Amplify Desmos Math
FLORIDA

Student Edition

6

Volume 1

Amplify Desmos Math FLORIDA

Grade 6

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math Florida! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

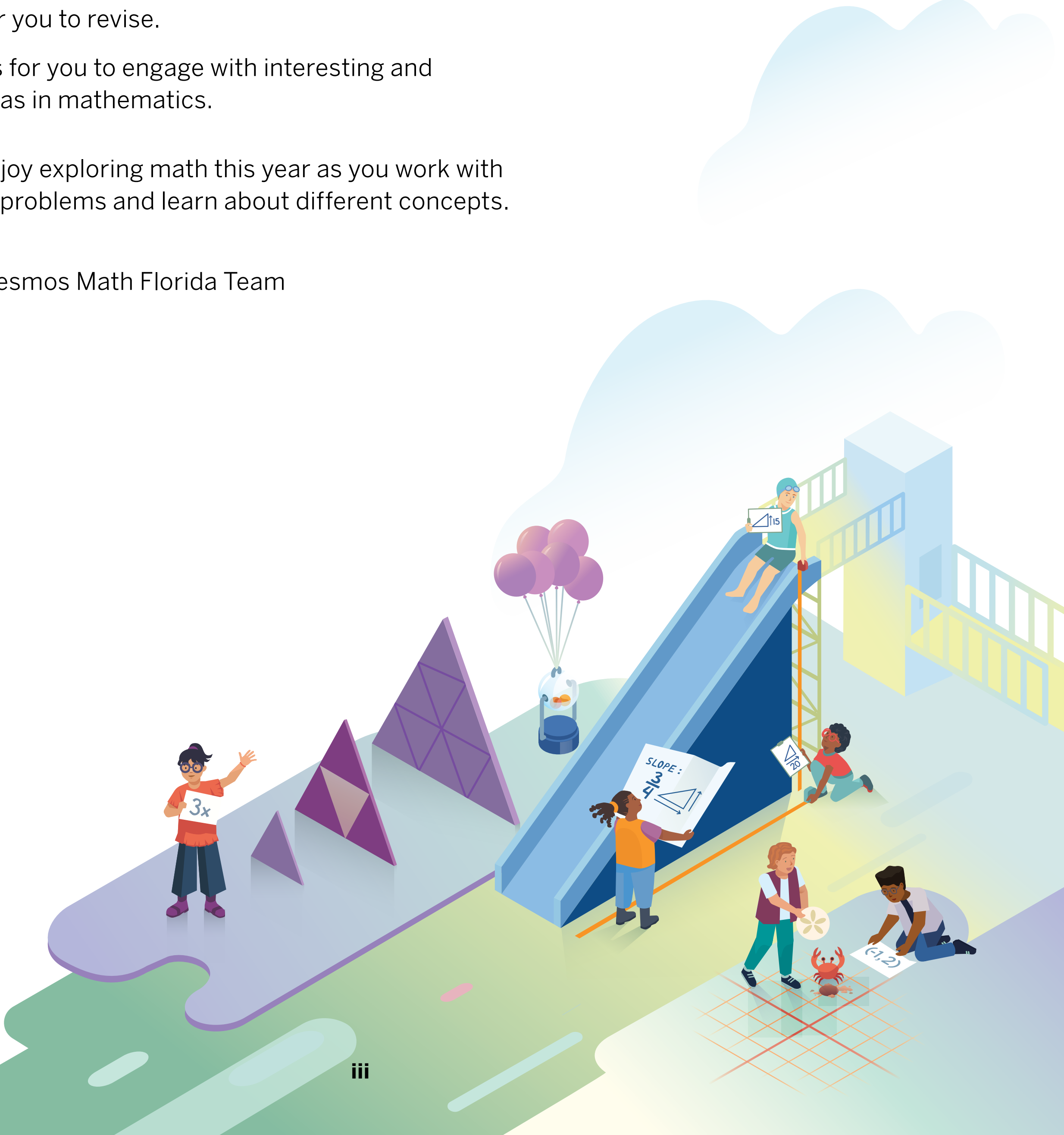
Amplify Desmos Math Florida is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- Lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important ideas in mathematics.

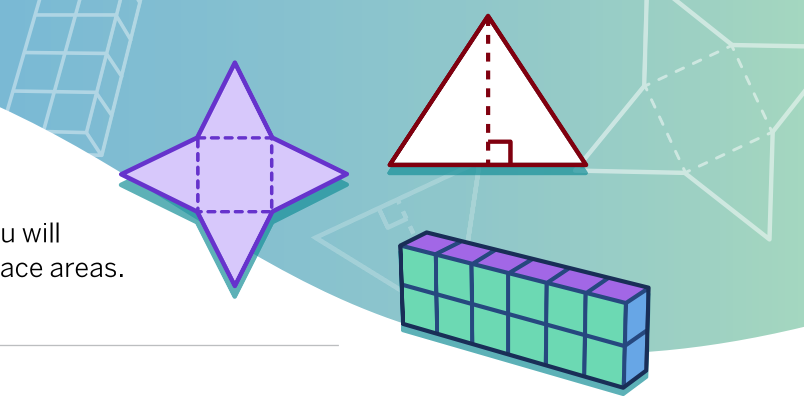
We hope you enjoy exploring math this year as you work with friends to solve problems and learn about different concepts.

–The Amplify Desmos Math Florida Team



Unit 1 Area and Surface Area

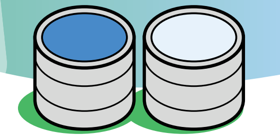
In this unit, you will learn to calculate areas of polygons. You will also represent polyhedra with nets and calculate their surface areas.



Sub-Unit 1 Area

Unit 2 Introducing Ratios

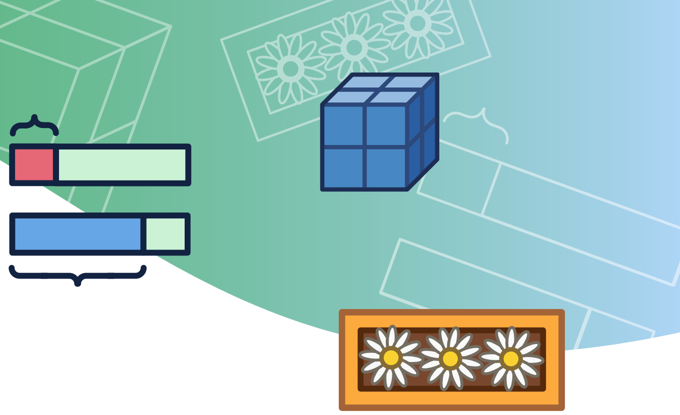
In this unit, you will be introduced to the concept of ratios. You will also represent ratios using double number lines, tables, and tape diagrams, and use ratio reasoning to solve problems.



Unit 3 Unit Rates and Percentages

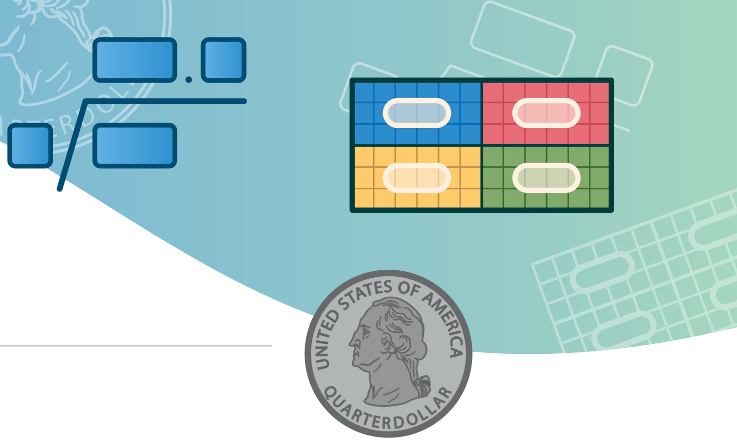
In this unit, you will apply ratio reasoning from Unit 2 to unit rates and recognize that equivalent ratios have the same unit rates. You will also use a variety of strategies and representations of percentages to determine missing percentages, parts, and wholes.

Unit 4 Multiplying and Dividing Fractions



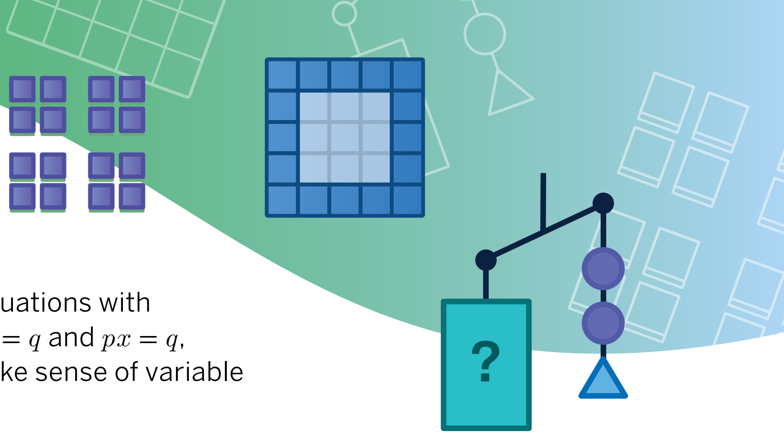
In this unit, you will extend what you learned about multiplying and dividing whole numbers to multiply and divide fractions by fractions. You will learn a variety of strategies, including making tape diagrams, creating common denominators, and rewriting equivalent multiplication problems using the reciprocal.

Unit 5 Decimal Arithmetic



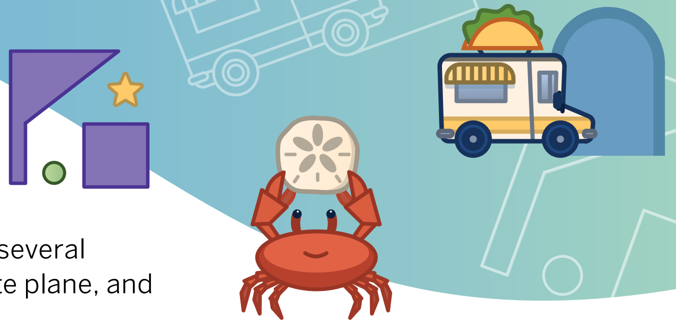
In this unit, you will develop and use a variety of strategies for multiplying and dividing multi-digit decimals.

Unit 6 Expressions and Equations



In this unit, you will reason about expressions and equations with variables. You will solve equations of the forms $x + p = q$ and $px = q$, write equivalent expressions using variables, and make sense of variable expressions involving exponents.

Unit 7 Positive and Negative Numbers



In this unit, you will explore positive and negative numbers in several contexts: on a number line, with inequalities, on the coordinate plane, and in equations.

Unit 8 Describing Data

In this unit, you will visualize data using dot plots, histograms, and box plots. You will also calculate measures of center and spread.

Florida's B.E.S.T. Standards for Mathematics

Benchmark	B.E.S.T Mathematics Benchmark
Number Sense and Operations	
MA.6.NSO.1.1	Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.
MA.6.NSO.1.2	Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.
MA.6.NSO.1.3	Given a mathematical or real-world context, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.
MA.6.NSO.1.4	Solve mathematical and real-world problems involving absolute value, including the comparison of absolute value.
MA.6.NSO.2.1	Multiply and divide positive multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.
MA.6.NSO.2.2	Extend previous understanding of multiplication and division to compute products and quotients of positive fractions by positive fractions, including mixed numbers, with procedural fluency.
MA.6.NSO.2.3	Solve multi-step real-world problems involving any of the four operations with positive multi-digit decimals or positive fractions, including mixed numbers.
MA.6.NSO.3.1	Given a mathematical or real-world context, find the greatest common factor and least common multiple of two whole numbers.
MA.6.NSO.3.2	Rewrite the sum of two composite whole numbers having a common factor, as a common factor multiplied by the sum of two whole numbers.
MA.6.NSO.3.3	Evaluate positive rational numbers and integers with natural number exponents.
MA.6.NSO.3.4	Express composite whole numbers as a product of prime factors with natural number exponents.
MA.6.NSO.3.5	Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.
MA.6.NSO.4.1	Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.
MA.6.NSO.4.2	Apply and extend previous understandings of operations with whole numbers to multiply and divide integers with procedural fluency.
Algebraic Reasoning	
MA.6.AR.1.1	Given a mathematical or real-world context, translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions.
MA.6.AR.1.2	Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$. Represent the inequality on a number line.
MA.6.AR.1.3	Evaluate algebraic expressions using substitution and order of operations.
MA.6.AR.1.4	Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.
MA.6.AR.2.1	Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false.
MA.6.AR.2.2	Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

Florida’s B.E.S.T. Standards for Mathematics

MA.6.AR.2.3	Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.
MA.6.AR.2.4	Determine the unknown decimal or fraction in an equation involving any of the four operations, relating three numbers, with the unknown in any position.
MA.6.AR.3.1	Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using appropriate notation: a/b, a to b, or $a{:}b$ where $b \neq 0$.
MA.6.AR.3.2	Given a real-world context, determine a rate for a ratio of quantities with different units. Calculate and interpret the corresponding unit rate.
MA.6.AR.3.3	Extend previous understanding of fractions and numerical patterns to generate or complete a two- or three-column table to display equivalent part-to-part ratios and part-to-part-to-whole ratios.
MA.6.AR.3.4	Apply ratio relationships to solve mathematical and real-world problems involving percentages using the relationship between two quantities.
MA.6.AR.3.5	Solve mathematical and real-world problems involving ratios, rates and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.

Geometric Reasoning

MA.6.GR.1.1	Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the x- or y-axis as the line of reflection when two ordered pairs have an opposite x- or y-coordinate.
MA.6.GR.1.2	Find distances between ordered pairs, limited to the same x-coordinate or the same y-coordinate, represented on the coordinate plane.
MA.6.GR.1.3	Solve mathematical and real-world problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle.
MA.6.GR.2.1	Derive a formula for the area of a right triangle using a rectangle. Apply a formula to find the area of a triangle.
MA.6.GR.2.2	Solve mathematical and real-world problems involving the area of quadrilaterals and composite figures by decomposing them into triangles or rectangles.
MA.6.GR.2.3	Solve mathematical and real-world problems involving the volume of right rectangular prisms with positive rational number edge lengths using a visual model and a formula.
MA.6.GR.2.4	Given a mathematical or real-world context, find the surface area of right rectangular prisms and right rectangular pyramids using the figure’s net.

Data Analysis and Probability

MA.6.DP.1.1	Recognize and formulate a statistical question that would generate numerical data.
MA.6.DP.1.2	Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.
MA.6.DP.1.3	Given a box plot within a real-world context, determine the minimum, the lower quartile, the median, the upper quartile and the maximum. Use this summary of the data to describe the spread and distribution of the data.
MA.6.DP.1.4	Given a histogram or line plot within a real-world context, qualitatively describe and interpret the spread and distribution of the data, including any symmetry, skewness, gaps, clusters, outliers and the range.
MA.6.DP.1.5	Create box plots and histograms to represent sets of numerical data within real-world contexts.
MA.6.DP.1.6	Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.

Mathematical Thinking and Reasoning Standards

MA.K12.MTR.1.1	Actively participate in effortful learning both individually and collectively.
MA.K12.MTR.2.1	Demonstrate understanding by representing problems in multiple ways.
MA.K12.MTR.3.1	Complete tasks with mathematical fluency.
MA.K12.MTR.4.1	Engage in discussions that reflect on the mathematical thinking of self and others.
MA.K12.MTR.5.1	Use patterns and structure to help understand and connect mathematical concepts.
MA.K12.MTR.6.1	Assess the reasonableness of solutions.
MA.K12.MTR.7.1	Apply mathematics to real-world contexts.

English Language Arts B.E.S.T. Standards

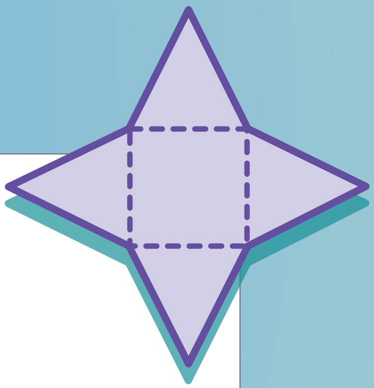
ELA.K12.EE.1.1	Cite evidence to explain and justify reasoning.
ELA.K12.EE.2.1	Read and comprehend grade-level complex texts proficiently.
ELA.K12.EE.3.1	Make inferences to support comprehension.
ELA.K12.EE.4.1	Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.
ELA.K12.EE.5.1	Use the accepted rules governing a specific format to create quality work.
ELA.K12.EE.6.1	Use appropriate voice and tone when speaking or writing.

English Language Development Standards

ELD.K12.ELL.MA.1	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
ELD.K12.ELL.SI.1	English language learners communicate for social and instructional purposes within the school setting.

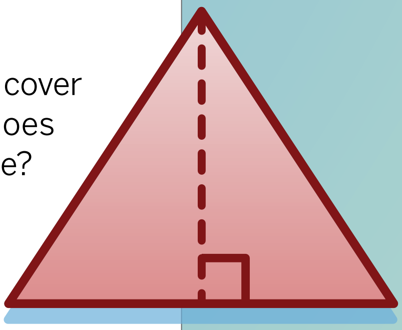
Unit 1

Area and Surface Area



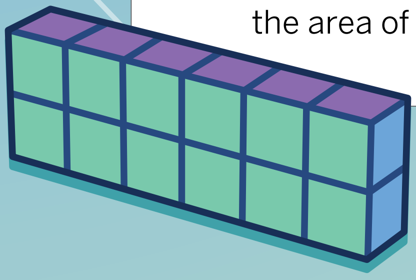
The area of a shape is the amount of space the shape covers. You know the names of many two-dimensional and some three-dimensional shapes, and have calculated the areas of rectangles.

How can you use what you have learned to cover other two-dimensional shapes? And what does it mean to cover a three-dimensional shape?



Essential Questions

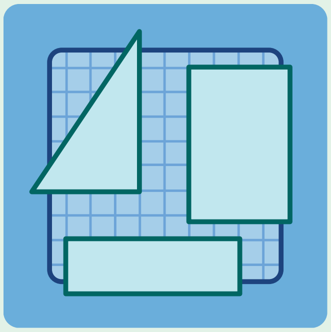
- What does it mean for two shapes to have the same area?
- How are the areas of rectangles and triangles related?
- How are the surface area of polyhedra and the area of polygons related?



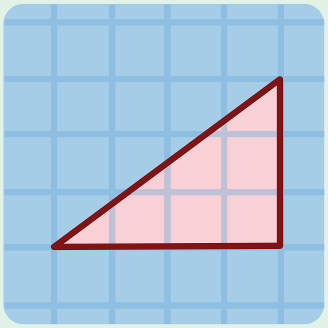
Sub-unit

1

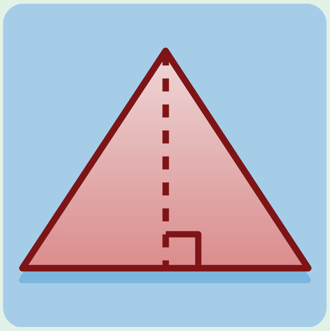
Area



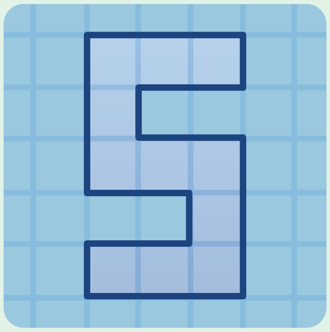
Lesson 1
Triangles and Rectangles



Lesson 2
Exploring Triangles



Lesson 3
Off the Grid



Lesson 4
Letters

Lesson 5
Breaking Down

Unit 1

Lesson

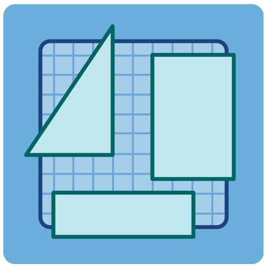
1

Name: ________ Date: ________ Period: ________

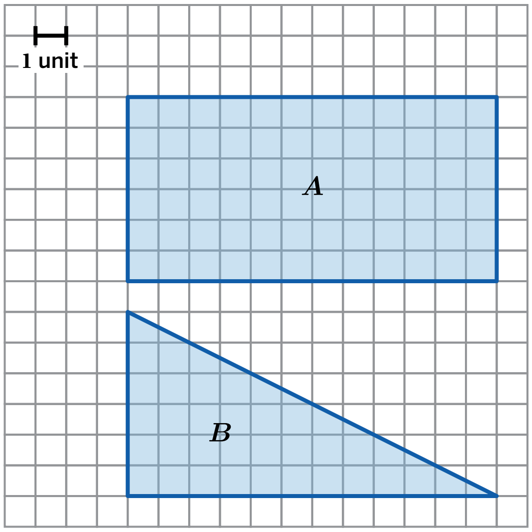
MA.6.GR.2.1, MTR.5.1

Triangles and Rectangles

Let's explore the relationship between triangles and rectangles.



Warm-Up



1. List two things that are the same about these figures.

List two things that are different.

Activity
1

Name: .... Date: .... Period: ....

Triangles and Rectangles

2. The **area** of a shape is one way to describe its size. Select *all* figures that have an area of 12 square units.

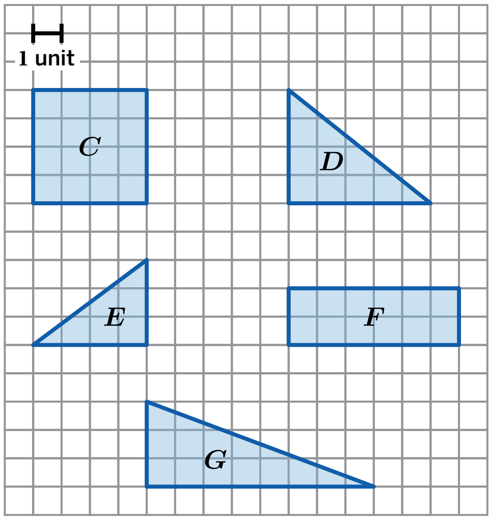
☐ *C*

☐ *D*

☐ *E*

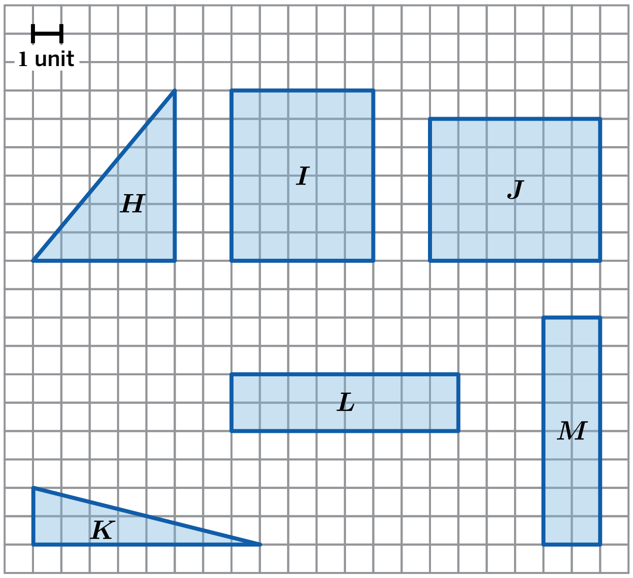
☐ *F*

☐ *G*



3. **a** Determine the **base**, **height**, and area of each shape.

Shape	Base (units)	Height (units)	Area (sq. units)
H			
I			
J			
K			
L			
M			



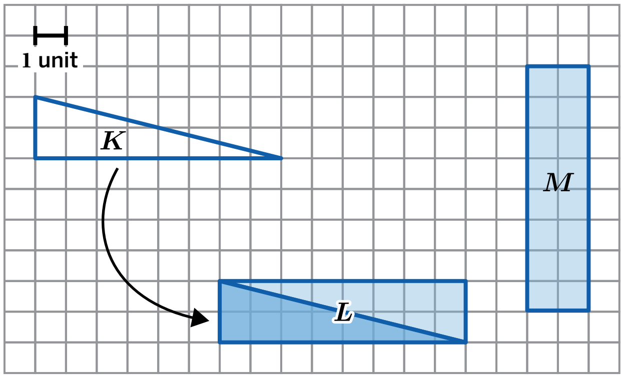
b **Discuss:** What patterns do you notice?

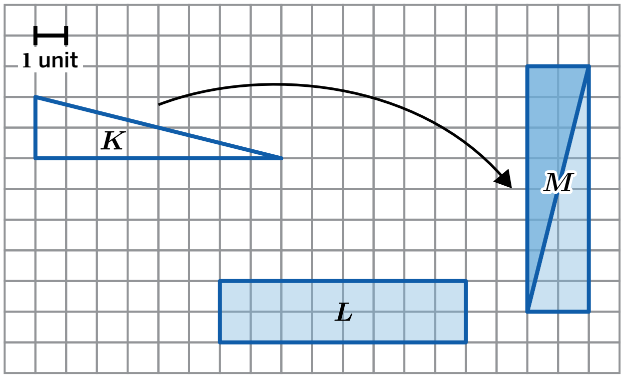
Name: .... Date: .... Period: ....

Triangles and Rectangles (continued)

4. Here are a **right triangle** and two rectangles from the previous problem.

a Take a look at how the shapes compare.





b What is the relationship between the areas of these three shapes?

Activity
2

Name: .... Date: .... Period: ....

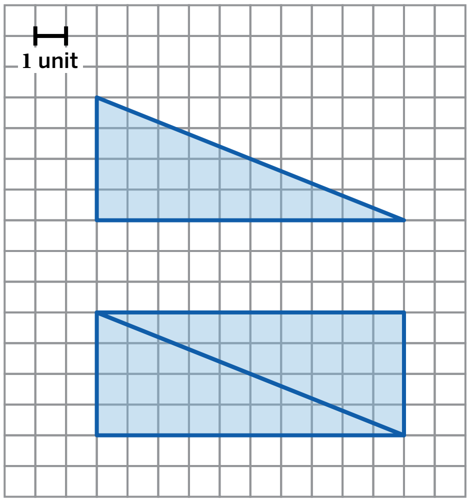
Generalizing Triangle Area

5. Let’s see if we can always combine two copies of a right triangle to form a rectangle.

a Draw a right triangle. Then draw a copy of the right triangle to make a rectangle.

b **Discuss:** How many different rectangles can you create using two copies of your right triangle?

6. Here are a right triangle and a rectangle.



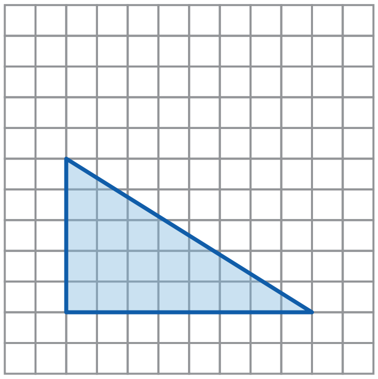
a What is the area of the right triangle?

b What is the area of the rectangle?

7. Here is the expression that Alisha entered to calculate the area of her triangle.

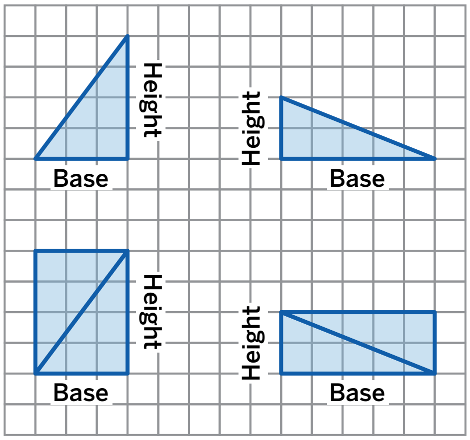
$8 \cdot 5 \cdot \frac{1}{2}$

Explain what each number represents in the expression.



Synthesis

8. How can you use the base and height of a right triangle to calculate its area?

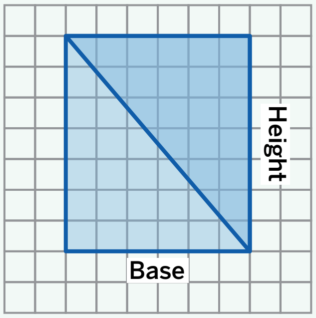


Lesson Practice 1.01

Lesson Summary

Area measures the space inside a two-dimensional figure and is expressed in square units. You can arrange two identical copies of any **right triangle** to create a rectangle with the same **base** and **height** measurements. This shows us that the area of a right triangle is equal to half the area of its related rectangle.

Here's an example of two identical right triangles that create a rectangle with the same base and height. The base is always 6 units long, and the height is always 7 units long. The area of the rectangle is $A = 6 \cdot 7 = 42$ square units. Since the area of the triangle is half the area of the rectangle, we can determine that the area of the right triangle is 21 square units.



Lesson Practice
1.01

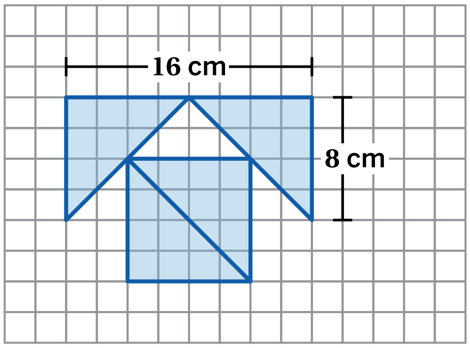
Name: .... Date: .... Period: ....

1. Determine whether each statement is true or false.

Statement	True	False
Two copies of a right triangle can be combined to make a rectangle.	○	○
Any two sides of a right triangle can be the base and height.	○	○
To determine the area of a right triangle, multiply the base by the height and then multiply by 2.	○	○

2. Choose one of the false statements in Problem 1 and explain why it is false.

3. Ali is drawing a logo for a "Guess the Logo" competition. There is a bonus round where contestants determine the shaded area of each logo. Help Ali create the answer key for the bonus round.



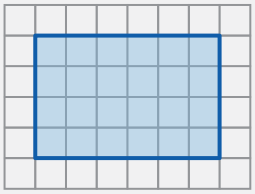
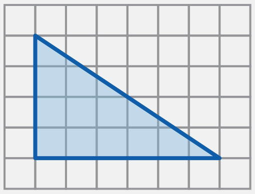
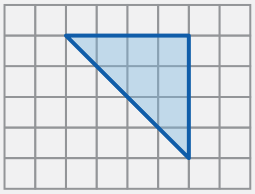
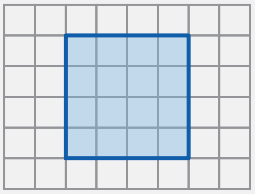
Lesson Practice

1.01

Name: .... Date: .... Period: ....

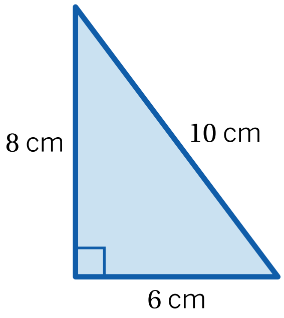
Problems 4–7: Determine the base, height, and area of each figure.

Each small square in the grid has an area of 1 square unit.

	4.	**5.**	**6.**	**7.**
				
Base				
Height				
Area				

FAST Practice

8. Which expression shows how to calculate the area of the right triangle?



A. $8 \cdot 6 \cdot 2$

B. $10 \cdot 6 \cdot \frac{1}{2}$

C. $8 \cdot 6 \cdot \frac{1}{2}$

D. $8 \cdot 10 \cdot 2$

Spiral Review

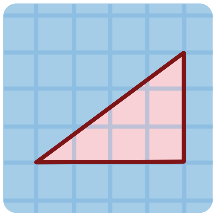
9. A store employee works for $2\frac{1}{4}$ hours stocking shelves. Then she works the register for $3\frac{1}{2}$ hours. If the employee will work 8 hours total, how much time does she have left in her day? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

MA.6.GR.2.1, MTR.5.1

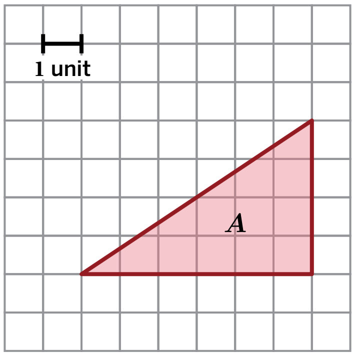
Exploring Triangles

Let’s explore the area of triangles.



Warm-Up

1. Determine the area of triangle A. Show or describe your thinking.

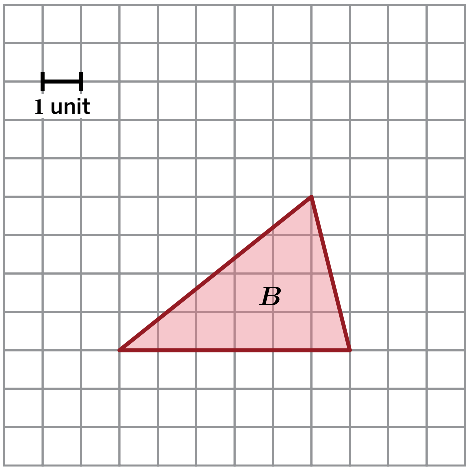


Activity
1

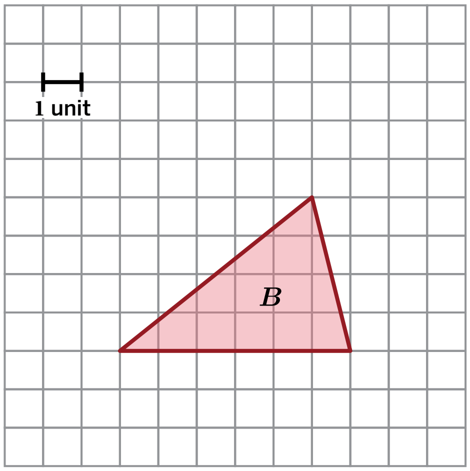
Name: .... Date: .... Period: ....

Area Strategies

2. Determine the area of triangle B. Show or describe your thinking.



3. Find a classmate who calculated the area of triangle B using a different strategy. Show or describe how your partner calculated the area.



4. Let's look at two strategies for calculating the area of triangle B.

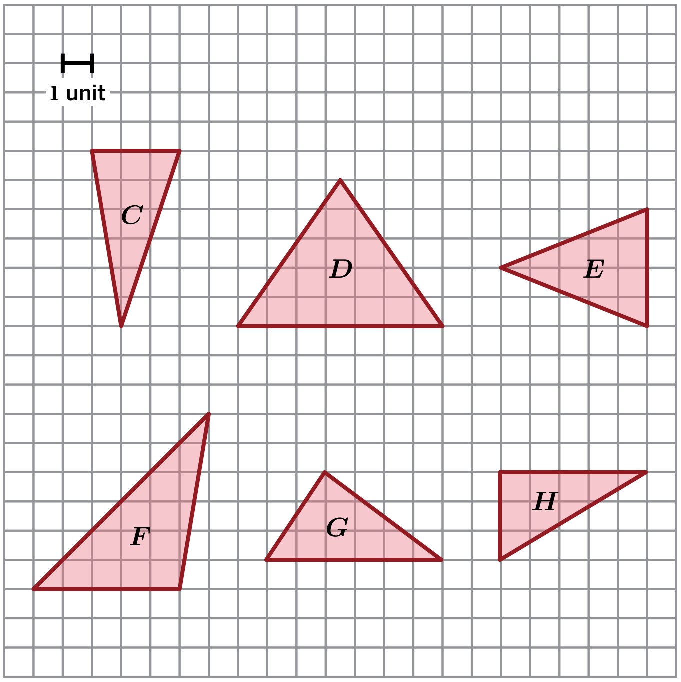
Discuss: How are these two strategies alike? How are they different?

Name: .... Date: .... Period: ....

Lots of Triangles

5. Determine the area of as many of these triangles as you can.

Triangle	Area (sq. units)
C	
D	
E	
F	
G	
H	

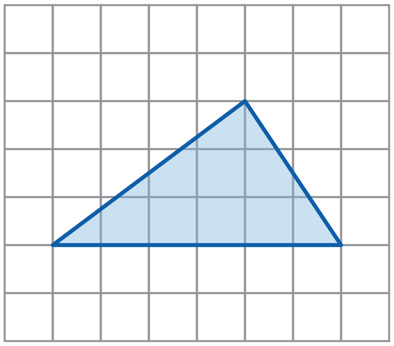


6. Describe the strategy that was most helpful to you. Did this strategy work for *all* the triangles?

7. Can you use the formula for the area of a right triangle to find the area of any triangle? Show or explain your thinking.

Synthesis

8. Describe a strategy to determine the area of a triangle. Use the example if it helps with your thinking.



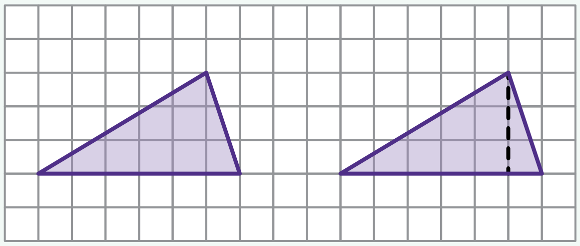
Lesson Practice 1.02

Lesson Summary

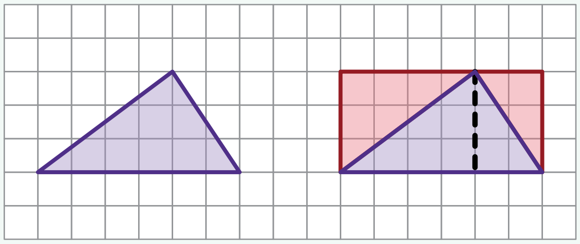
You can use what you know about the areas of rectangles and right triangles to help you determine the area of any triangle.

Here are two ways you can use rectangles and right triangles to help you determine the area of any triangle on a grid.

- Make a copy of the triangle and break it into two right triangles.
  - The sum of the areas of the two triangles is equal to the sum of the area of the original triangle.



- Enclose the triangle in a large rectangle that can be cut into two smaller rectangles.
  - This also cuts the triangle into two smaller triangles.
  - Each of these smaller triangles has half the area of its enclosed rectangle.
  - The sum of the two smaller triangles' areas is equal to the area of the original triangle.



You can also apply the formula for the area of a right triangle, $A = \frac{1}{2}bh$, to any triangle, where b is the length of the base and h is the height of the triangle.

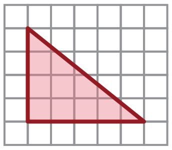
Lesson Practice
1.02

Name: .... Date: .... Period: ....

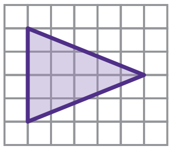
Problems 1–4: Determine the area of each triangle.

Each small square in the grid has an area of 1 square unit.

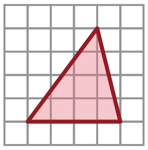
1.



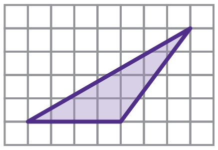
2.



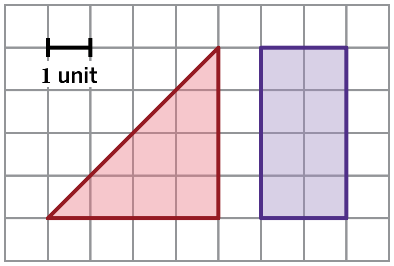
3.



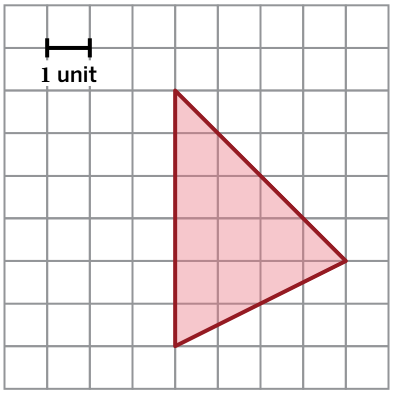
4.



5. Aki thinks that these two shapes have the same area. Is Aki's thinking correct?



6. Determine the area of this triangle. Show or explain your thinking.

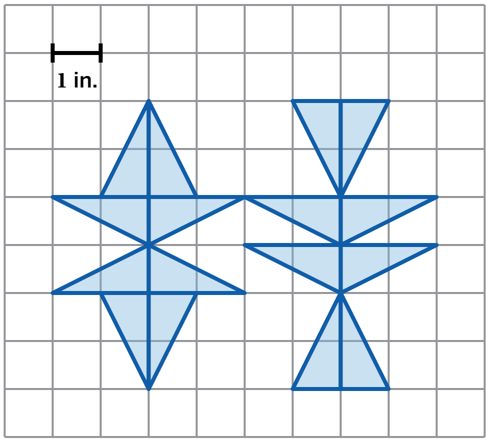


Lesson Practice

1.02

Name: .... Date: .... Period: ....

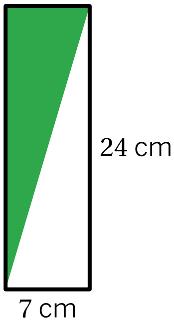
7. Alice used triangle tiles to make a design on an 8-by-8 inch area. Determine the total area, in square inches, that is covered by the triangle tiles in her design.



FAST Practice

8. What is the area, in square centimeters, of the shaded part of this rectangle?

- **A.** 15.5
- **B.** 62
- **C.** 84
- **D.** 168

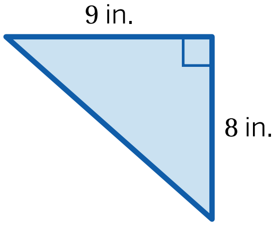


Spiral Review

9. Select *all* of the expressions that have the same value as $8 \div 2$.

- ☐ **A.** $\frac{8}{2}$
- ☐ **B.** $8 \bullet 2$
- ☐ **C.** $2 \div 8$
- ☐ **D.** $\frac{1}{2} \bullet 8$
- ☐ **E.** $\frac{2}{8}$

10. Use a formula to determine the area of the triangle. Show or describe your thinking.

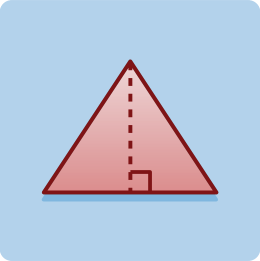


Name: .... Date: .... Period: ....

MA.6.GR.2.1, MTR.3.1, MTR.6.1

Off the Grid

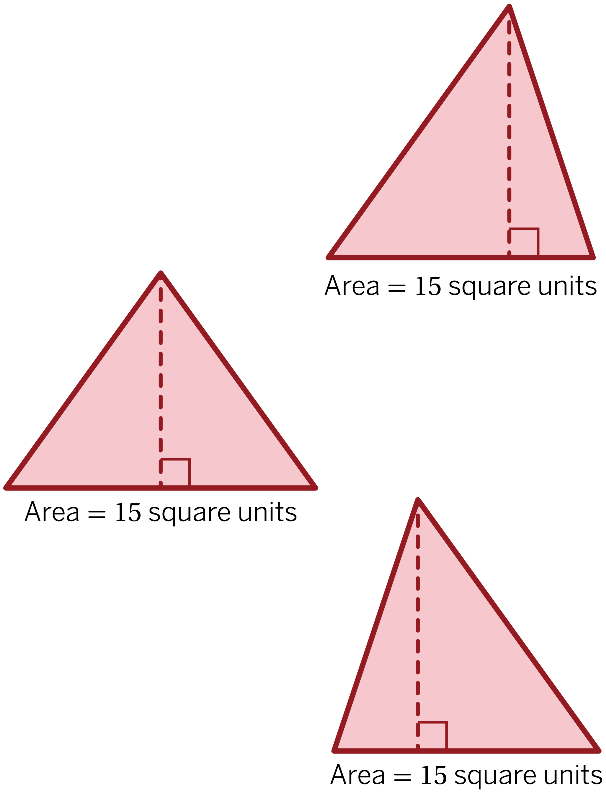
Let's practice calculating the area of triangles.



Warm-Up

1. Let's look at the different sides of a triangle.

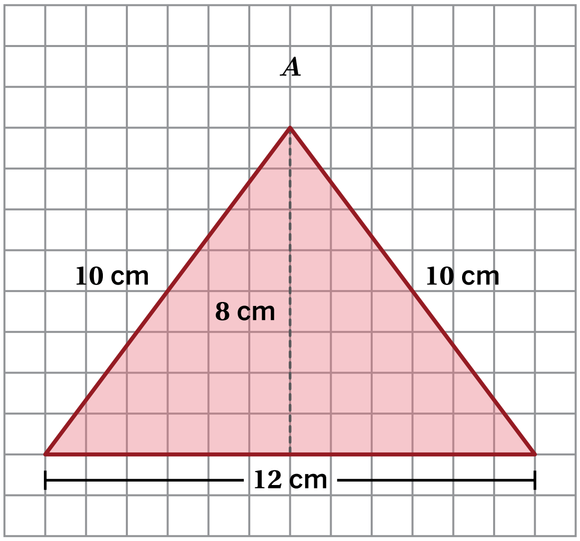
What is one thing that changes? What is one thing that stays the same?



Name: .... Date: .... Period: ....

Base, Height, and Area

2. Use any strategy to determine the area of triangle A.



3. Select *all* the expressions that could represent the area of this triangle. Draw on the triangle if it helps with your thinking.

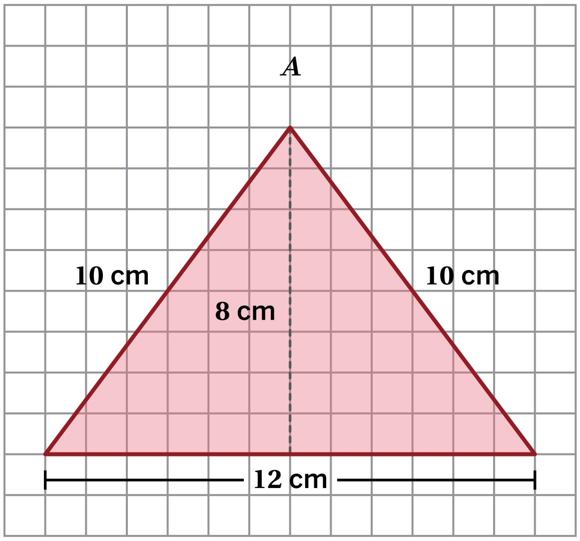
☐ **A.** $\frac{1}{2} \cdot 12 \cdot 8$

☐ **B.** $\frac{12 \cdot 10}{2}$

☐ **C.** $12 \cdot 8 \div 2$

☐ **D.** $6 \cdot 8$

☐ **E.** $6 \cdot 4$



4. Here is triangle A from the previous problem, along with a new triangle. Which triangle has the greater area? Circle one.

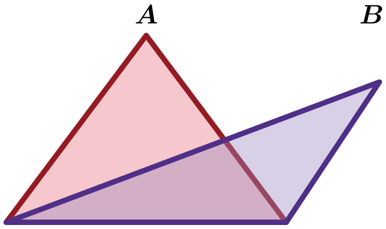
Triangle A

Triangle B

They are the same

Not enough information

Explain your thinking.

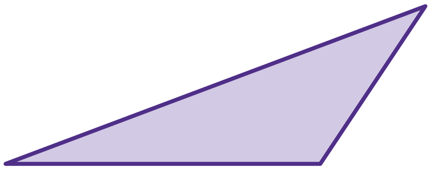


Name: .... Date: .... Period: ....

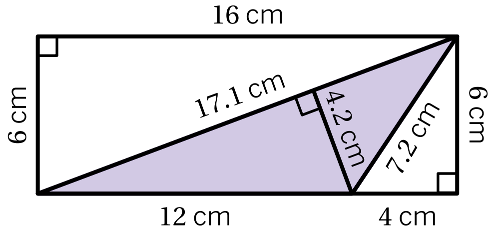
Choose Your Measurements

5. Ishaan wants to calculate the area of this triangle, but the measurements are not labeled.

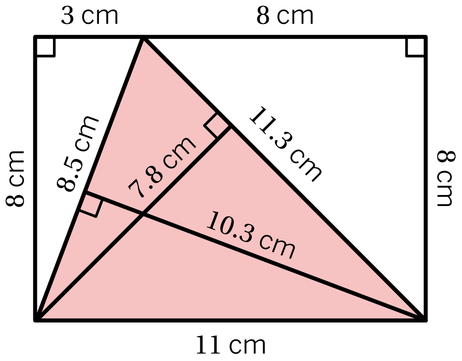
Draw on the triangle to show Ishaan what he should measure to calculate the area.



6. Use as many measurements as you want to calculate the area of the triangle to the nearest square centimeter.



7. Use as many measurements as you want to calculate the area of the triangle to the nearest square centimeter.

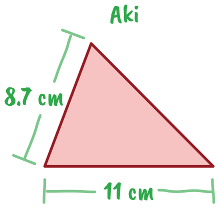


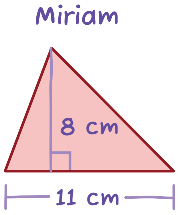
Name: .... Date: .... Period: ....

Choose Your Measurements (continued)

8. Aki and Miriam found different areas for the same triangle.

Here are the measurements they took.





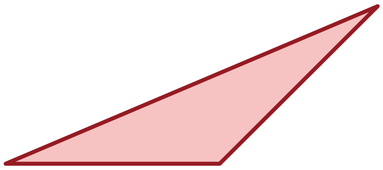
Whose measurements lead to the correct area? Circle one.

Aki's Miriam's Both Neither

Explain your thinking.

9. Sketch as many different triangles as you can with the same area as triangle C.

Triangle C
Area = 24 sq. cm

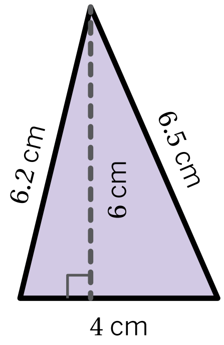


Name: .... Date: .... Period: ....

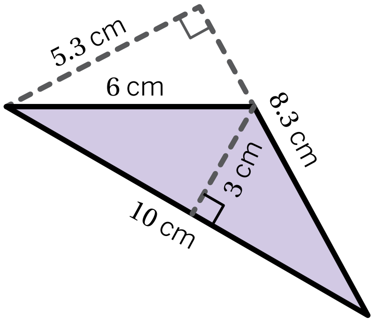
Repeated Challenges

10. Calculate the area of each triangle to the nearest square centimeter. Use as many measurements as you need.

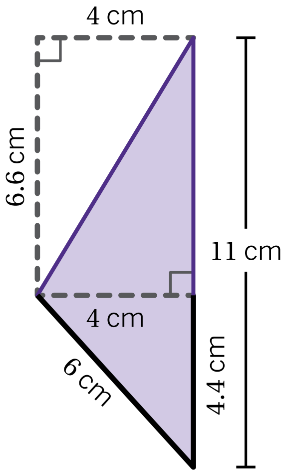
a



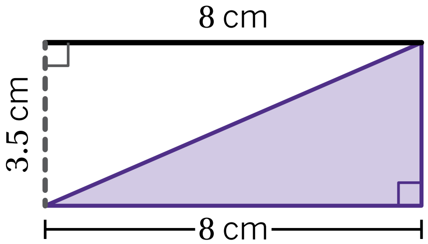
b



c

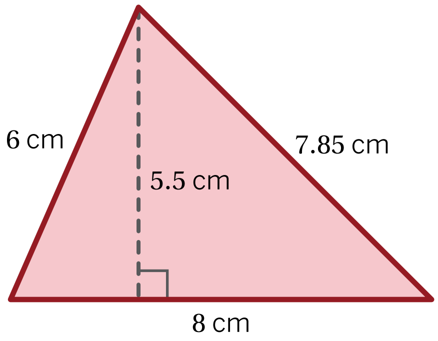


d



Synthesis

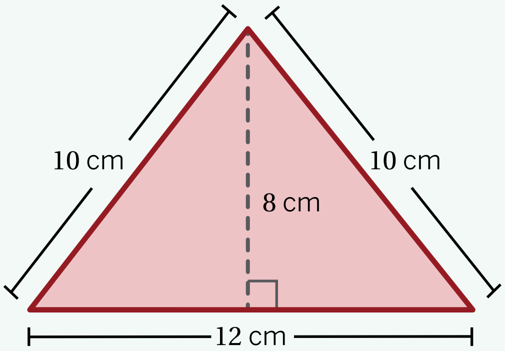
11. Describe how to calculate the area of a triangle. Use this example if it helps with your thinking.

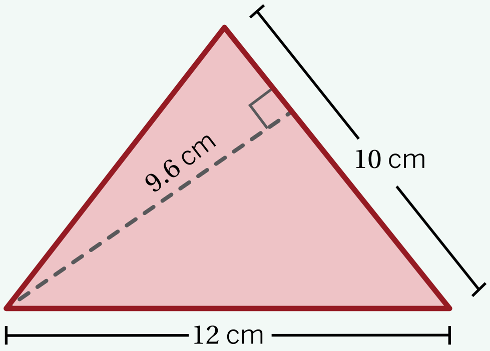


Lesson Practice 1.03

Lesson Summary

The area of any triangle is equal to half of the product of its base and height. You can select any side of the triangle to be the base. The height of a triangle is the *perpendicular* distance between a point on the base and the opposite corner of the triangle. The height is often shown with a dotted line.





$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(12)(8)$$

$A = 48$ square centimeters

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(9.6)$$

$A = 48$ square centimeters

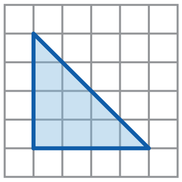
All sides of the triangle can be a base, but some base-height pairs are easier to measure and calculate with.

Lesson Practice
1.03

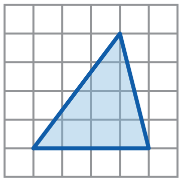
Name: .... Date: .... Period: ....

1. Select *all* of the triangles that have an area of 8 square units. Each small square in the grid has an area of 1 square unit.

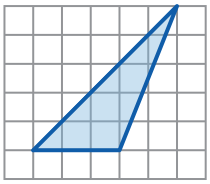
☐ **A.**



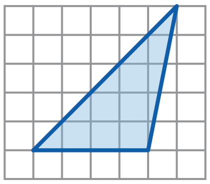
☐ **B.**



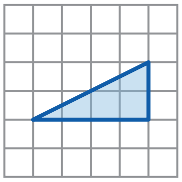
☐ **C.**



☐ **D.**

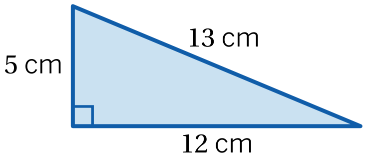


☐ **E.**

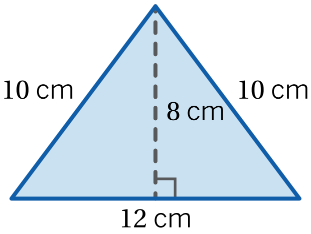


Problems 2–4: Determine the area of each triangle in square centimeters.

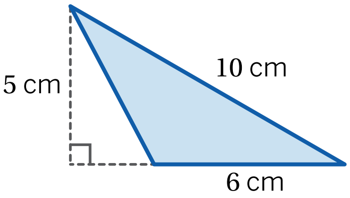
2.



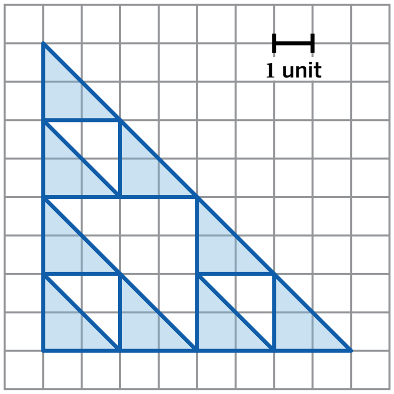
3.



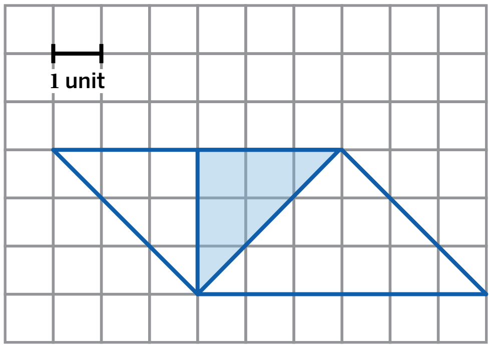
4.



5. Motion capture is a technology that records an actor's movements and transfers them onto a computer-generated character. Actors wear special suits covered in a specific pattern of sensors. One common pattern is called the Sierpiński triangle. Take a look at this Sierpiński triangle. Determine the area of the shaded region, and explain your thinking.



6. A company is designing a new logo that consists of a shaded triangle inside of a parallelogram. What fraction of the parallelogram's area is shaded?

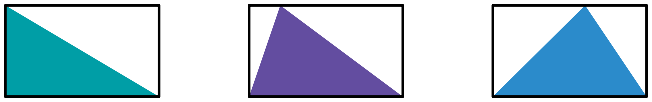


Lesson Practice
1.03

Name: .... Date: .... Period: ....

FAST Practice

7. Here are three triangles drawn inside three identical rectangles. Mariam states that all three triangles have the same area. Is Mariam correct? Explain your thinking.



Select **ONE** correct answer in each box to complete the sentences.

All three triangles have | Ⓐ. different Ⓑ. the same | base and height lengths,

so their areas are | Ⓐ. different Ⓑ. the same |.

Spiral Review

8. A rectangle has an area of 12 square units and a base of 9 units. What is the matching height?

9. A rectangle has an area of 7 square units and a height of $\frac{1}{4}$ units. What is the matching base?

Unit 1

Lesson

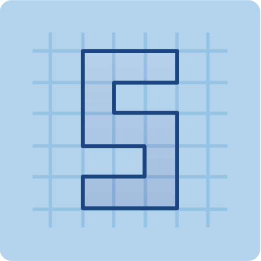
4

Name: ______________________ Date: __________ Period: __________

MA.6.GR.2.1, MA.6.GR.2.2, MTR.4.1, MTR.5.1

Letters

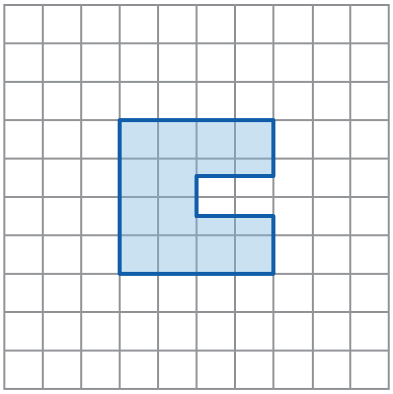
Let's explore the area of shapes.



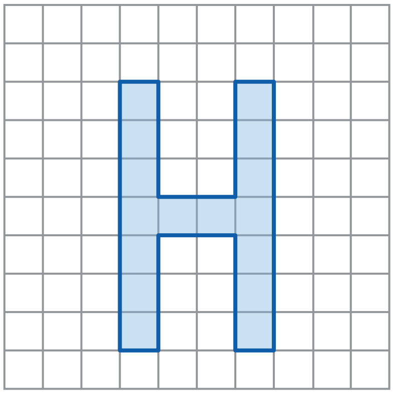
Warm-Up

1. Which figure doesn't belong? Explain your thinking.

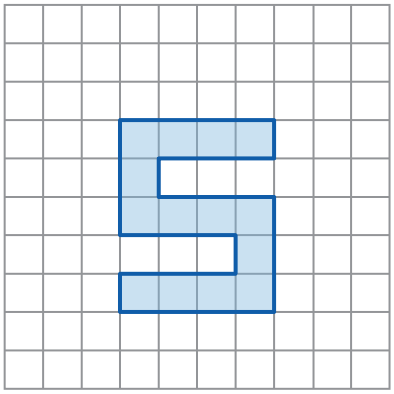
A.



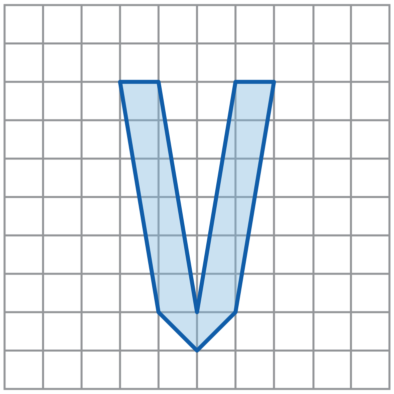
B.



C.



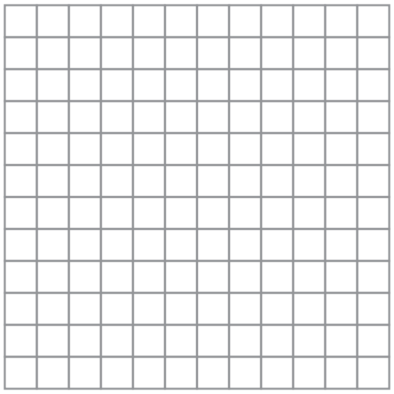
D.



Name: .... Date: .... Period: ....

Rearranging Shapes

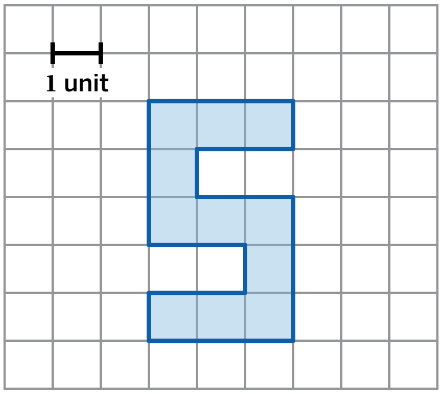
2. **a** Draw the first letter of your name on the grid.



b Tell a story about your name.

3. Saanvi colored in the "S" that she drew.

What is the area of the shape she colored?

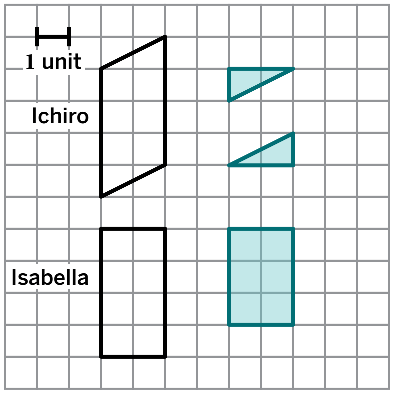


4. Ichiro and Isabella each drew an "I". Ichiro cut his "I" into pieces to see how much area to color.

Whose letter has a greater area? Circle one.

Ichiro Isabella They are the same

Explain your thinking.



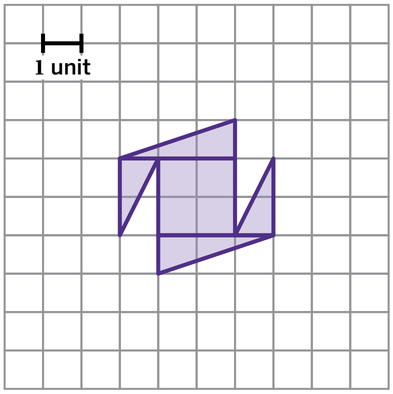
Name: .... Date: .... Period: ....

Rearranging Shapes (continued)

5. Zahra also cut up her "Z" to see how much area it covered.

   What is the area of the shape she covered?

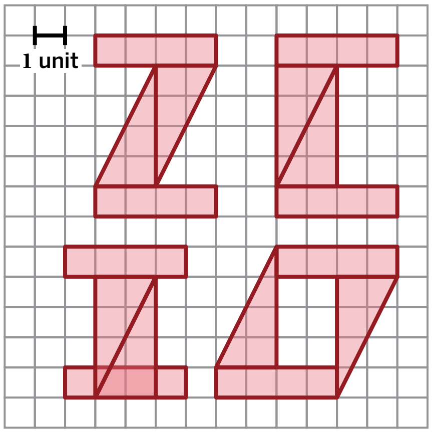
   Use arrows to show how you could rearrange the pieces, if it helps with your thinking.



6. Zola noticed she could cut up her "Z" and rearrange the pieces to make new letters.

   Select *all* the new letters that have the same area as the "Z."

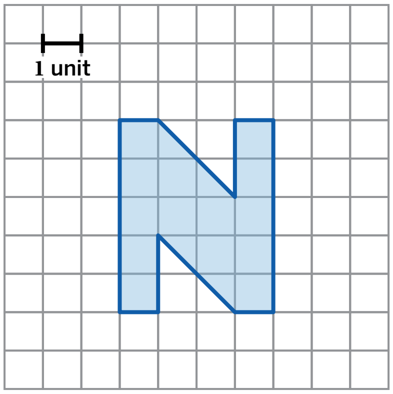
   ☐ C ☐ I ☐ O



7. Nathan made an "N."

   a What is the area of the shape he colored? Sketch on the grid if it helps to show your thinking.

   b **Discuss:** What strategy did you use?



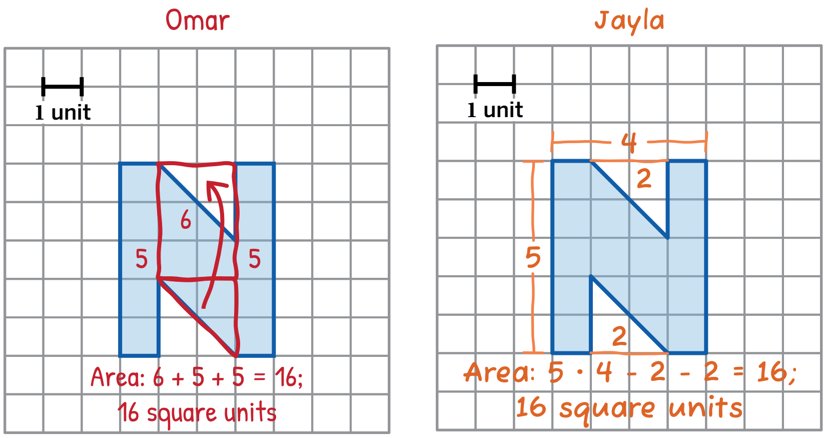
Activity 2

Name: .... Date: .... Period: ....

Area Strategies

8. Omar and Jayla used different strategies to determine the area of "N."

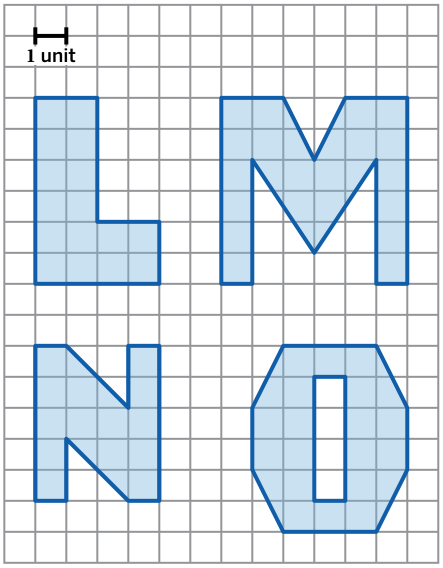
a Take a look at each student's work.



b Pick one student and explain how you think they determined the area.

9. Complete the table.

Letter	Area (sq. units)
L	
M	
N	16
O	

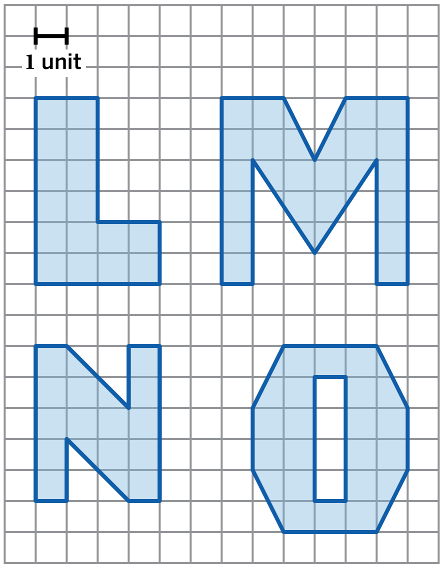


Synthesis

10. **a** Which area calculation are you most proud of? Circle one.

L M N O

b Write some advice for someone determining this area.

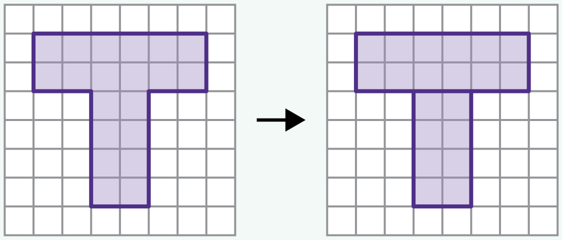


Lesson Practice 1.04

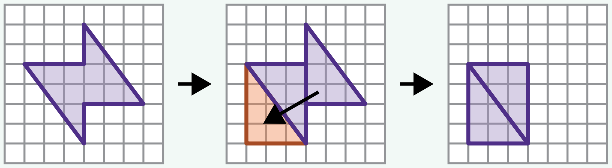
Lesson Summary

We can use shapes like rectangles, squares, and triangles to help us determine the area of more complex shapes. Here's how!

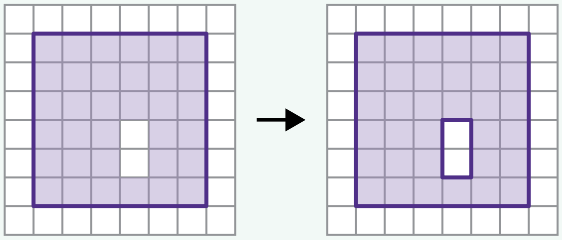
- *Decompose* the shape into two or more smaller shapes that have areas you know how to calculate.
- Add the smaller areas together.



- Decompose the shape and *rearrange* the pieces to form one or more other shapes that have areas you know how to calculate.
- Calculate the area of the new, simpler shape(s).



- If your shape has empty areas in it, determine its area as if it were a solid shape.
- Calculate the area of the empty space and subtract it from the total area.



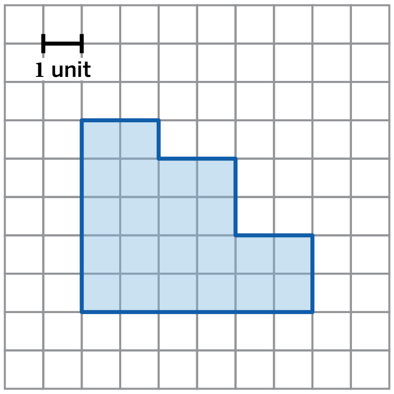
Lesson Practice

1.04

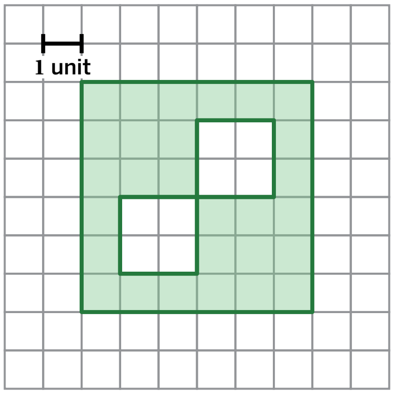
Name: .... Date: .... Period: ....

Problems 1–4: Determine the total area of each shaded region.

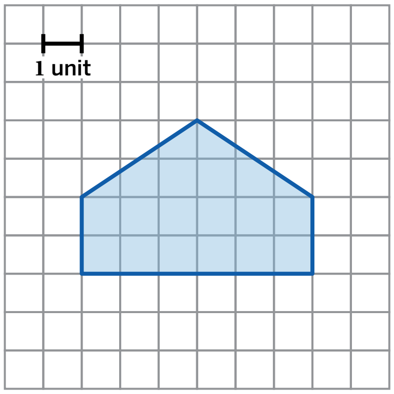
1.



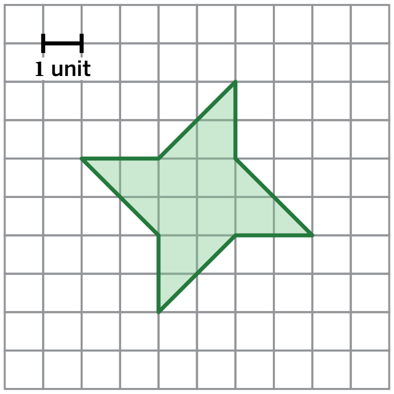
2.



3.



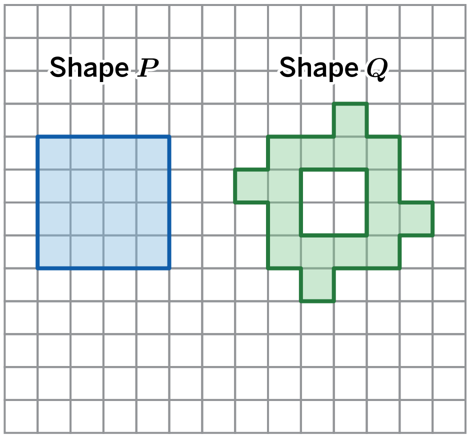
4.



5. Which shape has a greater area? Circle one.

Shape P Shape Q They have the same area.

Show or explain how you know.



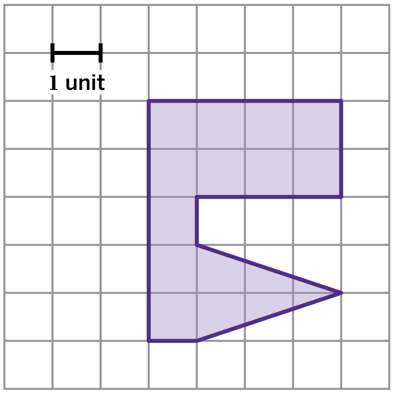
Lesson Practice
1.04

Name: .... Date: .... Period: ....

FAST Practice

6. Jasmyn drew this shape. Determine its area.

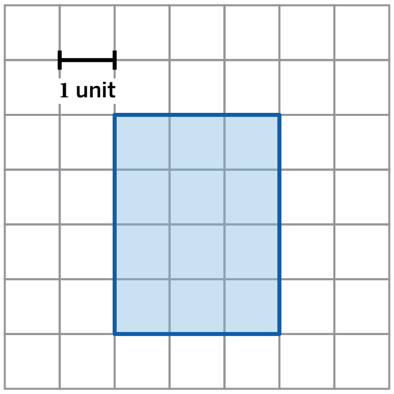
- **A.** 17 square units
- **B.** 16 square units
- **C.** 14 square units
- **D.** 11 square units



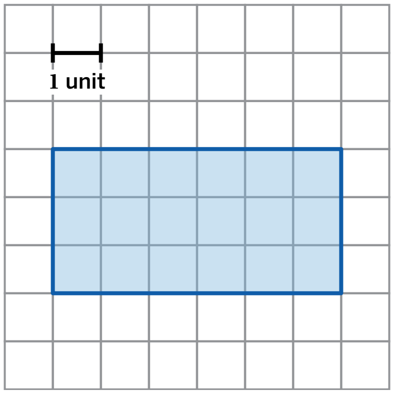
Spiral Review

7. Select *all* the rectangles with an area of 12 square units.

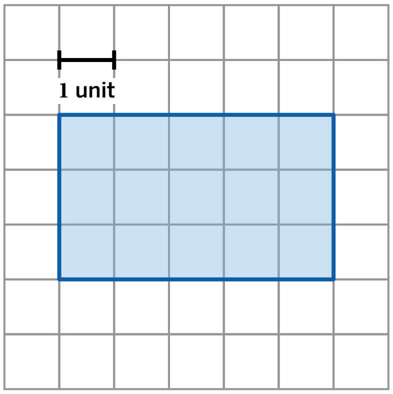
☐ **A.**



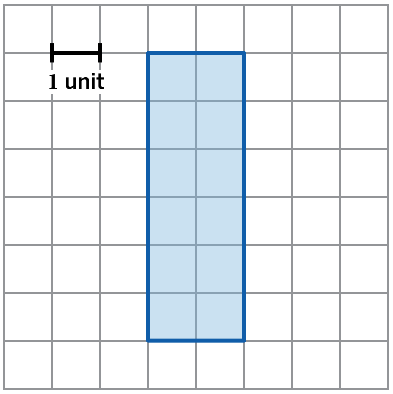
☐ **B.**



☐ **C.**



☐ **D.**



Name: .... Date: .... Period: ....

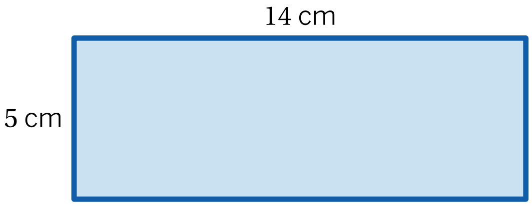
MA.6.GR.2.2, MTR.5.1

Breaking Down

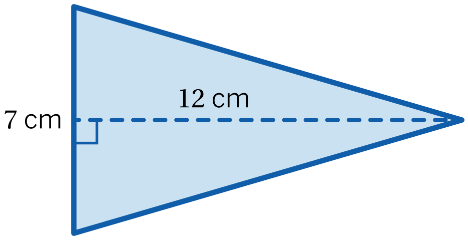
Let's practice using rectangles and triangles to determine the area of composite figures.

Warm-Up

1. Determine the area of the rectangle. Show or explain your thinking.



2. Determine the area of the triangle. Show or explain your thinking.



Name: .... Date: .... Period: ....

Challenge Creator

3. You will use the Challenge Creator Sheet to create your own area challenge.

a **Make It!** On the Challenge Creator Sheet, create your own area challenge.

b **Solve It!** On this page, determine the area of your figure.

My Area

c **Swap It!** Swap your challenge with one or more partners. Determine the area of each partner's figure.

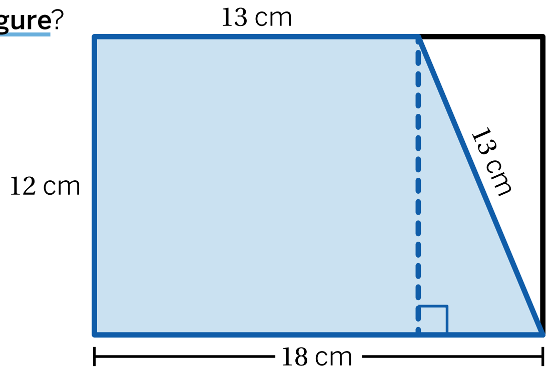
	Partners' Areas
Partner 1	
Partner 2	
Partner 3	

Activity
2

Name: ________ Date: ________ Period: ________

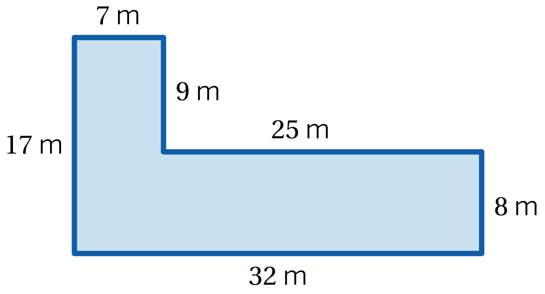
Composing and Decomposing

4. **a** How can you determine the area of this **composite figure**?



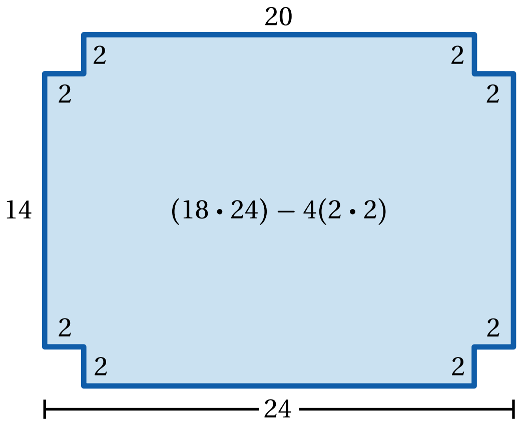
b What is the area of the figure?

5. Draw lines to show how you can break apart this composite figure to calculate its area. Then calculate its area using as many measurements as you need.

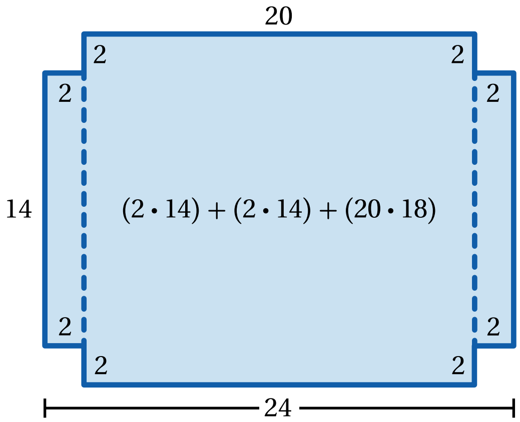


6. Kadeem and Sadia each wrote an expression for the area of the same shaded figure. Whose thinking is correct? Explain your thinking.

Kadeem



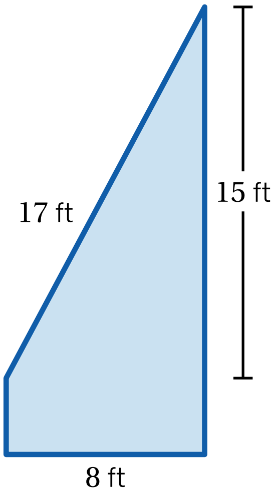
Sadia



Name: .... Date: .... Period: ....

Composing and Decomposing (continued)

7. The composite figure has an area of 84 square feet. What is the length of the entire right side of the figure? Show or explain your thinking.

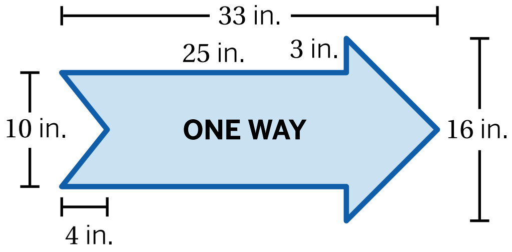


Name: Date: Period:

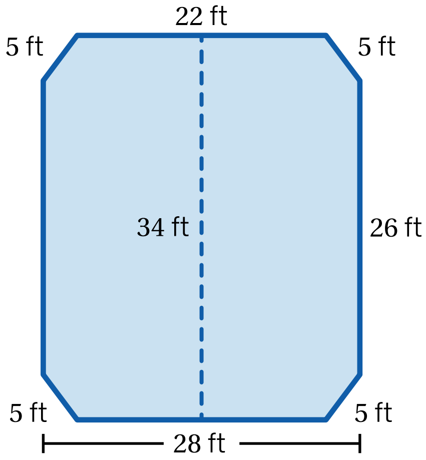
Repeated Challenges

8. Solve as many challenges as you have time for.

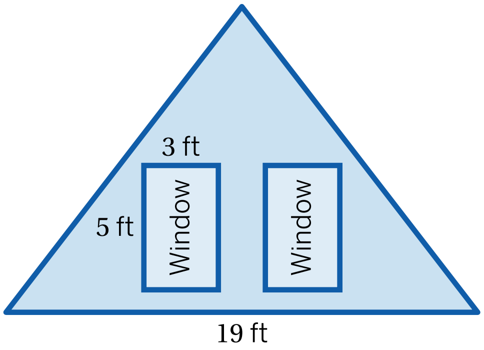
a The area of a road sign with these dimensions.



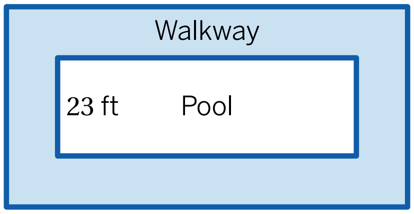
b The area of an octagonal stage with these dimensions.



c The area of a 12-foot-tall wall, not including its two identical windows.



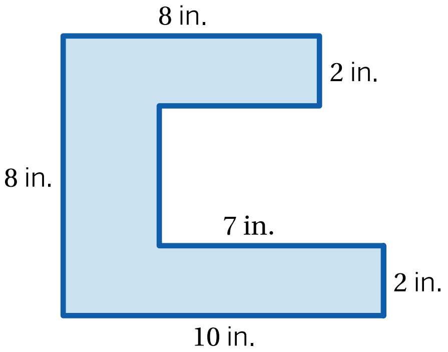
d The length of a pool with a 4-foot-wide walkway. The area of the walkway and pool together are 2,015 square feet.



Synthesis

9. Describe how you can determine the area of a composite figure.

Draw on this image if it helps to show your thinking.



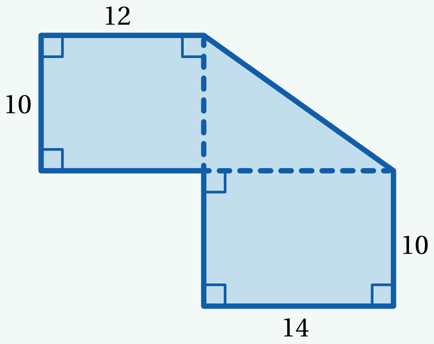
Lesson Practice 1.05

Lesson Summary

A **composite figure** is a two-dimensional figure that can be decomposed into smaller figures.

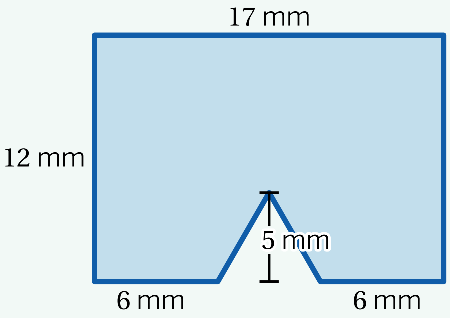
We can use different strategies to determine the area of a composite figure.

Decompose the figure into rectangles and triangles and then add the areas of the separate pieces.



This composite figure can be decomposed into two rectangles and a triangle.

Draw a rectangle around the figure and then subtract the areas that are not part of the figure from the area of the rectangle.



This composite figure can be composed into a rectangle with a missing triangle.

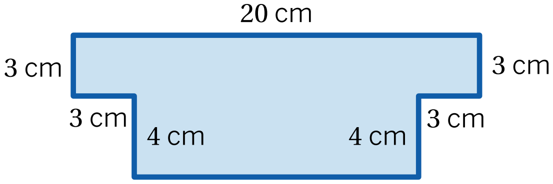
Lesson Practice

1.05

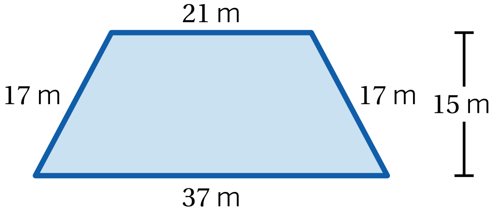
Name: .... Date: .... Period: ....

Problems 1–4: Calculate the area of each composite figure.

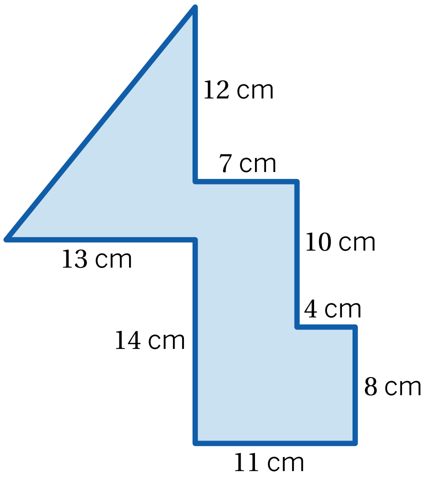
1.



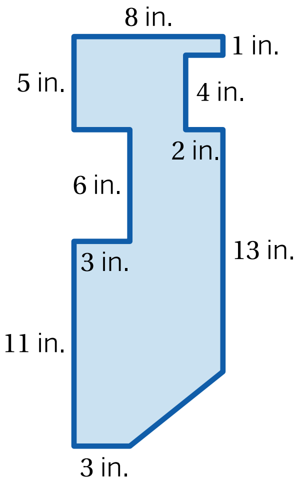
2.



3.



4.

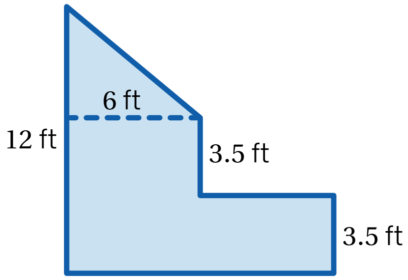


5. Describe two different strategies you can use to calculate the area of the figure in Problem 4.

Lesson Practice
1.05

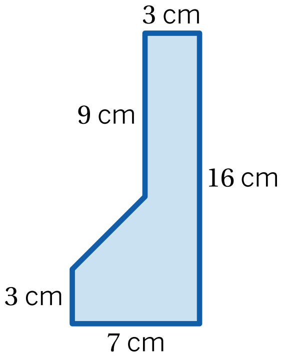
Name: .... Date: .... Period: ....

6. What is the length of the base of the composite figure if the area of the figure is 78 square feet?



FAST Practice

7. Which expression shows how to calculate the area of the composite figure?



A. $(16 \cdot 7) - (13 \cdot 4)$

B. $(16 \cdot 7) - (13 \cdot 4) - \left(\frac{1}{2} \cdot 4 \cdot 4\right)$

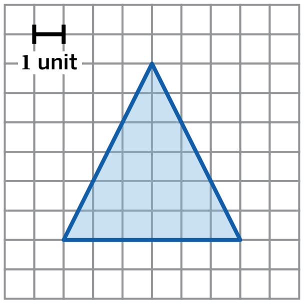
C. $(16 \cdot 3) + (3 \cdot 7) + \left(\frac{1}{2} \cdot 4 \cdot 4\right)$

D. $(16 \cdot 3) + (3 \cdot 4) + \left(\frac{1}{2} \cdot 4 \cdot 4\right)$

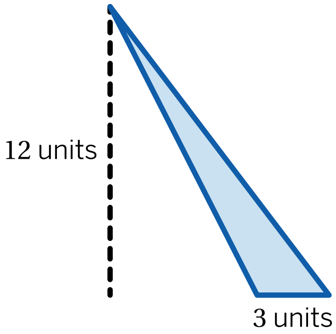
Spiral Review

8. Select *all* the triangles with an area of 18 square units.

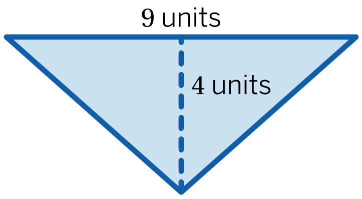
☐ **A.**



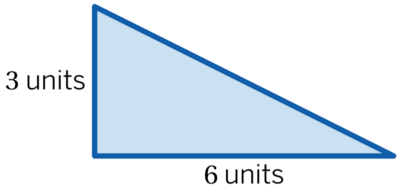
☐ **B.**



☐ **C.**



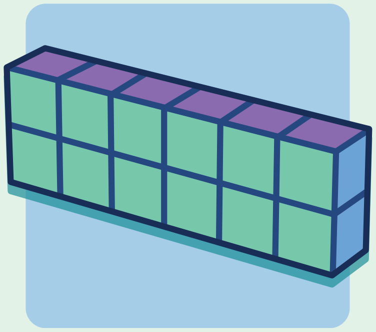
☐ **D.**



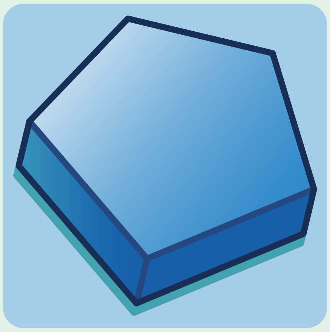
Sub-unit

2

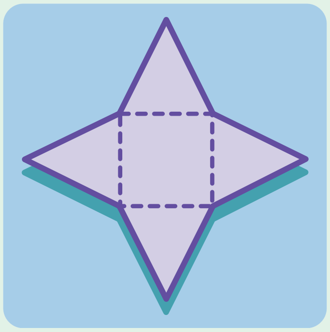
Surface Area



Lesson 6
Renata's Stickers



Lesson 7
Prisms and Pyramids



Lesson 8
Nothing But Nets

Unit 1

Lesson

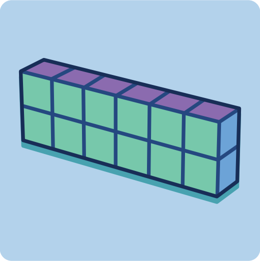
6

Name: ______ Date: ______ Period: ______

MA.6.GR.2.4, MTR.1.1, MTR.6.1

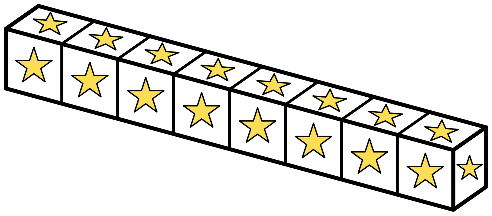
Renata's Stickers

Let's cover rectangular prisms with stickers.



Warm-Up

1. Renata loves to cover things with stickers.



a She adds stickers to all the visible faces of each block.

b If Renata bought a pack of 30 stickers, would she have enough to cover this figure? Circle one.

Yes No I'm not sure

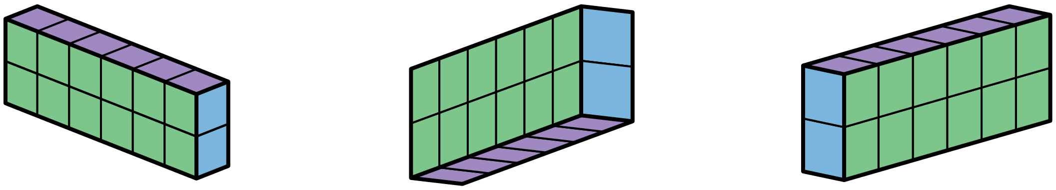
Explain your thinking.

Activity
1

Name: ________ Date: ________ Period: ________

Exploring Surface Area

2. Here are different views of a figure. How many stickers will cover this figure?



3. Each of these figures has a **volume** of 8 cubic units.

Which figure needs more stickers to cover it?
Circle one.

Figure A

Figure B

Both need the same number

Explain your thinking.

Figure A

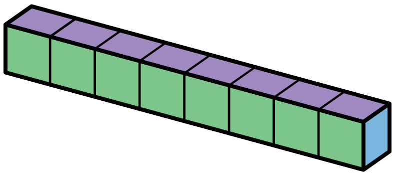
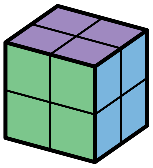


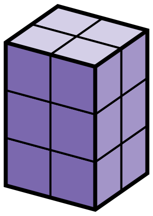
Figure B



Name: .... Date: .... Period: ....

Exploring Surface Area (continued)

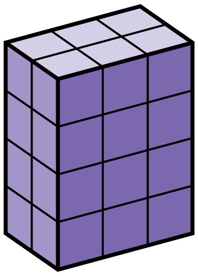
4. The volume of this rectangular prism is 12 cubic units.



Its **surface area** is the number of square units that cover its surface, like Renata's stickers do.

Determine the surface area of this rectangular prism.

5. Determine the surface area of this prism.



Name: .... Date: .... Period: ....

Surface Area Strategies

6. Jaleel determined the surface area of a new prism.

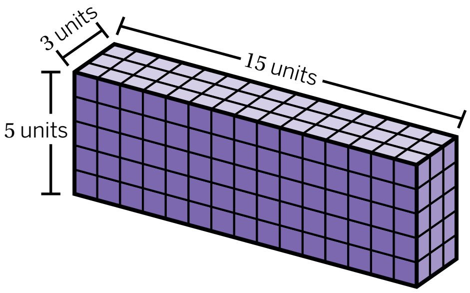
Jaleel

15 × 5 × 2 = 150

3 × 5 × 2 = 30

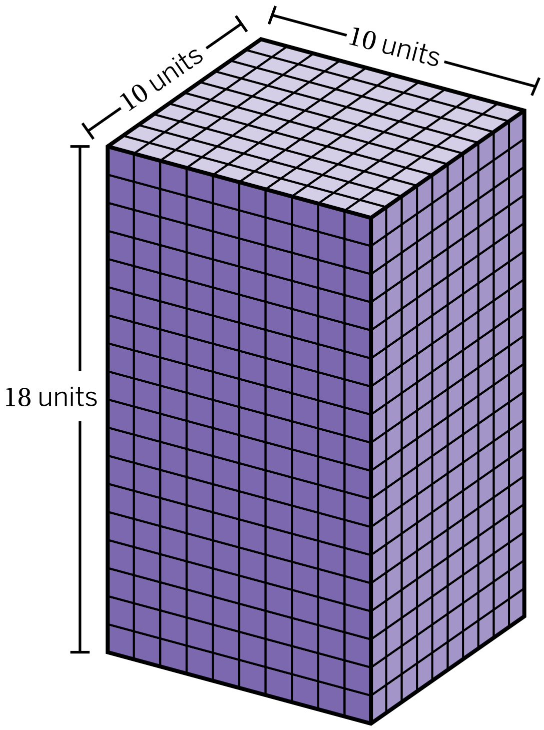
15 × 3 × 2 = 90

150 + 30 + 90 = 270 square units



Discuss: What did Jaleel do to determine the surface area?

7. Determine the surface area of this rectangular prism. Draw on the figure if it helps with your thinking.



Name: .... Date: .... Period: ....

Surface Area Strategies (continued)

8. The surface area of figure A is 22 square units.

Figure B is a *half unit* taller than figure A.

What is the surface area of figure B?
Explain your thinking.

Figure A
22 sq. units

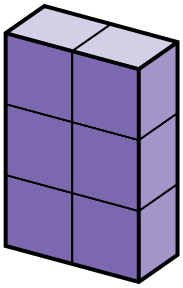
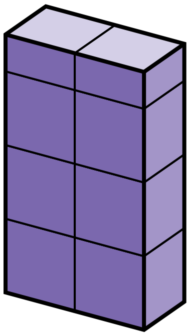


Figure B



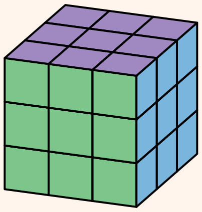
You're invited to explore more

9. Here is a prism made with 27 cubes that Renata covered with stickers.

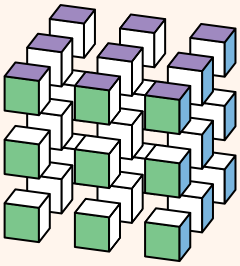
How many of the cubes have 0, 1, 2, and 3 stickers?

Stickers	Number of Cubes
0	
1	
2	
3	

Prism

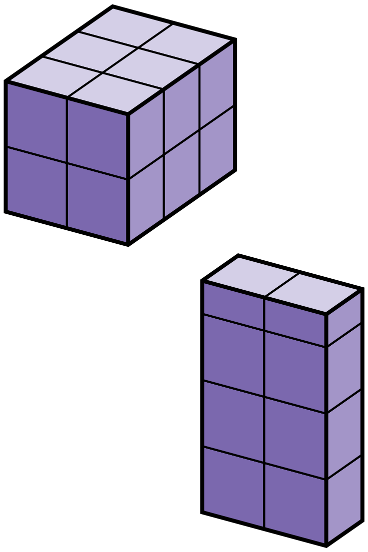


Pulled-Apart Prism



Synthesis

10. How do you determine the surface area of a rectangular prism?



Lesson Practice 1.06

Lesson Summary

The **surface area** of a rectangular prism is the sum of the areas of its surface. The **volume** of a rectangular prism measures the number of unit cubes that can be packed inside it without gaps or overlaps. Because volume is a three-dimensional measurement, it's measured in cubic units.

Here is a rectangular prism with a surface area of 52 square units and a volume of 24 cubic units.

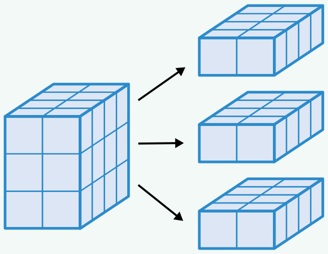
Surface Area

$(2 \cdot 3) \cdot 2 = 12$

$(4 \cdot 3) \cdot 2 = 24$

$(2 \cdot 4) \cdot 2 = 16$

$12 + 24 + 16 = 52$ square units



Volume

$8 + 8 + 8 = 24$

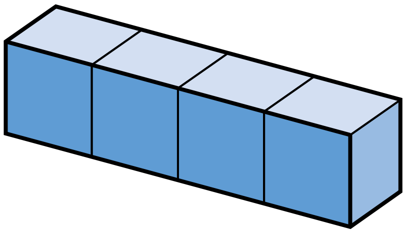
24 cubes = 24 cubic units

Lesson Practice
1.06

Name: .... Date: .... Period: ....

Problems 1–2: Determine the surface area and volume of each rectangular prism.

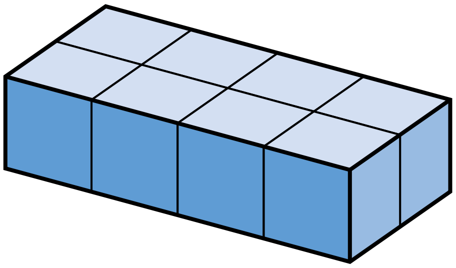
1.



Surface area: .... square units

Volume: .... cubic units

2.



Surface area: .... square units

Volume: .... cubic units

3. Explain the difference between *volume* and *surface area*.

4. Select *all* of the calculations that surface area might help with.

- ☐ **A.** How many blocks fit in a container
- ☐ **B.** How much wrapping paper a gift will need
- ☐ **C.** How heavy a gift is
- ☐ **D.** How much milk fits in a container
- ☐ **E.** How much cardboard it takes to make a milk container

5. Compare the surface areas of figure A and figure B. Explain your thinking.

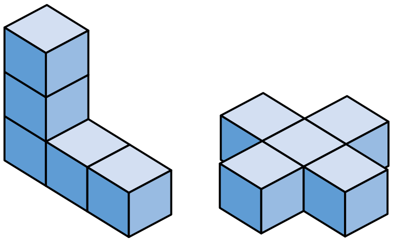


Figure A **Figure B**

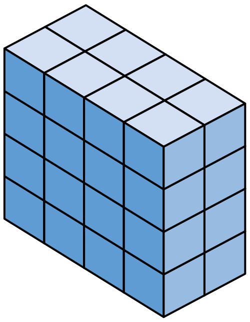
Lesson Practice

1.06

Name: ____________ Date: ________ Period: ________

FAST Practice

6. This rectangular prism is 4 units high, 4 units wide, and 2 units long. What is its surface area?

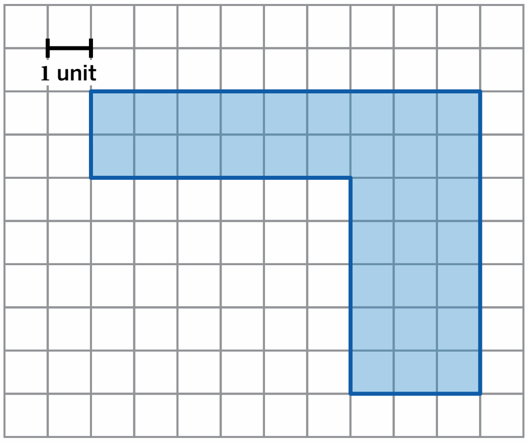


- **A.** 16 square units
- **B.** 32 square units
- **C.** 48 square units
- **D.** 64 square units

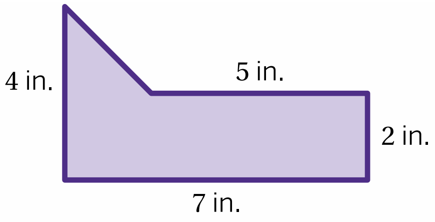
Spiral Review

Problems 7–8: Determine the area of each shaded figure.

7.



8.



Unit 1

Lesson

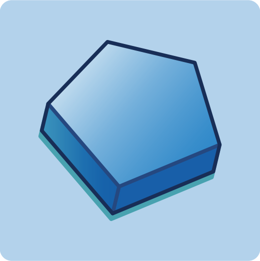
7

Name: .... Date: .... Period: ....

MA.6.GR.2.4, MTR.4.1, MTR.6.1

Prisms and Pyramids

Let’s play with polyhedra.



Warm-Up

1. Play a few rounds of Polygraph with your classmates!

You will use the Warm-Up Sheet with 3-D shapes to play for four rounds.
For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a 3-D shape from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating 3-D shapes until you’re ready to guess which shape the Picker chose.

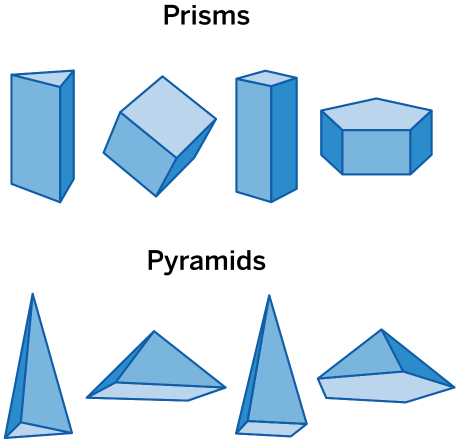
Record helpful questions from each round in this workspace:

Name: .... Date: .... Period: ....

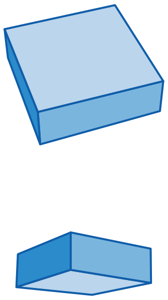
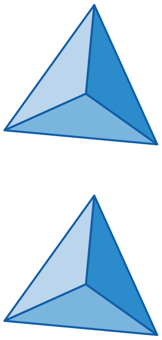
Prisms and Pyramids

2. **Polyhedra** are 3-D shapes with flat sides. **Prisms** and **pyramids** are two types of polyhedra.

How are prisms and pyramids alike? How are they different?



3. Each flat side of a polyhedron is called a **face**. Complete the table for each polyhedron.

	Figure A	Figure B
		
Number of Rectangular Faces		
Number of Triangular Faces		
Total Number of Faces		

Name: .... Date: .... Period: ....

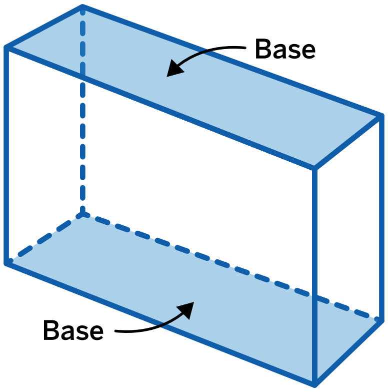
Prisms and Pyramids (continued)

4. Some of the faces of polyhedra are called **bases**.

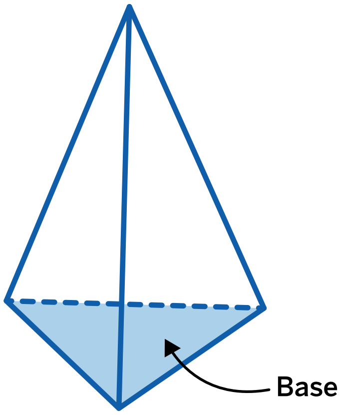
- A prism has two identical bases that are parallel.
- A pyramid has one base.

The base is what gives a polyhedron its name. For example, a prism with a triangular base is called a triangular prism.

Prism



Pyramid



Discuss: How would you name this prism and this pyramid?

Activity
2

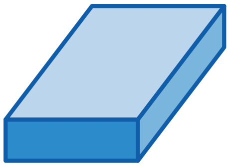
Name: .... Date: .... Period: ....

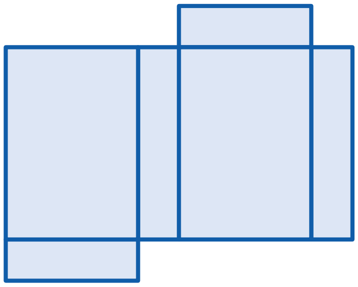
Nets of Prisms and Pyramids

5. When you unfold a polyhedron, you get a **net**.

Here is a rectangular prism and its net.

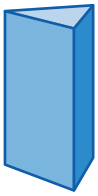
Discuss: What you notice? What do you wonder?



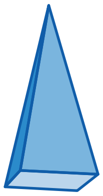


6. Which solid will this net form when it's folded? Circle one.

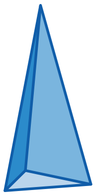
A.



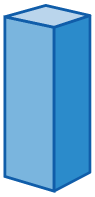
B.



C.



D.

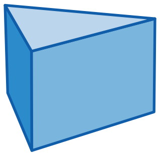


Name: Date: Period:

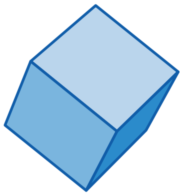
Nets of Prisms and Pyramids (continued)

7. Match each polyhedron with its net. One net will have no match.

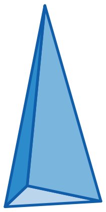
a

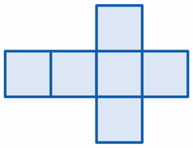


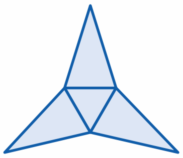
b

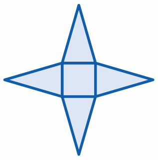


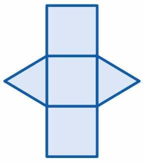
c





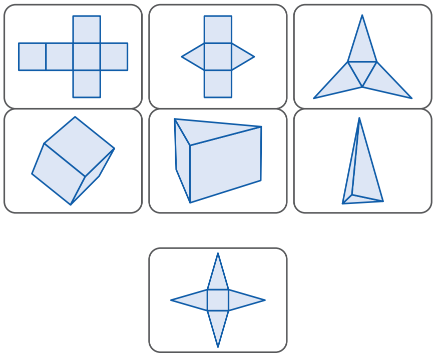






Synthesis

8. How can you decide whether a shape is a prism, a pyramid, or neither?



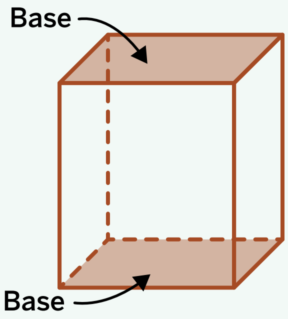
Lesson Practice 1.07

Lesson Summary

A **polyhedron** is a closed three-dimensional shape with flat sides. When we have more than one polyhedron, we call them *polyhedra*. Each flat side of a polyhedron is called a **face**, and the face that gives the polyhedron its name is called the **base**.

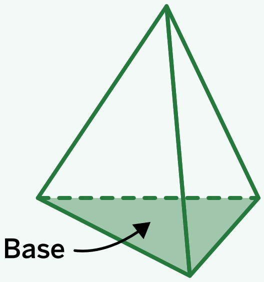
Prisms and pyramids are types of polyhedra.

A **prism** is a polyhedron that has two bases that are identical copies. The bases are connected by rectangles or parallelograms.



Prism

A **pyramid** is a polyhedron in which the base is a **polygon**, or a closed two-dimensional shape with straight sides. All the other faces are triangles that meet at a single point.



Pyramid

A **net** is a two-dimensional figure that can be folded to make a polyhedron. Nets show us what a polyhedron would look like if it was "unfolded" and allow us to see each face at the same time.

Lesson Practice

1.07

Name: .... Date: .... Period: ....

Problems 1–5: Here is a set of polyhedra.

Figure A

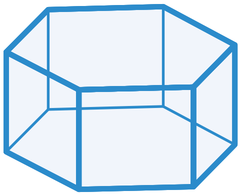


Figure B

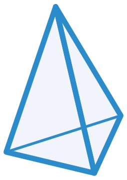


Figure C

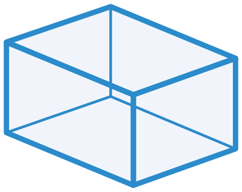


Figure D

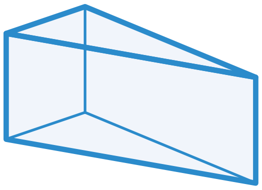
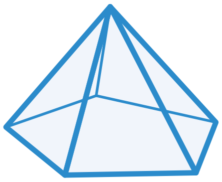


Figure E



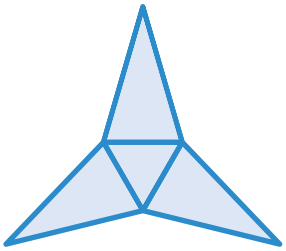
1. Which polyhedra are prisms?

2. Which polyhedra are pyramids?

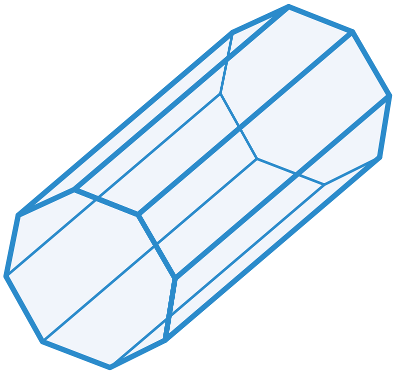
3. What type of polyhedron is figure C?

4. What type of polyhedron is figure B?

5. Which of these polyhedra could be created from this net?



6. Is this polyhedron a prism, a pyramid, or neither? Explain your thinking.

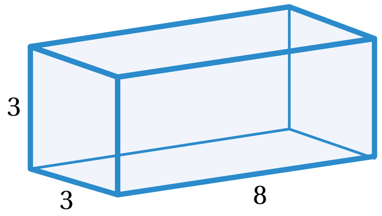


Lesson Practice
1.07

Name: .... Date: .... Period: ....

FAST Practice

7. Select *all* true statements about this polyhedron.



- ☐ **A.** The polyhedron has a total of 5 faces.
- ☐ **B.** The polyhedron has 2 square bases.
- ☐ **C.** The polyhedron has a total of 6 faces.
- ☐ **D.** The polyhedron is a square pyramid.
- ☐ **E.** The polyhedron is a square prism.

Spiral Review

Problems 8–9: Evaluate each expression. Show your thinking.

8. $5 + (4 \div 2)$

9. $(3 - 2) \cdot (4 + 1)$

Unit 1

Lesson

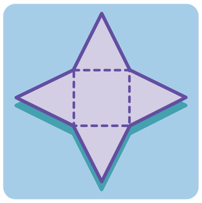
8

Name: .... Date: .... Period: ....

MA.6.GR.2.4, MTR.4.1

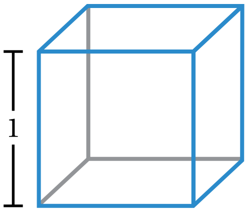
Nothing But Nets

Let's make connections between polyhedra and their nets.



Warm-Up

Here is a polyhedron.



1. What could you call this type of polyhedron?

2. Draw its net.

Activity
1

Name: .... Date: .... Period: ....

Nets and Polyhedra

3. You will use a set of cards for this activity. Match each polyhedron to its net. Record your matches in the table below and circle the name of the polyhedron.

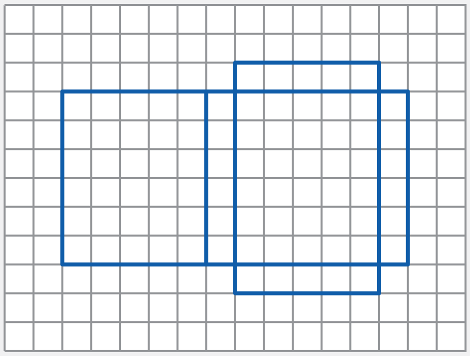
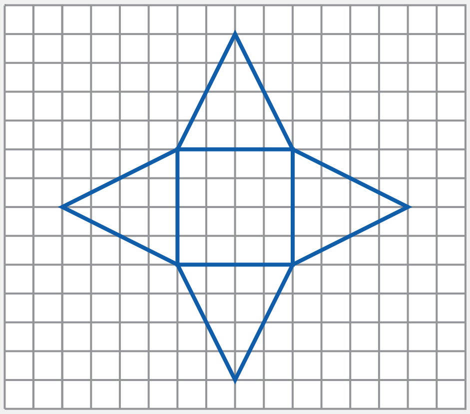
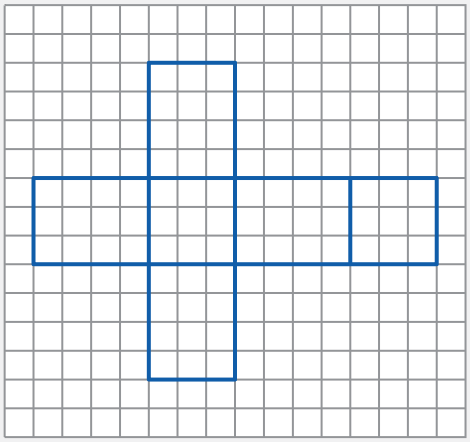
Polyhedron	Net	Name	
		Rectangular pyramid	Rectangular prism
		Rectangular pyramid	Rectangular prism
		Rectangular pyramid	Rectangular prism
		Rectangular pyramid	Rectangular prism

Activity
2

Name: Date: Period:

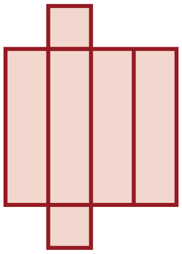
Make Polyhedra

4. You will use the Activity 2 Sheet for this activity. Take a look at the nets for polyhedra A, B, and C. Cut out, assemble, and name each polyhedron. Then calculate its surface area. Record your responses and show your thinking in the table below.

Net	Name	Surface Area
Polyhedron A 		
Polyhedron B 		
Polyhedron C 		

Synthesis

5. How can a net help you calculate surface area?

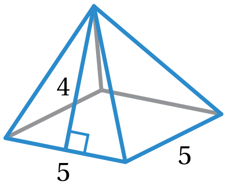
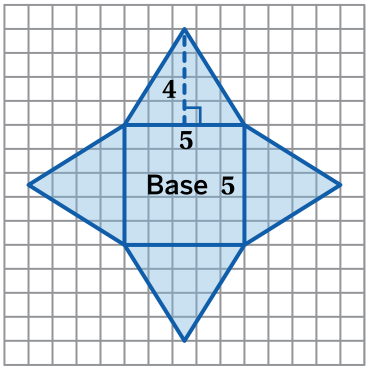
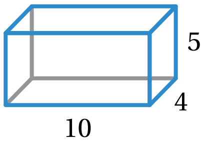
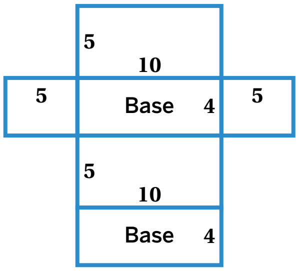


Lesson Practice 1.08

Lesson Summary

We can draw a net to create a two-dimensional representation of a three-dimensional figure. We can use the net to help us determine the surface area of a polyhedron because it shows every face at once.

Here are some examples of how a net can help us find the surface area of a pyramid or a prism.

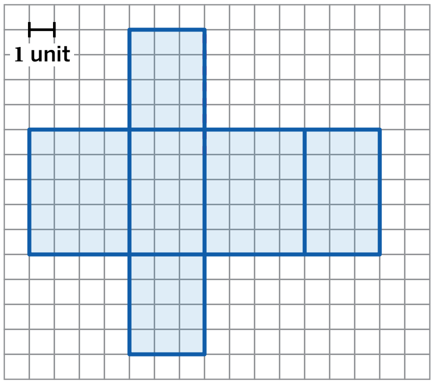
Polyhedron	Net	Surface Area
Rectangular pyramid 		$(5 \cdot 5) + 4\left(\frac{1}{2} \cdot 4 \cdot 5\right) =$ 65 square units
Rectangular prism 		$2(4 \cdot 10) + 2(4 \cdot 5) +$ $2(5 \cdot 10) = 220$ square units

Name: .... Date: .... Period: ....

1. Select *all* the units that can be used to describe surface area.

☐ **A.** Square meters
☐ **B.** Feet
☐ **C.** Centimeters
☐ **D.** Cubic inches
☐ **E.** Square inches
☐ **F.** Square feet

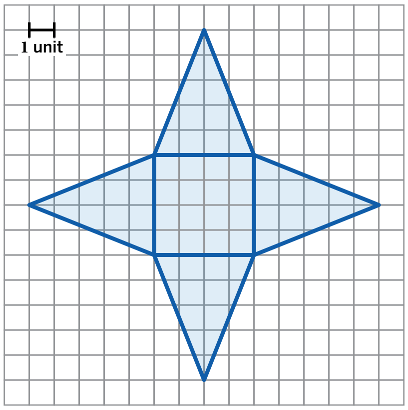
Problems 2–3: Here is a net.



2. Name the type of polyhedron that can be created from this net. Explain your thinking.

3. Determine the surface area of this polyhedron. Show or explain your thinking.

Problems 4–5: Here is a new net.



4. Name the type of polyhedron that can be created from this net. Show or explain your thinking.

5. Determine the surface area of this polyhedron. Show your thinking.

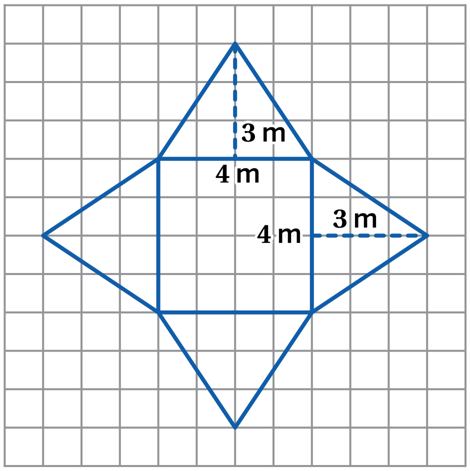
Lesson Practice

1.08

Name: ______________________ Date: __________ Period: __________

FAST Practice

6. The Pyramid Platform company would like to include a statue near the entrance of their building. A builder has designed this plan for the statue. What is the surface area of the statue?



A. 22 square meters
B. 36 square meters
C. 40 square meters
D. 64 square meters

Spiral Review

Problems 7–10: Complete each equation.

7. $3 \cdot \square = 15$

8. $4 \cdot \square = 24$

9. $15 \cdot \square = 5$

10. $24 \cdot \square = 4$

11. Take a look at the triangles in this pattern. What is the fewest number of these triangles needed to cover this pattern completely? Explain your thinking.

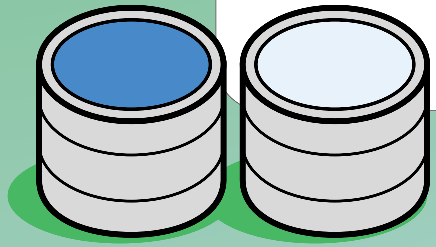
Unit 2

Introducing Ratios

You know how to use math to compare lengths, areas, and temperatures. But what if you wanted to compare price, taste, or color? In this unit, you'll use ratios to describe relationships between two quantities and compare them in real-world situations!

Essential Questions

- What does a ratio say about the relationship between quantities?
- How can ratios help you get the same taste, texture, or color every time you make a recipe?
- How can ratios help us consider issues in real-world situations?



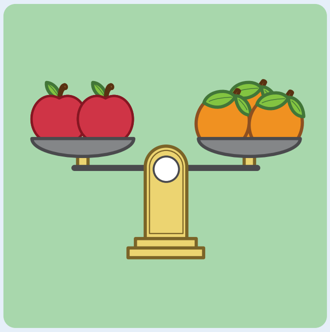
Sub-unit

1

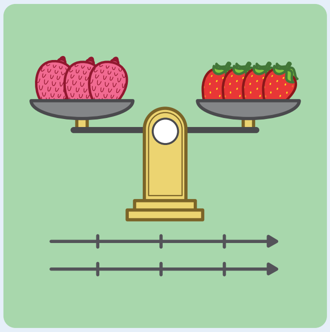
Ratios

Lesson 1
Ratio Relationships

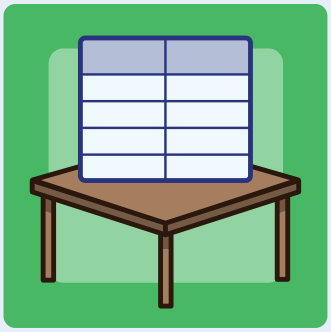
Lesson 2
Rice Ratios



Lesson 3
Fruit Lab



Lesson 4
Balancing Act



Lesson 5
Table It

Unit 2

Lesson

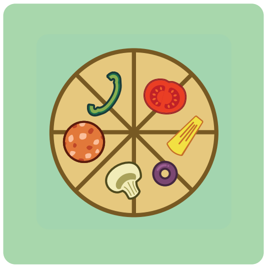
1

Name: .... Date: .... Period: ....

MA.6.AR.3.1, MTR.5.1

Ratio Relationships

Let's describe how to compare pizza toppings.



Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 15$

2. $4 \cdot 15$

3. $6 \cdot 15$

4. $12 \cdot 15$

Name: ________________ Date: ________ Period: ________

Ratio Rounds

5. You will use pizza cards to complete Rounds 1–3.

Round 1: Write down as many **ratios** as you can about your pizza card.

My Ratios

Complete as many statements as you can about your pizza card.

For every ________ ________, there are ________ ________.

For every ________ ________, there are ________ ________.

For every ________ ________, there are ________ ________.

For every ________ ________, there are ________ ________.

For every ________ ________, there are ________ ________.

For every ________ ________, there are ________ ________.

Round 2: Find a classmate whose card has a pizza that is *exactly the same* as one of your pizzas. Then write down the ratio relationship between the two toppings on each of your cards.

My Ratio

The ratio of ________ to mushroom is ____ to ____.

________'s Ratio (Classmate)

The ratio of ________ to mushroom is ____ to ____.

What is the same about your ratios?

What is different about your ratios?

Name: ______________ Date: ________ Period: ________

Ratio Rounds (continued)

Round 3: Form a group with 2–3 classmates whose cards each have the same *total number of mushrooms* as your card. Then write down the ratio relationship between the two toppings on each of your cards.

My Ratios

What is the ratio of mushrooms : ______________? ______ : ______

What is the ratio of ______________ : mushrooms? $\frac{\square}{\square}$

______________'s Ratio (Classmate)

What is the ratio of mushrooms : ______________? ______ : ______

What is the ratio of ______________ : mushrooms? $\frac{\square}{\square}$

______________'s Ratio (Classmate)

What is the ratio of mushrooms : ______________? ______ : ______

What is the ratio of ______________ : mushrooms? $\frac{\square}{\square}$

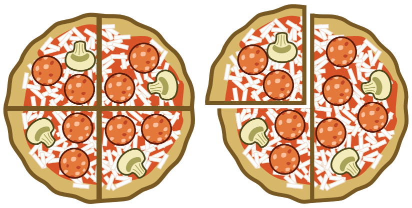
Activity
2

Name: .... Date: .... Period: ....

Two Truths and a Lie

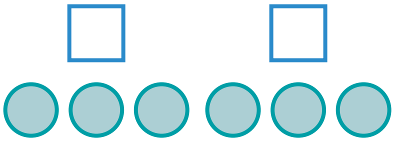
6. Which statement is false?

- **A.** The ratio of mushrooms to pepperoni is $2 : 1$.
- **B.** For every 4 mushrooms, there are 8 pepperoni.
- **C.** The ratio of pepperoni to mushrooms is 12 to 6.



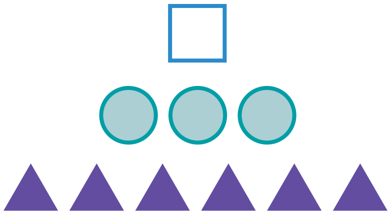
7. Which statement is false?

- **A.** The ratio of circles to squares is $1 : 3$.
- **B.** There are 2 squares for every 6 circles.
- **C.** For every square, there are 3 circles.



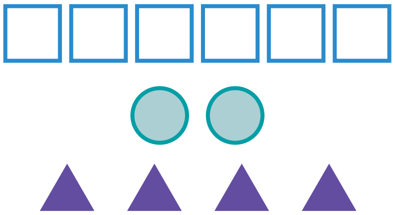
8. Which statement is false?

- **A.** For every circle, there are 2 triangles.
- **B.** The ratio of circles to squares is 3 to 1.
- **C.** The ratio of squares to triangles is $\frac{1}{2}$.



Name: .... Date: .... Period: ....

Two Truths and a Lie (continued)



9. Here is another set of shapes.

a Write three statements about these shapes: two that are true and one that is false.

b Trade your statements with a classmate. Which of his or her statements is false?

10. Now create your own challenge!

a Draw your own set of shapes.

b Write three statements about your drawing: two that are true and one that is false.

c Trade your challenge with a classmate. Which of his or her statements is false?

Synthesis

11. **a** Describe the ratio between these moons and stars in as many different ways as you can.

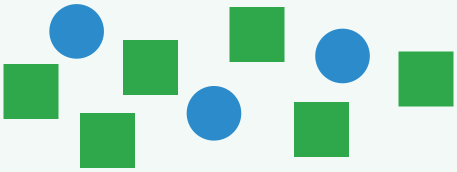
b Which way of describing a ratio is your favorite? Explain your reasoning.

Lesson Practice 2.01

Lesson Summary

A **ratio** is a relationship between two quantities. Two ways to write a ratio are $a : b$ and $\frac{a}{b}$ which means for every a of the first quantity, there are b of the second quantity.

There are many ways to describe a ratio in words.



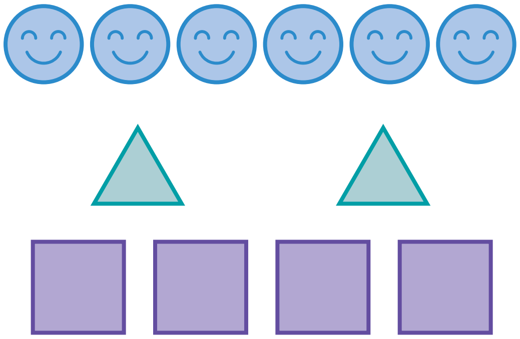
For example, here are some ways you can describe the ratio between circles and squares in this diagram.

- The ratio of circles to squares is 3 to 6.
- There are 6 squares for every 3 circles.
- There are 2 times as many squares as there are circles.
- For every 1 circle, there are 2 squares.
- The ratio of squares to circles is $\frac{6}{3}$.

Lesson Practice
2.01

Name: ______________ Date: ______ Period: ______

Problems 1–4: Here is a set of smiley faces, triangles, and squares.



1. The ratio of smiley faces to triangles is

______ to ______.

2. The ratio of squares to triangles is

______ : ______.

3. For every 2 triangles, there are ______ squares.

4. Which statement is false?

A. The ratio of smiley faces to squares is $4 : 6$.

B. The ratio of squares to triangles is $4 : 2$.

C. There are 3 smiley faces for every 1 triangle.

D. The ratio of triangles to squares is $\frac{2}{4}$.

Problems 5–8: There are 9 bananas, 4 apples, and 3 plums in a fruit basket.

5. The ratio of bananas to apples is ______ : ______.

6. The ratio of plums to apples is $\frac{\square}{\square}$.

7. For every ______ apples, there are ______ plums.

8. For every 3 bananas, there is ______ plum.

FAST Practice

9. Which **two** ratio statements are true for this situation?

For every 3 sports cars in the parking lot, there are 2 electric cars.

☐ **A.** The ratio of sports cars to electric cars is $2 : 3$.

☐ **B.** The ratio of sports cars to electric cars is $\frac{3}{2}$.

☐ **C.** The ratio of electric cars to sports cars $2 : 3$.

☐ **D.** The ratio of sports cars to the total number of cars is $\frac{3}{2}$.

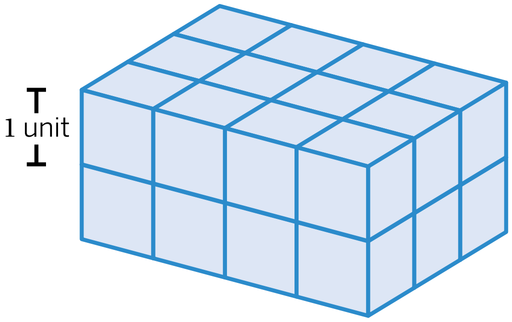
Lesson Practice

2.01

Name: ______ Date: ______ Period: ______

Spiral Review

Problems 10–11: Here is a rectangular prism.



10. Determine the volume of the prism. Show or explain your thinking.

11. Determine the surface area of the prism. Show or explain your thinking.

Unit 2

Lesson

2

Name: ________ Date: ________ Period: ________

MA.6.AR.3.1, MA.6.AR.3.5, MTR.5.1

Rice Ratios

Let’s explore ratios in recipes.

Warm-Up

Evaluate each expression mentally.

1. $4 \cdot 8$

2. $4 \cdot 10$

3. $4 \cdot 18$

4. $4 \cdot 30$

5. $4 \cdot 38$

Activity
1

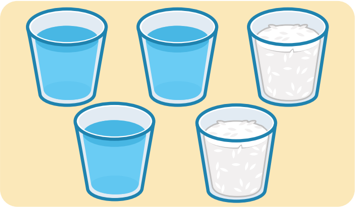
Name: .... Date: .... Period: ....

Rice Advice

6. Here are the cooking instructions for three different bags of basmati rice.

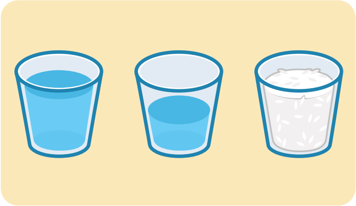
Bag A

Boil 3 cups of water for every 2 cups of rice.



Bag B

Boil $1\frac{1}{2}$ cups of water for every 1 cup of rice.



Bag C

Boil 4 cups of water for every 2 cups of rice.

a The ratios for Bag A and Bag B are called **equivalent ratios**. Why do you think they're called that?

b Marco wants to follow the directions for Bag A, but he wants to use more rice. What is another ratio of water to rice that he could use? Explain your thinking.

c The recipe for Bag A says it makes rice for 6 people. What ratio of water to rice would you use to feed 18 people?

Activity 2

Name: .... Date: .... Period: ....

Rice Around the World

Here are the recipes for four rice dishes from around the world.

7. Jamar invited a friend over for dinner. How much of each ingredient does he need to make 2 large bowls of jollof rice?

.... cups of rice

.... tablespoons of tomato paste

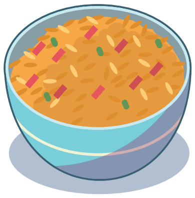
.... bell peppers

.... tomatoes

.... onions

.... cups of oil

Jollof Rice



Jollof rice is a tomato-based rice dish from Senegal, Ghana, and Nigeria.

Ingredients

Makes one large bowl

- 4 cups of rice
- 3 tablespoons of tomato paste
- 1 bell pepper
- 5 tomatoes
- 2 onions
- $\frac{1}{3}$ cup of oil

8. Nia wants to cook arroz con leche for 12 people.

a How much of each ingredient does she need?

.... cups of rice

.... cups of milk

.... cups of sugar

.... handfuls of raisins

.... cinnamon sticks

b Valeria wrote that Nia needs 9 cinnamon sticks. Why might Valeria think this?

c What advice would you give Valeria?

Arroz Con Leche

Arroz con leche is a creamy dessert from Mexico and Spain.

Ingredients

Serves 4 people

- 2 cups of rice
- 4 cups of milk
- $\frac{1}{3}$ cup of sugar
- 1 handful of raisins
- 1 cinnamon stick

Name: .... Date: .... Period: ....

Rice Around the World (continued)

9. Julian has 1 cup of sugar and wants to use all of it to make champorado.

a How much of the other ingredients does he need?

.... cups of rice

.... cups of water

.... cans of coconut milk

.... cups of cocoa powder

b How many people will Julian's champorado serve?

Champorado

Champorado is a chocolate rice porridge eaten in the Philippines.

Ingredients

Serves 4 people

- 1 cup of rice
- 4 cups of water
- 2 cans of coconut milk
- $\frac{1}{2}$ cup of cocoa powder
- 2 cups of sugar

10. Ariana says this recipe makes too much risotto.

a How much of each ingredient could she use to make a smaller amount of risotto?

.... cups of rice

.... cups of chicken broth

.... tablespoons of olive oil

.... tablespoons of butter

.... ounces of Parmesan cheese

b How many people will this serve?

Risotto

Risotto is an Italian rice dish that uses broth to create a creamy texture.

Ingredients

Serves 8 people

- 3 cups of rice
- 10 cups of chicken broth
- 4 tablespoons of olive oil
- 2 tablespoons of butter
- 8 ounces of Parmesan cheese

Synthesis

11. The cooking instructions on Bag A and Bag B call for equivalent ratios of water to rice.

Bag A

Boil 4 cups of water for every 2 cups of rice.

Bag B

Boil 2 cups of water for every 1 cup of rice.

a Explain what equivalent ratios are in your own words.

b Create a new ratio of water to rice that is equivalent to the ratios for Bag A and Bag B.

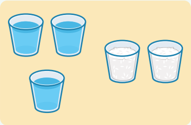
Lesson Practice 2.02

Lesson Summary

Recipes can help us understand **equivalent ratios**. Each recipe calls for a specific ratio of ingredients, but you can halve, double, or triple the ratio to make different amounts of the same recipe.

Original Recipe

Boil 3 cups of water
for every 2 cups
of rice.



3 to 2

Halved Recipe

$1\frac{1}{2}$ to 1

Doubled Recipe

6 to 4

Tripled Recipe

9 to 6

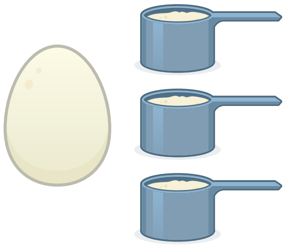
These ratios are equivalent because they all represent the same recipe. You can multiply or divide each of the values in the first ratio by the same number to get the values in each of the other ratios.

Lesson Practice

2.02

Name: .... Date: .... Period: ....

Problems 1–2: There are many recipes for pasta. Some suggest a ratio of 1 egg for every 3 ounces of flour.

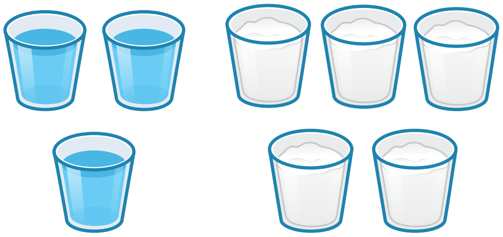


1. Draw a picture that shows how many ounces of flour you would need for 2 eggs. Then write the ratio of eggs to flour that represents your drawing.

2. Complete the table to create equivalent ratios.

Eggs	Flour (oz)
4	
	15

3. A bakery uses a ratio of 3 cups of water for every 5 cups of flour to bake bread. List 2 other ratios of water to flour they could use to bake the same bread.



Problems 4–7: Koharu's pie dough recipe uses 6 ounces of flour, 4 ounces of butter, and 2 ounces of water. Complete the sentences to describe the ratios in her recipe.

4. For every 2 ounces of ...., there are 6 ounces of ....

5. The ratio of .... to .... is 6 : 2.

6. The ratio of .... to .... is 2 : 3.

7. The ratio of .... to .... is 3 : 2.

Lesson Practice
2.02

Name: .... Date: .... Period: ....

FAST Practice

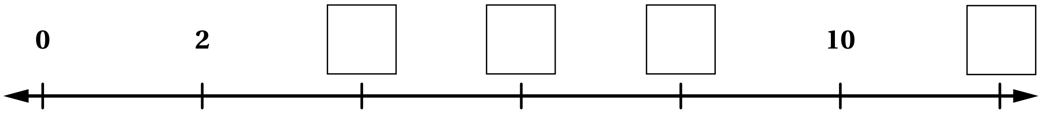
8. Koharu made a new batch of pie dough with 3 ounces of flour, 2 ounces of butter, and 1 ounce of water. Will her pie dough taste the same as the original recipe? Explain your thinking.

A. Yes. **B.** No.
The ratios of all the ingredients **A.** are **B.** are not
equivalent to the original recipe. Koharu used half the amount of
A. all **B.** some ingredients.

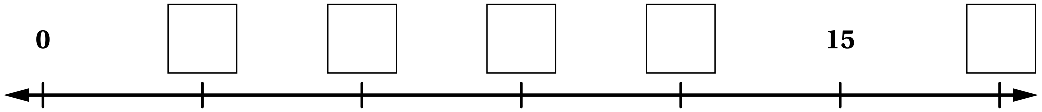
Spiral Review

Problems 9–10: Fill in the blanks on each number line.

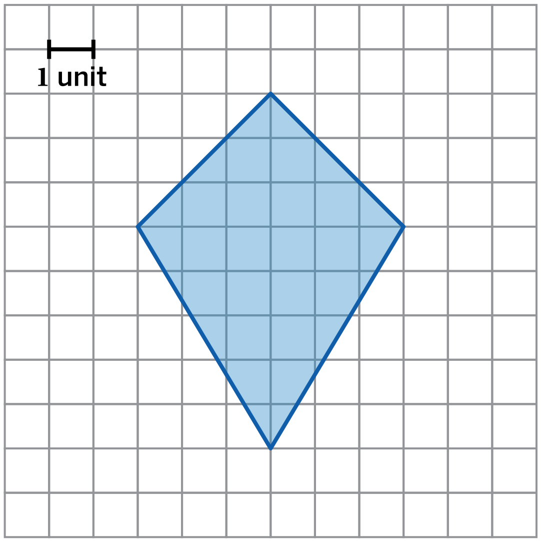
9.



10.



11. Determine the area of this polygon. Show or explain your thinking.



Unit 2

Lesson

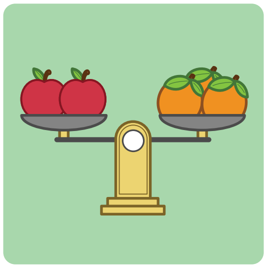
3

Name: ______________________ Date: __________ Period: __________

MA.6.AR.3.3, MTR.1.1, MTR.4.1, MTR.5.1

Fruit Lab

Let’s investigate equivalent ratios by balancing fruit on scales.

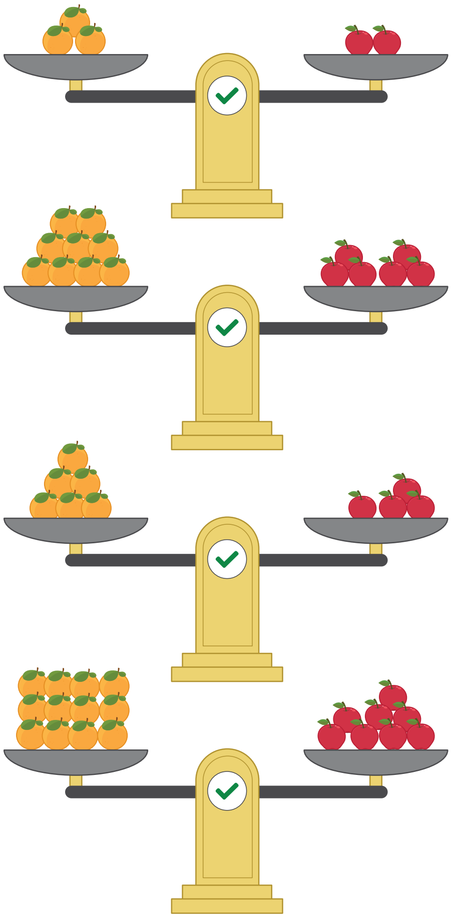


Warm-Up

1. Observe the balanced scales with apples and oranges.

Record the values in the **table**.

Number of Oranges	Number of Apples



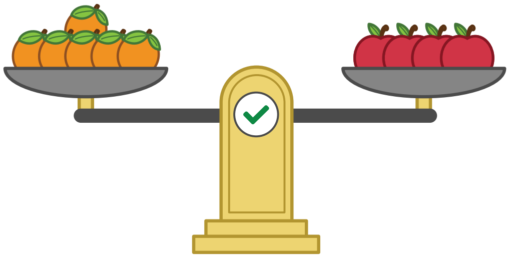
Name: .... Date: .... Period: ....

Apples to Oranges

2. Here is a table Victor created after using the scale to balance some apples and oranges. What do you notice about the table? What do you wonder?

I notice:

Number of Oranges	Number of Apples
15	10
3	2
6	4



I wonder:

3. Write another equivalent ratio in the last row. Try to find one that you think no one else will think of.

Activity
2

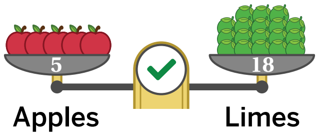
Name: ....

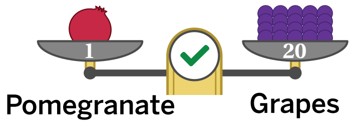
Date: ....

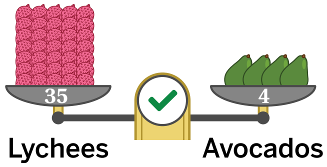
Period: ....

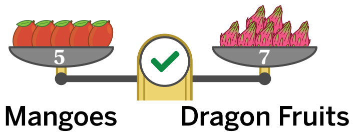
Fruit Lab

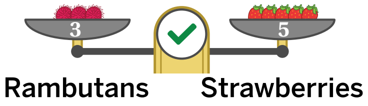
4. You will use the Activity 2 Sheet to complete this activity. Choose a pair of fruits to see how they balance. Then record several equivalent ratios for that pair of fruits. Repeat with different combinations of fruits.

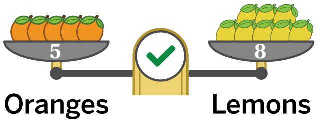


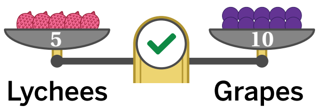


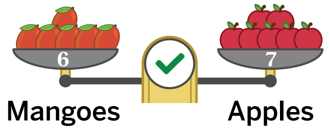


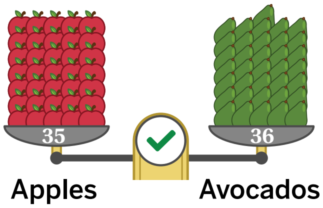


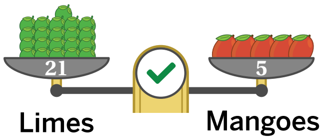


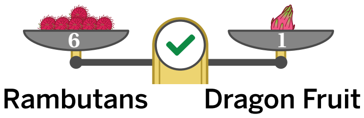


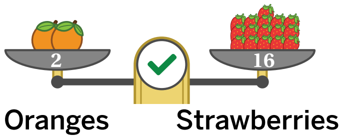








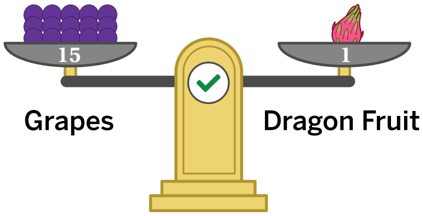




Name: .... Date: .... Period: ....

Fruit Lab (continued)

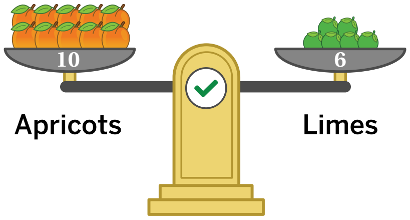
5. Ella knows that 15 grapes balance with 1 dragon fruit. She says 16 grapes will balance with 2 dragon fruits. Will this 16 : 2 ratio balance the scale? Circle one.



Yes No I'm not sure

Explain your thinking.

6. The scale balances with a ratio of 10 apricots to 6 limes. Select *all* of the equivalent ratios.



- ☐ **A.** 20 apricots to 16 limes
- ☐ **B.** 50 apricots to 30 limes
- ☐ **C.** 7 apricots to 3 limes
- ☐ **D.** 5 apricots to 3 limes
- ☐ **E.** 11 apricots to 7 limes

7. The table shows some ratios of limes to lychees that balance the scale. Dyani says that 22 limes will balance with 55 lychees. Will the 22 : 55 ratio balance? Circle one.

Number of Limes	Number of Lychees
2	5
20	50

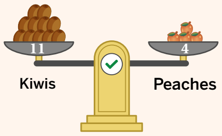
Yes No I'm not sure

Explain your thinking.

You're invited to explore more.

8. A ratio of 11 kiwis : 4 peaches balances. So does a ratio of 15 pears : 6 peaches.

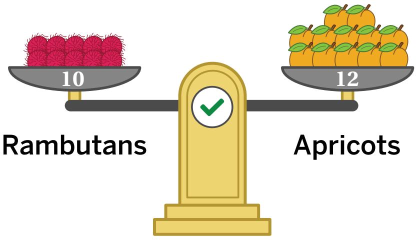
Write a ratio of kiwis to pears that would balance. Explain your thinking.



Synthesis

9. When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

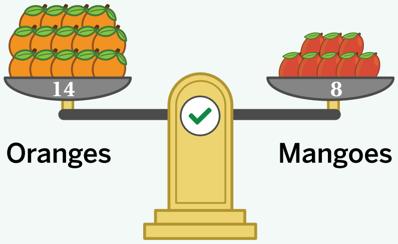
Use the example if it helps with your thinking.



Lesson Practice 2.03

Lesson Summary

We can use balance scales to help us understand equivalent ratios. When both quantities in a ratio are multiplied or divided by the same amount, the ratio relationship remains the same, and the scale stays balanced.



For example, the ratio of oranges to mangoes on this scale is $14 : 8$.

You can create an equivalent ratio of $7 : 4$ by dividing the number of each fruit by 2. This means that 7 oranges and 4 mangoes will also balance on the scale. 21 oranges to 12 mangoes would also be an equivalent ratio because you can get those values by multiplying 14 and 8 by $\frac{3}{2}$.

You can use a **table** to organize and keep track of equivalent values. Tables organize information into horizontal rows and vertical columns. The first row or column usually tells us what the numbers represent.

Here is a table that represents the different numbers of oranges and mangoes needed to balance the scale.

Number of Oranges	Number of Mangoes
14	8
7	4
21	12

Lesson Practice
2.03

Name: .... Date: .... Period: ....

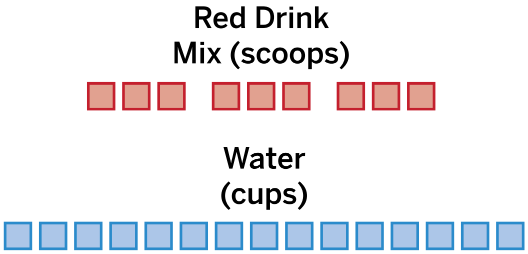
Problems 1–3: A package of red drink mix says to combine 3 scoops of red drink mix and 5 cups of water.

1. Complete the table with several ratios of red drink mix to water that are equivalent to the package instructions.

Red Drink Mix (scoops)	Water (cups)
3	5

2. Choose one of your ratios and explain how you know it's equivalent. Draw a diagram if it helps with your thinking.

3. Jaylin drew this diagram for one of her ratios. Will this mix taste the same as the original? Show or explain your thinking.



4. Select *all* of the ratios that are equivalent to 4 : 5.

☐ **A.** 3 : 4

☐ **B.** 8 : 10

☐ **C.** 1 : 2.5

☐ **D.** 9 : 10

☐ **E.** 20 : 25

5. Write a different ratio that is equivalent to 4 : 5.

Lesson Practice
2.03

Name: .... Date: .... Period: ....

Problems 6–7: You can make a certain color of green paint by mixing 10 ounces of green paint with 2 gallons of white paint.

6. Draw a diagram to represent this ratio.

FAST Practice

7. Select *all* the true statements.

☐ **A.** For every 5 ounces of green paint, you need 1 gallon of white paint.

☐ **B.** The ratio of green paint to white paint is $1 : 5$.

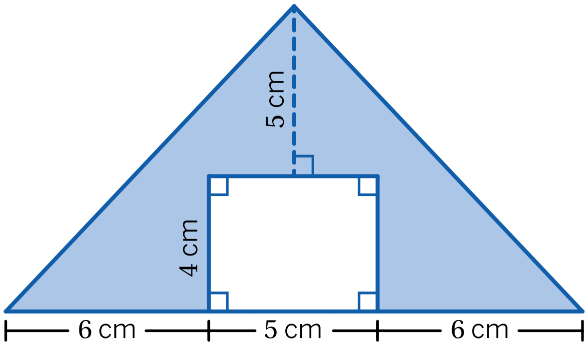
☐ **C.** For every gallon of white paint, you need 5 ounces of green paint.

☐ **D.** For every ounce of green paint, you need 5 gallons of white paint.

☐ **E.** The ratio of white paint to green paint is $10 : 2$.

Spiral Review

1. Determine the area of the shaded region. Explain your thinking.



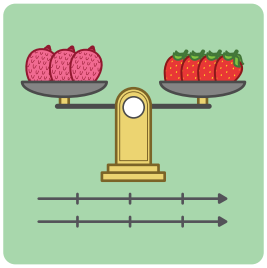
Unit 2
Lesson
4

Name: .... Date: .... Period: ....

MA.6.AR.3.3, MA.6.AR.3.5, MTR.4.1

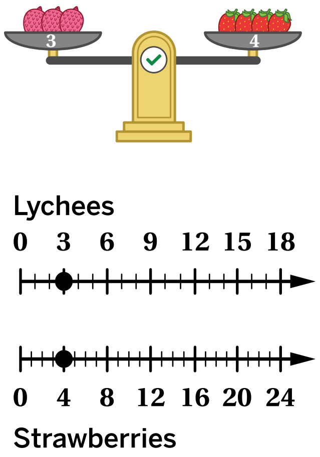
Balancing Act

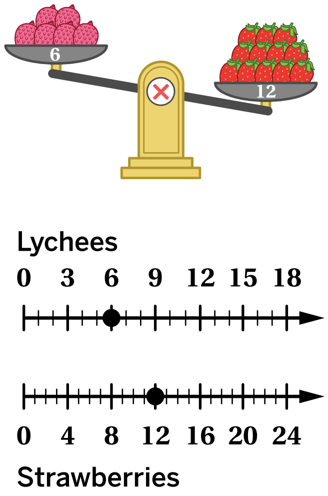
Let’s use double number lines to represent equivalent ratios.

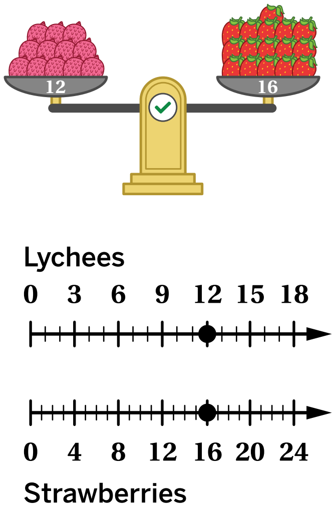


Warm-Up

1. Here are some **double number lines** that represent lychees and strawberries on a scale. What do you notice? What do you wonder?







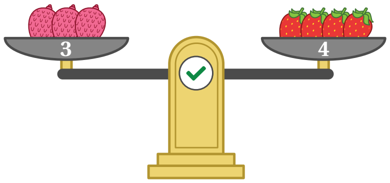
I notice . . .

I wonder . . .

Name: .... Date: .... Period: ....

Double Number Lines

2. This scale balances with a ratio of 3 lychees to 4 strawberries.



a Record several equivalent ratios in the table. Try to find ones that none of your classmates will.

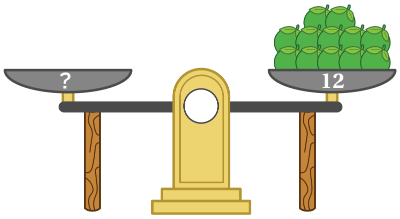
Number of Lychees	Number of Strawberries
3	4

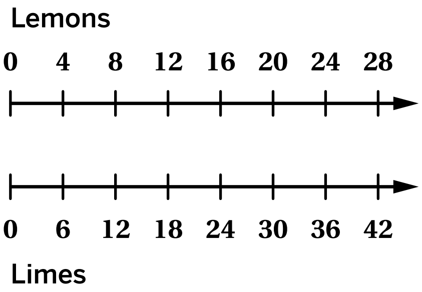
b 💬 **Discuss:** How can you use the double number line to determine the ratios that balance?

3. This scale balances with a ratio of 4 lemons to 6 limes.

How many lemons will balance with 12 limes?

Use the double number line if it helps to show your thinking.

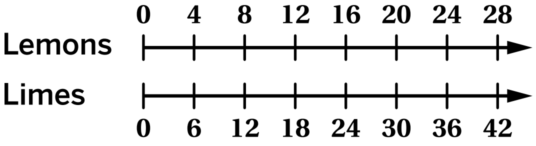




Name: .... Date: .... Period: ....

Double Number Lines (continued)

4. Complete the table. Use the double number line if it helps to show your thinking.



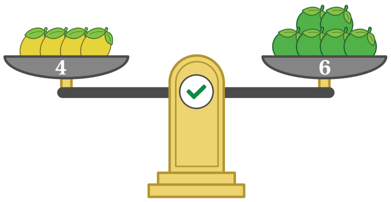
Number of Lemons	Number of Limes
4	6
20	
	42
32	

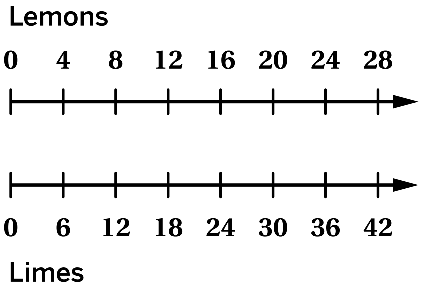
5. Kiri says that 16 lemons will balance with 24 limes. Lola says that 36 lemons will balance with 24 limes.

Whose thinking is correct? Circle one.

Kiri's Lola's Both Neither

Explain your thinking.





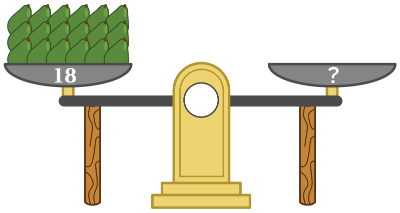
Activity
2

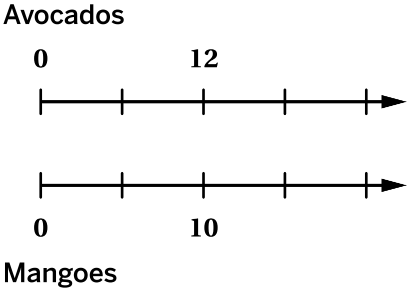
Name: .... Date: .... Period: ....

More Double Number Lines

6. A ratio of 12 avocados : 10 mangoes balances this scale. How many mangoes will balance with 18 avocados?

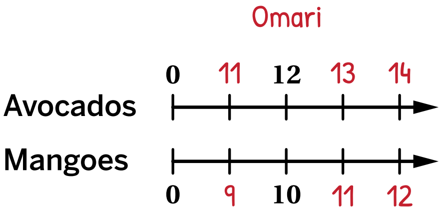
Use the double number line if it helps to show your thinking.

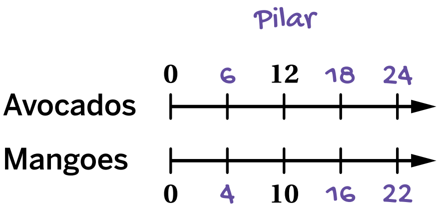




7. Omari and Pilar both labeled the rest of the diagram from Problem 6. They each made a mistake.

a Circle the person who made your favorite mistake.





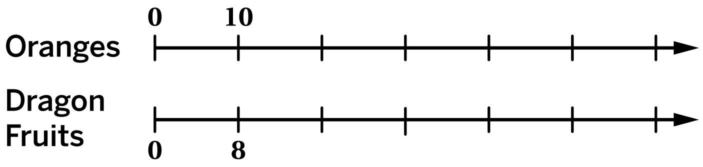
b What is something that person did well? What would you recommend this person change about work shown?

Name: .... Date: .... Period: ....

More Double Number Lines (continued)

8. 10 oranges balance with 8 dragon fruits.

Complete the table. Use the double number line if it helps to show your thinking.



Number of Oranges	Number of Dragon Fruits
10	8
	24
5	
	20

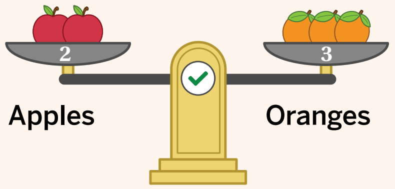
You're invited to explore more.

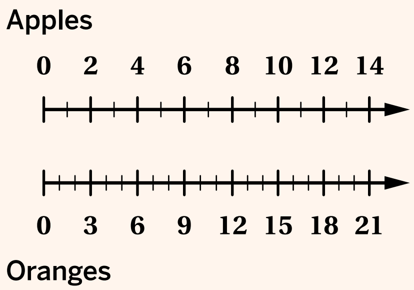
9. 2 apples balance with 3 oranges.

a. How many oranges will balance with 101 apples?

b. How many apples will balance with $25\frac{1}{2}$ oranges?

c. Create your own problem involving equivalent ratios of apples and oranges. Then trade problems with a classmate and solve them.

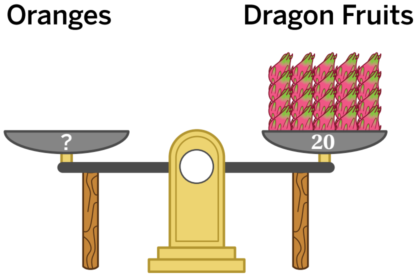


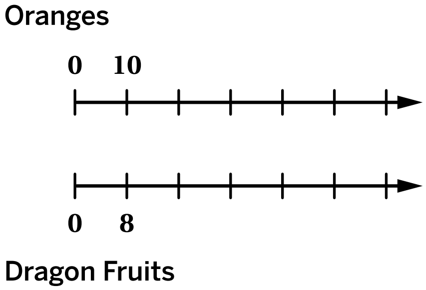


Synthesis

10. How can you use a double number line to solve problems with ratios?

Use the example if it helps to show your thinking.



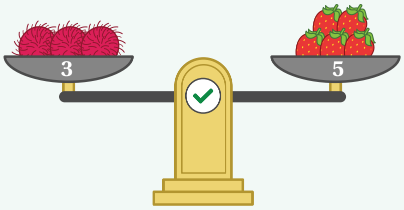


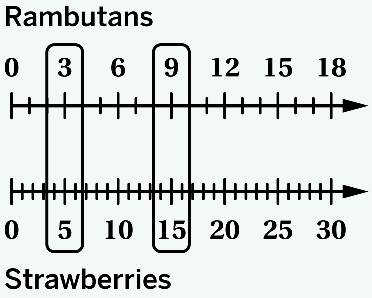
Lesson Practice 2.04

Lesson Summary

A **double number line** is another way to represent equivalent ratios. Each double number line is made up of a pair of parallel number lines. The tick marks are labeled so that the numbers that line up vertically make equivalent ratios.

For example, if a ratio of 3 rambutans to 5 strawberries will balance on a scale, you can use a double number line to determine how many strawberries will balance with 9 rambutans.





To represent a ratio of 3 : 5, you begin with 3 and 5 in the same location on each number line and then count up by units of 3 on one number line and units of 5 on the other line. In this example, you can determine that it will take 15 strawberries to balance 9 rambutans. Each pair of matching values represents an equivalent ratio to 3 : 5.

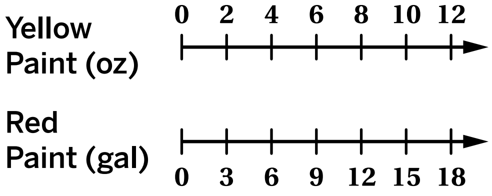
Lesson Practice

2.04

Name: .... Date: .... Period: ....

1. You can make a certain orange paint by mixing 2 ounces of yellow paint with 3 gallons of red paint.

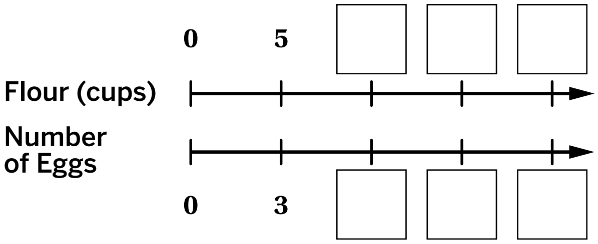
List two other combinations of yellow paint and red paint that can create this shade of orange.



Yellow Paint (oz)	Red Paint (gal)

Problems 2–5: This double number line represents the amount of flour and eggs needed for a cookie recipe.

2. Complete the double number line.



3. What is the ratio of cups of flour to number of eggs?

4. How much flour do you need for 12 eggs?

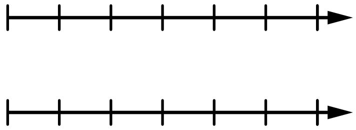
5. How many eggs do you need for 15 cups of flour?

Lesson Practice
2.04

Name: ______________ Date: ________ Period: ________

Problems 6–8: Metropolis Elementary recommends 2 adults join for every 15 students on a field trip.

6. Draw a double number line to represent this situation.



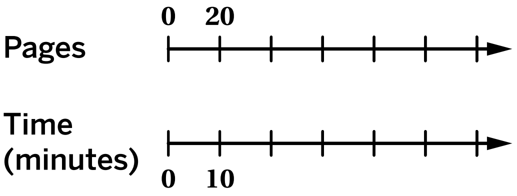
7. How many adults need to go on a field trip with 75 students?

8. How many adults need to go on a trip with 50 students? Explain your thinking.

FAST Practice

9. Neo can read 20 pages in 10 minutes. Use the double number line to determine how many pages he can read in 1 hour.

☐ pages



Spiral Review

10. Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select *all* the statements that represent this ratio.

☐ **A.** The ratio of kids to pizzas is 7 : 3.

☐ **B.** The ratio of pizzas to kids is 3 : 7.

☐ **C.** The ratio of kids to pizzas is 3 : 7.

☐ **D.** The ratio of pizzas to kids is 7 : 3.

☐ **E.** For every 7 kids, there needs to be 3 pizzas.

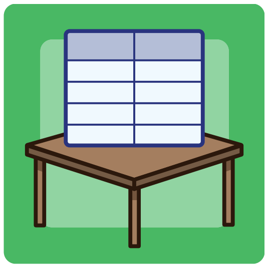
Unit 2
Lesson
5

Name: Date: Period:

MA.6.AR.3.3, MTR.4.1, MTR.5.1, MTR.7.1

Table It

Let’s use tables to display part-to-part-to-whole ratios.



Warm-Up

1. The table shows the relationship between the number of questions answered correctly, x, and the number of points scored, y, during a trivia game.

x	y
1	$\frac{1}{2}$
2	1
4	2

What do you notice about the relationship between x and y? Discuss your thinking with a partner.

Name: .... Date: .... Period: ....

Snack Time

2. The table shows a part-to-part relationship between the amount of pretzels and the amount of dried fruit to make in a snack mix recipe.

Recipe A

Cups of Pretzels	Cups of Dried Fruit
5	1

Create three different snack mix recipes that are equivalent to Recipe A.

Recipe B:

Recipe C:

Recipe D:

3. The table shows a part-to-part-to-whole ratio relationship.

a Record your recipes in the table and complete the additional information.

Recipe	Cups of Pretzels	Cups of Dried Fruit	Total Cups of Snack Mix
A	5	1	6
B			
C			
D			

b How did you find the total cups of snack mix for each recipe?

c 💬 **Discuss:** What do part-to-part-to-whole ratio tables include? What patterns do they have?

Name: ____________________ Date: __________ Period: __________

Put It in a Table

4. Lucia made an orange seltzer drink for a party. She mixed different amounts of seltzer and orange juice to make enough orange seltzer for everyone who attended. Lucia mixed 15 liters of juice with 12 liters of seltzer to make 27 liters of the orange seltzer drink.

a Complete the rest of the table.

Liters of Juice			15
Liters of Seltzer	4		12
Total Liters of Orange Seltzer		18	27

b Explain the strategies you used to determine the missing values.

Name: .... Date: .... Period: ....

Put It in a Table (continued)

5. Anand mixed red paint and blue paint together to make his favorite shade of purple. The relationship between the parts of red and blue paint that he used to make his favorite color is shown in the table below.

a Complete the table.

Parts of Red Paint	2.5	5		
Parts of Blue Paint	6	12	24	
Mixture of New Paint				51

b Oliver said that Anand can mix 17.5 parts of red paint with 42 parts of blue paint to make 59.5 parts of his favorite color. Explain why you think Oliver is correct or incorrect.

c Katie said that Anand can mix 20 parts of red paint with 27 parts of blue paint to get 47 parts of his favorite color. Explain why you think Katie is correct or incorrect.

Synthesis

6. A restaurant is preparing fruit punch for a birthday party. The recipe uses 3 cups of orange juice and 2 cups of pineapple juice to make 5 cups of fruit juice. How can you use a part-to-part-to-whole ratio table to determine the amount of each juice needed for different amounts of fruit punch?

Use the table if it helps with your thinking.

Cups of Orange Juice		
Cups of Pineapple Juice		
Total Cups of Fruit Punch		

Lesson Practice 2.05

Lesson Summary

A table can display equivalent part-to-part-to-whole ratios. You can use these tables to solve problems and make comparisons between quantities.

For example, the table shows the relationship between pink tulips and yellow tulips in flower arrangements. The ratio of pink tulips to yellow tulips to total tulips is $4 : 6 : 10$. An equivalent ratio is $8 : 12 : 20$. Each flower is a part, and the whole is the flower arrangement. You can create the equivalent ratio by multiplying the number of flowers in each category by 2.

Pink Tulips	4	8
Yellow Tulips	6	12
Flower Arrangement	10	20

You can apply what you know about equivalent ratios to extend the table and answer questions about the relationship.

Lesson Practice

2.05

Name: .... Date: .... Period: ....

Problems 1–3: For every 7 laps that a puppy completes in a pen, a rabbit completes 4 laps.

1. Complete the table with equivalent ratios.

Puppy Laps	Rabbit Laps	Total Laps
7	4	11

2. Choose one of your sets of equivalent ratios and explain how you know it's equivalent.

3. If the rabbit completes 16 laps and the total number of laps is 44, what are two strategies you can use to determine the number of laps the puppy completes?

Problems 4–7: A fruit salad recipe calls for $\frac{3}{4}$ cup of blueberries for every 1 cup of strawberries.

Cups of Strawberries	1	5		
Cups of Blueberries	$\frac{3}{4}$		6	$7\frac{1}{2}$
Cups of Fruit Salad				$17\frac{1}{2}$

4. Complete the table for different amounts of fruit salad.

5. Describe one strategy you used to complete the table.

Lesson Practice
2.05

Name: .... Date: .... Period: ....

6. Describe any patterns you notice in the table.

7. Binta uses 2 cups of strawberries to make 4 cups of fruit salad. Does Binta's fruit salad use the same recipe? Show or explain your thinking.

FAST Practice

8. Which statement about the table is true?

- **A.** For every 1.5 parts of black paint, you make 4 parts of new paint.
- **B.** For every 6 parts of white paint, you need 2.25 parts of black paint.
- **C.** For every 16.5 parts of new paint, you need 3.5 parts of black paint.
- **D.** For every 6 parts of white paint, you make 7.5 parts of new paint.

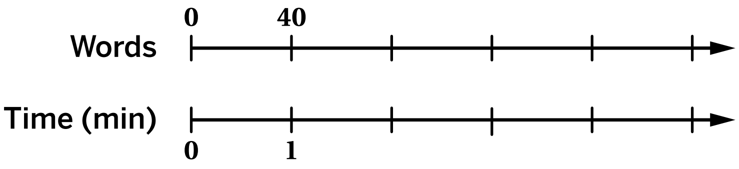
Parts of Black Paint	1.5		
Parts of White Paint	4	6	12
New Paint Mixture			16.5

Spiral Review

9. Which ratio is equivalent to 3 : 4?

- **A.** 3 : 7
- **B.** 4 : 3
- **C.** 9 : 12
- **D.** 30 : 44

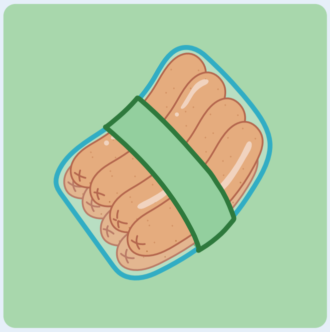
10. Tiam can type 40 words in 1 minute. Use the double number line to determine how many words Tiam can type in 5 minutes.



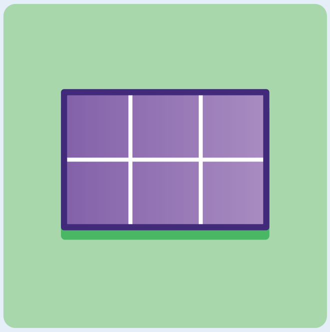
Sub-unit

2

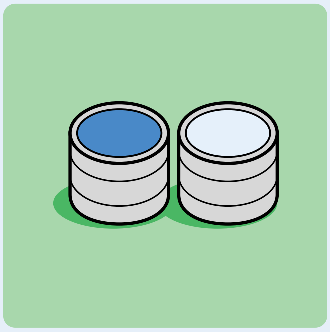
Common Factors and Multiples



Lesson 6
Common Multiples



Lesson 7
Common Factors

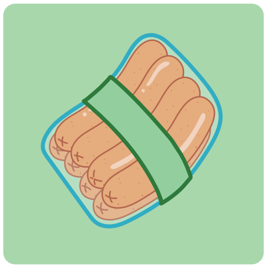


Lesson 8
Mixing Paint

Name: .... Date: .... Period: ....

MA.6.NSO.3.1, MTR.1.1

Common Multiples



Let's learn more about multiples.

Warm-Up

1. Abdel is grilling tofu dogs for his friends. His favorite tofu dogs come in packs of 8. His favorite buns come in packs of 6.

   What advice would you give to Abdel on how many packs to purchase?

2. Abdel bought 48 tofu dogs and 48 buns.

   What else could he buy if he only bought whole packs? Select *all* that apply.

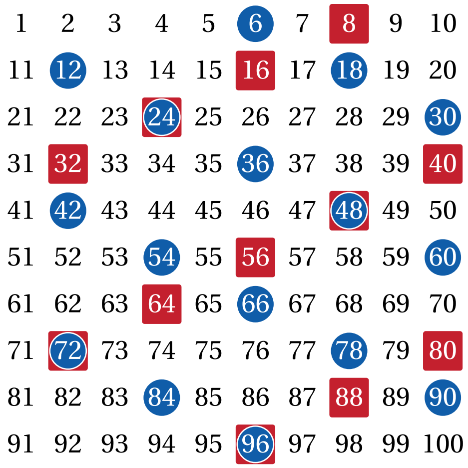
   ☐ **A.** 2 tofu dogs and 2 buns
   ☐ **B.** 12 tofu dogs and 12 buns
   ☐ **C.** 24 tofu dogs and 24 buns
   ☐ **D.** 32 tofu dogs and 32 buns
   ☐ **E.** 72 tofu dogs and 72 buns

Activity
1

Name: .... Date: .... Period: ....

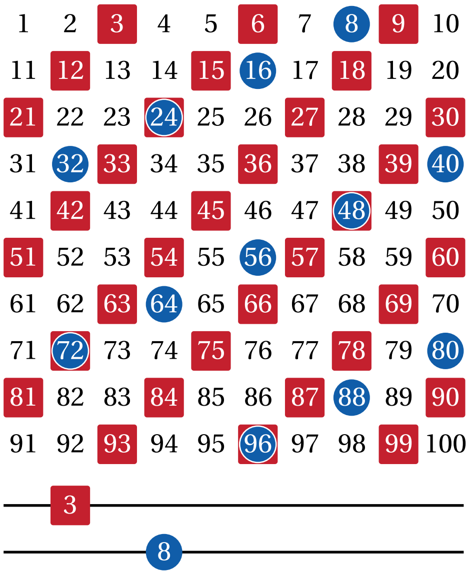
Least Common Multiples

3. 24 and 48 are **common multiples** of 8 and 6.

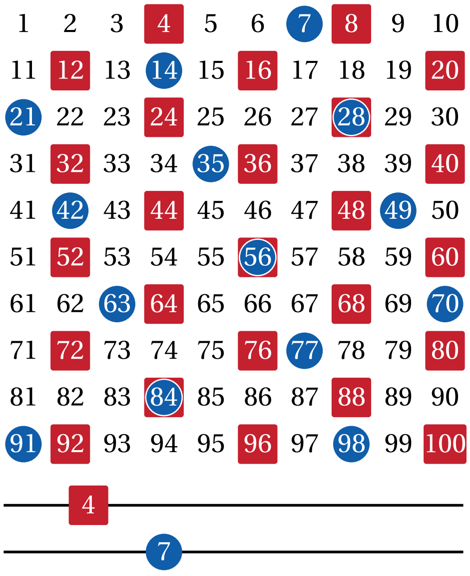


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

a Take a look at these other common multiples.

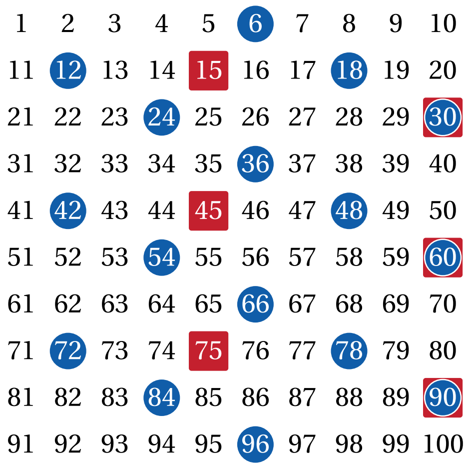


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b List some common multiples of 6 and 15.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

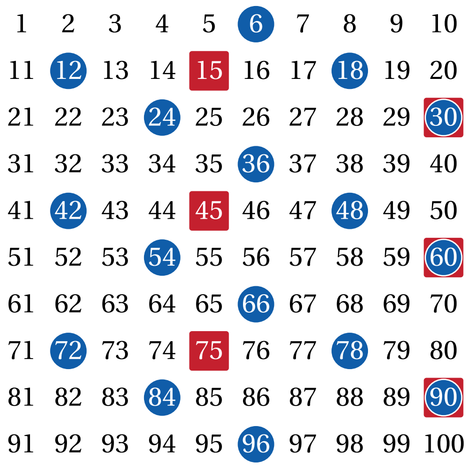
Activity
1

Name: ______________________ Date: __________ Period: __________

Least Common Multiples (continued)

4. The **least common multiple (LCM)** is the smallest number that is a common multiple of two numbers.

What is the least common multiple of 6 and 15?



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. **a** What is the least common multiple of 8 and 10?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b Describe how you could use the grid to determine the least common multiple of 8 and 10.

Activity
2

Name: .... Date: .... Period: ....

More Multiples

6. Determine the least common multiple for each pair of numbers in the table. Use the grid if it helps with your thinking.

Numbers	Least Common Multiple
10 and 15	
10 and 4	
6 and 4	
6 and 7	
21 and 7	
9 and 12	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

7. Abdel wants to get some dessert to go with his tofu dogs. He wants to buy an equal number of FreezyPops and DinoPops.

- FreezyPops come in packs of 9.
- DinoPops come in packs of 12.

a What is the least number of packages he will need to buy to have an equal number of each dessert?

FreezyPops: .... packages

DinoPops: .... packages

b **Discuss:** How did the LCM of 9 and 12 help you calculate the number of packages Abdel needs to buy?

Name: .... Date: .... Period: ....

More Multiples (continued)

8. **a** Select a pair of numbers whose least common multiple is *not* 24. Use the grid if it helps with your thinking.

☐ **A.** 4 and 6

☐ **B.** 6 and 8

☐ **C.** 6 and 18

☐ **D.** 12 and 24

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b What is the least common multiple of the numbers you chose?

9. *To find the least common multiple of two numbers, just multiply them together.*

Is this statement always, sometimes, or never true? Circle one.

Always Sometimes Never

Explain your thinking.

You're invited to explore more.

10. What are two numbers that have a least common multiple of 100?

List as many pairs as you can.

Synthesis

11. Describe a strategy for determining the least common multiple of two numbers, such as 8 and 12.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Lesson Practice 2.06

Lesson Summary

A **common multiple** of two numbers is a number that is a multiple of both numbers. Here's a chart that shows some multiples of 2 (marked with squares) and some multiples of 3 (marked with circles). We can see that some of the common multiples of 2 and 3 are 6, 12, and 18.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

The **least common multiple (LCM)** is the smallest number that is a common multiple of two numbers. In the example of 2 and 3, the LCM is 6. It's helpful to determine the LCM when solving problems like how many packages of tofu dogs and buns you need to buy or how often two trains stop at the same station. For example, if Train A stops at the station every 2 hours, and Train B stops at the station every 3 hours, then both trains will be at the station every 6 hours.

Lesson Practice

2.06

Name: .... Date: .... Period: ....

Problems 1–3: Here is a grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Circle the multiples of 12.

2. Draw a square around the multiples of 15.

3. What is the least common multiple of 12 and 15?

Problems 4–5: Lucia wants to buy the same number of cups and plates, but cups are sold in packs of 8 and plates are sold in packs of 12.

4. List *at least* three combinations of cups and plates that Lucia could buy so she has the same number of each.

5. What is the fewest number of cups she could buy while also buying an equal number of plates?

6. A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

Lesson Practice

2.06

Name: .... Date: .... Period: ....

FAST Practice

7. A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights blink at the same time?

Select *all* the correct values.

- ☐ **A.** Every 36 seconds
- ☐ **B.** Every 48 seconds
- ☐ **C.** Every 72 seconds
- ☐ **D.** Every 90 seconds
- ☐ **E.** Every 108 seconds

Spiral Review

8. 10 bananas cost $\$4.50$. What is the price for one banana?

9. If 20 cookies cost $\$5$, what is the price for 25 cookies?

10. 4.5 pounds of potatoes cost $\$12$. What is the price for 3 pounds of potatoes?

Unit 2
Lesson
7

Name: ______ Date: ______ Period: ______

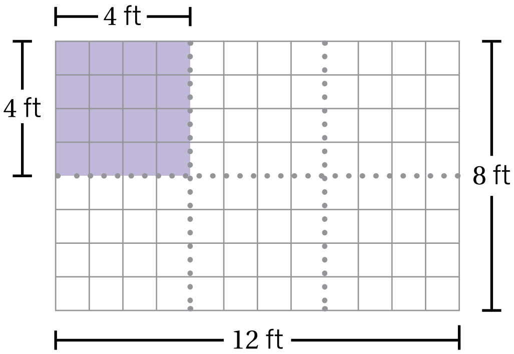
MA.6.NSO.3.1, MTR.4.1

Common Factors

Let's explore factors.

Warm-Up

1. A 4-by-4 foot square will tile the floor of this 8-by-12 foot room.



a Take a look at the diagram to see what we mean.

b Find other square sizes that tile the room.

c **Discuss:** What does it mean to *tile*?

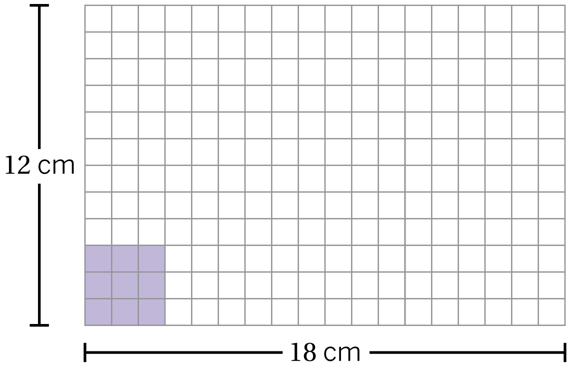
Name: .... Date: .... Period: ....

Greatest Common Factors

2. A 3-by-3 centimeter square will tile this 12-by-18 centimeter rectangle. This means that 3 is a **common factor** of 12 and 18.

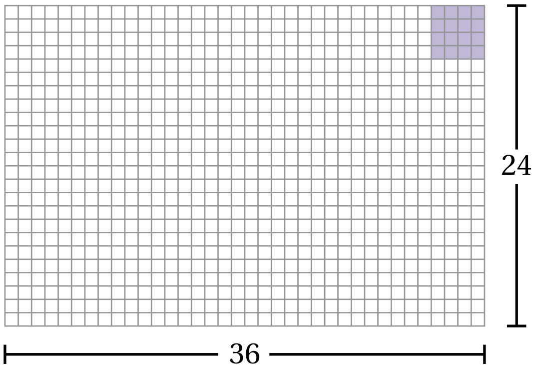
Select *all* the other common factors of 12 and 18.

- ☐ **A.** 1
- ☐ **B.** 2
- ☐ **C.** 4
- ☐ **D.** 6
- ☐ **E.** 12



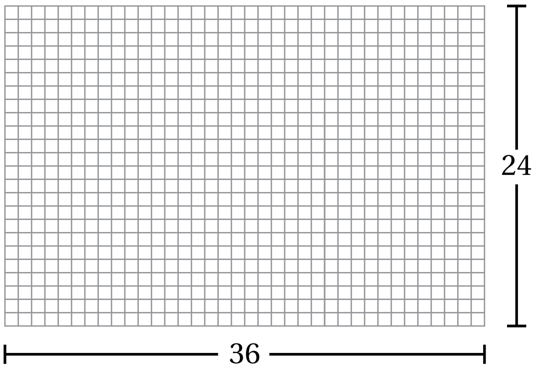
3. 4 is a common factor of 24 and 36.

Determine as many common factors of 24 and 36 as you can.



4. The **greatest common factor (GCF)** is the greatest number that is a common factor of two numbers.

What is the greatest common factor of 24 and 36?



Activity

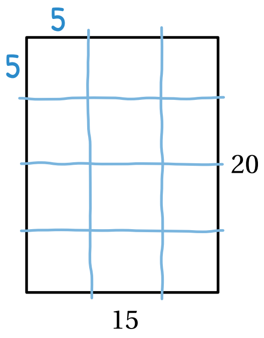
1

Name: .... Date: .... Period: ....

Greatest Common Factors (continued)

5. Here are Latifa's and Tameeka's strategies for determining the greatest common factor of 20 and 15.

Latifa



Tameeka

Factors of 15: 1, 3, 5, 15

Factors of 20: 1, 2, 4, 5, 10, 20

Discuss: What are the advantages and disadvantages of each strategy?

6. What is the greatest common factor of 27 and 36? Explain your thinking.

Activity
2

Name: .... Date: .... Period: ....

Common Factors and Multiples

7. The greatest common factor of 20 and 15 is 5.

What is the *least common multiple (LCM)* of 20 and 15?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8. How are *GCF* and *LCM* alike? How are they different?

Alike:

Different:

Name: .... Date: .... Period: ....

Repeated Challenges

9. Determine the least common multiple or greatest common factor. Use a 100-grid or draw a diagram if it helps with your thinking.

a What is the least common multiple of 6 and 4?

b What is the greatest common factor of 9 and 12?

c What is the least common multiple of 10 and 6?

d What is the least common multiple of 2 and 16?

e What is the greatest common factor of 16 and 2?

f What is the greatest common factor of 18 and 27?

You're invited to explore more.

10. Jamir and Kimaya each wrote a question about greatest common factor (GCF) and least common multiple (LCM).

Jamir	Kimaya
Does every pair of numbers have a GCF and a LCM?	Is the GCF of two numbers always smaller than the LCM?

Discuss your answer to at least one question with a classmate.

Synthesis

11. Discuss these questions with a classmate. Sketch a diagram if it helps with your thinking.

a What does *greatest common factor* mean?

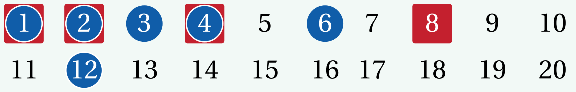
b Why do you think we don't study the *least* common factor?

Lesson Practice 2.07

Lesson Summary

A *factor* of a number is a whole number that divides evenly into the given number (with no remainder). A **common factor** of two numbers is a number that is a factor of both numbers.

Here's a chart that shows some factors of 8 (marked with squares) and some factors of 12 (marked with circles). We can see that some *common* factors of 8 and 12 are 1, 2, and 4.



The **greatest common factor (GCF)** is the largest number that is a common factor of two numbers. In the example of 8 and 12, the GCF is 4.

Lesson Practice

2.07

Name: .... Date: .... Period: ....

1. What is the greatest common factor of 12 and 44?

2. What is the greatest common factor of 4 and 6?

3. What is the least common multiple of 4 and 6?

Problems 4–6: Jayla's parents are replacing their bathroom floor with square tiles. The tiles will be laid side by side to cover the entire floor with no gaps, and none of the tiles can be cut. The floor is a rectangle that measures 48-by-60 inches.

4. What is the side length of the largest possible tile Jayla's parents could use?

5. How many of these tiles do they need?

6. List *three* other whole-number tile sizes that could cover the bathroom floor.

Problems 7–8: There are 90 sixth graders and 75 seventh graders in a school chorus. The music director wants to make groups of performers, with the same combination of sixth graders and seventh graders in each group. She wants to form as many groups as possible.

7. What is the greatest number of groups that could be formed? Show your thinking.

8. Using your answer from the previous problem, determine how many students of each grade would be in each group.

Lesson Practice
2.07

Name: .... Date: .... Period: ....

FAST Practice

9. Saanvi has two rolls of ribbon. The red-colored roll is 60 inches long, and the green-colored roll is 105 inches long. She wants to cut the rolls into equal lengths without wasting any ribbon. What is the greatest length she can cut each piece of ribbon?

A. 3 inches

B. 5 inches

C. 15 inches

D. 30 inches

Spiral Review

Problems 10–12: Circle the expression that has the greater value.

10. $5 \cdot 0.4$ $\quad$ $500 \cdot 0.04$ $\quad$ They have the same value.

11. $14.2 - 2.35$ $\quad$ $142 - 23.5$ $\quad$ They have the same value.

12. $1.82 + 33.3$ $\quad$ $18.2 + 3.33$ $\quad$ They have the same value.

Unit 2

Lesson

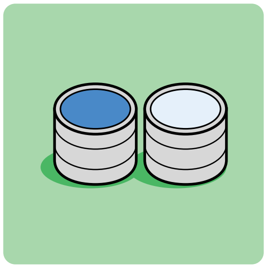
8

Name: ____________________ Date: __________ Period: __________

MA.6.AR.3.5, MTR.4.1, MTR.6.1

Mixing Paint

Let’s see how mixing colors relates to ratios.



Warm-Up

Mentally determine the missing value that makes each pair of fractions equivalent.

1. $\frac{1}{5} = \frac{\square}{10}$

2. $\frac{2}{5} = \frac{6}{\square}$

3. $\frac{4}{\square} = \frac{12}{15}$

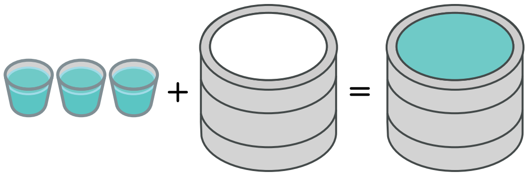
4. $\frac{\square}{8} = \frac{15}{12}$

Name: .... Date: .... Period: ....

Comparing Ratios

5. Paint stores create different colors by using different ratios of white paint to tint.

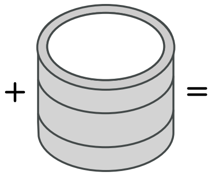
Example Paint Mix



a Choose *one* tint color.

b Circle the amount of tint you want to add to 2 gallons of white paint.

My Paint Mix



6. Write the ratio you created.

.... ounces tint : 2 gallons white paint

a Can you find two different ways to make a *darker* color?

.... ounces tint : .... gallons white

.... ounces tint : .... gallons white

b Can you find two different ways to make a *lighter* color?

.... ounces tint : .... gallons white

.... ounces tint : .... gallons white

Activity
1

Name: ______________________ Date: __________ Period: __________

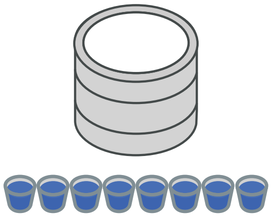
Comparing Ratios (continued)

7. Here are Luca's and Marc's ratios. Which will make a darker blue? Circle one.

Luca's ratio   Marc's ratio   They'll make the same blue

Explain your thinking.

Luca's Ratio
6 ounces blue
2 gallons white

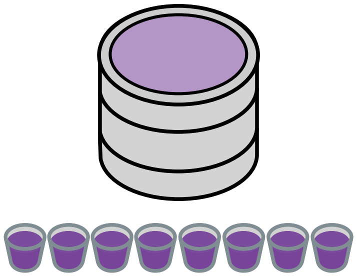


Marc's Ratio
8 ounces blue
4 gallons white

8. **Discuss:** What is a different strategy you could use to compare the ratios?

9. Here is Amoli's ratio:

8 ounces purple : 4 gallons white



Select *all* of the choices that will result in a darker purple.

☐ **A.** Adding white paint

☐ **B.** Using less white paint

☐ **C.** Adding purple tint

☐ **D.** Using less purple tint

☐ **E.** Adding 2 ounces of purple tint and 2 gallons of white paint

Activity
2

Name: ______________________ Date: __________ Period: __________

Lighter or Darker Paint

10. Order the ratios from *darkest* blue to *lightest* blue.

A. 5 ounces blue : 4 gallons white

B. 4 ounces blue : 3 gallons white

C. 10 ounces blue : 6 gallons white

D. 9 ounces blue : 6 gallons white

Darkest Blue **Lightest Blue**

11. Luca says that these two ratios make the same shade of blue.

≡ **4 ounces blue : 3 gallons white**

≡ **5 ounces blue : 4 gallons white**

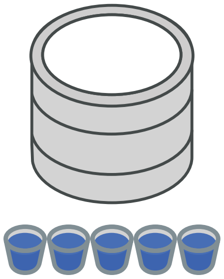
What would you recommend Luca change in his work?

12. Solve all six challenges. For each pair of ratios, choose which ratio makes a darker blue.

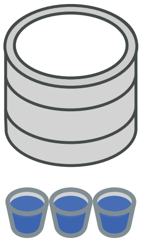
Ratio A	Ratio B	Ratio A	Ratio B	They make the same blue
2 oz blue : 4 gal white	3 oz blue : 4 gal white	○	○	○
4 oz blue : 3 gal white	4 oz blue : 5 gal white	○	○	○
3 oz blue : 2 gal white	5 oz blue : 4 gal white	○	○	○
5 oz blue : 2 gal white	15 oz blue : 6 gal white	○	○	○
7 oz blue : 3 gal white	5 oz blue : 2 gal white	○	○	○
5 oz blue : 4 gal white	9 oz blue : 7 gal white	○	○	○

Synthesis

13. Describe a strategy for comparing two ratios. Use the example if it helps with your thinking.



Ratio A
5 ounces blue
4 gallons white



Ratio B
3 ounces blue
2 gallons white

Lesson Practice 2.08

Lesson Summary

You can use different strategies to compare two ratios.

Let's compare the ratios of two cans of paint to see which will make a lighter shade of gray.

Ratio A	**Ratio B**
5 ounces black paint	7 ounces black paint
3 gallons white paint	4 gallons white paint

Strategy 1: Change both ratios so that they share one quantity.

- Multiply both ratios so they each have the same amount of black paint.
- The *LCM* for the number of ounces of black paint for both ratios is 35.
- Multiply Ratio A by 7 to get 35 ounces of black paint and 21 gallons of white paint.
- Multiply Ratio B by 5 to get 35 ounces of black paint and 20 gallons of white paint.

When both ratios have the same amount of black paint, Ratio A has more gallons of white paint, which means it will be a lighter shade of gray.

Strategy 2: Calculate how much each ratio is per 1 quantity.

- Calculate the number of ounces of black paint per gallon of white paint.
- Ratio A has $\frac{5}{3} = 1\frac{2}{3}$ ounces of black paint for every gallon of white paint.
- Ratio B has $\frac{7}{4} = 1\frac{3}{4}$ ounces of black paint for every gallon of white paint.

Ratio A has less black paint for 1 gallon of white paint, which means it will be a lighter shade of gray.

Lesson Practice
2.08

Name: Date: Period:

Problems 1–3: To make 1 can of sky blue paint, Ama mixes 2 ounces of blue tint with 3 gallons of white paint.

1. Write a ratio of blue tint to white paint that would make the *same* color blue.

2. Write a ratio of blue tint to white paint that would make a *darker* blue.

3. Write a ratio of blue tint to white paint that would make a *lighter* blue.

4. If you blend 2 scoops of chocolate frozen yogurt with 1 cup of milk, you will make a milkshake with a stronger chocolate flavor than if you blended 3 scoops of chocolate frozen yogurt with 2 cups of milk. Show or explain why this is true.

5. There are two mixtures of light purple paint.

- Mixture A is made with 5 cups of purple paint and 2 cups of white paint.
- Mixture B is made with 15 cups of purple paint and 8 cups of white paint.

Which mixture makes a lighter shade of purple? Explain your thinking.

Lesson Practice
2.08

Name: .... Date: .... Period: ....

FAST Practice

6. Order these mixtures from *lightest* green to *darkest* green.

A. 2 gallons white : 4 ounces green

B. 3 gallons white : 5 ounces green

C. 5 gallons white : 8 ounces green

Lightest green **Darkest green**

Spiral Review

Problems 7–9: Here are two recipes for lemonade.

- **Recipe A:** Mix 3 cups of lemon juice with 2 cups of water.
- **Recipe B:** Mix 3 cups of lemon juice with 3 cups of water.

7. What fraction of Recipe A is lemon juice?

8. What fraction of Recipe B is lemon juice?

9. Which recipe has a stronger lemon flavor? Explain your thinking.

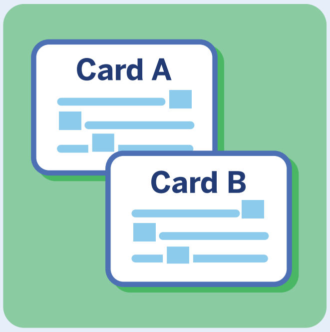
Sub-unit

3

Solving Problems With Ratios

Lesson 9
Disaster Preparation

Lesson 10
Balloons



Lesson 11
Community Life

Lesson 12
Lunch Waste

Unit 2

Lesson

9

Name: ____________ Date: ________ Period: ________

MA.6.AR.3.3, MTR.2.1, MTR.7.1

Disaster Preparation

Let’s use ratio tables to help prepare for disasters.

Warm-Up

1. Cities need to prepare for possible disasters.

What are *three* things a city should have for its people in case of a disaster?

1.

2.

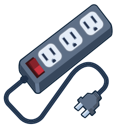
3.

2. Revise this sentence to read here are three items that are recommended for cities to stock up on in case of a disaster.

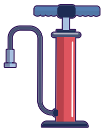
How many of each item do you think a city with a population of 100 should have?

Handheld Shower

Power Strip



Air Pump



Population	Handheld Showers	Power Strips	Air Pumps
100			

Name: .... Date: .... Period: ....

Shower, Power, and Air

3. Here are the recommendations for a city of 100 people.

How many of each item would you recommend for Lucas, Wisconsin?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Lucas, Wisconsin	700			

4. How many of each item would you recommend for Blue Ridge, Georgia?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Blue Ridge, Georgia	1,200			

5. How many of each item would you recommend for Hamlin City, Kansas?

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Hamlin City, Kansas	25			

Name: .... Date: .... Period: ....

Shower, Power, and Air (continued)

6. Taylor recommended that Hamlin City, Kansas should buy 1 handheld shower, 1 power strip, and 1 air pump.

	Population	Handheld Showers	Power Strips	Air Pumps
Recommendations	100	4	5	1
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	1	1

a 💬 **Discuss:** Why do you think Taylor made this recommendation?

b What do you agree with about Taylor's recommendations? What do you disagree with?

7. Here are actual recommendations for Lucas, Wisconsin; Blue Ridge, Georgia; and Hamlin City, Kansas.

💬 **Discuss:** What is the strategy for calculating the number of each item? Are there any recommendations you disagree with?

	Population	Handheld Showers	Power Strips	Air Pumps
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	2	1

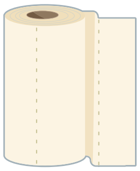
Name: ______ Date: ______ Period: ______

Poster

8. Here are some other examples of items to stock up on in case of disaster.

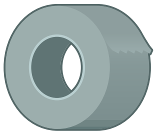
Paper Towels

For every 5 people, have 1 roll of paper towels.



Duct Tape

Have 3 rolls of duct tape for every 25 people.



Magnifying Glass

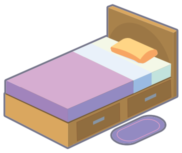
Have 1 magnifying glass for every 50 people.

Cotton Balls

For every 100 people, have 4 bags of 50 cotton balls each.

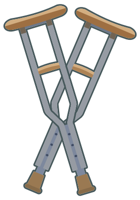
Beds

Have 1 bed for each person, plus 10 extra beds for volunteers.



Crutches

Have 6 pairs of crutches.



a Use the information above to make a recommendation for preparing these 3 cities for a disaster.

	Population	Rolls of Paper Towels	Magnifying Glasses	Cotton Balls	Pairs of Crutches
Branch City, Arkansas	300				
Bennington City, Nebraska	2,000				
Harrisburg, Pennsylvania	50,200				

b Is there anything you disagree with about the recommendations? If so, explain which numbers you think should change and why. If not, explain why not.

Name: .... Date: .... Period: ....

Poster (continued)

c Complete these steps and make a poster of your work:

☐ Choose a city or town that is meaningful to you and look up its population.

City, State	**Population (to the nearest 10 people)**
....	....

☐ Make recommendations about items that this city should stock up on. Choose *at least* four different supplies from the list. Then determine how many of each item the city should have on hand in case of a disaster.

Item 1:	**Item 2:**
Item 3:	**Item 4:**

☐ Show or explain how you determined the amount of each item your city will need.

☐ Explain *at least* two changes or additions you think should make to this guidance.

Synthesis

9. Explain how to use a table of equivalent ratios to determine unknown values. Use the example if it helps with your thinking.

	Population	Handheld Showers	Power Strips
Recommendations	100	4	5
Lucas, Wisconsin	700	28	35
Blue Ridge, Georgia	1,200	48	60

Lesson Practice 2.09

Lesson Summary

We can use ratio tables to help make plans for situations that we haven't experienced yet.

Here are some supply recommendations for a 50 person taco party:

- 10 pounds of carnitas
- 15 cups of pinto beans
- 125 tortillas

	People	Carnitas (lb)	Pinto Beans (cups)	Tortillas	
× 4	50	10	15	125	
	200	40	60	500	÷ 5
	10	2	3	25	

Let's use a table to determine the different amounts of each ingredient we might need for different-sized parties. For example, if we only had 10 people coming to the taco party, we would only need 2 pounds of carnitas, 3 cups of pinto beans, and 25 tortillas. We just have to multiply or divide all of the values in each row by the same number to preserve each ratio relationship.

Lesson Practice
2.09

Name: .... Date: .... Period: ....

Problems 1–3: A recipe for tropical fruit juice says to combine 4 cups of pineapple juice with 5 cups of orange juice.

1. Complete the table to determine how much of each type of juice you need for 1, 2, 3, and 4 batches of the recipe.

Batches	Pineapple Juice (cups)	Orange Juice (cups)
1	4	5
2		
3		
4		

2. The recipe also calls for $\frac{1}{3}$ cup of lime juice for every 5 cups of orange juice. Add an additional column of values to the table to represent the amount of lime juice for 1, 2, 3, and 4 batches of the recipe.

3. If you use 12 cups of pineapple juice with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

Problems 4–5: It takes about 9 kilograms of olives to make 2 liters of olive oil.

4. Complete the table to determine how much olive oil each orchard made.

	Olives (kg)	Olive Oil (L)
Ratio	9	2
Orchard A	9,000	
Orchard B	5,400	

5. Afia claims that to make 4 liters of olive oil, you need 11 kilograms of olives. Is she correct? Explain your thinking.

Lesson Practice
2.09

Name: .... Date: .... Period: ....

FAST Practice

6. Determine the unknown values in the table.

Number of Loaves	Bananas	Butter (cups)	Sugar (cups)	Eggs	Flour (cups)
4	12	2	3	8	6
2					

Spiral Review

Problems 7–8: Determine each product. Show your thinking.

7.
$$\begin{array}{r} 680 \\ \times\quad 502 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 401 \\ \times\quad 285 \\ \hline \end{array}$$

Unit 2

Lesson

10

Name: ________________ Date: ________ Period: ________

MA.6.AR.3.3, MTR.2.1, MTR.5.1

Balloons

Let’s develop and use tools to solve problems involving equivalent ratios.

Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 31$

2. $8 \cdot 31$

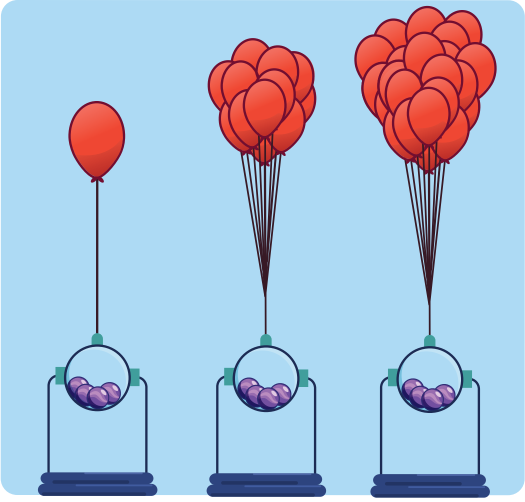
3. $9 \cdot 31$

4. $11 \cdot 31$

Name: ______________________ Date: __________ Period: __________

Balloon Float

Helium balloons can make objects float, but too many balloons will make objects fly away!

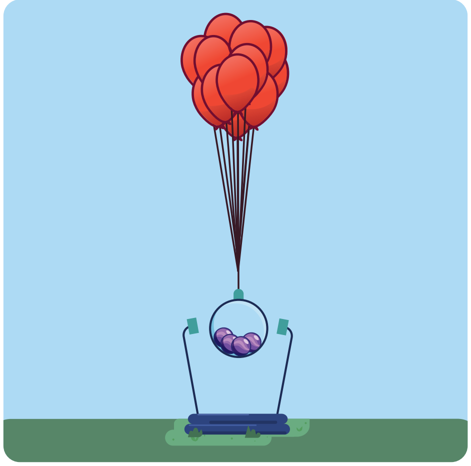


5. Red balloons float purple marbles at a ratio of $12 : 4$.

What will happen to the marbles if we add 1 balloon and 1 marble? Circle one.

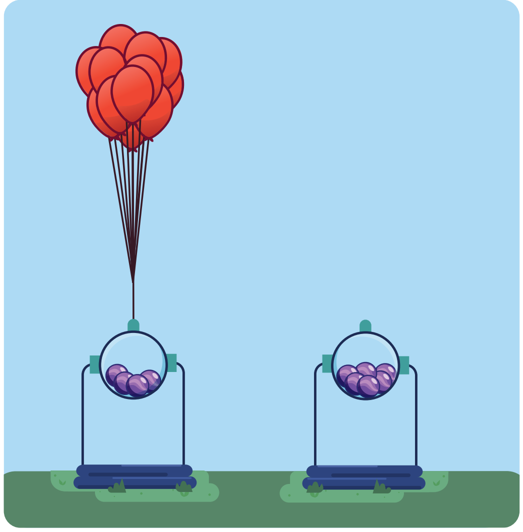
Sink down Float in place Fly up

Explain your thinking.



6. Red balloons float purple marbles at a ratio of $12 : 4$.

How many red balloons will float 6 purple marbles?

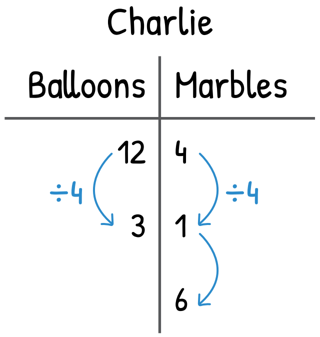


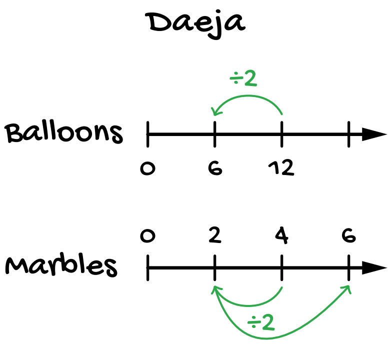
Name: .... Date: .... Period: ....

Balloon Float (continued)

7. Here are Charlie's and Daeja's strategies for determining how many red balloons will float 6 purple marbles.

a Look at each student's strategy.

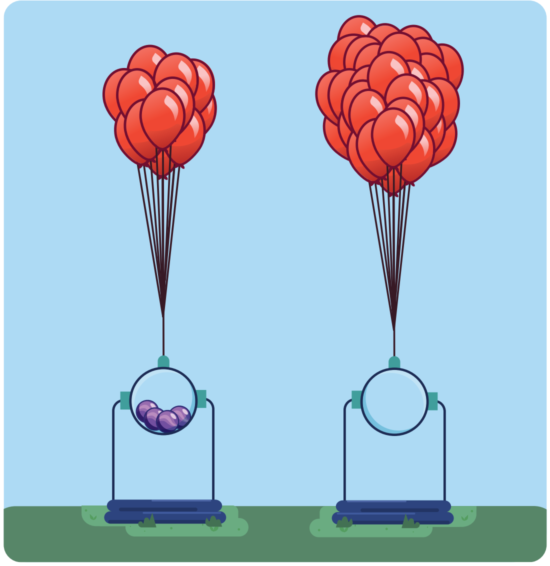




b Circle one student's strategy. Explain how either Charlie or Daeja could finish solving the problem.

8. Red balloons float purple marbles at a ratio of $12 : 4$.

How many purple marbles will 30 red balloons float?



Name: .... Date: .... Period: ....

Marble Float

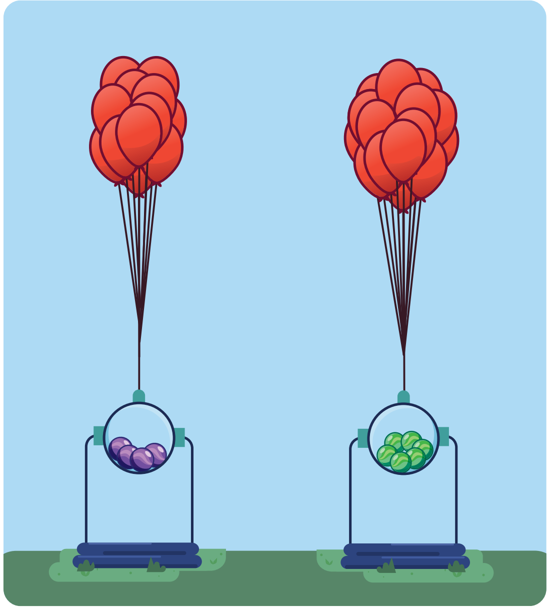
9. Red balloons float purple marbles at a ratio of $12 : 4$.

Red balloons float green marbles at a ratio of $15 : 6$.

Which is heavier: a purple marble or a green marble?

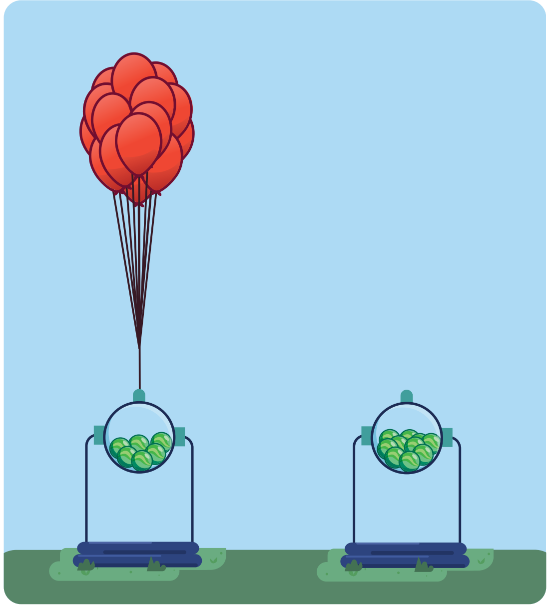
Purple Green They're the same

Explain your thinking.



10. Red balloons float green marbles at a ratio of $15 : 6$.

How many red balloons will float 10 green marbles?

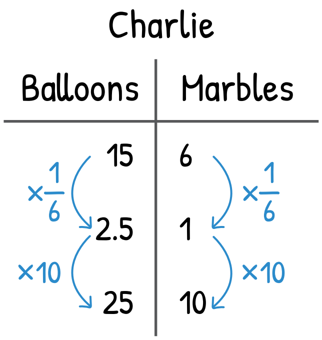


Name: .... Date: .... Period: ....

Marble Float (continued)

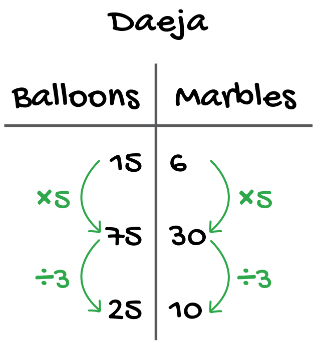
11. Here are Charlie's and Daeja's strategies for determining how many red balloons will float 10 green marbles.

Discuss: How are their strategies alike? How are they different?



Charlie

Balloons	Marbles
15	6
2.5	1
25	10



Daeja

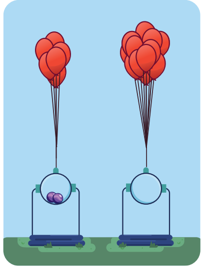
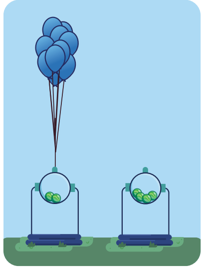
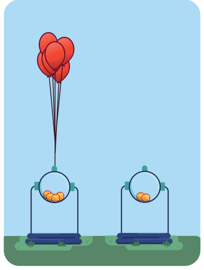
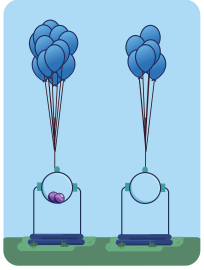
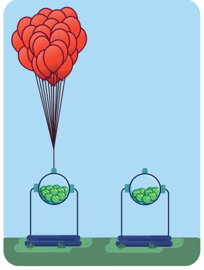
Balloons	Marbles
15	6
75	30
25	10

Activity
3

Name: .... Date: .... Period: ....

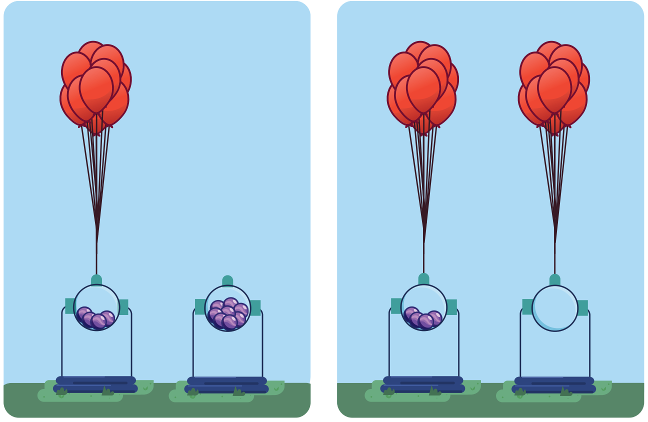
Repeated Challenges

12. For each ratio, create an equivalent ratio to make the balloons float.

	Ratio	Number of Balloons	Number of Marbles
	Red balloons float purple marbles at a ratio of 6 : 2.	12 red balloons	
	Blue balloons float green marbles at a ratio of 10 : 2.		4 green marbles
	Red balloons float orange marbles at a ratio of 6 : 4.		2 orange marbles
	Blue balloons float purple marbles at a ratio of 12 : 2.	6 blue balloons	
	Red balloons float green marbles at a ratio of 25 : 10.		8 green marbles

Synthesis

13. Describe a strategy for determining missing values in equivalent ratios, like an unknown number of balloons or marbles.



Lesson Practice 2.10

Lesson Summary

There are a few helpful strategies you can use to determine missing values in equivalent ratios. One strategy is to determine a new ratio where one of the quantities is equal to 1.

For example, if 6 balloons can make 3 marbles float, you can use the ratio $6 : 3$ and equivalent ratios to solve different problems.

To determine the number of balloons that can float 8 marbles:

- Determine the number of balloons that float 1 marble.
- Then you can multiply that ratio by 8 to determine that 16 balloons float 8 marbles.

	Number of Balloons	Number of Marbles	
	6	3	
÷3	2	1	÷3
×8	16	8	×8

To determine the number of marbles that 4 balloons can float:

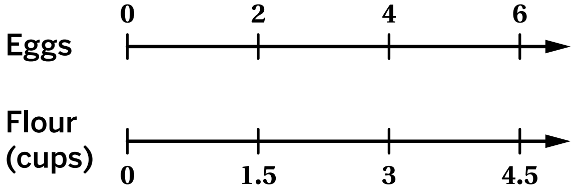
- Determine the number of marbles that 1 balloon can float.
- Then you can multiply that ratio by 4 to determine that 4 balloons float 2 marbles.

	Number of Balloons	Number of Marbles	
	6	3	
÷6	1	0.5	÷6
×4	4	2	×4

Lesson Practice
2.10

Name: .... Date: .... Period: ....

Problems 1–2: Here is a double number line showing the ratio of eggs to flour for different-sized cakes.



1. How much flour do you need for each egg in this recipe?

2. How many eggs would you need for 18 cups of flour?

Problems 3–5: The same cake recipe uses 2 cups of sugar for every 3 cups of flour.

3. Draw a double number line to represent this situation.

4. How much sugar do you need for 18 cups of flour?

5. Which representation do you prefer to help you answer the previous question: a table, a double number line, or some other tool? Explain your thinking.

6. Raven and Tiana are both training for a swimming competition in the same pool. Raven can swim 6 laps in 3 minutes. Tiana can swim 3 laps in 2 minutes. If both swimmers maintain their pace, which statement is *not* true?

- **A.** Raven can swim 2 laps per minute.
- **B.** Tiana can swim 1.5 laps in one minute.
- **C.** In 6 minutes, Raven can swim 3 more laps than Tiana.
- **D.** In 12 minutes, Tiana swims 8 fewer laps than Raven.

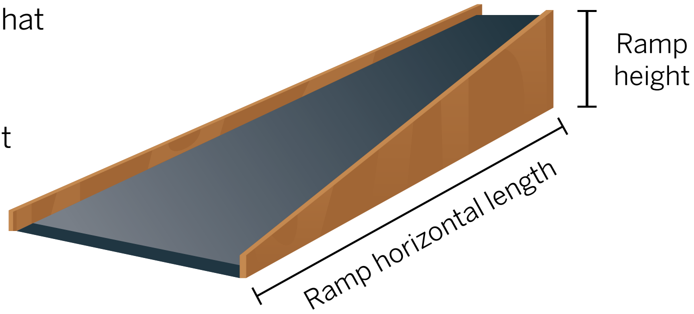
7. Inola is making personal pizzas for her birthday party. For 4 pizzas, she uses 10 ounces of cheese. Complete the table using this ratio.

Number of Pizzas	Cheese (oz)
12	
22	
11	

FAST Practice

8. A builder is following a set of instructions that states that the maximum height-to-length ratio of a curb ramp is $1 : 12$. That means for every 1 inch of ramp height, there must be at least 12 inches of ramp length.

Chloe measured the height of this ramp as 30 inches. What's the minimum horizontal length of the ramp?



☐ inches

Spiral Review

9. Fill in each blank using the numbers 1 to 12 only once to make each expression true.

$$\frac{\square}{\square} < \frac{\square}{\square} \qquad \frac{\square}{\square} = \frac{\square}{\square} \qquad \frac{\square}{\square} > \frac{\square}{\square}$$

Unit 2

Lesson

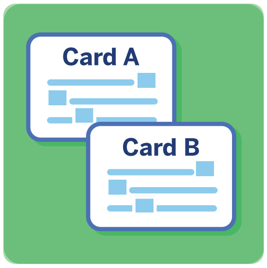
11

Name: ______ Date: ______ Period: ______

MA.6.AR.3.3, MTR.1.1

Community Life

Let’s practice solving ratio problems in context.



Warm-Up

A chef is making jollof rice for families at the Metropolis Community Center. His recipe requires 15 pounds of rice. The store sells 2 pounds of rice for $3. Customers can buy any amount of rice.

1. **Discuss:** What is this situation about?

2. Given the values, create a table or a double number line to represent the situation.

3. What question might you ask about the situation?

4. Use your diagram to answer this question.

Name: ______________________ Date: __________ Period: __________

Sort ’em

You will use the Activity 1 Sheet to complete this activity.

5. For each problem:

- Determine whether you plan to use equivalent ratios to solve. Do not attempt to solve.
- Explain or show your thinking.

Problem A

Yes No I’m not sure.

Problem B

Yes No I’m not sure.

Problem C

Yes No I’m not sure.

Problem D

Yes No I’m not sure.

Problem E

Yes No I’m not sure.

Problem F

Yes No I’m not sure.

Name: .... Date: .... Period: ....

Closer Look

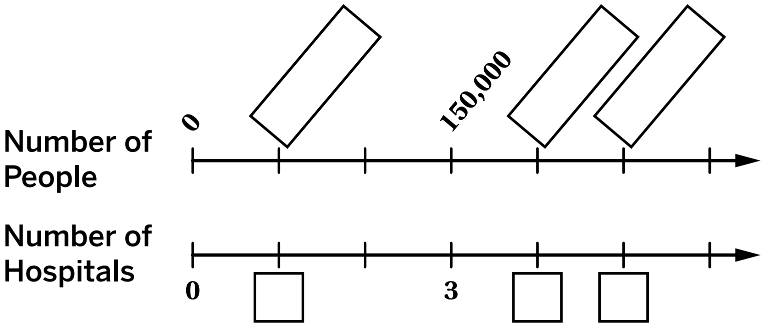
You will use the Activities 2 & 3 Sheet, which shows the problems from the Activity 1 Sheet with the numbers filled in.

6. Antwon and Riku started working on Problem D. Work with a partner to complete the missing pieces of their work.

Antwon

Number of People	Number of Hospitals
0	0
150,000	3

Riku



7. **Discuss:** How are the two representations similar? How are they different?

8. Which representation do you prefer for the situation in Problem D? Explain your thinking.

9. Answer Problem D: *If the population grows to* 250,000 *people, would you recommend that the city plan to have* 6 *hospitals?* Explain your thinking.

Activity

3

Name: .... Date: .... Period: ....

Solve ’em

10. Select *two* problems from the Activities 2 & 3 Sheet (other than Problem D) to solve. Show or explain your reasoning.

Problem ....

Problem ....

You're invited to explore more.

11. Write your own situation and problem that involves equivalent ratios. Trade problems with a partner and try to solve each other’s problems.

Synthesis

12. A shower uses 5 gallons of water every 2 minutes. How many gallons will it use during an 11-minute shower?

Without solving, describe your favorite strategy for solving a problem like this.

Lesson Practice 2.11

Lesson Summary

We can use equivalent ratios to help solve real-world problems that involve at least two quantities. When working with real-world problems, we may need to round numbers or think about the circumstances of the situation when determining which solutions make sense.

Let's say the Metropolis Delivery Service makes 15 deliveries every 2 hours. They need to make 100 deliveries tomorrow.

You can use a ratio table to determine that it will take approximately $13\frac{1}{3}$ hours, or 13 hours and 20 minutes, to make all 100 deliveries.

If you worked for the Metropolis Delivery Service, you might not report the exact value to customers. Instead, you might round to 13.5 or 14 hours to account for traffic and other delays.

Number of Deliveries	Number of Hours
15	2
5	$\frac{2}{3}$
100	$\frac{40}{3}$ or $13\frac{1}{3}$

Lesson Practice
2.11

Name: .... Date: .... Period: ....

Problems 1–2: Julian is paid $\$90$ for 8 hours of work at a restaurant.

1. Complete the table to determine the amount of money Julian is paid.

Hours of Work	Amount Paid ($)
24	
10	
3	

2. Wey Wey is paid $\$56$ for 5 hours of work. Is she being paid the same as Julian? Show or explain your thinking.

Problems 3–6: A chef needs 15 gallons of vinegar to make pickles. A store sells 2 gallons of vinegar for $\$3$ and allows customers to buy any amount of vinegar they want.

3. Make a table to represent this situation.

4. Draw a double number line to represent this situation.

5. Use any representation to determine how much the 15 gallons of vinegar will cost.

6. Which representation did you prefer in this situation? Explain your thinking.

Lesson Practice

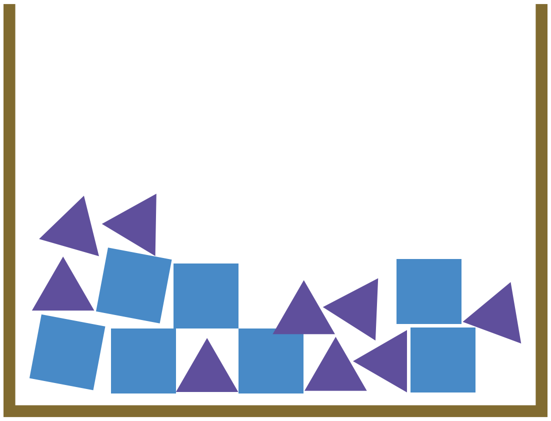
2.11

Name: .... Date: .... Period: ....

FAST Practice

7. Tyler has a box with square and triangle building blocks in it. How many blocks of each kind would Tyler need to add so that the ratio of square blocks to triangle blocks is 5 : 7?

Tyler would need ☐ square blocks and ☐ triangle blocks.



Spiral Review

Problems 8–11: Write in a number to make each equation true.

8. $4 \cdot \square = 24$

9. $24 \cdot \square = 4$

10. $\frac{1}{6} \cdot 6 = \square$

11. $6 \cdot \square = 1$

Unit 2

Lesson

12

Name: ______ Date: ______ Period: ______

MA.6.AR.3.5, MTR.4.1, MTR.7.1

Lunch Waste

Let's use ratios to solve problems about lunch waste.

Warm-Up

1. What gets thrown away during lunch time at your school?

2. Estimate how much lunch trash your school creates each day.

Activity
1

Name: ________ Date: ________ Period: ________

How Much Waste?

Maria and Hoang noticed that their school creates a lot of trash during lunch time. After lunch one day, they weighed the trash from all the students in their class. They determined that 25 students threw away 10 pounds of trash. Of that trash:

- 1 pound was styrofoam trays.
- 6 pounds were food.
- 3 pounds were other types of trash (wrappers, milk cartons, etc.).

Use these values to calculate the following:

3. How much lunch trash would the 400 students in their school create in a day? Show or explain your thinking.

4. Most schools have 180 school days each year. How much trash would the school generate in a school year? Show or explain your thinking.

5. How much of the yearly trash is styrofoam trays, food, and other trash? Show or explain your thinking.

Styrofoam Trays	**Food**	**Other Trash**
________ pounds	________ pounds	________ pounds

Activity
2

Name: .... Date: .... Period: ....

Cutting Waste

Maria and Hoang want to explore different ways to reduce the amount of trash their school creates.

6. One way to reduce trash is to switch from styrofoam to reusable trays.

How much yearly trash could the school reduce by switching from styrofoam to reusable trays?

7. Composting is another way to reduce food trash. Maria and Hoang want to cut the amount of food that gets thrown away each year by $5{,}000$ pounds.

How many students need to compost their food trash at lunch every day to meet this goal? Show or explain your thinking.

Maria and Hoang want to set a goal for reducing total yearly trash at the school.

8. What do you think is a reasonable goal? Explain your thinking.

9. Write a plan for how you think Maria and Hoang can achieve this goal. Include specific details about who they should share the recommendation with and how the school can achieve the goal.

Synthesis

10. How can you use ratios to explain ways to reduce trash in a school?

Lesson Practice 2.12

Lesson Summary

We can use ratios to solve problems.

For example, let's say Maria decided to do a food waste experiment at home. She determined that her family threw away 9 pounds of trash in 5 days. Of that 9 pounds of trash, 3 pounds were plastic, 4 pounds were food, and 2 pounds were other waste.

You can use ratios to learn more about her family's waste habits.

- You can determine how much trash Maria's family would throw out in a year (365 days).

365 days is 73 times as long as her experiment.

$9 \cdot 73 = 657$ pounds of trash

- Then you can determine how much of that yearly trash would be plastic, food, and other waste.

Plastics: $\frac{3}{9} \cdot 657 = 219$ pounds

Food: $\frac{4}{9} \cdot 657 = 292$ pounds

Other: $\frac{2}{9} \cdot 657 = 146$ pounds

Lesson Practice

2.12

Name: .... Date: .... Period: ....

Problems 1–3: Oscar wants to both spend and save the money he earns. For every $7 he puts in his wallet, he puts $3 in savings.

1. Draw a tape diagram to represent this situation.

2. If Oscar put $70 in his wallet, how much money would he put into savings? Show your thinking.

3. If Oscar earns $70 total, how much will be put into savings? Show your thinking.

4. The first floor of a doll house contains the living room, kitchen, and dining room. The combined area of these three rooms is 189 square inches. The areas of the living room, kitchen, and dining room are in the ratio $4 : 3 : 2$.

What is the area of each room? Show or explain your thinking.

Lesson Practice
2.12

Name: ____________ Date: ________ Period: ________

FAST Practice

5. Some friends are making a fruit salad for a party. The recipe for one family bowl calls for 3 cups of apples, 2 cups of grapes, and 4 cups of strawberries. A family bowl feeds 4 individuals. How many cups of each ingredient does the group need to feed 12 individuals?

☐ apples

☐ grapes

☐ strawberries

Spiral Review

Problems 6–9: Here are some pairs of equivalent ratios. Show or explain how you know they are equivalent.

6. 15 : 6 and 10 : 4

7. 40 : 15 and 8 : 3

8. 40 : 15 and 200 : 75

9. A class of 30 students shares a set of 150 crayons. Which statement is true on a day with 5 absent students?

- (A.) For every 5 students, there is 1 crayon.
- (B.) For every 6 students, there is 1 crayon.
- (C.) There are 5 crayons for each student.
- (D.) There are 6 crayons for each student.

Career Connection

How can ratios help drummers "keep the beat"?

AleksKo/Shutterstock.com

Music is measured by ratios called *time signatures*, such as $\frac{4}{4}$ or $\frac{3}{4}$. The first number tells how many beats to count. The second number tells the kind of note. If the second number is 4, the notes are quarter notes. A $\frac{4}{4}$ time signature would count 1, 2, 3, 4, 1, 2, 3, 4, and so on. A $\frac{3}{4}$ time signature would count 1, 2, 3, 1, 2, 3, and so on.

Drummers keep the beat of a song by using ratios to maintain a steady rhythm, which helps other musicians playing with them. They can change the beat to give different feelings to the sound.

B.E.S.T. Mathematics Benchmark Connection

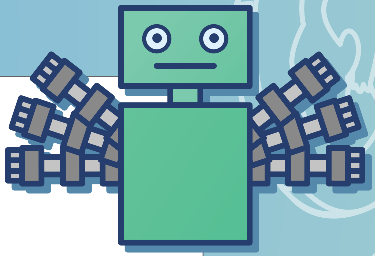
Believe it or not, musicians apply math extensively in their work. For example, the music scale and timing of different music arrangements are based on ratios (MA.6.AR.3.1, MA.6.AR.3.5) to create a certain sound, determine what speed to play certain songs, and more. Engineers and people who make musical instruments need to use their knowledge about geometry (MA.6.GR.2.1, MA.6.GR.2.4) as they cut out pieces for a musical instrument so that it makes the sound that a musician wants.

Mathematical Thinking and Reasoning Connection

People who are involved with music use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work with people they collaborate with, such as a writer, or fellow musicians in a band (MTR.1.1). When musicians study arrangements for performances such as a concert, they learn to complete the tasks of organizing and displaying the sheet music, attachments for their instruments, and other details with fluency (MTR.3.1).

Clayton Cameron

Born in Los Angeles, California, Clayton Cameron is a lecturer on percussion and Director of a Jazz Combo at the UCLA Herb Alpert School of Music. After receiving a degree in music from California State University at Northridge, he became a rising star in the music industry, performing as a percussionist with many award-winning acts. He led a TED Talk called "A-rhythm-etic. The Math Behind the Beats" to explain the mathematical ratios behind his drumming techniques.



Unit 3

Unit Rates and Percentages

In Unit 2, you discovered how to use ratios to compare quantities and determine unknown amounts. In this unit, you'll explore unit rates and percentages. They can help you answer questions like: *How much soft serve can I buy? How fast can a model train travel? How does someone make money selling T-shirts?* And even, *Which Florida county has the greatest percentage of young people?*

Essential Questions

- How are the terms *same rate*, *constant rate*, and *unit rate* alike and different?
- What is the relationship between unit rates and percentages?
- How are percentages used to estimate and compare quantities?

Sub-unit

1

Units and Measurement

Lesson 1
Many Measurements

Lesson 2
Counting Classrooms

Unit 3

Lesson

1

Name: ____________ Date: ________ Period: ________

MA.6.AR.3.5, MTR.4.1

Many Measurements

Let's connect units of measure with everyday objects.

Warm-Up

1. Which do you think is taller? Circle one.

A coconut

A pineapple

2. Which do you think is larger? Circle one.

A grapefruit

A plum

3. Which do you think is heavier? Circle one.

A cherry

A grape

Activity
1

Name: .... Date: .... Period: ....

Describe It

4. **Discuss:** Use words, drawings, hand gestures, familiar objects, or other strategies to answer the question: *How much is* ....?

- ☐ 1 foot
- ☐ 1 cup
- ☐ 1 meter
- ☐ 1 square foot
- ☐ 1 gallon
- ☐ 1 yard
- ☐ 1 millimeter
- ☐ 1 pound

5. Which measurements were *less* complicated to describe? Which measurements were *more* complicated to describe?

Less Complicated	More Complicated

6. Sort the Activity 1 Cards based on whether they measure length, volume, or weight. There will be four cards in each group.

Length	Volume	Weight

7. Sort the measurements in each group from the *smallest* unit to the *largest* unit.

	Smallest Unit			Largest Unit
Length				
Volume				
Weight				

Activity
2

Name: ____ Date: ____ Period: ____

Match It

8. Match each Activity 2 Card with the unit of measurement that best represents it.

1 Kilogram
Card ____

1 Mile
Card ____

1 Kilometer
Card ____

1 Milliliter
Card ____

1 Ounce
Card ____

1 Liter
Card ____

1 Pound
Card ____

1 Gallon
Card ____

1 Millimeter
Card ____

1 Gram
Card ____

1 Cup
Card ____

1 Centimeter
Card ____

9. **Discuss:** Choose *one* of the measurements from Problem 8. What else could you measure with this unit of measurement?

10. Here are four unit conversions arranged by what they're measuring. Add any other unit conversions you can think of for each category.

Length	Weight
1 foot = 12 inches	1 kilogram = 1000 grams

Volume	Time
1 gallon = 4 quarts	1 hour = 60 minutes

Synthesis

11. **a** List several things you could measure about this can.

b What units would you use to measure each of those things?

Lesson Practice 3.01

Lesson Summary

Units of measurement can be used to describe things like length, volume, and weight or mass. Certain units of measurement might be more appropriate to use than others, depending on what you're measuring. Here are some examples of units of measurement, arranged from the *smallest* unit to the *largest* unit.

Length	Volume	Weight
millimeter	milliliter	gram
centimeter	fluid ounce	ounce
inch	cup	pound
foot	quart	kilogram
yard	liter	ton
meter	gallon	
kilometer		
mile		

Lesson Practice
3.01

Name: .... Date: .... Period: ....

Problems 1–3: For each pair, circle the larger unit of measurement.

1. (A.) Meter
(B.) Kilometer

2. (A.) Yard
(B.) Foot

3. (A.) Pound
(B.) Ounce

4. Match each object with the unit you would most likely use to measure it.

a. The height of a building. .... Gallons
b. The length of a fingernail. .... Centimeters
c. The weight of a paper clip. .... Grams
d. The distance between two cities. .... Pounds
e. The weight of a bowling ball. .... Feet
f. The volume of a water cooler. .... Kilometers

Problems 5–6: Identify a unit that can be used to measure:

5. The length of a neighborhood road.

6. The volume of a car's gas tank.

Lesson Practice
3.01

Name: .... Date: .... Period: ....

FAST Practice

7. Determine whether each unit of measurement measures length, volume, or weight.

Unit	Length	Volume	Weight
Yard	☐	☐	☐
Milliliter	☐	☐	☐
Fluid Ounce	☐	☐	☐
Pound	☐	☐	☐
Ounce	☐	☐	☐

Spiral Review

Problems 8–9: Determine each quotient. Show or explain your thinking.

8. $1275 \div 15$

9. $1500 \div 25$

10. In a jazz orchestra, there is a horn section and a rhythm section. The ratio of horn players to rhythm players is 13 to 4. What is the ratio of rhythm players to total players?

A. $4 : 13$

B. $13 : 17$

C. $4 : 9$

D. $4 : 17$

Unit 3

Lesson

2

Name: ______ Date: ______ Period: ______

MA.6.AR.3.5, MTR.1.1, MTR.5.1

Counting Classrooms

Let’s measure using different units

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. $\frac{1}{3}$ of 15

2. $\frac{2}{3}$ of 15

3. $\frac{2}{3} \cdot 15$

4. $\frac{2}{5} \cdot 20$

Name: ________________ Date: ________ Period: ________

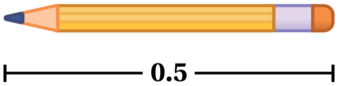
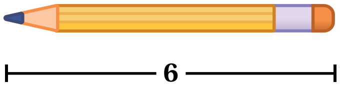
Classroom Lengths

5. **a** Choose a few objects in the classroom to measure. Try to use millimeters, centimeters, and meters. Record your findings below.

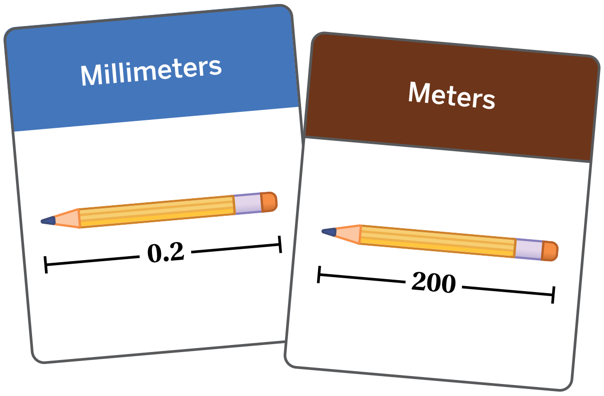
b **Discuss:** Which units were most useful for measuring each object?

6. Here are some measurements for Sahana's pencil. Match each unit to the appropriate value. One unit will not have a match.

Feet Inches Yards

7. Amara incorrectly matched these two pairs of cards. What would you say to help her understand her mistake?



Name: .... Date: .... Period: ....

Same Amount, Different Units

8. Sahana's classroom is 6 yards wide.

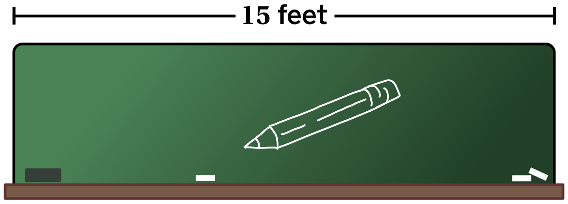
Her teacher wants to buy a new chalkboard that is 15 feet wide.

Will the new chalkboard fit on the wall? Circle one.

Yes No I'm not sure

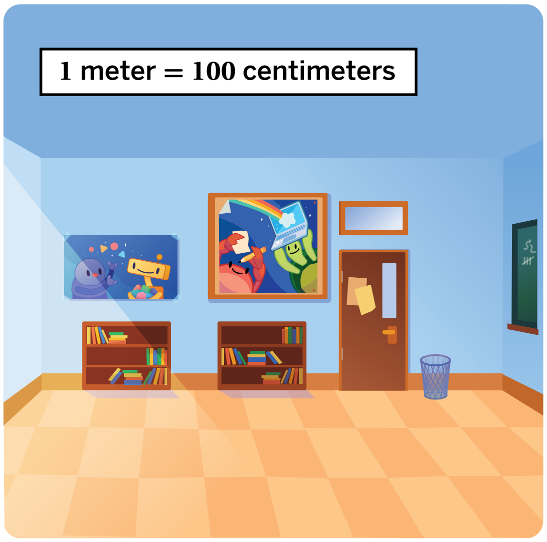
Explain your thinking.

1 yard = 3 feet



9. The classroom wall is 4 meters tall.

How many centimeters is that?



10. Ama tried to determine how many centimeters are in 4 meters and got stuck.

Discuss: What did Ama do? What might she do next?

Ama

m	cm
1	100
4	?

Activity

3

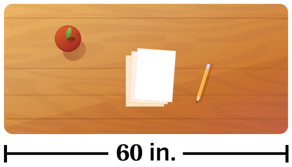
Name: .... Date: .... Period: ....

Repeated Challenges

11. Solve as many challenges as you have time for.

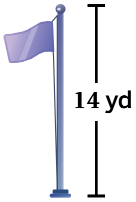
The desk is 60 inches wide. How many feet is that?

12 inches = 1 foot



The flagpole is 14 yards high. How many feet is that?

1 yard = 3 feet

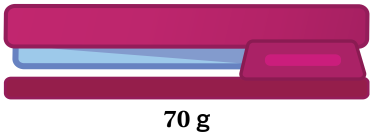


The mug holds 780 milliliters of liquid. How many liters is that?

1 liter = 1000 milliliters

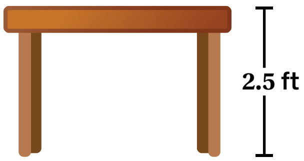
A stapler has a mass of 70 grams. How many milligrams is that?

1 gram = 1000 milligrams



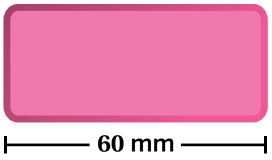
The desk is 2.5 feet tall. How many inches is that?

1 foot = 12 inches



The eraser is 60 millimeters long. How many centimeters is that?

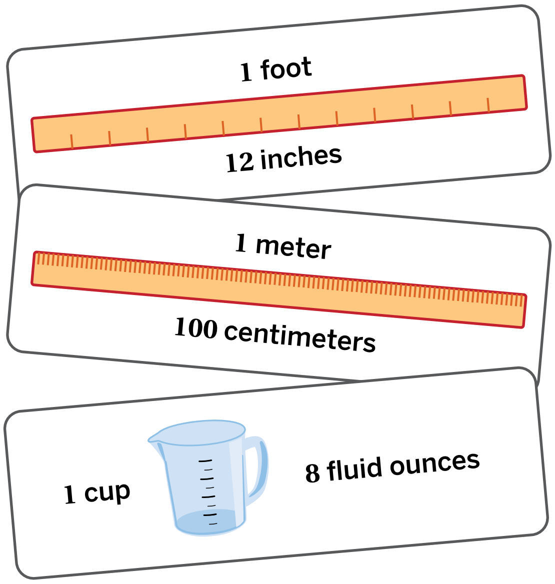
1 centimeter = 10 millimeters



Synthesis

12. Sahana says: *When you measure an object using different units, you need more of the smaller unit.*

What do you think about Sahana's claim? Explain your reasoning.



Lesson Practice 3.02

Lesson Summary

When you're measuring the same quantity with different units, you need more of the smaller unit and fewer of the larger unit to describe the measurement. So a room that's 10 feet long will measure 120 inches in length because a foot is longer than an inch.

The size of the object can help to determine the best unit of measurement.

For example, the length of the bottom edge of this notebook is 22 centimeters or 220 millimeters.

- It takes more millimeters to describe the length because millimeters are smaller than centimeters.
- You may choose to describe the length in centimeters instead of millimeters because of the size of the notebook.

Lesson Practice

3.02

Name: .... Date: .... Period: ....

1. Juana says: *This classroom is 11 yards long. A yard is longer than a foot, so if I measure the length of this classroom in feet, I will get less than 11 feet*. Is Juana correct? Explain your thinking.

Problems 2–3: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity.

1 foot = 12 inches

1 yard = 3 feet

1 meter = 100 centimeters

1 yard = 36 inches

2. A room is 30 feet wide. How many yards is that?

3. A door is 84 inches tall. How many feet is that?

Problems 4–5: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show or explain your thinking.

1 gallon = 4 quarts

1 liter = 1000 milliliters

4. 8 gallons = ... quarts

5. 5000 milliliters = ... liters

FAST Practice

6. Tyron wants to mail a package that weighs $4\frac{1}{2}$ pounds. Which of the following could be the weight of the package in ounces? 1 pound = 16 ounces

A. 16.5 ounces

B. 20.5 ounces

C. 54 ounces

D. 72 ounces

Lesson Practice

3.02

Name: ________ Date: ________ Period: ________

Spiral Review

7. Do each of these units measure length, volume, or weight?

Unit	Length	Volume	Weight
mile			
cup			
pound			
milliliter			
yard			
kilogram			
liter			
centimeter			

Problems 8–9: Identify a unit that can be used to measure each value.

8. The length of a calculator

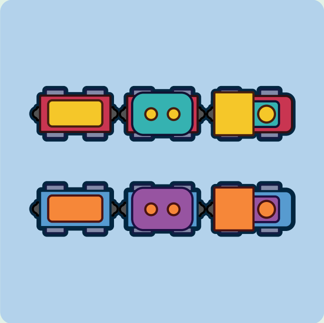
9. The volume of juice in a container

Sub-unit

2

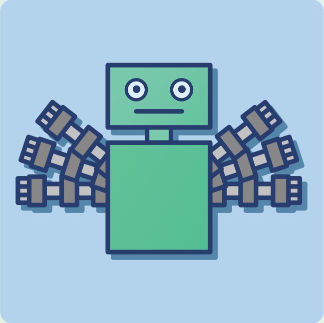
Unit Rates

Lesson 3
World Records



Lesson 4
Model Trains

Lesson 5
Soft Serve



Lesson 6
Welcome to the Robot Factory

Lesson 7
More Soft Serve

Unit 3
Lesson
3

Name: ______________________ Date: ____________ Period: ____________

MA.6.AR.3.2, MA.6.AR.3.5, MTR.4.1

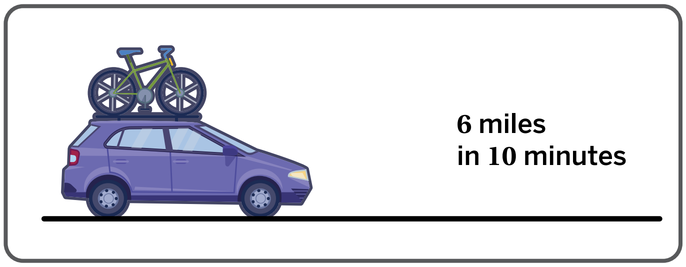
World Records

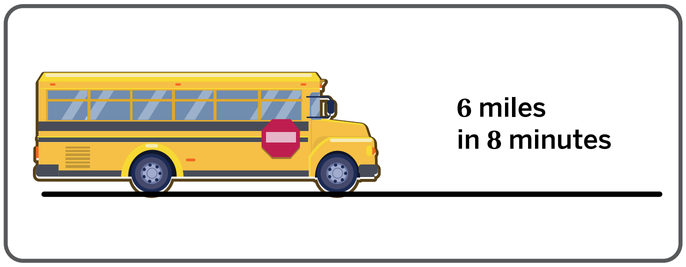
Let's calculate unit rates and use them to compare speeds.

Warm-Up

1. A car travels 6 miles in 10 minutes at a constant speed.

A bus travels 6 miles in 8 minutes at a constant speed.





Which travels faster? Circle one.

Car Bus They travel the same speed.

Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Moving 10 Meters

2. Let’s look at some instructions for determining your walking speed.

In groups of three or four, take turns as the “timer” and the “mover.”

Mover Instructions	Timer Instructions
1. Stand at the warm-up line.	**1.** Stand at the finish line.
2. Walk along the path. When you reach the starting line, say “Start!”	**2.** Start the stopwatch when the mover says “Start!”
3. Walk at a constant speed until you get to the finish line.	**3.** Stop the stopwatch when the mover reaches the finish line.
4. Record your rounded time on your worksheet.	**4.** Round their time to the nearest second.

Repeat these steps until each person has been the mover.

Record your data.

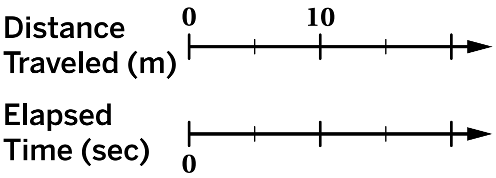
Distance: .... meters Time: .... seconds

3. A **unit rate** is a rate per 1. In this situation, the unit rate is the number of meters you can walk in 1 second.

Use your recorded data to calculate your unit rate and determine your walking speed.

Use the table or the double number line to help with your thinking.

Distance (m)	Time (sec)
10	
	1



4. At this rate, how long would it take you to walk 5 meters? 18 meters?

Activity
2

Name: .... Date: .... Period: ....

World Records

Here are three athletes who set records in different sports.

Danyil Boldyrev
15 m Climb

Keni Harrison
100 m Hurdle

César Cielo
50 m Swim

5. Estimate these athletes' speeds from *slowest* to *fastest*. Include your 10 meter walk in the rankings.

Slowest **Fastest**

6. What information do you need to determine the actual order?

Activity
2

Name: .... Date: .... Period: ....

World Records (continued)

7. Let's look at the approximate times for each event.

	Event	Distance (m)	Approximate Time (sec)
Danyil Boldyrev	Climbing	15	6
Keni Harrison	Hurdling	100	12.5
César Cielo	Swimming	50	20
You	Walking	10	

Use the approximate times to revise your speed estimates. Show your thinking.

Slowest **Fastest**

8. Let's look at the exact times for each record.

	Event	Distance (m)	Exact Time (sec)
Danyil Boldyrev	Climbing	15	5.556
Keni Harrison	Hurdling	100	12.195
César Cielo	Swimming	50	20.833
You	Walking	10	8.000

Use the exact times to calculate the speed of each athlete and determine the most accurate order. Show your thinking.

Slowest **Fastest**

Synthesis

9. Describe how unit rates can help you compare speeds. Use the tables if they help with your thinking.

Athlete 1

Distance (m)	Time (sec)
50	8

Athlete 2

Distance (m)	Time (sec)
60	10

Lesson Practice 3.03

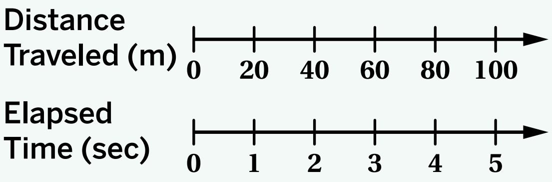
Lesson Summary

A **rate** is a number that compares two quantities with different units. For example, 16 meters *per* 2 seconds is a rate that compares distance and time.

A **unit rate** is a rate where one of the quantities is 1.

Let's say there's a train traveling 100 meters in 5 seconds. You can use a table of equivalent ratios or a double number line to calculate the unit rate, which is 20 meters per 1 second.

Distance (m)	Time (sec)
100	5
20	1



Now you can use the unit rate to answer other questions about the train. For example, to determine how far the train travels in 30 seconds, you can just multiply the unit rate of 20 meters per second by 30 to get 600. That means the train travels 600 meters in 30 seconds.

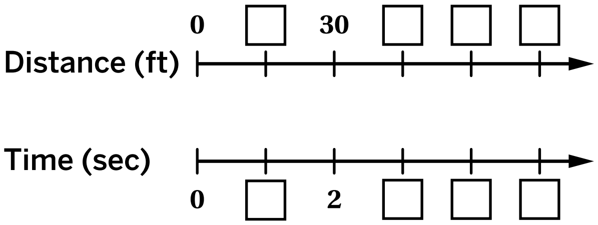
Lesson Practice

3.03

Name: .... Date: .... Period: ....

Problems 1–3: A person on a scooter travels 30 feet in 2 seconds at a constant rate.

1. Fill in the missing values on the double number line.



2. What is the speed of the scooter in feet per second?

3. At this rate, how long would it take the scooter to travel 105 feet?

4. Order these animals from *slowest* to *fastest*.

Galapagos Tortoise

16 meters in 3 minutes

Garden Snail

8 meters in 5 minutes

Three-Toed Sloth

9 meters in 2 minutes

Slowest **Fastest**

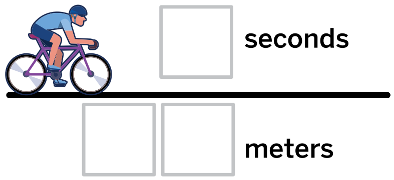
5. Alisha buys 6 muffins. She spends $24. If each muffin costs the same amount, how much does 1 muffin cost?

Lesson Practice
3.03

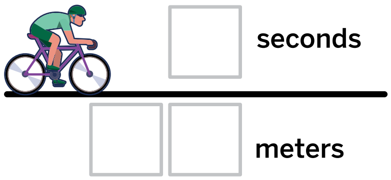
Name: .... Date: .... Period: ....

6. Fill in each blank using the numbers 0 to 9 only once so that Marc and Prisha have the same speed.

Marc



Prisha



FAST Practice

7. Ariana gets paid $90 for every 5 hours of work in her neighbor's garden. Last summer, Lucy got paid $36 for every 2 hours of work in the same garden. Are they paid at the same rate? Explain your thinking.

A. Yes. **B.** No. Ariana's rate is **A.** $12 **B.** $15 **C.** $18 per hour, and Lucy's rate is **A.** $16 **B.** $18 **C.** $22 per hour.

Spiral Review

Problems 8–9: A recipe for pasta dough calls for 150 grams of flour per large egg.

8. How much flour do you need for 6 large eggs?

9. How many large eggs do you need for 450 grams of flour?

Problems 10–11: Mentally calculate each quotient.

10. $34 \div 10$

11. $3.4 \div 10$

Unit 3

Lesson

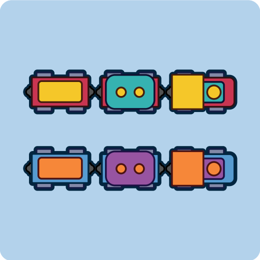
4

Name: .... Date: .... Period: ....

MA.6.AR.3.2, MA.6.AR.3.5, MTR.4.1

Model Trains

Let's use ratios to compare speeds.



Warm-Up

1. Which one doesn't belong? Circle one.

5 miles in 15 minutes	20 miles per 1 hour	3 minutes per mile	32 kilometers per 1 hour

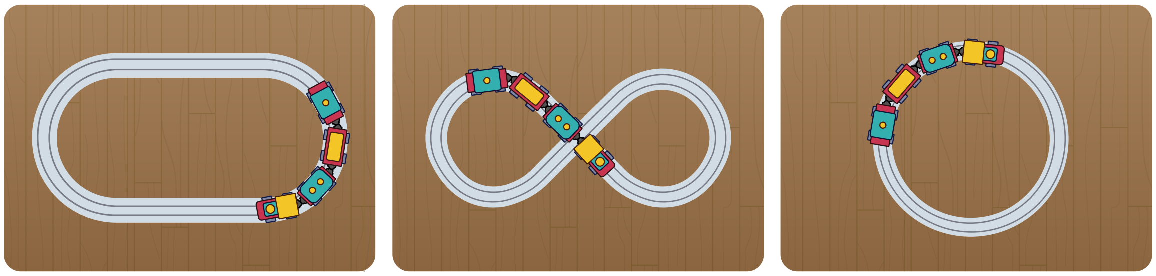
Explain your thinking.

Activity
1

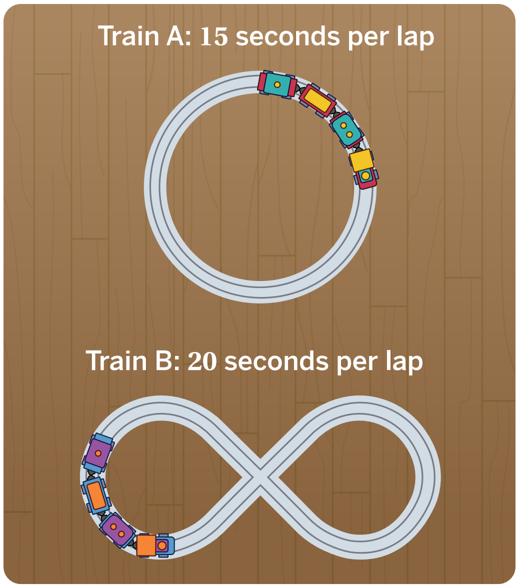
Name: ____________________ Date: __________ Period: __________

How Fast?

2. A children's museum has three types of model train sets for students to build and play with. Let's think about how the train moves on each track.



3. Here are trains from two students.



Which train is faster? Circle one.

Train A

Train B

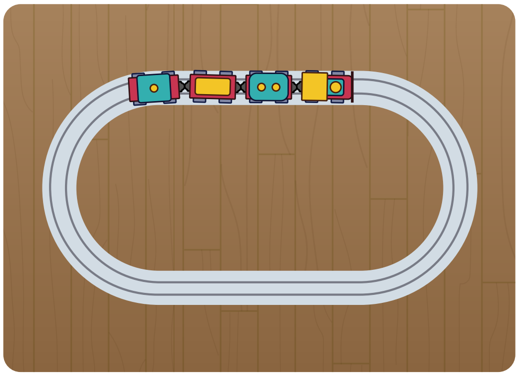
Not enough information

Explain your thinking. If you don't have enough information, what information would help you determine which train travels faster?

4. Here is a track. It is 325 centimeters long.

This train takes 10 seconds per lap.

What is its speed in centimeters per second?



Name: .... Date: .... Period: ....

How Fast? (continued)

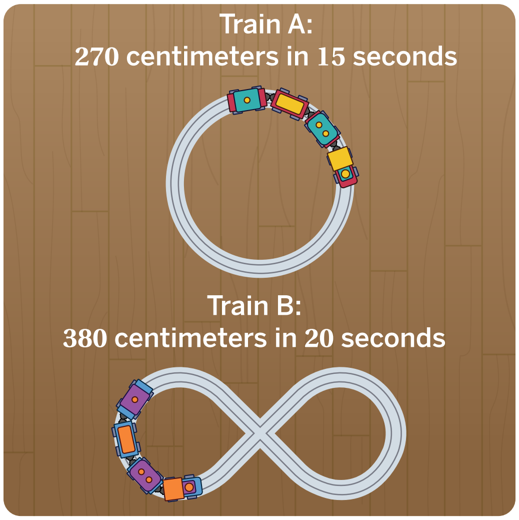
5. Which train is faster? Circle one.

Train A

Train B

They go the same speed

Explain your thinking.



Train A:
270 centimeters in 15 seconds

Train B:
380 centimeters in 20 seconds

6. Amoli and Tiam used different strategies to determine which train was faster.

💬 **Discuss:** How are their strategies alike? How are they different?

Amoli

Train A

270 ÷ 15 = 18 cm per sec

Train B

380 ÷ 20 = 19 cm per sec

Train B is faster.

Tiam

Train A		Train B	
cm	Sec	cm	Sec
270	15	380	20
1080	60	1140	60

Train B is faster.

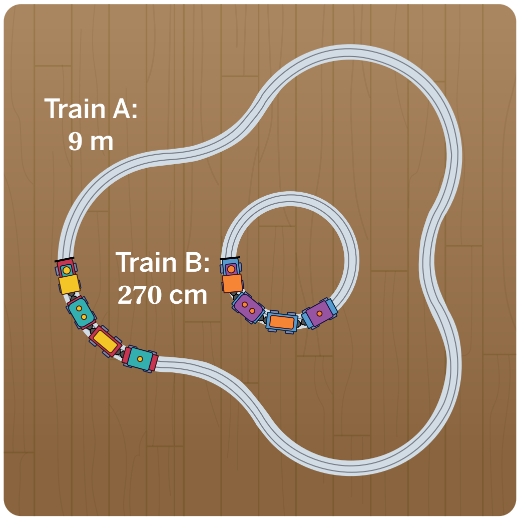
Name: .... Date: .... Period: ....

Which is Faster?

7. Here are two trains. They each complete a lap in 20 seconds.

What is each train's speed in centimeters per second?

	Speed (centimeters per second)
Train A	
Train B	



8. Here are distances and times for four model trains.

1 meter = 100 centimeters

1 minute = 60 seconds

Order the trains by speed.

A. 3.25 meters in 1 minute

B. 3.25 meters in 20 seconds

C. 270 centimeters in 20 seconds

D. 325 centimeters in 30 seconds

Slowest **Fastest**

You're invited to explore more.

9. A train's speed is 60 centimeters per second.

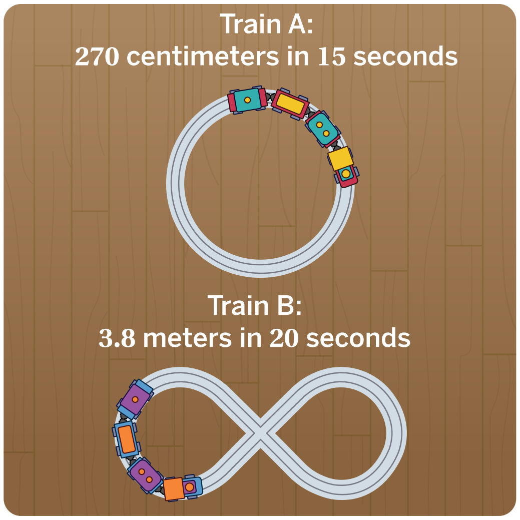
Write a track length. Then determine the number of laps the train can complete in 10 seconds.

Track Length (cm)	Laps in 10 Seconds

Synthesis

10. Describe *two* strategies for determining which of two trains is faster.

Use the examples if they help with your thinking.



Lesson Practice 3.04

Lesson Summary

When you're comparing different rates, like speeds, it's helpful to convert the rates to the same units of measurement. Then you can use equivalent ratios or unit rates to more accurately compare the rates.

Let's compare the speeds of two runners competing in different races.

- Runner A runs the 400-meter dash in 50 seconds.
- Runner B runs a 5-kilometer race in 20 minutes.

We can convert both of these speeds to meters per second.

Runner A

Seconds	Meters
50	400
1	8

8 meters per second

Runner B

5 kilometers = 5000 meters
20 minutes = 1200 seconds

Seconds	Meters
1,200	5,000
1	$4\frac{1}{6}$

$4\frac{1}{6}$ meters per second

Runner A runs at a faster rate because he ran a greater distance (8 meters) than Runner B ($4\frac{1}{6}$ meters) in the same amount of time (1 second).

Lesson Practice
3.04

Name: .... Date: .... Period: ....

Problems 1–2: Mia and Liam were trying out new remote control cars. Mia's car traveled 135 feet in 3 seconds. Liam's car traveled 228 feet in 6 seconds. Both cars traveled at a constant speed.

1. Determine the speed of each remote control car in feet per second.

Mia's Car's Speed	**Liam's Car's Speed**
.... feet per second	.... feet per second

2. Whose car traveled faster?

3. Emmanuel types 208 words in 4 minutes. Vihaan types 342 words in 6 minutes. Both type at a constant rate. Who types faster? Explain your thinking.

Problems 4–5: Penguin A walks 10 feet in 5 seconds. Penguin B walks 12 feet in 8 seconds. Each penguin keeps walking at those speeds.

4. How far does each penguin walk in 45 seconds?

5. If the two penguins start at the same place and walk in the same direction, how far apart will the two penguins be after 2 minutes? Show or explain your thinking.

Lesson Practice
3.04

Name: .... Date: .... Period: ....

FAST Practice

6. Here are the approximate distances and times for four Olympic swimmers in different events. Order the swimmers from *slowest* to *fastest*.

Swimmer A: 800 meters in 8 minutes

Swimmer B: 100 meters in 50 seconds

Swimmer C: 1.5 kilometers in 15.5 minutes

Swimmer D: 50 meters in 20 seconds

Slowest **Fastest**

Spiral Review

Problems 7–8: There are 16 cups in 1 gallon, and there are 4 quarts in 1 gallon.

7. How many cups are in 3 gallons? Show or explain your thinking.

8. How many cups are in 1 quart? Show or explain your thinking.

Unit 3

Lesson

5

Name: ____________ Date: ____________ Period: ____________

MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.5, MTR.4.1

Soft Serve

Let's compare soft serve prices using unit rates.

Warm-Up

1. a Take a look at the prices for different sizes of soft serve sold at a store.

$2	$2.80	$3.60
5 oz	7 oz	9 oz

b **Discuss:** Which size offers the best deal?

Activity
1

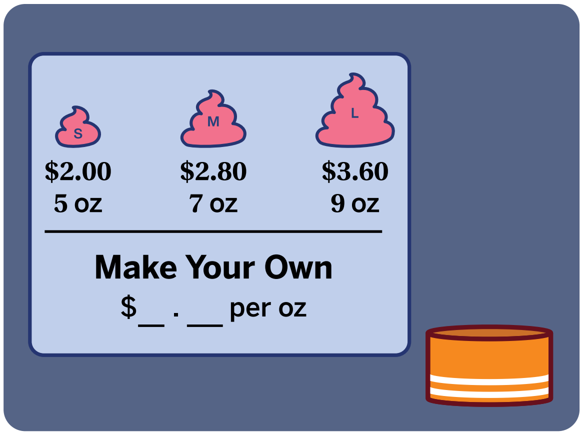
Name: ______ Date: ______ Period: ______

Two Unit Rates

2. Kala notices that soft serve costs the same per ounce no matter what size you get.

She suggests that the store put the rate on the menu.

How much does soft serve cost per ounce?



3. The store added the price per ounce, or **unit price**, to the menu.

A customer asks for 8 ounces of soft serve.

How much will this cost?

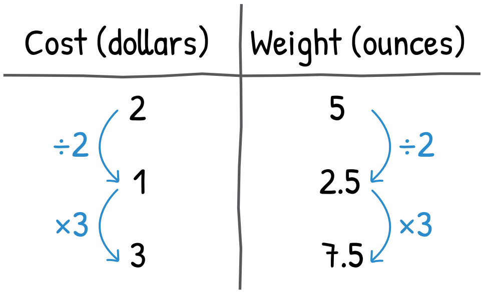
4. A new customer comes in with $3 and wants to spend it all on soft serve.

How many ounces can the customer get for $3?

Name: Date: Period:

Two Unit Rates (continued)

5. Here is how Neena figured out how much soft serve you can get for $3.



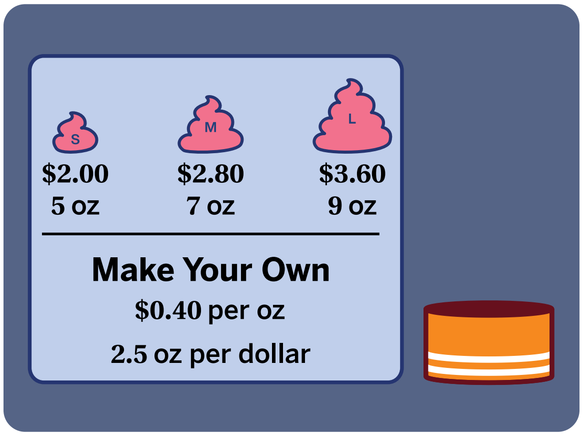
a **Discuss:** What was Neena's strategy?

b Explain or show where you can see *ounces per dollar* in Neena's work.

6. The store's menu now includes *both* unit rates.

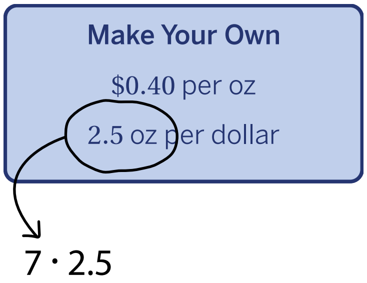
A new customer comes in with $7 and wants to spend it all on soft serve.

How much soft serve can the customer get for $7?



7. Here is how Jamal figured out how much soft serve you can get for $7.

How do you think Jamal knew which unit rate to use?



Activity
2

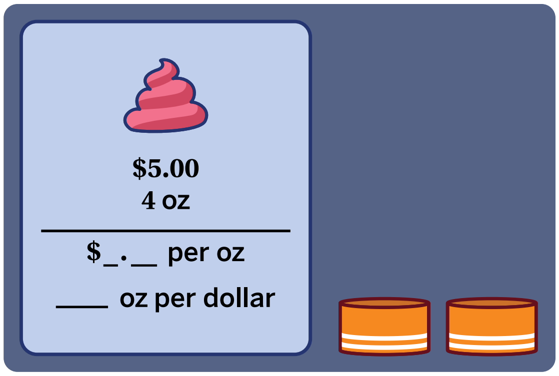
Name: .... Date: .... Period: ....

New Flavors

8. The store offers a new flavor, Swirl, with this pricing: $5 for every 4 ounces.

a How much does Swirl cost per ounce?

b How many ounces can you get per dollar?



9. How much does 7 ounces of Swirl cost?

Explain your thinking.

10. Match each rate with either chocolate or vanilla.

$2 per ounce	$0.50 per ounce	2 ounces per dollar	$\frac{1}{2}$ ounce per dollar	$9 for 4.5 ounces

Chocolate

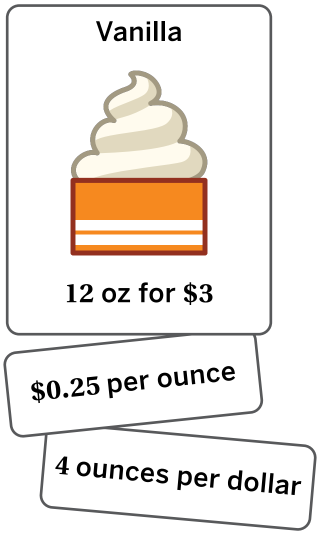
2 oz for $4

Vanilla

8 oz for $4

Synthesis

11. Explain how to calculate the two unit rates for vanilla soft serve.



Lesson Practice 3.05

Lesson Summary

When two quantities are related in a ratio, you can describe the relationship using two different unit rates.

For example, the ratio A : B can be represented as:

- The amount of Quantity A per 1 of Quantity B.
- The amount of Quantity B per 1 of Quantity A.

In situations that involve money, one of the two possible unit rates is the **unit price** (the price per unit of an item).

Let's say a store advertises 4 pounds of granola for $5.

You can use a table to determine the two different unit rates.

- Price per 1 pound: $1.25 per pound of granola. This is the unit price.
- Number of pounds per $1: 0.8 pounds of granola per dollar.

Granola (lb)	Price ($)
4	5.00
1	1.25
0.8	1.00

Lesson Practice
3.05

Name: ________ Date: ________ Period: ________

Problems 1–4: A copy machine can make 500 copies every 4 minutes.

1. How many copies can the copy machine make per minute?

2. How many minutes does it take per copy?

3. How many copies can the copy machine make in 10 minutes?

4. A teacher made 700 copies. How long did it take?

Problems 5–7: Jamar's class painted 50 square feet of a mural using 4 cans of paint.

5. How many square feet did they paint per can of paint?

6. How many cans did they use per square foot?

7. Jamar's class wants to paint a total of 310 square feet. Jamar calculated that they would need 3,875 cans of paint. Here is his work.

Jamar

12.5 square feet

$310 \cdot 12.5 = 3875$

Is Jamar correct? Circle one.

Yes No

Explain your thinking.

Lesson Practice
3.05

Name: .... Date: .... Period: ....

FAST Practice

8. Here are the prices for cans of juice at different stores. The cans are the same brand and size. Which store offers the best deal? Explain your thinking.

Store A	Store B	Store C
4 cans for $2.48	5 cans for $3.00	$0.59 per can

A. Store A **B.** Store B **C.** Store C offers the best deal. Cans are sold for

A. $0.60 **B.** $0.62 **C.** $1.60 each at Store A and **A.** $0.60 **B.** $0.62 **C.** $1.60

each at Store B.

Spiral Review

Problems 9–10: Evaluate each expression.

9. $\frac{1}{4}$ of 60

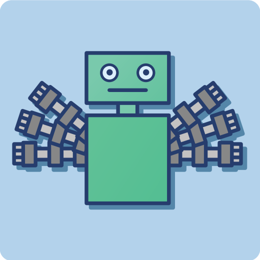
10. $\frac{3}{4}$ of 60

Name: .... Date: .... Period: ....

MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.5, MTR.5.1

Welcome to the Robot Factory

Let's determine unknown values using unit rates.



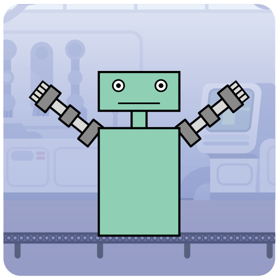
Warm-Up

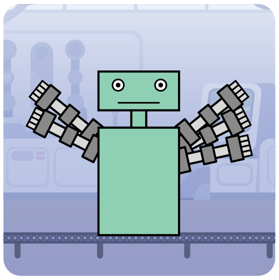
1. This table shows some lengths in both inches and feet.

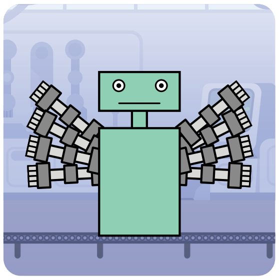
What are *three* things you notice about the table?

Length (ft)	Length (in.)
1	12
3	36
5	60
10	120

2. Welcome to the Robot Factory! Take a look at how many arms the robot has in each image.







Activity 1

Name: .... Date: .... Period: ....

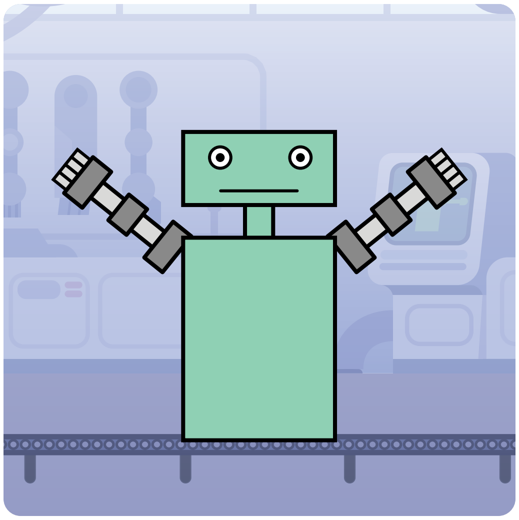
Arms and Fingers

3. This robot has 2 arms and 8 fingers.

Here are some other robots with different numbers of arms.

Complete the table to show the number of fingers on each robot.

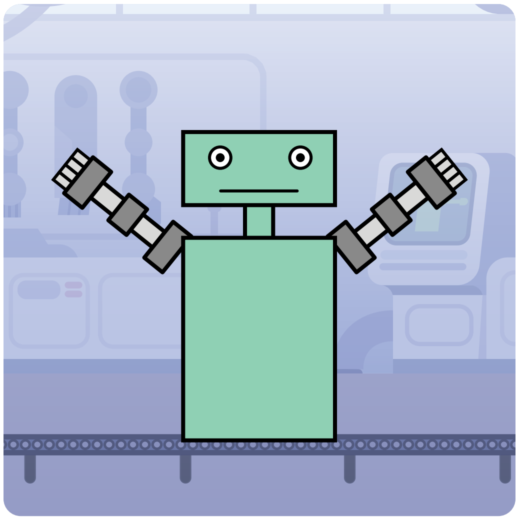
Arms	Fingers
2	8
7	
3	
9	



4. A new row has been added to the table.

How many arms go with this many fingers?

Arms	Fingers
2	8
	44



5. Choose *one* question and write your response.

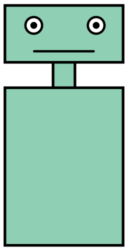
- ☐ If you know the number of *fingers*, how can you determine the number of arms?
- ☐ If you know the number of *arms*, how can you determine the number of fingers?

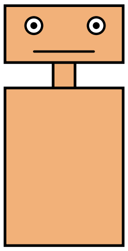
Activity
2

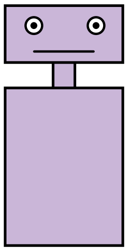
Name: .... Date: .... Period: ....

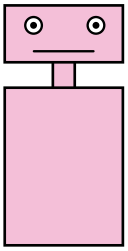
Painting Robots

6. Choose a color to paint your robot.





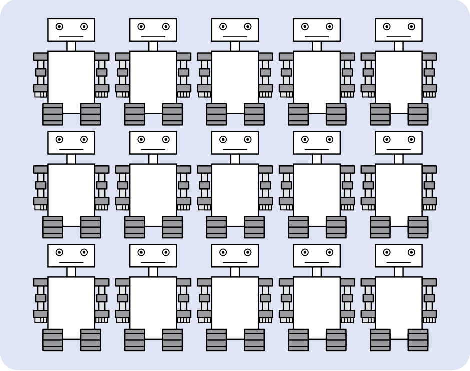




7. 6 robots need 2 gallons of paint.

Complete the table.

Number of Robots	Amount of Paint (gal)
6	2
15	
21	
11	

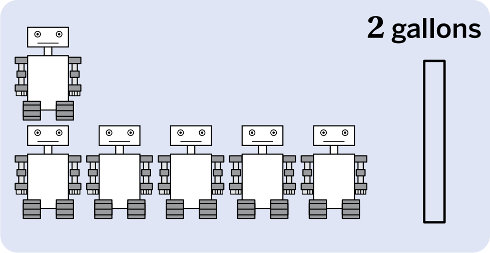


Name: .... Date: .... Period: ....

Painting Robots (continued)

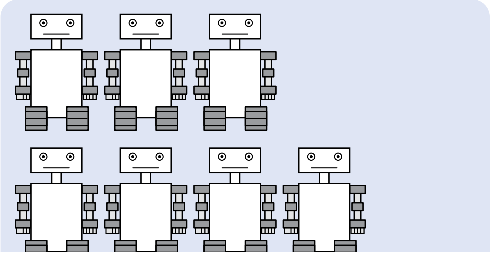
8. Write instructions for how you could determine the amount of paint needed for *any* number of robots.

Use your table from the previous problem if it helps with your thinking.



9. Here are some extra-large robots. 4 robots need 10 gallons of paint.

Complete the table.



Number of Robots	Amount of Paint (gal)
4	10
7	
9	
13	

You're invited to explore more.

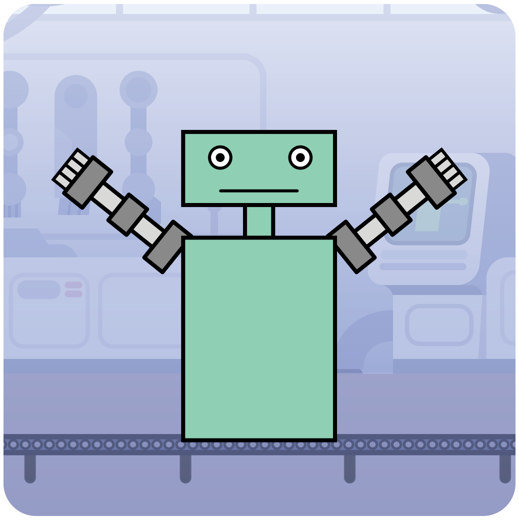
10. Lisa wrote down the amount of paint and the painting time needed for different numbers of robots. Some of the values are missing. Complete the table.

Number of Robots	Amount of Paint (gal)	Painting Time (min)
5	2	
	5	10
15		12
	1	

Name: .... Date: .... Period: ....

Heads and Bodies

11. This robot's head is 4 inches tall, and its body is 9 inches tall.



a Complete the table to show the number of fingers on each robot.

Length of Head (in.)	Length of Body (in.)
4	9
6	
7	
	27

b 💬 **Discuss:** How can you use a unit rate to create a robot of *any* body length? Of *any* head length?.

12. Create a table for the size of the robot's body if the width of the body is half the length.

Length of Body (in.)	Width of Body (in.)

Synthesis

13. Explain how you can use a table of equivalent ratios to determine unknown values, like the amount of paint needed for different numbers of robots.

Use this table if it helps with your thinking.

Number of Robots	Amount of Paint (gal)
1	$\frac{1}{3}$
6	2
33	11
18	6
9	3

Lesson Practice 3.06

Lesson Summary

Unit rates can help you determine missing values in a table.

For example, let's say 4 pounds of apples cost $10.

That means the cost of 1 pound of apples is $2.50. So you can calculate the cost of any amount of apples by multiplying the weight by 2.5.

That also means that for $1, you can buy 0.4 pounds of apples. So you can calculate the number of pounds of apples you can buy for any amount of money by multiplying the amount of money by 0.4.

$\div 0.4$

$\cdot 2.50$

Pounds	Dollars
4	10
2	5
1	2.5
0.4	1

$\div 2.5$

$\cdot 0.4$

Lesson Practice
3.06

Name: .... Date: .... Period: ....

Problems 1–4: This table shows how many onions and tomatoes you need to make different-sized batches of a salsa recipe.

Onions	Tomatoes
2	16
4	32
6	48

1. How many onions do you need for 40 tomatoes?

2. How many tomatoes do you need for 3.5 onions?

3. One unit rate in this situation is 8. What does that represent?

4. Another unit rate is $\frac{1}{8}$. What does that represent?

Problems 5–6: For every 10 meters you walk forward, you move 15 meters forward shuffling sideways.

Distance Walked (m)	Distance Shuffled (m)
10	15

5. How far should you shuffle sideways if you walk forward 15 meters? Use the table if it helps with your thinking.

6. How far should you walk forward if you shuffle sideways for 45 meters? Use the table if it helps with your thinking.

Lesson Practice
3.06

Name: ______________ Date: ________ Period: ________

7. Liam walks 1 mile in 20 minutes. At this rate, how many miles could Liam walk in 1 hour 30 minutes?

FAST Practice

8. A train is traveling at a constant rate. Complete the table to show the relationship between the train's travel time and its distance traveled.

Time (hr)	Distance Traveled (mi)
2	110
1	
	27.5
$1\frac{1}{2}$	
	165

Spiral Review

9. A sandwich is placed on a digital scale. The scale reads 4.3. What could be the unit of measurement?

A. Miles
B. Ounces
C. Pounds
D. Inches

10. Lola's family is planning to purchase a car that is 176.5 inches long. They have a parking space that is 16.25 feet long. Could this car fit in the parking space? Explain your thinking.

Unit 3

Lesson

7

Name: ______ Date: ______ Period: ______

MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.5, MTR.1.1, MTR.4.1, MTR.5.1

More Soft Serve

Let's compare ratios and calculate unknowns using unit rates.

Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $\frac{1}{5} \cdot 30$

2. $\frac{3}{5} \cdot 30$

3. $\frac{3}{5} \cdot 15$

4. $\frac{3}{5} \cdot 3$

Missing Orders

5. Which soft serve shop has the best price per ounce? Circle one.

Shop A (5 oz for $2.00)

Shop B (6 oz for $1.50)

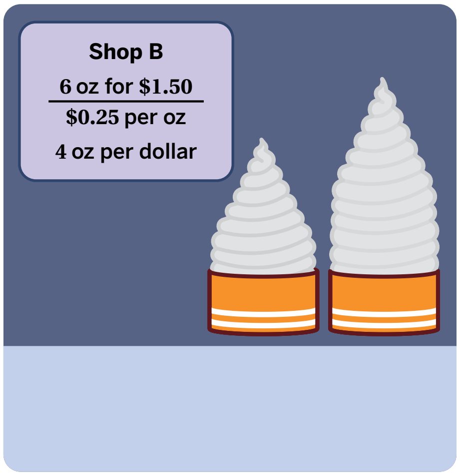
Shop C (4 oz for $1.20)

Explain your thinking.

6. Here are some new orders for Shop B.

Complete the table.

Weight (oz)	Cost (dollars)
6	1.50
8	
	3.50



Name: .... Date: .... Period: ....

Missing Orders (continued)

7. Riya completed the table in the previous problem by multiplying by the unit rates.

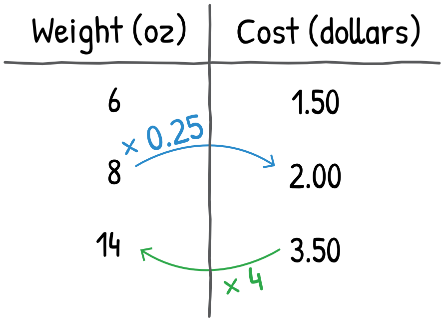
Explain how you could use Riya's strategy to calculate the cost of a 10.5-ounce soft serve.

Shop B

6 oz for $1.50

$0.25 per oz

4 oz per dollars

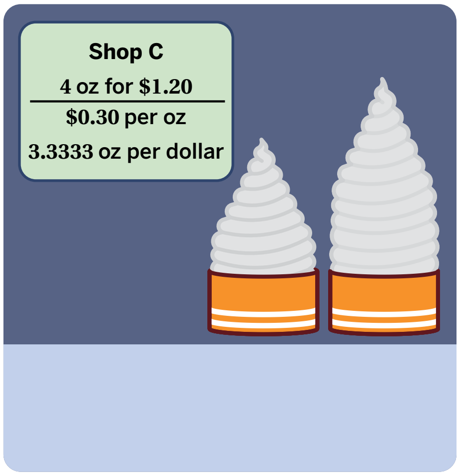


Weight (oz)	Cost (dollars)
6	1.50
8	2.00
14	3.50

8. Here are two new orders for Shop C.

Use Riya's strategy to calculate the missing values.

Weight (oz)	Cost (dollars)
4	1.20
	2.85
11	



Name: .... Date: .... Period: ....

Challenge Creator

9. **a** **Make It!**

- Choose a soft serve flavor.
- Choose the weight and the cost for a small cup of soft serve. Record them on your Challenge Sheet.

How much does your soft serve cost *per ounce*?

.... dollars per ounce

How many ounces can you get *per dollar*?

.... ounces per dollar

- Choose the *weight* of a medium soft serve and the *cost* of a large soft serve. Record them on your Challenge Sheet.

b **Swap It!**

- Swap your challenge with one or more partners.
- Fill in these tables with the weight and cost of each medium soft serve and large soft serve.

Partner 1

	Weight (oz)	Cost (dollars)
Medium		
Large		

Partner 2

	Weight (oz)	Cost (dollars)
Medium		
Large		

Partner 3

	Weight (oz)	Cost (dollars)
Medium		
Large		

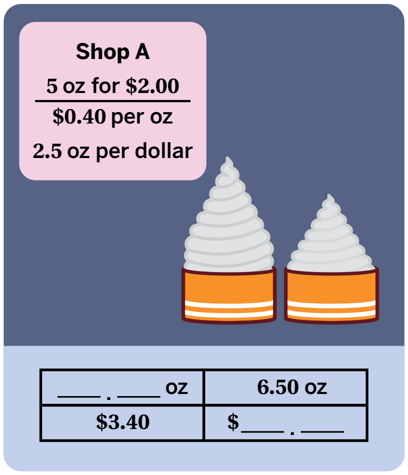
Partner 4

	Weight (oz)	Cost (dollars)
Medium		
Large		

Synthesis

10. Describe a strategy for calculating the unknown weights or costs of different soft serve orders.

Use the example if it helps with your thinking.



____.____ oz	6.50 oz
$3.40	$____.____

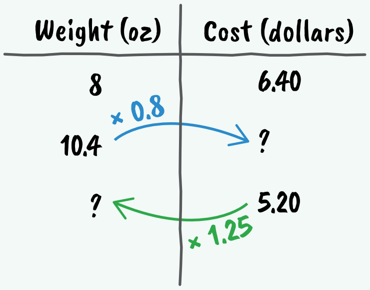
Lesson Practice 3.07

Lesson Summary

You can describe the relationship between the same two quantities using two different unit rates.

Let's say a shop charges $6.40 for 8 ounces of soft serve. The unit rates in this situation are:

- Dollars per ounce: $0.80 per ounce ($6.40 \div 8 = 0.80$)
- Ounces per dollar: 1.25 ounces per dollar ($8 \div 6.40 = 1.25$)



The unit rate you need for your calculations depends on what information you're given and what you want to determine.

So if you're given the number of ounces and want to determine the cost, you'll need to multiply by the dollars per ounce.

But if you're given the cost and want to determine the number of ounces you can get, you'll need to multiply by the ounces per dollar instead.

It's often helpful to determine *both* unit rates, so you can answer as many kinds of questions about the situation as possible!

Lesson Practice
3.07

Name: .... Date: .... Period: ....

Problems 1–2: A kangaroo hops 2 kilometers in 3 minutes.

1. How long does it take the kangaroo to hop 5 kilometers?

2. How far does the kangaroo hop in 2 minutes?

Problems 3–4: Neel buys 8 dog treats for $4.40.

3. What is the cost per dog treat?

4. Complete the table to show other numbers of dog treats he could buy at this rate.

Dog Treats	Cost ($)
8	4.40
18	
25	
	6.05

5. Haru and Victor are racing on scooters. Haru travels 15 meters in 6 seconds. Victor travels 22 meters in 10 seconds. Who is moving faster?

A. Haru

B. Victor

C. They are moving at the same speed

Explain your thinking.

6. Sothy plans to walk 10,000 steps. He starts his walk at 8:00 AM. At 8:23 AM, his phone tells him that he has taken 2,000 steps. If he continues at this rate, when will he reach 10,000 steps?

7. A corn vendor at a farmers market is selling a bag of 8 ears of corn for $\$2.56$. Another vendor is selling a bag of 12 ears of corn for $\$4.32$. Which bag is the better deal? Show or explain your thinking.

FAST Practice

8. Select *all* the statements that represent a unit rate.

☐ **A.** Cucumbers cost $\$2.00$ per pound.

☐ **B.** Eric buys 3 pairs of shorts for $\$75.00$.

☐ **C.** It takes Jose 4 hours to drive 180 miles.

☐ **D.** A recipe calls for 3 eggs for every cup of flour.

☐ **E.** A teacher groups her students so there are 5 students per group.

Spiral Review

Problems 9–10: Evaluate each expression.

9. $15 \div \frac{1}{5}$

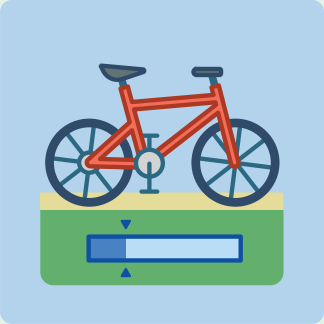
10. $\frac{1}{4} \div 12$

Sub-unit

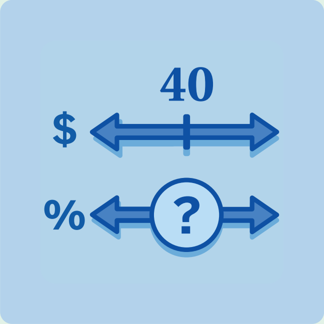
3

Percentages

Lesson 8
Lucky Duckies



Lesson 9
Bicycle Goals



Lesson 10
What’s Missing?

Lesson 11
Cost Breakdown

Lesson 12
More Bicycle Goals

Lesson 13
A County as a Village

Unit 3
Lesson
8

Name: Date: Period:

MA.6.NSO.3.5, MA.6.AR.3.4, MTR.1.1, MTR.3.1, MTR.4.1

Lucky Duckies

Let's learn about friendly percentages with rubber duckies.

Warm-Up

1. Here is a carnival game called *Duck, Duck, Choose*.

Players win a prize if they catch a rubber ducky with a star on the bottom.

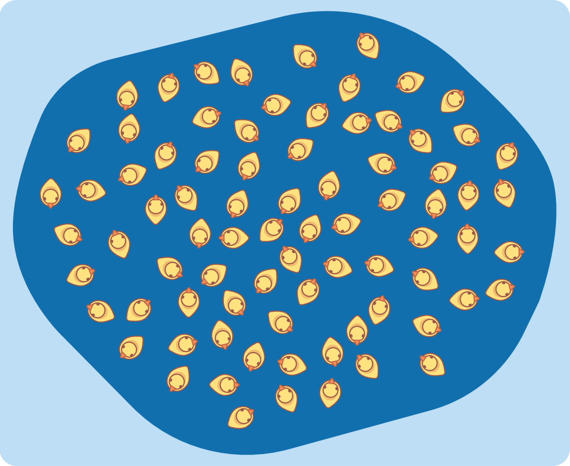
There are two games that both have 80 duckies. Which game has more duckies with stars?

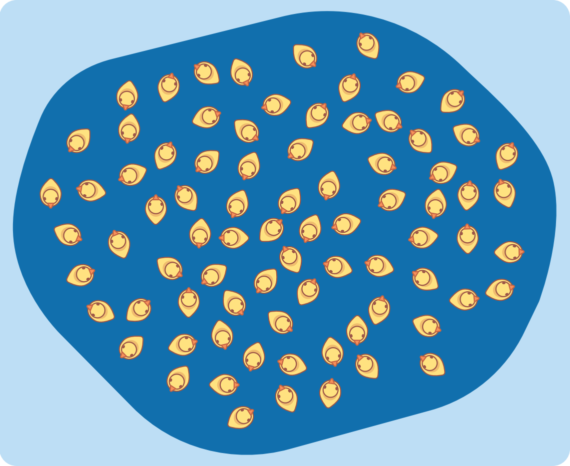
A. The game where 50% of duckies have stars

B. The game where 50 duckies have stars

C. They are the same

Explain your thinking.



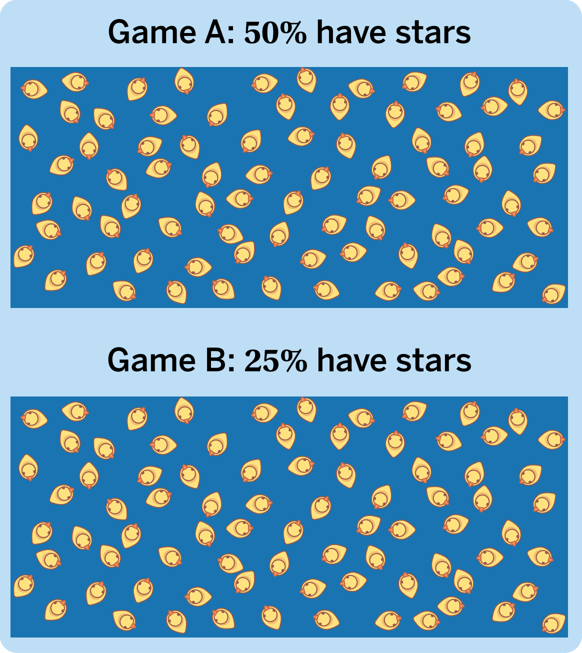


Name: ______________________ Date: __________ Period: __________

Ducky Game Design

2. **a** Draw a line to divide the pool of duckies so that each game has about the right number of duckies with stars.

b 💬 **Discuss:** How did you decide where to place each line?

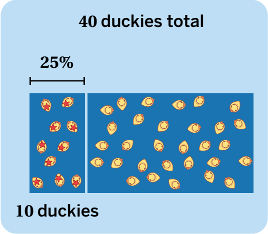


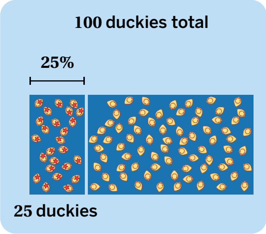
Game A: 50% have stars

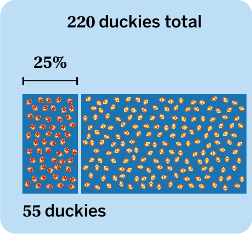
Game B: 25% have stars

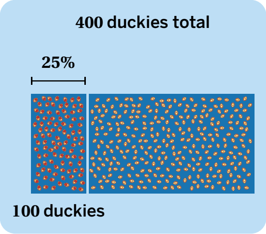
3. Here are some games where 25% of the duckies have stars.

a Take a look at the total number of duckies in each game.







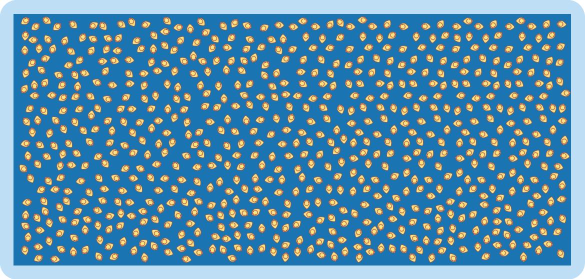


b Describe what 25% of a number means.

4. 10 **percent** (10%) means 10 for every 100.

This game has 800 duckies. 10% of them have stars.

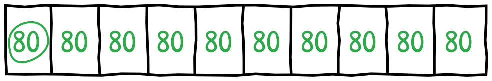
How many of the duckies have stars?



Name: .... Date: .... Period: ....

Ducky Game Design (continued)

5. Here is how Santiago figured out the number of duckies that are winners when 10% out of 800 duckies win.



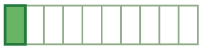
Show or explain what he may have been thinking.

6. Group these choices based on what percentage they represent. One choice will have no match.

a 10% is shaded green.

b $\frac{3}{4}$ is shaded green.

c



d $\frac{1}{2}$ is shaded green.

e 25% is shaded green.

f

g 75% is shaded green.

h $\frac{1}{4}$ is shaded green.

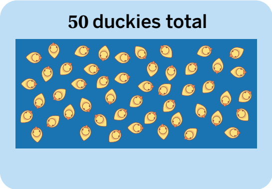
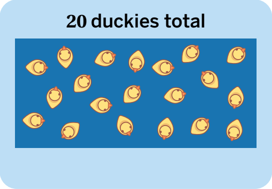
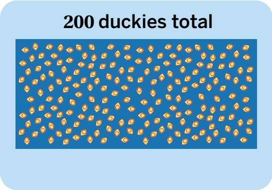
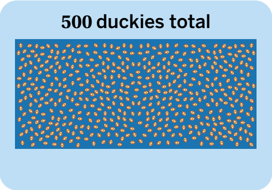
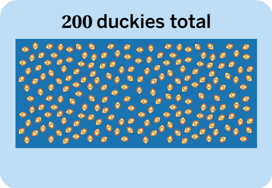
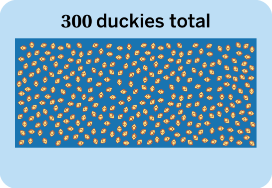
Group 1	Group 2	Group 3

Activity
2

Name: ____________________ Date: __________ Period: __________

Repeated Challenges

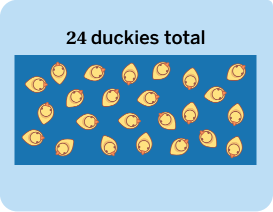
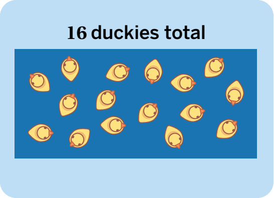
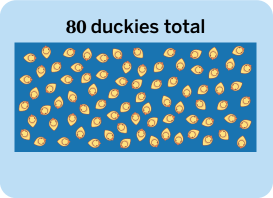
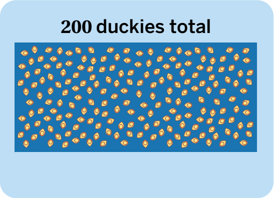
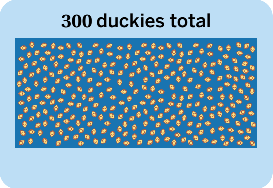
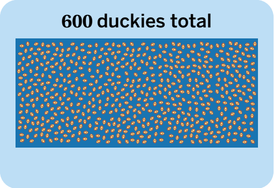
7.
- Pair up with a classmate. Decide who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
10% of 50 duckies have stars. How many duckies have stars?  	25% of 20 duckies have stars. How many duckies have stars? 
25% of 200 duckies have stars. How many duckies have stars?  	10% of 500 duckies have stars. How many duckies have stars? 
75% of 200 duckies have stars. How many duckies have stars?  	50% of 300 duckies have stars. How many duckies have stars? 

Activity
2

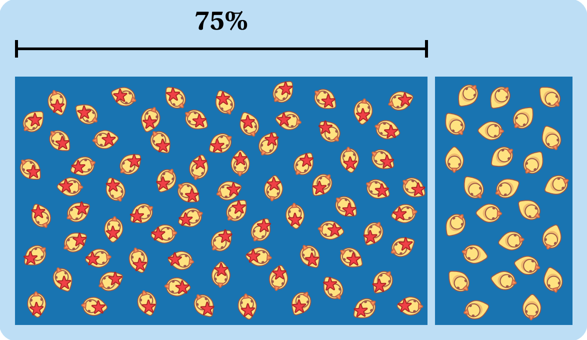
Name: .... Date: .... Period: ....

Repeated Challenges (continued)

Column A	Column B
50% of 24 duckies have stars. How many duckies have stars? 	75% of 16 duckies have stars. How many duckies have stars? 
25% of 80 duckies have stars. How many duckies have stars? 	10% of 200 duckies have stars. How many duckies have stars? 
50% of 300 duckies have stars. How many duckies have stars? 	25% of 600 duckies have stars. How many duckies have stars? 

Synthesis

8. In your own words, explain what 75% of a number means.

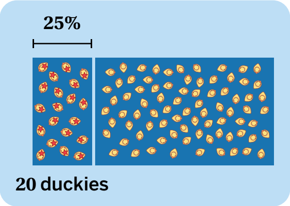
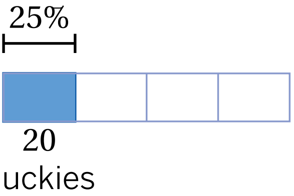
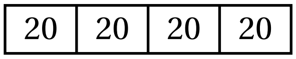


Lesson Practice 3.08

Lesson Summary

Percent means for every 100. It's represented by the percent symbol, %.

Each of the different Ducky Games in this lesson had a certain percentage of ducks with stars: 10%, 25%, 50%, or 75%. Fractions and tape diagrams can help us interpret these percentage problems.

Example Problem	Using Fractions	Using Tape Diagrams
25% of the 80 duckies have stars. 	25% of something means $\frac{25}{100}$ or $\frac{1}{4}$. $\frac{1}{4}$ of 80 duckies is 20 duckies. **80 total duckies** 	There are four 25s in 100, so the tape diagram can be split into 4 pieces. The total number of duckies can also be split into 4 parts, so there are 20 duckies in each section. **80 total duckies** 

Lesson Practice
3.08

Name: .... Date: .... Period: ....

1. Here are 24 stars. Circle 25% of these stars.

2. Evan made 40 muffins. 50% of the muffins are chocolate. How many muffins are chocolate?

3. Which is greater? Show or explain your thinking.

- **A.** 75% of 8
- **B.** 25% of 32
- **C.** They are the same.

Problems 4–5: Complete each statement. Make a tape diagram if it helps with your thinking.

4. 10% of 20 is ....

5. 25% of 60 is ....

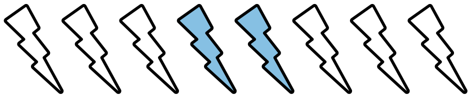
6. Explain how you could mentally calculate 10% of any number.

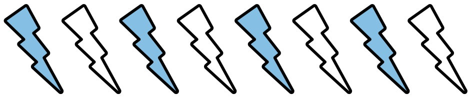
Lesson Practice
3.08

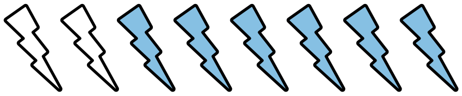
Name: .... Date: .... Period: ....

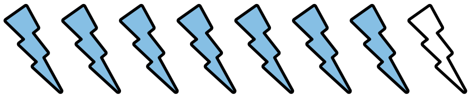
FAST Practice

7. Which group shows 75% of the lightning bolts shaded?

Ⓐ A.


B.


C.


D.


Spiral Review

8. Abdel paid $13 for 3 books. Jayden bought 12 books priced at the same rate. How much did Jayden pay for the 12 books? Explain your thinking.

Problems 9–11: Determine whether each product will be *less than*, *greater than*, or *equal to* 40.

9. $\left(\frac{6}{4}\right) \cdot 40$

10. $\left(\frac{8}{8}\right) \cdot 40$

11. $\left(\frac{1}{2}\right) \cdot 40$

Unit 3

Lesson

9

Name: .... Date: .... Period: ....

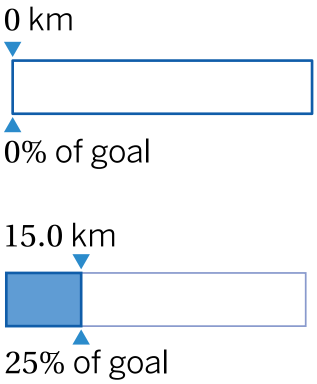
MA.6.NSO.3.5, MA.6.AR.3.4, MTR.2.1, MTR.4.1

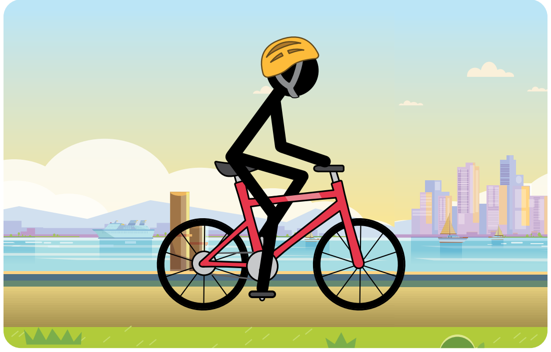
Bicycle Goals

Let's connect percentages and ratios.

Warm-Up

1. Study the percentage change as the biker tries to beat his goal.





2. What is this biker's goal distance?

Explain your thinking.

Activity
1

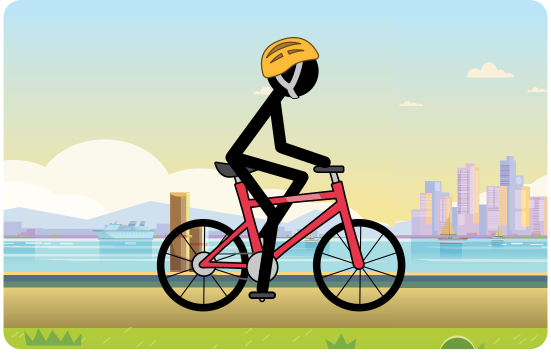
Name: .... Date: .... Period: ....

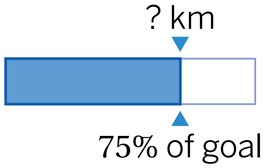
Bicycle Goals

3. Alejandro's goal was to ride 36 kilometers.

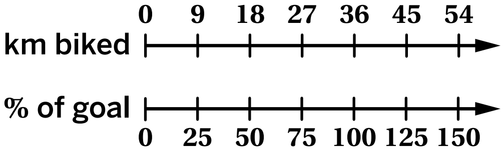
His app says he rode 75% of his goal.

How far did he ride?





4. **a** Take a look at the double number line that represents Alejandro's goal and progress.



b Describe how you can tell that the goal distance is 36 kilometers.

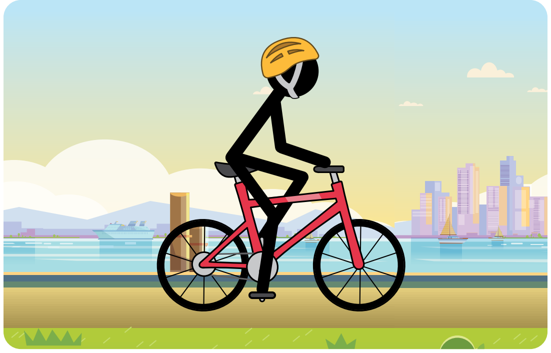
Name: ______________ Date: __________ Period: __________

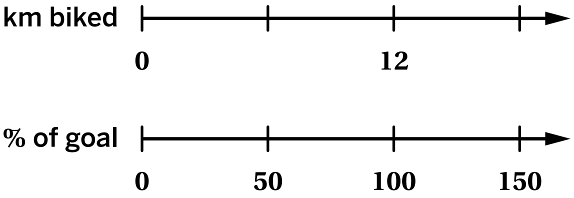
Bicycle Goals (continued)

5. Basheera's goal was to ride 12 kilometers.

Her app says she rode 150% of her goal.

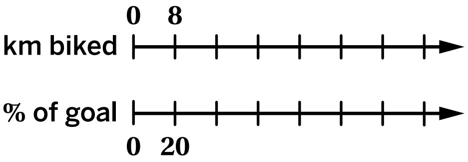
How far did she ride?





6. Callen's app says he biked 8 kilometers, which is 20% of his goal.

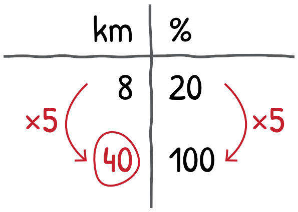
What was his goal distance?



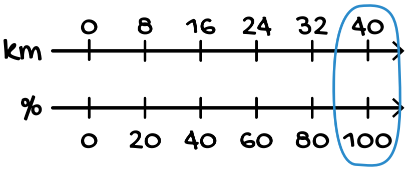
7. Here are two different strategies for calculating the goal when 20% of the goal is 8 kilometers.

💬 **Discuss:** How did each student use ratios to calculate the goal?

Melissa



Darryl



Name: .... Date: .... Period: ....

Percentages and Ratios

8. Match the strategies that represent the same percentage problem.

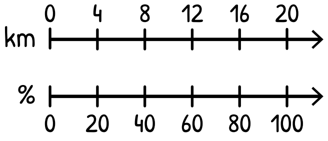
a

km	%
80	100
8	10
16	20

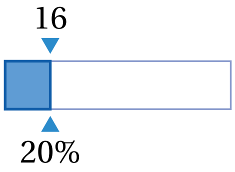
b

km	%
20	100
2	10
16	80

c



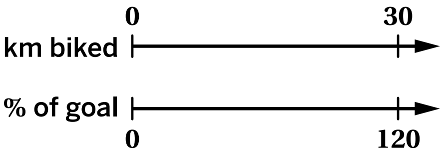
d



20% of 80 km	80% of 20 km

9. Miko's app says he biked 30 kilometers, which is 120% of his goal.

What was his goal distance?



You're invited to explore more.

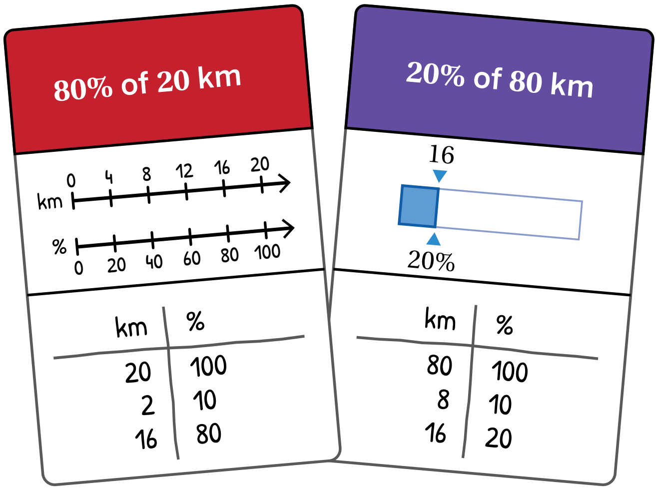
10. How many ways can you make a biker ride 10 kilometers?

Enter a goal and a percent of that goal for the biker to ride.

Goal				
Percent of Goal				

Synthesis

11. Describe how solving problems with percentages is like solving problems with ratios.



Lesson Practice 3.09

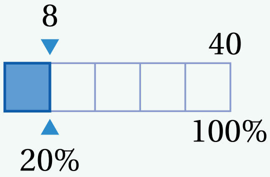
Lesson Summary

You can represent percentages using tape diagrams, double number lines, and tables. The strategies you've already used to solve ratio problems can help you think about and solve percentage problems, too!

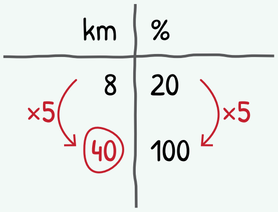
Let's say a biker traveled 8 kilometers, which is 20% of the biker's goal distance. What's the biker's goal distance?

Here are three ways to represent and solve this percentage problem.

Tape Diagram

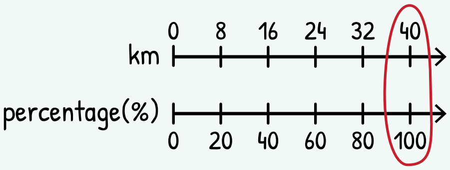


Table



km	%
8	20
40	100

Double Number Line



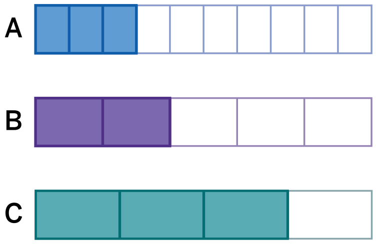
So the biker's goal distance is 40 kilometers.

Lesson Practice
3.09

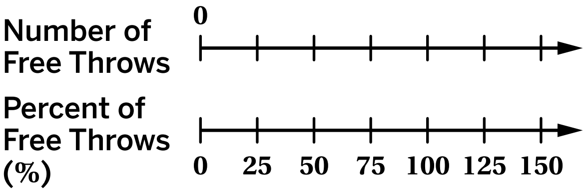
Name: .... Date: .... Period: ....

1. What percent of each figure is shaded? Record your answers in the table.

Figure A	
Figure B	
Figure C	



2. Martina shot 40 free throws at basketball practice. 25% of her free throws went into the basket. How many of them went into the basket? Use the double number line if it helps with your thinking.



Problems 3–4: Leonardo works as a server in a restaurant. He gets tipped 20% of the cost of each order.

3. What tip will he get if the food costs $60?

4. Leonardo got an $18 tip. What was the cost of the food for this order?

5. Nikhil says that to determine 20% of a number, you can divide the number by 5. For example, 20% of 60 is 12 because $60 \div 5 = 12$. Does Nikhil's method always work? Explain your thinking.

Lesson Practice
3.09

Name: .... Date: .... Period: ....

FAST Practice

6. On Tuesday, Parv made 12 cookies. On Wednesday, he made 150% as many cookies as he made on Tuesday. How many cookies did Parv make on Wednesday?

Parv made ☐ cookies on Wednesday.

Spiral Review

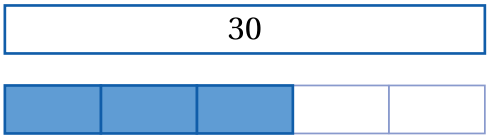
Problems 7–8: Light travels about 180,000,000 kilometers in 10 minutes.

7. How many kilometers per minute is that?

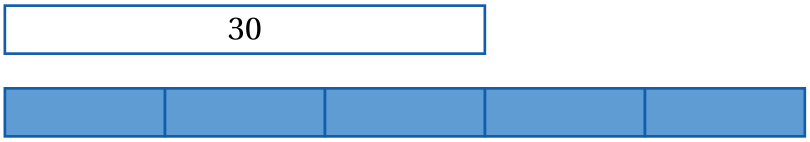
8. How many kilometers per *second* is that?

9. Match each expression with the tape diagram that represents it.

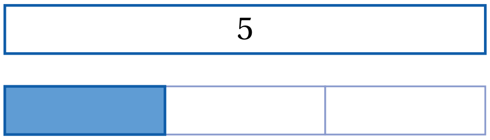
a



b



c



.... $\frac{3}{5} \cdot 30$

.... $\frac{1}{3} \cdot 5$

.... $\frac{5}{3} \cdot 30$

Unit 3

Lesson

10

Name: ______________________ Date: __________ Period: __________

MA.6.NSO.3.5, MA.6.AR.3.4, MTR.5.1

What's Missing?

Let's use ratio reasoning to find unknown quantities.

Warm-Up

Evaluate each expression mentally.

1. $\frac{3}{10} \cdot 20$

2. $\frac{3}{10} \cdot 25$

3. $\frac{3}{10} \cdot 5$

4. $\frac{3}{10} \cdot \frac{5}{2}$

Name: .... Date: .... Period: ....

Card Sort: What's Missing

5. You will use a set of cards for this activity. Match each card to its place in the table. Then fill in all the empty spaces that remain.

	Question	Representation	Answer
a	I have a 40% off coupon. If I use it to buy a shirt that costs $20, how much money would I save?		
b	I have a 20% off coupon. If I use it to buy a shirt and save $40, what was the original price of the shirt?		
c	I paid $40 for a jacket with an original price of $50. What percent of the original price did I pay?		
d			

Activity
2

Name: Date: Period:

Sale Price and Original Price

6. Complete the table.

	Question	Representation	Answer
a	Eliza bought a hat for \$21. The original price is \$30. What percent of the original price did she pay?		
b	A discount store sells items at 80% of the original price. If the original price of pants is \$55, what is the sale price?		
c	A discount store sells items at 80% of the original price. If the sale price of sneakers is \$96, what is the original price?		

You're invited to explore more.

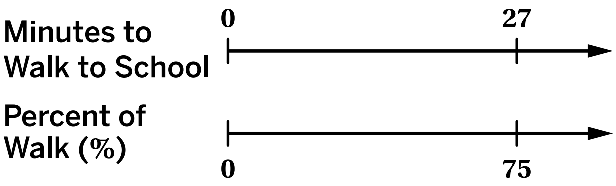
7. Precious biked 125% of her daily goal on Monday. If Precious met her weekly goal, what percent of her total weekly goal did she bike on Monday?

Precious's Biking Goals

Day	Su	M	T	W	Th	F	S
Goal (km)	0	8	4	10	0	8	20

Synthesis

8. Explain how this double number line can help you calculate the total time Eliza takes to walk to school.



Lesson Practice 3.10

Lesson Summary

You can use tables, tape diagrams, and double number lines to solve percentage problems.

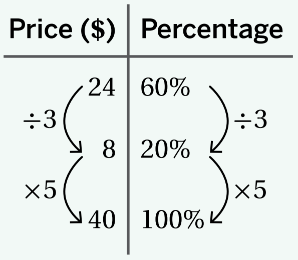
There are three main types of percentage problems.

- Determine the whole when you're given the part and the percentage.
- Determine the percentage when you're given the part and the whole.
- Determine the part when you're given the percentage and the whole.

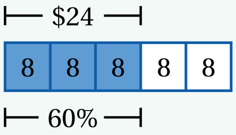
Let's say the sale price of a sweater is $24. The sweater is on sale for 60% of the original price. So how much did the sweater cost before the sale?

Here are three representations that you can use to find the whole (the original price of the sweater) given the part and the percentage.

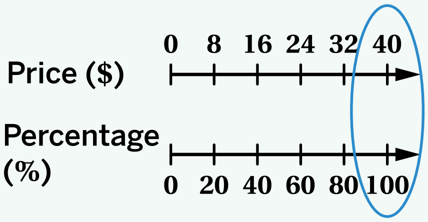
Table



Tape Diagram



Double Number Line



Lesson Practice
3.10

Name: .... Date: .... Period: ....

Problems 1–4: Evaluate each percentage problem.

1. 100% of 40

2. 50% of 40

3. 150% of 40

4. 10% of 40

Problems 5–6: A hardware store offers customers a coupon for $25 off.

5. The original price of a power drill is $125. If a customer uses the coupon, what percent will he save?

6. The original price of a ladder is $250. If a customer uses the coupon, what percent will he save?

Problems 7–8: Kiri is curious how many people think it will rain tomorrow. She asks 30 students in her class.

7. 12 students in Kiri's class say they think it will rain tomorrow. What percent of the class is that?

8. Kiri's older brother also asks the 25 students in his class. 11 students say they think it will rain tomorrow. Which class has a greater percent of students who think it will rain tomorrow?

A. Kiri's class

B. Kiri's brother's class

C. Same percent

Explain your thinking.

Lesson Practice

3.10

Name: ________ Date: ________ Period: ________

FAST Practice

9. Match the correct price with each situation.

Situation	$30	$50
A jacket is on sale for 25% off its original price of $40. What is the sale price?	☐	☐
A sweater costs $35, which is 30% off its original price. What was the original price?	☐	☐
A store has a 40% discount on all items. If the original price of a customer's items is $75, how much money will she save?	☐	☐

Spiral Review

Problems 10–11: Afia is 1,422 millimeters tall. Note: 10 millimeters = 1 centimeter.

10. What is Afia's height in centimeters?

11. What is her height in meters?

Unit 3

Lesson

11

Name: ______________________ Date: __________ Period: __________

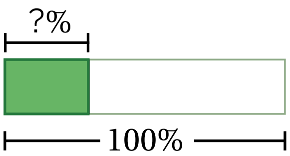
MA.6.NSO.3.5, MA.6.AR.3.4, MTR.3.1, MTR.4.1

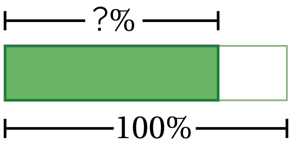
Cost Breakdown

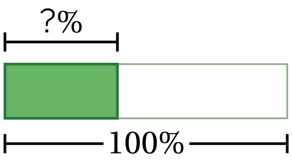
Let’s calculate any percentage of a number.

Warm-Up

1. For each challenge, write your best estimate of the missing percent.







Activity 1

Name: .... Date: .... Period: ....

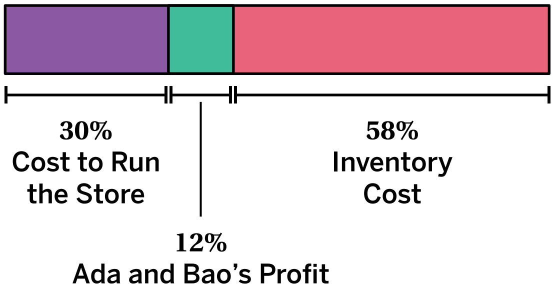
Break It Down

2. Ada and Bao run a clothing store.

The price of each item includes the profit that Ada and Bao make, the cost of inventory, and the cost to run the store.

The diagram shows where the money goes when Ada and Bao sell a T-shirt.

What do you notice? What do you wonder?



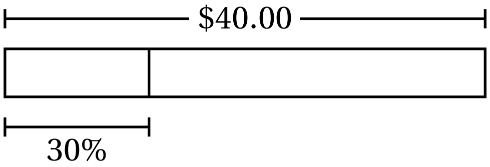
I notice:

I wonder:

3. 30% of the price of each shirt goes to running the store.

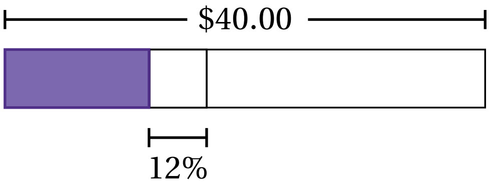
How much of a $40 shirt goes to running the store?

Use the tape diagram if it helps with your thinking.



4. Ada and Bao keep 12% of the price of each shirt as profit.

What is their profit on a $40 shirt?

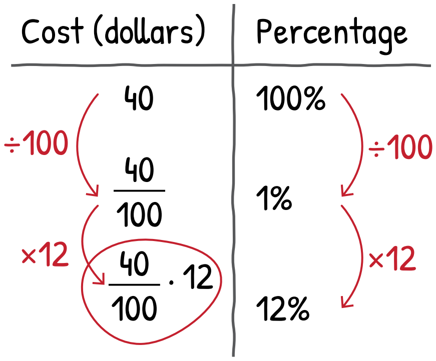


Name: .... Date: .... Period: ....

Break It Down (continued)

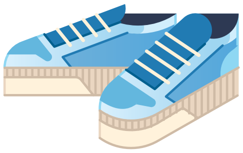
5. Here is how Bao calculated 12% of \$40.

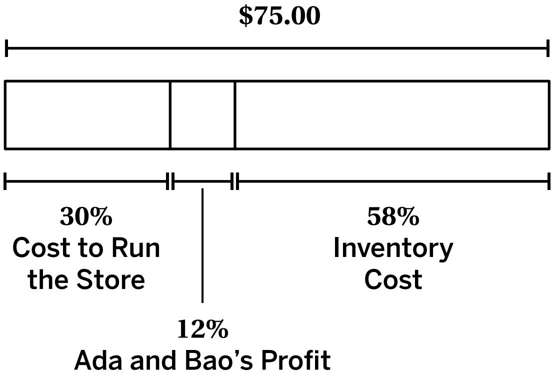
Explain how you could use Bao's strategy to calculate how much of the price of each shirt goes to inventory cost (58% of \$40).



6. Here is a \$75 pair of shoes. Calculate each value.

Cost to Run the Store	
Ada and Bao's Profit	
Inventory Cost	



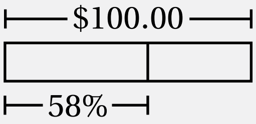
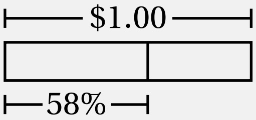
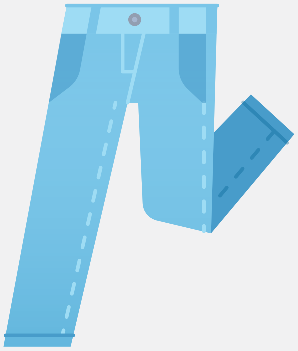
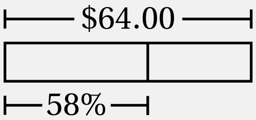
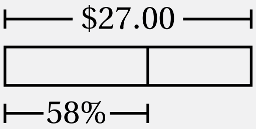


Activity
2

Name: .... Date: .... Period: ....

Another Strategy

7. Ada and Bao's inventory costs are 58% of the total cost. What is their inventory cost for each item in this table?

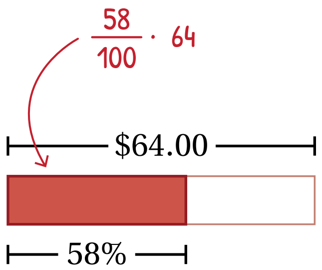
Item	Representation	Total Cost (dollars)	Inventory Cost (dollars)
Dress 		$100	
Sticker 		$1	
Jeans 		$64	
Hat 		$27	

Name: .... Date: .... Period: ....

Another Strategy (continued)

8. Ada thinks of 58% as 58 cents for every dollar. So he writes $\frac{58}{100} \cdot 64$ to calculate 58% of \$64.

Describe how Ada might calculate 36% of \$15.



9. Match each expression with a question. One expression will have no match.

a $\frac{36}{100} \cdot 70$

b $\frac{15}{100} \cdot 36$

c $\frac{100}{36} \cdot 48$

d $\frac{36}{100} \cdot 15$

e $70 \div 100 \cdot 36$

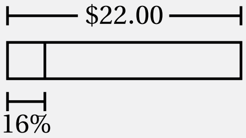
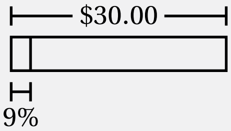
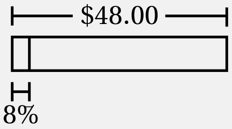
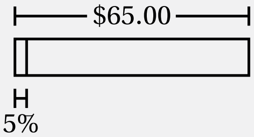
f $\frac{36}{100} \cdot 48$

What is 36% of \$15?	What is 36% of \$48?	What is 36% of \$70?

Name: ____________ Date: ________ Period: ________

Repeated Challenges

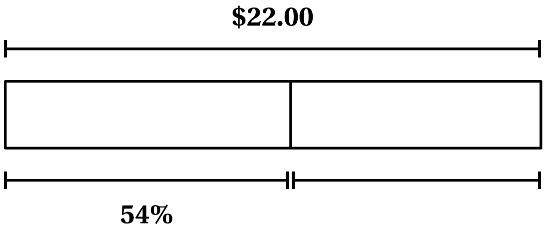
10. Solve as many challenges as you have time for.

Problem	Representation	Answer
The price of a space T-shirt is $22. 16% of every sale goes to material cost. Calculate the material cost.		
The price of a striped button-up shirt is $30. 9% of every sale goes to clothing company profit. Calculate the clothing company profit.		
The price of a striped long-sleeve T-shirt is $48. 8% of every sale goes to transport cost. Calculate the transport cost.		
The price of a blue pair of shoes is $65. 5% of every sale goes to factory profit. Calculate the factory profit.		

Synthesis

11. Describe a strategy for calculating a percentage of a number.

Use the example if it helps to explain your thinking.



Lesson Practice 3.11

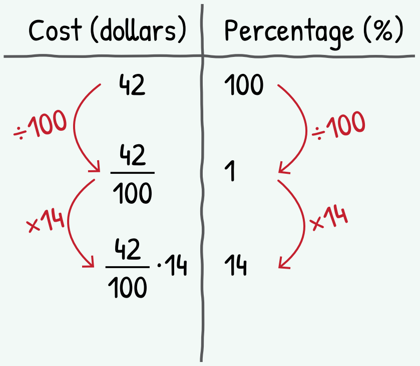
Lesson Summary

When solving percentage problems related to money, you can:

- Determine the value of 1% and multiply that by the percentage you're looking for.
- Determine how many cents per dollar a given percentage represents.

Let's say a pair of pants costs $42. If the factory makes a *profit* of 14% on the price of a pair of pants, how many dollars of profit does the factory make from each sale?

Strategy 1



$$\frac{42}{100} \cdot 14 = 5.88$$

The factory makes $5.88 of profit from each sale.

Strategy 2

- 14% profit means 14 cents of each dollar is profit.

  $\frac{14}{100} = 0.14$
- The price of the pants is $42.
- $\frac{14}{100} \cdot 42 = 5.88$

The factory makes $5.88 of profit from each sale.

Lesson Practice
3.11

Name: .... Date: .... Period: ....

Problems 1–4: Evaluate each expression.

1. 50% of 70

2. 10% of 70

3. 1% of 70

4. 2% of 70

5. A store is having a 30% off sale. The original price for a pair of headphones is $150. How much would a customer save with this sale?

6. Order the following expressions from *least* to *greatest* value.

55% of 180	300% of 26	12% of 700

Least **Greatest**

7. To find 40% of 75, Jamal calculates $\frac{2}{5} \cdot 75$. Does his calculation give the correct value for 40% of 75? Explain your thinking.

8. Emika has a monthly budget for her cell phone bill. Last month, she spent 120% of her budget, and the bill was $60. What is Emika's monthly budget?

9. Kyrie spent 75 minutes practicing the piano over the weekend. Yasmine practiced the violin for 152% as much time as Kyrie practiced the piano. How long did Yasmine practice?

Lesson Practice
3.11

Name: .... Date: .... Period: ....

10. Fill in each blank using the numbers 0 to 9 only once, so that the expression on the left is greater than the expression on the right.

Left	**Right**
☐☐ **% of 50**	**50% of** ☐☐

FAST Practice

11. Select *all* the expressions that could be used to calculate 45% of 60.

- ☐ **A.** $\frac{100}{45} \bullet 60$
- ☐ **B.** $\frac{60}{45} \bullet 100$
- ☐ **C.** $\frac{45}{100} \bullet 60$
- ☐ **D.** $\frac{100}{60} \bullet 45$
- ☐ **E.** $\frac{0.45}{100} \bullet 60$
- ☐ **F.** $\frac{60}{100} \bullet 45$

Spiral Review

12. Two stores sell identical sandwich rolls in different-sized packages. Store A sells a six-pack for $\$5.28$. Store B sells a four-pack for $\$3.40$. Which store offers the better price per roll?

(A.) Store A (B.) Store B (C.) They are the same

Show or explain your thinking.

Unit 3

Lesson

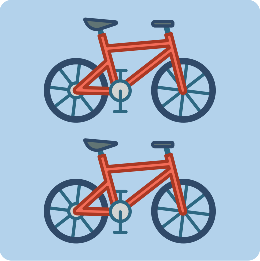
12

Name: ____________ Date: ________ Period: ________

MA.6.NSO.3.5, MA.6.AR.3.4, MTR.3.1, MTR.4.1

More Bicycle Goals

Let's calculate unknown percentages.



Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. $\frac{1}{3}$ of $\frac{1}{4}$

2. $\frac{1}{3} \cdot \frac{1}{4}$

3. $\frac{2}{3} \cdot \frac{1}{4}$

4. $\frac{2}{3} \cdot \frac{5}{4}$

Activity
1

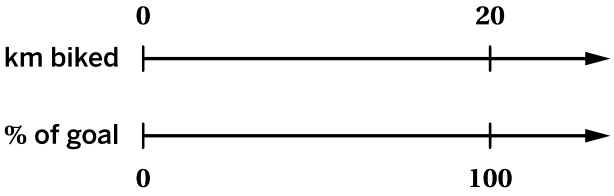
Name: .... Date: .... Period: ....

Chasing Goals

5. Alejandro's goal for Monday was to ride 20 kilometers.

His app says he rode 40% of his goal.

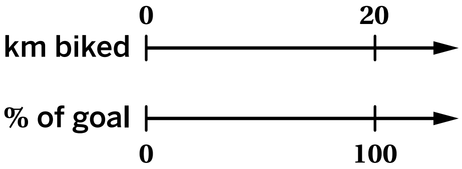
How far did he ride?



6. Alejandro's goal for Tuesday was to ride 20 kilometers.

His app says he rode 10 kilometers.

What percent of his goal did he ride?



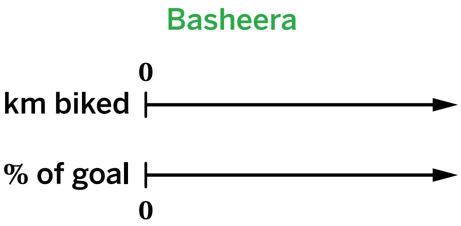
7. On Wednesday, Alejandro and Basheera rode 17 kilometers.

- Alejandro's goal was 20 kilometers.
- Basheera's goal was 50 kilometers.

Who rode a greater percent of their goal? Circle one.

Alejandro Basheera Same percent

Explain your thinking.



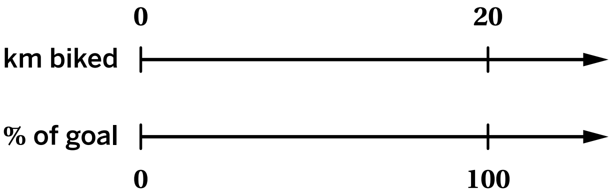
Name: .... Date: .... Period: ....

Chasing Goals (continued)

8. On Wednesday, Alejandro and Basheera rode 17 kilometers.

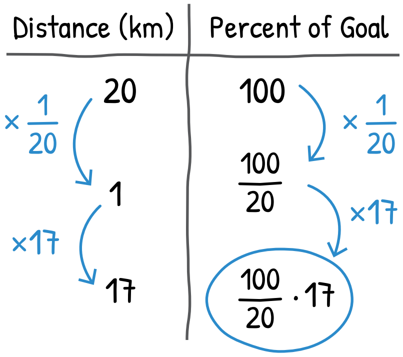
Alejandro's goal was 20 kilometers.

What percent of his goal did he ride?



9. Here is how Alejandro calculated 17 out of 20 as a percentage.

Explain how you could use Alejandro's strategy to calculate 13 out of 20 as a percentage.



Activity
2

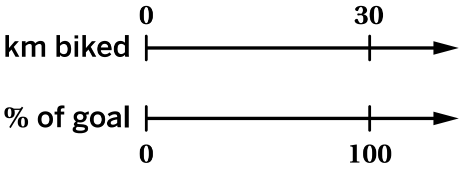
Name: .... Date: .... Period: ....

Reaching Goals

10. Alejandro and Basheera set a new goal for Saturday: 30 kilometers.

They rode 36 kilometers.

What percent of their goal did they ride?



11. Here are the expressions Alejandro and Basheera used to calculate 36 out of 30 as a percentage.

Whose expression is correct? Circle one.

Alejandro

Basheera

Both

Neither

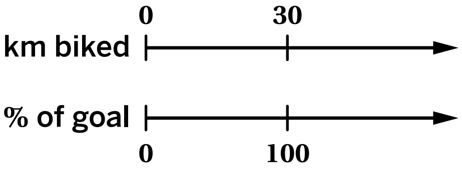
Explain your thinking.

Alejandro

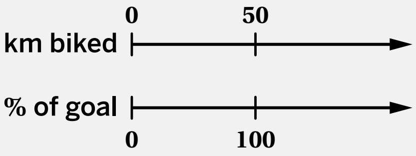
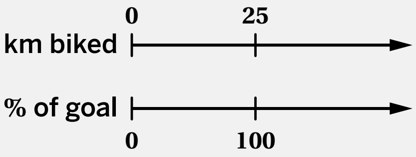
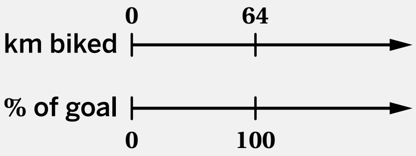
$$\frac{100}{30} \cdot 36$$

Basheera

$$\frac{36}{30} \cdot 100$$



12. On Sunday, Alejandro, Basheera, and Callen rode 40 kilometers. They each had different goals, as shown in the table. Calculate what percent of their goal each person rode.

Representation	Distance Traveled (km)	Goal (km)	Percent of Goal
Alejandro 	40	50	
Basheera 	40	25	
Callen 	40	64	

Name: .... Date: .... Period: ....

Reaching Goals (continued)

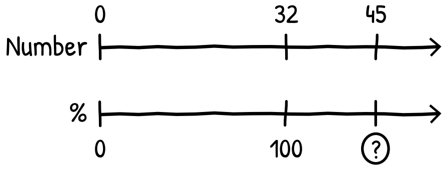
13. Match each question with a double number line and an expression. One choice will have no match.

a $\frac{32}{100} \cdot 45$

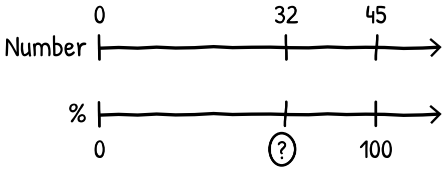
b $\frac{32}{45} \cdot 100$

c $\frac{45}{32} \cdot 100$

d



e

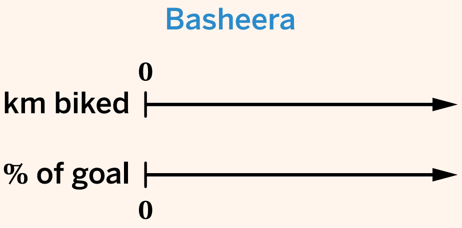


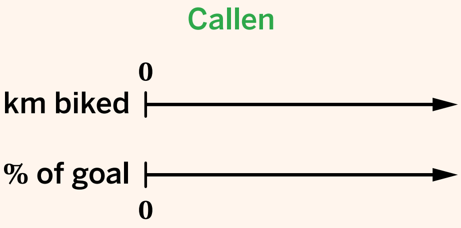
What is 32 out of 45 as a percentage?	What is 45 out of 32 as a percentage?

You're invited to explore more.

14. Basheera had a goal of riding 30 kilometers and rode 51 kilometers.
Callen had a goal of 20 kilometers and rode 41 kilometers.

Determine who rode a greater percent of their goal, in as many different ways as you can.





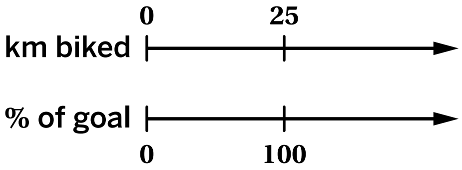
Explain your thinking.

Synthesis

15. Here is what Basheera wrote to solve a bicycle challenge.

What do 31, 25, and 124 represent in this scenario?

$$\frac{31}{25} \cdot 100 = 124$$



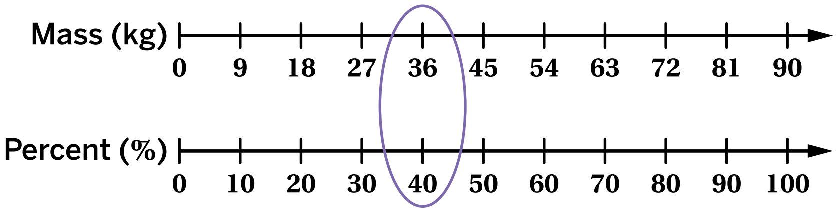
Lesson Practice 3.12

Lesson Summary

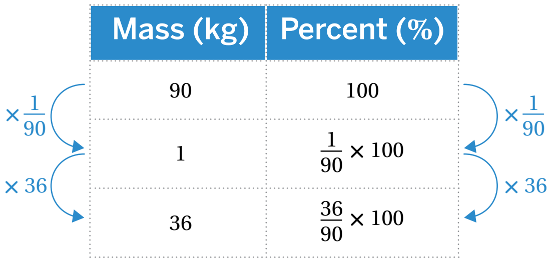
You can use ratios to determine what percent one amount is compared to another amount.

Let's say an adult giant panda weighs 90 kilograms and a giant panda cub weighs 36 kilograms. You can determine the cub's weight as a percent of the adult's weight using several strategies.

Double Number Line



Ratio Tables



Mass (kg)	Percent (%)
90	100
1	$\frac{1}{90} \times 100$
36	$\frac{36}{90} \times 100$

- Determine the unit rate (what percent matches 1 kilogram).
- Use the unit rate to determine what percent 36 kilograms is of 90 kilograms.

Expressions

$$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$$

Evaluate $\frac{p}{w} \cdot 100$ to determine what percent one value, p, is of the other value, w.

Lesson Practice
3.12

Name: .... Date: .... Period: ....

1. At a hardware store, a tool set normally costs $80. During a sale this week, the tool set costs $12 less than normal. What percent of the original price can a customer save? Show or explain your thinking.

Problems 2–4: A 6th grade class did a weekend fitness challenge. Each student set a goal for 75 minutes of exercise.

2. Luca exercised for 54 minutes. What percent of the goal did she complete?

3. Brianna completed 64% of her goal. How many minutes did she exercise for?

4. Amari exercised for 78 minutes. What percent of the goal did she complete?

FAST Practice

5. Select *all* the expressions that represent what percent 19 is of 20.

☐ **A.** $\frac{19}{20} \bullet 100$

☐ **B.** $\frac{19}{20} \div 100$

☐ **C.** $\frac{20}{19} \bullet 100$

☐ **D.** $19 \bullet \frac{100}{20}$

☐ **E.** $\frac{19}{100} \bullet 20$

Lesson Practice
3.12

Name: ________ Date: ________ Period: ________

Spiral Review

Problems 6–9: Determine each product.

6. $6.4 \bullet 0.1$

7. $6.4 \bullet 0.01$

8. $6.4 \div 0.1$

9. $6.4 \div 0.01$

Unit 3

Lesson

13

Name: ______________________ Date: __________ Period: __________

MA.6.NSO.3.5, MA.6.AR.3.4, MTR.1.1, MTR.7.1

A County as a Village

Let’s explore different Florida counties.

Warm-Up

1. Here are some facts about Broward County from the year 2020.

What do you notice? What do you wonder?

66 out of 100 people have a job.

400 thousand people are under 18 years old.

4% are military veterans

22 out of 25 people lived in the same house 1 year ago.

Activity
1

Name: .... Date: .... Period: ....

Exploring Broward County

2. The population of Broward County was about 1,944 thousand people in 2020.

How many people in Broward County have each of these characteristics?

Population of Broward County: 1,944 thousand

66 out of 100 people have a job.

400 thousand people are under 18 years old.

4% are military veterans

22 out of 25 people lived in the same house 1 year ago.

Characteristic	Number of People (thousands)
Have a job	
Lived in the same house 1 year ago	
Military veteran	
Under 18 years old	

3. It's hard to picture 1,944 thousand people.

Imagine Broward County was a village with just 100 people.

How many people would have each of these characteristics?

Characteristic	Number of People in Village
Have a job	
Lived in the same house 1 year ago	
Military veteran	
Under 18 years old	

Name: .... Date: .... Period: ....

A County as a Village

4. You and your partner will need the Activity 2 Sheet for this activity.

Pick a county. Use the information from 2020 on the Activity 2 Sheet to make a poster describing what this county would look like as a 100-person village.

Miami-Dade Escambia Lake Hillsborough Duval

Be sure to include these items in your poster:

☐ Your names and the name of the county you selected.

☐ An answer to the question: *If this county were a village of 100 people, how many of them would have each of these characteristics?*

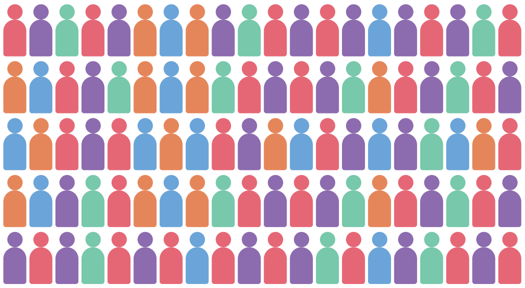
☐ Your thinking and calculations for each characteristic.

☐ Something that you are curious to learn more about.

☐ *At least two* other characteristics of this county you are interested in knowing about.

Synthesis

5. How is working with percentages like working with a village of 100 people?



Lesson Practice 3.13

Lesson Summary

Working with real-world data and information can be interesting, but it presents challenges, like working with very large numbers or information presented in different forms.

Ratios, rates, and percentages can help you make sense of real-world situations and compare very large numbers.

Some of the benefits of ratios, rates, and percentages are:

- They allow you to compare quantities that are on different scales because they describe things in terms of multiplying and dividing instead of adding and subtracting.
- They bring everything to the same scale, most commonly with a reference point of either 1 or 100, which makes comparing numbers more straightforward.

For example, percentages can help us compare different-sized groups of people around the state to see what the distribution of people really looks like.

Lesson Practice

3.13

Name: .... Date: .... Period: ....

Problems 1–3: The sale price of every item in a store is 85% of its original price. Complete the table to show the prices of each item.

	Item	Original Price ($)	Sale Price ($)
1.	Backpack	30.00	
2.	Soccer Ball		15.30
3.	Jacket		21.08

Problems 4–6: Last Sunday, an amusement park had 1,575 visitors.

4. 56% of the visitors were adults. Calculate the number of adults that visited the park.

5. 16% of the visitors were teenagers. Calculate the number of teenagers that visited the park.

6. 28% of the visitors were children ages 12 and under. Calculate the number of children ages 12 and under that visited the park.

Lesson Practice
3.13

Name: .... Date: .... Period: ....

FAST Practice

7. A veterinarian examined 45 animals on Wednesday. Of the animals she examined, 20% were cats. How many cats did the veterinarian examine on Wednesday?

- A. 18
- B. 9
- C. 5
- D. 14

Spiral Review

Problems 8–10: Fill in each blank to complete the sentence.

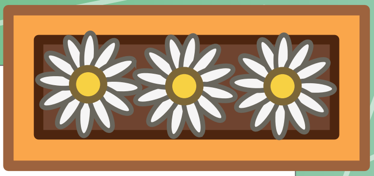
8. 5% of 70 is ....

9. 25% of .... is 6.

10. 12% of 700 is ....

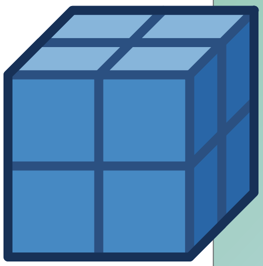
Unit 4

Multiplying and Dividing Fractions

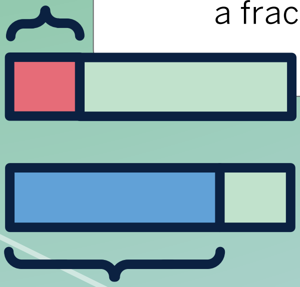


In multiplication, you determine the total amount from equal groups. Division is useful for answering questions like “How many groups?” or “How many in 1 group?” You’ve already multiplied and divided using whole numbers, but how do you multiply or divide when the number of groups is a fraction? Let’s find out!

Essential Questions



- What are two ways to think about multiplying by a fraction?
- How are division and multiplication related to each other?
- In what situations would there be fraction-sized groups or a number of groups that is a fraction?



Sub-unit

1

Multiplying Fractions

Lesson 1
Modeling Products

Lesson 2
Mixing It Up

Unit 4

Lesson

1

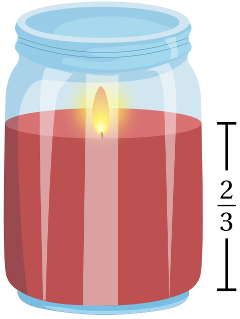
Name: ______ Date: ______ Period: ______

MA.6.NSO.2.2, MTR.2.1, MTR.5.1, MTR.6.1

Modeling Products

Let's explore strategies for multiplying fractions.

Warm-Up



1. As a candle burns, its wax turns to gas and evaporates. This candle has $\frac{2}{3}$ of its wax left in a jar. After lighting the candle, it burns $\frac{5}{6}$ of the remaining wax. How much of the wax burned away this time?

What do you notice? What do you wonder?

I notice:

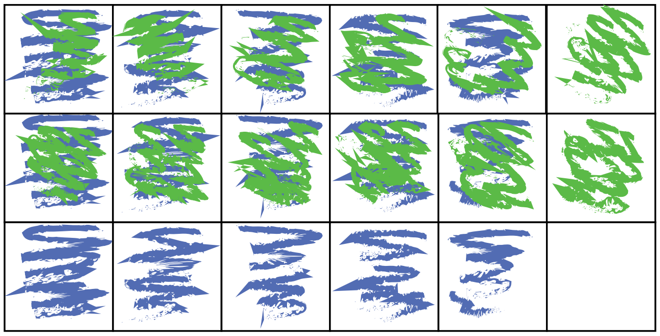
I wonder:

Name: .... Date: .... Period: ....

Making Scents

2. Ama and Kweku each created a representation to show the amount of candle wax that burned away.

Ama



Kweku

$\frac{5}{6} \times \frac{2}{3}$

Whose representation is correct? Circle one.

Ama Kweku Both Neither

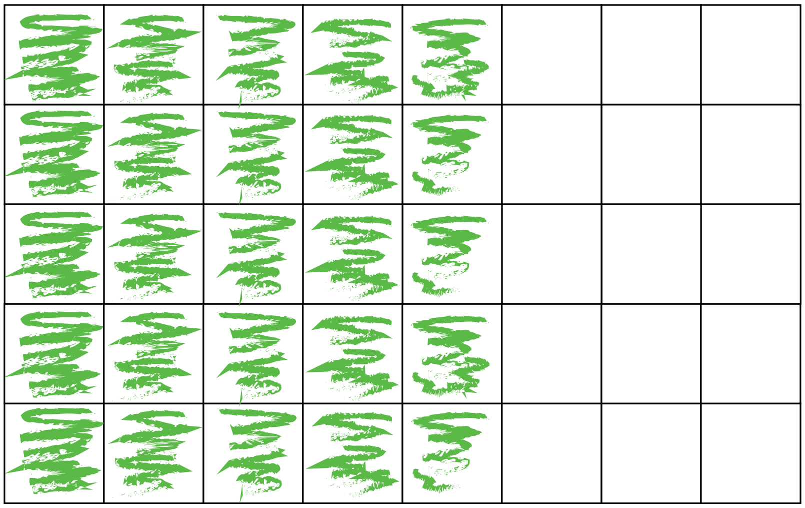
Explain your thinking.

3. Mauricio says the *product* of $\frac{5}{6} \bullet \frac{2}{3}$ is $\frac{10}{18}$. Show or explain why this answer makes sense.

4. Here is a partially completed area model for the expression $\frac{5}{8} \bullet \frac{2}{5}$.

a Draw on the model to represent the entire expression.

b What is the product?

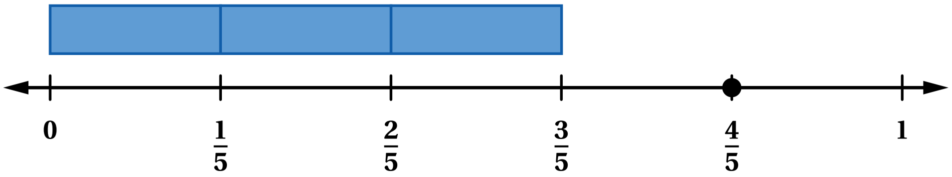


Activity 2

Name: .... Date: .... Period: ....

Many Models

5. This number line represents $\frac{4}{5} \cdot \frac{3}{4}$.



a What is the product?

b Create a tape diagram to represent this expression.

c **Discuss:** What are the advantages of using a model to represent fraction multiplication? How could you multiply fractions without using a model?

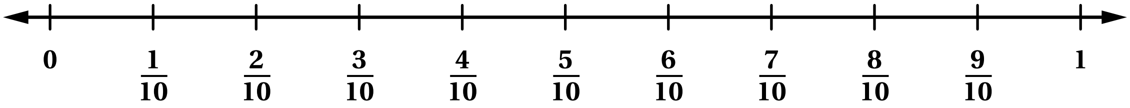
Activity
3

Name: .... Date: .... Period: ....

Multiplying Fractions

6. **a** Calculate $\frac{8}{10} \cdot \frac{3}{4}$.

Use a model if it helps with your thinking.



b 💬 **Discuss:** What was your strategy?

7. Solve as many challenges as you have time for.

a $\frac{7}{8} \cdot \frac{5}{6}$

b $\frac{2}{9} \cdot \frac{3}{5}$

c $\frac{3}{10} \cdot \frac{10}{11}$

d $\frac{2}{3} \cdot \frac{8}{9}$

e $\frac{5}{7} \cdot \frac{3}{4}$

f $\frac{5}{6} \cdot \frac{6}{11}$

g $\frac{3}{8} \cdot \frac{7}{10}$

h $\frac{5}{9} \cdot \frac{2}{7}$

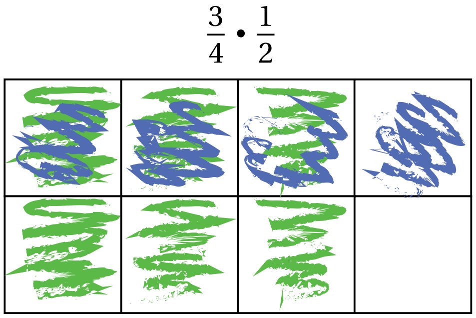
i $\frac{8}{11} \cdot \frac{7}{8}$

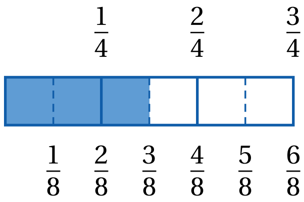
j $\frac{3}{4} \cdot \frac{9}{10}$

Synthesis

8. Describe how to multiply two fractions.

Use the examples if they help with your thinking.



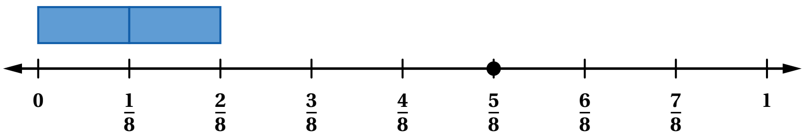


Lesson Practice 4.01

Lesson Summary

You can use models or a numerical expression to represent fraction multiplication and determine *products*.

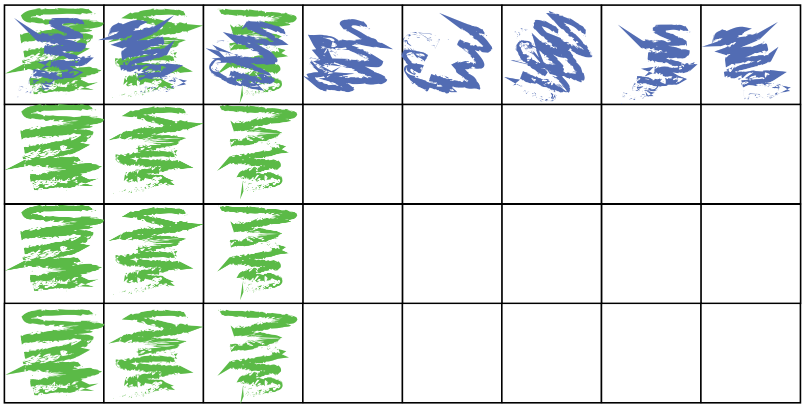
For example, a bag contains $\frac{2}{5}$ cup sesame seeds. You pour $\frac{5}{8}$ of the bag into a bowl. What fraction of sesame seeds are in the bowl?

Model	Numerical Expression
 $\frac{2}{5}$ of $\frac{5}{8}$ is $\frac{2}{8}$ or $\frac{1}{4}$	$\frac{2}{5} \cdot \frac{5}{8}$ $\frac{2 \cdot 5}{5 \cdot 8}$ $\frac{10}{40}$ or $\frac{1}{4}$

Lesson Practice
4.01

Name: .... Date: .... Period: ....

Problems 1–2: Here is an area model.



1. Write a numerical expression that the model represents.

2. Calculate the product.

3. Here is Mauricio's work for calculating $\frac{7}{9} \cdot \frac{4}{7}$. Explain why you think his work makes sense.

Mauricio

$$\frac{7}{9} \cdot \frac{4}{7}$$

$$\frac{7 \cdot 4}{9 \cdot 7}$$

$$\frac{28}{63} = \frac{4}{9}$$

Problems 4–7: Calculate each product. Use a model if it helps with your thinking.

4. $\frac{5}{6} \cdot \frac{3}{4}$

5. $\frac{2}{3} \cdot \frac{2}{3}$

6. $\frac{3}{10} \cdot \frac{3}{4}$

7. $\frac{4}{11} \cdot \frac{3}{4}$

Lesson Practice
4.01

Name: .... Date: .... Period: ....

FAST Practice

8. A car's gasoline tank is $\frac{3}{4}$ full. Then $\frac{2}{5}$ of this amount is used.

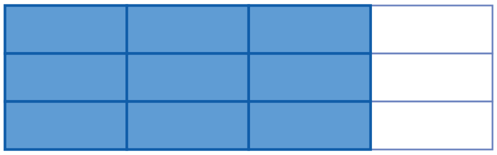
Write an expression to describe the amount of gasoline used.

Spiral Review

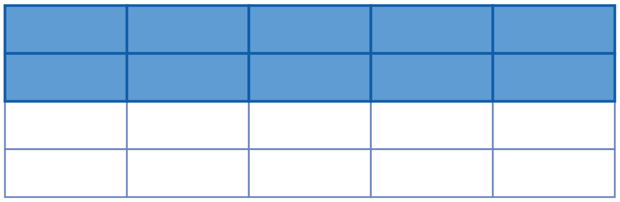
Problems 9–11: Determine the fraction of each model that is shaded.

9.

10.



11.



Unit 4

Lesson

2

Name: .... Date: .... Period: ....

MA.6.NSO.2.2, MTR.2.1, MTR.5.1, MTR.6.1

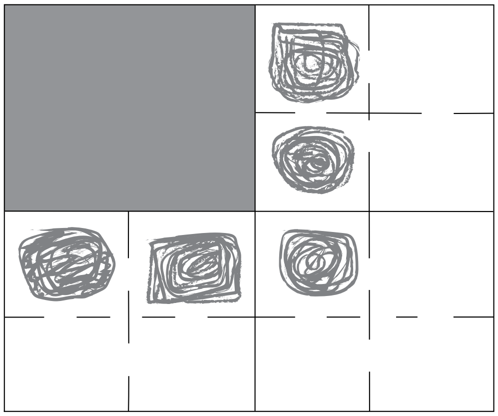
Mixing It Up

Let's explore multiplying mixed numbers.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.



B. $1\frac{1}{2} \cdot 1\frac{1}{2}$

C. $\frac{9}{4}$

D. $1\frac{1}{2}\left(1 + \frac{1}{2}\right)$

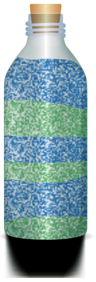
Activity
1

Name: ______ Date: ______ Period: ______

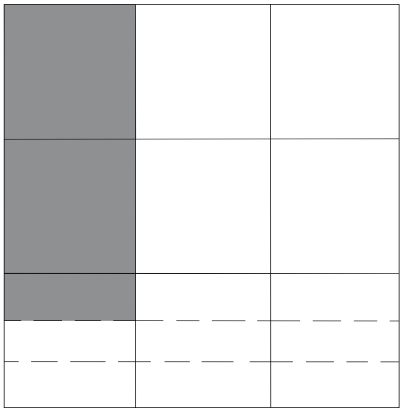
Mixing With Mixed Numbers

An activity offered at a local craft fair is creating sand art. There are also some pieces already made that people can buy.

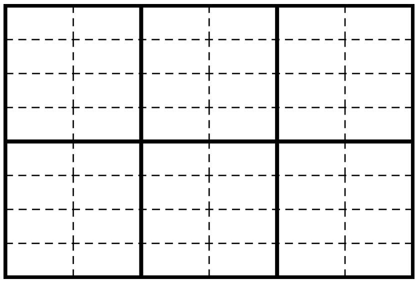
2. **a** Let's look at some of the art pieces.



b These art pieces each use $2\frac{1}{3}$ cups of blue sand. Draw on the model to show the amount of blue sand in all 3 bottles. Then calculate $2\frac{1}{3} \cdot 3$.

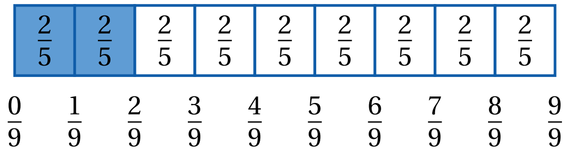


3. Calculate $1\frac{1}{4} \cdot 2\frac{1}{2}$.
Use the model if it helps with your thinking.



Mixing With Mixed Numbers (continued)

4. Mateo used a tape diagram to calculate $3\frac{3}{5} \cdot \frac{2}{9}$.



a **Discuss:** Why do you think Mateo used $\frac{2}{5}$-sized pieces?

b What is the product?

Activity
2

Name: .... Date: .... Period: ....

Making Sand Art

5. Mateo has $1\frac{3}{10}$ cups of orange sand and wants to use $\frac{1}{2}$ of the sand in the bottle.

Mateo says that he needs $\frac{1}{2} \bullet 1\frac{3}{10}$ cups of orange sand.

Precious says that Mateo needs $\left(1 + \frac{3}{10}\right)\frac{1}{2}$ cups of orange sand.

Are the values of the expressions the same? Show or explain your thinking.

6. Mateo and Precious used different strategies to calculate the amount of orange sand needed.

Mateo

$$\frac{1}{2} \bullet 1\frac{3}{10}$$

$$\frac{1}{2} \bullet \frac{13}{10}$$

$\frac{13}{20}$ cups of orange sand needed

Precious

$$\left(1 + \frac{3}{10}\right)\frac{1}{2}$$

$$\frac{1}{2} + \frac{3}{20}$$

$$\frac{10}{20} + \frac{3}{20}$$

$\frac{13}{20}$ cups of orange sand needed

Discuss: How are their strategies alike? How are they different?

Name: .... Date: .... Period: ....

Making Sand Art (continued)

7. Mateo has $2\frac{1}{5}$ cups of green sand and wants to use $\frac{1}{4}$ of this sand in the bottle.

a Use either strategy to calculate how much green sand Mateo should use.

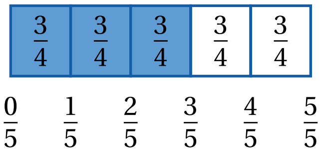
b How do you know your answer makes sense? Explain your thinking.

8. Calculate $1\frac{3}{4} \cdot 4\frac{1}{2}$.

Synthesis

9. Explain why $3\frac{3}{4} \cdot \frac{3}{5} = 2\frac{1}{4}$.

Use the model or the expression if it helps with your thinking.



$$\left(3 + \frac{3}{4}\right)\frac{3}{5}$$

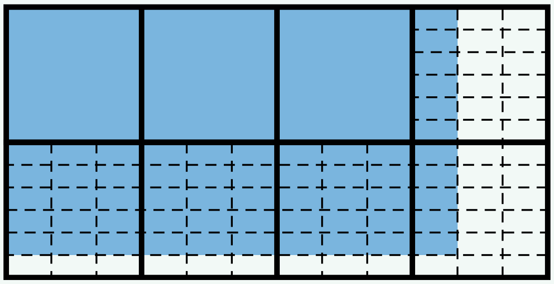
Lesson Practice 4.02

Lesson Summary

We can use a model or a numeric expression to help multiply with mixed numbers.

Area Model

$$1\frac{5}{6} \cdot 3\frac{1}{3}$$



$$6\frac{1}{9}$$

Improper Fractions

$$1\frac{5}{6} \cdot 3\frac{1}{3}$$
$$\frac{11}{6} \cdot \frac{10}{3}$$
$$\frac{110}{18}$$
$$6\frac{2}{18}$$
$$6\frac{1}{9}$$

Distributive Property

$$1\frac{5}{6} \cdot 3\frac{1}{3}$$
$$\left(1 + \frac{5}{6}\right)\frac{10}{3}$$
$$\frac{10}{3} + \frac{50}{18}$$
$$\frac{60}{18} + \frac{50}{18}$$
$$\frac{110}{18}$$
$$6\frac{2}{18}$$
$$6\frac{1}{9}$$

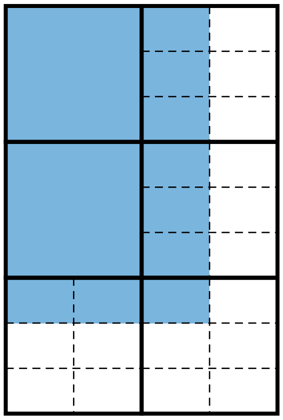
The third strategy uses the **distributive property**. The distributive property tells us that $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.

Lesson Practice

4.02

Name: .... Date: .... Period: ....

1. Select *all* the expressions that the model represents.



- ☐ **A.** $2\frac{1}{3} \cdot 1\frac{1}{2}$
- ☐ **B.** $\left(2 + \frac{1}{3}\right)\frac{3}{2}$
- ☐ **C.** $3\frac{2}{3}$
- ☐ **D.** $\left(2 \cdot \frac{1}{3}\right)\frac{3}{2}$
- ☐ **E.** $3\frac{3}{6}$

2. What is the product of $5\frac{1}{4} \cdot \frac{3}{7}$? Show or explain your thinking.

Problems 3–6: Calculate each product.

3. $3\frac{3}{4} \cdot \frac{2}{3}$

4. $6\frac{2}{5} \cdot \frac{5}{6}$

5. $4\frac{4}{5} \cdot 3\frac{5}{8}$

6. $3\frac{7}{10} \cdot 2\frac{3}{5}$

Lesson Practice
4.02

Name: .... Date: .... Period: ....

Problems 7–9: On Monday, Station 1 in a cooking class uses $2\frac{3}{4}$ cups of flour. Station 2 uses $3\frac{1}{2}$ times as much flour.

7. Use a model or a numeric expression to represent the problem.

8. How much flour does Station 2 use?

9. How do you know your answer makes sense? Explain your thinking.

FAST Practice

10. What is the product of $6\frac{1}{4} \cdot \frac{9}{10}$?

A. $5\frac{13}{20}$ **B.** $5\frac{1}{2}$ **C.** $5\frac{5}{8}$ **D.** $5\frac{2}{5}$

Spiral Review

Problems 11–13: Calculate each product.

11. $\frac{2}{5} \cdot \frac{1}{4}$

12. $\frac{5}{9} \cdot \frac{4}{5}$

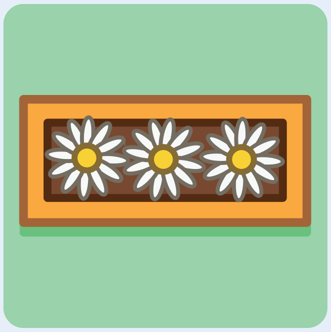
13. $\frac{3}{8} \cdot \frac{7}{10}$

Sub-unit

2

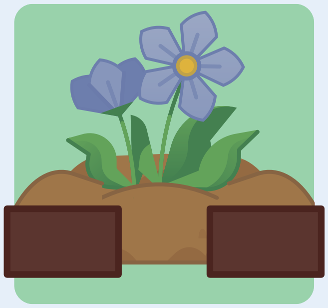
Dividing Fractions

Lesson 3
Flour Planner

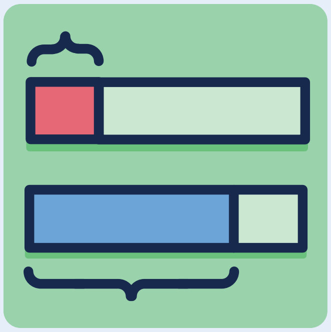


Lesson 4
Flower Planters

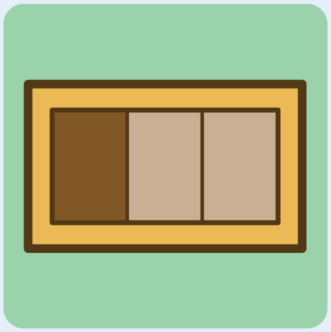
Lesson 5
Garden Bricks



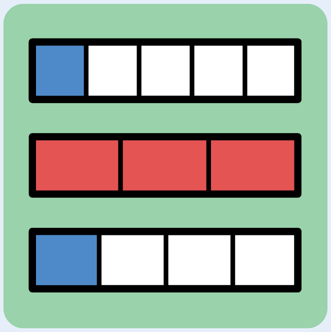
Lesson 6
Fill the Gap



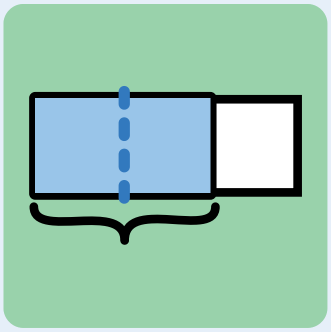
Lesson 7
Break It Down



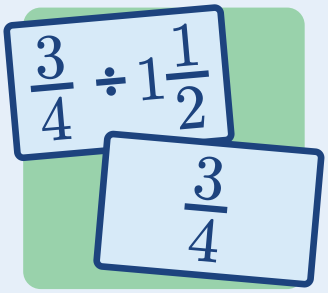
Lesson 8
Potting Soil



Lesson 9
Division Challenges



Lesson 10
Action Fractions



Lesson 11
Swap Meet

Unit 4

Lesson

3

Name: ______________________ Date: __________ Period: __________

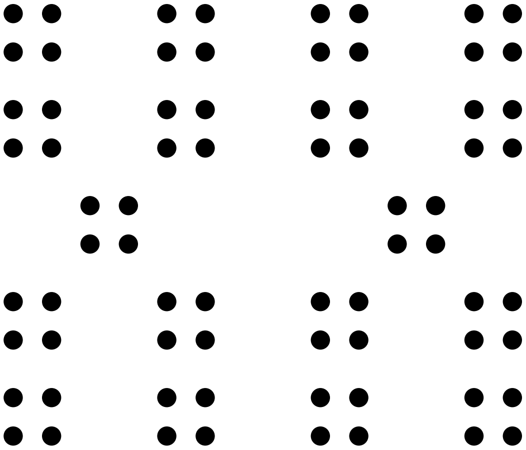
MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.2.1

Flour Planner

Let’s think about fractions by using drawings and diagrams to ask, “How many groups?”

Warm-Up

1. **a** How many dots are in this image?



b Explain or show how you saw them.

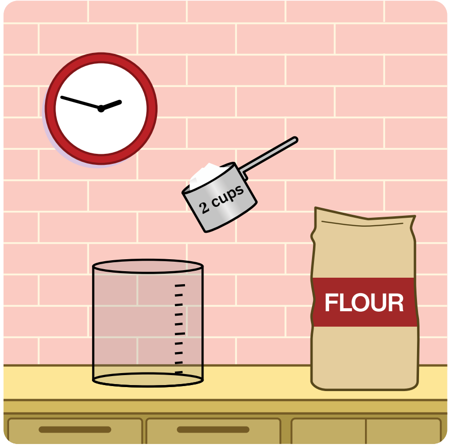
Name: .... Date: .... Period: ....

Fractional Scoops

2. Tres leches cake is a popular dessert in Mexico and Central America that's made with three kinds of milk.

Alexis needs 6 cups of flour to make tres leches cake, but he only has a 2-cup measuring scoop.

How many scoops does he need?



3. Circle an equation where the factor or **quotient** represents how many 2-cup scoops make 6 cups of flour.

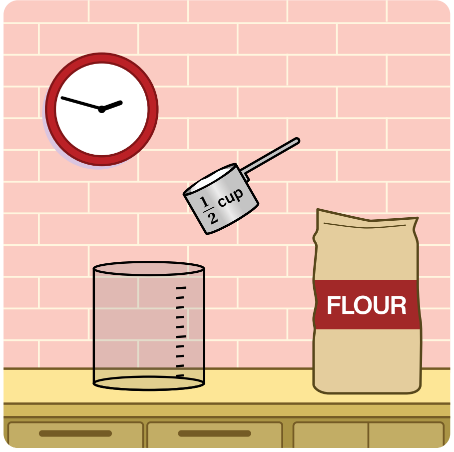
$6 \cdot ? = 2$ $\quad$ $6 \div 2 = ?$ $\quad$ $2 \div 6 = ?$ $\quad$ $2 \cdot ? = 6$

Explain your thinking.

4. LaShawn also needs 6 cups of flour to make tres leches cake.

He only has a $\frac{1}{2}$-cup measuring scoop.

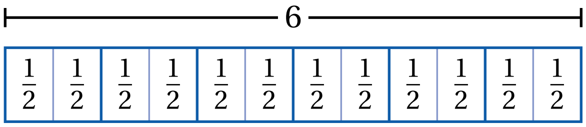
How many scoops does he need?



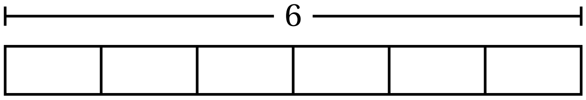
Name: .... Date: .... Period: ....

Fractional Scoops (continued)

5. **a** Consider how LaShawn determined the number of $\frac{1}{2}$-cup scoops needed for 6 cups of flour.



b Explain how this tape diagram helped LaShawn determine that he needs 12 scoops.



Activity
2

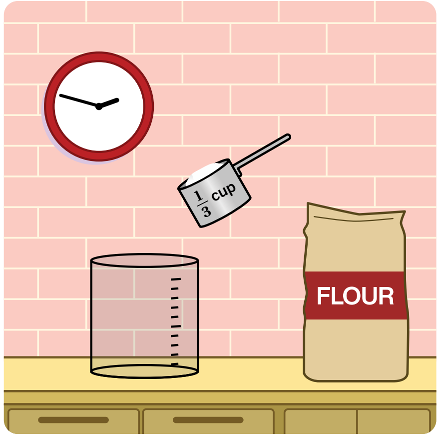
Name: .... Date: .... Period: ....

A Bigger Scoop

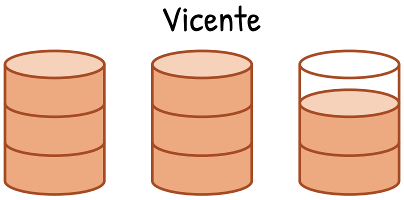
6. Sirnee is a sweet dish that is often made for celebratory feasts.

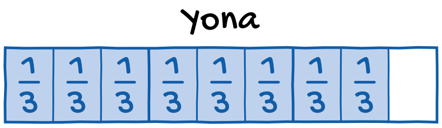
Hamza needs $2\frac{2}{3}$ cups of flour to make sirnee, but he only has a $\frac{1}{3}$-cup measuring scoop.

How many scoops does he need?



7. Vicente and Yona each sketched a diagram to determine how many $\frac{1}{3}$-cup scoops they need to measure $2\frac{2}{3}$ cups of flour.





Discuss: How could each diagram help us calculate the number of scoops needed?

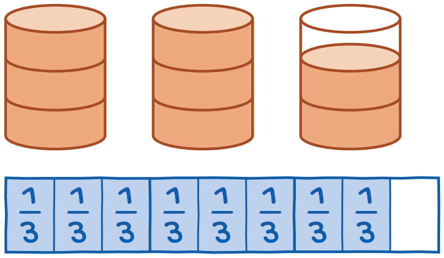
Name: .... Date: .... Period: ....

A Bigger Scoop (continued)

8. Hamza found a $\frac{2}{3}$-cup measuring scoop to use to make sirnee.

How many of these scoops would he need to measure $2\frac{2}{3}$ cups of flour?

Use the cups diagram and the tape diagram if they help with your thinking.



Explain your thinking.

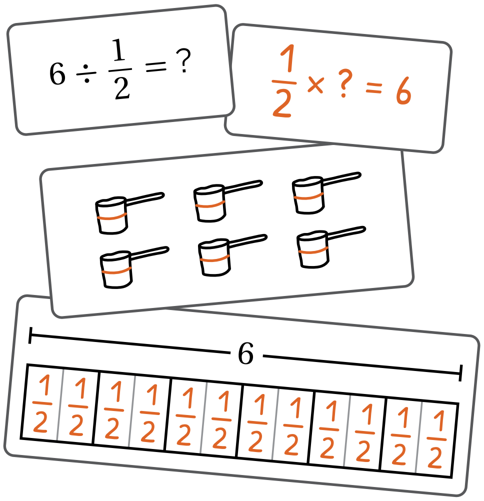
9. Group together the choices that represent the same situation. Two choices will have no match.

$3 \div \frac{3}{4} = ?$	$\frac{3}{4} \div 3 = ?$	$\frac{3}{4} \cdot ? = 3$	$3 \cdot ? = \frac{3}{4}$
$? \div \frac{3}{4} = 3$	$\frac{3}{4} \cdot 3 = ?$	4 scoops	$\frac{1}{4}$ scoops

Alexis needs 3 cups of flour. He has a $\frac{3}{4}$-cup measuring scoop.	LaShawn needs $\frac{3}{4}$ cups of flour. He has a 3-cup measuring scoop.

Synthesis

10. How can you use an equation or a diagram to determine how many $\frac{1}{2}$-cup scoops you need to make 6 cups?



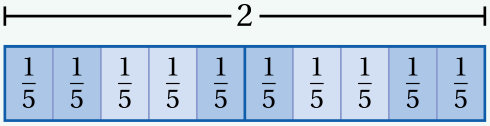
Lesson Practice 4.03

Lesson Summary

You can answer the question "How many groups?" using different representations that include both whole numbers and fractions.

Here's the problem "How many $\frac{2}{5}$s are in 2?" represented using a tape diagram, a multiplication equation, and a division equation.

Tape Diagram



Multiplication Equation

$$\frac{2}{5} \cdot ? = 2$$

Division Equation

$$2 \div \frac{2}{5} = ?$$

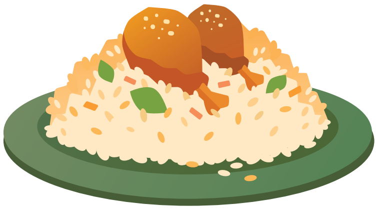
Because there are 5 groups of $\frac{2}{5}$ in 2, the value 5 can be used as the factor in $\frac{2}{5} \cdot 5 = 2$ and the **quotient** in $2 \div \frac{2}{5} = 5$ to make both equations true.

Lesson Practice

4.03

Name: .... Date: .... Period: ....

Problems 1–4: Biryani is a rice dish from South Asia. Three students made Alisha's biryani recipe using different-sized scoops. If the recipe calls for 4 cups of rice, how many scoops of rice does each student need?



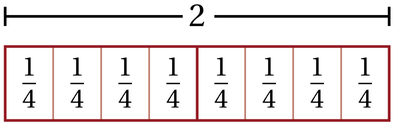
1. Alisha: 2-cup scoop

2. Lukas: $\frac{1}{2}$-cup scoop

3. Emma: $\frac{1}{3}$-cup scoop

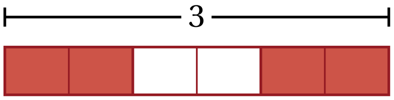
4. Explain how the equation $4 \div \frac{1}{3} = ?$ represents Emma's situation.

5. Lukas drew this diagram to represent the question "How many $\frac{1}{4}$s make 2?" Write a division equation to represent Lukas's diagram.



FAST Practice

6. Allison has a 2-pound bag of cat food. She has 6 cats. Her cats eat $\frac{2}{3}$ pounds of cat food per day. Which model best represents how many days her 2-pound bag of food will last?

Ⓐ


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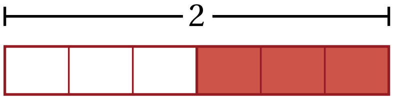

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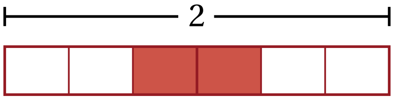

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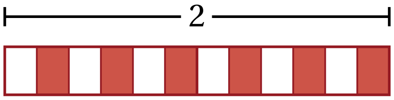


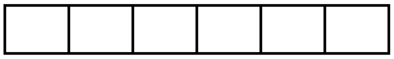
Lesson Practice
4.03

Name: .... Date: .... Period: ....

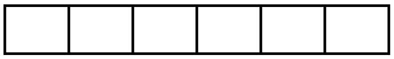
Spiral Review

Problems 7–9: Shade the boxes to represent each fraction.

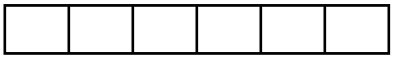
7. $\frac{1}{3}$



8. $\frac{1}{2}$



9. $\frac{5}{6}$



10. *When you multiply one number by another, the result will be larger than the first number.*

Is this statement *always*, *sometimes*, or *never* true? Circle one.

Always Sometimes Never

Explain your thinking.

Unit 4

Lesson

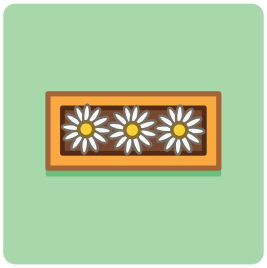
4

Name: .... Date: .... Period: ....

MA.6.NSO.2.3, MTR.1.1, MTR.3.1, MTR.4.1

Flower Planters

Let’s use flower planters to answer the question “How many in *one* group?”



Warm-Up

1. Order these expressions from *least* to *greatest* by the value of the quotient.

$12 \div 12$

$12 \div \frac{2}{3}$

$12 \div 1$

$12 \div 3$

$12 \div \frac{1}{4}$

Least

Greatest

Activity
1

Name: .... Date: .... Period: ....

Plenty of Planters

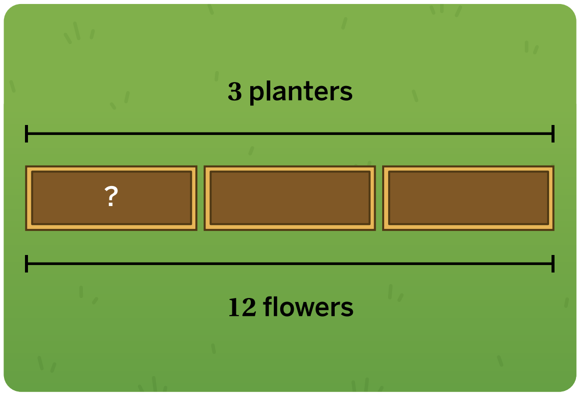
2. Write a story that could be represented by the expression $12 \div \frac{1}{3}$.

Draw a sketch if it helps you illustrate your story.

3. Brianna is planting flowers in her class garden.

12 flowers fill 3 small planters.

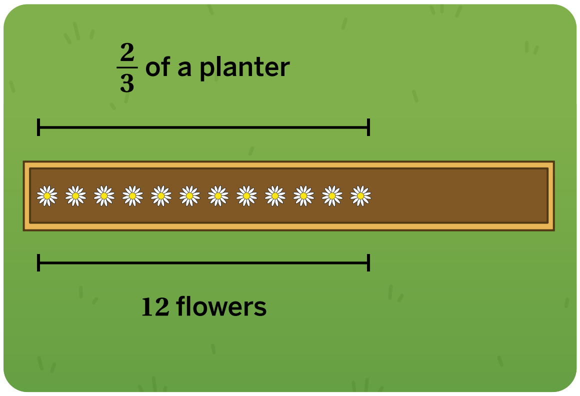
How many flowers fill 1 small planter?



4. Brianna also put flowers in a big planter.

12 flowers fill $\frac{2}{3}$ of a big planter.

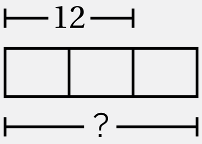
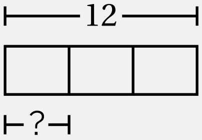
How many flowers fill 1 big planter?



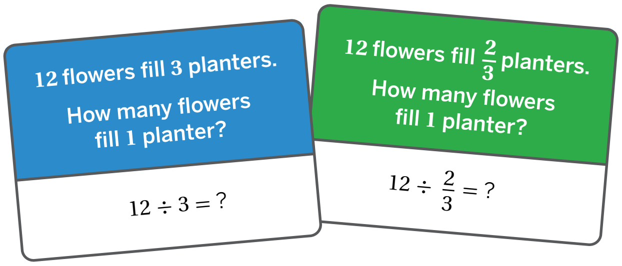
Name: .... Date: .... Period: ....

Plenty of Planters (continued)

5. Match each representation with a question.

	12 flowers fill 3 planters. How many flowers fill 1 planter?	12 flowers fill $\frac{2}{3}$ of a planter. How many flowers fill 1 planter?
$12 \div 3 = ?$		
$12 \div \frac{2}{3} = ?$		
$\frac{2}{3} \cdot ? = 12$		
$3 \cdot ? = 12$		
		
		

6. How are these expressions alike? How are they different?



12 flowers fill 3 planters.

How many flowers fill 1 planter?

$12 \div 3 = ?$

12 flowers fill $\frac{2}{3}$ planters.

How many flowers fill 1 planter?

$12 \div \frac{2}{3} = ?$

Alike:

Different:

Activity

2

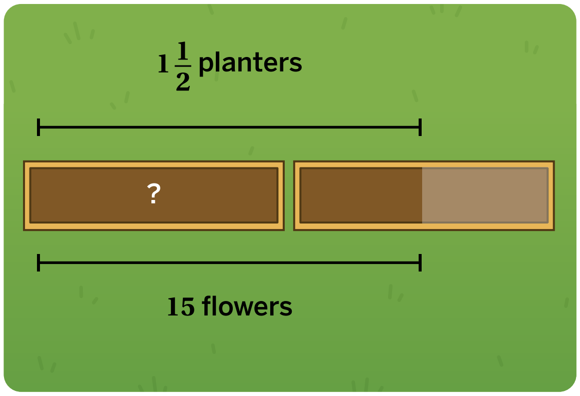
Name: ______ Date: ______ Period: ______

Practicing With Planters

7. Brianna has 15 flowers to put in her planters.

The flowers fill $1\frac{1}{2}$ planters.

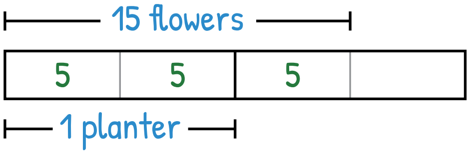
How many flowers fill 1 planter?



8. Here is a diagram Brianna made to calculate how many flowers fill 1 planter when 15 flowers fill $1\frac{1}{2}$ planters.

Explain how Brianna can use this diagram to help her answer the question.

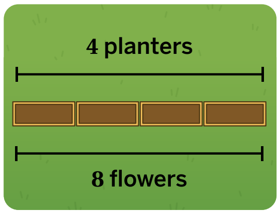
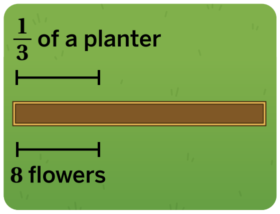
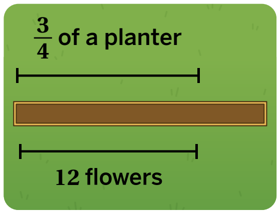
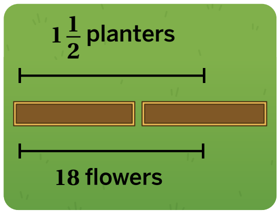
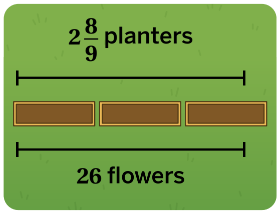
$$15 \div 1\frac{1}{2} = ?$$



Name: .... Date: .... Period: ....

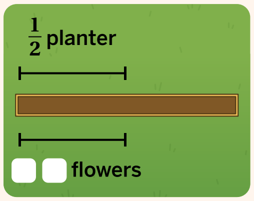
Practicing With Planters (continued)

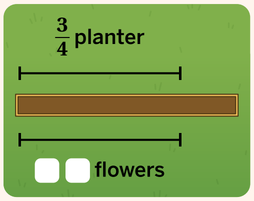
9. Solve as many challenges as you have time for.

	Situation	Diagram	How many flowers fill 1 planter?
a	8 flowers fill 4 planters.		
b	8 flowers fill $\frac{1}{3}$ of a planter.		
c	12 flowers fill $\frac{3}{4}$ of a planter.		
d	18 flowers fill $1\frac{1}{2}$ planters.		
e	26 flowers fill $2\frac{8}{9}$ of a planter.		

You're invited to explore more.

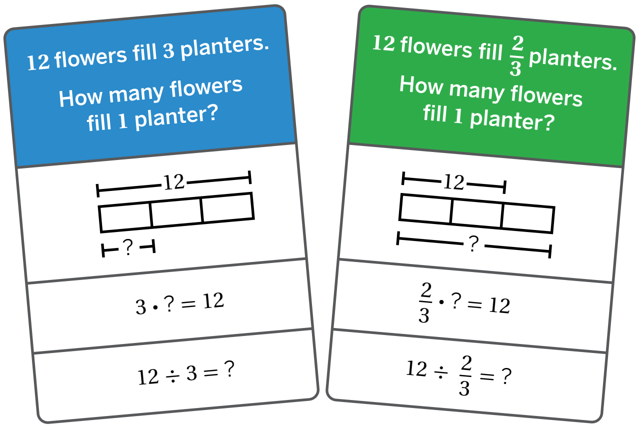
10. Fill in each blank using the numbers 0 to 9 only once, so that the same number of flowers fill each planter.





Synthesis

11. Describe how a tape diagram can represent a division problem.



12 flowers fill 3 planters. How many flowers fill 1 planter?

$3 \cdot ? = 12$

$12 \div 3 = ?$

12 flowers fill $\frac{2}{3}$ planters. How many flowers fill 1 planter?

$\frac{2}{3} \cdot ? = 12$

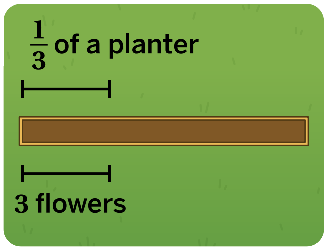
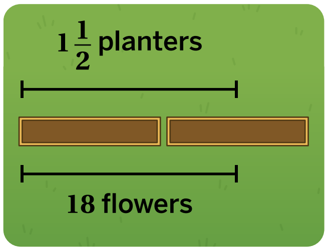
$12 \div \frac{2}{3} = ?$

Lesson Practice 4.04

Lesson Summary

You can answer "How many are in *one* group?" by:

- Evaluating division and multiplication expressions.
- Using tape diagrams that represent division and multiplication expressions.

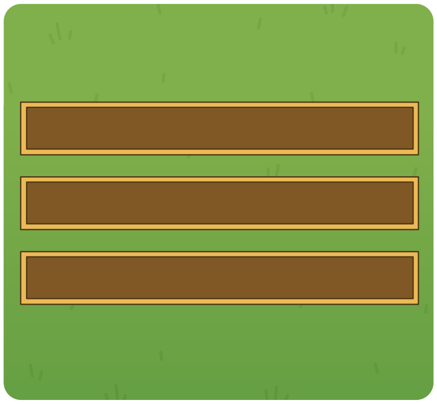
Situation	Diagram	Expressions	Number of Flowers in 1 Planter
3 flowers fill $\frac{1}{3}$ of a planter.		$3 \div \frac{1}{3} = ?$ or $\frac{1}{3} \cdot ? = 3$	9
18 flowers fill $1\frac{1}{2}$ planters.		$18 \div 1\frac{1}{2} = ?$ or $1\frac{1}{2} \cdot ? = 18$	12

Lesson Practice
4.04

Name: .... Date: .... Period: ....

Problems 1–4: Abena is planting vegetables in her backyard. Determine how many of each vegetable plant Abena can fit in 1 planter.

Use the diagrams if they help with your thinking.



1. Onion plants, if 10 onion plants fill $\frac{1}{2}$ of a planter

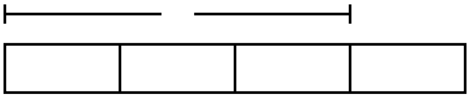
2. Asparagus crowns, if 8 asparagus crowns fill $\frac{2}{3}$ of a planter

3. Potato plants, if 6 potato plants fill $\frac{3}{4}$ of a planter

4. Abena wrote the expression $6 \div \frac{3}{4}$ to represent how many potato plants fill 1 planter. Describe a situation that represents the expression $8 \div \frac{4}{5}$.

Problems 5–6: Ashley picks 9 strawberries from her backyard, which fill $\frac{3}{4}$ of a cup.

5. Label the tape diagram to represent Ashley's situation.



6. Determine how many strawberries fill 1 cup. Use the tape diagram if it helps with your thinking.

FAST Practice

7. George is painting a mural. He uses 3 gallons of paint for $\frac{3}{8}$ of the mural. How many gallons of paint would he need to paint the whole mural?

☐ gallons

Lesson Practice
4.04

Name: .... Date: .... Period: ....

Spiral Review

Problems 8–9: Karima made 9 pairs of earrings in 6 hours.

8. How long will it take Karima to make 12 pairs of earrings?

9. How many pairs of earrings can Karima make in 10 hours?

Problems 10–12: Calculate each unknown number.

10. 5 is 50% of what number?

11. 300 is 10% of what number?

12. 18 is 150% of what number?

Unit 4

Lesson

5

Name: ______________________ Date: __________ Period: __________

MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.2.1, MTR.4.1

Garden Bricks

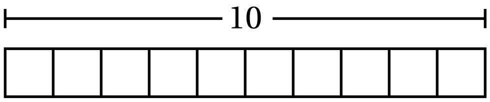
Let’s use tape diagrams to think about
“How many groups?”

Warm-Up

Question	Expression
How many groups of $2\frac{1}{2}$ are in 10?	$10 \div 2\frac{1}{2}$

1. **Discuss:** How do you know that the expression represents the question?

2. Use the tape diagram to answer the question.

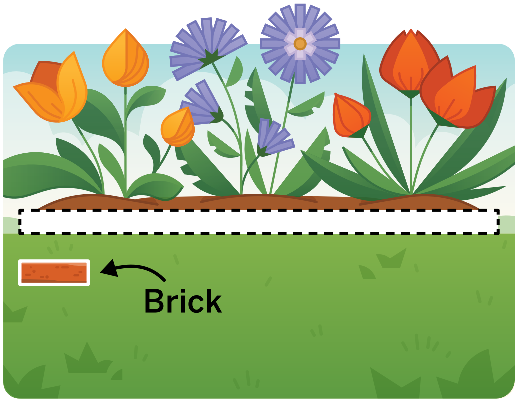


Activity
1

Name: .... Date: .... Period: ....

How Many Bricks?

Deja and Emma are upgrading their class gardens by placing bricks along the front of each garden.



3. The first garden is 4 feet long. Deja is using small bricks, which are $\frac{1}{3}$ of a foot long. How many small bricks does Deja need? Draw a tape diagram to show your thinking.

4. The second garden is also 4 feet long. Emma is using large bricks, which are $\frac{2}{3}$ of a foot long. How many large bricks does Emma need? Draw a tape diagram to show your thinking.

5. The third garden is 5 feet long. How many large bricks do Deja and Emma need? Draw a tape diagram to show your thinking.

Name: ____________ Date: ____________ Period: ____________

How Many Bricks? (continued)

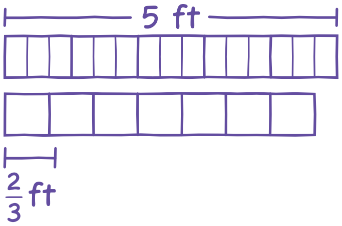
6. Deja and Emma are working on Problem 5. Each student's work contains accurate and inaccurate parts.

Deja

$5 \div \frac{2}{3}$

"$5 \div \frac{2}{3}$ is less than 5 because I'm dividing."

Emma



"I need $7\frac{1}{3}$ bricks because there are 7 whole bricks and $\frac{1}{3}$ left over."

a **Discuss:** How are their methods alike? How are they different?

b Pick *one* student's work. What do you think she did well? What question could you ask to help her understand her mistake?

7. Emma wrote $4\frac{1}{4} \div \frac{3}{4}$ to help answer a different question about bricks and gardens.

a Explain what $4\frac{1}{4}$ and $\frac{3}{4}$ mean in this situation.

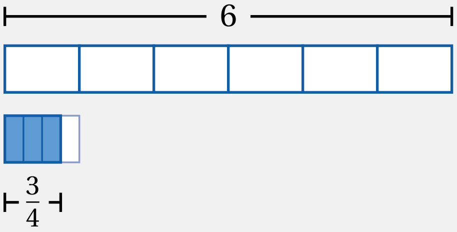
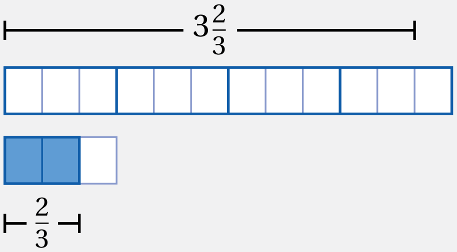
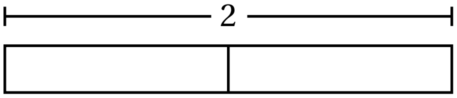
b Draw a tape diagram and use it to determine the value of $4\frac{1}{4} \div \frac{3}{4}$.

Activity
2

Name: ____________ Date: ________ Period: ________

What’s Missing?

8. Complete each row in the table.

	Expression	Tape Diagram	Quotient
a	$6 \div \frac{3}{4}$		
b			
c	$2 \div \frac{3}{5}$		
d			7

You’re invited to explore more.

9. **a** Write a division expression.

b On a separate piece of paper, draw a tape diagram that represents your expression.

c Trade tape diagrams with a partner. Determine their division expression and calculate its quotient.

Synthesis

10. **a** Draw a tape diagram to represent $3 \div \frac{2}{3}$.

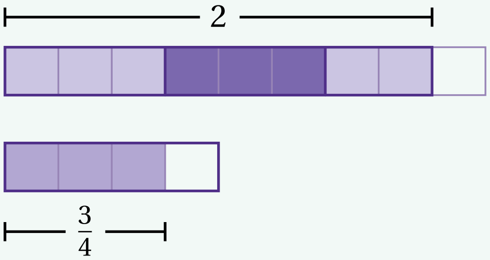
b Describe how you can use the tape diagram to help determine the value of $3 \div \frac{2}{3}$.

Lesson Practice 4.05

Lesson Summary

You can use division to determine how many groups fit into a whole. For example, the expression $2 \div \frac{3}{4}$ can represent how many $\frac{3}{4}$-foot-long bricks fit along a 2-foot garden wall. You can use tape diagrams or reasoning about equal groups to determine how many groups (bricks) fit into the whole (along the garden wall).

Tape Diagram



$$2 \div \frac{3}{4} = 2\frac{2}{3}$$

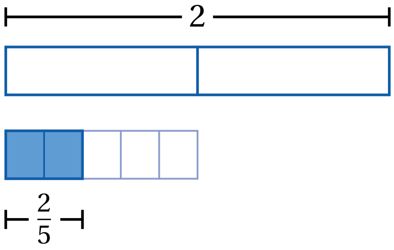
Reasoning About Equal Groups

- To calculate how many lengths of $\frac{3}{4}$ fit into 2, it would help to determine how many $\frac{1}{4}$s there are in 2 wholes.
- I can rewrite 2 as $\frac{8}{4}$.
- There are two groups of $\frac{3}{4}$ in $\frac{8}{4}$, with $\frac{2}{4}$ left over.
- The leftover $\frac{2}{4}$ has 2 of the 3 parts needed to complete a whole group of $\frac{3}{4}$. That means there are $2\frac{2}{3}$ groups of $\frac{3}{4}$ in 2.

Lesson Practice
4.05

Name: ______ Date: ______ Period: ______

1. How many $\frac{2}{5}$s are in 2? Use the diagram if it helps with your thinking.



Problems 2–3: Think about how many $\frac{1}{4}$s are in 3.

2. Draw a tape diagram to represent the situation.

3. Determine how many $\frac{1}{4}$s are in 3.

Problems 4–5: Think about the expression $3\frac{2}{5} \div \frac{4}{5}$.

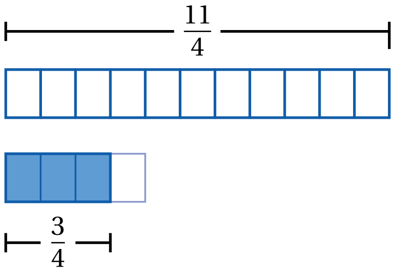
4. Draw a tape diagram to represent this expression.

5. Calculate the quotient.

Problems 6–7: Here is a tape diagram.

6. What expression does this tape diagram represent?

______ ÷ ______



7. Calculate the quotient for this expression.

Lesson Practice
4.05

Name: .... Date: .... Period: ....

Problems 8–9: Think about the expression $6\frac{1}{2} \div \frac{3}{4}$.

8. Draw a tape diagram to represent this expression.

9. Calculate the quotient.

FAST Practice

10. Kayleen buys one 3-pound bag of rice. Her family eats about $\frac{3}{4}$ of a pound every week. How many weeks does one bag last? Use a tape diagram if it helps you with your thinking.

One bag lasts ☐ weeks.

Spiral Review

11. Complete the table.

Fraction	Decimal	Percent
$\frac{1}{4}$	0.25	25%
	0.1	
$\frac{1}{5}$		
		140%

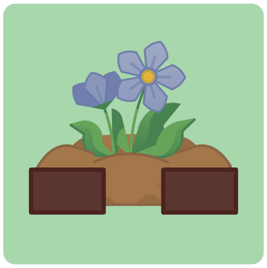
Unit 4
Lesson
6

Name: ______ Date: ______ Period: ______

MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.1.1, MTR.4.1, MTR.5.1

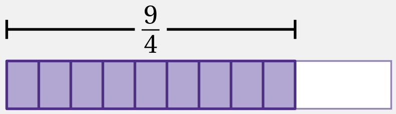
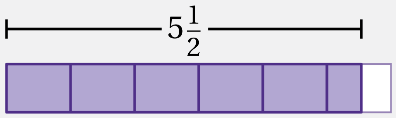
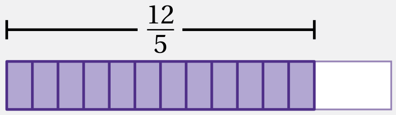
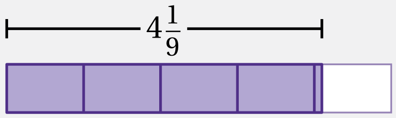
Fill the Gap

Let's use garden bricks to determine whether the number of groups is greater or less than 1.



Warm-Up

1. Complete the table.

Tape Diagram	Fraction	Mixed Number
	$\frac{9}{4}$	
		$5\frac{1}{2}$
	$\frac{12}{5}$	
		$4\frac{1}{9}$

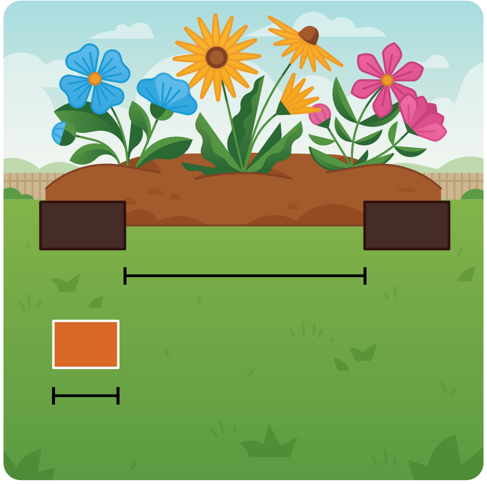
Activity 1

Name: ______ Date: ______ Period: ______

More or Less Than One Group

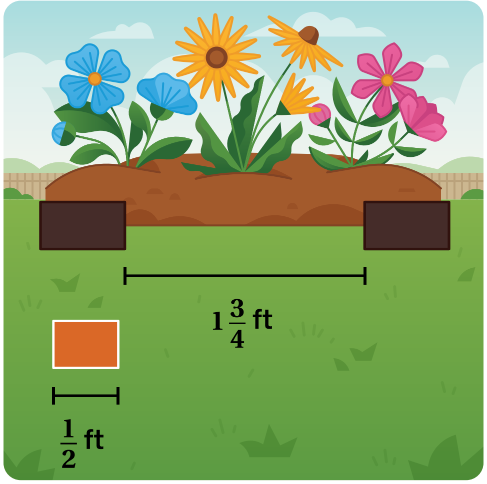
2. Deja is filling a gap along the front of this garden.

About how many bricks does she need?



3. The gap in Deja's garden is $1\frac{3}{4}$ feet long. Each brick is $\frac{1}{2}$ of a foot long.

How many bricks does Deja need to fill the gap?



4. Deja and Emma each wrote an expression to represent the number of bricks needed to fill the gap.

Deja wrote $1\frac{3}{4} \div \frac{1}{2}$. Emma wrote $\frac{1}{2} \div 1\frac{3}{4}$. Whose expression is correct? Circle one.

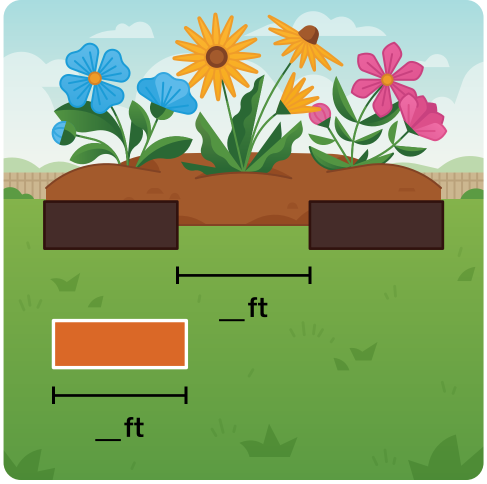
Deja's Emma's Both Neither

Explain your thinking.

Name: .... Date: .... Period: ....

More or Less Than One Group (continued)

5. Here is Emma's expression: $\frac{1}{2} \div 1\frac{3}{4}$.



a Draw a sketch to represent this expression in the garden situation.

b The value of $\frac{1}{2} \div 1\frac{3}{4}$ is:

Less than 1

Greater than 1

Equal to 1

6. Sort these expressions by the value of their quotient.

$2\frac{1}{4} \div \frac{3}{4}$	$\frac{1}{4} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{1}{4}$	$1 \div \frac{1}{4}$
$\frac{5}{4} \div 1\frac{1}{4}$	$\frac{3}{8} \div \frac{3}{8}$	$1 \div 4$	

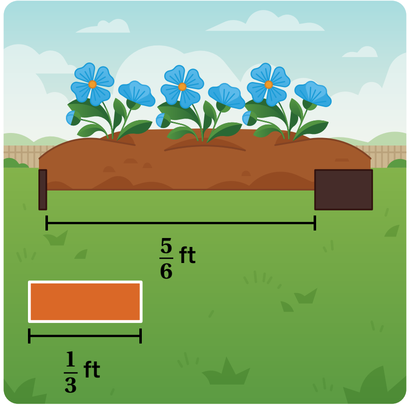
Less than 1	Greater than 1	Equal to 1

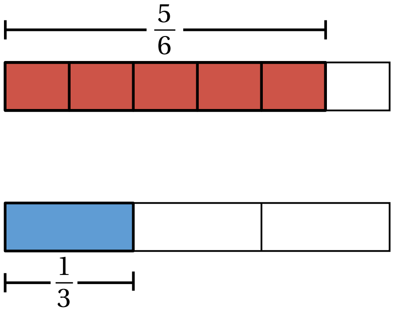
Name: ________ Date: ________ Period: ________

Equal-Sized Pieces

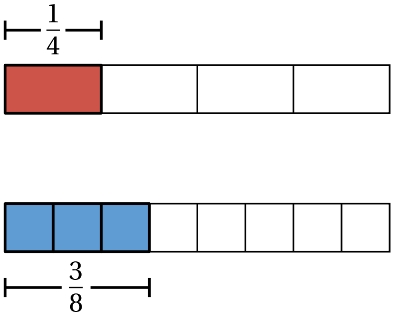
7. Here is a new expression: $\frac{5}{6} \div \frac{1}{3}$.

Use the garden or tape diagram to determine its value.



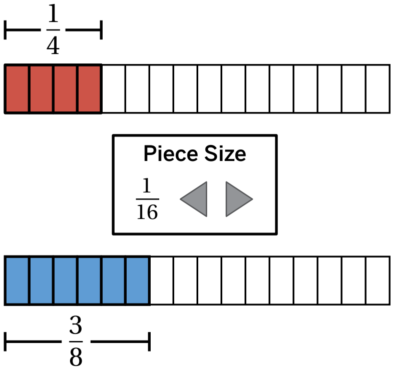


8. What is $\frac{1}{4} \div \frac{3}{8}$?



9. Deja broke $\frac{1}{4}$ and $\frac{3}{8}$ into $\frac{1}{16}$-sized pieces.

a **Discuss:** How does Deja's strategy show that $\frac{1}{4} \div \frac{3}{8} = \frac{4}{6}$?



b Let's determine other helpful ways to break up $\frac{1}{4}$ and $\frac{3}{8}$.

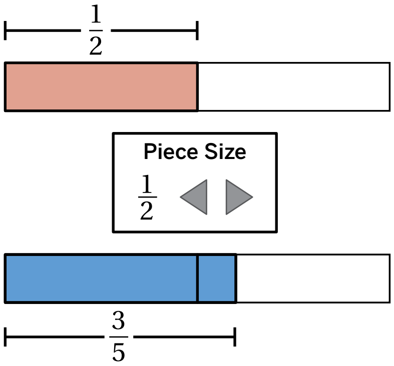
Activity
3

Name: .... Date: .... Period: ....

The Return of Common Denominators

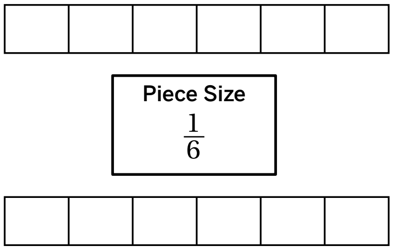
10. **a** Let's look at how we can break these fractions into equal pieces and set up a **common denominator**.

b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{2} \div \frac{3}{5}$.



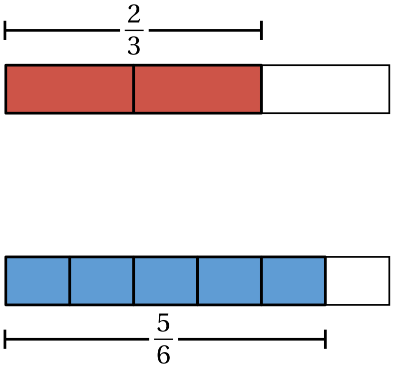
11. Calculate $\frac{2}{3} \div \frac{1}{2}$.

Use the diagram if it helps you with your thinking.



Synthesis

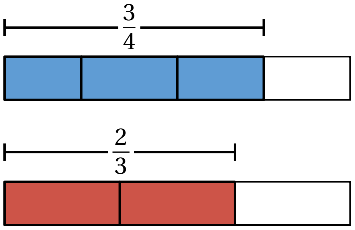
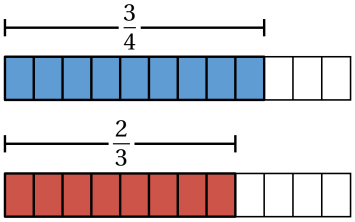
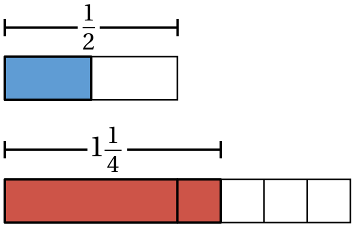
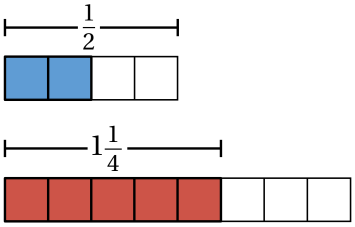
12. Explain how you can show that $\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$. Use the tape diagrams if they help with your thinking.



Lesson Practice 4.06

Lesson Summary

Creating equal-sized pieces, or using a **common denominator**, is a helpful strategy for calculating quotients involving fractions and determining when there is more or less than 1 group.

	Expression and Tape Diagram	Expression and Tape Diagram Using a Common Denominator
More Than 1 Group	$\frac{3}{4} \div \frac{2}{3}$ 	$\frac{9}{12} \div \frac{8}{12} = \frac{9}{8}$  A common denominator of 4 and 3 is 12.
Less than 1 Group	$\frac{1}{2} \div 1\frac{1}{4}$ 	$\frac{2}{4} \div \frac{5}{4} = \frac{2}{5}$  A common denominator of 2 and 4 is 4.

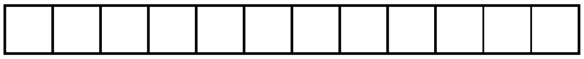
Lesson Practice
4.06

Name: .... Date: .... Period: ....

1. Afia uses a $\frac{1}{2}$-cup scoop for flour. How many scoops does Afia need for each amount of flour? Draw a diagram if it helps with your thinking.

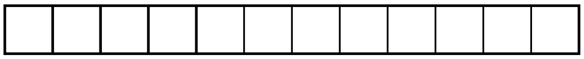
Flour (cups)	Number of Scoops
1	
$\frac{1}{4}$	
$\frac{3}{4}$	

Problems 2–3: Here is a diagram.



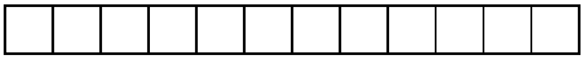
2. Determine if the value of $1\frac{1}{2} \div \frac{2}{3}$ is:

Less than 1 Greater than 1



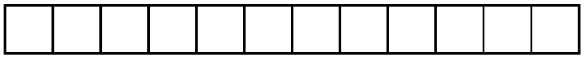
3. Calculate the value of the expression in Problem 2.

Problems 4–5: Here is a diagram.



4. Determine if the value of $\frac{4}{3} \div \frac{3}{2}$ is:

Less than 1 Greater than 1



5. Calculate the value of the expression in Problem 4.

FAST Practice

6. Select *all* the expressions whose value is greater than 1.

☐ **A.** $\frac{2}{3} \div 5$ ☐ **B.** $5 \div \frac{2}{3}$ ☐ **C.** $\frac{5}{3} \div 4$ ☐ **D.** $\frac{1}{3} \div \frac{4}{5}$ ☐ **E.** $\frac{4}{5} \div \frac{1}{3}$

Lesson Practice

4.06

Name: .... Date: .... Period: ....

Spiral Review

Problems 7–9: Determine the missing value that creates a pair of equivalent fractions. Draw diagrams if it helps with your thinking.

7. $\frac{2}{3} = \frac{\square}{9}$

8. $2\frac{1}{2} = \frac{\square}{8}$

9. $\frac{4}{\square} = \frac{10}{25}$

Problems 10–11: Your school is planning a Spring Sprung celebration and has a budget of $240 for all of the expenses.

10. The school wants to spend 40% of its budget on snacks. How much money will the school spend on snacks?

11. The school spent $36 on decorations for the celebration. What percent of the Spring Sprung celebration budget is that?

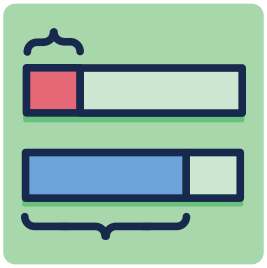
Unit 4
Lesson
7

Name: Date: Period:

MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.3.1, MTR.4.1, MTR.6.1

Break It Down

Let's divide fractions by rewriting with common denominators.



Warm-Up

1. Calculate the following:

a $12 \div 3$

b $\frac{12}{5} \div \frac{3}{5}$

How are these problems alike? How are they different?

Alike:

Different:

Activity
1

Name: .... Date: .... Period: ....

Common Denominators

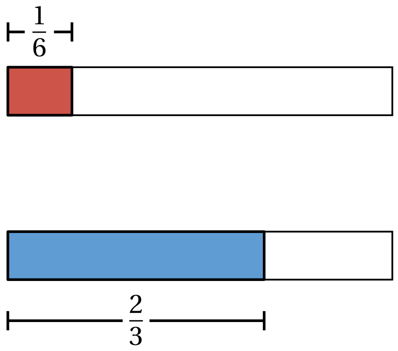
2. The value of $\frac{1}{6} \div \frac{2}{3}$ is:

Less than 1

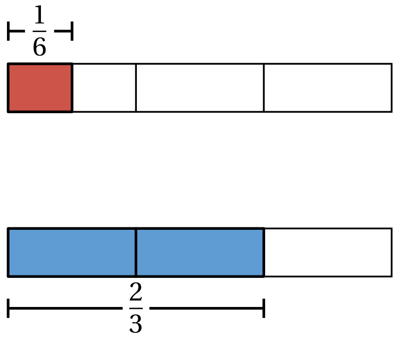
Greater than 1

Equal to 1

Explain your thinking.

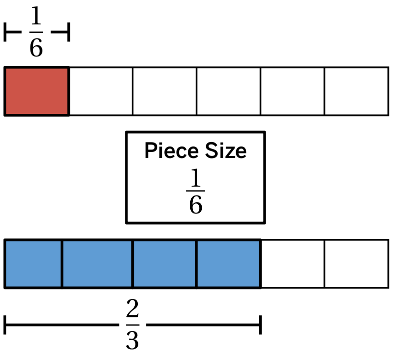


3. Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{6} \div \frac{2}{3}$.



4. Here's how Ahmed calculated $\frac{1}{6} \div \frac{2}{3}$.

💬 **Discuss:** Why do you think Ahmed used $\frac{1}{6}$-sized pieces?



$\frac{1}{6} \div \frac{2}{3}$

$\frac{1}{6} \div \frac{4}{6}$

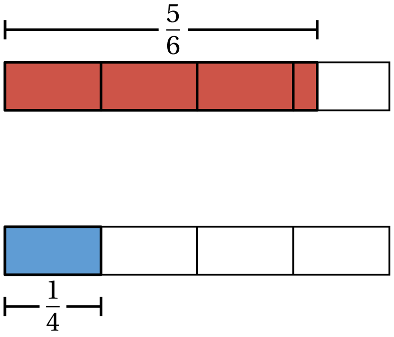
$1 \div 4$

$\boxed{\frac{1}{4}}$

Name: .... Date: .... Period: ....

Common Denominators (continued)

5. Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{5}{6} \div \frac{1}{4}$.



6. Ahmed and Zoe calculated the previous problem without a diagram. Their calculations are both correct.

How are their strategies alike? How are they different?

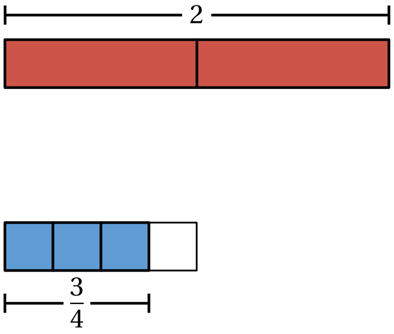
Ahmed	Zoe
$\frac{5}{6} \div \frac{1}{4}$	$\frac{5}{6} \div \frac{1}{4}$
$\frac{10}{12} \div \frac{3}{12}$	$\frac{20}{24} \div \frac{6}{24}$
$10 \div 3$	$20 \div 6$
$\frac{10}{3}$	$\frac{20}{6}$

Alike:

Different:

7. Zoe says she can't use common denominators to calculate $2 \div \frac{3}{4}$ because 2 is a whole number.

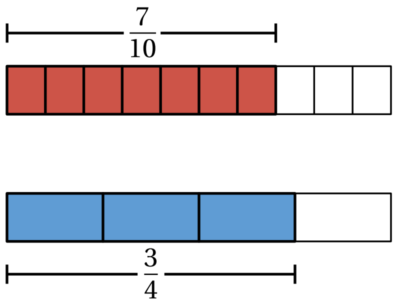
What advice would you give Zoe?



Name: .... Date: .... Period: ....

Dividing with Common Denominators

8. **a** Calculate $\frac{7}{10} \div \frac{3}{4}$.



b **Discuss:** What was your strategy?

9. Solve as many challenges as you have time for.

a $\frac{4}{3} \div \frac{2}{3}$

b $\frac{1}{6} \div \frac{5}{6}$

c $\frac{3}{8} \div \frac{1}{4}$

d $2 \div \frac{1}{3}$

e $\frac{3}{10} \div \frac{2}{5}$

f $\frac{5}{6} \div \frac{3}{4}$

g $4 \div \frac{3}{4}$

h $\frac{11}{4} \div \frac{2}{3}$

i $2\frac{1}{2} \div \frac{2}{3}$

j $1\frac{4}{5} \div \frac{1}{2}$

Synthesis

10. Describe how finding a common denominator can help you divide a fraction with another fraction.

$\frac{4}{3} \div \frac{2}{3}$ $\frac{5}{2} \div \frac{4}{3}$

Use these examples if they help you explain your thinking.

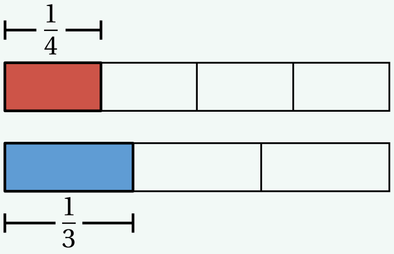
Lesson Practice 4.07

Lesson Summary

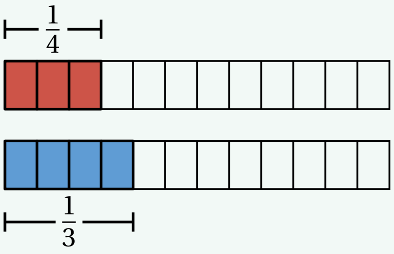
You can use common denominators to determine quotients involving fractions.

For example, in $\frac{1}{4} \div \frac{1}{3}$, you can use 12 as a common denominator of 4 and 3. Then you can rewrite the division expression as $\frac{3}{12} \div \frac{4}{12}$. This helps you determine that there are $\frac{3}{4}$ groups of $\frac{4}{12}$ in $\frac{3}{12}$.

Tape Diagram of Original Problem



Tape Diagram With Common Denominator



Equivalent Fractions With Common Denominator

$$\frac{1}{4} \div \frac{1}{3}$$

$$\frac{3}{12} \div \frac{4}{12}$$

$$3 \div 4$$

$$\frac{3}{4}$$

Lesson Practice

4.07

Name: .... Date: .... Period: ....

1. Here is Irelle's work for calculating $\frac{2}{3} \div \frac{3}{4}$. Explain what you think she did at each step.

Irelle

$\frac{2}{3} \div \frac{3}{4}$

Step 1: $\frac{8}{12} \div \frac{9}{12}$

Step 2: $\frac{8}{9}$

Problems 2–5: Calculate the value of each expression. Draw a diagram if it helps with your thinking.

2. $5 \div \frac{2}{3}$

3. $2\frac{1}{2} \div \frac{5}{8}$

4. $\frac{4}{3} \div \frac{5}{2}$

5. $\frac{10}{4} \div \frac{4}{5}$

6. Sahana's work for Problem 5 is incorrect. What advice would you give her?

Sahana

$\frac{10}{4} \div \frac{4}{5}$

$10 \div 5 = 2$ and $4 \div 4 = 1$

$\frac{2}{1} = 2$

FAST Practice

7. Kelly made $2\frac{1}{2}$ cups of slime. The shaded part of the rectangles show how many cups of slime she has.

Kelly is putting the slime into small containers. Each container holds $\frac{2}{3}$ of a cup of slime. What is the greatest number of containers Kelly can completely fill with slime?

A. 2

B. 4

C. 3

D. 5

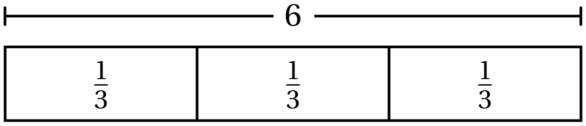
Lesson Practice
4.07

Name: .... Date: .... Period: ....

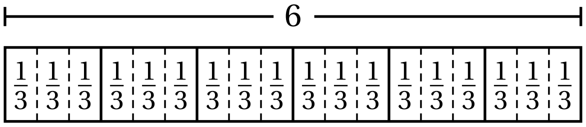
Spiral Review

8. Which of these tape diagrams represent the expression $6 \div \frac{1}{3}$?

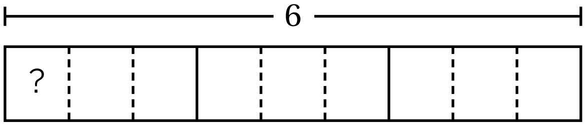
A.



B.



C.



9. Eliza and Isabella are running on a track. Isabella starts 10 meters ahead of Eliza. Eliza runs 120 meters in 24 seconds. Isabella runs 120 meters in 25 seconds. If they both continue running at this pace, how long will it take for Eliza to catch up to Isabella? Explain your thinking.

Problems 10–11: A rocking horse has a weight limit of 60 pounds.

10. What percent of the weight limit is 33 pounds?

11. What weight is 95% of the weight limit?

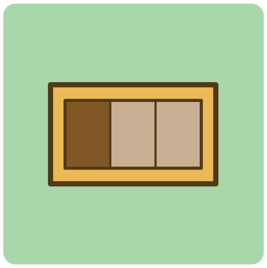
Unit 4
Lesson
8

Name: .... Date: .... Period: ....

MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.2.1, MTR.3.1

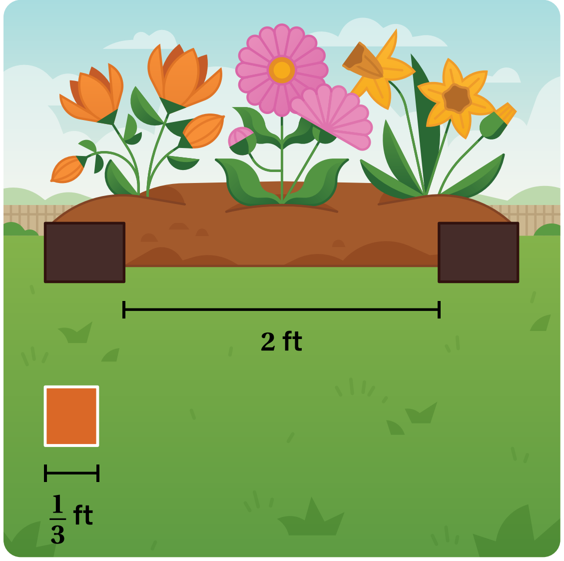
Potting Soil

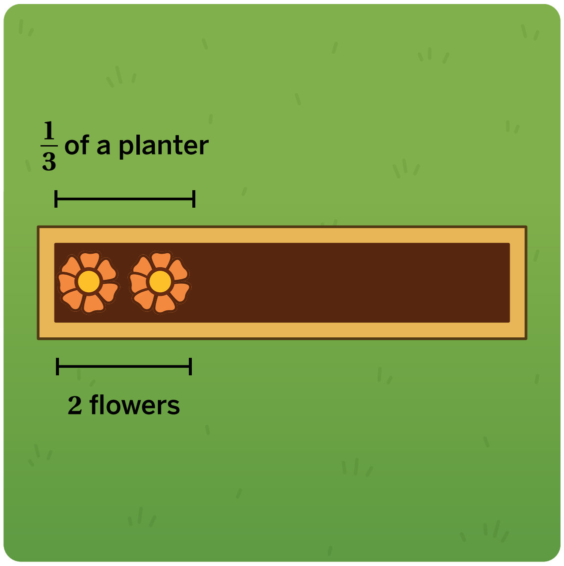
Let's explore another strategy for dividing fractions.



Warm-Up

1. Habib says $2 \div \frac{1}{3}$ represents the brick situation. Inola says $2 \div \frac{1}{3}$ represents the flower situation.





Discuss: Why are they both correct?

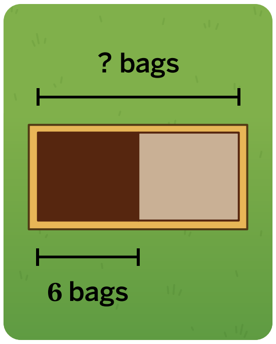
Name: ____________ Date: ____________ Period: ____________

Digging Into Fraction Division

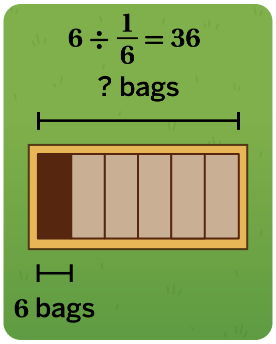
2. Habib and Inola are filling planters with potting soil so that their class can grow vegetables.

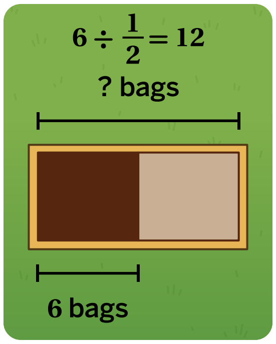
Let's take a look at how many bags of soil fill $\frac{1}{2}$ of a planter.

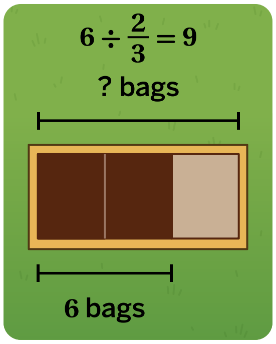
How many bags does it take to fill 1 planter?

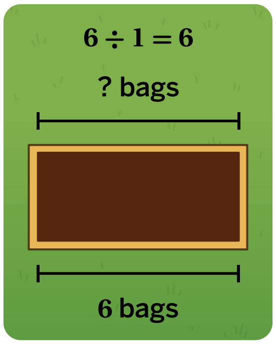


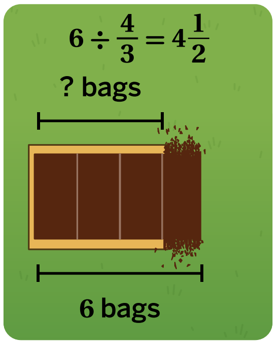
3. **a** Take a look at six different soil situations.

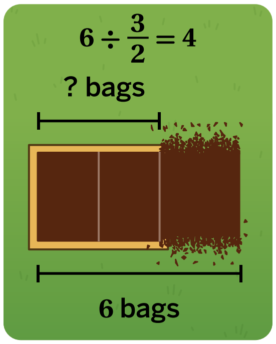












b What do you notice? What do you wonder?

I notice:

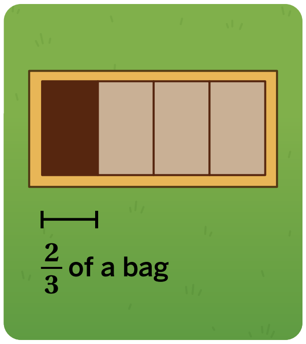
I wonder:

Name: .... Date: .... Period: ....

Digging Into Fraction Division (continued)

4. It takes $\frac{2}{3}$ of a bag of soil to fill $\frac{1}{4}$ of this planter.

How many bags does it take to fill 1 planter?



5. Habib wrote $\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$ to solve the previous problem.

What does each fraction mean in this situation?

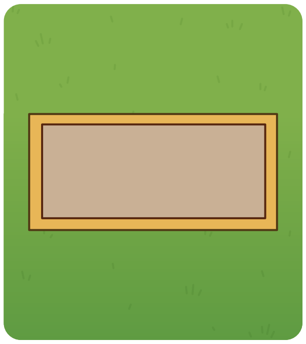
$\frac{2}{3}$ means . . .

$\frac{1}{4}$ means . . .

$\frac{8}{3}$ means . . .

6. Inola wrote $5\frac{1}{3} \div \frac{1}{2}$ to solve a new problem.

Draw or describe a situation about planters and potting soil that represents Inola's expression.



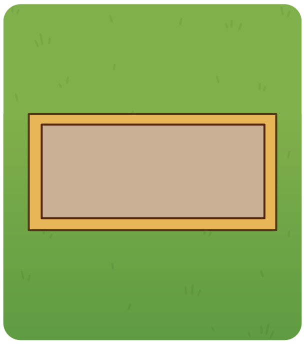
Activity
2

Name: ______ Date: ______ Period: ______

Different Operation, Same Value

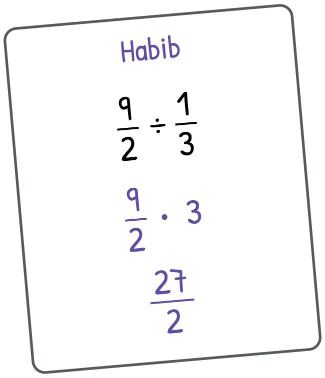
7. **Discuss:** How could you think about the expression $\frac{9}{2} \div \frac{1}{3}$ in terms of a planter?

Draw a diagram if it helps you with your thinking.



8. What is $\frac{9}{2} \div \frac{1}{3}$?

9. Habib says that $\frac{9}{2} \div \frac{1}{3}$ has the same value as $\frac{9}{2} \cdot 3$.



Habib

$$\frac{9}{2} \div \frac{1}{3}$$

$$\frac{9}{2} \cdot 3$$

$$\frac{27}{2}$$

a **Discuss:** How would you show Habib's strategy using a tape diagram?

b Use Habib's strategy to calculate $\frac{2}{3} \div \frac{1}{7}$.

Name: .... Date: .... Period: ....

Different Operation, Same Value (continued)

10. **a** Calculate the value of each expression.

Expression	Value
$\frac{4}{3} \div \frac{1}{3}$	
$\frac{4}{3} \div \frac{1}{6}$	
$\frac{4}{3} \div \frac{1}{5}$	
$1\frac{2}{3} \div \frac{1}{4}$	

b Discuss your answers and strategies with a classmate.

Synthesis

11. Describe a strategy for dividing a number by a unit fraction, such as $2\frac{1}{3} \div \frac{1}{5}$.

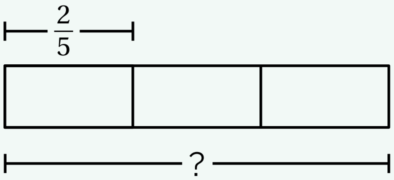
Lesson Practice 4.08

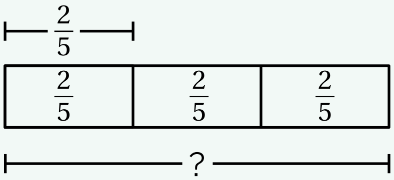
Lesson Summary

When you divide a number by a unit fraction $\frac{1}{b}$, it's generally the same as multiplying the number by b.

For example, think about the expression $\frac{2}{5} \div \frac{1}{3}$. In our planter and soil situation, this means it takes $\frac{2}{5}$ bags of soil to fill $\frac{1}{3}$ of a planter.

To fill the entire planter, you would need 3 times $\frac{2}{5}$ bags of soil, or $\frac{2}{5} \cdot 3$.





$$\frac{2}{5} \div \frac{1}{3}$$

$$= \frac{2}{5} \cdot 3$$

$$= \frac{6}{5}$$

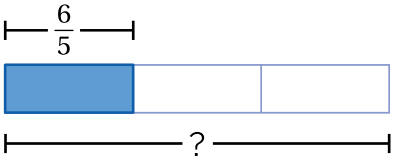
$$= 1\frac{1}{5}$$

Lesson Practice

4.08

Name: .... Date: .... Period: ....

1. Calculate $\frac{6}{5} \div \frac{1}{3}$. Use the tape diagram if it helps with your thinking.



Problems 2–3: Determine whether each statement is *always*, *sometimes*, or *never* true. Circle your answer and explain your thinking.

2. Dividing the same numbers in a different order keeps the value the same, like $2 \div 3 = 3 \div 2$.

Always Sometimes Never

3. Dividing a number by $\frac{1}{3}$ produces the same value as multiplying the number by 3.

Always Sometimes Never

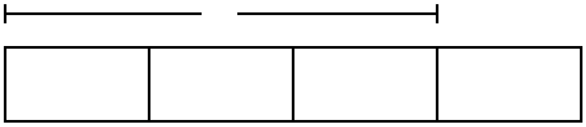
4. $\frac{2}{5}$ of the student population walked to school on a given Friday. If 150 students walked to school that day, how many total students go to the school?

Lesson Practice
4.08

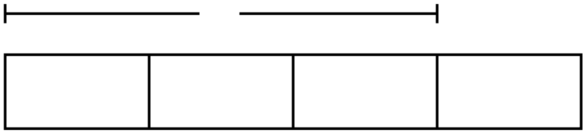
Name: .... Date: .... Period: ....

Problems 5–6: Complete the tape diagram to represent and solve each problem.

5. Mai picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a pan of strawberry oatmeal bars. How many cups does she need for a whole pan?



6. Prisha picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a loaf of raspberry bread. How many cups does she need for a whole loaf?



FAST Practice

7. $\frac{2}{3}$ cups of apple chips fill $\frac{1}{4}$ of a jar. Write and evaluate an expression to determine how many cups fill 1 jar.

Expression: ☐

Number of cups that fill 1 jar: ☐

Spiral Review

Problems 8–10: Determine each quotient.

8. $6 \div \frac{1}{3}$

9. $4 \div \frac{1}{9}$

10. $\frac{1}{10} \div 8$

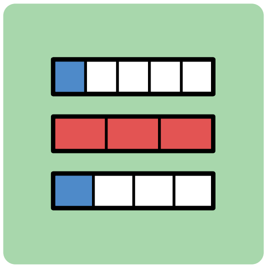
Unit 4
Lesson
9

Name: ______________ Date: ______ Period: ______

MA.6.NSO.2.2, MTR.3.1

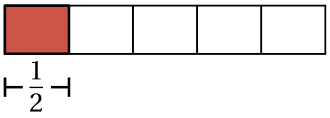
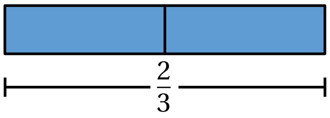
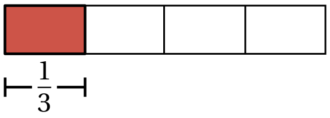
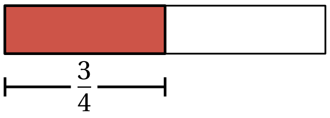
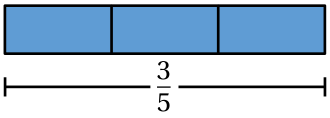
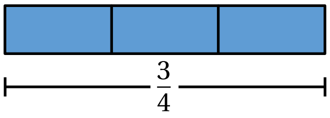
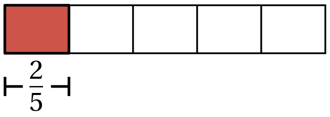
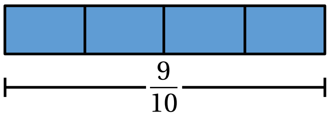
Division Challenges

Let’s compare strategies for dividing fractions with and without tape diagrams.



Warm-Up

1. Solve as many challenges as you have time for. Try some problems of each type.

Multiplying	Dividing	Surprise Me!
 What is the value of $5 \bullet \frac{1}{2}$?	 What is the value of $\frac{2}{3} \div 2$?	 What is the value of $4 \bullet \frac{1}{3}$?
 What is the value of $2 \bullet \frac{3}{4}$?	 What is the value of $\frac{3}{5} \div 3$?	 What is the value of $\frac{3}{4} \div 3$?
 What is the value of $5 \bullet \frac{2}{5}$?	 What is the value of $\frac{9}{10} \div 4$?	 What is the value of $3 \bullet \frac{2}{5}$?

Activity

1

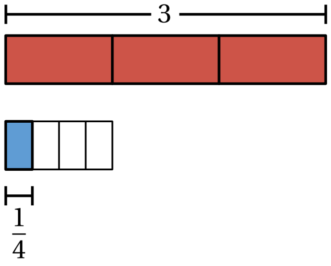
Name: ______________ Date: ______________ Period: ______________

Two Strategies With Tape Diagrams

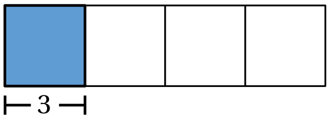
2. Jalen and Katie drew diagrams to calculate $3 \div \frac{1}{4}$.

a Take a look at each student's diagram.

Jalen



Katie



b Calculate $3 \div \frac{1}{4}$.

3. Here is a new expression: $\frac{4}{3} \div \frac{1}{5}$.

Jalen says the quotient is $\frac{20}{3}$. Katie says the quotient is $\frac{4}{15}$.

Diagram 1

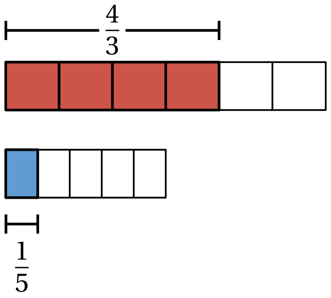
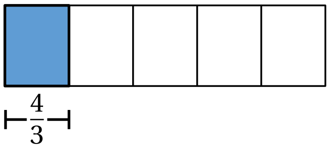


Diagram 2



a Whose quotient is correct? Circle one.

Jalen's Katie's Both Neither

b Use one of the diagrams to help explain your thinking.

Activity
2

Name: .... Date: .... Period: ....

Two Strategies Revisited

4. Here are four expressions.

a Order these expressions by value from *least* to *greatest*.

$\frac{4}{3} \div \frac{2}{5}$	$\frac{4}{3} \div 1$	$\frac{4}{3} \div \frac{1}{5}$	$\frac{4}{3} \div 2$

Least **Greatest**

b **Discuss:** How are these expressions alike? How are they different?

5. Here is an expression from the previous problem:

$$\frac{4}{3} \div \frac{2}{5}$$

Calculate its value.

Name: .... Date: .... Period: ....

Two Strategies Revisited (continued)

6. Here is how Jalen and Katie calculated $\frac{4}{3} \div \frac{2}{5}$.

a Take a look at each of their strategies.

Jalen

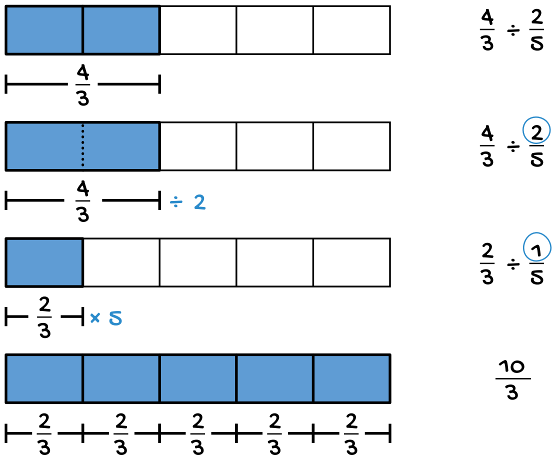
$\frac{4}{3} \div \frac{2}{5}$

$\frac{20}{15} \div \frac{6}{15}$

$20 \div 6$

$\frac{10}{3}$

Katie



b Which strategy would you use to calculate $\frac{9}{10} \div \frac{3}{4}$? Circle one.

Jalen's Katie's My own

c If you chose Jalen's or Katie's strategy, what would your first step be for the strategy you chose? Otherwise, describe your own strategy.

Name: .... Date: .... Period: ....

Fraction Fluency

7. Here is an expression from the previous problem:

$\frac{9}{10} \div \frac{3}{4}$

Calculate its value.

8. Here is a new expression: $\frac{6}{5} \div \frac{2}{3}$.

The three answers below are *not* correct.

$\frac{12}{15}$ $\frac{6}{5}$ 3

Circle your favorite (wrong) answer and explain why it cannot be correct.

Name: .... Date: .... Period: ....

Fraction Fluency (continued)

9. Solve as many challenges as you have time for. Calculate each expression.

a $5 \div \frac{2}{3}$

b $\frac{1}{2} \div \frac{3}{4}$

c $\frac{6}{5} \div \frac{2}{3}$

d $\frac{1}{2} \div \frac{5}{3}$

e $\frac{3}{4} \div \frac{3}{5}$

f $\frac{9}{4} \div \frac{7}{10}$

g $\frac{5}{9} \div \frac{5}{3}$

h $\frac{1}{9} \div \frac{3}{5}$

i $\frac{9}{7} \div \frac{1}{7}$

j $\frac{6}{7} \div \frac{4}{7}$

k $2 \div \frac{8}{9}$

l $\frac{6}{5} \div \frac{2}{5}$

Synthesis

10. Describe a strategy for calculating the quotient of two fractions, such as $\frac{2}{5} \div \frac{3}{4}$.

Draw a diagram if it helps you with your thinking.

Lesson Practice 4.09

Lesson Summary

You don't have to use tape diagrams to determine the quotient of two fractions!

Here are two ways to calculate the quotient of the expression $\frac{9}{10} \div \frac{3}{4}$: by using common denominators and by simplifying numerators.

Common Denominators

- Rewrite the expression using common denominators.
  $\frac{18}{20} \div \frac{15}{20}$
- Then divide the numerator of the first fraction by the numerator of the second fraction.
  $18 \div 15 = \frac{18}{15}$ or $\frac{6}{5}$

Simplifying Numerators

- Divide the first fraction by the numerator of the divisor to create a unit fraction.
  $\frac{3}{10} \div \frac{1}{4}$
- To divide by the unit fraction, multiply the dividend by the denominator of the divisor.
  $\frac{3}{10} \bullet 4 = \frac{12}{10}$ or $\frac{6}{5}$

Lesson Practice
4.09

Name: .... Date: .... Period: ....

Problems 1–4: Use any strategy to calculate each quotient.

1. $10 \div \frac{1}{5}$

2. $10 \div \frac{3}{5}$

3. $3\frac{3}{4} \div \frac{3}{8}$

4. $\frac{1}{2} \div \frac{5}{3}$

5. How many groups of $\frac{3}{4}$ are in $4\frac{1}{2}$?

6. How many groups of $\frac{3}{4}$ are in $2\frac{2}{3}$?

7. Use the equation $2\frac{1}{2} \div \frac{1}{8} = 20$ to determine $2\frac{1}{2} \div \frac{5}{8}$. Explain your thinking.

FAST Practice

8. A painter has $3\frac{1}{2}$ gallons of paint to paint some bathrooms. Each bathroom requires $\frac{2}{3}$ gallon of paint. How many bathrooms can he paint?

☐ bathrooms

Lesson Practice
4.09

Name: ______________________ Date: __________ Period: __________

Spiral Review

9. Basheera has 90 songs on a playlist. She listened to 40% of the songs. How many songs did Basheera listen to?

10. One batch of trail mix uses 2 cups of cereal, $\frac{1}{4}$ cups of raisins, and $\frac{2}{3}$ cups of almonds. Complete the table to show how much of each ingredient you would need to make 3 or 4 batches of trail mix.

	Cereal (cups)	Raisins (cups)	Almonds (cups)
3 Batches			
4 Batches			

Problems 11–12: Here are three expressions.

$56 \div 8$	$56 \div 800$	$56 \div 0.08$

11. Without calculating, order the quotients from *least* to *greatest*.

Least **Greatest**

12. Explain how you ordered the three quotients.

Unit 4
Lesson
10

Name: ______________________ Date: __________ Period: __________

MA.6.NSO.2.2, MTR.1.1, MTR.3.1, MTR.4.1

Action Fractions

Let’s rewrite fraction division as multiplication.

Warm-Up

1. Callen and Kiandra both evaluated $\frac{2}{3} \div \frac{3}{5}$. Here are their strategies.

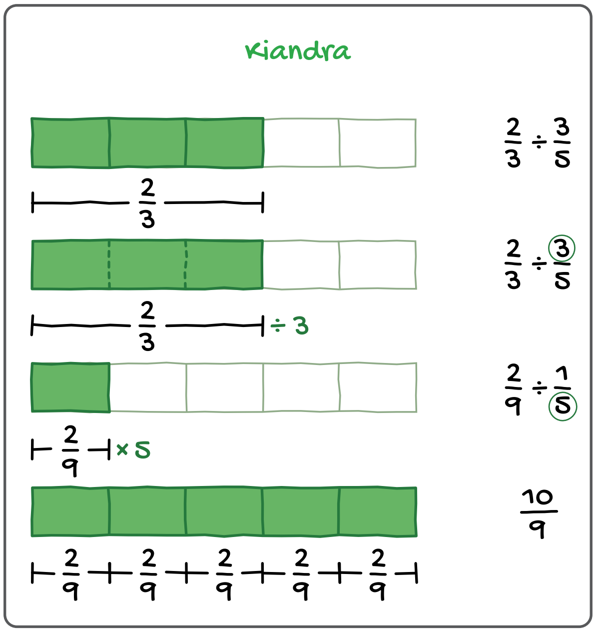
Callen

$\frac{2}{3} \div \frac{3}{5}$

$\frac{10}{15} \div \frac{9}{15}$

$10 \div 9$

$\frac{10}{9}$



What do you notice? What do you wonder?

I notice:

I wonder:

Activity
1

Name: .... Date: .... Period: ....

Creating New Strategies

2. Polina also evaluated $\frac{2}{3} \div \frac{3}{5}$. She said: *My work is similar to Kiandra's.*

Take a look at Polina's work. How is her strategy similar to Kiandra's?

Polina

$\frac{2}{3} \div \frac{3}{5}$

$\frac{2}{9} \div \frac{1}{5}$

$\frac{2}{9} \cdot 5$

$\frac{10}{9}$

3. Nia looked at Polina's work and said she could determine the answer in fewer steps.

Here is Nia's work.

Nia

$\frac{2}{3} \div \frac{3}{5}$

$\frac{2}{3} \cdot \frac{5}{3}$

$\frac{10}{9}$

a Is Nia's work correct? Explain your thinking.

b 💬 **Discuss:** Do you think you can use Nia's strategy on *all* division expressions?

Name: .... Date: .... Period: ....

Creating New Strategies (continued)

4. Nia multiplied by the **reciprocal**.

a Use Nia's strategy to evaluate $\frac{3}{5} \div \frac{7}{10}$.

b Then evaluate the same expression using a different strategy.

Nia's Strategy	**A Different Strategy**

5. 💬 **Discuss:** What are the advantages of Nia's strategy? What are the advantages of the other strategy you selected?

Activity
2

Name: .... Date: .... Period: ....

Card Sort: Equivalent Expressions

6. You will use a set of cards for this activity.

Sort the cards into groups of equivalent expressions.

Equivalent to $\frac{5}{4}$	Equivalent to $\frac{9}{5}$	Equivalent to $\frac{5}{9}$

7. Choose *one* group from the card sort. Then explain how you and your partner made these matches.

Activity
3

Name: .... Date: .... Period: ....

Partner Problems

Decide with your partner who will complete Column A and who will complete Column B. The solutions in each row should be the same.

Compare your solutions and strategies, then discuss and resolve any differences.

	Column A	Column B
8.	$\frac{1}{2} \div \frac{2}{3}$	$\frac{5}{8} \div \frac{5}{6}$
9.	$\frac{5}{9} \div \frac{7}{4}$	$\frac{10}{7} \div \frac{9}{2}$
10.	$1\frac{2}{3} \div \frac{8}{15}$	$1\frac{1}{4} \div \frac{2}{5}$
11.	$\frac{9}{10} \div 1\frac{1}{5}$	$\frac{5}{6} \div 1\frac{1}{9}$

Synthesis

12. Show that $\frac{3}{4} \div \frac{2}{3}$ is equivalent to $\frac{3}{4} \cdot \frac{3}{2}$.

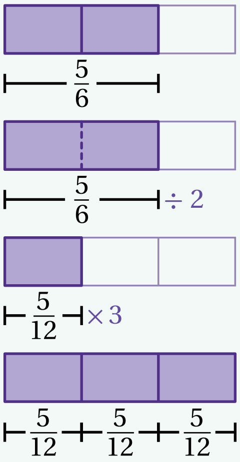
Lesson Practice 4.10

Lesson Summary

In general, when you divide a number by a unit fraction, $\frac{1}{b}$, it's the same as multiplying the number by b (which is the **reciprocal** of $\frac{1}{b}$).

Here are three strategies you can use when you divide a number by a fraction, $\frac{a}{b}$.

Tape Diagram



Simplifying Numerators

Divide by the numerator, a, then multiply the result by the denominator, b.

$$\begin{aligned} &\frac{5}{6} \div \frac{2}{3} \\ &= \frac{5}{12} \div \frac{1}{3} \\ &= \frac{5}{12} \cdot 3 \\ &= \frac{15}{12} \\ &= \frac{5}{4} \end{aligned}$$

Multiplying by the Reciprocal

Multiply by the reciprocal of the fraction $\left(\frac{b}{a}\right)$.

$$\begin{aligned} &\frac{5}{6} \div \frac{2}{3} \\ &= \frac{5}{6} \cdot \frac{3}{2} \\ &= \frac{15}{12} \\ &= \frac{5}{4} \end{aligned}$$

Lesson Practice

4.10

Name: .... Date: .... Period: ....

Problems 1–4: Determine the value of each expression. Show or explain your thinking.

1. $\frac{8}{9} \div 4$

2. $\frac{3}{4} \div \frac{1}{2}$

3. $\frac{9}{2} \div \frac{3}{8}$

4. $3\frac{1}{3} \div \frac{2}{9}$

5. Clare incorrectly said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$ because $\frac{4}{3} \bullet 5 = \frac{20}{3}$ and $\frac{20}{3} \div 2 = \frac{10}{3}$. Determine the correct quotient for $\frac{4}{3} \div \frac{5}{2}$. Then explain why Clare's quotient and reasoning are incorrect.

6. Determine the quotient of $3\frac{1}{5}$ divided by $\frac{8}{9}$.

FAST Practice

7. Select *all* the statements that provide the correct steps for evaluating the expression $\frac{14}{15} \div \frac{7}{5}$.

- ☐ **A.** Multiply $\frac{14}{15}$ by $\frac{1}{7}$, then multiply by 5.
- ☐ **B.** Divide $\frac{14}{15}$ by 5, then multiply by $\frac{1}{7}$.
- ☐ **C.** Multiply $\frac{14}{15}$ by 7, then multiply by $\frac{1}{5}$.
- ☐ **D.** Divide $\frac{14}{15}$ by 7, then multiply by 5.

Lesson Practice

4.10

Name: .... Date: .... Period: ....

Spiral Review

8. Without calculating, determine how the expressions $98 \cdot 25$ and $(100 \cdot 25) - (2 \cdot 25)$ are related. Explain your thinking.

9. Kiran and Nicolas are comparing the numbers $1{,}000$ and 10. Kiran says that $1{,}000$ is 100 times as large as 10. Nicolas says that 10 is $\frac{1}{100}$ times as large as $1{,}000$. Whose thinking is correct? Circle one.

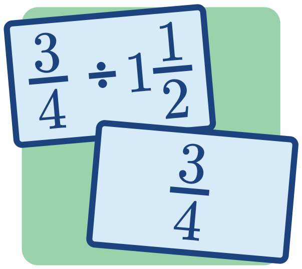
Kiran's Nicolas's Both Neither

Explain your thinking.

Name: ______ Date: ______ Period: ______

MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.1.1, MTR.2.1, MTR.5.1

Swap Meet



Let's solve division problems using different strategies.

Warm-Up

1. Write a division expression to represent each question. Then answer the question.

Question	Division Expression	Answer
How many groups of $2\frac{1}{3}$ are in 21?		
How many groups of 21 are in $2\frac{1}{3}$?		

Activity
1

Name: .... Date: .... Period: ....

Match and Solve

You will use a set of cards for this activity.

- Match each question to an expression and its answer.
- Write an expression and answer for the missing cards. Make sure to include a unit of measurement for each answer.

	Question	Expression	Answer
2.	Hailey lives $1\frac{1}{2}$ miles from school. Luis lives $\frac{3}{4}$ miles from school. How many more miles from school does Hailey live?		
3.	Darius is making a small garden that is $\frac{3}{4}$ feet long and $1\frac{1}{2}$ feet wide. What is the area of the garden?		
4.	A cookie recipe uses $1\frac{1}{2}$ cups of sugar per batch. Ana has $\frac{3}{4}$ cups of sugar. How many batches of cookies can she make?		
5.	Kanna biked $1\frac{1}{2}$ miles, which is $\frac{3}{4}$ of the distance between her home and school. What is the distance between her home and school?		

6. Choose *one* of the expressions from the table. Explain how you decided which question it represents.

Activity
2

Name: .... Date: .... Period: ....

Write, Trade, Solve!

A. $1\frac{1}{2} \div 3$	**B.** $\frac{5}{6} \cdot \frac{2}{3}$	**C.** $5 \div \frac{1}{4}$	**D.** $\frac{2}{5} \div \frac{9}{2}$	**E.** $4\frac{1}{2} \cdot \frac{1}{3}$
F. $\frac{3}{4} \div 1\frac{1}{2}$	**G.** $\frac{5}{6} \div \frac{2}{3}$	**H.** $5 \div \frac{3}{4}$	**I.** $\frac{9}{2} \div \frac{2}{5}$	**J.** $4\frac{1}{2} \div \frac{1}{3}$

7. Circle an expression from the table.

8. On a separate sheet, write a question that can be answered by the expression you chose.

9. Calculate the value of your expression and answer your question. Show your thinking. (Don't write any of this on the sheet of paper!)

a Expression: ....

b Value of expression:

c Answer to your question (include units):

Name: ____________________ Date: __________ Period: __________

Write, Trade, Solve! (continued)

Find a partner, then trade questions with them.

- Write an expression that can be used to represent their question.
- Calculate the value of the expression. Show your thinking.

Repeat with other partners.

	Partner's Name	Expression	Answer
10.			
11.			
12.			
13.			
14.			

Synthesis

15. Ricardo biked $1\frac{1}{4}$ miles, which is $\frac{3}{5}$ of the distance between his home and school. What is the distance between his home and school?

Circle the expression that represents this situation. Explain your thinking.

$1\frac{1}{4} \div \frac{3}{5}$

$\frac{3}{5} \div 1\frac{1}{4}$

Lesson Practice 4.11

Lesson Summary

There are many real-life situations where you can use fraction division.

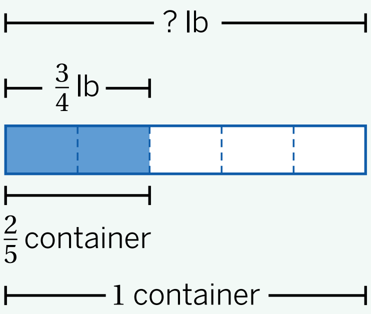
For example, let's say $\frac{3}{4}$ pounds of rice fills $\frac{2}{5}$ of a container.

There are two possible questions you can ask:

- How many pounds fill 1 container?
- How many containers for 1 pound?

Here's how you can use different division expressions and tape diagrams to answer each question.

How many pounds fill 1 container?

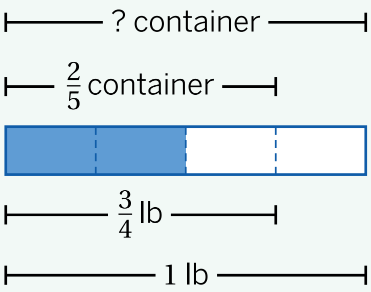


$1\frac{7}{8}$ pounds fill 1 container.

$\frac{3}{4} \div \frac{2}{5}$

$= \frac{15}{8}$ or $1\frac{7}{8}$

How many containers for 1 pound?



1 pound would fill $\frac{8}{15}$ of a container.

$\frac{2}{5} \div \frac{3}{4}$

$= \frac{8}{15}$

Lesson Practice

4.11

Name: .... Date: .... Period: ....

Problems 1–4: Use any strategy to calculate each quotient.

1. $2\frac{1}{2} \div \frac{5}{8}$

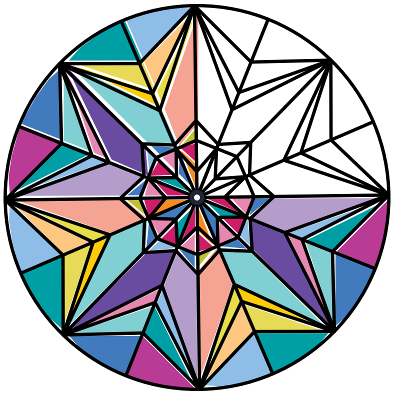
2. $\frac{4}{3} \div \frac{5}{2}$

3. $3\frac{1}{2} \div \frac{1}{3}$

4. $3 \div \frac{2}{3}$

5. Write a situation that could be represented by the expression $3 \div \frac{2}{3}$. Explain what the quotient means in your situation.

6. Elena has 5 tubes of blue glass paint to color her window. She used 4 tubes to paint $\frac{3}{4}$ of the window. Does she have enough blue paint to completely color her window? Show or explain your thinking.



7. Engineers designed superchargers to decrease the amount of time needed to charge the battery of an electric car. A supercharger can charge $\frac{3}{4}$ of a car's battery in 25 minutes. What fraction of the battery gets charged every 5 minutes? Show or explain your thinking.

Lesson Practice
4.11

Name: .... Date: .... Period: ....

FAST Practice

8. A recipe requires $1\frac{1}{3}$ cups of flour for every batch of cookies. Write and evaluate an expression to find how many full batches of cookies can be made with $6\frac{3}{4}$ cups of flour.

Expression:

Solution:

Number of full batches:

Spiral Review

Problems 9–10: Ethan works as a server in a restaurant. He gets a 15% tip on the cost of every order.

9. What tip would he get if the order costs $\$50$?

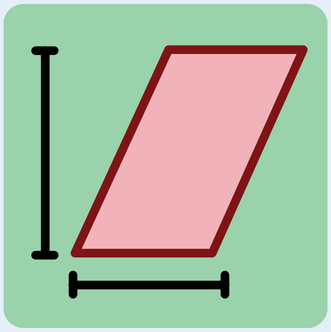
10. Ethan got a $\$9$ tip. What was the cost of the order?

Sub-unit

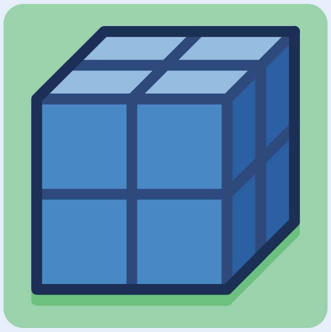
3

Area and Volume With Fractions

Lesson 12
Classroom Comparisons



Lesson 13
Puzzling Areas



Lesson 14
Volume Challenges

Lesson 15
Planter Planner

Unit 4

Lesson

12

Name: .... Date: .... Period: ....

MA.6.NSO.2.2, MA.6.NSO.2.3, MTR.1.1, MTR.4.1

Classroom Comparisons

Let's compare the size of familiar objects by asking, "How many times as long?"

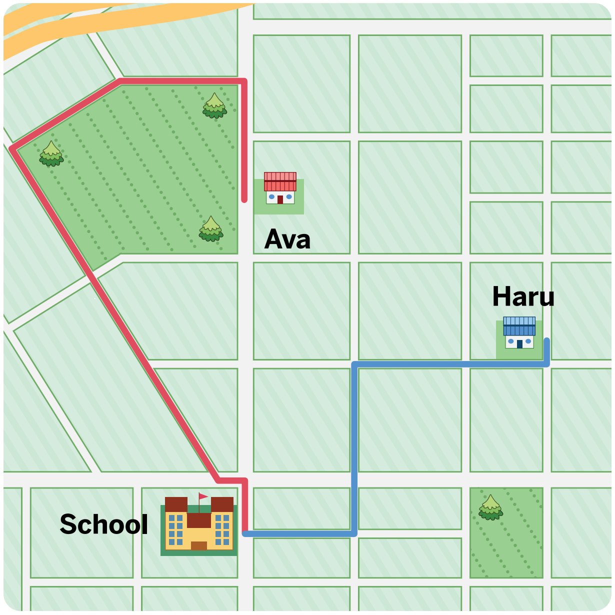
Warm-Up

1. Here is how Ava and Haru walked to school on Monday.

   What do you notice? What do you wonder?

   I notice:

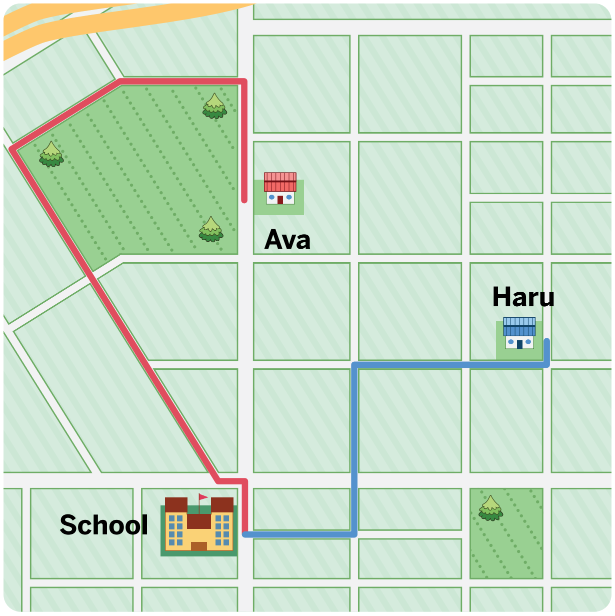
   I wonder:



Activity
1

Name: ______ Date: ______ Period: ______

Comparing Distances



2. Ava walked farther than Haru.

About how many times as far do you think Ava walked?

3. Ava walked $1\frac{1}{4}$ miles. Haru walked $\frac{3}{4}$ of a mile.

How many times as far did Ava walk?

4. Select *all* the expressions that represent how many times as far Ava walked compared to Haru.

☐ **A.** $\frac{5}{4} - \frac{3}{4}$

☐ **B.** $1\frac{1}{4} \cdot \frac{3}{4}$

☐ **C.** $1\frac{1}{4} \div \frac{3}{4}$

☐ **D.** $\frac{3}{4} \div 1\frac{1}{4}$

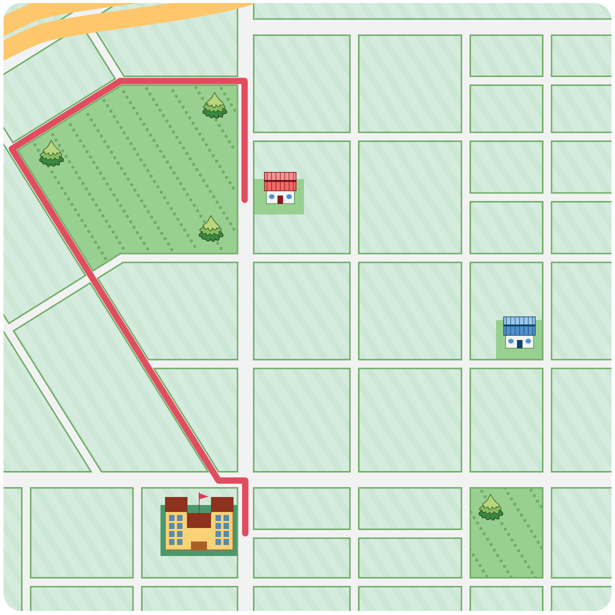
☐ **E.** $\frac{5}{4} \div \frac{3}{4}$

Name: .... Date: .... Period: ....

Comparing Distances (continued)

5. Ava decides to walk a different path to school on Tuesday.

Draw a path she could walk.

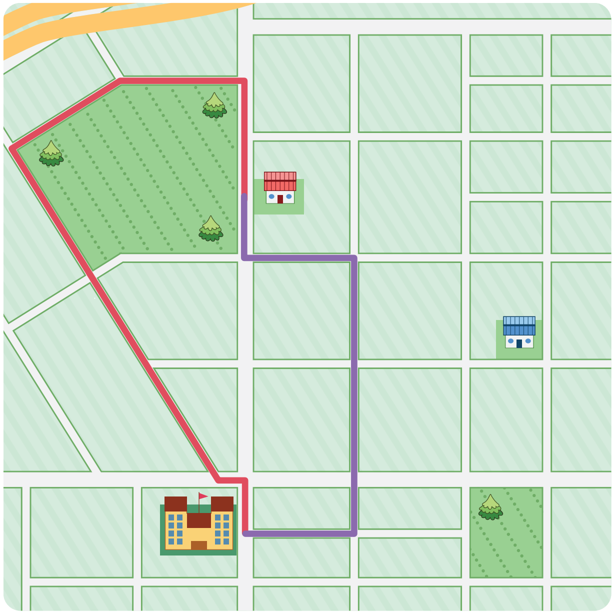


6. The map shows Ava's paths on Monday and Tuesday.

Monday: Ava walked $1\frac{1}{4}$ miles.

Tuesday: Ava walked $\frac{7}{8}$ of a mile.

How many times as far did Ava walk on Tuesday than on Monday?



Activity

2

Name: .... Date: .... Period: ....

Comparing Classroom Objects

7. Ava and Haru are comparing two objects in their classroom.

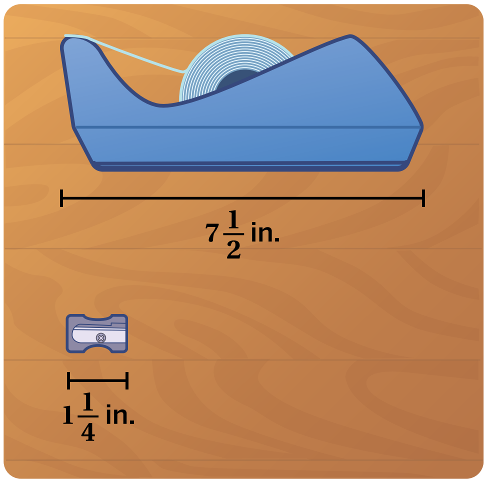
Ava says: *The tape dispenser is* 6 *times as long*.

Haru says: *The pencil sharpener is* $\frac{1}{6}$ *times as long*.

Whose thinking is correct? Circle one.

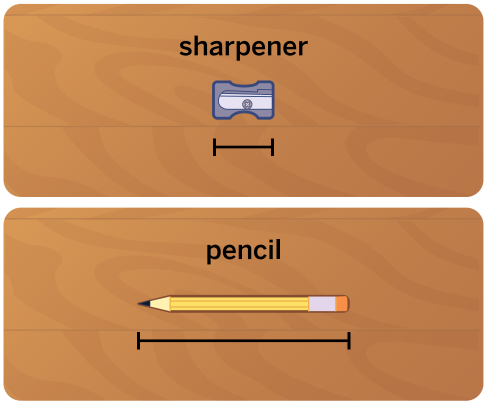
Ava's Haru's Both Neither

Explain your thinking.

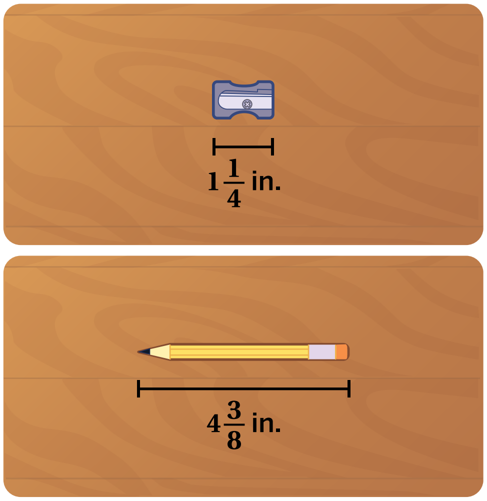


8. The sharpener is .... times as long as the pencil.

Estimate a value.



9. The sharpener is how many times as long as the pencil?



Activity
2

Name: ________________ Date: ________ Period: ________

Comparing Classroom Objects (continued)

10. Here is a collection of classroom objects, along with their lengths (in inches).

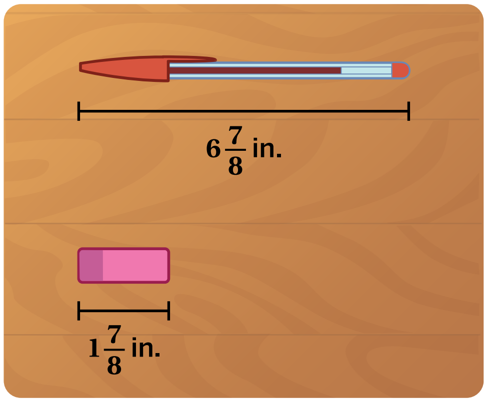
Scissors	$6\frac{1}{4}$	**Eraser**	$1\frac{7}{8}$
Marker	$5\frac{5}{8}$	**Stapler**	5
Red Pen	$6\frac{7}{8}$	**Pencil Sharpener**	$1\frac{1}{4}$
Tape Dispenser	$7\frac{1}{2}$	**Glue Bottle**	$3\frac{1}{8}$
Highlighter	$3\frac{3}{4}$	**Calculator**	$2\frac{1}{2}$
Large Pencil	$8\frac{1}{8}$	**Small Pencil**	$4\frac{3}{8}$

Choose pairs of objects to compare. Write an expression and a statement comparing their lengths. Solve as many challenges as you have time for!

	Objects	Expression	Statement
a	________ and ________		The ________ is ______ times as long as the ________.
b	________ and ________		The ________ is ______ times as long as the ________.
c	________ and ________		The ________ is ______ times as long as the ________.
d	________ and ________		The ________ is ______ times as long as the ________.

Synthesis

11. Describe a strategy for solving problems like this: *The pen is how many times as long as the eraser?*



Lesson Practice 4.12

Lesson Summary

You can use division to determine how many times as large one quantity is compared to another.

For example, let's say a song is $1\frac{1}{2}$ minutes long, and another song is $3\frac{3}{4}$ minutes long. You can compare the lengths of the two songs by answering either of these questions:

How many times longer is the second song than the first song?

$$3\frac{3}{4} \div 1\frac{1}{2} = ?$$
$$= \frac{15}{4} \div \frac{3}{2}$$
$$= \frac{15}{4} \bullet \frac{2}{3}$$
$$= \frac{30}{12} \text{ or } 2\frac{1}{2}$$

The second song is $2\frac{1}{2}$ times as long as the first song.

What fraction of the second song is the first song?

$$1\frac{1}{2} \div 3\frac{3}{4} = ?$$
$$= \frac{3}{2} \div \frac{15}{4}$$
$$= \frac{6}{4} \div \frac{15}{4}$$
$$= \frac{6}{15} \text{ or } \frac{2}{5}$$

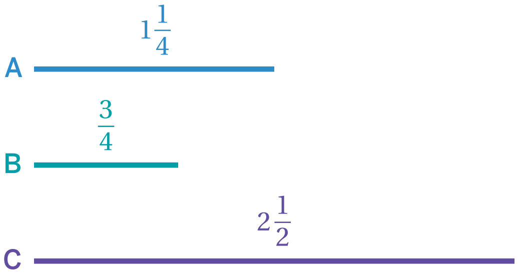
The first song is $\frac{2}{5}$ as long as the second song.

Lesson Practice
4.12

Name: .... Date: .... Period: ....

1. Segment A is $1\frac{1}{4}$ centimeters long. Segment B is $\frac{3}{4}$ centimeters long, and Segment C is $2\frac{1}{2}$ centimeters long.

Match each question with the expression that could be used to answer it.



a How much longer is Segment A than Segment B?

b Segment B is how many times as long as Segment A?

c Segment A is how many times as long as Segment B?

d What fraction of Segment A is Segment C?

e What fraction of Segment C is Segment A?

.... $\frac{3}{4} \div 1\frac{1}{4}$

.... $1\frac{1}{4} \div 2\frac{1}{2}$

.... $1\frac{1}{4} \div \frac{3}{4}$

.... $1\frac{1}{4} - \frac{3}{4}$

.... $2\frac{1}{2} \div 1\frac{1}{4}$

Problems 2–3: Deiondre set a daily goal to ride his bicycle $4\frac{1}{2}$ miles.

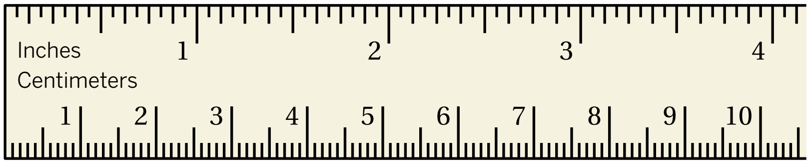
2. On Monday, he biked 6 miles. How many times his goal did Deiondre ride?

3. On Tuesday, he biked $1\frac{4}{5}$ miles. What fraction of his goal did Deiondre ride?

Lesson Practice
4.12

Name: .... Date: .... Period: ....

Problems 4–6: When taking measurements, engineers in the U.S. use the U.S. customary system, while engineers in Canada use the metric system. On an international project, it's important to convert measurements precisely. For example, one inch is the same length as $2\frac{27}{50}$ centimeters.



4. How many centimeters long is 3 inches? Show or explain your thinking.

5. What fraction of 1 inch is 1 centimeter? Show or explain your thinking.

6. What question can you answer about this situation by determining the value of $10 \div 2\frac{27}{50}$?

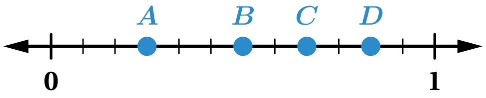
FAST Practice

7. A security guard works $9\frac{1}{2}$-hour-long shifts. At one point during a shift, the guard looks at the clock and realizes it has been $3\frac{3}{4}$ hours since the shift started. Calculate exactly how much of the shift the guard has worked.

The guard has worked ☐ of the shift.

Spiral Review

8. Here are four points plotted on a number line. Which point best represents $66\frac{2}{3}\%$ of the distance between 0 and 1?



A. Point A

B. Point B

C. Point C

D. Point D

Problems 9–10: Determine each product.

9. $\frac{2}{3} \cdot \frac{9}{20}$

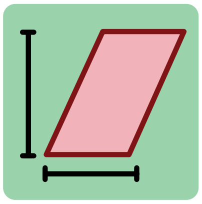
10. $1\frac{2}{5} \cdot \frac{3}{14}$

Name: .... Date: .... Period: ....

MA.6.NSO.2.2, MA.6.NSO.2.3, MA.6.GR.2.1, MA.6.GR.2.2, MTR.1.1

Puzzling Areas

Let's explore the areas of rectangles and triangles with fractional side lengths.



Warm-Up

Evaluate each expression mentally.

1. $3 \cdot 4$

2. $\frac{1}{3} \cdot 4$

3. $\frac{1}{3} \cdot \frac{1}{4}$

4. $2 \cdot \frac{1}{3} \cdot \frac{1}{4}$

5. $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$

Activity
1

Name: ____________________ Date: __________ Period: __________

Areas

6. Use any strategy to determine the area of as many figures as you can. Use the workspace below if it helps with your thinking.

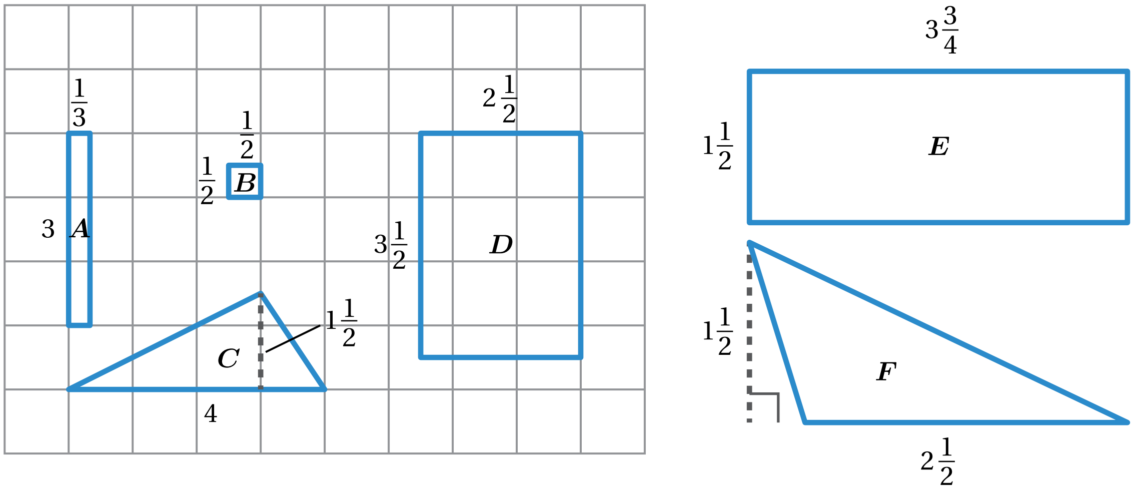


Figure	A	B	C	D	E	F
Area (sq. units)						

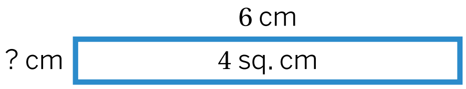
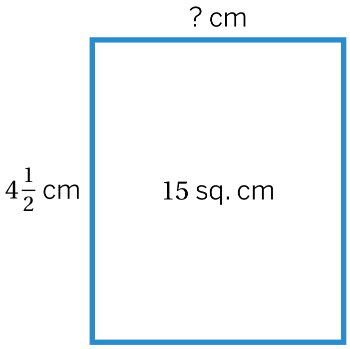
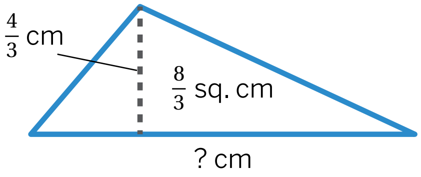
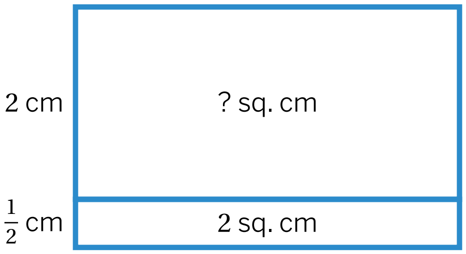
Workspace:

Activity 2

Name: ______________________ Date: __________ Period: __________

Level Up Area Puzzles

7. Use any strategy to determine the unknown side length or area.

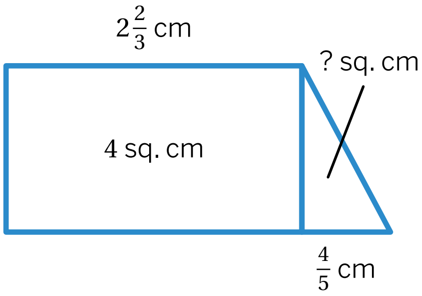
	Puzzle	Workspace
a		? = ______ centimeters
b		? = ______ centimeters
c		? = ______ centimeters
d		? = ______ square centimeters

Name: .... Date: .... Period: ....

Level Up Area Puzzles (continued)

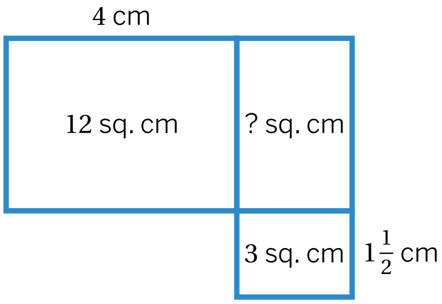
8. Solve as many puzzles as you have time for. You can work on them in any order.

Puzzle A



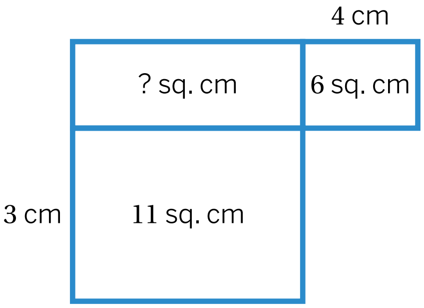
? = ... square centimeters

Puzzle B



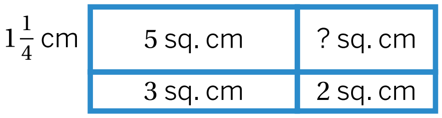
? = ... square centimeters

Puzzle C



? = ... square centimeters

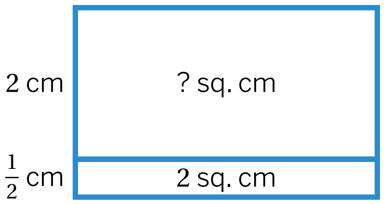
Puzzle D



? = ... square centimeters

Synthesis

9. **a** When is multiplication helpful for solving problems about areas?



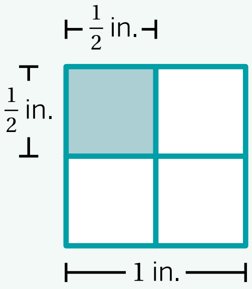
b When is division helpful for solving problems about areas?

Lesson Practice 4.13

Lesson Summary

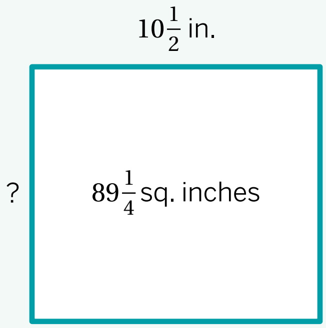
You can determine the area of a polygon that has fractional side lengths just like you would a polygon that has whole-number side lengths.

For example, you can calculate the area of the shaded square using the the formula $A = l \cdot w$. The area is equal to $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$ square inches.



You can also use area formulas to determine an unknown length. If you know the area and one side length of a rectangle, you can divide to determine the other side length.

For example, to determine the missing side length of this rectangle, you can calculate $89\frac{1}{4} \div 10\frac{1}{2} = 8\frac{1}{2}$. The missing side length is $8\frac{1}{2}$ inches.



Lesson Practice
4.13

Name: .... Date: .... Period: ....

1. A rectangular lawn has an area of $7\frac{1}{3}$ square yards and a width of $2\frac{1}{5}$ yards. What is the length of the lawn, in yards?

A. $9\frac{8}{15}$ **B.** $3\frac{1}{3}$ **C.** $\frac{3}{10}$ **D.** $5\frac{2}{15}$

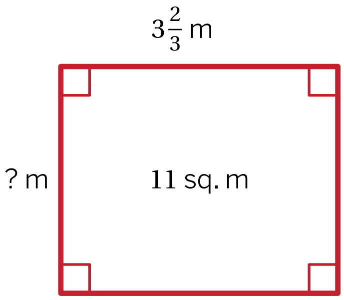
2. A television screen has a length of $16\frac{1}{2}$ inches, a width of w inches, and an area of 462 square inches. Select *all* the equations that represent the relationship between the dimensions of the television.

☐ **A.** $w \bullet 462 = 16\frac{1}{2}$ ☐ **B.** $16\frac{1}{2} \bullet w = 462$ ☐ **C.** $462 \div 16\frac{1}{2} = w$

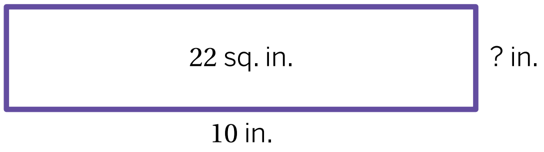
☐ **D.** $462 \div w = 16\frac{1}{2}$ ☐ **E.** $16\frac{1}{2} \bullet 462 = w$

Problems 3–6: Determine the missing length or lengths in each figure.

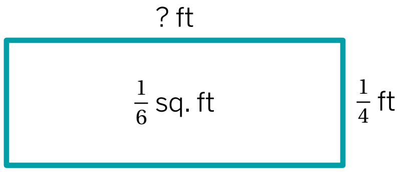
3.



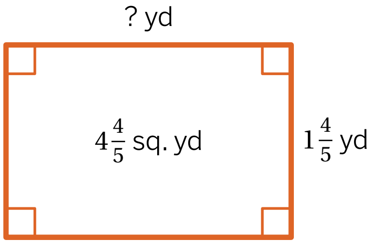
4.



5.



6.



Lesson Practice

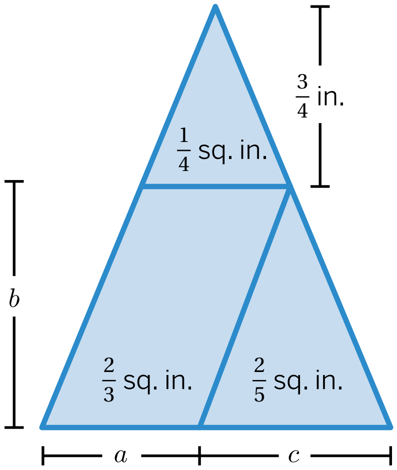
4.13

Name: ______ Date: ______ Period: ______

FAST Practice

7. Determine the missing lengths in this figure made up of a parallelogram and two triangles.

Unknown	Length (in.)
a	
b	
c	



Spiral Review

Problems 8–9: A bookshelf is 42 inches long.

8. How many books will fit on the bookshelf if each book is $1\frac{1}{2}$ inches wide? Show your thinking.

9. A bookcase has five of these 42-inch-long bookshelves. How many total feet of shelf space does the bookcase have? Show your thinking.

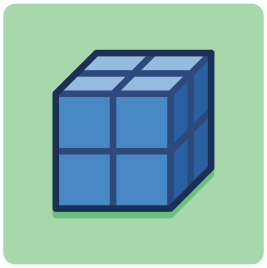
Unit 4
Lesson
14

Name: ______ Date: ______ Period: ______

MA.6.NSO.2.2, MA.6.NSO.2.3, MA.6.GR.2.3, MTR.4.1

Volume Challenges

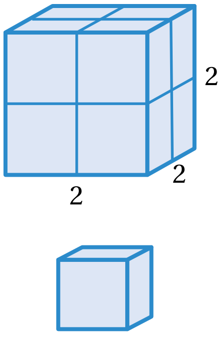
Let's explore the volume of prisms with fractional dimensions.



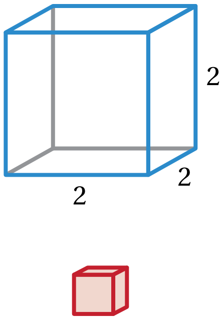
Warm-Up

1. A unit cube is a $1 \times 1 \times 1$ prism.

a **Discuss:** How do you know that it takes 8 unit cubes to fill a $2 \times 2 \times 2$ prism?



8 unit cubes



? small cubes

b A small cube measures $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ units. How many small cubes do you need to fill the $2 \times 2 \times 2$ prism?

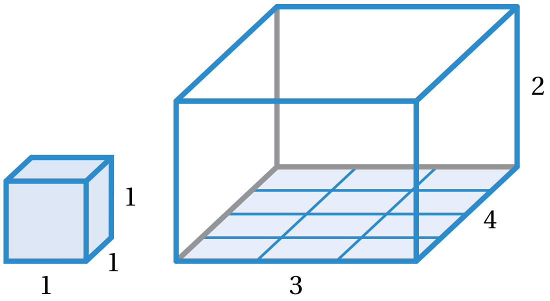
Activity
1

Name: .... Date: .... Period: ....

Volume Strategies

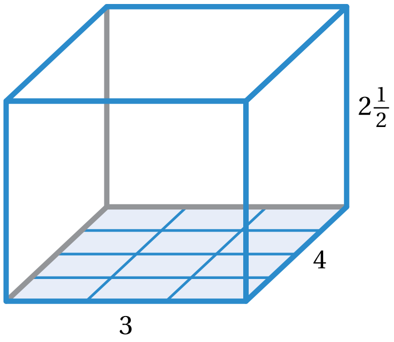
2. *Volume* is the number of unit cubes it takes to fill a container.

What is the volume of this rectangular prism?

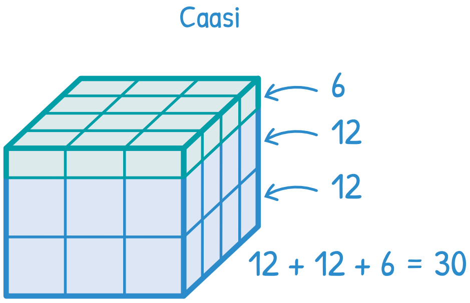


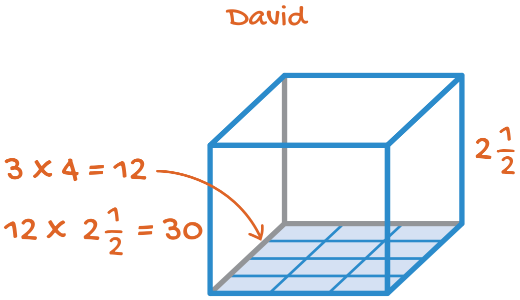
3. This prism has the same base as the one in the previous problem, but it's a $\frac{1}{2}$-unit taller.

What is the volume of this rectangular prism?



4. Here is the work that Caasi and David did to calculate the volume of the prism from the previous problem.





Discuss: What was each student's strategy?

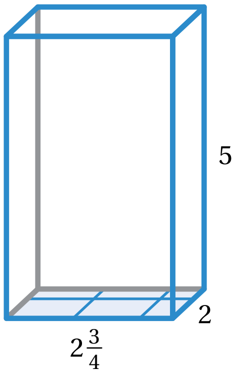
Activity
2

Name: .... Date: .... Period: ....

Calculating Volume

5. Here is another rectangular prism.

What is its volume?



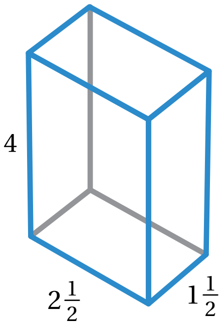
6. Caasi and David are calculating the volume of a new prism. Here are four different views of that prism.

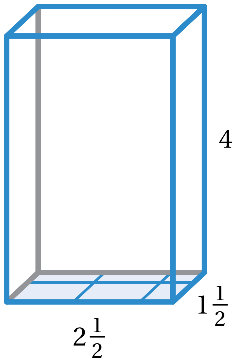
Caasi thinks the area of the base is 10 square units. David thinks the area of the base is 6 square units.

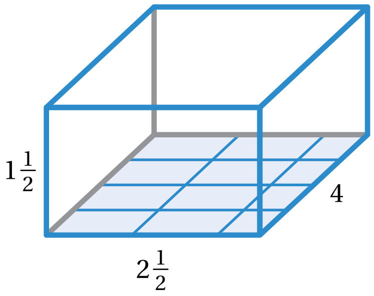
Whose thinking is correct?

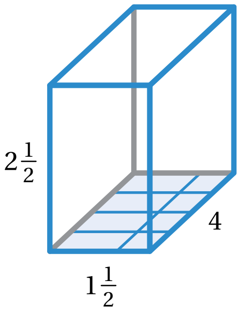
Caasi's David's Both Neither

Explain your thinking.



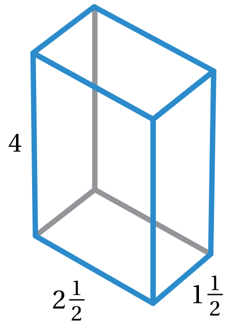






7. Here is the prism from the previous problem.

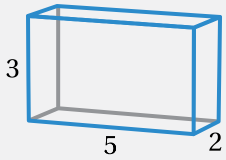
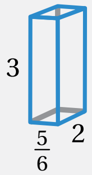
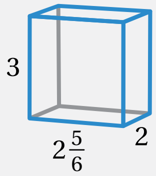
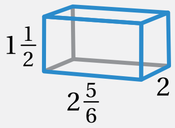
What is its volume?



Name: .... Date: .... Period: ....

Calculating Volume (continued)

8. Calculate the volume of each prism.

Prism	Dimensions (units)	Volume (cubic units)
	$5 \times 2 \times 3$	
	$\frac{5}{6} \times 2 \times 3$	
	$2\frac{5}{6} \times 2 \times 3$	
	$2\frac{5}{6} \times 2 \times 1\frac{1}{2}$	

You're invited to explore more.

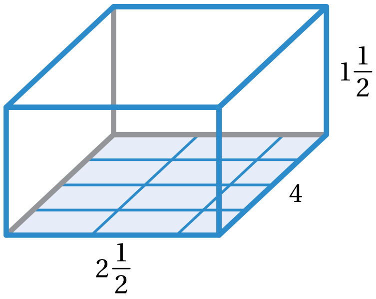
9. How many rectangular prisms can you make that have a volume of 12 cubic inches? List their dimensions. Only use dimensions measuring 24 inches or less.

The first one has been done for you.

Length (in.)	Width (in.)	Height (in.)	Volume (cu. in.)
2	2	3	12

Synthesis

10. Describe a strategy for calculating the volume of a rectangular prism like this one.

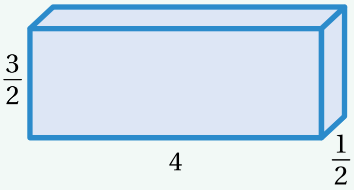


Lesson Practice 4.14

Lesson Summary

You can determine the *volume* of a prism by multiplying its dimensions.

For example, here is a rectangular prism with side lengths measuring 4 units, $\frac{3}{2}$ units, and $\frac{1}{2}$ units.

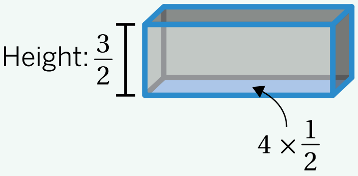


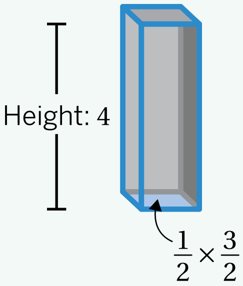
Volume

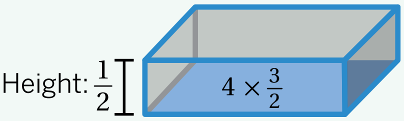
$4 \bullet \frac{3}{2} \bullet \frac{1}{2}$

$= 3$ cubic units

You can also calculate the volume of a prism as the product of its base area and the height. You can choose any of the rectangles as the base.







If you choose the 4-by-$\frac{1}{2}$ rectangle as the base, then the base area will be 2 square units.

The volume is $2 \bullet \frac{3}{2} = 3$ cubic units.

If you choose the $\frac{3}{2}$-by-$\frac{1}{2}$ rectangle as the base, then the base area will be $\frac{3}{4}$ square units.

The volume is $\frac{3}{4} \bullet 4 = 3$ cubic units.

If you choose the 4-by-$\frac{3}{2}$ rectangle as the base, then the base area will be 6 square units.

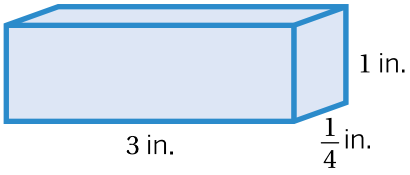
The volume is $6 \bullet \frac{1}{2} = 3$ cubic units.

Lesson Practice
4.14

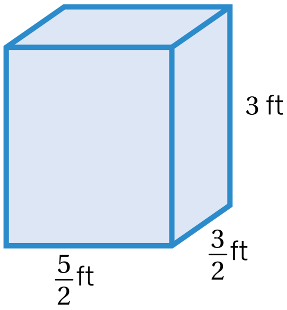
Name: .... Date: .... Period: ....

Problems 1–3: Determine the volume of these rectangular prisms.

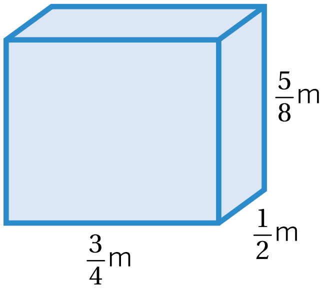
1.



2.



3.



Problems 4–5: Complete the table for each rectangular prism.

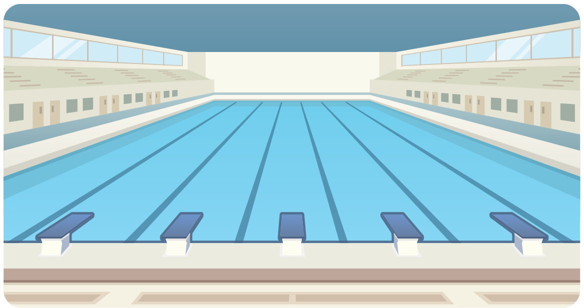
	Base Area (sq. in.)	Height (in.)	Volume (cu. in.)
4.	4	$1\frac{1}{3}$	
5.		$2\frac{2}{3}$	8

6. Diego says that he needs 108 cubes, each with an edge length of $\frac{1}{3}$ inches, to fill a rectangular prism measuring 3-by-1-by-$1\frac{1}{3}$ inches. Do you agree with Diego? Explain your thinking.

Lesson Practice
4.14

Name: .... Date: .... Period: ....

Problems 7–8: A swimming pool used in the Olympics is 25 meters wide, 50 meters long, and about $2\frac{1}{2}$ meters deep.



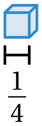
7. Determine the volume of this swimming pool. Show your thinking.

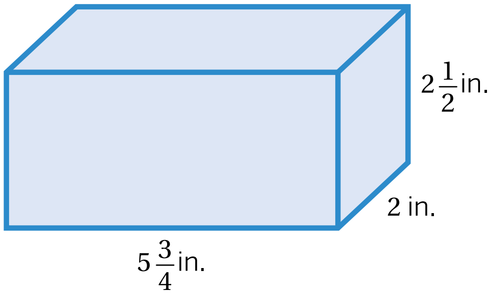
8. There are $1{,}000$ liters of water in 1 cubic meter. Calculate the number of liters needed to fill this pool.

FAST Practice

9. How many small cubes with edge lengths measuring $\frac{1}{4}$ inches can be packed into this right rectangular prism?

☐ small cubes





Spiral Review

10. Select *all* the fractions that are equivalent to $\frac{2}{3}$.

☐ **A.** $\frac{4}{6}$ ☐ **B.** $\frac{8}{15}$ ☐ **C.** $\frac{12}{13}$ ☐ **D.** $\frac{20}{30}$ ☐ **E.** $\frac{14}{21}$

11. A teacher wants to make a paste with flour and water. The table shows the ratio of the number of cups of flour to the number of cups of water she needs. Complete the table to show the other equivalent ratios.

Flour (cups)	Water (cups)
1	$\frac{1}{2}$
4	
	3
$\frac{1}{2}$	

Unit 4

Lesson

15

Name: ______ Date: ______ Period: ______

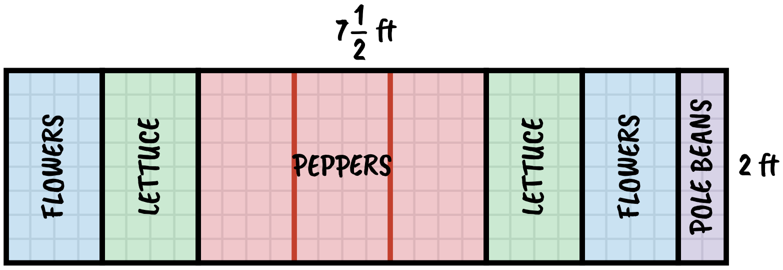
MA.6.NSO.2.2, MA.6.NSO.2.3, MA.6.GR.2.3, MTR.4.1, MTR.7.1

Planter Planner

Let’s apply division of fractions to real-world scenarios.

Warm-Up

Jin sketched a plan for her planter.



1. What do you notice? What do you wonder?

I notice:

I wonder:

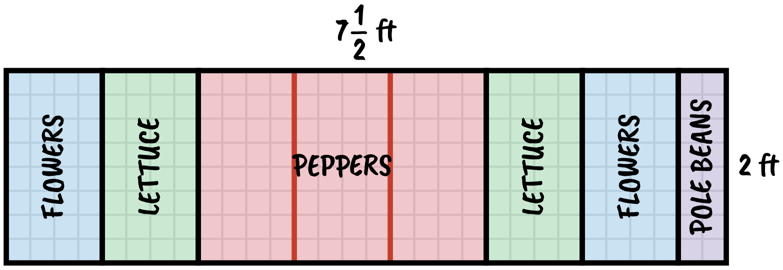
2. Jin says she can fit 8 lettuce plants in each lettuce section of the planter. Do you agree? Why or why not?

Activity 1

Name: .... Date: .... Period: ....

Jin's Planter

Here is Jin's planter diagram again.



3. Use the Activities 1 & 2 Sheet to help Jin figure out how many of each plant she can grow.

a .... flower plants

b .... lettuce plants

c .... pepper plants

d .... pole bean plants

4. How many servings of pole beans can Jin grow in her planter?

5. Will Jin get more servings of food from her pepper plants or her pole bean plants? Explain your thinking.

6. Jin needs to fill her planter with soil.

Help her figure out how many bags of soil she needs.

Activity
2

Name: .... Date: .... Period: ....

Build Your Own Planter

7. You are planning a planter for your school's greenhouse.

a Select *at least three* types of plants from the Activities 1 & 2 Sheet to grow in your planter.

b Select a planter to grow your plants in.

c Determine how many of each type of plant you can fit in your planter. Be sure each plant has enough space to grow! Show your work.

d Create a poster about your planter.

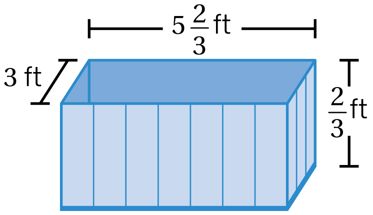
- ☐ Sketch your planter, showing the locations of each type of plant.
- ☐ List the types of plants in the planter.
- ☐ List your answer from part c.
- ☐ Calculate the amount of soil you will need to fill your planter. Show your work. (Each bag fills $\frac{3}{4}$ of a cubic foot.)
- ☐ Calculate how many servings of food and bunches of flowers you expect to grow.

You're invited to explore more.

8. Xavier has 20 cubic feet of soil. He wants to build a planter that is $2\frac{1}{2}$ feet wide and $1\frac{1}{4}$ feet deep. How many feet long should he make the planter so that he uses all of his soil? Explain your thinking.

Synthesis

9. What are some important things to remember when calculating volume using dimensions that are fractions?



Lesson Practice 4.15

Lesson Summary

Take a moment to review what you know about fractions and operations.

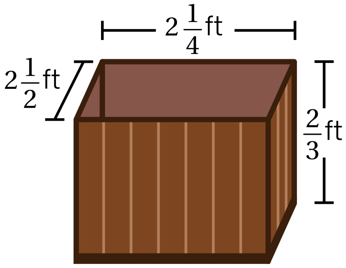
To multiply fractions . . .	To divide by a fraction, $\frac{a}{b}$ . . .
- Multiply the denominators as a way to determine a common denominator and make same-sized parts. - Then multiply the numerators to determine how many of those parts exist.	Multiply the dividend by the reciprocal, $\frac{b}{a}$.
Example: $\frac{3}{8} \cdot \frac{5}{9}$ $= \frac{3 \cdot 5}{8 \cdot 9}$ $= \frac{15}{72}$ $= \frac{5}{24}$	**Example:** $\frac{4}{7} \div \frac{5}{3}$ $= \frac{4}{7} \cdot \frac{3}{5}$ $= \frac{12}{35}$

Lesson Practice

4.15

Name: .... Date: .... Period: ....

Problems 1–3: Raven is building a new planter for her class garden.

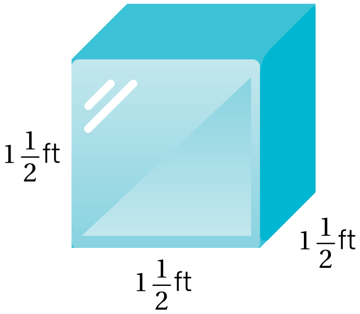


1. The base of her planter is $2\frac{1}{2}$ feet by $2\frac{1}{4}$ feet. What is its area?

2. If the planter is $\frac{2}{3}$ feet high, what volume of soil does she need to fill it?

3. Each bag holds $\frac{3}{4}$ cubic feet of soil. How many bags of soil does Raven need?

Problems 4–5: Before home freezers were introduced in the 1940s, some people had large blocks of ice delivered to their homes.



4. Delivery wagons held $7\frac{1}{2}$ feet by 6 feet by 6 feet of ice. What volume of ice could a delivery wagon hold?

5. Each ice block was a $1\frac{1}{2}$-foot cube. How many ice blocks could fit in a delivery wagon? Explain your thinking.

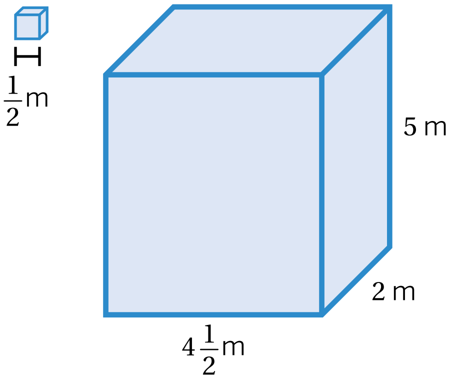
Lesson Practice
4.15

Name: ______________________ Date: __________ Period: __________

FAST Practice

6. A small cube has edge lengths measuring $\frac{1}{2}$ meter. How many small cubes could be packed into a rectangular prism that measures $4\frac{1}{2}$-by-2-by-5 meters?

- **A.** 45 small cubes
- **B.** 90 small cubes
- **C.** 180 small cubes
- **D.** 360 small cubes



Spiral Review

Problems 7–10: Determine each quotient.

7. $8 \div \frac{2}{9}$

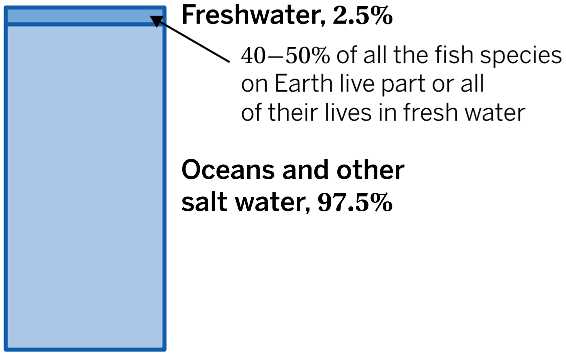
8. $\frac{5}{6} \div 15$

9. $\frac{4}{7} \div \frac{8}{12}$

10. $\frac{23}{2} \div \frac{14}{5}$

Career Connection

71% of Earth is covered with water, yet only a small fraction of that is fresh water.



Only about 2.5% of Earth's total water is fresh water. Despite this small percentage, about 40–50% of all of the species of fish on Earth live in fresh water for all or part of their lives. Many species of fresh water fish are endangered – due to pollution, overfishing, invasive species, and more.

Freshwater conservation ecologists collect data to study the fish and plants that live in freshwater regions. They develop strategies to conserve and restore these populations. They might use percentages to predict the number of fish in a certain freshwater region each year.

B.E.S.T. Mathematics Benchmark Connection

Ecologists use percentages in many aspects of their work. For example, they use percentages (MA.6.NSO.3.5) when they analyze data collected and they are describing results in a way that non-science people can understand. They may also look for unknown percentages (MA.6.AR.3.4) to set up an experiment.

Mathematical Thinking and Reasoning Connection

Ecologists and other scientists who work with them use thinking and reasoning skills like the ones you use for your math work! For example, they look for patterns when they study certain plants and animals in different parts of the world or at different times of the year (MTR.5.1). When they present their findings to others, they help and support others who may be doing similar research (MTR.1.1).

Meet Laura Máiz-Tomé

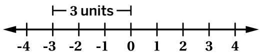
Laura Máiz-Tomé is a freshwater conservation ecologist who collaborates with other scientists to help preserve freshwater ecosystems around the world. She has led research to predict the extinction risk of freshwater species.

Glossary/Glosario

A

English

absolute value The distance from 0 to a number on a number line is its absolute value.

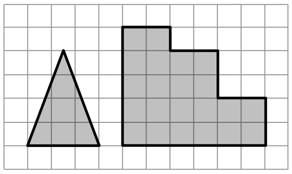


For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

additive inverses The additive inverse of a is $-a$. The sum of two additive inverses is 0.

For example, 5 and -5 are additive inverses because $5 + (-5) = 0$.

area Area measures the space inside a two-dimensional figure. It is expressed in square units.



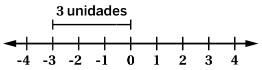
The area of the triangle is 6 square units. The area of the other shape is 22 square units.

associative property The property says $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. This means that expressions with addition or multiplication have the same sum or product no matter how the numbers in the expression are grouped.

For example, $(2 + 1) + 3 = 2 + (1 + 3)$ and $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$.

Español

valor absoluto La distancia del 0 a un número en una recta numérica es su valor absoluto.

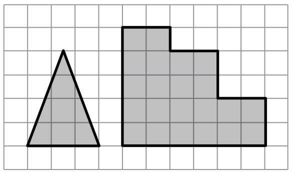


Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

inversos aditivos El inverso aditivo de a es $-a$. La suma de dos inversos aditivos es 0.

Por ejemplo, 5 y -5 son inversos aditivos porque $5 + (-5) = 0$.

área El área mide el espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.



El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.

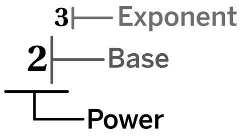
propiedad asociativa La propiedad indica que $a + (b + c) = (a + b) + c$ y $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente de cómo se agrupen los números en la expresión.

Por ejemplo, $(2 + 1) + 3 = 2 + (1 + 3)$ y $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$.

B

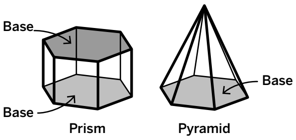
English

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.



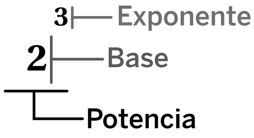
In the expression 2^3, 2 is the base.

base (of a pyramid or prism) The face that gives the solid its name. A prism has two identical bases that are parallel. A pyramid has one base.



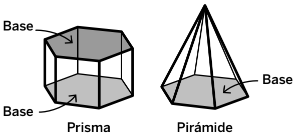
Español

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.



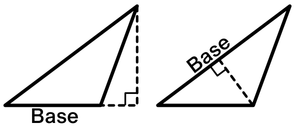
En la expresión 2^3, 2 es la base.

base (de una pirámide o un prisma) La cara que da el nombre al cuerpo geométrico. Un prisma tiene dos bases idénticas que son paralelas. Una pirámide tiene una base.



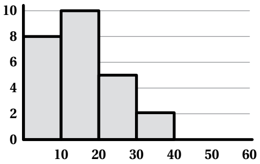
English

base (of a triangle) The base is one side of a triangle. We can choose any side to be the base.



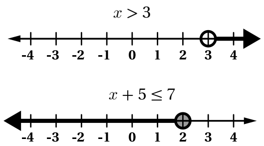
The base can also refer to the length of this side. The height of a shape is perpendicular to the base.

bin (of a histogram) The intervals used to group data values in a histogram.



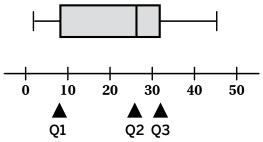
For example, this histogram shows bins of 10 units each.

boundary point The point on a number line or graph that separates solutions of an inequality from non-solutions. If the boundary point is a solution to the inequality (i.e., $\geq$ or $\leq$), it's represented with a closed circle on the graph. If it's not a solution (i.e., $>$ or $<$), it's represented with an open circle on the graph.



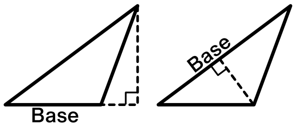
The boundary point of $x > 3$ is 3. The boundary point of $x \leq 2$ is 2.

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



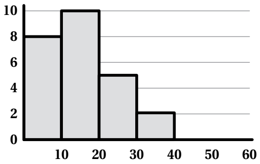
Español

base (de un triángulo) La base es un lado de un triángulo. Podemos elegir cualquier lado como base.



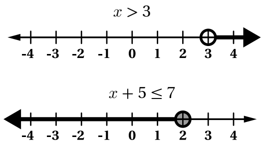
La base también puede referirse a la longitud de este lado. La altura de una figura es perpendicular a la base.

intervalo (de un histograma) Los intervalos que se usan para agrupar los valores de datos en un histograma.



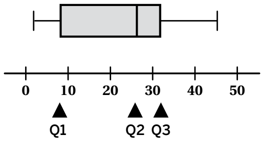
Por ejemplo, este histograma muestra intervalos de 10 unidades cada uno.

punto límite Punto en una recta numérica o una gráfica que separa las soluciones de una desigualdad de los valores que no son soluciones. Si el punto límite es una solución de la desigualdad (es decir, $\geq$ o $\leq$), se representa con un círculo cerrado en la gráfica. Si no es una solución (es decir, $>$ o $<$), se representa con un círculo abierto en la gráfica.



El punto límite de $x > 3$ es 3. El punto límite de $x \leq 2$ es 2.

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.

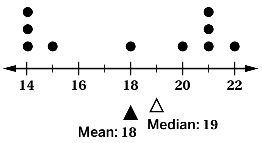


C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

"What kind of pet do you have?" is a question that would result in categorical data.

center A single value that can be used to summarize the typical value in a data set.

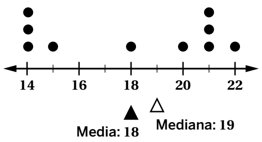


Mean and median are measures of center.

datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

centro Un solo valor que puede utilizarse para resumir el valor típico de un conjunto de datos.

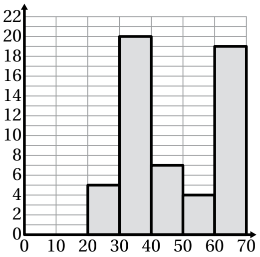


La media y la mediana son medidas de tendencia central, o sea, medidas de centro.

English

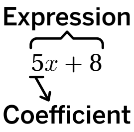
cluster A group of data values that are close together.

For example, this histogram shows a cluster between 30 and 40 and between 60 and 70.



coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression $5x + 8$, 5 is the coefficient of x.



common denominator The denominator is the bottom number in a fraction. Two fractions have a common denominator when their denominators are the same.

For example, $\frac{3}{4}$ and $\frac{5}{4}$ have a common denominator because they each split the whole into fourths.

common factor When two numbers have the same factor, we call that a common factor.

For example, the factors of 8 are 1, 2, 4, and 8 and the factors of 12 are 1, 2, 3, 4, 6, and 12.

Since 2 is a factor of 8 and also of 12, 2 is a common factor of 8 and 12.

common multiple When two numbers have the same multiple, we call that a common multiple.

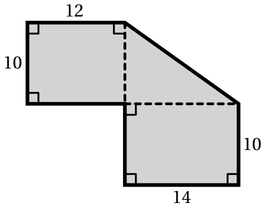
For example, 12 is a multiple of 2 and also of 3, so 12 is a common multiple of 2 and 3.

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

composite figure A two- or three-dimensional figure that can be decomposed into smaller figures.

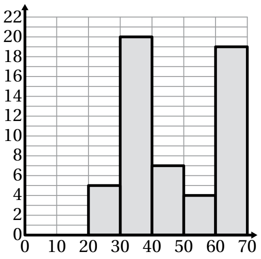
The composite figure can be decomposed into 2 rectangles and a triangle.



Español

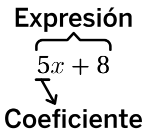
agrupación Un grupo de valores de datos que están próximos entre sí.

Por ejemplo, este histograma muestra una agrupación entre 30 y 40, y entre 60 y 70.



coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

En la expresión $5x + 8$, el coeficiente de x es 5.



denominador común El denominador es el número de abajo en una fracción. Dos fracciones tienen un denominador común cuando los denominadores son iguales.

Por ejemplo, $\frac{3}{4}$ y $\frac{5}{4}$ tienen un denominador común porque cada uno divide el todo en cuartos.

factor común Cuando dos números tienen el mismo factor, lo llamamos factor común.

Por ejemplo, los factores de 8 son 1, 2, 4 y 8 y los factores de 12 son 1, 2, 3, 4, 6 y 12.

Ya que 2 es un factor de 8 y también de 12, 2 es un factor común de 8 y 12.

múltiplo común Cuando dos números tienen el mismo múltiplo, lo llamamos múltiplo común.

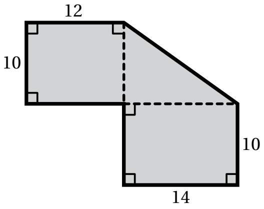
Por ejemplo, 12 es un múltiplo de 2 y también de 3, por lo tanto, 12 es un múltiplo común de 2 y 3.

propiedad conmutativa La propiedad indica que $a + b = b + a$ y $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto, independientemente del orden en el que estén los números.

Por ejemplo, $2 + 1 = 1 + 2$ o $3 \cdot 4 = 4 \cdot 3$.

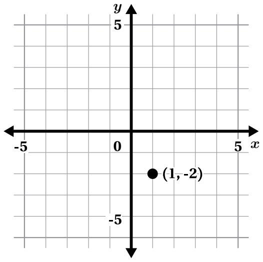
figura compuesta Figura bidimensional o tridimensional que puede descomponerse en figuras más pequeñas.

La figura compuesta se puede descomponer en 2 rectángulos y un triángulo.

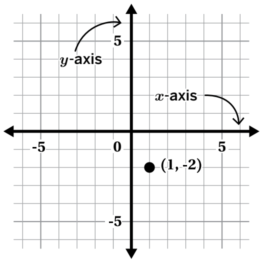


English

coordinate plane
The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



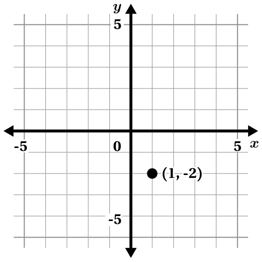
coordinates A pair of numbers that shows an exact position on the coordinate plane. The first number represents a position on the x-axis and is called the x-coordinate. The second number represents a position on the y-axis and is called the y-coordinate.



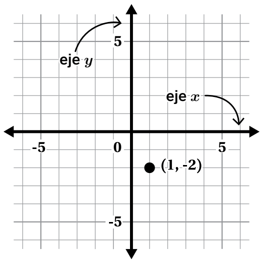
The coordinates of the point on the graph are $(1, -2)$.

Español

plano de coordenadas
El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).



coordenadas Un par de números que muestran una posición exacta en el plano de coordenadas. El primer número representa una posición en el eje x y se denomina coordenada x. El segundo número representa una posición en el eje y y se denomina coordenada y.



Las coordenadas del punto en la gráfica son $(1, -2)$.

D

decompose Decompose means "take apart." We use the word *decompose* to describe taking a figure apart to make more than one new shape.

distributive property The property that says $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.

Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dividend The number in a division statement that is being divided.

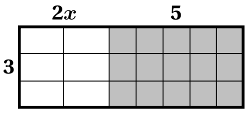
For example, in the equation $12 \div 3 = 4$, the dividend is 12.

divisor In a division statement, the divisor describes the number of equal-sized groups or the size of each group being created.

For example, in the equation $12 \div 3 = 4$, the divisor is 3.

descomponer Descomponer significa "desmontar." Usamos la palabra *descomponer* para describir que una figura se desmonta para formar más de una figura nueva.

propiedad distributiva
La propiedad que indica que $a(b + c) = ab + ac$. Significa que multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente y luego sumar los productos.



El área como un producto
$3(2x + 5)$

Por ejemplo, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dividendo El número que se está dividiendo en un enunciado de división.

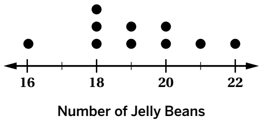
Por ejemplo, en la ecuación $12 \div 3 = 4$, el dividendo es 12.

divisor En un enunciado de división, el divisor describe la cantidad de grupos de igual tamaño o el tamaño de cada grupo que se produce.

Por ejemplo, esta ecuación $12 \div 3 = 4$, el divisor es 3.

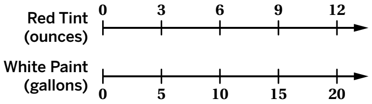
English

dot plot A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other. A dot plot is sometimes called a line plot.



For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

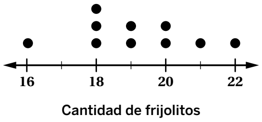
double number line A pair of parallel number lines showing equivalent ratios. The tick marks are labeled so that the marks that line up vertically are equivalent ratios.



This double number line shows a ratio of 3 ounces of red tint : 5 gallons of white paint.

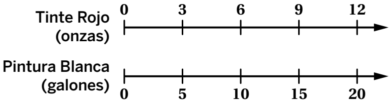
Español

diagrama de puntos Una forma de visualizar conjuntos de datos numéricos, en la que cada punto de datos se representa mediante un punto en una recta numérica. Los puntos de datos con el mismo valor se apilan unos sobre otros. Un diagrama de puntos a veces se conoce como gráfica de puntos.



Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

recta numérica doble Un par de rectas numéricas paralelas que muestran razones equivalentes. Las marcas indicadoras se denominan de forma tal que las marcas alineadas verticalmente sean razones equivalentes.



Esta recta numérica doble muestra una razón de 3 onzas de tinte rojo : 5 galones de pintura blanca.

E

English

edge Each straight side of a polygon is called an edge. An edge is also a line segment where two faces of a 3-D figure meet.

This rectangle has four edges.

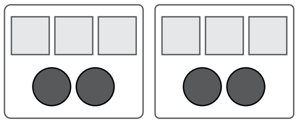
equation A mathematical statement made up of two expressions with an equal sign between them.

For example, $6m + 5 = 17$ and $12 - 15 = -3$ are equations, but $2n$ and $x > 5$ are not equations.

equivalent expressions Expressions that are equal for every value of a variable.

$x + x + x$ is equivalent to $3x$ because they both describe three copies of an unknown number, x.

equivalent ratio Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



$3 : 2$ is equivalent to $6 : 4$ because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. The second recipe will make twice as much lemonade but both recipes will taste the same.

Español

lado, arista Cada borde recto de un polígono se llama arista o lado. Una arista también es un segmento de recta donde se unen dos caras de una figura tridimensional.

Este rectángulo tiene cuatro aristas.

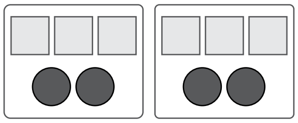
ecuación Un enunciado matemático formado por dos expresiones con un signo igual entre ellas.

Por ejemplo, $6m + 5 = 17$ y $12 - 15 = -3$ son ecuaciones, pero $2n$ y $x > 5$ no son ecuaciones.

expresiones equivalentes Expresiones que son iguales para cualquier valor de una variable.

$x + x + x$ equivale a $3x$ porque ambas describen tres copias de un número desconocido, x.

razón equivalente Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



$3 : 2$ es equivalente a $6 : 4$ porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

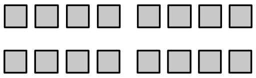
Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. La segunda receta rendirá el doble de limonada, pero ambas tendrán el mismo sabor.

English

evaluate To evaluate is to determine a single number that represents an expression's value.

For example, to evaluate $5x + 2$ when $x = 3$, we substitute 3 for x and then calculate $5(3) + 2 = 17$.

exponent A number used to describe repeated multiplication. Exponents are sometimes called powers.



$2^4 = 16$

For example, $2 \cdot 2 \cdot 2 \cdot 2 = 16$ can be represented by the equation $2^4 = 16$, where 4 is the exponent. We can read this equation as "2 to the power of 4 equals 16" or "2 to the fourth equals 16."

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

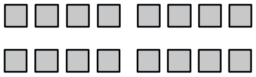
For example, $2n - 8$ and $21 + 37$ are expressions.

Español

evaluar Evaluar significa determinar el número individual que representa el valor de una expresión.

Por ejemplo, para evaluar $5x + 2$ cuando $x = 3$, sustituimos x por 3 y luego calculamos $5(3) + 2 = 17$.

exponente Un número que se usa para describir multiplicaciones repetidas. A los exponentes a veces se les conoce como potencias.



$2^4 = 16$

Por ejemplo, $2 \cdot 2 \cdot 2 \cdot 2 = 16$ puede representarse con la ecuación $2^4 = 16$, donde 4 es el exponente. Podemos leer esta ecuación como "2 a la potencia de 4 es igual a 16" o "2 a la cuarta potencia es igual a 16".

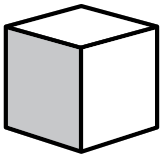
expresión Un conjunto de números, variables, operaciones y símbolos de agrupación que representan una cantidad.

Por ejemplo, $2n - 8$ y $21 + 37$ son expresiones.

F

face Each flat side of a polyhedron is called a face.

A cube has six faces and they are all squares.



factor (of a number) A whole number that divides evenly into the given number (with no remainder).

For example, 1, 2, 4, and 8 are all factors of the number 8.

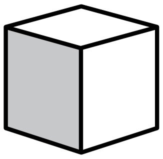
frequency table A table that shows the number of times each value or category occurs in a data set.

For example, this frequency table shows that 5 students selected red as their favorite color.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

cara Cada lado plano de un poliedro se llama cara.

Un cubo tiene seis caras y todas son cuadradas.



factor (de un número) Un número natural por el que se puede dividir en partes iguales el número dado (sin resto).

Por ejemplo, 1, 2, 4 y 8 son factores del número 8.

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

G

$\geq$ (greater-than-or-equal-to) This symbol means "greater than or equal to."

For example, the inequality $x \geq 3$ says the value of x is either exactly 3 or any value greater than 3.

$\geq$ (mayor que o igual a) Este símbolo significa "mayor que o igual a".

Por ejemplo, la desigualdad $x \geq 3$ indica que el valor de x es exactamente 3 o cualquier valor mayor que 3.

English

Español

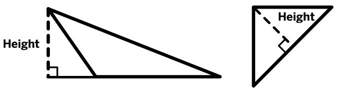
greatest common factor (GCF) The largest number that is a common factor of two numbers.

The common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is 4.

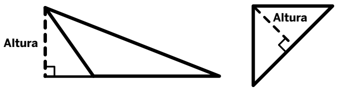
máximo común divisor (MCD) El número mayor que es factor común de dos números.

Los factores comunes de 8 y 12 son 1, 2 y 4. El máximo común divisor es 4.

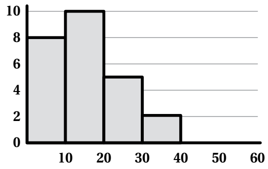
H



height (of a triangle) The shortest distance between a base and its opposite vertex. Sometimes the height falls outside the shape. The height is always perpendicular to the base.

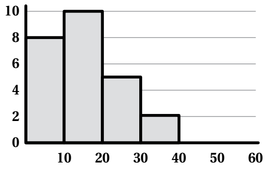


altura (de un triángulo) La distancia más corta entre una base y su vértice opuesto. A veces, la altura cae fuera de la figura. La altura siempre es perpendicular a la base.



histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.

For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.



histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.

Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

I

inequality A comparison statement that uses the symbols $<$ or $>$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y > 30$ means that the value of the expression y is any number greater than 30.

desigualdad Un enunciado de comparación que utiliza los símbolos $<$ o $>$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

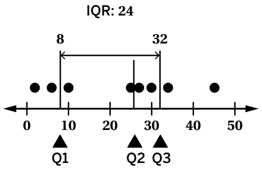
Por ejemplo, la desigualdad $y > 30$ significa que el valor de la expresión y es cualquier número mayor que 30.

integers All whole numbers and their opposites.

For example, 35, -15, and 1 are integers. 0.3 and $\frac{1}{3}$ are not.

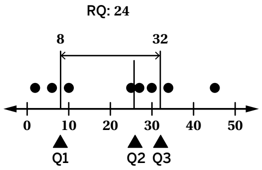
enteros Todos los números enteros y sus opuestos.

Por ejemplo, 35, -15 y 1 son números enteros. 0.3 y $\frac{1}{3}$ no lo son.



interquartile range (IQR) A measure of spread. The IQR is calculated as the distance from Q1 to Q3, or the width of the box in a box plot.

For example, the IQR of this data set is $32 - 8 = 24$.



rango intercuartílico (RQ) Una medida de dispersión. El RQ se calcula como la distancia de Q1 a Q3, o el ancho de la caja en un diagrama de caja.

Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.

L

least common multiple (LCM) The smallest number that is a common multiple of two numbers.

The common multiples of 2 and 3 are 6, 12, 18, . . .
The least common multiple is 6.

mínimo común múltiplo (MCM) El número menor que es múltiplo común de dos números.

Los múltiplos comunes de 2 y 3 son 6, 12, 18. . .
El mínimo común múltiplo es 6.

English

≤ (less-than-or-equal-to) This symbol means "less than or equal to."

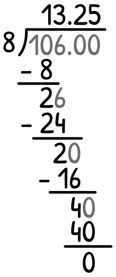
For example, the inequality $x \leq 9$ says the value of x is either exactly 9 or any value less than 9.

like terms Terms with variables and exponents that are the same.

For example, $8x$ and $12x$ are like terms because both terms have a variable of x. $3x$ and $3x^2$ are not like terms because they have different exponents.

long division A way to divide numbers. When we use long division, we determine the quotient one digit at a time, from left to right.

For example, here is the long division for $106 \div 8$.



Español

≤ (menor que o iqual a) Este símbolo significa "menor que o igual a".

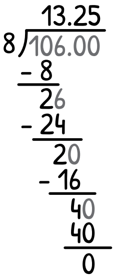
Por ejemplo, la desigualdad $x \leq 9$ indica que el valor de x es exactamente 9 o cualquier valor menor que 9.

términos semejantes Términos con variables y exponentes iguales.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable que incluye x. $3x$ y $3x^2$ no son términos semejantes porque tienen exponentes diferentes.

división larga Una forma de dividir números. Al usar la división larga, determinamos el cociente de izquierda a derecha, un dígito a la vez.

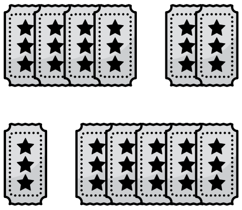
Por ejemplo, esta es la división larga de $106 \div 8$.



M

maximum The greatest value in a data set.

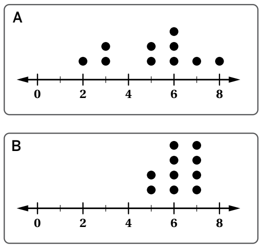
mean (average) A measure of center. If you equally distribute a set of items into different groups, the mean is the number of items in each group. It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.



The mean in this example is 3 tickets because that is the number each person would get if the tickets were distributed equally among four people.

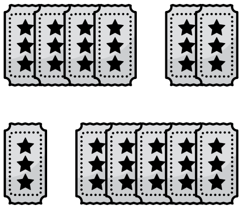
measure of center A single number that summarizes all of the values in a data set. It is usually a typical value for a data set. Mean and median are measures of center.

measure of spread A single number that describes the spread of a data set. Range and interquartile range are measures of spread.



máximo El mayor valor en un conjunto de datos.

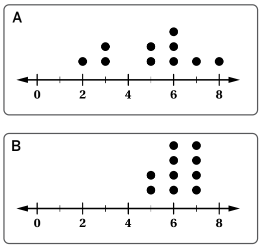
media (promedio) Una medida de tendencia central. Si se distribuye equitativamente un conjunto de elementos en diferentes grupos, la media es la cantidad de elementos en cada grupo. También es el punto de equilibrio de un diagrama de puntos. Para calcular la media, se pueden sumar los valores de todos los puntos de datos y luego dividir por la cantidad de puntos de datos.



La media en este ejemplo es 3 boletos porque es la cantidad que obtendría cada persona si los boletos se distribuyeran equitativamente entre cuatro personas.

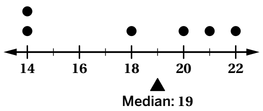
medida de tendencia central, medida de centro Un solo número que resume todos los valores de un conjunto de datos. Suele ser un valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central.

medida de dispersión Un solo número que describe la dispersión de un conjunto de datos. El rango y el rango intercuartílico son medidas de dispersión.



English

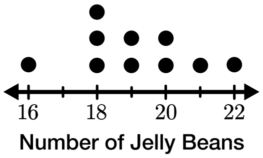
median A measure of center. It is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values.



The median of this data set is 19 because it is the average of the two middle values: 18 and 20.

minimum The least value in a data set.

mode A measure of center. It is the value that occurs most often in a data set. There may be no mode, one mode, or more than one mode.



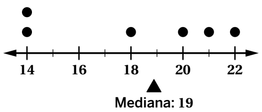
For example, the mode of this data set is 18 jelly beans because the value 18 occurs more times than any other data value.

multiple The result of multiplying a number by a whole number.

For example, 10, 15, and 20 are all multiples of the number 5.

Español

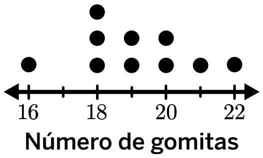
mediana Una medida de tendencia central. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay un número par de puntos de datos, la mediana es el promedio de los dos números centrales.



La mediana de este conjunto de datos es 19 porque es el promedio de los dos valores centrales: 18 y 20.

mínimo El menor valor en un conjunto de datos.

moda Una medida de centro. Es el valor que aparece con mayor frecuencia en un conjunto de datos. Puede no haber ninguna moda, haber una moda o más de una moda.



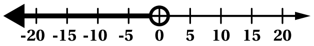
Por ejemplo, la moda de este conjunto de datos es 18 gomitas, porque el valor 18 aparece más veces que cualquier otro valor de datos.

múltiplo El resultado de multiplicar un número por un número natural.

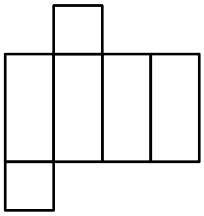
Por ejemplo, 10, 15 y 20 son múltiplos del número 5.

N

negative number
A number that is less than 0. On a horizontal number line, negative numbers are to the left of 0.



net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

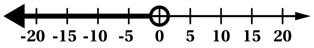


Here is a net for a rectangular prism.

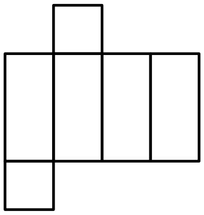
numerical data Data that includes values that are numbers and can be measured and meaningfully compared.

"How many pets do you have?" is a question that would result in numerical data.

número negativo
Un número que es menor que 0. En una recta numérica horizontal, los números negativos están a la izquierda del 0.



red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.



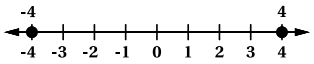
Esta es una red de un prisma rectangular.

datos numéricos Datos que incluyen valores numéricos que pueden medirse y compararse de forma significativa.

"¿Cuántas mascotas tienes?" es una pregunta que produciría datos numéricos.

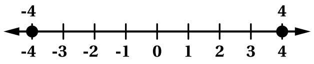
O

opposite Two numbers that are the same distance from 0 and on different sides of 0 on the number line are opposites.



For example, 4 and -4 are opposites.

opuesto Dos números son opuestos si están a la misma distancia del 0 y en diferentes lados del 0 en la recta numérica.



Por ejemplo, 4 y -4 son opuestos.

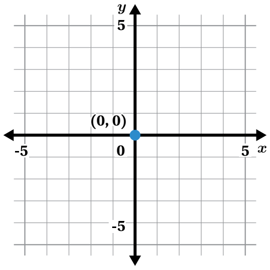
English

order of operations A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone. The standard order of operations is parentheses/grouping symbols, exponents/roots, multiplication/division, and then addition/subtraction.

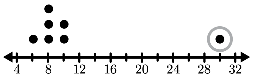
ordered pair Two values of x and y, written as (x, y), that represent a point on the coordinate plane.

For example, $(3, 5)$ represents the point where $x = 3$ and $y = 5$.

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



outlier A data value that is far from the other values in a data set.



The circled point is an outlier.

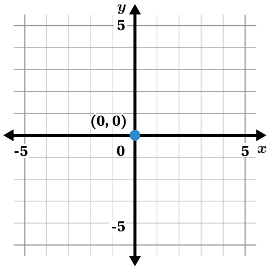
Español

orden de las operaciones Un orden coherente aplicado a una expresión con múltiples operaciones para que cualquiera pueda evaluar la expresión de la misma manera. El orden estándar de las operaciones es paréntesis/símbolos de agrupación, exponentes/raíces, multiplicación/división y, luego, suma/resta.

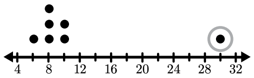
par ordenado Dos valores de x y y, escritos como (x, y), que representan un punto en el plano de coordenadas.

Por ejemplo, $(3, 5)$ representa el punto donde $x = 3$ y $y = 5$.

origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan los ejes x y y.



valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos.



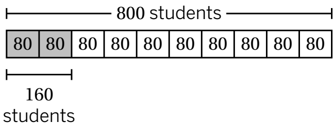
El punto encerrado en un círculo es un valor atípico.

P

per The word *per* means "for each."

For example, if the price is \$5 per ticket, that means that each ticket costs \$5. Buying 4 tickets would cost $5 \bullet 4 = \$20$.

percent (percentage) Percent means "for every 100." It is represented by the percent symbol, %. We use percentages to represent ratios and fractions.



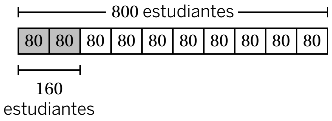
For example, 20% means 20 for every 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number. Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

perpendicular Describes a line that crosses or meets another line at a 90° angle.

por, por cada La palabra *por* puede significar "por cada".

Por ejemplo, si el precio es \$5 por boleto, esto significa que cada boleto cuesta \$5. Comprar 4 boletos costaría $5 \bullet 4 = \$20$.

por ciento (porcentaje) Por ciento significa "por cada 100". Se representa con el símbolo de porcentaje, %. Usamos porcentajes para representar razones y fracciones.

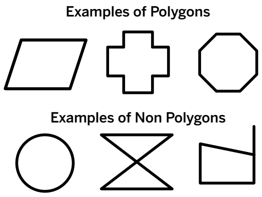


Por ejemplo, 20% significa 20 por cada 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número. Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos está en una excursión, eso significa 160 estudiantes porque 20 están de viaje por cada 100 estudiantes.

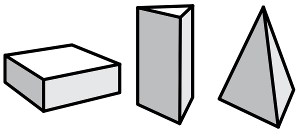
perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

English

polygon A closed two-dimensional shape with straight sides that do not cross each other.

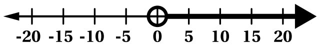


polyhedron A polyhedron is a closed three-dimensional shape with flat sides. The plural of polyhedron is polyhedra. Prisms and pyramids are types of polyhedra.



Here are some drawings of polyhedra.

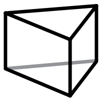
positive number A number that is greater than 0. On a horizontal number line, positive numbers are to the right of 0.



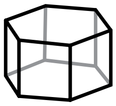
prime factorization The expression of a number as the product of prime factors.

The prime factorization of 28 is $2 \cdot 2 \cdot 7$.

prism A three-dimensional shape, or polyhedron, with two bases that are identical copies.

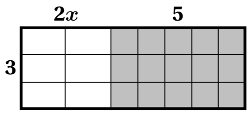


Triangular Prism



Hexagonal Prism

product The value of two or more quantities when multiplied.

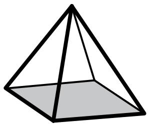


Area as a Product
$3(2x + 5)$

For example, the area of this rectangle is the product of 3 and $2x + 5$ or $3(2x + 5)$.

profit The money left after paying for business expenses.

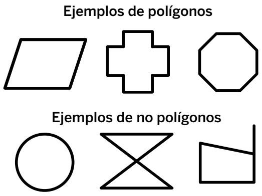
pyramid A three-dimensional shape, or polyhedron, that has one base. All of the other faces are triangles that meet at a single vertex.



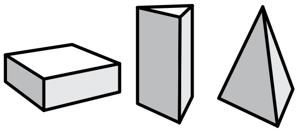
Pyramid

Español

polígono Una figura bidimensional cerrada con lados rectos que no se cruzan.

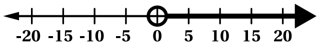


poliedro Un poliedro es una figura tridimensional cerrada con caras planas. En inglés, usamos las palabras polyhedron (singular) y polyhedra (plural). Los prismas y las pirámides son tipos de poliedros.



Estos son algunos dibujos de poliedros.

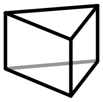
número positivo Un número que es mayor que 0. En una recta numérica horizontal, los números positivos están a la derecha del 0.



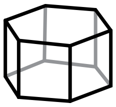
factorización prima La expresión de un número como producto de factores primos.

La factorización prima de 28 es $2 \cdot 2 \cdot 7$.

prisma Una figura tridimensional, o poliedro, con dos bases que son copias idénticas.

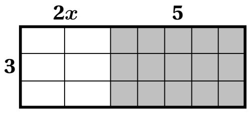


Prisma triangular



Prisma hexagonal

producto El valor de dos o más cantidades cuando se multiplican.

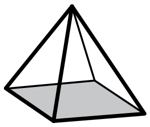


El área como un producto
$3(2x + 5)$

Por ejemplo, el área de este rectángulo es el producto de 3 y $2x + 5$ o $3(2x + 5)$.

ganancia El dinero que sobra después de pagar los gastos del negocio.

pirámide Una figura tridimensional, o poliedro, que tiene solo una base. Todas las demás caras son triángulos que se unen en un único vértice.



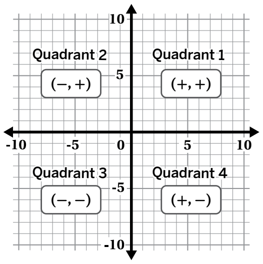
Pirámide

English

Español

Q

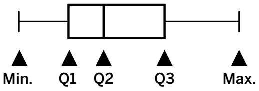
quadrant The coordinate plane is divided into 4 regions called quadrants.



The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.

quantity The amount or the number of a thing.

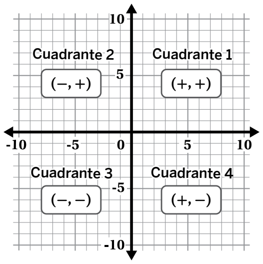
quartile Quartiles divide an ordered data set into four equal sections.
Quartile 1 (Q1) is the median of the lower half of the data.
Quartile 2 (Q2) is the median.
Quartile 3 (Q3) is the median of the upper half of the data.



quotient The result of dividing two numbers is called the quotient.

For example, in the equation $12 \div 3 = 4$, the quotient is 4.

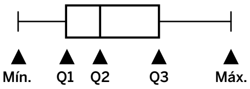
cuadrante El plano de coordenadas se divide en 4 regiones llamadas cuadrantes.



La imagen muestra cada cuadrante junto con el signo de los valores x y y del cuadrante correspondiente.

cantidad Cierta cantidad o número de algo.

cuartil Los cuartiles dividen un conjunto de datos ordenado en cuatro secciones iguales.
El cuartil 1 (Q1) es la mediana de la mitad inferior de los datos.
El cuartil 2 (Q2) es la mediana.
El cuartil 3 (Q3) es la mediana de la mitad superior de los datos.

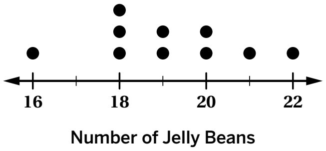


cociente Se denomina cociente al resultado de dividir dos números.

Por ejemplo, en la ecuación $12 \div 3 = 4$, el cociente es 4.

R

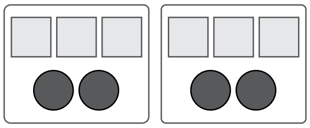
range A measure of spread. It is the difference between the maximum and minimum values in a data set.



For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.

rate A comparison, or ratio, that describes how two quantities change together.

ratio A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.

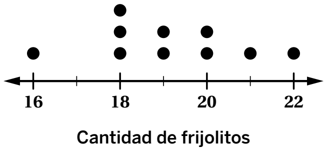


There are several ways to describe ratios.

- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is 3 : 2.

If the ratio of apples to oranges in a fruit bowl is 2 : 3, then for every 2 apples, there are 3 oranges.

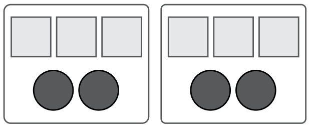
rango Una medida de dispersión. Es la diferencia entre los valores máximo y mínimo de un conjunto de datos.



Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

razón Una razón $a : b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.



Hay varias formas de describir razones.

- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es 3 : 2.

Si la razón de manzanas a naranjas en un frutero es 2 : 3, entonces por cada 2 manzanas hay 3 naranjas.

English

rational number Any number that can be written as a fraction, including whole numbers and negative numbers.

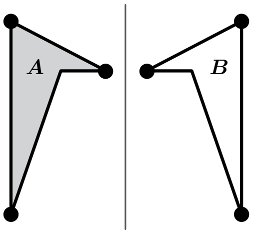
For example, -10, 2.5, $\frac{3}{7}$, and 82 are all rational numbers.

rearrange Change the position of something. We use the word *rearrange* to describe moving pieces of a figure to make a new shape.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

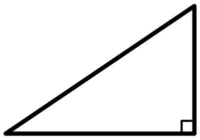
For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \cdot \frac{2}{3} = 1$.

reflection A reflection moves each point on a figure across a line of reflection to a point on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.



This diagram shows a reflection of A over a line that makes the mirror image B.

right triangle A triangle containing an interior right angle, or angle measuring exactly 90°.



Español

número racional Cualquier número que pueda escribirse como una fracción, incluidos los números enteros y los números negativos.

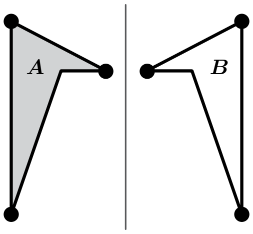
Por ejemplo, -10, 2.5, $\frac{3}{7}$ y 82 son números racionales.

reordenar Cambiar la posición de algo. Usamos la palabra *reordenar* para describir el movimiento de las partes de una figura para formar una nueva figura.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

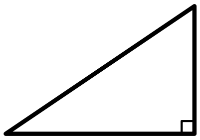
Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \cdot \frac{2}{3} = 1$.

reflexión Una reflexión mueve cada punto de una figura sobre una línea de reflexión a un punto en el lado opuesto de la línea. El nuevo punto está a la misma distancia de la línea que estaba en la figura original.



Este diagrama muestra una reflexión de A sobre una línea que produce la imagen espejo B.

triángulo recto Un triángulo que contiene un ángulo recto interior o un ángulo que mide exactamente 90°.



S

shape A description of the distribution (or pattern) of the data within a data set. Descriptions of the shape of a distribution can include: symmetry, skew, clusters, and outliers.

sign The sign of a number (other than 0) is either positive or negative.

For example, the sign of 4 or $+4$ is positive. The sign of -4 is negative.

skewed A data distribution where there are more values concentrated on one end of the data and few values on the other end.

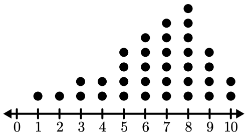
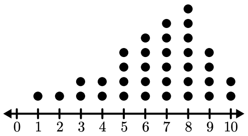


figura Una descripción de la distribución (o patrón) de los datos en un conjunto de datos. Las descripciones de la forma de una distribución pueden incluir: simetría, asimetría, agrupaciones y valores atípicos.

signo El signo de un número (que no sea 0) es positivo o negativo.

Por ejemplo, el signo de 4 o $+4$ es positivo. El signo de -4 es negativo.

asimétrico Una distribución de datos en la que hay valores concentrados en un extremo de los datos y pocos valores en el otro extremo.



English

solution to an equation A value of a variable that makes the equation true. “Solving an equation” is any work you do to answer the question “Which values make the equation true?”

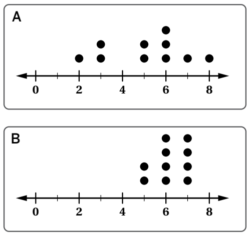
$$3x = 15$$
$$x = 15$$
$$3(5) = 15$$

For example, 5 is a solution to the equation $3x = 15$ because $3(5) = 15$ is true. 6 is not a solution to the equation $3x = 15$ because $3(6) = 15$ is not true.

solution to an inequality Any value of a variable that makes the inequality true.

For example, 5 is a solution to the inequality $x < 10$ because $5 < 10$. Some other solutions to $x < 10$ are 9.99, 0, and -4.

spread A description of how alike or different the values in a distribution are, often in relation to the center. The spread also describes the variability of a distribution.



For example, Dot Plot A has a larger spread than Dot Plot B.

statistic A single number that measures something about a data set.

Examples of statistics include: mean, median and IQR.

statistical question A question that requires more than one piece of data to answer.

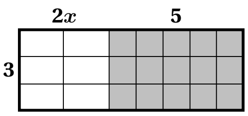
Here are some examples of statistical questions:
- What is the most popular band at your school?
- When do students in your class typically eat dinner?

substitute To replace a variable with a value or other expression.

$$4x = 4(5)$$
$$= 20$$

In this example, 5 is substituted for x in the expression $4x$.

sum The value of two or more quantities when added together.



Area as a Sum
$6x + 15$

For example, the area of this rectangle is the sum of $6x$ and 15, or $6x + 15$.

Español

solución de una ecuación Un valor de una variable que hace que la ecuación sea verdadera. “Resolver una ecuación” es cualquier trabajo que se hace para responder la pregunta: “¿Qué valores hacen que la ecuación sea verdadera?”.

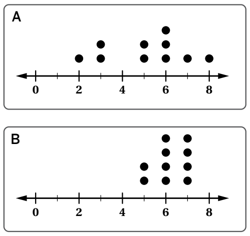
$$3x = 15$$
$$x = 15$$
$$3(5) = 15$$

Por ejemplo, 5 es una solución de la ecuación $3x = 15$ porque $3(5) = 15$ es verdadero. 6 no es una solución de la ecuación $3x = 15$ porque $3(6) = 15$ no es verdadero.

solución de una desigualdad Cualquier valor de una variable que hace que la desigualdad sea verdadera.

Por ejemplo, 5 es una solución de la desigualdad $x < 10$ porque $5 < 10$. Algunas otras soluciones de $x < 10$ son 9.99, 0 y -4.

dispersión Una descripción de las semejanzas o diferencias de los valores en una distribución, a menudo en relación con el centro. La dispersión también describe la variabilidad de una distribución.



Por ejemplo, el diagrama de puntos A tiene una dispersión mayor que el diagrama de puntos B.

dato estadístico, estadística Un número único que mide algo de un conjunto de datos.

Ejemplos de datos estadísticos: media, mediana y RQ.

pregunta estadística Una pregunta que requiere más de un dato para responder.

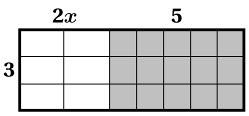
Estos son algunos ejemplos de preguntas estadísticas:
- ¿Cuál es la banda más popular en tu escuela?
- ¿Cuándo suelen cenar los estudiantes de tu clase?

sustituir Reemplazar una variable por un valor u otra expresión.

$$4x = 4(5)$$
$$= 20$$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

suma El valor de dos o más cantidades que se suman.



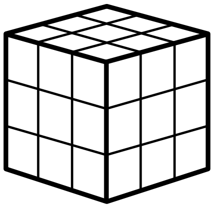
El área como una suma
$6x + 15$

Por ejemplo, el área de este rectángulo es la suma de $6x$ y 15, o $6x + 15$.

English

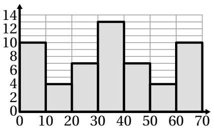
surface area The sum of the areas of a polyhedron's faces.

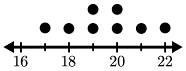
If the six faces of a cube each have an area of 9 square centimeters, then the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



symmetry (of a data distribution) A property of a data display where the left side of the distribution is a mirror image of the right side.

Here are some examples of data distributions that have symmetry.





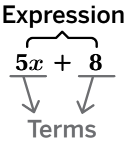
T

table A table organizes information into horizontal rows and vertical columns. The first row or column usually tells what the numbers represent.

Pet	Tail Length (in.)
Dog	22
Cat	12
Mouse	2

Here is a table showing the tail lengths of three different pets. This table has four rows and two columns.

term A part of an expression. A term can be a single number, a variable, or a number and variable multiplied together.



For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.

U

unit price The price per unit of an item. You can count how many units by counting the items or the weight.

For example, if 4 avocados cost \$12, then the unit price is $\frac{12}{4} = \$3$ per avocado.

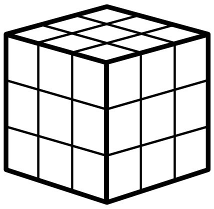
unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

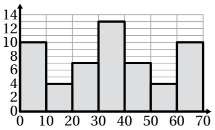
Español

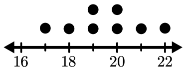
área de superficie La suma de las áreas de sus caras.

Si cada una de las seis caras de un cubo tiene un área de 9 centímetros cuadrados, el área de superficie del cubo mide $6 \cdot 9$, o 54 centímetros cuadrados.



simetría (de una distribución de datos) Una propiedad de una representación de datos donde el lado izquierdo de la distribución es una imagen reflejada del lado derecho.





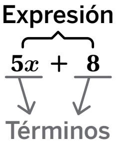
Estos son algunos ejemplos de distribuciones de datos que tienen simetría.

tabla Una tabla organiza la información en filas horizontales y columnas verticales. La primera fila o columna suele indicar lo que representan los números.

Mascota	Longitud de cola (pulg.)
Perro	22
Gato	12
Ratón	2

Esta es una tabla que muestra la longitud de la cola de tres mascotas diferentes. Esta tabla tiene cuatro filas y dos columnas.

término Una parte de una expresión. Un término puede ser un número individual, una variable, o una variable y un número multiplicados.



Por ejemplo, la expresión $5x + 8$ tiene dos términos. El primer término es $5x$ y el segundo término es 8.

precio unitario El precio por unidad de un artículo. Puedes contar la cantidad de unidades contando los artículos o el peso.

Por ejemplo, si 4 aguacates cuestan \$12, el precio unitario es $\frac{12}{4} = \$3$ por aguacate.

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

English

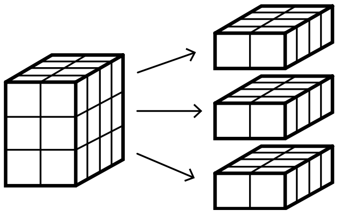
Español

V

variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

volume The number of unit cubes needed to fill a three-dimensional shape without gaps or overlaps.

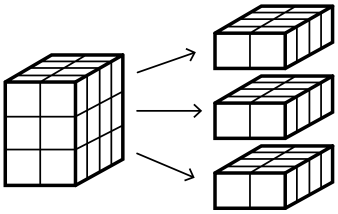


The volume of this rectangular prism is 24 cubic units because it is composed of 3 layers that are each 8 cubic units.

variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

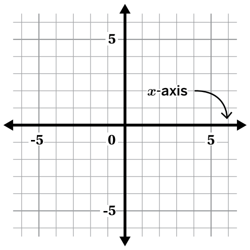
volumen La cantidad de cubos unitarios que se necesitan para llenar una figura tridimensional sin vacíos ni superposiciones.



El volumen de este prisma rectangular es de 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.

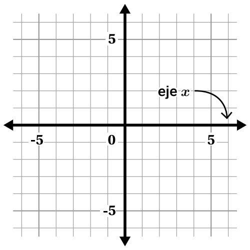
X

x-axis One of the perpendicular number lines that form the coordinate plane. The x-axis is the horizontal number line.



x-coordinate See *coordinates*.

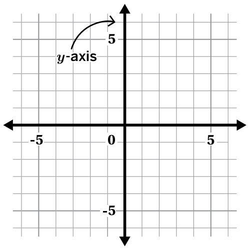
eje x Una de las rectas numéricas perpendiculares que forman el plano de coordenadas. El eje x es la recta numérica horizontal.



coordenada x Ver *coordenadas*.

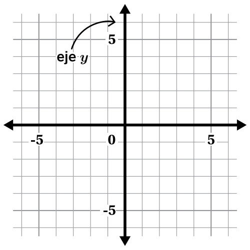
Y

y-axis One of the perpendicular number lines that form the coordinate plane. The y-axis is the vertical number line.



y-coordinate See *coordinates*.

eje y Una de las rectas numéricas perpendiculares que forman el plano de coordenadas. El eje y es la recta numérica vertical.



coordenada y Ver *coordenadas*.

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