Amplify Desmos Math
FLORIDA

Student Edition

ACC 7

Volume 1

Amplify Desmos Math FLORIDA

Accelerated 7

Volume 1: Units 1–5

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math Florida! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math Florida is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

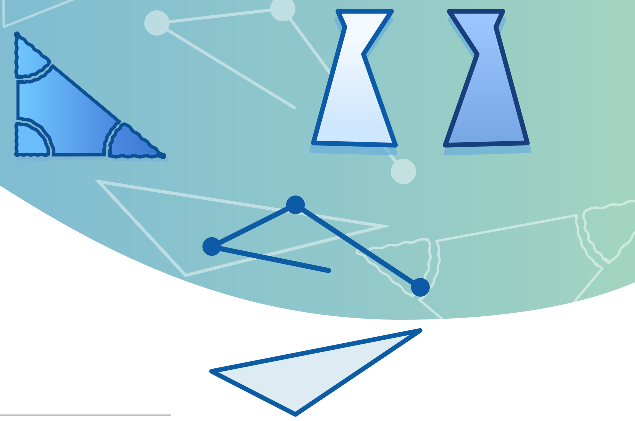
Here is what you can expect this year:

- Lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important ideas in mathematics.

We hope you enjoy exploring math this year as you work with friends to solve problems and learn about different concepts.

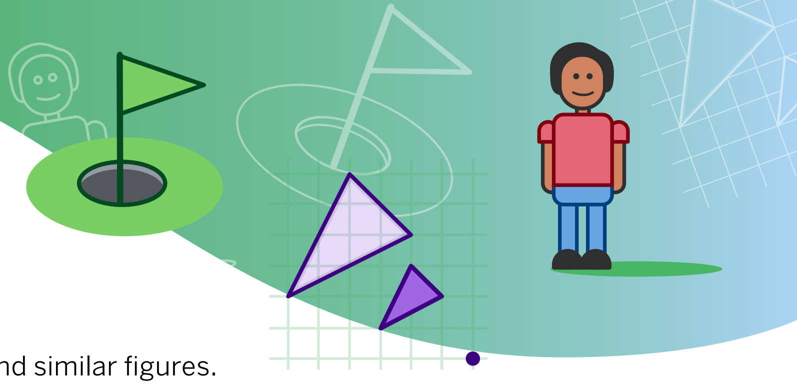
–The Amplify Desmos Math Florida Team

Unit 1 Rigid Transformations and Congruence



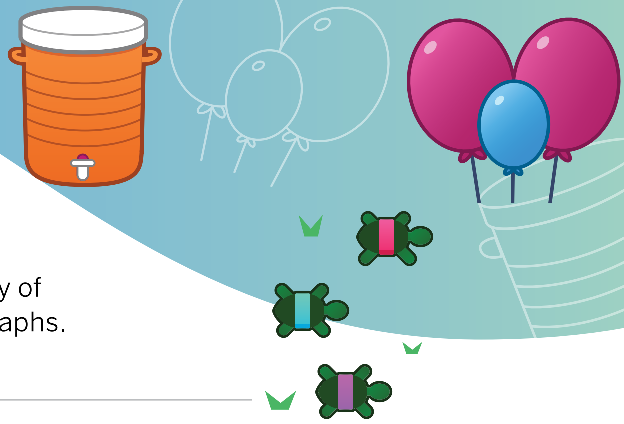
In this unit, you will investigate translations, reflections, and rotations, and use these transformations to make arguments about congruence. You will also explore angle relationships and the triangle sum theorem.

Unit 2 Scale Factors, Dilations, Similarity, and Slope



In this unit, you will study scaled copies, dilations, and similar figures. You will use similar triangles to explain, understand, and apply the concept of slope.

Unit 3 Introducing Proportional Relationships

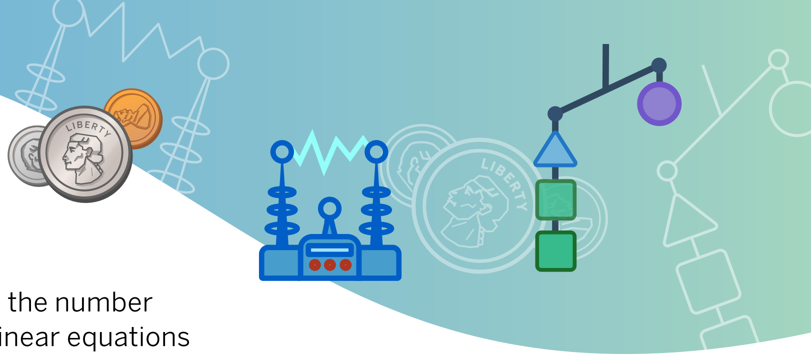


In this unit, you will explore proportional relationships in a variety of contexts and make comparisons using tables, equations, and graphs.

Unit 4 Linear Relationships and Equations

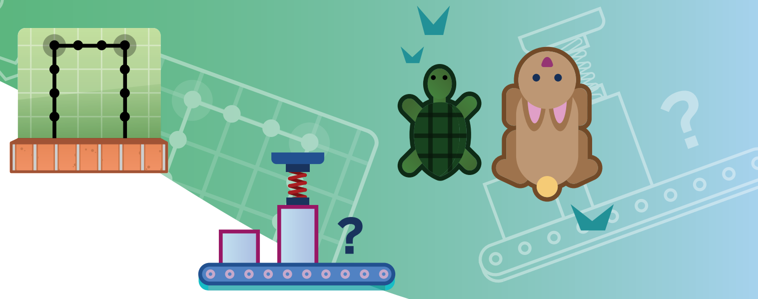
In this unit, you will compare representations of linear relationships and interpret them in context. You will also write equations for lines and determine solutions to two-variable linear equations.

Unit 5 Linear Equations and Linear Systems



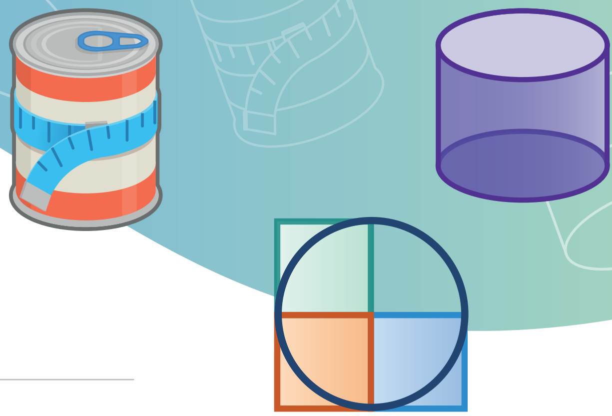
In this unit, you will solve linear equations and determine the number of possible solutions. You will also solve systems of two linear equations algebraically and graphically.

Unit 6 Functions



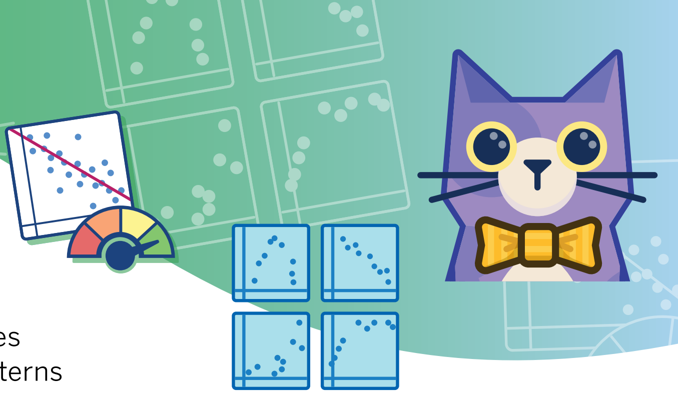
In this unit, you will learn about functions, analyze representations of functions, and examine functions in different contexts.

Unit 7 Circles and Cylinders



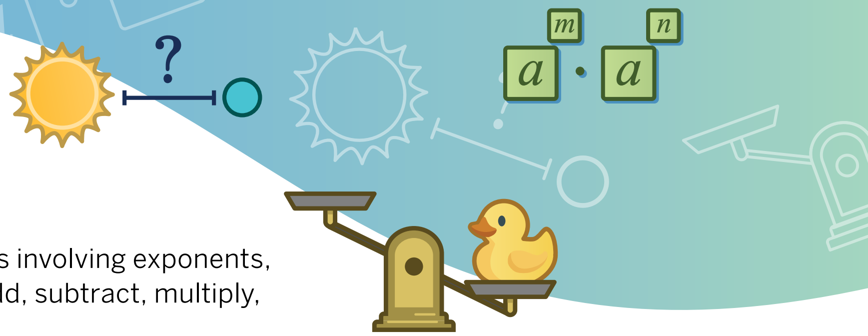
In this unit, you will explore the relationships between the radius, diameter, circumference, and area of a circle. You will also explore the volume and surface area of cylinders.

Unit 8 Associations in Data



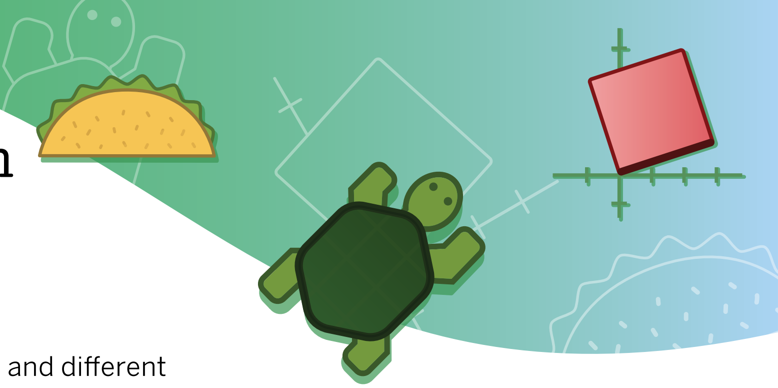
In this unit, you will decide the best display for numerical and categorical data. Then you will use scatter plots and fitted lines to analyze numerical data with two variables and observe patterns in the data.

Unit 9 Exponents and Scientific Notation



In this unit, you will develop fluency with expressions involving exponents, powers of 10, and scientific notation. You will also add, subtract, multiply, and divide numbers written in scientific notation.

Unit 10 The Pythagorean Theorem and Irrational Numbers



In this unit, you will explore the Pythagorean theorem and different types of numbers (square roots and cube roots, rational and irrational numbers).

Unit 11 Probability

In this unit, you will learn ways to display all of the possible outcomes for a repeated experiment. You will learn the difference between theoretical and experimental probability and calculate them.

Florida's B.E.S.T. Standards for Mathematics

Benchmark	B.E.S.T Mathematics Benchmark
Number Sense and Operations	
MA.7.NSO.1.1	Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.
MA.7.NSO.1.2	Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.
MA.8.NSO.1.1	Extend previous understanding of rational numbers to define irrational numbers within the real number system. Locate an approximate value of a numerical expression involving irrational numbers on a number line.
MA.8.NSO.1.2	Plot, order and compare rational and irrational numbers, represented in various forms.
MA.8.NSO.1.3	Extend previous understanding of the Laws of Exponents to include integer exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to integer exponents and rational number bases, with procedural fluency.
MA.8.NSO.1.4	Express numbers in scientific notation to represent and approximate very large or very small quantities. Determine how many times larger or smaller one number is compared to a second number.
MA.8.NSO.1.5	Add, subtract, multiply and divide numbers expressed in scientific notation with procedural fluency.
MA.8.NSO.1.6	Solve real-world problems involving operations with numbers expressed in scientific notation.
MA.8.NSO.1.7	Solve multi-step mathematical and real-world problems involving the order of operations with rational numbers including exponents and radicals.
Algebraic Reasoning	
MA.7.AR.2.2	Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.
MA.7.AR.3.3	Solve mathematical and real-world problems involving the conversion of units across different measurement systems.
MA.7.AR.4.1	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.
MA.7.AR.4.2	Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.
MA.7.AR.4.3	Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.
MA.7.AR.4.4	Given any representation of a proportional relationship, translate the representation to a written description, table or equation.
MA.7.AR.4.5	Solve real-world problems involving proportional relationships.
MA.8.AR.1.1	Apply the Laws of Exponents to generate equivalent algebraic expressions, limited to integer exponents and monomial bases.
MA.8.AR.1.2	Apply properties of operations to multiply two linear expressions with rational coefficients.

Florida’s B.E.S.T. Standards for Mathematics

MA.8.AR.1.3	Rewrite the sum of two algebraic expressions having a common monomial factor as a common factor multiplied by the sum of two algebraic expressions.
MA.8.AR.2.1	Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.
MA.8.AR.2.2	Solve two-step linear inequalities in one variable and represent solutions algebraically and graphically.
MA.8.AR.2.3	Given an equation in the form of $x^2 = p$ and $x^3 = q$, where p is a whole number and q is an integer, determine the real solutions.
MA.8.AR.3.1	Determine if a linear relationship is also a proportional relationship.
MA.8.AR.3.2	Given a table, graph or written description of a linear relationship, determine the slope.
MA.8.AR.3.3	Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.
MA.8.AR.3.4	Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form.
MA.8.AR.3.5	Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.
MA.8.AR.4.1	Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.
MA.8.AR.4.2	Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions.
MA.8.AR.4.3	Given a mathematical or real-world context, solve systems of two linear equations by graphing.
Functions	
MA.8.F.1.1	Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.
MA.8.F.1.2	Given a function defined by a graph or an equation, determine whether the function is a linear function. Given an input-output table, determine whether it could represent a linear function.
MA.8.F.1.3	Analyze a real-world written description or graphical representation of a functional relationship between two quantities and identify where the function is increasing, decreasing or constant.
Geometric Reasoning	
MA.7.GR.1.3	Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.
MA.7.GR.1.4	Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.
MA.7.GR.1.5	Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.
MA.7.GR.2.1	Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure’s net.

MA.7.GR.2.2	Solve real-world problems involving surface area of right circular cylinders.
MA.7.GR.2.3	Solve mathematical and real-world problems involving volume of right circular cylinders.
MA.8.GR.1.1	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving unknown side lengths in right triangles.
MA.8.GR.1.2	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving the distance between two points in a coordinate plane.
MA.8.GR.1.3	Use the Triangle Inequality Theorem to determine if a triangle can be formed from a given set of sides. Use the converse of the Pythagorean Theorem to determine if a right triangle can be formed from a given set of sides.
MA.8.GR.1.4	Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.
MA.8.GR.1.5	Solve problems involving the relationships of interior and exterior angles of a triangle.
MA.8.GR.1.6	Develop and use formulas for the sums of the interior angles of regular polygons by decomposing them into triangles.
MA.8.GR.2.1	Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.
MA.8.GR.2.2	Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.
MA.8.GR.2.3	Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.
MA.8.GR.2.4	Solve mathematical and real-world problems involving proportional relationships between similar triangles.
Data Analysis and Probability	
MA.7.DP.1.4	Use proportional reasoning to construct, display and interpret data in circle graphs.
MA.7.DP.1.5	Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.
MA.8.DP.1.1	Given a set of real-world bivariate numerical data, construct a scatter plot or a line graph as appropriate for the context.
MA.8.DP.1.2	Given a scatter plot within a real-world context, describe patterns of association.
MA.8.DP.1.3	Given a scatter plot with a linear association, informally fit a straight line.
MA.8.DP.2.1	Determine the sample space for a repeated experiment.
MA.8.DP.2.2	Find the theoretical probability of an event related to a repeated experiment.
MA.8.DP.2.3	Solve real-world problems involving probabilities related to single or repeated experiments, including making predictions based on theoretical probability.

Mathematical Thinking and Reasoning Standards

MTR 1.1	Actively participate in effortful learning both individually and collectively.
MTR 2.1	Demonstrate understanding by representing problems in multiple ways.
MTR 3.1	Complete tasks with mathematical fluency.
MTR 4.1	Engage in discussions that reflect on the mathematical thinking of self and others.
MTR 5.1	Use patterns and structure to help understand and connect mathematical concepts.
MTR 6.1	Assess the reasonableness of solutions.
MTR 7.1	Apply mathematics to real-world contexts.

Florida's B.E.S.T. Standards for English Language Arts

ELA.K12.EE.1.1	Cite evidence to explain and justify reasoning.
ELA.K12.EE.2.1	Read and comprehend grade-level complex texts proficiently.
ELA.K12.EE.3.1	Make inferences to support comprehension.
ELA.K12.EE.4.1	Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.
ELA.K12.EE.5.1	Use the accepted rules governing a specific format to create quality work.
ELA.K12.EE.6.1	Use appropriate voice and tone when speaking or writing.

English Language Development Standards

ELD.K12.ELL.MA.1	Communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
ELD.K12.ELL.SI.1	Read and comprehend grade-level complex texts proficiently.

Unit 1

Rigid Transformations and Congruence

If you look around, you'll notice shapes in art, architecture, and everyday objects. Shapes have parts that can be measured, like sides and angles. Will anything happen to these side lengths and angle measures when you *slide*, *flip*, or *turn* these shapes?

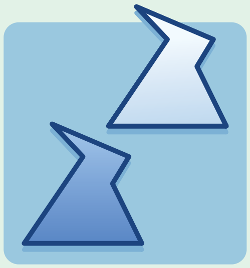
Essential Questions

- What are different ways to transform a figure?
- How can we use rigid transformations to decide whether two figures are congruent?
- How can transformations help make sense of angle relationships?

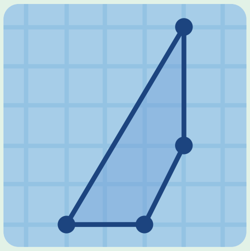
Sub-unit

1

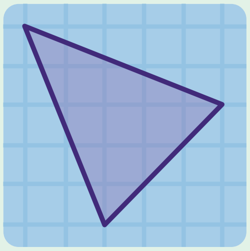
Rigid Transformations



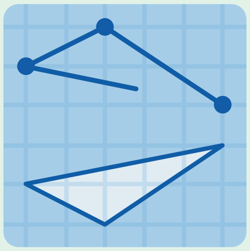
Lesson 1
Spinning, Flipping, Sliding



Lesson 2
Moving Day



Lesson 3
Getting Coordinated, Part 1



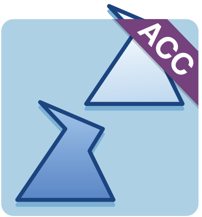
Lesson 4
Getting Coordinated, Part 2

Unit 1
Lesson
1

Name: .... Date: .... Period: ....

MA.8.GR.2.1, MTR.4.1

Spinning, Flipping, Sliding

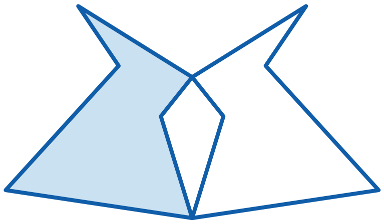


Let's learn some ways to describe how figures move.

Warm-Up

1. Annika started with this shaded figure and used a **transformation** to create the unshaded figure.

What happened to the figure?



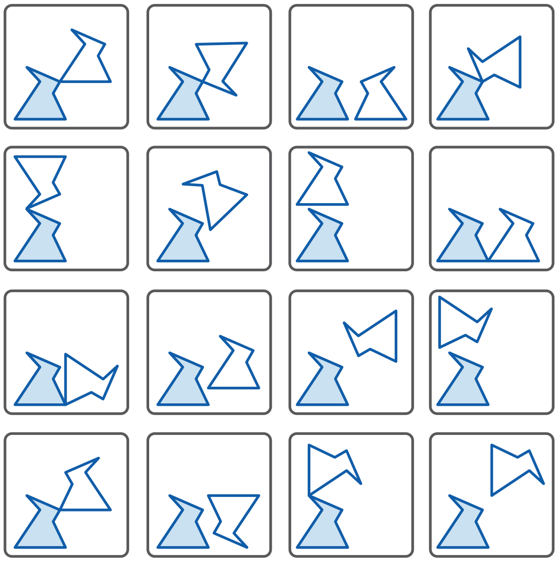
Name: .... Date: .... Period: ....

Describing Transformations

2. Play a few rounds of Polygraph with your classmates!

You will use an Activity 1 Sheet with different *transformation* images for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select an image from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating images until you're ready to guess which image the Picker chose.



Record helpful questions from each round in this workspace:

Activity
1

Name: ________ Date: ________ Period: ________

Describing Transformations (continued)

In Polygraph, you saw three types of transformations: **rotations**, **reflections**, and **translations**.

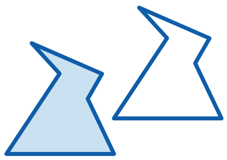
3. Match each word with one of these transformations.

Flip	Mirror	Slide	Spin	Turn

Rotation	Reflection	Translation

4. For each picture, circle the word that best describes how one figure can move onto the other in a single transformation.

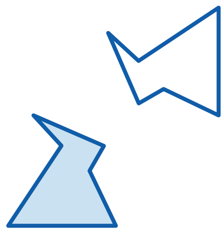
a



Rotation Reflection

Translation

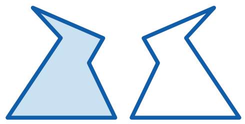
b



Rotation Reflection

Translation

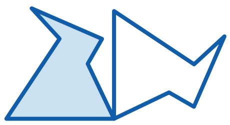
c



Rotation Reflection

Translation

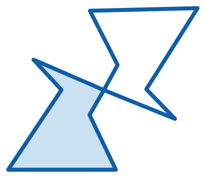
d



Rotation Reflection

Translation

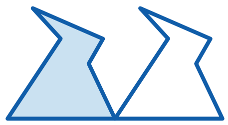
e



Rotation Reflection

Translation

f



Rotation Reflection

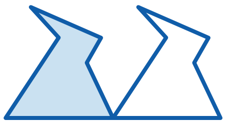
Translation

Name: .... Date: .... Period: ....

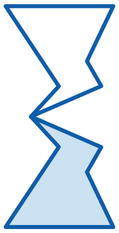
Rotations, Reflections, Translations

5. Circle an image. Then describe how to move the shaded figure onto the unshaded one. Use one of these words in your description: *reflection*, *rotation*, *translation*.

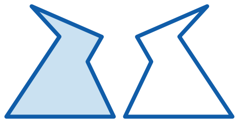
A.



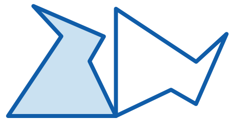
B.



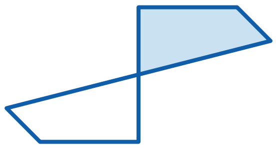
C.



D.



6. Matias says you can use one *reflection* to move the shaded figure onto the unshaded one.



Dyani says you can use *one rotation*.

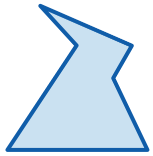
Whose claim is correct? Circle one.

Matias's   Dyani's   Both   Neither

Explain your thinking.

Synthesis

7. In your own words, describe what each *transformation* does to a figure.



Rotation:

Reflection:

Translation:

Lesson Practice ACC7.1.01

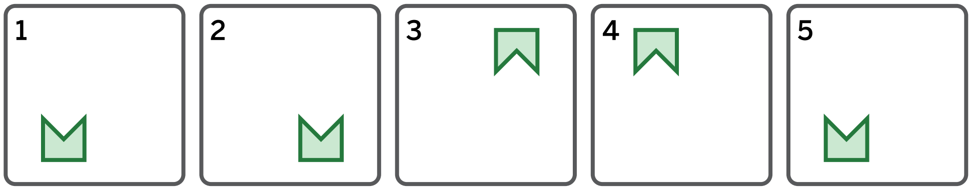
Lesson Summary

Transformations are actions that you can perform to change a figure. They are applied to every point on the figure. Here are some examples:

- A **rotation** turns or spins a figure to a new direction.
- A **reflection** flips or mirrors a figure over a line by moving every point to a point directly on the opposite side of the line.
- A **translation** slides a figure to a new location without turning it.

Name: .... Date: .... Period: ....

1. These five frames show a figure's different positions.

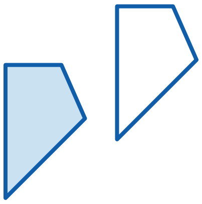


Describe how the figure moves to get from its position in each frame to the next.

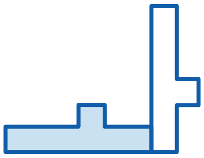
From	To	Description of Moves
Frame 1	Frame 2	
Frame 2	Frame 3	
Frame 3	Frame 4	
Frame 4	Frame 5	

Problems 2–4: Determine whether each transformation shows a translation, reflection, or rotation.

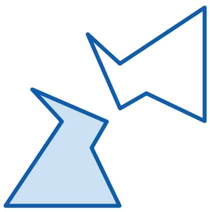
2.



3.



4.

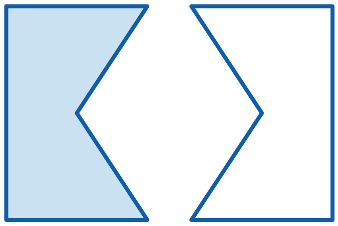


Lesson Practice
ACC7.1.01

Name: .... Date: .... Period: ....

FAST Practice

5. Which transformation could move the shaded figure to the unshaded one? Select *all* that apply.



- ☐ **A.** Reflection
- ☐ **B.** Rotation
- ☐ **C.** Translation
- ☐ **D.** None of these

Spiral Review

Problems 6–9: Evaluate each expression.

6. $-5 \cdot (-2.4)$

7. $-7.4 \div 10$

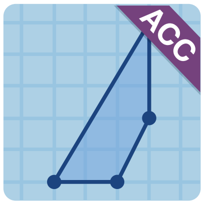
8. $-\frac{4}{7} \div (-2)$

9. $4 \cdot \left(-\frac{3}{8}\right)$

Name: .... Date: .... Period: ....

MA.8.GR.2.1, MTR.4.1, MTR.5.1

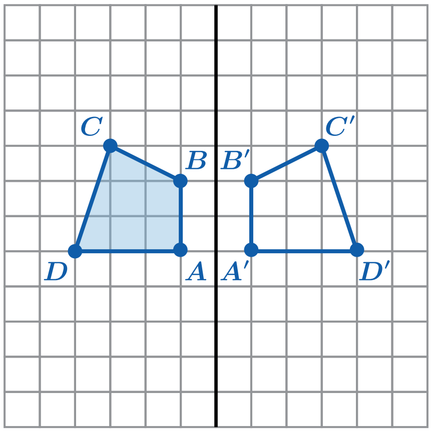
Moving Day



Let's do transformations by hand.

Warm-Up

1. Here is a transformation. The **pre-image** is shaded and the **image** is unshaded.



What do you notice? What do you wonder?

I notice:

I wonder:

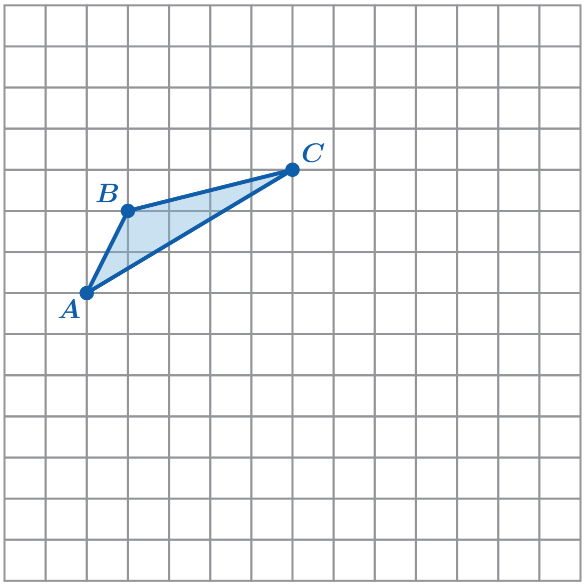
Activity 1

Name: Date: Period:

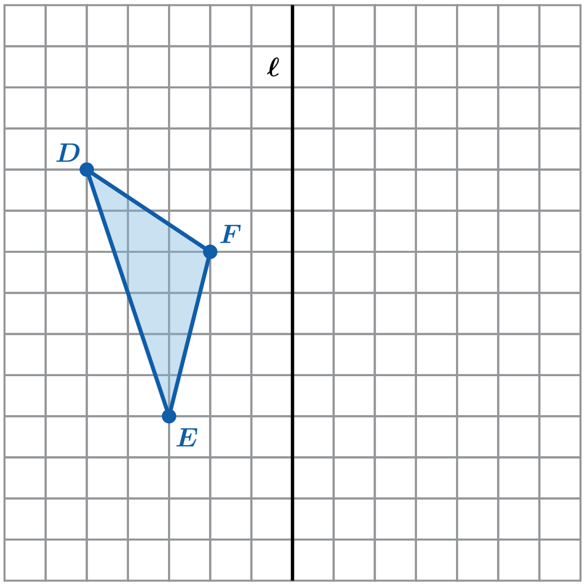
Move It

Perform each transformation. Then label the points in the image to **correspond** with the points in the pre-image.

2. Translate triangle ABC 3 units right.



3. Reflect triangle DEF over line ℓ.

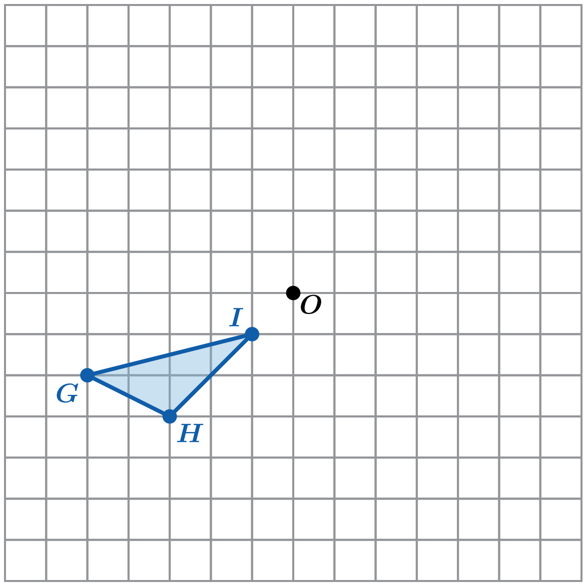


Activity
1

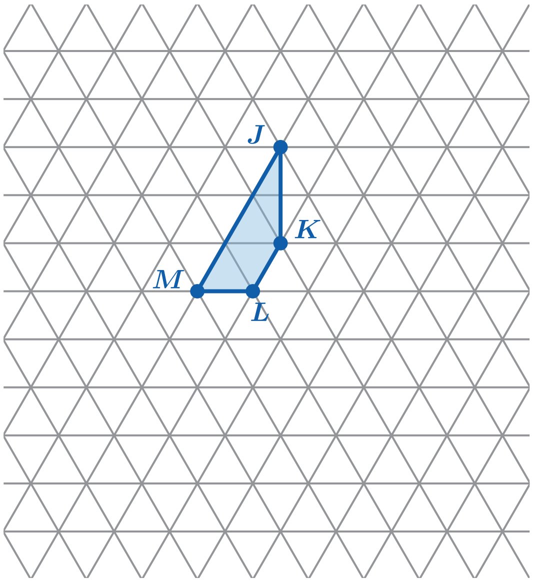
Name: .... Date: .... Period: ....

Move It (continued)

4. Rotate triangle GHI $180°$ **counterclockwise** around point O.



5. Rotate figure $JKLM$ $180°$ **clockwise** around point L.



Name: Date: Period:

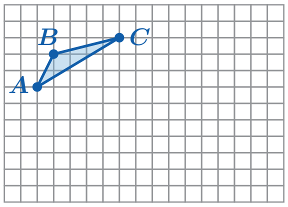
Transformation Information

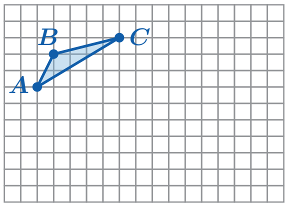
6. You and your partner will get a set of transformation cards. Place them grid-side down without looking at them.

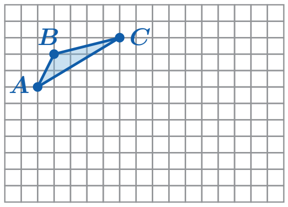
- Decide who will describe the transformation on a card and who will sketch the image. Start with Card 1.
- Describer: Give enough information about the transformation so that the Sketcher can sketch it.
- Sketcher: Pause after sketching and share what you think the transformation is.
- Together: Compare the card with the sketch and make adjustments as needed. Write a precise description of the transformation.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3 and 4.

a **Sketch 1:** Card 1 or Card 2 (Circle one.)

Use as many grids as you need to revise your work.



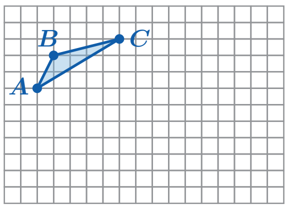


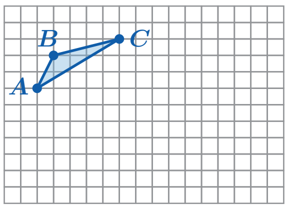


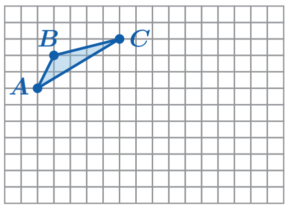
Description of Transformation:

b **Sketch 2:** Card 3 or Card 4 (Circle one.)

Use as many grids as you need to revise your work.







Description of Transformation:

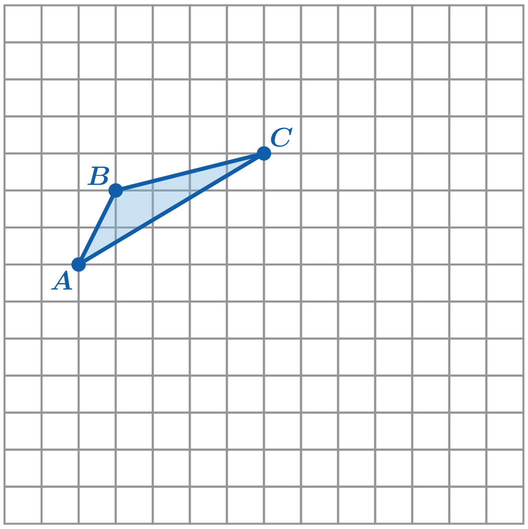
Synthesis

7. What details are helpful to include when precisely describing a transformation?

Translation:

Reflection:

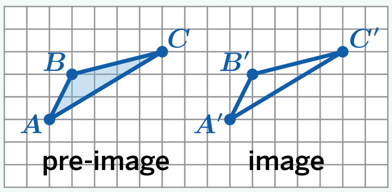
Rotation:



Lesson Practice ACC7.1.02

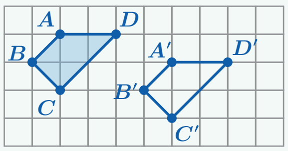
Lesson Summary

When you transform a figure, the original figure is called the **pre-image** and the new figure is called the **image**. All of the points in the image **correspond** to the points in the pre-image. The points in the image are named after the pre-image point they correspond to. For example, point A' corresponds to point A.



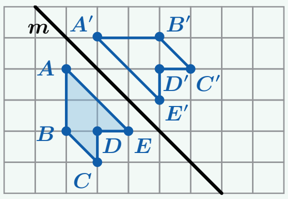
Here are important details to help you describe transformations:

Translations



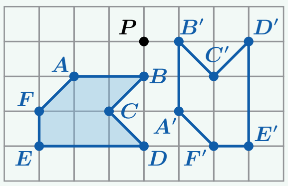
Describe the direction (up or down, left or right) and the number of units.
E.g., Figure $ABCD$ is translated 4 units right and 1 unit down.

Reflections



Describe the line of reflection.
E.g., Figure $ABCDE$ is reflected over line m.

Rotations

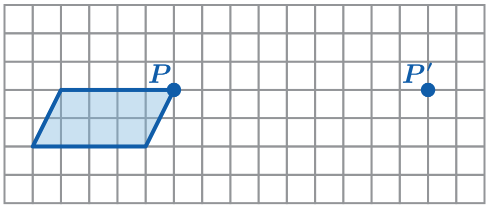


Describe the center of rotation, angle of rotation, and direction (**clockwise** or **counterclockwise**).
E.g., Figure $ABCDEF$ is rotated $90°$ counterclockwise around point P.

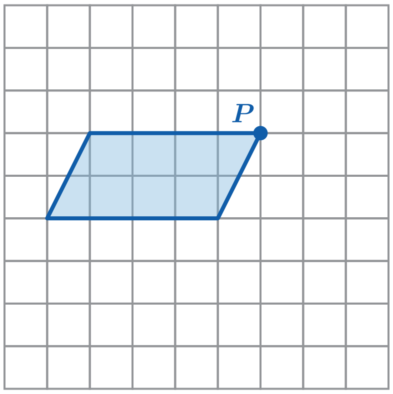
Lesson Practice
ACC7.1.02

Name: .... Date: .... Period: ....

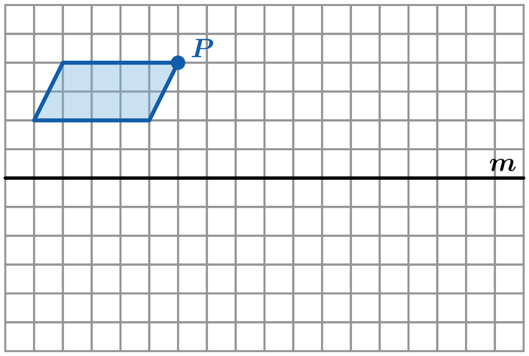
1. Translate this figure to take point P to P'.



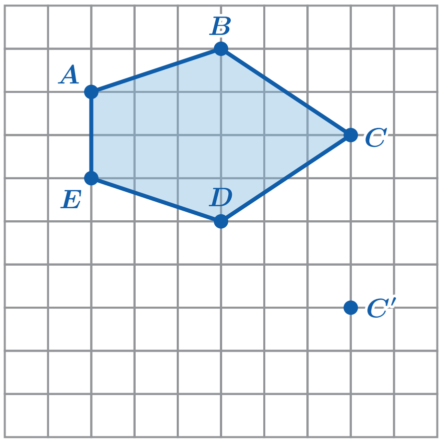
2. Rotate this figure $90°$ counterclockwise around point P.



3. Reflect this figure over line m.



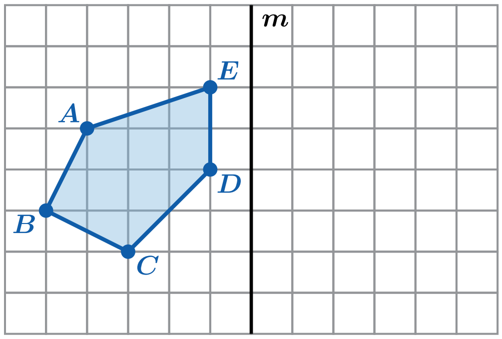
4. Translate figure $ABCDE$ to take point C to C'.



Lesson Practice
ACC7.1.02

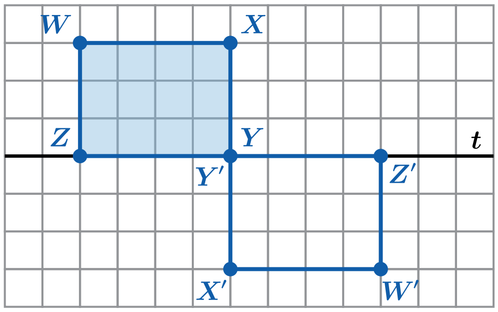
Name: .... Date: .... Period: ....

5. Reflect figure $ABCDE$ over line m.



FAST Practice

6. Figure $WXYZ$ was transformed to create figure $W'X'Y'Z'$. Which transformation took place?



A. A reflection over line t.

B. A translation 6 units down.

C. A rotation $90°$ clockwise around point Y.

D. A rotation $180°$ clockwise around point Y.

Spiral Review

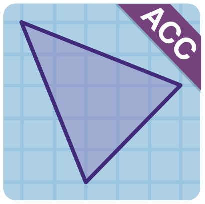
7. Match each expression with an equivalent expression.

Expressions	**Equivalent Expressions**
a $-3x - 7$	.... $-0.9 + 5.7x$
b $-3.4 + 5.7x + 2.5$	.... $-\frac{1}{2}(6x - 14)$
c $1.8x - 5.9 + 3.9x$	.... $5.7x - 5.9$
d $-3x + 7$	.... $\left(-\frac{7}{2} - \frac{3}{2}x\right) \cdot 2$

Name: .... Date: .... Period: ....

MA.8.GR.2.1, MA.8.GR.2.3, MTR.3.1, MTR.4.1

Getting Coordinated, Part 1



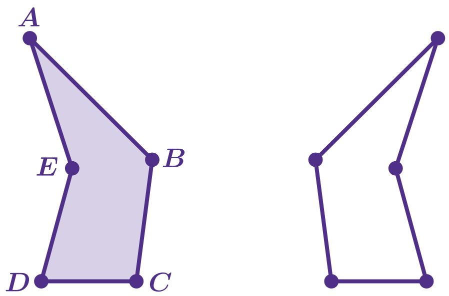
Let's explore how translations and reflections affect points on the coordinate plane.

Warm-Up

1. The pre-image (shaded) has been reflected to create the image (unshaded).

Label each corresponding point on the image.

A'	B'	C'	D'	E'

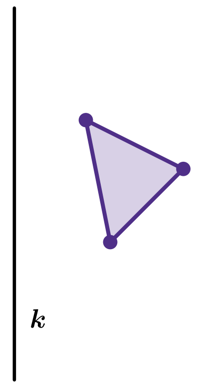


Activity 1

Name: .... Date: .... Period: ....

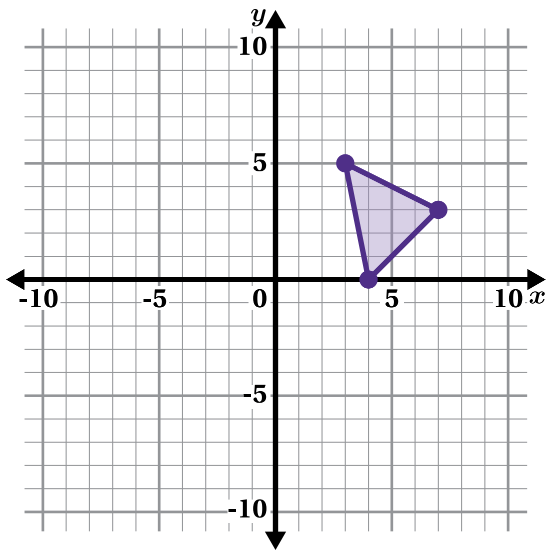
Do Coordinates Help?

2. Draw the image of the triangle after a reflection over line k.



3. Let's try that again, but with a coordinate plane.

   Draw the image of the triangle after a reflection over the y-axis.



4. Let's look at some responses to the previous two problems.

   Discuss: What do you notice about the two sets of reflections?

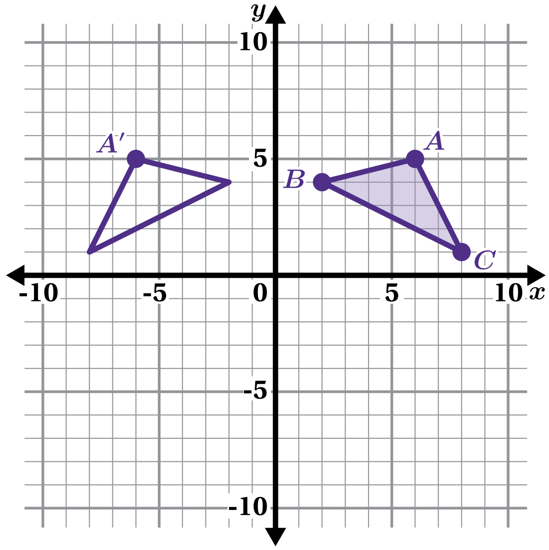
Activity 2

Name: ______ Date: ______ Period: ______

Coordinate Patterns, Part 1

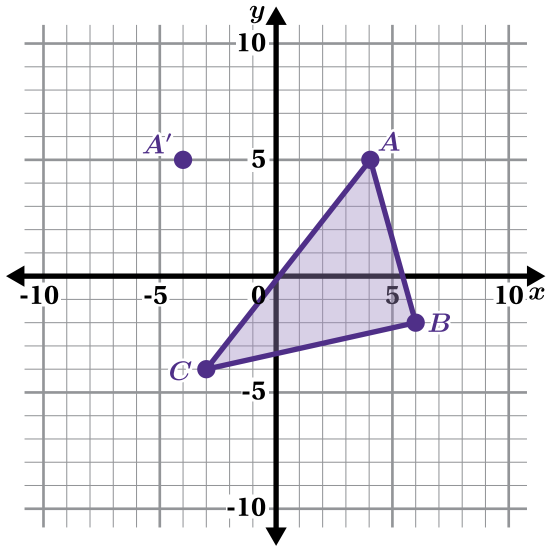
5. Identify the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(6, 5)	(-6, 5)
(2, 4)	
(8, 1)	



6. Determine the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(4, 5)	(-4, 5)
(6, -2)	
(-3, -4)	



7. Take a look at your tables from Problems 5 and 6. Those points show a reflection over the y-axis.

a. **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

b. Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	
(x, y)	

Activity 3

Name: .... Date: .... Period: ....

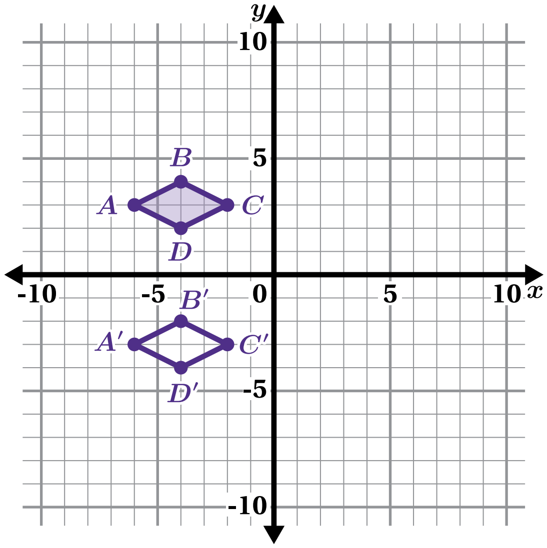
Coordinate Patterns, Part 2

8. Amari says that figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection over the x-axis.

Is Amari's claim correct? Circle one.

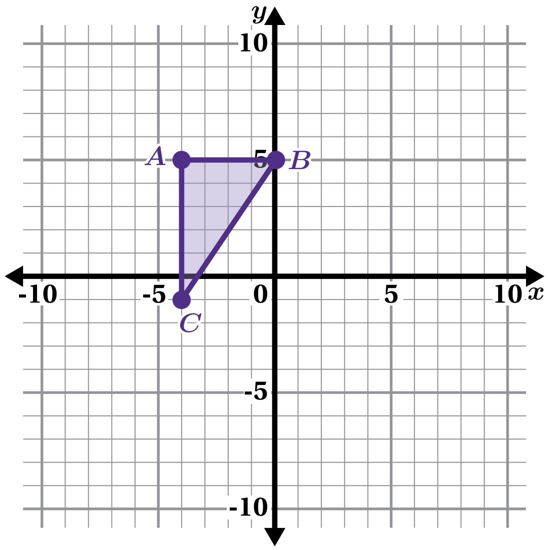
Yes No I'm not sure

Explain your thinking.



9. Determine the coordinates of each point after a translation 4 units right.

Pre-Image Coordinates	Image Coordinates
(-4, 5)	
(0, 5)	
(-4, -1)	



10. Take a look at your table from Problem 9. Those points show a translation 4 units right.

a **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

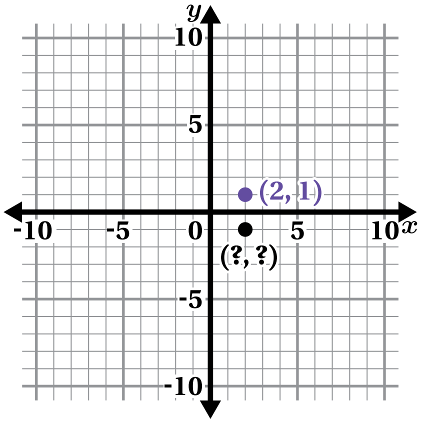
b Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	
(x, y)	

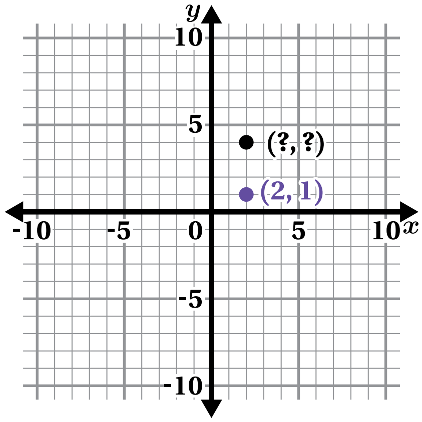
Synthesis

11. If you know the pre-image coordinates, how can you find the image coordinates for any reflection or translation?

Reflection



Translation



Use the examples if they help with your thinking.

Reflection:

Translation:

Lesson Practice ACC7.1.03

Lesson Summary

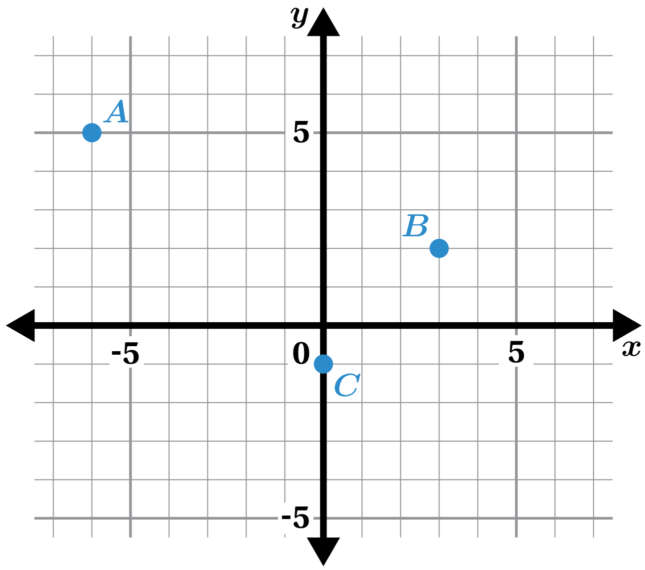
When you compare the coordinates of corresponding points in the image and pre-image, you might notice patterns in their values.

- When you translate a point to the left or right, it changes the value of the x-coordinate.
- When you translate a point up or down, it changes the value of the y-coordinate.
- When you reflect a point over the x-axis, it changes the sign of the y-coordinate. The x-coordinate remains the same.
- When you reflect a point over the y-axis, it changes the sign of the x-coordinate. The y-coordinate remains the same.

Name: ________ Date: ________ Period: ________

1. Plot the location of points A, B, and C after a translation 4 units to the right. Label the points A', B', and C'. Then write the coordinates in the table.

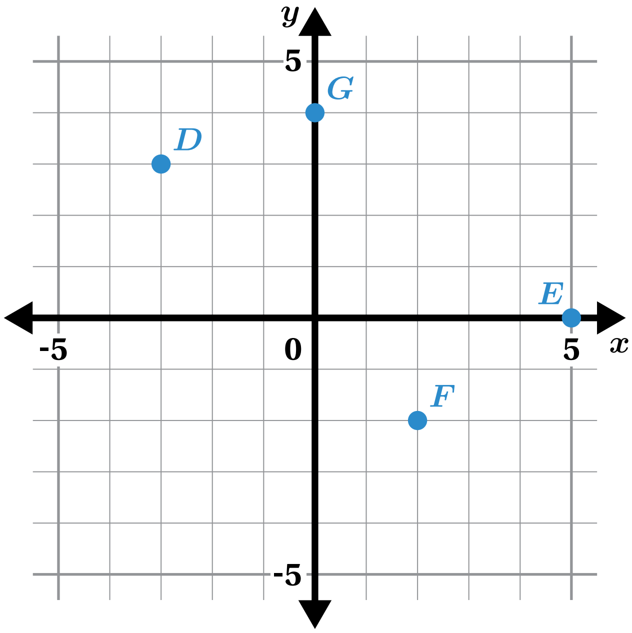
Point	Coordinates
A'	
B'	
C'	



Problems 2–3: Points D, E, F, and G are plotted on the coordinate plane.

2. Plot the coordinates of points D and E after a reflection over the y-axis. Label the images D' and E'. Include the coordinates.

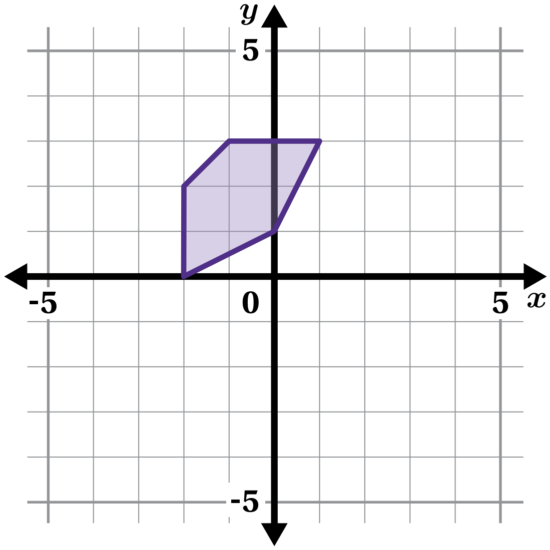
3. Plot the coordinates of points F and G after a reflection over the x-axis. Label the images F' and G'. Include the coordinates.



Name: .... Date: .... Period: ....

4. Determine the coordinates of each point after the shaded pre-image is reflected over the x-axis. Use the graph if it helps with your thinking.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	
(-2, 2)	
(-1, 3)	
(1, 3)	
(0, 1)	



FAST Practice

5. Point $H(5, 3)$ is translated 4 units down to get point H'. Which of the following are the coordinates of point H'?

A. (1, -1) **B.** (9, 3) **C.** (1, 3) **D.** (5, -1)

Spiral Review

Problems 6–11: Compare each of these values using the symbols <, =, or >.

6. -11 .... -15

7. 8.01 8

8. $\frac{2}{3}$ .... $-\frac{3}{2}$

9. -8.01 .... -8

10. -(-6) 6

11. -2.5 .... $-\frac{10}{4}$

Unit 1
Lesson
4

Name: .... Date: .... Period: ....

MA.8.GR.2.1, MA.8.GR.2.3, MTR.3.1, MTR.4.1

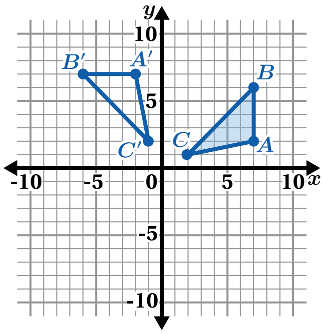
Getting Coordinated, Part 2

Let's explore how rotations affect coordinates.

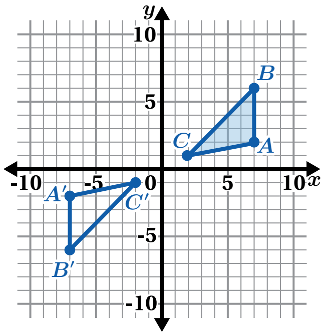
Warm-Up

1. **a** Take a look at several different rotations.

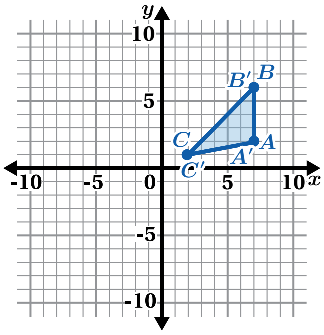
Counterclockwise 90°



Clockwise 180°



Counterclockwise 360°



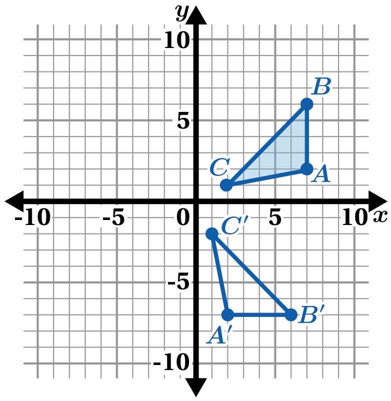
b **Discuss:** What do you notice? What do you wonder?

Activity 1

Name: ______ Date: ______ Period: ______

Coordinate Patterns

2. Tyani says this is a rotation of figure ABC $90°$ clockwise around center $(0, 0)$. Anushka says this is a rotation of figure ABC $270°$ counterclockwise around the *origin*.

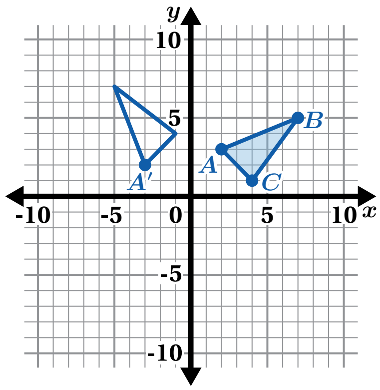


Whose claim is correct? Circle one.

Tyani's   Anushka's   Both   Neither

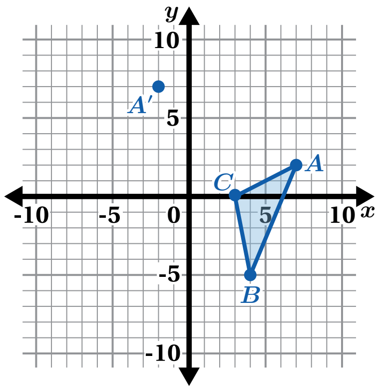
Explain your thinking.

3. Identify the coordinates for the image of figure ABC after a rotation $90°$ counterclockwise around center $(0, 0)$.



Pre-Image Coordinates	Image Coordinates
$(2, 3)$	$(-3, 2)$
$(7, 5)$	
$(4, 1)$	

4. Determine the coordinates for the image of figure ABC after a rotation $90°$ counterclockwise around center $(0, 0)$.



Pre-Image Coordinates	Image Coordinates
$(7, 2)$	$(-2, 7)$
$(4, -5)$	
$(3, 0)$	

Name: ______ Date: ______ Period: ______

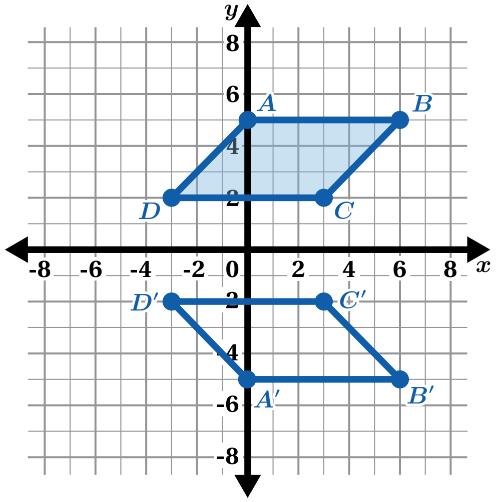
Coordinate Patterns (continued)

5. Binta says this is a $180°$ clockwise rotation of figure $ABCD$ around center $(0, 0)$. Chloe says this is a reflection over the x-axis.

Whose claim is correct? Circle one.

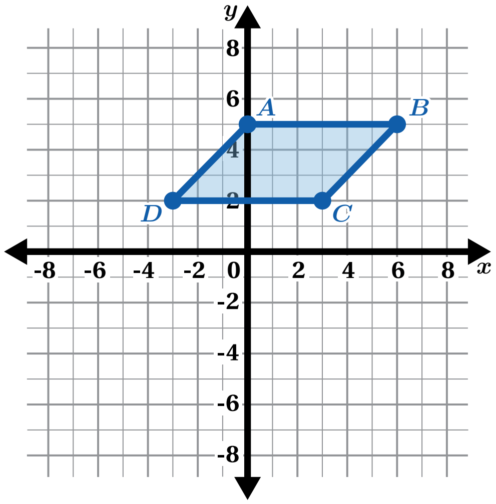
Binta's (Rotation) Chloe's (Reflection) Both Neither

Explain your thinking.



6. Determine the coordinates for the image of figure $ABCD$ after a rotation $180°$ clockwise around center $(0, 0)$.

Pre-Image Coordinates	Image Coordinates
$(0, 5)$	
$(6, 5)$	
$(3, 2)$	
$(-3, 2)$	



Name: ________ Date: ________ Period: ________

Challenge Creator

7. You will use the Activity 2 Sheet to create your own transformation challenge.

a **Make It!** On the Activity 2 Sheet, create a transformation challenge.

b **Solve It!** On this sheet, write the pre-image and image coordinates for your transformation.

Pre-Image Coordinates	Image Coordinates

c **Swap It!** Swap your challenge with one or more partners. Write the pre-image and image coordinates for their transformation.

Partner 1's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 2's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 3's Challenge

Pre-Image Coordinates	Image Coordinates

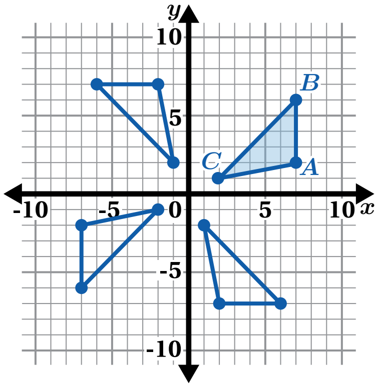
Partner 4's Challenge

Pre-Image Coordinates	Image Coordinates

Synthesis

8. Describe some patterns in the pre-image and image coordinates for rotations around center $(0, 0)$.

Use the examples if they help with your thinking.



Lesson Practice ACC7.1.04

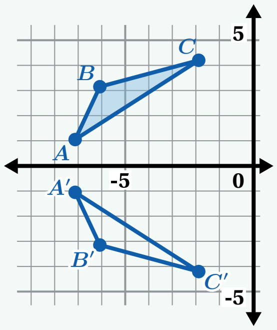
Lesson Summary

We can compare the coordinates of corresponding points in an image and pre-image to determine which transformations have been performed.

For triangles ABC and $A'B'C'$, the sign of the y-coordinate of each point changes but the x-coordinate remains the same.

This probably means there was a reflection over the x-axis.

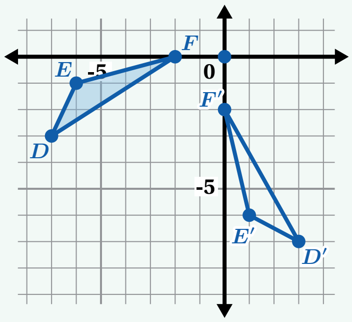
Pre-Image Coordinates	Image Coordinates
(-7, 1)	(-7, -1)
(-6, 3)	(-6, -3)
(-2, 4)	(-2, -4)



For triangles DEF and $D'E'F'$, the x- and y-coordinates of each point switch places, and some of the signs change.

This probably means there was a $90°$ rotation counterclockwise or a $270°$ rotation clockwise around the *origin*.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	(0, -2)
(-6, -1)	(1, -6)
(-7, -3)	(3, -7)

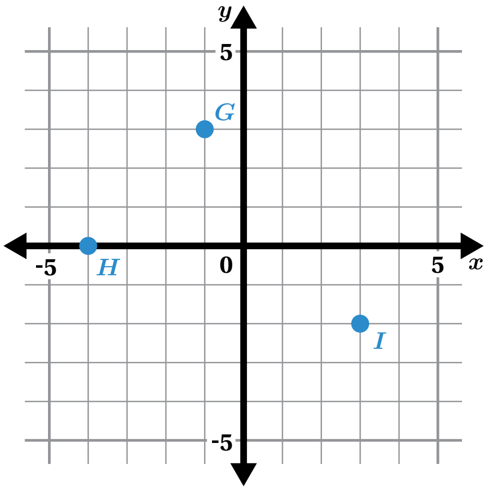


Lesson Practice
ACC7.1.04

Name: .... Date: .... Period: ....

1. Plot the location of points G, H, and I after a $180°$ rotation around the origin. Label the points G', H', and I'. Then write the coordinates in the table.

Point	Coordinates
G'	
H'	
I'	



2. Point $P(5, 3)$ is rotated $90°$ counterclockwise around the origin and the image is labeled P'. Which of the following are the coordinates of point P'?

A. (1, -3) B. (-3, -5) C. (3, -5) D. (-3, 5)

3. Here are the pre-image and image coordinates of points on a graph. Describe the transformation.

Pre-Image Coordinates	Image Coordinates
(0, 5)	(5, 0)
(-2, 1)	(1, 2)
(4, 3)	(3, -4)
(6, 0)	(0, -6)
(-5, -1)	(-1, 5)

Name: .... Date: .... Period: ....

FAST Practice

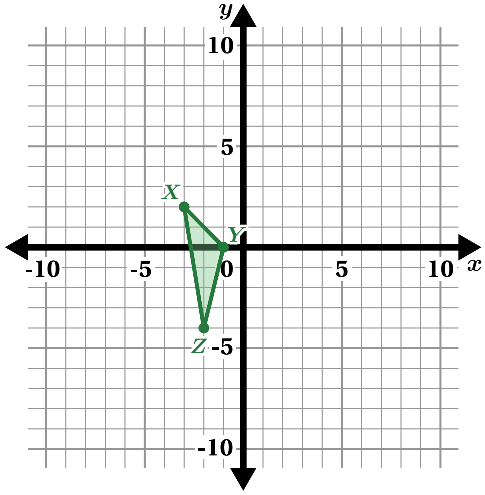
4. Triangle XYZ is rotated $90°$ clockwise around the origin. What are the coordinates of point X'?

A. $(3, 2)$

B. $(2, 3)$

C. $(-2, -3)$

D. $(-3, -2)$



Spiral Review

Problems 5–9: Write an expression that has a value of $\frac{3}{5}$ based on the given rule.

5. An expression that is a sum.

6. An expression that is a difference.

7. An expression that is a product.

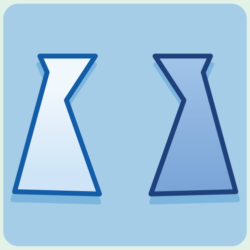
8. An expression that is a quotient.

9. An expression that involves at least two operations.

Sub-unit

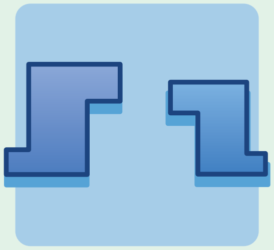
2

Congruence



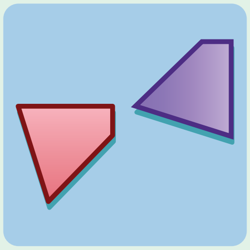
Lesson 5

No Bending, No Stretching



Lesson 6

Are They the Same?



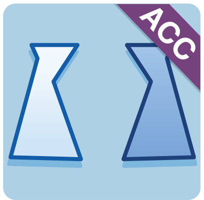
Lesson 7

Are They Congruent?

Name: Date: Period:

MA.8.GR.2.1, MA.8.GR.2.3, MTR.4.1, MTR.5.1

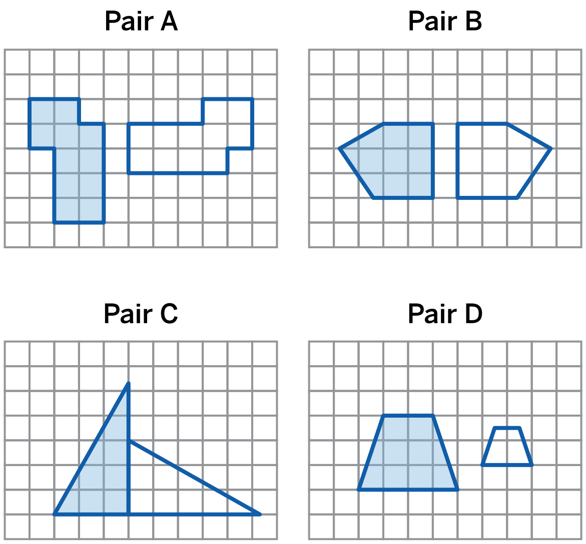
No Bending, No Stretching



Let's compare the measurements of transformed figures.

Warm-Up

1. Determine whether each image can be formed using a single transformation. If so, describe the transformation to move the pre-image (shaded) onto the corresponding image (unshaded).



Pair A:

Pair B:

Pair C:

Pair D:

Activity
1

Name: Date: Period:

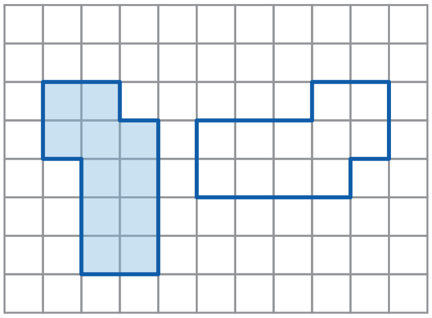
Sides and Angles

2. Here are the pairs from the Warm-Up.

For each pair:

- Use your tools to compare the measurements of the side lengths and interior angles.
- Determine which corresponding measurements are the same and which are different.

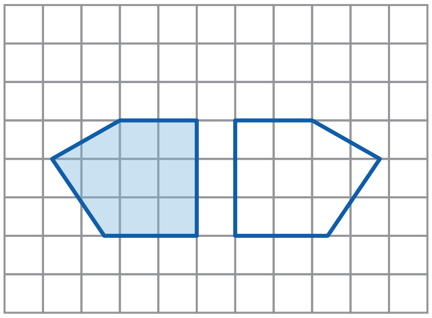
Pair A



Same:

Different:

Pair B



Same:

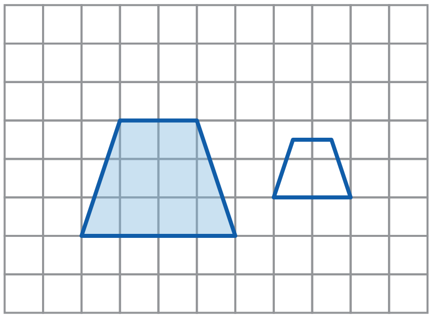
Different:

Pair C

Same:

Different:

Pair D



Same:

Different:

Activity
2

Name: ______________ Date: ______ Period: ______

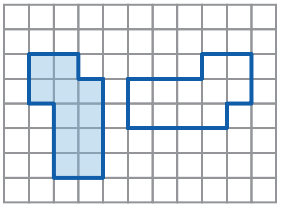
Rigid Transformations

3. A **rigid transformation** is a move that doesn't change any measurements in a figure.

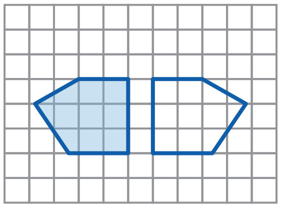
Select *all* the pairs that show a rigid transformation.

- ☐ Pair A
- ☐ Pair B
- ☐ Pair C
- ☐ Pair D

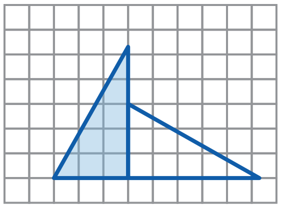
Pair A



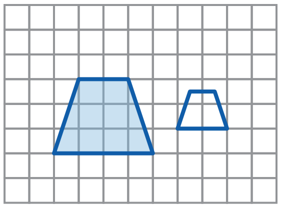
Pair B



Pair C



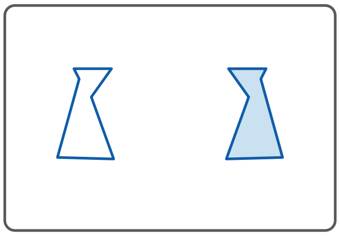
Pair D



4. You will use a set of cards. Sort the cards into two groups according to their type of transformation.

Rigid	Non-Rigid

5. Devon says that this card shows a rigid transformation.



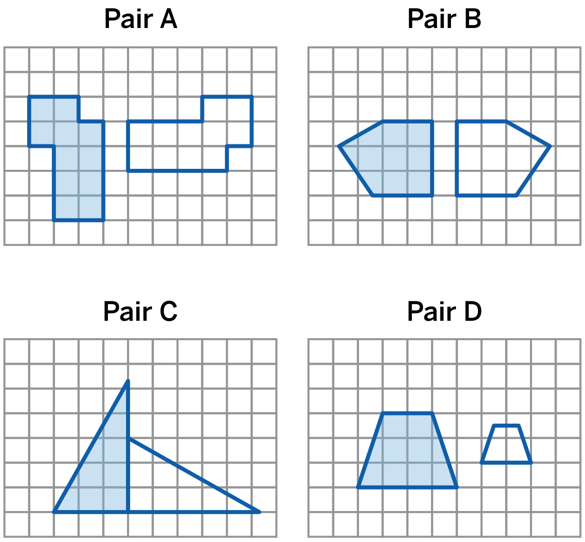
a Is her claim correct?

Explain your thinking.

b Describe a transformation that would move the pre-image (shaded) onto the image (unshaded).

Synthesis

6. Explain how you can determine whether a transformation is rigid. Use the examples if they help with your thinking.

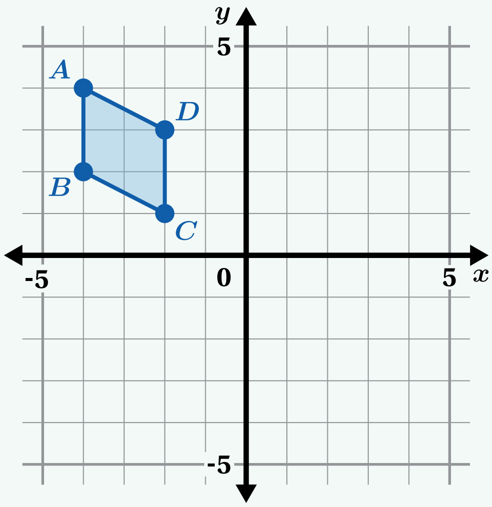


Lesson Practice ACC7.1.05

Lesson Summary

Translations, rotations, and reflections are all examples of **rigid transformations**.

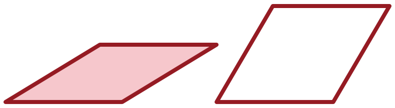
When a pre-image is transformed using a rigid transformation, corresponding sides will have the same length and corresponding angles will have the same measure. For example, figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection. Side AB has the same length as side $A'B'$ and angle C is the same measurement as angle C'.



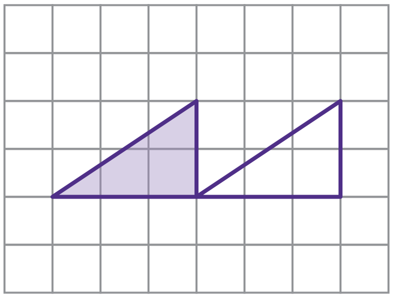
Name: .... Date: .... Period: ....

1. Is there a rigid transformation that can move the shaded figure onto the unshaded figure?

   Explain your thinking.

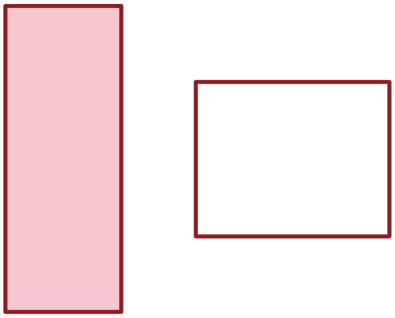


2. Describe a rigid transformation that moves the shaded figure onto the unshaded figure.

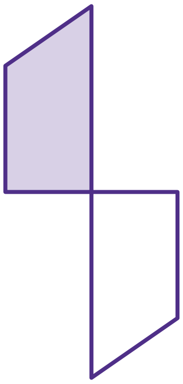


Problems 3–6: Determine whether a rigid transformation can move the shaded figure onto the unshaded figure. If so, describe the rigid transformation.

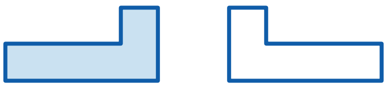
3.



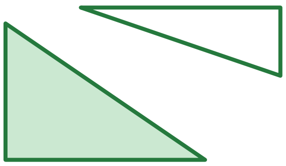
4.



5.



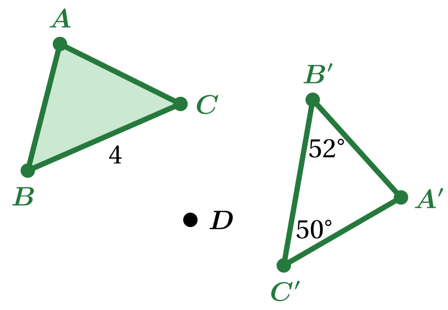
6.



Lesson Practice
ACC7.1.05

Name: .... Date: .... Period: ....

Problems 7–9: Triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is point D, which lies at the origin.



7. What is the length of side $B'C'$?

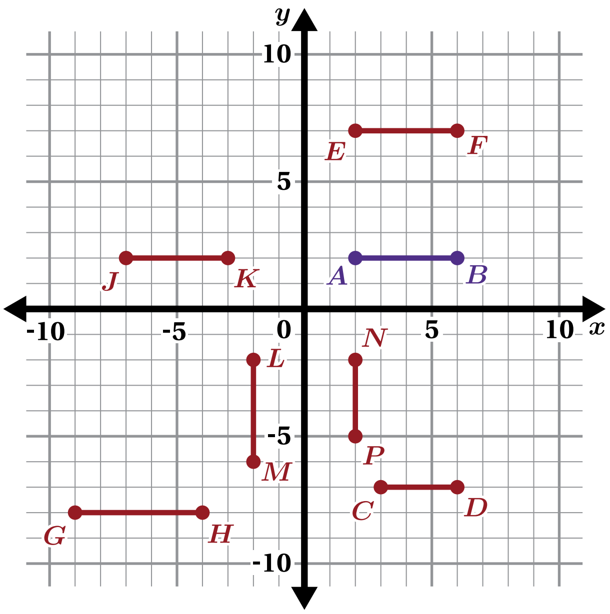
8. What is the measure of angle B?

9. What is the measure of angle C?

FAST Practice

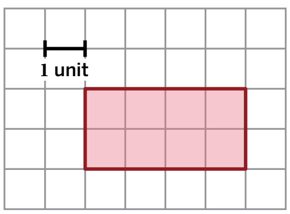
10. Which of the following segments are the result of a single rigid transformation of segment AB? Select *all* that apply.

- ☐ **A.** Line segment CD
- ☐ **B.** Line segment KJ
- ☐ **C.** Line segment NP
- ☐ **D.** Line segment LM
- ☐ **E.** Line segment GH
- ☐ **F.** Line segment EF



Spiral Review

Problems 11–12: Here is a rectangle.



11. What is the area of the rectangle?

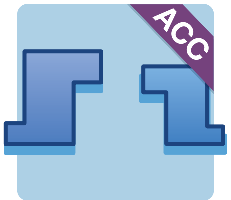
12. What is the perimeter of the rectangle?

Unit 1
Lesson
6

Name: ________ Date: ________ Period: ________

MA.8.GR.2.1, MTR.4.1

Are They the Same?



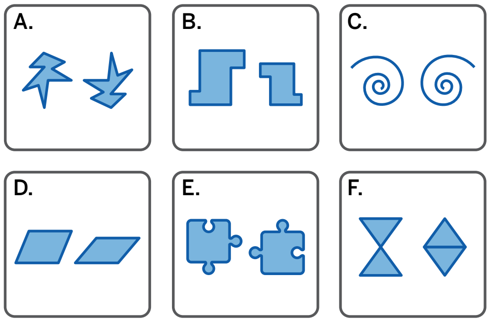
Let's explore a type of sameness.

Warm-Up

1. Here are six pairs of figures.

a Circle *all* the pairs with figures that are the same.

b **Discuss:** Which pairs did you choose? Why?



2. Why might someone say Pair B's figures are the same? Are *not* the same?

They are the same because . . .

They are not the same because . . .

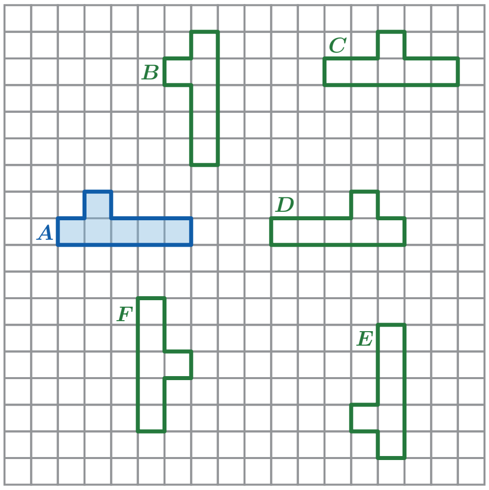
Activity
1

Name: Date: Period:

Defining Congruence

3. One figure is **congruent** to another if it has the exact same size and shape. Congruent figures have sides that are the same length and angles that are the same measure.

Circle *all* the figures that are congruent to figure A.



4. One figure is also congruent to another if it can be translated, rotated, or reflected to fit exactly over the other.

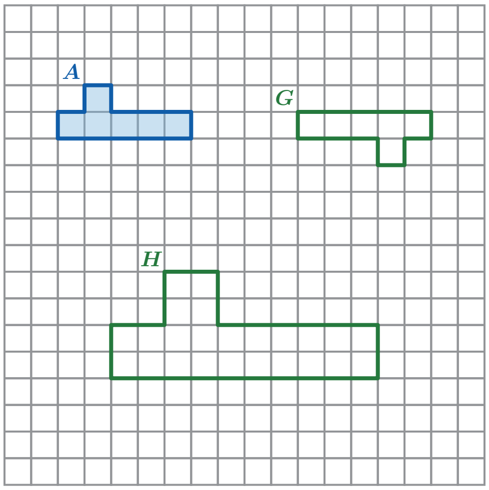
Which figure is the result of a single rigid transformation of figure A? Describe the transformation.

5. Kweku says figures A and G are congruent. Lan says figures A and H are congruent.

Whose claim is correct? Circle one.

Kweku's Lan's Both Neither

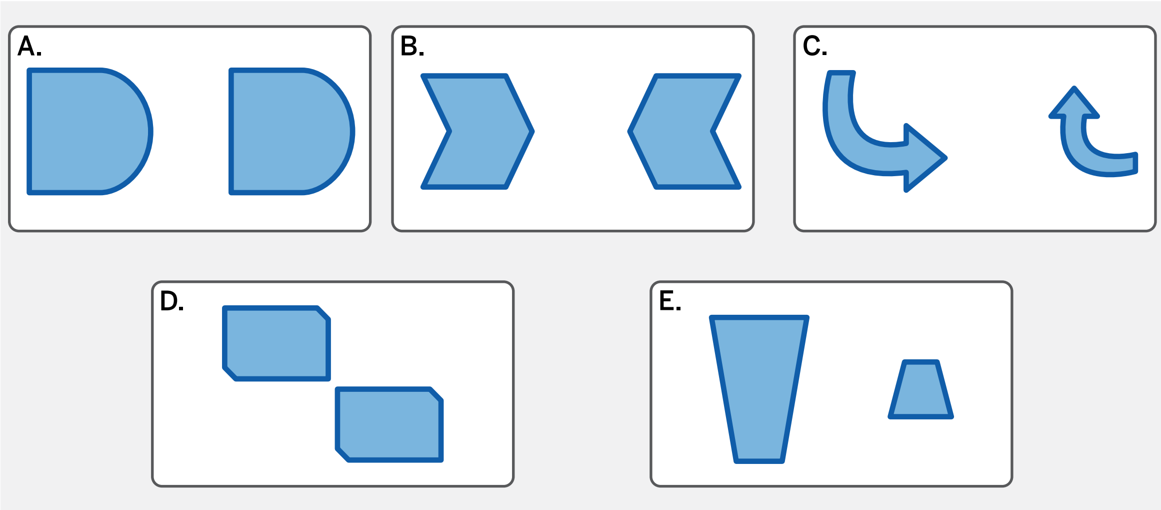
Explain your thinking.



Name: .... Date: .... Period: ....

Defining Congruence (continued)

6. Group the pairs of figures based on whether you think they are congruent or not congruent.



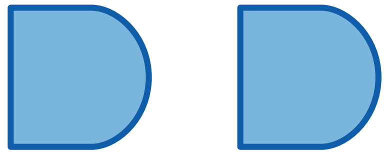
Congruent	Not Congruent

Activity 2

Name: .... Date: .... Period: ....

Are They Congruent?

7. Which rigid transformation could be used to show that these two figures are congruent?

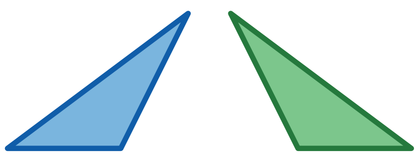
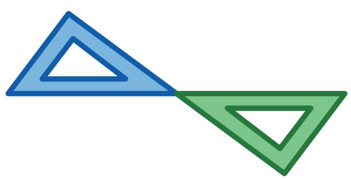


- **A.** A translation
- **B.** A rotation
- **C.** A reflection
- **D.** None. They're not congruent.

Explain your thinking.

8. For each pair of figures, name a rigid transformation that could be used to show that they are congruent or write that they aren't congruent. Decide with a partner who will complete Column A and who will complete Column B.

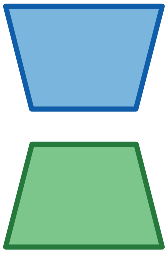
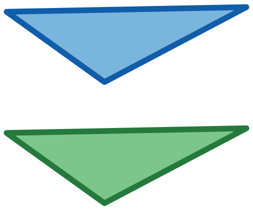
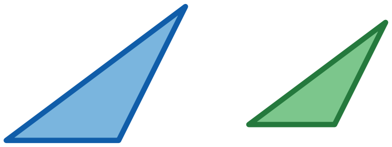
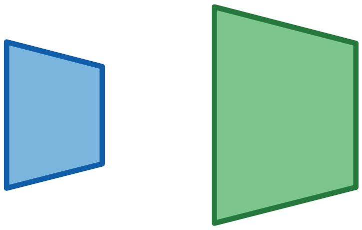
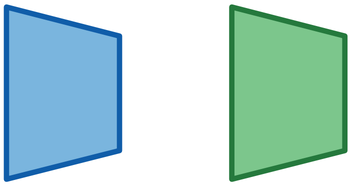
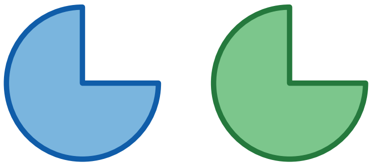
- After each problem, compare your answers. The answers in each row should be the same. Discuss and resolve any differences.
- Complete as many problems as you have time for.

Column A	Column B
 Sequence:	 Sequence:

Activity 2

Name: ____________ Date: ____________ Period: ____________

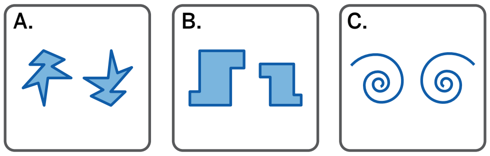
Are They Congruent? (continued)

Column A	Column B
 Sequence:	 Sequence:
 Sequence:	 Sequence:
 Sequence:	 Sequence:

Synthesis

9. How can you determine whether two figures are congruent?

Use the examples if they help with your thinking.



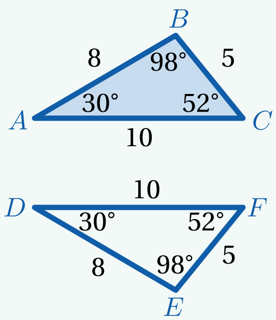
Lesson Practice ACC7.1.06

Lesson Summary

Two figures are **congruent** if they have exactly the same size and shape. They are also congruent if you can use a rigid transformation to move one exactly on top of the other.

You don't need to check that all corresponding angle measures and side lengths are equal if you can show a rigid transformation.

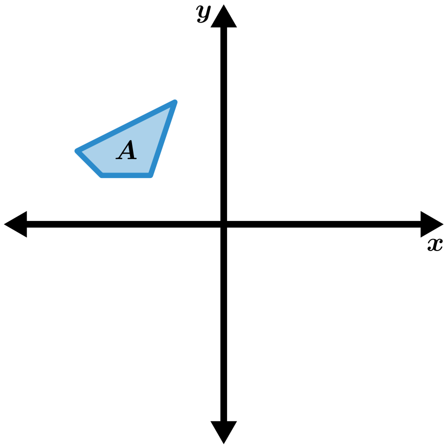
For example, figure DEF is congruent to figure ABC because you can reflect ABC over a horizontal line to fit exactly on top of figure DEF.



Lesson Practice
ACC7.1.06

Name: ______ Date: ______ Period: ______

Problems 1–2: This coordinate plane shows figure A.

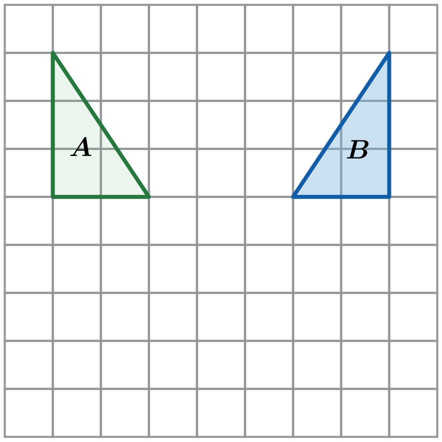


1. Reflect figure A over the x-axis. Label the image B.

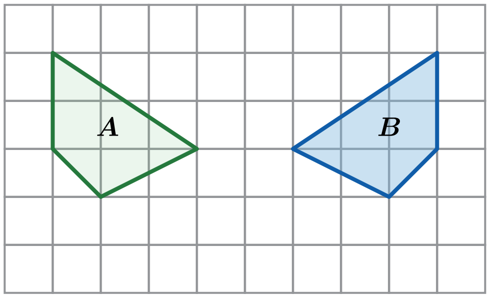
2. Are figures A and B congruent? Explain your thinking.

Problems 3–6: Determine whether figure A is congruent to figure B. Explain your thinking.

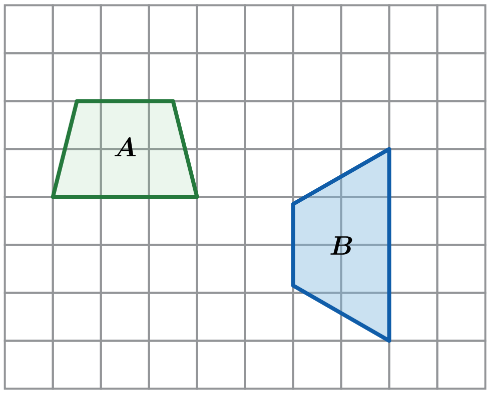
3.



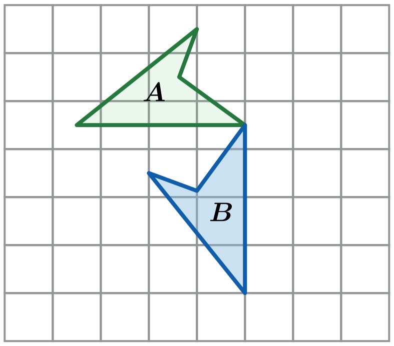
4.



5.



6.



Lesson Practice
ACC7.1.06

Name: ______ Date: ______ Period: ______

FAST Practice

7. Pentagon $ABCDE$ was graphed on a coordinate plane and then rotated $90°$ counterclockwise around the origin to form pentagon $A'B'C'D'E'$. Which statement is true?

A. Pentagon $A'B'C'D'E'$ is not congruent to pentagon $ABCDE$.

B. The area of pentagon $A'B'C'D'E'$ is not equal to the area of pentagon $ABCDE$.

C. The angle measures of pentagon $A'B'C'D'E'$ are congruent to the corresponding angle measures of pentagon $ABCDE$.

D. The perimeter of pentagon $A'B'C'D'E'$ is greater than the perimeter of pentagon $ABCDE$.

Spiral Review

Problems 8–9: Determine the coordinates of the image of point A (2, -5) after each transformation.

8. A reflection over the x-axis.

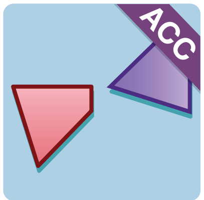
9. A reflection over the y-axis.

Unit 1
Lesson
7

Name: ______ Date: ______ Period: ______

MA.8.GR.2.1, MA.8.GR.2.3, MTR.4.1

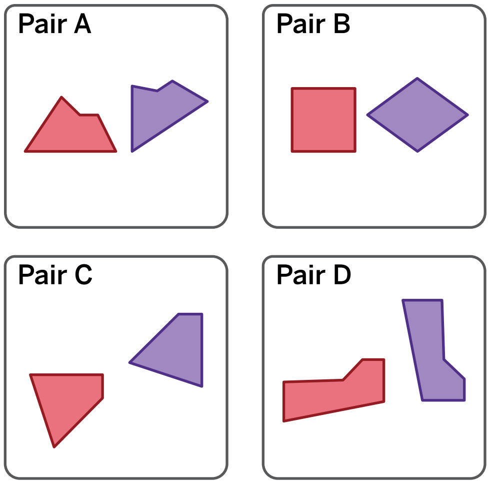
Are They Congruent?



Let's make arguments about whether two figures are congruent.

Warm-Up

1. Here are four pairs of figures.



a Make a prediction! Circle *all* pairs of figures that look congruent.

Pair A Pair B Pair C Pair D

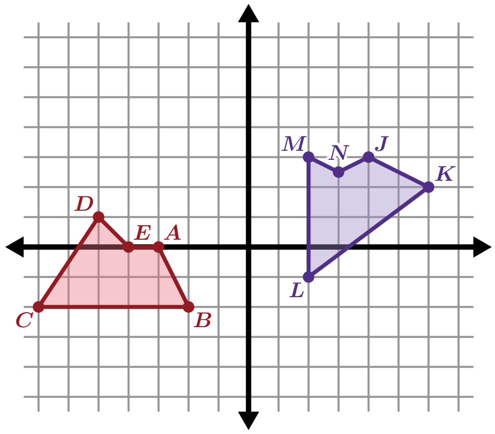
b **Discuss:** How can you be more sure which pairs of figures are congruent?

Name: ______ Date: ______ Period: ______

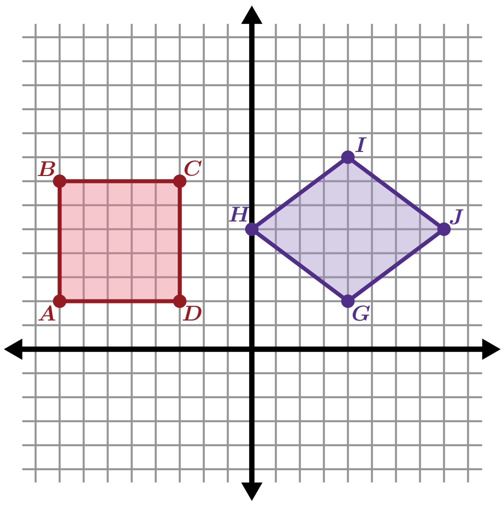
Congruent or Not?

Let's revisit some pairs from the Warm-Up.

2. Explain how you know the figures are not congruent.



3. Mauricio noticed that each side length in figure $ABCD$ is equal to each side length in figure $GHIJ$. He says this proves that $ABCD \cong GHIJ$ ($ABCD$ is congruent to $GHIJ$).



Is his claim correct? Circle one.

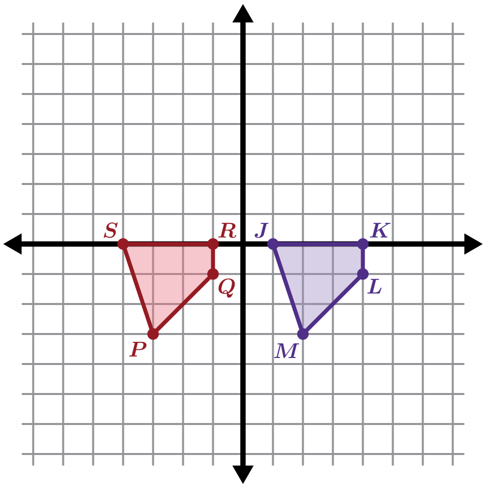
Yes No

Explain your thinking.

Name: ______________ Date: ________ P

Congruent or Not? (continued)

4. Students in another class were asked to convince their peers that the figures in Pair C are congruent. Which of their arguments do you think is most convincing?



(A.) Both figures have 4 sides and an area of 5.5 square units.

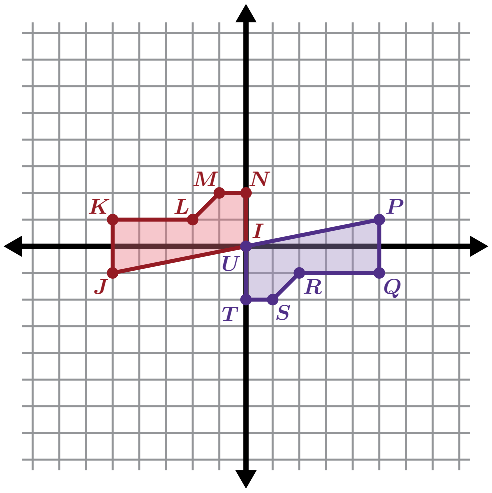
(B.) I can move the figures right on top of each other by translating figure $JKLM$ to the left 5 units.

(C.) When I measure the side lengths of figures $JKLM$ and $PQRS$, I get the same measurements.

Explain your thinking.

5. Describe a rigid transformation that will convince your classmates that the figures from Pair D are congruent.

Use tracing paper if it helps with your thinking.



Name: .... Date: .... Period: ....

Prove It!

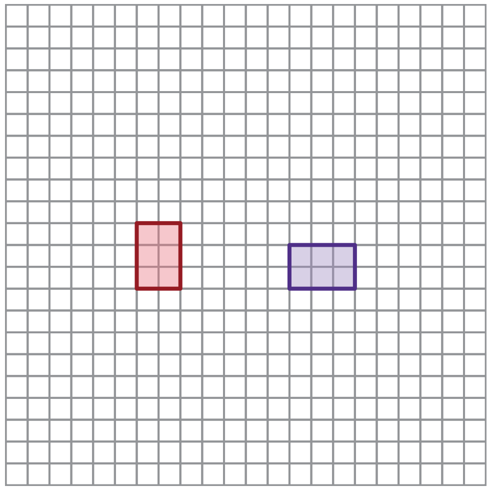
6. Angel says that any two rectangles with the same area are congruent.

Is his claim correct? Circle one.

Draw some rectangles to help you decide.

Yes No I'm not sure

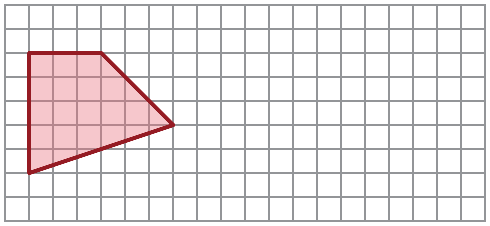
Explain your thinking.



7. Which statement(s) are enough to prove that two figures are congruent? Select *all* that apply.

- ☐ **A.** The figures are both right triangles.
- ☐ **B.** You can trace one figure on tracing paper and move it perfectly on top of the other.
- ☐ **C.** The figures that have areas of 8 square units are both rectangles.
- ☐ **D.** You can move one figure right on top of the other by translating 8 units left.
- ☐ **E.** The figures are both isosceles right triangles.

8. **a** Draw a second figure to create two figures that are *not* congruent but that someone might think are.

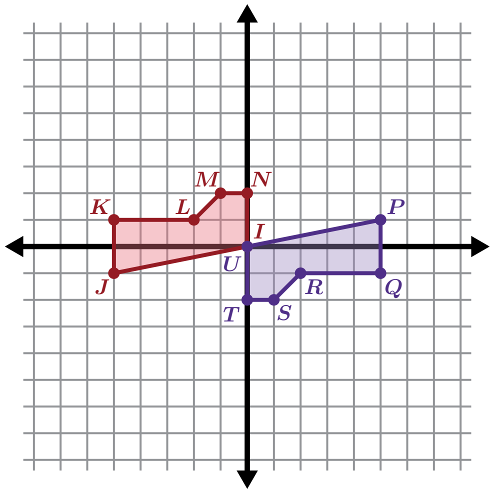


b Explain how you know the two figures are not congruent.

Synthesis

9. How can you prove that two figures are congruent?

Use the example if it helps with your thinking.



Lesson Practice ACC7.1.07

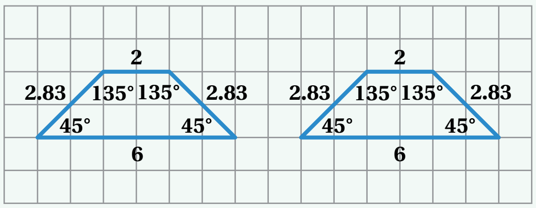
Lesson Summary

Here are two ways to determine whether two figures are congruent:

- You can determine a transformation that moves one exactly onto the other.
- You can determine that *all* the corresponding sides have the same length and *all* the corresponding angles have the same measure.

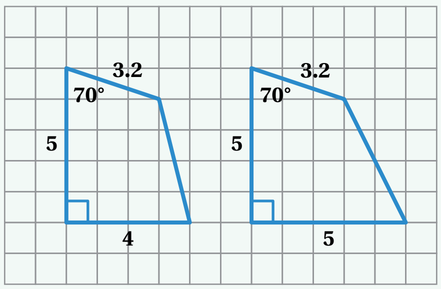
Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas.

Congruent



These figures are congruent because all the corresponding side lengths and angle measures are the same.

Not Congruent

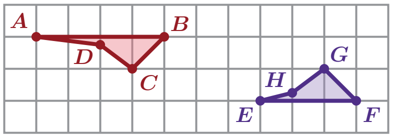


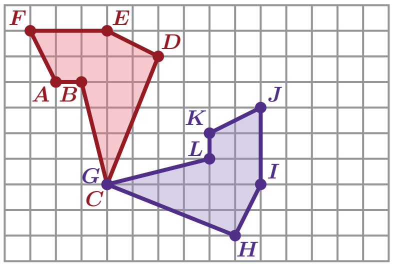
These figures are not congruent because *some* of the side lengths and measures are the same, but some are different.

Lesson Practice
ACC7.1.07

Name: .... Date: .... Period: ....

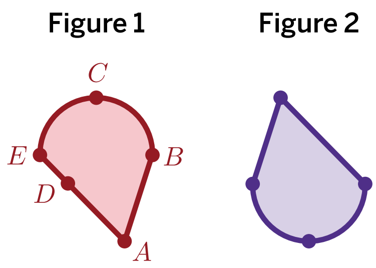
Problems 1–2: For each problem, determine whether the figures are congruent and explain how you know.

1. 

2. 

3. If two rectangles have the same perimeter, will they always be congruent? Explain your thinking.

Problems 4–6: Here are two congruent figures.



4. Label Figure 2 with points A', B', C', and E' so that they correspond to points A, B, C, and E in Figure 1.

5. If segment AB is 2 centimeters long, how long is segment $A'B'$? Explain your thinking.

6. Plot point D' where you think it is located. Explain your thinking.

Name: .... Date: .... Period: ....

FAST Practice

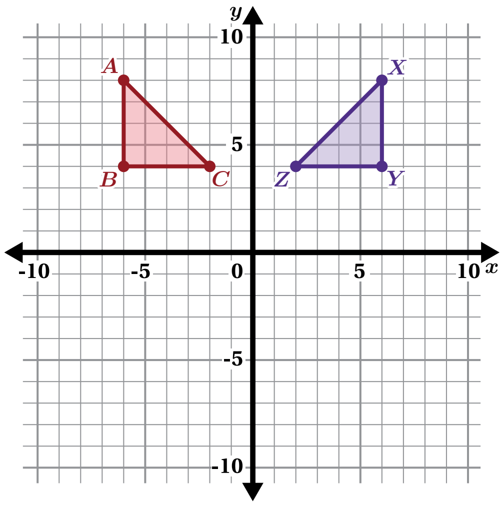
7. Triangle ABC and triangle XYZ are congruent. Which sequence of transformations will move triangle ABC onto triangle XYZ?

A. A reflection over the y-axis.

B. A reflection over the x-axis.

C. A translation 4 units right.

D. A translation 8 units right.



Spiral Review

8. Isaiah buys a shirt that costs $\$25.00$. He has a coupon for 15% off. The sales tax is 8%. What is the total cost of the shirt after tax?

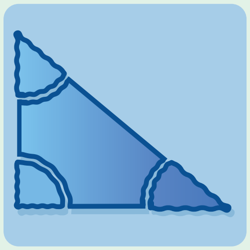
9. Sydney invests $\$500$ in a savings account. The simple interest rate is 3.75%. How much will Sydney have in the account after 3 years?

10. A manufacturer decreased the amount of shampoo in a bottle from 25 fluid ounces to 22.5 fluid ounces. What percent decrease is this?

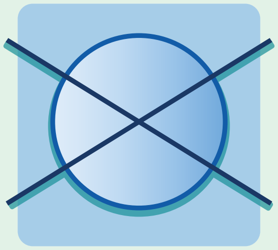
Sub-unit

3

Angle Relationships



Lesson 8
Tearing It Up



Lesson 9
Friendly Angles

Lesson 10
Sum It Up

Unit 1
Lesson
8

Name: ______ Date: ______ Period: ______

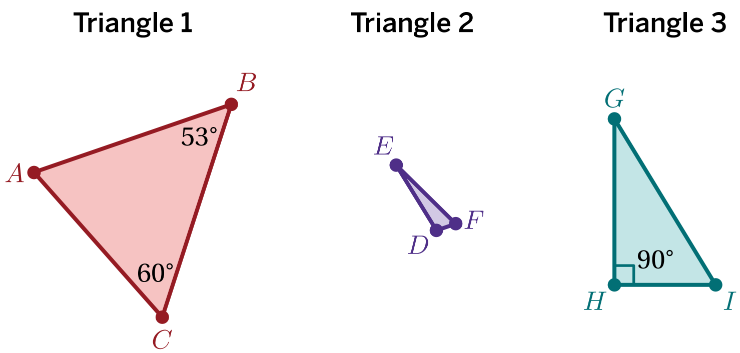
MA.8.GR.1.5, MTR.3.1

Tearing It Up

Let's explore the interior and exterior angles of triangles.

Warm-Up

1. Here are three triangles.



If you add up the three **interior angles** of each triangle, which triangle do you think has the greatest sum? Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Find All Three

You will get a card with a picture of a triangle.

2. The measurement of one of the angles is labeled. Estimate the measures of the other two angles.

Labeled angle measure: ....

Estimated angle measure: .... **Estimated angle measure:** ....

3. Find two other students with triangles that look congruent to yours, but with a different labeled angle.

Name: .... **Card number:** ....

Name: .... **Card number:** ....

4. Confirm that all three triangles are congruent and that each card has a different labeled angle. How did you know that the triangles were congruent?

5. Record the three angle measures for your triangle in the table.

Card Numbers	Angle 1	Angle 2	Angle 3	Angle Sum

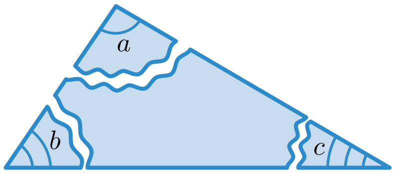
Activity 2

Name: ______ Date: ______ Period: ______

Tear It Up

6. You will use a blank sheet of paper to complete this activity. Use a straightedge to draw a triangle that you think will be different from the triangles your classmates will draw.

7. Label the angles of your triangle with the letters a, b, and c. Cut out the triangle, then tear the three angles off of the triangle like in this picture.

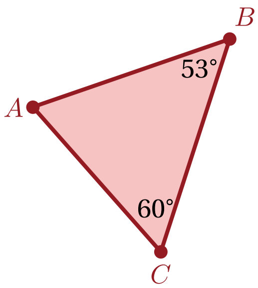


8. Rearrange the angles so that the three vertices meet with no overlap.

9. Compare your results with your classmates' results. What do you notice about your angles? What does this mean about the sum of the angles in a triangle?

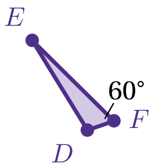
10. Here are the triangles from the Warm-Up, with some additional angle measurements labeled. For each triangle, determine a possible value for the angle listed.

Triangle 1



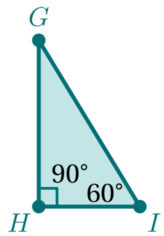
$m\angle A =$ ______

Triangle 2



$m\angle D =$ ______

Triangle 3

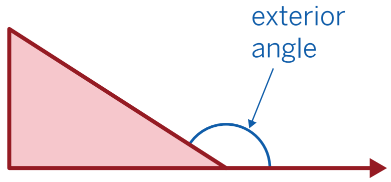


$m\angle G =$ ______

Name: .... Date: .... Period: ....

Tear It Up Again

An **exterior angle** of a triangle is formed between a side of the triangle and a line extended from the next side.



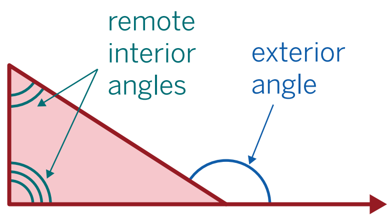
11. You will use the Activity 3 Sheet to explore.

Were you able to find a combination of angles that worked for each triangle? Explain.

12. Think about what you know about the sum of the measures of the interior angles of a triangle. How does that explain your observations?

13. When a triangle has an exterior angle, the angles inside the triangle that are not adjacent to it are called **remote interior angles**.

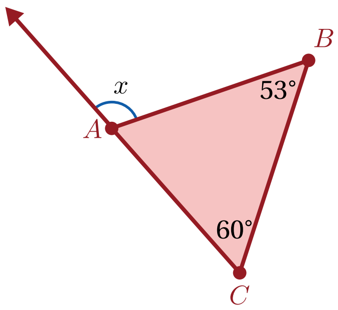
What conclusion can you draw about the relationship between an exterior angle and the remote interior angles?



Synthesis

14. What is true about the sum of the three angle measures in a triangle? What is true about the measure of an exterior angle of a triangle?

Use the example if it helps with your thinking.



Lesson Practice ACC7.1.08

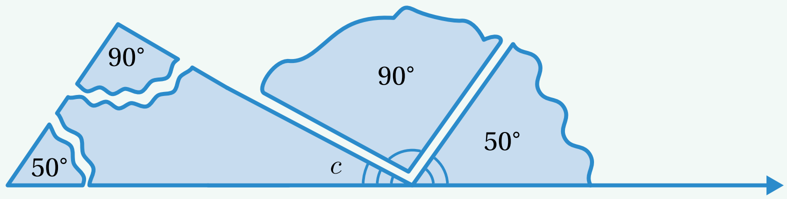
Lesson Summary

The **interior angle** measures of any triangle always sum to 180 degrees. We can show this by rearranging the angles of any triangle to form a straight line, which has a measure of 180°.

If you know the measures of two angles in a triangle, you can determine the third angle by subtracting the sum of the two known angle measures from 180°. Here is an example.

$$c = 180 - 90 - 50$$
$$c = 40$$

Similarly, the measure of an **exterior angle** of a triangle is equal to the sum of the measures of the **remote interior angles** because they also form a straight line.



Lesson Practice
ACC7.1.08

Name: .... Date: .... Period: ....

1. If triangle ABC is a right triangle and $m\angle A = 40°$, what are possible measures for angles B and C?

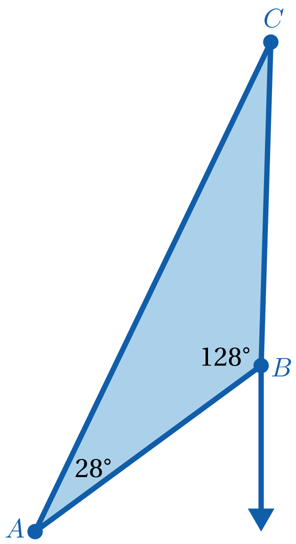
2. Triangle ABC is an isosceles triangle and the measure of one of the angles is $40°$. List a set of possible angle measures in this triangle.

3. Can there be a triangle with two right angles? Explain your thinking.

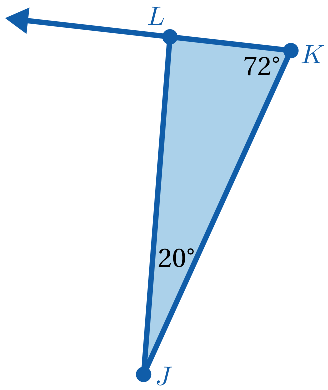
4. Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your thinking.

Problems 5–6: Determine the measure of each missing interior and exterior angle.

5.



6.



Lesson Practice
ACC7.1.08

Name: .... Date: .... Period: ....

FAST Practice

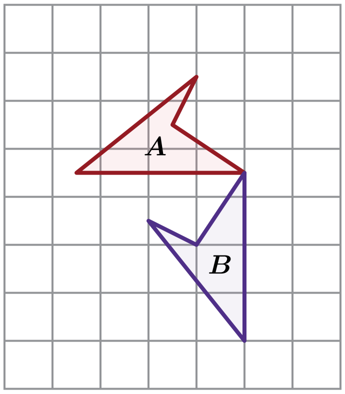
7. Select *all* of the sets of angles that could make a triangle.

- ☐ **A.** $60°, 60°, 60°$
- ☐ **B.** $90°, 90°, 45°$
- ☐ **C.** $30°, 40°, 50°$
- ☐ **D.** $90°, 45°, 45°$
- ☐ **E.** $120°, 30°, 30°$

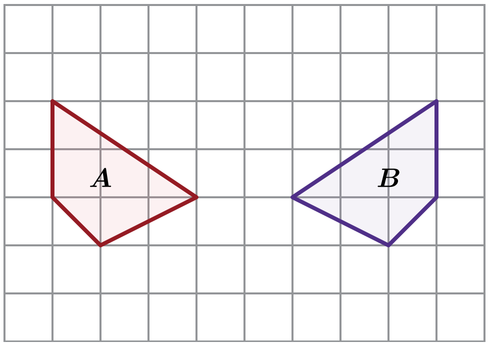
Spiral Review

Problems 8–9: For each pair of polygons, describe a transformation that shows polygon A is congruent to polygon B.

8.



9.

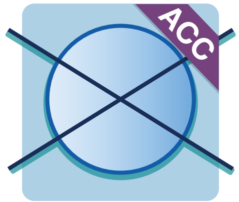


Name: ______________ Date: ________ Period: ________

MA.8.GR.1.4, MA.8.AR.2.1, MTR.1.1, MTR.3.1

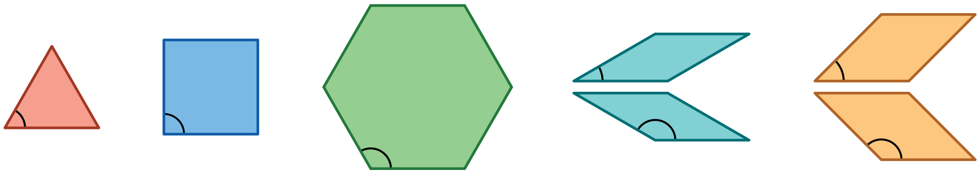
Friendly Angles

Let’s explore complementary, supplementary, vertical, and adjacent angles.



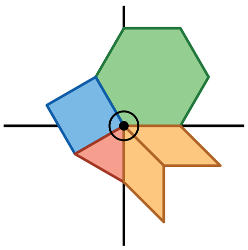
Warm-Up

1. These shapes were used to create four $360°$ designs.

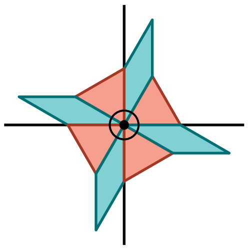


a Pick a design that you like.

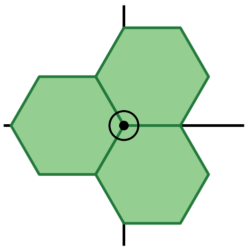
A.



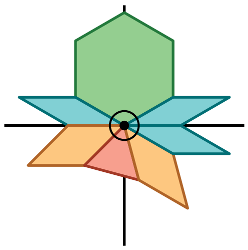
B.



C.



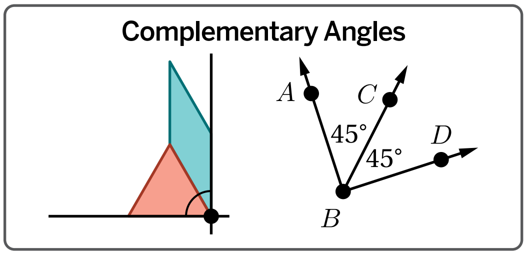
D.

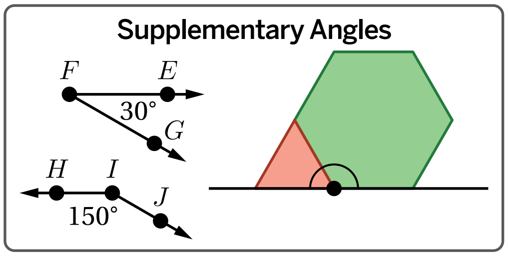


b **Discuss:** What do you like about the design you chose?

Relationships and Equations

2. The terms **complementary** and **supplementary** describe special pairs of angles.





Describe what you think these terms mean.

Complementary angles . . .

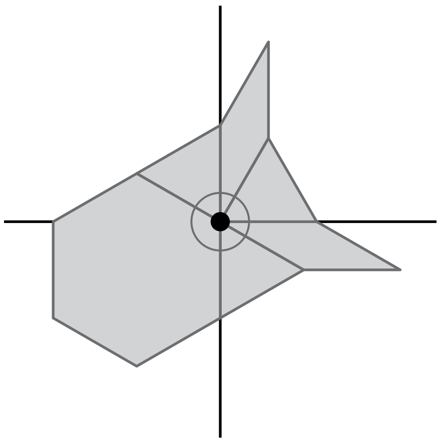
Supplementary angles . . .

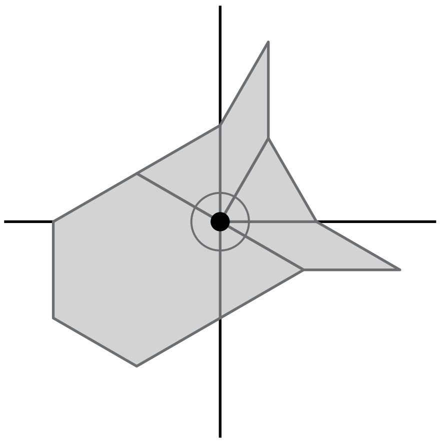
Adjacent angles . . .

3. Here is a new design.

a Shade in a pair of *complementary* angles.

b Shade in a pair of *supplementary* angles.



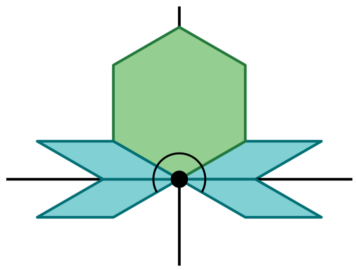


Name: .... Date: .... Period: ....

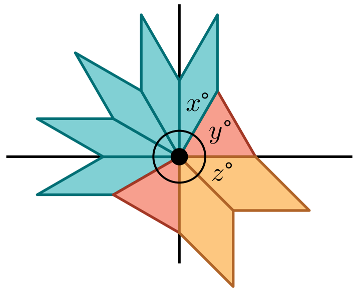
Relationships and Equations (continued)

4. Ivory used the equation $2x + 120 = 180$ to determine one angle measure in this diagram.

Explain or show what each part of Ivory's equation represents in the diagram.



5. Here is a new diagram.



a Select *all* the true equations.

- ☐ **A.** $3x = 90$
- ☐ **B.** $x + y = 90$
- ☐ **C.** $5x = 180$
- ☐ **D.** $x + y + 2z = 180$
- ☐ **E.** $x + y + z = 360$

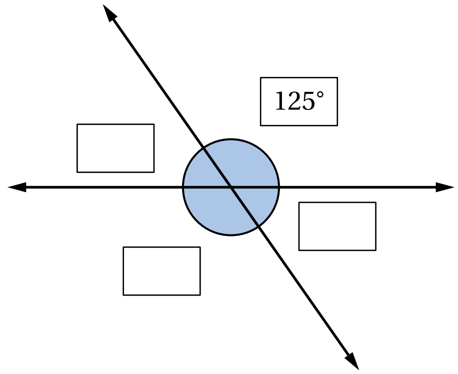
b **Discuss:** How did you decide which equations are true?

Activity
2

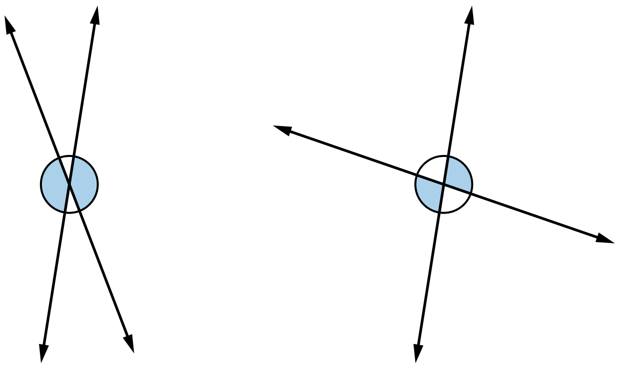
Name: ______ Date: ______ Period: ______

Vertical Angle Puzzles

6. Here is an angle puzzle. Use the given angle measure to determine all the angle measures.



7. Lola noticed that when two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.



Are the measures of vertical angles always, sometimes, or never the same? Circle one.

Always Sometimes Never

Explain your thinking.

Name: .... Date: .... Period: ....

Vertical Angle Puzzles (continued)

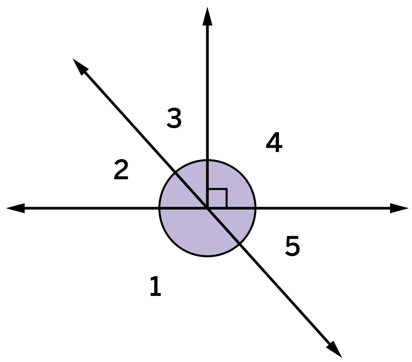
8. Here is a new angle puzzle. Which of these is a pair of vertical angles?

A. 1 and 4

B. 2 and 3

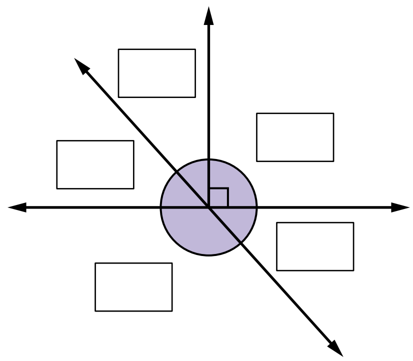
C. 2 and 5

D. 3 and 5



Discuss: Choose one of the other pairs. How do you know they are *not* vertical angles?

9. Here is a new angle puzzle. You can ask for the measure of an angle. Determine all the angle measures using as few hints as you can.



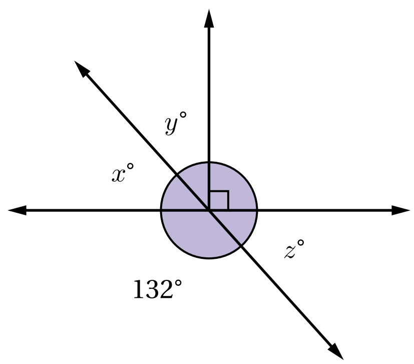
10. Kwasi and Lola wrote equations to help them solve the previous angle puzzle.

Kwasi's equation: $x + 132 = 180$

Lola's equation: $132 + z = 180$

Whose equation is correct? Circle one.

Kwasi Lola Both Neither

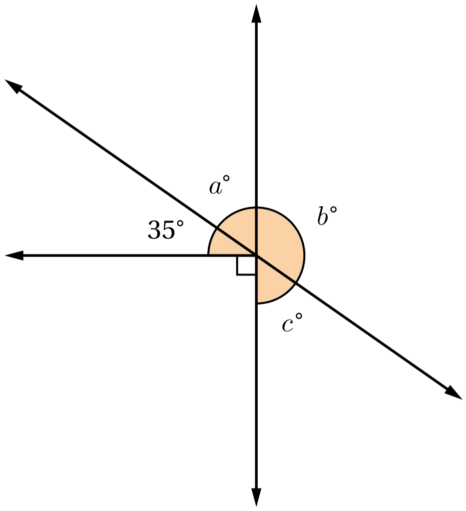


Explain your thinking.

Name: .... Date: .... Period: ....

Writing and Using Equations

Use the diagram for Problems 11 and 12.



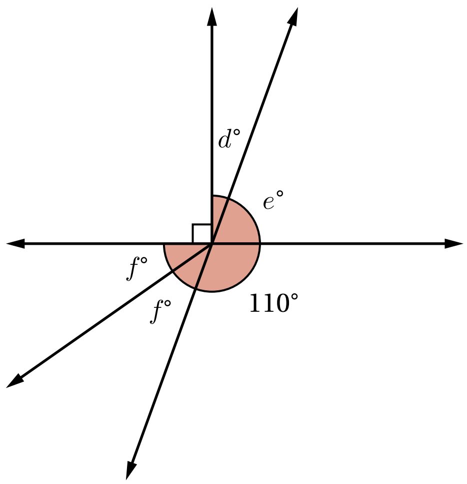
11. Write a true equation based on this angle puzzle. Try to write an equation none of your classmates will.

12. Determine the values of a, b, and c.

Writing and Using Equations (continued)

13. Here is a new angle puzzle.

Determine the values of d, e, and f.

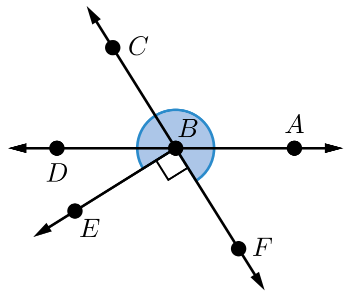


14. Kwasi wrote the equation $f + 110 = 180$ for the previous puzzle. Change Kwasi's equation to make it true.

Synthesis

15. Here is a diagram. Describe or show as many angle relationships as you can.

Use the terms *complementary*, *supplementary*, *vertical*, and *adjacent* in your description.

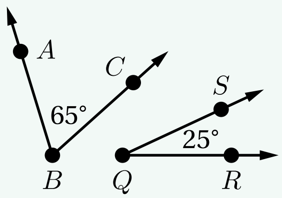


Lesson Practice ACC7.1.09

Lesson Summary

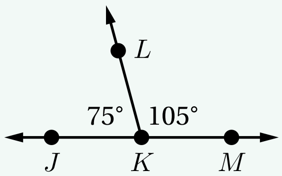
Here are four angle relationships that can help you determine missing angle measures.

Complementary angles have measures that add up to 90°.



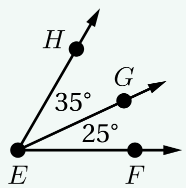
$\angle ABC$ and $\angle RQS$ are complementary angles.

Supplementary angles have measures that add up to 180°.



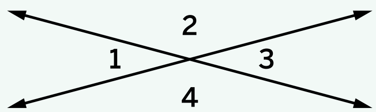
$\angle JKL$ and $\angle MKL$ are supplementary angles.

Adjacent angles share a side and a vertex.



$\angle HEG$ and $\angle FEG$ are adjacent angles.

Vertical angles are opposite angles formed when two lines cross. Vertical angles have the same measure.

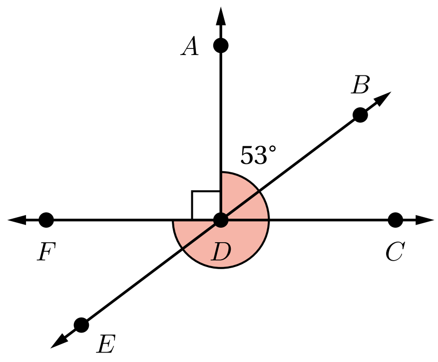


$\angle 1$ and $\angle 3$ are vertical angles.
$\angle 2$ and $\angle 4$ are vertical angles.

Lesson Practice
ACC7.1.09

Name: ________ Date: ________ Period: ________

Problems 1–3: Here is a diagram.



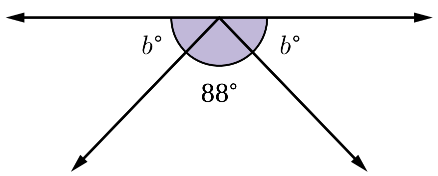
1. Determine the measure of each angle.

Angle	Measure (degrees)
ADB	53
BDC	
CDE	
FDE	
FDA	

2. Identify one pair of vertical angles, one pair of complementary angles, and one pair of supplementary angles in the diagram.

3. Explain how you know if a pair of angles are vertical angles.

Problems 4–5: Here is a new diagram.



4. Which equation represents the relationship between the angles in the figure?

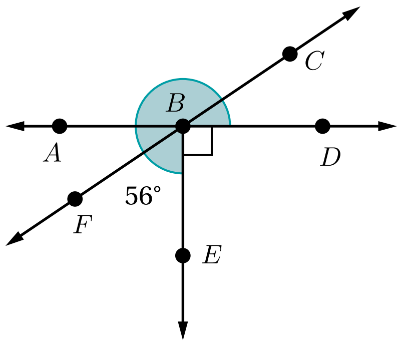
- A. $88 + b = 90$
- B. $88 + b = 180$
- C. $2b + 88 = 90$
- D. $2b + 88 = 180$

5. Dakota says that the angles marked b are vertical angles. Eva disagrees. Who is correct? Explain your thinking.

Name: .... Date: .... Period: ....

FAST Practice

6. This diagram includes supplementary, complementary, adjacent, and vertical angles. Determine each of the missing angle measures. Record your answers in the spaces provided.



Angle	Measure (degrees)
ABC	
CBD	
DBE	
FBA	

Spiral Review

Problems 7–10: Solve each equation.

7. $x + 40 = 180$ **8.** $x - 25 = 90$ **9.** $2x = 180$ **10.** $\frac{2}{3}x = 40$

Problems 11–12: A small dog gets fed $\frac{3}{4}$ of a cup of dog food twice a day.

11. Write an equation representing the relationship between the number of days, d, and the number of cups of food, f.

12. How many days will a large bag of dog food last if a new bag contains 210 cups of food?

Name: .... Date: .... Period: ....

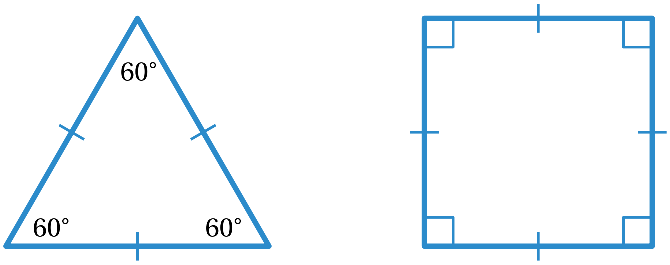
MA.8.GR.1.6, MTR.5.1

Sum It Up

Let's explore angle sums in polygons.

Warm-Up

Here are two polygons: an equilateral triangle and a square.



1. What is the sum of the interior angles in each polygon?

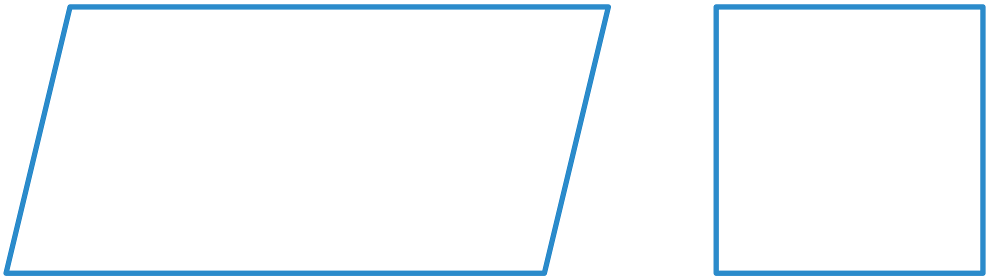
2. Does the angle sum depend on the number of sides of the polygon? Explain your thinking.

Activity
1

Name: Date: Period:

The Angle Sum of a Quadrilateral

3. Here are a parallelogram and a square.



a Choose one of the quadrilaterals and draw a diagonal to decompose it into 2 triangles. Then, label each angle of the triangles you created using a, b, c, d, e, and f.

b Write equations for the sum of the interior angles of each triangle.

c Write an equation for the sum of the interior angles of the parallelogram or square.

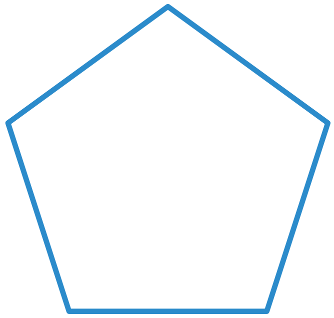
d **Discuss:** Find a partner who chose the other quadrilateral and compare your results. What did you discover?

Activity 2

Name: .... Date: .... Period: ....

Angle Sums Of Other Polygons

4. The polygon shown is a regular pentagon.

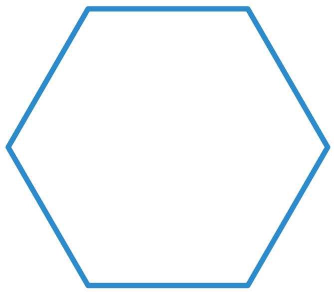


a Draw lines to decompose the pentagon into 3 triangles. All lines should be drawn from the same vertex.

b How can you use this decomposition to determine the angle sum of the pentagon?

c What is the angle sum of the pentagon?

5. The polygon below is a regular hexagon.



a Draw lines to decompose the hexagon into triangles. All lines should be drawn from the same vertex.

b What is the angle sum of the hexagon? Explain your reasoning.

Name: ______ Date: ______ Period: ______

Tying It All Together

6. Complete the table with the number of triangles and the sum of the interior angles of the different polygons you have investigated.

Polygon	Number of Sides	Number of Triangles	Interior Angle Measure Sum
Triangle	3	1	180°
Square			
Pentagon			
Hexagon			

7. Consider a polygon with n sides.

a Based on the pattern in the table, how many triangles would it decompose into?

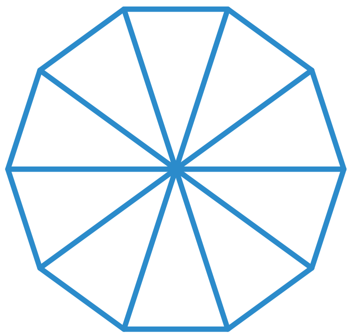
b Write an expression for the sum of the interior angles in a polygon with n sides.

c Use your expression to calculate the sum of the interior angles of a heptagon (7-sided polygon).

8. Polina tried to determine the sum of the interior angle measures in a decagon.

I decomposed the polygon into 10 triangles, so the angle sum is $10 \times 180^\circ = 1{,}800^\circ$.

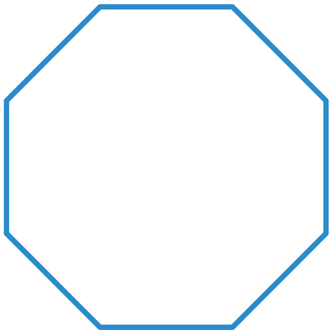
Describe the mistake in Polina's work.



Synthesis

9. Describe how you can use decomposition of a polygon into triangles to determine the sum of the interior angles.

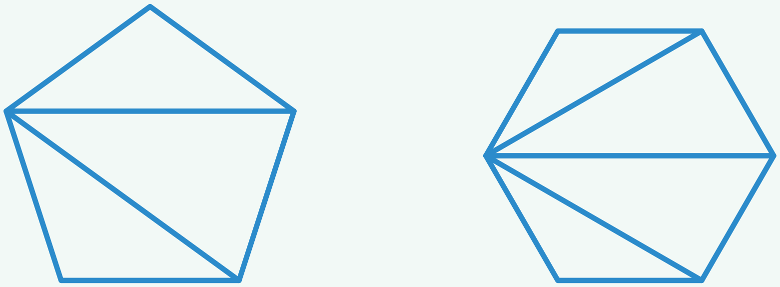
Use the polygon if it helps with your thinking.



Lesson Practice ACC7.1.10

Lesson Summary

You can determine the sum of the interior angle measures of a polygon by decomposing it into triangles. Because the angle sum of each triangle is $180°$, the angle sum of the polygon is equal to the number of triangles multiplied by $180°$.

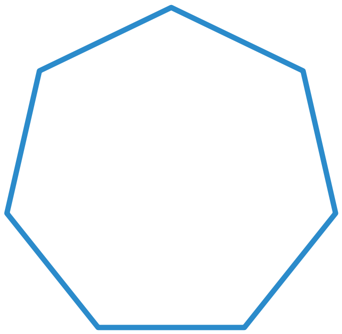


You can also calculate the sum of the interior angle measures of a polygon using the formula $(n - 2) \times 180°$, where n is the number of sides of the polygon.

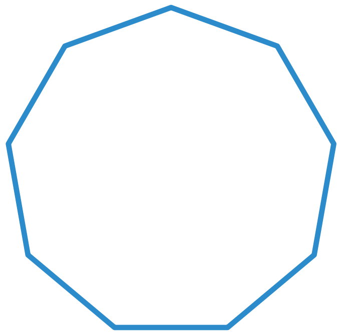
Name: .... Date: .... Period: ....

Problems 1–2: Decompose each polygon into triangles by drawing lines from the same vertex. Then calculate the sum of the interior angle measures.

1. Heptagon



2. Nonagon



3. A regular polygon has sides that are the same length and angles that are the same measure. A regular dodecagon has 12 sides. Determine the measure of each interior angle. Explain your reasoning.

4. Jamar says that he can calculate the sum of the interior angle measures of a 15-gon by multiplying $15 \times 180°$.

Do you agree with him? Explain your thinking.

FAST Practice

5. What is the sum of the interior angle measures of a polygon with 20 sides?

A. $180°$

B. $2{,}000°$

C. $3{,}240°$

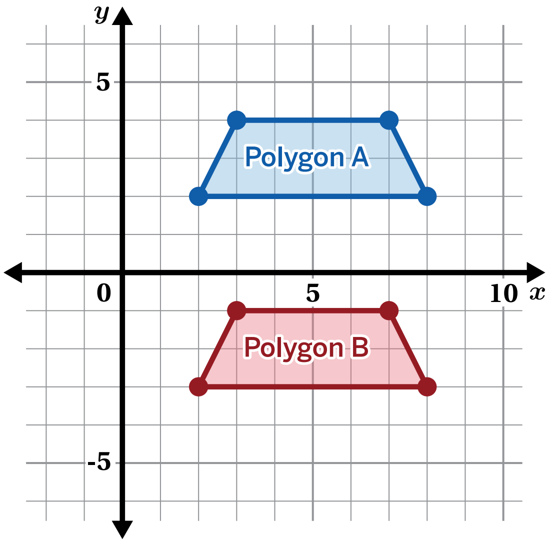
D. $3{,}600°$

Name: .... Date: .... Period: ....

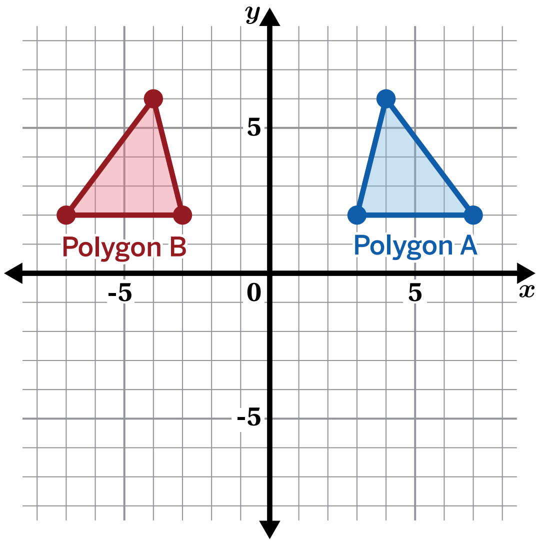
Spiral Review

Problems 6–9: For each pair of polygons, describe the transformation that maps polygon A onto polygon B.

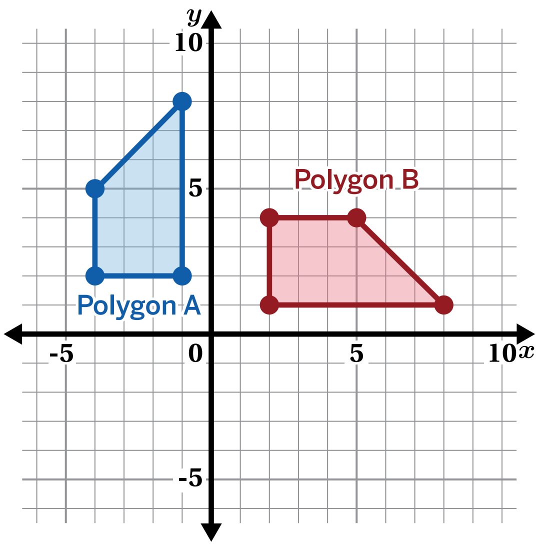
6.



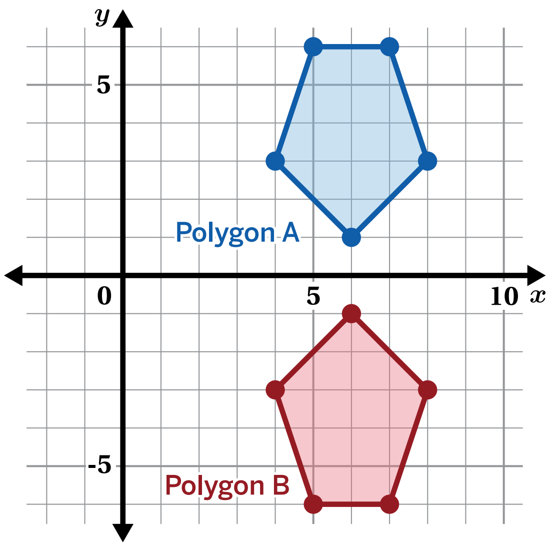
7.



8.

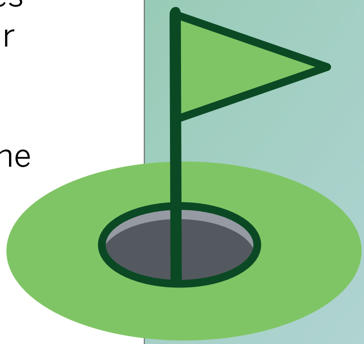


9.



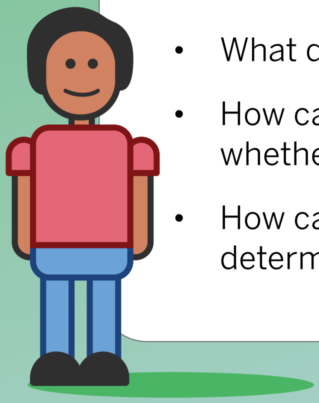
Unit 2

Scale Factors, Dilations, Similarity, and Slope



We use dilations in everyday life to make objects smaller or bigger, like printing pictures in different sizes or zooming in and out on our phone screens. Unlike rigid transformations, dilations change the dimensions of a shape. We'll learn how dilations can help us determine similarity and how similarity can help us understand slope.

Essential Questions

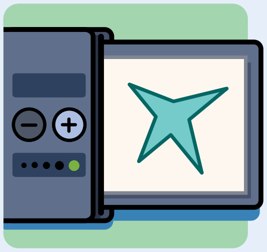


- What does it mean to dilate a figure?
- How can transformations be used to decide whether two figures are similar?
- How can similar triangles be used to determine the slope of a line?

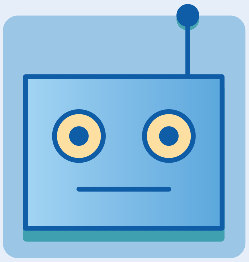
Sub-unit

1

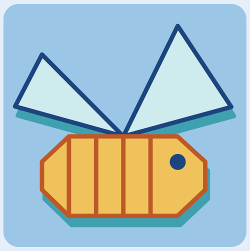
Scaled Copies



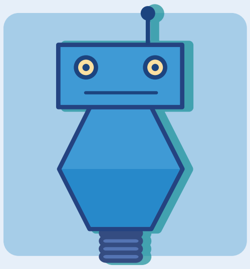
Lesson 1
Scaling Machines



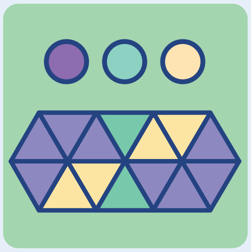
Lesson 2
Scaling Robots



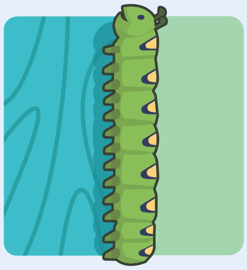
Lesson 3
Make It Scale



Lesson 4
Scale Factor Challenges



Lesson 5
Tiles



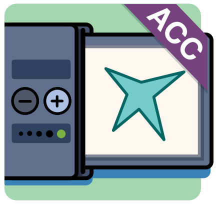
Lesson 6
Scale Drawings

Unit 2
Lesson
1

Name: ______________________ Date: __________ Period: __________

MA.7.GR.1.5, MTR.4.1, MTR.5.1

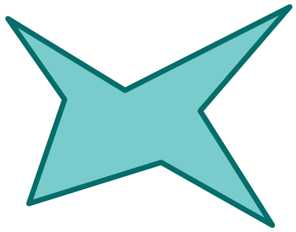
Scaling Machines



Let's describe the characteristics of scaled copies.

Warm-Up

1. What does this shape remind you of? Explain your thinking or tell a story about it.

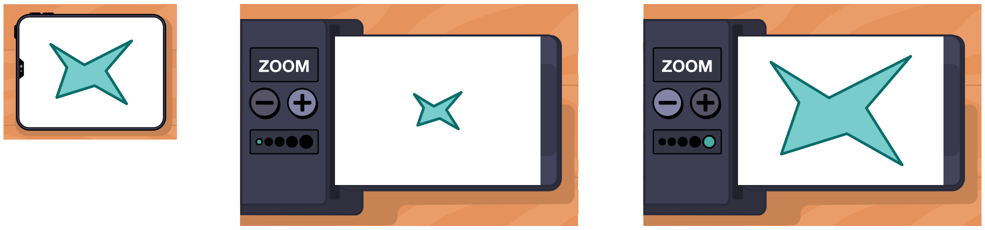


Activity
1

Name: .... Date: .... Period: ....

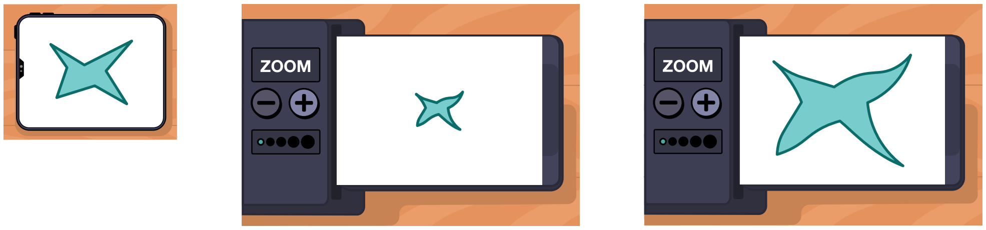
Printer Problems

2. Here is a working printer and printouts from two different zoom levels.



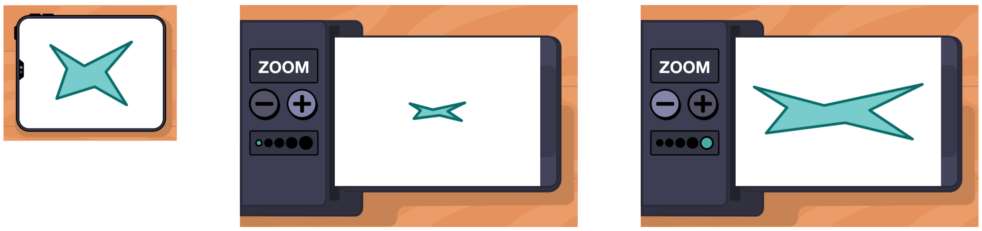
What stays the same no matter the zoom level? What changes?

3. Here is a *broken* printer and printouts from two different zoom levels.



Describe how the printer is broken.

4. Here is *another* broken printer that is broken in a *different* way.



Describe how the printer is broken.

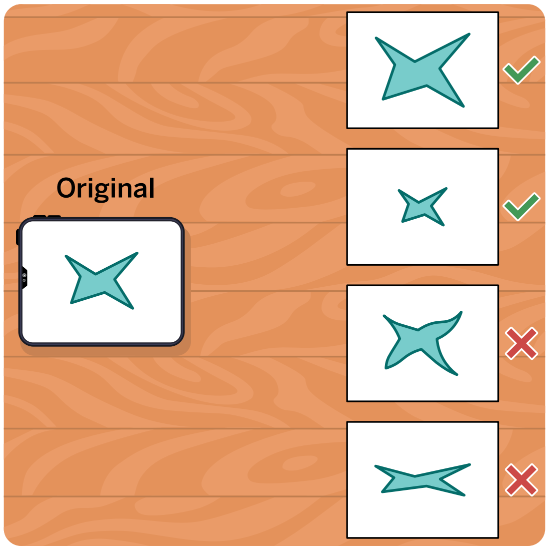
Name: .... Date: .... Period: ....

Printer Problems (continued)

5. Printouts 1 and 2 are **scaled copies**.

Printouts 3 and 4 are not.

Describe how you know if an image or a figure is a scaled copy.



Activity 2

Name: .... Date: .... Period: ....

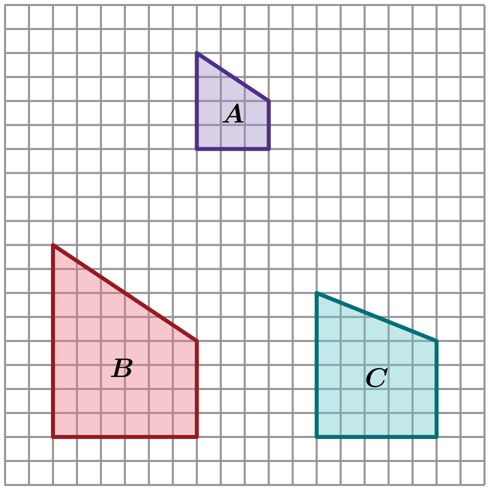
Scaled Copies

6. Here is figure A and two other figures.

   Which is a scaled copy of figure A? Circle one.

   Figure B Figure C Both Neither

   Explain your thinking.

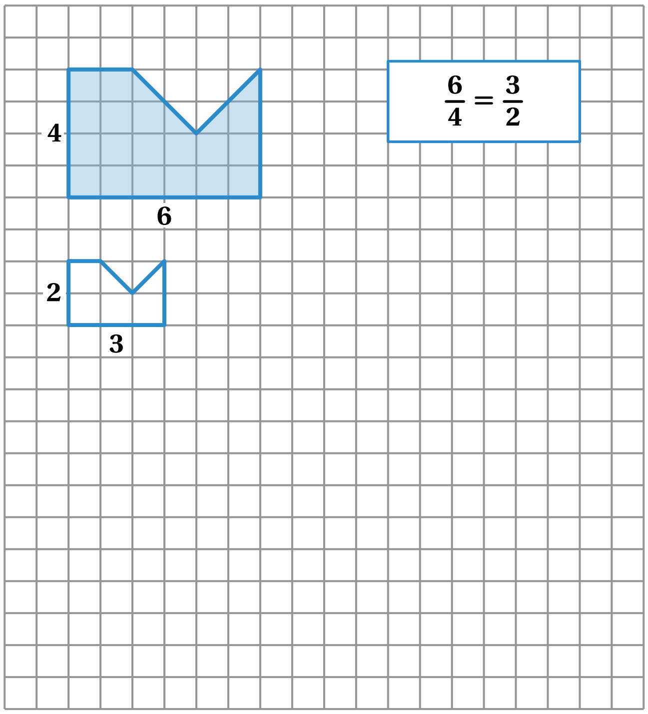


7. Scaled copies have **proportional** side lengths. This means that any two sides of the original shape form an *equivalent ratio* with the same two sides of the copy.

   Here is one scaled copy of the shaded figure, along with its equivalent ratio $\frac{3}{2}$.

   Sketch and label a *different* scaled copy of the shaded figure.

   Then sketch and label a figure that is *not* a scaled copy of the shaded figure.

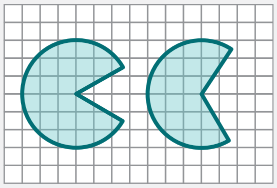
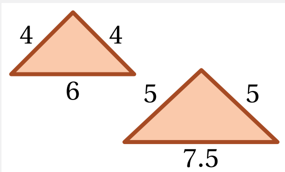
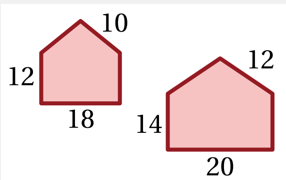
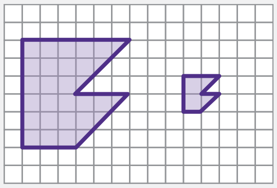
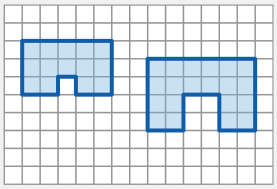


$\frac{6}{4} = \frac{3}{2}$

Name: .... Date: .... Period: ....

Scaled Copies (continued)

8. For each row, put a checkmark to show whether the pairs of figures are scaled copies.

	Scaled Copies	Not Scaled Copies
		
		
		
		
		

Name: ________ Date: ________ Period: ________

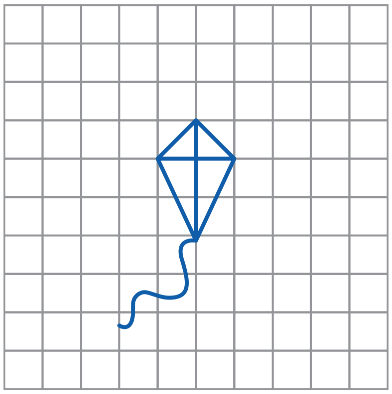
Machine Magic

9. Axel made a sketch.

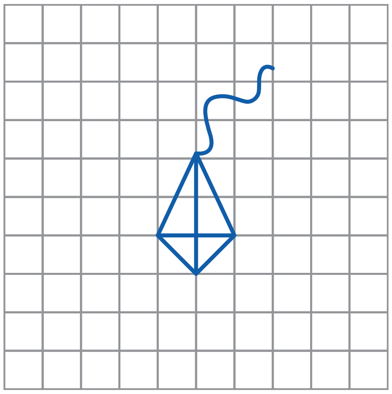
Here is what happened when four machines transformed his sketch.

Discuss: How does each machine work? Which machines make scaled copies?

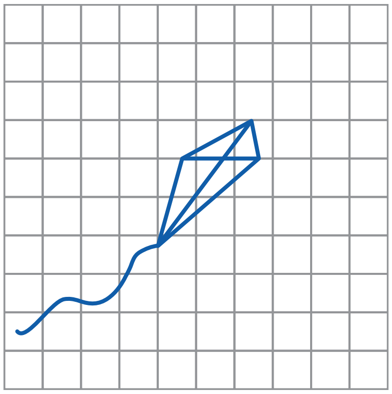
Axel's Sketch



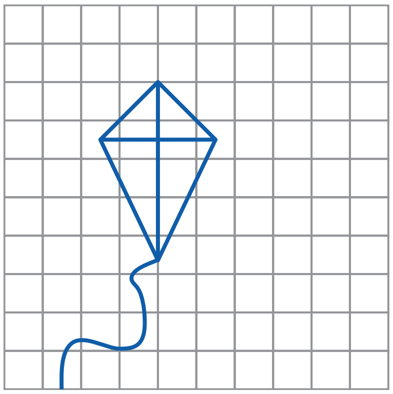
Machine 1



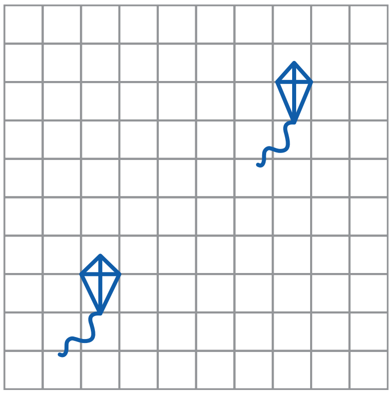
Machine 2



Machine 3

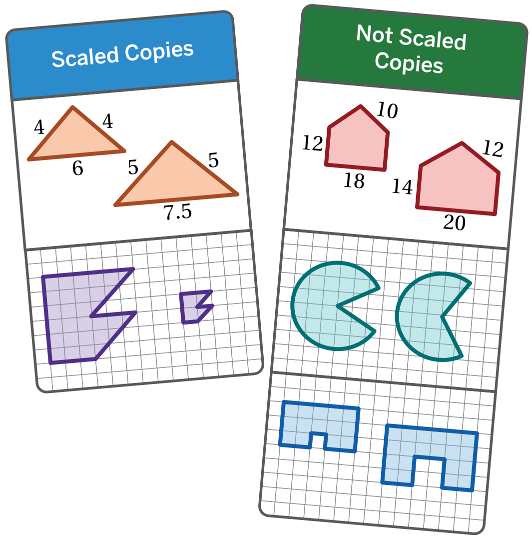


Machine 4



Synthesis

10. How can you tell whether a figure is a scaled copy of another figure?

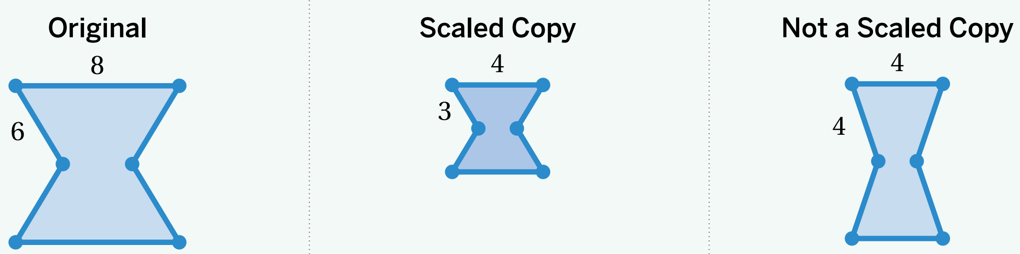


Lesson Practice ACC7.2.01

Lesson Summary

Scaled copies look similar to their originals. Though their overall size may change, their shape does not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

We know that two shapes are scaled copies when the lengths of their matching sides are **proportional** or form *equivalent ratios*. This means that the ratio of two sides on the original shape is equivalent to the ratio of the same two sides on the copy.



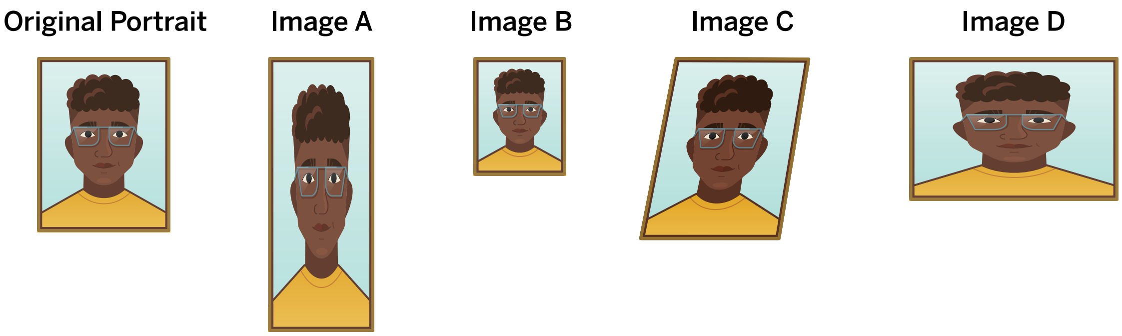
Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.

The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.

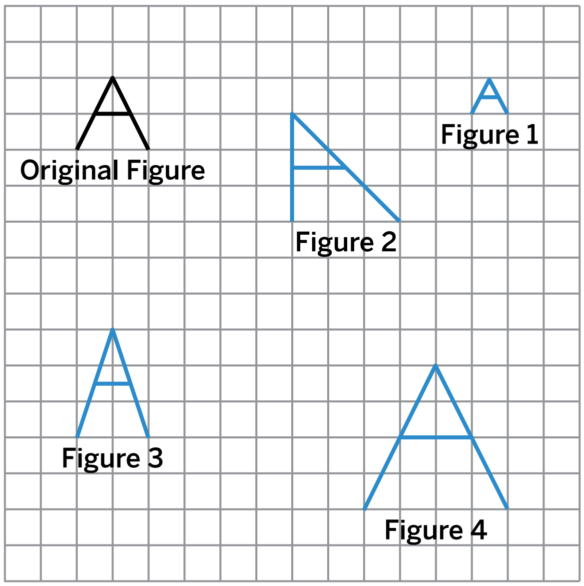
The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

Name: .... Date: .... Period: ....

1. Andre's grandma ordered school pictures. She thought she would receive the original portrait, but instead she got the following images. How is each image different from the original portrait of Andre?



Problems 2–4: Here is the original figure along with several other figures.



2. Which figures are scaled copies of the original figure?

3. How does each scaled copy compare to the original?

4. How do you know a figure is *not* a scaled copy?

Lesson Practice
ACC7.2.01

Name: ______________ Date: ________ Period: ________

FAST Practice

Problems 5–7: Figures 1, 2, and 3 are scaled copies of each other. Each small square has a side length of 1 unit and an area of 1 square unit.

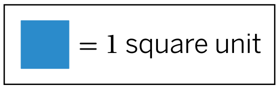



Figure 1

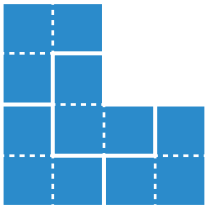


Figure 2

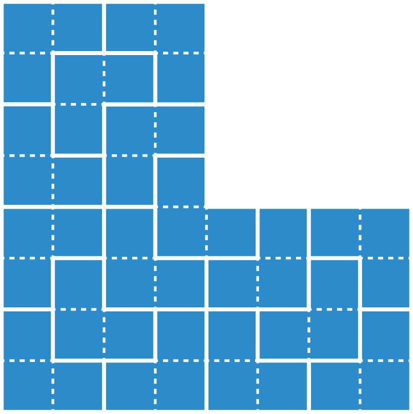


Figure 3

5. Complete the table.

	Figure 1	Figure 2	Figure 3
Perimeter (units)	8		
Area (sq. units)	3		

6. Describe any patterns you see.

7. If the pattern continues, what will be the perimeter and area of Figure 4?

Perimeter: ☐ units Area: ☐ sq. units

Spiral Review

Problems 8–11: Complete each equation with a number that makes it true.

8. $5 \cdot \square = 15$

9. $6 \cdot \square = 9$

10. $\square \cdot 4 = 32$

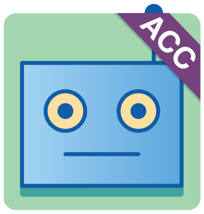
11. $\square \cdot 12 = 3$

Name: ______________________ Date: __________ Period: __________

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scaling Robots

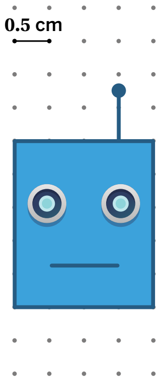
Let's explore scaled copies further.



Warm-Up

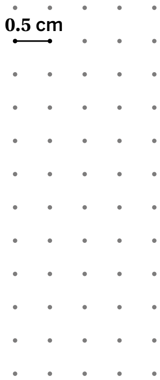
1. Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.

Example Robot



Height (cm)	2.5
Width (cm)	2
Eye Distance (cm)	1
Antenna (cm)	0.75

My Robot



Height (cm)	
Width (cm)	
Eye Distance (cm)	
Antenna (cm)	

Activity
1

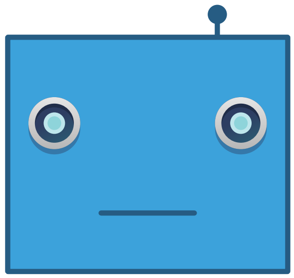
Name: .... Date: .... Period: ....

Scaling Robots

2. Here's Felipe's robot. He wants to make a scaled copy of his robot. Complete the table so that the new measurements represent a scaled copy of his original robot.

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	4	
Eye Distance (cm)	2.5	
Antenna (cm)	0.5	

Felipe's Robot



3. Scaled copies always have a **scale factor**.

Imani and Anushka built a robot and made a table for a scaled copy.

Imani

	Original Robot	New Robot
Height (cm)	5 $\xrightarrow{\times 2}$	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5 $\xrightarrow{\times 2}$	3

Anushka

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

$$\frac{10}{5} = \frac{6}{3} = \frac{4}{2} = \frac{3}{1.5}$$

Show or describe where in their work you can see the scale factor is 2.

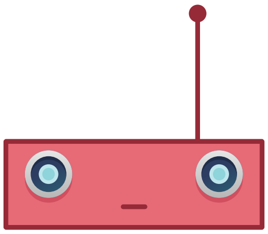
Activity
2

Name: .... Date: .... Period: ....

Analyzing Robots

4. Imani made another robot and tried to make a scaled copy. She recorded her thinking in this table.

Imani's Robot



	Original Robot	New Robot
Height (cm)	2	8
Width (cm)	6	12
Eye Distance (cm)	4	10
Antenna (cm)	1	7

Do you think the new robot will be a scaled copy?

If yes, explain your thinking.

If no, cross out and replace one or more measurements so that the new robot *is* a scaled copy.

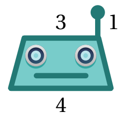
Activity 2

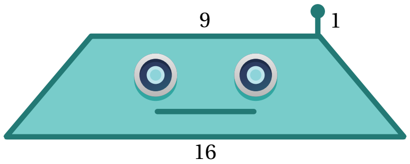
Name: .... Date: .... Period: ....

Analyzing Robots (continued)

Corresponding parts are parts of the original figure that match up with the parts of a copy of the figure.

5. Anushka made the small robot. Then she tried to make a scaled copy of the robot.

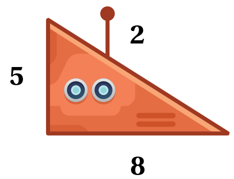




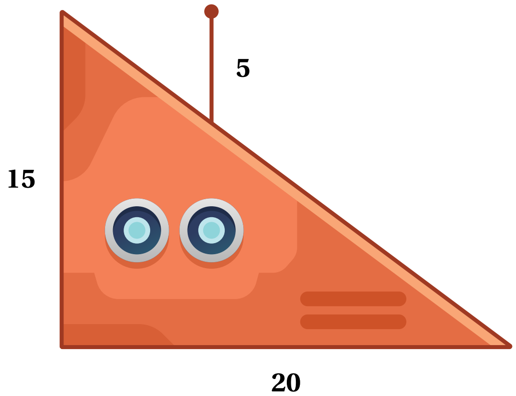
a. **Discuss:** What might Anushka's strategy have been?

b. How could you help Anushka revise her work?

6. Na'ilah drew a small and a large robot. Help her make the large robot a scaled copy.



a. Cross out and replace one or more measurements on the large robot that could make it a scaled copy.

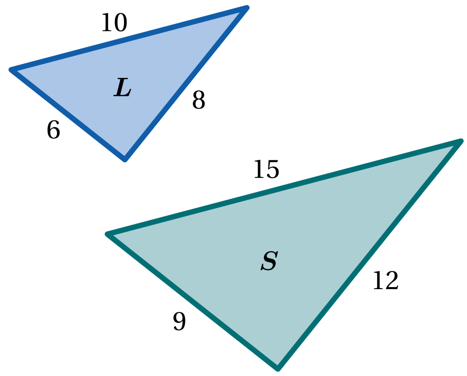


b. What is the scale factor from the small to the large robot?

Synthesis

7. How can you use corresponding parts to determine whether a figure is a scaled copy of another figure?

   Use the example if it helps with your thinking.

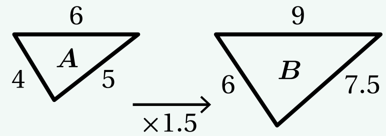


Lesson Practice ACC7.2.02

Lesson Summary

When we create scaled copies, the **scale factor** is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle A to triangle B is 1.5.



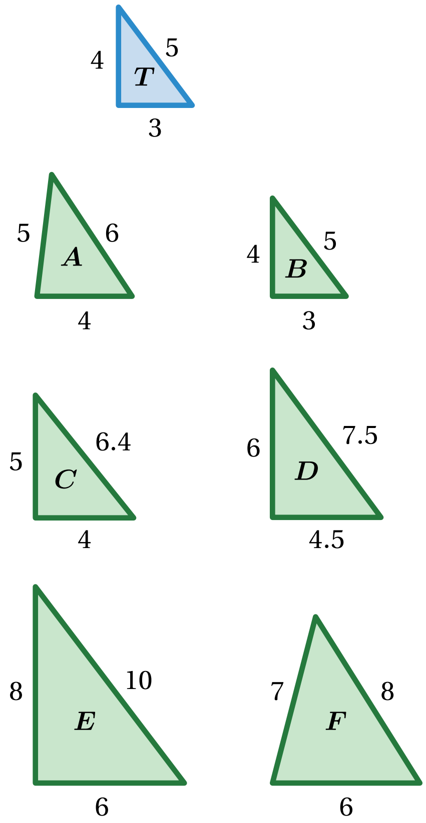
Another way to see the scale factor is to look for the ratio between **corresponding parts** in the two shapes. The ratio of the new length to the original length is the scale factor.

For triangle A and triangle B, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Lesson Practice
ACC7.2.02

Name: .... Date: .... Period: ....

Problems 1–3: Here are seven triangles.



1. Which triangles are scaled copies of triangle T?

2. For each scaled copy, write the scale factor that takes triangle T to that triangle. Leave blank if it is not a scaled copy.

Triangle	Scale Factor
A	
B	
C	
D	
E	
F	

3. List two triangles whose lengths can be represented by the equivalent ratios $\frac{4}{8} = \frac{5}{10} = \frac{3}{6}$.

4. Tyler says that figure K is a scaled copy of figure G. Is Tyler correct? Explain your thinking.

Figure G

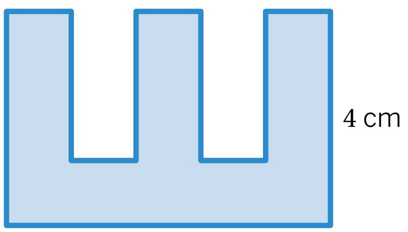
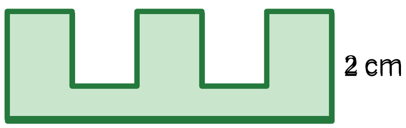


Figure K

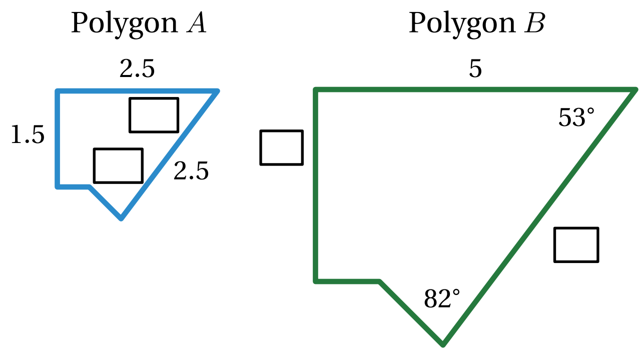


Lesson Practice
ACC7.2.02

Name: .... Date: .... Period: ....

Problems 5–7: Polygon B is a scaled copy of polygon A.

5. What is the scale factor that takes polygon A to polygon B? Explain your thinking.



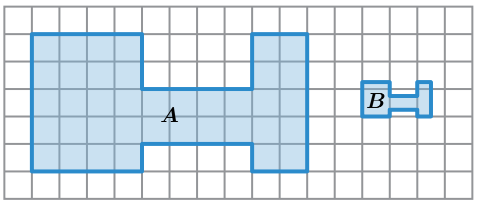
6. Enter the missing lengths in polygon B.

7. Enter the missing angle measurements in polygon A.

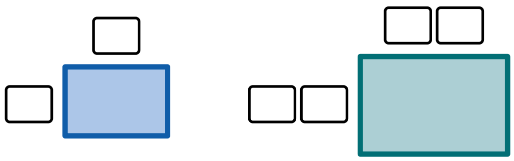
FAST Practice

8. Figures A and B are scaled copies.

What scale factor takes A to B?



9. Using the digits 0 to 9 without repetition, fill in the blanks to make rectangles that are scaled copies.



Spiral Review

Problems 10–13: Evaluate each expression.

10. $\frac{1}{4} \cdot 32$

11. $\frac{1}{4} \cdot 5.6$

12. $7.2 \cdot \frac{1}{9}$

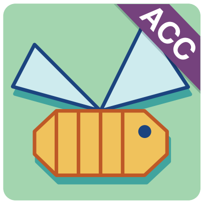
13. $2 \div \frac{1}{4}$

Name: .... Date: .... Period: ....

MA.7.GR.1.5, MTR.4.1

Make It Scale

Let’s draw scaled copies.

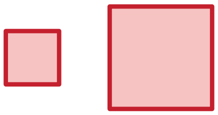


Warm-Up

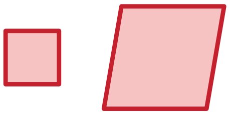
1. Which pair doesn’t belong? Explain your thinking.

A.

B.



C.



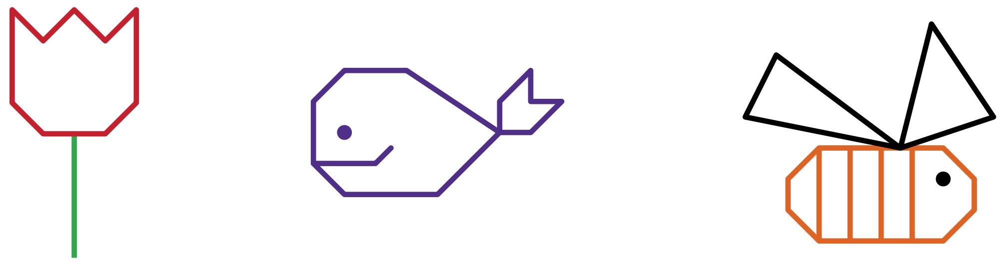
D.

Activity 1

Name: .... Date: .... Period: ....

Drawing Scaled Copies Without a Grid

2. Choose *one* figure that you'd like to make a scaled copy of. Complete the scaled copy of the figure you chose. Use a scale factor of 2.



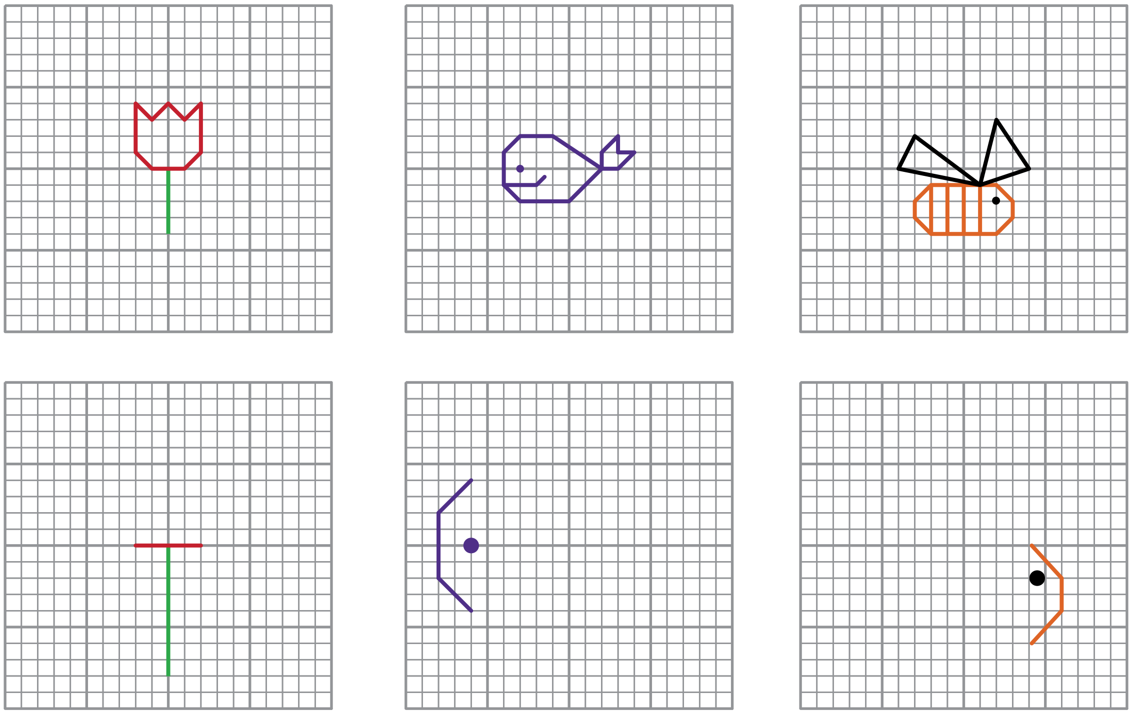
3. What might help you make a more accurate scaled copy?

Activity 2

Name: ______ Date: ______ Period: ______

Drawing Scaled Copies With a Grid

4. Make a new scaled copy of the figure you chose. Use a scale factor of 2.

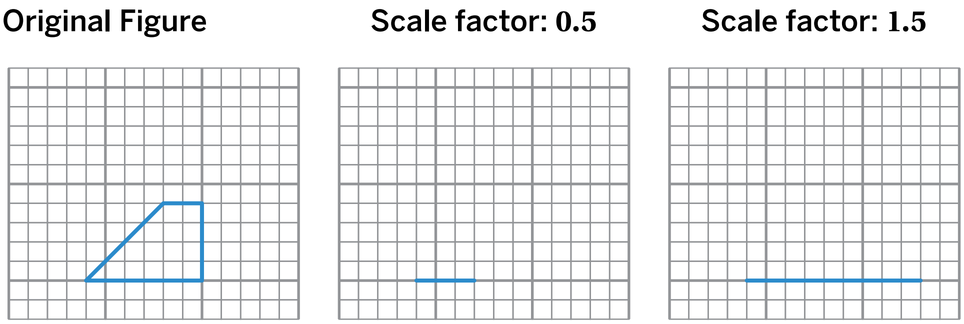


5. Explain the strategy you used to draw the new scaled copy.

Name: .... Date: .... Period: ....

Drawing Scaled Copies With a Grid (continued)

6. Choose a scale factor of 0.5 or 1.5. Then complete the scaled copy.



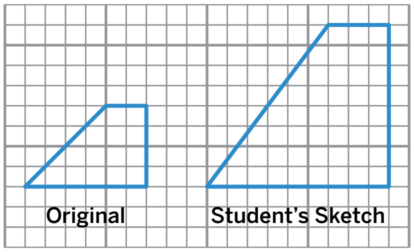
7. Here is one student's sketch. Sasha thinks the student used a scale factor of 2. Randy thinks the student used a scale factor of 1.5. Who is correct? Explain your thinking.

A. Sasha

B. Randy

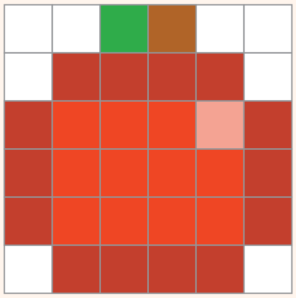
C. Both

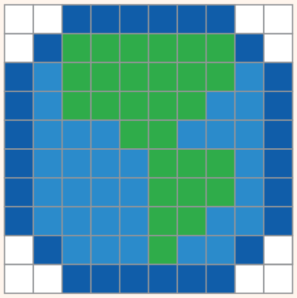
D. Neither

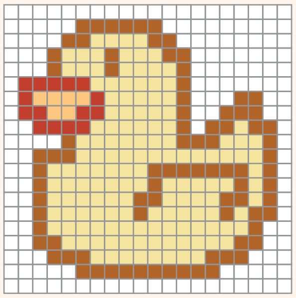


You're invited to explore more.

8. On a piece of graph paper, draw one of these images using a scale factor of 1.5. Or draw your own image and a scaled copy.



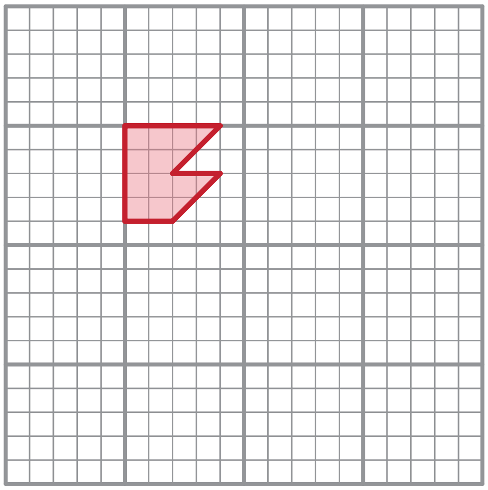




Synthesis

9. Describe how to draw a scaled copy.

Use the example if it helps with your thinking.



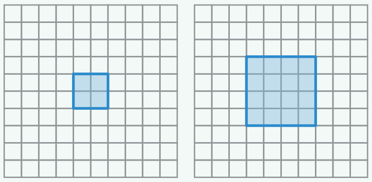
Lesson Practice ACC7.2.03

Lesson Summary

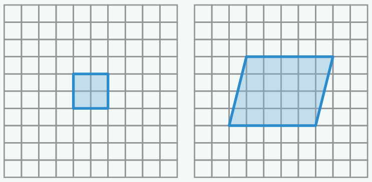
To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies



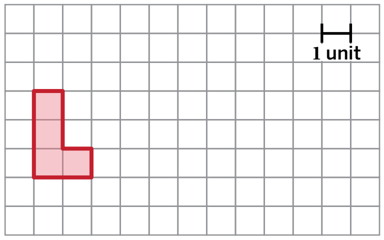
Not Scaled Copies



Lesson Practice
ACC7.2.03

Name: .... Date: .... Period: ....

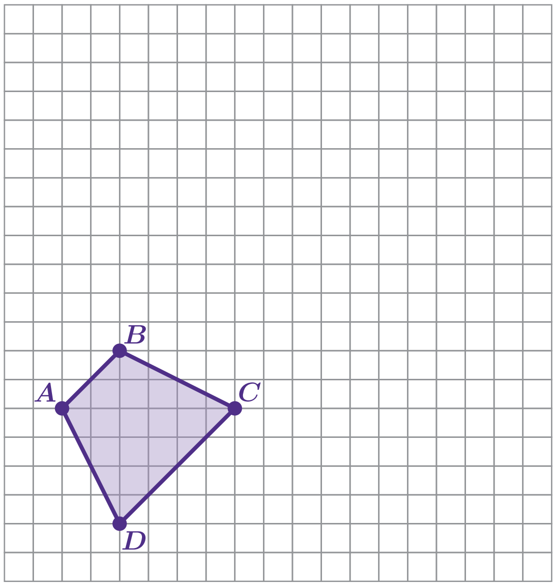
Problems 1–2: Here is a polygon.



1. Draw a scaled copy of the polygon using a scale factor of 2.

2. What is the area and perimeter of your scaled copy?

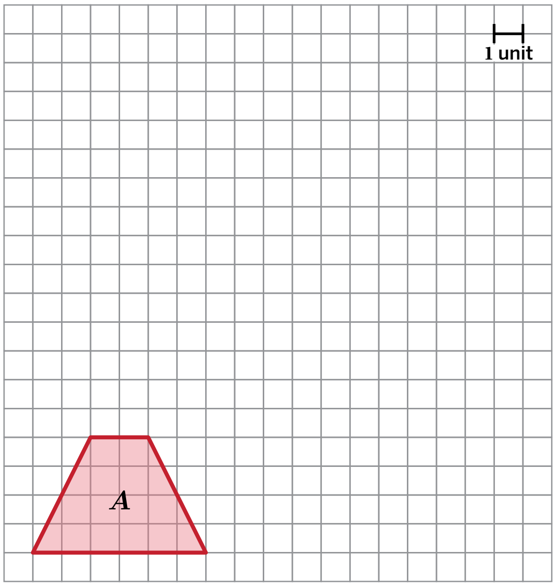
3. Draw a scaled copy of figure $ABCD$ using a scale factor of 1.5.



Problems 4–5: Imagine there is a quadrilateral B that is a scaled copy of quadrilateral A. Its shortest side is 5 units long.

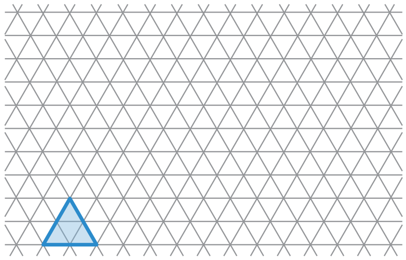
4. What is the scale factor from quadrilateral A to B?

5. Draw quadrilateral B.



Name: .... Date: .... Period: ....

Problems 6–7: Here is an equilateral triangle.



6. Draw a scaled copy of this equilateral triangle using a scale factor of 3.

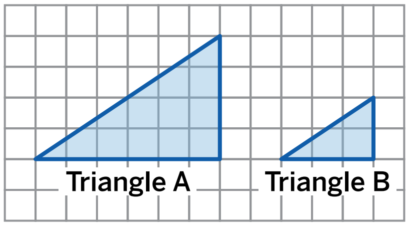
7. Equilateral triangles are always scaled copies. What other shapes are always scaled copies?

FAST Practice

8. Triangle B is a scaled copy of triangle A. Gabriela says the scale factor is 0.5. Is she correct?

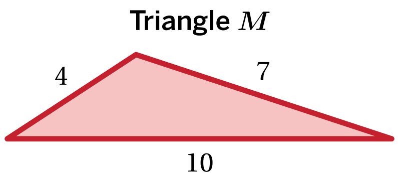
Select **ONE** correct answer in each box to complete the sentences.

Gabriela is **A.** correct **B.** incorrect. The height and base of triangle A is multiplied by **A.** 0.5 **B.** 2.



Spiral Review

9. Triangle Z is a scaled copy of triangle M. Select *all* sets of values that could be the side lengths of triangle Z.



- ☐ **A.** 8, 11, and 14
- ☐ **B.** 10, 17.5, and 25
- ☐ **C.** 6, 9, and 11
- ☐ **D.** 6, 10.5, and 15
- ☐ **E.** 8, 14, and 20

Problems 10–12: Solve each equation. Show or explain your thinking.

10. $6x = 156$

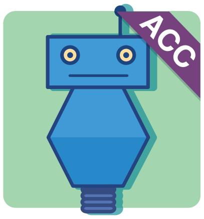
11. $16x = 8$

12. $\frac{1}{5}x = 1$

Name: ____________ Date: ________ Period: ________

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scale Factor Challenges



Let's explore how scale factors affect the size of scaled copies.

Warm-Up

1. **a** **Discuss:** How are these equations alike? How are they different?

A. $8x = 80$

B. $8x = 8$

C. $8x = 1$

D. $\frac{1}{8}x = 1$

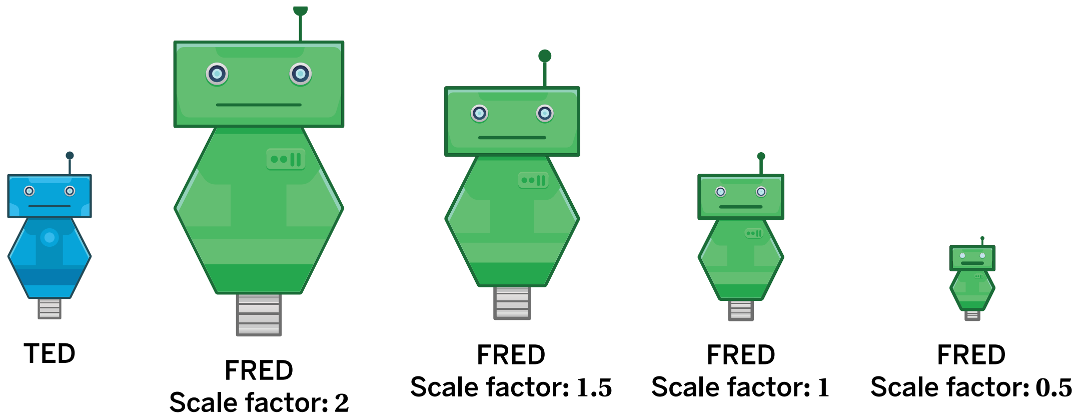
b Use what you noticed to solve each equation mentally.

Activity **1**

Name: .... Date: .... Period: ....

Exploring Scale Factors

2. Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.



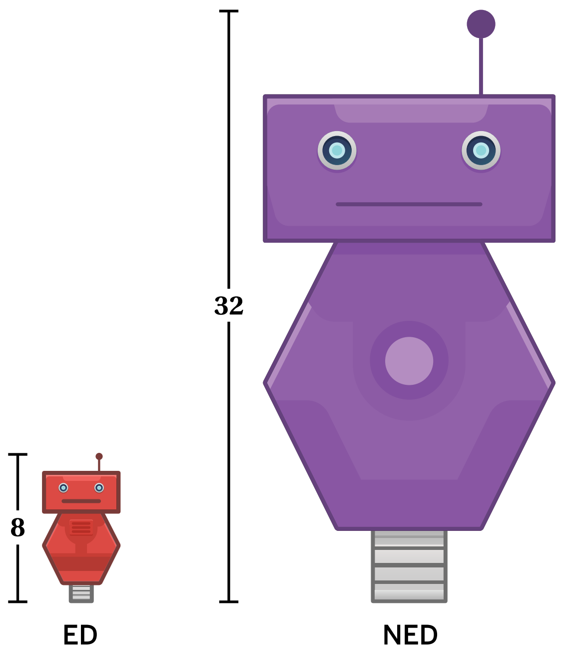
Discuss: What do you notice?

3. Here are two new robots: ED and NED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED?

4. What scale factor will make NED match ED?

Explain your thinking.

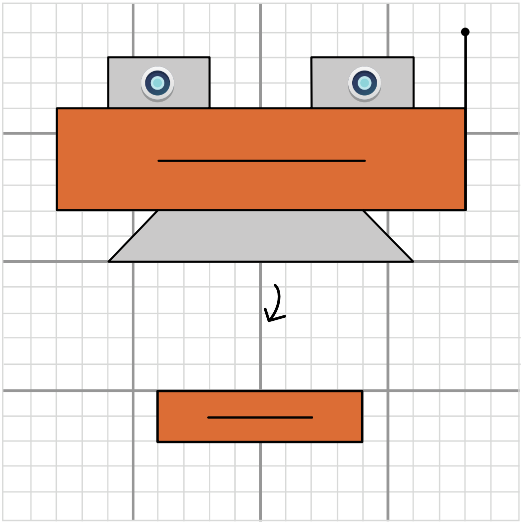


Name: ______ Date: ______ Period: ______

Scaled Down and Back Again

5. Here is a robot called ROVER.

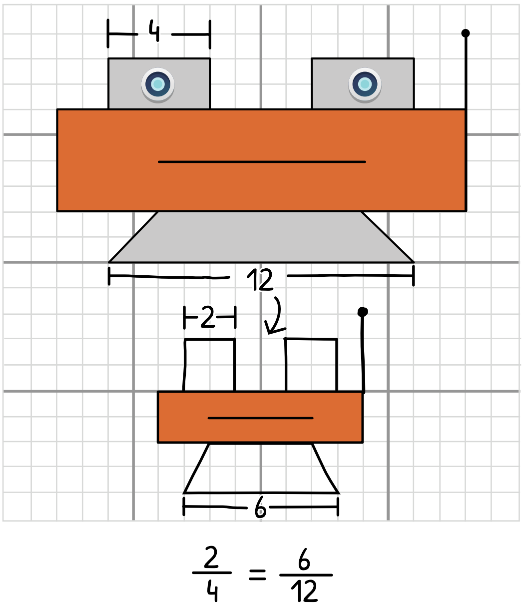
Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.



6. Here is Adhira's drawing from the previous problem.

a What is something Adhira did well?

b What is something Adhira can improve?



7. The scale factor from the original to the copy is $\frac{1}{2}$.

a What scale factor could you use to scale the copy back to the original?

b How are these two scale factors related?

Activity
3

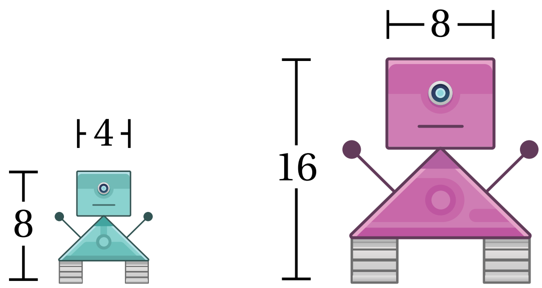
Name: .... Date: .... Period: ....

Practicing With Scale Factors

8. In this activity, all pairs of bots have corresponding measurements that are proportional.

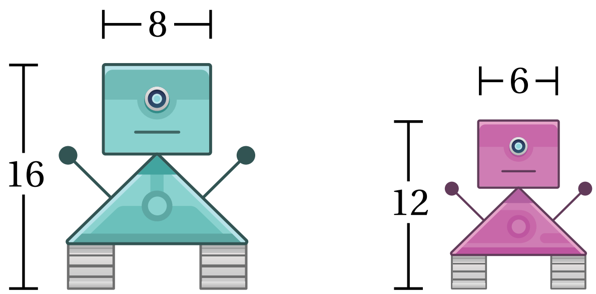
Determine a scale factor to make the bot on the left match the bot on the right.

a



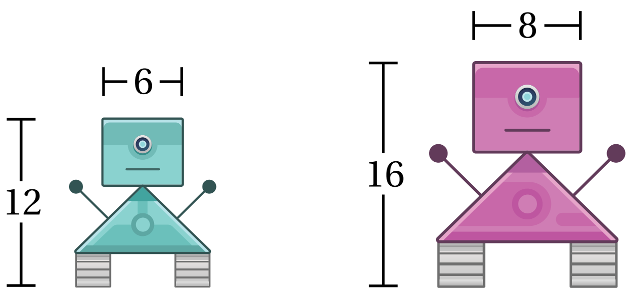
Scale factor: ....

b



Scale factor: ....

c



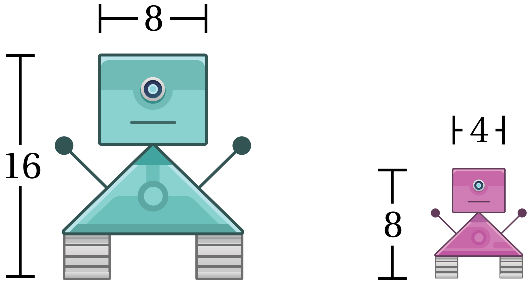
Scale factor: ....

Activity
3

Name: .... Date: .... Period: ....

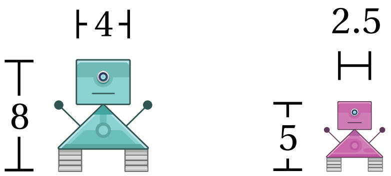
Practicing With Scale Factors (continued)

d



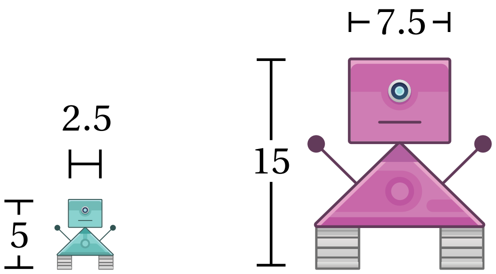
Scale factor: ....

e



Scale factor: ....

f



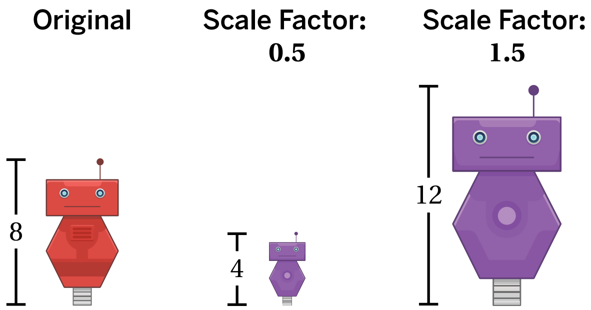
Scale factor: ....

You're invited to explore more.

9. Use the You're Invited to Explore More Sheet to design your own robot. Then complete its scaled copy.

Synthesis

10. Describe how you can tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original.



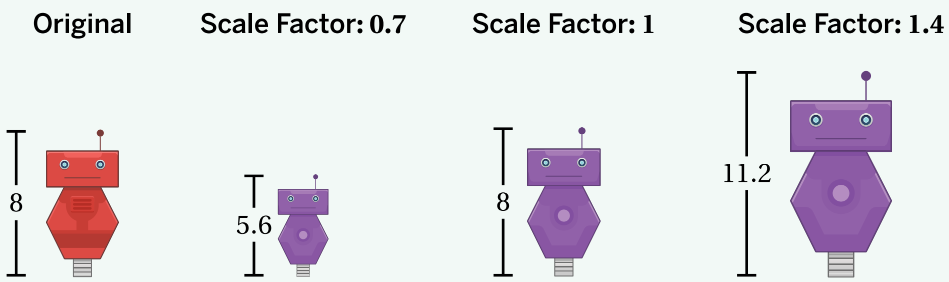
Lesson Practice ACC7.2.04

Lesson Summary

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:

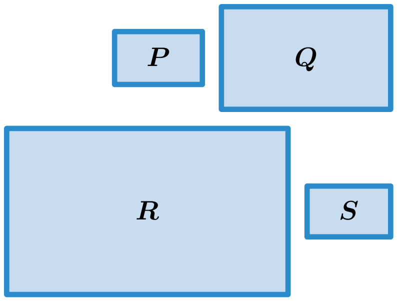
- *Less* than 1, the copy will be *smaller* than the original.
- *Equal to* 1, the copy will be *the same size* as the original.
- *Greater* than 1, the copy will be *larger* than the original.



Lesson Practice
ACC7.2.04

Name: .... Date: .... Period: ....

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, state whether the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.



Rectangle P to rectangle R ....

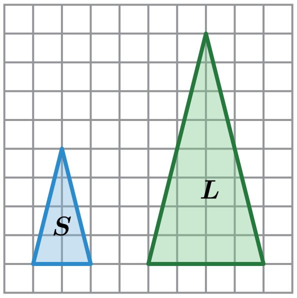
Rectangle Q to rectangle S ....

Rectangle Q to rectangle R ....

Rectangle S to rectangle P ....

Rectangle R to rectangle P ....

Problems 2–4: Triangle S and triangle L are scaled copies of one another.

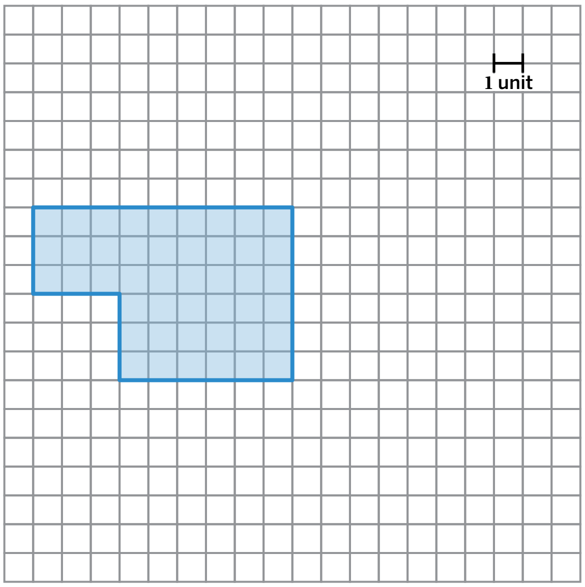


2. What is the scale factor that takes triangle S to triangle L?

3. What is the scale factor that takes triangle L to triangle S?

4. Triangle M (not shown) is also a scaled copy of triangle S. The scale factor that takes triangle S to triangle M is $\frac{3}{2}$. What is the scale factor that takes triangle M to triangle S?

Problems 5–7: Here is a polygon.



5. Draw a scaled copy of the polygon that has a perimeter of 10 units.

6. What is the scale factor from the original polygon to your scaled copy?

7. What is the scale factor from the scaled copy back to the original polygon?

8. Will any two squares always be scaled copies of one another?

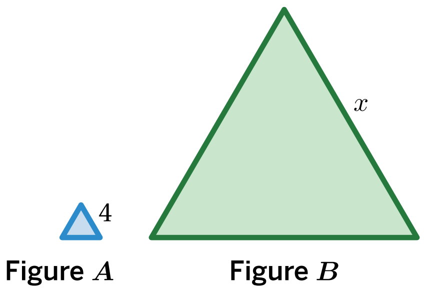
Explain your thinking.

FAST Practice

9. Figure B is a scaled copy of figure A with a scale factor of $5\frac{1}{2}$.

What is the value of x?

$x =$ ☐



Spiral Review

10. Select *all* the ratios that are equivalent to $12 : 3$.

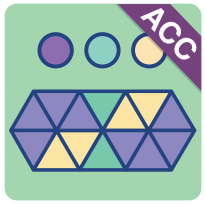
- ☐ **A.** $6 : 1$
- ☐ **B.** $1 : 4$
- ☐ **C.** $4 : 1$
- ☐ **D.** $24 : 6$
- ☐ **E.** $15 : 6$
- ☐ **F.** $1{,}200 : 300$
- ☐ **G.** $112 : 13$

Unit 2
Lesson
5

Name: Date: Period:

MA.7.GR.1.5, MTR.1.1, MTR.3.1, MTR.4.1

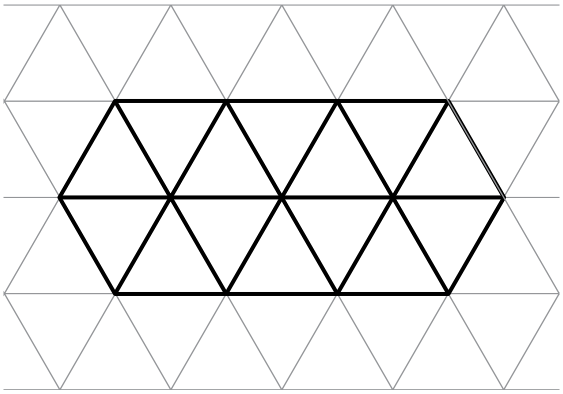
Tiles



Let's use tiles and mosaics to explore how *area* changes in scaled copies.

Warm-Up

1. Create your own tile mosaic by coloring the triangles. Use at least two colors.

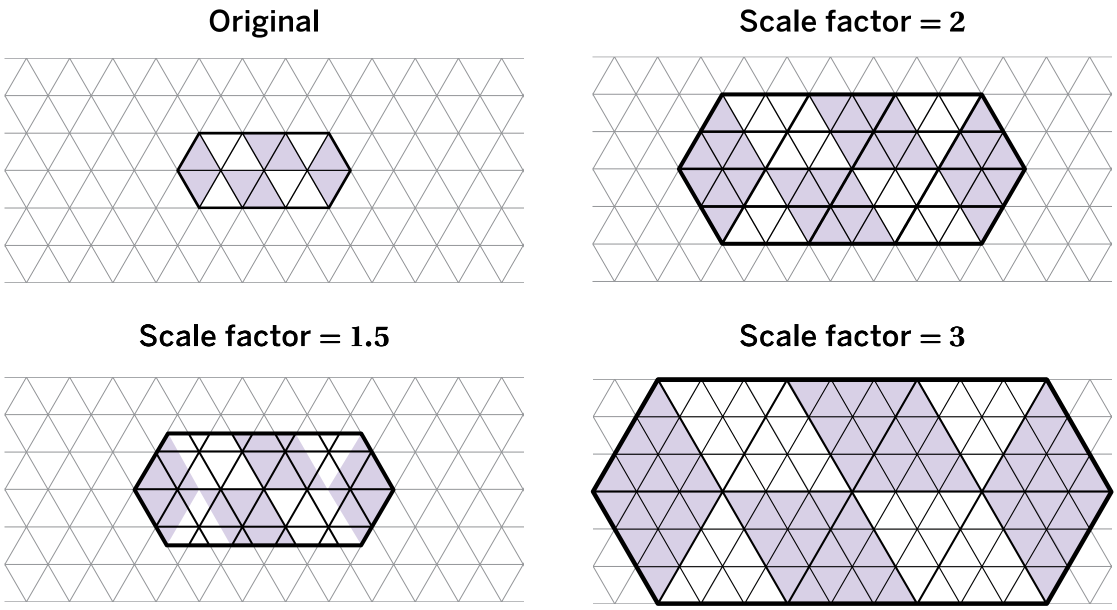


Activity 1

Name: Date: Period:

Scaling Mosaics

2. **a** Take a look at a mosaic and three scaled copies.



b What do you notice? What do you wonder?

I notice:

I wonder:

3. Here is a table about the mosaic and one of the scaled copies.

a Complete the table.

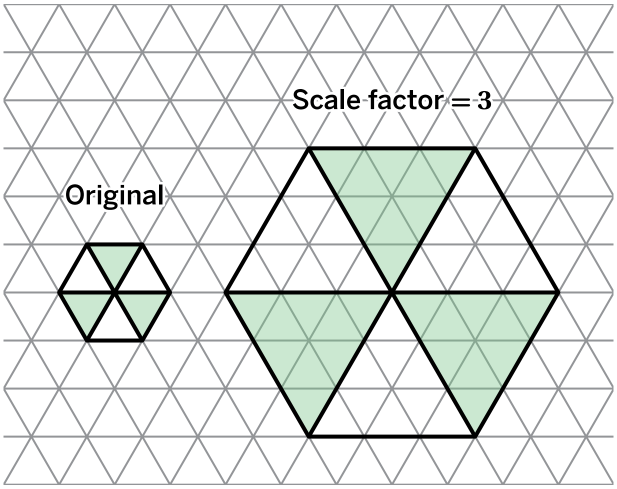
b Describe any patterns you see.

Color	Number of Small Tiles in Original	Number of Small Tiles When Scale Factor is 2
Purple	8	
White	6	

Name: Date: Period:

Scaling Mosaics (continued)

4. Here is a new mosaic and its scaled copy.



Lucy says the scaled copy has 3 times as many tiles as the original.

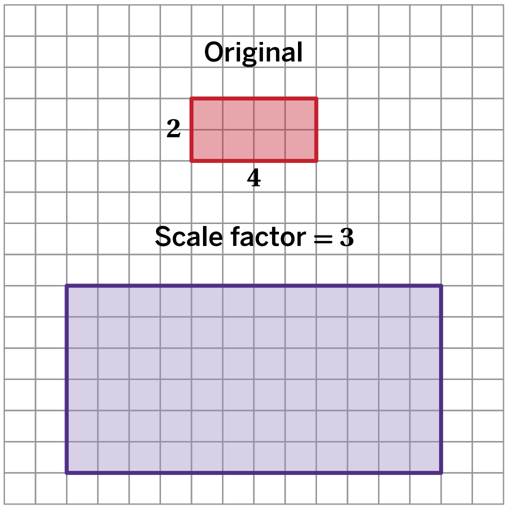
Malik claims the scaled copy has 6 times as many tiles as the original.

Whose claim is correct? Circle one.

Lucy Malik Both Neither

Explain your thinking.

5. Here is a rectangle and a scaled copy.



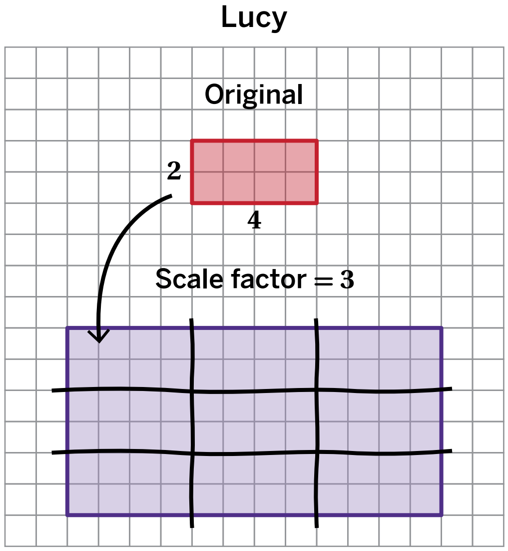
The *area* of the original rectangle is 8 square units.

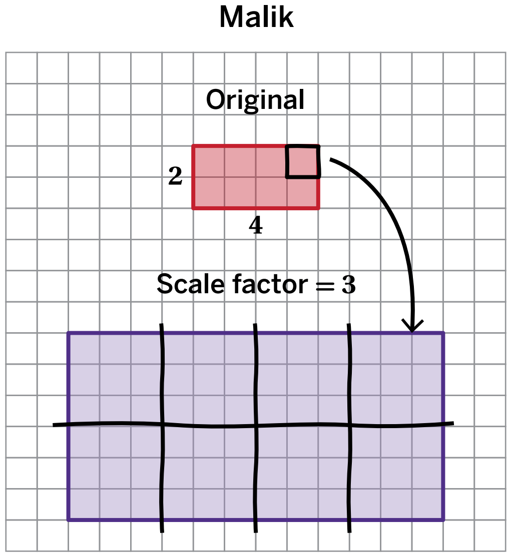
What is the area of the scaled copy? Show or explain your thinking.

Name: ________ Date: ________ Period: ________

Area of Scaled Copies

6. Here is Lucy's work and Malik's work for the previous problem.

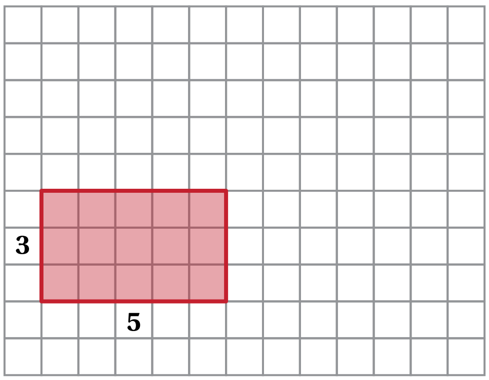




Explain how you think one student calculated the area.

7. Imagine scaling this rectangle using a scale factor of 2.

What is the area of the scaled copy?

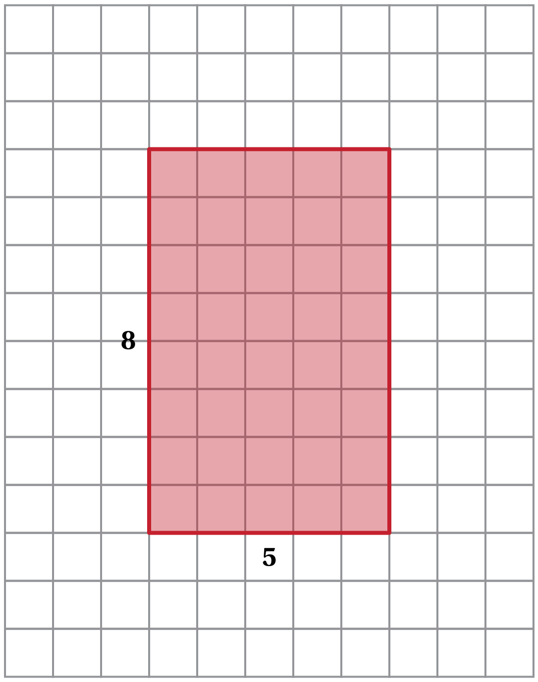


Name: .... Date: .... Period: ....

Area of Scaled Copies (continued)

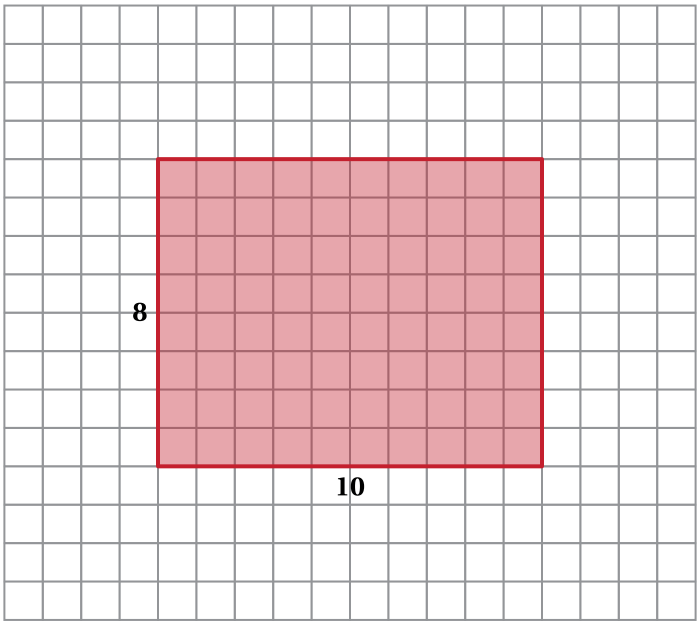
8. Imagine scaling this rectangle using a scale factor of 4.

What is the area of the scaled copy?



9. Imagine scaling this rectangle using a scale factor of $\frac{1}{2}$.

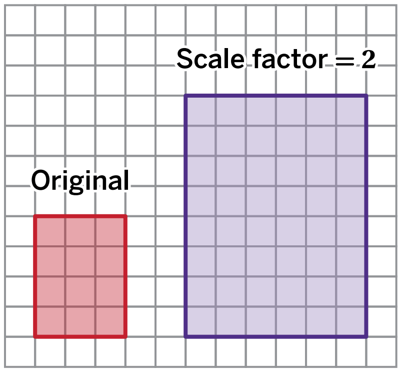
What is the area of the scaled copy?



Synthesis

10. Describe a strategy for determining the area of a rectangle's scaled copy.

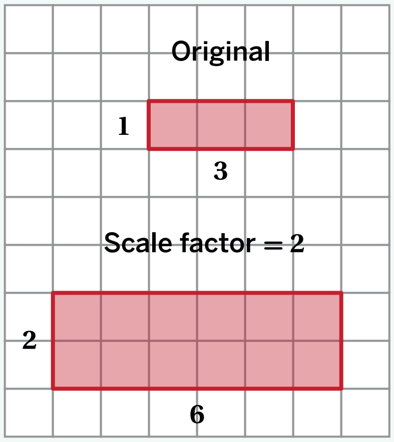
Use the example if it helps with your thinking.



Lesson Practice ACC7.2.05

Lesson Summary

The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the *area* will change. You can use reasoning about the scale factor to calculate the area of a scaled copy. Here are some strategies to calculate the area of a scaled copy:



Scale the side lengths

1. Multiply each side length by the scale factor.
$1 \times 2 = 2$
$3 \times 2 = 6$

2. Calculate the area.
$2 \times 6 = 12$

Square the scale factor

1. Determine the area of the original.
$1 \times 3 = 3$

2. Square the scale factor.
$2 \times 2 = 4$

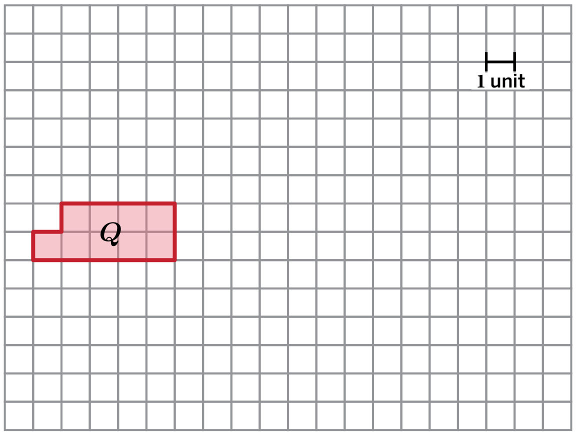
3. Multiply the original area by the squared scale factor. $3 \times 4 = 12$

We can use either strategy to determine that the area of this scaled copy is 12 square units.

Lesson Practice
ACC7.2.05

Name: .... Date: .... Period: ....

Problems 1–4: Here is polygon Q.



1. Draw a scaled copy of polygon Q using a scale factor of 2.

2. What is the area of the scaled copy?

3. Complete the table for different scaled copies of polygon Q.

Scale Factor	Perimeter (units)	Area (sq. units)
1	14	9
	28	
3		

4. How is the relationship between the scale factor and perimeter different from the relationship between the scale factor and area?

5. Imagine a right triangle with an area of 36 square units.

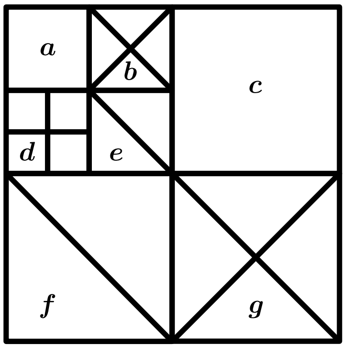
Scaled copies are made using different scale factors. Complete the table with the area of each scaled copy.

Scale Factor	Area (sq. units)
1	36
2	
3	
5	
$\frac{1}{2}$	
$\frac{2}{3}$	

Name: .... Date: .... Period: ....

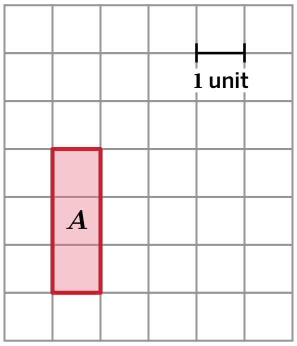
6. The square shown is made up of scaled copies of a triangle and a square. The area of region d is 1 square unit. Determine as many different areas as you can.

Region	a	b	c	d	e	f	g
Area (sq. units)				1			



FAST Practice

Problems 7–8: Ivory created a scaled copy of rectangle A. The area of Ivory's scaled copy is 48 square units.



7. How many times greater is the area of Ivory's scaled copy than the area of rectangle A?

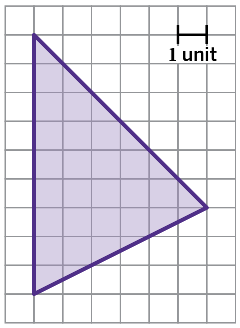
☐ times greater

8. What scale factor did Ivory use to create their scaled copy?

☐

Spiral Review

9. Determine the area of this triangle.



Name: ________________ Date: ________ Period: ________

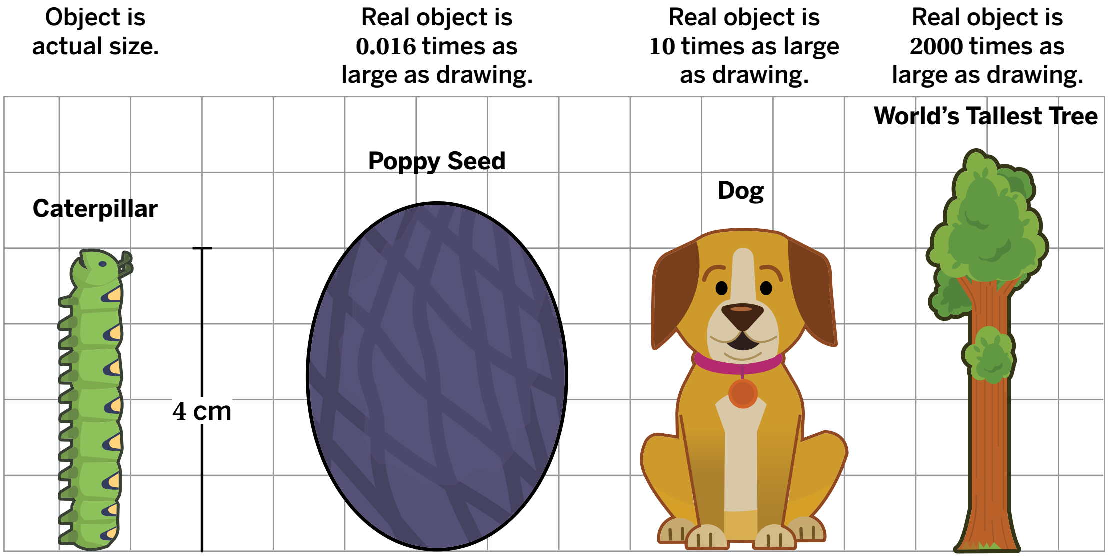
MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1

Scale Drawings

Let's explore scale diagrams.

Warm-Up

1. Each grid line is one centimeter.



What do you notice? What do you wonder?
I notice:

I wonder:

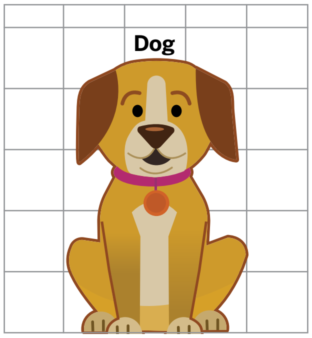
Activity 1

Name: ______ Date: ______ Period: ______

Scale Factor

2. Estimate the dog's actual height. Remember that each grid line is 1 centimeter.

Real object is 10 times as large as drawing.

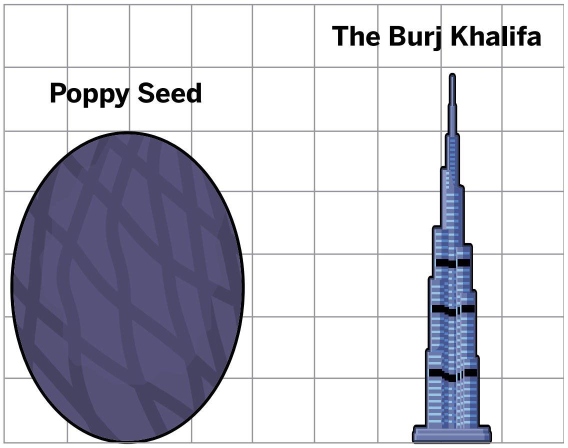


3. Estimate the actual heights of the poppy seed and the Burj Khalifa (the world's tallest building). Remember that each grid line is 1 centimeter.

Object	Actual Height (cm)
Poppy seed	
The Burj Khalifa	

Real object is 0.016 times as large as drawing.

Real object is 14000 times as large as drawing.



4. Using your estimate, how tall is the Burj Khalifa in meters? Remember that 1 meter is equal to 100 centimeters.

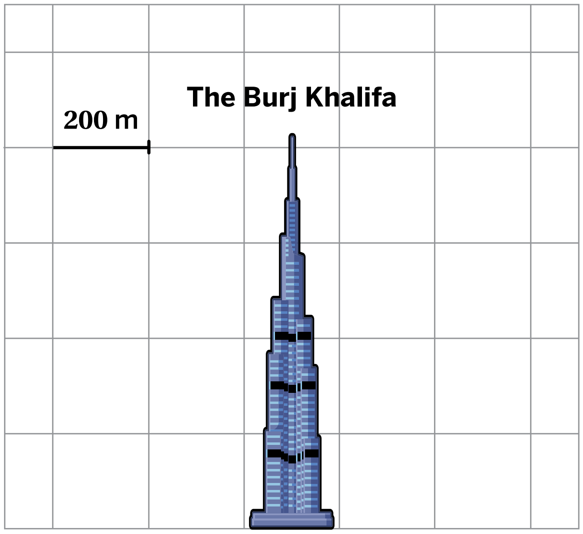
Activity
2

Name: ______ Date: ______ Period: ______

Scale

5. So far, the drawings in this lesson have used scale factor. Here is a new drawing that uses **scale**. The scale is 1 unit to 200 meters.

How can you tell from this new drawing that the Burj Khalifa is more than 800 meters tall?



6. The Activity 2 Sheet contains **scale drawings** of all the objects in the table.

Estimate the actual height of each object.

Object	Actual Height
Amoeba	
Poppy seed	
Ladybug	
Caterpillar	4 cm
Dog	
Person	
5-story building	
World's tallest tree	
The Burj Khalifa	828 m

You're invited to explore more.

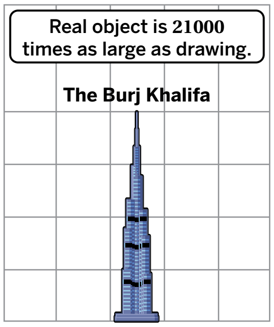
7. Use the You're Invited to Explore More Sheet to create your own scale drawing.

Synthesis

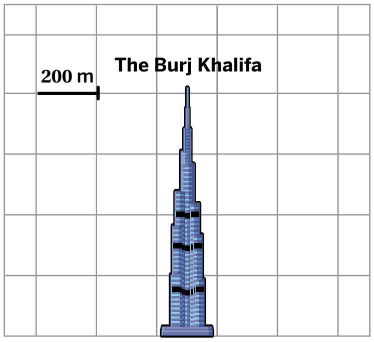
8. How are scale factor and scale alike?

How are they different?

Scale Factor



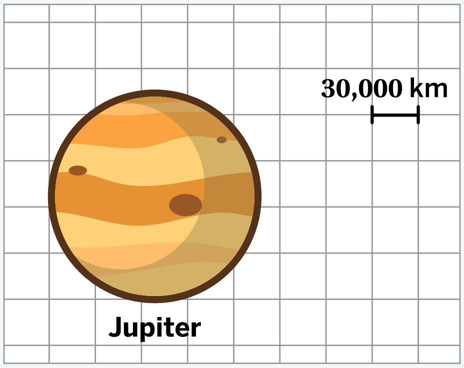
Scale



Lesson Practice ACC7.2.06

Lesson Summary

A **scale drawing** is a two-dimensional representation of an actual object or place. We can use a **scale** to tell how actual measurements are represented in the drawing.



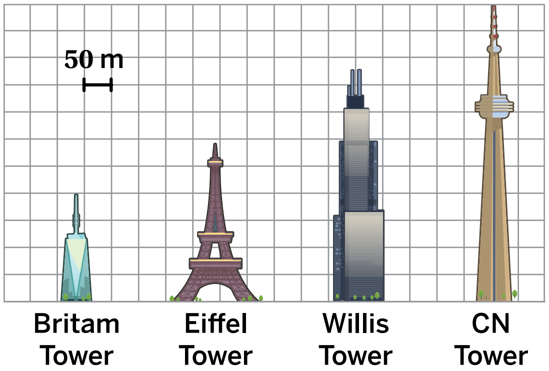
Scale is often shown with a line segment that indicates what distance 1 unit in the drawing represents in the actual object. Scales can also be written in units of measure, like inches or centimeters.

For example, this drawing of Jupiter uses a scale of 1 unit to 30,000 kilometers. This means that every 1 unit on the drawing represents 30,000 kilometers. Since the diameter of Jupiter is about 4.5 units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.

Lesson Practice
ACC7.2.06

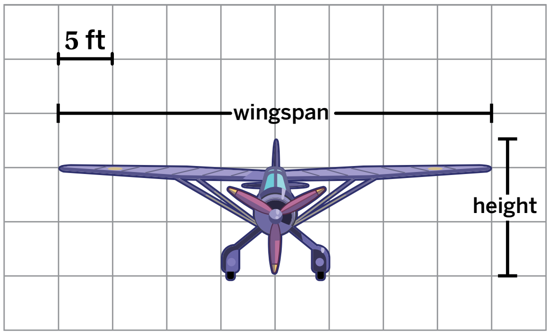
Name: .... Date: .... Period: ....

1. Here is a scale drawing of some towers in different cities. Use the scale to estimate how tall each tower is.



Tower	Estimated Height (m)
Britam Tower	
Eiffel Tower	
Willis Tower	
CN Tower	

Problems 2–3: The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here is a scale drawing that shows the Lysander.

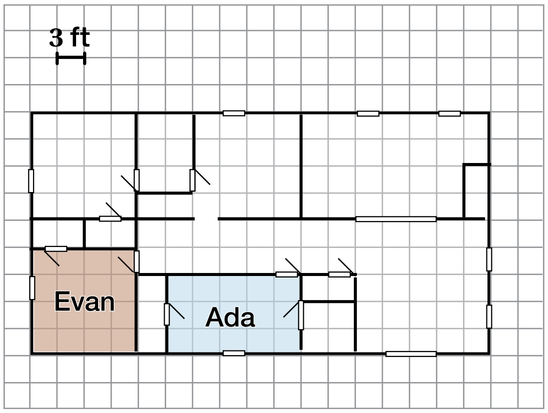


Approximate the actual lengths.

2. The wingspan of the plane:

3. The height of the plane:

Ada and Evan are siblings. Here is a scale drawing of their house that shows their rooms.



4. Complete the table with the dimensions of their rooms.

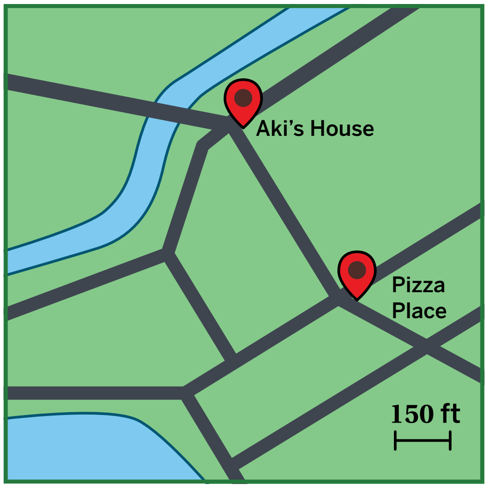
	Length (ft)	Width (ft)
Ada's Room	15	
Evan's Room		

Name: .... Date: .... Period: ....

FAST Practice

5. Here is a map of a town, with a scale at the bottom. Estimate how far Aki's house is from the pizza place.

About ☐ feet

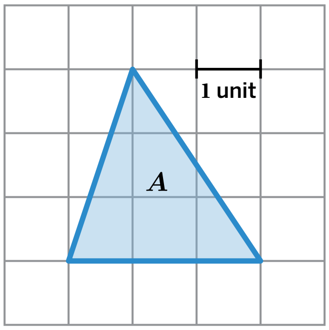


Spiral Review

6. Complete the table so that the two values in each row are related by the same ratio.

3	12
1.5	
4	16
	1

Problems 7–8: Tyrone created a scaled copy of triangle A with an area of 72 square units.



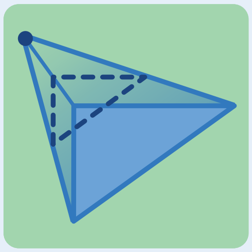
7. How many times greater is the area of the scaled copy compared to the area of triangle A?

8. What scale factor did Tyrone apply to triangle A to create the scaled copy?

Sub-unit

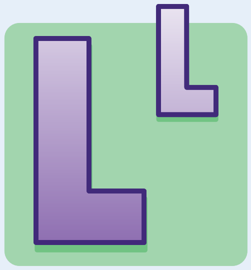
2

Dilations

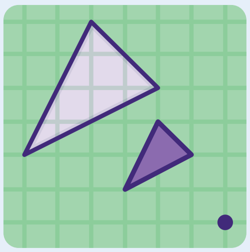


Lesson 7
Sketchy Dilations

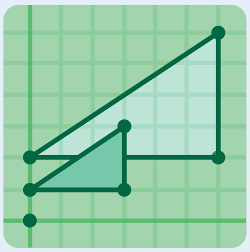
Lesson 8
Dilation Mini Golf



Lesson 9
Make It a Combo!



Lesson 10
Match My Dilation



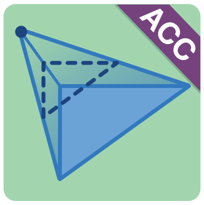
Lesson 11
Dilations on a Plane

Unit 2
Lesson
7

Name: .... Date: .... Period: ....

MA.8.GR.2.2, MTR 4.1

Sketchy Dilations

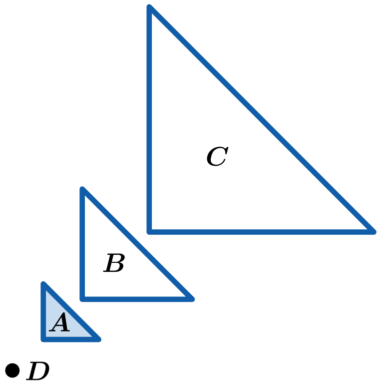


Let's explore scaled copies and dilations.

Warm-Up

1. Triangles B and C are images of triangle A.

What do you notice? What do you wonder?



I notice:

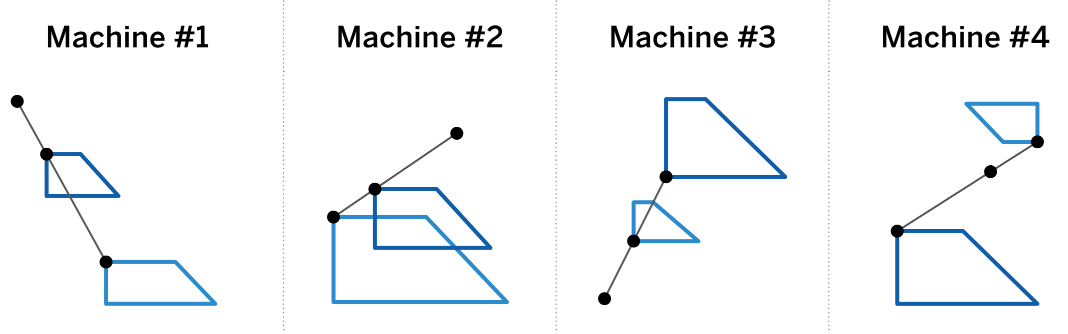
I wonder:

Activity 1

Name: .... Date: .... Period: ....

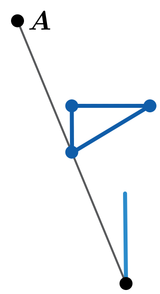
Dilations and Scaled Copies

2. **a** Take a look at these sketches, each created by a different stretching machine.



b **Discuss:** How does each stretching machine work?

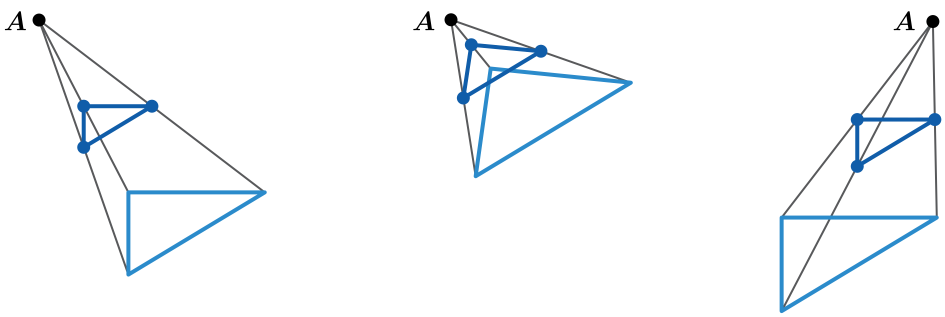
3. Here is a *pre-image* and part of its *image* in a stretching machine.



a Sketch the rest of its image.

b How would the image be different if point A were closer to the triangle?

4. Stretching machines create **dilations**. A dilation is a new type of transformation that creates a *scaled copy* from a given point.



Does moving point A change the size of the image, its location, or both? Circle one.

Size Location Both

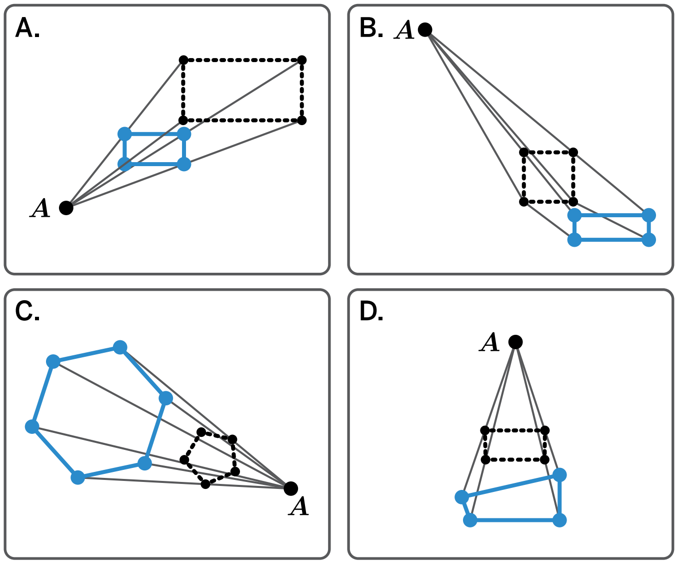
Explain your thinking.

Name: ______________________ Date: __________ Period: __________

Dilation Play

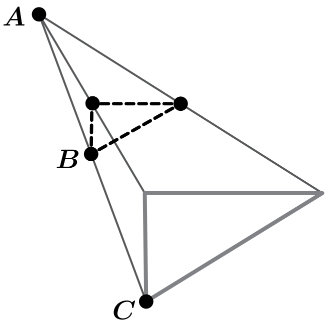
5. Circle one image that is *not* a dilation (in other words, it could not have been created using a stretching machine).

Explain your thinking.



6. Think about what might happen when points A and C move.

Here are several dilation challenges. Select *all* the ones you think are possible.

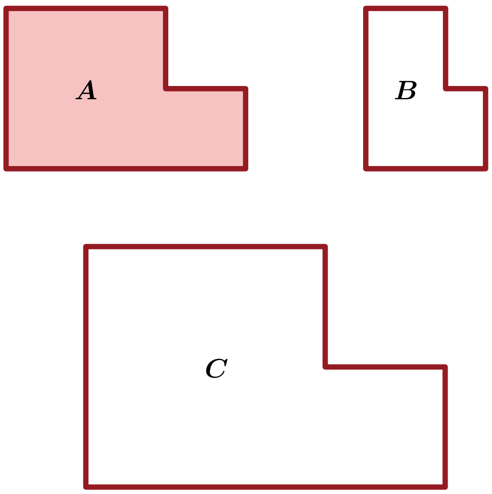


- ☐ **A.** Make the image smaller than the pre-image.
- ☐ **B.** Make the image the same size as the pre-image, but in a different location.
- ☐ **C.** Make the distance between A and B twice the distance between B and C.
- ☐ **D.** Make points A, B, and C form a triangle.

7. Which figure could be a *dilation* of figure A? Circle one.

Figure B Figure C Both   Neither

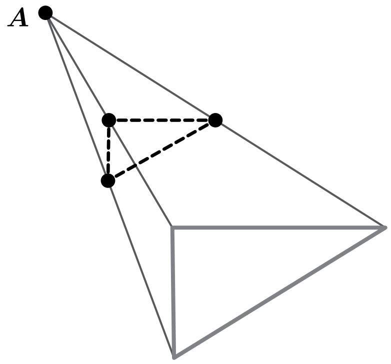
Explain your thinking.



Synthesis

8. Describe how dilations work.

Use this example if it helps with your thinking.

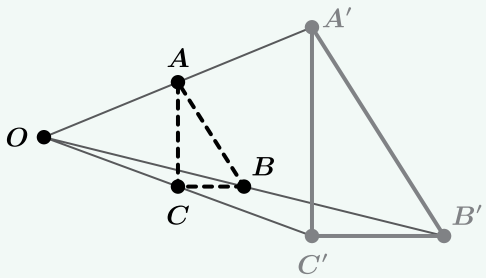


Lesson Practice ACC7.2.07

Lesson Summary

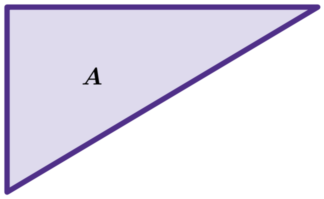
A **dilation** is a type of transformation that creates *scaled copies*. Dilating a figure means moving each of its vertices along a line that's extended from a given point. The original distance from the given point to each vertex on the *pre-image* is multiplied by the same number to create the dilated *image*.

For example, Triangle ABC was dilated from the point O to create triangle $A'B'C'$.



Name: .... Date: .... Period: ....

Problems 1–2: Here is triangle A.



1. Draw a dilation of triangle A where the image has a smaller area.

2. Draw a dilation of triangle A where the image has a larger area.

3. Rectangle A is 12 centimeters by 3 centimeters. Rectangle B is a scaled copy of rectangle A. Select *all* the pairs that could be the dimensions of rectangle B.

   ☐ **A.** 6 centimeters by 1.5 centimeters

   ☐ **B.** 10 centimeters by 1 centimeter

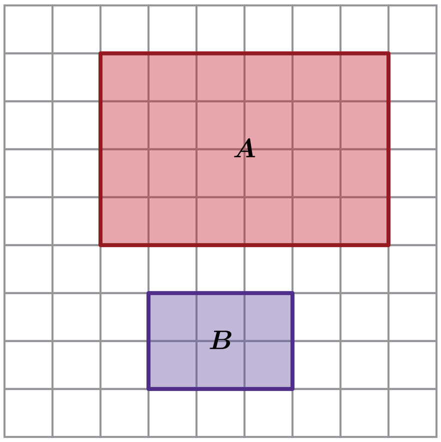
   ☐ **C.** 18 centimeters by 4.5 centimeters

   ☐ **D.** 6 centimeters by 1 centimeter

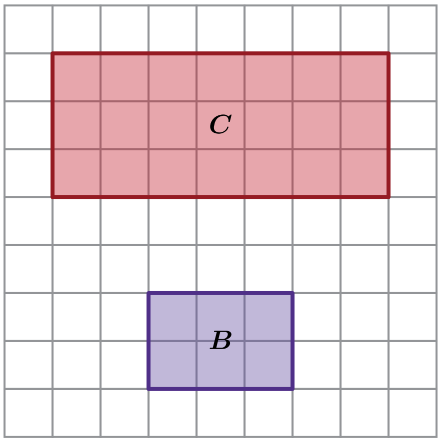
   ☐ **E.** 80 centimeters by 20 centimeters

Problems 4–5: Determine whether each pair of figures represents an original figure and its scaled copy. Explain your thinking.

4.



5.

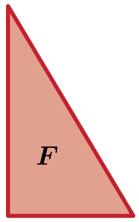


Lesson Practice
ACC7.2.07

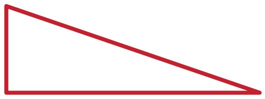
Name: ______________ Date: ________ Period: ________

FAST Practice

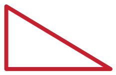
6. Select *all* the figures that could be a scaled copy of triangle F.



☐ A.

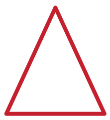


☐ B.

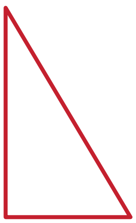


☐ C.

☐ D.



☐ E.



Spiral Review

7. Which of these sets of angle measures could be the three interior angle measures of a triangle?

A. $40°, 50°, 60°$

B. $50°, 60°, 70°$

C. $60°, 70°, 80°$

D. $70°, 80°, 90°$

8. What makes two figures congruent? Explain your thinking.

Unit 2
Lesson
8

Name: ______ Date: ______ Period: ______

MA.8.GR.2.2, MTR.4.1, MTR.7.1

Dilation Mini Golf

Let's dilate points using measurement tools.

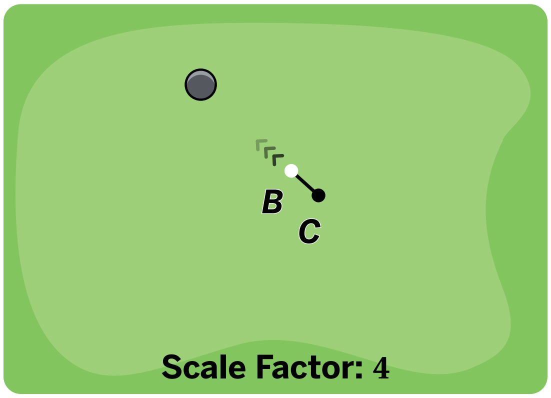
Warm-Up

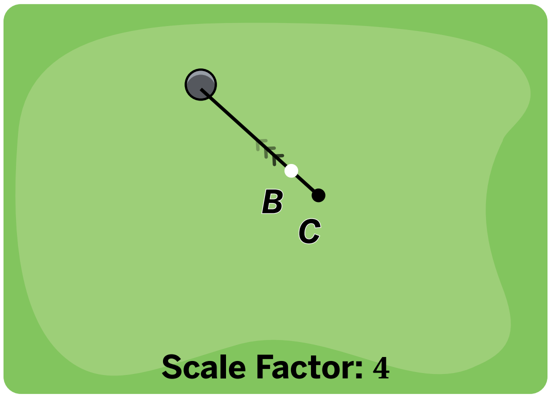
1. Welcome to Dilation Mini Golf.

Point B is the starting point of the golf ball. After a dilation, the ball is in the hole.

Discuss:

- In this situation, what is the pre-image? What is the image?
- What do you think *scale factor:* 4 means?



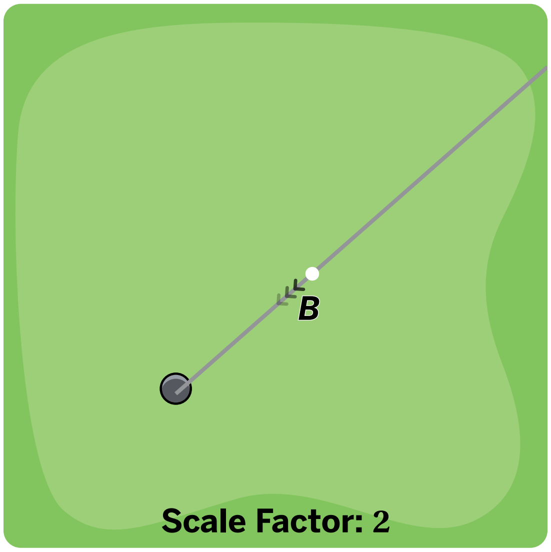


Name: .... Date: .... Period: ....

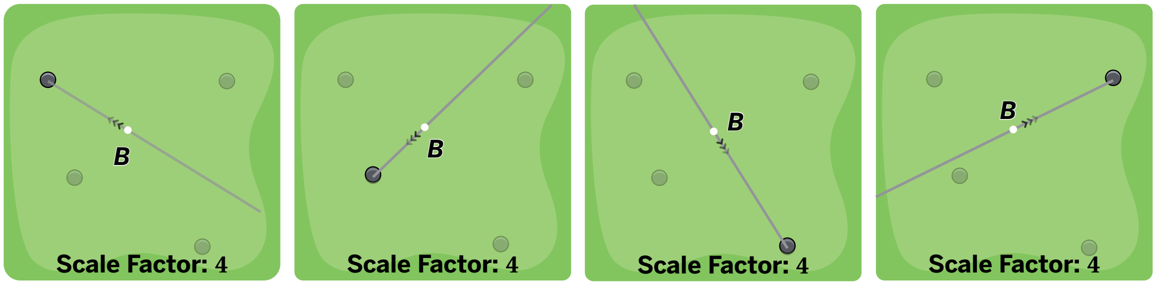
Rounds of Mini Golf

2. Point B will be dilated using point C as the **center of dilation** and a scale factor of 2.

Mark where point C should be so that the image of point B lands in the hole.



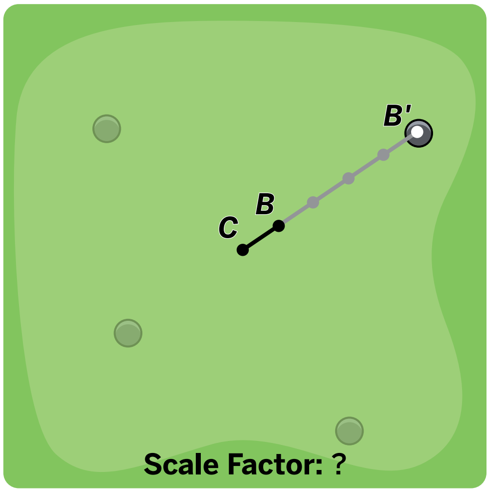
3. Point B will be dilated using point C as the center of dilation and a scale factor of 4. For each hole, mark where point C should be so that the image of point B lands in the hole.



4. Point B was dilated using point C as the center of dilation.

What is the scale factor?

Explain your thinking.



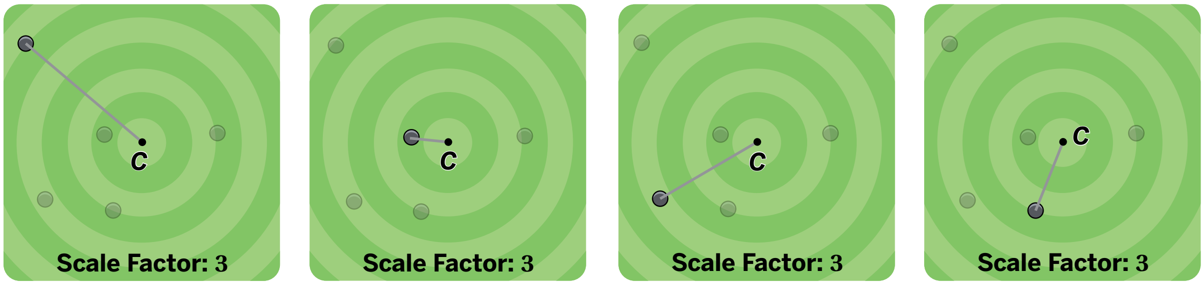
Name: .... Date: .... Period: ....

Dilation Distances

5. Now let's place the ball (point B). This will be your pre-image.

Point B will be dilated using point C as the center of dilation and a scale factor of 3.

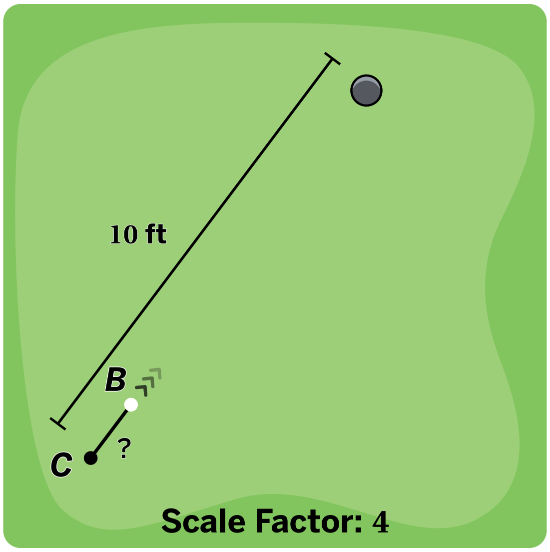
For each hole, mark point B so that its image lands in the hole.



6. Point B will be dilated using point C as the center of dilation and a scale factor of 4.

What should the distance between point C and point B be so that the image is in the hole?

Explain your thinking.

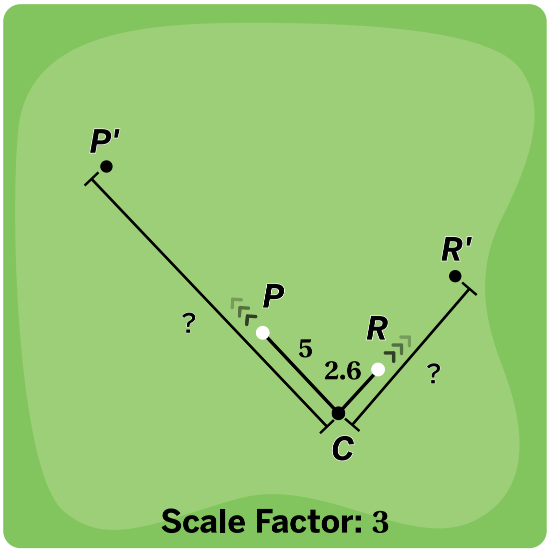


7. We can dilate more than one point at a time!

Points P and R will be dilated using point C as the center of dilation and a scale factor of 3.

What is the distance between point C and the image of point P?

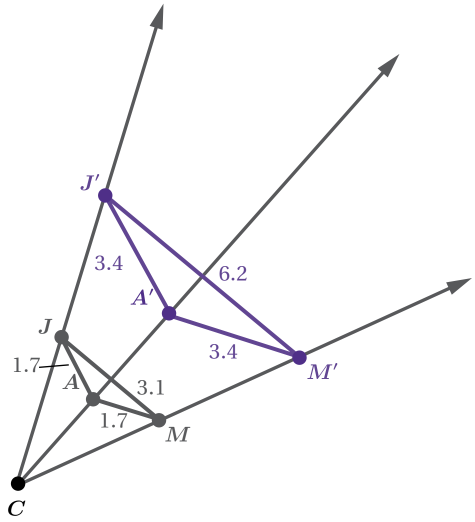
What is the distance between point C and the image of point R?



Name: ________ Date: ________ Period: ________

Dilating a Triangle

8. The diagram shows how triangle JAM is dilated using point C as the center of dilation and a scale factor of 2 to create triangle $J'A'M'$.



a Describe all of the ways you see a scale factor of 2 represented in this diagram.

b Select at least one more scale factor.

3 $\quad$ 1.5 $\quad$ $\frac{3}{4}$ $\quad$ Other: ________

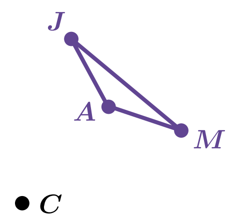
c On the same diagram, dilate triangle JAM using center C and each scale factor you chose.

d List everything that's alike about triangle JAM and its dilations.

Synthesis

9. Describe how to dilate a point or a figure given a center of dilation and a scale factor.

Use the example if it helps you with your thinking.



Scale Factor: 2

Lesson Practice ACC7.2.08

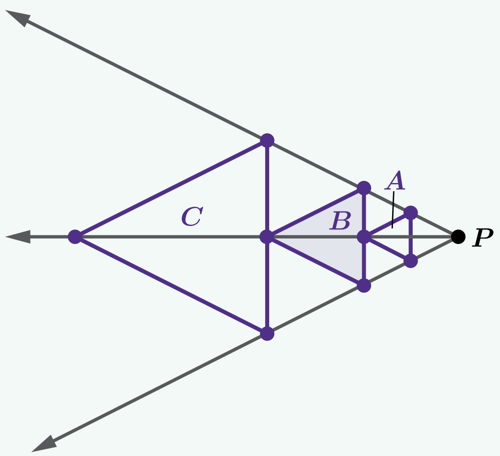
Lesson Summary

A *dilation* is a transformation that involves a **center of dilation** and a scale factor.

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.

In this example, triangle B is the pre-image.

- Triangle A is a dilation of triangle B using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
- Triangle C is a dilation of triangle B using point P as the center of dilation and a scale factor of 2.



Name: .... Date: .... Period: ....

Problems 1–2: Segment AB is 3 centimeters long. Point O is the center of dilation.

1. How long is the image of segment AB after a dilation with a scale factor of 5?

2. How long is the image of segment AB after a dilation with a scale factor of 3.7?

Problems 3–6: Here are points A and B. Plot the points after each dilation.

3. Point C is the image of point B using point A as the center of dilation and a scale factor of 2.

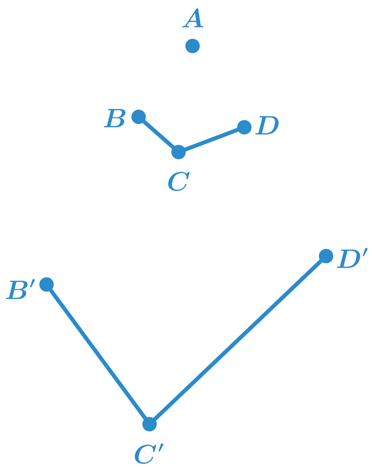
4. Point D is the image of point A using point B as the center of dilation and a scale factor of 2.

5. Point E is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{2}$.

6. Point F is the image of point A using point B as the center of dilation and a scale factor of $\frac{1}{2}$.

7. Isaiah claims that figure $B'C'D'$ is a dilation of figure BCD using point A as the center of dilation. Is Isaiah's claim correct?

   Explain your thinking.



Lesson Practice
ACC7.2.08

Name: .... Date: .... Period: ....

FAST Practice

Problems 8–9: Here is a diagram.

8. Point H is the image of point G using point F as the center of dilation. What is the scale factor?

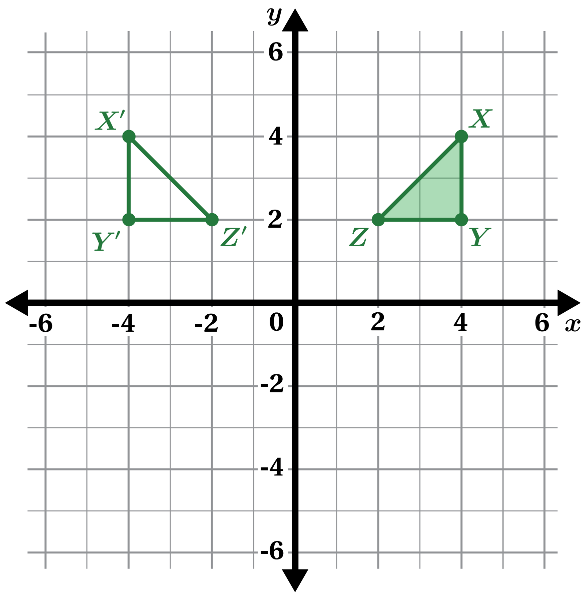
9. If the distance from point F to point H is 9 units, what is the distance from point F to point G? Record your answer in the space provided.

☐ units

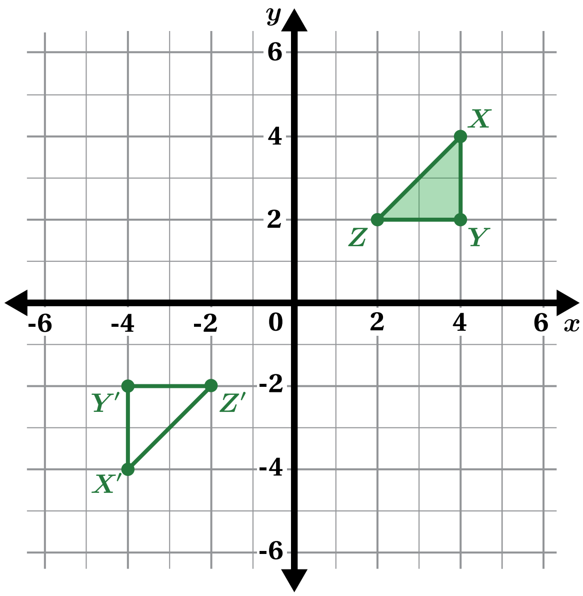
Spiral Review

Problems 10–11: Determine whether each statement is *true* or *false*. If false, write a correct statement about the transformation.

10. Triangle XYZ is translated 4 units to the left to form triangle $X'Y'Z'$.



11. Triangle XYZ is rotated 180° about the origin to form triangle $X'Y'Z'$.



Unit 2
Lesson
9

Name: Date: Period:

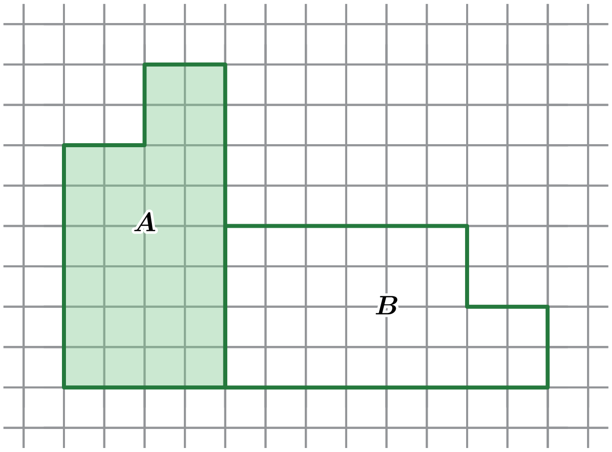
MA.8.GR.2.2, MTR.1.1, MTR.4.1

Make It a Combo!

Let's explore combining a rigid transformation with a dilation.

Warm-Up

1. Describe a transformation that could move polygon A onto polygon B.

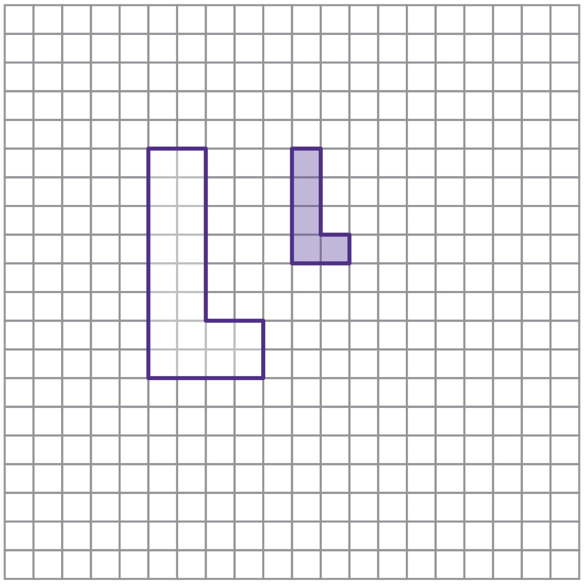


Activity
1

Name: .... Date: .... Period: ....

Transformation Challenges

2. **Challenge #1:** Show or describe a single transformation to move the pre-image (shaded) onto the image.

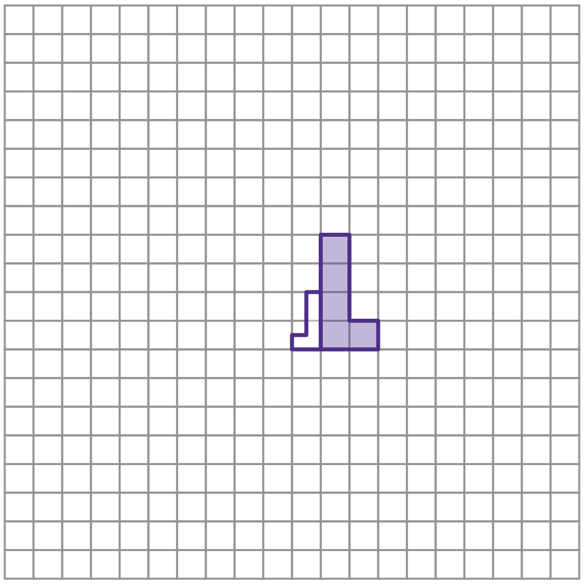


3. Aditi says you can complete the previous challenge with a dilation and a translation. Emiliano says you can complete this challenge with only a dilation. Whose claim is correct?

   Aditi's   Emiliano's   Both   Neither

   Explain your thinking.

4. **Challenge #2:** Show or describe a rigid transformation combined with a dilation to move the pre-image (shaded) onto the image.

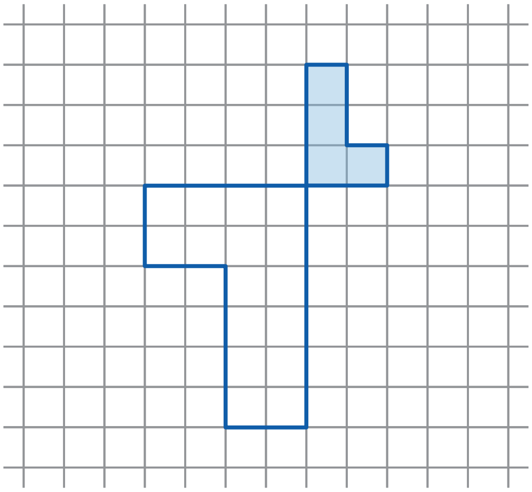


Activity 2

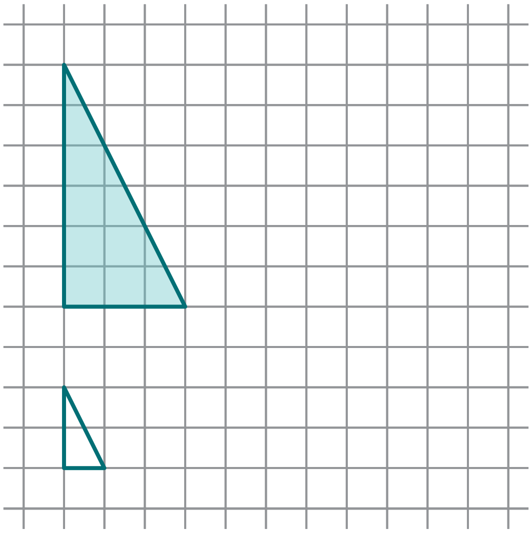
Name: .... Date: .... Period: ....

More Challenges!

5. **Challenge #3:** Describe a rigid transformation combined with a dilation to move the pre-image (shaded) onto the image.



6. **Challenge #4:** Describe a rigid transformation combined with a dilation to move the pre-image (shaded) onto the image.

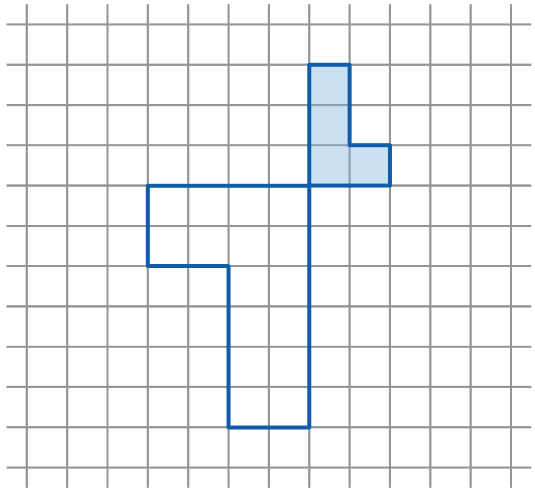


7. Based on your description in the previous problem, sketch the result of the first action you took (either the rigid transformation or the dilation).

Synthesis

8. Describe some strategies for determining a combination of a rigid transformation and a dilation that moves a pre-image onto an image.

Use the example if it helps you with your thinking.



Lesson Practice ACC7.2.09

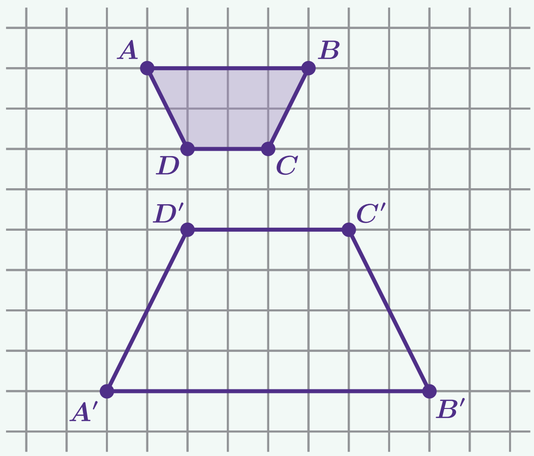
Lesson Summary

A dilation can be combined with a rigid transformation.

Here is a combination that moves figure $ABCD$ onto figure $A'B'C'D'$.

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

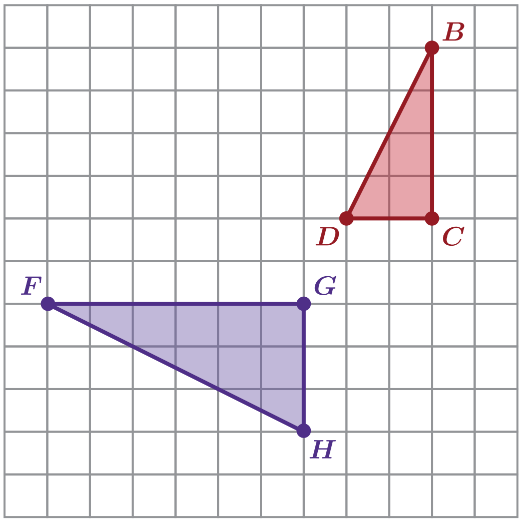
Step 2: Reflect the image across the horizontal line 1 unit below the figure.



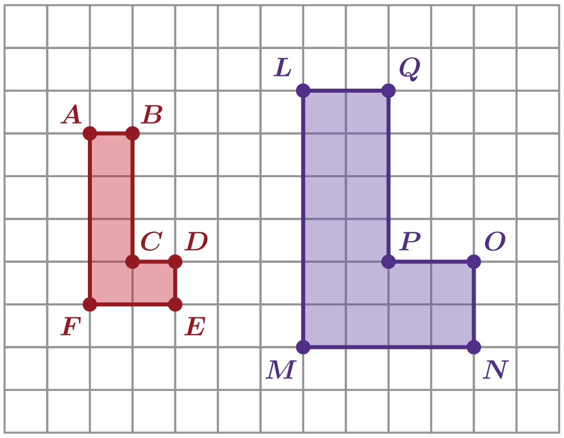
Name: .... Date: .... Period: ....

Problems 1–4: Is there a single rigid transformation or a rigid transformation combined with a dilation that moves one figure onto the other? (Write *yes* or *no*.) If you wrote *yes*, describe the transformation(s). If you wrote *no*, describe how you know it's not possible.

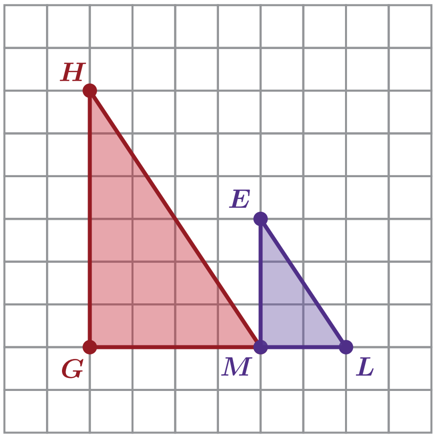
1.



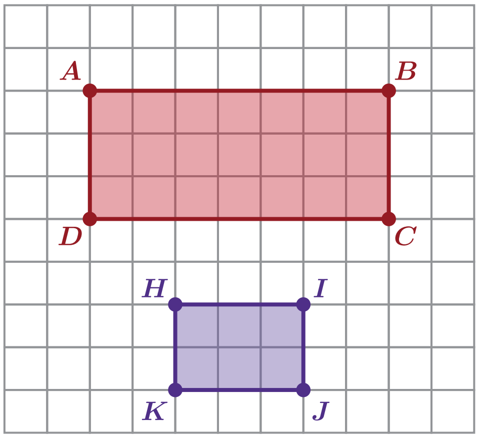
2.



3.



4.



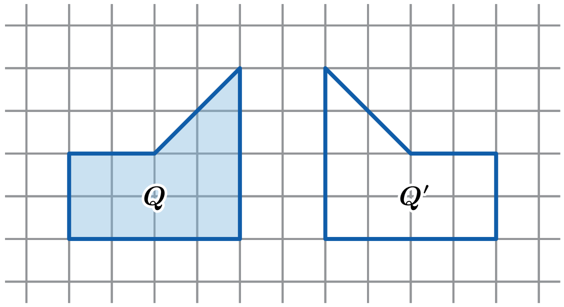
Lesson Practice
ACC7.2.09

Name: ______________________ Date: __________ Period: __________

5. Caasi says that dilating a figure by a scale factor of $\frac{3}{2}$ will produce an image that is smaller than the pre-image. Is Caasi's claim correct? Explain your thinking.

FAST Practice

6. Which transformation will move the pre-image onto the image?



A. Translating to the right.
B. Rotating $90°$.
C. Dilating by a scale factor of 4.
D. Reflecting across a vertical line.

Spiral Review

Problems 7–9: Point G is located at $(5, \text{-}2)$. Determine the coordinates of the image after each transformation.

7. A reflection over the y-axis.

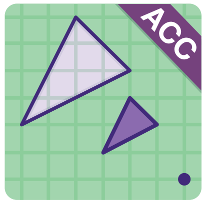
8. A reflection over the x-axis.

9. A $180°$ rotation around the origin.

Name: ______ Date: ______ Period: ______

MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1, MTR.5.1

Match My Dilation



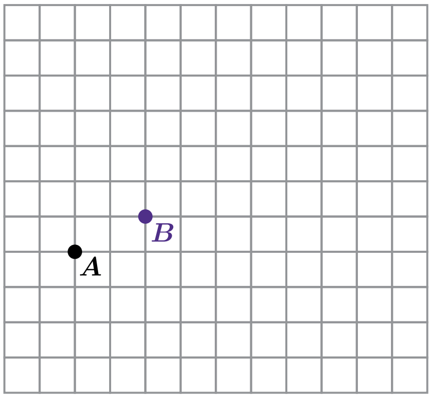
Let's dilate figures on a square grid.

Warm-Up

Plot a point C that is the image of point B dilated using point A as the center of dilation and a scale factor of 3.

1. Try it *without* a grid.

2. Try it *with* a grid.



Describe your strategy for dilating *with* a grid.

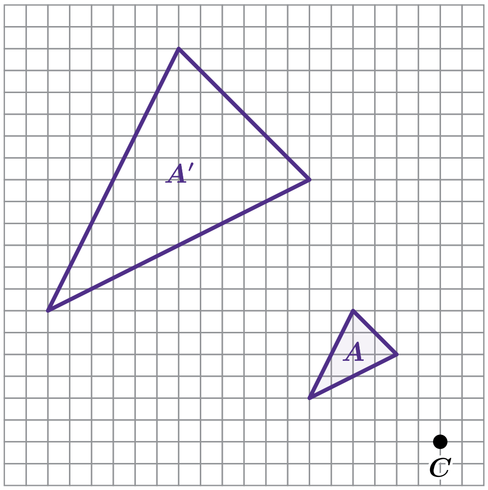
Activity 1

Name: ______ Date: ______ Period: ______

Dilation Challenges

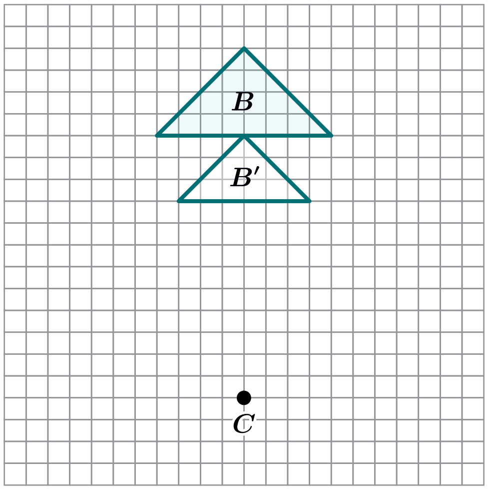
3. Triangle A' is the image of triangle A dilated using point C as the center of dilation.

What was the scale factor used in the dilation?

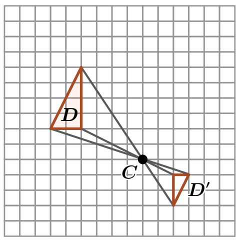


4. Triangle B' is the image of triangle B dilated using point C as the center of dilation.

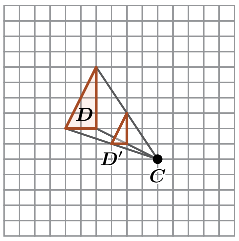
What was the scale factor used in the dilation?



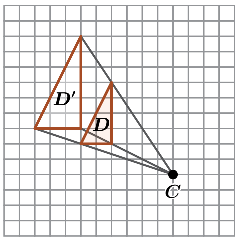
5. **a** Take a look at these dilations of triangle D.



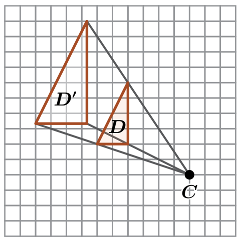
Scale factor: -0.5



Scale factor: 0.5



Scale factor: 1.5



Scale factor: 1.8

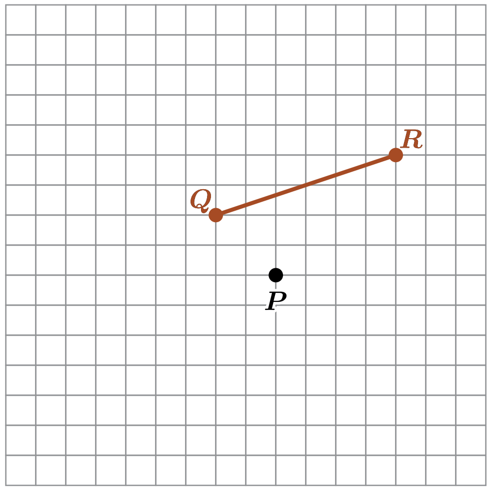
b How does the scale factor affect the size of the image? How does it affect the location of the image?

Activity
2

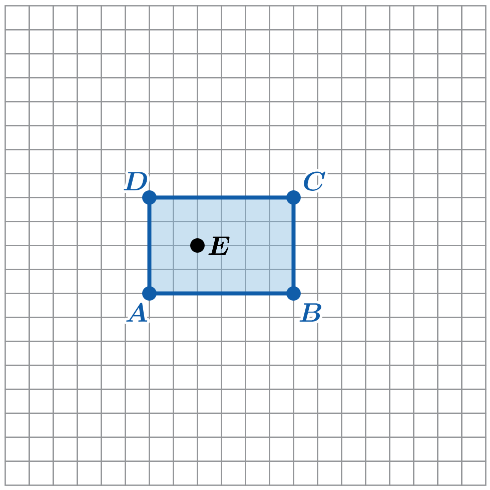
Name: .... Date: .... Period: ....

Match My Dilation

6. Show or explain how to dilate segment QR using point point P as the center of dilation and a scale factor of $\frac{1}{2}$.



7. **a** Dilate quadrilateral $ABCD$ using point E as the center of dilation and a scale factor of 2.



b **Discuss:** How would the size and location of the image change if point E were in a different location?

Activity
3

Name: ________ Date: ________ Period: ________

Challenge Creator

8. You will use the Activity 3 Sheet to complete this activity.

a. **Make It!** On the Activity 3 Sheet, create a dilation challenge.

b. **Solve It!** On this page, redraw your pre-image. Then draw your image using the center of dilation and scale factor you chose. Label the vertices A', B', and C'.

c. **Swap It!** Swap your challenge with one or more partners. Draw your partners' pre-images. Then draw each image using the center of dilation and scale factor.

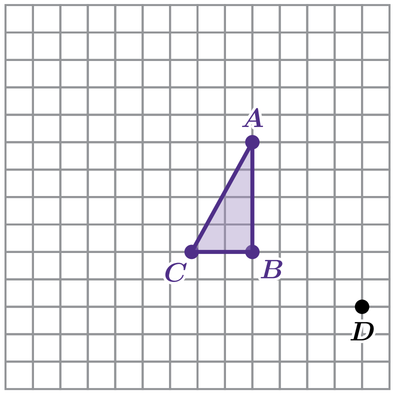
My Dilation Challenge	Partner 1
Scale Factor: ________	Scale Factor: ________

Partner 2	Partner 3
Scale Factor: ________	Scale Factor: ________

Synthesis

9. What is important to remember when dilating a figure using a center of dilation and a scale factor?

Use the example if it helps you with your thinking.



Lesson Practice ACC7.2.10

Lesson Summary

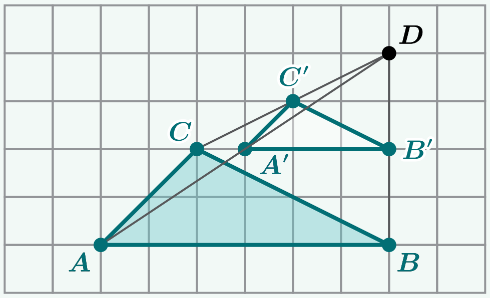
Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

If the scale factor of the dilation is:

- Greater than 1, the image will be *larger* than the pre-image and further from the center of dilation.
- Equal to 1, the image will be *the same size* as the pre-image and just as far from the center.
- Between 0 and 1, the image will be *smaller* than the pre-image and closer to the center.

For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.



Lesson Practice
ACC7.2.10

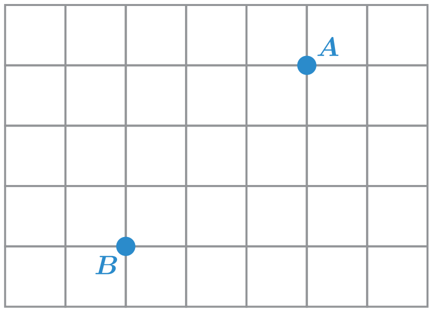
Name: ______________ Date: ________ Period: ________

Problems 1–2: Segment AB measures 3 centimeters. Point O is the center of dilation.

1. How long is the image of AB after a dilation with a scale factor of $\frac{1}{5}$?

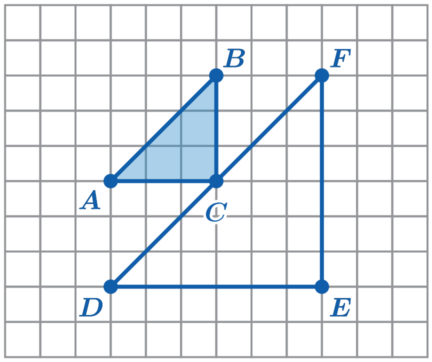
2. How long is the image of AB after a dilation with a scale factor of s?

3. Here are points A and B. Point F is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{3}$. Plot point F.



4. Triangle ABC was transformed into triangle DFE using a dilation.

Label the center of dilation P. Then determine the scale factor.

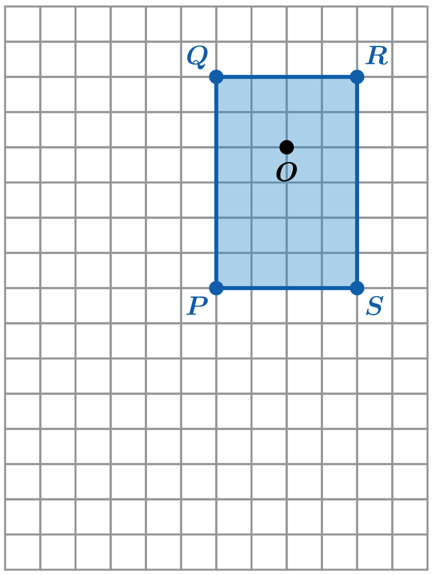


Problems 5–6: Here is rectangle $PQRS$.

Draw the image of rectangle $PQRS$ after each dilation.

5. A dilation using point R as the center of dilation and a scale factor of 2.

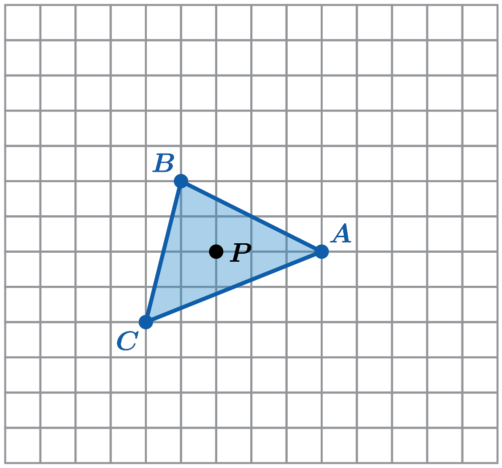
6. A dilation using point O as the center of dilation and a scale factor of $\frac{1}{2}$.



Name: .... Date: .... Period: ....

FAST Practice

Problems 7–9: Here is triangle ABC.



7. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of 2. Draw the image and label the vertices $A'B'C'$.

8. If the length of side AB is 4.5 units and the length of side BC is 4.1 units, what are the lengths of side $A'B'$ and side $B'C'$?

9. How would the image be affected if point P were in a different location? Select *all* the statements that are true.

- ☐ **A.** The location of the image would change based on where the center of dilation is located.
- ☐ **B.** The distance from point A' to point P will always be twice the distance from point A to point P.
- ☐ **C.** The size of the image would remain unchanged regardless of the new position of point P.
- ☐ **D.** The size of the image would change proportionally to the distance from the center of dilation.
- ☐ **E.** The location of the image would remain fixed even if point P were moved.

Spiral Review

Problems 10–11: Triangle ABC has vertices $A(3, 5)$, $B(6, 1)$, and $C(2, \text{-}2)$. Determine the coordinates of the vertices of the image for each transformation.

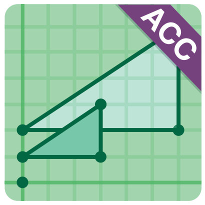
10. Triangle ABC is translated up 10 units.

11. Triangle ABC is rotated $90°$ clockwise around the origin.

Name: ____________ Date: ________ Period: ________

MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1, MTR.5.1

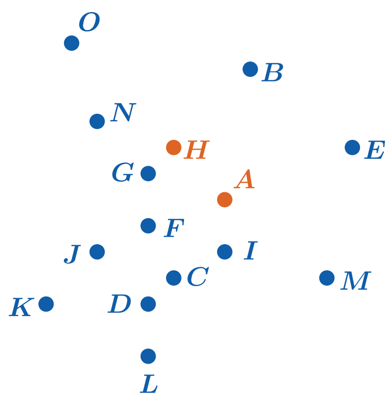
Dilations on a Plane



Let’s look at dilations on the coordinate plane.

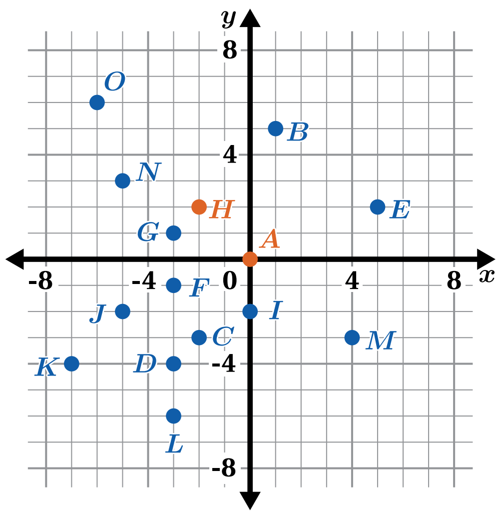
Warm-Up

1. Determine which point is a dilation of point H using point A as the center of dilation and a scale factor of 3.



2. Here is the same task, but with a coordinate plane.

Determine which point is a dilation of point H using point A as the center of dilation and a scale factor of 3.



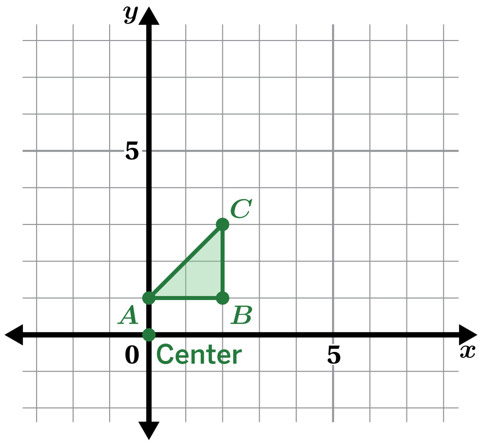
Activity 1

Name: .... Date: .... Period: ....

Dilate It!

Use any strategy or tool to perform each dilation. Label the corresponding points in the image using the ′ symbol.

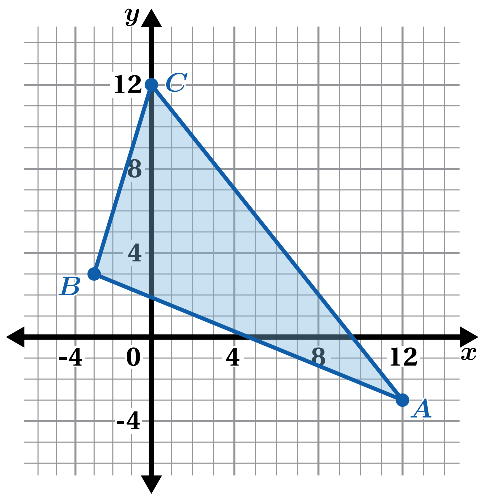
3. **a** Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of 2.



b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(0, 1)	
(2, 1)	
(2, 3)	

4. **a** Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{3}$.



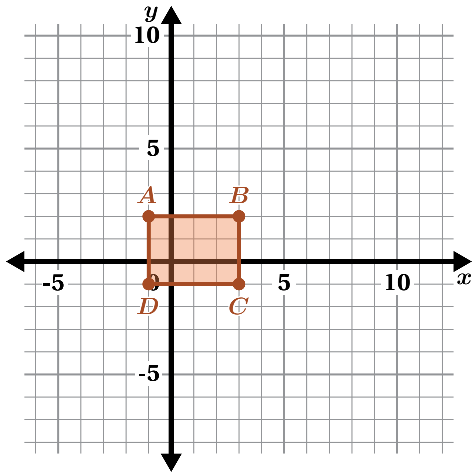
b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(12, -3)	
(-3, 3)	
(0, 12)	

Name: .... Date: .... Period: ....

Dilate It! (continued)

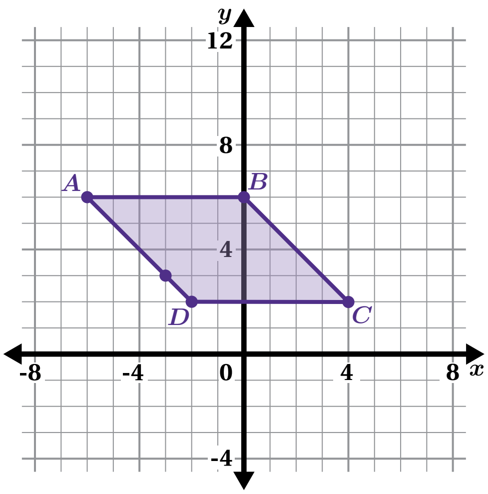
5. **a** Dilate rectangle $ABCD$ using $(0, 0)$ as the center of dilation and a scale factor of 3.



b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(-1, 2)	
(3, 2)	
(3, -1)	
(-1, -1)	

6. **a** Dilate parallelogram $ABCD$ using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{2}$.



b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(-6, 6)	
(0, 6)	
(4, 2)	
(-2, 2)	

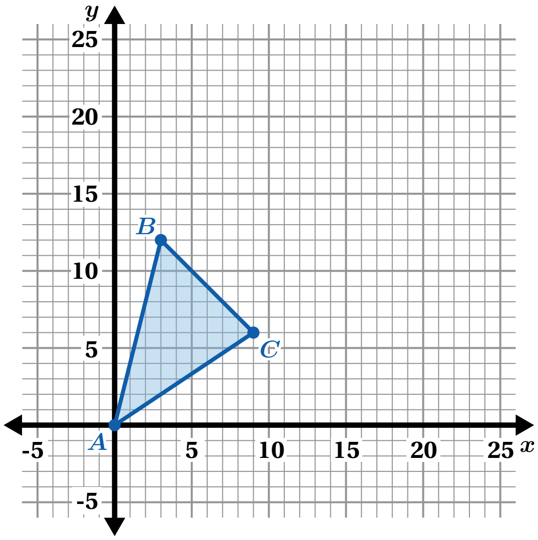
7. Review your work for Problems 2–6.

Discuss: What effect does the scale factor have on the coordinates of a figure?

Name: .... Date: .... Period: ....

Dilation Information

Problems 8–10: Here is triangle ABC.



8. Dilate triangle ABC using center $(0,0)$ and a scale factor of 2.

a Label the vertices A' B' C'.

b Explain how you determined the coordinates of triangle $A'B'C'$.

9. Dilate triangle ABC using center $(0,0)$ and a scale factor that is less than 1.

a Label the vertices $A''B''C''$.

b What scale factor did you use?

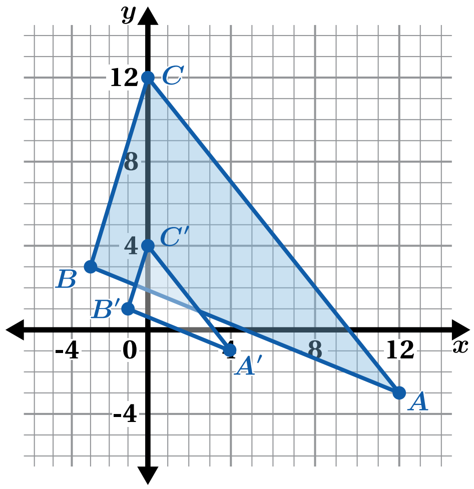
c Explain how you determined the coordinates of triangle $A''B''C''$.

10. Explain why A, A', and A'' must all be at the same coordinates.

Synthesis

11. What is important to remember when dilating figures on a coordinate plane?

Use the example if it helps with your thinking.



Lesson Practice ACC7.2.11

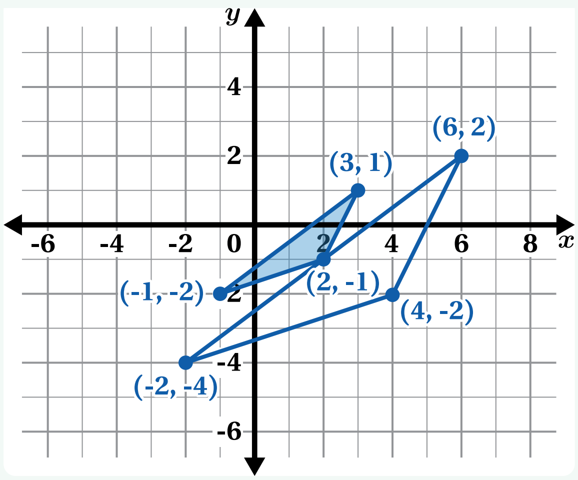
Lesson Summary

Coordinates can help us communicate geometric information, such as dilations.

Let's say you're dilating this shaded pre-image using the center of dilation $(0, 0)$ and a scale factor of 2.

When the center of dilation is $(0, 0)$, you can multiply the pre-image coordinates by the scale factor to get the image coordinates.

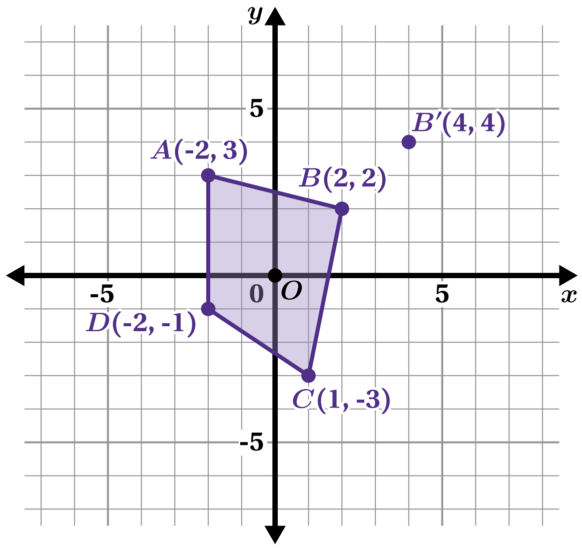
A scale factor greater than 1 will produce an image that is larger, with the points farther from the *origin*. A scale factor between 0 and 1 will produce an image that is smaller, with the points closer to the origin.



Lesson Practice
ACC7.2.11

Name: .... Date: .... Period: ....

Problems 1–3: Quadrilateral $ABCD$ is dilated with the origin as the center of dilation, moving point B onto point B'.

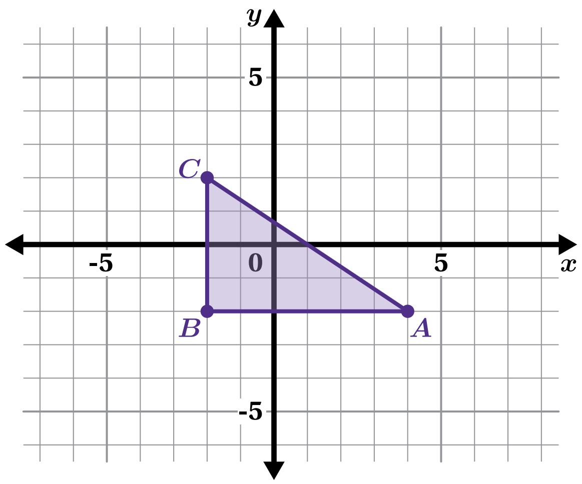


1. What is the scale factor of the dilation?

2. Draw quadrilateral $A'B'C'D'$.

3. Label the coordinate points for A', C', and D'.

Problems 4–6: Here is triangle ABC on the coordinate plane.



4. Using the origin as the center and a scale factor of 2, draw the dilation of triangle ABC. Label the image triangle DEF.

5. Using the origin as the center and a scale factor of $\frac{1}{2}$, draw the dilation of triangle ABC. Label the image triangle GHI.

6. Triangle GHI is a dilation of triangle DEF. Identify the center of dilation and the scale factor.

Lesson Practice
ACC7.2.11

Name: .... Date: .... Period: ....

FAST Practice

7. Here are the pre-image and image coordinates of points on a graph. Complete the statement so that it correctly describes the transformation.

Record your answers in the spaces provided.

This is a dilation with ☐ as the center of dilation and a scale factor of ☐.

Pre-Image Coordinates	Image Coordinates
(0, 4)	(0, 12)
(-1, 3)	(-3, 9)
(5, -6)	(15, -18)

Spiral Review

Problems 8–9: Use what you know about the interior angle measures of triangles to complete these problems.

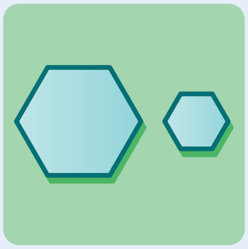
8. Triangle JKL is a right triangle, and the measure of angle J is 28°. What are the measures of the other two angles?

9. Triangle PQR is an obtuse triangle, and the measure of angle Q is 72°. What are possible measures of the other two angles?

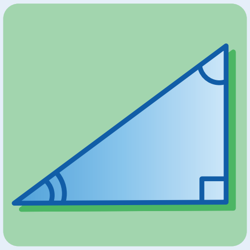
Sub-unit

3

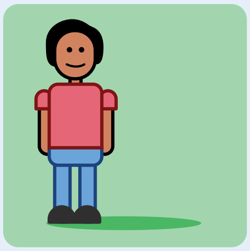
Similarity



Lesson 12
Scavenger Hunt



Lesson 13
Are Angles Enough?



Lesson 14
Shadows

Name: ____________ Date: ________ Period: ________

MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1

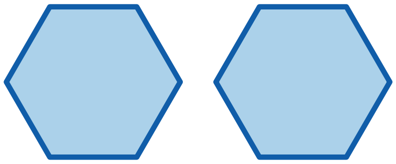
Scavenger Hunt

Let's explore what makes two figures similar.

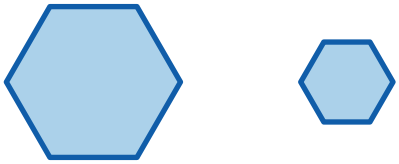
Warm-Up

1. Which one doesn't belong?

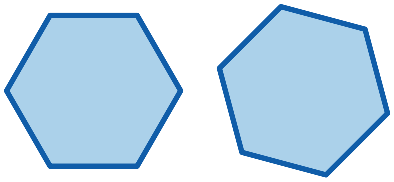
A.



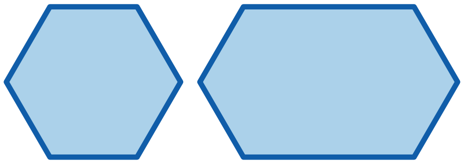
B.



C.



D.



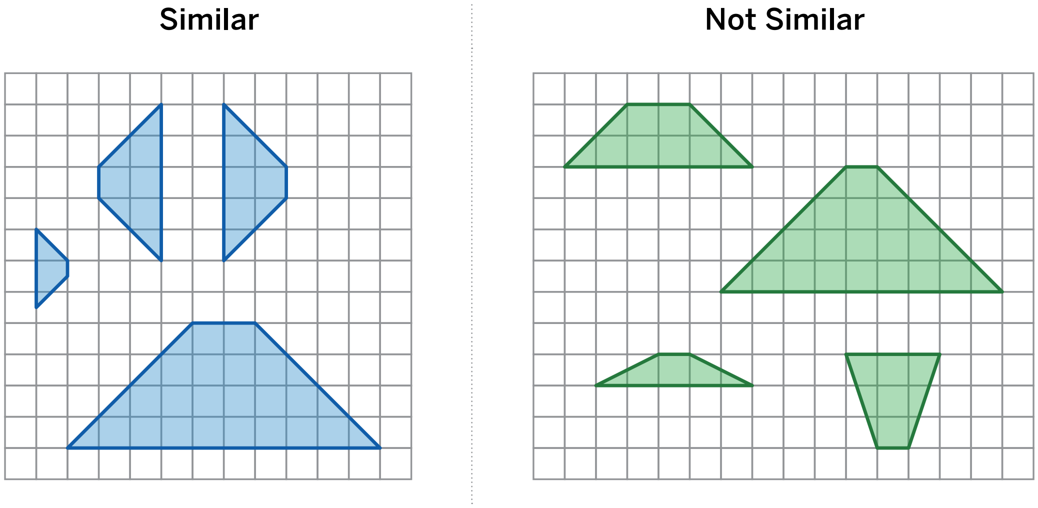
Explain your thinking.

Activity
1

Name: Date: Period:

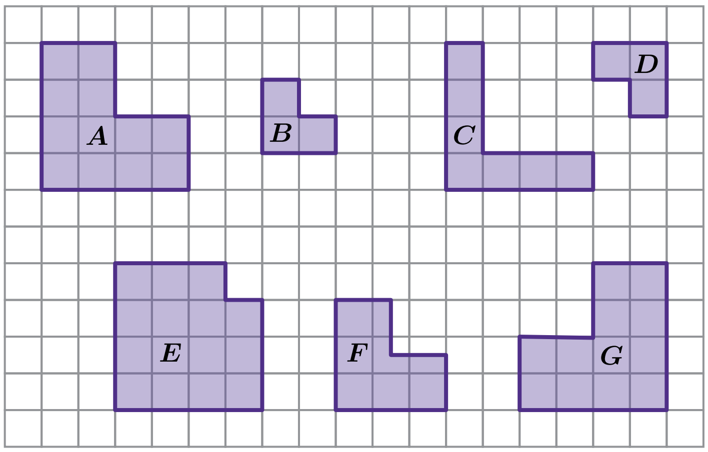
What Are Similar Figures?

2. Here are figures that are **similar** and not similar.



What do you think makes figures *similar*? How is this related to transformations?

3. Here are seven figures.



Choose two figures that fit each category. Explain your thinking.

Similar	Congruent	Not Similar

Activity 2

Name: ________ Date: ________ Period: ________

Scavenger Hunt

4. You will be given a card. For each round, you will have instructions for finding a partner. Once you have a partner, compare your cards and answer each question.

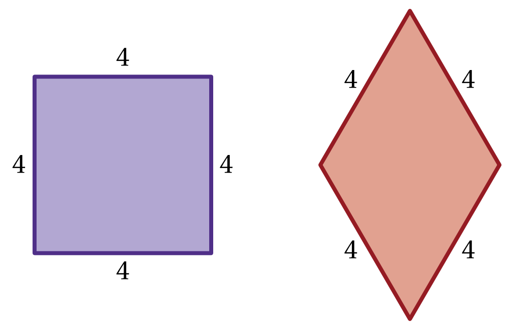
Round 1

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?	○	○
Are the figures similar?	○	○
Are the figures congruent?	○	○
Are the corresponding sides congruent?	○	○
Are the corresponding angles congruent?	○	○

Round 2

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?	○	○
Are the figures similar?	○	○
Are the figures congruent?	○	○
Are the corresponding sides congruent?	○	○
Are the corresponding angles congruent?	○	○

5. Here are two figures. Decide whether each statement is true or false.



a. Their corresponding sides are congruent.

b. Their corresponding angles are congruent.

c. The figures are similar.

d. The figures are congruent.

Name: .... Date: .... Period: ....

Scavenger Hunt (continued)

6. Here are two new rounds of the scavenger hunt. With each new partner, compare your cards and answer each question.

Round 3

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?	◯	◯
Are the figures similar?	◯	◯
Are the figures congruent?	◯	◯
Are the corresponding sides congruent?	◯	◯
Are the corresponding angles congruent?	◯	◯

Round 4

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?	◯	◯
Are the figures similar?	◯	◯
Are the figures congruent?	◯	◯
Are the corresponding sides congruent?	◯	◯
Are the corresponding angles congruent?	◯	◯

7. Decide whether each statement is *always*, *sometimes*, or *never* true.

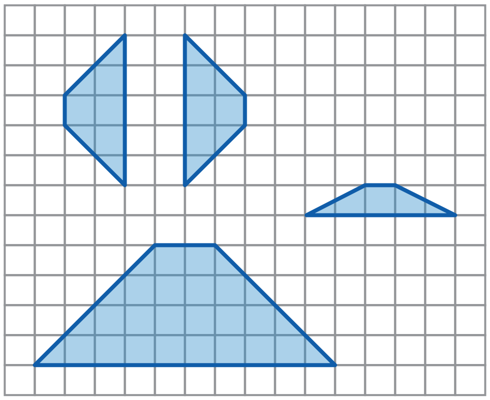
Statement	Always True	Sometimes True	Never True
a If figures are similar, they are also congruent.	◯	◯	◯
b If figures are similar, their corresponding angles are congruent.	◯	◯	◯
c If figures are similar, their corresponding sides are congruent.	◯	◯	◯
d If figures are similar, their corresponding side lengths have a common scale factor.	◯	◯	◯

Choose one statement and explain your thinking.

Synthesis

8. What does it mean for two figures to be similar? What does it tell you about their corresponding side lengths and angle measures?

Use the example if it helps with your thinking.



Lesson Practice ACC7.2.12

Lesson Summary

Congruent figures have exactly the same shape and size. Two figures are congruent if one figure can be moved exactly onto the other using a rigid transformation. **Similar** figures have exactly the same shape but not necessarily the same size. Two figures are similar if one figure can be moved exactly onto the other using a combination of a rigid transformation and a dilation.

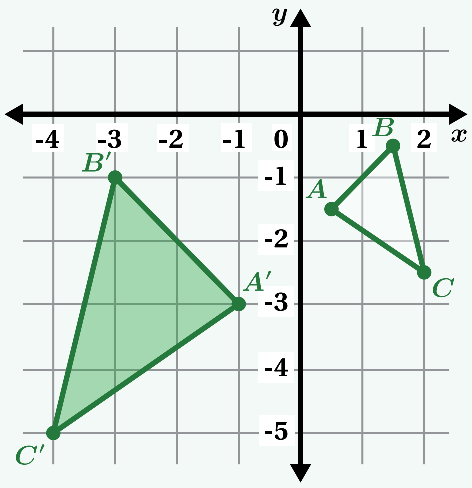
For example, triangle ABC can be mapped onto triangle $A'B'C'$ using a dilation with a scale factor of 2 and a reflection over the y-axis.

We use the symbol ~ to say that two figures are similar. For example, triangle ABC ~ triangle $A'B'C'$.

In congruent figures, the *corresponding* angles and corresponding sides are congruent.

In similar figures, the corresponding angles are congruent and the corresponding side lengths are proportional but *not* always congruent.

For example, $\angle A$ is congruent to $\angle A'$, but segment AB is not the same length as segment $A'B'$. Segment $A'B'$ is twice as long as segment AB because the scale factor from triangle ABC to triangle $A'B'C'$ is 2.

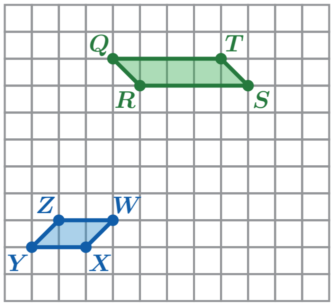


Lesson Practice
ACC7.2.12

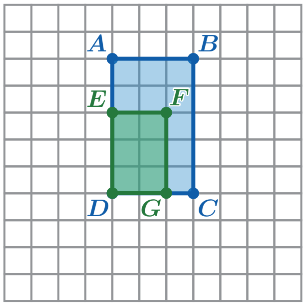
Name: .... Date: .... Period: ....

1. Which graph shows a pair of similar figures?

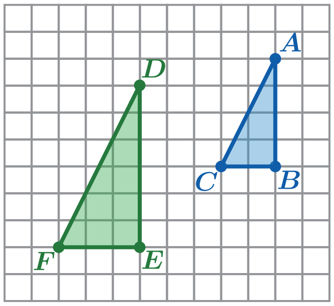
(A.)



B.

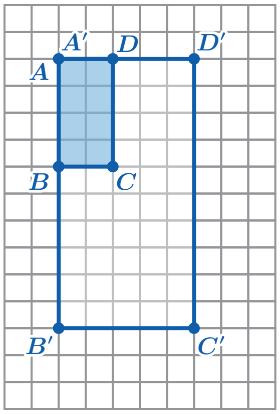


C.

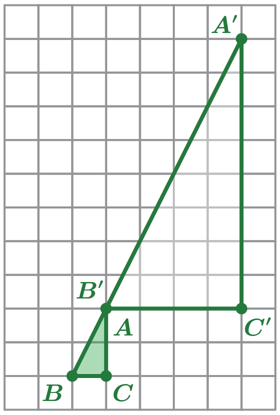


Problems 2–3: Show that the figures are similar by describing the rigid transformation, dilation, or rigid transformation and dilation that moves the shaded pre-image onto the unshaded image.

2.

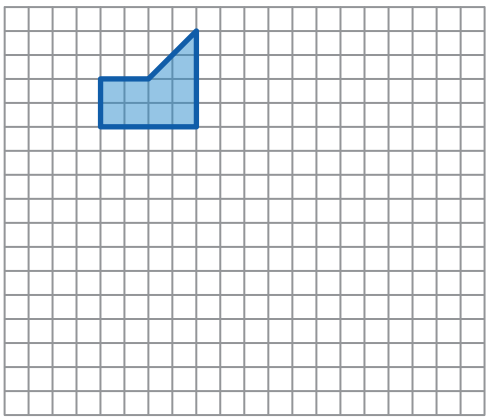


3.



Problems 4–5: Here is a polygon.

4. Draw a similar polygon that could be mistaken for being *not* similar.



5. Draw a polygon that is not similar, but could be mistaken for a similar polygon.

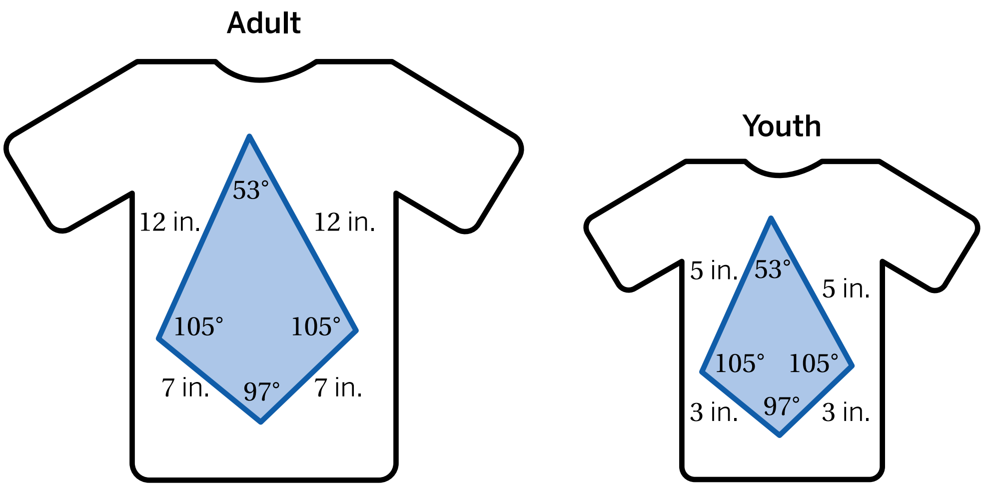
Name: .... Date: .... Period: ....

FAST Practice

6. A fashion designer is working with a supplier to print her design on different-sized shirts. She checks the quality of the shirts by using congruence and similarity.

Based on the measurements, do you think the designer should continue to work with the supplier? Explain your thinking using congruence and similarity.

Record your answer in the space provided.

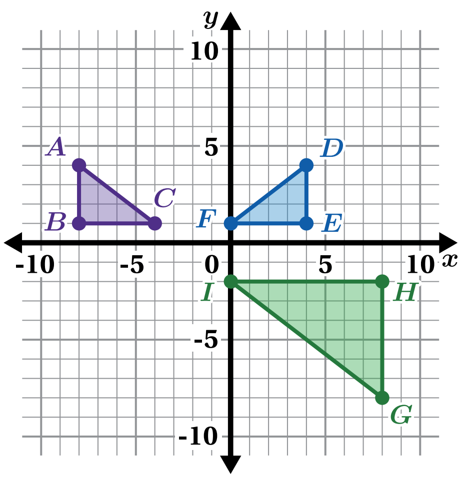


Spiral Review

Problems 7–8: Here are some triangles on a graph.

7. Describe a single transformation that shows that triangle DEF is congruent to triangle ABC.

8. Describe a sequence of transformations that shows that triangle DEF is similar to triangle GHI.



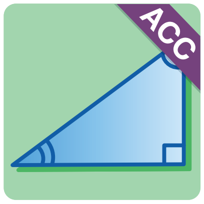
Unit 2
Lesson
13

Name: ________ Date: ________ Period: ________

MA.8.GR.1.4, MA.8.GR.1.5, MTR.4.1

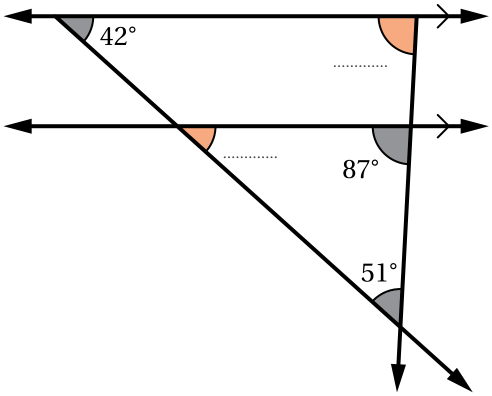
Are Angles Enough?

Let's determine whether congruent corresponding angles are enough to know whether triangles are similar.



Warm-Up

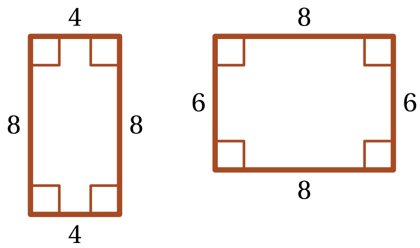
1. Determine the measure for each missing angle. Explain your reasoning.

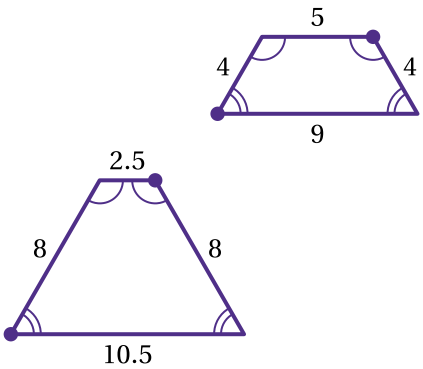


Name: ______ Date: ______ Period: ______

Are Angles Enough?

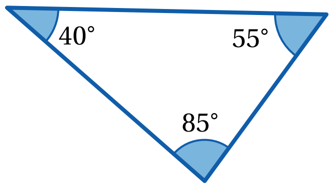
2. Take a look at these pairs of figures that have congruent corresponding angles.





Discuss: How do you know that the figures in each pair are *not* similar?

3. Take a look at this triangle with angle measures $40°$, $55°$, and $85°$. If all of your classmates made triangles with the same angle measures as this one, would all the triangles be similar? Circle one.

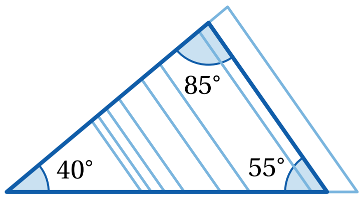


Yes No I'm not sure

Explain your thinking.

4. Here are some triangles that all have $40°$, $85°$, and $55°$ angles.

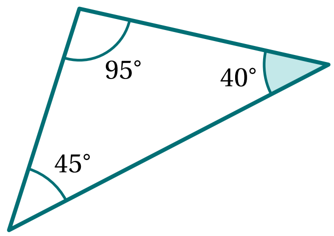
Discuss: Are these triangles all similar?



Name: .... Date: .... Period: ....

Are Angles Enough? (continued)

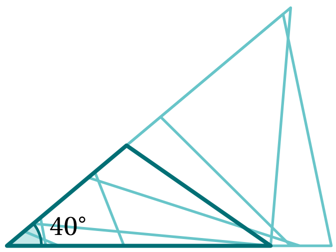
5. Take a look at this triangle with one angle measuring $40°$. If all of your classmates made triangles with one angle measuring $40°$, would all the triangles be similar? Circle one.



Yes No I'm not sure

Explain your thinking.

6. Here are some triangles that all have a $40°$ angle.



Discuss: Are these triangles all similar?

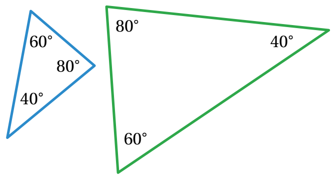
Activity 2

Name: ____________ Date: ________ Period: ________

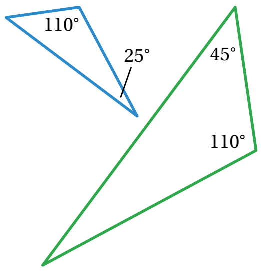
Similar Sort

7. Sort the pairs of figures into three groups. (Images are not to scale.)

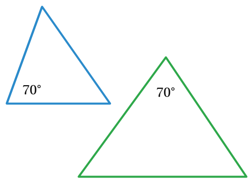
A.



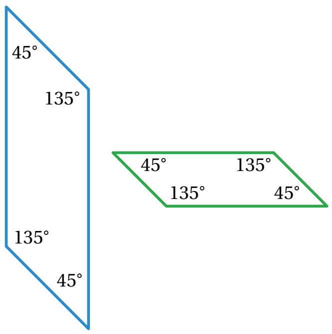
B.



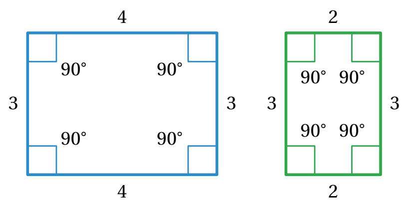
C.



D.



E.

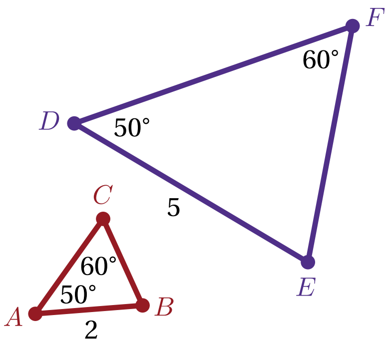


Similar	Not Similar	Not Enough Information

Synthesis

8. Describe how you can use angles to determine whether triangles are similar.

Use the example if it helps you with your thinking.

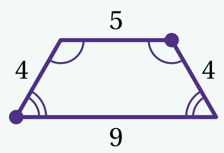


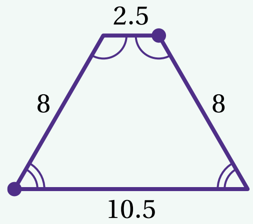
Lesson Practice ACC7.2.13

Lesson Summary

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

Here are two figures that have congruent corresponding angles but are not similar figures.

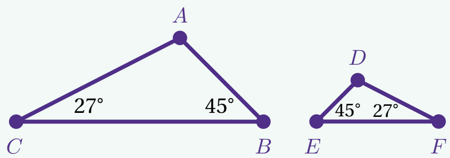




For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.

This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is $180°$ to figure out the third angle measure.

For example, the unknown angles in triangles CAB and FDE are each $108°$. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.



Lesson Practice
ACC7.2.13

Name: .... Date: .... Period: ....

Problems 1–3: Here are some angle measures for different pairs of triangles. Determine whether each pair is *similar*, *not similar*, or if there is *not enough information* to know.

1. Triangle A: $53°$, $71°$
Triangle B: $53°$, $71°$

2. Triangle E: $63°$, $45°$
Triangle F: $14°$, $71°$

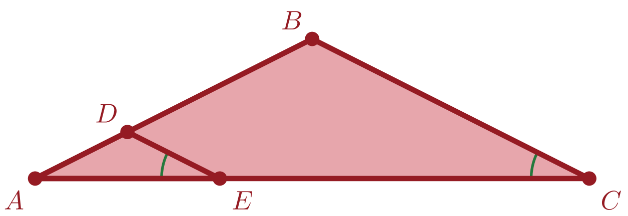
3. Triangle G: $100°$
Triangle H: $70°$

4. Draw two equilateral triangles to fit each category in the table. If it's not possible, write "Not possible."

Not Congruent	Not Similar

5. Do you think two equilateral triangles will be *always*, *sometimes*, or *never similar*? Explain your thinking.

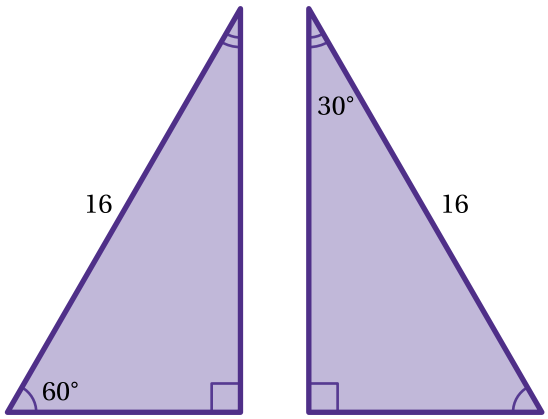
6. In this figure, $\angle AED$ and $\angle ACB$ are congruent. How do you know that triangle ABC is similar to triangle ADE?



7. Triangle A has two angles that measure $90°$ and $33°$. Triangle B has two angles that measure $90°$ and $57°$. Determine whether the triangles are *similar* or *not similar*. Explain your thinking.

FAST Practice

8. Which of the following *best* describes the triangles shown?



- **A.** Congruent but not similar
- **B.** Similar but not congruent
- **C.** Both similar and congruent
- **D.** Neither similar nor congruent

Spiral Review

9. A figure is translated 3 units to the right and then dilated by a scale factor of 3. Determine whether the pre-image and image are *congruent*, *similar*, or *neither*.

Name: .... Date: .... Period: ....

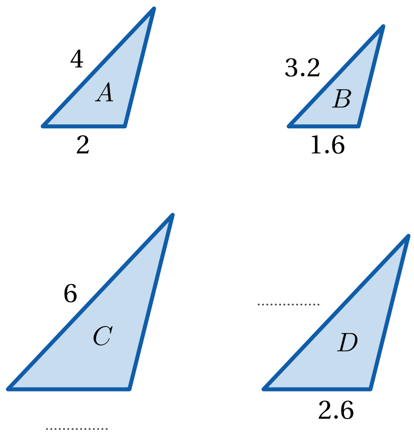
MA.8.GR.2.4, MTR.2.1

Shadows

Let's use what we know about similar triangles to determine missing side lengths.

Warm-Up

1. Here are four similar triangles.



a Examine the given side lengths. Then write in the missing values.

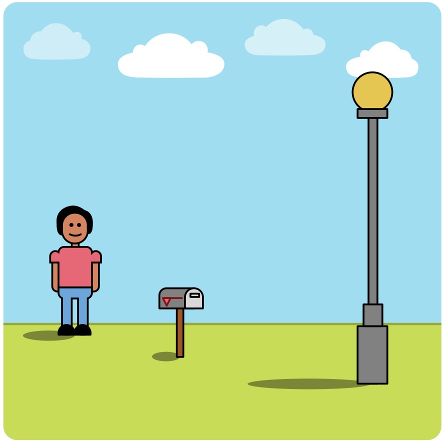
b **Discuss:** How did you determine these values?

Name: .... Date: .... Period: ....

Similar Triangles in Shadows

2. Kayla noticed she could form similar triangles using the shadows of these figures.

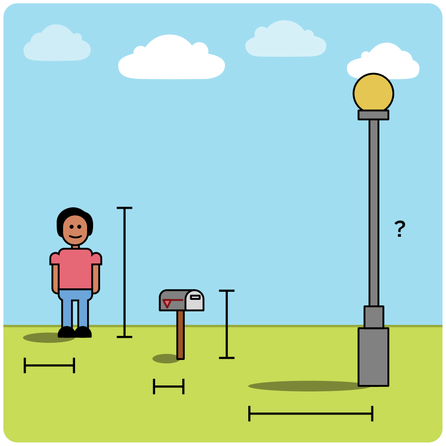
Where do you think she saw triangles? Why might she think these triangles are similar? Draw on the picture if it helps to show your thinking.



3. Your task is to determine the height of this lamppost.

a Decide which of these measurements you need and request them from your teacher:

- ☐ Height of the person
- ☐ Length of the person's shadow
- ☐ Height of the mailbox
- ☐ Length of the mailbox's shadow
- ☐ Length of the lamppost's shadow



b Once you have enough information, determine the height of the lamppost.

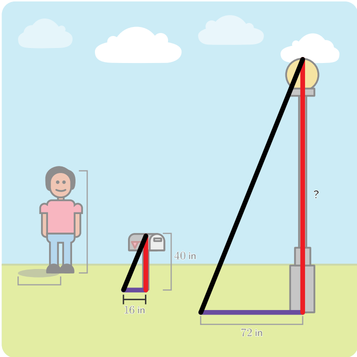
Activity 1

Name: ______________________ Date: __________ Period: __________

Similar Triangles in Shadows (continued)

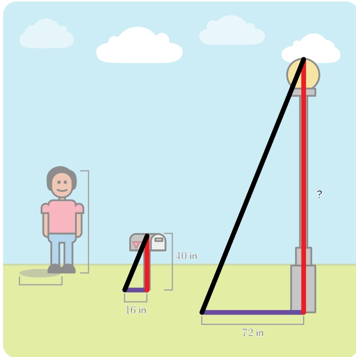
4. Here are Ama's and Neena's strategies for determining the height of the lamppost.

Ama



$$\frac{40}{?} = \frac{16}{72}$$

Neena



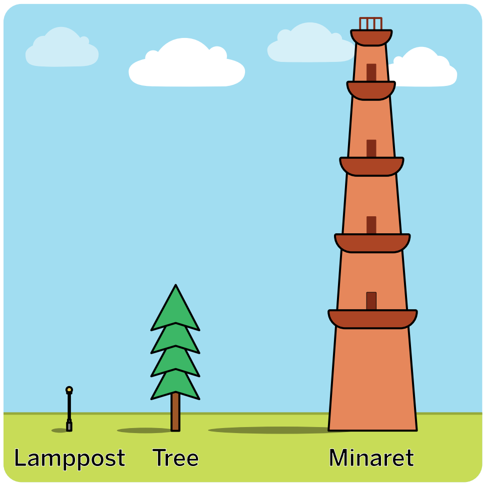
$$\frac{40}{16} = \frac{?}{72}$$

Discuss: How are their strategies alike? How are they different?

5. Here is the lamppost from the previous problem, as well as two new objects.

Determine the missing heights.

	Lamppost	Tree	Minaret
Height (ft)	15		
Shadow Length (ft)	6	20	56

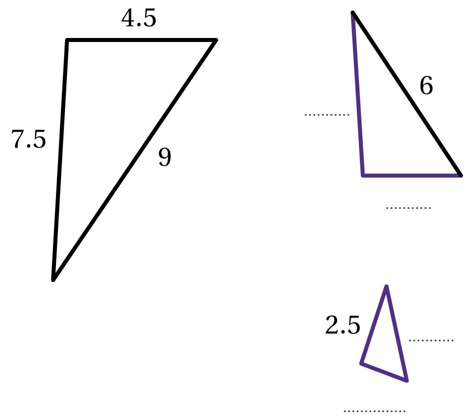


Activity 2

Name: Date: Period:

Similar Triangles Puzzle

6. Here are three similar triangles.



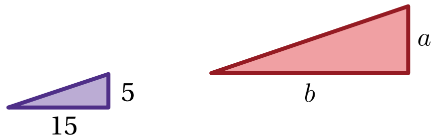
a Determine all the side lengths using as few hints as you can. You can ask for the measure of up to two side lengths, if needed.

b What was your strategy?

Synthesis

7. What are some strategies for determining unknown side lengths in similar triangles?

Use the example if it helps with your thinking.



Lesson Practice ACC7.2.14

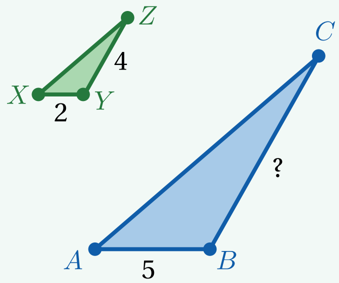
Lesson Summary

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

You can also determine the ratio of two side lengths in the same triangle, then apply that ratio to the corresponding sides in a similar triangle.

Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.



Using the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5. To calculate the length of side BC, you can multiply the length of side YZ by the scale factor. $4 \cdot 2.5 = 10$, so side BC is 10 units long.

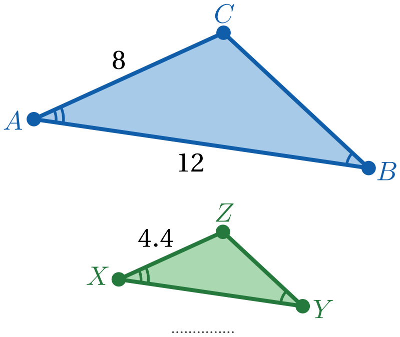
Using ratios of side lengths within one triangle.
The ratio of side YZ to side XY is $4 : 2$, or 2. That means the length of side BC is twice the length of side AB. $5 \cdot 2 = 10$, so side BC is 10 units long.

Lesson Practice
ACC7.2.14

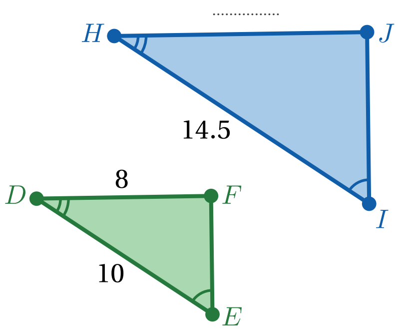
Name: .... Date: .... Period: ....

Problems 1–2: Triangle ABC is similar to triangle XYZ. Triangle DEF is similar to triangle HIJ. Determine the missing values. Explain your thinking. The figures may not be drawn to scale.

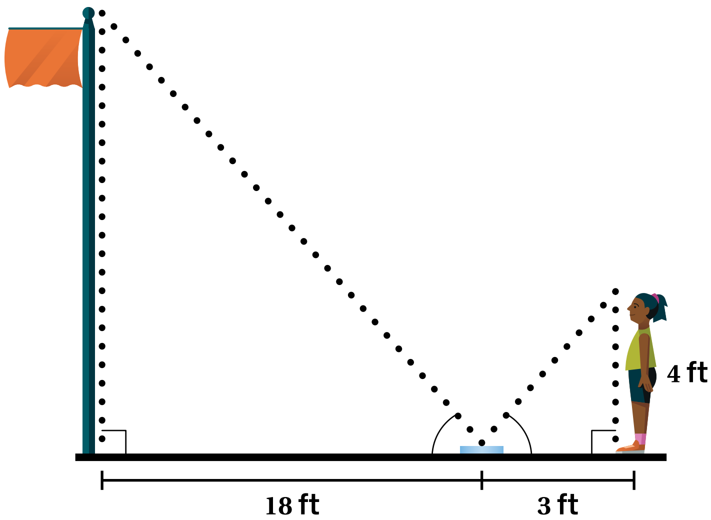
1.



2.



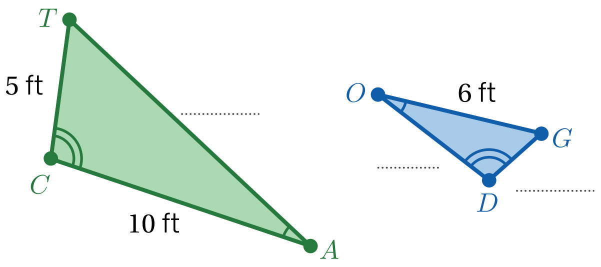
3. Zoe uses a mirror to measure the height of a flagpole. She places the mirror flat on the ground, then walks backward until she can see the top of the flagpole in the mirror. As the diagram shows, the way that light reflects in the mirror creates a set of congruent angles. Now she can use her own height, the mirror's distance from the flagpole, and her distance from the mirror to calculate the height of the flagpole.



Use the measurements in the diagram to calculate the flagpole's height. Show or explain your thinking.

Name: .... Date: .... Period: ....

4. Triangle CAT is similar to triangle DOG. Triangle CAT is dilated by a scale factor of $\frac{1}{2}$, creating triangle DOG. Determine the missing side lengths. Show or explain your thinking.



FAST Practice

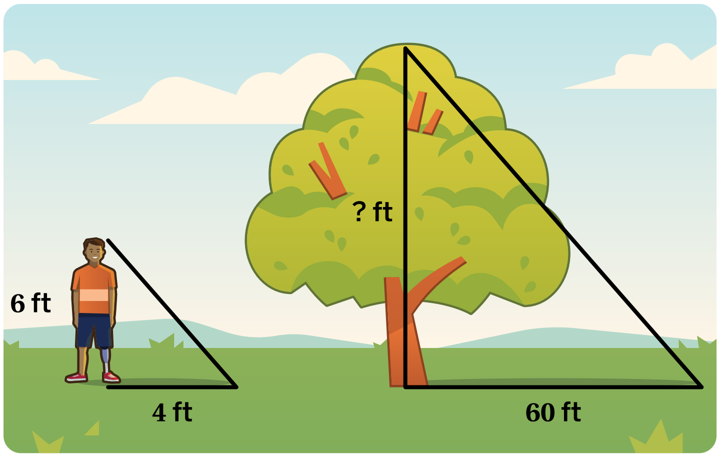
5. On a sunny day, a 6-foot-tall person has a shadow with a length of 4 feet. At the same time, a nearby tree has a shadow 60 feet long. What is the height of the tree?

A. 80 feet

B. 90 feet

C. 120 feet

D. 180 feet

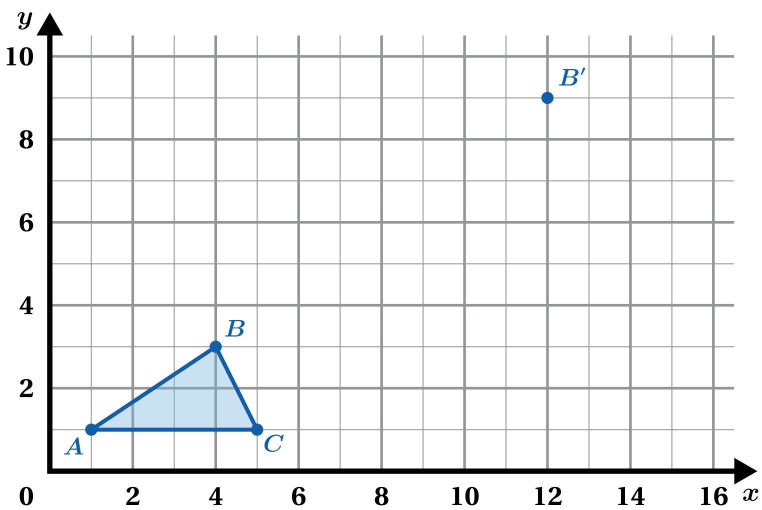


Spiral Review

Problems 6–7: Triangle ABC is dilated with the origin as the center of rotation, moving point B onto point B'.

6. What is the scale factor of the dilation?

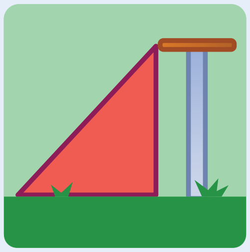
7. Draw and label triangle $A'B'C'$.



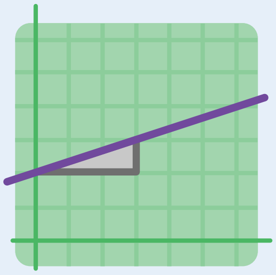
Sub-unit

4

Slope



Lesson 15
Water Slide



Lesson 16
Slope Challenges

Unit 2
Lesson
15

Name: .... Date: .... Period: ....

MA.8.AR.3.2, MA.8.GR.2.4, MTR.2.1, MTR.4.1, MTR.5.1

Water Slide

Let's look at similar triangles and lines.

Warm-Up

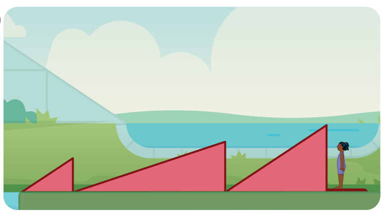
1. Use the Straight Water Slide Activity Sheet to create a variety of slides.

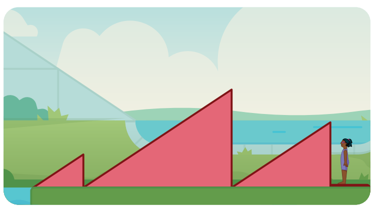
a. Which three triangles can you use to create a straight slide?

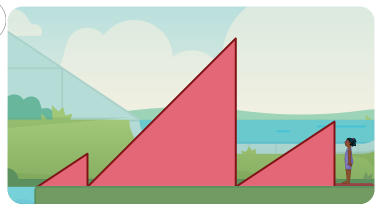
b. **Discuss:** What makes a smooth slide? What makes a bumpy slide?

2. Your goal is to create a smooth slide.

Which set of ramps do you think would make a smooth slide?

A.


B.


C.


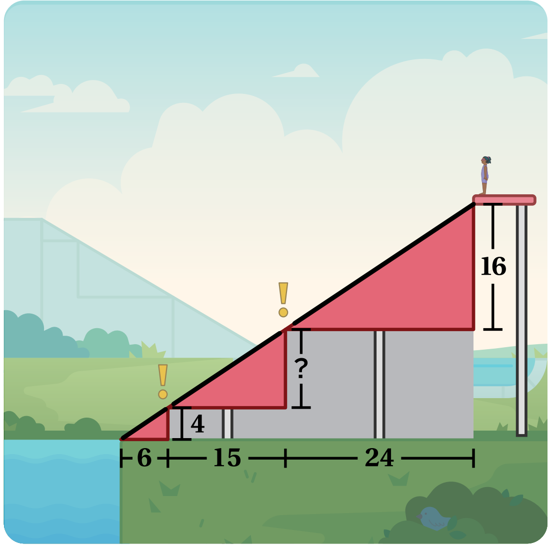
Name: ______ Date: ______ Period: ______

Smooth Slides

3. These ramps will make a bumpy slide!

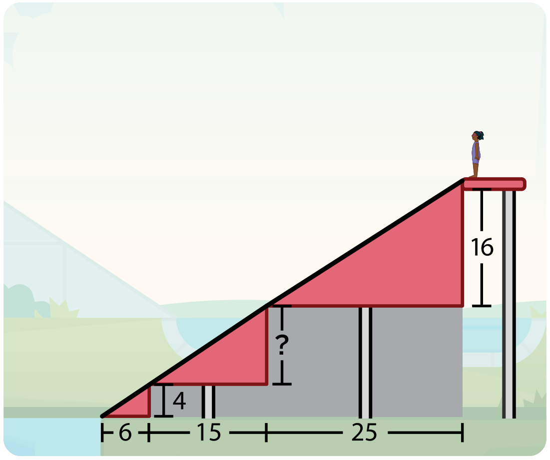
Fill in the height for Ramp 2 to make a smooth slide.

Ramp	Base (ft)	Height (ft)
Ramp 1	6	4
Ramp 2	15	
Ramp 3	24	16



4. Jada says: *The ramps are all similar triangles*.

How can she use the properties of similar triangles to find the height of the middle ramp?

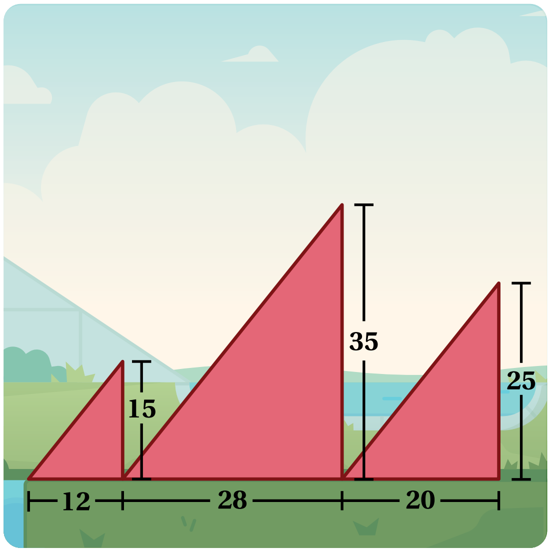


5. Here is a new set of ramps.

Will these ramps make a smooth slide? Circle one.

Yes No I'm not sure

Explain your thinking.



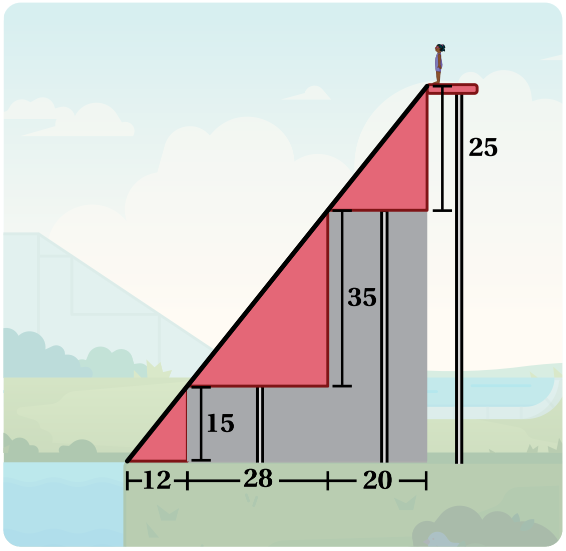
Name: ______ Date: ______ Period: ______

Introducing Slope

6. Slope measures the steepness of a line.

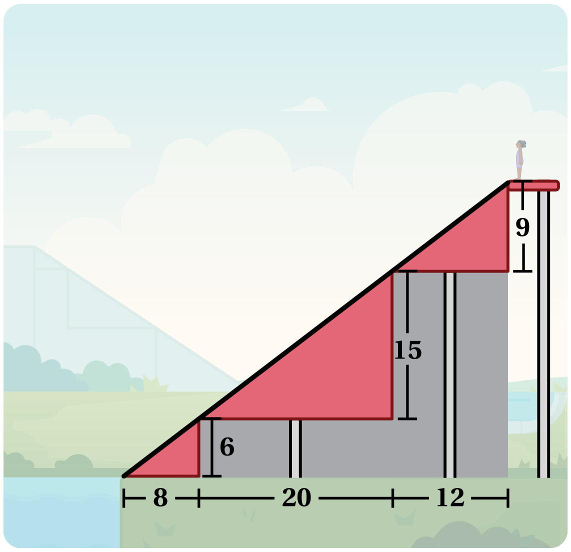
This slide forms a line with a slope of $\frac{5}{4}$.

How do you think slope is calculated?



7. What is the slope of this slide?

Explain your thinking.



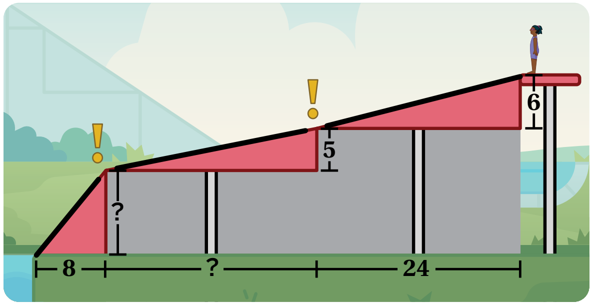
Name: ______ Date: ______ Period: ______

Introducing Slope (continued)

8. These ramps will make a bumpy slide!

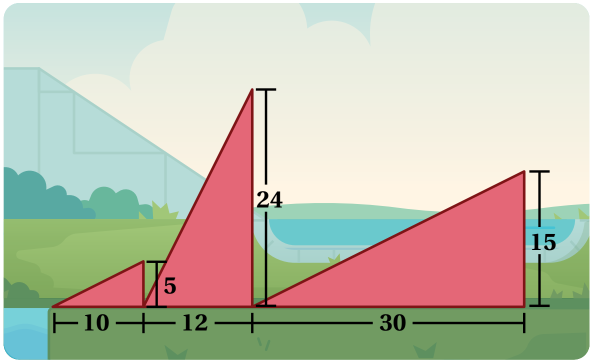
Fill in the missing values so that the slide has a slope of $\frac{1}{4}$.

Ramp	Base (ft)	Height (ft)
Ramp 1	8	
Ramp 2		5
Ramp 3	24	6



9. Habib tried to create a slide with a slope of $\frac{1}{2}$.

Ramp	Base (ft)	Height (ft)
Ramp 1	10	5
Ramp 2	12	24
Ramp 3	30	15



These ramps didn't create a smooth slide!

a **Discuss:** What do you think Habib did well?

b Describe what the mistake might be in Habib's work.

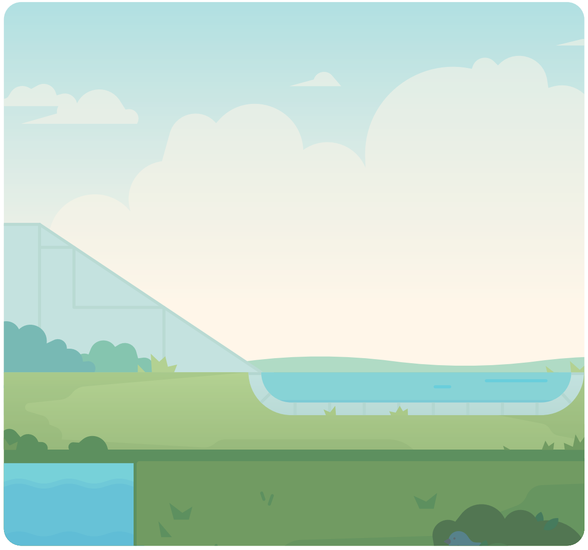
Activity 3

Name: ________ Date: ________ Period: ________

Your Water Slide

10. Draw a triangle to create a slide that will be fun, but not too scary.

Decide and write the slope of your slide.

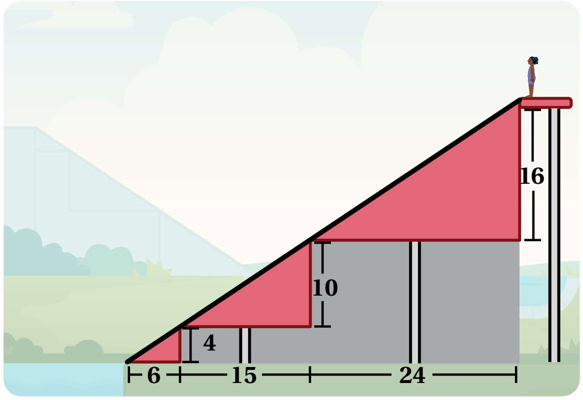


11. Now create three possible ramps for a smooth ride using the slope from the previous question.

Ramp	Base (ft)	Height (ft)
Ramp 1		
Ramp 2		
Ramp 3		

Synthesis

12. Define *slope* in your own words and describe how to calculate it.



Lesson Practice ACC7.2.15

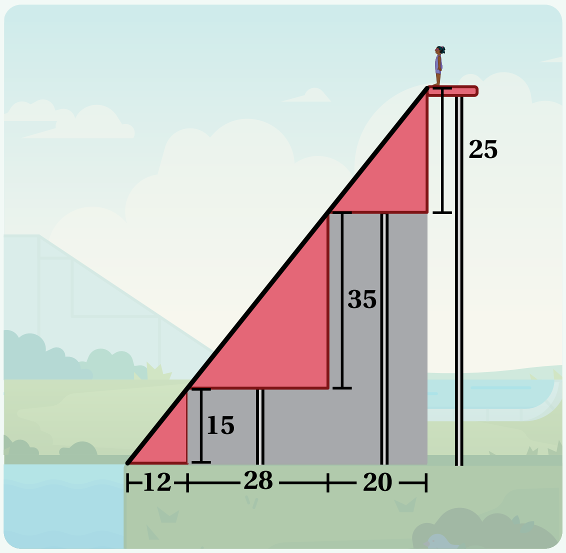
Lesson Summary

Here are three ramps made up of similar triangles with proportional corresponding side lengths. Because of this, they line up perfectly to create a smooth slide.

To line up the longest side of each triangle, the **slope** of each triangle must be the same. Slope is the height-to-base *ratio* of a triangle, which describes the steepness of its longest side.

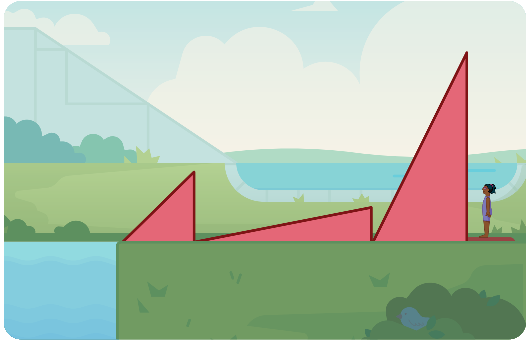
For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.

You could sketch infinite triangles on the same line that all have the same height-to-base ratio. Any of those triangles can be used to determine the slope.



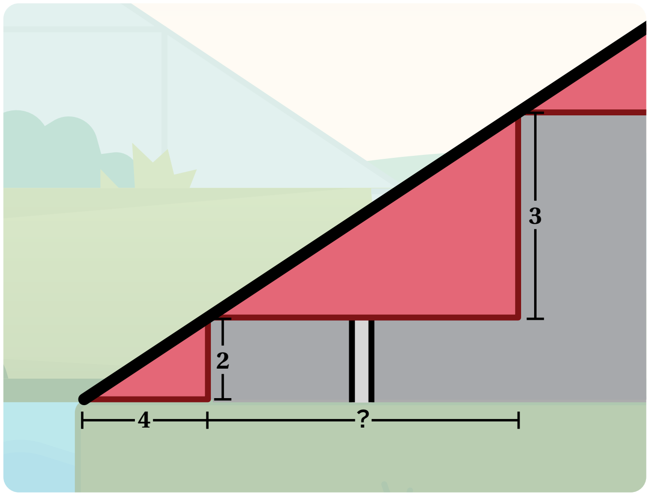
Name: ________ Date: ________ Period: ________

1. Here are three ramps. One ramp has a slope of 1, one ramp has a slope of 2, and one ramp has a slope of $\frac{1}{5}$. Label each ramp with its slope.



2. Jaylin created a slide with a slope of $\frac{1}{3}$. LaShawn created a slide with a slope of $\frac{1}{2}$. Whose slide was steeper? Explain your thinking.

Problems 3–5: Here are three ramps that make a smooth slide.



3. How long is the base of the second ramp?

4. Explain how you know the triangles that form the ramps are similar.

5. Which statement about the slopes of the ramps is true?

A. The slope of the first ramp is $\frac{2}{3}$ the slope of the second ramp because the height of the first ramp is $\frac{2}{3}$ the length of the second ramp.

B. The slope of the first ramp is the same as the slope of the second ramp because their triangles are similar.

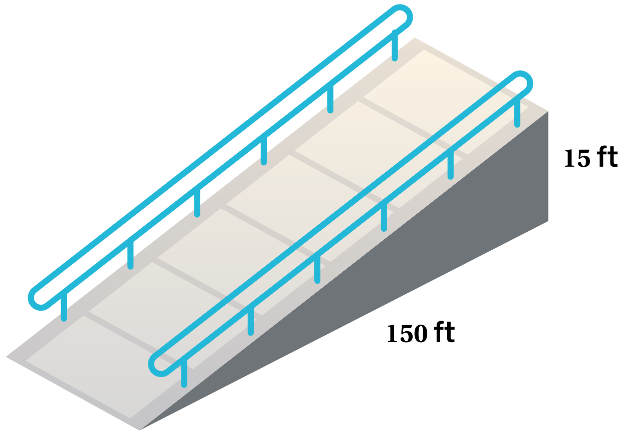
C. The slope of the first ramp is $\frac{4}{9}$ the slope of the second ramp because the area of the first ramp triangle is $\frac{4}{9}$ the area of the second ramp triangle.

D. The slope of the second ramp is 2 more than the slope of the first ramp because the difference between the bases of the ramps is 2.

Name: .... Date: .... Period: ....

FAST Practice

6. A builder is constructing a ramp for one of the entrances at the local library. He is checking the diagram before he starts construction. One requirement is that public wheelchair ramps must have a maximum slope of $\frac{1}{12}$.

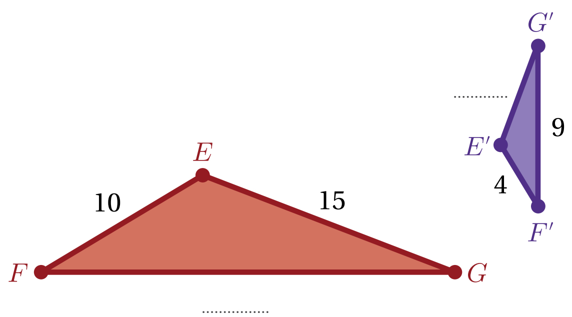


Does this ramp fit the requirement? Select ONE correct answer in each box to complete the sentences.

[**A.** Yes. **B.** No.] The slope of the line is [**A.** $\frac{1}{9}$ **B.** $\frac{1}{10}$ **C.** $\frac{1}{15}$], which is [**A.** greater **B.** less] than $\frac{1}{12}$.

Spiral Review

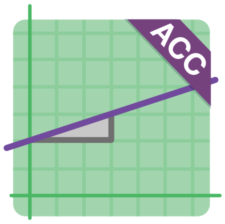
7. Triangle EFG is similar to triangle $E'F'G'$. Determine the missing values and explain your thinking.



Name: ________ Date: ________ Period: ________

MA.8.AR.3.2, MTR.3.1, MTR.4.1

Slope Challenges

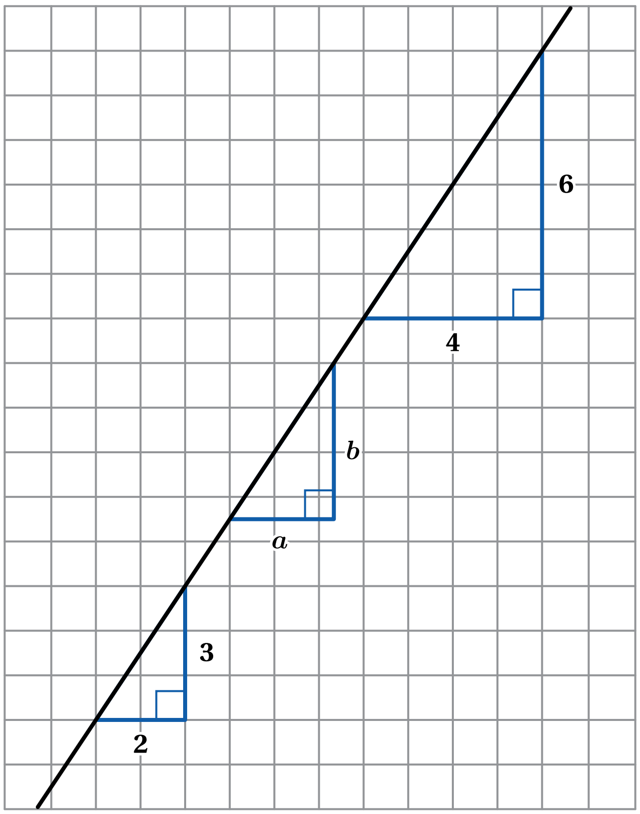


Let's figure out the slopes of lines.

Warm-Up

1. Here are three **slope triangles**.

What do you notice? What do you wonder?



I notice:

I wonder:

Activity
1

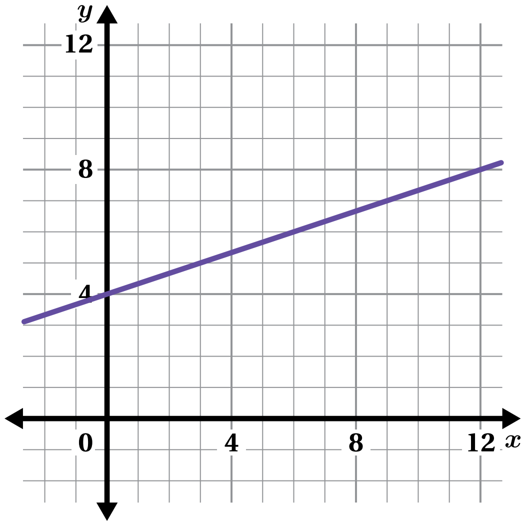
Name: ________ Date: ________ Period: ________

Determining Slope

2. Here is a line.

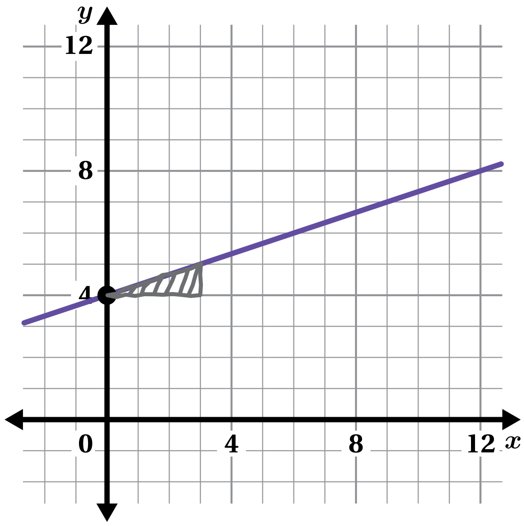
a Draw at least two *slope triangles* along the line.

b What is the slope of the line?



3. Liam says the slope of this line is 3.

What would you say to Liam to help him understand that the slope is $\frac{1}{3}$?

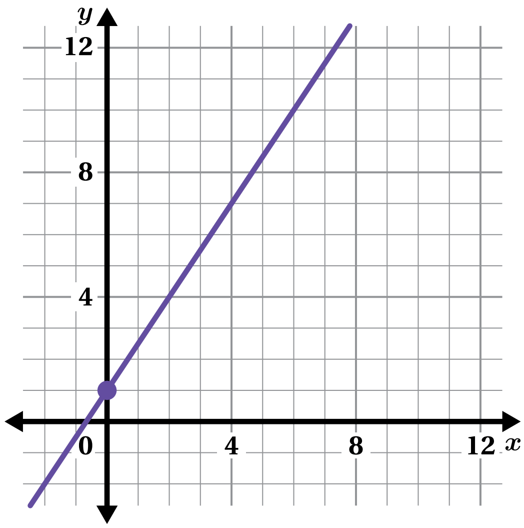


Activity 1

Name: .... Date: .... Period: ....

Determining Slope (continued)

4. What is the slope of this line?



5. Kweku says the slope of the line in the previous problem is $\frac{6}{4}$.

Liam says the slope is $\frac{2}{3}$.

Whose thinking is correct? Circle one.

Kweku's | Liam's | Both | Neither

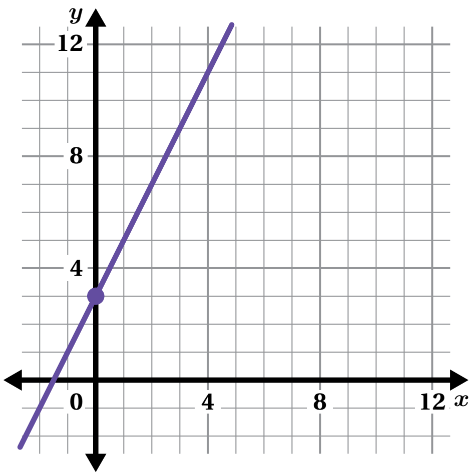
Explain your thinking.

Name: ________________ Date: ________ Period: ________

Repeated Challenges

6. What is the slope of each line? Solve as many challenges as you have time for. It's more important to make sense of the challenges than it is to work quickly!

a



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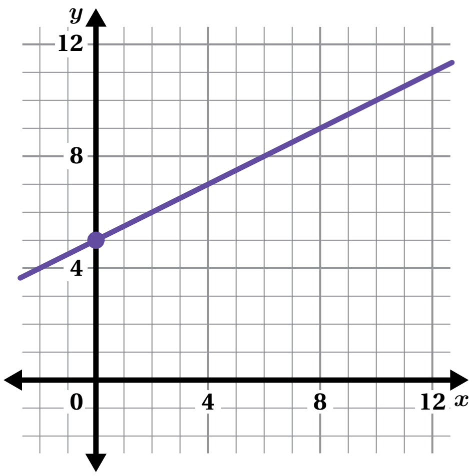

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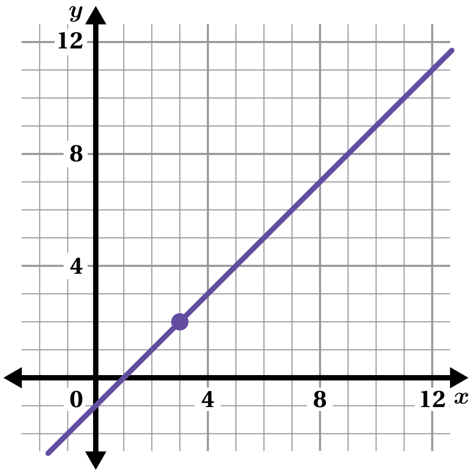

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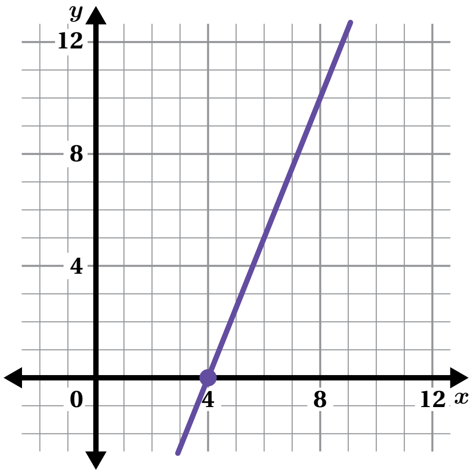

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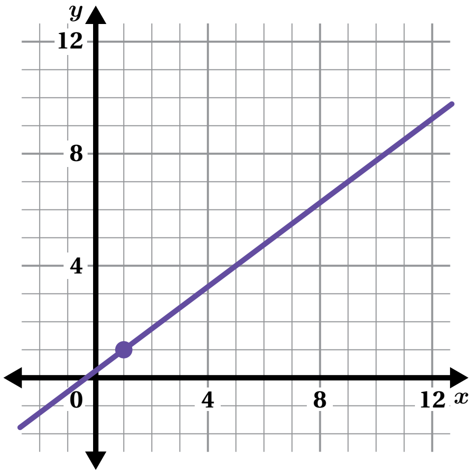

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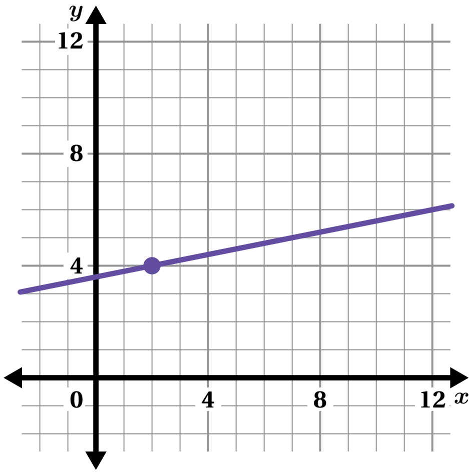
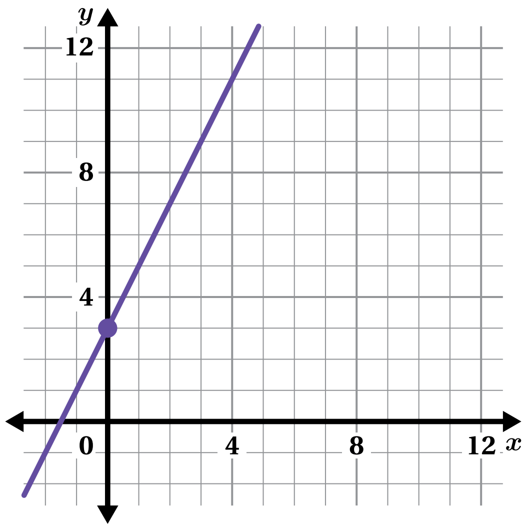


Synthesis

7. Describe a strategy for determining the slope of any line.

   Use the example if it helps you with your thinking.



Lesson Practice ACC7.2.16

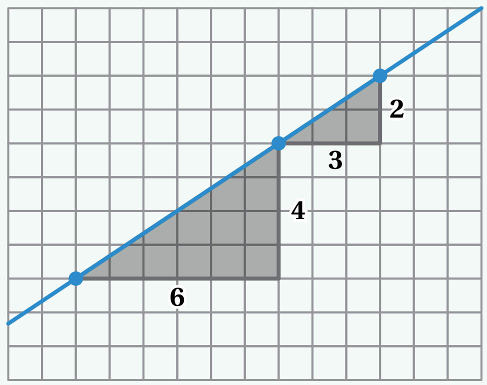
Lesson Summary

You can determine the slope of a line by drawing similar right triangles, called **slope triangles**, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height of a slope triangle to its base.

Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.

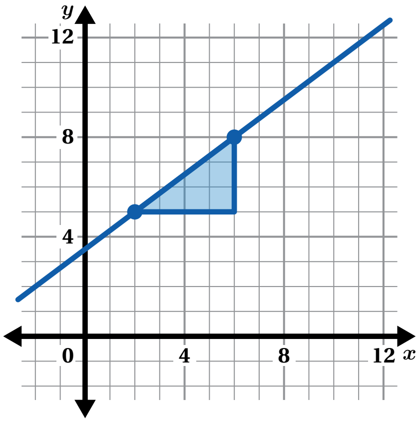


Lesson Practice
ACC7.2.16

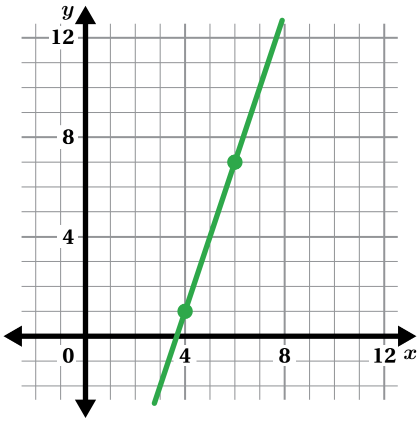
Name: .... Date: .... Period: ....

Problems 1–4: Determine the slope of each line.

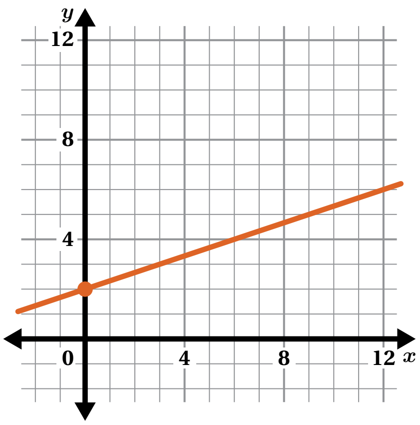
1. Slope: ....



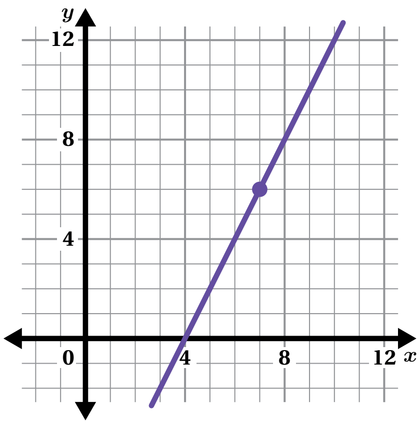
2. Slope: ....



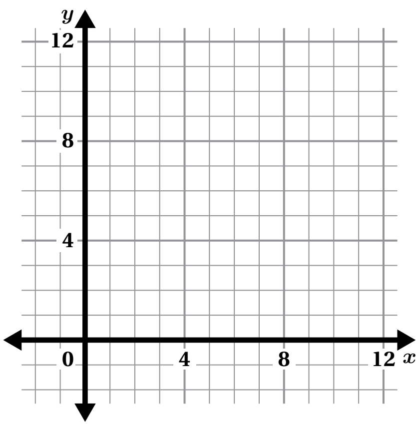
3. Slope: ....



4. Slope: ....

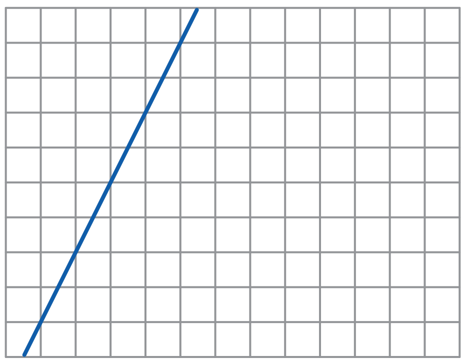


5. Draw a line with a slope of $\frac{2}{3}$.



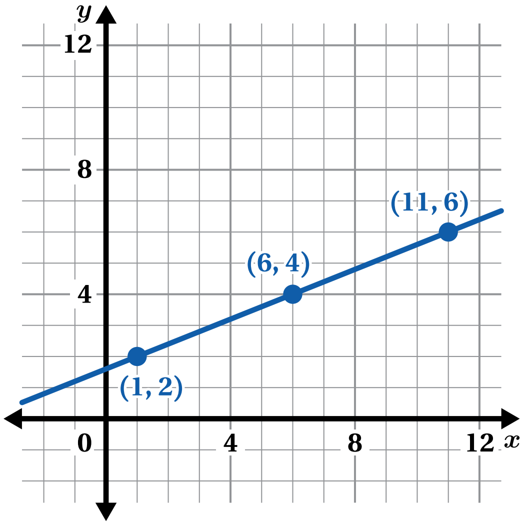
6. Here is a line. Draw a line that is parallel. What is the slope of each line?

Explain your thinking.



Name: .... Date: .... Period: ....

7. Fatima says the slope of this line is $\frac{2}{5}$. Daniela says the slope of this line is 2.5. Whose claim is correct? Explain your thinking.



FAST Practice

8. Select *all* the expressions that could represent the slope of this line.

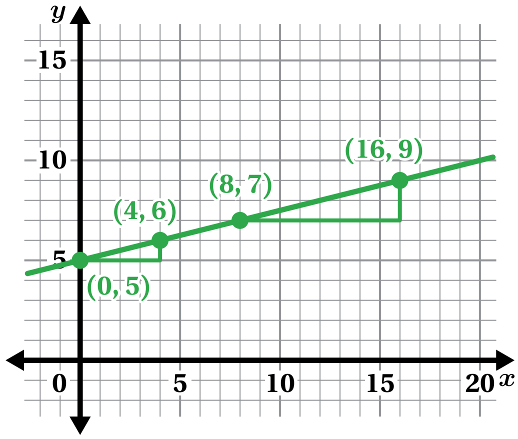
☐ A. $\frac{1}{4}$

☐ B. $\frac{4}{6}$

☐ C. 4

☐ D. $\frac{2}{8}$

☐ E. $\frac{8}{2}$



Spiral Review

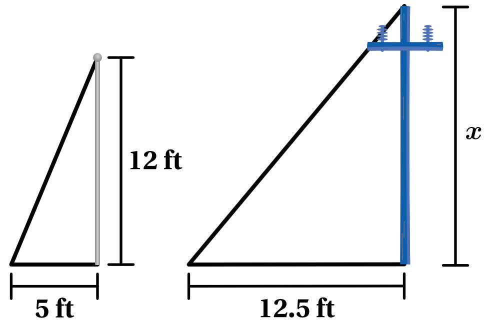
9. A 12-foot flagpole casts a shadow that is 5 feet long. At the same time, a nearby telephone pole casts a shadow that is 12.5 feet long. What is the height of the telephone pole, x?

A. 5.2 feet

B. 15 feet

C. 18.5 feet

D. 30 feet



Career Connection

When an online map is zoomed in or out, what math is at work?

Dilations and similarity are among the key players! For different zoom levels, street lengths on a map remain proportional to their real life counterparts, related by a scale factor. Topographic maps, which also show elevation, use the concept of slope to describe how land surfaces change vertically over given distances.

NicoElNino/Shutterstock.com

Cartographers collect data about different places on Earth to create and maintain print and interactive online maps. They use scale factors and coordinates to represent Earth's curved surfaces on a flat map.

B.E.S.T. Mathematics Benchmark Connection

Cartographers apply their geometry concepts extensively in their work. For example, they use dilations to control how a map displays on your laptop or other digital device (MA.8.GR.2.2, MA.8.GR.2.3) to make sure what you see is precise and usable. They also use their knowledge about angle relationships (MA.8.GR.1.4) as they study maps and design controls that let you plot a route to your favorite destination.

Mathematical Thinking and Reasoning Connection

Cartographers use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions within digital mapping systems to display your travel route in different forms, such as a map or a list of steps (MTR.2.1). Maps must be precise in order for them to be useful, so cartography teams frequently discuss the testing of map features (MTR.4.1) to make models more accurate, analyze customer feedback and error reports.

Meet Laura Escobar

Laura Escobar, from Bogotá, Colombia, is a professor of mathematics at Washington University in St. Louis, Missouri. Her research focuses on the connections between algebra and geometry. Mathematicians like Laura work with cartographers to make topographic maps.

Unit 3

Introducing Proportional Relationships

Have you ever wondered what car can drive the fastest, what recipe contains the most sugar, or how many balloons it would take to float an object? In past lessons, you've made comparisons when determining the unit rate to score the best deal, or when creating scaled copies of figures. In this unit, you'll explore proportionality even further. And you'll discover how it can be useful when making comparisons about everyday situations!

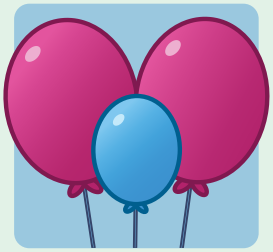
Essential Questions

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are these representations related?

Sub-unit

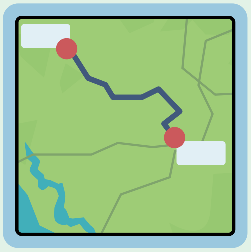
1

Proportional Relationships in Tables



Lesson 1
Balloon Float

Lesson 2
Sugar, Spice, and Everything Rice



Lesson 3
Pen Pals

Name: .... Date: .... Period: ....

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.4.1, MTR.5.1

Balloon Float

Let's explore proportional relationships in tables.

Warm-Up

1. This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

Name: .... Date: .... Period: ....

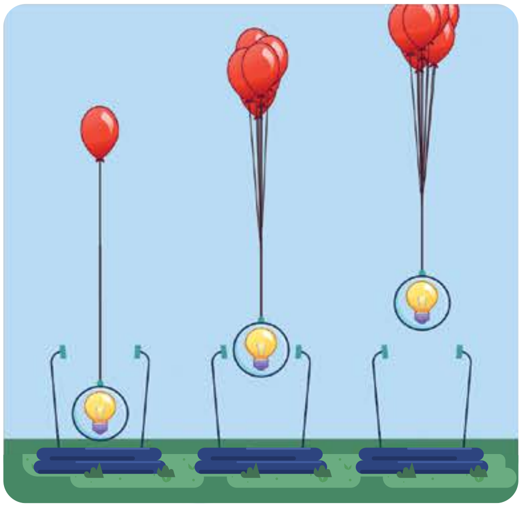
Balloon Float

Helium balloons can make objects float, but too many balloons will make objects fly away!

2. Marco tied helium balloons to three light bulbs.

- 1 balloon made the first light bulb fall to the ground.
- 6 balloons made the second light bulb float.
- 10 balloons made the third light bulb fly away.

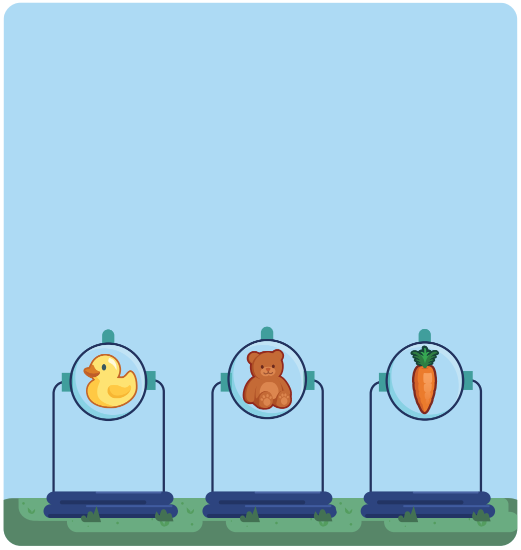
Discuss: Why do you think the number of balloons matters?



3. In the previous problem, the light bulb weighed 2 ounces and needed 6 balloons to float.

If each balloon carries the same weight, how many balloons would you need to float each object?

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	
Toy Bear	6	
Carrot	3	



Name: Date: Period:

Balloon Float (continued)

4. Here are two strategies for determining the number of balloons needed to make the rubber duck float.

Ariel

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	×5 10	30 ×5
Toy Bear	6	
Carrot	3	

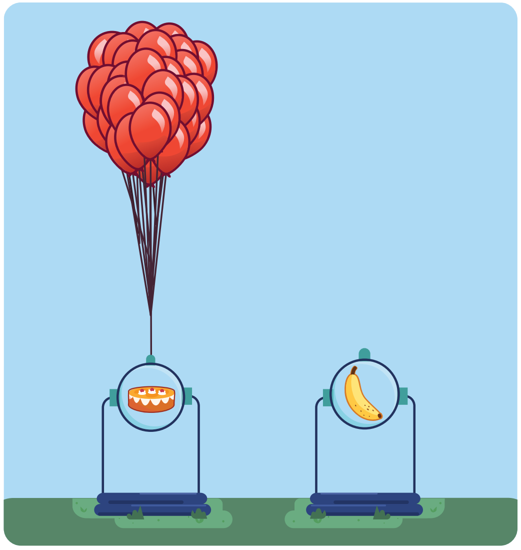
Emma

Object	Weight (oz)	Number of Balloons
Light Bulb	2 $\xrightarrow{\times 3}$	6
Rubber Duck	10 $\xrightarrow{\times 3}$	30
Toy Bear	6	
Carrot	3	

Discuss: How might Ariel and Emma use their strategies to finish their tables?

5. Here are some new objects. Complete the table so that each object floats.

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Cake		60
Banana	$3\frac{1}{3}$	



Activity 2

Name: Date: Period:

Proportional Relationships

When two quantities are always in an equivalent ratio, they have what's called a **proportional relationship**.

6. Here are two more tables.

Which of these two tables represents a proportional relationship? Circle one.

Table 1 Table 2 Both Neither

Explain your thinking.

Table 1

Weight (oz)	Number of Balloons
3	6
7	10
9	12
30	33

Table 2

Weight (oz)	Number of Balloons
4	12
6	18
42	126
8	24

7. Sort the tables into two groups based on whether they represent proportional relationships.

Table A

x	y
0	0
4	5
8	10
12	15

Table B

x	y
0	0
2	4
4	16
6	36

Table C

x	y
0	2
3	5
6	8
9	11

Table D

x	1	2	3	4
y	10	8	6	4

Table E

x	2	8	1	20
y	5	20	2.5	50

Proportional Relationship	Not a Proportional Relationship

Activity 2

Name: .... Date: .... Period: ....

Proportional Relationships (continued)

8. How did you decide whether this table represents a proportional relationship?

Proportional Relationship	Not a Proportional Relationship

x	y
0	0
2	4
4	16
6	36

9. Select *all* the relationships you think are proportional.

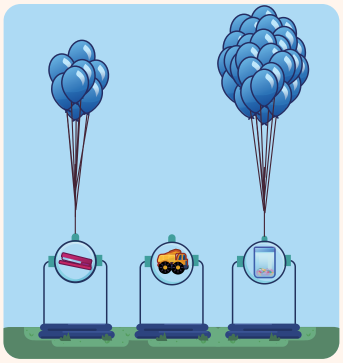
- ☐ **A.** A person’s height in feet and her height in inches
- ☐ **B.** The number of cookies baked and the number of minutes they were in the oven
- ☐ **C.** The amount of bread baked and the number of grams of flour needed to bake it
- ☐ **D.** A person’s time as he runs a marathon and his total distance covered
- ☐ **E.** The gallons of gasoline purchased and their total cost

You’re invited to explore more.

10. Blue balloons are different from red balloons. 8 blue balloons can float a 10-ounce stapler.

Complete the table so that each object floats.

Object	Weight (oz)	Blue Balloons
Stapler	10	8
Toy Truck	15	
Jelly Beans		28



Synthesis

11. Here are some relationships, some of which are proportional and some of which are not.

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

- A person's height in feet and her height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as he run a marathon and his total distance covered
- The gallons of gasoline purchased and their total cost

Lesson Practice ACC7.3.01

Lesson Summary

A **proportional relationship** is a set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by 8, the cost for them is multiplied by the same number.

Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Name: ______ Date: ______ Period: ______

Problems 1–4: Complete the tables so that the relationship is proportional.

1.

x	y
30	3
120	
	10

2.

x	y
1	1.5
3	
	12

3.

x	y
15	45
1	
	0

4.

x	y
0.2	1
1	
	20

5. Entrance to a state park costs $6 per vehicle, plus $2 per person. The table shows the entry cost for several recent groups of visitors. Is the relationship between the number of people and the total entrance cost a proportional relationship?

Explain your thinking.

Number of People in Vehicle	Total Cost ($)
2	10
3	12
4	14
10	26

6. A store charges $4.80 for 16 ounces of bubble tea. Complete the table so that it shows a proportional relationship between ounces of tea and cost.

Tea (oz)	Cost ($)
16	4.80
20	
	7.20

FAST Practice

7. A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Complete the table so that it shows a proportional relationship between the amount of honey and the amount of flour.

Honey (tbsp)	Flour (cups)
8	10
20	
13	
	12.5

Spiral Review

Problems 8–10: Solve each equation. Show your thinking.

8. $\frac{1}{2} + x = 2$

9. $\frac{2}{3}y = 6$

10. $3 = \frac{1}{4}b$

11. A student makes a scaled copy of a rectangle. The dimensions of the original rectangle are 3.5 by 2.5 meters. Select *all* the possible dimensions of the scaled copy.

- ☐ **A.** 1.75 meters and 1.25 meters
- ☐ **B.** 7 meters and 5 meters
- ☐ **C.** 7 meters and 6 meters
- ☐ **D.** 10 meters and 2.5 meters
- ☐ **E.** 10.5 meters and 7.5 meters

Name: ______________________ Date: __________ Period: __________

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.2.1, MTR.6.1

Sugar, Spice, and Everything Rice

Let's explore proportional relationships some more.

Warm-Up

1. This table shows the amount of sugar in different volumes of soda.

What do you notice? What do you wonder?

Soda (fl. oz)	Sugar (g)
1	3
7.5	22.5
12	36
20	60

I notice:

I wonder:

Activity 1

Name: ______ Date: ______ Period: ______

Sugary Drinks

A 12-ounce bottle of orange juice contains 33 grams of sugar.

2. Explain why the relationship between the volume of orange juice and the amount of sugar is proportional.

3. Complete the table to determine the amount of sugar in different-sized containers of orange juice.

	Orange Juice (fl. oz)	Sugar (g)
Glass	8	
Bottle	12	33
Carton	32	
Jug	128	

When two quantities have a *proportional relationship*, you can multiply the value of one by a **constant of proportionality** to get the value of the other.

4. What is a constant of proportionality in the relationship between volume of orange juice and amount of sugar? What does it tell us about this situation?

5. The label on an orange juice box claims that it contains only 11 grams of sugar. How many fluid ounces must be in the orange juice box? Show or explain your thinking.

6. A 25-ounce bottle of apple cider contains 31 grams of sugar. Which drink is more sugary: apple cider or orange juice? Explain how you know.

Activity 2

Name: .... Date: .... Period: ....

Rice and Spice

Here are the instructions for cooking instant rice in the microwave.

Ingredients

Rice (cups)	Water (cups)
1	$1\frac{1}{2}$
2	3
3	$4\frac{1}{2}$

Cook Time

Rice (cups)	Time (min)
1	7
2	11
3	15

7. Which relationships are proportional (if any)?

8. For any proportional relationships, determine a constant of proportionality and explain what it means.

When making graham crackers, the relationship between the amount of whole wheat flour and the amount of cinnamon is proportional.

9. Complete the table.

Whole Wheat Flour (cups)	2	4	5	
Cinnamon (tsp)	0.75	1.5		3.75

10. What is a constant of proportionality in this relationship?

11. Instead of following this recipe, Xavier decides to use 1 teaspoon of cinnamon for every 2 cups of whole wheat flour. How will that impact the taste of the graham crackers?

Synthesis

12. What does *constant of proportionality* mean?

How can you use a constant of proportionality to make sense of a proportional relationship?

Use the example if it helps with your thinking.

Corn (cups)	Protein (g)
1	5.1
3	15.3
5	25.5

Lesson Practice ACC7.3.02

Lesson Summary

In a proportional relationship, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

That means that you can multiply the number of feet by 12 to determine the number of inches.

Feet	1	2	6
Inches	12	24	72

Lesson Practice
ACC7.3.02

Name: ____________ Date: ________ Period: ________

Problems 1–2: When Deven makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. This table shows how to make batches of different sizes.

Milk (cups)	Chocolate Syrup (tbsp)
2	3
8	12
1	$\frac{3}{2}$
10	15

1. Is there a proportional relationship between cups of milk and tablespoons of chocolate syrup?

Show or explain your thinking.

2. What is a constant of proportionality for this relationship?

Problems 3–4: When you mix two colors of paint in equivalent ratios, the color you get is always the same.

3. Complete the table so that each row makes the same color purple.

Blue (cups)	Red (cups)
2	6
1	

4. What is a constant of proportionality for this relationship?

Problems 5–7: Here is some information about the side lengths of two scaled copies, triangle A and triangle B.

Side Length of Triangle A (in.)	Side Length of Triangle B (in.)
1	6
$\frac{1}{2}$	
	8

5. Complete the table to determine the missing side lengths of each triangle.

6. What is a constant of proportionality in this relationship?

7. What does that constant of proportionality tell you about the triangles?

Name: .... Date: .... Period: ....

8. Table A shows different amounts of blue and white paint that can be mixed together to create a color called "Blue Breeze." Sai wants to create a new color that is *bluer* than "Blue Breeze" and call it "Sea Glass Blue."

a Complete the first row of Table B to create a possible mixture for "Sea Glass Blue."

b Complete the rest of Table B so that the relationship between cups of blue and cups of white is proportional.

Table A
Blue Breeze

Blue (cups)	White (cups)
1	3
2	6
5	15
8	24

Table B
Sea Glass Blue

Blue (cups)	White (cups)
4	
7	
	1
12	

FAST Practice

9. Sarah orders a small poster that is 100 square inches and costs \$5.00. She then orders a large poster that is 500 square inches and costs \$25.00. What does the constant of proportionality tell you about the costs of the posters?

The cost is \$ ☐ for each square inch of poster.

Spiral Review

10. Select *all* the scales that are equivalent to the scale 1 cm to 5 m.

- ☐ **A.** $\frac{1}{2}$ cm to $2\frac{1}{2}$ m
- ☐ **B.** 1 cm to 5 km
- ☐ **C.** 5 cm to 1 km
- ☐ **D.** 5 cm to 25 m
- ☐ **E.** 10 cm to 50 m

11. Select *all* of the ratios that are equivalent to 4 : 7.

- ☐ **A.** 8 : 15
- ☐ **B.** 16 : 28
- ☐ **C.** 7 : 4
- ☐ **D.** 20 : 35
- ☐ **E.** 6 : 9

Name: ________________ Date: ________ Period: ________

MA.7.AR.3.3, MTR.2.1, MTR.4.1

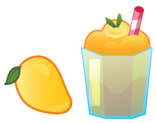
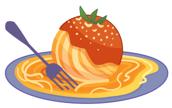
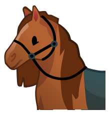
Pen Pals

Let's compare measurements in different systems.

Warm-Up

1. Four pen pals share letters with each other.

Discuss: What do you notice? What do you wonder?

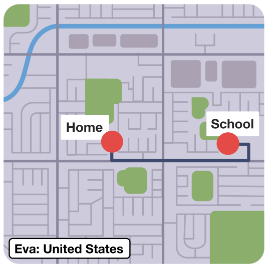
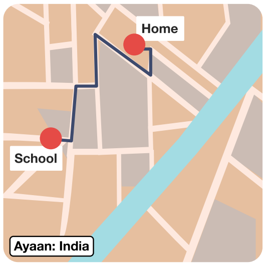
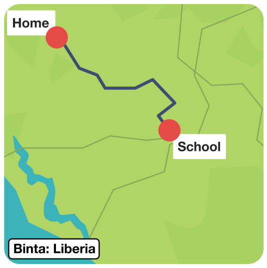
Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food	 Bubble tea	 Mango lassi	 Quindim	 Spaghetti
Favorite Animal	 Horse	 Dog	 Horse	 Bird
Favorite Sport	 Football	 Cricket	 Futebol	 Football

Activity **1**

Name: .... Date: .... Period: ....

Traveling to School

2. The pen pals discuss how far they each live from school. Use your best estimates to order the pen pals from *closest* to *farthest* from school.

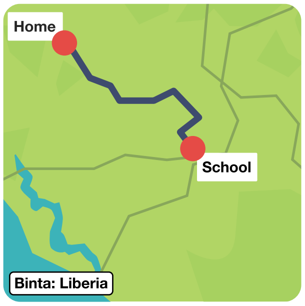
Thiago: Brazil	Eva: United States	Ayaan: India	Binta: Liberia
			
20 kilometers	2,000 feet	900 meters	15 miles

Closest **Farthest**

3. Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school? Circle one and explain your thinking.

Binta Thiago About the same distance

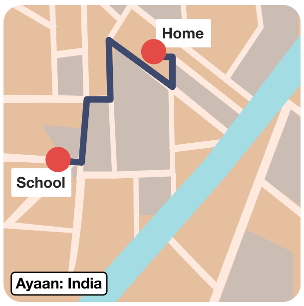


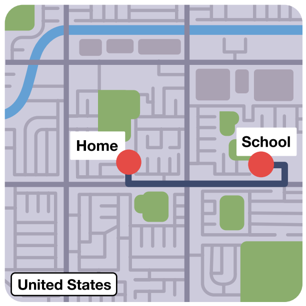
1 kilometer $\approx$ 0.62 miles

4. Ayaan lives 900 meters from his school in India. Eva lives 2,000 feet from her school in the United States.

Who lives closer to their school? Circle one and explain your thinking.

Ayaan Eva About the same distance





1 foot $\approx$ 0.305 meters

Activity 2

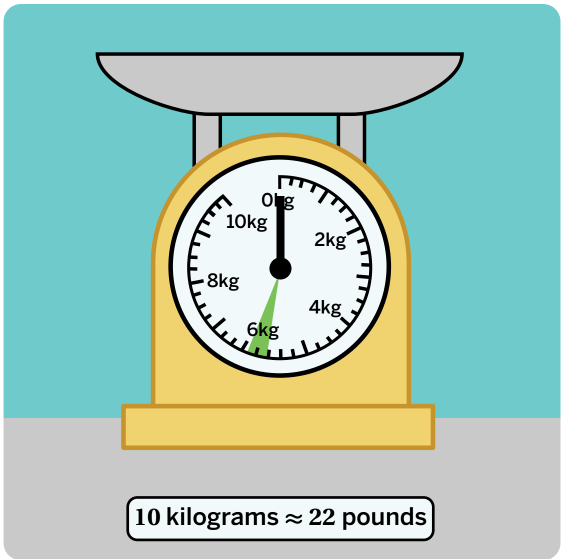
Name: .... Date: .... Period: ....

Weighing Strategies

5. Thiago's horse eats about 6 kilograms of hay per day.

Eva wants to know how many pounds that is.

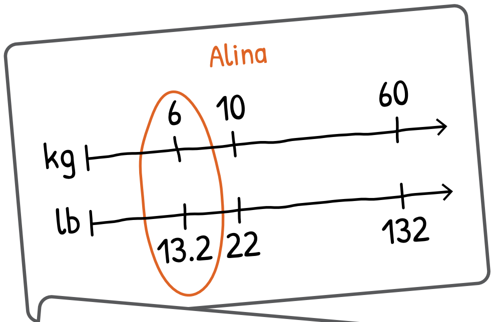
About how many pounds is 6 kilograms?

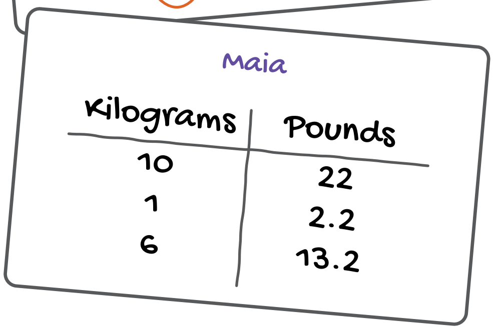


6. Alina and Maia both determined how many pounds of hay Thiago's horse eats per day.

Alina used a double number line and Maia used a table.

Choose a student and explain their thinking.





Kilograms	Pounds
10	22
1	2.2
6	13.2

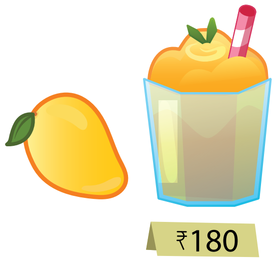
Activity 3

Name: .... Date: .... Period: ....

Favorite Things

7. Aayan can buy 1 liter of mango lassi for 180 Indian rupees (INR). Eva wants to know how much that is in U.S. dollars (USD).

About how many U.S. dollars is 180 Indian rupees?



1 USD = 84.40 INR

8. Thaigo and Binta have the same favorite sport.

In Brazil, Thaigo can buy a ball for 70 Brazilian reais (BRL), but in Liberia, Binta would pay about 3,000 Liberian dollars (LRD) for a similar ball.

Who pays more for their ball? Explain your thinking.

1 USD = 5.80 BRL

1 USD = 189.52 LRD

You're invited to explore more.

9. People have known for over 2,000 years that Earth is round, but it took a long time to discover how big it is.

A Greek mathematician named Eratosthenes was the first known person to calculate the distance around Earth's equator. In about 240 BCE, he calculated the distance around Earth's equator to be about 250,000 stadia using an estimated distance from Alexandria to Syene, along with the lengths of shadows.

The actual distance is about 24,901 miles.

What is the difference, in miles, between Eratosthenes's calculation and the actual distance? Explain your thinking.

Synthesis

10. Describe a strategy for converting a measurement from ovne unit to another.

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

30 milliliters ≈ 2 tablespoons

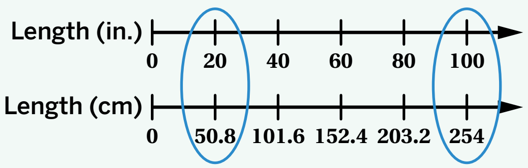
Lesson Practice ACC7.3.03

Lesson Summary

You can use equivalent ratios to convert between different measurement systems.

For example, if you know that 100 inches = 254 centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line



Ratio Table

	Inches	Centimeters	
	100	254	
÷ 100	1	2.54	÷ 100
× 20	20	50.8	× 20

Once you know the conversion ratio per 1 unit, you can use multiplication or division to convert between the units. Here are two ways you can use the conversion ratio 1 inch = 2.54 centimeters.

12 inches = ... centimeters

$12 \times 2.54 = 30.48$

So 12 inches = 30.48 centimeters.

76.2 centimeters = ... inches

$7.62 \div 2.54 = 30$

So 76.2 centimeters = 30 inches.

Lesson Practice
ACC7.3.03

Name: .... Date: .... Period: ....

1. Malik is 57 inches tall. If 1 inch = 2.54 centimeters, which value is closest to his height in centimeters?

A. 22.4 centimeters

B. 57 centimeters

C. 144.8 centimeters

D. 3,551 centimeters

Problems 2–3: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show or explain your thinking.

1 kilogram = 1000 grams	3 ounces ≈ 85 grams
1 pound ≈ 0.454 kilograms	4 kilograms ≈ 141 ounces

2. 15 ounces ≈ .... grams

3. 20 kilograms ≈ .... pounds

4. Dhruv's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Dollars	Pesos
250	4,250
25	
1	
3	
	510

Lesson Practice
ACC7.3.03

Name: .... Date: .... Period: ....

FAST Practice

5. A yard is equal to 3 feet, and there are 1,760 yards in 1 mile. How many feet are there in 4 miles?

A. 3,520

B. 7,040

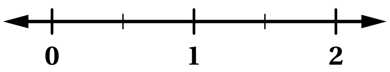
C. 5,280

D. 21,120

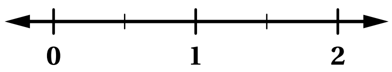
Spiral Review

Problems 6–9: Plot each value on the number line.

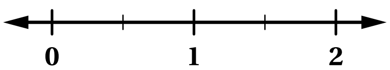
6. $\frac{2}{5}$



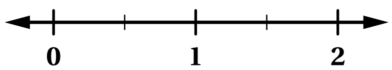
7. $1\frac{2}{5}$



8. $\frac{5}{8}$



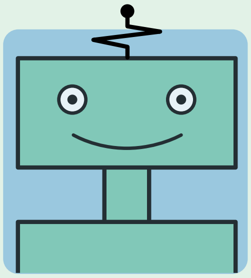
9. $\frac{12}{8}$



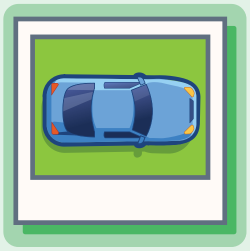
Sub-unit

2

Proportional Relationships in Equations

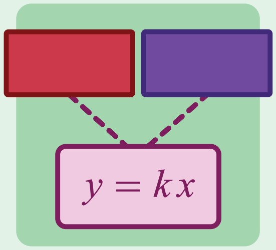


Lesson 4
Robot Factory



Lesson 5
Snapshots

Lesson 6
Two and Two



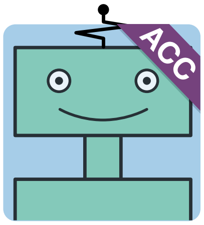
Lesson 7
All Kinds of Equations

Name: ____________ Date: ________ Period: ________

MA.7.AR.4.2, MA.7.AR.4.4, MTR.2.1, MTR.3.1

Robot Factory

Let's write equations for proportional relationships.



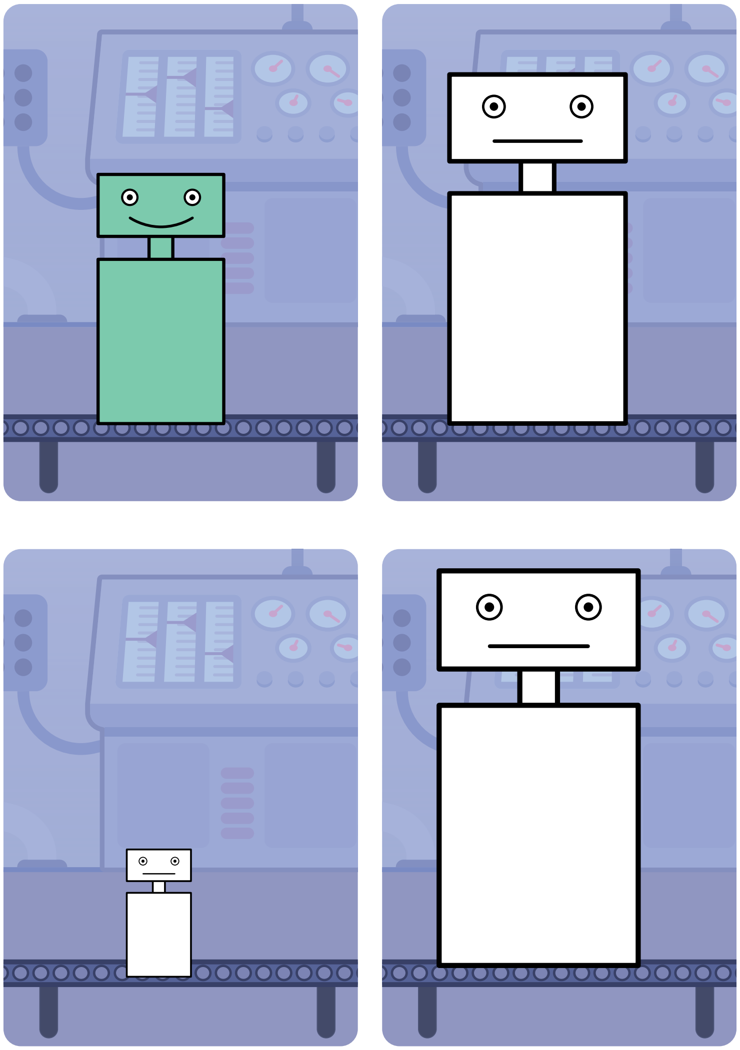
Warm-Up

1. The robot factory can make robots of all different sizes.

This robot's shade of green is made by mixing green and white paint using the amounts in the table.

Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	
1	
10	



Activity
1

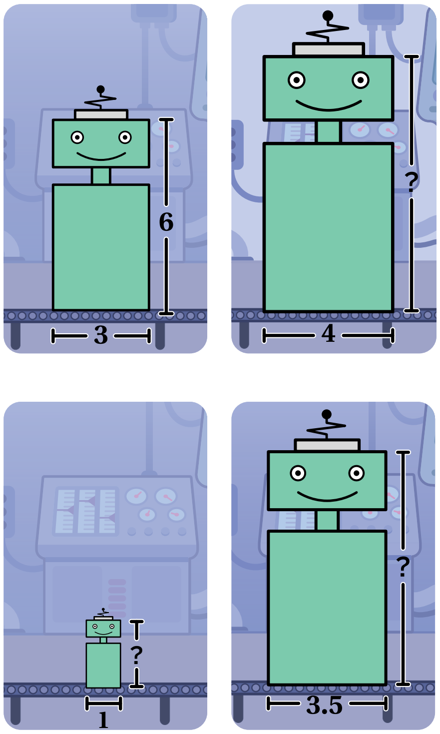
Name: ____________ Date: ________ Period: ________

Robot Hats

2. Let's make 30 *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.

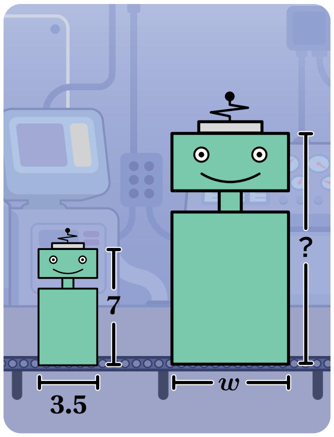


Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	
1	
3.5	
2.18	
1.76	
3.425	

3. Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
4	$4 \cdot 2$
3.5	$3.5 \cdot 2$

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?



4. Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?

A. $h = \frac{1}{2}w$ B. $h = w + 3$ C. $w = 2h$ D. $h = 2w$

Activity 2

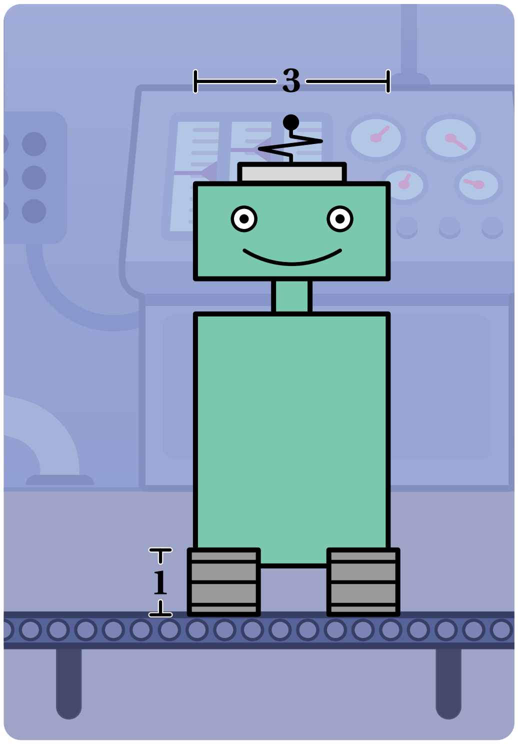
Name: .... Date: .... Period: ....

More Robot Parts

5. This robot is 3 inches wide. Its shoes are 1 inch tall.

Enter the shoe height for each robot.

Robot Width (in.), w	Shoe Height (in.), s
3	1
6	
5	
1	



6. Write an equation the factory could use to put shoes on the rest of the robots.

Use s for the shoe height and w for the robot's width.

$s =$ ....

7. This robot has a height of 9 inches. Its arms are 5 inches off the ground.

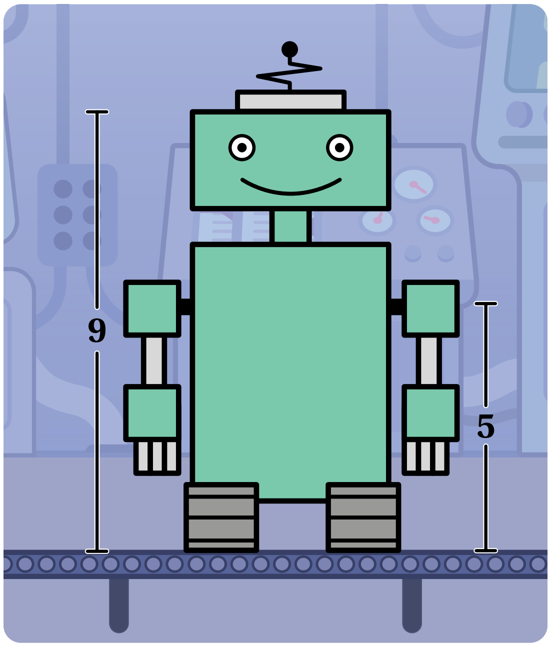
Write an equation that the factory could use to proportionally attach arms to the robots.

Use a for the height to place the arms and r for the robot height.

$a =$ ....

Use this table if it helps with your thinking.

Robot Height (in.), r	Height for Placing Arms (in.), a
9	5
5	2.78
1	0.56



Synthesis

8. Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionally is 2.

An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionally is $\frac{5}{9}$.

An equation for this relationship is $a = \frac{5}{9}r$.

Lesson Practice ACC7.3.04

Lesson Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality. For example, when you create a *scaled copy*, you multiply every length in the original figure by the same number, k.

The table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

Weight (lb), w	Cost ($), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Lesson Practice
ACC7.3.04

Name: .... Date: .... Period: ....

1. The ceilings in many basements are made up of rectangular tiles. For one basement, each square meter of ceiling requires 10.75 tiles. Complete the table.

Area of Ceiling (sq. m)	Number of Tiles
1	
10	
	53.75
x	

Problems 2–3: Each table represents a proportional relationship. Determine the constant of proportionality that completes each equation.

2.

s	P
2	8
3	12
5	20
10	40

$P =$ .... s

3.

d	C
2	6.28
3	9.42
5	15.7
10	31.4

$C =$ .... d

Problems 4–5: A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.

4. Complete the table.

Time (hr)	Distance (mi)
1	
1.5	915
2	
2.5	
t	

5. How far would the plane fly in 10 hours at this speed?

Lesson Practice
ACC7.3.04

Name: .... Date: .... Period: ....

FAST Practice

Problems 6–8: While she was traveling in 2023, Mai received 342.50 Norwegian kroner in exchange for 50 Australian dollars.

6. How many kroner would Mai have received in exchange for 1 Australian dollar?

☐ Norwegian kroner

7. Write an equation to represent the amount of kroner, k, received in exchange for a Australian dollars.

$k =$ ☐

8. Determine the number of kroner Mai would receive in exchange for 120 Australian dollars.

☐ Norwegian kroner

Spiral Review

Problems 9–10: A bicycle travels 21 meters in 3 seconds.

9. Complete the table.

Time (sec)	Distance (m)
3	21
$1\frac{1}{2}$	
	$6\frac{3}{10}$

10. What is a constant of proportionality in this relationship?

What does this represent in the situation?

Name: ______ Date: ______ Period: ______

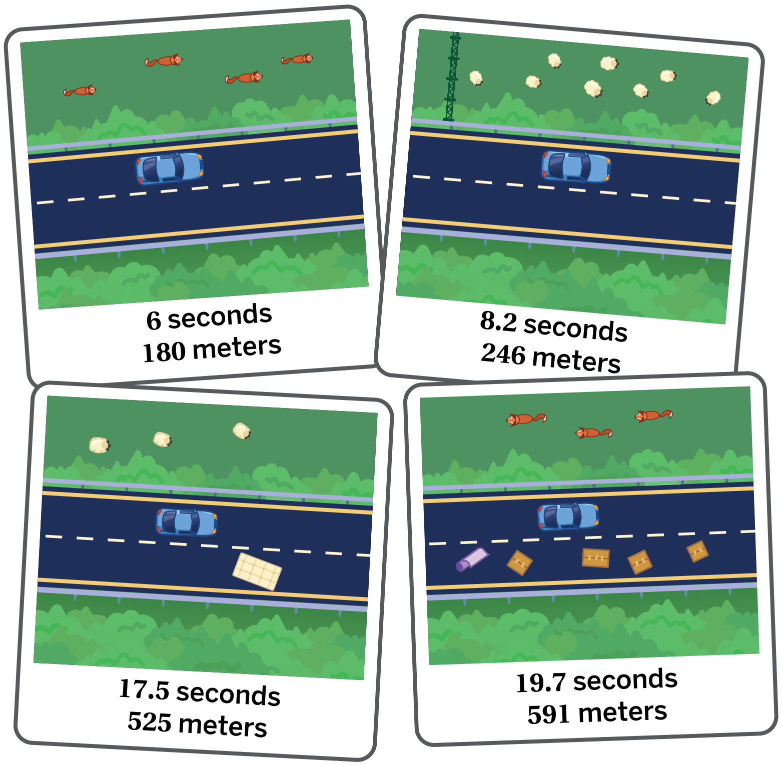
MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.7.1

Snapshots

Let's use equations to make sense of proportional relationships in the world.

Warm-Up

1. Write a story about this car's trip.

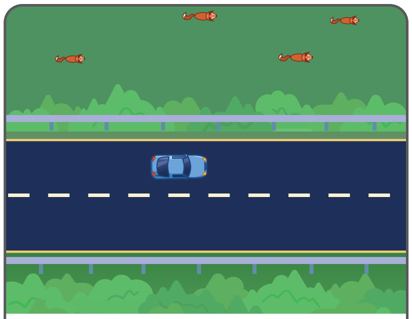


Activity 1

Name: .... Date: .... Period: ....

Travel Times

2. The car travels at a constant speed. After 6 seconds, it travels 180 meters.



a Write an equation for the car's distance, d, at any time, t.

$d =$ ....

b Enter the time and distance for three moments during the car's trip.

Time (sec), t	Distance (m), d

3. Ethan says that a row in a table is like a picture and an equation is like a video. Explain what he might be thinking.

4. Use the equation you wrote to complete the table.

Time (sec), t	Distance (m), d
6	180
3	
	60

Activity 2

Name: ______ Date: ______ Period: ______

Cakes

5. A cake recipe uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

Explain what the constant of proportionality means in this situation.

6. How many ounces of milk are needed to bake 12 cakes?

7. A cake recipe says to use 3 cups of flour for every 2 cakes.

a **Discuss:** How much flour do you need for 1 cake?

b Write an equation to calculate the amount of flour needed, f, for any number of cakes, c.

$f =$ ______

2 cakes
3 cups of flour

Activity
3

Name: .... Date: .... Period: ....

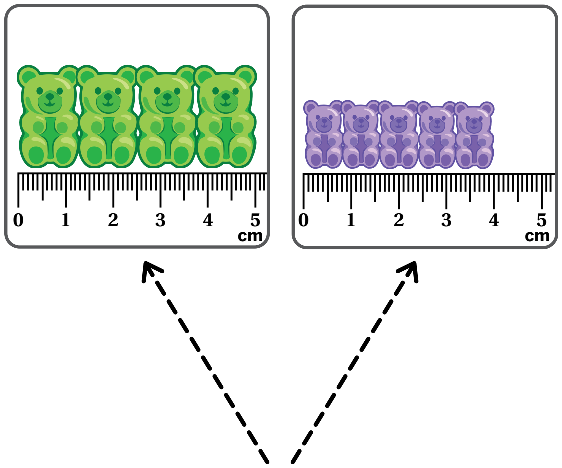
Gummy Bears

8. Here is an equation: $\ell = \frac{5}{4}g$. ℓ is the total length and g is the number of gummy bears.

Which size gummy bear does the equation represent? Circle one.

Large Small Both Neither

Explain your thinking.

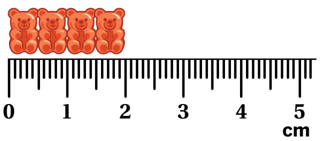


$$\ell = \frac{5}{4}g$$

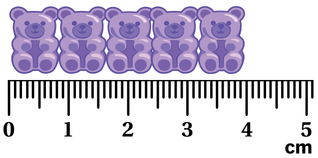
9. Match each image with one or more equations or descriptions. In each equation, ℓ is the total length and g is the number of gummy bears.

Image

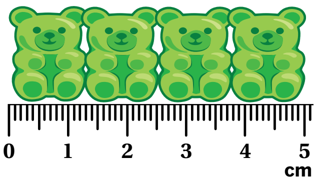
a.



b.



c.



Equation or Description

.... $\ell = \frac{4}{5}g$

.... $\ell = \frac{1}{2}g$

.... $\ell = \frac{5}{4}g$

.... A line of 80 of these gummy bears is 100 centimeters long.

You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to answer a question about a truck's trip.

Synthesis

11. Here is a snapshot from a situation represented by the equation $g = 8p$, where g represents a number of gallons and p represents a number of pints.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Lesson Practice ACC7.3.05

Lesson Summary

When a vehicle is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

The last row in the table shows that we can multiply the amount of time, t, with the constant of proportionality, 5, to determine the distance traveled, d.

The equation $d = 5t$ represents this relationship.

Time (sec)	Distance Traveled (m)	Speed (m per sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

Name: .... Date: .... Period: ....

Problems 1–3: A performer expects to sell $5{,}000$ tickets for an upcoming concert. He plans to make $\$311{,}000$ from the sales of these tickets. Assume that all tickets have the same price.

1. What is the price for one ticket? Show or explain your thinking.

2. Write an equation to represent the relationship between the number of tickets sold, x, and the total amount of money (in dollars) that he makes, y.

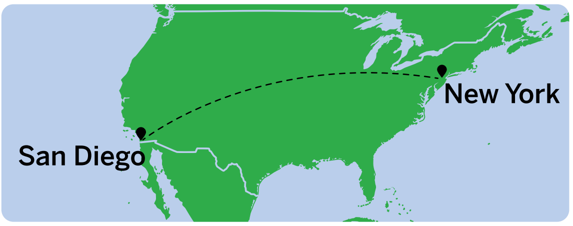
3. If he makes $\$379{,}420$, how many tickets has he sold?

Problems 4–5: A car is traveling on a highway at a constant speed. The equation that represents the distance traveled in miles, d, for t hours is $d = 65t$.

4. What does the value 65 represent in this situation?

5. At this rate, how many miles will the car travel in 1.5 hours? Show your thinking.

Problems 6–7: On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.



6. This table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Pittsburgh to Saint Louis	1	550	
Saint Louis to Albuquerque	1.7		
Albuquerque to Phoenix		330	

7. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.

Name: .... Date: .... Period: ....

8. Na'ilah is making a pitcher of a lemon-flavored sports drink. The drink mix container says to mix $\frac{1}{4}$ cups of powdered drink mix with 2 quarts of water. She prefers her sports drink to taste *more* lemon-y than the recipe on the container. Complete the equation to represent a mixture of cups of drink mix, c, and quarts of water, w, that would be *more* lemon-y than the original mixture.

$c =$ .... w

FAST Practice

9. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours. Complete the equation to represent the distance, d, that the train travels at any time, t.

$d = \square\, t$

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☐ **A.**

x	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

☐ **B.**

x	0	$\frac{1}{2}$	2	3
y	0	$\frac{1}{4}$	4	9

☐ **C.**

x	0	4	8	12
y	0	2	4	6

☐ **D.**

x	0	1	2	3
y	0	4	8	12

11. Is 4.5 a solution to the equation $1.5 + x = 6$?

Unit 3
Lesson
6

Name: ________ Date: ________ Period: ________

MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.5.1

Two and Two

Let's keep exploring constants of proportionality.

Warm-Up

1. Here are two tables. What do you notice? What do you wonder?

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

I notice:

I wonder:

Activity 1

Name: .... Date: .... Period: ....

Table Equation Match-Up

Here are the tables from the Warm-Up again.

2. Match each table to an equation that represents the situation. Some equations may not have a match.

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

Equations

$y = 100x$ ....

$y = \frac{1}{100}x$ ....

$x = 100y$ ....

$x = \frac{1}{100}y$ ....

3. Alexis thinks that the equation $y = 100x$ matches Table A.
Carlos thinks that the equation $x = \frac{1}{100}y$ matches Table A.
Whose claim is correct? Circle one.

Alexis's　　Carlos's　　Both　　Neither

Explain your thinking.

Activity 2

Name: ______________ Date: ______ Period: ______

Jayden's Cooler

4. It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

Complete either Part 1 or Part 2.

Part 1

a. Complete the table.

b. What is a constant of proportionality in this relationship?

c. Complete the equation for this proportional relationship.

$w =$ ______

Time (min), t	Water (gal), w
0	0
1	
2.5	
5	
t	

Part 2

a. Complete the table.

b. What is a constant of proportionality in this relationship?

c. Complete the equation for this proportional relationship.

$t =$ ______

Water (gal), w	Time (min), t
0	0
1	
4	
8	
w	

5. Find a partner who completed the part you *didn't* complete.

Discuss: How are your responses alike? How are they different?

Activity
2

Name: .... Date: .... Period: ....

Jayden’s Cooler (continued)

It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

6. What is the relationship between the constants of proportionality that you and your partner found?

7. What does $\frac{5}{8}$ tell you about the situation?

8. What does $\frac{8}{5}$ tell you about the situation?

You’re invited to explore more.

9. **a** Describe a situation where $\frac{10}{3}$ is a constant of proportionality.

b What would the other constant of proportionality, $\frac{3}{10}$, mean in the situation you described?

Synthesis

Here are the proportional relationships from this lesson.

10. Describe why there are two constants of proportionality in a proportional relationship.

Situation	Constants of Proportionality	Equations
There are 100 centimeters, y, in every meter, x.	100	$y = 100x$
	$\frac{1}{100}$	$x = \frac{1}{100}y$
It took Jayden 5 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.	$\frac{5}{8}$	$t = \frac{5}{8}w$
	$\frac{8}{5}$	$w = \frac{8}{5}t$

Lesson Practice ACC7.3.06

Lesson Summary

When two quantities x and y are in a proportional relationship, you can represent the relationship in two ways:

- The equation $y = kx$, with k as the constant of proportionality.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

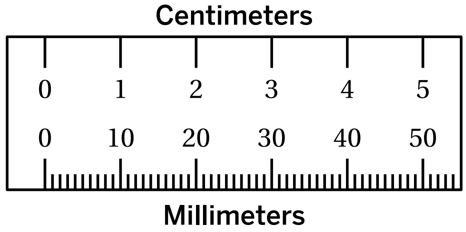
For example, if one pound of soybeans costs $\$2.00$, then:

- The cost, c, is proportional to the weight, w. The equation $c = 2w$ represents the situation because you can multiply the weight by 2 to get the cost.
- The weight, w, is proportional to the cost c. The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

Reciprocals are two numbers with a product of 1. For example, $\frac{1}{2}$ is the reciprocal of 2.

Name: .... Date: .... Period: ....

Problems 1–3: There is a proportional relationship between a length measured in centimeters and the same length measured in millimeters.



1. Complete the table and then determine a constant of proportionality.

Length (cm)	Length (mm)
4	40
12	
50	
88.49	

Constant of proportionality: ....

2. Complete the table and then determine a constant of proportionality.

Length (mm)	Length (cm)
25	2.5
240	
4	
699.1	

Constant of proportionality: ....

3. How are these two constants of proportionality related to each other?

4. Amoli and Emika are converting measurements between inches and feet.

Amoli says that the constant of proportionality for the relationship between inches and feet is 12. Emika says it is $\frac{1}{12}$.

Whose claim is correct?

A. Amoli's **B.** Emika's **C.** Both **D.** Neither

Explain your thinking.

Name: .... Date: .... Period: ....

FAST Practice

Problems 5–8: A recipe for granola calls for $\frac{5}{2}$ cups of rolled oats and $\frac{3}{4}$ cups of shredded coconut.

5. Complete the table to show how many cups of coconut should be used for different amounts of oats.

Oats (cups)	Coconut (cups)
1	
5	
12	

6. What is a constant of proportionality in this relationship? What does it represent?

7. What is the *other* constant of proportionality? What does it represent?

8. In a recipe, t represents the number of cups of oats, and c represents the number of cups of coconut. Select *all* equations that represent the relationship between t and c.

- ☐ A. $c = \frac{3}{10}t$
- ☐ B. $t = \frac{3}{10}c$
- ☐ C. $c = \frac{10}{3}t$
- ☐ D. $t = \frac{10}{3}c$
- ☐ E. $c = \frac{5}{2}t$
- ☐ F. $t = \frac{2}{5}c$

Spiral Review

9. The Wrangell-St. Elias National Park and Preserve in Alaska has an area of 20,625 square miles. A scale drawing of the Wrangell-St. Elias National Park and Preserve has an area of 8.25 square inches. What is the scale of the drawing?

10. Which equation has a constant of proportionality equal to 5?

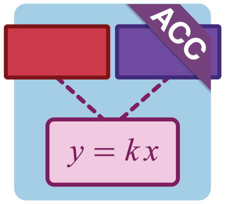
A. $5y = 5x$ B. $5y = 15x$ C. $3y = 5x$ D. $3y = 15x$

Name: .... Date: .... Period: ....

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1, MTR.1.1, MTR.4.1, MTR.5.1

All Kinds of Equations

Let's use equations to decide whether a relationship is proportional.



Warm-Up

1. This table represents the equation $y = 3x + 1$.

a Use the equation to complete the table.

b Does the equation represent a proportional relationship? Circle one.

Yes No Not enough information

Explain your thinking.

x	y
0	1
1	4
3	10
	16
2.5	

Activity
1

Name: ________ Date: ________ Period: ________

Stories, Equations, Tables

2. Use the story and equation to complete the table. Then decide whether the relationship is proportional. Complete either Story 1 or Story 2.

Story 1

Story: Lucia earns $12 per hour.

Equation: $y = 12x$

Is the relationship proportional?

Explain your thinking.

Time Worked (hr), x	Pay ($), y
0	
1	
	30

Story 2

Story: A recipe recommends 1 banana for every 2 smoothies.

Equation: $y = 0.5x$

Is the relationship proportional?

Explain your thinking.

Number of Smoothies, x	Number of Bananas, y
0	0
1	
	2.5

Find a partner who completed the other story.

Discuss: How are your responses alike? How are they different?

Name: Date: Period:

Stories, Equations, Tables (continued)

Complete either Story 3 or Story 4.

Story 3

Story: A cell phone costs $500, plus $35 per month for the plan.

Equation: $y = 35x + 500$

Is the relationship proportional?

Explain your thinking.

Number of Months, x	Total Cost ($), y
0	
1	
	605

Story 4

Story: The area of a square is the side length multiplied by itself.

Equation: $y = x^2$

Is the relationship proportional?

Explain your thinking.

Side Length (units), x	Area (sq. units), y
0	
1	
	100

3. Here are the equations that represent the four stories.

a Select *all* the equations that represent a proportional relationship.

- ☐ **A.** $y = 12x$
- ☐ **B.** $y = 500 + 35x$
- ☐ **C.** $y = \frac{1}{2}x$
- ☐ **D.** $y = x^2$

b Explain one way to decide if an equation represents a proportional relationship.

Activity 2

Name: ________ Date: ________ Period: ________

Equations and Proportionality

4. Decide whether each equation, table, or story represents a proportional relationship by placing a checkmark in the appropriate column.

<table>
<tr><th></th><th>Proportional</th><th>Not Proportional</th></tr>
<tr><td>$4 + x = y$</td><td></td><td></td></tr>
<tr><td>$y = 4x$</td><td></td><td></td></tr>
<tr><td>Jacy walked 4 miles in 100 minutes at a steady pace.</td><td></td><td></td></tr>
<tr><td>$0.04x = y$</td><td></td><td></td></tr>
<tr><td>$y = \frac{x}{4}$</td><td></td><td></td></tr>
<tr><td>$\frac{4}{x} = y$</td><td></td><td></td></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>$\frac{4}{3}$</td></tr><tr><td>4</td><td>1</td></tr></table></td><td></td><td></td></tr>
</table>

5. Mai thinks $\frac{4}{x} = y$ is proportional. Karima thinks $y = \frac{x}{4}$ is proportional. Whose thinking is correct? Circle one.

Mai's Karima's Both Neither

Explain your thinking.

Proportional

$\frac{4}{x} = y$ $y = \frac{x}{4}$

You're invited to explore more.

6. Use the You're Invited to Explore More Sheet to answer questions about a relationship and proportionality.

Synthesis

7. **a** Write two equations: one that represents a proportional relationship and one that does not.

Proportional Relationship	Not a Proportional Relationship

b Describe how you know whether an equation represents a proportional relationship.

Lesson Practice ACC7.3.07

Lesson Summary

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a constant of proportionality, k, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a constant of proportionality, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Name: .... Date: .... Period: ....

Problems 1–2: The relationship between a distance in yards, y, and the same distance in miles, x, is represented by the equation $y = 1760x$.

1. Complete the table.

Distance (mi), x	Distance (yd), y
1	
5	
	$3{,}520$
	$17{,}600$

2. Is there a proportional relationship between a distance in yards and the same distance in miles?

   Explain your thinking.

Problems 3–4: Determine whether or not each relationship is proportional. Explain your thinking.

3.

x	y
2	5
3	7.5
6	15

4. $y = 3.2x + 5$

Problems 5–6: Determine whether each relationship is proportional or not proportional. Explain your thinking.

5. The weight of a stack of standard 8.5-by-11-inch paper and the number of sheets of paper.

6. The weight of a stack of different-sized books (where each book weighs a different amount) and the number of books in a stack.

Lesson Practice
ACC7.3.07

Name: .... Date: .... Period: ....

7. Determine whether or not each relationship is proportional.

	Proportional	Not Proportional
The remaining length, L, of a 120-inch rope after x inches have been cut off: $120 - x = L$.	◯	◯
The total cost, t, after 8% sales tax is added to an item's price p: $1.08p = t$.	◯	◯
The number of marbles, x, each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$.	◯	◯
The volume, V, of a rectangular prism whose height is 12 centimeters and whose base is a square with side lengths of s centimeters: $V = 12s^2$.	◯	◯

FAST Practice

8. Liam and Sadia are running a 60-meter race. Each of their distances can be represented by an equation in the form $y = kx$ where y is the distance in meters and x is the time in seconds.

Use this information to complete the table.

- Liam's distance is represented with the equation $y = 6x$.
- At 6 seconds, Liam is 12 meters ahead of Sadia.

Time (sec)	Liam's Distance (m)	Sadia's Distance (m)
0		
2		
4		
6		

Spiral Review

9. The equation $y = 3.5x$ can be used to determine the total cost, y, in dollars, of x ounces of blueberries. What does the number 3.5 represent in the equation?

A. The number of blueberries that \$1 can buy.

B. The number of blueberries in x ounces.

C. The cost of 1 ounce of blueberries.

D. The cost of x ounces of blueberries.

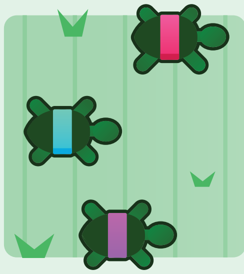
Sub-unit

3

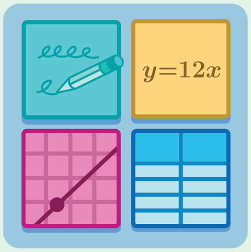
Proportional Relationships in Graphs

Lesson 8
DinoPops

Lesson 9
Gallon Challenge



Lesson 10
Three Turtles



Lesson 11
Four Representations

Name: ______________ Date: ________ Period: ________

MA.7.AR.4.1, MA.8.AR.3.1, MTR.2.1, MTR.5.1

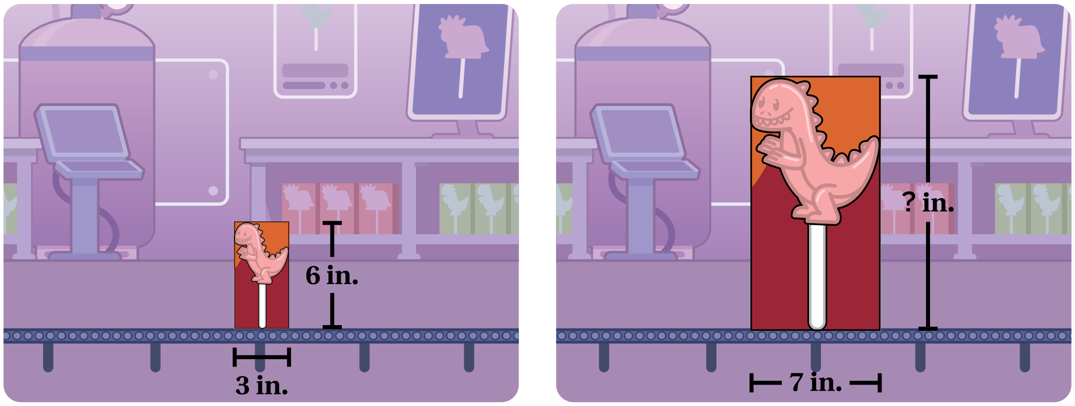
DinoPops

Let's explore what a proportional relationship looks like on a graph.

Warm-Up

1. Here are two DinoPops in their boxes.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.



Complete the table to determine the height of the box for the large DinoPop.

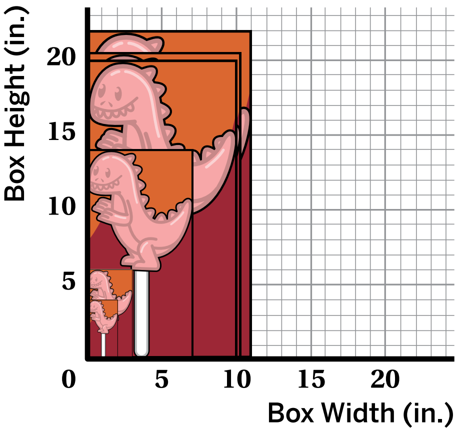
Box Width (in.)	Box Height (in.)
3	6
7	

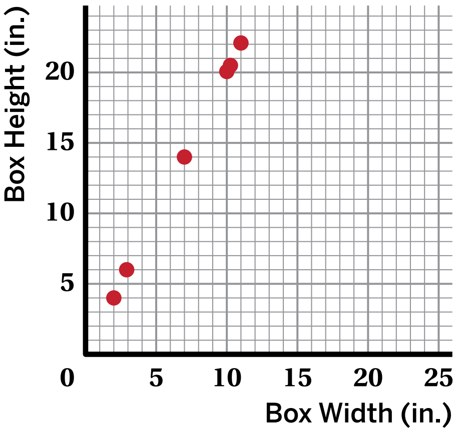
Activity 1

Name: ____________ Date: ________ Period: ________

DinoPops

2. Here are some DinoPop boxes and a graph of some points.



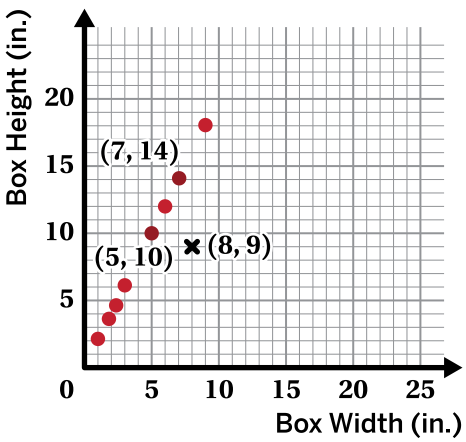


What do you notice? What do you wonder?

I notice:

I wonder:

3. A box that is 5 inches wide and 10 inches tall is a perfect fit for a DinoPop. A box that is 7 inches wide and 14 inches tall is also a perfect fit.



Write ordered pairs for two other boxes that are a perfect fit for a DinoPop.

(____, ____) (____, ____)

A box that is 8 inches wide and 9 inches tall is *not* a perfect fit for a DinoPop.

Write ordered pairs for two other boxes that are *not* a perfect fit for a DinoPop.

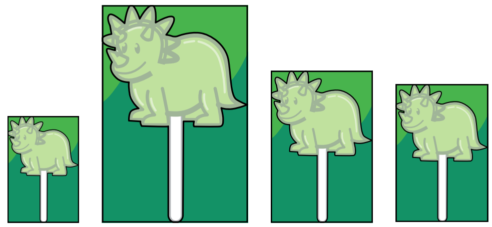
(____, ____) (____, ____)

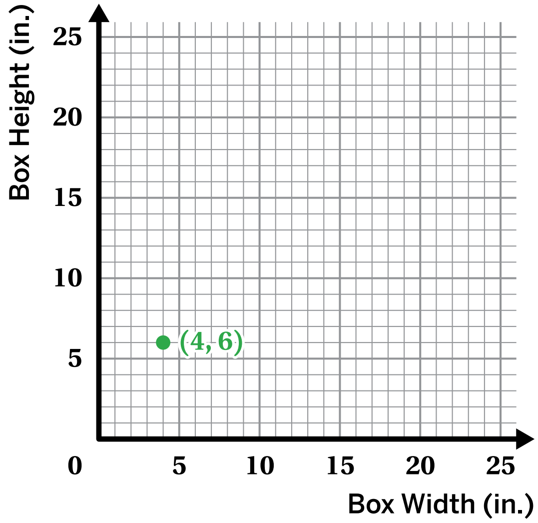
Name: .... Date: .... Period: ....

TriceraPops

4. Here are several TriceraPops in boxes.

The graphed point represents *one* of these boxes.





a **Discuss:** What do you know about this box?

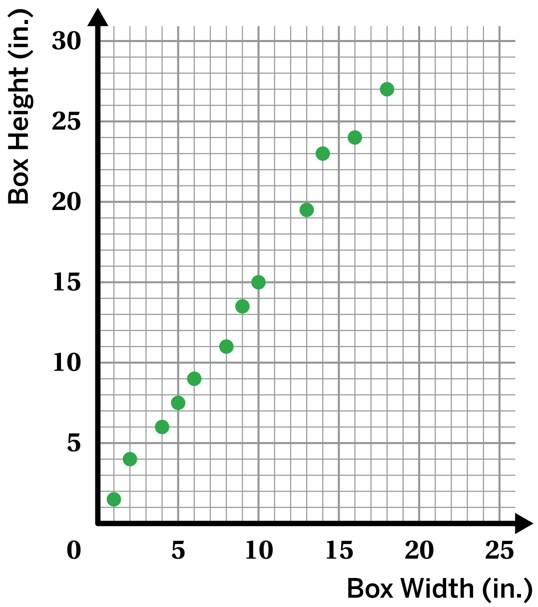
b Add at *least two* more points to the graph to represent other boxes that fit a TriceraPop.

5. A student made some TriceraPop boxes, as shown on this graph.

Some of the boxes were not a good fit.

Describe how to use the graph to identify the bad boxes.

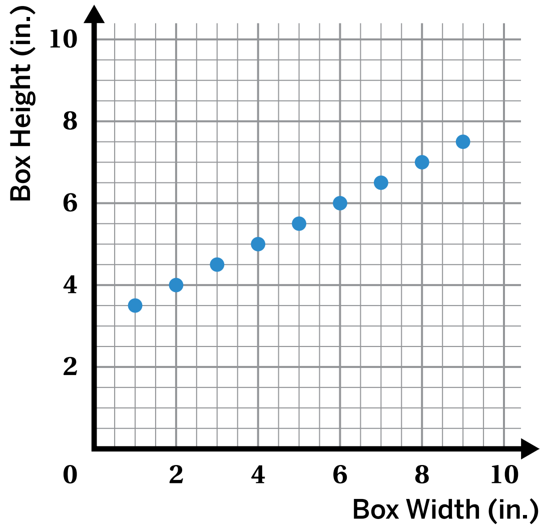
Draw on the graph if it helps to show your strategy.



Name: .... Date: .... Period: ....

TriceraPops (continued)

6. Here is a graph of box sizes for a new lollipop.



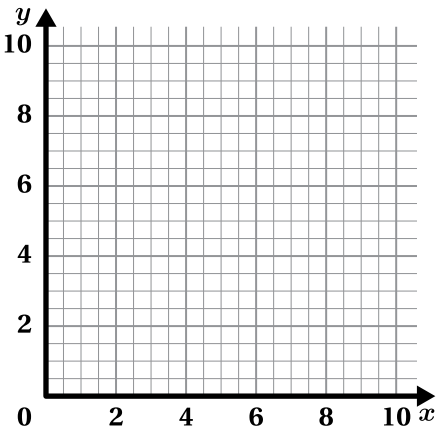
Is there a proportional relationship between the height and the width of these boxes? Circle one.

Yes No I'm not sure

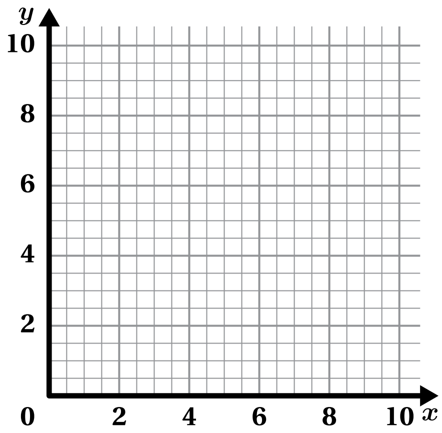
Explain your thinking.

7. Make one graph that represents a proportional relationship and one graph that does not.

Proportional



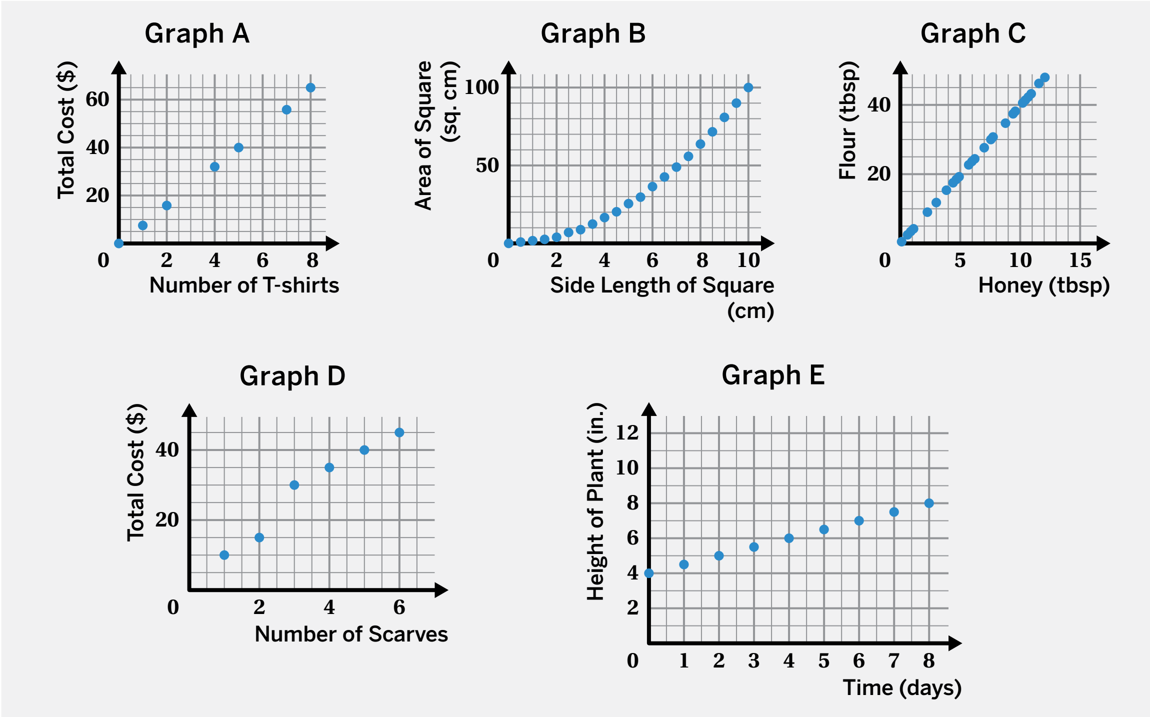
Not Proportional



Name: ________ Date: ________ Period: ________

Graphs

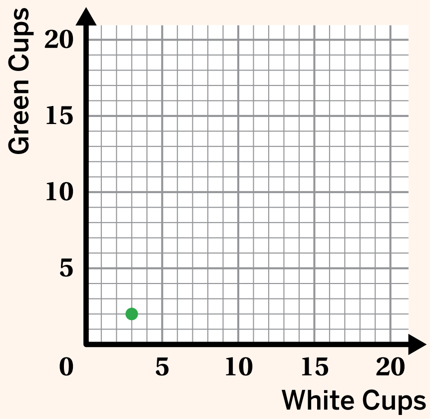
8. Decide whether each graph represents a proportional relationship.



Proportional	Not Proportional

You're invited to explore more.

9. This color green is made by mixing 3 cups of white paint and 2 cups of green paint.

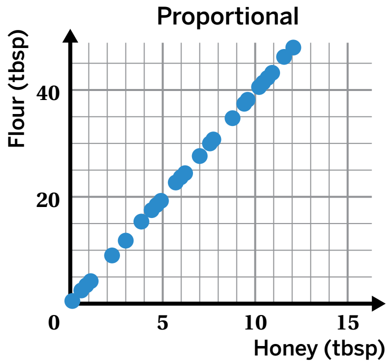


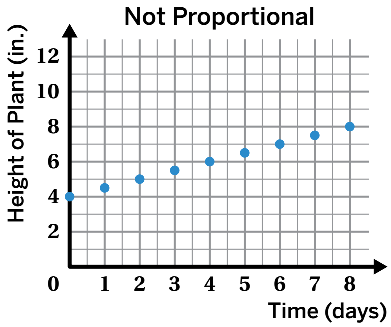
Graph points that represent *at least two* more ways to make this color.

Synthesis

10. How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.





Lesson Practice ACC7.3.08

Lesson Summary

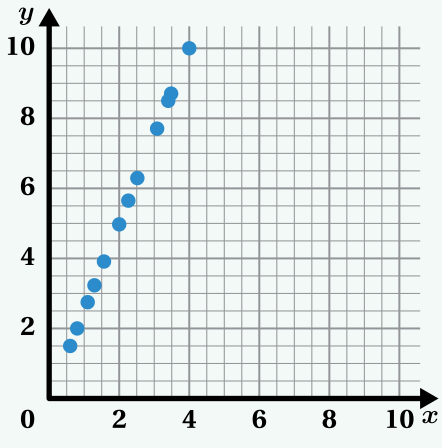
On a *coordinate plane*, if points all fall on a line that passes through $(0, 0)$, the relationship is proportional. The point $(0, 0)$ is known as the *origin*.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are $(2, 5)$ and $(4, 10)$.

$5 \div 2 = 2.5$ and $10 \div 4 = 2.5$

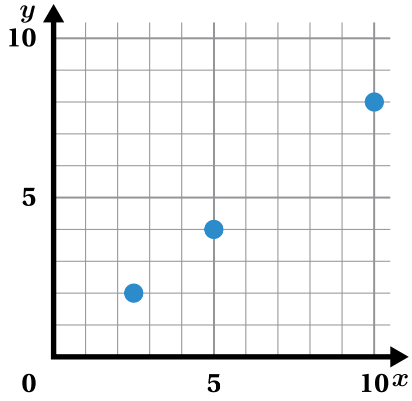
Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.



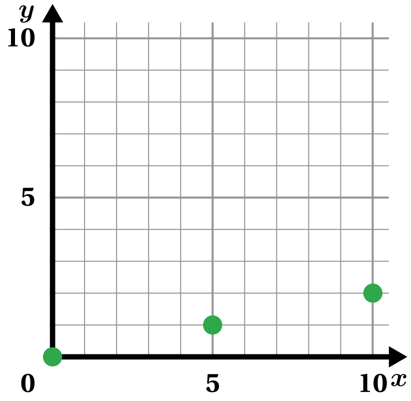
Name: .... Date: .... Period: ....

Problems 1–4: Determine if each graph represents a proportional relationship. Explain your thinking.

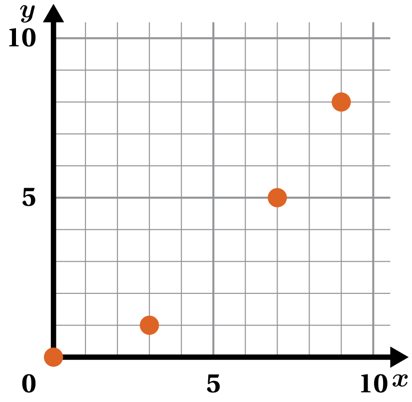
1.



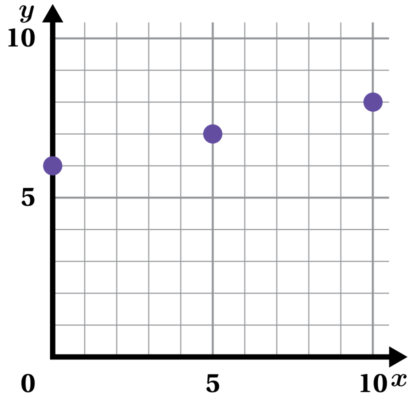
2.



3.

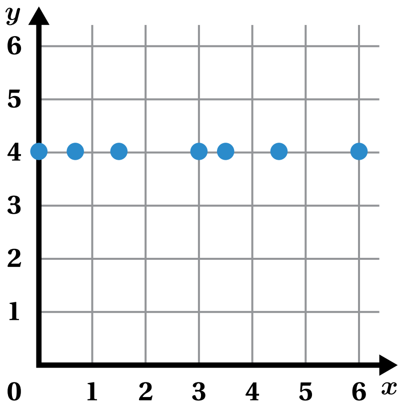


4.

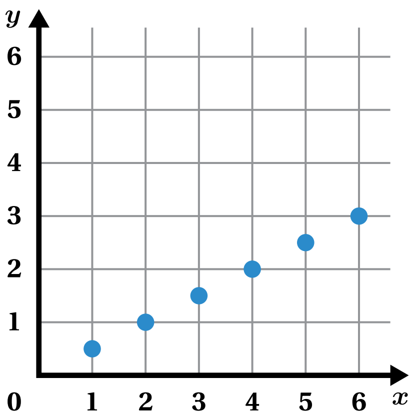


5. Select *all* the representations that show a proportional relationship.

☐ A.



☐ B.



☐ C.

x	0	2	4	6
y	0	3	9	27

☐ D.

x	0	2	4	6
y	0	12	24	36

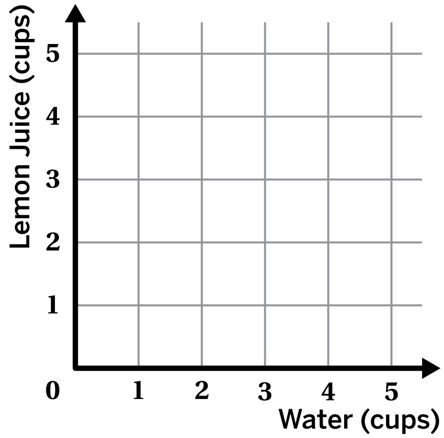
Lesson Practice
ACC7.3.08

Name: .... Date: .... Period: ....

FAST Practice

6. A lemonade recipe calls for a $\frac{1}{4}$ cup of lemon juice for every cup of water. The table shows different amounts of water and lemon juice you can use to make this recipe. Graph the ordered pairs to determine whether the relationship between water and lemon juice is proportional.

Water (cups)	Lemon Juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1



Select **ONE** correct answer in each box to complete the sentences.

The relationship [**A.** is **B.** is not] proportional. The points [**A.** fall **B.** do not fall] on a line that passes through the origin.

Spiral Review

7. Arturo earns $\$33.00$ for babysitting for 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your thinking.

8. A turtle is walking away from a rock. x represents the time in minutes that the turtle is walking. y represents the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* the true statements.

- ☐ **A.** The equation $y = 3x$ could represent the distance that the turtle walks.
- ☐ **B.** The turtle walks for a bit and then stops for a minute before walking again.
- ☐ **C.** The turtle walks away from the rock at a constant rate.
- ☐ **D.** The equation $y = x + 3$ could represent the distance that the turtle walks.
- ☐ **E.** After 6 minutes, the turtle walks 18 meters, and after 10 minutes, the turtle walks 20 meters.

Name: ______ Date: ______ Period: ______

MA.7.AR.4.2, MTR.3.1, MTR.7.1

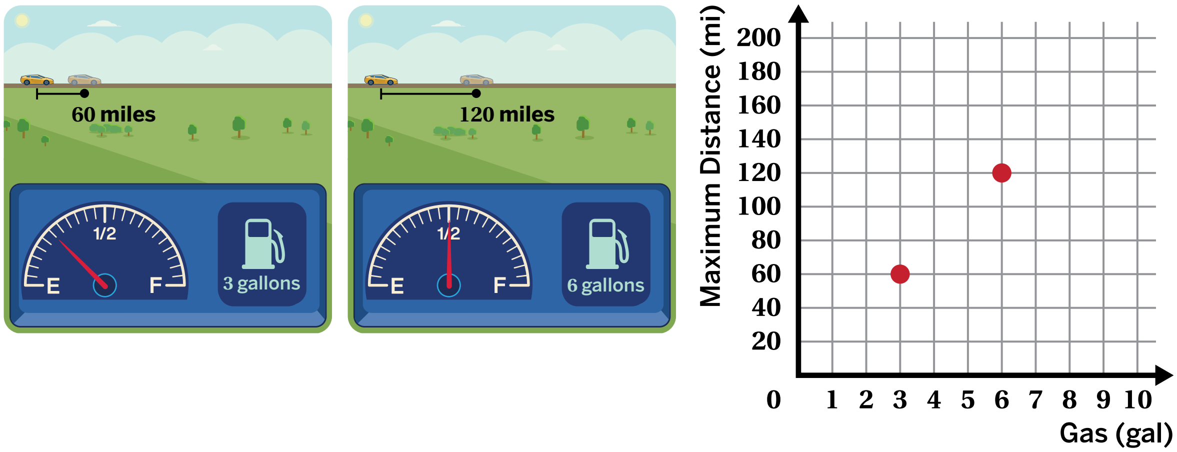
Gallon Challenge

Let's identify constants of proportionality using a graph.

Warm-Up

1. Here are two images showing the amount of gas in a car's tank.

The graph represents the maximum distance the car can go using each amount of gas.



Discuss: What do you notice? What do you wonder?

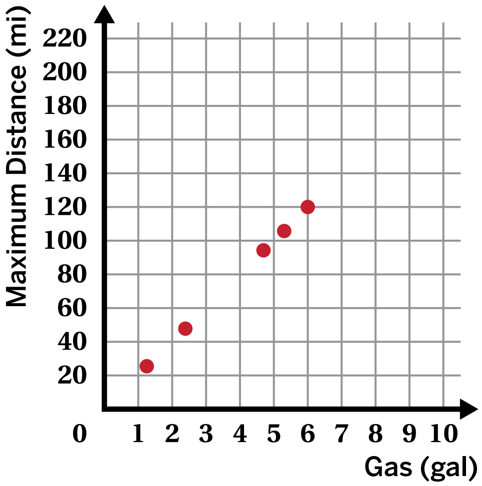
Activity 1

Name: ________ Date: ________ Period: ________

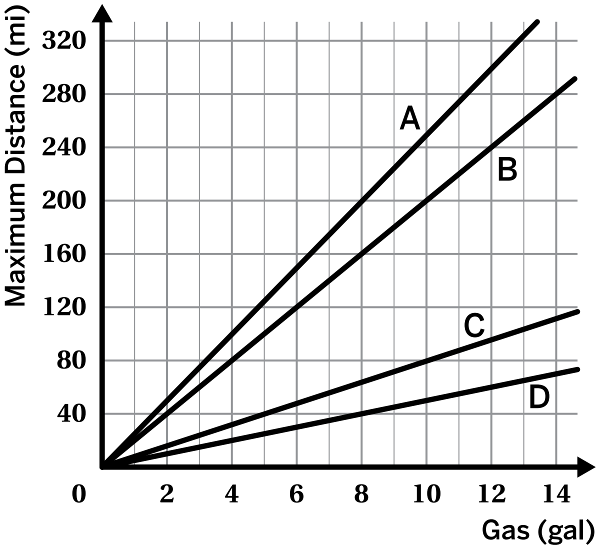
On the Road

2. Here are several points representing the maximum distance the car can go using different amounts of gas.

What would the graph look like if it included every possible point for this car?



3. The car can travel 240 miles with a full 12-gallon tank of gas. Which line represents this relationship?



4. A car's *gas mileage* is the maximum distance it can go using 1 gallon of gas (measured in miles per gallon).

Based on the graph, what is this car's gas mileage?

________ miles per gallon

Where do you see this number in the graph?

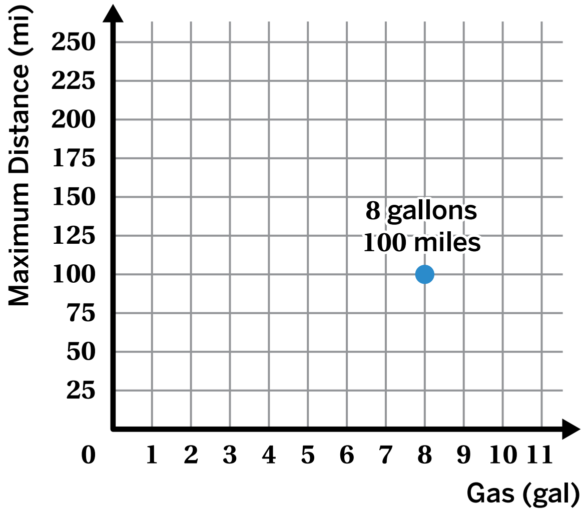
Activity
2

Name: Date: Period:

Gas Mileage

5. Kaya's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?

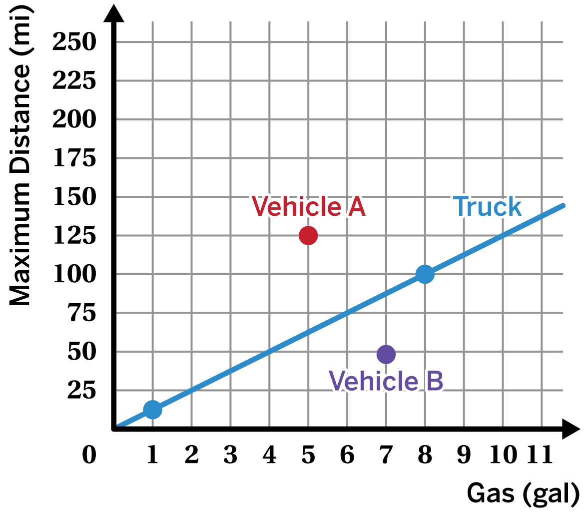


6. Kaya wants to buy a new vehicle that gets better gas mileage than her truck.

Which vehicle should she pick? Circle one.

Vehicle A Vehicle B Either Vehicle A or Vehicle B

Explain your thinking.



Activity 2

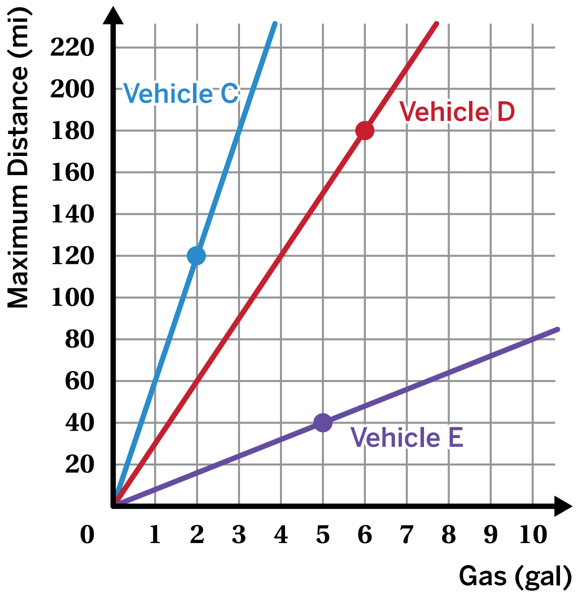
Name: .... Date: .... Period: ....

Gas Mileage (continued)

7. In the relationship between amount of gas and maximum distance, the vehicle's gas mileage is a constant of proportionality.

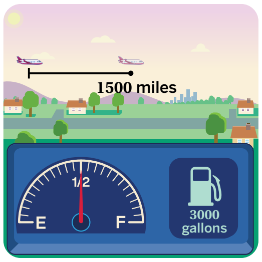
What is that constant of proportionality for each vehicle?

Vehicle	Constant of Proportionality (miles per gallon)
C	
D	
E	



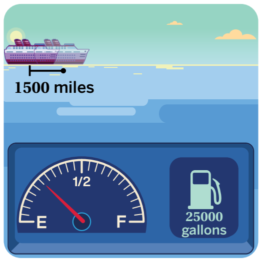
8. Here is the maximum distance traveled by three vehicles using a certain amount of gas.

Plane



Note: The average plane has 100 passengers.

Cruise Ship



Note: The average cruise ship has 3000 passenger.

Train

Note: The average train has 300 passengers.

Compare the fuel efficiency for each vehicle.

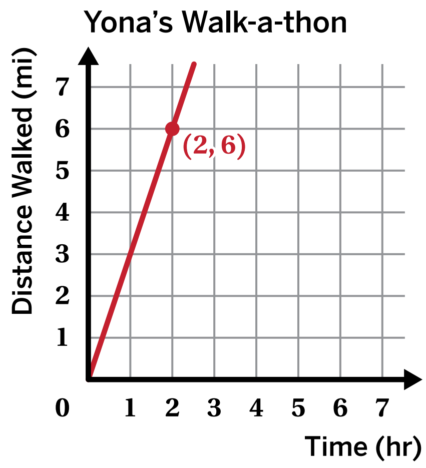
Synthesis

9. What are *two* different ways you can determine a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method:

Second method:



Lesson Practice ACC7.3.09

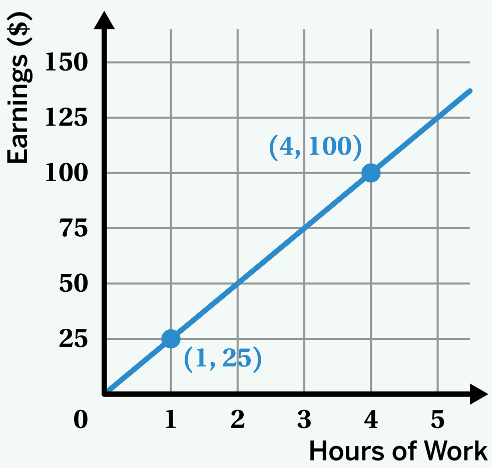
Lesson Summary

Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y. You can determine a constant of proportionality from a graph by using:

- The value of y when x is equal to 1.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, x, and money earned in dollars, y. One constant of proportionality is 25 because \$25 is earned for working 1 hour.

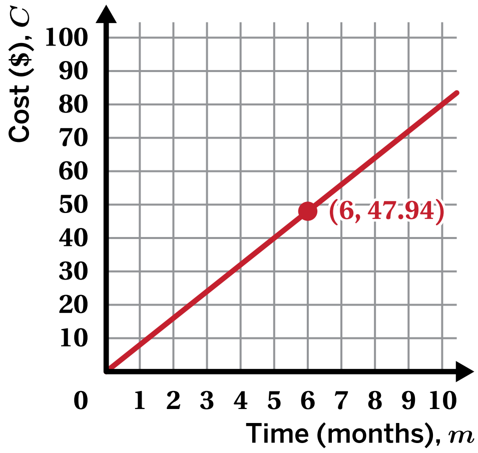
The ordered pair $(4, 100)$ shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.



Name: .... Date: .... Period: ....

Problems 1–2: Here is a graph that shows a proportional relationship between the number of months Tiara had a streaming service subscription and the total amount of money she paid for the subscription.

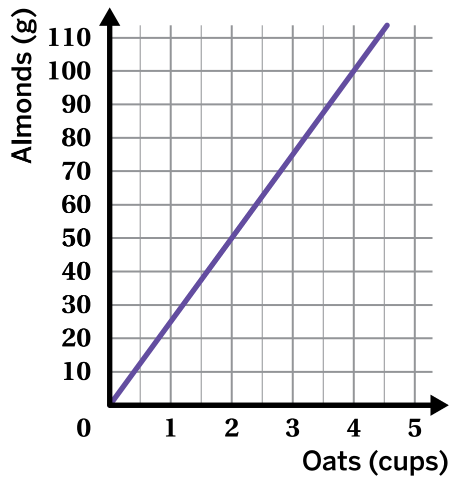
After 6 months, Tiara paid $47.94.



1. What is a constant of proportionality in this relationship?

2. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

Problems 3–4: A recipe for granola calls for a mix of almonds and oats. The graph shows the amount of almonds, in grams, for different amounts of oats, in cups.

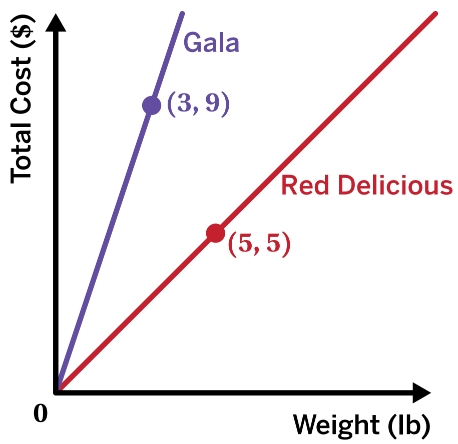


3. Determine a constant of proportionality for this relationship, then explain its meaning.

4. Label one place you see that constant of proportionality on the graph.

5. The graph shows the cost for two different varieties of apples. Which variety of apples has a higher cost per pound?

   Explain your thinking.



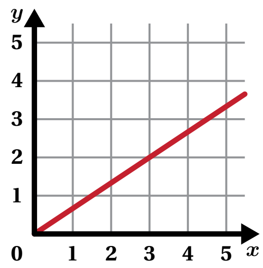
Lesson Practice
ACC7.3.09

Name: ______ Date: ______ Period: ______

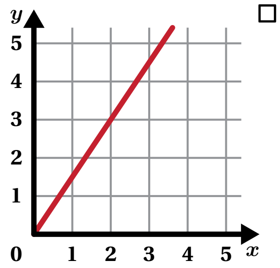
FAST Practice

6. Which of these relationships has the same constant of proportionality as the equation $y = \frac{2}{3}x$? Select *all* that apply.

☐ A.



☐ B.



☐ C.

x	y
0	0
3	2
6	4
9	6

☐ D.

x	y
0	0
2	3
4	6
6	9

Spiral Review

7. Write an expression that is equivalent to $8(n + 6)$.

Problems 8–10: Haru and Irene were running laps around the track. Their coach recorded their times at the end of laps 2, 4, and 6.

Haru's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	4	
4	9	
6	15	

Irene's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	5	
4	10	
6	15	

8. Complete the tables with the minutes per lap for each run.

9. Based on the table, is Haru running at a constant speed? Explain your thinking.

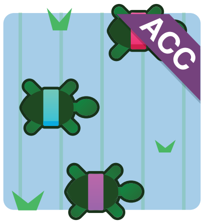
10. Based on the table, is Irene running at a constant speed? Explain your thinking.

Name: Date: Period:

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1, MTR.1.1, MTR.2.1, MTR.4.1

Three Turtles

Let's compare proportional relationships.

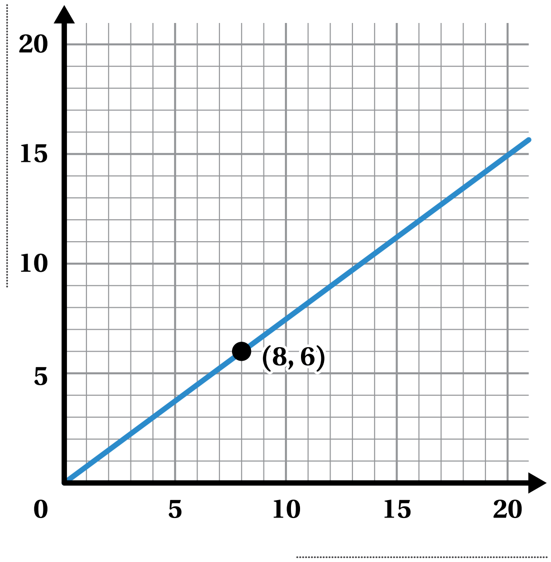


Warm-Up

1. Here is a graph that represents a proportional relationship.

a Label the axes with any quantities you'd like.

b Write a true statement about the quantities based on the graph.

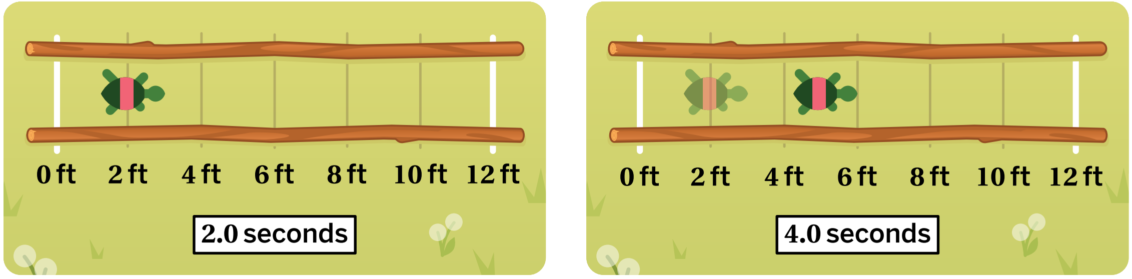


Activity 1

Name: ______ Date: ______ Period: ______

Traveling Turtles

2. This turtle walks at a constant rate. The turtle's distance is measured at the front of its head.



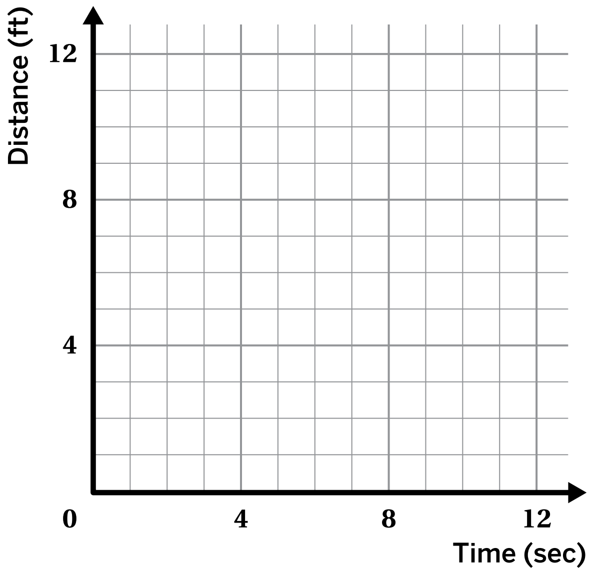
a Complete the table.

Time (sec)	Distance (ft)
2	3
4	6
6	
8	

b What is a constant of proportionality for this relationship?

3. Graph the points from the table in the previous problem.

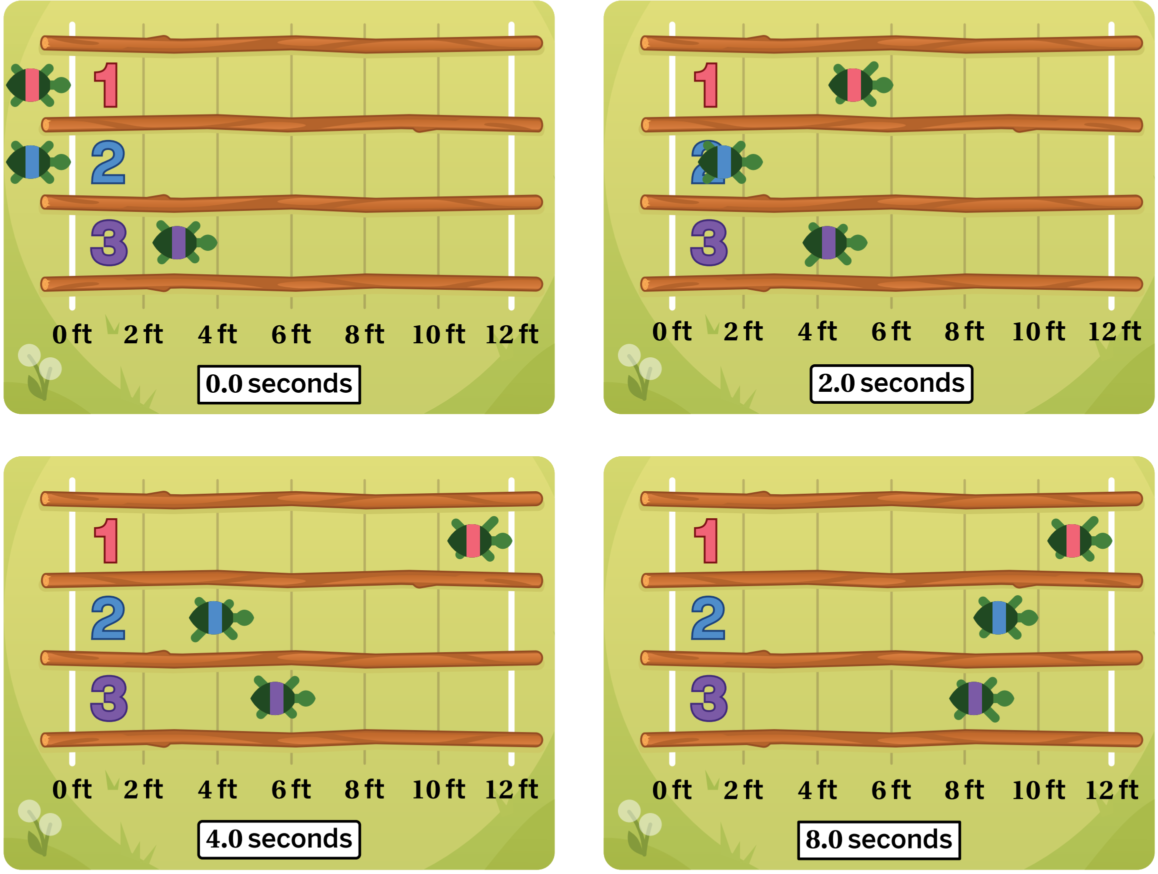
Write an equation for this relationship, using d for distance and t for time.



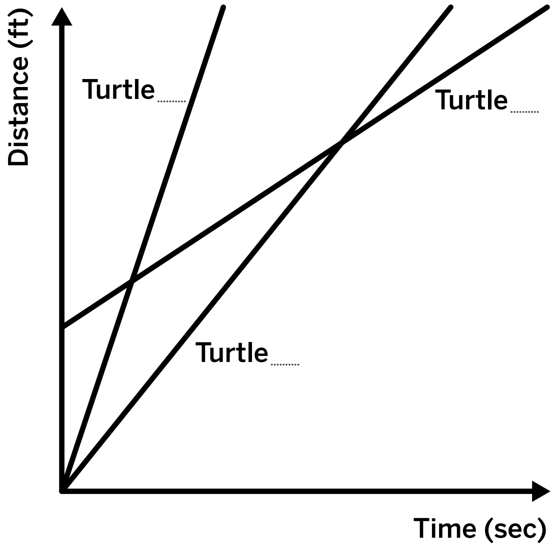
Name: .... Date: .... Period: ....

Three Turtles

4. These images show three turtles walking. Each turtle walks at a constant rate.



Label each line with the turtle it represents: Turtle 1, Turtle 2, or Turtle 3.



5. Match each equation to its graph.

Equation		Turtle
a. $d = 3t$	....	Turtle 1
b. $d = \frac{2}{3}t + 4$	....	Turtle 2
c. $d = 1.25t$	....	Turtle 3

Explain your thinking.

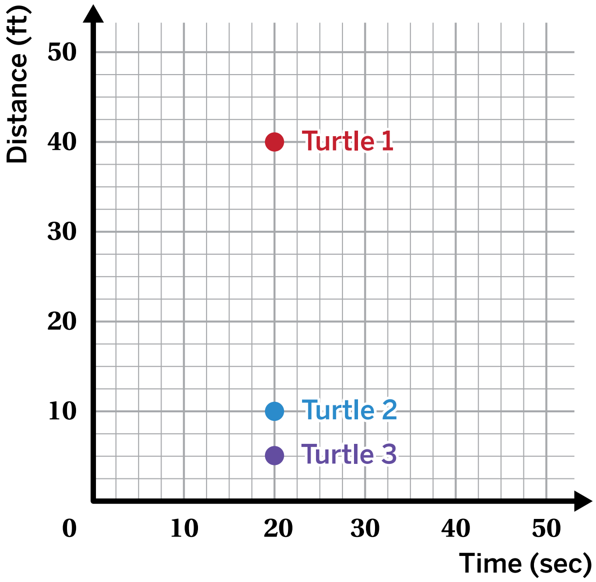
Name: ________ Date: ________ Period: ________

Turtle Challenges

6. Here are three new turtles and their distances at 20 seconds.

- Turtle 1 is 40 feet from the start.
- Turtle 2 is 10 feet from the start.
- Turtle 3 is 5 feet from the start.

Write an equation for each turtle, using d for distance and t for time. (One equation has been written for you.)



Turtle	Equation
Turtle 1	
Turtle 2	$d = \frac{1}{2}t$
Turtle 3	

7. Irelle wrote three equations for the new turtles.

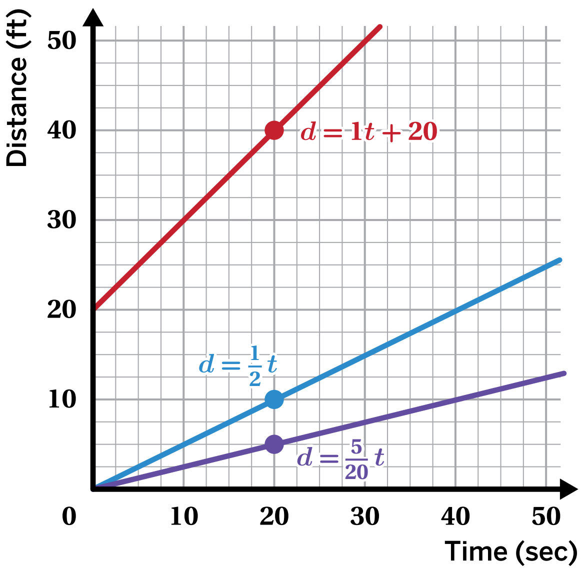
a Which of these relationships is *not* proportional?

A. $d = 1t + 20$

B. $d = \frac{1}{2}t$

C. $d = \frac{5}{20}t$

b Describe this turtle's race.

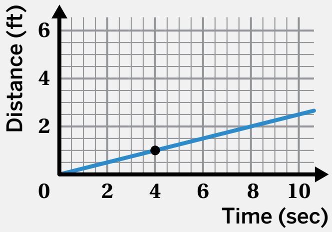


Name: ________ Date: ________ Period: ________

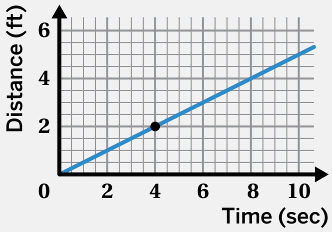
Turtle Challenges (continued)

8. Sort these cards into three groups that each represent the same turtle.

Card A



Card B



Card C

Time (sec)	Turtle Distance (ft)
0	0
1	4
2	8
4	16
8	32

Card D

Time (sec)	Turtle Distance (ft)
0	0
1	$\frac{1}{2}$
2	1
4	2
8	4

Card E

At 8 seconds, the turtle's distance is 2 feet.

Card F

At 2 seconds, the turtle's distance is 8 feet.

Card G

$d = 4t$

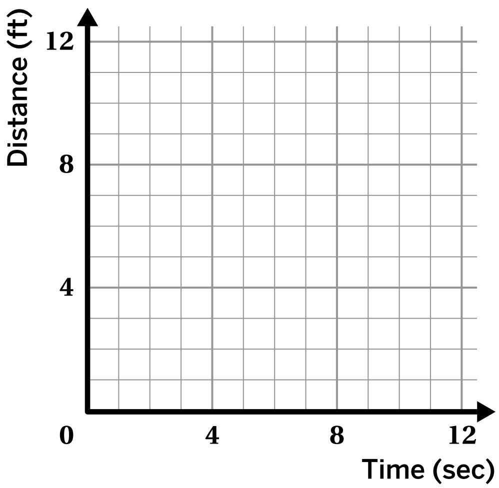
Card H

$d = \frac{1}{4}t$

Group 1	Group 2	Group 3

9. Create your own turtle race by sketching 3 lines that represent 3 different turtles. Your turtle race must include *at least two* of the following features:

- ☐ A turtle that stays still.
- ☐ A turtle that has a head start.
- ☐ Two turtles that finish at the exact same time.
- ☐ A turtle that travels backward.
- ☐ Two turtles that travel at the same pace.



You're invited to explore more.

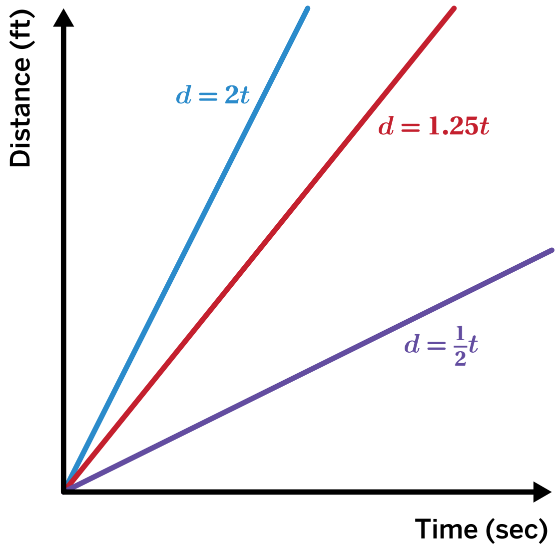
10. Use the You're Invited to Explore More Sheet to explore another turtle race.

Synthesis

11. This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- ☐ How can you tell from the graphs which turtle moved the fastest?
- ☐ How can you tell from the equations which turtle moved the fastest?



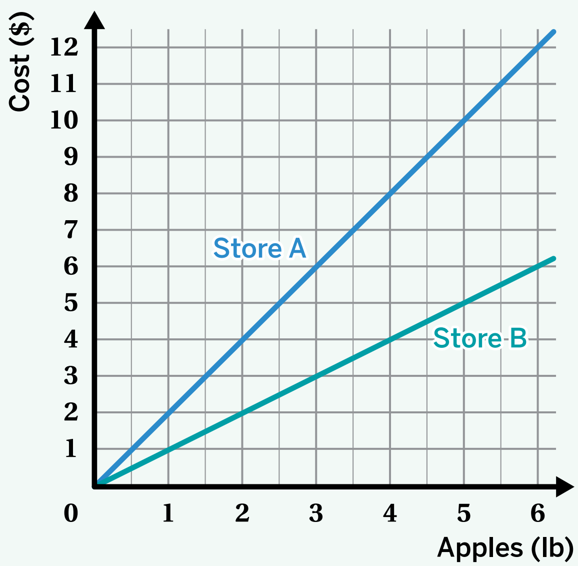
Lesson Practice ACC7.3.10

Lesson Summary

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.

- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges $2 for one pound ($k = 2$), while Store B charges $1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.



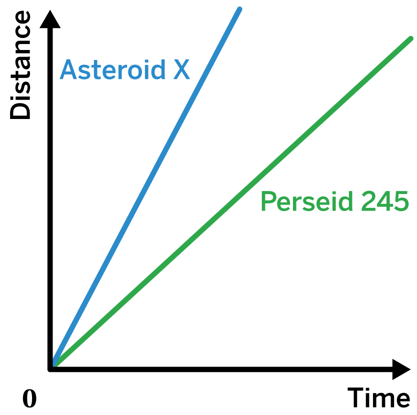
Lesson Practice
ACC7.3.10

Name: .... Date: .... Period: ....

1. Meteoroid Perseid 245 and Asteroid X travel through the solar system. This graph shows how much distance they travel over time.

   Does Asteroid X travel faster or slower than Perseid 245?

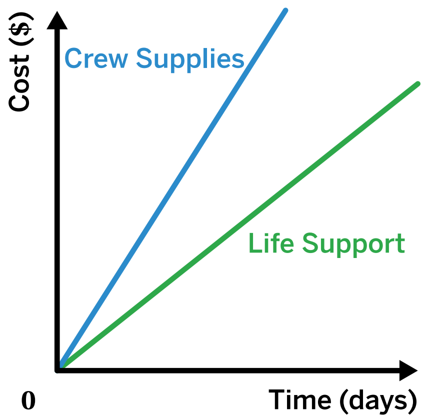
   Explain your thinking.



2. Having a crew on the International Space Station requires life support and crew supplies. The graph shows the relationship between the cost of life support and crew supplies, and the number of days spent in space.

   Which costs less, life support or crew supplies?

   Explain your thinking.



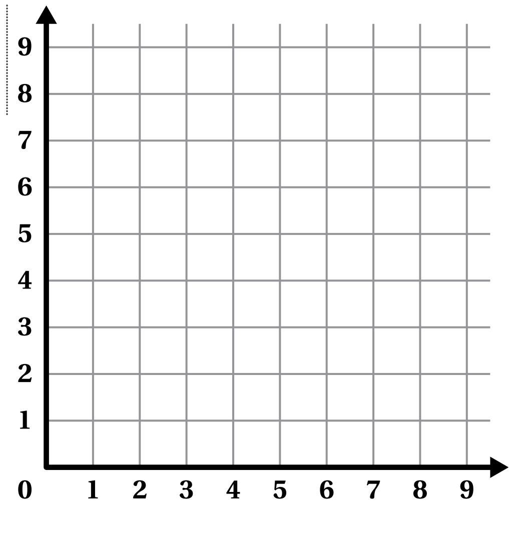
Problems 3–6: At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for $5.40.

3. How much does the honey cost per ounce? Show your thinking.

4. How much honey can be bought per dollar? Show your thinking.

5. Write *two* different equations representing this situation. Use *h* for ounces of honey and *c* for cost in dollars.

6. Choose *one* equation and draw its graph on the coordinate plane. Be sure to label the axes.

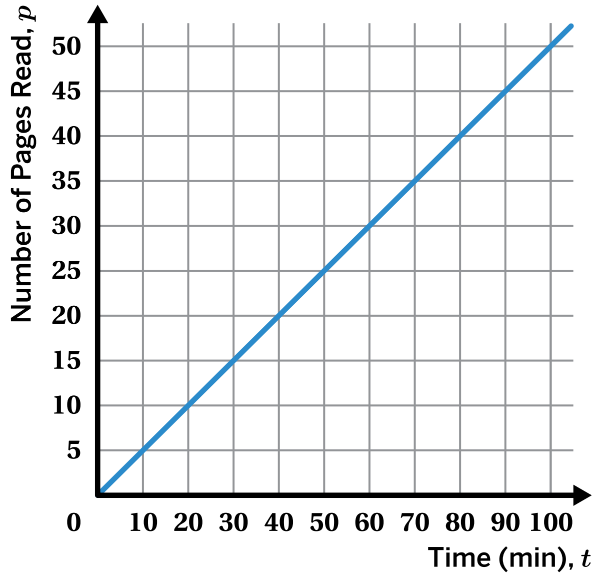


Lesson Practice
ACC7.3.10

Name: ________ Date: ________ Period: ________

FAST Practice

Problems 7–10: The graph shows the relationship between time in minutes, t, and the total number of pages Joel has read, p.



7. Is this relationship proportional?

Explain your thinking.

8. What is a constant of proportionality in this relationship?

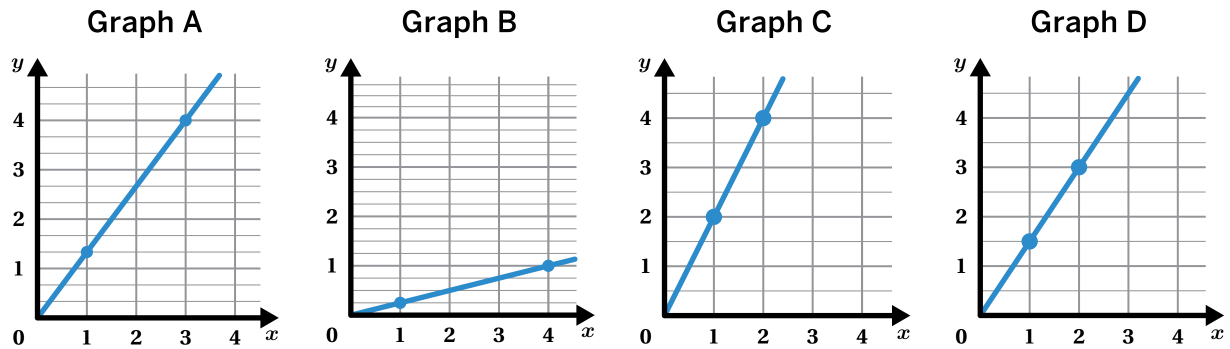
9. What does this number mean in this situation?

10. Write an equation that represents the relationship between the time in minutes, t, and the total number of pages Joel has read, p.

Equation: ________ (or equivalent)

Spiral Review

Problems 11–14: Match each equation with its graph.



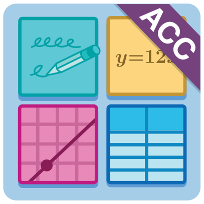
11. $y = \frac{1}{4}x$ ________ **12.** $y = \frac{3}{2}x$ ________ **13.** $y = 2x$ ________ **14.** $y = \frac{4}{3}x$ ________

Unit 3
Lesson
11

Name: ______ Date: ______ Period: ______

MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4, MTR.2.1, MTR.7.1

Four Representations



Let's explore connections between representations.

Warm-Up

1. Here are four representations of a proportional relationship.

Description

Lucia works at a sporting goods store where she earns $12 per hour.

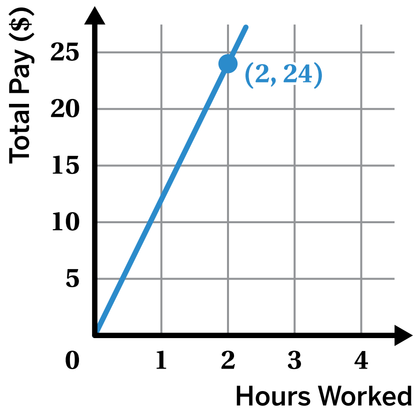
Equation

$y = 12x$

x represents the number of hours worked.

y represents the total pay in dollars.

Graph



Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96
10	120

Discuss: Show or explain the connections you see between the four representations.

Activity 1

Name: ______ Date: ______ Period: ______

Stronger and Clearer Each Time

2. Here are some lists of items and units of measurement.

Creatures		Units of Length		Units of Time	
starfish	centipedes	feet	centimeters	seconds	minutes
earthworms	dinosaurs	kilometers	miles	years	millennia

Body Parts		Units of Volume		Units of Weight	
legs	eyes	gallons	cups	ounces	pounds
toes	heads	cubic feet	teaspoons	kilograms	milligrams

a Choose *one* option from one list and *one* option from another list. You can also make up your own options.

______ and ______

b Write a description of a proportional relationship using your choices. Then use feedback from one or more partners to write a final draft of your description.

Description

First Draft:

Notes From Partner 1:

Notes From Partner 2:

Final Draft:

c Explain how you know the relationship you described is proportional.

Activity
2

Name: ______ Date: ______ Period: ______

Four Representations

3. Record the final draft of the description you wrote in Activity 1. Then create a table, a graph, and an equation for your relationship.

Description

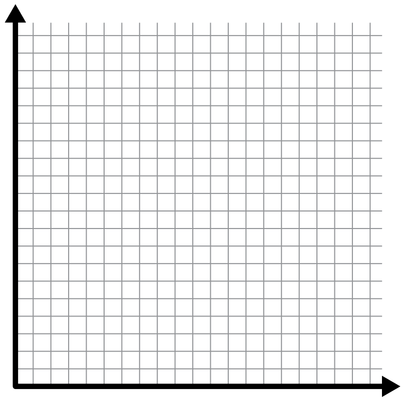
Table

Label each column. Then complete the table.

______	______

Graph

Label each axis. Then graph the relationship.



Equation

Write the equation.

Define the variables.

Synthesis

4. Here are the four representations from the Warm-Up.

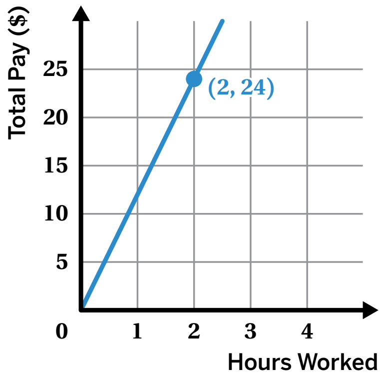
Equation

$y = 12x$

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Graph



Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96

Where do you see a constant of proportionality in each representation?

Lesson Practice ACC7.3.11

Lesson Summary

You can determine the constant of proportionality, k, in different ways using different representations.

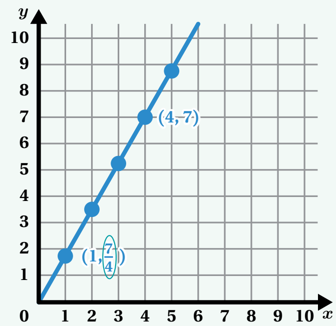
Equation

$y = \frac{7}{4}x$

Table

x	y
0	0
1	$\frac{7}{4}$
4	7

Graph



In a table or graph, the constant of proportionality is the y-value that is paired with the x-value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where (x, y) is any ordered pair.

In an equation in the form of $y = kx$, the number multiplying x (called the coefficient) is the constant of proportionality.

Name: .... Date: .... Period: ....

Problems 1–4: Determine a constant of proportionality in each representation. Show your thinking.

1. David walked 15 feet in 3 seconds.

2. $C = 4.8n$

3.

x	y
0	0
$\frac{7}{10}$	$\frac{28}{15}$

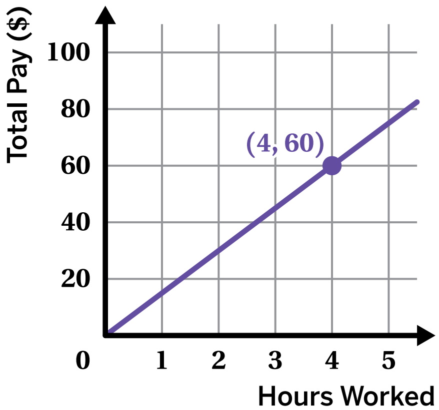
4.

5. Here is a description, a graph, and a table.

Description

Yosef has to work 12 hours in a week to earn $180.

Graph



Table

Hours Worked	Total Pay ($)
0	0
2	25
6	75
10	125
20	250

Which one represents a different relationship than the others? Explain your thinking.

Lesson Practice
ACC7.3.11

Name: ________ Date: ________ Period: ________

Problems 6–8: The equation $c = 2.95g$ gives the cost of gas on a particular day, where c represents the cost in dollars and g represents the number of gallons of gas purchased at a neighborhood gas station.

6. Write four ordered pairs that fit this relationship, where the x-value is the number of gallons of gas and the y-value is the cost.

7. What does the value 2.95 represent in this situation?

8. Hailey's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? Explain your thinking.

FAST Practice

9. $\left(\frac{1}{2}, 3\right)$ is a point on a line that represents a proportional relationship. Select *all* the points that are also on this line.

☐ **A.** $\left(9, \frac{3}{2}\right)$ ☐ **B.** $(1, 6)$ ☐ **C.** $(1, 3)$ ☐ **D.** $\left(\frac{3}{2}, \frac{1}{4}\right)$ ☐ **E.** $\left(\frac{7}{2}, 21\right)$

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☐ **A.**

x	y
0	0
$\frac{1}{4}$	$\frac{1}{8}$
$\frac{2}{4}$	$\frac{2}{8}$
$\frac{3}{4}$	$\frac{3}{8}$

☐ **B.**

x	y
0	0
$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{3}$	$\frac{1}{5}$
$\frac{1}{4}$	$\frac{1}{6}$

☐ **C.**

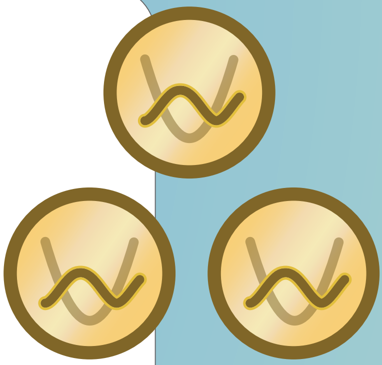
x	y
0	0
1	1
2	4
4	16

☐ **D.**

x	y
0	0
1	4
3	12
4	16

Unit 4

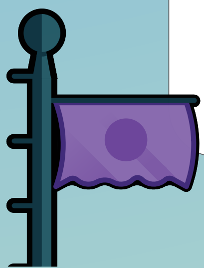
Linear Relationships and Equations



The slope of a line tells us about its steepness, but did you know it can reveal even more? A simple line on a graph holds *endless* information. A line can tell us what's faster: a tortoise or a hare. It can help us sort coins in a piggy bank. It can even help us determine how many cups a water cooler can fill. Let's explore proportional and linear relationships, and learn all about what a line can represent!

Essential Questions

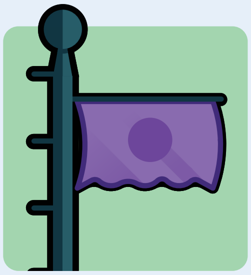
- What can proportional relationships and slope teach you about linear relationships?
- How do equations, tables, and graphs of linear relationships connect to one another?
- What does it mean for an ordered pair to be a solution to a linear equation?



Sub-unit

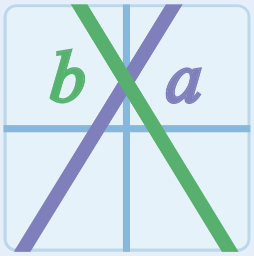
1

Linear Relationships



Lesson 1
Flags

Lesson 2
Water Cooler



Lesson 3
Ups and Downs

Lesson 4
Stacking Cups

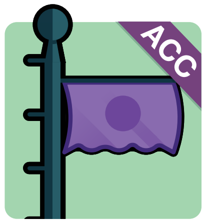
Unit 4
Lesson
1

Name: ____ Date: ____ Period: ____

MA.8.AR.3.1, MA.8.AR.3.2, MTR.1.1, MTR.2.1, MTR.4.1

Flags

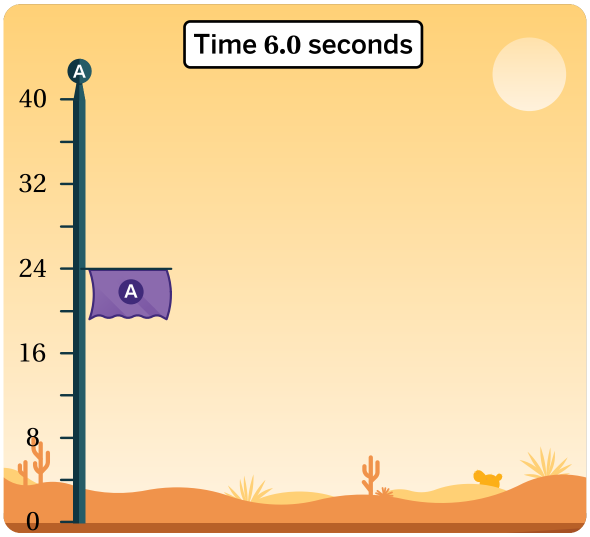
Let's explore non-proportional linear relationships.



Warm-Up

1. Let's think about the action of raising a team flag.

a Write a story about the picture.



b **Discuss:** How could the action of raising a flag be described as a proportional relationship?

Activity 1

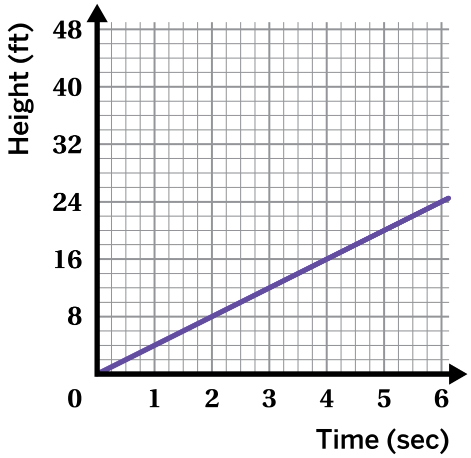
Name: .... Date: .... Period: ....

Forming an Equation

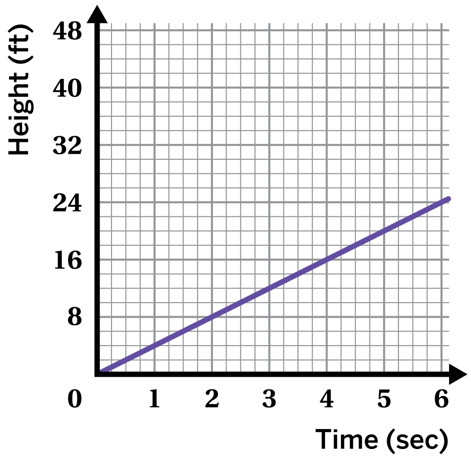
2. This line represents the relationship between height and time for the flag.

a What is the slope of the line?

b What does this number represent in the situation?



3. Write an equation for the flag's height, h, after t seconds.



Activity 2

Name: ________ Date: ________ Period: ________

Comparing Two Flags

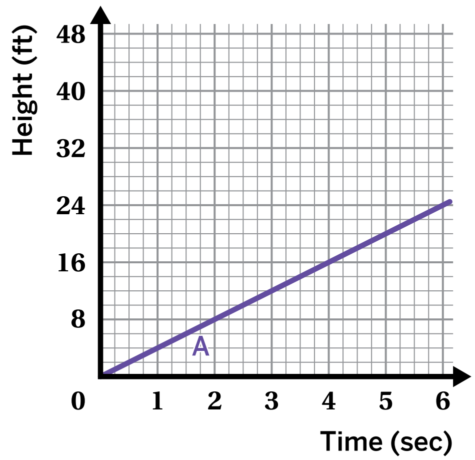
Let's explore the relationship between height and time for a new flag, Flag B.

4. Flag B begins at a height of 8 feet. It rises for 6 seconds and ends at a height of 20 feet.

Compare the behavior of Flag B to the behavior of Flag A.

5. Here is a graph representing the height of Flag A over time.

Draw a graph representing the height of Flag B over time.

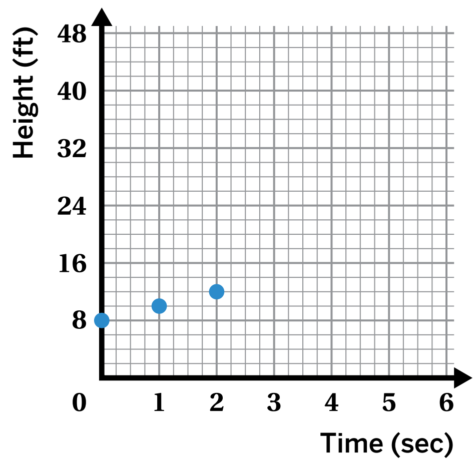


6. Here is a graph and a table with information about Flag B.

Cho claims: *Flag B rises at 2 feet per second, so its equation is $h = 2t$.*

Part of Cho's claim is correct and part of it is incorrect.

- What about Cho's claim is correct?



- What is incorrect?

Time (sec)	0	1	2
Height (ft)	8	10	12

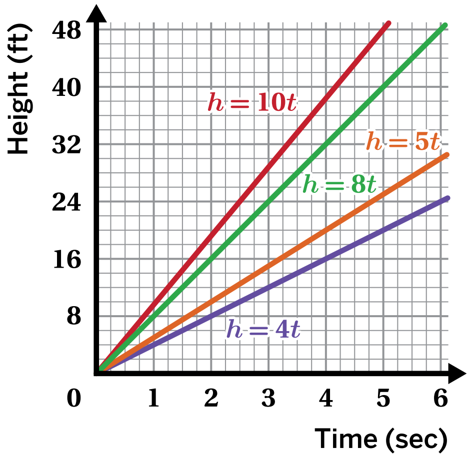
Name: .... Date: .... Period: ....

Exploring Linear Relationships

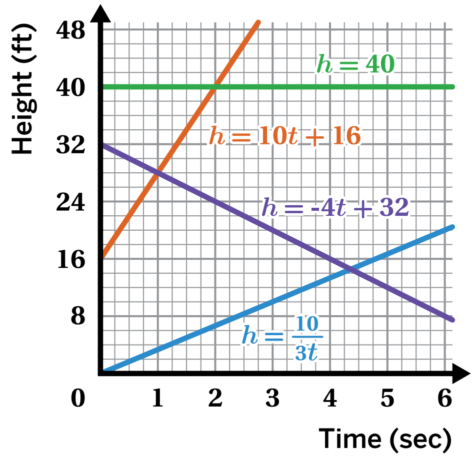
7. A relationship between two quantities is linear if there is a constant rate of change. It's called a **linear relationship** because its graph is a line.

Some linear relationships are proportional and some are non-proportional. Here are some examples.

Proportional Linear Relationships



Non-Proportional Linear Relationships



What are some differences between proportional and non-proportional linear relationships?

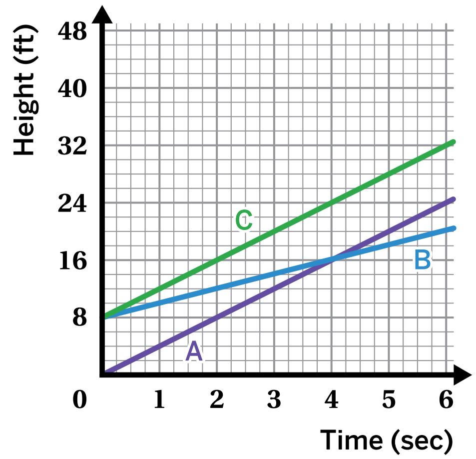
Name: .... Date: .... Period: ....

Exploring Linear Relationships (continued)

Now let's explore the relationship between height and time for a new flag, Flag C.

8. Based on the graph, select *all* the true statements.

- ☐ **A.** Flag C's graph is proportional.
- ☐ **B.** Flag C's graph is linear.
- ☐ **C.** The slope of Flag C's line is 8.
- ☐ **D.** Flag C rises at 8 feet per second.
- ☐ **E.** Flag C rises at 4 feet per second.



9. Write an equation for the height of Flag C.

Flag	Equation
A	$h = 4t$
B	$h = 2t + 8$
C	

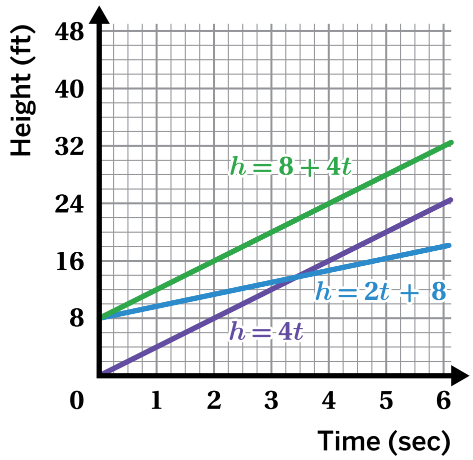
10. Can you write a new equation for Flag C so that the flag:

- Starts high?
- Starts low?
- Moves fast?
- Moves slow?

Synthesis

11. How can you identify a non-proportional linear relationship in a graph and in an equation?

Use the example if it helps with your thinking.



Lesson Practice ACC7.4.01

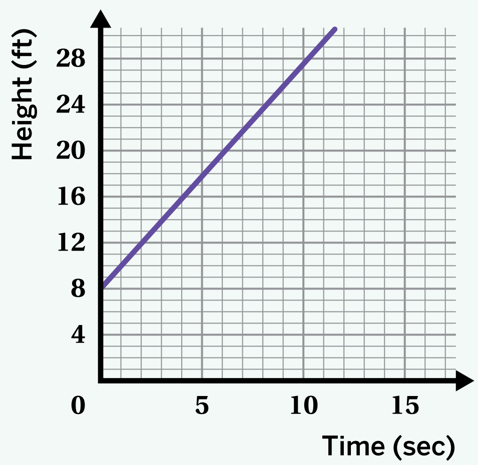
Lesson Summary

Linear relationships are graphs that are lines. Some linear relationships are *proportional relationships* and some are not. The graph of a proportional relationship passes through the points $(0, 0)$ and $(1, r)$, where r is the *unit rate*.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.



Lesson Practice
ACC7.4.01

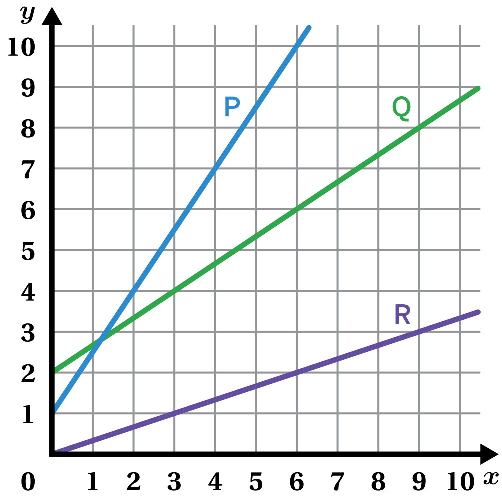
Name: ______________ Date: ________ Period: ________

1. Match each equation with the graph of its line.

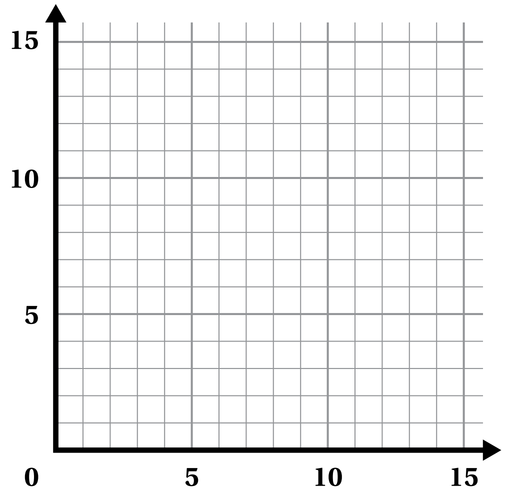
a $y = \frac{1}{3}x$ ____________

b $y = \frac{3}{2}x + 1$ ____________

c $y = \frac{2}{3}x + 2$ ____________



2. Create a graph that shows a non-proportional linear relationship with a slope of $\frac{1}{4}$. Then write an equation that represents the graph.



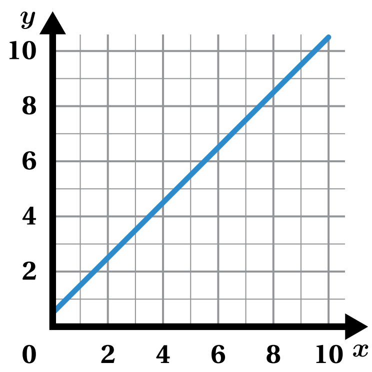
Problems 3–8: Determine whether each of these linear relationships is *proportional* or *non-proportional*.

3. From their initial step, a student walks at a constant speed of 4 miles per hour.

4. $y = 2x$

5. A giraffe at birth is 3 feet tall and grows 161 inches every month for a year.

6.



7.

x	y
2	4
3	5
5	7

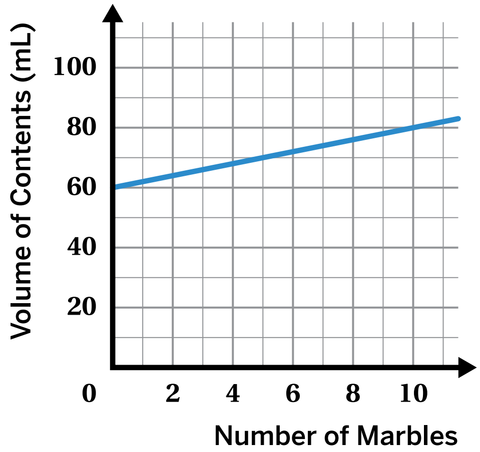
8. $y = 20 + 4x$

Name: .... Date: .... Period: ....

FAST Practice

9. In Aesop's fable "The Crow and the Pitcher," a thirsty crow discovers a pitcher of water but is unable to reach the water with its beak. So the crow adds pebbles to the pitcher to make the water rise higher and higher until it reaches the top. Then it can drink the water!

Thiago recreates this situation using a cylinder containing water and a bunch of marbles. The line represents the volume of the cylinder's contents, y, after x marbles are added.

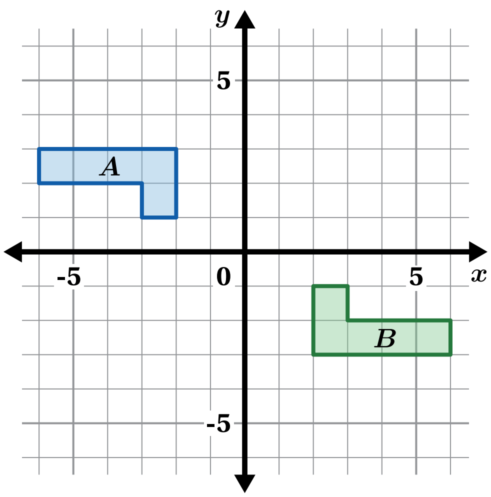


Determine whether each statement is *true* or *false*.

	True	False
The slope of the line is 2.	☐	☐
The volume of the contents increases by 2 milliliters for every 1 marble added.	☐	☐
The equation representing the relationship between x and y is $y = 60x + 2$.	☐	☐

Spiral Review

10. Are figures A and B congruent? Explain your thinking.



Name: ________ Date: ________ Period: ________

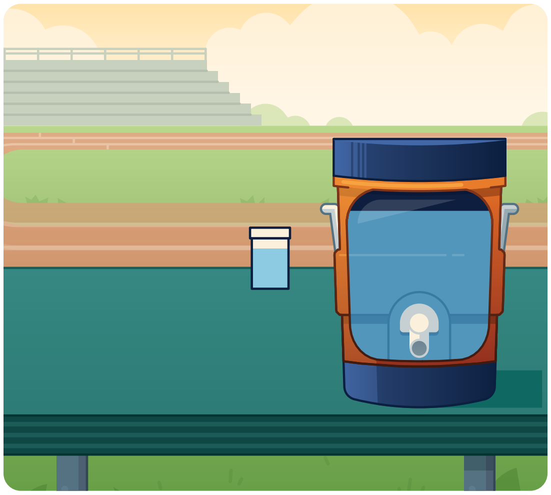
MA.8.AR.3.2, MA.8.AR.3.5, MTR.1.1, MTR.2.1, MTR.3.1

Water Cooler

Let's explore lines with different slopes.

Warm-Up

1. During a game, thirsty athletes pour water from a water cooler into cups.



a **Discuss:** What do you think happens to the water level in the cooler over time?

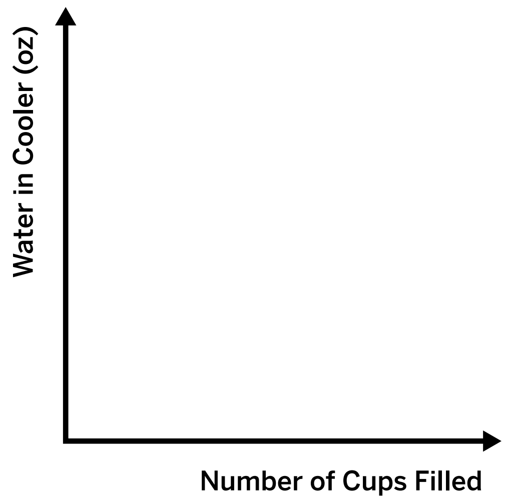
b Make an initial prediction. After how many cups do you think the cooler will run out of water?

Activity 1

Name: ______ Date: ______ Period: ______

Cooler Cups

2. **a** Sketch the relationship between the ounces of water remaining in the cooler and number of cups filled.



b **Discuss:** How can you see this relationship in your sketch?

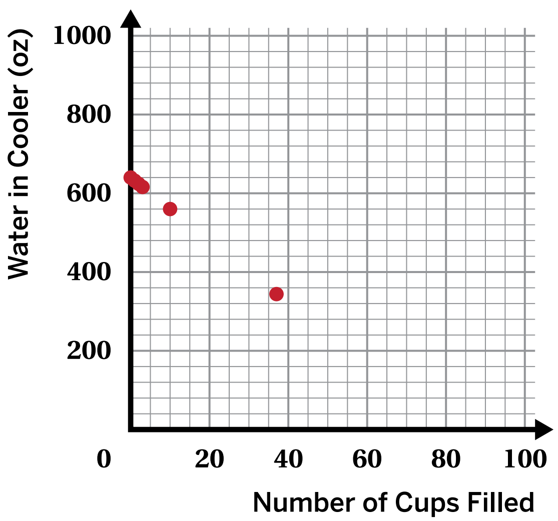
3. The table shows the amount of water remaining in the cooler after 0, 1, and 2 cups have been filled.

Determine the missing values.

Number of Cups Filled	0	1	2	3	. . .	10	37
Water in Cooler (oz)	640	632	624		. . .		

4. A classmate plotted their points from the table onto a graph.

Write an equation representing the amount of water in the cooler, y, after filling x cups.



Name: Date: Period:

Cooler Cups (continued)

5. Make a final prediction:

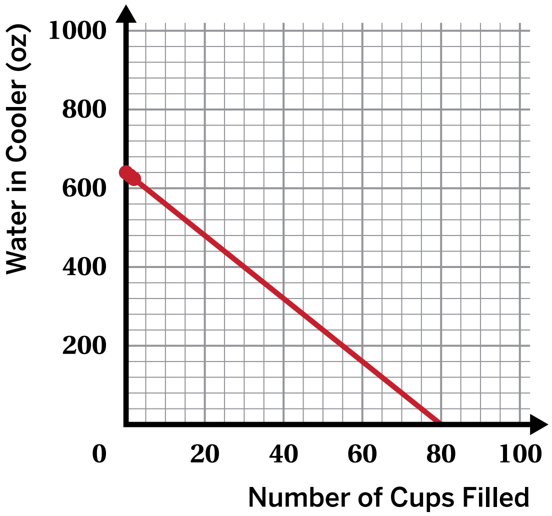
After how many cups will the cooler run out of water?

Explain your thinking.

6. Here is the equation that a classmate wrote for the relationship between the water in the cooler and the number of cups:

$y = 640 - 8x$

The **vertical intercept** is the point where the graph of a line crosses the vertical axis.



a What is the vertical intercept of this graph, and what does it represent in this situation?

b What is the slope of the line, and what does it represent in this situation?

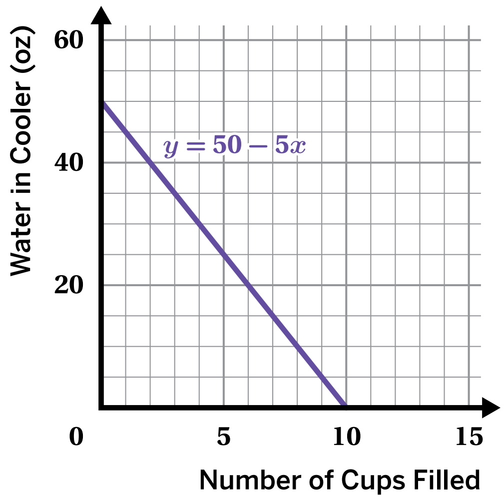
Activity 2

Name: .... Date: .... Period: ....

A New Water Cooler

The line $y = 50 - 5x$ represents a different water cooler situation.

7. Write a story that this graph could represent.



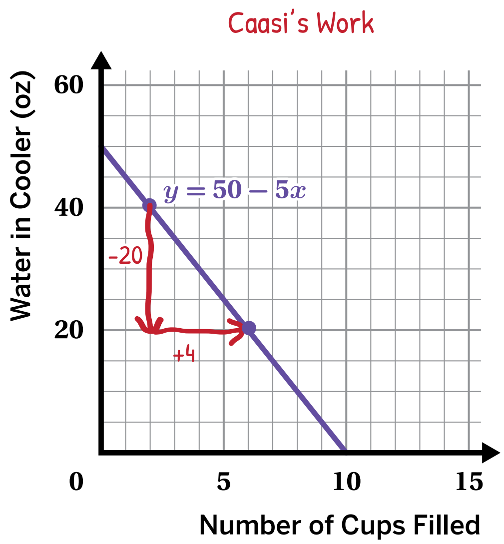
8. Caasi says that the water in the cooler decreases by 20 ounces for every 4 cups filled.

Jamar says that the water in the cooler decreases by 5 ounces for every 1 cup filled.

Whose claim is correct?

Caasi's Jamar's Both Neither

Explain your thinking.



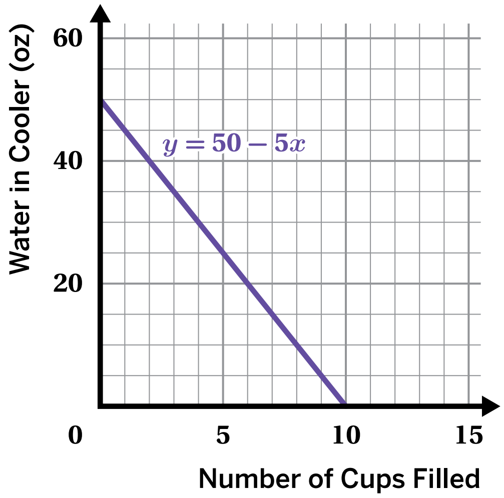
9. Look at the graph of $y = 50 - 5x$ from the previous problems. The line crosses the horizontal axis at the point $(10, 0)$. This point is called the **horizontal intercept**.

What does this intercept represent in this situation?

Name: .... Date: .... Period: ....

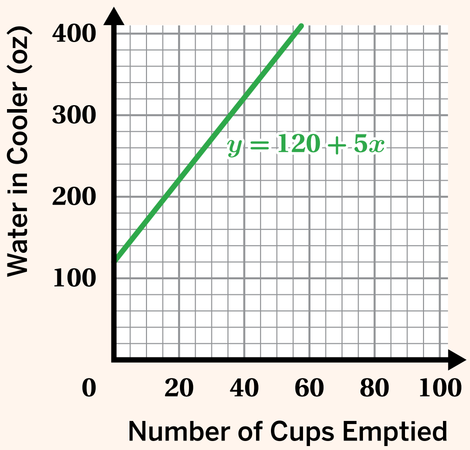
A New Water Cooler (continued)

10. Revise your original story so that it is stronger and clearer.



You're invited to explore more.

11. The line $y = 120 + 5x$ represents a third water cooler situation.



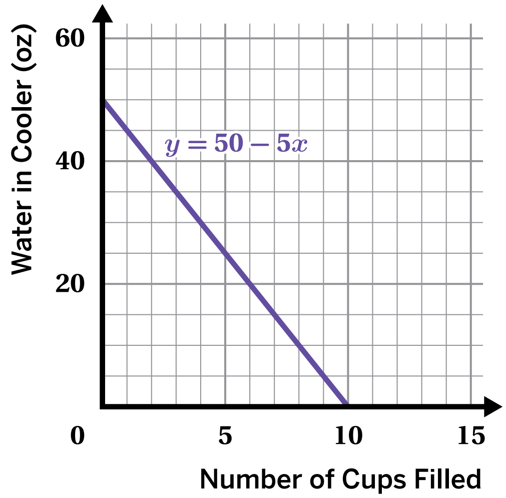
a. Write a story that this graph could represent.

b. How is this situation like the other two water cooler situations? How is it different?

Synthesis

12. What are 2–3 things you can determine about a situation from its graph?

Use the graph if it helps with your thinking.



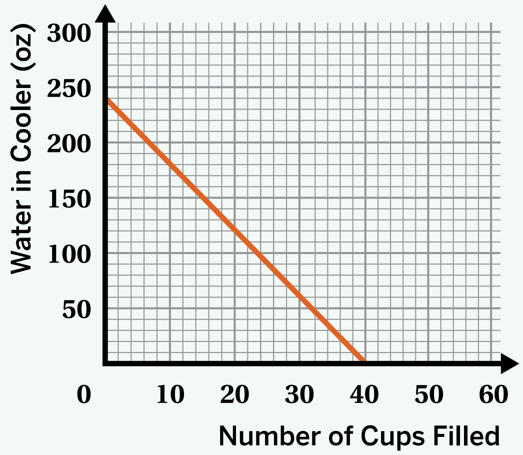
Lesson Practice ACC7.4.02

Lesson Summary

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

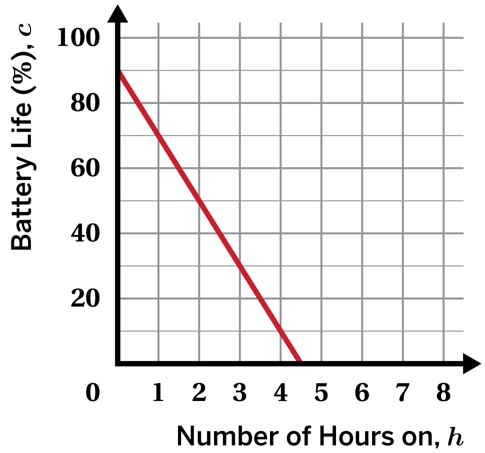
Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

- The **vertical intercept**, also called the y-*intercept*, is $(0, 240)$. In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the x-*intercept*, is $(40, 0)$. In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.



Name: .... Date: .... Period: ....

Problems 1–4: Tameeka is monitoring her computer's battery life. When the computer is on, the battery loses energy at a constant rate. This graph shows the percentage of battery life remaining, c, after the computer has been on for h hours.



1. What percent of battery life was left when Tameeka turned on the computer?

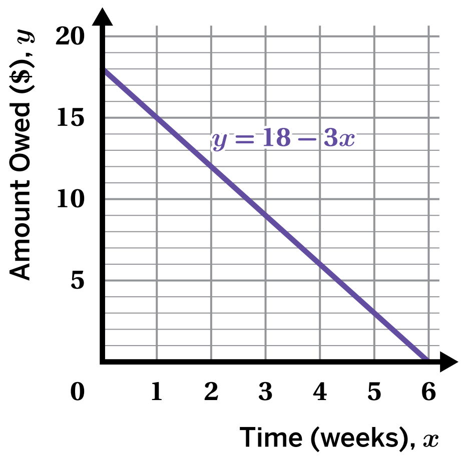
2. Complete the table.

Number of Hours On	0	1	2	. . .	3.5
Battery Life, %	90	70		. . .	

3. Write an equation that represents the percentage of battery life remaining, c, after the computer has been on for h hours.

4. After how many hours of being on will Tameeka's computer's battery life run out?

Problems 5–6: Juliana borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes him, y, after each week, x.



5. What is the vertical intercept and what does it represent in this situation?

6. What is the horizontal intercept and what does it represent in this situation?

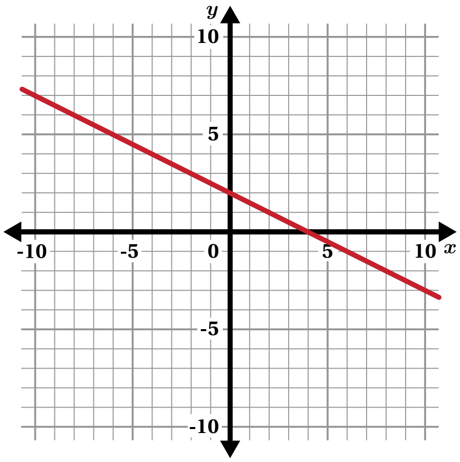
Lesson Practice
ACC7.4.02

Name: ______________ Date: ________ Period: ________

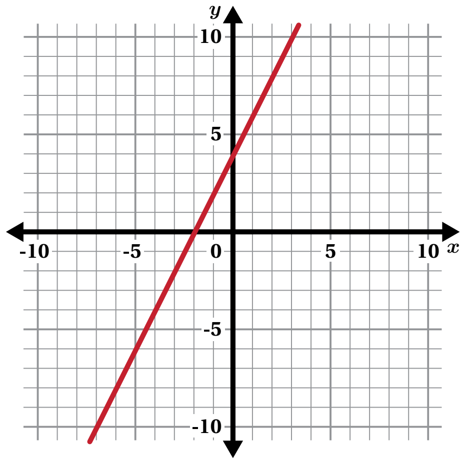
FAST Practice

7. Which graph has a slope of -2?

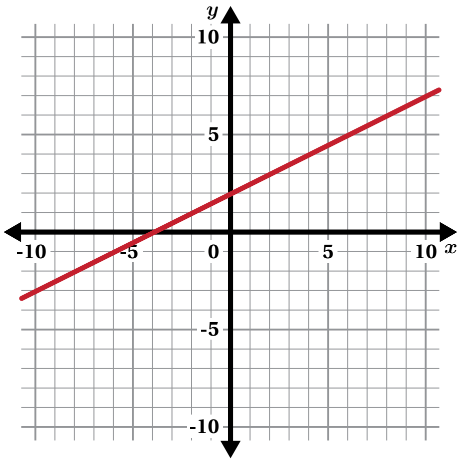
A.



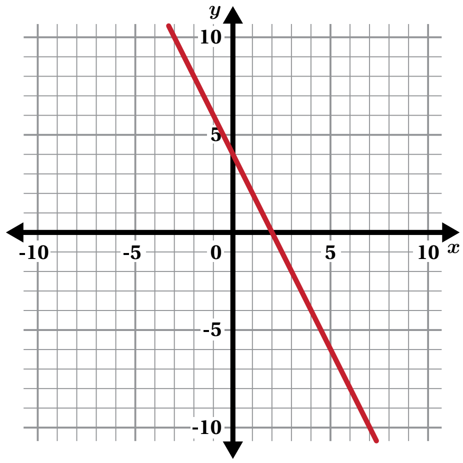
B.



C.



D.

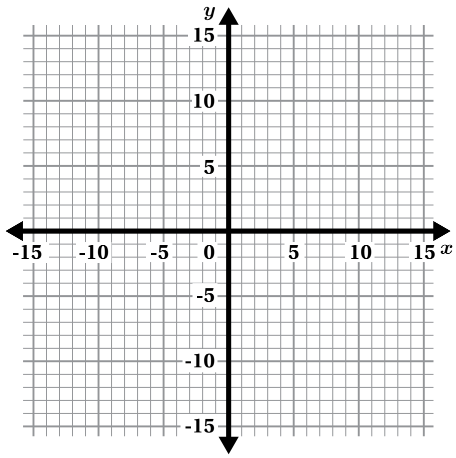


Spiral Review

Problems 8–9: Determine the coordinates of the image after the point is dilated using the origin as the center of dilation and a scale factor of 2.5.

8. $(1, 0)$

9. $(-3, 6)$



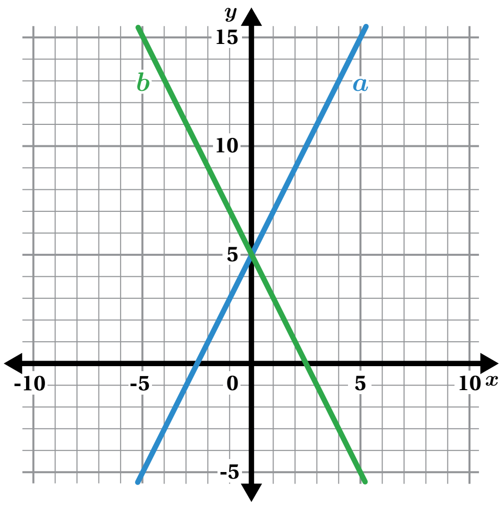
Name: .... Date: .... Period: ....

MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MTR.2.1, MTR.4.1

Ups and Downs

Let's explore more linear relationships in context.

Warm-Up



1. How are the two lines alike?

2. How are they different?

Activity 1

Name: .... Date: .... Period: ....

Two Transportation Cards

Two students have different cards they use to pay for public transportation.

3. Complete the table to show the amount of money remaining on the card for each student after several rides.

Student 1: The transportation card starts with $\$30$. Each ride costs $\$2$.

Number of Rides, x	Amount Remaining on Card ($), y
0	30
1	
2	
3	
4	

Student 2: The transportation card starts with $\$50$. Each ride costs $\$3$.

Number of Rides, x	Amount Remaining on Card ($), y
0	50
1	
2	
3	
4	

4. Write an equation that represents the amount on the card y, in dollars, after x rides.

Student 1	
Student 2	

5. How much money will *each* student have on their card after 10 rides? Show your thinking.

Activity 2

Name: .... Date: .... Period: ....

Matchmaker

6. You will get a set of graphs. Match each graph with its corresponding situation.

Situation	Graph
a A bike rental company charges a one-time fee of \$30 and then \$5 per hour to use a bike. Let y represent the total amount a customer pays in dollars after x hours.	
b Chef Kawbena purchases 30 gallons of tomato sauce for her restaurant. She uses 5 gallons of tomato sauce each day. Let y represent the number of gallons of tomato sauce remaining after x days.	
c A candle is 30 centimeters tall and burns at a constant rate of 1 centimeter every 5 minutes. Let y represent the height of the candle in centimeters after x minutes.	
d Ayaan starts with \$5 in his savings account. He adds \$30 to his account each month and doesn't take out any money. Let y represent the total amount of money in Ayaan's savings account after x months.	

Name: .... Date: .... Period: ....

Matchmaker (continued)

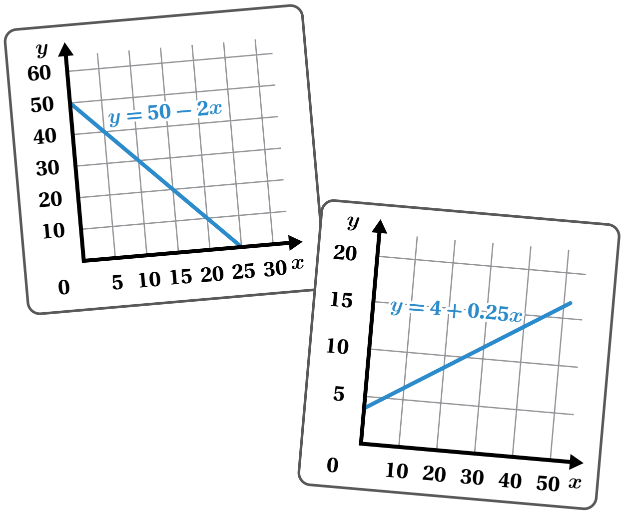
7. Complete the table.

Situation	What is the slope? What does it represent in the situation?	What is the vertical intercept? What does it represent in the situation?	Write an equation that represents the situation.
a			
b			
c			
d			

Synthesis

8. How can you determine whether a situation has a positive or negative slope?

Use the example if it helps with your thinking.



Lesson Practice ACC7.4.03

Lesson Summary

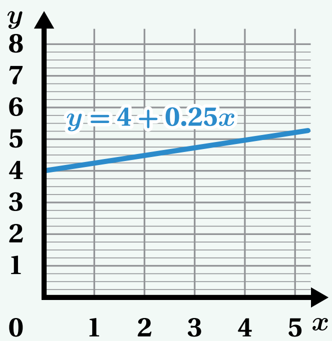
Equations of linear relationships, or linear equations, can be written in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.

For linear relationships with a *positive* slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a *negative* slope, the y-values decrease at a constant rate as the x-values increase.

Here are two examples.

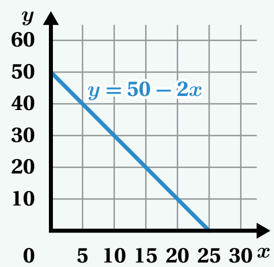
Positive slope: A medium-sized frozen yogurt costs $4, plus $0.25 per topping.

Let y represent the total cost of the frozen yogurt after adding x toppings.



Negative slope: A student loads an arcade game card with $50. Every time she plays a game, $2 is subtracted from the amount available on the game card.

Let y represent the amount in dollars on the card after the student plays x games.

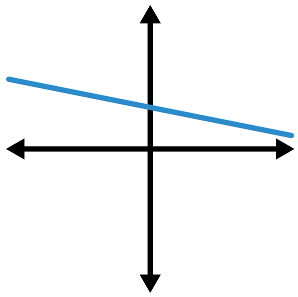


Lesson Practice
ACC7.4.03

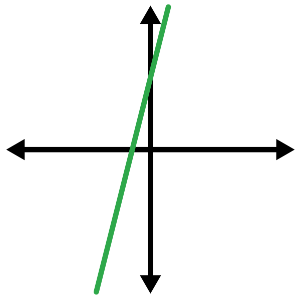
Name: .... Date: .... Period: ....

Problems 1–3: Determine whether the slope of each line is *positive* or *negative*.

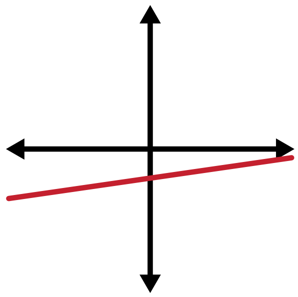
1.



2.



3.



Problems 4–5: The elevation of an airplane and its time since takeoff can be modeled by a linear equation. The horizontal axis represents time in minutes and the vertical axis represents elevation in feet. For each situation, determine whether the slope is positive or negative.

4. The plane descends at a rate of $1{,}000$ feet per minute.

5. The plane ascends at a rate of $2{,}000$ feet per minute.

Problems 6–7: Ella borrowed $\$40$ from her brother. She pays him back by giving him $\$5$ each week.

6. Complete the table to show the amount Ella owes each week.

7. Write an equation that represents the amount that Ella owes, in dollars, after x weeks.

Time (weeks)	Amount Owed ($)
0	40
1	
2	
3	

Name: ________ Date: ________ Period: ________

8. y is the total cost of grapes after purchasing x ounces of grapes.

Sketch a graph that could represent this situation.

FAST Practice

9. An acorn falls 20 feet from a tree to the ground at a rate of 4 feet every 2 seconds.

Determine whether each statement is *true* or *false*.

	True	False
The initial height of the acorn is 20 feet.	☐	☐
The acorn falls at a rate of $\frac{1}{2}$ foot every second.	☐	☐
The acorn is 5 feet above the ground after 10 seconds.	☐	☐

Spiral Review

10. The vertices of triangle ABC are $A(12,6)$, $B(1, 3)$, and $C(18, 0)$. Triangle ABC is dilated by a scale factor of $\frac{1}{3}$ using point $(0,0)$ as the center of dilation, resulting in triangle JKL. What are the coordinates of the vertices of triangle JKL?

Unit 4
Lesson
4

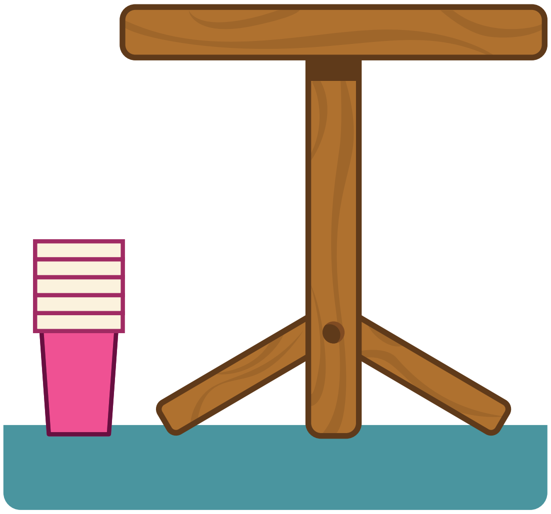
Name: ______ Date: ______ Period: ______

MA.8.AR.3.2, MA.8.AR.3.4, MA.8.AR.3.5, MTR.5.1, MTR.7.1

Stacking Cups

Let's use a linear relationship to make a prediction.

Warm-Up



1. **a** How many stacked cups do you think you need to reach the top of this table?

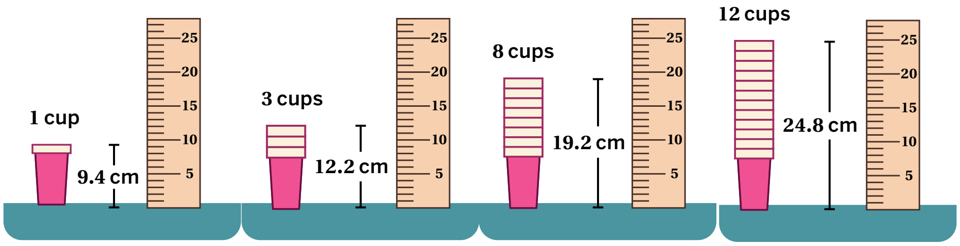
b What information would help you make a more precise prediction?

Activity
1

Name: Date: Period:

Stacking Cups

2. Let's look at different stacks of cups.



Record *at least two* data points.

Number of Cups	Height (cm)

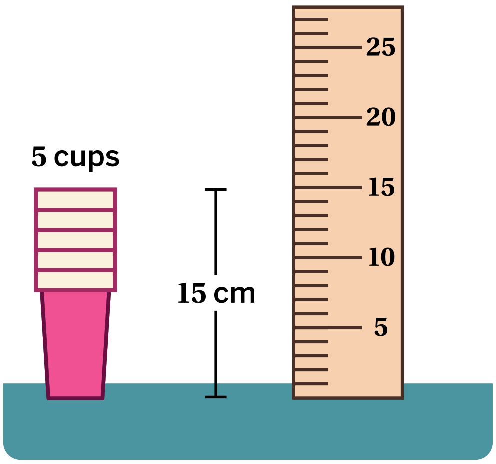
3. Sylvia found that a stack of 5 cups has a height of 15 centimeters.

She thinks that a stack of 10 cups will have a height of 30 centimeters.

Is she correct? Circle one.

Yes No

Explain your thinking.



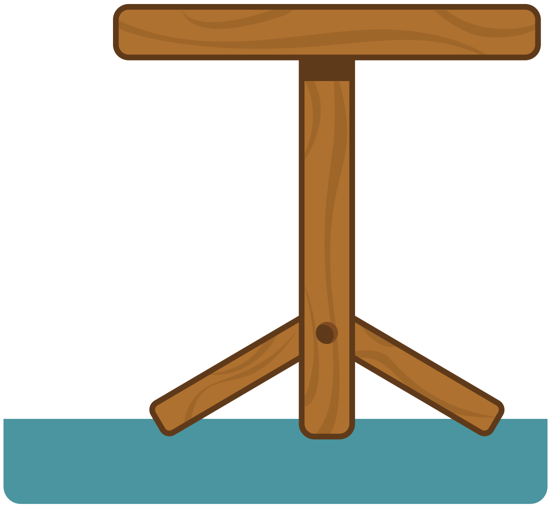
Name: Date: Period:

Stacking Cups (continued)

4. The height from the floor to the top of the table is 50 centimeters.

Previously, you predicted a number of cups to reach the top of the table.

Now that you have more information, calculate the exact number of stacked cups you need.



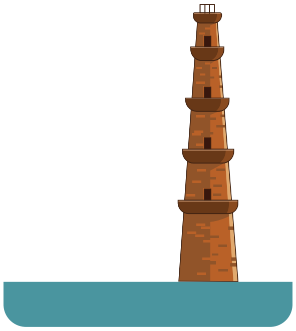
Activity 2

Name: .... Date: .... Period: ....

More and More Cups

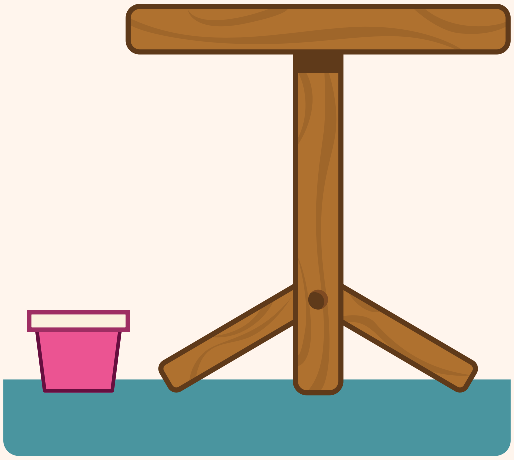
5. This minaret is 42.5 meters (or $4,250$ centimeters) tall.

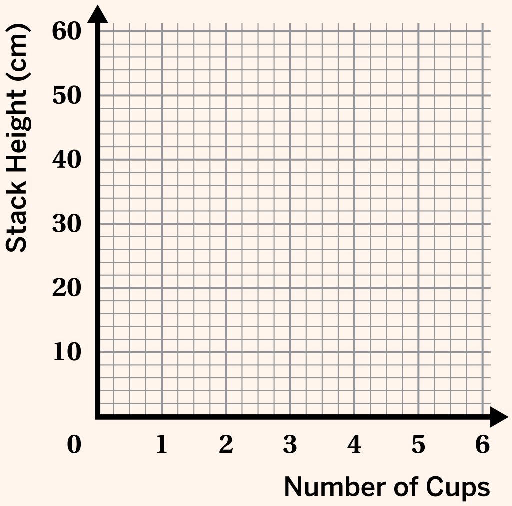
How many stacked cups do you need to reach the top of the minaret?



You're invited to explore more.

6. What cup and lip heights would you use so that 5 of your cups will reach the exact height of the table (50 centimeters)? Use the graph if it helps with your thinking.





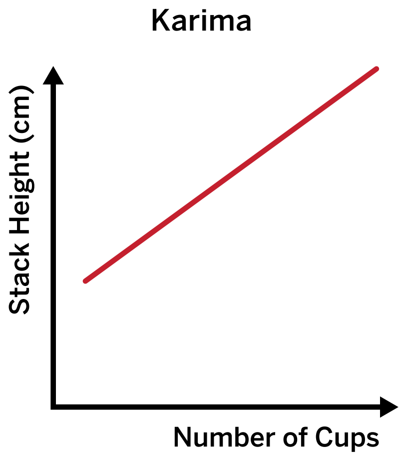
Activity 3

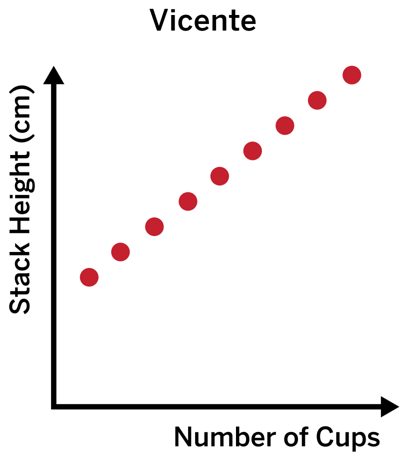
Name: Date: Period:

Modeling the Relationship

7. Graphs can help us understand a relationship better.

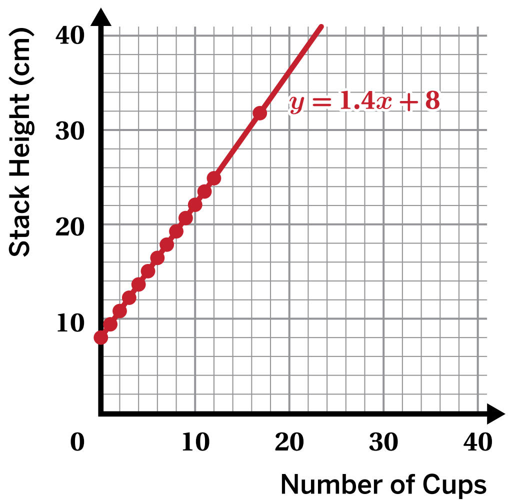
Karima and Vicente each made a graph.





When might Karima's graph be useful? When might Vicente's graph be useful?

8. Here is a graph with a line and an equation that represent the relationship between number of cups and stack height.

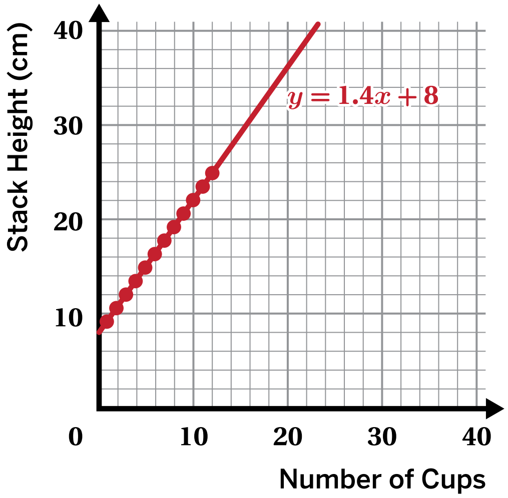


Discuss:

- What is the slope of the line? What does it represent in this situation?
- What is the y-intercept (vertical intercept) of the line? What does it represent in this situation?

Synthesis

9. How can you use a linear relationship to model a situation and make predictions?



Lesson Practice ACC7.4.04

Lesson Summary

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the other variable.

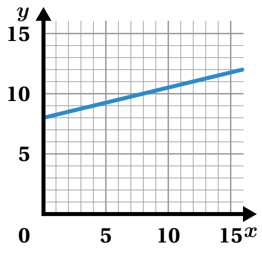
Let's say we want to know how many stacked cups will have a height of 50 centimeters. We can look at a graph and determine the number of cups, x, that correspond to $y = 50$. The slope of the line tells us how many centimeters the height of the stacked cups increases by every time we add a cup to the stack.

We might think of the y-intercept as the stack height when there are 0 cups, but this doesn't make sense in this situation. Instead, the y-value tells us the distance from the bottom of the first cup to its rim. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

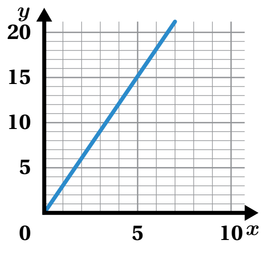
Name: .... Date: .... Period: ....

Problems 1–4: For each real-world situation, choose the graph that best represents it.

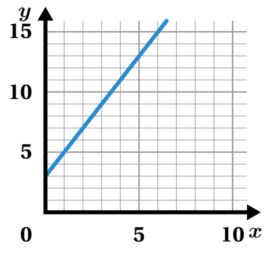
Graph A



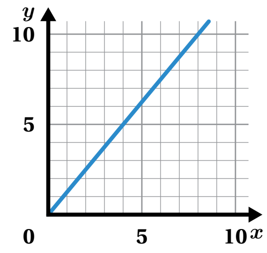
Graph B



Graph C



Graph D



1. y represents the perimeter of an equilateral triangle and x represents its side length. The slope of the line representing the relationship between x and y is 3.

2. y represents the amount of money collected after x raffle tickets are purchased. The slope of the line representing the relationship between x and y is 0.25.

3. y represents the number of chapters a student has read after x days. The slope of the line representing the relationship between x and y is $\frac{5}{4}$.

4. y represents the total cost, in dollars, of x blueberry muffins, including a delivery fee. The slope of the line representing the relationship between x and y is 2.

5. At a carnival, y represents the amount of money in a cash box after x tickets are purchased for carnival games. The line representing the relationship between x and y has a slope of $\frac{1}{4}$ and a y-intercept of 8. Explain what the slope and y-intercept represent in this situation.

Lesson Practice
ACC7.4.04

Name: ________________ Date: ________ Period: ________

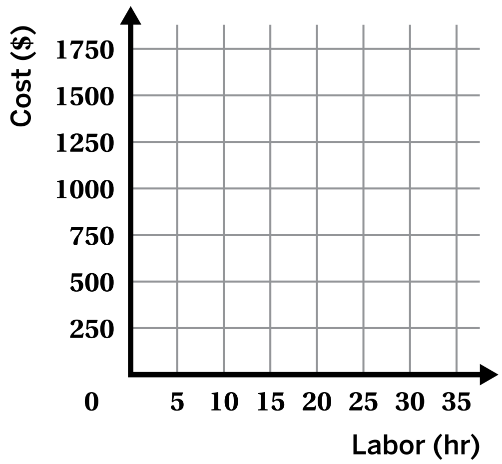
FAST Practice

Problems 6–9: To paint a house, a painting company charges a flat rate of $500 for supplies, plus $50 for each hour of labor.

6. How much would the painting company charge to paint a house that needs 20 hours of labor? 50 hours of labor? Write your answers in the table.

Labor (hr)	Cost ($)
20	
50	

7. Draw a line representing the relationship between the number of hours of labor needed to paint the house and the total cost of paint.



8. Plot a point to show the cost of 20 hours of labor.

9. What is the slope of the line? What does it represent?

The slope of the line is **A.** 5 **B.** 50 **C.** 500 .

It represents the **A.** cost per hour **B.** initial cost **C.** cost for 10 hours .

Spiral Review

10. Here is a table showing the values of a linear relationship. Write an equation to represent this relationship.

x	0	1	2
y	17	12	7

11. Write the equation of a line that has a slope of 2 and a vertical intercept of 8.

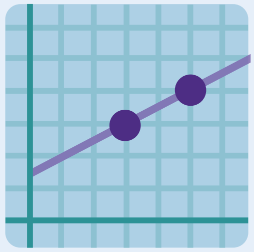
Sub-unit

2

Linear Equations and Points

Lesson 5
Landing Planes

Lesson 6
Coin Capture



Lesson 7
Why Intercepts?

Lesson 8
Pennies and Quarters

Name: ________ Date: ________ Period: ________

MA.8.AR.3.2, MTR.3.1, MTR.5.1

Landing Planes

Let's think about strategies for calculating slope.

Warm-Up

1. Determine possible values for each variable to make the equations true.

$$\frac{a}{b} = \text{-}2$$

$$\frac{m}{n} = 2$$

$$q - r = \text{-}2$$

Name: .... Date: .... Period: ....

Planes, Lines, and Slopes

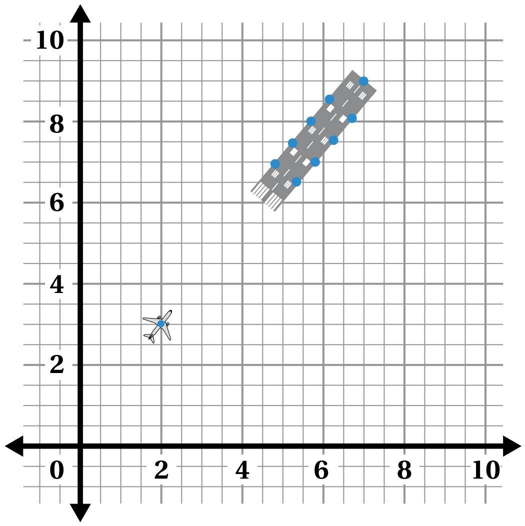
2. Predict whether the slope of the line through each set of points is positive, negative, or zero.

- $(600, 500)$ and $(400, 500)$
- $(7, 1)$ and $(12,7)$
- $(10, 40)$ and $(30, 20)$

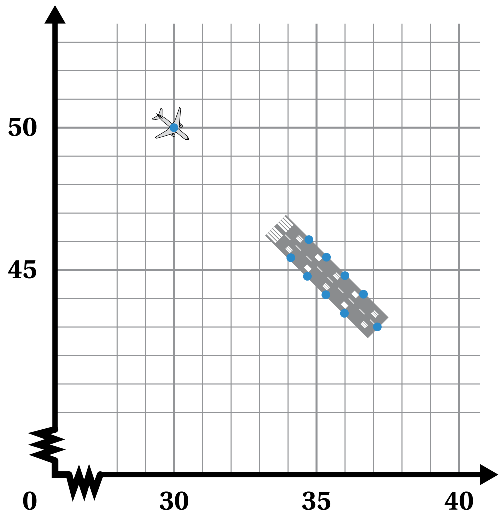
Positive Slope	Negative Slope	Zero Slope

3. Your task is to land the plane.

To do that, calculate the slope of the line that goes through $(2, 3)$ and $(7, 9)$.



4. To land the plane, calculate the slope of the line that goes through $(30, 50)$ and $(37, 43)$.

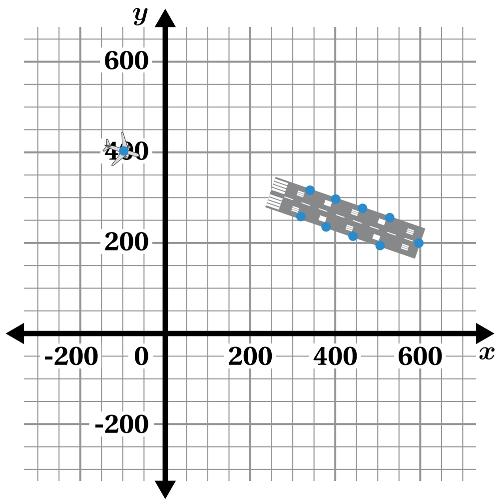


Activity 2

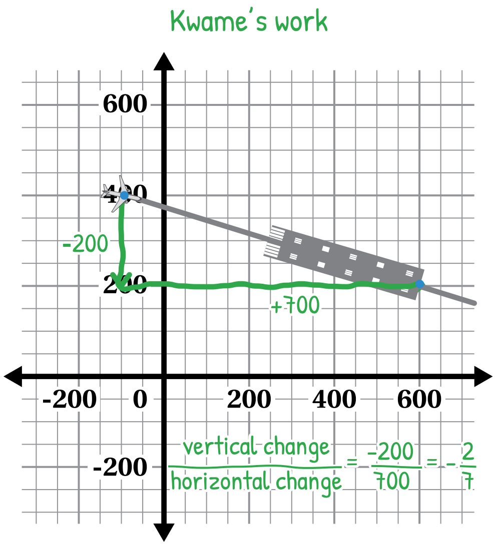
Name: ________ Date: ________ Period: ________

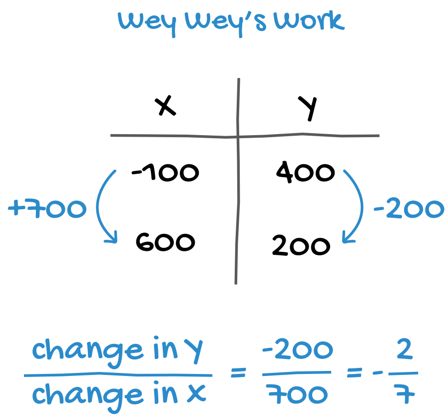
Strategies for Calculating Slope

5. To land the plane, calculate the slope of the line that goes through (-100, 400) and (600, 200).



6. Here are two students' strategies for calculating the slope of the line that goes through (-100, 400) and (600, 200).





Discuss: How are their strategies alike? How are they different?

Activity 3

Name: ____________ Date: ____________ Period: ____________

Challenge Creator

7. **a** **Make It!** Create your own plane-landing challenge.

- Choose a pair of coordinates to represent the starting position (Point 1) and ending position (Point 2) of your plane. Record the points on this page.
- On graph paper, choose a scale and label your axes, then plot your two points. Label the points with their coordinates.

b **Solve It!** On this page, calculate the slope that passes through your two points.

c **Swap It!** Trade graphs with one or more partners. On this page, calculate the slope of the line that passes through their two points.

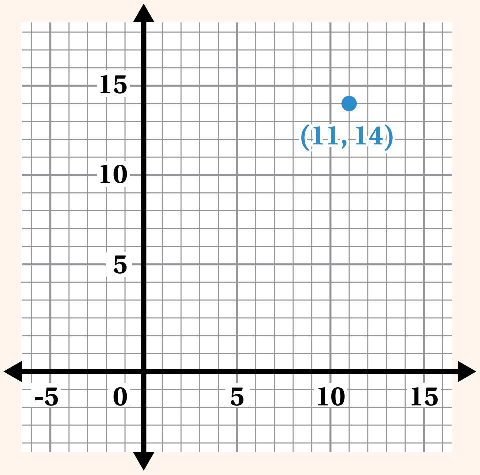
d With each partner, compare your strategies for calculating the slope of a line that passes through two given points.

My Challenge	Partner 1's Challenge	Partner 2's Challenge
Point 1: ____________	Point 1: ____________	Point 1: ____________
Point 2: ____________	Point 2: ____________	Point 2: ____________
Slope: ____________	Slope: ____________	Slope: ____________

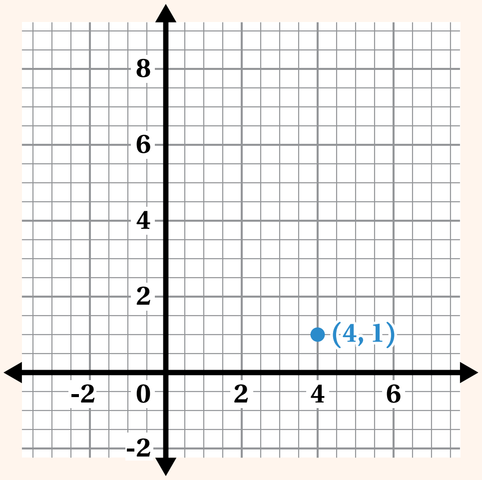
You're invited to explore more.

8. We know the slope of the path of each of these planes, but not their starting positions. Determine the value of p so that the line passing through the points has the indicated slope.

a Plane A starts at $(p, 2)$ and stops at $(11, 14)$. Its path has a slope of 2.



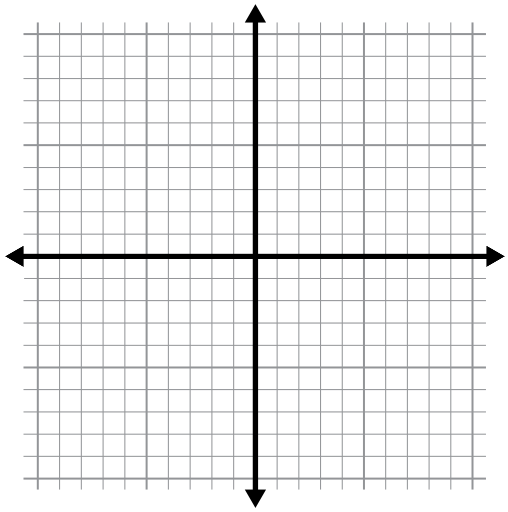
b Plane B starts at $(1, p)$ and stops at $(4, 1)$. Its path has a slope of -2.



Synthesis

9. What are some strategies for finding the slope of a line that goes through two given points?

Use the graph if it helps to show your thinking.



Lesson Practice ACC7.4.05

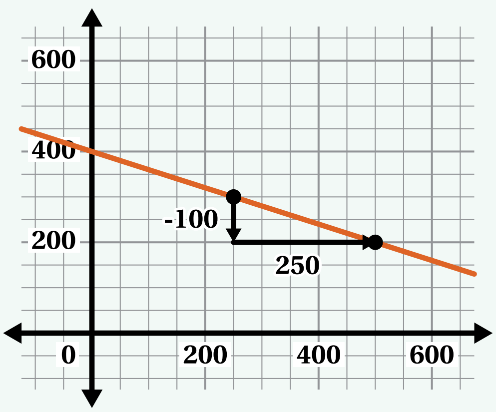
Lesson Summary

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane. You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

The slope is the ratio of the vertical change to the horizontal change.

Using Slope Triangles



$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= -\frac{100}{250}$$

$$= -\frac{2}{5}$$

Using Coordinates in a Table

	x	y	
+250	250	300	-100
	500	200	

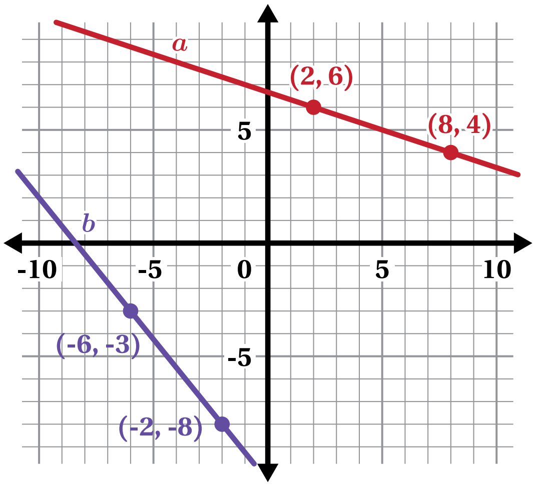
$$\frac{\text{change in } y}{\text{change in } x} = \frac{-100}{250} = -\frac{2}{5}$$

Name: .... Date: .... Period: ....

1. Calculate the slope of each line.

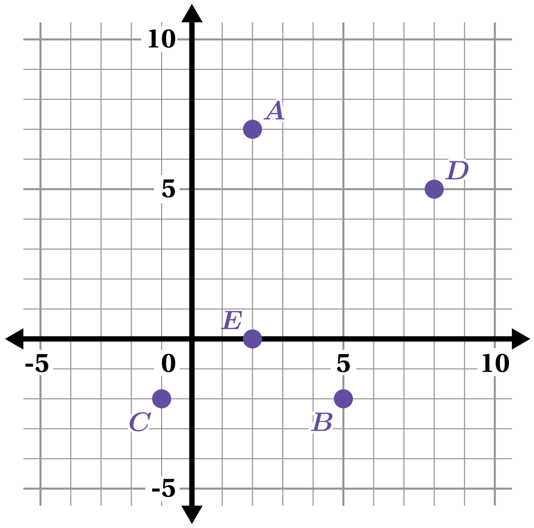
   line a: ....

   line b: ....



2. Draw a line with a slope of $-\frac{1}{3}$ that passes through point A.

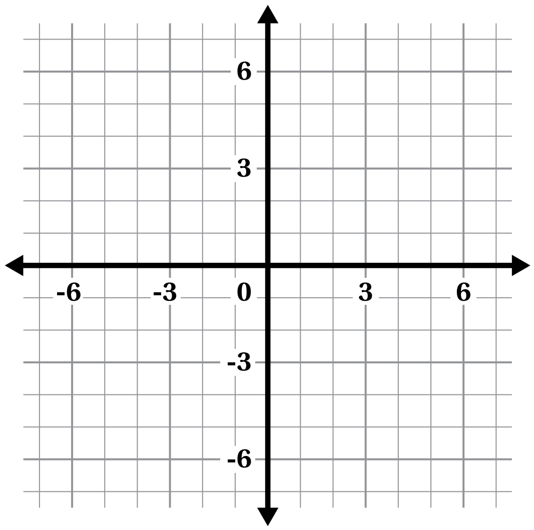
   What other point lies on that line?



Problems 3–4: Here is a blank graph.

3. Draw a line with a slope of 4 and a negative y-intercept.

4. Explain how you know the slope of your line is 4.

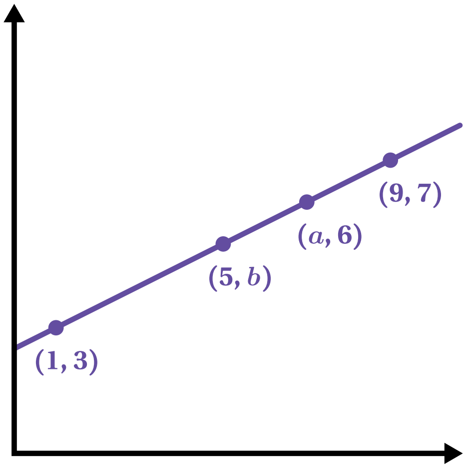


Lesson Practice
ACC7.4.05

Name: .... Date: .... Period: ....

FAST Practice

Problems 5–7: All the points in this graph are on the same line.



5. What is the slope of the line? Explain your thinking.

6. What are the values for a and b?

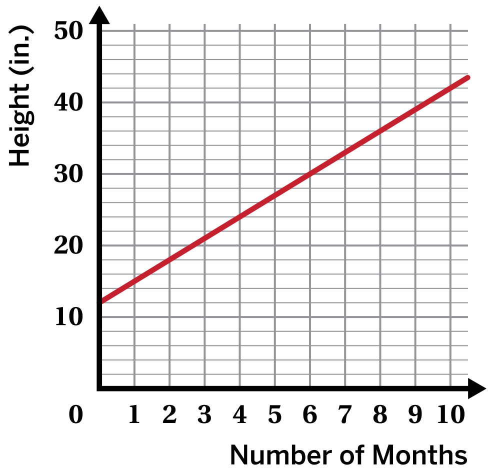
7. What is the x-value when $y = 0$?

Record your answer in the space provided.

$x = \boxed{}$

Spiral Review

Problems 8–9: The graph shows the height of a bamboo plant, h, n months after it has been planted.



8. Write an equation that gives the bamboo's height, h, after n months.

9. How many inches tall will the bamboo plant be after 18 months? Show your thinking.

Unit 4
Lesson
6

Name: ______ Date: ______ Period: ______

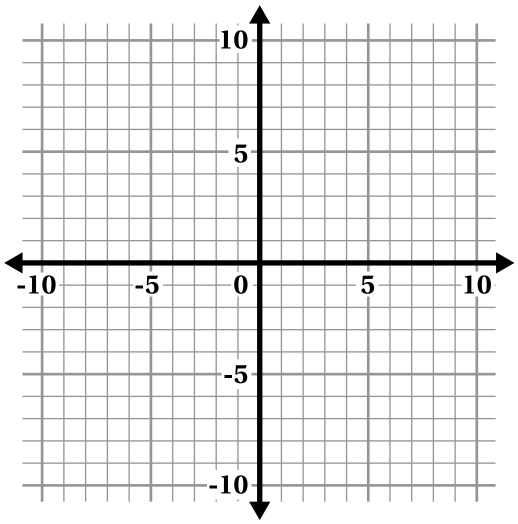
MA.8.AR.3.2, MA.8.AR.3.3, MTR.4.1, MTR.5.1

Coin Capture

Let's write equations for vertical and horizontal lines.

Warm-Up

1. **a** Plot four points in different locations. The x-coordinate of each point should be 7.



   b **Discuss:** What would your and your classmates' points look like if they were all on one graph?

2. Write an equation to represent all the points with an x-coordinate of 7.

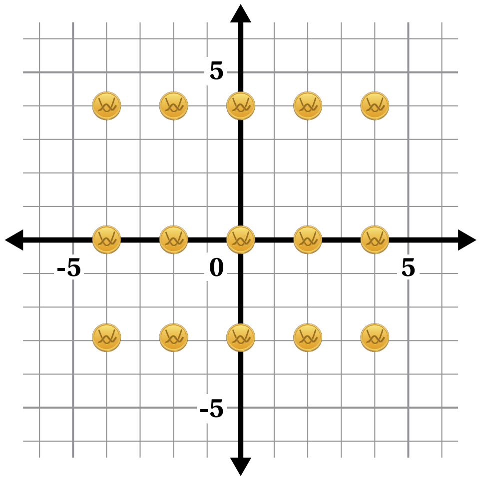
Name: .... Date: .... Period: ....

Capture the Coins

3. **a** Draw lines through the coins to “capture” them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

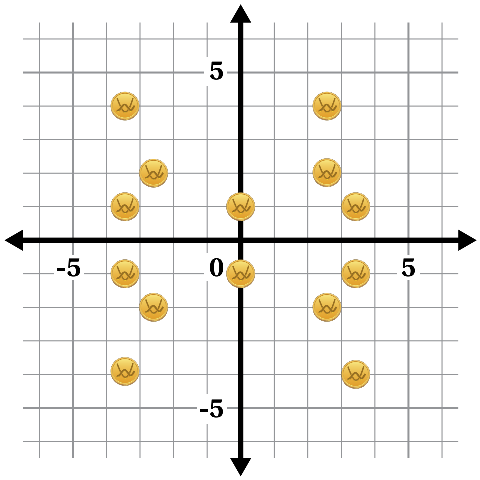
Equations:



4. **a** Draw lines through the coins to “capture” them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

Equations:



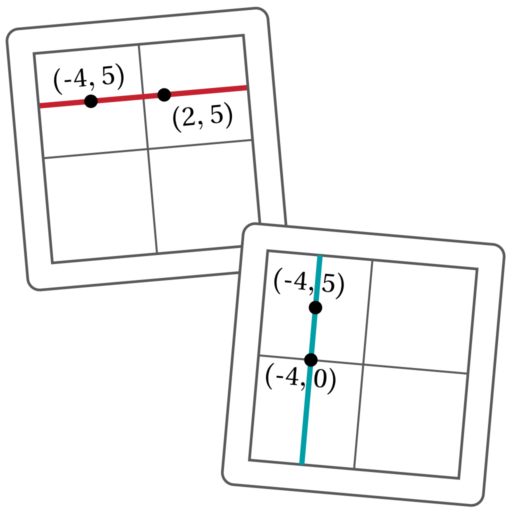
5. Juana says that vertical lines have a slope of zero.

Nekeisha says that horizontal lines have a slope of zero.

Whose claim is correct?

Juana’s Nekeisha’s Both Neither

Explain your thinking.



Activity 2

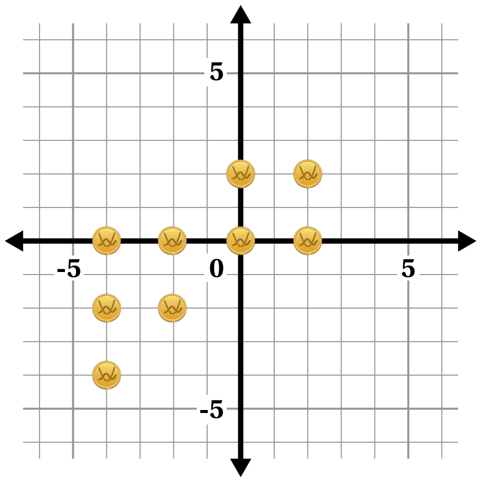
Name: ______ Date: ______ Period: ______

Challenge Creator

6. **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

Equations:



7. Create your own Coin Capture challenge!

a **Make It!** Use the Activity 2 Sheet to create your challenge.

b **Solve It!** On this page, write equations for the lines you would use to capture all the coins in your challenge. Try to use as few lines as you can.

c **Swap It!** Trade graphs with a partner and solve each other's challenges.

My Challenge

Equations:

Partner 1's Challenge

Equations:

Partner 2's Challenge

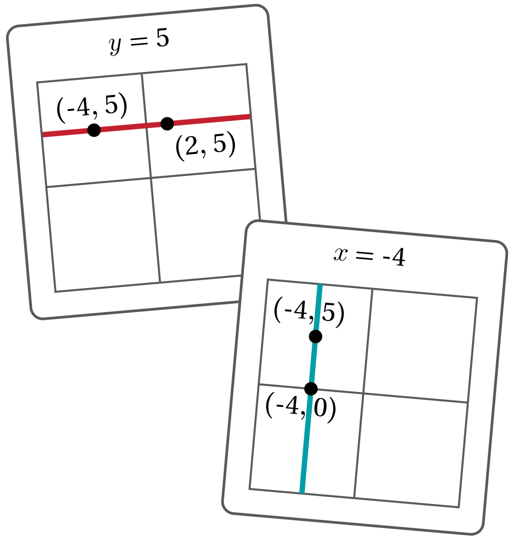
Equations:

Partner 3's Challenge

Equations:

Synthesis

8. How can you tell from looking at a linear equation if its graph is a horizontal or vertical line?



Lesson Practice ACC7.4.06

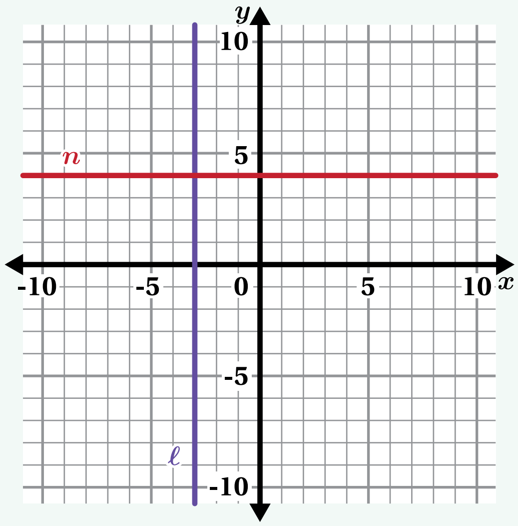
Lesson Summary

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

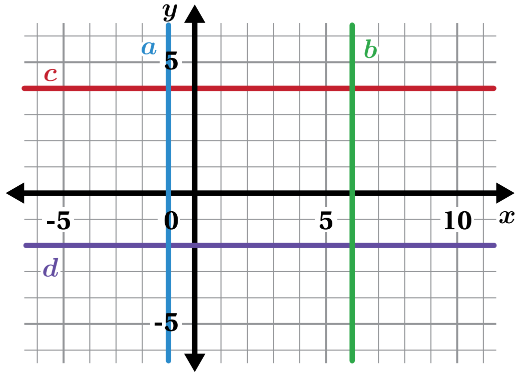
For example, the equation $y = 4$ represents the horizontal line n because every point on the line has the same y-coordinate, 4.

The equation $x = -3$ represents the vertical line ℓ because every point on the line has the same x-coordinate, -3.



Name: .... Date: .... Period: ....

Problems 1–4: Here are four lines on a coordinate plane. Write an equation for each line.



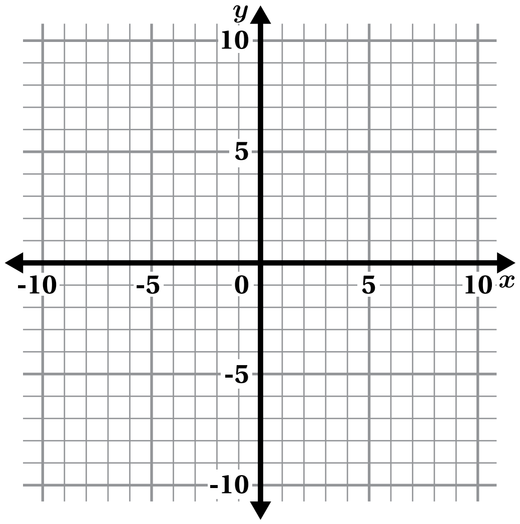
1. line a: ....

2. line b: ....

3. line c: ....

4. line d: ....

Problems 5–6: Use the coordinate plane to draw lines that meet the given slopes and y-intercepts. Then write an equation for each line.



5. Slope: 0
y-intercept: -2

Equation: ....

6. Slope: 2
y-intercept: -1

Equation: ....

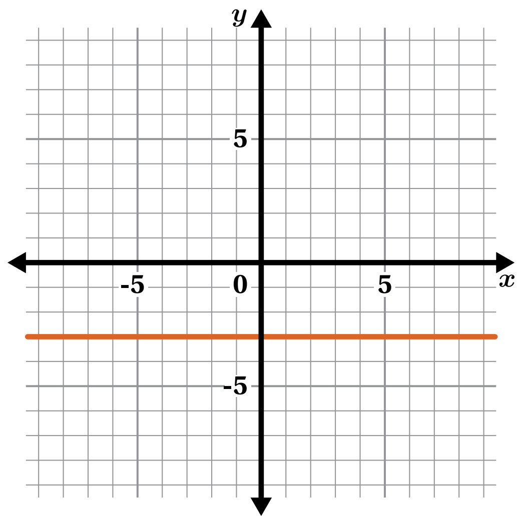
7. This graph represents a linear relationship. Choose the equation it matches.

A. $x = 3$

B. $x = -3$

C. $y = 3$

D. $y = -3$



Name: .... Date: .... Period: ....

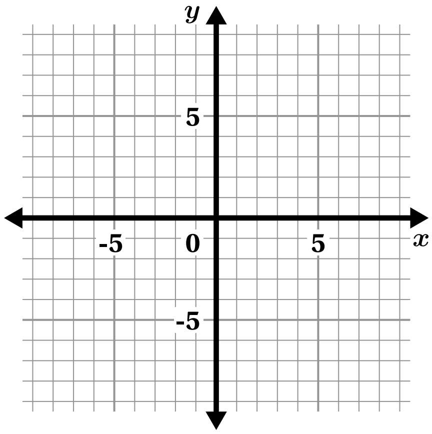
8. Write an equation for the line that passes through the points $(4, 3)$ and $(4, 15)$.

FAST Practice

9. Write an equation for the line that passes through the points $(1, -6)$ and $(-6, -6)$. Record your answer in the space provided.

Spiral Review

10. Graph the equation $y = -x - 5$.



11. Select *all* the pairs of points that have lines passing through them with a slope of $\frac{2}{3}$.

- ☐ **A.** $(0, 0)$ and $(2, 3)$
- ☐ **B.** $(0, 0)$ and $(3, 2)$
- ☐ **C.** $(1, 5)$ and $(4, 2)$
- ☐ **D.** $(-2, -2)$ and $(4, 2)$
- ☐ **E.** $(20, 30)$ and $(-20, -30)$

Name: ______________________ Date: __________ Period: __________

MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MTR.1.1, MTR.4.1

Why Intercepts?

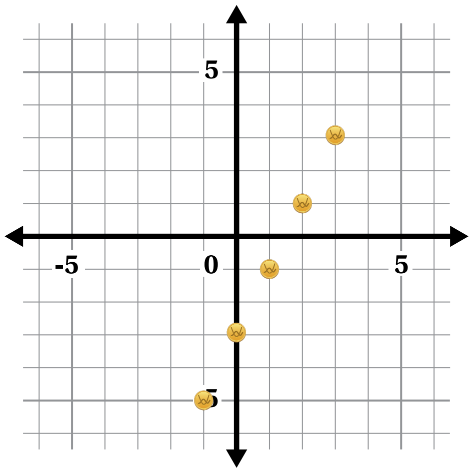
Let's write an equation for a line that passes through two given points.

Warm-Up

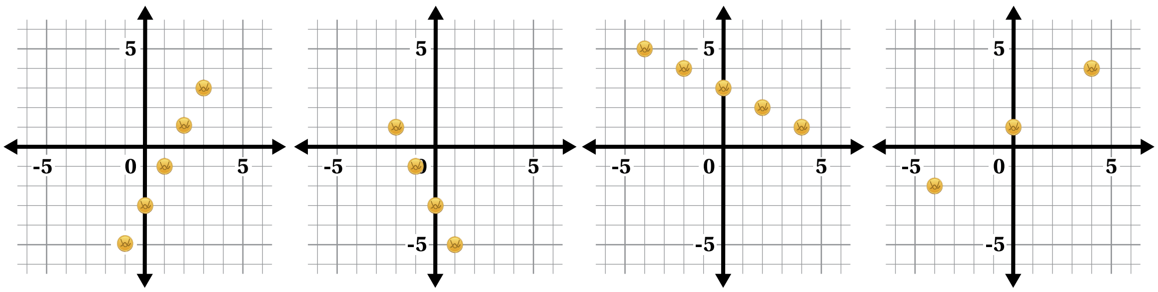
1. Darryl wants to "capture" these coins with just one line.

a What slope could Darryl use?

b What y-intercept could Darryl use?



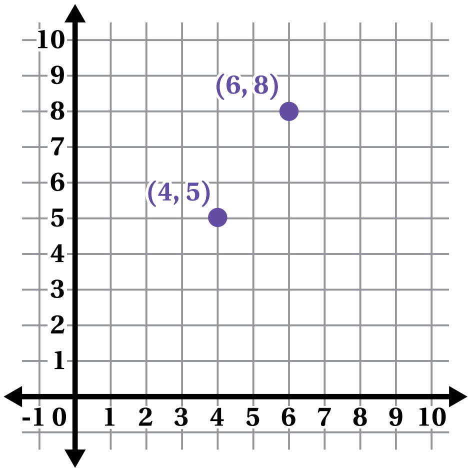
2. Write a single linear equation to capture all the coins for each challenge.



Name: .... Date: .... Period: ....

Determining the y-intercept

3. The points $(4, 5)$ and $(6, 8)$ represent the location of two coins.

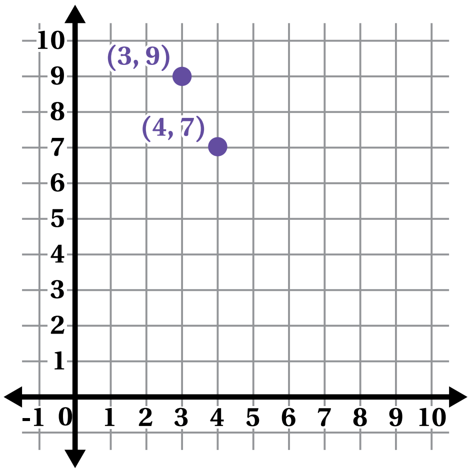


a Write an equation of the line that goes through both points.

b What was your strategy for determining the y-intercept?

4. Here are two new points: $(3, 9)$ and $(4, 7)$.

Describe how you could determine the y-intercept of the line going through these two points.



Name: .... Date: .... Period: ....

Determining the y-intercept (continued)

5. Here are two students' strategies for determining the y-intercept in the previous problem.

Tariq

x	y
3	9
4	7

+1 −2

slope: $\frac{-2}{1} = -2$

$y = -2x + b$

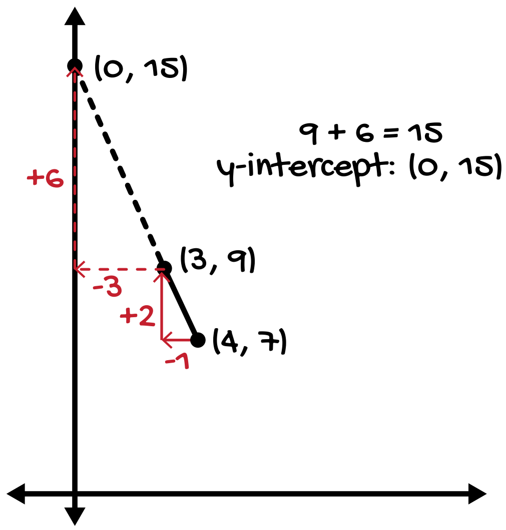
I'll substitute (3, 9) in for x and y!

$9 = -2(3) + b$

$9 = -6 + b$

$15 = b$

Nia



a Choose a student and explain their strategy to a classmate.

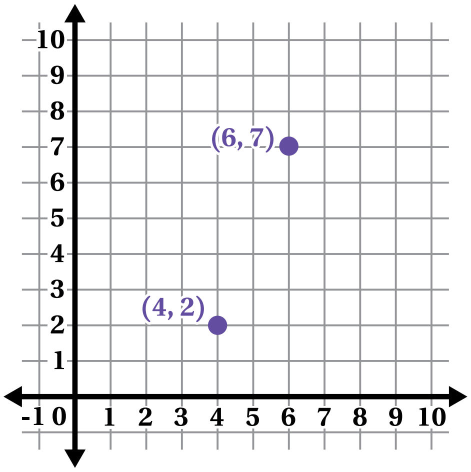
b **Discuss:** How are the students' strategies alike? How are they different?

Name: ______________________ Date: __________ Period: __________

Critiquing, Correcting, and Clarifying

6. Here are two new points: $(4, 2)$ and $(6, 7)$.

Write an equation of the line that goes through both points.



7. Victor made a mistake while writing the equation for the line that goes through $(4, 2)$ and $(6, 7)$.

a **Discuss:** What did Victor do well?

b Explain why Victor's work is incorrect.

Victor

	x	y	
+2	4	2	+5
	6	7	

slope: $\frac{5}{2}$

$$y = \frac{5}{2}x + b$$

$$4 = \frac{5}{2}(2) + b$$

$$4 = 5 + b$$

$$-1 = b$$

$$y = \frac{5}{2}x - 1$$

Activity 3

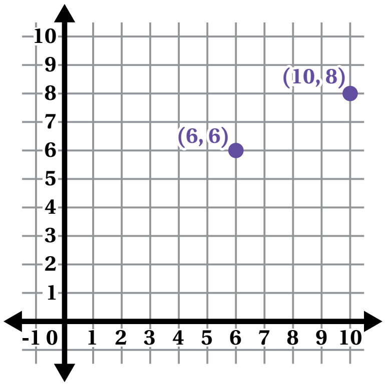
Name: ________ Date: ________ Period: ________

Repeated Challenges

8. For each problem, write an equation of the line that goes through both points.

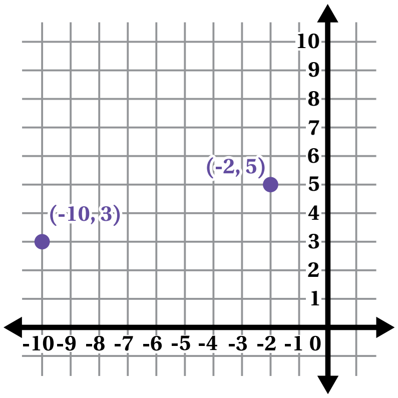
a Points: (6, 6) and (10, 8).

Equation: ________



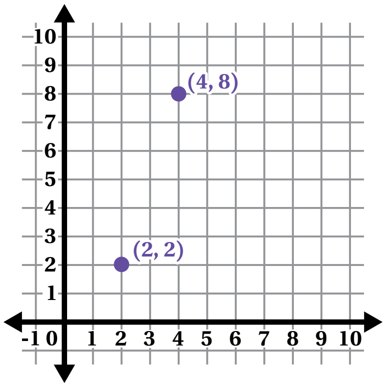
b Points: (-10, 3) and (-2, 5).

Equation: ________



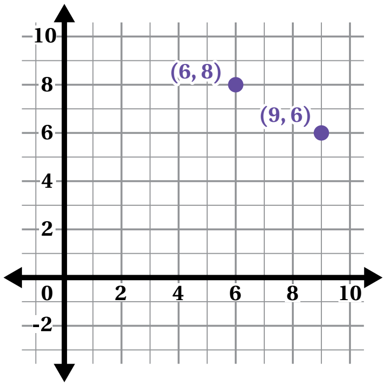
c Points: (2, 2) and (4, 8).

Equation: ________



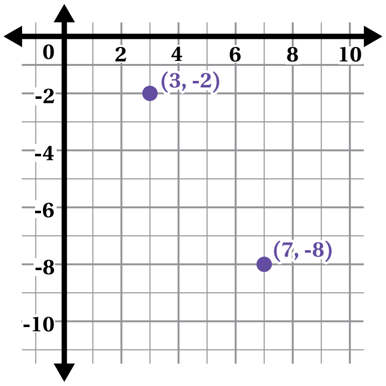
d Points: (6, 8) and (9, 6).

Equation: ________



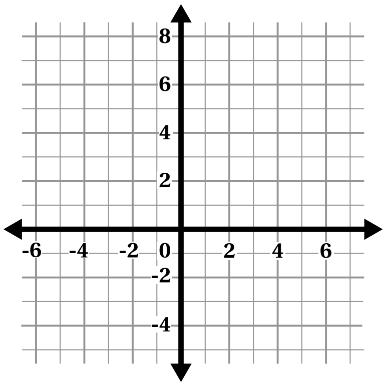
e Points: (3, -2) and (7, -8).

Equation: ________



f Points: (-1, 5) and (2, 2).

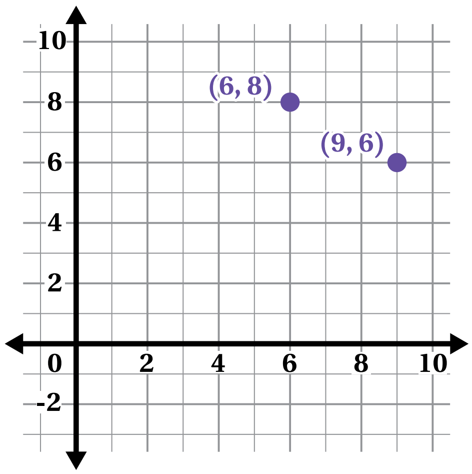
Equation: ________



Synthesis

9. Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.



Lesson Practice ACC7.4.07

Lesson Summary

Here are two different strategies for writing the equation of a line using two given points.

Strategy Using a Table

First, calculate the slope using a table. Next, substitute the coordinates of one of the points into the equation $y = mx + b$ to determine the y-intercept. Then write the equation in the form $y = mx + b$.

x	y
1	8
3	2

$+2$ $\quad$ -6 $\quad$ slope: $\frac{-6}{2} = -3$

$y = -3x + b$

Substitute (1, 8) in for x and y.

$8 = -3(1) + b$

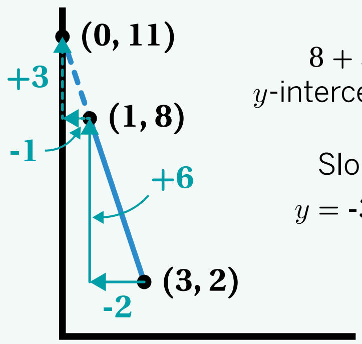
$8 = -3 + b$

$11 = b$

$y = -3x + 11$

Strategy Using Slope Triangles

Draw a line and use similar triangles to determine the slope and y-intercept of the line. Then write the equation in the form $y = mx + b$.



$8 + 3 = 11$

y-intercept: $(0, 11)$

Slope: -3

$y = -3x + 11$

Lesson Practice
ACC7.4.07

Name: ____________ Date: ________ Period: ________

1. Bao and Maia are each writing an equation of the line that passes through the points $(2, 9)$ and $(12, 14)$. They both calculate the slope as $\frac{1}{2}$.

- Bao substitutes the point $(2, 9)$ to determine the y-intercept.
- Maia substitutes the point $(12, 14)$ to determine the y-intercept.

Here is each student's work and their solutions for the y-intercept. Determine any mistakes in each student's work and explain how you would fix them.

Bao

$y = \frac{1}{2}x + b$

$9 = \frac{1}{2}(2) + b$

$9 = 1 + b$

$b = 8$

Maia

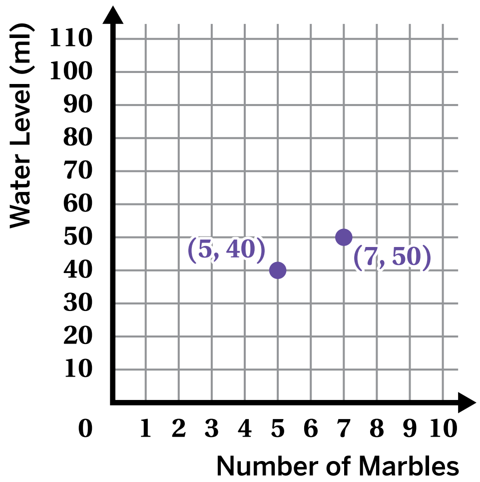
$y = \frac{1}{2}x + b$

$12 = \frac{1}{2}(14) + b$

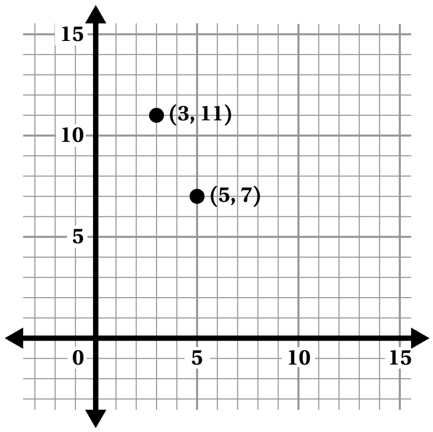
$12 = 7 + b$

$b = 5$

2. Chloe added marbles to a container of water. When she added 5 marbles, the water level was 40 millimeters. When she added 7 marbles, the water level was 50 millimeters. Write an equation for the water level, y, after x marbles are added. Show or explain your thinking.



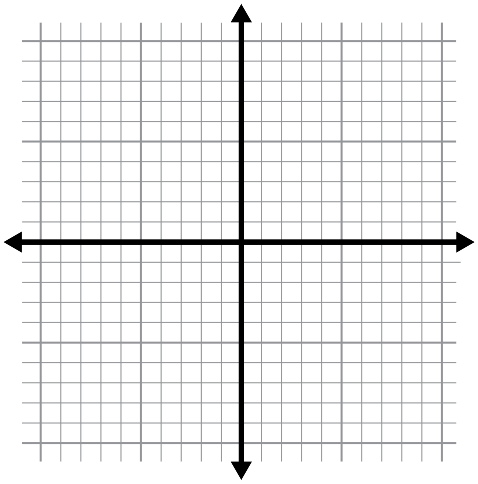
3. Here is a graph showing the points $(3, 11)$ and $(5, 7)$. What is the y-intercept of the line that passes through these points?



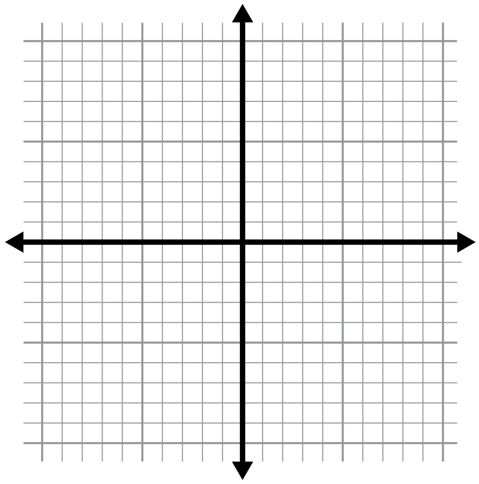
Name: .... Date: .... Period: ....

Problems 4–5: Write the equation of the line that passes through each pair of points. Show your work, and use the coordinate plane if it helps with your thinking.

4. $(2, 14)$ and $(6, 26)$



5. $(-5, 7)$ and $(1, 1)$



FAST Practice

6. A line has a slope of 2 and passes through the point $(-6, 1)$. Which is the equation of that line?

A. $y = 2x + 4$

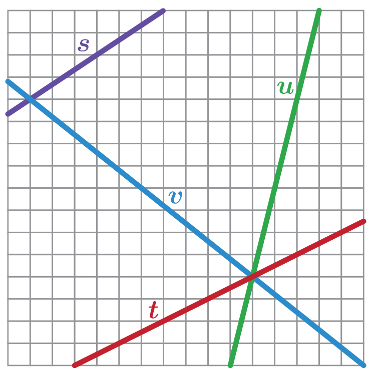
B. $y = 2x + 13$

C. $y = 2x + 8$

D. $y = 2x - 11$

Spiral Review

Problems 7–10: Determine if the slope of each line is positive or negative.



7. Line s

8. Line t

9. Line u

10. Line v

Unit 4
Lesson
8

Name: ________ Date: ________ Period: ________

MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MTR.1.1, MTR.5.1, MTR.7.1

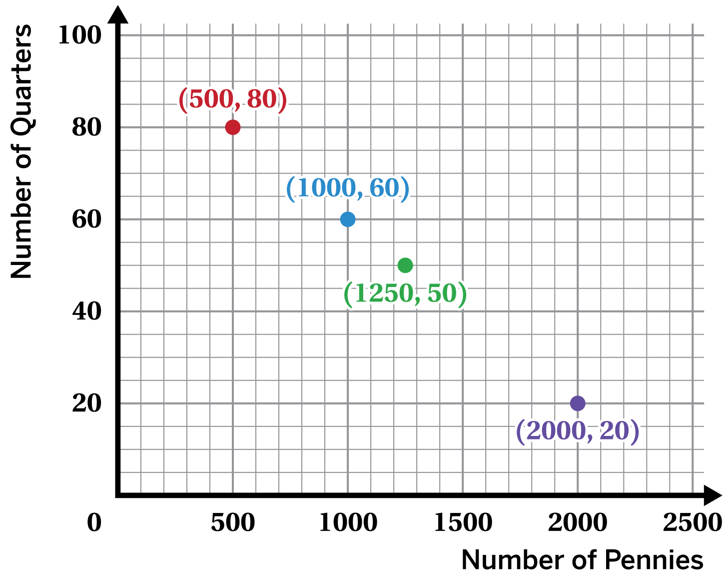
Pennies and Quarters

Let’s determine solutions to real-world linear relationships.

Warm-Up

1. The graph shows different combinations of pennies and quarters that total $25.

Look closely at the graph. Write a story about someone who has a mix of pennies and quarters. Consider who they are, why they have these coins, and what they might be doing with them.



Activity 1

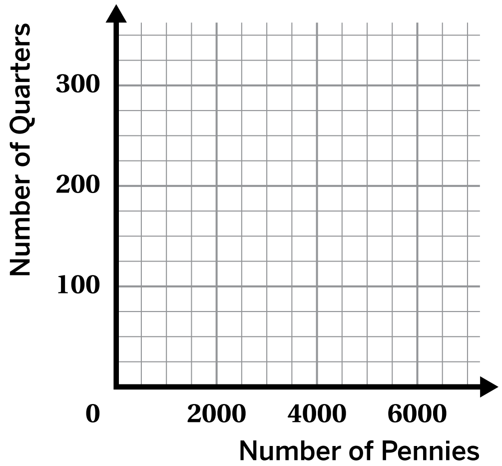
Name: Date: Period:

Pennies and Quarters

2. A piggy bank is filled with pennies and quarters worth a total of $62.00.

Write *four* possible combinations of pennies and quarters that are worth $62.00.

Number of Pennies	Number of Quarters
0	



Then graph the combinations of pennies and quarters you wrote.

3. Describe how you can tell if a combination of pennies and quarters is worth $62.00.

4. Write an equation that describes *all* the combinations of pennies, p, and quarters, q, that are worth $62.00.

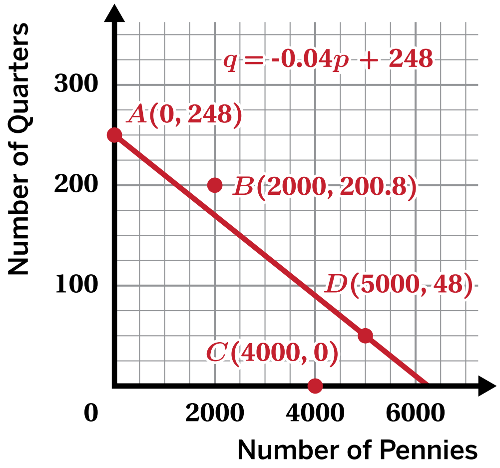
Name: .... Date: .... Period: ....

Pennies and Quarters (continued)

5. Ava wrote the equation $q = \text{-}0.04p + 248$ to represent all the combinations of pennies, p, and quarters, q, that are worth $62.00.

Select *all* the points that are **solutions to the equation**.

- ☐ **A.** Point A
- ☐ **B.** Point B
- ☐ **C.** Point C
- ☐ **D.** Point D



6. If there are 200 quarters, how many pennies do you need for a total of $62.00?

Use the graph if it helps with your thinking.

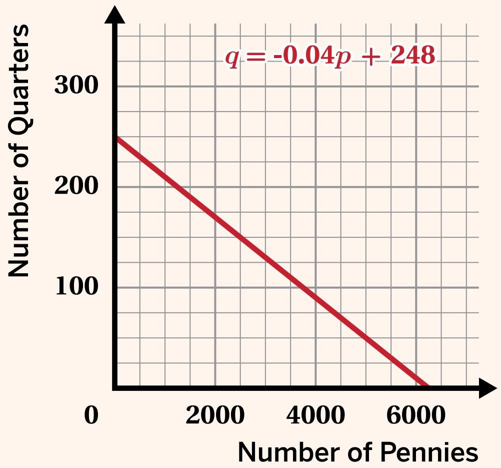
You're invited to explore more.

7. Here is a line and an equation describing all the combinations of pennies and quarters that are worth $62.00.

A different pile of pennies and quarters is worth $30.

a Draw the graph of this situation on the same plane.

b Describe the strategy you used.



Activity 2

Name: ________ Date: ________ Period: ________

Situation Sort

8. **a** Read each situation.

b **Discuss:**

- What is each situation about?
- What quantities or relationships do you see in each situation?

Situation 1

Sydney plans to buy 120 beverages for a picnic. Seltzers are sold in packs of 6. Waters are sold in packs of 8. x represents the number of packs of seltzers and y represents the number of packs of waters.

Situation 2

A coach has a \$120 budget to buy lunch for their team. He is ordering from a restaurant that charges \$8 per sandwich, plus a \$6 delivery fee. x represents the number of sandwiches and y represents the total cost of the lunch.

9. Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

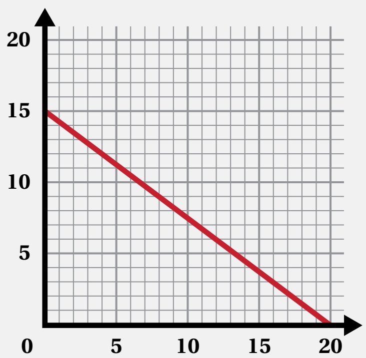
A

x	y
2	22
3	30
4	38

B

$y = -\frac{3}{4}x + 15$

C



D

$y = 8x + 6$

E

$y = 8x + 120$

F

$(5, 46)$

G

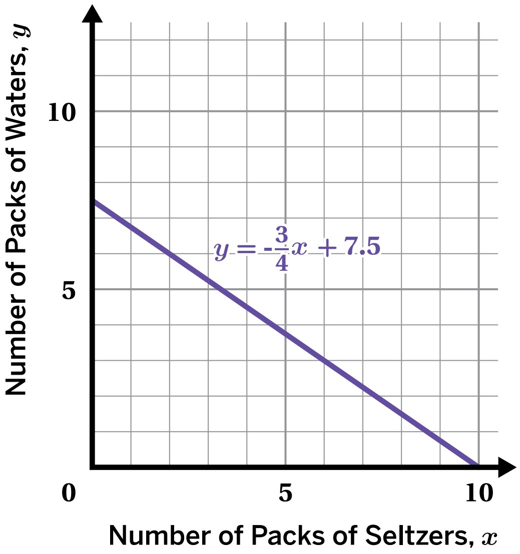
$(8, 9)$

Situation 1	Situation 2

Synthesis

10. Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.



Lesson Practice ACC7.4.08

Lesson Summary

The four representations of a linear relationship — table, graph, equation, and verbal description — are all useful when solving real-world problems.

Let's say a coach has a $120 budget to buy dinner for his team. Pizzas cost $20 and sandwiches cost $8. x represents the number of pizzas bought and y represents the number of sandwiches bought.

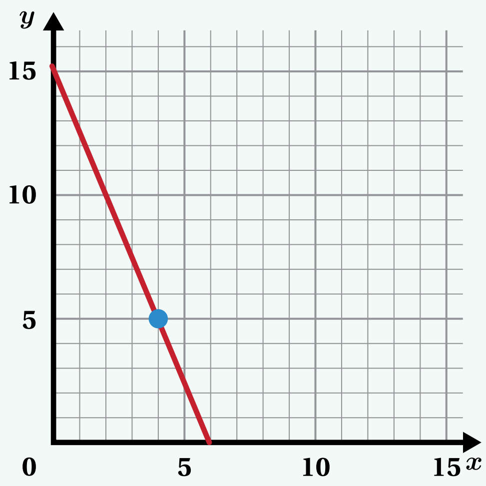
This situation can be modeled by the linear relationship $y = -\frac{5}{2}x + 15$.

Here are two ways to show that 4 pizzas and 5 sandwiches is one **solution to the equation**:

- The values $x = 4$ and $y = 5$ make the equation true.

$$5 = -\frac{5}{2}(4) + 15$$
$$5 = -10 + 15$$
$$5 = 5$$

- The point $(4, 5)$ is on the graph of the linear relationship.



Name: .... Date: .... Period: ....

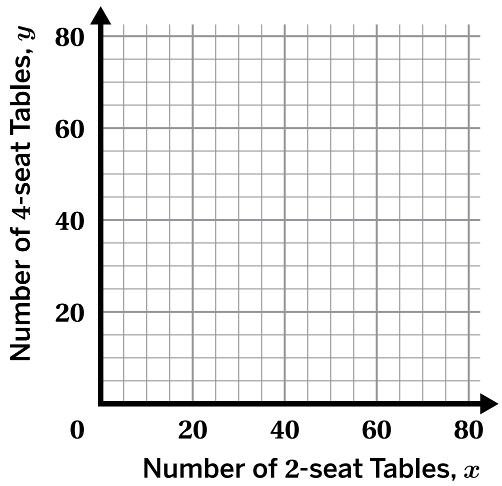
Problems 1–5: The owner of a new restaurant is ordering tables and chairs. She wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

1. Complete the table to show some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers.

Number of 2-Seat Tables	Number of 4-Seat Tables

2. Write an equation that represents the number of 2-seat tables, x, and the number of 4-seat tables, y, that the owner can order.

3. Graph all the possible combinations of 2-seat and 4-seat tables that will seat 120 customers.



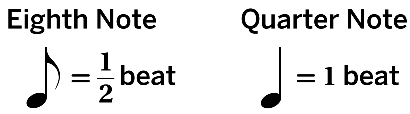
4. What is the slope of the line? What does it tell you about the situation?

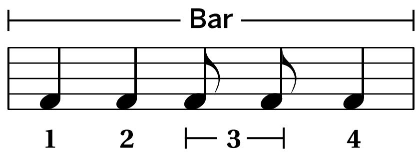
5. What are the x- and y-intercepts of the line? What do they represent in the situation?

Name: .... Date: .... Period: ....

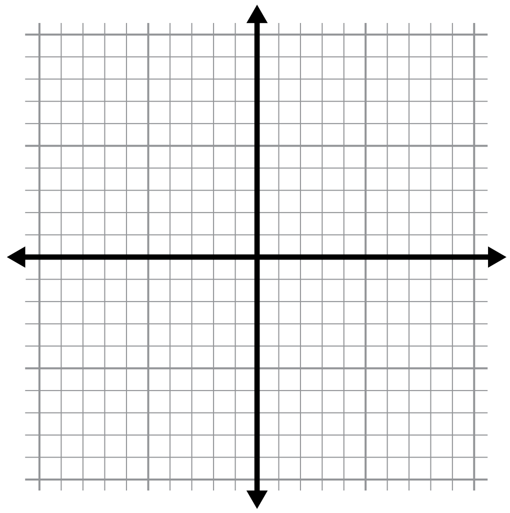
FAST Practice

Problems 6–7: A bar is a section of music made up of notes. A quarter note is 1 beat long. An eighth note is $\frac{1}{2}$ a beat long. A music composer wants to create a bar that's 4 beats long.





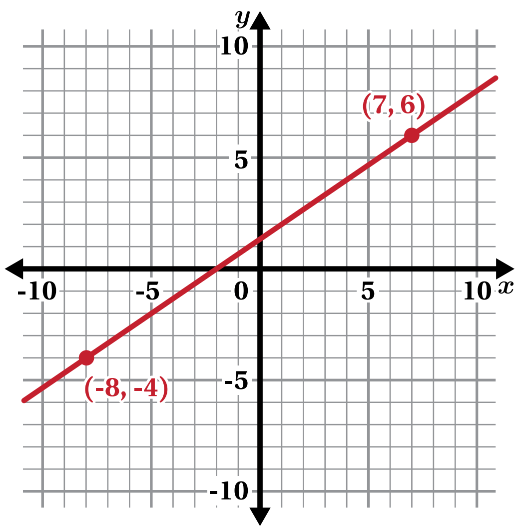
6. What are two possible combinations of eighth notes ($\frac{1}{2}$ a beat long) and quarter notes (1 beat long) that she could use to make a bar that's 4 beats long? Use a graph if it helps with your thinking.



7. Write an equation to represent the number of eighth notes, x, and number of quarter notes, y, in a bar that has a total of 4 beats. Record your answer in the space provided.

Spiral Review

Problems 8–10: Here is the graph of a linear relationship.



8. What is the y-intercept of the line?

A. $\frac{4}{3}$
B. $\frac{3}{4}$
C. $\frac{2}{3}$
D. $\frac{3}{2}$

9. What is the slope of the line?

A. $\frac{4}{3}$
B. $\frac{3}{4}$
C. $\frac{2}{3}$
D. $\frac{3}{2}$

10. Write an equation that represents the line.

Career Connection

This picture shows the Jet Propulsion Laboratory. It was founded in 1936 and includes laboratories where scientists specialize on constructing and testing the operation of robotic spacecraft. It is located in California, and is a part of NASA.

Robotic spacecraft are any crafts that do not have people on board. It may be operated by remote control, or it could be autonomous and have a set of pre-programmed instructions.

Jet Propulsion Laboratory" by NASA/JPL. Public Domain via WIkimedia Commons.

B.E.S.T. Mathematics Benchmark Connection

Engineers and physicists use variables, expressions, and equations extensively in their work For example, blueprints for the construction of robotic spacecraft being built involve complex equations and use scientific notation (MA.8.NSO.1) to describe very small measurements. They are also used to plan launches to ensure that the spacecraft can operate as successfully as possible from distances very far away!

Mathematical Thinking and Reasoning Connection

People involved in space exploration use patterns and structure to study the performance of robotic spacecraft (MTR.5.1). They use different forms, such as a list of steps, to describe the movements and tasks that an autonomous spacecraft will follow (MTR.2.1).

Meet William Pickering

New Zealand engineer William Pickering served as the director of the NASA Jet Propulsion Laboratory in California from 1954 until he retired in 1976. His work contributed to the launch of the first U.S. satellite and robotics that were used for Moon missions as well as ones to Venus and Mars. He is considered to be one of the pioneers in space exploration.

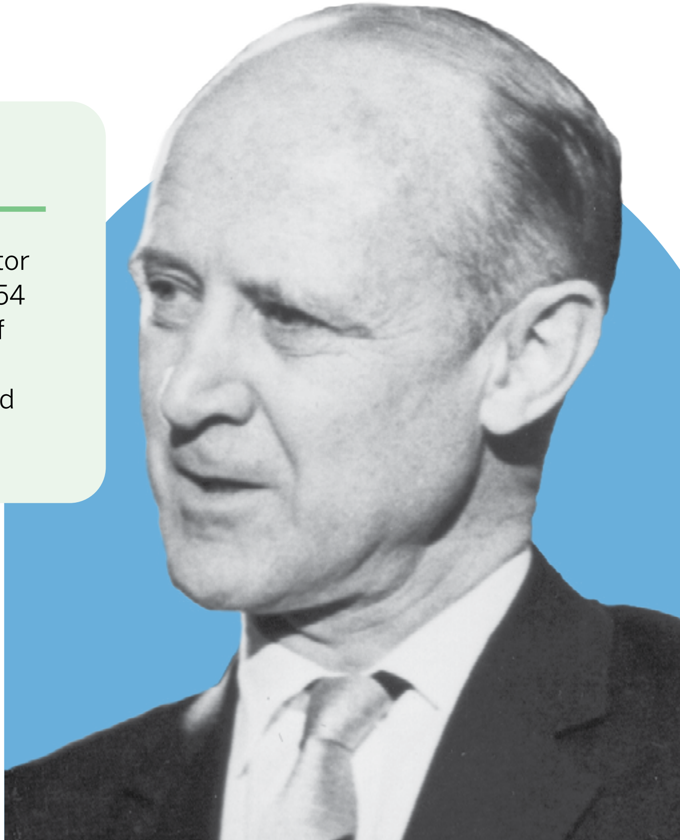
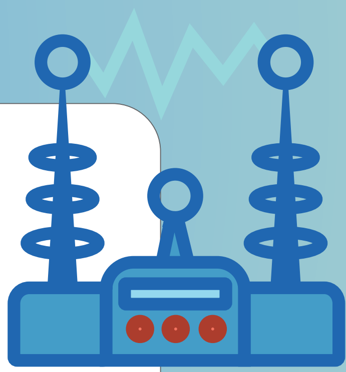


Photo credit text here (for photography only).

Unit 5

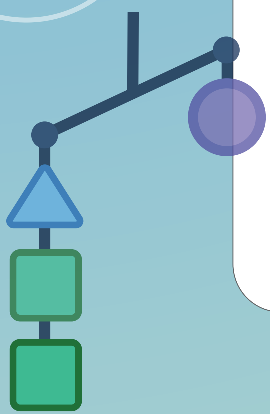
Linear Equations and Linear Systems



Equations can help you understand and solve problems. So far, you've solved single variable equations where the variable is on one side. In this unit, you'll solve equations with variables on both sides. You'll also solve two linear equations in a system and determine how many solutions there are.

Essential Questions

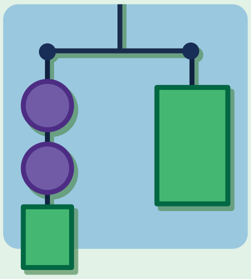
- How can you solve an inequality that has two operations?
- How can systems of equations be used to represent situations and solve problems?
- What does it mean for an equation or system of equations to have *no*, *one*, or *infinitely many* solutions?



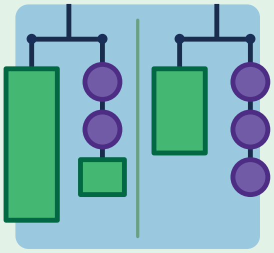
Sub-unit

1

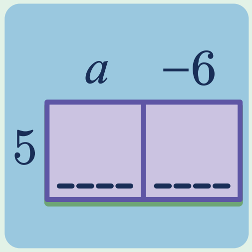
Two-Step Equations



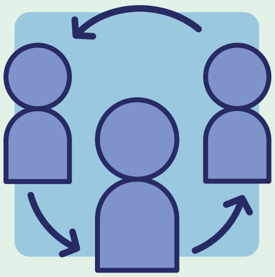
Lesson 1
Balancing Equations



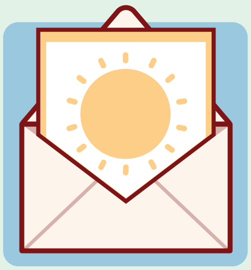
Lesson 2
Keeping It True



Lesson 3
Factoring and Expanding



Lesson 4
Pass the Equation

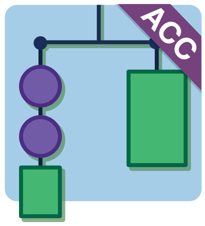


Lesson 5
Community Day

Name: ______ Date: ______ Period: ______

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Balancing Equations



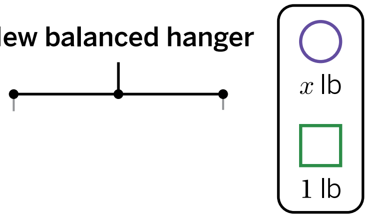
Let's use hanger diagrams to help us solve equations.

Warm-Up

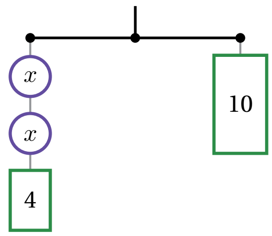
1. Hanger A is balanced.

Create a new balanced hanger that has the same weight for x.

New balanced hanger



Hanger A



$2x + 4 = 10$

Activity 1

Name: ______ Date: ______ Period: ______

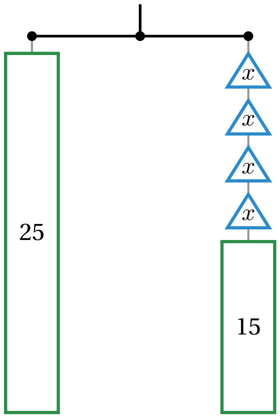
Connecting Hangers to Equations

2. The equation $25 = 4x + 15$ represents Hanger A.

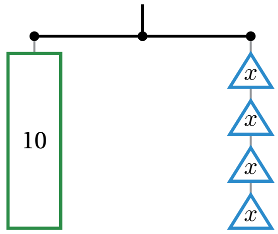
a Write an equation that represents Hanger B.

b What balanced move gets you from Hanger A to Hanger B?

Hanger A

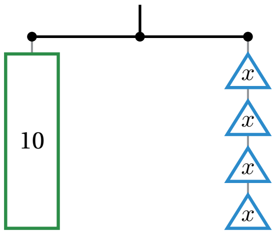


Hanger B

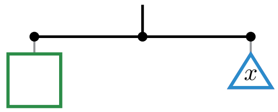


3. What is the weight of one triangle?

Hanger B



Hanger C



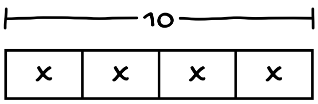
4. Here are Terrance's and Nikhil's strategies for determining the weight of one triangle on Hanger B.

Discuss: How are the two strategies alike? How are they different?

Terrance

$$\frac{10}{4} = \frac{4x}{4}$$

Nikhil

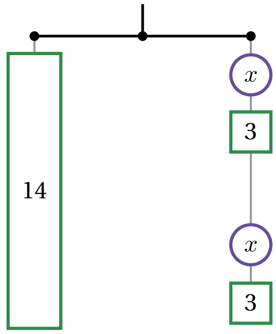


Activity
1

Name: .... Date: .... Period: ....

Connecting Hangers to Equations (continued)

5. Here is a new hanger. What is the value of x?



6. Anand and Darius used equations to determine the value of x in the previous problem.

Darius wrote the equation $14 = 2x + 6$.
Anand wrote the equation $14 = 2(x + 3)$.

Who is correct? Circle one.

Darius Anand Both Neither

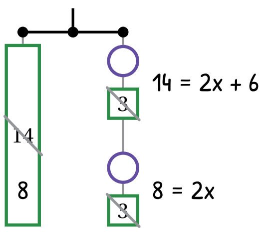
Explain your thinking.

7. Here are Darius's and Anand's work. Select a question to answer.

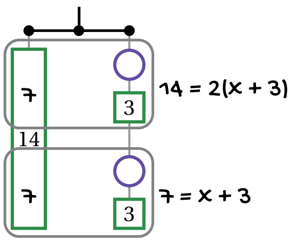
Why did Darius write $8 = 2x$?

Why did Anand write $7 = x + 3$?

Darius



Anand

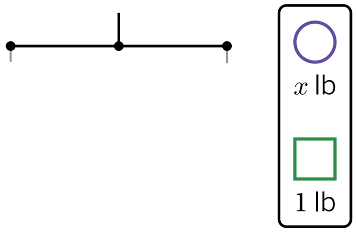


Activity 2

Name: ____________ Date: ____________ Period: ____________

Solving Equations

8. **a** Create a hanger to represent $7 = 4x + 2$.



b What value of x makes the equation true?

9. **a** Determine the value of x that makes the equation $4(x + 2) = 40.4$ true.

b Describe the steps you used to determine the value of x.

10. What value of x makes each equation true? Solve as many challenges as you have time for.

a $3x + 1 = 7$

b $2(x + 5) = 16$

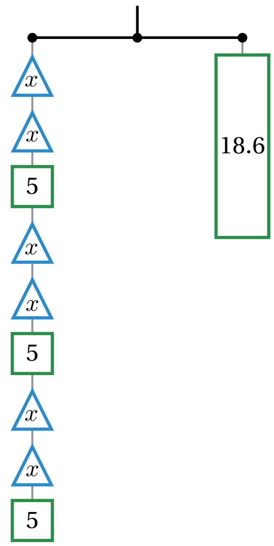
c $2x + 2.2 = 6.8$

d $4(x + 1.1) = 20.8$

e $4x + \frac{3}{2} = \frac{17}{2}$

Synthesis

11. Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.



$3(2x + 5) = 18.6$

Lesson Practice ACC7.5.01

Lesson Summary

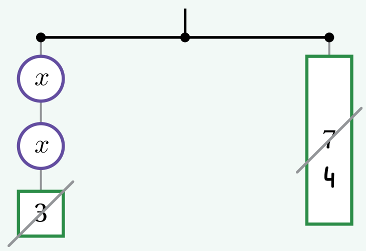
We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

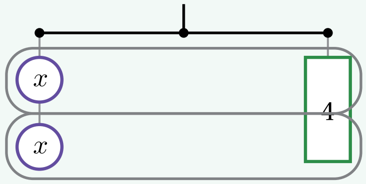
Remove 3 from both sides.



Divide both sides by 2.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Divide into two equal groups.



Name: .... Date: .... Period: ....

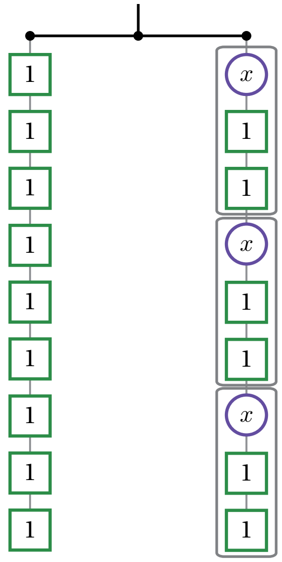
Problems 1–3: Solve each equation.

1. $x - 1 = 5$

2. $2(x - 1) = 10$

3. $500 = 100(x - 1)$

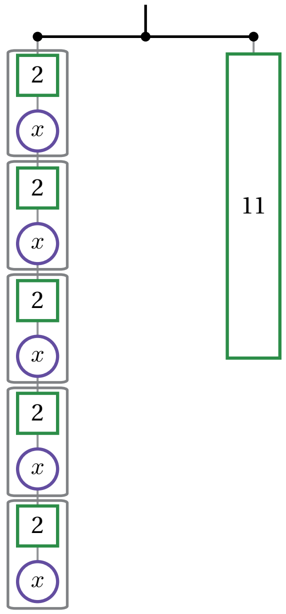
4. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.



Problems 5–6: Use the hanger.

5. Write an equation that represents this hanger.

6. What is the value of x that makes the equation true?



Lesson Practice
ACC7.5.01

Name: ____________ Date: ________ Period: ________

FAST Practice

Problems 7–8: Consider the equation $12.7 = 3x + 0.7$.

7. Draw a hanger to represent the equation in the space provided.

Your Diagram

8. What is the value of x that makes the equation true?

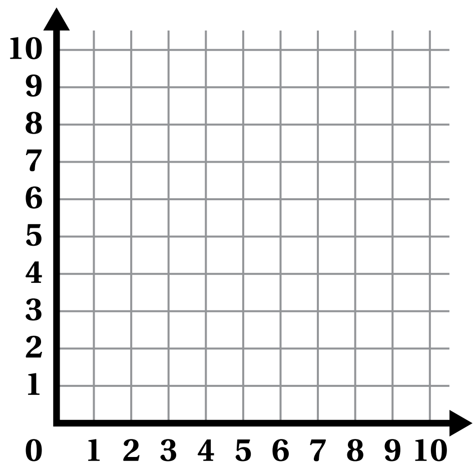
$x =$ ☐

Spiral Review

Problems 9–10: Draw a line for each slope. Then label the line.

9. Line a has a slope of $\frac{3}{5}$.

10. Line c has a slope of 5.



11. Which value is equivalent to $-9 + 5 + 3 - (-8)$?

A. -19

B. -9

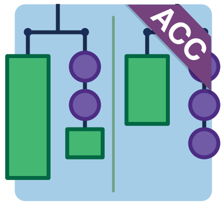
C. -1

D. 7

Name: ________________ Date: ________ Period: ________

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Keeping It True



Let's solve equations with positive and negative numbers.

Warm-Up

Solve each equation mentally. Try to think of more than one strategy.

1. $x + 4 = 6$

2. $x + 6 = 4$

3. $-2x = 4$

4. $-2x + 6 = 4$

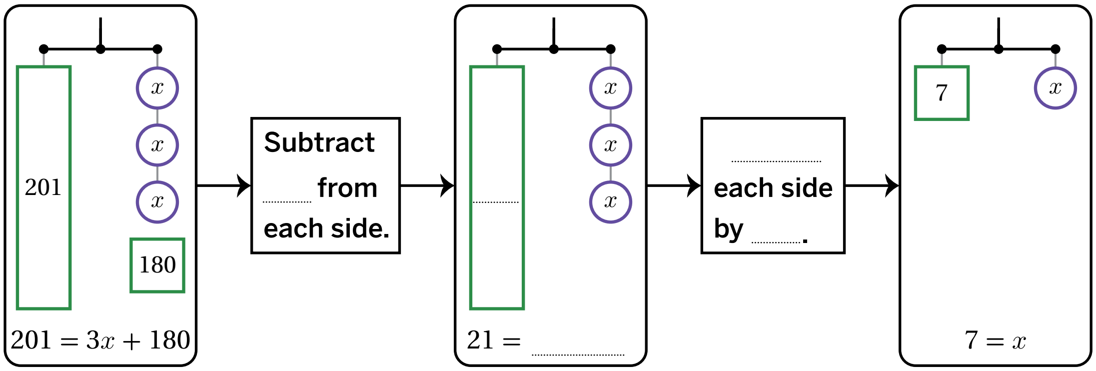
Activity 1

Name: ____________ Date: ________ Period: ________

Keep It True

Solve each equation by completing the blanks in the hangers, equations, and descriptions.

5.



6.

Equation	Moves
$5 = 2x + 8$	**Step 1:** ____________ from each side.
____________ $= 2x$	**Step 2:** ____________ each side ____________.
____________ $= x$	

7.

Equation	Moves
$2(x - 5) = -6$	**Step 1:** Divide each side ____________.
$x - 5 = -3$	**Step 2:** ______________________________.
____________ $=$ ____________	

8. How can you check if the *solutions to the equations* in Problems 5–7 are correct?

Activity 2

Name: ________ Date: ________ Period: ________

Solve 'em

Here are two groups of equations.

Group A	Group B
$x - (-4) = -6$	$2(x - 1) = -200$
$50x + 200 = 1700$	$900 = -100(x - 3)$
$8.6 = 3x - 3.4$	$3(x + 4.5) = 36$

9. **Discuss:** How are the equations in each group alike or different?

10. Which group do the equations $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ and $-\frac{1}{2}(2x - 6) = -2$ belong to? Explain your thinking.

11. Choose two equations from *each* group to solve.

	Group A	Group B
Equation 1		
Equation 2		

Synthesis

12. a. Write an equation that would belong in Group B.

b. What advice would you give to help someone solve an equation like yours?

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

Lesson Practice ACC7.5.02

Lesson Summary

When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✅	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✅

Lesson Practice
ACC7.5.02

Name: ______ Date: ______ Period: ______

Problems 1–3: Solve each equation by filling in the blanks.

1. $15x - 10 = 65$

$15x =$ ______

$x =$ ______

2. $3(x + 7) = -12$

$x + 7 =$ ______

$x =$ ______

3. $-100x - 100 = 0$

$-100x =$ ______

$x =$ ______

Problems 4–7: Solve each equation.

4. $-4x = -28$

5. $-4(x + 1) = -28$

6. $x - (-7) = -1$

7. $-3x + 7 = -1$

8. Using the numbers 0–9, complete the equations so that the solutions for x in both equations is the same. Use each number no more than once.

______ $x -$ ______ $=$ ______

______ $(x -$ ______ $) =$ ______

FAST Practice

9. Here is an equation: $\frac{1}{4}(x + 8) = -6$. What value of x makes the equation true?

$x = \boxed{}$

Spiral Review

10. Match each situation to an equation.

Situation	Equation
a. Mariana has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four equal pieces.	________ $8x + \frac{1}{4} = 4$
b. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 muffins.	________ $4 + \frac{1}{4}x = 8$
c. A stack of paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups adds $\frac{1}{4}$ inch to the height of the stack.	________ $\frac{1}{4} + 4x = 8$

11. Write an equation for the proportional relationship represented in this table.

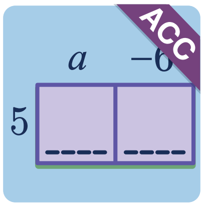
x	y
25	2000
40	3200

Unit 5
Lesson
3

Name: ______ Date: ______ Period: ______

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Factoring and Expanding

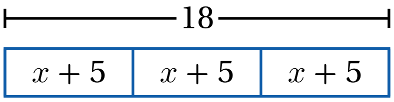


Let's think about efficient ways to solve equations with parentheses.

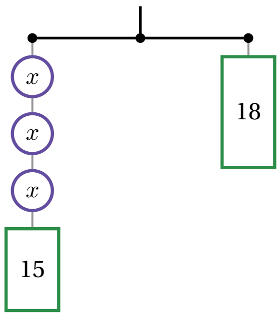
Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.



B.



C. $3(x+5)=18$

D. $3x+5=6$

Name: ______ Date: ______ Period: ______

Factoring Puzzles

Complete the missing information in each puzzle.

2.

	a	-6
5	$\square$	$\square$

Factored	Expanded
$5(a - 6)$	

3.

	$\square$	$\square$
2	$6a$	-2

Factored	Expanded
	$6a - 2$

4.

Factored	Expanded
	$-25x + 15$

5.

Factored	Expanded
$-(2c + 3)$	

Activity 2

Name: Date: Period:

Step by Step by Step by Step

6. Here are Amir's and Sadia's first steps for solving $2(x - 9) = 10$.

Amir	Sadia
$2(x - 9) = 10$	$2(x - 9) = 10$
$2x - 18 = 10$	$x - 9 = 5$

a Are each of their first steps correct? Explain your thinking.

b Finish solving each equation. Show your thinking.

Amir

Sadia

Name: ____________ Date: ________ Period: ________

Different First Steps

Solve these equations for x using both Amir's and Sadia's methods. Check the box when your solutions match.

7. $3(x+2) = 21$

Expand first:

Divide first:

☐

8. $200(x - 0.3) = 600$

Expand first:

Divide first:

☐

9. $\text{-}10\left(x - \frac{7}{10}\right) = \text{-}3$

Expand first:

Divide first:

☐

Synthesis

10. **a** What are two possible first steps you could use when solving an equation like $6(x+4)=30$?

b What are some advantages to having different ways to solve an equation?

Lesson Practice ACC7.5.03

Lesson Summary

Some equations have the form $p(x+q)=r$. We call this the **factored** form of the equation because it shows the product of two factors, p and $(x+q)$. In factored form, p is a common factor of the *coefficient* of x and the constant q.

You can solve these equations by **expanding** or dividing as a first step. When expanding first, you can use the *distributive property* to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x+1)=9$. The first steps are different, but the value of x, the solution to the equation, is the same.

Expanding First (using the distributive property)

$$\begin{aligned} 3(x+1)&=9\\ 3x+3&=9\\ 3x+3-3&=9-3\\ 3x&=6\\ 3x\div 3&=6\div 3\\ x&=2 \end{aligned}$$

Dividing First

$$\begin{aligned} 3(x+1)&=9\\ 3(x+1)\div 3&=9\div 3\\ x+1&=3\\ x+1-1&=3-1\\ x&=2 \end{aligned}$$

Name: .... Date: .... Period: ....

Problems 1–3: Write each expression in expanded form.

1. $-2(-6)$

2. $-2(-y)$

3. $-2(-6 - y)$

Problems 4–6: Complete the missing information in the puzzle and complete the table.

4.

	a	-5
4	$\square$	$\square$

Factored	Expanded
$4(a - 5)$	

5.

	$\square$	$\square$
-3	$12a$	-1

Factored	Expanded
	$12a - 1$

6.

	$\square$	$\square$
$\square$	$\square$	$\square$

Factored	Expanded
	$9x - 21$

7. Emmanuel and Mauricio started solving the equation $7(x - 2) = 91$. Finish solving each equation.

Emmanuel

$7(x - 2) = 91$
$7x - 14 = 91$

Mauricio

$7(x - 2) = 91$
$x - 2 = 13$

Name: ________ Date: ________ Period: ________

Problems 8–10: Solve each equation. Show your thinking.

8. $2(x - 3) = 14$

9. $-5(x - 1) = 40$

10. $\frac{5}{7}(x - 9) = 25$

FAST Practice

11. A student incorrectly solved the equation $-5(x - 1) = 40$ as shown.

Step 1: $-5x + 5 = 40$

Step 2: $-5x = 45$

Step 3: $x = -9$

Select **ONE** correct answer in each box to complete the sentence.

The student made the first error in [A. Step 1 B. Step 2]. The correct solution to the equation is [A. -7 B. 9].

Spiral Review

12. Use long division to write $\frac{2}{5}$ as a decimal.

13. Which equation is true when $x = 2$?

A. $3x + 4 = 8$ **B.** $5x - 3 = 12$ **C.** $6x + 4 = 16$ **D.** $4x + 5 = 17$

Unit 5
Lesson
4

Name: ______ Date: ______ Period: ______

MA.7.AR.2.2, MTR.2.1, MTR.3.1, MTR.5.1

Pass the Equation

Let’s practice solving equations multiple ways.

Warm-Up

1. Explain some possible first steps you could take to solve the equation.

$2(4x - 3) = 30$

Activity 1

Name: ______ Date: ______ Period: ______

Equation Roundtable, Round 1

2. Follow these instructions to solve the equations with your group.

- Choose one equation at a time. Write one step toward solving the equation, then pass your work.
- Check the work you received and ask your group mate to justify their thinking.
- Repeat these steps until each equation is solved.

$2x - 18 = 10$ $\qquad$ $2(3 - x) = -8$ $\qquad$ $5(x - 1) = 45$ $\qquad$ $-6 = \frac{1}{2}(x - 8)$

Equation 1	Equation 2
Equation: Check:	Equation: Check:

Equation 3	Equation 4
Equation: Check:	Equation: Check:

3. What do you think is important to remember when solving these types of equations?

Activity 2

Name: ______ Date: ______ Period: ______

Equation Roundtable, Round 2

4. Here are four new equations. Solve them using the same instructions as Activity 1.

$2x - 18 = 10$	$45 = 5(x - 1)$
$-10.5 = 6\left(x + \frac{1}{4}\right)$	$-2.8(x - 3) = 9\frac{4}{5}$

Equation 1	Equation 2
Equation: Check:	Equation: Check:

Equation 3	Equation 4
Equation: Check:	Equation: Check:

Synthesis

5. There are different ways to solve the equation $2(-3 + x) = -10$.

a List two different first steps you could take to solve this equation.

b Which first step do you prefer? Explain your thinking.

Lesson Practice ACC7.5.04

Lesson Summary

You can solve an equation in many different ways.

$-6(x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(x + (-5)) = 75$	Distribute the -6 to the x and -5.
$-6x + 30 = 75$	Subtract 30 from both sides.
$-6x = 45$	Divide each side by -6.
$x = -7.5$	

Here is a different strategy for solving the same equation.

$-6(x - 5) = 75$	Divide both sides of the equation by -6.
$x - 5 = -12.5$	Add 5 to both sides.
$x = -7.5$	

Lesson Practice
ACC7.5.04

Name: .... Date: .... Period: ....

Problems 1–3: Kwame and Maia each solved the same equation but used different strategies to do so. Show what they might have done to get from one step to another.

Kwame	Maia
$3.3(x - 10) = 66$	$3.3(x - 10) = 66$
$3.3x - 33 = 66$	$x - 10 = 20$
$3.3x = 99$	$x = 30$
$x = 30$	

1. How did Kwame begin? Maia?

2. Complete the missing steps for each student.

3. Which method do you prefer to use for this equation? Why?

Problems 4–7: Solve each equation. Show your thinking.

4. $4(x - 3) = 16$

5. $-5(x - 4) = 40$

6. $\frac{3}{7}(x - 8) = 15$

7. $\frac{1}{6}(x + 6) = 11$

Lesson Practice
ACC7.5.04

Name: .... Date: .... Period: ....

Problems 8–9: Alina and Naoki are each solving the equation $7(x + 2) = 91$.
Alina starts by using the distributive property.
Naoki starts by dividing each side by 7.

8. Show what Alina's and Naoki's full solution methods might look like.

Alina	**Naoki**

9. What is the same and what is different about their methods?

FAST Practice

10. What is the value of x in the equation $9 - \frac{x}{3} = 14$?

$x =$ ☐

Spiral Review

11. Vicente and Zwena are trying to write $9x - 2x + 4x$ using fewer terms.

- Vicente says that $9x - 2x + 4x = 3x$ because the subtraction sign tells us to subtract everything that comes after $9x$.
- Zwena says that $9x - 2x + 4x = 11x$ because the subtraction only applies to $2x$.

Who is correct? Explain your thinking.

12. Choose the expression that is equivalent to $(9x + 5) + (\text{-}7x + 2)$.

A. $\text{-}2x + 10$ B. $\text{-}63x + 10$ C. $16x + 7$ D. $2x + 7$

Name: ________ Date: ________ Period: ________

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.7.1

Community Day

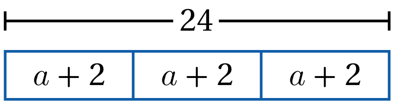
Let's represent and answer questions about situations in context.

Warm-Up

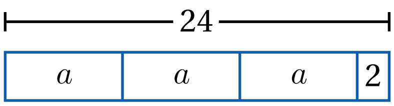
A baker put an equal number of cookies into 3 boxes. Then she put 2 more cookies in each box. She used 24 cookies total.

1. Which tape diagram best represents this situation?

A.



B.



2. Write an equation that represents the tape diagram you chose.

Name: .... Date: .... Period: ....

Three Reads

3. Here is a situation. Let's make sense of it together as a class.

Kyrie is making ____ invitations to his school's Community Day.

He has already made ____ invitations and wants to finish the rest of them within a week.

Kyrie plans to spread out the remaining work so that he makes the same number of invitations each day.

a **Discuss:** What is this situation about?

b Create a tape diagram or sketch that represents this situation.

c Let's look at the missing information. Adjust your diagram to represent that Kyrie is making 122 invitations and has already made 66 invitations.

d Determine how many invitations Kyrie should make each day.

Name: .... Date: .... Period: ....

Similar Problems

4. You will use the Activity 2 Sheet to explore a set of situations. Choose Set 1, Set 2, or Set 3.

Set ....

5. Create a poster. Here is what your poster should include *for each situation* in your set:

- ☐ The situation in words.
- ☐ A visual representation of the situation (tape diagram, hanger, etc.).
- ☐ An equation that represents the situation, along with your work for solving it.
- ☐ A check for the solution to the equation.
- ☐ The answer (with units) to the question in the situation.
- ☐ Connections between the visual representation, equation, and situation.

Synthesis

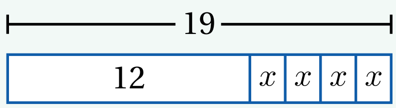
6. What do you think is important to remember when solving problems using visual representations and equations?

Lesson Practice ACC7.5.05

Lesson Summary

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for $\$12$. She pays a total of $\$19$. This tape diagram represents Zahra's situation.



Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$\begin{aligned} 12 + 4x &= 19 \\ 4x &= 7 \\ x &= 1.75 \end{aligned}$$

Zahra pays $\$1.75$ for each pen.

Lesson Practice
ACC7.5.05

Name: .... Date: .... Period: ....

Problems 1–4: Select an equation to represent the situation. Then solve the equation and explain the solution's meaning in context.

$5x - 7 = 3$ $\quad$ $7 = 3(5 - x)$ $\quad$ $3x + 5 = -7$ $\quad$ $x + 7 = 3 \cdot 5$

	Situation	Equation	Solution	Solution's Meaning
1.	The temperature outside is currently -7°C. Since midnight, the temperature tripled and then rose 5 degrees. What was the temperature at midnight?			
2.	Ama has 7 pink roses plus some white roses. She gives all of her roses away by giving 5 roses to each of her 3 favorite teachers. How many white roses does Ama give away?			
3.	A family of 3 goes to a fair. Tickets cost \$5 each, but each person has a coupon. They pay \$7 altogether. How much money does each person save on buying their ticket?			
4.	A club puts its members into 5 groups for an activity. 7 students leave early, so there are only 3 students left to finish the activity. How many students were in each group?			

Name: ________ Date: ________ Period: ________

FAST Practice

5. 6 soccer teams are practicing on a field. Each team has the same number of players, x. A coach asks 2 players from each team to leave the field to help move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

Equation: ________

Solution: $x =$ ____ players on each team

Spiral Review

Problems 6–9: Determine the value of the variable that makes each equation true.

6. $a \bullet 3 = -30$　**7.** $-9 \bullet b = -45$　**8.** $-89 \bullet 12 = c$　**9.** $d \bullet 88 = -88000$

10. Select *all* the expressions that show x increased by 35%.

- ☐ **A.** $1.35x$
- ☐ **B.** $\frac{35}{100}x$
- ☐ **C.** $x + \frac{35}{100}x$
- ☐ **D.** $(1 + 0.35)x$
- ☐ **E.** $(100 + 35)x$
- ☐ **F.** $\frac{100 + 35}{100}x$

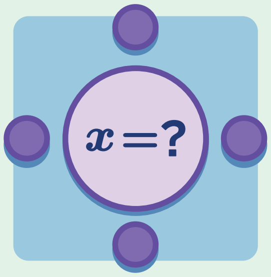
11. Alma has a loan of $12,589. The loan requires Alma to pay 2.1% of the loan amount in interest at the end of each year. About how much interest will Alma pay on this loan at the end of the year?

A. $2,644　**B.** $264　**C.** $15,233　**D.** $12,853

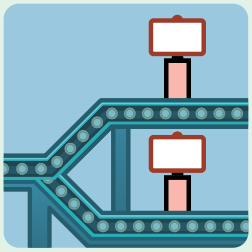
Sub-unit

2

Multi-Step Equations

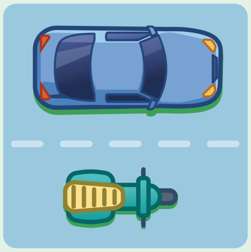


Lesson 6
Equation Roundtable

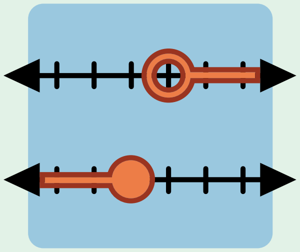


Lesson 7
All, Some, or None?
Part 1

Lesson 8
Strategic Solving



Lesson 9
When Will They Meet?



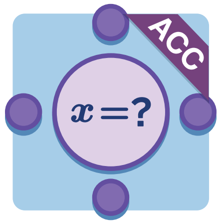
Lesson 10
Do the Two-Step

Name: .... Date: .... Period: ....

MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1

Equation Roundtable

Let's analyze balanced and unbalanced equation moves.



Warm-Up

Solve each equation mentally. Be prepared to share your strategy.

1. $5 - x = 8$

2. $-1 = x - 2$

3. $-3x = 9$

4. $-10 = -5x$

Name: ______ Date: ______ Period: ______

Find and Fix

5. Jordan solved three equations. Most of Jordan's solving moves keep the equations balanced, but some do not. Check his work for each equation.

- If all the solving steps keep the equations balanced, solve the equation a different way.
- If there is a move that creates an unbalanced equation, explain why. Then solve the equation correctly.

Equation 1:

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x - 3$

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x + -3$

$3x + 2 = 1x + -3$

$2x + 2 = -3$

$2x = -5$

$x = -\frac{5}{2}$

Equation 2:

$1.1(x - 3) = 0.1(2x - 6)$

$11(x - 3) = 1(2x - 6)$

$11x - 33 = 2x - 6$

$9x - 33 = -6$

$9x = 27$

$x = 3$

Equation 3:

$4 - 2(3x - 2) = 14 - x$

$2(3x - 2) = 14 - x$

$6x - 4 = 14 - x$

$6x - 4 = 14$

$6x = 18$

$x = \frac{18}{6}$

$x = 3$

Activity
2

Name: .... Date: .... Period: ....

Equation Roundtable

6. Your group will get a set of cards with equations on them. Each person will choose a card to start with and then you will collaboratively solve the equation on that card. Use this space to check the solution to your card's equation.

7. **Discuss:** What are some strategies your group used to check the solutions?

Synthesis

8. How might an equation become unbalanced?

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x - 3$

$1.1(x - 3) = 0.1(2x - 6)$

$4 - 2(3x - 2) = 14 - x$

Lesson Practice ACC7.5.06

Lesson Summary

You can use different steps to solve the same equation. For example, here are two ways to solve the equation $\frac{1}{3}(3x + 9) = 6x + 18$:

Start by distributing $\frac{1}{3}$ to $(3x + 9)$	**Start by multiplying both sides of the equation by 3**
$\frac{1}{3}(3x + 9) = 6x + 18$ $x + 3 = 6x + 18$ $-5x + 3 = 18$ $-5x = 15$ $x = -3$	$\frac{1}{3}(3x + 9) = 6x + 18$ $3x + 9 = 18x + 54$ $3x - 45 = 18x$ $-45 = 15x$ $-3 = x$

Sometimes you might unintentionally make a move that unbalances an equation. Here are some examples:

Description of Unbalanced Move	**Example**	**Alternative Balanced Move**
Distributing a factor to some terms in parentheses but not all of them	$\frac{1}{3}(3x + 9) = 5x$ $x + 9 = 5x$	$\frac{1}{3}(3x + 9) = 5x$ $x + 3 = 5x$
Multiplying some terms in an equation by a factor but not all of them	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 6$	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 30$
Adding or subtracting a term instead of distributing	$7 - 4(x + 1) = 2x + 5$ $3(x + 1) = 2x + 5$	$7 - 4(x + 1) = 2x + 5$ $7 - 4x - 4 = 2x + 5$

Problems 1–3: Solve each equation. Show your thinking.

1. $2(x+5)=3x+1$

2. $3y-4=6-2y$

3. $3(n+2)=9(6-n)$

4. Here is how Gabriela solved an equation. Circle Gabriela's mistake(s) and explain what she did wrong.

Gabriela

$$12(5+2y)=4y-(5-9y)$$
$$72+24y=4y-5-9y$$
$$72+24y=-5y-5$$
$$24y=-5y-77$$
$$29y=-77$$
$$y=-\frac{77}{29}$$

What is the correct solution?

Problems 5–6: Solve each equation. Show your thinking.

5. $\frac{1}{9}(2m-16)=\frac{1}{3}(2m+4)$

6. $1.5(5+0.2y)=0.4y-(0.6-0.9y)$

Name: ______ Date: ______ Period: ______

7. Complete the equation so that the solution is $n = 8$.

$$-4\left(\frac{2}{5}n - 6.1\right) = \_\_\_\_\_\_$$

FAST Practice

8. What is the solution to the equation $4.1(x + 2) = 8.1x - 1.8$?

A. $x = 1.6$

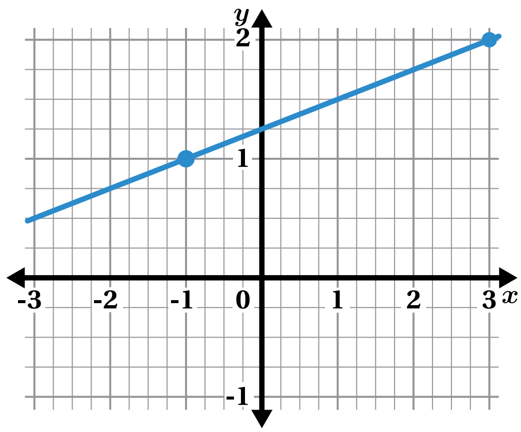
B. $x = 2.5$

C. $x = 0.4$

D. $x = -0.3$

Spiral Review

9. Here is the graph of a linear equation. Select *all* the true statements about this line and its equation.



☐ **A.** One solution to the equation is the ordered pair $(3, 2)$.

☐ **B.** One solution to the equation is the ordered pair $(-1, 1)$.

☐ **C.** One solution to the equation is the ordered pair $\left(\frac{3}{2}, 1\right)$.

☐ **D.** There are only two solutions.

☐ **E.** The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.

Problems 10–11: Evaluate each expression.

10. $8 + 5^2$

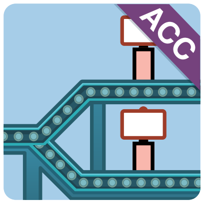
11. $(1 + 9)^3$

Unit 5
Lesson
7

Name: ____________ Date: ________ Period: ________

MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1, MTR.5.1

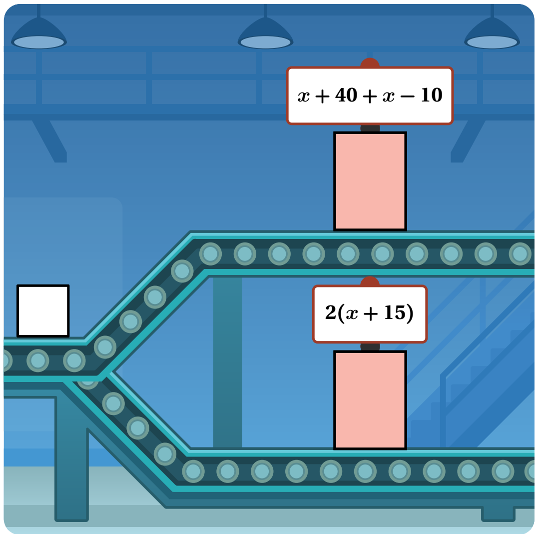
All, Some, or None? Part 1



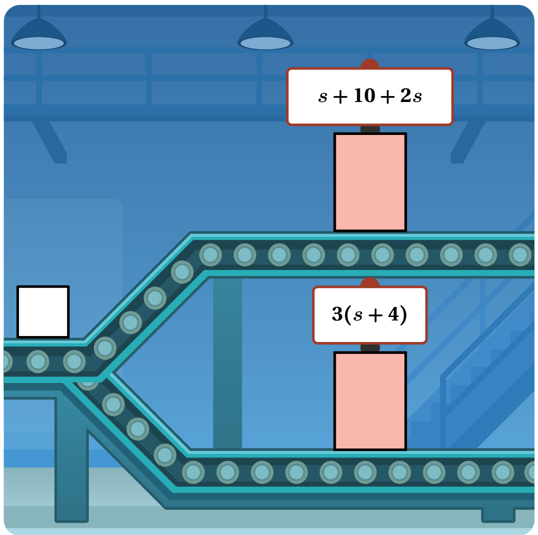
Let's think about how many solutions an equation can have.

Warm-Up

1. Here are two number machines. Tasia put a number into both machines, and the outputs were the same. What was Tasia's input?



2. Here are two new number machines. Can you find a number to put into both machines to get the same outputs?



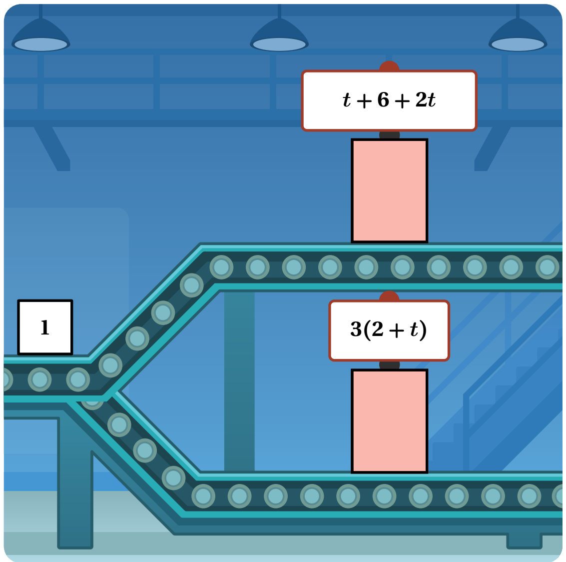
Activity 1

Name: ________ Date: ________ Period: ________

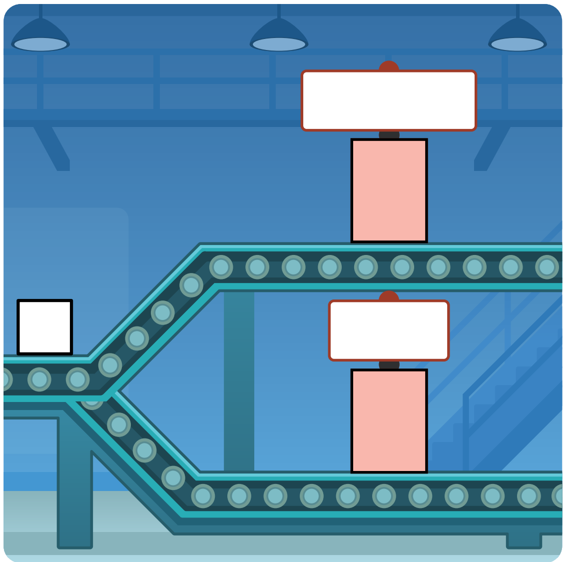
Always-Equal and Never-Equal

3. Let's look at two number machines that *always* give the same outputs for any input.

Use the two *expressions* to explain why both machines will give the same outputs for any input.



4. Write two expressions to create two new number machines that will give the same outputs for any input.

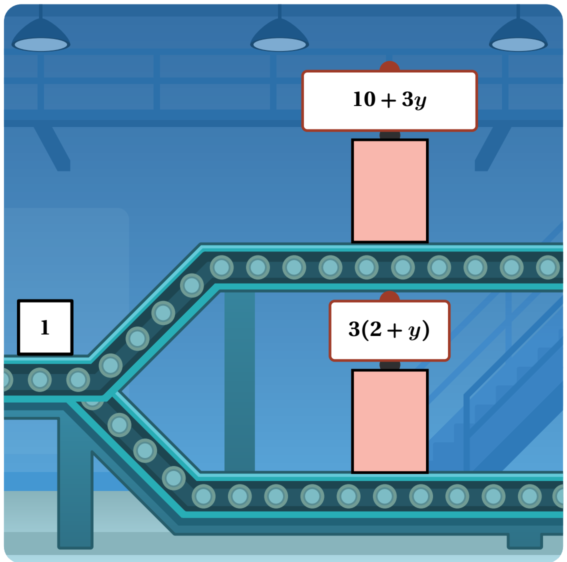


Name: .... Date: .... Period: ....

Always-Equal and Never-Equal (continued)

5. Let's look at two number machines that *never* give the same outputs for any input.

Use the two expressions to explain why both machines will *never* give the same outputs for any input.



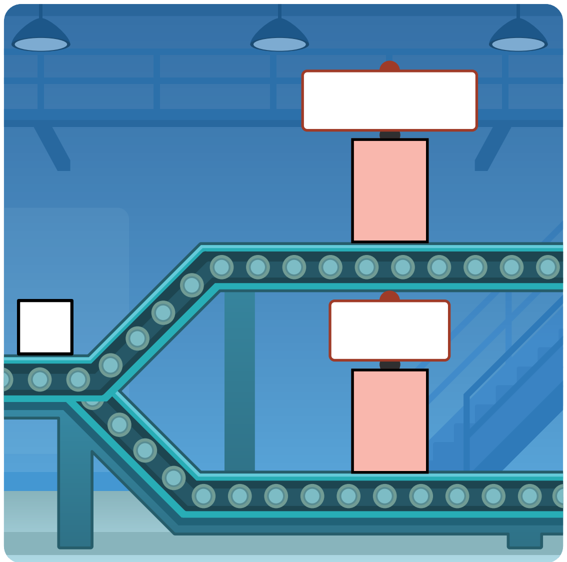
6. Select an equation made of two expressions that create number machines that will *never* give the same outputs for any input.

A. $2x + 3 = 3 + 2x$

B. $2x + 3 = 5 + 2x$

C. $2x + 3 = 2x + 3$

D. $2x + 2 = 3 + 3x$



Activity 2

Name: ______ Date: ______ Period: ______

Number of Solutions

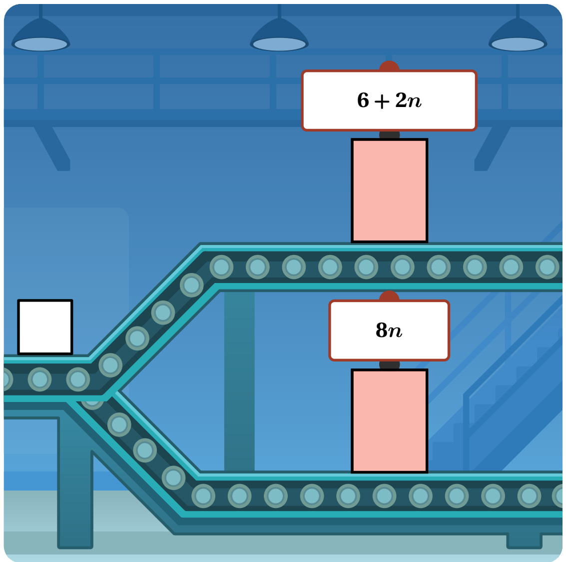
7. Here are two new number machines.

a. Write an equation to find an input number that will produce the same outputs for each machine.

b. For how many values of n will these machines produce the same output?

Circle one.

All values of n | One value of n | No values of n



8. Group the equations based on their number of solutions.

$v + 2 = v - 2$	$2n = n$	$7 - r = r - 7$	$\frac{1}{2} + x = \frac{1}{3} + x$
$y(-6) \cdot (-3) = 2 \cdot y \cdot 9$	$2t + 6 = 2(t + 3)$	$2n = 2n$	$3(n + 1) = 3n + 1$

No Solution (True for No Values)	One Solution (True for One Value)	Infinitely Many Solutions (True for All Values)

Name: Date: Period:

Number of Solutions (continued)

9. Kiandra looked at this equation and, without writing anything, said it must have no solution. What might she have noticed to lead her to this conclusion?

No Solution (True for No Values)
$\frac{1}{2} + x = \frac{1}{3} + x$

10. Write an equation for each number of solutions.

No solution:

One solution:

Infinitely many solutions:

Synthesis

11. How can you determine whether an equation has no solution, one solution, or infinitely many solutions?

Lesson Practice ACC7.5.07

Lesson Summary

Equations can have one solution, no solution, or **infinitely many solutions**.

Here are some examples.

One Solution

$$3x + 8 = 6 + 2 - 3x$$

This equation is only true when $x = 0$.

A linear equation has *one solution* when the *expressions* on either side of the equation have one value for the variable that makes them equal.

No Solution

$$3(x + 4) = 3x + 7$$

This equation is never true for any value of x.

A linear equation has *no solution* when the expressions on either side of the equation have no value for the variable that make them equal.

Infinitely Many Solutions

$$10 - 3x = 8 - 3x + 2$$

This equation is always true for any value of x.

A linear equation has *infinitely many solutions* when the expressions on either side of the equation are *equivalent*: always equal no matter the value of the variable.

Name: ________ Date: ________ Period: ________

Problems 1–4: Decide whether the given equation has *one solution, no solution,* or *infinitely many solutions*. Explain your thinking.

1. $x - 13 = x + 1$

2. $x + \frac{1}{2} = x - \frac{1}{2}$

3. $2(x + 3) = 5x + 6 - 3x$

4. $-(7 - 5x) = 6x - 3$

5. Ivory says that the equation $2x + 2 = x + 1$ has no solution because the left-hand side is double the right-hand side. Is Ivory's claim correct? Explain your thinking.

6. Write the other side of this equation so that it is true for all values of x.

$\frac{1}{2}(6x - 10) - x =$ ________

7. Write the other side of this equation so that it is true for no values of x.

$\frac{1}{2}(6x - 10) - x =$ ________

Lesson Practice
ACC7.5.07

Name: ______________ Date: ________ Period: ________

FAST Practice

8. Complete the statement so that it correctly identifies the number of solutions that exist for the given equation.

$5\left(0.2x + \frac{1}{25}\right) = x + \frac{1}{5}$

Select **ONE** correct answer in each box to complete the sentence.

The equation has **A.** no solution **B.** one solution **C.** infinitely many solutions .

Spiral Review

Problems 9–10: Solve each equation. Show your thinking.

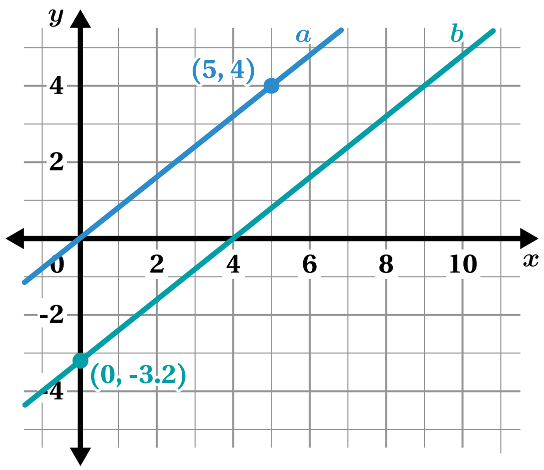
9. $-4(r + 2) = 4(2 - 2r)$

10. $1.3 + 6d = 2.7 - 8d$

11. These two lines are parallel. Write an equation for each line.

Line a: ______________

Line b: ______________



Unit 5
Lesson
8

Name: ________ Date: ________ Period: ________

MA.8.AR.1.2, MA.8.AR.2.1, MTR.1.1, MTR.5.1

Strategic Solving

Let's solve linear equations with no solution, one solution, and infinitely many solutions.

Warm-Up

1. Here are three equations:

- $13x = 3.25$
- $13x = 385x$
- $13x = 10x + 3x$

Choose one and write a situation that it could represent.

Activity
1

Name: Date: Period:

Predicting Solutions

2. Predict whether each equation has no solution, one solution, or infinite solutions. For equations with one solution, predict whether the solution will be *positive*, *negative*, or *zero*.

Equation	No Solution	One Solution			Infinite Solutions
$13x = 3.25$		$+$	$-$	0	
$13x = 385x$		$+$	$-$	0	
$13x = 10x + 3x$		$+$	$-$	0	
$13x + 42 = -584$		$+$	$-$	0	
$13x + 42 = 13x + -42$		$+$	$-$	0	

3. Choose one equation and explain how you made a prediction about its solution.

4. Why do you think it might be helpful to pause and try to predict the number of solutions or the sign of the solution before you start solving an equation?

Activity 2

Name: Date: Period:

What Happened?

5. Deven tried to solve the equation $13x + 42 = 13x + \text{-}42$. He thinks he made a mistake.

Deven

$$13x + 42 = 13x + \text{-}42$$
$$13x = 13x + \text{-}84$$
$$0 = \text{-}84$$

Do you agree? Circle one.

Yes   No   I'm not sure

Explain your thinking.

6. Write an equation with infinitely many solutions.

7. What happens when you try to solve the equation you wrote?

Activity 3

Name: ________ Date: ________ Period: ________

The Choice Is Yours

Equation A	Equation B	Equation C
$2r + 49 = \text{-}8(\text{-}r - 5)$	$\frac{n}{7} - 12 = 5n + 5$	$\frac{4m - 16}{4} = \frac{\text{-}16 + 8m}{8}$

Equation D	Equation E
$p - 5(p + 4) = p - (8 - p)$	$\text{-}3(c - 1) + 2(c - 1) = 5(c - 1)$

Equation F	Equation G	Equation H
$\text{-}\frac{1}{2}(t + 3) - 10 = \text{-}6.5$	$\frac{10 - v}{4} = 2(v + 17)$	$2(2q + 1.5) = 18 - q$

8. Examine these equations. Organize the equations into two or three groups based on the patterns you notice.

Group 1	Group 2	Group 3

9. **Discuss:** How did you group the equations?

10. Choose *three* equations to solve. (Choose at least one from each group.) Show your thinking.

Synthesis

11. What are some strategies for solving equations like these?

Equation B

$$\frac{n}{7} - 12 = 5n + 5$$

Equation C

$$\frac{4m - 16}{4} = \frac{-16 + 8m}{8}$$

Equation D

$$p - 5(p + 4) = p - (8 - p)$$

Lesson Practice ACC7.5.08

Lesson Summary

Equations can have many different features, including fractions, decimals, negative values, grouping symbols, and multiple terms. Based on the features, it can be helpful to think about what steps might be most useful in solving the equation.

When solving an equation with one solution, the goal is to end up with the variable isolated on one side of the equation and its value on the other. But this doesn't happen when there is no solution or infinitely many solutions.

One Solution

$$\begin{aligned} 3x + 8 &= 6 + 2 - 3x \\ 3x + 8 &= 8 - 3x \\ 6x + 8 &= 8 \\ 6x &= 0 \\ x &= 0 \end{aligned}$$

This equation is only true when $x = 0$.

No Solution

$$\begin{aligned} 3(x + 4) &= 3x + 7 \\ 3x + 12 &= 3x + 7 \\ 12 &= 7 \end{aligned}$$

This equation is never true for any value of x.

Infinitely Many Solutions

$$\begin{aligned} 10 - 3x &= 8 - 3x + 2 \\ 10 - 3x &= 10 - 3x \\ 10 &= 10 \end{aligned}$$

This equation is always true for any value of x.

Name: .... Date: .... Period: ....

Problems 1–3: Identify whether each equation has a solution that is *positive*, *negative*, or *zero* without solving.

1. $7x = 3.25$

2. $-7x = 32.5$

3. $3x + 11 = 11$

Problems 4–7: Solve each equation. Show your thinking.

4. $2b + 8 - 5b + 3 = -13 + 8b - 5$

5. $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

6. $3(3 - 3x) = 2(x + 3) - 30$

7. $\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$

8. Kadeem is solving the linear equation $x - 3(2 - 3x) = 2(5x + 3)$. Here are his final two steps. Select the statement that correctly describes Kadeem's solution.

Kadeem

$-6 + 10x = 10x + 6$

$-6 = 6$

A. The solution is the ordered pair $(-6, 6)$.

B. The solution is $x = 0$.

C. There are infinitely many solutions since $-6 = 6$ is a false statement.

D. There is no solution because $-6 = 6$ is a false statement.

Lesson Practice
ACC7.5.08

Name: .... Date: .... Period: ....

FAST Practice

9. What is the solution to this equation?

$-\frac{1}{5}(5x - 15) + 18 = 6x$

A. $x = \frac{7}{20}$

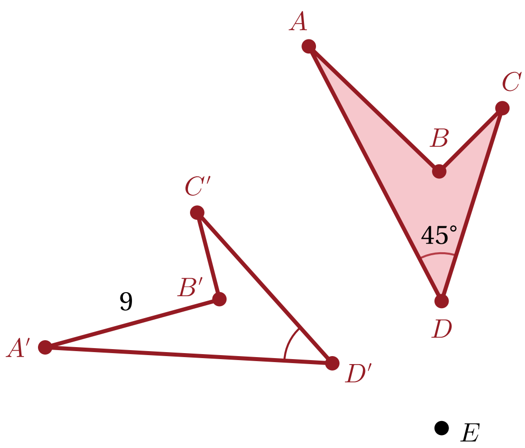
B. $x = 2$

C. $x = \frac{20}{7}$

D. $x = 3$

Spiral Review

Problems 10–11: Figure $A'B'C'D'$ is the image of figure $ABCD$ after a rotation around point E.



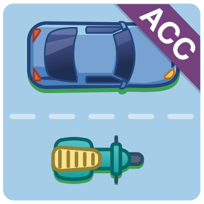
10. What is the length of side AB? Explain your thinking.

11. What is the measure of $\angle A'D'C'$? Explain your thinking.

Name: ________ Date: ________ Period: ________

MA.8.AR.1.2, MA.8.AR.2.1, MA.8.AR.3.3, MTR.2.1

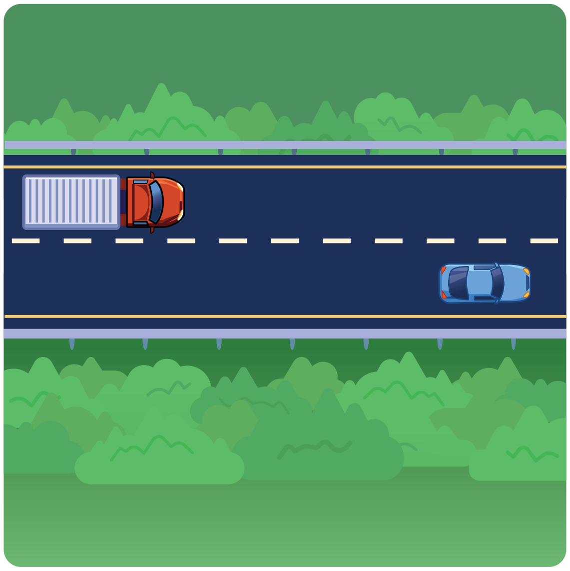
When Will They Meet?



Let's use equations to think about situations.

Warm-Up

1. A truck and a car are going the same direction on the same road. Write a story about what would need to occur for the truck to catch up to the blue car, if they are each moving at a constant speed.



Activity
1

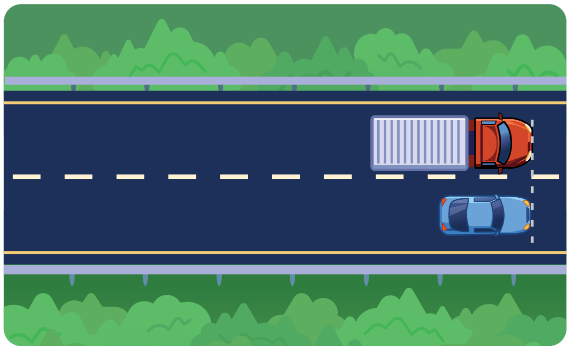
Name: ______ Date: ______ Period: ______

Distance and Time

2. The table shows each vehicle's position at certain times. The vehicles are moving at a constant *rate*. Fill in the missing information in the table.

Time (sec)	Truck Position (m)	Car Position (m)
0	0	18
1	15	29
2	30	40
3		
4		
...	...	...
t		

3. When will the truck meet the car?



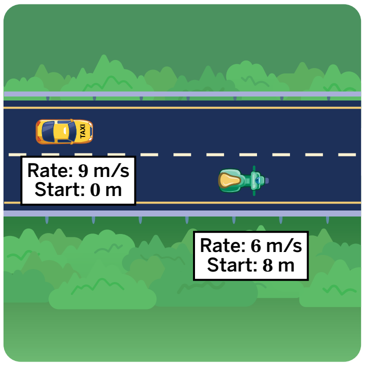
Activity 2

Name: Date: Period:

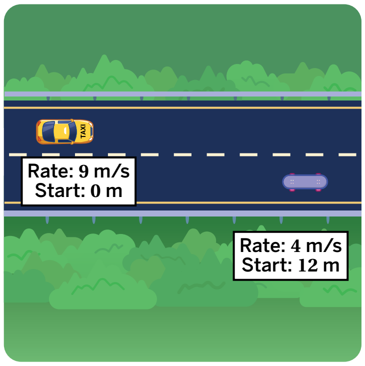
Choose Your Vehicle

4. Choose and circle a vehicle to compare to the taxi. The rate (in meters per second) and the starting point are displayed for each vehicle.

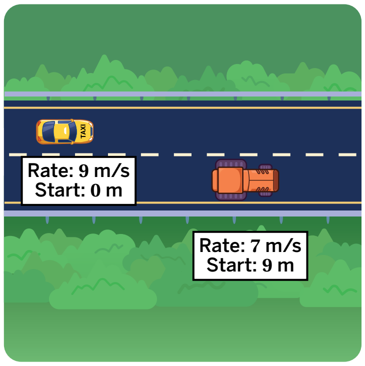
Scooter



Skateboard



Tractor



5. Let t represent time in seconds. For the vehicle you chose, which equation could you solve to determine when the two vehicles meet?

Scooter

A. $6 + 8t = 9t$

B. $8t + 6 = 9$

C. $6t + 8 = 9t$

Skateboard

A. $4t + 12 = 9t$

B. $12t + 4 = 9$

C. $4 + 12t = 9t$

Tractor

A. $7 + 9t = 9t$

B. $7t + 9 = 9t$

C. $9t + 7 = 9$

Explain your thinking.

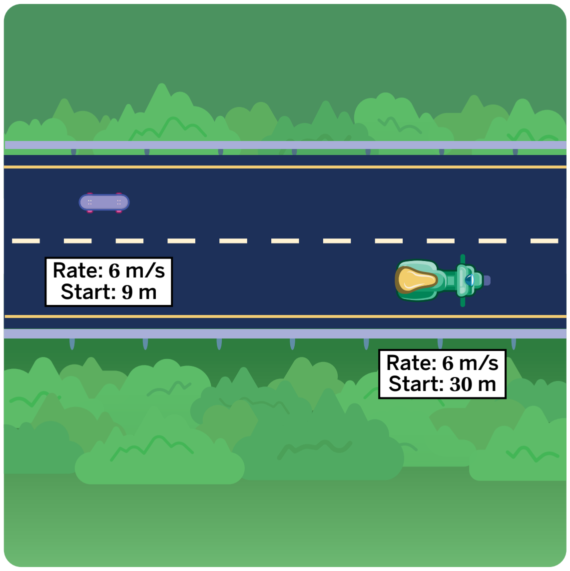
6. When will the vehicle you chose meet the taxi?

Name: ________ Date: ________ Period: ________

Meet Up

7. Demetrius wants to figure out when these vehicles will meet, so he wrote these expressions.

Skateboard Position (m)	Scooter Position (m)
$6t + 9$	$6t + 30$

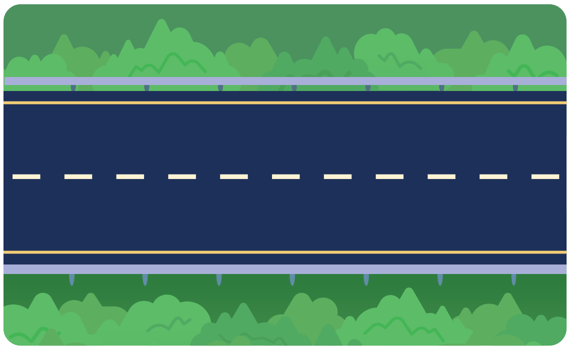


Without solving an equation, Demetrius knew the vehicles would never meet. How might he have figured this out?

Name: ______ Date: ______ Period: ______

Meet Up (continued)

8. Write expressions, in terms of t, that could represent two vehicles traveling at different rates with different starting positions that will eventually meet.



Truck position expression: ______

Car position expression: ______

9. When will the two vehicles meet?

You're invited to explore more.

10. A tractor and a scooter are in a race. Write expressions, in terms of t, for each vehicle so that the vehicles start separated and meet at 10 seconds.

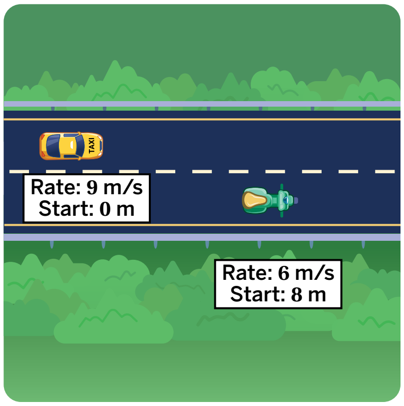
Tractor position expression: ______

Scooter position expression: ______

Synthesis

11. How can writing expressions to represent the position of vehicles at time t help you determine when they will meet?

Use the example if it helps with your thinking.



Lesson Practice ACC7.5.09

Lesson Summary

We can write two expressions in one variable and set them equal to each other to represent a scenario in which two conditions are equal. We can solve this equation to determine the unknown quantity.

For example, imagine two hikers walking in the same direction on a flat trail. The hikers will meet each other when they are at the same location on the trail at the same time.

To determine when this occurs, an expression can be used to represent the location and walking speed of each hiker.

	Location (ft)	Walking Speed (ft/s)	Expression
Hiker 1	30	4	$30 + 4t$
Hiker 2	10	7	$10 + 7t$

You can set these two expressions equal to each other to form one equation that can be solved.

$30 + 4t = 10 + 7t$

$20 = 3t$

$t = \frac{20}{3}$ or about 6.7 seconds

Name: .... Date: .... Period: ....

1. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

2. Which story could the equation $3x + 6 = 2 + 4x$ represent?

A. A truck starts 6 meters down a road and is moving at a speed of 3 meters per second. A car starts 2 meters down a road and is moving at a speed of 4 meters per second. After x seconds, the car and the truck meet.

B. A motorcycle starts 3 meters down a road and is moving at a speed of 6 meters per second. A bicycle starts 4 meters down a road and is moving at a speed of 2 meters per second. After x seconds, the motorcycle and the bike meet.

Problems 3–6: Tiana and Miko are biking in the same direction on the same path.

3. Miko rides at a constant speed of 16 miles per hour. Write an expression that represents the number of miles Miko travels after t hours.

4. Tiana starts riding a half hour before Miko. If Miko has been riding for t hours, how long has Tiana been riding?

5. Tiana rides at a constant speed of 12 miles per hour. Write an expression that represents the number of miles Tiana travels after Miko has been riding for t hours.

6. Use your expressions to determine when Miko and Tiana meet. Show or explain your thinking.

FAST Practice

7. Here are two cell phone plans.

- Plan A costs $\$70$ per month and comes with a free phone worth $\$500$.
- Plan B costs $\$50$ per month, but doesn't come with a free phone.

If you choose Plan B and buy a $\$500$ phone, after how many months will your total cost be the same as the cost of choosing Plan A?

Record your answer in the space provided.

The total cost will be the same after ☐ months.

Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $5(x+2)=30$

9. $5x+2=30$

10. A triangle is graphed on a coordinate grid. Which transformation will result in a triangle that is not congruent to the original triangle?

A. A reflection over the y-axis.

B. A translation 5 units to the left.

C. A dilation by a scale factor of 4.

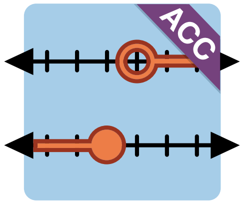
D. A $180°$ counterclockwise rotation around the origin.

Name: ______________ Date: ________ Period: ________

MA.8.AR.1.2, MA.8.AR.2.2, MTR.5.1, MTR.7.1

Do the Two-Step

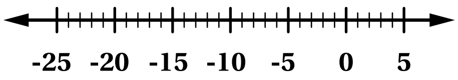
Let's solve two-step inequalities.



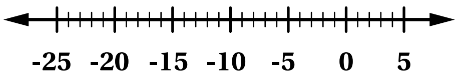
Warm-Up

1. Solve each *inequality* and graph the solution on the number line.

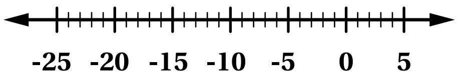
a $x + 5 \leq -10$



b $-\frac{1}{2}x > 5$



c $-5 \leq \frac{1}{2}x$



Name: .... Date: .... Period: ....

Comparing Equations and Inequalities

2. Jayden and Laila each solve a different problem.

Jayden	Laila
$2x - 3 = 11$	$2x - 3 > 11$
$2x = 14$	$2x > 14$
$x = 7$	$x > 7$

Compare Jayden's and Laila's solutions. How are they alike? How are they different?

3. Consider the inequality $\frac{3}{4}x - 1000 > 2000$.

a The inequality is solved in the table below. Justify each step.

Inequality	Explanation
$\frac{3}{4}x - 1000 > 2000$	Given inequality
$\frac{3}{4}x - 1000 + 1000 > 2000 + 1000$	
$\frac{3}{4}x > 3000$	Simplify
$\frac{3}{4}x \cdot \frac{4}{3} > 3000 \cdot \frac{4}{3}$	
$x > 4000$	Solution

b **Discuss:** How can you test if the solution is correct?

Activity
2

Name: ________ Date: ________ Period: ________

Negative Coefficients

4. The inequality $3 > -5$ is a true statement. Kai says, "If I multiply both sides of the inequality by -1, I have to reverse the inequality sign to keep the statement true."

a **Discuss:** Do you agree with Kai? Explain why or why not.

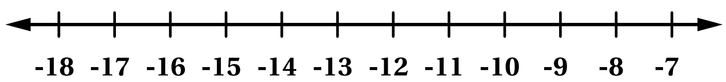
b What does that tell you about how to solve the inequality $-2x + 1 > 3$?

5. A solution for the inequality $-0.4(x + 2) \geq 4.4$ is shown.

a Justify each step.

Inequality	Explanation
$-0.4(x + 2) \geq 4.4$	Given inequality
$-0.4(x + 2) \div -0.4 \leq 4.4 \div -0.4$	
$x + 2 \leq -11$	Simplify
$x + 2 - 2 \leq -11 - 2$	
$x \leq -13$	Solution

b Graph the solution.



Name: .... Date: .... Period: ....

Negative Coefficients (continued)

6. Rudra made a mistake solving the inequality $5x + 10 < -50$.

Rudra

$5x + 10 < -50$

$5x < -60$

$x > -12$

a Circle his mistake and explain what he did wrong.

b Determine the correct solution.

7. Solve each inequality.

a $-\frac{1}{3}x + \frac{1}{6} > -\frac{5}{6}$

b $0.4 \geq 0.2(x - 7)$

Activity 3

Name: ______ Date: ______ Period: ______

Inequalities at the Movies

8. For a Saturday matinee, a movie theater has a goal of $\$2{,}000$ in revenue, which includes ticket sales and concessions. Last Saturday, the theater sold $\$1{,}278$ worth of concessions and x tickets for $\$13.50$ each. How many tickets did the theater need to sell to meet its goal?

a Solve the inequality. Show your work.

b Explain which types of numbers are a part of the solution.

9. Makayla is watching a movie with friends. She will buy a student ticket for $\$8.75$ and would like to buy some bags of popcorn to share. Each bag of popcorn costs $\$5.25$, and she has $\$25$ to spend.

a Write and solve an inequality to determine how many bags of popcorn Makayla can buy. Show you work.

b Explain which types of numbers are a part of the solution.

Synthesis

10. Compare and contrast the process for solving a two-step inequality with the process for solving a two-step equation.

$$3x - 5 > 10$$

$$\text{-}20 \geq \text{-}4(x + 3)$$

Use the inequalities if they help with your thinking.

Lesson Practice ACC7.5.10

Lesson Summary

When solving two-step *inequalities*, apply the same operations to both sides of the inequality at each step so that the inequality remains true.

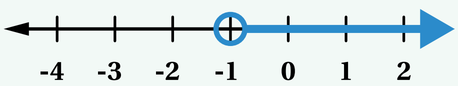
To keep an inequality true, you must reverse the direction of the inequality sign in any step that involves multiplying or dividing by a negative number.

Here are two examples:

Inequality 1	Inequality 2
$\text{-}10x + 20 < 30$	$\text{-}10(x - 2) < 30$
$\text{-}10x + 20 - 20 < 30 - 20$	$\text{-}10(x - 2) \div \text{-}10 > 30 \div \text{-}10$
$\text{-}10x < 10$	$x - 2 > \text{-}3$
$\text{-}10x \div \text{-}10 > 10 \div \text{-}10$	$x - 2 + 2 > \text{-}3 + 2$
$x > \text{-}1$	$x > \text{-}1$

Inequalities can have multiple solutions, so you often represent the solution on a graph.

- Use an open circle for an inequality sign of $<$ or $>$ because the solution does not include the boundary point.
- Use a closed circle for an inequality sign of $\leq$ or $\geq$ because the solution includes the boundary point.



Lesson Practice
ACC7.5.10

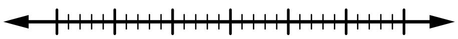
Name: ______________________ Date: __________ Period: __________

1. Explain the steps used to solve the inequality $-0.8x + 0.4 \geq 3.6$.

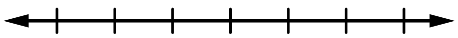
Inequality	Explanation
$-0.8x + 0.4 \geq 3.6$	Given inequality
$-0.8x + 0.4 - 0.4 \geq 3.6 - 0.4$	
$-0.8x \div -0.8 \leq 3.2 \div -0.8$	
$x \leq -4$	Solution

Problems 2–4: Solve the inequality. Graph each solution on a number line.

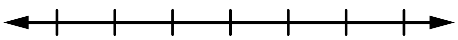
2. $3 < \frac{1}{4}(w - 8)$



3. $-\frac{1}{3}x + 2 \geq -2$



4. $0.25(y + 800) \geq -150$



5. Tasia has $\$40$ to spend at the fair. The admission is $\$12$, and the rides are $\$3$ each. Solve the inequality $3x + 12 \leq 40$ to determine the number of rides, x, she can ride. Explain the solution.

Name: ______ Date: ______ Period: ______

6. A group of 8th graders has raised $\$2{,}500$ for charity. If the group is selling raffle tickets for $\$5$ each, how many tickets must the group sell to raise at least $\$4{,}000$ in all? Write and solve an inequality to determine the number of tickets, x, the group must sell. Explain what types of numbers are included in the solution.

FAST Practice

7. Faaria has $\$800$ in her savings account and withdraws $\$50$ each week. She wants to keep a minimum of $\$200$ in the account. Solve the inequality $800 - 50x \geq 200$ to find the number of weeks she can withdraw money.

A. $x \geq 12$

B. $x \leq 12$

C. $x \geq 20$

D. $x \leq 20$

Spiral Review

Problems 8–9: Solve each of the following equations.

8. $5(t + 2) = 5t + 20$

9. $7t - 20 = 4(t + 10)$

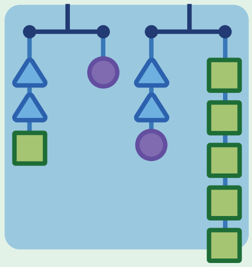
Sub-unit

3

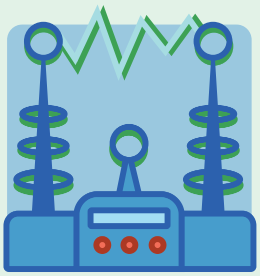
Systems of Linear Equations

Lesson 11
On or Off the Line?

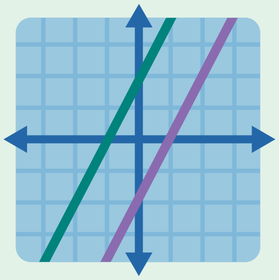
Lesson 12
On Both Lines



Lesson 13
Make Them Balance



Lesson 14
Line Zapper



Lesson 15
All, Some, or None? Part 2

Unit 5
Lesson
11

Name: ________ Date: ________ Period: ________

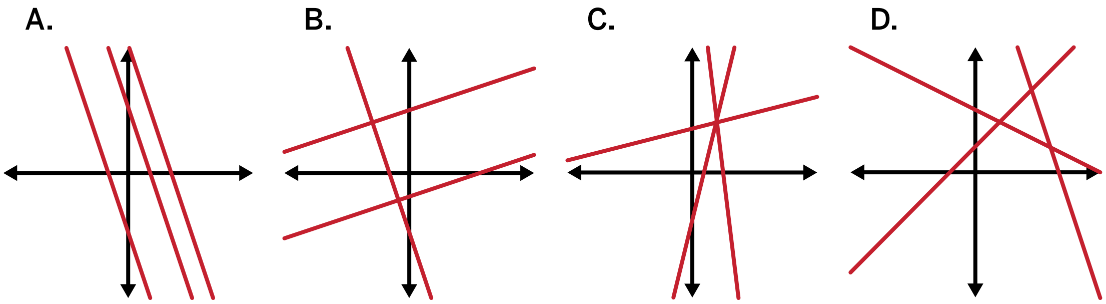
MA.8.AR.4.1, MTR.2.1, MTR.4.1

On or Off the Line?

Let's interpret the meaning of points on and off lines.

Warm-Up

1. Which one doesn't belong? Explain your thinking.



Activity 1

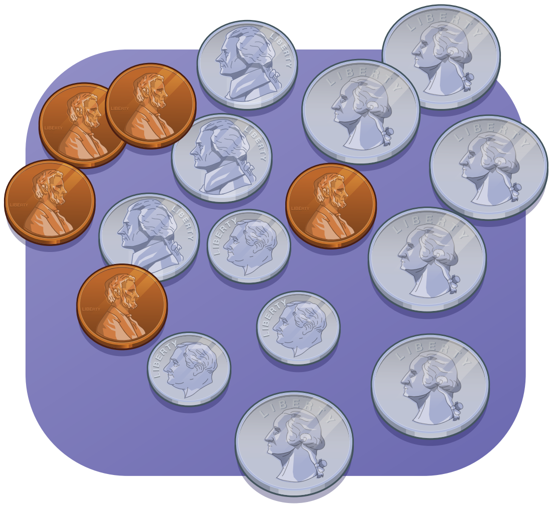
Name: ______ Date: ______ Period: ______

Two Dollars

2. I have $2 worth of coins in my pocket.

What is a combination of any coins that I could have?

Try to think of a combination that no one else in the class will write.

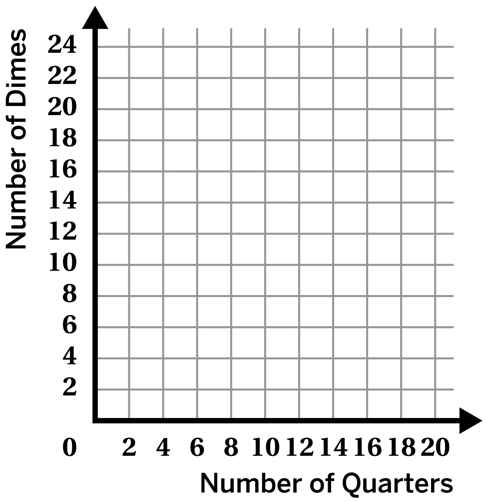


3. Here is more information about my coins:

- I only have quarters and dimes.

a Fill in three rows of possible combinations of quarters and dimes that are worth $2.

Number of Quarters	Number of Dimes



b Plot your points on the graph.

Activity 1

Name: ________ Date: ________ Period: ________

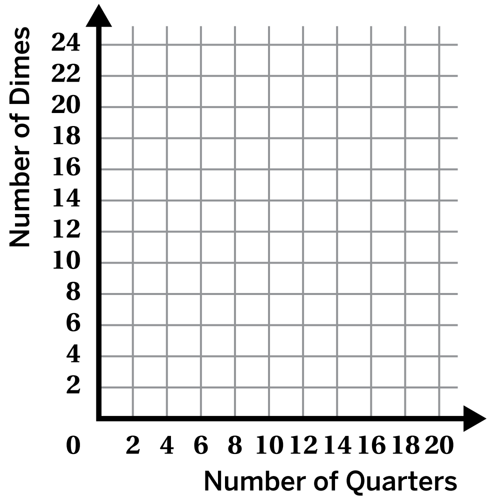
Two Dollars (continued)

4. Here is some more information about my coins:

- I have a total of 17 coins.

a Fill in at least three rows of possible combinations of 17 coins.

Number of Quarters	Number of Dimes

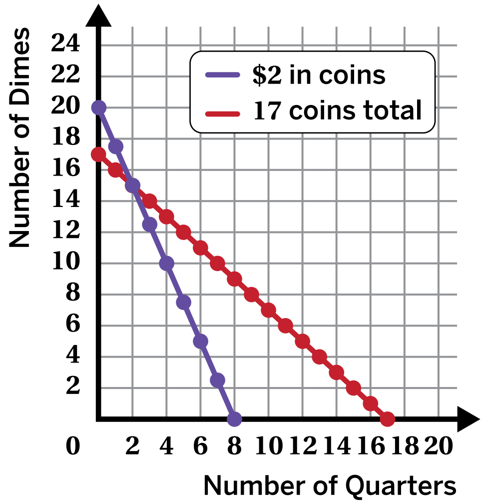


b Plot your points on the graph.

5. Let's look at the graphs of these conditions on the same coordinate plane.

I have \$2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

How many quarters and dimes must I have? Explain your thinking.



Activity 2

Name: ____ Date: ____ Period: ____

Challenge Creator

6. Let's look back at the graphs of both of these conditions on the same coordinate plane on the previous page.

I have $2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

In this situation, which conditions does the point $(12, 5)$ meet? Circle one.

I have $2 worth of coins in my pocket	I have a total of 17 coins	Both	Neither

Explain your thinking.

7. You will use the Activity 2 Sheet to create your own linear situations challenge.

a **Make it!** On the Activity 2 Sheet, create a linear situation challenge.

b **Solve it!** On this page, record the ordered pair that the statement represents and determine which line(s) your statement applies to.

Situation	Ordered Pair	Lines (Circle one.)
		Line 1 Line 2 Both Neither

c **Swap it!** Swap your challenge with one or more partners. On this page, record the ordered pair that the statement represents and determine which line(s) their statement applies to.

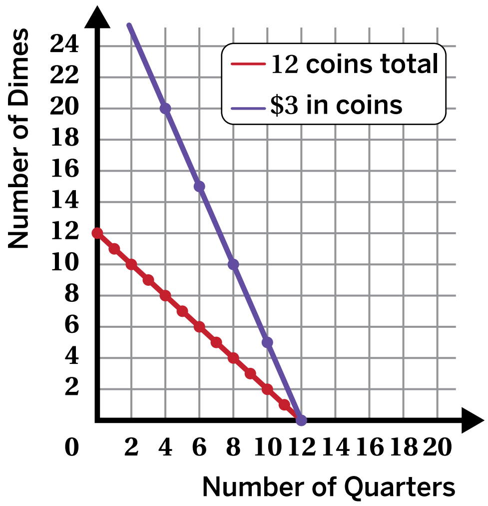
	Situation	Ordered Pair	Lines (Circle one.)
Partner 1			Line 1 Line 2 Both Neither
Partner 2			Line 1 Line 2 Both Neither
Partner 3			Line 1 Line 2 Both Neither
Partner 4			Line 1 Line 2 Both Neither

Synthesis

8. Here is a graph that represents two conditions:

- I have $3 worth of coins in my pocket.
- I have a total of 12 coins.

How can you use the graph to determine whether a combination of dimes and quarters meets both conditions, one condition, or neither condition?

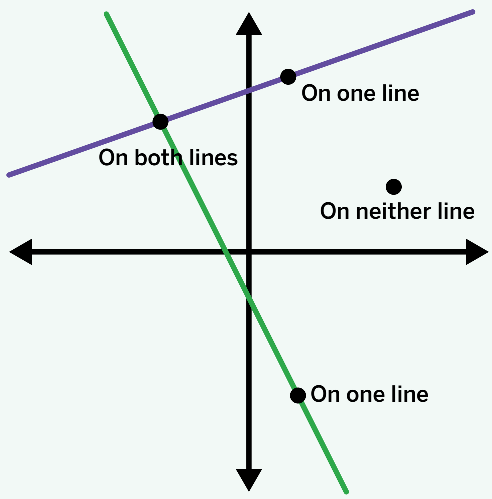


Lesson Practice ACC7.5.11

Lesson Summary

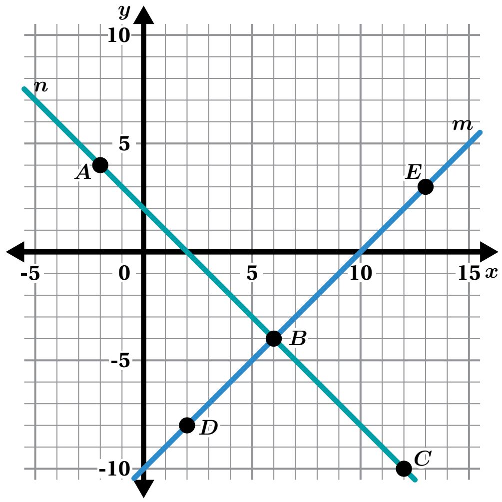
Linear relationships can represent many situations. Lines graphed on the same coordinate plane can simultaneously represent multiple conditions or relationships involving the same variables.

- The coordinates of a point that is on both lines make both relationships true.
- The coordinates of a point on only one line make only one relationship true.
- The coordinates of a point on neither line make neither relationship true.



Name: .... Date: .... Period: ....

Problems 1–5: Here is a coordinate plane.



1. Determine which line represents this condition: The coordinates of each point have a sum of 2.

2. Determine which line represents this condition: The y-coordinate of each point is 10 less than the x-coordinate.

3. Select *all* the points whose coordinates have a sum of 2.

- ☐ Point A
- ☐ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

4. Select *all* the points whose y-coordinate is 10 less than the x-coordinate.

- ☐ Point A
- ☐ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

5. Select *all* the points whose coordinates have a sum of 2 and the y-coordinate is 10 less than the x-coordinate.

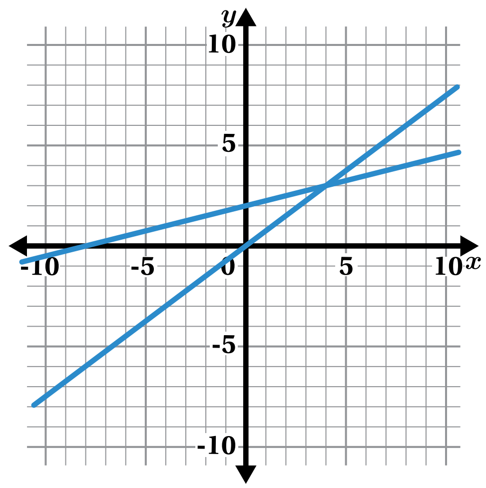
- ☐ Point A
- ☐ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

Lesson Practice
ACC7.5.11

Name: .... Date: .... Period: ....

FAST Practice

6. These two lines represent a system of equations. What is the y-coordinate of the point that makes both equations true?



A. 0

B. $\frac{1}{5}$

C. 3

D. 4

Spiral Review

Problems 7–9: Consider the equation $4x - 4 = 4x +$ .... What value or expression could you write in the blank so that the equation is true for:

7. No values of x?

8. All values of x?

9. One value of x?

Name: .... Date: .... Period: ....

MA.8.AR.4.1, MA.8.AR.4.3, MTR.1.1, MTR.4.1

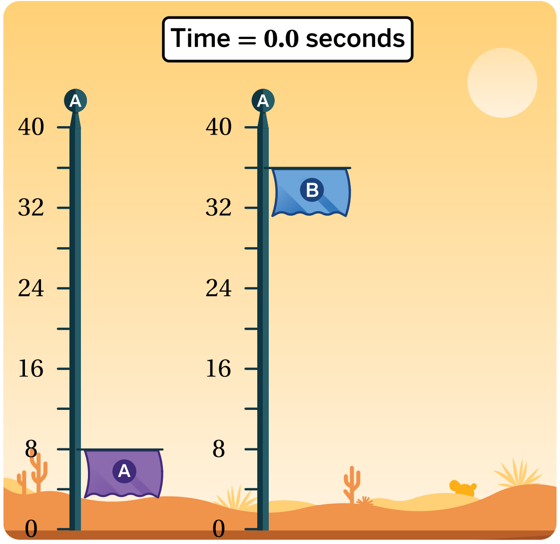
On Both Lines

Let's use lines to analyze real-world situations.

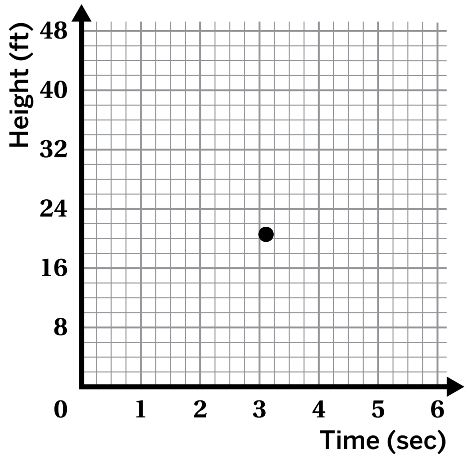
Warm-Up

1. Consider the image of two team flags on flagpoles. The heights on the poles are marked off in feet. Imagine that these two flags can move up or down their poles at constant speeds that may or may not be different from each other.

Describe a situation where Flag A and Flag B meet at the same height when Flag A is raised at a constant rate and Flag B is lowered at a constant rate.



2. A graph's **point of intersection** is a point where the lines meet. Graph a line to represent each flag's height over time. What does the point of intersection represent in the flag movement scenario?



Activity 1

Name: .... Date: .... Period: ....

On Both Lines

You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.

COMPANY #1

SET-UP FEE

$20

SERVICE PLAN

$25 /MONTH

PHONE PAYMENT

$40 /MONTH

COMPANY #2

SET-UP FEE

$30

NEW PHONE

$700

SERVICE PLAN

$35 /MONTH

COMPANY #3

PRICING INFORMATION TABLE (INCLUDES PLAN, PHONE, AND SET-UP FEE.)

Time (Months)	Total Cost ($)
0	30
9	885
12	1170

COMPANY #4

$y=95x+30$

y = TOTAL COST

x = MONTHS

INCLUDES PLAN, NEW PHONE, AND SET-UP FEE.

3. Choose *two* different companies and make a poster comparing them. Be sure to include these items in your poster:

- ☐ Your names
- ☐ The names of your selected companies
- ☐ A graph with both companies on the same set of axes
- ☐ An equation to represent each scenario
- ☐ Your conclusions about the situation
- ☐ The meaning of the point of intersection
- ☐ When each company has the better deal

4. Select a third phone provider. On your poster, explain when this third plan would be a better and worse deal than your two previously selected plans.

Activity 2

Name: .... Date: .... Period: ....

Gallery Tour

You will take a gallery tour as a class.

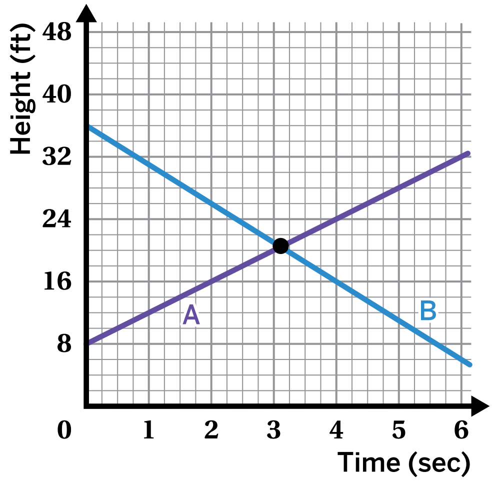
5. What features of your classmates' posters helped you understand their thinking?

6. Describe something you would change about your poster now that you have seen other groups' work.

Synthesis

7. Describe how a graph can help you compare different linear relationships.

Use the example if it helps with your thinking.



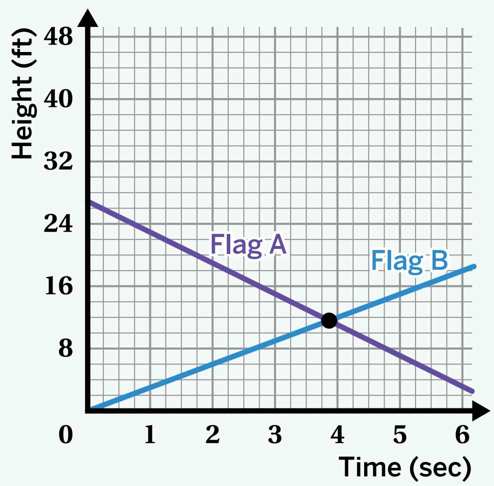
Lesson Practice ACC7.5.12

Lesson Summary

If there are two equations that share the same variables, you can find the solution that makes both equations true by locating the **point of intersection**, where the two lines meet on a graph.

For example, consider this graph.

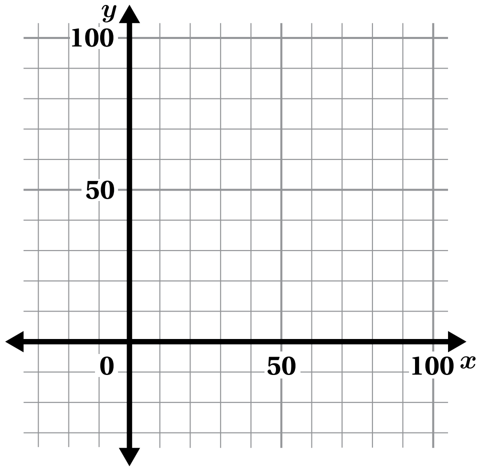
Although you can't see the exact values of the point of intersection, you can tell that the flags are the same height, at about 11.5 feet, just before 4 seconds.



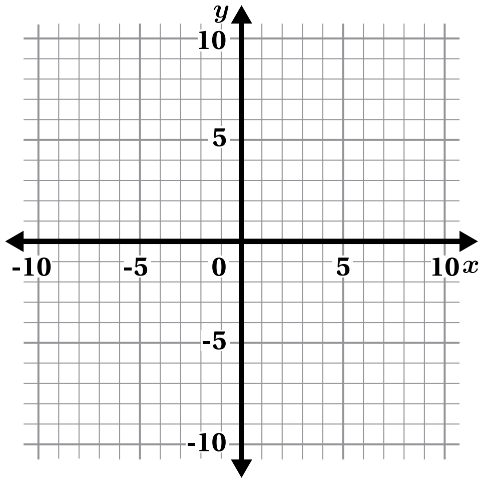
Lesson Practice
ACC7.5.12

Name: .... Date: .... Period: ....

1. Jayden has \$11 and begins saving \$5 each week to buy a new phone. At the same time that Jayden begins saving, Aditi has \$60 and begins spending \$2 per week on supplies for her art class. Is there a week when they will have the same amount of money? Explain your thinking.



2. Draw a graph to determine x- and y-values that make both of the equations $y = -\frac{2}{3}x + 3$ and $y = x - 2$ true.



Problems 3–4: Prisha and Mia agree to go to the movies after they have earned the same amount of money for the same number of hours worked.

Mia earns \$7 per hour mowing her neighbors' lawns. She also earned \$14 one time for hauling away bags of recyclables.

Prisha babysits her neighbor's children. The table shows the amount of money, m, she earns in h hours.

h	m
1	\$8.40
2	\$16.80
3	\$33.60

3. How many hours do they each have to work before they go to the movies?

4. How much will each of them have earned?

Name: .... Date: .... Period: ....

5. The graph of the equations $y = \square - x$ and $y = \square x - 3$ intersect at the point $(2, 1)$. Determine the missing values in the equations. Show or explain your thinking.

FAST Practice

6. The point where the graphs of two equations intersect has a y-coordinate of 2. One equation is $y = \text{-}3x + 5$.

Determine the other equation if its graph has a slope of 1.

Record your answer in the space provided.

Spiral Review

Problems 7–9: Determine whether each equation is true for one value, all values, or no values of x.

7. $10 + 3x = \text{-}4.2x + 9$

8. $5(4x + 1) - x = 19x + 5$

9. $\text{-}2(3x - 7) = \text{-}6x + 12$

Problems 10–11: Determine whether $x = \text{-}1$ is a solution to each equation.

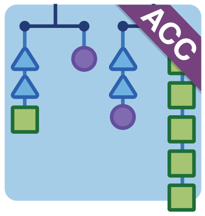
10. $1.5x + 2 = 8.5x - 4$

11. $\frac{1}{2} + 2x = \frac{1}{2}x - 1$

Name: .... Date: .... Period: ....

MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3, MTR.5.1

Make Them Balance



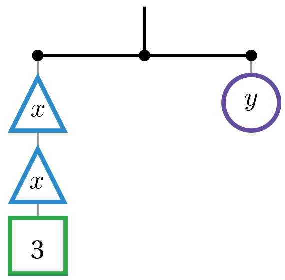
Let's explore solutions to more than one linear relationship.

Warm-Up

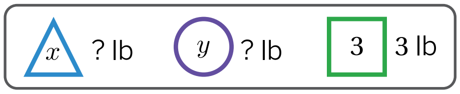
1. Determine values for x and y that will balance the hanger.

x	y

Hanger A

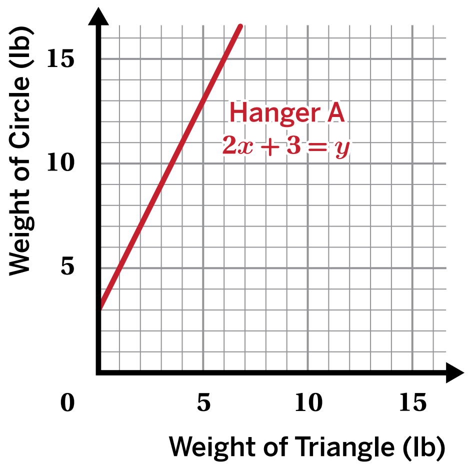


$2x + 3 = y$



2. Let's look at a graph that shows points whose x- and y-values balance the hanger.

Discuss: What do you notice? What do you wonder?



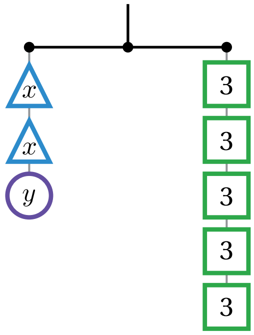
Activity 1

Name: .... Date: .... Period: ....

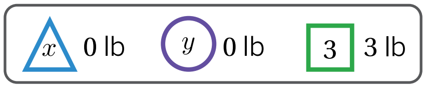
Two Hangers

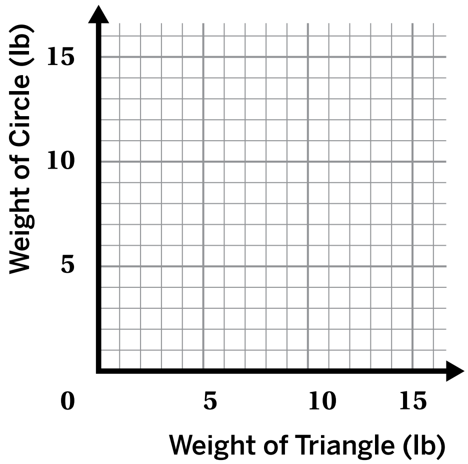
3. Here's another hanger. Plot three different pairs of x- and y-values that will balance this hanger.

Hanger B



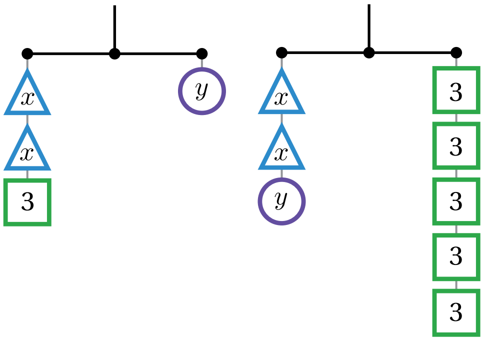
$2x + y = 15$



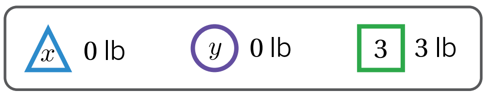


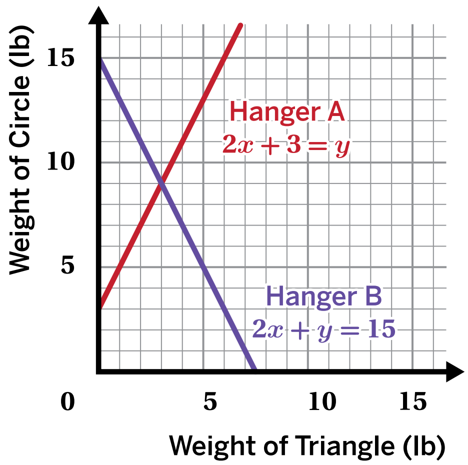
4. Here are both hangers' equations graphed on the same coordinate plane.

Hanger A   Hanger B



$2x + 3 = y$ $2x + y = 15$





Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

Two Hangers (continued)

5. You have been experimenting with two representations of this **system of equations**:

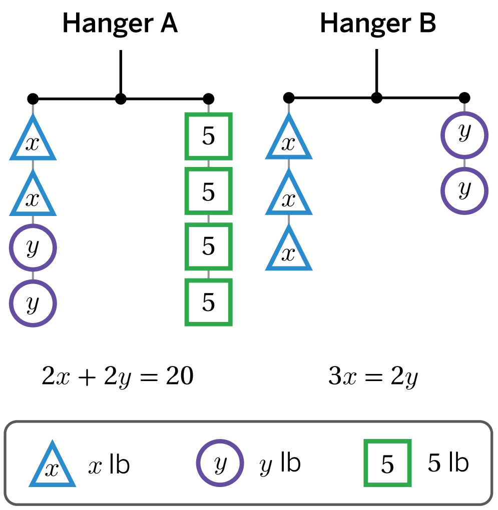
$2x + 3 = y$
$2x + y = 15$

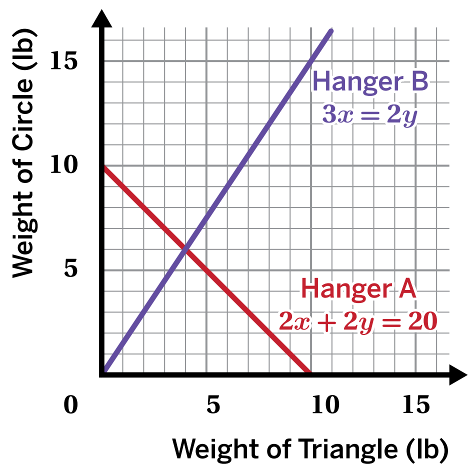
A **solution to a system of equations** is a set of values that makes both equations in that system true.

Write the solution to this system as an ordered pair. Explain your thinking.

6. This system represents another set of hangers:

$2x + 2y = 20$
$3x = 2y$





Which point is a solution to this system? Circle one.

$(1, 9)$ $(4, 6)$ $(6, 9)$ $(6, 4)$

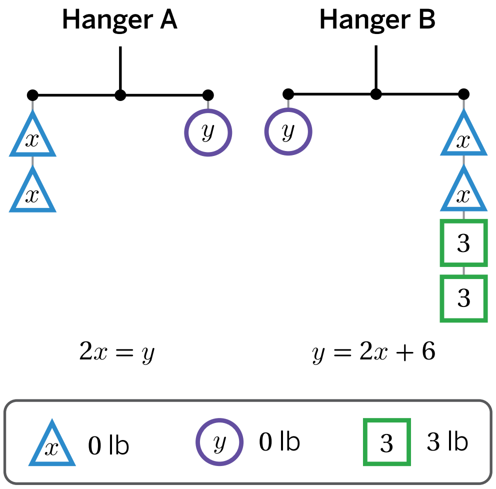
Explain your thinking.

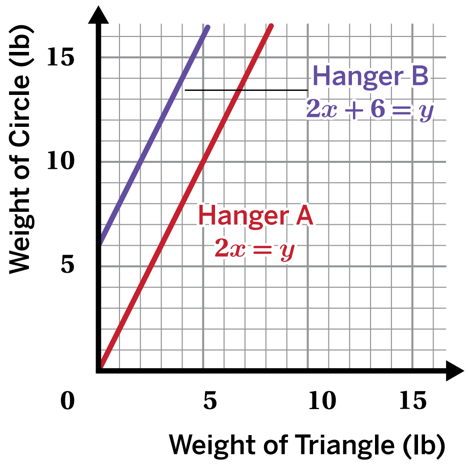
Activity 2

Name: ____________ Date: ________ Period: ________

Hanger Solutions

7. Here's another system of equations.





Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

8. This system of equations from the previous problem has no solution:

$2x = y$

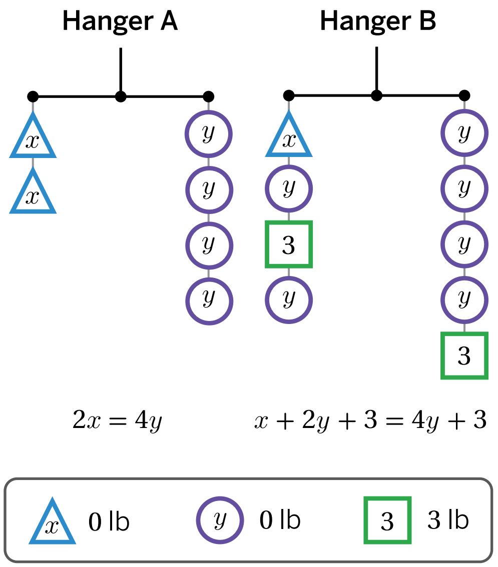
$y = 2x + 6$

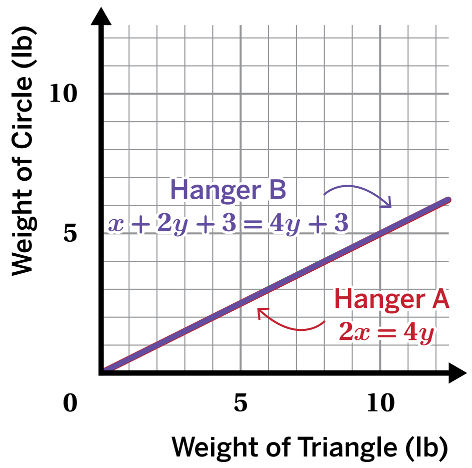
How can you use the hangers, graphs, or equations to tell that this system of equations has no solution?

Name: .... Date: .... Period: ....

Hanger Solutions (continued)

9. Here's one more system.

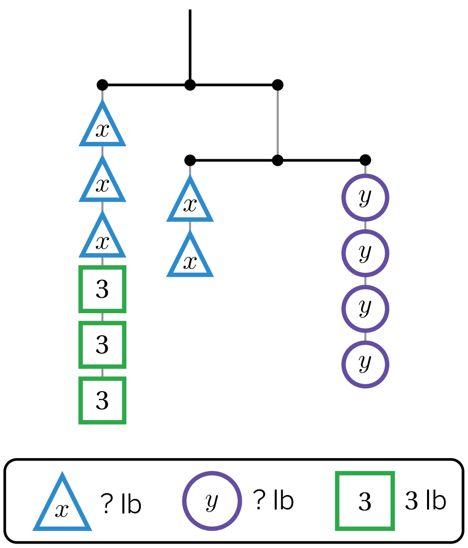




Discuss: How many solutions do you think this system has? How do you know?

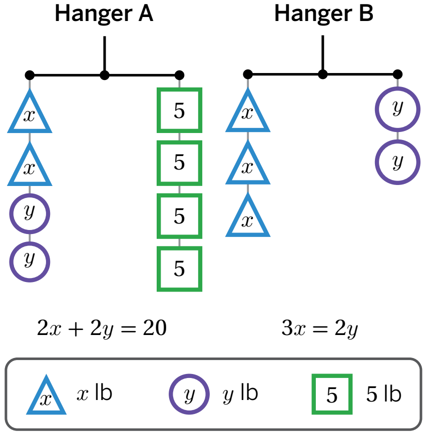
10. Determine values for x and y that will balance both hangers.

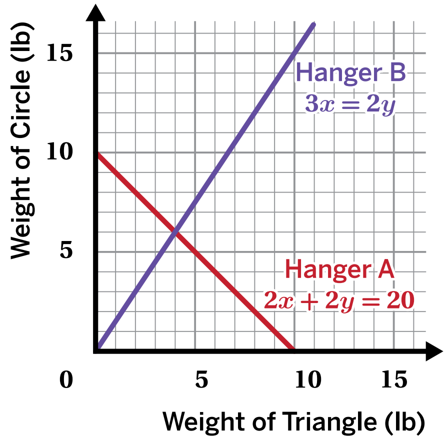
x	y



Synthesis

11. How can you tell if an ordered pair is a solution to a system of linear equations? Use the example if it helps with your thinking.





Lesson Practice ACC7.5.13

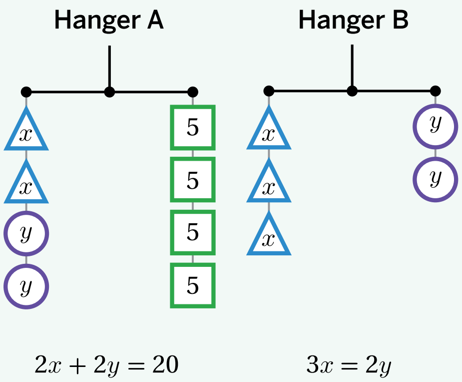
Lesson Summary

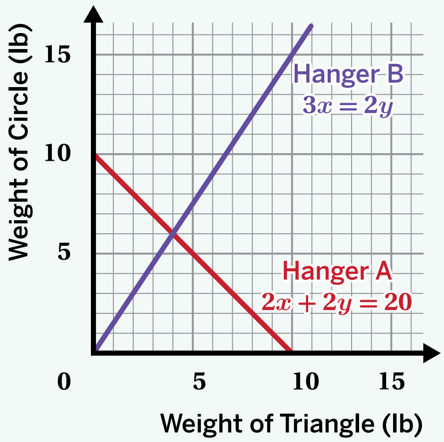
A **system of equations** is a set of two or more equations with the same variables and are meant to be solved together.

A **solution to a system** of equations is a set of values that makes all equations in that system true.

For example, here is a system of equations:

$$\begin{cases} 2x + 2y = 20 \\ 3x = 2y \end{cases}$$



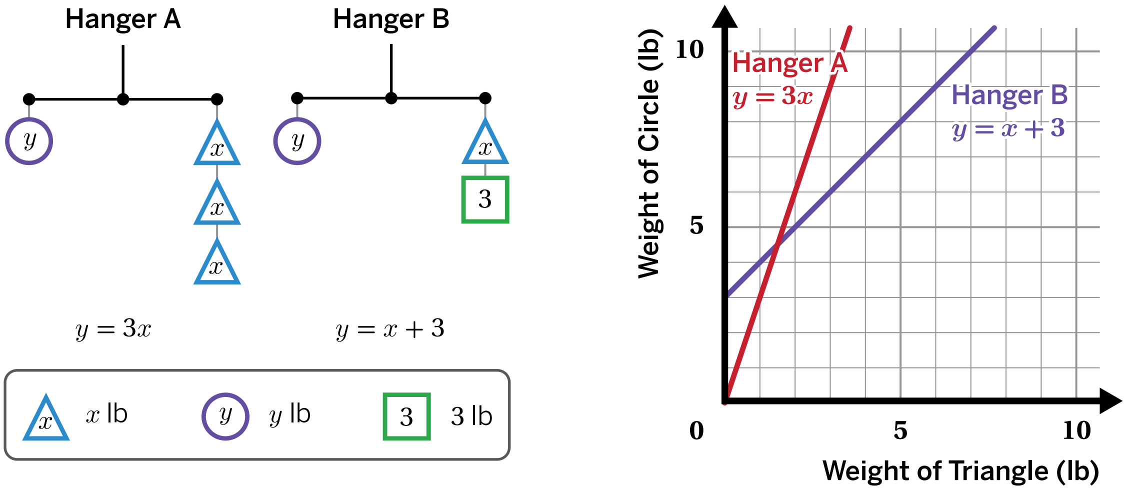


The ordered pair $(4, 6)$ is the point of intersection, which means that it will make both equations true. Both hangers will balance when the triangles weigh 4 pounds and the circles weigh 6 pounds.

Lesson Practice
ACC7.5.13

Name: .... Date: .... Period: ....

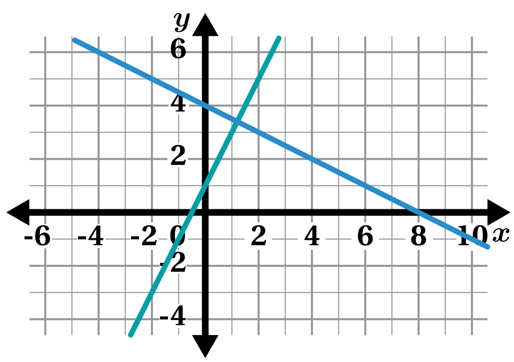
Problems 1–2: The hangers and the graph represent the same system of equations.



1. Determine the solution to the system of equations.

2. What does the solution tell you about the weight of a triangle and the weight of a circle that will balance the hanger?

Problems 3–4: Here is a graph.



3. Write an equation that can represent each line.

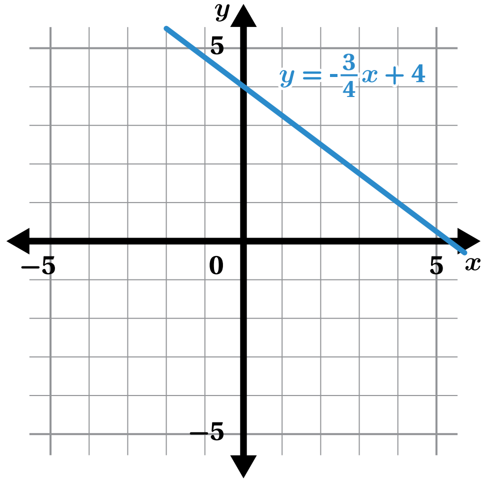
4. Estimate the solution to the system.

Lesson Practice
ACC7.5.13

Name: .... Date: .... Period: ....

FAST Practice

Problems 5–7: Here is a graph that represents one equation in a system of equations.



5. Write a second equation for the system so that it has infinitely many solutions.

6. Write a second equation whose graph goes through $(0, 1)$ so that the system has no solution.

7. Which equation creates a system with the graphed line that would have a solution of $(4, 1)$?

A. $y = -\frac{3}{4}x + 6$ B. $y = -\frac{1}{4}x + 6$ C. $y = \frac{1}{4}x - 4$ D. $y = -\frac{1}{4}x + 2$

Spiral Review

8. Select *all* the equations that have no solution.

- ☐ **A.** $2 + 4(4x + 5) = 8x + 2x - 11$
- ☐ **B.** $-x + 3x - 7 = 2(x - 7)$
- ☐ **C.** $7 - 5x(-3) = 5(3x - 2)$
- ☐ **D.** $6x + 3(2x - 1) = 5x - 4 + 7x + 1$

Problems 9–10: Solve the equation. Show or explain your thinking.

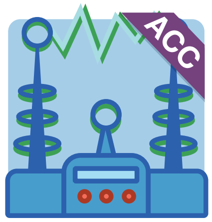
9. $\frac{15(x-3)}{5} = 3(2x - 3)$

10. $0.4(x + 7) = 0.2(x + 40) - 5.2 + 0.2x$

Name: .... Date: .... Period: ....

MA.8.AR.4.1, MA.8.AR.4.3, MTR.4.1, MTR.5.1

Line Zapper



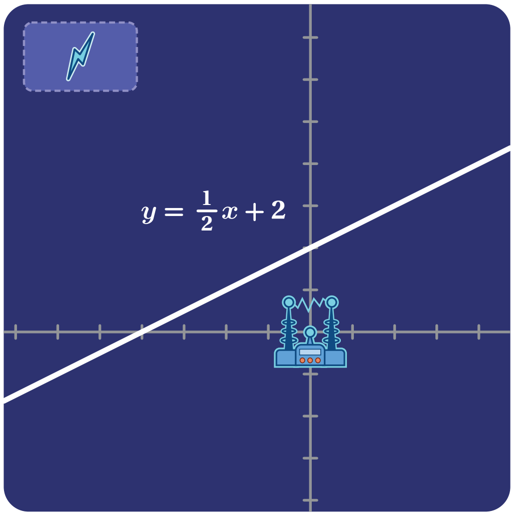
Let's solve systems of linear equations.

Warm-Up

1. In the game "Line Zapper," you can capture a line by writing the coordinates for a point that's on the line.

Capture the line shown.

Zap	Point
Zap 1	



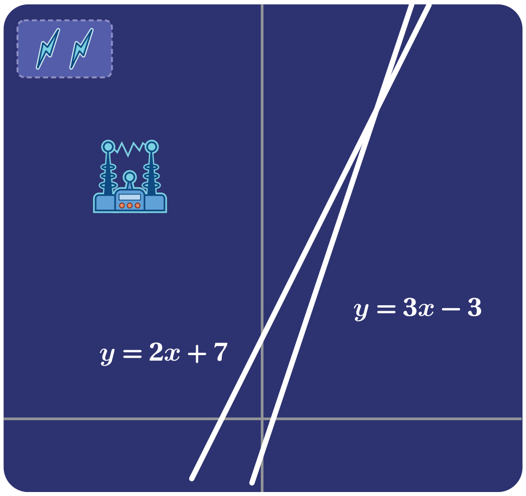
Name: .... Date: .... Period: ....

Two Lines, One Zap

2. Capture both of the lines by writing coordinates for points on these lines.

Use no more than two zaps!

Zap	Point
Zap 1	
Zap 2	



3. Cameron wanted to capture both lines with one zap.

He solved an equation to identify the point of intersection and determined that $x = 10$.

How could Cameron determine the y-value of the point of intersection?

Cameron

$y = 2x + 7$
$y = 3x - 3$

$2x + 7 = 3x - 3$
$-2x \qquad -2x$

$7 = 1x - 3$
$+3 \qquad +3$

$10 = 1x$
$\boxed{x = 10}$

Activity 2

Name: .... Date: .... Period: ....

Line Zapper

4. The following lines are hidden in a graph:

$y = -x + 10$
$y = 2x + 4$

Capture the lines by writing the coordinates for a point on both of these lines. You only have one zap!

Zap	Point
Zap 1	

5. **a** Select *all* the lines that would be captured if the point $(2, 4)$ were zapped.

- ☐ **A.** Line A: $y = \frac{1}{2}x$
- ☐ **B.** Line B: $y = 2x$
- ☐ **C.** Line C: $y = -x + 6$

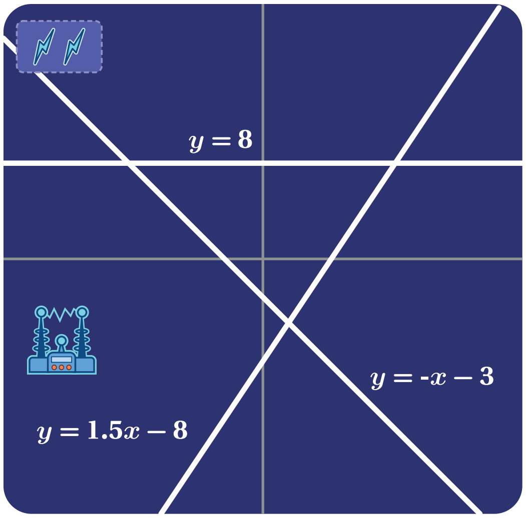
b Explain how you decided which lines to select.

Name: ________ Date: ________ Period: ________

Line Zapper (continued)

6. Capture all of the lines by writing coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	



7. The following lines are hidden in a graph:

$y = \text{-}2x + 9$

$y = 3x - 1$

$y = \frac{1}{2}x - 1$

Capture all of the lines by writing the coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	

You're invited to explore more.

8. The following lines are hidden in a graph:

$y = 3x + 6$

$y = 3(x - 5)$

$y = \text{-}\frac{1}{2}x - 15$

$\text{-}x + y = 12$

Capture all of the lines by writing coordinates for points on the lines. You only have three zaps!

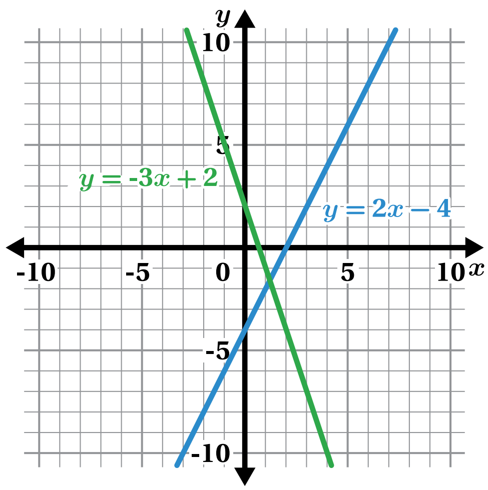
Zap	Point
Zap 1	
Zap 2	
Zap 3	

Synthesis

9. Here is the graph of this system of equations:

$y = \text{-}3x + 2$
$y = 2x - 4$

How can you determine the *exact* solution to this system of equations?



Lesson Practice ACC7.5.14

Lesson Summary

For a point to be a solution to a system of equations, the x- and y-coordinates must make both of the equations true. This ordered pair is the *point of intersection* when the system is graphed.

For example, here is a system of equations:

$$\begin{cases} y = 4x - 5 \\ y = \text{-}2x + 7 \end{cases}$$

To determine the solution to the system, you can write a single equation by taking the two expressions that are equal to y and setting them equal to each other.

$4x - 5 = \text{-}2x + 7$
$6x - 5 = 7$
$6x = 12$
$x = 2$

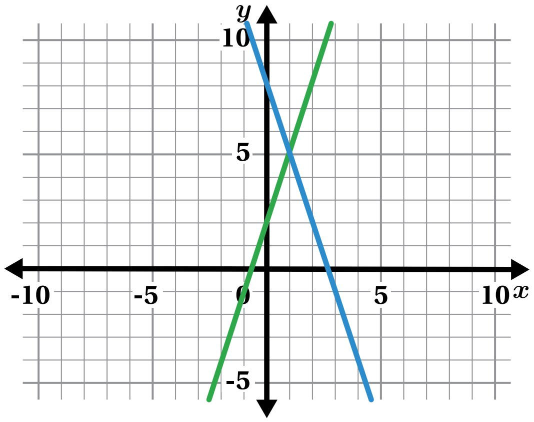
You can then substitute the solution for x into either of the original equations to determine the value of y.

$y = 4x - 5$
$y = 4(2) - 5$
$y = 8 - 5$
$y = 3$

The solution to this system of equations is the point $(2, 3)$.

Name: .... Date: .... Period: ....

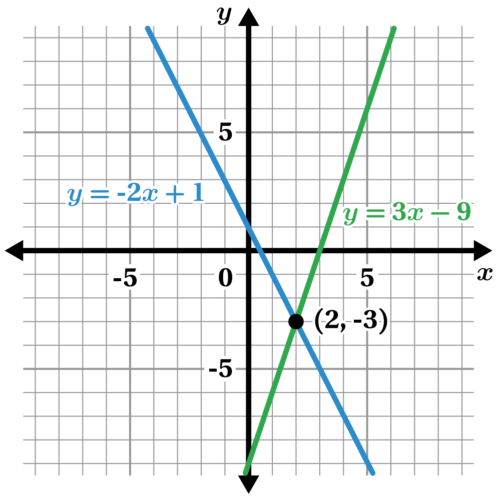
Problems 1–2: Here is the graph of this system of equations: $\begin{cases} y = \text{-}3x + 8 \\ y = 3x + 2 \end{cases}$



1. How can you determine the solution to this system of equations by looking at the graph?

2. What is the solution to the system of equations?

Problems 3–5: Use the lines in the graph to decide whether each statement is true or false.



3. The solution to the equation $\text{-}2x + 1 = 3x - 9$ is $x = 2$.

4. The point $(2, \text{-}3)$ is a solution to this system of equations: $\begin{cases} y = \text{-}2x + 1 \\ x = 2 \end{cases}$

5. The point $(0, 1)$ is a solution to the equation $y = \text{-}2x + 1$.

6. Here is how Haru tried to solve this system of equations. Did Haru solve the system of equations correctly? Explain your thinking.

Haru

$$\begin{cases} y = -3x + 1 \\ x = 4 \end{cases}$$

$-3x + 1 = 4$
$-3x = 3$
$x = -1$

$y = -3(-1) + 1$
$y = 4$

Solution: $(-1, 4)$

FAST Practice

7. What is the solution to the system of equations?

$$\begin{cases} y = \text{-}4x + 2 \\ y = 3x - 5 \end{cases}$$

A. (-1, -2)

B. (-1, 2)

C. (1, -2)

D. (1, 2)

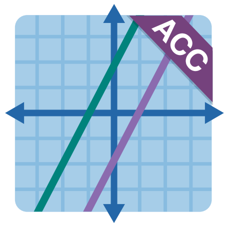
Spiral Review

8. The temperature in degrees Fahrenheit, F, is related to the temperature in degrees Celsius, C, which is represented by the equation $F = \frac{9}{5}C + 32$. There is one temperature where the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. What is that temperature? Show or explain your thinking.

Name: ________ Date: ________ Period: ________

MA.8.AR.4.2, MA.8.AR.4.3, MTR.5.1

All, Some, or None? Part 2

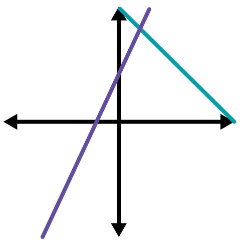


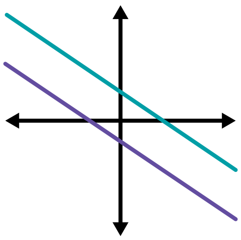
Let’s solve systems with no solution and infinitely many solutions.

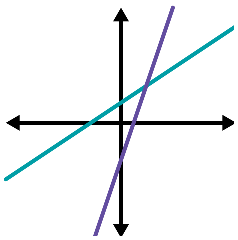
Warm-Up

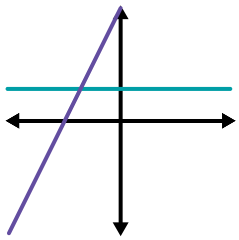
1. Which graph could represent this system of equations?

$y = 2x + 4$
$y = \text{-}x + 10$

A.


B.


C.


D.


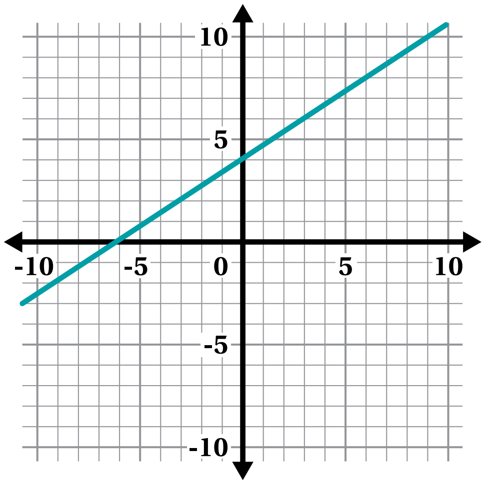
Explain your thinking.

Name: Date: Period:

Connecting Graphs and Equations

2. Here is the graph of $y = \frac{2}{3}x + 4$.

Graph another line to create a system of equations that has *no* solution.



3. Here are the equations for a system that has no solution:

$y = \frac{2}{3}x + 4$

$y = \frac{2}{3}x - 5$

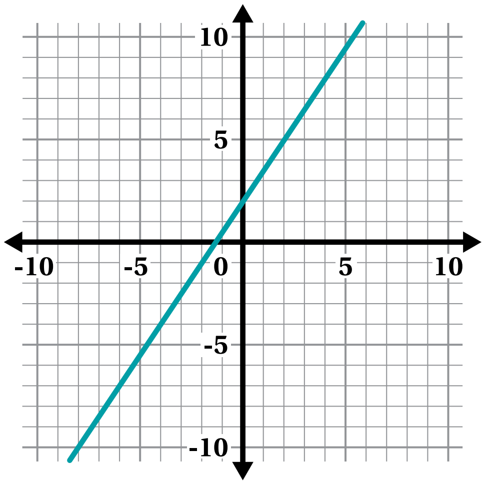
How can you determine from the equations that the system will have no solution?

Name: Date: Period:

Connecting Graphs and Equations (continued)

4. Here is the graph of $y = \frac{1}{2}(3x + 4)$.

Graph another line to create a system of equations that has *infinitely many* solutions.



5. Here are the equations for a system that has infinitely many solutions:

$y = \frac{1}{2}(3x + 4)$

$y = \frac{3}{2}x + 2$

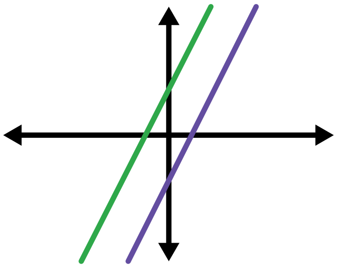
How can you determine from the equations that the system will have infinitely many solutions?

Name: .... Date: .... Period: ....

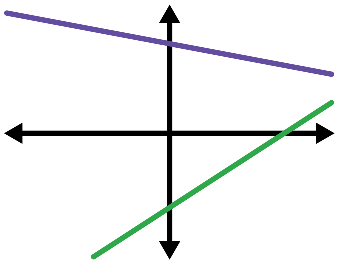
Sorting Systems

6. Group the systems of equations based on their number of solutions.

A.



B.



C. $\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$

D. $\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$

E. $\begin{cases} y = -x + 10 \\ y = -x - 3 \end{cases}$

F. $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

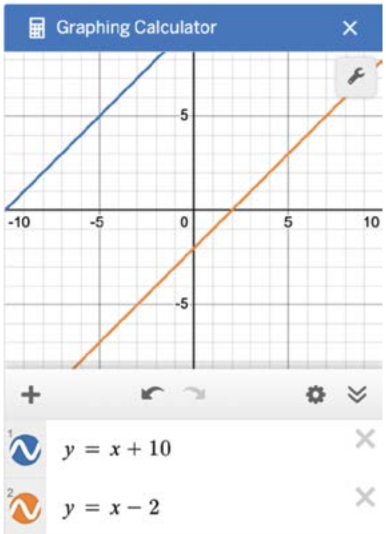
No Solution	One Solution	Infinitely Many Solutions

Name: .... Date: .... Period: ....

Sorting Systems (continued)

7. Here are Oscars's and Melanie's strategies to check whether System F goes in the "No Solution" group.

Oscar



Melanie

F $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

$$x + 10 = x - 2$$
$$-x \qquad\qquad -x$$
$$10 = -2$$

Discuss: How are their strategies alike? How are they different?

Synthesis

8. How can you determine the number of solutions for a system of equations?

C
$$\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$$

D
$$\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$$

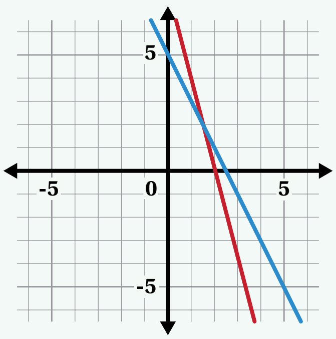
E
$$\begin{cases} y = -x + 10 \\ y = -x - 3 \end{cases}$$

Lesson Practice ACC7.5.15

Lesson Summary

Systems of two linear equations can have one solution, no solution, or infinitely many solutions. You can determine the number of solutions to a system of equations by graphing, comparing the slopes and y-intercepts, or solving the system algebraically.

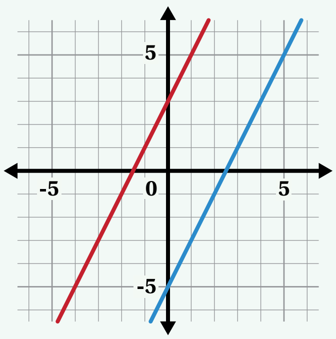
One solution:



$$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$$

- Different slopes

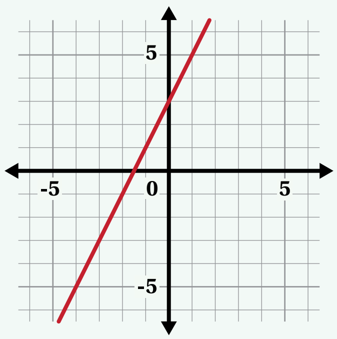
No solution:



$$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$$

- Same slopes
- Different y-intercepts
- Parallel lines

Infinitely many solutions:



$$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$$

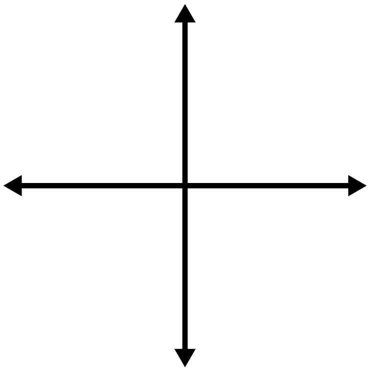
- Same slopes
- Same y-intercepts
- Same line

Lesson Practice
ACC7.5.15

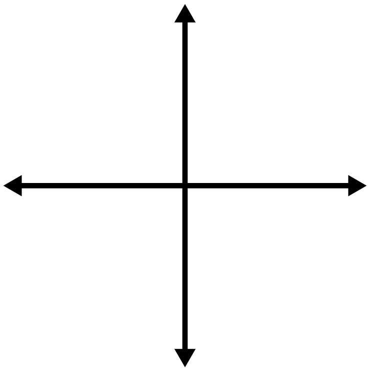
Name: .... Date: .... Period: ....

Problems 1–3: Sketch two lines that match each description. Then describe the number of solutions for each system of equations.

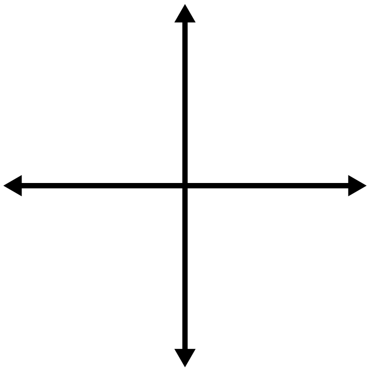
1. Two lines with the same slope and different y-intercepts.



2. Two lines with different slopes.



3. Two lines with the same slope and same y-intercept.

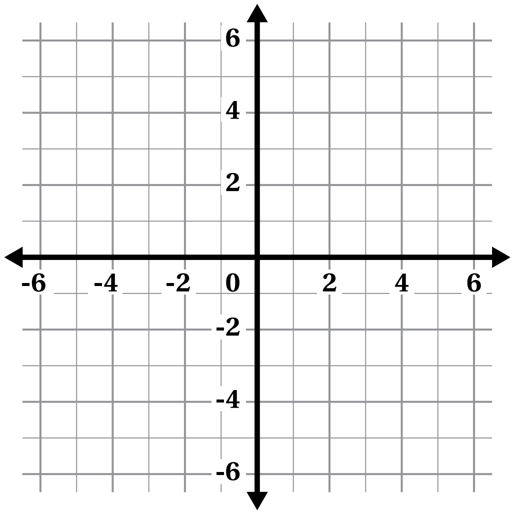


4. How many solutions does this system have?

$y = \frac{1}{4}x + 2$

$y = 2x - 5$

Show or explain your thinking.



Problems 5–6: Consider this system of equations: $\begin{cases} y = \frac{1}{6}x - \frac{2}{3} \\ y = \frac{1}{6}x - \frac{2}{9} \end{cases}$

5. Change one number to make a new system with one solution.

6. Change one number to make a new system with an infinite number of solutions.

Lesson Practice
ACC7.5.15

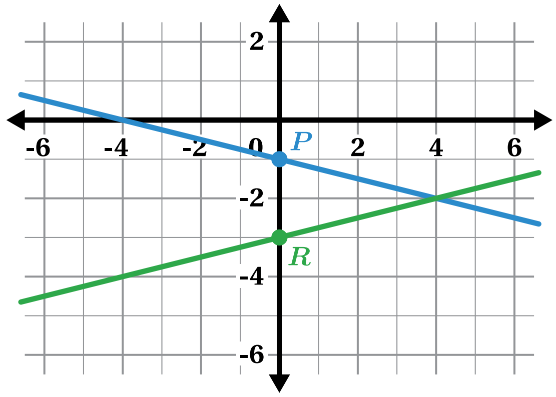
Name: ______________ Date: ________ Period: ________

FAST Practice

Problems 7–8: Ali graphed this system:

$$\begin{cases} y = \frac{1}{4}x - 1 \\ y = \frac{1}{4}x - 3 \end{cases}$$

He marked its solutions with points P and R.



7. Which statement describes Ali's solutions?

A. His solutions are correct.

B. He marked the y-intercepts instead of the intersection point of the two lines.

C. He marked the y-intercepts instead of the x-intercepts.

D. He marked only the y-intercepts instead of the x- and y-intercepts.

8. What is the solution to the system of equations?

Record your answer in the space provided.

Spiral Review

Problems 9–10: Solve each equation.

9. $\frac{3y - 6}{9} = \frac{4 - 2y}{-3}$

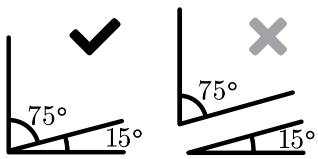
10. $0.3(x - 10) - 1.8 = 2.7x$

Glossary/Glosario

English

A

adjacent angles Angles that share a side and a vertex.

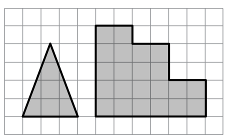


The image with the check mark shows a 75° angle and a 15° angle when they are adjacent.

approximation A rounded value that you can use to represent a number that may be difficult to work with, such as an irrational number or a repeating decimal.

For example, the exact value of pi (π) is an irrational number, so we often use the approximate value of 3.14 in calculations involving pi.

area The space inside a two-dimensional figure. It is expressed in square units.



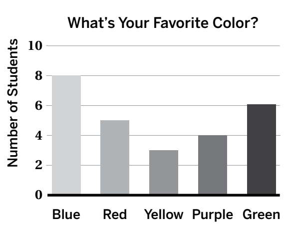
The area of the triangle is 6 square units. The area of the other shape is 22 square units.

association If two variables are related to one another, we say that there is an association between them. Associations can be described as positive or negative, linear or nonlinear, and strong or weak.

Some examples of associations are:
Positive: One variable increases as the other also increases.
Negative: One variable decreases as the other increases.
Linear: Can be modeled by a straight line.
Nonlinear: Cannot be modeled by a straight line.

B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.



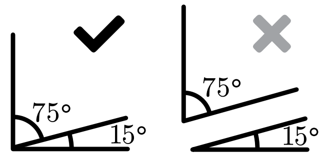
bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

For example, the repeating decimal $0.090909...$ can be written in bar notation as $0.\overline{09}$.

Español

A

ángulos adyacentes Ángulos que comparten un lado y un vértice.

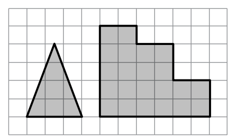


La imagen con la marca de verificación muestra un ángulo de 75° y un ángulo de 15° cuando son adyacentes.

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un número irracional o un decimal periódico.

Por ejemplo, el valor exacto de pi (π) es un número irracional, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

área El espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.



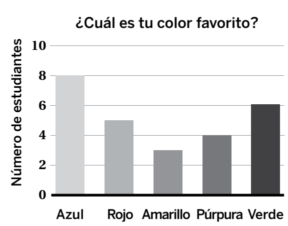
El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.

asociación Si dos variables están relacionadas entre sí, decimos que existe una asociación entre ambas. Las asociaciones pueden describirse como positivas o negativas, lineales o no lineales lineales y fuertes o débiles.

Algunos ejemplos de asociaciones son los siguientes:
Positiva: El valor de una variable aumenta a medida que el valor de la otra también aumenta. Negativa: El valor de una variable disminuye a medida que el valor de la otra aumenta.
Lineal: Puede modelarse con una línea recta.
No lineal: No puede modelarse con una línea recta.

B

gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.

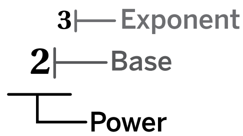


raya indicadora de decimales periódicos (vinculum) Una forma de representar los dígitos que se repiten en un número decimal. Sobre los dígitos que se repiten se traza una pequeña línea.

Por ejemplo, el decimal periódico $0.090909...$ puede escribirse con la raya indicadora de decimales periódicos (vinculum) como $0.\overline{09}$.

English

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.

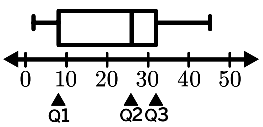


In this example, 2 is the base.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

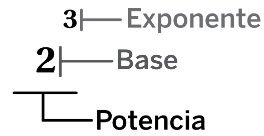
For example, a collection of students' heights and shoe sizes would be a bivariate data set.

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



Español

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.

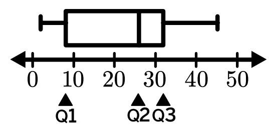


En este ejemplo, 2 es la base.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.

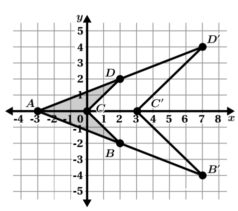


C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

"What kind of pet do you have?" is a question that would result in categorical data.

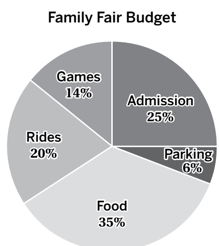
center of a dilation The point from which we measure distances in a dilation.



The center of dilation in this example is point A.

circle A shape made out of all the points that are the same distance from a center point.

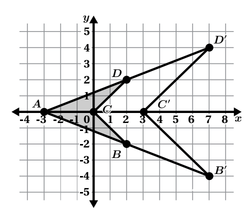
circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a pie chart.



datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

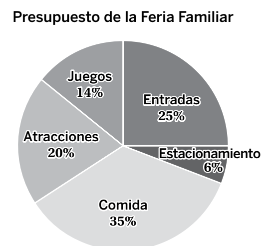
centro de una dilatación El punto desde el cual medimos las distancias en una dilatación.



El centro de dilatación en este ejemplo es el punto A.

círculo Una figura formada por todos los puntos que están a la misma distancia de un punto central.

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado gráfico de torta.

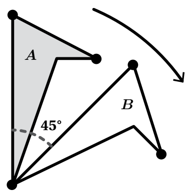


Glossary/Glosario

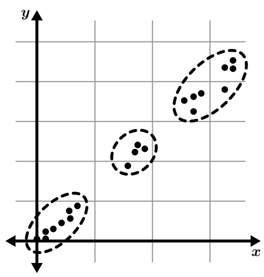
English

circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string. The circumference of a circle, C, can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

clockwise In the same direction as the hands of a clock; to the right (of a turn).



clusters Groups of data values that are close together.



coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

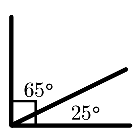
In the expression $5x + 8$, 5 is the coefficient of x.

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

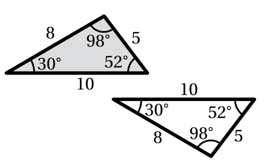
For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

complementary angles Two angles whose measures add up to $90°$.

For example, a $65°$ angle and a $25°$ angle are complementary.



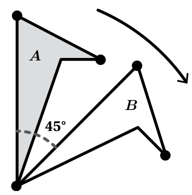
congruent One figure is congruent to another if they have exactly the same size and shape, or if one can be mapped to the other with a translation, rotation, or reflection.



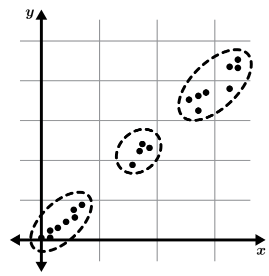
Español

circunferencia La distancia alrededor de un círculo. Si imaginamos el círculo como un trozo de cuerda, es la longitud de la cuerda. La circunferencia de un círculo, C, puede calcularse mediante la fórmula $C = \pi d$, donde d es el diámetro del círculo, o $C = 2\pi r$, donde r es el radio.

en el sentido de las manecillas del reloj Girar en la misma dirección que las manecillas de un reloj. En otras palabras, un giro a la derecha.



agrupaciones (clusters) Grupos de valores de datos que están próximos entre sí.



coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

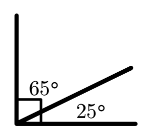
En la expresión $5x + 8$, el coeficiente de x es 5.

propiedad conmutativa La propiedad indica que $a + b = b + a$ and $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente del orden en el que estén los números.

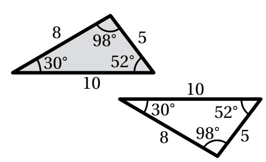
Por ejemplo, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

ángulos complementarios Dos ángulos cuyas medidas suman $90°$.

Por ejemplo, un ángulo de $65°$ y otro ángulo de $25°$ son complementarios.



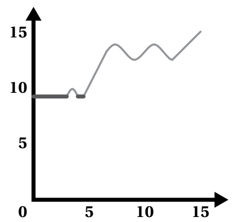
congruente Una figura es congruente con otra si tienen exactamente el mismo tamaño y forma, o si una puede ser transformada en la otra mediante una traslación, rotación o reflexión.



English

constant (function) A function or interval of a function is constant if the y-values stay the same when the x-values go up.

The bolded parts of this function are constant.



constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

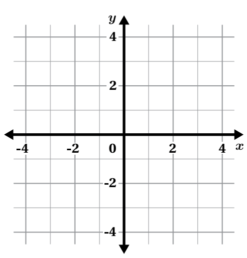
Carpet (sq. ft.)	Cost (dollars)
10 $\xleftarrow{\times\frac{2}{3}}$	15.00
20 $\xleftarrow{\times\frac{2}{3}}$	30.00
50 $\xleftarrow{\times\frac{2}{3}}$	75.00

In this table, one constant of proportionality is $\frac{2}{3}$.

converse A statement written in the opposite direction.

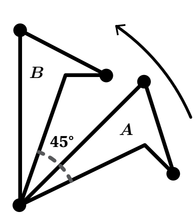
For example, the Pythagorean theorem states: If a triangle is right, it has side lengths such that $a^2 + b^2 = c^2$. The converse of the Pythagorean theorem is: If a triangle has side lengths such that $a^2 + b^2 = c^2$, it is a right triangle.

coordinate plane The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

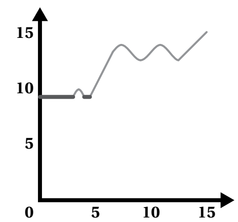
counterclockwise In the opposite direction as the hands of a clock; to the left (of a turn).



Español

constante (función) Una función o el intervalo de una función es constante si los valores de y se mantienen iguales cuando los valores de x aumentan.

Las partes resaltadas de esta función son constantes.



constante de proporcionalidad En una relación proporcional, una constante de proporcionalidad es el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.

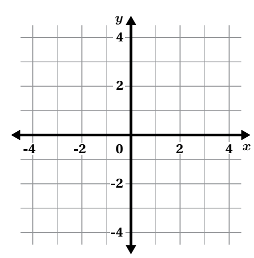
Alfombra (pies cuadrados)	Costo (dólares)
10 $\xleftarrow{\times\frac{2}{3}}$	15.00
20 $\xleftarrow{\times\frac{2}{3}}$	30.00
50 $\xleftarrow{\times\frac{2}{3}}$	75.00

En esta tabla, una constante de proporcionalidad es $\frac{2}{3}$.

recíproco Una expresión escrita en sentido contrario.

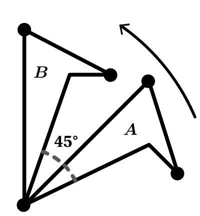
Por ejemplo, el teorema de Pitágoras indica lo siguiente: Si un triángulo es rectángulo, tiene longitudes de lado tales que $a^2 + b^2 = c^2$. El recíproco del teorema de Pitágoras sería el siguiente: Todo triángulo que tiene longitudes de lado tales que $a^2 + b^2 = c^2$, es un triángulo rectángulo.

plano de coordenadas, plano cartesiano El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).



corresponder (partes correspondientes) Corresponder es coincidir. Cuando una parte de una figura original corresponde con una parte de una copia, se llaman partes correspondientes. Pueden ser puntos, segmentos, ángulos o distancias.

en sentido contrario a las manecillas del reloj Girar en la dirección opuesta a las manecillas de un reloj.



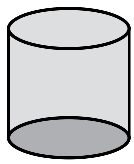
Glossary/Glosario

English

cube root The number that can be cubed to get n, written as $\sqrt[3]{n}$.
The cube root also describes the edge length of a cube with a volume of n.

For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

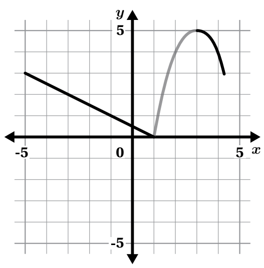
cylinder A 3-D figure that has two parallel circular bases connected by a curved surface.



D

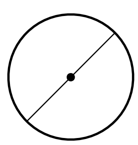
decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.

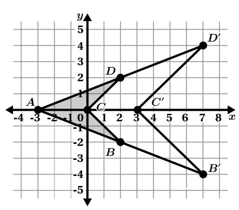


dependent variable (output) A variable whose value is based on the value of another variable or set of variables. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.
In a function, the dependent variable is sometimes called the output.

diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.



dilation A type of transformation that creates scaled copies. A dilation moves every point in a figure away from a center of dilation by the distance of a scale factor.



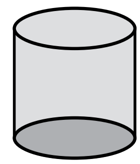
In this example, polygon $ABCD$ is dilated using point A as the center of dilation and a scale factor of 2. Point D' is twice as far away from point A as point D along the same ray.

Español

raíz cúbica El número que se puede elevar al cubo para obtener n. Se escribe $\sqrt[3]{n}$.
La raíz cúbica también describe la longitud de la arista de un cubo con un volumen de n.

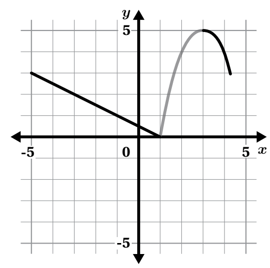
Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de lado de un cubo que tiene un volumen de 64 unidades cúbicas.

cilindro Una figura tridimensional que tiene dos bases circulares paralelas conectadas por una superficie curva.



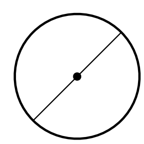
decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

Las partes resaltadas de esta función son decrecientes.

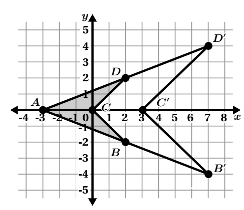


variable dependiente (salida) Una variable cuyo valor se basa en el valor de otra variable o un conjunto de variables. La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.
En una función, la variable dependiente a veces se denomina la salida.

diámetro La distancia entre un punto y otro en un círculo, pasando por el centro. También es la distancia mayor entre un punto y otro en un círculo.



dilatación Un tipo de transformación que produce copias a escala. Una dilatación aleja cada punto de una figura del centro de dilatación de acuerdo con un factor de escala.



Aquí, el polígono $ABCD$ se dilata usando punto A como centro de dilatación y un factor de escala de 2. El punto D' está al doble de la distancia del punto A que el punto D en la misma semirrecta.

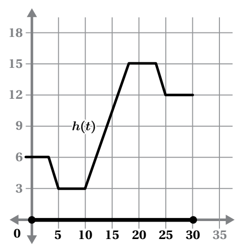
English

distributive property The distributive property says that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

Area as a Product
$3(2x+5)$

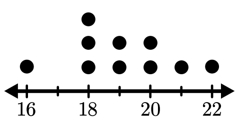
For example, $3(2x+5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as: All numbers from 0 to 30 or $0 \leq t \leq 30$.

dot plot One way to visualize data. Every data point is shown as a dot above its value on a number line. Data sets with repeating values will have multiple dots aligned with the same value. A dot plot is sometimes called a line plot.

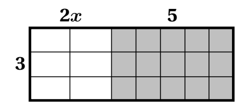


Number of Jelly Beans

For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

Español

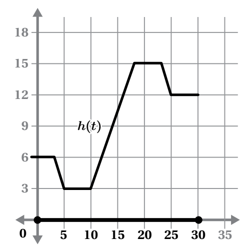
propiedad distributiva La propiedad distributiva indica que la multiplicación de un número por la suma de dos o más términos es igual a la suma de las multiplicaciones de dicho número por cada término individualmente.



El área como un producto
$3(2x+5)$

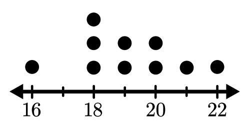
Por ejemplo: $3(2x+5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera: Todos los números del 0 al 30 o $0 \leq t \leq 30$.

diagrama de puntos Una forma de visualizar datos. Cada punto de datos se muestra como un punto encima de su valor en una recta numérica. Los conjuntos de datos con valores repetidos tienen múltiples puntos alineados en un mismo valor. Un diagrama de puntos a veces se llama gráfico de líneas.



Cantidad de frijolitos

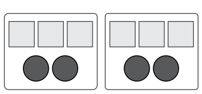
Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

E

equivalent expressions Expressions that are equal for every value of a variable.

For example, $6x + 2x$ is equivalent to $5x + 3x$. No matter what value x is, the two expressions are always equal.

equivalent ratios Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



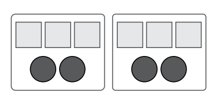
$3:2$ is equivalent to $6:4$ because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. Both recipes will taste the same.

expresiones equivalentes Expresiones que son iguales para todos los valores de una variable.

Por ejemplo, $6x + 2x$ es equivalente a $5x + 3x$. Independientemente del valor de x, las dos expresiones son siempre iguales.

razones equivalentes Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



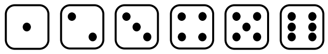
$3:2$ es equivalente a $6:4$ porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. Ambas recetas tendrán el mismo sabor.

Glossary/Glosario

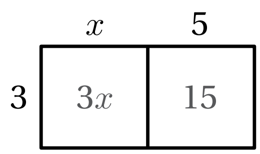
English

event A set of one or more outcomes in a chance experiment.



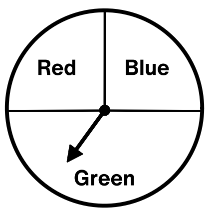
For example, if we roll a number cube, there are six possible outcomes.

expand To expand is to use the distributive property to multiply the factors in an expression and rewrite the expression as a sum. The new expression is equivalent to the original expression.



For example, we can expand $3(x + 5)$ to get the equivalent expression $3x + 15$.
The sum of the two boxes in the diagram show the expression in expanded form.

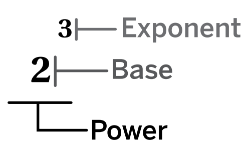
experiment An action or activity that you can do over and over again without knowing what the outcome will be each time.



For example, each time you spin the spinner, it could land on red, blue, or green.

experimental probability Experimental probability is the ratio of the number of times an event occurs to the total number of trials conducted. It is determined through actual experiments rather than theoretical calculations.

exponent A number used to describe repeated multiplication.

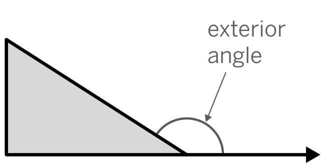


In this example, 3 is the exponent, which means $2^3 = 2 \cdot 2 \cdot 2$.

expression A set of numbers, symbols, and operators (such as $+$ and $\cdot$) grouped together to represent a value. Unlike equations, expressions do not include an equal sign ($=$).

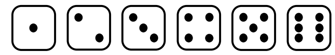
For example, $5x$, $m - 4$, $3y + 0.5$, and $2(n + 8)$ are all expressions.

exterior angle An exterior angle of a polygon is the angle that is formed between a side of the polygon and a line extended from the next side.



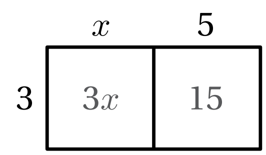
Español

evento Un conjunto de uno o más resultados en un experimento de azar.



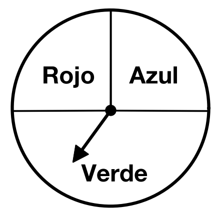
Por ejemplo, si tiramos un dado, hay seis resultados posibles.

expandir Expandir significa usar la propiedad distributiva para multiplicar los factores en una expresión y reescribir la expresión como una suma. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos expandir $3(x + 5)$ para obtener la expresión equivalente $3x + 15$.
La suma de las dos casillas en el diagrama muestra la expresión en forma expandida.

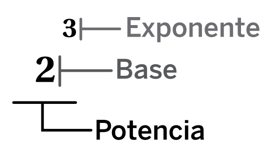
experimento Una acción o actividad que se puede llevar a cabo una y otra vez sin saber cuál será el resultado en cada ocasión.



Por ejemplo, cada vez que se gira la ruleta, podría caer en rojo, azul o verde.

probabilidad experimental La probabilidad experimental es la razón entre el número de veces que ocurre un suceso y el número total de ensayos realizados. Se determina mediante experimentos reales en vez de cálculos teóricos.

exponente Un número que se usa para describir multiplicaciones repetidas.

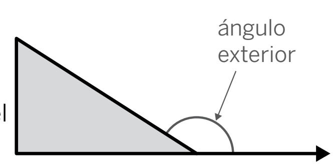


En este ejemplo, 3 es el exponente, lo cual significa que $2^3 = 2 \cdot 2 \cdot 2$.

expresión Un conjunto de números, símbolos y operadores (como $+$ y $\cdot$) agrupados para representar un valor. A diferencia de las ecuaciones, las expresiones no incluyen un signo igual ($=$).

Por ejemplo, $5x$, $m - 4$, $3y + 0.5$, y $2(n + 8)$ son expresiones.

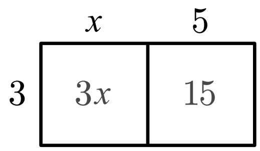
ángulo exterior Un ángulo exterior de un polígono es el ángulo que se forma entre un lado del polígono y una línea extendida desde el siguiente lado.



English / Español

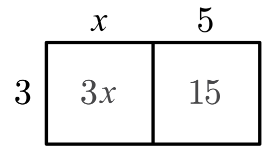
F

factor To factor is to use the distributive property to rewrite an expression as the product of two or more factors. The new expression is equivalent to the original expression.



For example, we can factor $3x + 15$ to get the equivalent expression $3(x + 5)$.
The product of the two sides in the diagram show the expression in factored form.

factorizar Factorizar significa usar la propiedad distributiva para reescribir una expresión como el producto de dos o más factores. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos factorizar $3x + 15$ para obtener la expresión equivalente $3(x + 5)$.
El producto de los dos lados del diagrama muestra la expresión en forma factorizada.

function A relationship that assigns exactly one output to each possible input.

función Una relación que asigna exactamente una salida a cada entrada posible.

G

greatest common factor (GCF) The largest number that is a common factor of two numbers.

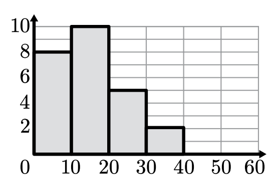
The common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is 4.

máximo común divisor (MCD) El número mayor que es factor común de dos números.

Los factores comunes de 8 y 12 son 1, 2 y 4. El máximo común divisor es 4.

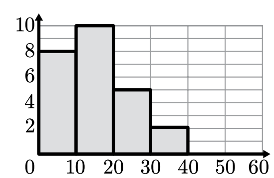
H

histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.



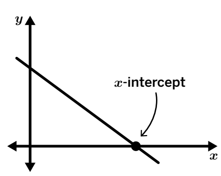
For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.

histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.

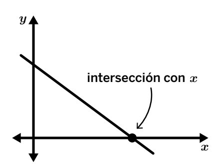


Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

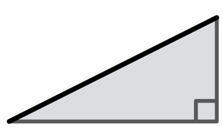
horizontal intercept The point where the graph of a line crosses the horizontal axis or when $y = 0$. The horizontal intercept is sometimes called the x-intercept.



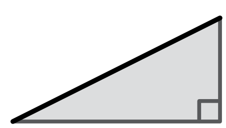
intersección horizontal El punto donde la gráfica de una recta se cruza con el eje horizontal o cuando $y = 0$. La intersección horizontal a veces se denomina intersección con el eje x.



hypotenuse The side of a right triangle that is opposite the right angle. The hypotenuse is always the longest side of a right triangle.



hipotenusa El lado del triángulo rectángulo que está opuesto al ángulo recto. La hipotenusa siempre es el lado más largo de un triángulo rectángulo.



I

image A new figure that is created after a transformation of an original figure (called the pre-image). Every part of the pre-image moves in the same way to match up with a part of the image.

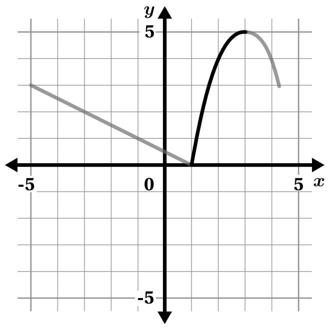
imagen Una nueva figura que se produce después de la transformación de una figura original (denominada preimagen). Todas las partes de la preimagen se mueven de la misma forma para coincidir con cada parte de la imagen.

Glossary/Glosario

English

increasing A function, or interval of a function, is increasing if the y-values go up when the x-values go up.

The bolded part of this function is increasing.



independent variable (input) A variable whose value is not based on the value of any other variable. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table. In a function, the independent variable is sometimes called the input.

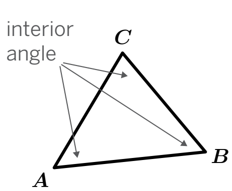
inequality A comparison statement that uses the symbols $<$ or $>$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y > 30$ means that the value of the expression y is any number greater than 30.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. Equivalent equations will create the same line on a graph, so every point on the line is a solution to each equation in the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

interior angle An angle between adjacent sides of a polygon, inside the polygon.



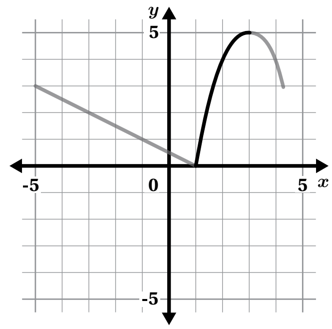
inverse operations Two operations that undo each other are called inverse operations.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

Español

creciente Una función, o un intervalo de una función, es creciente si los valores de y aumentan cuando los valores de x aumentan.

La parte resaltada de esta función es creciente.



variable independiente (entrada) Una variable cuyo valor no depende del valor de ninguna otra variable. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla. En una función, la variable independiente a veces se denomina la entrada.

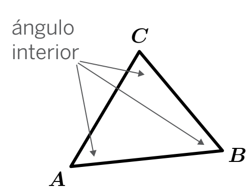
desigualdad Un enunciado de comparación que utiliza los símbolos $<$ o $>$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y > 30$ significa que el valor de la expresión y es cualquier número mayor que 30.

infinitas soluciones Una ecuación tiene un número infinito de soluciones si es verdadera independientemente del valor de la variable. Un sistema de ecuaciones tiene un número infinito de soluciones si las ecuaciones en el sistema son equivalentes. Las ecuaciones equivalentes producen la misma recta en una gráfica, por lo que cada punto en la recta es una solución de cada ecuación en el sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene un número infinito de soluciones porque la ecuación es verdadera para cualquier valor de x.

ángulo interior Un ángulo entre lados adyacentes de un polígono, dentro del polígono.



operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

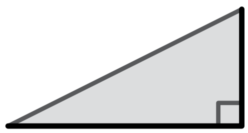
English

irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas pi (π) is irrational because it cannot be written as a fraction of two non-zero integers.

L

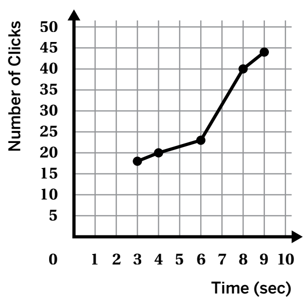
legs The two sides of a right triangle that are not the hypotenuse. The legs are the sides that form the right angle.



like terms Two or more terms that have the same variables and exponent values. Numbers, decimals, and fractions are all like terms.

For example, $8x$ and $12x$ are like terms because they both have a variable of x. $8x$ and 12 are not like terms. $8x$ and $12x^2$ are also not like terms because they have different exponents.

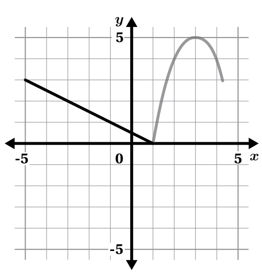
line graph A set of data points plotted on a coordinate plane and connected by lines. Line graphs allow us to investigate connections between two variables, including rates of change between consecutive points or groups of points.



line of fit See *linear model*.

linear A function is linear when its graph is a straight, non-vertical line.

The bolded part of this function is linear.



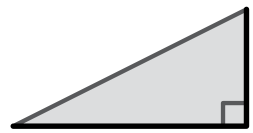
linear function A function that can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.
A vertical line is not a linear function because it has an input with different outputs.

Español

número irracional Un número que no se puede escribir como una fracción de números enteros, donde el denominador es diferente de cero.

Por ejemplo, 2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que pi (π) es irracional porque no se puede escribir como una fracción de dos números enteros distintos de cero.

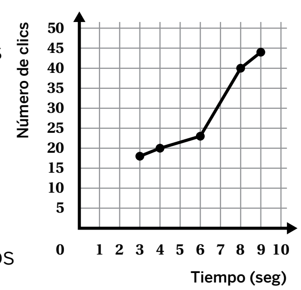
catetos Los dos lados de un triángulo rectángulo que no son la hipotenusa. Los catetos son los lados que forman el ángulo recto.



términos semejantes Dos o más términos que tienen variables y valores de exponentes iguales. Los números enteros, los decimales y las fracciones son términos semejantes.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable x. $8x$ y 12 no son términos semejantes. $8x$ y $12x^2$ tampoco son términos semejantes porque tienen exponentes diferentes.

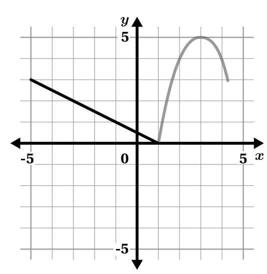
gráfico lineal Un conjunto de puntos de datos trazados en un plano de coordenadas y conectados por líneas. Los gráficos de líneas nos permiten investigar las conexiones entre dos variables, incluyendo las tasas de cambio entre puntos consecutivos o grupos de puntos.



línea de ajuste Ver *modelo lineal*.

lineal Una función es lineal cuando su gráfica es una línea recta que no es vertical.

La parte resaltada de esta función es lineal.



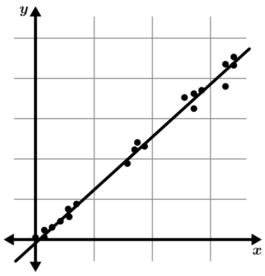
función lineal Una función que puede definirse mediante una ecuación de la forma $y = mx + b$, donde m representa la pendiente y b representa la intersección vertical.
Una línea vertical no es una función lineal porque tiene una entrada con diferentes salidas.

Glossary/Glosario

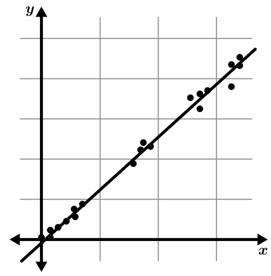
English

Español

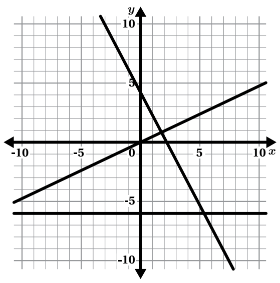
linear model A line that shows, or models, the general direction or trend of a group of points in a data set.
We can use linear models to make predictions about values related to a given data set.



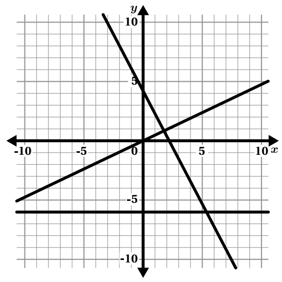
modelo lineal Una línea que muestra, o modela, la dirección o tendencia general de un grupo de puntos en un conjunto de datos.
Podemos emplear modelos lineales para hacer predicciones sobre valores relativos a un conjunto de datos determinado.



linear relationship A relationship is called linear when its graph is a line.



relación lineal Una relación se llama lineal cuando su gráfica es una recta.



M

mapping diagram A visual representation of a relation that shows a list of inputs and outputs and uses arrows to show how they are related.

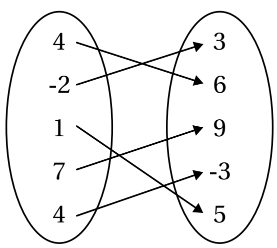
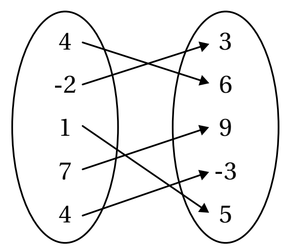


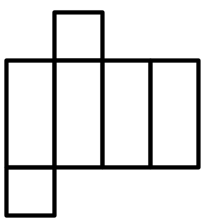
diagrama de mapeo Una representación visual de una relación que muestra una lista de entradas y salidas y utiliza flechas para mostrar cómo están relacionadas.



N

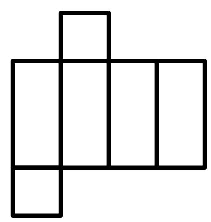
net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

Here is a net for a rectangular prism.



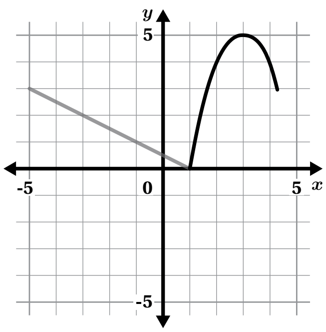
red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.

Esta es una red de un prisma rectangular.



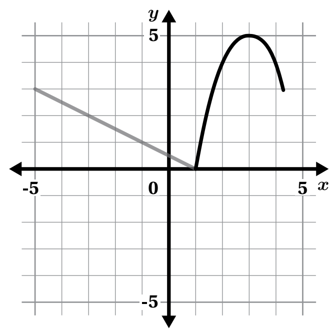
nonlinear A function is nonlinear when its graph is not a straight line.

The bolded part of this function is nonlinear.



no lineal Una función es no lineal cuando su gráfica no es una línea recta.

La parte resaltada de esta función es no lineal.



English	Español

numerical data Data presented as numbers, quantities, or measurements that can be meaningfully compared. It is sometimes called quantitative data.

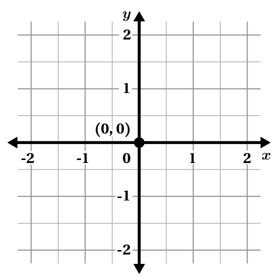
For example, data on a group of students' heights or arm spans.

datos numéricos Datos que se presentan como números, cantidades o medidas que pueden compararse de forma significativa. A veces se denominan datos cuantitativos.

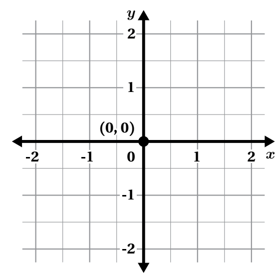
Por ejemplo, los datos de las estaturas o las longitudes de los brazos de un grupo de estudiantes.

O

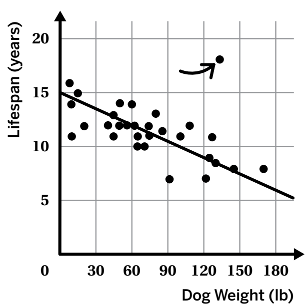
origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



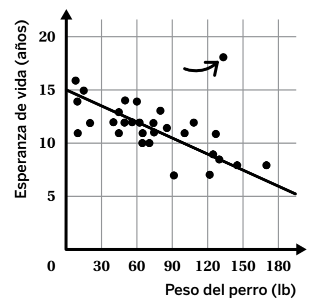
origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan el eje x y el eje y.



outlier A data value that is far from the other values in the data set.



valor atípico Un valor que está lejos de los demás valores del conjunto de datos.



P

perfect cubes The cube of an integer is called a perfect cube.

For example, 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 3^3$ and $3^3 = 27$.

cubo perfecto El cubo de un número entero se denomina cubo perfecto.

Por ejemplo, 27 es un cubo perfecto porque $3 \cdot 3 \cdot 3 = 3^3$ y $3^3 = 27$.

perfect square The square of an integer is called a perfect square.

For example, 49 is a perfect square because $7 \cdot 7 = 7^2$ and $7^2 = 49$.

cuadrado perfecto El cuadrado de un número entero se denomina cuadrado perfecto.

Por ejemplo, 49 es un cuadrado perfecto porque $7 \cdot 7 = 7^2$ y $7^2 = 49$.

pi A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π.

Some common approximations for π are 3.14 and $\frac{22}{7}$.

pi Un número que representa la constante de proporcionalidad entre el diámetro y la circunferencia de cualquier círculo. El símbolo de pi es π.

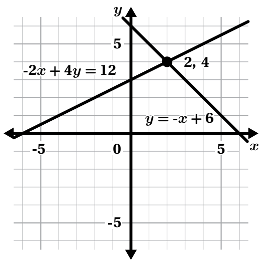
Algunas aproximaciones comunes de π son 3.14 y $\frac{22}{7}$.

Glossary/Glosario

English

point of intersection A point where two lines meet.

For example, $(2, 4)$ is the point of intersection for the lines $y = -x + 6$ and $-2x + 4y = 12$.



power of ten A number written in the form 10^n, where n represents the number of times 10 is multiplied.

pre-image The name of a figure before any transformations are performed.

prime notation Using the prime symbol (′) to name points in an image that correspond to points in the pre-image.

For example, if point A is transformed, the corresponding point would be named A'.

proportional Quantities are proportional if they form equivalent ratios.

For example, the heights and bases of these triangles are proportional because the ratio $6 : 12$ is equivalent to $4 : 8$.

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

Carpet (sq. ft.)	Cost (dollars)
10 $\xrightarrow{\times 1.5}$	15.00
20 $\xrightarrow{\times 1.5}$	30.00
50 $\xrightarrow{\times 1.5}$	75.00

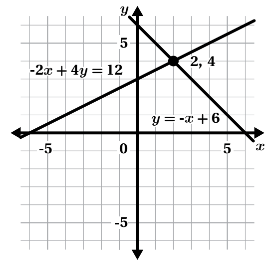
For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Pythagorean theorem The theorem that describes the relationship between the side lengths of a right triangle. The Pythagorean theorem says that the square of the hypotenuse is equal to the sum of the squares of the legs. We can write this as $a^2 + b^2 = c^2$.

Español

punto de intersección Un punto donde se cruzan dos rectas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



potencia de diez Un número escrito de la forma 10^n, donde n representa el número de veces que se multiplica el 10.

preimagen El nombre de una figura antes de realizar una transformación.

notación prima Uso del signo prima (′) para denominar los puntos de una imagen que corresponden a puntos de la preimagen.

Por ejemplo, si el punto A se transforma, el punto correspondiente se denominaría A'.

proporcional Diferentes cantidades son proporcionales si forman razones equivalentes.

Por ejemplo, las alturas y las bases de estos triángulos son proporcionales porque la relación $6 : 12$ es equivalente a $4 : 8$.

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
10 $\xrightarrow{\times 1.5}$	15.00
20 $\xrightarrow{\times 1.5}$	30.00
50 $\xrightarrow{\times 1.5}$	75.00

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

teorema de Pitágoras El teorema que describe la relación entre las longitudes de lado de los triángulos rectángulos. El teorema de Pitágoras dice que el cuadrado de la hipotenusa es igual a la suma de los cuadrados de los catetos. Podemos escribirlo como $a^2 + b^2 = c^2$.

Q

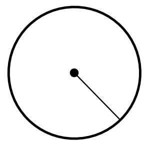
quantitative data See *numerical data*.

datos cuantitativos Ver *datos numéricos*.

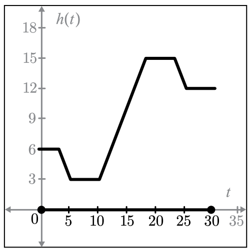
English

R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.



range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.

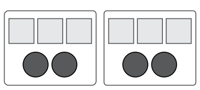


The range of this graph can be described as:
All numbers from 3 to 15.
$3 \leq h(t) \leq 15$.

rate A comparison, or ratio, that describes how two quantities change together.

rate of change See *slope*.

ratio A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.



If the ratio of apples to oranges in a fruit bowl is $2 : 3$, then for every 2 apples, there are 3 oranges.
There are several ways to describe ratios.

- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is $3 : 2$.

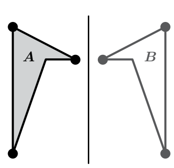
rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

Examples of rational numbers include 13, -74, 0, 0.2, and $\sqrt{9}$.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

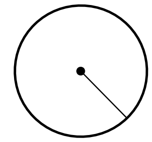
For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \times \frac{2}{3} = 1$.

reflection A reflection across a line moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.

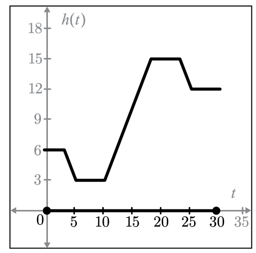


Español

radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.



rango (de una función) El conjunto de todos los posibles valores de salida de una función o relación. El rango puede describirse con palabras o como una desigualdad.

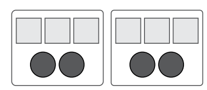


El rango de esta gráfica puede describirse de la siguiente manera:
Todos los números del 3 al 15.
$3 \leq h(t) \leq 15$

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

tasa de cambio Véase *pendiente*.

razón Una razón $a : b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.



Si la razón de manzanas a naranjas en un frutero es $2 : 3$, entonces por cada 2 manzanas hay 3 naranjas.
Hay varias formas de describir razones.

- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es $3 : 2$.

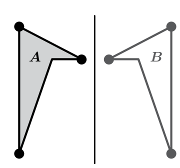
número racional Un número que se puede escribir como fracción de números enteros, donde el denominador es diferente de cero.

Algunos ejemplos de números racionales son 13, -74, 0, 0.2 y $\sqrt{9}$.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \times \frac{2}{3} = 1$.

reflexión Una reflexión con respecto a una línea mueve cada punto de una figura a un punto directamente en el lado opuesto de la línea. El nuevo punto está a la misma distancia de la línea que estaba en la figura original.

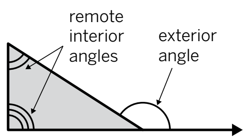


Glossary/Glosario

English

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a function.

remote interior angles The two angles that are inside the triangle and opposite from the exterior angle are the remote interior angles.



repeating decimal A decimal with one or more digits (not all zeros) that repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (...) at the end.

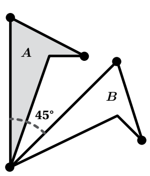
If the repeating digits are all zeros, it would be called a terminating decimal.

For example, the decimal representation of $\frac{1}{3}$ is $0.\overline{3}$, which means $0.33333...$

The decimal representation of $\frac{25}{22}$ is $1.1\overline{36}$, which means $1.1363636...$

rigid transformation A move that does not change any measurements of a figure. Translations, rotations, and reflections (or any sequence of these) are rigid transformations.

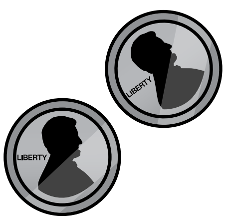
rotation A rotation moves every point on a figure around a center by a given angle in a specific direction.



S

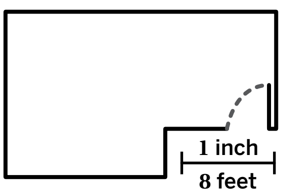
sample space The list of every possible outcome for a chance experiment.

For example, the sample space for tossing two coins is: heads-heads, tails-heads, heads-tails, tails-tails.



scale A scale tells us how the actual measurements of an object are represented in a drawing.

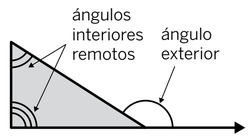
The scale of this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room.



Español

relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

ángulos interiores remotos Los dos ángulos que están dentro del triángulo y opuestos al ángulo exterior.



decimal periódico Un decimal con uno o más dígitos (no todos son ceros) que se repiten infinitamente. Un decimal periódico puede escribirse usando la raya indicadora de decimales periódicos (vinculum) encima de los dígitos que se repiten o con puntos suspensivos (...) al final.

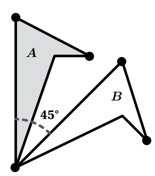
Si todos los dígitos periódicos son ceros, entonces se denomina decimal exacto.

Por ejemplo, la representación decimal de $\frac{1}{3}$ es $0.\overline{3}$, lo que significa $0.33333...$

La representación decimal de $\frac{25}{22}$ es $1.1\overline{36}$, o sea $1.1363636...$

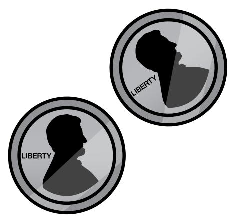
transformación rígida Un movimiento que no cambia ninguna de las medidas de una figura. Las traslaciones, rotaciones y reflexiones son transformaciones rígidas, así como lo es cualquier secuencia de ellas.

rotación Una rotación mueve cada punto en una figura alrededor de un centro hacia una dirección específica y con un ángulo determinado.



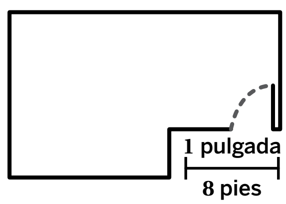
espacio muestral La lista de todos los posibles resultados de un experimento de azar.

Por ejemplo, el espacio muestral del lanzamiento de dos monedas es: cara-cara, sello-cara, cara-sello, sello-sello.



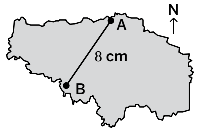
escala Una escala nos indica cómo están representadas en un dibujo o diagrama las medidas reales de un objeto.

La escala de este plano nos indica que 1 pulgada en el dibujo representa 8 pies en la habitación real.



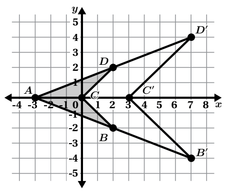
English

scale drawing A two-dimensional representation of an actual object or place. All the measurements in the drawing correspond to the measurements of the actual object by the same scale. Floor plans and maps are examples of scale drawings.



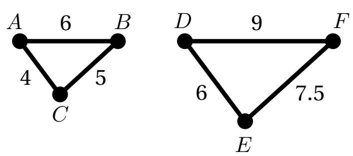
A scale may show that 1 centimeter on a map represents 30 miles on land. The distance marked on this map is 8 centimeters, or 240 miles.

scale factor The number used to create a dilation. All distances from the pre-image to the center of dilation are multiplied by the same number, called the *scale factor*.



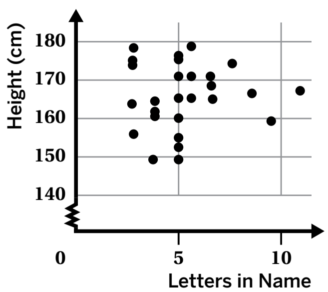
For example, the scale factor from polygon $ABCD$ to polygon $A'B'C'D'$ is 2.

scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a figure looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.



For example, triangle DEF is a scaled copy of triangle ABC.

scatter plot A set of disconnected data points plotted on a coordinate plane. Scatter plots allow us to investigate connections between two variables.

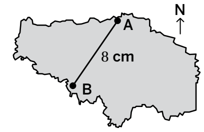


scientific notation A way to write very large or very small numbers. In scientific notation, a number between 1 and 10 is multiplied by a power of 10.

For example, the number 425,000,000 in scientific notation is $4.25 \cdot 10^8$. The number 0.0000000783 in scientific notation is $7.83 \cdot 10^{-8}$.

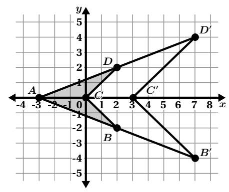
Español

dibujo a escala Una representación bidimensional de un objeto o lugar reales. Todas las medidas del dibujo corresponden a las medidas del objeto real usando la misma escala. Los planos y los mapas son ejemplos de dibujos a escala.



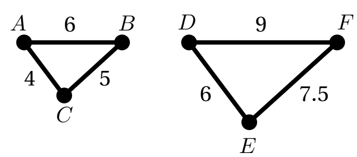
Una escala puede mostrar que 1 centímetro en un mapa representa 30 millas en tierra. La distancia mostrada en este mapa es 8 centímetros o 240 millas.

factor de escala El número que se usa para hacer una dilatación. Todas las distancias de la preimagen al centro de dilatación se multiplican por el mismo número, que se denomina *factor de escala*.



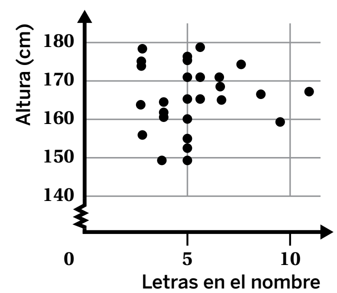
Por ejemplo, el factor de escala del polígono $ABCD$ al polígono $A'B'C'D'$ es 2.

copia a escala Una copia de una imagen que puede cambiar de tamaño, pero que siempre conserva la forma y las medidas de los ángulos de la imagen original. Si una figura luce aplastada o estirada en comparación con la original, no es una copia a escala. Para crear una copia a escala, multiplicamos todas las longitudes de la figura original por el mismo número.



Por ejemplo, el triángulo DEF es una copia a escala del triángulo ABC.

diagrama de dispersión Un conjunto de puntos de datos que no están conectados trazados en un plano de coordenadas. Los diagramas de dispersión nos permiten analizar las conexiones entre dos variables.



notación científica Una forma de escribir números muy grandes o muy pequeños. Cuando un número entre 1 y 10 está multiplicado por una potencia de 10, significa que está escrito en notación científica.

Por ejemplo, el número 425,000,000 en notación científica es $4.25 \cdot 10^8$. El número 0.0000000783 en notación científica es $7.83 \cdot 10^{-8}$.

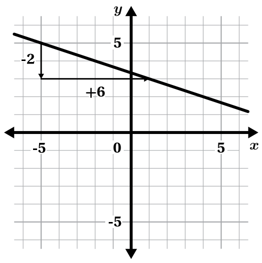
English

significant digits The important digits in a measurement that show how precise it is. Zeros in a number are only considered significant when they are between two non-zero numbers or when they are trailing zeros in a number that has a decimal point.

For example, 450 has only two significant digits, but 450.0 has four.

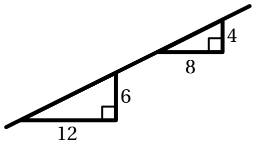
similar One figure is similar to another if translations, rotations, reflections, and dilations can be used to fit one exactly over the other. Two figures are similar if one is a scaled copy of the other.

slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.



The slope of this line is $-\frac{2}{6} = -\frac{1}{3}$.

slope triangle A triangle drawn to include two points on a line in order to determine the slope of that line. The longest side lies on the line and the other two sides are vertical and horizontal. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.



Both of these triangles are slope triangles.

solution to an equation The value or set of values that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y).

The solution to the equation $x + 15 = 8$ is $x = -7$ because $(-7) + 15 = 8$.
One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.

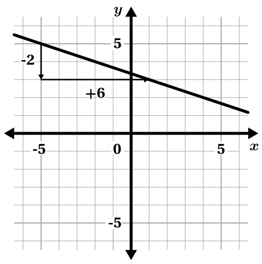
Español

dígitos significativos Los dígitos importantes en una medición que muestran cuán precisa es. Los ceros en un número solo se consideran significativos cuando están entre dos números distintos de cero o cuando son ceros finales en un número que tiene un punto decimal.

Por ejemplo, 450 tiene solo dos dígitos significativos, pero 450.0 tiene cuatro.

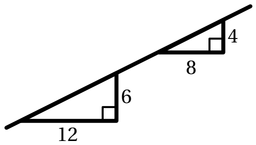
semejante Una figura es semejante a otra si se pueden emplear traslaciones, rotaciones, reflexiones y dilataciones para que coincida exactamente con la otra. Dos figuras son semejantes si una es una copia a escala de la otra.

pendiente Un número que describe la dirección e inclinación de una línea. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Es por eso que la pendiente de una recta a veces se denomina tasa de cambio. Para calcular la pendiente, la distancia vertical entre dos puntos cualesquiera en la recta se divide entre la distancia horizontal entre dichos puntos.



La pendiente de esta recta es $-\frac{2}{6} = -\frac{1}{3}$.

triángulo de pendiente Un triángulo que se traza para incorporar dos puntos de una recta y así determinar la pendiente de dicha recta. El lado más largo se sitúa sobre la recta y los dos lados restantes son uno vertical y otro horizontal. La altura del triángulo de pendiente representa la distancia vertical entre los puntos, mientras que la base del triángulo representa la distancia horizontal entre los puntos.



Estos dos triángulos son triángulos de pendiente.

solución de una ecuación El valor o conjunto de valores que hacen que la ecuación sea verdadera. Una solución de una ecuación con dos variables es un par de valores que hacen que la ecuación sea verdadera, y a menudo se escribe como un par ordenado, (x, y).

La solución de la ecuación $x + 15 = 8$ es $x = -7$ porque $(-7) + 15 = 8$.
Una solución de la ecuación $4x + 3y = 24$ es $(6, 0)$ porque $4(6) + 3(0) = 24$.

English

solution to a system of equations A set of values that makes all equations in that system true.
When the equations are graphed, the solution to the system is the point of intersection.

For example, $(2, 4)$ is the solution to this system of equations, and the point of intersection on the graph.
$y = \text{-}x + 6$
$\text{-}2x + 4y = 12$

square root A positive number that can be squared to get n. Written as $\sqrt{n}$.
The square root is also the side length of a square with an area of n.

The square root of 16 $(\sqrt{16})$ is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

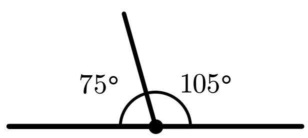
stem-and-leaf plot A table that organizes data by place value to compare data frequencies.

The stem-and-leaf plot shows the data set $(5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64)$.

Stem	Leaf
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

Key: $1 \mid 0 = 10$

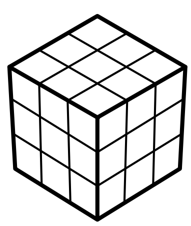
supplementary angles Two angles whose measures add up to $180°$.



For example, a $75°$ angle and a $105°$ angle are supplementary.

surface area The sum of the areas of the surfaces of a solid.

For example, the six faces of this cube each have an area of 9 square centimeters, so the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



system of equations Two or more equations that represent the constraints on a shared set of variables.

For example, these equations make up a system of equations:
$x + y = \text{-}2$
$x - y = 12$

Español

solución de un sistema de ecuaciones Un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.

Por ejemplo, $(2, 4)$ es la solución de este sistema de ecuaciones y el punto de intersección en la gráfica.
$y = \text{-}x + 6$
$\text{-}2x + 4y = 12$

raíz cuadrada Un número positivo que se puede elevar al cuadrado para obtener n. Se escribe $\sqrt{n}$.
La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 $(\sqrt{16})$ es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

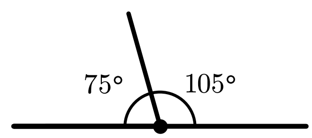
diagrama de tallo y hojas Una tabla que organiza los datos por valor posicional para comparar frecuencias de datos.

El diagrama de tallo y hojas muestra el conjunto de datos $(5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64)$.

Tallo	Hoja
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

Leyenda: $1 \mid 0 = 10$

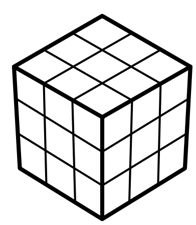
ángulos suplementarios
Dos ángulos cuyas medidas suman $180°$.



Por ejemplo, un ángulo de $75°$ y otro ángulo de $105°$ son suplementarios.

área de superficie La suma de las áreas de las superficies de un sólido.

Por ejemplo, cada una de las seis caras de este cubo tiene un área de 9 centímetros cuadrados, por lo tanto, el área de superficie del cubo mide $6 \cdot 9$ o 54 centímetros cuadrados.



sistema de ecuaciones Dos o más ecuaciones que representan las restricciones en un conjunto compartido de variables.

Por ejemplo, estas ecuaciones forman un sistema de ecuaciones:
$x + y = \text{-}2$
$x - y = 12$

Glossary/Glosario

English	Español

T

terminating decimal A decimal with a finite number of non-zero digits after the decimal point.

For example, 0.08, 1.5, and 0.2563 are all terminating decimals.

decimal exacto Un decimal con un número finito de dígitos distintos de cero después del punto decimal.

Por ejemplo: 0.08, 1.5 y 0.2563 son decimales exactos.

theoretical probability The probability that a certain outcome will occur, as determined through reasoning or calculation. It is equal to the ratio

$$\frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

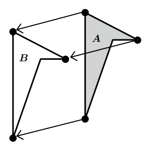
probabilidad teórica La probabilidad de que ocurra un determinado resultado, determinada mediante razonamiento o cálculo. Es igual a la razón

$$\frac{\text{número de resultados en el suceso}}{\text{número de resultados en el espacio de muestra}}$$

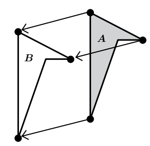
transformation An action or rule for moving or changing figures on a plane. Transformations include translations, reflections, rotations, and dilations.

transformación Una acción o regla para mover o cambiar figuras en un plano. Las transformaciones incluyen traslaciones, reflexiones, rotaciones y dilataciones.

translation A translation moves every point in a figure a given distance in a given direction.



traslación Una traslación mueve cada punto de una figura una determinada distancia en una determinada dirección.



tree diagram A diagram that represents all the possible outcomes in an experiment. It is often used to represent compound events.

For example, this tree diagram shows the outcomes of spinning a spinner and then flipping a coin.

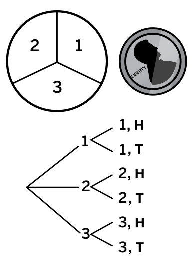
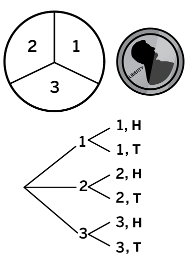


diagrama de árbol Diagrama que representa todos los resultados posibles de un experimento. A menudo se utiliza para representar eventos compuestos.

Por ejemplo, este diagrama de árbol muestra los resultados de girar una ruleta y luego lanzar una moneda.



triangle inequality theorem A theorem which states that in order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

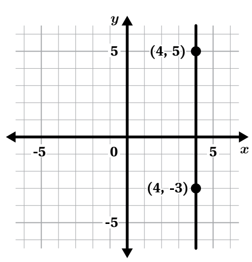
For example, a triangle can have side lengths of 4, 5, and 6, because $4 + 5 > 6$.

teorema de la desigualdad triangular Un teorema que establece que, para que tres segmentos de recta formen un triángulo, la suma de las longitudes de los dos segmentos más cortos debe ser mayor que la longitud del tercer segmento.

Por ejemplo, un triángulo puede tener lados de 4, 5 y 6, porque $4 + 5 > 6$.

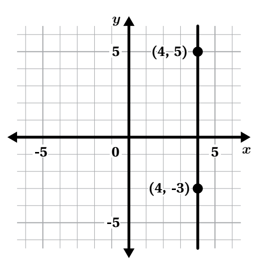
U

undefined An expression or term that has no understandable value. Dividing any (non-zero) value by 0 creates an undefined expression because we cannot divide by 0.



The slope of a vertical line is undefined because there is 0 horizonal change between any two points on the line. This means any vertical change would be divided by 0, which creates an undefined value.

indefinido Una expresión o un término que no tiene ningún valor comprensible. La división de un valor (diferente de cero) entre 0 produce una expresión indefinida porque no es posible dividir por 0.



La pendiente de una recta vertical es indefinida porque hay un cambio horizontal de 0 entre dos puntos cualesquiera en la recta. Esto significa que cualquier cambio vertical se dividiría por 0, lo cual produce un valor indefinido.

English

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

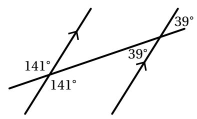
For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

univariate data A data set that involves one variable. Each data point contains one piece of information.

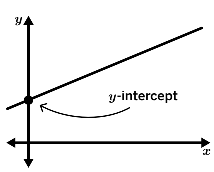
A collection of students' heights is a univariate data set. Tables, dot plots, and bar graphs are useful for displaying univariate data.

V

vertical angles Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal.

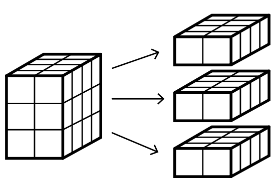


vertical intercept The point where the graph of a line crosses the vertical axis or when $x = 0$. The vertical intercept is sometimes called the y-intercept.



volume The number of cubic units that fill a 3-D region without any gaps or overlaps.

For example, the volume of this rectangular prism is 24 cubic units because it is made of 3 layers that are each 8 cubic units.



X

x-intercept See *horizontal intercept*.

Y

y-intercept See *vertical intercept*.

Español

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

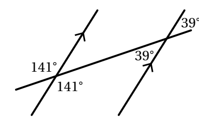
Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

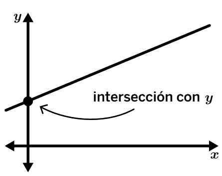
Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

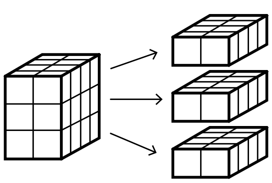
ángulos verticales Los ángulos verticales son ángulos opuestos que comparten el mismo vértice. Se forman con un par de rectas que se intersecan. Las medidas de sus ángulos son iguales.



intersección vertical El punto donde la gráfica de una recta se cruza con el eje vertical o cuando $x = 0$. La intersección vertical a veces se denomina intersección con el eje y.



volumen La cantidad de unidades cúbicas que llenan un espacio tridimensional, sin vacíos ni superposiciones.



Por ejemplo, el volumen de este prisma rectangular mide 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.

X

intersección con el eje *x* Véase *intersección horizontal*.

Y

intersección con el eje *y* Véase *intersección vertical*.

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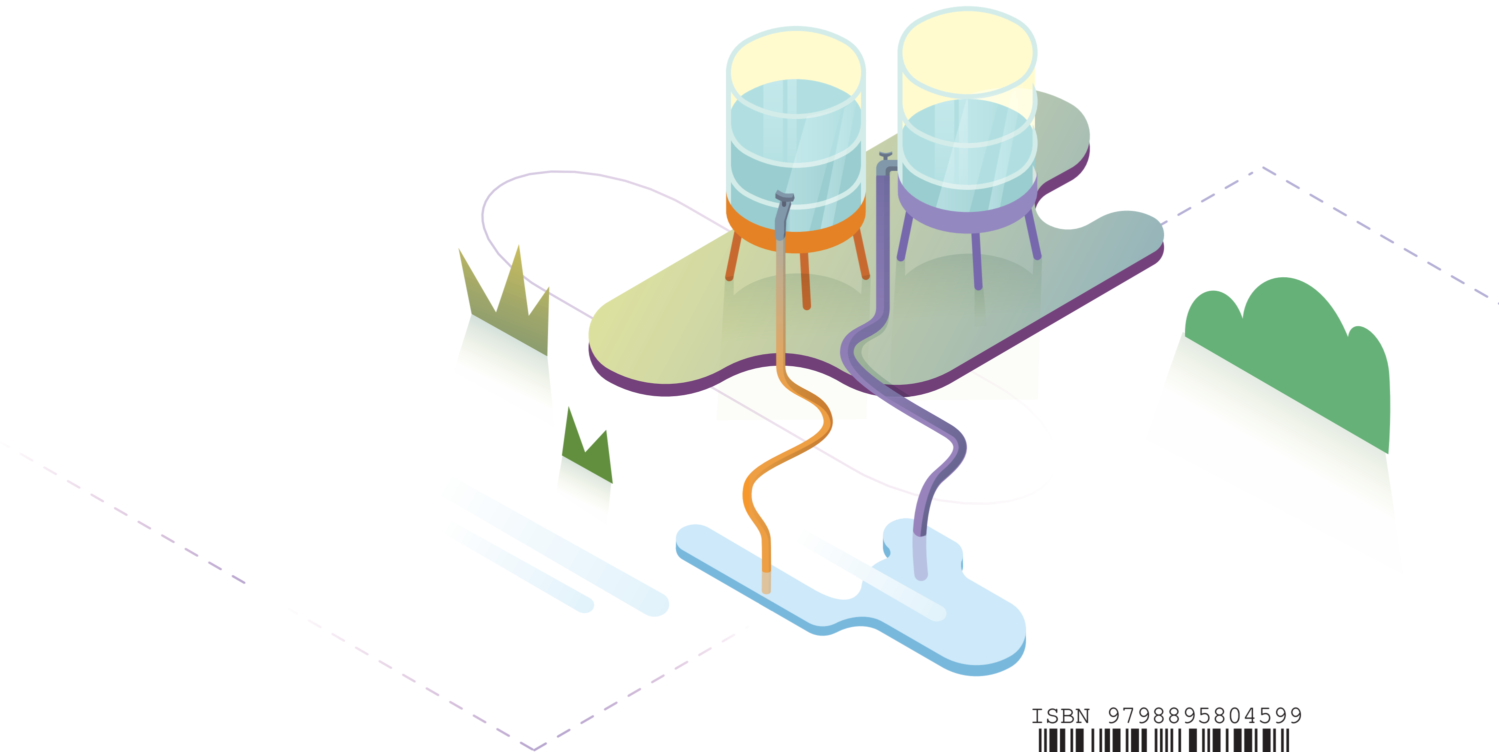

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