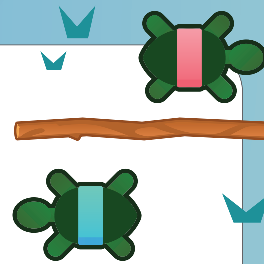
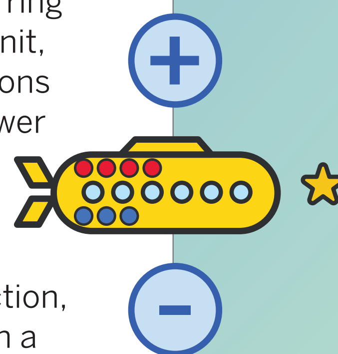


Unit 5

Operations With Positive and Negative Numbers

We can use positive and negative numbers to describe many situations in our everyday lives. How much warmer or cooler will it get as the day goes on? What will the new temperature be? How deep can a diver dive? What is the difference between the diver’s depth and a ring that fell to the bottom of the pool? In this unit, you will further explore performing operations with positive and negative numbers to answer everyday questions like these.



Essential Questions

- How do you represent addition, subtraction, multiplication, or division of numbers on a number line?
- How is solving problems with fractions or decimals the same or different from solving problems with only whole numbers?
- How can positive and negative numbers be used to represent real-world situations?

Summary | Lesson 1

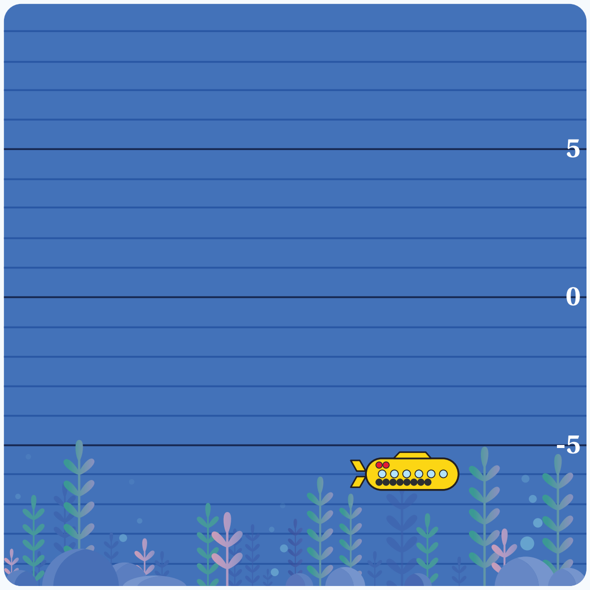
Using models such as floats and anchors on a vertical number line can be useful when representing addition and subtraction of positive and negative numbers.

For example, imagine a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- 3 floats being added represents moving up 3 units or $+3$.
- 2 anchors being removed represents moving up 2 units or $-(-2) = +2$.

The submarine's new position would be $-6 + 3 + 2 = -1$ units.

To move the submarine to 0 units from -1 units, 1 float can be added, represented by $+1$. -1 and $+1$ are an *opposite pair*, which means they add to 0.



Try This

A submarine is controlled by floats and anchors.

Enter the missing information in the table.

Starting Position	Action	Final Position
-3	Add 2 floats	-1
-3	Remove 2 anchors	
-3	Add 11 floats	
-3		0
-3		-7

Summary | Lesson 2

Different combinations of floats and anchors can give you the same result. Here are some examples:

- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. So adding a positive number is the same as subtracting a negative number.

Adding Floats	Removing Anchors
$4 + 2 = 6$	$4 - (-2) = 6$

- If a submarine starts at 5 units, removing 1 float or adding 1 anchor will both result in the submarine moving down to 4 units. So subtracting a number is the same as adding its opposite.

Removing Floats	Adding Anchors
$5 - 1 = 4$	$5 + (-1) = 4$

When you add two values that are opposites, the sum is always 0. These numbers are also called *additive inverses* of each other.

Try This

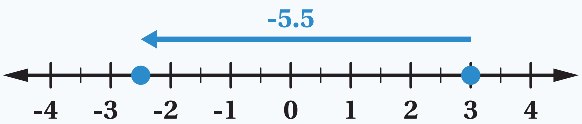
Complete the table for these four submarine situations.

Starting Position	Action	Expression	Final Position
-2	Add 6 floats	$-2 + 6$	4
	Remove 5 anchors	$1 - (-5)$	
3		$3 - 7$	
		$-1 + (-4)$	

Summary | Lesson 3

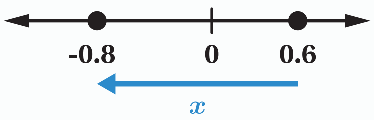
When you add positive and negative decimals and fractions, it might help to use a number line and think of each equation as representing start + change = end.

For example, in the equation $3 + (\text{-}5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving 5.5 units to the left), and x represents the end location.



Try This

a Write an equation that represents this challenge.

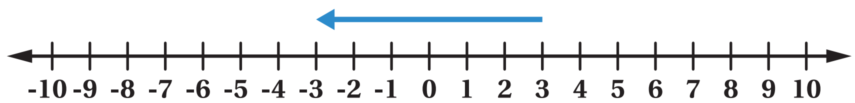
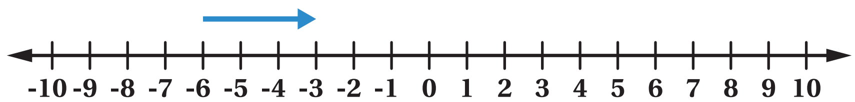


b What is the value of x that makes your equation true?

Summary | Lesson 4

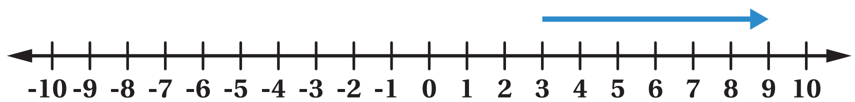
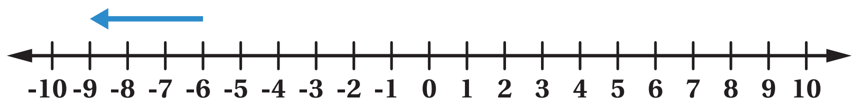
When determining the *sum* of terms in an expression, the order of the values does not change the final result. This is an example of the *commutative property*.

To represent addition on a number line, start at one of the values and use the other value for the direction and distance of the change.

3 + (-6)	
-6 + 3	

When determining the *difference*, the order of the values in an expression *does* affect the final result.

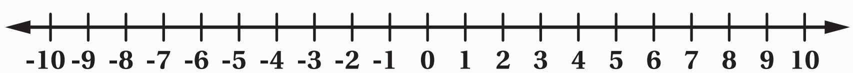
To represent subtraction on a number line, start at the first value and then use the second value to determine the direction and distance of the change. If subtracting a positive number, move to the left; if subtracting a negative number, move to the right.

3 – (-6)	
-6 – 3	

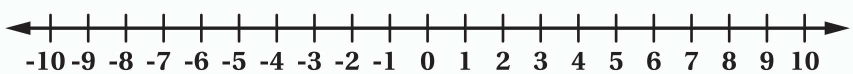
Try This

For each expression, complete the number line and determine the value of the expression.

a -5 – 2 = _____



b 2 – (-5) = _____



Summary | Lesson 5

When adding and subtracting **integers**, fractions, and decimals, there are multiple paths to the same value. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem as floats and anchors or think about it on a number line.
  For example, $(-3) + (-4)$ is like starting with (-3) and adding 4 anchors or moving 4 to the left.

- Rewrite subtraction as addition.
  For example, $-3 - 4$ can be rewritten as $-3 + (-4)$, which is -7.

- Combine numbers that add or subtract to make 0.
  For example, when adding -5 and 6, you can break 6 into $5 + 1$. Using properties of operations, we can add $-5 + 5 + 1$ in pieces. The $-5 + 5$ portion of the expression adds to 0 and $0 + 1 = 1$, so the final value is 1.

Try This

Fill in the blanks using the given numbers to make the equations true.

☐	−	☐	=	11
☐	+	9	=	☐

-1	-2	-3	-4
5	6	7	8

Summary | Lesson 6

We can use models, such as floats and anchors, to make sense of multiplying integers. Here are some examples for a submarine that starts at 0 units.

Action	Representation	Submarine's Direction	Final Value
Adding 2 groups of 3 floats	$2 \cdot 3$	Up	6
Removing 2 groups of 3 floats	$-2 \cdot 3$	Down	-6
Adding 2 groups of 4 anchors	$2 \cdot (-4)$	Down	-8
Removing 2 groups of 4 anchors	$-2 \cdot (-4)$	Up	8

Try This

Determine the value of each expression.

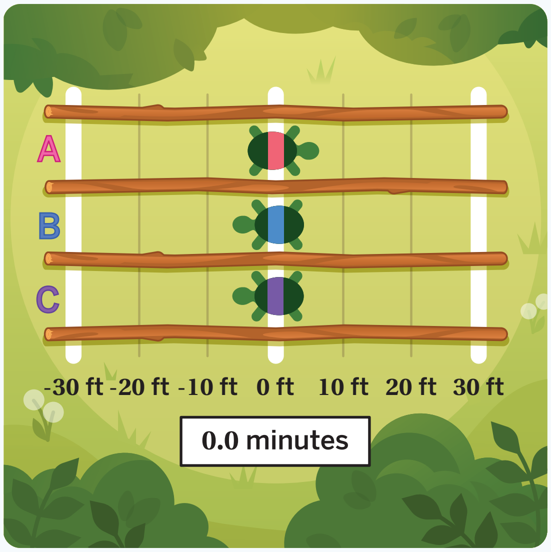
Action	Expression	Value
Add 3 groups of 5 anchors	$3 \cdot (-5)$	
Remove 4 groups of 6 floats	$(-4) \cdot 6$	
Remove 2 groups of 7 anchors	$(-2) \cdot (-7)$	

Summary | Lesson 7

Multiplying positive and negative numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or rate • time = change in position.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left 5 feet per minute. 6 minutes ago, Turtle B was at 30 feet because $(-5) \cdot (-6) = 30$.
- Turtle C walks to the left 3 feet per minute. In 4 minutes, Turtle C will be at -12 feet because $(-3) \cdot 4 = -12$.



Try This

A turtle travels at a constant rate.

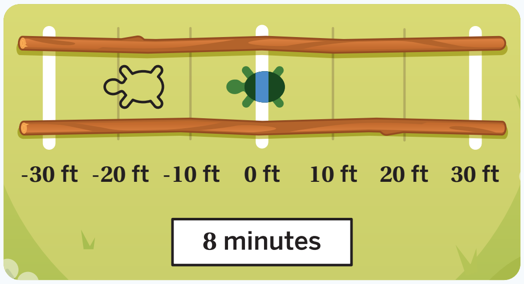
Complete the table to show the turtle's time and position.

Time	Position
-2	
0	0
1	-4
	-28

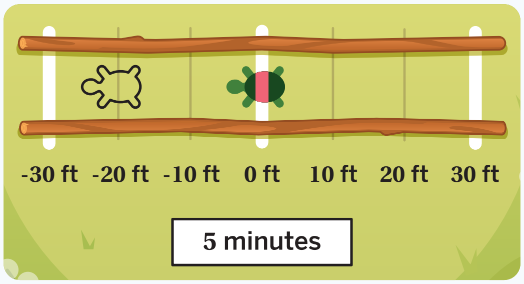
You can use your knowledge of position, rate, and time to multiply and divide positive and negative numbers.

Here are two examples.

Turtle A walks -2 feet per minute. If it starts at 0 feet, how long will it take to walk to -16 feet? Dividing position by walking rate tells us the time, so $\frac{-16}{-2} = 8$ minutes.



Turtle B takes 5 minutes to walk 20 feet to the left, or -20 feet. This means that each minute, Turtle B walks -4 feet. Dividing distance by time tells us the walking rate, so $\frac{-20}{5} = -4$.



Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Try This

Turtles A, B, and C are each traveling at a constant rate.

Complete the table to show the rate, time, and position of each turtle.

Turtle	Rate (ft/min)	Time (min)	Position (ft)
A	-3	2.5	
B	-2		-23
C		-2	11

Summary | Lesson 9

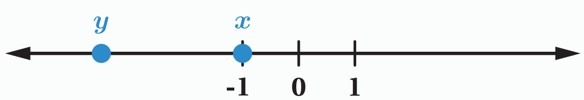
Thinking flexibly will help you reason about the value of expressions that contain variables. Using number lines and testing with example values are two strategies that can help us make sense of these types of expressions.

Some things that are familiar from using positive numbers are not the same when working with negative numbers.

For example, when x is a positive number, $10 - x$ will be less than 10. But if x is a negative number, $10 - x$ will be greater than 10, since subtracting a number is the same as adding its opposite.

Try This

This number line shows the positions of x and y.



Order the expressions from *least* to *greatest*.

$x - y$ $\qquad$ $x \cdot y$ $\qquad$ $2x$

Least $\qquad\qquad\qquad\qquad$ **Greatest**

Summary | Lesson 10

You can use properties of operations, order of operations, and multiplication and division of integers as strategies to solve integer puzzles.

For example, if we want to make this inequality true, it may be helpful to think about the signs of the sum inside the parentheses and the number outside of the parentheses.

Make the inequality true.

$\_\_\_\_ (\_\_\_\_ + \_\_\_\_) > 0$

$-8(-4 + 1) > 0$ is true because $-8(-3)$ is positive.

$-8(4 + 1) > 0$ is false because $-8(5)$ is negative.

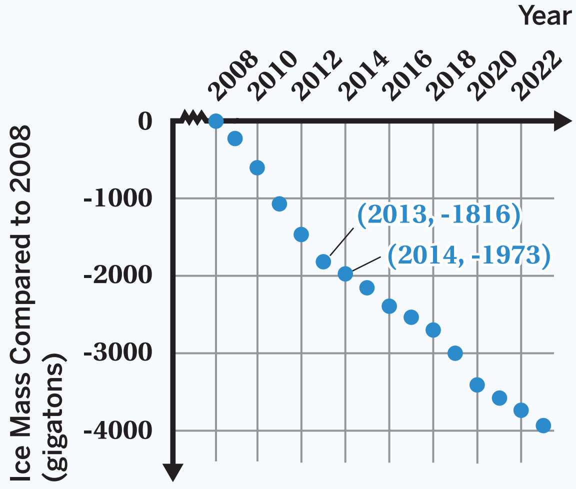
Try This

Fill in the blanks to make an expression with a negative value.

$\square \times \square + \square = ?$

Adding and subtracting positive and negative numbers can help you solve problems involving real-world situations.

You can tell from the graph that the ice mass in Greenland is decreasing each year. Subtraction lets you determine how much it decreased between any two years.



For example, to determine the change in the ice mass from 2013 to 2014, you need to find the difference between -1,973 and -1,816.

Since -1,973 is less than -1,816, the difference will be negative.

$-1973 - (-1816) = -1973 + 1816 = -157$

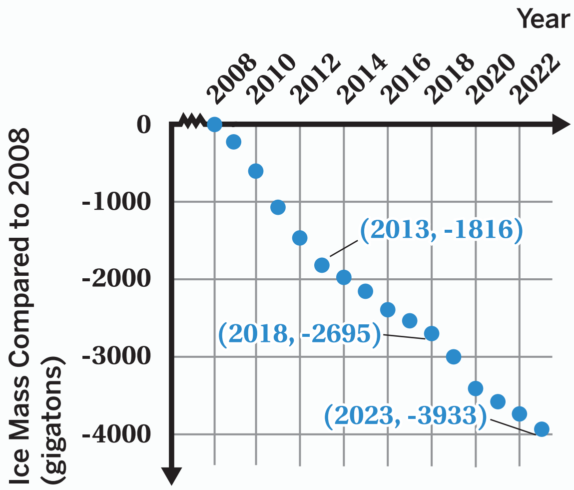
The change in ice mass from 2013 to 2014 is -157 gigatons. You can use this information to determine if the ice mass is continuing to change at the same rate in future years, or if the ice mass is changing at a faster or slower rate.

Try This

This graph shows the mass of ice in Greenland in different years.

What was the approximate change in Greenland's ice mass from 2013 to 2023?

Show whether the change is positive or negative.



Summary | Lesson 12

Analyzing data about sea ice and sea levels can help us make predictions about how these levels might change in the future. We can see patterns and make predictions by subtracting data values to get a rate and then extending that rate into the future using multiplication.

To calculate the average change in summer sea ice per year from 2010 to 2020, subtract the sea ice amounts for these years and then divide by the number of years (10).

$\frac{4158 - 4742}{10} = \frac{-584}{10} = -58.4$ cubic kilometers per year

To make a prediction about the amount of summer sea ice in 2040, multiply this rate by 20 and add that value to the sea ice amount from 2020.

$-58.4 \cdot 20 = -1168$

$4158 + (-1168) = 2990$ cubic kilometers

Making predictions like this can help us prepare for the future impact that changing sea ice and sea levels may have.

Year	Summer Sea Ice (cu. km)
1980	16,316
1990	13,815
2000	11,084
2010	4,742
2020	4,158

Try This

Use the table in the summary to answer the questions.

a What was the average change in summer sea ice per year from 1980 to 1990?

Show whether the change was positive or negative.

b During which decade was ice melting fastest? Show or explain your thinking.

Summary | Lesson 13

Performing operations with positive and negative numbers can help us represent and solve real-world problems.

Imagine a family of 5 whose current carbon footprint is 60.5 tons of carbon dioxide per year. This family decides to find ways to reduce this carbon footprint including:

- Installing solar panels (-4.4 tons of carbon).
- Recycling waste (-0.6 tons of carbon per person).
- Composting food waste (-0.5 tons of carbon per person).

We can represent the changes with this expression:

$$60.5 + (-4.4) + 5(-0.6) + 5(-0.5)$$

By making these changes, this family will bring their carbon footprint down to 50.6 tons per year.

Try This

A utility company charges customers \$0.19 per kilowatt-hour of energy they use.

It also gives customers who use solar panels a credit of -\$0.17 for every kilowatt-hour of electricity they generate.

The company sent a customer this bill.

Fill in the three missing values.

Bill

	Kilowatt Hours (kWh)	Charge/ Credit per kWh	Total Charge/ Credit
Electricity Used		\$0.19	\$180.50
Electricity Generated		-\$0.17	-\$136.85
Total Due			

Lesson 1

Starting Position	Action	Final Position
-3	Add 2 floats	-1
-3	Remove 2 anchors	**-1**
-3	Add 11 floats	**8**
-3	**Add 3 floats or remove 3 anchors**	0
-3	**Add 4 anchors or remove 4 floats**	-7

Lesson 2

Starting Position	Action	Expression	Final Position
-2	Add 6 floats	$-2 + 6$	4
1	Remove 5 anchors	$1 - (-5)$	**6**
3	**Remove 7 floats**	$3 - 7$	**-4**
-1	**Add 4 anchors**	$-1 + (-4)$	**-5**

Lesson 3

a **$0.6 + x = -0.8$ (or equivalent)**

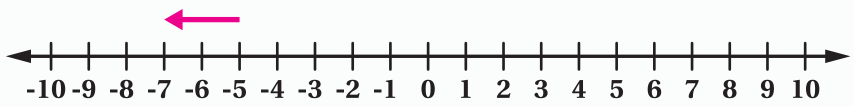
Explanation: One strategy is to think of 0.6 as the starting location, x as the change, and -0.8 as the end location.

b **-1.4**

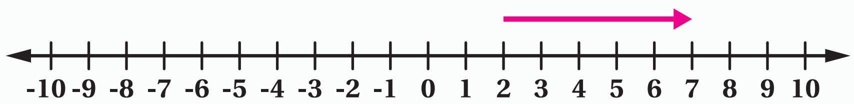
Explanation: Here is a strategy to calculate the value of the change: 0.6 is the starting location and -0.8 is the end location. The distance from 0.6 to 0 is 0.6, and the distance from 0 to -0.8 is 0.8, so the total distance is $0.6 + 0.8 = 1.4$. The arrow pointing from the start to the end points to the left, which shows that the change is negative.

Lesson 4

a -5 – 2 = <u>-7</u>



b 2 – (-5) = <u>7</u>



Lesson 5

Responses vary.

7	–	-4	=	11
-3	+	9	=	6

Lesson 6

Action	Expression	Value
Add 3 groups of 5 anchors	3 • (-5)	**-15**
Remove 4 groups of 6 floats	(-4) • 6	**-24**
Remove 2 groups of 7 anchors	(-2) • (-7)	**14**

Lesson 7

Time	Position
-2	**8**
0	0
1	-4
7	-28

Lesson 8

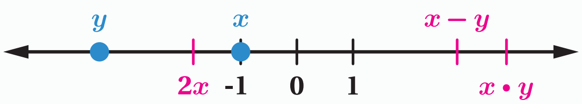
Turtle	Rate (ft/min)	Time (min)	Position (ft)
A	-3	2.5	**-7.5**
B	-2	**11.5**	-23
C	**-5.5**	-2	11

Explanation: One strategy to calculate time or rate is to divide the position by the known value. For example, if the rate is -2 and the position is -23, you can determine the time by calculating $\frac{-23}{-2} = 11.5$.

Lesson 9

$2x$	$x - y$	$x \cdot y$

Least — Greatest



Explanation:

- *Since* x *is equal to* -1, $2x = 2 \cdot (-1) = -2$.
- *Both* x *and* y *are to the left of* 0, *so they're both negative. Subtracting a negative is like taking away anchors, which moves the submarine up in the positive direction. To subtract* $x - y$, *start at* x (-1) *and move in the positive direction by the distance between* y *and* 0.
- *Because* $x = -1$, *multiplying* $x \cdot y$ *will result in a number that is the opposite of* y, *which is* 1 *more than* $x - y$.

Lesson 10

Responses vary.

$-6 \times 8 + -2 = -50$

Explanation: One strategy for making an expression with a negative value is to multiply numbers with opposite signs, then add a number that is also negative.

Lesson 11

-2,117 gigatons. Since -3,933 is less than -1,816, the change is negative and $-3933 - (-1816) = -3933 + 1816 = -2117$.

Lesson 12

a. -250.1 cubic kilometers per year

b. 2000 to 2010. *Explanations vary*. One strategy is to estimate.
From 2000 to 2010, the amount of ice decreased by at least 6,000 cubic kilometers ($11084 - 4742 > 6000$). During the other decades, the amount of ice decreased by less than 3,000 cubic kilometers.

Lesson 13

Bill			
	Kilowatt Hours (kWh)	Charge/Credit per kWh	Total Charge/Credit
Electricity Used	950	\$0.19	\$180.50
Electricity Generated	805	-\$0.17	-\$136.85
Total Due			\$43.65

Grade 7 Unit 5 Glossary/7.º grado Unidad 5 Glosario

English

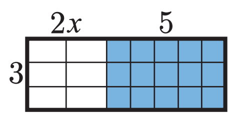
C

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

D

distributive property The property that says $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.



Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

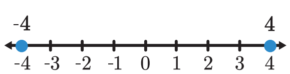
I

integer All positive and negative whole numbers and the number zero are called integers.

For example, 35, -15, and 1 are integers. 0.3 and $\frac{1}{3}$ are not.

O

opposite Two numbers that are the same distance from 0 and on different sides of the number line.



For example, 4 and -4 are opposites.

Español

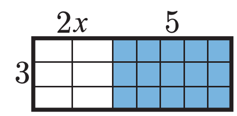
C

propiedad conmutativa La propiedad indica que $a + b = b + a$ y $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente del orden en el que estén los números.

Por ejemplo, $2 + 1 = 1 + 2$ o $3 \cdot 4 = 4 \cdot 3$.

D

propiedad distributiva La propiedad que indica que $a(b + c) = ab + ac$. Significa que multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente y luego sumar los productos.



El área como un producto
$3(2x + 5)$

Por ejemplo, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

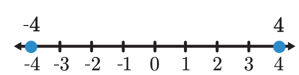
I

número entero Todos los números naturales, todos los números enteros negativos y el cero se llaman números enteros.

Por ejemplo, 35, -15 y 1 son números enteros. 0.3 y $\frac{1}{3}$ no lo son.

O

opuesto Dos números que están a la misma distancia del 0 y en diferentes lados de la recta numérica.



Por ejemplo, 4 y -4 son opuestos.