Amplify Desmos Math FLORIDA

ACC 7

Intervention and Extension Resources

Inside you'll find:

- Strategies for effective differentiation
- Mini-Lessons, including those from prior grades
- Extensions

Amplify Desmos Math **FLORIDA**

Accelerated 7

Intervention and Extension Resources

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

55 Washington Street, Suite 800
Brooklyn, NY 11201
www.amplify.com

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Contents Mini-Lessons

Log in to the digital curriculum to review all program Mini-Lessons and Extensions.

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Contents Extensions (continued)

Intervention and Extension

Amplify Desmos Math Florida provides a comprehensive suite of intervention and extension resources designed to meet the needs of all students and are intended to be used outside of the core lesson.

Assess and Respond: After the Lesson

Each lesson has a formative assessment called a ***Show What You Know***. This assessment illustrates students' progress toward key concepts in the lesson and is accompanied by a table with suggestions in three categories: students who need **support**, students who would benefit from more practice to **strengthen** their understanding, and students who are interested in a **stretch** to deepen their understanding.

Support

Provide targeted intervention for students.

Strengthen

Reinforce students' understanding of the concepts assessed.

Stretch

Challenge students and extend their learning.

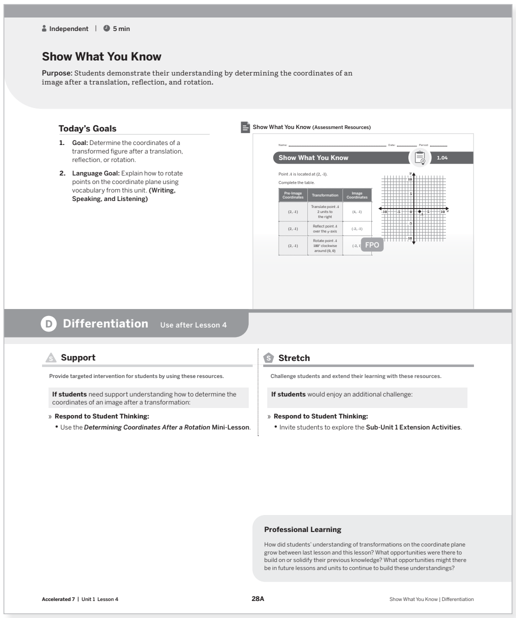
Differentiation Resources

The differentiation resources provide *beyond-the-lesson* support and challenge for all students.

Support, Strengthen, and Stretch print resources include:

- **Mini-Lessons:** Targeted intervention lessons to support students with a specific concept or skill.
- **Lesson Practices:** Practices to build and reinforce students' conceptual understanding, fluency, and application. Lesson Practices include fluency, test prep, and spiral review.
- **Extensions:** Problems aligned to the math of the sub-unit, designed for students who want to extend their thinking.

The differentiation table is available on the *Show What You Know* page of the Teacher Edition and in the *Differentiation Beyond the Lesson* tab of each lesson.



Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the coordinates of an image after a translation, reflection, and rotation.

Today's Goals

1. **Goal:** Determine the coordinates of a transformed figure after a translation, reflection, or rotation.
2. **Language Goal:** Explain how to rotate points on the coordinate plane using vocabulary from this unit. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: Date: Period:

Show What You Know 1.04

Point A is located at (2, -1).

Complete the table.

Pre-Image Coordinates	Transformation	Image Coordinates
(2, -1)	Translate point A 2 units to the right	(4, -1)
(2, -1)	Reflect point A over the y-axis	(-2, -1)
(2, -1)	Rotate point A 180° clockwise around (0, 0)	(-2, 1

FPO

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the coordinates of an image after a transformation:

» **Respond to Student Thinking:**
- Use the ***Determining Coordinates After a Rotation*** Mini-Lesson.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**
- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

How did students' understanding of transformations on the coordinate plane grow between last lesson and this lesson? What opportunities were there to build on or solidify their previous knowledge? What opportunities might there be in future lessons and units to continue to build these understandings?

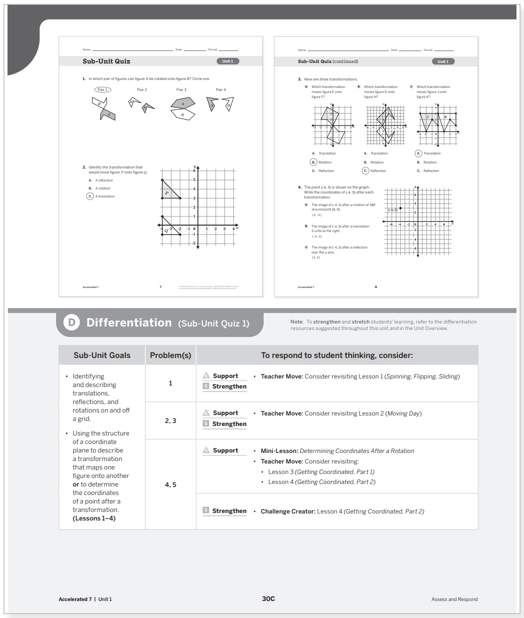
Accelerated 7 | Unit 1 Lesson 4 28A Show What You Know | Differentiation

Differentiation Beyond the Lesson table from Accelerated 7, Unit 1, Lesson 4.

Assess and Respond: After Unit Assessments

Embedded unit assessments offer key insights into students' understanding of the grade-level standards in the unit.

- Each unit includes an optional Pre-Unit Check, one or more Sub-Unit Quizzes, and an End-of-Unit Assessment.
- Each assessment is accompanied by an Assess and Respond Guide in the Teacher Edition, which includes responses to student thinking with resources that support, strengthen, and stretch learning.



Sub-Unit Quiz Differentiation table from Accelerated 7, Unit 1 Sub-Unit 1

About Mini-Lessons

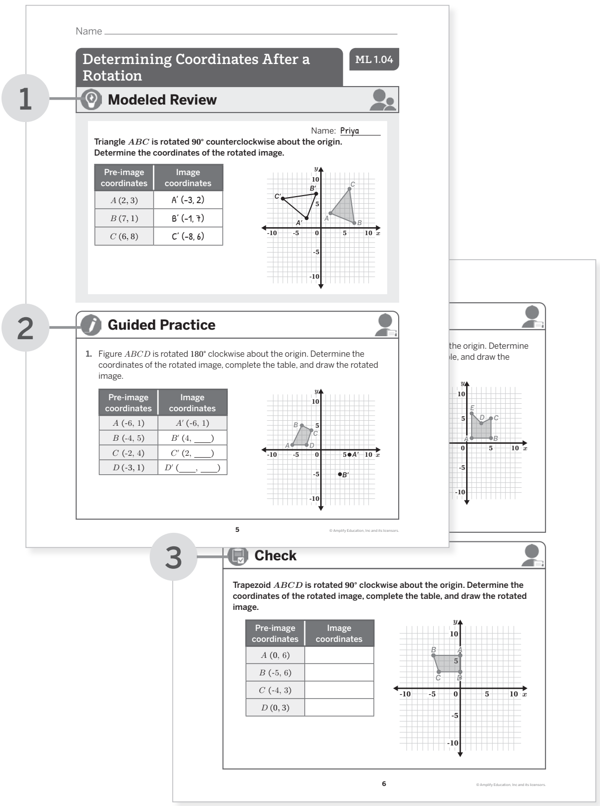
Amplify Desmos Math Florida Mini-Lessons are print activities aligned to the most critical content and skills in corresponding core lessons.

Mini-Lessons offer direct instruction and guided practice opportunities. These Mini-Lessons complement the problem-based approach, are ideal for small-group or whole-class instruction, as well as independent learning.

Student Page

Amplify Desmos Math Florida Mini-Lessons are designed based on extensive research around worked examples.[1]

The Student Page is organized to follow the flow of the Mini-Lesson: **Modeled Review**, **Guided Practice**, and **Check**.



Accelerated 7, Unit 1, Lesson 4 Mini-Lesson

1. **Modeled Review:** A worked example designed to be discussed as a group. Students make sense of a concept or process by examining the modeled student thinking.
2. **Guided Practice:** A series of problems that fades away scaffolding as students progress. Teachers can approach the Guided Practice in various ways based on their expertise and understanding of their students' needs.
3. **Check:** An opportunity for students to show what they've learned. We recommend all students complete this independently.

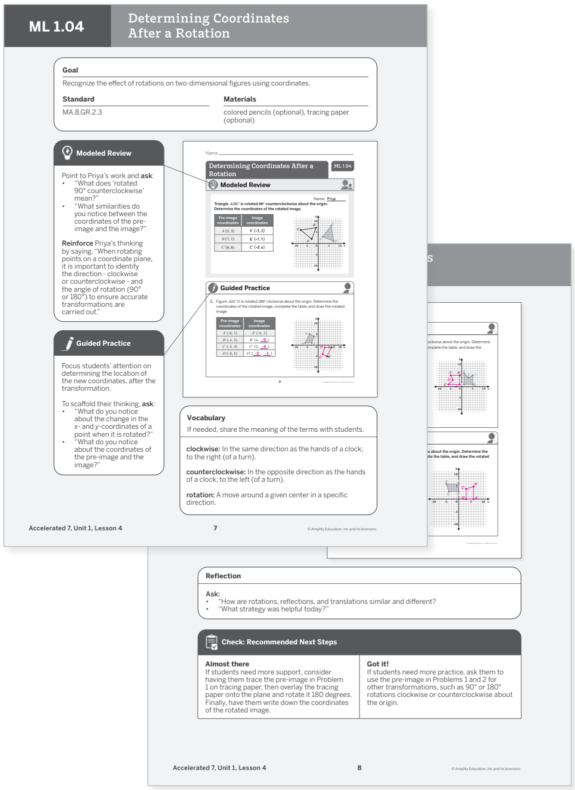
Considerations:

- Print or copy the Student Page in advance.
- Gather any needed materials. Any materials listed as optional on the Teacher Guide are not required to successfully implement the lesson.
- It may be helpful to have a whiteboard, but it's not required.

[1] Flores, R. and Inan, F. (2014). Examining the Impact of Adaptively Faded Worked Examples on Student Learning Outcomes. Journal of Interactive Learning Research, 467–485.

Teacher Guide

The Teacher Guide follows the same flow as the Student Page, with all the information needed to implement the Mini-Lesson.



Accelerated 7, Unit 1, Lesson 4 Mini-Lesson

Every Teacher Guide includes:

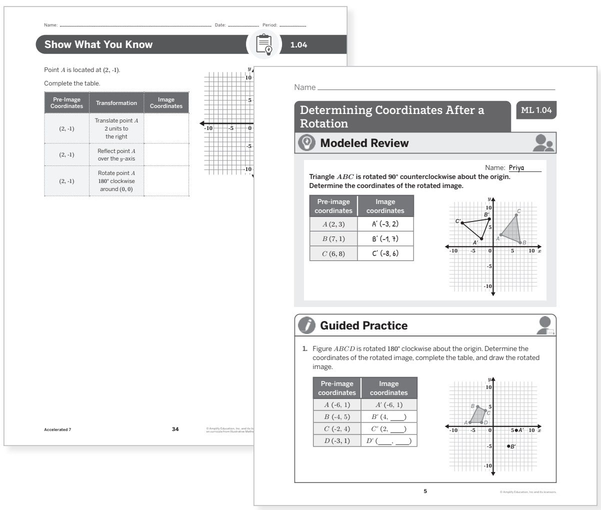
- The **lesson goal**, **materials**, and relevant **vocabulary terms**.
- Questions or statements to share with students in each part of the Mini-Lesson. **Reflection questions** are included as a way to close out the lesson.
- **Answer keys** with sample responses on the insets of the student pages.
- **Recommended next steps** for students needing more support or extra practice based on their performance on the Check problem(s).

Supporting All Learners

ELL This icon indicates suggestions for supporting **English Language Learners**.

A This icon appears at point-of-use and indicates suggestions for supporting the needs of all learners, based on the guidelines of **Universal Design for Learning (UDL)**.

Lesson and Mini-Lesson Alignment



Accelerated 7, Unit 1, Lesson 4 SWYK and Mini-Lesson

Mini-Lessons are closely aligned to the core lesson they are connected to. The **Mini-Lesson Modeled Review** is built using the *Show What You Know* (SWYK) from the lesson. Teachers can use student thinking on the *Show What You Know* to identify students who might benefit from the extra support of a Mini-Lesson.

About Extensions

Amplify Desmos Math Florida Extensions are sets of problems aligned to the math of the sub-unit. They are useful for students interested in an additional challenge or for the whole class.

Extensions build on our **student-led**, **problem-based** approach because they provide more opportunities for students to engage in creative and rigorous problems that can be approached using different strategies.

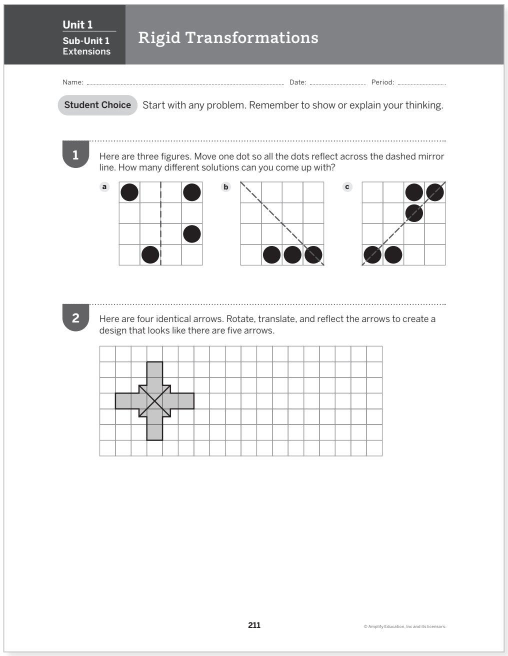
Student Page

Every sub-unit includes an Extensions problem set.

They are print-based, hands-on problems structured on the principle of student choice and designed to be student-led. The math is designed to be accessible to students at any time they are ready for more during the sub-unit.

Every sub-unit Extension includes:

- **Challenge:** Extensions focus on problem-solving and sharing thinking rather than answer-getting, with problems aligned to the math in the sub-unit.
- **Choice:** Extensions contain multiple open-ended problems, and students can start with what interests them.
- **Variety:** Some problems are designed with hands-on materials, others are discussion-based, and the rest require only a pencil and paper.


Unit 1
Sub-Unit 1
Extensions
Rigid Transformations

Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Here are three figures. Move one dot so all the dots reflect across the dashed mirror line. How many different solutions can you come up with?

a b c

2 Here are four identical arrows. Rotate, translate, and reflect the arrows to create a design that looks like there are five arrows.

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Accelerated 7, Unit 1, Sub-Unit 1 Extension Student Page

Considerations:

- Invite students to choose one problem to focus on at a time.
- Prepare enough Extensions at the start of the sub-unit for all students so you can be flexible with when different students work on them.
- Think about where you will want students to store their work on the Extensions for the duration of the sub-unit.

Teacher Guide

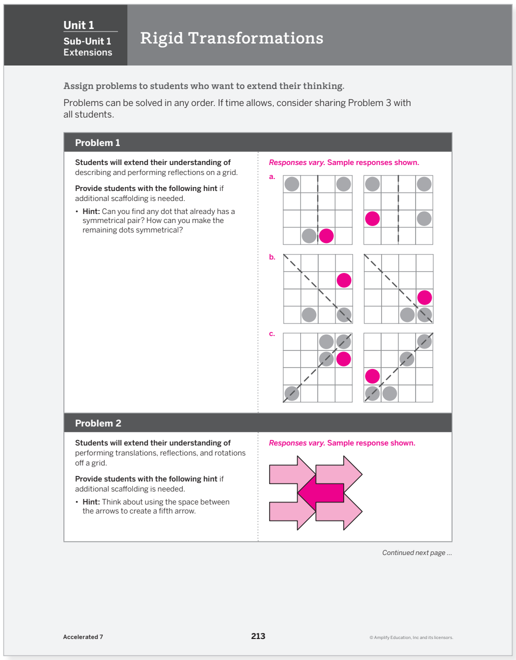
Extensions are designed to be a light lift for the teacher.

Every sub-unit Extension Teacher Guide includes:

- **Key background information** about the math in the problem.
- **Suggestions** for which problems to share with the whole class if time allows.
- **Hints** to share with students when needed.
- **Sample responses**.

Considerations:

- Look for opportunities to introduce an Extension problem with the whole class early in the sub-unit so all students can participate.
- Help students get started on the Extension task, then let them work independently or in pairs while you work with other groups of students who may benefit from more direct support from a teacher.



Unit 1
Sub-Unit 1
Extensions
Rigid Transformations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problem 3 with all students.

Problem 1

Students will extend their understanding of describing and performing reflections on a grid.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Can you find any dot that already has a symmetrical pair? How can you make the remaining dots symmetrical?

Responses vary. Sample responses shown.

a.

b.

c.

Problem 2

Students will extend their understanding of performing translations, reflections, and rotations off a grid.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Think about using the space between the arrows to create a fifth arrow.

Responses vary. Sample response shown.

Continued next page ...

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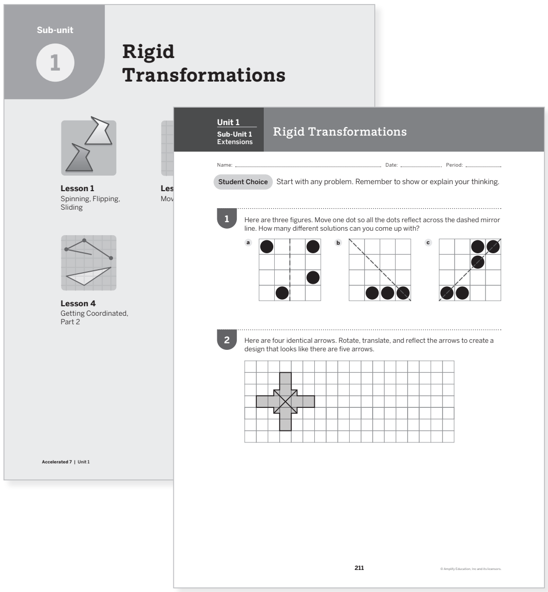
Accelerated 7, Unit 1, Sub-Unit 1 Extension Teacher Guide

Lesson and Extension Alignment



Sub-unit 1
Rigid Transformations

Lesson 1
Spinning, Flipping, Sliding

Lesson 4
Getting Coordinated, Part 2

Accelerated 7 | Unit 1

Unit 1
Sub-Unit 1
Extensions
Rigid Transformations

Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Here are three figures. Move one dot so all the dots reflect across the dashed mirror line. How many different solutions can you come up with?

a b c

2 Here are four identical arrows. Rotate, translate, and reflect the arrows to create a design that looks like there are five arrows.

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Accelerated 7, Unit 1, Sub-Unit 1 Opener and Extension

Extensions are aligned to the math of the **sub-unit**, but they also go deeper. The problems in Extensions are designed to make connections between the math of the sub-unit and other concepts. In some cases, problems will involve content from prior grades or units.

Mini-Lessons

Unit 1

Mini-Lessons

Name ______________________________

Determining Coordinates After a Rotation

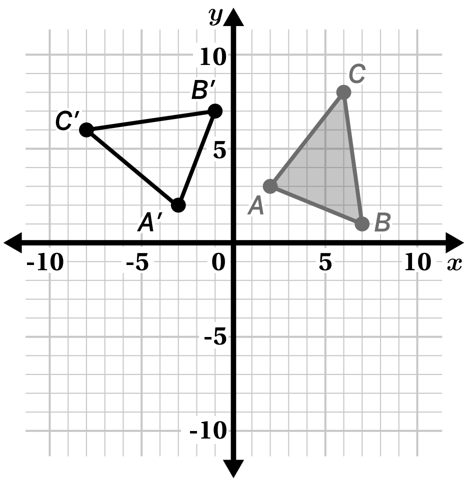
ML 1.04

Modeled Review

Name: Priya

Triangle ABC is rotated 90° counterclockwise about the origin. Determine the coordinates of the rotated image.

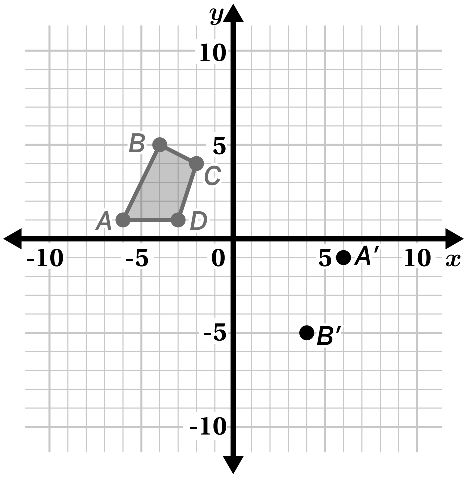
Pre-image coordinates	Image coordinates
$A\ (2, 3)$	A′ (-3, 2)
$B\ (7, 1)$	B′ (-1, 7)
$C\ (6, 8)$	C′ (-8, 6)



Guided Practice

1. Figure $ABCD$ is rotated 180° clockwise about the origin. Determine the coordinates of the rotated image, complete the table, and draw the rotated image.

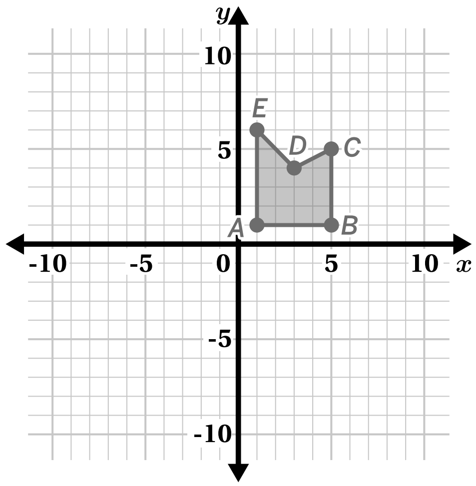
Pre-image coordinates	Image coordinates
$A\ (-6, 1)$	$A'\ (-6, 1)$
$B\ (-4, 5)$	$B'\ (4,$ ____$)$
$C\ (-2, 4)$	$C'\ (2,$ ____$)$
$D\ (-3, 1)$	$D'\ ($____, ____$)$



Guided Practice

2. Figure $ABCDE$ is rotated $90°$ counterclockwise about the origin. Determine the coordinates of the rotated image, complete the table, and draw the rotated image.

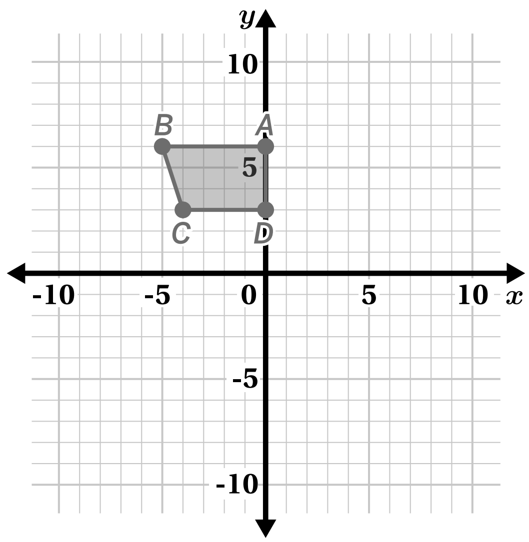
Pre-image coordinates	Image coordinates
A (1, 1)	A' (-1, 1)
B (5, 1)	
C (5, 5)	
D (3, 4)	
E (1, 6)	



Check

Trapezoid $ABCD$ is rotated $90°$ clockwise about the origin. Determine the coordinates of the rotated image, complete the table, and draw the rotated image.

Pre-image coordinates	Image coordinates
A (0, 6)	
B (-5, 6)	
C (-4, 3)	
D (0, 3)	



ML 1.04 Determining Coordinates After a Rotation

Goal

Recognize the effect of rotations on two-dimensional figures using coordinates.

Standard

MA.8.GR.2.3

Materials

colored pencils (optional), tracing paper (optional)

Modeled Review

Point to Priya's work and **ask**:

- "What does 'rotated 90° counterclockwise' mean?"
- "What similarities do you notice between the coordinates of the pre-image and the image?"

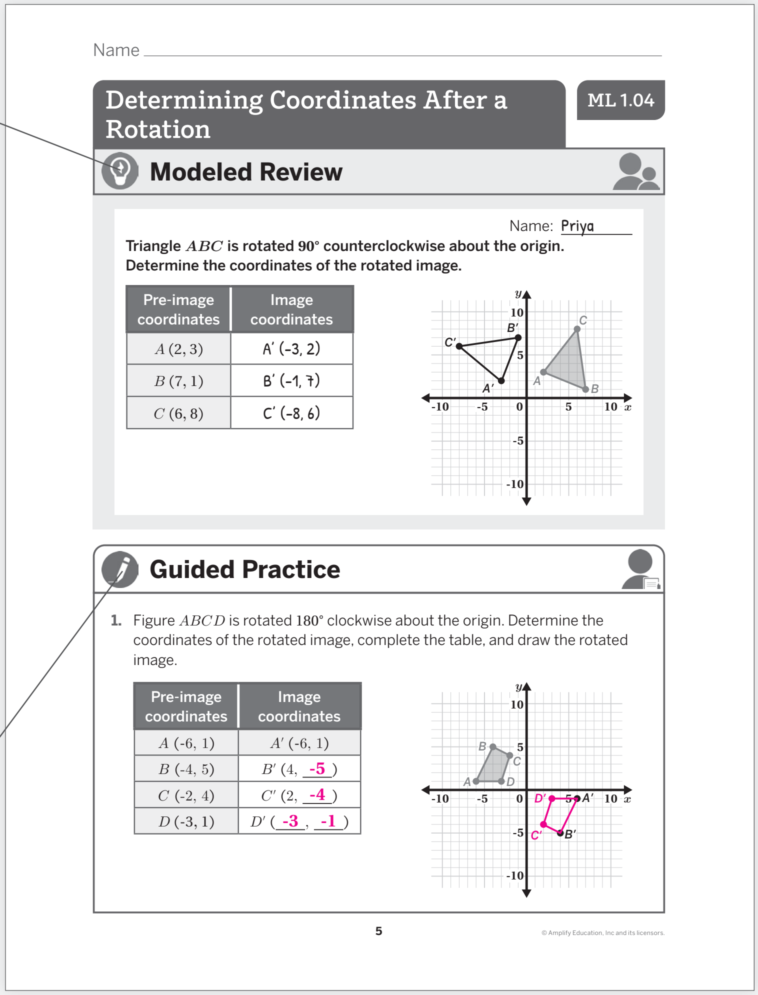
Reinforce Priya's thinking by saying, "When rotating points on a coordinate plane, it is important to identify the direction - clockwise or counterclockwise - and the angle of rotation (90° or 180°) to ensure accurate transformations are carried out."

Guided Practice

Focus students' attention on determining the location of the new coordinates, after the transformation.

To scaffold their thinking, **ask**:

- "What do you notice about the change in the *x*- and *y*-coordinates of a point when it is rotated?"
- "What do you notice about the coordinates of the pre-image and the image?"



Name

Determining Coordinates After a Rotation

ML 1.04

Modeled Review

Name: Priya

Triangle ABC is rotated 90° counterclockwise about the origin. Determine the coordinates of the rotated image.

Pre-image coordinates	Image coordinates
A (2, 3)	A′ (-3, 2)
B (7, 1)	B′ (-1, 7)
C (6, 8)	C′ (-8, 6)

Guided Practice

1. Figure $ABCD$ is rotated 180° clockwise about the origin. Determine the coordinates of the rotated image, complete the table, and draw the rotated image.

Pre-image coordinates	Image coordinates
A (-6, 1)	A' (-6, 1)
B (-4, 5)	B' (4, -5)
C (-2, 4)	C' (2, -4)
D (-3, 1)	D' (-3, -1)

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Vocabulary

If needed, share the meaning of the terms with students.

clockwise: In the same direction as the hands of a clock; to the right (of a turn).

counterclockwise: In the opposite direction as the hands of a clock; to the left (of a turn).

rotation: A move around a given center in a specific direction.

Determining Coordinates After a Rotation

Guided Practice

A As students interpret visual representations, provide access to colored pencils and suggest they color code the points that correspond between the pre-image and image using the same color. Invite them to annotate the pre-image with the word pre-image and the image with the word image to help reinforce the meanings of these terms.

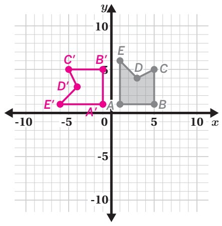
ML/EL Use gestures to illustrate the meaning of the term rotation, highlighting clockwise and counterclockwise motion.

Key Takeaway:
Say, "Coordinates can be used to describe the effect of rotations on figures in the coordinate plane."

Guided Practice

2. Figure $ABCDE$ is rotated 90° counterclockwise about the origin. Determine the coordinates of the rotated image, complete the table, and draw the rotated image.

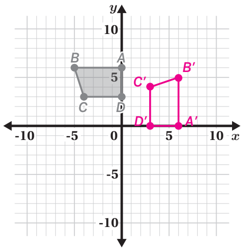
Pre-image coordinates	Image coordinates
A (1, 1)	A' (-1, 1)
B (5, 1)	**B' (-1, 5)**
C (5, 5)	**C' (-5, 5)**
D (3, 4)	**D' (-4, 3)**
E (1, 6)	**E' (-6, 1)**



Check

Trapezoid $ABCD$ is rotated 90° clockwise about the origin. Determine the coordinates of the rotated image, complete the table, and draw the rotated image.

Pre-image coordinates	Image coordinates
A (0, 6)	**A' (6, 0)**
B (-5, 6)	**B' (6, 5)**
C (-4, 3)	**C' (3, 4)**
D (0, 3)	**D' (3, 0)**



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Reflection

Ask:

- "How are rotations, reflections, and translations similar and different?
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them trace the pre-image in Problem 1 on tracing paper, then overlay the tracing paper onto the plane and rotate it 180 degrees. Finally, have them write down the coordinates of the rotated image.

Got it!
If students need more practice, ask them to use the pre-image in Problems 1 and 2 for other transformations, such as 90° or 180° rotations clockwise or counterclockwise about the origin.

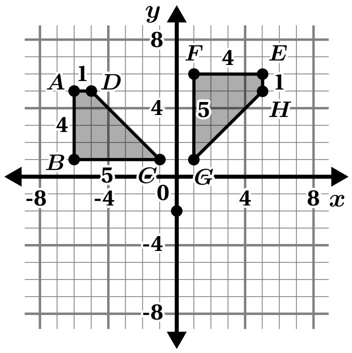
Name ______________________________

Justifying Congruent Figures

ML 1.07

Modeled Review

Name: Lola



Determine whether Figure $ABCD$ is congruent to figure $EFGH$. Explain your thinking.

Yes, ABCD is congruent to EFGH.
It can be rotated 90° CW about the origin..
The corresponding side lengths and angle measures are equal.

Guided Practice

For each pair of figures, determine whether they are congruent. Explain your thinking.

1.

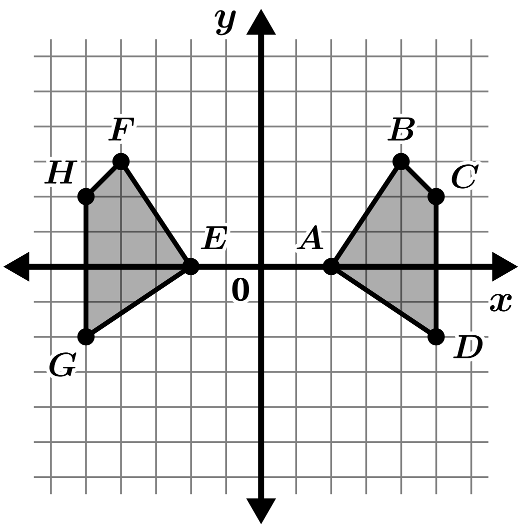


Figure $ABCD$ is __________ onto Figure $EFGH$.
The corresponding side lengths and angle measures are __________.

2.

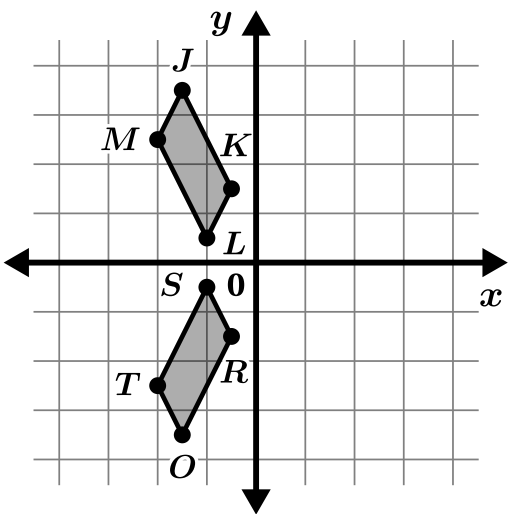
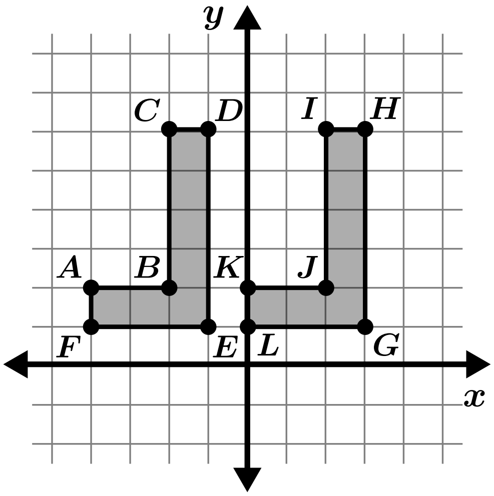


Figure $JKLM$ ______________________
____
____
____
____
____

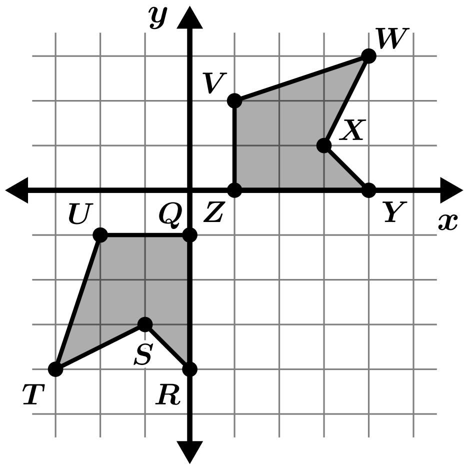
Guided Practice

For each pair of figures, determine whether they are congruent. Explain your thinking.

3.

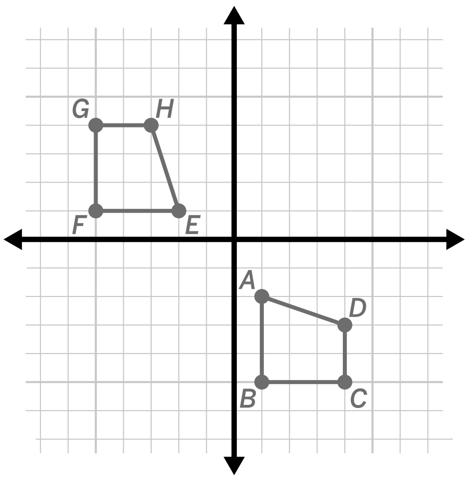


4.



Check

Determine whether Figure $ABCD$ is congruent to Figure $EFGH$. Explain your thinking.



Justifying Congruent Figures

Goal

Determine if two figures are congruent using a series of rigid transformations to move one figure onto another.

Standard

MA.8.GR.2.1 and MA.8.GR.2.3

Materials

straightedge (optional), tracing paper (optional)

Modeled Review

Point to Lola's work and **ask**:

- "What strategies did Lola use to determine if the figures were congruent?"
- "Which tools might she have used to determine her response?"

Reinforce the goal by saying, "You can determine if two figures are congruent by using a rigid transformation to move one figure onto another. If the figures are congruent, their side lengths and angle measures will be equal."

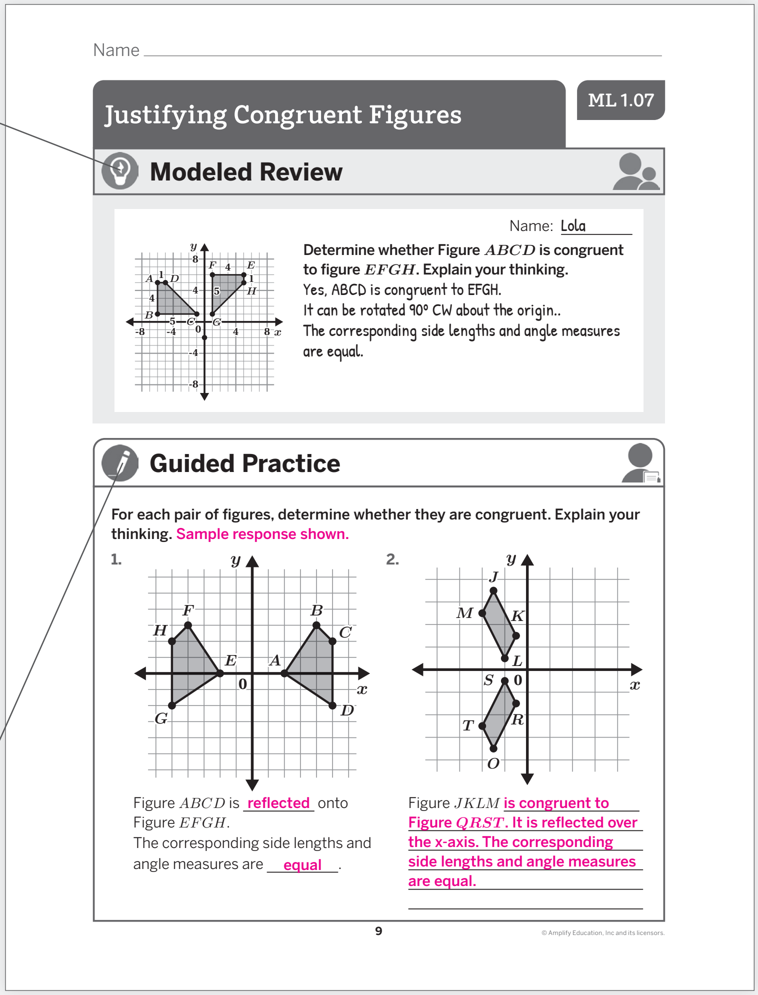
ML/EL Provide students with a straightedge and model how to translate one figure and connect the points.

Guided Practice

Focus students' attention on the transformations to determine whether each pair of figures are congruent.

To scaffold their thinking, **ask**:

- "How can you determine if the two figures are congruent?"
- "What transformation can be used to move one figure onto the other?"



Name

Justifying Congruent Figures

ML 1.07

Modeled Review

Name: Lola

Determine whether Figure $ABCD$ is congruent to figure $EFGH$. Explain your thinking.

Yes, ABCD is congruent to EFGH.
It can be rotated 90° CW about the origin..
The corresponding side lengths and angle measures are equal.

Guided Practice

For each pair of figures, determine whether they are congruent. Explain your thinking. Sample response shown.

1. Figure $ABCD$ is reflected onto Figure $EFGH$.
The corresponding side lengths and angle measures are equal.

2. Figure $JKLM$ is congruent to Figure $QRST$. It is reflected over the x-axis. The corresponding side lengths and angle measures are equal.

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Vocabulary

If needed, share the meaning of the terms with students.

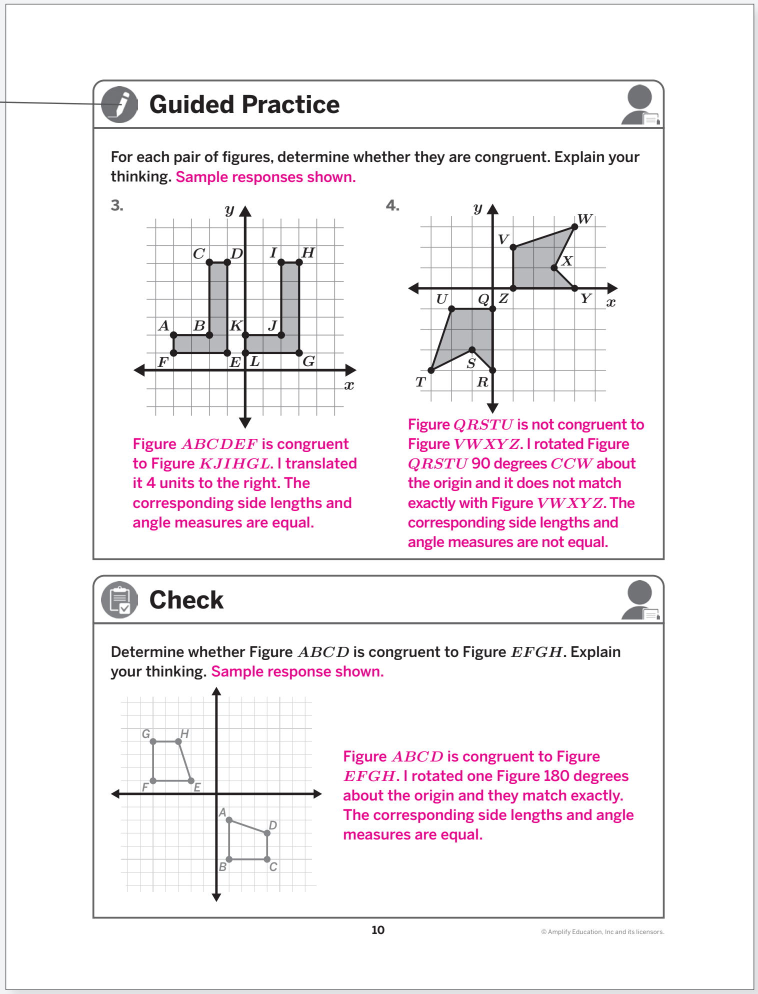
transformation: A rule for moving or changing figures on the plane. Transformations include translations, reflections, rotations, and dilations.

congruent: One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

Guided Practice

A As students interpret the visual representations, invite them to use tracing paper to trace a figure and then determine if they can move the traced figure directly on top of the other figure in the problem. Ask them to record the movements using transformation words such as reflection, translation, and rotation.

Key Takeaway:
Say, "Two figures are congruent when a rigid transformation. That means that in congruent figures, corresponding sides have the same length and corresponding angles have the same measure."



Guided Practice

For each pair of figures, determine whether they are congruent. Explain your thinking. Sample responses shown.

3. Figure $ABCDEF$ is congruent to Figure $KJIHGL$. I translated it 4 units to the right. The corresponding side lengths and angle measures are equal.

4. Figure $QRSTU$ is not congruent to Figure $VWXYZ$. I rotated Figure $QRSTU$ 90 degrees CCW about the origin and it does not match exactly with Figure $VWXYZ$. The corresponding side lengths and angle measures are not equal.

Check

Determine whether Figure $ABCD$ is congruent to Figure $EFGH$. Explain your thinking. Sample response shown.

Figure $ABCD$ is congruent to Figure $EFGH$. I rotated one Figure 180 degrees about the origin and they match exactly. The corresponding side lengths and angle measures are equal.

10

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Reflection

Ask:

- "How do you know whether two figures are congruent?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider providing them with tracing paper and review congruence through transformations of each figure in the mini-lesson.

Got it!
If students need more practice, ask them to plot the following points on the coordinate plane in Problem 4, and determine if *ABCDE* is congruent to *QRSTU*.

- *A* (1, −5)
- *B*(4, −5)
- *C*(4, −1)
- *D*(2, −3)
- *E*(1, −2)

Name ______________________________

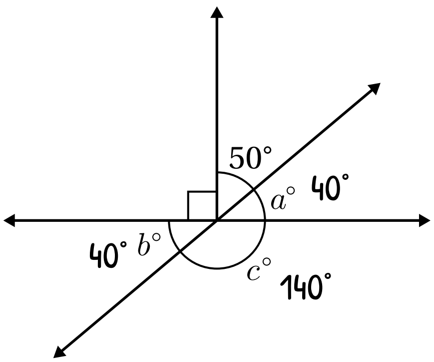
Determining Unknown Angle Measures

ML 1.10

Modeled Review

Name: Gabriel

Determine the values of a, b, and c.



$a + 50 = 90$
$-50 \quad -50$
$a = 40$

$a = b$
$b = 40$

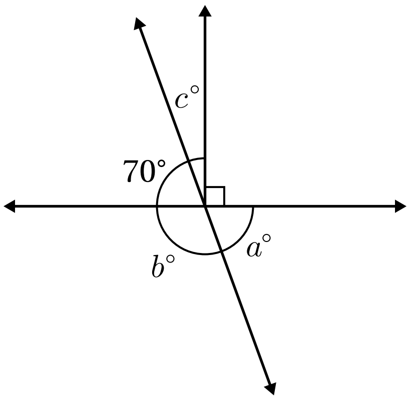
$c + a = 180$
$c + 40 = 180$
$-40 \quad -40$
$c = 140$

$a = 40$, $b = 40$, $c = 140$

Guided Practice

1. Fill in the missing values and solve the equations to determine the values of a, b, and c.

Equation	Relationship
$a = 70$	vertical angles
$c +$ ______ $= 90$	complementary angles
$b +$ ______ $= 180$	supplementary angles

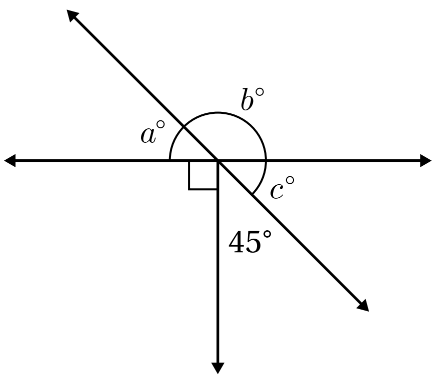


____

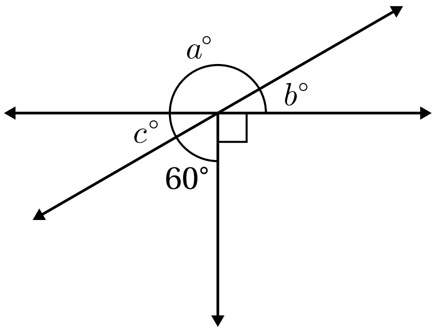
Guided Practice

Determine the values of a, b, and c for each diagram.

2.

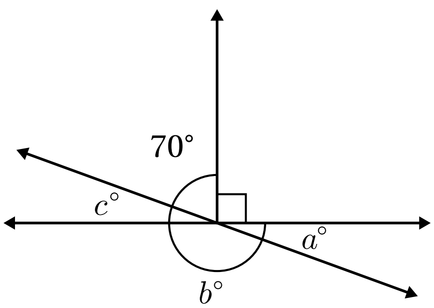


3.



Check

Determine the values of a, b, and c.



ML 1.10 Determining Unknown Angle Measures

Goal

Solve unknown angle measures involving complementary, supplementary, and vertical angles.

Standard

MA.8.GR.1.4

Materials

highlighter or coloring tools (optional)

Modeled Review

Point to Gabriel's work and **ask**:

- "How did Gabriel know which angles are supplementary? Complementary?"
- "How did Gabriel know angles *a* and *b* were equivalent?"
- "Why was it useful to identify if the angles were supplementary, complementary, or vertical?"
- "How could Gabriel check his work?"

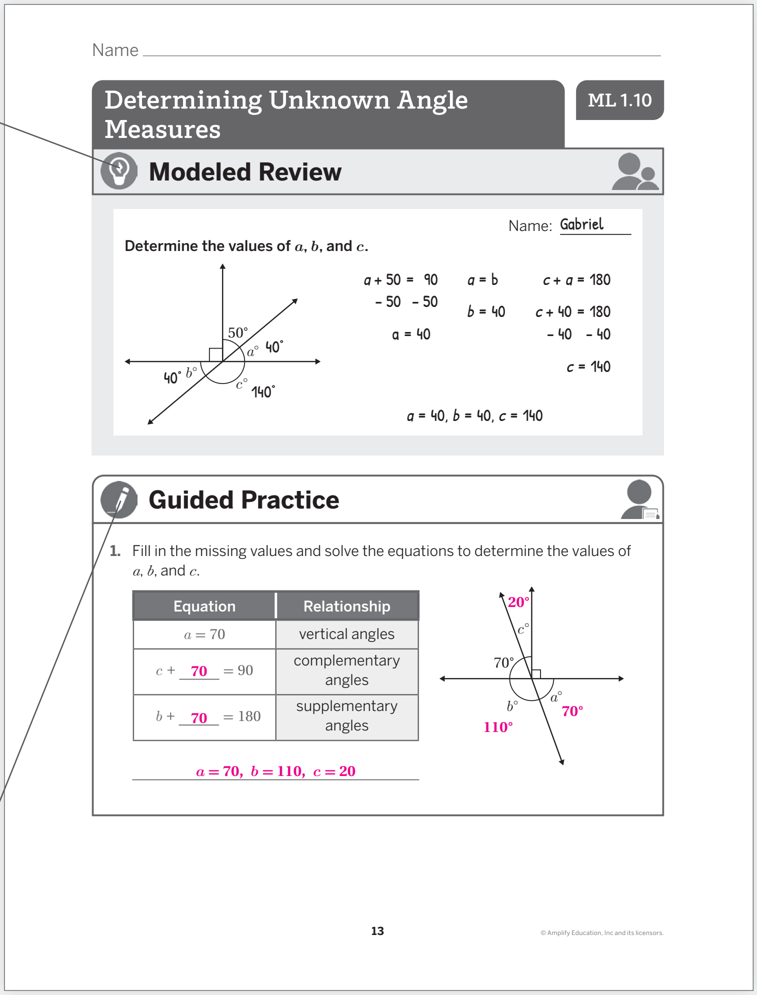
Reinforce Gabriel's thinking by saying, "Writing equations involving complementary, supplementary, and vertical angles can help efficiently calculate the missing angle measures in a diagram."

Guided Practice

Focus students' attention on writing equations to determine the missing angles in the diagram.

To scaffold their thinking, **say**:

- "First, identify any vertical angles."
- "Next, identify any supplementary and complementary angles."
- "Last, write and solve equations to represent these angle relationships."



Name

Determining Unknown Angle Measures

ML 1.10

Modeled Review

Name: Gabriel

Determine the values of a, b, and c.

$a + 50 = 90$
$-50 \quad -50$
$a = 40$

$a = b$
$b = 40$

$c + a = 180$
$c + 40 = 180$
$-40 \quad -40$
$c = 140$

$a = 40$, $b = 40$, $c = 140$

Guided Practice

1. Fill in the missing values and solve the equations to determine the values of a, b, and c.

Equation	Relationship
$a = 70$	vertical angles
$c +$ 70 $= 90$	complementary angles
$b +$ 70 $= 180$	supplementary angles

$a = 70$, $b = 110$, $c = 20$

Vocabulary

If needed, share the meaning of the terms with students.

complementary angles: Two angles whose measures add up to 90°.

supplementary angles: Two angles whose measures add up to 180°.

vertical angles: Angles that are opposite each other when two lines cross. Vertical angles have the same measure.

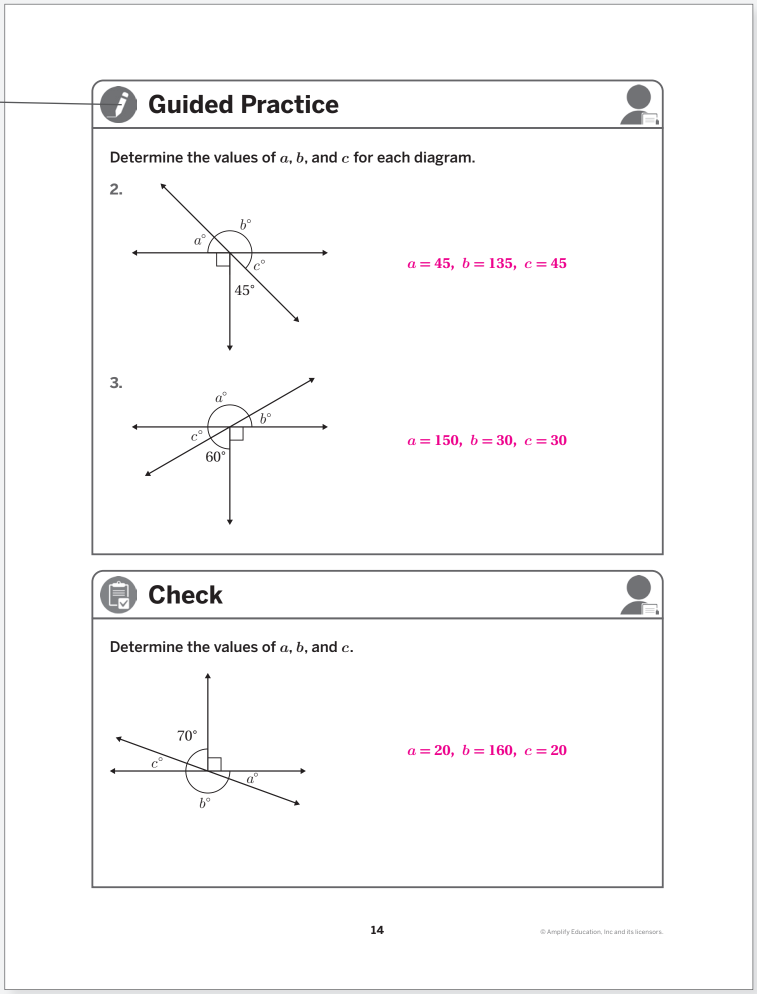
Determining Unknown Angle Measures

Guided Practice

A Invite students to color code the different angle relationships.

ML/EL Display sentence frames to support student use of the words complementary, supplementary, and vertical angles. For example, "Angles ___ and ____ are complementary angles because ___." or "Angles ___ and ___ are vertical angles because ___."

Key Takeaway:
Say, "Writing and solving equations that represent complementary, supplementary, and vertical angle pairs can help determine missing angle measures."



Guided Practice

Determine the values of a, b, and c for each diagram.

2.

$a = 45,\ b = 135,\ c = 45$

3.

$a = 150,\ b = 30,\ c = 30$

Check

Determine the values of a, b, and c.

$a = 20,\ b = 160,\ c = 20$

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Reflection

Ask:

- "What is important to remember when determining missing angle measures in a diagram?"
- "How does what you learned today connect to your prior learning?"

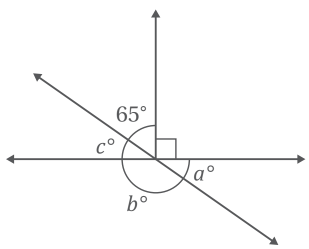
Check: Recommended Next Steps

Almost there

If students need more support, consider reviewing the definitions of complementary, supplementary, and vertical angles and their relationships to each other.

Got it!

If students need more practice, sketch the following diagram and have them determine the values of *a*, *b*, and *c*.



Unit 2

Mini-Lessons

Name ____________________

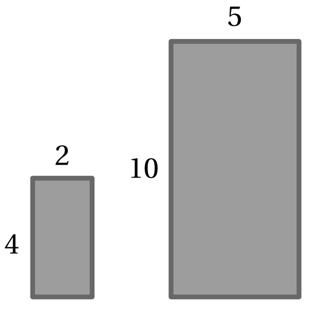
Connecting Scale Factors to Scaled Copies

ML 2.04

Modeled Review

Name: Shawn

What scale factor takes the original rectangle to its scaled copy? Show or explain your reasoning.



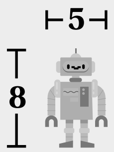
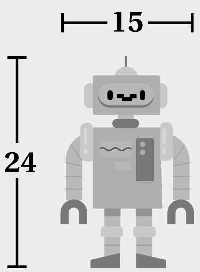
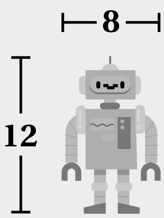
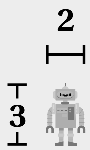
Original **Scaled Copy**

$\frac{\text{new length}}{\text{original length}}$ $\frac{5}{2}$ or $\frac{10}{4}$

$\frac{5}{2}$ is the scale factor.

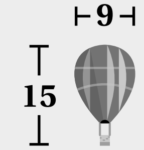
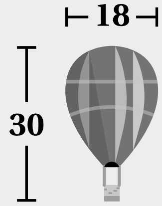
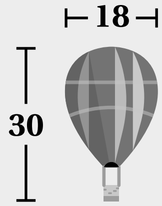
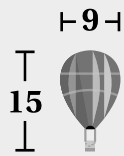
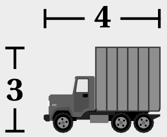
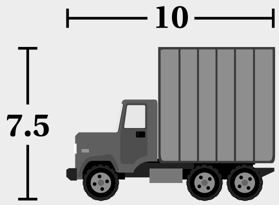
Guided Practice

1. Each original robot has a scaled copy. Complete the table by using the corresponding side lengths to determine the scale factor.

Original	Scaled copy	*Ratio* = $\frac{\textit{New Length}}{\textit{Original Length}}$	Scale factor
 5, 8	 15, 24	$\frac{15}{5}$ or $\frac{24}{8}$	
 8, 12	 2, 3	$\frac{\square}{\square}$ or $\frac{\square}{\square}$	

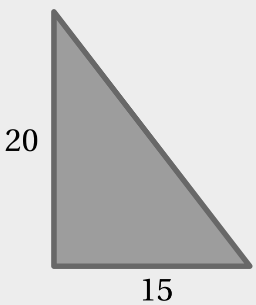
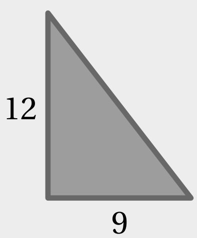
Guided Practice

2. For each pair of objects, determine the scale factor that takes the original to the scaled copy.

Original	Scaled copy	Scale factor
		
		
		

Check

What scale factor takes the original to the scaled copy?

Original	Scaled copy	Scale factor
		

ML 2.04 Connecting Scale Factors to Scaled Copies

Goal	Standard
Determine scale factors using corresponding side lengths.	MA.7.GR.1.5

Modeled Review

Point to Shawn's work and **ask**:

- "Which sides correspond between the original and the scaled copy?"
- "How do the ratios of corresponding measurements help to determine the scale factor?"
- "Why is the scale factor $\frac{5}{2}$ and not $\frac{2}{5}$?"
- "Why is the scale factor $\frac{5}{2}$ or $\frac{10}{4}$? How are these fractions related?"

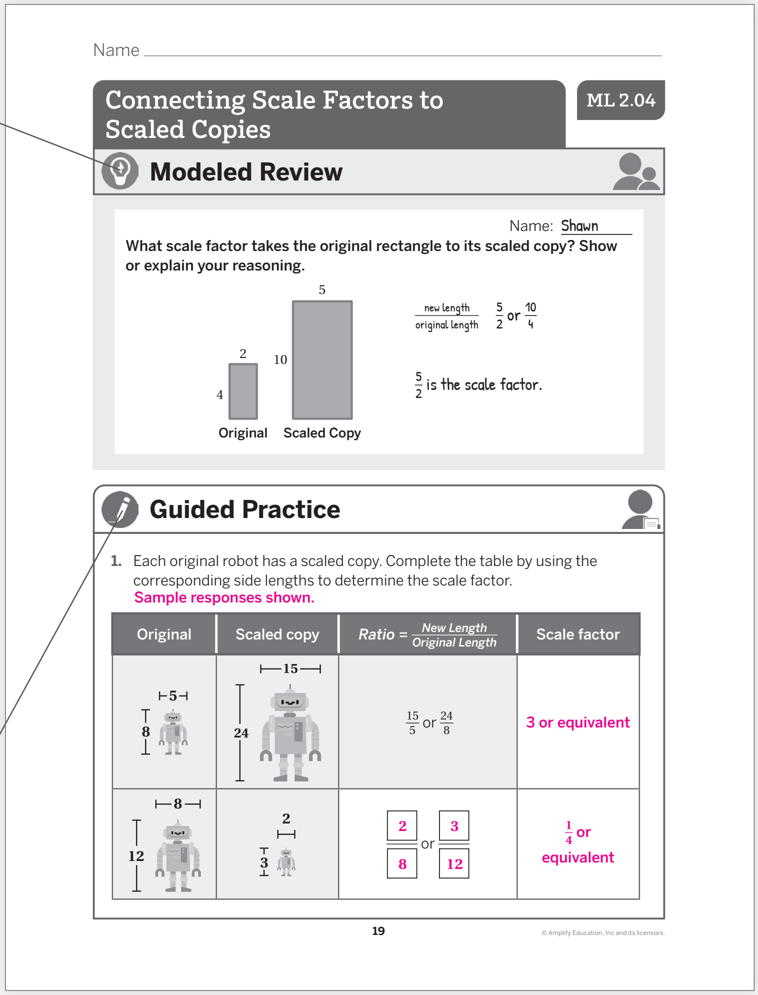
Reinforce the goal by saying, "Scaling an object requires scaling all dimensions."

Guided Practice

Focus students' attention on dividing the new length by the original length when setting up the ratio of corresponding measurements.

To scaffold their thinking, **ask**:

- "What are the corresponding sides of the original and the scaled copy?"
- "How do the ratios of corresponding measurements help to determine the scale factor?"
- "How do you determine the scale factor?"


Name

Connecting Scale Factors to Scaled Copies

ML 2.04

Modeled Review

Name: Shawn

What scale factor takes the original rectangle to its scaled copy? Show or explain your reasoning.

Guided Practice

1. Each original robot has a scaled copy. Complete the table by using the corresponding side lengths to determine the scale factor.
Sample responses shown.

Original	Scaled copy	Ratio = $\frac{\text{New Length}}{\text{Original Length}}$	Scale factor
5, 8	15, 24	$\frac{15}{5}$ or $\frac{24}{8}$	3 or equivalent
8, 12	2, 3	$\frac{2}{8}$ or $\frac{3}{12}$	$\frac{1}{4}$ or equivalent

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Vocabulary

If needed, share the meaning of the terms with students.

scale factor: The number that all side lengths are multiplied by to create a scaled copy.

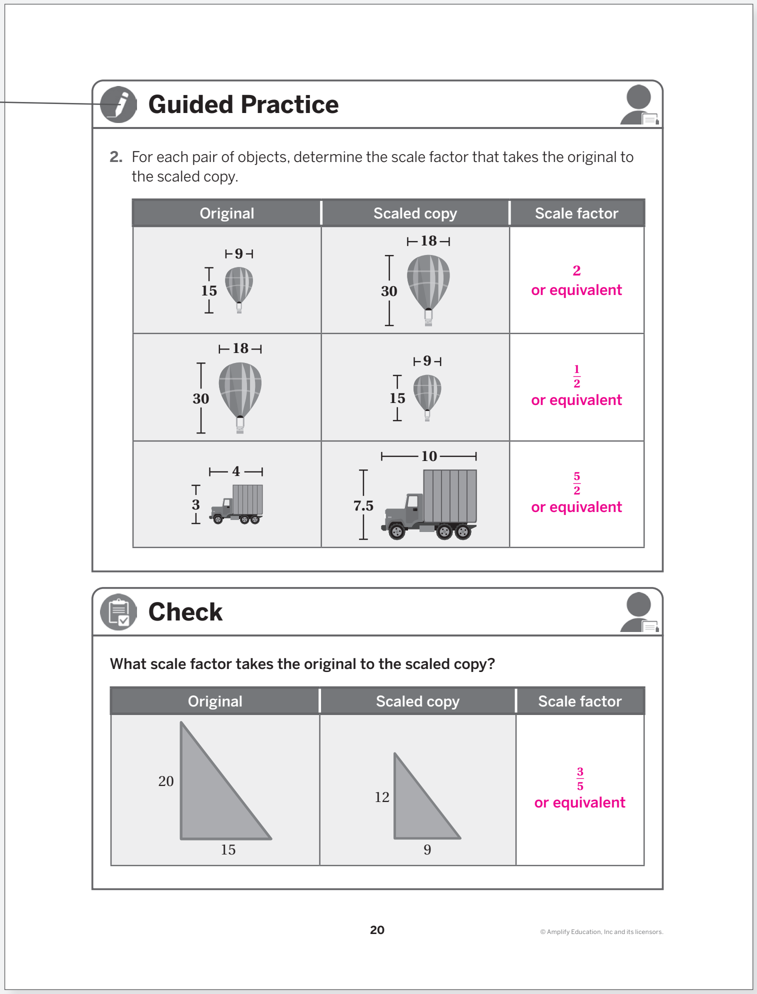
scaled copy: A copy of a figure where every length in the original figure is multiplied by the same number.

Connecting Scale Factors to Scaled Copies

Guided Practice

A If needed, model identifying the corresponding side lengths and determining the scale factor.

Key Takeaway:
Say, "The size of the scale factor affects the size of the scaled copy. If the scale factor is greater than 1, the scaled copy will be larger than the original figure. If the scale factor is between 0 and 1, the scaled copy will be smaller than the original figure. If the scale factor is equal to 1, the scaled copy will be the same size as the original figure."



Guided Practice

2. For each pair of objects, determine the scale factor that takes the original to the scaled copy.

Original	Scaled copy	Scale factor
9, 15	18, 30	2 or equivalent
18, 30	9, 15	$\frac{1}{2}$ or equivalent
4, 3	10, 7.5	$\frac{5}{2}$ or equivalent

Check

What scale factor takes the original to the scaled copy?

Original	Scaled copy	Scale factor
20, 15	12, 9	$\frac{3}{5}$ or equivalent

20

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Reflection

Ask:

- "What do you notice about the scale factor as the original image gets larger? Smaller?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling step-by-step how to determine the scale factor using corresponding side lengths. Break down each step and explain the reasoning behind each calculation.

Got it!
If students need more practice, sketch the following figures. Ask them to determine the scale factor that takes Figure A to Figure B and then takes Figure B to Figure A.

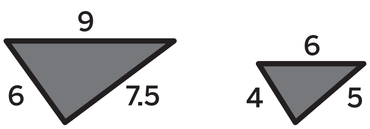


Figure A   Figure B

Name ______________________________

Dilating Figures on a Square Grid

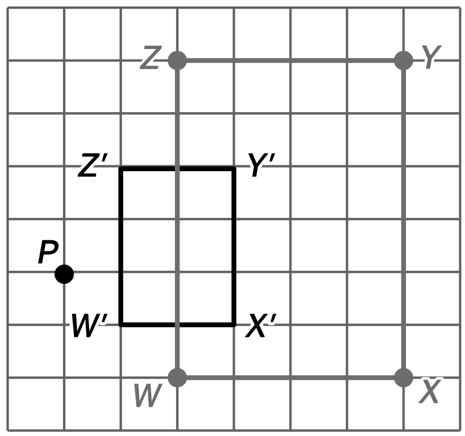
ML 2.10

Modeled Review

Name: Maya

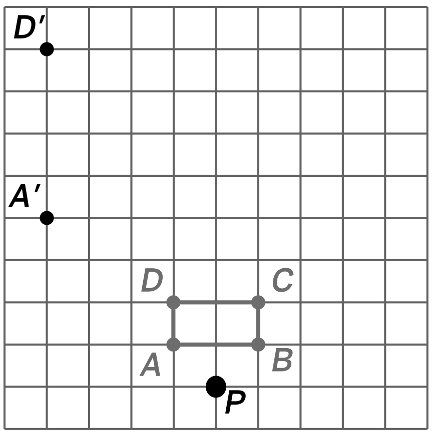
Dilate each vertex of rectangle $WXYZ$ using point P as the center of dilation with a scale factor of $\frac{1}{2}$. Draw the image and label the vertices of the image $W'X'Y'Z'$.

I counted up 4 and right 2 from point P to Z. Then multiplied the distances by $\frac{1}{2}$ to create Z'. I did the same steps to create the image W'X'Y'Z'.

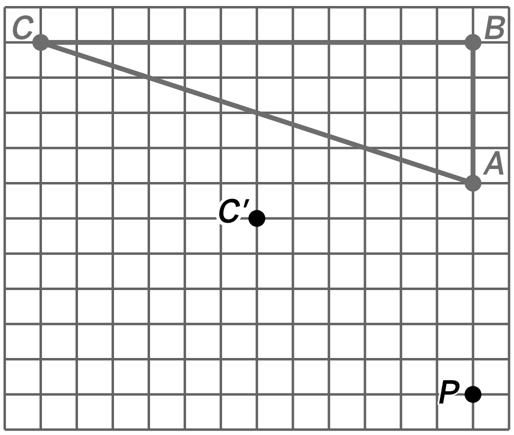


Guided Practice

1. Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of 4. Draw the image and label the missing vertices $B'C'$.

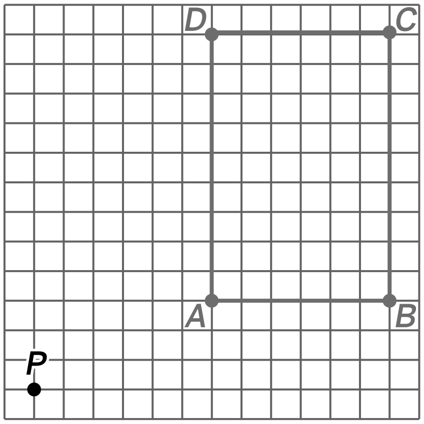


2. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the image and label the missing vertices $A'B'$.

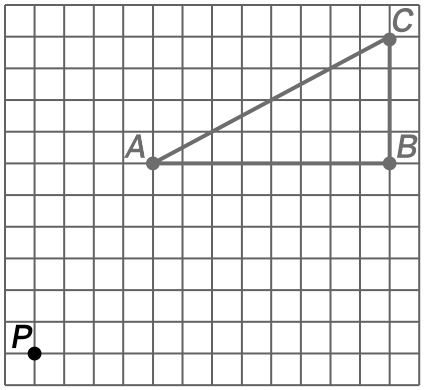


Guided Practice

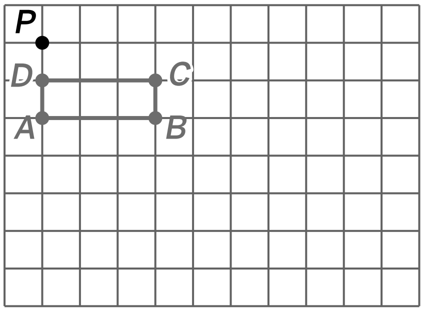
3. Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of $\frac{1}{3}$. Draw the image and label the vertices $A'B'C'D'$.



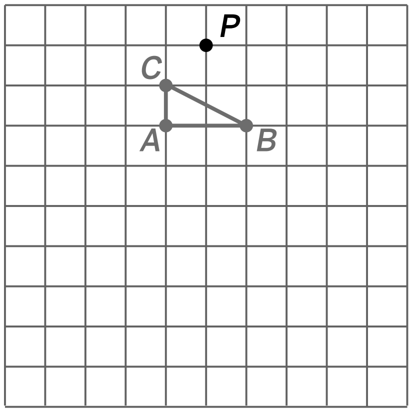
4. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the image and label the vertices $A'B'C'$.



5. Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of 3. Draw the image and label the vertices $A'B'C'D'$.

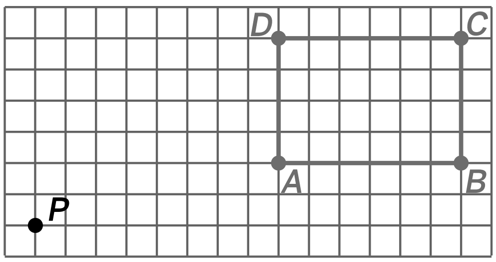


6. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of 4. Draw the image and label the vertices $A'B'C'$.



Check

Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the image and label the vertices $A'B'C'D'$.



Dilating Figures on a Square Grid

Goal

Perform dilations on a square grid.

Standard

MA.8.GR.2.1 and MA.8.GR.2.3

Materials

pen or highlighter (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "How did the scale factor affect the size of the image?"
- "How did Maya use the center of dilation to dilate the figure?"
- "How did Maya dilate the figure using a square grid?"

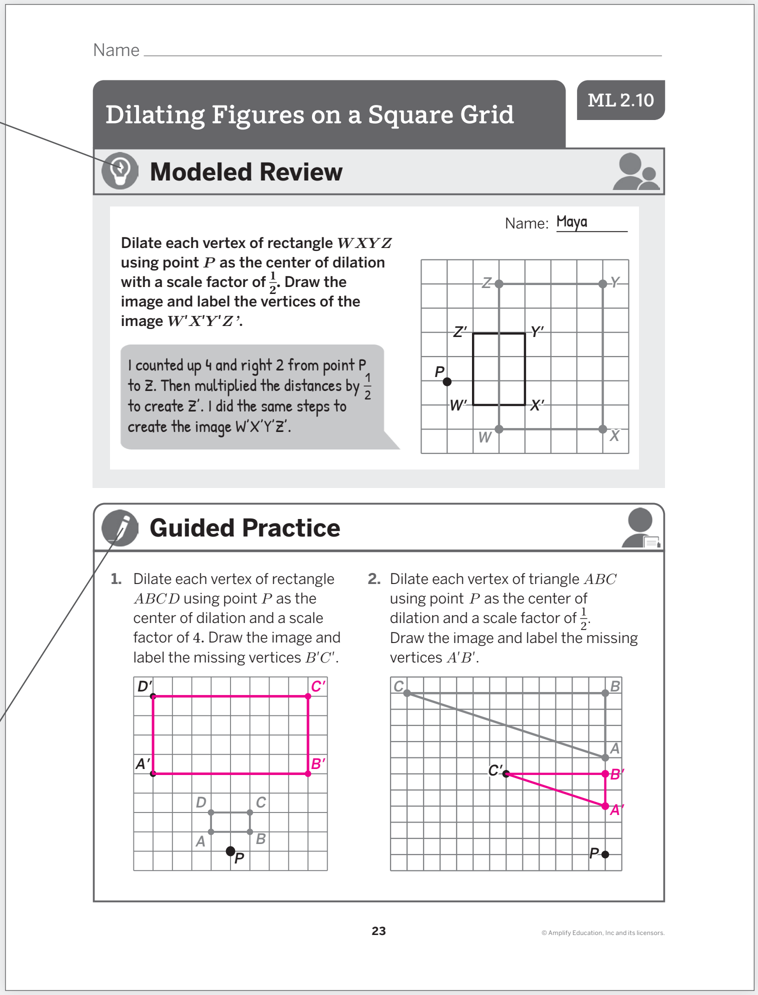
Reinforce the goal by saying, "It is important to be attentive when counting the vertical and horizontal distance from the center of dilation to each vertex on the pre-image."

Guided Practice

Focus students' attention on how to dilate the pre-image to create an image.

To scaffold their thinking, **ask**:

- "Where is the center of dilation?"
- "How do you use the scale factor to dilate the given figures?"



Name

Dilating Figures on a Square Grid

ML 2.10

Modeled Review

Name: Maya

Dilate each vertex of rectangle $WXYZ$ using point P as the center of dilation with a scale factor of $\frac{1}{2}$. Draw the image and label the vertices of the image $W'X'Y'Z'$.

I counted up 4 and right 2 from point P to Z. Then multiplied the distances by $\frac{1}{2}$ to create Z'. I did the same steps to create the image W'X'Y'Z'.

Guided Practice

1. Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of 4. Draw the image and label the missing vertices $B'C'$.

2. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the image and label the missing vertices $A'B'$.

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Vocabulary

If needed, share the meaning of the terms with students.

center of a dilation: The point from which we measure distances in a dilation.

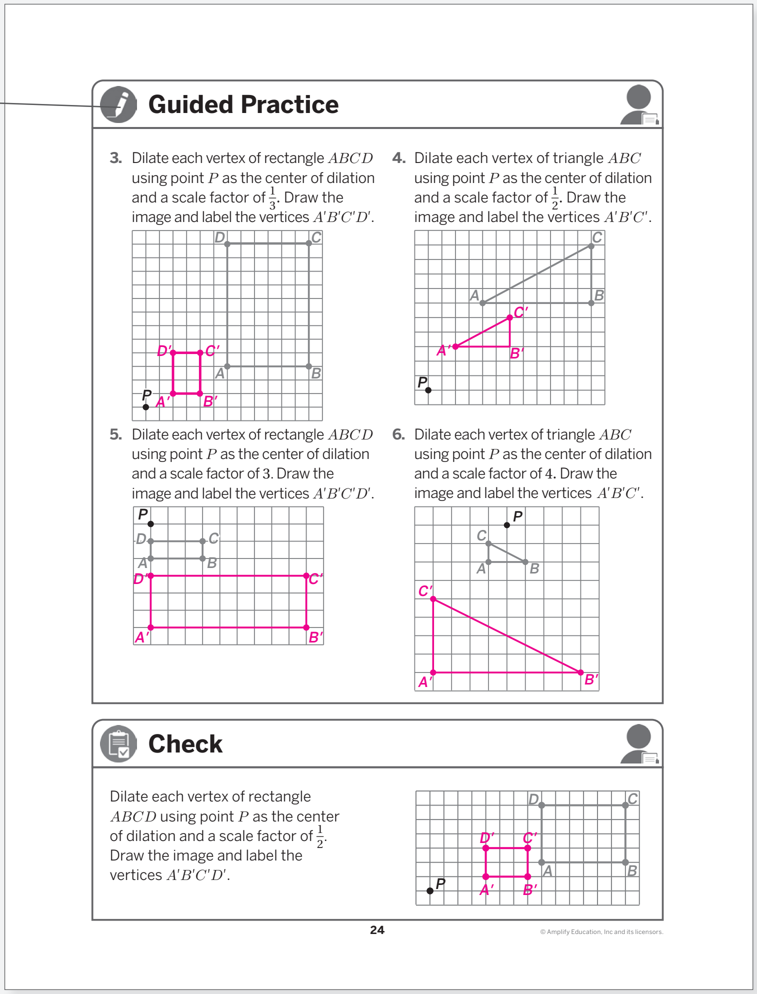
scale factor: To create a dilation, multiply all the distances from the pre-image to the center of dilation by the same number. This number is called the scale factor.

Guided Practice

A Chunk each problem into more manageable parts by inviting students to dilate the figure one vertex at a time.

ML/EL Provide students with pens or highlighters to outline specific points or lines with different colors so that they can track the changes and differentiate the original figure from the dilated figure.

Key Takeaway:
Say, "Descriptive measurements such as 'two up and two over' can be multiplied by the scale factor to create a dilation."



Guided Practice

3. Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of $\frac{1}{3}$. Draw the image and label the vertices $A'B'C'D'$.

4. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the image and label the vertices $A'B'C'$.

5. Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of 3. Draw the image and label the vertices $A'B'C'D'$.

6. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of 4. Draw the image and label the vertices $A'B'C'$.

Check

Dilate each vertex of rectangle $ABCD$ using point P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the image and label the vertices $A'B'C'D'$.

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Reflection

Ask:

- "What strategy is most helpful to you when solving dilating on a square grid?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, ask them to label the number of units as they count the vertical and horizontal distance each vertex is from the center of dilation. Then multiply the distances by the scale factor to create the dilated figure.	If students need more practice, ask them how the image of rectangle *ABCD* in Problem 3 would be similar and different if the center of dilation was changed to point *A* instead of point *P*.

Name ______________________________

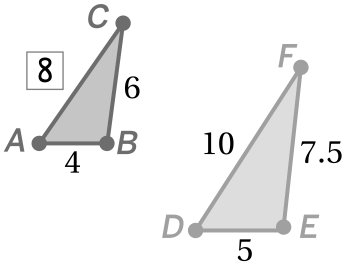
Determining Missing Side Lengths in Similar Triangles

ML 2.14

Modeled Review

Name: Santiago

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.



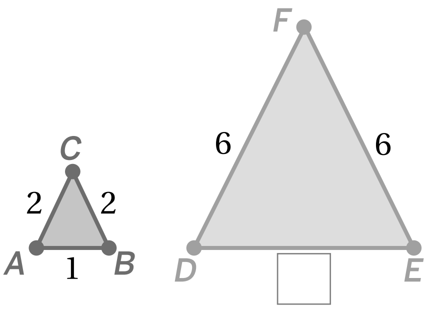
DE to AB is 5 : 4, so the scale factor is 1.25.
I divided DF by 1.5 to determine AC.

$10 \div 1.25 = 8$

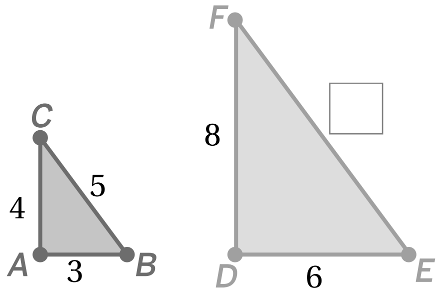
Guided Practice

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.

1.



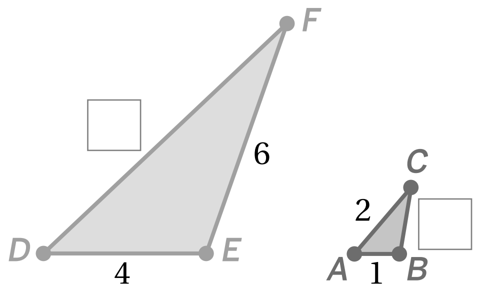
2.



Guided Practice

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.

3.



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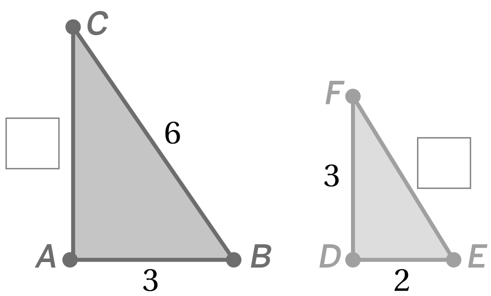
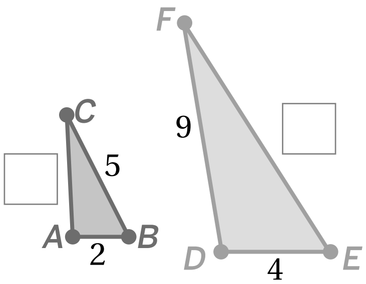


Check

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.



ML 2.14 Determining Missing Side Lengths in Similar Triangles

Goal

Determine missing side lengths in pairs of similar triangles using ratios.

Standard

MA.8.GR.2.4

Materials

colored pencil or highlighter (optional)

Modeled Review

Point to Santiago's work and **ask**:

- "How did Santiago know which side lengths were corresponding?"
- "How did Santiago use the given side lengths of the similar triangles to determine the missing lengths?"

Reinforce the goal by saying, "Use ratios with corresponding side lengths between similar triangles to determine the missing side lengths."

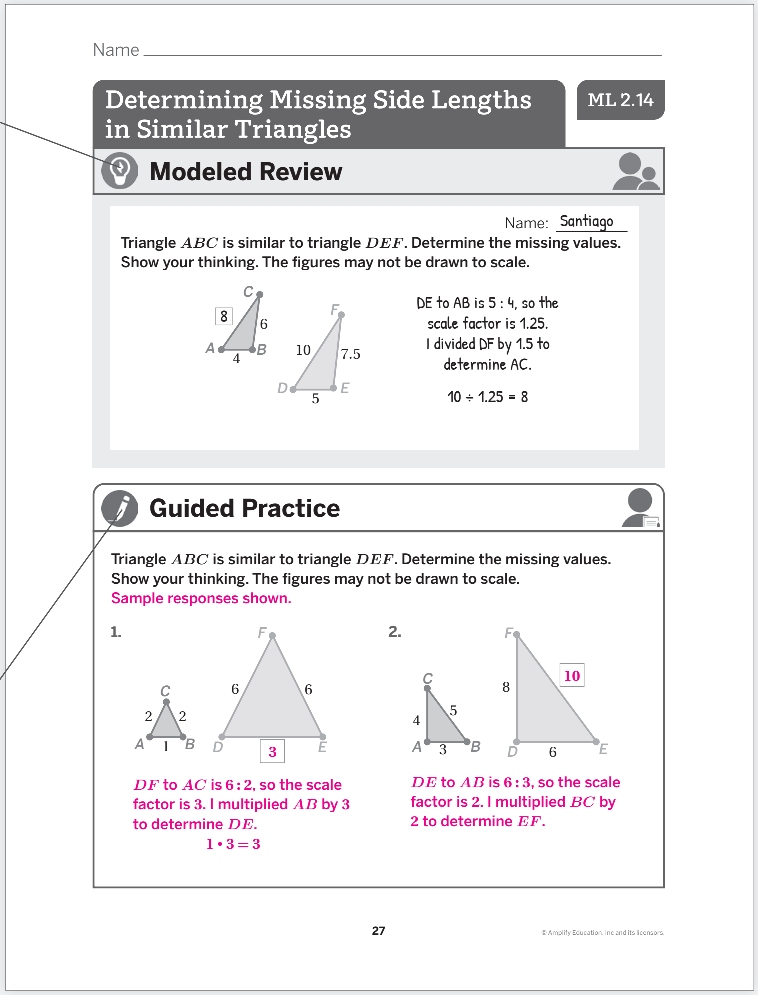
Guided Practice

Focus students' attention on how to use ratios to find the missing side lengths.

To scaffold their thinking, **ask**:

- "What are the corresponding sides that would be helpful to find the missing lengths?"
- "How can we determine the scale factor between the two triangles?"

Note: Problems 1 and 2 show different strategies for solving. Students can solve in the way that makes the most sense to them.



Name

Determining Missing Side Lengths in Similar Triangles

ML 2.14

Modeled Review

Name: Santiago

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.

DE to AB is 5 : 4, so the scale factor is 1.25.
I divided DF by 1.5 to determine AC.

10 ÷ 1.25 = 8

Guided Practice

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.

Sample responses shown.

1. DF to AC is 6 : 2, so the scale factor is 3. I multiplied AB by 3 to determine DE.
   $1 \cdot 3 = 3$

2. DE to AB is 6 : 3, so the scale factor is 2. I multiplied BC by 2 to determine EF.

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Vocabulary

If needed, share the meaning of the term with students.

similar: One figure is similar to another if it can be moved with translations, rotations, reflections, and dilations to fit exactly over the other.

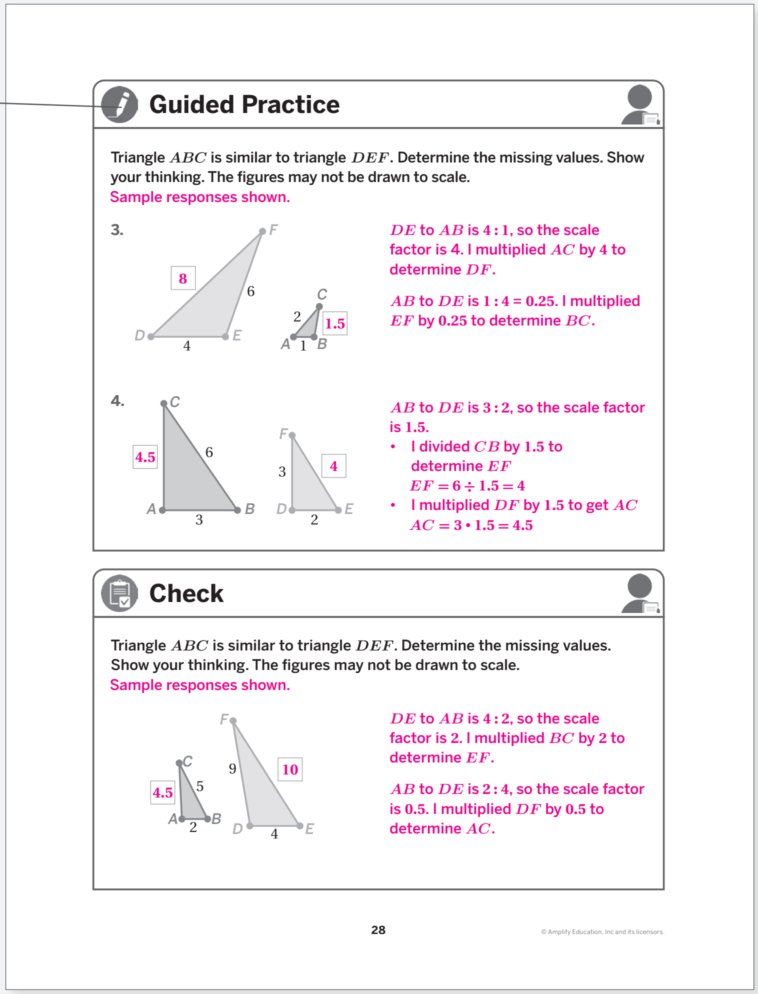
Determining Missing Side Lengths in Similar Triangles

Guided Practice

A As students interpret visual representations, invite them to use colored pencils or highlighters and annotate the lengths of the triangles to help determine the missing lengths.

ML/EL Provide sentence frames to support students as they explain their strategies. For example, "I noticed that ______, so I ______." or "First, I ________ because ______."

Key Takeaway:
Say, "Ratios of corresponding side lengths of a triangle can be used to determine unknown side lengths in similar triangles."



Guided Practice

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.
Sample responses shown.

3. DE to AB is $4:1$, so the scale factor is 4. I multiplied AC by 4 to determine DF.
AB to DE is $1:4 = 0.25$. I multiplied EF by 0.25 to determine BC.

4. AB to DE is $3:2$, so the scale factor is 1.5.
- I divided CB by 1.5 to determine EF
$EF = 6 \div 1.5 = 4$
- I multiplied DF by 1.5 to get AC
$AC = 3 \cdot 1.5 = 4.5$

Check

Triangle ABC is similar to triangle DEF. Determine the missing values. Show your thinking. The figures may not be drawn to scale.
Sample responses shown.

DE to AB is $4:2$, so the scale factor is 2. I multiplied BC by 2 to determine EF.
AB to DE is $2:4$, so the scale factor is 0.5. I multiplied DF by 0.5 to determine AC.

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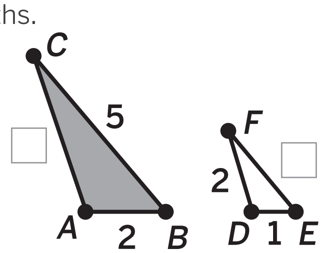
Reflection

Ask:
- "How are ratios helpful in determining unknown side lengths between similar triangles?"
- "After today's lesson, what questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, ask them to highlight corresponding side lengths to assist them in determining which sides should be compared when solving.

Got it!
If students need more practice, sketch these triangles and have them solve for the missing side lengths.



Name ______________________________

Calculating Slope By Drawing Triangles on a Coordinate Plane

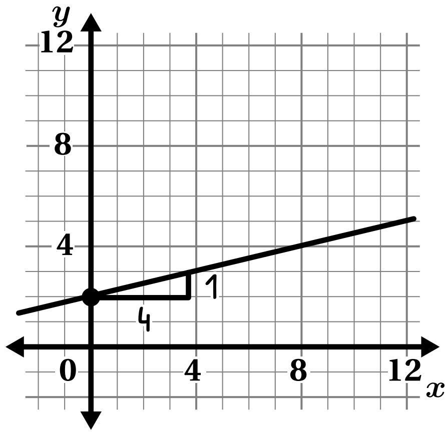
ML 2.16

Modeled Review

Name: Priya

Determine the slope of the line. Show your thinking.

The slope of the line is the ratio between the height of the triangle to its base.

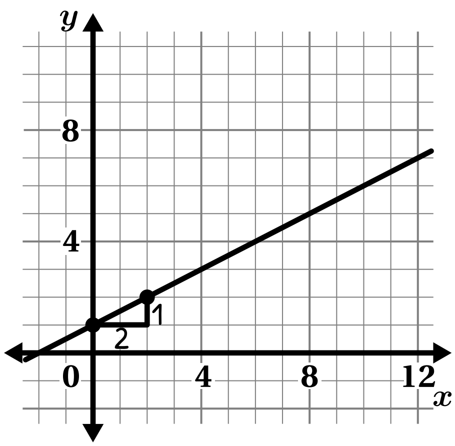


Slope is $\frac{1}{4}$.

Guided Practice

Determine the slope of the line using a slope triangle.

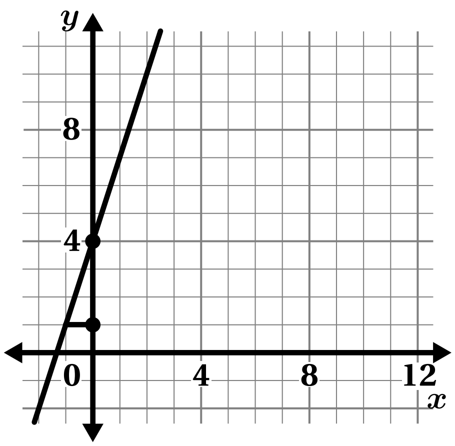
1.



slope = $\frac{\textit{height of slope triangle}}{\textit{base of slope triangle}} = \frac{\square}{\square}$

____

2.



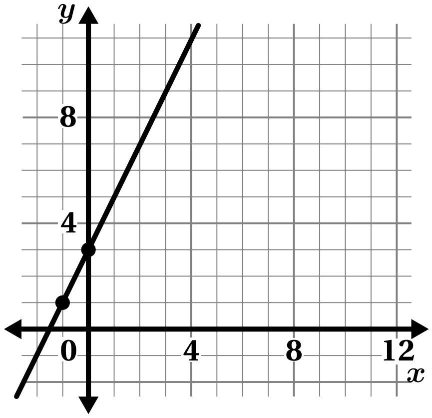
slope = $\frac{\textit{height of slope triangle}}{\textit{base of slope triangle}} = \frac{\square}{\square} = \square$

____

Guided Practice

Determine the slope of the line. Show your thinking.

3.



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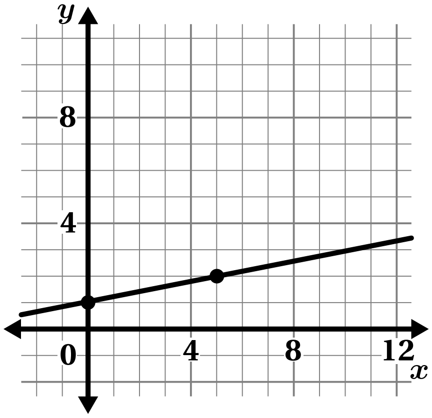

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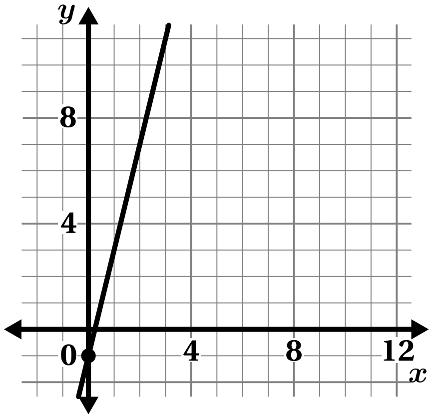

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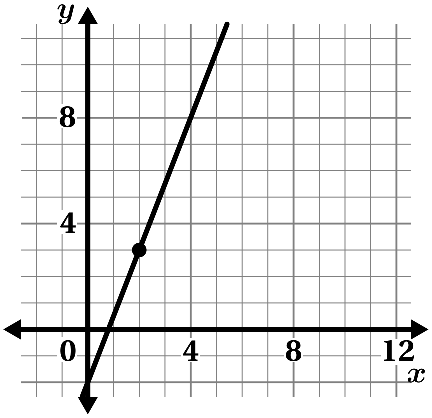
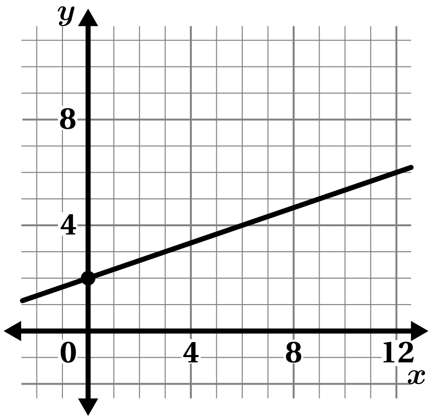


Check

Determine the slope of the line. Show your thinking.



ML 2.16 Calculating Slope By Drawing Triangles on a Coordinate Plane

Goal

Calculate the slope of a line using triangles on a coordinate plane

Standard

MA.8.GR.2.4 and MA.8.GR.3.2

Materials

straightedge (optional), highlighter (optional)

Modeled Review

Point to Priya's work and **ask**:
- "How did Priya use a triangle to find the slope of the line?"
- "Is there another similar triangle Priya could have drawn?"
- "How did Priya calculate the slope?"

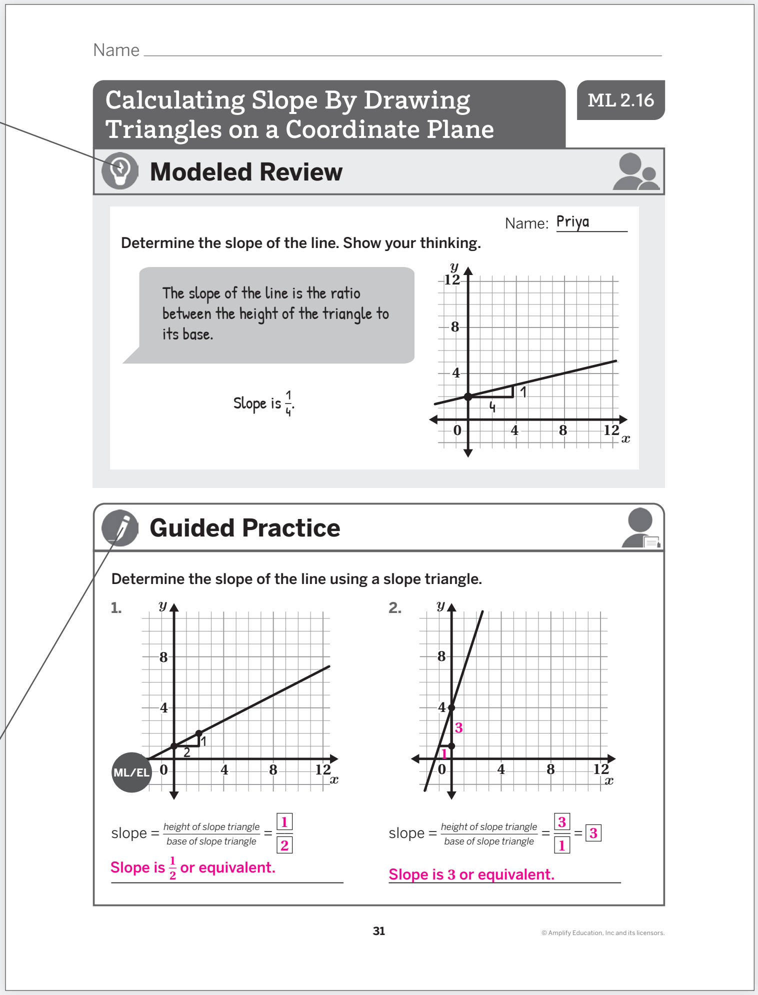
Reinforce the goal by saying, "The slope can be calculated by drawing similar right triangles, called slope triangles, between two points on the line."

ML/EL Model drawing a slope triangle on the coordinate plane.

Guided Practice

Focus students' attention on the line and points given.

To scaffold their thinking, **say**:
- "Identify the points given on the coordinate plane."
- "Create a slope triangle by connecting the two points."
- "Count the height and length of the base of the slope triangle."
- "Divide the height by the length of the base to find the slope."


Name

Calculating Slope By Drawing Triangles on a Coordinate Plane

ML 2.16

Modeled Review

Name: Priya

Determine the slope of the line. Show your thinking.

The slope of the line is the ratio between the height of the triangle to its base.

Slope is $\frac{1}{4}$.

Guided Practice

Determine the slope of the line using a slope triangle.

1. slope = $\frac{\text{height of slope triangle}}{\text{base of slope triangle}} = \frac{1}{2}$

Slope is $\frac{1}{2}$ or equivalent.

2. slope = $\frac{\text{height of slope triangle}}{\text{base of slope triangle}} = \frac{3}{1} = 3$

Slope is 3 or equivalent.

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Vocabulary

If needed, share the meaning of the terms with students.

slope: A number that describes the direction and steepness of a line.

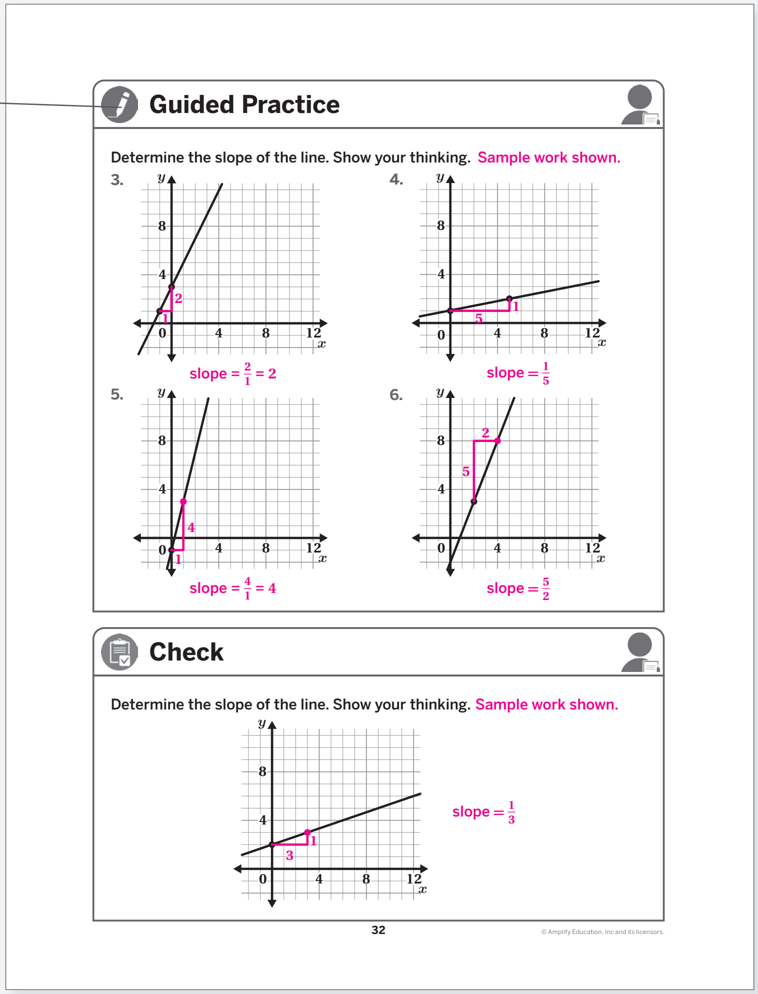
slope triangle: A slope triangle for a line is a triangle whose longest side lies on the line and whose other two sides are vertical and horizontal.

ML 2.16 Calculating Slope By Drawing Triangles on a Coordinate Plane

Guided Practice

A As students interpret visual representations, provide straightedges that students can use to create slope triangles. Invite students to count the number of squares on the height and the base to determine slope.

Key Takeaway:
Say, "Constructing a slope triangle and calculating its height to the length of its base is a strategy for determining slope."



Guided Practice

Determine the slope of the line. Show your thinking. Sample work shown.

3. slope = $\frac{2}{1} = 2$

4. slope = $\frac{1}{5}$

5. slope = $\frac{4}{1} = 4$

6. slope = $\frac{5}{2}$

Check

Determine the slope of the line. Show your thinking. Sample work shown.

slope = $\frac{1}{3}$

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Reflection

Ask:

- "How can you use a slope triangle to find the slope of a line?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, have them use two different colors to highlight the height and the base, and divide the height by the length of the base to determine the slope.

Got it!
If students need more practice, refer to Problem 2 and ask how the slope triangle and the slope would change if the coordinate (2, 10) was defined on the line.

Unit 3

Mini-Lessons

Name ______________________________

Determining the Constant of Proportionality

ML 3.02

Modeled Review

Name: Clare

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

The constant of proportionality is 5.

The paint is made with 5 cups of yellow paint for every 1 cup of blue paint.

$2 \cdot 5 = 10$
$6 \cdot 5 = 30$

Blue paint (cups)	Yellow paint (cups)
2	10
1	5
6	30
52	260

Guided Practice

Determine the constant of proportionality for the relationship and complete the table.

1. $8 \cdot 3 = 24$

$12 \cdot$ _____ = _____

$128 \cdot$ _____ = _____

The constant of proportionality is _____.

Lemonade

	Volume (oz)	Sugar (g)
glass	8	24
bottle	12	
carton	32	96
jug	128	

2. $8 \cdot$ _____ = _____

$12 \cdot$ _____ = _____

$128 \cdot$ _____ = _____

The constant of proportionality _____.

Apple juice

	Volume (oz)	Sugar (g)
glass	8	
bottle	12	
carton	32	80
jug	128	

Guided Practice

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

3. The constant of proportionality is_____.

The paint is made with _____ cups of blue paint for every _____ cup of red paint.

Red paint (cups)	Blue paint (cups)
2	3
1	
6	
40	60

4. The constant of proportionality is_____.

Red paint (cups)	Yellow paint (cups)
2	8
1	
	32
50	

Check

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

White paint (cups)	Red paint (cups)
2	4
1	
7	
	80

ML 3.02 Determining the Constant of Proportionality

Goal

Determine the constant of proportionality for a proportional relationship.

Standard

MA.7.AR.4.1

Materials

calculators (optional)

Modeled Review

Point to Clare's work and **ask**:

- "What did Clare do to find the constant of proportionality?"
- "How did Clare use the constant of proportionality to find the missing values?"

Reinforce the goal by saying, "When a table shows a proportional relationship, the values in one column can be multiplied by the same number, the constant of proportionality, to get the values in the other column."

Guided Practice

Focus students' attention on using the table to determine the constant of proportionality.

To scaffold their thinking, **ask**:

- "How could you determine the constant of proportionality?"
- "How does the constant of proportionality help you determine the missing values?"

Name ______________________________

Determining the Constant of Proportionality

ML 3.02

Modeled Review

Name: Clare

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

The constant of proportionality is 5.

The paint is made with 5 cups of yellow paint for every 1 cup of blue paint.

$2 \cdot 5 = 10$
$6 \cdot 5 = 30$

Blue paint (cups)	Yellow paint (cups)
2	10
1	5
6	30
52	260

×5

Guided Practice

Determine the constant of proportionality for the relationship and complete the table.

1. $8 \cdot 3 = 24$
$12 \cdot \underline{3} = \underline{36}$
$128 \cdot \underline{3} = \underline{384}$

The constant of proportionality is $\underline{3}$.

Lemonade

	Volume (oz)	Sugar (g)
glass	8	24
bottle	12	36
carton	32	96
jug	128	384

×3

2. $8 \cdot \underline{2.5} = \underline{20}$
$12 \cdot \underline{2.5} = \underline{30}$
$128 \cdot \underline{2.5} = \underline{320}$

The constant of proportionality $\underline{2.5}$.

Apple juice

	Volume (oz)	Sugar (g)
glass	8	20
bottle	12	30
carton	32	80
jug	128	320

37

Vocabulary

If needed, share the meaning of the term with students.

constant of proportionality: A number that the value of one quantity is multiplied by to get the value of the other quantity in a proportional relationship.

ML 3.02 Determining the Constant of Proportionality

Guided Practice

A As students identify patterns and relationships, invite them to annotate the tables to show how to obtain each value using multiplication and the constant of proportionality.

ML/EL Provide sentence frames to help students explain their strategy such as:

- "I know the paint is the same color because___."
- "I determined the missing paint amounts by___."

Key Takeaway:
Say, "In a proportional relationship, the values for one quantity are multiplied by the same number to obtain the values for the other quantity. This number is called the constant of proportionality."

Guided Practice

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation? Sample response shown for Problem 4.

3. The constant of proportionality is 1.5.

The paint is made with 1.5 cups of blue paint for every 1 cup of red paint.

Red paint (cups)	Blue paint (cups)
2	3
1	1.5
6	9
40	60

4. The constant of proportionality is 4.

The paint is made with 4 cups of yellow paint for every 1 cup of red paint.

Red paint (cups)	Yellow paint (cups)
2	8
1	4
8	32
50	200

Check

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation? Sample response shown.

The constant of proportionality is 2.

The paint is made with 2 cups of red paint for every 1 cup of white paint.

White paint (cups)	Red paint (cups)
2	4
1	2
7	14
40	80

38

Reflection

Ask:

- "How do you use the constant of proportionality to find missing values?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, explain that the constant of proportionality can also be found by dividing the value in the second column by the corresponding value in the first column.

Got it!
If students need more practice, have them revisit Problem 3 and continue completing the table to determine how much blue paint would be needed if the amount of the red paint is:

- 10 cups
- 15 cups
- 30 cups
- 100 cups

Name ______________________________

Finding More Than One Equation to Represent Proportional Relationships

ML 3.06

Modeled Review

Name: Jason

A car traveled 150 miles in 2 hours at a constant speed.

1. What are two constants of proportionality for the relationship between distance in miles, d, and number of hours, t, that the car traveled?

$\frac{150}{2}$ and $\frac{2}{150}$

2. Write two equations that relate d and t in this situation.

$d = \frac{150}{2}t$ and $t = \frac{2}{150}d$

Guided Practice

Complete the table and determine the constant of proportionality. Write an equation to represent the situation.

1.

x	y
1	
2	10
4	
t	$5t$

constant of proportionality: _______

equation: _______

2.

x	y
1	
4	
8	
w	$\frac{1}{4}w$

constant of proportionality: _______

equation: _______

Guided Practice

There are 400 centimeters in every 4 meters.

3. Determine two constants of proportionality for the relationship between meters, x, and centimeters, y.

4. Write two equations that relate x and y in this situation.

It took 6 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.

5. Determine two constants of proportionality for the relationship between time, t, and gallons of water, w.

6. Write two equations that relate t and w in this situation.

Check

A polar bear swam 426 miles across the Beaufort Sea, north of Alaska, in 9 days.

1. Determine two constants of proportionality for the relationship between distance in miles, d, and number of days, t, that the polar bear swam.

2. Write two equations that relate d and t in this situation.

ML 3.06 Finding More Than One Equation to Represent Proportional Relationships

Goal

Determine the constant of proportionality for a proportional relationship.

Standard

MA.7.AR.4.4

Materials

calculators (optional)

Modeled Review

Point to Jason's work and **ask**:

- "Why are you able to write two equations for any proportional relationship?"
- "How are the constants of proportionality related in the two equations?"

Reinforce Jason's thinking by saying, "Two equations can be used to represent a proportional relationship by using the reciprocal."

ML/EL Have students give examples of pairs of numbers that are reciprocals to formalize new vocabulary.

Guided Practice

Focus students' attention on writing an equation for the given proportional relationship.

To scaffold their thinking, **ask**:

- "How did you find an equation for each table?"
- "Where do you see the constant of proportionality in the table and in the equation?"
- "How are Problems 1 and 2 similar? Different?"

Name

Finding More Than One Equation to Represent Proportional Relationships ML 3.06

Modeled Review

Name: Jason

A car traveled 150 miles in 2 hours at a constant speed.

1. What are two constants of proportionality for the relationship between distance in miles, d, and number of hours, t, that the car traveled?

$\frac{150}{2}$ and $\frac{2}{150}$

2. Write two equations that relate d and t in this situation.

$d = \frac{150}{2}t$ and $t = \frac{2}{150}d$

Guided Practice

Complete the table and determine the constant of proportionality. Write an equation to represent the situation.

1.

x	y
1	5
2	10
4	20
t	$5t$

constant of proportionality: 5

equation: $y = 5x$

2.

x	y
1	$\frac{1}{4}$
4	1
8	2
w	$\frac{1}{4}w$

constant of proportionality: $\frac{1}{4}$

equation: $y = \frac{1}{4}x$

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Vocabulary

If needed, share the meaning of the term with students.

reciprocal: Two numbers whose product is 1.

ML 3.06 Finding More Than One Equation to Represent Proportional Relationships

Guided Practice

A To support organization in problem solving, chunk this activity into more manageable parts by inviting students to generate one constant of proportionality and/or equation at a time. Then have them compare the reciprocal.

Note: If students need additional support, consider asking:

- "What is the relationship between the two constants of proportionality?"
- "How could determining one equation help you to determine another equation to describe the relationship?"

Key Takeaway:
Say, "For every proportional relationship, there are two constants of proportionality that are reciprocals of each other, and two equations that can be written to represent each proportional relationship."

Guided Practice

There are 400 centimeters in every 4 meters. Sample responses shown.

3. Determine two constants of proportionality for the relationship between meters, x, and centimeters, y.

   $\frac{400}{4}$ (or equivalent) and $\frac{4}{400}$ (or equivalent)

4. Write two equations that relate x and y in this situation.

   $y = \frac{400}{4}x$ (or equivalent) and $x = \frac{4}{400}y$ (or equivalent)

It took 6 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate. Sample responses shown.

5. Determine two constants of proportionality for the relationship between time, t, and gallons of water, w.

   $\frac{6}{8}$ (or equivalent) and $\frac{8}{6}$ (or equivalent)

6. Write two equations that relate t and w in this situation.

   $t = \frac{6}{8}w$ (or equivalent) and $w = \frac{8}{6}t$ (or equivalent)

Check

A polar bear swam 426 miles across the Beaufort Sea, north of Alaska, in 9 days. Sample responses shown.

1. Determine two constants of proportionality for the relationship between distance in miles, d, and number of days, t, that the polar bear swam.

   $\frac{426}{9}$ (or equivalent) and $\frac{9}{426}$ (or equivalent)

2. Write two equations that relate d and t in this situation.

   $d = \frac{426}{9}t$ (or equivalent) and $t = \frac{9}{426}d$ (or equivalent)

42

Reflection

Ask:

- "What is the biggest takeaway you have from this lesson?"
- "What questions do you still have after this lesson?"

Check: Recommended Next Steps

Almost there
If students need more support, model and explain how multiplying by the reciprocal can be used to simplify the process of dividing fractions to create the second equation.

Got it!
If students need more practice, present them with the following problem. Have them determine the constants of proportionality and equations to represent the situation.

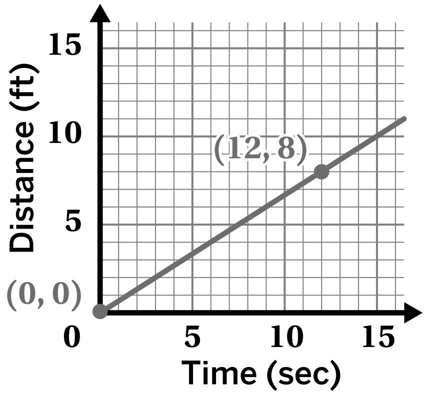
- A truck driving 480 miles, *d*, in 7 hours, *t*.
- A dog running 1600 feet, *d*, in 2 minutes, *t*.

Name ____________________

Comparing Proportional Relationships With Graphs

ML 3.10

Modeled Review



Name: Gabriela

This line represents a proportional relationship because it begins at the origin. Its constant of proportionality is $\frac{8}{12}$ or $\frac{2}{3}$. The equation for the line is $y = \frac{2}{3}x$.

Guided Practice

1. A red turtle moves _10_ feet in _5_ seconds.

Its speed is _2_ ft/sec. Equation: $y = 2x$

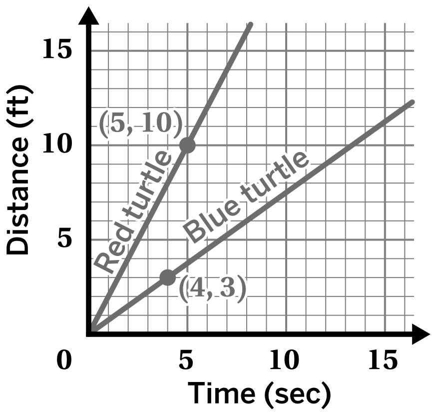
A blue turtle moves ____ feet in ____ seconds.

Its speed is $\frac{3}{4}$ ft/sec. Equation: $y = \frac{3}{4}x$

Which turtle moves *faster* ? How do you know?

____

____



2. A red turtle moves ____ feet in ____ seconds.

Its speed is ____ ft/sec. Equation: $y = \frac{5}{6}x$

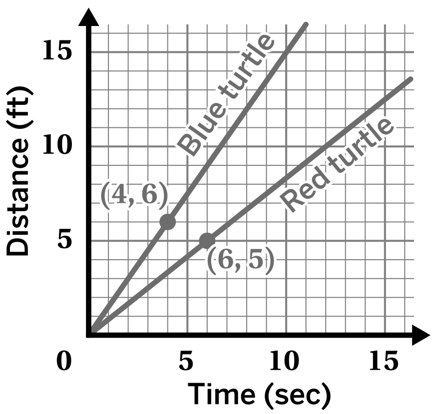
A blue turtle moves ____ feet in ____ seconds.

Its speed is ____ ft/sec. Equation: $y = \frac{3}{2}x$

Which turtle moves slower ? How do you know?

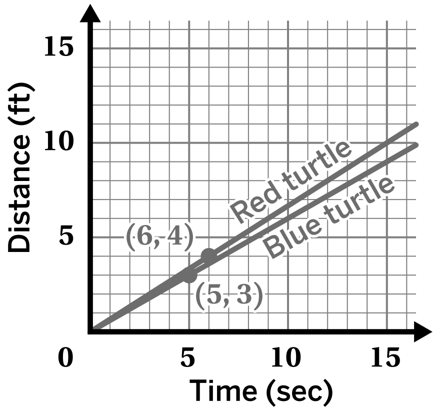
____

____

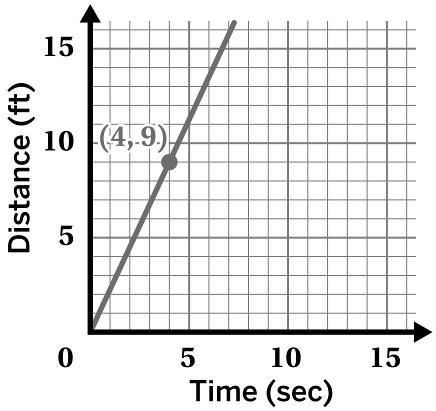


Guided Practice

3. A red turtle moves _____ feet in _____ seconds.
   Its speed is _____ ft/sec. Equation: ________
   A blue turtle moves _____ feet in _____ seconds.
   Its speed is _____ ft/sec. Equation: ________
   Which turtle moves *slower* ? How do you know?
   ____
   ____



4. A red turtle moves according to the equation $y = 2x$, where y represents the distance in feet and x represents the time in seconds.
   Here is a graph that shows the distance a blue turtle moves over time.
   Which turtle moves *faster*? How do you know?
   ____
   ____
   ____

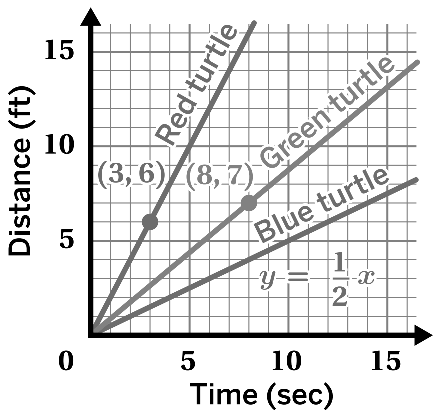


Check

Here is a graph that shows the distance three turtles move over time.

1. Which turtle moves the *fastest*? How do you know?
   ____
   ____

2. Write an equation for the turtle represented by the green line.



ML 3.10 Comparing Proportional Relationships With Graphs

Goal

Use constants of proportionality (as determined using points or equations) or graphs to compare proportional relationships.

Standard

MA.7.AR.4.3

Materials

graph paper (optional)

Modeled Review

Point to Gabriela's work and **ask**:

- "How did Gabriela calculate the constant of proportionality?"
- "How did Gabriela determine the equation for the line?"
- "How do you know this line and equation show a proportional relationship?"

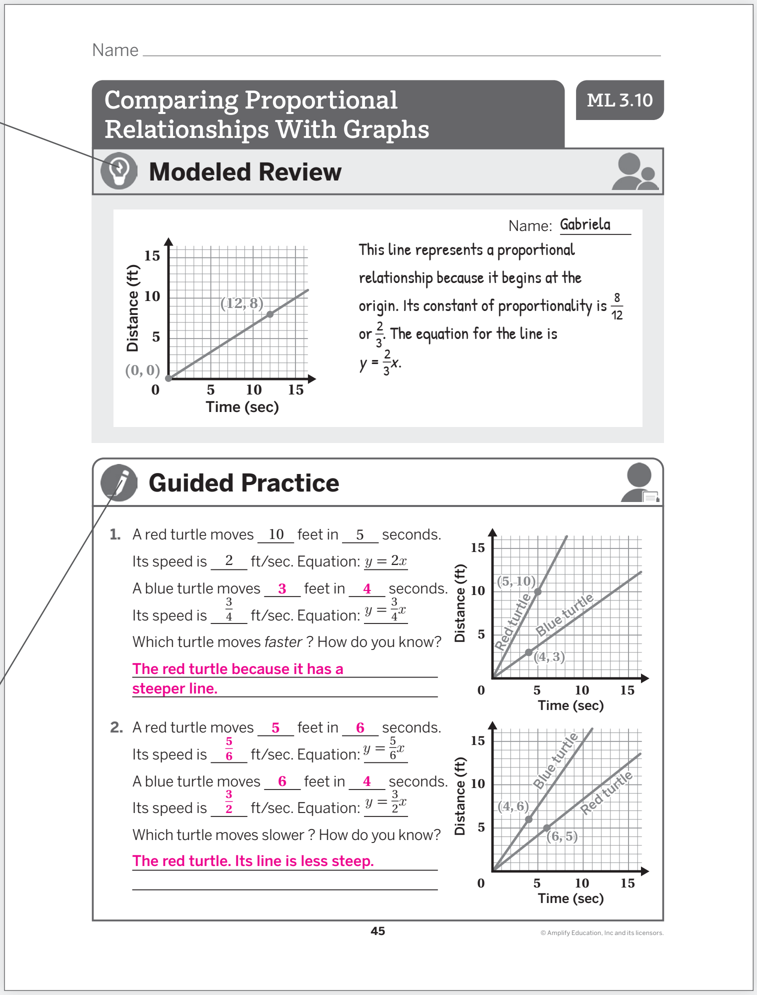
Reinforce Gabriela's thinking by saying, "Graphs and equations can be used to identify and compare proportional relationships."

Guided Practice

Focus students' attention on calculating the constant of proportionality and using the it to compare the speeds of the turtles.

To scaffold their thinking, **ask**:

- "What do you need to multiply the time by to calculate the distance?"
- "How could you determine the equation for a proportional relationship using the constant of proportionality?"



Name

Comparing Proportional Relationships With Graphs ML 3.10

Modeled Review

Name: Gabriela

This line represents a proportional relationship because it begins at the origin. Its constant of proportionality is $\frac{8}{12}$ or $\frac{2}{3}$. The equation for the line is $y = \frac{2}{3}x$.

Guided Practice

1. A red turtle moves 10 feet in 5 seconds. Its speed is 2 ft/sec. Equation: $y = 2x$
   A blue turtle moves 3 feet in 4 seconds. Its speed is $\frac{3}{4}$ ft/sec. Equation: $y = \frac{3}{4}x$
   Which turtle moves *faster*? How do you know?
   The red turtle because it has a steeper line.

2. A red turtle moves 5 feet in 6 seconds. Its speed is $\frac{5}{6}$ ft/sec. Equation: $y = \frac{5}{6}x$
   A blue turtle moves 6 feet in 4 seconds. Its speed is $\frac{3}{2}$ ft/sec. Equation: $y = \frac{3}{2}x$
   Which turtle moves slower? How do you know?
   The red turtle. Its line is less steep.

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Vocabulary

If needed, share the meaning of the term with students.

proportional relationship: A proportional relationship is a set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the value for the other quantity.

ML 3.10 Comparing Proportional Relationships With Graphs

Guided Practice

A Chunk this activity into smaller, more manageable parts by first writing the equations and then comparing the constants of proportionality.

Key Takeaway:
Say, "Graphs and equations can be helpful when comparing proportional relationships."

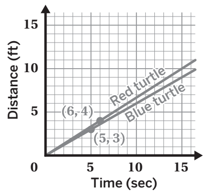
Guided Practice

3. A red turtle moves 4 feet in 6 seconds.
Its speed is $\frac{2}{3}$ ft/sec. Equation: $y = \frac{2}{3}x$

A blue turtle moves 3 feet in 5 seconds.
Its speed is $\frac{3}{5}$ ft/sec. Equation: $y = \frac{3}{5}x$

Which turtle moves *slower*? How do you know?

The blue turtle. Its line is less steep.

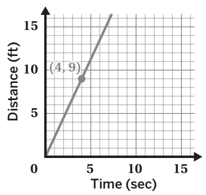


4. A red turtle moves according to the equation $y = 2x$, where y represents the distance in feet and x represents the time in seconds.

Here is a graph that shows the distance a blue turtle moves over time.

Which turtle moves *faster*? How do you know?

The blue turtle. The blue turtle's constant of proportionality (2.25) is greater than the red turtle's 2.



Check

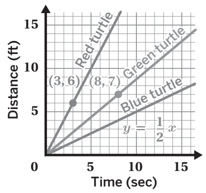
Here is a graph that shows the distance three turtles move over time.

1. Which turtle moves the *fastest*? How do you know?

   The red turtle. Its line is the steepest.

2. Write an equation for the turtle represented by the green line.

   $y = \frac{7}{8}x$



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Reflection

Ask:

- "How well did you understand the math in this lesson?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider reviewing the definition of constant of proportionality and its relationship to the steepness of a line on a graph.

Got it!
If students need more practice, add the coordinate (1,3) to the graph in Problem 3 to represent a green turtle. Ask students to find the speed of the green turtle and write an equation. Then compare the speeds of all three turtles and determine which turtle moves slowest.

Name ______________________________

Representing Proportional Relationships

ML 3.11

Modeled Review

Name: Priya

Here are four representations of a proportional relationship. Circle or show anywhere you can see the constant of proportionality.

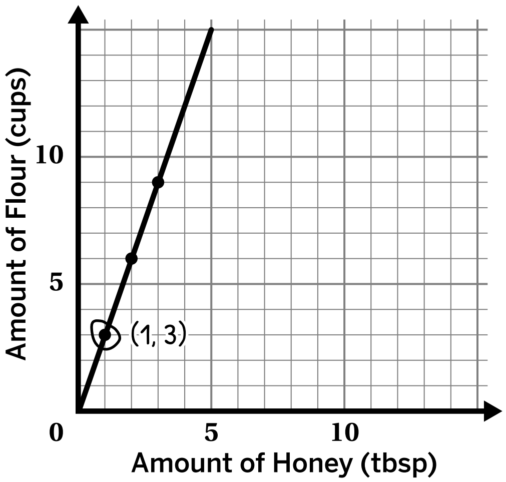
A baker uses 2 tablespoons of honey for every 6 cups of flour to make bread.

$k = \frac{y}{x}$ $k = \frac{6}{2}$

$f = 3h$

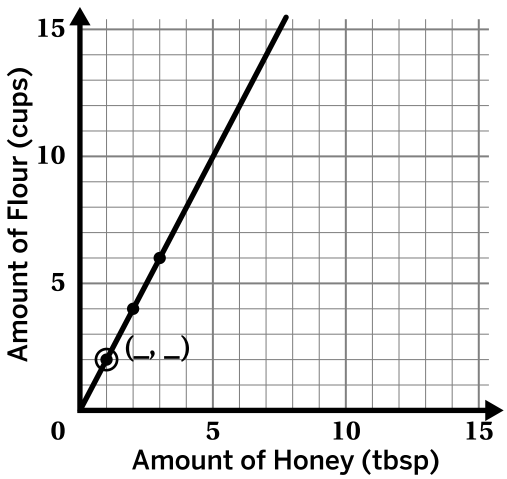
honey (tbsp), h	flour (cups), f
0	0
1	3
2	6

The honey represents the x-values and the flour represents the y-values.



Guided Practice

1. Analyze the graph and description of a proportional relationship to identify the constant of proportionality for each relationship.



A bakery uses 1 tablespoon of honey for every 2 cups of flour to make scones.

$k = \frac{2}{\square}$

$k =$

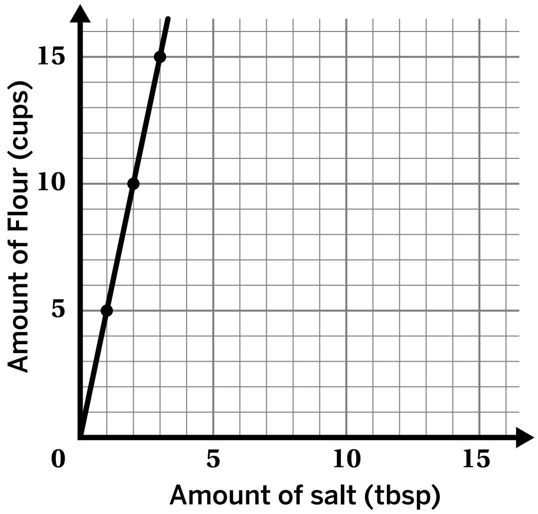
Guided Practice

2. Here are four representations of a proportional relationship. Circle or show where you can see the constant of proportionality in each representation.

A baker uses 3 tablespoons of salt for every 15 cups of flour to make pancakes.

$f = 5\ s$

salt (tbsp), s	flour (cups), f
0	0
1	5
2	10
3	15



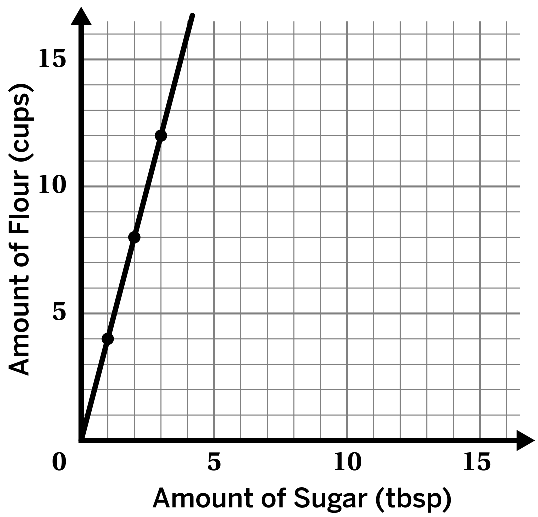
Check

Here are four representations of a proportional relationship. Circle or show anywhere you can see the constant of proportionality.

A baker uses 3 tablespoons of sugar for every 12 cups of flour to make muffins.

$f = 4\ s$

sugar (tbsp), s	flour (cups), f
0	0
1	4
2	8
3	12



ML 3.11 Representing Proportional Relationships

Goal

Use constants of proportionality (as determined using points or equations) or graphs to compare proportional relationships.

Standard

MA.7.AR.4.4

Materials

graph paper (optional)

Modeled Review

Point to Priya's work and **ask**:

- "How did Priya determine the constant of proportionality in the table and graph?"
- "How does Priya know to divide the amount of flour by the amount of honey?"

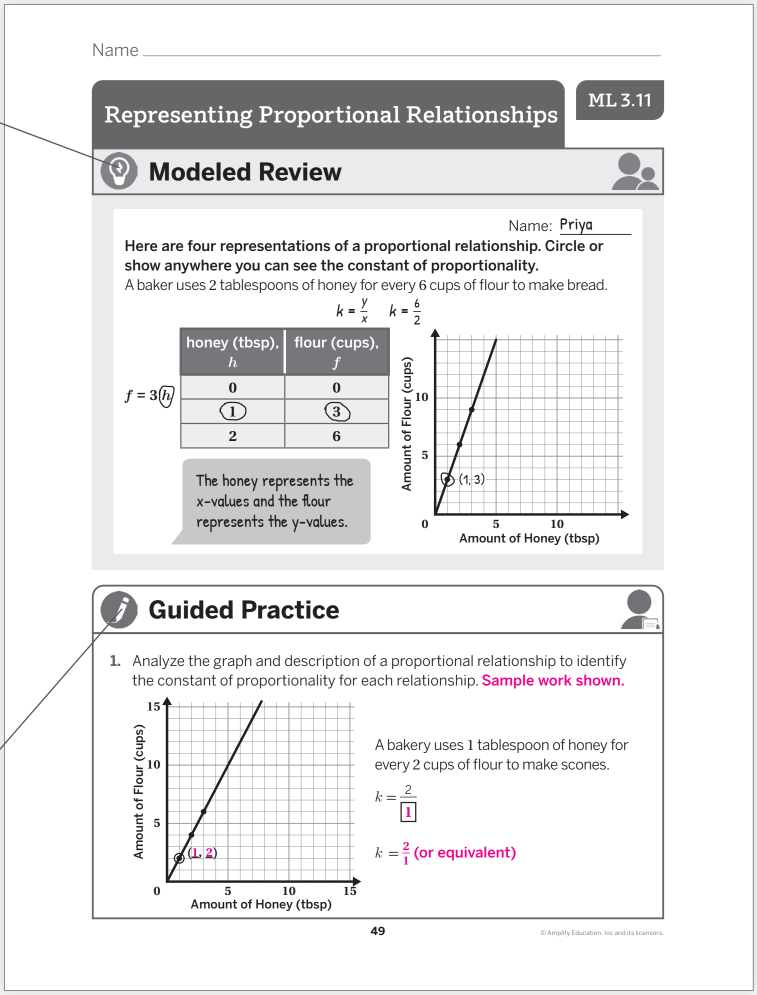
Reinforce Priya's thinking by saying, "Proportional relationships represent the rate at which one quantity changes in relation to another. You can find the constant of proportionality by using your knowledge of multiplication to find patterns in tables and graphs."

Guided Practice

Focus students' attention on determining the constant of proportionality.

To scaffold student thinking, **ask**:

- "How can you determine that the graph is proportional?"
- "Is it necessary for the graph to pass through the origin? Why or why not?"
- "How can you use the graph to help you calculate the constant of proportionality for the description?"



Name

Representing Proportional Relationships — ML 3.11

Modeled Review

Name: Priya

Here are four representations of a proportional relationship. Circle or show anywhere you can see the constant of proportionality.

A baker uses 2 tablespoons of honey for every 6 cups of flour to make bread.

$k = \frac{y}{x}$ $k = \frac{6}{2}$

$f = 3h$

honey (tbsp), h	flour (cups), f
0	0
1	3
2	6

The honey represents the x-values and the flour represents the y-values.

Guided Practice

1. Analyze the graph and description of a proportional relationship to identify the constant of proportionality for each relationship. Sample work shown.

A bakery uses 1 tablespoon of honey for every 2 cups of flour to make scones.

$k = \frac{2}{\boxed{1}}$

$k = \frac{2}{1}$ (or equivalent)

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Vocabulary

If needed, share the meaning of the term with students.

proportional relationship: A proportional relationship is a set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

Representing Proportional Relationships

Guided Practice

A Chunk this task into smaller, more manageable parts by reviewing the graph with students before they move on to identifying the constant of proportionality in the other representations.

ML/EL Model and annotate how to determine the constant of proportionality in both the graph and the table, and then connect those representations to the equation and description.

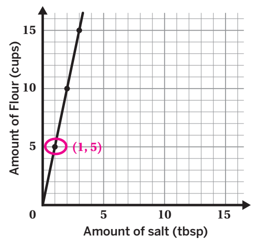
Key Takeaway:
Say, "A proportional relationship and its constant of proportionality can be identified in a variety of representations (verbal description, table, graph, and equation)."

Guided Practice

2. Here are four representations of a proportional relationship. Circle or show where you can see the constant of proportionality in each representation.
Sample work shown.
A baker uses 3 tablespoons of salt for every 15 cups of flour to make pancakes.
$k = \frac{15}{3}$ (or equivalent)

$f = 5s$

salt (tbsp), s	flour (cups), f
0	0
1	5
2	10
3	15



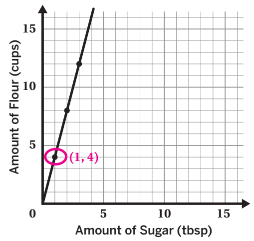
Check

Here are four representations of a proportional relationship. Circle or show anywhere you can see the constant of proportionality.

A baker uses 3 tablespoons of sugar for every 12 cups of flour to make muffins.
$k = \frac{12}{3}$ (or equivalent)

$f = 4s$

sugar (tbsp), s	flour (cups), f
0	0
1	4
2	8
3	12



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Reflection

Ask:
- "How could you determine if a relationship is proportional?"
- "Are there any representations that are still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, model annotating the units in the descriptions as *y* and *x* and then calculate the constant of proportionality.

Got it!
If students need more practice, have them determine the constant of proportionality and write an equation that matches the following descriptions:

- A baker uses 6 tablespoons of yogurt for every 4 cups of flour to make a pie crust.
- A baker uses 2 tablespoons of baking soda for every 12 cups of flour to make danishes.

Unit 4

Mini-Lessons

Name ______________________________

Interpreting Slope and Intercepts of Linear Relationships

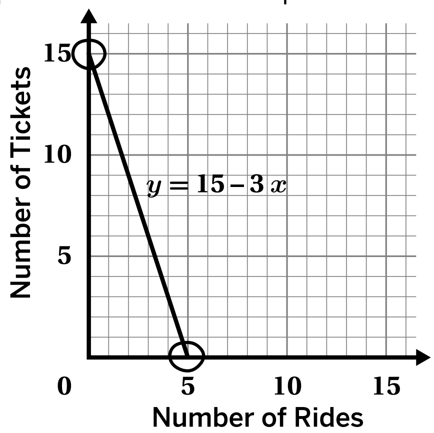
ML 4.02

Modeled Review

Name: Jack

The line $y = 15 - 3x$ represents Marc purchasing tickets at a carnival to go on rides.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,15); Marc started with 15 tickets.



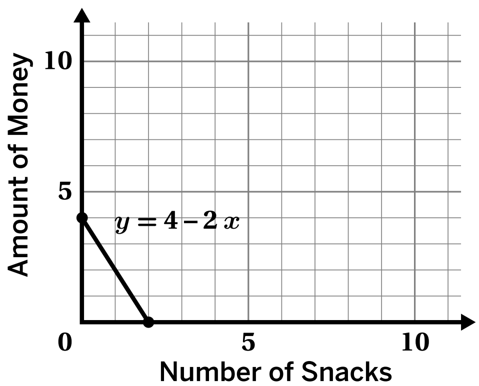
2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (5, 0); Marc went on 5 rides and then ran out of tickets.

3. What is the slope and what does it represent?
   -3; Marc uses 3 tickets for each ride.

Guided Practice

The line $y = 4 - 2x$ represents Mia purchasing snacks at the theater.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,__);



2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (0);

3. What is the slope, and what does it represent?
   ___; This means that Mia's ________ ____________ at a constant rate of ____________________________________

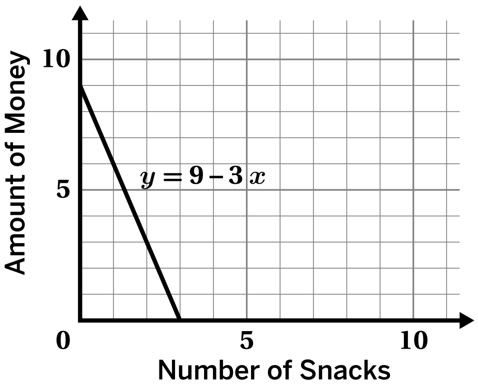
Guided Practice

The line $y = 9 - 3x$ represents Kai purchasing snacks at a soccer game.

4. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

5. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

6. What is the slope, and what does it represent?



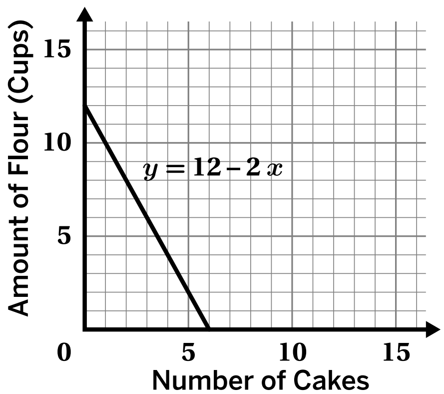
Check

The line $y = 12 - 2x$ represents Amy using flour to make cakes.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

3. What is the slope, and what does it represent?



ML 4.02 Interpreting Slope and Intercepts of Linear Relationships

Goal

Interpreting representations of the slope, vertical intercept, and horizontal intercept between a graph, equation, and situation.

Standard

MA.8.GR.3.2

Materials

colored pencils or highlighter (optional)

Modeled Review

Point to Jack's work and **ask**:

- "How did Jack use the situation and graph given to identify and describe the vertical intercept and horizontal intercept?"
- "How did Jack use the labels of the graph to help describe the slope?"

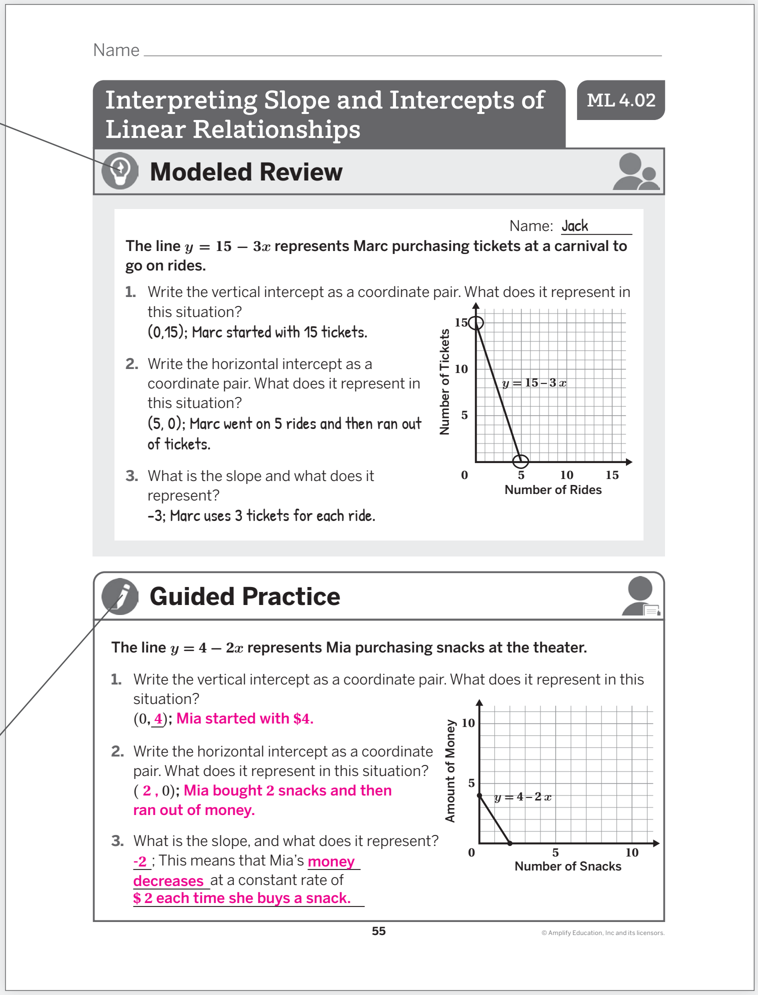
Reinforce he goal by saying, "The slope and intercepts can be interpreted from a graph by looking at the labels of the *x*-axis and *y*-axis and features of the graph."

Guided Practice

Focus students' attention on the equation and axes on the graph given.

To scaffold their thinking, **ask**:

- "Where do you see the vertical intercept in the equation?"
- "How is this shown on the graph?"


Name

Interpreting Slope and Intercepts of Linear Relationships ML 4.02

Modeled Review

Name: Jack

The line $y = 15 - 3x$ represents Marc purchasing tickets at a carnival to go on rides.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
(0,15); Marc started with 15 tickets.
2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
(5, 0); Marc went on 5 rides and then ran out of tickets.
3. What is the slope and what does it represent?
-3; Marc uses 3 tickets for each ride.

Guided Practice

The line $y = 4 - 2x$ represents Mia purchasing snacks at the theater.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
(0, 4); Mia started with $4.
2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
(2 , 0); Mia bought 2 snacks and then ran out of money.
3. What is the slope, and what does it represent?
-2 ; This means that Mia's money decreases at a constant rate of $ 2 each time she buys a snack.

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Vocabulary

If needed, share the meaning of the terms with students.

horizontal intercept: The horizontal intercept, sometimes called the *x*-intercept, is the point where the graph of a line crosses the horizontal axis or when $y = 0$.

vertical intercept: The vertical intercept, sometimes called the *y*-intercept, is the point where the graph of a line crosses the vertical axis or when $x = 0$.

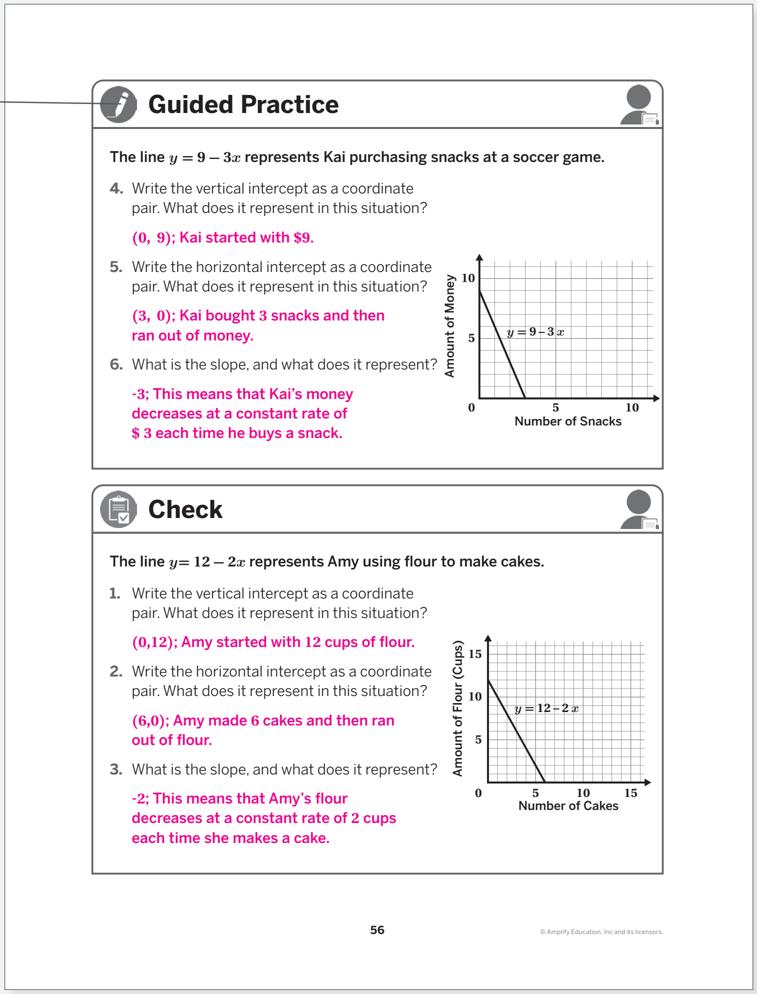
Interpreting Slope and Intercepts of Linear Relationships

Guided Practice

A As students connect visual representations to linear equations, provide colored pencils or highlighters for them to color-code elements from the equation and their corresponding representations on the graph.

ML/EL Provide sentence frames for students to interpret the slope and intercepts, such as "The vertical intercept means ________.", "The horizontal intercept means ________.", or "The slope shows the rate at which________changes for every ________."

Key Takeaway:
Say, "The slope, horizontal intercept, and vertical intercept of a graph representing a linear relationship can be used to determine important information."



Guided Practice

The line $y = 9 - 3x$ represents Kai purchasing snacks at a soccer game.

4. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

   $(0,\ 9)$; Kai started with \$9.

5. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

   $(3,\ 0)$; Kai bought 3 snacks and then ran out of money.

6. What is the slope, and what does it represent?

   -3; This means that Kai's money decreases at a constant rate of \$ 3 each time he buys a snack.

Check

The line $y = 12 - 2x$ represents Amy using flour to make cakes.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

   $(0, 12)$; Amy started with 12 cups of flour.

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

   $(6, 0)$; Amy made 6 cakes and then ran out of flour.

3. What is the slope, and what does it represent?

   -2; This means that Amy's flour decreases at a constant rate of 2 cups each time she makes a cake.

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Reflection

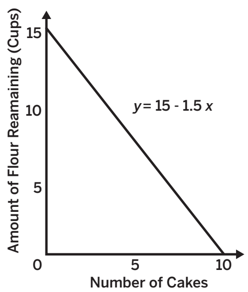
Ask:

- "How does a graph help identify the slope, vertical intercept, and horizontal intercept?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, have them highlight the x-axis and y-axis to guide in identifying the vertical intercept and horizontal intercept. Then have them highlight the labels of both axes to help interpret the slope and intercepts.

Got it!
If students need more practice, sketch the problem and have them interpret the vertical intercept, horizontal intercept, and slope.



Name ______________________________

Calculating Slope Given Two Points

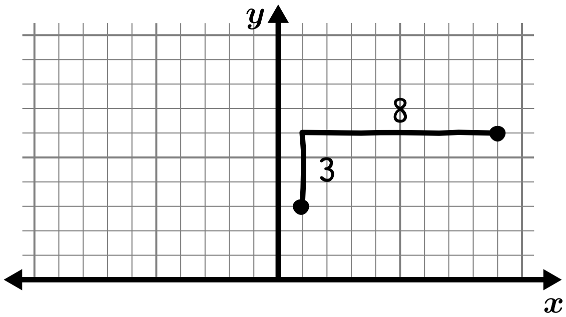
ML 4.06

Modeled Review

Name: Han

Calculate the slope of the line that passes through $(1, 3)$ and $(9, 6)$. Use the graph if it is helpful.

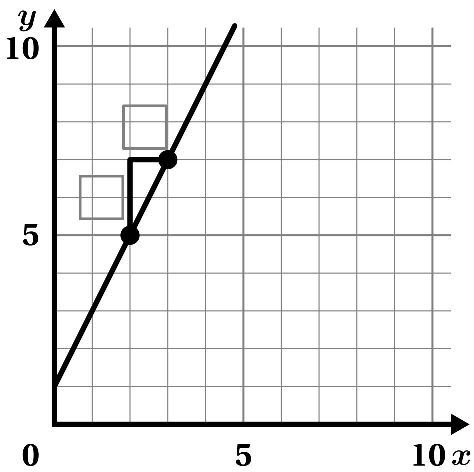
slope $= \frac{3}{8}$



Guided Practice

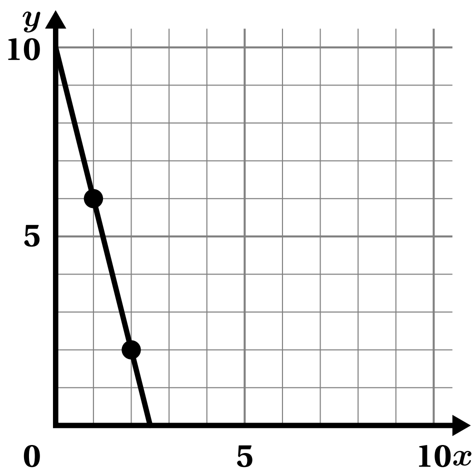
Calculate the slope of the line that passes through the given points.

1.



slope: ______________________

2.



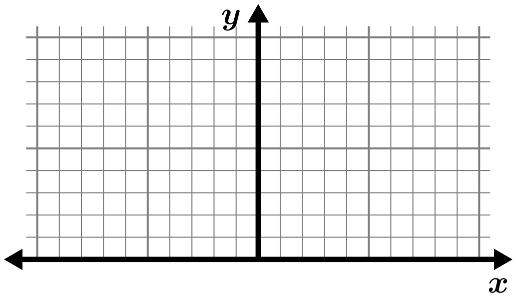
slope: ______________________

Guided Practice

Calculate the slope of the line that passes through the given points. Use the graph if it is helpful.

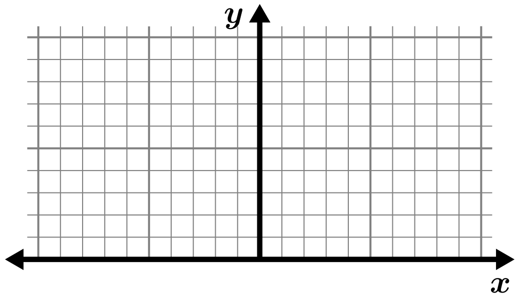
3. $(2, 4)$ and $(3, 1)$

slope: ______________________



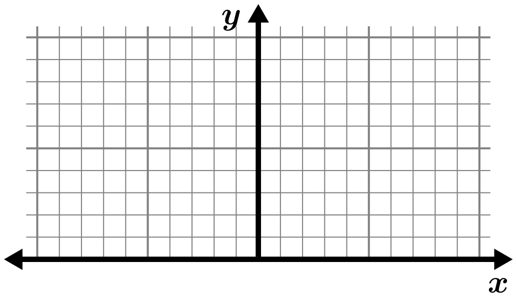
4. $(-1, 2)$ and $(4, 4)$

slope: ______________________



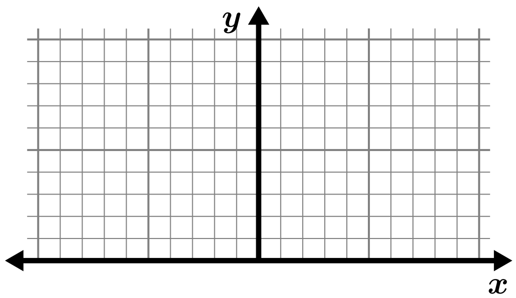
5. $(4, 2)$ and $(8, 5)$

slope: ______________________



6. $(6, 3)$ and $(9, 1)$

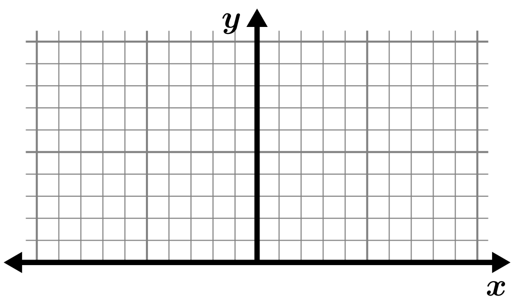
slope: ______________________



Check

Calculate the slope of the line that goes through $(2, 1)$ and $(4, 6)$. Use the graph if it is helpful.

slope: ______________________



Calculating Slope Given Two Points

Goal	**Standard**
Calculating the slope of a line through two given points.	MA.8.GR.3.2

Modeled Review

Point to Han's work and **ask**:

- "How did Han use a slope triangle to calculate the slope?"
- "How did Han know to label the side lengths as positive?"

Reinforce Han's thinking by saying, "Slope triangles can be used to calculate the slope of a line through two given points."

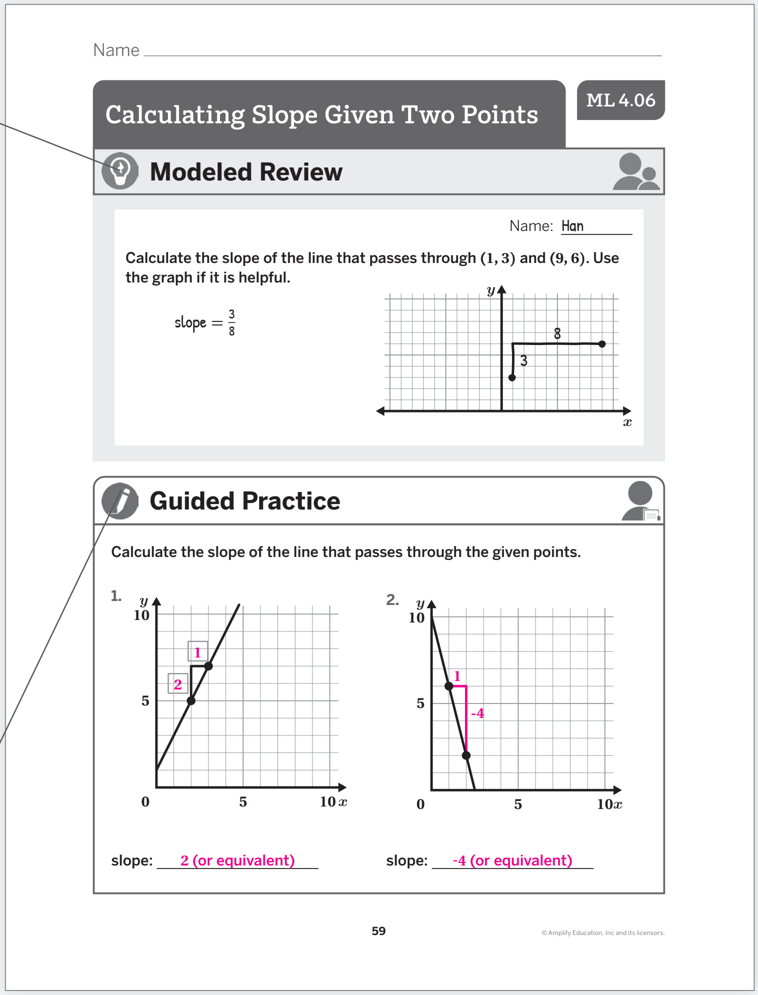
ML/EL Provide sentence frames to support students as they explain their strategies for calculating slope. For example, "I noticed that ______, so I ______." or "First, I ________ because ______."

Guided Practice

Focus students' attention on using slope triangles to calculate the slope.

To scaffold their thinking, **ask**:

- "Where are the points located on the coordinate plane?"
- "How do you create and label a slope triangle?"
- "How do you know if the side lengths are positive or negative?"



Name

Calculating Slope Given Two Points

ML 4.06

Modeled Review

Name: Han

Calculate the slope of the line that passes through $(1, 3)$ and $(9, 6)$. Use the graph if it is helpful.

slope $= \frac{3}{8}$

Guided Practice

Calculate the slope of the line that passes through the given points.

1.

slope: 2 (or equivalent)

2.

slope: -4 (or equivalent)

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Vocabulary

If needed, share the meaning of the terms with students.

slope: A number that describes the direction and steepness of a line.

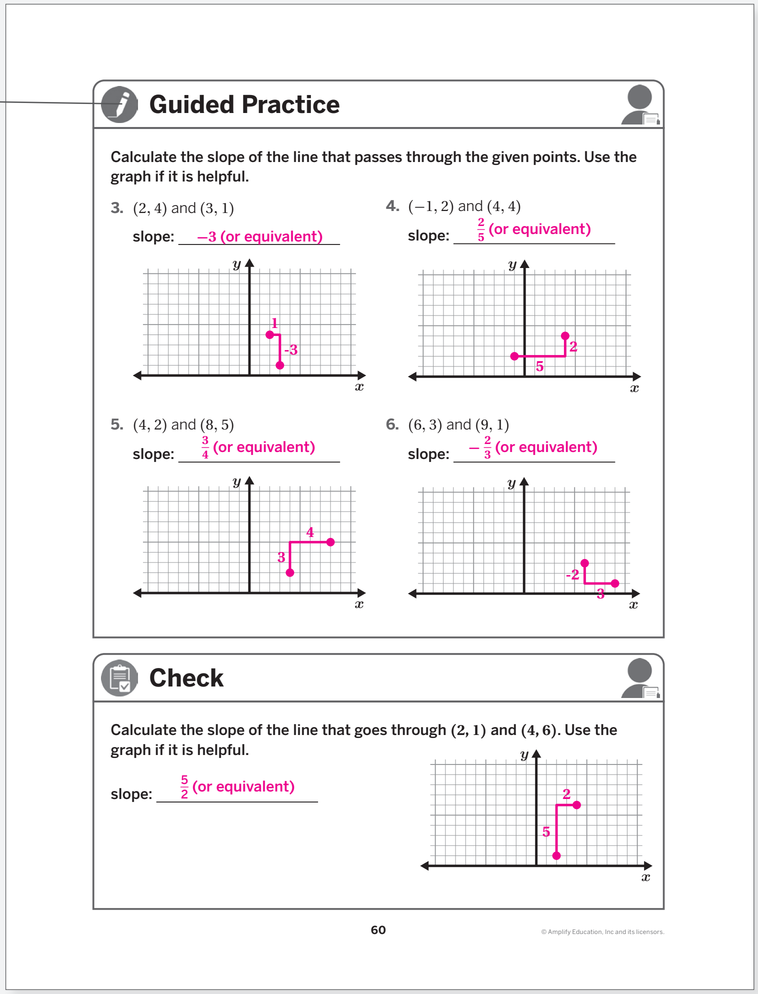
linear relationship: A relationship between two quantities is linear if there is a constant rate of change.

Guided Practice

A Chunk this task into smaller, more manageable parts by having students first number the scales then plot and label each point on the grid before drawing the slope triangle.

Note: Students may draw the slope triangle differently for each problem.

Key Takeaway:
Say, "The slope of a line can be calculated by dividing the difference in the values of *y* by the difference in the values of *x* using the coordinates of any two points on the line."



Guided Practice

Calculate the slope of the line that passes through the given points. Use the graph if it is helpful.

3. $(2, 4)$ and $(3, 1)$
slope: -3 (or equivalent)

4. $(-1, 2)$ and $(4, 4)$
slope: $\frac{2}{5}$ (or equivalent)

5. $(4, 2)$ and $(8, 5)$
slope: $\frac{3}{4}$ (or equivalent)

6. $(6, 3)$ and $(9, 1)$
slope: $-\frac{2}{3}$ (or equivalent)

Check

Calculate the slope of the line that goes through $(2, 1)$ and $(4, 6)$. Use the graph if it is helpful.

slope: $\frac{5}{2}$ (or equivalent)

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Reflection

Ask:
- "How is using a slope triangle helpful in calculating the slope of a line?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there If students need more support, consider asking them to draw an arrow to show if each length is positive or negative. Ask them to draw an arrow up or right to show a positive length and an arrow down or left to show a negative length. This will help them see which directions represent positive and negative values.	If students need more practice, invite them to look at Problem 3 and ask how the slope would change if the second point given was (5, 2).

Unit 5

Mini-Lessons

Name ______________________________

Solving Equations With Positive and Negative Numbers

ML 5.02

Modeled Review

Name: Jack

Solve the equation. Show your thinking.

$-4(x-2)=12$

$$\begin{aligned} \underline{-4}(x-2) &= \underline{12} \\ -4 \quad\quad &\quad -4 \\ x-2 &= -3 \\ +2 \quad &\quad +2 \\ x &= -1 \end{aligned}$$

Check:

$$\begin{aligned} -4(x-2) &= 12 \\ -4(-1-2) &= 12 \\ -4(-3) &= 12 \\ 12 &= 12 \end{aligned}$$

Guided Practice

1. Solve the equations by completing the blanks in the equations and descriptions.

Equation	Moves
$-2x+3=7$ $-2x=$ ____ $x=$ ____	**Step 1:** Subtract ____ from each side. **Step 2:** Divide each side by ____.
$2(x+1)=6$ $x+1=$ ____ $x=$ ____	**Step 1:** ________________ each side by ____. **Step 2:** ________________ from each side.

Guided Practice

For Problems 2–5, solve the equation. Show your thinking.

2. $-4x - 2 = 10$

3. $3(x - 2) = 9$

4. $-3x + 5 = -1$

5. $-4(x + 3) = 20$

Check

Solve each equation. Show your thinking.

1. $-4x - 3 = 13$

2. $-3(x - 4) = 27$

Solving Equations With Positive and Negative Numbers

Goal

Solve linear equations that involve positive and negative numbers.

Standard

MA.7.AR.2.2

Materials

highlighter (optional)

Modeled Review

Point to Jack's work and **ask**:

- "Why did Jack use inverse operations to solve for the variable?"
- "Why did Jack do the same operation on both sides of the equal sign?"
- "How did Jack check his solution?"

Reinforce Jack's thinking by saying, "When solving an equation, the same operations should be applied to both sides at each step, so that the equation remains true."

ML/EL Invite students to check the solution by substituting in the value for the variable and evaluating to determine whether the equation is true.

Guided Practice

Focus students' attention on solving equations by ensuring that both sides of the equation remain balanced while determining the value of the variable.

To scaffold their thinking, **say**:

- "Identify the variable that needs to be solved for."
- "Apply the inverse operation to both sides of the equation, repeating this process until the variable is isolated."

Name

Solving Equations With Positive and Negative Numbers ML 5.02

Modeled Review

Name: Jack

Solve the equation. Show your thinking.

$-4(x-2)=12$

$$\frac{-4(x-2)}{-4} = \frac{12}{-4}$$
$$x-2=-3$$
$$+2 \quad +2$$
$$x=-1$$

Check:
$$-4(x-2)=12$$
$$-4(-1-2)=12$$
$$-4(-3)=12$$
$$12=12$$

Guided Practice

1. Solve the equations by completing the blanks in the equations and descriptions.

Equation	Moves
$-2x+3=7$ $-2x=$ 4 $x=$ -2	**Step 1:** Subtract 3 from each side. **Step 2:** Divide each side by -2.
$2(x+1)=6$ $x+1=$ 3 $x=$ 2	**Step 1:** Divide each side by 2. **Step 2:** Subtract 1 from each side.

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ML 5.02 Solving Equations With Positive and Negative Numbers

Guided Practice

A Invite students to highlight the variable in each equation. Then have them make a step by step plan as to how they will isolate the variable. (For example, first I will divide. Next, I will add and finally, I will subtract.)

Key Takeaway:
Say, "The first step in solving equations may be different depending on the structure of the equation, but it is important that both sides of the equation remain balanced."

Guided Practice

For Problems 2–5, solve the equation. Show your thinking.
Sample work shown.

2. $-4x - 2 = 10$

$$\begin{aligned} -4x - 2 &= 10 \\ +2 \quad &\quad +2 \\ \frac{-4x}{-4} &= \frac{12}{-4} \\ x &= -3 \end{aligned}$$

3. $3(x - 2) = 9$

$$\begin{aligned} \frac{3(x-2)}{3} &= \frac{9}{3} \\ x - 2 &= 3 \\ +2 \quad &\quad +2 \\ x &= 5 \end{aligned}$$

4. $-3x + 5 = -1$

$$\begin{aligned} -3x + 5 &= -1 \\ -5 \quad &\quad -5 \\ \frac{-3x}{-3} &= \frac{-6}{-3} \\ x &= 2 \end{aligned}$$

5. $-4(x + 3) = 20$

$$\begin{aligned} \frac{-4(x+3)}{-4} &= \frac{20}{-4} \\ x + 3 &= -5 \\ -3 \quad &\quad -3 \\ x &= -8 \end{aligned}$$

Check

Solve each equation. Show your thinking. Sample work shown.

1. $-4x - 3 = 13$

$$\begin{aligned} -4x - 3 &= 13 \\ +3 \quad &\quad +3 \\ \frac{-4x}{-4} &= \frac{16}{-4} \\ x &= -4 \end{aligned}$$

2. $-3(x - 4) = 27$

$$\begin{aligned} \frac{-3(x-4)}{-3} &= \frac{27}{-3} \\ x - 4 &= -9 \\ +4 \quad &\quad +4 \\ x &= -5 \end{aligned}$$

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Reflection

Ask:
- "What suggestions would you share with a friend about solving equations that involve positive and negative numbers?"
- "Why is it helpful to check your solutions?"

Check: Recommended Next Steps

Almost there
If students need more support, invite them to draw a line separating the two sides of the equation to support them in organizing their work on each side of the equation.

Got it!
If students need more practice, ask them to solve the following equations:

- $-3\,(x - 5) = 21$
- $-5x + 6 = 21$

Name ______________________________

Solving Linear Equations With Parentheses

ML 5.06

Modeled Review

Name: Kayla

Solve the equation.

$-3(2x - 4) = 18$

$-3(2x - 4) = 18$
$-6x + 12 = 18$
$- 12 \quad - 12$
$-6x = 6$
$x = -1$

Name: Isaiah

Solve the equation.

$-3(2x - 4) = 18$

$\dfrac{-3(2x-4)}{-3} = \dfrac{18}{-3}$
$2x - 4 = -6$
$+ 4 = + 4$
$2x = -2$
$x = -1$

Guided Practice

Solve each equation. Complete the missing steps.

1.
$$\begin{aligned} 4x + 9 &= 5x + 7 \\ -\,4x \qquad & \quad -\,4x \\ 9 &= \_\_\_ + 7 \\ -\,7 \quad & \qquad -\,7 \\ \_\_\_ &= x \end{aligned}$$

solution: $x =$ ___

2.
$$\begin{aligned} x - 8 &= 10 - 5x \\ +\,\_\_\_ \quad & \quad +\,\_\_\_ \\ 6x - 8 &= 10 \\ +\,\_\_\_ \quad & +\,\_\_\_ \\ 6x &= 18 \\ x &= \_\_\_ \end{aligned}$$

solution: $x =$ ___

Solve the equation $\frac{1}{2}(4x - 2) = 11$ using two different methods.

3.
$$\begin{aligned} \tfrac{1}{2}(4x - 2) &= 11 \\ \times 2 \qquad & \quad \times 2 \\ 4x - 2 &= \_\_\_ \\ \_\_\_ \quad & \quad \_\_\_ \\ 4x &= 24 \\ x &= \_\_\_ \end{aligned}$$

solution: $x =$ ___

4.
$$\begin{aligned} \tfrac{1}{2}(4x - 2) &= 11 \\ 2x - \_\_\_ &= 11 \\ +\ 1 \quad & +\,\_\_\_ \\ 2x &= \_\_\_ \\ x &= \_\_\_ \end{aligned}$$

solution: $x =$ ___

Guided Practice

Solve each equation.

5. $4(3x + 5) = -2(-8x + 6)$

$\_\_\_\,x + 20 = \_\_\_\,x - \_\_\_$

$\_\_\_\_\_\_ \qquad \_\_\_\_\_\_$

$20 = \_\_\_\,x - 12$

$\_\_\_\_\_ \qquad \_\_\_\_\_$

$32 = \_\_\_\,x$

$\_\_\_ = x$

solution: x = ___

6. $-2(2x + 3) = 3(x + 5)$

solution: x = ___

7. $\frac{1}{3}(x - 5) = 2(x - 1)$

solution: x = ___

8. $0.5(x - 4) = -3(2x + 5)$

solution: x = ___

Check

Solve the equation.

$$-2(5x - 3) = 3(-4x + 8)$$

solution: x = ___

ML 5.06 Solving Linear Equations With Parentheses

Goal

Solve single-variable equations with parentheses using multiple balanced steps, including the distributive property.

Standard

MA.8.AR.2.1

Materials

calculator (optional)

Modeled Review

Point to Kayla's and Isaiah's work and **ask**:

- "What steps did each student take to solve the equation? What was each student's first step?"
- "What is the same about each student's work? What is different?"
- "Whose strategy do you think is more efficient for solving this equation?"

Reinforce the goal by saying, "Different strategies can be used to solve the same equation. When you have finished solving, you could substitute the value of *x* into the original equation to check that it is true."

Guided Practice

Focus students' attention on solving the linear equations.

To scaffold their thinking, **ask**:

- "What is the first step you could take to solve this equation?"
- "How could you check if your solution is correct?"

Name __________

Solving Linear Equations With Parentheses ML 5.06

Modeled Review

Name: Kayla

Solve the equation.

$-3(2x-4) = 18$

$-3(2x-4) = 18$
$-6x + 12 = 18$
$-12 \quad -12$
$-6x = 6$
$x = -1$

Name: Isaiah

Solve the equation.

$-3(2x-4) = 18$

$\frac{-3(2x-4)}{-3} = \frac{18}{-3}$
$2x - 4 = -6$
$+4 = +4$
$2x = -2$
$x = -1$

Guided Practice

Solve each equation. Complete the missing steps.

1. $4x + 9 = 5x + 7$
$-4x \quad -4x$
$9 = \underline{x} + 7$
$-7 \quad -7$
$\underline{2} = x$
solution: $x = \underline{2}$

2. $x - 8 = 10 - 5x$
$+\underline{5x} \quad +\underline{5x}$
$6x - 8 = 10$
$+\underline{8} \quad +\underline{8}$
$6x = 18$
$x = \underline{3}$
solution: $x = \underline{3}$

Solve the equation $\frac{1}{2}(4x-2) = 11$ using two different methods.

3. $\frac{1}{2}(4x-2) = 11$
$\times 2 \quad \times 2$
$4x - 2 = \underline{22}$
$\underline{+2} \quad \underline{+2}$
$4x = 24$
$x = \underline{6}$
solution: $x = \underline{6}$

4. $\frac{1}{2}(4x-2) = 11$
$2x - \underline{1} = 11$
$+1 \quad +\underline{1}$
$2x = \underline{12}$
$x = \underline{6}$
solution: $x = \underline{6}$

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Vocabulary

If needed, share the meaning of the term with students.

solution to an equation: The value or set of values that makes the equation true.

Solving Linear Equations With Parentheses

Guided Practice

A Invite students to use a calculator to support them with performing calculations.

Key Takeaway:
Say, "Different strategies can be used to solve the same equation. Using the structure of an equation can be helpful in determining effective steps to solving the equation."

Guided Practice

Solve each equation. Sample work shown for Problems 6–8.

5. $4(3x + 5) = -2(-8x + 6)$

$\underline{12}\,x + 20 = \underline{16}\,x - \underline{12}$

$-12x \qquad -12x$

$20 = \underline{4}\,x - 12$

$+12 \qquad +12$

$32 = \underline{4}\,x$

$\underline{8} = x$

solution: $x = \underline{8}$

6. $-2(2x + 3) = 3(x + 5)$

$-4x - 6 = 3x + 15$

$+4x \qquad +4x$

$-6 = 7x + 15$

$-15 \qquad -15$

$-21 = 7x$

$-3 = x$

solution: $x = \underline{-3}$

7. $\frac{1}{3}(x - 5) = 2(x - 1)$

$\times 3 \qquad \times 3$

$x - 5 = 6(x - 1)$

$x - 5 = 6x - 6$

$-x \qquad -x$

$-5 = 5x - 6$

$+6 = +6$

$1 = 5x$

$\frac{1}{5} = x$

solution: $x = \underline{\frac{1}{5}}$

8. $0.5(x - 4) = -3(2x + 5)$

$\times 2 \qquad \times 2$

$x - 4 = -6(2x + 5)$

$x - 4 = -12x - 30$

$+12x \qquad +12x$

$13x - 4 = -30$

$+4 \qquad +4$

$13x = -26$

$x = -2$

solution: $x = \underline{-2}$

Check

Solve the equation. Sample work shown.

$-2(5x - 3) = 3(-4x + 8)$

$-10x + 6 = -12x + 24$

$+12x \qquad +12x$

$2x + 6 = 24$

$-6 \qquad -6$

$2x = 18$

$x = 9$

solution: $x = \underline{9}$

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Reflection

Ask:

- "Which strategy was the most helpful when determining the solution for each equation?"
- "After today's lesson, what made sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Accelerated 6 Mini-Lesson 8.02: Writing and Solving One-Step Equations.

Got it!
If students need more practice, have them solve the following equations:

- $-5(x + 4) = -3(-2x - 8)$
- $\frac{1}{4}(8x - 4) = 3(-x + 1)$

Name ______________________________

Solving Systems of Linear Equations by Graphing

ML 5.13

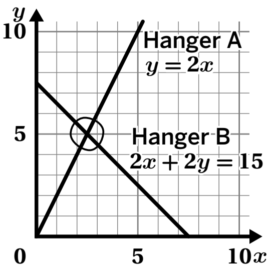
Modeled Review

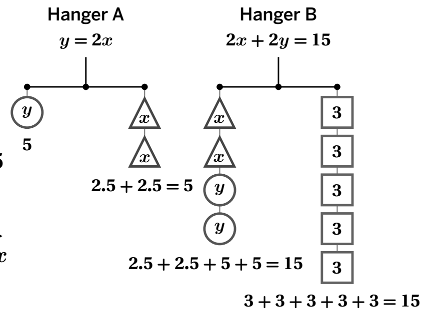
Name: Clare

Use these hanger diagrams and the related graph to complete the problems.

1. Determine the solution to the system of equations.
   (2.5, 5)

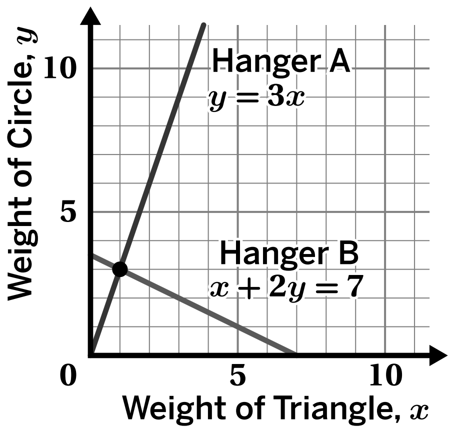
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
   Each triangle weighs 2.5 and each circle weighs 5.

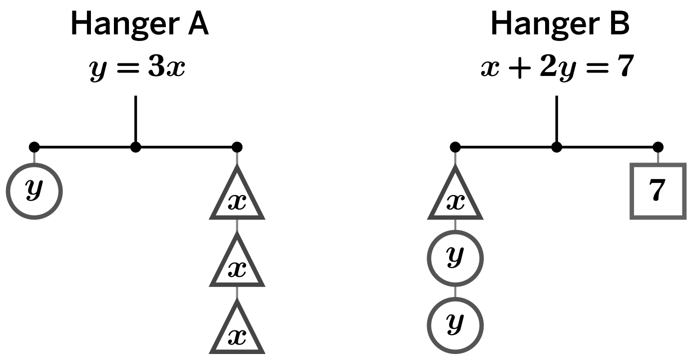




Guided Practice

Use these hanger diagrams and the related graph to complete the problems.





1. Determine the solution to the system of equations.

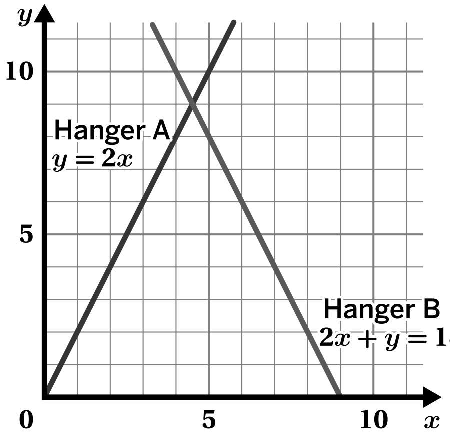
   ______

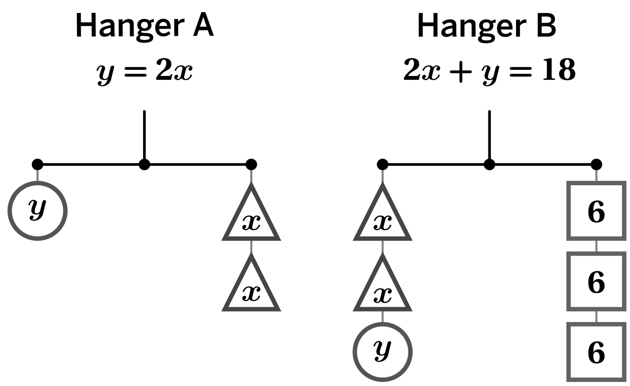
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

   Each triangle weighs 1 and each circle weighs ___.

Guided Practice

Use these hanger diagrams and the related graph to complete the problems.





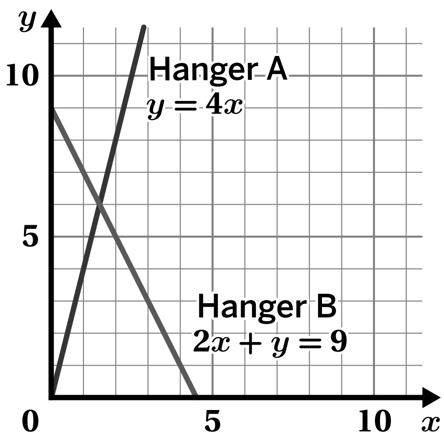
3. Determine the solution to the system of equations.

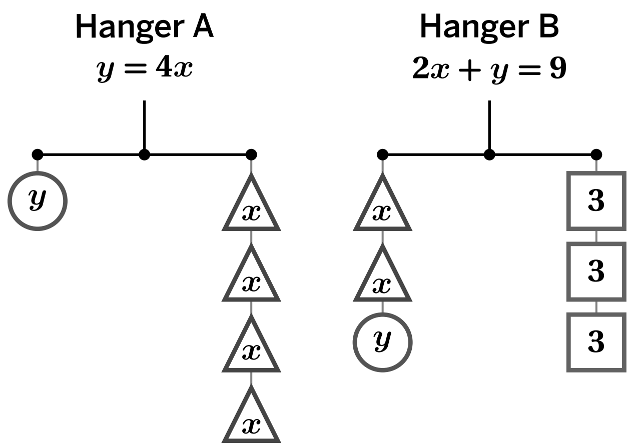
____

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

Check

Use these hanger diagrams and the related graph to complete the problems.





3. Determine the solution to the system of equations.

____

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

ML 5.13 Solving Systems of Linear Equations by Graphing

Goal

Solve systems of equations by determining the values that satisfy both equations.

Standard

MA.8.AR.4.1 and MA.8.AR.4.3

Modeled Review

Point to Clare's work and **ask**:

- "How did Clare identify the solution to the system of equations?"
- "What did Clare use from the graph to help determine how both hangers relate to the solution?"

Reinforce Clare's thinking by saying, "The solution to a system of equations is the point of intersection."

ML/EL To support students as they share their responses, provide the following sentence frames:

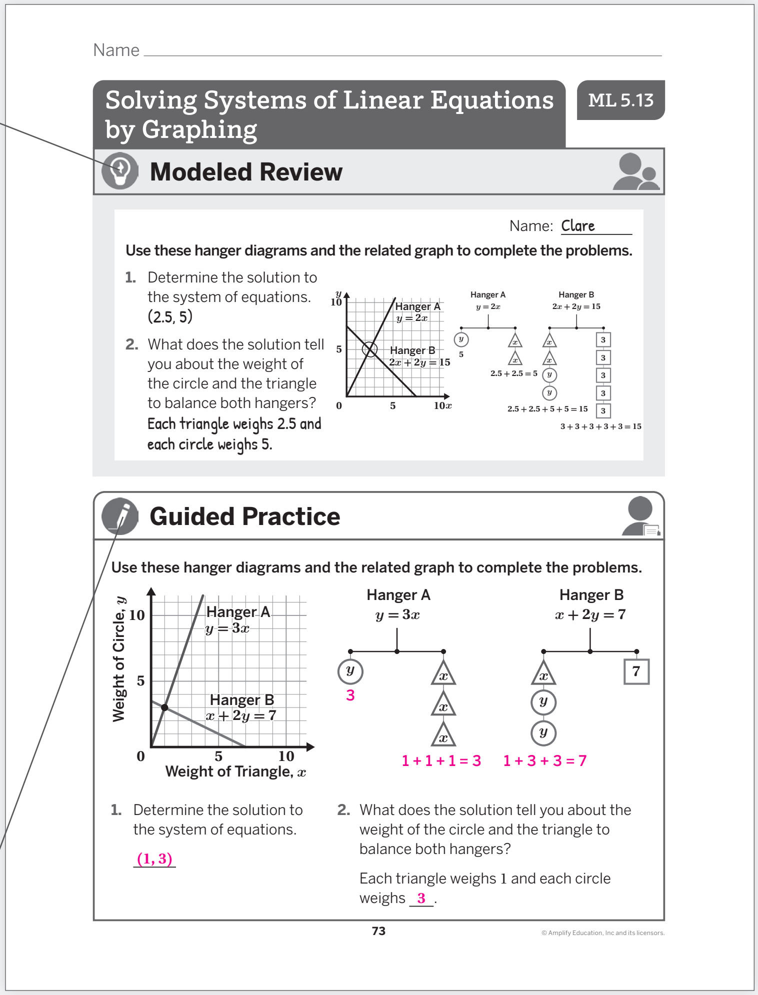
- "The point ___ balances just [Hanger A/B]."
- "When *x* is __ and *y* is __, both hangers are balanced."
- "The point __ will balance neither hanger."

Guided Practice

Focus students' attention on using the given graph to determine the solution to the system of equations.

To scaffold their thinking, **ask**:

- "Where is the point of intersection to the system of equations?"
- "Does this solution work to make both hangers true?"



Name

Solving Systems of Linear Equations by Graphing ML 5.13

Modeled Review

Name: Clare

Use these hanger diagrams and the related graph to complete the problems.

1. Determine the solution to the system of equations.
(2.5, 5)
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
Each triangle weighs 2.5 and each circle weighs 5.

Guided Practice

Use these hanger diagrams and the related graph to complete the problems.

1. Determine the solution to the system of equations.
(1, 3)
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
Each triangle weighs 1 and each circle weighs 3.

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Vocabulary

If needed, share the meaning of the terms with students.

system of equations: A system of equations is two or more equations that represent the constraints on a shared set of variables.

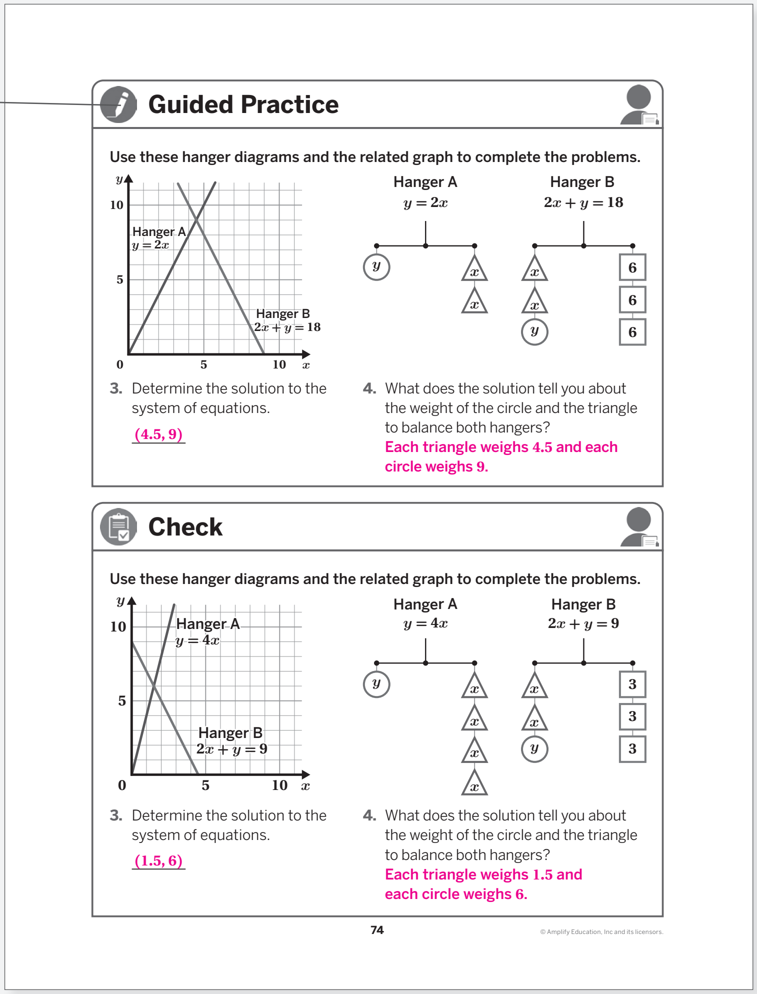
solution to a system of equations: A solution to a system of equations is a set of values that makes all equations in that system true.

Solving Systems of Linear Equations by Graphing

Guided Practice

A Chunk this task into smaller, more manageable parts by having students identify the point of intersection on the graph before looking at the hangers.

Key Takeaway:
Say, "A solution to a system of equations must satisfy all equations in the system."



Guided Practice

Use these hanger diagrams and the related graph to complete the problems.

3. Determine the solution to the system of equations.
(4.5, 9)

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
Each triangle weighs 4.5 and each circle weighs 9.

Check

Use these hanger diagrams and the related graph to complete the problems.

3. Determine the solution to the system of equations.
(1.5, 6)

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
Each triangle weighs 1.5 and each circle weighs 6.

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Reflection

Ask:

- "Where is the solution to a system of equations located on a graph?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider asking them to label both the *x*- and *y*-axes with all numbers to help them determine the correct solution.

Got it!
If students need more practice, refer to Problem 3 and ask them to write an ordered pair that would not be a solution to the system of equations.

Unit 6

Mini-Lessons

Name ______________________________

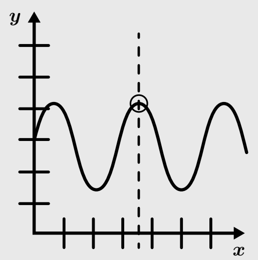
Justifying Whether a Graph Represents a Function

ML 6.03

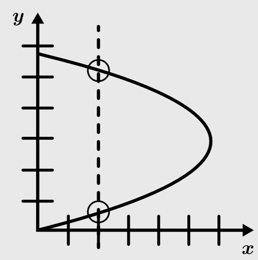
Modeled Review

Function: a rule that assigns exactly one output to each possible input.

Function



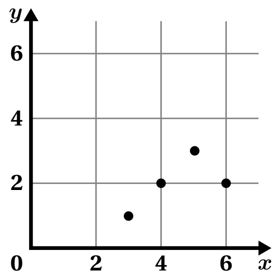
Not a function



Guided Practice

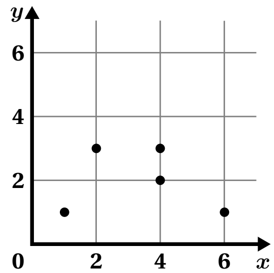
Determine whether the graph is a function. If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

1.



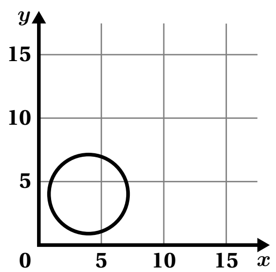
Function Not a function

2.



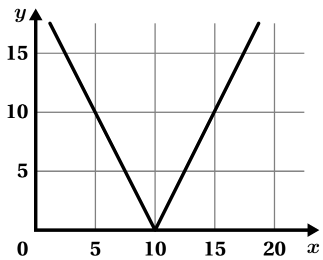
Function Not a function

3.



Function Not a function

4.

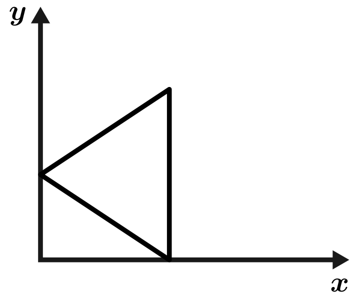


Function Not a function

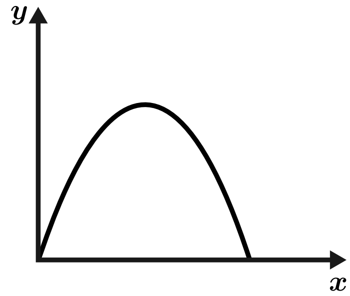
Guided Practice

Determine whether y is a function of x and write "function" or "not a function." If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

5.



6.



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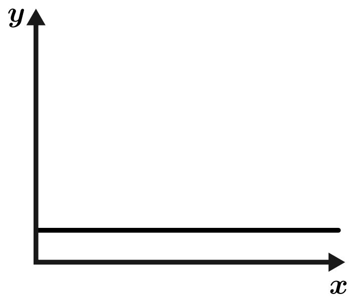

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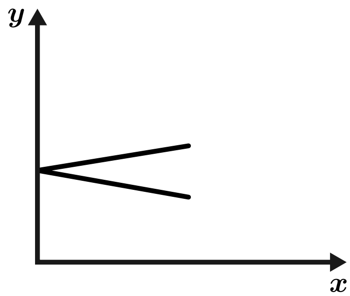
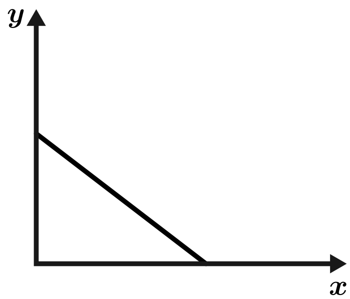


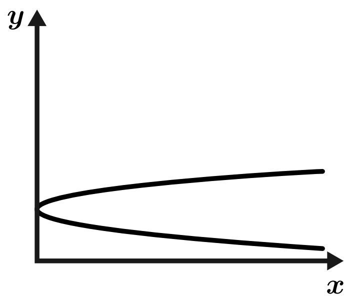
Check

Select the graph in which y is a function of x. On the graph that is not a function, draw a vertical line where the graph shows more than one output for the same input.

A.



B.



Justifying Whether a Graph Represents a Function

Goal

Justify whether or not y is a function of x using a graph.

Standard

MA.8.F.1.1

Materials

coloring tools (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "In your own words, what is a function?"
- "What makes the graph on the left a function?"
- "What makes the graph on the right *not* a function?"

Reinforce the goal by saying, "You can determine whether a graph is a function or not by analyzing whether each input value has exactly one output."

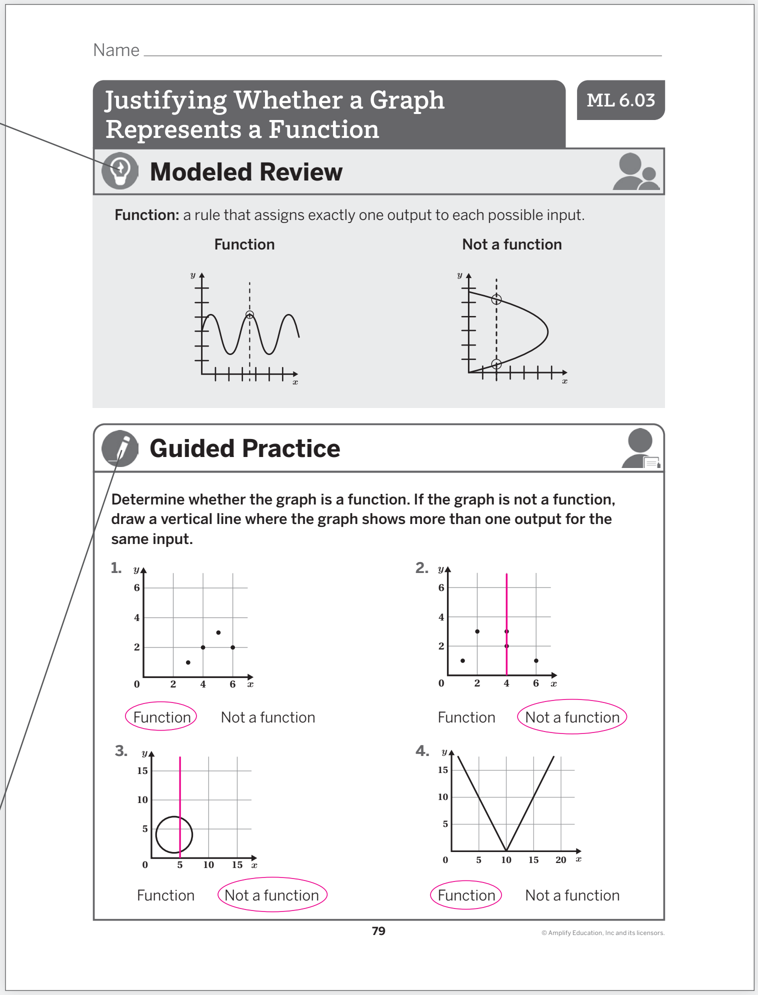
ML/EL Invite students to share what a new term reminds them of (e.g., "When I think of input, I think of someone giving advice or an opinion."). This may surface multiple meanings and support students as they connect to their prior knowledge.

Guided Practice

Focus students' attention on determining whether the graphs represent a function or not.

To scaffold their thinking, **ask**:

- "What is a function?"
- "What is not a function?"
- "How can you use a vertical line to help determine if the graph is a function or not?"



Name

Justifying Whether a Graph Represents a Function

ML 6.03

Modeled Review

Function: a rule that assigns exactly one output to each possible input.

Function   Not a function

Guided Practice

Determine whether the graph is a function. If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

1. Function   Not a function

2. Function   Not a function

3. Function   Not a function

4. Function   Not a function

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Vocabulary

If needed, share the meaning of the term with students.

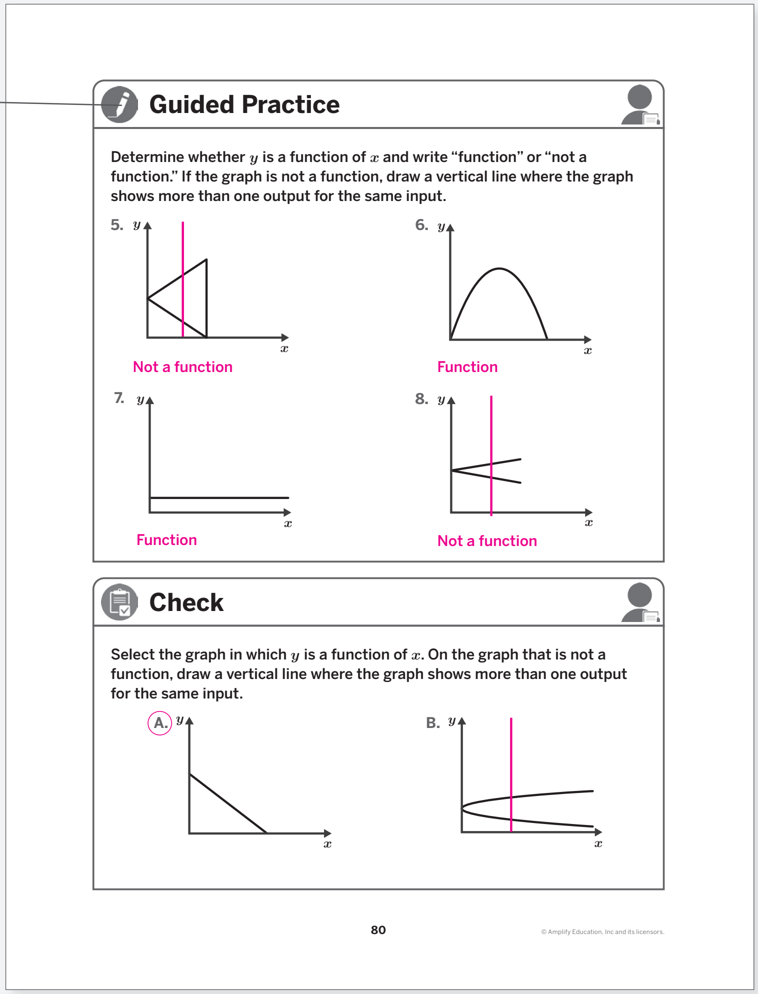
function: A function is a rule that assigns exactly one output to each possible input.

ML 6.03 Justifying Whether a Graph Represents a Function

Guided Practice

A Invite students to use coloring tools to determine if the graph is a function or not by having them denote the input in one color and the output in another color. Then have them use the colored graph to determine if each input has exactly one output.

Key Takeaway:
Say, "The graph of a function is a set of ordered pairs where each input has exactly one output."



Guided Practice

Determine whether y is a function of x and write "function" or "not a function." If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

5. Not a function

6. Function

7. Function

8. Not a function

Check

Select the graph in which y is a function of x. On the graph that is not a function, draw a vertical line where the graph shows more than one output for the same input.

A.

B.

80

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Reflection

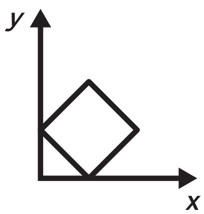
Ask:

- "How can you use a vertical line to determine if a graph is a function or not?"
- "What was easy for you? What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, model representing the ordered pairs from a graph in a table or mapping diagram. Then check that each input has exactly one output.

Got it!
If students need more practice, sketch the following graph and have them determine if the graph is a function or not.



Name ______________________________

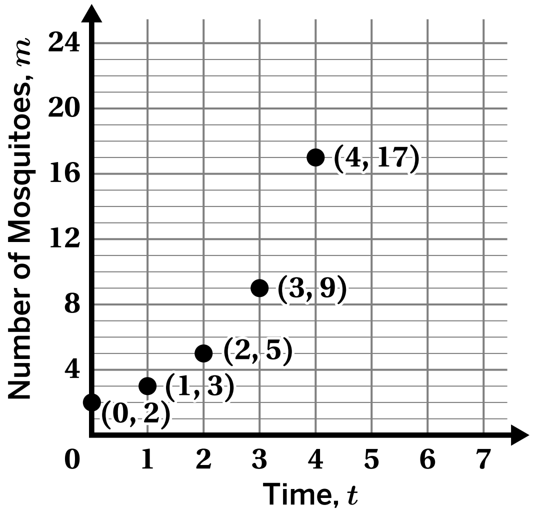
Interpreting Graphs

ML 6.04

Modeled Review

Name: Shawn

The graph represents the relationship between time, t, and the number of mosquitoes, m. Complete the table so it reflects the values in the graph.

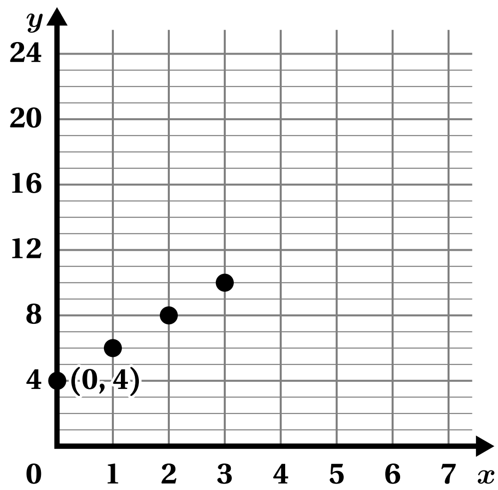


t	m
0	2
1	3
2	5
3	9
4	17

Time is the independent variable, so it represents the x-values. The number of mosquitos is the dependent variable, so it represents the y-values.

Guided Practice

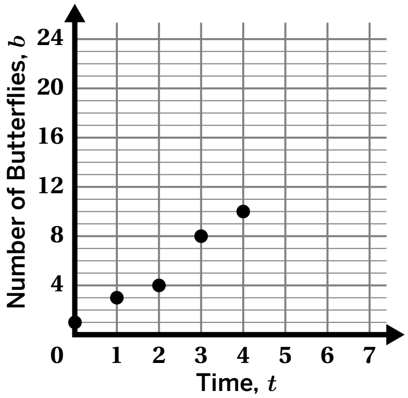
1. The graph represents the relationship between the independent variable, x, and the dependent variable, y. Complete the table so it reflects the values in the graph.



x	y
0	4

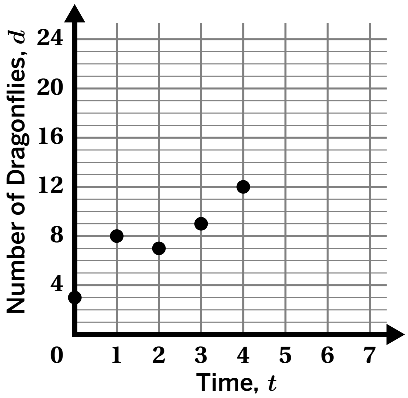
Guided Practice

2. This graph represents the relationship between time, t, and the number of butterflies, b. Complete the table so it reflects the values in the graph.



t	b

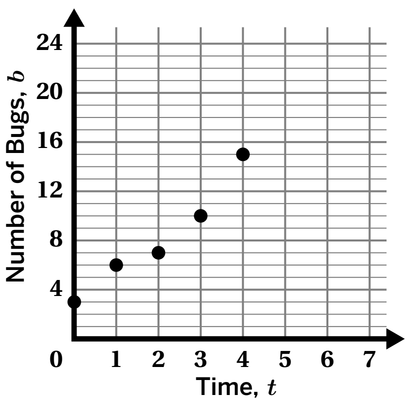
3. This graph represents the relationship between time, t, and the number of dragonflies, d. Complete the table so it reflects the values in the graph.



t	d

Check

This graph represents the relationship between time, t, and the number of bugs, b. Complete the table so it reflects the values in the graph.



t	b

Goal

Interpret a graph to create a table that represents a relationship between two quantities.

Standard

MA.8.F.1.1

Modeled Review

Point to Shawn's work and **ask**:

- "How does Shawn know how to write the ordered pairs?"
- "Why is *t* the independent variable and *m* the dependent variable?"
- "How are the graph and table similar? How are they different?"

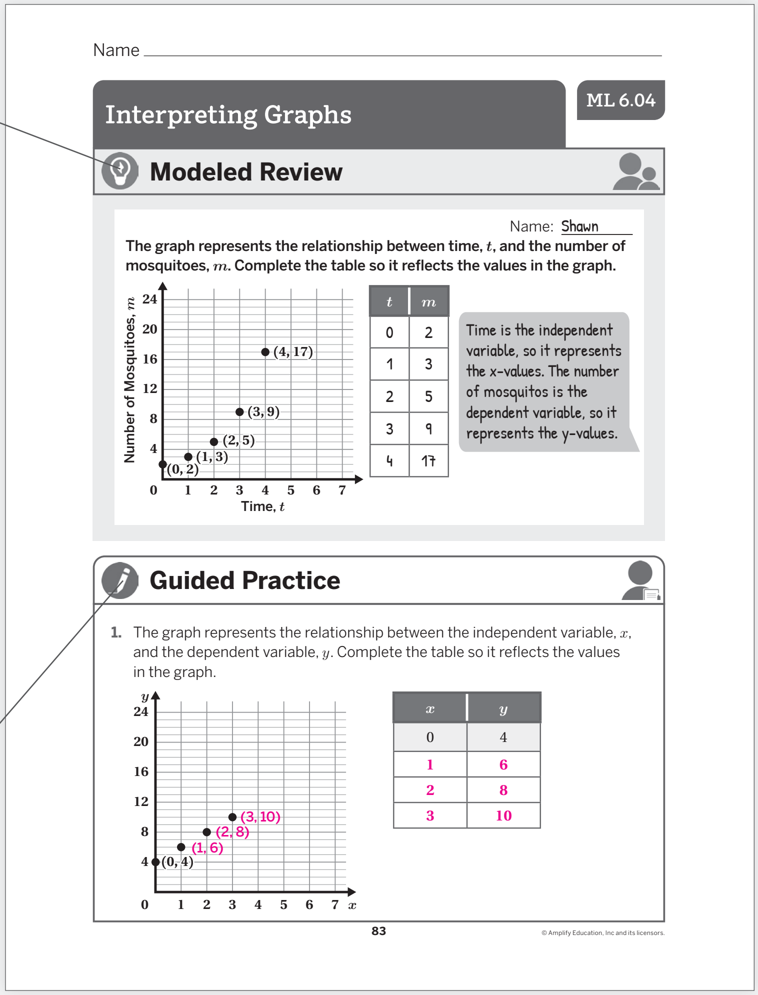
Reinforce Shawn's thinking by saying, "A graph can be used to create a table by focusing on the independent and dependent variables represented by each ordered pair."

Guided Practice

Focus students' attention on using the graph's ordered pairs to create a table that represents the data.

To scaffold their thinking, **say**:

- "Determine the independent and dependent variables."
- "Identify the ordered pairs on the graph."
- "Write the ordered pairs in the table by putting the *x*-values in the first column and *y*-values in the second column."



Name

Interpreting Graphs

ML 6.04

Modeled Review

Name: Shawn

The graph represents the relationship between time, t, and the number of mosquitoes, m. Complete the table so it reflects the values in the graph.

t	m
0	2
1	3
2	5
3	9
4	17

Time is the independent variable, so it represents the x-values. The number of mosquitos is the dependent variable, so it represents the y-values.

Guided Practice

1. The graph represents the relationship between the independent variable, x, and the dependent variable, y. Complete the table so it reflects the values in the graph.

x	y
0	4
1	6
2	8
3	10

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Vocabulary

If needed, share the meaning of the terms with students.

dependent variable: The variable in a relationship that is the effect or result. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

independent variable: The variable in a relationship that is the cause. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

Guided Practice

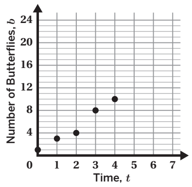
(A) Break down the task by covering points to focus on one ordered pair at a time, and then transfer them to the table.

(ML/EL) To increase accessibility, offer bilingual support materials by including key vocabulary in the students' home language to scaffold their understanding of the task.

Key Takeaway:
Say, "Each point on the graph represents the relationship between an independent variable (x) and the dependent variable (y)."

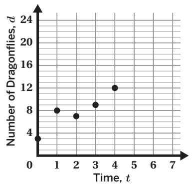
Guided Practice

2. This graph represents the relationship between time, t, and the number of butterflies, b. Complete the table so it reflects the values in the graph.



t	b
0	1
1	3
2	4
3	8
4	10

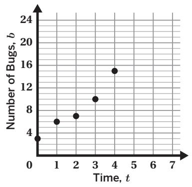
3. This graph represents the relationship between time, t, and the number of dragonflies, d. Complete the table so it reflects the values in the graph.



t	d
0	3
1	8
2	7
3	9
4	12

Check

This graph represents the relationship between time, t, and the number of bugs, b. Complete the table so it reflects the values in the graph.



t	b
0	3
1	6
2	7
3	10
4	15

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Reflection

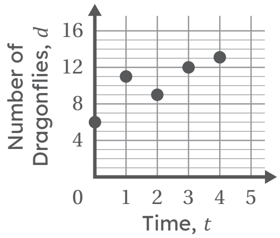
Ask:

- "How can you tell that a table and graph show the same relationship?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using the graph in the Check to model writing the ordered pairs by drawing a vertical line from each ordered pair to the x-axis and a horizontal line from each ordered pair to the y-axis.

Got it!
If students need more practice, sketch the graph and ask them to create a table that represents the data.



Name ______________________________

Comparing Properties of Linear Functions

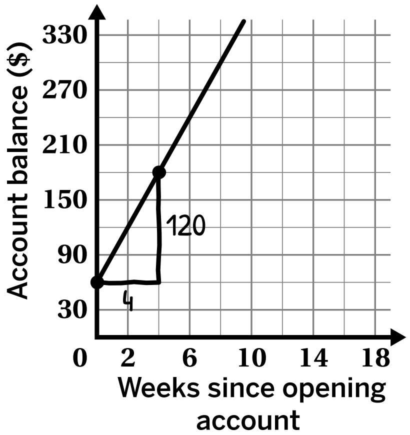
ML 6.07

Modeled Review

Name: Maya

The graph and table each show a different savings account with changes occurring at a constant rate.

Kai



Slope = $\frac{120}{4}$ = 30 y- intercept = 60

Jada

Number of weeks	Account balance ($)
1	75
2	60
3	45

+1 −15

Slope = $\frac{-15}{1}$ = −15

y = mx + b (1,75)

y = −15x + b

75 = −15(1) + b

75 = −15 + b

90 = b

1. Whose balance changes at a faster rate? Explain your thinking.
   Kai; Kai's account is increasing by $30 every week, while Jada's account is decreasing by $15 every week.
2. Who started with the larger amount in their account? Explain your thinking.
   Jada; At 0 weeks she had $90 in her account, while Kai at 0 weeks had $60.

Guided Practice

1. Which representation has the fastest rate of change?

   A. $y = -3x + 1$ **B.** $y = 2x + 1$ **C.** $y = -\frac{1}{2}x + 1$

2. Which representation has the least y-intercept?

   A. The account balance, *a*, starts at $30 and increases by $5 per week.

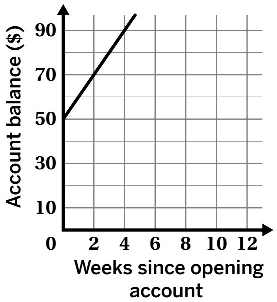
   B. The account balance, *a*, starts at $50 and increases by $8 per week.

   C. The account balance, *a*, starts at $20 and increases by $9 per week.

Guided Practice

3. The savings accounts of three customers are being compared. Circle the representation with the fastest rate of change. Explain your reasoning.

A.



B.

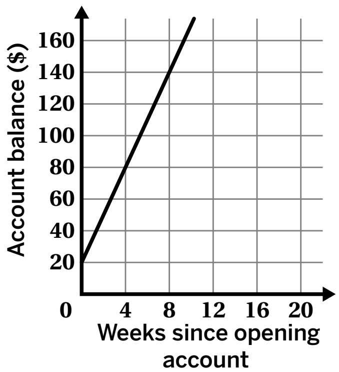
Number of weeks	Account balance ($)
1	90
2	70
3	50

C. The account balance, *a*, starts at $30 and increases $18 per week.

Check

The graph and table each show a different person's savings account.

Jack



Han

Number of weeks	Account balance ($)
1	100
2	80
3	60

1. Whose account balance changes at a faster rate? Explain your thinking.

2. Who started with the larger initial amount saved? Explain your thinking.

ML 6.07 Comparing Properties of Linear Functions

Goal

Compare y-intercepts and slopes of linear functions.

Standard

MA.8.AR.3.5

Modeled Review

Point to Maya's work and **ask**:

- "How did Maya use slope triangles to calculate the slope of the graph?"
- "How did Maya calculate the y-intercept from the table? The graph?"
- "How did Maya use the independent and dependent values to calculate the slope from the table?"

Note: Here, "faster rate" refers to the rate of change with the greatest magnitude and can be positive or negative.

Reinforce Maya's thinking by saying, "The slope and y-intercept can be used to compare linear functions in different representations."

Guided Practice

Focus students' attention on determining the slope and y-intercept to compare the linear functions.

To scaffold their thinking, **ask**:

- "How do you find the rate of change in a linear equation?"
- "How do you find the rate of change in a graph? The y-intercept?"
- "Which keywords in the description represent the slope and y-intercept?"

Name

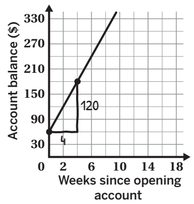
Comparing Properties of Linear Functions ML 6.07

Modeled Review

Name: Maya

The graph and table each show a different savings account with changes occurring at a constant rate.

Kai



Slope = $\frac{120}{4}$ = 30 y- intercept = 60

Jada

Number of weeks	Account balance ($)
1	75
2	60
3	45

+1, −15

Slope = $\frac{-15}{1}$ = −15

y = mx + b (1, 75)
y = −15x + b
75 = −15(1) + b
75 = −15 + b
90 = b

1. Whose balance changes at a faster rate? Explain your thinking.
   Kai; Kai's account is increasing by $30 every week, while Jada's account is decreasing by $15 every week.
2. Who started with the larger amount in their account? Explain your thinking.
   Jada; At 0 weeks she had $90 in her account, while Kai at 0 weeks had $60.

Guided Practice

1. Which representation has the fastest rate of change?
   A. $y = -3x + 1$ **B.** $y = 2x + 1$ **C.** $y = -\frac{1}{2}x + 1$
2. Which representation has the least y-intercept?
   A. The account balance, a, starts at $30 and increases by $5 per week.
   B. The account balance, a, starts at $50 and increases by $8 per week.
   C. The account balance, a, starts at $20 and increases by $9 per week.

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Vocabulary

If needed, share the meaning of the term with students.

linear function: A function that can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept. A vertical line is not a linear function because it has an input with different outputs.

ML 6.07 Comparing Properties of Linear Functions

Guided Practice

A Chunk the problem by evaluating one representation at a time for students to determine the slope or y-intercept, based on what is being asked.

ML/EL Display the graph and table to students without revealing the questions that follow. Then ask students to calculate the slope and y-intercept of each.

Note: Remind students that slope and rate of change can be used interchangeably.

Key Takeaway:
Say, "Multiple linear functions represented in different ways can be compared by determining the slope and the y-intercept of the functions."

Guided Practice

3. The savings accounts of three customers are being compared. Circle the representation with the fastest rate of change. Explain your reasoning.
Sample explanation shown.

A.

B.

Number of weeks	Account balance ($)
1	90
2	70
3	50

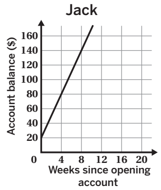
C. The account balance, a, starts at $30 and increases $18 per week.

B; The table decreases by $20 each week. The graph increases by $10 each week, and the description increases by $18 each week.

Check

The graph and table each show a different person's savings account.
Sample explanations shown.

Jack



Han

Number of weeks	Account balance ($)
1	100
2	80
3	60

1. Whose account balance changes at a faster rate? Explain your thinking.
Han; Han's account decreases by $20 each week. Jack's account increases by $15.

2. Who started with the larger initial amount saved? Explain your thinking.
Han; When the number of weeks is 0, the initial amount is $120, while Jack started with $20.

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Reflection

Ask:
- "How can you determine the slope and y-intercept from a graph, table, and description?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling how to find the y-intercept from a table when it is not given by counting backwards using the pattern in the table to find what y equals when x is zero.

Got it!
If students need more practice, have them look at Problem 3 part C and ask if changing the description to say, "The account balance, a, starts at $30 and increases $25 per week." would increase its current rate of change.

Unit 7

Mini-Lessons

Name ______________________________

Determining the Circumference of a Circle

ML 7.02

Modeled Review

Name: Shawn

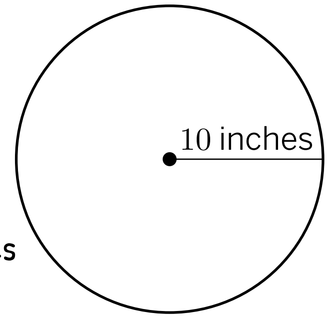
What is the approximate circumference of this circle? Show your thinking.

C = 2πr

C = 2π10

C = 20π

Circumference: 20π inches



The radius is given because it is the distance from the center to the edge.

Guided Practice

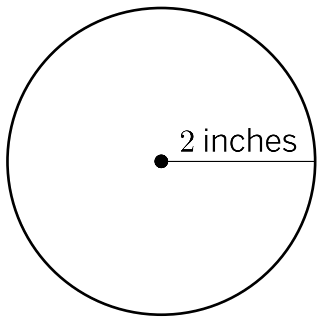
Determine the circumference using $C = 2\pi r$ or $C = \pi d$. Show your thinking.

1. $C = 2\pi r$ radius = 2

$C = 2\pi$ _______

$C =$ _______

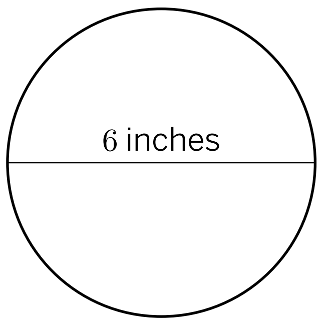
$C =$ ______________________ inches



2. $C = \pi d$ diameter = _______

$C =$ _______

$C =$ ______________________ inches



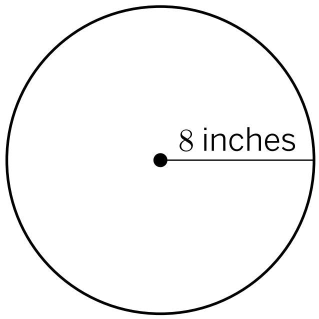
Guided Practice

3. Use the given information to determine the circumference of each object.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Magnifying glass	5	10	
Lid	4		
Salad Plate		14	
Hand Mirror	6		
Clock		18	

Check

What is the approximate circumference of this circle?



ML 7.02 Determining the Circumference of a Circle

Goal

Calculate the circumference of a circle given the radius or diameter.

Standard

MA.7.GR.1.3

Materials

highlighter (optional)

Modeled Review

Point to Shawn's work and **ask**:

- "How did Shawn know if the problem was giving the radius or diameter?"
- "Why did Shawn use $C = 2\pi r$ to determine the circumference?"
- "What are all the ways we can represent or approximate pi?"

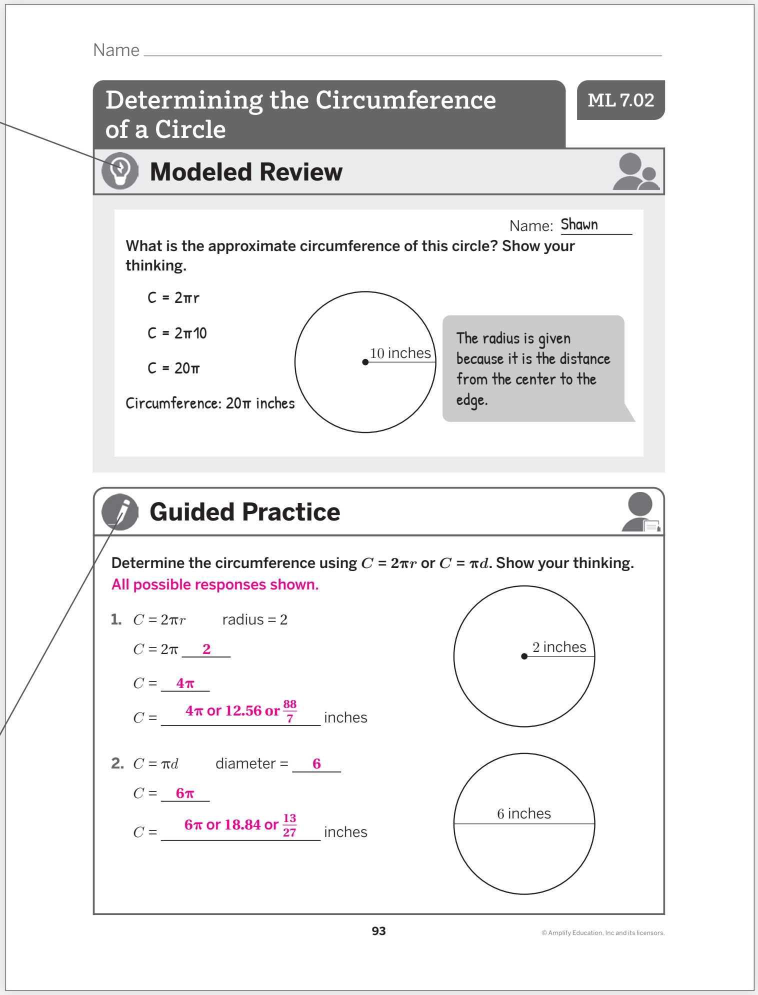
Reinforce the goal by saying, "The formula $C = 2\pi r$ can be used to calculate the circumference when given the radius. The formula $C = \pi d$ can be used to calculate the circumference of a circle when given the diameter."

Guided Practice

Focus students' attention on using the correct formula based on the information provided.

To scaffold their thinking, **ask**:

- "Are you given the radius or diameter?"
- "Which formula should be used?"



Name

Determining the Circumference of a Circle

ML 7.02

Modeled Review

Name: Shawn

What is the approximate circumference of this circle? Show your thinking.

C = 2πr

C = 2π10

C = 20π

Circumference: 20π inches

10 inches

The radius is given because it is the distance from the center to the edge.

Guided Practice

Determine the circumference using $C = 2\pi r$ or $C = \pi d$. Show your thinking.

All possible responses shown.

1. $C = 2\pi r$ radius = 2

$C = 2\pi$ 2

$C =$ 4π

$C =$ 4π or 12.56 or $\frac{88}{7}$ inches

2 inches

2. $C = \pi d$ diameter = 6

$C =$ 6π

$C =$ 6π or 18.84 or $\frac{13}{27}$ inches

6 inches

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Vocabulary

If needed, share the meaning of the terms with students.

circumference: The distance around a circle. (If you imagine the border of the circle as a piece of string, the circumference is the length of the string.)

diameter: The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

radius: A line segment that goes from the center to the edge of a circle.

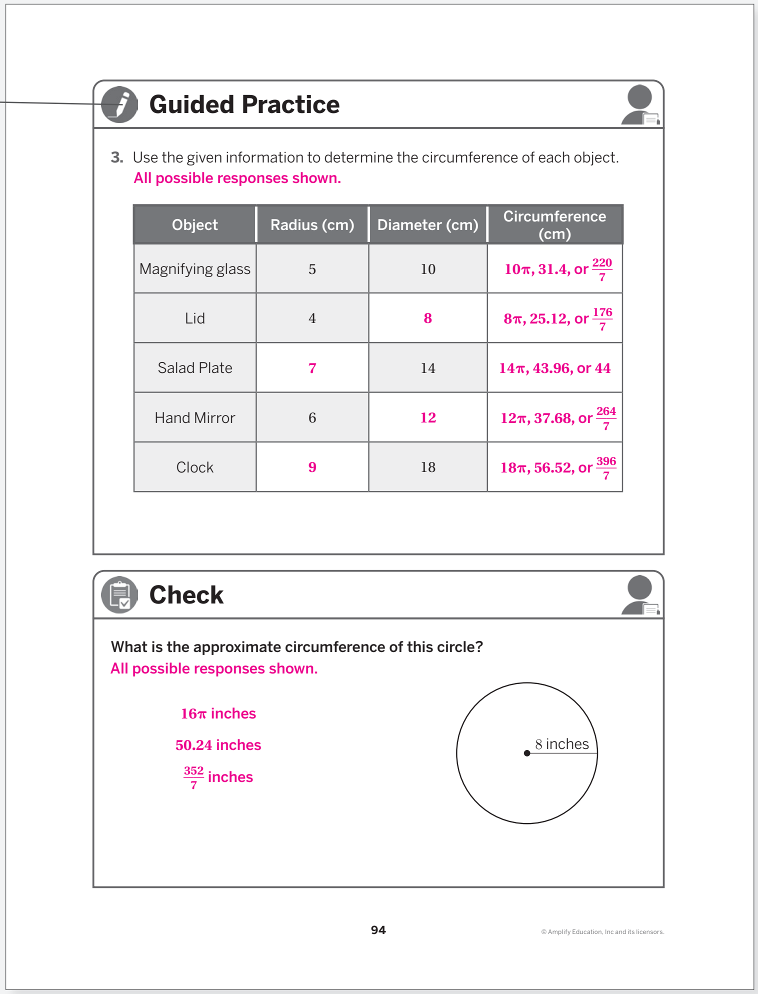
Determining the Circumference of a Circle

Guided Practice

A Create an anchor chart that describes the radius, diameter, and circumference of a circle for reference.

ML/EL Provide sentence frames to help students explain their reasoning. For example, "____ is given so I will use the ____ formula."

Key Takeaway:
Say, "The circumference of a circle is equal to the length of the diameter, or twice the length of the radius, multiplied by π."



Guided Practice

3. Use the given information to determine the circumference of each object.
All possible responses shown.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Magnifying glass	5	10	10π, 31.4, or $\frac{220}{7}$
Lid	4	8	8π, 25.12, or $\frac{176}{7}$
Salad Plate	7	14	14π, 43.96, or 44
Hand Mirror	6	12	12π, 37.68, or $\frac{264}{7}$
Clock	9	18	18π, 56.52, or $\frac{396}{7}$

Check

What is the approximate circumference of this circle?
All possible responses shown.

16π inches

50.24 inches

$\frac{352}{7}$ inches

94

Reflection

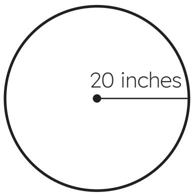
Ask:

- "How is it helpful to know the difference between radius and diameter when calculating the circumference?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, ask them to highlight the given radius or diameter to help them determine which equation to use. Then have them highlight the letter *r* or *d* in the formula that is needed to calculate the circumference for the circle given.

Got it!
If students need more practice, sketch the circle and ask them to determine the approximate circumference.



Name ______________________________

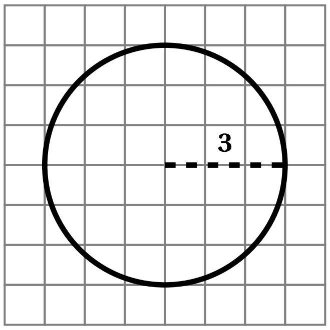
Determining the Area of a Circle

ML 7.04

Modeled Review

Name: Nick

Find the exact area of this circle.



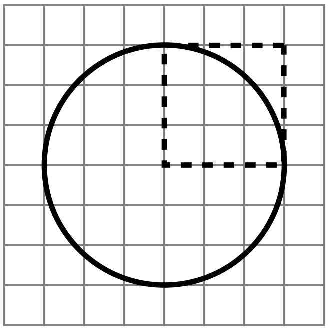
First, I drew the radius and labeled it 3 units. Then, I used the formula to calculate the area of the circle.

$$A = \pi \cdot r^2$$
$$A = \pi \cdot (3)^2$$
$$A = 9\pi$$

The area of the circle is 9π square units.

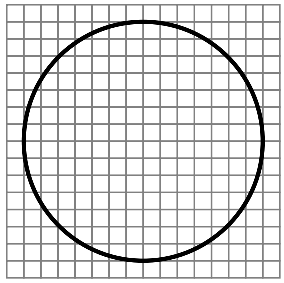
Guided Practice

1. Use the radius square to estimate the area of this circle. Show or explain your thinking.

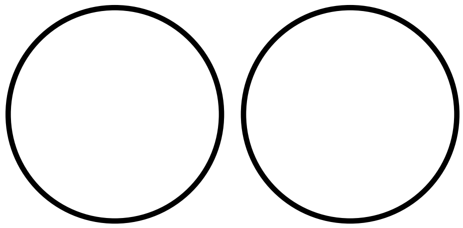


Guided Practice

2. Draw the radius for the circle and find its exact area.



3. Find the exact and approximate area of a circle with a diameter of 10 inches. (Use 3.14 for π.) Show your thinking. (A blank circle is provided to help with your thinking.)



4. Jenny is making 4 circular pizzas for her family. Each pizza will have a diameter of 12 inches. Approximately how much pizza dough will she need to buy? (Use 3.14 for π.) Show your thinking.

Check

Melanie is designing circular mirrors with a diameter of 20 inches. If she is making 3 mirrors, how much glass will she need? (Use 3.14 for π.)

ML 7.04 Determining the Area of a Circle

Goal

Apply a formula to find the area of a circle in problems of both mathematical and real word context.

Standard

MA.7.GR.1.4

Materials

Grid paper, scissors, colored pencils (optional)

Modeled Review

Point to Nick's work and **ask**:
- "How did Nick know where to draw the radius?"
- "What does *exact form* mean?"
- "Why is 3^2 equal to 9?"

Reinforce the goal by stating "If we know the radius of a circle, we can use the formula Area equals pi times the square of the radius to calculate its area."

Guided Practice

Focus students' attention on how radius squares can be used to estimate the area of a circle.

To scaffold their thinking, **ask/say**:
- "What is the area of the radius square?"
- "Approximately how many of those squares can fit inside the circle?"
- "How does the area of the 3 squares relate to the area of the circle?"

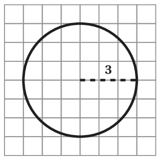
Name

Determining the Area of a Circle ML 7.04

Modeled Review

Name: Nick

Find the exact area of this circle.



First, I drew the radius and labeled it 3 units. Then, I used the formula to calculate the area of the circle.

$A = \pi \cdot r^2$

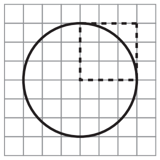
$A = \pi \cdot (3)^2$

$A = 9\pi$

The area of the circle is 9π square units.

Guided Practice

1. Use the radius square to estimate the area of this circle. Show or explain your thinking.



Answers vary. The area of the radius squared is $2 \cdot 2 = 4$. There are a little more than 3 radius squares in a circle. $4 \cdot 3 = 12$, so the area of the circle would be a little more than 12.

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Vocabulary

If needed, share the meaning of the terms with students.

diameter of a circle: the length from one side of the circle to the opposite side, passing through the center of the circle.

radius of a circle: the length from the center of a circle to any point on the circle.

radius square: a square with a side length is the same as the radius of a given circle.

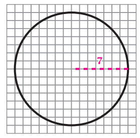
Guided Practice

A Provide students with grid paper, scissors, and highlighters to estimate the area of circles using radius squares. Help them to connect these approximations with the exact area found using the area of a circle formula.

Key Takeaway:
Say, "We can find the exact area of a circle by multiplying the square of the radius by pi. Because the value of pi is a little more than 3, we can replace pi with 3.14 and approximate the area, when solving problems in real world contexts."

Guided Practice

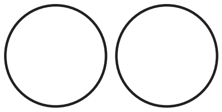
2. Draw the radius for the circle and find its exact area.



49π units². *Responses vary.*

$A = \pi \cdot r^2$
$A = \pi \cdot (7)^2$
$A = 49\pi$

3. Find the exact and approximate area of a circle with a diameter of 10 inches. (Use 3.14 for π.) Show your thinking. (A blank circle is provided to help with your thinking.)



25π or 78.5 inches². *Explanations vary.* Since the diameter of the circle is 10 inches, the radius is 5 inches.

$A = \pi \cdot r^2$
$A = \pi \cdot (5)^2$
$A = 25\pi$ $A = 25(3.14) = 78.5$

4. Jenny is making 4 circular pizzas for her family. Each pizza will have a diameter of 12 inches. Approximately how much pizza dough will she need to buy? (Use 3.14 for π.) Show your thinking.

About 452 square inches. *Explanations vary*. Since the diameter of each pizza is 12 inches, the radius is 6 inches.

$A = \pi \cdot r^2$
$A = \pi \cdot (6)^2$ There are 4 pizzas, so $4 \cdot 36\pi = 144\pi$
$A = 36\pi$ $144(3.14) = 452.16$

Check

Melanie is designing circular mirrors with a diameter of 20 inches. If she is making 3 mirrors, how much glass will she need? (Use 3.14 for π.)

About 942 square inches. Explanations vary. Since the diameter of one mirror is 20 inches, the radius is 10 inches.

$A = \pi \cdot r^2$
$A = \pi \cdot (10)^2$
$A = 100\pi$ There are 3 mirrors, so $3 \cdot 100\pi = 300\pi$
$300(3.14) = 942$

98

Reflection

Ask:

- "What is the relationship between the value of pi and the number of radius squares that fit inside a circle?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support with differentiating between the diameter and the radius of a circle, have them make their own visual model with a diameter drawn. Then, have them identify and label the two radii that make up the diameter, as well as sketch more radii of the same circle.

Got it!
If students need more practice, have them revisit Problem 4 and calculate the amount of pizza dough needed to make 6 circular pizzas with a diameter of 14 inches.

Name ______________________________

Calculating Areas of Complex Shapes

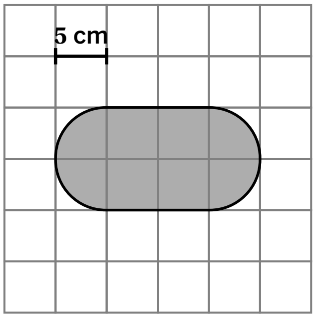
ML 7.05

Modeled Review

Name: Kai

What is the area of this shape?

Area of circle = πr^2
$\pi 5^2$
25π
Area of square = lw
10×10
100

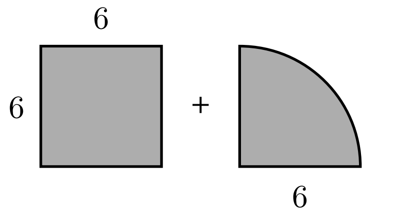


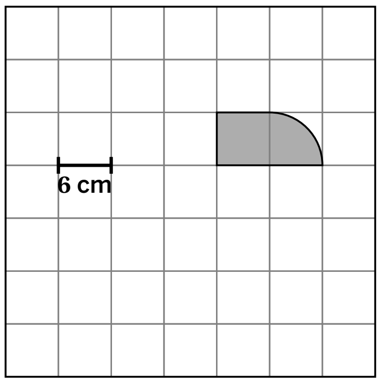
The shape is made up of a square and two semi-circles. Two semi-circles make one circle.

Area of the shape = $100 + 25\pi$ square centimeters

Guided Practice

1. Calculate the area of the shape.





Area of square = _______ square centimeters

Area of a quarter circle = $\frac{\pi r^2}{4}$

= _______ square centimeters

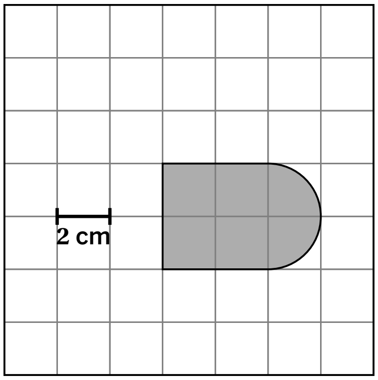
Area of shape = ____+____ square centimeters

Guided Practice

Calculate the area of the given shape.

Area of square = lw	Area of semi-circle = $\frac{\pi r^2}{2}$
Area of circle = πr^2	Area of quarter-circle = $\frac{\pi r^2}{4}$

2.



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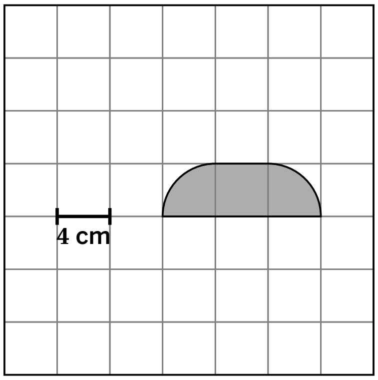
3.



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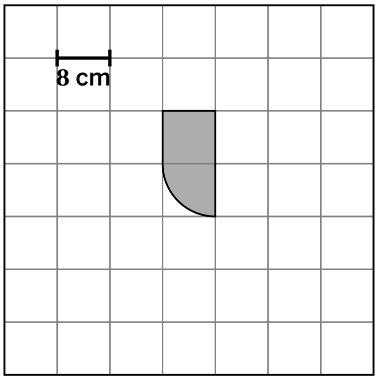
4.



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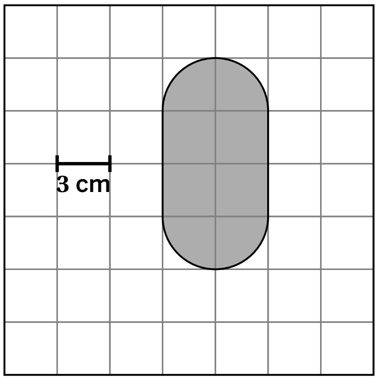
5.



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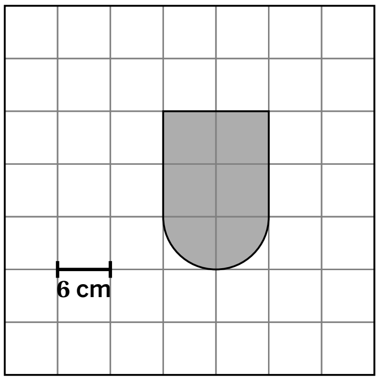
Check

Calculate the area of the shape.



ML 7.05 Calculating Areas of Complex Shapes

Goal

Calculate the area of a complex shape composed of fractions of circles.

Standard

MA.7.GR.1.4

Materials

coloring tools (optional)

Modeled Review

Point to Kai's work and **ask**:

- "How did Kai break up the complex shape?"
- "What shapes make up the complex shape?"
- "How did Kai calculate the area of the complex shape?"

Reinforce Kai's thinking by saying, "You can determine the area of complex shapes by decomposing them into squares and parts of circles."

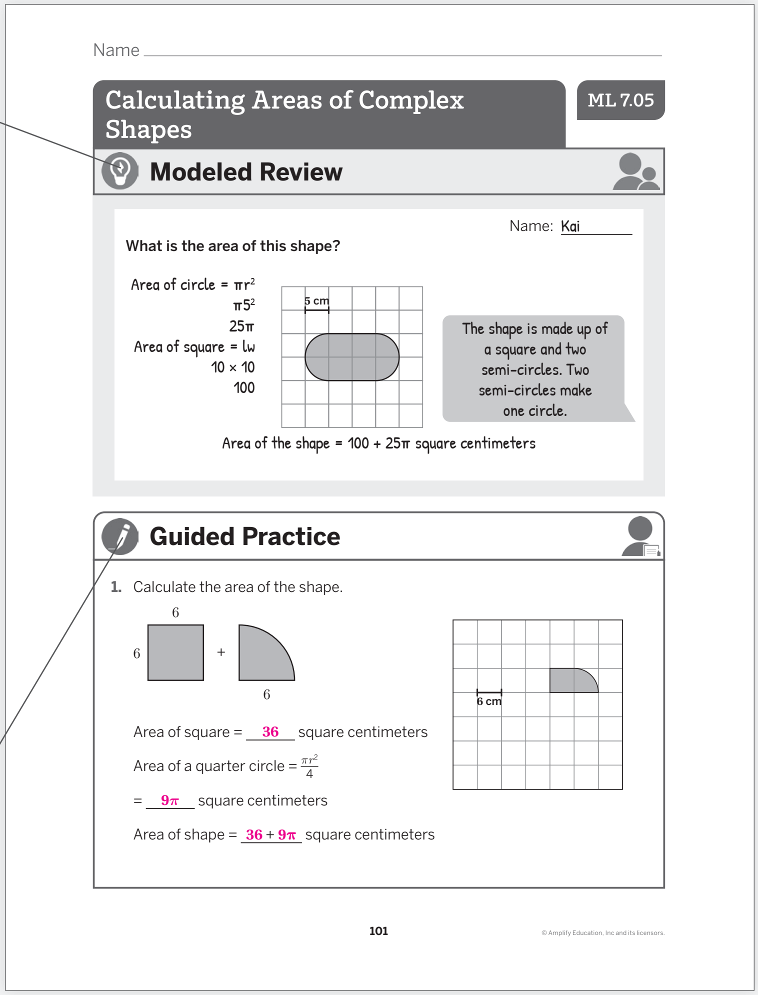
ML/EL Model decomposing the shape into a square and two semi-circles. Show students how two semi-circles can be combined to form a circle.

Guided Practice

Focus students' attention on decomposing the complex shape to calculate the area.

To scaffold their thinking, **say**:

- "The shape can be broken up into other shapes."
- "Calculate the area of each shape."
- "To find the area of the complex shape, add the areas of all the shapes."



Name

Calculating Areas of Complex Shapes — ML 7.05

Modeled Review

Name: Kai

What is the area of this shape?

Area of circle = πr^2
$\pi 5^2$
25π
Area of square = lw
10×10
100

5 cm

The shape is made up of a square and two semi-circles. Two semi-circles make one circle.

Area of the shape = $100 + 25\pi$ square centimeters

Guided Practice

1. Calculate the area of the shape.

6, 6, +, 6

6 cm

Area of square = 36 square centimeters

Area of a quarter circle = $\frac{\pi r^2}{4}$

= 9π square centimeters

Area of shape = $36 + 9\pi$ square centimeters

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Vocabulary

If needed, share the meaning of the term with students.

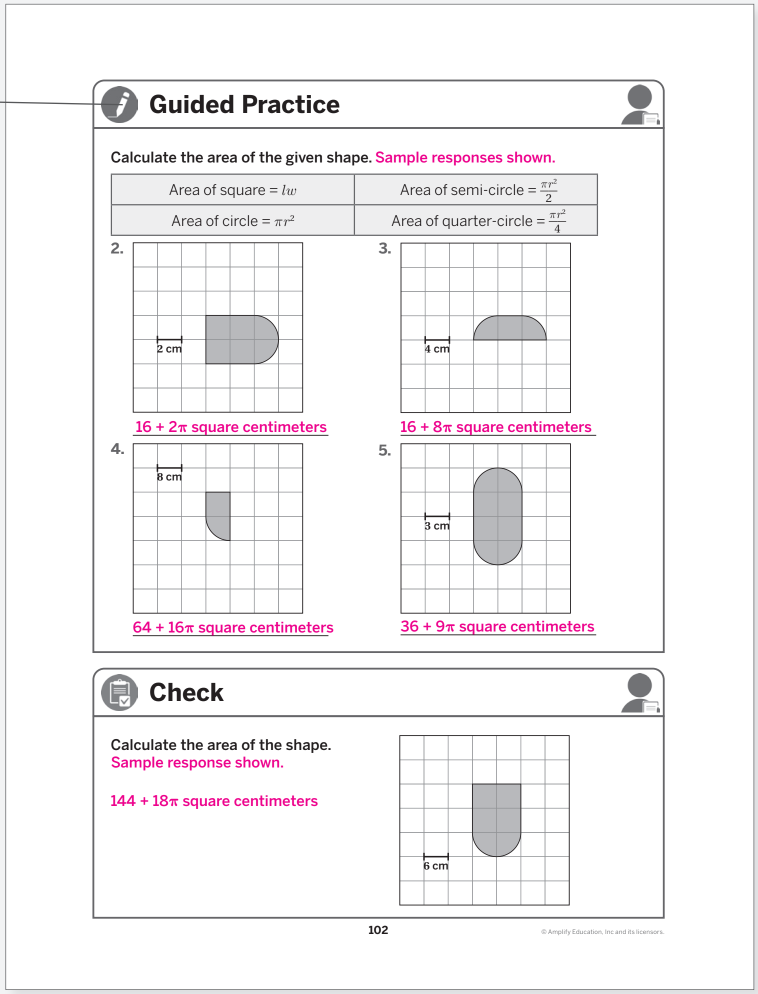
circle: A shape made out of all the points that are the same distance from a center.

Guided Practice

A Invite students to decompose the shapes into smaller, familiar shapes such as squares and semi-circles. Ask them to shade these smaller shapes in different colors.

Note: Students can leave their answer as an expression that includes pi, such as $50 + 16\pi$.

Key Takeaway:
Say, “Decomposing shapes into squares and parts of circles can help determine their total area.”



Guided Practice

Calculate the area of the given shape. Sample responses shown.

Area of square = lw	Area of semi-circle = $\frac{\pi r^2}{2}$
Area of circle = πr^2	Area of quarter-circle = $\frac{\pi r^2}{4}$

2. 2 cm — $16 + 2\pi$ square centimeters

3. 4 cm — $16 + 8\pi$ square centimeters

4. 8 cm — $64 + 16\pi$ square centimeters

5. 3 cm — $36 + 9\pi$ square centimeters

Check

Calculate the area of the shape.
Sample response shown.

$144 + 18\pi$ square centimeters

6 cm

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Reflection

Ask:

- “How does breaking up the complex shape help calculate the area?”
- “How did you overcome a hard problem today?”

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, ask them to create a t-chart for the shapes that make up the complex shape. Then have them use the t-chart to organize their work.	If students need more practice, ask them how the area in Problem 5 would change if the scale was 8 centimeters.

Name ______________________________

Calculating Volumes of Cylinders

ML 7.07

Modeled Review

Name: Clare

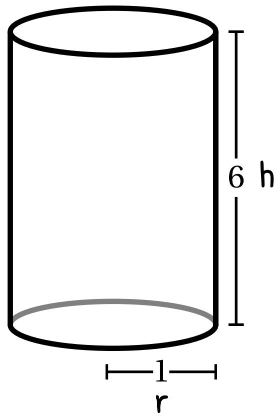
Calculate the volume of the cylinder.

$V = \pi r^2 h$

$V = \pi(1)^2(6)$

$V = 6\pi$

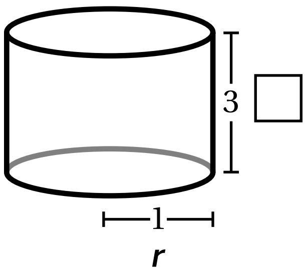
6π cubic units



Guided Practice

Write an equation to find the volume, V, of each cylinder.

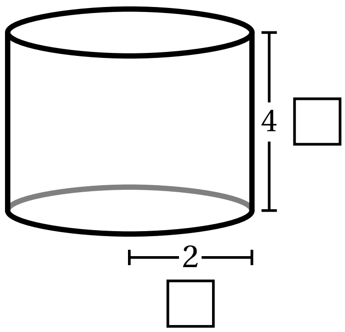
1.



$V = \pi r^2 h$

$V = \pi(1)^2(\_\_\_)$

2.

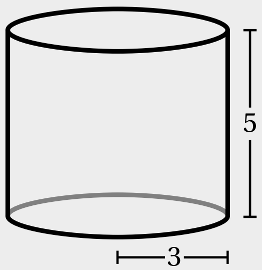
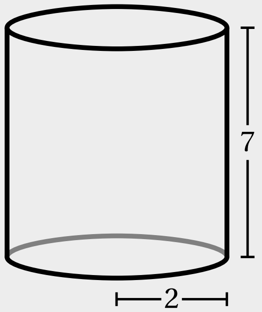
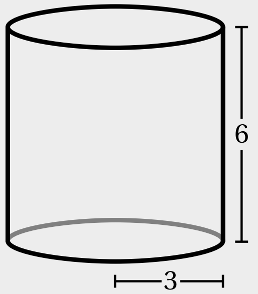


$V = \pi r^2 h$

$V =$ ________

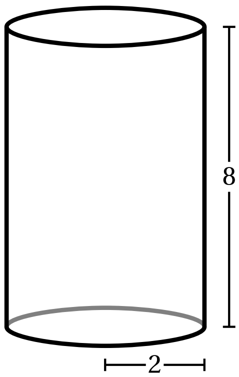
Guided Practice

3. Calculate the volume, V, of each cylinder using the formula.

Cylinder	Formula $V = \pi r^2 h$	Volume
		
		
		

Check

Calculate the volume of the cylinder.



Calculating Volumes of Cylinders

Goal

Calculate the volume of a cylinder given its height and radius.

Standard

MA.7.GR.2.3

Materials

image of a cylinder (optional)

Modeled Review

Point to Clare's work and **ask**:
- "How did Clare know which dimensions to label as the radius and height?"
- "What does it mean to square a number?"
- "Why does Clare square the radius first?"

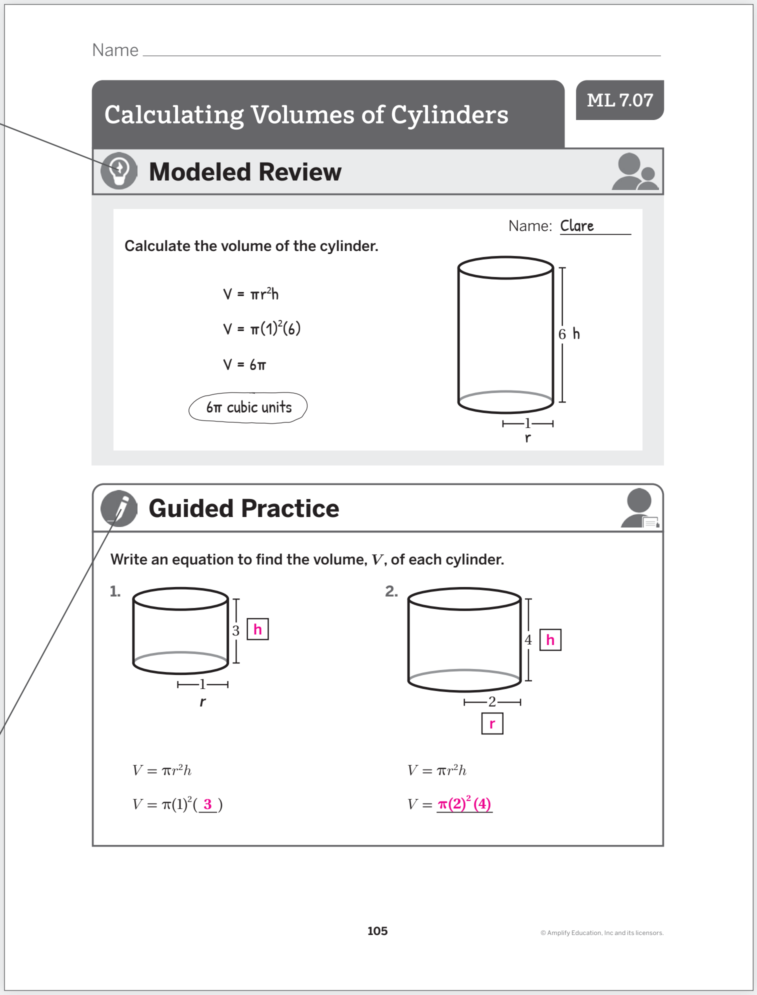
Reinforce Clare's thinking by saying, "Labeling the radius and height of the cylinder can help organize the dimensions given to efficiently calculate the volume of a cylinder."

ML/EL Consider modeling, or inviting a student to model, annotating the problem to make sense of it.

Guided Practice

Focus students' attention on writing the formula with the given radius and height to determine the volume of a cylinder.

To scaffold their thinking, **say**:
- "Label the radius and height of the cylinder."
- "Substitute the dimensions accordingly into the formula."
- "Leave your answer in terms of pi."



Name

Calculating Volumes of Cylinders

ML 7.07

Modeled Review

Name: Clare

Calculate the volume of the cylinder.

$V = \pi r^2 h$

$V = \pi(1)^2(6)$

$V = 6\pi$

6π cubic units

6 h

1

r

Guided Practice

Write an equation to find the volume, V, of each cylinder.

1. 3 $\boxed{h}$ 1 r

$V = \pi r^2 h$

$V = \pi(1)^2(\underline{3})$

2. 4 $\boxed{h}$ 2 $\boxed{r}$

$V = \pi r^2 h$

$V = \underline{\pi(2)^2(4)}$

105

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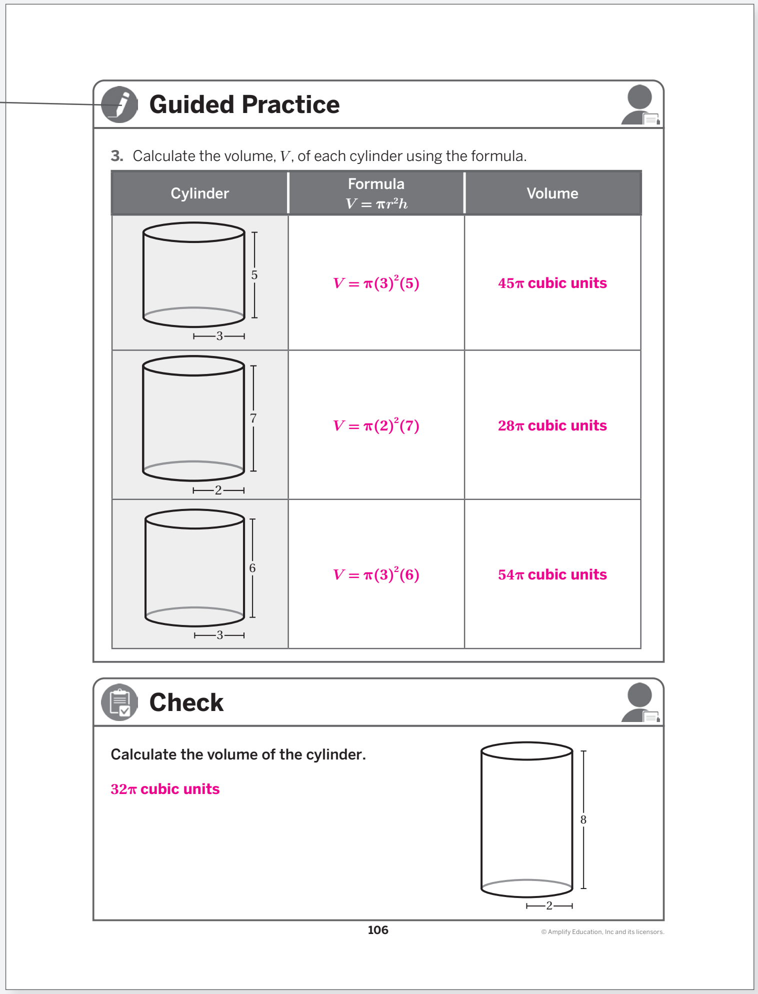
Guided Practice

A Consider sharing a picture of a cylinder to increase access to the task. For example, show students a can of soup and ask them to point out the height and radius.

Note: Students can use either $V = Bh$ or $V = \pi r^2 h$ to calculate the volume of a cylinder.

Key Takeaway:
Say, "The volume of a cylinder can be found by multiplying the area of its base by its height, and is represented by the formulas $V = \pi r^2 h$ and $V = Bh$."



Guided Practice

3. Calculate the volume, V, of each cylinder using the formula.

Cylinder	Formula $V = \pi r^2 h$	Volume
5 3	$V = \pi(3)^2(5)$	45π cubic units
7 2	$V = \pi(2)^2(7)$	28π cubic units
6 3	$V = \pi(3)^2(6)$	54π cubic units

Check

Calculate the volume of the cylinder.

32π cubic units

8
2

106

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Reflection

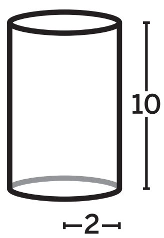
Ask:

- "How does knowing the area of a circular base help calculate the volume of a cylinder?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them write the radius twice in the formula for finding the volume of a cylinder, instead of squaring it. This approach helps students understand the meaning behind the calculation, preventing them from simply multiplying the radius by two.

Got it!
If students need more practice, sketch the following cylinder and ask them to calculate its volume.



Unit 8

Mini-Lessons

Name ____________________

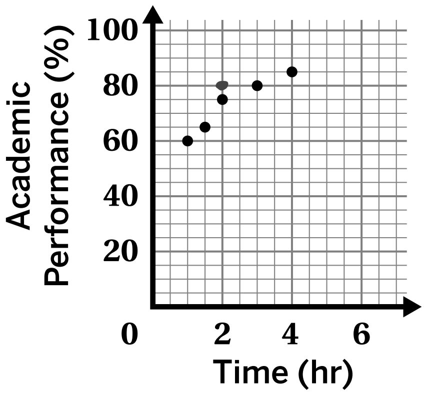
Interpreting Points on a Scatter Plot

ML 8.06

Modeled Review

Name: Diego

This scatter plot shows the academic performance and the number of hours students studied for a math test.

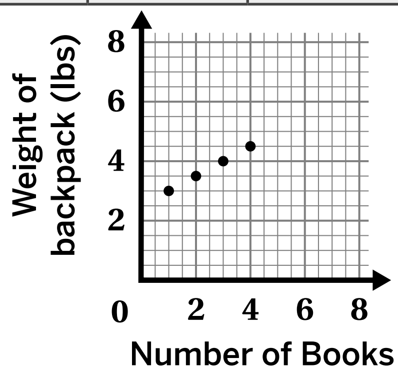


1. What is the highest academic performance a student received?
   85%
2. What is the academic performance of the student who studied the least?
   60%
3. Another student studied 2 hours and earned an academic performance of 80%. Plot a point on the graph that represents this student.

Guided Practice

The table and scatter plot show the number of books each student carried in their backpack and the corresponding weight of each backpack.

Student	Number of books	Backpack weight (lbs)
Isaiah	1	3
Kayla	3	4
Mia	2	3.5
Felipe	4	4.5



1. Circle the point on the scatter plot that represents the data for Mia.
2. What does the point $(3, 4)$ represent?
3. In the same study, the data showed that Arjun had a backpack weighing 5 pounds with 6 books. Add a point to the scatter plot to represent Arjun.

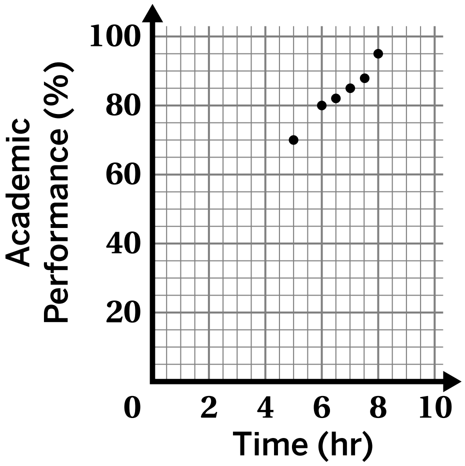
Guided Practice

This scatter plot illustrates the connection between the amount of sleep students get each night and their academic performance.

4. What is the highest academic performance a student received?

5. How many hours did the student who received the highest academic performance sleep?

6. The academic performance for a student who slept 5 hours is 50%. Plot a point on the graph that represents this student.



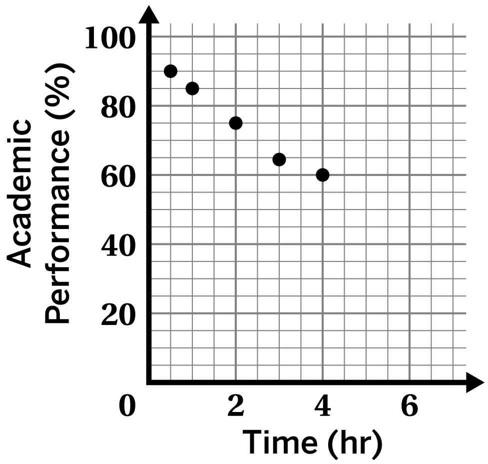
Check

This scatter plot shows the academic performance and the number of hours of screen time students have.

1. What is the highest academic performance observed in a student?

2. What is the lowest academic performance observed in a student?

3. The academic performance for a student who had 4 hours of screen time is 75%. Plot a point on the graph that represents this student.



ML 8.06 Interpreting Points on a Scatter Plot

Goal

Interpret points on a scatter plot in context.

Standard	Materials
MA.8.DP.1.1	highlighter (optional)

Modeled Review

Point to Diego's work and **ask**:

- "What specific information does each axis give you?"
- "What does a point represent?"
- "How did Diego find the academic performance of the student who studied the least?"

Reinforce the goal by saying, "The axis labels tell us how to interpret the coordinates of each point on the scatter plot."

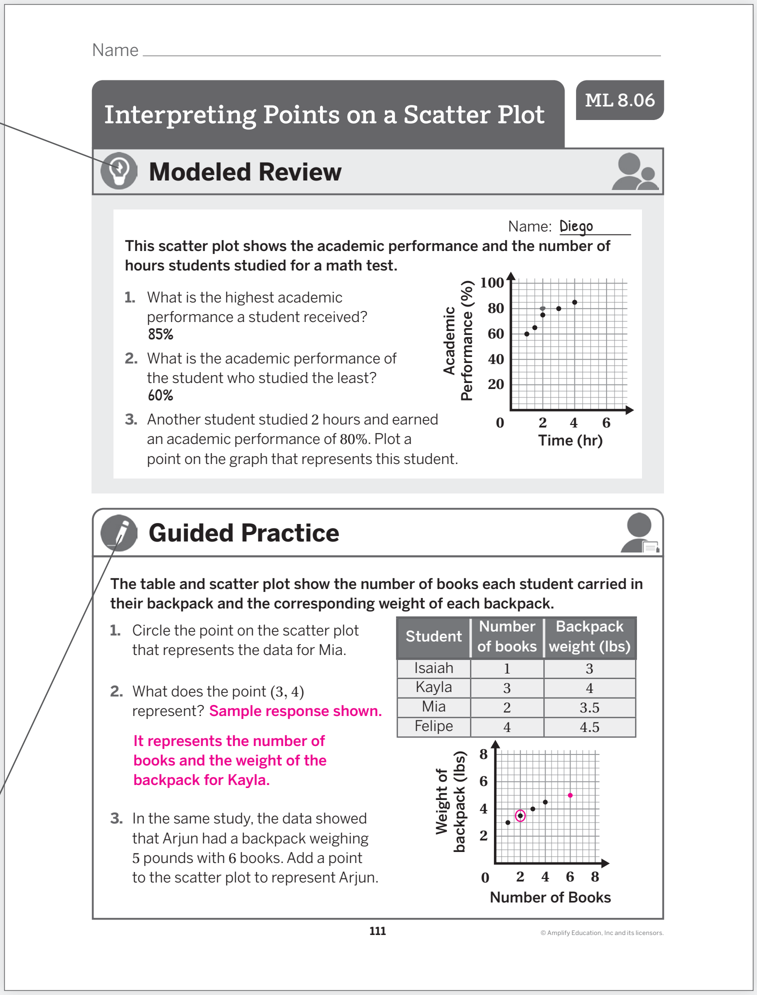
ML/EL Engage students in a discussion about everyday contexts where scatter plots might be useful, such as tracking hours studied vs. test scores or height vs. shoe size.

Guided Practice

Focus students' attention on using the table to interpret the scatter plot and answer the questions.

To scaffold their thinking, **ask**:

- "What information does the *x*-axis give you? Where do you see that in table?"
- "What information does the *y*-axis give you? Where do you see that in table?"
- "What does each point represent?"



Name

Interpreting Points on a Scatter Plot

ML 8.06

Modeled Review

Name: Diego

This scatter plot shows the academic performance and the number of hours students studied for a math test.

1. What is the highest academic performance a student received?
   85%
2. What is the academic performance of the student who studied the least?
   60%
3. Another student studied 2 hours and earned an academic performance of 80%. Plot a point on the graph that represents this student.

Guided Practice

The table and scatter plot show the number of books each student carried in their backpack and the corresponding weight of each backpack.

1. Circle the point on the scatter plot that represents the data for Mia.
2. What does the point $(3, 4)$ represent? **Sample response shown.**

   It represents the number of books and the weight of the backpack for Kayla.
3. In the same study, the data showed that Arjun had a backpack weighing 5 pounds with 6 books. Add a point to the scatter plot to represent Arjun.

Student	Number of books	Backpack weight (lbs)
Isaiah	1	3
Kayla	3	4
Mia	2	3.5
Felipe	4	4.5

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Vocabulary

If needed, share the meaning of the term with students.

scatter plot: A set of disconnected data points plotted on a coordinate plane. Scatter plots allow us to investigate connections between two variables.

Guided Practice

A Invite students to highlight the axis labels to help them interpret the scatter plot.

Key Takeaway:
Say, "The values of a point and the axis labels can be used to interpret the coordinates of each point on a scatter plot."

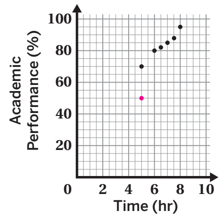
Guided Practice

This scatter plot illustrates the connection between the amount of sleep students get each night and their academic performance.

4. What is the highest academic performance a student received?
95%

5. How many hours did the student who received the highest academic performance sleep?
8

6. The academic performance for a student who slept 5 hours is 50%. Plot a point on the graph that represents this student.



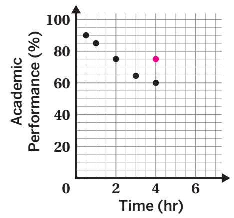
Check

This scatter plot shows the academic performance and the number of hours of screen time students have.

1. What is the highest academic performance observed in a student?
90%

2. What is the lowest academic performance observed in a student?
60%

3. The academic performance for a student who had 4 hours of screen time is 75%. Plot a point on the graph that represents this student.



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Reflection

Ask:
- "How do you know what a point represents on a scatter plot?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the scatter plot in the Check. Then have them represent the points as ordered pairs in a table.

Got it!
If students need more practice, have them revisit the scatter plot in the Check and ask the following questions:

- How many hours of screen time did the student who received the lowest academic performance have?
- What was the academic performance of the student who had 2 hours of screen time?

Name ____________________

Using Lines of Fit to Make Predictions

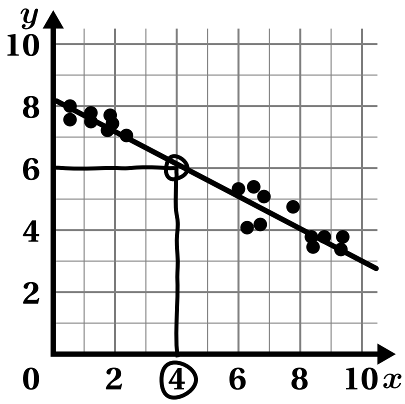
ML 8.12

Modeled Review

Name: Tristan

Use the line of fit to predict the y-value of a new data point whose x-value is 4.

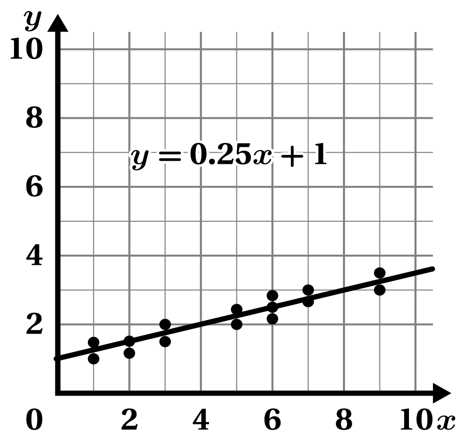
6



Guided Practice

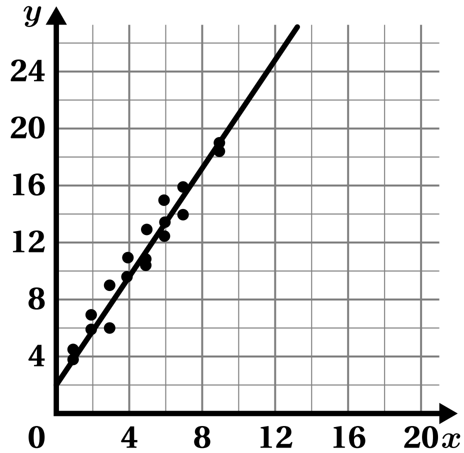
1. Use the line of fit to make predictions and complete the table.

x	y
4	
8	

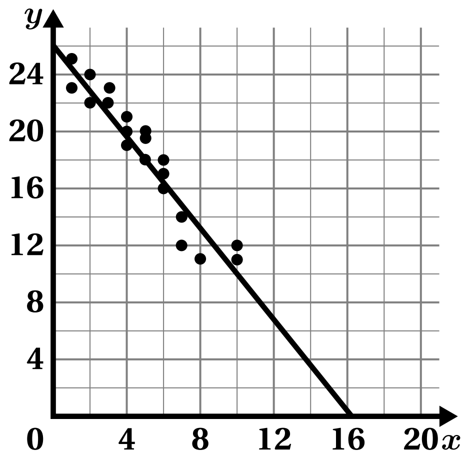


Guided Practice

2. Use the line of fit to predict the y-value of a new data point whose x-value is 8.

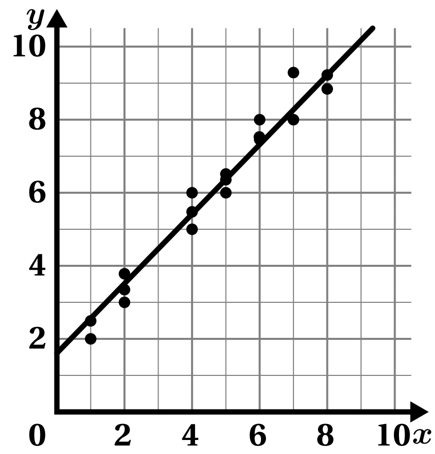


3. Use the line of fit to predict the y-value of a new data point whose x-value is 12.



Check

Use the line of fit to predict the y-value of a new data point whose x-value is 3.



ML 8.12 Using Lines of Fit to Make Predictions

Goal

Use the line of fit to make predictions about data points on a scatter plot.

Standard

MA.8.DP 1.3

Materials

highlighter (optional), ruler (optional)

Modeled Review

Point to Tristan's work and **ask**:

- "How did Tristan use the line of fit to make the prediction?"
- "How did Tristan use the given *x*-value of 4 to find the corresponding *y*-value?"

Reinforce Tristan's thinking by saying, "A line of fit can be used on scatter plots to make predictions for a given data set."

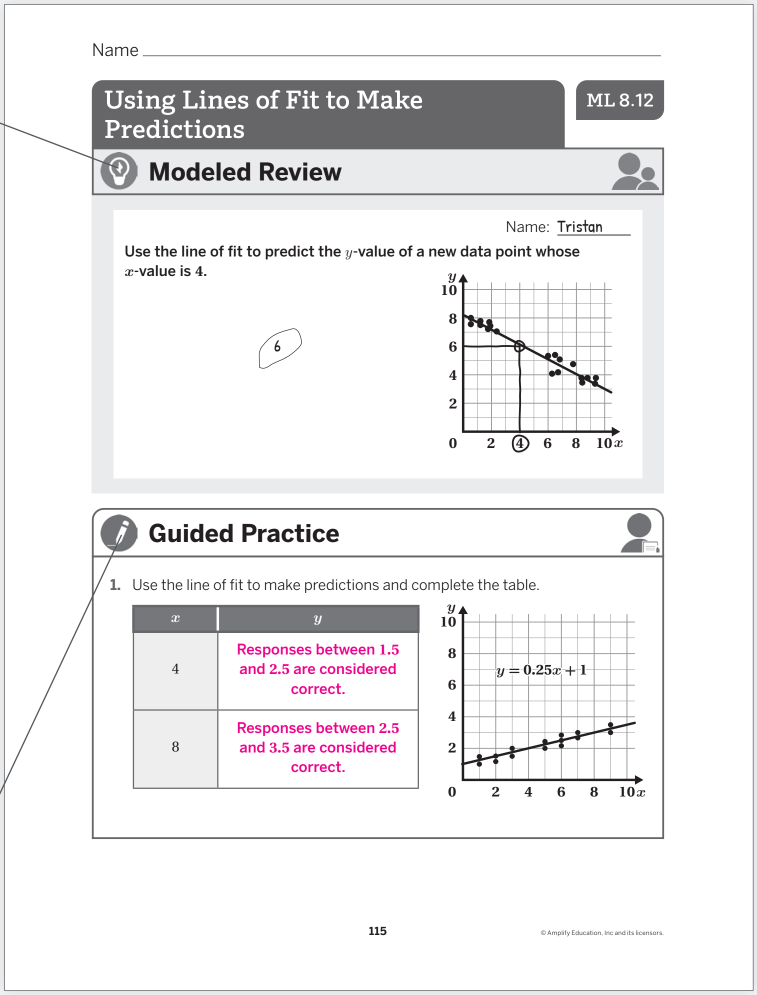
ML/EL Provide sentence frames to help students explain their strategy (e.g., "First, I see the *x*-value is ____, so I follow it vertically to meet the line of fit and predict the *y*-value is _____.").

Guided Practice

Focus students' attention on using the line of fit to make predictions.

Encourage students to annotate the graph as they find the corresponding *y*-value.

Note: Students can choose any number within the given range since it's an estimated value based on the line of fit.



Name

Using Lines of Fit to Make Predictions — ML 8.12

Modeled Review

Name: Tristan

Use the line of fit to predict the y-value of a new data point whose x-value is 4.

6

Guided Practice

1. Use the line of fit to make predictions and complete the table.

x	y
4	Responses between 1.5 and 2.5 are considered correct.
8	Responses between 2.5 and 3.5 are considered correct.

$y = 0.25x + 1$

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ML 8.12 Using Lines of Fit to Make Predictions

Guided Practice

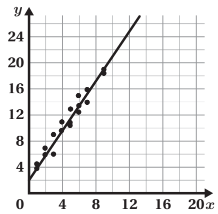
(A) Model highlighting the provided x-value and drawing a vertical line to assist students in identifying the corresponding y-value.

Key Takeaway:
Say, "A line of fit can be useful for making predictions for data being represented in scatter plots."

Guided Practice

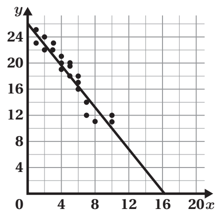
2. Use the line of fit to predict the y-value of a new data point whose x-value is 8.

Responses between 16 and 18 are considered correct.



3. Use the line of fit to predict the y-value of a new data point whose x-value is 12.

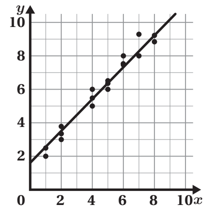
Responses between 6 and 8 are considered correct.



Check

Use the line of fit to predict the y-value of a new data point whose x-value is 3.

Responses between 4 and 5 are considered correct.



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Reflection

Ask:
- "What does a line of fit tell you about the data?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, model using a ruler to guide them in determining where to stop when making a prediction for a given x-value using the line of fit.

Got it!
If students need more practice, have them revisit the scatter plot in Problem 3. Then have them use the line of fit to predict the y-value for a data point whose x-value is 14.

Unit 9

Mini-Lessons

Name ___

Multiplying Powers

ML 9.03

Modeled Review

Name: Jack

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(5^5)^2$ $(5^5) \cdot (5^5)$ $(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)$	$5^4 \cdot 5^3$ $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	Yes No
$4^3 \cdot 2^5$ $(4 \cdot 4 \cdot 4) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ $(4 \cdot 2)(4 \cdot 2)(4 \cdot 2) \cdot 2 \cdot 2$	8^8 $(8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8)$	Yes No
$15^3 \cdot 2^3$ $(15 \cdot 15 \cdot 15) \cdot (2 \cdot 2 \cdot 2)$ $(15 \cdot 2)(15 \cdot 2)(15 \cdot 2)$	$(5 \cdot 2)^3 \cdot 3^3$ $(10 \cdot 10 \cdot 10) \cdot (3 \cdot 3 \cdot 3)$ $(10 \cdot 3)(10 \cdot 3)(10 \cdot 3)$	Yes No

Guided Practice

1. Match each expression to the correct expanded form on the right.

Expression	Expanded Form
$(3^4)^2$	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
3^9	$(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)$
$(3^3) \cdot (3^3)$	$(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$

Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(7^4)^2$	49^4	Yes No
$6^3 \cdot 2^4$	$(2 \cdot 2)^3 \cdot 3^3$	Yes No
$5^3 \cdot 2^3$	10	Yes No
$(6^4)^2$	$6^4 \cdot 6^2$	Yes No
$8^4 \cdot 3^3$	$(3 \cdot 2)^4 \cdot 4^3$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(4^3)^2$	$4^3 \cdot 4^3$	Yes No
$3^2 \cdot 2^3$	6^5	Yes No

ML 9.03 Multiplying Powers

Goal	Standard
Multiply expressions with exponents and determine equivalence.	MA.7.NSO.1.1

Modeled Review

Point to Jack's work and **ask**:

- "How did Jack show that $(5^5)^2$ is not equivalent to $5^4 \cdot 5^3$?"
- "How is expanded form helpful in determining whether the expressions are equivalent?"
- "Why is it helpful to rearrange the factors to show $15^3 \cdot 2^3$ is equivalent to $(5 \cdot 2)^3 \cdot 3^3$?"

Reinforce Jack's thinking by saying, "Expanded form is useful in determining whether expressions with exponents are equivalent."

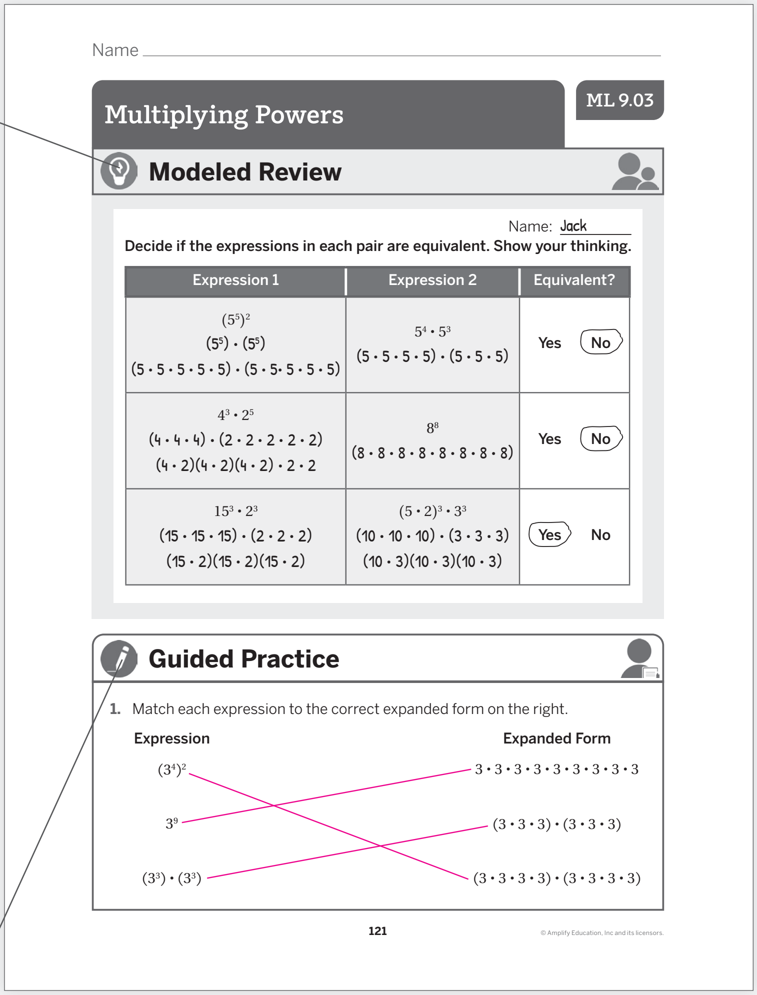
ML/EL Provide sentence frames to help students explain strategies for simplifying expressions with exponents. For example, "I need to simplify what's in parentheses first because ______." or "First, I need to write it in expanded form because ______."

Guided Practice

Focus students' attention on matching each expression to its equivalent expanded form.

To scaffold their thinking, **say**:

- "First, evaluate and simplify what is in parentheses."
- "Then, write each expression in expanded form and rearrange the factors if needed."



Name

Multiplying Powers ML 9.03

Modeled Review

Name: Jack

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(5^5)^2$ $(5^5) \cdot (5^5)$ $(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)$	$5^4 \cdot 5^3$ $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	Yes No
$4^3 \cdot 2^5$ $(4 \cdot 4 \cdot 4) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ $(4 \cdot 2)(4 \cdot 2)(4 \cdot 2) \cdot 2 \cdot 2$	8^8 $(8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8)$	Yes No
$15^3 \cdot 2^3$ $(15 \cdot 15 \cdot 15) \cdot (2 \cdot 2 \cdot 2)$ $(15 \cdot 2)(15 \cdot 2)(15 \cdot 2)$	$(5 \cdot 2)^3 \cdot 3^3$ $(10 \cdot 10 \cdot 10) \cdot (3 \cdot 3 \cdot 3)$ $(10 \cdot 3)(10 \cdot 3)(10 \cdot 3)$	Yes No

Guided Practice

1. Match each expression to the correct expanded form on the right.

Expression	Expanded Form
$(3^4)^2$	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
3^9	$(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)$
$(3^3) \cdot (3^3)$	$(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$

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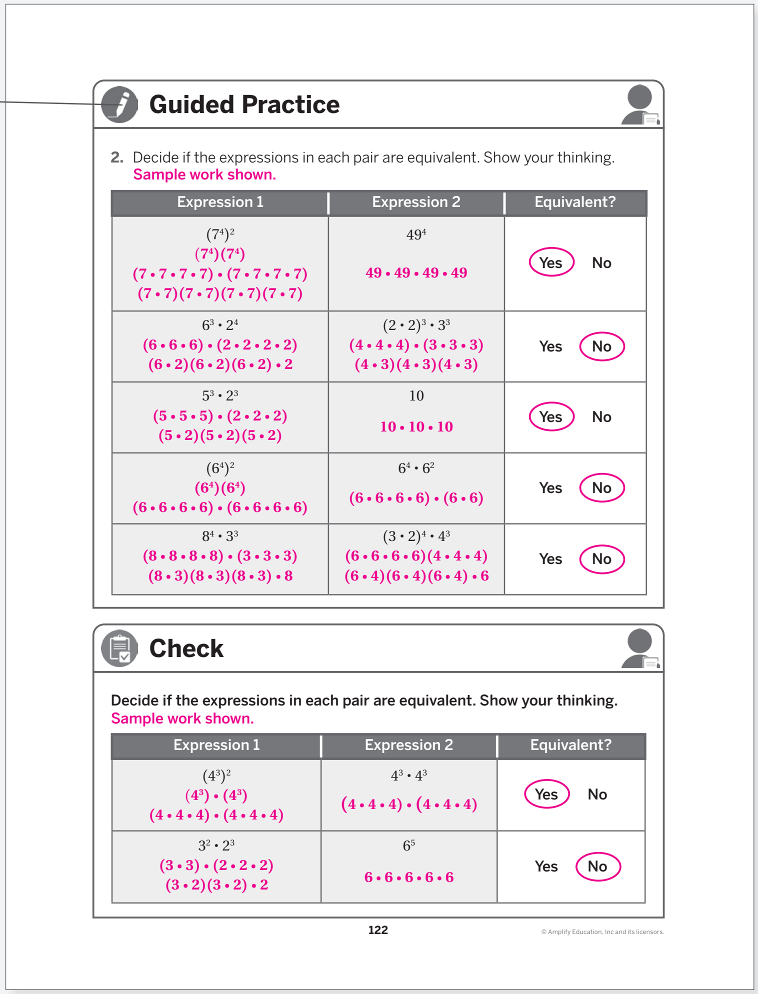
Guided Practice

A Chunk this task into smaller, more manageable parts by having students evaluate one expression at a time before determining if the expressions are equivalent.

Key Takeaway:
Say, "When determining if two expressions are equal, it helps to expand the exponents. You can also use the structure of the expression and rearrange the factors to see if they are equivalent."



Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.
Sample work shown.

Expression 1	Expression 2	Equivalent?
$(7^4)^2$ $(7^4)(7^4)$ $(7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7)$ $(7 \cdot 7)(7 \cdot 7)(7 \cdot 7)(7 \cdot 7)$	49^4 $49 \cdot 49 \cdot 49 \cdot 49$	Yes No
$6^3 \cdot 2^4$ $(6 \cdot 6 \cdot 6) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$ $(6 \cdot 2)(6 \cdot 2)(6 \cdot 2) \cdot 2$	$(2 \cdot 2)^3 \cdot 3^3$ $(4 \cdot 4 \cdot 4) \cdot (3 \cdot 3 \cdot 3)$ $(4 \cdot 3)(4 \cdot 3)(4 \cdot 3)$	Yes No
$5^3 \cdot 2^3$ $(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2)$ $(5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$	10 $10 \cdot 10 \cdot 10$	Yes No
$(6^4)^2$ $(6^4)(6^4)$ $(6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6 \cdot 6)$	$6^4 \cdot 6^2$ $(6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6)$	Yes No
$8^4 \cdot 3^3$ $(8 \cdot 8 \cdot 8 \cdot 8) \cdot (3 \cdot 3 \cdot 3)$ $(8 \cdot 3)(8 \cdot 3)(8 \cdot 3) \cdot 8$	$(3 \cdot 2)^4 \cdot 4^3$ $(6 \cdot 6 \cdot 6 \cdot 6)(4 \cdot 4 \cdot 4)$ $(6 \cdot 4)(6 \cdot 4)(6 \cdot 4) \cdot 6$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.
Sample work shown.

Expression 1	Expression 2	Equivalent?
$(4^3)^2$ $(4^3) \cdot (4^3)$ $(4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$	$4^3 \cdot 4^3$ $(4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$	Yes No
$3^2 \cdot 2^3$ $(3 \cdot 3) \cdot (2 \cdot 2 \cdot 2)$ $(3 \cdot 2)(3 \cdot 2) \cdot 2$	6^5 $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$	Yes No

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Reflection

Ask:
- "What are some important things to remember when determining whether expressions with exponents are equivalent?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using visual aids, such as anchor charts, to illustrate the order of operations and guide students through the correct sequence of steps to evaluate expressions with exponents.

Got it!
If students need more practice, ask them to decide if the following expressions are equivalent or not.

Expression 1	Expression 2
$(5^3)^2$	$5^4 \cdot 5$
$8^4 \cdot 2^4$	16^4
$12^2 \cdot 2^3$	$(3 \cdot 2)^2 \cdot 4^3$

Name ______________________________

Rules for Exponents

ML 9.08

Modeled Review

Name: Kira

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(3^2 \cdot 3)^3$ $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^3$ 3^9	$3^5 \cdot 3^{3}$ 3^8	Yes **No**
$\left(\frac{2}{3}\right)^{-3}$ $\left(\frac{3}{2}\right)^3$ $\frac{3^3}{2^3}$	$(3 \cdot 2^{-1})(3 \cdot 2^{-1})(3 \cdot 2^{-1})$ $\left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right)$ $\frac{3^3}{2^3}$	**Yes** No

Guided Practice

1. Match each expression to the correct equivalent form on the right.

Expression	Expanded Form
$\left(\frac{3}{5}\right)^{-2}$	$\frac{9}{25}$
$\left(\frac{5}{3}\right)^{-2}$	$-\frac{25}{9}$
$\left(-\frac{3^2}{5^2}\right)^{-1}$	$\frac{25}{9}$

Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\frac{4^2 \cdot 4^3}{4^{-1} \cdot 4^4}$	$\left(\frac{4^0}{4}\right)^{-2}$	Yes No
$\left(\frac{5}{3}\right)^{-2}$	$\frac{3 \cdot 3}{5 \cdot 5}$	Yes No
$\frac{(2^{-1} \cdot 3)^3}{(2^{-3}3^2)^2}$	$\left(\frac{3}{2 \cdot 2 \cdot 2}\right)^{-1}$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\left(\left(\frac{5}{7}\right)^{-2}\right)^{-1}$	$(5^4 \cdot 7^{-5}) \cdot (5^{-2}7^{-3})$	Yes No
$(3^0 7^{-2})^{-2}$	$\frac{3^3 7^{-1}}{3^{-3} 7^{-5}}$	Yes No

Goal

I can apply the rules of exponents to rewrite expressions in equivalent forms.

Standard	Materials
MA.8.AR.1.1	Whiteboards and dry erase markers (optional)

Modeled Review

Point to Kira's work and **ask**:

- "What strategy did Kira use to determine that $\left(\frac{2}{3}\right)^{-3}$ is equivalent to $(3 \cdot 2^{-1})(3 \cdot 2^{-1})(3 \cdot 2^{-1})$?"
- "How did Kira determine the expressions were equivalent without simplifying them completely?"

Reinforce Kira's thinking by saying, "Rewriting expressions using positive exponents is helpful when determining if they are equivalent."

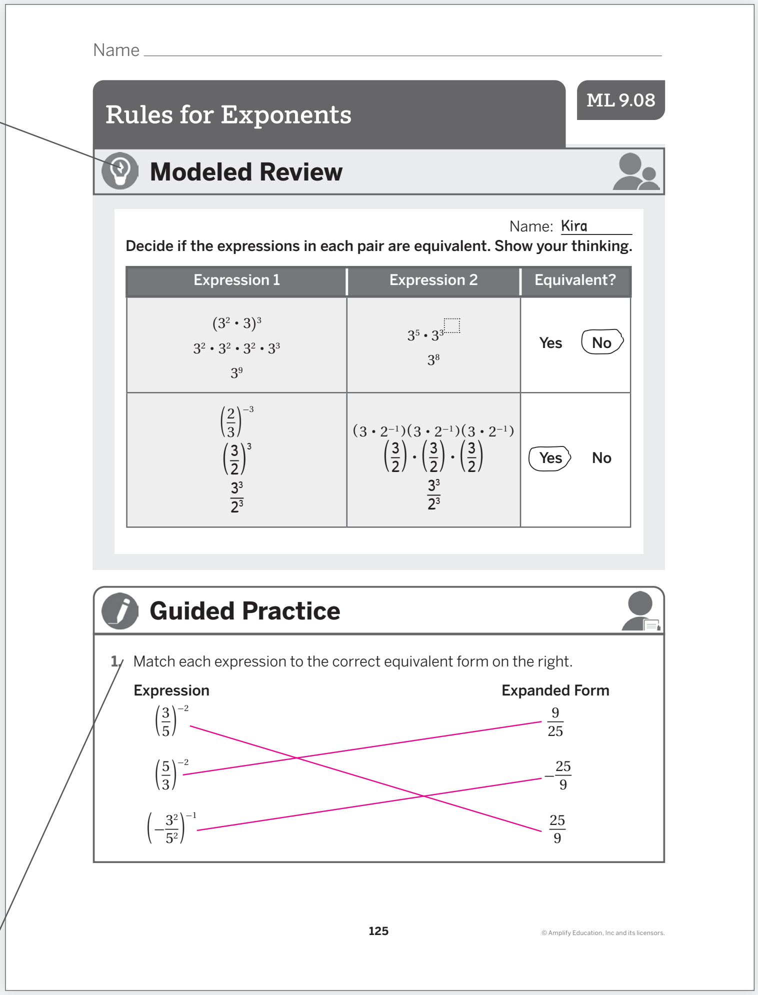
ML/EL Provide sentence frames to help students explain strategies for simplifying expressions with negative exponents. For example, "First, I need to rewrite expressions with negative exponents as their inverse with a positive exponent because _____."

Guided Practice

Focus students' attention on matching each expression to its equivalent expanded form.

To scaffold their thinking, **say**:

- "First, rewrite expressions using positive exponents."
- "Then, rewrite each expression in expanded form and rearrange the factors if needed."



Name

Rules for Exponents

ML 9.08

Modeled Review

Name: Kira

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(3^2 \cdot 3)^3$ $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^3$ 3^9	$3^5 \cdot 3^3$ 3^8	Yes No
$\left(\frac{2}{3}\right)^{-3}$ $\left(\frac{3}{2}\right)^3$ $\frac{3^3}{2^3}$	$(3 \cdot 2^{-1})(3 \cdot 2^{-1})(3 \cdot 2^{-1})$ $\left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right)$ $\frac{3^3}{2^3}$	Yes No

Guided Practice

1. Match each expression to the correct equivalent form on the right.

Expression	Expanded Form
$\left(\frac{3}{5}\right)^{-2}$	$\frac{9}{25}$
$\left(\frac{5}{3}\right)^{-2}$	$-\frac{25}{9}$
$\left(-\frac{3^2}{5^2}\right)^{-1}$	$\frac{25}{9}$

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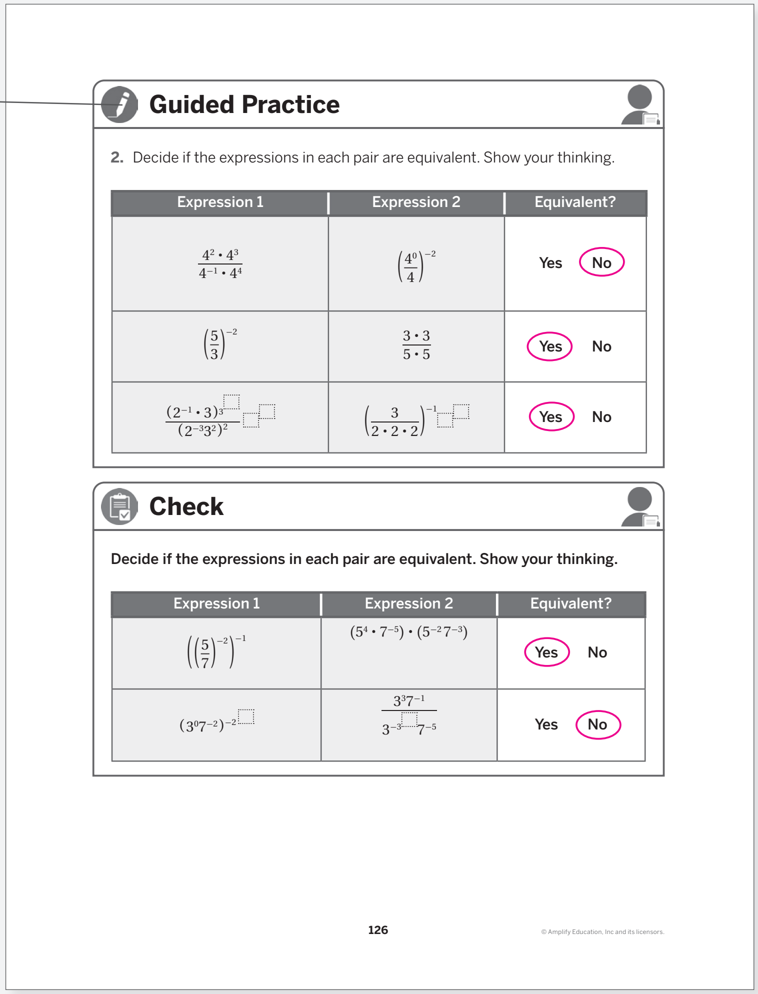
Guided Practice

A Chunk this task into smaller, more manageable parts by having students evaluate one expression at a time before determining if the expressions are equivalent. Allow students to work problems out on a whiteboard for extra room and ease of practice.

Key Takeaway:
Say, "Rewriting expressions using positive exponents is helpful when determining if they are equivalent."



Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\frac{4^2 \cdot 4^3}{4^{-1} \cdot 4^4}$	$\left(\frac{4^0}{4}\right)^{-2}$	Yes No
$\left(\frac{5}{3}\right)^{-2}$	$\frac{3 \cdot 3}{5 \cdot 5}$	Yes No
$\frac{(2^{-1} \cdot 3)^3}{(2^{-3}3^2)^2}$	$\left(\frac{3}{2 \cdot 2 \cdot 2}\right)^{-1}$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\left(\left(\frac{5}{7}\right)^{-2}\right)^{-1}$	$(5^4 \cdot 7^{-5}) \cdot (5^{-2}7^{-3})$	Yes No
$(3^07^{-2})^{-2}$	$\frac{3^37^{-1}}{3^{-3}7^{-5}}$	Yes No

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Reflection

Ask:

- "What are some important things to remember when determining whether expressions with exponents are equivalent?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using visual aids, such as anchor charts, to illustrate the order of operations and guide students through the correct sequence of steps to evaluate expressions with exponents.

Got it!
If students need more practice, ask them to decide if the following expressions are equivalent or not.

Expression 1	Expression 2
$\frac{3^6 \cdot 3^3 \cdot 3^{-2}}{3^{-1} \cdot 3^4}$	$\left(\frac{1}{3^2}\right)^{-2}$
$\frac{(4^2)^3}{(4^3)^{-1}}$	$4^2 \cdot 4^2 \cdot 4^2 \cdot 4^3$

Name ______________________________

Writing Numbers in Scientific Notation

ML 9.14

Modeled Review

Name: Priya

Write each number in scientific notation.

Number	Scientific Notation
48,200	$4.82 \cdot 10^4$
0.00099	$9.9 \cdot 10^{-4}$
$36 \cdot 10^5$	$3.6 \cdot 10^6$

Guided Practice

1. Write each number in scientific notation by completing the blanks for the numbers.

Number	Scientific Notation
23, 000, 000	$2.3 \cdot 10^{\square}$
0. 0012	______ $\cdot\ 10^{\square}$
24, 600	__________
0. 082	__________

Guided Practice

2. Write each number in scientific notation.

Number	Scientific Notation
2,250	
0.0065	
$23 \cdot 10^3$	
347	
0.056	
$78 \cdot 10^4$	
505	
0.00073	
$602 \cdot 10^5$	
85,600	

Check

Write each number in scientific notation.

Number	Scientific Notation
794,000	
0.000087	
$486 \cdot 10^6$	

ML 9.14 Writing Numbers in Scientific Notation

Goal

Write large and small numbers in scientific notation.

Standard

MA.8.NSO.1.4

Materials

highlighter (optional)

Modeled Review

Point to Priya's work and **ask**:

- "How did Priya determine the exponent when writing in scientific notation?"
- "Why does the second number (0.00099) have a negative exponent when written in scientific notation?"
- "How did Priya know the last number given is not already written in scientific notation?"

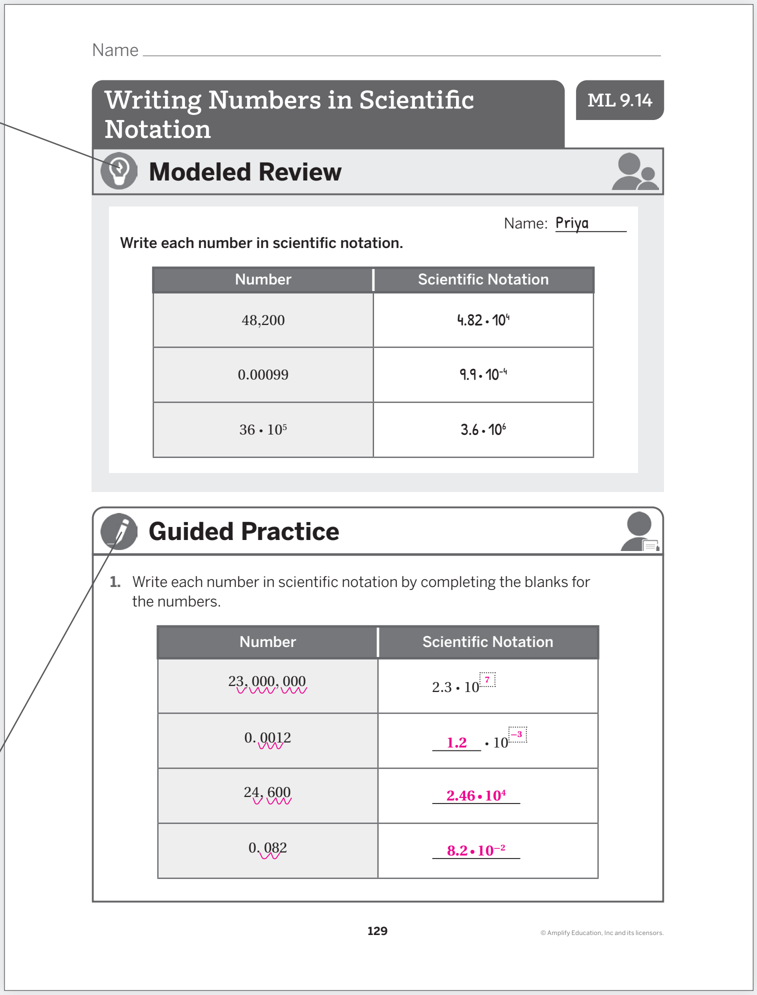
Reinforce Priya's thinking by saying, "Scientific notation can be used to write large and small numbers by using positive and negative exponents."

Guided Practice

Focus students' attention on writing numbers in scientific notation.

To scaffold their thinking, **say**:

- "First, count and move the decimal point until the first number is between 1 and 10."
- "Next, decide if the exponent is positive (if the number is greater than 1) or negative (if the number is less than 1)."
- "Last, write the number as a product of the adjusted number and 10 raised to the exponent you found."


Name

Writing Numbers in Scientific Notation

ML 9.14

Modeled Review

Name: Priya

Write each number in scientific notation.

Number	Scientific Notation
48,200	$4.82 \cdot 10^4$
0.00099	$9.9 \cdot 10^{-4}$
$36 \cdot 10^5$	$3.6 \cdot 10^6$

Guided Practice

1. Write each number in scientific notation by completing the blanks for the numbers.

Number	Scientific Notation
23, 000, 000	$2.3 \cdot 10^{\boxed{7}}$
0. 0012	$1.2 \cdot 10^{\boxed{-3}}$
24, 600	$2.46 \cdot 10^4$
0. 082	$8.2 \cdot 10^{-2}$

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Vocabulary

If needed, share the meaning of the term with students.

scientific notation: A way to write very large or very small numbers. In scientific notation, a number between 1 and 10 is multiplied by a power of 10.

ML 9.14 Writing Numbers in Scientific Notation

Guided Practice

A Model annotating the number of jumps it takes to move the decimal point to obtain a number between 1 and 10.

ML/EL To increase accessibility, provide students with questions they can ask themselves as they write numbers in scientific notation. For example: "How many places does the decimal need to move?" and "How can those moves be represented with an exponent?'"

Key Takeaway:
Say, "A number is written in scientific notation if it's shown as a number between 1 and 10 multiplied by a power of 10."

Guided Practice

2. Write each number in scientific notation.

Number	Scientific Notation
2,250	**$2.25 \cdot 10^3$**
0.0065	**$6.5 \cdot 10^{-3}$**
$23 \cdot 10^3$	**$2.3 \cdot 10^4$**
347	**$3.47 \cdot 10^2$**
0.056	**$5.6 \cdot 10^{-2}$**
$78 \cdot 10^4$	**$7.8 \cdot 10^5$**
505	**$5.05 \cdot 10^2$**
0.00073	**$7.3 \cdot 10^{-4}$**
$602 \cdot 10^5$	**$6.02 \cdot 10^7$**
85,600	**$8.56 \cdot 10^4$**

Check

Write each number in scientific notation.

Number	Scientific Notation
794,000	**$7.94 \cdot 10^5$**
0.000087	**$8.7 \cdot 10^{-5}$**
$486 \cdot 10^6$	**$4.86 \cdot 10^8$**

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Reflection

Ask:
- "Describe the steps for writing a number in scientific notation."
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then have them highlight the decimal point in the number to help them visually track how many places the decimal point needs to move when writing it in scientific notation.

Got it!
If students need more practice, ask them to write the following numbers in scientific notation:

- 78,400,000
- 0.0072
- $56 \cdot 10^6$

Unit 10

Mini-Lessons

Name ______________________________

Approximating Square Roots

ML 10.03

Modeled Review

Name: Evan

Approximate $\sqrt{18}$.

$\sqrt{18}$ is between 4.2 and 4.3

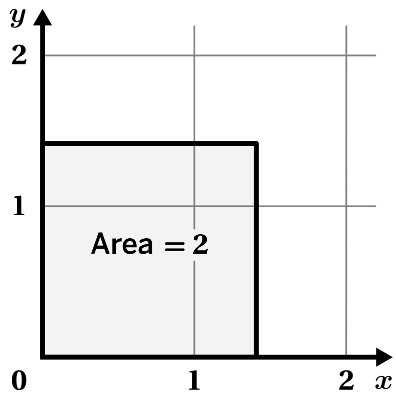
I multiplied 4.1, 4.2, and 4.3 by themselves to find their squares.

n	n^2
4.1	16.81
4.2	17.64
4.3	18.49

Guided Practice

Approximate the square root using the side lengths of the squares and the tables.

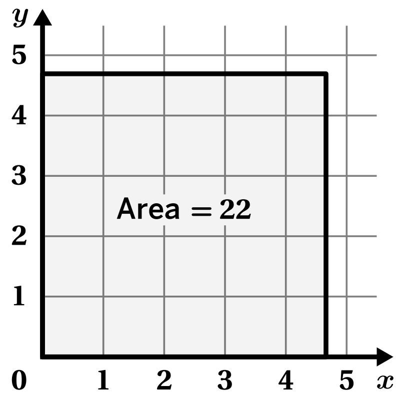
1. $\sqrt{2}$



n	n^2
1.4	1.96
1.5	

$\sqrt{2}$ is between ______ and ______

2. $\sqrt{22}$



n	n^2
4.6	

$\sqrt{22}$ is between ______ and ______

Guided Practice

For Problems 3–6, complete the table to approximate the square root.

3. $\sqrt{6}$ is between ______ and ______

n	n^2
2.3	5.29
2.4	

4. $\sqrt{10}$ is between ______ and ______

n	n^2
3.1	

5. $\sqrt{15}$ is between ______ and ______

n	n^2

6. $\sqrt{40}$ is between ______ and ______

n	n^2

Check

Approximate $\sqrt{28}$.

$\sqrt{28}$ is between ______ and ______

n	n^2

Approximating Square Roots

Goal

Approximate square roots as decimals.

Standard

MA.8.NSO.1.2 and MA.8.AR.2.3

Materials

calculator (optional), number line (optional)

Modeled Review

Point to Evan's work and **ask**:

- "Why is $\sqrt{18}$ not a perfect square root?"
- "How did Evan know to start with numbers slightly greater than 4?"
- "How did Evan calculate 4.1^2, 4.2^2, and 4.3^2?"

Reinforce Evan's thinking by saying, "Knowing perfect squares and calculating the nearest decimals helps you estimate square roots."

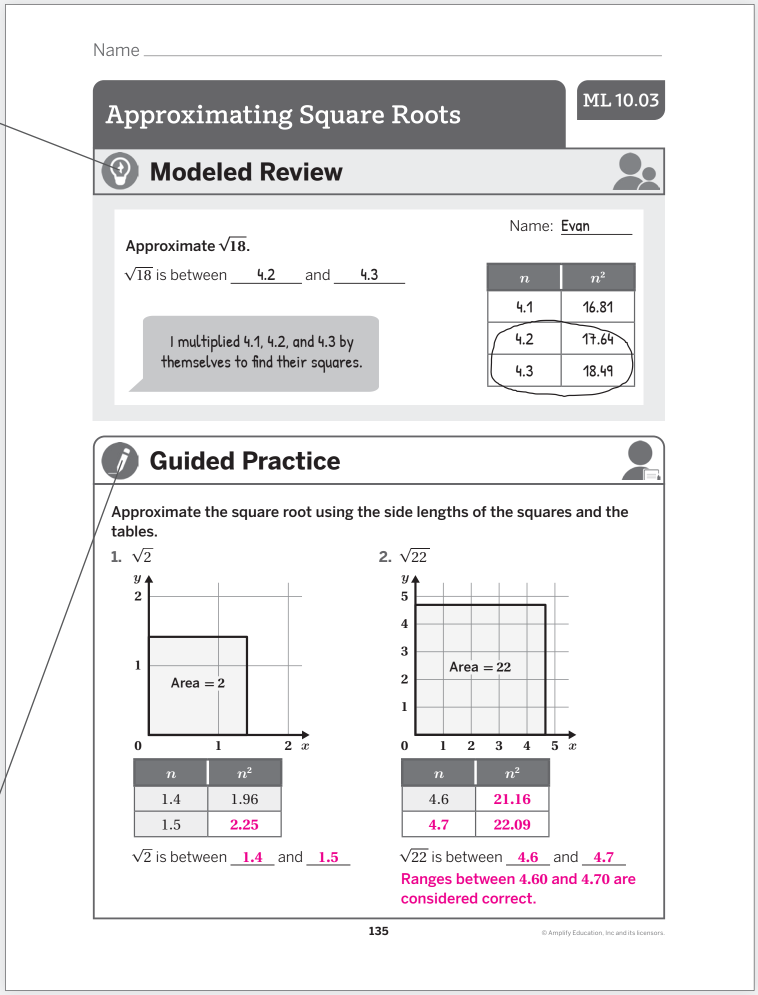
ML/EL Provide students with sentence frames to use when discussing how to approximate square roots. For example, "I need to find perfect squares near ______." or "I guess a number and square it to see if it's close to ______."

Guided Practice

Focus students' attention on estimating the square root.

To scaffold their thinking, **say**:

- "First, identify a perfect square closest to the number."
- "Next, select possible values for n and calculate n^2."
- "Last, find the range of n on either side of the square root."



Name

Approximating Square Roots ML 10.03

Modeled Review

Name: Evan

Approximate $\sqrt{18}$.

$\sqrt{18}$ is between 4.2 and 4.3

I multiplied 4.1, 4.2, and 4.3 by themselves to find their squares.

n	n^2
4.1	16.81
4.2	17.64
4.3	18.49

Guided Practice

Approximate the square root using the side lengths of the squares and the tables.

1. $\sqrt{2}$

n	n^2
1.4	1.96
1.5	2.25

$\sqrt{2}$ is between 1.4 and 1.5

2. $\sqrt{22}$

n	n^2
4.6	21.16
4.7	22.09

$\sqrt{22}$ is between 4.6 and 4.7

Ranges between 4.60 and 4.70 are considered correct.

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Vocabulary

If needed, share the meaning of the terms with students.

perfect square: A number that is the square of an integer. For example $\sqrt{16}$ is a perfect square because $4^2 = 16$.

square root: The square root of a positive number $\sqrt{p}$ is a positive solution to equations of the form $x^2 = p$. Write the square root of p as $\sqrt{p}$.

Guided Practice

A Provide students with a calculator to help them efficiently square decimal numbers.

Key Takeaway:
Say, "Calculating the nearest decimal approximations of square roots can help estimate the value of a square root."

Guided Practice

For Problems 3–6, complete the table to approximate the square root.

3. $\sqrt{6}$ is between 2.4 and 2.5

Ranges between 2.40 and 2.50 are considered correct.

n	n^2
2.3	5.29
2.4	5.76
2.5	6.25

4. $\sqrt{10}$ is between 3.1 and 3.2

Ranges between 3.10 and 3.20 are considered correct.

n	n^2
3.1	9.61
3.2	10.24
3.3	10.89

5. $\sqrt{15}$ is between 3.8 and 3.9

Ranges between 3.80 and 3.90 are considered correct.

n	n^2
3.7	13.69
3.8	14.44
3.9	15.21

6. $\sqrt{40}$ is between 6.3 and 6.4

Ranges between 6.30 and 6.40 are considered correct.

n	n^2
6.2	38.44
6.3	39.69
6.4	40.96

Check

Approximate $\sqrt{28}$.

$\sqrt{28}$ is between 5.2 and 5.3

Ranges between 5.20 and 5.30 are considered correct.

n	n^2
5.1	26.01
5.2	27.04
5.3	28.09

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Reflection

Ask:

- "Describe your favorite strategy to approximate a square root."
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a number line to visually estimate the position of the square root between two known whole numbers.

Got it!
If students need more practice, invite them to estimate each square root.

- $\sqrt{20}$
- $\sqrt{52}$
- $\sqrt{37}$

Name ____________________

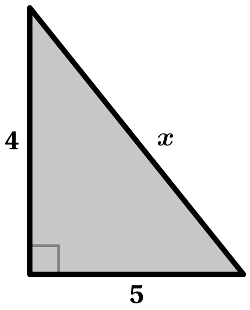
Calculating Unknown Side Lengths

ML 10.08

Modeled Review

Name: Anna

Calculate the *exact* value of the unknown side length, x, in the right triangle.



$x = \underline{\sqrt{41}}$

$a^2 + b^2 = c^2$

$4^2 + 5^2 = c^2$

$16 + 25 = c^2$

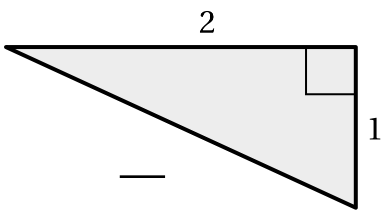
$\sqrt{41} = \sqrt{c^2}$

$\sqrt{41} = c$

Guided Practice

Calculate the *exact* value of the unknown side length, x, in each right triangle.

1.



$$a^2 + b^2 = c^2$$

$1^2 + \underline{(\quad)^2} = c^2$

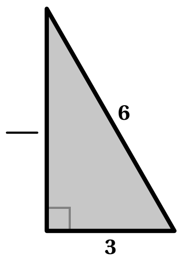
$1 + \_\_\_\_\_ = c^2$

$\_\_\_\_\_ = c^2$

$\_\_\_\_\_ = \sqrt{c^2}$

$\_\_\_\_\_ = c$

2.



$$a^2 + b^2 = c^2$$

$3^2 + b^2 = \underline{\quad}^2$

$\_\_\_\_\_ + b^2 = \_\_\_\_\_$

$\_\_\_\_\_ \qquad \_\_\_\_\_$

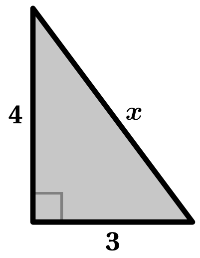
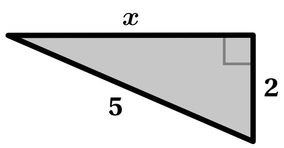
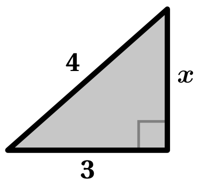
$b^2 = \_\_\_\_\_$

$b = \_\_\_\_\_$

Guided Practice

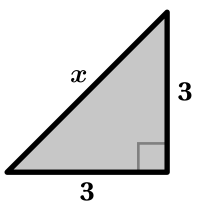
3. Calculate the *exact* value of the unknown side length, x, in each right triangle.

$$a^2 + b^2 = c^2$$

Triangle	Work	Side Length
		
		
		

Check

Calculate the *exact* value of the unknown side length, x, in the right triangle.



$x =$ __________

ML 10.08 Calculating Unknown Side Lengths

Goal

Use the Pythagorean theorem to calculate unknown side lengths in right triangles.

Standard

MA.8.GR.1.1

Materials

highlighter (optional)

Modeled Review

Point to Anna's work and **ask**:

- "Why did Anna use the Pythagorean theorem?"
- "How did Anna identify the legs and hypotenuse of the right triangle?"
- "How can Anna check her answer?"

Reinforce Anna's thinking by saying, "Using the Pythagorean theorem is an efficient way to calculate the unknown side length when given two sides of a right triangle."

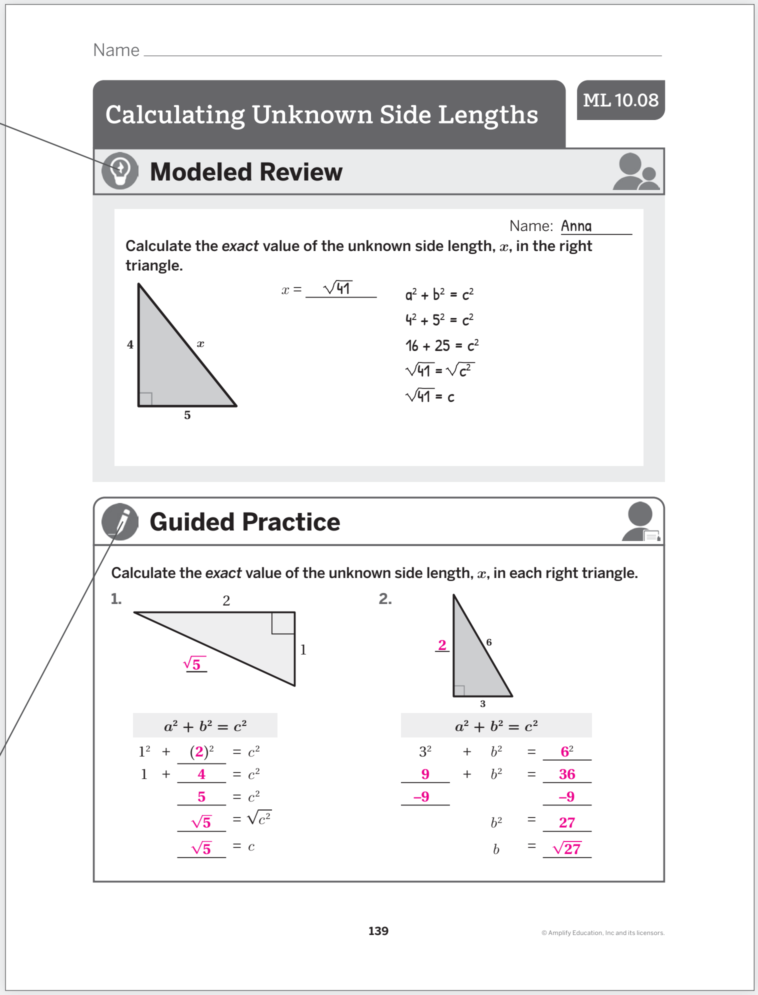
ML/EL Use a think aloud to model labeling a right triangle with *a*, *b*, and *c* to stay organized.

Guided Practice

Focus students' attention on using the Pythagorean theorem to calculate the unknown side lengths.

To scaffold their thinking, **say**:

- "First, identify the sides as *a*, *b*, and *c*."
- "Next, apply the Pythagorean theorem by substituting the known values into the equation $a^2 + b^2 = c^2$."
- "Finally, solve for the missing side length and double-check your work to make sure it's correct."



Name

Calculating Unknown Side Lengths ML 10.08

Modeled Review

Name: Anna

Calculate the *exact* value of the unknown side length, x, in the right triangle.

$x = \sqrt{41}$

$a^2 + b^2 = c^2$

$4^2 + 5^2 = c^2$

$16 + 25 = c^2$

$\sqrt{41} = \sqrt{c^2}$

$\sqrt{41} = c$

Guided Practice

Calculate the *exact* value of the unknown side length, x, in each right triangle.

1.

$a^2 + b^2 = c^2$

$1^2 + (2)^2 = c^2$

$1 + 4 = c^2$

$5 = c^2$

$\sqrt{5} = \sqrt{c^2}$

$\sqrt{5} = c$

2.

$a^2 + b^2 = c^2$

$3^2 + b^2 = 6^2$

$9 + b^2 = 36$

$-9 \qquad -9$

$b^2 = 27$

$b = \sqrt{27}$

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Vocabulary

If needed, share the meaning of the terms with students.

legs: The two sides of a right triangle that are not the hypotenuse. The legs are the sides that form the right angle.

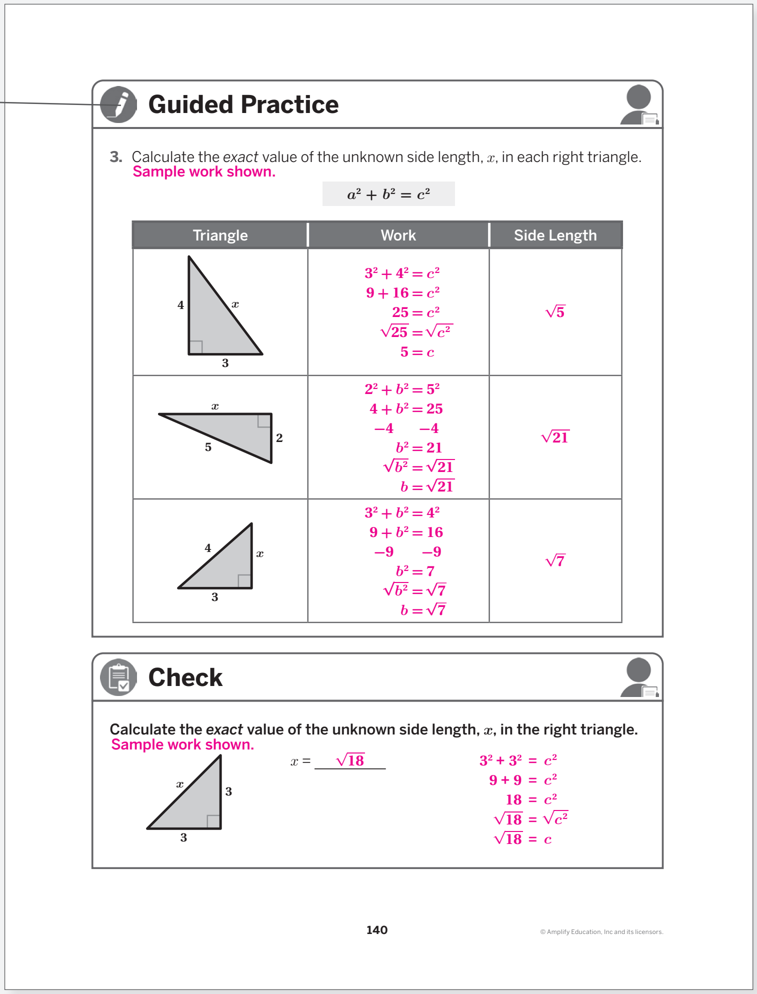
hypotenuse: The side of a right triangle that is opposite the right angle. The hypotenuse is always the longest side of a right triangle.

Guided Practice

A Consider using a highlighter to color-code the diagram and the substituted values in the formula to help organize the work.

Note: Students can substitute the value of a leg for either *a* or *b* in the Pythagorean theorem.

Key Takeaway:
Say, "When given two side lengths of a right triangle, identify whether the side lengths are for the legs or the hypotenuse. Then, use substitution and the Pythagorean theorem to calculate the third side length."



Guided Practice

3. Calculate the *exact* value of the unknown side length, x, in each right triangle.
Sample work shown.

$a^2 + b^2 = c^2$

Triangle	Work	Side Length
	$3^2+4^2=c^2$ $9+16=c^2$ $25=c^2$ $\sqrt{25}=\sqrt{c^2}$ $5=c$	$\sqrt{5}$
	$2^2+b^2=5^2$ $4+b^2=25$ $-4 \quad -4$ $b^2=21$ $\sqrt{b^2}=\sqrt{21}$ $b=\sqrt{21}$	$\sqrt{21}$
	$3^2+b^2=4^2$ $9+b^2=16$ $-9 \quad -9$ $b^2=7$ $\sqrt{b^2}=\sqrt{7}$ $b=\sqrt{7}$	$\sqrt{7}$

Check

Calculate the *exact* value of the unknown side length, x, in the right triangle.
Sample work shown.

$x = \sqrt{18}$

$3^2 + 3^2 = c^2$
$9 + 9 = c^2$
$18 = c^2$
$\sqrt{18} = \sqrt{c^2}$
$\sqrt{18} = c$

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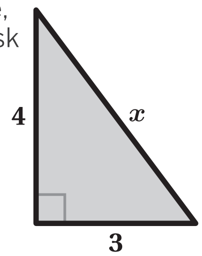
Reflection

Ask:
- "If you know two side lengths of a right triangle, how can you calculate the third side length?"
- "What advice would you give a friend learning to calculate the unknown side length of a right triangle?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling how to square numbers by rewriting the number twice, so they don't mistakenly multiply by 2 instead.

Got it!
If students need more practice, sketch the right triangle and ask them to calculate the exact value of the unknown side length, x.



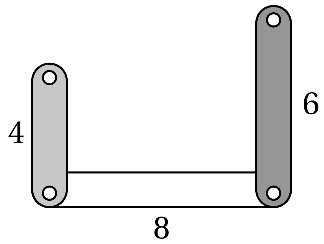
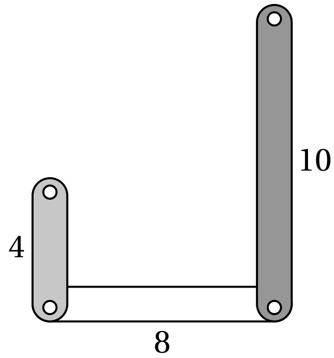
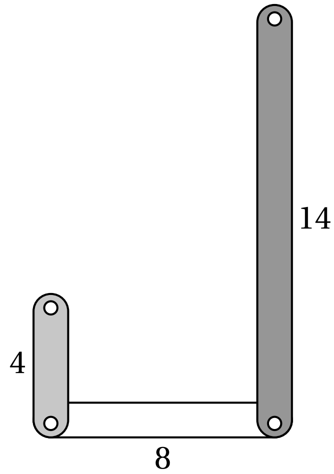
Name ______________________________

Determining If Three Segments Form a Triangle

ML 10.10

Modeled Review

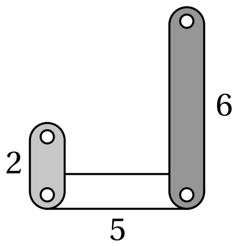
A triangle can be formed if the two shorter segments added together are longer than the third segment.

Triangle		Not a triangle
		

Guided Practice

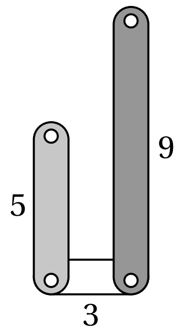
Determine whether or not the three segments will form a triangle.

1.



Yes No

2.



Yes No

Guided Practice

3. Select *all* the groups of segments that will form a triangle.

- ☐ 2, 3, 7
- ☐ 4, 2, 10
- ☐ 7, 10, 14
- ☐ 13, 8, 8
- ☐ 3, 4, 8

4. Select *all* the groups of segments that will form a triangle.

- ☐ 6, 5, 4
- ☐ 4, 7, 4
- ☐ 12, 5, 1
- ☐ 1, 8 ,6
- ☐ 6, 3, 2

5. Select *all* the groups of segments that will form a triangle.

- ☐ 9, 10, 19
- ☐ 14, 9, 10
- ☐ 12, 4, 6
- ☐ 7, 10, 16
- ☐ 8, 5, 14

6. Select *all* the groups of segments that will form a triangle.

- ☐ 11, 5, 4
- ☐ 4, 14, 6
- ☐ 16, 8, 19
- ☐ 3, 6, 3
- ☐ 9, 5, 12

Check

Select *all* the groups of segments that will form a triangle.

- ☐ 10, 5, 8
- ☐ 7, 4, 6
- ☐ 12, 5, 6
- ☐ 1, 6, 8
- ☐ 6, 2, 3

ML 10.10 Determining If Three Segments Form a Triangle

Goal

Determine whether or not three segments will form a triangle.

Standard

MA.8.GR.1.3

Materials

highlighter (optional)

Modeled Review

Point to the Modeled Review and ask:

- "When will three segments form a triangle? When will they not?"
- "What would happen if the sum of the lengths of two segments is exactly equal to the length of the third segment?"

Reinforce the goal by saying, "Three segments will form a triangle if the sum of the two shorter segments is greater than the length of the longest segment."

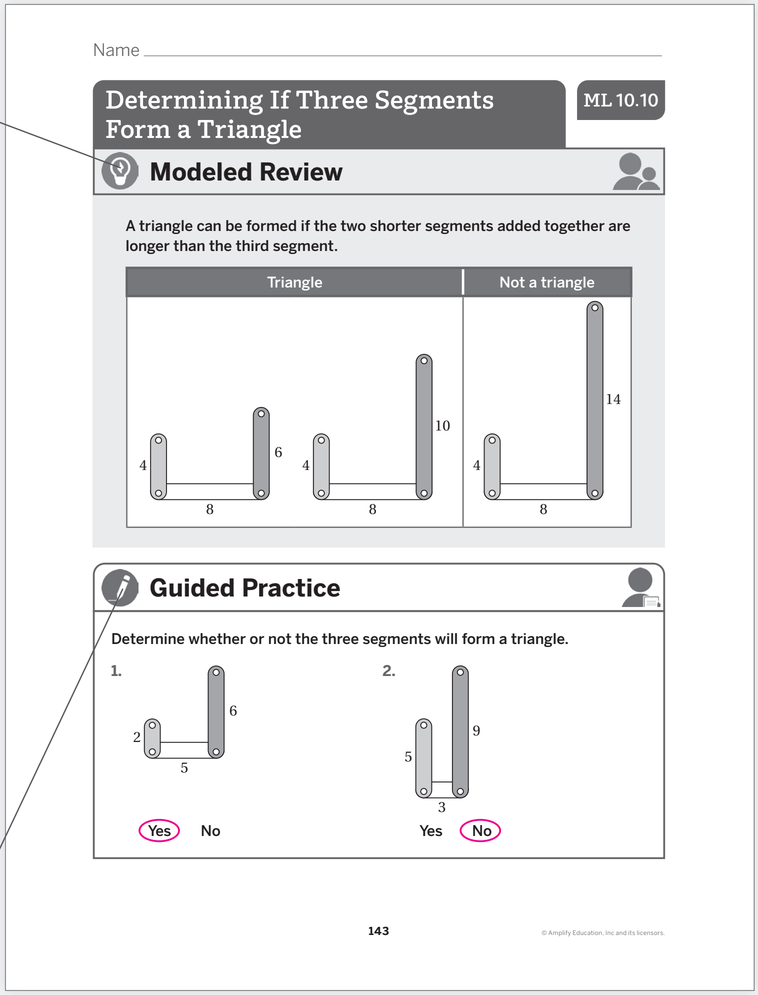
ML/EL Model identifying the longest segment and calculating the sum of the other segments to determine if the segments will form a triangle.

Guided Practice

Focus students' attention on determining if the three segments will form a triangle.

To scaffold their thinking, **say**:

- "First, identify the longest segment."
- "Then, calculate the sum of the other two segments."
- "Last, determine if that sum is greater than the longest segment. If so, the segments will form a triangle."



Name

Determining If Three Segments Form a Triangle

ML 10.10

Modeled Review

A triangle can be formed if the two shorter segments added together are longer than the third segment.

Guided Practice

Determine whether or not the three segments will form a triangle.

1.

Yes No

2.

Yes No

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ML 10.10 Determining If Three Segments Form a Triangle

Guided Practice

(A) Increase accessibility by covering all groups of segments except one to allow for students to focus on one group at a time.

Key Takeaway:
Say, “For three segments to form a triangle, the sum of the two shorter segments must be greater than the length of the third segment.”

Guided Practice

3. Select *all* the groups of segments that will form a triangle.

- ☐ 2, 3, 7
- ☐ 4, 2, 10
- ☑ 7, 10, 14
- ☑ 13, 8, 8
- ☐ 3, 4, 8

4. Select *all* the groups of segments that will form a triangle.

- ☑ 6, 5, 4
- ☑ 4, 7, 4
- ☐ 12, 5, 1
- ☐ 1, 8 ,6
- ☐ 6, 3, 2

5. Select *all* the groups of segments that will form a triangle.

- ☐ 9, 10, 19
- ☑ 14, 9, 10
- ☐ 12, 4, 6
- ☑ 7, 10, 16
- ☐ 8, 5, 14

6. Select *all* the groups of segments that will form a triangle.

- ☐ 11, 5, 4
- ☐ 4, 14, 6
- ☑ 16, 8, 19
- ☐ 3, 6, 3
- ☑ 9, 5, 12

Check

Select *all* the groups of segments that will form a triangle.

- ☑ 10, 5, 8
- ☑ 7, 4, 6
- ☐ 12, 5, 6
- ☐ 1, 6, 8
- ☐ 6, 2, 3

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Reflection

Ask:

- “Explain how you could determine if three line segments will form a triangle.”
- “How was the lesson helpful to you today?”

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, model highlighting the longest side in one color and the two shorter sides in a different color. Then write an expression to represent finding the sum of the two shorter sides.	If students need more practice, have them determine if the groups of three segments will form a triangle. • 3, 7, 8 • 1, 4, 5 • 2, 6, 9

Name ______________________________

Identifying Rational and Irrational Numbers

ML 10.14

Modeled Review

Rational	Irrational
all positive and negative numbers that can be written as fractions, including whole numbers	a number that cannot be written as a fraction of two non-zero integers
Examples	**Examples**
$\frac{3}{4}$; $2.5 = \frac{5}{2}$; $0.\overline{66}$; $\sqrt{16}$	π; $\sqrt{7}$; $\sqrt{3} = 1.732050...$

Guided Practice

1. Determine if the numbers in the bank are rational or irrational. Then add them to the correct column.

1.75	$\sqrt{3}$	9π	$\frac{1}{2}$
$-\frac{1}{4}$	$0.\overline{33}$	$\sqrt{10}$	$\sqrt{15} = 3.872983...$

Rational	Irrational
$\frac{1}{2}$	$\sqrt{15} = 3.872983...$

Guided Practice

2. Is the number rational or irrational? Add a check mark to the correct column.

Rational	Rational	Irrational
$\sqrt{36}$		
$0.\overline{72}$		
$1 + \sqrt{2} = 2.414213\ldots$		
$-\frac{7}{8}$		
$\sqrt{11}$		
$\pi^2 = 9.869604\ldots$		
8π		
$0.\overline{11}$		
4		

Check

Is the number rational or irrational? Add a check mark to the correct column.

Number	Rational	Irrational
$\sqrt{14}$		
5π		
$-\frac{11}{5}$		

ML 10.14 Identifying Rational and Irrational Numbers

Goal	Standard
Identify and categorize examples of rational and irrational numbers.	MA.8.NSO.1.1

Modeled Review

Point to the Modeled Review and **ask**:

- "How are rational and irrational numbers different?"
- "How can you tell if a square root is rational or irrational?"
- "What is the difference between repeating and terminating decimals?"

Reinforce the goal by saying, "An irrational number can't be written as a fraction of integers. It has a never-ending, non-repeating decimal."

ML/EL Provide sentence frames to help students explain their strategy. For example, "I noticed that ______ is an irrational number because _______." or "______ is a repeating decimal, so it is rational."

Guided Practice

Focus students' attention on determining whether each number is rational or irrational.

To scaffold their thinking, **say**:

- "Determine if the number is a fraction, repeating decimal, or terminating decimal."
- "If the number is none of these, it is irrational."

Name ______________________

Identifying Rational and Irrational Numbers — ML 10.14

Modeled Review

Rational	Irrational
all positive and negative numbers that can be written as fractions, including whole numbers	a number that cannot be written as a fraction of two non-zero integers
Examples	**Examples**
$\frac{3}{4}$, $2.5 = \frac{5}{2}$, $0.\overline{66}$, $\sqrt{16}$	π, $\sqrt{7}$, $\sqrt{3} = 1.732050...$

Guided Practice

1. Determine if the numbers in the bank are rational or irrational. Then add them to the correct column.

1.75	$\sqrt{3}$	9π	$\frac{1}{2}$
$-\frac{1}{4}$	$0.\overline{33}$	$\sqrt{10}$	$\sqrt{15} = 3.872983...$

Rational	Irrational
$\frac{1}{2}$	$\sqrt{15} = 3.872983...$
1.75	$\sqrt{3}$
$0.\overline{33}$	$\sqrt{10}$
$-\frac{1}{4}$	9π

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Vocabulary

If needed, share the meaning of the terms with students.

rational: All positive and negative numbers that can be written as fractions, including whole numbers, are called rational numbers.

irrational: A number that cannot be written as a fraction of two non-zero integers.

Identifying Rational and Irrational Numbers

Guided Practice

A Provide a checklist with the terms *fraction*, *repeating decimal*, and *terminating decimal*. Students can check the appropriate term as they go, helping them determine if the number is rational or irrational."

Key Takeaway:
Say, "An irrational number cannot be written as a fraction with integers. It goes on forever without repeating a pattern, unlike terminating or repeating decimals."

Guided Practice

2. Is the number rational or irrational? Add a check mark to the correct column.

Rational	Rational	Irrational
$\sqrt{36}$	✔	
$0.\overline{72}$	✔	
$1 + \sqrt{2} = 2.414213...$		✔
$-\frac{7}{8}$	✔	
$\sqrt{11}$		✔
$\pi^2 = 9.869604...$		✔
8π		✔
$0.\overline{11}$	✔	
4	✔	

Check

Is the number rational or irrational? Add a check mark to the correct column.

Number	Rational	Irrational
$\sqrt{14}$		✔
5π		✔
$-\frac{11}{5}$	✔	

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Reflection

Ask:
- "What are some characteristics of irrational numbers? Rational numbers?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider providing examples of a terminating decimal, a repeating decimal, and a non-terminating decimal for reference.

Got it!
If students need more practice, ask them to determine if each of the following numbers are rational or irrational:

- $\frac{1+\sqrt{5}}{2} = 1.618033...$
- $0.\overline{77}$
- $\sqrt{49}$

Unit 11

Mini-Lessons

Name ______________________________

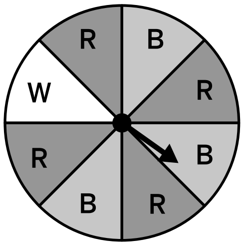
Theoretical Probability

ML 11.03

Modeled Review

Name: Luke

What is the probability of this spinner landing on each of the following colors?



a. P(R) $= \frac{4}{8}$ or 50%

b. P(W) $= \frac{1}{8}$ or 12.5%

c. P(B) $= \frac{3}{8}$ or 37.5%

Guided Practice

1. Which of the following events is most likely? Explain your thinking.

I. Flipping tails on a fair coin.

II. Spinning a spinner with 8 equal sections, labeled 1 to 8, and have it land on a number greater than 3.

III. Rolling an even number on a standard number cube.

Guided Practice

For Problems 2-3, you are given a spinner.

2. You spin the spinner one time. What is the probability of landing on a heart?

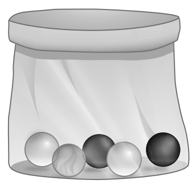
3. You spin the spinner one time. What is P(flower)?

4. You spin the spinner two times. What is the sample space for this experiment?

5. You spin the spinner two times. What is the probability of landing on the same figure each time?

Check

A bag of marbles contains 5 marbles. There are three different types of marbles in the bag - 2 clear (C), 2 dark (D), and 1 multi-colored (M). Denise chose a marble out of the bag, wrote down its type, and replaced it in the bag. What is P(D, C)? Show your thinking.



ML 11.03 Theoretical Probability

Goal

Determine the theoretical probability of an event related to a repeated experiment.

Standard

MA.8.DP.2.2

Materials

Whiteboards and dry erase markers

Modeled Review

Point to Luke's work and **ask**:

- "What does P(R) mean?"
- "How did Luke determine the probability of landing on a Blue space?"
- "Why is 8 the denominator of all of the fractions?"

Reinforce Luke's thinking by saying, "You can find the probability of an event by dividing the number of possible successful events by the total number of outcomes."

ML/EL Consider modeling, or inviting a student to model, the thinking for each probability.

Guided Practice

Focus students' attention on determining the likelihood of each event.

To scaffold their thinking, **ask/say**:

- "What are the total outcomes of this experiment?"
- "What are the possible successful events of this experiment?"
- "Write a ratio of the number of possible successful events to the total number of outcomes."

Name

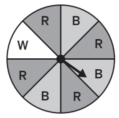
Theoretical Probability

ML 11.03

Modeled Review

Name: Luke

What is the probability of this spinner landing on each of the following colors?



a. $P(R) = \frac{4}{8}$ or 50%

b. $P(W) = \frac{1}{8}$ or 12.5%

c. $P(B) = \frac{3}{8}$ or 37.5%

Guided Practice

1. Which of the following events is most likely? Explain your thinking.

   I. Flipping tails on a fair coin.

   II. Spinning a spinner with 8 equal sections, labeled 1 to 8, and have it land on a number greater than 3.

   III. Rolling an even number on a standard number cube.

   Option II. *Explanations vary*. Flipping a fair coin $= \frac{1}{2}$. Spinning a number greater than 3 on a spinner with 8 equal sections is $\frac{5}{8}$. Rolling an even number on a standard number cube is $\frac{3}{6}$ or $\frac{1}{2}$. Option II is most likely because $\frac{5}{8} > \frac{1}{2}$.

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Vocabulary

If needed, share the meaning of the terms with students.

event: any possible outcome or a set of outcomes resulting from an experiment.

outcome: possible result of a single experiment

sample space: a list of the individual outcomes of an experiment

theoretical probability of an event: the ratio of the number of successful events to the number of outcomes in the sample space

Guided Practice

A Consider modeling (or having a student share) how a sample space of an experiment can be created multiple ways. Encourage students to use a whiteboard to create the tables for the sample spaces for Problem 4 and the Check.

Key Takeaway:
Say, "You can find the probability of an event in a repeated experiment by dividing the number of possible successful events by the total number of outcomes."

Guided Practice

For Problems 2-3, you are given a spinner.

2. You spin the spinner one time. What is the probability of landing on a heart?
$\frac{1}{4} = 25\%$

3. You spin the spinner one time. What is P(flower)?
$\frac{2}{4} = 50\%$

4. You spin the spinner two times. What is the sample space for this experiment?

	H	S	F1	F2
H	H, H	H, S	H, F1	H, F2
S	S, H	S, S	S, F1	S, F2
F1	F1, H	F1, S	F1, F1	F1, F2
F2	F2, H	F2, S	F2, F1	F2, F2

5. You spin the spinner two times. What is the probability of landing on the same figure each time?
25% *Explanations vary*. There are 4 of the 16 possible outcomes that would result in both spins landing on the same figure. Therefore, the probability of landing on the same figure twice is $\frac{4}{16} = 25\%$.

Check

A bag of marbles contains 5 marbles. There are three different types of marbles in the bag - 2 clear (C), 2 dark (D), and 1 multi-colored (M). Denise chose a marble out of the bag, wrote down its type, and replaced it in the bag. What is P(D, C)? Show your thinking.

16%. *Explanations vary*. There are 4 events out of 25 that would result in selecting a dark marble first, replacing it, and then selecting a clear marble. These are (D1, C1), (D1, C2), (D2, C1), and (D2, C2). Therefore, there is a $\frac{4}{25}$ or 16% probability.

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Reflection

Ask:

- "How is finding theoretical probability different from experimental probability?"
- "How did you overcome a challenging question today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider providing multiple opportunities to experience simulations of different situations, with physical or virtual manipulatives, in order to find and compare the experimental and theoretical probabilities.

Got it!
If students need more practice, ask them to revisit Problem 5 but find the probability of landing on a different figure on the second spin than on the first spin.

Prerequisite Skills and Concepts

Mini-Lessons

Name ______________________________

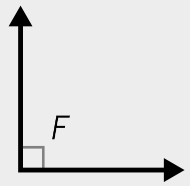
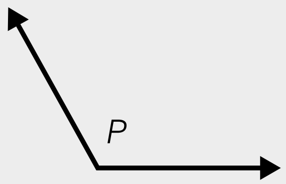
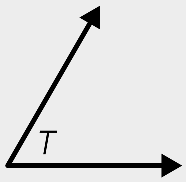
Identifying Acute, Obtuse, Right, and Straight Angles

ML 7.09

Modeled Review

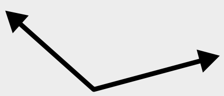
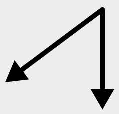
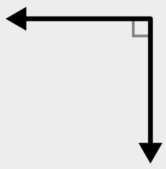
Name: Avery

Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

			
right 90°	obtuse 120°	straight 180°	acute 60°

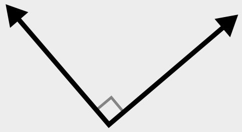
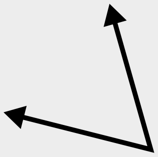
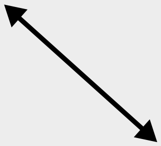
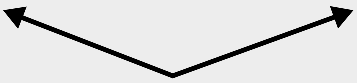
Guided Practice

1. Complete the table by labeling each angle. Use a protractor to measure if it is helpful.

	Acute, right, or obtuse	Greater than, less than, or equal to 90°
		greater than 90°
	acute	
		equal to 90°

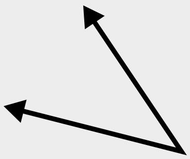
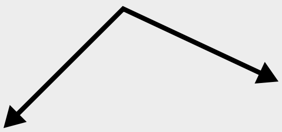
Guided Practice

2. Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

	Acute, right, obtuse, or straight	Measurement in degrees
		
		
		
		

Check

Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

	Acute, right, obtuse, or straight	Measurement in degrees
		
		

ML 7.09 Identifying Acute, Obtuse, Right, and Straight Angles

Goal

Identify acute, obtuse, right, and straight angles and record the angle measure in degrees.

Standard	Materials
MA.4.GR.1.1	protractor

Modeled Review

Point to Avery's work and **ask**:

- "What measures would classify an angle as acute? Obtuse? Right? Straight?"
- "What feature do you see on the right angle that is different from the other angles? What do you think that means?"

Reinforce the goal by saying, "Right angles measure exactly 90 degrees, and straight angles measure exactly 180 degrees. Acute angles measure less than 90 degrees, and obtuse angles measure greater than 90 degrees but less than 180 degrees."

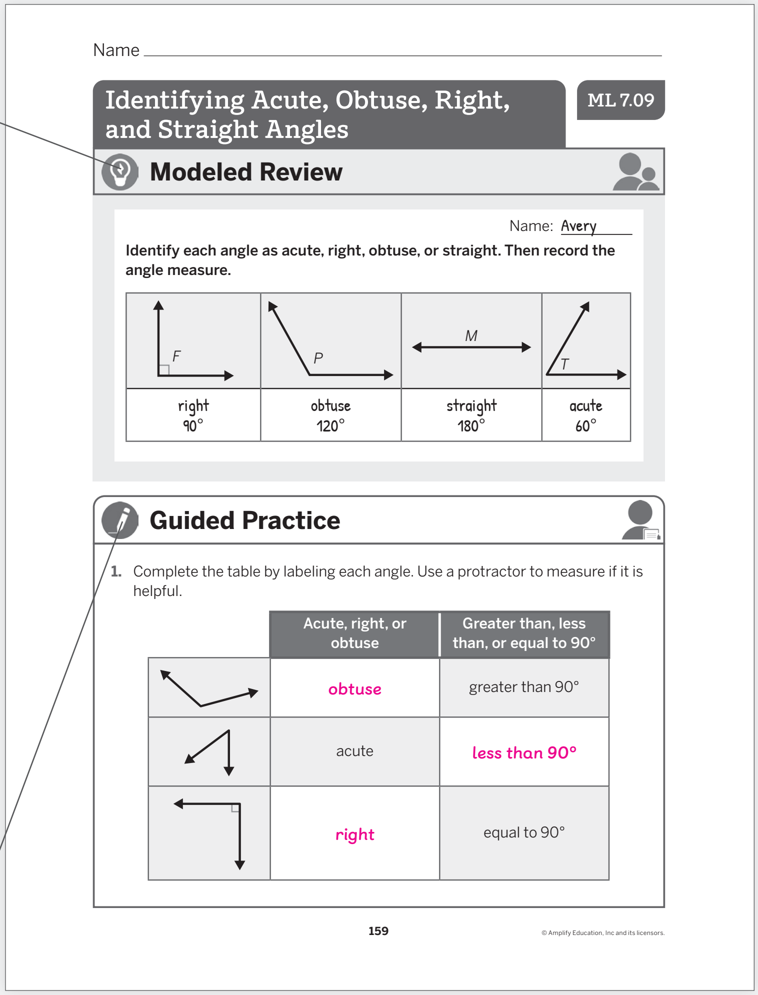
ML/EL Use arm gestures to represent the angles as you share the definition of each term.

Guided Practice

Focus students' attention on labeling each angle.

To scaffold their thinking, **ask**:

- "How can you locate 90° on the protractor?"
- "How will you determine if each angle is more, less than, or equal to 90° using a protractor?"
- "Is the angle acute, right, obtuse, or straight? How do you know?"



Name

Identifying Acute, Obtuse, Right, and Straight Angles

ML 7.09

Modeled Review

Name: Avery

Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

F	P	M	T
right 90°	obtuse 120°	straight 180°	acute 60°

Guided Practice

1. Complete the table by labeling each angle. Use a protractor to measure if it is helpful.

	Acute, right, or obtuse	Greater than, less than, or equal to 90°
	obtuse	greater than 90°
	acute	less than 90°
	right	equal to 90°

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Vocabulary

If needed, share the meaning of the terms with students.

acute angle: An angle with a measure of less than 90 degrees.

obtuse angle: An angle with a measure greater than 90 degrees but less than 180 degrees.

right angle: An angle that measures 90 degrees.

straight angle: An angle that measures 180 degrees.

ML 7.09 Identifying Acute, Obtuse, Right, and Straight Angles

Guided Practice

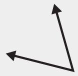
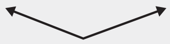
A Demonstrate to students how to properly align the protractor and determine which set of numbers to use to find the measurement of an angle before they complete Problem 2.

Note: Close, reasonable measurements are acceptable.

Key Takeaway:
Say, "Angles can be identified as acute, right, obtuse, or straight angles based on their size in degrees."

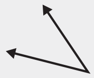
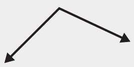
Guided Practice

2. Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

	Acute, right, obtuse, or straight	Measurement in degrees
	right	90°
	acute	60°
	straight	180°
	obtuse	140°

Check

Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

	Acute, right, obtuse, or straight	Measurement in degrees
	acute	40°
	obtuse	110°

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Reflection

Ask:

- "How could you use your arms to show an angle that is acute? Obtuse? Right? Straight?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them determine the measurement in degrees of the angles in Problem 1 using a protractor. Provide students with support and feedback as they measure each angle.

Got it!
If students need more practice, sketch a variety of angles. Then ask students to label each angle as acute, obtuse, right, or straight and measure each using a protractor.

Name ______________________________

Dividing by Powers of 10

ML 6.04

Modeled Review

Name: Jack

Evaluate each expression. Show your thinking.

1. $22 \div 10^3 =$ 0.022

$10^3 = 10 \times 10 \times 10 = 1{,}000$

$22 \div 1{,}000 = 0.022$

tens	ones	tenths	hundredths	thousandths
2	2			
	0	0	2	2

2. $45.7 \div 10^2 =$ 0.457

$10^2 = 10 \times 10 = 100$

$45.7 \div 100 = 0.457$

tens	ones	tenths	hundredths	thousandths
4	5	7		
	0	4	5	7

Guided Practice

Evaluate each equation.

1. $4{,}720 \div 10^1 = 4{,}720 \div 10 = 472.0$

2. $4{,}720 \div 10^2 = 4{,}720 \div 100 =$ __________

3. $4{,}720 \div 10^3 =$ ____________________ = __________

4. $4{,}720 \div 10^4 =$ ____________________ = __________

5. $4{,}720 \div 10^5 =$ ____________________ = __________

Guided Practice

Evaluate each equation.

6. $841 \div 10^2 =$ ____________________ = __________

7. $841 \div 10^3 =$ ____________________ = __________

8. $387.5 \div 10^1 =$ $387.5 \div 10$ $=$ 38.75

9. $387.5 \div 10^2 =$ ____________________ = __________

10. $387.5 \div 10^3 =$ ____________________ = __________

11. $387.5 \div 10^4 =$ ____________________ = __________

12. $152.1 \div 10^1 =$ ____________________ = __________

13. $152.1 \div 10^2 =$ ____________________ = __________

Check

Evaluate each equation.

1. $45{,}736 \div 10^4 =$ __________

2. $41.3 \div 10^2 =$ __________

Goal

Divide whole numbers by powers of 10.

Standard

MA.5.NSO.2.5

Materials

graph or lined paper (optional)

Modeled Review

Point to Jack's work and **ask**:

- "What similarities do you see in the dividend and quotient? Differences?"
- "How do Jack's place value charts show what is happening to the dividend?"
- "What conclusion can you make about dividing by a power of 10?"

Reinforce Jack's thinking by saying, "Dividing by a power of 10 shifts the digits of a number as many places to the right as the number of times you divide by 10. As you shift each place to the right, the value of the digit is $\frac{1}{10}$ as much."

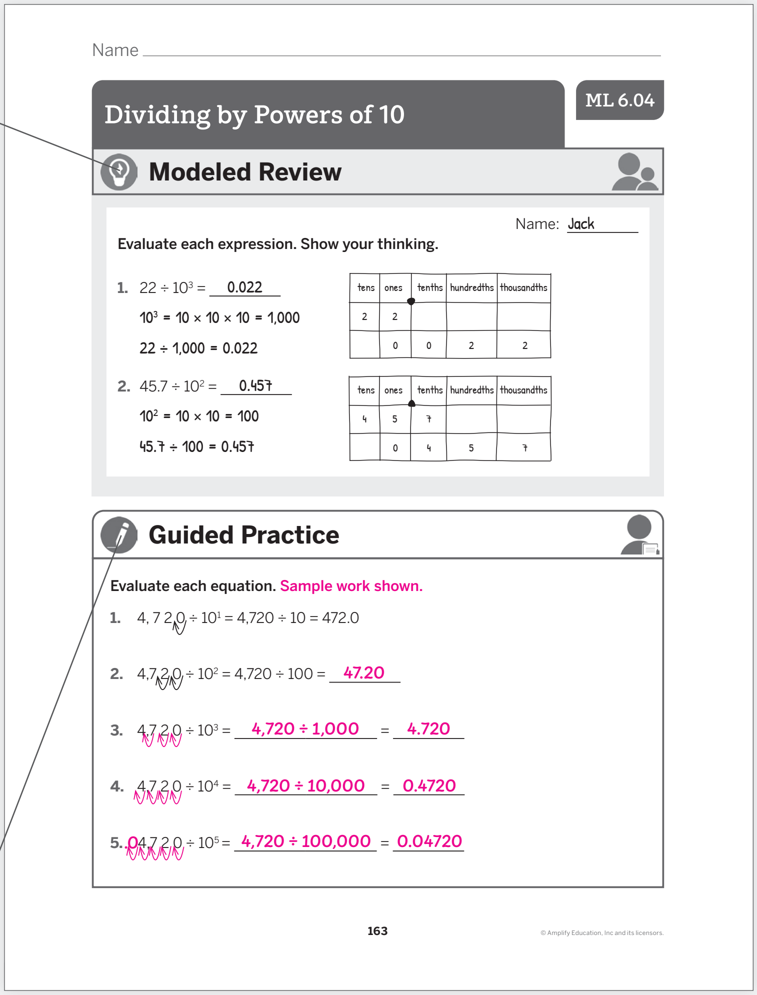
ML/EL Model annotating the place value charts to support place value understanding and dividing powers of 10.

Guided Practice

Focus students' attention on solving each division equation.

To scaffold their thinking, **ask**:

- "What is the standard form of the power of 10? Where do you see that in the division equation?"
- "How would you read your answer? How does that compare to the dividend in the problem?"


Name

Dividing by Powers of 10 — ML 6.04

Modeled Review

Name: Jack

Evaluate each expression. Show your thinking.

1. $22 \div 10^3 =$ 0.022

$10^3 = 10 \times 10 \times 10 = 1{,}000$

$22 \div 1{,}000 = 0.022$

tens	ones	tenths	hundredths	thousandths
2	2			
	0	0	2	2

2. $45.7 \div 10^2 =$ 0.457

$10^2 = 10 \times 10 = 100$

$45.7 \div 100 = 0.457$

tens	ones	tenths	hundredths	thousandths
4	5	7		
	0	4	5	7

Guided Practice

Evaluate each equation. Sample work shown.

1. $4{,}720 \div 10^1 = 4{,}720 \div 10 = 472.0$
2. $4{,}720 \div 10^2 = 4{,}720 \div 100 =$ 47.20
3. $4{,}720 \div 10^3 =$ 4,720 ÷ 1,000 = 4.720
4. $4{,}720 \div 10^4 =$ 4,720 ÷ 10,000 = 0.4720
5. .04,720 ÷ $10^5 =$ 4,720 ÷ 100,000 = 0.04720

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Vocabulary

If needed, share the meaning of the terms with students.

dividend: The number being divided, representing the total number being equally distributed or divided.

quotient: The result obtained by dividing two numbers representing either the number of equal-sized groups or the size of each group.

Guided Practice

A Invite students to use graph paper or lined paper oriented vertically to draw a place value chart to write out each dividend. Ask students how the digits will shift as they divide by the power of 10.

Key Takeaway:
Say, "When dividing a decimal by a power of 10, the exponent indicates how many places each of the digits shifts to the right, just like when dividing whole numbers by powers of 10."

Guided Practice

Evaluate each equation.

6. $841 \div 10^2 =$ 841 ÷ 100 = 8.41
7. $841 \div 10^3 =$ 841 ÷ 1,000 = 0.841
8. $387.5 \div 10^1 =$ 387.5 ÷ 10 = 38.75
9. $387.5 \div 10^2 =$ 387.5 ÷ 100 = 3.875
10. $387.5 \div 10^3 =$ 387.5 ÷ 1,000 = 0.3875
11. $387.5 \div 10^4 =$ 387.5 ÷ 10,000 = 0.03875
12. $152.1 \div 10^1 =$ 152.1 ÷ 10 = 15.21
13. $152.1 \div 10^2 =$ 152.1 ÷ 100 = 1.521

Check

Evaluate each equation. Sample work shown.

1. $45{,}736 \div 10^4 =$ 4.5736

   45,736 ÷ 10,000 = 4.5736

2. $41.3 \div 10^2 =$ 0.413

   41.3 ÷ 100 = 0.413

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Reflection

Ask:

- "What is different about dividing by a power of ten and multiplying by a power of ten?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there

If students need more support, consider having them write out the exponent in expanded form to connect to the equivalent expression before finding the quotient.

Got it!

If students need more practice, invite them to evaluate the following expressions:

- $3.2 \div 10^2$
- $3.2 \div 10^3$
- $48.9 \div 10^2$
- $48.9 \div 10^3$

Name ____________________

Calculating Areas of Triangles

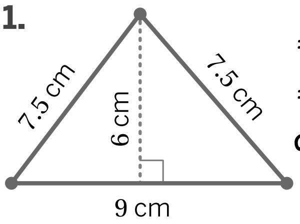
ML 1.03

Modeled Review

Name: Clare

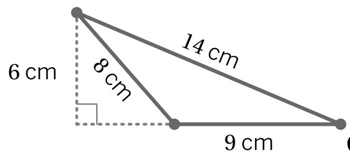
Determine the area of each triangle in square centimeters.

1.



$= \frac{1}{2}(9 \cdot 6)$
$= 27$ square centimeters

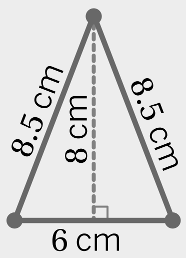
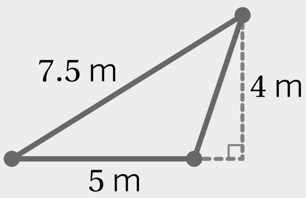
2.



$= \frac{1}{2}(9 \cdot 6)$
$= 27$ square centimeters

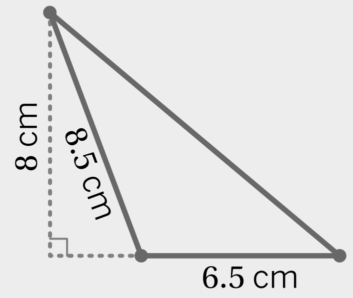
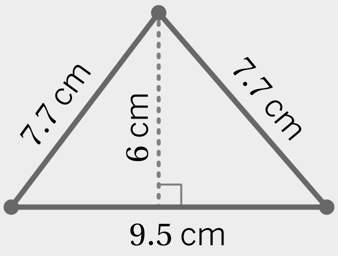
Guided Practice

1. Determine the base, height, and area of each triangle.

Triangle	Base	Height	Area
	6 cm	8 cm	
	5 m		

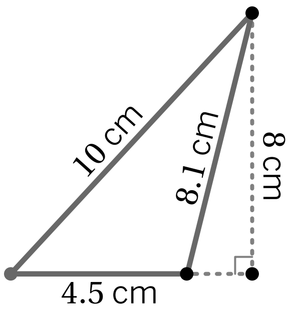
Guided Practice

2. Determine the base and height of each triangle and write an equation to represent the area of each triangle.

Triangle	Base	Height	Equation	Area
		8 cm		26 square centimeters
				28.5 square centimeters

Check

Determine the area of the triangle. Show or explain your thinking.



Calculating Areas of Triangles

Goal

Calculate the area of a triangle without a grid, given any side as its base.

Standard

MA.6.GR.2.1

Materials

coloring tools (optional), cut-out rectangles and triangles (optional)

Modeled Review

Point to Clare's work and **ask**:
- "How are the triangles alike? Different?"
- "Why did Clare use the inside height for one triangle and the outside height for the other?"

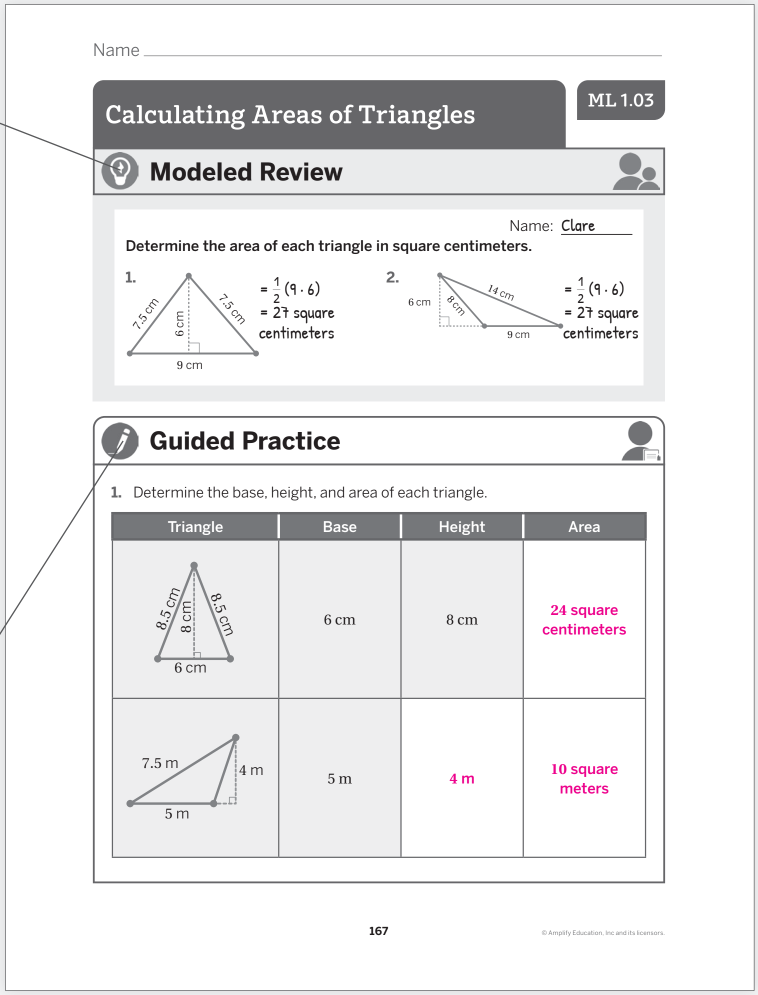
Reinforce Clare's thinking by saying, "The formula used to determine the area of any triangle is $A = \frac{1}{2} \cdot b \cdot h$."

ML/EL Model by tracing and labeling the given height and/or base of each triangle.

Guided Practice

Focus students' attention on determining which measurements represent the base and height of each triangle.

To scaffold their thinking, **ask**:
- "Why is the height measurement for the first triangle 8 centimeters and not 8.5 centimeters?"
- "How do you know which measurement is the base?"
- "Why does the formula for calculating the area of a triangle use $\frac{1}{2}$?"


Name

Calculating Areas of Triangles

ML 1.03

Modeled Review

Name: Clare

Determine the area of each triangle in square centimeters.

1. 7.5 cm, 7.5 cm, 6 cm, 9 cm
$= \frac{1}{2}(9 \cdot 6)$
$= 27$ square centimeters

2. 6 cm, 8 cm, 14 cm, 9 cm
$= \frac{1}{2}(9 \cdot 6)$
$= 27$ square centimeters

Guided Practice

1. Determine the base, height, and area of each triangle.

Triangle	Base	Height	Area
8.5 cm, 8.5 cm, 8 cm, 6 cm	6 cm	8 cm	24 square centimeters
7.5 m, 4 m, 5 m	5 m	4 m	10 square meters

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Vocabulary

If needed, share the meaning of the terms with students.

area: Area measures the space inside a two-dimensional figure. It is expressed in square units.

height of a triangle: The shortest distance between a base and its opposite vertex.

base: One side of a triangle. Any side of a triangle can be the base.

Calculating Areas of Triangles

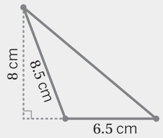
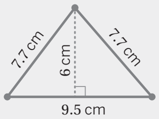
Guided Practice

A Provide coloring tools and suggest students use color coding to annotate the base-height pairs in each triangle by labeling or circling the base in one color and its corresponding height in another color.

Key Takeaway:
Say, "A triangle can have more than one base and height pair. The height of the triangle must be perpendicular to the base of the triangle. No matter which side of a triangle is chosen as the base, its area, A, is equal to $\frac{1}{2} \cdot b \cdot h$."

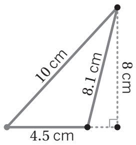
Guided Practice

2. Determine the base and height of each triangle and write an equation to represent the area of each triangle.

Triangle	Base	Height	Equation	Area
	6.5 cm	8 cm	$A = \frac{1}{2}(6.5 \cdot 8)$	26 square centimeters
	9.5 cm	6 cm	$A = \frac{1}{2}(9.5 \cdot 8)$	28.5 square centimeters

Check

Determine the area of the triangle. Show or explain your thinking.
Sample responses shown.



$A = \frac{1}{2}(4.5 \cdot 8)$

$A = 18$

The area of the triangle is 18 square centimeters.

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Reflection

Ask:

- "What is always true about the base and height of a triangle when calculating the area?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, use cut-out triangle and rectangle shapes to show how two identical triangles can form a rectangle or parallelogram. This will help reinforce the formula $\frac{1}{2} \cdot b \cdot h$.

Got it!
If students need more practice, have them solve the following problems. Then have them compare with a partner and discuss.

- Determine the area of a triangle with a base of 12.5 centimeters and a height of 9 centimeters.
- A triangle has an area of 30 square centimeters. If the height is 5 centimeters, what is the length of the base?

Name ____________________

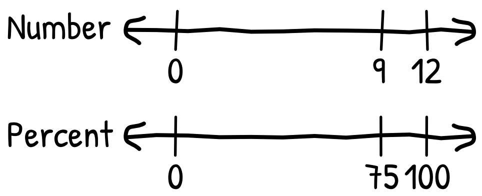
Calculating Unknown Percentages

ML 3.12

Modeled Review

Name: Tristan

What is 9 out of 12 as a percent?



75%

Name: Santiago

What is 9 out of 12 as a percent?

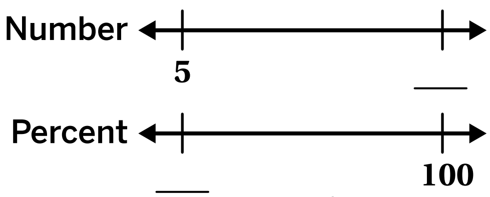
$\frac{9}{12} \cdot 100 = \underline{75}$

75%

Guided Practice

For Problems 1 and 2, calculate the unknown percentage. Show your thinking.

1. What is 5 out of 10 as a percent?



____%

2. What is 5 out of 50 as a percent?

$\frac{5}{50} \cdot 100 =$ ______

______ %

Guided Practice

Calculate the unknown percentage. Show your thinking.

3. What is 18 out of 24 as a percent?

$\frac{18}{24} \cdot 100 =$ _____

4. What is 4 out of 16 as a percent?

_____ $\cdot\ 100 =$ _____

5. What is 13 out of 26 as a percent?

6. What is 3 out of 15 as a percent?

7. What is 7 out of 70 as a percent?

8. What is 19 out of 38 as a percent?

Check

What is 16 out of 64 as a percent? Show your thinking.

ML 3.12 Calculating Unknown Percentages

Goal

Calculate an unknown percentage.

Standard

MA.6.AR.3.4

Materials

double number line template (optional)

Modeled Review

Point to the problem in the Modeled Review and **ask**:

- "How are Tristan's and Santiago's work alike? How are they different?"
- "How did Tristan use a double number line diagram to calculate the percent?"
- "How did Santiago use an equation to calculate the percent?"

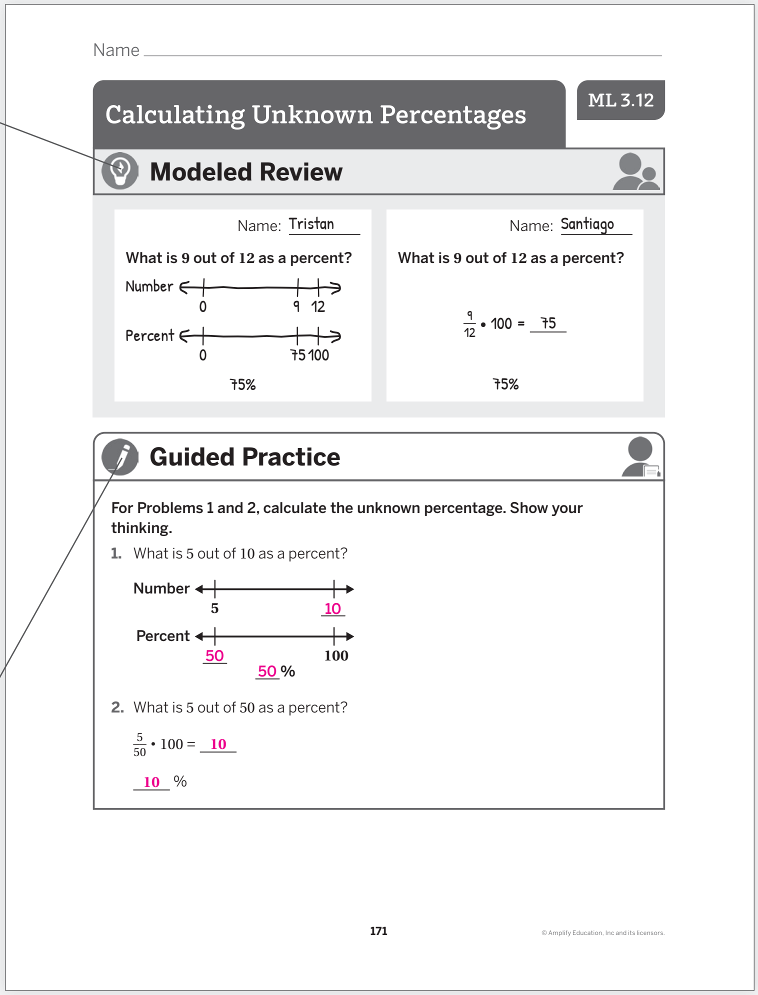
Reinforce the goal by saying, "The percent of a number can be calculated using double number line diagrams or equations."

Guided Practice

Focus students' attention on how the percent of a number can be calculated using a double number line an equation.

To scaffold their thinking, **ask**:

- "How can you use a double number line to calculate the percent?"
- "How can you use an equation to calculate the percent?"



Name

Calculating Unknown Percentages

ML 3.12

Modeled Review

Name: Tristan

What is 9 out of 12 as a percent?

Number 0 9 12

Percent 0 75 100

75%

Name: Santiago

What is 9 out of 12 as a percent?

$\frac{9}{12} \cdot 100 =$ 75

75%

Guided Practice

For Problems 1 and 2, calculate the unknown percentage. Show your thinking.

1. What is 5 out of 10 as a percent?

Number 5 10

Percent 50 100

50 %

2. What is 5 out of 50 as a percent?

$\frac{5}{50} \cdot 100 =$ 10

10 %

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Vocabulary

If needed, share the meaning of the term with students.

rate: A rate is a ratio that describes how two quantities change together.

Calculating Unknown Percentages

Guided Practice

A Provide students with access to double number line diagrams and support them with marking benchmark percentages (i.e., 50%, 25%, 10%) before starting the problems.

ML/EL Provide sentence frames to support students as they explain their strategies. For example, "I know that the percentage is _______ because _______."

Key Takeaway:
Say, "Percent of a number can be calculated using a variety of strategies, including double number line diagrams and equations."

Guided Practice

Calculate the unknown percentage. Show your thinking.
Sample work shown.

3. What is 18 out of 24 as a percent?

$\frac{18}{24} \cdot 100 = \underline{75}$

75%

4. What is 4 out of 16 as a percent?

$\frac{4}{16} \cdot 100 = \underline{25}$

25%

5. What is 13 out of 26 as a percent?

$\frac{13}{26} \cdot 100 = 50$

50%

6. What is 3 out of 15 as a percent?

$\frac{3}{15} \cdot 100 = 20$

20%

7. What is 7 out of 70 as a percent?

$\frac{7}{70} \cdot 100 = 10$

10%

8. What is 19 out of 38 as a percent?

$\frac{19}{38} \cdot 100 = 50$

50%

Check

What is 16 out of 64 as a percent? Show your thinking.
Sample work shown.

$\frac{16}{64} \cdot 100 = 25$

25%

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Reflection

Ask:
- "How are equations helpful in calculating the unknown percentage?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, ask them to circle the information given to help identify what they are solving for and set up their equation with the given information.

Got it!
If students need more practice, ask them to calculate the unknown percentage and show their thinking.

- What is 6 out of 30 as a percentage?
- What is 8 out of 32 as a percentage?

Name ____________________

Writing and Solving Equations

ML 6.04

Modeled Review

Name: Han

For Problems 1–3, use the equation $x + 2 = 10$.

1. Write a situation that represents this equation. Explain what x represents in your situation.

   Tristan spent \$10 on a box of tissues and medicine. The box of tissues cost \$2. The medicine cost x dollars.

   $$\begin{aligned} x + 2 &= 10 \\ -2 \quad & \;\; -2 \\ x &= 8 \end{aligned}$$

2. Determine the solution to the equation.

   $x = 8$

3. Explain what the solution means in your situation.

   The medicine cost \$8.

Guided Practice

For each situation, write the equation from the bank that best represents it.

$x + 6 = 13$	$6x = 13$	$x + 2 = 7$
$2x = 7$	$x - 2 = 7$	$2x = 10$

1. Santiago spent \$7 on a cup of coffee and a croissant. The cup of coffee cost \$2. The croissant cost x dollars.

   ____

2. Jada spent $\$10$ on two secondhand books. Each book cost x dollars.

   ____

3. Diego spent $\$13$ on an herb plant and a pot. The plant cost \$6. The pot cost x dollars.

   ____

Guided Practice

4. Complete the table for each equation shown.

Equation	Situation	Solution	Solution's meaning
$x + 6 = 15$	Rebecca spent $\$15$ on a deck of cards and a phone charger. The deck of cards cost $\$6$. The phone charger cost x dollars.	$x = 9$	
$4x = 20$	Esteban spent $\$20$ on four greeting cards. Each greeting card cost x dollars.		
$x + 3 = 14$			

Check

For Problems 1–3, use the equation $x + 5 = 12$.

1. Write a situation that represents the equation. Explain what x represents in your situation.

2. Determine the solution to the equation.

3. Explain what the solution means in your situation.

ML 6.04 Writing and Solving Equations

Goal	Standard
Write descriptions for and solve one-step equations.	MA.6.AR.2.2

Modeled Review

Point to Han's work and **ask**:

- "How did Han write a situation to represent the equation?"
- "Why did Han subtract 2 from both sides of the equal sign?"
- "How did Han know the solution represented the cost of the medicine?"

Reinforce Han's thinking by saying, "An equation can be written as situations by focusing on the operation and its solution."

ML/EL To increase accessibility throughout the lesson, provide students with the opportunity to share their situations orally rather than in writing.

Guided Practice

Focus students' attention on representing each situation with an equation from the bank.

To scaffold their thinking, **ask**:

- "Which operation is represented by the situation?"
- "Is there anything about the situation that could help you decide which equation it matches?"
- "What does the solution mean according to the situation?"

Name ________________________

Writing and Solving Equations

ML 6.04

Modeled Review

Name: Han

For Problems 1–3, use the equation $x + 2 = 10$.

1. Write a situation that represents this equation. Explain what x represents in your situation.

   Tristan spent \$10 on a box of tissues and medicine. The box of tissues cost \$2. The medicine cost x dollars.

$$\begin{aligned} x + 2 &= 10 \\ -2 \quad & \;\; -2 \\ x &= 8 \end{aligned}$$

2. Determine the solution to the equation.

   $x = 8$

3. Explain what the solution means in your situation.

   The medicine cost \$8.

Guided Practice

For each situation, write the equation from the bank that best represents it.

$x + 6 = 13$	$6x = 13$	$x + 2 = 7$
$2x = 7$	$x - 2 = 7$	$2x = 10$

1. Santiago spent \$7 on a cup of coffee and a croissant. The cup of coffee cost \$2. The croissant cost x dollars.

   $x + 2 = 7$

2. Jada spent \$10 on two secondhand books. Each book cost x dollars.

   $2x = 10$

3. Diego spent \$13 on an herb plant and a pot. The plant cost \$6. The pot cost x dollars.

   $x + 6 = 13$

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Guided Practice

A To support students with their written responses, consider providing sentence frames, such as "Jack spent ___ on ___ packs of gum. Each pack cost ___ dollars."

Key Takeaway:
Say, "Writing and solving equations can help us determine unknown information in a situation. A solution's meaning depends on the situation the equation represents."

Guided Practice

4. Complete the table for each equation shown. Sample situation shown for Row 3.

Equation	Situation	Solution	Solution's meaning
$x + 6 = 15$	Rebecca spent \$15 on a deck of cards and a phone charger. The deck of cards cost \$6. The phone charger cost x dollars.	$x = 9$	The phone charger cost \$9.
$4x = 20$	Esteban spent \$20 on four greeting cards. Each greeting card cost x dollars.	$x = 5$	Each greeting card cost \$5.
$x + 3 = 14$	Riya spent \$14 on a box of crayons and a pack of paint brushes. The box of crayons cost \$3. The pack of paint brushes cost x dollars.	$x = 11$	The pack of paint brushes cost \$11.

Check

For Problems 1–3, use the equation $x + 5 = 12$.
Sample responses shown for Problems 1 and 3.

1. Write a situation that represents the equation. Explain what x represents in your situation.

Jack spent \$12 on a pair of sunglasses and a shirt. The sunglasses cost \$5. The shirt cost x dollars.

2. Determine the solution to the equation.

$x = 7$

3. Explain what the solution means in your situation.

The shirt cost \$7.

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Reflection

Ask:
- "What do you think is important to remember when writing equations to represent situations?"
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider providing them with a list of topics they can use when generating situations to represent the equations.
- Grocery store
- Concession stand at a game

Got it!
If students need more practice, present them with the following problems. Ask them to write a situation and solve. Then describe what the solution means in their situation.

- $x + 7 = 20$
- $6x = 42$

Name ______________________________

Evaluating Expressions With Exponents

ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

$2 \cdot (3 \cdot 3)$
$2 \cdot (9)$
18

I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

$5 + (1)^2$
$5 + (1 \cdot 1)$
$5 + 1$
6

I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

$2 \cdot$ ___

______

2. $2 + 3^2$

$2 + ($___ $\cdot$ ___$)$

___ $+$ ___

______

3. $(6 - 2)^2$

$($___$)^2$

$($______$)$

______

4. $1 + (3 - 2)^2$

$1 + ($___$)^2$

____

______

______

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	
$(4 + 3)^2$	
$3 + (4 + 1)^2$	
$2 \cdot 4^2$	
$(7 - 1)^2 + 2$	
$6 + 7^2$	

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ ______

2. $7 + (5 - 2)^2$ ______

Evaluating Expressions With Exponents

Goal

Evaluate expressions with exponents using the order of operations.

Standard

MA.6.NSO.3.3

Materials

highlighter (optional)

Modeled Review

Point to Kai's work and **ask**:

- "How did Kai know which operation to evaluate first in each expression?"
- "Can you explain the order of operations Kai used to evaluate the expressions?"
- "What does the exponent 2 represent in each expression?"

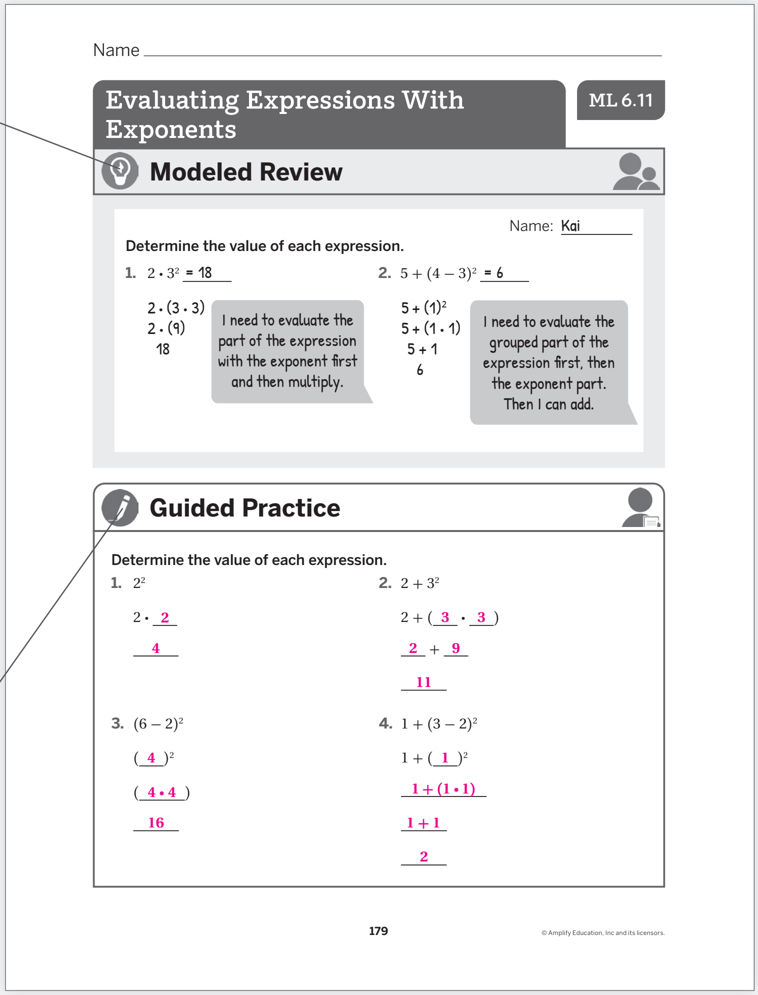
Reinforce Kai's thinking by saying, "The order of operations can be used to efficiently evaluate expressions with exponents."

Guided Practice

Focus students' attention on using the order of operations to evaluate each expression.

To scaffold their thinking, **say**:

- "First, evaluate brackets or parentheses."
- "Next, evaluate exponents."
- "Then, evaluate multiplication or division (left to right)."
- "Lastly, evaluate addition or subtraction (left to right)."



Name ____________________

Evaluating Expressions With Exponents ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

$2 \cdot (3 \cdot 3)$
$2 \cdot (9)$
18

I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

$5 + (1)^2$
$5 + (1 \cdot 1)$
$5 + 1$
6

I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

$2 \cdot$ 2

4

2. $2 + 3^2$

$2 + ($ 3 $\cdot$ 3 $)$

2 + 9

11

3. $(6 - 2)^2$

(4 $)^2$

($4 \cdot 4$)

16

4. $1 + (3 - 2)^2$

$1 + ($ 1 $)^2$

$1 + (1 \cdot 1)$

$1 + 1$

2

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Vocabulary

If needed, share the meaning of the terms with students.

order of operations: A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone.

exponent: A number used to describe repeated multiplication. Exponents are sometimes called powers.

ML 6.11 Evaluating Expressions With Exponents

Guided Practice

(A) Use visual aids, such as anchor charts or diagrams, to illustrate the order of operations and guide students in understanding the correct sequence of steps to evaluate expressions with exponents.

(ML/EL) Demonstrate the step-by-step process of evaluating expressions with exponents, narrating your thinking aloud to show students how to apply the order of operations correctly.

Key Takeaway:
Say, "When evaluating expressions, evaluate exponents first, unless there are grouping symbols, like parentheses or a fraction bar. When there are grouping symbols, perform the operation(s) inside them first."

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	15
$(4 + 3)^2$	49
$3 + (4 + 1)^2$	28
$2 \cdot 4^2$	32
$(7 - 1)^2 + 2$	38
$6 + 7^2$	55

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ = 75
2. $7 + (5 - 2)^2$ = 16

180

Reflection

Ask:
- "What are some things to remember when determining the value of expressions with exponents?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a highlighter to color-code different operations and parts of the expression.

Got it!
If students need more practice, have them evaluate the following expressions:

- $9 \cdot 5^2$
- $20 - (5 - 1)^2$
- $36 - 2 \cdot 3^2$

Name ______________________________

Calculating Percentages

ML 6.14

Modeled Review

Name: Dylan

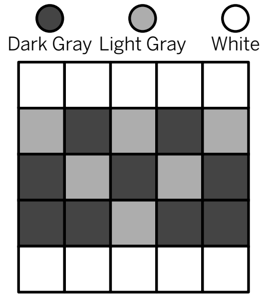
What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$\frac{100}{25} = 4\%$

$4 \cdot 10 = 40$

40%



Guided Practice

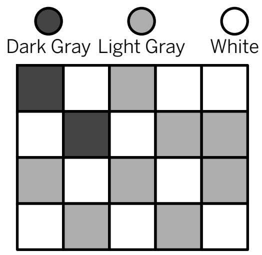
1. What percentage of the grid is light gray or white? Show your thinking.

Light Gray:

$\frac{100}{20} =$ ________

White:

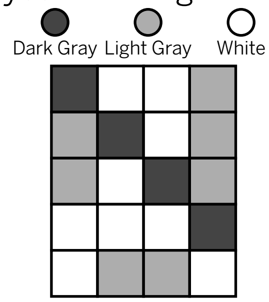
Color	Percentage
Dark Gray	10%
Light Gray	
White	



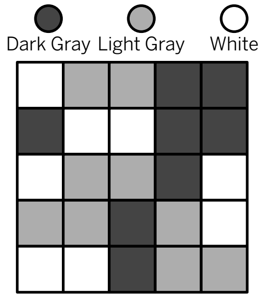
Guided Practice

Determine the percentages of the grids that are shaded each color.

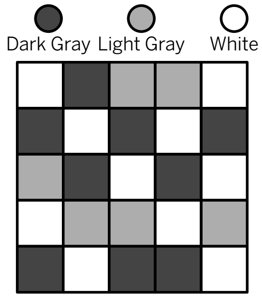
2. What percentage of the grid is dark gray? Show your thinking.



3. What percentage of the grid is white? Show your thinking.

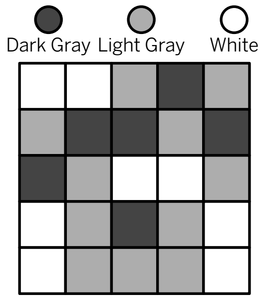


4. What percentage of the grid is light gray? Show your thinking.



Check

What percentage of the grid is white? Show your thinking.



Calculating Percentages

Goal	**Standard**
Calculate the percentage of a quantity on a grid.	MA.7.AR.3.1

Modeled Review

Point to Dylan's work and **ask**:

- "What percentage represents 1 whole?"
- "How did Dylan use the grid to calculate the percentage of the white squares?"

Reinforce Dylan's thinking by saying, "Finding the percentage each square represents can help you determine the total percentage of each color."

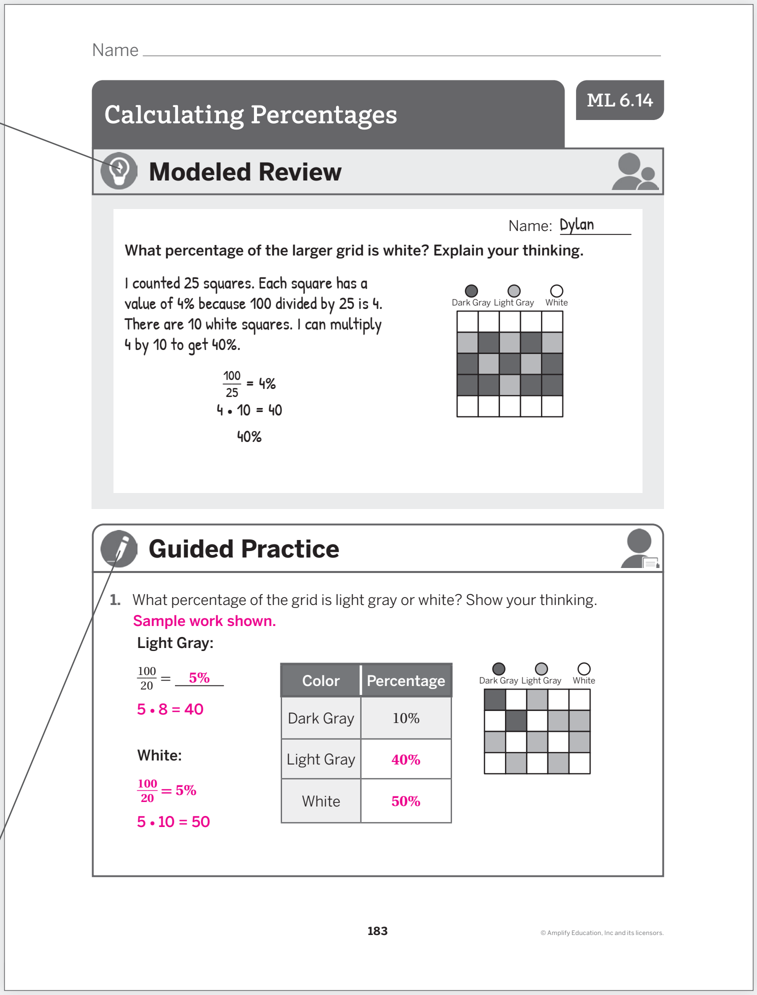
ML/EL Provide sentence frames to support students as they explain how they determined the percentage shaded for each colored section. For example, "I counted ________ sections in the larger square." or "There are ______ dark gray/light gray/white sections, so I ______."

Guided Practice

Focus students' attention on calculating the percentage of each color in the grid.

To scaffold their thinking, **ask**:

- "How many sections is the grid broken into?"
- "How many sections are there of each color?"
- "How do you calculate the percentage of each color?"



Name

Calculating Percentages

ML 6.14

Modeled Review

Name: Dylan

What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$\frac{100}{25} = 4\%$

$4 \cdot 10 = 40$

40%

Dark Gray Light Gray White

Guided Practice

1. What percentage of the grid is light gray or white? Show your thinking.

Sample work shown.

Light Gray:

$\frac{100}{20} = \underline{5\%}$

$5 \cdot 8 = 40$

White:

$\frac{100}{20} = 5\%$

$5 \cdot 10 = 50$

Color	Percentage
Dark Gray	10%
Light Gray	40%
White	50%

Dark Gray Light Gray White

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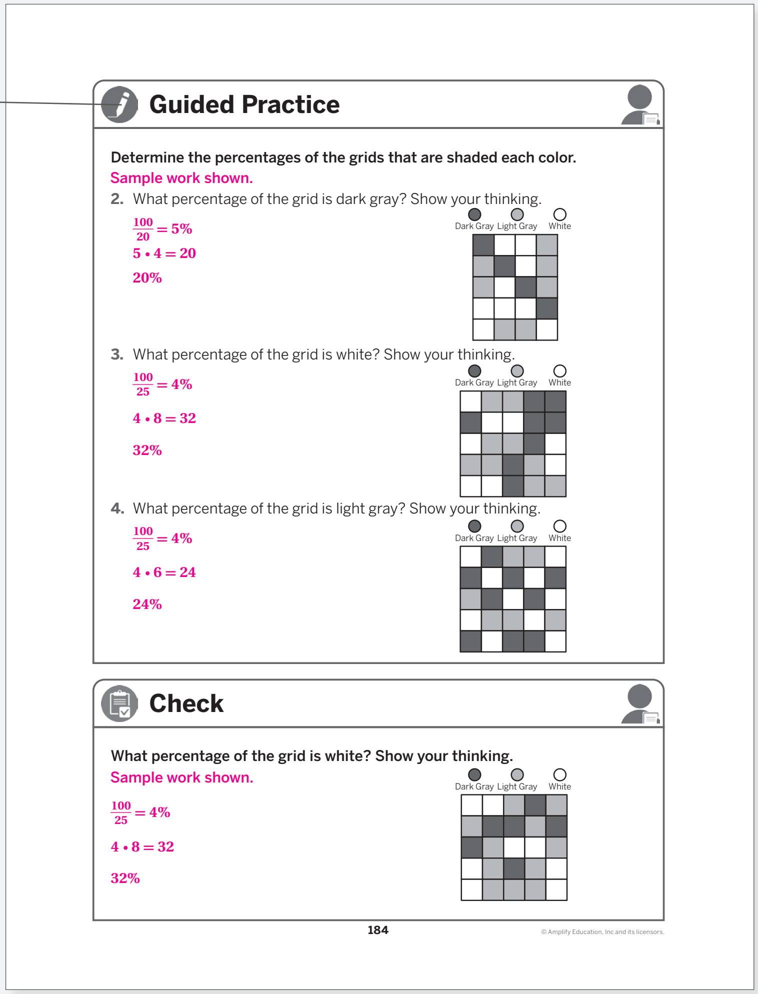
Guided Practice

A Chunk this task into smaller, more manageable parts by having students create a table that can be used to organize their data, including how many sections the grid is broken into and how many sections there are of each color.

Note: Students can check if they accurately determined the percentage for each color by ensuring all of the percentages add to 100%.

Key Takeaway:
Say, "Several strategies can be used to calculate the percent of a number such as multiplying the percentage by the whole, using benchmark fractions, or using a visual representation."



Guided Practice

Determine the percentages of the grids that are shaded each color.
Sample work shown.

2. What percentage of the grid is dark gray? Show your thinking.

$\frac{100}{20} = 5\%$
$5 \cdot 4 = 20$
20%

3. What percentage of the grid is white? Show your thinking.

$\frac{100}{25} = 4\%$
$4 \cdot 8 = 32$
32%

4. What percentage of the grid is light gray? Show your thinking.

$\frac{100}{25} = 4\%$
$4 \cdot 6 = 24$
24%

Check

What percentage of the grid is white? Show your thinking.
Sample work shown.

$\frac{100}{25} = 4\%$
$4 \cdot 8 = 32$
32%

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Reflection

Ask:

- "How is using a grid helpful in visualizing benchmark percentages?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider using Accelerated 6 Mini-Lesson 3.12: *Calculating Unknown Percentages*.	If students need more practice, ask them to look at Problem 2 and determine the percentage of the remaining colors in the grid.

Name ______________________________

Ordering and Comparing Positive and Negative Numbers

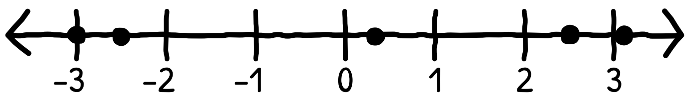
ML 7.04

Modeled Review

Name: Adam

Order the numbers from *least* to *greatest*.

−3	3.1	−2.5	2.5	0.25



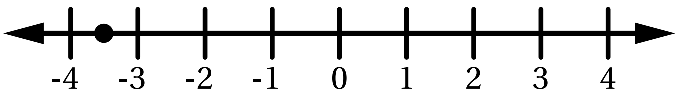
−3	−2.5	0.25	2.5	3.1
least				greatest

Guided Practice

Order each set of numbers from least to *greatest*. Use the number lines if they are helpful.

1.

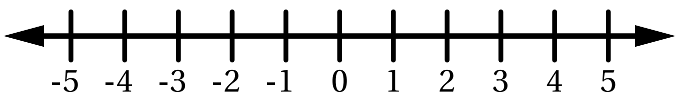
0	-3.5	3	-1



-3.5			
least			greatest

2.

4.5	-2	-4	0.25



least			greatest

Guided Practice

Order each set of numbers from *least* to *greatest.*

3.

0.2	-2	3.5	3	1.25

______ least ______ ______ ______ ______ greatest

4.

$\frac{1}{2}$	3	$1\frac{1}{2}$	-2	1

______ least ______ ______ ______ ______ greatest

5.

2.5	2	-2.5	-0.5	-3

______ least ______ ______ ______ ______ greatest

6.

$-3\frac{1}{2}$	$-\frac{1}{5}$	$2\frac{1}{2}$	-5	3

______ least ______ ______ ______ ______ greatest

Check

Order the numbers from *least* to *greatest.*

$-\frac{1}{2}$	1	-3	4	$-4\frac{1}{2}$

______ least ______ ______ ______ ______ greatest

Ordering and Comparing Positive and Negative Numbers

Goal

Order and compare positive and negative numbers represented as fractions and decimals.

Standards

MA.6.NSO.1.1

Modeled Review

Point to Adam's work and **ask**:

- "How could Adam tell if one number was greater than or less than another number?"
- "How could Adam tell if two numbers were opposites?"
- "How was using a number line helpful?"

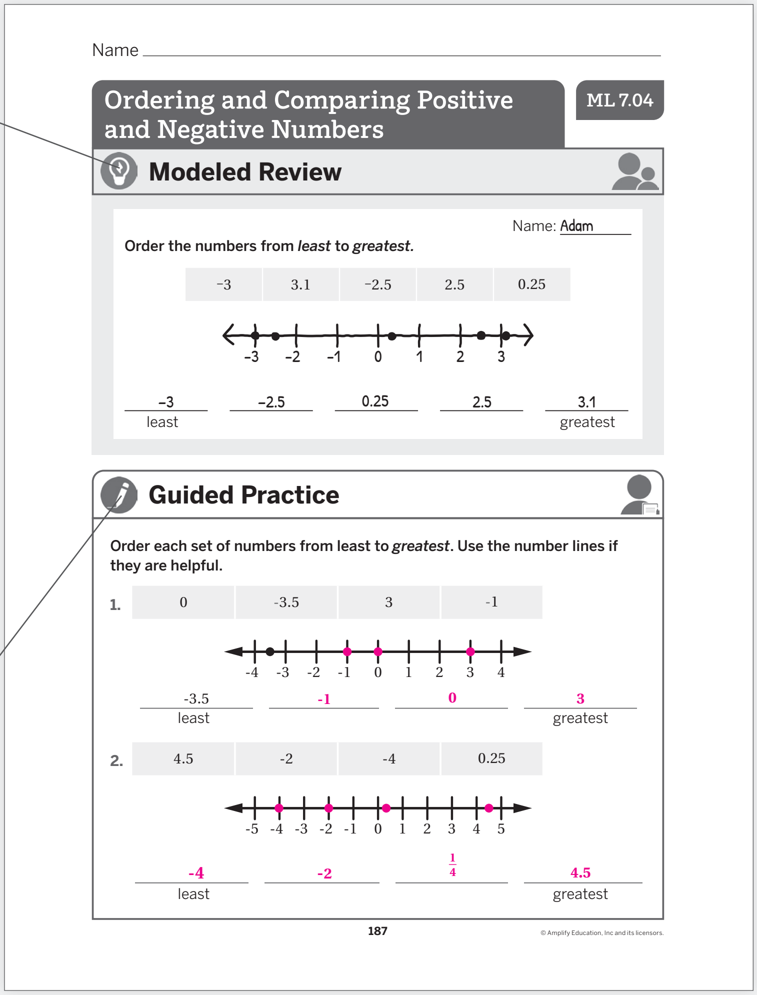
Reinforce Adam's thinking by saying, "Using a number line can help efficiently order and compare positive and negative numbers."

Guided Practice

Focus students' attention on ordering and comparing numbers using a number line.

To scaffold their thinking, **say**:

- "Find the smallest number and add it to the number line."
- "Then, look for the next smallest number and add it to the number line."
- "Continue this process until you have placed all of the numbers on the number line from least to the greatest."



Name

Ordering and Comparing Positive and Negative Numbers ML 7.04

Modeled Review

Name: Adam

Order the numbers from *least* to *greatest*.

−3	3.1	−2.5	2.5	0.25

−3 −2 −1 0 1 2 3

−3 (least), −2.5, 0.25, 2.5, 3.1 (greatest)

Guided Practice

Order each set of numbers from least to *greatest*. Use the number lines if they are helpful.

1.

0	-3.5	3	-1

-4 -3 -2 -1 0 1 2 3 4

-3.5 (least), -1, 0, 3 (greatest)

2.

4.5	-2	-4	0.25

-5 -4 -3 -2 -1 0 1 2 3 4 5

-4 (least), -2, $\frac{1}{4}$, 4.5 (greatest)

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Vocabulary

If needed, share the meaning of the terms with students.

sign: The sign of a number (other than '0') is either positive or negative.

opposite: Two numbers that are the same distance from 0 and on different sides of the number line. For example, 4 and -4 are opposites.

rational number: All positive and negative numbers that can be written as fractions, including whole numbers, are called rational numbers.

Ordering and Comparing Positive and Negative Numbers

Guided Practice

A Model crossing off the numbers in the list as they are ordered to ensure that all numbers have been included and none are overlooked.

ML/EL Guide processing by having students identify the decimal represented by each fraction before ordering the numbers from least to greatest.

Key Takeaway:
Say, "When ordering rational numbers, list them in the direction they appear on the number line: left to right for least to greatest."

Guided Practice

Order each set of numbers from *least* to *greatest*.

3.

0.2	-2	3.5	3	1.25
-2	0.2	1.25	3	3.5
least				greatest

4.

$\frac{1}{2}$	3	$1\frac{1}{2}$	-2	1
-2	0.5	1	$1\frac{1}{2}$	3
least				greatest

5.

2.5	2	-2.5	-0.5	-3
-3	-2.5	$-\frac{1}{2}$	2	2.5
least				greatest

6.

$-3\frac{1}{2}$	$-\frac{1}{5}$	$2\frac{1}{2}$	-5	3
-5	-3.5	$-\frac{1}{5}$	2.5	3
least				greatest

Check

Order the numbers from *least* to *greatest*.

$-\frac{1}{2}$	1	-3	4	$-4\frac{1}{2}$
-4.5	-3	$-\frac{1}{2}$	1	4
least				greatest

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Reflection

Ask:

- "What steps should you take when ordering positive and negative numbers from least to greatest?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then model drawing a number line and adding tick marks to support students with ordering the numbers.

Got it!
If students need more practice, present them with the following numbers and ask them to order these numbers from least to greatest.

-0.75	2	-5	3	-2.5

Name ______________________________

Adding and Subtracting Positive and Negative Numbers

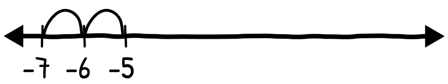
ML 7.08

Modeled Review

Name: Tristan

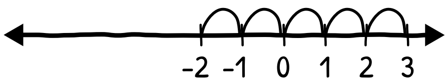
Evaluate each expression.

1. -5 + (-2) -7



I need to start at -5, and move left 2 times to represent adding -2.

2. -2 – (-5) 3

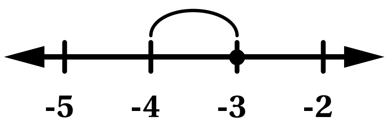


I need to start at -2, and move right 5 times.

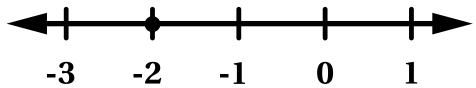
Guided Practice

Evaluate each expression using a number line.

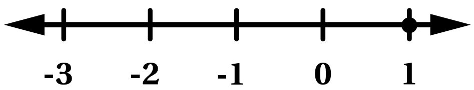
1. -3 – 2 ____



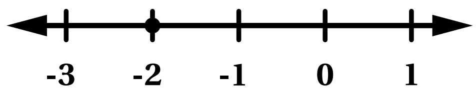
2. -2 + 3 ____



3. 1 – 3 ____



4. -2 – (-1) ____



Guided Practice

5. Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + 1 ____	4 – 1 ____
2 + (-3) ____	3 – (-2) ____
-1 + 5 ____	-5 – 1 ____
-3 + -3 ____	-3 – (-3) ____
-2 + 5 ____	5 – (-2) ____

Check

Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + (-2) ____	-2 – (-4) ____

ML 7.08

Adding and Subtracting Positive and Negative Numbers

Goal

Calculate sums and differences of positive and negative numbers.

Standard

MA.6.NSO.4.1

Materials

number line (optional)

Modeled Review

Point to Tristan's work and **ask**:

- "How did Tristan use a number line to add integers? Subtract?"
- "Why is subtracting negative numbers equivalent to adding positive numbers?"

Reinforce Tristan's thinking by saying, "Number lines can be used to add and subtract integers."

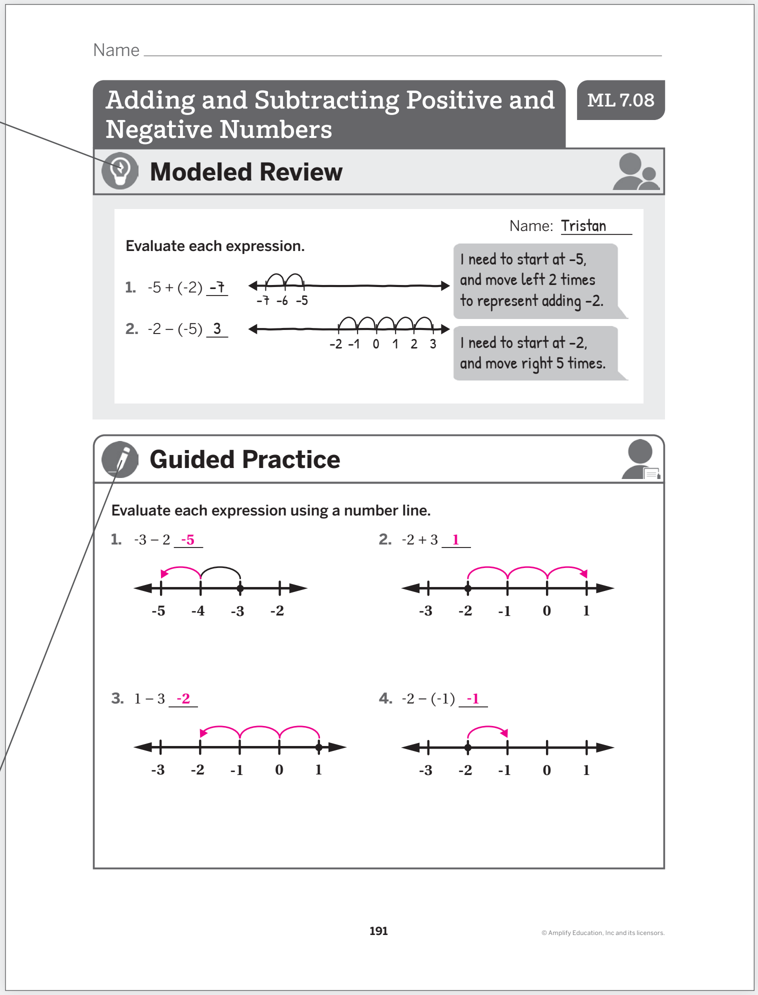
ML/EL Consider using a think aloud to reinforce that adding a negative number is equivalent to subtracting a positive number and subtracting a negative number is equivalent to adding a positive number.

Guided Practice

Focus students' attention on adding and subtracting integers.

To scaffold their thinking, **ask**:

- "What operation is in the expression?"
- "How can you use the number line to add or subtract integers?"


Name

Adding and Subtracting Positive and Negative Numbers ML 7.08

Modeled Review

Name: Tristan

Evaluate each expression.

1. -5 + (-2) -7

2. -2 − (-5) 3

Guided Practice

Evaluate each expression using a number line.

1. -3 − 2 -5

2. -2 + 3 1

3. 1 − 3 -2

4. -2 − (-1) -1

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Vocabulary

If needed, share the meaning of the term with students.

integers: Whole numbers, their opposites, and 0.

Adding and Subtracting Positive and Negative Numbers

Guided Practice

A Provide students with a number line to use as they evaluate each expression.

Key Takeaway:
Say, "Subtracting a value is equivalent to adding its opposite."

Guided Practice

5. Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + 1 _-3_	4 – 1 _3_
2 + (-3) _-1_	3 – (-2) _5_
-1 + 5 _4_	-5 – 1 _-6_
-3 + -3 _-6_	-3 – (-3) _0_
-2 + 5 _3_	5 – (-2) _7_

Check

Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + (-2) _-6_	-2 – (-4) _2_

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Reflection

Ask:
- "How does using a number line help add and subtract integers?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using the concepts of money to help make adding and subtracting integers less abstract. Positive numbers can represent having or receiving money. Negative numbers can represent situations involving expenses or debts.

Got it!
If students need more practice, have them evaluate the following expressions:

- -6 + (-2)
- -2 – (-6)
- -7 + (-3)
- -3 – (-7)

Name ____________________

Solving Real-World Problems Involving Positive and Negative Numbers

ML 7.09

Modeled Review

Name: Santiago

Use the table to determine the change.

Date	Temperature in Kazan, Russia (°F)
December 20, 2019	-11
July 27, 2020	60

Positive change

What is the change in Kazan's temperature from December 20th to July 27th?

$60 - (-11) = 60 + 11 = 71$

71 degrees Fahrenheit

Since I am calculating the change, I need to find the difference between the final temperature and the initial temperature.

Guided Practice

Use the tables to determine if each temperature change in Fitzgerald, Canada is positive or negative.

1.

Date	Temperature (°F)
January 29, 2020	10
May 30, 2020	50

Positive Change or Negative Change

2.

Date	Temperature (°F)
June 24, 2019	60
February 9, 2020	-5

Positive Change or Negative Change

3.

Date	Temperature (°F)
December 19, 2019	-4
January 28, 2020	-8

Positive Change or Negative Change

4.

Date	Temperature (°F)
December 14, 2019	-2
July 8, 2020	70

Positive Change or Negative Change

Guided Practice

Use the tables to determine the change in temperature.

5.

Date	Temperature in Siberia, Russia (°F)
July 24, 2019	50
February 11, 2020	-10

What is the change in Siberia's temperature from July 24th to February 11th?

6.

Date	Temperature in Fairbanks, Alaska (°F)
January 2, 2020	-5
September 25, 2020	45

What is the change in Fairbanks's temperature from January 2nd to September 25th?

7.

Date	Temperature in Harbin, China (°F)
December 13, 2019	-8
August 20, 2020	68

What is the change in Harbin's temperature from December 13th to August 20th?

Check

Use the table to determine the temperature change.

Date	Temperature in Prospect Creek, Alaska (°F)
February 26, 2020	-7
August 9, 2020	63

What is the change in Prospect Creek's temperature from February 26th to August 9th?

ML 7.09 Solving Real-World Problems Involving Positive and Negative Numbers

Goal

Add and subtract positive and negative numbers to solve problems involving real-world situations.

Standard

MA.6.NSO.4.1

Materials

highlighter (optional), number line (optional)

Modeled Review

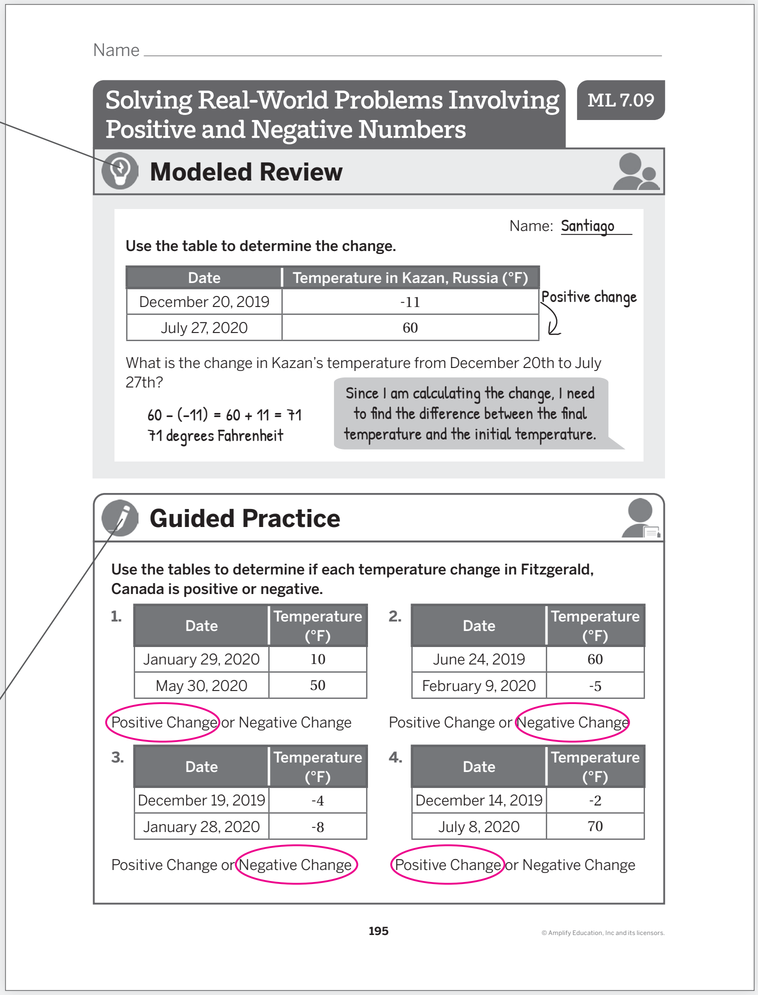
Point to Santiago's work and **ask**:
- "Why does Santiago say it is a positive change?"
- "Why does Santiago subtract the initial from the final temperature?"
- "How does knowing the table reflects a positive change help you check the calculated answer?"

Reinforce Santiago's thinking by saying, "Use your knowledge of adding and subtracting positive and negative numbers to solve real-world situations."

Guided Practice

Focus students' attention on determining if each temperature change is positive or negative.

To scaffold their thinking, **ask**:
- "What's the initial temperature?"
- "What's the final temperature?"
- "Is that change a positive (getting warmer) or negative (getting colder) change?"



Name

Solving Real-World Problems Involving Positive and Negative Numbers ML 7.09

Modeled Review

Name: Santiago

Use the table to determine the change.

Date	Temperature in Kazan, Russia (°F)
December 20, 2019	-11
July 27, 2020	60

Positive change

What is the change in Kazan's temperature from December 20th to July 27th?

60 – (–11) = 60 + 11 = 71

71 degrees Fahrenheit

Since I am calculating the change, I need to find the difference between the final temperature and the initial temperature.

Guided Practice

Use the tables to determine if each temperature change in Fitzgerald, Canada is positive or negative.

1.

Date	Temperature (°F)
January 29, 2020	10
May 30, 2020	50

Positive Change or Negative Change

2.

Date	Temperature (°F)
June 24, 2019	60
February 9, 2020	-5

Positive Change or Negative Change

3.

Date	Temperature (°F)
December 19, 2019	-4
January 28, 2020	-8

Positive Change or Negative Change

4.

Date	Temperature (°F)
December 14, 2019	-2
July 8, 2020	70

Positive Change or Negative Change

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Vocabulary

If needed, share the meaning of the term with students.

integers: Whole numbers and their opposites.

Solving Real-World Problems Involving Positive and Negative Numbers

Guided Practice

(A) Model color coding positive and negative numbers with a highlighter for students to understand whether it will be a positive or negative change.

ML/EL Consider modeling, or inviting a student to model, annotating the problem to make sense of it, such as labeling the first temperature as the initial and second as the final.

Key Takeaway:
Say, "Adding and subtracting positive and negative values help to solve problems involving real-world situations."

Guided Practice

Use the tables to determine the change in temperature.

5.

Date	Temperature in Siberia, Russia (°F)
July 24, 2019	50
February 11, 2020	-10

What is the change in Siberia's temperature from July 24th to February 11th?
-60 degrees Fahrenheit

6.

Date	Temperature in Fairbanks, Alaska (°F)
January 2, 2020	-5
September 25, 2020	45

What is the change in Fairbanks's temperature from January 2nd to September 25th?
50 degrees Fahrenheit

7.

Date	Temperature in Harbin, China (°F)
December 13, 2019	-8
August 20, 2020	68

What is the change in Harbin's temperature from December 13th to August 20th?
76 degrees Fahrenheit

Check

Use the table to determine the temperature change.

Date	Temperature in Prospect Creek, Alaska (°F)
February 26, 2020	-7
August 9, 2020	63

What is the change in Prospect Creek's temperature from February 26th to August 9th? **70 degrees Fahrenheit**

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Reflection

Ask:

- "What new questions do you have about adding and subtracting positive and negative numbers?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider orienting a number line vertically so they can understand whether a positive or negative change is occurring in the context of temperature, and then check it against their calculated solution.

Got it!
If students need more practice, present them with the following table and ask them to determine the change in Fairbanks' weather from February 29th to March 28th.

Month	Temperature in Fairbanks, Alaska (°F)
February 29, 2020	-33
March 28, 2020	-4

Name ______________________________

Solving Inequalities

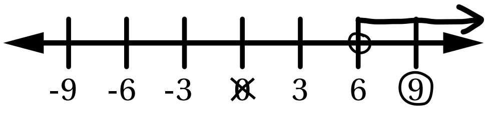
ML 8.07

Modeled Review

Name: Priya

Solve and graph the solution to $x - 1 > 5$.

x > 6



$x - 1 = 5$
$+1 \quad +1$
$x = 6$

$x < 6$	$x > 6$
$0 - 1 > 5$	$(9) - 1 > 5$
$-1 > 5$	$8 > 5$
False	True

Guided Practice

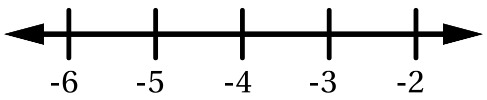
1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$3x = 9$ $x =$ ____
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	**Less than or equal to** 3(____) ≥ 9 ____ ≥ 9 __________ **Greater than or equal to** 3(____) ≥ 9 ____ ≥ 9 __________
Move 3: Graph the solution.	x ____ 3 

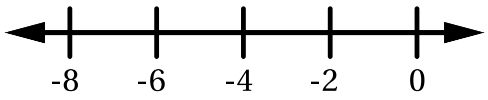
Guided Practice

Solve and graph the solution to each of the inequalities.

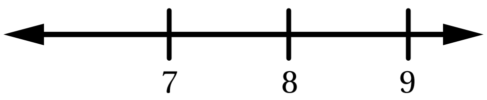
2. $2x \leq -8$ __________



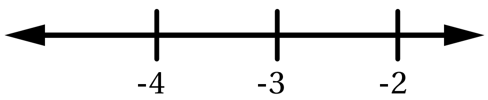
3. $x - 1 > -7$ __________



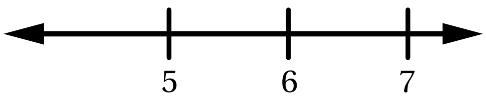
4. $x + 11 \geq 18$ __________



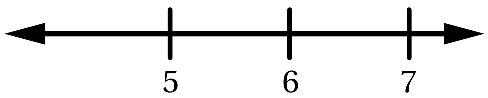
5. $-6x > 18$ __________



6. $x - 12 \geq -7$ __________



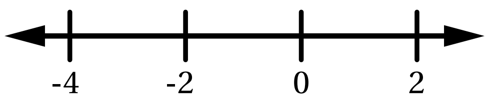
7. $-5x > -30$ __________



Check

Solve and graph the solution to $-4x < 8$**.**

____



ML 8.07 Solving Inequalities

Goal

Solve linear equations that involve positive and negative numbers.

Standard

MA.7.AR.2.1

Modeled Review

Point to Avery's work and **ask**:

- "How did Avery determine the boundary point?"
- "Why did Avery substitute 0 and 9 into the inequality?"
- "How does testing numbers help to determine which inequality symbol to use?"

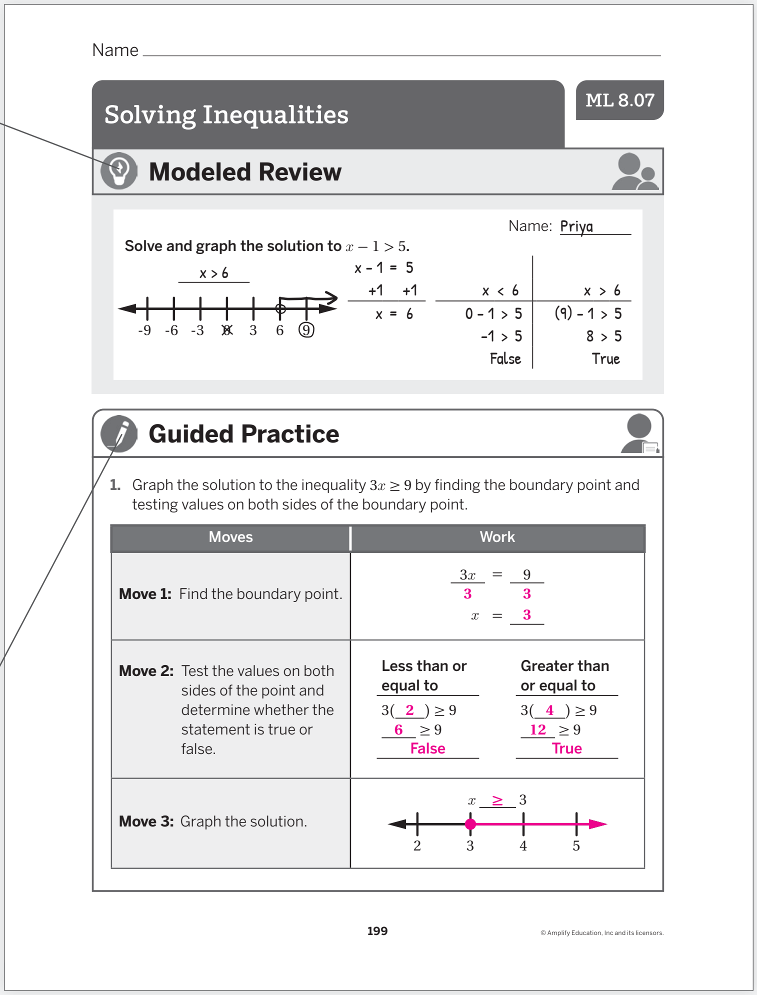
Reinforce Avery's thinking by saying, "After finding the boundary point, test values to determine which inequality symbol makes the solution statement true."

Guided Practice

Focus students' attention on finding the boundary point and testing values on both sides of the point.

To scaffold their thinking, **say**:

- "Determine the boundary point."
- "Choose numbers to the left and right of the boundary point and substitute them into the inequality."
- "Decide which number satisfies the inequality. Based on this, determine the correct inequality sign to use."



Name

Solving Inequalities ML 8.07

Modeled Review

Name: Priya

Solve and graph the solution to $x - 1 > 5$.

x > 6

-9 -6 -3 0 3 6 9

$x - 1 = 5$

$+1 \quad +1$

$x = 6$

x < 6	x > 6
0 - 1 > 5	(9) - 1 > 5
-1 > 5	8 > 5
False	True

Guided Practice

1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$\frac{3x}{3} = \frac{9}{3}$; $x = 3$
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	Less than or equal to: $3(2) \geq 9$; $6 \geq 9$; False. Greater than or equal to: $3(4) \geq 9$; $12 \geq 9$; True
Move 3: Graph the solution.	$x \geq 3$; 2 3 4 5

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Vocabulary

If needed, share the meaning of the terms with students.

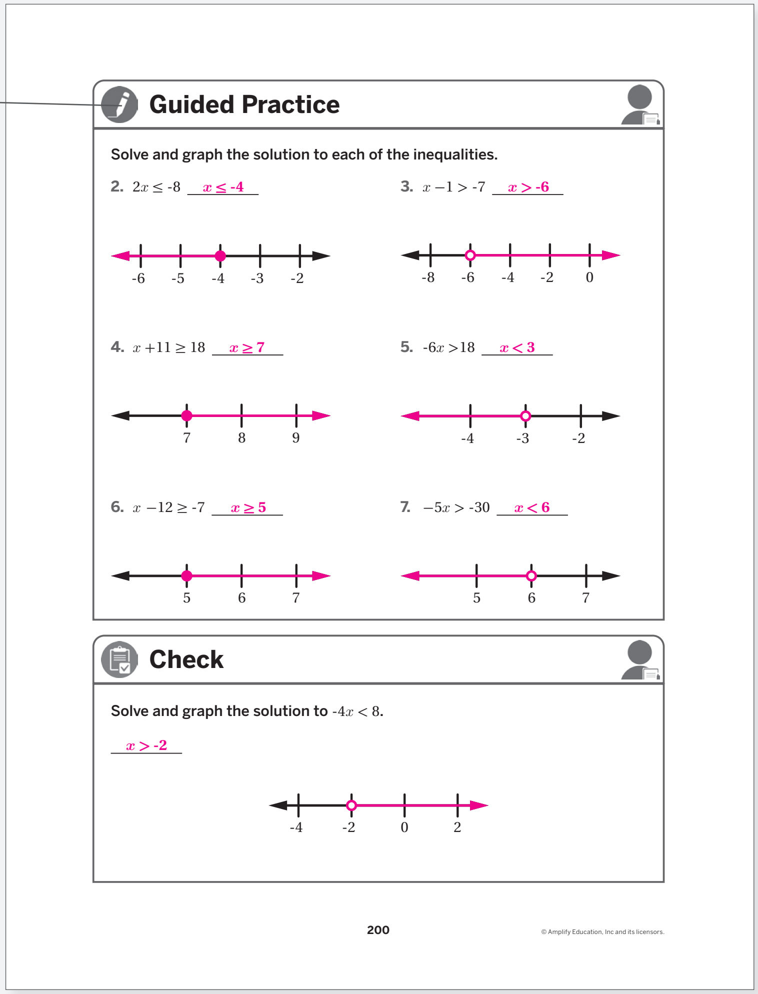
inequality: A comparison statement that uses the symbols < or >. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

solution to an inequality: All of the values of a variable that make that inequality true.

Guided Practice

A Emphasize that inequality signs that include 'equal to' should be represented with a shaded circle on the number line.

Key Takeaway:
Say, "Solving inequalities is similar to solving equations. The solutions to an inequality can be determined by first solving an equation, then testing whether the values on the left or right side of the boundary point make the inequality true."



Guided Practice

Solve and graph the solution to each of the inequalities.

2. $2x \le -8$ $x \le -4$

-6 -5 -4 -3 -2

3. $x - 1 > -7$ $x > -6$

-8 -6 -4 -2 0

4. $x + 11 \ge 18$ $x \ge 7$

7 8 9

5. $-6x > 18$ $x < 3$

-4 -3 -2

6. $x - 12 \ge -7$ $x \ge 5$

5 6 7

7. $-5x > -30$ $x < 6$

5 6 7

Check

Solve and graph the solution to $-4x < 8$.

$x > -2$

-4 -2 0 2

200

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Reflection

Ask:

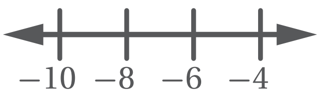
- "What steps can you take to graph the solutions to an inequality?"
- "How are solutions to an inequality different from an equation?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then model graphing the solution by marking off the value on the number line that does not make statement true and circling the one that does. Invite them to use that information to determine how they should draw the arrow.

Got it!
If students need more practice, present students with the following problem and ask them to graph the solution:

$-7x \le 42$



Name ______________________________

Creating Line Plots

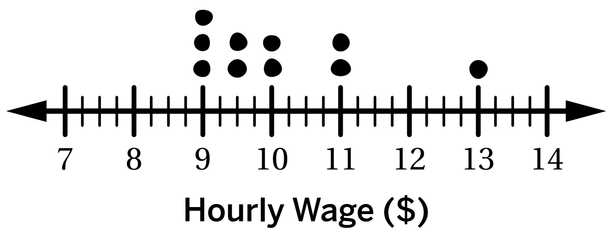
ML 9.03

Modeled Review

Name: Clare

10 fast food companies in Nebraska reported their hourly wages for new employees. Complete the line to display this data.

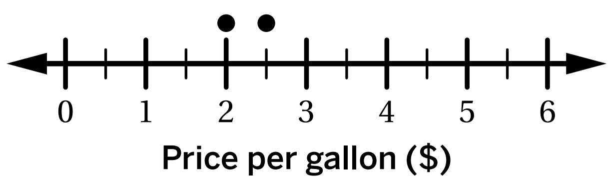
Hourly Wage				
$11.00	$9.00	$9.00	$10.00	$9.50
$10.00	$9.00	$11.00	$9.50	$13.00



Guided Practice

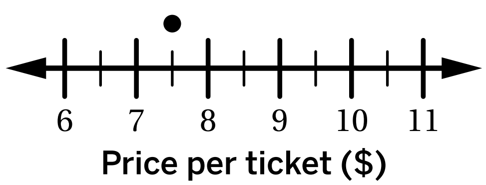
1. Here are the prices per gallon of gasoline at 6 gas stations in Georgia. Complete the line plotthat shows this data.

Price per gallon		
$2.00	$3.00	$2.50
$2.00	$3.50	$4.00



2. Here are the ticket prices for movies at 6 theaters in Texas. Complete the line plot to display this data.

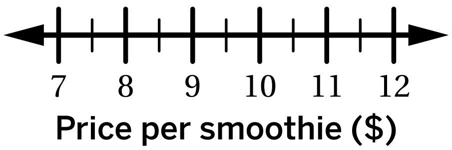
Price per ticket		
$10.00	$8.00	$7.50
$8.50	$7.00	$8.00



Guided Practice

3. 8 smoothie shops in Arizona reported the price of their large smoothie. Complete the line plot to display this data.

Price per smoothie			
$10.00	$9.50	$9.00	$10.50
$11.00	$10.00	$9.50	$11.00

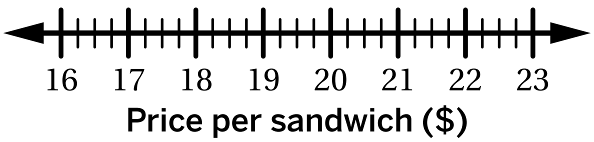


4. 10 parking lots in Chicago reported their daily parking fees. Complete the line plot to display this data.

Daily parking fee				
$12.50	$14.00	$18.00	$18.00	$12.00
$18.00	$15.00	$12.00	$15.50	$15.00

5. 10 deli shops in New York reported the price of a large sandwich. Complete the line plot to display this data.

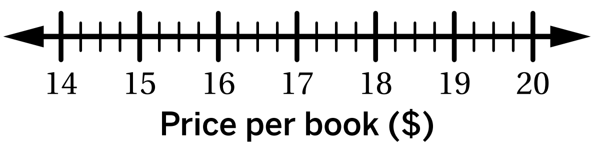
Price per sandwich				
$18.00	$21.00	$21.00	$23.00	$19.50
$20.50	$19.00	$20.00	$22.00	$20.00



Check

10 bookstores in Illinois reported the price of their best-selling book. Complete the line plot to display this data.

Price per book				
$15.00	$20.00	$16.00	$15.00	$16.00
$18.00	$18.50	$15.00	$17.00	$18.50



ML 9.03 Creating Line Plots

Goal	Standard
Create a line plot to visualize a data set.	MA.5.DP. 1.1

Modeled Review

Point to Clare's work and **ask**:

- "How did Clare know to place three dots above the number 9?"
- "Why is it important to stack the dots vertically on a line plot?"
- "How can Clare check that her line plot matches the data table?"

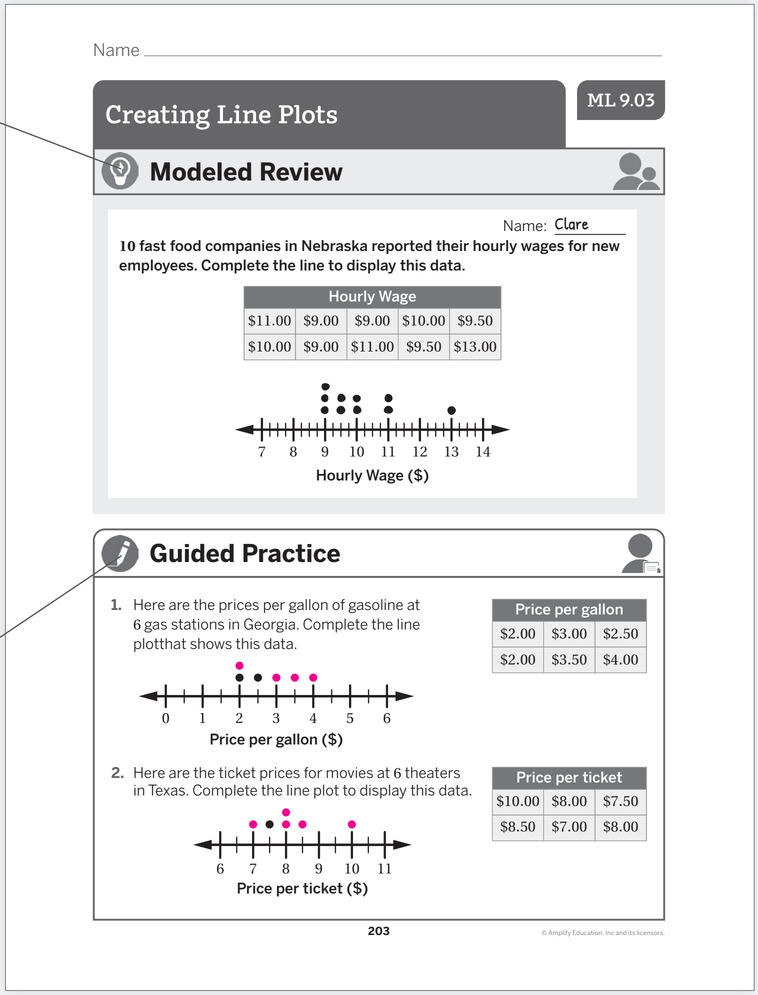
Reinforce the goal by saying, "A data set can be visualized by creating a line plot."

Guided Practice

Focus students' attention on creating a line plot that represents the data set provided.

To scaffold their thinking, **say**:

- "First, place a dot above the corresponding number on the *x*-axis for each data point."
- "Then, if a value repeats, stack the dots vertically above that number."
- "Last, check your line plot to make sure all values are plotted correctly and the number of dots matches your data."



Name

Creating Line Plots

ML 9.03

Modeled Review

Name: Clare

10 fast food companies in Nebraska reported their hourly wages for new employees. Complete the line to display this data.

Hourly Wage				
$11.00	$9.00	$9.00	$10.00	$9.50
$10.00	$9.00	$11.00	$9.50	$13.00

Guided Practice

1. Here are the prices per gallon of gasoline at 6 gas stations in Georgia. Complete the line plotthat shows this data.

Price per gallon		
$2.00	$3.00	$2.50
$2.00	$3.50	$4.00

2. Here are the ticket prices for movies at 6 theaters in Texas. Complete the line plot to display this data.

Price per ticket		
$10.00	$8.00	$7.50
$8.50	$7.00	$8.00

203

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Vocabulary

If needed, share the meaning of the term with students.

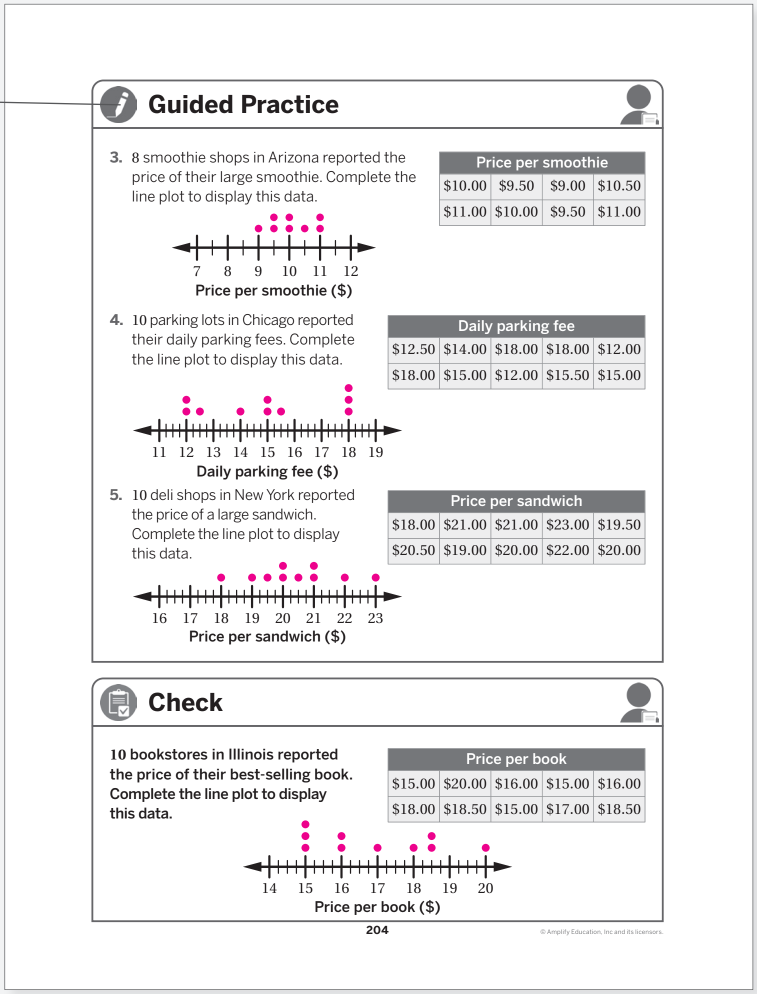
line plot: A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other.

Guided Practice

A Model crossing off values in the table so that students don't miss or double count any data.

ML/EL Use a think aloud as you model plotting the data points on the line plot.

Key Takeaway:
Say, "Line plots can be used to display and visualize numerical data."



Guided Practice

3. 8 smoothie shops in Arizona reported the price of their large smoothie. Complete the line plot to display this data.

Price per smoothie			
$10.00	$9.50	$9.00	$10.50
$11.00	$10.00	$9.50	$11.00

4. 10 parking lots in Chicago reported their daily parking fees. Complete the line plot to display this data.

Daily parking fee				
$12.50	$14.00	$18.00	$18.00	$12.00
$18.00	$15.00	$12.00	$15.50	$15.00

5. 10 deli shops in New York reported the price of a large sandwich. Complete the line plot to display this data.

Price per sandwich				
$18.00	$21.00	$21.00	$23.00	$19.50
$20.50	$19.00	$20.00	$22.00	$20.00

Check

10 bookstores in Illinois reported the price of their best-selling book. Complete the line plot to display this data.

Price per book				
$15.00	$20.00	$16.00	$15.00	$16.00
$18.00	$18.50	$15.00	$17.00	$18.50

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Reflection

Ask:

- "What do you think is important to remember when creating a line plot?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling how to label fractional values on the axis to help them with plotting points.

Got it!
If students need more practice, ask them to add the following sandwich prices to the line plot in Problem 5.

Price per sandwich				
$22.50	$21.00	$21.50	$16.50	$17.00

Extensions

Unit 1

Extensions

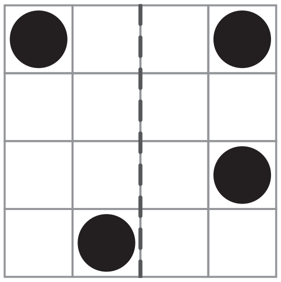
Rigid Transformations

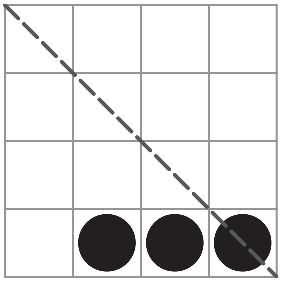
Name: .... Date: .... Period: ....

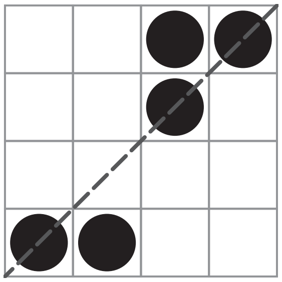
Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here are three figures. Move one dot so all the dots reflect across the dashed mirror line. How many different solutions can you come up with?

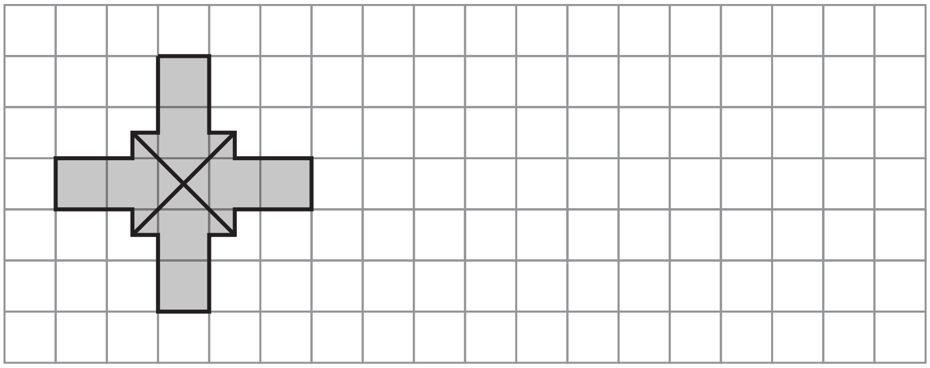
a


b


c


2

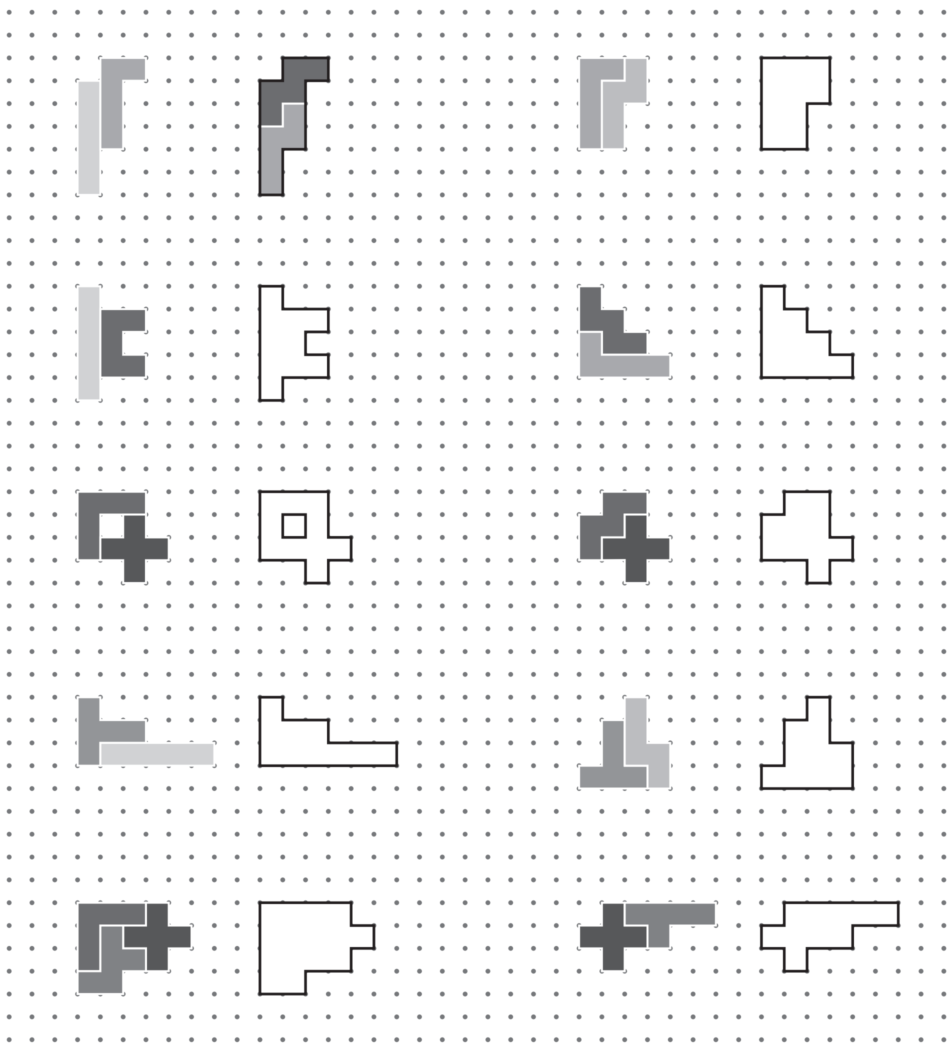
Here are four identical arrows. Rotate, translate, and reflect the arrows to create a design that looks like there are five arrows.



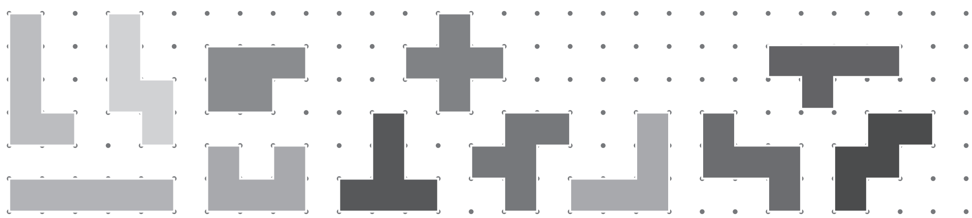
Rigid Transformations (continued)

Name: .... Date: .... Period: ....

3 A design formed from pentominoes is shown on the left. Create an identical design using pentominoes. The first pair is completed for you.



Set of Pentominoes



Rigid Transformations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problem 3 with all students.

Problem 1

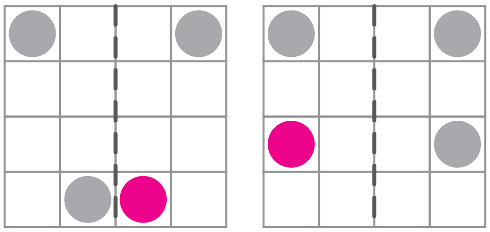
Students will extend their understanding of describing and performing reflections on a grid.

Provide students with the following hint if additional scaffolding is needed.

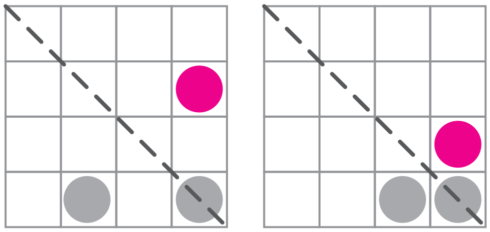
- **Hint:** Can you find any dot that already has a symmetrical pair? How can you make the remaining dots symmetrical?

Responses vary. Sample responses shown.

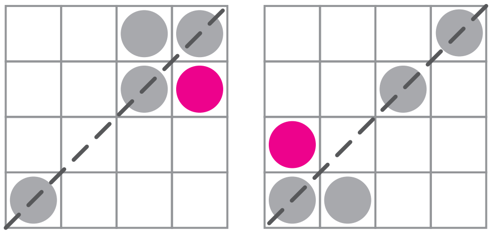
a.



b.



c.



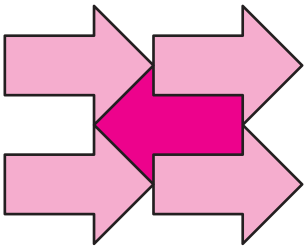
Problem 2

Students will extend their understanding of performing translations, reflections, and rotations off a grid.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Think about using the space between the arrows to create a fifth arrow.

Responses vary. Sample response shown.



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Rigid Transformations (continued)

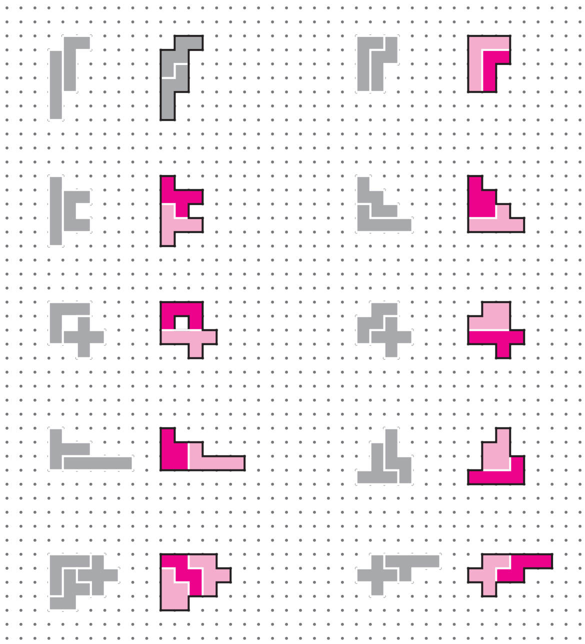
Problem 3

Students will extend their understanding of performing translations, reflections, and rotations on a grid by using pentominoes.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** One strategy is to try to fit each pentomino in the outline, one row at a time, from left to right. Can you find a pentomino that fits in the outline by reflecting or rotating?

Responses vary. Sample responses:

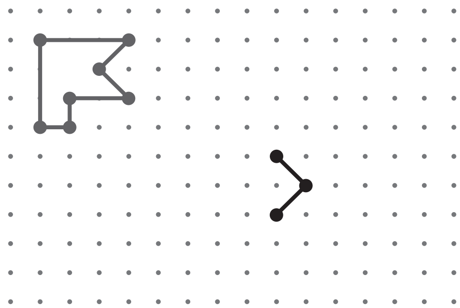


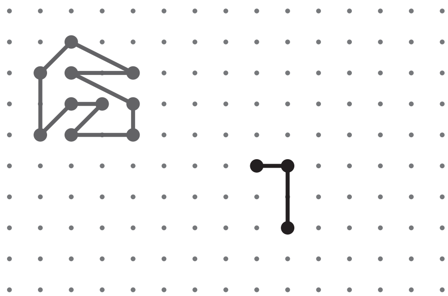
Congruence

Name: .... Date: .... Period: ....

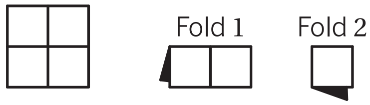
Student Choice Start with any problem. Remember to show or explain your thinking.

1 Here are two figures. A portion of a congruent figure is shown. Add lines to each drawing to finish the congruent figures.

a 

b 

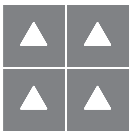
2 A square piece of paper is folded in half and half again, as shown.

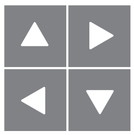


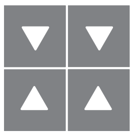
Two different-shaped figures are cut through the folded paper.

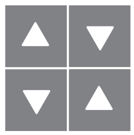
Circle the pattern that results from each folded cut.

a 

A. 

B. 

C. 

D. 

b 

A. 

B. 

C. 

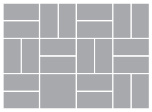
D. 

Congruence (continued)

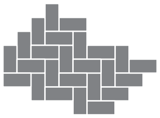
Name: ______ Date: ______ Period: ______

3 A *tessellation* is a pattern that tiles a plane without any gaps or overlaps. Here are some examples.

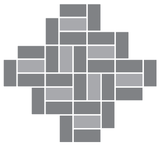
Design A



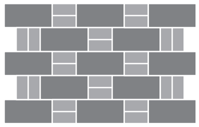
Design B



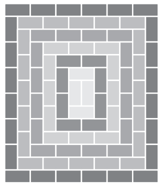
Design C



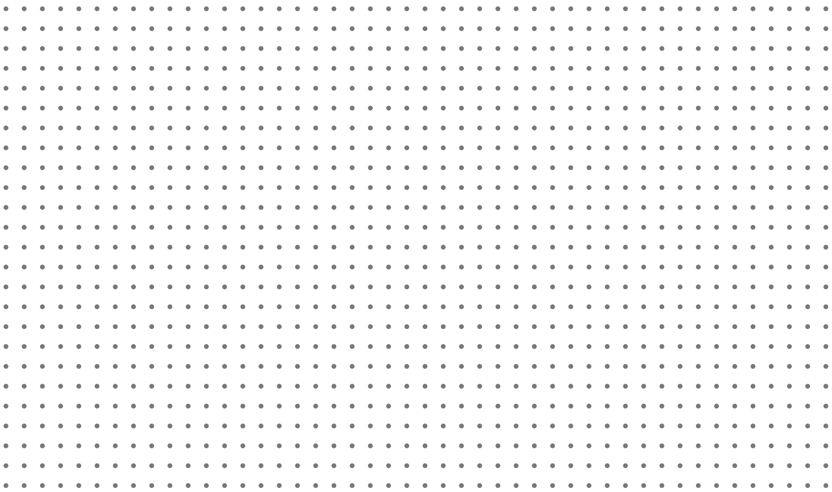
Design D



Design E



a Create a tessellation and identify the transformations you use to create your design.



b What do you notice about tessellations and transformations?

Congruence

Assign problems to students who want to extend their thinking.

Problems 1, 2, and 3 can be solved in any order. If time allows, consider sharing Problems 1–3 with all students.

Materials
- Scrap paper **(Problem 2)**

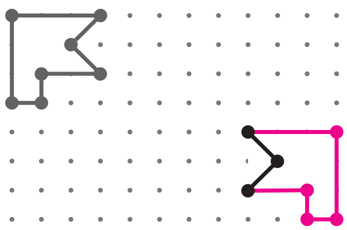
Problem 1

Students will extend their understanding of congruent shapes by drawing a congruent figure using rigid transformations.

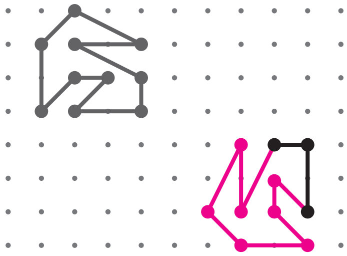
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Where does the given portion of the figure appear in the original figure? Shade or annotate it.
- **Hint 2:** How can you fold the paper to check if your figure is congruent to the original figure?

a.



b.



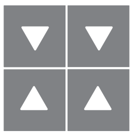
Problem 2

Students will extend their understanding of congruent shapes by exploring the effects of folding a paper.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can you fold, then cut a piece of paper to check your answers?

a. C



b. C

Continued next page ...

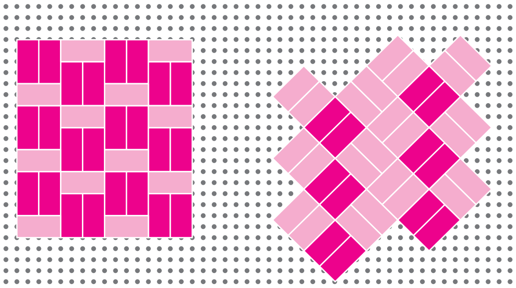
Problem 3

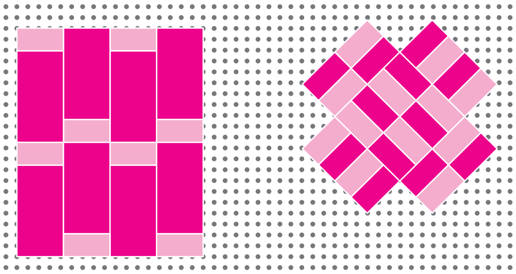
Students will extend their understanding of transformations by designing a tessellation.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What shape(s) are in each design? What transformations are used in each design to create the tessellation?

a. *Responses vary.* Sample tessellations shown.





b. *Responses vary.* I can use a rigid transformation or dilation to create a tessellation. As long as the sum of the angles is 360 degrees at any chosen point on the plane, without any gaps and overlaps, there are infinitely many different styles of tessellations I can design.

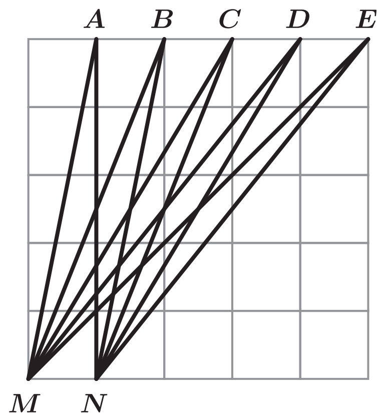
Angle Relationships

Name: .... Date: .... Period: ....

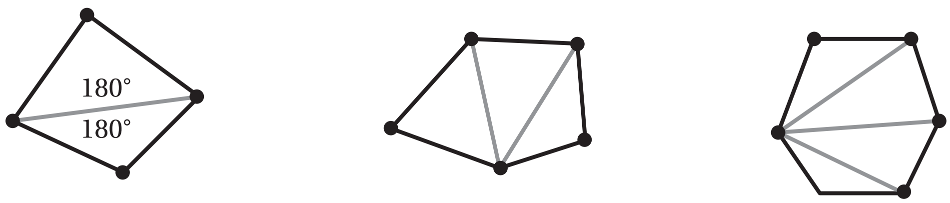
Student Choice Start with any problem. Remember to show or explain your thinking.

1 Use the figure to determine the sum of these five angles:

$m\angle MAN + m\angle MBN + m\angle MCN + m\angle MDN + m\angle MEN$



2 You can determine the sum of the interior angles of a polygon by dividing the polygon into triangles. Here are three examples for a polygon with 4, 5, and 6 sides.



a What is the sum of the interior angles of a 12-gon?

b What is the sum of interior angles of a n-gon?

Angle Relationships (continued)

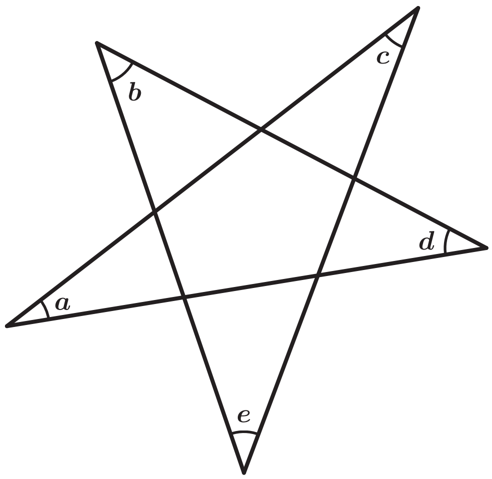
Name: .... Date: .... Period: ....

3 Start with a rectangular piece of paper. Fold one corner at any angle, and then fold the other corner so that the edges of the paper meet, as shown.

a Unfold the paper to determine the measure of the angle at the bottom.

b Explain why the bottom angle always has the same measure.

4 Draw and cut out a pentagonal star on a piece of paper.



a Cut out each vertex angle to align next to each other to determine the sum.

b Do you think no matter how a pentagonal star is drawn, and no matter how it is distorted, its five angles will always add up to the same number?

Angle Relationships

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problem 1. If time allows, consider sharing Problems 2 and 3 with all students.

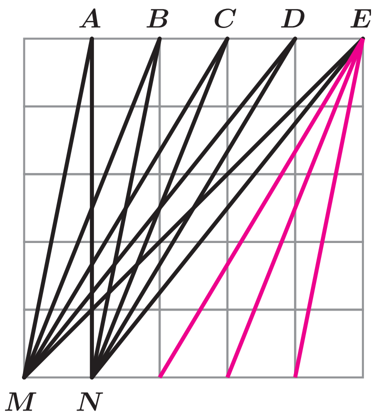
Problem 1

Students will extend their understanding of the interior angles of triangles by determining the sum of several angles in a coordinate plane.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Which angle measures can you determine in the diagram? Do you see any triangles with $90°$ angles? $45°$ angles?

$45°$. *Explanations vary.* I translated each triangle beneath $\angle MEN$ and noticed that combined, the angles form a triangle with two $45°$ angles and one $90°$ angle.



Problem 2

Students will extend their understanding of angle relationships by determining the sum of interior angles of different polygons.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What patterns do you notice? Can you create a table to organize your calculations?

a. $1800°$

b. $180 \cdot (n - 2)°$

Explanations vary.

Polygon	The number of triangles	The sum of interior angles
Triangle	1	$180°$
Quadrilateral	2	$180 \cdot 2 = 360°$
Pentagon	3	$180 \cdot 3 = 540°$
Hexagon	4	$180 \cdot 4 = 720°$
Heptagon	5	$180 \cdot 5 = 900°$
Octagon	6	$180 \cdot 6 = 1080°$
12-gon	10	$180 \cdot 10 = 1800°$
n-gon	$n - 2$	$180 \cdot (n - 2)°$

Continued next page ...

Angle Relationships (continued)

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problems 4 to students who have solved Problem 3. If time allows, consider sharing Problems 1–4 with all students.

Problem 3

Students will extend their understanding of complementary and supplementary angles by examining adjacent angle measures along a line.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Tag the angles with the same measurements.

a. 90°. *Explanations vary*.

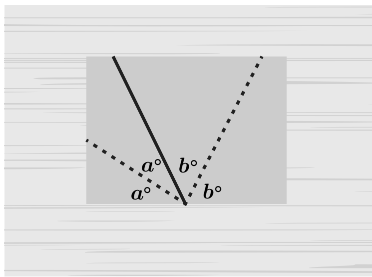
$a + a + b + b = 180$

$2a + 2b = 180$

$2(a + b) = 2 \cdot 90$

$a + b = 90$

b. *Explanations vary*. Because they were made by folding, there are two sets of angles with equal measures, $a°$ and $b°$. Because these four angles are adjacent and lie along a line, they are supplementary.



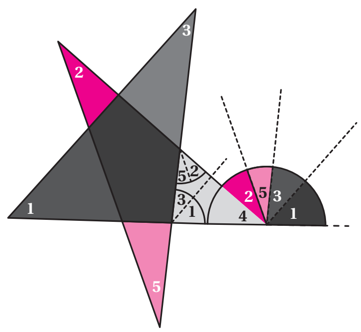
Problem 4

Students will extend their understanding of supplementary angles and vertical angles by examining corner angles of star shapes.

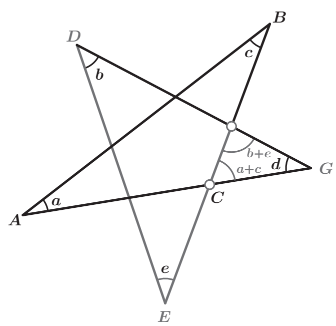
Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part b, try drawing and cutting multiple stars.

a. 180° *Explanations vary*. I used my pen and placed it along the horizontal line pointing left. I slid the pen to the end of the line and turned it along the angle and repeated. At the end I came back to the line I started from, with my pen pointing in the opposite direction, showing that it rotated 180°.



b. No matter how the star is drawn or how distorted it is, its five angles will always add up to 180°. *Explanations vary*.
For the triangle ABC, the sum of angles a and c is equal to the exterior angle of C. And for the triangle DEF, the sum of angles b and e is equal to the exterior angle of F. So in the triangle CFG, the sum of angles $a + c + b + d + e$ will always be 180°



Unit 2

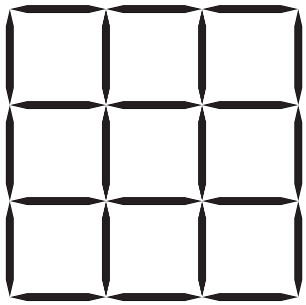
Extensions

Scaled Copies

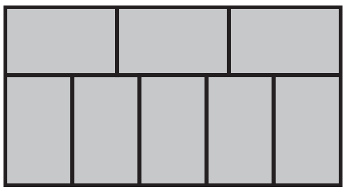
Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Remove eight toothpicks from the given design to create two squares.



2 Eight identical rectangles form a larger rectangle, as shown in the figure. Suppose you know that the length and width of the smaller rectangles are whole numbers, and the area of the larger rectangle is 480 square feet. Determine the perimeter of the larger rectangle.



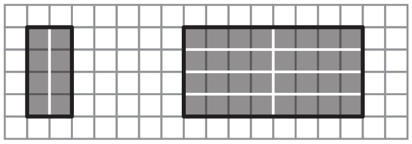
Scaled Copies (continued)

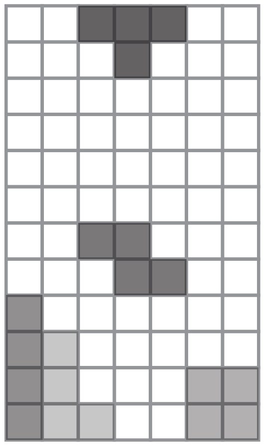
Name: .... Date: .... Period: ....

3 Tetrominoes are shapes made up of four squares. These are the tiles of the famous game Tetris.

All tetrominoes are examples of replicating tiles (*rep-tiles*), shapes that can tile an enlargement of themselves.

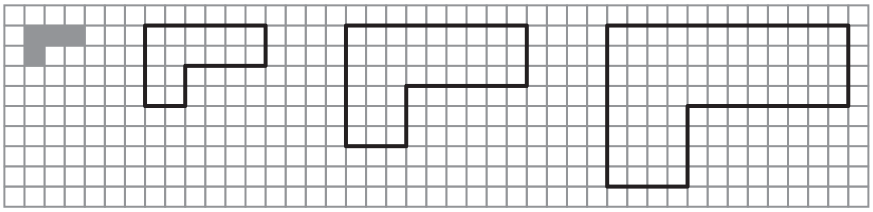
For example, the I tetromino can tile differently scaled copies of itself.





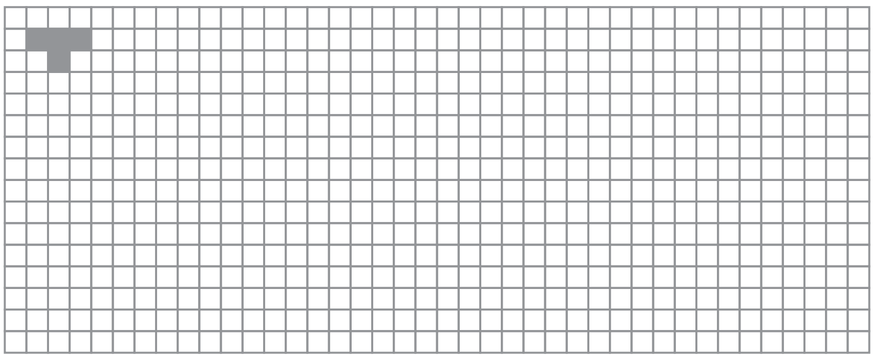
a Use the L shape tetromino to tile each of these scaled copies of itself and determine the scale factors.

$\times 1$ $\times$ ______ $\times$ ______ $\times$ ______



b Do you think any enlargement of the L tetromino can be tiled using the L tetromino?

c Is it possible to use the T tetromino to fill any scaled copies of itself?



Scaled Copies

Assign problems to students who want to extend their thinking.

Problems 1, 2, and 3 can be solved in any order. If time allows, consider sharing Problem 3 with all students.

Materials
- toothpicks (optional) **(Problem 1)**
- graph paper (optional) **(Problem 3)**

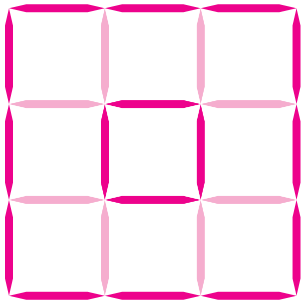
Problem 1

Students will extend their understanding of objects that are scaled copies of one another by solving the toothpick puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** You can use toothpicks to help you visualize the puzzle better.

Responses vary.



Problem 2

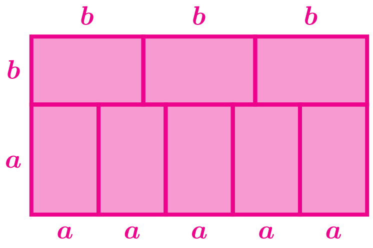
Students will extend their understanding of identical shapes, ratios, and both perimeter and area calculations by solving this problem.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** If you were to redraw this shape on grid paper, how would you arrange the length and width of the small rectangles so that three times the length would be the same as five times the width?

92 feet. *Explanations vary.*

I labeled the width of the small rectangle a and the length b. The top and bottom of the rectangle are the same length, so I know that $3b = 5a$.



8 rectangles fit in the area of the larger rectangle, so the area of each small rectangle is $480 \div 8 = 60$ sq. feet. I checked all the pairs of numbers with a product of 60, and the only pair that makes $3b = 5a$ is $a = 6$ and $b = 10$. So, the perimeter of the larger rectangle is $2 \cdot 5 \cdot 6 + 2 \cdot (10 + 6) = 92$ feet.

Continued next page ...

Scaled Copies (continued)

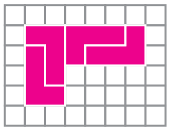
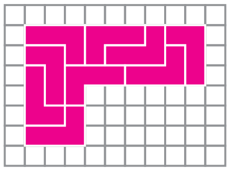
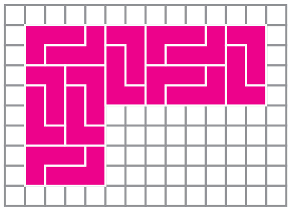
Problem 3

Students will extend their understanding of how scaling affects lengths, angles, and areas in scaled copies by working with tetrominoes.

Provide students with the following hint if additional scaffolding is needed.

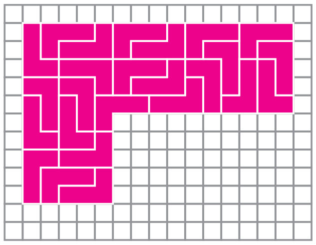
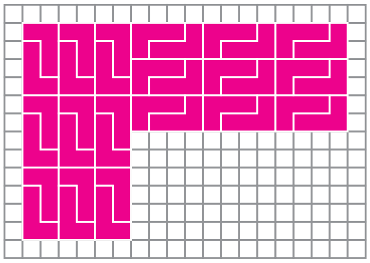
- **Hint:** Which polygons can always create scaled copies of themselves? How can you use the L tetromino to create one of those polygons?

a. *Responses vary.*

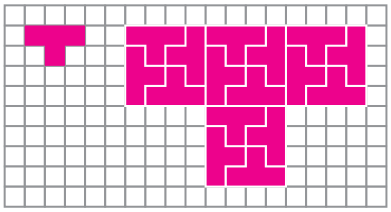
×2	×3	×4
		

The number of tetrominoes needed in the enlargements is always a square of the scale factor.

b. Any integer-sized enlargement of the L tetromino can be tiled using small copies of the L tetromino. For example:

×5	×6
	

c. Some, but not all, enlargements of the T tetromino can be rep-tiled. For example, I cannot create a T tetromino rep-tile that is 2 times larger, but I can create the rep-tile that is 4 times larger.

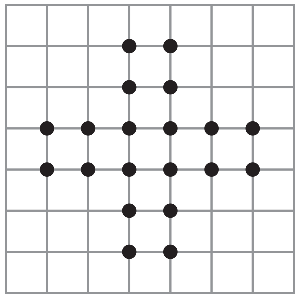


Dilations

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 20 points are shown on the square grid.



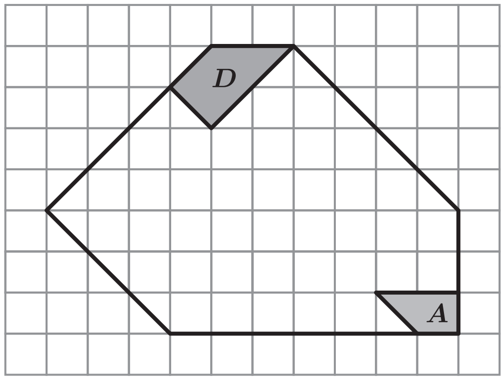
a How many squares can you form by connecting any 4 points on this shape?

b How many *different-sized* squares can you form by connecting any 4 points on this shape?

2 Here is a hexagon with trapezoids A and D labeled.

Divide the hexagon into five trapezoids labeled A, B, C, D, and E so that the scale factor between

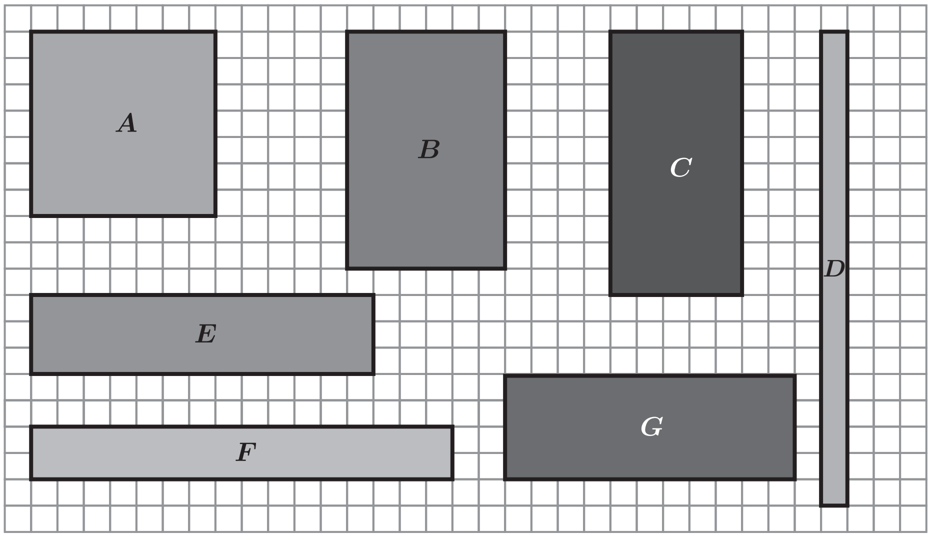
- A to B is $1:2$
- D to C is $1:2$
- D to E is $1:3$



Dilations (continued)

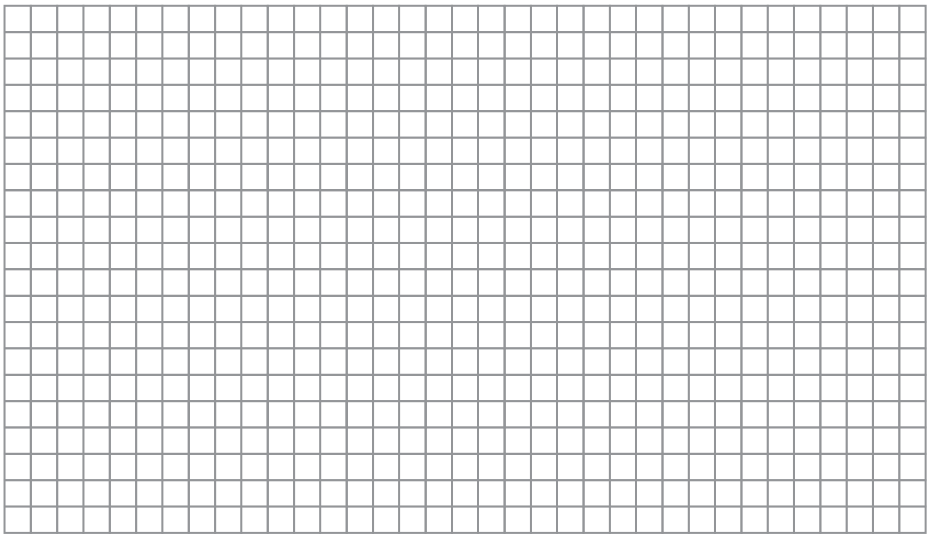
Name: ________ Date: ________ Period: ________

3 Here are several rectangles in different sizes.



a Are any of these rectangles scaled copies of each other?

b Arrange rectangles A–F to form one large rectangle.



c Are any rectangles A–F scaled copies of the newly formed large rectangle?

Dilations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1 and 3 with all students.

Materials
- Extension PDF, one per student, pre-cut (optional) **(Problem 3)**

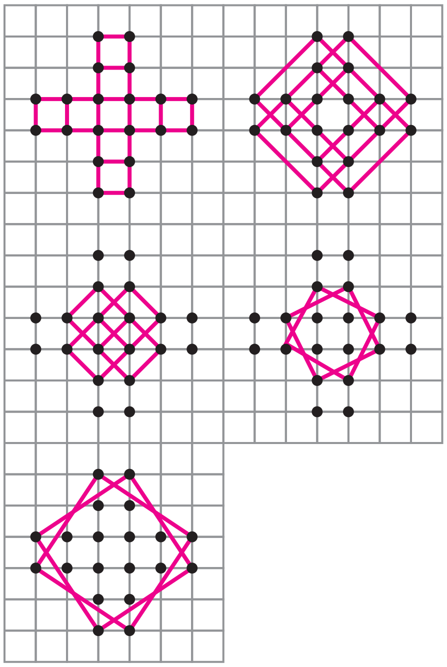
Problem 1

Students will extend their understanding of scaled copies of polygons by identifying squares hidden among the points.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Can you turn your paper to help you see the squares with different orientations?

a. 21 squares



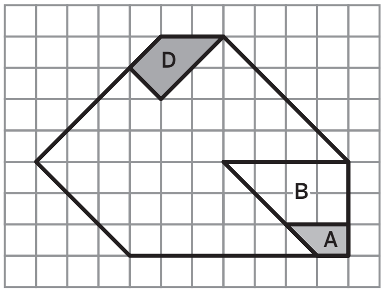
b. 5 different-sized squares

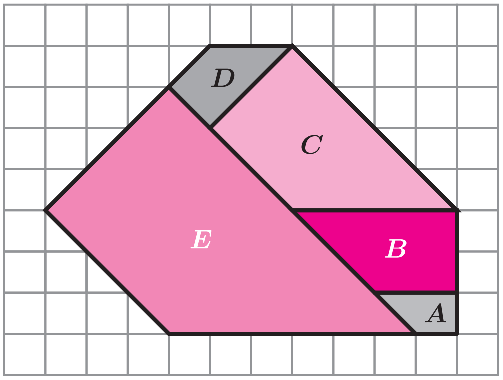
Problem 2

Students will extend their understanding of scaled copies of a trapezoid by dissecting a polygon into similar trapezoids.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Here is trapezoid B, it is adjacent to A and twice its size. How can you use the relationship between trapezoid A and B to help you draw trapezoid C?





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Dilations (continued)

Problem 3

Students will extend their understanding of scale factor by determining which rectangles are scaled copies of each other.

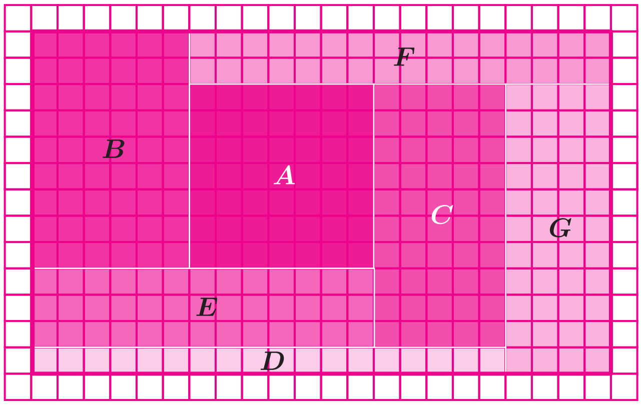
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In parts a and b, start by writing the dimensions of each rectangle.
- **Hint 2:** In part b, determine the area of the big rectangle and try to guess the dimensions based on the area.

a. No rectangle is a scaled copy of another. *Explanations vary.*

Rectangle	$\ell \times w$
A	7×7
B	6×9
C	5×10
D	1×18
E	3×13
F	2×16
G	4×11

b. *Responses vary*. I can form a 13×22 rectangle.



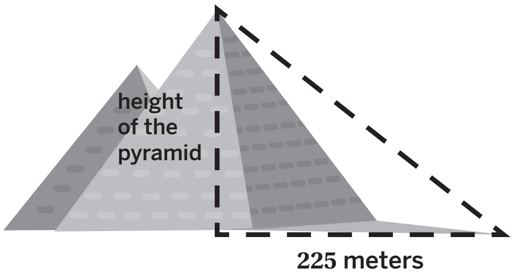
c. None of the smaller rectangles are a scaled copy of the larger rectangle formed.

Similarity

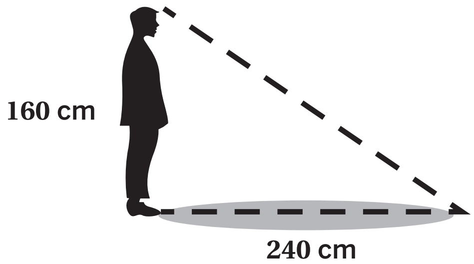
Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 According to historical sources, 2500 years ago, the Greek mathematician Thales successfully calculated the Great Pyramid's height.



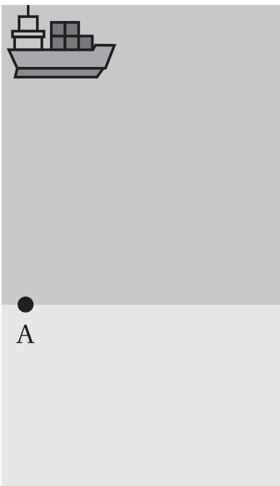
First, he measured the length of the Great Pyramid's shadow as 225 meters.



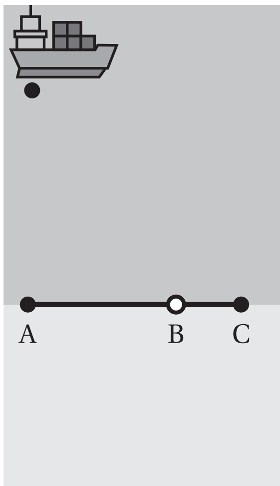
At the same time of day and in the same place, he measured the shadow of his 160-centimeter-tall friend to be 240 centimeters.

Use Thales' measurements to determine the height of the pyramid.

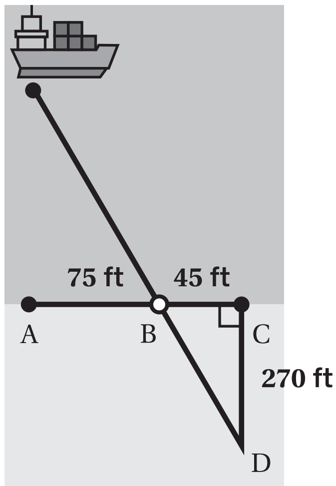
2 Thales also calculated the width of rivers and the distances of ships from the shoreline.



Here is a ship a certain distance from the shoreline, as shown.



Thales walked along the shoreline and placed sticks in the ground at points B and C.



From there, he walked directly away from the sea at a $90°$ angle until the stick he placed in the ground at point B lined up with the ship, point D.

Use Thales' measurements to determine the distance of the ship from the shoreline.

Similarity (continued)

Name: .... Date: .... Period: ....

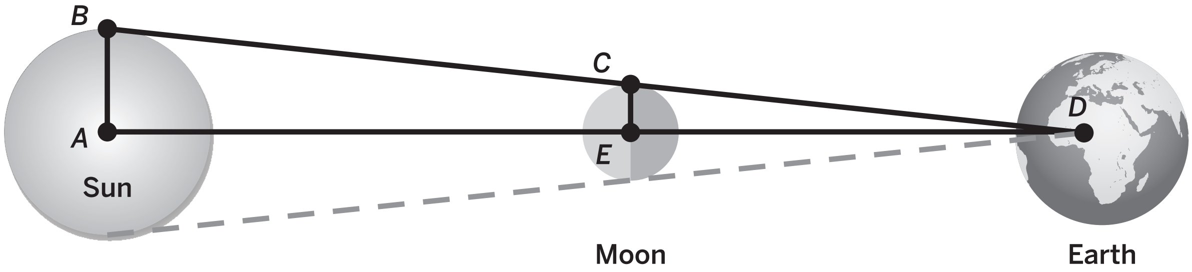
3

Years later, Galileo used the similarity of triangles to study solar eclipses and calculate the distances between planets.

The distance of the Sun from the Earth is about 150 million kilometers.

During a solar eclipse, the Moon is in line with the Sun and blocks the Sun's rays. The mighty Sun has a diameter of almost 1.4 million kilometers whereas the Moon's diameter is about 3500 kilometers.

How can the Moon block the Sun even though the Sun is much larger than the Moon?



Unit 2
Sub-Unit 3
Extensions

Similarity

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problems 1 and 2. If time allows, consider sharing Problems 1–3 with all students.

Problem 1

Students will extend their understanding of similar triangles by considering how the concept was used throughout history and determining missing side lengths in pairs of similar triangles.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What shape do each object and its shadow form? How can you use the relationship between these shapes to determine the height of the Great Pyramid?

150 meters. *Explanations vary*. Each object and its shadow form two sides of a right triangle. The triangles are all similar because the shadows have the same scale factor based on the time of day. I set up the relationship $\frac{1.60}{2.40} = \frac{\text{Pyramid's height}}{225}$, so the Great Pyramid's height is 150 meters.

Problem 2

Students will extend their understanding of similar triangles by considering how the concept was used throughout history and determining missing side lengths in pairs of similar triangles.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What do you think Thales tried to construct by walking in several directions and trying to align himself with the ship? How is that helpful for determining the distance?

450 feet. *Explanations vary.*
Thales created two similar right triangles. I set up the relationship $\frac{270}{x} = \frac{45}{75}$, so the distance of the ship from the shoreline is 450 feet.

Problem 3

Students will extend their understanding of similar triangles by considering how the concept was used throughout history and determining missing side lengths in pairs of similar triangles.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can you use the diameter of the Sun and the Moon to create similar triangles?

Responses vary.
Galileo used two similar triangles to explore how the Moon blocks the Sun during a solar eclipse. I can set up the relationship:

$$\frac{3500}{1{,}400{,}000} = \frac{\text{Moon's distance}}{150{,}000{,}000}$$

That means Moon's distance during the solar eclipse must be 0.375 million kilometers.
Therefore the Moon must be 400 times closer to the Earth to block the Sun.

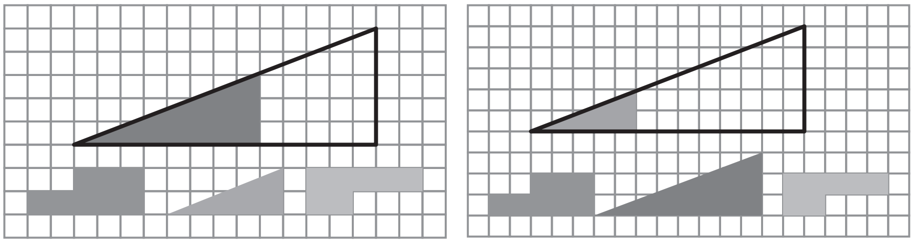
Slope

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 You will use a set of two identical triangles and four identical pieces to complete this problem.

a Each triangle has one piece placed inside, as shown. Use the remaining pieces to complete each triangle.



b What do you notice?

c This exploration is called *Curry's Paradox*. A paradox is a seemingly contradictory situation.

Here, we have the same four shapes that seem to have different areas based on how we arrange them. That does not fit with our understanding of area.

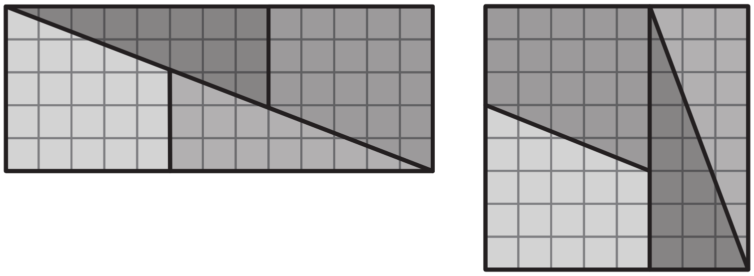
How do you explain the empty 1 square unit area?

Slope (continued)

Name: .... Date: .... Period: ....

2 Here is another geometrical paradox.

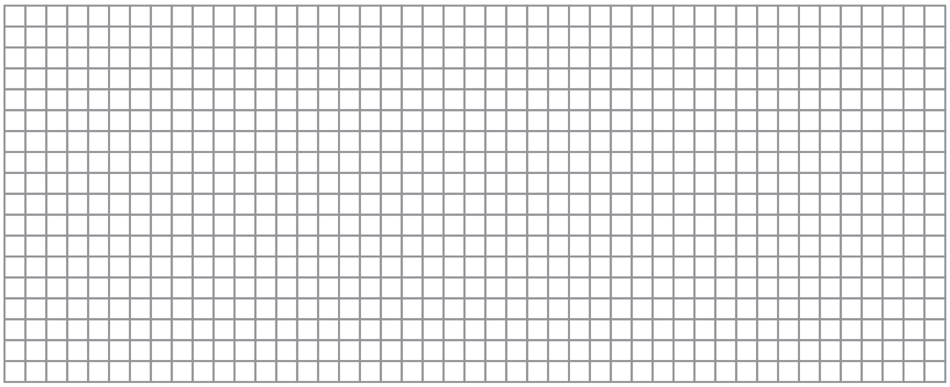
The two figures shown are composed of the same four pieces.



a Determine the area of each figure.

b What do you think causes this paradox?

3 Create your own geometric paradox.



Slope

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problem 1 and/or 2. If time allows, consider sharing Problem 1 with all students.

Materials

- Extension PDF, one per student, pre-cut **(Problem 1)**

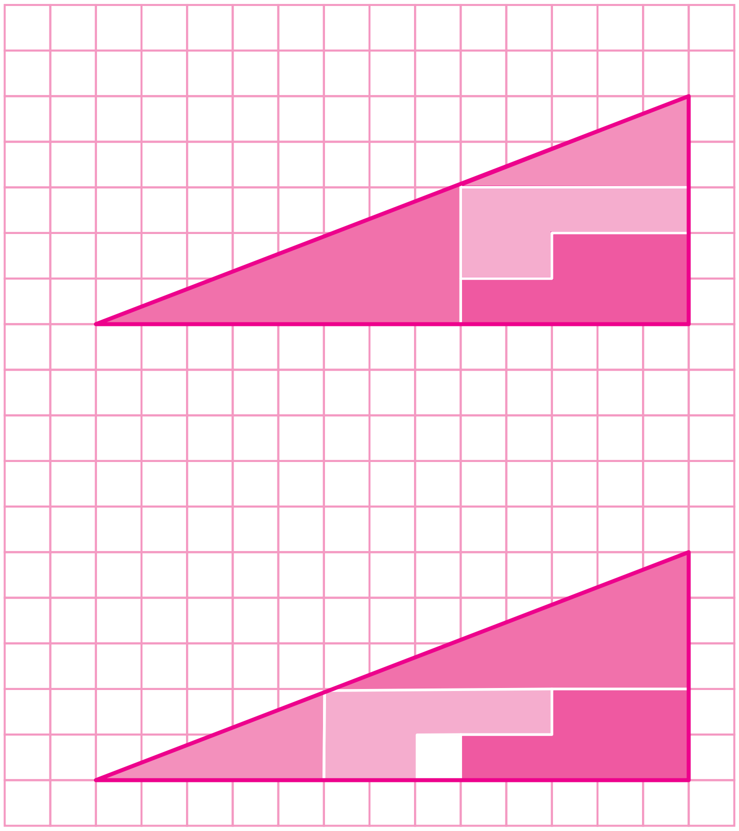
Problem 1

Students will extend their understanding of slope by solving Curry's Paradox.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part b, determine the area of the outlined triangle. Then determine the area of each piece. What do you notice?
- **Hint 2:** In part c, how can you use slope to help you?

a.



b. *Explanations vary*. I have the same four shapes that seem to have different area based on how I arrange them. The outlined triangle has an area of 32.5 square units while the four pieces have an area of 32 square units. That must mean when arranged in this manner, the four pieces don't quite fill up the triangle.

The second triangle's area must be 33 square units with the extra square unit. So they must be extending beyond the triangle slightly.

c. *Explanations vary*. The small triangle and the big triangle do not make a straight line. They don't line up exactly; The small triangle has a slope of $\frac{2}{5} = 0.4$, the big triangle has a slope of $\frac{3}{8} = 0.375$, and the main triangle's slope is $\frac{5}{13} \approx 0.385$.

Continued next page ...

Problem 2

Students will extend their understanding of slope by solving another paradox about area.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part b, determine the area of the square and the rectangle. Then determine the area of each piece. What do you notice?
- **Hint 2:** In part b, how can you use slope to help you?

a. **Each trapezoid has an area of 20 square units and each triangle has an area of 12 square units. The total area of the four pieces is 64 square units. The square has an area of 64 square units and the rectangle has an area of 65 square units. So when it is rearranged as a rectangle, the pieces must be slightly extending beyond the rectangle.**

b. **I checked the slopes of the slanted sides of each polygon and I see that they do not line up exactly:**

- **The slope of the trapezoid's slanted side is $\frac{2}{5} = 0.4$.**
- **The slope of the triangle's slanted side is $\frac{3}{8} = 0.375$.**
- **The slope of the rectangle's diagonal is $\frac{5}{13} \approx 0.385$.**

Problem 3

Students will extend their understanding of slope by creating their own paradox about area.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** You may research similar examples before you create yours.

Responses vary.

Unit 3

Extensions

Unit 3
Sub-Unit 1
Extensions

Proportional Relationships in Tables

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Farah used a blue marker to divide a plank of wood into into 5 equal parts. Raven then used a red marker to divide the same plank of wood into 7 equal parts.

a Name the colors on the plank from left to right. Draw a picture if it helps with your thinking.

b If the plank is cut at each marking, determine the lengths of the shortest and longest pieces.

2 If $a : b = 9 : 4$ and $b : c = 5 : 3$, determine the proportion $(a - b) : (b - c)$.

Proportional Relationships in Tables (continued)

Name: .... Date: .... Period: ....

3 Two thirds of Pablo's coins are nickels, one fourth are dimes, and the rest are quarters. If the total value of the coins is $6.65, how many of Pablo's coins are nickels? Dimes? Quarters?

Proportional Relationships in Tables

Assign problems to students who want to extend their thinking.

Problems 1–3 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Materials
- coloring tools **(Problem 1)**

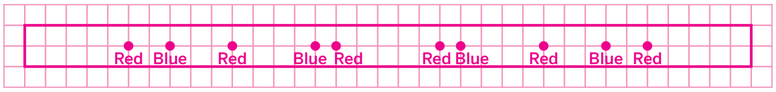
Problem 1

Students will extend their understanding of proportions by dividing the length of a plank into different pieces.

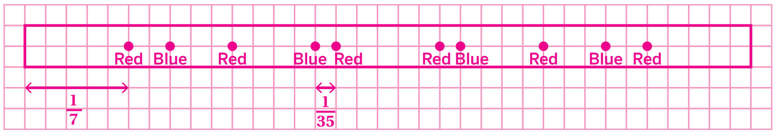
Provide students with the following hints if additional scaffolding is needed.

- **Hint:** Think about a possible length for the plank where you could easily divide by both 5 and 7.

a. Red, blue, red, blue, red, red, blue, red, blue, red.



b. The shortest length would be $\frac{1}{35}$ of the total length of the plank, and the longest length would be $\frac{1}{7}$ of the total length.



Problem 2

Students will extend their understanding of writing equations to describe proportional thinking in scenarios by solving proportional problems with variables.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** What is the common value in both proportions? How can having a common value help us write a proportion that includes all the variables?

$25 : 8$. *Explanations vary*. In both proportions, b is the common value. I created a table to find where the b value is the same for both proportions.

a	b
9	4
45	20

b	c
5	3
20	12

If I write the proportion $a : b : c$ as $45 : 20 : 12$, then I can subtract to find $(a - b) : (b - c) = 25 : 8$.

Continued next page ...

Problem 3

Students will extend their understanding of writing equations to describe proportional thinking in scenarios by connecting fractions and proportions.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** If there were 12 coins, how many of them would be nickels? Dimes? Quarters? What would be the total amount then?

56 nickels, 21 dimes, 7 quarters. *Explanations vary*. To determine the fraction of Pablo's coins that are quarters, I first added $\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$, which tells me that only $\frac{1}{12}$ of Pablo's coins are quarters.

If there were 12 coins, there would be 8 nickels, 3 dimes, and 1 quarter. That makes a total of 95 cents. 95 cents is $\frac{1}{7}$ of the actual amount \$6.65, so I can multiply all the numbers by 7 to find out how many of each coin Pablo has.

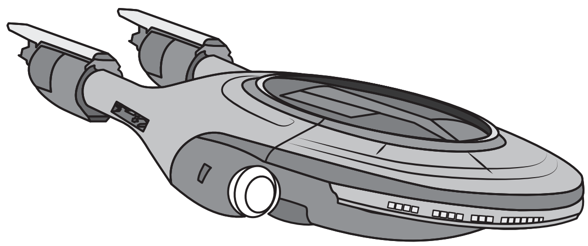
Proportional Relationships in Equations

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

The closest planet outside of our solar system orbits the star Proxima Centauri, which is 1.3 parsecs from Earth.



A *parsec* is about 3.26 light years.

A *light year* is the distance light travels in one year.

a How long does it take light from Proxima Centauri to reach Earth?

b How long would a voyage to Proxima Centauri take if a spaceship traveled at 90% the speed of light?

2

The International Space Station (ISS) is orbiting the Earth. The ISS is traveling in its orbit at a speed of 8 kilometers per second.

a What information do you need to determine how long it takes the ISS to orbit Earth?

b About how many hours does it take for the ISS to orbit Earth?

Proportional Relationships in Equations (continued)

Name: .... Date: .... Period: ....

3 A train traveling at 30 miles per hour reaches a tunnel that is 9 times as long as the train. The train takes 2 minutes to completely clear the tunnel. How long is the train?

Unit 3

Sub-Unit 2 Extensions

Proportional Relationships in Equations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. Assign Problem 3 to students who have solved Problem 1. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of speed and equations by extending their thinking to the speed of light.

a. About 4.24 years, because $1.3 \cdot 3.26 \approx 4.24$.

b. About 4.71 years, because $\frac{4.24}{0.9} \approx 4.71$.

Problem 2

Students will extend their understanding of equivalent ratios by determining how long it will take the International Space Station to orbit the Earth.

Provide students with the following hint if additional scaffolding is needed for part a, or if students request this information for part b.

- **Hint:** Earth's circumference is about $40{,}000$ km.

a. *Responses and explanations vary*. Since the ISS is orbiting around the Earth, I need to know the Earth's circumference.

b. *Responses and explanations vary*. About 1.5 hours. Earth's circumference is about $40{,}000$ kilometers. The orbit of the ISS is longer than this, but not a lot longer. The orbit will take a little more than $40000 \div 8 = 5000$ seconds. I can convert to minutes by dividing $5000 \div 60 \approx 83.3$. Because the orbit of the ISS has a slightly larger circumference, it will probably take closer to 90 minutes, which is 1.5 hours.

Continued next page ...

Problem 3

Students will extend their understanding of proportional relationships involving speed by using ratios and equations to determine an unknown distance.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What is the distance to completely clear the tunnel if the length of the train is x?

0.1 miles. *Explanations vary*. If the length of the train is x, then the distance to completely clear the tunnel is $10x$.

I need to convert 30 miles per hour into miles per minute, so I used a table to determine that the speed is equivalent to 0.5 miles per minute.

Miles	Minutes
30	60
0.5	1

Then I used the equation $d = rt$ to determine the length of the train x:

$$d = rt$$

$$10x = 0.5 \cdot 2$$

$$10x = 1$$

$$x = 0.1$$

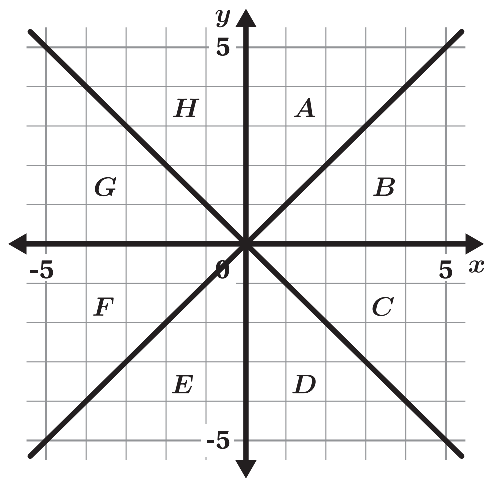
Proportionality Revisited

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

a What are the equations of the two lines in the graph?



b Consider a line in the form $y = mx$ that passes through the origin. Complete the table to show what values of m result in a line passing through the given regions

For the lines in the regions	A and E	B and F	C and G	D and H
m is	greater than 1			

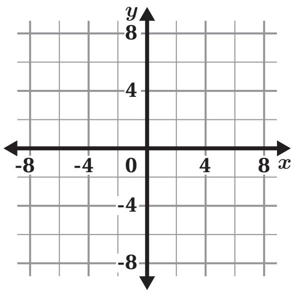
Proportionality Revisited (continued)

Name: .... Date: .... Period: ....

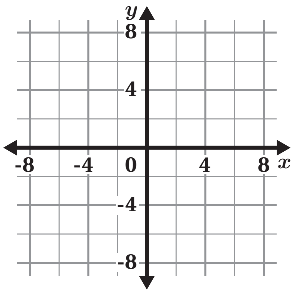
2

a Sketch the graph of equations on the given coordinate planes.

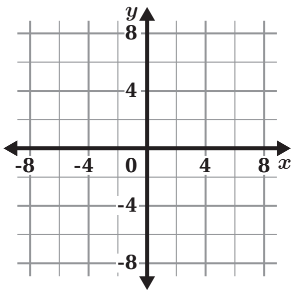
$y = 4x$ and $y = -\frac{1}{4}x$



$y = -2x$ and $y = \frac{1}{2}x$



$y = \frac{2x}{5}$ and $y = -\frac{5}{2}x$



b What do you notice?

c Write the equation of the line that is perpendicular to $y = -5x$ and passing through the origin.

Proportionality Revisited

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing all problems with all students.

Materials
- Protractor (optional) **(Problem 2)**

Problem 1

Students will extend their understanding of the relationship between the slope of a line and its equation by determining the value and sign of the slope.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** In part a, determine two ordered pairs that are on each of the lines. What is the relationship between x and y values of the ordered pairs?
- **Hint 2:** In part b, consider drawing line (s) passing through the given regions, choose a couple of ordered points on your line to determine their slope.

a. $y = x$

$y = -x$

b.

For the lines in the regions	A and E	B and F	C and G	D and H
m is	greater than 1	**between 0 and 1**	**between -1 and 0**	**less than -1**

Continued next page ...

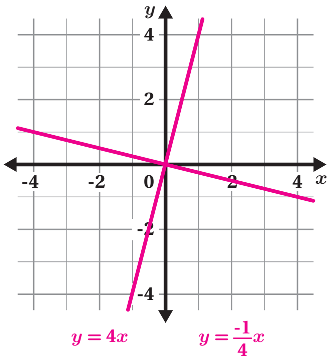
Problem 2

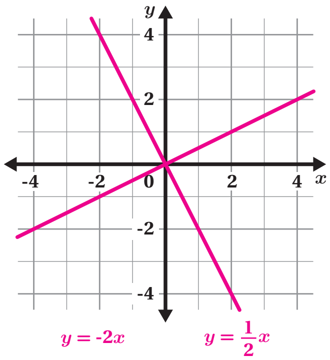
Students will extend their understanding of the relationship between the slope of a line and its equation by exploring the relationship between the slopes of the perpendicular lines.

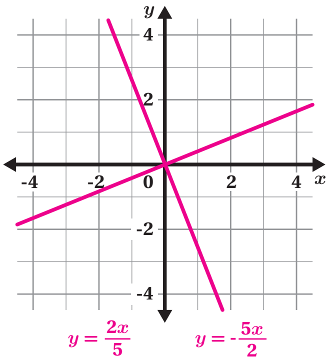
Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** In Part b, what are the slopes of each line? What is the relationship between the slopes of each pair?
- **Hint 2:** In Part b, are each pair of lines parallel or intersecting? Consider using a protractor to measure the angle at the intersection.

a.





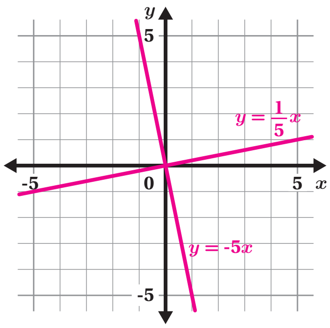


b. *Responses vary*.

- Each pair of equations intersect at one point.
- Each pair seems to be perpendicular to each other.
- For each pair, the product of their slopes is -1.

c. $y = \frac{1}{5}x$

Explanations vary. The slope of the given line is -5. The slope of the line perpendicular to it must have a slope of $\frac{1}{5}$. Because it passes through the origin, its equation is $y = \frac{1}{5}x$.



Unit 4

Extensions

Linear Relationships

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

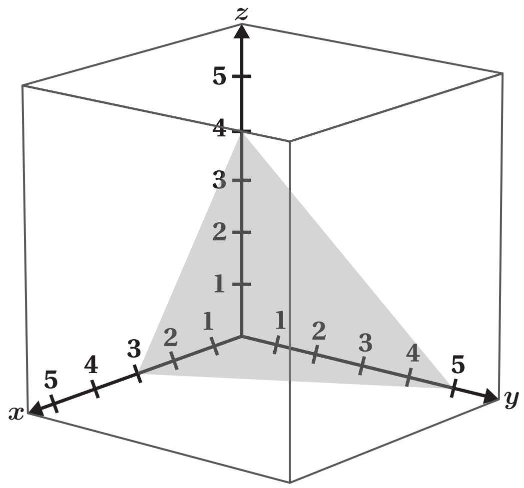
1 One form that a linear equation may be written in is the *intercept form*.

$$\frac{x}{a} + \frac{y}{b} = 1$$

The constants a and b are the x- and y- intercepts of the graph. In three dimensional space, the *equation of a plane* takes a similar form.

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

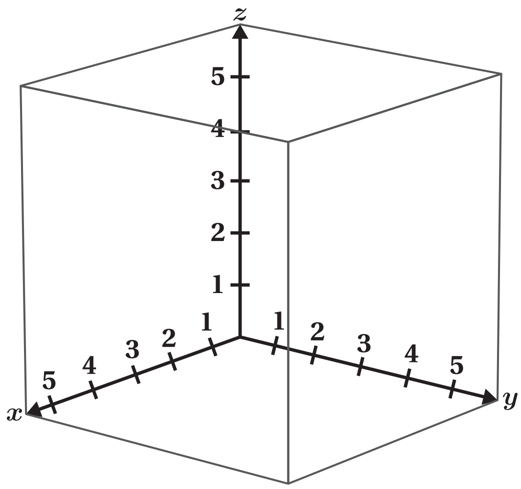
For example, plane of $\frac{x}{3} + \frac{y}{5} + \frac{z}{4} = 1$ intersects with x-axis at $(3, 0, 0)$, y-axis at $(0, 5, 0)$, z-axis at $(0, 0, 4)$



a Graph the equations

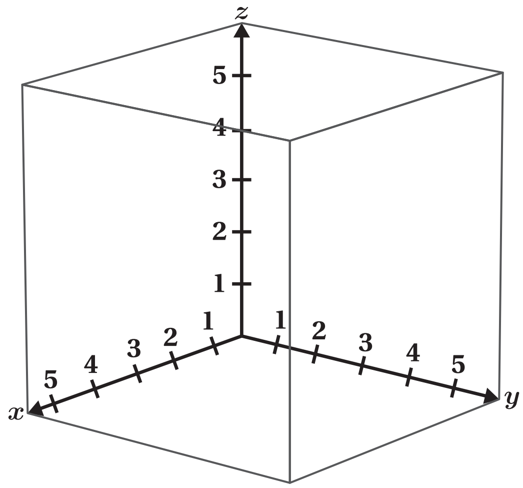
Graph A

$$\frac{x}{3} + \frac{y}{5} + \frac{z}{2} = 1$$



Graph B

$$\frac{x}{3} + \frac{y}{1} + \frac{z}{5} = 1$$



Linear Relationships (continued)

Name: .... Date: .... Period: ....

For Graph A, write the equation of the line where the plane ⋯

b intersects the xy-plane.

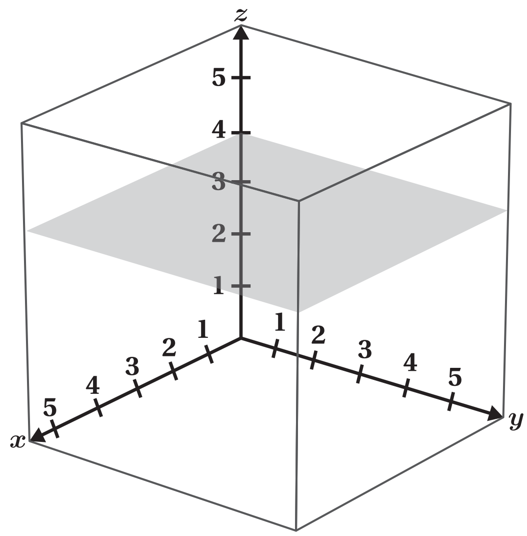
c intersects the xz-plane.

d intersects the yz-plane.

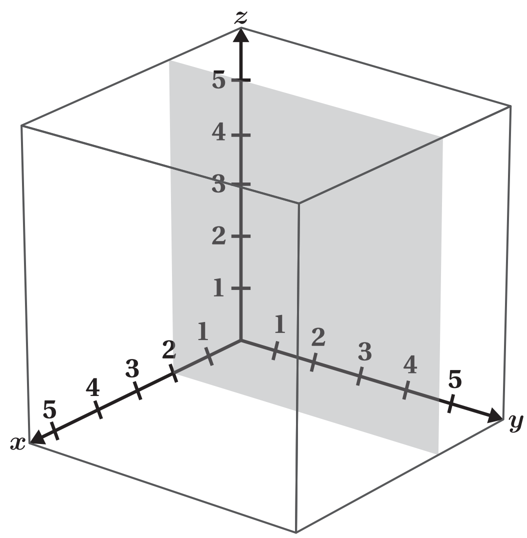
e In three dimensional space, the equation for the xy-plane is $z = 0$. Write an equation for the xz-plane.

f Write an equation for the yz-plane.

g Write an equation for a plane parallel to the xy-plane with a z-intercept of 4.



h Write an equation for a plane parallel to the yz-plane with an x-intercept of 2.



Linear Relationships

Assign problems to students who want to extend their thinking.

If time allows, consider sharing the problem with all students.

Materials

- Desmos 3D Graphing Calculator (optional) **(Problem 2)**

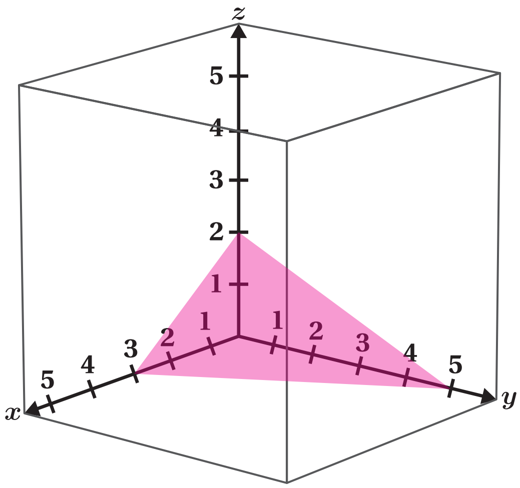
Problem 1

Students will extend their understanding of the equation, intercepts and the slope linear relationships by exploring graphing in 3D space.

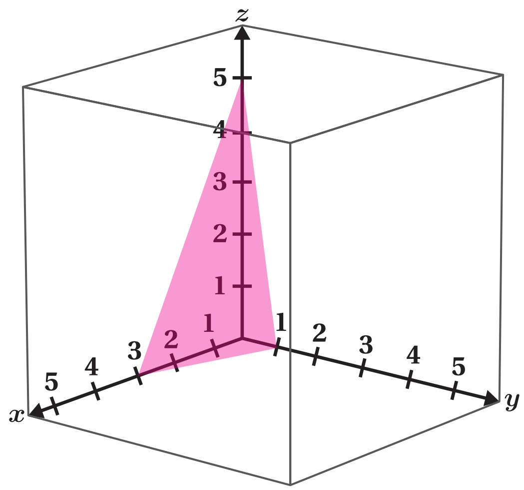
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Consider using Desmos 3D Graphing Calculator to sketch and explore $\frac{x}{3}+\frac{y}{5}+\frac{z}{4}=1$ in detail before solving the problems.
- **Hint 2:** In part a, which point on the x-axis should be a vertex of the plane? On the y-axis? On the z-axis?
- **Hint 3:** In part b, what is the x-intercept of the line that the plane intersects the xy-plane? What is its y-intercept? How can you use the intercepts to write the equation of the line?
- **Hint 4:** In part c, what is the x-intercept of the line that the plane intersects the xz-plane? What is its z-intercept?
- **Hint 5:** In parts c and d, on the xy-plane, a linear equation can be written as $\frac{x}{a}+\frac{y}{b}=1$ where a and b are the x-and y-intercepts of the graph. How can you adapt this form to xz-plane? To yz-plane?
- **Hint 6:** In parts e and f, in xy-plane, what is the equation of the y-axis? x-axis?

a. Graph A



Graph B



b. $\frac{x}{3}+\frac{y}{5}=1$

c. $\frac{x}{3}+\frac{z}{2}=1$

d. $\frac{y}{5}+\frac{z}{2}=1$

e. $z=0$

f. $x=0$

g. $z=4$

h. $x=2$

Linear Equations and Points

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 Use the digits $1–9$, without repeating, to fill each blank and create parallel lines.

$\square x + \square y = \square$

$\square x + \square y = \square$

2 Use the digits $1–9$, without repeating, to fill each blank and create parallel lines.

$y = \frac{\square}{\square}(x - \square) + \square$

$y = \frac{\square}{\square}(x - \square) + \square$

Linear Equations and Points (continued)

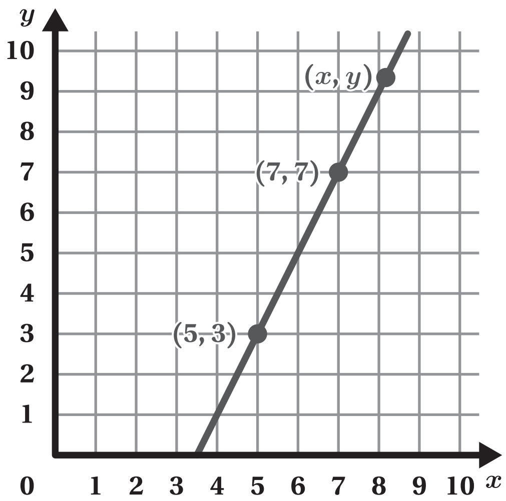
Name: .... Date: .... Period: ....

3 Consider the line shown.

a. Does $\dfrac{y-3}{x-6}=2$ represent the line?

b. Does $\dfrac{y-6}{x-4}=5$ represent the line?

c. Does $\dfrac{y+5}{x+1}=2$ represent the line?



4 The following table shows the freezing and boiling points of water in four temperature scales: Celsius, Fahrenheit, Kelvin, and Rankine.

	°C	°F	°K	°R
Boiling point	100	212	373	672
Freezing Point	0	32	273	492

Assume that the average human temperature is 97°F. Determine the average human temperature in the other three scales.

Linear Equations and Points

Assign problems to students who want to extend their thinking.

Problems 1, 3 and 4 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 1 and 4 with all students.

Problem 1

Students will extend their understanding of writing equations of a line by choosing coefficients and constants to create parallel lines.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What is the relationship between the slopes of the parallel lines?

Responses and explanations vary.

$2x + 3y = 5$

$4x + 6y = 7$

The slopes of the parallel lines are the same. Therefore the coefficients of x and y must be proportional. The ratios of the coefficients are: $\frac{2}{4} = \frac{3}{6} = \frac{1}{2}$

Problem 2

Students will extend their understanding of writing equations of a line by choosing coefficients and constants to create parallel lines.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can the pair of equations you wrote in the first problem help you create another pair for this problem?

Responses and explanations vary.

$y = \frac{2}{3}(x - 5) + 8$

$y = \frac{4}{6}(x - 7) + 9$

I chose the coefficients of x to be proportional, $\frac{2}{3} = \frac{4}{6}$, then picked the rest of the numbers randomly.

Continued next page ...

Linear Equations and Points (continued)

Problem 3

Students will extend their understanding of writing equations of a line by determining if different equations of a line are equivalent.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What is the slope and the equation of the given line?

a. No. *Responses vary.*

The equation represents a line with a slope of 2 containing the point $(6, 3)$. The line shown does not contain the point $(6, 3)$.

b. No. *Responses vary.*

The equation represents a line with a slope of 5 containing the point $(4, 6)$. The line shown has a slope of 2, not 5.

c. Yes. *Responses vary.*

The equation represents a line with a slope of 2 containing the point $(-1, -5)$. This is a possible equation for the line.

Problem 4

Students will extend their understanding of linear relationships by using rate of change to determine a temperature in three different scales.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What is the rate of change at each temperature point compared to Fahrenheit?

About $36.1°C$, $309.1°K$, and $557\ °R$.

Explanations vary.

I calculated the rate of change at each temperature point compared to Fahrenheit, then set up an equation to determine the unknown value at $97°F$.

For Celsius:

$$\frac{32-212}{0-100}=1.8$$
$$\frac{97-32}{x-0}=1.8$$
$$\frac{65}{x}=1.8$$
$$x\approx 36.1$$

For Kelvin:

$$\frac{32-212}{273-373}=1.8$$
$$\frac{97-32}{x-273}=1.8$$
$$\frac{65}{x-273}=1.8$$
$$65=1.8(x-273)$$
$$x\approx 309.1$$

For Rankine:

$$\frac{32-212}{492-672}=1$$
$$\frac{97-32}{x-492}=1$$
$$\frac{65}{x-492}=1$$
$$65=x-492$$
$$x=557$$

Unit 5

Extensions

Two-Step Equations

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1

Consider the calendar and the date highlighted, May 8.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

a Look at the numbers above, below, and to either side of the 8th. Calculate the mean of these numbers. What do you notice?

b Repeat part a by choosing a different date. Explain why the same thing will happen for any date in a location where it has a date above, below, and to either side.

c Assume you cover four dates next to May 8 with a square. Determine the difference of product of the corner numbers, i.e., $8 \bullet 2 - 1 \bullet 9$. Repeat by choosing different dates. What do you notice?

d Explain why the same thing will happen for any date.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

2

You have an unknown number, x. In the first step, you add 1 then halve the sum. In the second step, you add 1 to that and halve the sum again.

a Write an expression to represent the first two steps of this situation.

b Continue adding 1 and halving the previous sum for the steps below. What did you notice?

3rd step 4th step 5th step

Two-Step Equations (continued)

Name: .... Date: .... Period: ....

3 In an expression pyramid, each box is the sum of the two boxes under it. Complete the pyramids using the given expressions.

a

$2x + 1$	$2x$	$1 - x$
$3x - 1$	$3 - 2x$	$x - 2$

b

$-(y + 1)$	$2y$	$2(1 - x)$
$3(y - 1)$	$\frac{6 - 2y}{2}$	2

Two-Step Equations

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problem 1 or 2. If time allows, consider sharing Problems 3 with all students.

Problem 1

Students will extend their understanding of reasoning about solving equations by using algebra to investigate patterns.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part b, let the original date be x how can you express the date below in terms of x? The date above?
- **Hint 2:** In part d, consider using area models to help you multiply two two-term expressions.

a. 8. *Responses vary*. The mean of these four numbers is equivalent to the value of the date selected.

b. *Responses vary*. I chose the date of 21 and the mean for the values above, below, and on either side resulted in 21. The mean of these values is equivalent to the value of the date selected.

If the original date chosen is represented by x, then the data above is $x - 7$, because it is 7 days prior. The date below is $x + 7$, because it is 7 days after. The date to the left is $x - 1$ and the date to the right is $x + 1$. The sum of these four dates is $(x - 7) + (x + 7) + (x - 1) + (x + 1)$ which equals $4x$. To calculate the mean, I would divide this by 4, resulting in x, the original date chosen.

c. *Responses vary*. $8 \cdot 2 - 1 \cdot 9 = 7$. Then I chose May 10, the difference is again $10 \cdot 4 - 3 \cdot 11 = 7$. No matter which date I choose, the difference is always 7.

d. *Explanations vary*. If the original date chosen is represented by x, the difference of the products of the corner numbers is $[x \cdot (x - 6)] - [(x - 7)(x + 1)]$
$= [x^2 - 6x] - [x^2 - 6x - 7] = 7$

Problem 2

Students will extend their understanding of expanding and factoring by expanding a pattern using the fewest number of terms.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** One way to write the first step is $(x + 1) \cdot \frac{1}{2}$. The other way, which might be more helpful here, is using distributive property to write the result as a ratio, $\frac{x+1}{2}$. Consider using the same method for the following steps.

a. First step: $\frac{x+1}{2}$

Second step: $\frac{\frac{x+1}{2} + 1}{2} = \frac{x+3}{4}$

b.

3rd step	4th step	5th step
$\frac{\frac{x+3}{4} + 1}{2} = \frac{x+7}{8}$	$\frac{\frac{x+7}{8} + 1}{2} = \frac{x+15}{16}$	$\frac{\frac{x+15}{16} + 1}{2} = \frac{x+31}{32}$

Each step has the power of 2 as the denominator and one less than that as the constant in the numerator. For example, in step 8, it will be $\frac{x + (2^8 - 1)}{2^8}$.

Continued next page ...

Problem 3

Students will extend their understanding of expressions by adding, expanding, and factoring to complete the expression pyramid.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part b, writing each expression using fewer number of terms or as sum expressions can help determine the sums of two boxes.

a.

$2x$

$1 - x$ | $3x - 1$

$3 - 2x$ | $x - 2$ | $2x + 1$

b.

2

$2(1 - y)$ | $2y$

$-(y + 1)$ | $\frac{6-2y}{2}$ | $3(y - 1)$

Multi-Step Equations

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

A *magic square* is a square array of distinct numbers, arranged in a way that the sum of the numbers in every row, in every column and in each of the main diagonals is equal.

4	9	2
3	5	7
8	1	6

a Here is a different version of a magic square. Darryl thinks that 7 must go in the bottom right corner. Explain why it is true.

1		3
5		

b Place the numbers in the cells so that the sum of the numbers along both diagonals, columns, and rows are the same.

c How many solutions does this problem have?

Multi-Step Equations (continued)

Name: .... Date: .... Period: ....

2

a Take any two-digit number. Reverse the digits, and subtract your answer from your original number. Try several different two-digit numbers. What do you notice?

b How many positive two digit numbers increase by 36 when their digit is reversed?

c Take any two-digit number. Add its digits, and subtract the sum from your original number. What do you notice?

d Take any three-digit number. Reverse the digits, and subtract your answer from your original number. What do you notice?

3

The English mathematician Augustus de Morgan, who died in 1871, claimed that he became x years old in the year x^2. When was he born?

Multi-Step Equations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1, 2 and 3 with all students.

Problem 1

Students will extend their understanding of writing and solving equations with variables by solving a *Magic Square* puzzle

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Consider assigning variables to the empty cells.
- **Hint 2:** What is the sum of the top right corner and bottom left corner? What must be the sum of the top left corner and bottom right corner?
- **Hint 3:** What must be the sums of the middle numbers in opposite columns and rows?

a. *Explanations vary.*

Let the number in the middle cell be x. Because the sum of diagonals is equal, $3 + x + 5 = 1 + x + 7$. The bottom right corner must be 7.

b.

1	8	3
6	4	2
5	0	7

Explanations vary.

I have started by inserting variables to empty cells.

1	a	3
b	x	d
5	c	7

I know that $b + d = a + c = 8$, also $4 + a = 6 + b = 12 + c = 10 + d$. Using $4 + a = 12 + c$ and $a + c = 8$, I determined $a = 8$, and $c = 0$. Then I used $b + d = 8$, and $6 + b = 10 + d$ to determine $b = 6$, and $d = 2$.

c. Including rotation and mirroring there are 8 solutions for this problem.

1	8	3
6	4	2
5	0	7

5	6	1
0	4	8
7	2	3

7	0	5
2	4	6
3	8	1

3	2	7
8	4	0
1	6	5

3	8	1
2	4	6
7	0	5

1	6	5
8	4	0
3	2	7

5	0	7
6	4	2
1	8	3

7	2	3
0	4	8
5	6	1

Continued next page ...

Multi-Step Equations (continued)

Problem 2

Students will extend their understanding of writing and solving equations with variables by solving place value puzzles.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How can you use the place value of digits of a two digit number ab to write an equation?
- **Hint 2:** If any two digit number ab can be written as $10a + b$, how can you represent ba with an equation?

a. The differences are always a multiple of 9. *Explanations vary*. For example $73 - 37 = 36$, $54 - 45 = 9$, $91 - 19 = 72$. For a two digit number ab, when I reverse the digits and subtract from the original, the difference is $ab - ba = 10a + b - (10b + a) = 9a - 9b = 9(a - b)$.

b. Five numbers. *Explanations vary.* If $ba - ab = 36$, $9(b - a) = 36$, then $b - a = 4$, so the numbers are $15, 26, 37, 48, 59$.

c. The result is always nine times the tens digit. *Explanations vary.* For example $73 - (3 + 7) = 63$, $25 - (2 + 5) = 18$, $36 - (3 + 6) = 27$. For any two digit number ab, $10a + b - (a + b) = 9a$.

d. The differences are always a multiple of 99. *Explanations vary*. For example, $765 - 567 = 198$, and $601 - 106 = 495$. For any three digit number abc, $abc - cba = 100a + 10b + c - (100c + 10b + a) = 99(a - c)$.

Problem 3

Students will extend their understanding of writing and solving equations with variables by solving the age of a mathematician puzzle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How can we use his death year to help solve this problem?
- **Hint 2:** What are the square number years close to his death year?

1806. *Explanations vary*.

Square numbers near 1871 (year of death)

- $40 \cdot 40 = 1600$, but 1600 is too far away from 1871
- $42 \cdot 42 = 764$, but if he was 42 in 1764 then he would have been 149 when he died.
- $43 \cdot 43 = 1849$ is more reasonable because it would have been 22 years before his death in 1871.
- $44 \cdot 44 > 1871$ so no other squares will work. So he was born 43 years before 1849, in 1806.

Systems of Linear Equations

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

a Determine x, y, z and w.

$w + x + y = 49$

$x + y + z = 40$

$y + z + w = 37$

$z + x + w = 42$

b Determine $x + y + z$.

$-2x + 4y + z = 10$

$10x + 15y - 5z = 55$

$3x - y + 4z = 33$

2

A magician asked everyone in the audience to think of two whole numbers less than 10, and then follow these steps.

Take one of them and add 1.
Multiply by 5.
Add 1 again.
Double your answer.
Subtract 1.
Add your second number.
Add 2.
Double again.
Subtract 8.
Halve this number and tell me your answer.

a Try the magician's trick with at least two different pairs of answers.

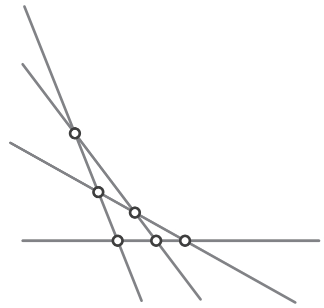
b Subtract 9 from both of your answers. What do you notice?

c How did the magician work out both of the numbers from the answers quickly?

Systems of Linear Equations (continued)

Name: .... Date: .... Period: ....

3 Two lines can intersect maximum in 1 point.



a What is the maximum point of intersections for 3 lines? 4 lines?

b Draw 5 lines in a way that they intersect with the maximum point of intersections.

c What is the maximum point of intersections for 6 lines? 7 lines?

d What is the maximum number of intersection points for n lines?

Systems of Linear Equations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1, 2 and 3 with all students.

Problem 1

Students will extend their understanding of systems of linear equations by solving systems with more than two variables.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, how many xs are there in the entire system of equations? How many ys? zs and ws?
- **Hint 2:** Is there a way to eliminate one of the variables by creating equivalent equations?

a. $x = 19, y = 14, z = 7, w = 16$

Explanations vary.

$3(x + y + w + z) = 168$
$x + y + w + z = 56$

Then I subtracted each equation to determine the values of each variable.

b. $x = 4, y = 3, z = 6$

Explanations vary. I multiplied the first equation by 5

$5(-2x + 4y + z) = 50$

I added the second equation to determine y as 3. Then I determined the values of the remaining variables.

Problem 2

Students will extend their understanding of systems of linear equations by exploring a think of a number trick.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1: In part b,** try several numbers to explore how subtracting 9 can help the magician to know the numbers.

a. *Responses vary.*

b. *Responses vary*. When I subtracted 9, I noticed the numbers I picked a and b were in the form of the two digit number ab.

c. For numbers a and b

$2(5(a + 1) + 1) - 1 = 10a + 11$ and

$$\frac{2(10a + 11 + b + 2) - 8}{2} = 10a + b + 9$$

The magician subtracts 9 from the final answer to get both of the numbers.

Continued next page ...

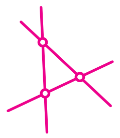
Problem 3

Students will extend their understanding of linear equations by exploring the famous String Art problem.

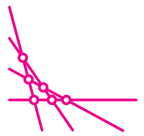
Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1: In Part a,** draw 3 and 4 lines with the maximum number of intersection points.
- **Hint 2:** In Parts c and d, organize your findings in a table to help you determine the number of intersection points with any number of lines.

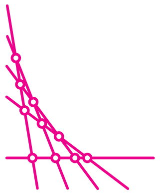
a. Three lines can intersect maximum in 3 points.



Four lines can intersect maximum in 6 points.



b. Five lines can intersect maximum in 10 points.



c. 15 points for 6 lines and 21 points for 7 lines.
Explanations vary.

- I noticed that when I added one more line, the number of intersection points increased by the number of lines from the previous stage.
- I noticed that the number of intersection points are triangular numbers.

Number of Lines	2	3	4	5	6	7
Maximum Number of Intersection Points	1	3	6	10	15	21

d. $\frac{1}{2}n(n-1)$
Explanations vary.

Unit 6

Extensions

Introduction to Functions

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1

a Use the given numbers only once to create three linear functions below.

-3	-2	-1	0	1	2	3

Input	Output
-2	
-1	-2
0	-1
1	0
2	

Input	Output
-2	
-1	2
0	1
1	
2	-1

Input	Output
-2	
-1	1
0	0
1	
2	

b Write y as a function of x for each table.

2

Determine the rule for each input-output table.

a

Input	Output
A	Y
B	X
E	U
M	M
Q	I

b

Input	Output
2	1
7	14
10	5
13	26
34	17

c

Input	Output
1	0
2	0
-4	1
-5	1
17	0

Introduction to Functions (continued)

Name: .... Date: .... Period: ....

3

Determine as many functions as possible that assign the y-value 4 to the x-value 1. Write as many functions which fit this model as you can.

4

Here are two more input-output tables.

Input (x)	Output (y)
	-12
-5	
-2	4
1	16
3	24
6	36

Input (x)	Output (y)
-4	-32
-1	
2	4
3	10
	34
9	46

a Describe each rule that allows you to get y from x.

b Use the rule to determine the missing numbers.

c Write y as a function of x.

Introduction to Functions

Assign problems to students who want to extend their thinking.

Problems 1 and 3 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. Assign Problem 4 to students who have solved Problem 3. If time allows, consider sharing Problems 1–3 with all students.

Problem 1

Students will extend their understanding of functions and non-function by completing input-output tables.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider determining the rules to help you complete the tables.

a.

Input	Output	Input	Output	Input	Output
-2	**-3**	-2	**3**	-2	**2**
-1	-2	-1	2	-1	1
0	-1	0	1	0	0
1	0	1	**0**	1	**-1**
2	**1**	2	-1	2	**-2**

b. **First table: $y = x - 1$ (or equivalent)**
Second table: $y = 1 - x$ (or equivalent)
Third table: $y = -x$

Problem 2

Students will extend their understanding of function rules by determining the rule for input-output tables.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part a, consider recording the position number of each letter in the alphabet.

a. ***Responses vary*. The output is the letter in the position of 26 minus the input's position number in the alphabet.**

b. ***Responses vary*. If the input value is even, the output divides the value of the input by 2. If the input value is odd, the output multiplies the value of the input by 2.**

c. ***Responses vary*. If the input value is positive, the output is 0, if the input value is negative, the output is 1.**

Continued next page ...

Problem 3

Students will extend their understanding of function rules by determining possible rules for a function machine that assigns 4 for 1.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Think about all the operations and exponents that can use the input of 1 to result in 4.

Responses vary.

- $y = 4x$
- $y = x + 3$
- $y = 5 - x$
- $y = \frac{x}{0.25}$
- $y = 2(3x - 1)$
- $y = x^2 + 3$
- $y = 4^x$
- $y = 2^{2x}$

Problem 4

Students will extend their understanding of function rules by determining the rule for input-output tables.

Provide students with the following hint if additional scaffolding is needed.

Hint: Consider using trial and error and a calculator to determine the missing numbers.

a. Table 1: Multiply x by 4 then add 12 to get y.

Table 2: Multiply x by 6 then subtract 8 to get y.

b.

Input (x)	Output (y)
-6	-12
-5	-8

Input (x)	Output (y)
-1	-14
7	34

c. Table 1: $y = 4x + 12$
Table 2: $y = 6x - 8$

Representing and Interpreting Functions

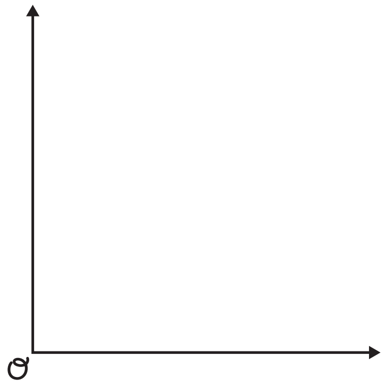
Name: ____________ Date: ____________ Period: ____________

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Han gets in his car to pick up his little brother from school.

- He starts the car and speeds up at a constant rate until he reaches the speed limit, then he drives the speed limit for a while.
- He thinks he might be able to get through the next light so he speeds up, but it turns yellow so he quickly slows down and comes to a complete stop for the duration of the light.
- When the light turns green, Han speeds up then drives the speed limit again.
- He slows down as he approaches his brother's school, and then comes to a complete stop to pick up his brother.

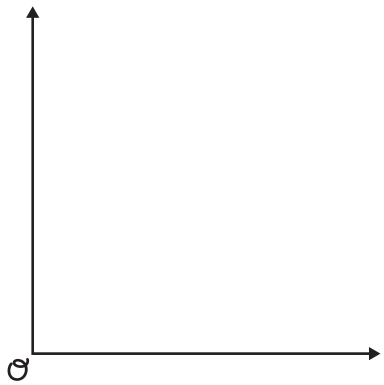


Sketch a graph that represents this situation. Be sure to label the axes.

2

In an experiment, a ball is launched straight up by some device and its height above the ground is recorded each second.

In the first and fifth seconds, the ball was 25 meters high. In the second and fourth seconds, it was 40 meters high. It reached its highest level, 45 meters high, at the third second. After six seconds, it was on the ground.



a Sketch a graph that represents this situation. Be sure to label the axes.

b Estimate the one-second interval(s) where the ball moved the fastest.

c Estimate the time(s) where the ball changed direction.

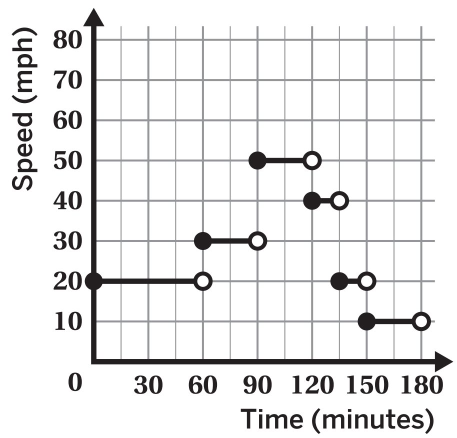
d Estimate the times the ball was at the following heights:

42 meters	35 meters	24 meters	10 meters

Representing and Interpreting Functions (continued)

Name: .... Date: .... Period: ....

3 The graph models the speed of a car over a function of time during a 3-hour trip.



a How far did the car travel over the course of the trip?

b How can you visualize the distance the car traveled over 3 hours on the graph?

Representing and Interpreting Functions

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problem 1 and 2 with all students.

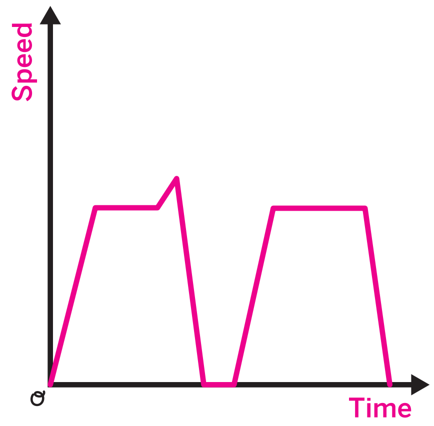
Problem 1

Students will extend their understanding of graphs of functions by drawing a graph from a story.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Evaluate each part of the story one by one. If needed, divide it into several sections to draw the graph.

Graphs vary.



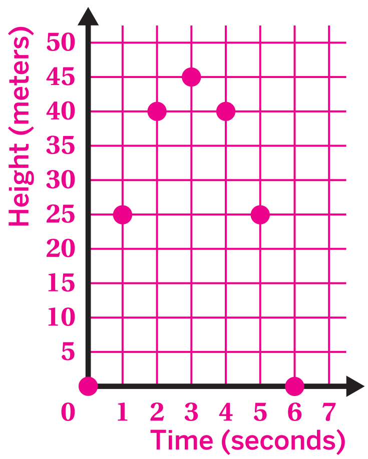
Problem 2

Students will extend their understanding of graphs of functions by drawing a graph from a story.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part d, how many time intervals are there for the ball to have the given height?

a.



b. Between 0–1, and 5–6

c. 3

d.

42 meters	35 meters	24 meters	10 meters
Between 2–3 and 3–4	Between 1–2 and 4–5	Between 0–1 and 5–6	Between 0–1 and 5–6

Continued next page …

Problem 3

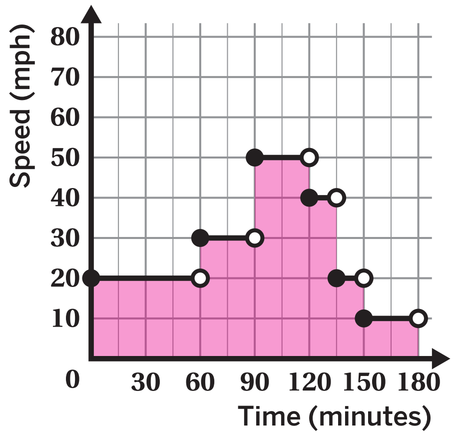
Students will extend their understanding of piecewise functions by reasoning about distance traveled in a speed vs. time graph.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** What is the speed of the car in the first segment of the trip? How long does it travel for 20 mph? Using the speed and time, how can you calculate the distance it traveled in the first segment?
- **Hint 2:** Record the time in hours and speed for each segment of the graph to determine the distance separately.

a. 80 miles. *Explanations vary*.
I divided the trip into segments over which the speed is constant and used the formula $distance = speed \cdot time$. Over the first hour, the car traveled 20 mph, for a total of 20 miles. Then for half an hour, the car traveled 30 mph, for a total of 15 miles.
Similarly, the remaining four segments correspond to distances traveled of 25 miles, 10 miles, 5 miles, and 5 miles, resulting in a total sum of 80 miles.

b. *Explanations vary*. I represented the quantity $distance, speed \cdot time$, graphically by a length measured along the x-axis times a height measured along the y-axis, resulting in the area of a rectangle. The distance traveled over each segment is the area of the rectangle under that segment, and the total distance is the total area of the shaded region.



Unit 7

Extensions

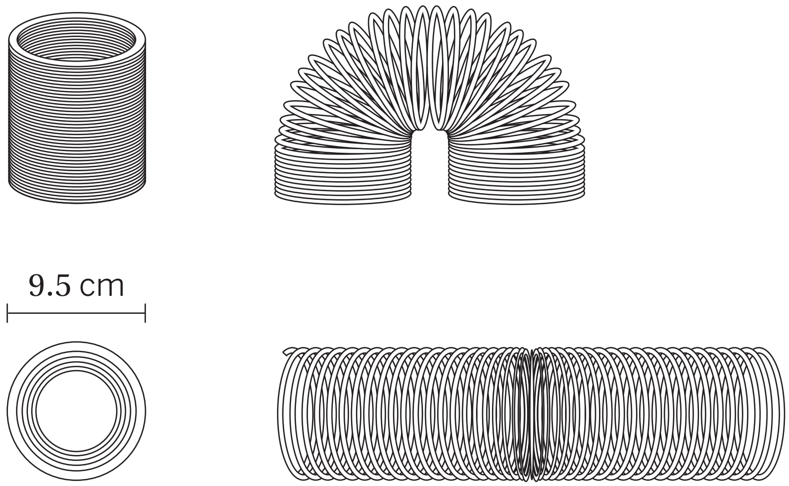
Circles

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Explain how to determine the center of a paper circle without a ruler or a compass.

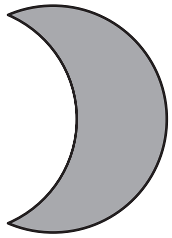
2 Consider the photos of a spring toy. If you could stretch out the spring completely straight, how long would it be?



Circles (continued)

Name: ______ Date: ______ Period: ______

3 The shaded region is bound by two arcs, or parts of a circle, with the same radius of 7 centimeters. Determine the perimeter of the shaded region.



4 The circumference of Earth is approximately $40,000$ kilometers. Imagine you wrapped a circle of wire around the planet that is only 10 meters longer than its circumference. Could a flea, a mouse, or even a person fit under it?

Unit 7

Sub-Unit 1
Extensions

Circles

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1 and 3 with all students.

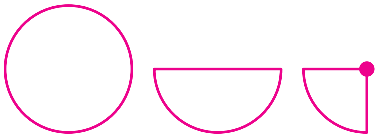
Problem 1

Students will extend their understanding of circles by explaining a way to determine the center of a circle without a ruler or compass.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can folding the paper help you determine the center?

Explanations vary. **If I fold the paper circle in half, and then fold it in half again, I would be able to see the center of the circle.**



Problem 2

Students will extend their understanding of circumference by calculating the length of a circular spring toy.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many loops can you count? How can the number of loops help you estimate the total length?

The length is about 12.5 meters or $1{,}253$ centimeters.
Explanations vary. **I can compute the approximate length if I know the diameter of the circle and the number of loops. The diameter of the toy is about 9.5 centimeters, and I counted 42 loops.**
$9.5 \cdot \pi \cdot 42 \approx 1252.8$.

Continued next page ...

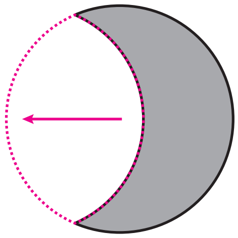
Problem 3

Students will extend their understanding of circumference by determining the perimeter of a circular shape.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Complete the outer arc of the region to a full circle. What do you notice?

14π or about 44 centimeters. *Explanations vary*. The inner arc can be repositioned to create a full circle with the outer arc since both arcs are part of a circle with the same radius.



The perimeter of the shaded region is equivalent to the circumference of a circle with a radius of 7.
$C = 2\pi \cdot 7 = 14\pi \approx 43.98$.

Problem 4

Students will extend their understanding of π by considering how changing the circumference changes the diameter.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine how the circumference of a circle changes with each meter added to its diameter.

Yes. *Explanations vary*.
A flea, mouse, or person would be able to fit underneath it.

Each meter added to the diameter of a circle adds about 3.1 meters to the circumference of the circle.

So if the circumference of Earth is increased by 10 meters, this means that a little more than 3 meters have been added to the diameter. So there would be about 1.5 meters of distance between the wire and Earth, making it easy for a flea, mouse, or person to go under the wire.

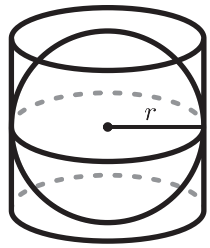
Solid Geometry

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

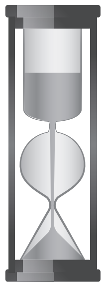
Archimedes made many significant mathematical discoveries, but he was most proud of his work involving a sphere inscribed in a cylinder. According to a legend, he even requested that his tombstone feature this discovery!



Determine the ratio of the volume of the sphere to the volume of the cylinder when the sphere is perfectly inscribed within the cylinder. ($V_{cylinder} = \pi \bullet r^2 \bullet h$, $V_{sphere} = \frac{4}{3}\pi \bullet r^3$)

2

Rod designs an hourglass that is made up of three parts: a cylinder, a sphere, and a cone, all with the same radius r.

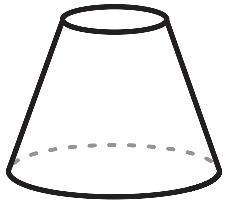


Determine the height h of the cylinder and the cone so that the sand from the cylinder completely fills the sphere and the cone.

($V_{cylinder} = \pi \bullet r^2 \bullet h$, $V_{sphere} = \frac{4}{3}\pi \bullet r^3$, $V_{cone} = \frac{1}{3}\pi r^2 h$)

3

A frustum is the result of taking a cone and slicing off a smaller cone using a cut parallel to the base. Determine a formula to calculate the volume of a frustum.



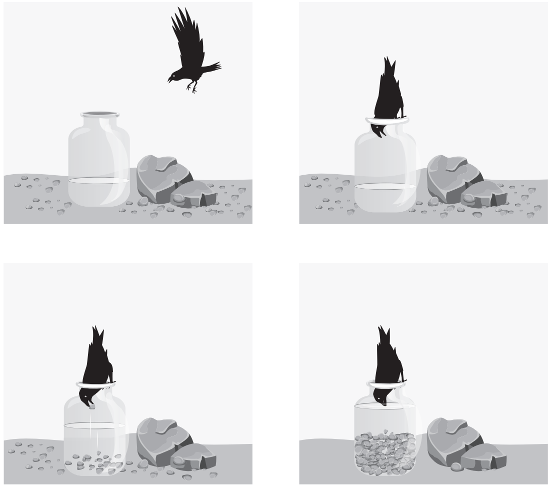
($V_{cone} = \frac{1}{3}\pi r^2 h$)

Name: .... Date: .... Period: ....

4

A thirsty crow wants to raise the level of water in a cylindrical container so that it can reach the water with its beak.

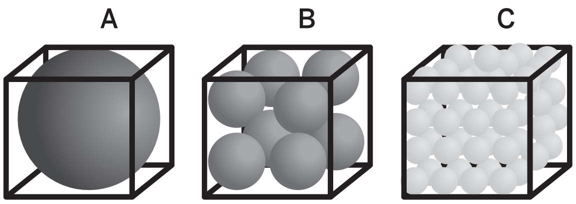
- The container has a diameter of 2 inches and a height of 9 inches.
- The crow needs to raise the water level from 6 inches to 8 inches in order to drink from it.



In order to raise the water level, the crow puts spherical pebbles in the container. If the pebbles are approximately $\frac{1}{2}$ inch in diameter, what is the fewest number of pebbles the crow needs to drop into the container in order to reach the water?

5

Here are three identical cubic tanks. The spheres in a given tank are the same size and packed wall-to-wall. If the tanks are filled to the top with water, which tank will hold the most water?



Solid Geometry

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing all problems with all students.

Problem 1

Students will extend their understanding of determining the volume of circular solids by investigating the volume of a sphere inscribed in a cylinder.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** When a sphere is inscribed in a cylinder, it means the sphere touches every face of the cylinder without any gaps. How do you think this helps you to determine the height of the cylinder?

2 : 3

***Explanations vary*. Because the sphere is inscribed in the cylinder, the height of the cylinder is the same as the diameter of the sphere.**

$V_{cylinder} = \pi \cdot r^2 h$

$V_{cylinder} = \pi \cdot r^2 \cdot 2r = 2 \cdot \pi \cdot r^3$

$V_{sphere} = \frac{4}{3}\pi \cdot r^3$

$\frac{V_{sphere}}{V_{cylinder}} = \frac{\frac{4}{3}\pi \cdot r^3}{2 \cdot \pi \cdot r^3} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$

Problem 2

Students will extend their understanding of determining the volume of circular solids by designing an hourglass made up of a cylinder, a cone and a sphere.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How does the volume of each shape relate?

$h_{cylinder} = h_{cone} = 2r$

***Explanations vary*. Because the volume of the cylinder must be the sum of the volumes of the cone and the sphere, I wrote:**

$V_{cylinder} = V_{sphere} + V_{cone}$

$\pi r^2 h = \frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$, then I canceled πr^2 from each term:

$h_{cylinder} = \frac{4}{3}r + \frac{1}{3}h_{cone}$

If the height of both the cone and the cylinder equals the diameter of the sphere, then the hourglass works perfectly.

Problem 3

Students will extend their understanding of the volume of a cone by applying the volume formula to determine a volume formula for a frustum.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Let R be the larger radius of the frustum, r be the smaller radius of the frustum, H be the height of the original cone, and h be the height of the conical piece cut off. Use these variables to help you figure out the formula.

***Responses vary*. I imagined the original cone before the top piece was cut off. Let R be the larger radius of the frustum, r be the smaller radius of the frustum, H be the height of the original cone, and h be the height of the conical piece which was cut off. Then the formula for the frustum is the volume: $V = \frac{1}{3}(\pi R^2 H - \pi r^2 h)$**

Continued next page ...

Problem 4

Students will extend their understanding of determining the volume of circular solids by solving a multi-step problem in a given context.

Provide students with the following hints if additional scaffolding is needed.

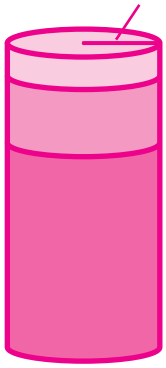
- **Hint 1:** The crow just needs to add a volume equal to a cylinder with a height of 2 in. and a radius of 1 in.
- **Hint 2:** What is the volume of each pebble?

96 pebbles. *Explanations vary*.

In order to raise the water level to 8 inches, the crow needs to add a volume of pebbles equivalent to a cylinder with a height of 2 inches and a radius of 1 inch. So the volume is:
$V = \pi r^2 \cdot h = \pi \cdot 1^2 \cdot 2 = 2\pi$

Each pebble has a volume of
$\frac{4}{3}\pi \cdot r^3 = \frac{4}{3}\pi \cdot \left(\frac{1}{4}\right)^3 = \frac{1}{48}\pi$

Therefore, the crow will need to add
$2\pi \div \frac{1}{48}\pi = 96$ **pebbles.**



Problem 5

Students will extend their understanding of the volume of a sphere by investigating the ratio of volumes of the spheres and a cube.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Start with representing the diameter of the large sphere in tank A by $2r$, then determine the side length of the tank.
- **Hint 2:** How many spheres are there in tank B? What are the radii of each of these spheres? What about tank C? How does the relation between the radius of the spheres and the number of spheres affect the total volume of the spheres?

All tanks will hold the same amount of water. *Explanations vary*.

Let the diameter of the sphere in tank A be $2r$. Then the cubic tanks have a volume of $V_{cube} = (2r)^3 = 8r^3$.

- **In tank A, sphere A has a volume of $V_A = \frac{4}{3}\pi r^3$.**
- **In tank B, The total volume of spheres is $V_B = 8 \cdot \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi r^3$.**
- **In tank C, The total volume of spheres is $V_C = 64 \cdot \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = \frac{4}{3}\pi r^3$.**

Therefore for each tank, the total volumes of the spheres are the same and the remaining volume for the water is the same.

Unit 8

Extensions

Organizing Numerical Data

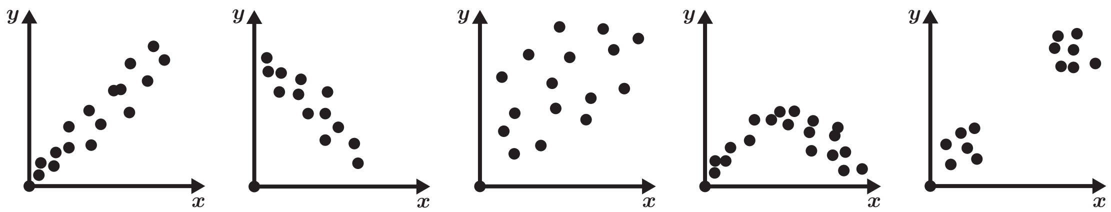
Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here are different scatter plot diagrams.

Scatter Plot A **Scatter Plot B** **Scatter Plot C** **Scatter Plot D** **Scatter Plot E**



Which scatter plot diagram would you expect to match each of the following?

a The physical fitness of a dog and the physical fitness of the owner.

b Number of salespersons and total dollar sales for real estate firms.

c Age of a person and their working memory.

d Total payroll and percent of wins of national league baseball teams.

e The amount spent on a week of TV advertising and sales of a cola.

f The number of swim suits owned and the distance to the nearest swimming area.

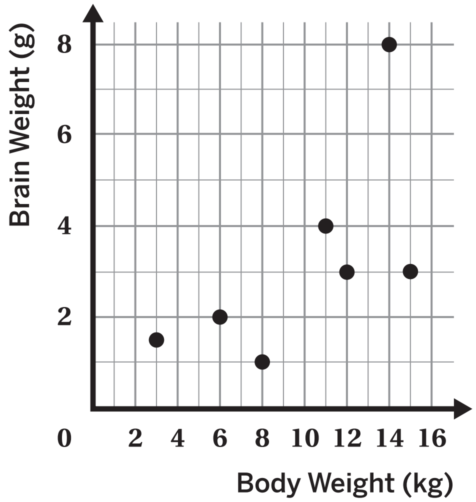
g Weight and the amount of physical activity for a thirty-year-old.

h The amount of chit-chat and guests' ages at a five-year-old's birthday party.

Organizing Numerical Data (continued)

Name: ________ Date: ________ Period: ________

2 The scatter plot shows the body weight and brain weight, in grams, of seven small animals.



a Compare the points $(3, 1.5)$ and $(12, 3)$. Which body weight to brain weight ratio is larger?

b Determine the average brain weight of the animals.

c Determine the average body weight of the animals.

d Circle the point that represents the animal with the smallest brain weight to body weight ratio.

Organizing Numerical Data

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of patterns in data by examining a different representation of data.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** The scatter plots provided don't need to be perfectly accurate; they just need to approximate the real data.

Responses vary.

a. Scatter Plot A
b. Scatter Plot A
c. Scatter Plot B
d. Scatter Plot C
e. Scatter Plot C
f. Scatter Plot C
g. Scatter Plot D
h. Scatter Plot E

Problem 2

Students will extend their understanding of patterns in data by examining a different representation of data.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider organizing the data using a table.

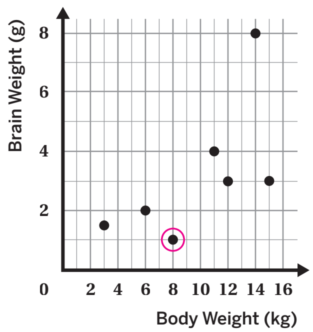
a. $(12, 3)$ *Explanations vary*. The animal represented by the point $(3, 1.5)$ has a body weight to brain weight ratio of $2000 : 1$. The animal represented by the point $(12, 3)$ has a body weight to brain weight ratio of $4000 : 1$.

b. 3.214 grams. Average brain weight is $\frac{1.5+2+1+4+3+8+3}{7} = \frac{22.5}{7} \approx 3.214$ g.

c. 9.857 kilograms. Average body weight is $\frac{3+6+8+11+12+14+15}{7} = \frac{69}{7} \approx 9.857$ kilograms.

d.

Brain weight (g)	Body weight (g)	Brain weight to body weight ratio
1.5	3000	0.0005
2	6000	0.00033
1	8000	0.000125
4	11000	0.00036
3	12000	0.00025
8	14000	0.000571
3	15000	0.0002



Analyzing Numerical Data

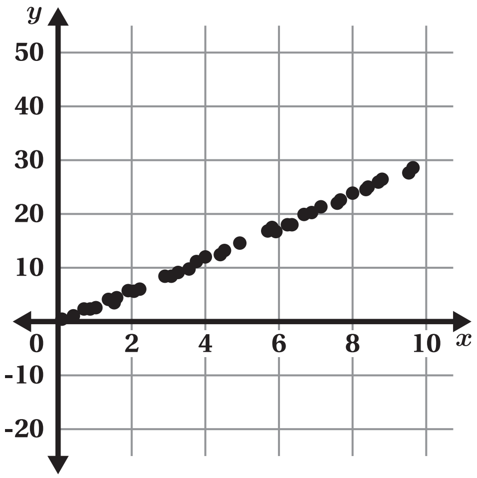
Name: .... Date: .... Period: ....

1

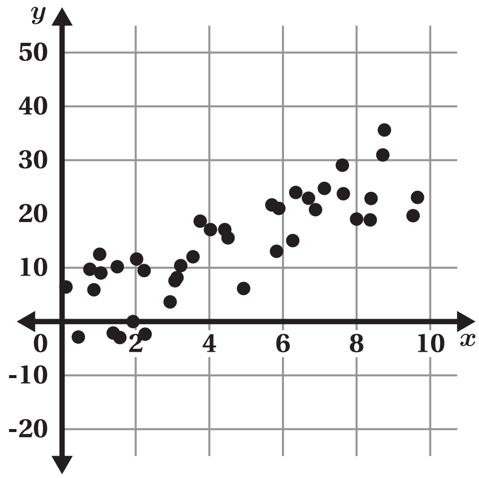
Each point on these scatterplots was created by randomly choosing an x-value between 0 and 10, then multiplying by 3 and adding a different random number to get the y-value.

- In Scatter Plot A, a random number between -0.5 and 0.5 was added to the y-coordinate.
- In Scatter Plot B, a random number between -8 and 8 was added to the y-coordinate.
- In Scatter Plot C, a random number between -20 and 20 was added to the y-coordinate.

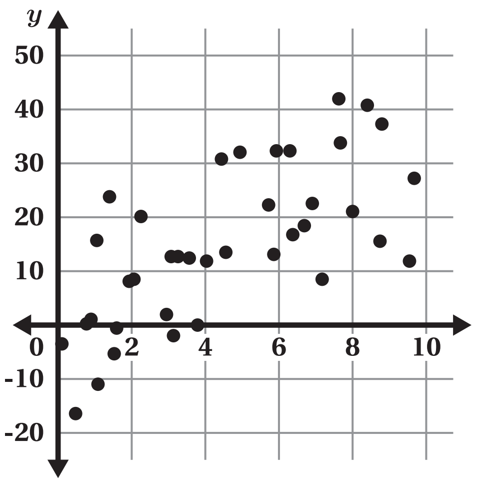
Scatter Plot A



Scatter Plot B



Scatter Plot C



a For each scatter plot, draw a line that fits the data.

Analyzing Numerical Data (continued)

Name: .... Date: .... Period: ....

b Why were some lines of fit more straightforward to draw than others?

c Write an equation in the form of $y = mx + b$ for each line of fit. What does the value of m tell you? The value of b?

d Create your own scatter plots using the same method, but this time multiply the x-coordinate by -4. Draw the lines of fit and determine the equation for each. What do you notice?

Analyzing Numerical Data

Assign the problem to students who want to extend their thinking.

Materials:
- graph paper

Problem 1

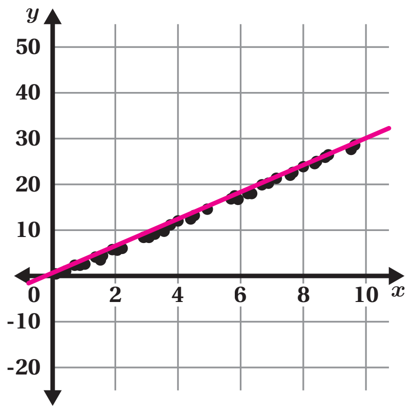
Students will extend their understanding of fitting lines to data by examining three different scatter plots.

Provide students with the following hint if additional scaffolding is needed.

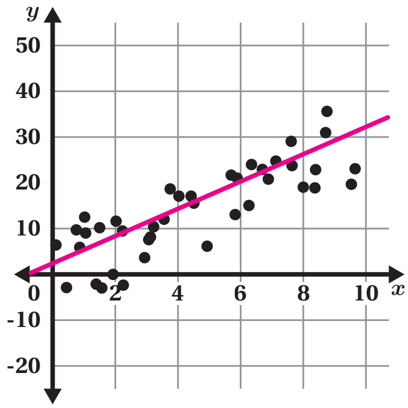
- **Hint:** In part c, What are the coordinates of two points you can use to write the equation of the line?

Correct response:

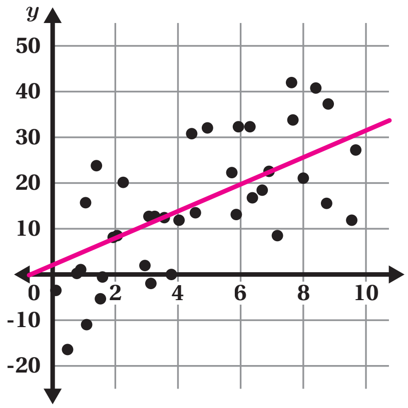
a. *Responses vary*. Samples shown on graph.
Scatter Plot A



Scatter Plot B



Scatter Plot C



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Problem 1 (continued)

b. *Responses vary*. Because the points were closer together in the first two scatter plots, it can be more straightforward to draw the line for those two graphs. Although there is some positive association in the third graph, it can be more challenging to draw a good line because the points are so spread out.

c. *Responses vary*.
Sample equations:
Scatter Plot A: $y = 3x + 0.2$
Scatter Plot B: $y = 3x + 3$
Scatter Plot C: $y = 3x + 1$
m represents the number that is multiplied by the x-coordinates and b represents the average of all the random numbers added to the product.

d. *Responses vary*. Equations should have -4 as the coefficient of x. All lines should have negative associations.
Sample equations:
Scatter Plot A: $y = -4x + 0.2$
Scatter Plot B: $y = -4x + 3$
Scatter Plot C: $y = -4x + 1$

Unit 9

Extensions

Exponent Properties

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 The Tower of Hanoi was invented by the French mathematician Édouard Lucas in 1883. The rules of this famous puzzle are:

- Only one disk may be moved at a time.
- Only the uppermost disk can be moved from one of the stacks and to the top of another stack or on an empty rod.
- No disk may be placed on top of a disk that is smaller than it.

In 2 moves, you can transfer 2 disks to another tower. In 7 moves, you can transfer 3 disks to another tower.

What is the minimum number of moves to transfer all the disks to another tower without placing a larger disk onto a smaller disk?

2 Consider this statement, $2^{12} = 4096$. What other whole numbers can you raise to a power to equal 4096?

Exponent Properties (continued)

Name: .... Date: .... Period: ....

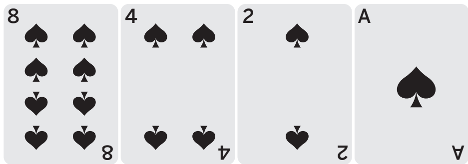
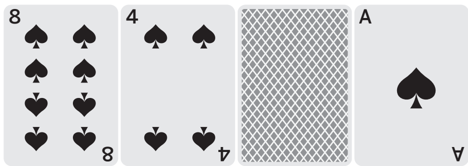
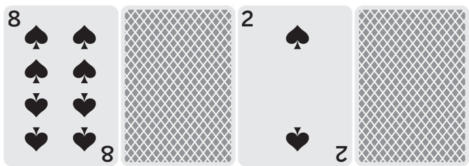
3 Without calculating, compare each pair of values to determine which value is greater.

a 3^{400} or 4^{300}

b π^{12} or 2^{25}

c 2^{20} or 10^{6}

4 Here is a way of coding numbers using specific playing cards.

15	
13	
10	

a How are the numbers coded?

b In *binary systems*, one can code situations like open and closed cards using 1 and 0. Ava writes 1111_2 for 15, 1101_2 for 13, and 1010_2 for 10. The subscript 2 is used to show that the number is coded in a binary system. How can Ava code 11 using binary coding?

c Compare the place value of the digits between the number system we use today (base-10) and the binary system (base-2).

d Complete the table.

Base-10	26	
Base-2 (binary)		100010_2

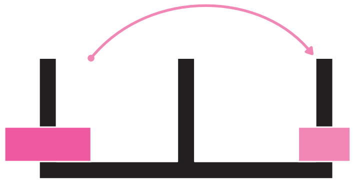
Exponent Properties

Name: ______________________ Date: ____________ Period: ____________

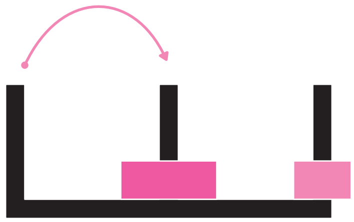
Hanoi Tower solution steps for 2 disks

Starting Position

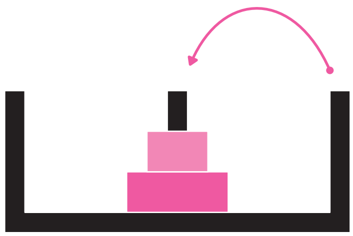
Move 1



Move 2



Move 3

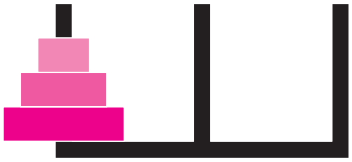


Exponent Properties (continued)

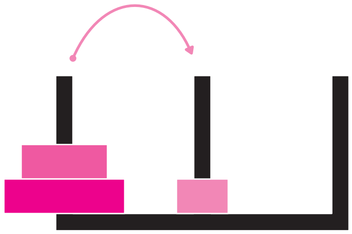
Name: ____________________ Date: __________ Period: __________

Hanoi Tower solution steps for 3 disks

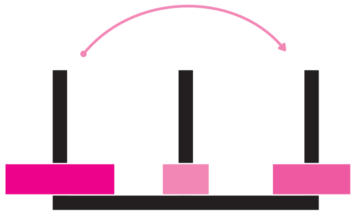
Starting Position



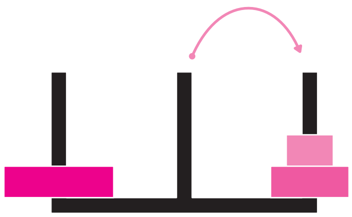
Move 1



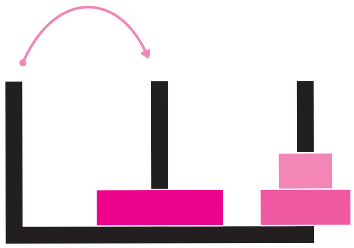
Move 2



Move 3

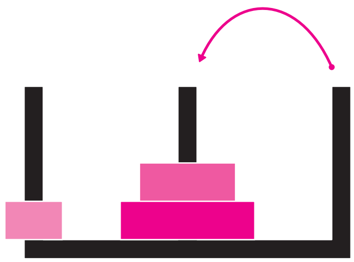


Move 4

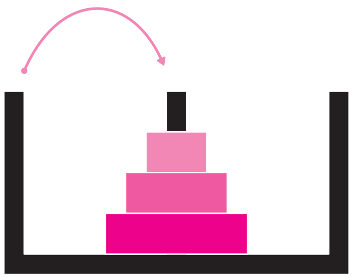


Move 5

Move 6



Move 7



Exponent Properties

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order. If time allows, consider sharing Problems 1–4 with all students.

Materials

- Sub Unit 1 Extension PDF **(Problem 1)**
- Hanoi Tower Game (optional) **(Problem 1)**

Problem 1

Students will extend their understanding of exponents by playing the Hanoi Tower game.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Use the Sub Unit 1 Extension PDF to explore the moves needed for 2 disks and 3 disks. Then determine the number of moves it would take for 4 disks? Organize your findings using a table.

127 moves. *Explanations vary.*

I first played with 2 disks and transferred them to a new bar in 3 moves. Then transferred 3 disks in 7 moves and 4 disks in 15 moves.

Number of disks	*Number of moves*
2	3
3	7
4	15
5	31
6	63
7	127

Then I noticed that for n disks the number of moves is $2^n - 1$. For 7 disks, it will be $2^7 - 1 = 128 - 1 = 127$.

Problem 2

Students will extend their understanding of the power rule by rewriting a power using different values.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Write 4,096 as a power of 2. How many different ways can you rewrite this power using the power rule?

Responses vary. Because $4096 = 2^{12}$, it can be broken down into other representations of the form $(2^m)^n$, so that $m \cdot n = 12$

- $(2^2)^6 = 4^6 = 4096$
- $(2^3)^4 = 8^4 = 4096$
- $(2^4)^3 = 16^3 = 4096$
- $(2^6)^2 = 64^2 = 4096$
- $(2^{12})^1 = 4096^1 = 4096$

Continued next page ...

Exponent Properties (continued)

Problem 3

Students will extend their understanding of multiplying powers with different bases by examining and comparing five different statements.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can exponent rules help you rewrite powers to make the comparisons easier?

a. 3^{400}. *Explanations vary*.
$(3^4)^{100} = 81^{100}$ and $81^{100} > 64^{100} = (4^3)^{100}$

b. 2^{25}. *Explanations vary*.
$\pi^{12} < 4^{12} = 2^{24}$ and $2^{24} < 2^{25}$

c. 2^{20}. *Explanations vary*.
$2^{10} = 1024$ and $1024 > 1000 = 10^3$

Problem 4

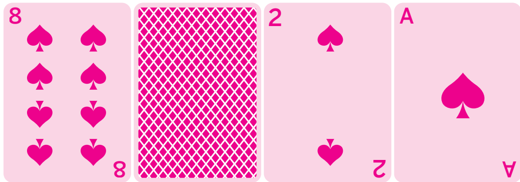
Students will extend their understanding of exponents by exploring the binary system.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** What pattern do you notice in the cards that are used in the first row $(8, 4, 2, 1)$?
- **Hint 2:** In part a, what is the sum of all four cards in each row?

a. *Explanations vary*. Four cards with powers of 2 are used. Their sums represent the number. If a card is closed, you do not add that card to the sum.

b. 1011_2. *Explanations vary*.



$8 + 2 + 1 = 11$

c. In the base-10 system, each digit represents a power of 10, with place values increasing from right to left (e.g., 1s, 10s, 100s, etc.). In the binary system (base-2), place values increase as powers of 2 (e.g., 1s, 2s, 4s, 8s, etc.). Binary uses only two digits (0 and 1), while base-10 uses ten digits (0 through 9).

d.

Base-10	26	34
Base-2 (binary)	11010_2	100010_2

Scientific Notation

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

This problem requires multiple steps to solve. You may not know all the facts needed to solve the problem. You may also need to make some assumptions. That is okay — make a conjecture or estimate about anything that you do not know.

If everyone alive on Earth right now stood next to each other, how much area would they cover?

2

Humans tend to calculate with numbers using powers of 10, but computers calculate with numbers using powers of 2.

Binary System (Powers of 2)	
Binary kilobyte	2^{10} bytes
Binary megabyte	2^{20} bytes
Binary gigabyte	2^{30} bytes
Binary terabyte	2^{40} bytes

Decimal System (Powers of 10)	
Kilobyte (KB)	10^3 bytes
Megabyte (MB)	10^6 bytes
Gigabyte (GB)	10^9 bytes
Terabyte (TB)	10^{12} bytes

a Which is greater: a binary gigabyte or a regular gigabyte? How many more bytes is it?

b Which is greater: a binary terabyte or a regular terabyte?

Scientific Notation (continued)

Name: .... Date: .... Period: ....

3 Here is a table showing the speed of light in different mediums. In which medium does light travel the fastest? Slowest?

Medium	Speed
Air	2.99×10^{8} m/s
Water	2.3×10^{10} cm/s
Ice	2.2×10^{5} km/s
Glass	6.6×10^{8} ft/s
Vacuum	3.0×10^{8} m/s

4 The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun?

Scientific Notation

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order. If time allows, consider sharing Problems 1–4 with all students.

Problem 1

Students will extend their understanding of using scientific notation by comparing quantities of different units.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** There are around 8×10^9 people on Earth. How many of them do you think are adults? How many are kids?
- **Hint 2:** Think about an adult standing on a rectangle comfortably. On average, what do you think the dimensions of that triangle are?

Responses vary.

There are around 8×10^9 people on Earth right now. Most adults can fit into a rectangle of about 1 meter by 0.5 meters when standing, so 0.5 square meters. Smaller children would take up less space — about half the space of an adult, so I assume children take up a 0.25 square meters. I am guessing that about a quarter of the people on Earth are small children. The space the adults will take up should be 6×10^9 times 0.5 square meters, which is 3×10^9 square meters. The children will take up 2×10^9 times 0.25 square meters, which is 0.5×10^9 square meters. In total, people will cover about 3.5×10^9 square meters.

Problem 2

Students will extend their understanding of multiplying powers by rewriting powers in various ways.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider starting with comparing kilobytes.

a. A binary gigabyte is about 74 million bytes (74 megabytes) more than a regular gigabyte. *Explanations vary*. A binary kilobyte is equal to $2^{10} = 1024$ bytes which is larger than $10^3 = 1000$ bytes. So $2^{10} \cdot 2^{10} > 10^3 \cdot 10^3$ and $2^{10} \cdot 2^{10} \cdot 2^{10} > 10^3 \cdot 10^3 \cdot 10^3$. So, a binary gigabyte is $(2^{10})^3$ (or $1,073,741,824$) bytes.

b. A binary terabyte is about 100 billion bytes (100 gigabytes) more than a regular terabyte. *Explanations vary*. A binary terabyte is $1,024$ times a binary gigabyte, so a binary terabyte would be $1,099,511,627,776$ bytes.

Continued next page ...

Scientific Notation (continued)

Problem 3

Students will extend their understanding of using scientific notation by comparing quantities of different units.

Provide students with the following hint if additional scaffolding is needed.

- **Hint 1:** 1 km $= 1000$ meters, 1 cm $= 0.01$ meters, and 1 ft ≈ 0.30 meters.

Fastest: Light travels fastest in a vacuum at approximately $3.0 \cdot 10^8$ meters per second

Slowest: Light travels slowest in glass, which is around $2.01 \cdot 10^8$ meters per second

Explanations vary. I converted all units to m/s:

Vacuum: $3.0 \cdot 10^8$ meters per second

Air: $2.99 \cdot 10^8$ meters per second

Water: $2.3 \cdot 10^8$ meters per second

Ice: $2.2 \cdot 10^8$ meters per second

Glass: $2.01 \cdot 10^8$ meters per second

Problem 4

Students will extend their understanding of using scientific notation by working with very large and very small quantities

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can we divide the exponents when dividing numbers in the scientific notation form?

Approximately 1.24×10^{57} protons.

Explanations vary.

Number of protons $=$ $\frac{\textit{Mass of a proton}}{\textit{Mass of the Sun}}$

Number of protons

$= \frac{1.98 \times 10^{33}}{1.6 \times 10^{-24}} = \frac{1.98}{1.6} \times 10^{33-(-24)}$

$\approx 1.24 \times 10^{57}$ protons

Unit 10

Extensions

Square Roots and Cube Roots

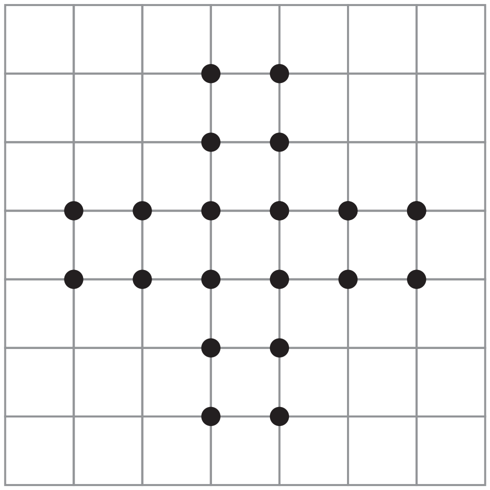
Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

a How many squares can you form by connecting any 4 points on this shape?

b How many different areas of squares can you form?



2

Determine the value of each expression.

a $\sqrt{1.21} - \sqrt{0.09} + \sqrt{0.25}$

b $\dfrac{\sqrt{1.44} - \sqrt{0.64}}{\sqrt{0.04}}$

c $\sqrt{14 + \sqrt{7 - \sqrt{1 + \sqrt{64}}}}$

d $\dfrac{\sqrt{6^2 + 8^2}}{\sqrt{5^2 + 12^2}}$

e $\sqrt{\frac{1}{24} \cdot \sqrt{\frac{1}{18} \cdot \sqrt{\frac{1}{4}}}}$

f $\sqrt{2 + \frac{1}{4}} - \sqrt{1 + \frac{7}{9}} + \sqrt{2 - \frac{8}{36}}$

Square Roots and Cube Roots (continued)

Name: .... Date: .... Period: ....

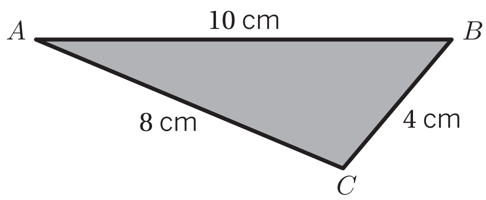
3

Heron of Alexandria, Egypt (60 B.C.E) derived a formula to determine the area of a triangle using only its side lengths. Here are his calculations with different triangles.

a For a triangle with side lengths a, b and c, can you figure out his formula?

Triangle	Half of the perimeter	Results of Heron's calculations	Area
$a = 3$, $b = 4$, $c = 5$	6	3, 2, 1	$\sqrt{6 \cdot 3 \cdot 2 \cdot 1} = 6$
$a = 8$, $b = 15$, $c = 17$	20	12, 5, 3	$\sqrt{20 \cdot 12 \cdot 5 \cdot 3} = 60$
$a = 7$, $b = 24$, $c = 25$	28	21, 4, 3	$\sqrt{28 \cdot 21 \cdot 4 \cdot 3} = 80$
a, b, c	s	____, ____, ____,	$\sqrt{s \cdot \text{..........}}$

b Determine the area of triangle ABC.



4

According to a legend, the ancient city of Delos faced a terrible plague. An oracle told them the plague would go away if they built a new temple that was exactly twice the volume of the existing one. Delians doubled the edges of the temple, but the plague did not stop. Instead of doubling the side lengths of the temple, what should Delians do to stop the plague?

Unit 10

Sub-Unit 1

Extensions

Square Roots and Cube Roots

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order. If time allows, consider sharing Problems 1–4 with all students.

Problem 1

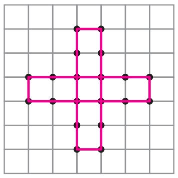
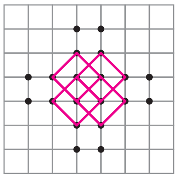
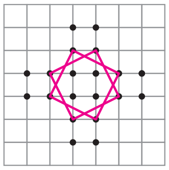
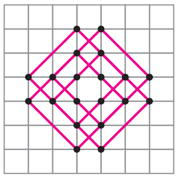
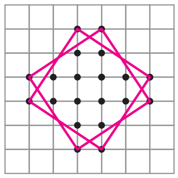
Students will extend their understanding of the area of a square with vertices at the intersections of grid lines by drawing squares and calculating their areas on a given grid.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many 1×1 squares can you draw? How many 2×2? What about the tilted squares?

a. 21 squares

b. 5 different areas

Number of Squares	9	4	2	4	2
Sizes of Squares	1×1	$\sqrt{2} \times \sqrt{2}$	$\sqrt{5} \times \sqrt{5}$	$\sqrt{8} \times \sqrt{8}$	$\sqrt{13} \times \sqrt{13}$
Drawings					

Problem 2

Students will extend their understanding of square roots by making calculations involving four operations.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In parts a, b, d, and e, write each decimal under the root sign as a fraction to determine the value.

a. $\sqrt{1.21} - \sqrt{0.09} + \sqrt{0.25} = \sqrt{\frac{121}{100}} - \sqrt{\frac{9}{100}} + \sqrt{\frac{25}{100}} = \frac{11}{10} - \frac{3}{10} + \frac{5}{10} = \frac{13}{10}$ (or equivalent)

b. $\frac{\sqrt{1.44} - \sqrt{0.64}}{\sqrt{0.04}} = \frac{\frac{12}{10} - \frac{8}{10}}{\frac{2}{10}} = \frac{\frac{4}{10}}{\frac{2}{10}} = 2$

c. $\sqrt{14 + \sqrt{7 - \sqrt{1 + \sqrt{64}}}} = \sqrt{14 + \sqrt{7 - \sqrt{1 + 8}}} = \sqrt{14 + \sqrt{7 - \sqrt{9}}} = \sqrt{14 + \sqrt{7 - 3}}$

$\sqrt{14 + \sqrt{4}} = \sqrt{14 + 2} = \sqrt{16} = 4$

d. $\frac{\sqrt{6^2 + 8^2}}{\sqrt{5^2 + 12^2}} = \frac{\sqrt{36 + 64}}{\sqrt{25 + 144}} = \frac{\sqrt{100}}{\sqrt{169}} = \frac{10}{13}$

e. $\sqrt{\frac{1}{24} \cdot \sqrt{\frac{1}{18} \cdot \sqrt{\frac{1}{4}}}} = \sqrt{\frac{1}{24} \cdot \sqrt{\frac{1}{18} \cdot \frac{1}{2}}} = \sqrt{\frac{1}{24} \cdot \sqrt{\frac{1}{36}}} = \sqrt{\frac{1}{24} \cdot \frac{1}{6}} = \frac{1}{12}$

f. $\sqrt{2 + \frac{1}{4}} - \sqrt{1 + \frac{7}{9}} + \sqrt{2 - \frac{8}{36}} = \sqrt{\frac{9}{4}} - \sqrt{\frac{16}{9}} + \sqrt{\frac{64}{36}} = \frac{3}{2} - \frac{4}{3} + \frac{8}{6} = \frac{3}{2}$ (or equivalent)

Continued next page ...

Problem 3

Students will extend their understanding of square roots by exploring Heron's formula.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the difference between the half of each triangle and side a. Then determine the differences with side b and side c. What did you notice?

a. Heron determined the differences between each side of the triangle and the half of the perimeter. Then he calculated the square root of the product of all these numbers.

Area: $\sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$

b. For the triangle with the side lengths 8, 4, and 10 centimeters, half of the perimeter is $\frac{8+4+10}{2} =$ 11 cm. Then I determined the area as

$\sqrt{11 \cdot (11-8) \cdot (11-4) \cdot (11-10)}$

$= \sqrt{11 \cdot 3 \cdot 7 \cdot 1}$

$\sqrt{231} \approx 15.2$ square centimeters.

Problem 4

Students will extend their understanding of the relationship between the edge length and the volume of a cube by solving an ancient mystery.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What happens to the volume when you double the sides?

To build a new temple that is twice the volume of the existing one, Delians should have multiplied the current side length by $\sqrt[3]{2}$ instead of doubling the side lengths.

Explanations vary. Let's assume the current side length of the temple is x units. Then the current volume of the temple is x^3 cubic units. When you double the side lengths, $2x$, the new volume will be $(2x)^3 = 8x^3$, which is 8 times as large as the original. To get a volume that is two times the original, $2x^3$, the side length should be $\sqrt[3]{2x^3} = \sqrt[3]{2}$ times larger than the original.

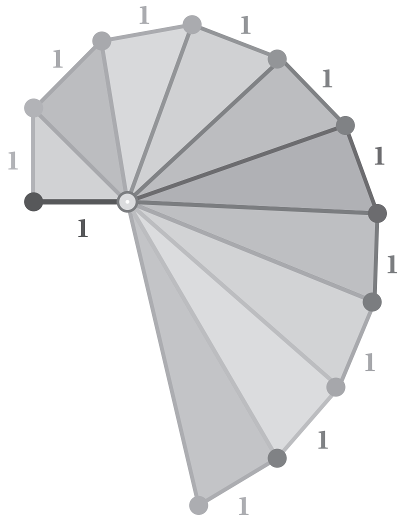
The Pythagorean Theorem

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here is a shape made up of right triangles called the *Spiral of Theodorus*.



a Determine the length of each hypotenuse.

b What do you notice? What do you wonder?

c Measure the length of each hypotenuse to write the decimal approximation for each square root.

2

A *Pythagorean triple* consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. For example, $(3, 4, 5)$ and $(5, 12, 13)$ are Pythagorean triples. There are infinitely many Pythagorean triples and there are many different ways to determine them.

Can you find the relationship between side length a and side lengths b and c.

a When a is an odd positive integer?

a	3	5	7	9	11
b and c	4, 5	12, 13	24, 25	40, 41	60, 61

b When a is an even positive integer?

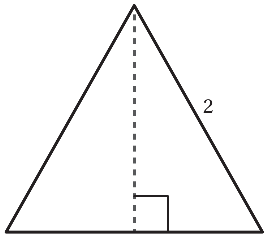
a	4	6	8	10	12
b and c	3, 5	8, 10	15, 17	24, 26	35, 37

The Pythagorean Theorem (continued)

Name: .... Date: .... Period: ....

3

Here is an equilateral triangle. The length of each side is 2 units and the height is drawn. In an equilateral triangle, the height divides the opposite side into two pieces of equal length.



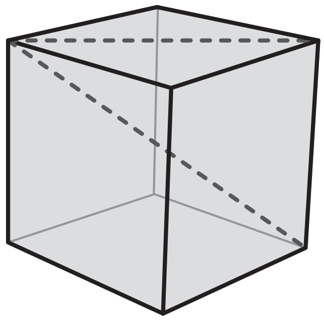
a Determine the exact height.

b Determine the area of the equilateral triangle.

c If a is the length of each side in the equilateral triangle, express its area in terms of a.

4

Determine the length of the solid diagonal (the line segment that connects two opposite vertices of the solid) of a cube . . .

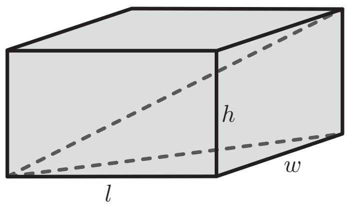


a When the side length is 2 cm.

b When the side length is 5 cm.

c When the side length is a cm.

d Express the length of the solid diagonal of a rectangular prism in terms its length, l, width, w, and height, h.



The Pythagorean Theorem

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order.If time allows, consider sharing Problems 1–4 with all students.

Material

- Ruler (Problem 1)

Problem 1

Students will extend their understanding of the Pythagorean theorem by exploring the spiral of Theodorus.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** First determine the hypotenuse of the isosceles right triangle with side lengths of 1 unit using the Pythagorean theorem. How can the hypotenuse of this first triangle help you determine the hypotenuse of the next one?

a. $\sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots, \sqrt{10}, \sqrt{11}$

b. Responses vary.

I notice:

- It's built by stacking right triangles so that each hypotenuse gives the square root of an integer.
- The spiral can be used to construct square roots like $\sqrt{2}, \sqrt{3}, \sqrt{5}$, and $\sqrt{6}$.

I wonder:

- Does the Spiral ever cross Itself?
- Do the hypotenuses ever form a straight line?
- Can it model natural spirals like a Nautilus shell?
- Is there a pattern in the triangle angles?

c.

$\sqrt{2} \approx 1.4$	$\sqrt{3} \approx 1.7$	$\sqrt{4} = 2$	$\sqrt{5} \approx 2.2$	$\sqrt{6} \approx 2.4$
$\sqrt{7} \approx 2.6$	$\sqrt{8} \approx 2.8$	$\sqrt{9} = 3$	$\sqrt{10} \approx 3.2$	$\sqrt{11} \approx 3.3$

Problem 2

Students will extend their understanding of Pythagorean Theorem by exploring Pythagorean triples.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Both in parts a and b, determine the square of a to explore its connection to the values b and c.

a. *Responses vary*. I determined a^2 for each example and noticed that $b + c = a^2$. Also, in all the examples b and c are consecutive integers. I can determine b and c by squaring a and then halving it. The largest integer less than its half and the smallest integer more than the half are b and c. For example, when $a = 11$, then $a^2 = 121$. By halving 121, I find 60.5. So the other two sides are 60 and 61.

b. *Responses vary*. I determined the square of a for each example and noticed that $b + c = \frac{a^2}{2}$. Also, in all the examples b and c are consecutive odd or consecutive even integers. I can determine b and c by calculating $\frac{a^2}{2} \div 2$. b is the largest integer less than the quotient and c is the smallest integer bigger than the quotient. For example, when $a = 10$, then $\frac{a^2}{2} = 50$. By halving 50, I find 25. So the other two sides are 24 and 26.

Continued next page ...

The Pythagorean Theorem (continued)

Problem 3

Students will extend their understanding of determining unknown side lengths of right triangles by expressing the area of an equilateral triangle with a side length of x, in terms of x.

Provide students with the following hints if additional scaffolding is needed.

Hint 1: In part a, focus on half of the equilateral triangle. How long is the hypotenuse of that right triangle? How long is the otherside? How can you use the lengths of two sides to determine the height?

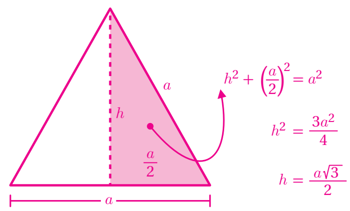
Hint 2: In part c, express the length of height in terms of the side length a using the Pythagorean theorem.

a. $\sqrt{3}$ units. *Explanations vary*. Because the height divides the side length into two equal parts, each leg of the resulting two right triangles is 1 unit. I used the Pythagorean theorem to determine the height of the triangle. $h^2 + 1^2 = 2^2$, so $h^2 = 3$ and $h = \sqrt{3}$.

b. $\sqrt{3}$ square units. *Explanations vary*. The base of the triangle is 2 units and the height is $\sqrt{3}$ units. I determined the area using the area of a triangle formula.

$$A = \frac{b \cdot h}{2} = \frac{2 \cdot \sqrt{3}}{2} = \sqrt{3}$$

c. $\frac{a^2\sqrt{3}}{4}$. *Explanations vary*. If each side length is a, then to determine the height, I would use the right triangle with leg $\frac{a}{2}$ and hypotenuse a.



Then the area,

$$\frac{a \cdot \frac{a\sqrt{3}}{2}}{2} = \frac{a^2\sqrt{3}}{4}$$

Problem 4

Students will extend their understanding of the Pythagorean theorem by determining the solid diagonals of a cube and a rectangular prism.

Provide students with the following hint if additional scaffolding is needed.

Hint: First determine the length of the face diagonal. Then, focus on the right triangle with right sides as the face diagonal and a side length of the cube.

a. $2\sqrt{3}$ (or equivalent). *Explanations vary*. The face diagonal of a cube is $\sqrt{2^2 + 2^2} = \sqrt{8}$. The solid diagonal of the cube is the hypotenuse of the right triangle with sides 2 and $\sqrt{8}$. I calculated the length of solid diagonal as $\sqrt{4 + 8} = \sqrt{12} = 2\sqrt{3}$.

b. $5\sqrt{3}$

c. $a\sqrt{3}$

d. $\sqrt{l^2 + w^2 + h^2}$. *Explanations vary*. I determined the face diagonal as $\sqrt{l^2 + w^2}$. Then to determine the length of the solid diagonal, I calculated the length of the hypotenuse of the right triangle with sides $\sqrt{l^2 + w^2}$ and h.

Unit 10
Sub-Unit 3
Extensions

Rational and Irrational Numbers

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 $\sqrt{1.\square\square}$ is a rational number. List *all* pairs of digits that would make the expression rational?

2 Give an example of a rational and an irrational number that is between the following numbers.

a 2 and 2.1

b 3.14 and 3.15

c $\frac{1}{7}$ and $\frac{2}{7}$

3 Estimate the value of $\sqrt[3]{75}$ to the nearest hundreds.

Rational and Irrational Numbers (continued)

Name: .... Date: .... Period: ....

4 Luca knows how to convert repeating decimals such as $0.\overline{7}$, $0.\overline{52}$, or $0.\overline{125}$ to fractions easily.

$0.\overline{7} = \frac{7}{9}$ $\quad$ $0.\overline{52} = \frac{52}{99}$ $\quad$ $0.\overline{125} = \frac{125}{999}$

Here are more repeating decimals Luca wrote as fractions using the equations.

$0.0\overline{7} = \frac{7}{90}$	$0.1\overline{7} = \frac{16}{90}$	$0.3\overline{5} = \frac{32}{90}$	$0.00\overline{2} = \frac{2}{900}$
$0.0\overline{14} = \frac{14}{990}$	$0.\overline{25} = \frac{25}{990}$	$0.1\overline{25} = \frac{124}{990}$	

What do you notice? Can you find a relationship between the repeating and non-repeating digits of the decimal and the numerator and the denominator of the fraction?

Rational and Irrational Numbers

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order. If time allows, consider sharing Problems 1–3 with all students.

Problem 1	
Students will extend their understanding of rational and irrational numbers by solving a number puzzle. **Provide students with the following hint** if additional scaffolding is needed. • **Hint:** Under what condition can $\sqrt{1.\square\square}$ be a rational number?	5 different values. *Explanations vary*. For $\sqrt{1.\square\square}$ be rational, then $1.\square\square$ must be a perfect square. I wrote $1.\square\square$ as a fraction $\frac{1.\square\square}{100}$. When the numerator is 100, 121, 144, 169, 196, then $\frac{1\square\square}{100}$ is a perfect square.

Problem 2	
Students will extend their understanding of rational and irrational numbers by finding an example for both in the given intervals. **Provide students with the following hint** if additional scaffolding is needed. • **Hint:** Think about the square roots and/or cube roots that have an approximation between the given values.	*Responses vary.*

	Rational	Irrational
a. 2 and 2.1	2.03	$\sqrt{4.1}$
b. 3.14 and 3.15	3.14	π
c. $\frac{1}{7}$ and $\frac{2}{7}$	$\frac{3}{14}$	$\sqrt{0.05}$

Problem 3	
Students will extend their understanding of rational and irrational numbers by finding the cube root of a number. **Provide students with the following hint** if additional scaffolding is needed. • **Hint:** Determine the perfect cube numbers less than and bigger than 75.	*Responses vary between 4.20 and 4.25.* I determined the perfect cube numbers less than and bigger than 75. Since $64 < 75 < 125$, then $\sqrt[3]{64} < \sqrt[3]{75} < \sqrt[3]{125}$. So $4 < \sqrt[3]{75} < 5$. Because 75 is much closer to 64, its cube root must be closer to 4. $(4.2)^3 = 74.088$, so $\sqrt[3]{75}$ must be bigger than but very close to 4.2. Therefore $4.20 < \sqrt[3]{75} < 4.25$.

Problem 4

Students will extend their understanding of converting repeating decimals to fractions by examining several conversions to look for patterns.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Start with focusing the denominator of the fractions. How does the number of repeating and non-repeating digits of the decimal affect the denominator?

I notice:

- For each digit in the non-repeating part of the decimal, a 0 is placed as a digit to the denominator.
- For each digit in the repeating block of the decimal, a 9 is placed as a digit to the denominator.

For example, $0.0\overline{25}$ has 2 digits repeating and 1 digit non-repeating, so the denominator of the fraction is 990.

- To determine the numerator, you write the whole number by writing out both the non-repeating and repeating parts as if they were a single number, without the decimal point. Then subtract the non-repeating digit from this total.

For example, for $0.\overline{35}$, I write the decimal part as 35 then subtract the non-repeating digit 3 to determine the numerator as 32. So $0.\overline{35} = \frac{32}{90}$.

If I want to write a repeating decimal such as $0.32\overline{5}$ in fraction form, I first determine the denominator as 900 since there are 1 repeating digit and 2 non-repeating digits. Then to determine the numerator, I write the whole decimal part as 325, then subtract the non-repeating digits $325 - 32 = 293$. So $0.\overline{35} = \frac{293}{900}$.

Unit 11

Extensions

Probability

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

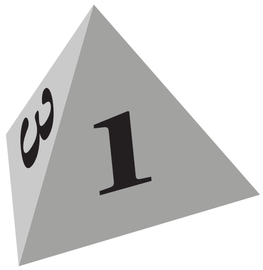
1 Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the theoretical probability that Angie and Carlos are seated opposite each other?

2 A fair six-sided die is rolled twice. What is the theoretical probability that the first number that comes up is greater than or equal to the second number?

Probability (continued)

Name: .... Date: .... Period: ....

Imagine a game in which two fair four-sided (tetrahedral) dice are rolled simultaneously. These dice are in the shape of a pyramid, and when a die is rolled, the outcome is determined by the side that lands face down. Suppose that for these two dice, the possible values (corresponding to the four sides of the die) that can be obtained from each die are as follows:



Die 1: 1, 2, 3, or 4   Die 2: 2, 4, 6, or 8

3 A certain game determines the movement of players' game pieces based on the SUM of the numbers on the face down sides when two dice are rolled. There are 10 distinct sum values that can occur, and some of those sums occur more often than others.

a Using an organized list, table, tree diagram, or method of your choosing, develop a list of all possible outcomes.

b Determine the 10 **distinct sum values** that are possible and calculate the probability of obtaining each sum value.

c Answer the following questions:

i. What is the probability of obtaining a sum of 5? ________

ii. What is the probability of obtaining a sum that is more than 5? ________

iii. What is the probability of obtaining a sum that is at most 5? ________

iv. What is the probability of obtaining a sum that is at least 5? ________

v. What is the probability of obtaining a sum that is no less than 5? ________

Probability

Assign problems to students who want to extend their thinking.

Problems 1, 2, and 3 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1	
Students will extend their understanding of calculating the theoretical probability of a repeated event involving the extra challenges of seating at a circular table. **Provide students with the following hints** if additional scaffolding is needed. • **Hint:** Start with one person in the first seat. What are the possible arrangements for the other three seats? • **Hint:** Repeat this process but with that same person in the second seat.	**$\frac{1}{3}$ (or equivalent). *Explanations vary*. Angie will be at one of the four seats of the table, leaving three open spots for Bridget, Carlos, and Diego. We can list the possible placements of the other three, proceeding clockwise from Angie: BCD, BDC, CBD, CDB, DCB, DCB. In 2 of these 6 possibilities Carlos is opposite Angie. This list can be repeated 4 times for the different four seats Angie might occupy. So there are 24 total possible seating arrangements and, in 8 of them, Carlos and Angie are opposite one another. Therefore the probability that Carlos and Angie will be seated opposite one another is $\frac{8}{24}$, or $\frac{1}{3}$.**

Problem 2	
Students will extend their understanding of calculating the theoretical probability of multiple successful events. **Provide students with the following hints** if additional scaffolding is needed. • **Hint:** Make a table for all possibilities that the first number is greater than or equal to the second number. • **Hint:** What is the sum of all of the possibilities in the table?	**$\frac{7}{12}$ (or equivalent). We can make a table for all possibilities that the first number is greater than or equal to the second. Adding up all of the possibilities in the table, there are 21 ways that the number on the second roll can be equal to or greater than the number on the first roll. Since there are $6 \times 6 = 36$ total possible outcomes, the probability that the first number rolled is greater than or equal to the second number rolled is $\frac{21}{36} = \frac{7}{12}$.**

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Problem 2 (Continued)

on 1st Throw	Possible on 2nd Throw
1	1
2	1, 2
3	1, 2, 3
4	1, 2, 3, 4
5	1, 2, 3, 4, 5
6	1, 2, 3, 4, 5, 6

Problem 3

Students will extend their understanding of calculating the theoretical probability of rolling a four-sided in a repeated experiment.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** "How can we organize the sample space of the sum of two dice?"

a.

SUM	Dice 2				
		2	4	6	8
Dice 1	1	3	5	7	9
	2	4	6	8	10
	3	5	7	9	11
	4	6	8	10	12

b.

SUM	Probability
3	$\frac{1}{16}$
4	$\frac{1}{16}$
5	$\frac{2}{16} = \frac{1}{8}$
6	$\frac{2}{16} = \frac{1}{8}$
7	$\frac{2}{16} = \frac{1}{8}$
8	$\frac{2}{16} = \frac{1}{8}$
9	$\frac{2}{16} = \frac{1}{8}$
10	$\frac{2}{16} = \frac{1}{8}$
11	$\frac{1}{16}$
12	$\frac{1}{16}$

Continued next page ...

Problem 3 (Continued)

c. i. $\frac{1}{8}$, or equivalent

ii. $\frac{3}{4}$, or equivalent

iii. $\frac{1}{4}$, or equivalent

iv. $\frac{7}{8}$, or equivalent

v. $\frac{7}{8}$, or equivalent

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