

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 3\}$

x-intercepts: none

y-intercept: $(0, 5)$

Vertex: $(-2, 3)$

Intervals of Increase: $x \geq -2$

Intervals of Decrease: $x < -2$

Symmetry: about the line $x = -2$

End Behavior: ∞

Positive Intervals: $-\infty < x < \infty$

Negative Intervals: none

Unit 3 Lesson 12 381 Synthesis | Lesson Practice

Lesson Takeaway: Graphs of absolute value functions have a V-shape. The structure of the function can help to identify the minimum value and make its graph.

Independent | 5 min

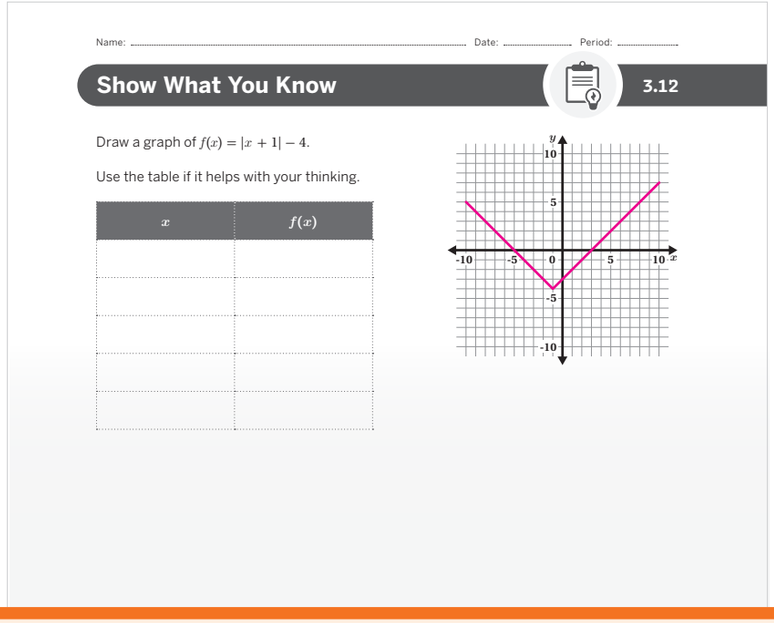
Show What You Know

Purpose: Students demonstrate their understanding by creating a graph of an absolute value function.

Today's Goals

1. **Goal:** Graph an absolute value function.
2. **Language Goal:** Analyze the key features (e.g., minimum) of an absolute value function. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.12

Draw a graph of $f(x) = |x + 1| - 4$.

Use the table if it helps with your thinking.

x	$f(x)$

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support creating a graph of an absolute value function:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problem 8**, as a class and discuss how to purposefully choose x-values to use in the table to generate ordered pairs for the function.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In what ways are students taking risks when trying out new strategies for graphing absolute value functions? What questions might encourage students in exploring new strategies such as using the structure of equations to create a graph after successfully using a table?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. What can you know about the graph of an absolute value function by looking at its table or equation?

Use the example if it helps with your thinking.

$f(x) = |x - 4| + 3$

x	$f(x)$
-2	9
0	7
2	5
4	3
6	5

Responses vary. I can know where the absolute value graph is increasing or decreasing by comparing values in a table.

I can know where the minimum value of the absolute value function will be by looking at its equation. Just plug in the number that makes the absolute value expression equal to zero. Then use the input and output values to plot a point.

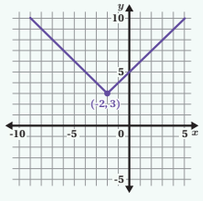
Lesson Practice 3.12

Lesson Summary

You can determine key features of the graph of an absolute value function by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = |x + 2| + 3$.

x	$f(x) = \|x + 2\| + 3$
-4	5
-2	3
0	5
2	7



Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 3\}$

x-ntercepts: none

y-ntercept: $(0, 5)$

Vertex: $(-2, 3)$

Intervals of Increase: $x \geq -2$

Intervals of Decrease: $x < -2$

Symmetry: about the line $x = -2$

End Behavior: ∞

Positive Intervals: $-\infty < x < \infty$

Negative Intervals: none

Lesson Practice 3.12

Name: _____ Date: _____ Period: _____

Problems 1–3: Write each expression as a single integer.

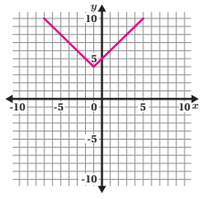
1. $|-4|$
4

2. $|7| - 2$
5

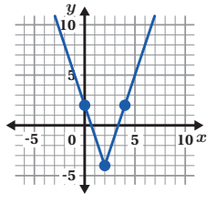
3. $|-8| + 1$
9

4. Graph $g(x) = |x + 1| + 4$. Use the table if it helps with your thinking.

x	$g(x)$
-3	6
-1	4
0	5
2	7
3	8



Problems 5–9: The function $g(x) = 3|x - 2| - 4$ is represented by the graph.



5. Identify the domain.
all real numbers

6. Identify the range.
$[-4, \infty)$

7. Identify the minimum value/vertex.
(2, -4)

8. Identify the increasing interval.
$(2, \infty)$

9. Identify the decreasing interval.
$(-\infty, 2)$

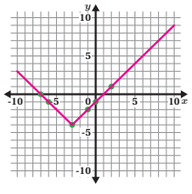
Lesson Practice 3.12

Name: _____ Date: _____ Period: _____

10. Here are some points on the graph of $h(x) = |x + 3| - 4$.

- Sketch a graph of $h(x)$.
- Describe the graph using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry	intercepts, vertex, end behavior		



Responses vary. This graph has a minimum of -4 when $x = -3$. The domain is all numbers and the range is all numbers greater than or equal to -4. The graph is increasing when $x > -3$ and decreasing when $x < -3$.

Test Practice

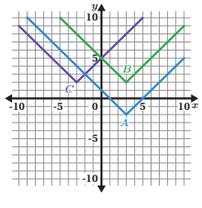
11. Choose the function that matches graph A.

A. $f(x) = |x + 3| + 2$

B. $g(x) = |x - 3| - 2$

C. $h(x) = |x - 3| + 2$

D. $k(x) = |x + 3| - 2$



Spiral Review

Problems 12–14: For each square root, write which two consecutive whole numbers the value is between.

12. $\sqrt{14}$
3 and 4

13. $\sqrt{60}$
7 and 8

14. $\sqrt{88}$
9 and 10

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.AR.4.3*
	4	2	MA.912.F.1.2, MA.912.AR.4.3
	5–9	2	MA.912.AR.4.3
	10	2	MA.912.AR.4.3
TEST Practice	11	2	MA.912.AR.4.3
Spiral Review			
Fluency	12–14	2	MA.8.NSO.1.1

*This problem builds toward the standard shown.

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students made sense of functions represented as graphs, tables, equations, and situations. They interpreted statements in function notation and described functions using their key features.

Career Connection | Meet Warren Washington

Invite students to read about the temperatures of the North and South Poles and how climate scientists study long-term weather patterns. Then invite them to read about Nobel Prize winner and presidential advisor Warren Washington.

Consider asking:

- "Why do you think the South Pole is a lot colder than the North Pole?" For interested students, consider mentioning that, according to NASA, the South Pole is colder because: (1) it is located over sea ice while the North Pole is located over land ice (Antarctica), (2) it has stronger winds than the North Pole, and (3) Antarctica is very dry with ice that can be 3 miles in thickness!

B.E.S.T. Benchmark Connection

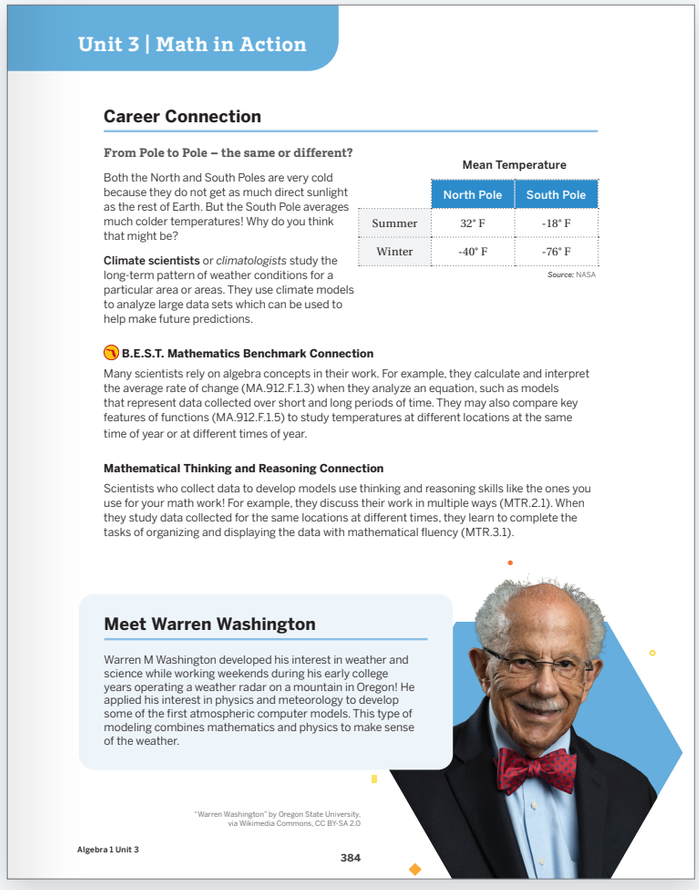
Consider asking:

- "What other math concepts have you studied in Units 1–3 that you could apply to the data shown?" Sample response: You can write inequalities to compare the mean temperatures at different times of year. (MA.912.AR.2.6)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "Suppose two teams of scientists in different parts of Florida are conducting research and gathering data about the mangroves in Everglades National Park. How would they benefit from utilizing the strategies of Mathematical Thinking and Reasoning standard MTR.4.1?" Sample response: They could compare their research and data to see if one team has a more efficient method than the other.



Unit 3 | Math in Action

Career Connection

From Pole to Pole – the same or different?

Both the North and South Poles are very cold because they do not get as much direct sunlight as the rest of Earth. But the South Pole averages much colder temperatures! Why do you think that might be?

Mean Temperature

	North Pole	South Pole
Summer	32° F	-18° F
Winter	-40° F	-76° F

Source: NASA

Climate scientists or *climatologists* study the long-term pattern of weather conditions for a particular area or areas. They use climate models to analyze large data sets which can be used to help make future predictions.

B.E.S.T. Mathematics Benchmark Connection

Many scientists rely on algebra concepts in their work. For example, they calculate and interpret the average rate of change (MA.912.F.1.3) when they analyze an equation, such as models that represent data collected over short and long periods of time. They may also compare key features of functions (MA.912.F.1.5) to study temperatures at different locations at the same time of year or at different times of year.

Mathematical Thinking and Reasoning Connection

Scientists who collect data to develop models use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work in multiple ways (MTR.2.1). When they study data collected for the same locations at different times, they learn to complete the tasks of organizing and displaying the data with mathematical fluency (MTR.3.1).

Meet Warren Washington

Warren M Washington developed his interest in weather and science while working weekends during his early college years operating a weather radar on a mountain in Oregon! He applied his interest in physics and meteorology to develop some of the first atmospheric computer models. This type of modeling combines mathematics and physics to make sense of the weather.

"Warren Washington" by Oregon State University, via Wikimedia Commons, CC BY-SA 2.0

Algebra 1 Unit 3 384

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Interpret a graph in context.	Lesson 5	2	MA.912.F.1.2, MA.912.F.1.6 MTR.3.1
2	Compare functions using statements in function notation.	Lesson 8	2	MA.912.F.1.2, MA.912.F.1.5
3	Determine the x-coordinates of the points of intersection of two functions.	Lesson 8	1	MA.912.AR.9.1
4	Calculate and interpret average rate of change.	Lesson 7	3	MA.912.F.1.3 MTR.6.1
5a	Determine the equation from the graph of an absolute value function.	Lesson 12	2	MA.912.AR.4.3
5b	Identify key features from a graph.	Lesson 6	2	MA.912.F.1.6
6a–6c	Identify and interpret key features from the graph of a function.	Lesson 6	1	MA.912.F.1.6
6d–6e	Determine and interpret the domain or range of a function.	Lesson 10	2	MA.912.F.1.5

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

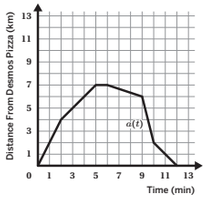
Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 3

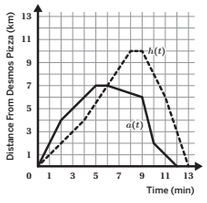
Adhira and Habib deliver pizzas for Desmos Pizza. They leave to deliver pizzas at the same time. $a(t)$ and $h(t)$ represent Adhira's and Habib's distance from Desmos Pizza (in kilometers) after t minutes.

1. Here is a graph of $a(t)$. *Select all* the true statements.



- ☑ **A.** Adhira's distance is 7 kilometers after 5 minutes.
- ☑ **B.** Adhira is the same distance from Desmos Pizza at 4 minutes and 9 minutes.
- ☐ **C.** $a(10) > a(2)$
- ☐ **D.** $a(0) = 12$
- ☑ **E.** $a(12) = 0$

2. Here is a graph of $a(t)$ and $h(t)$. Which statement is true?



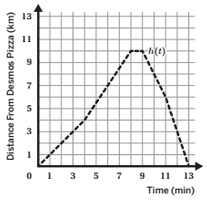
- (A.) $a(2) > h(2)$
- **B.** $a(2) < h(2)$
- **C.** $a(2) = h(2)$
- **D.** $a(2) = 2$

3. Write one value of t where $a(t) = h(t)$.

$t = 0$ or $t = 6$

4. Calculate Habib's *average rate of change* from 4 to 9 minutes.

1.2 kilometers per minute (or equivalent)



Explain what your answer means in this situation.

***Explanations vary*. Between 4 and 9 minutes, Habib traveled an average of 1.2 kilometers per minute.**

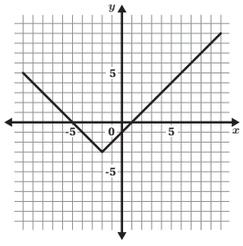
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Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 3

5. This is a graph of the function $f(x)$.



a Which equation represents this function?

- **A.** $f(x) = |x| - 1$
- **B.** $f(x) = |x - 3| + 2$
- **C.** $f(x) = |x - 2| - 3$
- (D.) $f(x) = |x + 2| - 3$

b When is $f(x)$ negative?

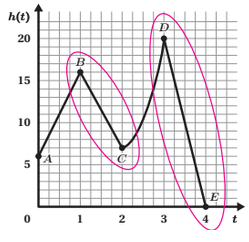
- **A.** $x < 0$
- **B.** $x < -2$
- (C.) $-5 < x < 1$
- **D.** $-2 < x < 1$

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 3

6. Here is a graph of $h(t)$, the height of a kite (in feet) after t minutes.



a Which point represents the minimum of $h(t)$? Circle one.

Point A Point B Point C

Point D (Point E)

b Which point represents when the kite is at its greatest height off the ground? Circle one.

Point A Point B Point C

(Point D) Point E

c Circle or describe *all* the intervals of the function that are decreasing.

***Responses vary*. Sample shown on graph.**
- **From B to C and from D to E.**
- **$1 < t < 2$ and $3 < t < 4$**

d Fill in the blanks to describe the *domain* of $h(t)$.

0 $\leq t \leq$ **4**

e Explain what the domain means in this situation.

***Responses vary*. The domain represents the minutes the kite was above the ground.**

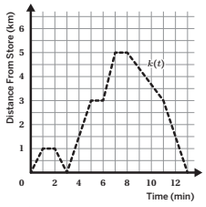
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End-of-Unit Assessment

Unit 3

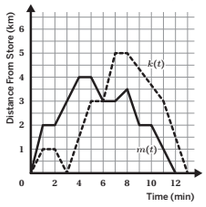
Kwasi and Mio deliver groceries for Descart. They leave to deliver groceries at the same time. $k(t)$ and $m(t)$ represent Kwasi's and Mio's distances from a store (in kilometers) after t minutes.

1. Here is a graph of $k(t)$. *Select all* the true statements.



- ☐ **A.** Kwasi's distance is 8 kilometers after 5 minutes.
- ☑ **B.** Kwasi is the same distance from the store at 5 minutes and 11 minutes.
- ☐ **C.** $k(11) > k(7)$
- ☑ **D.** $k(13) = 0$
- ☐ **E.** $k(0) = 3$

2. Here is a graph of $k(t)$ and $m(t)$. Which statement is true?

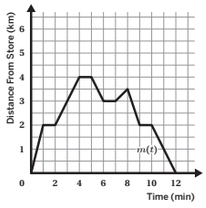


- **A.** $k(5) > m(5)$
- (B.) $k(5) < m(5)$
- **C.** $k(5) = m(5)$
- **D.** $k(5) = 5$

3. Write one value of t where $k(t) = m(t)$.

$t = 0$ or $t = 6$

4. Here is a graph of $m(t)$. Calculate Mio's average *rate of change* from 1 to 5 minutes.



0.5 kilometers per minute (or equivalent)

Explain what your answer means in this situation.

***Explanations vary*. Between 1 and 5 minutes, Mio traveled an average of 0.5 kilometers per minute.**

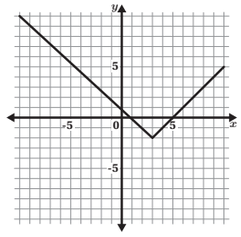
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Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 3

5. This is a graph of the function $f(x)$.



a Which equation represents this function?

- (A.) $f(x) = |x - 3| - 2$
- **B.** $f(x) = |x + 3| - 2$
- **C.** $f(x) = |x + 2| - 3$
- **D.** $f(x) = |x - 2| + 3$

b When is $f(x)$ increasing?

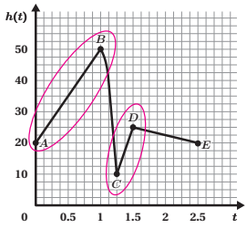
- **A.** $x > -2$
- (B.) $x > 3$
- **C.** $x > 5$
- **D.** $1 < x < 5$

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 3

6. Here is a graph of $h(t)$, the height above ground of a roller coaster (in meters) after t minutes.



a Which point represents the maximum of $h(t)$? Circle one.

Point A (Point B) Point C

Point D Point E

b Which point represents when the roller coaster is at its lowest height above the ground? Circle one.

Point A Point B (Point C)

Point D Point E

c Circle or describe *all* the intervals of the function that are increasing.

***Explanations vary*. Sample shown on graph.**

- **From A to B and from C to D.**
- **$0 < t < 1$ and $1.25 < t < 1.5$**

d Fill in the blanks to describe the *range* of $h(t)$.

10 $\leq h(t) \leq$ **50**

e Explain what the range means in this situation.

***Responses vary*. The range represents the heights at which the roller coaster was above the ground.**

D Differentiation (End-of-Unit Assessment)

Note: Consider inviting students to revise the corresponding assessment items after responding to student thinking.

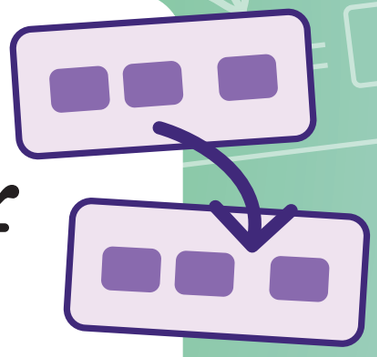
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
• Interpret statements in function notation. **(Lessons 1–4)**	1	**Support** • **Mini-Lesson:** Evaluating Function Notation • **Teacher Move:** Consider revisiting Lesson 5 (*Function Carnival*)
		Stretch • **You're invited to explore more:** • Lesson 5 (*Function Carnival*)

• Describe functions using their key features, including domain, range, and average rate of change. **(Lessons 5–10)**	4	**Support**
		• **Mini-Lessons:**
		  • Function Notation
		  • Interpreting Slope and y-Intercept in Context
		• **Teacher Move:** Consider revisiting Lesson 7 (*Planes, Trains, and Automobiles*)

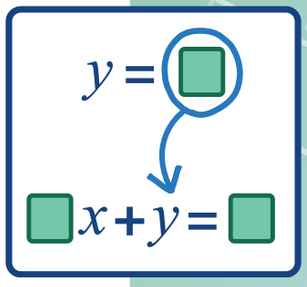
		Stretch • **You're invited to explore more:** • Lesson 7 (*Planes, Trains, and Automobiles*)
	5b, 6a–6c	**Support** • **Mini-Lesson:** Writing Domain and Range with Inequalities • **Teacher Move:** Consider revisiting Lesson 6 (*Craft-a-Graph*)
		Strengthen • **Repeated Challenges:** • Lesson 6 (*Craft-a-Graph*)
	6d–6e	**Support** • **Teacher Move:** Consider revisiting Lesson 10 (Elevator Stories)
		Stretch • **You're invited to explore more:** • Lesson 10 (*Elevator Stories*)
• Compare functions using function notation and their key features. **(Lessons 7–8)**	2, 3	**Support** • **Mini-Lesson:** Writing Domain and Range with Inequalities • **Teacher Move:** Consider revisiting Lesson 8 (*Elevator Stories*)
• Interpret, evaluate, graph, and write equations of absolute value functions. **(Lessons 11–12)**	5a	**Support** • **Mini-Lesson:** Absolute Value Functions
		Strengthen • **Repeated Challenges:** • Lesson 12 (*Absolute Value Machines*)
		Stretch • **Teacher Move:** Invite students to use a graphing calculator to create a design using the new types of functions they've learned about. • **You're invited to explore more:** • Lesson 11 (*What's Your Score?*)

UNIT 4

Systems of Linear Equations and Inequalities



Students write and solve systems of linear equations and inequalities, interpreting their solutions in context. They explore the connections between solving systems symbolically and graphically and discuss how they can strategically choose solving strategies for specific systems.



Essential Questions

- How can you solve systems of equations and inequalities symbolically and graphically?
- How can constraints be represented using systems of equations or inequalities?

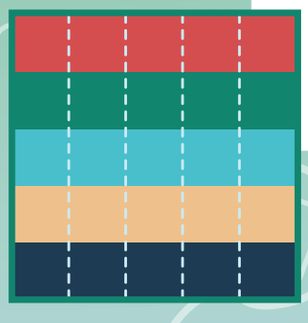
Sub-Unit Goals

Sub-Unit 1

- Solve systems of linear equations using elimination, substitution, and graphing.
- Write systems of linear equations to represent constraints or meet given criteria.
- Interpret solutions to systems of equations in context.

Sub-Unit 2

- Graph systems of linear inequalities and identify solutions.
- Write systems of linear inequalities to represent constraints or meet given criteria.
- Interpret solutions to systems of inequalities in context.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **(MA.912.AR.9.1)**

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. **(MA.912.AR.9.6)**

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **(MA.912.AR.9.4)**

Mathematical Thinking and Reasoning Standards

- Students represent constraints with systems of equations and inequalities and interpret their solutions in context. **(MTR.2.1)**
- Students use the structures of equations and inequalities to strategically choose solving methods. **(MTR.5.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.4.1, MTR.7.1**

Coherence Telling a story within and across grades

In This Unit

Students solve systems of linear equations and inequalities.

Prior Learning

- **Grade 8**: Solve one-variable equations **(MA.8.AR.2.1)**
- **Grade 8**: Solve systems of linear equations **(MA.8.AR.4.3)**
- **Algebra 1**: Write and solve one-variable equations and inequalities **(MA.912.AR.2.1, MA.912.AR.2.7)**
- **Algebra 1**: Write and graph linear equations and inequalities **(MA.912.AR.2.6)**

Future Learning

- **Algebra 1,** Unit 5: Write and evaluate exponential functions **(MA.912.AR.5.4, MA.912.F.1.2)**
- **Algebra 1,** Unit 7: Solve quadratic equations **(MA.912.AR.3.1)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Conceptual Understanding

In **Lesson 4**, students develop conceptual understanding of what no solution and infinite solutions means in the context of two lizards moving.

Strategic Competence

In **Lesson 5**, students develop strategic competence as they solve systems of equations using different solving methods.

Procedural Fluency

In **Lesson 9**, students develop procedural fluency as they graph the solution regions for a variety of systems of inequalities.

Adaptive Reasoning

In **Lesson 10**, students develop adaptive reasoning as they think logically about how systems can be used in real-world situations.

Application

In **Lesson 10**, students apply what they've learned about representing constraints with systems of inequalities to model a real-world situation.

Productive Disposition

In **Lesson 7**, students have opportunities to develop a productive disposition as they use their creativity to design quilts to represent constraints.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.7.AR.2.2 MA.8.AR.3.4 MA.8.AR.4.3 MA.912.AR.9.4 MA.912.AR.9.6
MTR.2.1 MTR.3.1 MTR.7.1

Sub-Unit 1

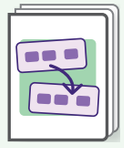


1 Eliminating Shapes

Introduction to Elimination

Solve systems of equations by adding or subtracting the equations to eliminate a variable.

MA.912.AR.9.1
MTR.2.1 MTR.3.1 MTR.4.1 MTR.5.1

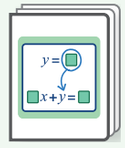


2 Process of Elimination

Elimination Using Equivalent Equations

Solve systems of equations by elimination (multiplying equation(s) by a factor and then adding or subtracting equations to create an equation with only one variable).

MA.912.AR.9.1
MTR.2.1 MTR.3.1 MTR.6.1 MTR.7.1



3 Solution by Substitution

Solving Systems by Substitution

Use substitution to solve systems of equations.

MA.912.AR.9.1
MTR.2.1 MTR.3.1 MTR.6.1 MTR.7.1

Sub-Unit 2



7 Quilts

Introduction to Systems of Inequalities

Make sense of a system of inequalities in a quilting context and explore what it means to be a solution to a system of inequalities.

MA.912.AR.9.4
MTR.2.1 MTR.3.1 MTR.6.1 MTR.7.1



8 Seeking Solutions

Solutions to Systems of Inequalities

Explain that the solution region is where two graphs overlap and discuss strategies for how to determine the location of the solution region.

MA.912.AR.9.4 MA.912.AR.9.6
MTR.2.1 MTR.3.1 MTR.6.1 MTR.7.1



9 Boundaries and Shading

Graphing Systems of Inequalities

Graph systems of linear inequalities by hand and identify the solution region.

MA.912.AR.9.4 MA.912.AR.9.6
MTR.4.1 MTR.5.1 MTR.6.1



10 Restaurant Meals

Using Systems of Inequalities to Make Decisions

Use systems of inequalities representing real-world constraints to make sense of a situation in society: preparing meals for a community.

MA.912.AR.9.4 MA.912.AR.9.6
MTR.2.1 MTR.3.1 MTR.6.1 MTR.7.1

Pacing: 13 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 10 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2



4 Lizard Lines

Graphing Systems of Linear Equations

Use the graphing calculator to solve systems of equations and use the structure of equations to determine if a system will have zero, one, or infinite solutions.

MA.912.AR.9.1
MTR.2.1 MTR.4.1 MTR.5.1



5 City Development

Solving Graphically and Symbolically

Solve systems of equations using multiple methods and interpret the solution in context.

MA.912.AR.9.1 MA.912.AR.9.6
MTR.1.1 MTR.6.1



6 Bus Systems

Writing and Solving Systems of Equations

Write and solve systems of equations about how many bus seats and handholds should be on the Metropolis Transit Association's new buses.

MA.912.AR.9.1 MA.912.AR.9.6
MTR.2.1 MTR.4.1 MTR.5.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.8.AR.4.1 MA.912.AR.9.1
MA.912.AR.2.4
MTR.2.1 MTR.4.1 MTR.5.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.AR.9.1 MA.912.AR.9.4
MA.912.AR.9.6
MTR.2.1 MTR.4.1 MTR.5.1

Pacing Considerations

Lesson 7: This lesson explores how to determine whether a coordinate is a solution to a system of inequalities. If most students demonstrate a strong understanding of solving systems of linear equalities by graphing and substitution, Lessons 4 and 5 can be combined.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1	Lesson 2	Lesson 3	Lesson 4
Copy: Show What You Know **Materials:** scientific calculators *(optional)*	**Copy:** Show What You Know, Activity 3 Cards, one per pair, pre-cut **Materials:** coloring tools *(as needed)*	**Copy:** Show What You Know **Materials:** coloring tools	**Copy:** Show What You Know **Materials:** graphing calculators
Lesson 5	**Lesson 6**	**Lesson 7**	**Lesson 8**
Copy: Show What You Know **Materials:** coloring tools *(optional)*, scientific calculators *(as needed)*, straightedges *(as needed)*	**Copy:** Show What You Know **Materials:** coloring tools *(as needed)*, graphing calculator *(as needed)*, graph paper *(as needed)*, straightedges	**Copy:** Show What You Know **Materials:** computers/ devices *(for research, optional)*	**Copy:** Show What You Know **Materials:** calculators *(optional)*, coloring tools *(optional)*
Lesson 9	**Lesson 10**		
Copy: Show What You Know, Activity 1 Cards, one set per pair of students, pre-cut **Materials:** coloring tools, straightedges *(optional)*	**Copy:** Show What You Know **Materials:** coloring tools *(optional)*		

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



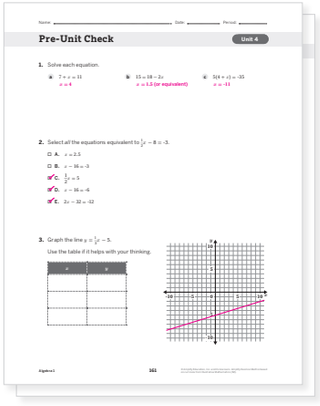
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.7.AR.2.2, MA.8.AR.3.4, MA.8.AR.4.3, MA.912.AR.9.4, MA.912.AR.9.6


Pre-Unit Check

Unit 4

1. Solve each equation.

a. $7 = x + 11$

$x = -4$

b. $15 = 18 - 2x$

$x = 1.5$ (or equivalent)

c. $5(4 + x) = -35$

$x = -11$

2. Select all the equations equivalent to $\frac{1}{2}x - 8 = -3$.

- A. $x = 2.5$
- B. $x - 16 = -3$
- C. $\frac{1}{2}x = 5$
- D. $x - 16 = -6$
- E. $2x - 32 = -12$

3. Graph the line $y = \frac{1}{2}x - 5$.

Use the table if it helps with your thinking.

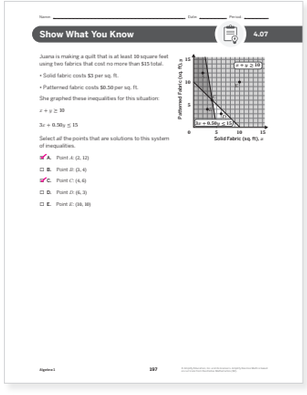
x	y

Algebra 1 161

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 7 Show What You Know: MA.912.AR.9.4


Show What You Know

4.07

Juana is making a quilt that is at least 30 square feet using two fabrics that cost no more than $15 total.

- Solid fabric costs $1 per sq. ft.
- Patterned fabric costs $0.50 per sq. ft.

She graphed these inequalities for this situation:

$x + y \geq 30$

$2x + 0.50y \leq 15$

Select all the points that are solutions to this system of inequalities.

- A. Point A: $(2, 12)$
- B. Point B: $(3, 6)$
- C. Point C: $(4, 4)$
- D. Point D: $(6, 3)$
- E. Point E: $(10, 10)$

Patterned Fabric (sq. ft), y

Solid Fabric (sq. ft), x

$x + y \geq 30$

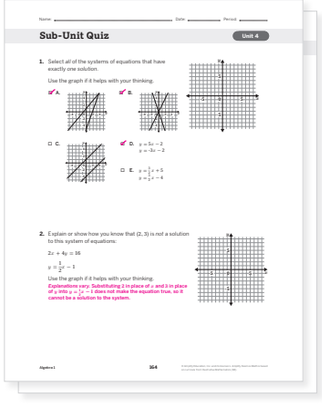
$2x + 0.50y \leq 15$

Algebra 1 197

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.8.AR.4.1, MA.912.AR.9.1, MA.912.AR.2.4


Sub-Unit Quiz

Unit 4

1. Select all of the systems of equations that have exactly one solution.

Use the graph if it helps with your thinking.

- A.
- B.
- C.
- D. $y = 5x - 2$, $y = -3x - 2$
- E. $y = \frac{1}{2}x + 5$, $y = \frac{1}{2}x - 4$

2. Explain or show how you know that $(2, 3)$ is not a solution to this system of equations.

$2x + 4y = 16$

$y = \frac{1}{2}x - 3$

Use the graph if it helps with your thinking.

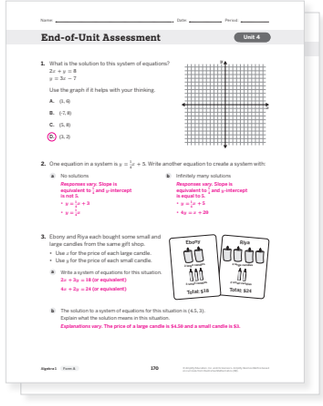
Explanations vary. Substituting 2 in place of x and 3 in place of y into $y = \frac{1}{2}x - 3$ does not make the equation true, so it cannot be a solution to the system.

Algebra 1 164

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.912.AR.9.1, MA.912.AR.9.4, MA.912.AR.9.6


End-of-Unit Assessment

Unit 4

1. What is the solution to this system of equations?

$2x + y = 8$

$y = 3x - 7$

Use the graph if it helps with your thinking.

- A. $(1, 6)$
- B. $(-7, 8)$
- C. $(5, 0)$
- D. $(3, -2)$

2. One equation in a system is $y = \frac{1}{2}x + 5$. Write another equation to create a system with:

a. No solutions

Responses vary. Slope is equivalent to $\frac{1}{2}$ and y-intercept is not 5.

- $y = \frac{1}{2}x + 3$
- $y = \frac{1}{2}x$

b. Infinitely many solutions

Responses vary. Slope is equivalent to $\frac{1}{2}$ and y-intercept is equal to 5.

- $y = \frac{1}{2}x + 5$
- $4y = x + 20$

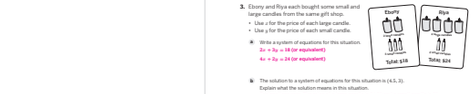
3. Ebony and Riya each bought some small and large candles from the same gift shop.

- Use x for the price of each large candle.
- Use y for the price of each small candle.

a. Write a system of equations for this situation.

$2x + 3y = 38$ (or equivalent)

$4x + 2y = 24$ (or equivalent)



b. The solution to a system of equations for this situation is $(4.5, 3)$. Explain what the solution means in this situation.

Explanations vary. The price of a large candle is $4.50 and a small candle is $3.

Algebra 1 Form A 170

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

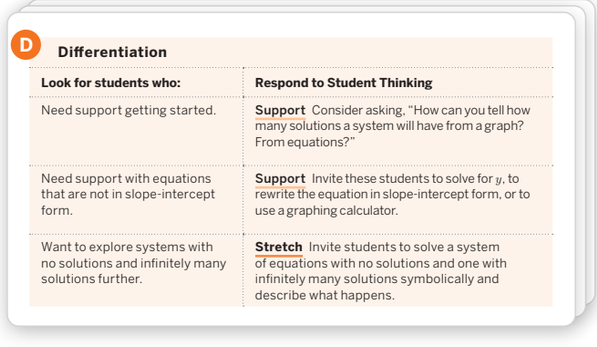
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–6) • Solve systems of linear equations using elimination, substitution, and graphing. • Write systems of linear equations to represent constraints or meet given criteria. • Interpret solutions to systems of equations in context.	**Mini-Lessons** • **Solving Systems of Linear Equations by Substitution**	**Repeated Challenges** • Lesson 1 (*Eliminating Shapes*) • Lesson 3 (*Solution by Substitution*) • Lesson 4 (*Lizard Lines*) **Practice** • Lessons 1–6	**You're invited to explore more.** • Lesson 2 (*Process of Elimination*)
Sub-Unit 2 (Lessons 7–10) • Graph systems of linear inequalities and identify solutions. • Write systems of linear inequalities to represent constraints or meet given criteria. • Interpret solutions to systems of inequalities in context.	**Mini-Lessons** • **Graphing Solutions to Systems of Inequalities**	**Repeated Challenges** • Lesson 8 (*Seeking Solutions*) **Practice** • Lessons 7–10	**DOK3 Practice** • Lessons 7–9 **Teacher Moves:** Invite students to create a design (e.g., heart or star) and write a system of inequalities that could model that design.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 4**, where students sort systems of equations and graphs of systems of equations based on how many solutions the system has: no solution, one solution, or infinitely many solutions.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you tell how many solutions a system will have from a graph? From equations?"
Need support with equations that are not in slope-intercept form.	**Support** Invite these students to solve for y, to rewrite the equation in slope-intercept form, or to use a graphing calculator.
Want to explore systems with no solutions and infinitely many solutions further.	**Stretch** Invite students to solve a system of equations with no solutions and one with infinitely many solutions symbolically and describe what happens.

Accessibility

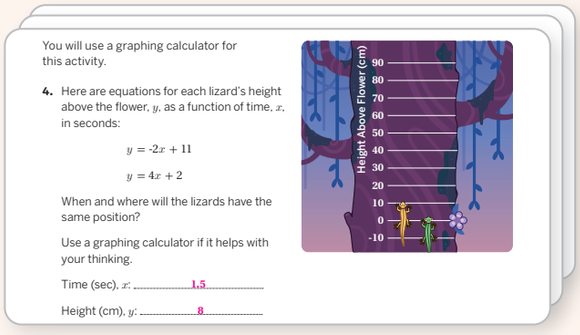
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action & Expression.

Spotlight on Expression & Communication

Lesson 4: In this lesson, students build fluency with a repeated challenge, which supports them in practicing graphing systems of equations to determine their solution and connecting it to their slope and y-intercept.


You will use a graphing calculator for this activity.

4. Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

$y = -2x + 11$

$y = 4x + 2$

When and where will the lizards have the same position?

Use a graphing calculator if it helps with your thinking.

Time (sec), x: 1.5

Height (cm), y: 8

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Memory and Attention. . . .

Here are two suggestions in this unit for supporting students with keeping track of information as they learn additional strategies for solving systems of equations.

Lesson 2, Activity 3

Accessibility: Memory and Attention To support students with keeping track of information, invite them to record any strategies they come up with in their Student Edition to refer back to.

Lesson 5, Activity 2

Accessibility: Memory and Attention Invite students to use a scientific calculator and refer to strategies in Activity 1 as needed.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
elimination **solution to a system of equations** **system of equations**	1
substitution	3
system of inequalities	7
solution region **solution to a system of inequalities**	8

Review Vocabulary

boundary line
coefficient
constraint
distribute
equivalent equations

Review Vocabulary

equivalent systems
half-plane
infinitely many solutions
no solution
opposite
parallel lines
point of intersection
region
slope (rate of change)
slope-intercept form
solution
standard form
substitute
sum
x-intercept
y-intercept

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	8
MLR2: Collect and Display	3–6, 8, 9
MLR3: Critique, Correct, Clarify	3, 8, 9
MLR6: Three Reads	6
MLR7: Compare and Connect	1, 2, 4

Sub-Unit Language Goals

- Interpret solutions to systems of equations in context.
- Interpret solutions to systems of inequalities in context.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 7**, where students describe if a point is a solution to the system. This suggestion supports students in developing language about systems of inequalities to help them determine if the point is a solution.

ELL **English Language Learners** Provide sentence frames to help students write their answers (e.g., "The point ___ a solution because it ___ both inequalities." **(Writing, Speaking, and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Write and Evaluate Exponential Functions

In this unit, students write systems of linear equations to model constraints. **In Unit 5**, students will write exponential functions to model situations and use them to solve problems about those situations.

FL B.E.S.T. Math Benchmarks:
MA.912.AR.5.4, MA.912.F.1.5

Example:

A brand new laptop battery will stay on for 480 minutes. The battery's capacity decreases by 8% each year.

Function Equation:

$b(x) = 480(1 - 0.08)^x$

$b(x)$ represents the battery's capacity after x years.

The function equation can be used to determine how long the laptop battery can be expected to stay on after 5 years.

$b(x) = 480(1 - 0.08)^5 = b(x) = 480(0.92)^5$
≈ 316 minutes

Solve Quadratic Equations

In this unit, students analyze the structure of equations to select strategies to solve systems. **In Unit 7**, students will use the structure of equations to select strategies and solve quadratic equations.

FL B.E.S.T. Math Benchmark:
MA.912.AR.3.1

Example:

Quadratic equations can be solved using strategies like factoring, the zero product property, taking the square root, completing the square, and the quadratic formula. Depending on the structure of the equation, one strategy might require less steps than another strategy.

The zero product property is helpful for equations in factored form.

$$(5x - 3)(2x + 3) = 0$$
$$5x - 3 = 0 \quad \text{and} \quad 2x + 3 = 0$$
$$5x = 3 \quad \text{and} \quad 2x = -3$$
$$x = \tfrac{3}{5} \quad \text{and} \quad x = -\tfrac{3}{2}$$

Taking the square root is helpful for equations in vertex form.

$$(x - 1)^2 = 36$$
$$x - 1 = \pm\sqrt{36}$$
$$x - 1 = \pm 6$$
$$x = 1 \pm 6$$
$$x = 7 \text{ and } x = -5$$

The quadratic formula is helpful for equations in standard form.

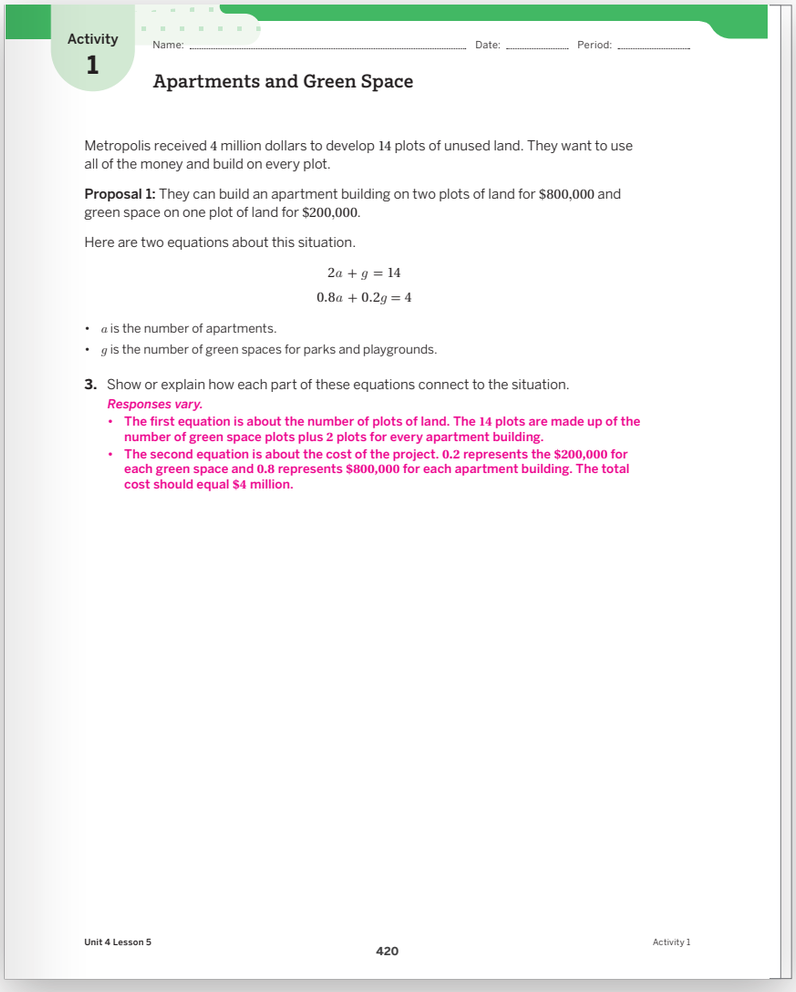
$$x^2 - 4x - 12 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{64}}{2}$$
$$x = -2 \text{ and } x = 6$$

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Connecting Representations

Lesson 5, Activity 1



Activity 1

Name: Date: Period:

Apartments and Green Space

Metropolis received 4 million dollars to develop 14 plots of unused land. They want to use all of the money and build on every plot.

Proposal 1: They can build an apartment building on two plots of land for $800,000 and green space on one plot of land for $200,000.

Here are two equations about this situation.

$$2a + g = 14$$
$$0.8a + 0.2g = 4$$

- a is the number of apartments.
- g is the number of green spaces for parks and playgrounds.

3. Show or explain how each part of these equations connect to the situation.

Responses vary.

- The first equation is about the number of plots of land. The 14 plots are made up of the number of green space plots plus 2 plots for every apartment building.
- The second equation is about the cost of the project. 0.2 represents the $200,000 for each green space and 0.8 represents $800,000 for each apartment building. The total cost should equal $4 million.

Unit 4 Lesson 5 420 Activity 1

Context

In this activity, students develop strategic competence as they move fluently between multiple strategies for solving systems of equations.

Try This

Put on your student hat and complete Problem 3.

Points to Ponder

- What do students need to understand about each strategy to support their development of strategic competence?
- How can strategic competence support students in selecting solving strategies?
- How does developing strategic competence build from their ability to analyze the structure of a system of equations?
- How will developing strategic competence support their understanding of solving systems of nonlinear equations like those in Units 7 and 8?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Solve one-variable equations	Lesson 1	1	MA.7.AR.2.2 MTR.3.1
2	Identify equivalent equations	Lesson 1	1	MA.7.AR.2.2 MTR.2.1
3–4	Graph linear equations	Lesson 4	2	MA.8.AR.3.4
5–6	Analyze systems of linear equations	Lesson 4	2	MA.8.AR.4.3
7	Graph linear inequalities	Lesson 7	2	MA.912.AR.9.4
8	Write linear inequalities from a context	Lesson 7	2	MA.912.AR.9.6 MTR.7.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 4

1. Solve each equation.

a. $7 + x = 11$
$x = 4$

b. $15 = 18 - 2x$
$x = 1.5$ (or equivalent)

c. $5(4 + x) = -35$
$x = -11$

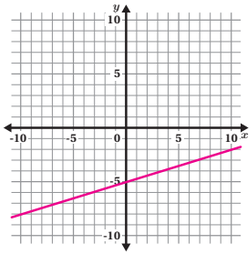
2. Select *all* the equations equivalent to $\frac{1}{2}x - 8 = -3$.

- ☐ A. $x = 2.5$
- ☐ B. $x - 16 = -3$
- ☑ C. $\frac{1}{2}x = 5$
- ☑ D. $x - 16 = -6$
- ☑ E. $2x - 32 = -12$

3. Graph the line $y = \frac{1}{3}x - 5$.

Use the table if it helps with your thinking.

x	y



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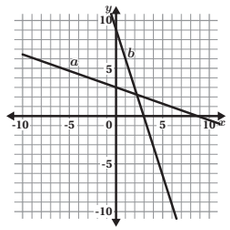
Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 4

4. Which line represents the equation $3x + y = 9$?

- A. Line a
- B. Line b
- C. Both
- D. Neither

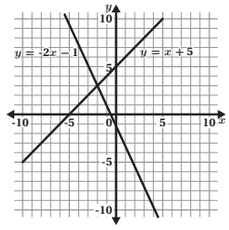


5. This graph shows a system of equations.

What are some things that you know about systems of equations?

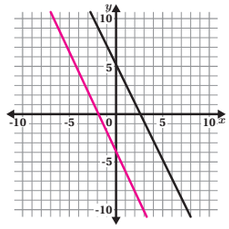
Use this graph if it helps with your thinking.

Responses vary.
- Systems of equations have two or more equations.
- The solution is where the lines intersect or the point that both lines have in common.
- Not all systems of equations have solutions. If the two lines are parallel there is no solution.
- You can solve systems by using substitution or looking at a graph of the system.



6. Add a line to the graph to create a system of equations that has no solutions.

Responses vary. Any line with a slope of -2, like $y = -2x - 3$ or $y = -2x$.



D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking

Solve one-variable equations. **(Preparation for Lessons 1–3)**	1–2	**Support**
		• **Teacher Move:** Consider creating an anchor chart of the strategies students use to solve equations. Then invite students to complete more challenges in Problem 1 of the Pre-Unit Check.
		• **Mini-Lesson:**
		  • **Solving Equations With Positive and Negative Numbers**
		  • **Solving Linear Equations with Parentheses**

Graph linear equations. **(Preparation for Lessons 4–6)**	3–4	**Support**
		• **Teacher Move:** Consider creating an anchor chart of student graphing strategies such as using the y-intercept and slope, identifying points on the line, or making a table. Then invite students to complete more challenges in Problem 3 of the Pre-Unit Check.
		• **Teacher Move:** Consider revisiting:
		  • Unit 1, Lesson 6 (*Shelley the Snail*)
		  • Grade 8, Unit 3, Lesson 4 (*Flags*)

Analyze systems of linear equations graphically. **(Preparation for Lessons 4–6)**	5–6	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Grade 8, Unit 4, Lesson 8 (*Make Them Balance*)
		  • Grade 8, Unit 4, Lesson 10 (*All, Some, or None? Part 2*)
		• **Mini-Lessons:**
		  • **Solving Systems of Linear Equations by Graphing**

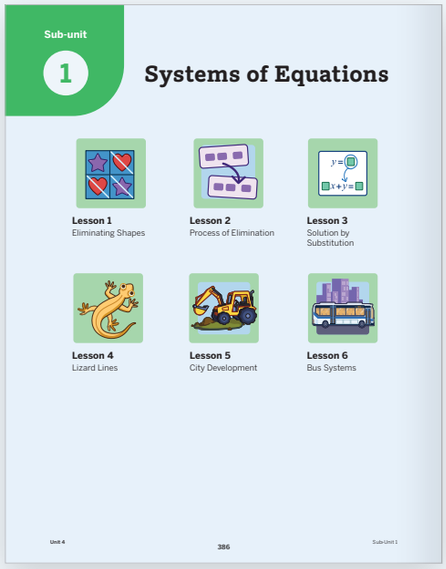
Graph and write linear inequalities. **(Preparation for Lessons 7–10)**	7–8	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Unit 1, Lesson 17 (*All the Solutions*)
		  • Unit 1, Lesson 18 (*Concert Planning*)
		• **Teacher Move:**
		  • Consider creating an anchor chart reviewing each of the inequality symbols — what they mean and how they are represented on a graph. Then invite students to complete more challenges in Problem 7 of the Pre-Unit Check.

Systems of Equations

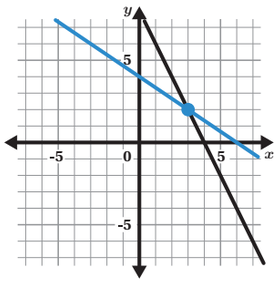
Sub-Unit 1 Goals

- **Goal:** Solve systems of linear equations using elimination, substitution, and graphing.
- **Goal:** Write systems of linear equations to represent constraints or meet given criteria.
- **Language Goal:** Interpret solutions to systems of equations in context.

In this sub-unit, students solve **systems of equations** using strategies like **elimination**, **substitution**, and graphing. They explore what it means to be a **solution to a system of equations** in multiple representations and interpret solutions in context. Students compare solving strategies and choose which to use strategically based on the structure of the equations in the system and their mathematical preferences. Finally, they write systems of equations to represent constraints and solve problems in context.



Math That Matters Most

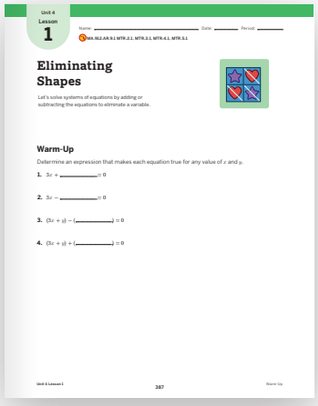
Concept or Skill	For example . . .
Solve systems of equations using elimination.	Solve this system of equations. $9x - 4y = 2$ $-3(3x + y = 10)$ $9x - 4y = 2$ $+ \ -9x - 3y = -30$ $0 - 7y = -28$ $y = 4$ $3x + (4) = 10$ $3x = 6$ $x = 2$
Solve systems of equations using substitution.	Solve this system of equations. $y = -4x + 6$ $y = 3x - 15$ $-4x + 6 = 3x - 15$ $-7x = -21$ $x = 3$ $y = 3(3) - 15$ $y = -6$
Solve systems of equations by graphing.	Finish graphing each system of equations and identify the solution. $2x + 3y = 12$ $4x + 2y = 16$ Solution: **(3, 2)** 

Concept or Skill	For example . . .
Determine if systems of linear equations have no solution, one solution, or infinitely many solutions.	How many solutions does the following system have? Explain your thinking. $y = \frac{2}{3}x + 10$ $y = \frac{2}{3}x - 7$ *This system has no solutions because the equations have the same slope and different y-intercepts. This means, when I graph them they will never intersect because they are parallel.*
Write systems of equations to represent constraints.	An office building has 900 square feet of space for 30 new parking spots. Accessible spots use 85 square feet of space and single spots use 10 square feet of space. Write a system of equations that represents this situation. *x: the number of accessible spots* *y: the number of single spots* *85x + 10y = 900* *x + y = 30*
Interpret solutions to a system of equations in context.	The solution to this system is (8, 22). Explain what it means in this situation. *To build exactly 30 spots with 900 square feet of space, they can create 8 accessible spots and 22 single spots.*

Unit 4

Lesson

1


Unit 4
Lesson
1
Name: _____ Date: _____ Period: _____
MA.912.AR.9.1, MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
Eliminating Shapes
Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.
Warm-Up
Determine an expression that makes each equation true for any value of x and y.
1. $3x + $ _____ $= 0$
2. $3x - $ _____ $= 0$
3. $(3x + y) - ($_____$) = 0$
4. $(3x + y) + ($_____$) = 0$
387
Warm-Up

Eliminating Shapes

Introduction to Elimination

Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.

Learning Outcomes

Today's Goals

1. **Goal:** Solve systems of equations by adding or subtracting the equations to eliminate a variable.
2. **Language Goal:** Identify when to use addition or subtraction to eliminate a variable. **(Writing, Speaking, and Listening)**

Students solve systems of equations by adding or subtracting the equations to eliminate a variable. The lesson begins with a Number Talk Routine exploring *coefficients* which are *opposites*, building the foundation of what it means to eliminate a variable. Students compare and connect strategies for solving shape puzzles with the strategy of **elimination** and debate whether adding or subtracting equations will eliminate a variable. Finally, students practice using elimination to solve a variety of systems of equations.

Coherence

Prior Learning

In Grade 8, students solved systems of equations using a variety of strategies, including graphing and substitution.

Future Learning

In Lesson 2, students will be introduced to solving systems of equations by multiplying one equation by a number, and then adding or subtracting the equations together to eliminate a variable.

Connections

- Students solve systems of equations using elimination with visuals of heart and star shapes representing the variables. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.1
Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.4.2	**MA.8.AR.4.3**
MA.8.AR.2.1	**MA.912.AR.9.6**

Vocabulary

New Vocabulary

elimination **system of equations**
solution to a system of equations

Review Vocabulary

coefficient *opposite*
standard form

Lesson at a Glance

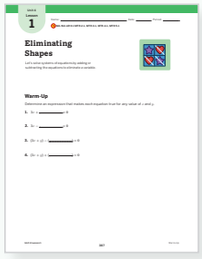
~ 45 min

MA.912.AR.9.1

Warm-Up

Whole Class | 5 min

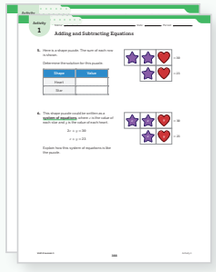
Students engage in a **Number Talk** routine to explore *opposites* and make connections between addition and subtraction, key ideas they will use later in this lesson.



Activity 1

Pairs | 12 min

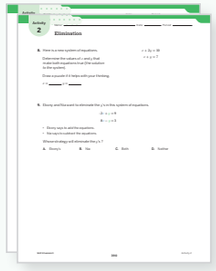
Students draw connections between strategies for solving shape puzzles and using **elimination** to solve systems of equations.
MLR7: Compare and Connect



Activity 2

Pairs | 10 min

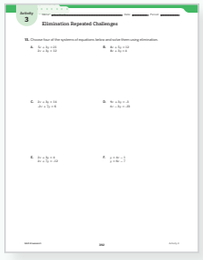
Students analyze the structures of equations to determine whether adding or subtracting equations will eliminate a variable.
Routine: **Decide and Defend** **(MTR.5.1)**



Activity 3

Pairs | 8 min

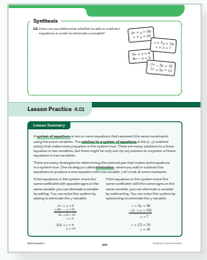
Students develop fluency in solving systems of equations by adding or subtracting to eliminate a variable.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to analyze the structure of a **system of equations** to know if adding or subtracting the equations will eliminate a variable.
(MTR.5.1)



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a system of equations using an elimination strategy.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students engage in a **Number Talk** routine to explore *opposites* and make connections between addition and subtraction, key ideas they will use later in this lesson.

1 Launch

Use the **Number Talk** routine to surface strategies for identifying *opposites* and making connections between addition and subtraction that students will need later in the lesson.

Invite students to determine an expression that makes the first equation true independently and then share their strategies aloud. Encourage them to consider how they can communicate their ideas in ways others can understand, and share that everyone is responsible for trying to understand each others' ideas.

Create a record of each strategy that students share, along with the name of the student who shared it.

Consider asking:
- "Who thought about it in a different way?"
- "Can someone explain ___'s strategy in their own words?"

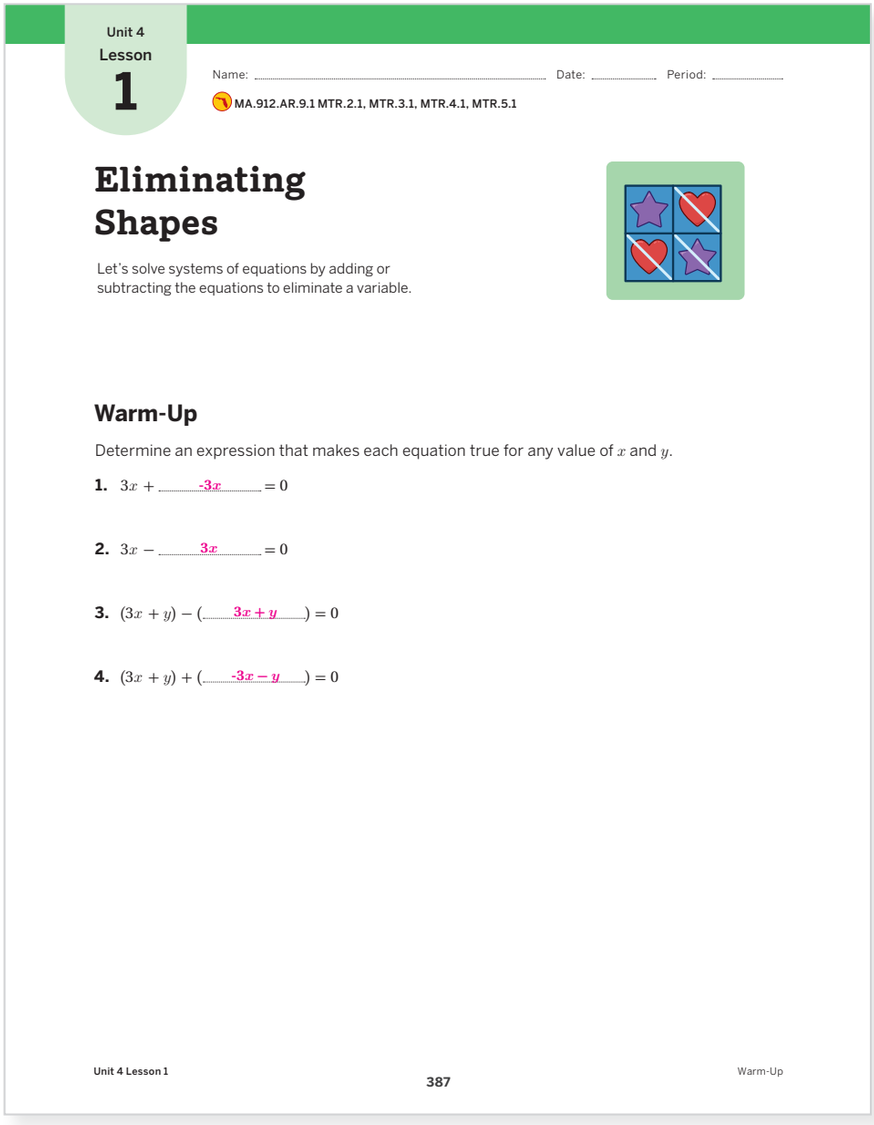
Note: Repeat this structure for each expression in the Number Talk.

Note: Students may use a variety of words to describe the relationship they see, including but not limited to: *opposites*, *zero pairs*, *additive inverses*. There isn't one specific term students need to use to be successful with the content in this lesson as the goal is to focus on the structure of the equations. Consider creating a list of terms to describe this relationship, and add to it as students generate more ideas. **(MTR.5.1)**

2 Connect

Invite students to compare and contrast the different strategies.

Consider asking:
- "What connections do you notice among the strategies we've discussed?"
- "What are some strategies for making *opposites*?"
- "When will adding result in 0? When will subtracting result in 0?"



Unit 4
Lesson
1

Name: _____ Date: _____ Period: _____

MA.912.AR.9.1 MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

Eliminating Shapes

Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.

Warm-Up

Determine an expression that makes each equation true for any value of x and y.

1. $3x +$ ___-3x___ $= 0$
2. $3x -$ ___3x___ $= 0$
3. $(3x + y) - ($ ___3x + y___ $) = 0$
4. $(3x + y) + ($ ___-3x − y___ $) = 0$

Unit 4 Lesson 1 387 Warm-Up

Pairs | 12 min

Activity 1 Adding and Subtracting Equations

Purpose: Students draw connections between strategies for solving shape puzzles and using **elimination** to solve systems of equations.

1 Launch

To surface connections, consider asking, "How does this puzzle in Problem 5 compare to the ones we solved in the previous lesson?"

A **Accessibility: Visual-Spatial Processing**
To support students in interpreting visual representations, invite them to say aloud what each row means (e.g., "Two stars and one heart is equal to 30.").

2 Monitor

Look for students who use a strategy like Ebony's. Invite them to share during the Connect.

Look for interesting strategies or responses to share during the Connect.

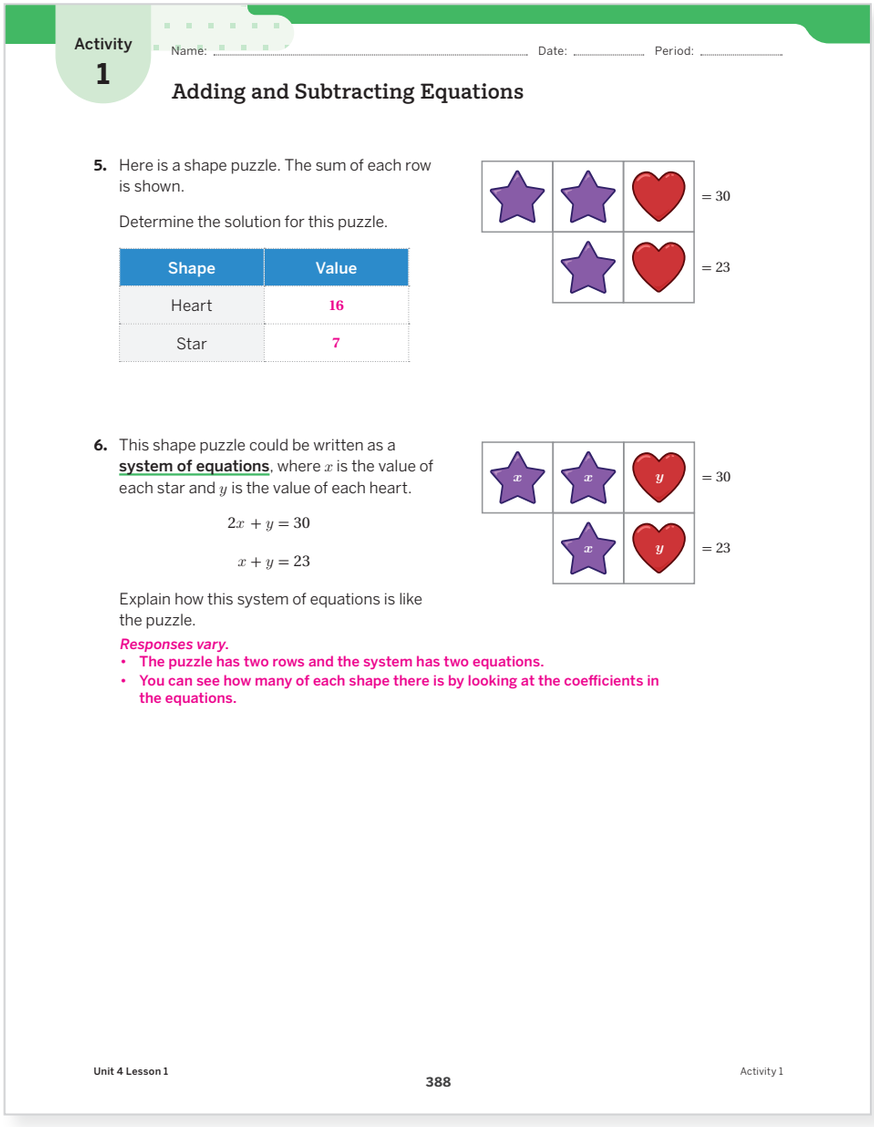
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to make connections between the system of equations and the shape puzzle by asking, "Where do you see the two stars in the system of equations? Where do you see the $x + y$ in the shape puzzle?"
Think that because both equations contain one y, they can create a new system with $2x = 30$ and $x = 23$.	**Strengthen** Celebrate that they tried an elimination strategy. Consider asking, "If you substitute 23 in for x in the first equation, does it make the equation true?"
Notice that the equations in this system are in *standard form*.	**Stretch** Celebrate this noticing. Consider asking, "What about the structure of equations written in standard form supports the strategy of elimination?" Invite them to share their thinking in the Connect.

Pause to introduce the term *system of equations*.

ELL **English Language Learners** Invite students to share what each word in the term *system of equations* reminds them of (e.g., "The word system reminds me of . . ."). Then invite students to share what they think the words used together mean. **(Reading, Listening and Speaking)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Adding and Subtracting Equations

5. Here is a shape puzzle. The sum of each row is shown.

Determine the solution for this puzzle.

Shape	Value
Heart	16
Star	7

6. This shape puzzle could be written as a **system of equations**, where x is the value of each star and y is the value of each heart.

$$2x + y = 30$$

$$x + y = 23$$

Explain how this system of equations is like the puzzle.

Responses vary.
- The puzzle has two rows and the system has two equations.
- You can see how many of each shape there is by looking at the coefficients in the equations.

Unit 4 Lesson 1 388 Activity 1

Pairs | 12 min

Activity 1 Adding and Subtracting Equations (continued)

Purpose: Students draw connections between strategies for solving shape puzzles and using **elimination** to solve systems of equations.

2 Monitor

Listen to student conversations as they compare and connect Ebony's and Nia's strategies with their own.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one student's work at a time. To promote student focus, consider covering up solving steps until students are ready to address them (e.g., In Nia strategy, cover up the work below the equivalence bar to support students in focusing on why Nia subtracted the two equations.).

ELL **English Language Learners** To support students in sharing their thinking, consider displaying sentence frames such as:

- First, they ___. Next, they ____. Finally, ____.
- Their strategy was to ___.
- I see ____ in both strategies.
- What is clear to me is ___. What is unclear to me is ___.

(Writing and Speaking)

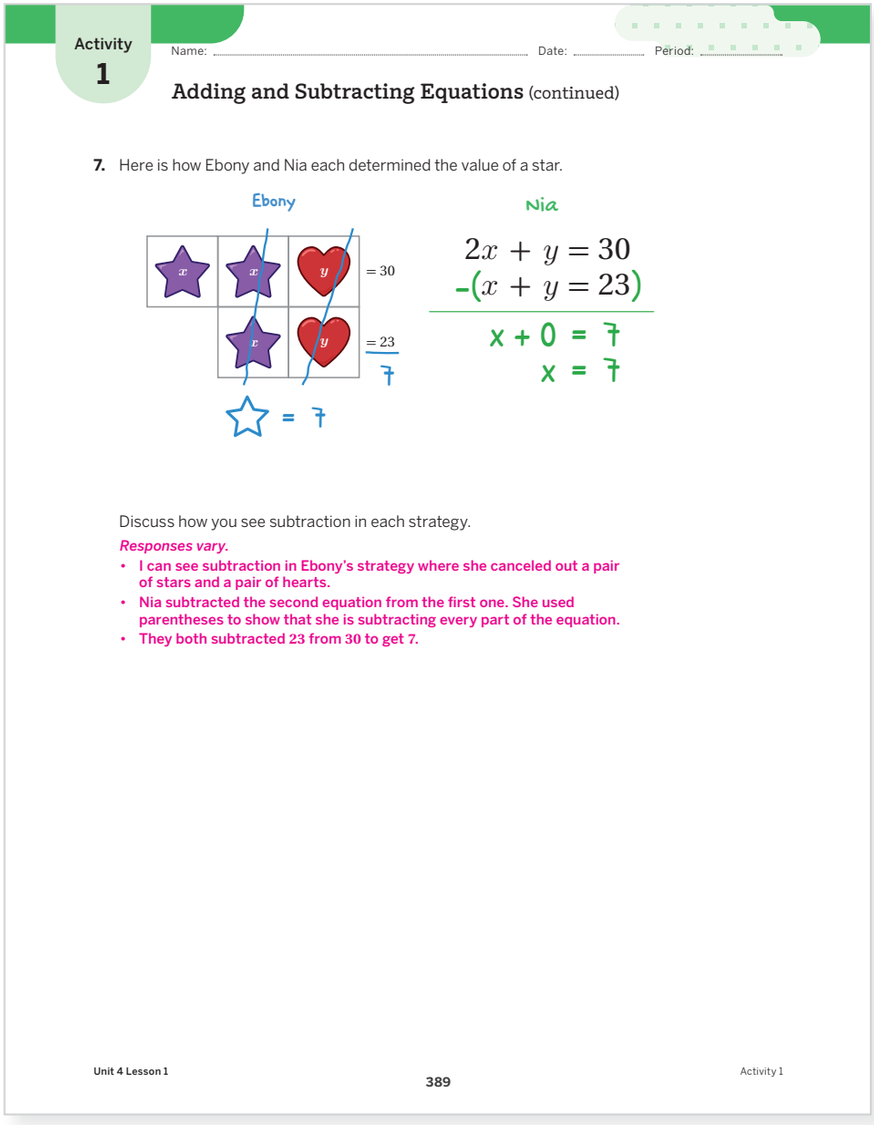
3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

Consider asking:

- "How are Ebony's and Nia's strategies alike? How are they different?"
- "How did each student determine the value of x?"
- "How could you use the value of the star (x) to find the value of the heart (y)?"
- "Why did different strategies lead to the same outcome?"
- "What is helpful about each strategy?"

Note: This is students' first introduction to the strategy of elimination. Students will dive deeper into adding and subtracting equations later in this lesson and systems that require multiplying equations in Lesson 3.



Activity 1

Name: Date: Period:

Adding and Subtracting Equations (continued)

7. Here is how Ebony and Nia each determined the value of a star.

Ebony

Nia

$$2x + y = 30$$
$$-(x + y = 23)$$
$$x + 0 = 7$$
$$x = 7$$

Discuss how you see subtraction in each strategy.

Responses vary.

- I can see subtraction in Ebony's strategy where she canceled out a pair of stars and a pair of hearts.
- Nia subtracted the second equation from the first one. She used parentheses to show that she is subtracting every part of the equation.
- They both subtracted 23 from 30 to get 7.

Unit 4 Lesson 1 389 Activity 1

Key Takeaway: A **system of equations** is two or more equations that represent the constraints on a shared set of variables. The **solution to a system of equations** is the set of all values that makes both equations true.

Pairs | 10 min

Activity 2 Elimination

Purpose: Students analyze the structures of equations to determine whether adding or subtracting equations will eliminate a variable. **(MTR.5.1)**

1 Launch

Consider asking, "What does it mean to determine a solution to a system of equations?" The solution to a system of equations is the set of values that makes both equations true.

Encourage students to use paper to support their thinking.

2 Monitor

Look for a variety of strategies students use to solve the system, including subtracting the equations or using reasoning to determine a value of x that makes both equations true.

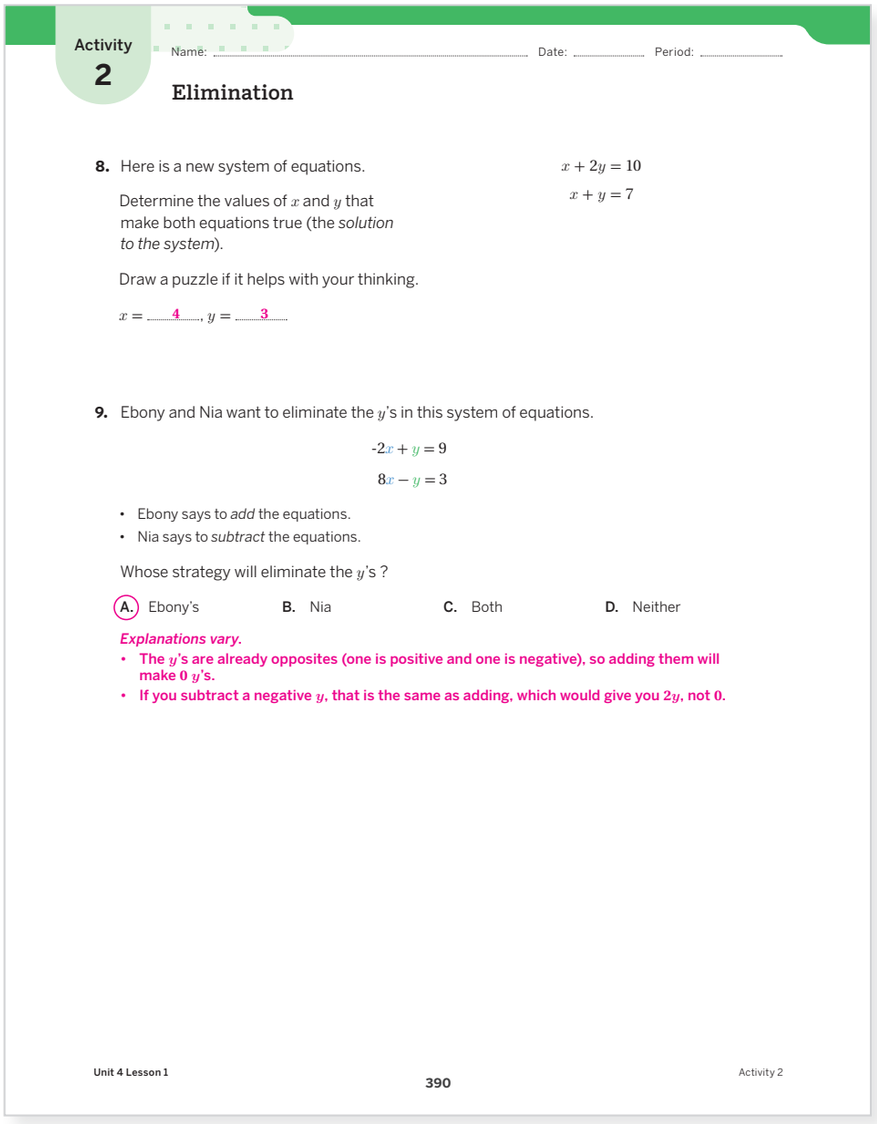
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw a shape puzzle to support them in visualizing the system of equations.
Subtract only the x-values and not the whole system.	**Strengthen** Invite them to review Nia's strategy on Problem 7. Consider asking, "When Nia subtracted the equations, what did he do with the y-values?
Solve for the value of either x or y in the system, and need support solving for the remaining variable.	**Strengthen** Celebrate the strategy they used to solve for the first variable. Invite students to look at the second equation in the system and consider asking, "If we know the value of $y = 3$ (or $x = 4$), what value of x (or y) would make this equation true? How can you use your values to see if they also make the first equation true?"

To support students in getting started, consider asking, "What does it mean to eliminate the y's? What are some strategies we used in the Warm-Up to make 0?"

ELL **English Language Learners** Invite students to share different meanings of the term *eliminate* (e.g., "When I hear eliminate, I think . . ."). Clarify which meaning this word will have for this unit. **(Reading and Listening)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Elimination

8. Here is a new system of equations.

$x + 2y = 10$

$x + y = 7$

Determine the values of x and y that make both equations true (the *solution to the system*).

Draw a puzzle if it helps with your thinking.

$x =$ 4, $y =$ 3

9. Ebony and Nia want to eliminate the y's in this system of equations.

$-2x + y = 9$

$8x - y = 3$

- Ebony says to *add* the equations.
- Nia says to *subtract* the equations.

Whose strategy will eliminate the y's ?

A. Ebony's **B.** Nia **C.** Both **D.** Neither

Explanations vary.

- The y's are already opposites (one is positive and one is negative), so adding them will make 0 y's.
- If you subtract a negative y, that is the same as adding, which would give you $2y$, not 0.

Unit 4 Lesson 1 390 Activity 2

Pairs | 10 min

Activity 2 Elimination (continued)

Purpose: Students analyze the structures of equations to determine whether adding or subtracting equations will eliminate a variable. **(MTR.5.1)**

2 Monitor

Encourage students to use a calculator or paper to support their thinking. Consider sharing that they can enter an expression in the table and the calculator will evaluate it.

D **Differentiation: (Support)** Some students may add some terms or subtract others. Consider reminding students to add or subtract every term in an equation to keep the system equivalent.

Look for a variety of student explanations for why elimination works to share during the Connect.

3 Connect

Display a variety of student responses on the board.

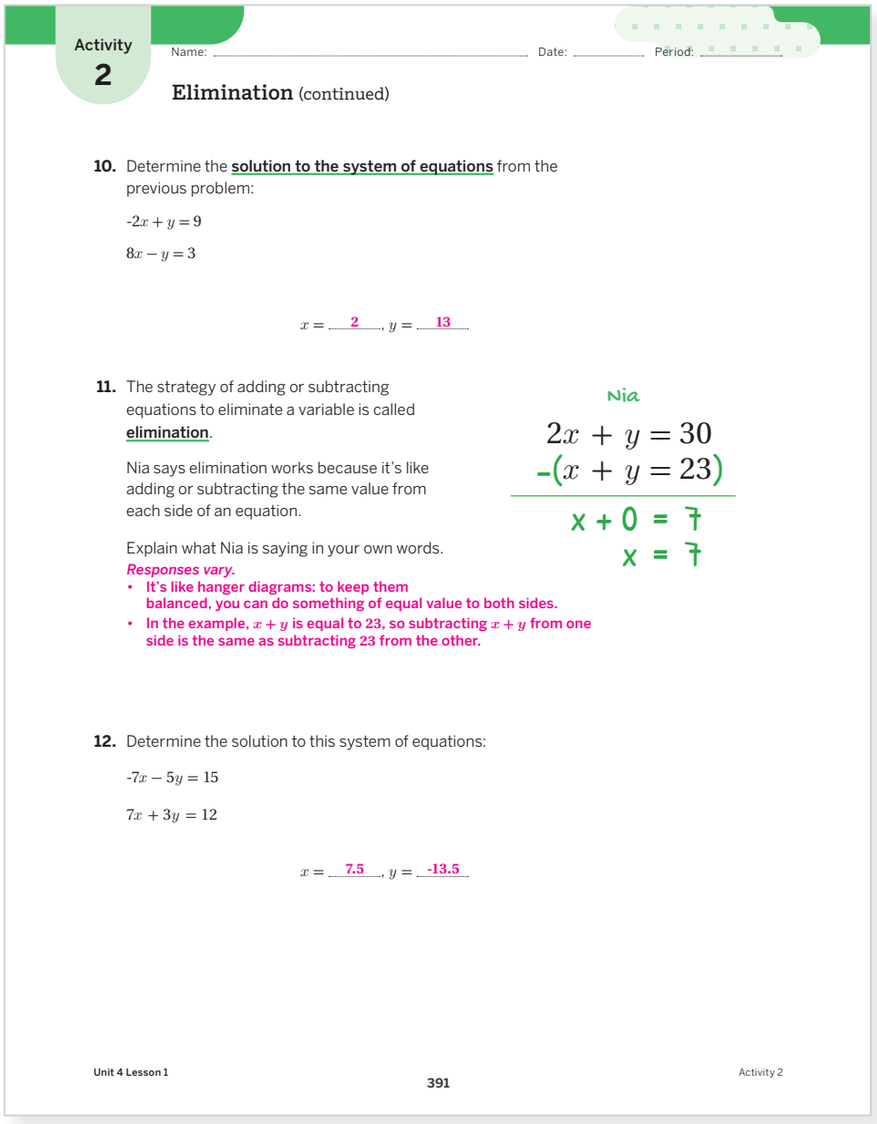
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:
- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How did you know . . .

(Writing and Speaking)

To surface the Key Takeaway, consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing?"
- "What can you look for to help you determine whether to add or subtract equations?" **(MTR.5.1)**

Share student explanations. Consider asking:
- "Where have you heard the word *elimination*? What does this word mean to you?"
- "What about the structure of the equations in a system must be true in order for elimination to be an appropriate strategy to try?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Elimination (continued)

10. Determine the **solution to the system of equations** from the previous problem:

$-2x + y = 9$

$8x - y = 3$

$x =$ 2, $y =$ 13

11. The strategy of adding or subtracting equations to eliminate a variable is called **elimination**.

Nia says elimination works because it's like adding or subtracting the same value from each side of an equation.

Nia

$2x + y = 30$
$-(x + y = 23)$
$x + 0 = 7$
$x = 7$

Explain what Nia is saying in your own words.

Responses vary.
- It's like hanger diagrams: to keep them balanced, you can do something of equal value to both sides.
- In the example, $x + y$ is equal to 23, so subtracting $x + y$ from one side is the same as subtracting 23 from the other.

12. Determine the solution to this system of equations:

$-7x - 5y = 15$

$7x + 3y = 12$

$x =$ 7.5, $y =$ -13.5

Unit 4 Lesson 1 391 Activity 2

Key Takeaway: One strategy for solving a system of equations is to eliminate a variable. When the *coefficients* of the same variable in a system are the same, subtracting the equations will eliminate a variable. When the coefficients are *opposites*, adding the equations will eliminate a variable.

Pairs | 8 min

Activity 3 Elimination Repeated Challenges

Purpose: Students develop fluency in solving systems of equations by adding or subtracting to eliminate a variable.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Invite students to set a goal for the number of challenges to solve. Share that understanding the challenges is more important than speed.

Invite students to work with a partner to solve and compare their solutions to each problem.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

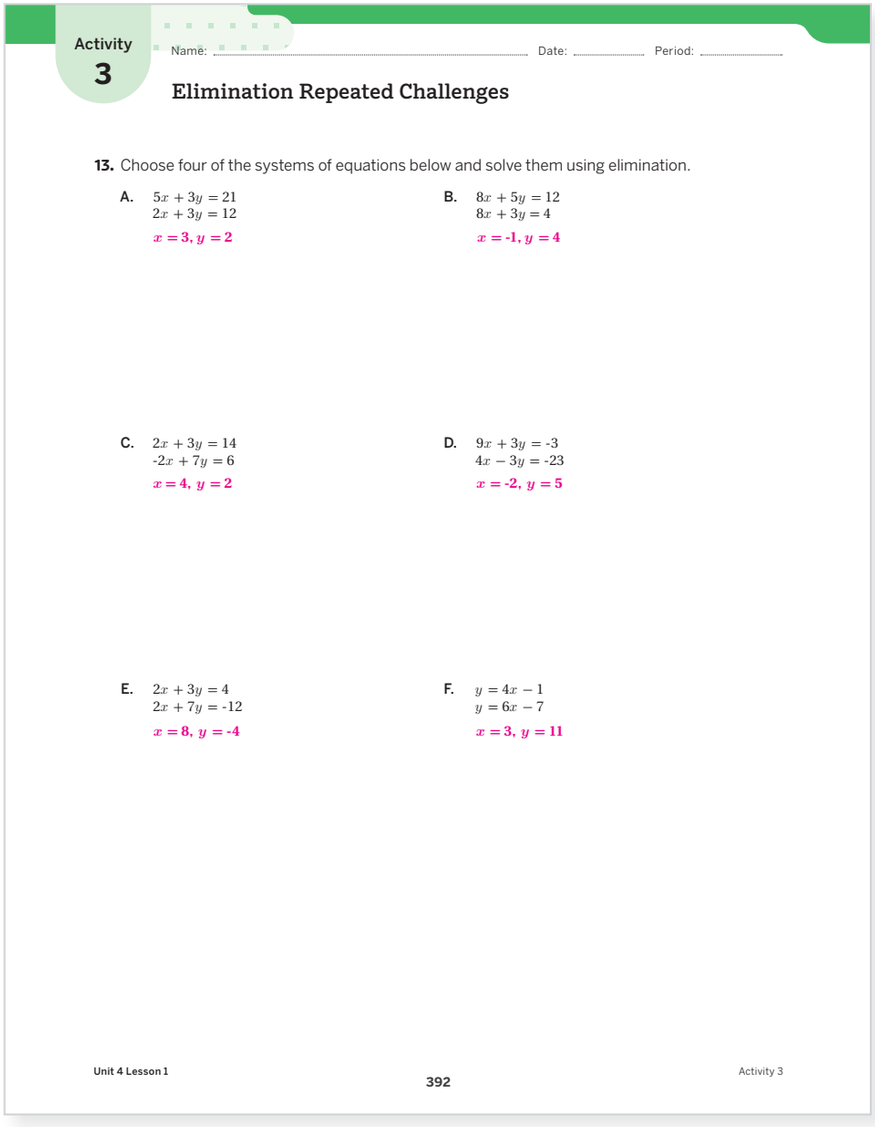
Listen for language students use in discussing strategies for elimination, such as adding opposite coefficients or subtracting same coefficients.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to analyze the structure of the equation to decide if they would like to add or subtract the equations. Consider asking, "How do you determine whether to add or subtract the equations?" **(MTR.5.1)**
Successfully solve for one variable by eliminating the other, and need support solving for the remaining variable.	**Strengthen** Invite students to substitute the value they found into either equation and see what happens.
Successfully solve for both variables, and switch the order of the variables.	**Strengthen** Consider asking, "How can you check your work to know that the ordered pair you determined makes both equations true?"

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 3

Name: Date: Period:

Elimination Repeated Challenges

13. Choose four of the systems of equations below and solve them using elimination.

A. $5x + 3y = 21$, $2x + 3y = 12$
$x = 3, y = 2$

B. $8x + 5y = 12$, $8x + 3y = 4$
$x = -1, y = 4$

C. $2x + 3y = 14$, $-2x + 7y = 6$
$x = 4, y = 2$

D. $9x + 3y = -3$, $4x - 3y = -23$
$x = -2, y = 5$

E. $2x + 3y = 4$, $2x + 7y = -12$
$x = 8, y = -4$

F. $y = 4x - 1$, $y = 6x - 7$
$x = 3, y = 11$

Unit 4 Lesson 1 392 Activity 3

Whole Class | 5 min

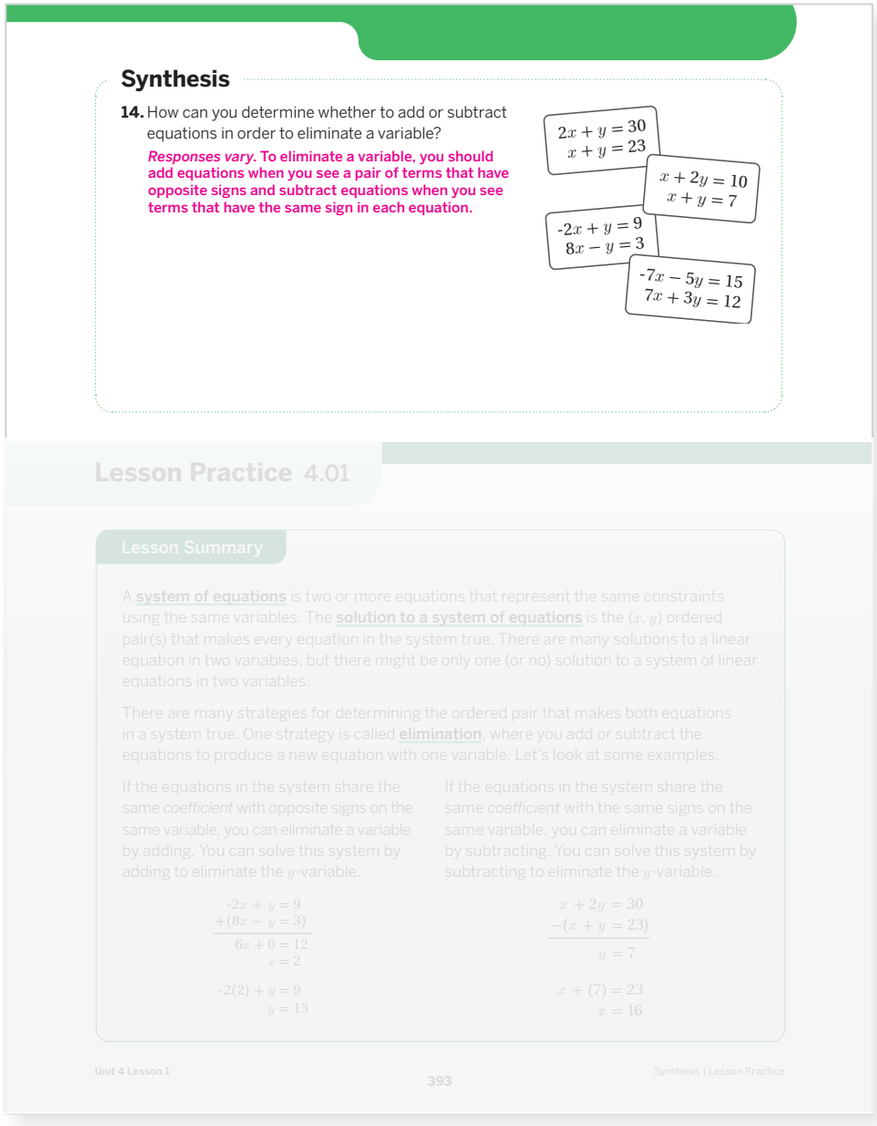
Synthesis

Purpose: Students synthesize their understanding of how to analyze the structure of a **system of equations** to know if adding or subtracting the equations will eliminate a variable. **(MTR.5.1)**

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Adding equations when you see opposite terms.
- Subtracting equations when you see the same terms.
- Remembering that the solution of the system of equations means we need to solve for both variables.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

14. How can you determine whether to add or subtract equations in order to eliminate a variable?

Responses vary. To eliminate a variable, you should add equations when you see a pair of terms that have opposite signs and subtract equations when you see terms that have the same sign in each equation.

$2x + y = 30$
$x + y = 23$

$x + 2y = 10$
$x + y = 7$

$-2x + y = 9$
$8x - y = 3$

$-7x - 5y = 15$
$7x + 3y = 12$

Lesson Practice 4.01

Lesson Summary

A **system of equations** is two or more equations that represent the same constraints using the same variables. The **solution to a system of equations** is the (x, y) ordered pair(s) that makes every equation in the system true. There are many solutions to a linear equation in two variables, but there might be only one (or no) solution to a system of linear equations in two variables.

There are many strategies for determining the ordered pair that makes both equations in a system true. One strategy is called **elimination**, where you add or subtract the equations to produce a new equation with one variable. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the y-variable.

$$\begin{array}{r} -2x + y = 9 \\ +(8x - y = 3) \\ \hline 6x + 0 = 12 \\ x = 2 \end{array}$$

$$\begin{array}{r} -2(2) + y = 9 \\ y = 13 \end{array}$$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the y-variable.

$$\begin{array}{r} x + 2y = 30 \\ -(x + y = 23) \\ \hline y = 7 \end{array}$$

$$\begin{array}{r} x + (7) = 23 \\ x = 16 \end{array}$$

Unit 4 Lesson 1 393 Synthesis | Lesson Practice

Lesson Takeaway: **Elimination** is a strategy for solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables. With adding or subtracting, the goal is to eliminate one variable by creating pairs of expressions that will make 0.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving a system of equations using an elimination strategy.

Today's Goals

1. **Goal:** Solve systems of equations by adding or subtracting the equations to eliminate a variable.
2. **Language Goal:** Identify when to use addition or subtraction to eliminate a variable. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.01

Determine the solution to this system of equations.

$2x + 2y = 9$

$8x - 2y = 16$

$x =$ 2.5, $y =$ 2

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine a solution to system of equations:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write a system that could be solved by adding or subtracting to eliminate a variable.

Professional Learning

Take a moment to reflect on how students engaged with looking for and making use of structure in today's lesson. How did analyzing the structure of equations support students in eliminating a variable using addition or subtraction? In the next lesson, students will be introduced to eliminating a variable using multiplication. How can they expand on their understanding of analyzing structure to make sense of this new **elimination** strategy?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

14. How can you determine whether to add or subtract equations in order to eliminate a variable?

Responses vary. To eliminate a variable, you should add equations when you see a pair of terms that have opposite signs and subtract equations when you see terms that have the same sign in each equation.

$2x + y = 30$
$x + y = 23$

$x + 2y = 10$
$x + y = 7$

$-2x + y = 9$
$8x - y = 3$

$-7x - 5y = 15$
$7x + 3y = 12$

Lesson Practice 4.01

Lesson Summary

A **system of equations** is two or more equations that represent the same constraints using the same variables. The **solution to a system of equations** is the (x, y) ordered pair(s) that makes every equation in the system true. There are many solutions to a linear equation in two variables, but there might be only one (or no) solution to a system of linear equations in two variables.

There are many strategies for determining the ordered pair that makes both equations in a system true. One strategy is called **elimination**, where you add or subtract the equations to produce a new equation with one variable. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the y-variable.

$$\begin{array}{r} -2x + y = 9 \\ +(8x - y = 3) \\ \hline 6x + 0 = 12 \end{array}$$

$x = 2$

$-2(2) + y = 9$

$y = 13$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the y-variable.

$$\begin{array}{r} x + 2y = 30 \\ -(x + y = 23) \\ \hline y = 7 \end{array}$$

$x + (7) = 23$

$x = 16$

Lesson Practice 4.01

Name: _____ Date: _____ Period: _____

Problems 1–2: Mateo made a mistake as he started to solve this system of equations.

1. Describe one thing Mateo did *correctly*.

Responses vary. Mateo correctly subtracted $2x - x$ and $19 - 11$. Also, trying to eliminate y is a good way to solve this system of equations.

$$\begin{array}{r} 2x + y = 19 \\ -(x - y = 11) \\ \hline x + 0 = 8 \\ x = 8 \end{array}$$

2. Describe one thing Mateo did incorrectly.

Responses vary. Mateo did not subtract correctly. Subtracting one equation from the other won't eliminate y; adding the equations will, because the two y-variables have opposite signs.

Problems 3–4: Determine the solution for each system of equations.

3. $3x + 4y = 6$
$3x + 2y = 18$

$x =$ 10 $y =$ -6

4. $5x + 6y = 26$
$-5x + 2y = -18$

$x =$ 4 $y =$ 1

Test Practice

5. Solve this system of equations. Use the shape puzzle if it helps with your thinking.

$2x + y = 10$

$x + y = 6$



$x =$ 4 $y =$ 2

Lesson Practice 4.01

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 6–8: Write an equivalent expression by combining like terms.

6. $5a + 3b - 2a$
$3a + 3b$

7. $3(c - 2) + 2c$
$5c - 6$

8. $5d - 2(7d + 3g)$
$-9d - 6g$

9. The function $f(t)$ models a hiker's elevation above or below sea level, in meters, t hours after noon. Circle the equation that represents this statement:
At 7 PM, the hiker was 3 meters below sea level.

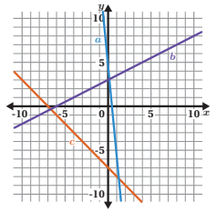
A. $f(7) = 3$ B. $f(19) = -3$ (C.) $f(7) = -3$ D. $f(-3) = 7$

10. Which line represents $x - 2y = -6$?

A. Line a
(B.) Line b
C. Line c

Explain your thinking.

Explanations vary. I found the x-intercept and y-intercept of the equation to help me pick the correct line. The x-intercept is -6 and the y-intercept is 3, so line b represents the equation $x - 2y = -6$.



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.AR.9.1
	3–4	2	MA.912.AR.9.1
Test Practice	5	2	MA.912.AR.9.1
Spiral Review			
Fluency	6–8	2	MA.7.AR.1.1
	9	1	MA.912.F.1.2
	10	2	MA.912.AR.2.2

Unit 4

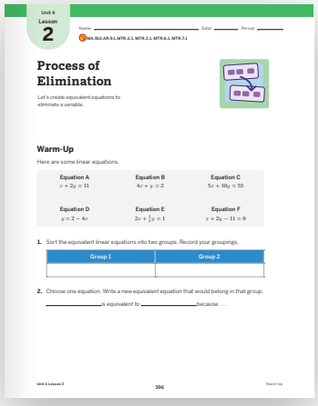
Lesson

2

Process of Elimination

Elimination Using Equivalent Equations

Let's create equivalent equations to eliminate a variable.


Unit 4 Lesson 2

Name _____ Date _____ Period _____

MA.912.AR.9.1, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Process of Elimination

Let's create equivalent equations to eliminate a variable.

Warm-Up

Here are some linear equations.

Equation A	Equation B	Equation C
$x + 2y = 11$	$4x + y = 2$	$5x + 10y = 55$
Equation D	**Equation E**	**Equation F**
$y = 2 - 4x$	$2x + \frac{1}{2}y = 1$	$x + 2y - 11 = 0$

1. Sort the equivalent linear equations into two groups. Record your groupings.

Group 1	Group 2

2. Choose one equation. Write a new equivalent equation that would belong in that group.

_____ is equivalent to _____ because . . .

Unit 4 Lesson 2 396 Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Solve a system of equations using an elimination strategy.
2. **Language Goal:** Explain how replacing an equation in a system with one that is equivalent creates a new system that has the same solution. **(Writing, Speaking, and Listening)**

Students solve systems of equations by using an elimination strategy (multiplying one or more equations by a number and then adding or subtracting equations to create an equation with only one variable). They consider whether or not the strategy of multiplying an equation in a system by a number will create an equivalent system with the same solution. Students develop fluency in solving systems of equations using elimination.

Coherence

Prior Learning

In Lesson 1, students solved systems of equations by adding or subtracting the equations to eliminate a variable and recognized that adding or subtracting the equations in a system creates a new equation that has the same solution as the original system.

Future Learning

In Lesson 3, students will use substitution strategies to solve systems of equations. In Lessons 4–6, students will solve systems of equations graphically as well as symbolically and interpret solutions to systems of equations in context.

Connections

- Students analyze visuals of student work examples for solving systems of equations by elimination. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.1
Given a mathematical or real-world context, write and solve a system of two variable linear equations algebraically or graphically

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.4.1	**MA.912.AR.9.6**

Vocabulary

Review Vocabulary

distributive property *equivalent equations*

opposite

Additional Vocabulary

equivalent systems

Lesson at a Glance

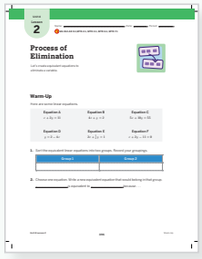
~ 45 min

MA.912.AR.9.1

Warm-Up

Pairs | 5 min

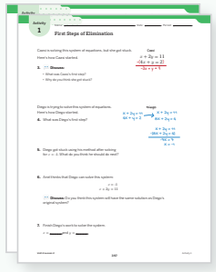
Students revisit how to create *equivalent equations* by multiplying by a number or by rearranging the equation in a different form.



Activity 1

Pairs | 10 min

Students analyze the structure of a system of equations as they are introduced to the strategy of multiplying an equation in a system by a number to eliminate a variable.



Activity 2

Pairs | 5 min

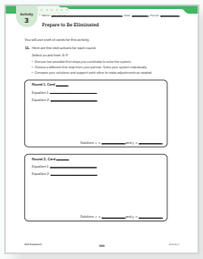
Students use **MLR7: Compare and Connect** to analyze two different first steps for solving a system of equations using elimination.



Activity 3

Pairs | 15 min

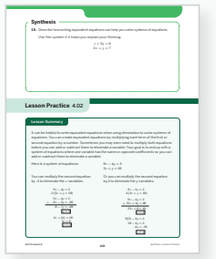
Students develop fluency in solving systems of equations using elimination.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how writing equivalent equations can support them in solving systems of equations.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a system of equations.



Prep Checklist

Prepare the Activity 3 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 cards, 1 set per pair of students, pre-cut.
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit how to create *equivalent equations* by multiplying by a number or by rearranging the equation in a different form.

1 Launch

To surface prior knowledge, consider asking, "How can you tell if two equations are equivalent?" If they have the same solutions, or if you can change one into the other using only balanced moves.

ELL **English Language Learners** Consider sharing a previous example from Unit 1 Lesson 2 Activity 1 to support students in accessing prior knowledge about equivalent equations and balanced moves. **(Reading, Writing, Speaking, and Listening)**

To support students getting started, consider choosing any two equations and ask the class to decide if they are equivalent before pairs work on their own.

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, chunk this activity by inviting them to choose one card at a time and comparing it to each of the other equations one at a time.

D **Differentiation: (Support)** To support students getting started, consider choosing equations A and C and asking what balanced moves could be used to decide if the two equations are equivalent. Multiplying by 5

2 Connect

Look for students who write new equations using a variety of balanced moves and invite them to share their strategies. Students may:

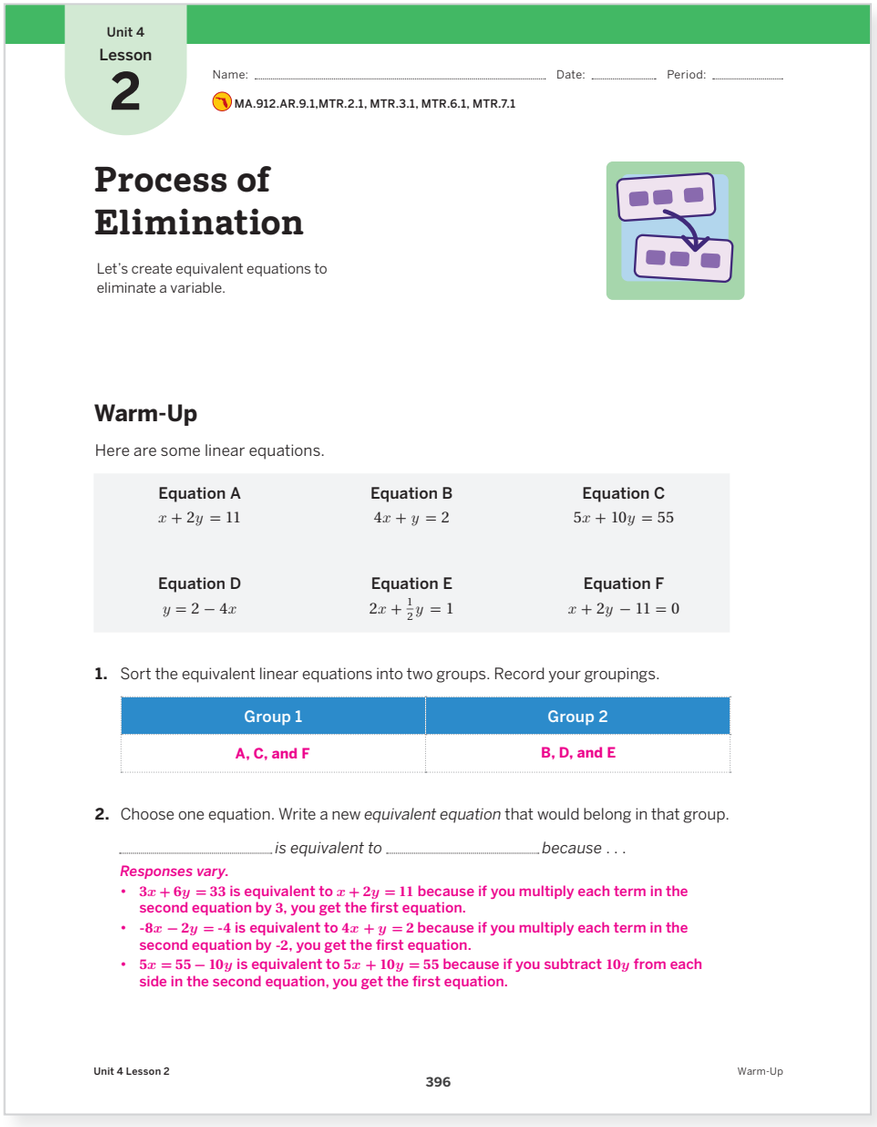
- Multiply all the terms in an equation by a number.
- Rearrange terms in the equations.
- Add or subtract terms to both sides of the equation.

Invite students to share the groups of equivalent equations they wrote and how they know they are equivalent to the original one.

Consider asking:

- "What balanced moves create equivalent equations?"
- "Why might creating an equivalent equation be helpful?"

Note: If it does not come up naturally, consider explicitly reviewing that multiplying both sides of an equation by a number creates an equivalent equation, and when multiplying an equation by a number it is necessary to use the *distributive property* to distribute each term in both sides of the equation to keep it balanced.



Unit 4
Lesson
2

Name: _____ Date: _____ Period: _____

MA.912.AR.9.1,MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Process of Elimination

Let's create equivalent equations to eliminate a variable.

Warm-Up

Here are some linear equations.

Equation A	Equation B	Equation C
$x + 2y = 11$	$4x + y = 2$	$5x + 10y = 55$
Equation D	**Equation E**	**Equation F**
$y = 2 - 4x$	$2x + \frac{1}{2}y = 1$	$x + 2y - 11 = 0$

1. Sort the equivalent linear equations into two groups. Record your groupings.

Group 1	Group 2
A, C, and F	B, D, and E

2. Choose one equation. Write a new *equivalent equation* that would belong in that group.

_____ *is equivalent to* _____ *because . . .*

Responses vary.

- $3x + 6y = 33$ is equivalent to $x + 2y = 11$ because if you multiply each term in the second equation by 3, you get the first equation.
- $-8x - 2y = -4$ is equivalent to $4x + y = 2$ because if you multiply each term in the second equation by -2, you get the first equation.
- $5x = 55 - 10y$ is equivalent to $5x + 10y = 55$ because if you subtract $10y$ from each side in the second equation, you get the first equation.

Unit 4 Lesson 2 396 Warm-Up

Pairs | 10 min

Activity 1 First Steps of Elimination

Purpose: Students analyze the structure of a system of equations as they are introduced to the strategy of multiplying an equation in a system by a number to eliminate a variable.

1 Launch

Invite students to share why they think Caasi got stuck. Consider asking, "Why didn't the variables eliminate when Caasi subtracted the equations?"

To surface prior knowledge, consider asking, "What do you remember about solving systems of equations from the last lesson?"

ELL **English Language Learners** Invite students to share their thinking in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

A **Accessibility: Visual-Spatial Processing** As students interpret visual representations, invite them to use coloring tools to connect the original system and Diego's new system.

2 Monitor

D **Differentiation** *(Problems 4–5)*

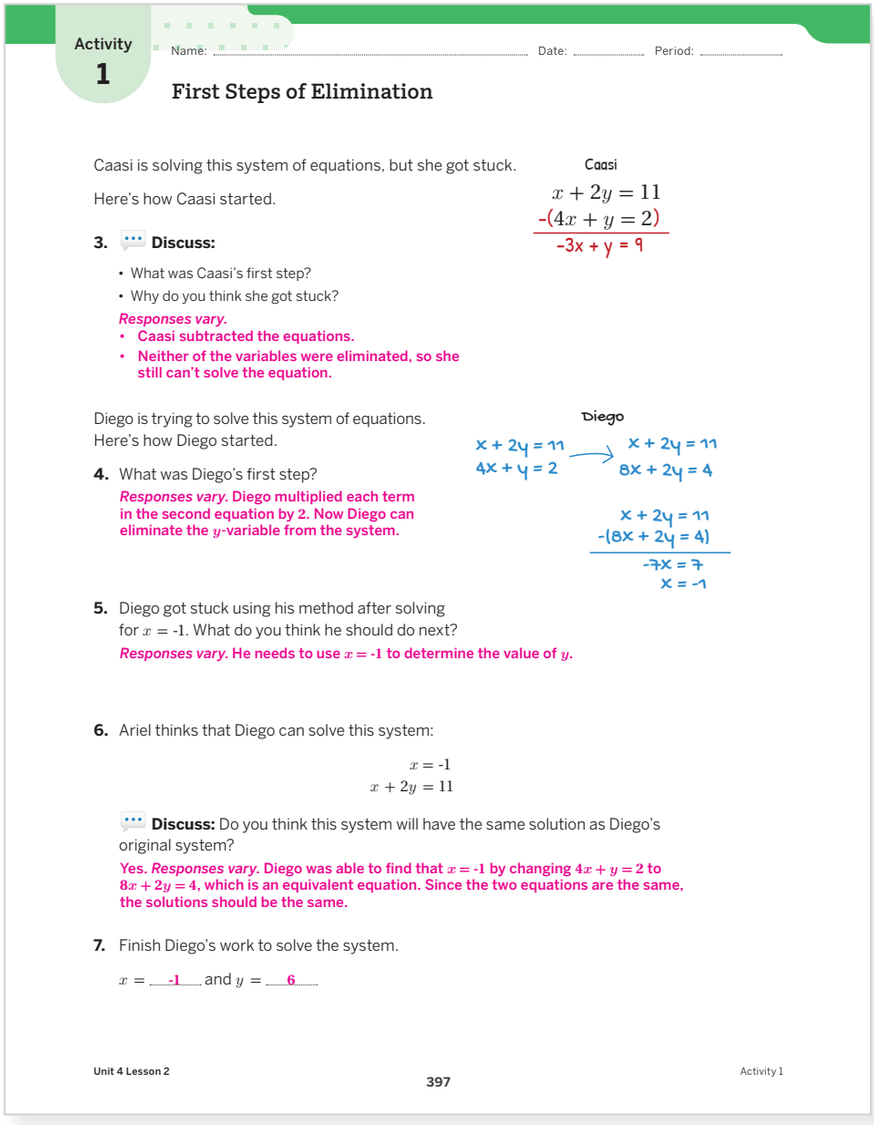
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Why might Diego have multiplied by 2 as a first step? What might be a next step in solving this system?"
Notice the structure of the equation and use it to explain why Diego might have multiplied by 2.	**Strengthen** Celebrate their noticing and invite them to share during the Connect.
Would like an additional challenge.	**Stretch** Invite them to solve Diego's system, then substitute their solution back into the equation to check.

3 Connect

Consider asking:

- "How did you decide what Diego's first step might be?"
- "What are some ways to decide what number is useful to multiply an equation by?"
- "How can we check if a value is a solution to a system of equations?"

To surface the Key Takeaway, consider sharing a graph of Ariel's system and a graph of Diego's system. Show the point of intersection for each system. Consider asking, "What do you notice about the solution for each of these systems? Why do you think they have the same solution?"



Activity 1

Name: Date: Period:

First Steps of Elimination

Caasi is solving this system of equations, but she got stuck.

Here's how Caasi started.

Caasi

$x + 2y = 11$

$-(4x + y = 2)$

$-3x + y = 9$

3. **Discuss:**

- What was Caasi's first step?
- Why do you think she got stuck?

Responses vary.
- Caasi subtracted the equations.
- Neither of the variables were eliminated, so she still can't solve the equation.

Diego is trying to solve this system of equations. Here's how Diego started.

Diego

$x + 2y = 11$, $4x + y = 2$ → $x + 2y = 11$, $8x + 2y = 4$

$x + 2y = 11$

$-(8x + 2y = 4)$

$-7x = 7$

$x = -1$

4. What was Diego's first step?

Responses vary. Diego multiplied each term in the second equation by 2. Now Diego can eliminate the y-variable from the system.

5. Diego got stuck using his method after solving for $x = -1$. What do you think he should do next?

Responses vary. He needs to use $x = -1$ to determine the value of y.

6. Ariel thinks that Diego can solve this system:

$$x = -1$$
$$x + 2y = 11$$

Discuss: Do you think this system will have the same solution as Diego's original system?

Yes. *Responses vary.* Diego was able to find that $x = -1$ by changing $4x + y = 2$ to $8x + 2y = 4$, which is an equivalent equation. Since the two equations are the same, the solutions should be the same.

7. Finish Diego's work to solve the system.

$x =$ -1 and $y =$ 6

Unit 4 Lesson 2 397 Activity 1

Key Takeaway: Multiplying an equation in a system by a number creates an equivalent equation. An equivalent system of equations will have the same solution as the original system of equations.

Pairs | 5 min

Activity 2 More Than One Way?

Purpose: Students use **MLR7: Compare and Connect** to analyze two different first steps for solving a system of equations using elimination.

Short on time? Consider facilitating a whole-class discussion for Problem 8 instead of discussing in pairs.

1 Launch

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, encourage students to use their Student Edition as they write one step at a time to solve the system of equations.

2 Monitor

Invite students in a pair each to choose Caasi's or Kwabena's first step and continue solving the system, and then to compare their solutions with each other.

Look for students who describe similarities and differences between Caasi's and Kwabena's strategies and invite them to share during the Connect.

D **Differentiation: (Stretch)** Invite students to solve the same system of equations in a different way. Students could multiply by a different number, use substitution, or try another strategy.

3 Connect

Note: Consider inviting students to write each step of Caasi's and Kwabena's ways of solving in a public place for students to refer to throughout the lesson. If time permits, share a version of each solving method that was completed by your students.

MLR **MLR7: Compare and Connect** Look for two students' work. Invite students to analyze and compare each strategy, and then connect to Diego's strategy from Activity 1.

To surface the Key Takeaway, consider asking:

- "Why do you think Kwabena multiplied the second equation by -4 instead of 4?"
- "Where do you see the same information in Caasi and Kwabena's strategies?"
- "Why did the different strategies lead to the same outcome?"

Activity 2

Name: Date: Period:

More Than One Way?

8. Caasi and Kwabena started solving this system in different ways.

$$4x - y = 5$$
$$x + 2y = 8$$

With a partner, solve the system both ways. Compare your solutions.

Caasi: *Multiply the first equation by 2.*

$$\begin{array}{r} 8x - 2y = 10 \\ +\ \ x + 2y = 8 \\ \hline 9x = 18 \\ x = 2 \end{array} \qquad \begin{array}{r} (2) + 2y = 8 \\ 2y = 6 \\ y = 3 \end{array}$$

Kwabena: *Multiply the second equation by -4.*

$$\begin{array}{r} 4x - y = 5 \\ +\ -4x - 8y = -32 \\ \hline -9y = -27 \\ y = 3 \end{array} \qquad \begin{array}{r} x + 2(3) = 8 \\ x + 6 = 8 \\ x = 2 \end{array}$$

9. **Discuss:**

- What is similar about Caasi's and Kwabena's methods for solving the linear system?
- What is different about their methods?

Responses vary.

- Similar: They both used elimination. They both solved for one variable and then had to plug it back into the equation to get the other.
- Different: Caasi solved for x first. Kwabena solved for y first. Caasi multiplied by a positive number, Kwabena multiplied by a negative number.

10. Constanza is trying to solve this system of equations.

$$-7x + 5y = -19$$
$$-3x + 3y = -9$$

Review her process. What do you notice about the steps taken to solve the system?

I notice that instead of eliminating right away, or multiplying one equation, Constanza had to modify both equations.

```
3(-7x + 5y) = 3(-19)
-5(-3x + 3y) = -5(-9)

-21x + 15y = -57
15x - 15y = 45

-6x = -12
x = 2

-3(2) + 3y = -9
-6 + 3y = -9
3y = -3
y = -1

(2,1)
```

Unit 4 Lesson 2 398 Activity 2

Key Takeaway: There are multiple ways to use the strategy of elimination to solve a system of equations. You can add or subtract equations together to create new equations, or multiply equations by a number. Sometimes you will need to multiply both equations. As long as the equations in the system are equivalent, then the solution will be the same.

Pairs | 15 min

Activity 3 Prepare to Be Eliminated

Purpose: Students develop fluency in solving systems of equations using elimination.

Short on time? Consider inviting students to complete as many rounds as time allows.

1 Launch

Distribute the Activity 3 cards.

Invite students to choose one card at a time, discuss the first steps, each solve the system a different way, and then compare their solutions and support each other as needed.

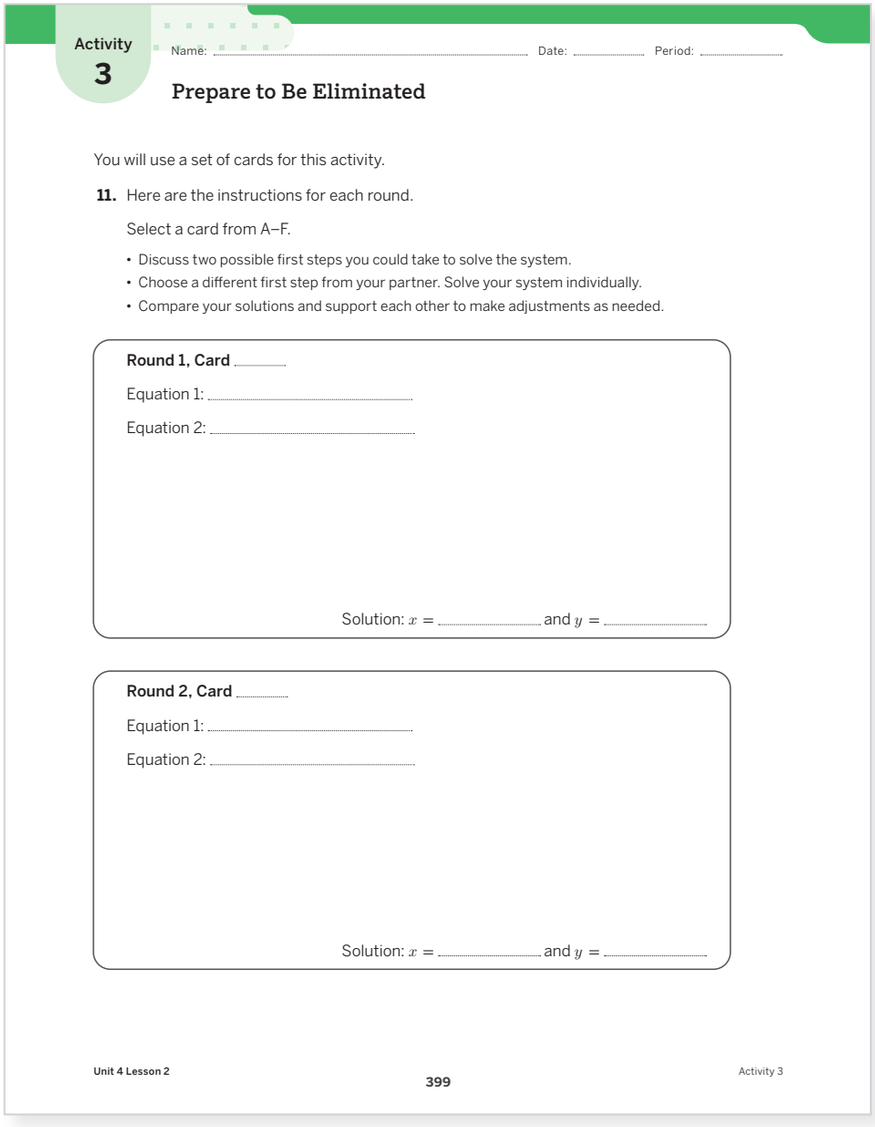
Note: There are multiple options for facilitating this activity: students can stay with the same partner for all four rounds, or students can switch partners between rounds.

2 Monitor

D Differentiation *(Problem 10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What variable do you want to try to eliminate? Would it be helpful to multiply the first or second equation by a number? Why?"
Find a solution for a system.	**Strengthen** Consider asking, "What was your first step? What did it help you do next? How can you check your solution?"

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Prepare to Be Eliminated

You will use a set of cards for this activity.

11. Here are the instructions for each round.

Select a card from A–F.

- Discuss two possible first steps you could take to solve the system.
- Choose a different first step from your partner. Solve your system individually.
- Compare your solutions and support each other to make adjustments as needed.

Round 1, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Round 2, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Unit 4 Lesson 2 399 Activity 3

Pairs | 15 min

Activity 3 Prepare to Be Eliminated (continued)

Purpose: Students develop fluency in solving systems of equations using elimination.

Short on time? Consider inviting students to complete as many rounds as time allows.

2 Monitor

A **Accessibility: Memory and Attention** To support students with keeping track of information, invite them to record any strategies they come up with in their Student Edition to refer back to.

ELL **English Language Learners** As students share in partner discussions, provide sentence frames such as:

- I agree/disagree with you because _____.
- I think that makes sense because _____.
- How do you know?

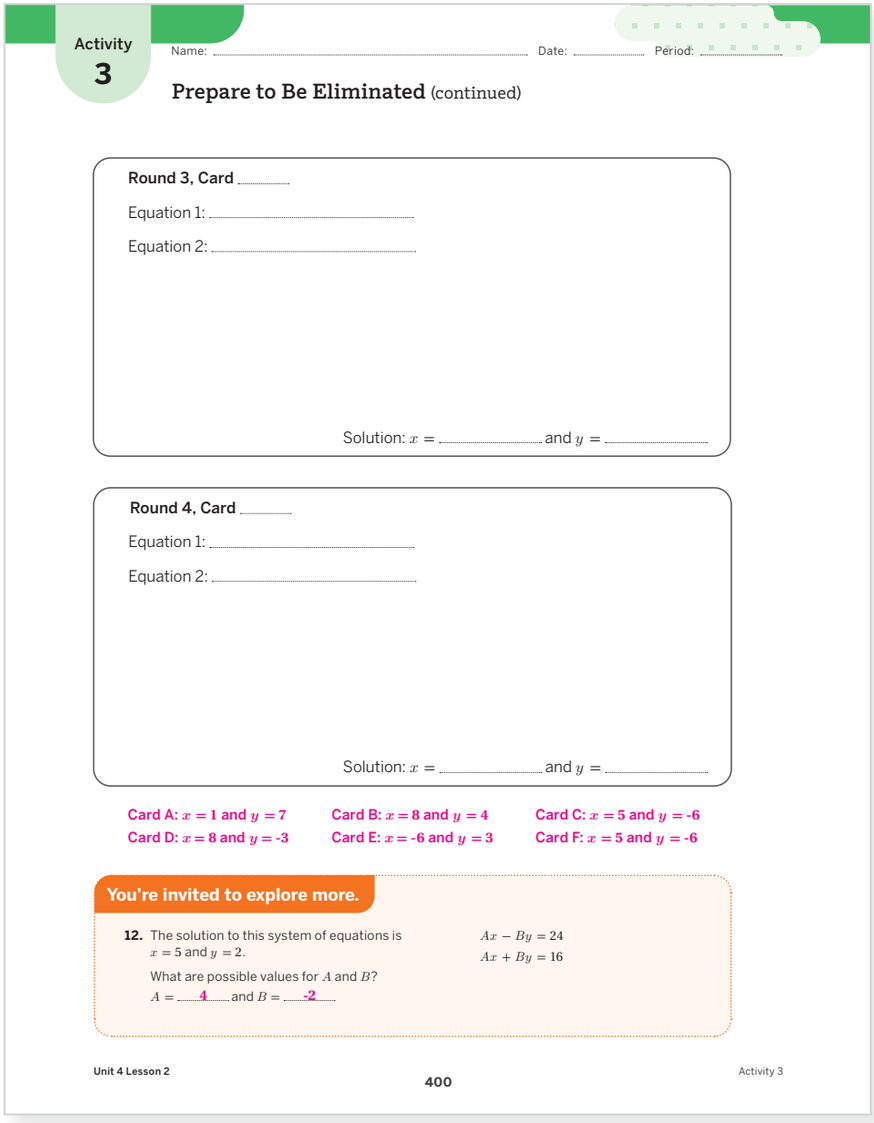
(Speaking and Listening)

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore solving systems of equations using multiplication to create equivalent systems further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share strategies that were useful in solving the systems in their pairs.

Note: If it does not come up naturally, consider asking, "Are there any cards that you would group together? Which ones and why?" Consider pairing any suggestions using the card sort. Cards C and F because they have the same solution for x and y.



Activity 3

Name: _____ Date: _____ Period: _____

Prepare to Be Eliminated (continued)

Round 3, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Round 4, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Card A: $x = 1$ and $y = 7$ Card B: $x = 8$ and $y = 4$ Card C: $x = 5$ and $y = -6$
Card D: $x = 8$ and $y = -3$ Card E: $x = -6$ and $y = 3$ Card F: $x = 5$ and $y = -6$

You're invited to explore more.

12. The solution to this system of equations is $x = 5$ and $y = 2$.

$Ax - By = 24$
$Ax + By = 16$

What are possible values for A and B?

$A =$ 4 and $B =$ -2

Unit 4 Lesson 2 400 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how writing equivalent equations can support them in solving systems of equations.

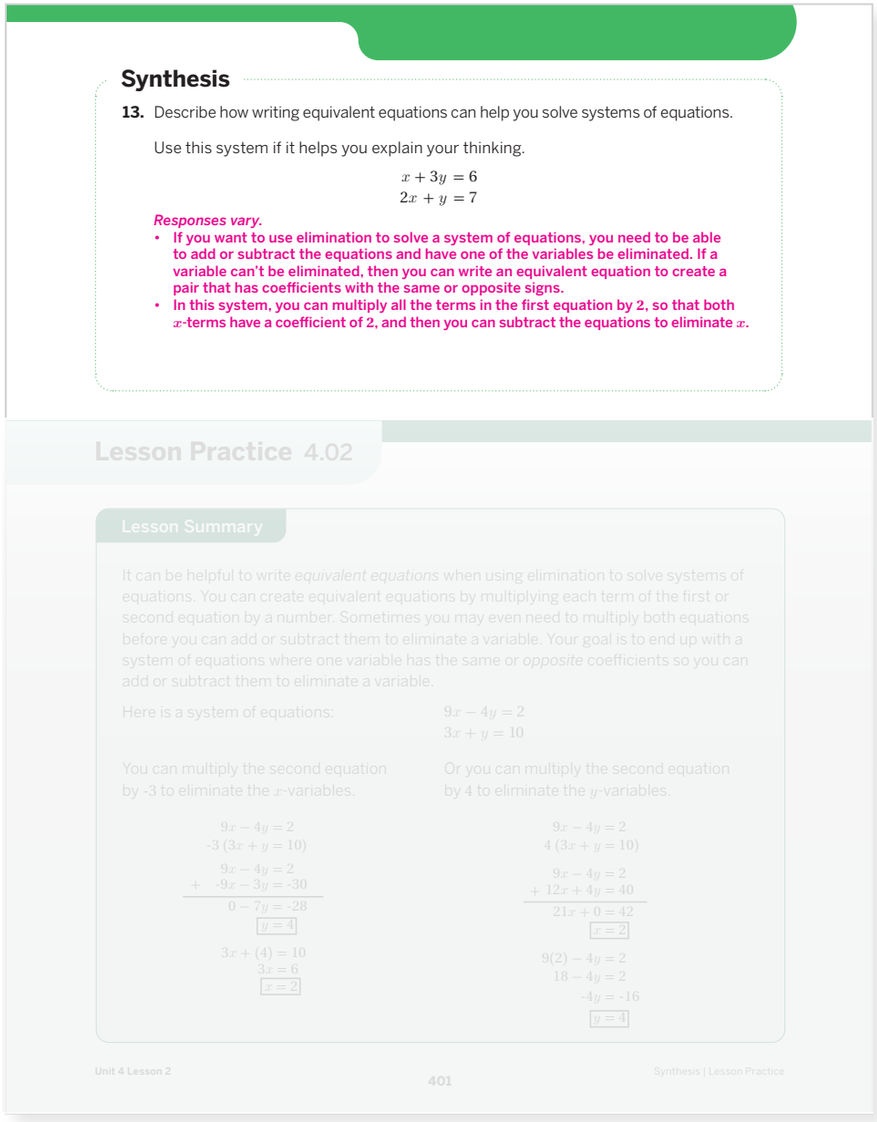
Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *eliminate*, *variable*, *coefficient*, *term*, *equivalent equation*, etc. **(Speaking and Writing)**

Look for and share a variety of ideas, such as:

- Writing equivalent equations helps you eliminate a variable.
- Multiplying by a negative number may help to make the coefficients opposites of one another so you can add the equations together.
- As long as the equations in the system remain equivalent, they will have the same solution.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. Describe how writing equivalent equations can help you solve systems of equations.

Use this system if it helps you explain your thinking.

$$x + 3y = 6$$
$$2x + y = 7$$

Responses vary.

- If you want to use elimination to solve a system of equations, you need to be able to add or subtract the equations and have one of the variables be eliminated. If a variable can't be eliminated, then you can write an equivalent equation to create a pair that has coefficients with the same or opposite signs.
- In this system, you can multiply all the terms in the first equation by 2, so that both x-terms have a coefficient of 2, and then you can subtract the equations to eliminate x.

Lesson Practice 4.02

Lesson Summary

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Sometimes you may even need to multiply both equations before you can add or subtract them to eliminate a variable. Your goal is to end up with a system of equations where one variable has the same or *opposite* coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$9x - 4y = 2$$
$$3x + y = 10$$

You can multiply the second equation by -3 to eliminate the x-variables.

$$9x - 4y = 2$$
$$-3(3x + y = 10)$$
$$9x - 4y = 2$$
$$+\ -9x - 3y = -30$$
$$0 - 7y = -28$$
$$\boxed{y = 4}$$
$$3x + (4) = 10$$
$$3x = 6$$
$$\boxed{x = 2}$$

Or you can multiply the second equation by 4 to eliminate the y-variables.

$$9x - 4y = 2$$
$$4(3x + y = 10)$$
$$9x - 4y = 2$$
$$+\ 12x + 4y = 40$$
$$21x + 0 = 42$$
$$\boxed{x = 2}$$
$$9(2) - 4y = 2$$
$$18 - 4y = 2$$
$$-4y = -16$$
$$\boxed{y = 4}$$

Unit 4 Lesson 2 401 Synthesis | Lesson Practice

Lesson Takeaway: Creating *equivalent equations* is one way to use elimination as a strategy for solving a system of equations. Adding or subtracting equations together that have the same or *opposite* coefficients will eliminate a variable. You can also multiply an equation by a number to make the coefficients the same or opposite of each other so they will eliminate.

Independent | 5 min

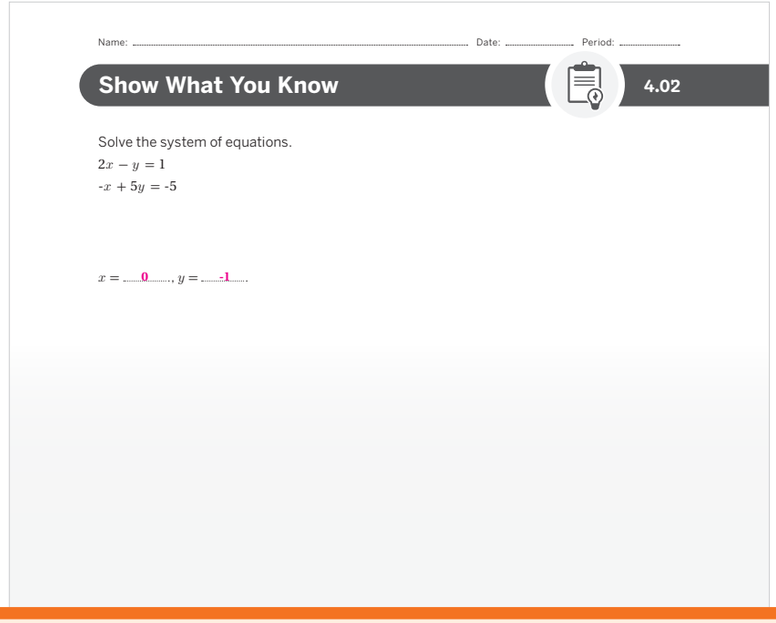
Show What You Know

Purpose: Students demonstrate their understanding by solving a system of equations.

Today's Goals

1. **Goal:** Solve a system of equations using an elimination strategy.
2. **Language Goal:** Explain how replacing an equation in a system with one that is equivalent creates a new system that has the same solution. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.02

Solve the system of equations.

$2x - y = 1$

$-x + 5y = -5$

$x =$ 0, $y =$ -1

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve a system of equations:

» **Respond to Student Thinking:**

- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **You're invited to Explore More** task and discuss their thinking with a partner.

Professional Learning

Consider the different elimination strategies students were introduced to today. How did you support students in identifying the pattern of eliminating a variable even when the details of the strategy were different? How does framing elimination as a strategy support students in choosing an appropriate strategy by analyzing the structure of the equations in a system? How can you continue to support students with finding meaning in multiple strategies as they move on to substitution and graphing strategies later in this unit?

Lesson Practice

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. Describe how writing equivalent equations can help you solve systems of equations.

Use this system if it helps you explain your thinking.

$$x + 3y = 6$$
$$2x + y = 7$$

Responses vary.

- If you want to use elimination to solve a system of equations, you need to be able to add or subtract the equations and have one of the variables be eliminated. If a variable can't be eliminated, then you can write an equivalent equation to create a pair that has coefficients with the same or opposite signs.
- In this system, you can multiply all the terms in the first equation by 2, so that both x-terms have a coefficient of 2, and then you can subtract the equations to eliminate x.

Lesson Practice 4.02

Lesson Summary

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Sometimes you may even need to multiply both equations before you can add or subtract them to eliminate a variable. Your goal is to end up with a system of equations where one variable has the same or *opposite* coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$9x - 4y = 2$$
$$3x + y = 10$$

You can multiply the second equation by -3 to eliminate the x-variables.

$$9x - 4y = 2$$
$$-3(3x + y = 10)$$
$$9x - 4y = 2$$
$$+\ -9x - 3y = -30$$
$$0 - 7y = -28$$
$$\boxed{y = 4}$$
$$3x + (4) = 10$$
$$3x = 6$$
$$\boxed{x = 2}$$

Or you can multiply the second equation by 4 to eliminate the y-variables.

$$9x - 4y = 2$$
$$4(3x + y = 10)$$
$$9x - 4y = 2$$
$$+\ 12x + 4y = 40$$
$$21x + 0 = 42$$
$$\boxed{x = 2}$$
$$9(2) - 4y = 2$$
$$18 - 4y = 2$$
$$-4y = -16$$
$$\boxed{y = 4}$$

Lesson Practice 4.02

Name: _____ Date: _____ Period: _____

1. Select *all* expressions that are equivalent to $6x + 8y = 10$.

- ☑ **A.** $12x + 16y = 20$
- ☐ **B.** $2x + 6y = 10$
- ☑ **C.** $3x + 4y = 5$
- ☑ **D.** $1.5x + 2y = 2.5$
- ☐ **E.** $4x + 5y = 4$

2. Arnav and Omari are solving this system of equations. They disagree about what the first step should be to eliminate a variable.

$$4x + 2y = 62$$
$$-8x - y = 59$$

Arnav's strategy: Multiply $4x + 2y = 62$ by 2

Omari's strategy: Multiply $-8x - y = 59$ by 2

Whose strategy will eliminate a variable once the equations are added? Circle your choice.

A. Arnav's **B.** Omari's **(C.)** Both **D.** Neither

Explain your thinking.

Explanations vary. Both strategies will create a term that can be eliminated by adding the equations together. Arnav's strategy will create $8x + (-8x)$ to eliminate x. Omari's strategy will create $2y - (2y)$ to eliminate y.

Problems 3–4: Determine the solution to each system of equations.

3. $2x - 4y = 10$
$x + 5y = 40$

$x =$ 15 $y =$ 5

4. $5x + 2y = 20$
$2x - 3y = -11$

$x =$ 2 $y =$ 5

Lesson Practice 4.02

Name: _____ Date: _____ Period: _____

Test Practice

5. Which ordered pair represents the solution to the system of linear equations?

$$9x + 8y = -5$$
$$4x + 10y = -28$$

A. $(-3, -4)$ **B.** $(-4, 3)$ **(C.)** $(3, -4)$ **D.** $(4, -3)$

Spiral Review

Problems 6–8: Solve each equation.

6. $x - 2 = 5$
$x = 7$

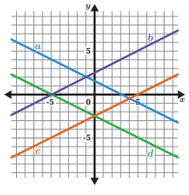
7. $3(x - 2) = 3 \cdot 5$
$x = 7$

8. $8x - 16 = 40$
$x = 7$

9. Solve the equation $3x - 9y = 72$ for y.
$y = \frac{1}{3}x - 8$

10. Which line represents $5x + 10y = 15$?

(A.) Line a
B. Line b
C. Line c
D. Line d



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.AR.9.1
	3–4	2	MA.912.AR.9.1
Test Practice	5	2	MA.912.AR.9.1
Spiral Review			
Fluency	6–8	1	MA.7.AR.2.2
	9	2	MA.912.AR.1.2
	10	2	MA.912.AR.2.4

Unit 4

Lesson

3

Solution by Substitution

Solving Systems by Substitution

Let's use substitution to solve systems of equations.


Unit 4
Lesson
3
Solution by Substitution
Let's use substitution to solve systems of equations.
Warm-Up
1. Here are two scales showing the weights of foxes and alligators.
What do you notice? What do you wonder?
2. Let's look at the scale. Use the information from the scales to find the weights of each animal.
404

Learning Outcomes

Today's Goals

1. **Goal:** Solve systems of equations using substitution.
2. **Language Goal:** Describe a strategy for solving systems of equations using substitution. **(Writing, Speaking, and Listening)**

Students use **substitution** to solve systems of equations, with a strong focus on the first step of the process of substituting for a value in an equation. They analyze student work to revisit the strategy of substitution and identify common errors using this method. Students then discuss a variety of systems, explaining whether they'd prefer to use the tool of elimination or substitution to solve them based on their analysis of the structure of the equations in the system. **(MTR.7.1)**

Coherence

Prior Learning

In Grade 8, students were introduced to the strategy of solving systems of equations by graphing (MA.8.AR.4.3). In Lessons 1–2, students were introduced to solving systems of equations using an elimination strategy.

Future Learning

In Lesson 4, students will gain another solving strategy for systems of equations: solving by graphing.

Connections

- Students use the visual scales showing the weights of foxes and alligators to solve systems of equations using substitutions. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.1
Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.1	MA.912.AR.9.1

Vocabulary

New Vocabulary

substitution

Review Vocabulary

distributive property

Lesson at a Glance

~ 45 min

MA.912.AR.9.1

Warm-Up

Independent | 5 min

Students use the **Notice and Wonder** routine to make sense of a see-saw context that will be used throughout this lesson.



Activity 1

Pairs | 15 min

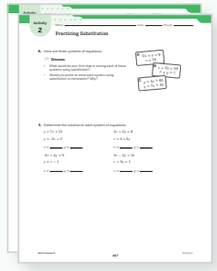
Students are introduced to the strategy of **substitution** to solve systems of equations as they analyze a student's work and identify common errors when using this strategy. **MLR3: Critique, Correct, Clarify**



Activity 2

Pairs | 15 min

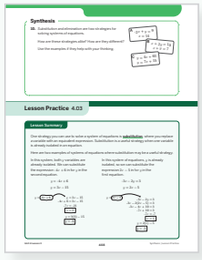
Students develop fluency with solving systems of equations as they explain which solving strategy they would prefer to use based on the structure of the equations. **Routine: Think-Pair-Share**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the two solving strategies they have learned so far: substitution and elimination. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a system of equations using any strategy.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator
- colored pencils (optional)

Independent | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of a see-saw context that will be used throughout this lesson.

1 Launch

To surface connections, consider asking, "Have you ever been on a see-saw before? What happens on a see-saw?"

ELL **English Language Learners** Consider sharing a picture or a video of a see-saw to increase access to the task. **(Reading and Listening)**

Consider sharing that *lb* is an abbreviation of the word *pounds*.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the image. **(MTR.6.1)**

2 Connect

Create a record of what students noticed and wondered, along with the name of the students who shared them.

Consider asking, "Did anyone notice that the see-saw is balanced? What does that mean? Does this make you wonder about how much the alligator and the foxes weigh?"

Invite students to share what they think the weight of each animal is, as well as their strategy for determining the weight.

Consider asking:

- "Why does replacing the alligator with three foxes keep the see-saw balanced?"
- "How does replacing the alligator with three foxes help us determine the weights of each animal?"
- "What would happen if the alligator was replaced by two foxes?"

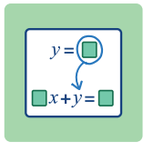
Unit 4
Lesson
3

Name: Date: Period:

MA.912.AR.9.1, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Solution by Substitution

Let's use substitution to solve systems of equations.

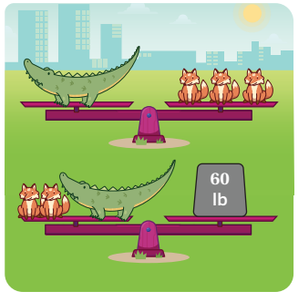


Warm-Up

1. Here are two scales showing the weights of foxes and alligators.

   What do you notice? What do you wonder?

   Responses vary.
   - I notice that an alligator weighs the same as three foxes.
   - I notice that two foxes and an alligator together weigh 60 pounds.
   - I wonder if all the foxes weigh the same amount.
   - I wonder if both see-saws are balanced.
   - I wonder how much a fox weighs. I wonder how much an alligator weighs.



2. Let's look at the scale. Use the information from the scales to find the weights of each animal.

Unit 4 Lesson 3 404 Warm-Up

Pairs | 15 min

Activity 1 Introducing Substitution

Purpose: Students are introduced to the strategy of **substitution** to solve systems of equations as they analyze a student's work and identify common errors when using this strategy.

1 Launch

To surface connections, consider asking, "How are the equations in Problem 3 connected to the see-saws?"

Invite students who used a strategy like Riya's to share their thinking. Consider asking, "How did using equations support your thinking?"

Consider asking:

- "What did Riya do first?"
- "How do you think Riya could use her strategy to solve the new system?"
- "What is another strategy you could use to solve this system?"

Demonstrate how to annotate a problem or type math expressions using colored chalk or dry-erase markers. Invite students to use colored pencils to support student thinking throughout the lesson.

2 Monitor

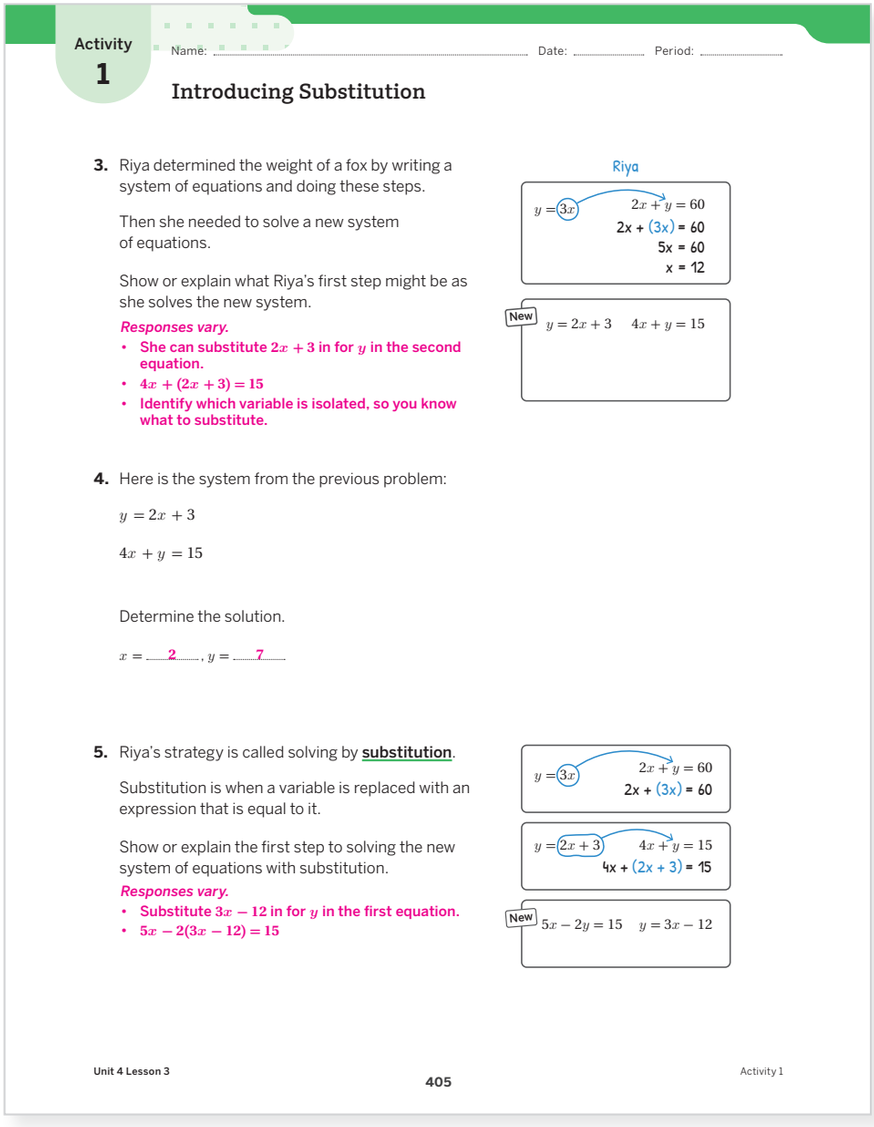
Invite students to review the answers with a classmate to help them revise their responses.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to discuss Riya's strategy with a partner and work together to solve the new system.
Substitute $2x + 3$ for y in the second equation, writing the new equation without parentheses.	**Strengthen** Consider sharing that parentheses support us in attending to precision when substituting, and ask students why they think that is.
Solve for x, and need support solving for y.	**Strengthen** Consider asking, "Where could you substitute this value of x?"
Would like a challenge.	**Stretch** Consider asking, "How would you solve this system using an elimination strategy?"

Notice student responses that include terms like *replace, plug in, isolated variable*, or *variable by itself* to share during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Introducing Substitution

3. Riya determined the weight of a fox by writing a system of equations and doing these steps.

Then she needed to solve a new system of equations.

Show or explain what Riya's first step might be as she solves the new system.

Responses vary.

- She can substitute $2x + 3$ in for y in the second equation.
- $4x + (2x + 3) = 15$
- Identify which variable is isolated, so you know what to substitute.

Riya

$y = 3x$ $2x + y = 60$

$2x + (3x) = 60$

$5x = 60$

$x = 12$

New $y = 2x + 3$ $4x + y = 15$

4. Here is the system from the previous problem:

$y = 2x + 3$

$4x + y = 15$

Determine the solution.

$x =$ 2, $y =$ 7

5. Riya's strategy is called solving by **substitution**.

Substitution is when a variable is replaced with an expression that is equal to it.

Show or explain the first step to solving the new system of equations with substitution.

Responses vary.

- Substitute $3x - 12$ in for y in the first equation.
- $5x - 2(3x - 12) = 15$

$y = 3x$ $2x + y = 60$

$2x + (3x) = 60$

$y = 2x + 3$ $4x + y = 15$

$4x + (2x + 3) = 15$

New $5x - 2y = 15$ $y = 3x - 12$

Unit 4 Lesson 3 405 Activity 1

Pairs | 15 min

Activity 1 Introducing Substitution (continued)

Purpose: Students are introduced to the strategy of **substitution** to solve systems of equations as they analyze a student's work and identify common errors when using this strategy.

2 Monitor

Look for students who notice in their responses that while Natalia's solution for the problem was incorrect, part of her process was correct. Consider sharing that there is often correct mathematical thinking in every strategy, even those with incorrect solutions.

Encourage students to verify their solutions by plugging their work in. **(MTR.6.1)**

3 Connect

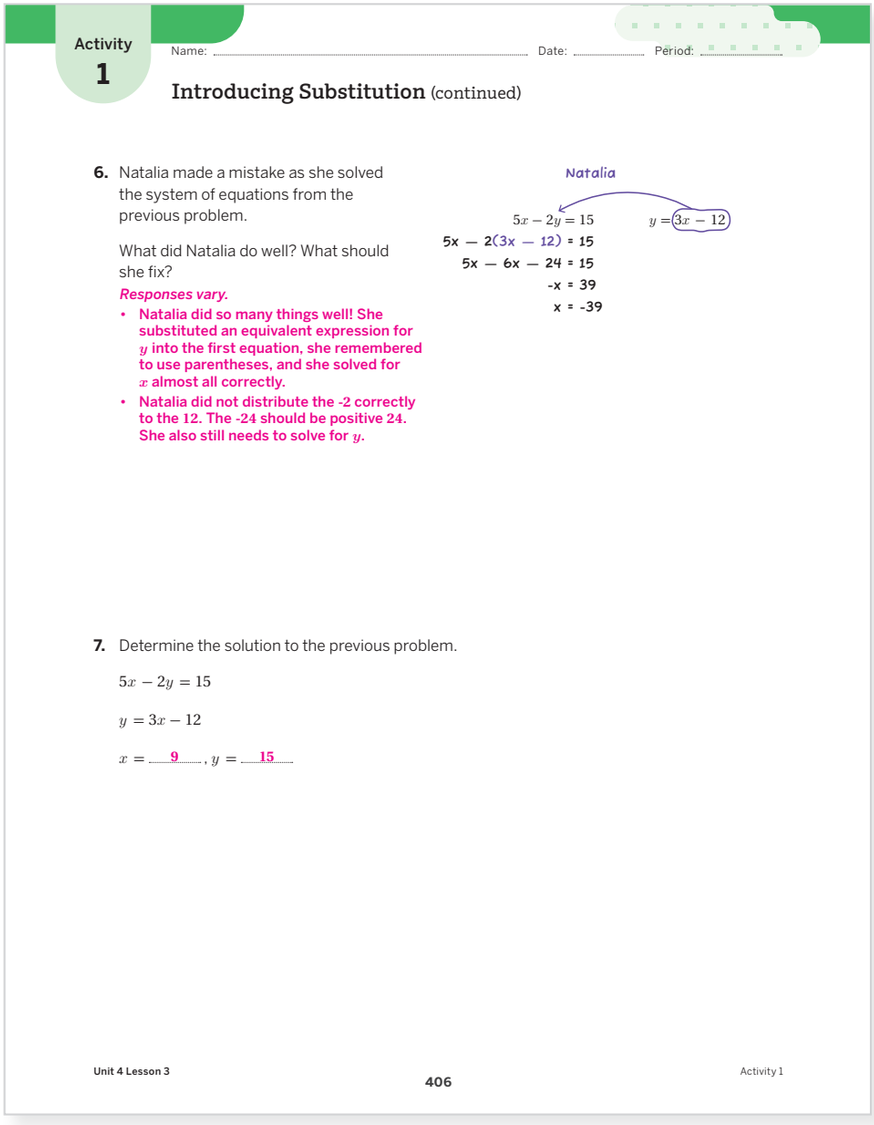
Share student responses. Consider asking:

- "Where else have you heard the word *substitution*?"
- "How did you know which variable to substitute?"
- "How did you know where to substitute the expression?"
- "What happened after you substituted? How did it help your solution process?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Natalia's work. As a class:

- Discuss what is *correct* about her response.
- Discuss what is *incorrect* or *unclear.*
- Create a *correct and clear* solution.

Consider asking, "What does it mean to use the *distributive property*? How can we check if Natalia distributed the coefficient -2 correctly?"



Activity 1

Name: Date: Period:

Introducing Substitution (continued)

6. Natalia made a mistake as she solved the system of equations from the previous problem.

What did Natalia do well? What should she fix?

Responses vary.

- Natalia did so many things well! She substituted an equivalent expression for y into the first equation, she remembered to use parentheses, and she solved for x almost all correctly.
- Natalia did not distribute the -2 correctly to the 12. The -24 should be positive 24. She also still needs to solve for y.

Natalia

$5x - 2y = 15$ $\quad y = 3x - 12$

$5x - 2(3x - 12) = 15$

$5x - 6x - 24 = 15$

$-x = 39$

$x = -39$

7. Determine the solution to the previous problem.

$5x - 2y = 15$

$y = 3x - 12$

$x =$ 9 , $y =$ 15

Unit 4 Lesson 3 406 Activity 1

Key Takeaway: **Substitution** is a method for solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

Pairs | 15 min

Activity 2 Practicing Substitution

Purpose: Students develop fluency with solving systems of equations as they explain which solving strategy they would prefer to use based on the structure of the equations.

Short on time? Consider having students complete fewer Repeated Challenges.

1 Launch

Use the Think-Pair-Share routine to support students as they shape their ideas about how to solve each system using substitution. See the Routine Facilitation Guide for more info.

Listen for student language that describes how they are making use of the structure of each system of equations to identify first steps and pick a solving strategy.

Consider asking:

- "Which system's structure is best set up to solve using substitution? Using elimination?"
- "What evidence do you have to support your thinking?"

2 Monitor

D **Differentiation: (Stretch)** Invite students to write one system they would want to solve using substitution and one they would not.

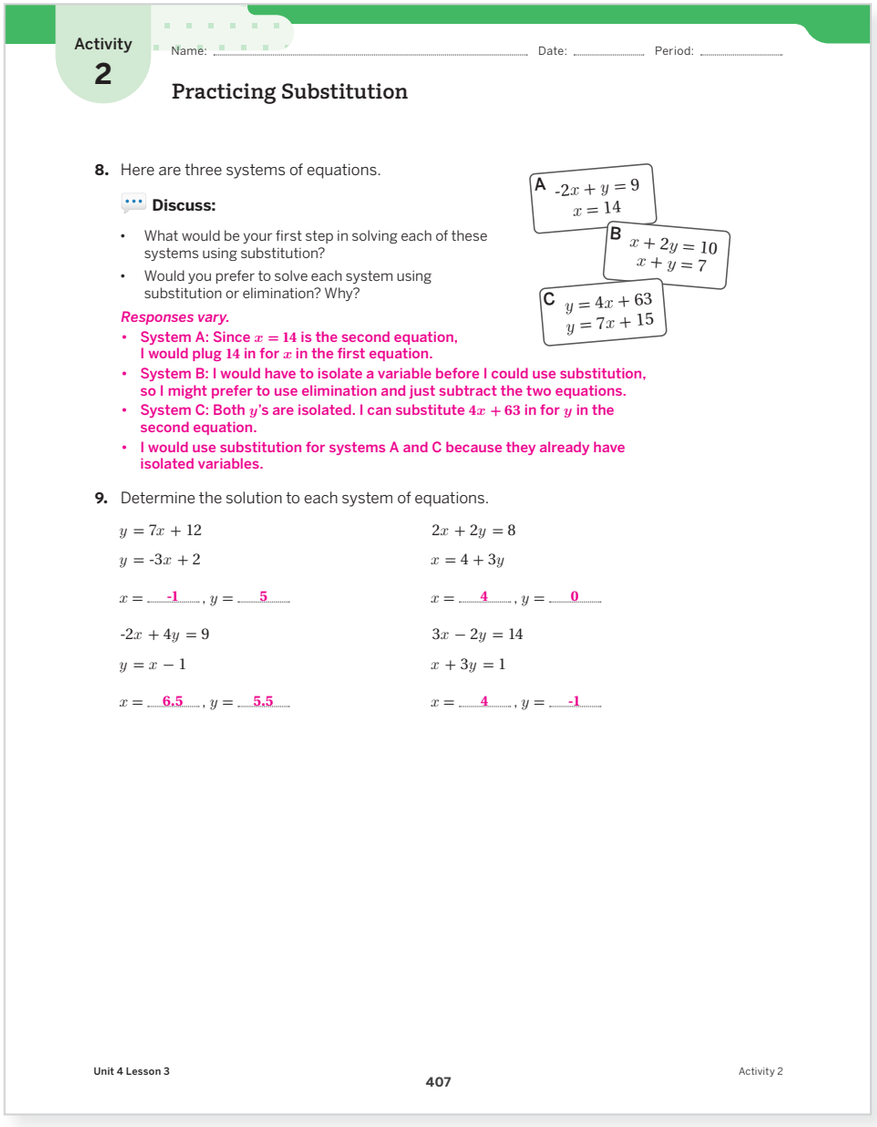
Invite students to set a goal for how many challenges they want to solve. Share that it's more important to make sense of the challenges than it is to work quickly.

Students using print have four problems to solve instead of receiving responsive feedback.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

3 Connect

Invite students to share strategies they found helpful while solving challenges.



Activity 2

Name: Date: Period:

Practicing Substitution

8. Here are three systems of equations.

A $-2x + y = 9$, $x = 14$

B $x + 2y = 10$, $x + y = 7$

C $y = 4x + 63$, $y = 7x + 15$

Discuss:

- What would be your first step in solving each of these systems using substitution?
- Would you prefer to solve each system using substitution or elimination? Why?

Responses vary.

- System A: Since $x = 14$ is the second equation, I would plug 14 in for x in the first equation.
- System B: I would have to isolate a variable before I could use substitution, so I might prefer to use elimination and just subtract the two equations.
- System C: Both y's are isolated. I can substitute $4x + 63$ in for y in the second equation.
- I would use substitution for systems A and C because they already have isolated variables.

9. Determine the solution to each system of equations.

$y = 7x + 12$
$y = -3x + 2$

$x =$ -1, $y =$ 5

$2x + 2y = 8$
$x = 4 + 3y$

$x =$ 4, $y =$ 0

$-2x + 4y = 9$
$y = x - 1$

$x =$ 6.5, $y =$ 5.5

$3x - 2y = 14$
$x + 3y = 1$

$x =$ 4, $y =$ -1

Unit 4 Lesson 3 407 Activity 2

Key Takeaway: Systems of equations can be structured in ways that make solving by substitution or by elimination an easier strategy to start with. Substitution is an easier strategy when one variable is already isolated in an equation. Elimination is an easier strategy when the coefficients of one of the variables are the same or opposite.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the two solving strategies they have learned so far: substitution and elimination.

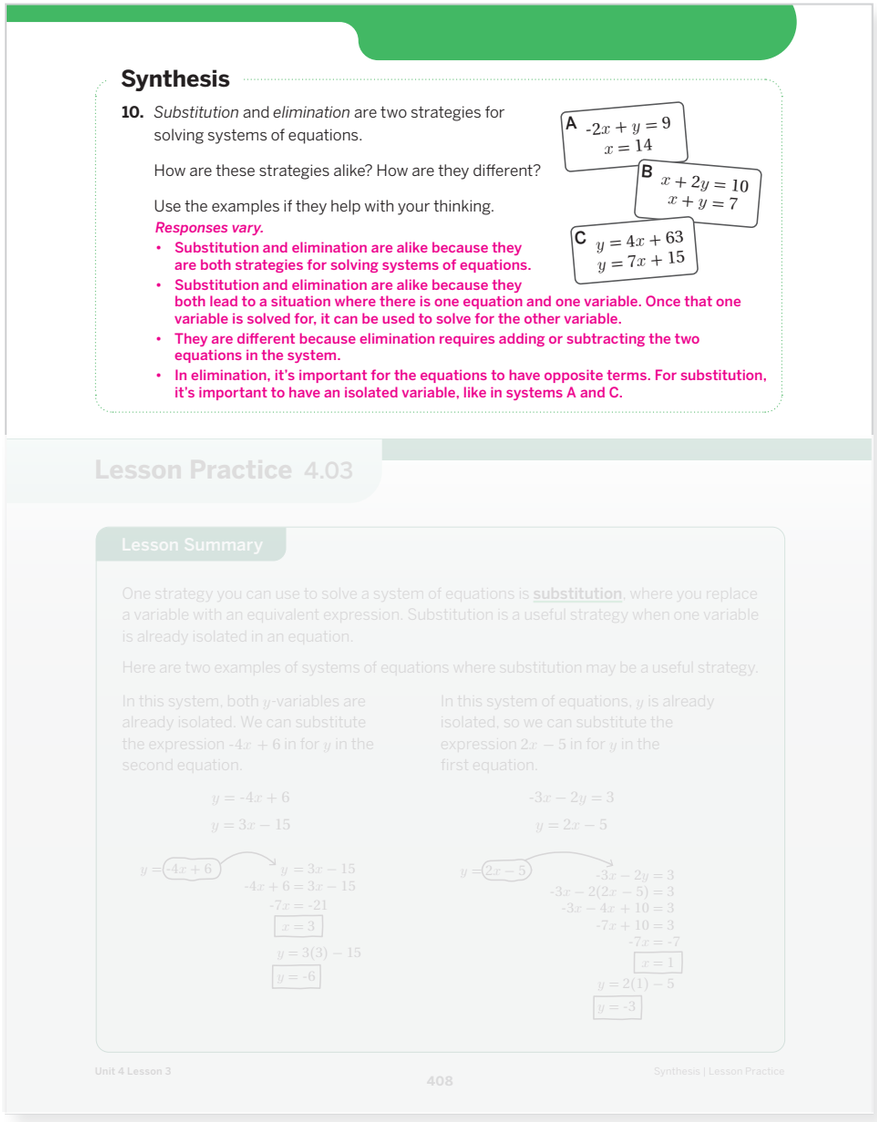
Invite students to respond independently, after discussing each question with their partner.

Look for and share a variety of ideas, such as:

- When a system has a variable that is already isolated, the structure may make substitution an easier strategy to try first.
- When a system has variables in standard form, the structure may make elimination an easier strategy to try first.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the Connect. Create an anchor chart comparing the strategies of solving systems of equations by substitution and elimination. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. *Substitution* and *elimination* are two strategies for solving systems of equations.

How are these strategies alike? How are they different?

Use the examples if they help with your thinking.

A $-2x + y = 9$, $x = 14$

B $x + 2y = 10$, $x + y = 7$

C $y = 4x + 63$, $y = 7x + 15$

Responses vary.

- Substitution and elimination are alike because they are both strategies for solving systems of equations.
- Substitution and elimination are alike because they both lead to a situation where there is one equation and one variable. Once that one variable is solved for, it can be used to solve for the other variable.
- They are different because elimination requires adding or subtracting the two equations in the system.
- In elimination, it's important for the equations to have opposite terms. For substitution, it's important to have an isolated variable, like in systems A and C.

Lesson Practice 4.03

Lesson Summary

One strategy you can use to solve a system of equations is **substitution**, where you replace a variable with an equivalent expression. Substitution is a useful strategy when one variable is already isolated in an equation.

Here are two examples of systems of equations where substitution may be a useful strategy.

In this system, both y-variables are already isolated. We can substitute the expression $-4x + 6$ in for y in the second equation.

$y = -4x + 6$

$y = 3x - 15$

$y = (-4x + 6)$

$y = 3x - 15$

$-4x + 6 = 3x - 15$

$-7x = -21$

$x = 3$

$y = 3(3) - 15$

$y = -6$

In this system of equations, y is already isolated, so we can substitute the expression $2x - 5$ in for y in the first equation.

$-3x - 2y = 3$

$y = 2x - 5$

$y = (2x - 5)$

$-3x - 2y = 3$

$-3x - 2(2x - 5) = 3$

$-3x - 4x + 10 = 3$

$-7x + 10 = 3$

$-7x = -7$

$x = 1$

$y = 2(1) - 5$

$y = -3$

Unit 4 Lesson 3 408 Synthesis | Lesson Practice

Lesson Takeaway: There are many ways to solve a system of equations. **Substitution** and using elimination are two strategies that can be used to solve systems symbolically. These strategies can be used to solve any system of equations, although the structure of some systems may make one strategy easier than the other.

Independent | 5 min

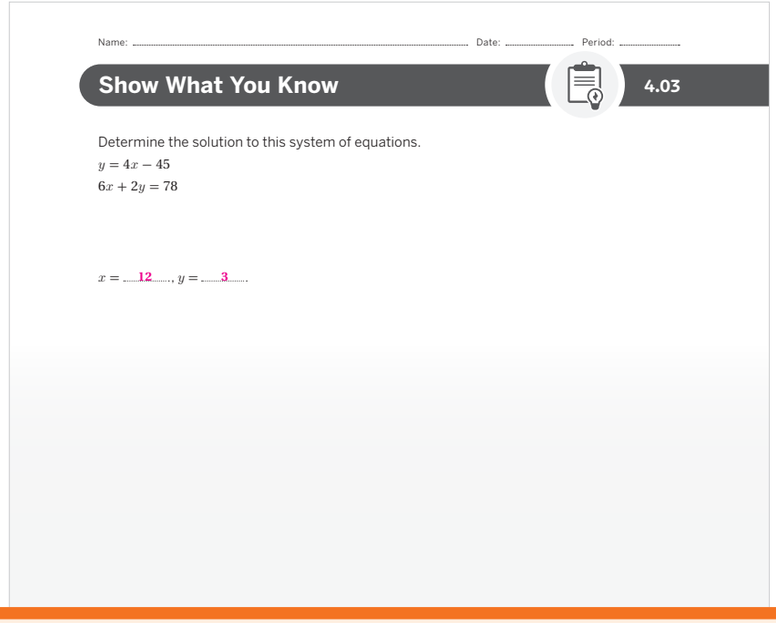
Show What You Know

Purpose: Students demonstrate their understanding by solving a system of equations using any strategy.

Today's Goals

1. **Goal:** Solve systems of equations using substitution.
2. **Language Goal:** Describe a strategy for solving systems of equations using substitution. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.03

Determine the solution to this system of equations.

$y = 4x - 45$

$6x + 2y = 78$

$x =$ 12, $y =$ 3

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve systems of equations using substitution:

» **Respond to Student Thinking:**

- Use the **Solving Systems of Linear Equations by Substitution Mini-lesson**.
- Revisit **Grade 8, Unit 4, Lesson 8** (*Make Them Balance*).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to construct an argument about whether substitution or elimination is the best strategy, encourage students who support different strategies to share their thinking with the class.

Professional Learning

Take a moment to reflect on how students engaged with choosing appropriate tools strategically in today's lesson. What opportunities were there to help students see the value in having more than one tool and strategy to solve the same problem? What space was there to honor that students might prefer one strategy over another?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. *Substitution* and *elimination* are two strategies for solving systems of equations.

How are these strategies alike? How are they different?

Use the examples if they help with your thinking.

A $-2x + y = 9$, $x = 14$

B $x + 2y = 10$, $x + y = 7$

C $y = 4x + 63$, $y = 7x + 15$

Responses vary.

- Substitution and elimination are alike because they are both strategies for solving systems of equations.
- Substitution and elimination are alike because they both lead to a situation where there is one equation and one variable. Once that one variable is solved for, it can be used to solve for the other variable.
- They are different because elimination requires adding or subtracting the two equations in the system.
- In elimination, it's important for the equations to have opposite terms. For substitution, it's important to have an isolated variable, like in systems A and C.

Lesson Practice 4.03

Lesson Summary

One strategy you can use to solve a system of equations is **substitution**, where you replace a variable with an equivalent expression. Substitution is a useful strategy when one variable is already isolated in an equation.

Here are two examples of systems of equations where substitution may be a useful strategy.

In this system, both y-variables are already isolated. We can substitute the expression $-4x + 6$ in for y in the second equation.

$y = -4x + 6$

$y = 3x - 15$

$y = (-4x + 6)$

$-4x + 6 = 3x - 15$

$-7x = -21$

$\boxed{x = 3}$

$y = 3(3) - 15$

$\boxed{y = -6}$

In this system of equations, y is already isolated, so we can substitute the expression $2x - 5$ in for y in the first equation.

$-3x - 2y = 3$

$y = 2x - 5$

$y = (2x - 5)$

$-3x - 2y = 3$

$-3x - 2(2x - 5) = 3$

$-3x - 4x + 10 = 3$

$-7x + 10 = 3$

$-7x = -7$

$\boxed{x = 1}$

$y = 2(1) - 5$

$\boxed{y = -3}$

Lesson Practice 4.03

Name: _____ Date: _____ Period: _____

Problems 1–2: Show or explain what your first step would be for solving each system of equations.

1. $4x - y = 20$, $x + y = 5$

Responses vary. I would add the equations to eliminate y. I would get a new equation with just one variable: $5x = 25$.

2. $6x - 12y = 24$, $y = 2x - 1$

Responses vary. I would substitute the second equation into the first equation. I would have a new equation with just one variable: $6x - 12(2x - 1) = 24$.

3. Determine the solution to this system of equations:

$7x - y = -3$

$y = x - 3$

$x = -1$ $y = -4$

Test Practice

Problems 4–5: Alma made a mistake as she started to solve this system of equations.

$y = \frac{1}{2}x - 1$

$4x - 2y = 11$

Alma

$4x - 2(\frac{1}{2}x - 1) = 11$

$4x - x - 2 = 11$

$3x - 2 = 11$

$3x = 13$

$x = \frac{13}{3}$

4. What is the error in Alma's work?

Responses vary. The error is in the second line of Alma's work, where she is distributing the -2. The operation in front of the 2 should be + instead of -.

5. Determine the solution to this system of equations.

$x = 3$

$y = 0.5$

Lesson Practice 4.03

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 6–7: Kadeem made a mistake as he started to solve this system of equations.

$5x - 4y = 6$

$5(x + y = 25)$

Kadeem

$5x - 4y = 6$

$5x + 5y = 125$

$-1y = 131$

$y = -131$

6. Show or explain one thing Kadeem did correctly.

Responses vary. He multiplied the second equation by 5, which makes it so that both equations have a $5x$.

7. Show or explain Kadeem's mistake.

Responses vary. It isn't clear if Kadeem is adding or subtracting the two equations. He subtracted some terms but added others.

Problems 8–10: Solve each equation for the given variable.

8. Solve for k. $2t + k = 6$

$k = 6 - 2t$ (or equivalent)

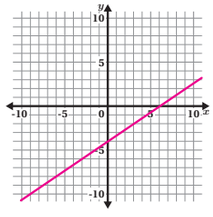
9. Solve for x. $4x + 3y = 12$

$x = 3 - \frac{3}{4}y$ (or equivalent)

10. Solve for y. $4x + 3y = 12$

$y = -\frac{4}{3}x + 4$ (or equivalent)

11. Graph the equation $4x - 6y = 24$.



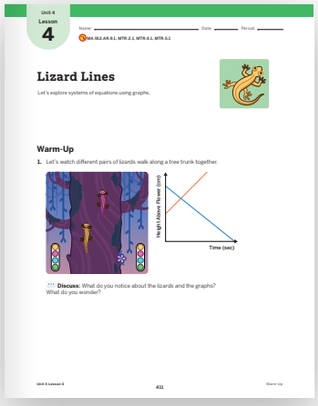
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.AR.9.1
	3	2	MA.912.AR.9.1
Test Practice	4-5	2	MA.912.AR.9.1
Spiral Review			
	6–7	2	MA.912.AR.9.1
Fluency	8–10	2	MA.912.AR.1.2
	11	2	MA.912.AR.2.5

Unit 4

Lesson

4


Unit 4
Lesson
4
Name: Date: Period:
MA.912.AR.9.1, MTR.2.1, MTR.4.1, MTR.5.1
Lizard Lines
Let's explore systems of equations using graphs.
Warm-Up
1. Let's watch different pairs of lizards walk along a tree trunk together.
Height Above the Ground (cm)
Time (sec)
Discuss: What do you notice about the lizards and the graphs? What do you wonder?
Unit 4 Lesson 4
411
Warm-Up

Lizard Lines

Graphing Systems of Linear Equations

Let's explore systems of equations using graphs.

Learning Outcomes

Today's Goals

1. **Goal:** Solve a system of linear equations using a graph.
2. **Language Goal:** Analyze the structure of a system of equations to justify whether the system has zero, one, or infinitely many solutions. **(Writing, Speaking, and Listening)**

Students use a graphing calculator to solve systems of equations and use the structure of the equations to determine whether a system will have zero, one, or *infinitely many solutions*. They describe that the *point of intersection* on a graph represents the solution to the system of equations. Students make sense of systems that have no solutions and infinitely many solutions within a context of lizard walks. **(MTR.2.1, MTR5.1)**

Coherence

Prior Learning

In Unit 1, students represented linear situations with multiple representations and learned that a graph is the set of all solutions to an equation plotted on a coordinate plane. In Lessons 2–3, students solved systems of equations using elimination and substitution.

Future Learning

In Unit 7, students will solve systems that include nonlinear functions, which allows for more possibilities than just no solutions, one solution, or infinitely many solutions.

Connections

- Students explore systems of equations by interpreting the graph of a line representing the time it takes a lizard to reach a certain height. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.1

Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.4.1	**MA.912.AR.3.8**
MA.8.AR.4.3	
MA.912.AR.9.6	

Vocabulary

Review Vocabulary

infinitely many solutions	*no solution*
parallel lines	*point of intersection*
slope (rate of change)	*slope-intercept form*
y-intercept	

Lesson at a Glance

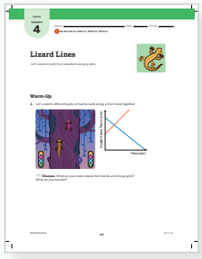
~ 45 min

MA.912.AR.9.1

Warm-Up

Pairs | 5 min

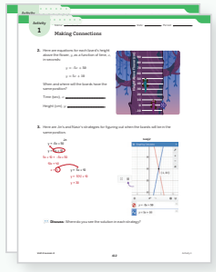
Students use the Notice and Wonder routine to make connections between linear situations involving lizards and their graphs. **(MTR.2.1)**



Activity 1

Pairs | 5 min

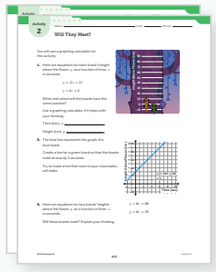
Students use MLR7: Compare and Connect to relate symbolic and graphing strategies for solving systems of linear equations.



Activity 2

Pairs | 5 min

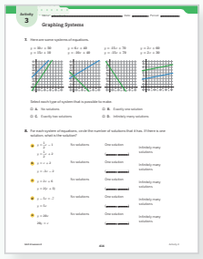
Students interpret what solutions to systems mean in context and make sense of what it means for a system to have *no solutions*.
Routine: Decide and Defend



Activity 3

Pairs | 20 min

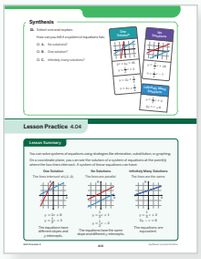
Students explore equations and graphs of various systems to describe how to use the structure of the equations to determine the number of solutions a system has. **Routine:** Decide and Defend **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize how to determine the number of solutions a system has by analyzing the structure of equations. MLR2: Collect and Display **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining how many solutions a system has and explaining their reasoning.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make connections between linear situations involving lizards and their graphs. **(MTR.2.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the context and graphs.

Note: Numbers are purposely removed from the Warm-Up so students can focus on the concepts of *slope* and y-intercepts rather than specific quantities. Listen for the different vocabulary students use. Students may use language like y-intercept, starting height above the flower on the tree, or the initial value, and they might refer to slope, rise over run, or rate of change.

2 Connect

Invite students to share things that they noticed and wondered.

To surface connections between the lizards and the graph, consider asking, "How can you see that in the graph?" **(MTR.2.1)**

Note: If it doesn't come up naturally, consider asking:

- "What is true about the lizards at the *point of intersection* in the graph?"
- "When there is no intersection point in the graph, what does that mean about the lizards?"
- "Which pair(s) of lizards will never be at the same height at the same time? Always be at the same height?"

ELL **English Language Learners** Invite several students to share what a new term reminds them of (e.g., When I hear *point of intersection*, I think _____.) to help surface multiple meanings and invite students to connect to their prior knowledge. It may also be helpful to research how to say the new terms in other languages. **(Reading, Writing, Speaking, and Listening)**

Unit 4
Lesson
4

Name: _____ Date: _____ Period: _____

MA.912.AR.9.1, MTR.2.1, MTR.4.1, MTR.5.1

Lizard Lines

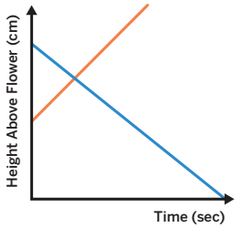
Let's explore systems of equations using graphs.



Warm-Up

1. Let's watch different pairs of lizards walk along a tree trunk together.





Discuss: What do you notice about the lizards and the graphs? What do you wonder?

Responses vary.

- I notice that you can compare the starting heights of the lizards on the y-axis of the graph.
- I notice that the green and yellow lizards will never be in the same place at the same time. Their graphs never intersect.
- I notice that the blue lizard is going down the tree, so the blue line on the graph is decreasing.
- I notice that when you pair two lizards of the same color, there is only one line on the graph.
- I wonder why there is only one line on the graph when there are two lizards of the same color.
- I wonder which lizard is moving the fastest.

Unit 4 Lesson 4 411 Warm-Up

Pairs | 5 min

Activity 1 Making Connections

Purpose: Students use **MLR7: Compare and Connect** to relate symbolic and graphing strategies for solving systems of linear equations.

1 Launch

Consider asking, "What can you tell about these lizards by looking at their equations?" **(MTR.2.1, MTR.5.1)**

Encourage students to use paper and/or a calculator to help with their thinking.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What strategies have we used in previous lessons that might help us now?"
Are using a guess and check strategy.	**Support** Celebrate their persistence. After they explore two different strategies on the next problem (Problem 3), invite them to reflect on if they would have preferred either of those strategies.
Use strategies similar to Jin's (substitution) and Nasir's (graphing).	Consider renaming Jin's and Nasir's strategies after the students who used them. **Strengthen** Invite these students to prepare to share their thinking during the Connect.
Determine the intersection point.	**Strengthen** Consider asking: • "What does the solution to this system mean about the lizards?" **(MTR.2.1)** • "What would the graph of this system look like?" • "How did you determine when they have the same position?" • "Is there another strategy for this problem?"

3 Connect

MLR MLR7: Compare and Connect Invite students to compare how they see the solution in each strategy. Consider asking:

- "What strategy did Jin use? Nasir?"
- "What are the advantages/disadvantages of each strategy?"
- "What does the solution to the system mean about the lizards?" **(MTR.2.1)**

To surface the Key Takeaway, discuss how the graph of a line is the set of all solutions to that equation and any point that lies on both lines is the solution to the system of equations.

Activity 1

Name: _____ Date: _____ Period: _____

Making Connections

2. Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

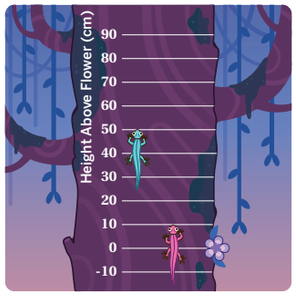
$y = -5x + 50$

$y = 5x + 10$

When and where will the lizards have the same position?

Time (sec), x: 4

Height (cm), y: 30



3. Here are Jin's and Nasir's strategies for figuring out when the lizards will be in the same position.

Jin

$y = -5x + 50$

$y = 5x + 10$

$5x + 10 = -5x + 50$

$10x = 40$

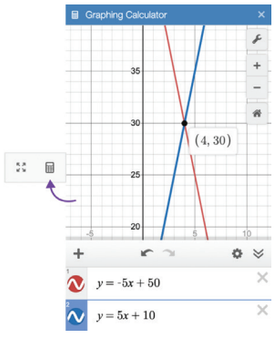
$x = 4$

$y = 5x + 10$

$y = 5(4) + 10$

$y = 30$

Nasir



Discuss: Where do you see the solution in each strategy?

Responses vary.

- In Jin's strategy, I can see the solution where Jin solved for both x and y: $x = 4$ and $y = 30$.
- On the graph, I can see the solution as the intersection point of the two lines. The first value in the coordinate pair is the x-value and the second is the y-value.

Unit 4 Lesson 4 412 Activity 1

Key Takeaway: On a graph, the solution to a system of equations is the *point of intersection*. Substituting this point into each equation will make both equations true. In this situation, the solution to the system means that at 4 seconds both lizards will be 30 centimeters above the flower.

Pairs | 5 min

Activity 2 Will They Meet?

Purpose: Students interpret what solutions to systems mean in context and make sense of what it means for a system to have *no solutions*.

Short on time? Consider skipping the discussion about Problem 5 if students understand that the intersection point is the solution to the system of equations.

1 Launch

Encourage students to use a graphing calculator and refer to Nasir's strategy if needed. Consider sharing that graphing calculators are tools for solving systems that allow us to focus more on interpreting the solution.

2 Monitor

To surface connections, consider asking pairs:
- "What can the equations tell us about the lizards?" **(MTR.5.1)**
- "What does the solution mean about the lizards?" **(MTR.2.1)**

To support students getting started, consider asking, "Where is the blue lizard at 5 seconds?"

To encourage learning connections, consider asking pairs, "Could you make a solution where the green lizard goes down the tree or one where the green lizard doesn't move at all?" **(MTR.2.1)**

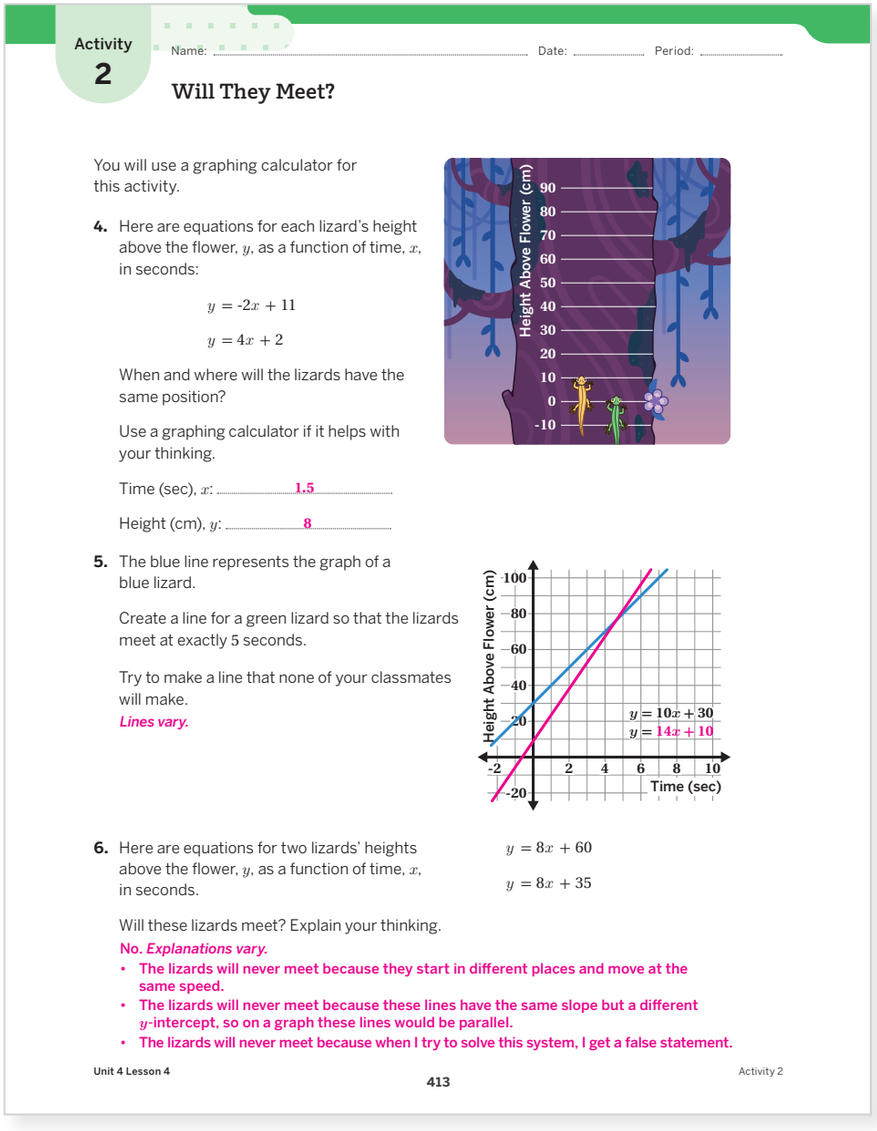
Select and sequence a variety of responses:
- They start at different heights, but move in the same direction at the same pace. **(MTR.2.1)**
- The lines are parallel using the graphing calculator.
- The equations have the same slope, but different y-intercepts. **(MTR.5.1)**
- Solve the system symbolically and get a false statement.

3 Connect

Consider asking (for Problem 6):
- "What is similar and different about the lines?"
- "Where is the solution on the graph?"
- "What does each part of the equation tell us about the lizards?" **(MTR.5.1)** The slope tells me the direction and speed of the lizard. The y-intercept tells me the lizard's height above the flower at 0 seconds.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Consider asking:
- "If the lizards never meet, what does this mean about the graph of the system of equations?"
- "If the lines are parallel, what does it mean about the solution to the system of equations?"



Activity 2

Name: Date: Period:

Will They Meet?

You will use a graphing calculator for this activity.

4. Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

$y = -2x + 11$

$y = 4x + 2$

When and where will the lizards have the same position?

Use a graphing calculator if it helps with your thinking.

Time (sec), x: 1.5

Height (cm), y: 8

5. The blue line represents the graph of a blue lizard.

Create a line for a green lizard so that the lizards meet at exactly 5 seconds.

Try to make a line that none of your classmates will make.

Lines vary.

6. Here are equations for two lizards' heights above the flower, y, as a function of time, x, in seconds.

$y = 8x + 60$

$y = 8x + 35$

Will these lizards meet? Explain your thinking.

No. *Explanations vary.*
- The lizards will never meet because they start in different places and move at the same speed.
- The lizards will never meet because these lines have the same slope but a different y-intercept, so on a graph these lines would be parallel.
- The lizards will never meet because when I try to solve this system, I get a false statement.

Unit 4 Lesson 4 413 Activity 2

Pairs | 20 min

Activity 3 Graphing Systems

Purpose: Students explore equations and graphs of various systems to describe how to use the structure of the equations to determine the number of solutions a system has. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer repeated challenges.

1 Launch

Consider asking, "What does the word *infinite* remind you of?" Ensure that all students understand the term *infinitely many solutions*.

ELL **English Language Learners** Invite students to research how to say the new term in other languages. **(Reading and Listening)**

2 Monitor

To support students getting started, consider asking, "What does a solution to a system look like on the graph?"

To use patterns and structure, consider asking:

- "Can you make a system that someone might think has no solutions, but it actually does?"
- "Can you make a system with a solution in the third quadrant? How do you know the solution is there?"
- "Which of these situations are possible? Not possible?"

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, chunk this activity into more manageable parts by inviting them to focus on one type of system at a time and check it off if it is possible.

Select and sequence interesting examples of each of the possible types of systems. **(MTR.5.1)**

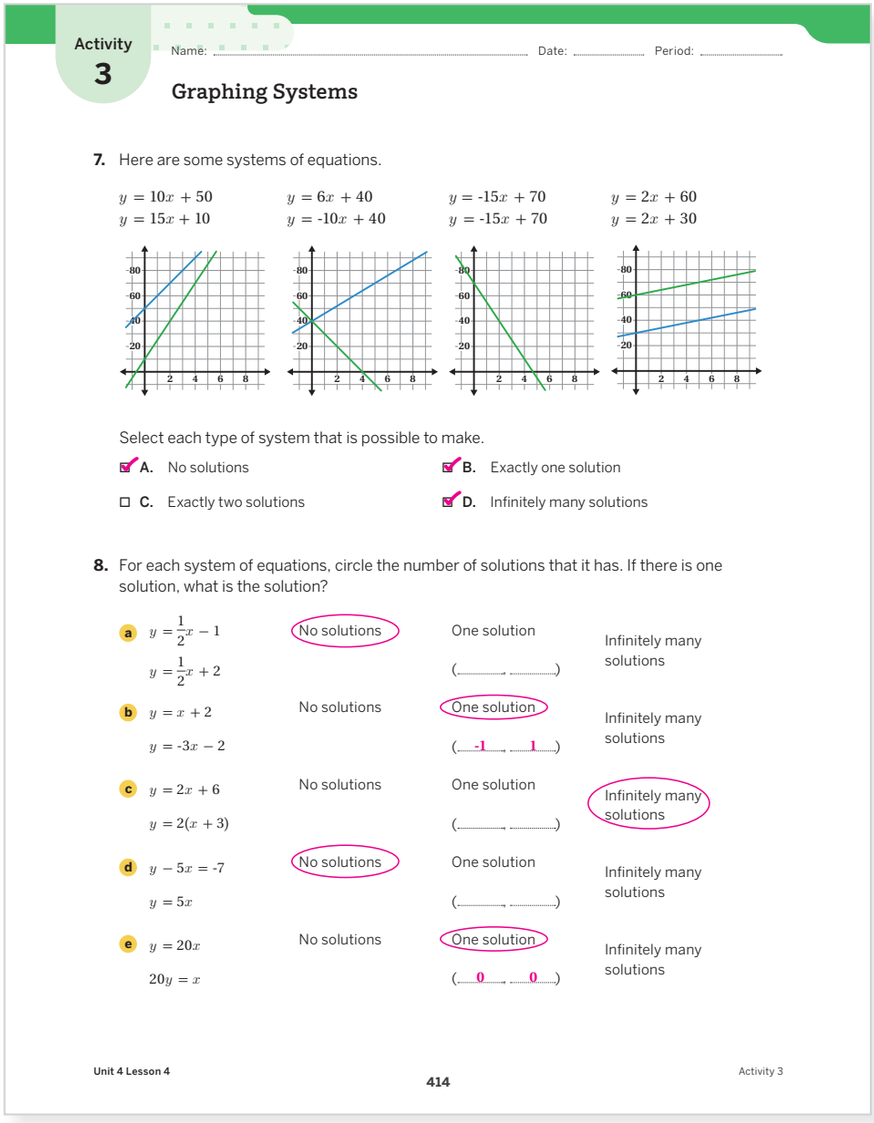
Pause to share and discuss the displayed systems one at time, asking questions like:

- "How many solutions does this system have?"
- "How can you tell from the graphs? the equations?"
- "What would this system mean in the lizard situation?"

Invite students to solve as many challenges in a set amount of time. Share that understanding the challenges is more important than speed.

Encourage students to start by analyzing the structure of the equations and use a graphing calculator if Language needed. **(MTR.5.1)**

Activity 3 continued >



Activity 3

Name: Date: Period:

Graphing Systems

7. Here are some systems of equations.

$y = 10x + 50$, $y = 15x + 10$

$y = 6x + 40$, $y = -10x + 40$

$y = -15x + 70$, $y = -15x + 70$

$y = 2x + 60$, $y = 2x + 30$

Select each type of system that is possible to make.

- ☑ A. No solutions
- ☑ B. Exactly one solution
- ☐ C. Exactly two solutions
- ☑ D. Infinitely many solutions

8. For each system of equations, circle the number of solutions that it has. If there is one solution, what is the solution?

a	$y = \frac{1}{2}x - 1$, $y = \frac{1}{2}x + 2$	(No solutions)	One solution (____, ____)	Infinitely many solutions
b	$y = x + 2$, $y = -3x - 2$	No solutions	(One solution) (-1, 1)	Infinitely many solutions
c	$y = 2x + 6$, $y = 2(x + 3)$	No solutions	One solution (____, ____)	(Infinitely many solutions)
d	$y - 5x = -7$, $y = 5x$	(No solutions)	One solution (____, ____)	Infinitely many solutions
e	$y = 20x$, $20y = x$	No solutions	(One solution) (0, 0)	Infinitely many solutions

Unit 4 Lesson 4 414 Activity 3

Pairs | 20 min

Activity 3 Graphing Systems (continued)

Purpose: Students explore equations and graphs of various systems to describe how to use the structure of the equations to determine the number of solutions a system has. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer repeated challenges.

2 Monitor

Encourage students to share their reasoning and come to a consensus about how to sort the systems with a partner.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you tell how many solutions a system will have from a graph? From equations?"
Need support with equations that are not in slope-intercept form.	**Support** Invite these students to solve for y, rewrite the equation in slope-intercept form, or use a graphing calculator.
Want to explore systems with no solutions and infinitely many solutions further.	**Stretch** Invite students to solve a system of equations with no solutions and one with infinitely many solutions symbolically and describe what happens.

To support students getting started, invite them to describe each statement's graph.

A Accessibility: Conceptual Processing To support students thinking abstractly, invite them to answer more concrete questions like, "In the system on the left, what are the slopes? the y-intercepts? What will the graph look like?"

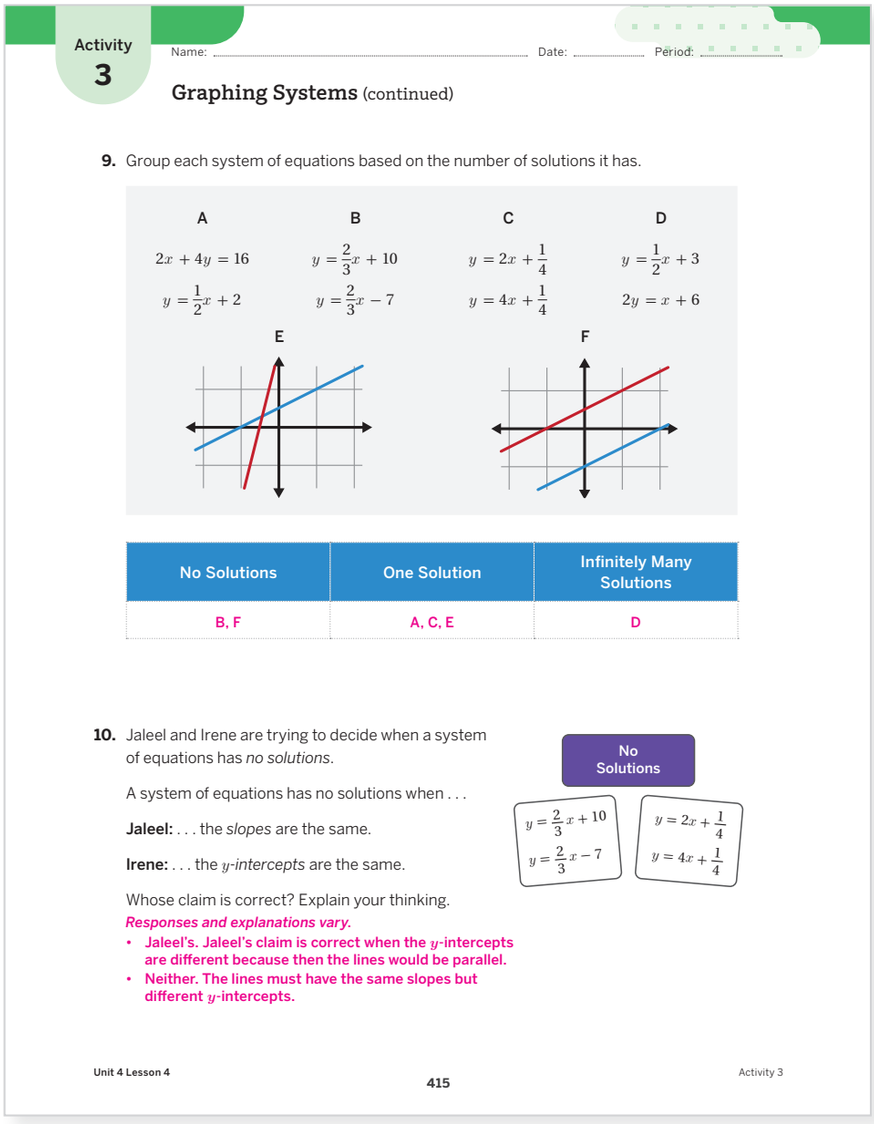
ELL English Language Learners Consider displaying an anchor chart of a slope-intercept form equation identifying the slope and y-intercept. Consider including translations of words like slope and y-intercept. **(Reading, Writing, Speaking, and Listening)**

Select and sequence a variety of correct and incorrect responses and explanations.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "How can you tell from the equations if a system has no solutions?" **(MTR.5.1)**



Activity 3

Name: Date: Period:

Graphing Systems (continued)

9. Group each system of equations based on the number of solutions it has.

A	B	C	D
$2x + 4y = 16$ $y = \frac{1}{2}x + 2$	$y = \frac{2}{3}x + 10$ $y = \frac{2}{3}x - 7$	$y = 2x + \frac{1}{4}$ $y = 4x + \frac{1}{4}$	$y = \frac{1}{2}x + 3$ $2y = x + 6$

E

F

No Solutions	One Solution	Infinitely Many Solutions
B, F	A, C, E	D

10. Jaleel and Irene are trying to decide when a system of equations has *no solutions*.

A system of equations has no solutions when . . .

Jaleel: . . . the *slopes* are the same.

Irene: . . . the *y-intercepts* are the same.

Whose claim is correct? Explain your thinking.

No Solutions

$y = \frac{2}{3}x + 10$
$y = \frac{2}{3}x - 7$

$y = 2x + \frac{1}{4}$
$y = 4x + \frac{1}{4}$

Responses and explanations vary.

- Jaleel's. Jaleel's claim is correct when the y-intercepts are different because then the lines would be parallel.
- Neither. The lines must have the same slopes but different y-intercepts.

Unit 4 Lesson 4 415 Activity 3

Key Takeaway: A system of two linear equations can have one solution, *no solutions*, or *infinitely many solutions*. It is possible to determine which of these cases is true for a given system by analyzing the lines' *slopes* and y-intercepts.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize how to determine the number of solutions a system has by analyzing the structure of equations. **(MTR.5.1)**

Invite students to respond independently, after discussing each option with their partner.

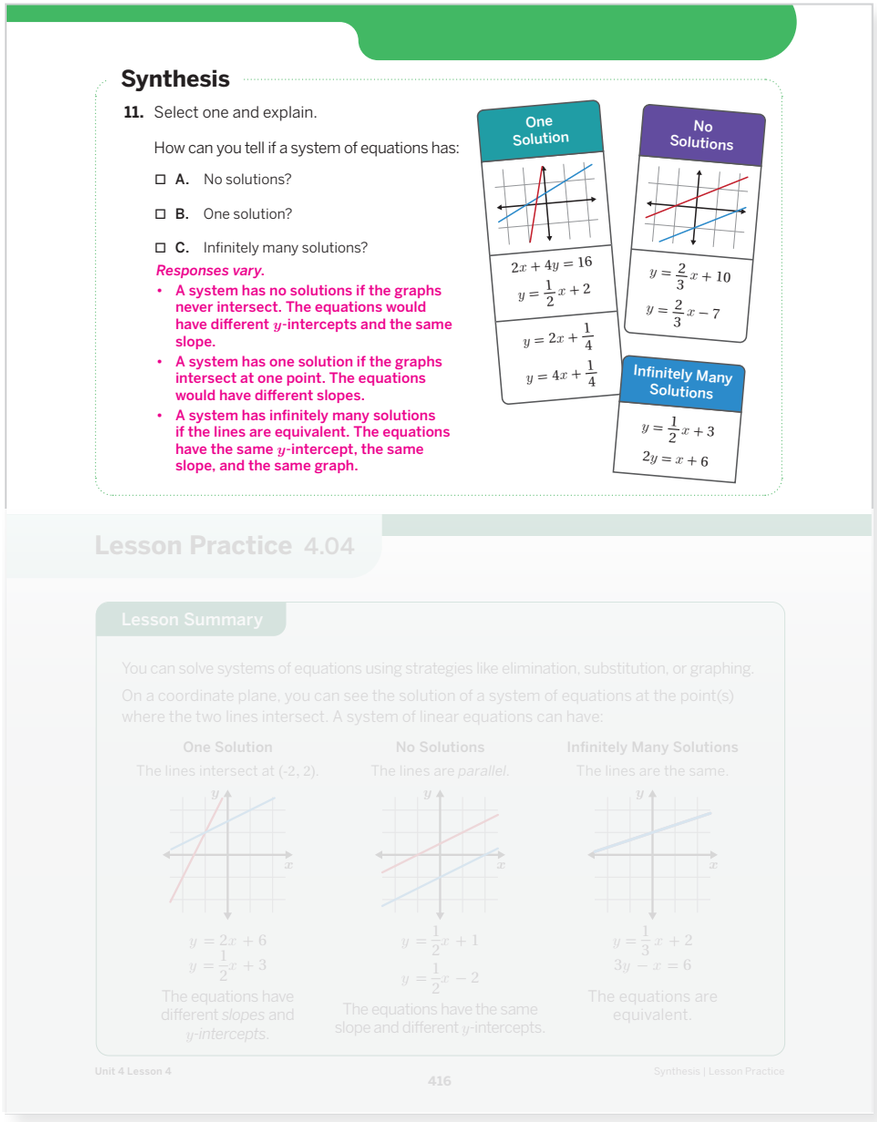
ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *graphs*, *equations*, *same*, *different*, *slope*, y*-intercept*, *because*, *parallel*, *intersect*, etc. **(Writing, Speaking, and Listening)**

Look for and share a variety of ideas, such as:

- Looking at a graph to see how many times the lines intersect.
- Analyzing the slopes and y-intercepts of the equation.
- Looking for equivalent equations.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and create an anchor chart about systems that have no solutions, one solution, or infinitely many solutions. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Select one and explain.

How can you tell if a system of equations has:

- ☐ **A.** No solutions?
- ☐ **B.** One solution?
- ☐ **C.** Infinitely many solutions?

Responses vary.

- A system has no solutions if the graphs never intersect. The equations would have different y-intercepts and the same slope.
- A system has one solution if the graphs intersect at one point. The equations would have different slopes.
- A system has infinitely many solutions if the lines are equivalent. The equations have the same y-intercept, the same slope, and the same graph.

One Solution

$2x + 4y = 16$

$y = \frac{1}{2}x + 2$

$y = 2x + \frac{1}{4}$

$y = 4x + \frac{1}{4}$

No Solutions

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

Infinitely Many Solutions

$y = \frac{1}{2}x + 3$

$2y = x + 6$

Lesson Practice 4.04

Lesson Summary

You can solve systems of equations using strategies like elimination, substitution, or graphing.

On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

One Solution

The lines intersect at (-2, 2).

$y = 2x + 6$

$y = \frac{1}{2}x + 3$

The equations have different *slopes* and y*-intercepts*.

No Solutions

The lines are *parallel*.

$y = \frac{1}{2}x + 1$

$y = \frac{1}{2}x - 2$

The equations have the same slope and different y-intercepts.

Infinitely Many Solutions

The lines are the same.

$y = \frac{1}{3}x + 2$

$3y - x = 6$

The equations are equivalent.

Unit 4 Lesson 4 416 Synthesis | Lesson Practice

Lesson Takeaway: On a graph, the solution to a system of equations is the *point of intersection*. In a system of two linear equations, the *slopes* and y*-intercepts* can be analyzed to determine the number of solutions a system has.

- If they have the same slope and different y-*intercepts*, they are *parallel*. This means they never intersect and therefore the system of equations has *no solutions* because no point lies on both lines.
- If they have the same slope and same y-intercept, then the equations are equivalent and their system has *infinitely many solutions*.
- If they have different slopes, they intersect one time, and therefore have one solution.

Independent | 5 min

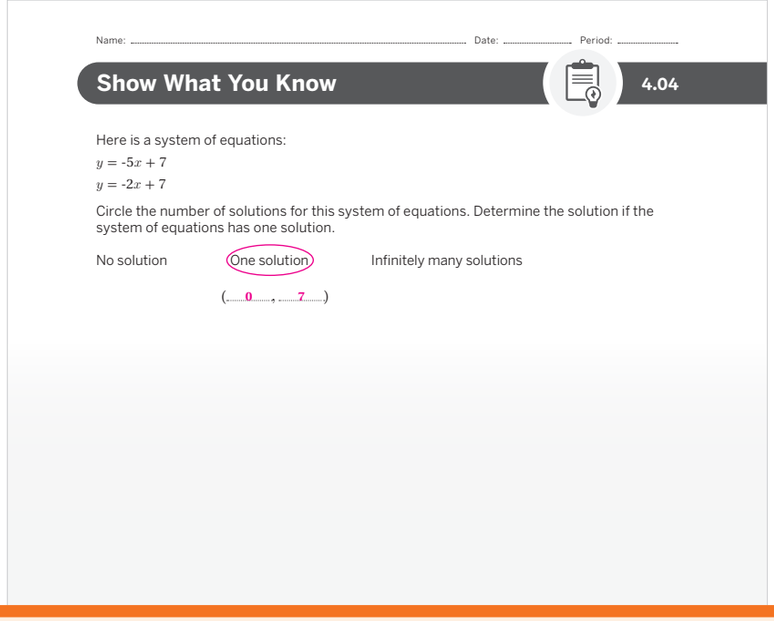
Show What You Know

Purpose: Students demonstrate their understanding by determining how many solutions a system has and explaining their reasoning.

Today's Goals

1. **Goal:** Solve a system of linear equations using a graph.
2. **Language Goal:** Analyze the structure of a system of equations to justify whether the system has zero, one, or infinitely many solutions. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.04

Here is a system of equations:

$y = -5x + 7$

$y = -2x + 7$

Circle the number of solutions for this system of equations. Determine the solution if the system of equations has one solution.

No solution (One solution) Infinitely many solutions

(0, 7)

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding that all points on a graph are solutions to that equation:

» **Respond to Student Thinking:**

- Revisit **Unit 1, Lesson 6** (*Shelley the Snail*).

If students need support understanding how to of graph a system of equations:

» **Respond to Student Thinking:**

- Revisit **Grade 8, Unit 4, Lesson 8** (*Make Them Balance*).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write systems of equations that have one solution, no solutions, and infinitely many solutions.

Professional Learning

Reflect on your students' use of the graphing calculator in today's lesson. How did the graphing calculator support students in being able to interpret systems and their solutions in context? What were students' strengths and obstacles when using the graphing calculator? Were students comfortable adjusting the window and scale? How can you continue to support students in using this tool strategically?

Lesson Practice

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Select one and explain.

How can you tell if a system of equations has:

- ☐ A. No solutions?
- ☐ B. One solution?
- ☐ C. Infinitely many solutions?

Responses vary.

- A system has no solutions if the graphs never intersect. The equations would have different y-intercepts and the same slope.
- A system has one solution if the graphs intersect at one point. The equations would have different slopes.
- A system has infinitely many solutions if the lines are equivalent. The equations have the same y-intercept, the same slope, and the same graph.

Lesson Practice 4.04

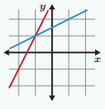
Lesson Summary

You can solve systems of equations using strategies like elimination, substitution, or graphing.

On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

One Solution

The lines intersect at (-2, 2).



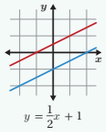
$y = 2x + 6$

$y = \frac{1}{2}x + 3$

The equations have different *slopes* and *y-intercepts*.

No Solutions

The lines are *parallel*.



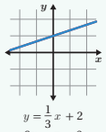
$y = \frac{1}{2}x + 1$

$y = \frac{1}{2}x - 2$

The equations have the same slope and different y-intercepts.

Infinitely Many Solutions

The lines are the same.



$y = \frac{1}{3}x + 2$

$3y - x = 6$

The equations are equivalent.

Lesson Practice 4.04

Name: _____ Date: _____ Period: _____

1. The point (-2, 2) is on the line $y = x + 4$. Explain how you can determine if this point is the solution to this system of equations:

$y = x + 4$

$y = 2x - 1$

Explanations vary. **If the point (-2, 2) is also on the line $y = 2x - 1$, then it is the solution to the system.**

2. Solve this system of equations. Write the solution as a coordinate pair.

$y = -\frac{1}{2}x - 8$

$y = 3x + 6$

(-4, -6)

3. Match each system of equations to the number of solutions it has.

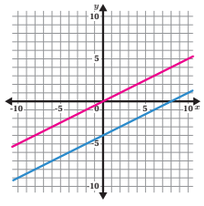
(a) $y = -2x + 1$, $2y = -4x + 2$ — b — No solutions

(b) $y = -2x + 1$, $y = -2x + 4$ — c — One solution

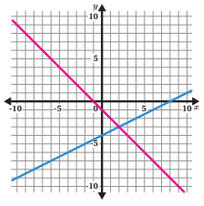
(c) $y = -2x + 1$, $y = 2x + 1$ — a — Infinitely many solutions

Problems 4–5: Here is a graph of $y = \frac{1}{2}x - 4$. Graph a second line to make this system of equations have:

4. No solutions. ***Lines vary.*** **Line should have a slope of $\frac{1}{2}$ and a y-intercept different than -4.**



5. One solution at (2, -3). ***Lines vary.*** **Line should include the point (2, -3).**



6. Here is one equation in a system of equations: $y = 3x - 2$. Write a different second equation so that the system of equations has infinite solutions.

Equations vary. **Equation should have a slope of 3 and y-intercept of -2.**

Lesson Practice 4.04

Name: _____ Date: _____ Period: _____

Test Practice

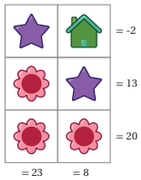
7. A system of equations has no solutions. Select *all* of the statements that must be true about the equations in this system.

- ☐ A. The equations have different slopes
- ☑ B. The equations have the same slope
- ☑ C. The equations have different y-intercepts
- ☐ D. The equations have the same y-intercepts

Spiral Review

8. Here is a shape puzzle. What is the value of each shape?

Shape	Value
Star	3
House	-5
Flower	10



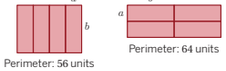
9. Solve this inequality: $3(x - 3) > 2x - 6$

$x > 3$

10. Here are two arrangements of identical rectangles. Determine the dimensions a and b.

$a =$ 4 units

$b =$ 12 units



Perimeter: 56 units

Perimeter: 64 units

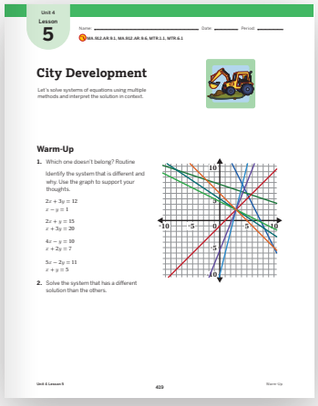
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4, 6	2	MA.912.AR.9.1
	2–3, 5	2	MA.912.AR.9.1
Test Practice	7	2	MA.912.AR.9.1
Spiral Review			
	8	2	MA.8.AR.4.3
Fluency	9	1	MA.8.AR.2.2
	10	3	MA.8.AR.4.3

Unit 4

Lesson

5



City Development

Solving Graphically and Symbolically

Let's solve systems of equations using multiple methods and interpret the solution in context.

Learning Outcomes

Today's Goals

1. **Goal:** Solve systems of equations graphically and symbolically.
2. **Language Goal:** Interpret the solution to a system of equations in context. **(Reading, Writing, Speaking, and Listening)**

Students solve systems of equations using multiple methods and interpret the solution in context. Students make sense of systems of equations by describing connections between the equations and the context. They solve systems graphically (by hand) and symbolically, and describe how to see the solution using each method. **(MTR.6.1)**

Note: This lesson is designed for students to graph by hand, instead of using the Desmos Graphing Calculator or other technology for graphing. Consider encouraging students to graph by hand throughout the lesson.

Coherence

Prior Learning

In Lessons 1–2, students solved systems of equations using elimination strategies. In Lesson 3, students solved systems of equations using substitution strategies. In Lesson 4, students solved systems using graphs.

Future Learning

In Lesson 6, students will represent a situation with a system of equations and explain how to use a system of equations and its solution to make recommendations in context. In Lesson 10, students will represent constraints in a situation using a system of inequalities.

Connections

- Students solve systems of equations using multiple methods and interpreting the solution in context of building apartments on unused plots of land. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.1
Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.

MA.912.AR.9.6
Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.6.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.4.3	**MA.912.AR.9.6**

Vocabulary

Review Vocabulary

- *constraint*
- *slope-intercept form*
- *standard form (of a linear equation)*
- x*-intercept*
- y*-intercept*

Lesson at a Glance

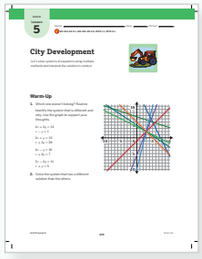
~ 45 min

MA.912.AR.9.1, MA.912.AR.9.6

Warm-Up

Small Groups | 5 min

Students make sense of four different systems and compare the graphs. Then, they solve one.
Routine: Which One Doesn't Belong
(MTR.1.1, MTR.6.1)



Activity 1

Small Groups | 12 min

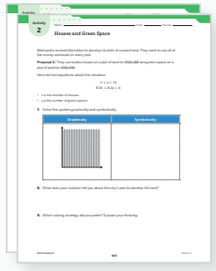
Students complete and compare different strategies for solving a system of equations in context.



Activity 2

Small Groups | 10 min

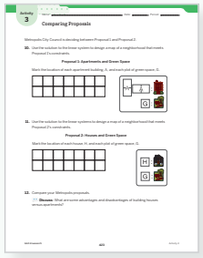
Students use two different strategies to solve a second system of equations and interpret the solution in context. **(MTR.6.1)**



Activity 3

Small Groups | 8 min

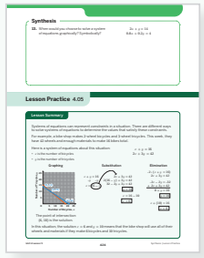
Students create two proposals, one for building apartment buildings and green spaces and the other for building houses and green spaces, and compare them.



Synthesis

Whole Class | 5 min

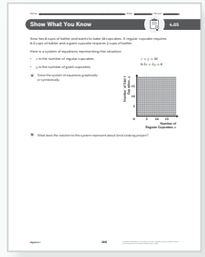
Students synthesize their strategies for solving systems of equations by describing when they would choose to use each strategy.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a system of equations graphically or symbolically and interpreting the solution in context. **(MTR.6.1)**



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Small Groups | 5 min

Warm-Up

Purpose: Students make sense of the context for this lesson, in which a city develops unused plots of land to meet community needs. **(MTR.6.1)**

1 Launch

Display the discussion prompts.

ELL **English Language Learners** Invite students to read the discussion prompts aloud and paraphrase what they mean, clarifying any unknown words or concepts before responding.

Consider asking, looking at the graph what do you notice is different? What do you notice is similar?

Use the Which One Doesn't Belong? routine to support student-to-student discourse. Give students one minute to think quietly. Ask them to indicate when they have noticed one letter that does not belong and can explain why. **(MTR.1.1)**

2 Connect

Invite students to share their responses to the prompts. Consider inviting students to share what someone else in their group thought a partner said rather than what they said during the conversation. **(MTR.1.1)**

Unit 4
Lesson
5

Name: Date: Period:

MA.912.AR.9.1, MA.912.AR.9.6, MTR.1.1, MTR.6.1

City Development

Let's solve systems of equations using multiple methods and interpret the solution in context.



Warm-Up

1. Which one doesn't belong? Routine

   Identify the system that is different and why. Use the graph to support your thoughts.

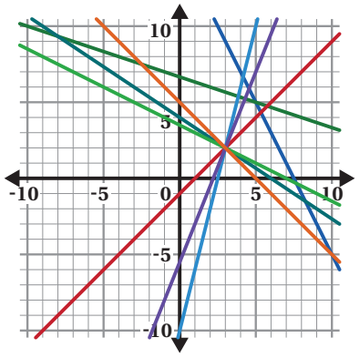
   $2x + 3y = 12$
   $x - y = 1$

   $2x + y = 15$
   $x + 3y = 20$

   $4x - y = 10$
   $x + 2y = 7$

   $5x - 2y = 11$
   $x + y = 5$

   I think the second system is different because all the others share the same solution. You can tell by where they intersect on the graph.



2. Solve the system that has a different solution than the others.

   $2x + y = 15$
   $x + 3y = 20$

   $x = 5\ y = 5$

Unit 4 Lesson 5 419 Warm-Up

Small Groups | 12 min

Activity 1 Apartments and Green Space

Purpose: Students complete and compare different strategies for solving a system of equations in context.

1 Launch

Display the details of Metropolis Proposal 1.

ELL **English Language Learners** Create a list of words about the context that students might not know (e.g., *grant*, *plot*, *apartment*, *green space*), along with visuals or translations of each word in languages your students use. **(Reading and Listening)**

Consider asking, "What does green space mean in this situation?" Parks, playgrounds, etc.

Encourage students to use symbols or to color code parts of the situation to match parts of the system of equations. **(MTR.6.1)**

A **Accessibility: Conceptual Processing** Invite students to say aloud what each equation means with the variable definitions in place of a and g (e.g., The 14 plots are made up of the number of green space plots plus 2 plots for every apartment building.).

2 Monitor

D **Differentiation: (Support)** To support students getting started on Problem 2, consider asking, "Which equation is about the plots of land? Which is about the budget?"

ELL **English Language Learners** As students share in group and whole-class discussions, provide sentence frames such as:
The _____ in the equation represents . . .
The _____ in the situation represents . . .
(Writing and Speaking)

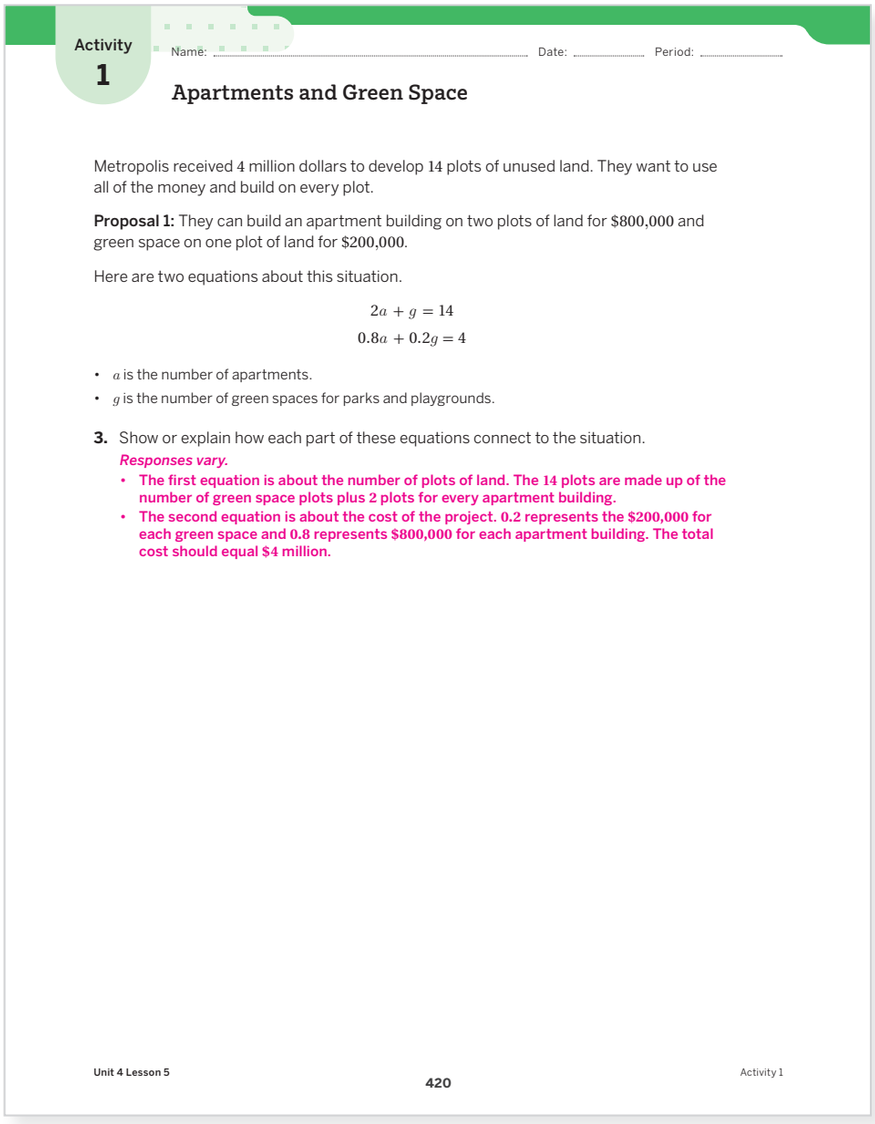
Pause briefly to discuss student interpretations of each part of the equation.

Consider asking, "How would you solve this system of equations? What would your first step be?" I would solve it by graphing because both equations are in standard form.

To surface connections, consider rewriting $.8a + .2g = 4$ as $800000\,a + 200000g = 4000000$ and discussing why someone might choose to work with one or the other of these equations.

Invite students to revise their interpretations based on the discussion if time allows.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Apartments and Green Space

Metropolis received 4 million dollars to develop 14 plots of unused land. They want to use all of the money and build on every plot.

Proposal 1: They can build an apartment building on two plots of land for $800,000 and green space on one plot of land for $200,000.

Here are two equations about this situation.

$$2a + g = 14$$
$$0.8a + 0.2g = 4$$

- a is the number of apartments.
- g is the number of green spaces for parks and playgrounds.

3. Show or explain how each part of these equations connect to the situation.

Responses vary.

- The first equation is about the number of plots of land. The 14 plots are made up of the number of green space plots plus 2 plots for every apartment building.
- The second equation is about the cost of the project. 0.2 represents the $200,000 for each green space and 0.8 represents $800,000 for each apartment building. The total cost should equal $4 million.

Unit 4 Lesson 5 420 Activity 1

Activity 1 Apartments and Green Space (continued)

Purpose: Students complete and compare different strategies for solving a system of equations in context.

2 Monitor

Encourage students in their groups to each choose a different student's work to focus on.

Invite students to engage in a jigsaw structure:
- Invite students to form new groups with classmates that chose the same student's work.
- Encourage students to think together in this group and complete the student's work they chose.
- Invite students to return to their original group to explain their thinking and compare across the different students' strategies.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose a step of one person's work at a time and ask, "What did they do? Why would they do that?"

D **Differentiation**

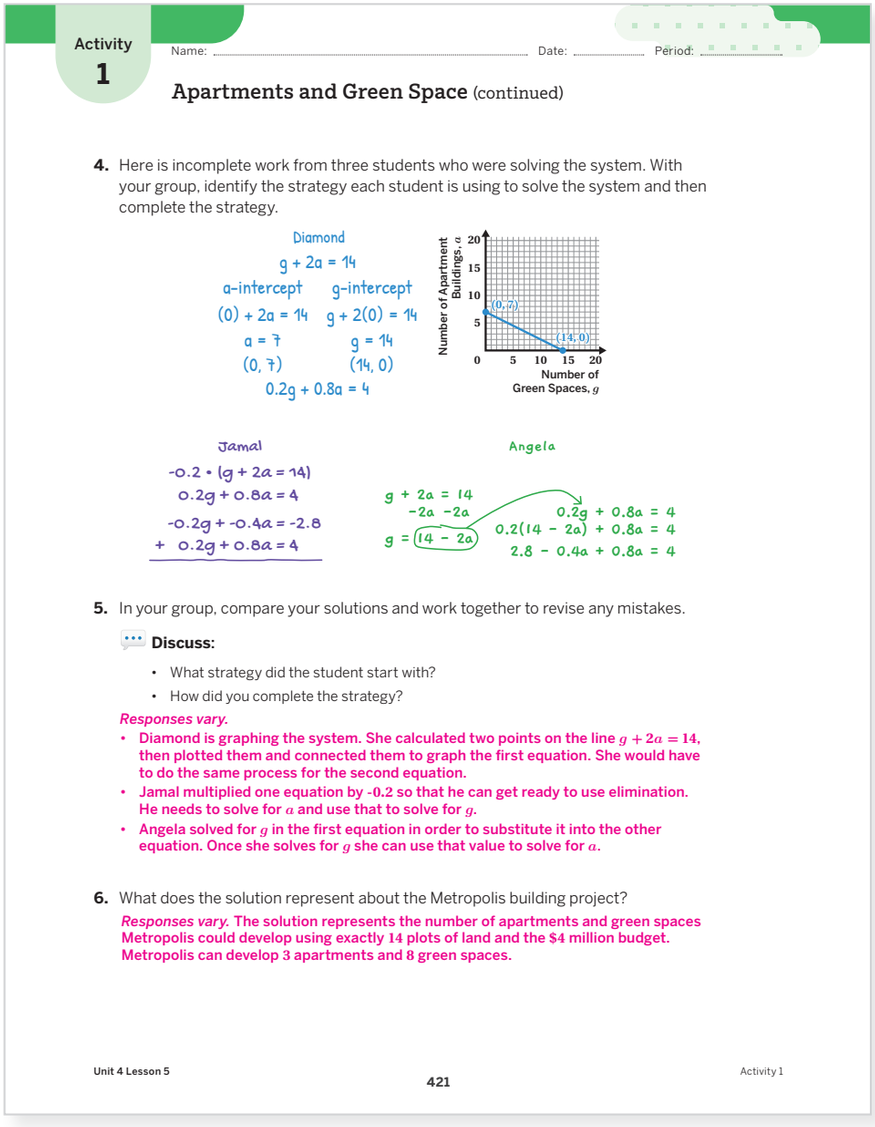
Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 4)*	**Support** Consider asking, "What solving strategy is this student using?"
Stop after solving correctly for one variable. *(Problem 4)*	**Support** Consider asking, "What could you do to find the value of the other variable?"
Solve the system by approximating using the graph in Diamond's work. *(Problem 4)*	**Support** Invite them to make their graph more precise with a straightedge and substitute their approximation into the system to check, then solve the system. Consider asking, "How does the solution compare to your approximation? When is each strategy useful?"
Are not sure how to interpret the solution in context of the building project as 8 green spaces and 3 apartment buildings. *(Problem 6)*	**Strengthen** Invite them to look back at how a and g were defined. Consider asking, "What do these values mean? What do they tell us about the budget and plots of land?"

3 Connect

Display the three strategies.

Invite students to share how they completed each strategy and where they saw the solution.

To surface the Key Takeaway, consider asking:
- "Did all three strategies give the same solution?"
- "What does the solution mean for the situation?"



Activity 1

Name: Date: Period:

Apartments and Green Space (continued)

4. Here is incomplete work from three students who were solving the system. With your group, identify the strategy each student is using to solve the system and then complete the strategy.

Diamond

$g + 2a = 14$

a-intercept: $(0) + 2a = 14$, $a = 7$, $(0, 7)$

g-intercept: $g + 2(0) = 14$, $g = 14$, $(14, 0)$

$0.2g + 0.8a = 4$

Jamal

$-0.2 \cdot (g + 2a = 14)$

$0.2g + 0.8a = 4$

$-0.2g + -0.4a = -2.8$

$+ \ 0.2g + 0.8a = 4$

Angela

$g + 2a = 14$

$-2a \ -2a$

$g = 14 - 2a$

$0.2g + 0.8a = 4$

$0.2(14 - 2a) + 0.8a = 4$

$2.8 - 0.4a + 0.8a = 4$

5. In your group, compare your solutions and work together to revise any mistakes.

Discuss:
- What strategy did the student start with?
- How did you complete the strategy?

Responses vary.
- Diamond is graphing the system. She calculated two points on the line $g + 2a = 14$, then plotted them and connected them to graph the first equation. She would have to do the same process for the second equation.
- Jamal multiplied one equation by -0.2 so that he can get ready to use elimination. He needs to solve for a and use that to solve for g.
- Angela solved for g in the first equation in order to substitute it into the other equation. Once she solves for g she can use that value to solve for a.

6. What does the solution represent about the Metropolis building project?

Responses vary. The solution represents the number of apartments and green spaces Metropolis could develop using exactly 14 plots of land and the $4 million budget. Metropolis can develop 3 apartments and 8 green spaces.

Unit 4 Lesson 5 421 Activity 1

Key Takeaway: Multiple constraints can be represented using a system of equations, which can be solved using symbolic strategies (like substitution or elimination) and graphing strategies. Each strategy can be used to find the solution, if there is one, which is an ordered pair that satisfies both constraints. In this case, the solution describes a number of apartment buildings and green spaces that manages to use exactly 14 plots and the full 4 million budget.

Small Groups | 10 min

Activity 2 Houses and Green Space

Purpose: Students use two different strategies to solve a second system of equations and interpret the solution in context. **(MTR.6.1)**

Short on time? Invite students to solve the system using their method from Activity 1 and then share with their group.

1 Launch

Display the details of Metropolis 2 and read aloud with students.

A **Accessibility: Memory and Attention** Invite students to use a scientific calculator and refer to strategies in Activity 1 as needed.

ELL **English Language Learners** Invite students to share different meanings of the word *symbolically* in this situation. Clarify which meanings this word will have for this activity, e.g., solving with substitution and elimination strategies. **(Reading and Listening)**

2 Monitor

Select and sequence strategies to share during the Connect.

D **Differentiation** *(Problem 7)*

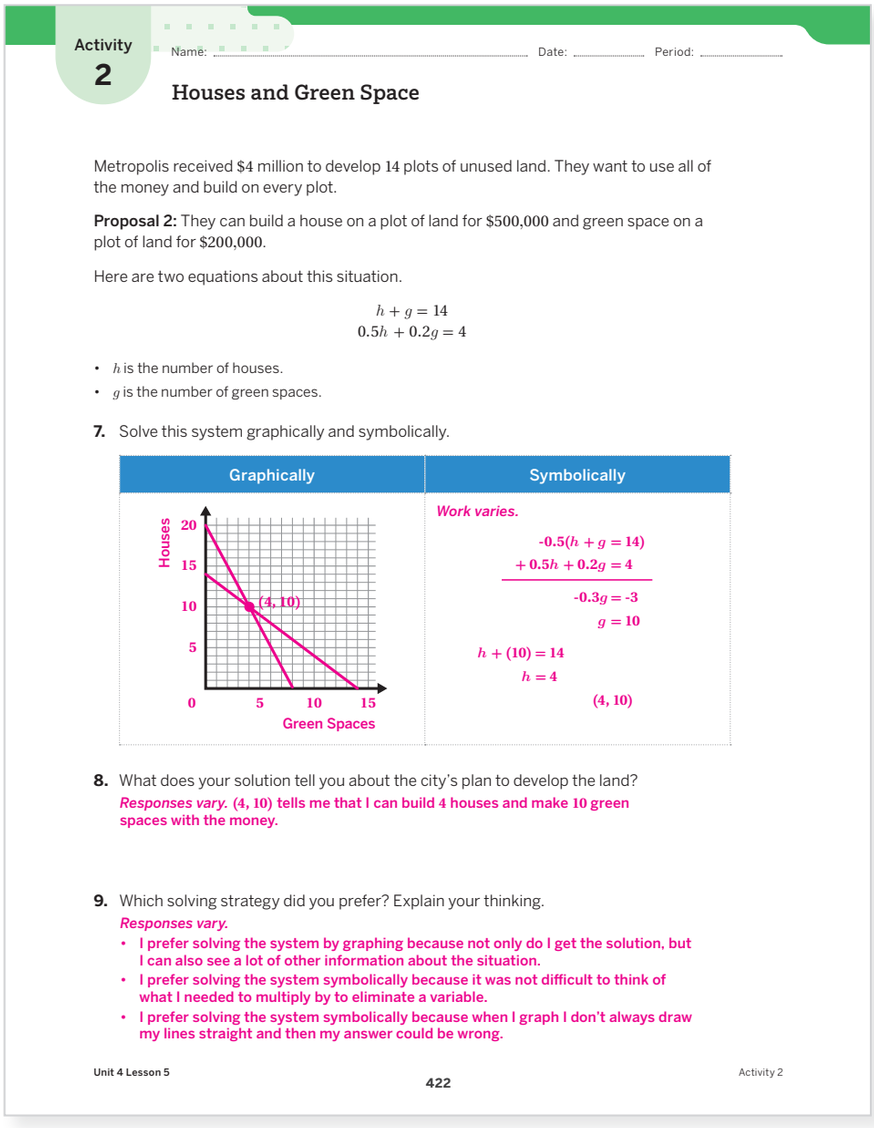
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which solving strategy do you like the most?"
Need support graphing.	**Support** Invite students to share strategies they use to graph. (E.g., solving for intercepts and connecting them, solving for h to determine the slope and y-intercept, making a table of points on the line).
Wonder why the variable h is being used instead of a.	**Support** Consider asking, "What did a represent in Activity 1? Why do we need a new variable for this situation?"
Choose to use elimination to solve symbolically, or do not believe it is possible to use a substitution strategy.	**Strengthen** Consider asking, "Could you isolate a variable? could you use substitution if you did that?" **(MTR.6.1)**
Would like an additional challenge.	**Stretch** Consider asking, "Evergreen City has 20 plots of land to develop with green spaces, houses, and apartment buildings. How many of each would you build? What needs would this meet? How much would it cost?"

3 Connect

Invite students to share which strategy they prefer and why.

Consider asking:

- "How did you graph the system of equations?"
- "What strategy did you use to solve the problem symbolically? Why?"
- "If the equations had looked different, would you prefer a different strategy? Why or why not?"



Activity 2

Name: Date: Period:

Houses and Green Space

Metropolis received $4 million to develop 14 plots of unused land. They want to use all of the money and build on every plot.

Proposal 2: They can build a house on a plot of land for $500,000 and green space on a plot of land for $200,000.

Here are two equations about this situation.

$$h + g = 14$$
$$0.5h + 0.2g = 4$$

- h is the number of houses.
- g is the number of green spaces.

7. Solve this system graphically and symbolically.

Graphically	Symbolically
	Work varies. $-0.5(h + g = 14)$ $+ 0.5h + 0.2g = 4$ $-0.3g = -3$ $g = 10$ $h + (10) = 14$ $h = 4$ $(4, 10)$

8. What does your solution tell you about the city's plan to develop the land?

Responses vary. (4, 10) tells me that I can build 4 houses and make 10 green spaces with the money.

9. Which solving strategy did you prefer? Explain your thinking.

Responses vary.

- I prefer solving the system by graphing because not only do I get the solution, but I can also see a lot of other information about the situation.
- I prefer solving the system symbolically because it was not difficult to think of what I needed to multiply by to eliminate a variable.
- I prefer solving the system symbolically because when I graph I don't always draw my lines straight and then my answer could be wrong.

Unit 4 Lesson 5 422 Activity 2

Small Groups | 8 min

Activity 3 Comparing Proposals

Purpose: Students create two proposals, one for building apartment buildings and green spaces and the other for building houses and green spaces, and compare them.

Short on time? Consider facilitating a whole-class discussion for Problem 9 instead of students discussing in groups.

1 Launch

Display the proposals to discuss the prompt.

Demonstrate how to place apartment buildings (two plots each) and green spaces for Proposal 1, and how to place houses (one plot each) and green spaces for Proposal 2.

Consider sharing that students will be invited to show their designs during the Connect.

2 Monitor

Look for a variety of student designs for both Proposals to discuss during the Connect.

D Differentiation *(Problems 7–8)*

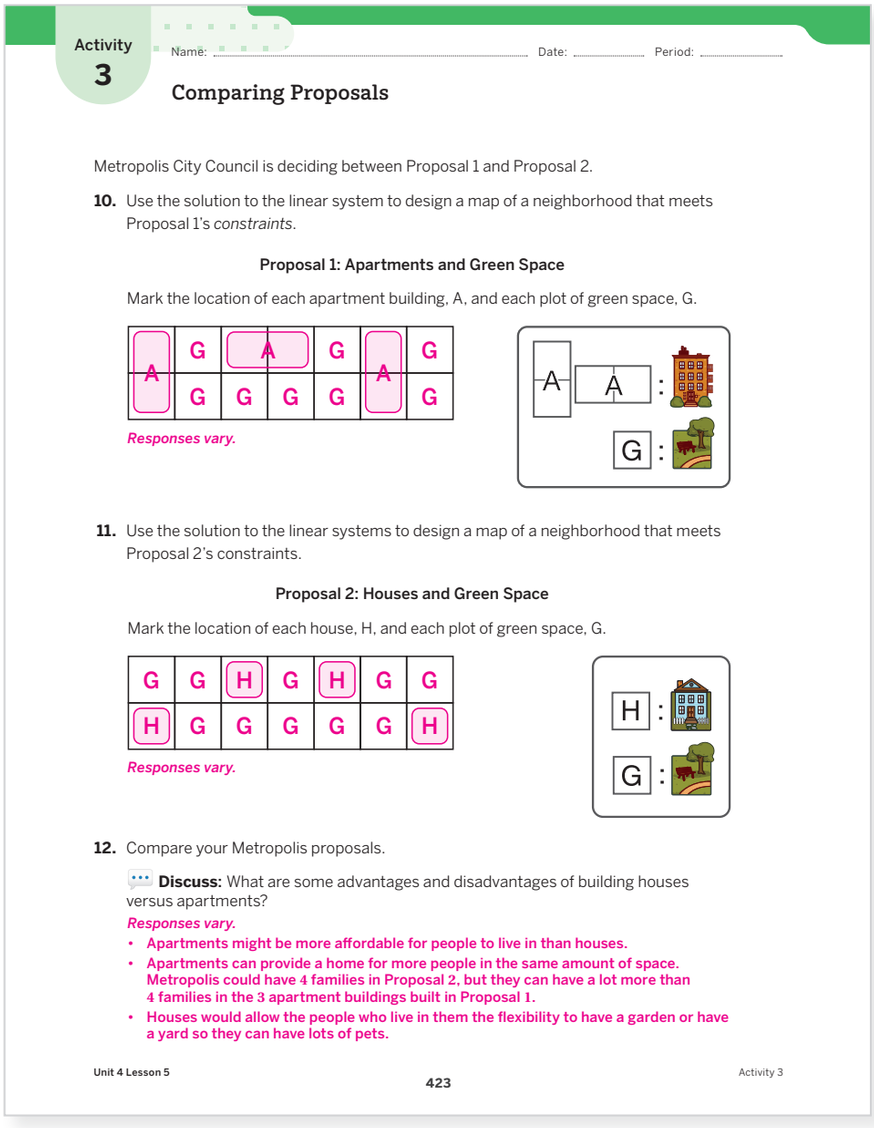
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to review their solutions from Activities 1 and 2. Consider asking, "How many green spaces would you build for Proposal 1? For Proposal 2?"
Place apartment buildings on single plots of land instead of double plots (i.e., two consecutive plots).	**Support** Consider asking, "How many plots of land does each apartment building take to develop?"
Would like a challenge.	**Stretch** Invite students to decide if there is a possible plan using a mix of houses and apartment buildings to satisfy both constraints. One house, one apartment building, and 11 green spaces will stay under budget and use all the plots of land. There is no way to mix houses and apartment buildings using all funds and plots.

3 Connect

Invite students to share their designs for each proposal.

To surface the Key Takeaway, consider asking:

- "How did you choose your design for each proposal? What community needs might your design meet? What needs might it not meet?"



Activity 3

Name: Date: Period:

Comparing Proposals

Metropolis City Council is deciding between Proposal 1 and Proposal 2.

10. Use the solution to the linear system to design a map of a neighborhood that meets Proposal 1's *constraints*.

Proposal 1: Apartments and Green Space

Mark the location of each apartment building, A, and each plot of green space, G.

A	G	A	G	A	G
G	G	G	G	G	G

Responses vary.

A A :

G :

11. Use the solution to the linear systems to design a map of a neighborhood that meets Proposal 2's constraints.

Proposal 2: Houses and Green Space

Mark the location of each house, H, and each plot of green space, G.

G	G	H	G	H	G	G
H	G	G	G	G	G	H

Responses vary.

H :

G :

12. Compare your Metropolis proposals.

Discuss: What are some advantages and disadvantages of building houses versus apartments?

Responses vary.

- Apartments might be more affordable for people to live in than houses.
- Apartments can provide a home for more people in the same amount of space. Metropolis could have 4 families in Proposal 2, but they can have a lot more than 4 families in the 3 apartment buildings built in Proposal 1.
- Houses would allow the people who live in them the flexibility to have a garden or have a yard so they can have lots of pets.

Unit 4 Lesson 5 423 Activity 3

Key Takeaway: Mathematical correctness is not the only important consideration when solving a system of equations that represents a situation. For example, in this situation the community may have reasons for deciding to build apartment buildings with the grant money instead of houses (or vice versa), even though both proposals develop all plots and use all the grant money.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for solving systems of equations by describing when they would choose to use each strategy.

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** As students process their ideas in words, invite them to talk through their thinking with a partner before writing it down.

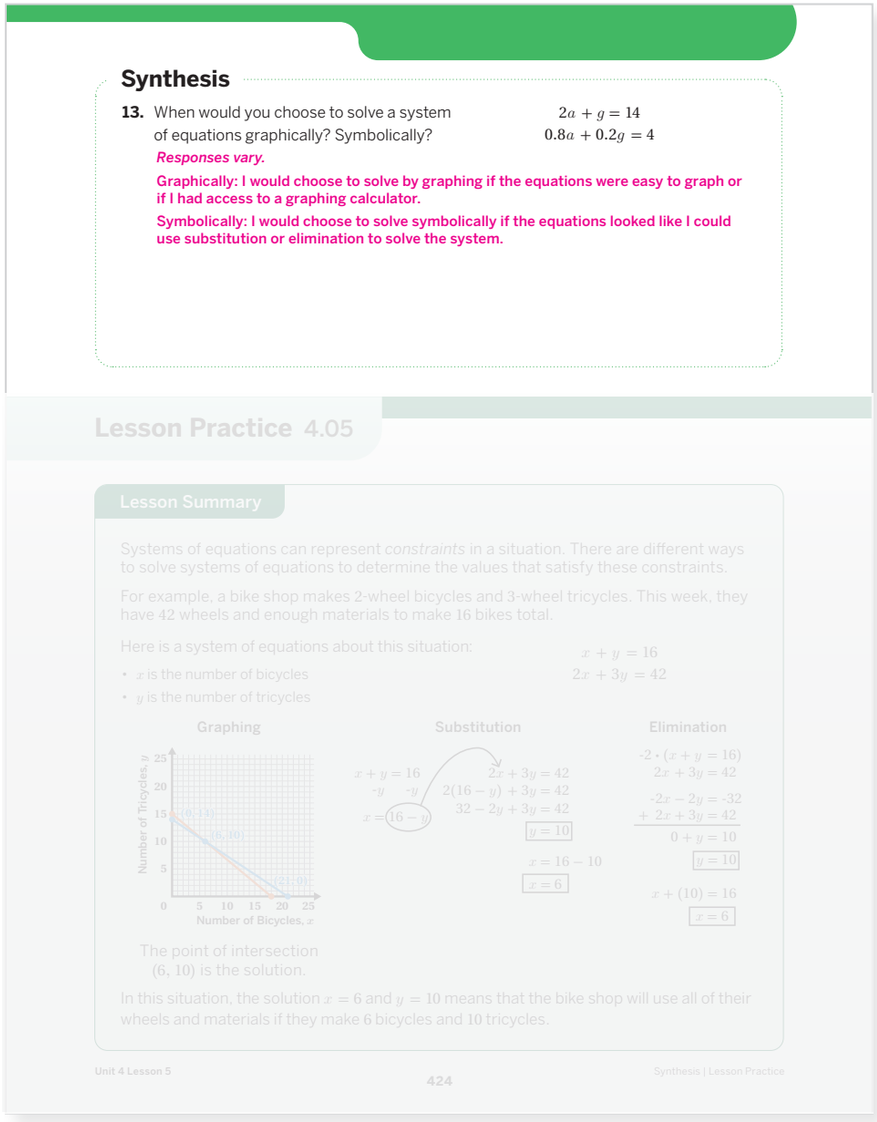
Collect and share a variety of ideas, such as:

- Choosing to graph when both equations are in slope-intercept form.
- Choosing to graph when you have a graphing calculator.
- Graphing is easier to do accurately with a graphing calculator, or I can graph hand to estimate an intersection point (e.g., by substituting 0 for x and y to find the intercepts and connecting them with a straight line).
- Symbolic strategies like substitution or elimination can make sense based on the structure of the equations. (e.g., one equation in slope-intercept form can help with substitution, or equations that already have matching or opposite coefficients on one variable, such as one with $2x$ and another with $2x$ or $-2x$, can help with elimination).

Consider asking:

- "What are some strategies for graphing lines when you are solving by graphing?" You can substitute 0 in for x and y to find the intercepts, or you can rewrite the equation in slope-intercept form.
- "What are some strategies that help you solve symbolically using substitution? Elimination?" Solving for one variable in one of the equations for substitution, or multiplying one equation to create matching/opposite coefficients for elimination.
- "Why is it helpful to have more than one strategy for solving systems?"

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about when they prefer to use substitution and elimination. If you made a display in Lesson 4 comparing substitution and elimination, consider reviewing it and adding to it.



Synthesis

13. When would you choose to solve a system of equations graphically? Symbolically?

$$2a + g = 14$$
$$0.8a + 0.2g = 4$$

Responses vary.

Graphically: I would choose to solve by graphing if the equations were easy to graph or if I had access to a graphing calculator.

Symbolically: I would choose to solve symbolically if the equations looked like I could use substitution or elimination to solve the system.

Lesson Practice 4.05

Lesson Summary

Systems of equations can represent *constraints* in a situation. There are different ways to solve systems of equations to determine the values that satisfy these constraints.

For example, a bike shop makes 2-wheel bicycles and 3-wheel tricycles. This week, they have 42 wheels and enough materials to make 16 bikes total.

Here is a system of equations about this situation:

$$x + y = 16$$
$$2x + 3y = 42$$

- x is the number of bicycles
- y is the number of tricycles

Graphing

Substitution

$x + y = 16$
$-y \quad -y$
$x = (16 - y)$
$2x + 3y = 42$
$2(16 - y) + 3y = 42$
$32 - 2y + 3y = 42$
$y = 10$
$x = 16 - 10$
$x = 6$

Elimination

$-2 \cdot (x + y = 16)$
$2x + 3y = 42$
$-2x - 2y = -32$
$+ \; 2x + 3y = 42$
$0 + y = 10$
$y = 10$
$x + (10) = 16$
$x = 6$

The point of intersection (6, 10) is the solution.

In this situation, the solution $x = 6$ and $y = 10$ means that the bike shop will use all of their wheels and materials if they make 6 bicycles and 10 tricycles.

Unit 4 Lesson 5 424 Synthesis | Lesson Practice

Lesson Takeaway: Solving a system of equations that represents a situation means finding a solution that works for both equations (or, in other words, satisfies both *constraints*) simultaneously (e.g., finding a number of apartment buildings and green spaces that uses all available lots and makes use of all grant funding).

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving a system of equations graphically or symbolically and interpreting the solution in context. **(MTR.6.1)**

Today's Goals

1. **Goal:** Solve systems of equations graphically and symbolically.
2. **Language Goal:** Interpret the solution to a system of equations in context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.05

Ama has 8 cups of batter and wants to bake 10 cupcakes. A regular cupcake requires 0.5 cups of batter and a giant cupcake requires 2 cups of batter.

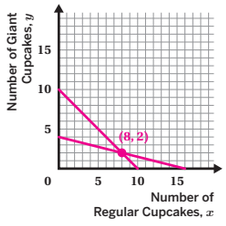
Here is a system of equations representing this situation.

- x is the number of regular cupcakes.
- y is the number of giant cupcakes.

$$x + y = 10$$
$$0.5x + 2y = 8$$

a. Solve the system of equations graphically or symbolically. *Responses vary.*

$$-0.5(x + y = 10)$$
$$+ 0.5x + 2y = 8$$
$$1.5y = 3$$
$$y = 2$$
$$x + (2) = 10$$
$$x = 8$$



b. What does the solution to the system represent about Ama's baking project?

Responses vary. Ama has enough batter to make 8 regular cupcakes and 2 giant cupcakes.

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to interpret solutions to systems of equations in context:

» **Respond to Student Thinking:**

- Revisit **Activity 1, Problem 6** and invite students to share strategies for interpreting solutions in context.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to research actual grants given to support city development. Encourage students to contact local organizations and government officials to learn about how grants are applied for and how grant money is allocated, then share what they learn.

Professional Learning

Take a moment to reflect on the ways students developed strategic competence in this lesson by using multiple strategies to solve a system of equations representing a situation. In the remaining lessons and assessments in this unit, solving methods are not prescribed. How did students choose between and among strategies? How can you continue to support students in seeing the value of multiple strategies as they continue their work in this unit with more freedom to make their own decisions about solving methods?

Lesson Practice

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. When would you choose to solve a system of equations graphically? Symbolically?

$2a + y = 14$
$0.8a + 0.2y = 4$

Responses vary.

Graphically: I would choose to solve by graphing if the equations were easy to graph or if I had access to a graphing calculator.

Symbolically: I would choose to solve symbolically if the equations looked like I could use substitution or elimination to solve the system.

Lesson Practice 4.05

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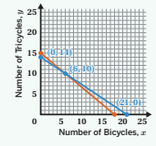
For example, a bike shop makes 2-wheel bicycles and 3-wheel tricycles. This week, they have 42 wheels and enough materials to make 16 bikes total.

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Substitution

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Elimination

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$0 + y = 10$
$y = 10$
$x + (10) = 16$
$x = 6$

The point of intersection $(6, 10)$ is the solution.

In this situation, the solution $x = 6$ and $y = 10$ means that the bike shop will use all of their wheels and materials if they make 6 bicycles and 10 tricycles.

Lesson Practice 4.05

Name: ..... Date: ..... Period: .....

1. Match each system of equations to its solution.

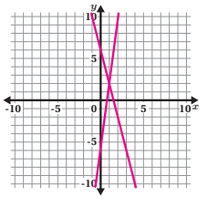
a $y = 3x + 1$, $y = -3x + 25$ — c — $(1, 4)$

b $y = -2x$, $y = x - 12$ — b — $(4, -8)$

c $4x + 2y = 12$, $y = 3 + x$ — a — $(4, 13)$

Problems 2–3: Here is a system of equations: $y = 8x - 6$, $8x + 2y = 12$

2. Graph the system of equations.



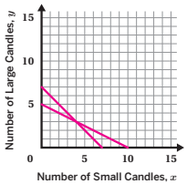
3. Determine the solution to the system of equations.

(1, 2)

Problems 4–6: Dylan needs to use 40 ounces of wax to make 7 candles. A small candle requires 4 ounces of wax and a large candle requires 8 ounces of wax. Dylan wrote this system of equations:

$x + y = 7$
$4x + 8y = 40$

4. Graph the system of equations.



5. Determine the solution to the system.

(4, 3)

6. What does the solution to the system tell you?

Dylan should make 4 small candles and 3 large candles.

Lesson Practice 4.05

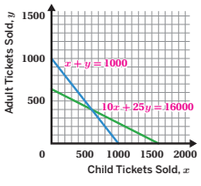
Name: ..... Date: ..... Period: .....

Test Practice

Problems 7–8: A child ticket to a state fair costs \$10. An adult ticket costs \$25. The fair sold 1,000 tickets and made \$16,000 total. Here is a system of equations and a graph that represent the situation.

$10x + 25y = 16000$
$x + y = 1000$

7. Label each line with the equation it represents.



8. Determine the solution to the system.

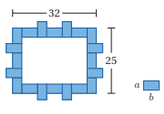
(600, 400)

Spiral Review

9. All the small rectangles are identical. Determine the dimensions, a and b, of one rectangle.

$a =$ 3.5

$b =$ 6



Problems 10–11: Using only the number 4 exactly four times and the four basic operations, write an expression to create each number. *Responses vary.*

10. 1

$\frac{(4 + 4)}{(4 + 4)}$

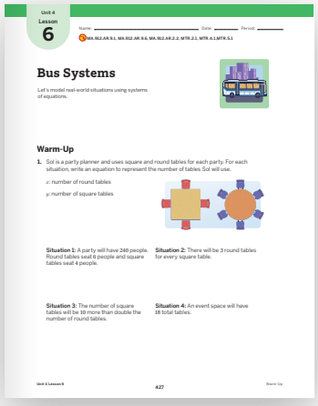
11. 15

$\frac{44}{4} + 4$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 2, 3, 5	2	MA.912.AR.9.1
	4, 6	2	MA.912.AR.9.6
Test Practice	7-8	2	MA.912.AR.9.1, MA.912.AR.9.6
Spiral Review			
	9	3	MA.8.AR.4.3
Fluency	10–11	1	MA.7.NSO.1.2

Unit 4
Lesson
6


Unit 4
Lesson
6

Name _____ Date _____ Period _____

MA.912.AR.9.1, MA.912.AR.9.6, MA.912.AR.2.2, MTR.2.1, MTR.4.1,MTR.5.1

Bus Systems

Let's model real-world situations using systems of equations.

Warm-Up

1. Sol is a party planner and uses square and round tables for each party. For each situation, write an equation to represent the number of tables Sol will use.

x: number of round tables

y: number of square tables

Situation 1: A party will have 249 people. Round tables seat 6 people and square tables seat 4 people.

Situation 2: There will be 3 round tables for every square table.

Situation 3: The number of square tables will be 19 more than double the number of round tables.

Situation 4: An event space will have 18 total tables.

Unit 4 Lesson 6 427 Warm-Up

Bus Systems

Writing and Solving Systems of Equations

Let's model real-world situations using systems of equations.

Learning Outcomes

Today's Goals

1. **Goal:** Represent a situation with a system of equations.
2. **Goal:** Solve systems of equations graphically and symbolically.
3. **Language Goal:** Interpret the meaning of a system of equations and its solutions in context in order to make recommendations. **(Reading and Writing)**

Students write and solve systems of equations to represent how many bus seats and handholds should be on the new buses of the Metropolis Transit Association. They use information about different bus lines to consider which combination of bus seats and handholds will work best for each community.

Note: Anywhere the abbreviation "MTA" is used in this lesson, it stands for "Metropolis Transit Association."

Coherence

Prior Learning

In Lesson 5, students solved systems of equations using multiple methods and interpreted the solutions in context.

Future Learning

In Lessons 8–10, students will apply their understanding in solving systems of linear equations to solving systems of linear inequalities.

Connections

- Students model real world situations using systems of equations to decide how many seats and handholds will be on new buses for Metropolis Transit Association. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.1
Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.

MA.912.AR.9.6
Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.8.AR.4.3
MA.912.AR.2.1

Vocabulary

Review Vocabulary

slope-intercept form

standard form (of a linear equation)

Lesson at a Glance

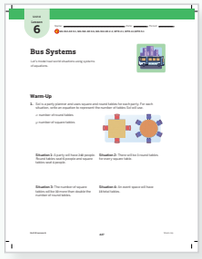
~ 45 min

MA.912.AR.9.1, MA.912.AR.9.6

Warm-Up

Pairs | 10 min

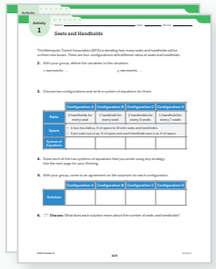
Students write equations to represent a situation about tables for a party.



Activity 1

Small Groups | 15 min

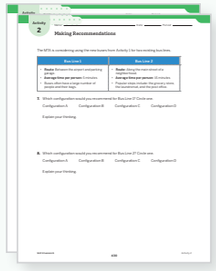
Students use MLR6: Three Reads to make sense of how to determine the number of seats and handholds on new buses and write and solve systems to represent the situation.



Activity 2

Small Groups | 10 min

Students make and revise recommendations to the MTA about which combination of bus seats and handholds will work best for different bus lines. **Routine:** Think-Pair-Share



Synthesis

Whole Class | 5 min

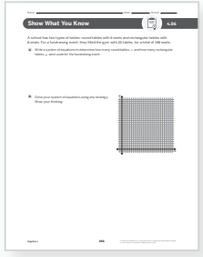
Students synthesize their strategies for how to write a system of equations to represent a situation. MLR2: Collect and Display **(MTR.2.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing and solving a system of equations to represent a situation about a fundraiser.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 10 min

Warm-Up

Purpose: Students write equations to represent a situation about tables for a party.

Short on time? Consider inviting pairs to complete Situations 1 and 3 or Situations 2 and 4.

1 Launch

Display the situations.

ELL **English Language Learners** Consider sharing a picture of a party and differently-shaped tables to increase access to the task. **(Reading and Listening)**

Invite students to read each situation aloud.

Consider asking:

- "How can we use the information in each situation to help us write an equation?"
- "Is it okay to use the same variables, x and y, in each situation? Why or why not?" It can help to use the same variables if multiple constraints apply to the same situation and the variables mean the same things. If you needed a variable to represent a different thing, like a third kind of table, it might make sense to use a different variable.

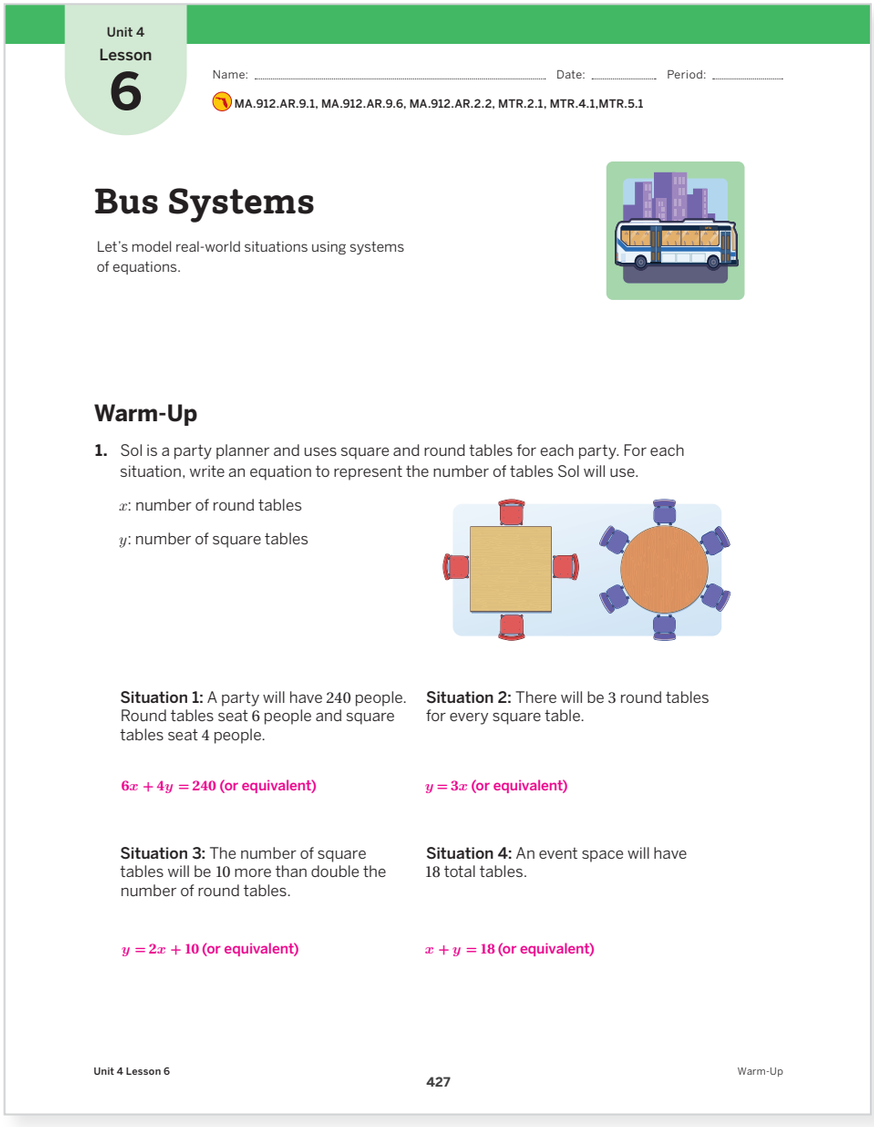
A **Accessibility: Conceptual Processing** Invite students to say aloud what each equation means with the variable definitions in place of x and y (e.g., The total number of people equals 6 people at each round table plus 4 people at each square table.).

2 Connect

Consider asking:

- "For which situations was *standard form* more helpful? For which situations was *slope-intercept form* more helpful?" **(MTR.5.1)**
- "What about a situation can help you determine which structure to write an equation in?" If there's a starting amount in the situation, slope-intercept form might be an easier structure to use. If there's a combination of items, standard form might be an easier structure to use. **(MTR.5.1)**

Note: Consider spending adequate time discussing how to set up Situation 2, since proportional relationships will be important for setting up configurations in Activity 1.



Unit 4
Lesson
6

Name: Date: Period:

MA.912.AR.9.1, MA.912.AR.9.6, MA.912.AR.2.2, MTR.2.1, MTR.4.1,MTR.5.1

Bus Systems

Let's model real-world situations using systems of equations.

Warm-Up

1. Sol is a party planner and uses square and round tables for each party. For each situation, write an equation to represent the number of tables Sol will use.

x: number of round tables

y: number of square tables

Situation 1: A party will have 240 people. Round tables seat 6 people and square tables seat 4 people.

$6x + 4y = 240$ (or equivalent)

Situation 2: There will be 3 round tables for every square table.

$y = 3x$ (or equivalent)

Situation 3: The number of square tables will be 10 more than double the number of round tables.

$y = 2x + 10$ (or equivalent)

Situation 4: An event space will have 18 total tables.

$x + y = 18$ (or equivalent)

Unit 4 Lesson 6 427 Warm-Up

Small Groups | 15 min

Activity 1 Seats and Handholds

Purpose: Students use MLR6: Three Reads to make sense of how to determine the number of seats and handholds on new buses and write and solve systems to represent the situation.

Short on time? Consider inviting students to choose one configuration instead of two.

1 Launch

Direct students to read the problem statement from their student edition pages while you read it aloud.

MLR **MLR6: Three Reads** Use this routine to support students in making sense of the situation, reading the situation aloud and then discussing the prompt. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Consider asking:

- "What does the word *configuration* mean here?"
- "Why might the MTA consider different ratios of seats to handhelds on their buses?"
- "Why is it important for the group to define the variables the same way?"

Note: Students' equations will depend on how they define the variables x and y. In the sample response, x is defined as the number of seats and y is defined as the number of handholds. Students who switch the definitions will have different equations (e.g., if x is the number of handholds, then Configuration A's equation would be $4y = x$ instead). Spend adequate time assuring that students write their equation to correctly represent the proportional relationship between the variables as they defined them (e.g., $y = 4x$ represents 4 handholds for every seat *if* the variables are defined as above because there are 4 times as many handholds as seats).

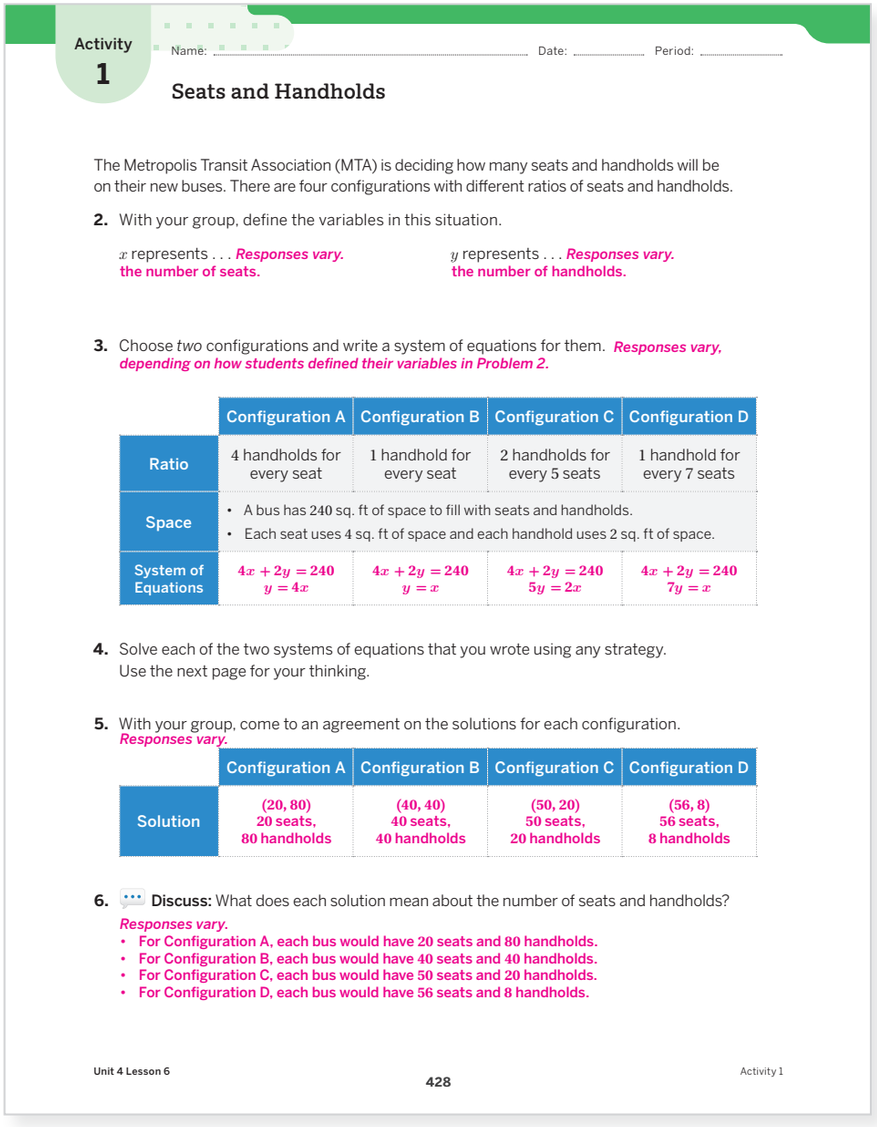
2 Monitor

Invite students to choose two systems of equations so that all four systems will be solved by one member of the group. Invite students to consider the equations and predict the combination of seats and handholds for their system before solving it exactly. Invite students to compare their work.

A **Accessibility: Conceptual Processing** Invite students to use resources to support their reasoning (e.g., a table, graph paper.

D **Differentiation: (Support)** To support students getting started, consider asking, "Are there more handholds or seats in the relationship of 4 handholds for every seat? How can we use that information to help us write the equation?" **(MTR.2.1)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Seats and Handholds

The Metropolis Transit Association (MTA) is deciding how many seats and handholds will be on their new buses. There are four configurations with different ratios of seats and handholds.

2. With your group, define the variables in this situation.

x represents . . . *Responses vary.* the number of seats.

y represents . . . *Responses vary.* the number of handholds.

3. Choose *two* configurations and write a system of equations for them. *Responses vary, depending on how students defined their variables in Problem 2.*

	Configuration A	Configuration B	Configuration C	Configuration D
Ratio	4 handholds for every seat	1 handhold for every seat	2 handholds for every 5 seats	1 handhold for every 7 seats
Space	• A bus has 240 sq. ft of space to fill with seats and handholds. • Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.			
System of Equations	$4x + 2y = 240$ $y = 4x$	$4x + 2y = 240$ $y = x$	$4x + 2y = 240$ $5y = 2x$	$4x + 2y = 240$ $7y = x$

4. Solve each of the two systems of equations that you wrote using any strategy. Use the next page for your thinking.

5. With your group, come to an agreement on the solutions for each configuration. *Responses vary.*

	Configuration A	Configuration B	Configuration C	Configuration D
Solution	(20, 80) 20 seats, 80 handholds	(40, 40) 40 seats, 40 handholds	(50, 20) 50 seats, 20 handholds	(56, 8) 56 seats, 8 handholds

6. **Discuss:** What does each solution mean about the number of seats and handholds?

Responses vary.

- For Configuration A, each bus would have 20 seats and 80 handholds.
- For Configuration B, each bus would have 40 seats and 40 handholds.
- For Configuration C, each bus would have 50 seats and 20 handholds.
- For Configuration D, each bus would have 56 seats and 8 handholds.

Unit 4 Lesson 6 428 Activity 1

Activity 1 Seats and Handholds (continued)

Purpose: Students use MLR6: Three Reads to make sense of how to determine the number of seats and handholds on new buses and write and solve systems to represent the situation.

Short on time? Consider inviting students to choose one configuration instead of two.

2 Monitor

Consider asking:

- "How did you organize the information to write your system of equations?" **(MTR.2.1)**
- What solving strategy are you planning to use? **(MTR.5.1)**
- "How can you scale and label your axes to help graph the configuration?"

Invite students to approximate the solution to the system before solving. After students solve, encourage them to compare to their approximated solution. Consider asking, "How precise was your approximation? How precise is your solution?"

Note: Consider inviting students to use coloring tools to label parts of their system of equations and use a graphing calculator to check their solutions graphically.

D **Differentiation: (Strengthen)** Encourage students to use tools strategically (e.g., using coloring tools to label their system of equations or the graphing tool to check work). Invite them to share their thinking during the Connect.

Invite students to give their group members feedback before sharing during the Connect.

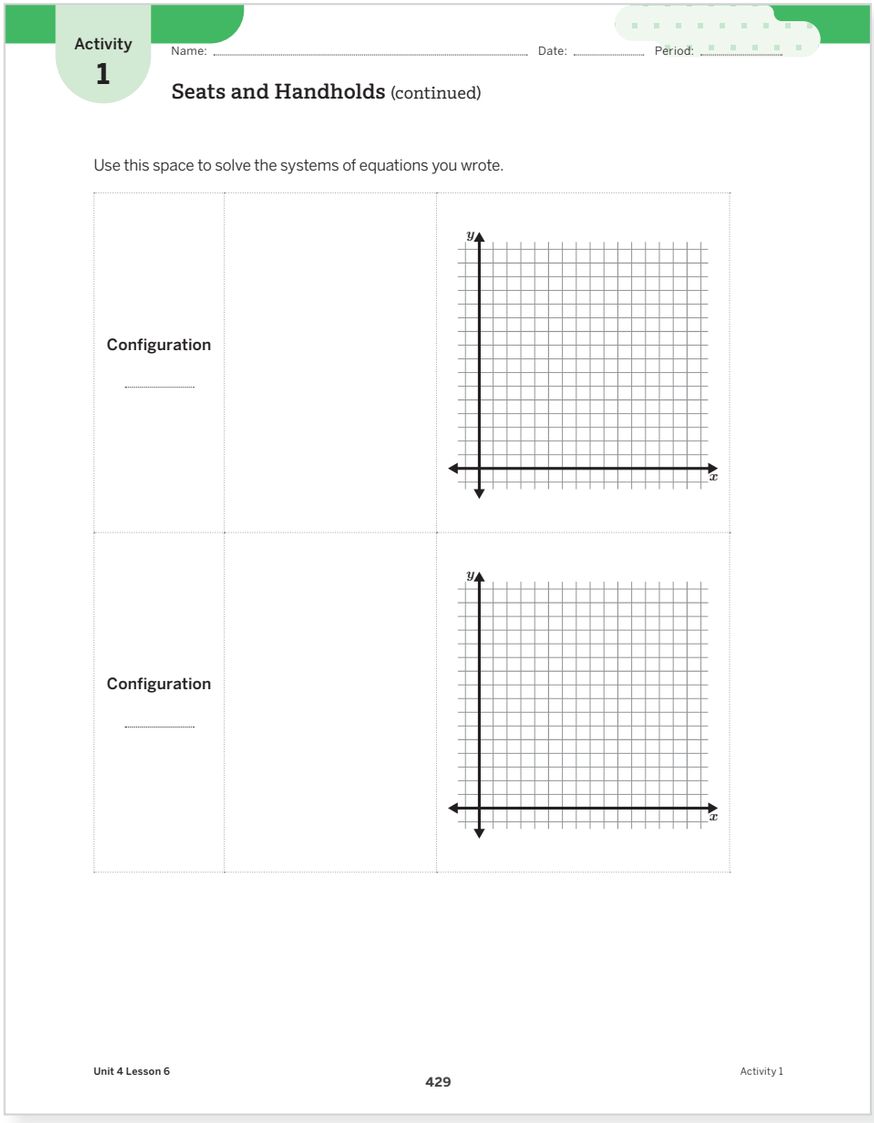
A **Accessibility: Affective Functioning** To support students in receiving constructive feedback from their group members, consider modeling how to give and receive constructive feedback.

3 Connect

Consider asking:

- "How are the four systems alike? How are they different?"
- "How close was your prediction to the actual solutions for the system?"
- "Which configuration would have the most seats available? The most handholds?"

To surface the Key Takeaway, consider asking, "What else might you need to consider if you were deciding between the different configurations?"
A bus might need more seats, rather than less, to be accessible for more passengers.



Activity 1

Name: Date: Period:

Seats and Handholds (continued)

Use this space to solve the systems of equations you wrote.

Configuration

Configuration

Unit 4 Lesson 6 429 Activity 1

Key Takeaway: Systems of equations can help us to model real-world situations, but not every aspect of the modeling process can be reflected in just one system. For example, in this situation about seats and handholds on buses, there are multiple possible configurations, each represented by their own system and which produce solutions that would mean different combinations of seats and handholds.

Small Groups | 10 min

Activity 2 Making Recommendations

Purpose: Students make and revise recommendations to the MTA about which combination of bus seats and handholds will work best for different bus lines.

1 Launch

Display the information about the bus lines. Consider sharing that students will consider new constraints and make recommendations based on their preferences.

ELL **English Language Learners** To help students access background knowledge, invite them to share what they know about seating and handholds on buses, and consider showing a video or image before students begin the task. **(Speaking and Listening)**

Use the Think-Pair-Share routine to help students shape their thinking. See the Routine Facilitation Guide for more information.

Consider asking, if it does not come up naturally:

- "Which line would benefit from the most seats? The most handholds?"
- "Which line typically has the most people on it?"
- "How might the amount of time spent on a bus impact the ideal number of seats and handholds?"

2 Monitor

A **Accessibility: Executive Functioning** Consider chunking this activity into more manageable parts by inviting students to consider one bus line and one configuration at a time.

Look for students who consider different configurations for each bus line:

- The total number of people who would have a seat or handhold on each bus
- The best quality ride for the community members
- The duration of the ride
- The community's needs

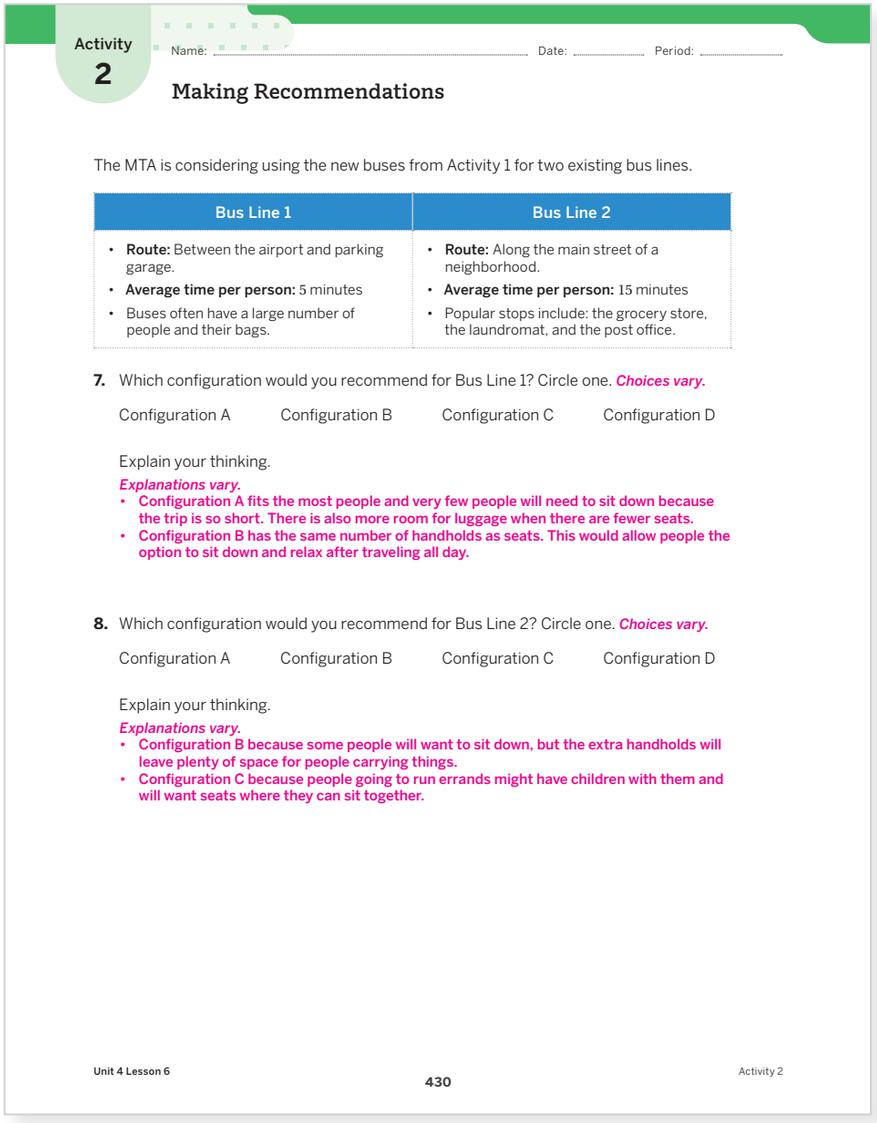
Invite students to share their reasoning for each choice. Encourage students to revise their thinking based on their group conversation.

3 Connect

Display the bus line and configuration information.

Consider asking:

- "What were your priorities when creating your proposal?"
- "How do your priorities impact other constraints?"



Activity 2

Name: Date: Period:

Making Recommendations

The MTA is considering using the new buses from Activity 1 for two existing bus lines.

Bus Line 1	Bus Line 2
• **Route:** Between the airport and parking garage. • **Average time per person:** 5 minutes • Buses often have a large number of people and their bags.	• **Route:** Along the main street of a neighborhood. • **Average time per person:** 15 minutes • Popular stops include: the grocery store, the laundromat, and the post office.

7. Which configuration would you recommend for Bus Line 1? Circle one. ***Choices vary.***

Configuration A Configuration B Configuration C Configuration D

Explain your thinking.

Explanations vary.
- **Configuration A fits the most people and very few people will need to sit down because the trip is so short. There is also more room for luggage when there are fewer seats.**
- **Configuration B has the same number of handholds as seats. This would allow people the option to sit down and relax after traveling all day.**

8. Which configuration would you recommend for Bus Line 2? Circle one. ***Choices vary.***

Configuration A Configuration B Configuration C Configuration D

Explain your thinking.

Explanations vary.
- **Configuration B because some people will want to sit down, but the extra handholds will leave plenty of space for people carrying things.**
- **Configuration C because people going to run errands might have children with them and will want seats where they can sit together.**

Unit 4 Lesson 6 430 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for how to write a system of equations to represent a situation. **(MTR.2.1)**

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** As students process their ideas in words, invite them to talk through their thinking with a partner before writing it down.

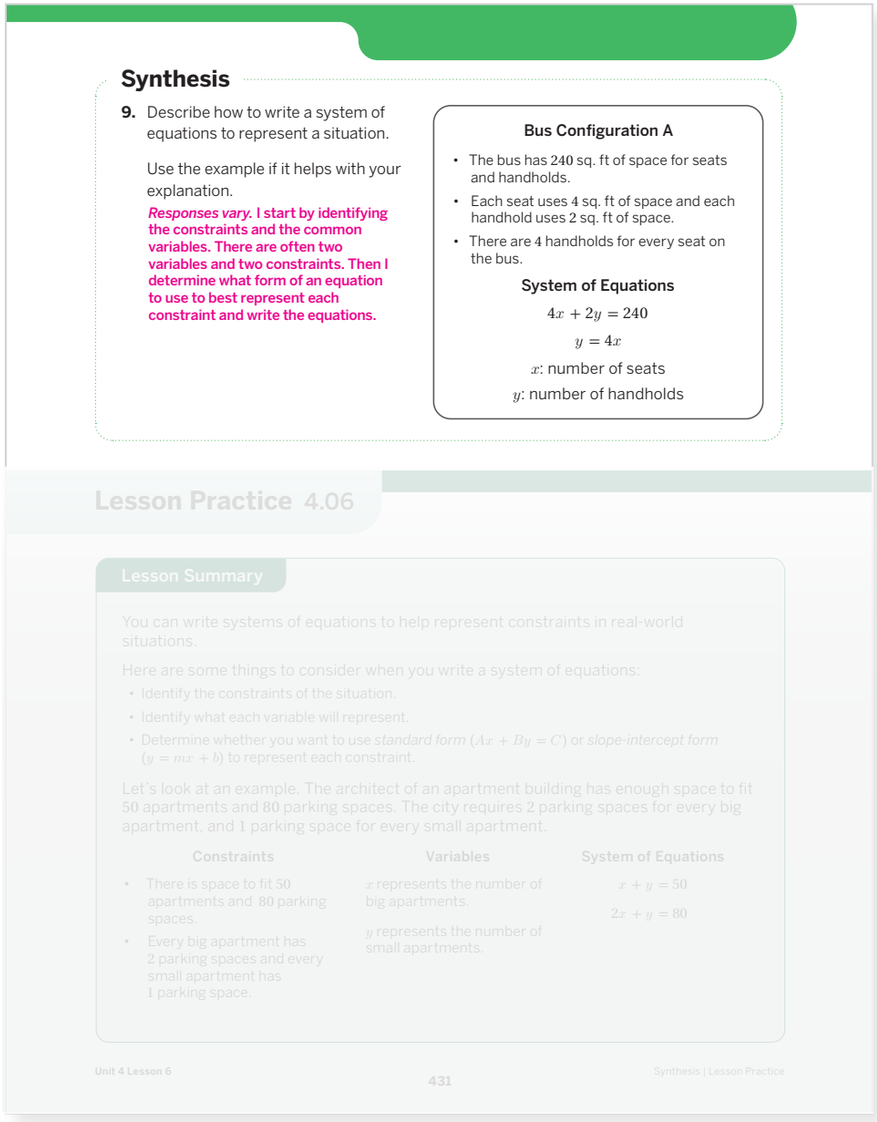
Look for and share a variety of ideas, such as:

- Recognizing that the situation is a system of equations.
- Defining the unknown quantities with variables. **(MTR.2.1)**
- Using the same variables to write both equations. **(MTR.2.1)**
- Writing an equation to represent both constraints in the situation. **(MTR.2.1)**
- Choosing a strategy to solve the system. **(MTR.5.1)**

Create a record of students' strategies for writing systems that represent a situation.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they share their thinking. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. Describe how to write a system of equations to represent a situation.

Use the example if it helps with your explanation.

Responses vary. I start by identifying the constraints and the common variables. There are often two variables and two constraints. Then I determine what form of an equation to use to best represent each constraint and write the equations.

Bus Configuration A

- The bus has 240 sq. ft of space for seats and handholds.
- Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.
- There are 4 handholds for every seat on the bus.

System of Equations

$4x + 2y = 240$

$y = 4x$

x: number of seats

y: number of handholds

Lesson Practice 4.06

Lesson Summary

You can write systems of equations to help represent constraints in real-world situations.

Here are some things to consider when you write a system of equations:

- Identify the constraints of the situation.
- Identify what each variable will represent.
- Determine whether you want to use *standard form* ($Ax + By = C$) or *slope-intercept form* ($y = mx + b$) to represent each constraint.

Let's look at an example. The architect of an apartment building has enough space to fit 50 apartments and 80 parking spaces. The city requires 2 parking spaces for every big apartment, and 1 parking space for every small apartment.

Constraints	Variables	System of Equations
• There is space to fit 50 apartments and 80 parking spaces. • Every big apartment has 2 parking spaces and every small apartment has 1 parking space.	x represents the number of big apartments. y represents the number of small apartments.	$x + y = 50$ $2x + y = 80$

Unit 4 Lesson 6 431 Synthesis | Lesson Practice

Lesson Takeaway: Systems of equations can be used to represent real-world situations with multiple constraints. By writing a system of equations to represent constraints, we can choose a strategy to solve the system and interpret the solution to make recommendations for the situation. **(MTR.5.1)**

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing and solving a system of equations to represent a situation about a fundraiser.

Today's Goals

1. **Goal:** Represent a situation with a system of equations.
2. **Goal:** Solve systems of equations graphically and symbolically.
3. **Language Goal:** Interpret the meaning of a system of equations and its solutions in context in order to make recommendations. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.06

A school has two types of tables: round tables with 6 seats and rectangular tables with 8 seats. For a fundraising event, they filled the gym with 25 tables, for a total of 190 seats.

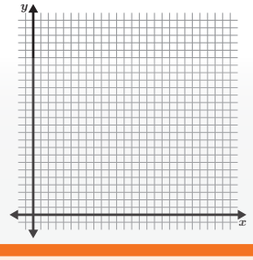
a. Write a system of equations to determine how many round tables, x, and how many rectangular tables, y, were used for the fundraising event.

$x + y = 25$

$6x + 8y = 190$

b. Solve your system of equations using any strategy. Show your thinking.

$x = 5$ and $y = 20$



D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write systems of linear equations to represent constraints or meet given criteria:

» **Respond to Student Thinking:**

- Revisit **Activity 1** from this lesson and give students individualized feedback.
- Revisit **Unit 1, Lesson 7** (Five Representations).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on your facilitation of **MLR6: Three Reads** in this lesson. How did students use their knowledge and creativity to help them make sense of the situation? How did the way you facilitated the routine support students in reasoning about the quantities involved in the situation and their equations? Students will use this routine again in Lesson 10. How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how to write a system of equations to represent a situation.

Use the example if it helps with your explanation.

Responses vary. I start by identifying the constraints and the common variables. There are often two variables and two constraints. Then I determine what form of an equation to use to best represent each constraint and write the equations.

Bus Configuration A

- The bus has 240 sq. ft of space for seats and handholds.
- Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.
- There are 4 handholds for every seat on the bus.

System of Equations

$4x + 2y = 240$

$y = 4x$

x: number of seats

y: number of handholds

Lesson Practice 4.06

Lesson Summary

You can write systems of equations to help represent constraints in real-world situations.

Here are some things to consider when you write a system of equations:

- Identify the constraints of the situation.
- Identify what each variable will represent.
- Determine whether you want to use *standard form* ($Ax + By = C$) or *slope-intercept form* ($y = mx + b$) to represent each constraint.

Let's look at an example. The architect of an apartment building has enough space to fit 50 apartments and 80 parking spaces. The city requires 2 parking spaces for every big apartment, and 1 parking space for every small apartment.

Constraints	Variables	System of Equations
• There is space to fit 50 apartments and 80 parking spaces. • Every big apartment has 2 parking spaces and every small apartment has 1 parking space.	x represents the number of big apartments. y represents the number of small apartments.	$x + y = 50$ $2x + y = 80$

Lesson Practice 4.06

Name: _____ Date: _____ Period: _____

1. Match the system of equations to the situation.

(a) $2x + 2y = 17$, $y = 5 + x$ — b — The sum of two numbers is 17. The difference between two numbers is 5.

(b) $x + y = 17$, $x - y = 5$ — a — A rectangle has a perimeter of 17 units. The length of the rectangle is 5 more units than the width.

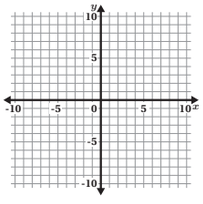
(c) $x + y = 17$, $x = 5$ — c — The sum of two numbers is 17. One of the numbers is 5.

2. Solve this system of equations. Sketch a graph if it helps with your thinking.

$3x + 4y = 24$

$y = 2x - 5$

(4, 3)



Problems 3–5: A school choir has a concert coming up. They are figuring out what prices to set for student tickets, s, and adult tickets, a.

- They estimate that 150 students and 50 adults will buy tickets.
- They want to make $1,000 in ticket sales.
- They would like the adult ticket price to be double the student ticket price.

3. Write a system of equations that represents this situation.

$150s + 50a = 1000$

$a = 2s$

4. Solve the system.

$s =$ 4 $a =$ 8

5. Explain the solution in context.

The prices that will meet their criteria are $4 for student tickets and $8 for adult tickets.

Lesson Practice 4.06

Name: _____ Date: _____ Period: _____

6. Determine values for A and B so that the system has infinitely many solutions.

$12x + Ay = 8$

$Ax + By = 4$

A: 6

B: 3

Test Practice

7. Determine the solution to the system of equations.

$y = 2x - 8$

$3x + y = 2$

(2, -4)

Spiral Review

Problems 8–10: For each equation, identify the slope of its graph.

8. $y = 5 + 2x$ — **2**

9. $2x + 4y = 4$ — **$-\frac{1}{2}$**

10. $y + 1 = 9x$ — **9**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 2, 4, 6	2	MA.912.AR.9.1
	3, 5	1	MA.912.AR.9.6
	3	2	MA.912.AR.2.2
Test Practice	7	3	MA.912.AR.9.1
Spiral Review			
Fluency	8–10	1	MA.8.AR.3.2

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determine the number of solutions for systems of equations	Lesson 4	2	MA.912.AR.9.1 MTR.5.1
2	Explain if an ordered pair is a solution to a system of equations	Lessons 1–6	2	MA.8.AR.4.1 MTR.4.1
3a–3b	Solve systems of linear equations	Lessons 1–3	2	MA.912.AR.9.1 MTR.2.1
4a	Write a system of equations from a context	Lesson 6	2	MA.912.AR.9.1 MTR.6.1
4b	Interpret the solution to a system of equations in context	Lessons 5–6	2	MA.912.AR.9.6 MTR.6.1
5a	Graph a linear equation	Lesson 5	2	MA.912.AR.2.4 MTR.3.1
5b	Solve a system of linear equations	Lessons 4–5	2	MA.912.AR.9.1 MTR.7.1

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

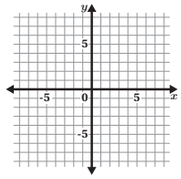
Name: Date: Period:

Sub-Unit Quiz

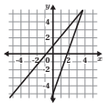
Unit 4

1. Select *all* of the systems of equations that have exactly *one solution*.

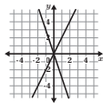
Use the graph if it helps with your thinking.



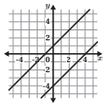
☑ A.



☑ B.



☐ C.



☑ D. $y = 5x - 2$
$y = -3x - 2$

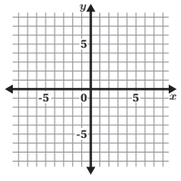
☐ E. $y = \frac{3}{2}x + 5$
$y = \frac{3}{2}x - 4$

2. Explain or show how you know that (2, 3) is *not* a solution to this system of equations:

$2x + 4y = 16$

$y = \frac{1}{2}x - 1$

Use the graph if it helps with your thinking.



Explanations vary. Substituting 2 in place of x and 3 in place of y into $y = \frac{1}{2}x - 1$ does not make the equation true, so it cannot be a solution to the system.

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Name: Date: Period:

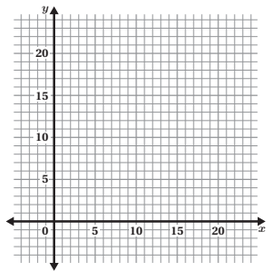
Sub-Unit Quiz (continued)

Unit 4

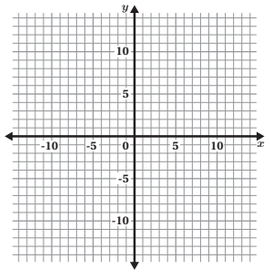
3. Solve each system of equations. Write the solution as an ordered pair.

Use the graphs if they help with your thinking.

a. $3x + y = 21$
$3x + 2y = 27$
(5, 6)



b. $-3x - 3y = 6$
$y = 5x - 14$
(2, -4)



D Differentiation (Sub-Unit Quiz)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
• Solve systems of linear equations using elimination, substitution, and graphing. **(Lessons 1–6)**	1–3, 5a	**Support**	• **Mini-Lessons:** • **Solving Systems of Linear Equations by Substitution** • **Solving Systems of Linear Equations by Graphing**
		Strengthen	• **Repeated Challenges:** • Lesson 1 (*Eliminating Shapes*) • Lesson 4 (*Lizard Lines*)
		Stretch	• **You're invited to explore more:** • Lesson 2 (*Process of Elimination*)
• Write systems of linear equations to represent constraints or meet given criteria. • Interpret solutions to systems of equations in context. **(Lessons 5–6)**	4a, 4b, 5b	**Support**	• **Teacher Move:** Consider revisiting: • Lesson 5 (*City Development*) • Lesson 6 (*Bus Systems*)
		Stretch	• **Teacher Move:** Invite students to write systems of three equations that have no solution, one solution, and infinitely many solutions.

Systems of Inequalities

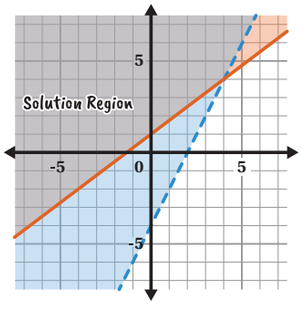
Sub-Unit 2 Goals

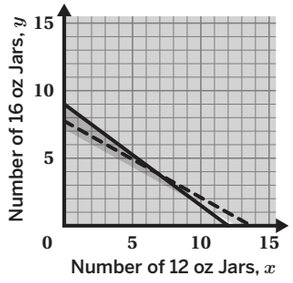
- **Goal:** Graph systems of linear inequalities and identify solutions.
- **Goal:** Write systems of linear inequalities to represent constraints or meet given criteria.
- **Language Goal:** Interpret solutions to systems of inequalities in context.

In this sub-unit, students graph **systems of inequalities** in order to identify solutions by hand and using technology. They explain how to determine the **solution region** to a system of inequalities and interpret solutions in context. Students write systems of inequalities to represent constraints and use them to solve problems.



Math That Matters Most

Concept or Skill	For example . . .
Determine if a point is a solution to a system of inequalities.	Is $(0,0)$ a solution to this system of inequalities? $x + y < 1$ $y \geq \frac{1}{2}x + 4$ $x + y < 1$ $0 + 0 < 1$ $0 < 1$ True ✓ $y \geq \frac{1}{2}x + 4$ $0 \geq \frac{1}{2}(0) + 4$ $0 \geq 4$ False ✗ No, it is not a solution to the system because it only makes one inequality true.
Graph a system of inequalities and identify the solution region.	Graph the system of inequalities. Mark the solution region on the graph. $y \geq \frac{3}{4}x + 1$ $2x - y < 4$ 

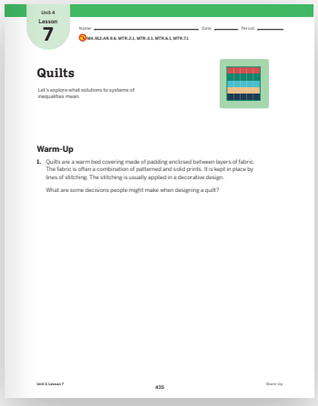
Concept or Skill	For example . . .
Define variables to represent the situation.	A juice shop has 144 oz of juice to fit in 12 oz and 16 oz jars. They need to earn more than \$33.50 from the juice. They earn \$2.50 for each 12 oz jar. They earn \$4.50 for each 16 oz jar. Write a system of inequalities that represents this situation. Be sure to define your variables. x represents the number of 12 oz jars. y represents the number of 16 oz jars.
Write a system of inequalities to represent constraints.	$12x + 16y \leq 144$ $2.5x + 4.5y \geq 33.5$
Interpret solutions to systems of inequalities in context.	The graph of this situation is shown.  Identify a solution and explain what it means about the situation. Selling two 12 oz jars and seven 16 oz jars is one possible combination that would meet the shop's constraints.

Unit 4
Lesson
7

Quilts

Introduction to Systems of Inequalities

Let's explore what solutions to a system of inequalities mean.


Unit 4
Lesson
7

Name _____ Date _____ Period _____

MA.912.AR.9.4, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Quilts

Let's explore what solutions to systems of inequalities mean.

Warm-Up

1. Quilts are a warm bed covering made of padding enclosed between layers of fabric. The fabric is often a combination of patterned and solid prints. It is kept in place by lines of stitching. The stitching is usually applied in a decorative design.

   What are some decisions people might make when designing a quilt?

Unit 4 Lesson 7 435 Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Determine if a point is a solution to a system of inequalities using a graph.
2. **Language Goal:** Describe how ordered pairs relate to constraints in a situation. **(Writing)**

Students make sense of a **system of inequalities** in a quilting context and explore what a solution to a system of inequalities means. First, they identify solutions that meet each *constraint* individually, and then they identify solutions that satisfy both constraints as a system of inequalities. Students interpret points that are solutions and non-solutions to the system and make connections between the inequalities, the context, and the graph.

Coherence

Prior Learning

In Unit 1, students represented constraints by creating two-variable inequalities and graphing their solutions. In Lessons 5–6, students represented constraints by creating systems of equations and interpreting their solutions in context.

Future Learning

In Lesson 8, students will identify the location of the solutions to a system of inequalities by looking for where the two solution regions for each individual inequality in a system overlap.

Connections

- Students explore what solutions to systems of inequalities means when designing a quilt. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.4
Graph the solution set of a system of two-variable linear inequalities.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.NSO.1.7 **MA.912.AR.2.6**	**MA.912.AR.2.8**

Vocabulary

New Vocabulary

system of inequalities

Review Vocabulary

boundary line
constraint
region

Lesson at a Glance

~ 45 min

MA.912.AR.9.4

Warm-Up

Whole Class | 5 min

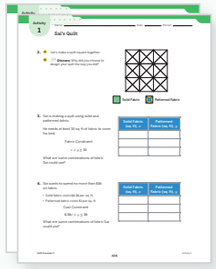
Students are introduced to the context of this lesson as they explore a variety of quilts and discuss the decisions people might make when designing a quilt.



Activity 1

Pairs | 15 min

Students make sense of two individual inequalities by determining combinations of fabric to meet each *constraint*.

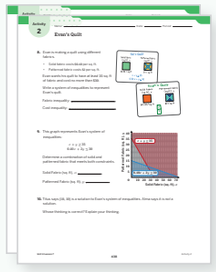


Activity 2

Pairs | 15 min

Students write a new system of inequalities, determine a solution, and settle a dispute about whether another point is a solution or not.

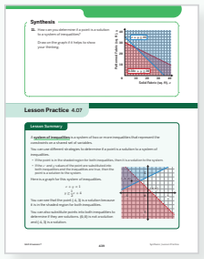
Routine: Decide and Defend **(MTR.2.1)**



Synthesis

Whole Class | 5 min

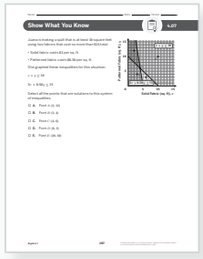
Students synthesize their understanding of how to determine if a point is a solution to a system of inequalities.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying all the points within a given list that are solutions of a system of inequalities.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students are introduced to the context of this lesson as they explore a variety of quilts and discuss the decisions people might make when designing a quilt.

Short on time? Consider doing a quick share out with a few students.

1 Launch

To surface connections, consider asking:

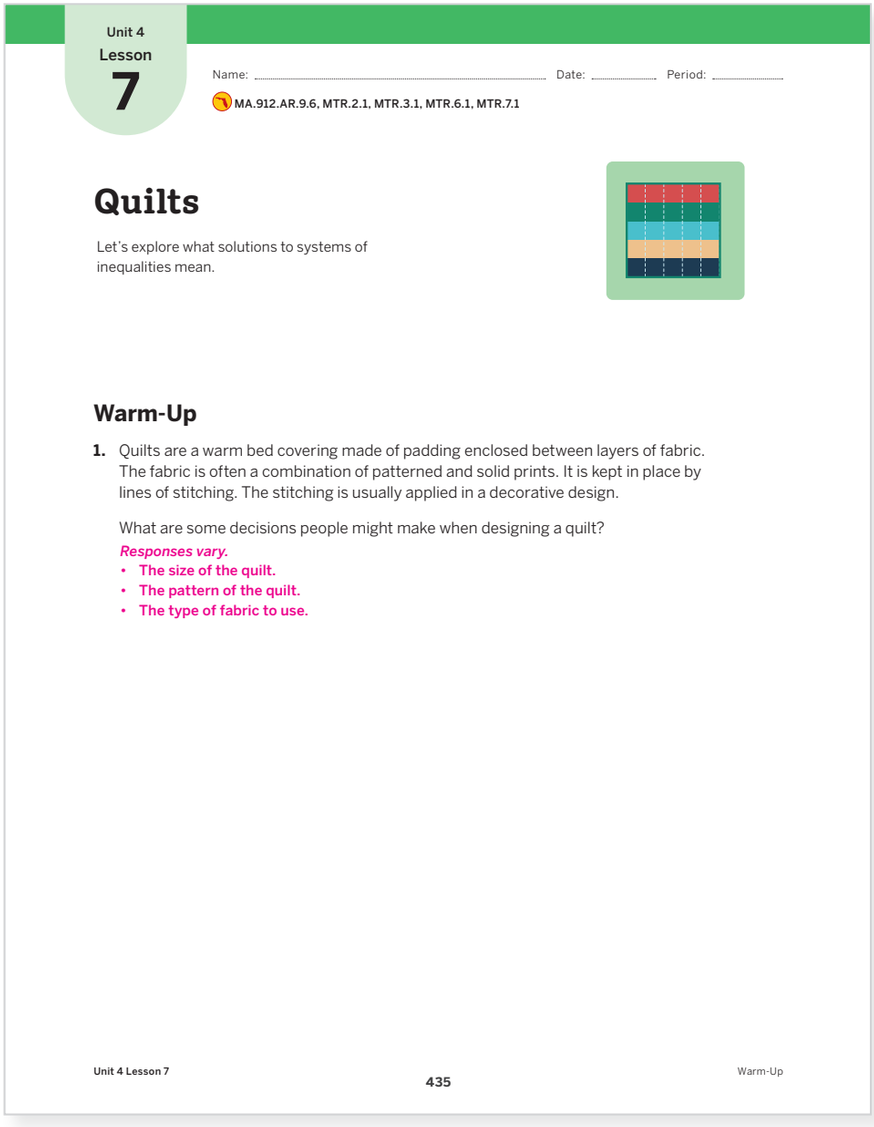
- "Have you or anyone you know ever made a quilt or a blanket?"
- "Do you have a favorite quilt or blanket? What makes it special? Does it hold any special meaning for you or your family?"

ELL **English Language Learners** Invite students to read the text aloud and paraphrase what it means. Clarify any words they don't know before they respond to the question (e.g., *quilt*, *warmth*, *storytelling*, *political involvement*, *income*). **(Reading)**

2 Connect

Collect and share several students' responses using a display in the classroom.

Consider asking, "What constraints might a quilter encounter in their work?" Amount of fabric available, size of the quilt, amount of time to sew the quilt, price of fabric.



Unit 4
Lesson
7

Name: _____ Date: _____ Period: _____

MA.912.AR.9.6, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Quilts

Let's explore what solutions to systems of inequalities mean.

Warm-Up

1. Quilts are a warm bed covering made of padding enclosed between layers of fabric. The fabric is often a combination of patterned and solid prints. It is kept in place by lines of stitching. The stitching is usually applied in a decorative design.

   What are some decisions people might make when designing a quilt?

   Responses vary.
   - The size of the quilt.
   - The pattern of the quilt.
   - The type of fabric to use.

Unit 4 Lesson 7 435 Warm-Up

Pairs | 15 min

Activity 1 Sai's Quilt

Purpose: Students make sense of two individual inequalities by determining combinations of fabric to meet each *constraint*.

1 Launch

Invite students to work together to design a quilt in small groups using the solid and patterned fabric squares.

Encourage students to be creative as they build their quilt. Look for and share several different quilt designs, including ones that are interesting, unique, or surprise you.

2 Monitor

A **Accessibility: Conceptual Processing** Invite students to answer a concrete question first (e.g., "If Sai used 10 square feet of solid fabric, how much patterned fabric could he could use?").

ELL **English Language Learners** Invite students to share different meanings of the terms *solid, patterned,* and *constraint*. Clarify which meaning each of these words will have for this lesson. **(Listening and Speaking)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking: • "Where do you see *at least* in the inequality?" • "Where do you see the amount of solid fabric in the inequality?" • "What does the 35 in the inequality mean in this situation?"
Choose all combinations that equal exactly 35 sq. ft.	**Strengthen** Consider asking, "What's the difference between *at least* and *equal*?"
Create combinations, including those that: • Are equal or greater than 35. • Include only one fabric or decimal amounts of fabric.	**Stretch** Encourage variety in the combinations students create. Consider asking, "How are your and your partner's combinations similar and different?"

Consider asking, "Where do you see:
- "no more than" in the inequality?"
- the costs of each fabric in the inequality?"
- the 30 from the inequality in this situation?"

Activity 1 continued >

Activity 1

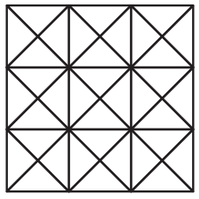
Name: Date: Period:

Sai's Quilt

2. a Let's make a quilt square together.

b **Discuss:** Why did you choose to design your quilt the way you did?

Responses vary.
- Our design uses more patterned fabric because we thought the pattern was fun.
- We made our design symmetrical.



Solid Fabric Patterned Fabric

3. Sai is making a quilt using solid and patterned fabric.

He needs *at least* 35 sq. ft of fabric to cover his bed.

Fabric Constraint

$$x + y \geq 35$$

What are some combinations of fabric Sai could use?

Responses vary.

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y
25	10
20	20
12	41

4. Sai wants to spend no more than $30 on fabric.
- Solid fabric costs $0.50 per sq. ft.
- Patterned fabric costs $1 per sq. ft.

Cost Constraint

$$0.50x + y \leq 30$$

What are some combinations of fabric Sai could use?

Responses vary.

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y
25	10
20	20
23	13

Unit 4 Lesson 7 436 Activity 1

Pairs | 15 min

Activity 1 Sai's Quilt (continued)

Purpose: Students make sense of two individual inequalities by determining combinations of fabric to meet each *constraint*.

2 Monitor

Consider sharing that a **system of inequalities** is two or more inequalities that represent the constraints on a shared set of variables. In this case, both inequalities are about the amounts of solid fabric (x) and patterned fabric (y).

A **Accessibility: Executive Functioning** Consider chunking this question into more manageable parts by inviting students to focus on one constraint at a time to determine if Sai's design meets it.

Collect a variety of explanations and sketches to discuss during the Connect by asking for volunteers.

Encourage students to make connections between the equations, the graph, and the point $(10, 28)$. Consider asking:

- "What does the point $(10, 28)$ mean in this situation?"
- How is the cost constraint represented on the graph? The fabric constraint?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Use the context or reasoning to determine if Sai's design meets the constraints.	**Strengthen** Encourage their use of reasoning. Consider asking, "How can you use the point $(10, 28)$ to support your thinking?"
Substitute the point $(10, 28)$ into the cost or fabric inequality, and not both.	**Strengthen** Support their use of substitution. Consider asking, "Does the point $(10, 28)$ make both inequalities true? How do you know?"

To support students getting started, consider asking, "Does this combination of fabrics meet both constraints? How can you see this in the graph? In the inequalities?"

"How can we test the ordered pairs?"

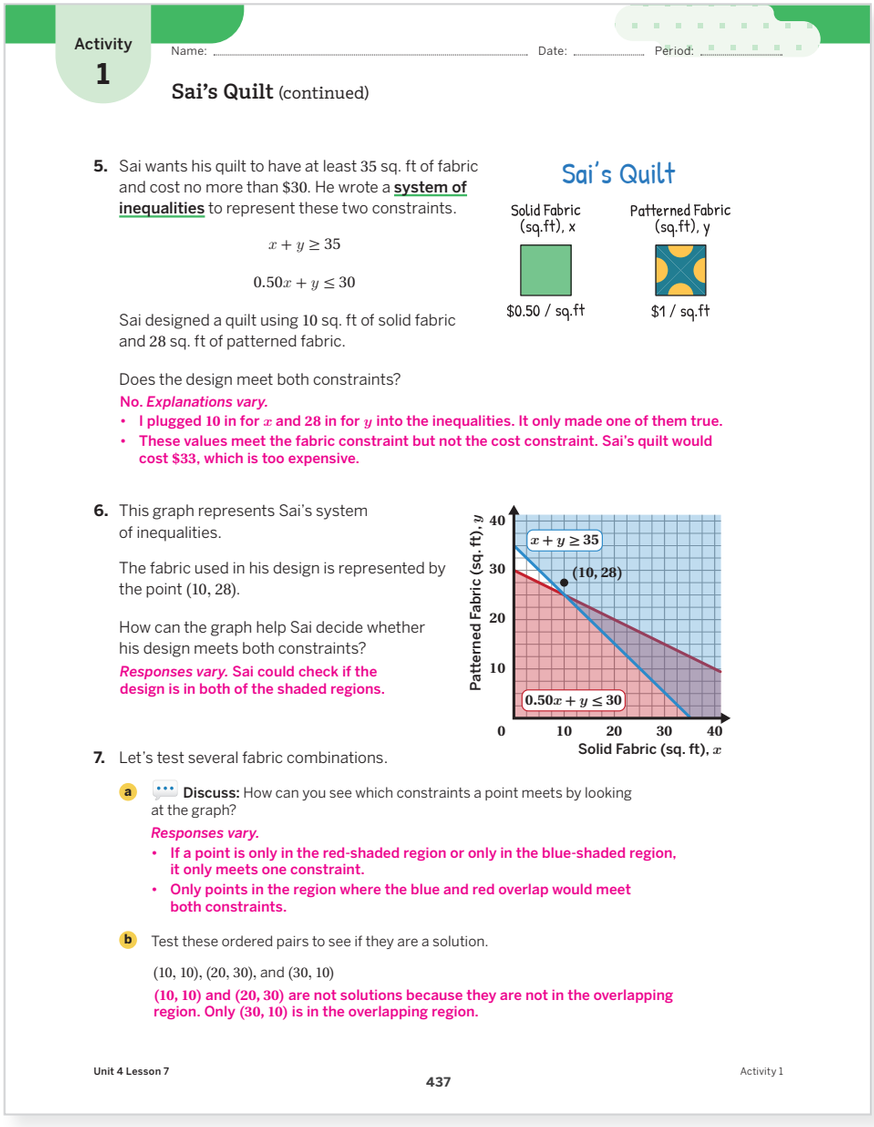
ANSWER: (10,10) and (20,30) are not solutions because they are not in the overlapping region. Only (30,10) is in the overlapping region.

D **Differentiation: (Stretch)** Invite students to determine if the quilt they designed would meet both constraints.

3 Connect

To surface the Key Takeaway, consider asking:

- "How can we determine which constraints a point meets using the inequalities? The graphs?"
- "How can viewing the same situation using different representations be helpful?"



Activity 1

Name: Date: Period:

Sai's Quilt (continued)

5. Sai wants his quilt to have at least 35 sq. ft of fabric and cost no more than \$30. He wrote a **system of inequalities** to represent these two constraints.

$$x + y \geq 35$$

$$0.50x + y \leq 30$$

Sai designed a quilt using 10 sq. ft of solid fabric and 28 sq. ft of patterned fabric.

Sai's Quilt

Solid Fabric (sq.ft), x — \$0.50 / sq.ft

Patterned Fabric (sq.ft), y — \$1 / sq.ft

Does the design meet both constraints?

No. *Explanations vary.*

- I plugged 10 in for x and 28 in for y into the inequalities. It only made one of them true.
- These values meet the fabric constraint but not the cost constraint. Sai's quilt would cost \$33, which is too expensive.

6. This graph represents Sai's system of inequalities.

The fabric used in his design is represented by the point $(10, 28)$.

How can the graph help Sai decide whether his design meets both constraints?

Responses vary. Sai could check if the design is in both of the shaded regions.

7. Let's test several fabric combinations.

a. **Discuss:** How can you see which constraints a point meets by looking at the graph?

Responses vary.

- If a point is only in the red-shaded region or only in the blue-shaded region, it only meets one constraint.
- Only points in the region where the blue and red overlap would meet both constraints.

b. Test these ordered pairs to see if they are a solution.

$(10, 10)$, $(20, 30)$, and $(30, 10)$

$(10, 10)$ and $(20, 30)$ are not solutions because they are not in the overlapping region. Only $(30, 10)$ is in the overlapping region.

Unit 4 Lesson 7 437 Activity 1

Lesson Takeaway: You can determine graphically if a point is a solution to a **system of inequalities** by looking to see if the point lies in the shaded region for both inequalities. You can determine symbolically if a point is a solution by substituting the x- and y-values into both inequalities and seeing if both statements are true.

Pairs | 15 min

Activity 2 Evan's Quilt

Purpose: Students write a new system of inequalities, determine a solution, and settle a dispute about whether another point is a solution or not.

1 Launch

To surface connections, consider asking, "How is Evan's quilt similar to Sai's? How is it different?"

2 Monitor

A **Accessibility: Executive Functioning** Invite students to input Sai's inequalities and change one value at a time to meet the constraints for Evan's quilt (e.g., start with $0.50x + y \leq 30$ and change 0.50 to 0.40 for the change in cost).

Note: Students will get an error if they write a compound inequality like $0 \leq 0.4x + 2y \leq 30$. Discuss their symbolic representation. Consider asking, "How does the cost inequality compare to a domain restriction?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to enter a solution and use the feedback to revise their response.
Switch the x- and y-values in their combinations.	**Strengthen** Consider asking students to share their ordered pair and explain what it means for the situation in terms of solid and patterned fabric. **(MTR.2.1)**
Create a combination of fabric that falls on a boundary line or point of intersection.	

Collect a variety of explanations to share during the Connect, including those that discuss substituting $(10, 10)$ into both inequalities to see if it makes them true, using the point's location relative to the shaded regions, and using context and reasoning to support their thinking.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Consider asking:

- "How can you use the graph to support your reasoning? The inequalities?"
- "How many solutions does this system have?"

Activity 2

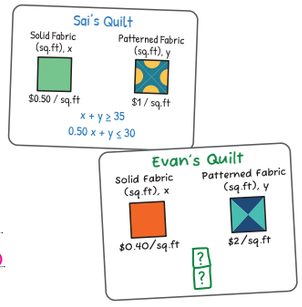
Name: _____ Date: _____ Period: _____

Evan's Quilt

8. Evan is making a quilt using different fabrics.

- Solid fabric costs $0.40 per sq. ft.
- Patterned fabric costs $2 per sq. ft.

Evan wants his quilt to have *at least* 35 sq. ft of fabric and cost *no more than* $30.



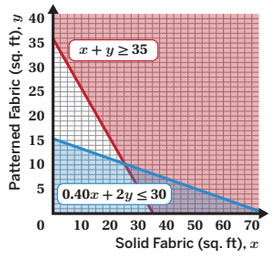
Write a system of inequalities to represent Evan's quilt.

Fabric inequality: $x + y \geq 35$ (or equivalent)

Cost inequality: $0.4x + 2y \leq 30$ (or equivalent)

9. This graph represents Evan's system of inequalities:

$$x + y \geq 35$$
$$0.40x + 2y \leq 30$$



Determine a combination of solid and patterned fabric that meets both constraints.

Responses vary.

Solid Fabric (sq. ft), x: 40

Patterned Fabric (sq. ft), y: 5

10. Titus says $(10, 10)$ *is* a solution to Evan's system of inequalities. Alma says it *is not* a solution.

Whose thinking is correct? Explain your thinking.

Alma's. *Explanations vary.*

- $(10, 10)$ is only a solution to one of the inequalities, so it is not a solution to the system.
- $(10, 10)$ is only in the blue-shaded region. It needs to be in both the blue and red-shaded regions to be a solution to the system of inequalities.

Unit 4 Lesson 7 438 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to determine if a point is a solution to a system of inequalities.

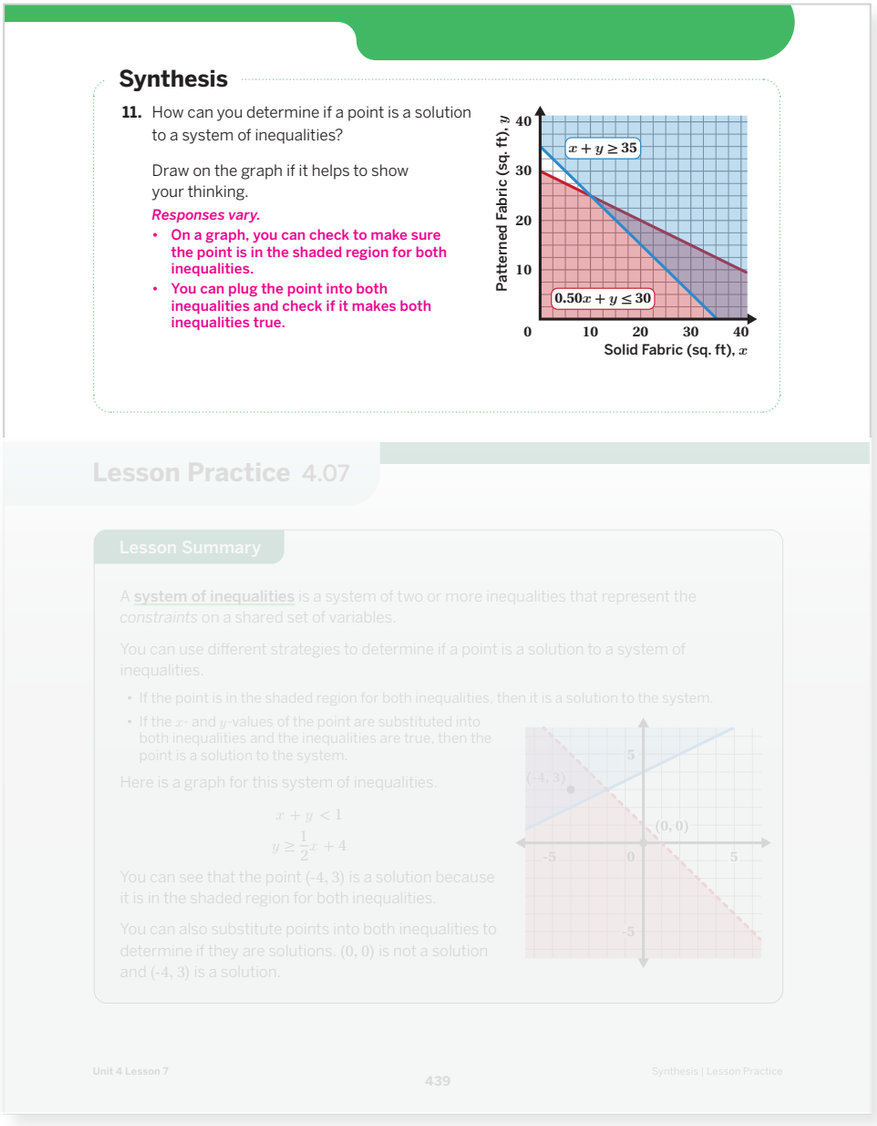
Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and inviting them to synthesize their ideas in their primary language. **(Writing, Speaking, and Listening)**

Collect and share a variety of ideas, such as:

- The point being in both shaded regions on the graph.
- The solution region being where both inequalities overlap.
- Substituting the point in both inequalities to see if it makes both inequalities true.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. How can you determine if a point is a solution to a system of inequalities?

Draw on the graph if it helps to show your thinking.

Responses vary.

- On a graph, you can check to make sure the point is in the shaded region for both inequalities.
- You can plug the point into both inequalities and check if it makes both inequalities true.

Lesson Practice 4.07

Lesson Summary

A system of inequalities is a system of two or more inequalities that represent the *constraints* on a shared set of variables.

You can use different strategies to determine if a point is a solution to a system of inequalities.

- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the x- and y-values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

Here is a graph for this system of inequalities.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$

You can see that the point $(-4, 3)$ is a solution because it is in the shaded region for both inequalities.

You can also substitute points into both inequalities to determine if they are solutions. $(0, 0)$ is not a solution and $(-4, 3)$ is a solution.

Unit 4 Lesson 7 439 Synthesis | Lesson Practice

Lesson Takeaway: A **system of inequalities** can be used to represent *constraints* in a real-world situation, like making quilts. There can be many solutions to a system of inequalities. Every solution is an ordered pair that lies in the shaded *region* of both graphs and which has x- and y-values that make both inequalities true.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying all the points within a given list that are solutions for a system of inequalities.

Today's Goals

1. **Goal:** Determine if a point is a solution to a system of inequalities using a graph.
2. **Language Goal:** Describe how ordered pairs relate to constraints in a situation. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.07

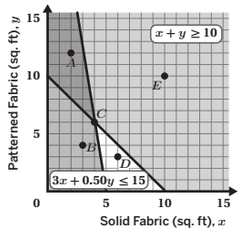
Juana is making a quilt that is at least 10 square feet using two fabrics that cost no more than $15 total.

- Solid fabric costs $3 per sq. ft.
- Patterned fabric costs $0.50 per sq. ft.

She graphed these inequalities for this situation:

$x + y \geq 10$

$3x + 0.50y \leq 15$



Select *all* the points that are solutions to this system of inequalities.

- ☑ **A.** Point A: (2, 12)
- ☐ **B.** Point B: (3, 4)
- ☑ **C.** Point C: (4, 6)
- ☐ **D.** Point D: (6, 3)
- ☐ **E.** Point E: (10, 10)

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to graph the solution region for a system of inequalities:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to return to the Warm-Up and research other quilters. Encourage them to share their findings with the class.

Professional Learning

In today's lesson, students explored the topic of quilting to support them in making sense of how a system of inequalities can represent constraints. While the constraints students explored in this lesson were focused on types of fabric (e.g., solid or patterned), what other constraints might the quilters have experienced in their efforts to design and make quilts? What questions might your students have about the quilters? What resources might you and the class use to learn more about these constraints?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you determine if a point is a solution to a system of inequalities?

Draw on the graph if it helps to show your thinking.

Responses vary.

- On a graph, you can check to make sure the point is in the shaded region for both inequalities.
- You can plug the point into both inequalities and check if it makes both inequalities true.

Lesson Practice 4.07

Lesson Summary

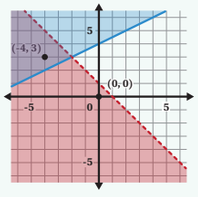
A **system of inequalities** is a system of two or more inequalities that represent the *constraints* on a shared set of variables.

You can use different strategies to determine if a point is a solution to a system of inequalities.

- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the x- and y-values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

Here is a graph for this system of inequalities.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$



You can see that the point (-4, 3) is a solution because it is in the shaded region for both inequalities.

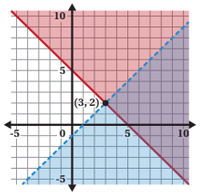
You can also substitute points into both inequalities to determine if they are solutions. (0, 0) is not a solution and (-4, 3) is a solution.

Lesson Practice 4.07

Name: _____ Date: _____ Period: _____

1. The graph shows this system of inequalities.

$$x + y \geq 5$$
$$x - y > 1$$



Is the point (3, 2) a solution to the system?

No.

Explain your thinking.

Explanations vary. **The point (3, 2) satisfies the first inequality, but not the second one.**

Problems 2–4: It costs Lukas $5.00 to mail a package. Lukas has *postcard stamps*, p, that are worth $0.34 each and *first-class stamps*, f, that are worth $0.49 each.

2. Lukas wrote the inequality $0.34p + 0.49f \geq 5$. What does this inequality represent?

Responses vary. **The inequality represents the combinations of postcard stamps and first-class stamps that add up to a total value $5 or more.**

3. Lukas wrote another inequality: $p + f \leq 12$. What does this inequality represent?

Responses vary. **The inequality represents the combinations of postcard stamps and first-class stamps that add up to a total of 12 stamps or fewer.**

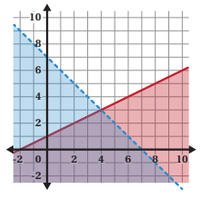
4. If Lukas uses 1 postcard stamp and 9 first-class stamps, will this satisfy both constraints? Explain your thinking.

No. *Explanations vary.* 1 postcard stamp and 9 first-class stamps would satisfy the second constraint about having 12 or fewer stamps. But $0.34(1) + 0.49(9) = 4.75$, which is less than $5.

5. Using the digits 0–9 without repeating, fill in each blank such that each statement is true:

- A is a solution to both inequalities
- B is a solution to only one inequality
- C is a solution to only the other inequality
- D is not a solution to either inequality

Responses vary.



$A = (\boxed{4}, \boxed{2})$ $B = (\boxed{0}, \boxed{3})$

$C = (\boxed{9}, \boxed{5})$ $D = (\boxed{8}, \boxed{6})$

Lesson Practice 4.07

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Test Practice

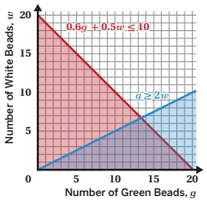
6. Arjun is making a bracelet.

He has $10 to spend on beads.

Green beads cost $0.60 each and white beads cost $0.50 each.

His bracelet design needs at least twice as many green beads as white beads.

The graph shows the system of inequalities that represents this situation.



What is a combination of green and white beads that meets both constraints?

_____ green beads and _____ white beads

Responses vary, 10 green beads and 4 white beads.

Spiral Review

7. Solve this system of equations. Write your solution as a coordinate pair.

$2x + y = 8$
$y = 2x + 4$

(1, 6)

8. Jayla is at the market with $14 to buy fruit. She decides to buy apples and grapes. Apples, a, cost $1.67 per pound and grapes, g, cost $1.87 per pound. Write an inequality to represent this situation.

$1.67a + 1.87g \leq 14$ (or equivalent)

9. Esteban claims that $(5, 10)$ is *not* a solution to the inequality $y < 3x - 5$. Do you agree with Esteban's claim? Explain your thinking.

Yes. *Explanations vary.* If you substitute the values into the inequality you get $10 < 3(5) - 5$. This simplifies to $10 < 10$ which is not a true statement.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.2.8
	2–4	2	MA.912.AR.9.6
	5	3	MA.912.AR.2.8
Test Practice	6	2	MA.912.AR.9.6
Spiral Review			
Fluency	7	2	MA.912.AR.9.1
	8	2	MA.912.AR.9.6
	9	2	MA.912.AR.9.6

Unit 4

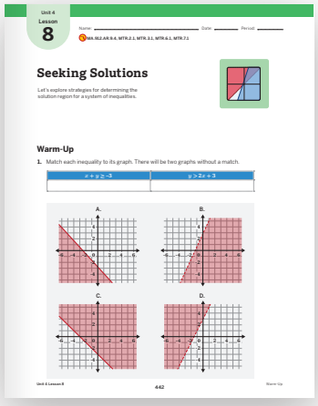
Lesson

8

Seeking Solutions

Solutions to Systems of Inequalities

Let's explore strategies for determining the solution region for a system of inequalities.


Unit 4
Lesson
8
Seeking Solutions
Let's explore strategies for determining the solution region for a system of inequalities.
Warm-Up
1. Match each inequality to its graph. There will be two graphs without a match.
$y \geq y \geq -3$
$y > 2x + 2$
A.
B.
C.
D.
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Learning Outcomes

Today's Goals

1. **Goal:** Determine the solution region of a system of inequalities given the boundary lines.
2. **Language Goal:** Describe how to determine the solution region of a system of inequalities on a graph. **(Speaking, Listening, and Writing)**

Students describe the **solution region** of a system of inequalities as where the graphs of the two inequalities overlap. They discuss strategies for how to determine the location of the solution region when given boundary lines. Students interpret points in different regions of the graph of a system of inequalities, and test points to identify the solution region and determine where to shade.

Coherence

Prior Learning

In Unit 1, students graphed a two-variable inequality by using the inequality symbol to decide if points along the boundary line were included in the solution and tested points to determine the solution region. In Lesson 7, students explored what it means for a single point to be a solution to a system of inequalities.

Future Learning

In Lesson 9, students will develop fluency in graphing systems of linear inequalities by hand and identifying the solution region. In Lesson 10, they will write a system of inequalities to represent real-world constraints.

Connections

- Students explore strategies for determining the solution region for a system of inequalities when interpreting visuals of graphs. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.4
Graph the solution set of a system of two-variable linear inequalities.

Also addressing: **MA.912.AR.9.6**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.912.AR.2.8

Vocabulary

New Vocabulary

solution (to a system of inequalities)
solution region

Review Vocabulary

boundary line
no solution
region

Lesson at a Glance

~ 45 min

MA.912.AR.9.4, MA.912.AR.9.6

Warm-Up

Pairs | 5 min

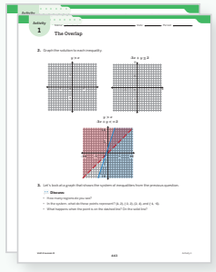
Students revisit how to graph a two-variable inequality, including when to use a solid or dashed *boundary line* and how to determine where to shade. MLR2: Collect and Display



Activity 1

Pairs | 10 min

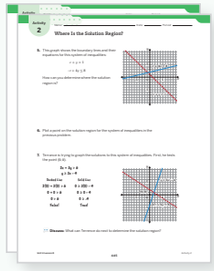
Students analyze a graph of a system of inequalities and determine that the **solution region** is where the two graphs overlap. MLR2: Collect and Display



Activity 2

Pairs | 10 min

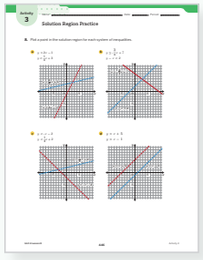
Students discuss strategies for determining the solution region. MLR3: Critique, Correct, Clarify



Activity 3

Pairs | 10 min

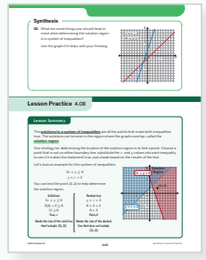
Students determine the solution region for a variety of systems of inequalities, including systems with *no solutions*. MLR1: Stronger and Clearer Each Time



Synthesis

Whole Class | 5 min

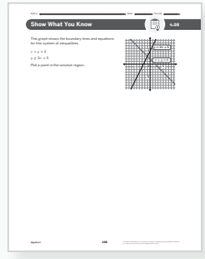
Students synthesize strategies for determining the solution region of a system of inequalities, given the boundary lines. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the solution region for a system of inequalities.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit how to graph a two-variable inequality, including when to use a solid or dashed *boundary line* and how to determine where to shade.

1 Launch

To surface prior knowledge, consider asking, "What do you remember about graphing a two-variable inequality?"

Encourage students to share their reasoning with a partner and work together to agree on how to sort the graphs and inequalities.

A **Accessibility: Executive Function** Invite students to focus on one graph at a time to support them in carrying out multiple steps.

ELL **English Language Learners** Encourage students to discuss how to group the graphs and inequalities in their primary language, clarifying any unknown words, symbols, or graphical representations (e.g., inequality symbols, dashed or solid lines, shading to indicate solution regions). **(Speaking and Reading)**

D **Differentiation: (Stretch)** Encourage students who complete their matching to write inequalities to match the two remaining graphs.

2 Connect

Display the answers to support students in seeing if they matched the equation and graph correctly. If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

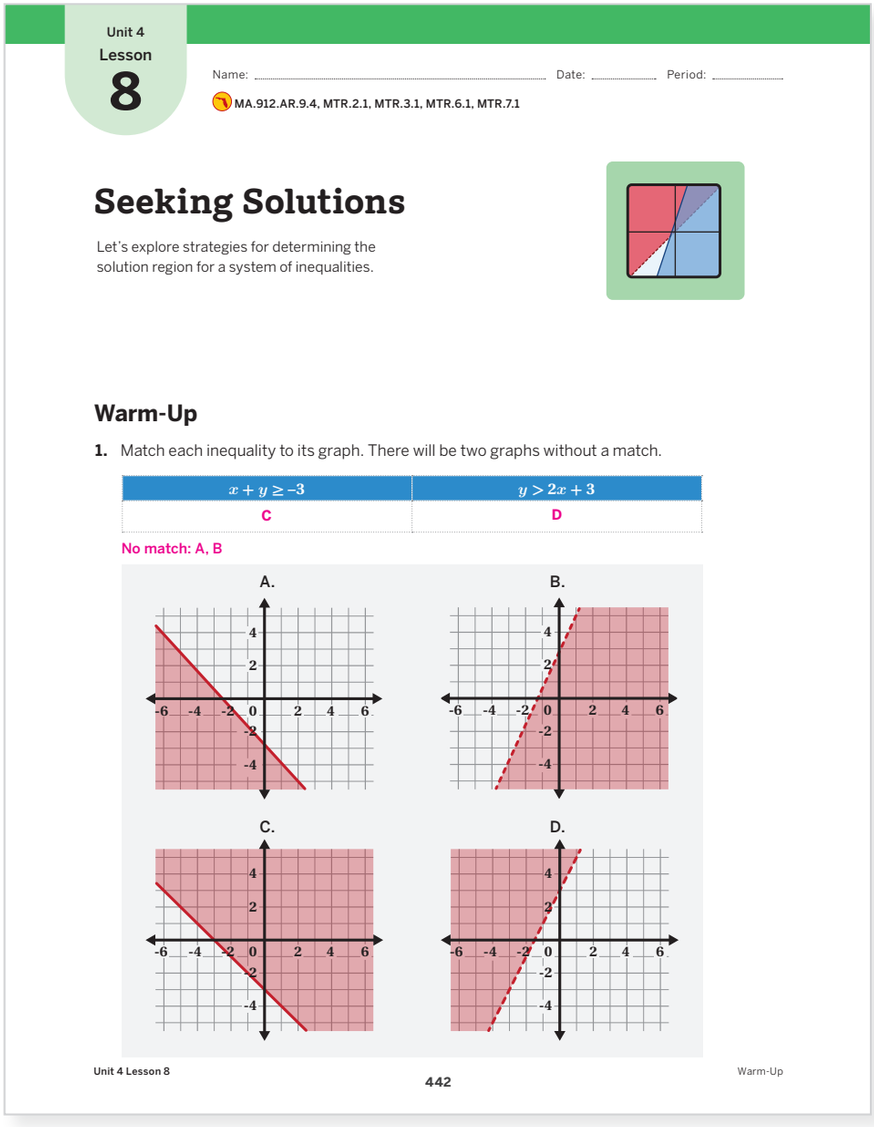
Invite students to share their strategies for matching the graphs and inequalities.

Consider asking:

- "How did you determine whether the inequality would have a dashed or solid boundary line?"
- "How did you decide which region should be shaded?"

Note: Consider reviewing how to test a point to determine the shaded region.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for graphing two-variable inequalities, including how to test a point that is not on the boundary line to determine the solution region for an inequality. Add student ideas, language, and strategies to the chart throughout the lessons.



Unit 4
Lesson
8

Name: Date: Period:

MA.912.AR.9.4, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Seeking Solutions

Let's explore strategies for determining the solution region for a system of inequalities.

Warm-Up

1. Match each inequality to its graph. There will be two graphs without a match.

$x + y \geq -3$	$y > 2x + 3$
C	D

No match: A, B

A. B. C. D.

Unit 4 Lesson 8 442 Warm-Up

Pairs | 10 min

Activity 1 The Overlap

Purpose: Students analyze a graph of a system of inequalities and determine that the **solution region** is where the two graphs overlap.

1 Launch

Identify different parts of the graphs and invite students to ask questions or clarifications.

A **Accessibility: Visual-Spatial Processing**
Invite students to use coloring tools when graphing by hand to support interpreting visual representations (e.g., by graphing each inequality in a different color).

Consider asking, "How is solving a system of inequalities different from solving one inequality?"

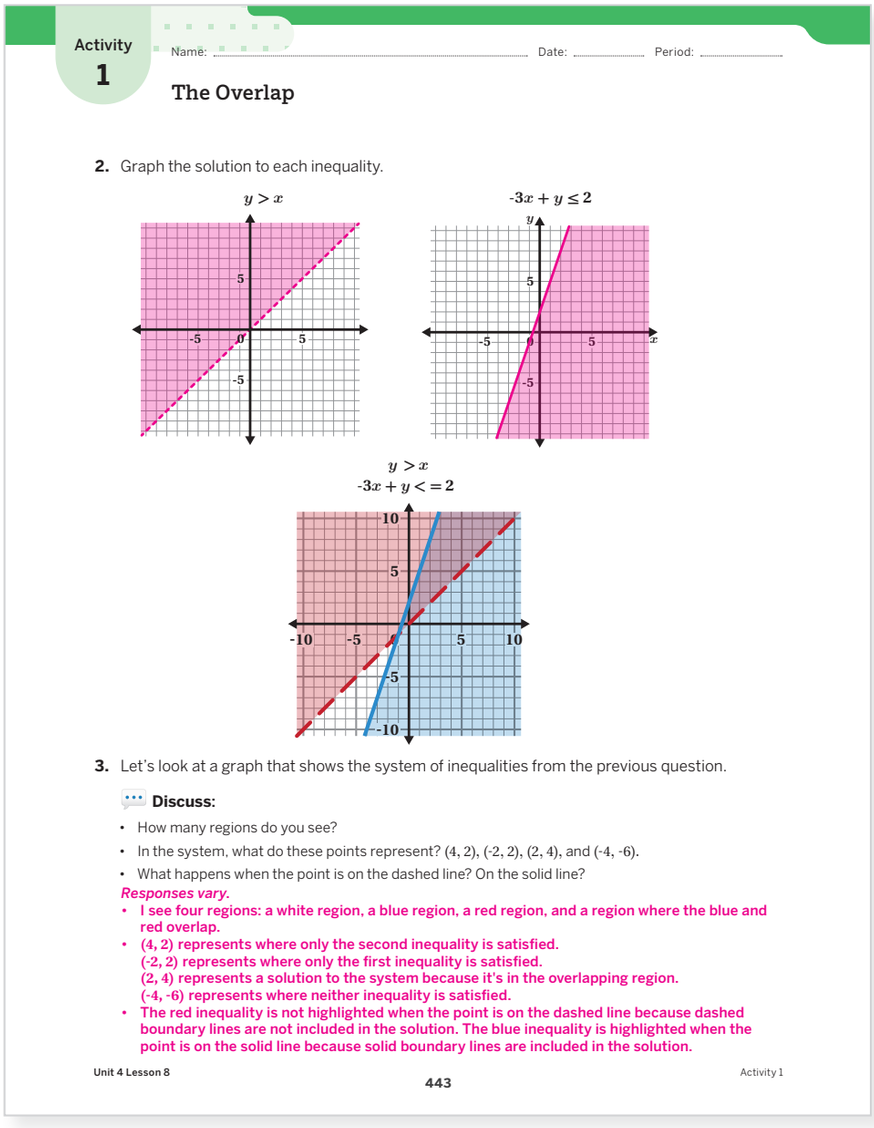
Consider sharing with students that when graphing systems of inequalities, it can be helpful to use different tools to support their thinking, such as paper to test points or a calculator to evaluate specific points. Encourage students to use these tools as needed throughout the lesson.

2 Monitor

Pause briefly to discuss the different regions on the graph. Consider asking:

- "How can you see the different regions?"
- "Which points are solutions to only one inequality?"
- "Which points are solutions to both inequalities?"
- "When are the points on the boundary lines included in the solution region? When are they not included?"

Activity 1 continued >



Activity 1

Name: Date: Period:

The Overlap

2. Graph the solution to each inequality.

$y > x$

$-3x + y \leq 2$

$y > x$
$-3x + y < = 2$

3. Let's look at a graph that shows the system of inequalities from the previous question.

Discuss:

- How many regions do you see?
- In the system, what do these points represent? (4, 2), (-2, 2), (2, 4), and (-4, -6).
- What happens when the point is on the dashed line? On the solid line?

Responses vary.

- I see four regions: a white region, a blue region, a red region, and a region where the blue and red overlap.
- (4, 2) represents where only the second inequality is satisfied.
  (-2, 2) represents where only the first inequality is satisfied.
  (2, 4) represents a solution to the system because it's in the overlapping region.
  (-4, -6) represents where neither inequality is satisfied.
- The red inequality is not highlighted when the point is on the dashed line because dashed boundary lines are not included in the solution. The blue inequality is highlighted when the point is on the solid line because solid boundary lines are included in the solution.

Unit 4 Lesson 8 443 Activity 1

Pairs | 10 min

Activity 1 The Overlap (continued)

Purpose: Students analyze a graph of a system of inequalities and determine that the **solution region** is where the two graphs overlap.

2 Monitor

Invite students to share several points that are solutions to the system, Identifying the points on the graph as they state their points.

Encourage students to use the feedback to revise their thinking.

ELL **English Language Learners** Consider using a think-aloud to model making sense of the feedback. **(Reading and Listening)**

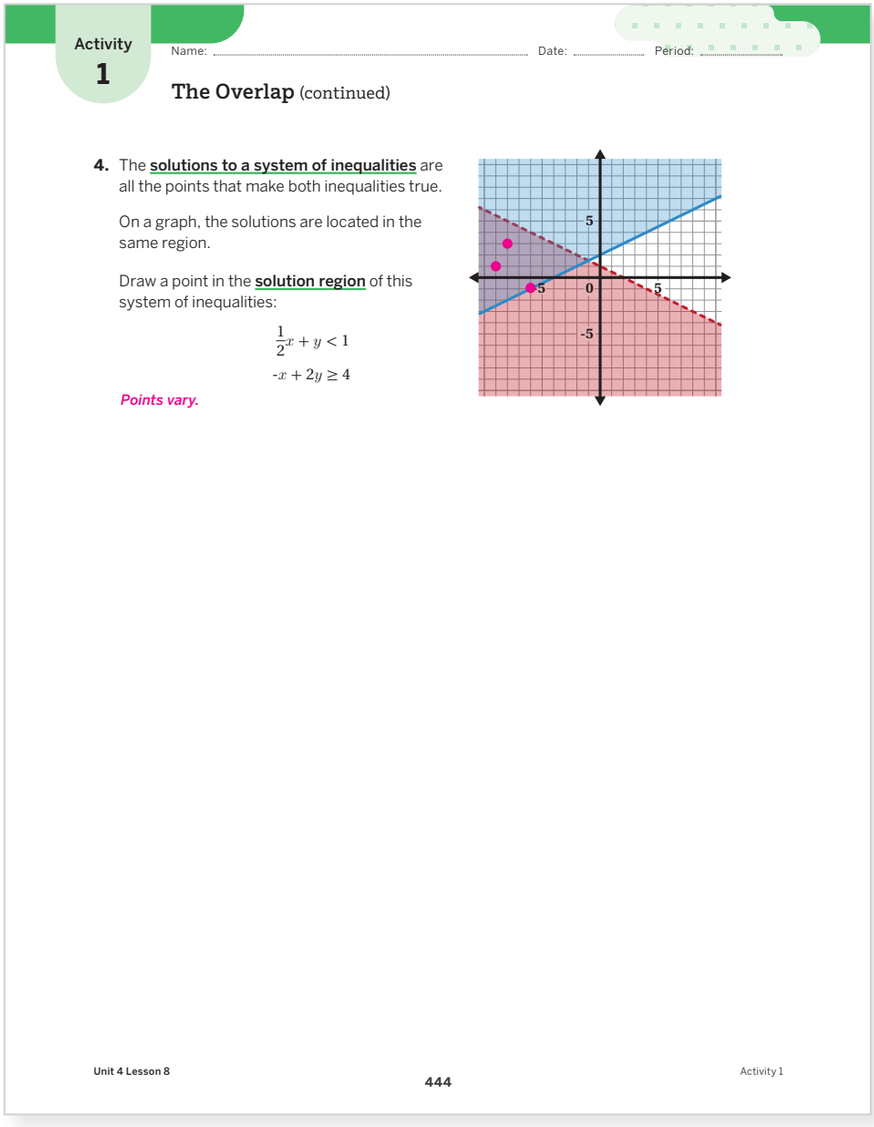
D **Differentiation: (Stretch)** Consider asking, "Is the point of intersection of the two boundary lines included in the solution region? Why or why not?" **(MTR.4.1)**

3 Connect

Invite students to share what the term *solution region* means in their own words (e.g., "When I hear *solution region*, I think ___.").

To surface the Key Takeaway, consider asking, "How can you tell if a point is a solution to a system of inequalities?"

MLR **MLR2: Collect and Display** Use this routine to revisit students' strategies for graphing a system of inequalities. Add any new strategies to the display for students to reference for the rest of the unit.



Activity 1

Name: Date: Period:

The Overlap (continued)

4. The **solutions to a system of inequalities** are all the points that make both inequalities true.

On a graph, the solutions are located in the same region.

Draw a point in the **solution region** of this system of inequalities:

$$\frac{1}{2}x + y < 1$$

$$-x + 2y \geq 4$$

Points vary.

Unit 4 Lesson 8 444 Activity 1

Key Takeaway: The **solutions to a system of inequalities** are all of the points that make both inequalities true. On the graph, the solutions make up the **solution region**, which contains all the points that are located within both shaded *regions*.

Pairs | 10 min

Activity 2 Where is the Solution Region?

Purpose: Students discuss strategies for determining the solution region.

1 Launch

Collect and share a variety of student responses.

Consider asking:

- "How are these strategies alike? How are they different?"
- "How would you describe the solution region in words?" Where the two shaded regions from each inequality overlap, or the place where both inequalities are shaded.

Note: It is okay – even desirable – to lack consensus at this stage. The activity will build toward consensus later.

2 Monitor

Encourage students to use paper or a graphing tool to help them test points.

D **Differentiation: (Support)** Some students may use a guess and check strategy. Consider asking, "How can we use the coordinate pair to determine what the feedback will say?"

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Terrance's work. Then invite students to:

- Discuss what is *correct* about the response.
- Discuss what is *incorrect, unclear, or unfinished.*
- Write a question to help Terrance see what he can do next.

To surface the Key Takeaway, consider asking:

- "Does Terrance have to test more than one point to determine the solution region?"
- "What points do you like to use when determining the solution region? What points might be less helpful?" The origin, intercepts, and any interesting points might all be helpful.

Activity 2

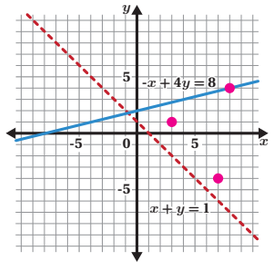
Name: Date: Period:

Where Is the Solution Region?

5. This graph shows the boundary lines and their equations for this system of inequalities:

$$x + y > 1$$

$$-x + 4y \leq 8$$



How can you determine where the solution region is?

Responses vary.

- I can plug in (0, 0) to both inequalities.
- I can test a point in each region to see which one makes both inequalities true.
- For each inequality, I can test a point that's not on the boundary line, then shade both inequalities and see where they overlap.

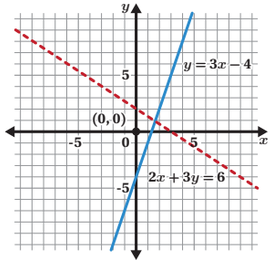
6. Plot a point on the solution region for the system of inequalities in the previous problem.

Points vary.

7. Terrance is trying to graph the solutions to this system of inequalities. First, he tests the point (0, 0).

$2x + 3y > 6$
$y \geq 3x - 4$

Dashed Line	Solid Line
$2(0) + 3(0) > 6$	$0 \geq 3(0) - 4$
$0 + 0 > 6$	$0 \geq 0 - 4$
$0 > 6$	$0 \geq -4$
False!	True!



Discuss: What can Terrence do next to determine the solution region?

Responses vary. Since (0, 0) makes the first inequality false, Terrence could shade the side of the dashed boundary line that doesn't include (0, 0). Since (0, 0) makes the second inequality true, Terrence could shade the side of the solid boundary line that does include (0, 0). Then he could look for where the shading overlaps.

Unit 4 Lesson 8 445 Activity 2

Key Takeaway: To determine the solution region for a system of inequalities, one strategy is to test points until finding an (x, y) ordered pair that when substituted makes both inequalities true. The solution region will be the region containing that point.

Pairs | 10 min

Activity 3 Solution Region Practice

Purpose: Students determine the solution region for a variety of systems of inequalities, including systems with *no solutions*.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Invite students to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly.

Students will need a partner for this activity. They will solve and compare their solutions to problems instead of receiving responsive feedback.

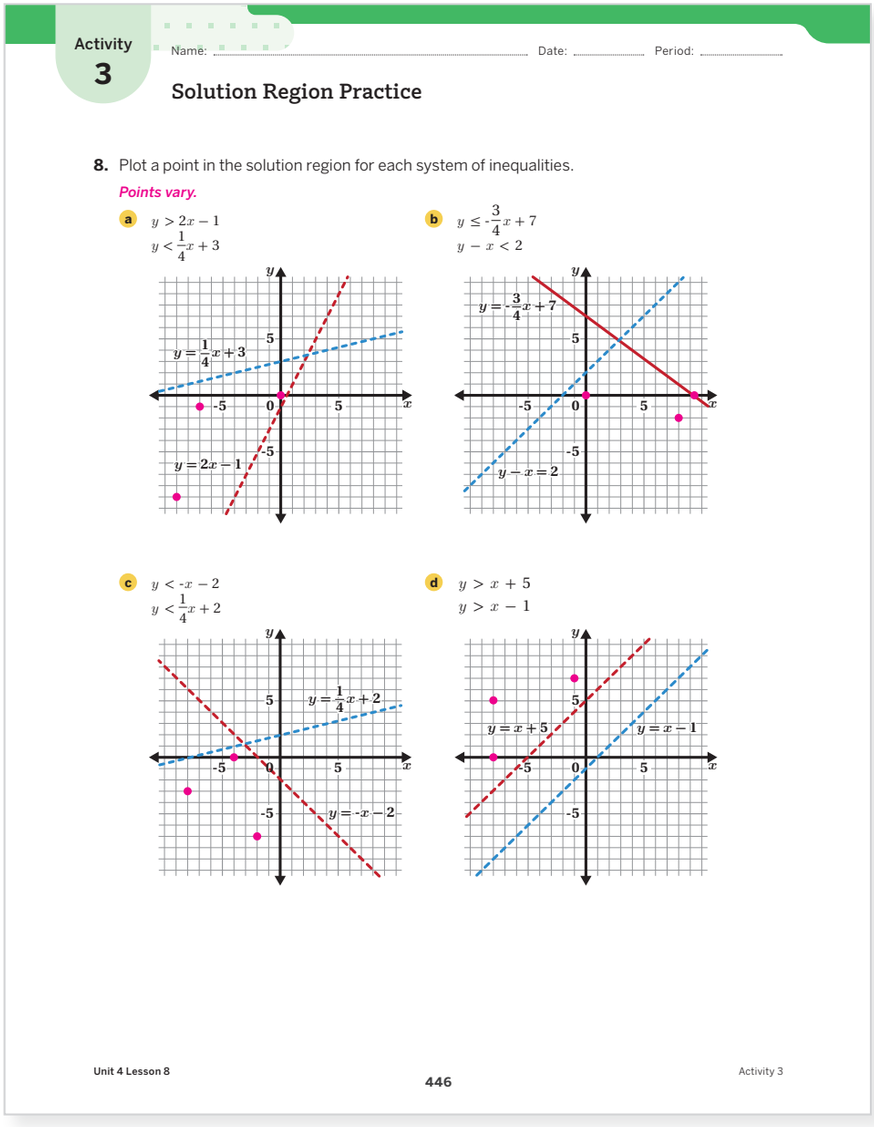
Invite students to use a calculator to support them with performing calculations.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

Listen for mathematical language students use in discussing how to solve the problems, such as how the solution region is located where both graphs overlap or testing specific points to determine the solution region.

Activity 3 continued >



Activity 3

Name: Date: Period:

Solution Region Practice

8. Plot a point in the solution region for each system of inequalities.

Points vary.

a $y > 2x - 1$, $y < \frac{1}{4}x + 3$

b $y \leq -\frac{3}{4}x + 7$, $y - x < 2$

c $y < -x - 2$, $y < \frac{1}{4}x + 2$

d $y > x + 5$, $y > x - 1$

Unit 4 Lesson 8 446 Activity 3

Pairs | 10 min

Activity 3 Solution Region Practice (continued)

Purpose: Students determine the solution region for a variety of systems of inequalities, including systems with *no solutions*.

Short on time? Consider inviting students to complete fewer challenges.

2 Monitor

Look for a variety of reasoning students share in their responses.

D Differentiation

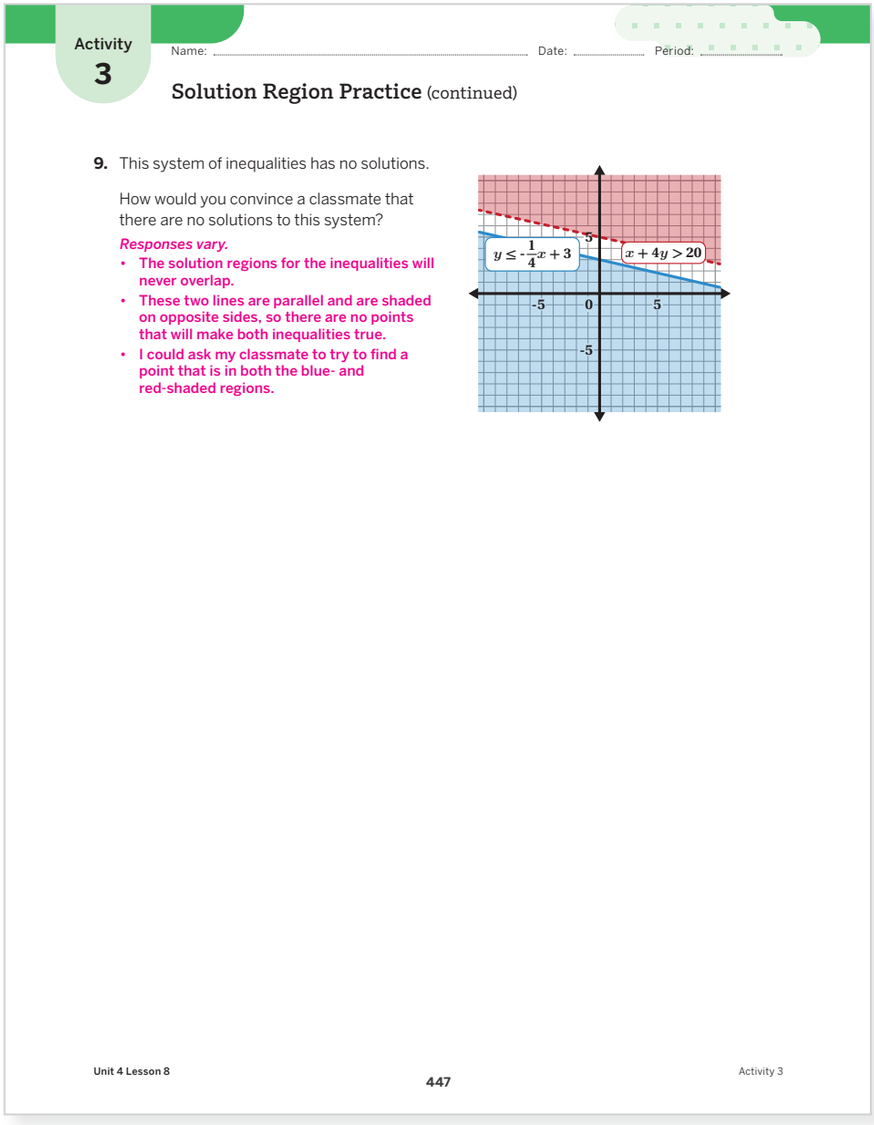
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to make sense of the graphical representation. Consider asking, "What does it look like for a system of inequalities to have a solution? Where do these two inequalities show the solution region?"
Describe how the shaded regions do not overlap.	**Strengthen** Listen for student's use of language. Invite them to share during the Connect.
Recognize that the lines are parallel.	**Strengthen** Encourage this student's noticing. Consider asking, "How could we rewrite the inequalities to show that the lines are parallel?" Invite them to share during the Connect.
Recognize the lines have the same slope and different y-intercepts by looking at the structure of the inequalities.	**Strengthen** Support this student's noticing. Consider asking, "How does the graph confirm your observation?"
Would like a challenge.	**Stretch** Invite students to write a system of inequalities that is parallel and has solutions or describe what a system of inequalities with infinitely many solutions would look like. Encourage these students to share their thinking with a partner.

3 Connect

MLR1: Stronger and Clearer Each Time Use this routine to support students in developing their reasoning. Invite students to write a first draft and use feedback to make their thinking stronger and clearer.

Consider asking:

- "How can you see from the graph that there are *no solutions* to this system of inequalities?"
- "Is there anything in someone else's explanation that you can use to make your writing stronger and clearer?"
- "How does using multiple representations support your argument?"



Activity 3

Name: Date: Period:

Solution Region Practice (continued)

9. This system of inequalities has no solutions.

How would you convince a classmate that there are no solutions to this system?

Responses vary.

- The solution regions for the inequalities will never overlap.
- These two lines are parallel and are shaded on opposite sides, so there are no points that will make both inequalities true.
- I could ask my classmate to try to find a point that is in both the blue- and red-shaded regions.

Unit 4 Lesson 8 447 Activity 3

Whole Class | 5 min

Synthesis

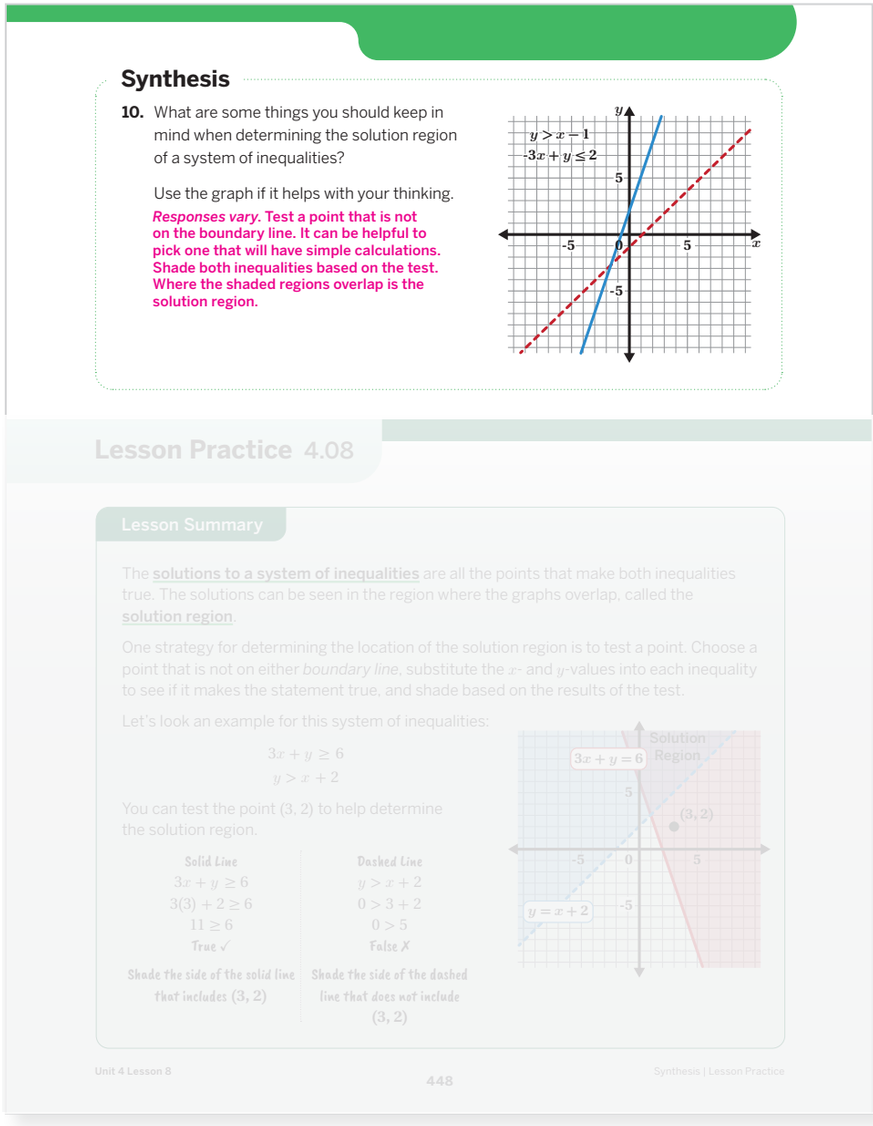
Purpose: Students synthesize strategies for determining the solution region of a system of inequalities, given the boundary lines.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Test a point (or points) not on a boundary line.
- Shade both inequalities based on the tested point(s).
- A solution to the system of inequalities is where the x- and y-values make both inequalities true.
- The solution region is where the shaded regions overlap for both graphs.

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas about graphing a system of inequalities. Add any new strategies or examples to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. What are some things you should keep in mind when determining the solution region of a system of inequalities?

Use the graph if it helps with your thinking.

Responses vary. Test a point that is not on the boundary line. It can be helpful to pick one that will have simple calculations. Shade both inequalities based on the test. Where the shaded regions overlap is the solution region.

Lesson Practice 4.08

Lesson Summary

The solutions to a system of inequalities are all the points that make both inequalities true. The solutions can be seen in the region where the graphs overlap, called the solution region.

One strategy for determining the location of the solution region is to test a point. Choose a point that is not on either *boundary line*, substitute the x- and y-values into each inequality to see if it makes the statement true, and shade based on the results of the test.

Let's look an example for this system of inequalities:

$3x + y \geq 6$
$y > x + 2$

You can test the point (3, 2) to help determine the solution region.

Solid Line	Dashed Line
$3x + y \geq 6$	$y > x + 2$
$3(3) + 2 \geq 6$	$0 > 3 + 2$
$11 \geq 6$	$0 > 5$
True ✓	False ✗
Shade the side of the solid line that includes (3, 2)	Shade the side of the dashed line that does not include (3, 2)

Unit 4 Lesson 8 448 Synthesis | Lesson Practice

Lesson Takeaway: The **solution region of a system of inequalities** is the *region* that contains ordered pairs that make both inequalities true. One strategy for determining the region is to select a point that is not on the *boundary lines* and test to see if the values make the inequalities true. Another strategy is to graph the solution region for each inequality and look for where the shaded regions overlap.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the solution region for a system of inequalities.

Today's Goals

1. **Goal:** Determine the solution region of a system of inequalities given the boundary lines.
2. **Language Goal:** Describe how to determine the solution region of a system of inequalities on a graph. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.08

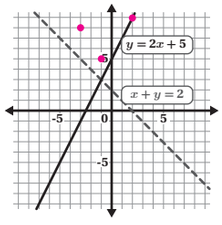
This graph shows the boundary lines and equations for this system of inequalities.

$x + y > 2$

$y \geq 2x + 5$

Plot a point in the solution region.

Points vary.



Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to graph systems of inequalities:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their thinking about graphing solutions to systems of inequalities in the next lesson. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write a system of inequalities with parallel lines that has solutions and discuss their thinking with a partner.

Professional Learning

In this lesson, students expanded on their prior knowledge of graphing two-variable inequalities to support their understanding of graphing a system of inequalities and determining the location of the **solution region**. How were students able to apply their thinking to this new concept? How can you support students in recognizing the connection between these two key math concepts as they continue to explore systems and inequalities in their future learning?

Lesson Practice

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. What are some things you should keep in mind when determining the solution region of a system of inequalities?

Use the graph if it helps with your thinking.

Responses vary. Test a point that is not on the boundary line. It can be helpful to pick one that will have simple calculations. Shade both inequalities based on the test. Where the shaded regions overlap is the solution region.

Lesson Practice 4.08

Lesson Summary

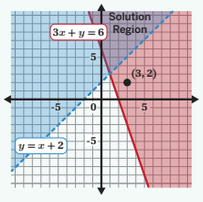
The **solutions to a system of inequalities** are all the points that make both inequalities true. The solutions can be seen in the region where the graphs overlap, called the **solution region**.

One strategy for determining the location of the solution region is to test a point. Choose a point that is not on either *boundary line*, substitute the x- and y-values into each inequality to see if it makes the statement true, and shade based on the results of the test.

Let's look an example for this system of inequalities:

$$3x + y \geq 6$$
$$y > x + 2$$

You can test the point (3, 2) to help determine the solution region.



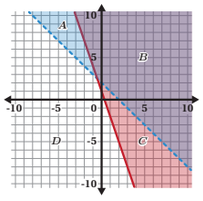
Solid Line	Dashed Line
$3x + y \geq 6$	$y > x + 2$
$3(3) + 2 \geq 6$	$0 > 3 + 2$
$11 \geq 6$	$0 > 5$
True ✓	False ✗
Shade the side of the solid line that includes (3, 2)	Shade the side of the dashed line that does not include (3, 2)

Lesson Practice 4.08

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is the graph of a system of inequalities.

$$y > -x + 2$$
$$3x + y \geq 1$$



1. Which letter represents the solution region to the system of inequalities? Circle one:

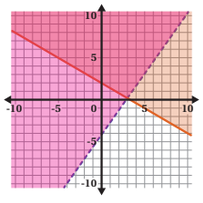
A (B) C D

2. Is the point (5, -4) a solution to the system? Circle one:

Yes (No)

Problems 3–4: Javier graphed the first inequality and the boundary line of the second inequality.

$$3x + 5y \geq 10$$
$$4x - 3y < 9$$



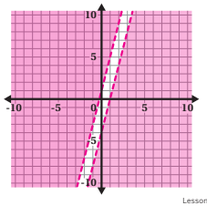
3. Complete the graph of the second inequality.

4. Explain how you knew where to shade the second inequality.

Responses vary. I plugged (0, 0) into $4x - 3y < 9$. Since (0, 0) makes the inequality true, that side of the boundary line should be shaded.

5. Make a graph of a system of inequalities that has no solutions.

Graphs vary.



6. Explain how you know it has no solutions.

Responses vary. The system has no solutions because the regions of each inequality do not overlap. Because the boundary lines are parallel, they won't intersect.

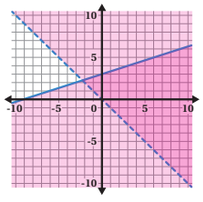
Lesson Practice 4.08

Name: _____ Date: _____ Period: _____

Problems 7–8: Nyanna graphed the boundary lines of this system of inequalities:

$$x + y > 0$$
$$-x + 3y \leq 9$$

7. Complete the graph of the system of inequalities.



8. Identify a coordinate pair that is in the solution region.

Responses vary. (2, 1)

Test Practice

Problems 9–10: Fill in each blank with an inequality symbol such that:

9. The system has no solutions.

$x - y$ [<] 0

$x - y$ [>] 0

10. Only points with matching x- and y-coordinates are a solution.

$x - y$ [≤] 0

$x - y$ [≥] 0

Spiral Review

Problems 11–13: Determine the value of each expression.

11. $\frac{4}{6} \cdot 8$

$\frac{16}{3}$ (or equivalent)

12. $\frac{3}{5} \cdot 2 \cdot \frac{10}{9}$

$\frac{4}{3}$ (or equivalent)

13. $\frac{3}{6} \cdot 3 \cdot \frac{6}{9} \cdot 2$

2 (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.9.4
	2, 8	1	MA.912.AR.9.4
	3–7	2	MA.912.AR.9.4
Test Practice	9–10	3	MA.912.AR.9.4
Spiral Review			
Fluency	11–13	1	MA.5.NSO.2.2

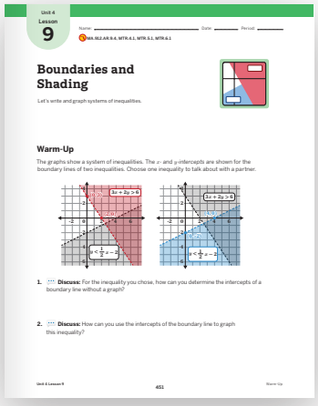
Unit 4

Lesson 9

Boundaries and Shading

Graphing Systems of Inequalities

Let's write and graph systems of inequalities.


Unit 4
Lesson 9
Name: Date: Period:
MA.912.AR.9.4, MTR.4.1, MTR.5.1, MTR.6.1
Boundaries and Shading
Let's write and graph systems of inequalities.
Warm-Up
The graphs show a system of inequalities. The x- and y-intercepts are shown for the boundary lines of two inequalities. Choose one inequality to talk about with a partner.
$3x + 2y > 5$
$3x + 2y \geq 5$
$x < \frac{5}{3}$, $y < -\frac{3}{2}$
$x < \frac{5}{3}$, $y \leq -\frac{3}{2}$
1. Discuss: For the inequality you chose, how can you determine the intercepts of a boundary line without a graph?
2. Discuss: How can you use the intercepts of the boundary line to graph this inequality?
Unit 4 Lesson 9
451
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Write a system of inequalities when given a graph.
2. **Goal:** Graph the solution region to a system of inequalities.
3. **Language Goal:** Describe a strategy for graphing the solution region to a system of inequalities. **(Writing, Speaking, and Listening)**

Students graph systems of linear inequalities by hand and identify the solution region. With a partner, students select inequalities and graph them to create systems. Then they create graphs for systems of inequalities and write equations for given graphs.

Coherence

Prior Learning

In Lesson 7, students made sense of systems of inequalities in a quilting context and explored what it means to be a solution to a system of inequalities. In Lesson 8, students explained that the solution region is where two graphs overlap and discussed strategies for how to determine the location of the solution region.

Future Learning

In Lesson 10, students will use systems of inequalities to model real-world situations.

Connections

- Students explore visual graphs to write and graph systems of inequalities. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.4

Graph the solution set of a system of two-variable linear inequalities.

Also addressing: **MA.912.AR.9.6**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Vocabulary

Review Vocabulary

slope-intercept form

standard form (of a linear equation)

x-intercept

y-intercept

Additional Vocabulary

half-plane

Lesson at a Glance

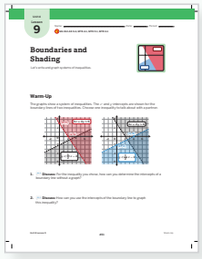
~ 45 min

MA.912.AR.9.4, MA.912.AR.9.6

Warm-Up

Pairs | 5 min

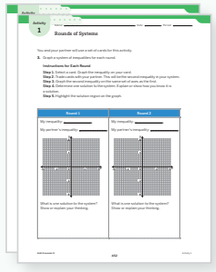
Students revisit how to determine intercepts for an inequality and discuss how intercepts can be helpful for creating a graph.



Activity 1

Pairs | 15 min

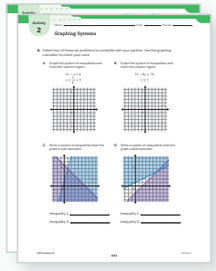
Students make use of structure as they graph systems of inequalities with a partner. **(MTR.5.1)**



Activity 2

Pairs | 15 min

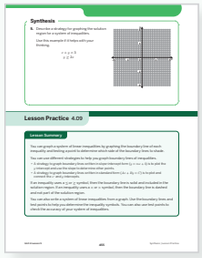
Students create graphs of systems of inequalities and write systems of inequalities when given graphs. **MLR3: Critique, Correct, Clarify**



Synthesis

Whole Class | 5 min

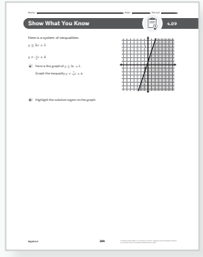
Students synthesize their strategies for graphing the solution region of a system of inequalities. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by completing the graph of a system of inequalities and highlighting the solution region.



Prep Checklist

Prepare the Activity 1 cards.

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 1 Cards, one set per pair, pre-cut.
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit how to determine intercepts for an inequality and discuss how intercepts can be helpful for creating a graph.

1 Launch

Display the graph of the system of inequalities.

ELL **English Language Learners** To help students access prior knowledge and increase access to the task, invite them to share where they've heard the word *intercept* before and what they think it means.

To surface prior knowledge, consider asking:

- "How do we know if a point is a solution to a system of inequalities?"
- "How can we identify where the solution region will be for a system of inequalities?"
- "What is an intercept? Where would we find an intercept on the graph?"

A **Accessibility: Executive Functioning** Consider chunking this activity into more manageable parts by discussing and recording responses to one discussion question at a time.

Look for students who choose different inequalities and invite them to share their strategy for finding the x- and y-intercepts.

2 Connect

Consider asking:

- "What strategies can you use to determine the x-and y-intercepts without a graph?"
- "How is determining the x- and y-intercepts different for an inequality in standard form versus an inequality in slope-intercept form?" **(MTR.5.1)**
- "How do you use the intercepts to make the graph?"

Note: The term *half-plane* (a set of points in the coordinate plane on one side of a boundary line) may help students understand that any boundary line splits the coordinate plane into two equal halves. The solution region for a system of inequalities is the intersection of the corresponding half-planes. We use the term solution region instead of *half-plane* in this lesson to be consistent with our language for graphing inequalities in Unit 1, and because the solutions to systems of inequalities exist in a region that is rarely a half-plane.

Unit 4
Lesson
9

Name: ..... Date: ..... Period: .....

MA.912.AR.9.4, MTR.4.1, MTR.5.1, MTR.6.1

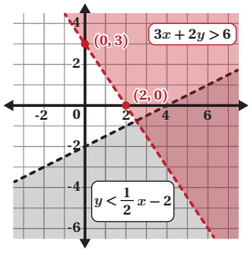
Boundaries and Shading

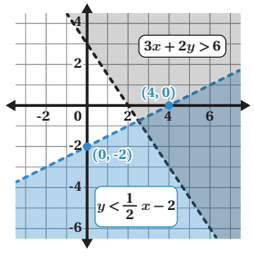


Let's write and graph systems of inequalities.

Warm-Up

The graphs show a system of inequalities. The x- and y-*intercepts* are shown for the boundary lines of two inequalities. Choose one inequality to talk about with a partner.





1. **Discuss:** For the inequality you chose, how can you determine the intercepts of a boundary line without a graph?

   Responses vary. I can determine the x-intercept by plugging $y = 0$ into the related equation and solving for x. I can use the same process to determine the y-intercept if I plug $x = 0$ into the related equation and solve for y.

2. **Discuss:** How can you use the intercepts of the boundary line to graph this inequality?

   Responses vary. I need points to graph the boundary line. If I know both intercepts, I can draw the boundary line through both points and shade on the appropriate side of the boundary line.

Unit 4 Lesson 9 451 Warm-Up

Pairs | 15 min

Activity 1 Rounds of Systems

Purpose: Students make use of structure as they graph systems of inequalities with a partner. **(MTR.5.1)**

1 Launch

Review the instructions.

Distribute the Activity 1 cards to each pair.

Consider asking:

- "Which of these inequalities are in *slope-intercept form?* Which are in *standard form*?" **(MTR.5.1)**
- "Which inequalities will have solid boundary lines? Dashed boundary lines?" **(MTR.6.1)**
- "Which might you graph using the intercepts?"

2 Monitor

A **Accessibility: Memory and Attention** Consider displaying or giving students a reference sheet containing inequality symbols, their meanings, and how they can look on a graph.

Encourage students to use different coloring tools/ways of shading to make their solution regions clear. Invite students with difficulties distinguishing differences in color to use different shadings to clearly portray the solution region.

Invite students who have graphed their first inequality to discuss strategies with their partner, and work together to revise mistakes. For the second round, encourage students to choose an inequality to graph with a different strategy. Invite students to check their graphs are the same.

Consider asking:

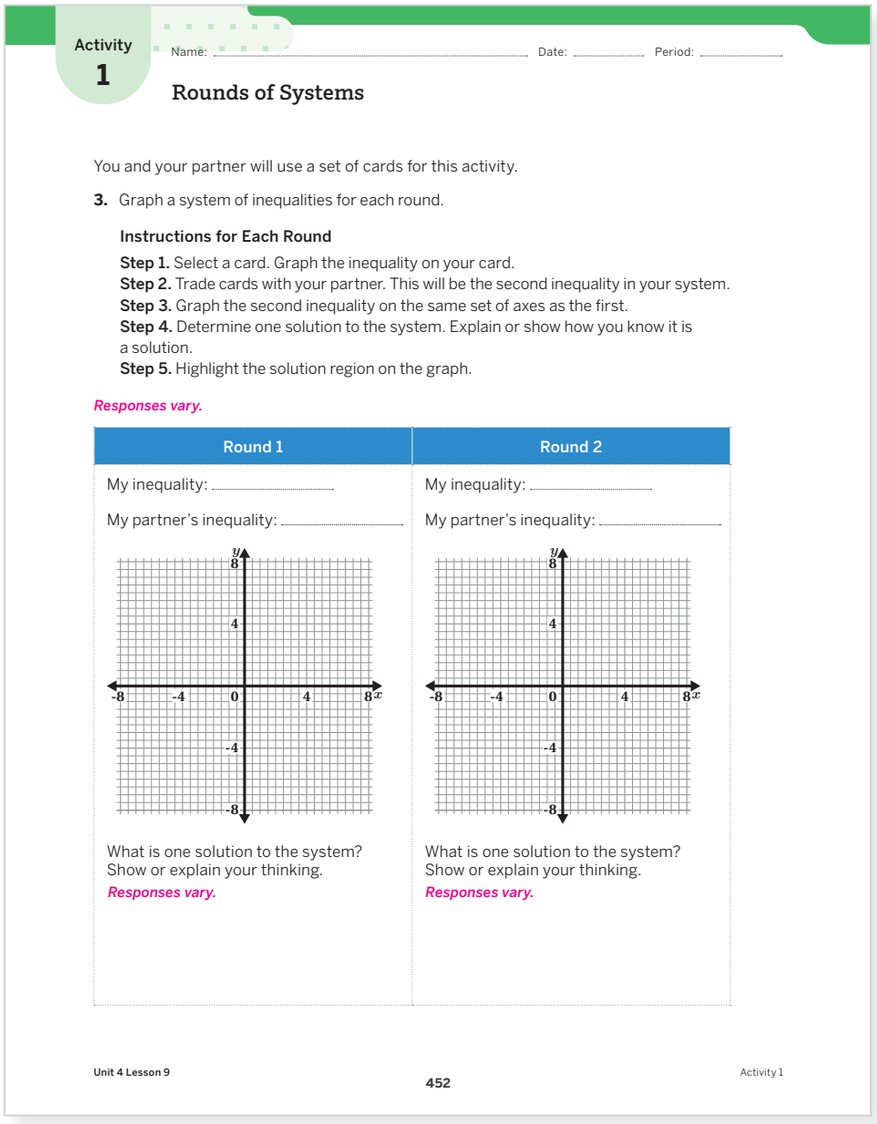
- "How does the inequality indicate whether the boundary line should be dashed or solid?"
- "How did you decide which region to shade?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can we determine a point that we know will be on the boundary line?"
Are not sure which region to shade.	**Support** Invite them to use the strategy of testing a point to determine whether it makes the inequalities true or false.

3 Connect

To surface the Key Takeaway, consider asking, "What strategies did you use to graph inequalities?"



Activity 1

Name: _____ Date: _____ Period: _____

Rounds of Systems

You and your partner will use a set of cards for this activity.

3. Graph a system of inequalities for each round.

Instructions for Each Round

Step 1. Select a card. Graph the inequality on your card.
Step 2. Trade cards with your partner. This will be the second inequality in your system.
Step 3. Graph the second inequality on the same set of axes as the first.
Step 4. Determine one solution to the system. Explain or show how you know it is a solution.
Step 5. Highlight the solution region on the graph.

Responses vary.

Round 1	Round 2
My inequality: _____	My inequality: _____
My partner's inequality: _____	My partner's inequality: _____
What is one solution to the system? Show or explain your thinking. *Responses vary.*	What is one solution to the system? Show or explain your thinking. *Responses vary.*

Unit 4 Lesson 9 452 Activity 1

Key Takeaway: There are many helpful strategies for graphing a system of inequalities and determining its solution region. Some strategies, like graphing using the intercepts, make use of the structure of an equation, such as whether it is in *standard form* or *slope-intercept form*. **(MTR.5.1)**

Pairs | 15 min

Activity 2 Graphing Systems

Purpose: Students create graphs of systems of inequalities and write systems of inequalities when given graphs.

Short on time? Consider inviting pairs of students to choose fewer than four problems to complete.

1 Launch

Invite students to browse all six problems and come to a consensus with a partner about which four problems they will solve together.

Consider asking, "How are these systems different from each other? How are they similar?"

A **Accessibility: Executive Functioning** Consider chunking this activity into more manageable parts by inviting students to select one inequality at a time and identify key features of the boundary line before creating the graph or writing the inequality.

2 Monitor

A **Accessibility: Executive Functioning** Consider asking, "What do you think a first step could be?"
I could pick one boundary line and try to write its equation in slope-intercept form.

Note: Consider encouraging students to use a straightedge or other tool to make their graphs as accurate as possible.

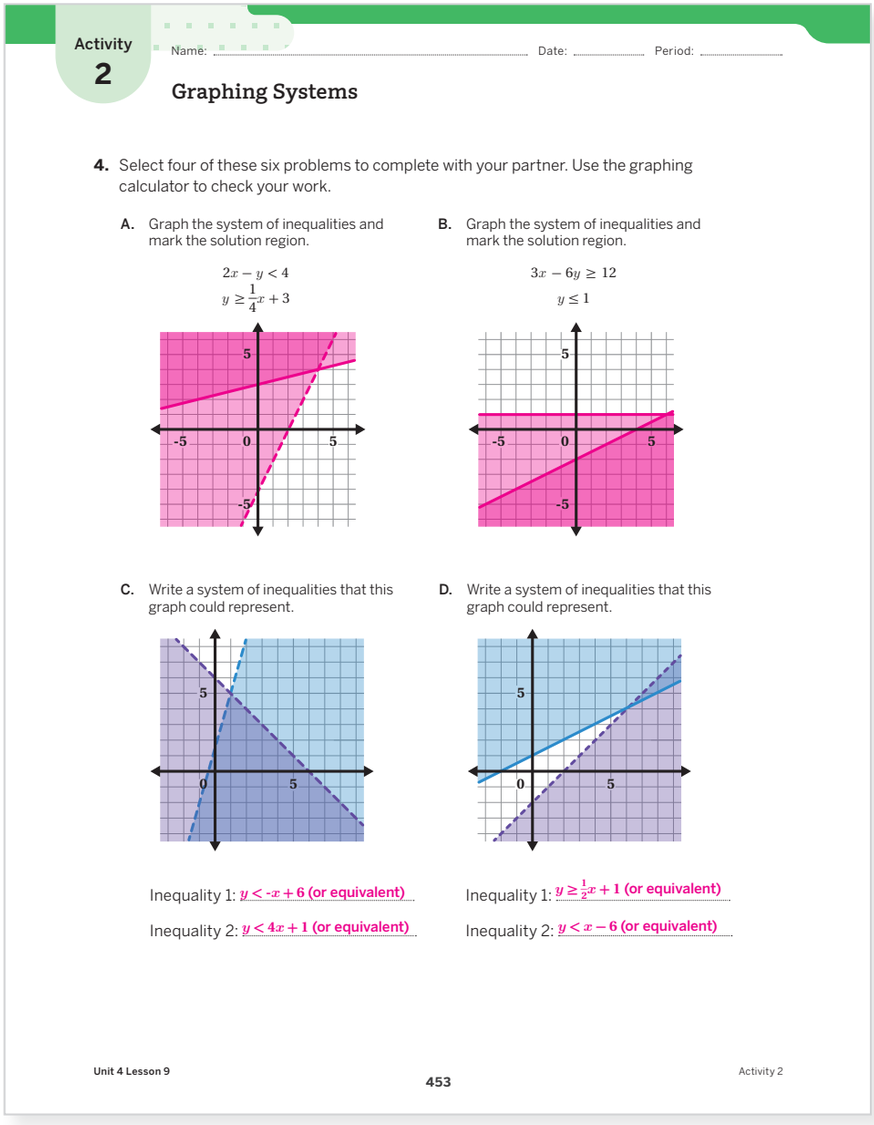
Encourage students to check in with another pair to discuss which problems they chose, their strategies for solving them, and work together to revise any mistakes.

Look for student systems of inequalities to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Do you want to write the inequality in *slope-intercept* or *standard form*? What information do you need to write the inequality in that form? What inequality symbol do you need to write the inequality?" **(MTR.5.1, MTR.6.1)**
Write all or most of their inequalities in the same form.	**Strengthen** Celebrate their work. Choose one of the systems they graphed correctly and consider asking: • "What about this form do you prefer over the other?" • "How would you write one or both of these inequalities in another form?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Graphing Systems

4. Select four of these six problems to complete with your partner. Use the graphing calculator to check your work.

A. Graph the system of inequalities and mark the solution region.

$2x - y < 4$

$y \geq \frac{1}{4}x + 3$

B. Graph the system of inequalities and mark the solution region.

$3x - 6y \geq 12$

$y \leq 1$

C. Write a system of inequalities that this graph could represent.

Inequality 1: $y < -x + 6$ (or equivalent)

Inequality 2: $y < 4x + 1$ (or equivalent)

D. Write a system of inequalities that this graph could represent.

Inequality 1: $y \geq \frac{1}{2}x + 1$ (or equivalent)

Inequality 2: $y < x - 6$ (or equivalent)

Unit 4 Lesson 9 453 Activity 2

Pairs | 15 min

Activity 2 Graphing Systems (continued)

Purpose: Students create graphs of systems of inequalities and write systems of inequalities when given graphs.

Short on time? Consider inviting pairs of students to choose fewer than four problems to complete.

2 Monitor

ELL **English Language Learners** Invite students to ensure that both students in the pair can explain their strategy for writing a system of inequalities. Vary who is called on to represent the work of the group and encourage students to plan out how they might explain their thinking to others. **(Writing, Speaking, and Listening)**

3 Connect

Discuss the systems of inequalities students created.

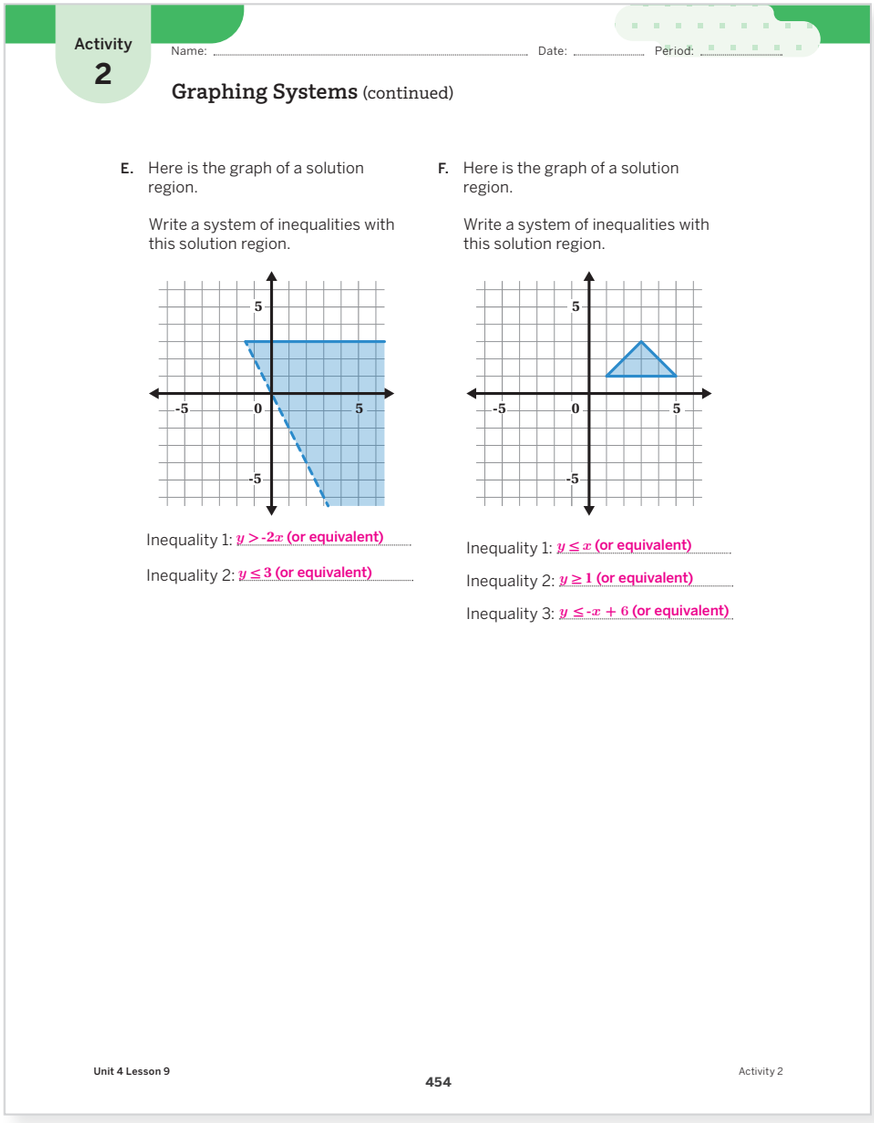
MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in student work from this Activity. If possible, use an anonymous student response from one of the problems. If not, consider using the response $y > -x + 6$ and $y \leq 4x + 1$ for Graph C. Then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or unclear.
c. Write a question to help the student see how their work can be improved, or create a revised response. **(MTR.4.1, MTR.6.1)**

Invite students to share the systems of inequalities they created for problems D, E, and/or F and share their strategies for how they wrote the inequalities.

Consider asking:

- "What strategy did you use to write the equation for the boundary line?"
- "How did you decide which symbol to use in your inequality?" **(MTR.6.1)**
- "How did you decide whether to use slope-intercept or standard form in your inequality?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Graphing Systems (continued)

E. Here is the graph of a solution region.

Write a system of inequalities with this solution region.

Inequality 1: $y > -2x$ (or equivalent)

Inequality 2: $y \leq 3$ (or equivalent)

F. Here is the graph of a solution region.

Write a system of inequalities with this solution region.

Inequality 1: $y \leq x$ (or equivalent)

Inequality 2: $y \geq 1$ (or equivalent)

Inequality 3: $y \leq -x + 6$ (or equivalent)

Unit 4 Lesson 9 454 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for graphing the solution region of a system of inequalities.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- Identifying intercepts to graph a boundary line if an inequality is written in standard form.
- Identifying the slope and y-intercept to graph a boundary line if an inequality is written in slope-intercept form.
- Rewriting an inequality in a form that a student finds more helpful.
- Using the inequality symbols to determine whether the points on the boundary line are included or not included in the solution region.
- Plugging in $(0, 0)$ as a helpful point for determining the solution region.
- Understanding that the solution region is where the shaded regions overlap.
- Understanding that a solution to the system of inequalities is located where the shaded regions for both inequalities overlap.

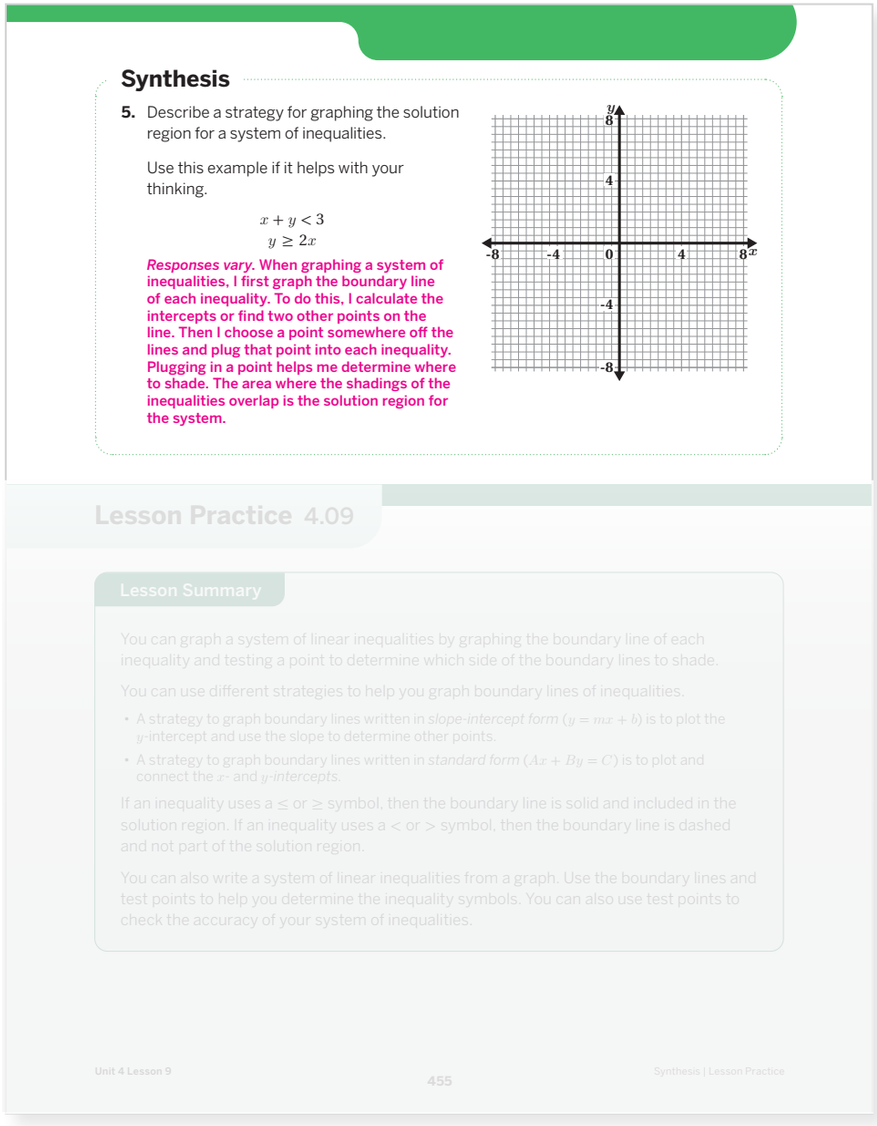
Create a record of students' strategies for graphing the solution region of a system of inequalities.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display for students to reference and add to for the rest of the unit.

Consider asking:

- "What makes sense to you about each strategy? What does not make sense?"
- "What connections do you see between your classmates' strategies?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

5. Describe a strategy for graphing the solution region for a system of inequalities.

Use this example if it helps with your thinking.

$$x + y < 3$$
$$y \geq 2x$$

Responses vary. **When graphing a system of inequalities, I first graph the boundary line of each inequality. To do this, I calculate the intercepts or find two other points on the line. Then I choose a point somewhere off the lines and plug that point into each inequality. Plugging in a point helps me determine where to shade. The area where the shadings of the inequalities overlap is the solution region for the system.**

Lesson Practice 4.09

Lesson Summary

You can graph a system of linear inequalities by graphing the boundary line of each inequality and testing a point to determine which side of the boundary lines to shade.

You can use different strategies to help you graph boundary lines of inequalities.

- A strategy to graph boundary lines written in *slope-intercept form* ($y = mx + b$) is to plot the y-intercept and use the slope to determine other points.
- A strategy to graph boundary lines written in *standard form* ($Ax + By = C$) is to plot and connect the x- and y-*intercepts*.

If an inequality uses a $\leq$ or $\geq$ symbol, then the boundary line is solid and included in the solution region. If an inequality uses a $<$ or $>$ symbol, then the boundary line is dashed and not part of the solution region.

You can also write a system of linear inequalities from a graph. Use the boundary lines and test points to help you determine the inequality symbols. You can also use test points to check the accuracy of your system of inequalities.

Unit 4 Lesson 9 455 Synthesis | Lesson Practice

Lesson Takeaway: There are many helpful strategies for graphing a system of inequalities and determining its solution region. There are also many helpful strategies for writing a system of inequalities when given a solution region. In either case, it is important to attend to precision when determining the type of boundary line and where to shade. **(MTR.6.1)**

Independent | 5 min

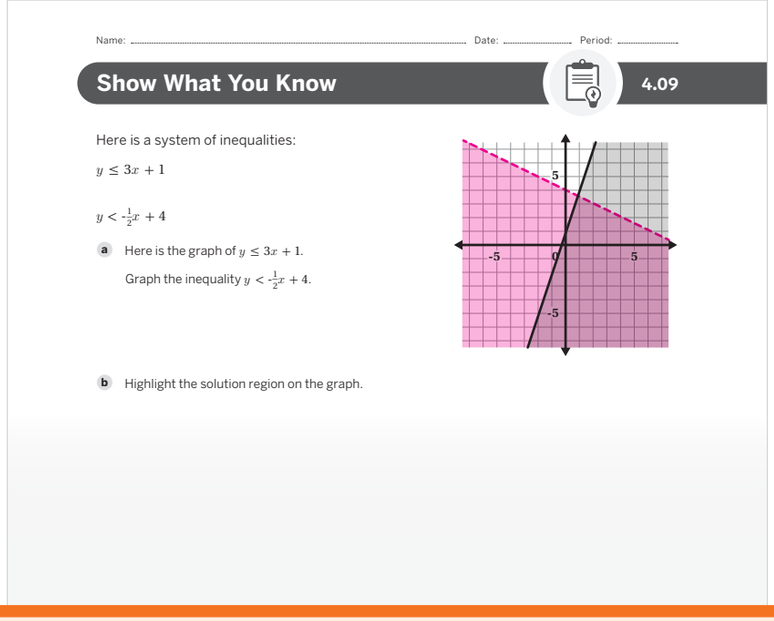
Show What You Know

Purpose: Students demonstrate their understanding by completing the graph of a system of inequalities and highlighting the solution region.

Today's Goals

1. **Goal:** Write a system of inequalities when given a graph.
2. **Goal:** Graph the solution region to a system of inequalities.
3. **Language Goal:** Describe a strategy for graphing the solution region to a system of inequalities. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.09

Here is a system of inequalities:

$y \leq 3x + 1$

$y < -\frac{1}{2}x + 4$

a. Here is the graph of $y \leq 3x + 1$.
Graph the inequality $y < -\frac{1}{2}x + 4$.

b. Highlight the solution region on the graph.

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write and graph systems of inequalities:

» **Respond to Student Thinking:**

- Use the **Graphing Solutions to Systems of Inequalities Mini-Lesson**.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create additional shapes by graphing a system of inequalities.

Professional Learning

In today's lesson, students spent extended time working with a partner to engage in complex math tasks involving systems of linear inequalities. Take a moment to reflect on students' collaboration. Who participated and who did not participate in the work of graphing and writing systems of linear inequalities during Activities 1 and 2 today? What overall trends do you see in participation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

5. Describe a strategy for graphing the solution region for a system of inequalities.

Use this example if it helps with your thinking.

$$x + y < 3$$
$$y \geq 2x$$

Responses vary. When graphing a system of inequalities, I first graph the boundary line of each inequality. To do this, I calculate the intercepts or find two other points on the line. Then I choose a point somewhere off the lines and plug that point into each inequality. Plugging in a point helps me determine where to shade. The area where the shadings of the inequalities overlap is the solution region for the system.

Lesson Practice 4.09

Lesson Summary

You can graph a system of linear inequalities by graphing the boundary line of each inequality and testing a point to determine which side of the boundary lines to shade.

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If an inequality uses a $\leq$ or $\geq$ symbol, then the boundary line is solid and included in the solution region. If an inequality uses a $<$ or $>$ symbol, then the boundary line is dashed and not part of the solution region.

You can also write a system of linear inequalities from a graph. Use the boundary lines and test points to help you determine the inequality symbols. You can also use test points to check the accuracy of your system of inequalities.

Lesson Practice 4.09

Name: _____ Date: _____ Period: _____

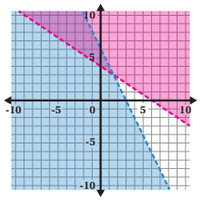
Problems 1–3: The first inequality of this system of inequalities is graphed below.

$$5y < 30 - 10x$$
$$2x + 3y > 12$$

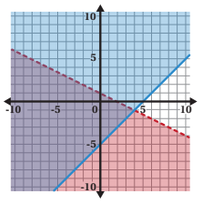
1. Graph the second inequality.

2. Write a point that is *not* a solution to this system.
Responses vary. (5, -1)

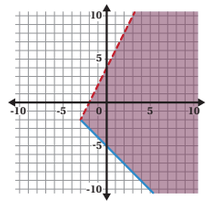
3. Write a point that *is* a solution to this system.
Responses vary. (0, 5)



4. Write a system of inequalities to represent this graph.
$y \geq x - 5$ (or equivalent)
$y < -\frac{1}{2}x + 1$ (or equivalent)



5. Here is the graph of a solution region. Write a system of inequalities that has this solution region.
$y < 2x + 4$ (or equivalent)
$y \geq -x - 5$ (or equivalent)



Lesson Practice 4.09

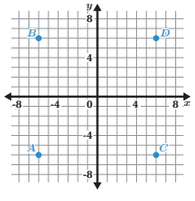
Name: _____ Date: _____ Period: _____

6. Using the digits 0–9 without repeating, fill in each blank such that all of the following statements are true.

- A is a solution to both inequalities.
- B is a solution to only one inequality.
- C is a solution to only the other inequality.
- D is not a solution to either inequality.

Responses vary.

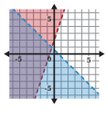
$$\boxed{3}x + \boxed{1}y < \boxed{4}$$
$$\boxed{2}x + \boxed{8}y < \boxed{0}$$

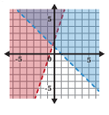


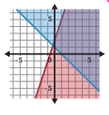
Test Practice

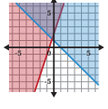
7. Which graph matches this system of inequalities?

$$-3x + y \geq 2$$
$$2x + 2y \geq 2$$

A. 

B. 

C. 

D. 

Spiral Review

Problems 8–10: Consider the line $f(x) = 3x - 4$ and the point $(5, c)$. Write a value for c so that the point is: *Responses vary.*

8. On the line. $c =$ 11

9. Above the line. $c =$ 15

10. Below the line. $c =$ 6

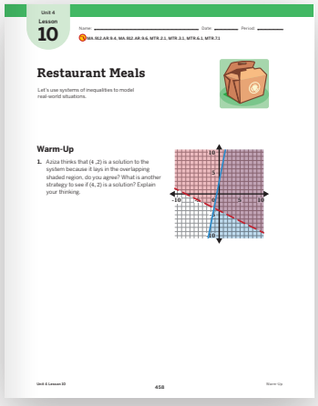
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.AR.9.4
	4–5	2	MA.912.AR.9.4
	6	3	MA.912.AR.9.4
Test Practice	7	1	MA.912.AR.9.4
Spiral Review			
Fluency	8–10	2	MA.912.AR.2.2

Unit 4

Lesson

10



Restaurant Meals

Using Systems of Inequalities to Make Decisions

Let's use systems of inequalities to model real-world situations.

Learning Outcomes

Today's Goals

1. **Goal:** Write systems of inequalities to represent constraints in context.
2. **Language Goal:** Interpret the solutions to a system of inequalities in context in order to make decisions. **(Reading, Writing, Speaking, Listening)**

Students use systems of inequalities representing real-world *constraints* to make sense of a situation: preparing meals for a restaurant. Students use mathematics as a tool to help them make decisions as they consider the advantages and disadvantages of different solutions to the system of inequalities they write.

Coherence

Prior Learning

In Unit 1, students created, graphed, and solved inequalities in one variable. In Lessons 7–9, students learned how to graph a system of inequalities by hand and with technology, determined the solution region, and interpreted the meaning of a solution in context.

Future Learning

In Algebra 2, students will create and graph systems of nonlinear inequalities in order to solve problems and represent real-world situations.

Connections

- Students use systems of inequalities to model real-world situations in the context of preparing vegetarian meals and non vegetarian meals. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.9.4
Graph the solution set of a system of two-variable linear inequalities.

MA.912.AR.9.6
Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.4	MA.912.NSO.1.1
MA.912.AR.2.5	MA.912.NSO.1.2
	MA.912.NSO.1.3

Vocabulary

Review Vocabulary

constraint

Lesson at a Glance

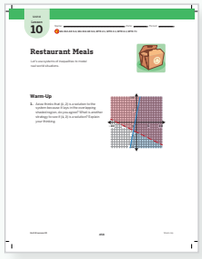
~ 45 min

MA.912.AR.9.4, MA.912.AR.9.6

Warm-Up

Pairs | 10 min

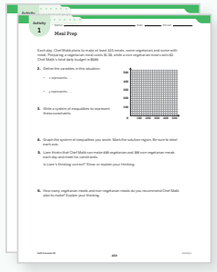
Students make sense of the lesson's context: making decisions about preparing meals for a restaurant based on real-world *constraints*.
Routine: Decide and Defend **(MTR.6.1)**



Activity 1

Pairs | 10 min

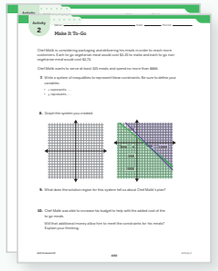
Students create and graph a system of inequalities, determine the solution region, and consider how many of each kind of meal Chef Malik can make for his customers.



Activity 2

Pairs | 10 min

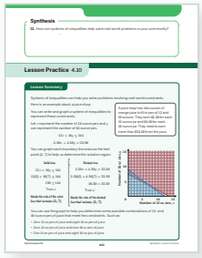
Students solve a system of inequalities for a new set of constraints, using math as a tool to analyze the system in order to serve more customers.



Synthesis

Whole Class | 5 min

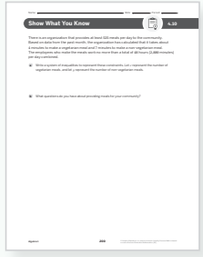
Students reflect on how systems of inequalities can be used as a mathematical tool to solve problems with real-world constraints.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing a system of inequalities to represent constraints given in context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 10 min

Warm-Up

Purpose: Students make sense of the lesson's context: making decisions about preparing meals for customers based on real-world *constraints*. **(MTR.6.1)**

1 Launch

Display the graph and have students think to themselves. Then, share distribution of responses by polling the class.

To surface connections, invite students to share how they felt about previous lessons with graphing inequalities or linear equations.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

Consider asking:

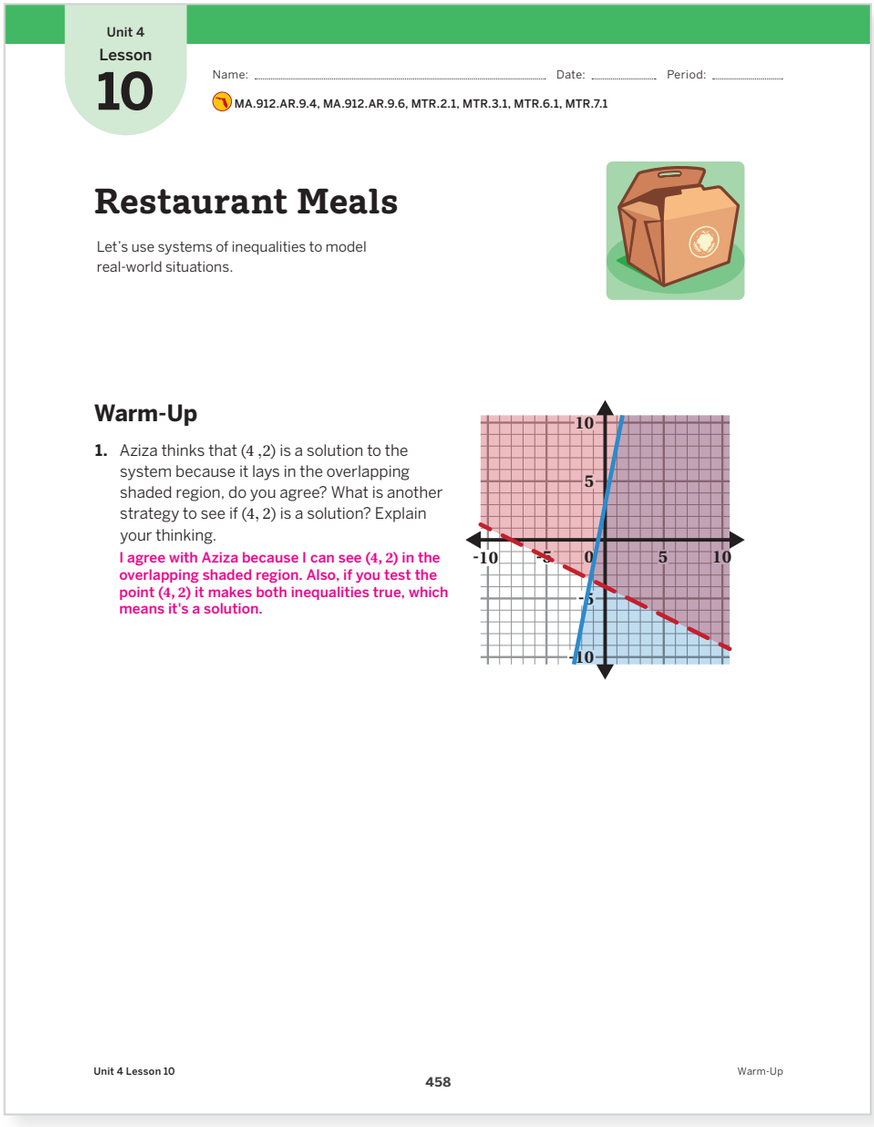
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"
- "What point is not a solution and how do you know?"

ELL **English Language Learners** Invite students to read [each claim, the claim] aloud and paraphrase what it means. Clarify any unknown words. **(Reading)**

2 Connect

Consider asking:

- "What might Chef Malik consider about his customers as he makes sense of this situation?" The community is largely vegetarian. **(MTR.3.1)**
- "What does the word *constraint* mean to you?"
- "What are the different constraints that Chef Malik has to think about in order to prepare meals for his customers?"
- "Why is this a system of inequalities and not a system of equations?" **(MTR.3.1)**



Unit 4
Lesson
10

Name: Date: Period:

MA.912.AR.9.4, MA.912.AR.9.6, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Restaurant Meals

Let's use systems of inequalities to model real-world situations.

Warm-Up

1. Aziza thinks that (4 ,2) is a solution to the system because it lays in the overlapping shaded region, do you agree? What is another strategy to see if (4, 2) is a solution? Explain your thinking.
   I agree with Aziza because I can see (4, 2) in the overlapping shaded region. Also, if you test the point (4, 2) it makes both inequalities true, which means it's a solution.

Unit 4 Lesson 10 458 Warm-Up

Pairs | 10 min

Activity 1 Meal Prep

Purpose: Students create and graph a system of inequalities, determine the solution region, and consider how many of each kind of meal Chef Malik can make for his restaurant.

1 Launch

Read the text aloud.

Consider asking, "Why might Chef Malik want to prepare both vegetarian and nonvegetarian meals?"

2 Monitor

Encourage students to label their axes according to how they defined their variables, and to check if the scale for the graph makes sense for the situation.

Note: Students may have different inequalities, and therefore also different graphs, depending on how they define their variables.

Note: Encourage students to use coloring tools as they graph.

D Differentiation

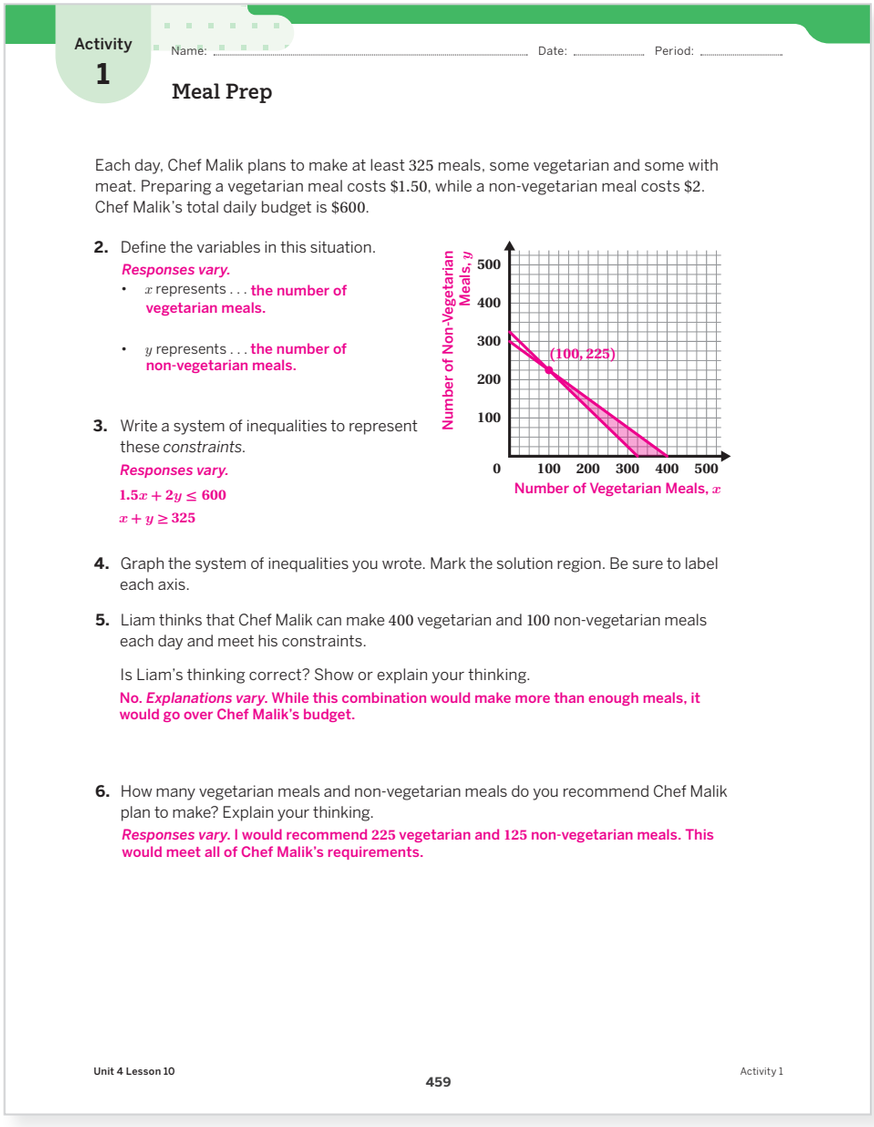
Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 3)*	**Support** Encourage them to organize the information into two inequalities. Consider asking, "How did you decide to define your variables? How did you organize the information to create two inequalities?"
Need support labeling and scaling their axes. *(Problem 4)*	**Support** Consider asking, "How can we use the total budget and Chef Malik's goal to choose a scale?"
Graph their inequality by: • Using intercepts. • Changing from standard to slope-intercept form. *(Problem 4)*	**Strengthen** Discuss the strategy each student chose. Invite these students to share their strategy during the Connect.

3 Connect

Invite students to share their recommendations for Chef Malik and their strategies for graphing the system.

To surface the Key Takeaway, consider asking:

- "The points $(325, 0)$ and $(0, 400)$ are within the solution region. Why might Chef Malik not choose these combinations?"
- "What do you think is the best option for Chef Malik?



Activity 1

Name: Date: Period:

Meal Prep

Each day, Chef Malik plans to make at least 325 meals, some vegetarian and some with meat. Preparing a vegetarian meal costs \$1.50, while a non-vegetarian meal costs \$2. Chef Malik's total daily budget is \$600.

2. Define the variables in this situation.
   Responses vary.
   - x represents . . . **the number of vegetarian meals.**
   - y represents . . . **the number of non-vegetarian meals.**

3. Write a system of inequalities to represent these *constraints*.
   Responses vary.
   $1.5x + 2y \leq 600$
   $x + y \geq 325$

4. Graph the system of inequalities you wrote. Mark the solution region. Be sure to label each axis.

5. Liam thinks that Chef Malik can make 400 vegetarian and 100 non-vegetarian meals each day and meet his constraints.
   Is Liam's thinking correct? Show or explain your thinking.
   No. *Explanations vary.* While this combination would make more than enough meals, it would go over Chef Malik's budget.

6. How many vegetarian meals and non-vegetarian meals do you recommend Chef Malik plan to make? Explain your thinking.
   ***Responses vary.* I would recommend 225 vegetarian and 125 non-vegetarian meals. This would meet all of Chef Malik's requirements.**

Unit 4 Lesson 10 459 Activity 1

Key Takeaway: Mathematics is a helpful, powerful tool that can be used to help make decisions and consider the advantages and disadvantages of different options. Using mathematics to model *constraints* in a system of inequalities supports us in considering different solutions to the system, and their impact on people.

Pairs | 10 min

Activity 2 Make It To-Go

Purpose: Students solve a system of inequalities for a new set of constraints, using math as a tool to analyze the system in order to serve more customers.

1 Launch

Read the new situation aloud.

Consider asking, "Why does changing the packaging and delivery impact the cost of the meals?" Chef Malik will need to pay for the to-go packages and the delivery cost.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 7)*	**Support** Encourage them to define their variables. Consider asking, "What are the two quantities being measured? What quantities do we already know?" **(MTR.7.1)**
Need support labeling and scaling their axes. *(Problem 8)*	**Support** Encourage students to use the units in the situation to support them in defining their variables and choosing a scale (e.g., whole number meals, maximum budget of $600). **(MTR.3.1)**
Need support graphing inequalities. *(Problem 8)*	**Support** Encourage students to check their work with their partner.

3 Connect

Consider asking:

- "Why are there no solutions for Chef Malik's plan?"
- "How does seeing the graph help you make a decision about what to recommend to Chef Malik?"

Activity 2

Name: Date: Period:

Make It To-Go

Chef Malik is considering packaging and delivering his meals in order to reach more customers. Each to-go vegetarian meal would cost $2.25 to make and each to-go non-vegetarian meal would cost $2.75.

Chef Malik wants to serve at least 325 meals and spend no more than $600.

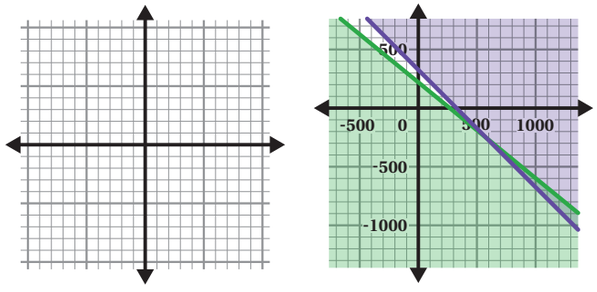
7. Write a system of inequalities to represent these constraints. Be sure to define your variables.

- x represents . . . **the number of vegetarian meals.**
- y represents . . . **the number of non-vegetarian meals.**

$2.25x + 2.75y \leq 600$

$x + y \geq 325$

8. Graph the system you created.



9. What does the solution region for this system tell us about Chef Malik's plan?

Responses vary. **There is no combination of vegetarian and non-vegetarian meals that Chef Malik can make with these constraints.**

10. Chef Malik was able to increase his budget to help with the added cost of the to-go meals.

Will that additional money allow him to meet the constraints for his meals? Explain your thinking.

Responses vary. **I changed the first inequality in my system to $2.25x + 2.75y \leq 800$ to represent this new situation. When I graphed this system, I had an overlapping solution region. So any point in that region, like (290, 40), would represent a combination of to-go meals that meets Chef Malik's constraints.**

Unit 4 Lesson 10 460 Activity 2

Key Takeaway: Real-world constraints like budget and the way meals are packaged can greatly impact the possible solutions to a system of inequalities. Changing one constraint within the system will change the set of possible solutions for that system.

Whole Class | 5 min

Synthesis

Purpose: Students reflect on how systems of inequalities can be used as a mathematical tool to solve problems with real-world constraints.

Invite students to respond independently, and then share their thinking with a partner.

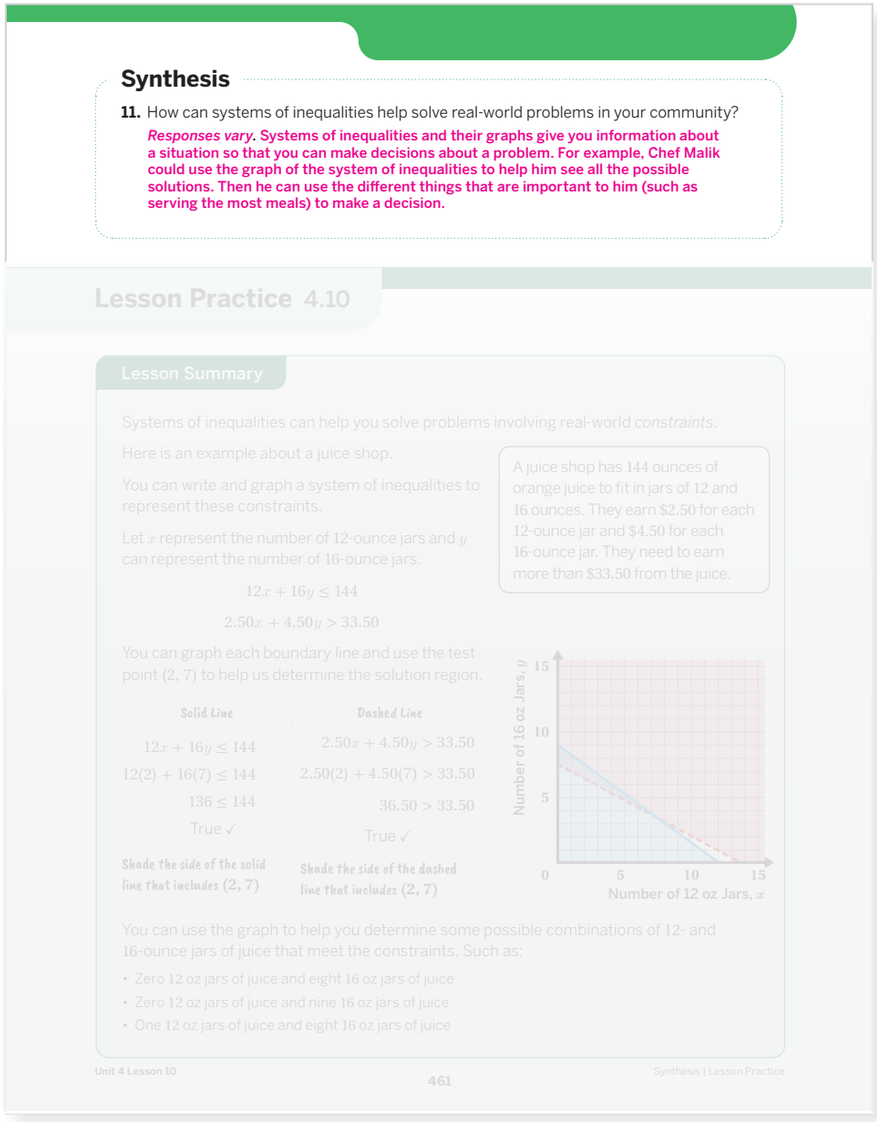
Collect and share a variety of ideas, such as:

- How changing one constraint impacts the whole system (e.g., total cost impacts number of meals and price per meal).
- How situations can help us think about the scale of a problem. **(MTR.5.1)**
- Systems of inequalities are a mathematical tool we can use to represent constraints.
- There is more than one decision that can be made when considering a system of inequalities.
- Sometimes there are mathematical reasons for not choosing an option (e.g., the option does not satisfy the constraints) and sometimes there are nonmathematical reasons (e.g., Chef Malik might not want to make only vegetarian or only nonvegetarian meals).

Consider asking, "How can we use mathematics as a tool to provide information that helps us make decisions?"

ELL **English Language Learners** As students process their ideas in words, invite them to use a specific problem from Activity 1 or 2 to help them express their ideas.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. How can systems of inequalities help solve real-world problems in your community?

Responses vary. Systems of inequalities and their graphs give you information about a situation so that you can make decisions about a problem. For example, Chef Malik could use the graph of the system of inequalities to help him see all the possible solutions. Then he can use the different things that are important to him (such as serving the most meals) to make a decision.

Lesson Practice 4.10

Lesson Summary

Systems of inequalities can help you solve problems involving real-world *constraints*.

Here is an example about a juice shop.

You can write and graph a system of inequalities to represent these constraints.

Let x represent the number of 12-ounce jars and y can represent the number of 16-ounce jars.

$$12x + 16y \leq 144$$

$$2.50x + 4.50y > 33.50$$

A juice shop has 144 ounces of orange juice to fit in jars of 12 and 16 ounces. They earn \$2.50 for each 12-ounce jar and \$4.50 for each 16-ounce jar. They need to earn more than \$33.50 from the juice.

You can graph each boundary line and use the test point (2, 7) to help us determine the solution region.

Solid Line	Dashed Line
$12x + 16y \leq 144$	$2.50x + 4.50y > 33.50$
$12(2) + 16(7) \leq 144$	$2.50(2) + 4.50(7) > 33.50$
$136 \leq 144$	$36.50 > 33.50$
True ✓	True ✓
Shade the side of the solid line that includes (2, 7)	Shade the side of the dashed line that includes (2, 7)

You can use the graph to help you determine some possible combinations of 12- and 16-ounce jars of juice that meet the constraints. Such as:

- Zero 12 oz jars of juice and eight 16 oz jars of juice
- Zero 12 oz jars of juice and nine 16 oz jars of juice
- One 12 oz jars of juice and eight 16 oz jars of juice

Unit 4 Lesson 10 461 Synthesis | Lesson Practice

Lesson Takeaway: Systems of inequalities can be used to represent real-world *constraints* to make sense of real issues in our world.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing a system of inequalities to represent constraints given in context.

Today's Goals

1. **Goal:** Write systems of inequalities to represent constraints in context.
2. **Language Goal:** Interpret the solutions to a system of inequalities in context in order to make decisions. **(Reading, Writing, Speaking, Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.10

There is an organization that provides at least 525 meals per day to the community. Based on data from the past month, the organization has calculated that it takes about 4 minutes to make a vegetarian meal and 7 minutes to make a non-vegetarian meal. The employees who make the meals work no more than a total of 48 hours (2,880 minutes) per day combined.

a. Write a system of inequalities to represent these constraints. Let x represent the number of vegetarian meals, and let y represent the number of non-vegetarian meals.

$x + y \geq 525$

$4x + 7y \leq 2880$

b. What questions do you have about providing meals for your community?

Responses vary.

- I wonder if there are organizations in my community that serve free meals.
- Are there ways I can help provide meals in my community?

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of creating, graphing, and solving systems of inequalities to represent constraints:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how students engaged in modeling with mathematics in this lesson. How did students use mathematics as a tool to inform and make decisions? How did the concepts of defining variables, labeling axes, and choosing scale based on the situation support students in engaging with modeling? In what ways did mathematics serve as a tool to support students in making connections between math ideas they learned in this unit? **(MA.912.NSO.1.2, MTR.7.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can systems of inequalities help solve real-world problems in your community?

Responses vary. Systems of inequalities and their graphs give you information about a situation so that you can make decisions about a problem. For example, Chef Malik could use the graph of the system of inequalities to help him see all the possible solutions. Then he can use the different things that are important to him (such as serving the most meals) to make a decision.

Lesson Practice 4.10

Lesson Summary

Systems of inequalities can help you solve problems involving real-world *constraints*.

Here is an example about a juice shop.

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A juice shop has 144 ounces of orange juice to fit in jars of 12 and 16 ounces. They earn $2.50 for each 12-ounce jar and $4.50 for each 16-ounce jar. They need to earn more than $33.50 from the juice.

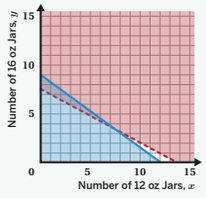
Let x represent the number of 12-ounce jars and y can represent the number of 16-ounce jars.

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You can graph each boundary line and use the test point (2, 7) to help us determine the solution region.

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$136 \leq 144$	$36.50 > 33.50$
True ✓	True ✓
Shade the side of the solid line that includes (2, 7)	Shade the side of the dashed line that includes (2, 7)



You can use the graph to help you determine some possible combinations of 12- and 16-ounce jars of juice that meet the constraints. Such as:

- Zero 12 oz jars of juice and eight 16 oz jars of juice
- Zero 12 oz jars of juice and nine 16 oz jars of juice
- One 12 oz jars of juice and eight 16 oz jars of juice

Lesson Practice 4.10

Name: _____ Date: _____ Period: _____

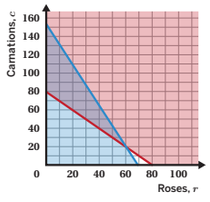
Problems 1–4: Victor has $100 to spend on flowers for a school celebration. Roses, r, cost $1.45 each and carnations, c, cost $0.65 each. Victor wants to buy enough flowers so that all 80 people can take home at least one flower.

1. Select *all* inequalities that represent the constraints in this situation.

- ☐ A. $r + c \leq 80$
- ☑ B. $r + c \geq 80$
- ☐ C. $1.45r + 0.65c \leq 80$
- ☐ D. $1.45r + 0.65c \geq 100$
- ☑ E. $1.45r + 0.65c \leq 100$

2. Victor created a graph using inequalities. What is a combination of carnations and roses that meets the constraints?

Responses vary. 40 roses and 50 carnations



3. What is a combination of carnations and roses that *does not* meet the constraints?

Responses vary. 100 roses and 100 carnations

4. Victor wants to get *the most roses possible* while still meeting the constraints. How many roses and carnations should he get?

60 roses and 20 carnations

Explain your thinking.

Explanations vary. Of all the points within the solution region, this is the point that has the most roses.

Problems 5–6: Cho needs at least 3 pounds of fruit to make a fruit salad for a family gathering. She decides to buy blueberries and apples. She has $12 to spend. Blueberries cost $4.85 per pound and apples cost $1.31 per pound.

- b represents pounds of blueberries.
- a represents pounds of apples.

5. Write a system of inequalities to represent Cho's constraints.

$b + a \geq 3$

$4.85b + 1.31a \leq 12$

Test Practice

6. Can Cho buy 1.5 pounds of blueberries and 2.5 pounds of apples? Circle your choice.

(Yes) No

Lesson Practice 4.10

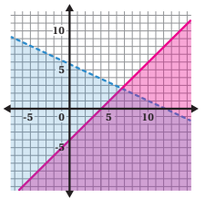
Name: _____ Date: _____ Period: _____

Spiral Review

7. The first inequality of this system of inequalities is graphed. Graph the second inequality.

$x + 2y < 12$

$y \leq x - 4$



Problems 8–9: Solve each system. Write the solution as a coordinate pair.

8. $2x - 3y = 14$
$2x + y = 6$
(4, -2)

9. $y = 9x + 17$
$y = 6 - 2x$
(-1, 8)

Problems 10–12: Evaluate each expression.

10. 25% of 200
50

11. 12% of 200
24

12. 8% of 200
16

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.912.AR.9.6
	2–3	1	MA.912.AR.9.6
	4–5	2	MA.912.AR.9.6
Test Practice	6	1	MA.912.AR.9.6
Spiral Review			
	7	2	MA.912.AR.9.4
	8–9	2	MA.912.AR.9.1
Fluency	10–12	1	MA.6.AR.3.4

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Solve a system of linear equations	Lessons 3–6	2	MA.912.AR.9.1 MTR.2.1
2a–2b	Write a system of equations with no solutions and one with infinitely many solutions	Lesson 4	2	MA.912.AR.9.1 MTR.5.1
3a	Write a system of equations from a context	Lesson 6	2	MA.912.AR.9.1 MTR.6.1
3b	Interpret the meaning of a solution to a system of equations in context	Lesson 5	2	MA.912.AR.9.1 MTR.6.1
4a–4b	Critique a student's reasoning about solving a system of equations	Lesson 2	3	MA.912.AR.9.1 MTR.4.1
4c	Solve a system of linear equations	Lesson 2	2	MA.912.AR.9.1
5a–5b	Determine which points are solutions to a system of linear inequalities	Lessons 7–10	1	MA.912.AR.9.4
6	Represent a system of inequalities graphically	Lesson 9	2	MA.912.AR.9.4 MTR.3.1
7a	Identify a solution to a system of inequalities	Lessons 7–10	2	MA.912.AR.9.6
7b	Determine and explain whether a set of values is a solution to a system of inequalities	Lessons 7 and 10	2	MA.912.AR.9.6 MTR.6.1
7c	Write a system of inequalities from a context	Lessons 9–10	2	MA.912.AR.9.6 MTR.6.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

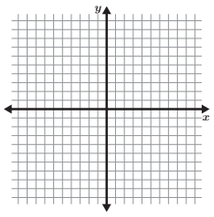
Name: Date: Period:

End-of-Unit Assessment

Unit 4

1. What is the solution to this system of equations?
$2x + y = 8$
$y = 3x - 7$

Use the graph if it helps with your thinking.



A. (1, 6)

B. (-7, 8)

C. (5, 8)

D. (3, 2)

2. One equation in a system is $y = \frac{1}{4}x + 5$. Write another equation to create a system with:

a. No solutions

Responses vary. Slope is equivalent to $\frac{1}{4}$ and y-intercept is not 5.

- $y = \frac{1}{4}x + 3$
- $y = \frac{1}{4}x$

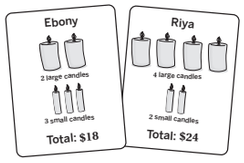
b. Infinitely many solutions

Responses vary. Slope is equivalent to $\frac{1}{4}$ and y-intercept is equal to 5.

- $y = \frac{1}{4}x + 5$
- $4y = x + 20$

3. Ebony and Riya each bought some small and large candles from the same gift shop.

- Use x for the price of each large candle.
- Use y for the price of each small candle.



a. Write a system of equations for this situation.

$2x + 3y = 18$ (or equivalent)

$4x + 2y = 24$ (or equivalent)

b. The solution to a system of equations for this situation is $(4.5, 3)$.
Explain what the solution means in this situation.

Explanations vary. The price of a large candle is $4.50 and a small candle is $3.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

4. Victor made a mistake as he started to solve this system of equations:
$5x + 3y = 12$
$2x + y = 5$

Victor
$5x + 3y = 12$
$-3(2x + y = 5)$
$5x + 3y = 12$
$-6x - 3y = 5$

a. Show or explain why the work is incorrect.

Responses vary. Victor did not multiply -3 to the right side of the equation $2x + y = 5$. His new equation is not equivalent to the original one.

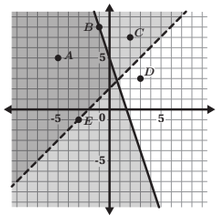
b. Describe one thing that Victor did correctly.

Responses vary. Victor chose to multiply by -3, which will make the y-terms opposites, which will get the system of equations ready for elimination.

c. Solve the system of equations. Write the solution as an ordered pair.

(3, -1)

5. Here is a system of inequalities and its graph.
$y > x + 2$
$y \leq -3x + 5$



a. Select *all* of the labeled points that are in the solution region.

☑ A ☑ B ☐ C ☐ D ☐ E

b. Circle a point that is a solution to only *one* of the inequalities.

A B C D E

Responses vary. C or E

Explain how you know.

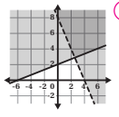
Explanations vary. Point C is only in one shaded region.

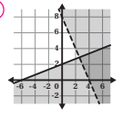
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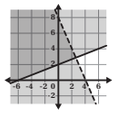
End-of-Unit Assessment (continued)

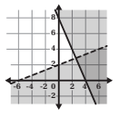
Unit 4

6. Which graph represents this system of inequalities?
$y \leq \frac{1}{3}x + 2$
$4x + 2y > 16$

A. 

B. 

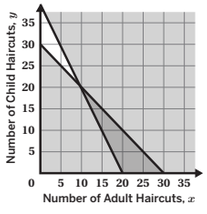
C. 

D. 

7. A barber can do up to 30 haircuts per week.

- He earns $18 for each adult haircut, x.
- He earns $9 for each child haircut, y.

He wants to earn at least $360 this week.



a. Write the coordinates of a point in the solution region.

Responses vary. Coordinate point is in the solution region. (20, 5)

b. Will the barber meet both constraints if he does 5 adult and 25 child haircuts this week?

No

Explain your thinking.

Explanations vary. (5, 25) is not in the solution region for both inequalities.

c. Write a system of inequalities to represent this situation.

$x + y \leq 30$ (or equivalent)

$18x + 9y \geq 360$ (or equivalent)

Name: Date: Period:

End-of-Unit Assessment

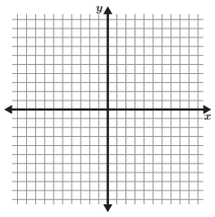
Unit 4

1. What is the solution to this system of equations?

$y = -2x + 1$

$-3x + y = 6$

Use the graph if it helps with your thinking.



A. $(2, 7)$

B. $(1, 9)$

C. $(-1, 3)$

D. $(5, -9)$

2. One equation in a system is $y = -\frac{1}{2}x + 5$. Write another equation to create a system with:

a. Infinitely many solutions

Responses vary. Slope is equivalent to $-\frac{1}{2}$ and y-intercept is equal to 5.

- $y = -\frac{1}{2}x + 5$
- $2y = -x + 10$

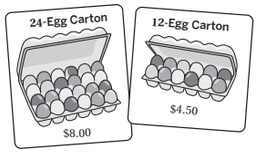
b. No solutions

Responses vary. Slope is equivalent to $-\frac{1}{2}$ and y-intercept is not 5.

- $y = -\frac{1}{2}x + 10$
- $y = -\frac{1}{2}x$

3. A local farm sells 12-egg cartons for \$4.50 and 24-egg cartons for \$8. The farmer has 408 eggs to place in these cartons and hopes to make \$138.

- Use x for the number of 12-egg cartons.
- Use y for the number of 24-egg cartons.



a. Write a system of equations for this situation.

$12x + 24y = 408$ (or equivalent)

$4.5x + 8y = 138$ (or equivalent)

b. The solution to a system of equations for this situation is $(4, 15)$. Explain what the solution means in this situation.

Explanations vary. The farmer needs to package the eggs into four 12-egg cartons and fifteen 24-egg cartons.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

4. Matias made a mistake as he started to solve this system of equations:

$2x - 3y = -6$

$x + 4y = 19$

Matias

$2x - 3y = -6$

$2(x + 4y = 19)$

$2x - 3y = -6$

$2x + 8y = 38$

$5y = 32$

a. Show or explain why the work is incorrect.

Responses vary. Matias subtracted $2x$ from the first equation but did not subtract $8y$ or 38.

b. Describe one thing that Matias did correctly.

Responses vary. Matias chose to multiply by 2, which will make the x-terms the same, which will get the system of equations ready for elimination.

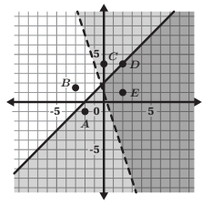
c. Solve the system of equations. Write the solution as an ordered pair.

(3, 4)

5. Here is a system of inequalities and its graph.

$y \leq x + 2$

$y > -3x + 1$



a. Select *all* of the labeled points that are in the solution region.

☐ A ☐ B ☐ C ☑ D ☑ E

b. Circle a point that is *not* a solution to any of the inequalities.

A (B) C D E

Explain how you know.

Explanations vary. Point B is not in any of the shaded regions.

Name: Date: Period:

End-of-Unit Assessment (continued)

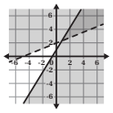
Unit 4

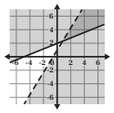
6. Which graph represents this system of inequalities?

$-3x + 2y \leq 2$

$y < \frac{2}{5}x + 2$

A. 

B. 

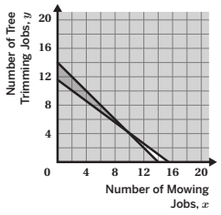
C. 

D. 

7. A landscaper can do up to 14 jobs per week.

- She earns \$60 for each mowing job, x.
- She earns \$80 for each tree-trimming job, y.

She wants to earn at least \$940 this week.



a. Write the coordinates of one point in the solution region.

Responses vary. Coordinate point is in the solution region. (2, 11)

b. Will the landscaper meet both constraints if she does 4 mowing jobs and 9 tree-trimming jobs this week?

Yes

Explain your thinking.

Explanations vary. (4, 9) is in the solution region.

c. Write a system of inequalities to represent this situation.

$60x + 80y \geq 940$ (or equivalent)

$x + y \leq 14$ (or equivalent)

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

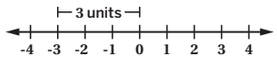
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
• Solve systems of linear equations using elimination, substitution, and graphing. **(Lessons 1–6)**	**1, 4**	**Support**	• **Mini-Lessons:** • **Solving Systems of Linear Equations by Substitution** • **Solving Systems of Linear Equations by Graphing**
		Strengthen	• **Repeated Challenges:** • Lesson 1 (*Eliminating Shapes*) • Lesson 4 (*Lizard Lines*)
		Stretch	• **You're invited to explore more:** • Lesson 2 (*Process of Elimination*)
• Write systems of linear equations to represent constraints or meet given criteria. • Interpret solutions to systems of equations in context. **(Lessons 4–6)**	**2, 3a, 3b**	**Support**	• **Teacher Move:** Consider revisiting: • Lesson 5 (*City Development*) • Lesson 6 (*Bus Systems*)
		Stretch	• **Teacher Move:** Invite students to write systems of three equations that have no solution, one solution, and infinitely many solutions.
• Graph systems of linear inequalities and identify solutions **(Lessons 7–9)**	**5–7a**	**Support**	• **Mini-Lesson: Graphing Solutions to Systems of Inequalities** • **Teacher Move:** Invite students to complete the rest of the problems in **Lesson 9, Activity 2** (*Boundaries and Shading*).
		Strengthen	• **Repeated Challenges:** • Lesson 8 (*All the Solutions*)
• Write systems of linear inequalities to represent constraints or meet given criteria. **(Lessons 9–10)**	**7c**	**Support**	• **Teacher Move:** Consider revisiting: • Lesson 9 (*Boundaries and Shading*) • Lesson 10 (*Restaurant Meals*)
		Stretch	• **Teacher Move:** Invite students to create a design (e.g., heart or star) and write a system of inequalities that could model that design.
• Interpret solutions to systems of inequalities in context. **(Lessons 8 and 10)**	**7b**	**Support**	• **Teacher Move:** Consider reviewing Problem 7b and then invite students to identify a solution to the system and interpret it in context.

Glossary/Glosario

English

A

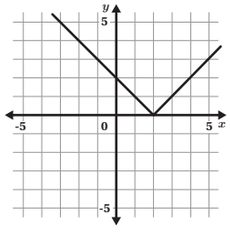
absolute value The absolute value of a number is its distance from 0 on the number line.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

$|4| = 4$ and $|-4| = 4$. They are both 4 units away from 0.

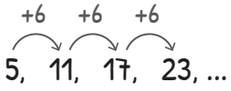
absolute value function A function that is defined using absolute value symbols. When written in the form $f(x) = |x - h|$, its output is the distance of its input from a given value, h.



For example, $f(x) = |x - 2|$ outputs the distance from 2 for every input value.

area The amount of space inside a two-dimensional shape. It is measured in square units, such as square inches or square centimeters.

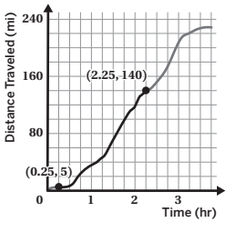
arithmetic sequence A sequence that changes by a constant difference.



In this arithmetic sequence, the first term is 5 and the constant difference is 6.

association If two variables are statistically related to one another, we say that there is an association between the two variables. A *correlation* is a type of association.

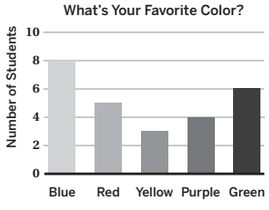
average rate of change A measure of how much a function changes, on average, over an interval. To calculate the average rate of change over an interval, find the slope between the point on the graph of the function where the interval begins and the point where it ends.



To calculate the average rate of change over the interval from 0.25 hours to 2.25 hours, calculate the change in y-values $(140 - 5)$ and divide by the change in x-values $(2.25 - 0.25)$. The average rate of change from 0.25 to 2.25 is 67.5, which means the average speed on that trip was 67.5 miles per hour in that interval.

B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.



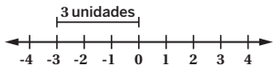
base A number or expression that is raised to an exponent. The exponent describes the number of times to multiply the base by itself.

For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

Español

A

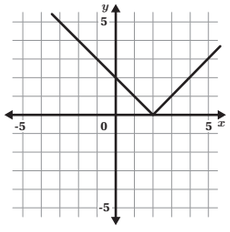
valor absoluto El valor absoluto de un número es su distancia al 0 en la recta numérica.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

$|4| = 4$ y $|-4| = 4$. Ambos están a 4 unidades del 0.

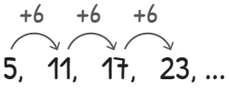
función de valor absoluto Una función que se define utilizando signos de valor absoluto. Cuando se escribe en la forma $f(x) = |x - h|$, su salida es la distancia de su entrada a un valor determinado, h.



Por ejemplo, $f(x) = |x - 2|$ arroja la distancia de cada valor de entrada al 2.

área La cantidad de espacio dentro de una figura bidimensional. Se mide en unidades cuadradas, como pulgadas cuadradas o centímetros cuadrados.

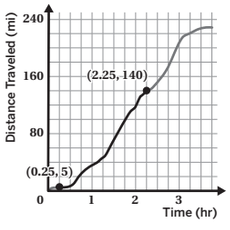
secuencia aritmética Una secuencia que cambia con una diferencia constante.



En esta secuencia aritmética, el primer término es 5 y la diferencia constante es 6.

asociación Si dos variables están relacionadas estadísticamente, decimos que existe una asociación entre ambas variables. Una *correlación* es un tipo de asociación.

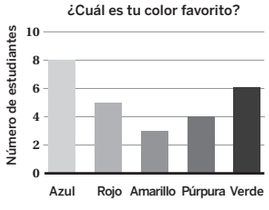
tasa de cambio promedio Una medida de cuánto cambia una función, en promedio, en un intervalo. Para calcular la tasa de cambio promedio a lo largo de un intervalo, se halla la pendiente entre el punto donde empieza el intervalo y el punto donde termina en la gráfica de la función.



Para calcular la tasa de cambio promedio en el intervalo de 0.25 horas a 2.25 horas, se calcula el cambio en los valores de y $(140 - 5)$ y se divide por el cambio en los valores de x $(2.25 - 0.25)$. La tasa de cambio promedio de 0.25 a 2.25 es 67.5, lo que significa que la velocidad media en ese trayecto fue de 67.5 millas por hora en ese intervalo.

B

gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.

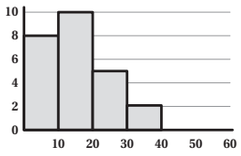


base Un número o una expresión que se eleva a un exponente. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

English

bin (of a histogram) The intervals used to group data values in a histogram are called *bins*. The distance of each interval is called the *bin width*.

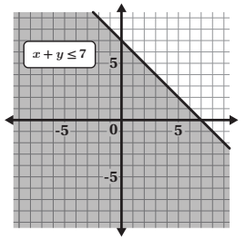


For example, this histogram shows 4 bins, each with a bin width of 10 units.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

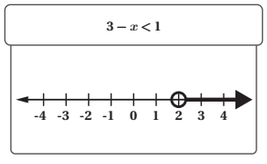
For example, a collection of students' heights and shoe sizes would be a bivariate data set.

boundary line The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \leq x$). A dashed boundary line indicates that they are not (e.g., $y < x$).



The solution region to $x + y \leq 7$ has a boundary line at $x + y = 7$. The line is solid because the points on the line $x + y = 7$ are included in the solution region.

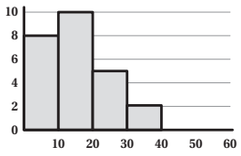
boundary point The value that separates the solution set of an inequality from non-solutions. A solid boundary point indicates that the point is included in the solution set. An empty boundary point indicates the point is not included in the solution set.



The solution set to $3 - x < 1$ has a boundary point at $x = 2$. The point at 2 is empty because 2 is not included in the solution set.

Español

intervalo (de un histograma) Se denominan intervalos a los espacios que se usan para agrupar los valores de datos en un histograma. La distancia de cada intervalo se denomina *ancho del intervalo*.

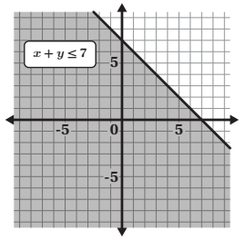


Por ejemplo, este histograma muestra 4 intervalos, cada uno con un ancho de intervalo de 10 unidades.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

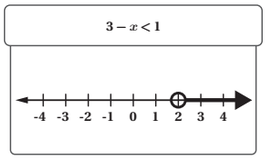
Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

recta límite La línea que separa la región solución de una desigualdad lineal de todos los valores que no son soluciones. Una desigualdad lineal (p. ej., $y < 2x + 5$) tiene una recta límite representada simbólicamente por la ecuación correspondiente (p. ej., $y = 2x + 5$). Una recta límite continua indica que esos puntos están incluidos en el conjunto de soluciones (p. ej., $y \leq x$). Una recta límite discontinua indica que no lo están (p. ej., $y < x$).



La región solución de $x + y \leq 7$ tiene una recta límite en $x + y = 7$. La recta es continua porque los puntos en la recta $x + y = 7$ se encuentran dentro de la región solución.

punto límite El valor que separa el conjunto de soluciones de una desigualdad de todos los valores que no son soluciones. Un punto límite sólido (relleno) indica que el punto está incluido en el conjunto de soluciones. Un punto límite vacío indica que el punto no está incluido en el conjunto de soluciones.



El conjunto de soluciones de $3 - x < 1$ tiene un punto límite en $x = 2$. El punto en 2 no está relleno porque 2 no está incluido en el conjunto de soluciones.

C

categorical data A type of data that is divided into groups or categories. Categorical data describes qualities or characteristics, such as colors, names, or zip codes, rather than numerical values.

For example, the breeds of 10 dogs are categorical data. Another example is the colors of 100 flowers.

causation Describes a relationship between two variables in which a change in one variable causes a change in the other variable. A special type of *correlation*.

datos categóricos Un tipo de datos que se dividen en grupos o categorías. Los datos categóricos describen cualidades o características, como colores, nombres o códigos postales, en vez de valores numéricos.

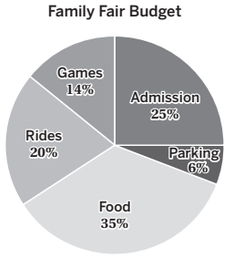
Por ejemplo, las razas de 10 perros diferentes son datos categóricos. Otro ejemplo son los colores de 100 flores diferentes.

causalidad Describe una relación entre dos variables en la que un cambio en una variable provoca un cambio en la otra variable. Un tipo especial de *correlación*.

Glossary/Glosario

English

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a *pie chart*.



coefficient The number that multiplies a variable in an algebraic expression. If no number is shown, the coefficient is understood to be 1.

For example, in the expression $5x$, the coefficient is 5. In y, the coefficient is 1.

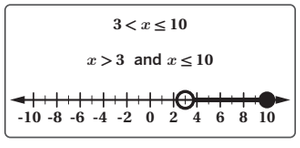
completing the square The process of rewriting a quadratic expression or equation to include a perfect square.

$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

compound To increase in value by applying an operation repeatedly. In mathematics, compounding can involve repeated multiplication or growth over time.

For example, when bacteria multiply by splitting in half every hour, the number of bacteria compounds, doubling each hour.

compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or."

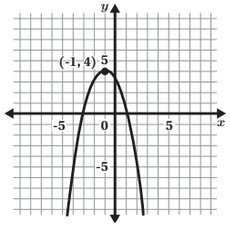


The numbers greater than 3 and less than or equal to 10 can be written as: $x > 3$ and $x \leq 10$ or $3 < x \leq 10$.

compound interest Interest calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.). The balance in an account that earns compound interest can be modeled by an exponential function.

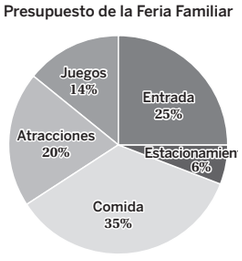
concave down A parabola that opens downward is described as *concave down*. A function with a negative a-value will produce a concave down parabola.

This parabola is concave down. Two ways to write the equation are $f(x) = -1(x + 1)^2 + 4$ and $f(x) = -x^2 - 2x + 3$.



Español

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado *gráfico de torta*.



coeficiente El número que multiplica una variable en una expresión algebraica. Si no se muestra ningún número, se entiende que el coeficiente es 1.

Por ejemplo, en la expresión $5x$, el coeficiente es 5. En y, el coeficiente es 1.

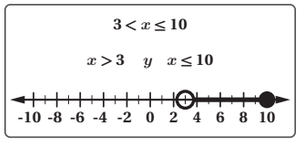
completación del cuadrado El proceso de reescribir una expresión o ecuación cuadrática para incluir un cuadrado perfecto.

$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

crecimiento compuesto Aumentar en valor aplicando una operación repetidamente. En matemáticas, el crecimiento compuesto puede implicar la multiplicación repetida o el crecimiento a lo largo del tiempo.

Por ejemplo, cuando las bacterias se multiplican dividiéndose por la mitad cada hora, el número de bacterias crece de forma compuesta y se duplica cada hora.

desigualdad compuesta Dos o más desigualdades juntas. Una desigualdad compuesta puede escribirse con signos o las palabras "y" u "o".

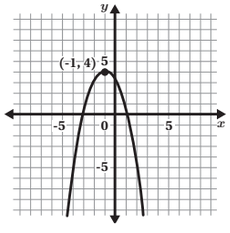


Los números mayores que 3 y menores o iguales que 10 pueden escribirse de la siguiente forma:
$x > 3$ y $x \leq 10$ o $3 < x \leq 10$.

interés compuesto Interés que se calcula según una cantidad inicial y el interés de los períodos anteriores. Se calcula en intervalos regulares (diariamente, mensualmente, anualmente, etc.). El saldo de una cuenta que produce interés compuesto puede modelarse con una función exponencial.

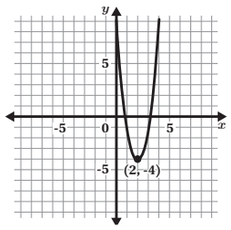
cóncava hacia abajo Una parábola que se abre hacia abajo se describe como cóncava hacia abajo. Una función con un valor a negativo producirá una parábola cóncava hacia abajo.

Esta parábola es cóncava hacia abajo. Dos formas de escribir la ecuación son $f(x) = -1(x + 1)^2 + 4$ y $f(x) = -x^2 - 2x + 3$.



English

concave up A parabola that opens upward is described as *concave up*. A function with a positive a-value will produce a concave up parabola.



This parabola is concave up. Two ways to write the equation are $f(x) = 4(x-2)^2 - 4$ and $f(x) = 4x^2 - 16x + 12$.

confidence interval A range of values used to estimate an unknown population value. It shows how likely it is that the true value is within the range.

A survey estimates that 60% of students prefer online learning, with a confidence interval of 55% to 65%. This means the true percentage is likely between 55% and 65%.

constant difference When the difference between every two consecutive values in a pattern is the same, there is a *constant difference*.

x	y	
0	5	
1	7	+2
2	9	+2
3	11	+2

The pattern in the table has a constant difference of 2.

constant ratio When the ratio between every two consecutive values in a pattern is the same, there is a *constant ratio*.

x	y	
0	1	
1	3	·3
2	9	·3
3	27	·3

The pattern in the table has a constant ratio of 3.

constraint A limitation on the possible values of variables in a model. Equations and inequalities are often used to represent constraints.

The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality $h \geq 36$.

correlation A statistical relationship between two or more variables. Also called an *association*.

cube root The cube root of a number n (written as $\sqrt[3]{n}$) is the number that can be cubed to get n. The cube root is also the edge length of a cube with a volume of n.

For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

data A collection of information, usually in the form of numbers, words, or measurements, that can be analyzed.

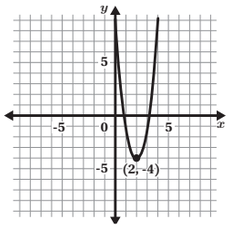
A list of students' test scores is an example of numerical data.

data sets A collection of values gathered for reference, analysis, or interpretation.

A list of test scores collected from a class is an example of data.

Español

cóncava hacia arriba Una parábola que se abre hacia arriba se describe como cóncava hacia arriba. Una función con un valor a positivo producirá una parábola cóncava hacia arriba.



Esta parábola es cóncava hacia arriba. Dos formas de escribir la ecuación son $f(x) = 4(x-2)^2 - 4$ y $f(x) = 4x^2 - 16x + 12$.

intervalo de confianza Un rango de valores utilizado para estimar un valor poblacional desconocido. Muestra la probabilidad de que el valor real esté dentro del rango.

Una encuesta estima que el $60\ \%$ de los estudiantes prefieren el aprendizaje en línea, con un intervalo de confianza del $55\ \%$ al $65\ \%$. Esto significa que el porcentaje real probablemente esté entre el $55\ \%$ y el $65\ \%$.

diferencia constante Cuando la diferencia entre dos valores consecutivos en un patrón permanece igual, hay una diferencia constante.

x	y	
0	5	
1	7	+2
2	9	+2
3	11	+2

El patrón en la tabla tiene una diferencia constante de 2.

razón constante Cuando la razón entre dos valores consecutivos en un patrón permanece igual, hay una razón constante.

x	y	
0	1	
1	3	·3
2	9	·3
3	27	·3

El patrón en la tabla tiene una razón constante de 3.

restricción Una limitación de los posibles valores de las variables en un modelo. Suelen usarse ecuaciones o desigualdades para representar restricciones.

La restricción "debes medir 36 pulgadas o más para subirte a la rueda de la fortuna" puede representarse con la desigualdad $h \geq 36$.

correlación Una relación estadística entre dos o más variables. También se denomina *asociación*.

raíz cúbica La raíz cúbica de un número n (se escribe $\sqrt[3]{n}$) es el número que puede elevarse al cubo para obtener n. La raíz cúbica también es la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de arista de un cubo que tiene un volumen de 64 unidades cúbicas.

D

datos Una recopilación de información, generalmente en forma de números, palabras o medidas, que se puede analizar.

Una lista de las calificaciones de los exámenes de unos estudiantes es un ejemplo de datos numéricos.

conjuntos de datos Una recopilación de valores reunidos para referencia, análisis o interpretación.

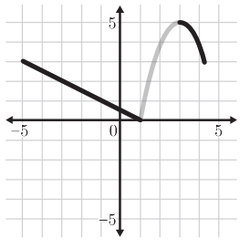
Una lista de las calificaciones de exámenes recopiladas de una clase es un ejemplo de datos.

Glossary/Glosario

English

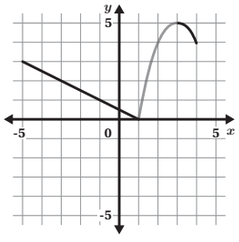
decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.



decreasing (interval or function) A function is decreasing when its outputs decrease as its inputs increase. A function can be decreasing for its entire domain or over an interval.

For example, the function $f(x)$ is decreasing when $-5 < x < 1$ and when $3 < x < 4$.



dependent variable The value of a dependent variable is based on the value of another variable or set of variables. In a function, the value of the dependent variable represents the output. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

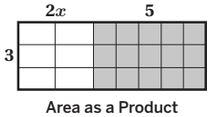
difference of squares An expression that can be written as one perfect square subtracted from another perfect square. It has the structure $r^2 - s^2$ in standard form and $(r - s)(r + s)$ in factored form.

These expressions are differences of squares: $x^2 - 25$, $9x^2 - 100$, and $16x^2 - 1$.

discrete A set of values is discrete if there is separation between the values. A graph can be described as discrete when it consists of unconnected points or intervals.

The set of numbers $1, 2, 3$ is discrete, while the set of all values between 1 and 3 is not discrete.

distributive property Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

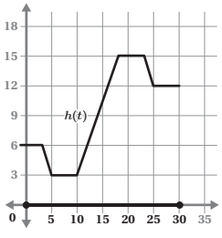


Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.

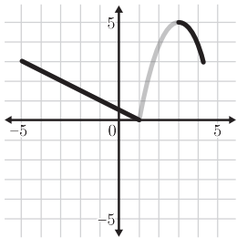
The domain of this graph can be described as: All numbers from 0 to 30 or $0 \leq t \leq 30$.



Español

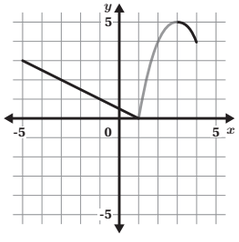
decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

Las partes resaltadas de esta función son decrecientes.



decreciente (intervalo o función) Una función es decreciente cuando sus valores de salida decrecen a medida que crecen sus valores de entrada. Una función puede ser decreciente en todo su dominio o en un intervalo.

Por ejemplo, la función $f(x)$ es decreciente cuando $-5 < x < 1$ y cuando $3 < x < 4$.



variable dependiente El valor de una variable dependiente se basa en el valor de otra variable o un conjunto de variables. En una función, el valor de la variable dependiente representa la salida. La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

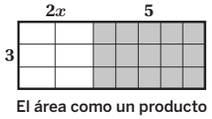
diferencia de cuadrados Una expresión que puede escribirse como un cuadrado perfecto que se resta a otro cuadrado perfecto. Tiene la estructura $r^2 - s^2$ en forma estándar y $(r - s)(r + s)$ en forma factorizada.

Estas expresiones son diferencias de cuadrados: $x^2 - 25$, $9x^2 - 100$ y $16x^2 - 1$.

discreto Un conjunto de valores es discreto si hay separación entre los valores. Una gráfica puede describirse como discreta cuando consta de puntos o intervalos que no se conectan.

El conjunto de números $1, 2, 3$ es discreto, mientras que el conjunto de todos los valores entre 1 y 3 no es discreto.

propiedad distributiva Multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente antes de sumarlos.

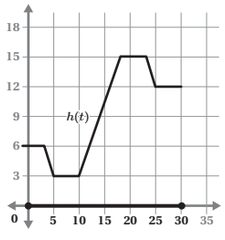


El área como un producto
$3(2x + 5)$

Ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.

El dominio de esta gráfica puede describirse de la siguiente manera:
Todos los números del 0 al 30 o $0 \leq t \leq 30$.



English

E

Español

elimination A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

In the example, subtraction is used to eliminate y and create an equation that can be solved for x.

eliminación Un método para resolver sistemas de ecuaciones en el que se suman o restan las ecuaciones para producir una nueva ecuación con menos variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

En el ejemplo, se usa la resta para eliminar y y producir una ecuación que puede resolverse para determinar el valor de x.

equation Two expressions with an equal sign between them.

$-3(2x + 5) = 5 + x$ is an example of an equation.

ecuación Dos expresiones con un signo igual entre ambas.

$-3(2x + 5) = 5 + x$ es un ejemplo de una ecuación.

equivalent equations Equations that have the same solution(s).

$3x + 4 = 10$ and $9x + 12 = 30$ are equivalent equations because if you multiply the first equation by 3, you create the second. The solution to each equation is $x = 2$.

ecuaciones equivalentes Ecuaciones que tienen exactamente la misma o las mismas soluciones.

$3x + 4 = 10$ y $9x + 12 = 30$ son ecuaciones equivalentes porque si se multiplica la primera por 3, se forma la segunda. La solución de cada ecuación es $x = 2$.

equivalent systems Two or more systems of equations that have the same set of solutions.

sistemas equivalentes Dos o más sistemas de ecuaciones que tienen el mismo conjunto de soluciones.

exponent Exponents describe repeated multiplication. The exponent describes the number of times to multiply the base by itself.

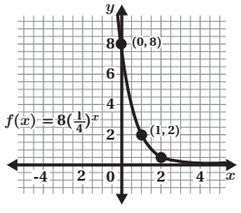
For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

exponente Los exponentes describen una multiplicación repetida. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

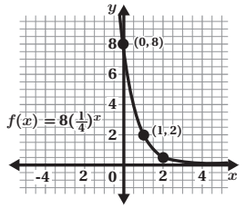
exponential decay An exponential relationship that decreases. An exponential decay relationship has a growth factor between 0 and 1.

For example, $f(x) = 8\left(\frac{1}{4}\right)^x$ is an example of exponential decay because the growth factor, $\frac{1}{4}$, is between 0 and 1.



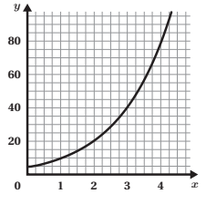
decaimiento exponencial Una relación exponencial que decrece. Una relación de decaimiento exponencial tiene un factor de crecimiento entre 0 y 1.

Por ejemplo, $f(x) = 8\left(\frac{1}{4}\right)^x$ es un ejemplo de decaimiento exponencial porque el factor de crecimiento, $\frac{1}{4}$, está entre 0 y 1.



exponential function A function that increases or decreases by a constant percent rate of change. A function that changes by equal factors over equal intervals.

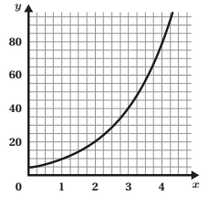
x	$f(x)$
0	5
1	10
2	20
3	40



For example, the table and graph show the exponential function $f(x) = 5 \cdot (2)^x$, which has an initial value of 5 and has a growth factor of 2.

función exponencial Una función que crece o decrece con una tasa porcentual de cambio constante. Una función que cambia por factores iguales en intervalos iguales.

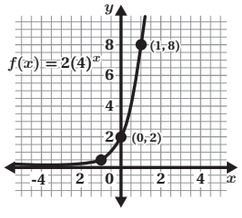
x	$f(x)$
0	5
1	10
2	20
3	40



Por ejemplo, la tabla y la gráfica muestran la función exponencial $f(x) = 5 \cdot (2)^x$, que tiene un valor inicial de 5 y un factor de crecimiento de 2.

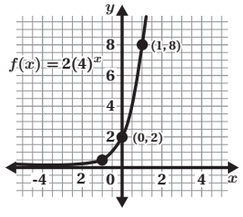
exponential growth An exponential relationship that increases. An exponential growth relationship has a growth factor greater than 1.

For example, $f(x) = 2(4)^x$ is an example of exponential growth because the growth rate, 4, is greater than 1.



crecimiento exponencial Una relación exponencial que crece. Una relación con crecimiento exponencial tiene un factor de crecimiento mayor que 1.

Por ejemplo, $f(x) = 2(4)^x$ es un ejemplo de crecimiento exponencial porque la tasa de crecimiento, 4, es mayor que 1.



exponential relationship See *exponential function*.

relación exponencial Ver *función exponencial*.

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

5, $2x$, $7 - 3^p$, and $4(2 - 5x^2) + 8$ are all examples of expressions.

expresión Un conjunto de números, variables, operaciones y signos de agrupación que representan una cantidad.

5, $2x$, $7 - 3^p$ y $4(2 - 5x^2) + 8$ son ejemplos de expresiones.

Glossary/Glosario

English

Español

F

factor (of a number or expression) A number or expression multiplied with other numbers or expressions to make a product.

For example, 1, 2, 4, and 8 are all factors of the number 8 because $1 \cdot 8 = 8$ and $2 \cdot 4 = 8$. Also, $(x + 3)$ and $(x - 5)$ are factors of $x^2 - 2x - 15$ because $(x + 3)(x - 5) = x^2 - 2x - 15$.

factor (de un número o una expresión) Un número o una expresión que se multiplica por otros números o expresiones para dar como resultado un producto.

Por ejemplo, 1, 2, 4 y 8 son factores del número 8 porque $1 \cdot 8 = 8$ y $2 \cdot 4 = 8$. Además, $(x + 3)$ y $(x - 5)$ son factores de $x^2 - 2x - 15$ porque $(x + 3)(x - 5) = x^2 - 2x - 15$.

factored form One of three common forms of a quadratic equation. A quadratic equation in factored form looks like $f(x) = a(x - m)(x - n)$.

These equations are in factored form:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

forma factorizada Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma factorizada tiene el siguiente orden: $f(x) = a(x - m)(x - n)$.

Estas ecuaciones están en forma factorizada:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

frequency table A table that shows the number of times each value or category occurs in a data set.

For example, this frequency table shows that 5 students selected red as their favorite color.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

function A rule, or relation, that assigns exactly one output to each possible input. Every function is a relation, but not every relation is a function. In a function, the value of the output variable depends on the value of the input variable.

función Una regla, o relación, que asigna exactamente una salida a cada entrada posible. Toda función es una relación, pero no toda relación es una función. En una función, el valor de la variable de salida depende del valor de la variable de entrada.

function notation A notation used to represent the inputs and outputs of a function. For a function f, when x is an input (or domain value), the symbol $f(x)$ shows the corresponding output (or range value).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the independent variable is 4, the value of the dependent variable is 9.

notación de función Una notación utilizada para representar las entradas y las salidas de una función. Para una función (x), cuando x es una entrada (o valor de dominio), el símbolo $f(x)$ muestra la salida correspondiente (o valor de rango).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

Por ejemplo, $f(4) = 9$ es una expresión escrita en notación de funciones. Indica que cuando la entrada de la función f es 4, la salida es 9. En otras palabras, cuando el valor de la variable independiente es 4, el valor de la variable dependiente es 9.

G

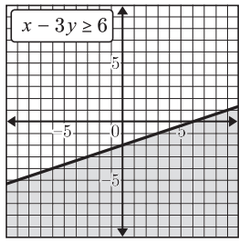
growth factor The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.

factor de crecimiento La razón constante (o el factor común) que multiplica a cada término para generar un patrón exponencial.

H

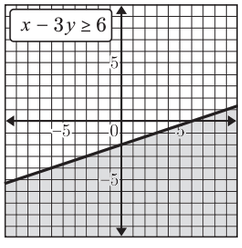
half-plane A region that defines the solution set of a single two-variable linear inequality. The boundary line of the inequality splits the coordinate plane into two equal half-planes.

The graph shows the solution region to $x - 3y \geq 6$. The boundary line $x - 3y = 6$ divides the coordinate plane into two equal half-planes.



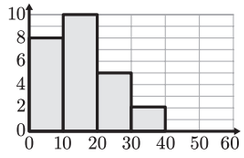
semiplano Una región que define el conjunto de soluciones de una desigualdad lineal con dos variables. La recta límite de la desigualdad divide el plano de coordenadas en dos semiplanos iguales.

La gráfica muestra la región solución de $x - 3y \geq 6$. La recta límite $x - 3y = 6$ divide el plano de coordenadas en dos semiplanos iguales.



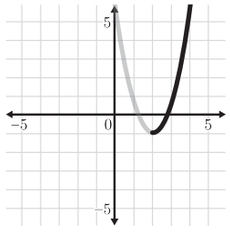
English

histogram A visual display of quantitative data that groups values into intervals, called *bins*. Each bin is represented by a rectangle, where the height shows the frequency or relative frequency of the data in that bin.



For example, this histogram shows that there are 8 values between 0 and 10.

increasing (interval or function) A function is increasing when its outputs increase as its inputs increase. A function can be increasing for its entire domain or over an interval.



For example, the function $f(x)$, whose graph is shown, is increasing when $x > 2$.

independent variable The value of an independent variable is not based on the value of any other variable. In a function, the value of the independent variable represents the input. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

index In a radical $\sqrt[n]{x}$ the quantity n is called the *index*. (See *radical*.)

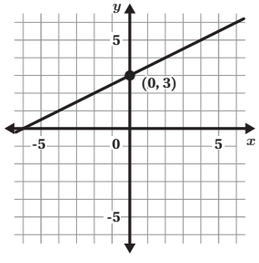
inequality A comparison statement that uses the symbols $<$, $>$, $\leq$, or $\geq$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y + 2 \geq 30$ means that the value of the expression $y + 2$ will be greater than or equal to 30.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. In a system of equations with infinitely many solutions, every point on the graph is a solution to the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

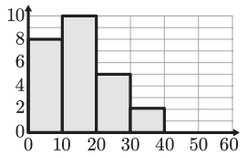
initial value A point where the graph of an equation or function crosses the y-axis or when $x = 0$.



The initial value of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.

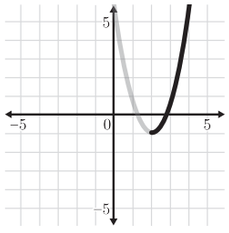
Español

histograma Una representación visual de datos cuantitativos que agrupa los valores en intervalos, llamados *contenedores*. Cada contenedor está representado por un rectángulo, donde la altura muestra la frecuencia o la frecuencia relativa de los datos en ese contenedor.



Por ejemplo, este histograma muestra que hay 8 valores entre 0 y 10.

creciente (intervalo o función) Una función es creciente cuando sus valores de salida crecen a medida que crecen sus valores de entrada. Una función puede ser creciente en todo su dominio o en un intervalo.



Por ejemplo, la función $f(x)$, cuya gráfica se muestra, es creciente cuando $x > 2$.

variable independiente El valor de una variable independiente no depende del valor de ninguna otra variable. En una función, el valor de la variable independiente representa la entrada. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

índice En un radical $\sqrt[n]{x}$, la cantidad n se llama índice. (Ver *radical*).

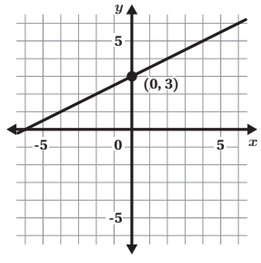
desigualdad Un enunciado de comparación que utiliza los signos $<$, $>$, $\leq$ o $\geq$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y + 2 \geq 30$ significa que el valor de la expresión $y + 2$ será mayor o igual que 30.

infinitas soluciones Una ecuación tiene infinitas soluciones si es verdadera sea cual sea el valor de la variable. Un sistema de ecuaciones tiene infinitas soluciones si las ecuaciones del sistema son equivalentes. En un sistema de ecuaciones con infinitas soluciones, cada punto de la gráfica es una solución del sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene infinitas soluciones porque la ecuación es verdadera con cualquier valor de x.

valor inicial Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.



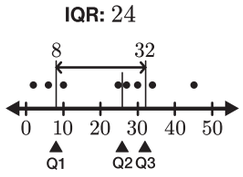
El valor inicial de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.

Glossary/Glosario

English

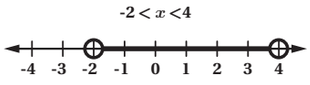
input In a function, any value that you substitute for x is called an input. The input is sometimes called the *independent variable*. The input typically appears on the horizontal axis of a graph and in the left-hand column of a table.

interquartile range (IQR) A measure of variation in a set of numerical data. The interquartile range is the distance between the first quartile (Q1) and the third quartile (Q3) of the data set.



For example, the IQR of this data set is $32 - 8 = 24$.

interval A set of values between two points.



In words: All numbers between -2 and 4.
Inequality: $-2 < x < 4$.

interval notation A way to represent a range of values on a number line using brackets and parentheses.

$(2, 6]$ represents all values between 2 and 6, not including 2 but including 6. $[-3, 5)$ represents all values between -3 and 5, including -3 but not including 5.

inverse operations Two operations that undo each other are called *inverse operations*.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

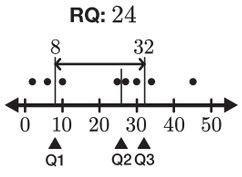
irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

2 is a rational number because it can be written as $\frac{2}{1}$, whereas $\sqrt{3}$ is irrational because it cannot be written as a fraction made up of two integers.

Español

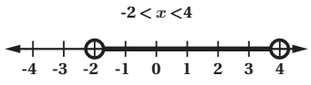
entrada En una función, todo valor que sustituya a x se denomina entrada. La entrada a veces se denomina variable independiente. La entrada suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

rango intercuartil (RIC) Una medida de variación en un conjunto de datos numéricos. El rango intercuartil es la distancia entre el primer cuartil (Q1) y el tercer cuartil (Q3) del conjunto de datos.



Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.

intervalo Un conjunto de valores entre dos puntos.



Con palabras: Todos los números entre el -2 y el 4. Desigualdad: $-2 < x < 4$.

notación de intervalo Una forma de representar un rango de valores en una recta numérica usando corchetes y paréntesis.

$(2, 6]$ representa todos los valores entre 2 y 6, excluyendo 2 pero incluyendo 6. $[-3, 5)$ representa todos los valores entre -3 y 5, incluyendo -3 pero excluyendo 5.

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

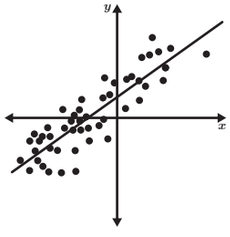
La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

número irracional Un número que no se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

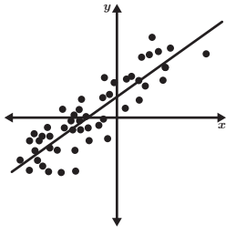
2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que $\sqrt{3}$ es irracional porque no se puede escribir como una fracción de dos números enteros.

L

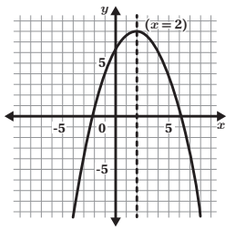
line of fit The line on a scatter plot that best represents the trend created by the points in a data set. There are many lines of fit for a single data set, but there is only one line of best fit.



línea de (mejor) ajuste La línea de un diagrama de dispersión que mejor representa la tendencia que definen los puntos de un conjunto de datos. Existen muchas líneas de ajuste para un mismo conjunto de datos, pero solo hay una línea de mejor ajuste.

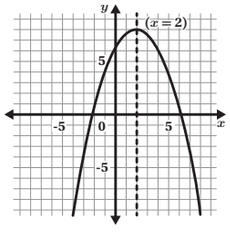


line of symmetry A line that divides a figure or the graph of a function into two halves. For every point (except the vertex), there is a corresponding point on the other side of the line that is the same distance from the line.



The equation of this line of symmetry is $x = 2$.

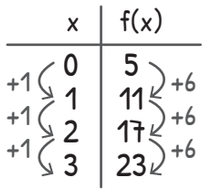
eje de simetría Una línea que divide una figura o la gráfica de una función en dos mitades. Cada punto (excepto el vértice) tiene un punto correspondiente en el otro lado de la línea, el cual está a la misma distancia de la línea.



La ecuación de este eje de simetría es $x = 2$.

English

linear function A function that increases or decreases by a constant rate of change, or shows a linear relationship. A function that changes by equal differences over equal intervals. The graph of a linear function is a line.



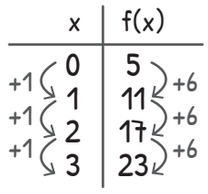
For example, $f(x) = 5 + 6x$ represents a linear function that has an initial value of 5 and increases by a constant difference of 6.

linear relationship See *linear function*.

lurking variable An additional variable that has an effect on the other variables being analyzed. Lurking variables can lead us to make conclusions about data and relationships that are inaccurate.

Español

función lineal Función que crece o decrece con una tasa de cambio constante o muestra una relación lineal. Una función que cambia con diferencias iguales en intervalos iguales. La gráfica de una función lineal es una recta.



Por ejemplo, $f(x) = 5 + 6x$ representa una función lineal que tiene un valor inicial de 5 y crece con una diferencia constante de 6.

relación lineal Ver *función lineal*.

variable de confusión Una variable adicional que tiene un efecto sobre las demás variables que se analizan. Las variables de confusión pueden producir conclusiones imprecisas en torno a los datos y las relaciones.

M

margin of error A measure that shows how much the results of a survey or study might differ from the true population value.

If a survey shows that 60% of people like a product with a margin of error of ±3%, the true percentage is likely between 57% and 63%.

margen de error Una medida que muestra cuánto pueden diferir los resultados de una encuesta o un estudio del valor real de la población.

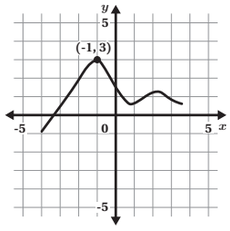
Si una encuesta muestra que al 60 % de las personas les gusta un producto con un margen de error de ±3 %, el porcentaje real probablemente esté entre el 57 % y el 63 %.

maximum The greatest value in a data set.

máximo El mayor valor en un conjunto de datos.

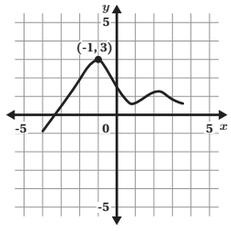
maximum (of a function) The highest point on the graph.

The maximum of this function is $(-1, 3)$.

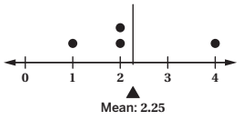


máximo (de una función) El punto más alto en una gráfica.

El máximo de esta función está en $(-1, 3)$.

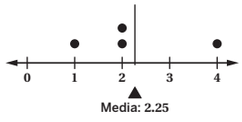


mean A statistic used to describe the typical value of a data set. To calculate the mean, you can add up all the data values and divide by the number of data points. The mean is a measure of center and is also called the *average*.



In this data set, the mean is 2.25.
$1 + 2 + 2 + 4 = 9$
$\frac{9}{4} = 2.25$

media Una estadística que se usa para describir el valor típico de un conjunto de datos. Para calcular la media, se suman todos los valores y el resultado se divide por el número de puntos de datos. La media es una medida de tendencia central y también se denomina promedio.



En este conjunto de datos, la media es 2.25.
$1 + 2 + 2 + 4 = 9$
$\frac{9}{4} = 2.25$

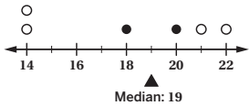
measure of center A numerical value that describes the overall center or clustering of data in a set. The three most common measures of central tendency are the mean, median, and mode.

medida de centro Un valor numérico que describe el centro general o la agrupación de datos en un conjunto. Las tres medidas de tendencia central más comunes son la media, la mediana y la moda.

Glossary/Glosario

English

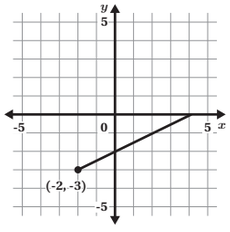
median A statistic used to describe the typical value of a data set. It is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the mean of those two values. The median is a measure of center.



maximum The greatest value in a data set.

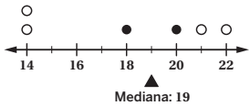
minimum (of a function) The lowest point on the graph.

The minimum of this function is (-2, -3).



Español

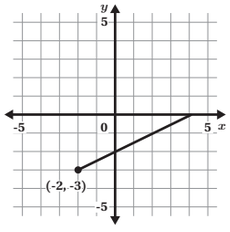
mediana Una estadística que se usa para describir el valor típico de un conjunto de datos. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay dos valores en el medio del conjunto de datos, la mediana es la media de esos dos valores. La mediana es una medida de tendencia central.



máximo El mayor valor en un conjunto de datos.

mínimo (de una función) El punto más bajo en una gráfica.

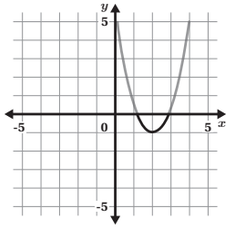
El mínimo de esta función está en (-2, -3).



N

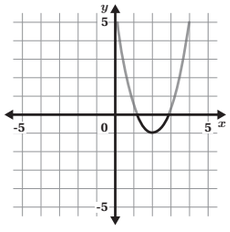
negative (interval or function) A function is negative when its outputs are negative and its graph is below the x-axis. A function can be negative for its entire domain or over an interval.

This function is negative when $1 < x < 3$.

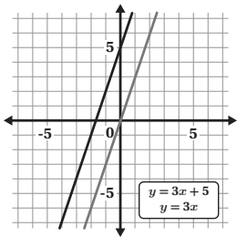


negativo (intervalo o función) Una función es negativa cuando sus salidas son negativas y su gráfica está por debajo del eje x. Una función puede ser negativa en todo su dominio o en un intervalo.

Esta función es negativa cuando $1 < x < 3$.

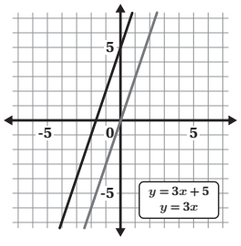


no solution An equation has no solution if there is no value of the variable that will make the equation true. A system of equations has no solution if there is no set of values that makes all the equations in the system true. In a system of equations with no solution, there is no point that is on the graph of every equation in the system.



For example, the system of equations containing $y = 3x + 5$ and $y = 3x$ has no solution because the graphs are parallel and never intersect.

sin solución Una ecuación no tiene solución si no hay ningún valor de la variable que haga que la ecuación sea verdadera. Un sistema de ecuaciones no tiene solución si no hay ningún conjunto de valores que haga que todas las ecuaciones de ese sistema sean verdaderas. En un sistema de ecuaciones sin solución, no hay ningún punto que esté en la gráfica de cada una de las ecuaciones del sistema.



Por ejemplo, el sistema de ecuaciones que contiene $y = 3x + 5$ y $y = 3x$ no tiene solución porque las gráficas son paralelas y nunca se intersecan.

non-linear relationship A pattern, table, scenario, graph, or equation that does not have a constant rate of change. A non-linear relationship is called non-linear because its graph is not a line.

Day	Teal Globs
0	1
1	2
2	5
3	10

(+1, +1, +3, +5)

The equation $y = x^2 + 1$ represents a non-linear relationship because its rate of change is not constant and its graph is not a line.

relación no lineal Patrón, tabla, contexto, gráfica o ecuación que no tiene una tasa de cambio constante. Una relación no lineal se denomina no lineal porque su gráfica no es una recta.

Día	Manchitas turquesa
0	1
1	2
2	5
3	10

(+1, +1, +3, +5)

La ecuación $y = x^2 + 1$ representa una relación no lineal porque su tasa de cambio no es constante y su gráfica no es una recta.

numerical data See *quantitative data*.

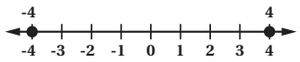
datos numéricos Ver *datos cuantitativos*.

English

Español

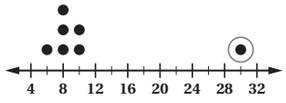
O

opposite Two numbers that are the same distance from 0 and on different sides of the number line. Two terms that are opposites are also referred to as zero pairs and additive inverses.



For example, 4 and -4 are opposites.

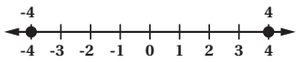
outlier A data value that is far from the other values in the data set. Values that are 1.5 times the IQR from either quartlile 1 or quartile 3 are outliers.



The circled point is an outlier.

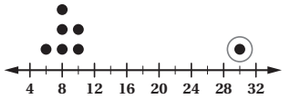
output The value of a function after it has been evaluated for an input value of x is called the *output*. The output is sometimes called the *dependent variable*. The output typically appears on the vertical axis of a graph and in the right-hand column of a table.

opuestos Dos números que están a la misma distancia del 0 y en diferentes lados de la recta numérica. Dos términos que son opuestos también se conocen como inversos aditivos.



Por ejemplo, 4 y -4 son opuestos.

valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos. Son valores atípicos aquellos que están a 1.5 veces el RQ del cuartil 1 o del cuartil 3.

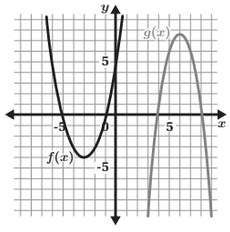


El punto encerrado en un círculo es un valor atípico.

salida El valor de una función tras haber sido evaluada con un valor de entrada x se denomina salida. La salida a veces se denomina variable dependiente. La salida suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

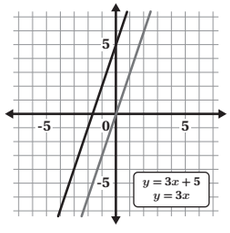
P

parabola The graph of a quadratic function, which is a U-shaped curve.



parallel lines Lines that never cross or intersect. On a graph, two lines with the same slope and different y-intercepts are parallel.

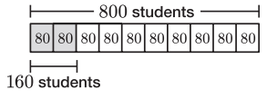
For example, the lines $y = 3x + 5$ and $y = 3x$ are parallel and never intersect.



percent decrease Describes how much a quantity goes down, expressed as a percent of the starting amount.

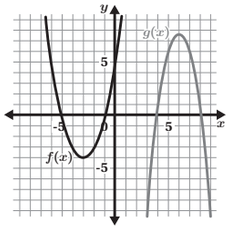
For example, the function $f(x) = 1000(.78)^x$ represents the value of a phone with an initial value of $1,000 with a percent decrease in value of 22% each year, x.

percent (percentage) Percent means for every 100. It is represented by the percent symbol: %. We use percentages to represent ratios and fractions.



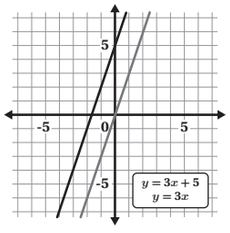
For example, 20% means 20 : 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number.
Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

parábola La gráfica de una función cuadrática, que es una curva en forma de U.



líneas (o rectas) paralelas Líneas que nunca se cruzan o intersecan. En una gráfica, dos rectas con la misma pendiente e intersecciones diferentes con el eje y son paralelas.

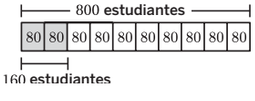
Por ejemplo, las rectas $y = 3x + 5$ y $y = 3x$ son paralelas y nunca se intersecan.



disminución porcentual Describe cuánto disminuye una cantidad y se expresa como un porcentaje de la cantidad inicial.

Por ejemplo, la función $f(x) = 1000(.78)^x$ representa el valor de un teléfono con un valor inicial de $1,000 y una disminución porcentual del 22% de su valor cada año, x.

por ciento (porcentaje) Por ciento significa por cada 100. Se representa con el símbolo de porcentaje: %. Usamos porcentajes para representar razones y fracciones.

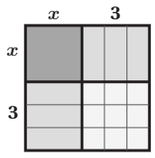


Por ejemplo, 20% significa 20 : 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número.
Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos están en una excursión, entonces eso es 160 estudiantes porque 20 están de viaje por cada 100 estudiantes en total.

Glossary/Glosario

English

perfect square An expression that can be written as something multiplied by itself.



These expressions are perfect squares:
3^2, 9, $(x + 3)^2$, and $x^2 + 6x + 9$.

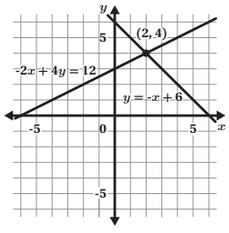
perimeter The sum of the lengths of all the sides of a polygon.

perpendicular Describes a line that crosses or meets another line at a 90° angle.

± (plus/minus symbol) A symbol used to represent both the positive and negative values of a number. It also can be used to represent two expressions.

± 9 represents -9 and +9.
2 ± 3 represents $2 - 3$ and $2 + 3$.

point of intersection A point where two lines or curves meet.



For example, $(2, 4)$ is the point of intersection for the lines $-2x + 4y = 12$ and $y = -x + 6$.

polynomial The sum or difference of terms that include variables raised to non-negative integer powers, with coefficients that can be real or complex numbers.

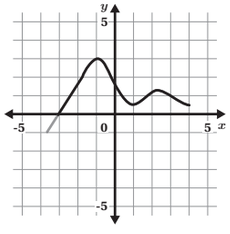
$2x - 7$ and $3x^2 - x + 4$ are polynomial expressions.
$f(x) = 2x - 7$ and $g(x) = 3x^2 - x + 4$ are polynomial functions.

population A set of people or things that are being studied.

For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

population mean The average of all the values in a population. It is calculated by adding all the values and dividing by the total number of values.

positive (interval or function) A function is positive when its outputs are positive and its graph is above the x-axis. A function can be positive for its entire domain or over an interval.

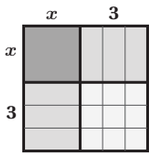


This function is positive when $-3 < x < 4$.

principal The total amount (initial value) of money borrowed or invested, not including any interest.

Español

cuadrado perfecto Una expresión que puede escribirse como algo multiplicado por sí mismo.



Estas expresiones son cuadrados perfectos:
3^2, 9, $(x + 3)^2$ y $x^2 + 6x + 9$.

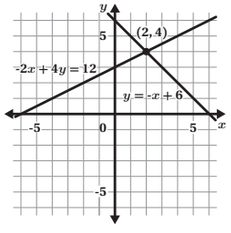
perímetro La suma de las longitudes de todos los lados de un polígono.

perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

± (signo más menos) Un símbolo que se usa para representar los valores positivos y negativos de un número. También puede usarse para representar dos expresiones.

± 9 representa -9 and +9.
2 ± 3 representa $2 - 3$ y $2 + 3$.

punto de intersección Un punto donde se cruzan dos rectas o curvas.



Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.

polinomio La suma o la diferencia de términos que incluyen variables elevadas a potencias enteras no negativas, con coeficientes que pueden ser números reales o complejos.

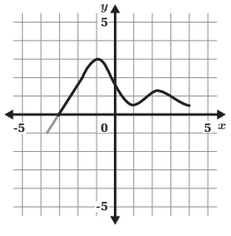
$2x - 7$ y $3x^2 - x + 4$ son expresiones polinómicas.
$f(x) = 2x - 7$ y $g(x) = 3x^2 - x + 4$ son funciones polinómicas.

población Conjunto de personas o cosas que se estudian.

Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

media poblacional El promedio de todos los valores en una población. Se calcula sumando todos los valores y dividiendo por el número total de valores.

positivo (intervalo o función) Una función es positiva cuando sus salidas son positivas y su gráfica está por encima del eje x. Una función puede ser positiva en todo su dominio o en un intervalo.



Esta función es positiva cuando $-3 < x < 4$.

principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, sin incluir los intereses.

English

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Carpet (sq. ft.)	Cost (dollars)
10 $\xrightarrow{\times 1.5}$	15.00
20 $\xrightarrow{\times 1.5}$	30.00
50 $\xrightarrow{\times 1.5}$	75.00

Q

quadratic formula A formula that can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function A function with output values that change by a constant second difference. Equations of quadratic functions have a squared term when written in standard form. The graph of a quadratic function is a parabola.

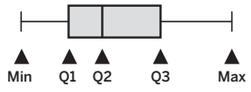
x	f(x)		
1	5		
2	8	+3	
3	14	+6	+3
4	23	+9	+3
5	35	+12	+3

quadratic relationship See *quadratic function*.

quantitative data Data represented as numbers, quantities, or measurements that can be meaningfully compared is called *quantitative data*, or *numerical data*.

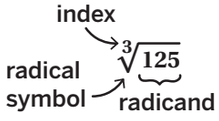
How many pets do you have? is a question that produces quantitative data. This is different from categorical data.

quartile Quartiles divide a data set into four sections.
Quartile 1 is the median of the lower half of the data.
Q2 is the median.
Q3 is the median of the upper half of the data.
Q4 is the maximum.



R

radical A square root, cube root, fourth root, etc. The radical symbol is $\sqrt{\ }$.



radicand The expression under a radical symbol. In the radical $\sqrt[n]{x}$ the quantity x is called the *radicand*. (See *radical*).

random Happening without a predictable pattern or order. In mathematics, a random event has outcomes that occur by chance.

Rolling a die is a random event because any number from 1 to 6 can appear with equal probability.

Español

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

Alfombra (pies cuadrados)	Costo (dólares)
10 $\xrightarrow{\times 1.5}$	15.00
20 $\xrightarrow{\times 1.5}$	30.00
50 $\xrightarrow{\times 1.5}$	75.00

Q

fórmula cuadrática Una fórmula que puede usarse para determinar las soluciones de una ecuación cuadrática $ax^2 + bx + c = 0$, donde $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

función cuadrática Una función con valores de salida que cambian de acuerdo con una segunda diferencia constante. Las ecuaciones de las funciones cuadráticas tienen un término elevado al cuadrado cuando se escriben en forma estándar. La gráfica de una función cuadrática es una parábola.

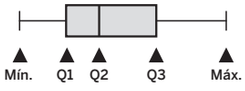
x	f(x)		
1	5		
2	8	+3	
3	14	+6	+3
4	23	+9	+3
5	35	+12	+3

relación cuadrática Ver *función cuadrática*.

datos cuantitativos Los datos que se representan como números, cantidades o medidas y que pueden compararse de forma significativa se denominan datos cuantitativos o datos numéricos.

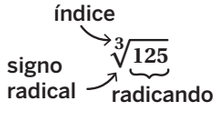
¿Cuántas mascotas tienes? es una pregunta que produce datos cuantitativos, que son diferentes de los datos categóricos.

cuartil Los cuartiles dividen un conjunto de datos en cuatro secciones.
El cuartil 1 es la mediana de la mitad inferior de los datos.
Q2 es la mediana.
Q3 es la mediana de la mitad superior de los datos.
Q4 es el máximo.



R

radical Una raíz cuadrada, raíz cúbica, raíz cuarta, etc. El signo radical es $\sqrt{\ }$.



radicando La expresión bajo un signo radical. En un radical $\sqrt[n]{x}$, la cantidad x se llama radicando. (Ver *radical*).

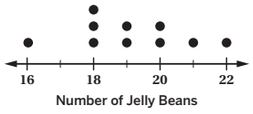
aleatorio Algo que ocurre sin un patrón o un orden predecibles. En matemáticas, un suceso aleatorio tiene resultados que ocurren por casualidad.

Lanzar un dado es un suceso aleatorio porque cualquier número del 1 al 6 puede aparecer con la misma probabilidad.

Glossary/Glosario

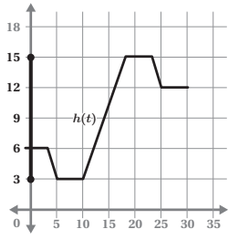
English

range The difference between the maximum and minimum values in a data set. Range is a measure of spread.



For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.

range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.



The range of this graph can be described as: All numbers from 3 to 15 or $3 \leq h(t) \leq 15$.

rate of change The ratio of the change in one quantity to the corresponding change in another quantity. In a linear relationship, the rate of change is the change in y divided by the change in x, which is also the slope of the graph.

rational exponent A real number that can be expressed as the ratio of two integers, where the denominator is not zero. Rational numbers can also appear as fractional exponents in mathematical expressions.

In the expression $5^{\frac{1}{2}}$, $\frac{1}{2}$ is a rational exponent.

rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5, and $\sqrt{9}$ are rational numbers.

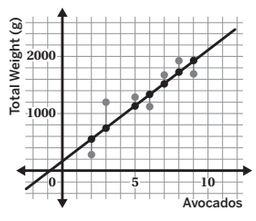
reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a *function*.

relative frequency table A type of two-way table used to compare data across two categorical variables. Relative frequency tables present the fraction or percent of the data that is in that category, instead of the actual number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

residual The difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.



The short lines connecting each point to the line of fit show the residual value for that point.

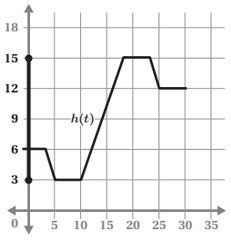
Español

rango La diferencia entre los valores máximo y mínimo de un conjunto de datos. El rango es una medida de dispersión.



Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.

fluctuación (de una función) El conjunto de todos los posibles valores de salida de una función o relación. La fluctuación puede describirse con palabras o como una desigualdad.



La fluctuación de esta gráfica puede describirse de la siguiente manera: Todos los números del 3 al 15 o $3 \leq h(t) \leq 15$.

tasa de cambio La razón entre el cambio en una cantidad y el cambio correspondiente en otra cantidad. En una relación lineal, la tasa de cambio es el cambio en (x) dividido por el cambio en x, que también es la pendiente del gráfico.

exponente racional Un número real que puede expresarse como la razón de dos números enteros, donde el denominador no es cero. Los números racionales también pueden aparecer como exponentes fraccionarios en expresiones matemáticas.

En la expresión $5^{\frac{1}{2}}$, $\frac{1}{2}$ es un exponente racional.

número racional Un número que se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5 y $\sqrt{9}$ son números racionales.

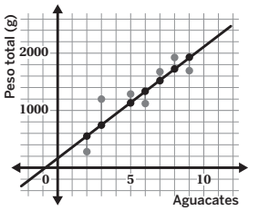
recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

tabla de frecuencia relativa Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa presentan la fracción o el porcentaje de los datos que están en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

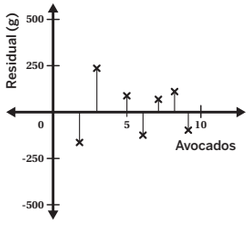
residuo La diferencia entre el valor y de un punto en un diagrama de dispersión y el valor que predice la línea de mejor ajuste.



Las líneas cortas que conectan cada punto con la línea de ajuste muestran el valor del residuo del punto en cuestión.

English

residual plot A scatter plot of residual values for a data set. The x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.



The residual plot shows how the total weight of different numbers of avocados vary from their predicted values.

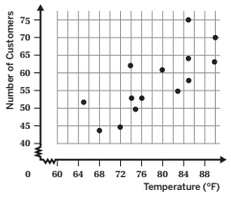
revenue The amount of money generated by selling a product or service.

S

sample A part of a population.

For example, a population could be all the seventh-grade students at one school. One sample of that population is all the seventh-grade students who are in band.

scatter plot A graph in the coordinate plane that shows the relationship between two variables by plotting individual data points. Scatter plots are used to observe patterns in bivariate numerical data.



second difference The differences between consecutive output values in the table of a function are called *first differences*. The differences between those values are called *second differences*. Quadratic functions have constant second differences.

x	f(x)		
1	5		
		+3	
2	8		+3
		+6	
3	14		+3
		+9	
4	23		+3
		+12	
5	35		

In this example, the first differences are 3, 6, 9, and 12. The second differences are constant at 3, so $f(x)$ is a quadratic function.

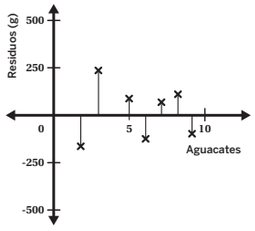
segmented bar graph A bar graph where each bar is divided into segments that represent different categories, showing how the total is split into parts.

set-builder notation A shorthand used to describe sets, often those with an infinite number of elements. It is written in the form $\{x : x > 5\}$, which is read as "the set of all x such that x is greater than 5." The colon (:) can also be replaced by a vertical line (|), as in $\{x \mid x > 5\}$, and is read the same way.

The set of all numbers greater than 8 can be written in set-builder notation as: $\{x \mid x>8\}$
This is read as "the set of all x such that x is greater than 8."

Español

diagrama de residuos Un diagrama de dispersión de los valores de los residuos de un conjunto de datos. El eje x representa el valor que predice la línea de mejor ajuste, y el valor y de cada punto representa el valor del residuo.



El diagrama de residuos muestra que el peso total de diferentes cantidades de aguacates varía con respecto a sus valores esperados.

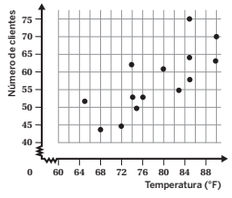
ingresos La cantidad de dinero que genera la venta de un producto o servicio.

S

muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

diagrama de dispersión Un gráfico en el plano de coordenadas que muestra la relación entre dos variables mediante el trazado de puntos de datos individuales. Los diagramas de dispersión se utilizan para observar patrones en datos numéricos bivariados.



segunda diferencia Las diferencias entre valores de salida consecutivos en la tabla de una función se llaman primeras diferencias. Las diferencias entre dichos valores se llaman segundas diferencias. Las funciones cuadráticas tienen segundas diferencias constantes.

x	f(x)		
1	5		
		+3	
2	8		+3
		+6	
3	14		+3
		+9	
4	23		+3
		+12	
5	35		

En este ejemplo, las primeras diferencias son 3, 6, 9 y 12. Las segundas diferencias son constantes, de 3, por lo que $f(x)$ es una función cuadrática.

gráfico de barras segmentadas Un gráfico de barras donde cada barra está dividida en segmentos que representan diferentes categorías y muestran cómo el total se divide en partes.

notación de construcción de conjuntos Una abreviatura utilizada para describir conjuntos, a menudo aquellos con un número infinito de elementos. Se escribe en la forma $\{x : x > 5\}$, que se lee como "el conjunto de todas las x, de modo que x es mayor que 5". Los dos puntos (:) también se pueden reemplazar por una línea vertical (|), como en $\{x \mid x > 5\}$, y se leen de la misma manera.

El conjunto de todos los números mayores que 8 se puede escribir en notación de construcción de conjuntos como: $\{x \mid x > 8\}$
Esto se lee como "el conjunto de todas las x, de modo que x es mayor que 8".

Glossary/Glosario

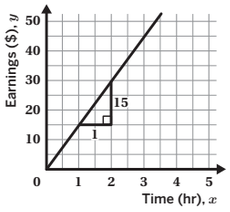
English

simple interest Interest that is calculated solely based on the initial amount. The balance in an account with simple interest is modeled by a linear function.

simulation An approximate imitation of a statistical experiment, often performed using a computer program to analyze the results of a large number of trials.

A simulation can be used to predict the likelihood of getting heads or tails by flipping a virtual coin many times.

slope (rate of change) The ratio of the change in the vertical direction (y-axis) to the change in the horizontal direction (x-axis).



In this graph, y increases by 15 dollars when x increases by 1 hour. The slope of the line is 15, and the rate of change is 15 dollars per hour.

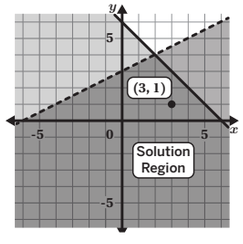
slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where m represents the slope, b represents the y-intercept of the line, and x and y are variables.

The equations $y = 2x + 4$ and $y = -5x - 10$ are in slope-intercept form. The equation $2x + 5y = 20$ is not in slope-intercept form.

solution A value or set of values that makes an equation or inequality true.

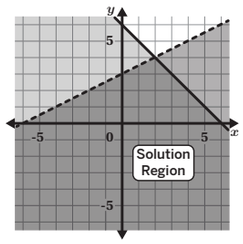
For example, $x = 2$ is a solution to the equation $3x + 4 = 10$.
$x > 2$ are the solutions to the inequality $3x + 4 > 10$.
The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.

solution (to a system of inequalities) An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the solution region where the graphs overlap.



For system $y \leq -x + 6$ and $-2x + 4y < 12$, (3,1) is a solution to this system of inequalities because it makes both inequalities true and falls in the region where the inequalities overlap.

solution region The set of all ordered pairs that make an inequality or each inequality in a system true. For a two-variable linear inequality, the solution region is a half-plane. For a system of inequalities, the solution region is located where the graphs overlap.



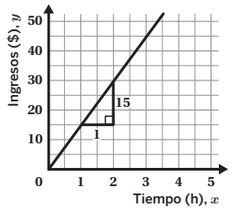
Español

interés simple Interés que se calcula únicamente en función de la cantidad inicial. El saldo de una cuenta con interés simple se modela con una función lineal.

simulación Una imitación aproximada de un experimento estadístico, a menudo realizada mediante un programa informático para analizar los resultados de una gran cantidad de ensayos.

Se puede utilizar una simulación para predecir la probabilidad de obtener cara o cruz lanzando una moneda virtual muchas veces.

pendiente (tasa de cambio) La razón entre el cambio en la dirección vertical (eje y) y el cambio en la dirección horizontal (eje x).



En esta gráfica, y incrementa en 15 dólares cuando x incrementa en 1 hora. La pendiente de la recta es 15 y la tasa de cambio es de 15 dólares por hora.

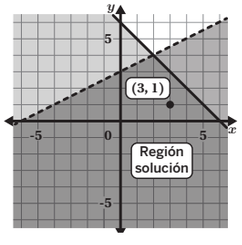
forma pendiente-intersección, forma pendiente-ordenada al origen Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje y (o la ordena al origen) de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben $y = mx + b$, donde m representa la pendiente, b representa la intersección con el eje y de la recta, y tanto x como y son variables.

Las ecuaciones $y = 2x + 4$ y $y = -5x - 10$ están en la forma pendiente-intersección. La ecuación $2x + 5y = 20$ no está en la forma pendiente-intersección.

solución Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

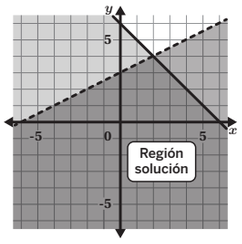
Por ejemplo, $x = 2$ es una solución de la ecuación $3x + 4 = 10$.
$x > 2$ son las soluciónes de la desigualdad $3x + 4 > 10$.
El par ordenado $(1, 2)$ es una solución de la ecuación $3x + 4y = 11$.

solución (de un sistema de desigualdades) Un par ordenado que hace que cada desigualdad de un sistema sea verdadera. Cada par ordenado que sea una solución de un sistema se encuentra en la región solución donde se superponen las gráficas.



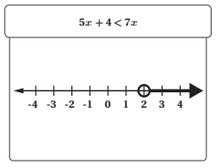
Por ejemplo, en el sistema de desigualdades $y \leq -x + 6$ y $-2x + 4y < 12$, la solución es (3,1) porque hace que ambas desigualdades sean verdaderas y se ubica en la región donde las desigualdades se suporponen.

región solución El conjunto de todos los pares ordenados que hacen que una desigualdad, o cada desigualdad de un sistema, sean verdaderas. En una desigualdad lineal de dos variables, la región solución es un semiplano. En un sistema de desigualdades, la región solución se encuentra donde se superponen las gráficas.



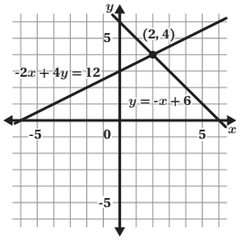
English

solution set The set of all values that makes an inequality true. To describe a solution set symbolically, we often use inequalities. To describe a solution set graphically, we often shade a portion of a number line or a region of the coordinate plane.



The solution set for the inequality $5x + 4 < 7x$ includes all of the values that are larger than 2. 2 is not included in the solution set.

solution to a system of equations A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the point of intersection.



For the system $y = -x + 6$ and $-2x + 4y = 12$, $(2, 4)$ is the solution to this system of equations and the point of intersection on the graph.

square root The square root of a number n (written as $\sqrt{n}$) is the positive number that can be squared to get n. The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

standard form (of a linear equation) Linear equations that are written in the form $ax + by = c$, where a, b, and c are constants and x and y are variables.

The equations $2x + 5y = 20$ and $3x - 4y = -10$ are in standard form. The equation $y = 2x + 4$ is not in standard form.

standard form (of a quadratic equation) One of three common forms of a quadratic equation. A quadratic equation in standard form looks like $f(x) = ax^2 + bx + c$.

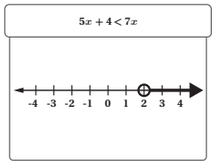
These equations are in standard form:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

substitute Replace a variable or expression with a value or another expression. $4x = 4(5)$

In this example, 5 is substituted for x in the expression $4x$.

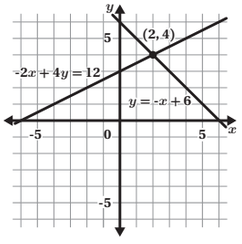
Español

conjunto de soluciones El conjunto de todos los valores que hacen verdadera una desigualdad. Para describir con signos un conjunto de soluciones suelen emplearse desigualdades. Para describir con gráficas un conjunto de soluciones suele colorearse o sombrearse una parte de una recta numérica o una región del plano de coordenadas.



El conjunto de soluciones de la desigualdad $5x + 4 \geq 7x$ incluye todos los valores mayores que 2. El 2 no está incluido en el conjunto de soluciones.

solución de un sistema de ecuaciones Una solución de un sistema de ecuaciones es un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.



Por ejemplo, en el sistema de ecuaciones $y = -x + 6$ y $-2x + 4y = 12$, la solución y el punto de intersección de la gráfica es $(2, 4)$.

raíz cuadrada La raíz cuadrada de un número n (se escribe $\sqrt{n}$) es el número positivo que puede elevarse al cuadrado para obtener n. La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

forma estándar (de una ecuación lineal) Las ecuaciones lineales que se escriben en la forma $ax + by = c$, donde a, b y c son constantes, y tanto x como y son variables, se conocen como ecuaciones en forma estándar.

Las ecuaciones $2x + 5y = 20$ y $3x - 4y = -10$ están en forma estándar. La ecuación $y = 2x + 4$ no está en forma estándar.

forma estándar (de una ecuación cuadrática) Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma estándar tiene el siguiente orden: $f(x) = ax^2 + bx + c$.

Estas ecuaciones están en forma estándar:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

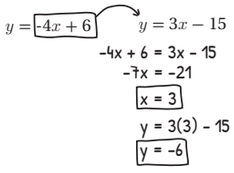
sustituir Reemplazar una variable o expresión por un valor u otra expresión. $4x = 4(5)$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

Glossary/Glosario

English

substitution A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.



For example, we can substitute $-4x + 6$ in for y in $y = 3x - 15$ because they are equivalent.

survey A method of collecting data by asking people to answer a given set of questions.

system of equations Two or more equations that represent the constraints on a shared set of variables form a system of equations.

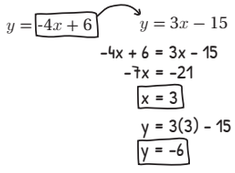
These equations make a system:
$3b + c = -2$
$b - 5c = 12$

system of inequalities Two or more inequalities that represent the constraints on a shared set of variables form a system of inequalities.

These inequalities make a system:
$10m + 5n > -2$
$m - 5n \leq 12$

Español

sustitución Un método para resolver sistemas de ecuaciones donde una variable se reemplaza con una expresión equivalente para producir una nueva ecuación con menos variables.



Por ejemplo, podemos introducir $-4x + 6$ en lugar de y en $y = 3x - 15$ porque son equivalentes.

encuesta Un método de recopilación de datos que consiste en pedir a personas que respondan una serie de preguntas.

sistema de ecuaciones Dos o más ecuaciones que representan las restricciones de un conjunto compartido de variables forman un sistema de ecuaciones.

Estas ecuaciones forman un sistema:
$3b + c = -2$
$b - 5c = 12$

sistema de desigualdades Dos o más desigualdades que representan las restricciones de un conjunto compartido de variables forman un sistema de desigualdades.

Estas desigualdades forman un sistema:
$10m + 5n > -2$
$m - 5n \leq 12$

T

total relative frequency table A type of two-way table used to compare data across two categorical variables. Total relative frequency tables present the fraction or percentage of the data that is in that category, instead of the number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

	Meditated	Did Not Meditate	Total
Calm	46%	8%	54%
Agitated	24%	22%	46%
Total	70%	30%	100%

translation A transformation that moves every point in a function a given distance in a given direction. A translation changes the location of a function, but does not change its shape.

two-way table A way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

	Meditated	Did Not Meditate	Total
Calm	45	8	53
Agitated	23	21	44
Total	68	29	97

tabla de frecuencia relativa total Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa total presentan la fracción o el porcentaje de los datos que se encuentran en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

	Meditaron	No meditaron	Total
Calmados	46%	8%	54%
Agitados	24%	22%	46%
Total	70%	30%	100%

traslación Una transformación que mueve cada punto de una función una determinada distancia en una determinada dirección. Una traslación cambia la ubicación de una función, pero no su forma.

tabla de doble entrada Una manera de comparar dos variables categóricas. Muestra una de las variables en la fila superior y la otra variable en la columna de un costado. Cada entrada de la tabla es la frecuencia o frecuencia relativa de la categoría que describen los títulos de la columna y la fila.

	Meditaron	No meditaron	Total
Calmados	45	8	53
Agitados	23	21	44
Total	68	29	97

English

Español

U

univariate data A data set that involves one variable. Each data point contains one piece of information.

A collection of students' heights is a univariate data set.
Tables, dot plots, and bar graphs are useful for displaying univariate data.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

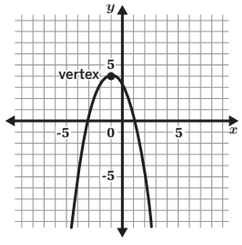
variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

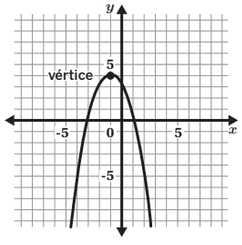
variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

vertex On the graph of a quadratic or absolute value function, the vertex is the maximum or minimum point. The vertex is also where the function changes from increasing to decreasing, or vice versa.



vértice En la gráfica de una función cuadrática o una función de valor absoluto, el vértice es el punto máximo o mínimo. El vértice también es donde la función cambia de creciente a decreciente, o viceversa.



vertex form One of three common forms of a quadratic equation. A quadratic equation in vertex form looks like $f(x) = a(x - h)^2 + k$.

These equations are in vertex form:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

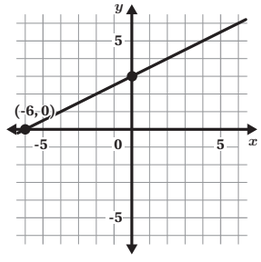
forma de vértice Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma de vértice tiene el siguiente orden: $f(x) = a(x - h)^2 + k$.

Estas ecuaciones están en forma de vértice:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

X

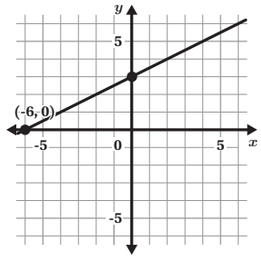
x-intercept A point where the graph of an equation or function crosses the x-axis or when $y = 0$.

The x-intercept of the graph of $-2x + 4y = 12$ is $(-6, 0)$, or just -6.



intersección con el eje x, abscisa al origen Un punto donde la gráfica de una ecuación o función cruza el eje x, o cuando $y = 0$.

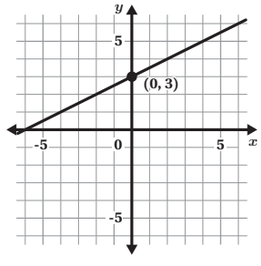
La intersección con el eje x de la gráfica de $-2x + 4y = 12$ es $(-6, 0)$, o simplemente -6.



Y

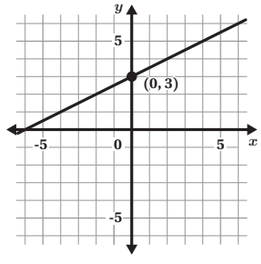
y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

The y-intercept of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.



intersección con el eje y, ordenada al origen Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.

La intersección con el eje y de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.



Glossary/Glosario

Z

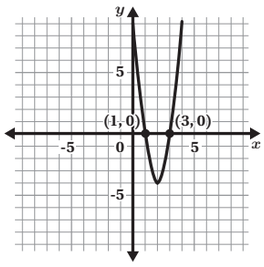
English

zero-product property A property which states that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to help solve equations.

If $(2x - 3)(x + 1) = 0$, then either $2x - 3 = 0$ or $x + 1 = 0$.

zeros The x-values that make a function equal zero, or $f(x) = 0$.

The zeros of $f(x) = 4(x - 1)(x - 3)$ are 1 and 3.



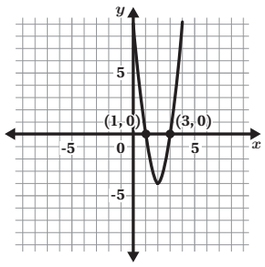
Español

propiedad del producto cero Una propiedad que establece que si el producto de dos o más factores es 0, entonces al menos uno de los factores es 0. Esta propiedad puede usarse como ayuda para resolver ecuaciones.

Si $(2x - 3)(x + 1) = 0$, entonces $2x - 3 = 0$ o $x + 1 = 0$.

ceros Los valores x que hacen que una función sea igual a cero, o $f(x) = 0$.

Los ceros de $f(x) = 4(x - 1)(x - 3)$ son 1 y 3.



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