Amplify Desmos Math FLORIDA

Algebra 1

A1

Volume 2

Teacher Edition



Amplify Desmos Math FLORIDA

Algebra 1

Volume 2: Units 5–7

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Teacher,

Welcome to Amplify Desmos Math Florida! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

- A program designed around the **Florida B.E.S.T. Standards for Mathematics**, grounded in the strands, standards, and benchmarks, and the importance of student exploration and investigation.
- **Deep and intentional alignment** to the Florida B.E.S.T. standards, Florida English Language Arts standards for K-12 Mathematics, and Florida English Language Development Standards related to the Language of Mathematics.
- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Manipulatives, visual models, discussions, and drawings to encourage perseverance and revision.
  - » Easy-to-follow lesson plans, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency, help students review previous topics, and gain experience with FAST Assessment problem types.
- **Recommended differentiation moves** that meet the needs of all learners.
- **Diagnostic, formative, and summative assessments** along with lesson-level checks for understanding.
- **A caregiver resource** for each unit that includes explanations of key math concepts and problems to try with your student.

We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math Florida team



Unit 1 Linear Equations and Inequalities

Students solve one-variable equations and inequalities to make sense of multi-variable equations and two-variable inequalities.



Pre-Unit Check 1L

Sub-Unit 1 One-Variable Equations 2

				B.E.S.T. Benchmark Connection
1.01	Working Backwards	Solving Equations with Inverse Operations	3A	1.01: MA.912.AR.2.1, MTR.1.1, MTR.2.1, MTR.5.1
1.02	Solving Strategies	More Solving with One-Variable Equations	10A	1.02: MA.912.AR.2.1, MTR.4.1, MTR.5.1
1.03	Same Position	No Solution and Infinitely Many Solutions	17A	1.03 MA.8.AR.2.1, MA.912.AR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Sub-Unit 2 Multi-Variable Equations 26

				B.E.S.T. Benchmark Connection
1.04	Subway Seats	Representing Situations with Two-Variable Equations	27A	1.04: MA.912.AR.1.2, MA.912.AR.2.2 MA.912.AR.2.5, MA.912.AR.2.6, MTR.2.1, MTR.3.1, MTR.4.1
1.05	Various Variables	Solving Multi-Variable Equations	34A	1.05: MA.912.AR.1.2, MA.912.AR.2.2 MA.912.AR.2.5, MA.912.AR.2.6, MTR.3.1, MTR.4.1
1.06	Shelley the Snail	Connecting Graphs and Linear Equations	42A	1.06: MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MA.912.AR.2.5, MA.912.AR.2.6, MTR.1.1, MTR.2.1
1.07	Five Representations	Linear Relationships in Equations, Tables, and Graphs	49A	1.07: MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.3, MA.912.AR.2.4, MA.912.AR.2.5, MTR.4.1, MTR.6.1, MTR.7.1
1.08	Whale Growth Rate	Finding Equations of Parallel Lines	56A	1.08: MA.912.AR.2.3, MTR.4.1, MTR.7.1
1.09	House Design	Finding Equations of Perpendicular Lines	64A	1.09: MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.3, MA.912.AR.2.4, MA.912.AR.2.5, MTR.4.1, MTR.6.1, MTR.7.1

Sub-Unit Quiz 1 ..... 71A

Sub-Unit 3 One-Variable and Two-Variable Inequalities 72

				B.E.S.T. Benchmark Connection
1.10	Pizza Delivery	Representing Situations with One-Variable Inequalities	73A	1.10: MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1
1.11	Graphing Inequalities	Inequalities on the Number Line	81A	1.11: MA.K12.MTR.1.1, MA.912.AR.2.6, MA.912.AR.2.8, MTR.1.1, MTR.2.1, MTR.4.1
1.12	Solutions and Sheep	Solving One-Variable Inequalities	88A	1.12: MA.912.AR.2.6, MTR.1.1, MTR.4.1, MTR.5.1
1.13	Tick Tock	One-Variable Compound Linear Inequalities	96A	1.13: MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1
1.14	Absolute Value Solutions	Solving Absolute Value Equations and Inequalities	106A	1.14: MA.912.AR.4.1, MTR.2.1, MTR.5.1
1.15	Getting Absolute	Creating and Solving Absolute Value Equations	114A	1.15: MA.912.AR.4.1, MTR.1.1, MTR.4.1, MTR.5.1
1.16	Bracelet Budgets	Introduction to Two-Variable Inequalities	123A	1.16: MA.912.AR.2.8, MTR.1.1, MTR.2.1
1.17	All of the Solutions	Graphing Solutions to Two-Variable Inequalities	132A	1.17: MA.912.AR.2.8, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1
1.18	Concert Planning	Graphing Two-Variable Inequalities in Context	140A	1.18: MA.912.AR.2.7, MA.912.AR.2.8, MTR.1.1, MTR.5.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 148A

Unit 2 Describing Data



Students use dot plots, histograms, box plots, line graphs, and measures of center and spread to analyze and describe one-variable data sets. Students use two-way tables to make decisions and predictions based on data. Students also use scatter plots, correlation coefficients, and lines of best fit to describe two-variable data sets.

Pre-Unit Check	149L	
Sub-Unit 1 Representing Categorical Data	150	**B.E.S.T. Benchmark Connection**
2.01 Survey Says | What Kinds of Data Can I Collect?	151A	**2.01: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.4.1
2.02 Data Dimensions | Univariate and Bivariate Vocabulary	159A	**2.02: MA.912.DP.1.2,** MTR.3.1, MTR.4.1, MTR.5.1
Sub-Unit 2 Summarizing One-Variable Data	166	**B.E.S.T. Benchmark Connection**
2.03 Data Driven | Revisiting Dot Plots and Histograms	167A	**2.03: MA.912.DP.1.1,** MTR.4.1, MTR.5.1, MTR.7.1
2.04 Better Weather? | Revisiting Box Plots	176A	**2.04: MA.912.DP.1.1,** MTR.2.1, MTR.4.1, MTR.7.1
2.05 Quick Pick | Revisiting Measures of Center	184A	**2.05: MA.912.DP.1.2,** MTR.3.1, MTR.4.1
2.06 Far Out | Identifying Outliers	192A	**2.06: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.2.1, MTR.4.1, MTR.6.1
2.07 Dynamic Decades | Comparing Data Using Measures of Center and Spread	200A	**2.07: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.1.1, MTR.4.1, MTR.7.1
2.08 How Big? | Estimate a Population Using Sample Survey Data	206A	**2.08: MA.912.DP.1.4,** MTR.4.1, MTR.5.1, MTR.7.1
2.09 How Many? | Estimate Mean or Percentage Using Survey Data	212A	**2.09: MA.912.DP.1.4,** MTR.1.1, MTR.4.1, MTR.7.1
2.10 Dribble, Draw, and Decide | Margin of Error through Simulation	218A	**2.10: MA.912.DP.1.4,** MTR.1.1, MTR.4.1, MTR.7.1
Sub-Unit Quiz 1	223A	
Sub-Unit 3 Summarizing Two-Variable Data	224	**B.E.S.T. Benchmark Connection**
2.11 Trains and Traffic | Two-Way Tables and Relative Frequency Tables	225A	**2.11: MA.912.DP.3.1,** MTR.4.1, MTR.7.1
2.12 Remodel Choices | Making Decisions with Frequency Tables	233A	**2.12: MA.912.DP.3.1,** MTR.2.1, MTR.4.1, MTR.7.1
2.13 Connecting the Dots | Numerical Bivariate Data	241A	**2.13: MA.912.DP.1.1, MA.912.DP.1.2,** MTR.1.1, MTR.3.1, MTR.6.1
2.14 City Slopes | Interpreting Slope and y-intercept in Context	250A	**2.14: MA.912.DP.2.4, MA.912.DP.2.6,** MTR.1.1, MTR.4.1, MTR.6.1, MTR.7.1
2.15 Residual Fruit | Residuals and Residual Plots	257A	**2.15: MA.912.DP.2.4, MA.912.DP.2.6,** MTR.1.1, MTR.4.1, MTR.7.1
2.16 Penguin Populations | Using Technology to Generate the Line of Best Fit	265A	**2.16: MA.912.DP.2.4, MA.912.DP.2.6,** MTR.1.1, MTR.4.1, MTR.7.1
2.17 Behind the Headlines | Causation vs. Correlation	274A	**2.17: MA.912.DP.1.3,** MTR.1.1, MTR.4.1, MTR.7.1
End-of-Unit Assessment	280A	

Unit 3 Describing Functions



Students explore what makes a relationship a function and how functions can model situations and tell stories. Students represent functions using tables, graphs, equations, and words. They use function notation to describe key features of functions, compare different functions, and define functions. Finally, students explore new types of functions that model situations with different rules for different inputs.

Pre-Unit Check 281L

Sub-Unit 1 Function Notation 282

B.E.S.T. Benchmark Connection
3.01 Mystery Rule | What Is a Function? ..... 283A — **3.01: MA.912.F.1.1,** MTR.3.1, MTR.4.1
3.02 Pricing Pizzas | Introducing Function Notation ..... 291A — **3.02: MA.912.F.1.2, MA.912.AR.2.2,** MTR.2.1, MTR.4.1
3.03 Rule Breakers | Domain and Range Notation ..... 299A — **3.03: MA.912.F.1.1, MA.912.F.1.5, MA.912.F.1.6,** MTR.1.1, MTR.4.1, MTR.5.1
3.04 Toy Factory | Function Notation and Equations ..... 306A — **3.04: MA.912.F.1.2, MA.912.AR.2.2,** MTR.3.1, MTR.4.1, MTR.5.1

Sub-Unit Quiz 1 ..... 313A

Sub-Unit 2 Key Features of Functions 314

B.E.S.T. Benchmark Connection
3.05 Function Carnival | Creating and Interpreting Graphs of Functions ..... 315A — **3.05: MA.912.F.1.2, MA.912.F.1.6,** MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1
3.06 Craft-a-Graph | Key Features of Graphs ..... 323A — **3.06: MA.912.F.1.6,** MTR.4.1
3.07 Plane, Train, and Automobile | Average Rate of Change ..... 331A — **3.07: MA.912.F.1.3, MA.912.F.1.5,** MTR.1.1, MTR.2.1, MTR.4.1
3.08 Space Race | Comparing Graphs ..... 340A — **3.08: MA.912.F.1.5, MA.912.F.1.6,** MTR.4.1
3.09 Ins and Outs | Introducing Domain and Range ..... 348A — **3.09: MA.8.F.1.1, MA.912.F.1.2, MA.912.F.1.5,** MTR.4.1
3.10 Elevator Stories | Describing Domain and Range with Inequalities ..... 356A — **3.10: MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6,** MTR.1.1, MTR.2.1, MTR.4.1

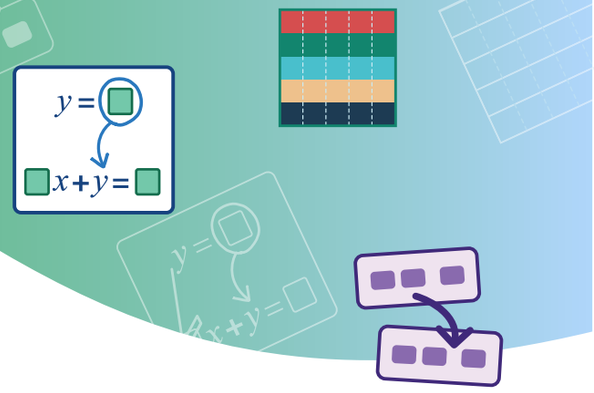
Sub-Unit 3 Special Types of Functions 366

B.E.S.T. Benchmark Connection
3.11 What's Your Score? | Absolute Value Functions, Part 1 ..... 367A — **3.11: MA.912.AR.4.3,** MTR.2.1, MTR.5.1
3.12 Absolute Value Machines | Absolute Value Functions, Part 2 ..... 375A — **3.12: MA.912.AR.4.3,** MTR.4.1, MTR.5.1

End-of-Unit Assessment 384B

Unit 4 Systems of Linear Equations and Inequalities

Students explore how to solve systems of equations and inequalities using different strategies. Students also analyze the structure of the equations in a system to strategize about which solving method to choose.



Pre-Unit Check 385L

Sub-Unit 1 Systems of Equations 386

		B.E.S.T. Benchmark Connection
4.01	Eliminating Shapes | Introduction to Elimination 387A	**4.01: MA.912.AR.9.1,** MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
4.02	Process of Elimination | Elimination Using Equivalent Equations 396A	**4.02: MA.912.AR.9.1,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.03	Solution by Substitution | Solving Systems by Substitution 404A	**4.03: MA.912.AR.9.1,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.04	Lizard Lines | Graphing Systems of Linear Equations 411A	**4.04: MA.912.AR.9.1,** MTR.2.1, MTR.4.1, MTR.5.1
4.05	City Development | Solving Graphically and Symbolically 419A	**4.05: MA.912.AR.9.1, MA.912.AR.9.6,** MTR.1.1, MTR.6.1
4.06	Bus Systems | Writing and Solving Systems of Equations 427A	**4.06: MA.912.AR.9.1, MA.912.AR.9.6,** MTR.2.1, MTR.4.1, MTR.5.1

Sub-Unit Quiz 1 ..... 433A

Sub-Unit 2 Systems of Inequalities 434

		B.E.S.T. Benchmark Connection
4.07	Quilts | Introduction to Systems of Inequalities 435A	**4.07: MA.912.AR.9.4,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.08	Seeking Solutions | Solutions to Systems of Inequalities 442A	**4.08: MA.912.AR.9.4, MA.912.AR.9.6,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1
4.09	Boundaries and Shading | Graphing Systems of Inequalities 451A	**4.09: MA.912.AR.9.4, MA.912.AR.9.6,** MTR.4.1, MTR.5.1, MTR.6.1
4.10	Restaurant Meals | Using Systems of Inequalities to Make Decisions 458A	**4.10: MA.912.AR.9.4, MA.912.AR.9.6,** MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 464A

Unit 5 Exponential Functions

Students learn about relationships that grow quickly over time. Students learn how to distinguish between linear and exponential relationships, and create equations to represent them. Students model situations that increase or decrease by a percentage. Students also model compound interest with exponential functions.

Pre-Unit Check 465L

Sub-Unit 1 Comparing Linear and Exponential Functions 466

		B.E.S.T. Benchmark Connection
5.01	Growing Globs | Patterns of Growth 467A	5.01: MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1
5.02	Going Viral | Graphs of Exponential Relationships 475A	5.02: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1
5.03	Return of the Globs | Connecting Representations of Linear and Exponential Functions 483A	5.03: MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1
5.04	Carlos and Corals | Evaluating Exponential Functions 489A	5.04: MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.F.1.2, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1
5.05	Differences and Factors | Changes Over Equal Intervals 497A	5.05: MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.F.1.2, MA.912.F.1.8, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.5.1, MTR.7.1

Sub-Unit 2 Exponential Growth and Decay 504

		B.E.S.T. Benchmark Connection
5.06	Growing Mold | Percent Increase and Decrease, Part 1 ..... 505A	5.06: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.NSO.1.1, MTR.2.1, MTR.6.1, MTR.7.1
5.07	At a Loss | Percent Increase and Decrease, Part 2 ..... 513A	5.07: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.NSO.1.1, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

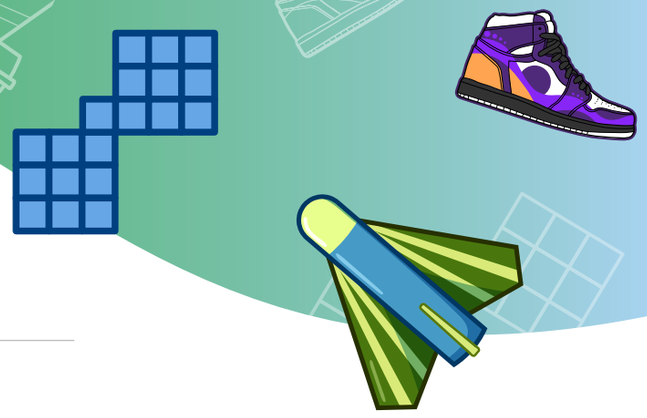
Sub-Unit Quiz 1 ..... 521A

Sub-Unit 3 Modeling with Exponentials 522

		B.E.S.T. Benchmark Connection
5.08	Thinking Rationally | Writing Expressions Using Radicals and Rational Exponents 523A	5.08: MA.912.NSO.1.1, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1
5.09	Writing Radicals | Writing Equivalent Expressions Using Rational Exponents and Radicals 532A	5.09: MA.912.NSO.1.1, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1
5.10	Rule the Roots | Operations with Radicals 538A	5.10: MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
5.11	Tame the Terms | Adding and Subtracting Radicals 547A	5.11: MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.1.1, MTR.2.1, MTR.4.1, MTR.5.1
5.12	Bank Accounts | Introducing Simple and Compound Interest 555A	5.12: MA.912.FL.3.2, MA.912.FL.3.4, MA.912.F.1.6, MA.912.F.1.8, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1
5.13	Payday Loan | Revisiting Compound Interest 563A	5.13: MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1
5.14	Credit Card Compounding | Different Compound Intervals 571A	5.14: MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1
5.15	Exploring Interest | Relating Interest to Growth 580A	5:15: MA.912.FL.3.4, MTR.2.1, MTR.4.1, MTR.7.1

End-of-Unit Assessment 587A

Unit 6 Quadratic Functions



Students learn about how quadratic relationships are different from linear and exponential relationships, and graph them. Students model situations with quadratic functions and make predictions and decisions based on your models.

Pre-Unit Check 589L

Sub-Unit 1 Introduction to Quadratic Functions 590

6.01	Revisiting Visual Patterns	A New Type of Pattern 591A	**6.01: MA.912.F.1.8,** MTR.2.1, MTR.5.1
6.02	Quadratic Visual Patterns	Expressions for Quadratic Patterns 599A	**6.02: MA.912.F.1.8,** MTR.2.1, MTR.5.1
6.03	Sorting Relationships	Comparing Linear, Exponential, and Quadratic Relationships 608A	**6.03: MA.912.F.1.1, MA.912.F.1.8,** MTR.2.1, MTR.5.1
6.04	On the Fence	Quadratics in Context 615A	**6.04: MA.912.F.1.1, MA.912.F.1.8, MA.912.AR.3.7,** MTR.2.1, MTR.5.1, MTR.7.1
6.05	Stomp Rockets	Projectiles and Predictions 623A	**6.05: MA.912.AR.3.7, MA.912.AR.3.8,** MTR.1.1, MTR.2.1, MTR.3.1, MTR.5.1
6.06	Plenty of Parabolas	Key Features of Parabolas 631A	**6.06: MA.912.AR.3.7,** MTR.2.1, MTR.4.1, MTR.5.1
6.07	Robot Launch	Key Features of Graphs in Context 639A	**6.07: MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.4.1, MTR.5.1

Sub-Unit 2 Standard Form and Factored Form 646

6.08	What's My Graph?	Creating Graphs of Quadratics 647A	**6.08: MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.4.1
6.09	Two for One	Standard Form and Factored Form 653A	**6.09: MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.5.1
6.10	Interesting Intercepts	Intercepts in Factored and Standard Forms 659A	**6.10: MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.11	Break Through: Parabolas	Building Quadratics in Factored Form 667A	**6.11: MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1,** MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1
6.12	Sneaker Drop	Exploring Revenue 675A	**6.12: MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2,** MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

Sub-Unit Quiz 1 ..... 681A

Sub-Unit 3 Vertex Form 682

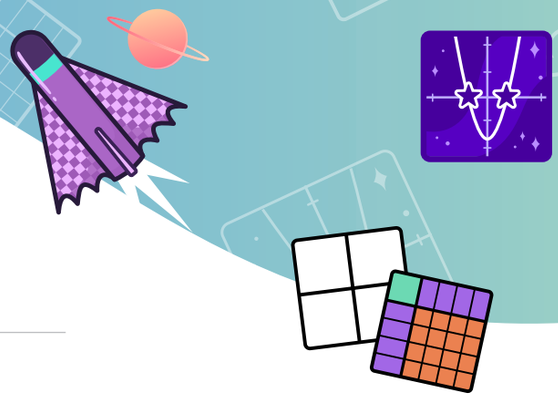
6.13	Vertex Form	Translating Quadratic Functions 683A	**6.13: MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1,** MTR.2.1, MTR.4.1, MTR.5.1
6.14	Stretch It Out	Vertical Stretches and Vertex Form 692A	**6.14: MA.912.AR.3.4, MA.912.AR.3.7, MA.912.F.2.1,** MTR.2.1, MTR.5.1
6.15	Predicting Sales	Linear, Exponential, and Quadratic Modeling 701A	**6.15: MA.912.F.1.2, MA.912.F.1.8, MA.912.AR.3.8,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 708B

B.E.S.T. Benchmark Connection

Unit 7 Quadratic Equations

Students learn different methods for solving quadratic equations and develop strategies for choosing your solving method strategically. Students also connect your solving process to the graphs and key features of quadratic functions.



Pre-Unit Check 709L

Sub-Unit 1 Multiplying and Factoring 710

		B.E.S.T. Benchmark Connection
7.01	Sums and Differences | Adding and Subtracting Linear and Quadratic Expressions 711A	**7.01: MA.912.AR.1.3, MA.912.NSO.1.1,** MTR.1.1, MTR.4.1, MTR.5.1
7.02	Two-Factor Multiplication | Rewriting Factored-Form Expressions in Standard Form 719A	**7.02: MA.912.AR.1.3, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.1.1, MTR.3.1, MTR.4.1, MTR.5.1
7.03	Standard Feature | Patterns in Factored-Form and Standard-Form Expressions 727A	**7.03: MA.912.AR.1.3, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.1.1, MTR.3.1, MTR.5.1
7.04	X-Factor | Factoring Quadratic Expressions 735A	**7.04: MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.3.1, MTR.5.1, MTR.6.1
7.05	Form Up | More Factoring Quadratic Expressions 743A	**7.05: MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
7.06	Divide and Conquer | Dividing Polynomial Expressions 750A	**7.06: MA.912.AR.1.4, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.3.1, MTR.4.1, MTR.5.1
7.07	Consider the Factors | Factoring Polynomial Expressions 757A	**7.07: MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.5.1
7.08	Shooting Stars | Determining the x-intercepts of Quadratic Functions 766A	**7.08: MA.912.AR.1.7, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
7.09	Make It Zero | Solving Quadratic Equations Using the Zero-Product Property 773A	**7.09: MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2,** MTR.2.1, MTR.4.1, MTR.5.1

Sub-Unit Quiz 1 ..... 780A

Sub-Unit 2 Solving Equations and Completing the Square 782

		B.E.S.T. Benchmark Connection
7.10	Zero, One, or Two? | Solving Equations by Reasoning 783A	**7.10: Building toward: MA.912.AR.3.1,** MTR.4.1, MTR.5.1, MTR.6.1
7.11	Graph to Solve | Solving Quadratic Equations by Graphing 791A	**7.11: MA.912.AR.3.1, MA.912.AR.3.6,** MTR.2.1, MTR.4.1, MTR.5.1
7.12	Couldn't Square Less | Solving by Taking the Square Root 799A	**7.12: MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1
7.13	Square Dance | Perfect Square Expressions 807A	**7.13: MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1,** MTR.2.1, MTR.4.1, MTR.5.1
7.14	Square Tactic | Solving by Completing the Square 815A	**7.14: MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.5.1, MTR.6.1
7.15	Back and Forth | Rewriting Quadratic Expressions in Vertex Form 823A	**7.15: MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

Sub-Unit 3 The Quadratic Formula and More 830

		B.E.S.T. Benchmark Connection
7.16	Formula Foundations | Introducing the Quadratic Formula 831A	**7.16: MA.912.AR.1.2, MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.3.1, MTR.4.1, MTR.5.1
7.17	Formula Fluency | Solving Quadratic Equations Using the Quadratic Formula 839A	**7.17: MA.912.AR.3.1, MA.912.NSO.1.4,** MTR.3.1, MTR.4.1, MTR.5.1
7.18	Stomp Rockets in Space | Solving Quadratic Equations in Context 846A	**7.18: MA.912.AR.3.1, MA.912.AR.3.8, MA.912.NSO.1.4,** MTR.3.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 854A

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math Florida, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

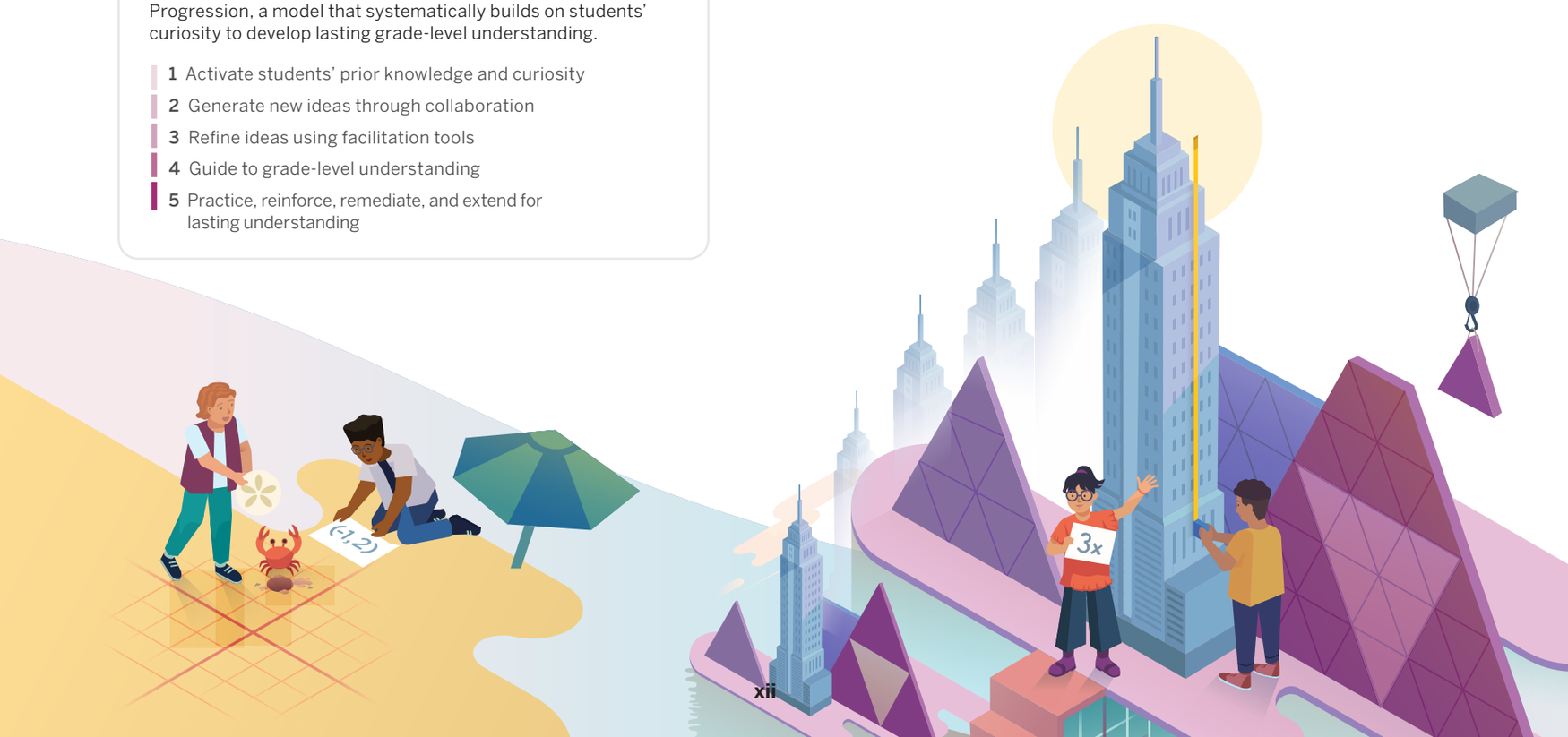
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.





Student thinking is valuable and can be made evident

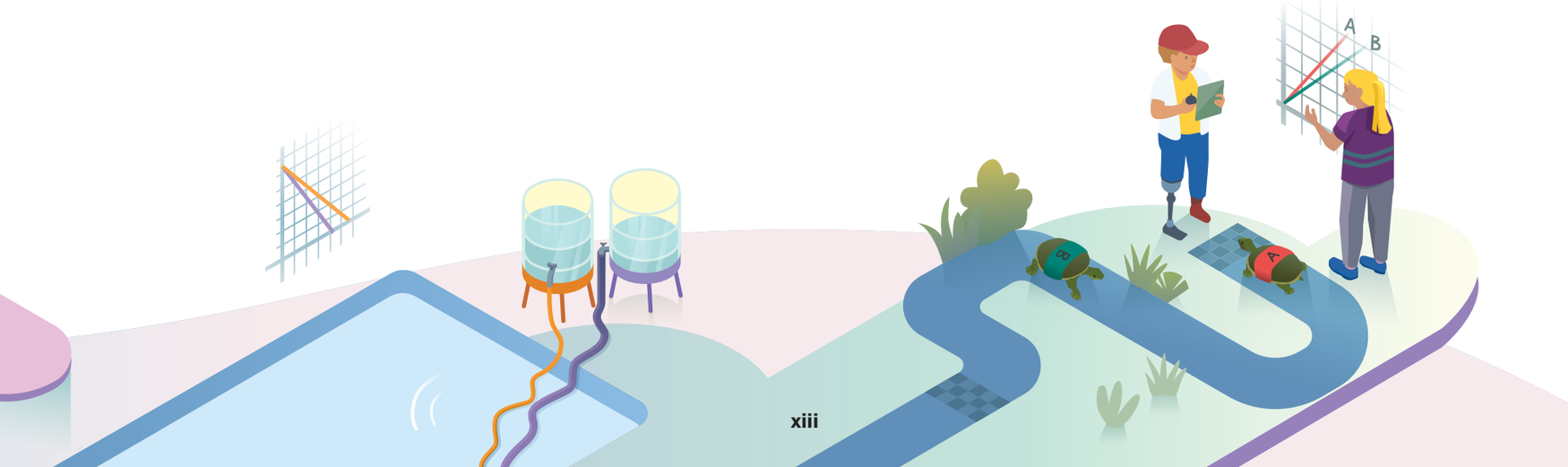
Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Math that motivates students

Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity.

Our curriculum supports collaborative classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.



Scope and Sequence | Grade 6–Algebra 1

Volume 1

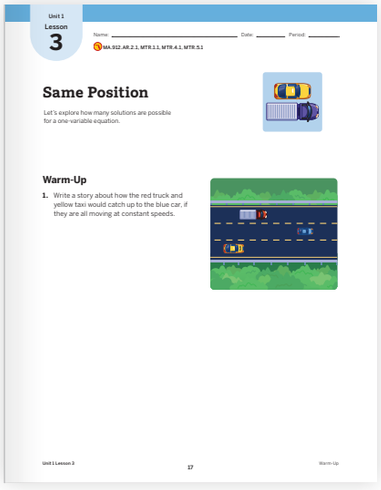
	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 127 days	**Area and Surface Area** 8 instructional days 3 assessment days 11 days total	**Introducing Ratios** 12 instructional days 3 assessment days, Math in Action 16 days total	**Unit Rates and Percentages** 13 instructional days 3 assessment days 16 days total	**Multiplying and Dividing Fractions** 15 instructional days 3 assessment days, Math in Action 19 days total
Grade 7 116 days	**Scale Drawings** 10 instructional days 3 assessment days 13 days total	**Introducing Proportional Relationships** 13 instructional days 4 assessment days, Math in Action 17 days total	**Measuring Circles** 6 instructional days 3 assessment days 9 days total	**Proportional Relationships and Percentages** 12 instructional days 4 assessment days, Math in Action 16 days total
Grade 8 120 days	**Rigid Transformations and Congruence** 10 instructional days 3 assessment days 13 days total	**Dilations, Similarity, and Slope** 10 instructional days 4 assessment days, Math in Action 14 days total	**Proportional and Linear Relationships** 11 instructional days 3 assessment days 14 days total	**Linear Equations and Linear Systems** 10 instructional days 4 assessment days, Math in Action 14 days total
Algebra 1 130 days	**Linear Equations and Inequalities** 18 instructional days 4 assessment days, Math in Action 22 days total	**Describing Data** 17 instructional days 3 assessment days 20 days total	**Describing Functions** 12 instructional days 4 assessment days, Math in Action 16 days total	**Describing Functions** 10 instructional days 3 assessment days 13 days total

Volume 2

Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Decimal Arithmetic 10 instructional days 3 assessment Days 13 days total	**Expressions and Equations** 11 instructional days 4 assessment Days, Math in Action 15 days total	**Positive and Negative Numbers** 18 instructional days 3 assessment Days 21 days total	**Describing Data** 12 instructional days 4 assessment Days, Math in Action 16 days total	
Positive and Negative Rational Numbers 10 instructional days 3 assessment days 13 days total	**Equations, Expressions, and Inequalities** 16 instructional days 4 assessment days, Math in Action 20 days total	**Polygons and Cylinders** 8 instructional days 3 assessment days 11 days total	**Probability and Sampling** 13 instructional days 4 assessment days, Math in Action 17 days total	
Functions 7 instructional days 3 assessment days 10 days total	**Associations in Data** 9 instructional days 4 assessment days, Math in Action 13 days total	**Exponents and Scientific Notation** 13 instructional days 3 assessment days 16 days total	**The Pythagorean Theorem and Irrational Numbers** 15 instructional days 4 assessment days, Math in Action 19 days total	**Probability** 5 instructional days 2 assessment days 7 days total
Exponential Functions 15 instructional days 4 assessment days, Math in Action 17 days total	**Quadratic Functions** 15 instructional days 3 assessment days 18 days total	**Quadratic Equations** 18 instructional days 4 assessment days, Math in Action 22 days total		

Program Resources

For Students


Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.AR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Same Position

Let's explore how many solutions are possible for a one-variable equation.

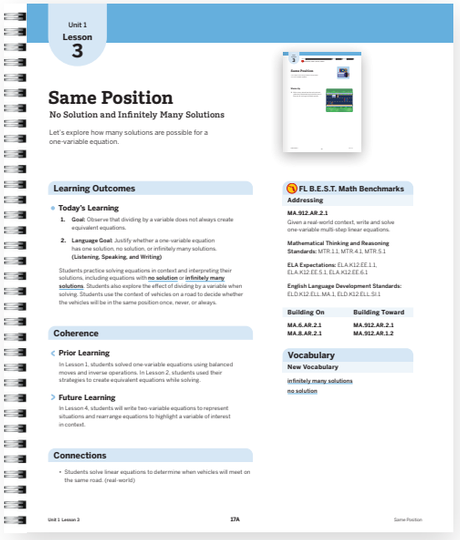
Warm-Up

1. Write a story about how the red truck and yellow taxi would catch up to the blue car, if they are all moving at constant speeds.

Unit 1 Lesson 3 17 Warm-Up

- Student Edition (two-volume)

For Teachers


Unit 1
Lesson
3

Same Position

No Solution and Infinitely Many Solutions

Let's explore how many solutions are possible for a one-variable equation.

Learning Outcomes

Today's Learning

1. **Goal:** Observe that dividing by a variable does not always create equivalent equations.
2. **Language Goal:** Justify whether a one-variable equation has one solution, no solution, or infinitely many solutions. **(Listening, Speaking, and Writing)**

Students practice solving equations in context and interpreting their solutions, including equations with no solution or infinitely many solutions. Students also explore the effect of dividing by a variable when solving. Students use the context of vehicles on a road to decide whether the vehicles will be in the same position once, never, or always.

Coherence

Prior Learning

In Lesson 1, students solved one-variable equations using balanced moves and inverse operations. In Lesson 2, students used their strategies to create equivalent equations while solving.

Future Learning

In Lesson 4, students will write two-variable equations to represent situations and rearrange equations to highlight a variable of interest in context.

Connections

- Students solve linear equations to determine when vehicles will meet on the same road. (real-world)

FL B.E.S.T. Math Benchmarks Addressing

MA.912.AR.2.1

Given a real-world context, write and solve one-variable multi-step linear equations.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.1	MA.912.AR.2.1
MA.8.AR.2.1	MA.912.AR.1.2

Vocabulary

New Vocabulary

infinitely many solutions
no solution

Unit 1 Lesson 3 27A Same Position

- Teacher Edition (two-volume)

Additional Resources



Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior grade Mini-Lessons
- Extensions

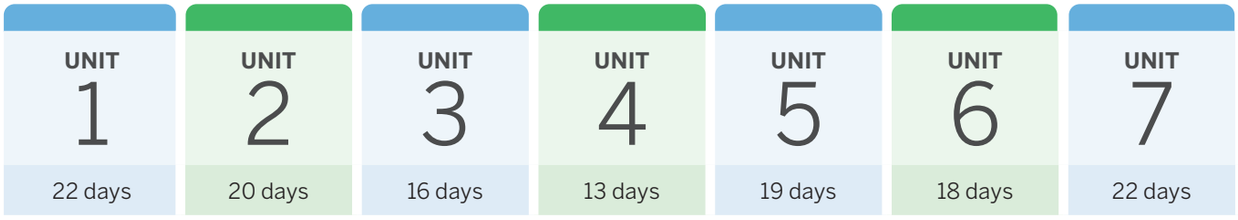
Optional: Manipulative Kits



hand2mind

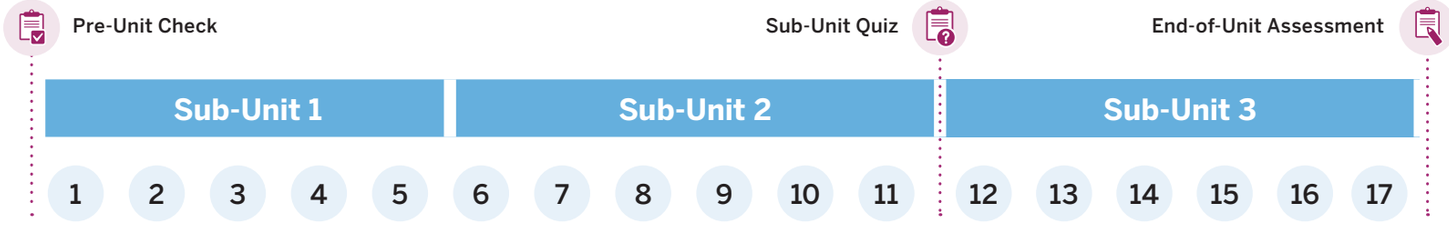
Program Architecture

Course



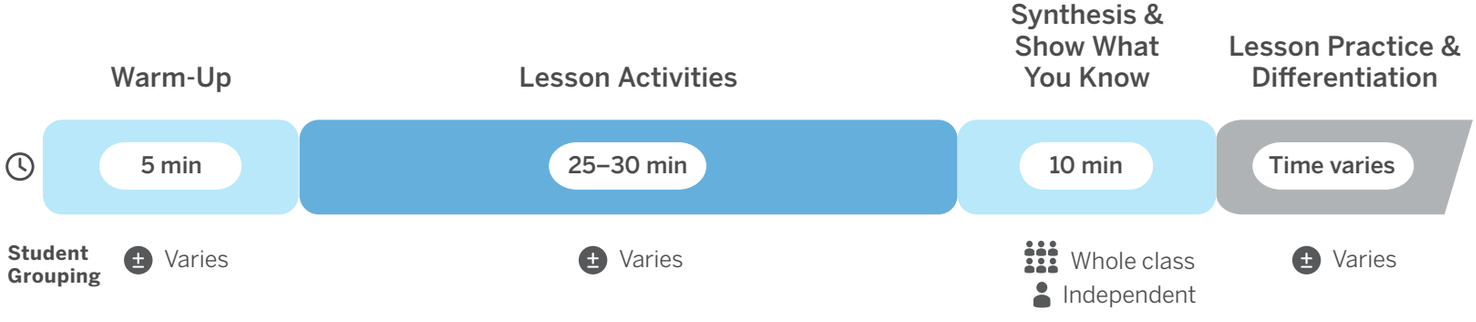
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

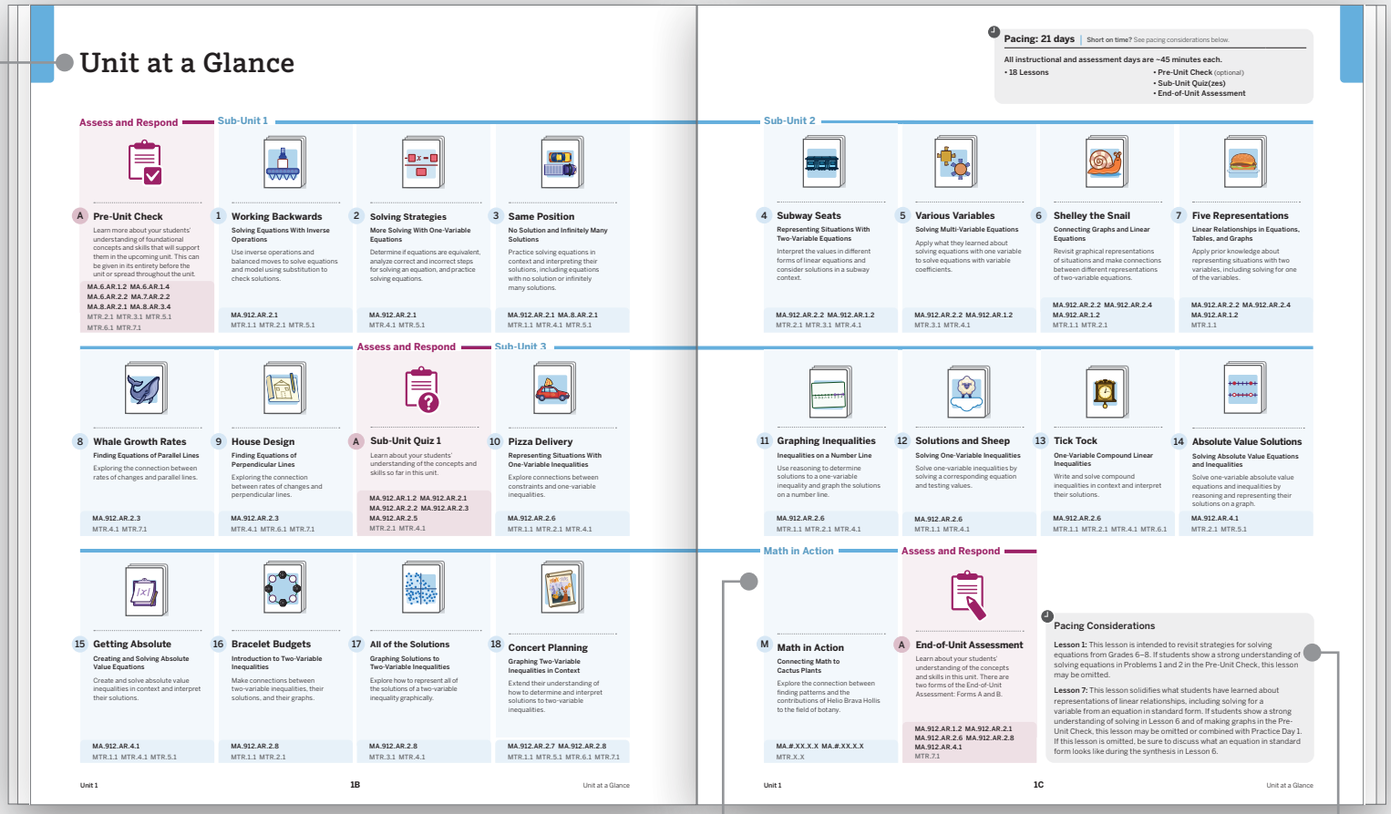
Lesson



Navigating This Program

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

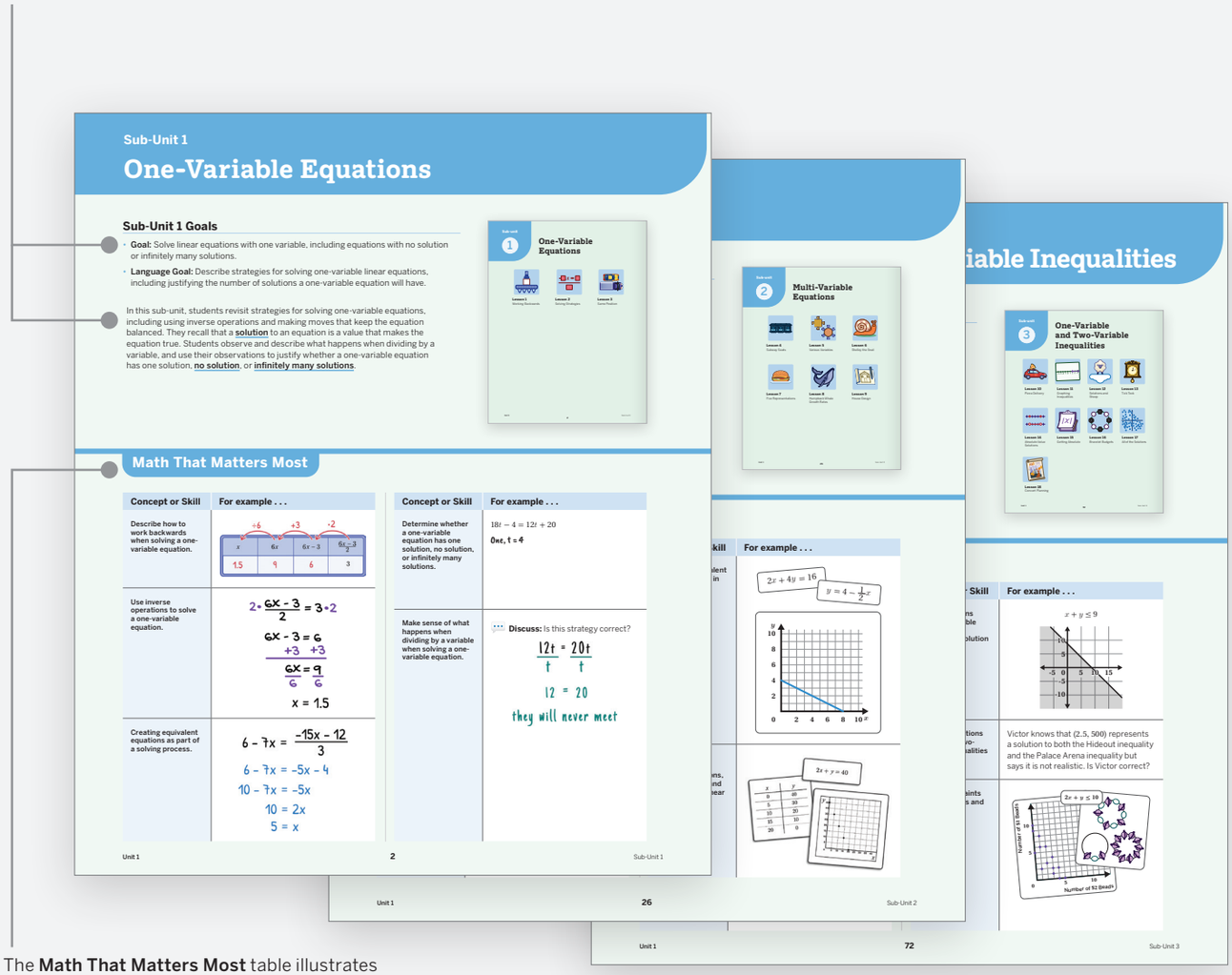
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, a preview of each of the unit assessments, and guidance about differentiation, accessibility, language development, materials, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

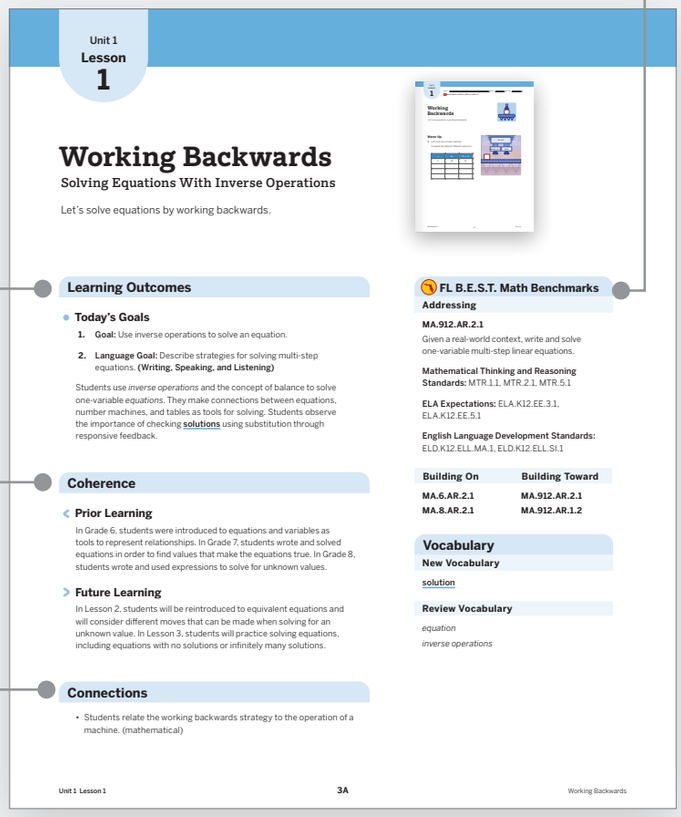
The lessons within each unit are grouped into sub-units that address a related group of concepts. All sub-units include lesson practice and Show What You Know formative assessments, and targeted sub-units include a quiz.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

The **Florida B.E.S.T. Math Benchmarks** section will list all standards addressed in this lesson, including the **Mathematical Thinking and Reasoning Standards,** the **ELA Expectations,** and the **English Language Development Standards.** This section shows the standards the lesson builds on and the standards the lesson builds toward.



The **Learning Outcome**s section is where teachers will find the goals of the lesson, including language goals

In the **Coherence** section, teachers will find information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

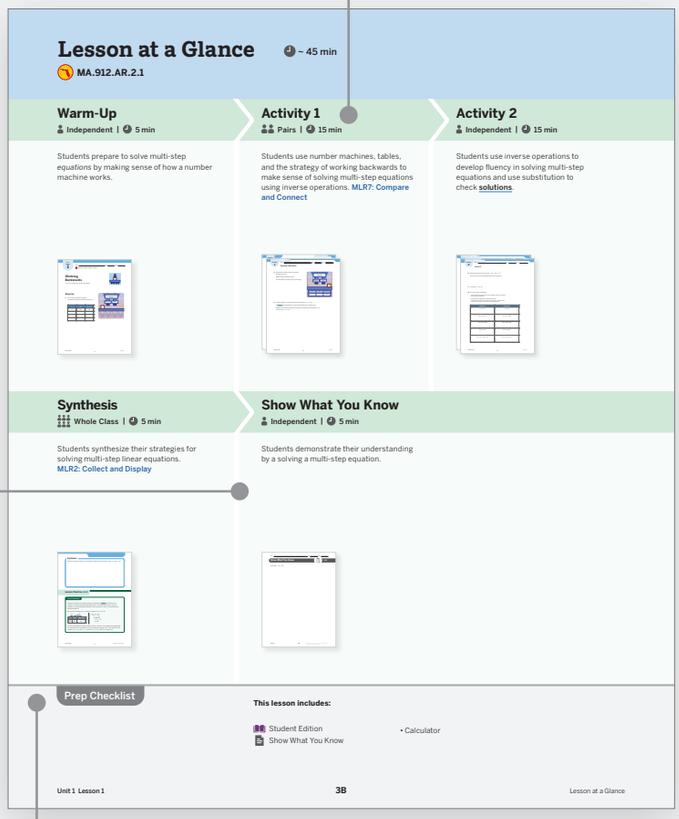
In the **Connections** section, teachers will find opportunities to help their students make connections cross-curricularly, to the real world, or to other mathematical learning.

Lesson Overview

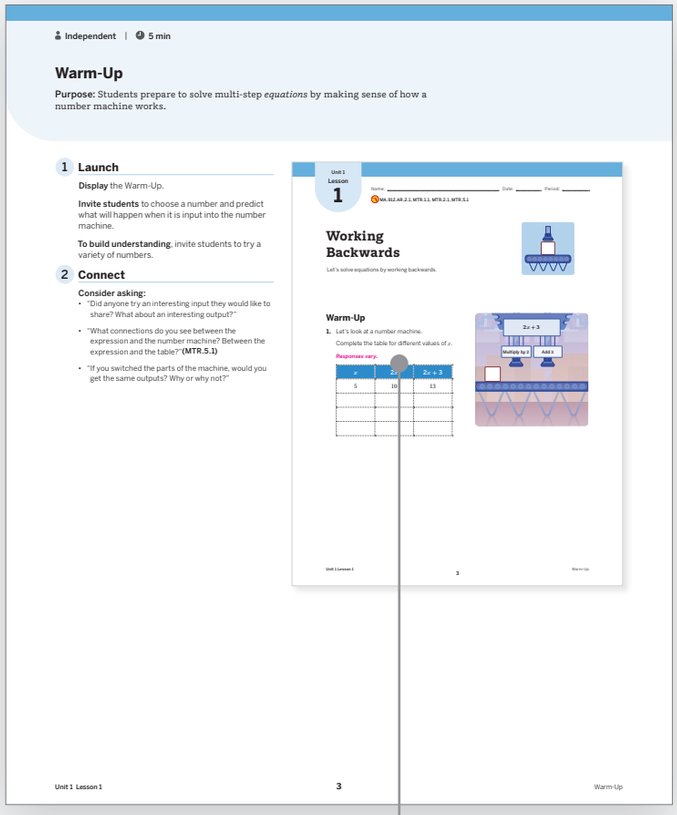
This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.



The **Prep Checklist** displays all materials students will need for the lesson.



Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class. The page also lists any hands-on materials that may be needed.

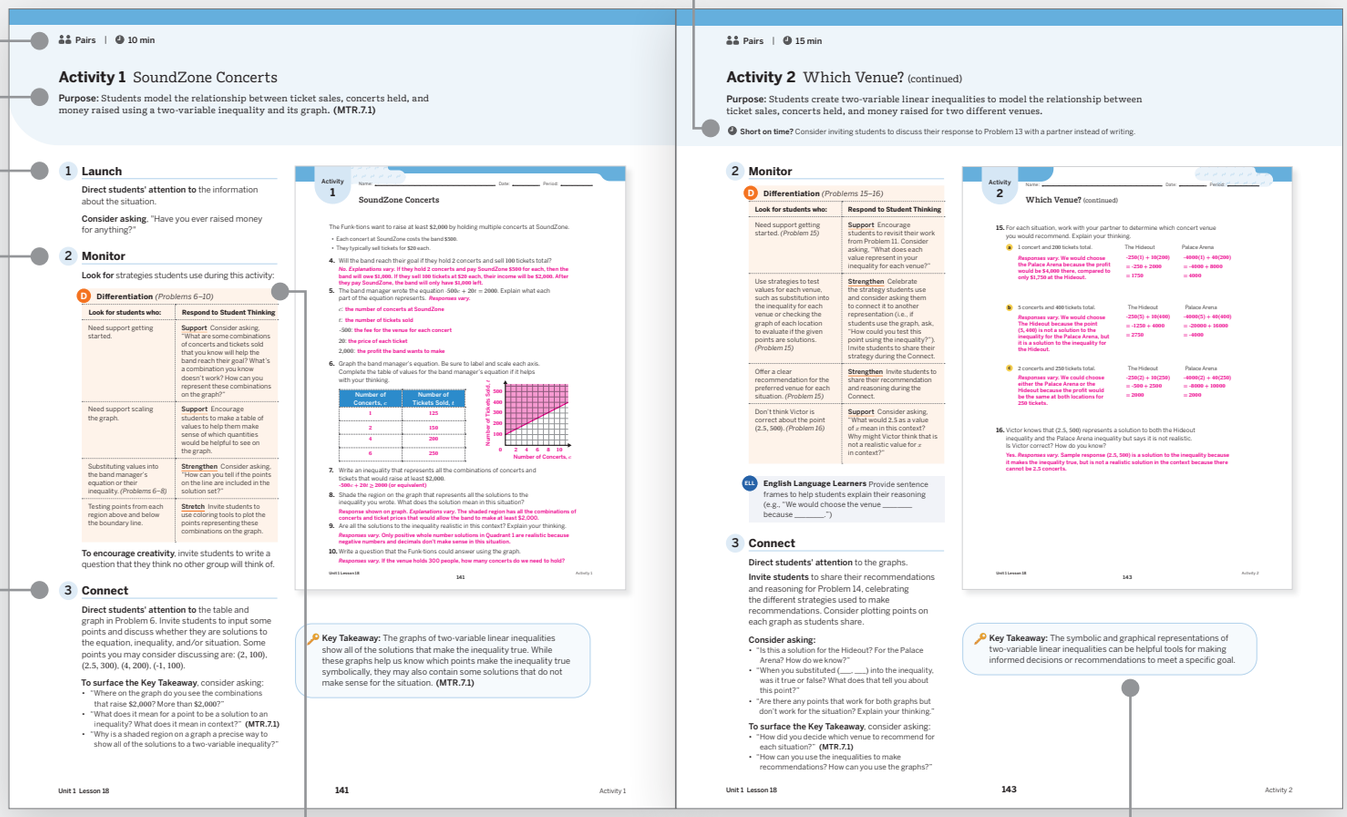
Warm-Up

Every Amplify Desmos Math Florida lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping** and **time frame** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.



Pairs | 10 min

Activity 1 SoundZone Concerts

Purpose: Students model the relationship between ticket sales, concerts held, and money raised using a two-variable inequality and its graph. **(MTR.7.1)**

1 Launch

Direct students' attention to the information about the situation.

Consider asking, "Have you ever raised money for anything?"

2 Monitor

Look for strategies students use during this activity:

D Differentiation *(Problems 6–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are some combinations of concerts and tickets sold that you know will help the band reach their goal? What's a combination you know doesn't work? How can you represent these combinations on the graph?"
Need support scaling the graph.	**Support** Encourage students to make a table of values to help them make sense of which quantities would be helpful to see on the graph.
Substituting values into the band manager's equation or their inequality. *(Problems 6–8)*	**Strengthen** Consider asking, "How can you tell if the points on the line are included in the solution set?"
Testing points from each region above and below the boundary line.	**Stretch** Invite students to use coloring tools to plot the points representing these combinations on the graph.

To encourage creativity, invite students to write a question that they think no other group will think of.

3 Connect

Direct students' attention to the table and graph in Problem 6. Invite students to input some points and discuss whether they are solutions to the equation, inequality, and/or situation. Some points you may consider discussing are: (2, 100), (2.5, 300), (4, 200), (-1, 100).

To surface the Key Takeaway, consider asking:
- "Where on the graph do you see the combinations that raise $2,000? More than $2,000?"
- "What does it mean for a point to be a solution to an inequality? What does it mean in context?" **(MTR.7.1)**
- "Why is a shaded region on a graph a precise way to show all of the solutions to a two-variable inequality?"

Activity 1 — SoundZone Concerts

The Funk-tions want to raise at least $2,000 by holding multiple concerts at SoundZone.
- Each concert at SoundZone costs the band $300.
- They typically sell tickets for $20 each.

4. Will the band reach their goal if they hold 2 concerts and sell 100 tickets total?
No. Explanations vary. If they hold 2 concerts and pay SoundZone $300 for each, then the band will owe $1,000. If they sell 100 tickets at $20 each, their income will be $2,000. After they pay SoundZone, the band will only have $1,000 left.
5. The band manager wrote the equation $-500c + 20t \geq 2000$. Explain what each part of the equation represents. *Responses vary.*
c: the number of concerts at SoundZone
t: the number of tickets sold
-500: the fee for the venue for each concert
20: the price of each ticket
$2{,}000$: the profit the band wants to make
6. Graph the band manager's equation. Be sure to label and scale each axis. Complete the table of values for the band manager's equation if it helps with your thinking.

Number of Concerts, c	Number of Tickets Sold, t
1	125
2	150
4	200
6	250

7. Write an inequality that represents all the combinations of concerts and tickets that would raise at least $2,000.
$-500c + 20t \geq 2000$ (or equivalent)
8. Shade the region on the graph that represents all the solutions to the inequality you wrote. What does the solution mean in this situation?
Response shown on graph. Explanations vary. The shaded region has all the combinations of concerts and ticket prices that would allow the band to make at least $2,000.
9. Are all the solutions to the inequality realistic in this context? Explain your thinking.
Responses vary. Only positive whole number solutions in Quadrant I are realistic because negative numbers and decimals don't make sense in this situation.
10. Write a question that the Funk-tions could answer using the graph.
Responses vary. If the venue holds 300 people, how many concerts do we need to hold?

Unit 1 Lesson 18 — 141 — Activity 1

Key Takeaway: The graphs of two-variable linear inequalities show all of the solutions that make the inequality true. While these graphs help us know which points make the inequality true symbolically, they may also contain some solutions that do not make sense for the situation. **(MTR.7.1)**

Pairs | 15 min

Activity 2 Which Venue? (continued)

Purpose: Students create two-variable linear inequalities to model the relationship between ticket sales, concerts held, and money raised for two different venues.

Short on time? Consider inviting students to discuss their response to Problem 13 with a partner instead of writing.

2 Monitor

D Differentiation *(Problems 15–16)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 15)*	**Support** Encourage students to revisit their work from Problem 11. Consider asking, "What does each value represent in your inequality for each venue?"
Use strategies to test values for each venue, such as substitution into the inequality for each venue or checking the graph of each location to evaluate if the given points are solutions. *(Problem 15)*	**Strengthen** Celebrate the strategy students use and consider asking them to connect it to another representation (i.e., if students use the graph, ask, "How could you test this point using the inequality?"). Invite students to share their strategy during the Connect.
Offer a clear recommendation for the preferred venue for each situation. *(Problem 15)*	**Strengthen** Invite students to share their recommendation and reasoning during the Connect.
Don't think Victor is correct about the point (2.5, 500). *(Problem 16)*	**Support** Consider asking, "What would 2.5 as a value of x mean in this context? Why might Victor think that is not a realistic value for x in context?"

ELL English Language Learners Provide sentence frames to help students explain their reasoning (e.g., "We would choose the venue _____ because _____.")

3 Connect

Direct students' attention to the graphs.

Invite students to share their recommendations and reasoning for Problem 14, celebrating the different strategies used to make recommendations. Consider plotting points on each graph as students share.

Consider asking:
- "Is this a solution for the Hideout? For the Palace Arena? How do we know?"
- "When you substituted (___, ___) into the inequality, was it true or false? What does that tell you about this point?"
- "Are there any points that work for both graphs but don't work for the situation? Explain your thinking."

To surface the Key Takeaway, consider asking:
- "How did you decide which venue to recommend for each situation?" **(MTR.7.1)**
- "How can you use the inequalities to make recommendations? How can you use the graphs?"

Activity 2 — Which Venue? (continued)

15. For each situation, work with your partner to determine which concert venue you would recommend. Explain your thinking.

a. 1 concert and 200 tickets total.

	The Hideout	Palace Arena
	$-250(1) + 18(200)$	$-4000(1) + 46(200)$
	$= -250 + 2900$	$= -4000 + 8000$
	$= 1750$	$= 4000$

Responses vary. We would choose the Palace Arena because the profit would be $4,000 there, compared to only $1,750 at the Hideout.

b. 5 concerts and 400 tickets total.

	The Hideout	Palace Arena
	$-250(5) + 18(400)$	$-4000(5) + 46(400)$
	$= -1250 + 4000$	$= -20000 + 18400$
	$= 2750$	$= -4000$

Responses vary. We would choose The Hideout because the point (5, 400) is not a solution to the inequality for the Palace Arena, but it is a solution to the inequality for the Hideout.

c. 2 concerts and 250 tickets total.

	The Hideout	Palace Arena
	$-250(2) + 18(250)$	$-4000(2) + 46(250)$
	$= -500 + 2500$	$= -8000 + 10000$
	$= 2000$	$= 2000$

Responses vary. We could choose either the Palace Arena or the Hideout because the profit would be the same at both locations for 250 tickets.

16. Victor knows that (2.5, 500) represents a solution to both the Hideout inequality and the Palace Arena inequality but says it is not realistic. Is Victor correct? How do you know?
Yes. Responses vary. Sample response (2.5, 500) is a solution to the inequality because it makes the inequality true, but is not a realistic solution in the context because there cannot be 2.5 concerts.

Unit 1 Lesson 18 — 143 — Activity 2

Key Takeaway: The symbolic and graphical representations of two-variable linear inequalities can be helpful tools for making informed decisions or recommendations to meet a specific goal.

In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a **D Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

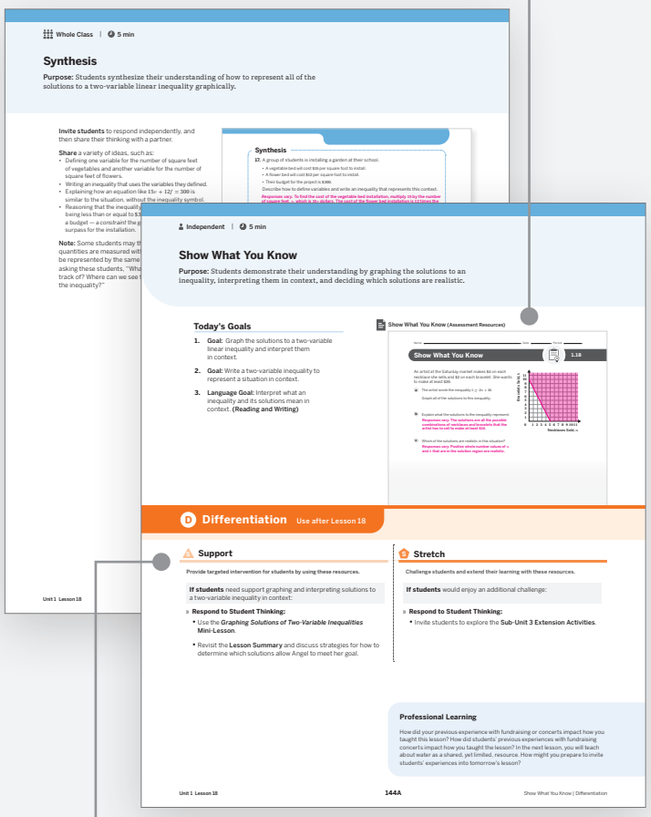
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

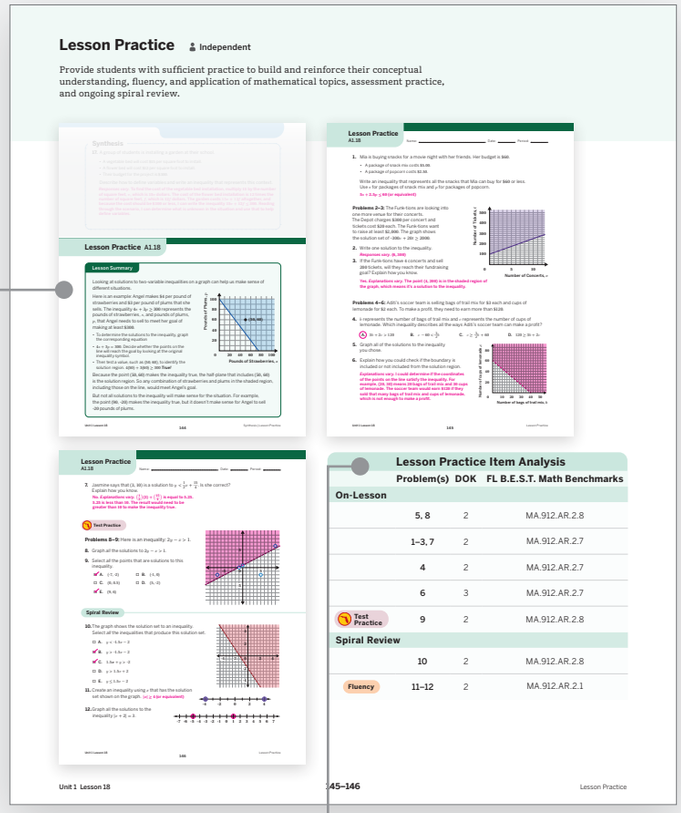
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, and ways to increase access to the language and content. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** begins with a **Summary** of the big ideas in the lesson, often including a worked example. Students can highlight parts of the summary or share it with a caregiver or classmate.



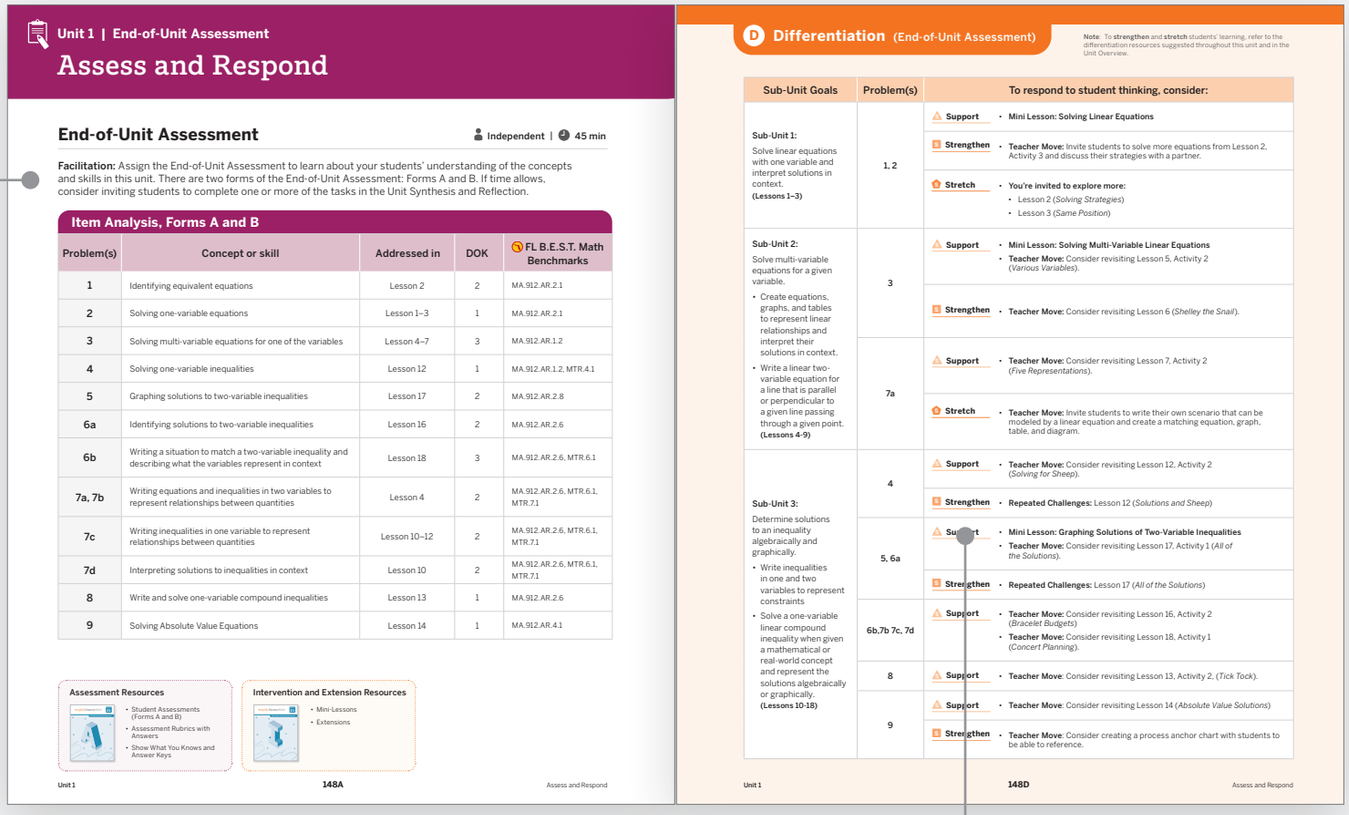
A **Practice Problem Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding B.E.S.T. Math Benchmarks.

Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

Lesson Practice

Daily practice problems for the day's lesson are included both online and in the print Student Edition, including Fluency, FAST Practice, and Spiral Review.



Unit 1 | End-of-Unit Assessment

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying equivalent equations	Lesson 2	2	MA.912.AR.2.1
2	Solving one-variable equations	Lesson 1–3	1	MA.912.AR.2.1
3	Solving multi-variable equations for one of the variables	Lesson 4–7	3	MA.912.AR.1.2
4	Solving one-variable inequalities	Lesson 12	1	MA.912.AR.1.2, MTR.4.1
5	Graphing solutions to two-variable inequalities	Lesson 17	2	MA.912.AR.2.8
6a	Identifying solutions to two-variable inequalities	Lesson 16	2	MA.912.AR.2.6
6b	Writing a situation to match a two-variable inequality and describing what the variables represent in context	Lesson 18	3	MA.912.AR.2.6, MTR.6.1
7a, 7b	Writing equations and inequalities in two variables to represent relationships between quantities	Lesson 4	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
7c	Writing inequalities in one variable to represent relationships between quantities	Lesson 10–12	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
7d	Interpreting solutions to inequalities in context	Lesson 10	2	MA.912.AR.2.6, MTR.6.1, MTR.7.1
8	Write and solve one-variable compound inequalities	Lesson 13	1	MA.912.AR.2.6
9	Solving Absolute Value Equations	Lesson 14	1	MA.912.AR.4.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Unit 1 148A Assess and Respond

Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: Solve linear equations with one variable and interpret solutions in context. **(Lessons 1–3)**	1, 2	**Support** • **Mini Lesson: Solving Linear Equations** **Strengthen** • **Teacher Move:** Invite students to solve more equations from Lesson 2, Activity 3 and discuss their strategies with a partner. **Stretch** • **You're invited to explore more:** Lesson 2 (*Solving Strategies*); Lesson 3 (*Same Position*)
Sub-Unit 2: Solve multi-variable equations for a given variable. • Create equations, graphs, and tables to represent linear relationships and interpret their solutions in context. • Write a linear two-variable equation for a line that is parallel or perpendicular to a given line passing through a given point. **(Lessons 4-9)**	3	**Support** • **Mini Lesson: Solving Multi-Variable Linear Equations** • **Teacher Move:** Consider revisiting Lesson 5, Activity 2 (*Various Variables*). **Strengthen** • **Teacher Move:** Consider revisiting Lesson 6 (*Shelley the Snail*).
	7a	**Support** • **Teacher Move:** Consider revisiting Lesson 7, Activity 2 (*Five Representations*). **Stretch** • **Teacher Move:** Invite students to write their own scenario that can be modeled by a linear equation and create a matching equation, graph, table, and diagram.
Sub-Unit 3: Determine solutions to an inequality algebraically and graphically. • Write inequalities in one and two variables to represent constraints • Solve a one-variable linear compound inequality when given a mathematical or real-world concept and represent the solutions algebraically or graphically. **(Lessons 10-18)**	4	**Support** • **Teacher Move:** Consider revisiting Lesson 12, Activity 2 (*Solving for Sheep*). **Strengthen** • **Repeated Challenges:** Lesson 12 (*Solutions and Sheep*)
	5, 6a	**Support** • **Mini Lesson: Graphing Solutions of Two-Variable Inequalities** • **Teacher Move:** Consider revisiting Lesson 17, Activity 1 (*All of the Solutions*). **Strengthen** • **Repeated Challenges:** Lesson 17 (*All of the Solutions*)
	6b,7b 7c, 7d	**Support** • **Teacher Move:** Consider revisiting Lesson 16, Activity 2 (*Bracelet Budgets*) • **Teacher Move:** Consider revisiting Lesson 18, Activity 1 (*Concert Planning*).
	8	**Support** • **Teacher Move**: Consider revisiting Lesson 13, Activity 2, (*Tick Tock*).
	9	**Support** • **Teacher Move**: Consider revisiting Lesson 14 (*Absolute Value Solutions*) **Strengthen** • **Teacher Move**: Consider creating a process anchor chart with students to be able to reference.

Unit 1 148D Assess and Respond

An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math Florida is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions*.

1 Launch



The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor



As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect



Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

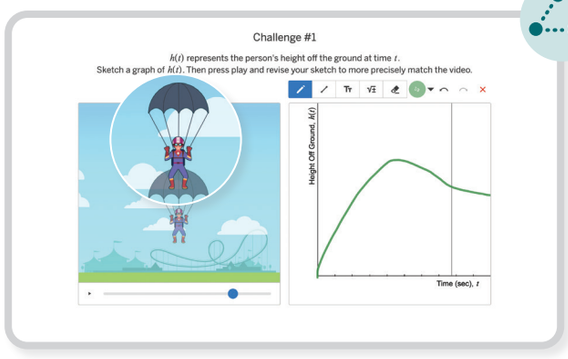
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

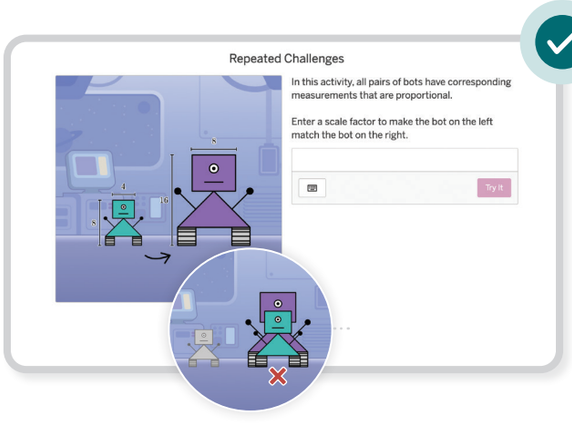
The Power of Digital

Amplify Desmos Math Florida provides the option for teachers to deliver lessons using the power of digital technology. This includes the use of visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. These interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



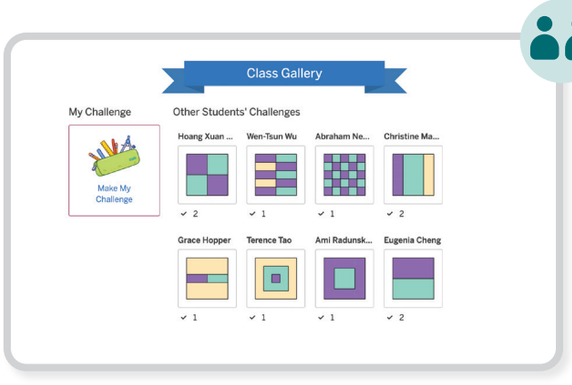
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math Florida, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

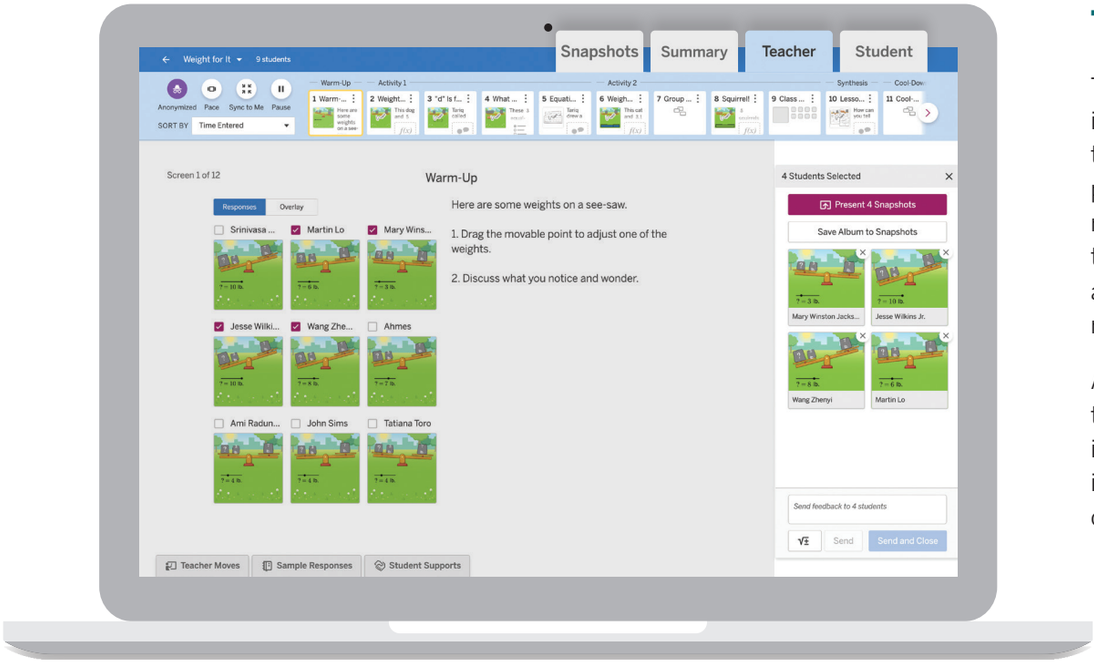


Collaborative experiences

Digital tools allow students to interact with each other's mathematical ideas in a variety of ways. Students can use the "Share With Class" button to exchange mathematical thinking with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Facilitation Tools

Amplify Desmos Math Florida includes a suite of digital facilitation tools. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

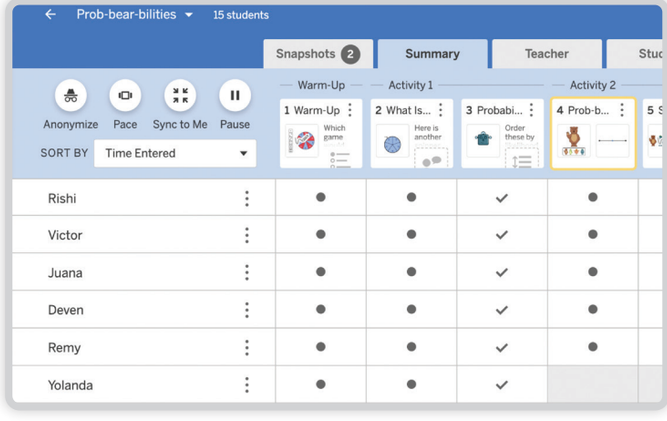
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math in Action

Each Math in Action feature highlights an individual whose work in the field of mathematics, or a connected field, have made an impact on American society. These features can be used to help make connections between the mathematical content of the grade and how math shows up in a wide variety of careers.

Career Connections

Amplify Desmos Math Florida intentionally celebrates traditional mathematics fields as well as elevates work that engages mathematics in unique ways.

Representation of Mathematicians

Students can read about the contributions and work of a diverse range of individuals with varying backgrounds, ages, careers, and ethnicities.



Unit 5 | Math in Action

Career Connection

How is city planning related to solving a puzzle?

Solving a puzzle can involve thinking about different strategies, trying out ideas, and thinking about constraints. One famous historical puzzle, the "Hundred Fowls Problem," was created by 5th century Chinese mathematician Zhang Quijian.

KieferPix/Shutterstock.com.

City planners think about constraints as they design cities to be attractive and functional. Some constraints might be geographic location, access to natural water sources, or available budget to spend. City planners might create systems of equations to help decide where to place buildings, public transit systems, and parks.

B.E.S.T. Mathematics Benchmark Connection

Many city planners rely on algebra concepts in their work. For example, they solve systems of linear equations (MA.912.AR.9.1) when they analyze various costs for materials and labor. They may also represent constraints as systems of equations or inequalities (MA.912.F.1.5) to determine what combination of supplies to use when they want to stay within budget.

Mathematical Thinking and Reasoning Connection

People who work on city planning use thinking and reasoning skills like the ones you use for your math work! For example, teams have different areas of expertise, so when they start a new project, they have to learn about the location, building materials, community requirements, and other key elements as individuals and as a group (MTR.1.1). They use multiple representations to explain budgets, schedules, supplies, and equipment needs so that the other teams involved in the planning understand their work (MTR.2.1).

Meet Zhang Qiujian

While little is known about Chinese mathematician Zhang Quijian's biography, he wrote a famous puzzle called the "Hundred Fowls" problem in his 5th century book, *Zhang Qiujian Suanjing*. Can you solve it? How can a system of equations help you get started?

"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens and chicks are there?"

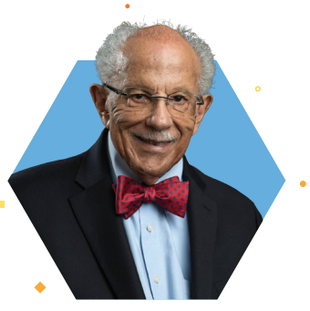
Mathematicians and others often study problems as if they are puzzles. They use equations and graphs to analyze the information they are given, and test values to find succesful solutions.

Algebra 1 Unit 5 587



Helia Bravo Hollis

Mathematics and botany? Helia Bravo Hollis found a way to combine her interest in both, studying documenting and writing books about different types of cacti.



Warren Washington

Mathematics and weather? Warren Washington found a way to combine her interest in both, developing atmospheric computer models to better understand weather patterns.



Zhang Qiujian

Mathematics and puzzles? Zhang Qiujian found a way to combine his interest in both by creating a famous puzzle called the "Hundred Fowls" that can be solved using systems of equations.



Po-Shen Loh

Mathematics and history? Po-Shen Loh found a way to combine his interest in both, researching ways mathematicians solved problems hundreds and thousands of years ago.

Facilitating Meaningful Conversations

Amplify Desmos Math Florida is designed to support interactive and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the ideas of others, and appreciate different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the real-world and mathematical experiences that students bring to the classroom.

Use the Tell a Story routine to help students make sense of the turtle race animation. Invite students to write a story about the animation using their unique experiences, perspectives, identities, and creativity. **(MP1)**



In the lesson **Turtle Time Trials**, students are invited to use the **Tell a Story** routine to create their own narratives about what the turtles in the animation are doing and why. This is a chance for students to use their creativity while engaging deeply in the math.

1 Launch

Direct students' attention to the images of the rice dishes.

To surface connections, consider asking, "What is your favorite rice dish? When or where do you eat it?"



In the lesson **Rice Ratios**, students are introduced to the concept of equivalent ratios in the context of making rice. The Teacher Edition provides teachers with prompts they can use to help students connect the rice dishes in the activities to their own lives.

Supporting Student Collaboration

Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.

How will you support student collaboration?

- Discuss what supporting each other without giving answers could look or sound like.
- Invite a student from each group to read the task aloud.

The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

In the lesson **Subway Seats**, students use the **Which One Doesn't Belong?** routine to begin exploring two-variable equations. Classroom conversations about activities like this are a way for teachers to elicit different kinds of student brilliance. The Teacher Edition provides suggestions for how teachers can engage students in conversation and reinforce the idea that each student has something uniquely valuable to contribute.

Accessibility

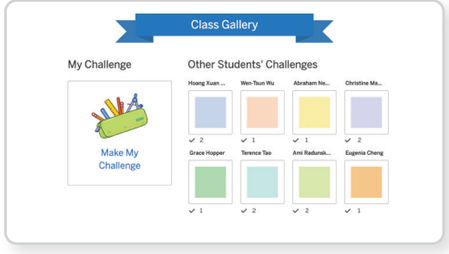
Amplify Desmos Math Florida is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

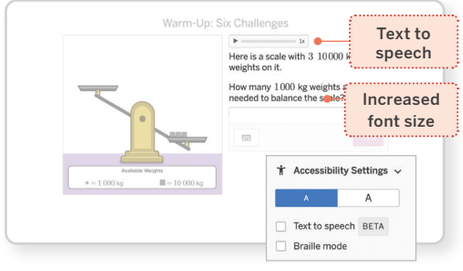
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome interests and identities, support sustaining effort and persistence, and develop emotional capacity.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

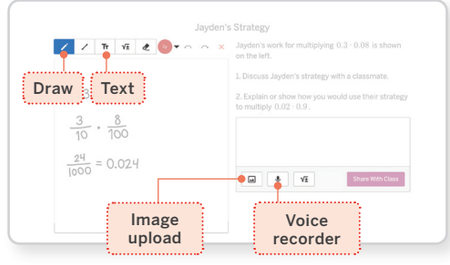
Learners make sense of information differently. Amplify Desmos Math Florida includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math Florida ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

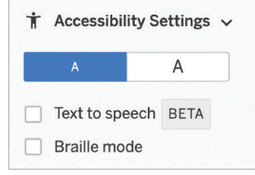
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math Florida enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

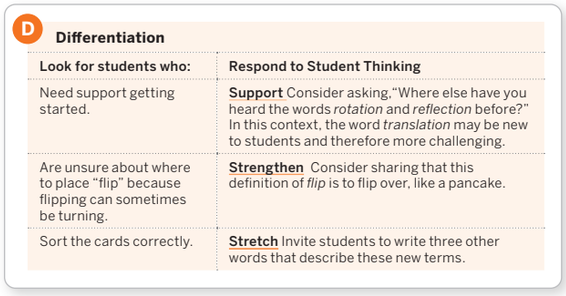
Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.



Differentiation

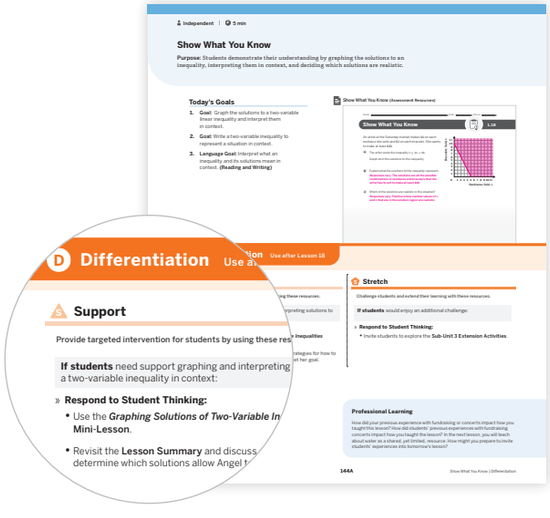
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Differentiation: Beyond the Lesson

In each lesson, students' understanding of the learning goals is broken down for teachers into three categories: students who need support to understand the learning goals, students who need to continue strengthening their understanding of the learning goals, and students who are ready to stretch their learning.

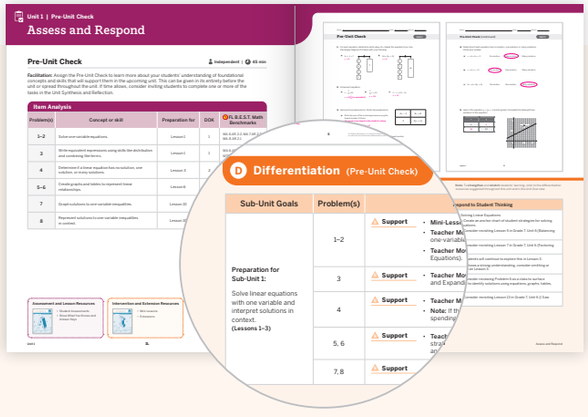
Each **Support**, **Strengthen**, and **Stretch** resource is designed to take 15 minutes:

- **Mini-Lessons:** Targeted intervention for students who need additional support or need more time.
- **Extensions:** Small group or independent challenges.



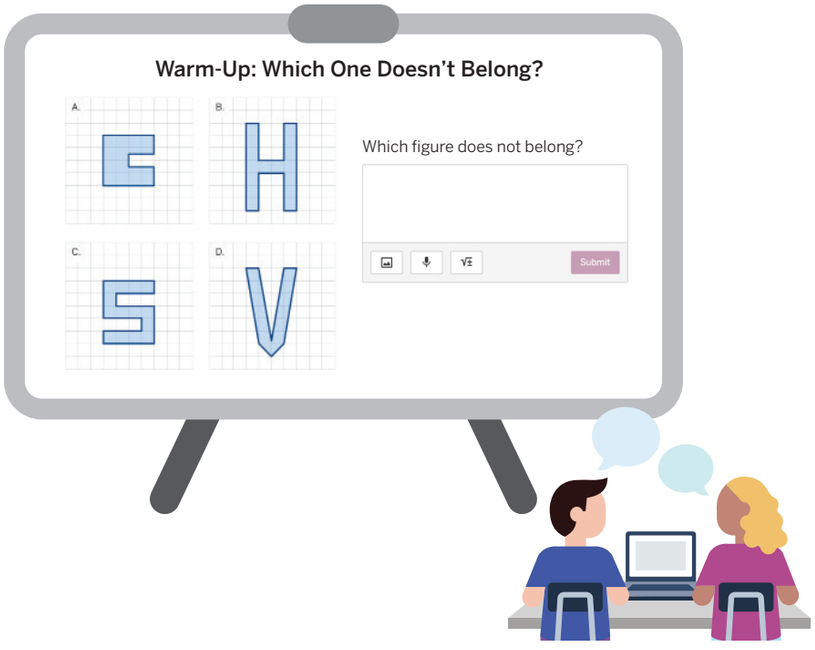
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

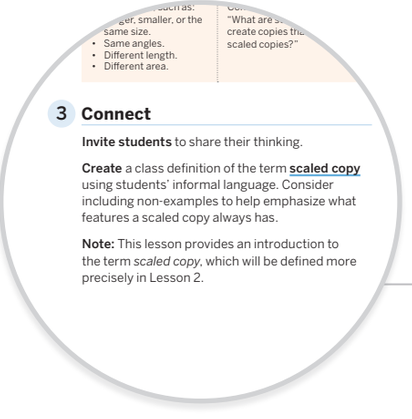
- The **Which One Doesn't Belong?** routine communicates to students that their ideas have value, that there are many ways to be correct in math, and that they can learn math by sharing their math thinking with each other.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

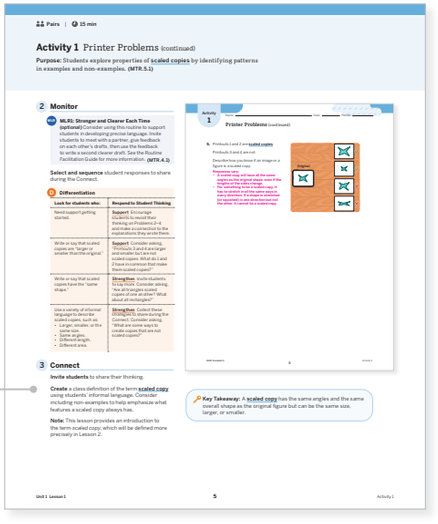
Each instructional routine included in an Amplify Desmos Math Florida lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' mathematical thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math Florida curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?





For more information about the Instructional Routines embedded in the Amplify Desmos Math Florida curriculum, see the Routine Facilitation Guide in the Professional Development Library.

Math Language Development

At Amplify Desmos Math Florida, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of English Language Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math Florida includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for English Language Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* (′). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does *pre* mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Calculate the area of a parallelogram on a grid using a variety of strategies.
  2. **Language Goal:** Compare and contrast strategies for determining the area of a parallelogram. **(Writing, Speaking, and Listening)**
  3. **Language Goal:** Describe parallelograms using precise mathematical language. **(Speaking and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

English Language Learners Support

Supports for ELL **English Language Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for English Language Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. English Language Learner supports may also be attached to MLRs.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Developing Fluency

Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another*.

In Algebra 1, Florida students are expected to perform all four operations with integers, positive decimals and positive fractions with procedural fluency.

Expectation

Exploration

To develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.

Procedural Reliability

To utilize skills to develop an accurate, reliable method that aligns with the student's understanding and learning style.

Procedural Fluency

To utilize skills to become fluent with an efficient, generalizable and accurate procedure, including a standard algorithm.

How Amplify Desmos Math Florida Supports the Expectation

- Manipulatives and visual models and diagrams are used extensively to support student's exploration of new concepts.
- Discussions are strongly encouraged in each lesson, including 💬 Discuss prompts in the student edition as well as suggested discussion prompts in the teacher's edition.
- Students are encouraged to estimate and compare estimations to actual answers to support reasonableness of problem solving solutions.
- Students are often asked to analyze a drawing or to make a drawing to explain their reasoning and strengthen their understanding of the math concepts being studied.

- Frequently in each course, students are asked to analyze and evaluate work presented in their student edition that represents work done by other students for accuracy, preference to learning style, and for efficiency.

- Program features such as practice, spiral reviews, mini lessons, and pre-unit assessments are all designed to assist students with achieving procedural fluency.

*National Research Council [2001, 2005, 2012]; Star [2005]

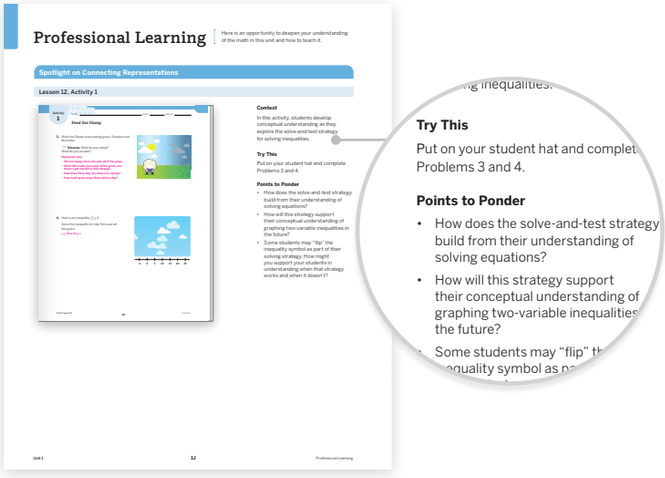
Professional Learning

The Amplify Desmos Math Florida program includes regular opportunities for teachers to deepen their understanding while they teach.

Embedded Professional Development

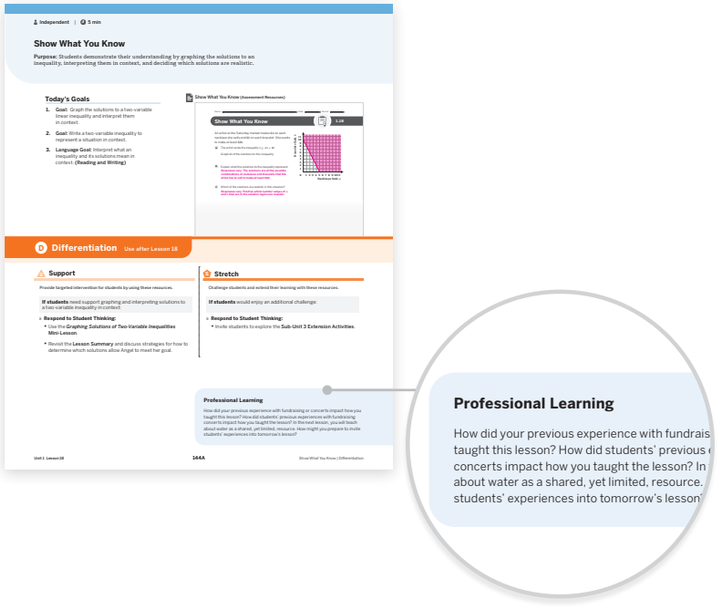
Unit-Level Professional Development

The Unit Overview for each unit includes a Professional Development page, which lists opportunities for teachers to deepen their understanding of the math in the unit and how to teach it.



Lesson-Level Professional Development

Each lesson provides a moment of professional learning as an opportunity for teachers to reflect on how well the lesson went. These professional learning moments can relate to principles, instruction, pedagogy, or program content.



Professional Development Sessions

Amplify's professional development team supports both teachers and administrators throughout their entire implementation journey. These sessions are available as on-site or virtual options.

Launch

Launch sessions introduce Amplify programs and support strong implementation. After learning about the program's foundational principles, you'll practice implementing the program in a collaborative environment.

Strengthen

Strengthen sessions deepen understanding of the program. These offerings are targeted and are meant to take your practice – and your students' learning to the next level.

Coach

These sessions are tailored to elevate instructional practices and meet the unique needs of teachers and leaders. These sessions can be customized leveraging our menu of supports.

Professional learning on your schedule



The PD Library serves as a centralized location where educators can access an ever-growing collection of professional learning resources and courses designed to support the implementation of Amplify Desmos Math Florida. Educators will find bite-sized how to videos, unit and lesson planning guides and templates, resources for leaders, and so much more!

Program Assessments

A variety of performance data in Amplify Desmos Math Florida provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math Florida, understanding is made continually visible.

Lesson-Level Assessments

Amplify Desmos Math lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.



Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.



Pre-Unit Check

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.



Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.



End-of-Unit Assessment

Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight. Two forms are included to help facilitate assessment administration in the classroom.

Florida's B.E.S.T. Standards for Mathematics

The following correlations show the alignment of Amplify Desmos Math Florida to the FL B.E.S.T. Mathematics Standards for Algebra 1.

Benchmark	B.E.S.T Mathematics Benchmark	Amplify Desmos Math Lesson(s)
	Number Sense and Operations	
MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.	**Unit 5** Lessons 5, 6, 10, 11, 12, 13, 15 **Unit 7** Lessons 3, 4, 5, 6, 7, 8, 9, 10
MA.912.NSO.1.2	Generate equivalent algebraic expressions using the properties of exponents.	**Unit 5** Lessons 5, 6, 12, 13, 15 **Unit 7** Lessons 4, 5, 6, 7, 8, 9, 10
MA.912.NSO.1.4	Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.	**Unit 5** Lessons 10, 11, 12, 13 **Unit 7** Lessons 14, 18
	Algebraic Reasoning	
MA.912.AR.1.1	Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.	**Unit 7** Lessons 3, 4, 5, 6
MA.912.AR.1.2	Rearrange equations or formulas to isolate a quantity of interest.	**Unit 1** Lessons 5, 6, 7, 8, 9 **Unit 7** Lessons 10, 14, 15, 16, 18
MA.912.AR.1.3	Add, subtract and multiply polynomial expressions with rational number coefficients.	**Unit 7** Lessons 3, 4, 5, 15, 16
MA.912.AR.1.4	Divide a polynomial expression by a monomial expression with rational number coefficients.	**Unit 7** Lessons 7, 8, 9
MA.912.AR.1.7	Rewrite a polynomial expression as a product of polynomials over the real number system.	**Unit 7** Lessons 6, 7, 9, 10, 14, 15, 16
MA.912.AR.2.1	Given a real-world context, write and solve one-variable multi-step linear equations.	**Unit 1** Lessons 2, 3
MA.912.AR.2.2	Write a linear two-variable equation to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.	**Unit 1** Lessons 5, 6, 7, 8, 9 **Unit 4** Lesson 6
MA.912.AR.2.3	Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.	**Unit 1** Lesson 9
MA.912.AR.2.4	Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.	**Unit 1** Lessons 7, 8
MA.912.AR.2.5	Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.	**Unit 1** Lessons 5, 6, 7, 8, 9
MA.912.AR.2.6	Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.	**Unit 1** Lessons 5, 6, 7, 8, 11, 12, 13, 14
MA.912.AR.2.7	Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.	**Unit 1** Lesson 18

Florida's B.E.S.T. Standards for Mathematics

MA.912.AR.2.8	Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.	**Unit 1** Lessons 12, 17, 18
MA.912.AR.3.1	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.	**Unit 7** Lessons 10, 13, 14, 15, 16, 18
MA.912.AR.3.4	Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.	**Unit 6** Lesson 15
MA.912.AR.3.5	Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function.	**Unit 6** Lesson 11
MA.912.AR.3.6	Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.	**Unit 6** Lessons 12, 13, 15 **Unit 7** Lessons 10, 13, 16
MA.912.AR.3.7	Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.	**Unit 6** Lessons 6, 7, 8, 10, 11, 12, 13
MA.912.AR.3.8	Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.	**Unit 6** Lessons 7, 13
MA.912.AR.4.1	Given a mathematical or real-world context, write and solve one-variable absolute value equations.	**Unit 1** Lessons 15, 16
MA.912.AR.4.3	Given a table, equation or written description of an absolute value function, graph that function and determine its key features.	**Unit 3** Lesson 12
MA.912.AR.5.3	Given a mathematical or real-world context, classify an exponential function as representing growth or decay.	**Unit 5** Lessons 5, 6
MA.912.AR.5.4	Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.	**Unit 5** Lesson 4
MA.912.AR.5.6	Given a table, equation or written description of an exponential function, graph that function and determine its key features.	**Unit 5** Lessons 4, 7
MA.912.AR.9.1	Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.	**Unit 4** Lessons 3, 4, 5, 6, 7
MA.912.AR.9.4	Graph the solution set of a system of two-variable linear inequalities.	**Unit 4** Lessons 9, 10
MA.912.AR.9.6	Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.	**Unit 4** Lessons 7, 10
	Functions	
MA.912.F.1.1	Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.	**Unit 3** Lesson 4 **Unit 6** Lessons 5, 6
MA.912.F.1.2	Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.	**Unit 3** Lessons 3, 6, 10 **Unit 5** Lesson 6 **Unit 6** Lessons 8, 10, 11, 12, 13

Florida's B.E.S.T. Standards for Mathematics

MA.912.F.1.3	Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.	**Unit 3** Lesson 8
MA.912.F.1.5	Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.	**Unit 3** Lessons 4, 8, 9, 10
MA.912.F.1.6	Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.	**Unit 3** Lessons 4, 6, 7, 9 **Unit 5** Lessons 3, 4, 14
MA.912.F.1.8	Determine whether a linear, quadratic or exponential function best models a given real-world situation.	**Unit 5** Lessons 3, 4, 14 **Unit 6** Lessons 3, 4, 5, 6
MA.912.F.2.1	Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.	**Unit 6** Lesson 15
	Financial Literacy	
MA.912.FL.3.2	Solve real-world problems involving simple, compound and continuously compounded interest.	**Unit 5** Lessons 14, 15
MA.912.FL.3.4	Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.	**Unit 5** Lessons 14, 15
	Data Analysis & Probability	
MA.912.DP.1.1	Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.	**Unit 2** Lessons 2, 4, 5, 6, 7, 8, 14
MA.912.DP.1.2	Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.	**Unit 2** Lessons 2, 6, 7, 8, 14
MA.912.DP.1.3	Explain the difference between correlation and causation in the contexts of both numerical and categorical data.	**Unit 2** Lesson 17
MA.912.DP.1.4	Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.	**Unit 2** Lessons 9, 10
MA.912.DP.2.4	Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.	**Unit 2** Lessons 15, 16, 17
MA.912.DP.2.6	Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.	**Unit 2** Lessons 15, 16, 17
MA.912.DP.3.1	Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.	**Unit 2** Lessons 12, 13

Mathematical Thinking and Reasoning Standards

The following correlations show the alignment of Amplify Desmos Math Florida to the Mathematical Thinking and Reasoning Standards.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Unit 1 Lessons 1, 3, 6, 7, 10, 11, 12, 13, 15, 16, 18
Unit 2 Lessons 7, 9, 10, 13, 14, 15, 16, 17
Unit 3 Lessons 3, 7, 10
Unit 4 Lesson 5
Unit 5 Lesson 11
Unit 6 Lessons 5, 11
Unit 7 Lessons 1, 2, 3

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Unit 1 Lessons 1, 4, 6, 7, 10, 11, 13, 14, 16, 17
Unit 2 Lessons 4, 6, 12
Unit 3 Lessons 2, 5, 7, 10, 11
Unit 4 Lessons 1, 2, 3, 4, 6, 7, 8, 10
Unit 5 Lessons 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15
Unit 7 Lessons 4, 5, 7, 8, 9, 11, 12, 13, 15

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Unit 1 Lessons 4, 5
Unit 2 Lessons 2, 5, 13
Unit 3 Lessons 1, 4
Unit 4 Lessons 1, 2, 3, 7, 8, 10
Unit 5 Lessons 3, 10
Unit 6 Lessons 5, 11
Unit 7 Lessons 2, 3, 4, 6, 16, 17, 18

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Unit 1 Lessons 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15, 17
Unit 2 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17
Unit 3 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12
Unit 4 Lessons 1, 4, 6, 9
Unit 5 Lessons 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15
Unit 6 Lessons 6, 7, 8, 10, 13, 15
Unit 7 Lessons 1, 2, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Unit 1 Lessons 1, 2, 3, 12, 14, 15, 17, 18
Unit 2 Lessons 2, 3, 8
Unit 3 Lessons 3, 4, 11, 12
Unit 4 Lessons 1, 4, 6, 9
Unit 5 Lessons 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15
Unit 7 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Unit 1 Lessons 9, 10, 13, 18
Unit 2 Lessons 6, 13, 14
Unit 3 Lesson 5
Unit 4 Lessons 2, 3, 5, 7, 8, 9, 10
Unit 5 Lessons 4, 6, 13, 14
Unit 6 Lessons 11, 12, 15
Unit 7 Lessons 4, 10, 12, 14, 15, 18

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Unit 1 Lessons 8, 9, 18
Unit 2 Lessons 3, 4, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17
Unit 3 Lesson 5
Unit 4 Lessons 2, 3, 7, 8, 10
Unit 5 Lessons 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 15
Unit 6 Lessons 4, 12, 15
Unit 7 Lesson 18

English Language Arts B.E.S.T. Standards

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

Unit 1 Lessons 3, 8, 11, 12, 13
Unit 2 Lessons 1, 5, 6, 11
Unit 3 Lessons 1, 2, 4, 10
Unit 4 Lessons 1, 4, 7, 10
Unit 5 Lessons 1, 6, 8, 13, 15
Unit 6 Lessons 1, 2, 10
Unit 7 Lessons 2, 3, 13, 15, 18

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

Unit 1 Lesson 18
Unit 2 Lesson 13
Unit 3 Lessons 2, 3, 5, 6, 9, 10, 11
Unit 4 Lesson 6
Unit 5 Lessons 7, 12, 13, 14, 15
Unit 6 Lesson 15
Unit 7 Lesson 16, 18

ELA.K12.EE.3.1 Make inferences to support comprehension.

Unit 1 Lessons 1, 4, 8, 12, 13, 14, 15, 16, 17
Unit 2 Lessons 1, 2, 4, 5, 7, 9, 10, 11, 13, 15, 17
Unit 3 Lessons 4, 5, 7, 8, 11
Unit 4 Lessons 1, 3, 4, 5, 7
Unit 5 Lessons 2, 3, 4, 6, 9
Unit 6 Lessons 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 14, 15
Unit 7 Lessons 3, 5, 8, 16, 18

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

Unit 1 Lessons 2, 4, 5, 8, 9, 13, 14, 15, 16, 17
Unit 2 Lessons 3, 4, 9, 10, 11, 12, 14, 16, 17
Unit 3 Lessons 1, 6, 7, 9, 12
Unit 4 Lessons 2, 3, 5, 6, 10
Unit 5 Lessons 3, 5, 7, 8, 10, 11, 12, 15
Unit 6 Lessons 2, 4, 5, 6, 13
Unit 7 Lessons 1, 2, 5, 7, 8, 9, 11, 13, 14

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

Unit 1 Lessons 1, 2, 3, 5, 6, 7, 10, 11, 14, 15, 16, 17
Unit 2 Lessons 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17
Unit 3 Lessons 1, 11
Unit 4 Lessons 1, 2, 3, 4, 5, 6, 8, 9
Unit 5 Lessons 1, 2, 3, 9
Unit 6 Lessons 3, 5, 6, 7, 8, 9, 10, 11, 13
Unit 7 Lessons 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 15, 16, 17

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

Unit 1 Lessons 3, 4, 8, 9, 10, 13
Unit 2 Lessons 1, 4, 5, 8, 12, 14, 15, 16
Unit 3 Lesson 1
Unit 4 Lessons 8, 9
Unit 5 Lessons 1, 2, 14
Unit 6 Lesson 12
Unit 7 Lessons 9, 11

English Language Development Standards

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

Unit 1 Lessons - All Lessons
Unit 2 Lessons - All Lessons
Unit 3 Lessons - All Lessons
Unit 4 Lessons - All Lessons
Unit 5 Lessons - All Lessons
Unit 6 Lessons - All Lessons
Unit 7 Lessons - All Lessons

ELD.K12.ELL.SI.1 English language learners communicate for social and instructional purposes within the school setting.

Unit 1 Lessons - All Lessons
Unit 2 Lessons - All Lessons
Unit 3 Lessons - All Lessons
Unit 4 Lessons - All Lessons
Unit 5 Lessons - All Lessons
Unit 6 Lessons - All Lessons
Unit 7 Lessons - All Lessons



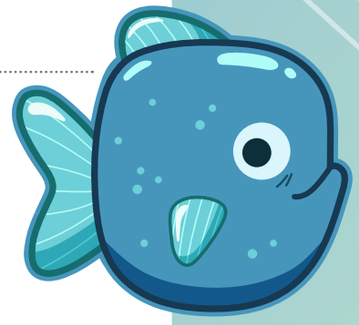
UNIT 5

Exponential Functions

Students use exponential functions to make sense of how changing by a small percentage can make a big impact over time.

Essential Questions

- How do exponential and linear functions compare?
- What type of function models repeated percent increase or decrease?
- How can you model situations involving compound interest and population growth?

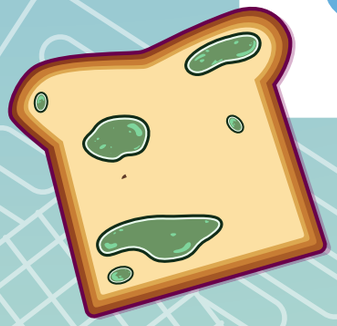


Sub-Unit Goals

Sub-Unit 1
- Compare and contrast linear and exponential functions.
- Create equations, tables, and graphs of linear and exponential functions.

Sub-Unit 2
- Interpret and write exponential functions to model situations involving percent increase or decrease.
- Use exponential functions to solve problems in context.

Sub-Unit 3
- Use the properties of exponents to interpret expressions for exponential functions.
- Performing operations with radicals.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Interpret parts of an expression, such as terms, factors, and coefficients. **(MA.912.AR.1.1)**

Distinguish between situations that can be modeled with linear functions and with exponential functions. **(MA.912.AR.5.3)**

Other Standards Addressed in This Unit: MA.912.AR.5.4 MA.912.AR.5.6 MA.912.F.1.2 MA.912.F.1.6 MA.912.F.1.8 MA.912.FL.3.2 MA.912.FL.3.4 MA.912.NSO.1.1 MA.912.NSO.1.2 MA.912.NSO.1.4

Mathematical Thinking and Reasoning Standards

- Students make connections between different representations of an exponential function (e.g., table, graph, equation, situation). **(MTR.2.1)**
- Students rewrite equivalent forms of exponential functions to highlight key features. **(MTR.5.1)**
- Students use exponential functions to model situations that occur in everyday life. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.4.1**

Coherence Telling a story within and across grades

In This Unit

Students use exponential functions to make sense of how changing by a small percentage can make a big impact over time.

Prior Learning

- **Grade 7:** Calculating percent increase and decrease **(MA.7.AR.3.1)**
- **Grade 7:** Properties of exponents **(MA.7.NSO.1.1, MA.7.NSO.1.2)**

Future Learning

- **Algebra 1, Unit 6:** Comparing exponentials and quadratics **(MA.912.F.1.1)**
- **Algebra 1, Unit 6:** Transformations of quadratics **(MA.912.F.2.1)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 11,** students develop conceptual understanding as they relate simple and compound interest to linear and exponential functions.

Procedural Fluency

In **Lesson 3,** students develop procedural fluency as they make connections between graphs, tables, situations, and equations for exponential functions.

Application

In **Lesson 2,** students apply their understanding of key features like initial value and growth factor to write equations of exponential functions.

Strategic Competence

In **Lesson 4,** students develop strategic competence as they use tables, graphs, and equations to evaluate exponential functions.

Adaptive Reasoning

In **Lesson 10,** students develop adaptive reasoning as they work to perform operations with radicals, adapting and reflecting on their strategies with each new problem.

Productive Disposition

In **Lesson 12,** students have opportunities to develop a productive disposition as they develop financial literacy of how real-life loans work.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.AR.3.2 MA.7.AR.3.1 MA.7.NSO.1.1 MA.7.NSO.1.2
MTR.2.1 MTR.3.1 MTR.5.1

Sub-Unit 1



1 Growing Globs

Patterns of Growth

Identify and compare two different patterns of growth.

MA.912.AR.1.1 MA.912.F.1.6 MA.912.F.1.8
MTR.4.1 MTR.5.1 MTR.7.1



2 Going Viral

Graphs of Exponential Relationships

Describe connections between graphs and equations and use graphs to write equations of exponential functions.

MA.912.AR.1.1 MA.912.AR.5.4 MA.912.F.1.6 MA.912.F.1.8
MTR.4.1 MTR.5.1 MTR.7.1



3 Return of the Globs

Connecting Representations of Linear and Exponential Functions

Make connections between different representations of linear and exponential functions.

MA.912.AR.1.1 MA.912.AR.5.3 MA.912.AR.5.4 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.3.1 MTR.4.1 MTR.5.1 MTR.7.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.912.AR.3.6 MA.912.AR.3.7 MA.912.AR.3.8 MA.912.F.1.8
MTR.2.1 MTR.5.1 MTR.7.1

Sub-Unit 3



8 Thinking Rationally

Writing Expressions Using Radicals and Rational Exponents

Write equivalent expressions using radicals and rational exponents.

MA.912.NSO.1.1 MA.912.NSO.1.4
MTR.2.1 MTR.4.1 MTR.5.1



9 Writing Radicals

Writing Equivalent Expressions Using Radicals and Rational Exponents

Explore rational exponents with numerators other than 1.

MA.912.NSO.1.1 MA.912.NSO.1.4
MTR.2.1 MTR.4.1 MTR.5.1



10 Rule the Roots

Operations with Radicals

Multiply and divide radical expressions.

MA.912.NSO.1.1 MA.912.NSO.1.2 MA.912.NSO.1.4
MTR.2.1 MTR.3.1 MTR.4.1 MTR.5.1

Assess and Respond



15 Exploring Interest

Compare Different Types of Interest

Use functions to model the population growth of Detroit.

MA.912.FL.3.4
MTR.2.1 MTR.4.1 MTR.7.1



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.AR.3.4 MA.912.AR.3.5 MA.912.AR.3.6 MA.912.AR.3.7 MA.912.F.1.1 MA.912.F.1.2 MA.912.F.1.8
MTR.2.1 MTR.4.1 MTR.5.1 MTR.7.1

Pacing: 18 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 15 Lessons
- Pre-Unit Check
- Sub-Unit Quiz(zes)
- End-of-Unit Assessment

Sub-Unit 2



4 Carlos and Corals

Evaluating Exponential Functions

Evaluate exponential functions with positive, negative, and zero inputs.

MA.912.AR.1.1 MA.912.AR.5.3 MA.912.AR.5.4 MA.912.AR.5.6 MA.912.F.1.2 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.4.1 MTR.5.1 MTR.6.1 MTR.7.1

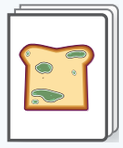


5 Differences and Factors

Changes Over Equal Intervals

Generalize how linear and exponential functions change over equal intervals

MA.912.AR.1.1 MA.912.AR.5.3 MA.912.AR.5.4 MA.912.F.1.2 MA.912.F.1.8 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.5.1 MTR.7.1



6 Growing Mold

Percent Increase and Decrease, Part 1

Explore how to model situations that change by a percent increase with exponential functions.

MA.912.AR.1.1 MA.912.AR.5.4 MA.912.NSO.1.1
MTR.2.1 MTR.6.1 MTR.7.1



7 At a Loss

Percent Increase and Decrease, Part 2

Make connections between different representations of exponential decay functions.

MA.912.AR.1.1 MA.912.AR.5.4 MA.912.AR.5.6 MA.912.NSO.1.1
MTR.2.1 MTR.4.1 MTR.5.1 MTR.7.1



11 Tame the Terms

Adding and Subtracting Radicals

Add and subtract radical expressions.

MA.912.NSO.1.1 MA.912.NSO.1.2 MA.912.NSO.1.4
MTR.1.1 MTR.2.1 MTR.4.1 MTR.5.1



12 Bank Accounts

Introducing Simple and Compound Interest

Learn how to model situations involving simple and compound interest.

MA.912.FL.3.2 MA.912.FL.3.4 MA.912.F.1.6 MA.912.F.1.8
MTR.2.1 MTR.4.1 MTR.5.1 MTR.7.1



13 Payday Loan

Revisiting Compound Interest

Analyze exponential functions that represent different compound interest scenarios.

MA.912.FL.3.2 MA.912.FL.3.4 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.4.1 MTR.5.1 MTR.6.1 MTR.7.1



14 Credit Card Compounding

Different Compounding Intervals

Explore how to calculate and compare account balances with interest rates that compound at different intervals.

MA.912.FL.3.2 MA.912.FL.3.4 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.4.1 MTR.6.1 MTR.7.1

Pacing Considerations

Lessons 8–9: These lessons can be omitted because the content will be addressed again in a later course.

Lessons 12–14: If time runs short, these lessons can be consolidated into two by omitting the Warm-Up of Lesson 12, Activity 3 of Lesson 13, and Activity 3 of Lesson 14.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

Copy: Show What You Know

Materials: calculators (*as needed*)

Lesson 2

Copy: Show What You Know, You're Invited to Explore More Sheet (*as needed*)

Materials: scientific calculators (*optional*)

Lesson 3

Copy: Show What You Know

Materials: coloring tools (*as needed*), graph paper (*as needed*), calculators (*optional*)

Lesson 4

Copy: Show What You Know

Materials: scientific calculators

Lesson 5

Copy: Show What You Know

Materials: chips, coins, blocks, or other counters (*as needed*)

Lesson 6

Copy: Show What You Know

Materials: graphing calculators

Lesson 7

Copy: how What You Know, Activity 2 Cards, one set per student, pre-cut

Materials: graphing calculators, coloring tools (*optional*)

Lesson 8

Copy: Show What You Know

Materials: coloring tools (optional), scientific calculators (*as needed*)

Lesson 9

Copy: Show What You Know, Activity 2 Cards, one set per student, pre-cut

Materials: coloring tools (*optional*), scientific calculators (*as needed*)

Lesson 10

Copy: Show What You Know

Materials: *no required materials*

Lesson 11

Copy: Show What You Know, Activity 3 Cards

Materials: *no required materials*

Lesson 12

Copy: Show What You Know, You're Invited to Explore More Sheet (*as needed*)

Materials: graphing calculators

Lesson 13

Copy: Show What You Know

Materials: scientific calculators

Lesson 14

Copy: Show What You Know

Materials: scientific calculators

Lesson 15

Copy: Show What You Know

Materials: *no required materials*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



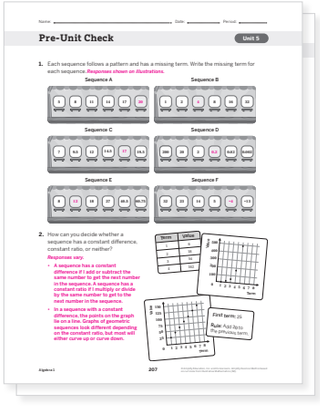
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.AR.3.2, MA.7.AR.3.1, MA.7.NSO.1.1, MA.7.NSO.1.2


Pre-Unit Check

Unit 5

1. Each sequence follows a pattern and has a missing term. Write the missing term for each sequence. *Responses shown on illustrations.*

Sequence A

Sequence B

Sequence C

Sequence D

Sequence E

Sequence F

2. How can you decide whether a sequence has a constant difference, constant ratio, or neither?

Responses vary.

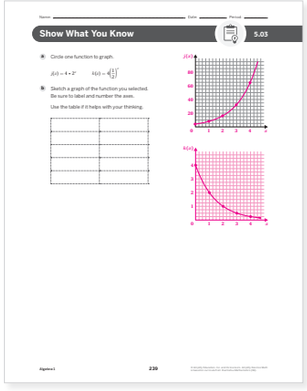
- *A sequence has a constant difference if I add or subtract the same number to get the next number in the sequence. A sequence has a constant ratio if I multiply or divide by the same number to get to the next number in the sequence.*
- *In a sequence with a constant difference, the points on the graph lie on a line. Graphs of geometric sequences look different depending on the constant ratio, but most will either curve up or curve down.*

Algebra 1 207

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed: on Lesson 3 Show What You Know: MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4


Show What You Know

5.03

1. Circle one function to graph.

$f(x) = 4 \cdot 2^x$ $g(x) = 4\left(\frac{1}{2}\right)^x$

2. Sketch a graph of the function you selected. Be sure to label and number the axes.

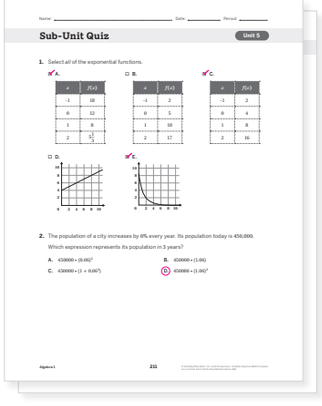
Use the table if it helps with your thinking.

Algebra 1 239

Sub-Unit Quizzes

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.912.AR.1.1, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.F.1.2, MA.912.F.1.8


Sub-Unit Quiz

Unit 5

1. Select all of the exponential functions.

A. | x | f(x) | | -1 | 18 | | 0 | 12 | | 1 | 8 | | 2 | 5.3 |

B. | x | f(x) | | -1 | 2 | | 0 | 5 | | 1 | 10 | | 2 | 17 |

C. | x | f(x) | | -1 | 2 | | 0 | 4 | | 1 | 8 | | 2 | 16 |

D.

E.

2. The population of a city increases by 6% every year. Its population today is 450,000.

Which expression represents its population in 3 years?

A. $450000 + (0.06)^3$

B. $450000 + (1.06)$

C. $450000 + (1 + 0.06^3)$

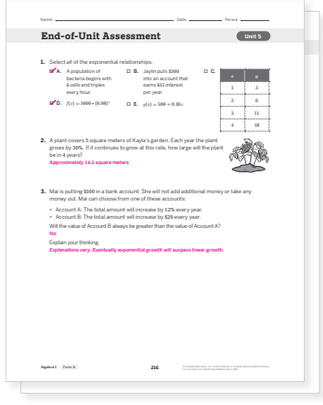
D. $450000 \cdot (1.06)^3$

Algebra 1 231

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.9.12.NSO.1.1, MA.9.12.NSO.1.2, MA.9.12.NSO.1.4, MA.9.12.AR.1.1, MA.9.12.AR.5.4, MA.9.12.F.1.2, MA.9.12.F.1.6, MA.9.12.F.1.8, MA.9.12.F.3.2


End-of-Unit Assessment

Unit 5

1. Select all of the exponential relationships.

A. A population of bacteria begins with 6 cells and triples every hour.

B. Jaylin puts $300 into an account that earns $15 interest per year.

C. | x | y | | 1 | 3 | | 2 | 6 | | 3 | 11 | | 4 | 18 |

D. $f(x) = 1000 \cdot (0.98)^x$

E. $g(x) = 500 + 0.10x$

2. A plant covers 5 square meters of Kayla's garden. Each year the plant grows by 30%. If it continues to grow at this rate, how large will the plant be in 4 years?

Approximately 14.3 square meters

3. Mai is putting $1000 in a bank account. She will not add additional money or take any money out. Mai can choose from one of these accounts:

- Account A: The total amount will increase by 12% every year.
- Account B: The total amount will increase by $20 every year.

Will the value of Account B always be greater than the value of Account A?

No

Explain your thinking.

Explanations vary. Eventually exponential growth will surpass linear growth.

Algebra 1 Form A 216

D Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

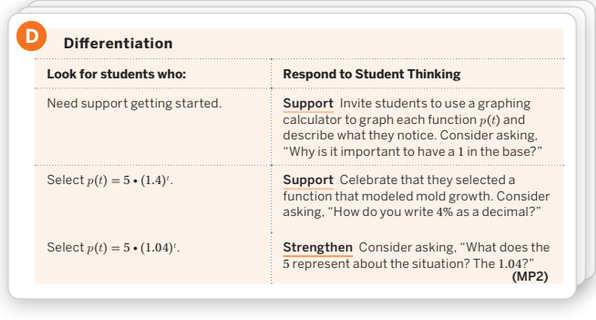
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Compare and contrast linear and exponential functions. • Create equations, tables, and graphs of linear and exponential functions.	**Mini-Lesson** • Grade 6, Unit 6, *(Evaluating Expressions with Exponents)* **Previous Lessons** • Grade 6, Unit 6, Lesson 11 *(Exponent Expressions)* • Grade 7, Unit 6, Lesson 5 *(Combining Exponents)* • Grade 7, Unit 5, Lesson 10 *(Negative Bases)* **Teacher Moves**	**Practice** • Lessons 1–5 **Teacher Moves**	**You're invited to explore more.** • Lesson 2 *(Going Viral)* **DOK3 Practice** • Lesson 3 **Teacher Moves**
Sub-Unit 2 (Lessons 6–7) • Interpret and write exponential functions to model situations that involve percent increase or decrease. • Use exponential functions to solve problems in context.	**Mini-Lesson** • Grade 7, Unit 4, *(Calculating Percentages)* **Previous Lessons** • Grade 6, Unit 3, Lesson 11 (*Cost Breakdown*) • Grade 7, Unit 4, Lesson 2 (*All the Equations*) **Teacher Moves**	**Practice** • Lessons 6–7 **Teacher Moves**	**You're invited to explore more.** • Lesson 6 (*Growing Mold*) **DOK3 Practice** • Lesson 7 **Teacher Moves**
Sub-Unit 3 (Lessons 8–15) • Use properties of exponents to interpret expressions for exponential functions. • Perform operations with radicals.	**Mini-Lessons** • Grade 8, Unit 6, *(Using Lines of Best Fit to Make Predictions)* • Grade 8, Unit 7, *(Multiplying Powers)* **Previous Lessons** • Grade 7, Unit 5, Lesson 7 (*Power Pairs*) • Grade 7, Unit 5, Lesson 8 (*Rewriting Powers*) **Teacher Moves**	**Repeated Challenges** • Lesson 12 (*Bank Accounts*) • Lesson 14 (*Credit Card Compounding*) **Practice** • Lessons 8–15 **Teacher Moves**	**You're invited to explore more.** • Lesson 12 (*Bank Accounts*) • Lesson 13 (*Payday Loan*) **DOK3 Practice** • Lessons 12–13 **Teacher Moves**

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 6**, where students are writing exponential functions to represent situations involving percent increase.



D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to use a graphing calculator to graph each function $p(t)$ and describe what they notice. Consider asking, "Why is it important to have a 1 in the base?"
Select $p(t) = 5 \cdot (1.4)^t$.	**Support** Celebrate that they selected a function that modeled mold growth. Consider asking, "How do you write 4% as a decimal?"
Select $p(t) = 5 \cdot (1.04)^t$.	**Strengthen** Consider asking, "What does the 5 represent about the situation? The 1.04?" **(MP2)**

Accessibility

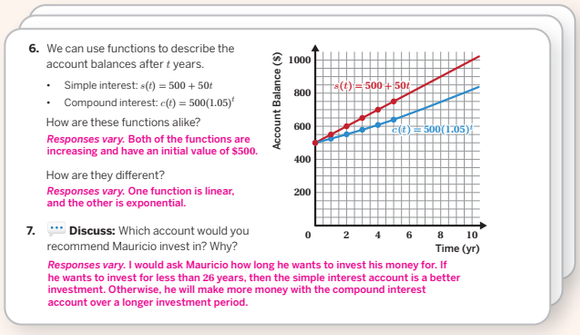
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Recruiting Interest . . .

In **Lesson 12:** This lesson highlights the utility and relevance of the math related to simple and compound interest and linear and exponential functions through meaningful activities.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Fine Motor Skills . . .

Here are two suggestions in this unit for supporting students by varying task demands.

Lesson 1, Warm-Up

A **Accessibility: Fine Motor Skills** Consider offering students the option to use a scribe or to verbally share their response instead of writing it.

Lesson 13, Activity 3

A **Accessibility: Fine Motor Skills** Consider varying the task demands by providing students with a pre- or partially-completed table.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
exponential function **growth factor** **linear function**	1
exponential decay **exponential growth**	3
radical **rational exponent**	8
compound interest **simple interest**	12

Review Vocabulary

base
constant difference
constant ratio
cube root

Review Vocabulary

domain
exponent
factor
initial value
interval
model
percent decrease
percent increase
range
rate of change
reciprocal
slope
square root
transformation
translation
y-intercept

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	1-2, 4, 6-9, 12, 14
MLR3: Critique, Correct, Clarify	7, 14
MLR7: Compare and Connect	4, 9-11

Sub-Unit Language Goals

- Describe exponential functions by their key features and statements written in function notation.
- Critique the fit of a model.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 4**, where students are exploring a situation that models the decay of coral reef populations.

ML/EL **Multilingual/English Language Learners**
Create a list of words about the context that students might not know (e.g. *coral*, *reef*, *marine biologist*, *volume*), along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Comparing Exponentials and Quadratics

In this unit, students observed that exponential functions will always exceed linear functions. **In Unit 6**, students will observe that exponential functions always exceed quadratic functions as well.

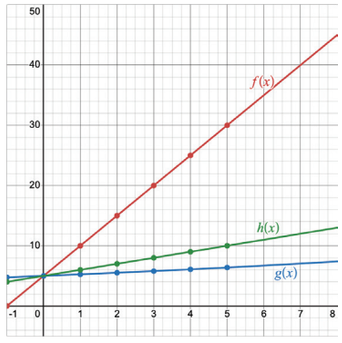
FL B.E.S.T. Math Benchmarks:
MA.912.F.1.1

Example:

How do exponential functions compare to linear and quadratic functions?

Functions	At $x = 5$	At $x = 150$
$f(x) = 5 + 5x$	$f(5) = 30$	$f(20) = 755$
$g(x) = 5(1.05)^x$	$g(5) = 6.38$	$g(5) = 7539.9$
$h(x) = 5 + \frac{1}{5}x^2$	$h(5) = 10$	$h(5) = 4505$

Eventually, exponential functions will exceed linear and quadratic functions.



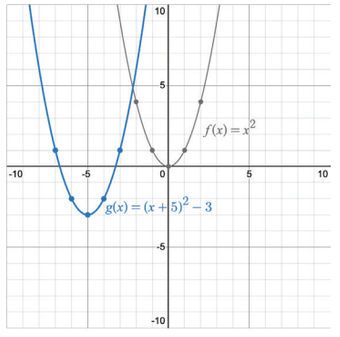
Transformations of Quadratics

In this unit, students translate exponential functions by writing equations in the form $f(x) = a \cdot b^{(x-h)} + k$. **In Unit 6**, students will translate quadratic functions by writing them in vertex form, $f(x) = (x - h)^2 + k$.

FL B.E.S.T. Math Benchmarks:
MA.912.F.2.1

Example:

The function $g(x) = (x + 5)^2 - 3$ is written in vertex form and translates the function $f(x) = x^2$ five units to the left and three units down.



Exponential and Logarithmic Functions

In this unit, students are introduced to exponential relationships. **In Algebra 2**, students will be introduced to the inverse relationship between exponential and logarithmic functions.

FL B.E.S.T. Math Benchmarks:
MA.912.F.1.6

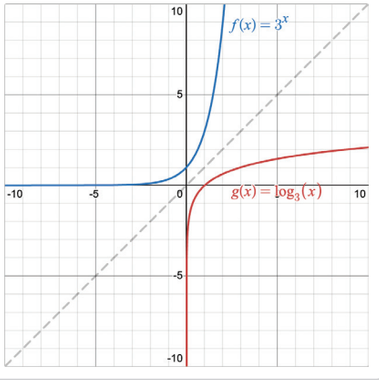
Example:

$f(x) = 3^x$ and $g(x) = \log_3 x$ are inverse functions. This means:

- Their graphs reflect over the line $y = x$.
- Every (x, y) ordered pair on $f(x)$ is a (y, x) ordered pair on $g(x)$.
- When composed, their operations undo one another, equaling x.

$f(g(x)) = 3^{\log_3 x}$ $\quad$ $g(f(x)) = \log_3(3^x)$

$f(g(x)) = x$ $\quad$ $g(f(x)) = x$

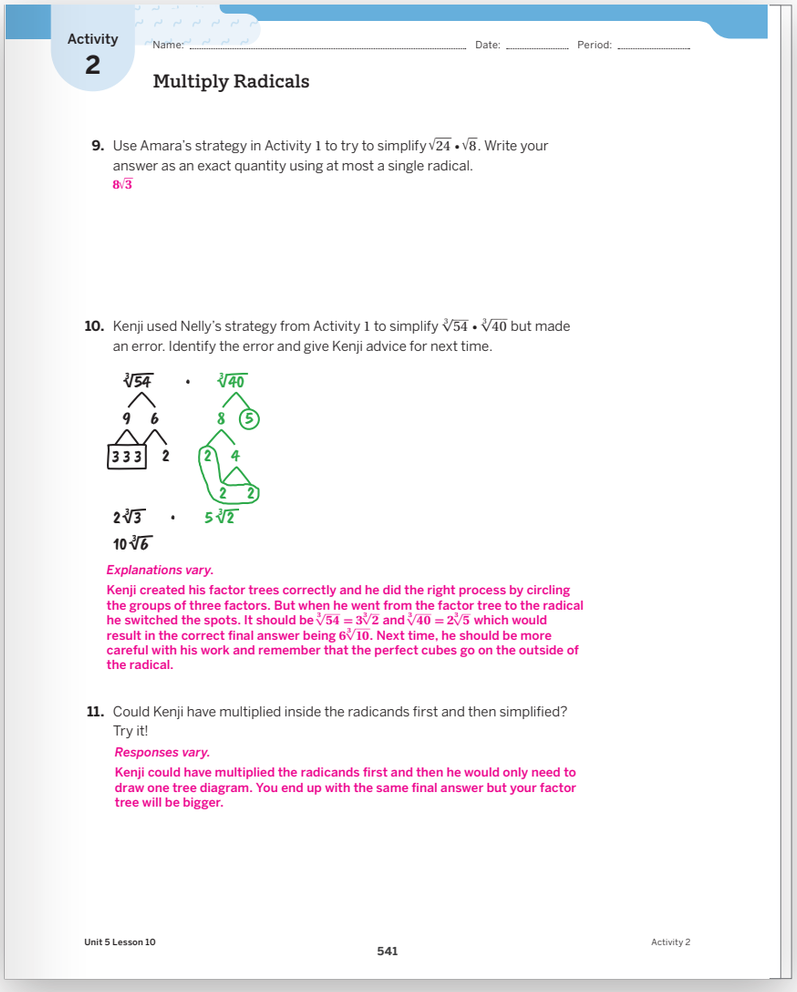


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Connecting Representations

Lesson 10, Activity 2



Activity 2

Name: Date: Period:

Multiply Radicals

9. Use Amara's strategy in Activity 1 to try to simplify $\sqrt{24} \cdot \sqrt{8}$. Write your answer as an exact quantity using at most a single radical.

$8\sqrt{3}$

10. Kenji used Nelly's strategy from Activity 1 to simplify $\sqrt[3]{54} \cdot \sqrt[3]{40}$ but made an error. Identify the error and give Kenji advice for next time.

Explanations vary.

Kenji created his factor trees correctly and he did the right process by circling the groups of three factors. But when he went from the factor tree to the radical he switched the spots. It should be $\sqrt[3]{54} = 3\sqrt[3]{2}$ and $\sqrt[3]{40} = 2\sqrt[3]{5}$ which would result in the correct final answer being $6\sqrt[3]{10}$. Next time, he should be more careful with his work and remember that the perfect cubes go on the outside of the radical.

11. Could Kenji have multiplied inside the radicands first and then simplified? Try it!

Responses vary.

Kenji could have multiplied the radicands first and then he would only need to draw one tree diagram. You end up with the same final answer but your factor tree will be bigger.

Unit 5 Lesson 10 541 Activity 2

Context

In this activity, students develop adaptive reasoning as they adapt and reflect on their strategies to perform operations with radicals.

Try This

Put on your student hat and complete Problems 9-11 in Activity 2.

Points to Ponder

- What do students need to understand about each strategy to support their development of strategic competence?
- What do students need to understand about operations with radicals to support their development of adaptive reasoning?
- How will using adaptive reasoning support student understanding of operations with radicals in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Determine missing terms in sequences with a constant difference or a constant ratio	Lesson 1	1	MA.912.AR.5.3 MTR.3.1
2	Describe how to determine if a sequence has a constant difference, constant ratio, or neither	Lesson 1	2	MA.912.F.1.8 MTR.3.1
3	Match relationships in multiple representations	Lesson 1	2	MA.912.F.1.1 MTR.2.1
4a	Interpret expressions in context	Lesson 1	2	MA.912.AR.1.1 MTR.2.1
4b	Make predictions about relationships in context	Lesson 1	2	MA.912.F.1.2 MTR.3.1
5–6	Write equivalent exponential expressions using exponent properties	Lesson 4	2	MA.7.NSO.1.1, MA.7.NSO.1.2 MTR.5.1
7	Calculate percentages	Lesson 6	1	MA.6.AR.3.2 MTR.3.1
8	Calculate percent increase and decrease	Lesson 6	2	MA.7.AR.3.1 MTR.3.1
9	Represent percent increase and decrease with expressions	Lesson 6	2	MA.7.AR.3.1 MTR.2.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 5

1. Each sequence follows a pattern and has a missing term. Write the missing term for each sequence. *Responses shown on illustrations.*

Sequence A



Sequence B



Sequence C



Sequence D



Sequence E



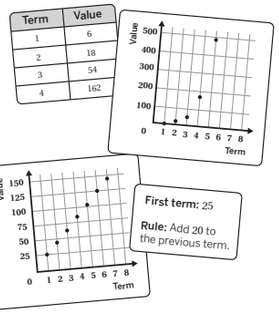
Sequence F



2. How can you decide whether a sequence has a constant difference, constant ratio, or neither?

Responses vary.

- A sequence has a constant difference if I add or subtract the same number to get the next number in the sequence. A sequence has a constant ratio if I multiply or divide by the same number to get to the next number in the sequence.
- In a sequence with a constant difference, the points on the graph lie on a line. Graphs of geometric sequences look different depending on the constant ratio, but most will either curve up or curve down.



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Name: Date: Period:

Pre-Unit Check (continued)

Unit 5

3. Match each representation with a description.

Card A

Term	Value
0	80
1	240
2	720
3	2160
4	6480

Card B

Term	Value
0	80
1	83
2	86
3	89
4	92

Card C

$y = 80 \cdot 3^x$

Card D

$y = 80 + 3x$

Card E

$y = 80 + 3 + 3 + 3 + \ldots$

Card F

$y = 80 \cdot 3 \cdot 3 \cdot 3 \cdot \ldots$

A pond has 80 fish. The number of fish multiplies by 3 each year.	A tank has 80 gallons of water and fills by 3 gallons each minute.
Card A Card C Card F	Card B Card D Card E

4. The population of frogs in a lake is represented by the explicit expression $8 \cdot 3^t$, where t is the number of years since they were first counted.



a Explain what the 8 and the 3 represent in this situation. *Responses vary.*

The 8 represents. . . **that there were 8 frogs when they were first counted.**

The 3 represents. . . **that the number of frogs is multiplying by 3 each year.**

b How many frogs are predicted 2 years after they were first counted?

72 frogs

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Prior Knowledge	Problem(s)	Respond to Student Thinking
Sequences with constant differences and ratios **(Preparation for Lessons 1–5)**	1–2	**Support** • **Teacher Move:** Emphasize these skills during Lesson 1, Activity 1.
Represent relationships **(Preparation for Lessons 1–4)**	3–4	**Support** • **Teacher Move:** Emphasize these skills in Lesson 1, Activity 1. Invite students to share multiple interpretations and strategies in Activity 1 and checking their understanding during Activity 2.
Write equivalent exponential expressions using exponent properties **(Preparation for Lessons 4 and 13)**	5	**Support** • **Teacher Move:** Emphasize strategies for rewriting expressions with integer exponents in the Warm-Up of Lesson 4. • **Mini-Lesson:** Grade 6, Unit 6 *(Evaluating Expressions with Exponents)* • **Lesson:** Grade 6, Unit 6, Lesson 10 *(Powers)* • **Lesson:** Grade 7, Unit 5, Lesson 10 *(Negative Bases)*
	6	**Support** • **Teacher Move:** Emphasize strategies for rewriting expressions using exponent properties in the Warm-Up of Lesson 13. • **Mini-Lesson:** Grade 7, Unit 5 *(Multiplying Powers)* • **Lesson:** Grade 7, Unit 5, Lesson 6 *(Combining Exponents)* • **Lesson:** Grade 7, Unit 5, Lesson 7 *(Power Pairs)* • **Lesson:** Grade 7, Unit 5, Lesson 8 *(Rewriting Powers)*
Calculate percent increase and decrease **(Preparation for Lessons 6–7)**	7–9	**Support** • **Teacher Move:** Emphasize strategies for calculating percentages in the Warm-Up of Lesson 6 during the Number Talk. • **Mini-Lesson:** Grade 7, Unit 4 *(Calculating Percentages)* • **Lesson:** Grade 6, Unit 3, Lesson 11 *(Cost Breakdown)* • **Lesson:** Grade 7, Unit 4, Lesson 2 *(All the Equations)*

Sub-Unit 1

Comparing Linear and Exponential Functions

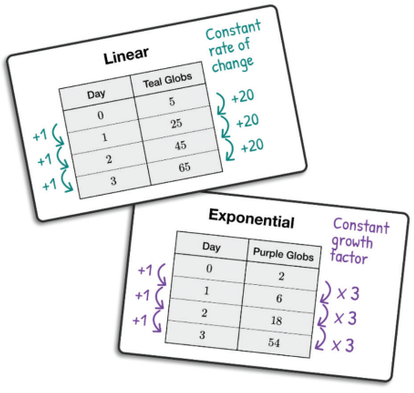
Sub-Unit 1 Goals

- **Goal:** Compare and contrast linear and exponential functions.
- **Goal:** Create equations, tables, and graphs of linear and exponential functions.
- **Language Goal:** Describe exponential functions by their key features and statements written in function notation.

In this sub-unit, students distinguish between situations that can be modeled by linear and exponential functions. They compare and contrast their key features using different representations.



Math That Matters Most

Concept or Skill	Example
Distinguish between linear and exponential relationships.	
Evaluate exponential functions and interpret the meaning in context.	The function $v(t) = 320 \cdot \left(\frac{4}{5}\right)^x$ represents the volume of the coral reef t years after 2010. Determine the value of $v(2)$ and explain what the answer tells you about the volume of the coral reef. $v(2) = 320 \cdot \left(\frac{4}{5}\right)^2$ $v(2) = 204.8$ **In 2012, the volume of the coral reef decayed to 204.8 cubic meters.**

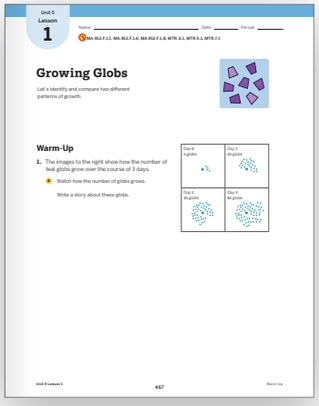
Concept or Skill	Example
Create functions, tables, and graphs of linear and exponential functions.	**Linear** x | y 0 | 2 1 | 6 2 | 10 3 | 14 $y = 4x + 2$ **Exponential** x | y 0 | 160 1 | 40 2 | 10 3 | 2.5 $160 \cdot 0.25^x$
Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.	For $f(x) = 2x + 5$, how can you know that $f(5009) - f(5008)$ must be 2? **For any two consecutive values of x, f(x) increases by 2.** For g(x) = 3x, how can you know that $\frac{g(5009)}{g(5008)}$ must be 3? **For any two consecutive values of x, g(x) multiplies by 3.**

Unit 5
Lesson
1

Growing Globs

Patterns of Growth

Let's identify and compare two different patterns of growth.


Unit 5
Lesson
1
Name _____ Date _____ Period _____
MA.912.F.1.1, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1
Growing Globs
Let's identify and compare two different patterns of growth.
Warm-Up
1. The images to the right show how the number of teal globs grow over the course of 3 days.
Watch how the number of globs grows.
Write a story about these globs.
Day 0
6 globs
Day 1
18 globs
Day 2
54 globs
Day 3
162 globs
467
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Identify functions that grow by constant differences as *linear functions* and functions that grow by constant ratios as *exponential functions*.
2. **Goal:** Compare linear and exponential functions by their key features.
3. **Language Goal:** Explain that quantities that grow exponentially eventually exceed quantities that grow linearly. **(Speaking and Listening)**

Students explore the differences between **linear functions** that grow by a constant *rate of change* and **exponential functions** that grow by a constant **growth factor**. They compare these relationships in tables and graphs and observe that functions growing exponentially will always exceed those growing linearly.

Coherence

Prior Learning

In Lesson 1, students matched equations to situations that have a constant ratio and interpreted their parameters in context. In Unit 1, students compared sequences that changed by constant differences and constant ratios.

Future Learning

In Lesson 2, students will compare graphs of linear and exponential functions in context and write exponential functions to represent graphs and situations.

Connections

- Students identify and compare two different patterns of growth using a visual representation of a linear function and exponential function. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.6
Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

MA.912.F.1.8
Determine whether a linear, quadratic or exponential function best models a given real-world situation.

Also Addressing: **MA.912.AR.1.1**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.912.F.1.1 **MA.7.AR.2.2** **MA.8.F.1.2**	**MA.912.AR.5.4**

Vocabulary

New Vocabulary

exponential function
growth factor
linear function

Review Vocabulary

constant difference
constant ratio
rate of change
slope

Lesson at a Glance

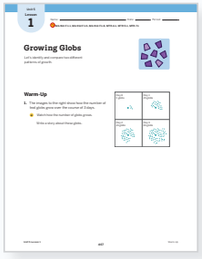
~ 45 min

MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8

Warm-Up

Pairs | 5 min

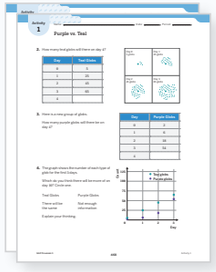
Students use the **Tell a Story** routine to make sense of one of the patterns they will be exploring in the lesson. **(MTR.4.1)**



Activity 1

Pairs | 15 min

Students make predictions about two patterns and compare their growth.
MLR2: Collect and Display



Activity 2

Pairs | 15 min

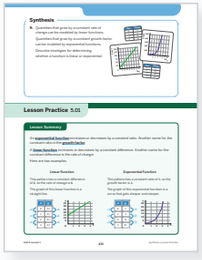
Students use the **Decide and Defend** routine to justify that globs that grow by a constant growth rate will eventually exceed globs that grow by a constant *rate of change*.
Routine: Notice and Wonder **(MTR.7.1)**



Synthesis

Whole Class | 5 min

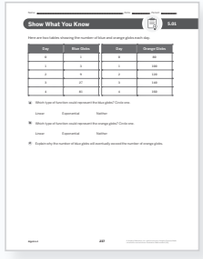
Students synthesize their understanding of why quantities that grow by a constant **growth factor** will always exceed those that grow by a constant rate of change. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by making predictions about two different patterns.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use the Tell a Story routine to make sense of one of the patterns they will be exploring in the lesson. **(MTR.4.1)**

1 Launch

Invite students to observe as you display the globs on Days 0–3.

Consider asking, "What do these globs remind you of?"

Use the Tell a Story routine to help students make sense of the pattern. Invite students to write a story about the pattern using their own experiences and creativity. **(MTR.4.1)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, with words such as: *first*, *in the beginning*, *then*, *finally*, *increase*, *decrease*, etc. **(Writing and Speaking)**

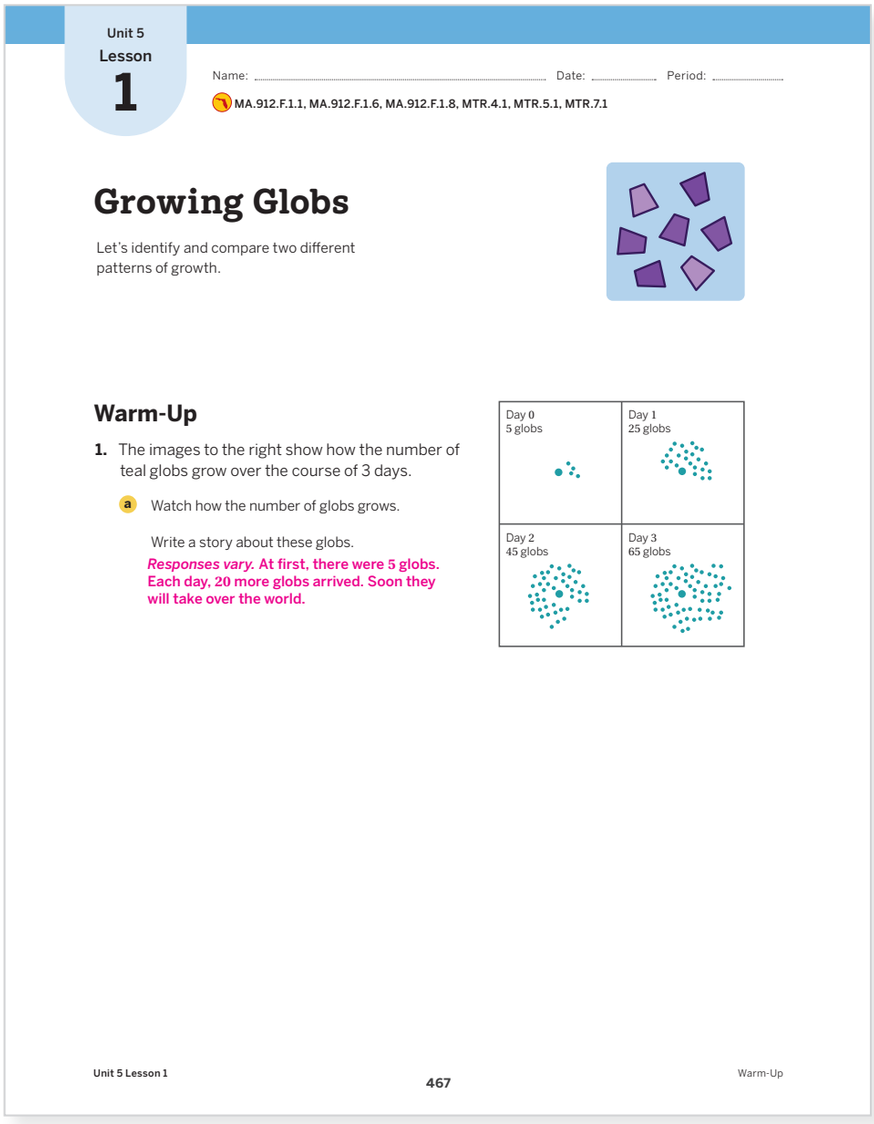
A **Accessibility: Fine Motor Skills** Consider offering students the option to use a scribe or to verbally share their response instead of writing it.

2 Connect

Listen for and share a few different stories, encouraging students to ask questions about each other's stories.

Consider asking:

- "Where can we see the starting number of globs in the story?"
- "Where can we see how the globs are growing in the story?"
- "What are some ways to describe how the number of globs are growing?" Their growth could be described with a slope, a constant rate of change, or a constant difference of 20.
- "What parts of each of our stories are alike? How are they different?"
- "What questions do you have about each other's stories?"



Unit 5
Lesson
1

Name: ..... Date: ..... Period: .....

MA.912.F.1.1, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1

Growing Globs

Let's identify and compare two different patterns of growth.

Warm-Up

1. The images to the right show how the number of teal globs grow over the course of 3 days.
   a. Watch how the number of globs grows.

   Write a story about these globs.

   Responses vary. At first, there were 5 globs. Each day, 20 more globs arrived. Soon they will take over the world.

Day 0
5 globs

Day 1
25 globs

Day 2
45 globs

Day 3
65 globs

Unit 5 Lesson 1 467 Warm-Up

Pairs | 15 min

Activity 1 Purple vs. Teal

Purpose: Students make predictions about two patterns and compare their growth.

Short on time? Omit the brief discussion.

1 Launch

To reveal connections, consider asking, "What does the table row with 1 and 25 tell us about the globs?" **(MTR.5.1)**

Encourage students to estimate the number of globs first.

A **Accessibility: Memory and Attention** Invite students to record patterns they notice and use a calculator or paper to support them with performing calculations throughout this lesson.

2 Monitor

To support getting started, consider asking, "Will there be more than 100 globs or fewer?"

Listen for and celebrate how students describe the patterns of growth. Students may use words like *slope, rate of change, constant difference, constant ratio*, etc.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., I think there will be more _____ globs on day 10 because _____.). **(Reading and Writing)**

Select and sequence a variety of responses to share.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How are the teal globs changing? The purple globs?"
Want to explore these patterns further.	**Strengthen** Encourage students to read their peers' responses and revise their response to make it stronger. **Stretch** Consider asking, "Which will have more on day 100?"

Pause briefly to emphasize the range of student responses. It's okay — even desirable — to lack consensus at this stage.

Invite students to make arguments and justify their reasoning for each response. **(MTR.4.1)**

Activity 1 continued >

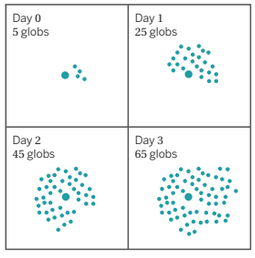
Activity 1

Name: _____ Date: _____ Period: _____

Purple vs. Teal

2. How many teal globs will there on day 4?

Day	Teal Globs
0	5
1	25
2	45
3	65
4	85



3. Here is a new group of globs.

How many purple globs will there be on day 4?

Day	Purple Globs
0	2
1	6
2	18
3	54
4	162

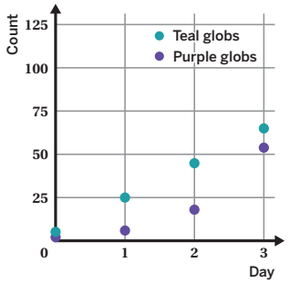
4. The graph shows the number of each type of glob for the first 3 days.

Which do you think there will be more of on day 10? Circle one.

Teal Globs Purple Globs

There will be the same Not enough information

Explain your thinking.



Responses and explanations vary.

- Purple globs. They are growing faster. They are already almost caught up to the teal globs by day 3.
- Not enough information. I don't know if the globs will keep growing in the same way every day. What if the purple globs stop growing after day 3?

Unit 5 Lesson 1 468 Activity 1

Pairs | 15 min

Activity 1 Purple vs. Teal (continued)

Purpose: Students make predictions about two patterns and compare their growth.

Short on time? Omit the brief discussion.

2 Monitor

Invite students to observe as you display the globs on Days 1–10.

D **Differentiation: (Support)** As students explore past day 8, they may think that the teal globs eventually stop growing or that their growth slows because of the change in scale. Consider asking, "How does the diagram show that the teal globs are still growing? What about the graph's scale might make it seem like they aren't? What's that mean about the purple globs?"

Consider asking pairs of students, "What was surprising?"

Listen for a variety of ways that students describe rate of change and **growth factor** to surface during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Describe the change in the number of globs but not the number of days.	**Strengthen** Celebrate their descriptions and invite students to strengthen their descriptions by also discussing how the number of days is changing.
Make connections to terms from previous units like: *slope*, *constant difference*, and *constant ratio*.	**Strengthen** Consider asking, "Which types of functions have constant differences? Constant ratios?" Invite them to share during the Connect.
Would like to explore linear and exponential functions further.	**Stretch** Invite students to create linear and exponential functions showing the rate of change and growth factor.

3 Connect

MLR **MLR2: Collect and Display** Use this routine to create class definitions of *linear functions* and *exponential functions* using students' language and review vocabulary. Display the examples for students to reference and add to for the rest of the unit.

To reveal the Key Takeaway, consider asking:
- "What is another word for *constant rate of change*? *Growth factor*?"
- "How can you identify a rate of change? Growth factor?"
- "How can you tell if a function is linear or exponential?"

Activity 1

Name: _____ Date: _____ Period: _____

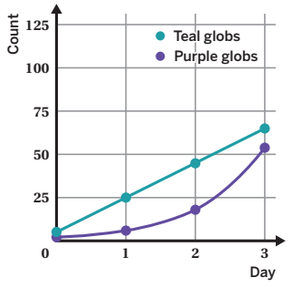
Purple vs. Teal (continued)

5. The graph below shows how the number of teal and purple globs grow over the course of three days.

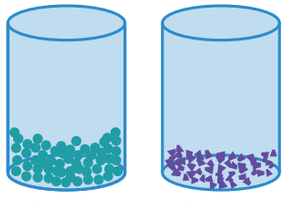
a. What do you notice? What do you wonder?

Responses vary. I notice the teal globs are growing at a constant rate, but the purple globs are not. I wonder when there will be more purple globs than teal globs.

b. Share your observations with a partner.



Day 3



Count: 65 Count: 54

6. Here are tables for the teal and purple growing globs.

- Teal globs grow are modeled by a **linear function** and have a constant *rate of change*.
- Purple globs are modeled by an **exponential function** and have a constant **growth factor**.

Linear

Day	Teal Globs	
0	5	
1	25	+20
2	45	+20
3	65	+20

Constant rate of change

Exponential

Day	Purple Globs	
0	2	
1	6	×3
2	18	×3
3	54	×3

Constant growth factor

Discuss:
- How are *rate of change* and *growth factor* alike?
- How are they different?

Responses vary.
- Rate of change and growth factor are alike because they both tell you how the y-values of a function are changing for a constant change in x-values.
- They are different because rate of change is adding to each previous value of a function and growth factor is multiplying each previous value in a function.

Unit 5 Lesson 1 469 Activity 1

Key Takeaway: **Linear functions** have a constant *rate of change*. **Exponential functions** have a constant **growth factor**.

Pairs | 15 min

Activity 2 Comparing Growth

Purpose: Students use the **Decide and Defend** routine to justify that globs that grow by a constant growth rate will eventually exceed globs that grow by a constant *rate of change*. **(MTR.4.1)**

Short on time? Consider inviting students to share what they notice and wonder without a whole class discussion.

1 Launch

Invite students to nominate different values and observe as you graph the glob growth.

Encourage students to predict what the growth will look like. **(MTR.7.1)**

Use the Notice and Wonder routine to promote curiosity and help students compare linear and exponential functions.

2 Monitor

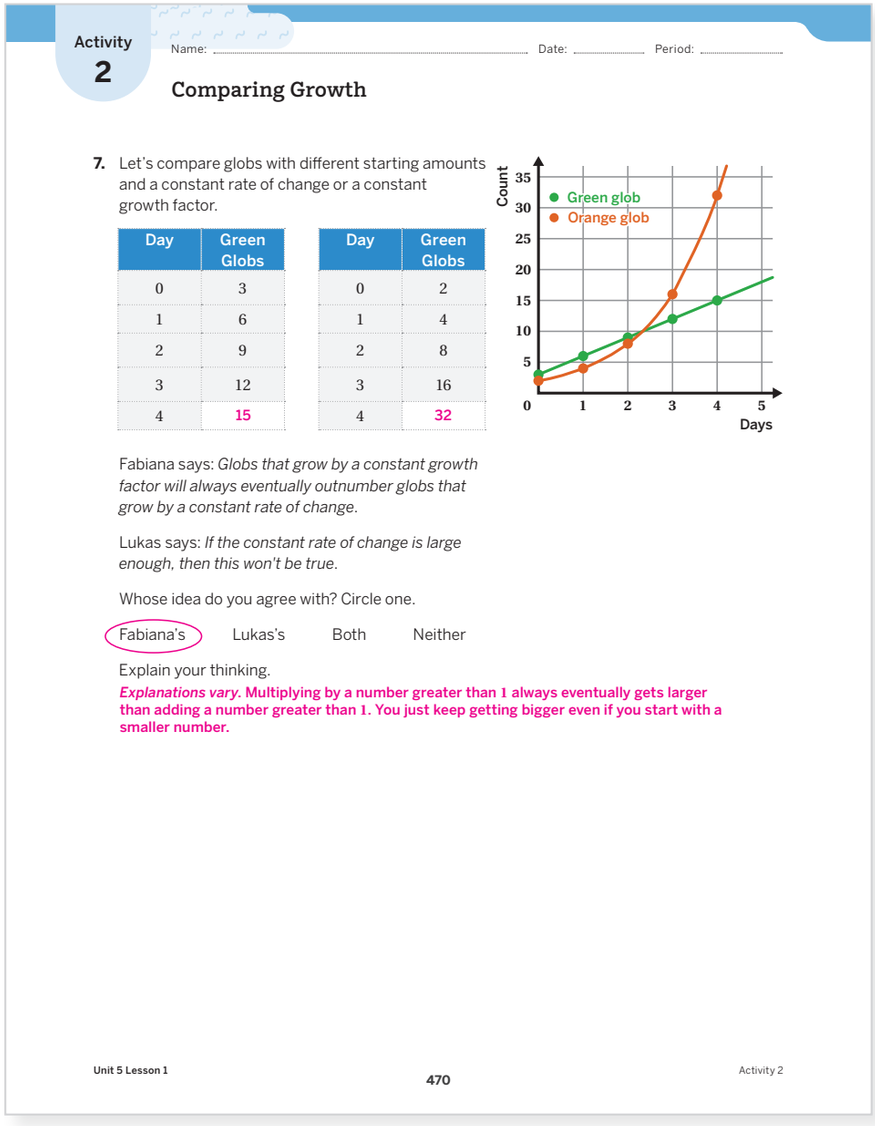
D **Differentiation: (Stretch)** Encourage students to try to create a situation where:
- The number of teal globs is always greater than the number of purple.
- The teal globs never grow or the purple globs never grow.
- The amount of globs is exactly the same on day 4.

ELL **English Language Learners** Invite students to read each claim aloud and paraphrase what it means. **(Reading)**

Select and sequence a variety of student response.

Encourage students to record their thinking about the patterns in the tables and work together to reach an agreement about how to sort the cards.

Activity 2 continued >



Activity 2

Name: Date: Period:

Comparing Growth

7. Let's compare globs with different starting amounts and a constant rate of change or a constant growth factor.

Day	Green Globs
0	3
1	6
2	9
3	12
4	15

Day	Green Globs
0	2
1	4
2	8
3	16
4	32

Fabiana says: *Globs that grow by a constant growth factor will always eventually outnumber globs that grow by a constant rate of change.*

Lukas says: *If the constant rate of change is large enough, then this won't be true.*

Whose idea do you agree with? Circle one.

Fabiana's Lukas's Both Neither

Explain your thinking.

Explanations vary. **Multiplying by a number greater than 1 always eventually gets larger than adding a number greater than 1. You just keep getting bigger even if you start with a smaller number.**

Unit 5 Lesson 1 470 Activity 2

Key Takeaway: Quantities that grow exponentially eventually exceed quantities that grow linearly.

Pairs | 15 min

Activity 2 Comparing Growth (continued)

Purpose: Students use the **Decide and Defend** routine to justify that globs that grow by a constant growth rate will eventually exceed globs that grow by a constant *rate of change*. **(MTR.4.1)**

Short on time? Consider inviting students to share what they notice and wonder without a whole class discussion.

3 Connect

Display the distribution of responses.

Use the Decide and Defend routine to support students in making arguments and critiquing reasoning. Invite students to go to different locations of the classroom based on their choice. As students discuss their reasoning, encourage students to move if they change their mind. **(MTR.4.1)**

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**



Activity 2

Name: Date: Period:

Comparing Growth (continued)

8. Group these cards by their function type.

Card A

x	y
0	2
1	4
2	6
3	8
4	10

Card B

x	y
0	0
1	1
2	4
3	9
4	16

Card C

x	y
0	1
1	2
2	4
3	8
4	16

Card D

x	y
0	0
1	4
2	8
3	12
4	16

Linear	Exponential	Neither

Unit 5 Lesson 1 471 Activity 2

Whole Class | 5 min

Synthesis

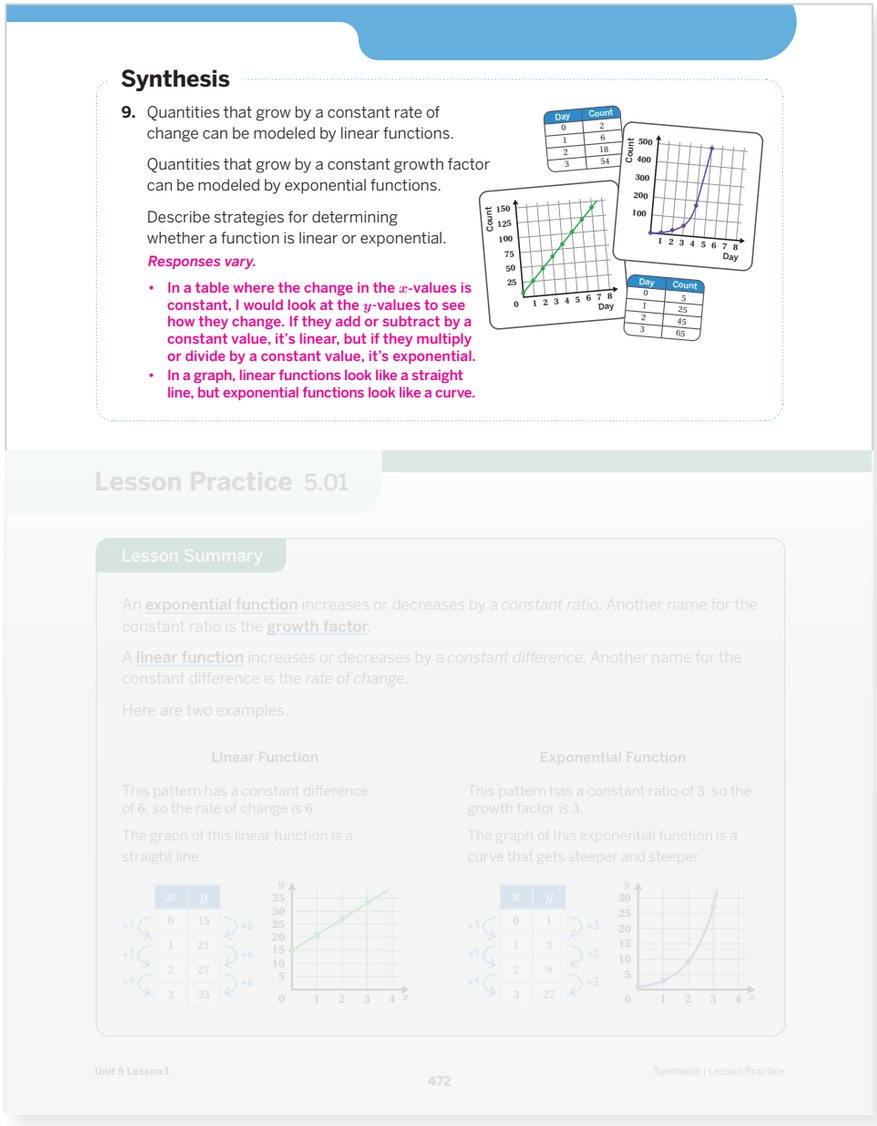
Purpose: Students synthesize their understanding of why quantities that grow by a constant **growth factor** will always exceed those that grow by a constant rate of change. **(MTR.4.1)**

Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:
- The outputs of exponential functions change by multiplication (or division).
- The outputs of linear functions change by addition (or subtraction).
- Graphs of linear functions look like straight lines, while exponential functions curve as they grow.

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas about linear and exponential functions. Add any new examples or student language to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Quantities that grow by a constant rate of change can be modeled by linear functions.

Quantities that grow by a constant growth factor can be modeled by exponential functions.

Describe strategies for determining whether a function is linear or exponential.

Responses vary.

- In a table where the change in the x-values is constant, I would look at the y-values to see how they change. If they add or subtract by a constant value, it's linear, but if they multiply or divide by a constant value, it's exponential.
- In a graph, linear functions look like a straight line, but exponential functions look like a curve.

Lesson Practice 5.01

Lesson Summary

An **exponential function** increases or decreases by a *constant ratio*. Another name for the constant ratio is the **growth factor**.

A **linear function** increases or decreases by a *constant difference*. Another name for the constant difference is the *rate of change*.

Here are two examples.

Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

The graph of this linear function is a straight line.

Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.

Unit 5 Lesson 1 472 Synthesis | Lesson Practice

Lesson Takeaway: **Linear functions** have a constant *rate of change*. **Exponential functions** have a constant **growth factor**. Functions that grow exponentially (constant growth factor) will eventually exceed those that grow linearly (constant rate of change).

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by making predictions about two different patterns.

Today's Goals

1. **Goal:** Identify functions that grow by constant differences as *linear functions* and functions that grow by constant ratios as *exponential functions*.
2. **Goal:** Compare linear and exponential functions by their key features.
3. **Language Goal:** Explain that quantities that grow exponentially eventually exceed quantities that grow linearly. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.01

Here are two tables showing the number of blue and orange globs each day.

Day	Blue Globs
0	1
1	3
2	9
3	27
4	81

Day	Orange Globs
0	80
1	100
2	120
3	140
4	160

a. Which type of function could represent the blue globs? Circle one.

Linear (Exponential) Neither

b. Which type of function could represent the orange globs? Circle one.

(Linear) Exponential Neither

c. Explain why the number of blue globs will eventually exceed the number of orange globs.

Explanations vary. Exponential functions grow by multiplying the previous value by a constant value. Linear functions grow by adding a constant value to the previous value. Multiplying by a number greater than 1 will always eventually get larger than adding a

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding identifying linear and exponential functions:

» **Respond to Student Thinking:**

- Use the ***Identifying Linear and Exponential Functions Using Tables*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to write their own tables or scenarios, trading them with a partner, and determining if they are linear, exponential, or neither.

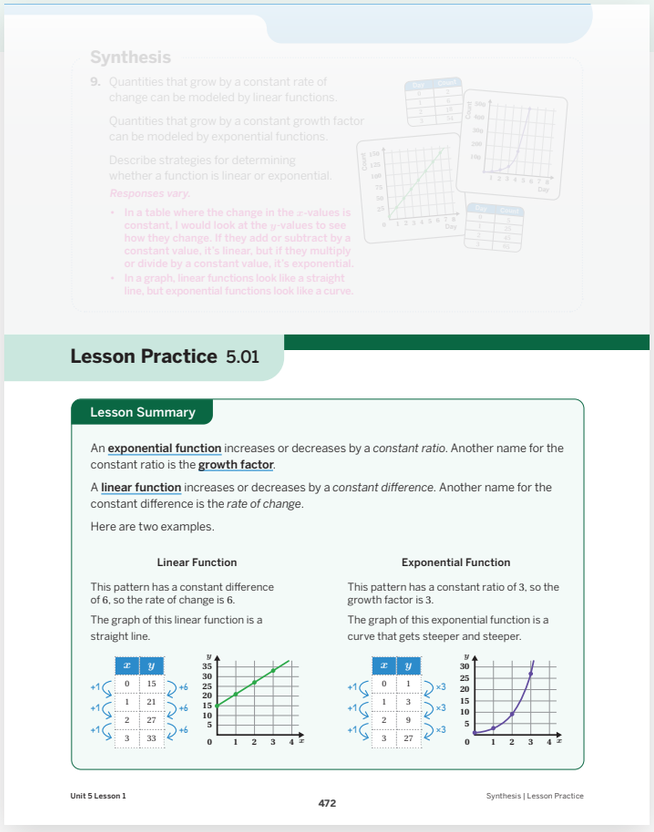
Professional Learning

Reflect on the language students used to describe the different types of relationships in this lesson. Did students use language from Unit 1 like *arithmetic, geometric, constant difference*, or *constant ratio*? Did students have their own unique and creative ways of describing **linear** and **exponential functions**? How can you continue to honor and build connections between student language, vocabulary from previous units, and new vocabulary?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

9. Quantities that grow by a constant rate of change can be modeled by linear functions.

Quantities that grow by a constant growth factor can be modeled by exponential functions.

Describe strategies for determining whether a function is linear or exponential.

Responses vary.

- In a table where the change in the x-values is constant, I would look at the y-values to see how they change. If they add or subtract by a constant value, it's linear, but if they multiply or divide by a constant value, it's exponential.
- In a graph, linear functions look like a straight line, but exponential functions look like a curve.

Lesson Practice 5.01

Lesson Summary

An **exponential function** increases or decreases by a *constant ratio*. Another name for the constant ratio is the **growth factor**.

A **linear function** increases or decreases by a *constant difference*. Another name for the constant difference is the *rate of change*.

Here are two examples.

Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

The graph of this linear function is a straight line.

x	y
0	15
1	21
2	27
3	33

Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.

x	y
0	1
1	3
2	9
3	27

Lesson Practice 5.01

Name: _____ Date: _____ Period: _____

Problems 1–3: These tables show the number of red and yellow globs each day.

Day	0	1	2	3	4
Red Globs	50	70	90	110	130

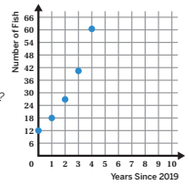
Day	0	1	2	3	4
Yellow Globs	5	10	20	40	80

1. How many of each type of glob will there be on day 4?

2. Will there be more red or yellow globs on day 10? Show or explain your thinking.
   Yellow globs. *Explanations vary.* There will be more yellow globs on day 10. Even though there are fewer yellow globs in the beginning, they are growing faster.

3. Which group of globs changes by a constant growth factor? Show or explain how you know.
   Yellow globs. *Explanations vary.* The number of yellow globs is multiplied by 2 each day. $10 \div 5 = 2$ and $40 \div 20 = 2$, which shows a constant ratio. The red globs grow by adding 20 every day, which represents a constant difference.

Problems 4–5: This graph shows the number of fish in a pond from 2019–2023.

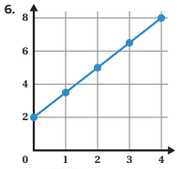


4. How many fish are in the pond in 2019?
   12 fish

5. Does the number of fish grow by a *constant difference*? Show or explain how you know.
   No. *Explanations vary.* A constant difference would mean that the same number of fish were added every year. There were 6 fish added the first year and more than 6 fish added the second year, so there is no constant difference.

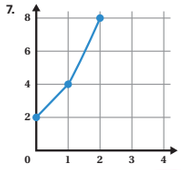
Problems 6–7: Determine whether each graph shows a constant rate of change or a constant growth rate. Circle your choice.

6.



(Constant rate of change) Constant growth rate

7.



Constant rate of change (Constant growth rate)

Lesson Practice 5.01

Name: _____ Date: _____ Period: _____

Test Practice

8. Determine whether each table shows a linear or exponential function.

Table A

x	y
0	4
1	8
2	16
3	32

Table B

x	y
0	4
1	8
2	12
3	16

A. Both represent exponential functions.

B. Both represent linear functions.

(C.) Table A is exponential and Table B is linear.

D. Table A is linear and Table B is exponential.

Spiral Review

9. Determine the value of each expression when $n = 4$.

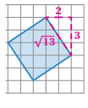
(a) $n^2 - 5$ $= 11$ (b) $n(n + 6)$ $= 40$ (c) $3n^2$ $= 48$

10. Approximate the value of each radical expression.

(a) $\sqrt{10}$ (b) $\sqrt{54}$ (c) $\sqrt{41}$

Between 3 & 4 Between 7 & 8 Between 6 & 7

11. Each grid square represents 1 square unit. What is the *exact* side length of the shaded square?
    $\sqrt{13}$ units



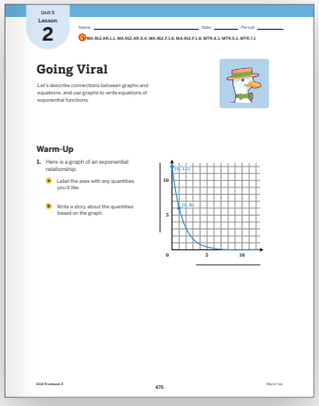
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.F.1.6
	4	2	MA.912.AR.1.1
	5	2	MA.912.F.1.6
	6–7	2	MA.912.F.1.6
Test Practice	8	2	MA.912.F.1.8
Spiral Review			
	9	1	MA.6.AR.1.3
	10	1	MA.8.NSO.1.7
Fluency	11	1	MA.8.GR.1.1

Unit 5

Lesson

2



Going Viral

Graphs of Exponential Relationships

Let's describe connections between graphs and equations and use graphs to write equations of exponential functions.

Learning Outcomes

Today's Goals

1. **Goal:** Write exponential functions to represent graphs or situations.
2. **Language Goal:** Compare and contrast graphs of exponential and linear function in context. **(Writing and Listening)**

Students describe connections between graphs and equations and use graphs to write equations. They compare and contrast linear and exponential functions in the context of meme and app popularity over time.

Coherence

Prior Learning

In Lesson 1, students compared the tables and graphs of linear and exponential functions.

Future Learning

In Lesson 4, students will create tables, graphs, and equations of linear and exponential functions and describe connections between each of the representations.

Connections

- Students explore connections between graphs and equations on the number of likes on a meme for every hour after it goes viral. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.5.4

Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

MA.912.F.1.6

Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

Also Addressing: **MA.912.AR.1.1, MA.912.F.1.8**

Mathematical Thinking and Reasoning: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1.2	
MA.8.F.1.1	
MA.912.AR.1.1	

Vocabulary

Review Vocabulary

initial value	*y-intercept*

Lesson at a Glance

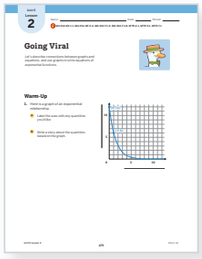
~ 45 min

MA.912.AR.1.1, MA.912.AR.5.4, MA.912.F.1.6, MA.912.F.1.8

Warm-Up

Pairs | 5 min

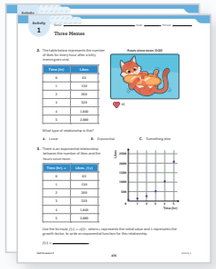
Students use their creativity while attending to the key features of an exponential relationship to write a story about a graph. **Routine:** Tell a Story



Activity 1

Pairs | 15 min

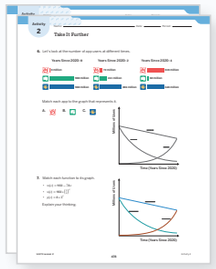
Students connect graphs to situations and equations using their knowledge of *initial values*, rates of change, and growth factors. **(MTR.2.1)**



Activity 2

Pairs | 15 min

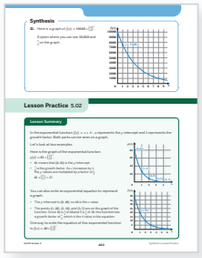
Students match equations to graphs and write equations to represent graphs of exponential functions. **Routine:** Notice and Wonder



Synthesis

Whole Class | 5 min

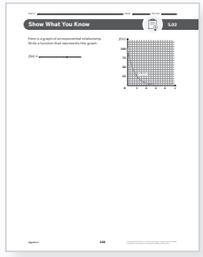
Students describe connections between the graph and the equation of an exponential function. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equation to represent an exponential graph.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use their creativity while attending to the key features of an exponential relationship to write a story about a graph.

Short on time? Consider inviting students to read their classmates' stories instead of facilitating a whole-class discussion.

1 Launch

To reveal prior knowledge, consider asking:

- "How do you know this is an exponential relationship and not a linear one?"
- "What are some examples of relationships that can be modeled using exponential functions?"

Encourage students to choose units for their axis labels (e.g., "Distance From School (miles)" instead of just "Distance From School").

Use the Tell a Story routine to help students make sense of the graph. Invite students to write a story about the graph using their own experiences and creativity.

Consider sharing that stories do not have to be complete or perfect to be valuable.

ELL **English Language Learners** Invite students to write stories in their primary language. **(Writing)**

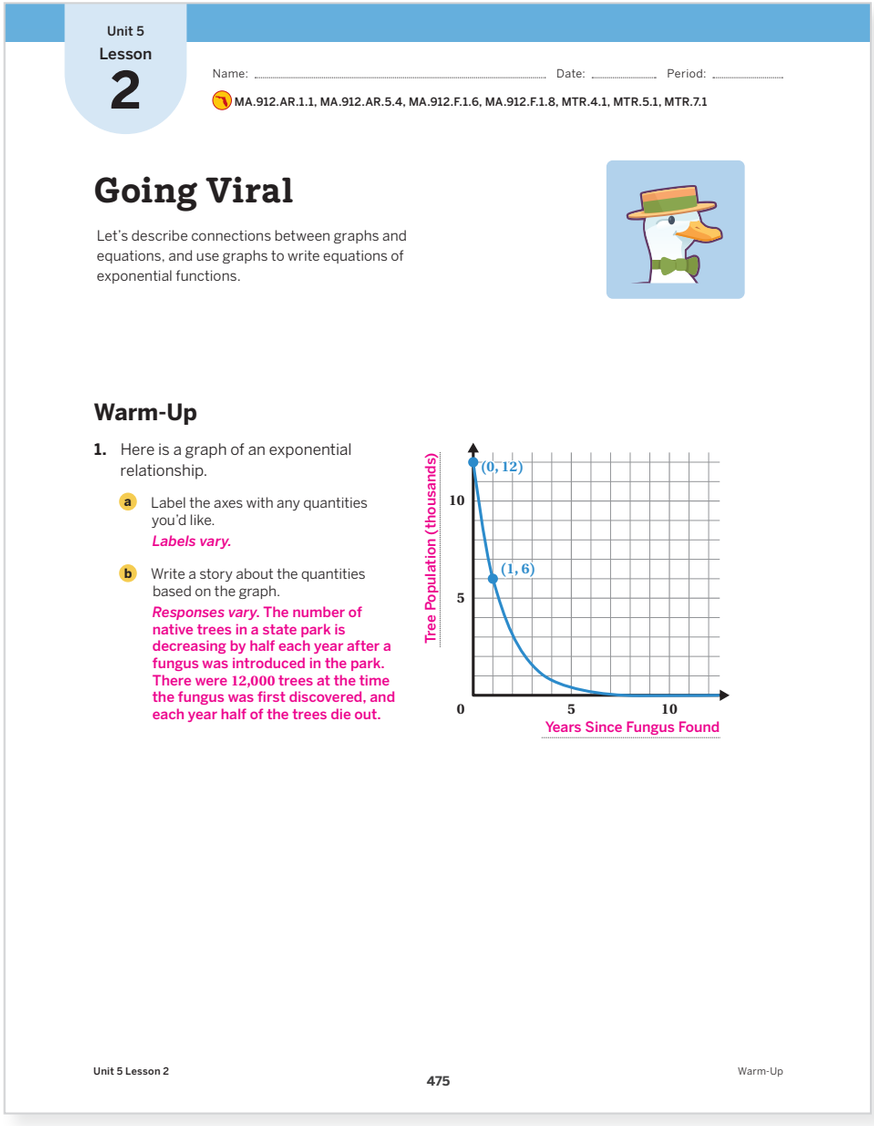
2 Connect

Look for and share several student graphs and stories. Consider highlighting stories that:

- Mention an initial/starting value of 12.
- Use the values 1 and 6.
- Describe a growth factor of $\frac{1}{2}$.

Encourage students to ask questions about each other's stories. Consider asking:

- "How do you see the parts of ___'s story in the graph?"
- "What parts of each of our stories are alike? How are they different?"
- "What questions do you have about each other's stories?"



Unit 5
Lesson
2

Name: Date: Period:

MA.912.AR.1.1, MA.912.AR.5.4, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1

Going Viral

Let's describe connections between graphs and equations, and use graphs to write equations of exponential functions.

Warm-Up

1. Here is a graph of an exponential relationship.

   a. Label the axes with any quantities you'd like.
   Labels vary.

   b. Write a story about the quantities based on the graph.
   Responses vary. The number of native trees in a state park is decreasing by half each year after a fungus was introduced in the park. There were 12,000 trees at the time the fungus was first discovered, and each year half of the trees die out.

Unit 5 Lesson 2 475 Warm-Up

Pairs | 15 min

Activity 1 Three Memes

Purpose: Students connect graphs to situations and equations using their knowledge of *initial values*, rates of change, and growth factors. **(MTR.2.1)**

1 Launch

To generate excitement, consider inviting students to describe their favorite memes.

To reveal connections, consider asking, "When something 'goes viral,' what does that mean mathematically about its growth?"

Encourage students to classify the relationship. Then share the distribution of responses. If there is not yet consensus, invite students to make arguments and justify their reasoning. **(MTR.2.1)**

A **Accessibility: Conceptual Processing** Invite students to describe what patterns they notice in the table before responding to the prompt. Consider asking, "How are the number of likes changing each hour?"

ELL **English Language Learners** Review an anchor chart (possibly from Lesson 2) that displays student ideas about linear and exponential functions. **(Reading, Speaking, and Listening)**

2 Monitor

To support students getting started, consider asking, "What is the initial value? What is the growth factor?"

Listen for a variety of strategies that students use to create an equation for the exponential function.

Encourage students to use the feedback given to help them revise their thinking.

A **Accessibility: Visual-Spatial Processing** Encourage students to touch on each point on the graph and say its coordinates to support them in making connections between the graph and the table.

Activity 1 continued >

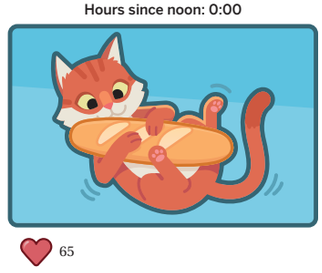
Activity 1

Name: _____ Date: _____ Period: _____

Three Memes

2. The table below represents the number of likes for every hour after a kitty meme goes viral.

Time (hr)	Likes
0	65
1	130
2	260
3	520
4	1,040
5	2,080

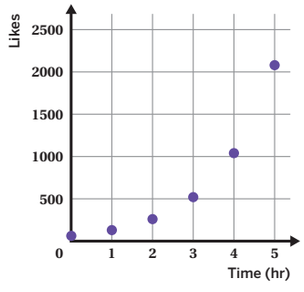


What type of relationship is this?

A. Linear (B.) Exponential C. Something else

3. There is an exponential relationship between the number of likes and the hours since noon.

Time (hr), x	Likes, $f(x)$
0	65
1	130
2	260
3	520
4	1,040
5	2,080



Use the formula $f(x) = a(b)^x$, where a represents the initial value and b represents the growth factor, to write an exponential function for this relationship.

$f(x) =$ $65 \cdot (2)^x$ (or equivalent)

Unit 5 Lesson 2 476 Activity 1

Pairs | 15 min

Activity 1 Three Memes (continued)

Purpose: Students connect graphs to situations and equations using their knowledge of *initial values*, rates of change, and growth factors. **(MTR.5.1)**

2 Monitor

Listen for a variety of strategies students use to connect graphs and equations to share during the Connect, such as:

- Distinguishing between linear and exponential functions using the graph shape and equation structures. **(MTR.5.1)**
- Comparing initial values/y-intercepts.
- Connecting the rate of change or growth factor to the steepness of each graph.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to revisit Problem 3 and ask, "What did each number in the equation tell you about the graph?"
Would like an additional challenge.	**Stretch** Consider asking, "What would the graph of $y = 10 \cdot 2^x$ look like?" or "What do the intersection points tell us about the situation?" **(MTR.2.1)**

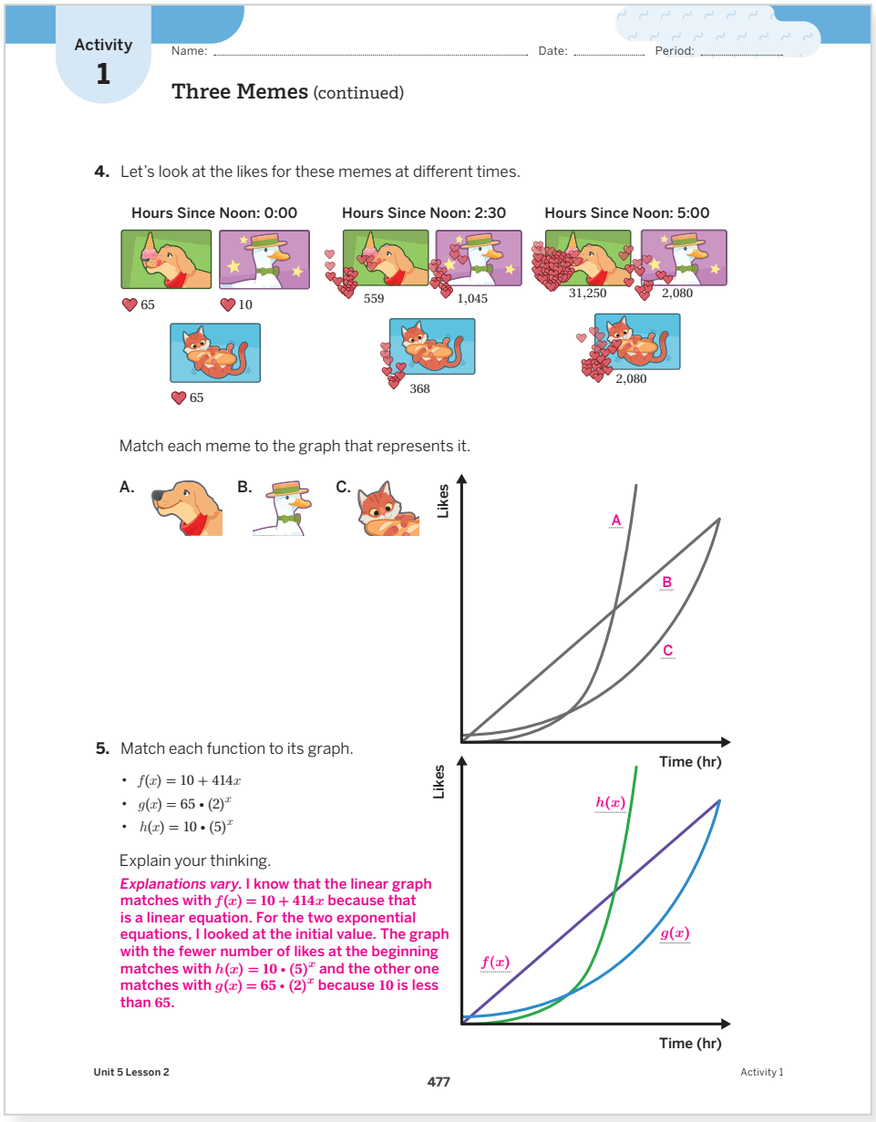
ELL **English Language Learners** Invite students to show their thinking by identifying connections between each equation and its graph in the Student Edition instead of describing it. **(Writing)**

3 Connect

Invite students to share connections they see between graphs and equations of linear and exponential functions.

To reveal the Key Takeaway, consider asking:

- "What did you pay attention to when you were matching?"
- "How can you tell if a graph is linear or exponential? An equation?"
- "How could you convince a skeptic that the two exponential relationships are correctly matched?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Three Memes (continued)

4. Let's look at the likes for these memes at different times.

Hours Since Noon: 0:00 — 65, 10, 65

Hours Since Noon: 2:30 — 559, 1,045, 368

Hours Since Noon: 5:00 — 31,250, 2,080, 2,080

Match each meme to the graph that represents it.

A. B. C.

5. Match each function to its graph.

- $f(x) = 10 + 414x$
- $g(x) = 65 \cdot (2)^x$
- $h(x) = 10 \cdot (5)^x$

Explain your thinking.

Explanations vary. I know that the linear graph matches with $f(x) = 10 + 414x$ because that is a linear equation. For the two exponential equations, I looked at the initial value. The graph with the fewer number of likes at the beginning matches with $h(x) = 10 \cdot (5)^x$ and the other one matches with $g(x) = 65 \cdot (2)^x$ because 10 is less than 65.

Unit 5 Lesson 2 477 Activity 1

Key Takeaway: Equations of linear functions can be written in the form $f(x) = a + bx$, where a represents the initial value and b represents the constant rate of change. Equations of exponential functions can be written in the form $f(x) = a \cdot b^x$, where a represents the initial value and b represents the constant growth factor. **(MTR.5.1)**

Pairs | 15 min

Activity 2 Take It Further

Purpose: Students match equations to graphs and write equations to represent graphs of exponential functions.

Short on time? Consider only discussing one of the two graphs during the Connect.

1 Launch

To generate excitement, consider inviting students to describe their favorite apps.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the visual displays.

2 Monitor

Look for student matches and explanations. Offer partner support where needed.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *initial value*, y-*intercept*, *rate of change*, *growth rate*, *linear*, *exponential*, *base*, *increasing*, *decreasing*, etc. **(Writing and Speaking)**

Activity 2 continued >

Activity 2

Name: Date: Period:

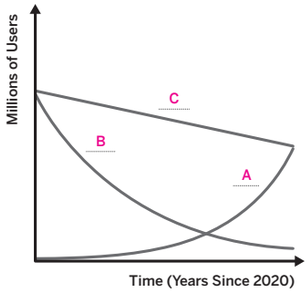
Take It Further

6. Let's look at the number of app users at different times.

Years Since 2020: 0	Years Since 2020: 2	Years Since 2020: 4
8 million	72 million	618 million
960 million	241 million	60 million
960 million	804 million	648 million

Match each app to the graph that represents it.

A. B. C.

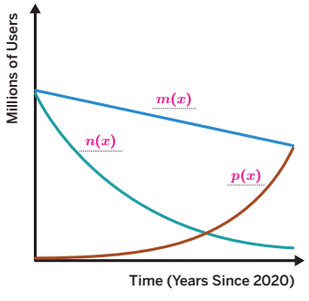


7. Match each function to its graph.

- $m(x) = 960 - 78x$
- $n(x) = 960 \cdot \left(\frac{1}{2}\right)^x$
- $p(x) = 8 \cdot 3^x$

Explain your thinking.

Explanations vary. The equation $n(x)$ matches the exponential graph that decreases over time because the growth factor is $\frac{1}{2}$. The equation $m(x)$ matches the linear graph. The equation $p(x)$ matches the exponential graph that increases because the growth factor is 3.



Unit 5 Lesson 2 478 Activity 2

Pairs | 10 min

Activity 2 Take It Further (continued)

Purpose: Students match equations to graphs and write equations to represent graphs of exponential functions.

Short on time? Consider only discussing one of the two graphs during the Connect.

2 Monitor

To support students getting started, consider asking,

- "What will be the same? What will be different?"
- "What will be the value of y when $x = 1$?"

Encourage students to use a calculator to support them with their thinking.

ELL **English Language Learners** Invite students to read the instructions aloud and paraphrase what it means. **(Reading)**

Look for a variety of strategies students use to write equations for exponential functions. Invite them to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to choose one equation at a time and ask, "What is the initial value? What is the growth factor?"
Start creating an equation with the correct a-value, but need support determining the growth factor.	**Strengthen** Consider sharing that the growth factor $\frac{3}{2}$ for Graph 2 can be found from the points $(0, 20)$ and $(1, 30)$. Consider asking, "How could you use those points to get $\frac{3}{2}$? Can you use this strategy for Graph 1?"

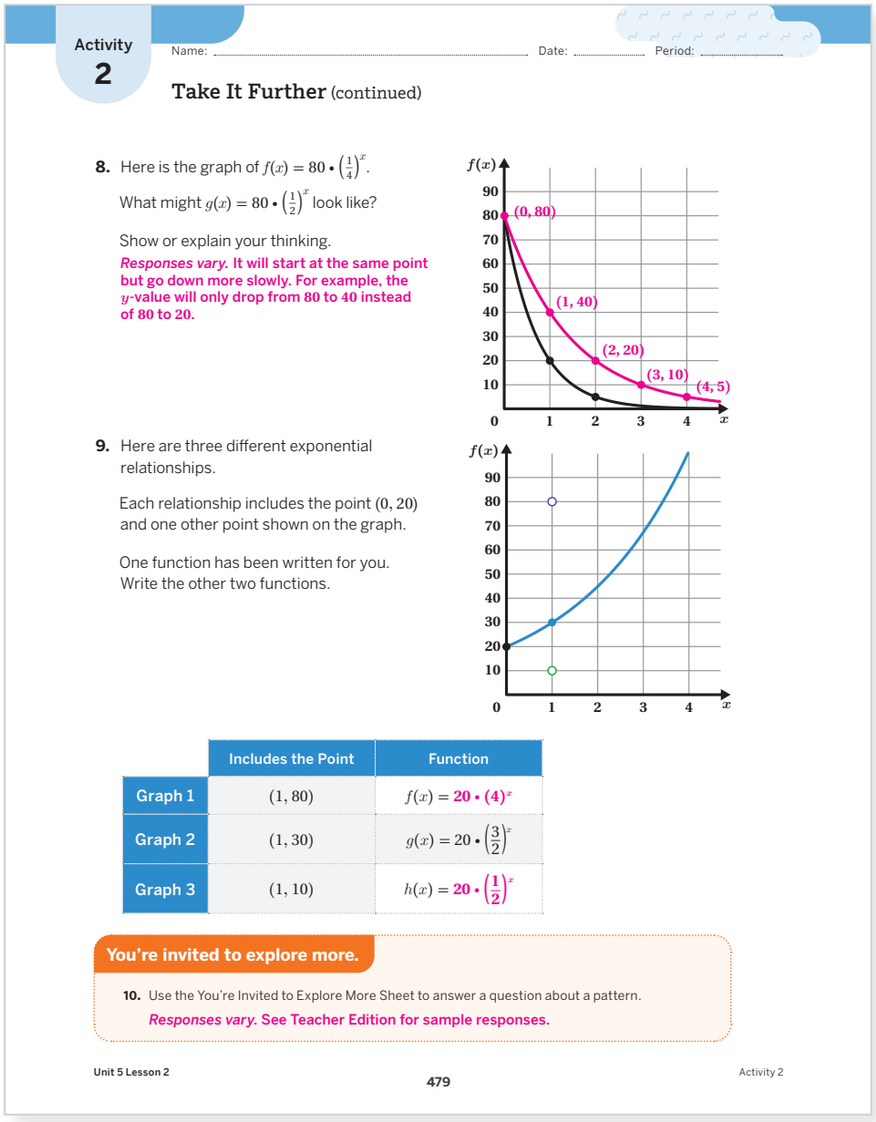
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore exponential relationships further to complete this optional task with a partner.

3 Connect

Invite students to share their strategies for writing equations of exponential functions.

Consider asking:

- "What structure can you use to write equations of exponential functions?" **(MTR.5.1)**
- "How can you determine each part of the equation when given a graph?"



Activity 2

Name: _____ Date: _____ Period: _____

Take It Further (continued)

8. Here is the graph of $f(x) = 80 \cdot \left(\frac{1}{4}\right)^x$.

What might $g(x) = 80 \cdot \left(\frac{1}{2}\right)^x$ look like?

Show or explain your thinking.

Responses vary. It will start at the same point but go down more slowly. For example, the y-value will only drop from 80 to 40 instead of 80 to 20.

9. Here are three different exponential relationships.

Each relationship includes the point $(0, 20)$ and one other point shown on the graph.

One function has been written for you. Write the other two functions.

	Includes the Point	Function
Graph 1	$(1, 80)$	$f(x) = 20 \cdot (4)^x$
Graph 2	$(1, 30)$	$g(x) = 20 \cdot \left(\frac{3}{2}\right)^x$
Graph 3	$(1, 10)$	$h(x) = 20 \cdot \left(\frac{1}{2}\right)^x$

You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to answer a question about a pattern.

Responses vary. See Teacher Edition for sample responses.

Unit 5 Lesson 2 479 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students describe connections between the graph and the equation of an exponential function.

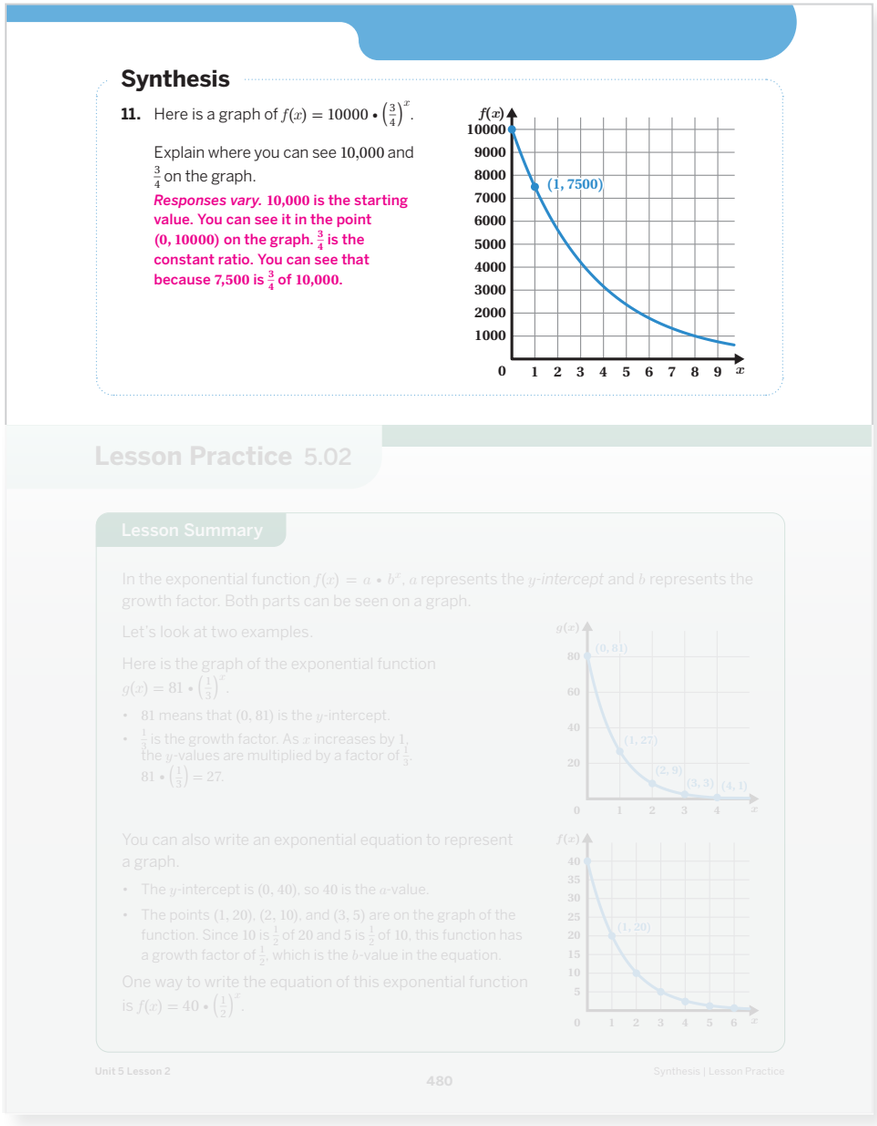
Invite students to respond independently, and then share their thinking with a partner.

Encourage students to sketch on paper to show their thinking.

Listen for and share a variety of ideas, such as:
- The $10{,}000$ is the initial value/starting value/y-intercept.
- The $10{,}000$ is the value of y when $x = 0$.
- The $\frac{3}{4}$ is the growth factor/constant ratio.
- You can see the $\frac{3}{4}$ in the graph because $\frac{7500}{10000} = \frac{3}{4}$.

Consider asking:
- "How can you use the $7{,}500$ to see that the growth factor is $\frac{3}{4}$?"
- "How can you determine the growth factor when given a graph of an exponential function?"
- "How can you tell from the equation if the graph will be increasing or decreasing?" **(MTR.2.1)**
- "How can you tell from the equation if the graph will be linear, exponential, or something else?" **(MTR.5.1)**
- "What makes sense to you about this equation? What does not make sense yet?"

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas about linear and exponential functions. Add any new examples or student language about graphs and equations of exponential functions to the display for students to reference for the rest of the unit.



Synthesis

11. Here is a graph of $f(x) = 10000 \cdot \left(\frac{3}{4}\right)^x$.

Explain where you can see $10{,}000$ and $\frac{3}{4}$ on the graph.

Responses vary. **10,000 is the starting value. You can see it in the point (0, 10000) on the graph. $\frac{3}{4}$ is the constant ratio. You can see that because 7,500 is $\frac{3}{4}$ of 10,000.**

Lesson Practice 5.02

Lesson Summary

In the exponential function $f(x) = a \cdot b^x$, a represents the y-*intercept* and b represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function $g(x) = 81 \cdot \left(\frac{1}{3}\right)^x$.

- 81 means that $(0, 81)$ is the y-intercept.
- $\frac{1}{3}$ is the growth factor. As x increases by 1, the y-values are multiplied by a factor of $\frac{1}{3}$. $81 \cdot \left(\frac{1}{3}\right) = 27$.

You can also write an exponential equation to represent a graph.

- The y-intercept is $(0, 40)$, so 40 is the a-value.
- The points $(1, 20)$, $(2, 10)$, and $(3, 5)$ are on the graph of the function. Since 10 is $\frac{1}{2}$ of 20 and 5 is $\frac{1}{2}$ of 10, this function has a growth factor of $\frac{1}{2}$, which is the b-value in the equation.

One way to write the equation of this exponential function is $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$.

Unit 5 Lesson 2 480 Synthesis | Lesson Practice

Lesson Takeaway: When exponential functions are written in the form $f(x) = a \cdot b^x$, a represents the y-intercept on a graph and b represents the constant growth factor. One way to determine the growth factor is to divide any two consecutive function values $\frac{f(x+1)}{f(x)}$. **(MTR.5.1)**

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing an equation to represent an exponential graph.

Today's Goals

1. **Goal:** Write exponential functions to represent graphs or situations.
2. **Language Goal:** Compare and contrast graphs of exponential and linear function in context. **(Writing and Listening)**

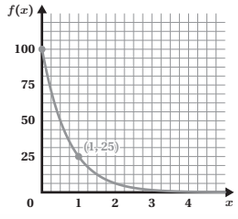
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.02

Here is a graph of an exponential relationship. Write a function that represents this graph.

$f(x) =$ $100 \cdot \left(\frac{1}{4}\right)^x$ (or equivalent)



Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to create equations for exponential functions:

» **Respond to Student Thinking:**

- Use the ***Writing Exponential Functions From Graphs*** **Mini-lesson.**
- Students will have more opportunities to develop this skill in Lessons 3.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to research the change in users over time for an app they enjoy. Encourage them to try to create an equation to model the data.

Professional Learning

Reflect on participation in today's lesson. What are different ways participation can be observed? Who participated and who did not participate in written work, partner discussions, and whole-class discussions today? What trends do you see in participation? How can you continue to support student participation in various forms as students continue their learning about exponential functions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here is a graph of $f(x) = 10000 \cdot \left(\frac{3}{4}\right)^x$.

Explain where you can see 10,000 and $\frac{3}{4}$ on the graph.

Responses vary. 10,000 is the starting value. You can see it in the point (0, 10000) on the graph. $\frac{3}{4}$ is the constant ratio. You can see that because 7,500 is $\frac{3}{4}$ of 10,000.

Lesson Practice 5.02

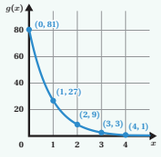
Lesson Summary

In the exponential function $f(x) = a \cdot b^x$, a represents the *y-intercept* and b represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function $g(x) = 81 \cdot \left(\frac{1}{3}\right)^x$.

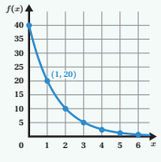
- 81 means that (0, 81) is the y-intercept.
- $\frac{1}{3}$ is the growth factor. As x increases by 1, the y-values are multiplied by a factor of $\frac{1}{3}$. $81 \cdot \left(\frac{1}{3}\right) = 27$.



You can also write an exponential equation to represent a graph.

- The y-intercept is (0, 40), so 40 is the a-value.
- The points (1, 20), (2, 10), and (3, 5) are on the graph of the function. Since 10 is $\frac{1}{2}$ of 20 and 5 is $\frac{1}{2}$ of 10, this function has a growth factor of $\frac{1}{2}$, which is the b-value in the equation.

One way to write the equation of this exponential function is $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$.



Lesson Practice 5.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine the value of each expression when $x = 2$.

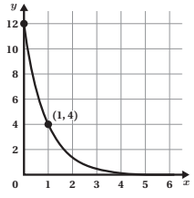
1. 4^x
= 16

2. $\left(\frac{1}{3}\right)^x$
$= \frac{1}{9}$ (or equivalent)

3. $5(6^x)$
= 180

4. Here is a graph of $y = 12 \cdot \left(\frac{1}{3}\right)^x$. Explain where you can see the 12 and the $\frac{1}{3}$ in the graph.

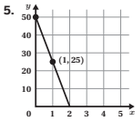
Responses vary. The 12 is the starting value. You can see it in the point (0, 12) in the graph. The $\frac{1}{3}$ is the constant ratio. You can see that because 4 is $\frac{1}{3}$ of 12.



Problems 5–7: Match each equation to the graph that represents it.

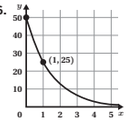
Equation A	Equation B	Equation C
$y = 50 \cdot \left(\frac{1}{2}\right)^x$	$y = 50 \cdot 2^x$	$y = 50 - 25x$

5.



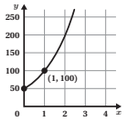
Equation C

6.



Equation A

7.



Equation B

8. Explain how you determined which equation matches with which graph in Problems 5–7.

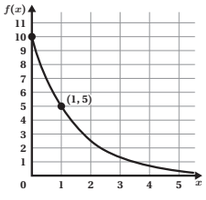
Responses vary.
- The middle graph starts at 50 and has a constant difference of 25. $y = 50 - 25x$ is the only equation with a constant difference.
- The first graph shows a linear relationship, and I know that linear equations do not have exponents. $y = 50 - 25x$ is the only equation without an exponent.

Lesson Practice 5.02

Name: _____ Date: _____ Period: _____

Test Practice

9. Which equation represents the exponential graph?



A. $f(x) = 10 \cdot \left(\frac{1}{2}\right)^x$

B. $f(x) = 10 \cdot (2)^x$

C. $f(x) = 10 \cdot (x)^{\frac{1}{2}}$

D. $f(x) = 10 \cdot (x)^2$

Spiral Review

Problems 10–13: Rewrite each expression as a single power.

10. $4^4 \cdot 4^8$
4^{12}

11. $\frac{3^5}{3^8}$
3^{-3} or $\frac{1}{3^3}$

12. $(12^3)^5$
12^{15}

13. $\frac{7^3 \cdot 7^5}{7^5}$
7^3

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.AR.5.6*
	4	2	MA.912.F.1.6
	5–7	2	MA.912.AR.5.4, MA.912.F.1.8
	8	2	MA.912.AR.5.4
Test Practice	9	2	MA.912.AR.5.4
Spiral Review			
Fluency	10–13	1	MA.8.NSO.1.3

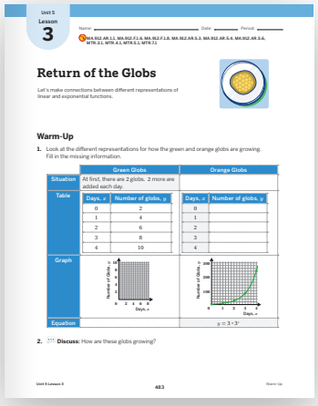
*This problem builds toward the standard shown.

Unit 5
Lesson
3

Return of the Globs

Connecting Representations of Linear and Exponential Functions

Let's make connections between different representations of linear and exponential functions.


Unit 5
Lesson
3

Return of the Globs

Let's make connections between different representations of linear and exponential functions.

Warm-Up

1. Look at the different representations for how the green and orange globs are growing. Fill in the missing information.

	Green Globs	Orange Globs
Situation	At first, there are 2 globs. 2 more are added each day.	
Table	Days, x / Number of globs, y: 0 → 2; 1 → 4; 2 → 6; 3 → 8; 4 → 10	Days, x / Number of globs, y: 0; 1; 2; 3; 4
Graph	Number of Globs, y / Days, x	Number of Globs, y / Days, x
Equation		$y = 3 \cdot 3^x$

2. Discuss: How are these globs growing?

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Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Create functions, tables, and graphs of linear and exponential relationships.
2. **Language Goal:** Describe connections between functions, tables, graphs, and situations that represent an exponential or linear relationship. **(Speaking and Listening)**

In this lesson, students make connections between different representations of linear and exponential functions: graphs, tables, situations, and equations. They compare and contrast linear and exponential growth in the context of growing globs, then apply this knowledge to other situations involving **exponential growth** and **decay**. Students also create graphs of exponential functions by hand.

Coherence

Prior Learning

In Lesson 1, students compared linear and exponential functions and explored how quantities that grow exponentially eventually exceed quantities that grow linearly.

Future Learning

In Lesson 4, students will evaluate exponential functions with positive, negative, and zero inputs and describe the domain of an exponential function in context. In Lessons 6–8, students will solve problems with percent increase and decrease in the context of exponential functions.

Connections

- Students make connections between different representations of exponential growth and decay for bacteria in a petri dish. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.5.3
Given a mathematical or real-world context, classify an exponential function as representing growth or decay.

MA.912.AR.5.4
Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1**

Mathematical Thinking and Reasoning: MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1.2	**MA.912.F.1.6**
MA.8.F.1.3	**MA.912.F.1.8**

Vocabulary

New Vocabulary

exponential decay	exponential growth

Review Vocabulary

rate of change	*slope*

Lesson at a Glance

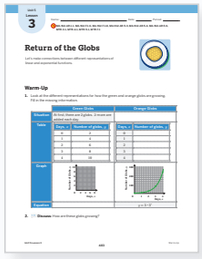
~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4

Warm-Up

Pairs | 5 min

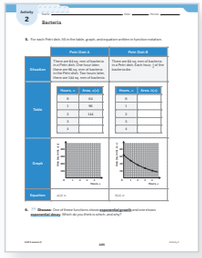
Students describe the patterns they will be exploring in this lesson. **Routine:** Think-Pair-Share



Activity 1

Pairs | 15 min

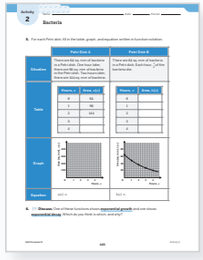
Students compare linear and **exponential growth** by analyzing features of growing globs.



Activity 2

Pairs | 15 min

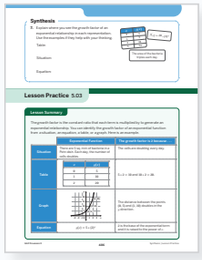
Students create graphs, tables, and equations of situations representing **exponential decay** and growth.



Synthesis

Whole Class | 5 min

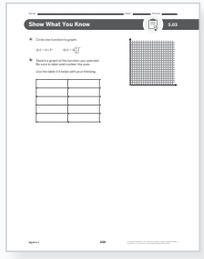
Students synthesize their understanding of where they see the growth factor of an exponential relationship in each representation. **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by sketching the graph of an exponential function by hand.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students describe the patterns they will be exploring in this lesson.

1 Launch

Display the globs and the prompt.

Consider asking, "What do you remember about these globs from Lesson 2?"

ELL **English Language Learners** Consider providing sentence frames (e.g., "At first there are _____ [color] blobs. The number of globs _____ each day.") **(Writing and Speaking)**

Use the Think-Pair-Share routine to help students shape their ideas about the patterns. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

A **Accessibility: Memory and Attention** To support students with keeping track of information, invite them to record the number of globs for each day on paper as they are working.

2 Connect

Consider asking, "Do you think these globs grow linearly, exponentially, or in some other way? How did you decide?"

Unit 5
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1

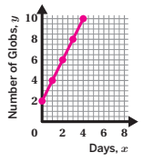
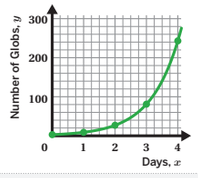
Return of the Globs

Let's make connections between different representations of linear and exponential functions.



Warm-Up

1. Look at the different representations for how the green and orange globs are growing. Fill in the missing information.

	Green Globs	Orange Globs
Situation	At first, there are 2 globs. 2 more are added each day.	At first, there are 3 globs. Each day, the number of globs triples.
Table	Days, x / Number of globs, y: 0 / 2; 1 / 4; 2 / 6; 3 / 8; 4 / 10	Days, x / Number of globs, y: 0 / 3; 1 / 9; 2 / 27; 3 / 81; 4 / 243
Graph		
Equation	$y = 2x + 2$ (or equivalent)	$y = 3 \cdot 3^x$

2. **Discuss:** How are these globs growing?

Responses vary.

- At first there are 2 green globs. 2 more globs are added each day.
- At first there are 3 orange globs. The number of globs is three times the number of globs the day before.

Unit 5 Lesson 3 483 Warm-Up

Pairs | 15 min

Activity 1 Return of the Globs

Purpose: Students compare linear and **exponential growth** by analyzing features of growing globs.

1 Launch

Display the images and invite students to predict which kind of globs will be greater in number on day 10.

A **Accessibility: Visual-Spatial Processing** To support students in creating graphs, invite them to use graph paper with a scale of 1 or to discuss with a partner where to put each value beforehand.

2 Monitor

Look for and celebrate student strategies for making tables, graphs, and equations.

D **Differentiation**

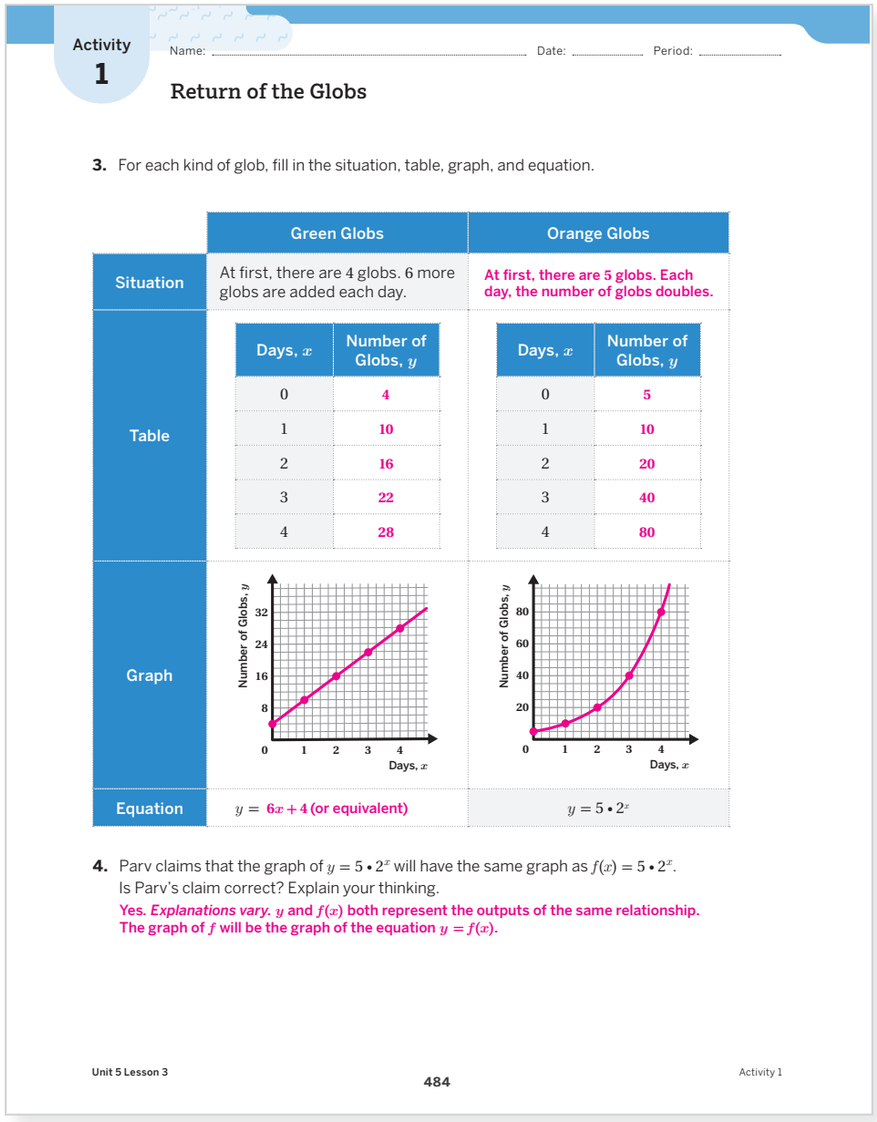
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What representation (situation, table, graph, or equation) would you like to start with?"
Need support writing an equation.	**Support** Consider inviting students to discuss briefly with their partner what equations of linear and exponential functions look like. It might be helpful to post $y = a + bx$ and $y = a \cdot b^x$ in a place where students can reference it.
Use the rate of change, growth factor, and starting value to help write equations.	**Strengthen** Celebrate their thinking and invite them to share during the Connect. Encourage students to use coloring tools to show how they see these key features in each representation.
Would like an additional challenge.	**Stretch** Invite students to determine when each population will reach 300. Green: between 49 and 50 days. Orange: between 5 and 6 days.

3 Connect

Consider asking:

- "How are the tables/graphs/equations of each situation alike? How are they different?"
- "One of these functions shows linear growth; one shows **exponential growth**. Which do you think is which? Why?"
- "When is writing y = helpful? When is $f(x)$ = helpful?"

Note: If it does not come up naturally, consider color coding or using another way to show how the starting value and rate of change/growth rate can be seen in each representation.



Activity 1

Name: _____ Date: _____ Period: _____

Return of the Globs

3. For each kind of glob, fill in the situation, table, graph, and equation.

	Green Globs	Orange Globs
Situation	At first, there are 4 globs. 6 more globs are added each day.	At first, there are 5 globs. Each day, the number of globs doubles.
Table	Days, x / Number of Globs, y: 0 / 4; 1 / 10; 2 / 16; 3 / 22; 4 / 28	Days, x / Number of Globs, y: 0 / 5; 1 / 10; 2 / 20; 3 / 40; 4 / 80
Graph	Number of Globs, y (8, 16, 24, 32); Days, x (0, 1, 2, 3, 4)	Number of Globs, y (20, 40, 60, 80); Days, x (0, 1, 2, 3, 4)
Equation	$y = 6x + 4$ (or equivalent)	$y = 5 \cdot 2^x$

4. Parv claims that the graph of $y = 5 \cdot 2^x$ will have the same graph as $f(x) = 5 \cdot 2^x$. Is Parv's claim correct? Explain your thinking.

Yes. *Explanations vary.* y and $f(x)$ both represent the outputs of the same relationship. The graph of f will be the graph of the equation $y = f(x)$.

Unit 5 Lesson 3 484 Activity 1

Pairs | 15 min

Activity 2 Bacteria

Purpose: Students create graphs, tables, and equations of situations representing exponential decay and growth.

1 Launch

Display the Petri dish and describe how the bacteria population grows.

Consider asking, "What are bacteria? What's a Petri dish? Have you seen one?"

A **Accessibility: Conceptual Processing** As students apply concepts to new situations, invite them to revisit their work in Activity 1 and ask questions like, "How is this similar to what we did before? How is it different?"

2 Monitor

Invite students to use color or arrows to make connections between the situation, table, graph, and equation. **(MTR.5.1)**

A **Accessibility: Executive Functioning** As students carry out multiple steps, consider chunking this activity into more manageable parts by inviting students to choose one petri dish at a time (A or B) and complete its table, graph, and equation.

To reveal connections, consider asking, "Which one is increasing? Decreasing? How can you tell from each of the representations?"

D **Differentiation: (Stretch)** Encourage students to create their own situation and equation/table/graph. Invite students to trade situations and create all the representations for their classmate's situation.

3 Connect

ELL **English Language Learners** Consider asking students where they may have heard the words *growth* or *decay* before, and consider researching how to say those words in the primary languages used by your students. **(Writing and Speaking)**

To reveal the Key Takeaway, consider asking:

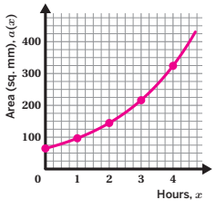
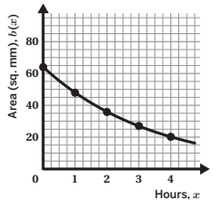
- "What is the same about each petri dish? Different?"
- "Where do you see the starting value in the table? In the equation? In the graph?" **(MTR.5.1)**
- "One of these functions shows exponential growth and one shows exponential decay. Which do you think is which, and why?"

Activity 2

Name: _____ Date: _____ Period: _____

Bacteria

5. For each Petri dish, fill in the table, graph, and equation written in function notation.

	Petri Dish A	Petri Dish B
Situation	There are 64 sq. mm of bacteria in a Petri dish. One hour later, there are 96 sq. mm of bacteria in the Petri dish. Two hours later, there are 144 sq. mm of bacteria.	There are 64 sq. mm of bacteria in a Petri dish. Each hour, $\frac{1}{4}$ of the bacteria die.
Table	Hours, x / Area, $a(x)$: 0 → 64; 1 → 96; 2 → 144; 3 → 216; 4 → 324	Hours, x / Area, $b(x)$: 0 → 64; 1 → 48; 2 → 36; 3 → 27; 4 → 20.25
Graph		
Equation	$a(x) = 64 \cdot 1.5^x$ (or equivalent)	$b(x) = 64 \cdot \left(\frac{3}{4}\right)^x$ (or equivalent)

6. **Discuss:** One of these functions shows exponential growth and one shows exponential decay. Which do you think is which, and why?

Responses vary.

- Petri dish A shows exponential growth because the area of bacteria is growing.
- Petri dish B shows exponential decay because some of the bacteria are dying.

Unit 5 Lesson 3 485 Activity 2

Key Takeaway: Exponential functions can model exponential growth, where a quantity increases exponentially over time, or exponential decay where a quantity decreases exponentially over time. Functions with a growth factor greater than 1 show exponential growth, and functions with a growth factor between 0 and 1 show exponential decay.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of where they see the growth factor of an exponential relationship in each representation. **(MTR.5.1)**

Short on time? Consider omitting the partner conversation and leading a full-class discussion instead.

Invite students to respond independently, and then share their thinking with a partner.

Accessibility: Visual-Spatial Processing
As students interpret visual representations, invite them to use coloring tools to make connections between the table, situation, and equation.

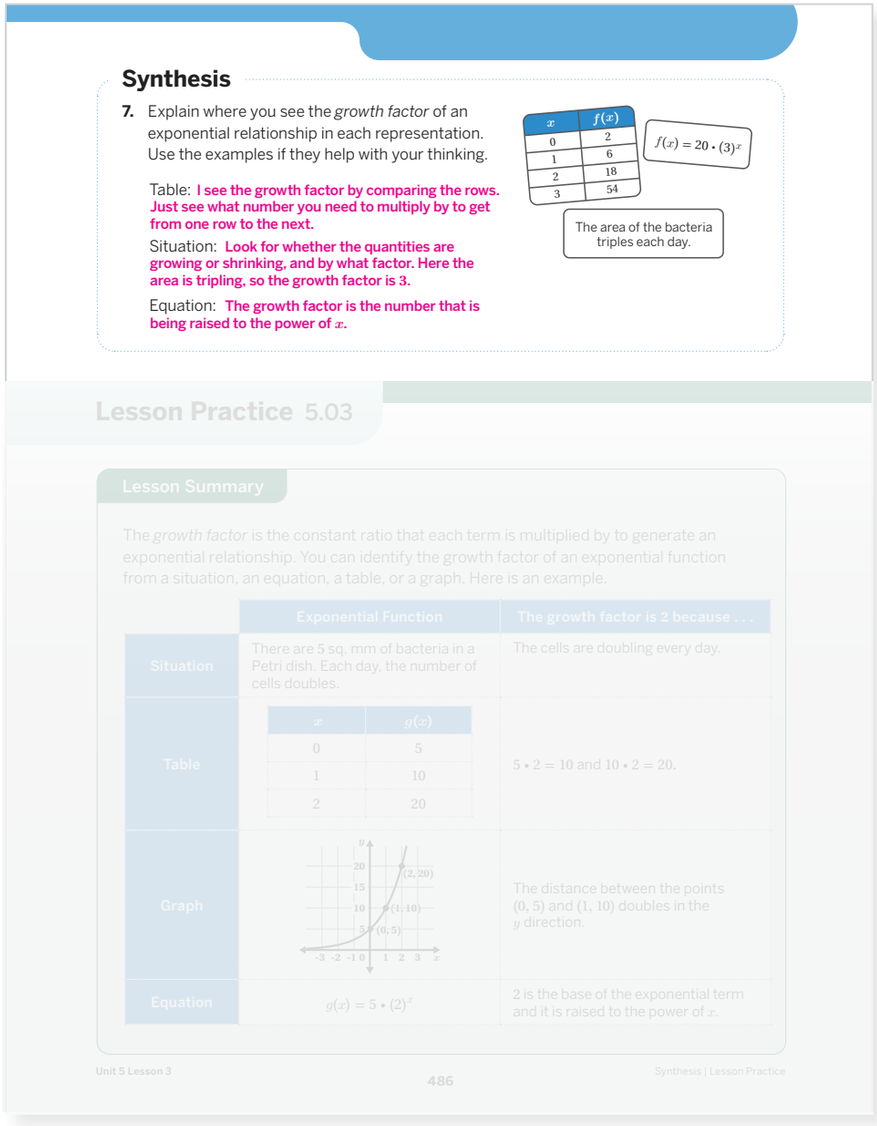
English Language Learners As students process their ideas in words, invite them to talk through their thinking with a partner before writing it down. **(Listening, Writing, and Speaking)**

Listen for and share a variety of ideas, such as:

- Looking for words like *multiply*, *divide*, *constant ratio*, *double*, *triple*, or a percentage.
- Determining the constant ratio between rows of a table.
- Looking for the number raised to a power. **(MTR.5.1)**
- Remembering that the growth factor will be between 0 and 1 if it is exponential decay and the growth factor will be greater than 1 if it is exponential growth.

Consider asking:

- "What makes sense to you about determining growth factors from different representations? What does not make sense yet?"
- "What other information can you know from the different representations?"
- "How can you tell if a situation will be modeled by an exponential or a linear function?"



Synthesis

7. Explain where you see the *growth factor* of an exponential relationship in each representation. Use the examples if they help with your thinking.

x	$f(x)$
0	2
1	6
2	18
3	54

$f(x) = 20 \cdot (3)^x$

The area of the bacteria triples each day.

Table: I see the growth factor by comparing the rows. Just see what number you need to multiply by to get from one row to the next.

Situation: Look for whether the quantities are growing or shrinking, and by what factor. Here the area is tripling, so the growth factor is 3.

Equation: The growth factor is the number that is being raised to the power of x.

Lesson Practice 5.03

Lesson Summary

The *growth factor* is the constant ratio that each term is multiplied by to generate an exponential relationship. You can identify the growth factor of an exponential function from a situation, an equation, a table, or a graph. Here is an example.

	Exponential Function	The growth factor is 2 because . . .
Situation	There are 5 sq. mm of bacteria in a Petri dish. Each day, the number of cells doubles.	The cells are doubling every day.
Table	x | $g(x)$: 0 | 5; 1 | 10; 2 | 20	$5 \cdot 2 = 10$ and $10 \cdot 2 = 20$.
Graph		The distance between the points (0, 5) and (1, 10) doubles in the y direction.
Equation	$g(x) = 5 \cdot (2)^x$	2 is the base of the exponential term and it is raised to the power of x.

Unit 5 Lesson 3 486 Synthesis | Lesson Practice

Lesson Takeaway: Exponential functions can be represented using tables, equations, and graphs, each of which shows the growth factor in a different way. **Exponential growth** describes situations with a growth factor greater than 1 and **exponential decay** describes situations with a growth factor between 0 and 1.

Independent | 5 min

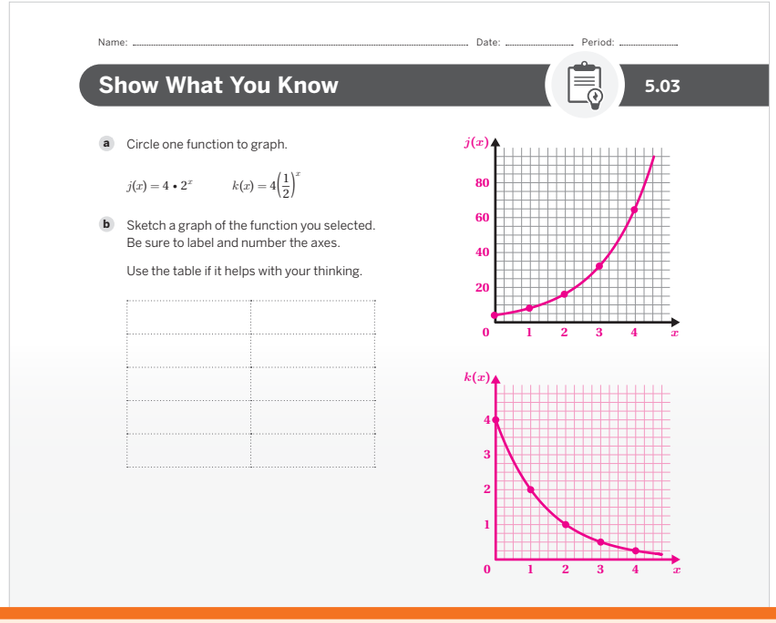
Show What You Know

Purpose: Students demonstrate their understanding by sketching the graph of an exponential function by hand.

Today's Goals

1. **Goal:** Create functions, tables, and graphs of linear and exponential relationships.
2. **Language Goal:** Describe connections between functions, tables, graphs, and situations that represent an exponential or linear relationship. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.03

a. Circle one function to graph.

$j(x) = 4 \cdot 2^x$ $k(x) = 4\left(\frac{1}{2}\right)^x$

b. Sketch a graph of the function you selected. Be sure to label and number the axes.

Use the table if it helps with your thinking.

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to make connections between situations, tables, graphs, and equations:

» **Respond to Student Thinking:**

- Invite students to discuss the Summary with a partner.
- Students will also have more opportunities to develop this concept in future lessons, so intervention is not necessary at this time.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students discovered how to determine if a situation is an example of **exponential growth** or **exponential decay** using the growth factor. How will that work support their thinking about percent growth and decay in Lessons 6–7?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Explain where you see the *growth factor* of an exponential relationship in each representation. Use the examples if they help with your thinking.

x	$f(x)$
0	2
1	6
2	18
3	54

$f(x) = 20 \cdot (3)^x$

The area of the bacteria triples each day.

Table: I see the growth factor by comparing the rows. Just see what number you need to multiply by to get from one row to the next.

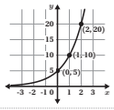
Situation: Look for whether the quantities are growing or shrinking, and by what factor. Here the area is tripling, so the growth factor is 3.

Equation: The growth factor is the number that is being raised to the power of x.

Lesson Practice 5.03

Lesson Summary

The *growth factor* is the constant ratio that each term is multiplied by to generate an exponential relationship. You can identify the growth factor of an exponential function from a situation, an equation, a table, or a graph. Here is an example.

	Exponential Function	The growth factor is 2 because . . .
Situation	There are 5 sq. mm of bacteria in a Petri dish. Each day, the number of cells doubles.	The cells are doubling every day.
Table	x | $g(x)$: 0 | 5; 1 | 10; 2 | 20	$5 \cdot 2 = 10$ and $10 \cdot 2 = 20$.
Graph		The distance between the points $(0, 5)$ and $(1, 10)$ doubles in the y direction.
Equation	$g(x) = 5 \cdot (2)^x$	2 is the base of the exponential term and it is raised to the power of x.

Lesson Practice 5.03

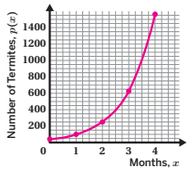
Name: ..... Date: ..... Period: .....

Problems 1–2: A scientist monitors a population of termites that is growing exponentially. She writes the equation $p(x) = 40 \cdot 2.5^x$ to model the total number of termites after x months.

1. Complete the table.

Months, x	Number of Termites, $p(x)$
0	40
1	100
2	250
3	625
4	1562.5

2. Graph the situation.



Problems 3–4: The table models an exponential function with a growth factor of $\frac{1}{3}$.

3. Fill in the table with values for $f(x)$ that could represent this function. *Responses vary.*

x	0	1	2	3
$f(x)$	90	30	10	$3\frac{1}{3}$

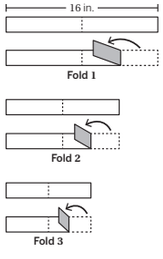
4. Write an equation to model your function.
Responses vary. Equations should include the term $\left(\frac{1}{3}\right)^x$.
$y = 90 \cdot \left(\frac{1}{3}\right)^x$

5. There are 50 sq. mm of bacteria in a Petri dish. The bacteria grow exponentially. 1 hour later, there are 70 sq. mm of bacteria in the Petri dish.
What is the growth factor that models the hourly growth of this bacteria?
1.4

Lesson Practice 5.03

Name: ..... Date: ..... Period: .....

6. One side of a strip of paper is folded to the middle several times. The new length of the paper is recorded after each fold. Determine the length of the strip of paper after each fold.



Folds	Length (in.)
0	16
1	12
2	9
3	6.75
4	5.0625

Test Practice

7. Neena writes the function $f(x) = 4 \cdot (-0.5)^x$ to model a population that has an initial value of 4 and decreases by half every x days. Is she correct? Explain your thinking.

A. The population has an initial value of 4 and decreases by half every x days.

B. The population has an initial value of 4 and increases by half on every even day, and decreases by half every odd day.

C. The population has an initial value of 4 and increases by .5 every x days.

D. The population has an initial value of 4 and decreases by .5 every x days.

Spiral Review

Problems 8–10: Here is a data set: [1, 2, 3, 3, 3, 5, 5, 6, 7, 8, 22].

8. Complete the table. Use a calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.
1	3	5	7	22

9. What number is an outlier of the data set?
22

10. What number could be added to the data set that wouldn't change the 5-number summary?
7

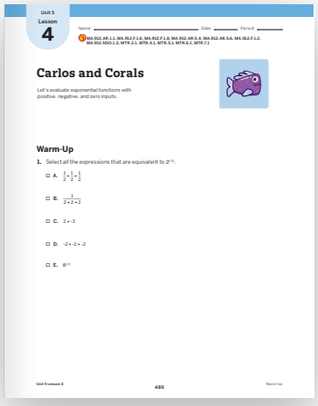
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.AR.5.6
	3–4	2	MA.912.AR.5.4
	5	2	MA.912.AR.5.4
	6	2	MA.912.AR.5.3
Test Practice	7	2	MA.912.AR.5.3
Spiral Review			
Fluency	8	1	MA.6.DP.1.2 MA.6.DP.1.3
	9–10	2	MA.7.DP.1.1

Unit 5

Lesson

4



Carlos and Corals

Evaluating Exponential Functions

Let's evaluate exponential functions with positive, negative, and zero inputs.

Learning Outcomes

Today's Goals

1. **Goal:** Evaluate exponential functions with positive, zero, and negative inputs.
2. **Language Goal:** Describe the domain of an exponential function in context. **(Speaking and Listening)**

Students evaluate exponential functions for positive, negative, and zero inputs. They use these values to help them describe the *domain* of an exponential function in the context of a growing toy fish and a decaying coral population.

Coherence

Prior Learning

In Unit 3, students evaluated functions using function notation and interpreted statements written in function notation in context. They also described the *domain* of a function using words and symbols.

Future Learning

In Unit 6, students will evaluate quadratic functions and interpret the solutions in context. In Algebra 2, students will use function notation to evaluate different types of functions and interpret their solutions in context.

Connections

- Students evaluate exponential functions with positive, negative, and zero inputs through the lens of marine biology. (mathematical, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.5.6

Given a table, equation or written description of an exponential function, graph that function and determine its key features.

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Also addressed: **MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4**

Mathematical Thinking and Reasoning: MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.912.F.1.1

Vocabulary

Review Vocabulary

base
domain
exponent
range
reciprocal

Lesson at a Glance

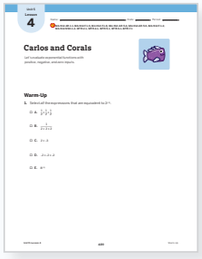
~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.F.1.2

Warm-Up

Pairs | 5 min

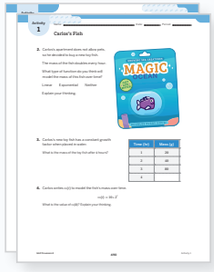
Students use properties of *exponents* to select equivalent expressions to represent an expression with a negative exponent. **MLR2: Collect and Display**


Unit 5 Lesson 4
Carlos and Corals
Warm-Up

Activity 1

Pairs | 15 min

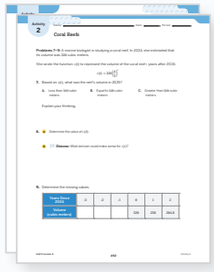
Students revisit the context of Carlos's fish to evaluate an exponential function using different inputs and then use the information to help them describe the *domain* of the function.


Activity 1
Carlos's Fish
MAGIC

Activity 2

Pairs | 15 min

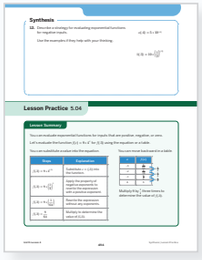
Students compare different solving strategies for evaluating an exponential function with a fractional *base* for negative inputs.
MLR7: Compare and Connect


Activity 2
Coral Reefs

Synthesis

Whole Class | 5 min

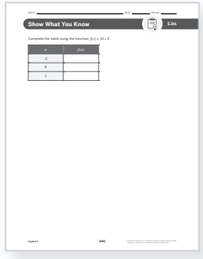
Students synthesize their strategies for evaluating exponential functions for negative inputs in their domains.


Synthesis
Lesson Practice 5.04
Lesson Summary

Show What You Know

Independent | 5 min

Students demonstrate their understanding by evaluating an exponential function for various inputs in its domain.


Show What You Know
5.04a

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use properties of *exponents* to select equivalent expressions to represent an expression with a negative exponent.

1 Launch

Consider asking, "What do you remember about negative exponents?"

A **Accessibility: Memory and Attention** Invite students to use additional paper, or a sketch to write out their calculations and what each answer choice evaluates in order to compare with the original expression. Support them with performing calculations and determining equivalence.

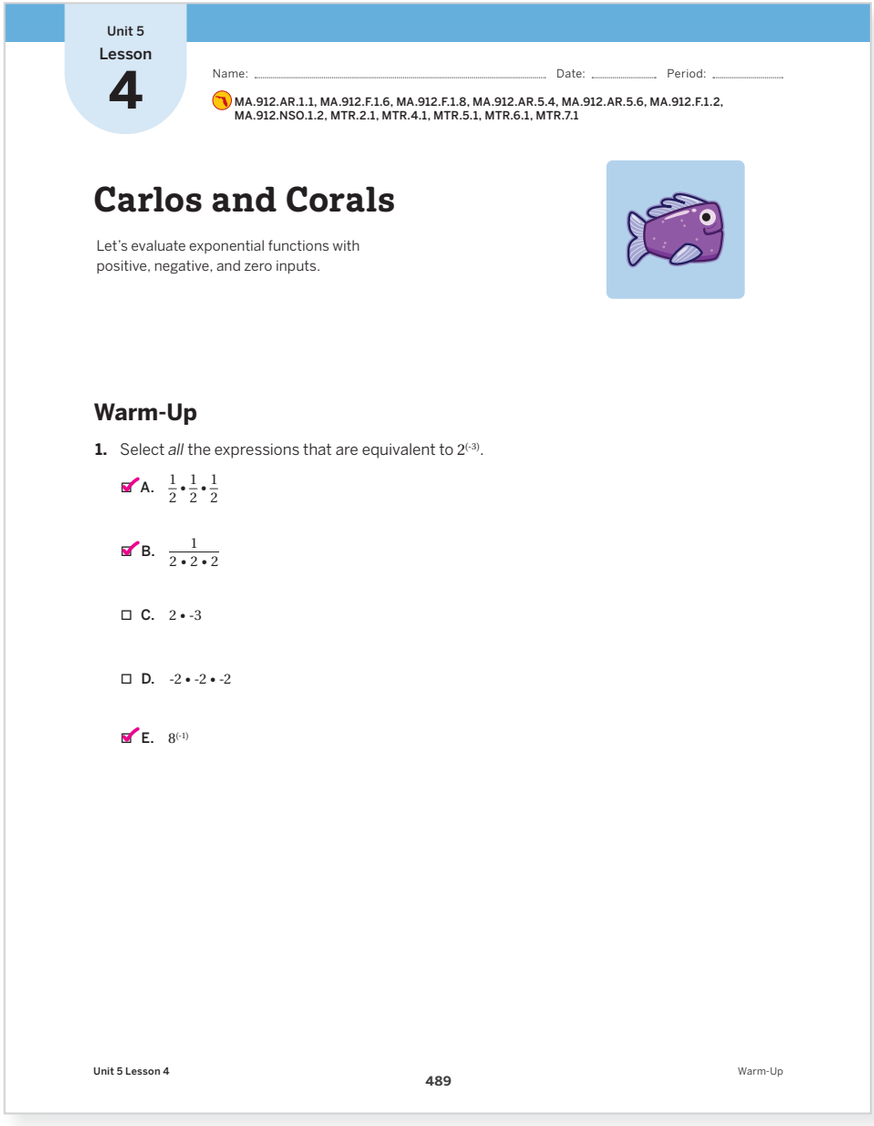
Note: This lesson uses parentheses around negative exponents to make the exponent distinct from the base. Consider sharing that this is one way to write an expression with a negative exponent and share others if there is time.

2 Connect

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and language as they discuss the expression with a negative exponent. Consider labeling an exponential expression with the terms *base* and *exponent*. Display for students to reference and add to for the rest of the lesson.

Consider asking:

- "How did you decide which expressions were equivalent?"
- "What properties hold true for negative exponents?" Negative exponents create a fraction with the base of the exponent becoming the denominator of the fraction. If a positive exponent means repeated multiplication, then a negative exponent means repeated division.



Unit 5
Lesson
4

Name: ..... Date: ..... Period: .....

MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.F.1.2, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Carlos and Corals

Let's evaluate exponential functions with positive, negative, and zero inputs.

Warm-Up

1. Select *all* the expressions that are equivalent to $2^{(-3)}$.

- ☑ A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
- ☑ B. $\frac{1}{2 \cdot 2 \cdot 2}$
- ☐ C. $2 \cdot -3$
- ☐ D. $-2 \cdot -2 \cdot -2$
- ☑ E. $8^{(-1)}$

Unit 5 Lesson 4 489 Warm-Up

Pairs | 15 min

Activity 1 Carlos's Fish

Purpose: Students revisit the context of Carlos's fish to evaluate an exponential function using different inputs and then use the information to help them describe the *domain* of the function.

1 Launch

To reveal connections, consider asking:

- "What do you remember about Carlos's fish?"
- "What is something you've seen double over a short period of time?" Likes on a social media post.

Invite students to share their reasoning for which type of function best models the situation. Consider asking, "What information would indicate that a relationship is linear? Exponential? Neither?"

2 Monitor

D **Differentiation: (Support)** Students may think that the relationship is adding the previous value. Consider asking, "Is there a way to move from each value to the next using the same operation and value?"

A **Accessibility: Conceptual Processing** Invite students to create a hypothesis to support identifying patterns. Consider asking, "What patterns do you see? Does it hold true for for hour 5?"

Look for several student explanations for what $m(0)$ means for Carlos's fish, including that:

- The fish hasn't been put in the water yet.
- The fish's mass at time 0 or $t = 0$.
- The fish has a starting weight of 10 grams.

Pause briefly to discuss the zero property of exponents. Consider using the Pre-Unit Check or a table like it to support students in identifying patterns in exponential relationships to make sense of why $2^0 = 1$. Consider asking:

- "Is 0 an input or an output for $m(t)$?"
- "Where would we expect to see $m(0)$ on the graph of the function $m(t)$? How does that help you explain what it means for Carlos's fish?"
- "What is the zero property of exponents? Why does any base raised to the zero power equal one?"

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Carlos's Fish

2. Carlos's apartment does not allow pets, so he decided to buy a new toy fish.

The mass of the fish doubles every hour.

What type of function do you think will model the mass of this fish over time?

Linear (Exponential) Neither

Explain your thinking.

Explanations vary. I think the function that models the mass of the fish is exponential because the packaging says "doubles in size." To double means to multiply by 2, and we can use an exponent to show multiplying over and over again.



3. Carlos's new toy fish has a constant growth factor when placed in water.

What is the mass of the toy fish after 4 hours?

Time (hr)	Mass (g)
1	20
2	40
3	80
4	160

4. Carlos writes $m(t)$ to model the fish's mass over time.

$$m(t) = 10 \cdot 2^t$$

What is the value of $m(0)$? Explain your thinking.

10. *Explanations vary.*

- I substituted 0 into the equation: $m(0) = 10 \cdot 2^{(0)}$. This equals 10.
- The toy fish was 10 grams when Carlos first put it in the water or when he first opened the package.

Unit 5 Lesson 4 490 Activity 1

Pairs | 15 min

Activity 1 Carlos's Fish (continued)

Purpose: Students revisit the context of Carlos's fish to evaluate an exponential function using different inputs and then use the information to help them describe the *domain* of the function.

2 Monitor

To support students getting started, consider asking, "What is the growth factor for Carlos's fish? How do you know?"

Collect student responses or sketches.

Encourage students to use paper to support them in evaluating each input in the domain.

Note: Consider sharing that a calculator is available to support their solution strategy.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to use paper to substitute 5 in for t (e.g., $m(5) = 10 \cdot 2^5$). Consider asking, "What operation should we complete first: 2^5 or $10 \cdot 2$?"
Use a variety of methods, such as doubling the height of the previous y-value, continuing the table, or evaluating using exponents.	**Strengthen** Invite them to share their strategy during the Connect. Consider asking, "What's another strategy you could use to evaluate the function?" at $t = 5$?"
Use reasoning to notice that evaluating $m(t)$ for $t = -1$ doesn't make sense.	**Stretch** Celebrate their noticing. Consider asking, "What does that mean about the domain of $m(t)$?"

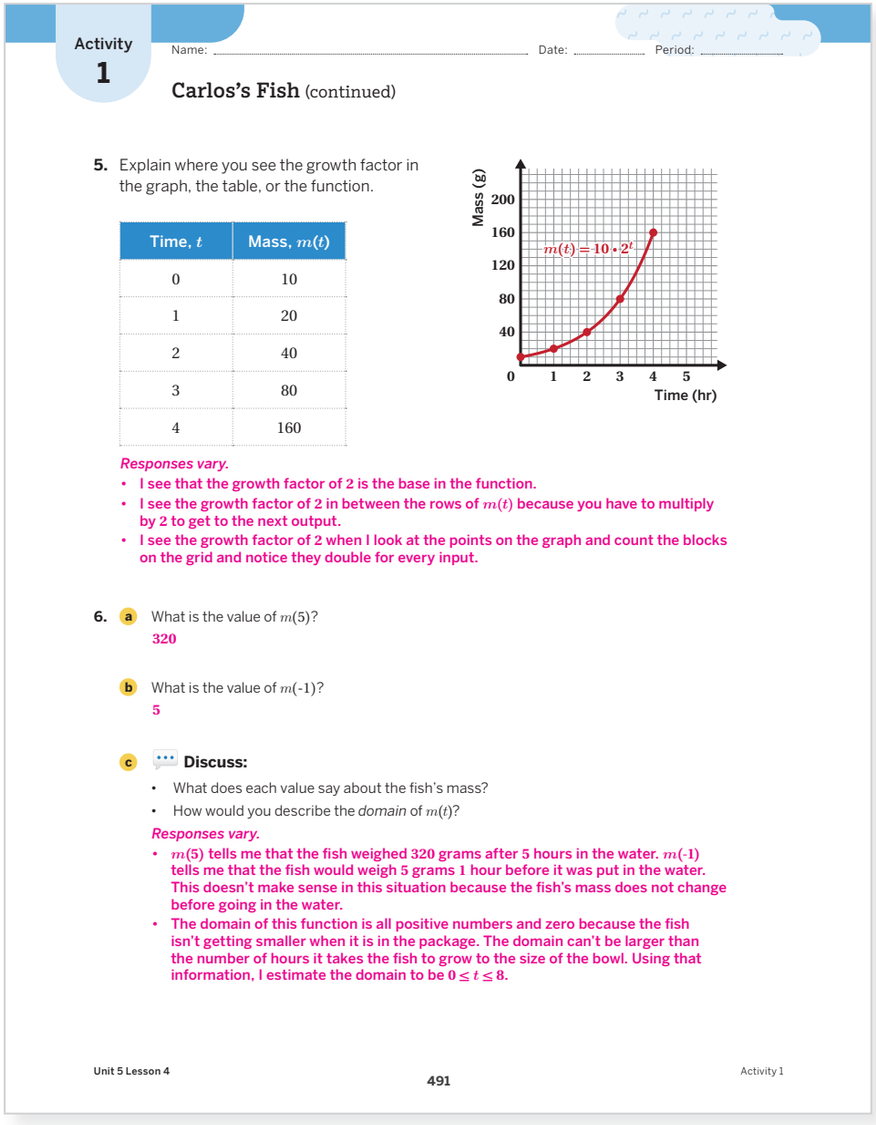
3 Connect

Share the sketches to make connections across representations about how outputs on the graph double in height as the inputs increase by 1. Consider asking:

- "Where do you see the fish's mass before it was placed in the water on the graph? In the table? In the function?" **(MTR.2.1)**
- "Where do you see the growth factor of the fish in the table? On the graph? In the function?" **(MTR.2.1)**

Consider asking:

- "What is the *domain* of the function? Is 320 a value in the *domain* or in the *range*?"
- "Why is the value of $m(-1)$ less than 10?"
- "Can the domain of $m(t)$ include any positive number? A fraction? A decimal? A number larger than 100?"



Activity 1

Name: _____ Date: _____ Period: _____

Carlos's Fish (continued)

5. Explain where you see the growth factor in the graph, the table, or the function.

Time, t	Mass, $m(t)$
0	10
1	20
2	40
3	80
4	160

Responses vary.

- I see that the growth factor of 2 is the base in the function.
- I see the growth factor of 2 in between the rows of $m(t)$ because you have to multiply by 2 to get to the next output.
- I see the growth factor of 2 when I look at the points on the graph and count the blocks on the grid and notice they double for every input.

6. **a** What is the value of $m(5)$?

320

b What is the value of $m(-1)$?

5

c **Discuss:**

- What does each value say about the fish's mass?
- How would you describe the *domain* of $m(t)$?

Responses vary.

- $m(5)$ tells me that the fish weighed 320 grams after 5 hours in the water. $m(-1)$ tells me that the fish would weigh 5 grams 1 hour before it was put in the water. This doesn't make sense in this situation because the fish's mass does not change before going in the water.
- The domain of this function is all positive numbers and zero because the fish isn't getting smaller when it is in the package. The domain can't be larger than the number of hours it takes the fish to grow to the size of the bowl. Using that information, I estimate the domain to be $0 \leq t \leq 8$.

Unit 5 Lesson 4 491 Activity 1

Key Takeaway: The *domain* of an exponential function can include values like zero, all positive, or all negative values. It is important to use the information in the situation to decide which values make sense and which do not (e.g., negative inputs don't make sense for Carlos's fish because the fish does not shrink when it's in the package).

Pairs | 15 min

Activity 2 Coral Reefs

Purpose: Students compare different solving strategies for evaluating an exponential function with a fractional *base* for negative inputs.

Short on time? Consider omitting Evaluating Functions if students demonstrate understanding of evaluating functions out of context.

1 Launch

To reveal connections, consider asking, "What do you know about coral reef decay?"

Consider sharing a video that explains the rate of decay of coral reefs globally to increase access to the context of the activity.

ELL **English Language Learners** Create a list of words about the context that students might not know (e.g. *coral*, *reef*, *marine biologist*, *volume*), along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

Display the responses collected calling attention to any conflict or consensus you see. Invite students to make arguments and justify their reasoning for each option. **(MTR.4.1)**

Consider asking, "How is $\frac{4}{5}$ related to 1? How does this information help us understand if the coral reefs are growing or decaying?"

2 Monitor

To support students getting started, encourage students to write the statement in function notation before evaluating (e.g. $v(2) = 320\left(\frac{4}{5}\right)^2$).

D **Differentiation:** (Support) Students may think that the domain of $v(t)$ is only positive numbers like the function of Carlos's fish. Consider asking, "In this situation, what would a negative input mean? Would it make sense for this situation? Why or why not?"

Encourage students to use a calculator to support them in evaluating.

A **Accessibility: Executive Functioning** To support students in initiating complex tasks, consider asking, "What do you think a first step could be?"

Look for students who use strategies similar to Angel's or Sora's. Listen for their strategies and invite them to share their thinking during the Connect.

Activity 2 continued >

Activity 2

Name: Date: Period:

Coral Reefs

Problems 7–9: A marine biologist is studying a coral reef. In 2024, she estimated that its volume was 320 cubic meters.

She wrote the function $v(t)$ to represent the volume of the coral reef t years after 2024:

$$v(t) = 320\left(\frac{4}{5}\right)^t$$

7. Based on $v(t)$, what was the reef's volume in 2025?

A. Less than 320 cubic meters
B. Equal to 320 cubic meters
C. Greater than 320 cubic meters

Explain your thinking.

Explanations vary. The amount of coral is decreasing because $\frac{4}{5}$ is less than 1, which means the original amount is getting smaller. This means that the reef's volume in 2025 was less than it was the year before.

8. a. Determine the value of $v(2)$.

204.8

b. **Discuss:** What domain could make sense for $v(t)$?

Responses vary.

- The domain for $v(t)$ could be all real numbers because coral reefs have been around for a long time, so it makes sense to think about how big they were before 2024.
- The domain could be from $-10 \leq t \leq 10$ because I don't think the coral reef will keep changing this way. This domain is from the years 2014 to 2034.

9. Determine the missing values.

Years Since 2024	-3	-2	-1	0	1	2
Volume (cubic meters)	625	500	400	320	256	204.8

Unit 5 Lesson 4 492 Activity 2

Pairs | 15 min

Activity 2 Coral Reefs (continued)

Purpose: Students compare different solving strategies for evaluating an exponential function with a fractional *base* for negative inputs.

Short on time? Consider omitting Evaluating Functions if students demonstrate understanding of evaluating functions out of context.

2 Monitor

ELL **English Language Learners** Invite students to share why it makes sense to use negative inputs to represent past time to increase access to the task. **(Reading, Listening, and Speaking)**

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one student's work at a time. To promote student focus, consider covering up steps of Sora's work until students are ready to address them.

D **Differentiation**

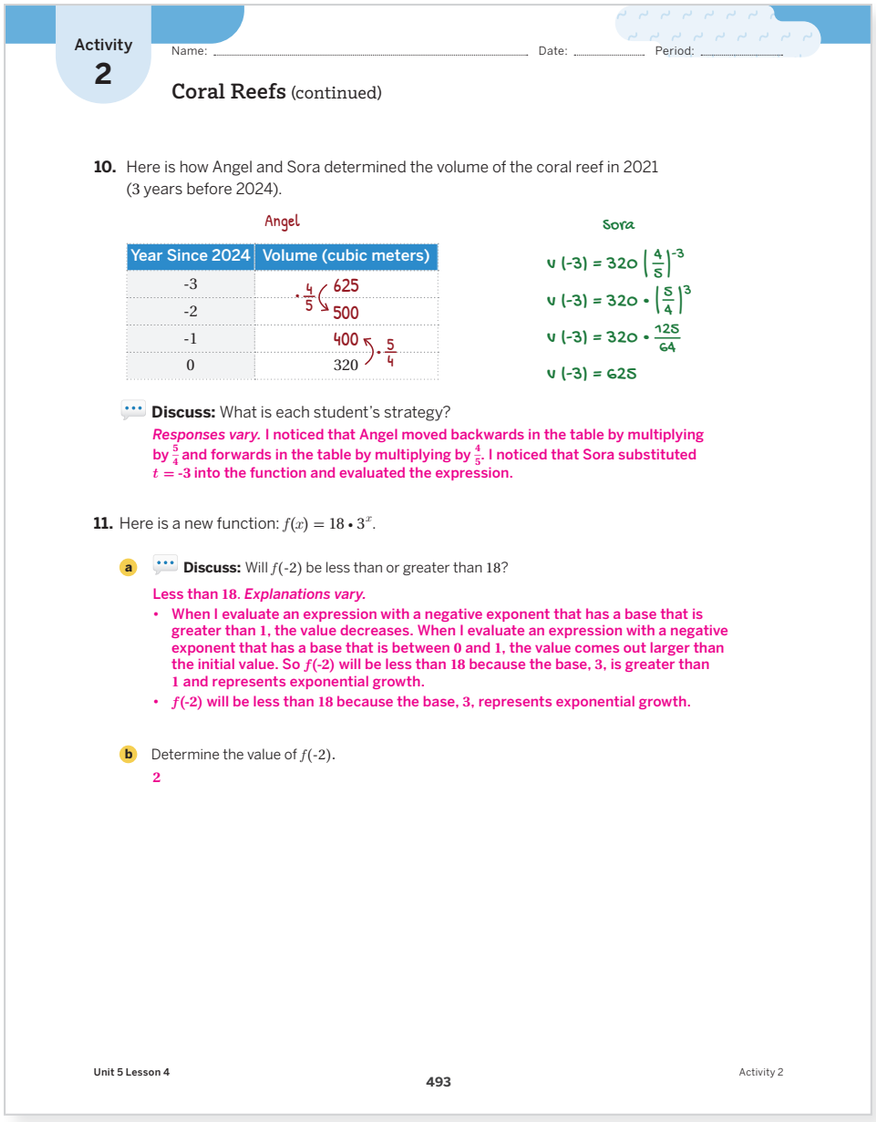
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to return to Problem 10 to revisit Angel's and Sora's strategies, or to try their strategy from a previous problem.
Think that $f(-2)$ will be larger than 18 because the base of the expression is 3.	**Strengthen** Consider asking, "Will 3^{-2} be greater than or less than 1? How will that help us determine if $f(-2)$ will be greater than or less than 18?"
Would like an additional challenge.	**Stretch** Invite these students to describe the domain of $f(x)$.

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

To reveal the Key Takeaway, consider asking:

- "How are Angel's and Sora's strategies alike? How are they different?"
- "Why did Sora use the fraction $\frac{5}{4}$ in her strategy?"
- "Is dividing by $\frac{4}{5}$ equivalent to multiplying by $\frac{5}{4}$? Why or why not?"
- "What is a *reciprocal*?"
- "When might each strategy be useful?"
- "How does each strategy compare to your own?"



Activity 2

Name: _____ Date: _____ Period: _____

Coral Reefs (continued)

10. Here is how Angel and Sora determined the volume of the coral reef in 2021 (3 years before 2024).

Angel

Year Since 2024	Volume (cubic meters)
-3	625
-2	500
-1	400
0	320

Sora

$v(-3) = 320\left(\frac{4}{5}\right)^{-3}$

$v(-3) = 320 \cdot \left(\frac{5}{4}\right)^{3}$

$v(-3) = 320 \cdot \frac{125}{64}$

$v(-3) = 625$

Discuss: What is each student's strategy?

Responses vary. I noticed that Angel moved backwards in the table by multiplying by $\frac{5}{4}$ and forwards in the table by multiplying by $\frac{4}{5}$. I noticed that Sora substituted $t = -3$ into the function and evaluated the expression.

11. Here is a new function: $f(x) = 18 \cdot 3^x$.

a. **Discuss:** Will $f(-2)$ be less than or greater than 18?

Less than 18. *Explanations vary.*

- When I evaluate an expression with a negative exponent that has a base that is greater than 1, the value decreases. When I evaluate an expression with a negative exponent that has a base that is between 0 and 1, the value comes out larger than the initial value. So $f(-2)$ will be less than 18 because the base, 3, is greater than 1 and represents exponential growth.
- $f(-2)$ will be less than 18 because the base, 3, represents exponential growth.

b. Determine the value of $f(-2)$.

2

Unit 5 Lesson 4 493 Activity 2

Key Takeaway: When evaluating exponential functions with negative inputs, a negative *exponent* represents repeated division. If the base is a fraction and the exponent is negative, multiplying by the *reciprocal* of the base repeatedly is equivalent to dividing by the *base* repeatedly.

Whole Class | 5 min

Synthesis

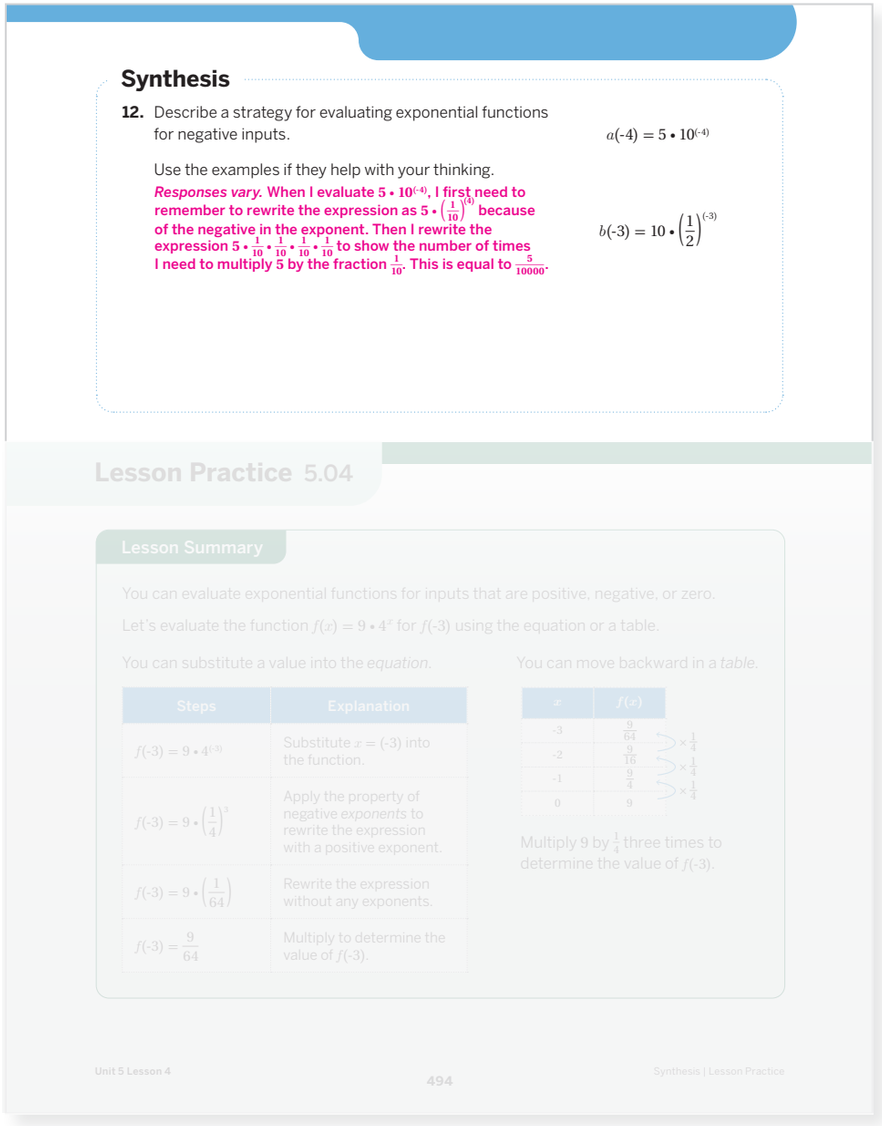
Purpose: Students synthesize their strategies for evaluating exponential functions for negative inputs in their domains.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- Using a table to apply repeated division.
- Using a graph to divide the height of the y-values proportionally.
- Multiplying by the reciprocal if the base is a fraction.
- Negative exponents representing repeated division.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. Describe a strategy for evaluating exponential functions for negative inputs.

$a(-4) = 5 \cdot 10^{(-4)}$

Use the examples if they help with your thinking.

Responses vary. When I evaluate $5 \cdot 10^{(-4)}$, I first need to remember to rewrite the expression as $5 \cdot \left(\frac{1}{10}\right)^{(4)}$ because of the negative in the exponent. Then I rewrite the expression $5 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$ to show the number of times I need to multiply 5 by the fraction $\frac{1}{10}$. This is equal to $\frac{5}{10000}$.

$b(-3) = 10 \cdot \left(\frac{1}{2}\right)^{(-3)}$

Lesson Practice 5.04

Lesson Summary

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function $f(x) = 9 \cdot 4^x$ for $f(-3)$ using the equation or a table.

You can substitute a value into the *equation*.

Steps	Explanation
$f(-3) = 9 \cdot 4^{(-3)}$	Substitute $x = (-3)$ into the function.
$f(-3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative *exponents* to rewrite the expression with a positive exponent.
$f(-3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(-3) = \frac{9}{64}$	Multiply to determine the value of $f(-3)$.

You can move backward in a *table*.

x	$f(x)$	
-3	$\frac{9}{64}$	$\times \frac{1}{4}$
-2	$\frac{9}{16}$	$\times \frac{1}{4}$
-1	$\frac{9}{4}$	$\times \frac{1}{4}$
0	9	

Multiply 9 by $\frac{1}{4}$ three times to determine the value of $f(-3)$.

Unit 5 Lesson 4 494 Synthesis | Lesson Practice

Lesson Takeaway: A negative *exponent* represents repeated division. On a graph, a negative exponent can be seen as the change in the height of the y-values proportional to the *base*. In a table, we can apply repeated division one step at a time. Using a function, we can evaluate at any input using properties of exponents, but we should also consider what types of inputs in the *domain* make sense to evaluate for the situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by evaluating an exponential function for various inputs in its domain.

Today's Goals

1. **Goal:** Evaluate exponential functions with positive, zero, and negative inputs.
2. **Language Goal:** Describe the domain of an exponential function in context. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.04

Complete the table using the function $f(x) = 12 \cdot 3^x$.

x	$f(x)$
-2	$\frac{4}{3}$ (or equivalent)
0	12
1	36

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students' need support understanding how to evaluate with negative exponents:

» **Respond to Student Thinking:**

- Invite students to discuss the Summary with a partner.
- Students will also have more opportunities to develop this concept in future lessons, so intervention is not necessary at this time.

Stretch

Challenge students and extend their learning with these resources.

If students' would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students used their prior knowledge of the properties of negative *exponents* to support their understanding of evaluating exponential functions. How were students able to apply their thinking to this new concept? How can you support students in recognizing the connection between these two key math concepts? How can you continue to help students extend their understanding of key concepts (like evaluating functions) to new types of functions and number systems (e.g., students will study complex numbers and properties of exponents and radicals in Algebra 2)?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe a strategy for evaluating exponential functions for negative inputs.

Use the examples if they help with your thinking.

Responses vary. When I evaluate $5 * 10^{-4}$, I first need to remember to rewrite the expression as $5 * \left(\frac{1}{10}\right)^4$ because of the negative in the exponent. Then I rewrite the expression $5 * \frac{1}{10} * \frac{1}{10} * \frac{1}{10} * \frac{1}{10}$ to show the number of times I need to multiply 5 by the fraction $\frac{1}{10}$. This is equal to $\frac{5}{10000}$.

$a(-4) = 5 * 10^{(-4)}$

$b(-3) = 10 * \left(\frac{1}{2}\right)^{(-3)}$

Lesson Practice 5.04

Lesson Summary

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function $f(x) = 9 \cdot 4^x$ for $f(-3)$ using the equation or a table.

You can substitute a value into the *equation*.

Steps	Explanation
$f(-3) = 9 \cdot 4^{(-3)}$	Substitute $x = (-3)$ into the function.
$f(-3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative *exponents* to rewrite the expression with a positive exponent.
$f(-3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(-3) = \frac{9}{64}$	Multiply to determine the value of $f(-3)$.

You can move backward in a *table*.

x	$f(x)$	
-3	$\frac{9}{64}$	$\times \frac{1}{4}$
-2	$\frac{9}{16}$	$\times \frac{1}{4}$
-1	$\frac{9}{4}$	$\times \frac{1}{4}$
0	9	

Multiply 9 by $\frac{1}{4}$ three times to determine the value of $f(-3)$.

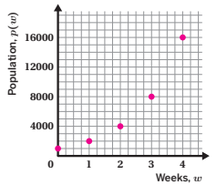
Lesson Practice 5.04

Name: _____ Date: _____ Period: _____

Problems 1–3: The equation $p(w) = 1000 \cdot 2^w$ models a population of mosquitos, $p(w)$, where w is the number of weeks after the population was first measured.

1. Complete the table and plot the values on the graph.

Weeks, w	Population, $p(w)$
0	1,000
1	2,000
2	4,000
3	8,000
4	16,000



2. Where on the graph do you see the 1,000 from the equation?

Responses vary. I see that 1,000 is the y-intercept of the graph.

3. Determine the value of $p(-2)$ and explain what it means in this situation.

$p(-2) = 250$. *Explanations vary.* This means that there were 250 mosquitos 2 weeks before the population was first measured.

Problems 4–6: The equation $f(t) = 800 \cdot \left(\frac{1}{5}\right)^t$ models a fish population, $f(t)$, where t is time in years since the beginning of 2015.

4. What is the population of fish at the beginning of 2015?

800

5. What is the population of fish at the beginning of 2018?

100

6. What is the population of fish at the beginning of 2012? Do you think this makes sense in this situation?

6,400. *Responses vary.* Maybe, it depends on how big the lake is. For one of the Great Lakes, this number makes sense. If it is a small lake then it is unlikely that there were 6,400 fish in it.

Lesson Practice 5.04

Name: _____ Date: _____ Period: _____

Test Practice

7. Which equation best models the data in the table?

A. $f(x) = 80(1.25)^x$

B. $f(x) = 64(1.25)^x$

C. $f(x) = 64 + 1.25x$

D. $f(x) = 60 + 20x$

x	$f(x)$
1	80
2	100
3	125
4	156.25

Spiral Review

Problems 8–9: Charlie's gaming club wants to make at least \$300 selling boxes of cookies and pies. The gaming club makes \$9 for each box of cookies and \$15 for each pie.

8. Write an inequality to represent the number of boxes of cookies, c, and pies, p, the club can sell to make at least \$300.

$9c + 15p \geq 300$

9. If the club sells 5 boxes of cookies, what is the minimum number of pies they need to sell in order to meet their goal?

17 pies

Problems 10–13: Determine the value of each expression.

10. $2^4 = 16$

11. $-2^4 = -16$

12. $(-2)^4 = 16$

13. $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.AR.5.6 MA.912.F.1.2
	3	2	MA.912.AR.5.6 MA.912.F.1.2
	4–5	1	MA.912.F.1.2
	6	2	MA.912.F.1.2
Test Practice	7	2	MA.912.F.1.2
Spiral Review			
	8–9	2	MA.912.AR.2.7
Fluency	10–13	1	MA.8.AR.1.1

Unit 5

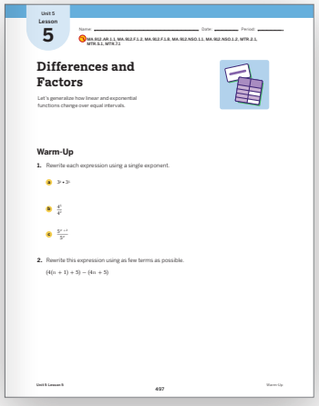
Lesson

5

Differences and Factors

Changes Over Equal Intervals

Let's generalize how linear and exponential functions change over equal intervals.


Unit 5
Lesson
5
Name _____ Date _____ Period _____
Differences and Factors
Let's generalize how linear and exponential functions change over equal intervals.
Warm-Up
1. Rewrite each expression using a single exponent.
2. Rewrite this expression using as few terms as possible.
$(4(x+1)+5)-(4x+5)$
497

Learning Outcomes

Today's Goals

1. **Goal:** Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
2. **Language Goal:** Explain how exponential functions change by a factor and linear functions change by a difference. **(Writing, Speaking, and Listening)**

Students explore and then generalize how linear and exponential functions change over equal *intervals*. Students use the structure of equations to help them show that $f(x+1)-f(x)$ is constant for linear functions and $\frac{g(x+1)}{g(x)}$ is constant for exponential functions. **(MTR.2.1, MTR.7.1)**

Coherence

Prior Learning

In Lessons 1–3, students compared linear functions, which have a constant rate of change or a *constant difference*, and exponential functions, which have a constant growth factor.

Future Learning

In Unit 7, students will be introduced to quadratic functions, which do not grow by constant rates of change or growth factors over equal intervals.

Connections

- Students generalize how linear and exponential functions change over equal intervals with function tables. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

MA.912.AR.5.4

Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1, MA.912.AR.5.3, MA.912.F.1.8**

Mathematical Thinking and Reasoning: MTR.2.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.7.AR.1.1
MA.8.NSO.1.3

Vocabulary

Review Vocabulary

constant difference	*constant ratio*
factor	*interval*

Lesson at a Glance

~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.F.1.2, MA.912.F.1.8

Warm-Up

Whole Class | 5 min

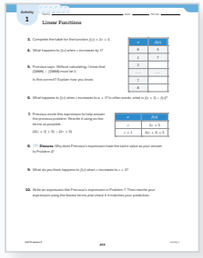
Students write equivalent expressions using a single exponent and as few terms as possible.
Routine: Think-Pair-Share


Unit 5 Lesson 5
Differences and Factors
Warm-Up

Activity 1

Pairs | 15 min

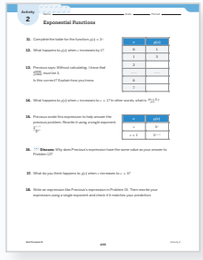
Students analyze the structure of $f(x+1)-f(x)$ to generalize and prove that linear functions grow by *constant differences* over equal *intervals*. **(MTR.2.1, MTR.5.1)**


Activity 1
Linear Functions

Activity 2

Pairs | 15 min

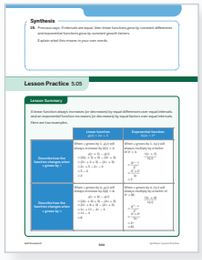
Students analyze the structure of $f(x+1)-f(x)$ to generalize and prove that exponential functions grow by constant *factors* over equal intervals. **(MTR.2.1, MTR.5.1)**


Activity 2
Exponential Functions

Synthesis

Whole Class | 5 min

Students synthesize their understanding of how linear and exponential functions change over equal intervals.


Synthesis
Lesson Practice 5.05
Lesson Summary

Show What You Know

Independent | 5 min

Students demonstrate their understanding by describing how a linear function and an exponential function change when x increases by 2.


Show What You Know
5.05

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students write equivalent expressions using a single exponent and as few terms as possible.

1 Launch

Display the expressions and the prompts.

ELL **English Language Learners** Consider creating a list of words about expressions to support students (e.g., *combine*, *simplify*, *distribute*), along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

Use the Think-Pair-Share routine to support student-to-student discourse about the expressions. Invite students to think independently and then take turns sharing with a partner. Invite students to share their or their partner's strategies with the class.

To support students getting started, consider inviting students to write out the exponents (e.g., by expanding $3^2 \cdot 3^5$ to $(3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$), or review properties of the products and quotients for exponents as a class before students begin.

A **Accessibility: Conceptual Processing** As students manipulate symbols, consider using objects like chips, coins, blocks, or other counters to represent the symbols, or explain what the expressions mean aloud (e.g., 5^{x+2} is 5 multiplied by itself an unknown number of times and then 2 more times).

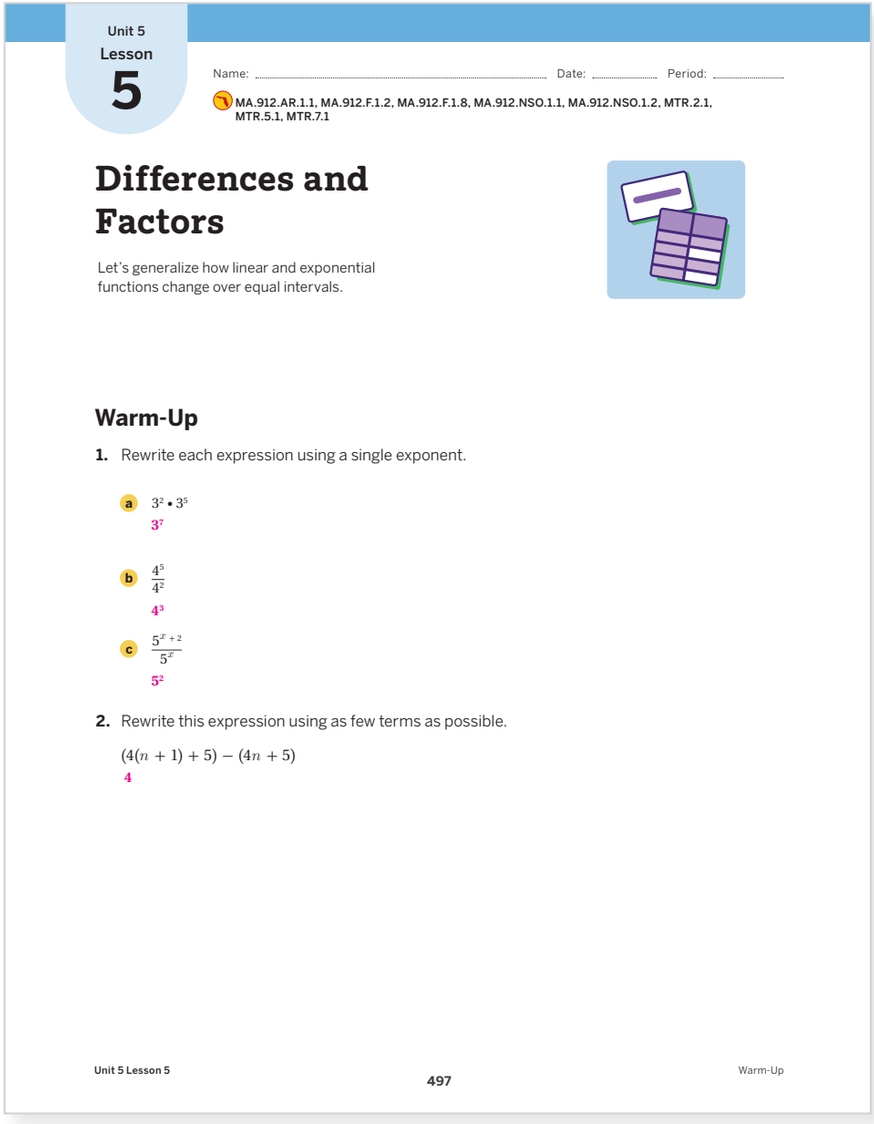
2 Connect

Invite students to rewrite each expression publicly.

Consider asking, after discussing each expression:

- "What exponent rules were helpful in these problems?"
- "How was your thinking in Problem 2 and Problem 3 alike? How was it different?"
- "What rules or strategies were most helpful in rewriting these expressions?"
- "How can we check to make sure the expression we wrote is equivalent to the original?"

Note: Consider leaving student work up for students to refer back to throughout the lesson.



Unit 5
Lesson
5

Name: Date: Period:

MA.912.AR.1.1, MA.912.F.1.2, MA.912.F.1.8, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.5.1, MTR.7.1

Differences and Factors

Let's generalize how linear and exponential functions change over equal intervals.

Warm-Up

1. Rewrite each expression using a single exponent.

a. $3^2 \cdot 3^5$
3^7

b. $\frac{4^5}{4^2}$
4^3

c. $\frac{5^{x+2}}{5^x}$
5^2

2. Rewrite this expression using as few terms as possible.

$(4(n+1)+5)-(4n+5)$
4

Unit 5 Lesson 5 497 Warm-Up

Pairs | 15 min

Activity 1 Linear Functions

Purpose: Students analyze the structure of $f(x+1)-f(x)$ to generalize and prove that linear functions grow by *constant differences* over equal *intervals*. **(MTR.2.1, MTR.5.1)**

1 Launch

Display the function and invite students to share what they know about linear functions.
They have a constant rate of change, a slope, and they look like lines on a graph. They can be written as $f(x) = mx + b$.

Encourage students to spend adequate time thinking and working with their partner through Problem 6, then invite them to check in with another pair before continuing.

2 Monitor

ELL **English Language Learners** As students process complex language, consider comparing different questions that have the same meaning (e.g., "What happens to $f(x)$ when x increases by 1?" and "What happens to $f(x)$ when x increases to $x+1$?").

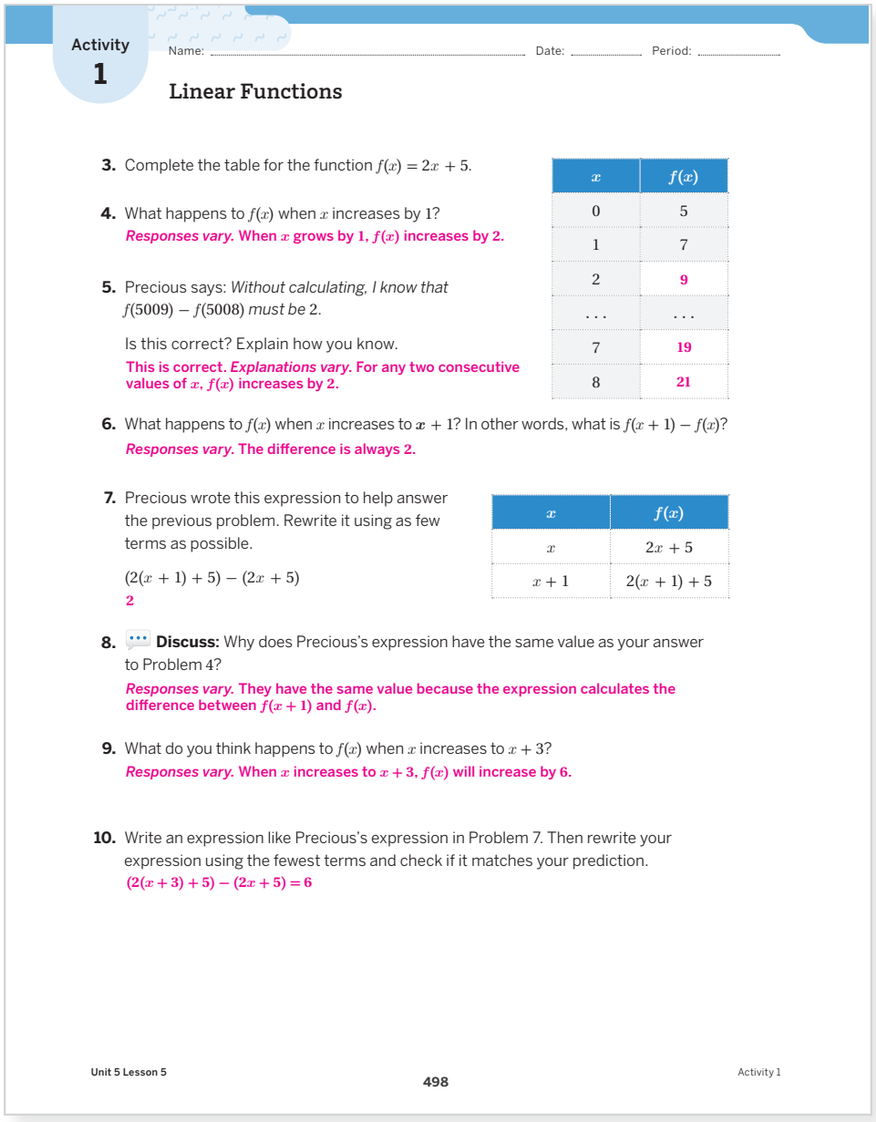
D **Differentiation** (*Problems 4–8*)

Look for students who:	Respond to Student Thinking
Are unsure whether the behavior of increasing by 2 happens for every possible value of x. *(Problems 4–6)*	**Support** Consider asking, "Can you find an example when $f(x)$ *does not* increase by the same number when x increases by 1? If not, why not?" **(MTR.2.1)**
Need support getting started or rewriting Precious's expression. *(Problem 7)*	**Support** Consider inviting them to revisit Problem 2 from the Warm-Up or working through the problem as a class.
Recognize that Precious's expression has the same value as their answer to Problem 4.	**Strengthen** Celebrate their generalization about the difference between $f(x+1)$ and $f(x)$ and invite them to share during the Connect. **(MTR.7.1)**
Would like an additional challenge.	**Stretch** Consider asking, "What happens to $f(x)$ when x increases by 2? 4? a?"

3 Connect

To surface the Key Takeaway, consider asking:

- "Where is $f(x)$ in Precious's expression? $f(x+1)$?" **(MTR.5.1)**
- "How did Precious know $f(5009) - f(5008)$ would be 2?"
- "$f(x)$ is linear. Is this true for all linear functions?" **(MTR.7.1)**
- "Is this true for intervals larger than 1? Why or why not?"
- "Can we say we *proved* linear functions grow by constant differences over equal intervals? Why or why not?"
- "When you prove something, why is it important to look for both cases that work *and* cases that might not work?"



Activity 1

Name: _____ Date: _____ Period: _____

Linear Functions

3. Complete the table for the function $f(x) = 2x + 5$.

x	$f(x)$
0	5
1	7
2	9
...	...
7	19
8	21

4. What happens to $f(x)$ when x increases by 1?
Responses vary. When x grows by 1, $f(x)$ increases by 2.

5. Precious says: *Without calculating, I know that* $f(5009) - f(5008)$ *must be* 2.
Is this correct? Explain how you know.
This is correct. *Explanations vary.* For any two consecutive values of x, $f(x)$ increases by 2.

6. What happens to $f(x)$ when x increases to $x+1$? In other words, what is $f(x+1) - f(x)$?
Responses vary. The difference is always 2.

7. Precious wrote this expression to help answer the previous problem. Rewrite it using as few terms as possible.
$(2(x+1)+5) - (2x+5)$
2

x	$f(x)$
x	$2x+5$
$x+1$	$2(x+1)+5$

8. **Discuss:** Why does Precious's expression have the same value as your answer to Problem 4?
Responses vary. They have the same value because the expression calculates the difference between $f(x+1)$ and $f(x)$.

9. What do you think happens to $f(x)$ when x increases to $x+3$?
Responses vary. When x increases to $x+3$, $f(x)$ will increase by 6.

10. Write an expression like Precious's expression in Problem 7. Then rewrite your expression using the fewest terms and check if it matches your prediction.
$(2(x+3)+5) - (2x+5) = 6$

Unit 5 Lesson 5 498 Activity 1

Key Takeaway: For a linear function $f(x)$, the expression $f(x+1) - f(x)$ has the same value as the rate of change of the function. Since this is true for any value of x, this proves that linear functions grow by *constant differences* over equal *intervals*.

Pairs | 15 min

Activity 2 Exponential Functions

Purpose: Students analyze the structure of $f(x+1) - f(x)$ to generalize and prove that exponential functions grow by constant *factors* over equal intervals. **(MTR.2.1, MTR.5.1)**

Short on time? Consider leading a whole-class discussion on Problems 17–18 and their connection to Problems 9–10.

1 Launch

Display the function and invite students to share what they know about exponential functions compared to linear functions. They grow by multiplication, not addition.

Encourage students to spend adequate time thinking and working with their partner through Problem 14, then invite them to check in with another pair before continuing.

A **Accessibility: Conceptual Processing** As students apply concepts to new situations, invite them to share with a partner how the problems in Activity 2 are different from the problems in Activity 1 before responding. **(MTR.2.1)**

2 Monitor

Note: Students may look for constant differences in this activity because of their work on Activity 1. Consider asking, "Do exponential functions change by constant differences? How would you describe how they change?" Constant ratios

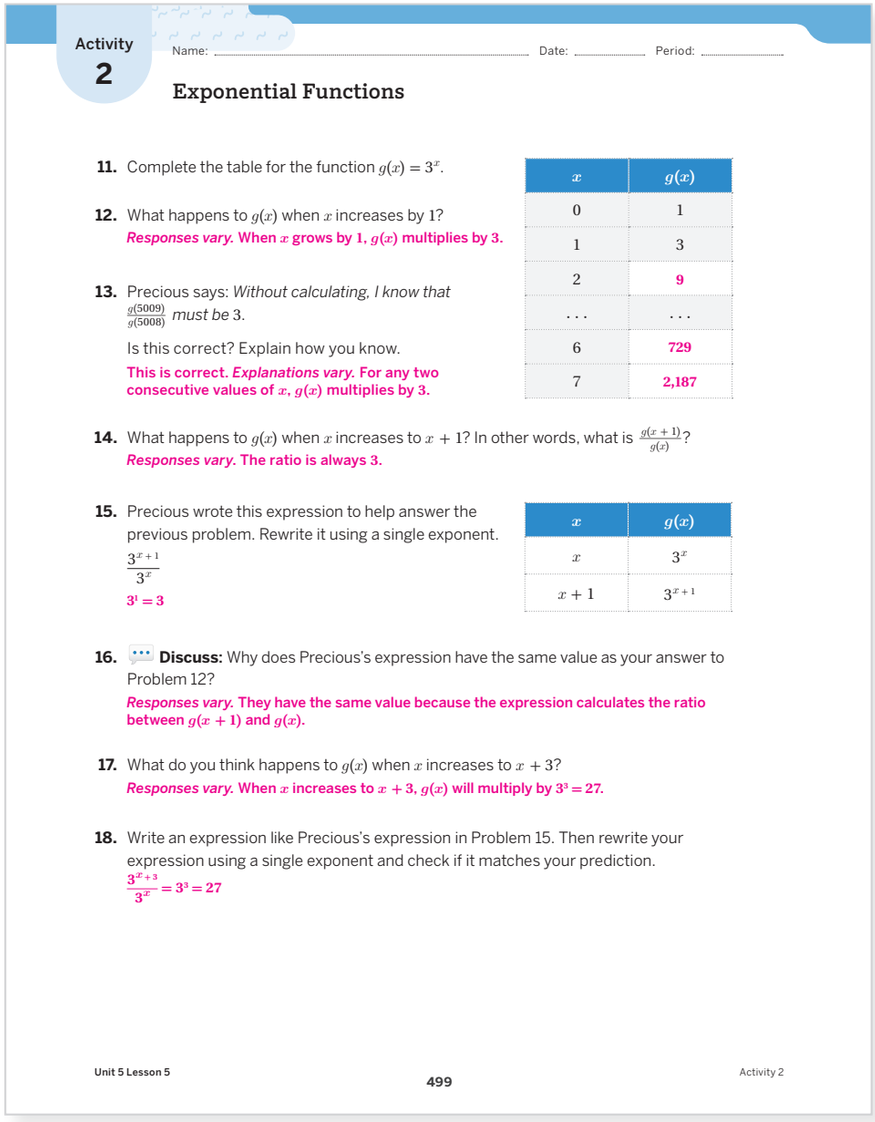
D **Differentiation** (*Problems 15–16*)

Look for students who:	Respond to Student Thinking
Need support getting started or rewriting Precious's expression. *(Problem 15)*	**Support** Consider inviting them to revisit Problem 1 from the Warm-Up or working through the problem as a class.
Recognize that Precious's expression has the same value as their answer to Problem 12.	**Strengthen** Celebrate their generalization about the ratio between $g(x+1)$ and $g(x)$ and invite them to share during the Connect. **(MTR.7.1)**
Would like an additional challenge.	**Stretch** Consider asking, "What happens to $g(x)$ when x increases by 2? 4? a?"

3 Connect

To surface the Key Takeaway, consider asking:

- "Where is $g(x)$ in Precious's expression? $g(x+1)$?" **(MTR.5.1)**
- "What was the value of Precious's expression?"
- "How is Activity 2 with exponential functions like Activity 1 with linear functions? How is it different?" **(MTR.5.1)**
- "$g(x)$ is exponential. Is this true for all exponential functions?" **(MTR.7.1)**
- "Is this true for intervals larger than 1? Why or why not?"
- "Can we say we *proved* exponential functions grow by a constant factor over equal intervals? Why or why not?"



Activity 2

Name: _____ Date: _____ Period: _____

Exponential Functions

11. Complete the table for the function $g(x) = 3^x$.

x	$g(x)$
0	1
1	3
2	9
...	...
6	729
7	2,187

12. What happens to $g(x)$ when x increases by 1?

Responses vary. When x grows by 1, $g(x)$ multiplies by 3.

13. Precious says: *Without calculating, I know that* $\frac{g(5009)}{g(5008)}$ *must be* 3.

Is this correct? Explain how you know.

This is correct. *Explanations vary.* For any two consecutive values of x, $g(x)$ multiplies by 3.

14. What happens to $g(x)$ when x increases to $x+1$? In other words, what is $\frac{g(x+1)}{g(x)}$?

Responses vary. The ratio is always 3.

15. Precious wrote this expression to help answer the previous problem. Rewrite it using a single exponent.

$\frac{3^{x+1}}{3^x}$

$3^1 = 3$

x	$g(x)$
x	3^x
$x+1$	3^{x+1}

16. **Discuss:** Why does Precious's expression have the same value as your answer to Problem 12?

Responses vary. They have the same value because the expression calculates the ratio between $g(x+1)$ and $g(x)$.

17. What do you think happens to $g(x)$ when x increases to $x+3$?

Responses vary. When x increases to $x+3$, $g(x)$ will multiply by $3^3 = 27$.

18. Write an expression like Precious's expression in Problem 15. Then rewrite your expression using a single exponent and check if it matches your prediction.

$\frac{3^{x+3}}{3^x} = 3^3 = 27$

Unit 5 Lesson 5 499 Activity 2

Key Takeaway: For an exponential function $g(x)$, the expression $\frac{g(x+1)}{g(x)}$ has the same value as the growth *factor* of the function. Since this is true for any value of x, this proves that exponential functions grow by constant growth factors over equal intervals.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how linear and exponential functions change over equal intervals.

Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:

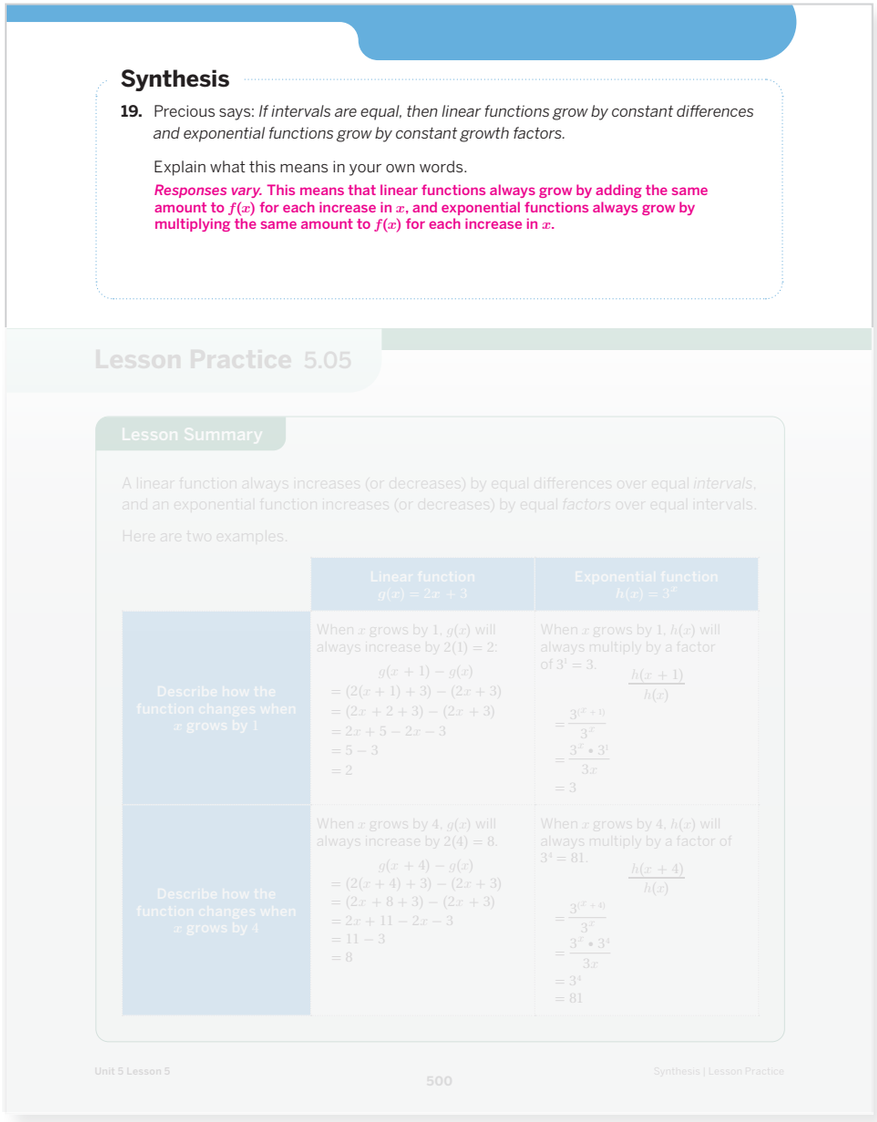
- How important it is that x be growing by the same number each time, or that the intervals are equal. **(MTR.2.1)**
- Exponential functions change by a constant ratio/growth factor and linear functions change by a constant difference/rate of change.
- Exponents are an equivalent way to write repeated multiplication and multiplication is an equivalent way to write repeated addition.
- You can use the change from $f(x)$ to $f(x+1)$ to make predictions about longer equal intervals (e.g., from $f(x)$ to $f(x+3)$).

ELL **English Language Learners** As students share in whole-class discussions, encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit.

Consider asking:

- "What connections do you see between your and your classmates' descriptions?"
- "What would you say if you had to explain this idea to a 6th grader?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

19. Precious says: *If intervals are equal, then linear functions grow by constant differences and exponential functions grow by constant growth factors.*

Explain what this means in your own words.

Responses vary. This means that linear functions always grow by adding the same amount to $f(x)$ for each increase in x, and exponential functions always grow by multiplying the same amount to $f(x)$ for each increase in x.

Lesson Practice 5.05

Lesson Summary

A linear function always increases (or decreases) by equal differences over equal *intervals*, and an exponential function increases (or decreases) by equal *factors* over equal intervals.

Here are two examples.

	Linear function $g(x) = 2x + 3$	Exponential function $h(x) = 3^x$
Describe how the function changes when x grows by 1	When x grows by 1, $g(x)$ will always increase by $2(1) = 2$: $g(x+1) - g(x)$ $= (2(x+1)+3) - (2x+3)$ $= (2x+2+3) - (2x+3)$ $= 2x+5-2x-3$ $= 5-3$ $= 2$	When x grows by 1, $h(x)$ will always multiply by a factor of $3^1 = 3$. $\frac{h(x+1)}{h(x)}$ $= \frac{3^{(x+1)}}{3^x}$ $= \frac{3^x \cdot 3^1}{3^x}$ $= 3$
Describe how the function changes when x grows by 4	When x grows by 4, $g(x)$ will always increase by $2(4) = 8$. $g(x+4) - g(x)$ $= (2(x+4)+3) - (2x+3)$ $= (2x+8+3) - (2x+3)$ $= 2x+11-2x-3$ $= 11-3$ $= 8$	When x grows by 4, $h(x)$ will always multiply by a factor of $3^4 = 81$. $\frac{h(x+4)}{h(x)}$ $= \frac{3^{(x+4)}}{3^x}$ $= \frac{3^x \cdot 3^4}{3^x}$ $= 3^4$ $= 81$

Unit 5 Lesson 5 500 Synthesis | Lesson Practice

Lesson Takeaway: Linear and exponential functions change in a predictable, consistent way: over equal *intervals*, linear functions grow by a *constant difference* and exponential functions grow by equal *factors*.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by describing how a linear function and an exponential function change when x increases by 2.

Today's Goals

1. **Goal:** Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
2. **Language Goal:** Explain how exponential functions change by a factor and linear functions change by a difference. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.05

Here is a linear function: $h(x) = 3x + 2$.

x	0	1	2	3	4
$h(x)$	2	5	8	11	14

Here is an exponential function: $k(x) = 4^x$.

x	0	1	2	3	4
$k(x)$	1	4	16	64	256

a. How does $h(x)$ change when x increases by 2?
Responses vary. When x grows by 2, $h(x)$ always increases by 6.

b. How does $k(x)$ change when x increases by 2?
Responses vary. When x grows by 2, $k(x)$ always multiplies by 16.

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of how linear and exponential functions grow over equal intervals:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to investigate what happens to $f(x)$ when x decreases by 1, or increases by values that are not whole numbers (e.g., 0.5). Consider asking them, "Does the proof still hold true? What is different about these cases?"

Professional Learning

Take a moment to reflect on how students engaged with making generalizations based on repeated reasoning in today's lesson. How did students engage in generalizing? What did students understand about the difference between analyzing, justifying, and proving a mathematical truth? As students continue to progress, how can you continue to support them when they begin to look at proofs more deeply in their next math course?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

19. Precious says: *If intervals are equal, then linear functions grow by constant differences and exponential functions grow by constant growth factors.*

Explain what this means in your own words.

Responses vary. This means that linear functions always grow by adding the same amount to $f(x)$ for each increase in x, and exponential functions always grow by multiplying the same amount to $f(x)$ for each increase in x.

Lesson Practice 5.05

Lesson Summary

A linear function always increases (or decreases) by equal differences over equal *intervals*, and an exponential function increases (or decreases) by equal *factors* over equal intervals.

Here are two examples.

	Linear function $g(x) = 2x + 3$	Exponential function $h(x) = 3^x$
Describe how the function changes when x grows by 1	When x grows by 1, $g(x)$ will always increase by $2(1) = 2$: $g(x+1) - g(x)$ $= (2(x+1)+3) - (2x+3)$ $= (2x+2+3) - (2x+3)$ $= 2x + 5 - 2x - 3$ $= 5 - 3$ $= 2$	When x grows by 1, $h(x)$ will always multiply by a factor of $3^1 = 3$. $\frac{h(x+1)}{h(x)}$ $= \frac{3^{(x+1)}}{3^x}$ $= \frac{3^x \cdot 3^1}{3x}$ $= 3$
Describe how the function changes when x grows by 4	When x grows by 4, $g(x)$ will always increase by $2(4) = 8$. $g(x+4) - g(x)$ $= (2(x+4)+3) - (2x+3)$ $= (2x+8+3) - (2x+3)$ $= 2x + 11 - 2x - 3$ $= 11 - 3$ $= 8$	When x grows by 4, $h(x)$ will always multiply by a factor of $3^4 = 81$. $\frac{h(x+4)}{h(x)}$ $= \frac{3^{(x+4)}}{3^x}$ $= \frac{3^x \cdot 3^4}{3x}$ $= 3^4$ $= 81$

Lesson Practice 5.05

Name: _____ Date: _____ Period: _____

Problems 1–2: Here are two functions. $f(x)$ is a linear function and $g(x)$ is an exponential function.

x	0	1	2	3	4	5
$f(x) = 2x + 1$	1	3	5	7	9	11

x	0	1	2	3	4	5
$g(x) = 2 \cdot 3^x$	2	6	18	54	162	486

1. How does $f(x)$ change when x grows by 1?
   Responses vary. When x grows by 1, $f(x)$ increases by 2.

2. How does $g(x)$ change when x grows by 1?
   Responses vary. When x grows by 1, $g(x)$ multiplies by 3.

Problems 3–4: Here is a function: $h(x) = 4^x$.

3. Calculate $\frac{h(x+1)}{h(x)}$.
   4

4. Calculate $\frac{h(x+2)}{h(x)}$.
   16

5. Here is a function: $g(x) = 2^x$. Select *all* of the true statements.
   - ☐ A. When the input x is increased by 1, the value of $g(x)$ increases by 2.
   - ☑ B. When the input x is increased by 1, the value of $g(x)$ multiplies by 2.
   - ☐ C. When the input x is increased by 3, the value of $g(x)$ increases by 8.
   - ☑ D. When the input x is increased by 3, the value of $g(x)$ multiplies by 8.
   - ☐ E. When the input x is increased by 3, the value of $g(x)$ multiplies by 6.

6. Whenever the x-value increases by 1, the value of $j(x)$ increases by 5. Which of these functions could be $j(x)$?
   A. $j(x) = 3x + 5$ (B.) $j(x) = 5x + 3$ C. $j(x) = 5^x$ D. $j(x) = x^5$

7. Whenever the x-value increases by 2, the value of $m(x)$ multiplies by 9. Which of these functions could be $m(x)$?
   A. $m(x) = 3x + 9$ B. $m(x) = 9x + 3$ (C.) $m(x) = 3^x$ D. $m(x) = 9^x$

Lesson Practice 5.05

Name: _____ Date: _____ Period: _____

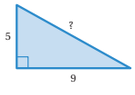
Test Practice

8. Select the description for the function $f(x) = 0.19^x$.
   A. Decreases by 19% when x increases by 1.
   B. Increases by 81% when x increases by 1.
   C. Increases by 19% when x increases by 1.
   (D.) Decreases by 81% when x increases by 1.

Spiral Review

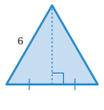
9. Determine the value of $f(x) = 15x + 4$ for each function statement.
   $f(3) =$ 49 $f(-1) =$ -11 $f\left(\frac{1}{3}\right) =$ 9

10. Here is a right triangle. What is the *exact* length of the hypotenuse?
    $\sqrt{106}$



11. Here is an equilateral triangle. The length of each side is 6 units.

    Find the *exact* height of the triangle.
    $\sqrt{27}$



Lesson Practice Item Analysis

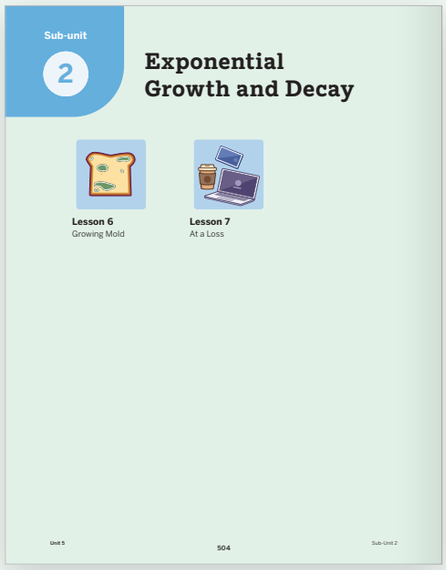
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.F.1.2
	3–5	2	MA.912.F.1.2
	6–7	2	MA.912.F.1.2
Test Practice	8	2	MA.912.F.1.2
Spiral Review			
Fluency	9	1	MA.912.F.1.2
	10	1	MA.8.GR.1.1
	11	2	MA.8.GR.1.1

Exponential Growth and Decay

Sub-Unit 2 Goals

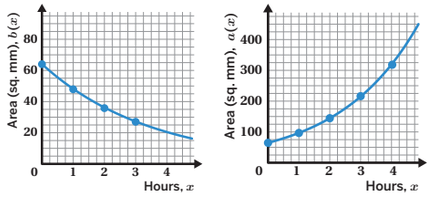
- **Goal:** Interpret and write exponential functions to model situations involving percent increase or decrease.
- **Goal:** Use exponential functions to solve problems in context.

In this sub-unit, students interpret and write functions to model situations involving repeated percent increase in order to solve problems. They learn to translate exponential functions by writing them in the form $f(x) = a \cdot b^{(x-h)} + k$, and interpret the parameters h and k in context.



Math That Matters Most

Concept or Skill	Example
Model repeated percent increase with exponential functions.	Tyler made potato salad and forgot to put it in the refrigerator. The amount of bacteria in the potato salad increases by 4% every minute and had an initial 5 cells of bacteria. Write a function $p(t)$ that represents the amount of bacteria in the potato salad after t minutes. $p(t) = 5 \cdot 1.04^t$
Write exponential functions that represent exponential decay.	Aaliyah bought a Des-Phone for $1,000. The value of the phone depreciates by 22% each year, retaining 78% of its value. Write two equivalent exponential functions to represent this situation. $f(x) = 1000 \cdot (0.78)^x$ $f(x) = 1000 \cdot (1 - .22)^x$

Concept or Skill	Example
Distinguish between situations that model exponential growth and exponential decay.	One of these functions shows exponential growth and one shows exponential decay. Which do you think is which, and why?  The graph on the left shows exponential decay because the total amount is decreasing over time. The graph on the right shows exponential growth because the total is increasing over time.
Translate exponential functions by writing them in the form $f(x) = a \cdot b^{(x-h)} + k$.	Write a function that will translate $f(x) = 2^x$ two units right and four units down. $g(x) = 2^{(x-2)} - 4$

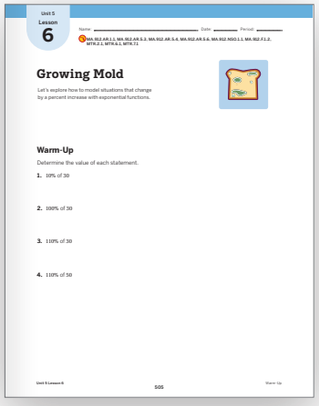
Unit 5

Lesson 6

Growing Mold

Percent Increase and Decrease, Part 1

Let's explore how to model situations that change by a percent increase with exponential functions.


Unit 5
Lesson 6
Growing Mold
Let's explore how to model situations that change by a percent increase with exponential functions.
Warm-Up
Determine the value of each statement.
1. 10% of 30
2. 100% of 30
3. 110% of 30
4. 110% of 50
505

Learning Outcomes

Today's Goals

1. **Goal:** Model repeated percent increase with exponential functions.
2. **Language Goal:** Interpret the parameters of an exponential function $f(x) = a \cdot b^x$ that represents a percent increase. **(Writing, Speaking, and Listening)**

Students use equations and tables to reason about exponential relationships that involve *percent increase* in the context of mold growth. They relate the structure of exponential function equations involving percent increase to a percent change larger than 100% and learn to rewrite the value of b in $f(x) = a \cdot b^x$ to highlight the percent increase. **(MTR.2.1, MTR.6.1)**

Coherence

Prior Learning

In Lessons 1–2, students wrote and interpreted exponential functions in the form $(x) = a \cdot b^x$, where b was either an integer or a fraction.

Future Learning

In Lesson 7, students will write functions that represent situations modeled by exponential decay involving percent decrease.

Connections

- Students explore how to model situations that change by a percent increase with exponential functions by growing mold. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.1
Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.

Also Addressing: **MA.912.NSO.1.1, MA.912.AR.5.4**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3	**MA.912.NSO.1.6**
MA.8.NSO.1.3	**MA.912.AR.5.5**
MA.8.AR.2.1	**MA.912.AR.5.7**
MA.8.AR.2.2	

Vocabulary

Review Vocabulary

base

initial value

percent increase

Lesson at a Glance

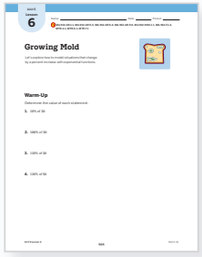
~ 45 min

MA.912.NSO.1.1, MA.912.AR.1.1, MA.912.AR.5.4

Warm-Up

Whole Class | 10 min

Students engage in a **Number Talk** routine to surface strategies for computing percentages in order to support students in determining the growth factor later in this lesson.


Unit 5 Lesson 6

6

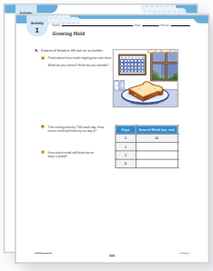
Growing Mold

Warm-Up

Activity 1

Pairs | 15 min

Students explore an exponential relationship in multiple representations that models the *percent increase* in the amount of mold that grows on a piece of bread over time. **Routine: Notice and Wonder (MTR.2.1, MTR.6.1, MTR.7.1)**

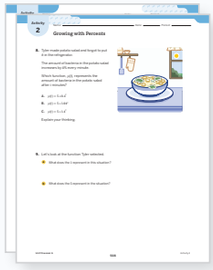

Activity 1

Growing Mold

Activity 2

Pairs | 10 min

Students create exponential function equations to model situations involving percent increase and interpret parts of the equation in context. **Routine: Decide and Defend (MTR.2.1, MTR.6.1, MTR.7.1)**

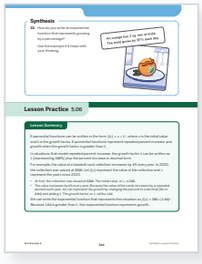

Activity 2

Growing with Percents

Synthesis

Whole Class | 5 min

Students synthesize their strategies for writing exponential functions that represent growing by a percentage. **MLR2: Collect and Display**


Synthesis

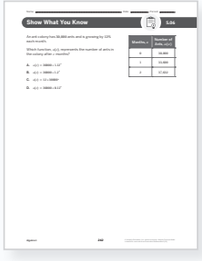
Lesson Practice 5.06

Lesson Summary

Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting an equation that models a situation and table.


Show What You Know

Edit

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Whole Class | 10 min

Warm-Up

Purpose: Students engage in a Number Talk routine to surface strategies for computing percentages in order to support students in determining the growth factor later in this lesson.

Short on time? Consider discussing fewer of the statements if students did well on Problems 7–9 on the Pre-Unit Check.

1 Launch

Draw students' attention to each problem one at a time.

Use the Number Talk routine to support students' ability to think flexibly about calculating percentages.

ELL **English Language Learners** Consider inviting students to read the statements out loud to support students with interpreting symbols. Consider discussing where students see percents in their lives. **(Reading, Speaking, and Listening)**

Invite students to determine the value of the first statement independently and then share their strategies aloud.

Create a record of each strategy that students share, along with the name of the student who shared it.

Consider asking:
- "Who thought about it in a different way?"
- "Can someone explain ___'s strategy in their own words?"

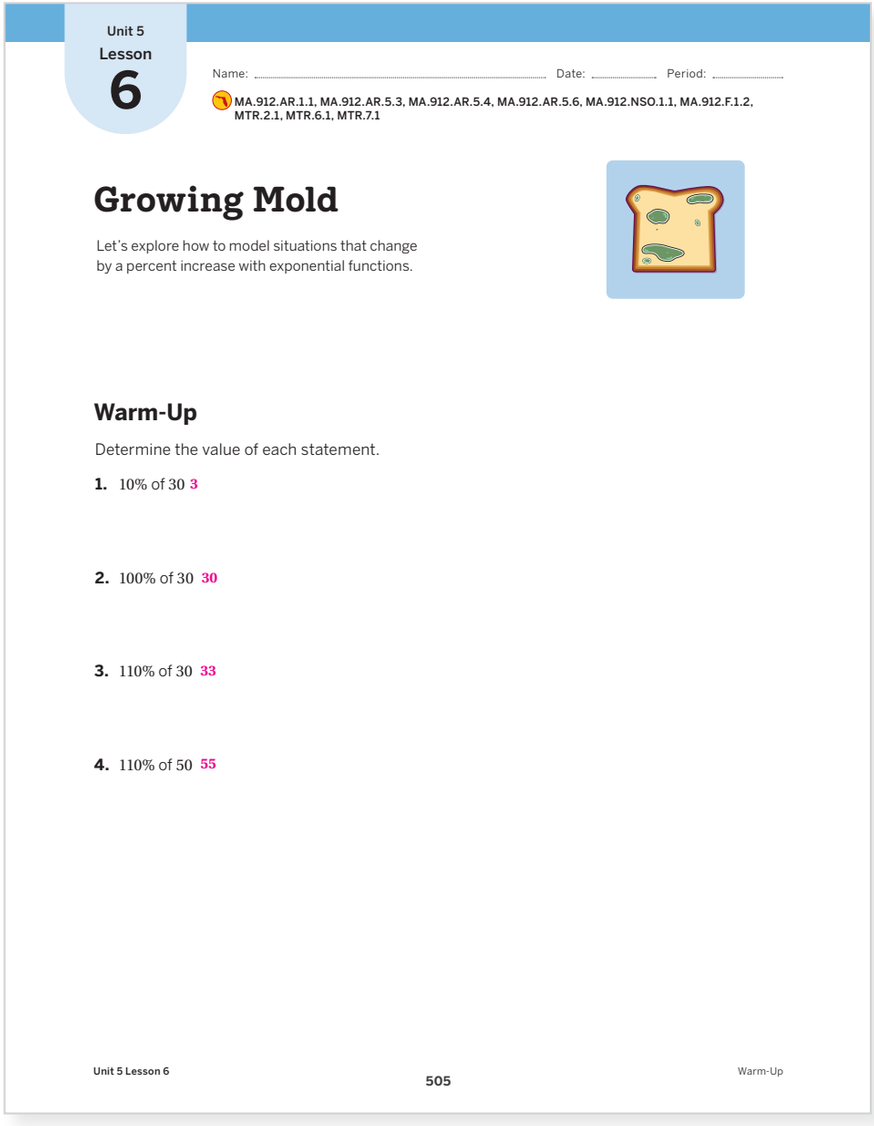
Note: Repeat this structure for each statement in the Number Talk.

2 Connect

Invite students to celebrate, compare, and contrast each other's strategies.

Consider asking:
- "What are some strategies for calculating percent change?"
- "For what percentages will the value be larger than the original value? When will it be smaller?"
- "What does taking 100% of a number do to the original value?"

To reveal prior knowledge, consider asking: "How do you write 10% in decimal form? 1%? 8%? 80%?" **(MTR.2.1)**



Unit 5
Lesson
6

Name: Date: Period:

MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.NSO.1.1, MA.912.F.1.2, MTR.2.1, MTR.6.1, MTR.7.1

Growing Mold

Let's explore how to model situations that change by a percent increase with exponential functions.

Warm-Up

Determine the value of each statement.

1. 10% of 30 3
2. 100% of 30 30
3. 110% of 30 33
4. 110% of 50 55

Unit 5 Lesson 6 505 Warm-Up

Pairs | 15 min

Activity 1 Growing Mold

Purpose: Students explore an exponential relationship in multiple representations that models the *percent increase* in the amount of mold that grows on a piece of bread over time. **(MTR.2.1, MTR. 6.1, MTR.7.1)**

Short on time? Consider encouraging students to read their classmates' responses for Problem 5 instead of facilitating a class discussion.

1 Launch

Consider asking, "Have you ever seen mold on food? What might make it grow faster or slower?"

Invite students to share their own observations of food that has molded.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the context.

ELL **English Language Learners** Create a list of words about the context that students might be unfamiliar with (e.g., *mold, bacteria, growth*), along with translations in languages used by your students. **(Reading and Listening)**

Consider asking, "Do you think mold might grow linearly, exponentially, or in some other way?"

2 Monitor

Consider using the Think-Pair- Share **routine** as students complete their calculations for the table. After individually calculating, encourage pairs to work together to confirm their answers. Validate answers as a large group.

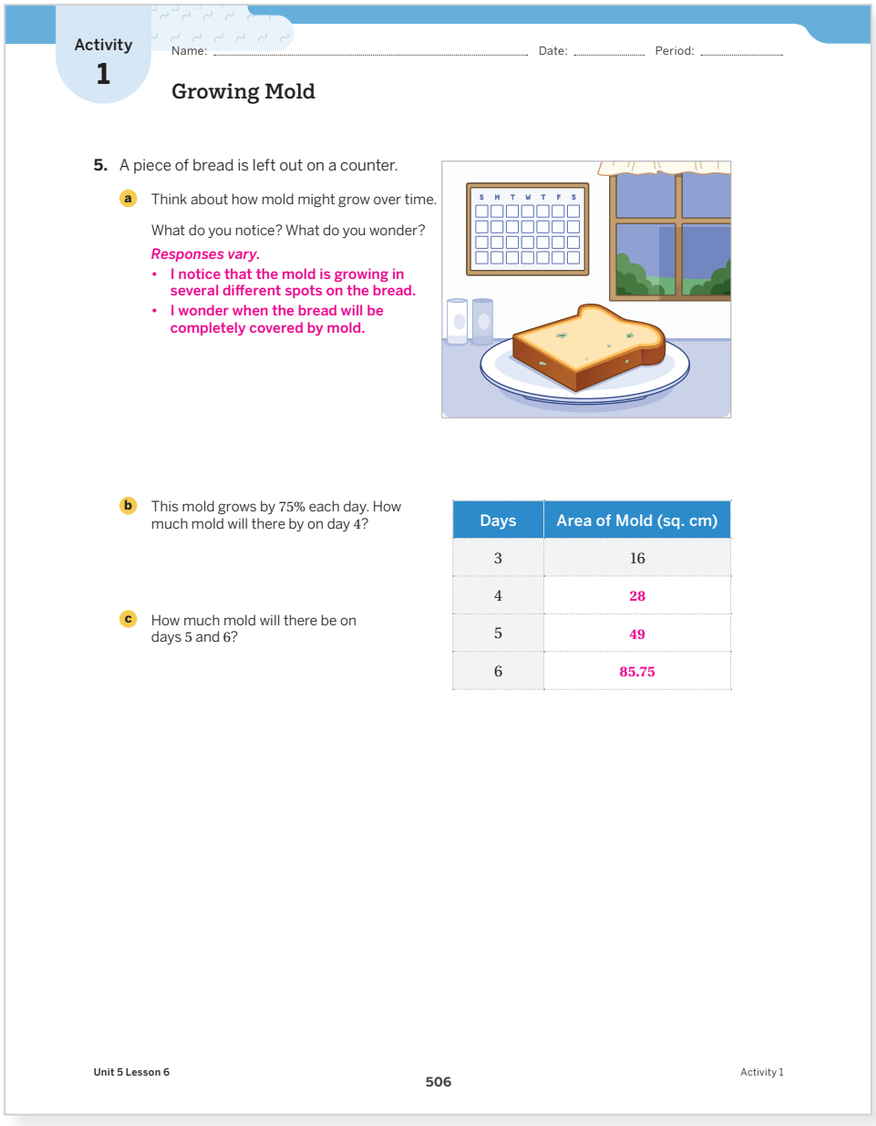
A **Accessibility: Conceptual Processing** As students think abstractly, invite them to answer a more concrete question first, like, "How can you determine 75% of 16?"

Look for and celebrate the different ways that students explain how to calculate the amount of mold there will be on day 4, such as: $(75\% \text{ of } 16) + 16$, $\left(\frac{16}{4}\right) 3 + 16$, $\left(16 \cdot \frac{3}{4}\right) + 16$ or $16 \cdot 1.75$. **(MTR.7.1)**

D **Differentiation: (Strengthen)** Consider asking pairs:

- "What strategy did you use to calculate the amount of mold?"
- "What growth factor can we multiply the current amount by to get the amount of mold on the next day every time? **(MTR.2.1)**

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Growing Mold

5. A piece of bread is left out on a counter.

a. Think about how mold might grow over time. What do you notice? What do you wonder?

Responses vary.

- I notice that the mold is growing in several different spots on the bread.
- I wonder when the bread will be completely covered by mold.

b. This mold grows by 75% each day. How much mold will there by on day 4?

c. How much mold will there be on days 5 and 6?

Days	Area of Mold (sq. cm)
3	16
4	28
5	49
6	85.75

Unit 5 Lesson 6 506 Activity 1

Pairs | 15 min

Activity 1 Growing Mold (continued)

Purpose: Students explore an exponential relationship in multiple representations that models the *percent increase* in the amount of mold that grows on a piece of bread over time. **(MTR.6.1)**

Short on time? Consider encouraging students to read their classmates' responses for Problem 5 instead of facilitating a class discussion.

2 Monitor

Encourage students to use paper and pencil to sketch as they reason through their responses.

Select and sequence a variety of student responses and sketches to share during the Connect.

Differentiation

Look for students who:	Respond to Student Thinking
Need support identifying the 75% in the table.	**Support** Invite them to use a calculator to help look for a constant growth factor.
Found 1.75 in the equation and/or the table and are confused how this relates to 75%.	**Strengthen** Celebrate that connection. Invite students to graph the equation with .75 as the base instead of 1.75 and describe what happens. **(MTR.2.1)** **Stretch** Consider asking, "What situation could $m(x) = 2.985(.75)^x$ represent?" **(MTR.6.1)**

Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which variable represents days?" **(MTR.6.1)**
Think that mold can grow forever.	**Support** Invite students to predict the average surface area of a slice of bread and compare that to what the function predicts the amount of mold will be on a day far in the future (e.g., day 43).

3 Connect

Share and discuss student responses.

To reveal the Key Takeaway, consider asking:

- "Why does there need to be a 1 in front of the 0.75?"
- "What happens when we multiply a number by 1? Why?"
- "When we multiply by 1.75, why does the amount of mold increase? What types of numbers would make it decrease?"
- "How can you see the initial amount of mold in the table, equation, and graph?"

Consider asking:

- "Is there a limit to how much mold will grow? Why?"
- "Mathematics is a tool we use to model real-world situations. What should we be mindful of when using mathematics to model real-world problems, especially those modeling exponential growth?"

Activity 1

Name: _____ Date: _____ Period: _____

Growing Mold (continued)

6. Arnav made a table to help him write a function to represent the area of mold, $m(x)$, after x days.

$m(x) = 2.985(1.75)^x$

Days	Area of Mold (sq. cm)
0	2.985
1	5.224
2	9.14
3	16
4	28

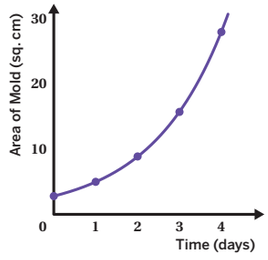
Where do you see the 75% increase represented in the function's equation? In the table?

Responses vary.

Equation: I see the 75% represented as the decimal 0.75 after the 1 in the base of the exponential equation.

Table: I found 1.75 in the table by dividing the amount of mold on day 4 by the amount of mold on day 3. The growth factor is constant when I continue to divide an amount by the previous amount.

7. Here is the graph of $m(x) = 2.985(1.75)^x$.



a. Determine how much mold there will be after 10 days.

Approximately 804.127 sq. cm

b. **Discuss:** Do you think the mold could grow according to $m(x)$ forever?

Responses vary. No, I think eventually there won't be any more area on the bread, so the graph will just be horizontal.

Unit 5 Lesson 6 507 Activity 1

Key Takeaway: Repeated percent increase scenarios are modeled by exponential functions. The percent increase is defined by the base, b, of an equation in the form $f(x) = a \cdot b^x$, which is the constant growth factor.

Pairs | 10 min

Activity 2 Growing with Percents

Purpose: Students create exponential function equations to model situations involving percent increase and interpret parts of the equation in context. **(MTR.2.1, MTR.6.1, MTR.7.1)**

Short on time? Consider skipping the Connect if students demonstrate strong understanding on Problem 10.

1 Launch

To reveal connections, consider asking:

- "How long do you think it takes potato salad to begin growing bacteria?"
- "What ingredients in potato salad could create mold given the right conditions?" Perishable items such as mayonnaise, eggs, or potatoes all develop mold quickly when left in a warm environment.

2 Monitor

Select and sequence a variety of student responses to share during the brief discussion.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to use a graphing calculator to graph each function $p(t)$ and describe what they notice. Consider asking, "Why is it important to have a 1 in the base?"
Select $p(t) = 5 \cdot (1.4)^t$.	**Support** Celebrate that they selected a function that modeled mold growth. Consider asking, "How do you write 4% as a decimal?"
Select $p(t) = 5 \cdot (1.04)^t$.	**Strengthen** Consider asking, "What does the 5 represent about the situation? The 1.04?" **(MTR.6.1)**

Pause briefly to discuss the functions chosen by students. **(MTR.2.1, MTR.6.1)**

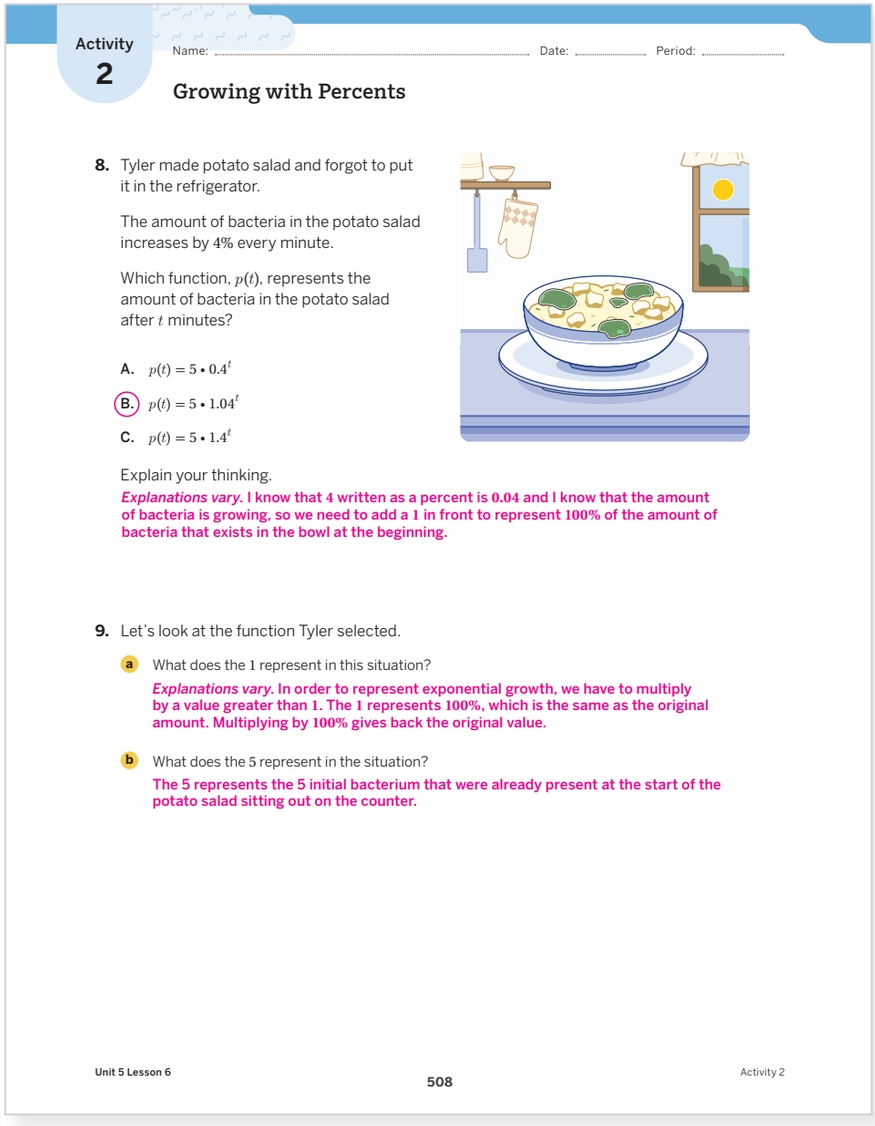
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information.

Consider asking pairs of students "What is the percent increase for each equation?"

Draw students' attention to Problem 9. Confirm all students agree with the correct function that models the scenario in Problem 8.

A Accessibility: Conceptual Understanding
Invite students to answer a more concrete question first, like, "What would happen if you multiplied the initial amount of bacteria by 1?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Growing with Percents

8. Tyler made potato salad and forgot to put it in the refrigerator.

The amount of bacteria in the potato salad increases by 4% every minute.

Which function, $p(t)$, represents the amount of bacteria in the potato salad after t minutes?

A. $p(t) = 5 \cdot 0.4^t$

B. $p(t) = 5 \cdot 1.04^t$

C. $p(t) = 5 \cdot 1.4^t$

Explain your thinking.

Explanations vary. I know that 4 written as a percent is 0.04 and I know that the amount of bacteria is growing, so we need to add a 1 in front to represent 100% of the amount of bacteria that exists in the bowl at the beginning.

9. Let's look at the function Tyler selected.

a. What does the 1 represent in this situation?

Explanations vary. In order to represent exponential growth, we have to multiply by a value greater than 1. The 1 represents 100%, which is the same as the original amount. Multiplying by 100% gives back the original value.

b. What does the 5 represent in the situation?

The 5 represents the 5 initial bacterium that were already present at the start of the potato salad sitting out on the counter.

Unit 5 Lesson 6 508 Activity 2

Pairs | 10 min

Activity 2 Growing with Percents (continued)

Purpose: Students create exponential function equations to model situations involving percent increase and interpret parts of the equation in context. **(MTR.2.1, MTR.6.1, MTR.7.1)**

Short on time? Consider skipping the Connect if students demonstrate strong understanding on Problem 10.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement.

ELL **English Language Learners** Invite students to read the two situations aloud and paraphrase what they mean. Clarify any unfamiliar words. **(Reading)**

A **Accessibility: Executive Functioning** Invite students to reflect on how the functions are similar and different before responding.

Listen for connections students make between the structure of the equations to the situations and tables like:

- The coefficient in the equation with *starts with* in the situation or the y-value when $x = 0$ in the table.
- The base of the equation with *grows by* in the situation or the growth factor in the table. **(MTR.2.1, MTR.6.1)**

To reveal connections, consider asking pairs of students:

- "Where can you see the percent increase?"
- "Which equation has the largest percent increase?"
- "Which equation has the largest/smallest initial amount?"
- "How would the equation change if the percent were 8.5% instead of 85%?"

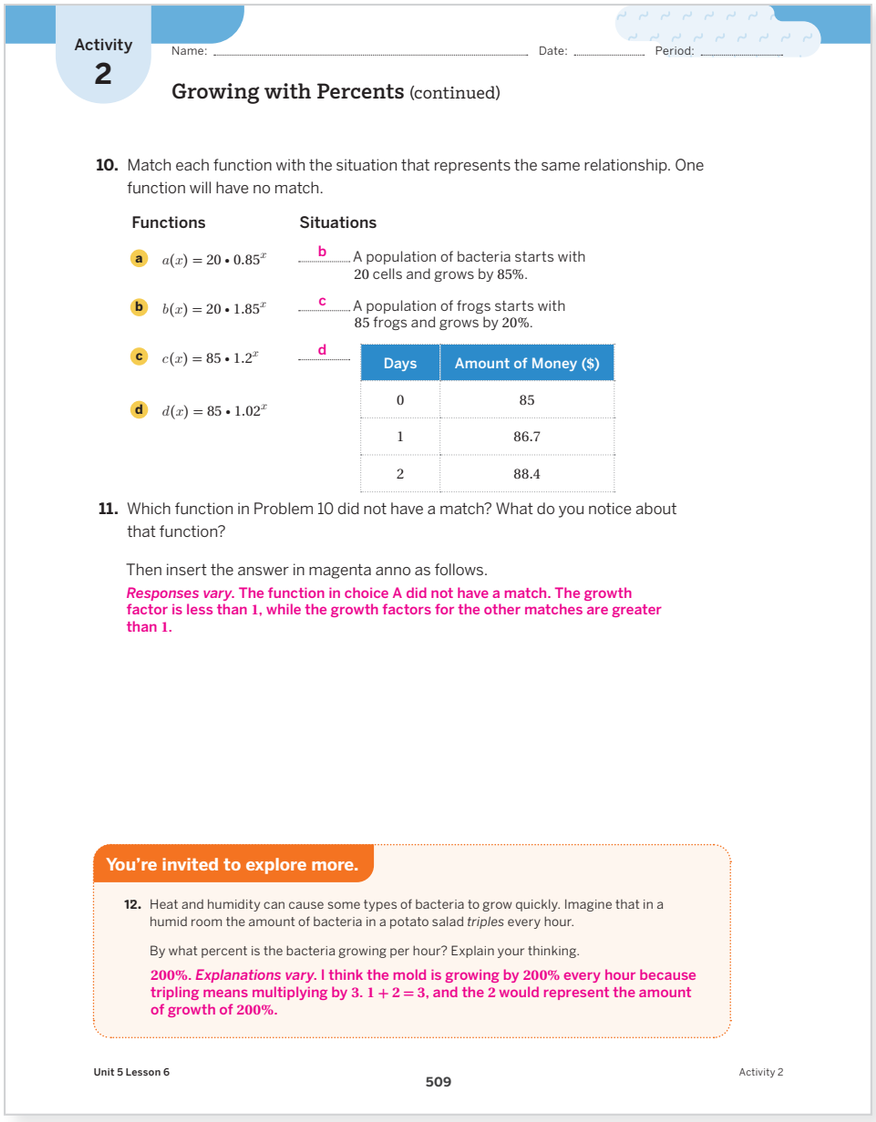
D **Differentiation: (Stretch)** Invite students to create a table and situation to match the left over card.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore exponential relationships further to complete this optional task and discuss their thinking with a partner.

3 Connect

Invite students to share the connections they see between tables, situations, and equations of functions.

Consider asking, "For what values of b in the equation $f(x) = a \cdot b^x$ will we see exponential growth?"



Activity 2

Name: _____ Date: _____ Period: _____

Growing with Percents (continued)

10. Match each function with the situation that represents the same relationship. One function will have no match.

Functions	Situations
a $a(x) = 20 \cdot 0.85^x$	b A population of bacteria starts with 20 cells and grows by 85%.
b $b(x) = 20 \cdot 1.85^x$	c A population of frogs starts with 85 frogs and grows by 20%.
c $c(x) = 85 \cdot 1.2^x$	d
d $d(x) = 85 \cdot 1.02^x$	

Days	Amount of Money ($)
0	85
1	86.7
2	88.4

11. Which function in Problem 10 did not have a match? What do you notice about that function?

Then insert the answer in magenta anno as follows.

Responses vary. The function in choice A did not have a match. The growth factor is less than 1, while the growth factors for the other matches are greater than 1.

You're invited to explore more.

12. Heat and humidity can cause some types of bacteria to grow quickly. Imagine that in a humid room the amount of bacteria in a potato salad *triples* every hour.

By what percent is the bacteria growing per hour? Explain your thinking.

200%. *Explanations vary.* I think the mold is growing by 200% every hour because tripling means multiplying by 3. $1 + 2 = 3$, and the 2 would represent the amount of growth of 200%.

Unit 5 Lesson 6 509 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for writing exponential functions that represent growing by a percentage.

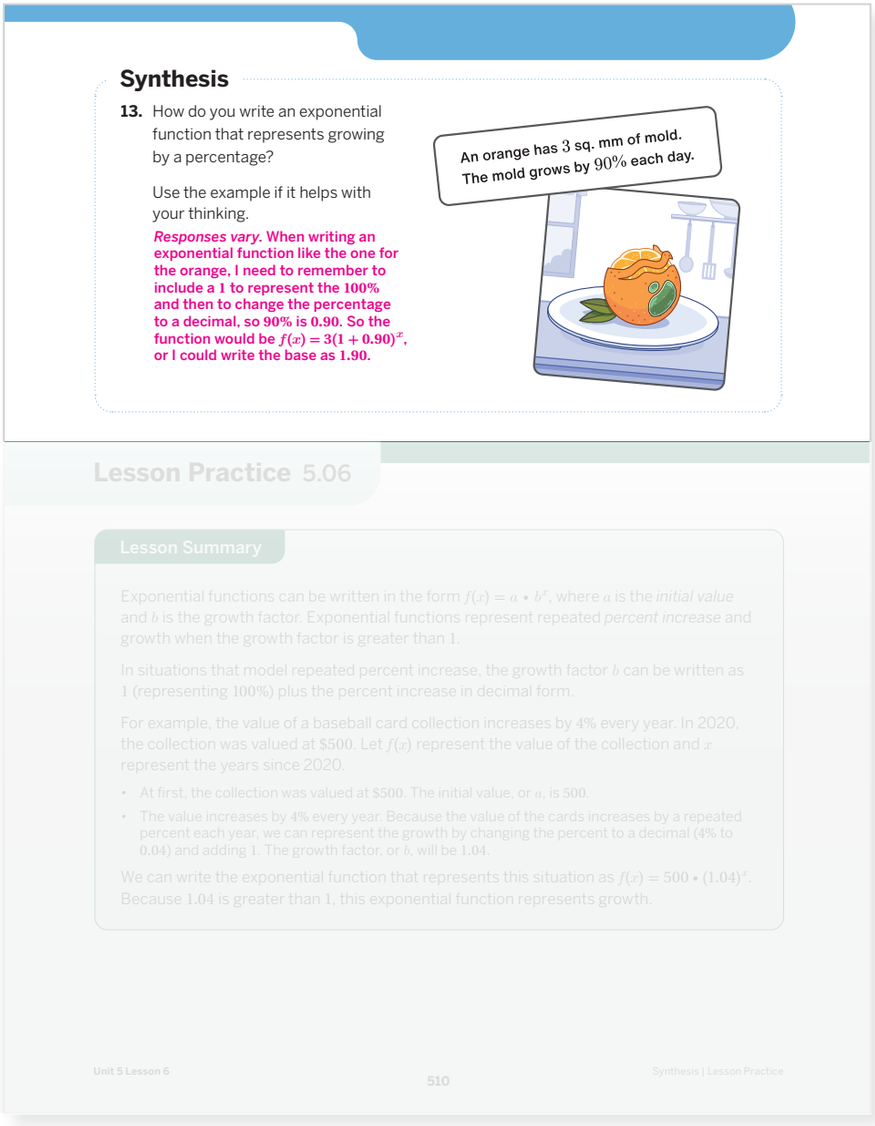
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- For percent increase, include a 1 to represent the 100%.
- Rewrite a percentage as a decimal.
- Multiplying by the same value each time (growth factor) is modeled by an exponential function.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas, examples, and equation structures for writing an exponential function that represents growing by a percentage. Consider showing connections between the equation structure, the situation, graph, and table. Display for students to reference and add to for the rest of the unit. **(MTR.2.1, MTR.6.1)**

D **Differentiation: (Stretch)** Invite students to think about the equation $f(x) = 5 \cdot 2^x$. Consider asking, "How could you rewrite this function to show the percent increase?" **(MTR.2.1)**

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. How do you write an exponential function that represents growing by a percentage?

Use the example if it helps with your thinking.

Responses vary. When writing an exponential function like the one for the orange, I need to remember to include a 1 to represent the 100% and then to change the percentage to a decimal, so 90% is 0.90. So the function would be $f(x) = 3(1 + 0.90)^x$, or I could write the base as 1.90.

Lesson Practice 5.06

Lesson Summary

Exponential functions can be written in the form $f(x) = a \cdot b^x$, where a is the *initial value* and b is the growth factor. Exponential functions represent repeated *percent increase* and growth when the growth factor is greater than 1.

In situations that model repeated percent increase, the growth factor b can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at $\$500$. Let $f(x)$ represent the value of the collection and x represent the years since 2020.

- At first, the collection was valued at $\$500$. The initial value, or a, is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or b, will be 1.04.

We can write the exponential function that represents this situation as $f(x) = 500 \cdot (1.04)^x$. Because 1.04 is greater than 1, this exponential function represents growth.

Unit 5 Lesson 6 510 Synthesis | Lesson Practice

Lesson Takeaway: Exponential functions in the form form $f(x) = a \cdot b^x$ represent situations with percent increase when $b > 1$. The growth factor b can be written as 1 (representing 100%) plus plus the percent increase in decimal form.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting an equation that models a situation and table. **(MTR.6.1)**

Today's Goals

1. **Goal:** Model repeated percent increase with exponential functions.
2. **Language Goal:** Interpret the parameters of an exponential function $f(x) = a \cdot b^x$ that represents a percent increase. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.06

An ant colony has 30,000 ants and is growing by 12% each month.

Which function, $a(x)$, represents the number of ants in the colony after x months?

A. $a(x) = 30000 \cdot 1.12^x$

B. $a(x) = 30000 \cdot 1.2^x$

C. $a(x) = 12 \cdot 30000^x$

D. $a(x) = 30000 \cdot 0.12^x$

Months, x	Number of Ants, $a(x)$
0	30,000
1	33,600
2	37,632

Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of calculating percentages:

» **Respond to Student Thinking:**

- Revisit the **Pre-Unit Check**, Problems 7–9.
- Revisit **Grade 6, Unit 3, Lesson 11** (Cost Breakdown).
- Revisit **Grade 7, Unit 4, Lesson 2** (All the Equations).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Inviting them to revisit one of the situations from the lesson and sketch what they think a realistic graph of the mold might look like since it cannot grow exponentially forever.

Professional Learning

Take a moment to reflect on how students looked for and made use of structure today. What previous math skills did students utilize in class (e.g., knowledge of *percent increase*, writing percentages as decimals, writing exponential expressions to represent geometric sequences)? What connections did students make between their prior knowledge and the equation structure $f(x) = a \cdot b^x$, especially when thinking about how the value of b can be used to determine percent increase? How can you continue to help students make sense of this equation structure as they model exponential situations involving percent decrease tomorrow? **(MTR.2.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. How do you write an exponential function that represents growing by a percentage?

Use the example if it helps with your thinking.

Responses vary. When writing an exponential function like the one for the orange, I need to remember to include a 1 to represent the 100% and then to change the percentage to a decimal, so 90% is 0.90. So the function would be $f(x) = 3(1 + 0.90)^x$, or I could write the base as 1.90.

Lesson Practice 5.06

Lesson Summary

Exponential functions can be written in the form $f(x) = a \cdot b^x$, where a is the *initial value* and b is the growth factor. Exponential functions represent repeated *percent increase* and growth when the growth factor is greater than 1.

In situations that model repeated percent increase, the growth factor b can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at \$500. Let $f(x)$ represent the value of the collection and x represent the years since 2020.

- At first, the collection was valued at \$500. The initial value, or a, is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or b, will be 1.04.

We can write the exponential function that represents this situation as $f(x) = 500 \cdot (1.04)^x$. Because 1.04 is greater than 1, this exponential function represents growth.

Lesson Practice 5.06

Name: _____ Date: _____ Period: _____

Problems 1–3: A group of biologists tracked the number of deer in a forest over several years. There were 600 deer when they first counted. The population has increased by 15% each year.

1. How many deer are in the forest 1 year after the biologists first counted?
690 deer

2. Write an expression that represents the deer population after 3 years.
$600 \cdot 1.15^3$ (or equivalent)

3. Write an expression that represents the deer population after t years.
$600 \cdot 1.15^t$ (or equivalent)

Problems 4–5: Sai gets a \$500 loan from the bank with an annual interest rate of 6%.

4. Write a function, $f(t)$, to represent the amount Sai will owe, in dollars, after t years.
$f(t) = 500 \cdot 1.06^t$

5. Complete the table to determine how much money Sai will owe over time if no payments are made.

Time (yr)	Amount Owed (\$)
0	500
1	**530**
2	**561.80**
3	**595.51**
4	**631.24**

Problems 6–8: Three cities have the same initial population and different percent increases each year. Match each function $p(t)$, representing the population after t years, with its correct description.

$p(t) = 5000 \cdot (1.20)^t$ $p(t) = 5000 \cdot (1.02)^t$ $p(t) = 5000 \cdot (1.002)^t$

6. City A has a 0.2% annual increase in population.
$p(t) = 5000 \cdot (1.002)^t$

7. City B has a 20% annual increase in population.
$p(t) = 5000 \cdot (1.20)^t$

8. City C has a 2% annual increase in population.
$p(t) = 5000 \cdot (1.02)^t$

Lesson Practice 5.06

Name: _____ Date: _____ Period: _____

Test Practice

9. Sothy's family paid \$1,300 in property tax last year. This year, the county will increase the property tax by 2.1%.

Select *all* the expressions that represent Sothy's family's property taxes this year.

- ☐ **A.** $1300 + (1.021)$
- ☐ **B.** $1300(1.21)$
- ☑ **C.** $1300(1.021)$
- ☐ **D.** $1300(1.0021)$
- ☑ **E.** $1300 + 1300(0.021)$

Spiral Review

10. Order these values from least to greatest.

75% of 12 25% of 32 50% of 20 10% of 95

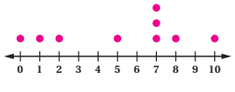
25% of 32	**75% of 12**	**10% of 95**	**50% of 20**

Least **Greatest**

11. Create a dot plot that has:

- At least 5 data points.
- A median of 7.
- A mean that is less than the median.

Responses vary.



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.AR.1.1
	4–5	2	MA.912.AR.5.4
	6–8	3	MA.912.AR.5.4
Test Practice	9	2	MA.912.AR.5.4
Spiral Review			
	10	1	MA.6.AR.3.4
Fluency	11	2	MA.7.DP.1.1 MA.7.DP.1.5

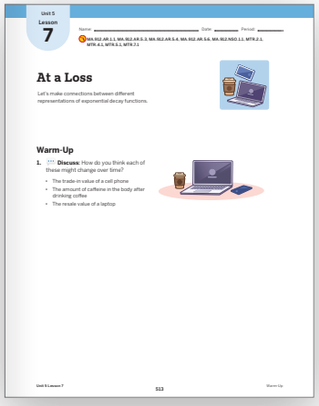
Unit 5

Lesson 7

At a Loss

Percent Increase and Decrease, Part 2

Let's make connections between different representations of exponential decay functions.



Learning Outcomes

Today's Goals

1. **Goal:** Calculate percent change of exponential functions from a graph.
2. **Goal:** Write equivalent functions that represent exponential decay.
3. **Language Goal:** Compare and contrast equivalent equations of functions that represent exponential decay. **(Writing, Speaking, and Listening)**

Students make connections between different representations of functions that model exponential decay. They compare and contrast equivalent forms of equations that represent exponential decay, one written to highlight *percent decrease* and the other written to highlight percent remaining. Students also write equations from graphs of functions that represent exponential decay.

Coherence

Prior Learning

In Grade 7, students calculated the percent change for situations that involve adding or subtracting by a percent. In Lesson 7, students analyzed equations for exponential functions that involve percent increase.

Future Learning

In Lesson 9, students will use equations of exponential functions that involve percent change to solve problems. In Lessons 10–11, students will investigate the impact on financial accounts of repeatedly applying percent change in the context of compound interest.

Connections

- Students explore representations of exponential decay functions through the value of phones over time. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.5.4

Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

MA.912.AR.1.1

Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.

Also Addressing: **MA.912.NSO.1.1, MA.912.AR.5.6**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3	**MA.912.AR.5.5**
MA.8.AR.3	**MA.912.AR.5.7**
	MA.912.AR.8.2

Vocabulary

Review Vocabulary

base

initial value

percent decrease

Lesson at a Glance

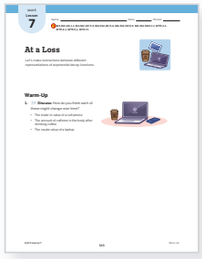
~ 45 min

MA.912.NSO.1.1, MA.912.AR.1.1, MA.912.AR.5.4, MA.912.AR.5.6

Warm-Up

Pairs | 5 min

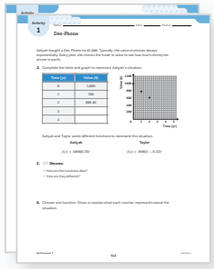
Students make predictions about how different quantities will change over time.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

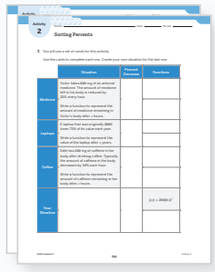
Students compare and contrast equivalent equations that represent exponential decay, one written to highlight *percent decrease* and the other to highlight percent remaining.



Activity 2

Pairs | 15 min

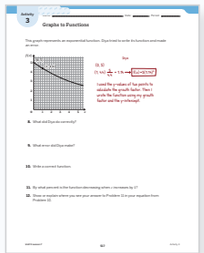
Students make connections between exponential situations that involve percent decrease and the equivalent forms of equations that represent them.



Activity 3

Pairs | 5 min

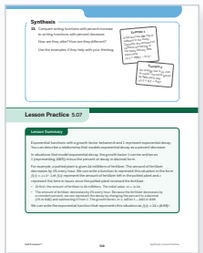
Students analyze a common mistake when writing equations of exponential functions.
MLR3: Critique, Correct, Clarify



Synthesis

Whole Class | 5 min

Students synthesize their understanding by comparing and contrasting writing an equation representing percent increase and an equation representing percent decrease.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an exponential function that represents percent decrease and using that function to make a prediction.



Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students make predictions about how different quantities will change over time.

1 Launch

Display the image and the prompt.

A **Accessibility: Memory and Attention** Encourage students to record their ideas in their Student Edition to help show their thinking.

Use the Think-Pair-Share routine to help students shape their ideas about the situations. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How do you know?

(Writing and Speaking)

Invite students to take two minutes of quiet work time, followed by a partner discussion.

D **Differentiation: (Stretch)** Consider challenging students to draw a graph of what they think each situation would look like.

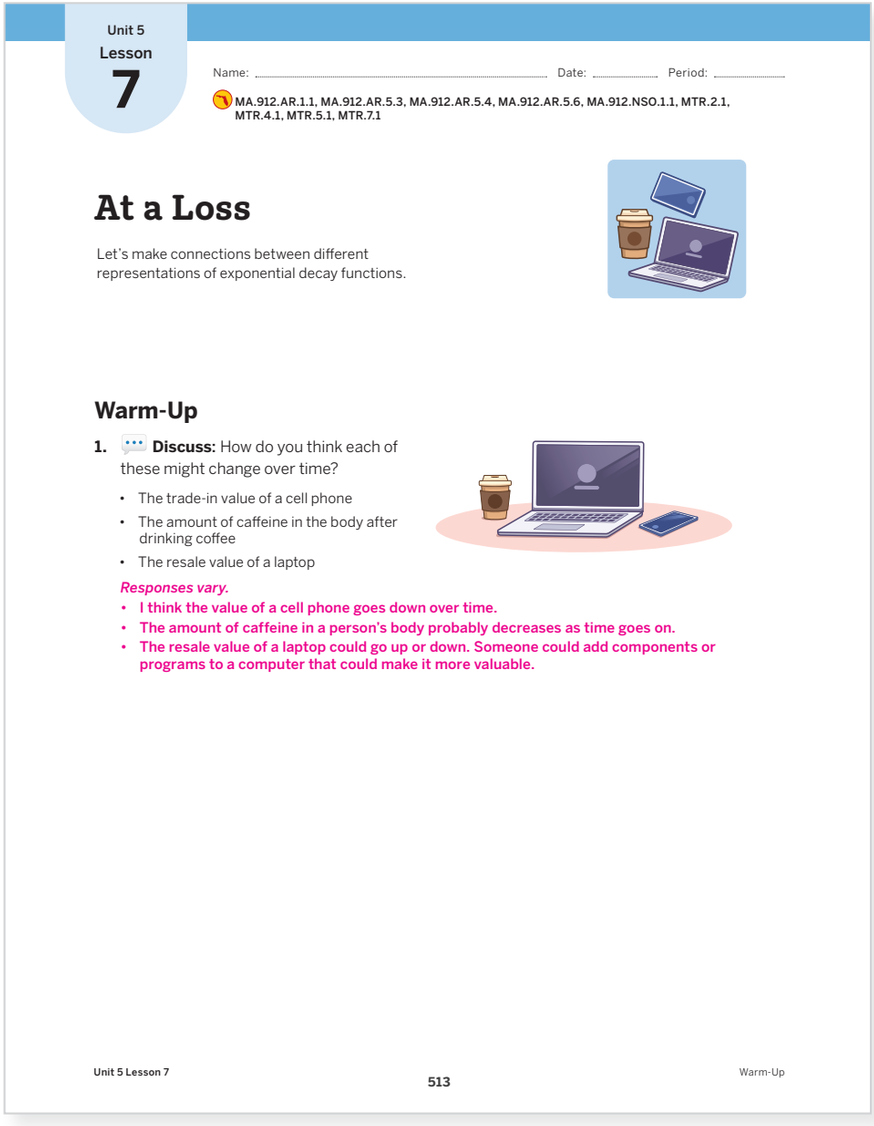
2 Connect

Invite students to share their ideas and graphs with the class.

Consider asking:

- "How are each of these situations alike? How are they different?"
- "What information did you use to help you think about each situation?"

Note: It's okay — even desirable — to lack consensus at this stage. The lesson will build toward consensus throughout the later activities.



Unit 5
Lesson
7

Name: _____ Date: _____ Period: _____

MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.NSO.1.1, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

At a Loss

Let's make connections between different representations of exponential decay functions.

Warm-Up

1. **Discuss:** How do you think each of these might change over time?
   - The trade-in value of a cell phone
   - The amount of caffeine in the body after drinking coffee
   - The resale value of a laptop

Responses vary.

- I think the value of a cell phone goes down over time.
- The amount of caffeine in a person's body probably decreases as time goes on.
- The resale value of a laptop could go up or down. Someone could add components or programs to a computer that could make it more valuable.

Unit 5 Lesson 7 513 Warm-Up

Pairs | 10 min

Activity 1 Des-Phone

Purpose: Students compare and contrast equivalent equations that represent exponential decay, one written to highlight *percent decrease* and the other to highlight percent remaining.

1 Launch

Display the table, graph, and situation.

Consider asking:
- "What could be the current price of a phone?"
- "What happens when you trade in a phone to get an upgrade?" **(MTR.7.1)**

Invite students to use a graphing calculator to support them with performing calculations.

ELL **English Language Learners** To help students access background knowledge, invite them to share what they know about trading in goods. Consider explaining phrases like "trade-in value" or "retains __% of its value" to increase access to the task.

2 Monitor

Look for and celebrate the different ways that students solve Problem 2. Some students may:
- Use a graphing calculator.
- Determine the growth factor to continue the table of values.
- Divide two consecutive values to determine the growth factor.
- Evaluate using either Aaliyah's or Taylor's equation.

Pause briefly after students have had time to discuss Problem 3 and complete Problem 4. Consider asking:
- "Are Aaliyah's and Taylor's equations equivalent? How do you know?"
- "Where do you see the 0.78 in Aaliyah's equation on the graph? In the table? What about the 0.22 in Taylor's equation?" **(MTR.2.1, MTR.5.1)**
- "One of these two equations shows the percent remaining and one shows the percent decrease. Which do you think is which and why?" **(MTR.2.1, MTR.5.1)**
- "How is Taylor's equation like the ones we saw in Lesson 7 about mold? How is it different?"

Activity 1 continued >

Activity 1

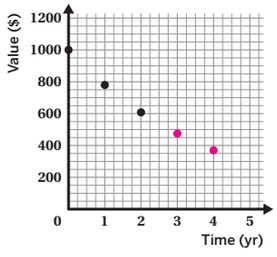
Name: Date: Period:

Des-Phone

Aaliyah bought a Des-Phone for $1,000. Typically, the value of phones decays exponentially. Every year, she checks the trade-in value to see how much money her phone is worth.

2. Complete the table and graph to represent Aaliyah's situation.

Time (yr)	Value ($)
0	1,000
1	780
2	608.40
3	474.55
4	370.15



Aaliyah and Taylor wrote different functions to represent this situation.

Aaliyah

$f(x) = 1000(0.78)^x$

Taylor

$f(x) = 1000(1 - 0.22)^x$

3. **Discuss:**
- How are the functions alike?
- How are they different?

Responses vary.
- Both equations have the same initial value.
- Both equations show that the phone retains 78% of its value.
- Taylor's equation shows the percent decrease of the value of the phone each year.

4. Choose one function. Show or explain what each number represents about the situation.

Responses vary.
- In Aaliyah's function, the 1,000 represents the initial value of the phone. 0.78 represents the value the phone is retaining each year.
- In Taylor's function, the 1,000 represents the initial value of the phone. The $1 - 0.22$ represents 100% of the original amount minus the percentage the value of the phone is decreasing each year.

Unit 5 Lesson 7 514 Activity 1

Pairs | 10 min

Activity 1 Des-Phone (continued)

Purpose: Students compare and contrast equivalent equations that represent exponential decay, one written to highlight *percent decrease* and the other to highlight percent remaining.

2 Monitor

Consider asking, to support students starting Problem 5, "How could you use the function you discussed in Problem 4 to calculate the value after 10 years?"

Look for a variety of strategies students use to calculate the trade-in value of the phone after 10 years. Invite students with different strategies to share during the Connect.

Consider asking, as you check in with groups after Problem 5, "Is the value you calculated close to what you expected?"

D Differentiation *(Problem 6)*

Look for students who:	Respond to Student Thinking
Think the statement "The value of the phone is decreasing by 78% each year." is true because of the 0.78 in Aaliyah's function.	**Support** Celebrate their identification of the 0.78 as an important piece of information. Consider sharing that decreasing by 78% would mean the value is decreasing by more than half each time; consider asking, "Is that what is happening here? How might the $1 - 0.22$ in Taylor's function be helpful to consider to understand the statements?"
Identify the true statements.	**Stretch** Encourage students to write their own statements about the situation – one true and one false – and share them with a partner.

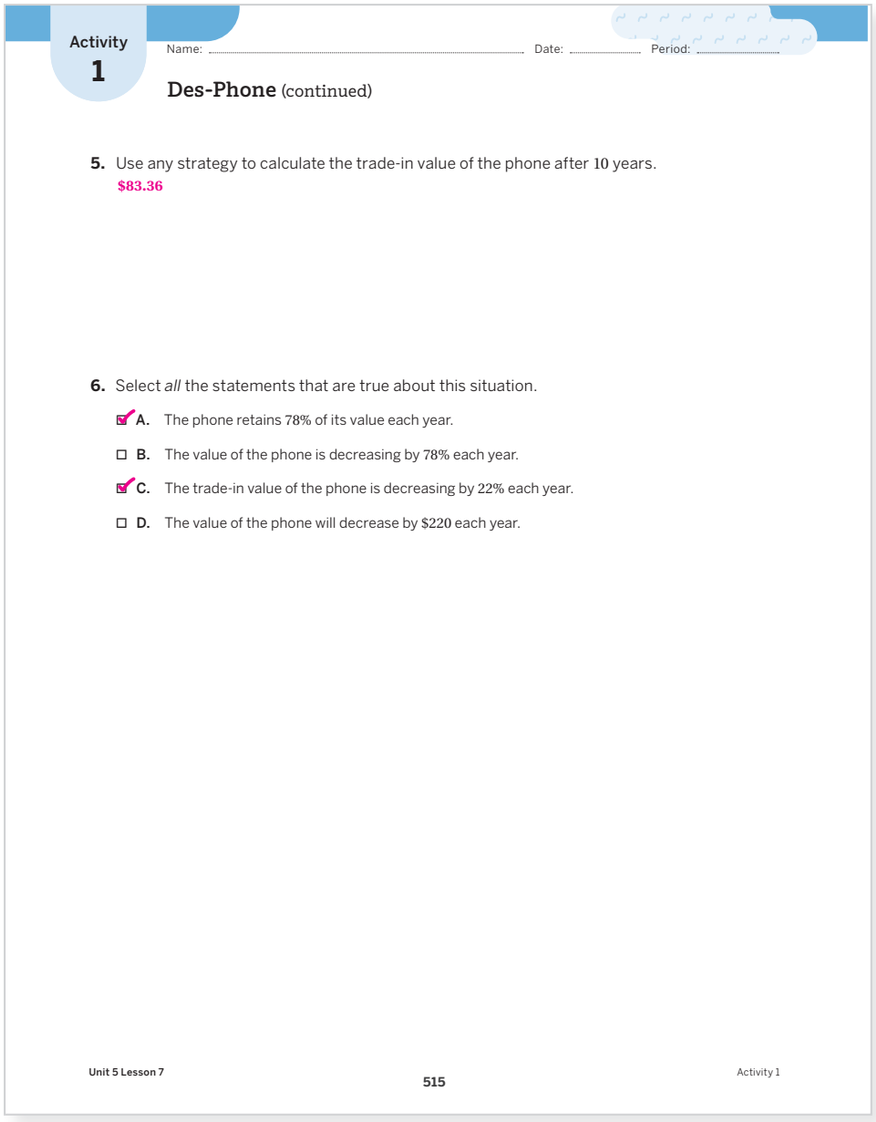
3 Connect

Display the table, graph, equations, and the statements.

Invite students to share their strategies for calculating the trade-in value of the phone after 10 years. Then encourage students to share which statements they think are true and their reasoning.

To reveal the Key Takeaway, consider asking:

- "Should we get the same value when we use Aaliyah's equation or Taylor's equation to calculate the trade-in value after 10 years? Why or why not?"
- "One of these phrases better matches Aaliyah's equation and one better matches Taylor's. Which do you think is which? Why?"
- "Why might it be helpful to have more than one way to describe how the value of the phone changes over time?"



Activity 1

Name: Date: Period:

Des-Phone (continued)

5. Use any strategy to calculate the trade-in value of the phone after 10 years.
$83.36

6. Select *all* the statements that are true about this situation.

- ☑ **A.** The phone retains 78% of its value each year.
- ☐ **B.** The value of the phone is decreasing by 78% each year.
- ☑ **C.** The trade-in value of the phone is decreasing by 22% each year.
- ☐ **D.** The value of the phone will decrease by $220 each year.

Unit 5 Lesson 7 515 Activity 1

Key Takeaway: Exponential decay situations can be described in terms of the *percent decrease* of a quantity over time. For example, the exponential decay function $f(x) = 10(0.6)^x$, in which a quantity with an *initial value* of 10 is multiplied by 0.6 each time, can be written as $f(x) = 10(1 - 0.40)^x$, showing instead a *percent decrease* of 40%.

Pairs | 15 min

Activity 2 Sorting Percents

Purpose: Students make connections between exponential situations that involve percent decrease and the equivalent forms of equations that represent them.

Short on time? Invite each pair of students to work on different rows of the table, then share out as a whole class.

1 Launch

Display the directions.

Distribute one set of cards to each student.

ELL **English Language Learners** Consider inviting a student to model strategies for annotating the text. **(Reading) (MTR.4.1)**

A **Accessibility: Executive Functioning** Chunk this task by inviting students to first sort the cards into functions and percent decreases before selecting one card a time to match to a situation.

2 Monitor

Encourage students to use coloring tools or arrows to make connections between the situation, the percent decrease, and the two equivalent forms of the equation. **(MTR.5.1)**

D **Differentiation** *(Problem 7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to revisit their work on Activity 1 and consider asking, "How is this similar to Aaliyah's or Taylor's equation? How is it different?"
Match all the cards and complete each row.	**Stretch** Consider asking, "What other real life situations might show a percent decrease? Why?" Invite them to record their ideas and share during the Connect.

3 Connect

Display the Student Edition and poll the class to decide which card(s) and situation(s) students would like to discuss further.

Consider asking about the chosen situation(s)/card(s):

- "How is this situation similar to the other situations? What is different about it?"
- "How are the two functions for this situation alike? How are they different?"
- "It is more common to write an equation for the function that shows the percent remaining. For this situation, which equation is that? Why do you think that is more common?"

Activity 2

Name: Date: Period:

Sorting Percents

7. You will use a set of cards for this activity.

Use the cards to complete each row. Create your own situation for the last row.

	Situation	Percent Decrease	Functions
Medicine	Victor takes 800 mg of an antiviral medicine. The amount of medicine left in his body is reduced by 25% every hour. Write a function to represent the amount of medicine remaining in Victor's body after x hours.	Card 11 25%	Card 2 $b(x) = 800(0.75)^x$ Card 7 $f(x) = 800(1 - 0.25)^x$
Laptops	A laptop that was originally \$800 loses 75% of its value each year. Write a function to represent the value of the laptop after x years.	Card 6 75%	Card 1 $a(x) = 800(0.25)^x$ Card 4 $c(x) = 800(1 - 0.75)^x$
Coffee	Aditi has 200 mg of caffeine in her body after drinking coffee. Typically, the amount of caffeine in the body decreases by 10% each hour. Write a function to represent the amount of caffeine remaining in her body after x hours.	Card 8 10%	Card 9 $g(x) = 200(0.90)^x$ Card 5 $d(x) = 200(1 - 0.1)^x$
Your Situation	*Responses vary.* A colony of mold originally contained 200 spores. After being treated with a mold remover solution, the population is reduced by 90% each hour. Write a function to represent the population of mold spores remaining after x hours.	Card 3 90%	$j(x) = 200(0.1)^x$ Card 10 $h(x) = 200(1 - 0.9)^x$

Unit 5 Lesson 7 516 Activity 2

Pairs | 5 min

Activity 3 Graphs to Functions

Purpose: Students analyze a common mistake when writing equations of exponential functions.

1 Launch

Display the graph and Diya's work to discuss Problems 8–9 as a class.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Diya's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

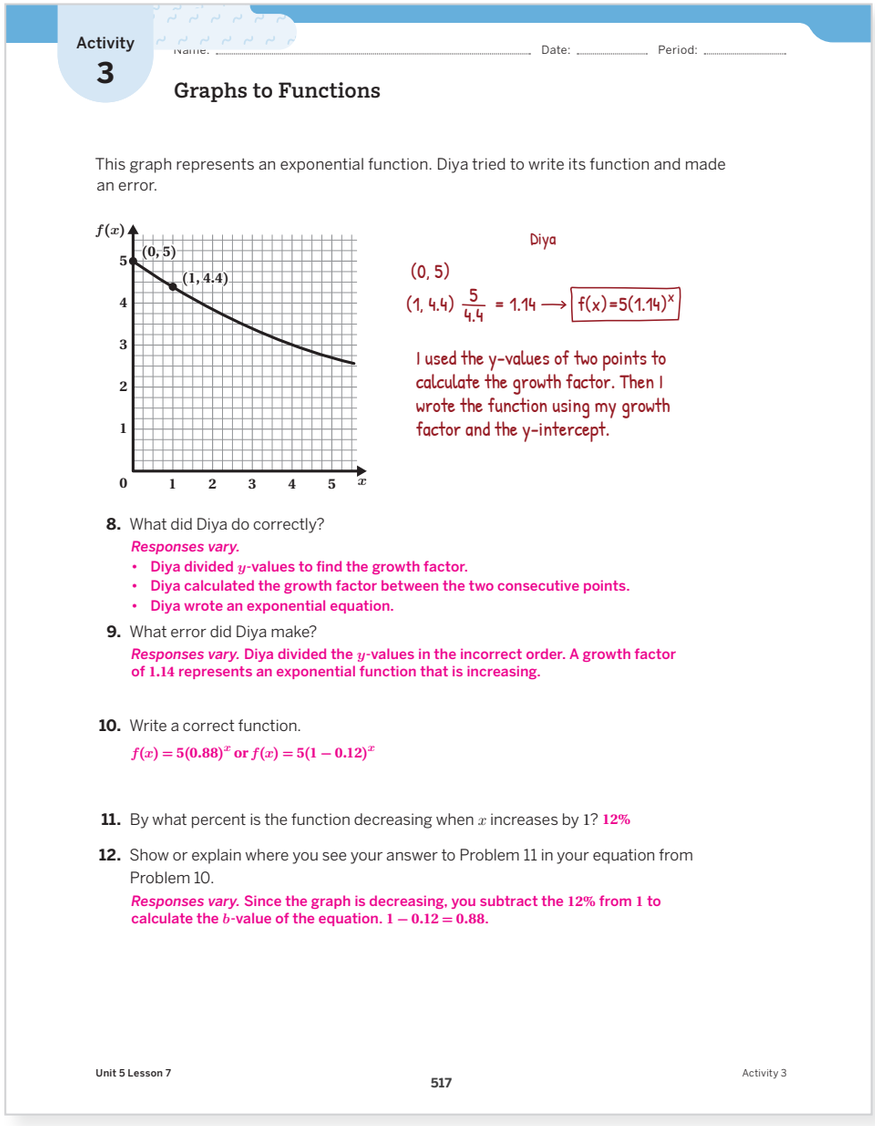
2 Monitor

D **Differentiation** *(Problems 9–12)*

Look for students who:	Respond to Student Thinking
Do not think Diya made an error. *(Problem 9)*	**Support** Consider asking, "Is the graph increasing or decreasing? Does that match Diya's function?" Encourage them to try substituting 1 in for x in Diya's function and seeing if their calculation matches what they see in the graph.
Write a function like $f(x) = 5(0.88)^x$ that shows the percent remaining and a function like $f(x) = 5(1 - 0.12)^x$ that shows the percent decrease. *(Problem 10)*	**Strengthen** Invite students who wrote each kind of function to share during the Connect. To surface connections, consider asking pairs of students, "Does your equation show the percent decrease or the percent remaining? Can you write another equivalent equation to represent this situation?" **(MTR.5.1)**
Write a correct function for Problem 10 but need support to write the percent decrease. *(Problems 11–12)*	**Strengthen** Invite them to revisit the forms of Aaliyah's and Taylor's functions from Activity 1 and invite them to try writing their function in another form. Consider asking, "Where do you see the percent decrease in these functions?" **(MTR.5.1)**

3 Connect

Consider asking, "What do you think is helpful to remember about Diya's work?"



Activity 3

Name: Date: Period:

Graphs to Functions

This graph represents an exponential function. Diya tried to write its function and made an error.

Diya

(0, 5)

(1, 4.4) $\frac{5}{4.4} = 1.14 \longrightarrow f(x)=5(1.14)^x$

I used the y-values of two points to calculate the growth factor. Then I wrote the function using my growth factor and the y-intercept.

8. What did Diya do correctly?

Responses vary.

- Diya divided y-values to find the growth factor.
- Diya calculated the growth factor between the two consecutive points.
- Diya wrote an exponential equation.

9. What error did Diya make?

Responses vary. Diya divided the y-values in the incorrect order. A growth factor of 1.14 represents an exponential function that is increasing.

10. Write a correct function.

$f(x) = 5(0.88)^x$ or $f(x) = 5(1 - 0.12)^x$

11. By what percent is the function decreasing when x increases by 1? 12%

12. Show or explain where you see your answer to Problem 11 in your equation from Problem 10.

Responses vary. Since the graph is decreasing, you subtract the 12% from 1 to calculate the b-value of the equation. $1 - 0.12 = 0.88$.

Unit 5 Lesson 7 517 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding by comparing and contrasting writing an exponential growth function (representing percent increase) and an exponential decay function (representing percent decrease)

Invite students to respond independently, and then share their thinking with a partner.

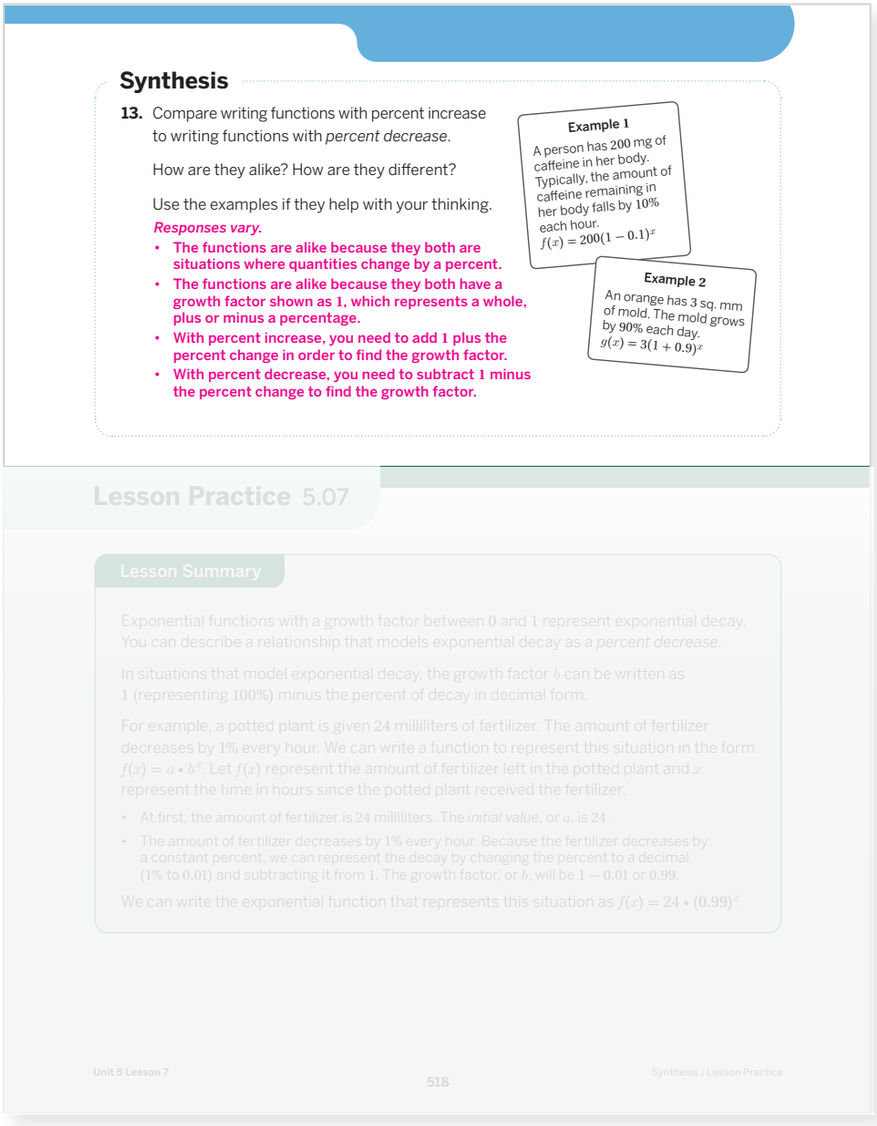
Look for and share a variety of ideas, such as:
- You add the percent increase to 1 for the base of an exponential growth function.
- You subtract the percent decrease from 1 for the base of an exponential decay function.
- Both equations use the value of 1 to represent 100% of the original amount.
- Both equations are in the form $y = a \cdot b^x$.
- Both equations have an initial value and a base.

Consider asking,
- "What connections do you see between your classmates' ideas?"
- "For what values of b in the function $f(x) = a \cdot b^x$ will we see an exponential decay?"

Note: Consider displaying the Desmos Graphing Calculator, typing in the equation $y = a \cdot b^x$, and creating a slider for the variable b to investigate with students what values of b show exponential growth and decay.

MLR **MLR2: Collect and Display** Use this routine to add to the display created during Lesson 7 by collecting students' ideas as they share with the class. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. Compare writing functions with percent increase to writing functions with *percent decrease*.

How are they alike? How are they different?

Use the examples if they help with your thinking.

Responses vary.
- The functions are alike because they both are situations where quantities change by a percent.
- The functions are alike because they both have a growth factor shown as 1, which represents a whole, plus or minus a percentage.
- With percent increase, you need to add 1 plus the percent change in order to find the growth factor.
- With percent decrease, you need to subtract 1 minus the percent change to find the growth factor.

Example 1
A person has 200 mg of caffeine in her body. Typically, the amount of caffeine remaining in her body falls by 10% each hour.
$f(x) = 200(1 - 0.1)^x$

Example 2
An orange has 3 sq. mm of mold. The mold grows by 90% each day.
$g(x) = 3(1 + 0.9)^x$

Lesson Practice 5.07

Lesson Summary

Exponential functions with a growth factor between 0 and 1 represent exponential decay. You can describe a relationship that models exponential decay as a *percent decrease*.

In situations that model exponential decay, the growth factor b can be written as 1 (representing 100%) minus the percent of decay in decimal form.

For example, a potted plant is given 24 milliliters of fertilizer. The amount of fertilizer decreases by 1% every hour. We can write a function to represent this situation in the form $f(x) = a \cdot b^x$. Let $f(x)$ represent the amount of fertilizer left in the potted plant and x represent the time in hours since the potted plant received the fertilizer.

- At first, the amount of fertilizer is 24 milliliters. The *initial value*, or a, is 24.
- The amount of fertilizer decreases by 1% every hour. Because the fertilizer decreases by a constant percent, we can represent the decay by changing the percent to a decimal (1% to 0.01) and subtracting it from 1. The growth factor, or b, will be $1 - 0.01$ or 0.99.

We can write the exponential function that represents this situation as $f(x) = 24 \cdot (0.99)^x$.

Unit 5 Lesson 7 518 Synthesis | Lesson Practice

Key Takeaway: Situations with repeated *percent decreases* can be modeled with exponential functions in the form $f(x) = a(1 - r)^x$, where a represents the *initial value*, 1 represents 100%, and r represents the percent decrease in decimal form. These equations can be written to highlight the percent of the quantity remaining, such as $y = 100(0.75)^x$, or to highlight the percent decrease, like $y = 100(1 - 0.25)^x$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing an exponential function that represents percent decrease and using that function to make a prediction.

Today's Goals

1. **Goal:** Calculate percent change of exponential functions from a graph.
2. **Goal:** Write equivalent functions that represent exponential decay.
3. **Language Goal:** Compare and contrast equivalent equations of functions that represent exponential decay. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.07

A town has a population of 200,000 people. The population is decreasing by 3% each year.

a. Write a function that represents the population of the town, $p(x)$, after x years.
$p(x) = 200000(0.97)^x$ or $p(x) = 200000(1 - 0.03)^x$

b. Use the function you wrote to determine how many people will be in the town after 10 years.
About 147,485 people

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of writing equations that represent percent increase and percent decrease:

» **Respond to Student Thinking:**

- Use the ***Calculating Percent Change in Exponential Functions*** **Mini-lesson.**
- Revisit **Activity 3, Problems 8–10** as a class and invite students to share strategies for writing equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to research more examples of situations with percent decrease, such as medicine dissolving in the body or the value of a car over time. Encourage them to share their findings with the class.

Professional Learning

Reflect on how students analyzed student work today and wrote functions for situations with *percent decrease*. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of Diya's work? How can you continue to support students to value learning from both correct and incorrect thinking?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

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How are they alike? How are they different?

Use the examples if they help with your thinking.

Responses vary.

- The functions are alike because they both are situations where quantities change by a percent.
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Example 1

A person has 200 mg of caffeine in her body. Typically, the amount of caffeine remaining in her body falls by 10% each hour. $f(x) = 200(1 - 0.1)^x$

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An orange has 3 sq. mm of mold. The mold grows by 90% each day. $g(x) = 3(1 + 0.9)^x$

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We can write the exponential function that represents this situation as $f(x) = 24 \cdot (0.99)^x$.

Lesson Practice 5.07

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine the percent decrease for each function.

1. $f(x) = 20(0.9)^x$ — 10%

2. $g(x) = 45(1 - 0.75)^x$ — 75%

3. $h(t) = 0.65\left(1 - \frac{25}{100}\right)^t$ — 25%

4. Select *all* the functions with a 30% decrease.

- [x] A. $g(t) = 22(0.7)^t$
- [ ] B. $h(x) = 22(0.3)^x$
- [ ] C. $f(x) = 30(1 - 0.7)^x$
- [x] D. $r(t) = 30(1 - 0.3)^x$
- [x] E. $v(x) = 70\left(1 - \frac{30}{100}\right)^x$

5. A cell phone is worth $800 when it is first made, but loses 25% of its value after each year. Write an equation to represent the value, $v(t)$, of the cell phone, t years after it is made.

$v(t) = 800 \cdot (0.75)^t$

6. Jamir says that $r(x) = 26(0.3)^x$ and $r(x) = 26\left(1 - \frac{7}{10}\right)^x$ are equivalent equations.

a. Is Jamir correct? Explain your thinking.

Yes. *Responses vary.* I agree because $1 - \frac{7}{10} = 1 - 0.7 = 0.3$.

b. Write another equivalent equation that uses fractions.

$r(x) = 26\left(\frac{3}{10}\right)^x$

7. Fill in each blank using the digits 0 to 9 only once each to create the smallest possible value.

$$\boxed{2}\boxed{4}\left(\boxed{0}.\boxed{1}\boxed{3}\right)^3$$

Lesson Practice 5.07

Name: _____ Date: _____ Period: _____

Test Practice

8. There are 1,024 players in a tennis tournament. After each round, *half* of the players are eliminated. Which function represents the number of players remaining after x rounds?

A. $f(x) = 1024(1.50)^x$

B. $f(x) = 1024(1.05)^x$

C. $f(x) = 1024(0.50)^x$

D. $f(x) = 1024(0.05)^x$

Spiral Review

Problems 9–10: A factory makes boxes. Each box's height, $h(x)$, is 2 inches more than 3 times its width, x.

9. Complete the table

x	1	2	3	4
$h(x)$	5	8	11	14

10. Write an equation for the function $h(x)$.

$h(x) = 3x + 2$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.912.AR.1.1
	5–6	2	MA.912.AR.1.1 MA.912.AR.5.4
	7	3	MA.912.AR.5.4
Test Practice	8	2	MA.912.AR.5.4
Spiral Review			
	9–10	2	MA.8.AR.3.3

Assess and Respond

Sub-Unit Quiz

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify exponential functions	Lessons 1–3	1	MA.912.F.1.8 MTR.3.1
2	Write expressions representing percent increase	Lesson 6	2	MA.912.AR.3.7 MTR.2.1
3a	Write functions representing percent decrease	Lesson 7	2	MA.912.AR.3.7 MTR.2.1
3b	Use exponential functions to solve problems in context	Lesson 7	1	MA.912.AR.3.7 MTR.3.1
4a–b	Match linear and exponential functions to their graphs	Lessons 2–3	2	MA.912.AR.3.8 MTR.5.1
5a	Interpret parameters of exponential functions in context	Lessons 6–7	2	MA.912.AR.3.6 MTR.2.1, MTR.7.1
5b	Evaluate exponential functions	Lesson 4, 6–7	1	MA.912.AR.3.6 MTR.3.1
5c	Interpret function notation statements in context	Lesson 4, 6–7	2	MA.912.AR.3.6 MTR.2.1

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 5

1. Select *all* of the exponential functions.

☑ A.

x	$f(x)$
-1	18
0	12
1	8
2	$5\frac{1}{3}$

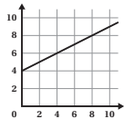
☐ B.

x	$f(x)$
-1	2
0	5
1	10
2	17

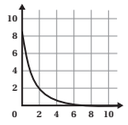
☑ C.

x	$f(x)$
-1	2
0	4
1	8
2	16

☐ D.



☑ E.



2. The population of a city increases by 6% every year. Its population today is 450,000.

Which expression represents its population in 3 years?

A. $450000 \cdot (0.06)^3$

B. $450000 \cdot (1.06)$

C. $450000 \cdot (1 + 0.06^3)$

D. $450000 \cdot (1.06)^3$

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 5

3. A new refrigerator costs $1,500. The value of the refrigerator will decrease 40% each year.



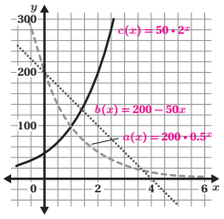
a. Write an equation to represent this relationship.
- Use x to represent the time in years.
- Use $r(x)$ to represent the value of the refrigerator.

$r(x) = 1500 \cdot 0.6^x$ (or equivalent)

b. Determine the value of the refrigerator after 5 years.

$116.64

4. Here is a graph with three functions.



a. Label each graph with its function.
- $a(x) = 200 \cdot 0.5^x$
- $b(x) = 200 - 50x$
- $c(x) = 50 \cdot 2^x$

b. Explain your thinking.

Explanations vary. The line has to be $b(x)$ because that's a linear function. The decreasing exponential graph is $a(x)$ because the growth factor is less than 1. The increasing exponential graph is $c(x)$ because the growth factor is greater than 1.

5. In 1995, wolves were reintroduced to Yellowstone National Park.

The function $w(x) = 14 \cdot 1.08^x$ models the number of wolves, w, in the years since 1995, x.

a. Select *all* the true statements.

☐ A. There were 8 wolves in 1995.

☑ B. There were 14 wolves in 1995.

☐ C. The number of wolves increased by 1.08% each year.

☑ D. The number of wolves increased by 8% each year.

☐ E. The number of wolves increased by 14% each year.

b. Determine the value of $w(25)$.

$w(25) \approx 96$

c. What does the value of $w(25)$ say about the wolf population?

Responses vary. According to the model, there will be about 96 wolves in the park in 2020, which is 25 years after 1995.

D Differentiation (Sub-Unit Quiz)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Compare and contrast linear and exponential functions. **(Lessons 1–3, 5)**	1	**Support** **Strengthen** • **Teacher Move:** Review Problem 6 of Lesson 1 *(Growing Globs)*, then encourage students to revisit Problems 8–9. • **Practice Day 1:** Cards 5–6, 12 • **Practice Day 2:** Task D • **Practice:** Lessons 1–3, 5
Create equations, tables, and graphs of linear and exponential functions. **(Lessons 1–3)**	4	**Support** **Strengthen** • **Teacher Move:** Consider reviewing Activity 2 of Lesson 2 *(Going Viral)* and then encourage students to revise this quiz problem. • **Practice Day 1:** Cards 5–6 • **Practice:** Lessons 1–3
Interpret and write exponential equations to model situations that involve percent increase or decrease. **(Lessons 6–7)**	2, 3a, 5a, 5c	**Support** **Strengthen** **Stretch** • **Teacher Move:** Invite students to revisit Activity 2 of Lesson 6 *(Growing Mold)* and then revise this quiz problem. • **Practice Day 1:** Cards 7–10 • **Practice:** Lessons 6–7 • **You're invited to explore more:** Lesson 6 *(Growing Mold)*
Use exponential functions to solve problems in context. **(Lessons 1, 4, 6–7)**	3b, 5b	**Support** **Stretch** • **Teacher Move:** Review Lesson 8, Activity 7 *(At a Loss)*, then encourage students to revise this quiz problem. • **Practice Day 1:** Cards 1, 7, 9, 11 • **Practice Day 2:** Task C • **Practice:** Lessons 1, 4, 6–7

Sub-Unit 3

Modeling With Exponentials

Sub-Unit 3 Goals

- **Goal:** Use properties of exponents to interpret expressions for exponential functions.
- **Goal:** Perform operations with radicals.
- **Language Goal:** Use operations to simplify radical expressions.

In this sub-unit, students engage in modeling with exponential functions. They use properties of exponents to develop financial literacy regarding accounts and loans that earn compound and simple interest. They learn how to add, subtract, multiply, and divide with radical expressions.



Math That Matters Most

Concept or Skill	Example
Write equivalent expressions involving radicals and rational exponents.	Rewrite the expression $4^{\frac{1}{3}}$ in an equivalent radical form. Rewrite the expression $\sqrt{9}$ in an equivalent form with a rational exponent. $4^{\frac{1}{3}} = \sqrt[3]{4^1}$ $\sqrt{9} = 9^{\frac{1}{2}}$
Compare and contrast accounts that earn compound and simple interest.	Mauricio can invest \$500 in an account that earns simple or compound interest: $s(t) = 500 + 5t$ $c(t) = 500(1.05)^t$ Which type of account is a better investment? When? After 5 Years: s(5) = \$750.00; c(5) = \$638.14 After 30 Years: s(30) = \$2000.00; c(30) = \$2160.97 If Mauricio has less time, the simple interest account is a better choice. With at least 30 years, Mauricio should choose the account that earns compound interest.
Write expressions modeling account balances that earn interest at different compounding intervals.	Write an expression for an \$800 loan with a 12% annual interest rate compounded annually, semi-annually, and monthly for 2 years. • Annually: $800 \cdot (1.12)^2$ • Semi-annually: $800 \cdot \left(1 + \frac{.12}{2}\right)^4$ • Monthly: $800 \cdot \left(1 + \frac{.12}{12}\right)^{24}$

Concept or Skill	Example
Interpret equivalent exponential expressions in context using the properties of exponents.	Emmanuel wrote three equivalent expressions to represent how much he will owe on a \$100 payday loan that charges a monthly interest rate of 15%. What does each interest rate represent? Expression A: $100 \cdot 1.15^{36}$ Expression B: $100 \cdot (1.15^{12})^3$ Expression C: $100 \cdot (5.35)^3$ • Expression A shows the 15% monthly interest rate over 36 months. • Expression B shows the 15% monthly interest rate calculated for 3 years, 12 months each year. • Expression C shows the annual interest rate of 435% over 3 years.
Perform operations with radicals.	Simplify the following radical expressions. $3\sqrt{5} + 2\sqrt{5} + 7\sqrt{45} = 26\sqrt{5}$ $9\sqrt{2} - \sqrt{8} - \sqrt{2} = 6\sqrt{2}$ $\sqrt{3} \cdot \sqrt{12} = 6$ $\frac{\sqrt{18}}{\sqrt{2}} = 3$

Unit 5

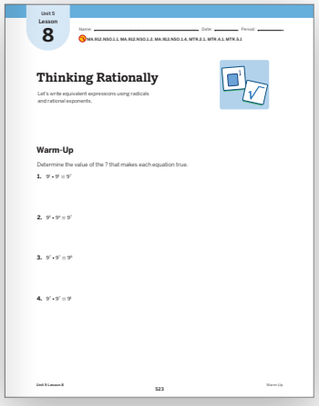
Lesson

8

Thinking Rationally

Writing Expressions Using Radicals and Rational Exponents

Let's write equivalent expressions using radicals and rational exponents.


Unit 5
Lesson
8
Thinking Rationally
Let's write equivalent expressions using radicals and rational exponents.
Warm-Up
Determine the value of the ? that makes each equation true.
1. $9^{?} \cdot 9^{?} = 9^{?}$
2. $9^{?} \cdot 9^{?} = 9^{?}$
3. $9^{?} \cdot 9^{?} = 9^{?}$
4. $9^{?} \cdot 9^{?} = 9^{?}$
523

Learning Outcomes

Today's Goals

1. **Goal:** Use the properties of exponents to write equivalent expressions with *radicals* and *rational exponents*.
2. **Goal:** Explain how to match expressions with rational exponents to equivalent radical expressions.
3. **Language Goal:** Justify the equivalence of $x^{\frac{1}{n}}$ and $\sqrt[n]{x}$ using the properties of exponents. **(Writing, Speaking, and Listening)**

Students explore the equivalent relationship between expressions written using exponents, including **rational exponents**, and expressions written using **radicals.** They generalize the patterns in the relationships to justify the equivalence of $x^{\frac{1}{n}}$ and $\sqrt[n]{x}$. Students then write equivalent expressions using radicals or exponents.

Coherence

Prior Learning

In Grade 8, students applied the properties of integer exponents to generate equivalent numerical expressions.

Future Learning

In Lesson 12, students will write equivalent expressions involving rational exponents and radicals and justify the equivalence of $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$.
In Algebra 2, students will study radical functions in more depth.

Connections

- Students write equivalent expressions using radicals and rational exponents as the explore and analyze visuals of student work examples. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.NSO.1.1
Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

MA.912.NSO.1.4
Divide a polynomial expression by a monomial expression with rational number coefficients.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.NSO.1.3	MA.912.NSO.1.5
MA.8.NSO.1.5	MA.912.NSO.1.6
	MA.912.AR.7.1
	MA.912.AR.7.4

Vocabulary

New Vocabulary

radical	rational exponent

Review Vocabulary

cube root	*square root*

Additional Vocabulary

index	*radicand*

Lesson at a Glance

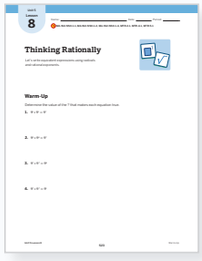
~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

Students use the **Number Talk** routine to determine missing values in exponential equations that make them true.



Activity 1

Pairs | 10 min

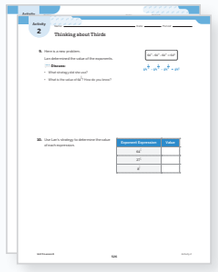
Students use exponent rules and the structure of expressions to justify the value of exponential expressions with the **rational exponents** $\frac{1}{2}$ and $\frac{1}{3}$. **Routines:** Think-Pair-Share, MLR2: Collect and Display



Activity 2

Pairs | 10 min

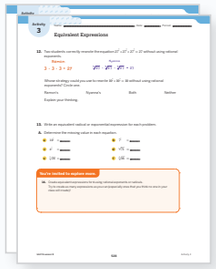
Students make connections between **radical** expressions and expressions with rational exponents of the form $\frac{1}{n}$.



Activity 3

Pairs | 10 min

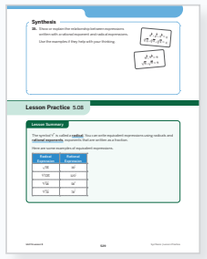
Students write equivalent radical and exponential expressions. **Routine:** Decide and Defend



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the relationship between expressions written with a rational exponent and radical expressions. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by rewriting a radical expression and an exponential expression.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use the **Number Talk** routine to determine missing values in exponential equations that make them true.

1 Launch

Display each problem one at a time.

Use the Number Talk routine to support students' ability to access prior knowledge about exponent rules and evaluating expressions with exponents.

Invite students to determine the value of the first question mark independently and then share their strategies aloud. Encourage them to consider how they can communicate their ideas in ways others can understand, and share that everyone is responsible for trying to understand each others' ideas.

A **Accessibility: Conceptual Processing** To support abstract thinking, invite students to write an expanded equation for each step (e.g., $9^3 \cdot 9^4 = (9 \cdot 9 \cdot 9)(9 \cdot 9 \cdot 9 \cdot 9) = 9^?$).

ELL **English Language Learners** Consider creating an anchor chart about exponents and exponent rules. Add student ideas and language to the chart throughout the lesson.

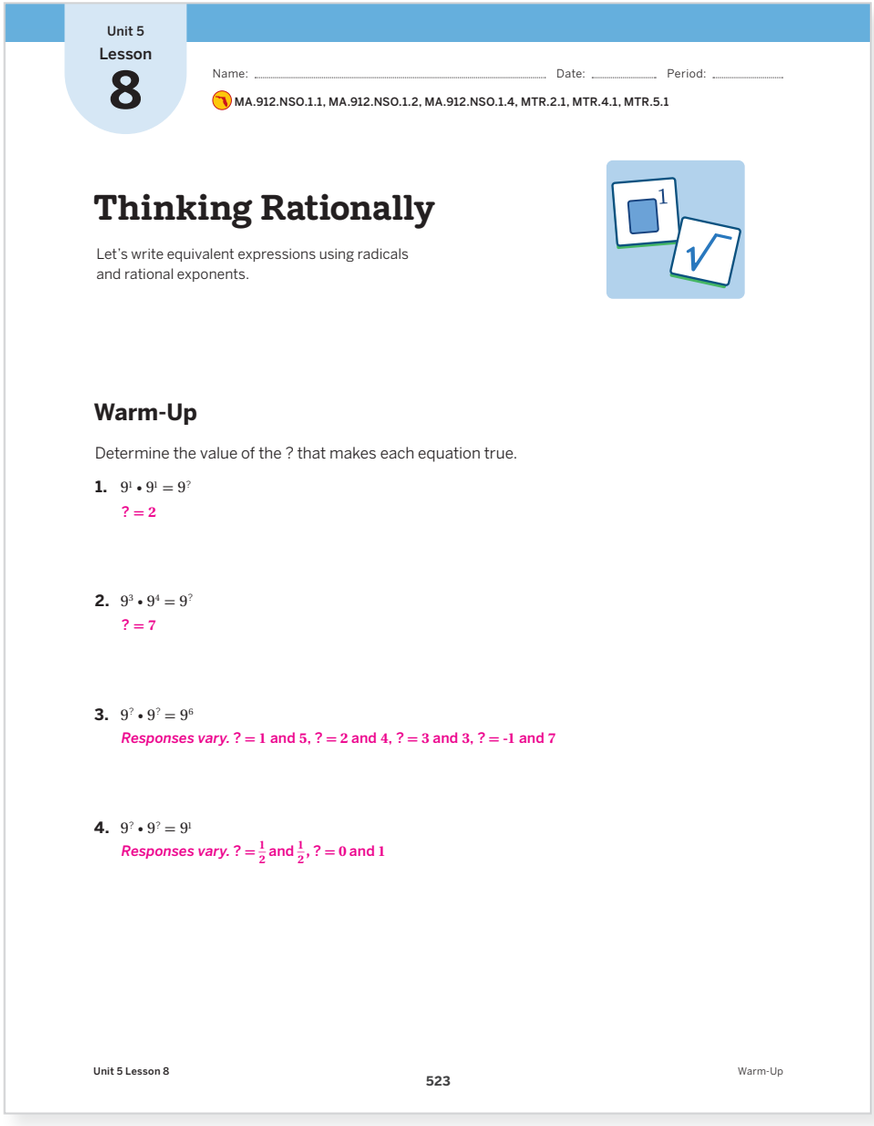
2 Connect

Create a record of each strategy that students share, along with the name of the student who shared it.

Consider asking:
- "Who thought about it in a different way?"
- "Explain _____'s strategy in your own words."

Note: Repeat this structure for each equation/expression in the Number Talk.

Consider asking, "How is this last equation similar to the first three? How is it different?" It is similar because there are two of the same exponents, like the first and third equation. It is different because these exponents have to have a value $\left(\frac{1}{2} \text{ or } 0.5\right)$ that is not a whole number.



Unit 5
Lesson
8

Name: ..... Date: ..... Period: .....

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1

Thinking Rationally

Let's write equivalent expressions using radicals and rational exponents.

Warm-Up

Determine the value of the ? that makes each equation true.

1. $9^1 \cdot 9^1 = 9^?$
   $? = 2$

2. $9^3 \cdot 9^4 = 9^?$
   $? = 7$

3. $9^? \cdot 9^? = 9^6$
   Responses vary. $? = 1$ and 5, $? = 2$ and 4, $? = 3$ and 3, $? = -1$ and 7

4. $9^? \cdot 9^? = 9^1$
   Responses vary. $? = \frac{1}{2}$ and $\frac{1}{2}$, $? = 0$ and 1

Unit 5 Lesson 8 523 Warm-Up

Pairs | 10 min

Activity 1 Express Yourself

Purpose: Students use exponent rules and the structure of expressions to justify the value of exponential expressions with the **rational exponents** $\frac{1}{2}$ and $\frac{1}{3}$.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse about Lan's equations. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to highlight just the exponents in each of Lan's equations and consider what they notice.

2 Monitor

Invite students to check in with their partners and share their explanations about what they think the value of $9^{\frac{1}{2}}$ is.

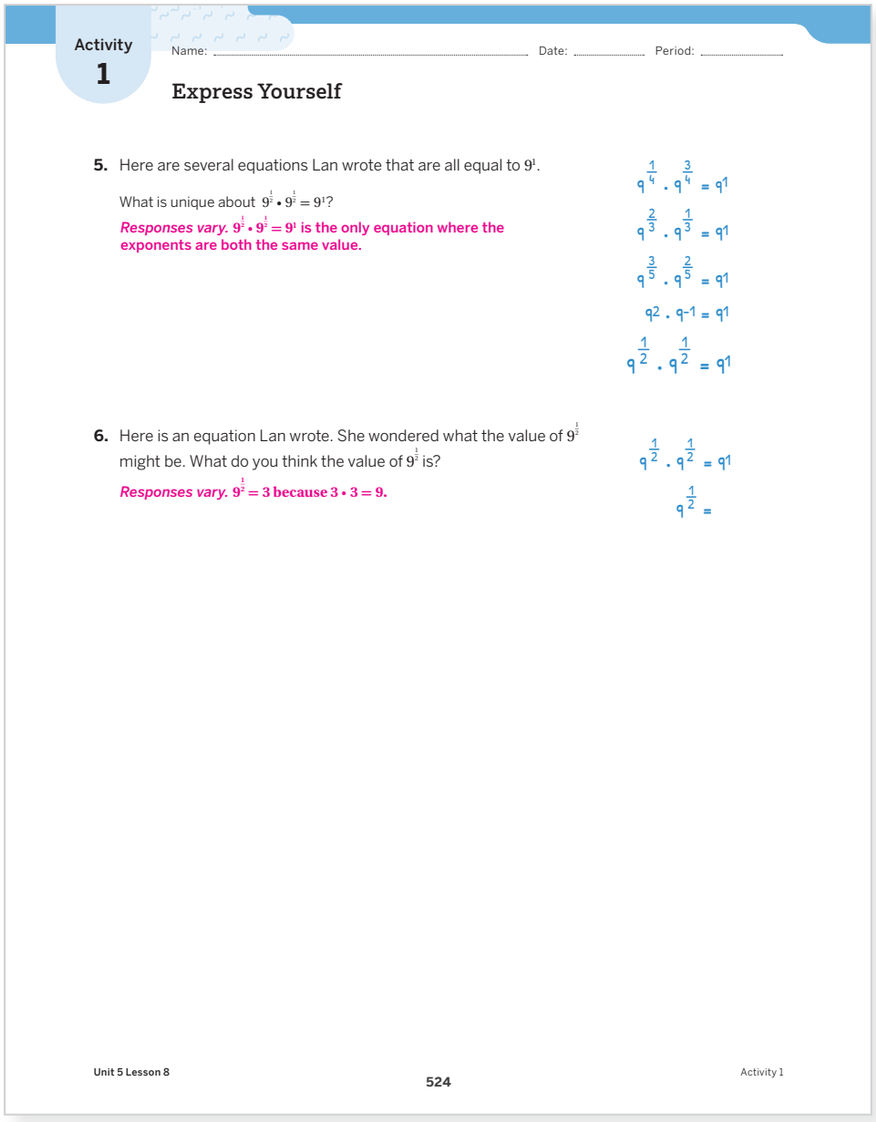
ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Pause briefly to poll the class about what they think the value of $9^{\frac{1}{2}}$ is. If there is not yet a consensus, invite students to share their explanations about why the value should be 3.

Listen for student strategies and explanations to highlight during the pause. Some ideas to highlight might include:

- Anything raised to the power of 1 stays the same (e.g., $9^1 = 9$).
- 3 is the *square root* of 9.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Express Yourself

5. Here are several equations Lan wrote that are all equal to 9^1.

$q^{\frac{1}{4}} \cdot q^{\frac{3}{4}} = q^1$

$q^{\frac{2}{3}} \cdot q^{\frac{1}{3}} = q^1$

$q^{\frac{3}{5}} \cdot q^{\frac{2}{5}} = q^1$

$q^2 \cdot q^{-1} = q^1$

$q^{\frac{1}{2}} \cdot q^{\frac{1}{2}} = q^1$

What is unique about $9^{\frac{1}{4}} \cdot 9^{\frac{3}{4}} = 9^1$?

Responses vary. $9^{\frac{1}{4}} \cdot 9^{\frac{3}{4}} = 9^1$ **is the only equation where the exponents are both the same value.**

6. Here is an equation Lan wrote. She wondered what the value of $9^{\frac{1}{2}}$ might be. What do you think the value of $9^{\frac{1}{2}}$ is?

$q^{\frac{1}{2}} \cdot q^{\frac{1}{2}} = q^1$

$q^{\frac{1}{2}} =$

Responses vary. $9^{\frac{1}{2}} = 3$ **because** $3 \cdot 3 = 9$.

Unit 5 Lesson 8 524 Activity 1

Pairs | 10 min

Activity 1 Express Yourself (continued)

Purpose: Students use exponent rules and the structure of expressions to justify the value of exponential expressions with the **rational exponents** $\frac{1}{2}$ and $\frac{1}{3}$.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to write out equations like Lan's equation from Problem 6 for each problem (e.g., $25^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 25^1$).
Determine the value of $25^{\frac{1}{2}}$ and $64^{\frac{1}{2}}$ but are not sure about $1^{\frac{1}{2}}$, or who think their response cannot be 1.	**Strengthen** Consider sharing that, just like $9^1 = 9$, it is possible for an exponential expression to have the same value as its original base.
Would like an additional challenge.	**Stretch** Consider asking, "What do you think the value of $0^{\frac{1}{2}}$ is? What about $\left(\frac{1}{4}\right)^{\frac{1}{2}}$? Do you think every number can be raised to the power $\frac{1}{2}$ and still have an answer? Why or why not?"

Invite students in their pairs to generalize from their repeated reasoning in this activity about any number raised to the $\frac{1}{2}$ power. Once they have built consensus about an explanation, invite them to then check in with another pair to share their explanations with each other.

Listen for student explanations to share during the Connect, including mentions of *square roots* or use of radical symbols.

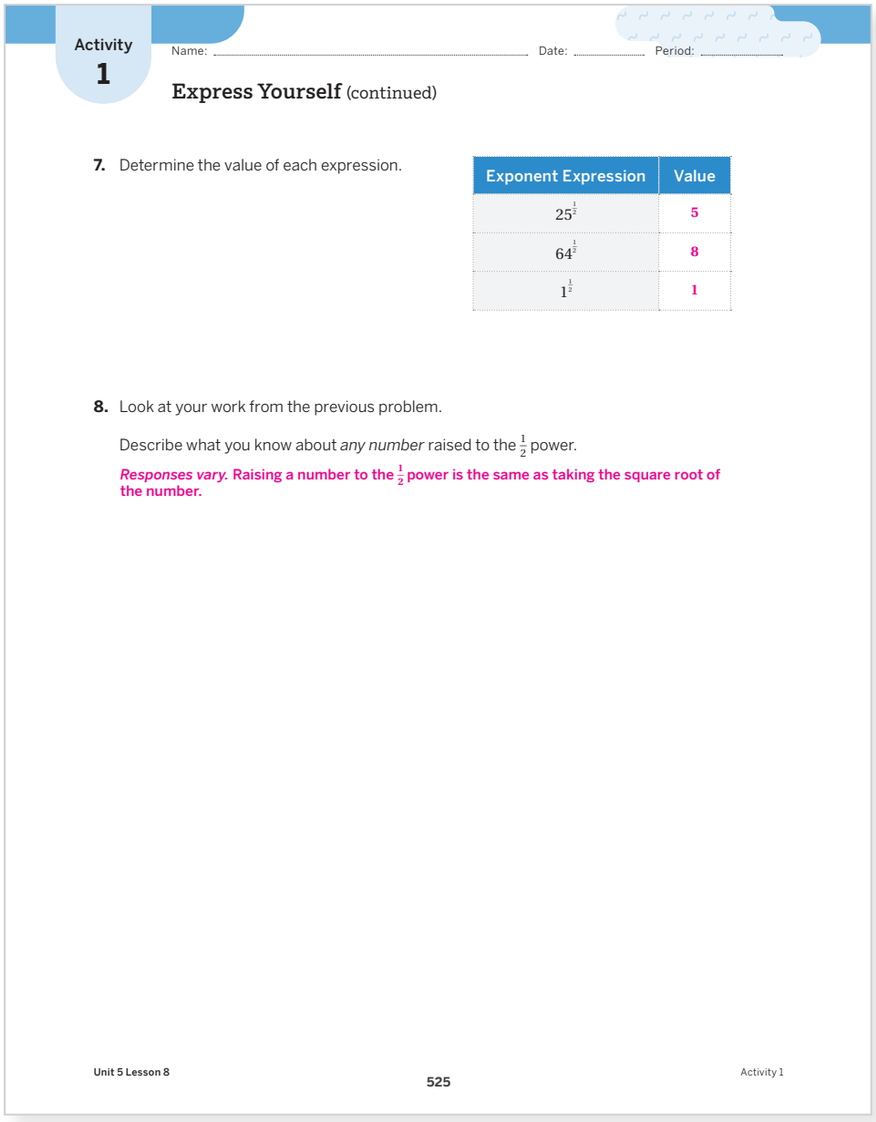
3 Connect

Invite students to share their explanations of how to determine the value of any number raised to the $\frac{1}{2}$ power.

To reveal the Key Takeaway, consider asking:

- "How do you know this explanation will work for every number raised to the $\frac{1}{2}$ power?"
- "How is an exponent of $\frac{1}{2}$ connected to the *square root*?"

MLR MLR2: Collect and Display Use this routine to collect students' explanations as they share during the Connect. Display for students to reference and add to for the rest of this lesson and the next lesson.



Activity 1

Name: _____ Date: _____ Period: _____

Express Yourself (continued)

7. Determine the value of each expression.

Exponent Expression	Value
$25^{\frac{1}{2}}$	5
$64^{\frac{1}{2}}$	8
$1^{\frac{1}{2}}$	1

8. Look at your work from the previous problem.

Describe what you know about *any number* raised to the $\frac{1}{2}$ power.

Responses vary. Raising a number to the $\frac{1}{2}$ power is the same as taking the square root of the number.

Unit 5 Lesson 8 525 Activity 1

Key Takeaway: Raising a number or expression to the $\frac{1}{2}$ power is equivalent to taking its *square root*. Written symbolically, this means that $x^{\frac{1}{2}} = \sqrt{x}$.

Pairs | 10 min

Activity 2 Thinking about Thirds

Purpose: Students make connections between **radical** expressions and expressions with rational exponents of the form $\frac{1}{n}$.

1 Launch

D **Differentiation: (Support)** To support students getting started, invite them to revisit Lan's equations. Consider asking, "How is this equation similar? How is it different?"

Encourage students to justify their explanations for what the value of $64^{\frac{1}{3}}$ is using evidence.

Invite students to work with their partner to determine the value of each expression with an exponent of $\frac{1}{3}$.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

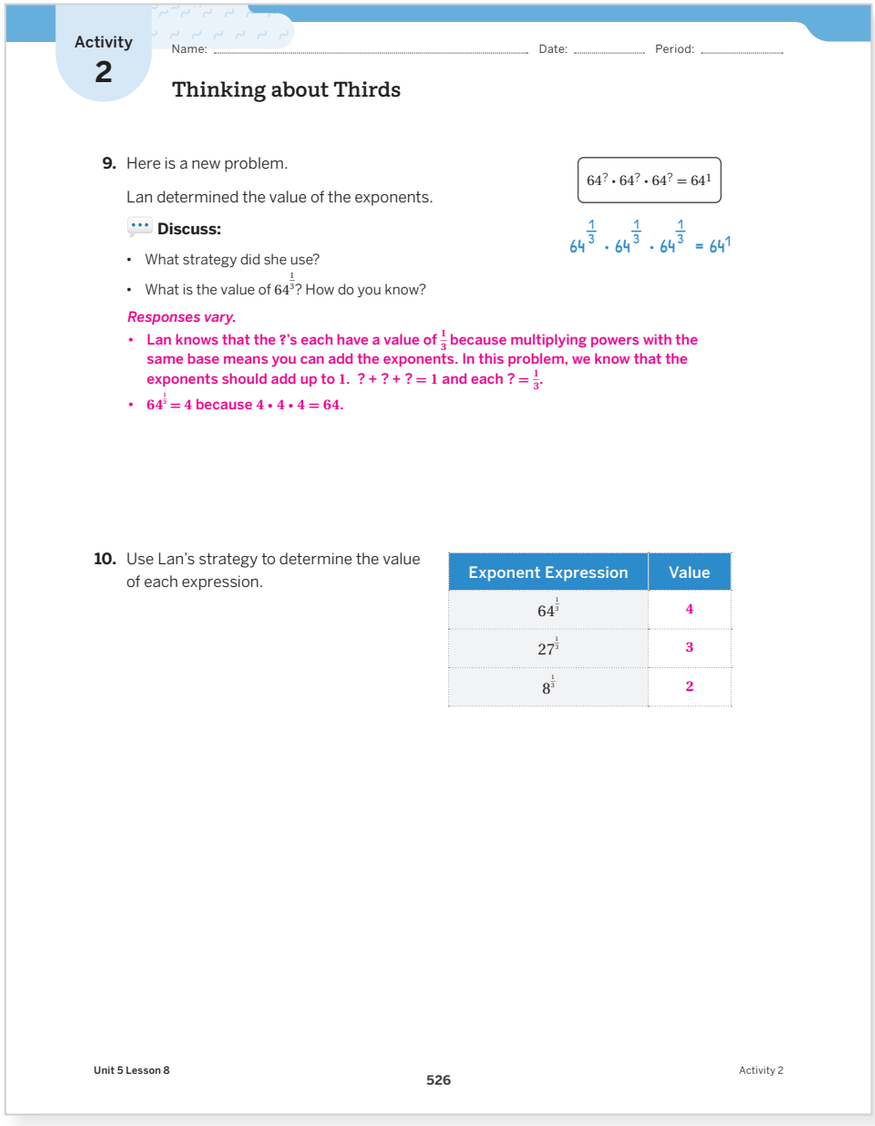
2 Monitor

D **Differentiation: (Support)** To support students getting started, invite them to revisit Problem 9. Consider asking, "Could you use Lan's strategy to write a similar equation that would help here?"

Listen for students who make connections to Lan's strategy or to the thinking they did in Activity 1 about the rational exponent $\frac{1}{2}$. Encourage students to use those connections as part of their explanation on Problem 11.

D **Differentiation: (Stretch)** Invite students to continue the pattern in the table. Consider asking, "What do you think the next rows in the table would be?" $1^{\frac{1}{3}} = 1$, $0^{\frac{1}{3}} = 0$, $(-1)^{\frac{1}{3}} = -1$, etc.

Activity 2 continued >



Activity 2

Name: Date: Period:

Thinking about Thirds

9. Here is a new problem.

$64^{?} \cdot 64^{?} \cdot 64^{?} = 64^{1}$

Lan determined the value of the exponents.

$64^{\frac{1}{3}} \cdot 64^{\frac{1}{3}} \cdot 64^{\frac{1}{3}} = 64^{1}$

Discuss:

- What strategy did she use?
- What is the value of $64^{\frac{1}{3}}$? How do you know?

Responses vary.

- Lan knows that the ?'s each have a value of $\frac{1}{3}$ because multiplying powers with the same base means you can add the exponents. In this problem, we know that the exponents should add up to 1. $? + ? + ? = 1$ and each $? = \frac{1}{3}$.
- $64^{\frac{1}{3}} = 4$ because $4 \cdot 4 \cdot 4 = 64$.

10. Use Lan's strategy to determine the value of each expression.

Exponent Expression	Value
$64^{\frac{1}{3}}$	4
$27^{\frac{1}{3}}$	3
$8^{\frac{1}{3}}$	2

Unit 5 Lesson 8 526 Activity 2

Pairs | 10 min

Activity 2 Thinking about Thirds (continued)

Purpose: Students make connections between **radical** expressions and expressions with rational exponents of the form $\frac{1}{n}$.

2 Monitor

Listen for student explanations to share during the Connect. Look for the language students use to describe the *index* and the *radicand*, which may include:

- The number inside/under the radical.
- The number on/next to the radical.

ELL **English Language Learners** Invite students to research how to say the new terms in other languages. Consider creating an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**

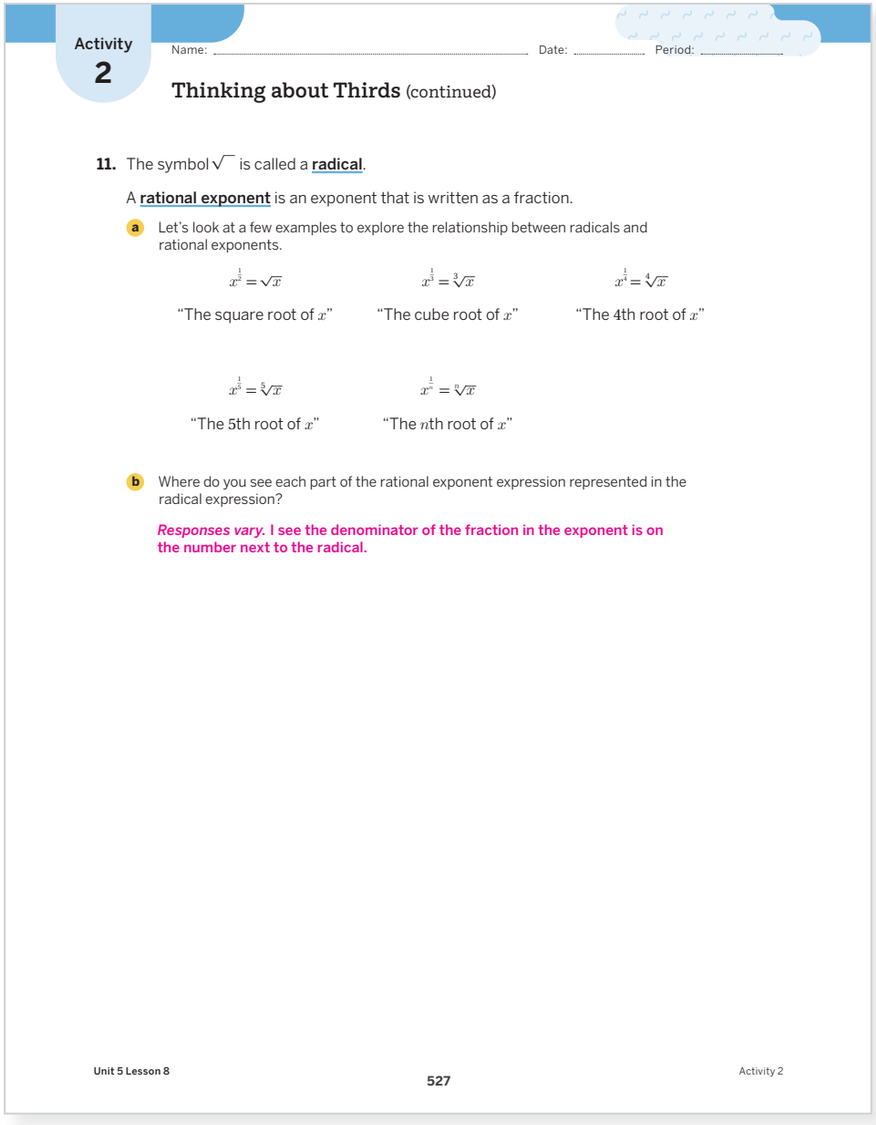
3 Connect

Invite students to share their explanations.

Consider sharing that there is more than one way to write the square root of a number, and. that $\sqrt[2]{\ }$ and $\sqrt{\ }$ are equivalent.

To reveal the Key Takeaway, consider asking:

- "How is an exponent of $\frac{1}{3}$ connected to the *cube root*?"
- "Will this explanation work for every number raised to the $\frac{1}{3}$ power? The $\frac{1}{n}$ power? How do you know?"
- "Where is the 1 in the rational exponent? In the radical expression?"
- "If radicals and rational exponents are equivalent, why have more than one way to express them?"
- "When are radicals helpful? When are rational exponents helpful?"



Activity 2

Name: _____ Date: _____ Period: _____

Thinking about Thirds (continued)

11. The symbol $\sqrt{\ }$ is called a **radical**.

A **rational exponent** is an exponent that is written as a fraction.

a. Let's look at a few examples to explore the relationship between radicals and rational exponents.

$x^{\frac{1}{2}} = \sqrt{x}$ "The square root of x"

$x^{\frac{1}{3}} = \sqrt[3]{x}$ "The cube root of x"

$x^{\frac{1}{4}} = \sqrt[4]{x}$ "The 4th root of x"

$x^{\frac{1}{5}} = \sqrt[5]{x}$ "The 5th root of x"

$x^{\frac{1}{n}} = \sqrt[n]{x}$ "The nth root of x"

b. Where do you see each part of the rational exponent expression represented in the radical expression?

Responses vary. I see the denominator of the fraction in the exponent is on the number next to the radical.

Unit 5 Lesson 8 527 Activity 2

Key Takeaway: just like with the **rational exponent** of $\frac{1}{2}$ and the *square root*, raising a number or expression to the $\frac{1}{3}$ power is equivalent to taking its *cube root*. Written symbolically, this means that $x^{\frac{1}{3}} = \sqrt[3]{x}$. In general, $x^{\frac{1}{n}} = \sqrt[n]{x}$.

Pairs | 10 min

Activity 3 Equivalent Expressions

Purpose: Students write equivalent radical and exponential expressions.

Short on time? Consider inviting students to complete fewer of the repeated challenges on Problem 13.

1 Launch

Display the distribution of responses.

Use the Decide and Defend routine to support students in making arguments and critiquing others' reasoning.

Invite students who made different choices to use reasoning to convince their classmates.

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing?"
- "If the base in the equation $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}} = 10$ was different from 10, would that change your thinking?" If it was a perfect square, like 16, I could use either strategy.

2 Monitor

Invite students to complete the repeated challenges.

Look for student solutions of at least two challenges, one where students write the radical expression from an exponent and vice versa, to share during the Connect.

Consider sharing how to navigate to the $\sqrt{\ }$ and $\sqrt[n]{\ }$ symbols using the Keypad, or that they appear when you type "sqrt" or "nthroot" into the math input field.

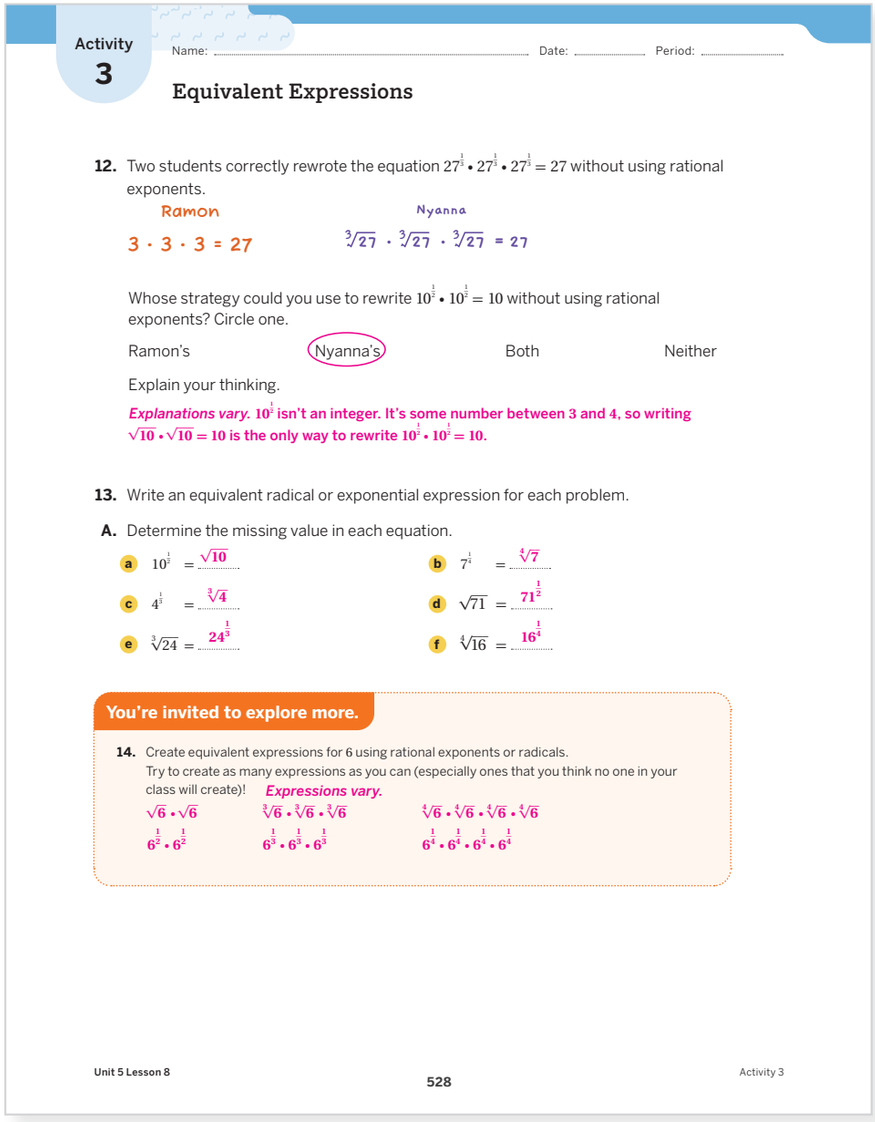
D **Differentiation: You're invited to explore more.** **(Stretch)** Invite students who would like to explore rational exponents and radicals further to complete this optional task. Encourage them to discuss their thinking with a partner.

A **Accessibility: Memory and Attention** Invite students to refer to Problem 11 to support writing exponential expressions with radicals.

3 Connect

Invite students to share which repeated challenges they might like to discuss further by polling the class.

Display the solutions and invite students to share.



Activity 3

Name: Date: Period:

Equivalent Expressions

12. Two students correctly rewrote the equation $27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} = 27$ without using rational exponents.

Ramon: $3 \cdot 3 \cdot 3 = 27$

Nyanna: $\sqrt[3]{27} \cdot \sqrt[3]{27} \cdot \sqrt[3]{27} = 27$

Whose strategy could you use to rewrite $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}} = 10$ without using rational exponents? Circle one.

Ramon's | Nyanna's | Both | Neither

Explain your thinking.

Explanations vary. $10^{\frac{1}{2}}$ isn't an integer. It's some number between 3 and 4, so writing $\sqrt{10} \cdot \sqrt{10} = 10$ is the only way to rewrite $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}} = 10$.

13. Write an equivalent radical or exponential expression for each problem.

A. Determine the missing value in each equation.

a. $10^{\frac{1}{2}} = \sqrt{10}$

b. $7^{\frac{1}{4}} = \sqrt[4]{7}$

c. $4^{\frac{1}{3}} = \sqrt[3]{4}$

d. $\sqrt{71} = 71^{\frac{1}{2}}$

e. $\sqrt[3]{24} = 24^{\frac{1}{3}}$

f. $\sqrt[4]{16} = 16^{\frac{1}{4}}$

You're invited to explore more.

14. Create equivalent expressions for 6 using rational exponents or radicals. Try to create as many expressions as you can (especially ones that you think no one in your class will create)! *Expressions vary.*

$\sqrt{6} \cdot \sqrt{6}$ $\quad 6^{\frac{1}{2}} \cdot 6^{\frac{1}{2}}$

$\sqrt[3]{6} \cdot \sqrt[3]{6} \cdot \sqrt[3]{6}$ $\quad 6^{\frac{1}{3}} \cdot 6^{\frac{1}{3}} \cdot 6^{\frac{1}{3}}$

$\sqrt[4]{6} \cdot \sqrt[4]{6} \cdot \sqrt[4]{6} \cdot \sqrt[4]{6}$ $\quad 6^{\frac{1}{4}} \cdot 6^{\frac{1}{4}} \cdot 6^{\frac{1}{4}} \cdot 6^{\frac{1}{4}}$

Unit 5 Lesson 8 528 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the relationship between expressions written with a rational exponent and radical expressions.

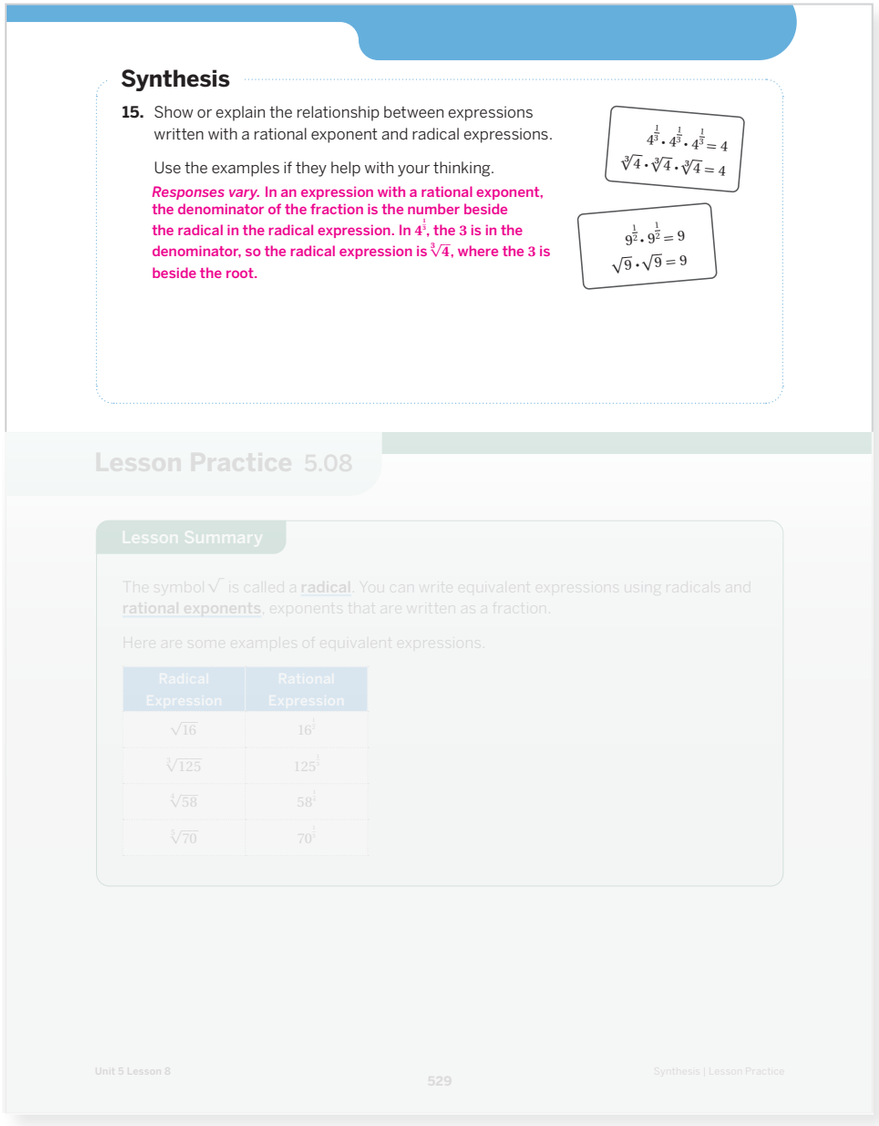
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- A rational exponent of $\frac{1}{2}$ is equivalent to taking the *square root*.
- A rational exponent of $\frac{1}{3}$ is equivalent to taking the *cube root*.
- A rational exponent of $\frac{1}{n}$ is equivalent to taking the nth root.
- You can rewrite an expression with a rational exponent as an expression with a radical, and vice versa.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the whole-class discussion Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each aspect of the relationship and what questions they still have.



Synthesis

15. Show or explain the relationship between expressions written with a rational exponent and radical expressions.

Use the examples if they help with your thinking.

Responses vary. In an expression with a rational exponent, the denominator of the fraction is the number beside the radical in the radical expression. In $4^{\frac{1}{3}}$, the 3 is in the denominator, so the radical expression is $\sqrt[3]{4}$, where the 3 is beside the root.

$4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} = 4$

$\sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4} = 4$

$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9$

$\sqrt{9} \cdot \sqrt{9} = 9$

Lesson Practice 5.08

Lesson Summary

The symbol $\sqrt{\ }$ is called a **radical**. You can write equivalent expressions using radicals and **rational exponents**, exponents that are written as a fraction.

Here are some examples of equivalent expressions.

Radical Expression	Rational Expression
$\sqrt{16}$	$16^{\frac{1}{2}}$
$\sqrt[3]{125}$	$125^{\frac{1}{3}}$
$\sqrt[5]{58}$	$58^{\frac{1}{5}}$
$\sqrt[7]{70}$	$70^{\frac{1}{7}}$

Unit 5 Lesson 8 529 Synthesis | Lesson Practice

Lesson Takeaway: Raising a number or expression to the $\frac{1}{n}$ power is equivalent to taking its nth root. Written symbolically, this means that $x^{\frac{1}{n}} = \sqrt[n]{x}$.

Independent | 5 min

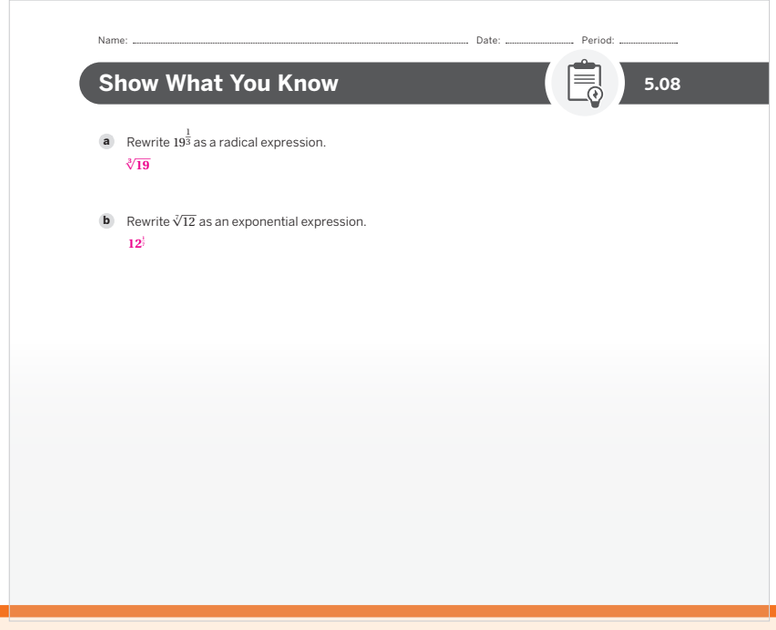
Show What You Know

Purpose: Students demonstrate their understanding by rewriting a radical expression and an exponential expression.

Today's Goals

1. **Goal:** Use the properties of exponents to write equivalent expressions with *radicals* and *rational exponents*.
2. **Goal:** Explain how to match expressions with rational exponents to equivalent radical expressions.
3. **Language Goal:** ustify the equivalence of $x^{\frac{1}{n}}$ and $\sqrt[n]{x}$ using the properties of exponents. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.08

a. Rewrite $19^{\frac{1}{3}}$ as a radical expression.

$\sqrt[3]{19}$

b. Rewrite $\sqrt{12}$ as an exponential expression.

$12^{\frac{1}{2}}$

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of radicals and evaluating square and cube roots:

» **Respond to Student Thinking:**

- Revisit **Grade 8, Unit 8, Lesson 4** (Root Down) or **Lesson 5** (Filling Cubes).
- Use the ***Writing Equivalent Expressions Using Radicals and Rational Exponents*** **Mini-Lesson.**

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

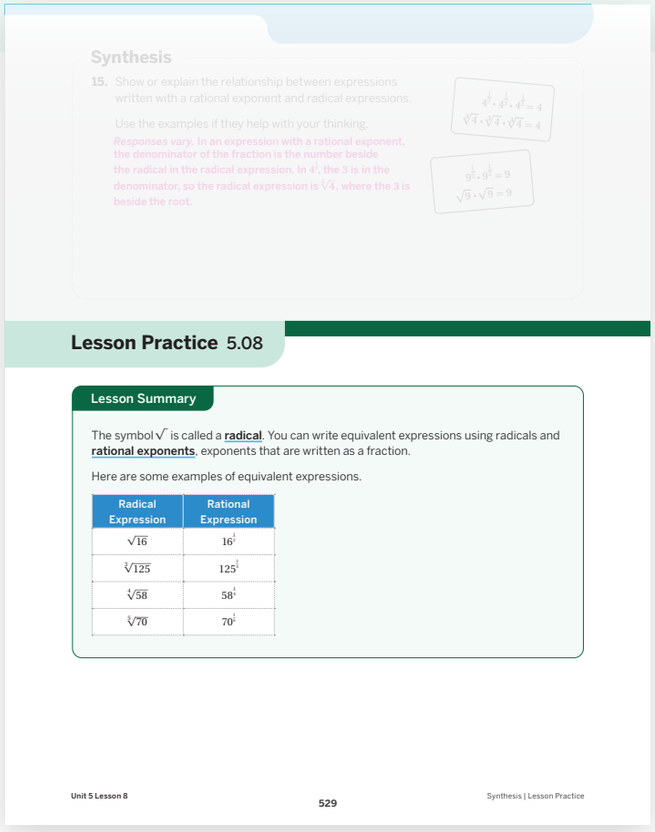
Professional Learning

In this lesson, students used their prior knowledge of exponent rules to support their understanding of **rational exponents** and **radical** expressions. How were students able to apply their thinking to this new concept? How can you support students in recognizing the connection between these key math concepts?
In the next lesson, students will expand their learning from today to justify the equivalent of $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$, where m is larger than 1. How will students' work today prepare them for their learning in Lesson 12?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

15. Show or explain the relationship between expressions written with a rational exponent and radical expressions.

Use the examples if they help with your thinking.

Responses vary. In an expression with a rational exponent, the denominator of the fraction is the number beside the radical in the radical expression. In $4^{\frac{1}{3}}$, the 3 is in the denominator, so the radical expression is $\sqrt[3]{4}$, where the 3 is beside the root.

$4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} = 4$

$\sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4} = 4$

$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9$

$\sqrt{9} \cdot \sqrt{9} = 9$

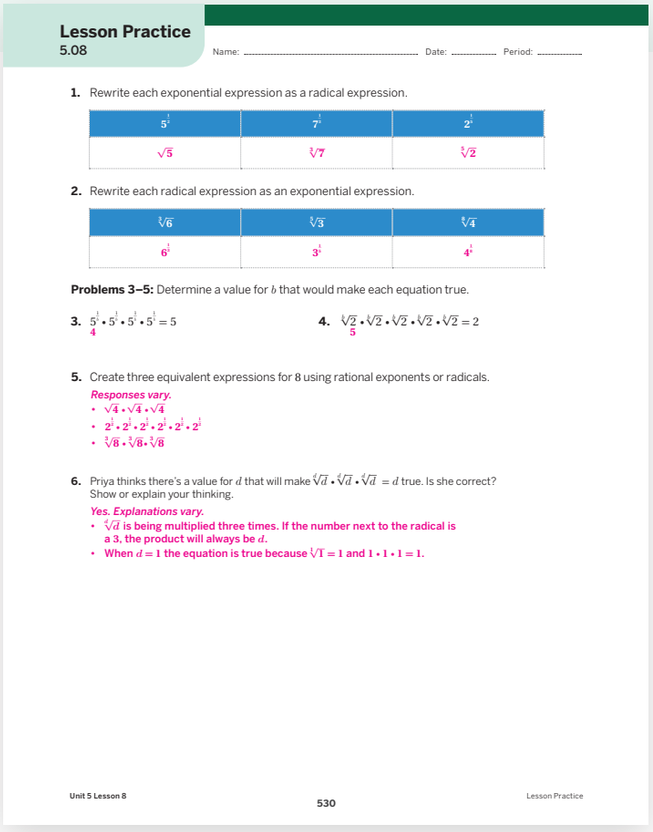
Lesson Practice 5.08

Lesson Summary

The symbol $\sqrt{\ }$ is called a **radical**. You can write equivalent expressions using radicals and **rational exponents**, exponents that are written as a fraction.

Here are some examples of equivalent expressions.

Radical Expression	Rational Expression
$\sqrt{16}$	$16^{\frac{1}{2}}$
$\sqrt[3]{125}$	$125^{\frac{1}{3}}$
$\sqrt[4]{58}$	$58^{\frac{1}{4}}$
$\sqrt[5]{70}$	$70^{\frac{1}{5}}$



Lesson Practice 5.08

Name: Date: Period:

1. Rewrite each exponential expression as a radical expression.

$5^{\frac{1}{2}}$	$7^{\frac{1}{3}}$	$2^{\frac{1}{2}}$
$\sqrt{5}$	$\sqrt[3]{7}$	$\sqrt[3]{2}$

2. Rewrite each radical expression as an exponential expression.

$\sqrt[3]{6}$	$\sqrt[3]{3}$	$\sqrt[3]{4}$
$6^{\frac{1}{3}}$	$3^{\frac{1}{3}}$	$4^{\frac{1}{3}}$

Problems 3–5: Determine a value for b that would make each equation true.

3. $5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 5$

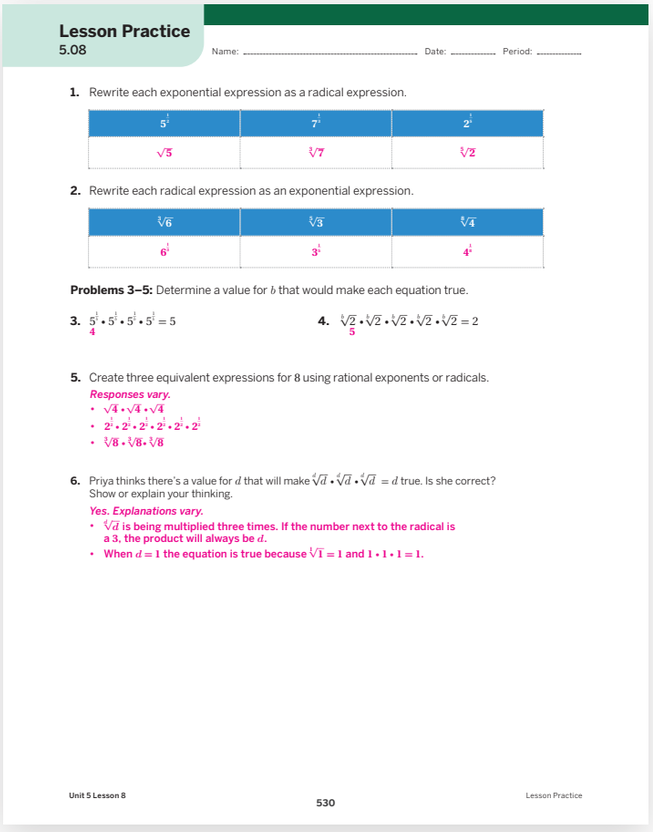
4. $\sqrt[5]{2} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2} = 2$

5. Create three equivalent expressions for 8 using rational exponents or radicals.

Responses vary.
- $\sqrt{4} \cdot \sqrt{4} \cdot \sqrt{4}$
- $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$
- $\sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{8}$

6. Priya thinks there's a value for d that will make $\sqrt[4]{d} \cdot \sqrt[4]{d} \cdot \sqrt[4]{d} = d$ true. Is she correct? Show or explain your thinking.

Yes. Explanations vary.
- $\sqrt[4]{d}$ is being multiplied three times. If the number next to the radical is a 3, the product will always be d.
- When $d = 1$ the equation is true because $\sqrt[4]{1} = 1$ and $1 \cdot 1 \cdot 1 = 1$.



Lesson Practice 5.08

Name: Date: Period:

1. Rewrite each exponential expression as a radical expression.

$5^{\frac{1}{2}}$	$7^{\frac{1}{3}}$	$2^{\frac{1}{2}}$
$\sqrt{5}$	$\sqrt[3]{7}$	$\sqrt[3]{2}$

2. Rewrite each radical expression as an exponential expression.

$\sqrt[3]{6}$	$\sqrt[3]{3}$	$\sqrt[3]{4}$
$6^{\frac{1}{3}}$	$3^{\frac{1}{3}}$	$4^{\frac{1}{3}}$

Problems 3–5: Determine a value for b that would make each equation true.

3. $5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} = 5$

4. $\sqrt[5]{2} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2} = 2$

5. Create three equivalent expressions for 8 using rational exponents or radicals.

Responses vary.
- $\sqrt{4} \cdot \sqrt{4} \cdot \sqrt{4}$
- $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$
- $\sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{8}$

6. Priya thinks there's a value for d that will make $\sqrt[4]{d} \cdot \sqrt[4]{d} \cdot \sqrt[4]{d} = d$ true. Is she correct? Show or explain your thinking.

Yes. Explanations vary.
- $\sqrt[4]{d}$ is being multiplied three times. If the number next to the radical is a 3, the product will always be d.
- When $d = 1$ the equation is true because $\sqrt[4]{1} = 1$ and $1 \cdot 1 \cdot 1 = 1$.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.NSO.1.1
	3–5	2	MA.912.NSO.1.1
	6	3	MA.912.NSO.1.1
Test Practice	7	2	MA.912.NSO.1.1
Spiral Review			
	8	1	MA.8.AR.1.1
	9–10	2	MA.8.AR.1.1

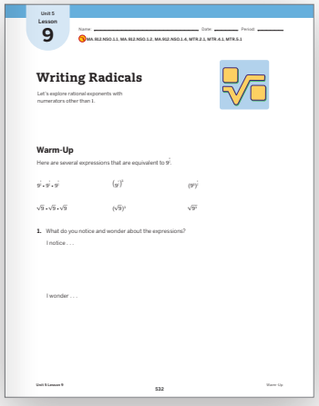
Unit 5

Lesson 9

Writing Radicals

Writing Equivalent Expressions Using Rational Exponents and Radicals

Let's explore rational exponents with numerators other than 1.


Unit 5
Lesson 9
Writing Radicals
Let's explore rational exponents with numerators other than 1.
Warm-Up
1. What do you notice and wonder about the expressions?
I notice . . .
I wonder . . .
532

Learning Outcomes

Today's Goals

1. **Goal:** Use the properties of exponents to write equivalent expressions in the form of $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$.
2. **Language Goal:** Justify the equivalence $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$ using the properties of exponents. **(Writing, Speaking, and Listening)**

Students analyze and make use of the structure of expressions written using rational exponents and expressions written using radicals. They consider exponential expressions of the form $b^{\frac{m}{n}}$ and radical expressions of the form $\sqrt[n]{b^m}$, where m is a whole number larger than 1. Students develop fluency in writing equivalent expressions using radicals and rational exponents. **(MTR.2.1)**

Coherence

Prior Learning

In Grade 8, students applied the properties of integer exponents to generate equivalent numerical expressions. In Lesson 11, students explored the relationship between expressions written using rational exponents, and expressions written using radicals.

Future Learning

In Algebra 2, students will study radical functions in more depth.

Connections

- Students explore rational exponents with numerators other than 1 by analyzing visuals of student work examples. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.NSO.1.1
Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

MA.912.NSO.1.4
Divide a polynomial expression by a monomial expression with rational number coefficients.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.NSO.1.3	**MA.912.NSO.1.5**
MA.8.NSO.1.5	**MA.912.NSO.1.6**
	MA.912.AR.7.1
	MA.912.AR.7.4

Vocabulary

Additional Vocabulary

index

radicand

Lesson at a Glance

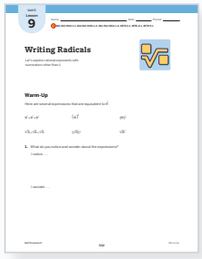
~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

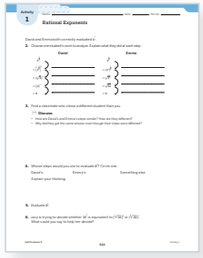
Students use the **Notice and Wonder** routine to make sense of expressions written using rational exponents and radicals. **(MTR.5.1)**



Activity 1

Pairs | 15 min

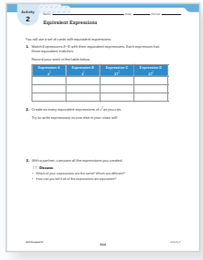
Students analyze two strategies for evaluating expressions of the form $b^{\frac{m}{n}}$. **MLR2: Collect and Display; MLR7: Compare and Connect** **(MTR.2.1)**



Activity 2

Pairs | 15 min

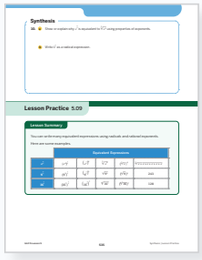
Students match expressions including rational exponents to equivalent expressions written using radicals, and then create equivalent expressions. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of equivalent expressions of the form $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$. **MLR2: Collect and Display** **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing at least two expressions that are equivalent to a rational exponent expression.



Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of expressions written using rational exponents and radicals. **(MTR.5.1)**

1 Launch

Display the expressions and the prompt.

A **Accessibility: Visual-Spatial Processing** Eliminate extraneous visual stimuli by inviting students to use their hands to cover up all but two or three of the expressions at a time.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the expressions. **(MTR.5.1)**

ELL **English Language Learners** Consider reviewing the anchor chart about exponents and exponent rules created in Lesson 11, or creating an anchor chart to review exponent rules, including the powers of powers rule and multiplication rule. Add student ideas and language to the chart throughout the lesson.

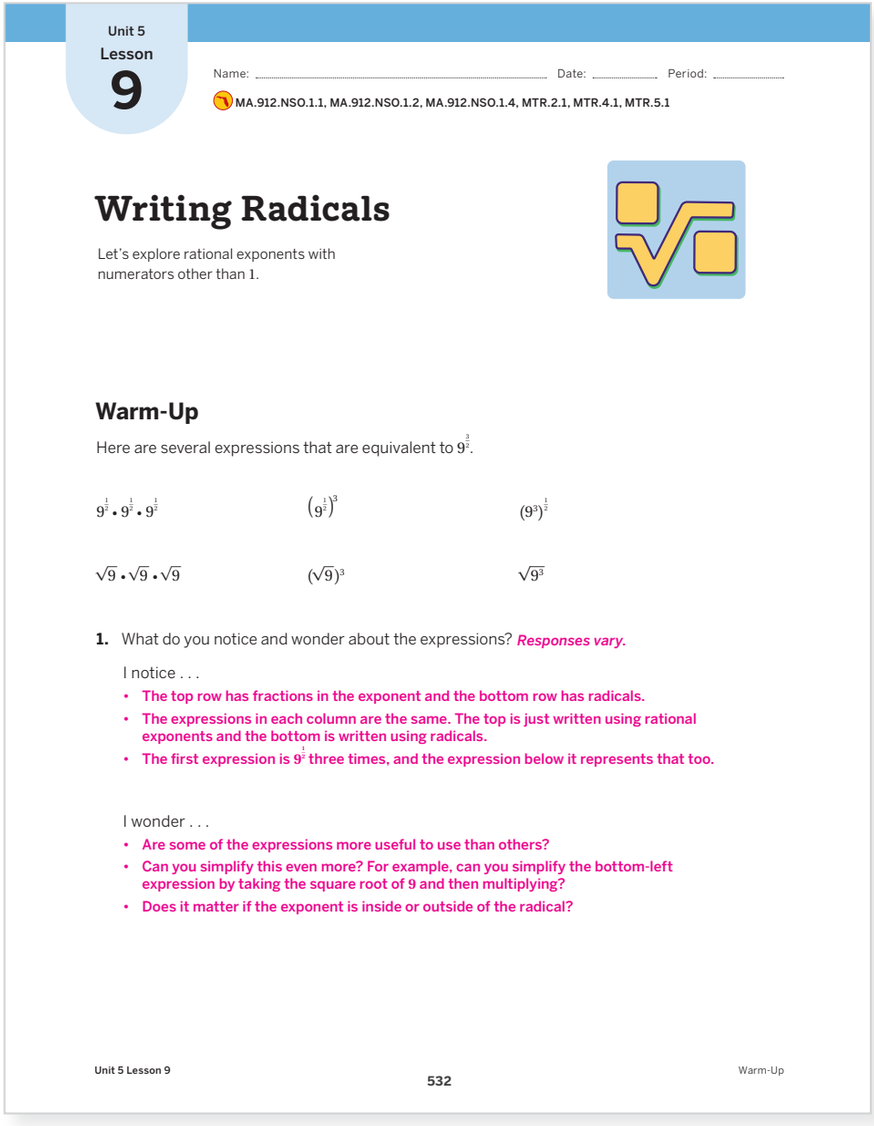
Create a record of the things students noticed and wondered, along with the name of the students who shared them.

Consider asking, "Did anyone notice what the top row of expressions has in common? What about the bottom row?"

Note: If it doesn't come up naturally, consider asking, "Do you think it matters if the exponent is inside or outside of the radical? Why or why not?"

2 Connect

Invite students to share their noticings and wonderings.



Unit 5
Lesson
9

Name: _____ Date: _____ Period: _____

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1

Writing Radicals

Let's explore rational exponents with numerators other than 1.

Warm-Up

Here are several expressions that are equivalent to $9^{\frac{3}{2}}$.

$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}}$ $\quad$ $\left(9^{\frac{1}{2}}\right)^3$ $\quad$ $(9^3)^{\frac{1}{2}}$

$\sqrt{9} \cdot \sqrt{9} \cdot \sqrt{9}$ $\quad$ $(\sqrt{9})^3$ $\quad$ $\sqrt{9^3}$

1. What do you notice and wonder about the expressions? *Responses vary.*

I notice . . .

- The top row has fractions in the exponent and the bottom row has radicals.
- The expressions in each column are the same. The top is just written using rational exponents and the bottom is written using radicals.
- The first expression is $9^{\frac{3}{2}}$ three times, and the expression below it represents that too.

I wonder . . .

- Are some of the expressions more useful to use than others?
- Can you simplify this even more? For example, can you simplify the bottom-left expression by taking the square root of 9 and then multiplying?
- Does it matter if the exponent is inside or outside of the radical?

Unit 5 Lesson 9 532 Warm-Up

Pairs | 15 min

Activity 1 Rational Exponents

Purpose: Students analyze two strategies for evaluating expressions of the form $b^{\frac{m}{n}}$. **(MTR.2.1)**

1 Launch

Display the prompt.

Encourage students to work with their partner to create as many equivalent expressions of $4^{\frac{3}{2}}$ as they can. Invite students to share their equivalent expressions publicly. **(MTR.4.1)**

Invite students to work on one student's steps, and then check in with someone who worked on different steps.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to the new prompt in Problem 4. See the Routine Facilitation Guide for more information.

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to highlight key information in Emma's/David's steps.

2 Monitor

D **Differentiation: (Support)** To support students getting started with Problem 5, invite them to write out the steps for the student they chose in Problem 4 but for the new exponential expression $8^{\frac{4}{3}}$.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Listen for responses to Problem 6 to invite students to share with other pairs. Consider asking, "How could it help Jacy to rewrite the expression? Is there another way to rewrite it?"

3 Connect

Display the Student Edition. Consider asking, "How do you know where to put the denominator? Numerator?" **(MTR.5.1)**

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas for writing expressions with rational exponents. Generalize about the case of $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$. Add to the display for students to reference, labeling the numerator as equivalent to the power and the denominator as equivalent to the index. **(MTR.5.1)**

Activity 1

Name: _____ Date: _____ Period: _____

Rational Exponents

David and Emma both correctly evaluated $4^{\frac{3}{2}}$.

2. Choose one student's work to analyze. Explain what they did at each step. *Responses vary.*

David

$4^{\frac{3}{2}}$
$= \left(4^{\frac{1}{2}}\right)^3$ — Rewrite $\frac{3}{2}$ as $\left(\frac{1}{2}\right)^3$
$= (\sqrt{4})^3$ — Rewrite $4^{\frac{1}{2}}$ as $\sqrt{4}$
$= (2)^3$ — Simplify $\sqrt{4} = 2$
$= 8$ — Cube 2

Emma

$4^{\frac{3}{2}}$
$= (4^3)^{\frac{1}{2}}$ — Rewrite $\frac{3}{2}$ as $3^{\frac{1}{2}}$
$= \sqrt{4^3}$ — Rewrite $(4^3)^{\frac{1}{2}}$ as $\sqrt{4^3}$
$= \sqrt{64}$ — Multiply $4^3 = 64$
$= 8$ — Simplify $\sqrt{64} = 8$

3. Find a classmate who chose a different student than you.

Discuss:

- How are David's and Emma's steps similar? How are they different?
- Why did they get the same answer even though their steps were different?

Responses vary.

- David used the powers of powers law to rewrite the exponent of $\frac{3}{2}$ as $\frac{1}{2}$ raised to the third power, which allowed him to take the square root of 4. Emma used the powers of powers law to rewrite the exponent of 3 raised to the $\frac{1}{2}$ power, which let her evaluate 4^3 under the radical.
- Both Emma's and David's solving strategies created equivalent expressions at every step, so their answers are the same. David took the square root and then raised to the third power, and Emma raised to the third power and then took the square root.

4. Whose steps would you use to evaluate $8^{\frac{4}{3}}$? Circle one. *Responses vary.*

David's Emma's Something else

Explain your thinking. *Explanations vary.*

- I would use David's method. It seems easier to me to first break the number down into something smaller before applying the exponent.
- I would use Emma's method. It feels easier to me to keep the exponent inside the radical and apply the exponent first before taking the square root.

5. Evaluate $8^{\frac{4}{3}}$.

16

6. Jacy is trying to decide whether $10^{\frac{6}{5}}$ is equivalent to $(\sqrt[5]{10})^6$ or $(\sqrt[6]{10})^7$. What could you say to help her decide?

Responses vary. Jacy could rewrite $10^{\frac{6}{5}}$ as $(10^{\frac{1}{5}})^6$. Both of those expressions are equivalent to $(\sqrt[5]{10})^6$ (the 7th root of 10^6).

Unit 5 Lesson 9 533 Activity 1

Key Takeaway: Using the powers of powers law, you can write equivalent expressions with rational exponents of the form $b^{\frac{m}{n}}$ as $(bm)^{\frac{1}{n}}$ or $(b^{\frac{1}{n}})m$. Since $b^{\frac{1}{n}} = \sqrt[n]{b}$, you can write these expressions in radical form as $\sqrt[n]{bm}$ or $(\sqrt[n]{b})m$.

Pairs | 15 min

Activity 2 Equivalent Expressions

Purpose: Students match expressions including rational exponents to equivalent expressions written using radicals, and then create equivalent expressions. **(MTR.5.1)**

Short on time? Consider inviting students to write fewer equivalent expressions in Problem 8.

1 Launch

Display the expressions and the directions.

Distribute the Activity 2 Cards to each student pair.

A **Accessibility: Executive Functioning** Invite students to organize the cards to support them in initiating a complex task.

2 Monitor

D **Differentiation** *(Problem 7)*

Look for students who:	Respond to Student Thinking
Think they have to find the square root of 27 or the fifth root of 4 to sort the cards.	**Support** Invite them to revisit David's and Emma's steps. Consider asking, "Can you rewrite the expression differently?"
Match cards to Expression B and Expression C by calculating the value of each.	**Strengthen** Celebrate their strategy. Consider asking, "Can you use the structure of the expressions to compare them another way?" **(MTR.2.1)**
Would like an additional challenge.	**Stretch** Invite students to write expressions with bases or radicands other than 4 or 27 that are equivalent to Expressions A–D. A: $16^{\frac{1}{2}}$ and $\sqrt[5]{2^4}$

Look for many expressions that are equivalent to $x^{\frac{5}{4}}$ and invite students to share them during the Connect.

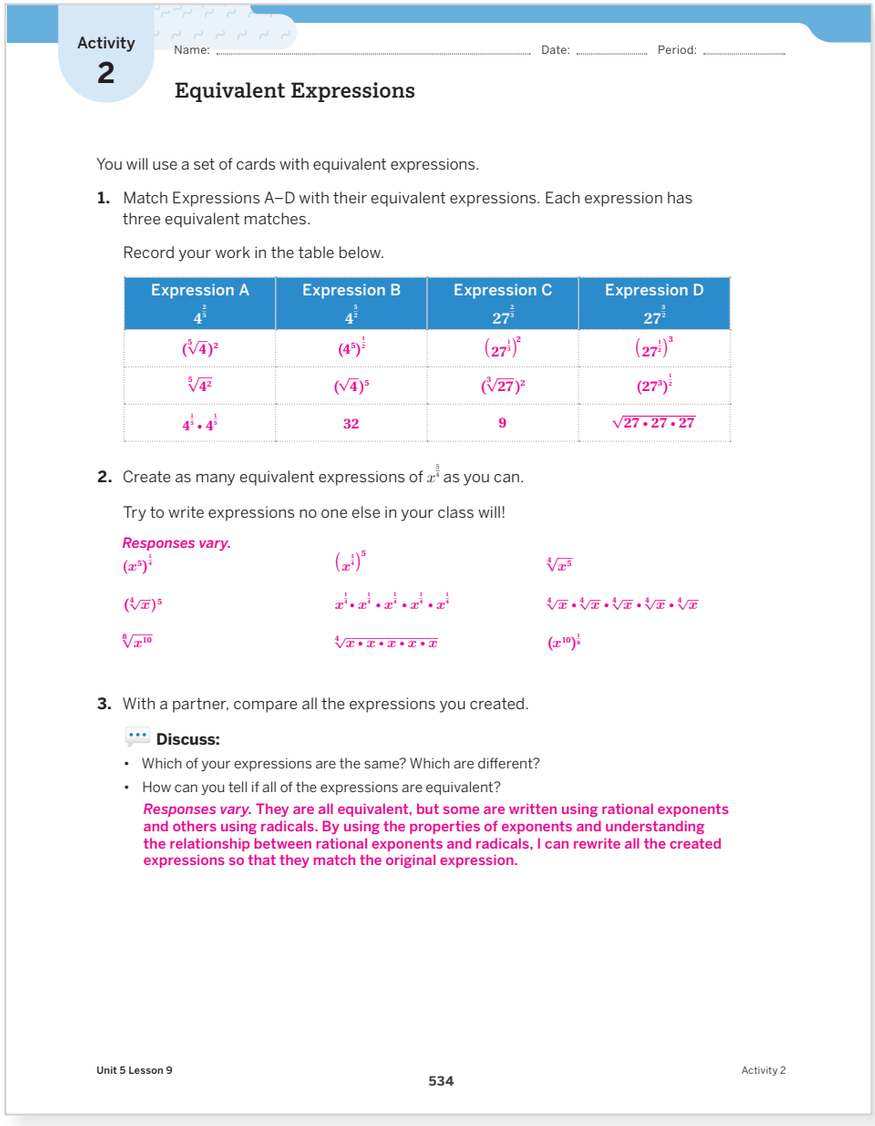
ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as, "The expressions ___ and ___ are the same/different because . . ." **(Writing and Speaking)**

3 Connect

Display the Student Edition and invite students to share their expressions from Problem 8.

Consider asking, "What are some strategies you could use to check that all of these expressions are equivalent?"

Encourage students to determine if every expression they wrote is equivalent to $x^{\frac{5}{4}}$.



Activity 2

Name: _____ Date: _____ Period: _____

Equivalent Expressions

You will use a set of cards with equivalent expressions.

1. Match Expressions A–D with their equivalent expressions. Each expression has three equivalent matches.

   Record your work in the table below.

Expression A $4^{\frac{2}{5}}$	Expression B $4^{\frac{5}{2}}$	Expression C $27^{\frac{2}{3}}$	Expression D $27^{\frac{3}{2}}$
$(\sqrt[5]{4})^2$	$(4^5)^{\frac{1}{2}}$	$\left(27^{\frac{1}{3}}\right)^2$	$\left(27^{\frac{1}{2}}\right)^3$
$\sqrt[5]{4^2}$	$(\sqrt{4})^5$	$(\sqrt[3]{27})^2$	$(27^3)^{\frac{1}{2}}$
$4^{\frac{1}{5}} \cdot 4^{\frac{1}{5}}$	32	9	$\sqrt{27 \cdot 27 \cdot 27}$

2. Create as many equivalent expressions of $x^{\frac{5}{4}}$ as you can.

   Try to write expressions no one else in your class will!

   Responses vary.

   $(x^5)^{\frac{1}{4}}$ $\left(x^{\frac{1}{4}}\right)^5$ $\sqrt[4]{x^5}$

   $(\sqrt{x})^5$ $x^{\frac{1}{4}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{4}}$ $\sqrt[4]{x} \cdot \sqrt[4]{x} \cdot \sqrt[4]{x} \cdot \sqrt[4]{x} \cdot \sqrt[4]{x}$

   $\sqrt[8]{x^{10}}$ $\sqrt[4]{x \cdot x \cdot x \cdot x \cdot x}$ $(x^{10})^{\frac{1}{4}}$

3. With a partner, compare all the expressions you created.

   Discuss:

   - Which of your expressions are the same? Which are different?
   - How can you tell if all of the expressions are equivalent?

   Responses vary. They are all equivalent, but some are written using rational exponents and others using radicals. By using the properties of exponents and understanding the relationship between rational exponents and radicals, I can rewrite all the created expressions so that they match the original expression.

Unit 5 Lesson 9 534 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of equivalent expressions of the form $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$.

Invite students to respond independently, and then share their thinking with a partner. **(MTR.4.1)**

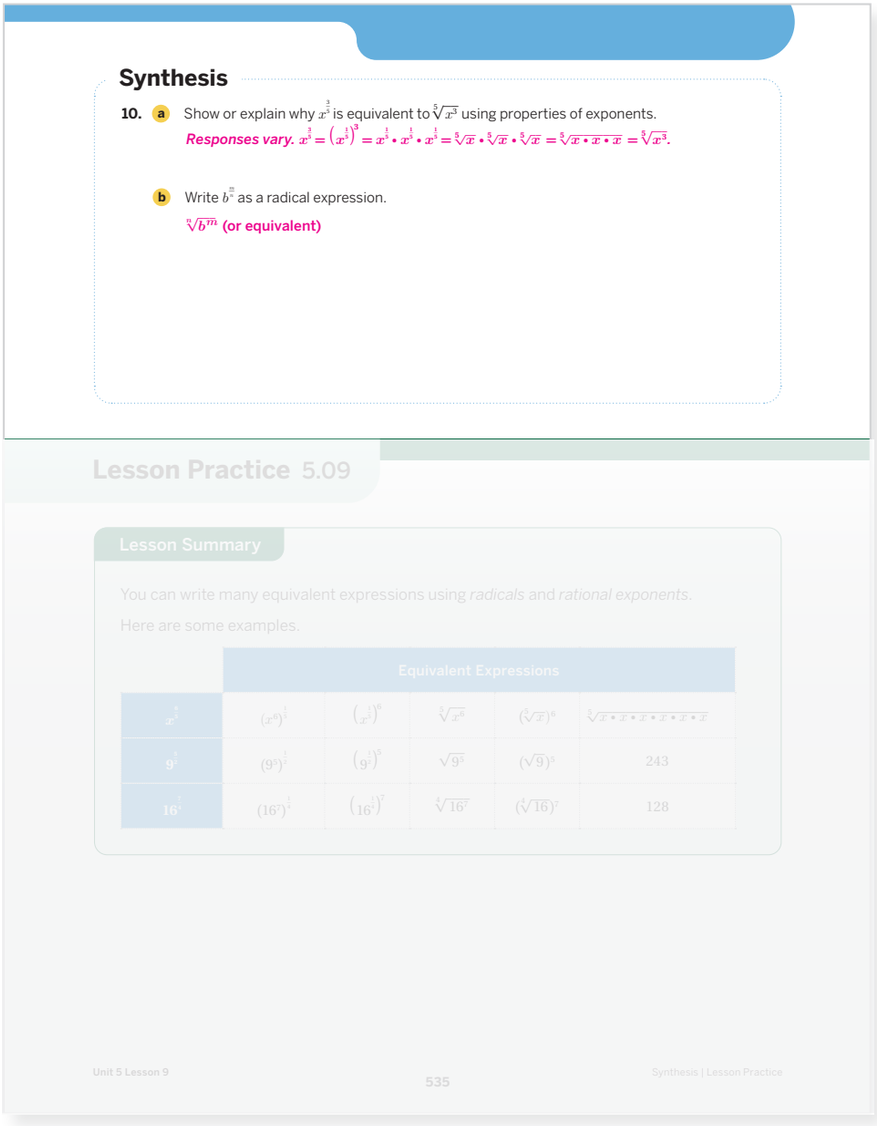
Look for and share a variety of ideas, such as:

- $x^{\frac{3}{5}} = x^{\frac{1}{5}} \cdot x^{\frac{1}{5}} \cdot x^{\frac{1}{5}}$.
- $x^{\frac{3}{5}} = (x^3)^{\frac{1}{5}}$ or $x^{\frac{3}{5}} = (x^{\frac{1}{5}})^3$.
- $x^{\frac{1}{5}}$ is equivalent to $\sqrt[5]{x}$.
- $\sqrt[5]{x} \cdot \sqrt[5]{x} \cdot \sqrt[5]{x} = (\sqrt[5]{x})^3$.
- It doesn't matter if the exponent is inside the radical or outside, so we can write $(\sqrt[5]{x})^3$ or $\sqrt[5]{x^3}$.

Create a record of students' ideas, or add to an existing one if it was created for Lesson 11.

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas about writing equivalent expressions with radicals and rational exponents. Add students' ideas and strategies to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. **a** Show or explain why $x^{\frac{3}{5}}$ is equivalent to $\sqrt[5]{x^3}$ using properties of exponents.

Responses vary. $x^{\frac{3}{5}} = \left(x^{\frac{1}{5}}\right)^3 = x^{\frac{1}{5}} \cdot x^{\frac{1}{5}} \cdot x^{\frac{1}{5}} = \sqrt[5]{x} \cdot \sqrt[5]{x} \cdot \sqrt[5]{x} = \sqrt[5]{x \cdot x \cdot x} = \sqrt[5]{x^3}$.

b Write $b^{\frac{m}{n}}$ as a radical expression.

$\sqrt[n]{b^m}$ (or equivalent)

Lesson Practice 5.09

Lesson Summary

You can write many equivalent expressions using *radicals* and *rational exponents*.

Here are some examples.

	Equivalent Expressions				
$x^{\frac{6}{5}}$	$(x^6)^{\frac{1}{5}}$	$\left(x^{\frac{1}{5}}\right)^6$	$\sqrt[5]{x^6}$	$(\sqrt[5]{x})^6$	$\sqrt[5]{x \cdot x \cdot x \cdot x \cdot x \cdot x}$
$9^{\frac{5}{2}}$	$(9^5)^{\frac{1}{2}}$	$\left(9^{\frac{1}{2}}\right)^5$	$\sqrt{9^5}$	$(\sqrt{9})^5$	243
$16^{\frac{7}{4}}$	$(16^7)^{\frac{1}{4}}$	$\left(16^{\frac{1}{4}}\right)^7$	$\sqrt[4]{16^7}$	$(\sqrt[4]{16})^7$	128

Unit 5 Lesson 9 535 Synthesis | Lesson Practice

Lesson Takeaway: Raising a number or expression to the $\frac{m}{n}$ power is equivalent to raising it to the mth power and taking its nth root, in either order. Written symbolically, this means that $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing at least two expressions that are equivalent to a rational exponent expression.

Today's Goals

1. **Goal:** Use the properties of exponents to write equivalent expressions in the form of $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$.
2. **Language Goal:** Justify the equivalence $b^{\frac{m}{n}}$ and $\sqrt[n]{b^m}$ using the properties of exponents. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.09

Write at least two expressions that are equivalent to $49^{\frac{3}{2}}$.
Use a radical in at least one of your expressions.

Responses vary.

343 $\sqrt{49^3}$ $(\sqrt{49})^3$ $(49^{\frac{1}{2}})^3$

$(49^3)^{\frac{1}{2}}$ $\sqrt{49 \cdot 49 \cdot 49}$ $\sqrt{49} \cdot \sqrt{49} \cdot \sqrt{49}$ $49^{\frac{1}{2}} \cdot 49^{\frac{1}{2}} \cdot 49^{\frac{1}{2}}$

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students' need support understanding exponent properties or writing equivalent expressions with exponents:

» **Respond to Student Thinking:**

- Revisit **Grade 8, Unit 8, Lessons 4–5** (Root Down or Filling Cubes).
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students' would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to create their own equivalent expressions with radicals and rational exponents and share with a partner.

Professional Learning

The following lessons in this sub-unit have a financial literacy theme, exploring simple and compound interest. Some students may already be familiar with the concept of interest-bearing accounts. Consider inviting your students to share any knowledge or experience they may have with accounts that earn interest. What might you want to think or learn more about before facilitating Lessons 13–15?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. a. Show or explain why $x^{\frac{3}{2}}$ is equivalent to $\sqrt[3]{x^2}$ using properties of exponents.

Responses vary. $x^{\frac{3}{2}} = \left(x^{\frac{1}{2}}\right)^3 = x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = \sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x} = \sqrt{x \cdot x \cdot x} = \sqrt{x^3}$.

b. Write $b^{\frac{m}{n}}$ as a radical expression.

$\sqrt[n]{b^m}$ (or equivalent)

Lesson Practice 5.09

Lesson Summary

You can write many equivalent expressions using *radicals* and *rational exponents*.

Here are some examples.

	Equivalent Expressions				
$x^{\frac{5}{2}}$	$(x^5)^{\frac{1}{2}}$	$\left(x^{\frac{1}{2}}\right)^5$	$\sqrt[2]{x^5}$	$(\sqrt[2]{x})^5$	$\sqrt[2]{x \cdot x \cdot x \cdot x \cdot x}$
$9^{\frac{5}{2}}$	$(9^5)^{\frac{1}{2}}$	$\left(9^{\frac{1}{2}}\right)^5$	$\sqrt{9^5}$	$(\sqrt{9})^5$	243
$16^{\frac{7}{4}}$	$(16^7)^{\frac{1}{4}}$	$\left(16^{\frac{1}{4}}\right)^7$	$\sqrt[4]{16^7}$	$(\sqrt[4]{16})^7$	128

Lesson Practice 5.09

Name: _____ Date: _____ Period: _____

Problems 1–3: Write an equivalent expression for each problem.

1. $\sqrt{2^5}$ *Responses vary.* $2^{\frac{5}{2}}$
2. $8^{\frac{5}{3}}$ *Responses vary.* $\sqrt[3]{8^5}$
3. $27^{\frac{1}{3}}$ *Responses vary.* $\sqrt[3]{27}$

4. Write two different expressions that are equivalent to $\sqrt[3]{6^2}$.

Responses vary.

- $6^{\frac{2}{3}}$
- $(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)^{\frac{1}{3}}$

5. Write or sketch a problem that has $4^{\frac{3}{2}}$ as an answer.

Responses vary.

- $4^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} = ?$
- Rewrite $\sqrt{4 \cdot 4 \cdot 4}$ using a rational exponent.
- What is the side length of a square that has an area of 64 square units?

6. Show or explain why $a^{\frac{3}{2}}$ is equivalent to $\sqrt[2]{a^3}$ using properties of exponents.

Responses vary.
$a^{\frac{3}{2}} = \left(a^{\frac{1}{2}}\right)^3 = a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = \sqrt{a} \cdot \sqrt{a} \cdot \sqrt{a} = \sqrt{a \cdot a \cdot a} = \sqrt[2]{a^3}$.

Lesson Practice 5.09

Name: _____ Date: _____ Period: _____

Test Practice

7. Match each expression with the equivalent term.

a. $9^{\frac{1}{2}}$ — c — $\sqrt{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

b. $9^{\frac{5}{2}}$ — d — $\sqrt[2]{9}$

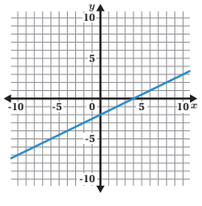
c. $3^{\frac{3}{2}}$ — b — 27

d. $3^{\frac{1}{2}}$ — a — $\sqrt[2]{3^3}$

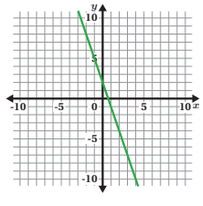
Spiral Review

Problems 8–9: Write an equation for each line.

8. Equation: $y = \frac{1}{2}x - 2$ (or equivalent)



9. Equation: $y = -3x + 2$ (or equivalent)



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.NSO.1.1
	4–6	3	MA.912.NSO.1.1
Test Practice	7	2	MA.912.NSO.1.1
Spiral Review			
	8–9	1	MA.8.AR.3.2

Unit 5

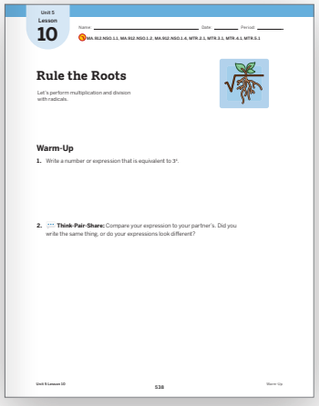
Lesson

10

Rule the Roots

Operations with Radicals

Let's perform operations with radicals.


Unit 5
Lesson
10
Name _____ Date _____ Period _____
MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
Rule the Roots
Let's perform multiplication and division with radicals.
Warm-Up
1. Write a number or expression that is equivalent to 3^4.
2. Think-Pair-Share: Compare your expression to your partner's. Did you write the same thing, or do your expressions look different?
538

Learning Outcomes

Today's Goals

1. **Goal:** Students will simplify and perform operations on radical expressions.
2. **Goal:** Students will analyze and correct common errors in radical operations.
3. **Language Goal:** Students will explain their reasoning and preferred strategies for simplifying radicals. **(Speaking and Listening)**

Students apply properties of radicals to simplify, multiply, divide, add, and subtract radical expressions, ensuring they recognize when terms can be combined. They analyze errors in radical operations and justify their reasoning through structured discussions and written explanations. **(MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In 8th Grade Unit 8, students learned about square roots with triangles and the Pythagorean theorem. In Algebra 1, Unit 5 Lessons 8-9 students explored writing radicals as numbers with rational exponents.

Future Learning

In future math courses, students will extend their understanding of radical operations to work with algebraic expressions containing radicals, solve radical equations, and explore how simplifying radicals relates to simplifying exponential and logarithmic expressions.

Connections

- Students perform multiplication and division with radicals as they analyze visuals of student work and explain their mathematical thinking. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.NSO.1.4
Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.

MA.912.NSO.1.2
Generate equivalent algebraic expressions using the properties of exponents.

Also Addressing: **MA.912.NSO.1.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.912.AR.1.1	**MA.912.NSO.1.5**
MA.912.AR.1.2	**MA.912.AR.7.1**
	MA.912.AR.7.4

Vocabulary

Review Vocabulary

radical
radicand
index

Lesson at a Glance

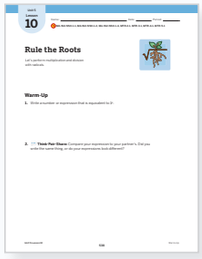
~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

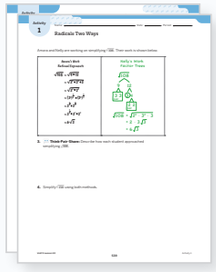
Students compare equivalent expressions using factors, prime factorization, and exponents.



Activity 1

Pairs | 10 min

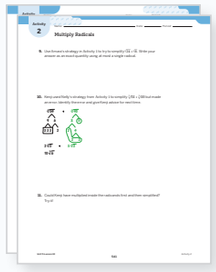
Students use the Think-Pair-Share routine to compare two methods for simplifying radicals, using factorization and exponent notation. **Routine(s):** Think-Pair-Share **(MTR.2.1, MTR.4.1, MTR.5.1)**



Activity 2

Pairs | 10 min

Students explore multiplying with radicals.



Activity 3

Pairs | 10 min

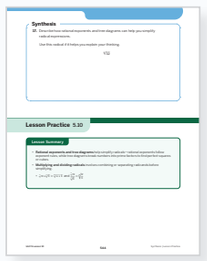
Students will learn how to divide radicals by applying the quotient rule for radicals.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of simplifying radicals and operations with radicals.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by completing practice problems simplifying radicals.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students compare equivalent expressions using factors, prime factorization, and exponents. **(MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1)**

1 Launch

To access prior knowledge, consider asking:

- "Is there only one correct answer? Or many?" **(MTR.2.1)**
- "Did you only write one answer? Or did you try others?"
- "What's the longest or shortest expression you can think of to represent 3^4?"

A **Accessibility: Conceptual Processing** Invite students to make connections to familiar contexts to support abstract thinking, such as an example with a smaller exponent.

Look for students recalling information about factors and different equivalent expressions.

2 Connect

Share student work on the board, showcasing different ways to express the same power, highlighting that all forms are equivalent.

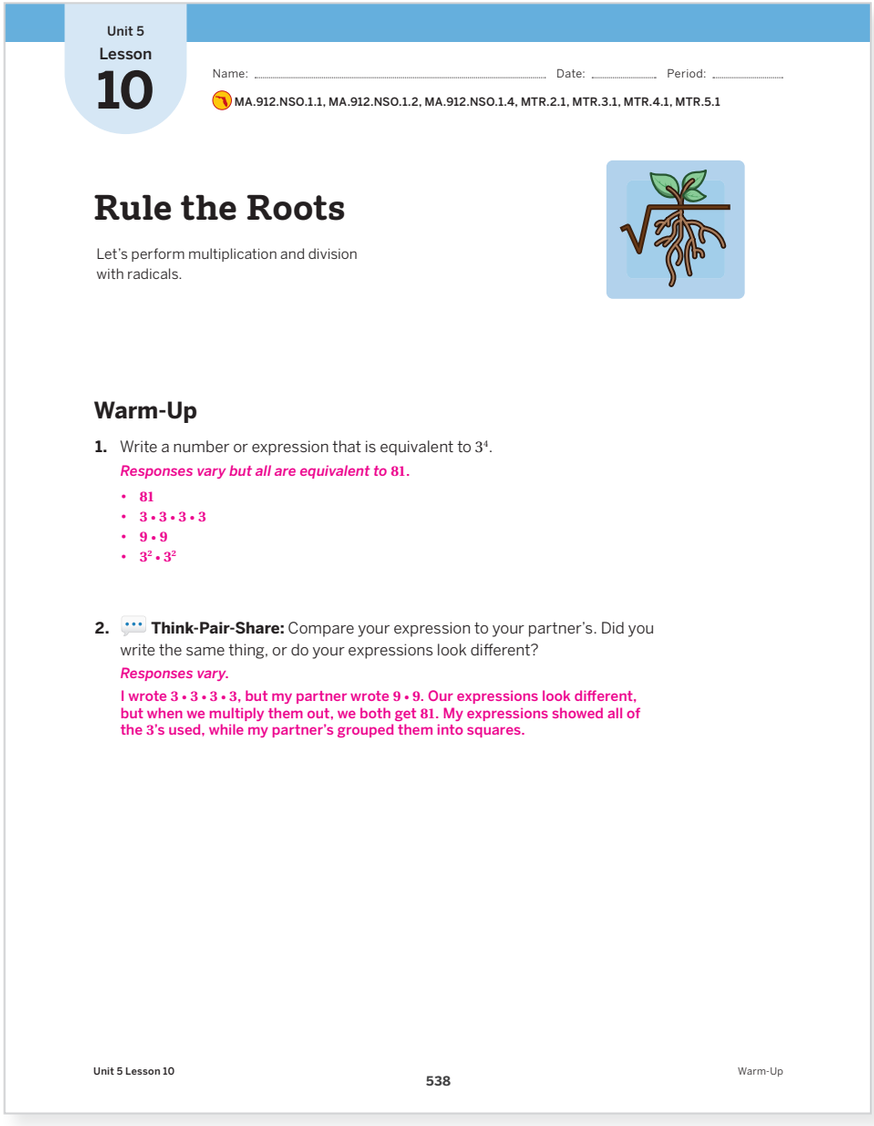
Invite students to revise their responses to make them stronger and clearer after the discussion. **(MTR.4.1)**

ELL **English Language Learners** Consider creating an anchor chart about equivalent expressions. Add student ideas and language to the chart throughout the lesson.

To reveal the Key Takeaway, Consider asking:

- "How do you know all of these expressions are equal to the same number?"
- "What patterns do you notice when breaking down the number into factors or exponents?" **(MTR.5.1)**
- "Why might someone choose to write an expression using exponents instead of repeated multiplication?" **(MTR.2.1)**

Note: Consider leaving student work up for students to refer back to throughout the lesson.



Unit 5
Lesson
10

Name: _____ Date: _____ Period: _____

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

Rule the Roots

Let's perform multiplication and division with radicals.

Warm-Up

1. Write a number or expression that is equivalent to 3^4.

   Responses vary but all are equivalent to 81.

   - 81
   - $3 \cdot 3 \cdot 3 \cdot 3$
   - $9 \cdot 9$
   - $3^2 \cdot 3^2$

2. **Think-Pair-Share:** Compare your expression to your partner's. Did you write the same thing, or do your expressions look different?

   Responses vary.

   I wrote $3 \cdot 3 \cdot 3 \cdot 3$, but my partner wrote $9 \cdot 9$. Our expressions look different, but when we multiply them out, we both get 81. My expressions showed all of the 3's used, while my partner's grouped them into squares.

Unit 5 Lesson 10 538 Warm-Up

Pairs | 10 min

Activity 1 Radicals Two Ways

Purpose: Students use the Think-Pair-Share routine to compare two methods for simplifying radicals, using factorization and exponent notation. **(MTR.2.1, MTR.4.1, MTR.5.1)**

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse about the two strategies used. Invite students to think independently and then take turns sharing with a partner. Invite students to share their or their partner's ideas with the class. **(MTR.4.1)**

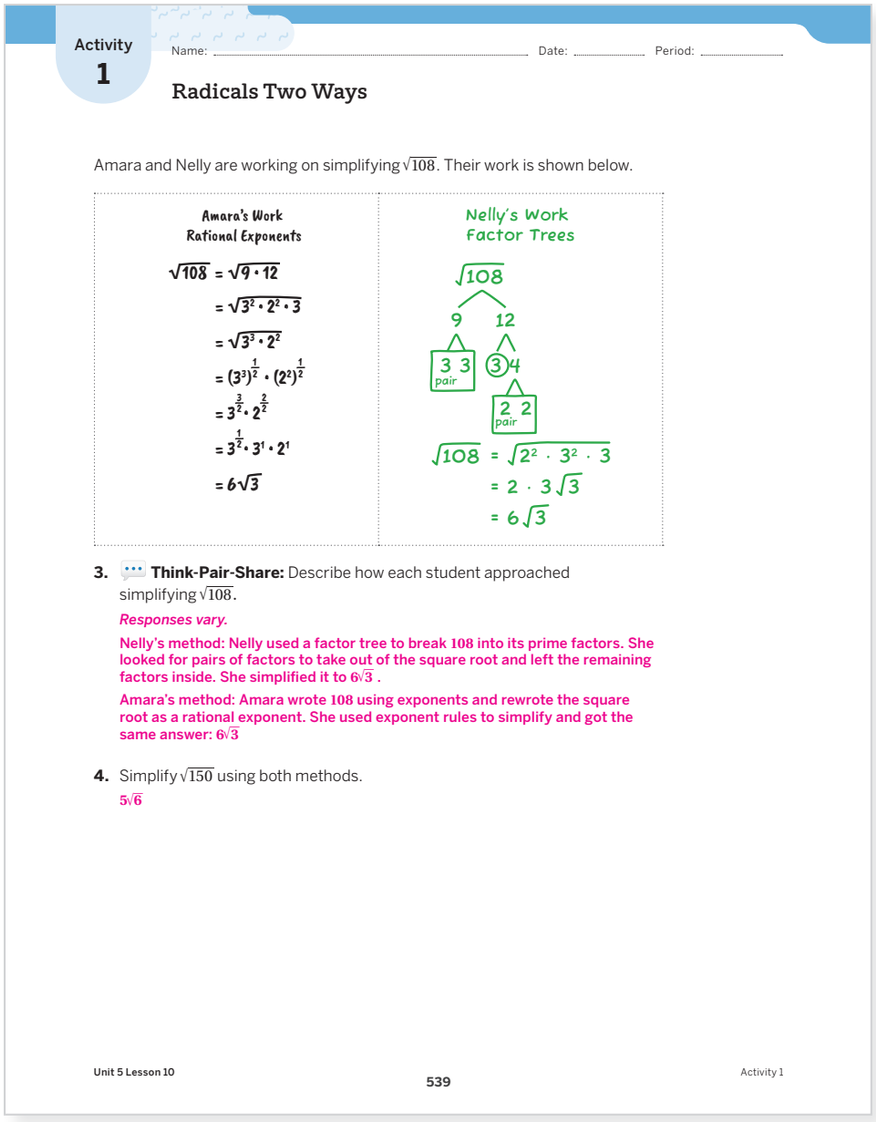
To support students in getting started, consider inviting them to re-write the work on a separate sheet of paper and ask them to make connections to prior lessons and the warm-up.

2 Monitor

D Differentiation *(Problems 3–4)*

Look for students who:	Respond to Student Thinking
Need support getting started. (Problem 3)	**Support** Encourage them to highlight key features of the student work and relate it to their work in the warm-up.
Recognize that factor trees show perfect squares. (Problems 3-4)	**Strengthen** Celebrate their generalization about prime factorization and discuss how squares are related to square roots.
Recognize that rational exponents can help simplify radicals.	**Strengthen** Consider asking, "Is this a strategy you would like to use all the time?"
Need support trying a new problem. (Problem 4)	**Support** Encourage them to start with the strategy that makes the most sense to them. Help them get started.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Radicals Two Ways

Amara and Nelly are working on simplifying $\sqrt{108}$. Their work is shown below.

Amara's Work
Rational Exponents

$\sqrt{108} = \sqrt{9 \cdot 12}$
$= \sqrt{3^2 \cdot 2^2 \cdot 3}$
$= \sqrt{3^3 \cdot 2^2}$
$= (3^3)^{\frac{1}{2}} \cdot (2^2)^{\frac{1}{2}}$
$= 3^{\frac{3}{2}} \cdot 2^{\frac{2}{2}}$
$= 3^{\frac{1}{2}} \cdot 3^1 \cdot 2^1$
$= 6\sqrt{3}$

Nelly's Work
Factor Trees

$\sqrt{108}$
9 12
3 3 pair, 3 4
2 2 pair
$\sqrt{108} = \sqrt{2^2 \cdot 3^2 \cdot 3}$
$= 2 \cdot 3\sqrt{3}$
$= 6\sqrt{3}$

3. **Think-Pair-Share:** Describe how each student approached simplifying $\sqrt{108}$.

Responses vary.

Nelly's method: Nelly used a factor tree to break 108 into its prime factors. She looked for pairs of factors to take out of the square root and left the remaining factors inside. She simplified it to $6\sqrt{3}$.

Amara's method: Amara wrote 108 using exponents and rewrote the square root as a rational exponent. She used exponent rules to simplify and got the same answer: $6\sqrt{3}$

4. Simplify $\sqrt{150}$ using both methods.

$5\sqrt{6}$

Unit 5 Lesson 10 539 Activity 1

Pairs | 15 min

Activity 1 Radicals Two Ways (continued)

Purpose: Students use the Think-Pair-Share routine to compare two methods for simplifying radicals, using factorization and exponent notation. **(MTR.2.1, MTR.4.1, MTR.5.1)**

2 Monitor

Look for students that:

- Identify patterns in factorization and recognize exponent rules.
- Compare and contrast both strategies. **(MTR.5.1)**
- Make connections between methods. **(MTR.2.1)**

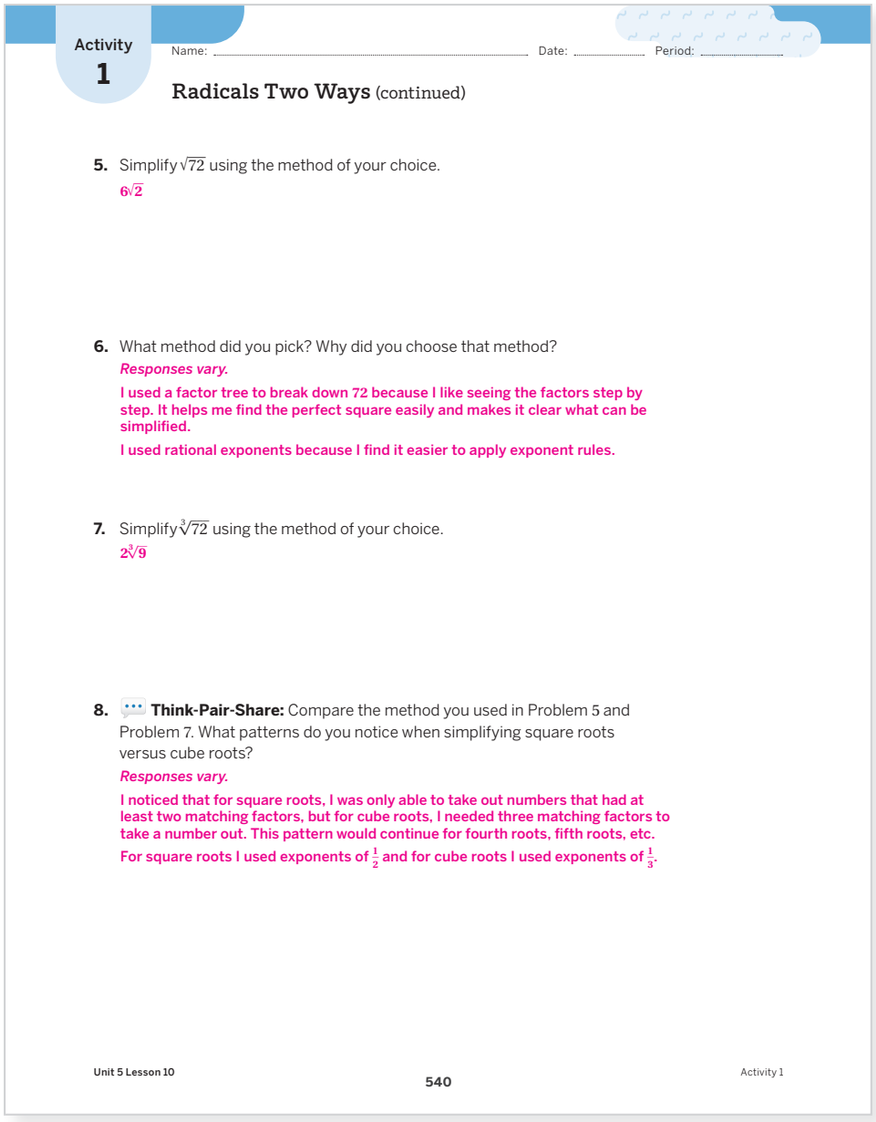
Note: You must mention the connection between cubes and cube roots as well. The factor tree identifies factors that can be written as a square of a factor, or a cube of a factor. Ask a guiding question, "Do you think the factor tree works with identifying cubed factors? Would we still be looking for pairs?" **(MTR.5.1)**

3 Connect

Invite students to share the strategies and methods they used to simplify the radical expressions.

To reveal the Key Takeaway, ask:

- "What are two ways to simplify radicals?"
- "When simplifying a square root, how many matching factors do you need to take a number out of the radical? What about for a cube root?" **(MTR.5.1)**
- "How does the root index tell you how to group factors when simplifying?" **(MTR.2.1)**
- "If you were simplifying a fourth root, how do you think the process would change? What about a fifth root?" **(MTR.5.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Radicals Two Ways (continued)

5. Simplify $\sqrt{72}$ using the method of your choice.

$6\sqrt{2}$

6. What method did you pick? Why did you choose that method?

Responses vary.

I used a factor tree to break down 72 because I like seeing the factors step by step. It helps me find the perfect square easily and makes it clear what can be simplified.

I used rational exponents because I find it easier to apply exponent rules.

7. Simplify $\sqrt[3]{72}$ using the method of your choice.

$2\sqrt[3]{9}$

8. **Think-Pair-Share:** Compare the method you used in Problem 5 and Problem 7. What patterns do you notice when simplifying square roots versus cube roots?

Responses vary.

I noticed that for square roots, I was only able to take out numbers that had at least two matching factors, but for cube roots, I needed three matching factors to take a number out. This pattern would continue for fourth roots, fifth roots, etc.

For square roots I used exponents of $\frac{1}{2}$ and for cube roots I used exponents of $\frac{1}{3}$.

Unit 5 Lesson 10 540 Activity 1

Key Takeaway: Both prime factorization and rational exponents can simplify radicals.

Independent | 10 min

Activity 2 Multiply Radicals

Purpose: Students explore multiplying with radicals. **(MTR.2.1, MTR.4.1, MTR.5.1)**

Short on time? Consider doing Problem 12a-c together as a class.

1 Launch

Encourage students to make a strong attempt before asking for help. Remind them that this process is similar to multiplying without radicals.

Encourage students to try both methods of simplifying (rational exponents or factor tree) before picking which one they like best. **(MTR.2.1)**

A **Accessibility: Fine Motor Skills** Consider varying the task demands by providing students with a pre- or partially completed factor tree.

Remind students to review the strategies from Activity 1 carefully. Encourage them to use a highlighter to mark up Amara's and Nelly's work if needed.

2 Monitor

ELL **English Language Learners** Give students time to think independently before sharing their ideas in writing. **(Writing)**

Invite students to check their work with a peer and make any necessary revisions. Then, invite the students to write their answers on the board.

Encourage students to make sure their radicals have matching indexes. **(MTR.5.1)**

Note: Some students may need additional support when simplifying their radicals to obtain the final answer.

Invite students to revise their responses to make them stronger and clearer after the discussion. **(MTR.4.1)**

Activity 2 continued >

Activity 2

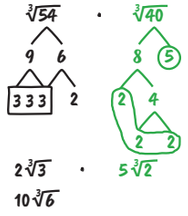
Name: Date: Period:

Multiply Radicals

9. Use Amara's strategy in Activity 1 to try to simplify $\sqrt{24} \cdot \sqrt{8}$. Write your answer as an exact quantity using at most a single radical.

$8\sqrt{3}$

10. Kenji used Nelly's strategy from Activity 1 to simplify $\sqrt[3]{54} \cdot \sqrt[3]{40}$ but made an error. Identify the error and give Kenji advice for next time.



Explanations vary.

Kenji created his factor trees correctly and he did the right process by circling the groups of three factors. But when he went from the factor tree to the radical he switched the spots. It should be $\sqrt[3]{54} = 3\sqrt[3]{2}$ and $\sqrt[3]{40} = 2\sqrt[3]{5}$ which would result in the correct final answer being $6\sqrt[3]{10}$. Next time, he should be more careful with his work and remember that the perfect cubes go on the outside of the radical.

11. Could Kenji have multiplied inside the radicands first and then simplified? Try it!

Responses vary.

Kenji could have multiplied the radicands first and then he would only need to draw one tree diagram. You end up with the same final answer but your factor tree will be bigger.

Unit 5 Lesson 10 541 Activity 2

Independent | 10 min

Activity 2 Multiply Radicals (continued)

Purpose: Students explore multiplying with radicals. **(MTR.2.1, MTR.4.1, MTR.5.1)**

Short on time? Consider doing Problem 12a-c together as a class.

2 Monitor

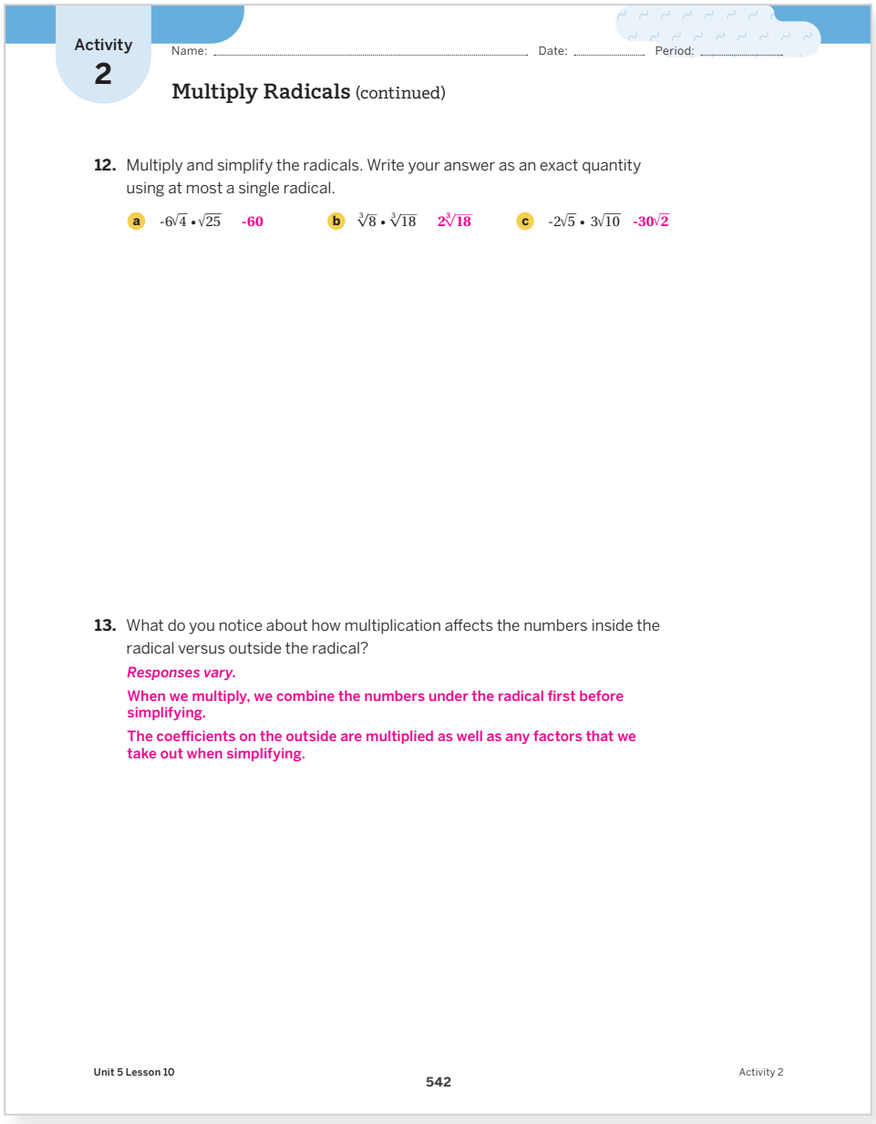
D Differentiation *(Problem 12)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to combine the numbers inside the radicals first.
Struggle to recognize the connection between radicals and coefficients.	**Strengthen** Ask, "What happens to the numbers outside the radical when we multiply? How does that relate to the numbers inside?" Guide them to see that coefficients are multiplied just like factors inside the radical. **(MTR.5.1)**
Notice that cube roots and square roots follow similar patterns.	**Strengthen** Celebrate their insight and ask them to explain how the grouping process (pairs for square roots, triples for cube roots) works in each case. **(MTR.2.1)**
Need support simplifying their final answers.	**Support** Encourage them to carefully check for perfect squares or cubes in the product and simplify fully. Remind them of the factor tree strategy to spot these.

3 Connect

To reveal the Key Takeaway, consider asking:

- "What do you notice about how multiplication affects the numbers inside the radical?"
- "If you had to describe the rule for multiplying radicals in one sentence, what would you say? What about for division?"
- "Can you think of a situation where these properties might make simplifying easier? What would happen if we tried to multiply without these rules?"



Activity 2

Name: _____ Date: _____ Period: _____

Multiply Radicals (continued)

12. Multiply and simplify the radicals. Write your answer as an exact quantity using at most a single radical.

a. $-6\sqrt{4} \cdot \sqrt{25}$ -60

b. $\sqrt[3]{8} \cdot \sqrt[3]{18}$ $2\sqrt[3]{18}$

c. $-2\sqrt{5} \cdot 3\sqrt{10}$ $-30\sqrt{2}$

13. What do you notice about how multiplication affects the numbers inside the radical versus outside the radical?

Responses vary.

When we multiply, we combine the numbers under the radical first before simplifying.

The coefficients on the outside are multiplied as well as any factors that we take out when simplifying.

Unit 5 Lesson 10 542 Activity 2

Key Takeaway: When multiplying radicals use the property $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$.

Pairs | 10 min

Activity 3 Divide Radicals

Purpose: Students will learn how to divide radicals by applying the quotient rule for radicals.

Short on time? Consider completing Problem 15a-c together as a class.

1 Launch

Show the properties of dividing somewhere everyone can see. Consider adding an example for students to refer to.

After Activity 1, you might have noticed that some students preferred exponent notation, while others preferred factor trees. Pairing these students can encourage discussion about different approaches to simplifying before adding or subtracting. **(MTR.2.1, MTR.4.1)**

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

2 Monitor

D **Differentiation** *(Problems 14–16)*

Look for students who:	Respond to Student Thinking
Need support simplifying their final answers.	**Support** Encourage them to review Problem 12 and make the connection between similarities of multiplication and division.
Misunderstand that the problem is talking about cube roots, not square roots. (Problem 15a and Problem 15c)	**Strengthen** Invite students to circle the index of every radical simplifying problem. Remind them they are looking for triples or re-writing to the power of ⅓. **(MTR.5.1)**
Would like an additional challenge.	**Stretch** Create a three-term simplifying problem for the student to work on.

3 Connect

Invite students to share their ideas about extending these concepts to adding and subtracting.

To reveal the Key Takeaway, consider asking:

- "How can you rewrite a division problem with two radicals as a single radical expression?" **(MTR.2.1)**
- "What happens when you divide the numbers inside the radical before simplifying? Does it make the problem easier?" **(MTR.5.1)**
- "If you simplify first versus dividing first, do you get the same result? Why or why not?"

Activity 3

Name: Date: Period:

Divide Radicals

14. Use the information you discovered in Problem 13 to simplify $\frac{\sqrt{100}}{\sqrt{4}}$.

5

15. Divide and simplify the radicals. Write your answer as an exact quantity using, at most, a single radical.

a. $\frac{\sqrt[3]{54}}{\sqrt[3]{6}}$ $\sqrt[3]{9}$

b. $\frac{15\sqrt{24}}{5\sqrt{6}}$ 6

c. $\frac{-\sqrt[3]{216}}{\sqrt[3]{8}}$ -3

16. Do you think these properties will work the same way when adding or subtracting radicals? Why or why not?

Responses vary.

I'm not sure yet, but I think addition and subtraction might work differently because, so far, we've been multiplying and dividing the numbers inside/outside the square root. Maybe with adding and subtracting we have to think about the numbers differently.

Unit 5 Lesson 10 543 Activity 3

Key Takeaway: Radical division uses the property $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of simplifying radicals and operations with radicals.

Invite students to respond independently, and then share their thinking with a partner.

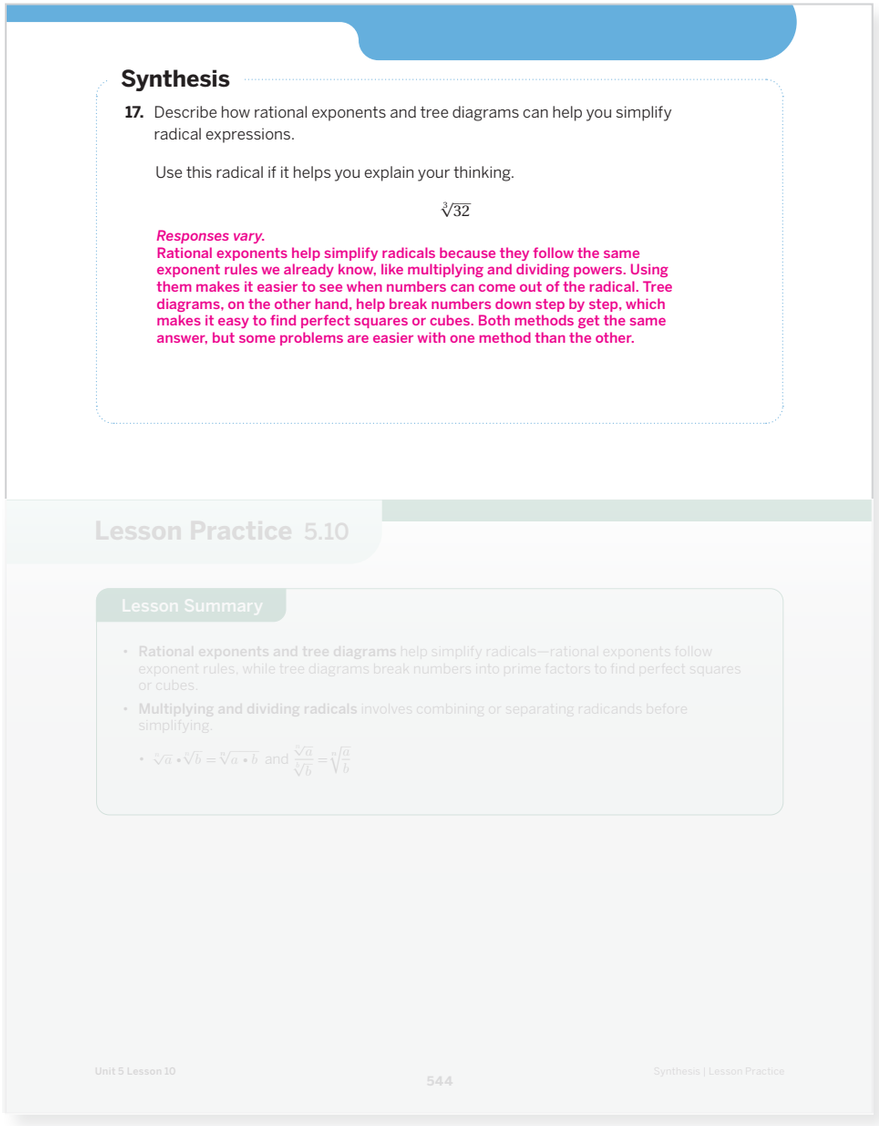
Listen for and share a variety of ideas, such as:
- Factor trees help you see pairs and triples when simplifying radicals.
- Rational exponents are another way we can simplify radicals.
- Multiplying and dividing radicals can be made into one term.

Create an anchor chart of all the ideas talked about in today's lesson.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and examples of simplifying radicals.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

17. Describe how rational exponents and tree diagrams can help you simplify radical expressions.

Use this radical if it helps you explain your thinking.

$$\sqrt[3]{32}$$

Responses vary.
Rational exponents help simplify radicals because they follow the same exponent rules we already know, like multiplying and dividing powers. Using them makes it easier to see when numbers can come out of the radical. Tree diagrams, on the other hand, help break numbers down step by step, which makes it easy to find perfect squares or cubes. Both methods get the same answer, but some problems are easier with one method than the other.

Lesson Practice 5.10

Lesson Summary

- **Rational exponents and tree diagrams** help simplify radicals—rational exponents follow exponent rules, while tree diagrams break numbers into prime factors to find perfect squares or cubes.
- **Multiplying and dividing radicals** involves combining or separating radicands before simplifying.
  - $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ and $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Unit 5 Lesson 10 544 Synthesis | Lesson Practice

Lesson Takeaway: Radicals can be simplified using factor trees or rational exponents. Operations with radicals have their own rules and properties.

Independent | 5 min

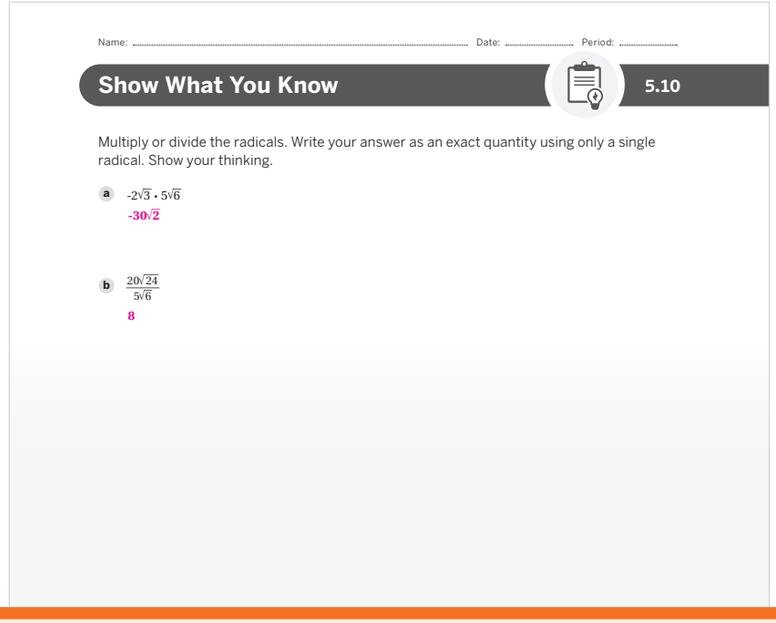
Show What You Know

Purpose: Students demonstrate their understanding by completing practice problems simplifying radicals.

Today's Goals

1. **Goal:** Students will simplify and perform operations on radical expressions.
2. **Goal:** Students will analyze and correct common errors in radical operations.
3. **Language Goal:** Students will explain their reasoning and preferred strategies for simplifying radicals. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.10

Multiply or divide the radicals. Write your answer as an exact quantity using only a single radical. Show your thinking.

a. $-2\sqrt{3} \cdot 5\sqrt{6}$

$-30\sqrt{2}$

b. $\frac{20\sqrt{24}}{5\sqrt{6}}$

8

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to simplify radicals:

» **Respond to Student Thinking:**

- Review multiplication tables and make the connection to factor trees.
- Review prime factorization.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities.**
- Invite students to create their own expressions with more than two terms.

Professional Learning

Consider how efficiently your students were able to recall factors of large numbers. How did they do? Were they able to find factors with fluency? If not, what can be done in the future to strengthen their skills?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

17. Describe how rational exponents and tree diagrams can help you simplify radical expressions.

Use this radical if it helps you explain your thinking.

$\sqrt[4]{32}$

Responses vary.
Rational exponents help simplify radicals because they follow the same exponent rules we already know, like multiplying and dividing powers. Using them makes it easier to see when numbers can come out of the radical. Tree diagrams, on the other hand, help break numbers down step by step, which makes it easy to find perfect squares or cubes. Both methods get the same answer, but some problems are easier with one method than the other.

Lesson Practice 5.10

Lesson Summary

- **Rational exponents and tree diagrams** help simplify radicals—rational exponents follow exponent rules, while tree diagrams break numbers into prime factors to find perfect squares or cubes.
- **Multiplying and dividing radicals** involves combining or separating radicands before simplifying.
  - $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ and $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Lesson Practice 5.10

Name: ..... Date: ..... Period: .....

Problems 1–6: Simplify the radical expressions. Write your answer as an exact quantity using only a single radical.

1. $\sqrt[3]{4} \cdot \sqrt[3]{16}$
4

2. $2\sqrt{6} \cdot -3\sqrt{2}$
$-12\sqrt{3}$

3. $\frac{\sqrt[3]{250}}{\sqrt[3]{5}}$
$\sqrt[3]{50}$

4. $-4\sqrt{27} \cdot 8\sqrt{18}$
$-288\sqrt{6}$

5. $\sqrt[4]{9} \times \sqrt[4]{27}$
$3\sqrt[4]{9}$

6. $\frac{10\sqrt{50}}{-2\sqrt{2}}$
-25

7. Boris was solving a problem. Review his work, find the mistake, and correct it.

$\frac{8\sqrt{75}}{-2\sqrt{5}}$

$\frac{4\sqrt{75}}{-1\sqrt{5}}$

$= -4\sqrt{75 - 5}$

$= -4\sqrt{70}$

Responses vary. Boris made a big mistake by subtracting the radicands instead of dividing them. You can't do $\sqrt{75} - \sqrt{5}$; that is not how radicals work. I think he is getting confused with the Laws of Exponents.
The correct answer is $-4\sqrt{15}$.

Lesson Practice 5.10

Name: ..... Date: ..... Period: .....

Test Practice

8. Which of the following is equivalent to $\frac{6\sqrt{72}}{-3\sqrt{2}}$?

A. $-6\sqrt{6}$
B. -12
C. $-6\sqrt{2}$
D. $-12\sqrt{3}$

Spiral Review

9. Is $x = 4$ a solution to the inequality $40 + 5x > 10 + 8x$?
Yes, $x = 4$ does fall within the solution $x < 10$.

Problems 10–12: Solve the equations. Check your work by plugging your answers back in.

10. $4(2x - 3) = 3(x + 5) + x$
$x = \frac{27}{4}$

11. $5(x - 4) + 2x = 3(2x + 1) + 6$
$x = 29$

12. $6(3x + 2) - \frac{1}{2}(4x - 3) = 5x$
$x = -\frac{27}{22}$

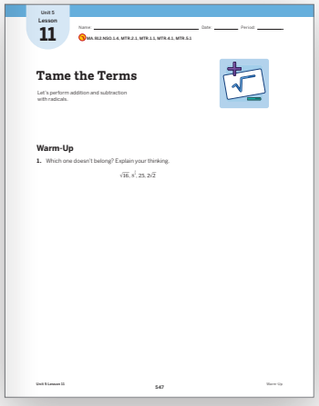
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	1	MA.912.NSO.1.4
	7	2	MA.912.NSO.1.4
Test Practice	8	1	MA.912.NSO.1.4
Spiral Review			
	9	1	MA.912.AR.2.6
	10–12	1	MA.8.AR.2.1

Unit 5

Lesson

11



Tame the Terms

Adding and Subtracting Radicals

Let's perform addition and subtraction with radicals.

Learning Outcomes

Today's Goals

1. **Goal:** Students will simplify and perform operations on numerical radical expressions.
2. **Goal:** Students will analyze and correct common errors when simplifying radical expressions.
3. **Language Goal:** Students will explain their reasoning and preferred strategies for simplifying radicals. **(Speaking and Listening)**

Students add and subtract radicals by identifying like terms and simplifying when necessary. They recognize structural similarities to algebraic expressions and refine their understanding through discussion, problem-solving, and partner activities. **(MTR.2.1, MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In 8th Grade Unit 8, students learned about square roots with triangles and the Pythagorean theorem. In Algebra 1, Unit 5 Lessons 8–10, students explored writing radicals as numbers with rational exponents and learned how to use factor trees to simplify radicals.

Future Learning

In future math courses, students will extend their understanding of radical operations to work with algebraic expressions containing radicals, solve radical equations, and explore how simplifying radicals relates to simplifying exponential and logarithmic expressions.

Connections

- Students analyze a visual of student work of addition and subtraction with radicals by discussing notices and wonders. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.NSO.1.4
Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
MA.912.AR.1.1	**MA.912.NSO.1.5**
MA.912.AR.1.2	**MA.912.AR.7.1**
	MA.912.AR.7.4

Vocabulary

Review Vocabulary

radical
radicand
index

Lesson at a Glance

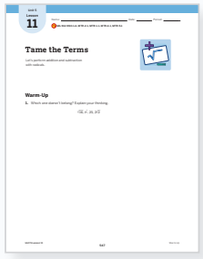
~ 45 min

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

Students use the **Which One Doesn't Belong?** routine to analyze numerical expressions to recognize patterns and differences. **(MTR.1.1, MTR.2.1, MTR.4.1, MTR.5.1)**


Unit 5 Lesson 11

Tame the Terms

Let's perform addition and subtraction with radicals.

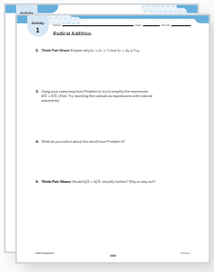
Warm-Up

1. Which one doesn't belong? Explain your thinking.

Activity 1

Pairs | 10 min

Students will explore adding radicals by recognizing like terms, simplifying expressions when needed, and applying structure to radical operations.

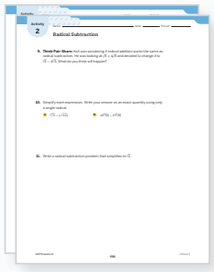

Activity 1

Radical Addition

Activity 2

Pairs | 10 min

Students will learn to subtract radicals by recognizing like terms, and simplifying when necessary.

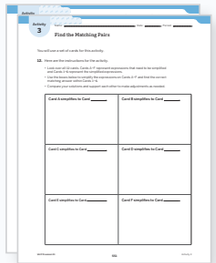

Activity 2

Radical Subtraction

Activity 3

Pairs | 10 min

Students work in pairs to apply their knowledge of simplifying, multiplying, dividing, adding, and subtracting radicals. **(MTR.4.1)**

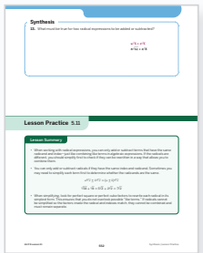

Activity 3

Find the Matching Pairs

Synthesis

Whole Class | 5 min

Students synthesize their understanding of simplifying radicals and operations with radicals.


Synthesis

Lesson Practice 5.11

Lesson Summary

Show What You Know

Independent | 5 min

Students demonstrate their understanding of adding and subtracting radicals.


Show What You Know

5.11

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to analyze numerical expressions to recognize patterns and differences. **(MTR.2.1, MTR.4.1, MTR.5.1)**

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing and precisely describing properties of expressions comparing and contrasting radicals, whole numbers, and exponents. While looking for structure in the expressions and justifying their thinking precisely. **(MTR.2.1, MTR.4.1, MTR.5.1)**

Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____is the only one that has/doesn't have _____."). (Writing and Speaking)

2 Connect

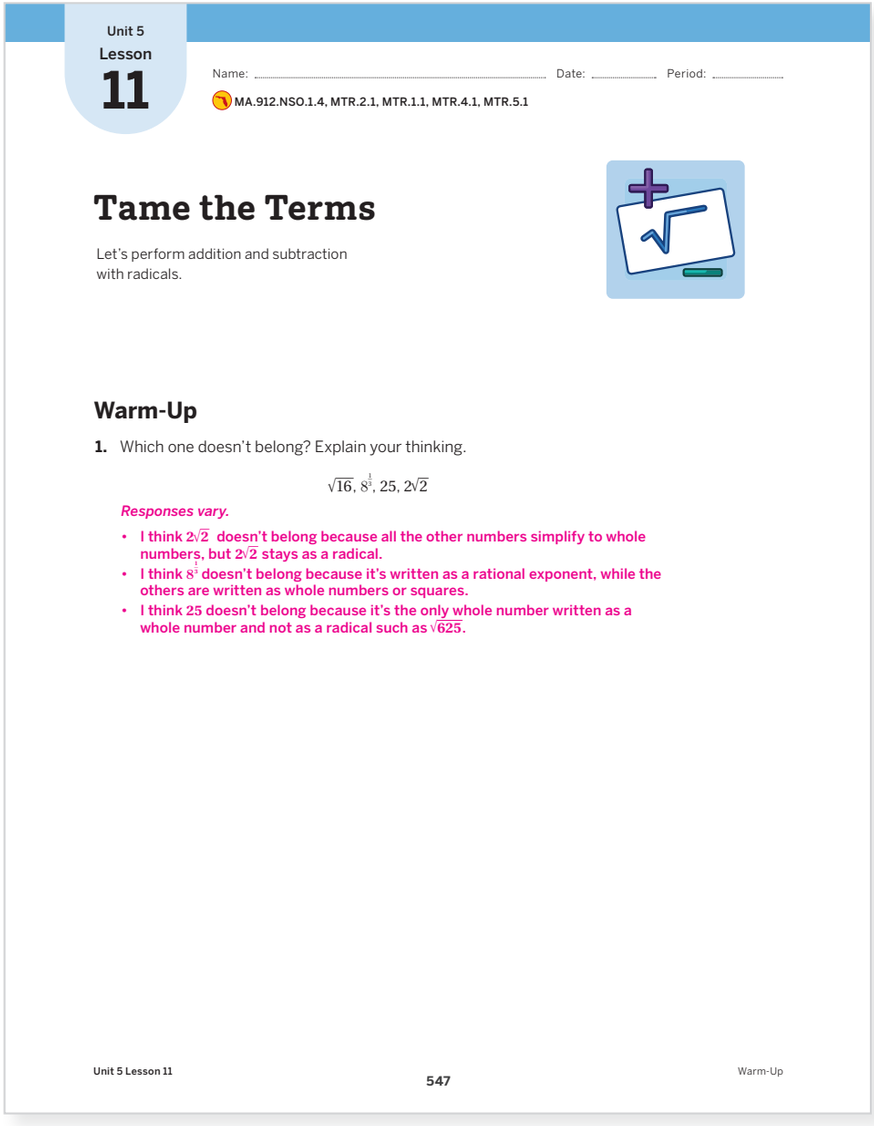
Invite students to share at least one reason why each expression doesn't belong.

Create a record of why each representation doesn't belong, along with the name of the student who shared the reasoning.

Consider asking:

- "How is $\sqrt{16}$ alike/different from the other expressions?"
- "What is another way to say why $8^{\frac{1}{2}}$ doesn't belong?"
- "Is there another reason why 25 doesn't belong?"

Note: If it doesn't come up naturally, consider asking, "Are there other ways to write these expressions so they are more similar?"



Unit 5
Lesson
11

Name: _____ Date: _____ Period: _____

MA.912.NSO.1.4, MTR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Tame the Terms

Let's perform addition and subtraction with radicals.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

$\sqrt{16}$, $8^{\frac{1}{2}}$, 25, $2\sqrt{2}$

Responses vary.

- I think $2\sqrt{2}$ doesn't belong because all the other numbers simplify to whole numbers, but $2\sqrt{2}$ stays as a radical.
- I think $8^{\frac{1}{2}}$ doesn't belong because it's written as a rational exponent, while the others are written as whole numbers or squares.
- I think 25 doesn't belong because it's the only whole number written as a whole number and not as a radical such as $\sqrt{625}$.

Unit 5 Lesson 11 547 Warm-Up

Pairs | 10 min

Activity 1 Radical Addition

Purpose: Students will explore adding radicals by recognizing like terms, simplifying expressions when needed, and applying structure to radical operations. **(MTR.1.1, MTR.2.1, MTR.4.1, MTR.5.1)**

Short on time? Consider going over Problem 6 as a class.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse about combining like terms. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thoughts with the class.

To encourage movement, invite students to find a partner in a different location.

A **Accessibility: Memory and Attention** Invite students to record strategies to support students with keeping track of information.

2 Monitor

Look for students making connections between simplifying radicals and adding radicals.

D **Differentiation** *(Problems 3–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to identify what makes two terms "like terms" in algebra before applying it to radicals. **(MTR.2.1)**
Struggle to see why only the coefficients change. (Problem 3)	**Strengthen** Ask, "What part of the expression stayed the same? What part changed? Why?" **(MTR.5.1)**
Notice that adding radicals follows the same structure as combining like terms. (Problem 4)	**Strengthen** Invite them to share their reasoning and explain how this connects to algebraic expressions. **(MTR.4.1)**
Recognize that cube roots follow the same pattern. (Problem 5)	**Stretch** Ask them to extend their reasoning: "Would this work the same way with fourth roots? Why or why not?" **(MTR.5.1)**

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Radical Addition

2. **Think-Pair-Share:** Explain why $5x + 2x = 7x$ but $5x + 2y \neq 7xy$.

Responses vary.

- In $5x + 2x = 7x$, both terms have the same variable x, which makes them like terms. So, we can add the coefficients. But in $5x + 2y \neq 7xy$, the terms have different variables, so they are not like terms and cannot be combined.
- Some people may think $5x + 2y \neq 7xy$ because they see both terms and try to multiply them, but addition and multiplication are different operations. You can only add terms if they are the same types.

3. Using your reasoning from Problem 2, try to simplify the expression $6\sqrt{5} + 9\sqrt{5}$. (Hint: Try rewriting the radicals as expressions with rational exponents) $15\sqrt{5}$ or $15\ (5)^{\frac{1}{2}}$

4. What do you notice about the result from Problem 3?

Responses vary. I notice that the coefficients add together but the radicands do not change. I also notice that the index doesn't change and because the original indices and radicands match I am able to combine them just like other like terms.

5. **Think-Pair-Share:** Would $3\sqrt[3]{2} + 3\sqrt[3]{4}$ simplify further? Why or why not?

Responses vary.

- No, this does not simplify further because $2^{\frac{1}{3}}$ and $4^{\frac{1}{3}}$ do not have matching bases. That tells me that when the radicands do not match, they cannot be added.
- I thought maybe they could because they both have a 3 in the front but thinking back to Problem 2, I do not think they are like terms and the 3s are just the coefficients that match.

Unit 5 Lesson 11 548 Activity 1

Pairs | 10 min

Activity 1 Radical Addition (continued)

Purpose: Students will explore adding radicals by recognizing like terms, simplifying expressions when needed, and applying structure to radical operations. **(MTR.1.1, MTR.2.1, MTR.4.1, MTR.5.1)**

2 Monitor

Use the Notice and Wonder routine to promote curiosity and help students make sense of adding radicals with like and unlike radicands. **(MTR.1.1)**

Consider asking, "Did anyone notice sometimes the radicands don't always match? Does this make you wonder about how to identify like terms with radicals?"

Note: If it doesn't come up naturally, consider asking, "Is there another way to write radicals that may have perfect squares or cubes as factors?"

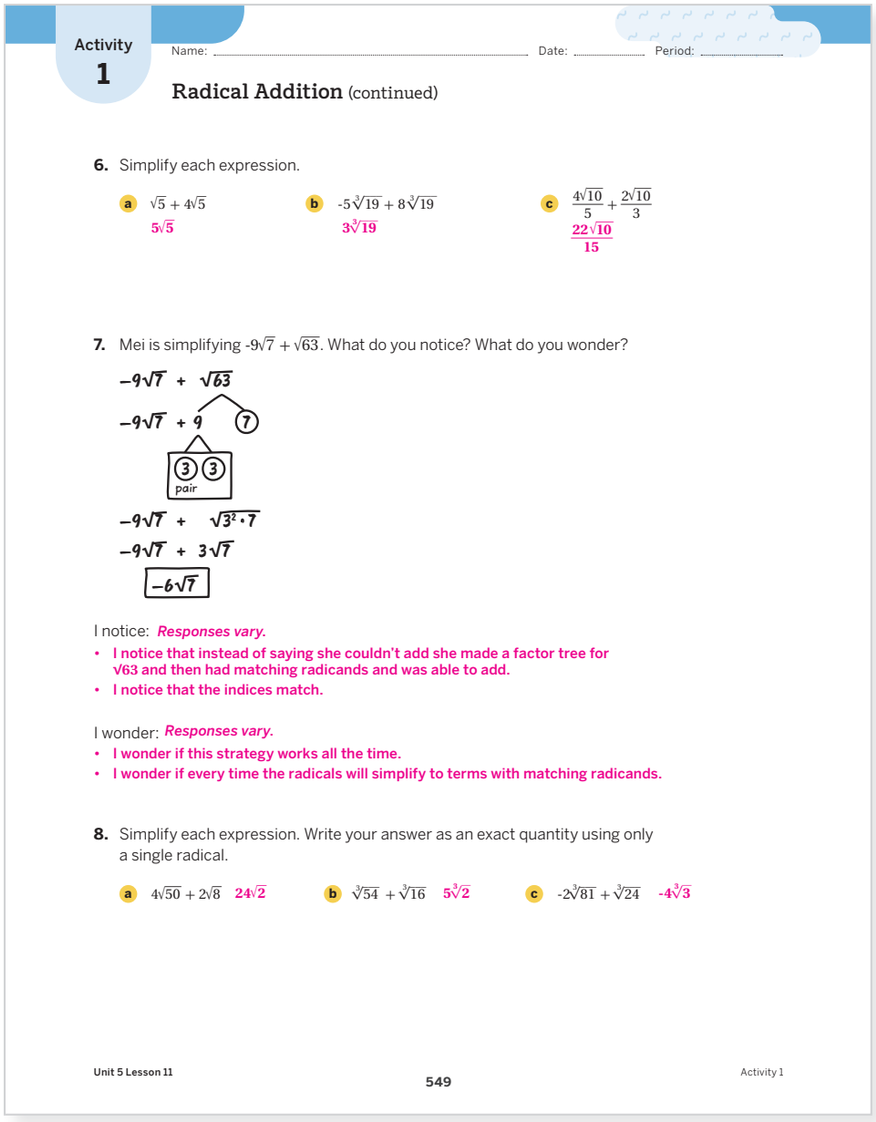
D Differentiation *(Problems 6–8)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to review Problem 2. **(MTR.2.1)**
Struggle to recognize when radicals can be added. (Problem 7, Mei's work)	**Strengthen** Ask: "What did Mei do before adding? Why did that work?" Help them see when simplifying first is useful. **(MTR.5.1)**
Notice that error analysis helps refine strategies. (Problem 7, Mei's work)	**Strengthen** Invite students to discuss how identifying mistakes helps deepen understanding of radical operations. **(MTR.4.1)**
Can generalize the process of simplifying before adding. (Problem 8)	**Stretch** Ask, "Does this strategy work for all radical operations? What happens when we subtract instead of add?" **(MTR.5.1)**

3 Connect

To reveal the Key Takeaway, consider asking,

- "What must be the same in order to add radical expressions?" **(MTR.5.1)**
- "How does simplifying a radical first sometimes allow you to add or subtract terms that didn't look alike at first?" **(MTR.2.1)**
- "What happens when you try to add two radicals that don't have the same radicand? Does anything change?" **(MTR.5.1)**
- "How is adding radicals similar to combining like terms in algebra? How is it different?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Radical Addition (continued)

6. Simplify each expression.

a. $\sqrt{5} + 4\sqrt{5}$ $5\sqrt{5}$

b. $-5\sqrt[3]{19} + 8\sqrt[3]{19}$ $3\sqrt[3]{19}$

c. $\frac{4\sqrt{10}}{5} + \frac{2\sqrt{10}}{3}$ $\frac{22\sqrt{10}}{15}$

7. Mei is simplifying $-9\sqrt{7} + \sqrt{63}$. What do you notice? What do you wonder?

$-9\sqrt{7} + \sqrt{63}$

$-9\sqrt{7} + 9 \cdot 7$

(3)(3) pair

$-9\sqrt{7} + \sqrt{3^2 \cdot 7}$

$-9\sqrt{7} + 3\sqrt{7}$

$-6\sqrt{7}$

I notice: *Responses vary.*

- I notice that instead of saying she couldn't add she made a factor tree for √63 and then had matching radicands and was able to add.
- I notice that the indices match.

I wonder: *Responses vary.*

- I wonder if this strategy works all the time.
- I wonder if every time the radicals will simplify to terms with matching radicands.

8. Simplify each expression. Write your answer as an exact quantity using only a single radical.

a. $4\sqrt{50} + 2\sqrt{8}$ $24\sqrt{2}$

b. $\sqrt[3]{54} + \sqrt[3]{16}$ $5\sqrt[3]{2}$

c. $-2\sqrt[3]{81} + \sqrt[3]{24}$ $-4\sqrt[3]{3}$

Unit 5 Lesson 11 549 Activity 1

Key Takeaway: Radicals must have the same radicand and index to be added or subtracted. Simplify terms first, if needed.

Pairs | 10 min

Activity 2 Radical Subtraction

Purpose: Students will learn to subtract radicals by recognizing like terms, and simplifying when necessary. **(MTR.2.1, MTR.4.1, MTR.5.1)**

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse/help students shape their ideas about whether radical subtraction behaves like radical addition. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

A **Accessibility: Memory and Attention** Use MLR2: Collect and Display to collect students' words and sketches for them to reference to activate prior knowledge.

2 Monitor

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, with words such as: *factor, index, radical, radicand*. **(Writing and Speaking)**

D **Differentiation** *(Problems 9–11)*

Look for students who:	Respond to Student Thinking
Need support recognizing that radical subtraction follows the same rules as radical addition. (Problem 9)	**Support** Encourage them to compare radical subtraction to algebraic subtraction. Ask, "What stays the same? What changes?" **(MTR.4.1)**
Struggle to see when radicals can be combined in subtraction. (Problem 10)	**Strengthen** Ask: "Before subtracting, what should you check first?" Guide them to identify matching radicands and indices. **(MTR.5.1)**
Confidently recognize when radicals can be combined. (Problem 10)	**Stretch** Challenge them to explain why cube roots follow the same rule but require checking different factors. **(MTR.5.1)**
Need help creating a correct subtraction problem for a target value. (Problem 11)	**Strengthen** Encourage them to first work forward (simplifying radicals) before trying to work backward (writing a problem). **(MTR.2.1)**

2 Connect

To reveal the Key Takeaway, consider asking,

- "What do you need to check before subtracting two radical expressions?" **(MTR.5.1)**
- "If two radicals don't look the same at first, what should you do before deciding if they can be combined?" **(MTR.2.1)**
- "How is subtracting radicals similar to subtracting algebraic expressions with variables? How is it different?" **(MTR.4.1)**

Activity 2

Name: _____ Date: _____ Period: _____

Radical Subtraction

9. **Think-Pair-Share:** Kofi was wondering if radical addition works the same as radical subtraction. He was looking at $\sqrt{5} + 4\sqrt{5}$ and decided to change it to $\sqrt{5} - 4\sqrt{5}$. What do you think will happen?

Responses vary. I think the only difference is the subtraction sign. Addition and subtraction are usually grouped together. I think we just subtract the coefficients for an answer of $-3\sqrt{5}$.

10. Simplify each expression. Write your answer as an exact quantity using only a single radical.

a. $\sqrt{75} - (-\sqrt{12})$ $7\sqrt{3}$

b. $-4\sqrt[3]{54} - 2\sqrt[3]{16}$ $-16\sqrt[3]{2}$

11. Write a radical subtraction problem that simplifies to $\sqrt{2}$.

Any equivalent expressions:

$\sqrt{18} - \sqrt{8}$

$\sqrt{50} - 4\sqrt{2}$

$5\sqrt{2} - 4\sqrt{2}$

Unit 5 Lesson 11 550 Activity 2

Key Takeaway: Radicals can only be subtracted when they have the same radicand and index. Simplify terms in the expression first, then subtract like terms.

Pairs | 10 min

Activity 3 Find the Matching Pairs

Purpose: Students work in pairs to apply their knowledge of simplifying, multiplying, dividing, adding, and subtracting radicals.

1 Launch

Distribute the cut-out cards to the pairs. Encourage students to review them all before they start solving.

Invite a student to read the directions aloud to the class. Give opportunities for students to ask clarifying questions about the activity.

ELL **English Language Learners** Invite students to review their notes from the previous activities to support their success in card sorting.

2 Monitor

Look for pairs that are actively writing down their work and not just trying to match cards quickly.

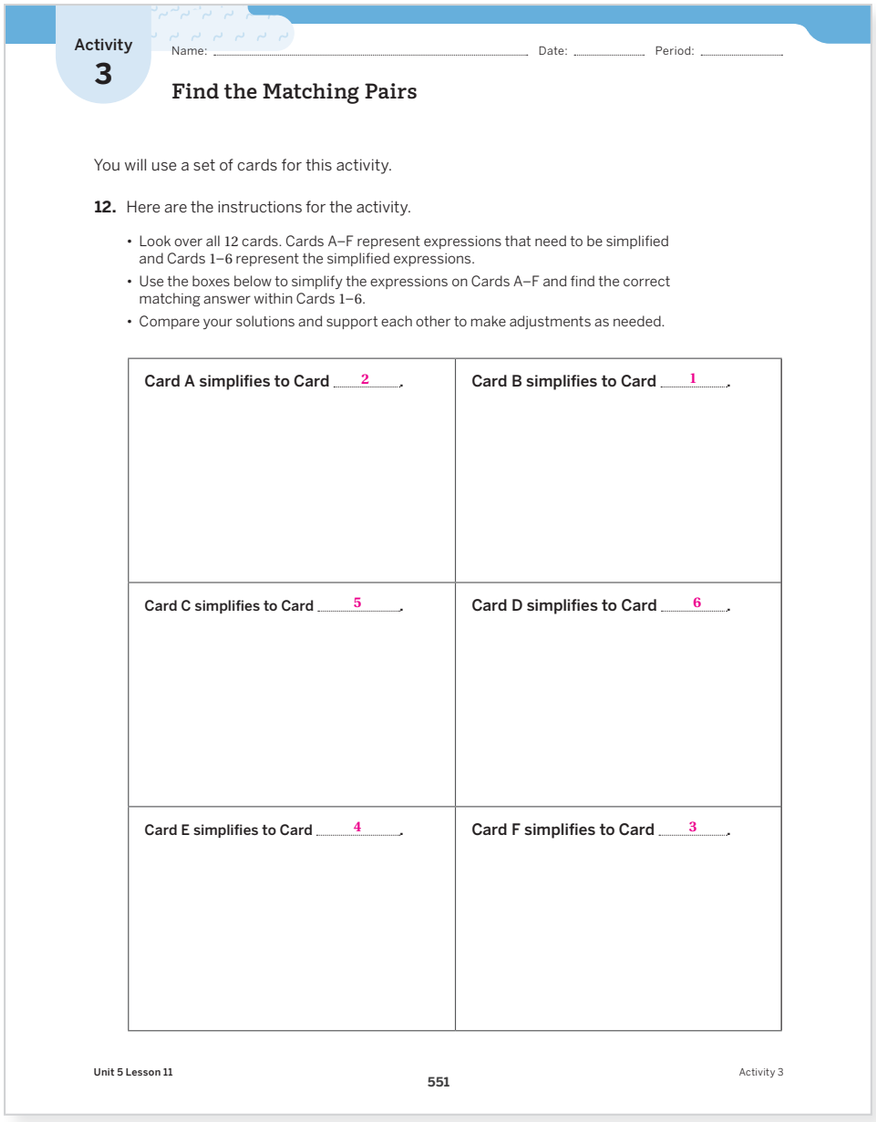
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look at one card at a time and not all six at once.
Think Card E does not simplify to Card 4.	**Strengthen** Remind them to review the rules of adding and subtracting radicals.
Are struggling with rational exponents.	**Strengthen** Consider showing them the tree diagram again and see if that strategy feels better for them.
Finish quickly.	**Stretch** Create a challenge problem for the pair and have them simplify using both strategies.

3 Connect

Invite students to share their answers to the card sort once the activity is over.

Encourage students to revise and edit their answers if necessary.



Activity 3

Name: _____ Date: _____ Period: _____

Find the Matching Pairs

You will use a set of cards for this activity.

12. Here are the instructions for the activity.

- Look over all 12 cards. Cards A–F represent expressions that need to be simplified and Cards 1–6 represent the simplified expressions.
- Use the boxes below to simplify the expressions on Cards A–F and find the correct matching answer within Cards 1–6.
- Compare your solutions and support each other to make adjustments as needed.

Card A simplifies to Card 2.	Card B simplifies to Card 1.
Card C simplifies to Card 5.	Card D simplifies to Card 6.
Card E simplifies to Card 4.	Card F simplifies to Card 3.

Unit 5 Lesson 11 551 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of simplifying radicals and operations with radicals.

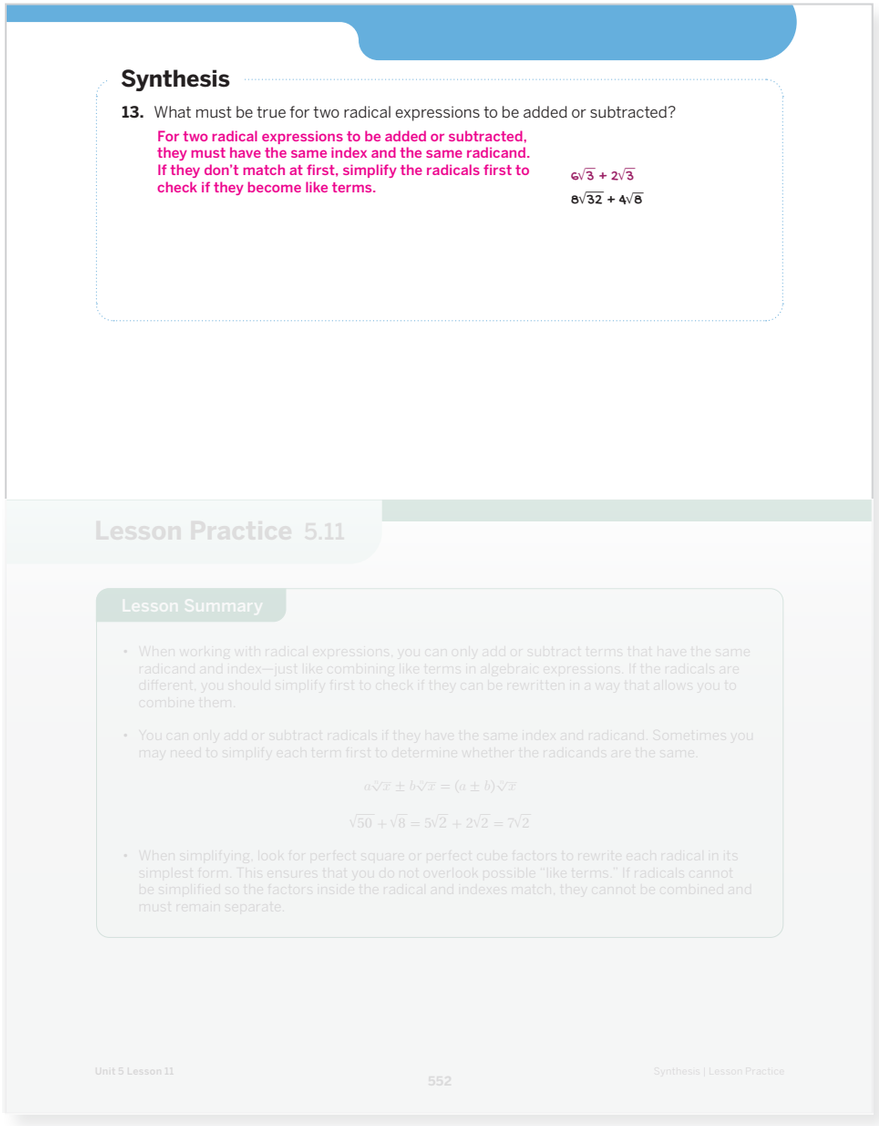
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Factor trees help see pairs and triples when simplifying radicals.
- Rational exponents are another way we can simplify radicals.
- To add and subtract radicals the radicands must match.

Create an anchor chart of all the ideas talked about in today's lesson.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and examples of simplifying radicals.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

13. What must be true for two radical expressions to be added or subtracted?

For two radical expressions to be added or subtracted, they must have the same index and the same radicand. If they don't match at first, simplify the radicals first to check if they become like terms.

$6\sqrt{3} + 2\sqrt{3}$

$8\sqrt{32} + 4\sqrt{8}$

Lesson Practice 5.11

Lesson Summary

- When working with radical expressions, you can only add or subtract terms that have the same radicand and index—just like combining like terms in algebraic expressions. If the radicals are different, you should simplify first to check if they can be rewritten in a way that allows you to combine them.
- You can only add or subtract radicals if they have the same index and radicand. Sometimes you may need to simplify each term first to determine whether the radicands are the same.

$$a\sqrt[n]{x} \pm b\sqrt[n]{x} = (a \pm b)\sqrt[n]{x}$$

$$\sqrt{50} + \sqrt{8} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$$

- When simplifying, look for perfect square or perfect cube factors to rewrite each radical in its simplest form. This ensures that you do not overlook possible "like terms." If radicals cannot be simplified so the factors inside the radical and indexes match, they cannot be combined and must remain separate.

Unit 5 Lesson 11 552 Synthesis | Lesson Practice

Lesson Takeaway: Radicals can be simplified using factor trees or rational exponents. Operations with radicals have their own rules and properties.

Independent | 5 min

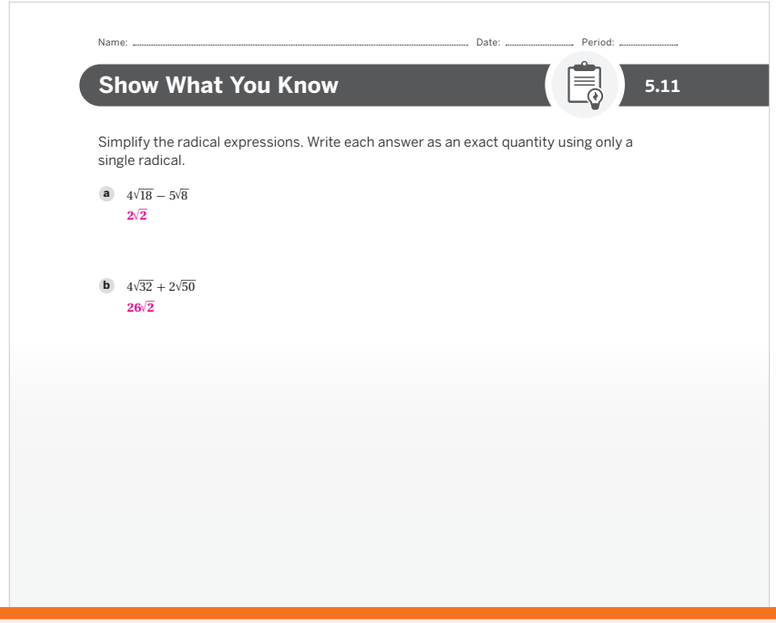
Show What You Know

Purpose: Students demonstrate their understanding of adding and subtracting radicals.

Today's Goals

1. **Goal:** Students will simplify and perform operations on numerical radical expressions.
2. **Goal:** Students will analyze and correct common errors when simplifying radical expressions.
3. **Language Goal:** Students will explain their reasoning and preferred strategies for simplifying radicals. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.11

Simplify the radical expressions. Write each answer as an exact quantity using only a single radical.

a. $4\sqrt{18} - 5\sqrt{8}$

$2\sqrt{2}$

b. $4\sqrt{32} + 2\sqrt{50}$

$26\sqrt{2}$

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to add and subtract with radicals:

» **Respond to Student Thinking:**

- Review multiplication tables and make the connection to factor trees.
- Review prime factorization.
- Review combining like terms.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 3 Extension Activities.
- Invite students to create their own expressions with more than two terms.

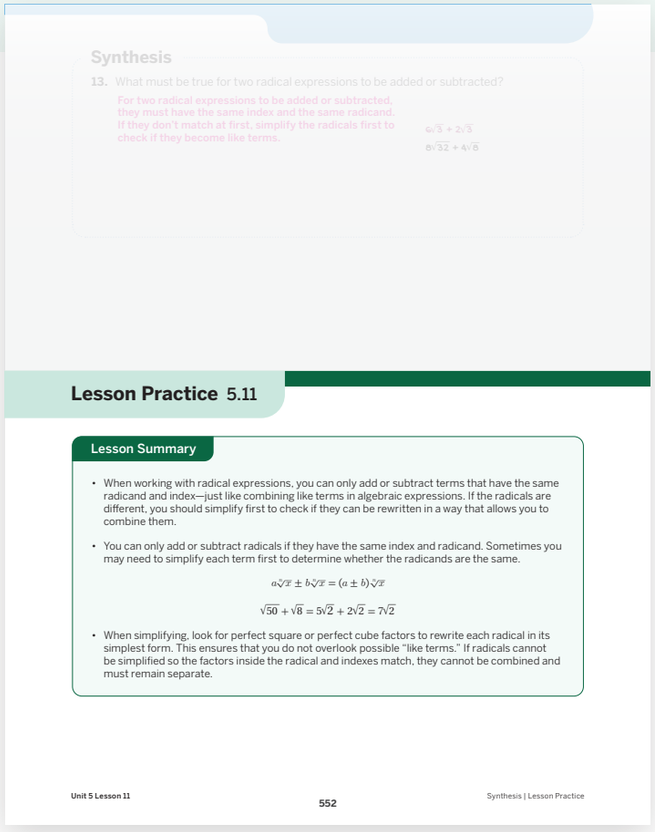
Professional Learning

Consider how well students recognized when radicals could be added or subtracted. Did they fluently simplify before combining like terms? If not, how can future lessons reinforce these skills through additional practice or connections to algebraic expressions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

13. What must be true for two radical expressions to be added or subtracted?

For two radical expressions to be added or subtracted, they must have the same index and the same radicand. If they don't match at first, simplify the radicals first to check if they become like terms.

$a\sqrt{3} + 2\sqrt{3}$

$8\sqrt{32} + 4\sqrt{8}$

Lesson Practice 5.11

Lesson Summary

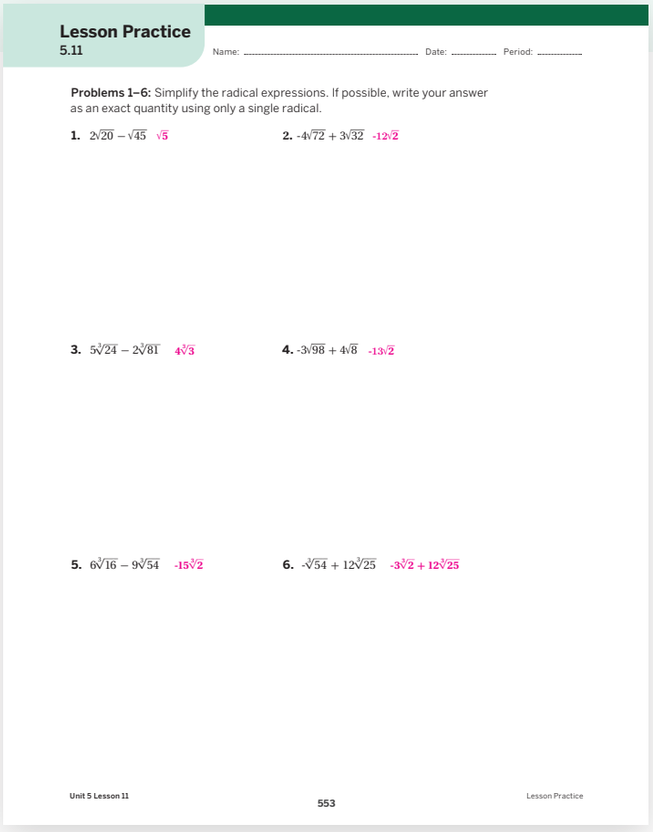
- When working with radical expressions, you can only add or subtract terms that have the same radicand and index—just like combining like terms in algebraic expressions. If the radicals are different, you should simplify first to check if they can be rewritten in a way that allows you to combine them.
- You can only add or subtract radicals if they have the same index and radicand. Sometimes you may need to simplify each term first to determine whether the radicands are the same.

$$a\sqrt[n]{x} \pm b\sqrt[n]{x} = (a \pm b)\sqrt[n]{x}$$

$$\sqrt{50} + \sqrt{8} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$$

- When simplifying, look for perfect square or perfect cube factors to rewrite each radical in its simplest form. This ensures that you do not overlook possible "like terms." If radicals cannot be simplified so the factors inside the radical and indexes match, they cannot be combined and must remain separate.

Unit 5 Lesson 11 552 Synthesis | Lesson Practice



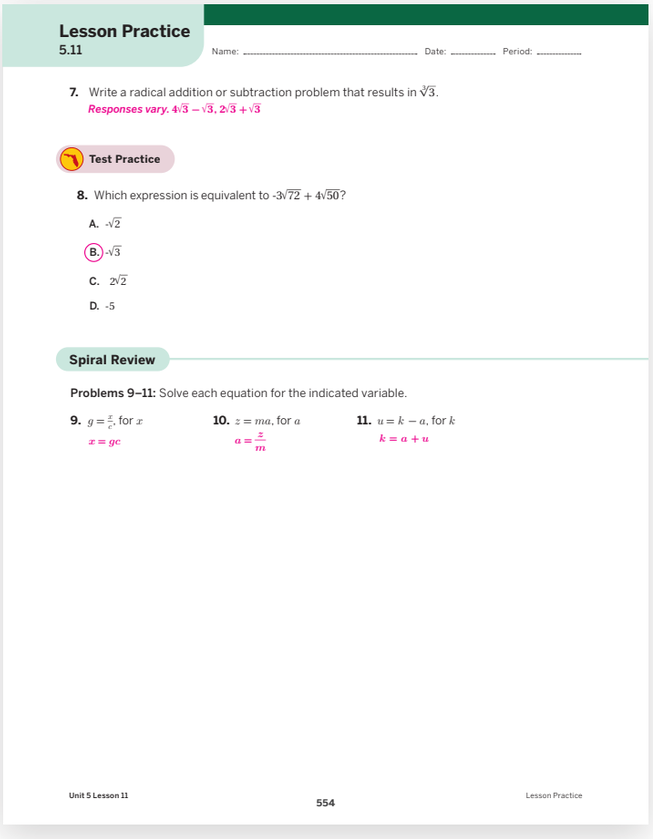
Lesson Practice 5.11

Name: Date: Period:

Problems 1–6: Simplify the radical expressions. If possible, write your answer as an exact quantity using only a single radical.

1. $2\sqrt{20} - \sqrt{45}$ $\sqrt{5}$
2. $-4\sqrt{72} + 3\sqrt{32}$ $-12\sqrt{2}$
3. $5\sqrt[3]{24} - 2\sqrt[3]{81}$ $4\sqrt[3]{3}$
4. $-3\sqrt{98} + 4\sqrt{8}$ $-13\sqrt{2}$
5. $6\sqrt[3]{16} - 9\sqrt[3]{54}$ $-15\sqrt[3]{2}$
6. $-\sqrt[3]{54} + 12\sqrt[3]{25}$ $-3\sqrt[3]{2} + 12\sqrt[3]{25}$

Unit 5 Lesson 11 553 Lesson Practice



Lesson Practice 5.11

Name: Date: Period:

7. Write a radical addition or subtraction problem that results in $\sqrt[3]{3}$.

Responses vary. $4\sqrt{3} - \sqrt{3}$, $2\sqrt{3} + \sqrt{3}$

Test Practice

8. Which expression is equivalent to $-3\sqrt{72} + 4\sqrt{50}$?

A. $-\sqrt{2}$

B. $-\sqrt{3}$

C. $2\sqrt{2}$

D. -5

Spiral Review

Problems 9–11: Solve each equation for the indicated variable.

9. $g = \frac{z}{c}$, for x $x = gc$
10. $z = ma$, for a $a = \frac{z}{m}$
11. $u = k - a$, for k $k = a + u$

Unit 5 Lesson 11 554 Lesson Practice

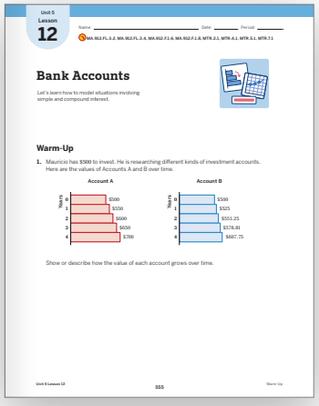
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	1	MA.912.NSO.1.4
	7	2	MA.912.NSO.1.4
Test Practice	8	1	MA.912.NSO.1.4
Spiral Review			
	9–11	1	MA.912.AR.1.2

Unit 5

Lesson

12


Unit 5
Lesson
12
Name: _____ Date: _____ Period: _____
MA.912.FL.3.2, MA.912.FL.3.4, MA.912.F.1.6, MA.912.F.1.8, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1
Bank Accounts
Let's learn how to model situations involving simple and compound interest.
Warm-Up
1. Mauricio has $500 to invest. He is researching different kinds of investment accounts. Here are the values of Accounts A and B over time.
Account A
Account B
Year
$500
$550
$600
$650
$700
$500
$525
$551.25
$578.81
$607.75
Show or describe how the value of each account grows over time.
555
Warm-Up

Bank Accounts

Introducing Simple and Compound Interest

Let's learn how to model situations involving simple and compound interest.

Learning Outcomes

Today's Goals

1. **Goal:** Use a graphing calculator to solve problems involving linear and exponential functions.
2. **Language Goal:** Describe connections between *simple* and *compound interest*, and linear and exponential functions. **(Speaking and Listening)**

Students make connections between **simple** and **compound interest**, and between linear and exponential functions. They also use a variety of tools, including a graphing calculator, to solve problems.

Coherence

Prior Learning

In Lessons 1–5, students distinguished between linear and exponential functions. In Lessons 6–7, students modeled exponential functions that grow by a repeated percent increase or decrease.

Future Learning

In Lessons 12–13, students will continue to explore compound interest by looking at exponential expressions that reveal information about how the interest compounds given different scenarios.

Connections

- Students model situations involving simple and compound interest through visual representations for two accounts earning interest. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.FL.3.2
Solve real-world problems involving simple, compound and continuously compounded interest.

MA.912.FL.3.4
Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.

Also Addressing: **MA.912.F.1.6, MA.912.F.1.8**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Towards
A.912.AR.1.1	**A.912.FL.2.2**
A.912.AR.5.4	**A.912.AR.5.5**
	A.912.NSO.1.2

Vocabulary

New Vocabulary

compound interest	simple interest

Lesson at a Glance

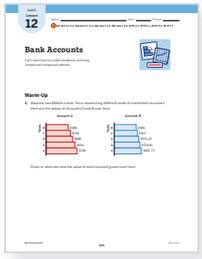
~ 45 min

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.F.1.6, MA.912.F.1.8

Warm-Up

Pairs | 5 min

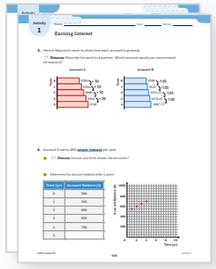
Students describe how the two different bank accounts they will explore in Activity 1 are growing.



Activity 1

Pairs | 15 min

Students are introduced to **simple** and **compound interest** and use what they know about linear and exponential functions to make a recommendation. **Routine:** Think-Pair-Share Routine



Activity 2

Pairs | 15 min

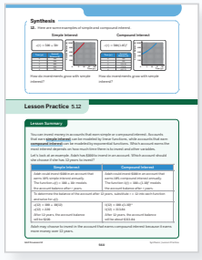
Students learn two strategies for solving problems involving exponential functions and compound interest using a graphing calculator. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

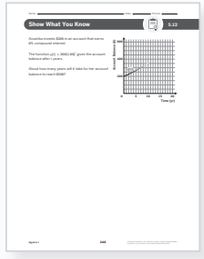
Students synthesize their understanding of how investments grow with simple and compound interest. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining when an account balance will reach a specific amount.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students describe how the two different bank accounts they will explore in Activity 1 are growing.

1 Launch

To access connections, consider asking:

- "What does it mean to make an investment?"
- "Why might someone want to invest their money into an account?"

ELL **English Language Learners** Create a list of words about the context that students might not know (e.g., *investment*, *account*, *value*, *account balance*), along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

Encourage students to use a graphing calculator to investigate how each investment grows.

2 Connect

Listen for and share student descriptions that use a variety of language including terms from this unit and Unit 3 such as *linear, exponential, constant ratio, constant difference, rate of change, growth factor, percent increase,* etc. (e.g., "Account A is growing linearly at a constant rate of change of $50." or "Account B has a constant ratio of 1.05 and is growing exponentially.").

Consider asking:

- "How is Account A growing? How do you know?"
- "How is Account B growing? How do you know?"
- "How is Account A growing differently than Account B?" **(MTR.4.1, MTR.7.1)**

Unit 5
Lesson
12

Name: ..... Date: ..... Period: .....

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.F.1.6, MA.912.F.1.8, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

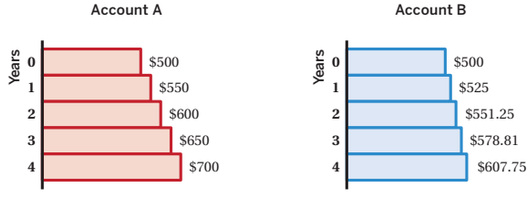
Bank Accounts

Let's learn how to model situations involving simple and compound interest.



Warm-Up

1. Mauricio has $500 to invest. He is researching different kinds of investment accounts. Here are the values of Accounts A and B over time.



Show or describe how the value of each account grows over time. *Responses vary.*

- The value of Account A increases by $50 each year.
- Account A has a constant rate of change of $50 and is growing linearly.
- The value of Account B is increasing by a growth factor of 1.05.
- Account B is growing exponentially since each year gets bigger by a factor of 1.05.

Unit 5 Lesson 12 555 Warm-Up

Pairs | 15 min

Activity 1 Earning Interest

Purpose: Students are introduced to **simple** and **compound interest** and use what they know about linear and exponential functions to make a recommendation.

1 Launch

To generate excitement, consider asking, "If it were you, which account would you invest in? Why?"

Use the Think-Pair-Share routine to support student-to-student discourse about making a recommendation for a specific type of interest-bearing account. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class. **(MTR.2.1, MTR.4.1)**

Consider asking:

- "What do you think Mauricio noticed about how each account is growing?"
- "How does your recommendation for Mauricio change if you know he only wants to invest his money for 5 years? 10 years? 30 years?"
- "What information or strategies are you using to decide which account is a better choice?"

A **Accessibility: Executive Functioning** Invite students to describe how each account is growing first before making a recommendation about which account they think is best for Mauricio.

2 Monitor

To support students getting started, consider asking:

- "Where do you see the 10% increase in the table? In the graph?" **(MTR.2.1)**
- "How can you tell by the graph and the table that Account A is increasing by the same percent each year?"

Look for students who make the connection that **simple interest** can be modeled by a linear function. Consider inviting these students to share during the Connect.

D **Differentiation: (Strengthen)** Some students may make the connection that in Lessons 6–7 they used exponential functions to model situations with repeated percent increase. To support their sensemaking, consider asking, "How is adding the same percent increase repeatedly differently than multiplying the percent increase repeatedly?" **(MTR.2.1, MTR.7.1)**

Activity 1 continued >

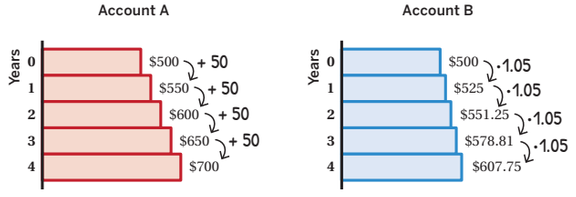
Activity 1

Name: _____ Date: _____ Period: _____

Earning Interest

3. Here is Mauricio's work to show how each account is growing.

Discuss: Describe his work to a partner. Which account would you recommend he invest in?



Responses vary.

- Mauricio noticed that Account A grows by a constant rate of change of $50, and Account B changes by a constant growth factor of 1.05.
- I recommend Mauricio invest in Account A because he will get more interest quickly.
- I recommend Mauricio invest in Account B. Since Account B is growing exponentially, eventually the balance will earn more interest than Account A and grow very quickly.

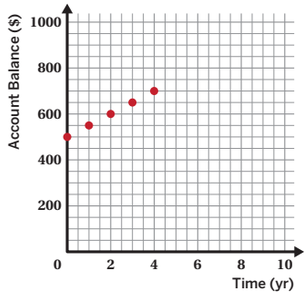
4. Account A earns 10% **simple interest** per year.

a. **Discuss:** How do you think *simple interest* works?

Responses vary. Simple interest adds a constant amount to the account each year.

b. Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700
5	750



Unit 5 Lesson 12 556 Activity 1

Activity 1 Earning Interest (continued)

Purpose: Students are introduced to **simple** and **compound interest** and use what they know about linear and exponential functions to make a recommendation.

2 Monitor

To support students getting started, consider asking:

- "Where do you see the 5% increase in the table? In the graph?" **(MTR.4.1)**
- "How can you tell by the graph and the table that Account B is increasing by the same percent each year?"

Encourage students to use the calculator to support their thinking.

Look for students who make the connection that compound interest can be modeled by an exponential relationship.

Listen for ideas to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Describe the functions using language from prior lessons (e.g., *linear*, *exponential*, *rate of change*, *growth factor*).	**Strengthen** Consider asking, "How is this situation similar to sequences that grow by a constant difference or ratio?"
Notice both functions use the variable t and have the same y-intercept.	**Strengthen** Consider asking, "Why do you think this is the same for both account types?"
Suggest Mauricio should always choose the simple interest account.	**Support** Consider asking, "Does your choice change if Mauricio has 30 years to invest?"
Suggest that their recommendation depends on the amount of time Mauricio has to invest.	**Stretch** Celebrate their attention to precision. Consider asking, "When would you recommend Mauricio invest in each account?" **(MTR.4.1, MTR.7.1)**

Invite students to discuss as they analyze the display the account balance in future years.

3 Connect

To reveal the Key Takeaway, consider asking:

- "Where do you see the 10% represented in the simple interest function? The 5% represented in the compound interest function?" **(MTR.2.1)**
- "What type of function can we use to model simple interest? Compound interest?"

Activity 1

Name: _____ Date: _____ Period: _____

Earning Interest (continued)

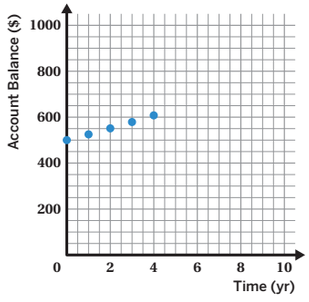
5. Account B earns 5% **compound interest** per year.

a. **Discuss:** How do you think *compound interest* works?

Responses vary. Compound interest adds a percentage of the previous balance to the account each year.

b. Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75
5	638.14



6. We can use functions to describe the account balances after t years.

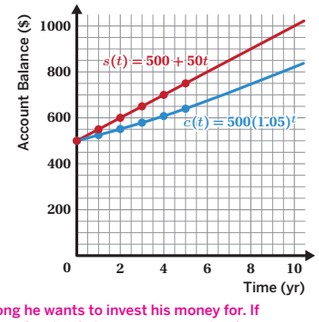
- Simple interest: $s(t) = 500 + 50t$
- Compound interest: $c(t) = 500(1.05)^t$

How are these functions alike?

Responses vary. Both of the functions are increasing and have an initial value of $500.

How are they different?

Responses vary. One function is linear, and the other is exponential.



7. **Discuss:** Which account would you recommend Mauricio invest in? Why?

Responses vary. I would ask Mauricio how long he wants to invest his money for. If he wants to invest for less than 26 years, then the simple interest account is a better investment. Otherwise, he will make more money with the compound interest account over a longer investment period.

Unit 5 Lesson 12 557 Activity 1

Key Takeaway: Accounts earning **simple interest** grow at a constant rate of change and can be modeled by a linear function. Accounts earning **compound interest** grow by a constant growth factor and can be modeled by an exponential function.

Pairs | 15 min

Activity 2 Simple and Compound Interest

Purpose: Students learn two strategies for solving problems involving exponential functions and compound interest using a graphing calculator. **(MTR.2.1, MTR.4.1, MTR.5.1)**

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

To access connections, consider asking, "How does this account's interest compare to the previous ones Mauricio was considering?"

To generate excitement, invite students to a precision challenge using the calculator by asking them to substitute values for t into the function $a(t)$ to see who can get as close as possible to $\$1{,}000$. **(MTR.5.1)**

Consider sharing that if Mauricio started with an initial investment of $\$500$ that grows to $\$1{,}000$, this represents a situation that can be described as *doubling his money*.

Consider asking:

- "Does substituting values for t into the function give us an exact or approximate answer for when Mauricio's account reaches $\$1{,}000$?" **(MTR.4.1)**
- "When might an approximate solution be useful? When might an exact solution be useful?" **(MTR.4.1)**
- "When might it matter that our solution is approximate and not exact?"

2 Monitor

Demonstrate how to use the graphing calculator to determine the point of intersection by zooming out or changing the scale of the x- and y-axis.

Consider asking:

- "How does each strategy help you solve the problem differently?" **(MTR.2.1)**
- "What are other strategies you might use to solve this problem in a different way?"

Invite students to continue to the next problems.

Activity 2 continued >

Activity 2

Name: Date: Period:

Simple and Compound Interest

8. Mauricio decided to invest in an account that offers 6% compound interest per year.

$a(t) = 500(1.06)^t$ represents its balance after t years.

About how many years will it take for the balance to reach $\$1{,}000$? Explain your thinking.

Use this space or a graphing calculator to help with your thinking.

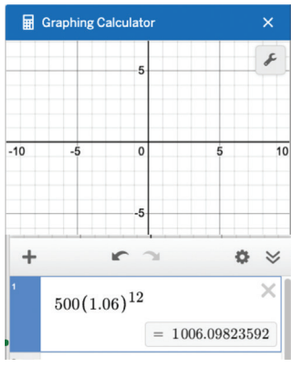
Responses between 11 and 12 years are considered correct. *Explanations vary.*

- I plugged in different numbers for t until the balance got close to $\$1{,}000$. The t-value represents the number of years until the account balance is about $\$1{,}000$.
- I graphed the function $a(t) = 500(1.06)^t$ and found the point where the y-value is $\$1{,}000$. This happens after about 11.9 years

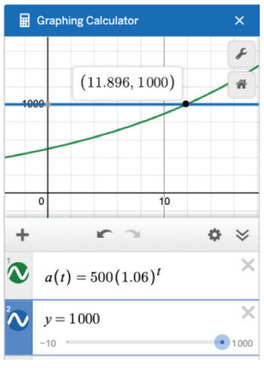
9. Below are different methods used by two students to determine the time it would take to reach $\$1{,}000$.

Discuss: Which method helps get a more precise answer?

Fabiana



Antwon



Responses vary. Antwon's method is more accurate. Graphing the function $a(t) = 500(1.06)^t$ and the line $y = 1000$ and determining the x-coordinate where they intersect will help you get a more precise answer.

Unit 5 Lesson 12 558 Activity 2

Pairs | 15 min

Activity 2 Simple and Compound Interest (continued)

Purpose: Students learn two strategies for solving problems involving exponential functions and compound interest a graphing calculator. **(MTR.2.1, MTR.4.1, MTR.5.1)**

Short on time? Consider inviting students to complete fewer challenges.

2 Monitor

Invite students to solve as many challenges as they have time for or set a goal for how many challenges they want to solve. Share that it's more important to make sense of the challenges than it is to work quickly.

Encourage students to use the graphing calculator to solve the problems.

Consider using Think-Pair-Share routine, encouraging students to begin their challenge with individual think time, then regroup students into pairs to reflect and revise their work **(MTR.4.1)**

A **Accessibility: Memory and Attention** Invite students who would benefit from more time to think deeply to complete fewer challenges.

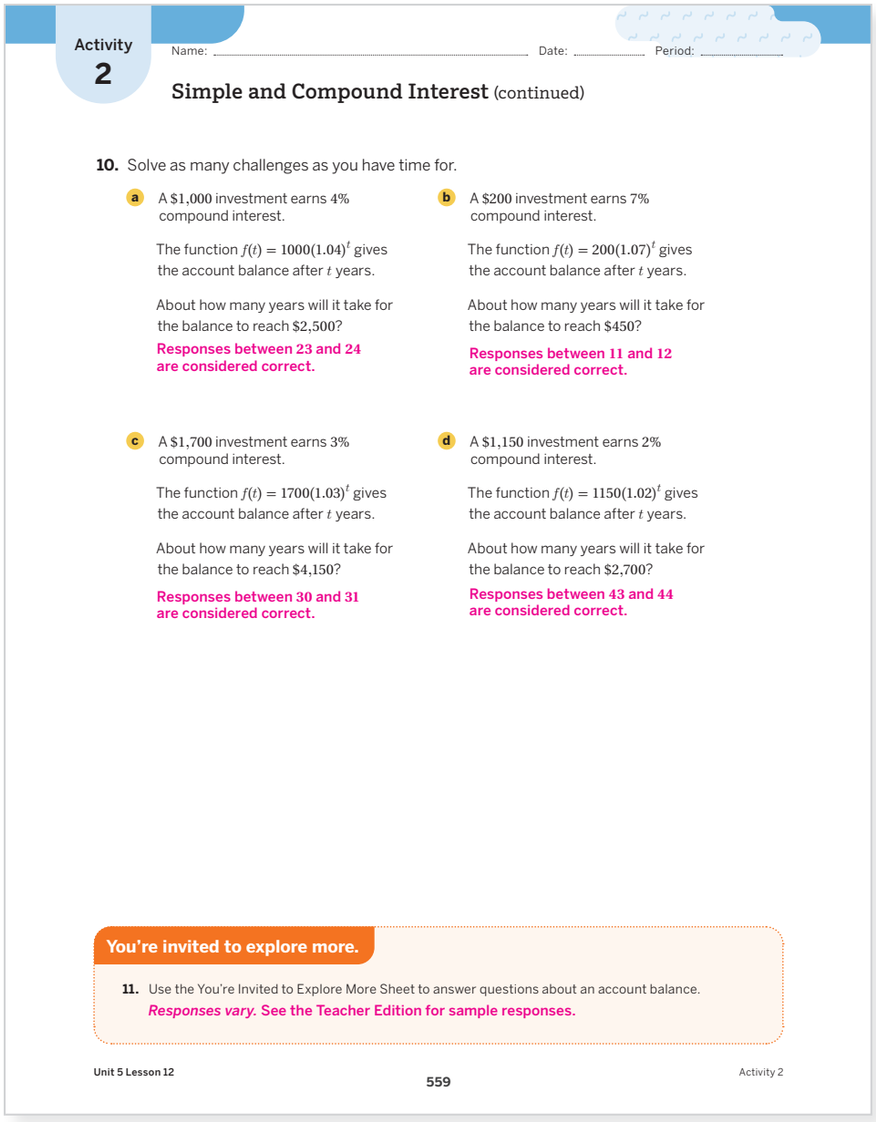
Listen for strategies students use in discussing how to solve the problems, such as whether to determine the point of intersection or approximate solution by substituting values into the function.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore simple and compound interest further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges. **(MTR.4.1)**

Consider asking, "Was there anything new you learned about the graphing calculator today? How might you be able to use that skill in the future?"



Activity 2

Name: _____ Date: _____ Period: _____

Simple and Compound Interest (continued)

10. Solve as many challenges as you have time for.

a A $1,000 investment earns 4% compound interest.

The function $f(t) = 1000(1.04)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $2,500?

Responses between 23 and 24 are considered correct.

b A $200 investment earns 7% compound interest.

The function $f(t) = 200(1.07)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $450?

Responses between 11 and 12 are considered correct.

c A $1,700 investment earns 3% compound interest.

The function $f(t) = 1700(1.03)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $4,150?

Responses between 30 and 31 are considered correct.

d A $1,150 investment earns 2% compound interest.

The function $f(t) = 1150(1.02)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $2,700?

Responses between 43 and 44 are considered correct.

You're invited to explore more.

11. Use the You're Invited to Explore More Sheet to answer questions about an account balance.

Responses vary. **See the Teacher Edition for sample responses.**

Unit 5 Lesson 12 559 Activity 2

Key Takeaway: There are many strategies to determine an input for a given output of a function. These include substituting values, guessing and checking, and graphing the output and the function to find the point of intersection. **(MTR.2.1)**

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how investments grow with simple and compound interest.

Invite students to respond independently, and then share their thinking with a partner.

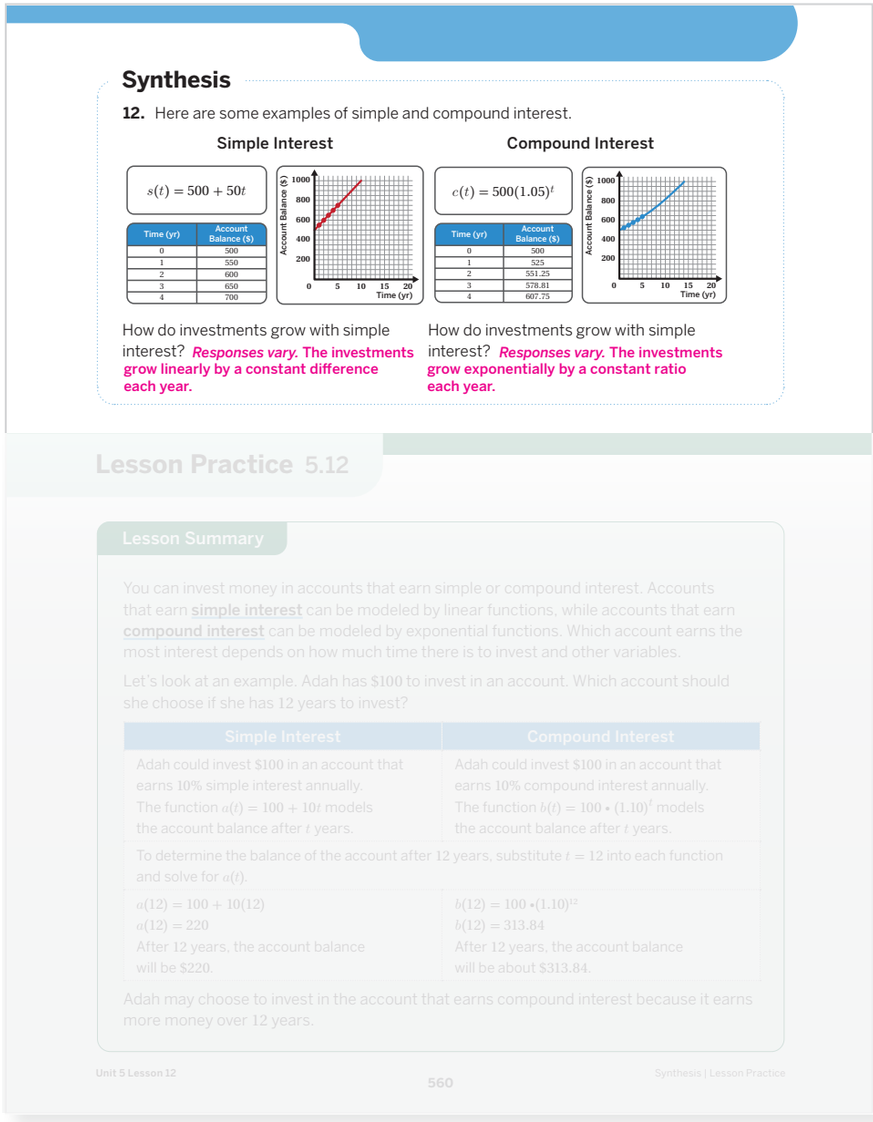
Look for and share a variety of ideas, such as:

- Compound and simple interest both grow by a percent.
- Simple interest repeatedly adds the same percent.
- Compound interest repeatedly multiples the same percent.
- Simple interest can be modeled by linear functions.
- Compound interest can be modeled by exponential functions.
- Compound interest accounts will always surpass simple interest accounts, given enough time.
- Simple interest accounts are still a good investment but it depends on the amount of time one has to invest.

Consider asking:

- "In what ways can accounts earning interest be helpful?" Accounts that earn interest can be helpful in several cases, including on the side of the earner. If someone invests money in an account that earns interest, the account balance will increase beyond what they invest.
- "In what ways can accounts earning interest be harmful?" Accounts that earn interest can be harmful on the side of the borrower. If someone takes out a loan in an account that earns interest, they will owe more money than they were lent due to the interest earned on the account.

MLR **MLR2: Collect and Display** Use this routine to collect students' language as they discuss the difference between simple and compound interest with a partner. Display for students to reference and add to for Lessons 11–13.



Synthesis

12. Here are some examples of simple and compound interest.

Simple Interest

$s(t) = 500 + 50t$

Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700

How do investments grow with simple interest? *Responses vary.* The investments grow linearly by a constant difference each year.

Compound Interest

$c(t) = 500(1.05)^t$

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75

How do investments grow with simple interest? *Responses vary.* The investments grow exponentially by a constant ratio each year.

Lesson Practice 5.12

Lesson Summary

You can invest money in accounts that earn simple or compound interest. Accounts that earn simple interest can be modeled by linear functions, while accounts that earn compound interest can be modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables.

Let's look at an example. Adah has $100 to invest in an account. Which account should she choose if she has 12 years to invest?

Simple Interest	Compound Interest
Adah could invest $100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after t years.	Adah could invest $100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \cdot (1.10)^t$ models the account balance after t years.
To determine the balance of the account after 12 years, substitute $t = 12$ into each function and solve for $a(t)$.	
$a(12) = 100 + 10(12)$ $a(12) = 220$ After 12 years, the account balance will be $220.	$b(12) = 100 \cdot (1.10)^{12}$ $b(12) = 313.84$ After 12 years, the account balance will be about $313.84.

Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.

Unit 5 Lesson 12 560 Synthesis | Lesson Practice

Lesson Takeaway: An account earning **compound interest** is modeled by an exponential function. One earning simple interest is modeled by a linear function.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining when an account balance will reach a specific amount.

Today's Goals

1. **Goal:** Use a graphing calculator to solve problems involving linear and exponential functions.
2. **Language Goal:** Describe connections between *simple* and *compound interest*, and linear and exponential functions. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

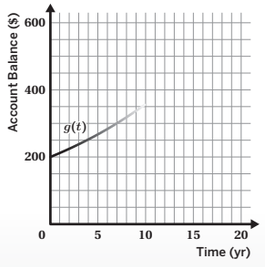
Show What You Know 5.12

Anushka invests $200 in an account that earns 6% compound interest.

The function $g(t) = 200(1.06)^t$ gives the account balance after t years.

About how many years will it take for her account balance to reach $500?

Responses from 15 and 16 years are considered correct.



Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students' need support understanding of compound interest:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their conceptual understanding in Lessons 13–14. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students' would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

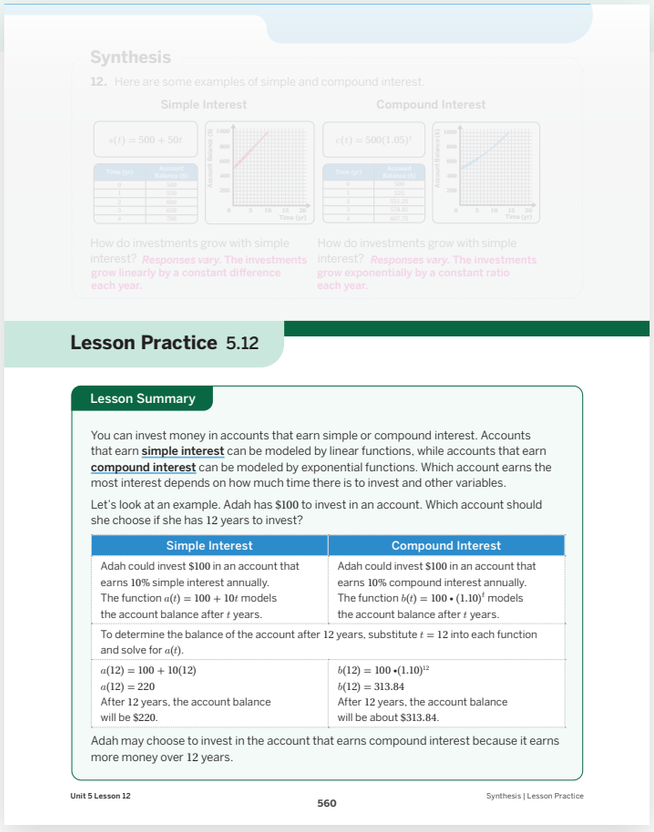
Professional Learning

Take a moment to reflect on the context introduced in this lesson. Students developed early ideas about the terms **simple interest** and **compound interest**, including that accounts that earn compound interest eventually surpass those earning simple interest. In Lessons 12–13, students will explore the topic of payday loans and credit cards.What questions might your students have about how compound interest can be used in helpful and harmful ways? What resources might you and the class use to help you learn more about these questions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

12. Here are some examples of simple and compound interest.

Simple Interest

$a(t) = 500 + 50t$

Time (yr)	Account Balance ($)
0	500
1	550
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Time (yr)	Account Balance ($)
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How do investments grow with simple interest? *Responses vary. The investments grow exponentially by a constant ratio each year.*

Lesson Practice 5.12

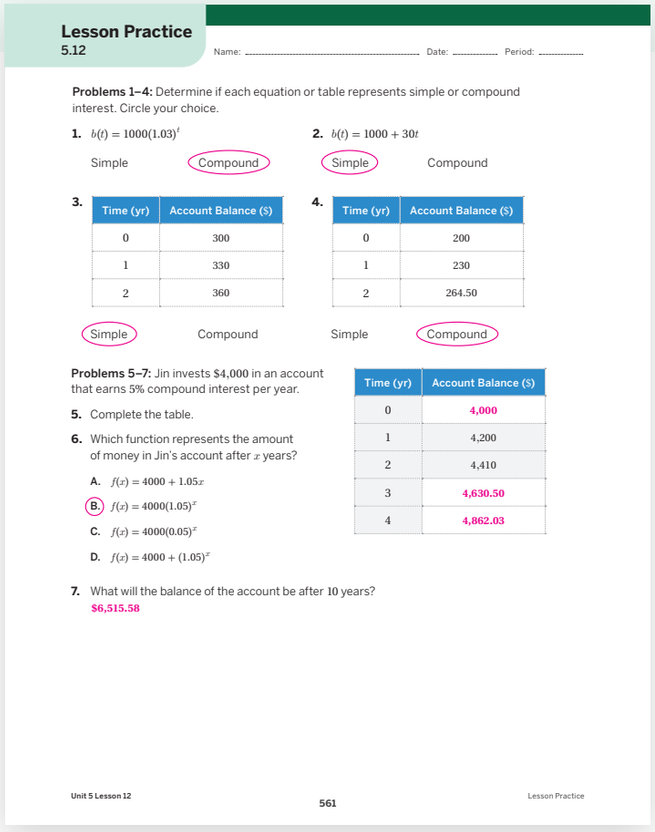
Lesson Summary

You can invest money in accounts that earn simple or compound interest. Accounts that earn **simple interest** can be modeled by linear functions, while accounts that earn **compound interest** can be modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables.

Let's look at an example. Adah has $100 to invest in an account. Which account should she choose if she has 12 years to invest?

Simple Interest	Compound Interest
Adah could invest $100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after t years.	Adah could invest $100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \cdot (1.10)^t$ models the account balance after t years.
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Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.



Lesson Practice 5.12

Name: Date: Period:

Problems 1–4: Determine if each equation or table represents simple or compound interest. Circle your choice.

1. $b(t) = 1000(1.03)^t$

Simple (Compound)

2. $b(t) = 1000 + 30t$

(Simple) Compound

3.

Time (yr)	Account Balance ($)
0	300
1	330
2	360

(Simple) Compound

4.

Time (yr)	Account Balance ($)
0	200
1	230
2	264.50

Simple (Compound)

Problems 5–7: Jin invests $4,000 in an account that earns 5% compound interest per year.

5. Complete the table.

Time (yr)	Account Balance ($)
0	4,000
1	4,200
2	4,410
3	4,630.50
4	4,862.03

6. Which function represents the amount of money in Jin's account after x years?

A. $f(x) = 4000 + 1.05x$

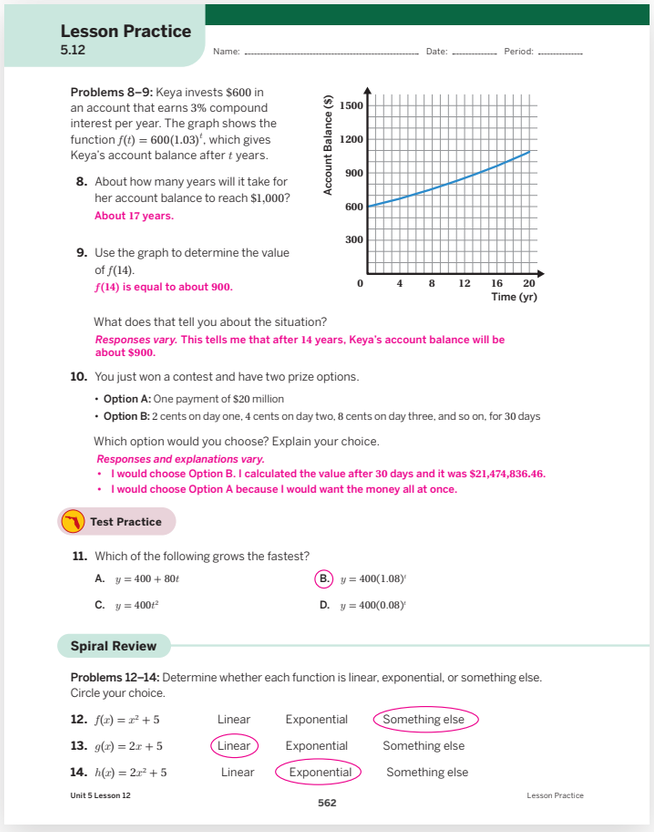
(B.) $f(x) = 4000(1.05)^x$

C. $f(x) = 4000(0.05)^x$

D. $f(x) = 4000 + (1.05)^x$

7. What will the balance of the account be after 10 years?

$6,515.58



Lesson Practice 5.12

Name: Date: Period:

Problems 8–9: Keya invests $600 in an account that earns 3% compound interest per year. The graph shows the function $f(t) = 600(1.03)^t$, which gives Keya's account balance after t years.

8. About how many years will it take for her account balance to reach $1,000?

About 17 years.

9. Use the graph to determine the value of $f(14)$.

$f(14)$ is equal to about 900.

What does that tell you about the situation?

Responses vary. This tells me that after 14 years, Keya's account balance will be about $900.

10. You just won a contest and have two prize options.

- **Option A:** One payment of $20 million
- **Option B:** 2 cents on day one, 4 cents on day two, 8 cents on day three, and so on, for 30 days

Which option would you choose? Explain your choice.

Responses and explanations vary.

- *I would choose Option B. I calculated the value after 30 days and it was $21,474,836.46.*
- *I would choose Option A because I would want the money all at once.*

Test Practice

11. Which of the following grows the fastest?

A. $y = 400 + 80t$

(B.) $y = 400(1.08)^t$

C. $y = 400t^2$

D. $y = 400(0.08)^t$

Spiral Review

Problems 12–14: Determine whether each function is linear, exponential, or something else. Circle your choice.

12. $f(x) = x^2 + 5$ — Linear, Exponential, (Something else)

13. $g(x) = 2x + 5$ — (Linear), Exponential, Something else

14. $h(x) = 2x^2 + 5$ — Linear, (Exponential), Something else

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.912.FL.3.4 MA.912.F.1.6
	5–7	2	MA.912.FL.3.2 MA.912.FL.3.4 MA.912.F.1.8
	8–9	2	MA.912.FL.3.2
	10	3	MA.912.FL.3.2 MA.912.FL.3.4 MA.912.F.1.8
Test Practice	11	2	MA.912.F.1.6
Spiral Review			
Fluency	12–14	1	MA.912.F.1.6

Unit 5

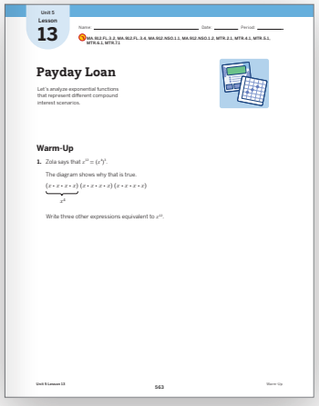
Lesson

13

Payday Loan

Revisiting Compound Interest

Let's analyze exponential functions that represent different compound interest scenarios.


Unit 5
Lesson
13
Name _____ Date _____ Period _____
MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1
Payday Loan
Let's analyze exponential functions that represent different compound interest scenarios.
Warm-Up
1. Zola says that $x^{10} = (x^5)^2$.
The diagram shows why that is true.
$\underbrace{(x \cdot x \cdot x \cdot x \cdot x)\ (x \cdot x \cdot x \cdot x \cdot x)}_{x^{10}}$
Write three other expressions equivalent to x^{10}.
Unit 5 Lesson 13
563
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Write exponential functions that represent situations involving compound interest.
2. **Language Goal:** Interpret equivalent exponential expressions in context using properties of exponents. **(Reading, Speaking, and Listening)**

Students write and analyze functions representing compound interest scenarios. They use exponent properties, specifically the powers of powers law, to write equivalent expressions that reveal different information to help them identify the interest rate per year given a monthly interest rate. **(MTR.5.1)**

Coherence

Prior Learning

In Grade 8, students created equivalent expressions using exponent properties. In Lesson 11, students made connections between simple and compound interest, and linear and exponential functions.

Future Learning

In Lesson 13, students will explore expressions that represent situations for different compounding intervals.

Connections

- Students analyze exponential functions that represent different compound interest scenarios through payday loans and credit card offers. (real world, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.FL.3.2
Solve real-world problems involving simple, compound and continuously compounded interest.

MA.912.NSO.1.2
Generate equivalent algebraic expressions using the properties of exponents.

Also Addressing: **MA.912.FL.3.4, MA.912.NSO.1.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.1.1	**MA.912.NSO.1.3**
MA.912.AR.1.2	

Lesson at a Glance

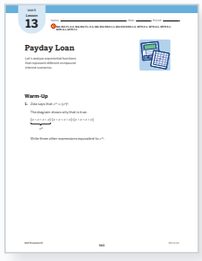
~ 45 min

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

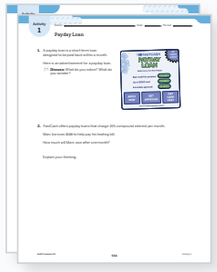
Students revisit the powers of powers law to create equivalent exponential expressions.



Activity 1

Pairs | 10 min

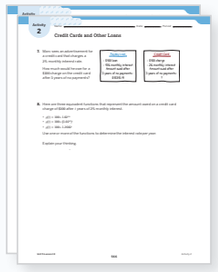
Students explore a payday loan with a monthly interest rate and analyze equivalent expressions to determine the interest rate per year. **Routines:** Notice and Wonder, Decide and Defend **(MTR.5.1, MTR.7.1)**



Activity 2

Pairs | 12 min

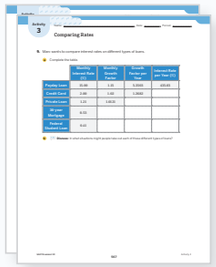
Students compare and contrast other types of loans with monthly interest rates, including credit cards and student loans, with a payday loan.



Activity 3

Pairs | 8 min

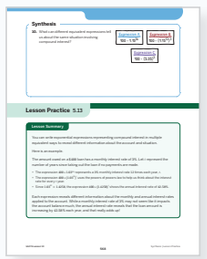
Students compare and contrast other types of loans with monthly interest rates, including credit cards and student loans, with a payday loan. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

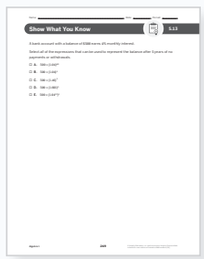
Students synthesize their understanding of using exponent properties to write equivalent expressions that reveal different information.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting all of the equivalent expressions that would represent an account balance given an initial investment and monthly interest rate.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit the powers of powers law to create equivalent exponential expressions.

Short on time? Consider omitting this Warm-Up if students show strong understanding of exponent properties on Problems 5-6 in the Pre-Unit Check.

1 Launch

To reveal prior knowledge, consider asking, "What do you remember about using exponent properties to write equivalent expressions?"

Consider asking "What do you see when $x^{12} = (x^4)^3$?" I see three groups of four x's, which is written as x^4.

Invite students to make suggestions of expressions they think are equivalent.

D **Differentiation: (Support)** Some students may need support understanding what it means for expressions to be equivalent. Consider showing how two expressions can be expanded to be equal to x^{12}. Encourage these students to reflect on what makes Zola's equation true before writing a new expression.

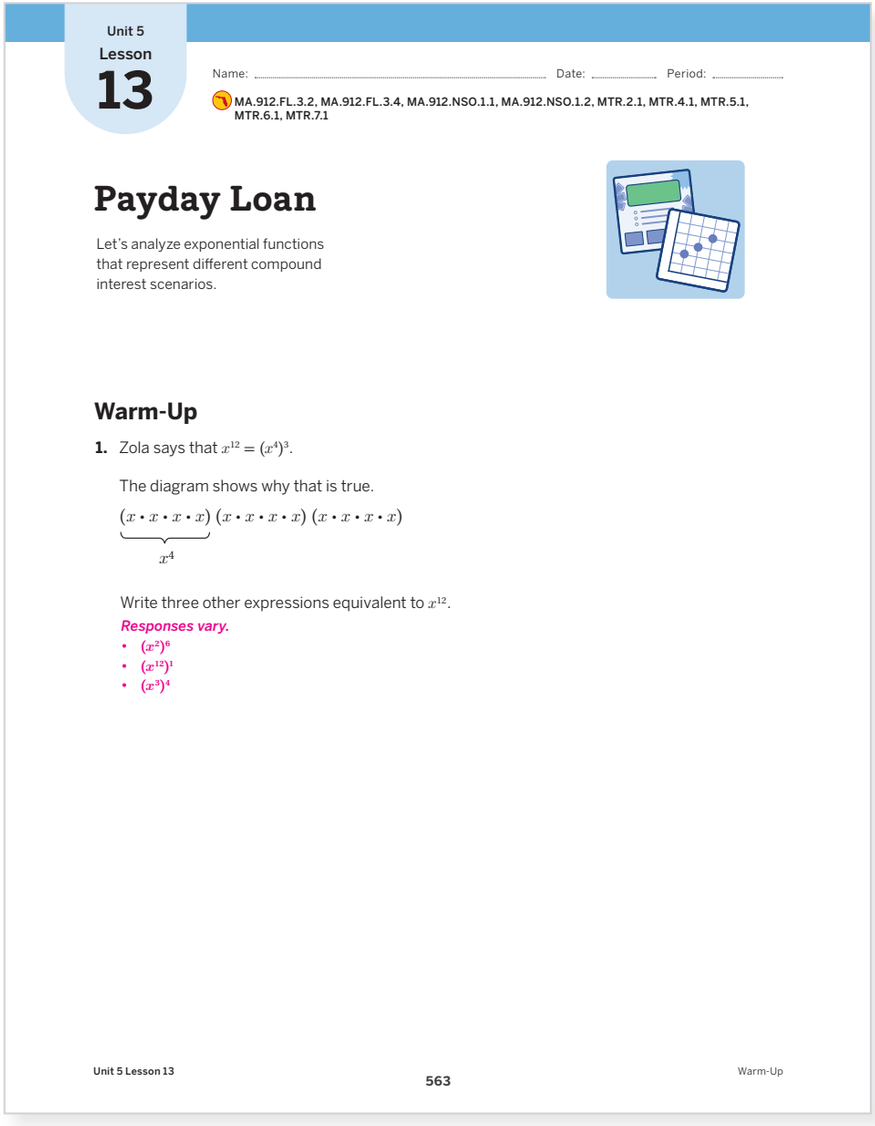
A **Accessibility: Visual-Spatial Processing** To support students in interpreting visual representations, invite them to drag the point to create a random expression and asking, "How do you see x^{12} in this expression? What changes about the expression? What stays the same?".

ELL **English Language Learners** Invite students to share different meanings of the term equivalent. Clarify which meaning this word will have for this activity. **(Reading and Listening)**

2 Connect

Look for and share equivalent expressions that are unique or interesting (e.g., $x^7 \cdot x^5$). Invite these students to share their thinking.

To reveal the Key Takeaway, consider asking, "How can we know if an expression is equivalent using the powers of powers law of exponents?"



Unit 5
Lesson
13

Name: ..... Date: ..... Period: .....

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Payday Loan

Let's analyze exponential functions that represent different compound interest scenarios.

Warm-Up

1. Zola says that $x^{12} = (x^4)^3$.

The diagram shows why that is true.

$\underbrace{(x \cdot x \cdot x \cdot x)}_{x^4}\ (x \cdot x \cdot x \cdot x)\ (x \cdot x \cdot x \cdot x)$

Write three other expressions equivalent to x^{12}.

Responses vary.

- $(x^2)^6$
- $(x^{12})^1$
- $(x^3)^4$

Unit 5 Lesson 13 563 Warm-Up

Key Takeaway: The powers of powers law of exponents states that $(x^a)^b = x^{ab}$. The exponents can be multiplied together to create an equivalent expression with the same base.

Pairs | 10 min

Activity 1 Payday Loan

Purpose: Students explore a payday loan with a monthly interest rate and analyze equivalent expressions to determine the interest rate per year. **(MTR.5.1, MTR.7.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the context of payday loans they will be learning about in this lesson. **(MTR.7.1)**

ELL **English Language Learners** Create a list of words about the context that students might not know (e.g., *loan*, *payday*, *fast cash*, *bad credit*, *credit check*, etc.), along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

Consider asking:

- "What is a loan? How does a loan work?"
- "What is a payday loan? How do you think a payday loan works differently than other types of loans?"
- "What are some reasons someone might take out a payday loan?"
- "What important details are included in the advertisement for the payday loan?" The advertisement makes it seem like getting a payday loan is fast and easy. Also, there's no credit check, but proof of regular paychecks is required.
- "Why might someone want to get a payday loan?" If someone needed cash quickly, they may take out a short-term loan like a payday loan to help cover expenses.

2 Monitor

Encourage students to use a calculator to support them in evaluating throughout this lesson.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Payday Loan

1. A payday loan is a short-term loan designed to be paid back within a month.

Here is an advertisement for a payday loan.



Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that you can get up to $1,000.
- I notice that you need proof of a regular paycheck to apply.
- I wonder what the highest amount is that someone could take out in a payday loan.
- I wonder what happens if you cannot settle your loan by your next payday.

2. FastCash offers payday loans that charge 15% compound interest per month.

Marc borrows $100 to help pay his heating bill.

How much will Marc owe after one month?

$115

Explain your thinking.

Explanations vary. After one month he will owe the original $100 plus 15% of $100, so $15 of interest. $100 + $15 = $115.

Unit 5 Lesson 13 564 Activity 1

Pairs | 10 min

Activity 1 Payday Loan (continued)

Purpose: Students explore a payday loan with a monthly interest rate and analyze equivalent expressions to determine the interest rate per year. **(MTR.7.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to determine the initial value and percent increase.
Write a linear function.	**Strengthen** Celebrate the function they wrote. Consider asking, "How can you tell if months and amount owed are in a linear or exponential relationship?"
Write an exponential function.	**Strengthen** Consider asking, "How do you know the function you wrote matches the table? The graph?"

Listen for how students describe and make use of structure, including how the powers of powers law makes the two expressions equivalent, interpreting parts of each expression, or noticing the initial loan of $100. Consider waiting to discuss these ideas until the Connect. **(MTR.7.1)**

Encourage students to evaluate Marc's account balance after three years of no payments for Expression A.

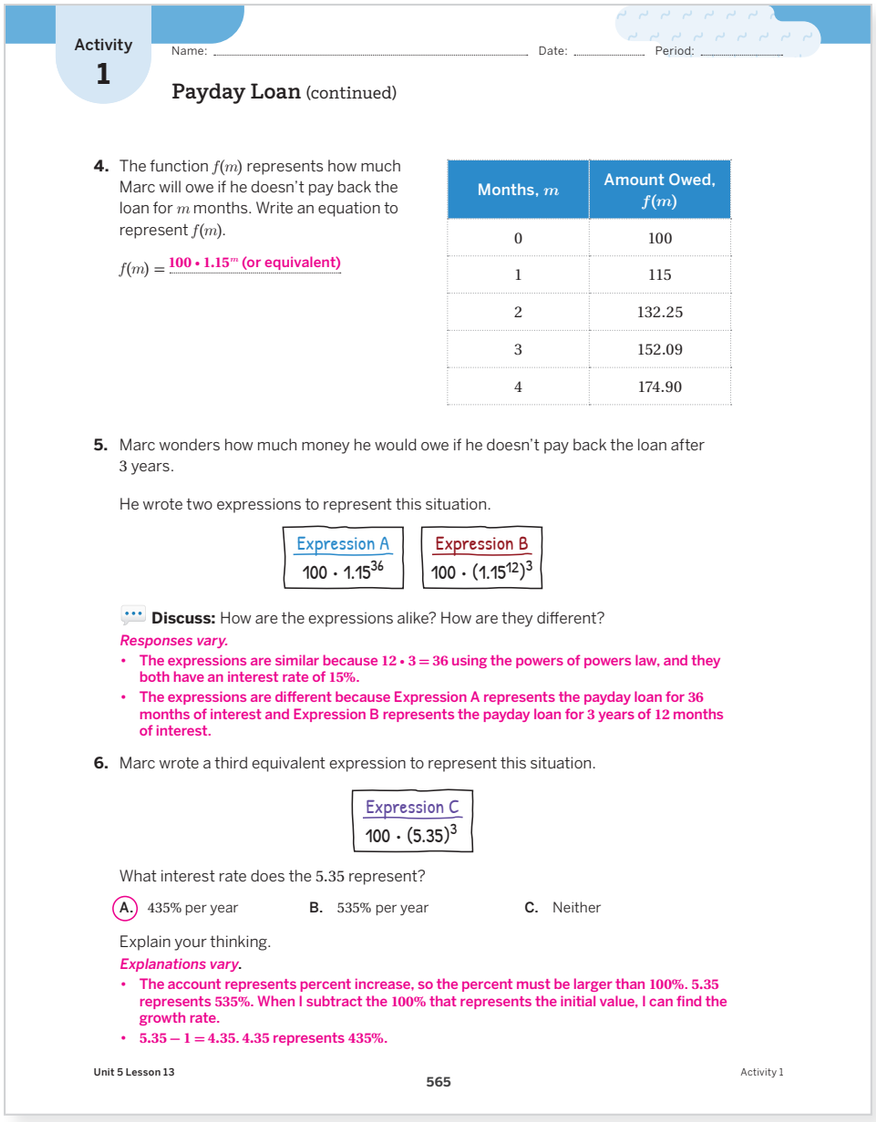
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to analyze the structure to see they are all equivalent. Consider asking, "Where do you think the 5.35 in Expression C came from?"
Think that 535% represents the interest rate 5.35.	**Strengthen** Consider asking, "What would the growth factor be for an interest rate of 50%? 100%? What is the difference between growth factor and interest rate?"

3 Connect

Use the Decide and Defend routine to support students in making and critiquing arguments. See the Routine Facilitation Guide for more information.

To reveal the Key Takeaway, consider asking:
- "How do you know that the expressions are equivalent?" **(MTR.6.1)**
- "What does each expression represent?" **(MTR.4.1)**
- "How could you rewrite expression C to show that the interest rate per year is 435%?" **(MTR.2.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Payday Loan (continued)

4. The function $f(m)$ represents how much Marc will owe if he doesn't pay back the loan for m months. Write an equation to represent $f(m)$.

$f(m) =$ $100 \cdot 1.15^m$ (or equivalent)

Months, m	Amount Owed, $f(m)$
0	100
1	115
2	132.25
3	152.09
4	174.90

5. Marc wonders how much money he would owe if he doesn't pay back the loan after 3 years.

He wrote two expressions to represent this situation.

Expression A: $100 \cdot 1.15^{36}$

Expression B: $100 \cdot (1.15^{12})^3$

Discuss: How are the expressions alike? How are they different?

Responses vary.
- The expressions are similar because $12 \cdot 3 = 36$ using the powers of powers law, and they both have an interest rate of 15%.
- The expressions are different because Expression A represents the payday loan for 36 months of interest and Expression B represents the payday loan for 3 years of 12 months of interest.

6. Marc wrote a third equivalent expression to represent this situation.

Expression C: $100 \cdot (5.35)^3$

What interest rate does the 5.35 represent?

A. 435% per year B. 535% per year C. Neither

Explain your thinking.

Explanations vary.
- The account represents percent increase, so the percent must be larger than 100%. 5.35 represents 535%. When I subtract the 100% that represents the initial value, I can find the growth rate.
- $5.35 - 1 = 4.35$. 4.35 represents 435%.

Unit 5 Lesson 13 565 Activity 1

Key Takeaway: Exponential functions can be written in equivalent ways using properties of exponents to reveal different information. The different information revealed can help inform decision-making.

Pairs | 12 min

Activity 2 Credit Cards and Other Loans

Purpose: Students compare and contrast other types of loans with monthly interest rates, including credit cards and student loans, with a payday loan.

1 Launch

Distribute how to calculate the balance of the payday loan with 15% monthly interest after 3 years of no payments.

Consider asking:

- "Why might someone think that it's not fair to have to pay back $15,315.19 for a loan of $100?"
- "Why might Marc look for an alternative to a payday loan?"

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Write an expression using 2% as the interest rate per year.	**Support** Encourage them to think about the time intervals. Consider asking, "How many times would monthly interest be collected in 3 years?"
Write an expression with: • One exponent: $100(1.02)^{36}$. • More than one exponent: $100(1.02^{12})^3$. • Using the interest rate per year: $100(1.268)^3$.	**Stretch** Look for the expression they wrote to discuss during the Connect. Consider asking, "How can you use the powers of powers law to write this expression another way?"

To support students getting started, encourage them to choose a function and interpret the interest rate.

D Differentiation: (Strengthen) If students need support seeing equivalent expressions, invite them to use the graphing calculator to graph all three expressions and share what they notice.

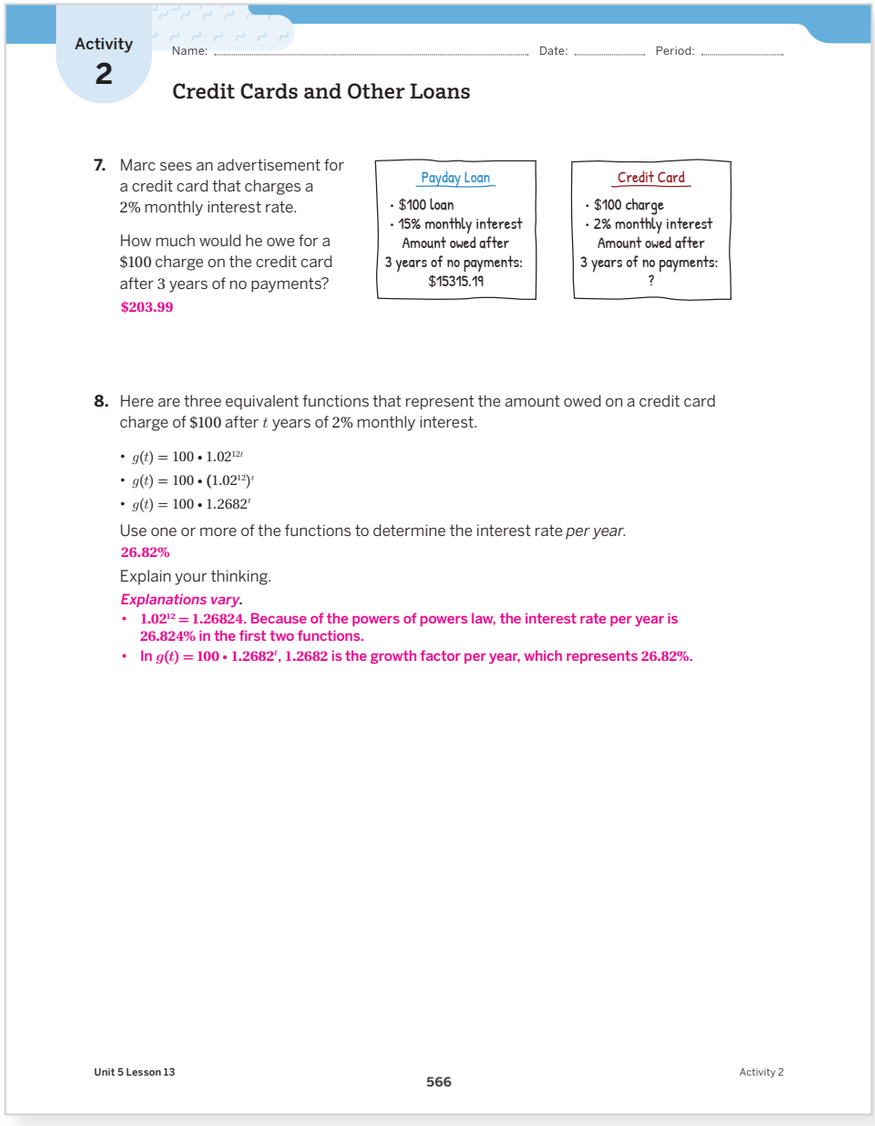
3 Connect

Consider asking:

- "Why is there a difference in the amount owed for the payday loan and the credit card loan?"
- "Which option would you recommend for Marc?"

Consider asking:

- "How can we see the interest rate per year in each function?" **(MTR.5.1)**
- "Which function matches the graph? How do you know?" **(MTR.6.1)**
- "Which function is most helpful for determining the interest rate per year? Why?"



Activity 2

Name: Date: Period:

Credit Cards and Other Loans

7. Marc sees an advertisement for a credit card that charges a 2% monthly interest rate.

How much would he owe for a $100 charge on the credit card after 3 years of no payments?

$203.99

Payday Loan	Credit Card
• $100 loan • 15% monthly interest Amount owed after 3 years of no payments: $15315.19	• $100 charge • 2% monthly interest Amount owed after 3 years of no payments: ?

8. Here are three equivalent functions that represent the amount owed on a credit card charge of $100 after t years of 2% monthly interest.

- $g(t) = 100 \cdot 1.02^{12t}$
- $g(t) = 100 \cdot (1.02^{12})^t$
- $g(t) = 100 \cdot 1.2682^t$

Use one or more of the functions to determine the interest rate *per year*.

26.82%

Explain your thinking.

Explanations vary.

- **$1.02^{12} = 1.26824$. Because of the powers of powers law, the interest rate per year is 26.824% in the first two functions.**
- **In $g(t) = 100 \cdot 1.2682^t$, 1.2682 is the growth factor per year, which represents 26.82%.**

Unit 5 Lesson 13 566 Activity 2

Pairs | 8 min

Activity 3 Comparing Rates

Purpose: Students compare and contrast other types of loans with monthly interest rates, including credit cards and student loans, with a payday loan. **(MTR.7.1)**

1 Launch

Consider sharing that there are many different types of loans and that each loan type is used for a specific situation.

ELL **English Language Learners** Consider inviting students to share what they know about each of the loan types. Discuss when someone might apply for each type of loan to increase access to the task (e.g., someone might apply for a 30-Year mortgage loan to purchase a house, or a Federal Student loan in order to attend college). **(Reading and Listening)**

Consider asking:

- "Why do you think the interest rates are different for different types of loans?" Interest rates are determined based on many variables, one being the risk that the person receiving the loan will pay it back. The more risk, the larger the interest rate.
- "Who has access to each kind of loan?" Federal student loans are only available for students enrolled in college, career, or trade schools, whereas credit cards are available based on criteria (e.g., income level, credit score, etc.)

2 Monitor

To access connections, consider asking:

- "How is the monthly growth factor related to the monthly interest rate?" **(MTR.4.1)**
- "How is the interest rate per year related to the monthly growth factor?" **(MTR.4.1)**
- "How does the growth factor per year help us identify the interest rate per year?" **(MTR.4.1)**

A **Accessibility: Fine Motor Skills** Consider varying the task demands by providing students with a pre- or partially-completed table.

3 Connect

Note: Here are two options for Connects based on student needs and interests.

- If students need additional support analyzing the relationship between the monthly and yearly interest rate, consider spending time discussing connections and interpretations in Problem 9.
- If students show a strong understanding of the equivalent relationships between monthly and yearly interest rates, consider discussing the Explore in Problem 10.

Consider asking, "How is the monthly interest rate related to the growth factor per year? The interest rate per year?"

Activity 3

Name: _____ Date: _____ Period: _____

Comparing Rates

9. Marc wants to compare interest rates on different types of loans.

a. Complete the table.

	Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
Payday Loan	15.00	1.15	5.3503	435.03
Credit Card	2.00	1.02	1.2682	26.82
Private Loan	1.21	1.0121	1.1553	15.53
30-year Mortgage	0.53	1.0053	1.0655	6.55
Federal Student Loan	0.41	1.0041	1.0503	5.03

b. **Discuss:** In what situations might people take out each of these different types of loans?

Responses vary.

- A college student might take out a private or a federal student loan to pay for college.
- A person or family might take out a mortgage when they have a down payment and can't immediately pay the full price for a house.

Unit 5 Lesson 13 567 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using exponent properties to write equivalent expressions that reveal different information.

Invite students to respond independently, and then share their thinking with a partner.

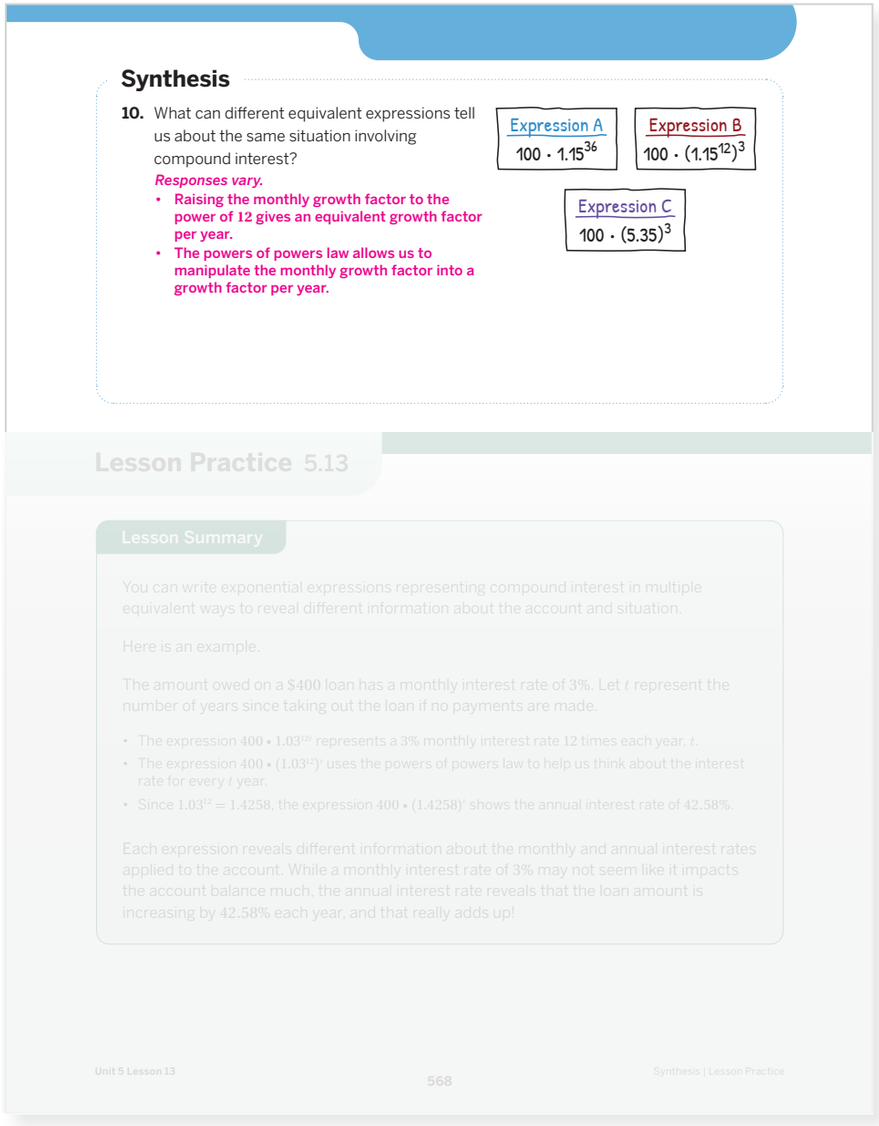
Look for and share a variety of ideas, such as:

- All three expressions have a monthly interest rate of 15%, and a yearly interest rate of 435%.
- Expression A shows 36 months of 15% interest.
- Expression B shows 3 years each of 12 months of 15% interest.
- Expression A and B are equivalent because of the powers of powers law of exponents.
- Expression B and C are equivalent because $1.15^{12} = 5.35$.
- Expression C most directly reveals the yearly interest rate of 435%.

Consider asking:

- "In what ways can accounts earning interest be helpful?" Accounts that earn interest can be helpful on the side of the earner. If someone invests money in an account that earns interest, the account balance will increase beyond what they are investing.
- "In what ways can accounts earning interest be harmful?" Accounts that earn interest can be harmful on the side of the borrower. If someone takes out a loan in an account that earns interest, they will owe more money than they were lent due to the interest earned on the account.

Note: If time allows, invite students to share with a partner something new they learned about loans in this lesson and how it may impact their thinking about loans for their future.



Synthesis

10. What can different equivalent expressions tell us about the same situation involving compound interest?

Responses vary.

- Raising the monthly growth factor to the power of 12 gives an equivalent growth factor per year.
- The powers of powers law allows us to manipulate the monthly growth factor into a growth factor per year.

Expression A: $100 \cdot 1.15^{36}$

Expression B: $100 \cdot (1.15^{12})^3$

Expression C: $100 \cdot (5.35)^3$

Lesson Practice 5.13

Lesson Summary

You can write exponential expressions representing compound interest in multiple equivalent ways to reveal different information about the account and situation.

Here is an example.

The amount owed on a $400 loan has a monthly interest rate of 3%. Let t represent the number of years since taking out the loan if no payments are made.

- The expression $400 \cdot 1.03^{12t}$ represents a 3% monthly interest rate 12 times each year, t.
- The expression $400 \cdot (1.03^{12})^t$ uses the powers of powers law to help us think about the interest rate for every t year.
- Since $1.03^{12} = 1.4258$, the expression $400 \cdot (1.4258)^t$ shows the annual interest rate of 42.58%.

Each expression reveals different information about the monthly and annual interest rates applied to the account. While a monthly interest rate of 3% may not seem like it impacts the account balance much, the annual interest rate reveals that the loan amount is increasing by 42.58% each year, and that really adds up!

Unit 5 Lesson 13 568 Synthesis | Lesson Practice

Lesson Takeaway: Writing exponential functions in different, yet equivalent forms can help compare the differences in loans and aid in decision making in different situations.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting all of the equivalent expressions that would represent an account balance given an initial investment and monthly interest rate.

Today's Goals

1. **Goal:** Write exponential functions that represent situations involving compound interest.
2. **Language Goal:** Interpret equivalent exponential expressions in context using properties of exponents. **(Reading, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.13

A bank account with a balance of $500 earns 4% monthly interest.

Select *all* of the expressions that can be used to represent the balance after 5 years of no payments or withdrawals.

- ☑ A. $500 \cdot (1.04)^{60}$
- ☐ B. $500 \cdot (1.04)^{5}$
- ☐ C. $500 \cdot (1.46)^{5}$
- ☑ D. $500 \cdot (1.601)^{5}$
- ☑ E. $500 \cdot (1.04^{12})^{5}$

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of using the powers of powers law of exponents:

» **Respond to Student Thinking:**

- Use the ***Writing Exponential Functions Involving Compound Interest*** Mini-lesson.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

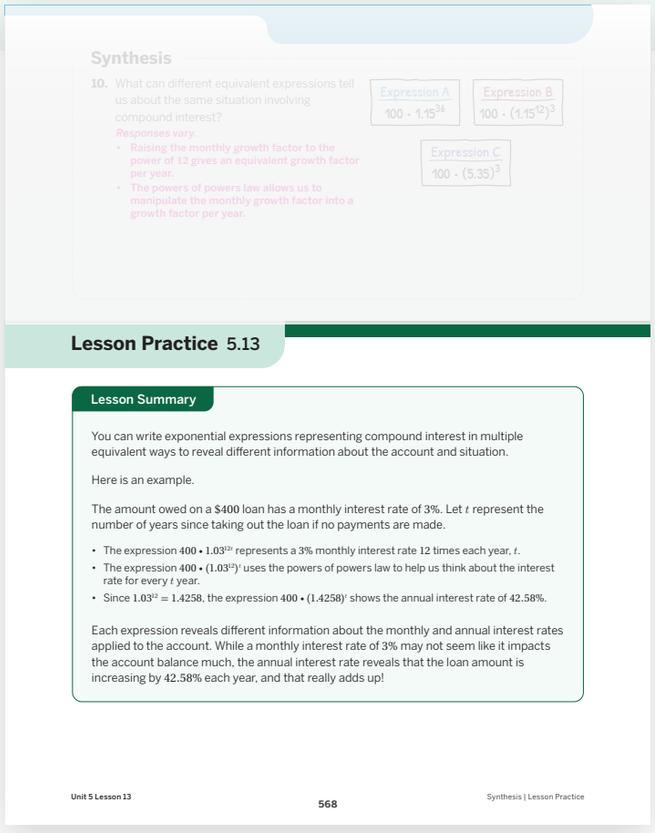
Professional Learning

Consider the important issues raised in this lesson. What aspects of payday loans are you more knowledgeable about? Less knowledgeable? In what ways were students able to make connections between math ideas and their community in this lesson? How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. What can different equivalent expressions tell us about the same situation involving compound interest?

Responses vary.

- Raising the monthly growth factor to the power of 12 gives an equivalent growth factor per year.
- The powers of powers law allows us to manipulate the monthly growth factor into a growth factor per year.

Expression A: $100 \cdot 1.15^{36}$

Expression B: $100 \cdot (1.15^{12})^3$

Expression C: $100 \cdot (5.35)^3$

Lesson Practice 5.13

Lesson Summary

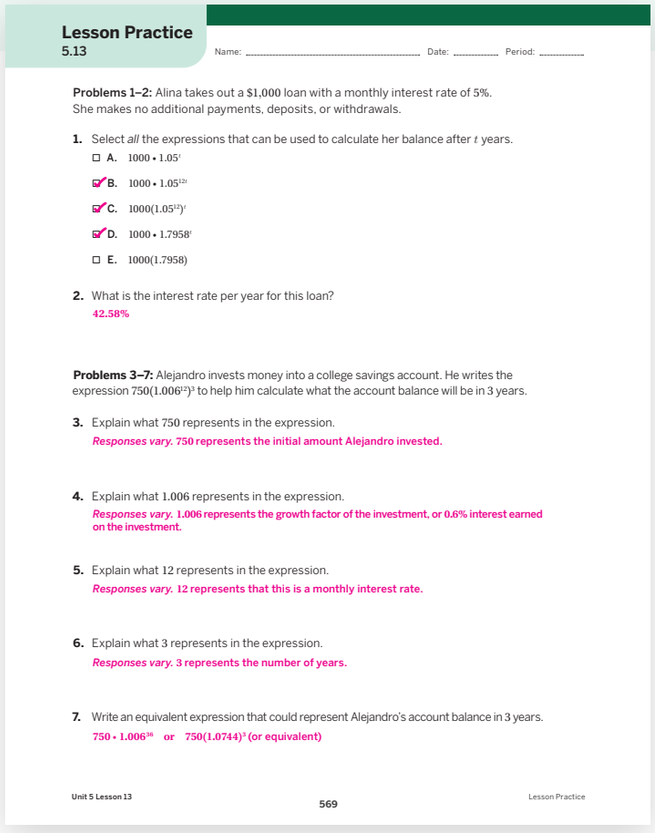
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Lesson Practice 5.13

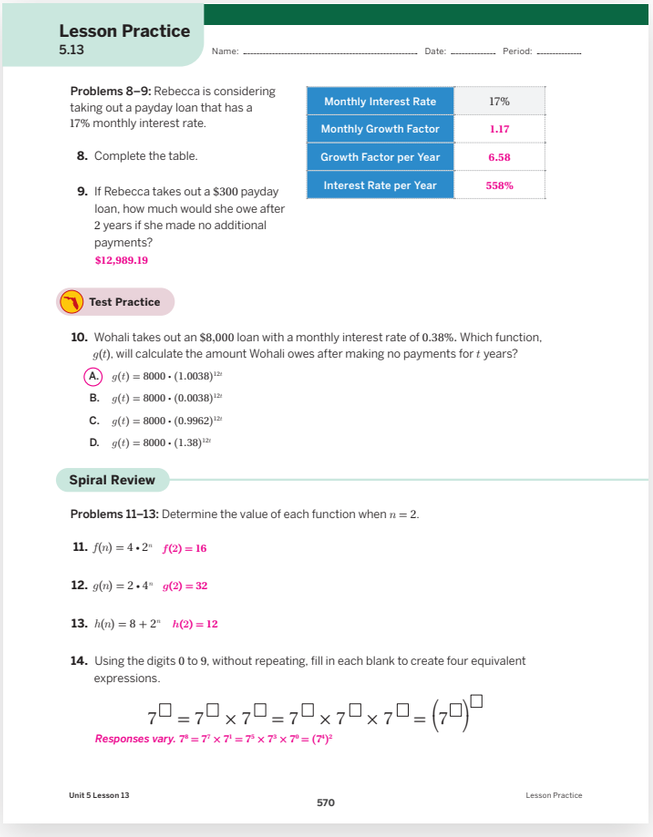
Name: _____ Date: _____ Period: _____

Problems 1–2: Alina takes out a $1,000 loan with a monthly interest rate of 5%. She makes no additional payments, deposits, or withdrawals.

1. Select *all* the expressions that can be used to calculate her balance after t years.
   - ☐ A. $1000 \cdot 1.05^t$
   - ☑ B. $1000 \cdot 1.05^{12t}$
   - ☑ C. $1000(1.05^{12})^t$
   - ☑ D. $1000 \cdot 1.7958^t$
   - ☐ E. $1000(1.7958)$
2. What is the interest rate per year for this loan?
   42.58%

Problems 3–7: Alejandro invests money into a college savings account. He writes the expression $750(1.006^{12})^3$ to help him calculate what the account balance will be in 3 years.

3. Explain what 750 represents in the expression.
   Responses vary. **750 represents the initial amount Alejandro invested.**
4. Explain what 1.006 represents in the expression.
   Responses vary. **1.006 represents the growth factor of the investment, or 0.6% interest earned on the investment.**
5. Explain what 12 represents in the expression.
   Responses vary. **12 represents that this is a monthly interest rate.**
6. Explain what 3 represents in the expression.
   Responses vary. **3 represents the number of years.**
7. Write an equivalent expression that could represent Alejandro's account balance in 3 years.
   $750 \cdot 1.006^{36}$ or $750(1.0744)^3$ (or equivalent)



Lesson Practice 5.13

Name: _____ Date: _____ Period: _____

Problems 8–9: Rebecca is considering taking out a payday loan that has a 17% monthly interest rate.

Monthly Interest Rate	17%
Monthly Growth Factor	**1.17**
Growth Factor per Year	**6.58**
Interest Rate per Year	**558%**

8. Complete the table.
9. If Rebecca takes out a $300 payday loan, how much would she owe after 2 years if she made no additional payments?
   $12,989.19

Test Practice

10. Wohali takes out an $8,000 loan with a monthly interest rate of 0.38%. Which function, $g(t)$, will calculate the amount Wohali owes after making no payments for t years?
    - (A.) $g(t) = 8000 \cdot (1.0038)^{12t}$
    - B. $g(t) = 8000 \cdot (0.0038)^{12t}$
    - C. $g(t) = 8000 \cdot (0.9962)^{12t}$
    - D. $g(t) = 8000 \cdot (1.38)^{12t}$

Spiral Review

Problems 11–13: Determine the value of each function when $n = 2$.

11. $f(n) = 4 \cdot 2^n$ $f(2) = 16$
12. $g(n) = 2 \cdot 4^n$ $g(2) = 32$
13. $h(n) = 8 + 2^n$ $h(2) = 12$
14. Using the digits 0 to 9, without repeating, fill in each blank to create four equivalent expressions.

$$7^{\square} = 7^{\square} \times 7^{\square} = 7^{\square} \times 7^{\square} \times 7^{\square} = \left(7^{\square}\right)^{\square}$$

Responses vary. $7^8 = 7^7 \times 7^1 = 7^5 \times 7^3 \times 7^0 = (7^4)^2$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.912.FL.3.2
	2	2	MA.912.FL.3.2
	3–6	1	MA.912.FL.3.2, MA.912.FL.3.4
	7	2	MA.912.FL.3.2, MA.912.NSO.1.2
	8	1	MA.912.FL.3.2
	9	2	MA.912.FL.3.2
Test Practice	**10**	1	MA.912.FL.3.2
Spiral Review			
	11–13	1	MA.912.F.1.2
	14	2	MA.912.NSO.1.2

Unit 5

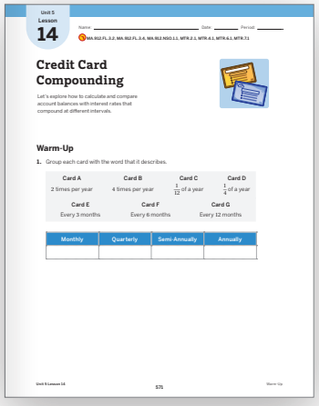
Lesson

14

Credit Card Compounding

Different Compounding Intervals

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.


Unit 5
Lesson
14

Credit Card Compounding

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.

Warm-Up

1. Group each card with the word that it describes.

Card A	Card B	Card C	Card D
2 times per year	4 times per year	$\frac{1}{12}$ of a year	$\frac{1}{4}$ of a year
Card E	**Card F**	**Card G**	
Every 3 months	Every 6 months	Every 12 months	

Monthly	Quarterly	Semi-Annually	Annually

Learning Outcomes

Today's Goals

1. **Goal:** Calculate account balances given an initial amount, annual interest rate, compounding interval, and period of time.
2. **Language Goal:** Interpret exponential expressions that represent annual interest rates with different compounding intervals (semi-annually, quarterly, monthly). **(Reading, Writing, Speaking, and Listening)**

Students explore credit cards with the same interest rate that compound at different *intervals* to generalize how more frequent compounding results in a higher account balance over time. **(MTR.7.1)**

Coherence

Prior Learning

In Lessons 7–9, students wrote exponential equations that represented percent increase or decrease. In Lesson 13–14, they compared and wrote functions to model simple and compound interest scenarios.

Future Learning

In Algebra 2, students will learn about compounding continuously, the constant e, and how to solve exponential equations algebraically.

Connections

- Students explore how credit card debt and its compounding interest demonstrates exponential growth and how it can significantly impact personal finances. (real world, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.FL.3.2

Solve real-world problems involving simple, compound and continuously compounded interest.

MA.912.FL.3.4

Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.1.1	**MA.912.AR.5.7**
MA.8.NSO.1.7	

Vocabulary

Review Vocabulary

interval

Additional Vocabulary

principal

Lesson at a Glance

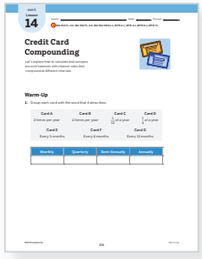
~ 45 min

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MTR.2.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

Students reason quantitatively to match language used to express different periods of time with equivalent phrases with quantities.



Activity 1

Pairs | 10 min

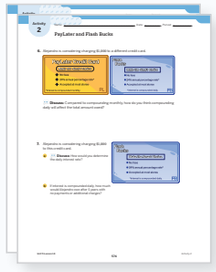
Students write and critique expressions for calculating the balance of an account with an interest rate that compounds monthly.
MLR3: Critique, Correct, Clarify



Activity 2

Pairs | 5 min

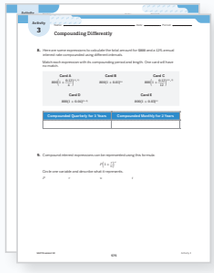
Students explore another credit card with the same rate compounded daily to generalize that compounding more often results in a higher account balance over time. **(MTR.7.1)**



Activity 3

Pairs | 15 min

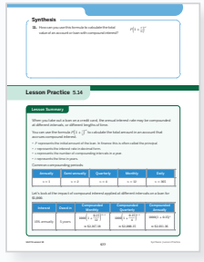
Students practice writing expressions to represent compound interest scenarios and make sense of the generalized formula $P\left(1+\frac{r}{n}\right)^{nt}$. MLR2: Collect and Display



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how the formula $P\left(1+\frac{r}{n}\right)^{nt}$ can be used to calculate account balances for different compound interest scenarios. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating an account balance with an interest rate that compounds quarterly.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students reason quantitatively to match language used to express different periods of time with equivalent phrases with quantities.

Short on time? Consider omitting the whole-class discussion if the class shows a strong understanding of the time interval terms.

1 Launch

To access prior knowledge, consider asking, "How many months are in a year? Half a year? A quarter of a year?"

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.

A **Accessibility: Conceptual Processing** To support students in thinking abstractly, consider asking a more concrete question like, "If something occurs every 6 months, how many times does it occur within one year?"

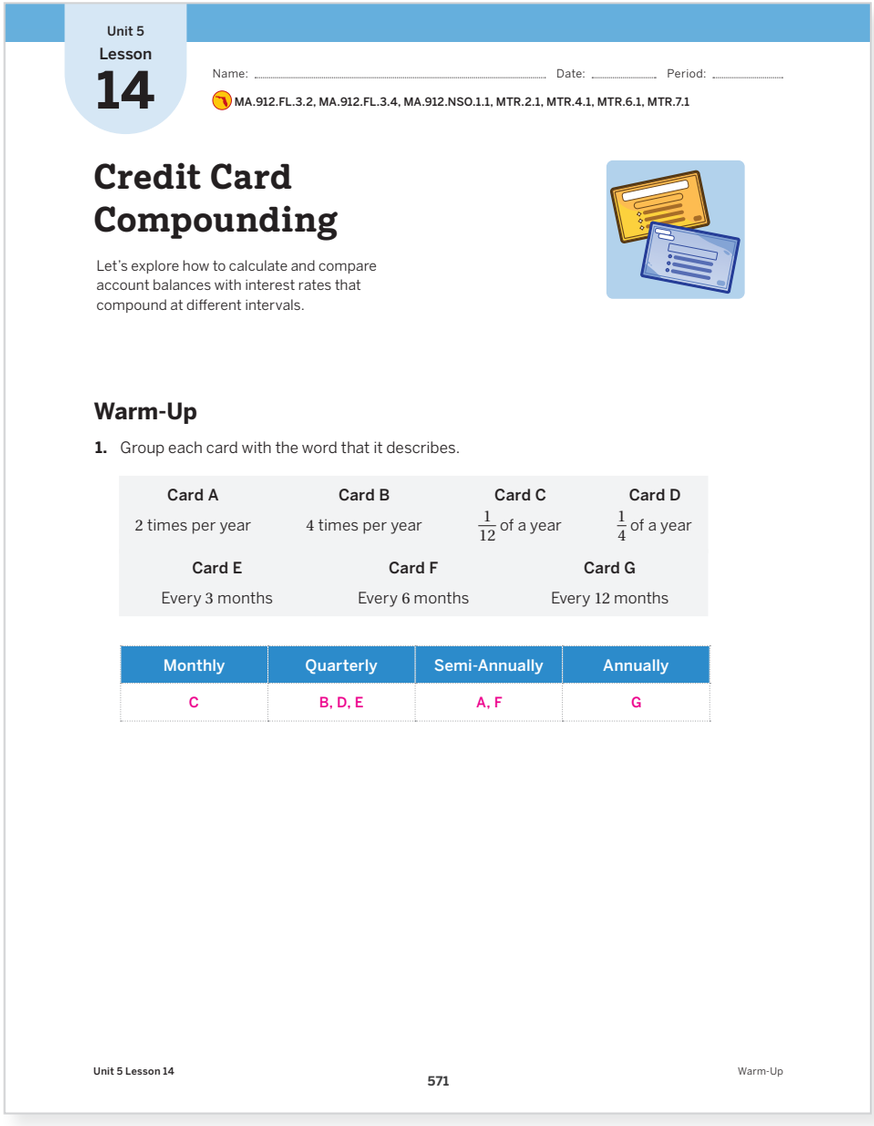
ELL **English Language Learners** Invite students to research how to say the new terms (e.g., *semi-annually, quarterly, monthly, daily*) in other languages and to share where they have heard the prefix *semi-* used in other terms (e.g. semicircle, semisweet, etc.). Create an anchor chart with the new term in multiple languages, along with examples. **(Reading)**

2 Connect

Invite students to share the strategies they used to match the cards.

Consider asking:

- "Which word would mean the event occurs the most times within a year? The least?"
- "What are other ways to describe how often an event occurs?"



Unit 5
Lesson
14

Name: _____ Date: _____ Period: _____

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

Credit Card Compounding

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.

Warm-Up

1. Group each card with the word that it describes.

Card A	Card B	Card C	Card D
2 times per year	4 times per year	$\frac{1}{12}$ of a year	$\frac{1}{4}$ of a year

Card E	Card F	Card G
Every 3 months	Every 6 months	Every 12 months

Monthly	Quarterly	Semi-Annually	Annually
C	B, D, E	A, F	G

Unit 5 Lesson 14 571 Warm-Up

Pairs | 10 min

Activity 1 PayLater

Purpose: Students write and critique expressions for calculating the balance of an account with an interest rate that compounds monthly.

1 Launch

To access prior knowledge, consider asking:

- "How do credit cards work?"
- "Why might someone use a credit card?"

2 Monitor

Encourage students to read others' responses and use them to make their descriptions stronger and clearer.

To access connections, consider asking pairs:

- "What does the 1 tell us about the account balance?"
- "Why is the 5 written in the exponent?"
- "Why might Alejandro consider charging to this card? Why might he not?"

D **Differentiation: (Stretch)** Invite students to write an equivalent expression and explain what it means for this situation.

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means. Clarify any words they don't know (e.g., *fine print*). **(Reading)**

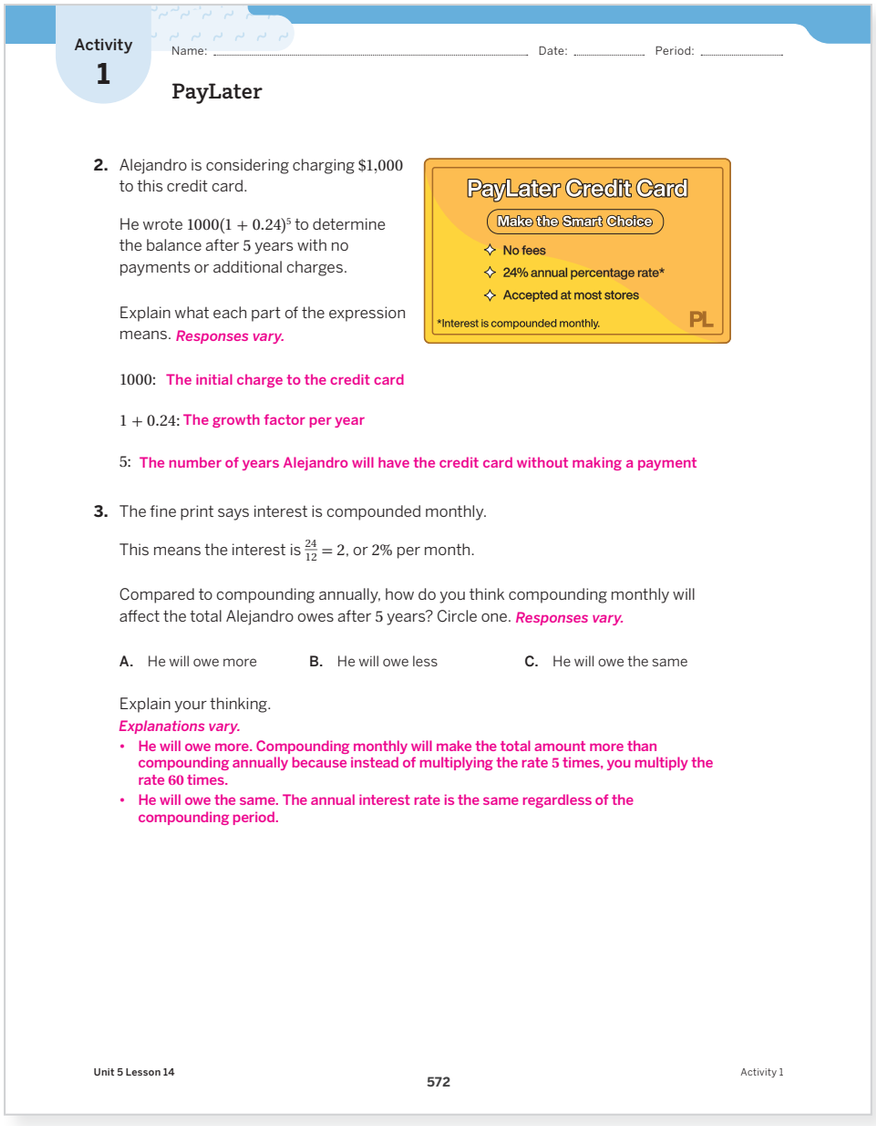
Listen for a variety of student responses and explanations.

Pause briefly and discuss the difference between simple and compound interest. Consider asking, "What does it mean to compound?"

Display the distribution of student responses, emphasizing the range of student responses. It's okay — even desirable — to lack consensus at this stage. The activity will build toward consensus in Activity 2.

Invite students to share their reasoning for their predictions. **(MTR.4.1)**

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

PayLater

2. Alejandro is considering charging $1,000 to this credit card.

He wrote $1000(1 + 0.24)^5$ to determine the balance after 5 years with no payments or additional charges.

Explain what each part of the expression means. ***Responses vary.***

PayLater Credit Card
Make the Smart Choice
- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded monthly.
PL

1000: **The initial charge to the credit card**

$1 + 0.24$: **The growth factor per year**

5: **The number of years Alejandro will have the credit card without making a payment**

3. The fine print says interest is compounded monthly.

This means the interest is $\frac{24}{12} = 2$, or 2% per month.

Compared to compounding annually, how do you think compounding monthly will affect the total Alejandro owes after 5 years? Circle one. ***Responses vary.***

A. He will owe more **B.** He will owe less **C.** He will owe the same

Explain your thinking.

Explanations vary.

- **He will owe more. Compounding monthly will make the total amount more than compounding annually because instead of multiplying the rate 5 times, you multiply the rate 60 times.**
- **He will owe the same. The annual interest rate is the same regardless of the compounding period.**

Unit 5 Lesson 14 572 Activity 1

Pairs | 10 min

Activity 1 PayLater (continued)

Purpose: Students write and critique expressions for calculating the balance of an account with an interest rate that compounds monthly.

2 Monitor

Encourage students to enter their responses as expressions and the computer will calculate the value for them.

Invite students using print to use a calculator to support them.

Listen for and celebrate different ways that students write expressions to solve this problem. Some students may write:

- $1000\,(1.02)^{60}$
- $1000\,(1+.02)^{60}$
- $1000\left(1+\frac{0.24}{12}\right)^{60}$
- $1000\left(1+\frac{0.24}{12}\right)^{(12\cdot 5)}$

Note: Consider waiting until Problem 9 to discuss how the expressions connect to the formula $P\left(1+\frac{r}{n}\right)^{nt}$.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support interpreting Alejando's expression.	**Support** Consider asking, "Where in the expression do you see the annual interest rate? That the account is compounding monthly?"
Think that Alejandro did not make an error.	**Support** Consider asking, "If the interest is compounding monthly for 5 years, how many times will it be applied?"
Explain Alejandro's error.	**Strengthen** Listen for their responses to share during the Connect and ask, "What question could you ask Alejandro to help him recognize his mistake?"

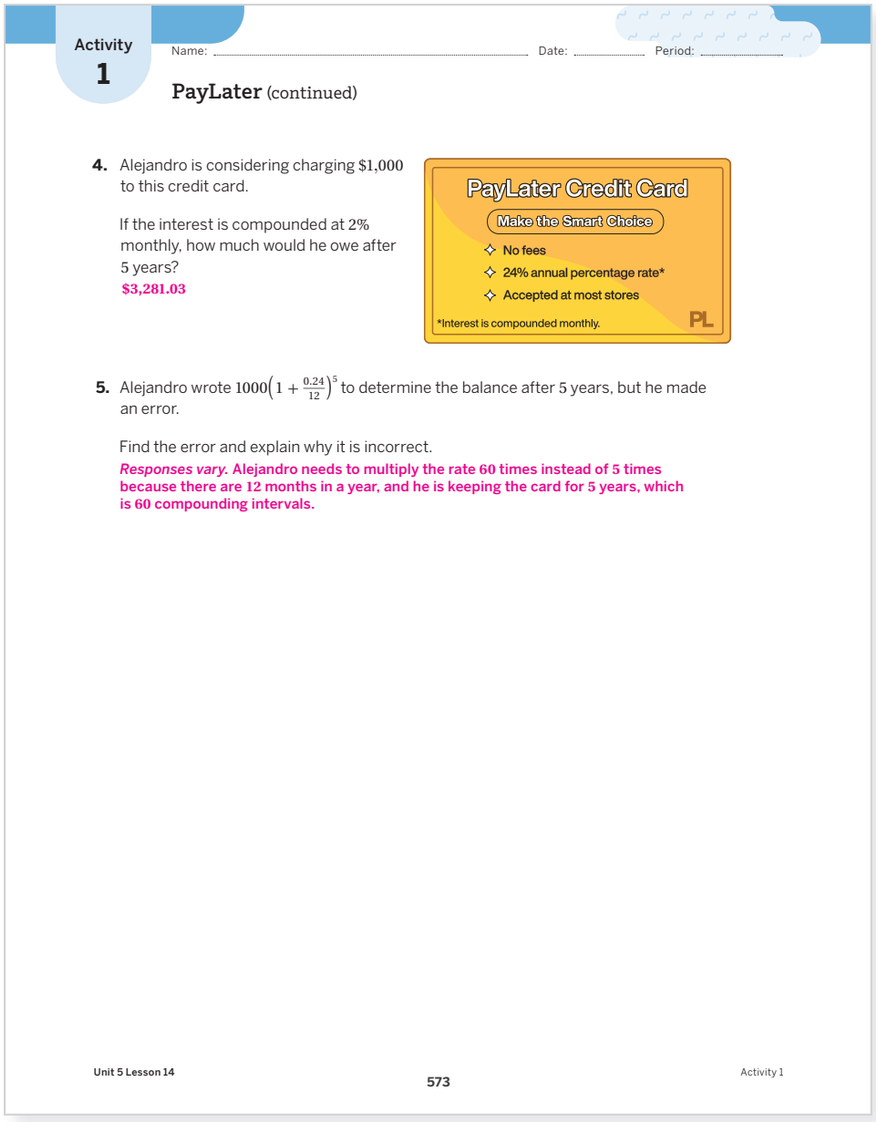
3 Connect

Consider asking:

- "How do the values of compounding annually versus monthly compare?"
- "If the yearly interest rates are equivalent, why does compounding differently matter?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths and errors in Alejandro's work. Share the collected errors and explanations. **(MTR.6.1)**

Consider asking, "Why do you think we need to divide the rate by 12 *and* multiply the number of years by 12?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

PayLater (continued)

4. Alejandro is considering charging $1,000 to this credit card.

If the interest is compounded at 2% monthly, how much would he owe after 5 years?

$3,281.03

PayLater Credit Card
Make the Smart Choice
- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded monthly.
PL

5. Alejandro wrote $1000\left(1+\frac{0.24}{12}\right)^5$ to determine the balance after 5 years, but he made an error.

Find the error and explain why it is incorrect.

Responses vary. Alejandro needs to multiply the rate 60 times instead of 5 times because there are 12 months in a year, and he is keeping the card for 5 years, which is 60 compounding intervals.

Unit 5 Lesson 14 573 Activity 1

Pairs | 5 min

Activity 2 PayLater and FlashBucks

Purpose: Students explore another credit card with the same rate compounded daily to generalize that compounding more often results in a higher account balance over time. **(MTR.7.1)**

1 Launch

Consider asking, "How are the two credit cards alike? How are they different?"

Note: Consider sharing that compounding daily is based on a standard 365-day year and usually excludes leap years.

2 Monitor

Consider asking pairs of students:

- "How many times will the Flashbucks card earn interest each year?"
- "Does the difference in compounding matter? Why or why not?" **(MTR.4.1)**

Look for different ways that students write expressions.

D **Differentiation: (Stretch)** Consider asking, "How would the account balance be different if it compounded by the minute? By the second?"

3 Connect

A **Accessibility: Conceptual Processing** To support students in making generalizations, create a display that visually compares the 5 year account balance for each compounding interval.

Example:

Yearly	Monthly	Daily
$2,932	$3,281	$3,319

Consider asking, "What patterns do you see? Do you think this pattern will always be true?"

To reveal the Key Takeaway, consider asking:

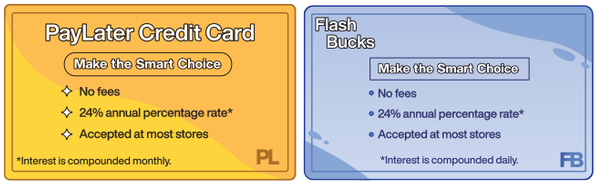
- "Why does compounding more frequently result in a greater total amount?"
- "Which card would you recommend to Alejandro? Why?" **(MTR.4.1)**
- "Which types of accounts would you want to compound more frequently? Compound less frequently?" **(MTR.6.1)**

Activity 2

Name: Date: Period:

PayLater and Flash Bucks

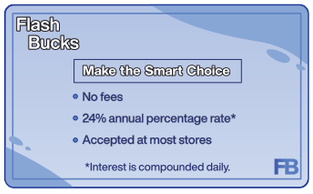
6. Alejandro is considering charging $1,000 to a different credit card.



Discuss: Compared to compounding monthly, how do you think compounding daily will affect the total amount owed?

Responses vary. One card is compounding monthly, or 12 times a year. The other card is compounding daily, or 365 times a year. They both have no additional fees and a 24% annual interest rate. I think compounding daily means applying the rate 365 • 5 or 1825 times, so I think it will make the total amount owed increase.

7. Alejandro is considering charging $1,000 to this credit card.



a. **Discuss:** How would you determine the daily interest rate?

Responses vary. I would divide the 24% annual rate by the number of days in a year. So $0.24 \div 365$.

b. If interest is compounded daily, how much would Alejandro owe after 5 years with no payments or additional charges?

$1000\left(1+\frac{0.24}{365}\right)^{(5\cdot 365)} \approx \$3{,}318.81$

Unit 5 Lesson 14 574 Activity 2

Key Takeaway: Accounts with interest rates that compound more often lead to higher account balances over time. When considering whether to charge an amount to an account that earns compound interest, the amount of time and the number of compounding *intervals* matters! Researching the interest rates and how they compound can save someone a lot of money over a period of time.

Pairs | 15 min

Activity 3 Compounding Differently

Purpose: Students practice writing expressions to represent compound interest scenarios and make sense of the generalized formula $P\left(1+\frac{r}{n}\right)^{nt}$.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Invite students to read the situation aloud and paraphrase it in their own words.

Encourage students to share their reasoning with a partner and work together to reach an agreement for sorting.

2 Monitor

Look for students who are using a variety of strategies and justifying their thinking. **(MTR.2.1)**

To access connections, consider asking:
- "Which cards are equivalent? How do you know?"
- "In which expressions can you see the rate? The time, in years?"

Note: If time allows, invite pairs to compare their groups and make revisions based on their conversation.

A **Accessibility: Executive Functioning** Invite students to describe how the expressions are alike and different before sorting them.

D **Differentiation: (Stretch)** Encourage students to determine the compounding interval for the card with no match.

Listen for and share student descriptions of each variable.

Pause to support students in making sense of the formula.

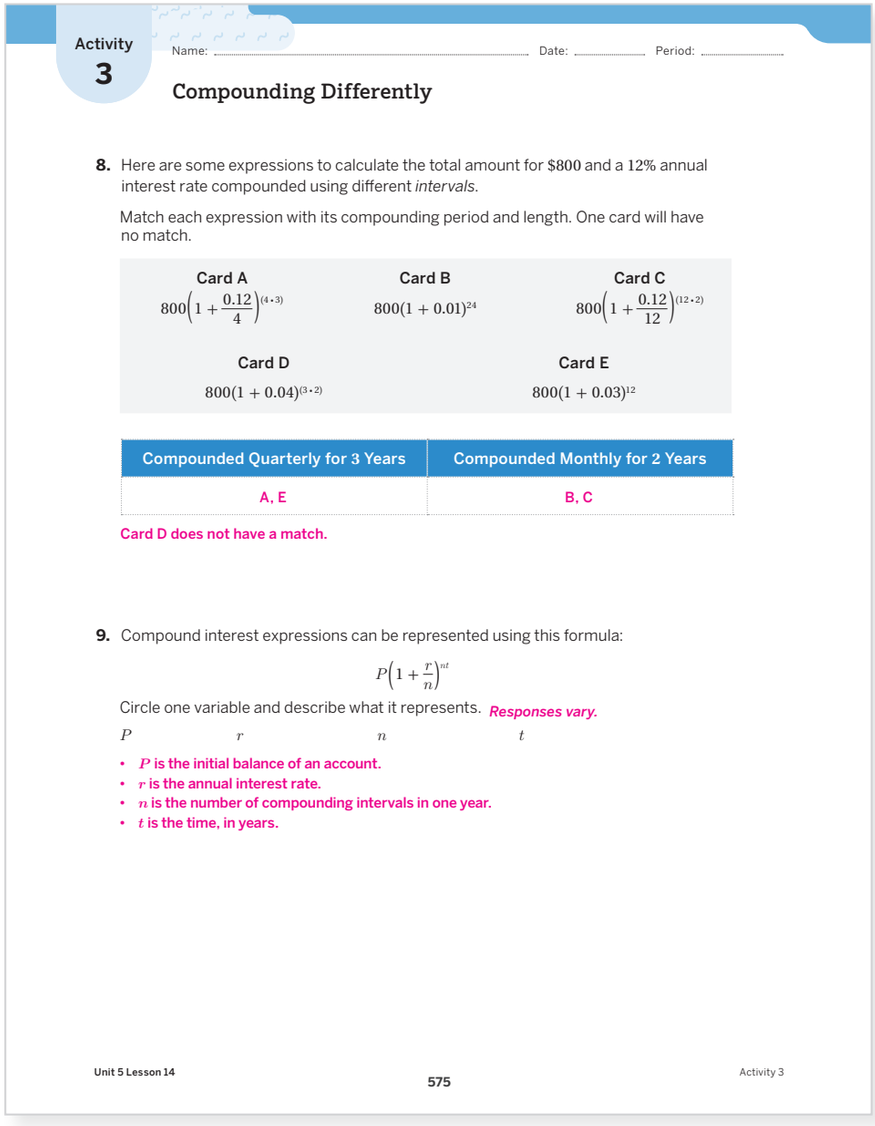
MLR **MLR2: Collect and Display** Use this routine to collect students' language for making sense of the formula $P\left(1+\frac{r}{n}\right)^{nt}$. Display for students to reference and add to for the rest of the unit.

To access connections, consider sharing that in finance the initial account balance is often called the *principal*, which is why there is a P in the formula.

Consider asking, "How is this formula helpful? How may it not be helpful?"

Consider sharing that students solved problems 4, 7, and 8 without the formula, so they only have to use it if they find it helpful.

Activity 3 continued >



Activity 3

Name: Date: Period:

Compounding Differently

8. Here are some expressions to calculate the total amount for \$800 and a 12% annual interest rate compounded using different *intervals*.

Match each expression with its compounding period and length. One card will have no match.

Card A	Card B	Card C
$800\left(1+\frac{0.12}{4}\right)^{(4\cdot 3)}$	$800(1+0.01)^{24}$	$800\left(1+\frac{0.12}{12}\right)^{(12\cdot 2)}$
Card D		**Card E**
$800(1+0.04)^{(3\cdot 2)}$		$800(1+0.03)^{12}$

Compounded Quarterly for 3 Years	Compounded Monthly for 2 Years
A, E	B, C

Card D does not have a match.

9. Compound interest expressions can be represented using this formula:

$$P\left(1+\frac{r}{n}\right)^{nt}$$

Circle one variable and describe what it represents. *Responses vary.*

P r n t

- P is the initial balance of an account.
- r is the annual interest rate.
- n is the number of compounding intervals in one year.
- t is the time, in years.

Unit 5 Lesson 14 575 Activity 3

Pairs | 15 min

Activity 3 Compounding Differently (continued)

Purpose: Students practice writing expressions to represent compound interest scenarios and make sense of the generalized formula $P\left(1+\frac{r}{n}\right)^{nt}$.

Short on time? Consider inviting students to complete fewer challenges.

2 Monitor

Invite students to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly.

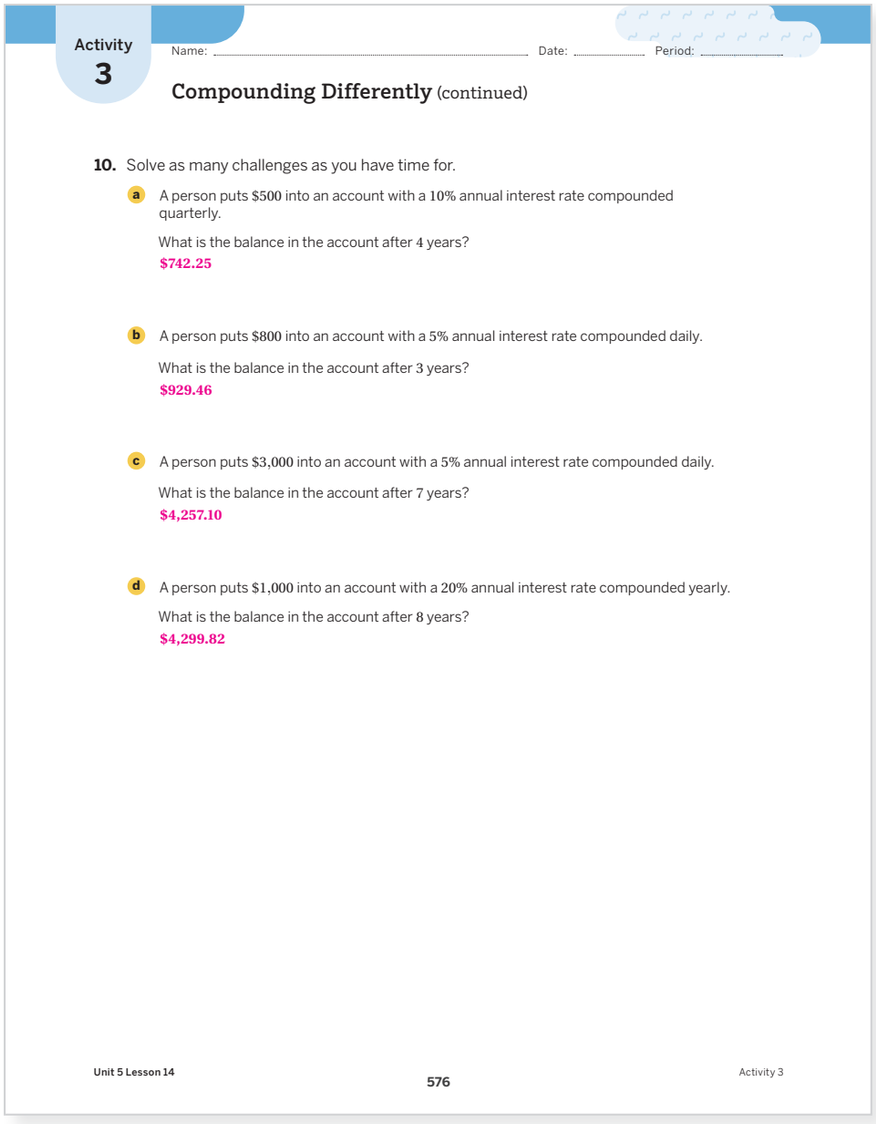
Students using print will need a partner for this activity. They will solve and compare their solutions to problems instead of receiving responsive feedback.

ELL **English Language Learners** To make sense of the situation, consider modeling or inviting a student to model strategies for annotating the text. **(Reading)**

A **Accessibility: Conceptual Processing** To support students in applying concepts to new situations, invite students to share how each problem is different from the previous one.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 3

Name: _____ Date: _____ Period: _____

Compounding Differently (continued)

10. Solve as many challenges as you have time for.

a. A person puts $500 into an account with a 10% annual interest rate compounded quarterly.

What is the balance in the account after 4 years?

$742.25

b. A person puts $800 into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 3 years?

$929.46

c. A person puts $3,000 into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 7 years?

$4,257.10

d. A person puts $1,000 into an account with a 20% annual interest rate compounded yearly.

What is the balance in the account after 8 years?

$4,299.82

Unit 5 Lesson 14 576 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how the formula $P\left(1+\frac{r}{n}\right)^{nt}$ can be used to calculate account balances for different compound interest scenarios.

Invite students to respond independently, and then share their thinking with a partner.

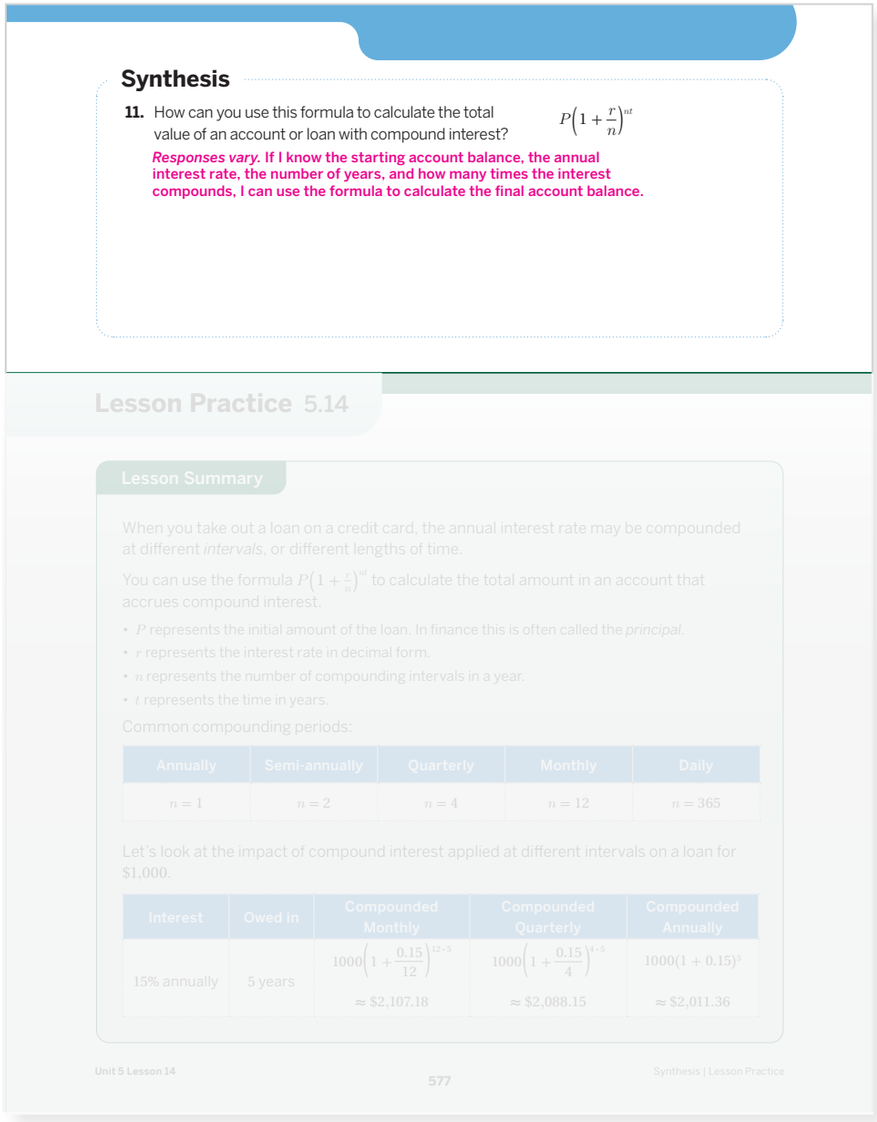
Listen for and share a variety of ideas, such as:

- The formula helps me organize the information for a compound interest situation.
- Calculating the balance for any compounding interval.
- The n in the formula shows that the interest rate gets divided by the number of intervals and the amount of times the interest occurs gets multiplied.
- The 1 shows that the account balance is growing.

MLR **MLR2: Collect and Display** Use this routine to revisit student ideas about the formula. Add any new student ideas to the display. Display for students to reference and add to for the rest of the unit.

To access connections, consider asking:

- "Credit cards are powerful tools. When might you want to use a credit card? When might you not want to use a credit card?" People might want to use a credit card to build credit to qualify for different types of loans. Some credit cards have benefits such as travel points, cash back, or discounts at stores. You may not want to use a credit card that has a high interest rate, a frequent compound interval, or benefits that don't match your lifestyle.
- "Would compound interest work the same way in savings accounts as it does for a credit card? Why or why not?"



Synthesis

11. How can you use this formula to calculate the total value of an account or loan with compound interest? $P\left(1+\frac{r}{n}\right)^{nt}$

Responses vary. If I know the starting account balance, the annual interest rate, the number of years, and how many times the interest compounds, I can use the formula to calculate the final account balance.

Lesson Practice 5.14

Lesson Summary

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula $P\left(1+\frac{r}{n}\right)^{nt}$ to calculate the total amount in an account that accrues compound interest.

- P represents the initial amount of the loan. In finance this is often called the *principal*.
- r represents the interest rate in decimal form.
- n represents the number of compounding intervals in a year.
- t represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n=1$	$n=2$	$n=4$	$n=12$	$n=365$

Let's look at the impact of compound interest applied at different intervals on a loan for $1,000.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1+\frac{0.15}{12}\right)^{12\cdot 5}$ $\approx \$2{,}107.18$	$1000\left(1+\frac{0.15}{4}\right)^{4\cdot 5}$ $\approx \$2{,}088.15$	$1000(1+0.15)^5$ $\approx \$2{,}011.36$

Unit 5 Lesson 14 577 Synthesis | Lesson Practice

Lesson Takeaway: Account balances can be calculated using the formula $P\left(1+\frac{r}{n}\right)^{nt}$ where P: the initial balance of an account. In finance this is often called the principal, r: the annual interest rate in decimal form, n: the number of compounding *intervals* in a year, and t: the time, in years. For accounts with the same interest rate, the more frequently the interest compounds, the higher the account balance will be. Accounts that earn compound interest, like credit cards, are powerful tools that can be used in helpful and harmful ways. Understanding how interest and compounding intervals impact the account balance over time can support someone in making sense of how to make compound interest helpful for them.

Independent | 5 min

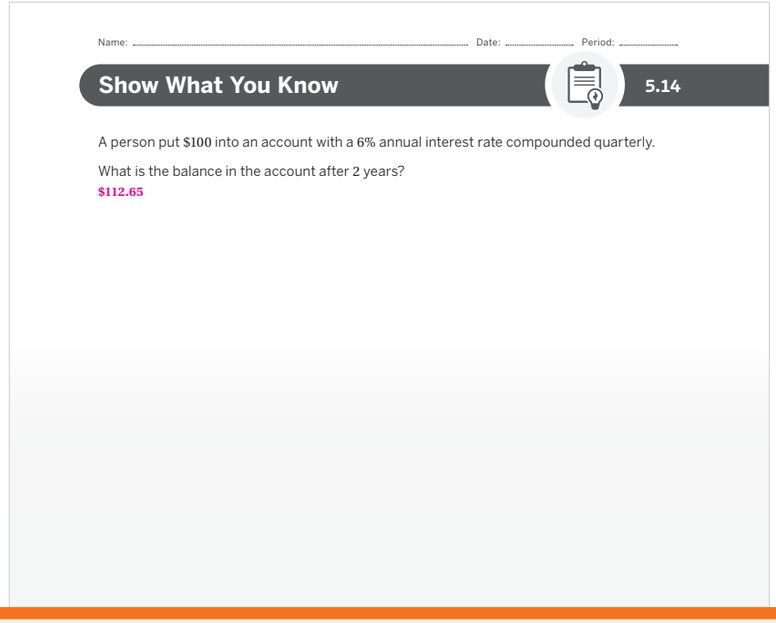
Show What You Know

Purpose: Students demonstrate their understanding by calculating an account balance with an interest rate that compounds quarterly.

Today's Goals

1. **Goal:** Calculate account balances given an initial amount, annual interest rate, compounding interval, and period of time.
2. **Language Goal:** Interpret exponential expressions that represent annual interest rates with different compounding intervals (semi-annually, quarterly, monthly). **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.14

A person put $100 into an account with a 6% annual interest rate compounded quarterly.

What is the balance in the account after 2 years?

$112.65

D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate account balances with compound interest:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Inviting them to research a real credit card plan and writing a function to model how an account balance with an initial amount of $1000 would grow over time.

Professional Learning

Take a moment to reflect on this lesson and how the math of this lesson can be applied in your and your students' lives. In what ways were students able to connect the math they learned about to their world? How might you continue to support students in making connections between compound interest and their life? How has students' understanding of compound interest developed over the course of Lessons 13–15?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you use this formula to calculate the total value of an account or loan with compound interest? $P\left(1+\frac{r}{n}\right)^{nt}$

Responses vary. If I know the starting account balance, the annual interest rate, the number of years, and how many times the interest compounds, I can use the formula to calculate the final account balance.

Lesson Practice 5.14

Lesson Summary

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula $P\left(1+\frac{r}{n}\right)^{nt}$ to calculate the total amount in an account that accrues compound interest.

- P represents the initial amount of the loan. In finance this is often called the *principal*.
- r represents the interest rate in decimal form.
- n represents the number of compounding intervals in a year.
- t represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n = 1$	$n = 2$	$n = 4$	$n = 12$	$n = 365$

Let's look at the impact of compound interest applied at different intervals on a loan for $1,000.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1+\frac{0.15}{12}\right)^{12\cdot 5}$ $\approx \$2{,}107.18$	$1000\left(1+\frac{0.15}{4}\right)^{4\cdot 5}$ $\approx \$2{,}088.15$	$1000(1+0.15)^5$ $\approx \$2{,}011.36$

Lesson Practice 5.14

Name: ..... Date: ..... Period: .....

1. Tyrone puts $2,500 into a savings account with a 1.2% annual interest rate, compounded semi-annually. He makes no additional payments, deposits, or withdrawals.

Select *all* the expressions that can be used to calculate his balance after 3 years.

- ☑ A. $2500\left(1+\frac{0.012}{2}\right)^{3\cdot 2}$
- ☐ B. $2500\left(1+\frac{0.012}{6}\right)^{6}$
- ☐ C. $2500(1+0.012)^3$
- ☑ D. $2500(1+0.006)^6$
- ☐ E. $2500\left(1+\frac{0.012}{3}\right)^{3\cdot 2}$

Problems 2–4: Maneli wants to take out a $5,000 loan to help pay for a new washing machine and dryer. The bank offers her the loan with an 18% annual interest rate, compounded quarterly.

Maneli wrote this expression to calculate the balance of the loan in 2 years, but she made an error.

$$5000\left(1+\frac{0.18}{2}\right)^{(4\cdot 2)}$$

2. Find the error and explain why it is incorrect.

Explanations vary. Since the interest is compounded quarterly, the 0.18 should be divided by 4.

3. Write a correct expression to represent Maneli's balance after 2 years.

$5000\left(1+\frac{0.18}{4}\right)^{4\cdot 2}$ (or equivalent)

4. What will her balance be in 2 years?

$7,110.50

Problems 5–6: A payday loan company offers a $1,000 loan with a 25% annual interest rate.

5. If no other charges or payments are made, what will the balance of the loan be after 1 year at each compounding period?

Compounding Period	Balance ($)
Annually	1,250
Monthly	1,280.73
Daily	1,283.92

6. Describe how changing the compounding period affects the balance of the loan.

Responses vary. The more often a loan is compounded, the higher the balance will be.

Lesson Practice 5.14

Name: ..... Date: ..... Period: .....

Problems 7–8: Xavier has $5,000 to invest and has to choose between three investment options.

- Option A: 2.25% interest applied each quarter
- Option B: 3% interest applied every 4 months
- Option C: 4.5% interest applied twice each year

7. Write an expression for each account to represent Xavier's balance after 5 years.

$a(x) = 5000 \cdot \left(1+\frac{0.0225}{4}\right)^{4\cdot 5}$

$b(x) = 5000 \cdot \left(1+\frac{0.03}{3}\right)^{3\cdot 5}$

$c(x) = 5000 \cdot \left(1+\frac{0.045}{2}\right)^{2\cdot 5}$

Test Practice

8. How much would Xavier have with Option C after 5 years?

- A. $5,450
- (B.) $6,246.02
- C. $26,137.66
- D. $38,047.92

Spiral Review

Problems 9–10: Irene needs to make at least 25 dinners for a party, including chicken dinners and vegetarian dinners. She has $250 to spend. Chicken dinners cost $8.75 each and vegetarian dinners cost $5.50 each.

- c represents the number of chicken dinners.
- v represents the number of vegetarian dinners.

9. Write a system of inequalities that represents Irene's constraints.

$c + v \geq 25$

$8.75c + 5.50v \leq 250$

10. Can Irene make 5 chicken dinners and 20 vegetarian dinners? Explain your thinking.

Yes. *Explanations vary.*

- 5 chicken dinners and 20 vegetarian dinners cost $153.75, which is less than $250.
- When $c = 5$ and $v = 20$, the inequality $8.75(5) + 5.50(20) \leq 250$ is true.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
Fluency	1	2	MA.912.FL.3.2
	2–4	2	MA.912.FL.3.2
	5–6	2	MA.912.FL.3.2
	7	2	MA.912.FL.3.2
Test Practice	8	2	MA.912.FL.3.2
Spiral Review			
	9–10	2	MA.912.AR.9.1

Unit 5

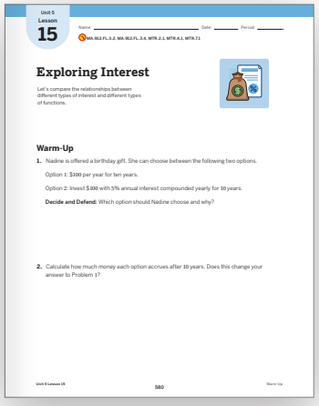
Lesson

15

Exploring Interest

Compare Different Types of Interest

Let's compare the relationships between different types of interest and different types of functions.


Unit 5
Lesson
15
Name: _____ Date: _____ Period: _____
MA.912.FL.3.2, MA.912.FL.3.4, MTR.2.1, MTR.4.1, MTR.7.1
Exploring Interest
Let's compare the relationships between different types of interest and different types of functions.
Warm-Up
1. Nadine is offered a birthday gift. She can choose between the following two options.
Option 1: $100 per year for ten years.
Option 2: Invest $100 with 5% annual interest compounded yearly for 10 years.
Decide and Defend: Which option should Nadine choose and why?
2. Calculate how much money each option accrues after 10 years. Does this change your answer to Problem 1?
Unit 5 Lesson 15
580
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that simple interest is connected to linear functions.
2. **Goal:** Recognize that compound interest is connected to exponential functions.
3. **Language Goal:** Interpret formulas to match the correct functions and explain why. **(Speaking, and Listening)**

Students explore linear and exponential growth in the forms of interest using real-world examples. They look at and explain different scenarios and identify linear and exponential patterns. **(MTR.4.1, MTR.7.1)**

Coherence

Prior Learning

In Lessons 13–15, students explored simple interest and compound interest.

Future Learning

In future courses, students will calculate the finance charges and total amount due on a bill using various forms of credit. They will also compare the advantages and disadvantages of different types of student loans by manipulating a variety of variables.

Connections

- Students compare the relationships between different types of interest and different types of functions through simple interest savings accounts. (real world, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.FL.3.4
Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3.1	**MA.912.FL.3.1, MA.912.FL.3.3**

Vocabulary

Review Vocabulary

linear
exponential
simple interest
compound interest
compounded continuously

Lesson at a Glance

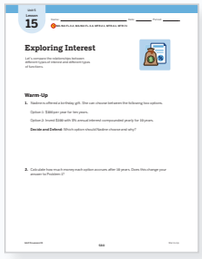
~ 45 min

MA.912.FL.3.4

Warm-Up

Whole Class | 5 min

Students use the **Decide and Defend** routine to compare simple and compound interests. **Routine(s): Decide and Defend, MLR3: Critique, Correct, Clarify.** **(MTR.4.1)**



Activity 1

Pairs | 15 min

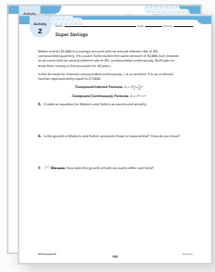
Students use the **Think-Pair-Share** routine to support discourse in relating simple interest to linear growth. **Routine(s): Think-Pair-Share** **(MTR.2.1, MTR.4.1, MTR.7.1)**



Activity 2

Pairs | 15 min

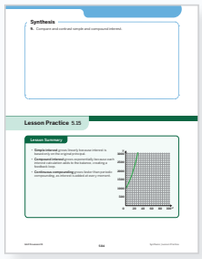
Students use the Compare and Connect routine to support understanding in relating compounding interest to exponential growth. **Routine(s): Compare and Connect MLR7** **(MTR.4.1, MTR.7.1)**



Synthesis

Whole Class | 5 min

Students synthesize the differences between simple interest and compound interest.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating an account balance with an interest rate that compounds semiannually.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students use the Decide and Defend routine to compare simple and compound interests. **(MTR.4.1)**

Short on time? Consider only polling a few students and doing the calculations for Problem 2 together as a class.

1 Launch

Display the distribution of responses by polling the class.

Accessibility: Conceptual Processing Invite students to answer a more concrete question first to support abstract thinking, like: What do you like to do to celebrate your birthday?

Use the Decide and Defend routine to support students in making arguments and critiquing others' reasoning. **(MTR.4.1)**

2 Connect

Invite students who made different choices to use reasoning to convince their classmates.

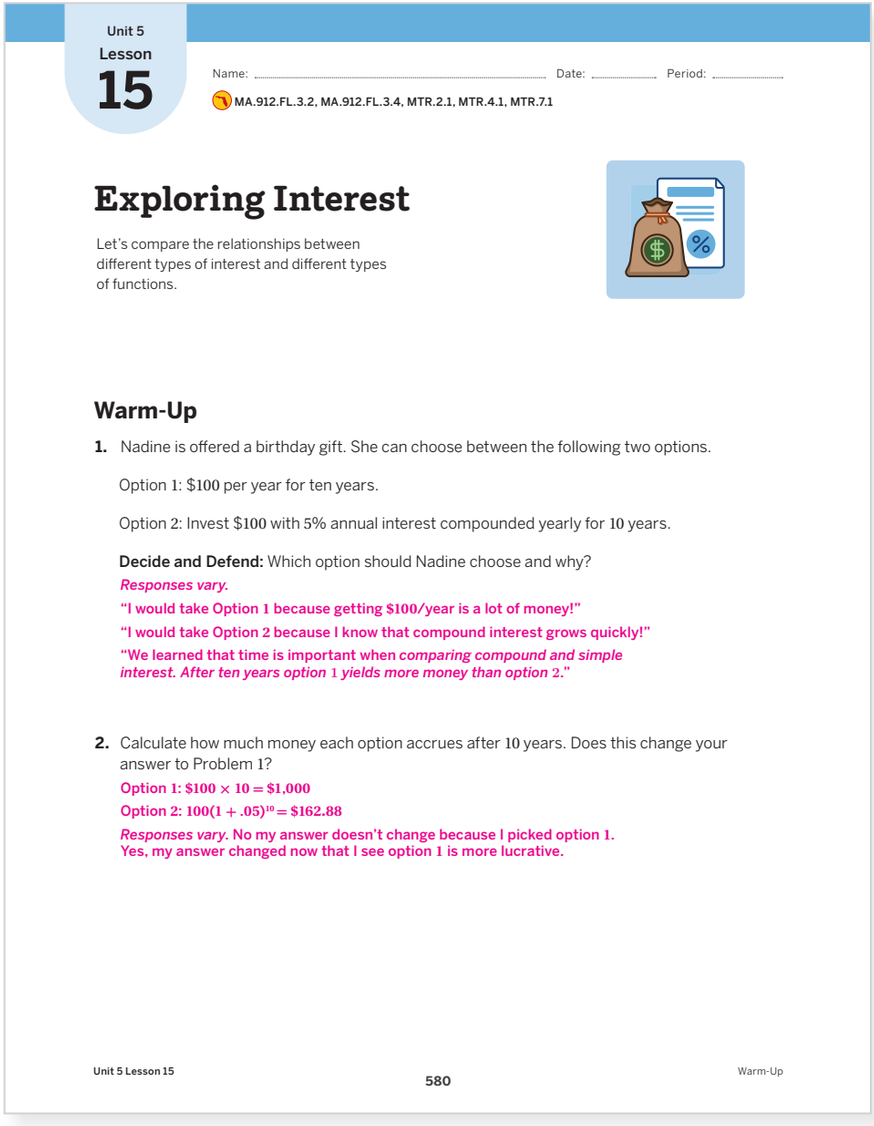
English Language Learners Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Note: If everyone agrees, consider making a class argument using different students' reasoning.

Note: It's okay — even desirable — to lack consensus at this stage. The activity will build toward consensus when students learn the key differences between simple interest and compound interest and the importance of time.

Consider asking:

- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"
- "Why do you feel Option 1 is better than Option 2?"
- "Why do you feel Option 2 is better than Option 1?"



Unit 5
Lesson
15

Name: Date: Period:

MA.912.FL.3.2, MA.912.FL.3.4, MTR.2.1, MTR.4.1, MTR.7.1

Exploring Interest

Let's compare the relationships between different types of interest and different types of functions.

Warm-Up

1. Nadine is offered a birthday gift. She can choose between the following two options.

   Option 1: $100 per year for ten years.

   Option 2: Invest $100 with 5% annual interest compounded yearly for 10 years.

   Decide and Defend: Which option should Nadine choose and why?

   Responses vary.

   "I would take Option 1 because getting $100/year is a lot of money!"

   "I would take Option 2 because I know that compound interest grows quickly!"

   "We learned that time is important when *comparing compound and simple interest. After ten years option 1 yields more money than option 2.*"

2. Calculate how much money each option accrues after 10 years. Does this change your answer to Problem 1?

   Option 1: $\$100 \times 10 = \$1{,}000$

   Option 2: $100(1 + .05)^{10} = \$162.88$

   Responses vary. No my answer doesn't change because I picked option 1.
   Yes, my answer changed now that I see option 1 is more lucrative.

Unit 5 Lesson 15 580 Warm-Up

Pairs | 15 min

Activity 1 Laptop Savings

Purpose: Students use the Think-Pair-Share routine to support discourse in relating simple interest to linear growth. **(MTR.2.1, MTR.4.1)**

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse as students shape their ideas about simple interest and linear growth. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

2 Monitor

Look for a variety of understanding of the linear pattern in this real-world problem. **(MTR.7.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to think about which formula they will need to use for the problem. **(MTR.2.1)**
Say "This has a percentage which means it has to be exponential growth."	**Support** Invite them to look back at the previous lesson's notes to look at the simple interest formula.
Say "The equation looks like a linear equation!"	**Strengthen** Consider asking, what are the components of a linear function? **(MTR.2.1)** *(Slope and a point or a y-intercept)*
Need a challenge.	**Stretch** Inquire how much money Amara would have if she saved for 15 years. *($650).*

Look for/Listen for students making connections between the starting principal and the y-intercept. Also, listen for students discussing the linear patterns.

ELL English Language Learners Provide a word bank to support students in expressing their ideas, with words such as: *simple interest, linear, growth, function, slope*, and *y-intercept*. **(Writing and Speaking)**

3 Connect

To reveal the Key Takeaway, consider asking: **(MTR.2.1, MTR.4.1)**

- "What do you notice about the graph?"
- "Does the formula look like a function we have seen before?"
- "Why is there a constant change each year?"

Activity 1

Name: _____ Date: _____ Period: _____

Laptop Savings

Amara wants to save money to eventually buy a new laptop. She finds a bank that offers a simple interest savings account with an annual interest rate of 2%. Amara deposits $500 into the account and plans to leave it there for several years without adding or withdrawing money.

3. Amara tracks her account balance over time and notices that the total amount increases by the same amount each year.

a. Use the simple interest formula, $A = P(1 + rt)$, to write the growth equation for Amara's money.

$A = 500(1 + .02t)$ or $A = 500 + 10t$

b. Compare your formula from part A to a linear equation.

The formula $A = 500 + 10t$ looks similar to $y = mx + b$ and has a y-intercept of (0, 500) and a slope of 10.

c. Explain why the growth in her account is linear and describe what her account balance will look like over time.

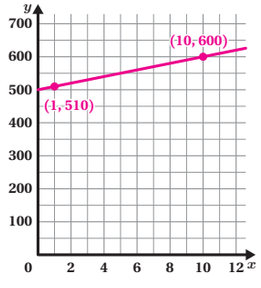
The growth in Amara's account is linear because the interest earned each year is based only on the original principal, not on the total amount in the account. This means the amount of interest added to her account remains the same each year. This is a linear pattern.

4. Amara wants to visualize the progress of her account over the next ten years.

a. What will the graph look like?

The graph will be a linear function, starting at (0, 500) and increasing by $10 every year.

b. Graph the function and label two points.



Unit 5 Lesson 15 581 Activity 1

Key Takeaway: Simple interest grows linearly because the interest earned is based only on the original principal, not the total amount in the account. The amount of interest is constant by year, which represents a linear function.

Pairs | 15 min

Activity 2 Super Savings

Purpose: Students use the Compare and Connect routine to support understanding in relating compounding interest to exponential growth. **(MTR.4.1, MTR.7.1)**

Short on time? Consider creating the equations together as a class to get started quickly.

1 Launch

Rigor and Balance

- Students develop **strategic competence** as they compare different strategies and then identify the subtle differences between compounding continuously and quarterly.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to the previous lessons on exponential growth.

A **Accessibility: Executive Function** Invite students to focus on one fund at a time to support students in carrying out multiple steps.

2 Monitor

Consider asking:

- "What did each student do?"
- "How did _____ solve the problem?"
- "What is a key difference between Mateo's account and Sofia's account?" **(MTR.4.1)**
- "Have you thought about saving money for something in the future, like college or a car?" **(MTR.7.1)**

Look for students noticing that both equations are exponential, but have some key differences.

3 Connect

Compare Questions

- "How are Mateo's and Sofia's strategies alike? How are they different?" **(MTR.4.1)**
- "Where do you see the same information in Mateo's and Sofia's strategy?" **(MTR.4.1)**

Connect Questions

- "What do you think was helpful about each strategy?"
- "When might you use each strategy?"

Activity 2

Name: Date: Period:

Super Savings

Mateo invests $\$1{,}000$ in a savings account with an annual interest rate of 4%, compounded quarterly. His cousin Sofia invests the same amount of $\$1{,}000$, but chooses an account with an annual interest rate of 4%, compounded continuously. Both plan to leave their money in the accounts for 10 years.

In the formula for interest compounded continuously, e is a constant. It is an irrational number approximately equal to 2.71828.

Compound Interest Formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$

Compound Continuously Formula: $A = P \cdot e^{rt}$

5. Create an equation for Mateo's and Sofia's accounts and simplify.

Mateo (compounded quarterly):

$A = P\left(1+\frac{r}{n}\right)^{nt}$ therefore, $A = 1000\left(1+\frac{0.04}{4}\right)^{4 \cdot 10} = 1000(1.01)^{40} \approx \$1{,}488.86$

Sofia (compounded continuously):

$A = P \cdot e^{rt}$ therefore, $A = 1000 \cdot e^{0.04 \cdot 10} = 1000e^{0.40} \approx \$1{,}491.82$

6. Is the growth in Mateo's and Sofia's accounts linear or exponential? How do you know?

Responses vary. The growth in Mateo's and Sofia's accounts is exponential, not linear. This is because the amount of interest added to their accounts increases over time.

7. **Discuss:** How does the growth of both accounts differ over time?

Responses vary. The growth in both accounts is exponential, but Sofia's account grows slightly faster than Mateo's over time. This happens because Sofia's interest is compounded continuously, meaning her balance is updated infinitely often, while Mateo's is compounded quarterly, so it's only updated four times a year.

Unit 5 Lesson 15 582 Activity 2

Pairs | 15 min

Activity 2 Super Savings (continued)

Purpose: Students use the Compare and Connect routine to support understanding in relating compounding interest to exponential growth. **(MTR.4.1, MTR.7.1)**

Short on time? Consider creating the equations together as a class to get started quickly.

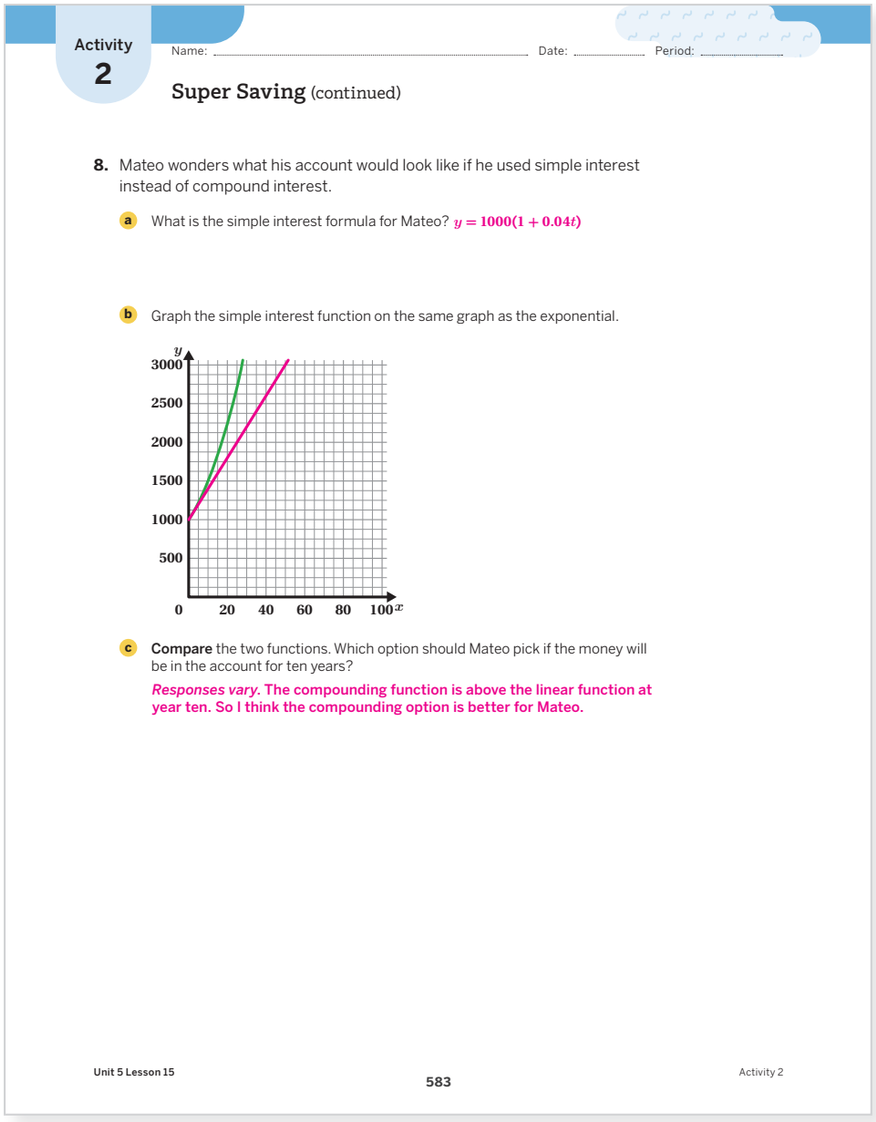
3 Connect

Compare Questions

- "How are these functions different?" **(MTR.4.1)**
- "How do you know which is the better account?"
- "Is time an important factor here?"

To reveal the Key Takeaway, consider asking,

- "Are these strategies really that different? If so, what are the key differences?"
- "Whose strategy do you like the best? Mateo or Sofia? Why?"
- "Is this better than the linear growth we saw in activity 1?"
- "Why is their money growing so quickly?"



Activity 2

Name: _____ Date: _____ Period: _____

Super Saving (continued)

8. Mateo wonders what his account would look like if he used simple interest instead of compound interest.

a. What is the simple interest formula for Mateo? $y = 1000(1 + 0.04t)$

b. Graph the simple interest function on the same graph as the exponential.

c. **Compare** the two functions. Which option should Mateo pick if the money will be in the account for ten years?

Responses vary. The compounding function is above the linear function at year ten. So I think the compounding option is better for Mateo.

Lesson Takeaway: Compound interest grows exponentially because interest is calculated on both the principal and any previously earned interest, causing the balance to grow faster over time. Continuous compounding amplifies this effect by adding interest infinitely often, resulting in even faster exponential growth.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize the differences between simple interest and compound interest.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- Simple interest is related to linear growth.
- Compound interest is related to exponential growth.
- A simple interest graph is represented by a line.
- The compound interest graph is represented by an increasing curve.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Compare and contrast simple and compound interest.

Simple interest shows linear growth because the interest earned each year is based only on the original principal, not on any accumulated interest. This means the amount of interest added is constant each year, resulting in a straight-line increase over time.

Compound interest results in exponential growth because the interest is calculated on both the original principal and the accumulated interest. As the balance grows, each new interest calculation is based on a larger amount, leading to faster increases over time. The more frequently interest is compounded, the faster the balance grows, because the interest is updated more often.

Lesson Practice 5.15

Lesson Summary

- **Simple interest** grows linearly because interest is based only on the original principal.
- **Compound interest** grows exponentially because each interest calculation adds to the balance, creating a feedback loop.
- **Continuous compounding** grows faster than periodic compounding, as interest is added at every moment.

Unit 5 Lesson 15 584 Synthesis | Lesson Practice

Lesson Takeaway: Simple interest represents linear growth because the interest earned is constant and based only on the initial principal. In contrast, compound interest demonstrates exponential growth, as interest is calculated on both the principal and previously earned interest. Continuous compounding enhances this effect by adding interest infinitely often, showcasing how compounding frequency impacts the rate of growth over time.

Independent | 5 min

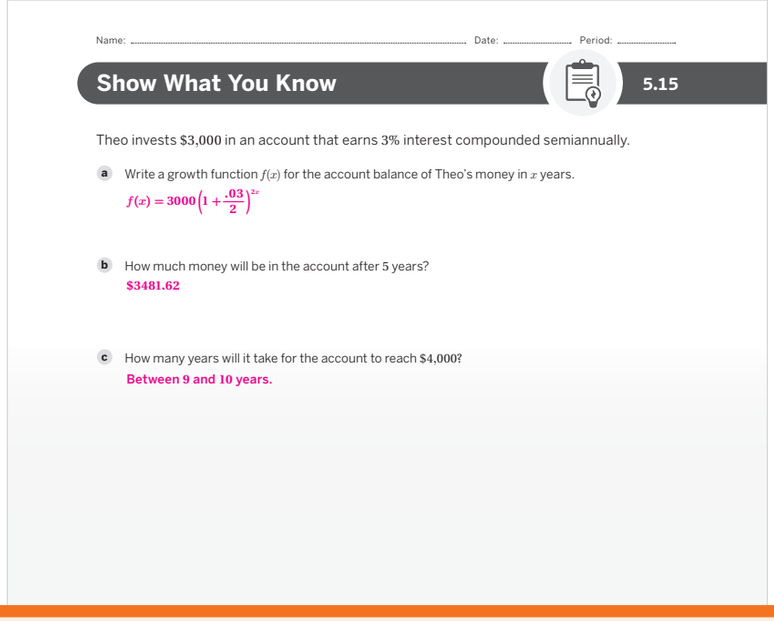
Show What You Know

Purpose: Students demonstrate their understanding by completing the practice problems.

Today's Goals

1. **Goal:** Recognize that simple interest is connected to linear functions.
2. **Goal:** Recognize that compound interest is connected to exponential functions.
3. **Language Goal:** Interpret formulas to match the correct functions and explain why. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.15

Theo invests $3,000 in an account that earns 3% interest compounded semiannually.

a. Write a growth function $f(x)$ for the account balance of Theo's money in x years.

$f(x) = 3000\left(1+\frac{.03}{2}\right)^{2x}$

b. How much money will be in the account after 5 years?

$3481.62

c. How many years will it take for the account to reach $4,000?

Between 9 and 10 years.

Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of linear and exponential functions:

» **Respond to Student Thinking:**

- Refer back to **Unit 1 Lesson 6** on linear functions.
- Refer back to **Unit 5 Lesson 2** on exponential functions.
- Refer back to **Unit 5 Lesson 3** to help students compare linear and exponential.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Create a project around different types of savings funds and have students analyze them.
- Research different types of accounts or investments (e.g., savings accounts, CDs, or stock portfolios) to find their compounding rates and annual percentage yields (APY). Have them compare how these affect long term growth.
- Create a table or graph to compare the growth of an investment with different compounding frequencies (e.g., annually, quarterly, monthly,and continuously) over 20 years.

Professional Learning

Take a moment to reflect on this lesson. How did Compare and Connect support students in understanding the differences between simple interest and compound interest?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Compare and contrast simple and compound interest.

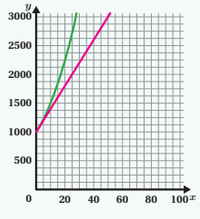
Simple interest shows linear growth because the interest earned each year is based only on the original principal, not on any accumulated interest. This means the amount of interest added is constant each year, resulting in a straight-line increase over time.

Compound interest results in exponential growth because the interest is calculated on both the original principal and the accumulated interest. As the balance grows, each new interest calculation is based on a larger amount, leading to faster increases over time. The more frequently interest is compounded, the faster the balance grows, because the interest is updated more often.

Lesson Practice 5.15

Lesson Summary

- **Simple interest** grows linearly because interest is based only on the original principal.
- **Compound interest** grows exponentially because each interest calculation adds to the balance, creating a feedback loop.
- **Continuous compounding** grows faster than periodic compounding, as interest is added at every moment.



Lesson Practice 5.15

Name: _____ Date: _____ Period: _____

1. A savings account offers simple interest at a rate of 5% annually. If you deposit $800, how much will your total balance be after 3 years?

$A = 800(1 + 0.05 \times 3) = \$920$

2. Why does simple interest result in a straight-line graph when plotted over time?

Responses vary. Because the interest earned each year is constant, leading to a fixed increase each year.

3. Which of the following situations best represents linear growth?

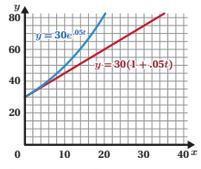
(A.) A savings account with simple interest

B. A population doubles every five years

C. A stock portfolio growing by 6% per year

D. An allowance that doubles every week

4. Write a story about the two graphs.



Responses vary. These graphs represent two different saving options for someone interested in putting down $30 at a growth rate of 5%. The blue graph represents the exponential growth option and the red represents the linear growth option.

5. If $500 is invested in an account with 6% annual interest compounded quarterly, what is the balance after 2 years? Write the equation and then solve it.

$A = 500\left(1 + \frac{0.06}{4}\right)^{4 \cdot 2} \approx 563.25$

Lesson Practice 5.15

Name: _____ Date: _____ Period: _____

6. A student claims that compound interest grows linearly over time. Explain why this is incorrect.

Compound interest grows exponentially because each interest calculation adds to the principal, causing the balance to grow faster over time.

7. Calculate the balance after 5 years if $1,000 is invested in an account with 4% annual interest compounded continuously.

$A = 1000 \cdot e^{0.04 \cdot 5} \approx 1221.40$

Test Practice

8. Which of the following statements correctly describes the relationship between types of interest and their growth patterns? Select **two** correct answers.

☑ A. Simple interest represents linear growth because the total interest added is constant over time.

☐ B. Continuously compounded interest represents linear growth because the interest rate is constant.

☑ C. Compound interest represents exponential growth because the interest is calculated on the increasing principal.

☐ D. The formula for simple interest can be written as $y = Pe^{rt}$.

Spiral Review

Problems 9–12. Find the solutions to the systems of equations.

9. $5x + y = -19$

$-4x + 3y = 0$

(-3, -4)

10. $7x + 2y = 20$

$x - 6y = -16$

(2, 3)

11. $6x + 9y = 15$

$4x - 6y = 22$

(4, -1)

12. $4x + 6y = -28$

$-10x + 10y = 20$

(-4, -2)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3, 5, 7	1	MA.912.F.3.2
	2, 4, 6	2	MA.912.F.3.4
Test Practice	8	1	MA.912.F.3.4
Spiral Review			
	9–12	1	MA.912.AR.9.1

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students made sense of functions represented as graphs, tables, equations, and situations. They interpreted statements in function notation and described functions using their key features.

Career Connection | Meet Zhang Qiujian

Invite students to read about how puzzles are related to city planning. Then invite them to read about Chinese mathematician Zhang Quijian and the "Hundred Fowls" problem.

Consider asking:

- "Are there any unique features of the design of your town or city? Could any of these be viewed as constraints during the design process?"

B.E.S.T. Benchmark Connection

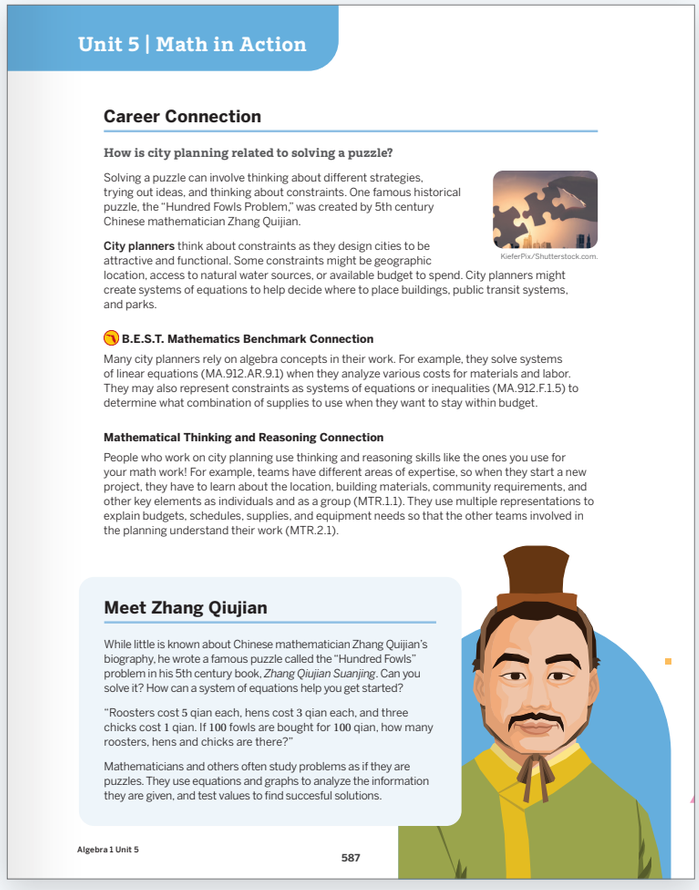
Consider asking:

- "What other math concepts have you studied in Units 1–5 that you could apply to city planning?" Sample response: You could analyze what portion of the development is completed each month and determine if it represents a linear or exponential relationship. (MA.912.AR.5.3)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "City planning is a specialized field of work. People who work in this field advise their clients on what materials to use or how long a project might take to complete. How are these actions similar to how you engage with math through Mathematical Thinking and Reasoning?" Sample response: City planners use patterns and structure to help clients understand and connect the many aspects of the project (MTR.5.1).



Unit 5 | Math in Action

Career Connection

How is city planning related to solving a puzzle?

Solving a puzzle can involve thinking about different strategies, trying out ideas, and thinking about constraints. One famous historical puzzle, the "Hundred Fowls Problem," was created by 5th century Chinese mathematician Zhang Quijian.

KieferPix/Shutterstock.com.

City planners think about constraints as they design cities to be attractive and functional. Some constraints might be geographic location, access to natural water sources, or available budget to spend. City planners might create systems of equations to help decide where to place buildings, public transit systems, and parks.

B.E.S.T. Mathematics Benchmark Connection

Many city planners rely on algebra concepts in their work. For example, they solve systems of linear equations (MA.912.AR.9.1) when they analyze various costs for materials and labor. They may also represent constraints as systems of equations or inequalities (MA.912.F.1.5) to determine what combination of supplies to use when they want to stay within budget.

Mathematical Thinking and Reasoning Connection

People who work on city planning use thinking and reasoning skills like the ones you use for your math work! For example, teams have different areas of expertise, so when they start a new project, they have to learn about the location, building materials, community requirements, and other key elements as individuals and as a group (MTR.1.1). They use multiple representations to explain budgets, schedules, supplies, and equipment needs so that the other teams involved in the planning understand their work (MTR.2.1).

Meet Zhang Qiujian

While little is known about Chinese mathematician Zhang Quijian's biography, he wrote a famous puzzle called the "Hundred Fowls" problem in his 5th century book, *Zhang Qiujian Suanjing*. Can you solve it? How can a system of equations help you get started?

"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens and chicks are there?"

Mathematicians and others often study problems as if they are puzzles. They use equations and graphs to analyze the information they are given, and test values to find succesful solutions.

Algebra 1 Unit 5

587

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify exponential functions	Lessons 1–3	1	MA.912.F.1.8
2	Use exponential functions to solve problems in context	Lesson 6	2	MA.912.AR.5.4
3	Compare linear and exponential growth	Lessons 1, 12	3	MA.912.F.1.6 MTR.4.1
4a	Calculate percentages from a graph	Lessons 2–3, 7	2	MA.912.AR.5.4 MTR.6.1
4b	Write functions for situations with percent decrease	Lesson 7	2	MA.912.AR.5.4
5a	Identify equivalent exponential functions to model account balances with compound interest	Lesson 3	2	MA.912.FL.3.2 MTR.5.1
5b	Calculate account balances with compound interest	Lessons 13–14	2	MA.912.F.1.8
5c	Compare accounts with different interest rates and compounding intervals	Lessons 13–14	3	MA.912.F.1.6 MTR.4.1
6a	Determine percent decrease from an equation	Lessons 6–7	2	MA.912.AR.5.4 MTR.6.1
6b	Interpret parameters of exponential functions in context	Lessons 6–7	2	MA.912.AR.1.1 MTR.6.1, MTR.7.1
6c	Evaluate and interpret exponential functions with negative inputs	Lesson 4	2	MA.912.F.1.2 MTR.6.1
7a	Writing a radical as a rational exponent	Lesson 8	2	MA.912.NSO.1.1
7b	Writing a rational exponent as a radical	Lesson 9	2	MA.912.NSO.1.2
8a	Adding and subtracting with radicals	Lesson 11	2	MA.912.NSO.1.4
8b	Multiplying with radicals	Lesson 10	2	MA.912.NSO.1.4

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 5

1. Select *all* of the exponential relationships.

- ☑ **A.** A population of bacteria begins with 6 cells and triples every hour.
- ☐ **B.** Jaylin puts $300 into an account that earns $15 interest per year.
- ☐ **C.**

x	y
1	3
2	6
3	11
4	18

- ☑ **D.** $f(x) = 1000 \cdot (0.98)^x$
- ☐ **E.** $g(x) = 500 + 0.10x$

2. A plant covers 5 square meters of Kayla's garden. Each year the plant grows by 30%. If it continues to grow at this rate, how large will the plant be in 4 years?



Approximately 14.3 square meters

3. Mai is putting $100 in a bank account. She will not add additional money or take any money out. Mai can choose from one of these accounts:

- Account A: The total amount will increase by 12% every year.
- Account B: The total amount will increase by $20 every year.

Will the value of Account B always be greater than the value of Account A?

No

Explain your thinking.

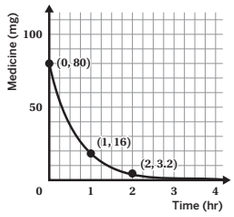
***Explanations vary*. Eventually exponential growth will surpass linear growth.**

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End-of-Unit Assessment (continued)

Unit 5

4. This graph shows the number of milligrams of medicine in a person's body after she takes it.



a What *percent* of the medicine is in the person's body after 1 hour?

20%

b Write an equation of the form $m(x) = a \cdot b^x$ to represent this situation.

- Use x to represent the time in hours.
- Use $m(x)$ to represent the milligrams of medicine remaining.

$m(x) = 80 \cdot (0.2)^x$ (or equivalent)

5. Adnan puts $200 in an account with a 1.5% monthly interest rate. The account earns interest 12 times per year. He does not add money into or take money out of the account.

a Select *all* of the equations that represent Adnan's account balance, $a(t)$, after t years.

- ☐ **A.** $a(t) = 200 \cdot (1.5)^t$
- ☑ **B.** $a(t) = 200 \cdot (1.015^{12})^t$
- ☑ **C.** $a(t) = 200 \cdot (1.015)^{12t}$
- ☑ **D.** $a(t) = 200 \cdot (1.1956)^t$
- ☐ **E.** $a(t) = 200 \cdot (1.1956)^{12t}$

b How much money will be in Adnan's account after 3 years?

Approximately $341.83

c Kwabena says: *Adnan could earn more money if he puts the $200 into an account that earns 8% annual interest (1 time per year).*

Is Kwabena correct? **No**

Explain your thinking.

***Explanations vary*. The interest rate per year of the first option is 19.6%, since $1.015^{12} \approx 1.196$. This is higher than the 8% annual interest rate Kwabena mentioned.**

End-of-Unit Assessment (continued)

Unit 5

6. The population of elk in a park x years after 1980 is represented by $f(x) = 1280 \cdot (0.75)^x$.

Years Since 1980	0	1	2	3	4
Number of Elk	1,280	960	720	540	405

a By what percent is the population decreasing each year?

25%

b Explain what the 1,280 and 0.75 in the equation represent about the situation.

Responses vary.

- 1,280 represents. . . **the number of elk in the park in 1980.**
- 0.75 represents. . . **that 75% of the elk are left each year.**

c Calculate $f(-2)$.

Responses from 2,275 to 2,276 are considered correct.

Explain what $f(-2)$ represents in this situation.

***Explanations vary.* This means that 2 years before 1980, there were about 2,276 elk in the park.**

End-of-Unit Assessment (continued)

Unit 5

7. **a** Which expression is an equivalent form of $\sqrt[3]{4^5}$?

- **A.** $4^{\frac{3}{5}}$
- (**B.**) $4^{\frac{5}{3}}$
- **C.** $2^{\frac{3}{5}}$
- **D.** $2^{\frac{5}{3}}$

b Which expression is an equivalent form of $5^{\frac{2}{3}}$?

- **A.** $\sqrt[3]{2^5}$
- **B.** $\sqrt[2]{5^3}$
- (**C.**) $\sqrt[3]{5^2}$
- **D.** $\sqrt{5^2}$

8. Complete the operations. Choose the option that is an equivalent expression.

b $3\sqrt{8} + 2\sqrt{6} - \sqrt{32}$

- **A.** $2\sqrt{6} - 2\sqrt{2}$
- **B.** $10\sqrt{2} - 2\sqrt{6}$
- **C.** $10\sqrt{2} + 2\sqrt{6}$
- (**D.**) $2\sqrt{2} + 2\sqrt{6}$

b $3\sqrt{12}\ (2\sqrt{8})$

- **A.** $12\sqrt{6}$
- (**B.**) $24\sqrt{6}$
- **C.** $12\sqrt{2}$
- **D.** $24\sqrt{2}$

Name: Date: Period:

End-of-Unit Assessment

Unit 5

1. Select *all* of the exponential relationships.

- ☐ **A.** $f(x) = 1.1x^2$
- ☑ **B.** $g(x) = 200(0.75)^x$
- ☑ **C.**

x	y
1	200
2	100
3	50
4	25

- ☐ **D.** A population of deer begins with 100 deer and decreases by 10 every year.
- ☑ **E.** Dalia puts $300 into an account that earns 2% annual interest.

2. A petri dish contains 100 cells of bacteria and is increasing by 20% per hour. If it continues to grow at this rate, how many bacteria cells will be present in 4 hours?



Approximately 207 cells

3. Jaleel is putting $200 in a bank account. He will not add additional money or take any money out. Jaleel can choose from one of these accounts:

- Account A: The total amount will increase by $15 every year.
- Account B: The total amount will increase by 5% every year.

Will the value of Account A always be greater than the value of Account B?

No

Explain your thinking.

***Explanations vary*. Eventually exponential growth will surpass linear growth.**

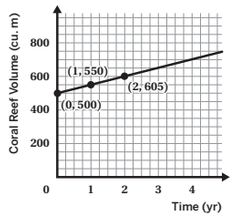
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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

4. This graph shows the volume of a section of the ocean's coral reef under restoration from 2010 to 2015.



a What percent of the coral reef's volume is present after 1 year?

110%

b Write an equation of the form $c(x) = a \cdot b^x$ to represent this situation.

- Use x to represent the time in years.
- Use $c(x)$ to represent the cubic meters of coral reef present.

$c(x) = 500 \cdot (1.1)^x$ (or equivalent)

5. Darius puts $400 in an account with a 1.1% weekly interest rate. The account earns interest 52 times per year. He does not add money into or take money out of the account.

a Select *all* of the equations that could represent Darius' account balance, $d(t)$, after t years.

- ☑ **A.** $d(t) = 400 \cdot (1.7663)^t$
- ☐ **B.** $d(t) = 400 \cdot (1.7663)^{52t}$
- ☑ **C.** $d(t) = 400 \cdot (1.011)^{52t}$
- ☐ **D.** $d(t) = 400 \cdot (1.011)^t$
- ☐ **E.** $d(t) = 400 \cdot (1.7663^{52})^t$

b How much money will be in Darius's account after 4 years?

Approximately $3,893.14

c Fabiana says: *Darius could earn more money if he puts the $400 into an account that earns 6% annual interest (1 time per year).*

Is Fabiana correct? **No**

Explain your thinking.

***Explanations vary*. The interest rate per year of the first option is 76.6%, since $1.011^{52} \approx 1.766$. This is higher than the 6% annual interest rate Fabiana mentioned.**

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

6. The population of sea urchins in a region of an ocean x years after 1990 is represented by $f(x) = 2500 \cdot (0.6)^x$.

Years Since 1990	0	1	2	3	4
Number of Sea Urchins	2,500	1,500	900	540	324

a By what percent is the population decreasing each year?

40%

b Explain what the 2,500 and 0.6 in the equation represent about the situation.

***Responses vary*.**

- 2,500 represents . . . **the number of sea urchins in a part of an ocean in 1990.**
- 0.6 represents . . . **that 60% of the sea urchins are left each year.**

c Calculate $f(-3)$.

Responses from 11,574 to 11,575 are considered correct.

Explain what $f(-3)$ represents in this situation.

***Explanations vary*. This means that 3 years before 1990, there were about 11,574 sea urchins in that part of the ocean.**

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

7. **a** Which expression is an equivalent form of $\sqrt[3]{9^5}$?

- **A.** $3^{\frac{3}{5}}$
- **B.** $3^{\frac{5}{3}}$
- **C.** $9^{\frac{3}{5}}$
- **D.** $9^{\frac{5}{3}}$

b Which expression is an equivalent form of $7^{\frac{2}{3}}$?

- **A.** $\sqrt[3]{2^7}$
- **B.** $\sqrt[3]{7^2}$
- **C.** $\sqrt{7^2}$
- **D.** $\sqrt{7^3}$

8. Complete the operations. Choose the option that is an equivalent expression.

a $6\sqrt{12} + 2\sqrt{6} - \sqrt{27}$

- **A.** $15\sqrt{3} - 2\sqrt{6}$
- **B.** $9\sqrt{3} - 2\sqrt{6}$
- **C.** $9\sqrt{3} + 2\sqrt{6}$
- **D.** $15\sqrt{3} + 2\sqrt{6}$

b $2\sqrt{18}\,(2\sqrt{12})$

- **A.** $24\sqrt{6}$
- **B.** $48\sqrt{6}$
- **C.** $24\sqrt{5}$
- **D.** $48\sqrt{5}$

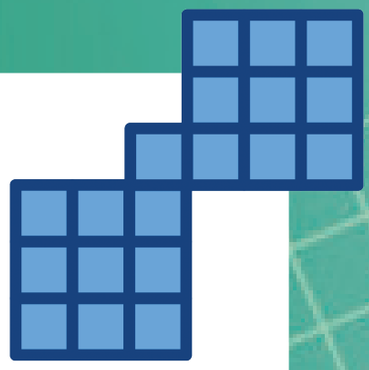
D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Compare and contrast linear and exponential functions. **(Lessons 1–3, 5)**	1, 3	**Support** • **Teacher Move:** Review Problem 6 of Lesson 1 *(Growing Globs)*, then encourage students to revisit Problems 8–9. **Strengthen** • **Practice Day 1:** Cards 5–6, 12 • **Practice Day 2:** Task D • **Practice:** Lessons 1–3, 5
Interpret and write exponential equations to model situations that involve percent increase or decrease. **(Lessons 6–7)**	4, 6a–b	**Support** • **Teacher Move:** Invite students to revisit Lesson 6, Activity 2 *(Growing Mold)* and then revise this assessment problem. **Strengthen** • **Practice Day 1:** Cards 7–10 • **Practice:** Lessons 6–7 **Stretch** • **You're invited to explore more:** Lesson 6 *(Growing Mold)*
Use exponential functions to solve problems in context. **(Lessons 4, 6–7)**	2, 6c	**Support** • **Teacher Move:** Review Lesson 7, Activity 2 *(At a Loss)*, then encourage students to revise this assessment problem. **Strengthen** • **Practice Day 1:** Cards 1, 7, 9, 11 • **Practice Day 2:** Task C • **Practice:** Lessons 4, 6–7
Use the properties of exponents to interpret expressions for exponential functions. **(Lessons 12–14)**	5	**Support** • **Lesson:** Lesson 13, Activity 2 *(Payday Loan)* • **Lesson:** Lesson 14, Activity 1 *(Credit Card Compounding)* **Strengthen** • **Repeated Challenges:** Lesson 12 *(Bank Accounts)* • **Repeated Challenges**: Lesson 14 *(Credit Card Compounding)* • **Practice Day 2:** Task A • **Practice:** Lessons 12–14 **Stretch** • **You're invited to explore more:** Lesson 12 *(Bank Accounts)* • **You're invited to explore more:** Lesson 13 *(Payday Loan)* • **DOK3 Practice:** Lessons 12–14 • **Teacher Move:** Invite students to research a credit card plan and write a function modeling how an account balance with an initial amount of $1,000 would grow over time.
Perform operations with radicals. **(Lessons 10–11)**	7a–b, 8a–b	**Support** • **Teacher Move:** Review Lesson 10, Activity 2 (*Rule the Roots*), then encourage students to revise this assessment problem. • **Teacher Move:** Review Lesson 11, Activities 1 and 2 (*Tame the Terms*), then encourage students to revise this assessment problem. **Strengthen** • **Practice:** Lessons 10–11

UNIT 6

Quadratic Functions



Students analyze graphs, tables, and equations in three forms to identify and interpret key features of quadratic functions.

Essential Questions

- What are the important features of quadratic relationships?
- How can we graph and write equations for quadratic functions in different forms?
- What situations can be modeled by quadratic functions?

Sub-Unit Goals

Sub-Unit 1

- Justify whether a function is linear, quadratic, exponential, or none of these.
- Identify and interpret key features of quadratic functions in graphs and tables.

Sub-Unit 2

- Determine key features and graph quadratic functions from standard- and factored-form equations.
- Write quadratic equations from a graph, description, or criteria.

Sub-Unit 3

- Create quadratic models to solve problems in context.
- Describe the effect of transformations on key features of a parabola and its equation in vertex form.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Build a function that models a relationship between two quantities. **(MA.912.F.1.8)**

Analyze functions using different representations. **(MA.912.AR.3.7)**

Other Standards Addressed in This Unit: MA.912.AR.3.4 MA.912.AR.3.5 MA.912.AR.3.6 MA.912.AR.3.8 MA.912.F.1.1 MA.912.F.1.2 MA.912.F.2.1

Mathematical Thinking and Reasoning Standards

- Students interpret key features of parabolas to answer questions about a context. **(MTR.2.1)**
- Students use precise language to describe the key features of parabolas. **(MTR.4.1)**
- Students use the structure of equations in factored form to determine x- and y-intercepts, and in vertex form to determine the coordinates of a parabola's vertex. **(MTR.5.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.7.1**

Coherence Telling a story within and across grades

In This Unit

Students learn about quadratic functions in tables, graphs, and equations. They learn the properties of quadratic functions, including their key features, and then consider how to determine these key features from equations in standard, factored, and vertex form.

Prior Learning

- **Grade 6:** Evaluating expressions **(MA.6.NSO.3.3)**
- **Grade 8:** Modeling functions **(MA.8.F.1.1, MA.8.F.1.3)**
- **Algebra 1:** Function notation **(MA.912.F.1.2)**
- **Algebra 1:** Interpreting and comparing functions **(MA.912.F.1.5)**

Future Learning

- **Algebra 1, Unit 7:** Quadratic equations **(MA.912.AR.3.2)**
- **Algebra 2:** Transformations of functions **(MA.912.F.2.1)**
- **Algebra 2:** Polynomial functions **(MA.912.AR.3.7)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 13–14**, students develop conceptual understanding for how transformations affect the graph and equation of a quadratic function.

Strategic Competence

In **Lesson 9**, students develop strategic competence as they compare standard form and factored form.

Procedural Fluency

In **Lesson 8**, students develop procedural fluency as they evaluate quadratic functions for various inputs and determine points on their graphs.

Adaptive Reasoning

In **Lesson 10**, students develop adaptive reasoning as they adjust their strategies for finding the x-intercepts of a parabola depending upon if it is written in standard form or factored form.

Application

In **Lesson 7**, students apply their understanding of key features of quadratic functions to the motion of a launched ball.

Productive Disposition

In **Lesson 12,** students have opportunities to develop a productive disposition as they experience using math to determine what shoe price would maximize revenue for a company.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.AR.1.4 MA.8.F.1.3 MA.912.AR.3.7 MA.912.AR.5.3 MA.912.F.1.2 MA.912.F.1.5
MTR.2.1 MTR.3.1 MTR.7.1

Sub-Unit 1



1 Revisiting Visual Patterns

A New Type of Pattern

Explore a new type of pattern in a familiar context: visual patterns.

MA.912.F.1.8
MTR.2.1 MTR.5.1



2 Quadratic Visual Patterns

Expressions for Quadratic Patterns

Describe a new type of pattern using expressions.

MA.912.F.1.8
MTR.2.1 MTR.5.1



3 Sorting Relationships

Comparing Linear, Exponential, and Quadratic Relationships

Compare and contrast linear, exponential, and quadratic relationships in tables, patterns, and equations.

MA.912.F.1.1 MA.912.F.1.2
MTR.2.1 MTR.5.1

Sub-Unit 2

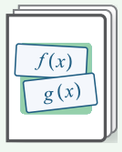


8 What's My Graph?

Creating Graphs of Quadratics

Develop strategies for graphing quadratic functions by hand using order of operations and tables.

MA.912.AR.3.7, MA.912.F.1.2
MTR.2.1, MTR.4.1



9 Two for One

Standard Form and Factored Form

Explore two forms of quadratic equations: standard form and factored form, and make connections between quadratic equations in these forms with their tables and graphs.

MA.912.AR.3.7, MA.912.F.1.2
MTR.2.1, MTR.5.1

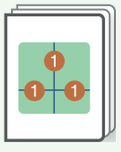


10 Interesting Intercepts

Intercepts in Factored and Standard Forms

Learn and use the relationship between the intercepts of a parabola and the structure of its equation written in factored or standard form.

MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2
MTR.2.1, MTR.4.1, MTR.5.1



11 Break Through: Parabolas

Building Quadratics in Factored Form

Use information about a parabola to write an equation in factored form.

MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1
MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1

Assess and Respond



15 Predicting Sales

Linear, Exponential, and Quadratic Modeling

Use linear, exponential, and quadratic models to make sense of real-life data about sales and make predictions.

MA.912.AR.3.8, MA.912.F.1.2, MA.912.F.1.8
MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1



A End-of-Unit Assessment

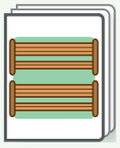
Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.8
MTR.4.1, MTR.5.1, MTR.6.1

Pacing: 20 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 14 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment



4 On the Fence

Quadratics in Context

Use the concept of building fences to explore symmetry in quadratic relationships.

MA.912.AR.3.7 MA.912.F.1.1 MA.912.F.1.8
MTR.2.1 MTR.5.1 MTR.7.1

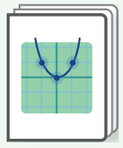


5 Stomp Rockets

Projectiles and Predictions

Use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.

MA.912.AR.3.7 MA.912.AR.3.8
MTR.1.1 MTR.2.1 MTR.3.1 MTR.5.1



6 Plenty of Parabolas

Key Features of Parabolas

Use informal language to distinguish between parabolas, and learn the terms *vertex*, *concave up*, and *concave down* to describe and compare parabolas.

MA.912.AR.3.7
MTR.2.1 MTR.4.1 MTR.5.1



7 Robot Launch

Key Features of Graphs in Context

Identify features of quadratic functions in graphs, tables, and equations and determine their meanings in context.

MA.912.AR.3.7 MA.912.F.1.2
MTR.2.1 MTR.4.1 MTR.5.1

Assess and Respond

Sub-Unit 3



12 Sneaker Drop

Exploring Revenue

Use quadratic equations to make sense of revenue models, considering how changing the price of an item affects the number of customers willing to buy it, and thus the maximum revenue a product can generate.

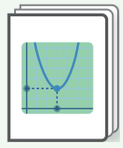
MA.912.AR.3.6 MA.912.AR.3.7 MA.912.AR.3.8 MA.912.F.1.2
MTR.2.1 MTR.5.1 MTR.6.1 MTR.7.1



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.912.AR.3.6 MA.912.AR.3.7 MA.912.AR.3.8 MA.912.F.1.8
MTR.3.1 MTR.5.1 MTR.6.1



13 Vertex Form

Translating Quadratic Functions

Describe the effect of vertical and horizontal translations on the graph of a quadratic function and use that understanding to write equations in the form $f(x) = (x - h)^2 + k$.

MA.912.AR.3.4 MA.912.AR.3.6 MA.912.AR.3.7 MA.912.F.2.1
MTR.2.1 MTR.4.1 MTR.5.1



14 Stretch It Out

Vertical Stretches and Vertex Form

Make connections between vertical stretches of a parabola and the structure of its equation written in vertex form.

MA.912.AR.3.4, MA.912.AR.3.7, MA.912.F.2.1
MTR.2.1, MTR.5.1

Pacing Considerations

Lesson 1: This lesson can be omitted because students will have opportunities to work with quadratic visual patterns in Lesson 2.

Lesson 5: In this lesson, students use what they have learned about quadratic functions to make predictions from tables and graphs. If students show a strong understanding of identifying and using the constant second difference in earlier lessons, this lesson may be omitted.

Lessons 8–9: These lessons invite students to use what they know about evaluating functions to make sense of standard and factored forms of equations. If students show a strong understanding of order of operations in the Pre-Unit Check, consider consolidating these two lessons into one class period. If Lesson 9 is omitted, spend adequate time discussing the differences between standard form and factored form of quadratic equations in other lessons.

Lesson 11: Activity 2 of this lesson mostly consists of opportunities for students to practice and play with writing quadratic functions in factored form. If time runs short, consider omitting part of this lesson or encouraging students to use it as extra practice.

Lesson 12: This lesson gives students an opportunity to apply their understanding of quadratic equations in factored form to make sense of a revenue context. There is no new math content introduced in this lesson.

Lesson 15: This lesson gives students an opportunity to use quadratic functions as models of real-world data sets and (optionally) to create a poster of their findings. There is no new math content introduced in this lesson.

Lesson Materials and Prep

Here are the materials and resources used in this unit and where to find them.

Lesson 1

Copy: Show What You Know

Materials: coloring tools *(as needed)*

Lesson 2

Copy: Show What You Know

Materials: highlighters *(as needed)*, calculators *(as needed)*

Lesson 3

Copy: Show What You Know, Activity 2 Cards, one per pair, pre-cut

Materials: calculators *(as needed)*, coloring tools *(as needed)*

Lesson 4

Copy: Show What You Know

Materials: calculators, graph paper *(optional)*, algebra tiles *(as needed)*

Lesson 5

Copy: Show What You Know

Materials: calculators *(optional)*

Lesson 6

Copy: Show What You Know, Warm-Up Sheet, one per pair

Materials: coloring tools

Lesson 7

Copy: Show What You Know, Activity 1 Cards, one per pair, precut

Materials: *no required materials*

Lesson 8

Copy: Show What You Know, Activity 1 Cards, one per pair, pre-cut

Materials: calculators

Lesson 9

Copy: Show What You Know, Activity 1 Cards, one per pair, pre-cut

Materials: calculators *(as needed)*

Lesson 10

Copy: Show What You Know

Materials: graph paper *(as needed)*, graphing calculators *(as needed)*

Lesson 11

Copy: Show What You Know

Materials: *no required materials*

Lesson 12

Copy: Show What You Know, Activity 2 Cards, one per pair

Materials: graphing calculators, coloring tools

Lesson 13

Copy: Show What You Know

Materials: graph paper *(as needed)*, graphing calculators

Lesson 14

Copy: Show What You Know

Materials: coloring tools, graphing calculators

Lesson 15

Copy: Show What You Know

Materials: *no required materials*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



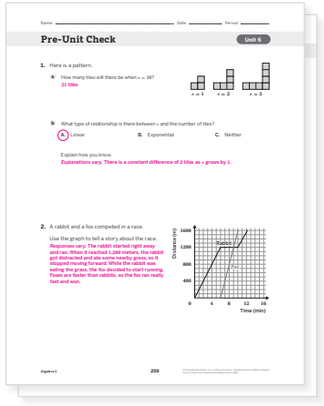
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.AR.1.4, MA.8.F.1.3, MA.912.AR.3.7, MA.912.AR.5.3, MA.912.F.1.2, MA.912.F.1.5


Pre-Unit Check

Unit 6

1. Here is a pattern.

a. How many tiles will there be when $n = 10$?

21 tiles

b. What type of relationship is there between n and the number of tiles?

A. Linear B. Exponential C. Neither

Explain how you know.

Explanations vary. There is a constant difference of 2 tiles as n grows by 1.

2. A rabbit and a fox competed in a race.

Use the graph to tell a story about the race.

Responses vary. The rabbit started right away and ran. When it reached 1,200 meters, the rabbit got distracted and ate some nearby grass, so it stopped moving forward. While the rabbit was eating the grass, the fox decided to start running. Foxes are faster than rabbits, so the fox ran really fast and won.

Distance (m)

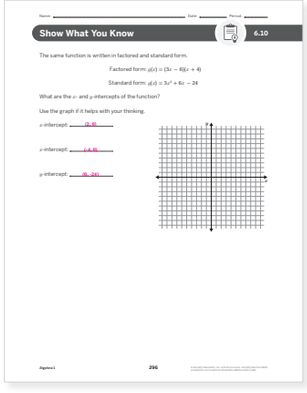
Time (min)

Algebra 1 259

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goals. Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed: on Lesson 10 Show What You Know: MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2


Show What You Know

6.10

The same function is written in factored and standard form.

Factored form: $g(x) = (3x - 6)(x + 4)$

Standard form: $g(x) = 3x^2 + 6x - 24$

What are the x- and y-intercepts of the function?

Use the graph if it helps with your thinking.

x-intercept: (2, 0)

x-intercept: (-4, 0)

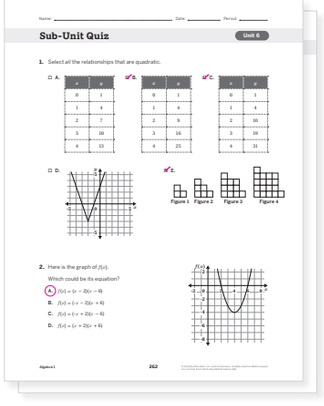
y-intercept: (0, -24)

Algebra 1 296

Sub-Unit Quizzes

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.8


Sub-Unit Quiz

Unit 6

1. Select all the relationships that are quadratic.

A.

x	y
0	1
1	4
2	7
3	10
4	13

B.

x	y
0	1
1	4
2	9
3	16
4	25

C.

x	y
0	1
1	4
2	16
3	19
4	34

D.

E.

Figure 1 Figure 2 Figure 3 Figure 4

2. Here is the graph of $f(x)$.

Which could be its equation?

A. $f(x) = (x - 2)(x - 4)$

B. $f(x) = (x - 2)(x + 4)$

C. $f(x) = (x + 2)(x - 4)$

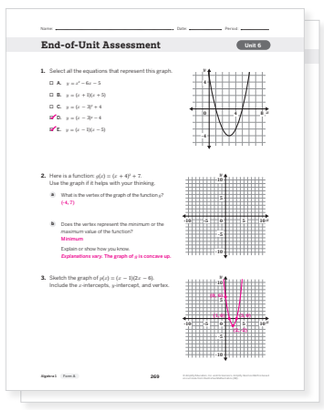
D. $f(x) = (x + 2)(x + 4)$

Algebra 1 262

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.8


End-of-Unit Assessment

Unit 6

1. Select all the equations that represent this graph.

A. $y = x^2 - 6x - 5$

B. $y = (x + 1)(x + 5)$

C. $y = (x - 3)^2 + 4$

D. $y = (x - 3)^2 + 4$

E. $y = (x - 1)(x - 5)$

2. Here is a function: $g(x) = (x + 4)^2 + 7$.

Use the graph if it helps with your thinking.

a. What is the vertex of the graph of the function? (-4, 7)

b. Does the vertex represent the minimum or the maximum value of the function?

Minimum

Explain or show how you know.

Explanations vary. The graph of g is concave up.

3. Sketch the graph of $p(x) = (x - 1)(2x - 6)$.

Include the x-intercepts, y-intercept, and vertex.

Algebra 1 Form A 269

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

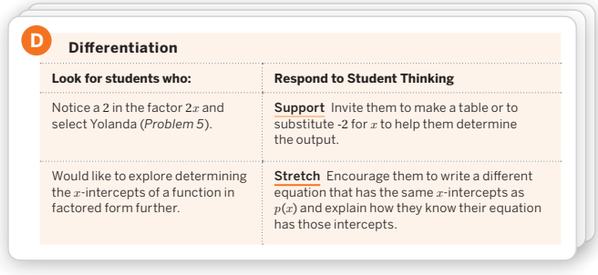
Differentiation Across the Unit

	Support Provide targeted intervention for students by using these resources.	Strengthen Reinforce students' understanding of the concepts assessed by using these resources.	Stretch Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–7) • Justify whether a function is linear, quadratic, exponential, or none of these. • Identify and interpret key features of quadratic functions in graphs and tables.	**Previous Lessons** • Unit 5, Lesson 1, Problems 6–11 *(Growing Globs)* • Unit 3, Lesson 6 *(Craft-a-Graph)* • Unit 3, Lesson 8 *(Space Race)* **Teacher Moves**	**Practice** • Lessons 1–7 **Teacher Moves**	**You're invited to explore more.** • Lesson 1 *(Revisiting Visual Patterns)* • Lesson 5 *(Stomp Rockets)* • Lesson 6 *(Plenty of Parabolas)* **DOK3 Practice** • Lessons 3, 6–7 **Teacher Moves**
Sub-Unit 2 (Lessons 8–12) • Determine key features and graph quadratic functions from standard and factored form equations. • Write quadratic equations from a graph, description, or criteria.	**Mini-Lesson** • Grade 6, Unit 6 *(Evaluating Expressions With Exponents)* **Previous Lessons** • Unit 1, Lesson 7, Problems 3–4 *(Five Representations)* **Teacher Moves** Consider creating a class display of standard and factored form equations and their connections to graphical key features.	**Practice** • Lessons 8–12 **Teacher Moves**	**DOK3 Practice** • Lessons 8–12 **Teacher Moves** Invite students to create their own graph and equation for a quadratic function in factored and/or standard form.
Sub-Unit 3 (Lessons 13–15) • Create quadratic models to solve problems in context. • Describe the effect of transformations on key features of a parabola and its equation in vertex form.	**Previous Lessons** • Unit 3, Lesson 12 *(Absolute Value Machines)* **Teacher Moves** Consider creating a class display of quadratic equations in vertex form and how each part is connected to transformations and key features of parabolas.	**Repeated Challenges** • Lesson 13 *(Vertex Form)* • Lesson 14 *(Stretch It Out)* **Practice** • Lessons 13–15 **Teacher Moves**	**You're invited to explore more.** • Lesson 13 *(Vertex Form)* **DOK3 Practice** • Lesson 13 **Teacher Moves** Invite students to transform any function by entering it as $f(x)$ in the Desmos Graphing Calculator, and then entering $af(x - h) + k$ with sliders. Encourage them to explore and discuss with a partner.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 10**, where students note what they notice about the intercepts.



Differentiation

Look for students who:	Respond to Student Thinking
Notice a 2 in the factor $2x$ and select Yolanda (*Problem 5*).	**Support** Invite them to make a table or to substitute -2 for x to help them determine the output.
Would like to explore determining the x-intercepts of a function in factored form further.	**Stretch** Encourage them to write a different equation that has the same x-intercepts as $p(x)$ and explain how they know their equation has those intercepts.

Accessibility

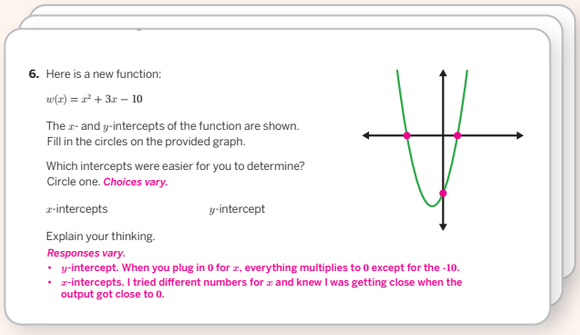
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action & Expression.

Spotlight on Executive Functioning . . .

Lesson 10: In this lesson, students have opportunities for strategizing, planning, and monitoring progress when they are challenged to find the intercepts of the quadratic function.



6. Here is a new function:

$w(x) = x^2 + 3x - 10$

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph.

Which intercepts were easier for you to determine? Circle one. ***Choices vary.***

x-intercepts y-intercept

Explain your thinking.

Responses vary.

- **y-intercept. When you plug in 0 for x, everything multiplies to 0 except for the -10.**
- **x-intercepts. I tried different numbers for x and knew I was getting close when the output got close to 0.**

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Affective Functioning . . .

Here are two suggestions in this unit for supporting students with collaboration and sharing their thinking.

Lesson 12, Warm-Up

A **Accessibility: Affective Functioning** IConsider setting a timer or playing music while students look for a partner to support students with collaboration.

Lesson 2, Synthesis

A **Accessibility: Affective Functioning** To support students in sharing their thinking, check in with students in advance to let them know you will ask for their thoughts.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
quadratic relationship	2
quadratic function **second difference**	3
line of symmetry **parabola**	4
concave down **concave up** **vertex**	6
factored form **standard form (of a quadratic equation)**	9
vertex form	13
vertical stretch	14

Review Vocabulary

constant difference
domain
exponential relationship
factor
linear relationship
maximum
mean
median
minimum
non-linear relationship
translation
x-intercept
y-intercept

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	3, 6
MLR2: Collect and Display	2, 4, 6–15
MLR3: Critique, Correct, Clarify	10, 13
MLR7: Compare and Connect	1–2, 8–9, 12

Sub-Unit Language Goals

- Compare and contrast properties of quadratic functions.
- Identify and interpret key features of quadratic functions from equations.
- Describe the effect of transformations on key features of a parabola and its equation in vertex form.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 12**, where students design their own sneaker to make sense of a context involving revenue models for sneaker companies.

ML/EL **Multilingual/English Language Learners** Invite students to share what they know about sneakers. Consider sharing that sneakers may be a regional term in some areas, and that students may know other ways of referring to shoes like these (e.g., "tennis shoes" or "gym shoes"). Consider inviting students to share the word(s) for these shoes in their primary language and record these words publicly for the class to reference. **(Reading and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Quadratic Equations

In this unit, students explore functions that represent quadratic relationships. **In Unit 7**, students will solve quadratic equations and systems of equations using reasoning, factoring, the zero-product property, completing the square, and the quadratic formula.

FL B.E.S.T. Math Benchmark:
MA.912.AR.3.1

Example:

There are many strategies for solving the equation $x^2 + 12x = 24$. One strategy is to rewrite the equation as a perfect square, then solve by taking square roots. There are two solutions because both $\sqrt{60}$ and $-\sqrt{60}$ can be squared to make 60.

$$x^2 + 12x = 24$$
$$x^2 + 12x + 36 = 24 + 36$$
$$(x + 6)^2 = 60$$
$$x + 6 = \pm\sqrt{60}$$
$$x = -6 \pm \sqrt{60}$$

Transformations of Functions

In this unit, students analyzed transformations of quadratic functions. **In Algebra 2,** students will analyze transformations of several families of functions and generalize the effects of different transformations.

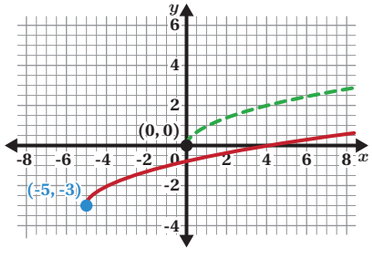
FL B.E.S.T. Math Benchmark:
MA.912.F.2.1

Example:

The function $g(x) = \sqrt{x}$ is translated 5 units to the left and 3 units down.

Its critical point used to be $(0, 0)$ and is now $(-5, -3)$.

We can record this transformation using function notation: $g(x + 5) - 3 = \sqrt{x + 5} - 3$.



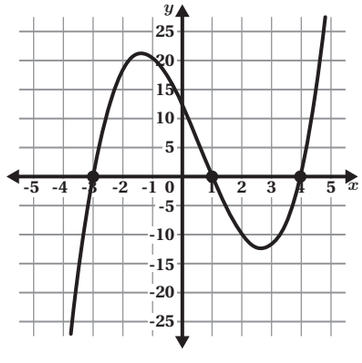
Polynomial Functions

In this unit, students use quadratic functions in standard and factored form to identify key features of parabolas. **In Algebra 2**, students will extend this understanding to higher order polynomials.

FL B.E.S.T. Math Benchmark:
MA.912.AR.6.6

Example:

The function $f(x) = (x + 3)(x - 1)(x - 4)$ has three factors with no repeated factors. The zero product property explains why the graph of $f(x)$ has x-intercepts at $(-3, 0)$, $(1,0)$, and $(4, 0)$. We can test large and small values and values close to each of the intercepts to determine what the rest of the graph of $f(x)$ looks like.

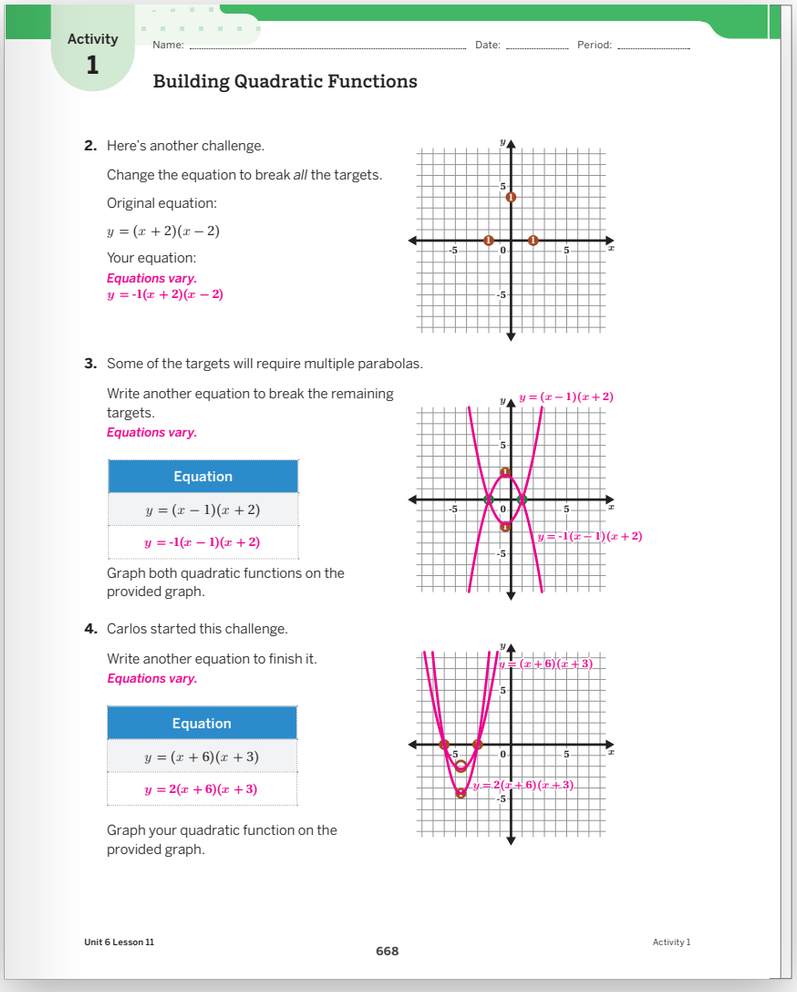


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Procedural Fluency

Lesson 11, Activity 1



Activity 1

Name: Date: Period:

Building Quadratic Functions

2. Here's another challenge.

Change the equation to break *all* the targets.

Original equation:

$y = (x + 2)(x - 2)$

Your equation:

Equations vary.

$y = -1(x + 2)(x - 2)$

3. Some of the targets will require multiple parabolas.

Write another equation to break the remaining targets.

Equations vary.

Equation
$y = (x - 1)(x + 2)$
$y = -1(x - 1)(x + 2)$

Graph both quadratic functions on the provided graph.

4. Carlos started this challenge.

Write another equation to finish it.

Equations vary.

Equation
$y = (x + 6)(x + 3)$
$y = 2(x + 6)(x + 3)$

Graph your quadratic function on the provided graph.

Unit 6 Lesson 11 668 Activity 1

Context

In this activity, students develop procedural fluency as they use key features or other strategies to write equations of parabolas that will break the targets shown on the graphs.

Try This

Put on your student hat and complete Problems 1–5.

Points to Ponder

- What do students need to understand about key features and factored form to break the targets on the graph?
- Some students may be able to break one target faster than the other(s), but then get stuck. How might you support students in finding other points using the structure of the factored form equation?
- How will students' work with factored form in this lesson support their understanding of factored form of other polynomial equations in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Extend and describe patterns	Lesson 1	2	MA.912.AR.5.3 MTR.3.1
2	Interpret functions	Lesson 4	3	MA.912.AR.3.7 MTR.2.1
3	Compare functions	Lesson 4	2	MA.912.F.1.5 MTR.3.1
4	Evaluate quadratic expressions	Lesson 8	1	MA.6.AR.1.4 MTR.3.1
5	Evaluate expressions in function notation	Lesson 8	2	MA.912.F.1.2 MTR.3.1
6	Model situations	Lesson 12	2	MA.8.F.1.3 MTR.7.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



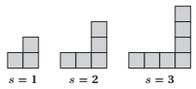
- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 6

1. Here is a pattern.



a. How many tiles will there be when $s = 10$?
21 tiles

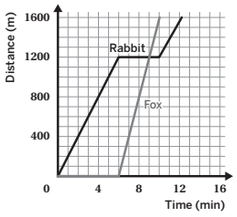
b. What type of relationship is there between s and the number of tiles?

A. Linear B. Exponential C. Neither

Explain how you know.
Explanations vary. **There is a constant difference of 2 tiles as s grows by 1.**

2. A rabbit and a fox competed in a race.

Use the graph to tell a story about the race.
Responses vary. **The rabbit started right away and ran. When it reached 1,200 meters, the rabbit got distracted and ate some nearby grass, so it stopped moving forward. While the rabbit was eating the grass, the fox decided to start running. Foxes are faster than rabbits, so the fox ran really fast and won.**



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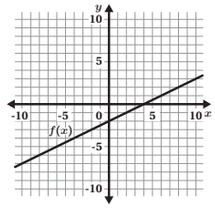
Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 6

3. $f(x)$ and $g(x)$ are two functions.

- $f(x)$ is shown in the graph.
- $g(x) = \frac{1}{2}x + 3$



a. Which function has a larger y-intercept? Circle one.

$f(x)$ $g(x)$ They have the same y-intercept

b. Which function is increasing at a faster rate? Circle one.

$f(x)$ $g(x)$ They are increasing at the same rate

Explain how you know.
Explanations vary. **The slope of both lines is $\frac{1}{2}$.**

4. Determine the value of each expression.

a. $(x + 5)^2$ when $x = 2$
49

b. $3x^2$ when $x = 5$
75

c. $(2x)^2$ when $x = 4$
64

5. Here is a function: $h(x) = 4 + x^2$.

a. What is the value of $h(5)$?
$h(5) = 29$

b. What is the value of $h(-3)$?
$h(-3) = 13$

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

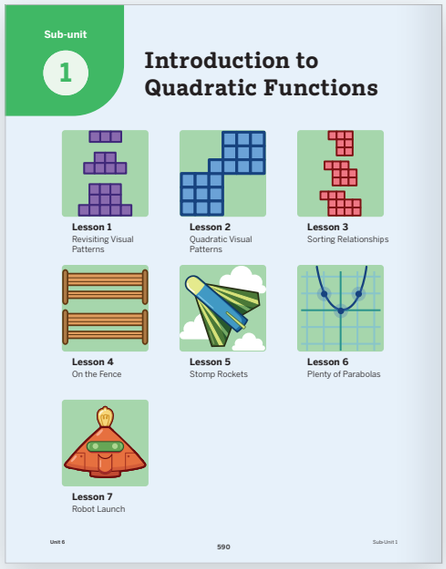
Prior Knowledge	Problem(s)	Respond to Student Thinking
Extend and describe patterns **(Preparation for Lessons 1–5)**	1	**Support** • **Teacher Move:** During Lesson 1 or 2, review how to determine if a relationship is linear, exponential, or something else. Consider making a display for students to reference. • **Lesson:** Unit 5, Lesson 1, Problems 6–9 (*Growing Globs*)
Interpret and compare functions **(Preparation for Lessons 5–7)**	2–3	**Support** • **Lesson:** Unit 3, Lesson 8 (*Space Race*) • **Lesson:** Grade 8, Unit 5, Lesson 5 (*The Tortoise and the Hare*) • **Mini-Lesson:** Grade 8, Unit 5 (*Comparing Properties of Linear Functions*)
Evaluate quadratic expressions **(Preparation for Lessons 8–10)**	4–5	**Support** • **Teacher Move:** Before Lesson 8, consider reviewing order of operations as a class. Consider making a display for students to reference. • **Mini-Lesson:** Grade 6, Unit 6 (*Evaluating Expressions With Exponents*) • **Lesson:** Grade 6, Unit 6, Lesson 12 (*Squares and Cubes*)
Model situations **(Preparation for Lessons 12)**	6	**Support** • **Teacher Move:** Consider reviewing this problem before Lesson 12. • **Lesson:** Grade 8, Unit 3, Lesson 5 (*Water Cooler*)

Introduction to Quadratic Functions

Sub-Unit 1 Goals

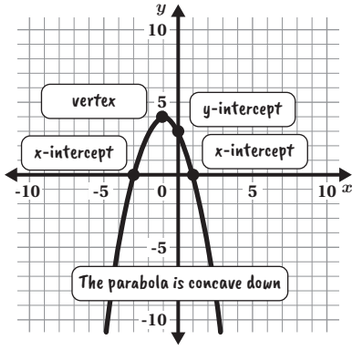
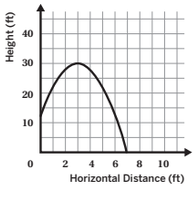
- **Goal:** Justify whether a function is linear, quadratic, exponential, or none of these.
- **Goal:** Identify and interpret key features of quadratic functions in graphs and tables.
- **Language Goal:** Compare and contrast properties of quadratic functions.

In this sub-unit, students explore **quadratic functions** and compare them to linear and exponential functions. They use graphs and tables to identify and interpret key features. They model area and projectile motion with quadratic functions and analyze these situations by interpreting key features.



Math That Matters Most

Concept or Skill	For example . . .
Determine whether a pattern represents a linear, quadratic, or exponential relationship	Linear x | y 1 | 3 (+3) 2 | 6 (+3) 3 | 9 (+3) 4 | 12 Quadratic x | y 1 | 3 (+3) 2 | 6 (+4, +1) 3 | 10 (+5, +1) 4 | 15 Exponential x | y 1 | 3 (×2) 2 | 6 (×2) 3 | 12 (×2) 4 | 24 How can you determine if a pattern represents a linear, quadratic, or exponential relationship? *Linear relationships have a constant difference, quadratic relationships have a constant second difference, and exponential relationships have a constant ratio.*
Observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically	Nyanna says that exponential growth functions will eventually exceed all of the other function types. When is Nyanna's statement true? *Always true. Multiplication makes the function grow more quickly than addition. Even if you multiply by a small number, like 3, the outputs of the function get really big. For example, d(4) is equal to 324.*

Concept or Skill	For example . . .
Describe the key features of a parabola from its graph (maximum, minimum, intercepts, concavity)	Label the key features of this parabola. 
Compare properties of quadratic functions represented in different ways	Balloon A  Balloon B Horizontal Distance (ft) | Height (ft) 1 | 28 2 | 29 3 | 28 4 | 25 5 | 20 Which balloon went higher? *Balloon A. The y-coordinate of the vertex in the graph is 30 feet, but the highest value in the table is 29 feet.*

Unit 6

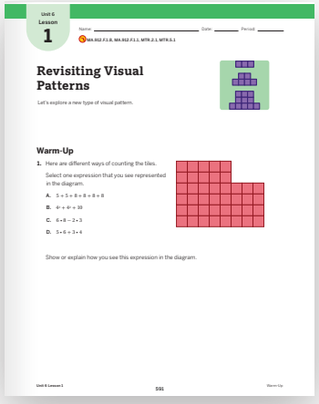
Lesson

1

Revisiting Visual Patterns

A New Type of Pattern

Let's explore a new type of visual pattern.


Unit 6 Lesson 1

Revisiting Visual Patterns

Let's explore a new type of visual pattern.

Warm-Up

1. Here are different ways of counting the tiles.
   Select one expression that you see represented in the diagram.
   A. $5 + 5 + 8 + 8 + 8 + 8$
   B. $4 \cdot 4 + 4 \cdot 10$
   C. $6 \cdot 8 - 2 \cdot 3$
   D. $5 \cdot 6 + 3 \cdot 4$

   Show or explain how you see this expression in the diagram.

591

Learning Outcomes

Today's Goals

1. **Goal:** Analyze patterns to determine the output value for a given input value.
2. **Language Goal:** Describe patterns using words or by writing an expression. **(Writing, Speaking, and Listening)**
3. **Language Goal:** Compare and contrast linear and non-linear relationships. **(Writing, Speaking, and Listening)**

Students describe and extend visual patterns that are neither linear nor exponential. **Note:** Students will be introduced to the term *quadratic relationship* in Lesson 2.

Coherence

Prior Learning

In Unit 5, students described and extended patterns with constant differences and constant ratios. They also defined sequences recursively and explicitly. They also classified relationships in multiple representations as linear or exponential. **(MTR.5.1)**

Future Learning

In Lesson 2, students will be introduced to the term *quadratic relationship* and practice using expressions to represent quadratic visual patterns.

Connections

- Students extend their understanding of visual patterns of linear and exponential relationships in an exploration of visual patterns of quadratic relationships. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.8
Determine whether a linear, quadratic or exponential function best models a given real-world situation.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.3.1	**MA.8.AR.3.1**

Vocabulary

Review Vocabulary

exponential relationship *linear relationship*

non-linear relationship

Lesson at a Glance

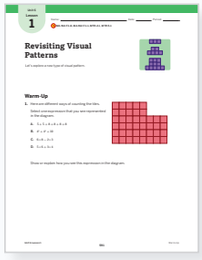
~ 45 min

MA.912.F.1.8

Warm-Up

Pairs | 5 min

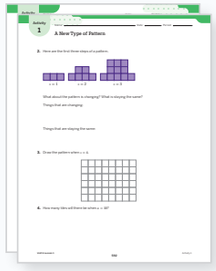
Students describe connections between the structure of a figure and the parts of expressions that represent it. **(MTR.2.1, MTR.5.1)**



Activity 1

Pairs | 15 min

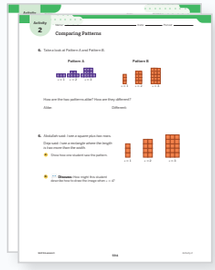
Students describe and extend a pattern that is neither linear nor exponential. **(MTR.5.1)**
Routine: Decide and Defend



Activity 2

Pairs | 15 min

Students use **MLR7: Compare and Connect** to explore how different descriptions of the structures of patterns might lead towards different strategies for extending those patterns. **(MTR.2.1, MTR.5.1)**



Synthesis

Whole Class | 5 min

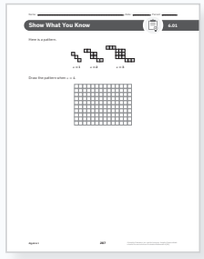
Students synthesize their understanding of strategies for describing patterns. **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by extending a visual pattern.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students describe connections between the structure of a figure and the parts of expressions that represent it. **(MTR.2.1, MTR.5.1)**

1 Launch

Draw students' attention to Problem 1. Ask, "How do you see this figure? What shapes, parts, or structures do you see?"

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas throughout this lesson: *length, width, height, area, side, square, rectangle, rows, columns*, etc. **(Writing and Speaking)**

A **Accessibility: Visual-Spatial Processing** Encourage students to use different coloring tools in their Student Edition to show connections between the figure and the expressions. **(MTR.2.1, MTR.5.1)**

2 Connect

Collect and share a variety of different student responses and sketches.

Consider asking:

- "How did describing what you see in words help you select an expression?"
- "Why might it be helpful to describe the same figure in different ways?"

Unit 6
Lesson
1

Name: ..... Date: ..... Period: .....

MA.912.F.1.8, MA.912.F.1.1, MTR.2.1, MTR.5.1

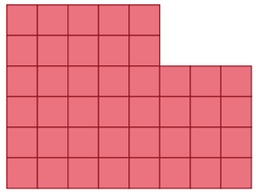
Revisiting Visual Patterns

Let's explore a new type of visual pattern.



Warm-Up

1. Here are different ways of counting the tiles.



Select one expression that you see represented in the diagram.

A. $5+5+8+8+8+8$

B. 4^2+4^2+10

C. $6 \cdot 8-2 \cdot 3$

D. $5 \cdot 6+3 \cdot 4$

Show or explain how you see this expression in the diagram.

Responses vary.

- Choice A: I see two rows of 5 at the top and four rows of 8 at the bottom.
- Choice B: I see two 4-by-4 squares at the bottom of the shape, plus 10 small squares on the top.
- Choice C: I can draw a large 6-by-8 rectangle around the entire shape, then subtract a 2-by-3 rectangle from the corner.
- Choice D: I see two rectangles put together. One 5-by-6 rectangle on the left and a smaller 3-by-4 rectangle on the right.

Unit 6 Lesson 1 591 Warm-Up

Key Takeaway: There are many different ways to be correct in math class.

Pairs | 15 min

Activity 1 A New Type of Pattern

Purpose: Students describe and extend a pattern that is neither linear nor exponential. **(MTR.5.1)**

Short on time? Consider omitting the discussion in the Launch.

1 Launch

To access prior knowledge, consider asking:
- "Where have we seen visual patterns before?"
- "What strategies have you used to extend and describe visual patterns in the past?"

To encourage creativity, remind students that there are many ways to describe patterns and that more variety will result in a deeper class understanding.

2 Monitor

A **Accessibility: Memory and Attention** Invite students to use the Student Edition to compare all three figures simultaneously.

D **Differentiation:** (Strengthen) Consider asking pairs of students, "What do you think s represents?"

Encourage students to read their classmates' responses and describe how they are similar and different before continuing.

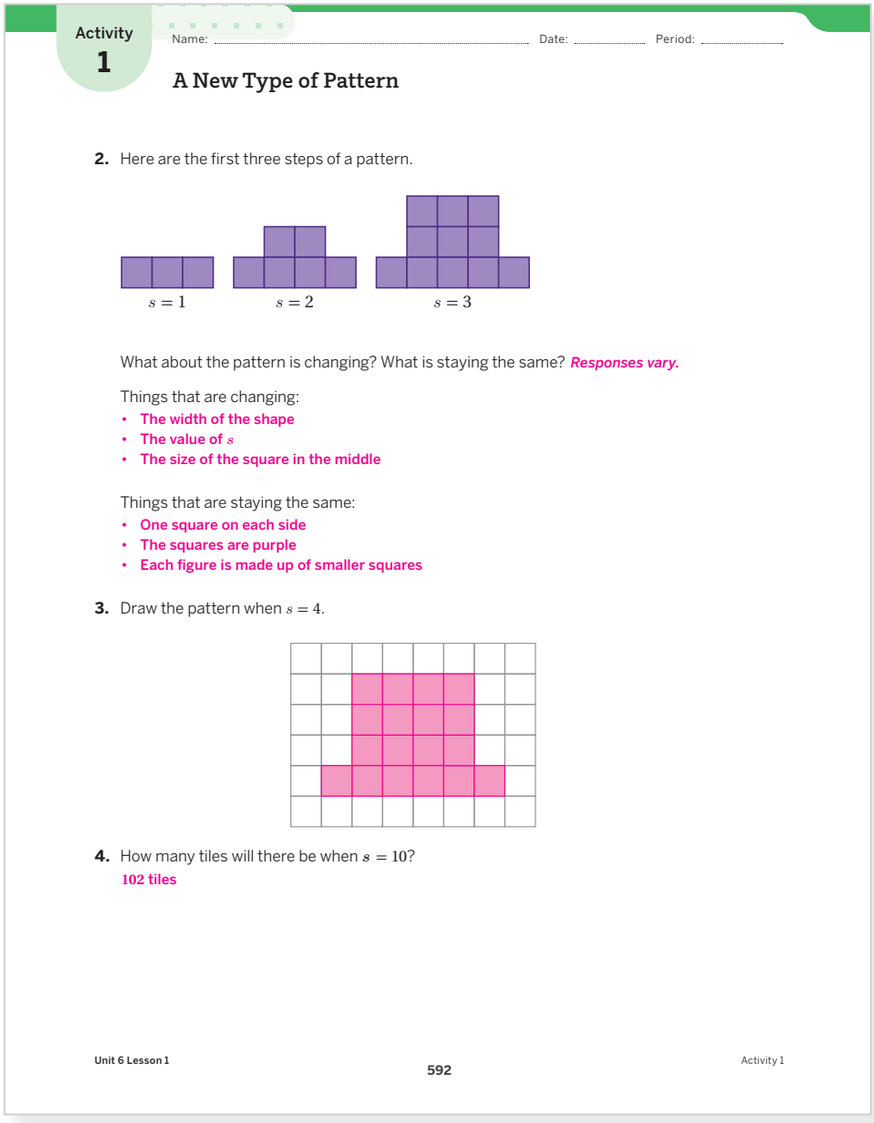
To access connections, consider asking pairs of students, "What structure do you notice in this pattern?" **(MTR.5.1)**

Select and sequence a variety of student strategies to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What would the image for $s = 10$ look like?"
Use a table similar to the strategy in the next problem.	**Strengthen** Invite them to share during the Connect. Consider asking, "What patterns do you notice in the table?"
Use strategies that make use of structure like: • See a square with side length s plus 2 more. • Draw a picture with labeled dimensions rather than individual tiles. **(MTR.5.1)**	**Strengthen** Invite them to share during the Connect and compare with another pair who used a different strategy.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

A New Type of Pattern

2. Here are the first three steps of a pattern.

What about the pattern is changing? What is staying the same? ***Responses vary.***

Things that are changing:
- **The width of the shape**
- **The value of s**
- **The size of the square in the middle**

Things that are staying the same:
- **One square on each side**
- **The squares are purple**
- **Each figure is made up of smaller squares**

3. Draw the pattern when $s = 4$.

4. How many tiles will there be when $s = 10$?
102 tiles

Unit 6 Lesson 1 592 Activity 1

Pairs | 15 min

Activity 1 A New Type of Pattern (continued)

Purpose: Students describe and extend a pattern that is neither linear nor exponential. **(MTR.5.1)**

Short on time? Consider omitting the discussion in the Launch.

2 Monitor

ELL **English Language Learners** Consider creating or reviewing an anchor chart that displays definitions of *linear relationship* and *exponential relationship* to support students in their explanations and reasoning.

Select and sequence a variety of responses and explanations to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you tell if a pattern is linear? Exponential?"
Think the pattern is linear because the first differences grow linearly.	**Support** Celebrate this special pattern they found and invite them to share it during the Connect. Encourage them to graph the table to see if it makes a line or remind them that the first difference is constant for linear relationships.
Would enjoy an additional challenge.	**Stretch** Encourage students to create a rule for the number of tiles for any value of s.

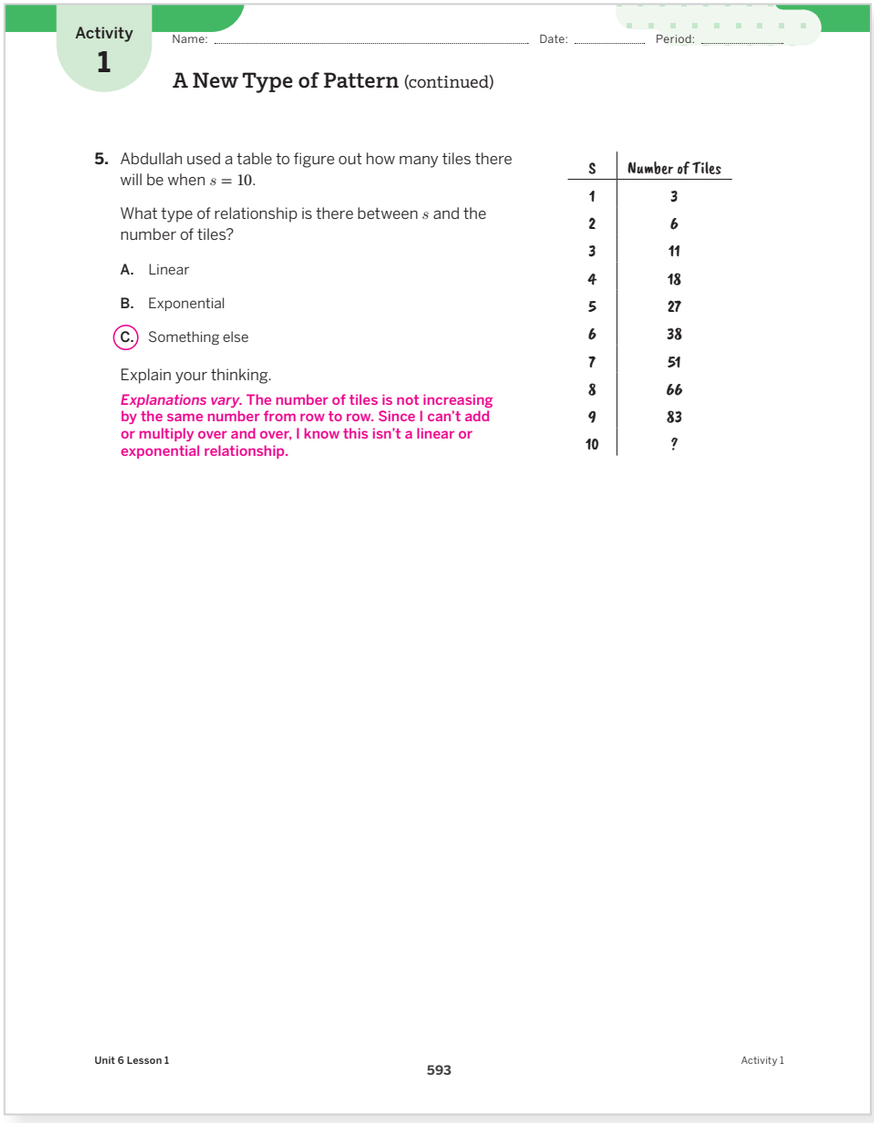
3 Connect

Invite students to share their strategies.

Consider using the **Think-Pair-Share routine**. After students have had individual think time to develop their responses, regroup students to share, reflect and revise their work with a partner. Then, share a selection of student responses to discuss as a class. **(MTR.4.1)**

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

To reveal the Key Takeaway, consider asking:
- "How do you know that the pattern is not linear? Not exponential?"
- "What other patterns do you see in the table?"



Activity 1

Name: _____ Date: _____ Period: _____

A New Type of Pattern (continued)

5. Abdullah used a table to figure out how many tiles there will be when $s = 10$.

What type of relationship is there between s and the number of tiles?

A. Linear

B. Exponential

C. Something else

Explain your thinking.

Explanations vary. The number of tiles is not increasing by the same number from row to row. Since I can't add or multiply over and over, I know this isn't a linear or exponential relationship.

s	Number of Tiles
1	3
2	6
3	11
4	18
5	27
6	38
7	51
8	66
9	83
10	?

Unit 6 Lesson 1 593 Activity 1

Key Takeaway: *Linear relationships* have a constant rate of change. *Exponential relationships* have a constant difference. Some patterns are not linear or exponential, but change in a predictable way.

Pairs | 15 min

Activity 2 Comparing Patterns

Purpose: Students use MLR7: Compare and Connect to explore how different descriptions of the structures of patterns might lead towards different strategies for extending those patterns. **(MTR.2.1, MTR.5.1)**

1 Launch

Encourage students to describe to their partner what shapes, parts, or structures they see in the visual patterns before starting. **(MTR.5.1)**

A **Accessibility: Memory and Attention** Invite students to use the Student Edition to compare all three figures simultaneously.

2 Monitor

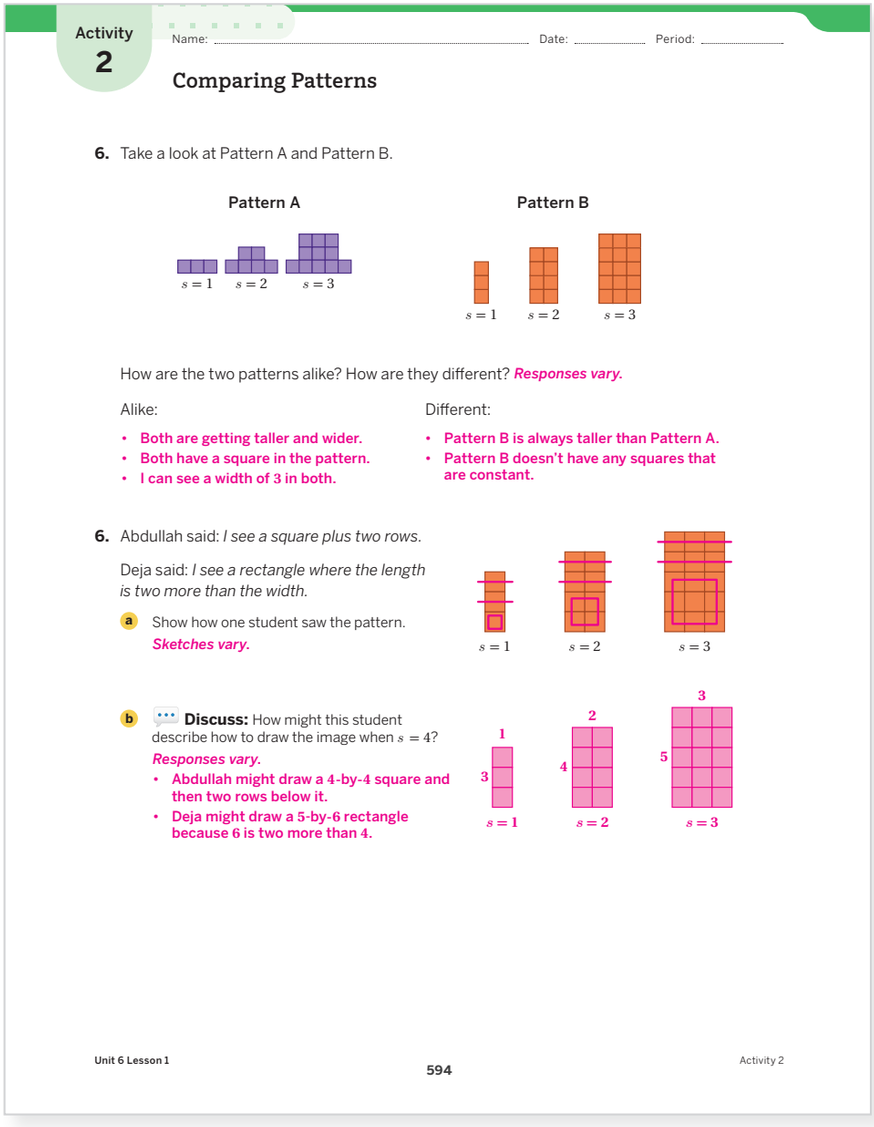
Encourage students to read others' responses and decide if others' noticings were similar to or different from their own.

ELL **English Language Learners** Invite students to read each description aloud in Problem 6 and paraphrase what it means. Clarify any unknown words. **(Reading, Speaking, and Listening)**

Select and sequence a variety of student sketches for each description to share during the Connect. **(MTR.2.1)**

D **Differentiation: (Stretch)** Invite students to think of a different way to describe the pattern in Problem 6.

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Comparing Patterns

6. Take a look at Pattern A and Pattern B.

Pattern A

Pattern B

How are the two patterns alike? How are they different? *Responses vary.*

Alike:

- Both are getting taller and wider.
- Both have a square in the pattern.
- I can see a width of 3 in both.

Different:

- Pattern B is always taller than Pattern A.
- Pattern B doesn't have any squares that are constant.

6. Abdullah said: *I see a square plus two rows.*

Deja said: *I see a rectangle where the length is two more than the width.*

a Show how one student saw the pattern.

Sketches vary.

b **Discuss:** How might this student describe how to draw the image when $s = 4$?

Responses vary.

- Abdullah might draw a 4-by-4 square and then two rows below it.
- Deja might draw a 5-by-6 rectangle because 6 is two more than 4.

Unit 6 Lesson 1 594 Activity 2

Pairs | 15 min

Activity 2 Comparing Patterns (continued)

Purpose: Students use MLR7: Compare and Connect to explore how different descriptions of the structures of patterns might lead towards different strategies for extending those patterns. **(MTR.2.1, MTR.5.1)**

Short on time? Consider facilitating Problem 6 as a whole class instead of individually.

2 Monitor

D **Differentiation: (Support)** Encourage students to use a strategy that their classmates shared in Activity 1.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore visual patterns further to complete this optional task. Encourage them to discuss their thinking with a partner.

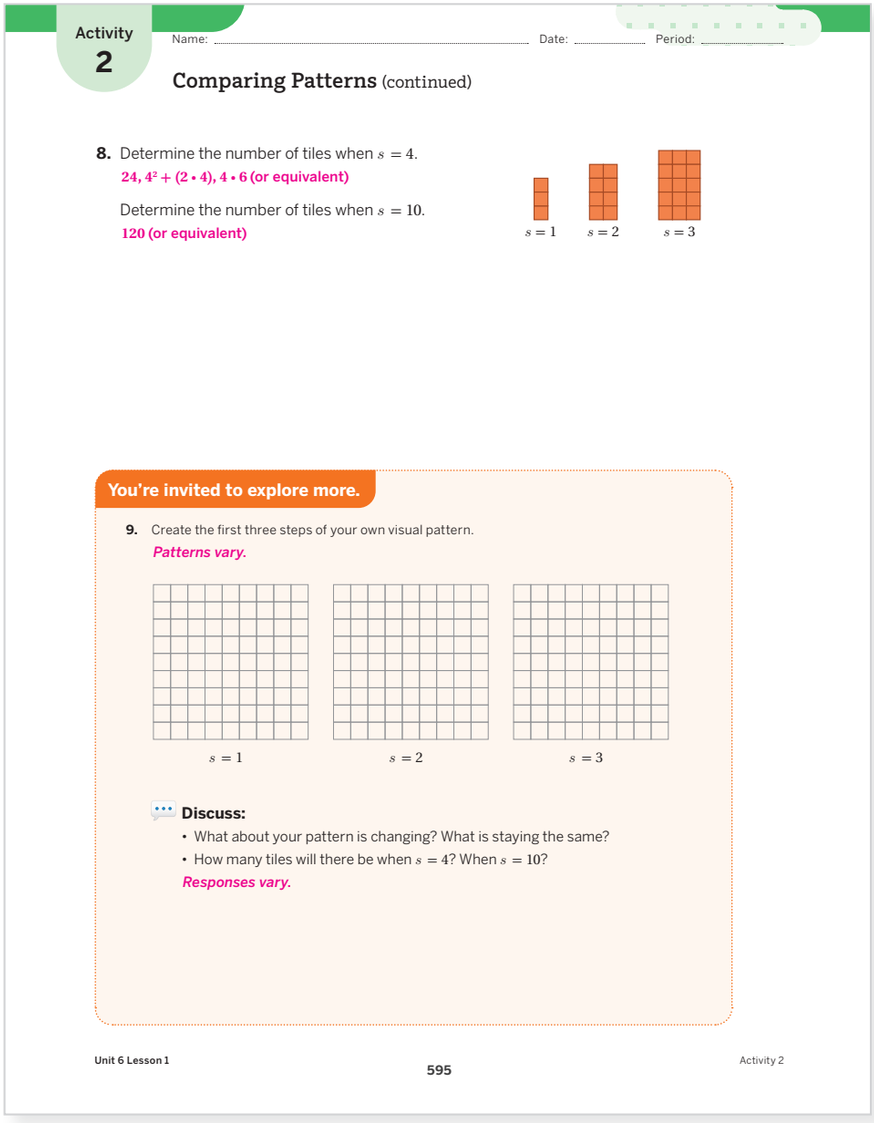
3 Connect

MLR **MLR7: Compare and Connect** Share student sketches to support students in analyzing and comparing each strategy.

Consider asking:

- "Whose description was closer to the way that you see it?"
- "How are Abdullah's and Deja's descriptions helpful for thinking about other values of s?" **(MTR.2.1)**
- "How might Abdullah determine the number of tiles when $s = 10$? Deja? What strategy would you use?"
- "How would each student see s in the figures?" **(MTR.2.1, MTR.5.1)**

D **Differentiation: (Stretch)** Consider asking, "How would each student write a rule to determine the number of tiles for any value of s?"



Activity 2

Name: _____ Date: _____ Period: _____

Comparing Patterns (continued)

8. Determine the number of tiles when $s = 4$.

24, $4^2 + (2 \cdot 4)$, $4 \cdot 6$ (or equivalent)

Determine the number of tiles when $s = 10$.

120 (or equivalent)

You're invited to explore more.

9. Create the first three steps of your own visual pattern.

Patterns vary.

Discuss:

- What about your pattern is changing? What is staying the same?
- How many tiles will there be when $s = 4$? When $s = 10$?

Responses vary.

Unit 6 Lesson 1 595 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for describing patterns. **(MTR.5.1)**

Invite students to respond independently, and then share their thinking with a partner.

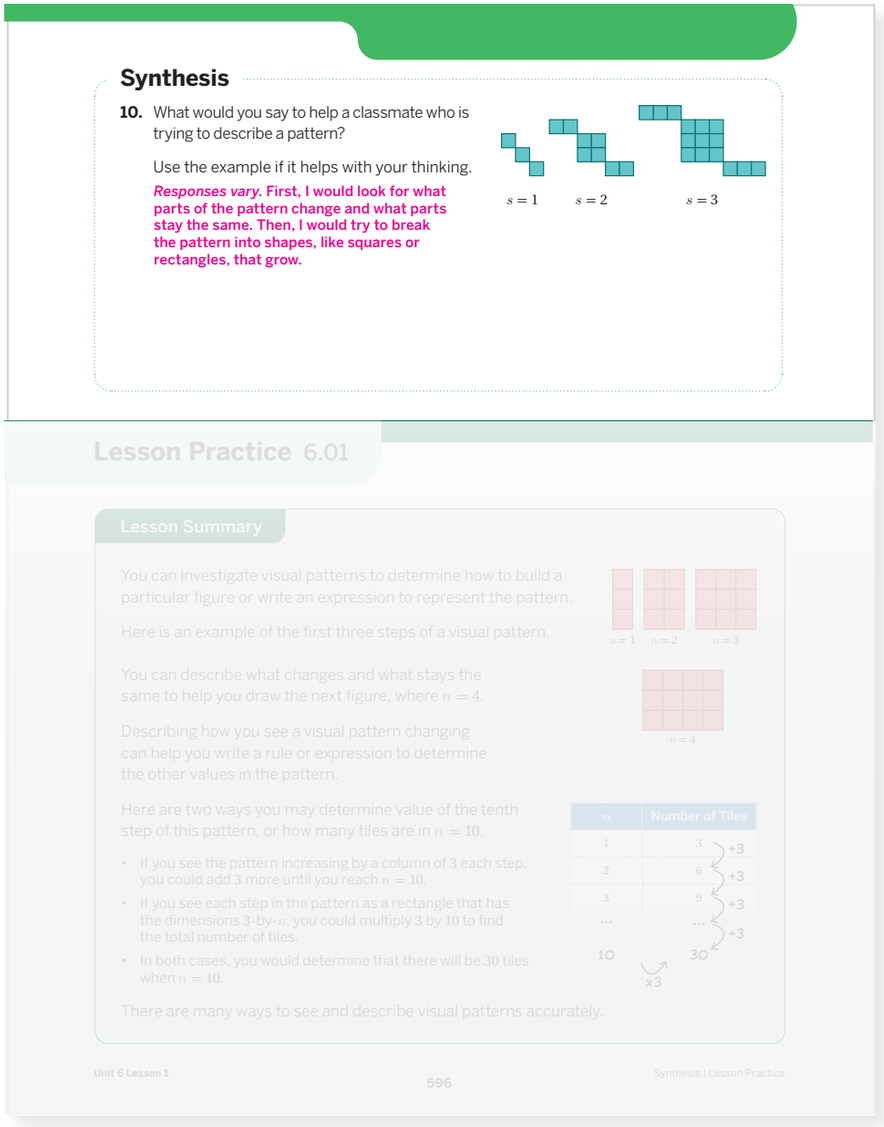
Listen for and share a variety of ideas, such as:

- Identifying which parts of the pattern stay the same and which change. **(MTR.5.1)**
- Looking for different shapes or structures in the pattern.
- Looking for where you can see the figure number in the pattern.
- Breaking the pattern into different parts.

Consider asking:

- "What words might you use to discuss this pattern? Why are you choosing those words?"
- "Why can describing a pattern in words be helpful?"

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. What would you say to help a classmate who is trying to describe a pattern?

Use the example if it helps with your thinking.

Responses vary. First, I would look for what parts of the pattern change and what parts stay the same. Then, I would try to break the pattern into shapes, like squares or rectangles, that grow.

$s = 1$ $s = 2$ $s = 3$

Lesson Practice 6.01

Lesson Summary

You can investigate visual patterns to determine how to build a particular figure or write an expression to represent the pattern.

Here is an example of the first three steps of a visual pattern.

$n = 1$ $n = 2$ $n = 3$

You can describe what changes and what stays the same to help you draw the next figure, where $n = 4$.

$n = 4$

Describing how you see a visual pattern changing can help you write a rule or expression to determine the other values in the pattern.

Here are two ways you may determine value of the tenth step of this pattern, or how many tiles are in $n = 10$.

- If you see the pattern increasing by a column of 3 each step, you could add 3 more until you reach $n = 10$.
- If you see each step in the pattern as a rectangle that has the dimensions 3-by-n, you could multiply 3 by 10 to find the total number of tiles.
- In both cases, you would determine that there will be 30 tiles when $n = 10$.

n	Number of Tiles
1	3
2	6
3	9
...	...
10	30

+3 +3 +3 +3 ×3

There are many ways to see and describe visual patterns accurately.

Unit 6 Lesson 1 596 Synthesis | Lesson Practice

Lesson Takeaway: Patterns can be described in many ways. Often, different descriptions can help identify different structures within patterns, which may be helpful for extending them and making predictions.

Independent | 5 min

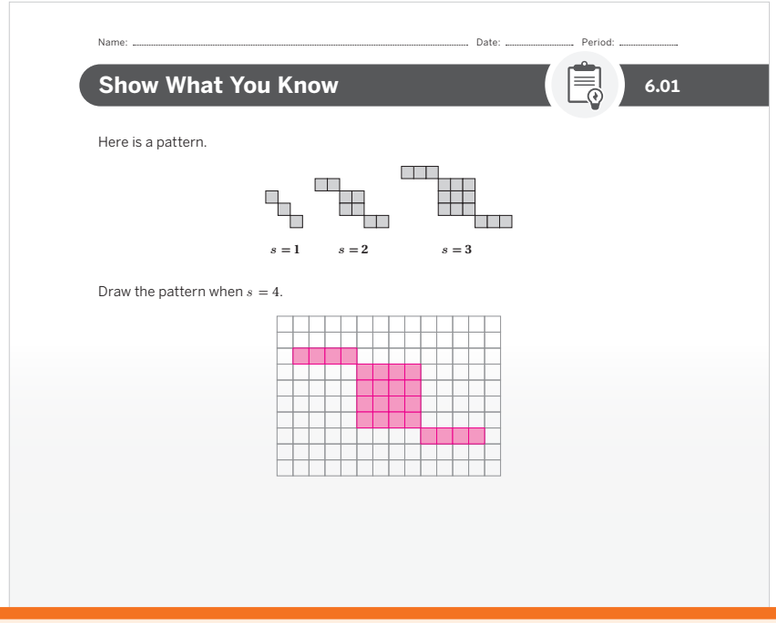
Show What You Know

Purpose: Students demonstrate their understanding by extending a visual pattern.

Today's Goals

1. **Goal:** Analyze patterns to determine the output value for a given input value.
2. **Language Goal:** Describe patterns using words or by writing an expression. **(Writing, Speaking, and Listening)**
3. **Language Goal:** Compare and contrast linear and non-linear relationships. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.01

Here is a pattern.

$s = 1$ $s = 2$ $s = 3$

Draw the pattern when $s = 4$.

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to describe and extend patterns:

» **Respond to Student Thinking:**

- Students will have more opportunities to describe and extend patterns in Lesson 2.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.
- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

Professional Learning

Reflect on the strategies students used today. What opportunities were there to share specific students' strategies with the class and celebrate them? What facilitation techniques (e.g., naming strategies after the students) can help make this a part of your classroom culture? How might you continue to reference student strategies from today in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. What would you say to help a classmate who is trying to describe a pattern?

Use the example if it helps with your thinking.

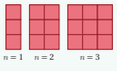
Responses vary. First, I would look for what parts of the pattern change and what parts stay the same. Then, I would try to break the pattern into shapes, like squares or rectangles, that grow.

Lesson Practice 6.01

Lesson Summary

You can investigate visual patterns to determine how to build a particular figure or write an expression to represent the pattern.

Here is an example of the first three steps of a visual pattern.



You can describe what changes and what stays the same to help you draw the next figure, where $n = 4$.



Describing how you see a visual pattern changing can help you write a rule or expression to determine the other values in the pattern.

Here are two ways you may determine value of the tenth step of this pattern, or how many tiles are in $n = 10$.

- If you see the pattern increasing by a column of 3 each step, you could add 3 more until you reach $n = 10$.
- If you see each step in the pattern as a rectangle that has the dimensions 3-by-n, you could multiply 3 by 10 to find the total number of tiles.
- In both cases, you would determine that there will be 30 tiles when $n = 10$.

n	Number of Tiles
1	3
2	6
3	9
...	...
10	30

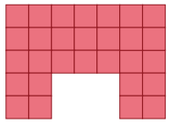
+3, +3, +3, +3, x3

There are many ways to see and describe visual patterns accurately.

Lesson Practice 6.01

Name: _____ Date: _____ Period: _____

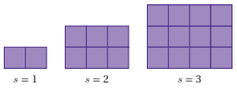
1. *Select all* of the expressions that could represent the number of tiles in this diagram.



- ☐ A. $5 \cdot 7$
- ☑ B. $5 \cdot 7 - 6$
- ☐ C. $3 \cdot 7 + 2 \cdot 2$
- ☑ D. $5 \cdot 7 - 2 \cdot 3$
- ☑ E. $2 \cdot 5 + 3 \cdot 3 + 2 \cdot 5$

2. Here are the first three steps in a pattern. How many tiles will there be when $s = 10$?

110 tiles



Explain your thinking.

Explanations vary. **I saw that each step is a square with the side length of s and an extra column with s squares. So $s = 10$ would be a 10-by-10 square with an extra column of 10. $10 \cdot 10 + 10 = 110$ tiles.**

3. What type of relationship does the pattern in the table represent? Circle one.

Linear (Exponential) Neither

s	Number of Tiles
1	3
2	9
3	27
4	81

Explain your thinking.

Explanations vary. **As s increases by 1, the number of tiles changes by a common ratio of 3.**

Lesson Practice 6.01

Name: _____ Date: _____ Period: _____

Test Practice

Problems 4–6: A teacher gives her class a table with only the first two rows in the pattern.

s	Number of Tiles
1	5
2	25

4. Rishi says the pattern is an exponential relationship.

Ichiro says there is not enough information to be sure.

Whose thinking is correct? Explain your thinking.

Ichiro's. *Explanations vary.* If the number of tiles is increasing by 20, then the pattern is linear. If the number of tiles is being multiplied by 5, then the pattern is exponential.

5. How many tiles would be in the next step if the relationship were *linear*?

45 tiles

6. How many tiles would be in the next step if the relationship were *exponential*?

125 tiles

Spiral Review

7. Juana began hiking at 6:00 AM. At noon, she had hiked 12 miles. At 4:00 PM, Juana finished her hike with a total distance of 26 miles.

On average, during which time interval was Juana hiking faster? Circle one.

6:00 AM to noon (Noon to 4:00 PM)

Explain your thinking.

Explanations vary. **From 6:00 AM to noon, Juana hiked 12 miles in 6 hours, so her average speed was 2 miles per hour. From noon to 4:00 PM, Juana hiked another 14 miles in 4 hours. I knew that because $26 - 12 = 14$. That means her average speed was 3.25 miles per hour.**

Problems 8–10: Use the distributive property to write an equivalent expression for each.

8. $3(n + 5)$
$3n + 15$ (or equivalent)

9. $16x + 20$
$4(4x + 5)$ (or equivalent)

10. $2(4m - 6)$
$8m - 12$
$4(2m - 3)$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.912.F.1.8
	2–3	2	MA.912.F.1.8
Test Practice	**4–6**	2	MA.912.F.1.8
Spiral Review			
	7	1	MA.8.AR.3.5
	8–10	1	MA.6.NSO.3.2

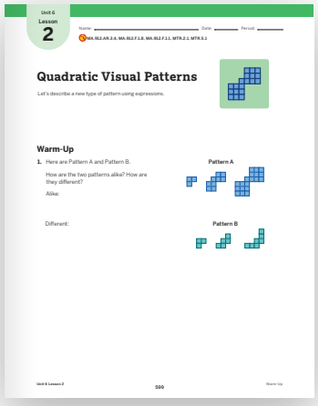
Unit 6

Lesson 2

Quadratic Visual Patterns

Expressions for Quadratic Patterns

Let's describe a new type of pattern using expressions.


Unit 6
Lesson 2
Name: Date: Period:
Quadratic Visual Patterns
Let's describe a new type of pattern using expressions.
Warm-Up
1. Here are Pattern A and Pattern B.
How are the two patterns alike? How are they different?
Alike:
Different:
Pattern A
Pattern B
599

Learning Outcomes

Today's Goals

1. **Goal:** Write expressions to represent *quadratic relationships* from patterns.
2. **Language Goal:** Explain that a quadratic expression contains a squared term. **(Writing, Speaking, and Listening)**

Students write expressions to represent quadratic patterns. They generalize by writing expressions and connect those expressions to the structure of the figures. Finally, they are introduced to the term quadratic relationship to describe patterns that grow in two dimensions and expressions with a quantity squared. **(MTR.2.1, MTR.5.1)**

Coherence

Prior Learning

In Lesson 1, students described and extended visual patterns that were neither linear nor exponential. In Unit 1, students described and extended patterns with constant differences and constant ratios. They also defined them recursively and explicitly. In Unit 5, students classified relationships in multiple representations as linear or exponential.

Future Learning

In Lesson 3, students will learn how to identify quadratic relationships in tables using a constant second difference.

Connections

- Students relate visual models of quadratic relationships to equivalent algebraic expressions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.8
Determine whether a linear, quadratic or exponential function best models a given real-world situation.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.3.1	MA.8.AR.3.1

Vocabulary

New Vocabulary

quadratic relationship

Lesson at a Glance

~ 45 min

MA.912.F.1.8

Warm-Up

Pairs | 5 min

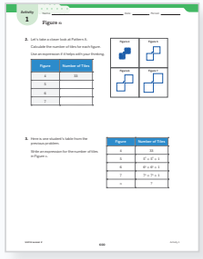
Students compare and contrast the structures of two visual patterns. **(MTR.2.1, MTR.5.1)**
Routine: Think-Pair-Share



Activity 1

Pairs | 8 min

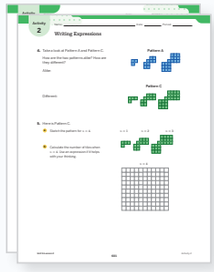
Students reason abstractly and quantitatively to determine the number of tiles in different figures and write an expression to represent the number of tiles in any figure. **(MTR.2.1)**



Activity 2

Pairs | 12 min

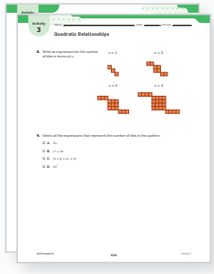
Students use the structure of visual patterns involving squares to write expressions that represent the number of tiles. **(MTR.2.1, MTR.5.1)**
Routines: Think-Pair-Share, MLR7: Compare and Connect



Activity 3

Pairs | 10 min

Students write an expression to represent a new visual pattern and describe how to determine if an expression or a pattern represents a quadratic relationship. **(MTR.2.1)**
Routines: Decide and Defend, MLR2: Collect and Display



Synthesis

Whole Class | 5 min

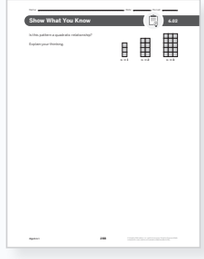
Students synthesize their understanding of how to write an expression to represent a quadratic visual pattern.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining if a pattern is quadratic or not and explaining why.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity Sheet
  - Calculator

Pairs | 5 min

Warm-Up

Purpose: Students compare and contrast the structures of two visual patterns. **(MTR.2.1, MTR.5.1)**

1 Launch

To access prior knowledge, consider asking, "What strategies can be helpful for analyzing visual patterns? What were some key ideas from Lesson 1?"

- Look for shapes, parts, or structures in the pattern.
- Identify which parts stay the same and which parts change.
- Look for where you can see the figure number in the pattern.

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner.

A **Accessibility: Visual-Spatial Processing** To support students in making visual connections between the two patterns, invite students to draw or highlight in their Student Edition.

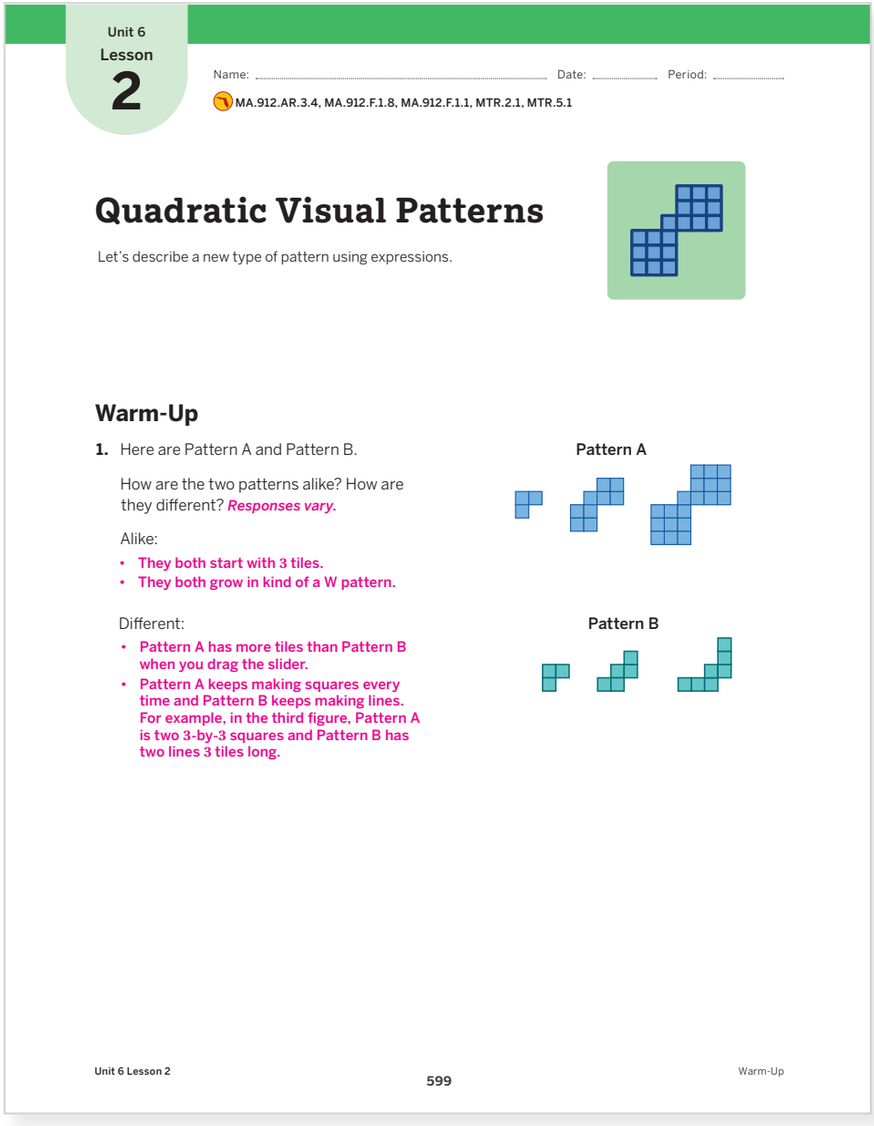
ELL **English Language Learners** Provide a word bank to support students in expressing their ideas throughout this lesson: *length, width, height, area, side, square, rectangle, rows, columns, figure,* etc. **(Writing and Speaking)**

2 Connect

Invite students to share their or their partner's thinking with the class.

Note: If it does not come up naturally, consider asking:

- "What shapes (e.g., lines, squares, etc.) do you see growing in Pattern A? Pattern B?"
- "Is anything staying the same?"



Unit 6
Lesson
2

Name: _____ Date: _____ Period: _____

MA.912.AR.3.4, MA.912.F.1.8, MA.912.F.1.1, MTR.2.1, MTR.5.1

Quadratic Visual Patterns

Let's describe a new type of pattern using expressions.

Warm-Up

1. Here are Pattern A and Pattern B.

How are the two patterns alike? How are they different? ***Responses vary.***

Alike:

- **They both start with 3 tiles.**
- **They both grow in kind of a W pattern.**

Different:

- **Pattern A has more tiles than Pattern B when you drag the slider.**
- **Pattern A keeps making squares every time and Pattern B keeps making lines. For example, in the third figure, Pattern A is two 3-by-3 squares and Pattern B has two lines 3 tiles long.**

Pattern A

Pattern B

Unit 6 Lesson 2 599 Warm-Up

Pairs | 8 min

Activity 1 Figure n

Purpose: Students reason abstractly and quantitatively to determine the number of tiles in different figures and write an expression to represent the number of tiles in any figure. **(MTR.2.1)**

1 Launch

Invite students to describe Figure 4 in words.

Consider asking, "Without counting one by one, how could you convince someone there are 33 tiles?"

Encourage students to share their responses as expressions and use a calculator to calculate the total. Invite student pairs to confirm each others' totals, revising as needed.

2 Monitor

To support students getting started, encourage students to label side lengths they know.

Look for students who use the strategy in the next problem ($5^2 + 5^2 + 1$) or write other creative expressions. Invite them to sketch where they see each part of the expression in the figures. **(MTR.2.1, MTR.5.1)**

A **Accessibility: Visual-Spatial Processing**
Encourage students to sketch where they see each term of $5^2 + 5^2 + 1$ on Figure 5. **(MTR.2.1, MTR.5.1)**

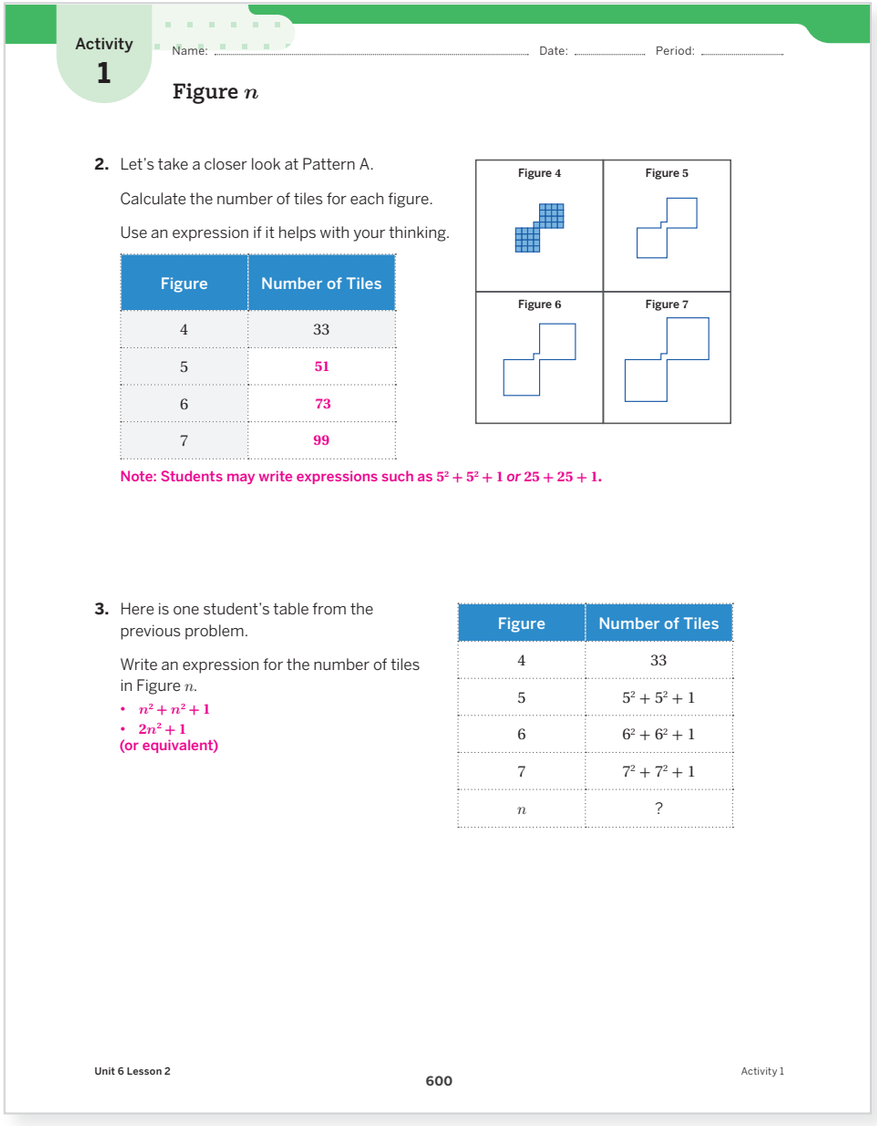
Look for a variety of expressions students write to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support determining where to put the variable in the expression.	**Support** Consider asking: • "Where do you see the figure number in each expression?" • "What stays the same in each expression? What changes?" **(MTR.2.1)**
Want to explore this pattern further.	**Stretch** Encourage students to write different expressions for Figure n and show how they are equivalent to their original expression.

3 Connect

Invite students to share a variety of expressions they wrote to represent the tiles in Figure n. For each expression, consider asking, "How do you see the parts of this expression in the pattern?" **(MTR.2.1, MTR.5.1)**



Activity 1

Name: Date: Period:

Figure n

2. Let's take a closer look at Pattern A.

Calculate the number of tiles for each figure.

Use an expression if it helps with your thinking.

Figure	Number of Tiles
4	33
5	51
6	73
7	99

Note: Students may write expressions such as $5^2 + 5^2 + 1$ *or* $25 + 25 + 1$.

3. Here is one student's table from the previous problem.

Write an expression for the number of tiles in Figure n.

- $n^2 + n^2 + 1$
- $2n^2 + 1$

(or equivalent)

Figure	Number of Tiles
4	33
5	$5^2 + 5^2 + 1$
6	$6^2 + 6^2 + 1$
7	$7^2 + 7^2 + 1$
n	?

Unit 6 Lesson 2 600 Activity 1

Pairs | 13 min

Activity 2 Writing Expressions

Purpose: Students use the structure of visual patterns involving squares to write expressions that represent the number of tiles. **(MTR.2.1, MTR.5.1)**

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

Note: If it does not come up naturally, consider asking, "What shapes do you see growing in Pattern A? Pattern C?"

A Accessibility: Visual-Spatial Processing
To support students in making visual connections between the two patterns, invite students to draw or highlight in their Student Edition.

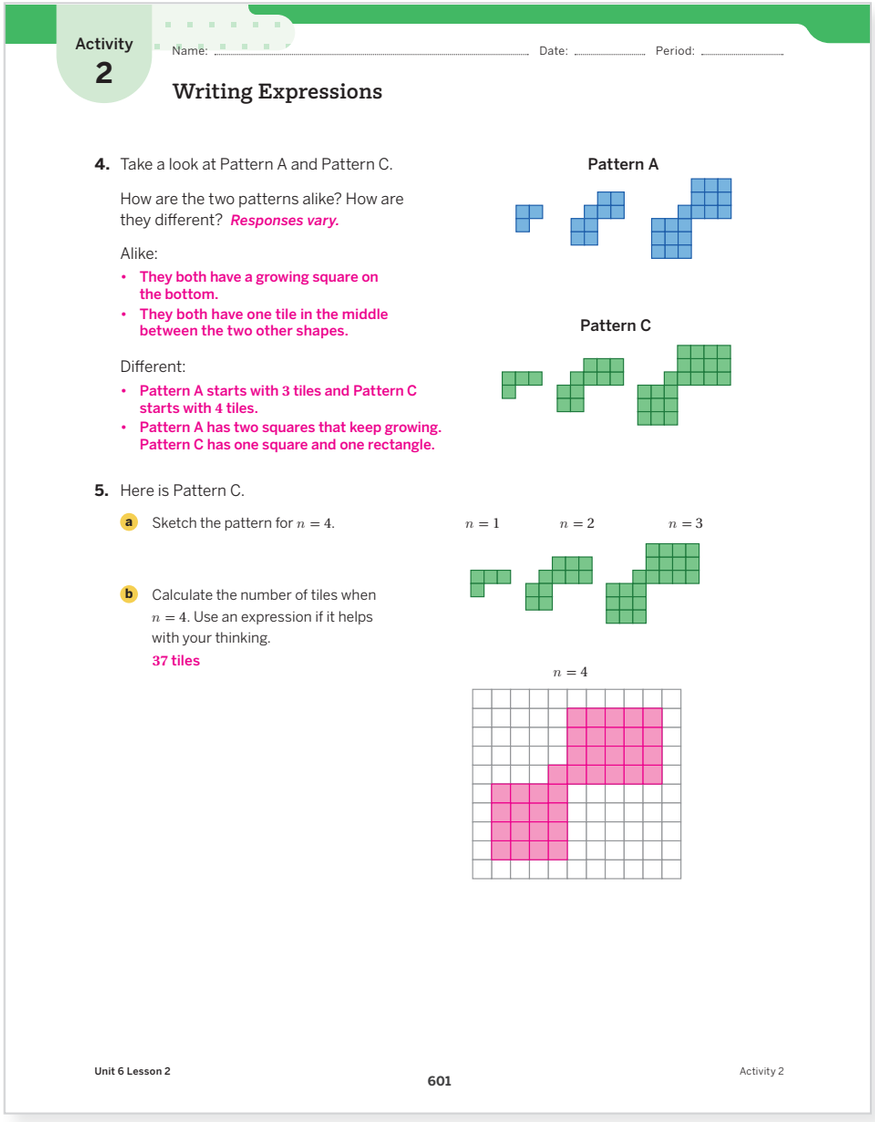
2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to describe in words what the pattern for $n = 4$ would look like before responding.
Think each figure is symmetrical and that there are 33 tiles when $n = 4$.	**Support** Consider asking, "Is the figure for $n = 2$ symmetrical? How do you know?"
Sketch the pattern using structure by labeling the dimensions rather than drawing each individual tile.	**Strengthen** Celebrate how these students made sense of the structure. **(MTR.2.1, MTR.5.1)**
Write creative expressions like: • $4^2 + 1 + 4^2 + 4$ • $4(4) + 1 + 4(5)$ • $2(4^2) + 4 + 1$	**Strengthen** Invite them to show where they see each part of the expression in their sketch. Celebrate how these students made sense of the structure. **(MTR.2.1, MTR.5.1)**

A Accessibility: Executive Functioning
To support organization in problem solving, consider chunking this activity by inviting students to calculate the number of tiles for $n = 2$ and $n = 3$ without counting one by one before they write an expression for $n = 4$.

Activity 2 continued >



Activity 2

Name: Date: Period:

Writing Expressions

4. Take a look at Pattern A and Pattern C.

How are the two patterns alike? How are they different? *Responses vary.*

Alike:

- They both have a growing square on the bottom.
- They both have one tile in the middle between the two other shapes.

Different:

- Pattern A starts with 3 tiles and Pattern C starts with 4 tiles.
- Pattern A has two squares that keep growing. Pattern C has one square and one rectangle.

Pattern A

Pattern C

5. Here is Pattern C.

a. Sketch the pattern for $n = 4$.

$n = 1$ $n = 2$ $n = 3$

b. Calculate the number of tiles when $n = 4$. Use an expression if it helps with your thinking.

37 tiles

$n = 4$

Unit 6 Lesson 2 601 Activity 2

Pairs | 13 min

Activity 2 Writing Expressions (continued)

Purpose: Students use the structure of visual patterns involving squares to write expressions that represent the number of tiles. **(MTR.2.1, MTR.5.1)**

2 Monitor

Encourage students to consider making a sketch to show their thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to describe or sketch what they think $n = 10$ looks like on their paper and write an expression for the number of tiles when $n = 10$.
Need support writing an expression.	**Support** Consider displaying students' expressions from Problem 3 (e.g., $n^2 + n^2 + 1$) and asking, "How can we change the expression so it shows Pattern C instead of Pattern A?"
Write expressions similar to Luis's and Ishaan's in Problem 7 or other creative expressions.	**Strengthen** Invite these students to share their thinking during the Connect.

Invite students to think about Luis and Ishaan's strategies.

Encourage students to sketch to show where they can see each part of Luis's and Ishaan's expressions in the figures.

Select and sequence student sketches for both expressions to share during the Connect.

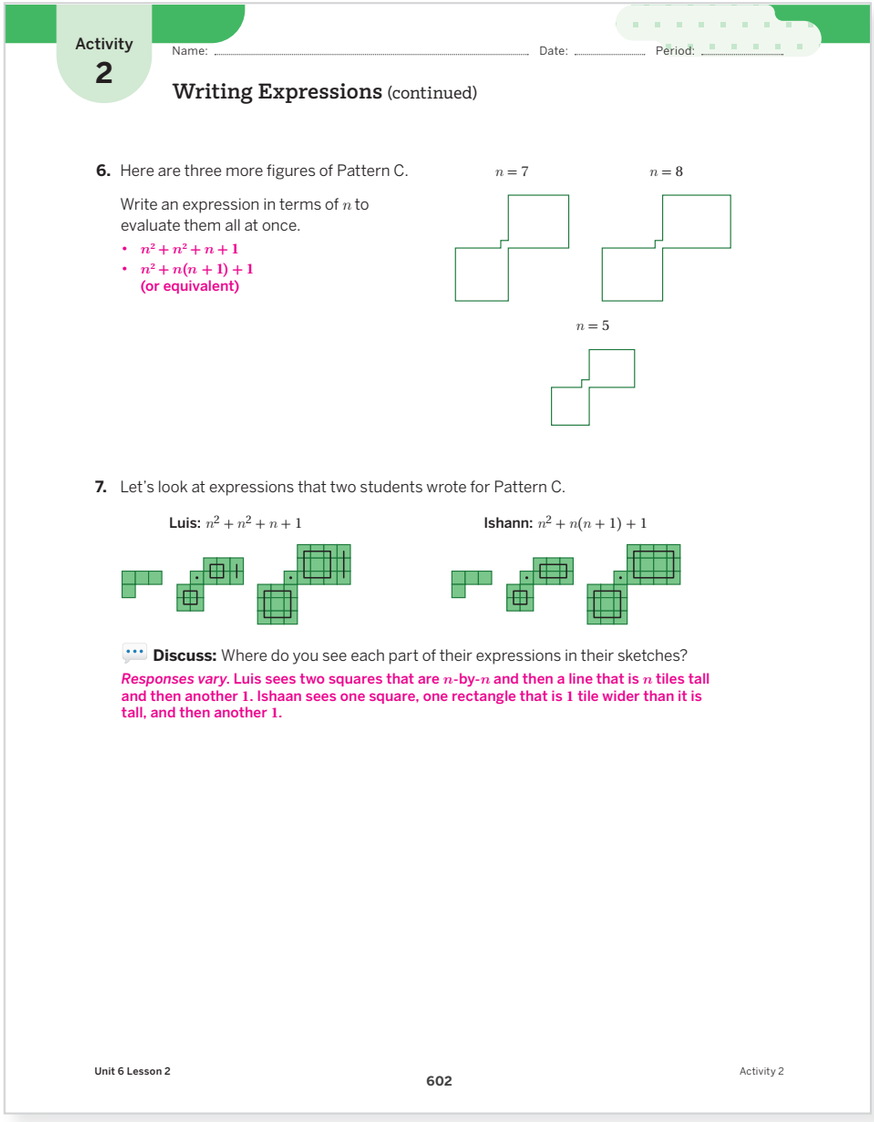
ELL English Language Learners Provide sentence frames to help students explain their thinking (e.g., I see the _____ in Luis's/Ishaan's expression in the _____ in the figure.). **(Speaking and Listening)**

3 Connect

MLR MLR7: Compare and Connect Share student sketches to support students in analyzing and comparing the two strategies.

To reveal the Key Takeaway, consider asking:

- "Where do you see the n^2 in Luis's sketch? The n? The 1?"
- "Where do you see the $n(n + 1)$ in Ishaan's sketch?"
- "What about the pattern requires the expressions to have n^2 (or equivalent) in them?"
- "Whose way of seeing the pattern is closer to the way you saw it? Did anyone see it in a different way?"



Activity 2

Name: Date: Period:

Writing Expressions (continued)

6. Here are three more figures of Pattern C.

Write an expression in terms of n to evaluate them all at once.

- $n^2 + n^2 + n + 1$
- $n^2 + n(n + 1) + 1$ (or equivalent)

$n = 7$ $n = 8$ $n = 5$

7. Let's look at expressions that two students wrote for Pattern C.

Luis: $n^2 + n^2 + n + 1$ **Ishann:** $n^2 + n(n + 1) + 1$

Discuss: Where do you see each part of their expressions in their sketches?

Responses vary. Luis sees two squares that are n-by-n and then a line that is n tiles tall and then another 1. Ishaan sees one square, one rectangle that is 1 tile wider than it is tall, and then another 1.

Unit 6 Lesson 2 602 Activity 2

Key Takeaway: Analyzing the structure of a pattern is a helpful strategy for writing an expression. Some interpretations may result in different, but equivalent, expressions. Patterns that grow in two dimensions are modeled by expressions with a variable squared.

Pairs | 10 min

Activity 3 Quadratic Relationships

Purpose: Students write an expression to represent a new visual pattern and describe how to determine if an expression or a pattern represents a **quadratic relationship**. **(MTR.2.1)**

Short on time? Consider omitting the discussion in Problem 10 if the class shows a strong understanding.

1 Launch

Consider asking, "Why is writing an expression in terms of n helpful?"

2 Monitor

To support students getting started, consider asking:

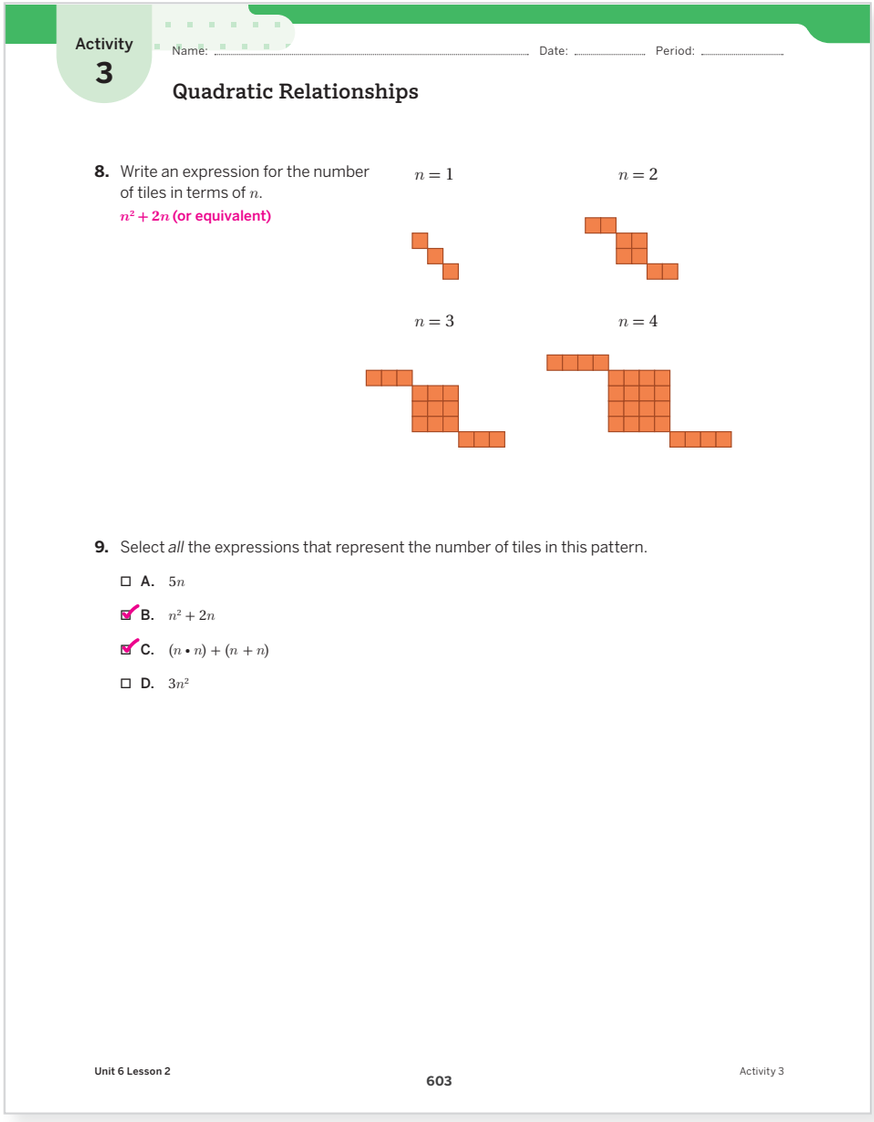
- "What is growing? What is staying the same?"
- "Where do you see the figure number show up in each figure?" **(MTR.5.1)**
- "How would you describe the structure of the figure using words?" **(MTR.2.1)**

To access connections, consider asking,

- "For the expressions you chose, where do you see each term in the figures?" **(MTR.5.1)**
- "Which expressions are equivalent? How do you know?"

A **Accessibility: Executive Functioning**
Consider chunking this activity by inviting students to choose any expression and decide if it represents the tiles in the pattern, then repeat. It may be helpful to invite them to use a calculator to evaluate the expression at each value of n.

Activity 3 continued >



Activity 3

Name: Date: Period:

Quadratic Relationships

8. Write an expression for the number of tiles in terms of n.

$n^2 + 2n$ (or equivalent)

$n = 1$ $n = 2$ $n = 3$ $n = 4$

9. Select *all* the expressions that represent the number of tiles in this pattern.

- ☐ A. $5n$
- ☑ B. $n^2 + 2n$
- ☑ C. $(n \cdot n) + (n + n)$
- ☐ D. $3n^2$

Unit 6 Lesson 2 603 Activity 3

Pairs | 10 min

Activity 3 Quadratic Relationships (continued)

Purpose: Students write an expression to represent a new visual pattern and describe how to determine if an expression or a pattern represents a **quadratic relationship**. **(MTR.2.1)**

2 Monitor

Encourage students to analyze both the expressions and the patterns.

Select and sequence student responses to share during the Connect.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to focus on just the quadratic expressions or just the quadratic patterns to start. Consider asking, "What is the same about all of them? What is different?"

ELL **English Language Learners** Consider inviting them to research how to say quadratic in their home language. **(Reading and Writing)**

D **Differentiation: (Stretch)** Encourage students to make their own pattern or expression and trade with a partner to determine if it is a quadratic relationship or not.

3 Connect

Invite students to share their responses with a peer or small group. Listen for mathematical dialog and make a selection of responses to share with the large group. Poll students to share views of conflict or consensus, to develop further understanding.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Invite students to share their reasoning for whether or not this expression could represent the number of tiles. Encourage students to connect the parts of the expression to the figures. **(MTR.2.1)**

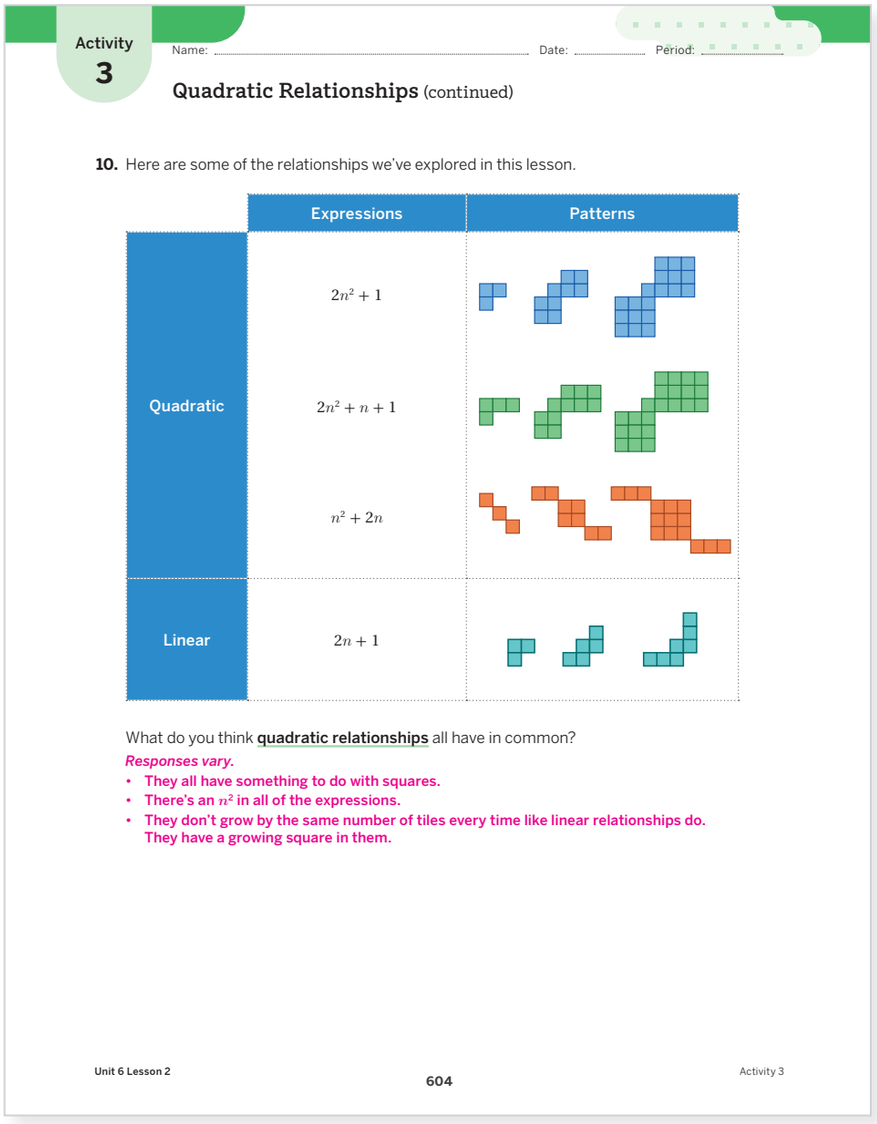
Note: If it doesn't come up naturally, consider asking:

- "What advice would you give to someone who selected $3n^2$?"
- "Which parts of the pattern are growing linearly? Which parts are not growing linearly? How can you tell?" **(MTR.5.1)**

MLR **MLR2: Collect and Display** Use this routine to collect students' language to create a class definition for **quadratic relationships**. Display for students to reference and add to for the rest of the unit.

To reveal the Key Takeaway, consider asking:

- "How do you know if an expression shows a quadratic relationship?"
- "How do you know if a pattern shows a quadratic relationship?"
- "Which expressions on the previous problem are quadratic?"



Activity 3

Name: Date: Period:

Quadratic Relationships (continued)

10. Here are some of the relationships we've explored in this lesson.

	Expressions	Patterns
Quadratic	$2n^2 + 1$	
	$2n^2 + n + 1$	
	$n^2 + 2n$	
Linear	$2n + 1$	

What do you think **quadratic relationships** all have in common?

Responses vary.

- They all have something to do with squares.
- There's an n^2 in all of the expressions.
- They don't grow by the same number of tiles every time like linear relationships do. They have a growing square in them.

Unit 6 Lesson 2 604 Activity 3

Key Takeaway: Patterns that grow in two dimensions can be represented with expressions that involve multiplying a variable times itself. They are called **quadratic relationships**.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to write an expression to represent a quadratic visual pattern.

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening. **(Reading, Speaking, and Listening)**

A **Accessibility: Affective Functioning** To support students in sharing their thinking, check in with students in advance to let them know you will ask for their thoughts.

Listen for and share a variety of ideas, such as:

- Looking for where you can see the figure number in the pattern. **(MTR.5.1)**
- Identifying which parts of the pattern stay the same and which change.
- Describing the pattern in words first.
- Looking for different shapes or structures in the pattern. **(MTR.5.1)**
- Breaking the pattern into different parts. **(MTR.2.1, MTR.5.1)**

Consider asking, "Why is it helpful to write an expression for a pattern?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. What would you say to help a classmate who is trying to write an expression for a quadratic relationship?

Use this pattern if it helps with your thinking.

Responses vary. To write an expression for a quadratic relationship, first look for what is growing and what is staying the same. If what is growing is a square, then maybe there is an n^2 in the expression. It can be helpful to look for places where you see the value of n in the expression. For example, in this pattern, there is always a square whose sides are the value of n and then a line that is n tiles long, so the expression is $n^2 + n$.

Lesson Practice 6.02

Lesson Summary

You can determine if a visual pattern represents a linear, exponential, or quadratic relationship, or something else.

If you can observe a square that is changing throughout a pattern, the relationship might be quadratic and you can use a squared term to write a quadratic expression.

Here are some strategies you might use to write an expression to represent a visual pattern:

You can look for what is changing and staying the same.

- The 3 outside tiles stay the same.
- The interior square is growing from 1 to 4 to 9.

Figure 1 Figure 2 Figure 3

You can look for where you see the figure number, n, in each diagram.

- You can see the figure number as the side length of the growing square.
- You can write each number of tiles as an expression.

Figure 1 $1^2 + 3$ Figure 2 $2^2 + 3$ Figure 3 $3^2 + 3$

You can write a quadratic expression to represent the number of tiles in Figure n as $n^2 + 3$.

Unit 6 Lesson 2 605 Synthesis | Lesson Practice

Lesson Takeaway: Analyzing the structure of a pattern is a strategy used to write an expression. Patterns that grow in two dimensions (e.g., a growing square) are expressions with a variable squared, called **quadratic relationships**.

Independent | 5 min

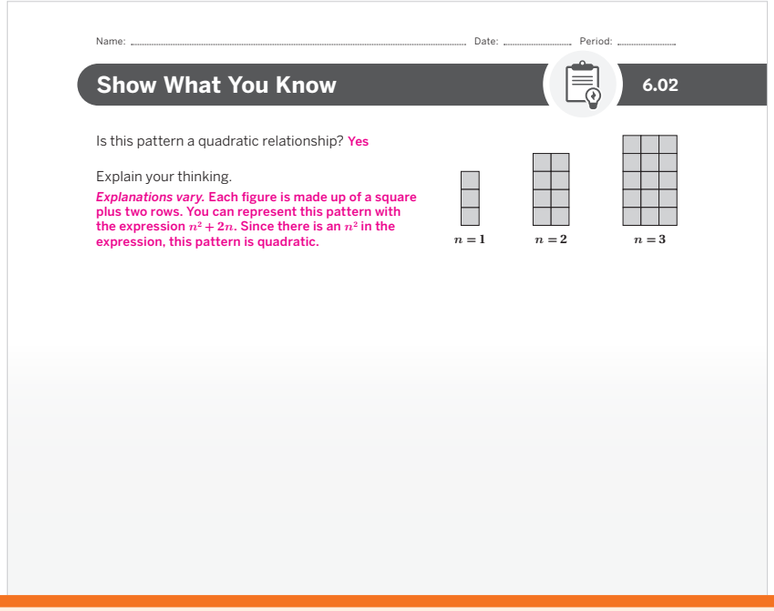
Show What You Know

Purpose: Students demonstrate their understanding by determining if a pattern is quadratic or not and explaining why.

Today's Goals

1. **Goal:** Write expressions to represent quadratic relationships from patterns.
2. **Language Goal:** Explain that a quadratic expression contains a squared term. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.02

Is this pattern a quadratic relationship? Yes

Explain your thinking.

Explanations vary. Each figure is made up of a square plus two rows. You can represent this pattern with the expression $n^2 + 2n$. Since there is an n^2 in the expression, this pattern is quadratic.

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write expressions to describe patterns:

» **Respond to Student Thinking:**

- Revisiting **Unit 6, Lesson 1** (Revisiting Visual Patterns).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.
- Invite students to create their own visual pattern. Encourage them to trade with a partner to determine if they are quadratic or not, extend the pattern, and write an expression to model it.

Professional Learning

Take a moment to reflect on how students looked for and made use of structure in visual patterns in order to write expressions. How did students describe structures they saw? How did they connect these structures to the expressions they wrote? How can understanding the structure of a quadratic pattern help students understand conceptually where squared variables come from in quadratic expressions? **(MTR.2.1, MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. What would you say to help a classmate who is trying to write an expression for a quadratic relationship?

Use this pattern if it helps with your thinking.

Responses vary. To write an expression for a quadratic relationship, first look for what is growing and what is staying the same. If what is growing is a square, then maybe there is an n^2 in the expression. It can be helpful to look for places where you see the value of n in the expression. For example, in this pattern, there is always a square whose sides are the value of n and then a line that is n tiles long, so the expression is $n^2 + n$.

Lesson Practice 6.02

Lesson Summary

You can determine if a visual pattern represents a linear, exponential, or **quadratic relationship**, or something else.

If you can observe a square that is changing throughout a pattern, the relationship might be quadratic and you can use a squared term to write a quadratic expression.

Here are some strategies you might use to write an expression to represent a visual pattern:

You can look for what is changing and staying the same.

- The 3 outside tiles stay the same.
- The interior square is growing from 1 to 4 to 9.

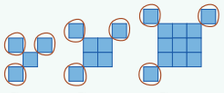


Figure 1 Figure 2 Figure 3

You can look for where you see the figure number, n, in each diagram.

- You can see the figure number as the side length of the growing square.
- You can write each number of tiles as an expression.

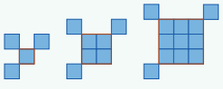


Figure 1 $1^2 + 3$ Figure 2 $2^2 + 3$ Figure 3 $3^2 + 3$

You can write a quadratic expression to represent the number of tiles in Figure n as $n^2 + 3$.

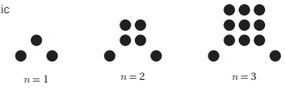
Lesson Practice 6.02

Name: ..... Date: ..... Period: .....

Problems 1–4: Circle whether each expression represents a quadratic relationship, linear relationship, or neither.

1. $2^n + n$	Quadratic	Linear	(Neither)
2. $2n^2 + 2$	(Quadratic)	Linear	Neither
3. $2(n - 5)$	Quadratic	(Linear)	Neither
4. $2n(n + 3)$	(Quadratic)	Linear	Neither

5. Does this pattern show a quadratic relationship?

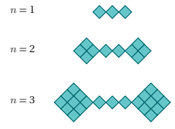


Yes

Explain your thinking.

Explanations vary. This pattern is quadratic because each figure contains a square that is related to the figure number (1^2, 2^2, 3^2).

6. Write an expression to represent the relationship between the figure number, n, and the total number of tiles.

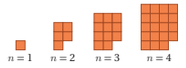


Responses vary.

- $2n^2 + n$
- $n^2 + n + n^2$
- $n(2n + 1)$

7. Three students each wrote different, but correct, expressions to represent this pattern.

Ethan	Ama	Nasir
$n^2 + (n - 1)$	$n(n + 1) - 1$	$n^2 + n - 1$



Choose a student's expression and describe how it matches the pattern.

Responses vary.

- Ethan sees a square, n^2, and an extra row that is 1 tile shorter than the square, $n - 1$.
- Ama sees a rectangle that is n units on one side and 1 tile longer on the other side, $n + 1$, but the corner piece is removed, -1.
- Nasir sees a square, n^2, and an extra row, n, but a piece is removed from the bottom row, -1.

Lesson Practice 6.02

Name: ..... Date: ..... Period: .....

Test Practice

8. Karima says that she sees a square plus one more row in each pattern.



Circle *all* the expressions Karima could use to represent the number of tiles in this pattern.

$n^2 + 1$ (circled) $n^2 + n$ (circled) $n(n + 1)$

Spiral Review

9. *Select all* the expressions that are equivalent to $6m + 3q$.

- [x] **A.** $4m + 2m + 5q - 2q$
- [x] **B.** $3(2m + q)$
- [ ] **C.** $(6 + 3)(m + q)$
- [ ] **D.** $3q + 2m + 3q + m$
- [x] **E.** $q + 15m + 2q - 9m$

10. Complete the table for the function $h(x) = 5(2)^x$.

x	-2	-1	0	1	2
$h(x)$	1.25	2.5	5	10	20

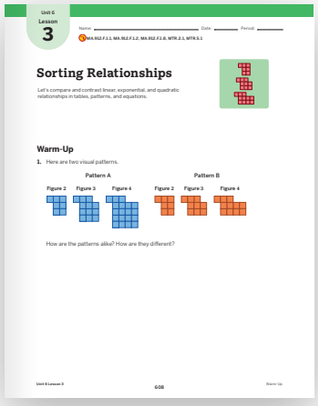
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.912.F.1.8
	5–7	2	MA.912.F.1.8
Test Practice	8	2	MA.912.F.1.8
Spiral Review			
	9	2	MA.6.AR.1.4
	10	2	MA.912.F.2.5

Unit 6

Lesson

3



Sorting Relationships

Comparing Linear, Exponential, and Quadratic Relationships

Let's compare and contrast linear, exponential, and quadratic relationships in tables, patterns, and equations.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether a pattern represents a linear, quadratic, or exponential relationship.
2. **Goal:** Observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
3. **Language Goal:** Explain that quadratic relationships have a constant *second difference*. **(Speaking and Listening)**

Students categorize relationships as linear, exponential, quadratic, or something else. They analyze, compare, and contrast these relationships in tables, patterns, and equations. They are introduced to the term **second difference** to describe what is constant in **quadratic functions**.

Coherence

Prior Learning

In Lesson 2, students wrote expressions for quadratic visual patterns. In Units 2 and 5, they explored linear and exponential relationships.

Future Learning

In Lesson 4, students will create tables and graphs to represent quadratic relationships in context.

Connections

- Students compare and contrast linear, exponential, and quadratic relationships in tables, patterns, and equations. (mathematical, cross-curricular)
- In a sorting activity, partners determine which type of relationship (linear, exponential, quadratic, or other) is represented by each table and defend their reasoning. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.1

Given an equation or graph that defines a function, classify the function type. Given an input-output table, determine a function type that could represent it.

Also Addressing: **MA.912.F.1.8**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3	**MA.912.AR.5.5**
MA.8.F.1	**MA.912.AR.6**
	MA.912.AR.7

Vocabulary

New Vocabulary

quadratic function

second difference

Review Vocabulary

constant difference

exponential relationship

linear relationship

Lesson at a Glance

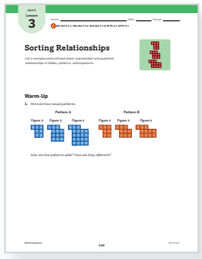
~ 45 min

MA.912.F.1.1, MA.912.F.1.8

Warm-Up

Pairs | 5 min

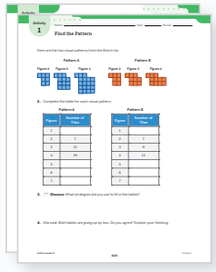
Students compare and contrast two visual patterns, which will be used in Activity 1.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

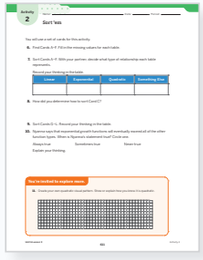
Students explore numeric patterns in tables of linear and quadratic relationships to identify a quadratic relationship by a constant **second difference**. **(MTR.2.1, MTR.5.1)**



Activity 2

Pairs | 20 min

Students identify quadratic relationships in tables, patterns, and equations and notice that a quantity growing exponentially will always exceed one growing quadratically.
MLR1: Stronger and Clearer Each Time



Synthesis

Whole Class | 5 min

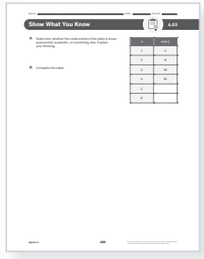
Students summarize how to decide if a relationship is linear, exponential, or quadratic.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether a table represents a quadratic relationship.



Prep Checklist

Prepare the Activity 2 cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students compare and contrast two visual patterns, which will be used in Activity 1.

1 Launch

Display the two visual patterns.

Use the Think-Pair-Share routine to help students shape their ideas about the quadratic relationships represented in the patterns. See the Routine Facilitation Guide for more information.

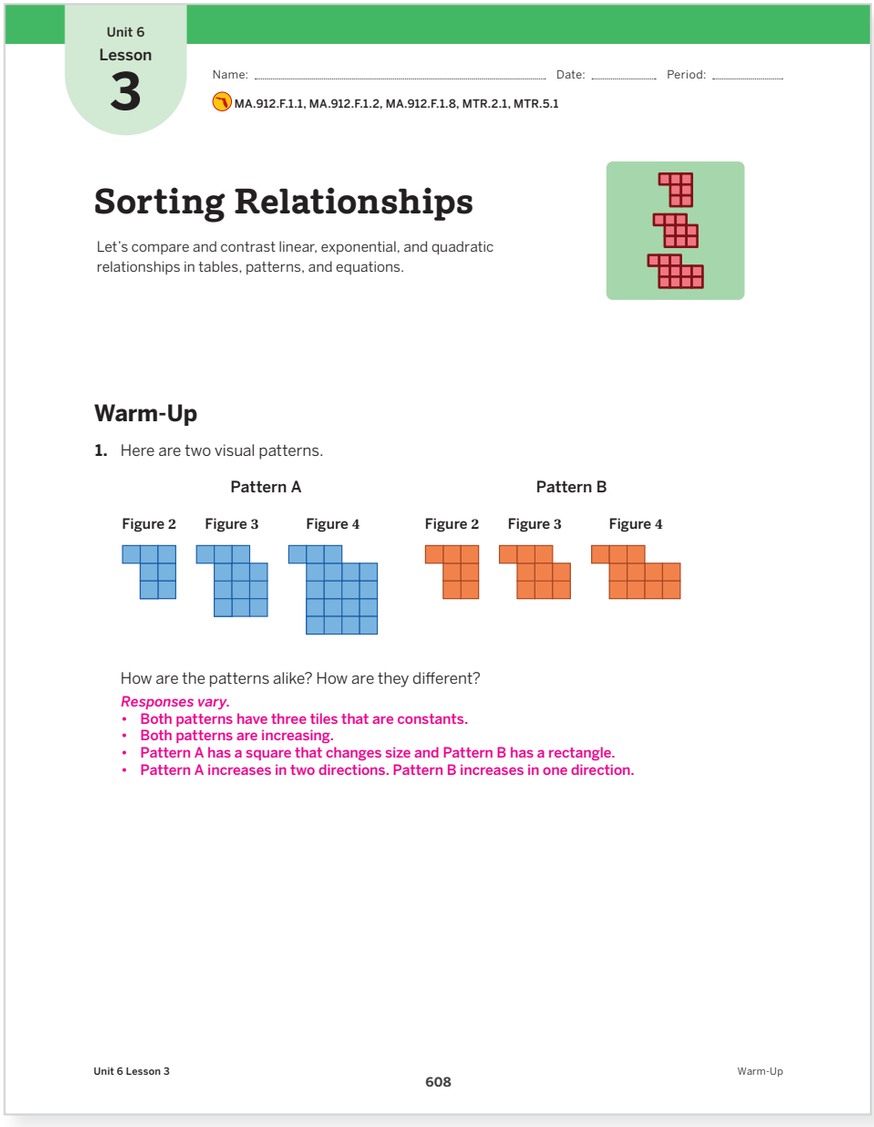
ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "The patterns are alike because they both _____." "The patterns are different because Pattern A _____ and Pattern B _____."). **(Writing and Speaking)**

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to support showing similarities and differences between visual representations.

2 Connect

Invite students to share what they notice as they compare and contrast the two visual patterns.

To access connections, consider asking, "Where in these patterns do we see a growing square?"



Unit 6
Lesson
3

Name: _____ Date: _____ Period: _____

MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.8, MTR.2.1, MTR.5.1

Sorting Relationships

Let's compare and contrast linear, exponential, and quadratic relationships in tables, patterns, and equations.

Warm-Up

1. Here are two visual patterns.

Pattern A

Figure 2 Figure 3 Figure 4

Pattern B

Figure 2 Figure 3 Figure 4

How are the patterns alike? How are they different?

Responses vary.
- Both patterns have three tiles that are constants.
- Both patterns are increasing.
- Pattern A has a square that changes size and Pattern B has a rectangle.
- Pattern A increases in two directions. Pattern B increases in one direction.

Unit 6 Lesson 3 608 Warm-Up

Pairs | 10 min

Activity 1 Find the Pattern

Purpose: Students explore numeric patterns in tables of linear and quadratic relationships to identify a quadratic relationship by a constant **second difference**. **(MTR.2.1, MTR.5.1)**

1 Launch

Display the two visual patterns.

Invite students to complete the table independently before discussing their strategies and the remaining problems with a partner.

Encourage students to use a calculator to support them with performing calculations.

2 Monitor

Listen for students who look for and make use of structure. **(MTR.2.1)**

D Differentiation *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to look at the visual patterns and consider asking: • "How does the number of tiles increase as you add 1 to the figure number?" **(MTR.2.1)** • "What patterns do you see?"
Use one strategy to fill in both tables.	**Strengthen** Celebrate their strategy. Consider asking them, "How might someone else have filled in the tables a different way?"
Do not agree with Afia. *(Problem 4)*	**Support** Invite them to annotate the first differences in each table and consider asking: • "What patterns do you see?" • "How does the difference in Pattern A change by 2?" **(MTR.2.1)**

Activity 1 continued >

Activity 1

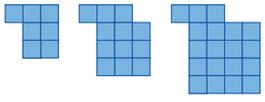
Name: Date: Period:

Find the Pattern

Here are the two visual patterns from the Warm-Up.

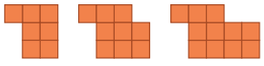
Pattern A

Figure 2 Figure 3 Figure 4



Pattern B

Figure 2 Figure 3 Figure 4



2. Complete the table for each visual pattern.

Pattern A

Figure	Number of Tiles
1	4
2	7
3	12
4	19
5	28
6	39
7	52

Pattern B

Figure	Number of Tiles
1	5
2	7
3	9
4	11
5	13
6	15
7	17

3. **Discuss:** What strategies did you use to fill in the tables?

Responses vary.
- For Pattern A, I noticed that each figure is a square plus 3. I squared the figure number, then added 3 to find the number of tiles.
- For Pattern B, I noticed that as the figure number increased by 1, the number of tiles increased by 2 each time.

4. Afia said: *Both tables are going up by two.* Do you agree? Explain your thinking.

Responses vary. Yes, but each pattern is going up by 2 in different ways. In Pattern B, the difference is equal to 2 each time. In Pattern A, the difference is increasing by 2 each time.

Unit 6 Lesson 3 609 Activity 1

Activity 1 Find the Pattern (continued)

Purpose: Students explore numeric patterns in tables of linear and quadratic relationships to identify a quadratic relationship by a constant **second difference**. **(MTR.2.1, MTR.5.1)**

2 Monitor

Listen for student strategies from Problem 5 to highlight during the Connect.

D Differentiation *(Problem 5)*

Look for students who:	Respond to Student Thinking
Need support determining the type of relationship for each pattern.	**Support** Consider referencing or creating an anchor chart about how to decide if a relationship is linear, exponential, or neither.
Use the terms *constant difference* and *constant ratio* from Unit 1.	**Stretch** Celebrate their noticing. Consider asking, "How can you use what you've remembered about linear relationships changing by a constant difference to help you make sense of quadratic relationships changing by a constant second difference?"
Would like to explore identifying linear, quadratic, and exponential relationships further.	**Stretch** Encourage students to determine the number of tiles in figure n of each pattern.

3 Connect

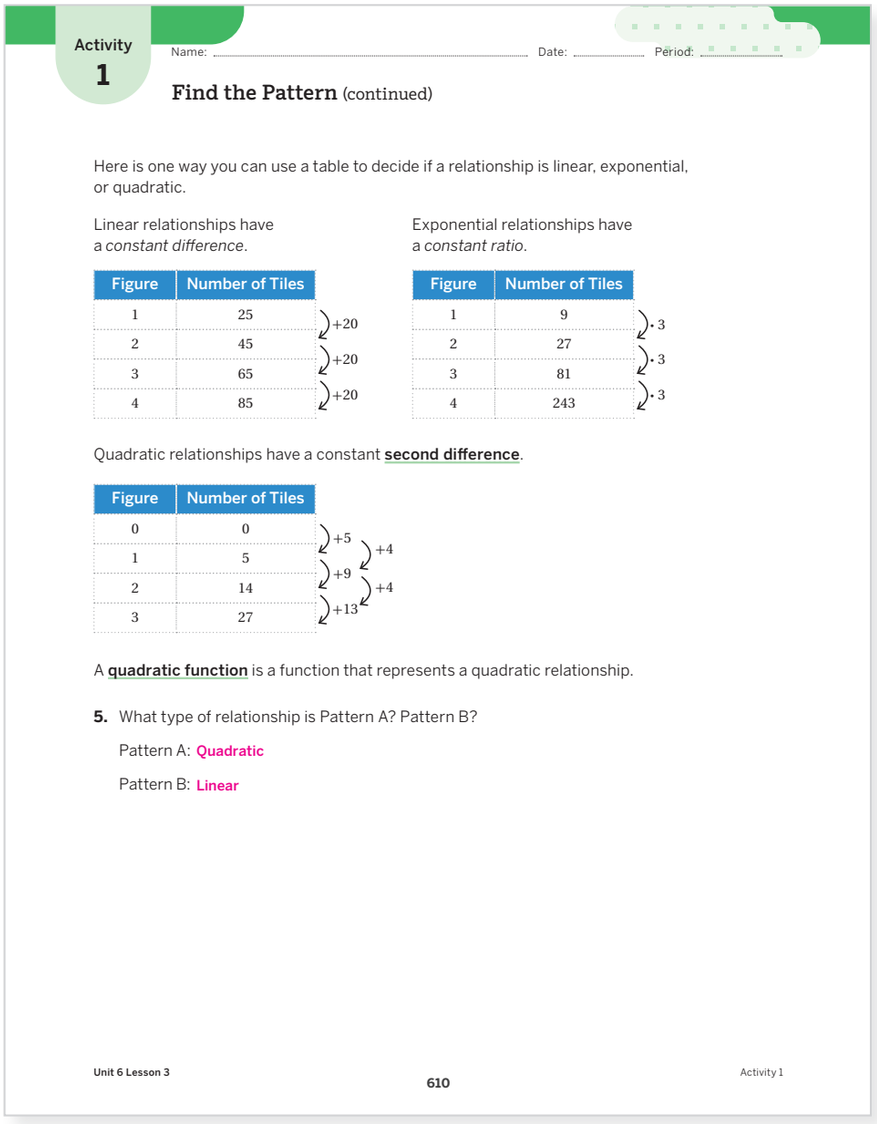
Display the problem and create a class definition for the term **second difference** using students' language and the glossary.

To reveal the Key Takeaway, consider asking:

- "What is the constant second difference in the quadratic table?"
- "How do you know that Pattern A represents a quadratic relationship?"**(MTR.2.1, MTR.5.1)**
- "How is the squared term in a **quadratic function** related to the constant second difference in quadratic relationships?" **(MTR.2.1)**
- "What is the constant second difference for Pattern A? Pattern B?"

Note: Invite students to generalize their thinking to make a rule about the first and second differences of linear functions. If it does not come up naturally, emphasize how linear relationships have a constant second difference of zero, and quadratic relationships have a nonzero constant second difference. **(MTR.5.1)**

ELL English Language Learners Invite students to share what each word in the new term *second difference* reminds them of (e.g., "When I hear the word *second*, I think . . .". "When I hear the word *difference*, I think of . . ."). Then invite students to share what they think it means when the two words are used together. **(Listening and Speaking)**



Activity 1

Name: Date: Period:

Find the Pattern (continued)

Here is one way you can use a table to decide if a relationship is linear, exponential, or quadratic.

Linear relationships have a *constant difference*.

Figure	Number of Tiles	
1	25	
2	45	+20
3	65	+20
4	85	+20

Exponential relationships have a *constant ratio*.

Figure	Number of Tiles	
1	9	
2	27	·3
3	81	·3
4	243	·3

Quadratic relationships have a constant **second difference**.

Figure	Number of Tiles		
0	0		
1	5	+5	
2	14	+9	+4
3	27	+13	+4

A **quadratic function** is a function that represents a quadratic relationship.

5. What type of relationship is Pattern A? Pattern B?

Pattern A: Quadratic

Pattern B: Linear

Unit 6 Lesson 3 610 Activity 1

Key Takeaway: The differences between consecutive output values in the table of a function are called first differences. The differences between those values are called **second differences**. Quadratic relationships have a constant second difference. A **quadratic function** is a function that represents a quadratic relationship.

Pairs | 20 min

Activity 2 Sort 'em

Purpose: Students identify quadratic relationships in tables, patterns, and equations and notice that a quantity growing exponentially will always exceed one growing quadratically.

Short on time? Consider omitting Cards H, F, J, and K.

1 Launch

Distribute one set of cards to each pair. Invite students to work together to sort the cards.

A **Accessibility: Executive Functioning** Invite students to choose one card at a time to determine the type of relationship.

2 Monitor

Pause after Problem 7 to review the type of relationship represented in each table.

Look for different sorting strategies to share during the Connect.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore quadratic relationships further to complete this optional task. Encourage students to write an expression to match their visual pattern, and discuss their thinking with a partner.

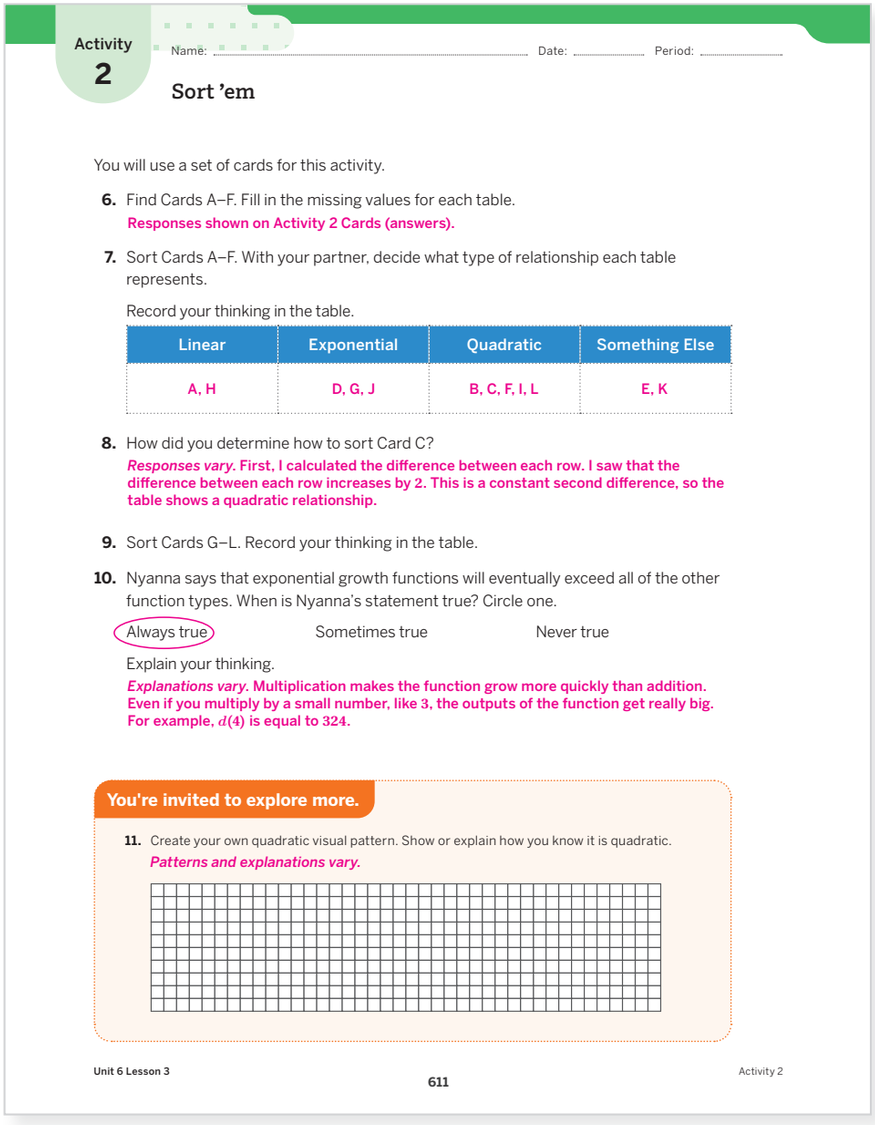
3 Connect

Invite students to share how they determined the type of relationship for each card.

Display one card with an exponential relationship (D, G, or J) and one card with a quadratic relationship (B, C, F, I, or L).

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft in Problem 10 and use their partner's feedback to write a second draft that is stronger and clearer. Consider asking, "How can you use quadratic relationships growing by second differences to support your reasoning?" **(MTR.5.1)**

Note: Students may use different strategies to determine that exponential functions will eventually exceed all other function types. Students do not need to prove this idea at this time.



Activity 2

Name: Date: Period:

Sort 'em

You will use a set of cards for this activity.

6. Find Cards A–F. Fill in the missing values for each table.
   Responses shown on Activity 2 Cards (answers).

7. Sort Cards A–F. With your partner, decide what type of relationship each table represents.
   Record your thinking in the table.

Linear	Exponential	Quadratic	Something Else
A, H	D, G, J	B, C, F, I, L	E, K

8. How did you determine how to sort Card C?
   Responses vary. First, I calculated the difference between each row. I saw that the difference between each row increases by 2. This is a constant second difference, so the table shows a quadratic relationship.

9. Sort Cards G–L. Record your thinking in the table.

10. Nyanna says that exponential growth functions will eventually exceed all of the other function types. When is Nyanna's statement true? Circle one.
    Always true Sometimes true Never true
    Explain your thinking.
    Explanations vary. Multiplication makes the function grow more quickly than addition. Even if you multiply by a small number, like 3, the outputs of the function get really big. For example, $d(4)$ is equal to 324.

You're invited to explore more.

11. Create your own quadratic visual pattern. Show or explain how you know it is quadratic.
    Patterns and explanations vary.

Unit 6 Lesson 3 611 Activity 2

Key Takeaway: A quantity that grows exponentially will always exceed a quantity that grows quadratically.

Whole Class | 5 min

Synthesis

Purpose: Students summarize how to decide if a relationship is linear, exponential, or quadratic.

Invite students to respond independently, and then share their thinking with a partner.

Accessibility: Memory and Attention Invite students to refer back to Activity 2 to support their reasoning.

English Language Learners Encourage students to use diagrams, visuals, and coloring tools to help them express their thinking.

Listen for and share a variety of ideas, such as:

- Looking for a constant difference or constant ratio between each row.
- Looking for a constant second difference if there is not a constant difference or constant ratio.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. How can you tell if a table is linear, exponential, or quadratic? *Responses vary.*

Linear: There is a constant difference in the y-values when the x-values change by a constant value.

Exponential: The y-values are being multiplied by the same value.

Quadratic: There is a constant second difference, which means the difference between the y-values increases or decreases by the same number when the x-values increase by 1.

Lesson Practice 6.03

Lesson Summary

You can analyze a table of values, a pattern, or an equation to help you determine whether a relationship is linear, quadratic, exponential, or something else.

- Linear relationships have a constant first difference in the y-values when the x-values change by a constant value.
- Quadratic relationships change by a constant **second difference**, the difference between the first differences. A **quadratic function** is a function that represents a quadratic relationship.
- Exponential relationships have a constant ratio.

Here are some examples.

	Linear	Quadratic	Exponential
Table	x | $f(x)$: 1 | 3; 2 | 6; 3 | 9; 4 | 12 (+3, +3, +3)	x | $g(x)$: 1 | 2; 2 | 5; 3 | 10; 4 | 17 (+3, +5, +7; +2, +2)	x | $h(x)$: 1 | 6; 2 | 12; 3 | 24; 4 | 48 (·2, ·2, ·2)
Equation	$f(x) = 3x$	$g(x) = x^2 + 1$	$h(x) = 3 \cdot 2^x$

Unit 6 Lesson 3 612 Synthesis | Lesson Practice

Lesson Takeaway: Quadratic relationships change by a constant **second difference**, the differences between the first differences. When you write an equation to represent a quadratic relationship, it will have a squared term.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether a table represents a quadratic relationship.

Today's Goals

1. **Goal:** Determine whether a pattern represents a linear, quadratic, or exponential relationship.
2. **Goal:** Observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
3. **Language Goal:** Explain that quadratic relationships have a constant second difference. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.03

a. Determine whether the relationship in this table is linear, exponential, quadratic, or something else. Explain your thinking.

Quadratic. *Explanations vary.* The second differences are constant, 4.

b. Complete the table.

x	$m(x)$
1	2
2	8
3	18
4	32
5	50
6	72

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine whether a relationship in a table is quadratic:

» **Respond to Student Thinking:**

- Revisiting **Activity 1, Problem 5** from this lesson and giving students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.
- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

Professional Learning

In this lesson, students expanded on their prior knowledge of constant differences for linear relationships to support their understanding of constant second differences for quadratic relationships. How were students able to apply their thinking to this new concept? How can you support students in recognizing the connection between these two key math concepts?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can you tell if a table is linear, exponential, or quadratic? *Responses vary.*

Linear: There is a constant difference in the y-values when the x-values change by a constant value.

Exponential: The y-values are being multiplied by the same value.

Quadratic: There is a constant second difference, which means the difference between the y-values increases or decreases by the same number when the x-values increase by 1.

Lesson Practice 6.03

Lesson Summary

You can analyze a table of values, a pattern, or an equation to help you determine whether a relationship is linear, quadratic, exponential, or something else.

- Linear relationships have a constant first difference in the y-values when the x-values change by a constant value.
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- Exponential relationships have a constant ratio.

Here are some examples.

	Linear	Quadratic	Exponential
Table	x | $f(x)$: 1 | 3; 2 | 6; 3 | 9; 4 | 12 (+3, +3, +3)	x | $g(x)$: 1 | 2; 2 | 5; 3 | 10; 4 | 17 (+3, +5, +7; +2, +2)	x | $h(x)$: 1 | 6; 2 | 12; 3 | 24; 4 | 48 (·2, ·2, ·2)
Equation	$f(x) = 3x$	$g(x) = x^2 + 1$	$h(x) = 3 \cdot 2^x$

Lesson Practice 6.03

Name: _____ Date: _____ Period: _____

Problems 1–3: Here are two functions: $g(x) = 3x^2$ and $h(x) = 3^x$.
Determine which function is greater for each value of x. Circle your choice.

1. $g(1)$ $h(1)$ (They are the same value)
2. ($g(2)$) $h(2)$ They are the same value
3. $g(5)$ ($h(5)$) They are the same value

Problems 4–5: Use the table of $f(x)$.

x	1	2	3	4	5	6	7
$f(x)$	3	12	27	48	75	108	147

4. What type of relationship does the table represent? Circle one.

Linear Exponential (Quadratic) Something Else

Explain your thinking.

Explanations vary. I know this is quadratic because the table has a constant second difference of 6.

5. Complete the table. *Responses in table.*

6. The function $m(x)$ represents a quadratic relationship. Complete the missing values in the table of $m(x)$.

x	0	1	2	3	4	5
$m(x)$	7	12	27	52	87	132

Problems 7–9: Use the pattern shown.

7. Explain how you see this pattern growing.

Explanations vary. I see a square that has a side length of the figure number + 1, and there are 2 corner tiles missing from the top row.

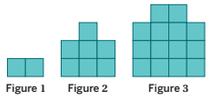


Figure 1 Figure 2 Figure 3

8. How many tiles will there be in Figure 10? 119 tiles

9. Write an expression for the number of tiles in Figure n.

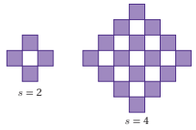
$(n + 1)(n + 1) - 2$ (or equivalent)

Lesson Practice 6.03

Name: _____ Date: _____ Period: _____

Test Practice

10. Complete the table for each type of function to show the number of tiles when $s = 3$ and $s = 5$. Choose from the given values to complete the table.



$s = 2$

$s = 4$

6	8	9	12	15	16	18
20	22	24	25	26	28	32

Choose from the given values to complete the table.

s	Linear	Quadratic	Exponential
2	4	4	4
3	10	9	8
4	16	16	16
5	22	25	32

Spiral Review

11. The function $C(x)$ gives the percentage of students at a high school who have cell phones x years after 2004. Explain what $C(10) = 35$ means in this context.

Responses vary. $C(10) = 35$ means that 35% of students have cell phones 10 years after 2004 (2014).

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.F.1.2
	4–6	2	MA.912.F.1.1
	7–8	1	MA.912.F.1.1
	9	2	MA.912.F.1.1
Test Practice	10	3	MA.912.F.1.1
Spiral Review			
	11	1	MA.8.F.1.3

Unit 6

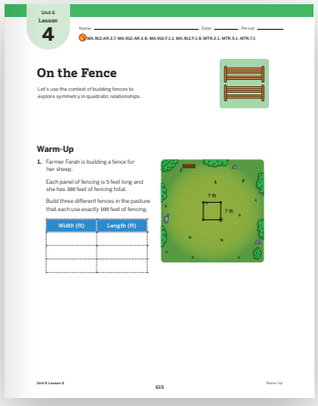
Lesson

4

On the Fence

Quadratics in Context

Let's use the context of building fences to explore symmetry in quadratic relationships.


Unit 6
Lesson
4
Name: Date: Period:
On the Fence
Let's use the context of building fences to explore symmetry in quadratic relationships.
Warm-Up
1. Farmer Farah is building a fence for her sheep.
Each panel of fencing is 5 feet long and she has 300 feet of fencing total.
Build three different fences in the pasture that each use exactly 100 feet of fencing.
Width (ft)
Length (ft)
7 ft
Unit 6 Lesson 4
615
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Create tables and graphs to represent a quadratic relationship in context.
2. **Goal:** Understand that the graphs of quadratic functions (called *parabolas*) are symmetrical around a *line of symmetry*.
3. **Language Goal:** Justify whether the relationship represented in a table or graph is quadratic. **(Writing, Speaking, and Listening)**

Students use the context of building fences to explore symmetry in quadratic relationships. They use graphs and tables to explore all of the areas that can be created with a specific amount of fencing, which highlights that as the width of the fence increases, the area increases until it reaches a maximum point, and then begins to decrease. Students are also introduced to the **line of symmetry**.

Coherence

Prior Learning

In Lesson 3, students modeled quadratic relationships with tables, graphs, and equations.

Future Learning

In Lesson 5, students will use tables and graphs to make predictions about quadratic relationships in context.

Connections

- Within the context of building fences, students explore the symmetry found in quadratic relationships. (real world, mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

MA.912.F.1.8
Determine whether a linear, quadratic or exponential function best models a given real-world situation.

Also Addressing: **MA.912.F.1.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3	**MA.912.AR.5.5**
MA.8.AR.3.4	**MA.912.AR.5.6**
	MA.912.AR.6.4

Vocabulary

New Vocabulary

line of symmetry **parabola**

Additional Vocabulary

area *perimeter*

Lesson at a Glance

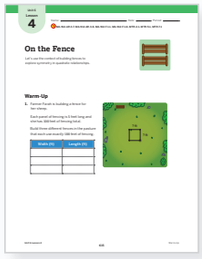
~ 45 min

MA.912.AR.3.7, MA.912.F.1.8, MA.912.F.1.1

Warm-Up

Independent | 5 min

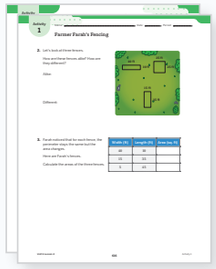
Students reason about how to build different fences with the same amount of fencing.



Activity 1

Pairs | 15 min

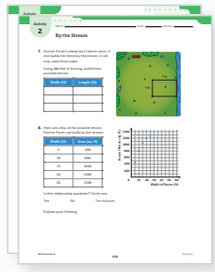
Students use the context of building a fence to identify key features of quadratic relationships in graphs and tables, including the maximum and **line of symmetry**.
Routines: Notice and Wonder, Think-Pair-Share, MLR2: Collect and Display (MTR.2.1, MTR.7.1)



Activity 2

Pairs | 15 min

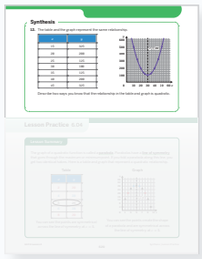
Students use the symmetry in quadratic relationships to make predictions and write the equation for the line of symmetry for different quadratic relationships. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

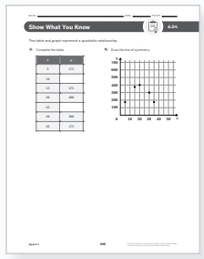
Students synthesize their understanding of how to identify key features of quadratic relationships in tables and graphs.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining missing values and the line of symmetry for a quadratic relationship using a table and a graph.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students reason about how to build different fences with the same amount of fencing.

Short on time? Consider inviting students to build one fence instead of three and compare with a partner.

1 Launch

To access connections, consider asking:

- "What experiences do you have with farms?"
- "Why might Farmer Farah be interested in building different fences?"

Consider asking, "Will you use exactly 100 feet of fencing? How do you know?"

Invite students using print to suggest a width or length as you build their suggested fence using the dashboard's Student View.

ELL **English Language Learners** Create a list of words about the context that students might not know (e.g., *width, length, fencing, panel, farm, pasture*), along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

2 Connect

Invite students to share their strategies for determining different combinations of width and length to build a fence around a pasture using exactly 100 feet of fencing.

Note: If it doesn't come up naturally, consider asking, "One strategy is to find two numbers that add up to 50. Why do you think the number 50 is important in this situation?"

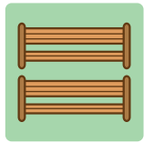
Unit 6
Lesson
4

Name: Date: Period:

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.1, MA.912.F.1.8, MTR.2.1, MTR.5.1, MTR.7.1

On the Fence

Let's use the context of building fences to explore symmetry in quadratic relationships.

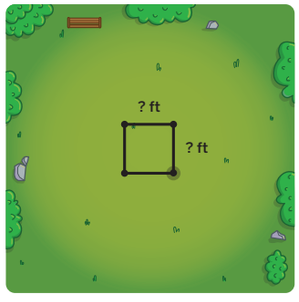


Warm-Up

1. Farmer Farah is building a fence for her sheep.

   Each panel of fencing is 5 feet long and she has 100 feet of fencing total.

   Build three different fences in the pasture that each use *exactly* 100 feet of fencing.



Width (ft)	Length (ft)
40	10
25	25
15	35

Fences vary. In all correct fences, the sum of the width and the length is 50 feet.

Unit 6 Lesson 4 615 Warm-Up

Pairs | 15 min

Activity 1 Farmer Farah's Fencing

Purpose: Students use the context of building a fence to identify key features of quadratic relationships in graphs and tables, including the maximum and line of symmetry. (MTR.2.1, MTR.7.1)

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then share their thinking with a partner before recording their response.

Listen for and share several student responses that highlight similarities and differences between the three fences.

Consider asking:

- "What is the *perimeter*? How is it measured?"
- "What is the *area*? How is it measured?"

A **Accessibility: Conceptual Processing** Use graph paper or algebra tiles to review how to calculate the area of a rectangle.

2 Monitor

Consider sharing that students can input an expression (e.g., $40 \cdot 10$) into the table and the calculator will compute the area for them.

Invite students using print to use a calculator to support them in calculating the area.

ELL **English Language Learners** Review the terms *perimeter* and *area* and invite students to share what they know about the term. **(Reading and Listening)**

D **Differentiation: (Stretch)** Consider asking, "How do the units of measure support you in knowing that perimeter is a linear relationship while area is a quadratic relationship?" Or, invite students to write an expression in terms of a variable, such as w or l.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Farmer Farah's Fencing

2. Let's look at three fences.

How are these fences alike? How are they different?

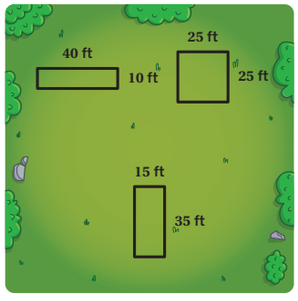
Responses vary.

Alike:

- All the fences are rectangles.
- All the numbers in the fences are multiples of 5.
- The length and the width add to 50.
- They all have the same perimeter.

Different:

- Some fences are skinnier than others.
- They have different areas.
- Some fences are more wide than long. Others are more long than wide.



3. Farah noticed that for each fence, the perimeter stays the same but the area changes.

Here are Farah's fences.

Calculate the areas of the three fences.

Note: Students using digital will see the widths and lengths of the fences they made in the Warm-Up.

Width (ft)	Length (ft)	Area (sq. ft)
40	10	400
15	35	525
5	45	225

Unit 6 Lesson 4 616 Activity 1

Pairs | 15 min

Activity 1 Farmer Farah's Fencing (continued)

Purpose: Students use the context of building a fence to identify key features of quadratic relationships in graphs and tables, including the maximum and line of symmetry. **(MTR.2.1, MTR.7.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Select one row and consider asking, "What will this fence look like?"
Use symmetry, patterns, or second differences to determine the values.	**Strengthen** Celebrate their strategy and invite them to share during the Connect.
Would like an additional challenge.	**Stretch** Encourage students to consider how the table would be different if Farah had 200 feet of fencing.

A Accessibility: Executive Functioning Consider chunking this activity by inviting students to choose one row or column and look for patterns before moving to the next one.

Display the distribution of responses using the dashboard's Teacher View and invite students to discuss how to distinguish between linear, exponential, quadratic, and other functions.

Encourage students to use graph paper to choose a point on the left side of the line of symmetry, draw its fence, then draw the fence for the corresponding point on the right side and compare.

3 Connect

Use the Notice and Wonder routine to support students in making connections between the context and quadratic relationships in tables. **(MTR.2.1)**

Consider asking:
- "What strategy did you use to complete the table?"
- "What do you notice when the width is 15 feet and when it is 35 feet? Why does this make sense?"

Consider asking:
- "Why does it make sense that this graph is symmetrical?"
- "Why is the equation of the line of symmetry $x = 25$ instead of $y = 25$?"
- "What does the maximum point mean?" **(MTR.7.1)**
- "How does the domain of possible fences compare to the domain of the quadratic?" The domain of the possible fences is discrete because the panels are 5 feet in length, but the domain of the quadratic is continuous from $0 \leq x \leq 50$.

MLR MLR2: Collect and Display Use this routine to create an anchor chart with the annotated features of a parabola on a graph and in a table, including the line of symmetry, maximum, and intercepts.

Activity 1

Name: _____ Date: _____ Period: _____

Farmer Farah's Fencing (continued)

4. The table represents all the possible fences Farmer Farah can build in the pasture.

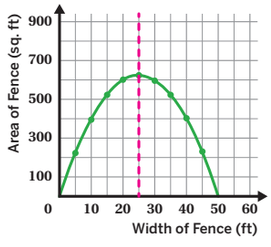
Width (ft)	Length (ft)	Area (sq. ft)
5	45	225
10	40	400
15	35	525
20	30	600
25	25	625
30	20	600
35	15	525
40	10	400
45	5	225

a. Complete the missing values in the table.

b. **Discuss:** What do you notice? What do you wonder?

Responses vary.
- I notice the widths are going up by 5 and the lengths are going down by 5.
- I notice that there are a lot of the same numbers in the table.
- I wonder if Farah could have a width of 22 feet.
- I wonder if 625 is the biggest area you could make.

5. Here is a graph of a parabola that includes all the possible fences.



What type of relationship is represented?

A. Linear B. Exponential

C. Quadratic D. Something else

Explain your thinking. *Explanations vary.*
- $400 - 225 = 175$, $525 - 400 = 125$, and $600 - 525 = 75$, so there is a constant second difference of -50.
- The graph is shaped like a quadratic. The difference between the y-values decreases by 50 between each point.

6. The graph of a quadratic function is called a parabola.

Parabolas have a line of symmetry. If you fold a parabola along this line, you get two identical halves.

a. Draw the line of symmetry on the graph of possible fences. $x = 25$

b. **Discuss:** What does this line mean in the context of the sheep fence?

Responses vary. It is the width of the fence when the area is the largest. The width of the fence is 25 feet.

Unit 6 Lesson 4 617 Activity 1

Key Takeaway: The graph of a quadratic function is called a **parabola**. Parabolas have a **line of symmetry** that goes through the maximum (or minimum) and divides the parabola into two identical halves.

Pairs | 15 min

Activity 2 By the Stream

Purpose: Students use the symmetry in quadratic relationships to make predictions and write the equation for the line of symmetry for different quadratic relationships. **(MTR.2.1)**

1 Launch

To access connections, consider beginning with the activity paused and asking, "Do you think a fence by the stream will have a larger area? Why or why not?"

Invite students using print to suggest a width or length as you build their suggested fence using the dashboard's Student View.

Encourage students to label the width and length in their Student Edition. Consider asking, "Which side is the width? The length? How do you know?"

2 Monitor

To support students getting started, consider inviting pairs of students to build any fence and determine how many feet of fencing they used.

Look for a variety of strategies students use, including using the structure of the table or the graph to decide if the relationship is quadratic. **(MTR.2.1)**

Display the distribution of responses using the dashboard's Teacher View.

Pause briefly to invite students who used different strategies to share.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

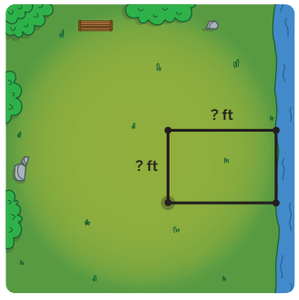
By the Stream

7. Farmer Farah's sheep don't like to swim. If she builds her fence by the stream, it will only need three sides.

Using 100 feet of fencing, build three possible fences.

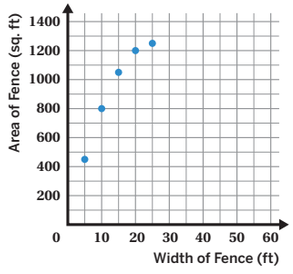
Width (ft)	Length (ft)
40	20
20	60
45	10

Fences vary.



8. Here are a few of the possible fences Farmer Farah can build by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250



Is this relationship quadratic? Circle one.

(Yes) No I'm not sure

Explain your thinking.

Explanations vary. There is a constant second difference of -100.

Unit 6 Lesson 4 618 Activity 2

Pairs | 15 min

Activity 2 By the Stream (continued)

Purpose: Students use the symmetry in quadratic relationships to make predictions and write the equation for the line of symmetry for different quadratic relationships. **(MTR.2.1)**

2 Monitor

To support students getting started, consider asking pairs of students, "Why does it matter that each panel of fencing is 5 feet in length?"

A **Accessibility: Conceptual Processing** Invite students to revisit their work to help apply concepts to new situations.

To support students getting started, invite students to enter a random number, press "Try It" and use the feedback to revise their response.

ELL **English Language Learners** Invite students to use gestures or describe where they think the line of symmetry is with a partner before writing the equation. **(Writing and Speaking)**

Select and sequence several student responses to share during the Connect.

3 Connect

Consider asking:

- "How is the domain of each graph alike? How is it different?"
- "How is the range of each graph alike? How is it different?"
- "If you were Farmer Farah, where would you build the fence? Why?"

Activity 2

Name: _____ Date: _____ Period: _____

By the Stream (continued)

9. What are all the possible fences this parabola could represent?

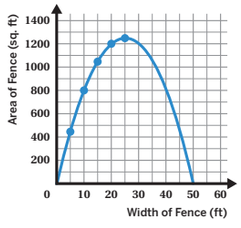
Complete the table to show all the possible fences Farmer Farah could make by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250
30	1200
35	1050
40	800
45	450

10. Here is a graph of a parabola that includes all the possible areas of fences along the stream.

Write the equation for the line of symmetry for this parabola.

$x = 25$

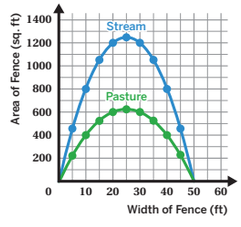


11. Here are the graphs of parabolas that include all the possible fences Farmer Farah could build in the pasture and by the stream.

How are these relationships alike? How are they different? ***Responses vary.***

Alike:

- They are both parabolas that have a highest point.
- They both start at 0 feet and go until 50 feet.
- They both have a line of symmetry at $x = 25$.



Different:

- The stream has a higher maximum than the pasture. It looks about twice as high.

Unit 6 Lesson 4 619 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to identify key features of quadratic relationships in tables and graphs.

Invite students to respond independently, and then share their thinking with a partner.

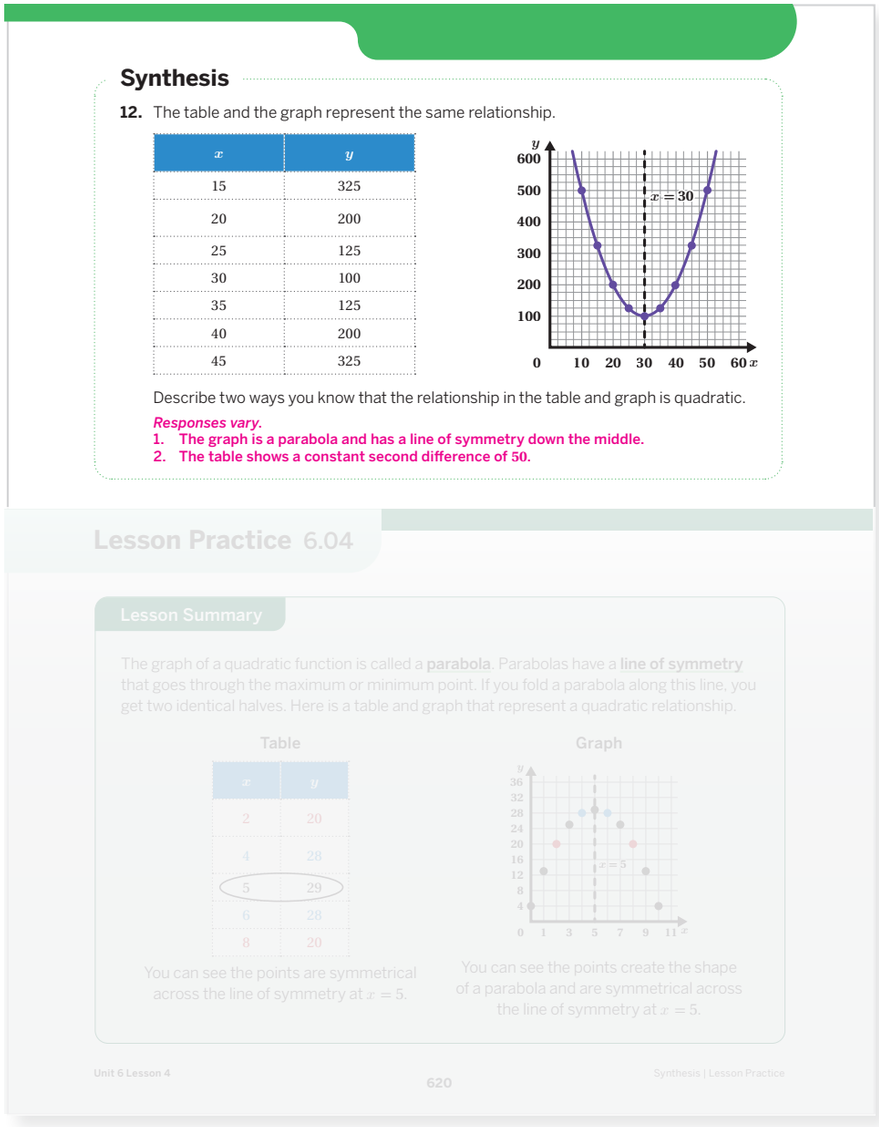
ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., "The table/graph shows that this relationship is quadratic because _____." or "I noticed _____, so I know _____.") **(Writing and Speaking)**

Listen for and share a variety of ideas, such as:

- Using the table to determine the second differences.
- Noticing that the graph is a parabola.
- Looking for symmetry in both the graph and the table.

MLR **MLR2: Collect and Display** Use this routine to review and revise students' ideas about identifying key features of quadratic relationships in graphs and tables. Add any new ideas to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. The table and the graph represent the same relationship.

x	y
15	325
20	200
25	125
30	100
35	125
40	200
45	325

Describe two ways you know that the relationship in the table and graph is quadratic.

Responses vary.

1. The graph is a parabola and has a line of symmetry down the middle.
2. The table shows a constant second difference of 50.

Lesson Practice 6.04

Lesson Summary

The graph of a quadratic function is called a **parabola**. Parabolas have a **line of symmetry** that goes through the maximum or minimum point. If you fold a parabola along this line, you get two identical halves. Here is a table and graph that represent a quadratic relationship.

Table

x	y
2	20
4	28
5	29
6	28
8	20

You can see the points are symmetrical across the line of symmetry at $x = 5$.

Graph

You can see the points create the shape of a parabola and are symmetrical across the line of symmetry at $x = 5$.

Unit 6 Lesson 4 620 Synthesis | Lesson Practice

Lesson Takeaway: **Parabolas** come in all shapes and sizes. Some have maximum values while some have minimum values. All parabolas have a **line of symmetry** that folds the parabola into two identical halves.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining missing values and the line of symmetry for a quadratic relationship using a table and a graph.

Today's Goals

1. **Goal:** Create tables and graphs to represent a quadratic relationship in context.
2. **Goal:** Understand that the graphs of quadratic functions (called parabolas) are symmetrical around a line of symmetry.
3. **Language Goal:** Justify whether the relationship represented in a table or graph is quadratic. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

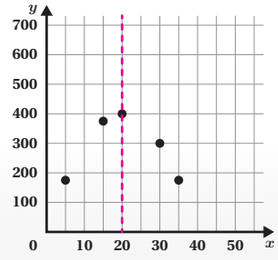
Show What You Know 6.04

The table and graph represent a quadratic relationship.

a Complete the table.

x	y
5	175
10	300
15	375
20	400
25	375
30	300
35	175

b Draw the line of symmetry.



D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use tables to make predictions about quadratic relationships or writing the equation of the line of symmetry:

» **Respond to Student Thinking:**

- Students will have more opportunities to use tables to make predictions and write the equation of the line of symmetry in the next lesson.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.
- Invite students to write expressions for area in terms of variables, such as w for width and/or l for length.

Professional Learning

Take a moment to reflect on how students used units of measure (e.g. feet or square feet) to make sense of problems in this lesson and across the course. What connections did students make to the units of measure and the types of functions they relate to? How can you support students in continuing to reflect on making use of a reasonable domain and the context of problems to support them in their reasoning?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. The table and the graph represent the same relationship.

x	y
15	325
20	200
25	125
30	100
35	125
40	200
45	325

Describe two ways you know that the relationship in the table and graph is quadratic.

Responses vary.

1. The graph is a parabola and has a line of symmetry down the middle.
2. The table shows a constant second difference of 50.

Lesson Practice 6.04

Lesson Summary

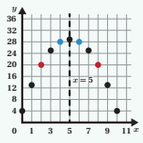
The graph of a quadratic function is called a **parabola**. Parabolas have a **line of symmetry** that goes through the maximum or minimum point. If you fold a parabola along this line, you get two identical halves. Here is a table and graph that represent a quadratic relationship.

Table

x	y
2	20
4	28
5	29
6	28
8	20

You can see the points are symmetrical across the line of symmetry at $x = 5$.

Graph



You can see the points create the shape of a parabola and are symmetrical across the line of symmetry at $x = 5$.

Lesson Practice 6.04

Name: _____ Date: _____ Period: _____

Problems 1–3: For each pair of symmetrical points on a parabola, determine the equation for the line of symmetry.

1. (0, 0) and (13, 0) $x =$ 6.5

2. (4, 2) and (20, 2) $x =$ 12

3. (7, 10) and (28, 10) $x =$ 17.5

Problems 4–6: Here are a few points that belong to a function $g(x)$.

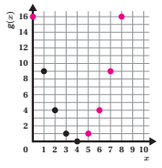
4. Does $g(x)$ represent a quadratic relationship? Circle your response and explain your thinking.

(Yes) No Not enough information

Explanations vary. I know that $g(x)$ is quadratic because the second differences of the values in the table are all 2. Quadratic relationships have constant second differences.

x	$g(x)$
0	16
1	9
2	4
3	1
4	0
5	1
6	4
7	9
8	16

5. Complete the table for $g(x)$ and plot the new points on the graph.



6. Write the equation for the line of symmetry.

$x =$ 4

Problems 7–8: Here is an incomplete table that could represent several types of functions.

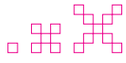
Figure	Number of Tiles
1	1
2	*Responses vary.*
3	9

7. Select a function type and determine the number of tiles that would be in Figure 2. Circle one. *Responses vary.*

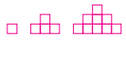
Linear Quadratic Exponential

8. Draw three figures to match the pattern in the table. *Responses vary.*

Linear



Quadratic



Exponential

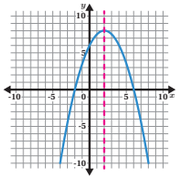


Lesson Practice 6.04

Name: _____ Date: _____ Period: _____

Test Practice

Problems 9–10: Here is a graph of a parabola.



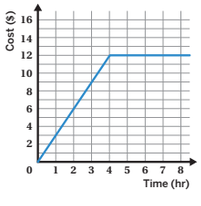
9. Draw the *line of symmetry* where you think it is located on this parabola.

10. Write the equation for the line of symmetry.

$x =$ 2

Spiral Review

Problems 11–12: The graph represents the relationship between the amount of time a car is parked, in hours, and the cost of parking, in dollars.



11. Is the relationship a function?

Yes

12. Describe the relationship between the amount of time a car is parked and the cost of parking.

Explanations vary. The cost of parking is $3 per hour, until 4 hours. For any additional time after 4 hours, you will pay $12 total.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
Fluency	1–3	1	MA.912.AR.3.7
	4, 7	2	MA.912.F.1.8
	5–6, 8	2	MA.912.AR.3.7
Test Practice	9–10	2	MA.912.AR.3.7
Spiral Review			
	11	1	MA.8.F.1.1
	12	1	MA.8.F.1.3

Unit 6

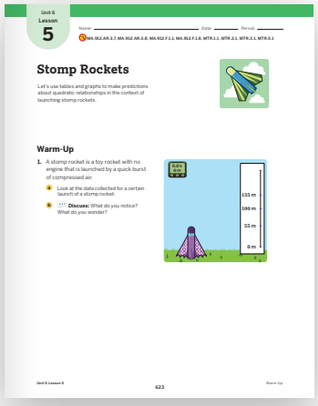
Lesson

5

Stomp Rockets

Projectiles and Predictions

Let's use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.


Unit 6
Lesson
5
Stomp Rockets
Let's use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.
Warm-Up
1. A stomp rocket is a toy rocket with no engine that is launched by a quick burst of compressed air.
Look at the data collected for a certain launch of a stomp rocket.
Discuss: What do you notice? What do you wonder?
135 m
100 m
55 m
0 m
623

Learning Outcomes

Today's Goals

1. **Goal:** Use tables and graphs to determine values in quadratic relationships in context.
2. **Language Goal:** Compare and contrast the graphs of different quadratic relationships in context. **(Speaking and Listening)**

Students use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets. Students use second differences to extend a quadratic pattern in a table. They also informally identify key features of quadratic functions in tables and graphs related to a context. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 3, students explained that quadratic relationships have a constant second difference. In Lesson 4, students created tables and graphs to model quadratic relationships in context.

Future Learning

In Lesson 6, students will create graphs of parabolas given a description. In Lesson 7, students will compare properties of quadratic functions represented in different ways and describe connections between equations, tables, descriptions, and graphs of quadratic functions.

Connections

- Students use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets. (real world, mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7

Given a table, equation, or written description of a quadratic function, graph that function and determine and interpret its key features.

MA.912.AR.3.8

Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.3.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.8.F.1.3

Lesson at a Glance

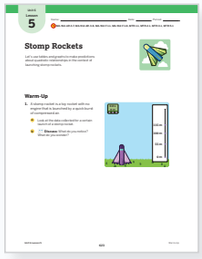
~ 45 min

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.8

Warm-Up

Pairs | 5 min

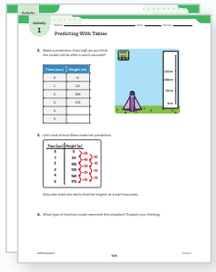
Students use the **Notice and Wonder** routine to make sense of the context of the lesson: stomp rockets and their flight paths over time.



Activity 1

Pairs | 15 min

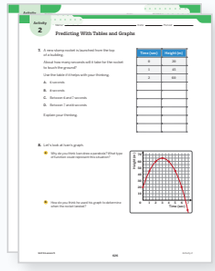
Students use the structure of quadratic relationships represented in tables to make predictions about stomp rockets. **(MTR.5.1)**



Activity 2

Pairs | 15 min

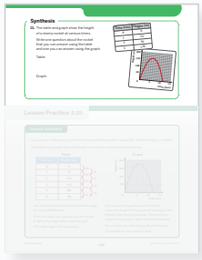
Students use tables and graphs to compare the flight paths of several different stomp rockets. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

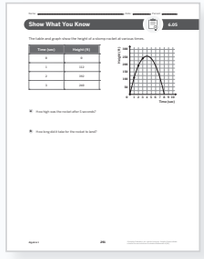
Students synthesize their understanding of using tables and graphs of quadratic relationships to answer questions about stomp rockets.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using tables and graphs of quadratic relationships to answer questions about stomp rockets.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of the context of the lesson: stomp rockets and their flight path over time.

1 Launch

Consider asking, "Have you ever seen a stomp rocket before? How high do you think it went?"

ELL **English Language Learners** Use gestures to illustrate the meaning of the word *stomp* and the phrase *compressed air* if they are unfamiliar to students. Consider showing pictures or videos of real-life stomp rockets to help them make sense of the context. **(Reading and Listening)**

Use the Notice and Wonder routine to promote curiosity and help students make sense of the stomp rocket context. **(MTR.1.1)**

Encourage students to use the feedback to help them revise their thinking. Invite them to use the calculator to support their thinking.

2 Connect

Invite students to share what they notice and wonder with the class.

Note: If it doesn't come up naturally, consider asking, "Did anyone notice when the rocket turned around in the air? What about when it hit the ground?"

Unit 6
Lesson
5

Name: ..... Date: ..... Period: .....

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.1, MA.912.F.1.8, MTR.1.1, MTR.2.1, MTR.3.1, MTR.5.1

Stomp Rockets

Let's use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.



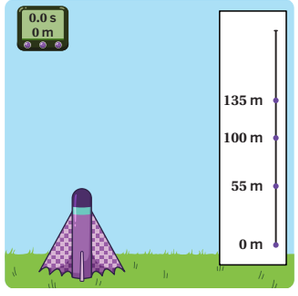
Warm-Up

1. A stomp rocket is a toy rocket with no engine that is launched by a quick burst of compressed air.

   a. Look at the data collected for a certain launch of a stomp rocket.

   b. **Discuss:** What do you notice? What do you wonder?

   Responses vary.
   - I notice this stomp rocket is going very high.
   - I wonder how high the rocket goes.
   - I wonder if the rocket always takes the same amount of time to go up as it does to come down.
   - I wonder how high the highest stomp rocket ever went.



Unit 6 Lesson 5 623 Warm-Up

Pairs | 15 min

Activity 1 Predicting With Tables

Purpose: Students use the structure of quadratic relationships represented in tables to make predictions about stomp rockets. **(MTR.5.1)**

1 Launch

Invite students to test their predictions with a partner.

Encourage students to revise their thinking with a partner. Invite them to use a calculator to support their thinking.

A **Accessibility: Executive Functioning** Consider inviting students to annotate the table on additional paper to help with their thinking.

2 Monitor

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What patterns do you notice in the table?" Invite them to calculate the differences and look for patterns.
Think the height continues to increase by 35 meters for 4 and 5 seconds.	**Strengthen** Celebrate their noticing and consider asking, "How did the height change between 0 and 1 second? Between 1 and 2 seconds? How can that help us make predictions?"
Would like an additional challenge.	**Stretch** Consider asking, "Will the numbers in the table keep increasing? Why or why not?"

Select and share several student responses.

Pause the class when most students have responded for a discussion. Consider asking:

- "How could Maia know this was a quadratic relationship?"
- "Where do you see the first differences in the table? Second differences?"
- "How can we use Maia's work to predict the height of the stomp rocket at 6 seconds?"

D **Differentiation: (Stretch)** Invite students to discuss how the situation would be different if the second differences were 10 instead of -10.

Activity 1 continued >

Activity 1

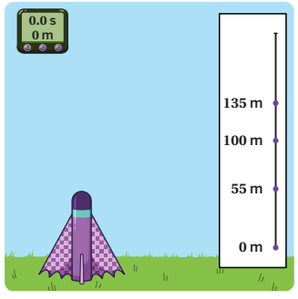
Name: _____ Date: _____ Period: _____

Predicting With Tables

2. Make a prediction: How high do you think the rocket will be after 4 and 5 seconds?

Predictions vary.

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	160
5	600



3. Let's look at how Maia made her prediction.

Time (sec)	Height (m)	
0	0	
		+55
1	55	
		+45
2	100	
		+35
3	135	
		+25
4	160	
		+15
5	175	

-10 -10 -10 -10

Describe what she did to find the heights at 4 and 5 seconds.

Responses vary. Maia noticed the first difference was going down by 10 each time. That means the height at 4 seconds is 25 more than the height at 3 seconds, and the height at 5 seconds is 15 more than the height at 4 seconds.

4. What type of function could represent this situation? Explain your thinking.

Responses vary. A quadratic function could represent this situation because the second differences in the table are constant.

Unit 6 Lesson 5 624 Activity 1

Pairs | 15 min

Activity 1 Predicting With Tables (continued)

Purpose: Students use the structure of quadratic relationships represented in tables to make predictions about stomp rockets. **(MTR.5.1)**

2 Monitor

Invite students to use additional paper to support their thinking.

To support students getting started, consider encouraging them to use Maia's strategy to extend the pattern. Then ask, "What does it mean about a parabola when one of the first differences is a negative number?"

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Select and share student responses.

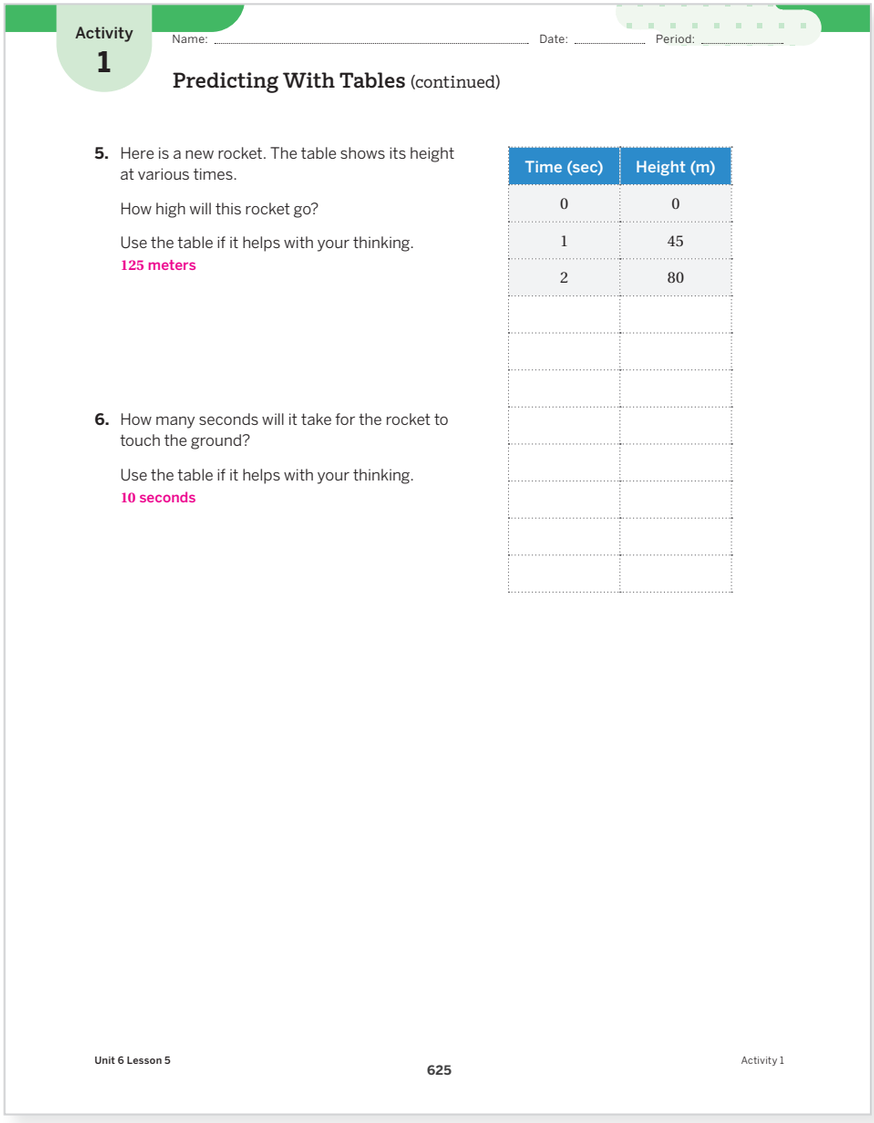
D **Differentiation: (Support)** To support students getting started, consider asking, "The first differences are 45, 35, 25, 15, 5. What do you think is the next first difference?"

3 Connect

Invite students to explain their thinking for their predictions.

To reveal the Key Takeaway, consider asking:

- "How can we use the table to determine when the rocket touches the ground?"
- "How did Maia's strategy help us determine when the rocket touches the ground?"



Activity 1

Name: _____ Date: _____ Period: _____

Predicting With Tables (continued)

5. Here is a new rocket. The table shows its height at various times.

How high will this rocket go?

Use the table if it helps with your thinking.

125 meters

Time (sec)	Height (m)
0	0
1	45
2	80

6. How many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

10 seconds

Unit 6 Lesson 5 625 Activity 1

Key Takeaway: We can use a table to identify and extend the pattern of a quadratic relationship. The first and second differences can help us make predictions about the situation represented by the table, such as an object's maximum height or how long it takes to reach the ground.

Pairs | 15 min

Activity 2 Predicting With Tables and Graphs

Purpose: Students use tables and graphs to compare the flight paths of several different stomp rockets. **(MTR.3.1)**

Short on time? Consider facilitating Problem 9 as a whole-class demonstration and discussion.

1 Launch

To access connections, consider asking, "How is this rocket similar to the one from the previous problem? How is it different?"

A **Accessibility: Executive Functioning** Consider inviting students to write out and annotate the table on additional paper to help make sense of how the height changes.

2 Monitor

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit Maia's strategy on Problem 3 to support their thinking.
Think the stomp rocket takes 6 seconds to touch the ground because the table gives a negative height for 7 seconds.	**Support** Ask them, "What does it mean for the height to go from a positive to a negative value? What does the table say about the height at 6 seconds?"
Use a graph to sketch the rocket's path, like Ivan's strategy on Problem 8.	**Strengthen** Celebrate their strategy. Invite these students to share their thinking with their partners and other pairs of students.

Display the graph for students.

Encourage students to read others' responses and decide if others' responses were similar to or different from their own.

Consider asking pairs of students:

- "Where is the line of symmetry? What does that tell us about the stomp rocket?"
- "Where on the graph can we see when the rocket landed?" **(MTR.2.1)**

Encourage students to share their responses. If enough pairs of students are struggling, consider pausing briefly to discuss Ivan's graph as a class.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Predicting With Tables and Graphs

7. A new stomp rocket is launched from the top of a building.

About how many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

Time (sec)	Height (m)
0	20
1	45
2	60

A. 6 seconds

B. 8 seconds

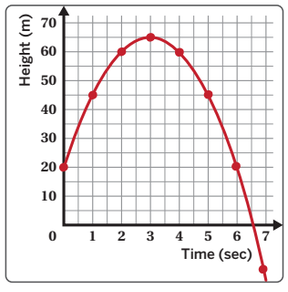
C. Between 6 and 7 seconds

D. Between 7 and 8 seconds

Explain your thinking.

Explanations vary. If I use the second difference to extend the table, I get 20 meters for 6 seconds and -15 meters for 7 seconds. This means the stomp rocket must have touched the ground and stopped moving sometime between 6 and 7 seconds.

8. Let's look at Ivan's graph.



a Why do you think Ivan drew a parabola? What type of function could represent this situation?

Responses vary.

- Ivan probably plotted the points and noticed that when you connect them, it makes a parabola.
- The table showed a constant second difference, so the relationship is quadratic. That means the graph is a parabola.
- All graphs of projectiles like the stomp rocket make a parabola.

b How do you think he used his graph to determine when the rocket landed?

Responses vary.

- Ivan looked for when the parabola touches the x-axis.
- The x-intercept will show when the rocket landed.
- Look for where the y-value equals 0. That's how long it takes for the rocket to land.

Unit 6 Lesson 5 626 Activity 2

Pairs | 15 min

Activity 2 Predicting With Tables and Graphs (continued)

Purpose: Students use tables and graphs to compare the flight paths of several different stomp rockets. **(MTR.3.1)**

Short on time? Consider facilitating Problem 9 as a whole-class demonstration and discussion.

2 Monitor

Invite students to discuss their noticings with a partner.

A **Accessibility: Visual-Spatial Processing** Invite students to choose two flight paths to focus on comparing.

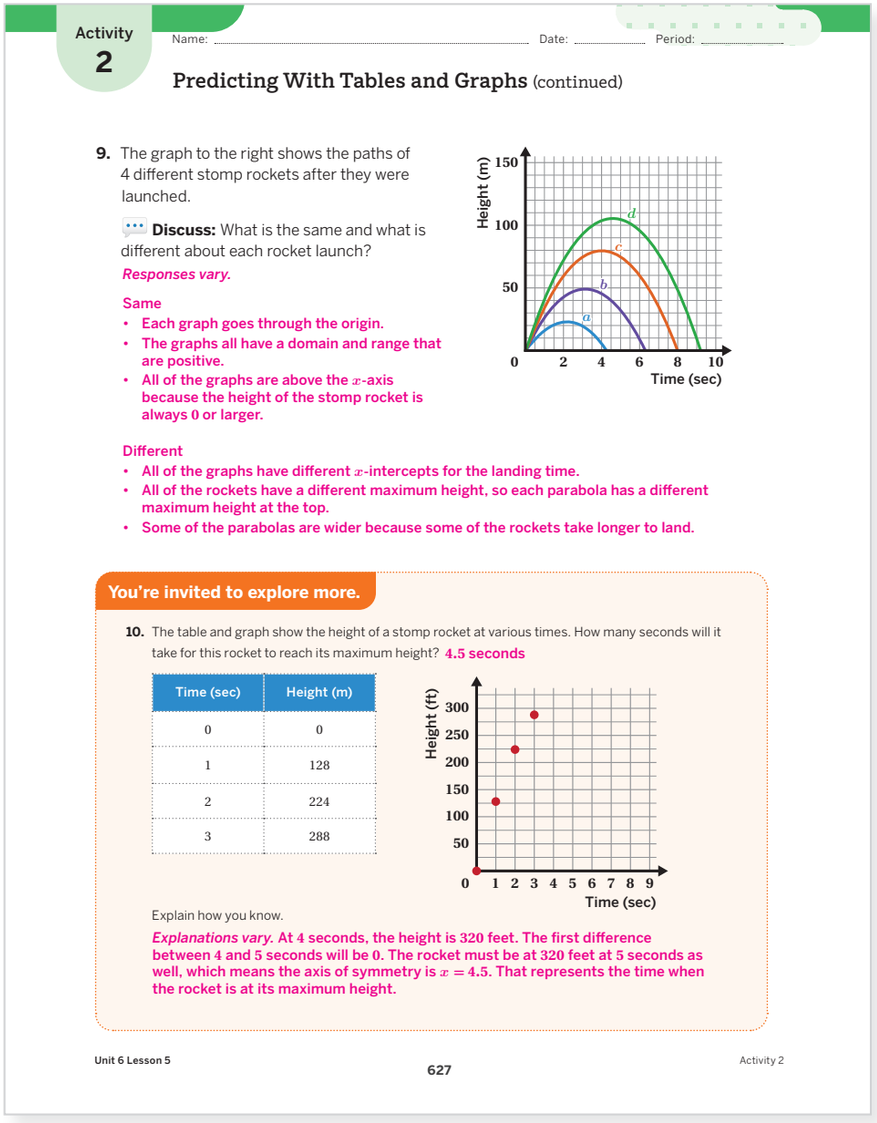
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore making predictions using graphs and tables further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their and their partners' noticings.

To reveal the Key Takeaway, consider asking:

- "Where can you see the highest point of the rocket's path?"
- "How can you see when the rocket lands?"
- "What is the line of symmetry for each graph? What does that tell us about each rocket?"



Activity 2

Name: _____ Date: _____ Period: _____

Predicting With Tables and Graphs (continued)

9. The graph to the right shows the paths of 4 different stomp rockets after they were launched.

Discuss: What is the same and what is different about each rocket launch?

Responses vary.

Same

- Each graph goes through the origin.
- The graphs all have a domain and range that are positive.
- All of the graphs are above the x-axis because the height of the stomp rocket is always 0 or larger.

Different

- All of the graphs have different x-intercepts for the landing time.
- All of the rockets have a different maximum height, so each parabola has a different maximum height at the top.
- Some of the parabolas are wider because some of the rockets take longer to land.

You're invited to explore more.

10. The table and graph show the height of a stomp rocket at various times. How many seconds will it take for this rocket to reach its maximum height? **4.5 seconds**

Time (sec)	Height (m)
0	0
1	128
2	224
3	288

Explain how you know.

Explanations vary. At 4 seconds, the height is 320 feet. The first difference between 4 and 5 seconds will be 0. The rocket must be at 320 feet at 5 seconds as well, which means the axis of symmetry is $x = 4.5$. That represents the time when the rocket is at its maximum height.

Unit 6 Lesson 5 627 Activity 2

Key Takeaway: We can also make a graph of a quadratic relationship that represents a situation. The graph can help us to make other predictions about the situation that may not be easy to determine from a table alone, like approximate values that fall between the table entries.

Whole Class | 5 min

Synthesis

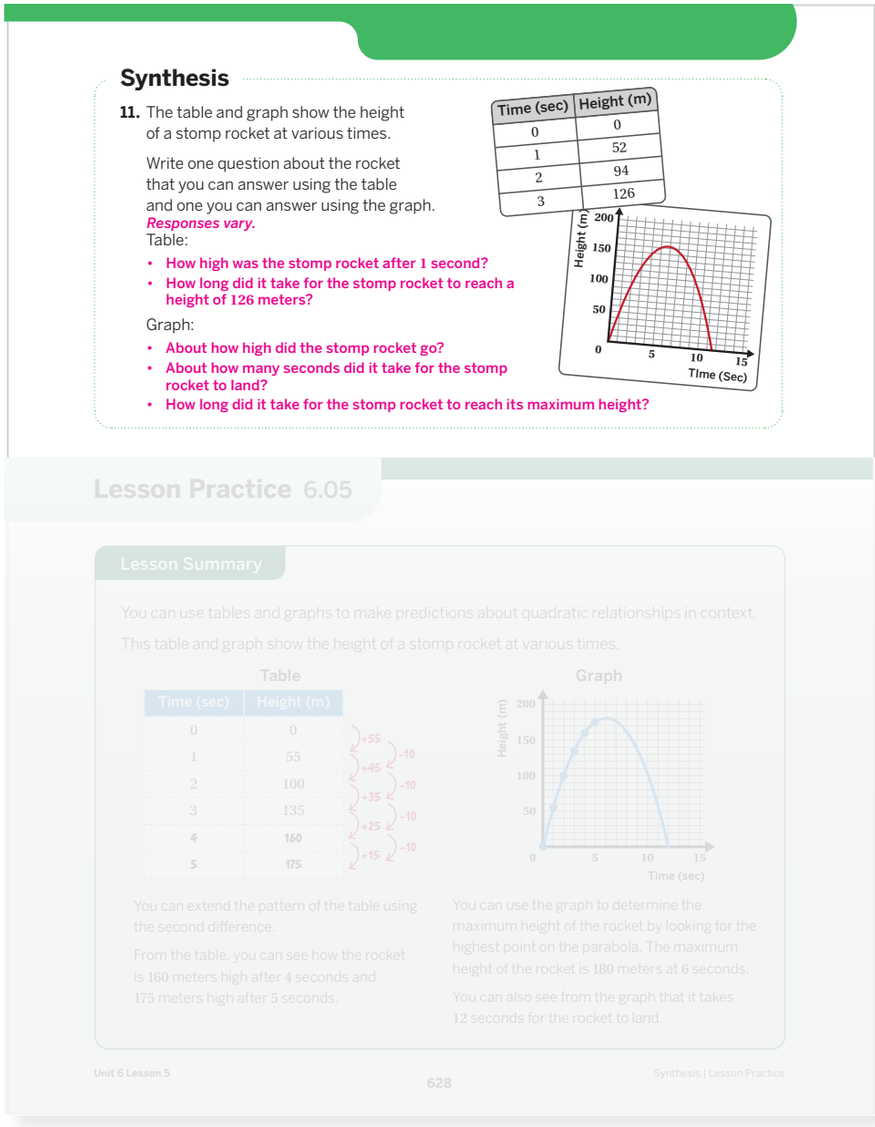
Purpose: Students synthesize their understanding of using tables and graphs of quadratic relationships to answer questions about stomp rockets.

Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:

- A question about specific heights in the table or graph.
- A question about specific times in the table or graph.
- A question about the highest point of the rocket's flight path.
- A question about when the rocket hits the ground.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. The table and graph show the height of a stomp rocket at various times.

Time (sec)	Height (m)
0	0
1	52
2	94
3	126

Write one question about the rocket that you can answer using the table and one you can answer using the graph.

Responses vary.

Table:

- How high was the stomp rocket after 1 second?
- How long did it take for the stomp rocket to reach a height of 126 meters?

Graph:

- About how high did the stomp rocket go?
- About how many seconds did it take for the stomp rocket to land?
- How long did it take for the stomp rocket to reach its maximum height?

Lesson Practice 6.05

Lesson Summary

You can use tables and graphs to make predictions about quadratic relationships in context.

This table and graph show the height of a stomp rocket at various times.

Table

Time (sec)	Height (m)		
0	0		
1	55	+55	
2	100	+45	−10
3	135	+35	−10
4	160	+25	−10
5	175	+15	−10

Graph

You can extend the pattern of the table using the second difference.

From the table, you can see how the rocket is 160 meters high after 4 seconds and 175 meters high after 5 seconds.

You can use the graph to determine the maximum height of the rocket by looking for the highest point on the parabola. The maximum height of the rocket is 180 meters at 6 seconds.

You can also see from the graph that it takes 12 seconds for the rocket to land.

Unit 6 Lesson 5 628 Synthesis | Lesson Practice

Lesson Takeaway: Tables and graphs of quadratic relationships that represent situations can help us make predictions about those relationships.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using tables and graphs of quadratic relationships to answer questions about stomp rockets.

Today's Goals

1. **Goal:** Use tables and graphs to determine values in quadratic relationships in context.
2. **Language Goal:** Compare and contrast the graphs of different quadratic relationships in context. **(Speaking and Listening)**

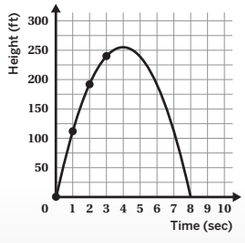
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.05

The table and graph show the height of a stomp rocket at various times.

Time (sec)	Height (ft)
0	0
1	112
2	192
3	240



a. How high was the rocket after 5 seconds?
240 feet

b. How long did it take for the rocket to land?
8 seconds

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to represent a relationship using a table:

If students need support understanding how to represent a relationship using a graph:

» **Respond to Student Thinking:**

- Revisiting **Grade 8, Unit 5, Lesson 5, Activity 1** (The Tortoise and the Hare).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.
- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

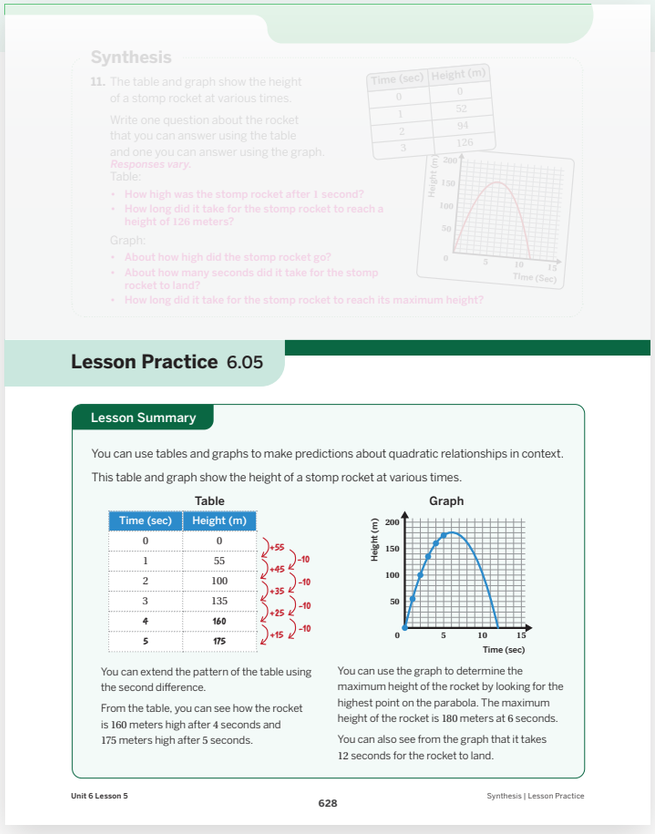
Professional Learning

Take a moment to reflect on how students reasoned abstractly and quantitatively in this lesson. How did the context of stomp rockets support students in making sense of the key features of quadratic functions? How did students engage in approximating values to support their reasoning? How did students' understanding of first and second differences support them in reasoning about the parabolic nature of quadratic functions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. The table and graph show the height of a stomp rocket at various times.

Time (sec)	Height (m)
0	0
1	52
2	94
3	126

Write one question about the rocket that you can answer using the table and one you can answer using the graph.

Responses vary.

Table:

- How high was the stomp rocket after 1 second?
- How long did it take for the stomp rocket to reach a height of 126 meters?

Graph:

- About how high did the stomp rocket go?
- About how many seconds did it take for the stomp rocket to land?
- How long did it take for the stomp rocket to reach its maximum height?

Lesson Practice 6.05

Lesson Summary

You can use tables and graphs to make predictions about quadratic relationships in context.

This table and graph show the height of a stomp rocket at various times.

Table

Time (sec)	Height (m)	
0	0	
1	55	+55
2	100	+45, −10
3	135	+35, −10
4	160	+25, −10
5	175	+15, −10

Graph

You can extend the pattern of the table using the second difference.

From the table, you can see how the rocket is 160 meters high after 4 seconds and 175 meters high after 5 seconds.

You can use the graph to determine the maximum height of the rocket by looking for the highest point on the parabola. The maximum height of the rocket is 180 meters at 6 seconds.

You can also see from the graph that it takes 12 seconds for the rocket to land.

Lesson Practice 6.05

Name: _____ Date: _____ Period: _____

Problems 1–3: This table and graph show the height of a stomp rocket at various times.

Time (sec)	0	1	2	3
Height (ft)	0	144	256	336

1. What is the maximum height that the stomp rocket reached?

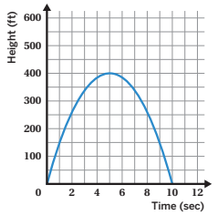
400 feet

2. How long did it take for the stomp rocket to land?

10 seconds

3. How high was the stomp rocket after 4 seconds?

384 feet



Problems 4–5: Oliver jumps off a diving board into a swimming pool. The relationship between his height and time can be modeled by a quadratic relationship.

Time (sec)	0	0.2	0.4	0.6	0.8	1	1.2	1.4
Height (m)	3	4.8	6.2	7.2	**7.8**	**8**	**7.8**	**7.2**

4. Complete the table.

5. After how many seconds will Oliver reach his maximum height?

1 second

Lesson Practice 6.05

Name: _____ Date: _____ Period: _____

6. The table shows the height of a stomp rocket from the time it is launched. How many seconds will it take for the rocket to land? Circle one.

Time (sec)	Height (m)
0	0
1	42
2	74
3	96

8 seconds 9 seconds Between 8 and 9 seconds **Between 9 and 10 seconds**

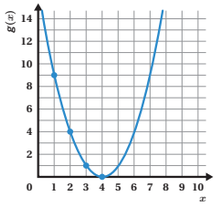
Explain your thinking.

Explanations vary. **There is a constant second difference of -10. If I continue the pattern using the constant second differences, then I get heights of 108, 110, 100, 80, 50, 10, and -40 meters. The rocket will land when the height is 0 meters, which is between 10 and -40. That is between 9 and 10 seconds.**

Test Practice

7. Here is the graph of the function $g(x)$. Write the equation for the line of symmetry of $g(x)$.

$x =$ **4**



Spiral Review

Problems 8–10: Mohamed can run a mile in 9 minutes. The number of miles Mohamed runs is a function of the amount of time he has been running.

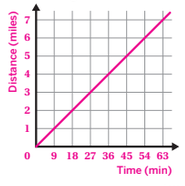
8. Graph this function. Be sure to label the axes.

9. Describe the domain of this function.

Domain: $x \geq 0$

10. Describe the range of this function.

Range: $y \geq 0$



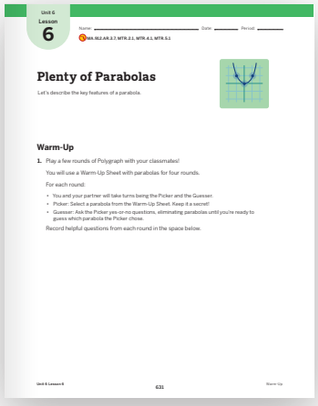
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.AR.3.8
	4–5	2	MA.912.AR.3.8
	6	2	MA.912.AR.3.8
Test Practice	7	1	MA.912.AR.3.7
Spiral Review			
	8–10	2	MA.912.AR.3.8

Unit 6

Lesson

6


Unit 6
Lesson
6
Name: _____ Date: _____ Period: _____
MA.912.AR.3.7, MTR.2.1, MTR.4.1, MTR.5.1
Plenty of Parabolas
Let's describe the key features of a parabola.
Warm-Up
1. Play a few rounds of Polygraph with your classmates!
You will use a Warm-Up Sheet with parabolas for four rounds.
For each round:
• You and your partner will take turns being the Picker and the Guesser.
• Picker: Select a parabola from the Warm-Up Sheet. Keep it a secret!
• Guesser: Ask the Picker yes-or-no questions, eliminating parabolas until you're ready to guess which parabola the Picker chose.
Record helpful questions from each round in the space below.
Unit 6 Lesson 6
631
Warm-Up

Plenty of Parabolas

Key Features of Parabolas

Let's describe the key features of a parabola.

Learning Outcomes

Today's Goals

1. **Goal:** Create graphs of parabolas given a description.
2. **Language Goal:** Describe the key features of a parabola from its graph (maximum, minimum, intercepts, concavity). **(Writing)**

Students describe and create graphs of parabolas using key features. Students make sense of the terms **vertex**, **concave up**, and **concave down** by playing a game and analyzing examples. They match graphs to descriptions and create graphs to fit descriptions.

Coherence

Prior Learning

In Lessons 4–5, students used graphs of parabolas in context to explore the symmetry of parabolas and used them to make predictions.

Future Learning

In Lesson 7, students compare quadratic functions and their key features in multiple representations.

Connections

- Students recognize the importance of common vocabulary when communicating with others about key features of a parabola. (mathematical, cross-curricular)
- Students create artistic designs by graphing multiple parabolas on the same coordinate plane. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

Mathematical Thinking and Reasoning Standard: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1.3	**MA.912.AR.2.4**

Vocabulary

New Vocabulary

concave down
concave up
vertex

Review Vocabulary

maximum (of a function)
minimum (of a function)
x-intercept
y-intercept

Lesson at a Glance

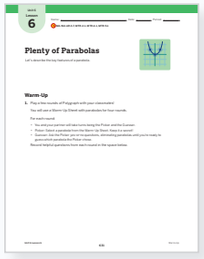
~ 45 min

MA.912.AR.3.7

Warm-Up

Pairs | 10 min

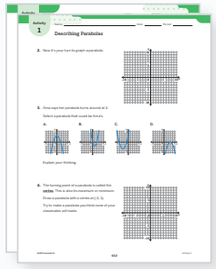
Students develop language to describe key features of parabolas by playing a game where they use yes-or-no questions to try to guess which function their classmate has chosen.



Activity 1

Pairs | 15 min

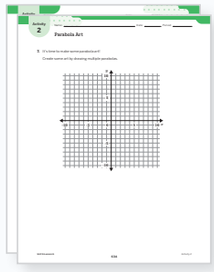
Students use descriptions to create parabolas and create class definitions of the terms **vertex**, **concave up**, and **concave down**. MLR2: Collect and Display



Activity 2

Pairs | 10 min

Students use their creativity to make parabola art and use key features to describe another student's artwork.



Synthesis

Whole Class | 5 min

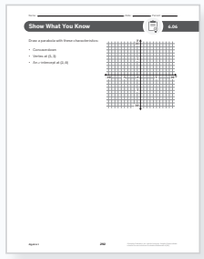
Students synthesize their understanding of key features of parabolas by precisely describing a parabola. MLR1: Stronger and Clearer **(MTR.4.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating a parabola from a description of its key features.



Prep Checklist

Prepare the Warm-Up Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Warm-Up Sheets
  - Calculator

Pairs | 10 min

Warm-Up

Purpose: Students develop language to describe key features of parabolas by playing a game where they use yes-or-no questions to try to guess which function their classmate has chosen.

1 Launch

Invite students to work on their own device. Students can play a practice round against the computer, then the platform will assign them a random partner from class for each round of the game.

Students using print will need one or more partners. Distribute the Warm-Up Sheet so that one partner has Set A and one has Set B. Invite one partner to be the Picker and the other to be the Guesser in the first round, and then switch roles for the next round.

A **Accessibility: Executive Functioning** To support multi-step activities, invite students to model an example round of the activity.

Encourage students to try and guess the function using as few questions as possible.

Note: Students will learn the terms *vertex*, *concave up*, and *concave down* to describe parabolas in Activity 1.

Pause briefly to discuss common questions and terms that students are using. Some examples of language to highlight might include: smile, frown, U-shape, upside down U, x-intercept, y-intercept, maximum, minimum, line of symmetry, etc.

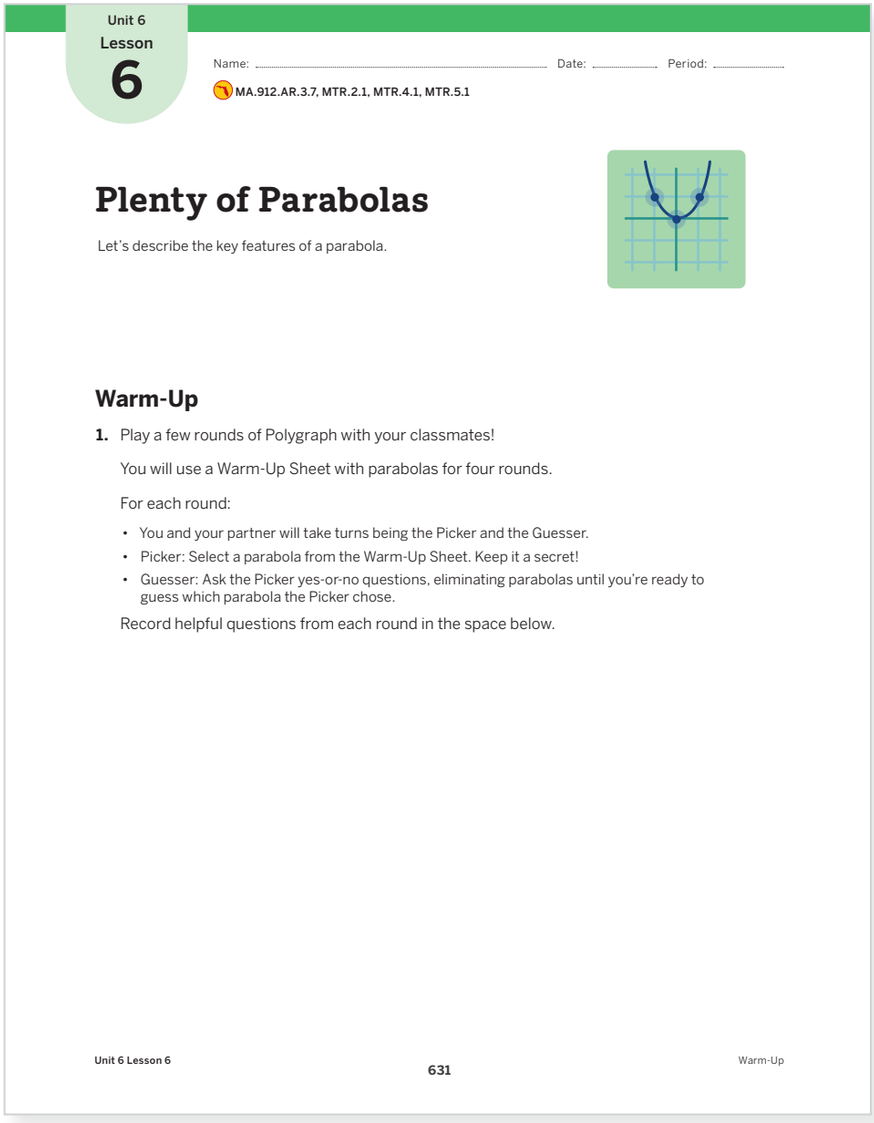
Encourage students to continue playing using language from the discussion if it is helpful for them.

2 Connect

Consider asking:

- "What were you paying attention to as you wrote your questions?"
- "What words or phrases did you find particularly helpful for distinguishing between parabolas?" **(MTR.4.1)**

Create a record of interesting or effective questions, including ones that describe key features of parabolas. Consider inviting students to add this language to the anchor chart during the MLR2: Collect and Display in Activity 1.



Unit 6
Lesson
6

Name: Date: Period:

MA.912.AR.3.7, MTR.2.1, MTR.4.1, MTR.5.1

Plenty of Parabolas

Let's describe the key features of a parabola.

Warm-Up

1. Play a few rounds of Polygraph with your classmates!

   You will use a Warm-Up Sheet with parabolas for four rounds.

   For each round:

   - You and your partner will take turns being the Picker and the Guesser.
   - Picker: Select a parabola from the Warm-Up Sheet. Keep it a secret!
   - Guesser: Ask the Picker yes-or-no questions, eliminating parabolas until you're ready to guess which parabola the Picker chose.

   Record helpful questions from each round in the space below.

Unit 6 Lesson 6 631 Warm-Up

Key Takeaway: Listening to and connecting different ways of describing parabolas can help us better understand the concept of a function and strengthen our mathematical community.

Pairs | 15 min

Activity 1 Describing Parabolas

Purpose: Students use descriptions to create parabolas and create class definitions of the terms **vertex**, **concave up**, and **concave down**.

1 Launch

To access connections, consider asking, "Where have you seen parabolas in real life?"

ELL **English Language Learners** For each problem in this activity, encourage a partner from each pair to read the problem aloud before responding. **(Reading, Speaking, and Listening)**

To encourage creativity, invite students to make a parabola they think no one else in the class will create.

2 Monitor

Collect a variety of student parabolas to share during the Connect.

ELL **English Language Learners** Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words. It may be helpful to invite students to share what they think the phrase "turn around" could mean about a parabola. **(Reading and Writing)**

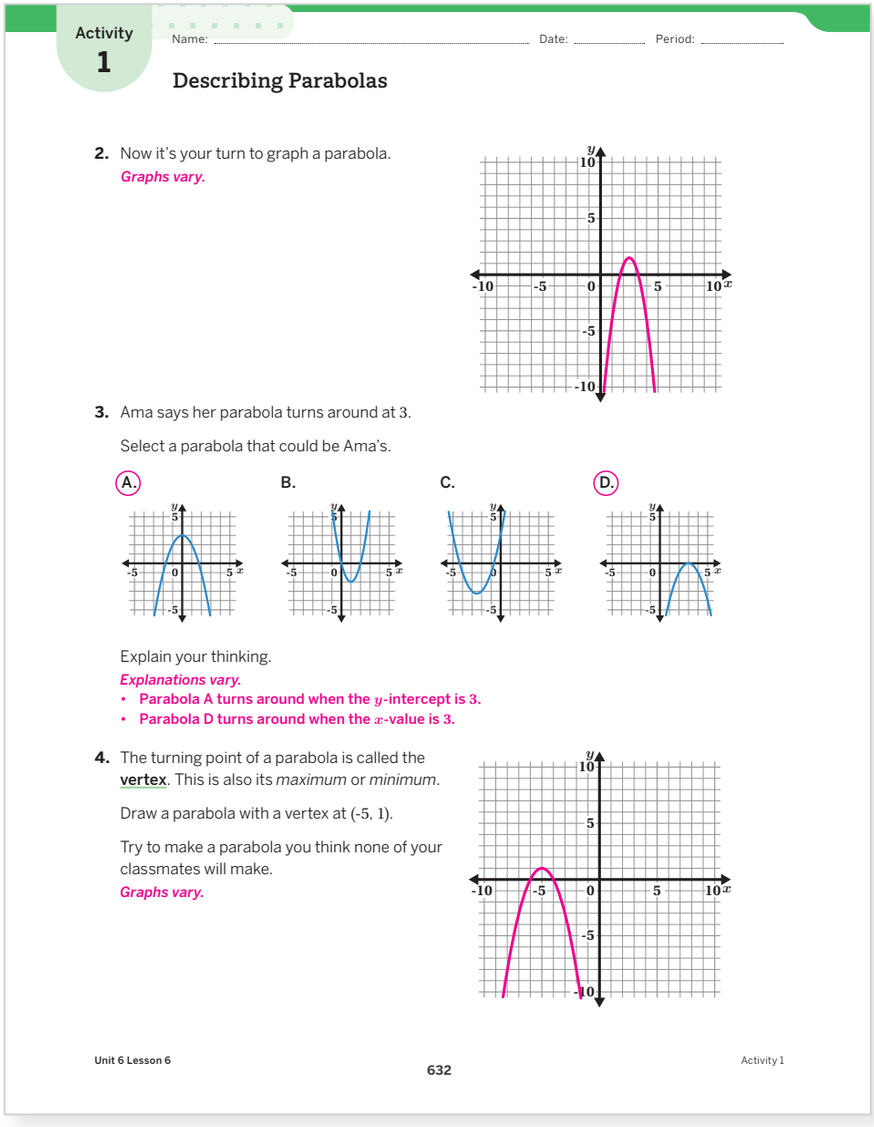
To support students getting started, encourage students to identify where they think each parabola "turns around."

ELL **English Language Learners** Consider inviting students to research how to say the term *vertex* in their primary language. **(Reading)**

D **Differentiation: (Stretch)** Consider asking, "What other types of functions have a vertex? What types of functions do not have a vertex?"

Look for student progress using the dashboard's Summary View. Check in with pairs of students as necessary.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Describing Parabolas

2. Now it's your turn to graph a parabola.

Graphs vary.

3. Ama says her parabola turns around at 3.

Select a parabola that could be Ama's.

A.

B.

C.

D.

Explain your thinking.

Explanations vary.

- Parabola A turns around when the y-intercept is 3.
- Parabola D turns around when the x-value is 3.

4. The turning point of a parabola is called the **vertex**. This is also its *maximum* or *minimum*.

Draw a parabola with a vertex at $(-5, 1)$.

Try to make a parabola you think none of your classmates will make.

Graphs vary.

Unit 6 Lesson 6 632 Activity 1

Pairs | 15 min

Activity 1 Describing Parabolas (continued)

Purpose: Students use descriptions to create parabolas and create class definitions of the terms vertex, concave up, and concave down.

2 Monitor

To encourage creativity, invite students to make a parabola they think no one else in the class will create.

Look for students who create parabolas that meet only one criterion in the description. Celebrate the correct key feature and invite them to reread for the second key feature.

ELL **English Language Learners** Consider inviting students to research how to say the terms *concave up* or *concave down* in their primary language. **(Reading)**

D **Differentiation: (Stretch)** Consider asking, "Is it possible for a parabola to have more than one vertex? Why or why not?"

3 Connect

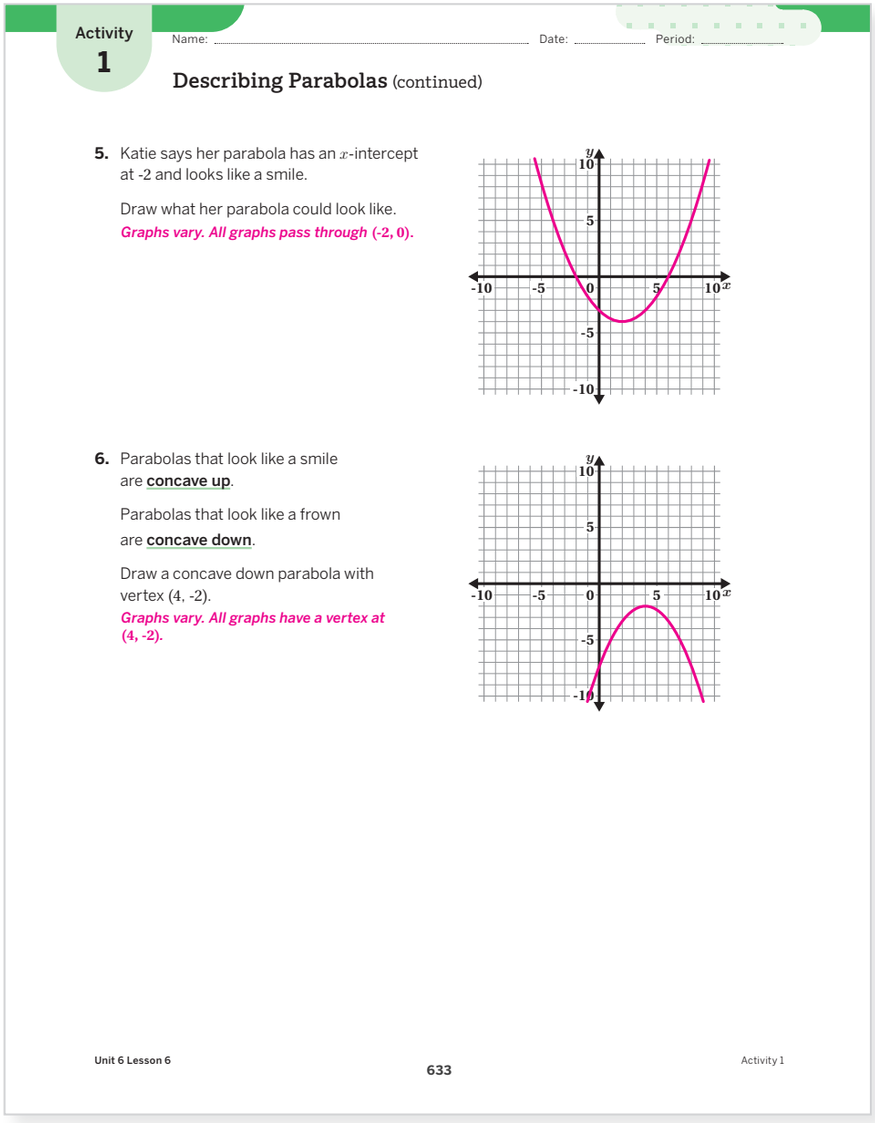
Share student parabolas one at a time and ask:

- "What is the vertex? How do you know?" It's the maximum/minimum. It's where the function switches from increasing to decreasing or vice versa.
- "Is this parabola concave up or concave down? How do you know?"

MLR **MLR2: Collect and Display** Use this routine to create class definitions of **vertex**, **concave up**, and **concave down** using students' language. Display the language, examples, and diagrams for students to reference throughout the unit.

To access connections, consider asking:

- "If a parabola is concave up, will its vertex be a maximum or a minimum? How do you know?"
- "How could you describe the key features of the parabolas you used to model stomp rockets in the previous lesson?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Describing Parabolas (continued)

5. Katie says her parabola has an x-intercept at -2 and looks like a smile.

Draw what her parabola could look like.

Graphs vary. All graphs pass through (-2, 0).

6. Parabolas that look like a smile are **concave up**.

Parabolas that look like a frown are **concave down**.

Draw a concave down parabola with vertex (4, -2).

Graphs vary. All graphs have a vertex at (4, -2).

Unit 6 Lesson 6 633 Activity 1

Key Takeaway: The terms **vertex**, **concave up**, and **concave down** can be used to describe key features of parabolas.

- The vertex of a parabola is its maximum or minimum point.
- A parabola that is concave up opens upward. Its vertex is a minimum.
- A parabola that is concave down opens downward. Its vertex is a maximum.

Pairs | 10 min

Activity 2 Parabola Art

Purpose: Students use their creativity to make parabola art and use key features to describe another student's artwork.

Short on time? Consider limiting the amount of time students spend on Problem 7.

1 Launch

To encourage creativity, invite students to make a design that is personal to them utilizing different colors. Consider generating excitement by creating your own parabola art or asking, "What kinds of things can you make using parabolas?"

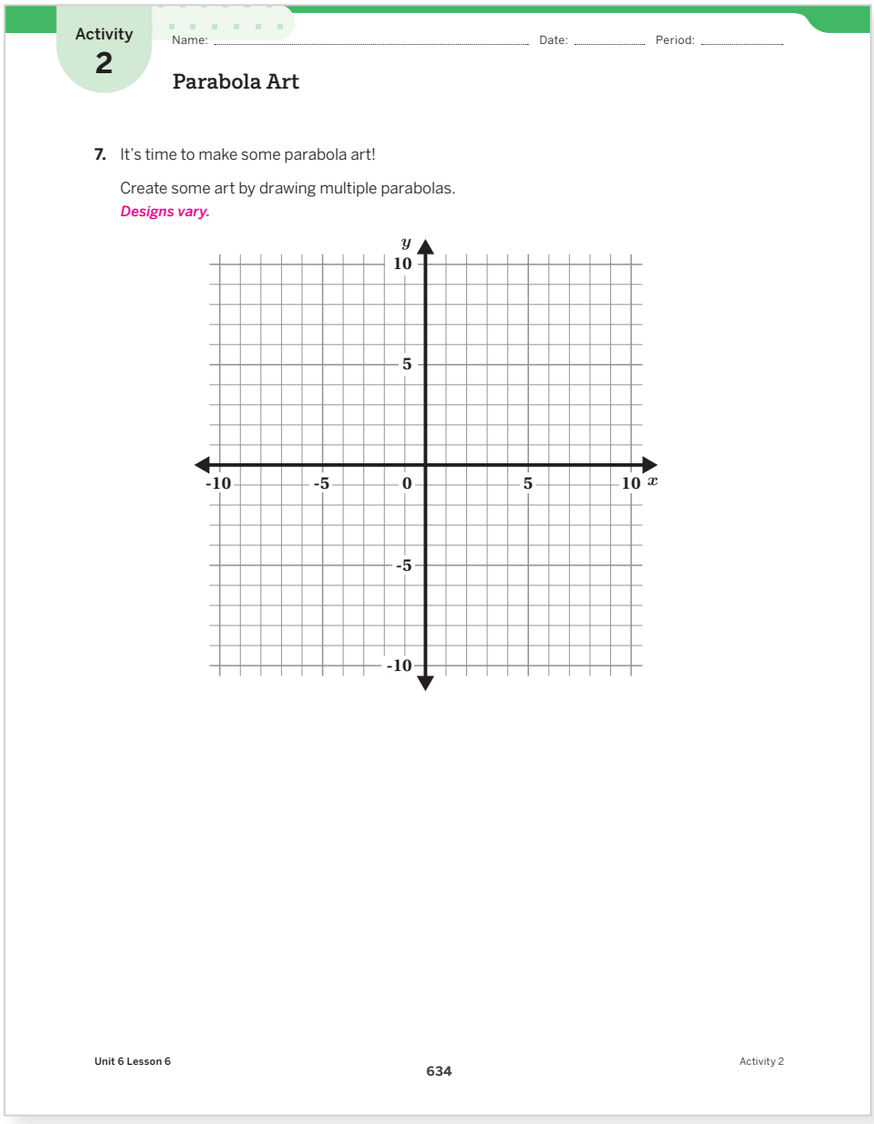
Invite students using print to use coloring tools to create their parabola art. To encourage creativity, they can even add shading and domain or range restrictions to their designs.

2 Monitor

Note: At this point in the unit, it is not expected that students will draw precise parabolas by hand. The goal is for students to understand that parabolas are symmetric and have a U-shape. Each parabola should have a clear vertex, concavity, and line of symmetry. Students may draw functions that aren't quadratic, such as graphs that look like a V or a sideways U. Celebrate their design and creativity. Consider asking, "Which of these functions are quadratic? Which ones are not?"

Collect a variety of student responses to share during the Connect.

Activity 2 continued >



Activity 2

Name: Date: Period:

Parabola Art

7. It's time to make some parabola art!

Create some art by drawing multiple parabolas.

Designs vary.

Unit 6 Lesson 6 634 Activity 2

Pairs | 10 min

Activity 2 Parabola Art (continued)

Purpose: Students use their creativity to make parabola art and use key features to describe another student's artwork.

Short on time? Consider limiting the amount of time students spend on Problem 7.

2 Monitor

Encourage students to use key feature vocabulary to describe the parabolas in Manuel's design.

Select and sequence student responses of various levels of precision to share during the Connect.

ELL **English Language Learners** Consider displaying an anchor chart or word bank of key features such as: *vertex*, *concave up*, *concave down*, x-*intercept*, y-*intercept*, *line of symmetry*, *maximum*, and *minimum*. **(Writing)**

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore key features of parabolas further to complete this optional task. Encourage them to discuss their thinking with a partner. Students may sketch parabolas that open to the left or to the right. Consider inviting them to discuss if those are quadratic functions or not.

3 Connect

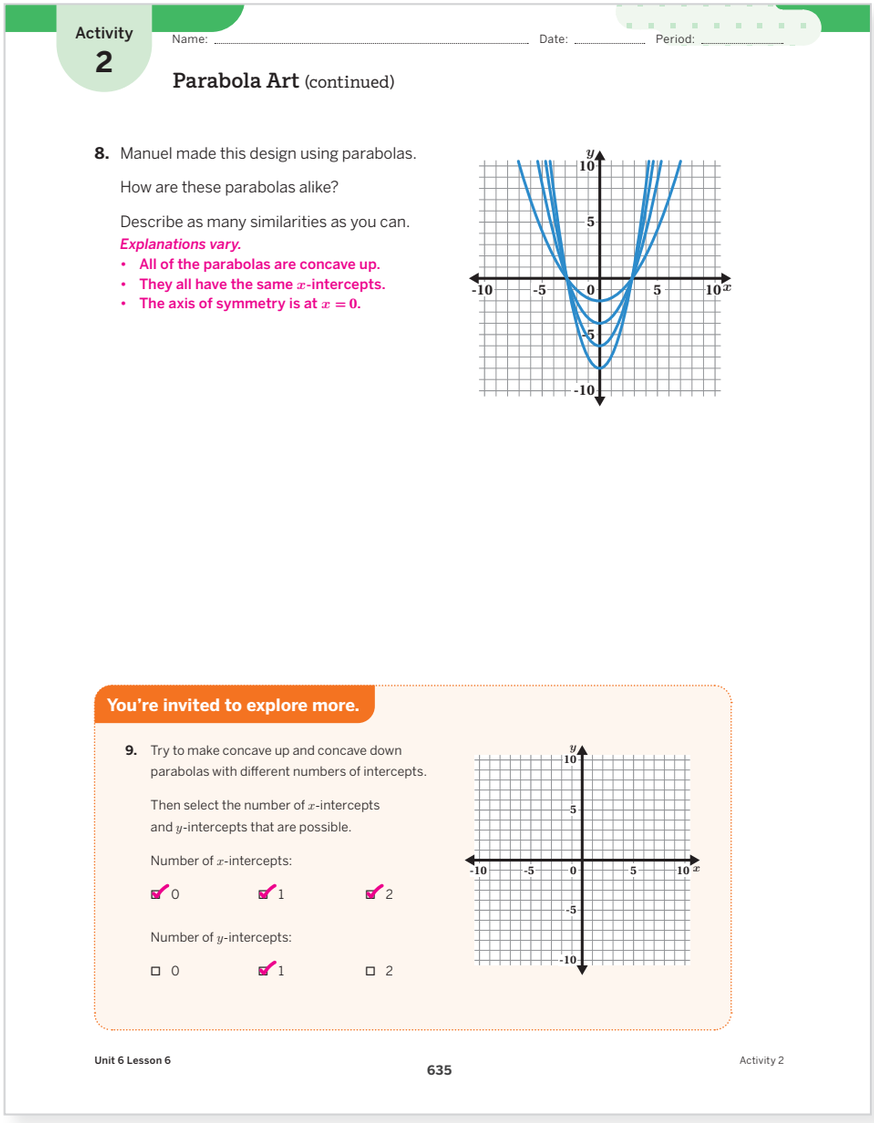
Share student parabola art one at a time, inviting students to describe the design using key feature vocabulary.

Consider asking, "Which design uses x-intercepts in an interesting way? The vertex?"

Share student responses one at a time, inviting students to paraphrase responses to incorporate key feature vocabulary.

Note: If it doesn't come up naturally, consider surfacing connections by asking: "How is the vertex related to a parabola's x-intercepts? To the line of symmetry?"

D **Differentiation: (Stretch)** Consider sharing that there are artists who use lots of math, like functions and geometry, in their work. Encourage students to research and explore some mathematical works of art and how they were created.



Activity 2

Name: _____ Date: _____ Period: _____

Parabola Art (continued)

8. Manuel made this design using parabolas.

How are these parabolas alike?

Describe as many similarities as you can.

Explanations vary.

- All of the parabolas are concave up.
- They all have the same x-intercepts.
- The axis of symmetry is at $x = 0$.

You're invited to explore more.

9. Try to make concave up and concave down parabolas with different numbers of intercepts.

Then select the number of x-intercepts and y-intercepts that are possible.

Number of x-intercepts:

☑ 0 ☑ 1 ☑ 2

Number of y-intercepts:

☐ 0 ☑ 1 ☐ 2

Unit 6 Lesson 6 635 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of key features of parabolas by precisely describing a parabola. **(MTR.4.1)**

Short on time? Consider limiting the number of rounds of feedback students get before revising their draft.

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft and use feedback to make their thinking stronger and clearer.

Consider sharing that first drafts do not have to be complete or perfect to be valuable.

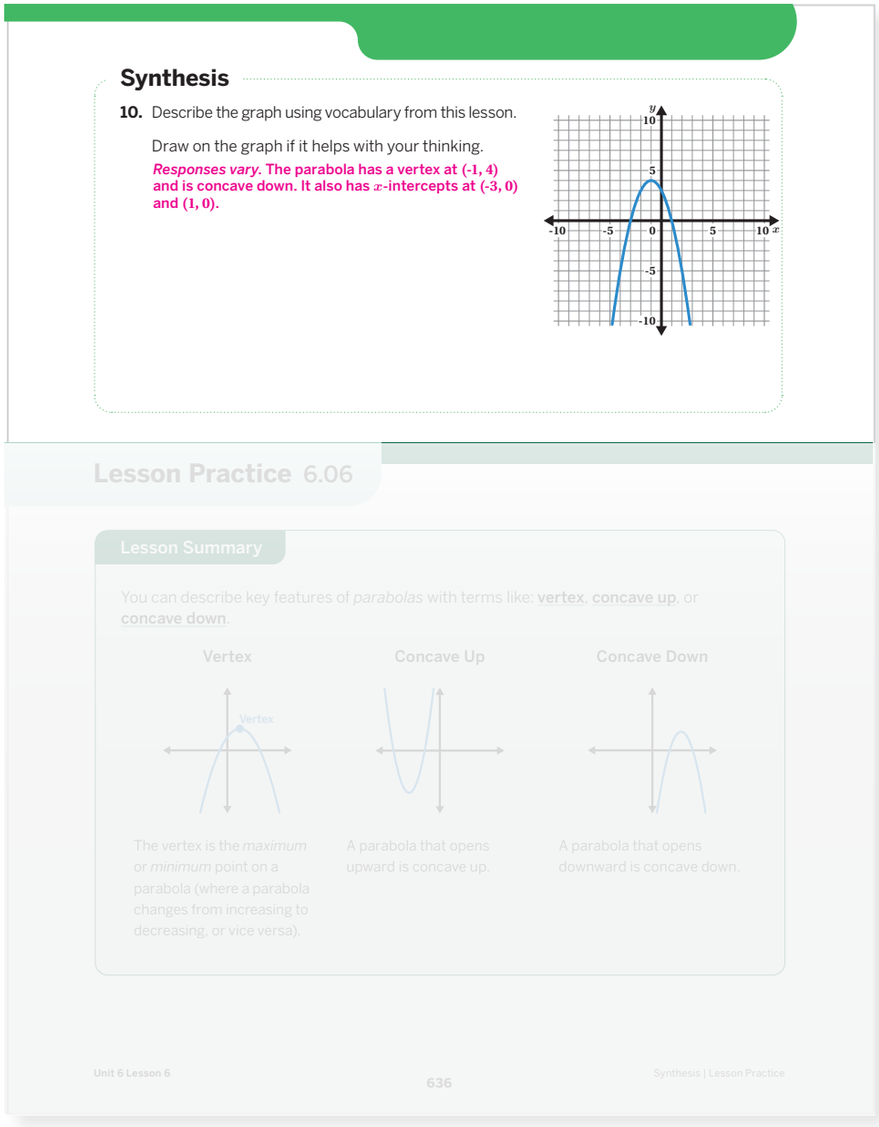
Invite students to share their drafts with a partner and give each other feedback. Repeat this partner feedback process for as many rounds as there is time for.

Encourage students to use the feedback they got from their partners to write a second draft that is stronger or clearer.

Look for and share a variety of student descriptions and sketches. Acknowledge and celebrate where you see precision of language. **(MTR.4.1)**

Consider asking:

- "What is helpful about this person's description?"
- "How is describing a parabola similar to describing other functions we've learned about? How is it different?"
- "What does the vertex tell us about the parabola? What does a concave up or concave down parabola tell us?"
- "Which key features does a parabola share with linear functions? Exponential functions?"



Synthesis

10. Describe the graph using vocabulary from this lesson.

Draw on the graph if it helps with your thinking.

Responses vary. The parabola has a vertex at (-1, 4) and is concave down. It also has x-intercepts at (-3, 0) and (1, 0).

Lesson Practice 6.06

Lesson Summary

You can describe key features of *parabolas* with terms like: **vertex**, **concave up**, or **concave down**.

Vertex

The vertex is the *maximum* or *minimum* point on a parabola (where a parabola changes from increasing to decreasing, or vice versa).

Concave Up

A parabola that opens upward is concave up.

Concave Down

A parabola that opens downward is concave down.

Unit 6 Lesson 6 636 Synthesis | Lesson Practice

Lesson Takeaway: There are many ways to describe parabolas and often many parabolas that will fit a single description. Using key feature language like **vertex**, **concave up**, **concave down**, x-*intercept*, y-*intercept*, *line of symmetry*, *maximum*, and *minimum* can be helpful for describing parabolas.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by creating a parabola from a description of its key features.

Today's Goals

1. **Goal:** Create graphs of parabolas given a description.
2. **Language Goal:** Describe the key features of a parabola from its graph (maximum, minimum, intercepts, concavity). **(Writing)**

Show What You Know (Assessment Resources)

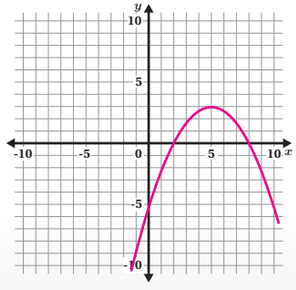
Name: _____ Date: _____ Period: _____

Show What You Know 6.06

Draw a parabola with these characteristics:

- Concave down
- Vertex at $(5, 3)$
- An x-intercept at $(2, 0)$

Responses vary.



Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to describe and creating parabolas:

» **Respond to Student Thinking:**

- Use the *Describing Key Features of Parabolas* **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.
- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

Professional Learning

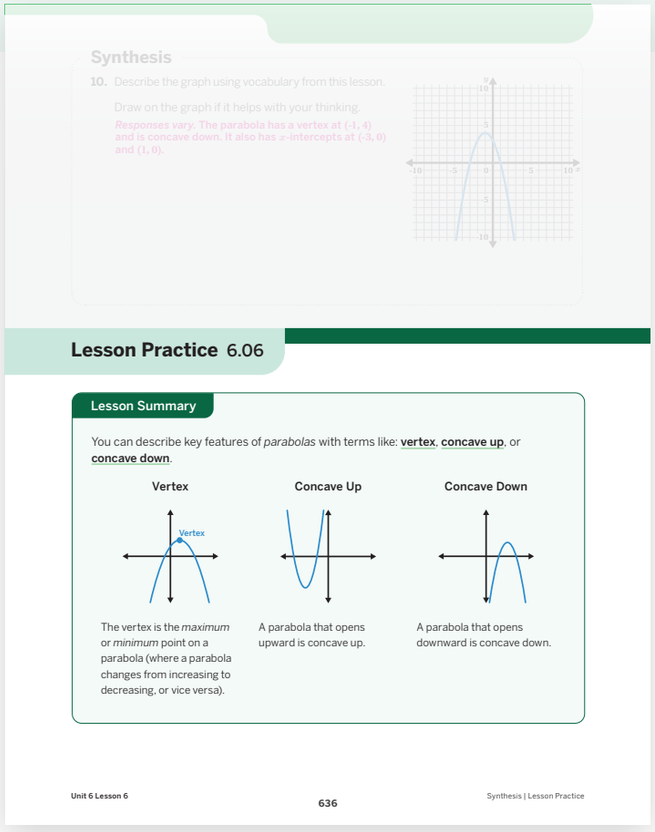
Reflect on how student language for describing parabolas developed throughout this lesson.

- How did your use of Polygraph and **MLR2: Collect and Display** support students in making sense of the new key feature vocabulary?
- How did you honor student language for describing parabolas today?
- How can you continue to support students in attending to precision with key feature language throughout this unit?
- How do you imagine your class definitions of the new terms might grow when students study polynomial functions in Algebra 2?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. Describe the graph using vocabulary from this lesson.

Draw on the graph if it helps with your thinking.

Responses vary. The parabola has a vertex at (-1, 4) and is concave down. It also has x-intercepts at (-3, 0) and (1, 0).

Lesson Practice 6.06

Lesson Summary

You can describe key features of *parabolas* with terms like: **vertex**, **concave up**, or **concave down**.

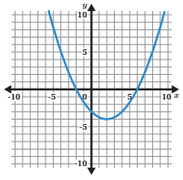
Vertex	Concave Up	Concave Down
The vertex is the *maximum* or *minimum* point on a parabola (where a parabola changes from increasing to decreasing, or vice versa).	A parabola that opens upward is concave up.	A parabola that opens downward is concave down.

Lesson Practice 6.06

Name: ..... Date: ..... Period: .....

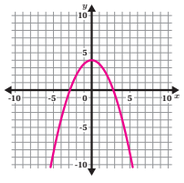
Problems 1–3: Use the graph to determine the coordinates of each key feature.

1. vertex: **(2, -4)**
2. x-intercept(s): **(-2, 0) and (6,0)**
3. y-intercept: **(0, -3)**

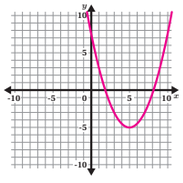


Problems 4–5: Graph a parabola that fits each description. ***Responses vary.***

4. Concave down with a positive y-intercept



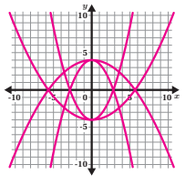
5. Concave up and vertex at (5, -5)



6. A parabola has a vertex at $(4, 2)$. Give two possible coordinates for its x-intercepts.
***Responses vary.* (2, 0) and (6, 0). Correct responses will be an equal distance from $x = 4$.**

7. Using only parabolas, make a design that:
   - Looks the same when reflected over the x-axis.
   - Looks the same when reflected over the y-axis.

Designs vary.



Lesson Practice 6.06

Name: ..... Date: ..... Period: .....

Test Practice

8. The key features of this parabola are labeled A, B, C, and D.

Match each key feature with a term from the word bank.

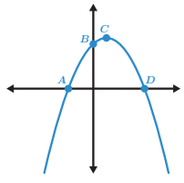
vertex x-intercept y-intercept

A: ...*x-intercept*...

B: ...*y-intercept*...

C: ...*vertex*...

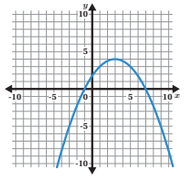
D: ...*x-intercept*...



Spiral Review

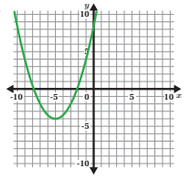
Problems 9–10: Write an equation for the line of symmetry of each parabola.

9.



Equation: ...*x = 3*...

10.



Equation: ...*x = -5*...

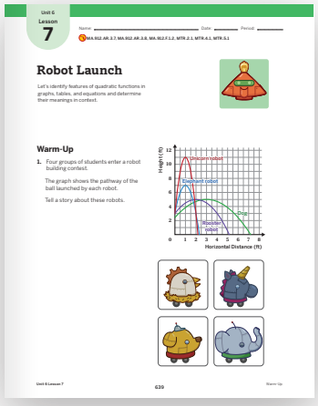
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.AR.3.7
	4–6	2	MA.912.AR.3.7
	7	3	MA.912.AR.3.7
Test Practice	8	2	MA.912.AR.3.7
Spiral Review			
Fluency	9–10	2	MA.912.AR.3.7

Unit 6

Lesson

7



Robot Launch

Key Features of Graphs in Context

Let's identify features of quadratic functions in graphs, tables, and equations and determine their meanings in context.

Learning Outcomes

Today's Goals

1. **Goal:** Compare properties of quadratic functions represented in different ways.
2. **Language Goal:** Describe connections between equations, tables, descriptions, and graphs that represent the same quadratic function. **(Speaking and Listening)**

Students make sense of quadratic relationships in context by identifying and comparing key features in graphs, tables, and equations. In particular, they will interpret the vertex and intercepts of quadratic functions in multiple representations.

Coherence

Prior Learning

In Lesson 5, students used tables and graphs of parabolas in context to make sense of the key features of a stomp rocket. In Lesson 6, students created graphs of parabolas using key features.

Future Learning

In Lesson 8, students will graph a quadratic function by evaluating its equation at different x-values.

Connections

- Students identify features of quadratic functions in multiple representations (tables, graphs, equations) and determine their meaning in the context of different robots launching a ball. (real world, mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7

Given a table, equation or written description of a quadratic function, graph that function and determine and interpret its key features.

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.F.1.3	**MA.912.AR.3.8**
MA.912.F.1.7	

Vocabulary

Review Vocabulary

x-intercept *y-intercept*

Lesson at a Glance

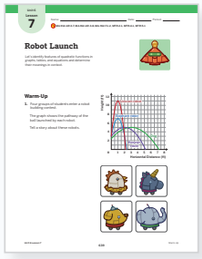
~ 45 min

MA.912.AR.3.7, MA.912.F.1.2

Warm-Up

Pairs | 5 min

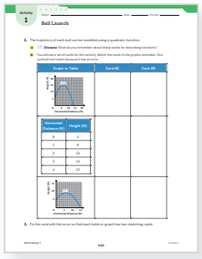
Students are introduced to the context of a robot ball-launching contest. **Routine:** Tell A Story



Activity 1

Pairs | 10 min

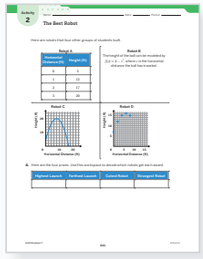
Students make sense of the vertex, intercepts, and other key features of quadratic functions in the context of launching a ball.



Activity 2

Pairs | 20 min

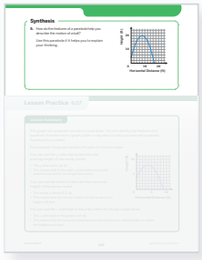
Students use the structure of different representations of quadratic functions to compare their properties. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

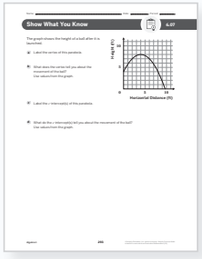
Students synthesize their understanding of how features of a parabola can help answer questions in context. **Routines:** Think-Pair-Share, MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying and interpreting key features of a quadratic function in context.



Prep Checklist

Prepare the Activity 1 cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of a robot ball-launching contest.

1 Launch

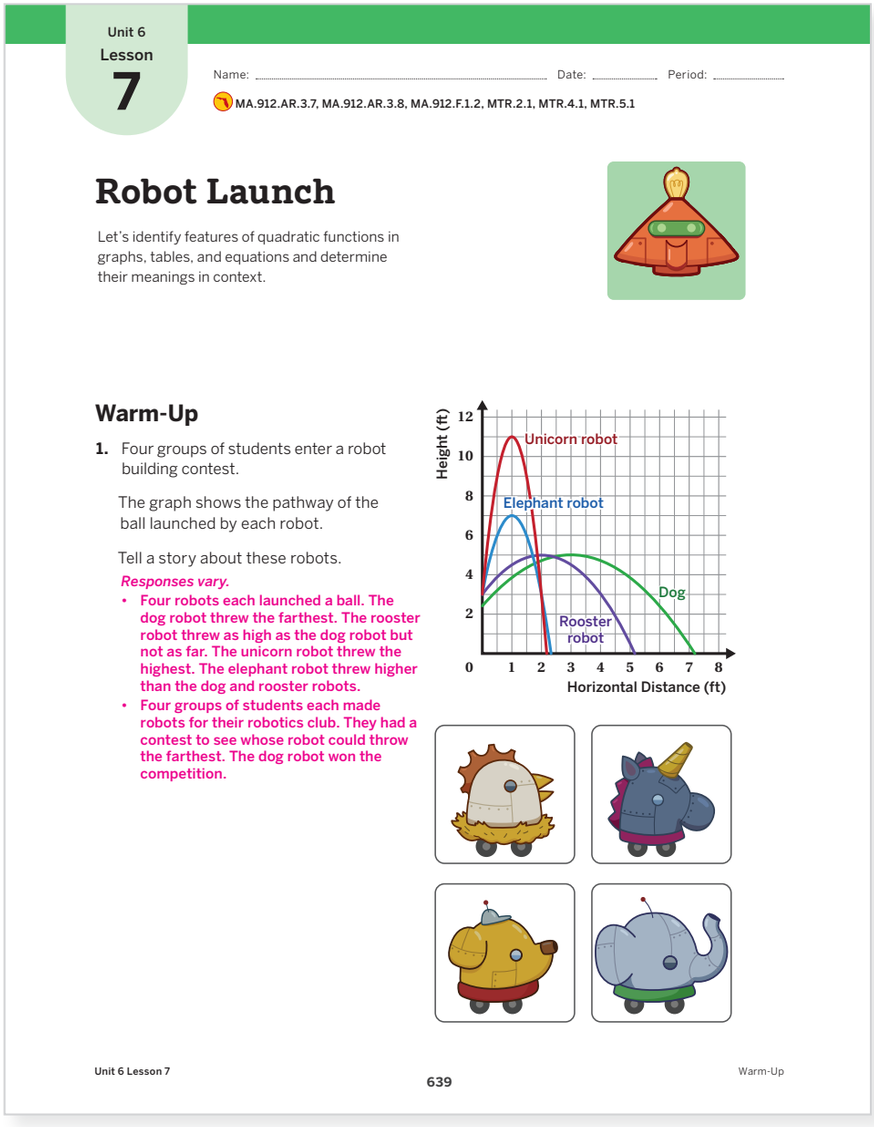
Display the robots.

To generate excitement, consider asking, "Have you ever built something? What was it?"

Use the Tell a Story routine. Invite students to write a story about the robots using their unique experiences, perspectives, and creativity. See the Routine Facilitation Guide for more information.

2 Connect

To access connections, consider asking, "What do you notice about the path of each ball?" If it does not come up naturally, discuss how the path of the ball can be modeled by a parabola.



Unit 6
Lesson
7

Name: _____ Date: _____ Period: _____

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Robot Launch

Let's identify features of quadratic functions in graphs, tables, and equations and determine their meanings in context.

Warm-Up

1. Four groups of students enter a robot building contest.

   The graph shows the pathway of the ball launched by each robot.

   Tell a story about these robots.

   Responses vary.
   - Four robots each launched a ball. The dog robot threw the farthest. The rooster robot threw as high as the dog robot but not as far. The unicorn robot threw the highest. The elephant robot threw higher than the dog and rooster robots.
   - Four groups of students each made robots for their robotics club. They had a contest to see whose robot could throw the farthest. The dog robot won the competition.

Unit 6 Lesson 7 639 Warm-Up

Pairs | 10 min

Activity 1 Ball Launch

Purpose: Students make sense of the vertex, intercepts, and other key features of quadratic functions in the context of launching a ball.

Short on time? Consider discussing Cards A and F as a whole class instead of in pairs.

1 Launch

Create a class definition of the term **quadratic function** using students' language and the glossary.

To surface prior knowledge, invite students to discuss what they remember about the key features of a parabola.

ELL **English Language Learners** Consider reviewing and adding to an anchor chart that publicly displays each term along with class definitions from earlier lessons. **(Writing and Speaking)**

Distribute one set of cards to each pair. Invite students to work together to sort the cards.

A **Accessibility: Executive Functioning** Invite students to choose one card at a time and decide which table or graph it represents to support students in carrying out multiple steps.

2 Monitor

Encourage students to annotate the graphs and the table to help make connections.

Look for strategies for sorting the cards that students can share during the Connect.

D **Differentiation** *(Problems 2–3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where would you look to see the maximum height? To see the ball hit the ground?" **(MTR.2.1)**
Would like to explore key features of quadratic functions further.	**Stretch** Invite students to discuss another way they could fix Card F.

3 Connect

Display the cards and Problem 2. Invite students to share their strategies for sorting the cards.

To reveal the Key Takeaway, consider asking:

- "Where can you see the key features of a quadratic function in a graph? In a table?" **(MTR.2.1)**
- "What can the vertex of a parabola tell us about the position of the ball? The y-intercept? The x-intercept?" **(MTR.2.1)**

Activity 1

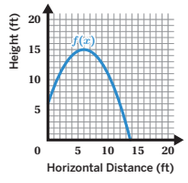
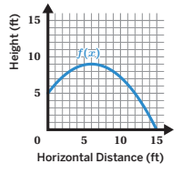
Name: _____ Date: _____ Period: _____

Ball Launch

2. The trajectory of each ball can be modeled using a quadratic function.

a. **Discuss:** What do you remember about these words for describing functions?

b. You will use a set of cards for this activity. Match the cards to the graphs and table. One card will not match because it has an error.

Graph or Table	Card #1	Card #2
	Card B The maximum height of the ball is 15 feet.	Card D The range of this graph is $0 \le f(x) \le 15$.
Horizontal Distance (ft) / Height (ft): 0 / 4; 1 / 9; 2 / 12; 3 / 13; 4 / 12	Card C Vertex at (3, 13)	Card E $f(2) = 12$
	Card A The ball hits the ground 15 feet from where it is launched.	

3. Fix the card with the error so that each table or graph has two matching cards.

Incorrect card: Card F. *Responses vary.* It should say the y-intercept is at $(0, 5)$.

Unit 6 Lesson 7 640 Activity 1

Key Takeaway: The path of a launched ball can be represented by a *quadratic function*. The y-intercept represents the height of the ball at the initial launch. The vertex represents the maximum height the ball reaches. The x-intercept represents the horizontal distance the ball has traveled when it hits the ground.

Pairs | 20 min

Activity 2 The Best Robot

Purpose: Students use the structure of different representations of quadratic functions to compare their properties. **(MTR.5.1)**

Short on time? Consider omitting the Cutest Robot and/or Strongest Robot award.

1 Launch

Invite students to share how they might decide which robot is the best.

Encourage students to look at the robots and prize categories. What are some awards you would give out to these robots?

ELL **English Language Learners** Consider using the graphs from the Warm-Up to demonstrate what the terms "highest launch" and "farthest launch" mean. **(Reading and Listening)**

2 Monitor

Look for and celebrate the different ways that students explain how to award the prizes.

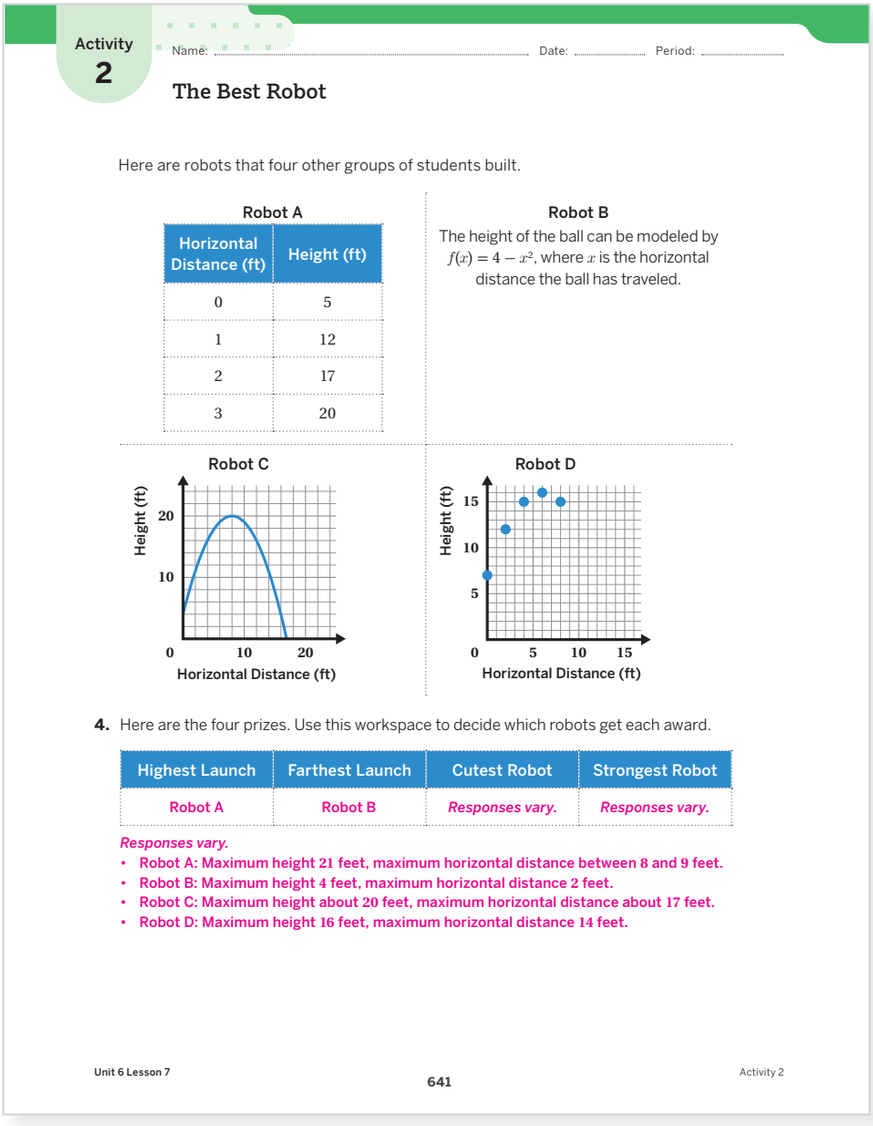
D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to think about one prize at a time and ask, "How can you find the ball's height with a graph? A table?" Invite them to annotate each representation. **(MTR.5.1)**
Need support determining the vertex or intercepts from a table or graph.	**Support** Consider asking, "How can we use second differences to make predictions? How can we use symmetry?"
Think Robot A and Robot C tie for the highest launch because each throws 16 feet higher than their launch point.	**Strengthen** Celebrate their noticing. Consider asking, "Where can you see the vertex? How can this help you think about the highest launch?"
Would like to explore quadratic functions further.	**Stretch** Invite students to discuss how they know each ball launch is quadratic.

3 Connect

Invite students to share which robot should get each award and why. Consider asking, "How did you decide which robot launched the ball the highest and the farthest?" **(MTR.4.1, MTR.5.1)**

To access connections, consider asking, "What strategies did you use to make sense of the key features of the quadratic functions?" **(MTR.5.1)**



Activity 2

Name: _____ Date: _____ Period: _____

The Best Robot

Here are robots that four other groups of students built.

Robot A

Horizontal Distance (ft)	Height (ft)
0	5
1	12
2	17
3	20

Robot B

The height of the ball can be modeled by $f(x) = 4 - x^2$, where x is the horizontal distance the ball has traveled.

Robot C

Robot D

4. Here are the four prizes. Use this workspace to decide which robots get each award.

Highest Launch	Farthest Launch	Cutest Robot	Strongest Robot
Robot A	Robot B	*Responses vary.*	*Responses vary.*

Responses vary.

- Robot A: Maximum height 21 feet, maximum horizontal distance between 8 and 9 feet.
- Robot B: Maximum height 4 feet, maximum horizontal distance 2 feet.
- Robot C: Maximum height about 20 feet, maximum horizontal distance about 17 feet.
- Robot D: Maximum height 16 feet, maximum horizontal distance 14 feet.

Unit 6 Lesson 7 641 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how features of a parabola can help answer questions in context.

Invite students to respond independently, and then share their thinking with a partner.

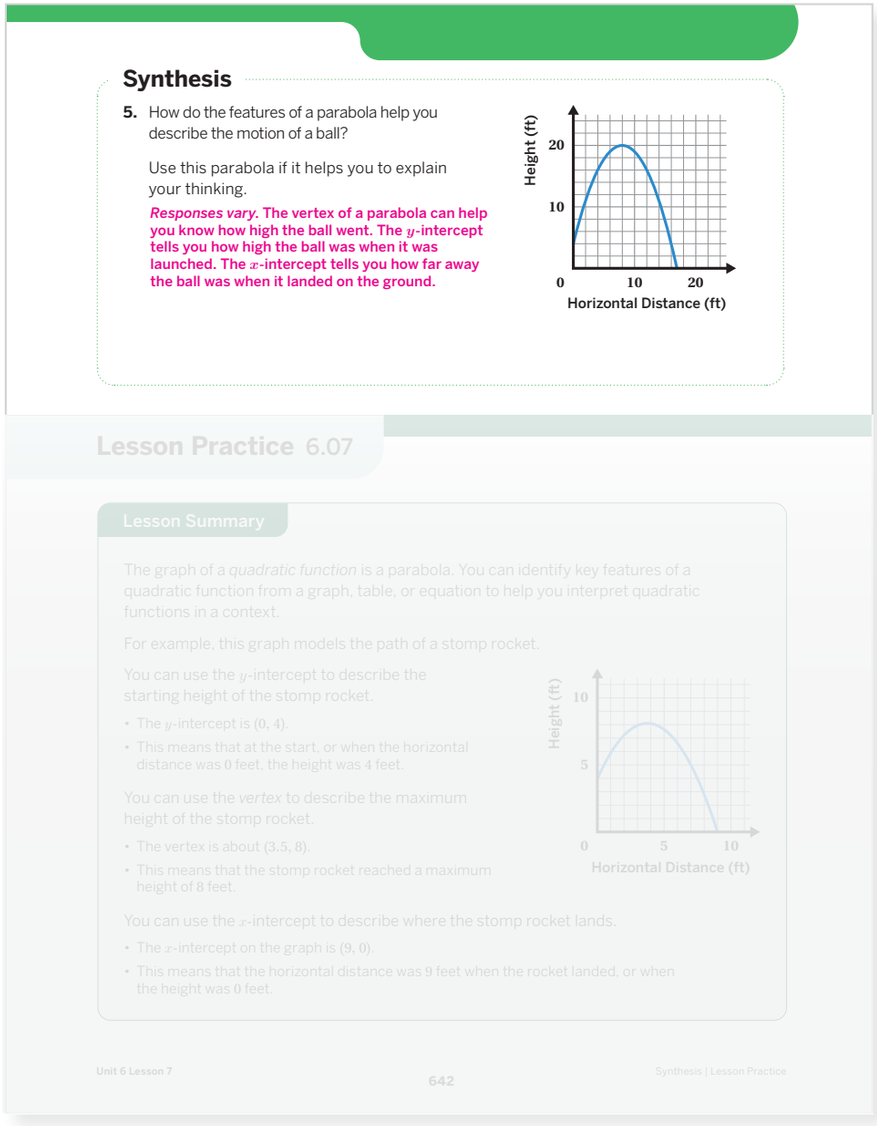
ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., "The vertex/y-intercept/x-intercept tells you _____."). **(Writing and Speaking)**

MLR **MLR2: Collect and Display** Use this routine to revise the definition of **quadratic function** and to synthesize strategies for determining key features of a parabola using students' language. Display any new language, examples, or diagrams for students to reference later.

Share a variety of ideas, such as:

- The vertex represents the maximum height of the ball.
- The x-intercept represents the horizontal distance the ball has traveled when it hits the ground.
- The y-intercept represents the height of the ball before it is launched.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

5. How do the features of a parabola help you describe the motion of a ball?

Use this parabola if it helps you to explain your thinking.

Responses vary. The vertex of a parabola can help you know how high the ball went. The y-intercept tells you how high the ball was when it was launched. The x-intercept tells you how far away the ball was when it landed on the ground.

Lesson Practice 6.07

Lesson Summary

The graph of a *quadratic function* is a parabola. You can identify key features of a quadratic function from a graph, table, or equation to help you interpret quadratic functions in a context.

For example, this graph models the path of a stomp rocket.

You can use the y-intercept to describe the starting height of the stomp rocket.

- The y-intercept is $(0, 4)$.
- This means that at the start, or when the horizontal distance was 0 feet, the height was 4 feet.

You can use the *vertex* to describe the maximum height of the stomp rocket.

- The vertex is about $(3.5, 8)$.
- This means that the stomp rocket reached a maximum height of 8 feet.

You can use the x-intercept to describe where the stomp rocket lands.

- The x-intercept on the graph is $(9, 0)$.
- This means that the horizontal distance was 9 feet when the rocket landed, or when the height was 0 feet.

Unit 6 Lesson 7 642 Synthesis | Lesson Practice

Lesson Takeaway: The key features of a parabola can help us make sense of a quadratic function in a context.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying and interpreting key features of a quadratic function in context.

Today's Goals

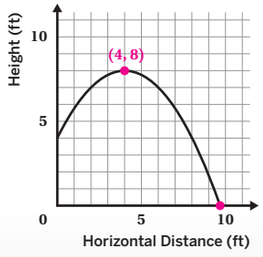
1. **Goal:** Compare properties of quadratic functions represented in different ways.
2. **Language Goal:** Describe connections between equations, tables, descriptions, and graphs that represent the same quadratic function. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 6.07

The graph shows the height of a ball after it is launched.



a. Label the vertex of this parabola.

b. What does the vertex tell you about the movement of the ball?
Use values from the graph.
Responses vary. The ball is at its maximum height of 8 feet when it is horizontally 4 feet away from where it launched.

c. Label the x-intercept(s) of this parabola.
Coordinates vary. The y-coordinate should be 0.

d. What do the x-intercept(s) tell you about the movement of the ball?
Use values from the graph.
Responses vary. The x-intercept tells me that the ball lands between 9 and 10 feet away from where it was launched.

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to interpret the vertex and x-intercepts of a parabola in context:

» **Respond to Student Thinking:**

- Revisit **Activity 2,** Cards A and B as a class and invite students to share strategies for interpreting key features in context.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 1 Extension Activities.

Professional Learning

Take a moment to reflect on how students' understanding of the key features of a parabola has developed throughout this sub-unit. How have their definitions of *vertex*, *x-intercept*, and *y-intercept* expanded over the last few lessons? What new examples of each have they seen?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

5. How do the features of a parabola help you describe the motion of a ball?

Use this parabola if it helps you to explain your thinking.

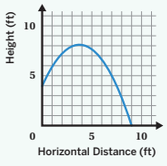
Responses vary. The vertex of a parabola can help you know how high the ball went. The y-intercept tells you how high the ball was when it was launched. The x-intercept tells you how far away the ball was when it landed on the ground.

Lesson Practice 6.07

Lesson Summary

The graph of a *quadratic function* is a parabola. You can identify key features of a quadratic function from a graph, table, or equation to help you interpret quadratic functions in a context.

For example, this graph models the path of a stomp rocket.



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- The y-intercept is (0, 4).
- This means that at the start, or when the horizontal distance was 0 feet, the height was 4 feet.

You can use the *vertex* to describe the maximum height of the stomp rocket.

- The vertex is about (3.5, 8).
- This means that the stomp rocket reached a maximum height of 8 feet.

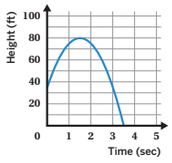
You can use the x-intercept to describe where the stomp rocket lands.

- The x-intercept on the graph is (9, 0).
- This means that the horizontal distance was 9 feet when the rocket landed, or when the height was 0 feet.

Lesson Practice 6.07

Name: _____ Date: _____ Period: _____

Problems 1–4: The graph shows the height of a bird over time.



1. Which key feature tells you the maximum height the bird flew? Circle one:

x-intercept y-intercept (vertex)

2. When did the bird reach its maximum height?

1.5 seconds

3. Which key feature tells you when the bird landed? Circle one:

(x-intercept) y-intercept vertex

4. How long was the bird in flight?

3.5 seconds

5. Here is a table of values for the function $f(x)$. Circle *two* values of $f(x)$ to change so that the table represents a quadratic function. *Responses vary.*

x	-3	-2	-1	0	1	2	3
$f(x)$	20	12	6	2	0	(2)	(6)

Show or explain what they should change to.

Explanations vary. $f(2) = 0$ and $f(3) = 2$.

Lesson Practice 6.07

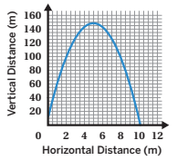
Name: _____ Date: _____ Period: _____

6. Ava and Mariam launch their stomp rockets at the same time. Ava launches her rocket from the ground. Mariam launches her rocket from a platform.

Ava's Table

Horizontal Distance (m)	Vertical Distance (m)
0	0
1	45
2	80
3	105

Mariam's Graph

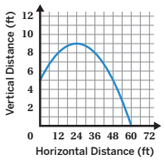


Whose rocket goes higher? Describe the key feature that helped you decide.

Mariam's. *Explanations vary.* I continued Ava's table using a constant second difference of -10 and the highest her rocket goes is 125 meters. Mariam's graph shows that the highest her rocket goes is around 150 meters, so Mariam's rocket went higher.

Test Practice

7. Isaiah throws a ball. The graph shows its height as a function of the horizontal distance from where it was thrown.



Select *all* true statements about the situation.

- ☑ **A.** The maximum height the ball reaches is 9 feet.
- ☐ **B.** The maximum height the ball reaches is 24 feet.
- ☐ **C.** The ball lands 24 feet from where it is thrown.
- ☑ **D.** The ball lands 60 feet from where it is thrown.
- ☑ **E.** The ball is thrown from a height of 5 feet.

Spiral Review

Problems 8–10: Evaluate each expression for $x = 3$.

8. $(x + 2)(x + 3)$ 30

9. $x^2 + 2x + 1$ 16

10. $2(x - 4)^2 - 5$ -3

Lesson Practice Item Analysis

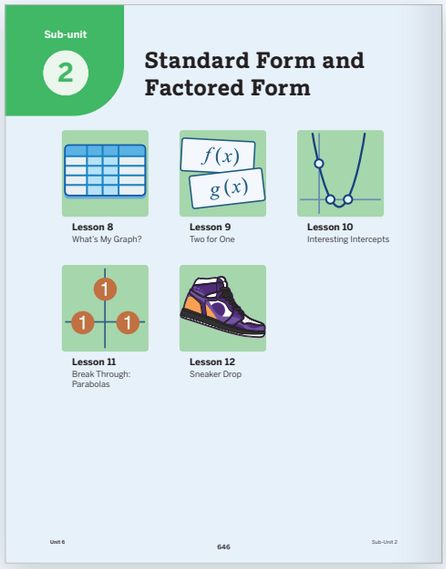
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.912.AR.3.7
	5	3	MA.912.F.1.2
	6	2	MA.912.AR.3.7
Test Practice	7	2	MA.912.AR.3.7
Spiral Review			
Fluency	8–10	1	MA.6.AR.1.3

Standard Form and Factored Form

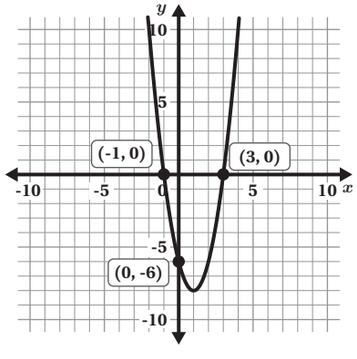
Sub-Unit 1 Goals

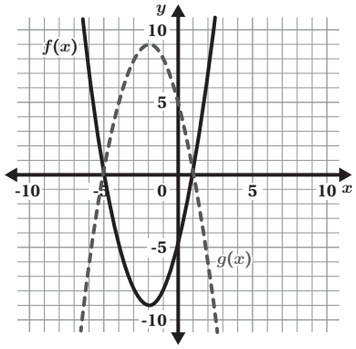
- **Goal:** Determine key features and graph quadratic functions from standard- and factored-form equations.
- **Goal:** Write quadratic equations from a graph, description, or criteria.
- **Language Goal:** Identify and interpret key features of quadratic functions from equations.

In this sub-unit, students create graphs and analyze the key features of quadratic functions written in **factored form** and **standard form**. They write quadratic functions using different pieces of information, such as a verbal description, a graph, or a table.



Math That Matters Most

Concept or Skill	For example . . .
Graph quadratic functions written in factored form using their key features	Sketch a graph of $g(x)= (x - 3)(2x + 2)$ on the coordinate plane. Include the x- and y-intercepts. 
Identify the x- and y-intercepts of quadratic functions from their equations written in factored or standard form	What are the x-intercepts of $p(x) = (2x - 10)(x + 3)$? **(5, 0) because 2x - 10 = 0 when x = 5, and (-3, 0) because x + 3 = 0 when x = -3.** What is the y-intercept of $p(x) = 2x^2 - 4x - 30$? **(0, -30) because p(0) = -30.**

Concept or Skill	For example . . .
Create a table and graph from an equation in standard form or factored form	Determine three points on the graph of $p(x) = (x - 3)(2x + 4)$. x | g(x) | Point -2 | 0 | (-2, 0) 0 | -12 | (0, -12) 2 | -8 | (2, -8)
Describe how the value of a in the equation $y = a(x - m)(x - n)$ affects the graph of its corresponding parabola	 How is the graph of $g(x) = -(x + 5)(x - 1)$ different from the graph of $f(x) = (x + 5)(x - 1)$? How are they alike? **g(x) is concave down and f(x) is concave up, but they have the same x-intercepts.**

Unit 6

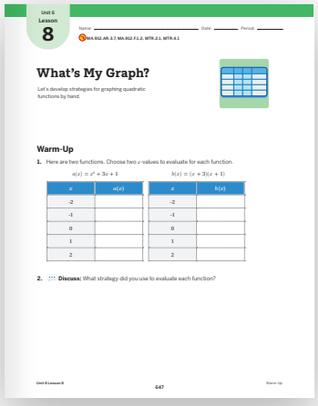
Lesson

8

What's My Graph?

Creating Graphs of Quadratics

Let's develop strategies for graphing quadratic functions by hand.


Unit 6
Lesson
8
What's My Graph?
Let's develop strategies for graphing quadratic functions by hand.
Warm-Up
1. Here are two functions. Choose two x-values to evaluate for each function.
$a(x) = x^2 + 3x + 1$
$h(x) = (x + 3)(x + 1)$
x	$a(x)$
-2	
-1	
0	
1	
2	
x	$h(x)$
-2	
-1	
0	
1	
2	
2. Discuss: What strategy did you use to evaluate each function?
647

Learning Outcomes

Today's Goals

1. **Goal:** Graph parabolas by hand by evaluating functions at different x-values.
2. **Language Goal:** Describe a strategy to determine if a point is on the graph of a quadratic function. **(Writing, Speaking, and Listening)**

Students use order of operations to support them in evaluating and graphing quadratic functions. Students use tables to support their evaluation strategies for both standard and factored form equations of quadratic functions, making connections to the specific points on the graph corresponding to rows of the table.

Note: The terms *standard form (of a quadratic equation)* and *factored form* will be introduced in the next lesson. Throughout this lesson and the remainder of this unit, students will use extended tables (sometimes called story tables) to support them in evaluating functions and making connections between key features of quadratic functions and their graphs.

Coherence

Prior Learning

In Lesson 6, students created graphs given a description of a parabola. In Unit 3, students evaluated functions by substituting inputs into equations.

Future Learning

In Lesson 9, students will create a table and graph from an equation in standard form or factored form.

Connections

- Students collaborate to explore strategies for graphing quadratic functions by hand using a table of values. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.3 **MA.6.AR.1.3**	**MA.912.AR.3.8**

Lesson at a Glance

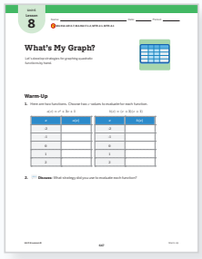
~ 45 min

MA.912.AR.3.7, MA.912.F.1.2

Warm-Up

Small Groups | 10 min

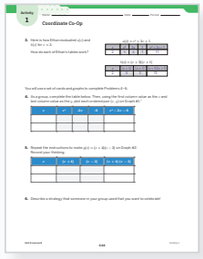
Students evaluate functions and organize their thinking in a table.



Activity 1

Small Groups | 15 min

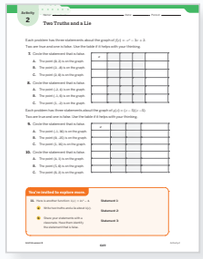
Students use tables to graph quadratic functions collaboratively and identify strategies for evaluating quadratics written in standard and factored form. MLR7: Compare and Connect



Activity 2

Small Groups | 10 min

Students attend to precision in using evaluation strategies to determine which statements are true and false about the graphs of functions. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

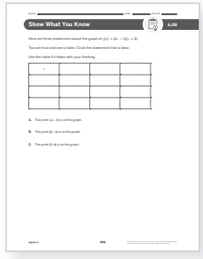
Students synthesize their understanding of how tables can support evaluating quadratic functions. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which statement about the graph of a function is false.



Prep Checklist

Prepare the Activity 1 Cards and Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards and Sheets
- Calculator

Small Groups | 10 min

Warm-Up

Purpose: Students evaluate functions and organize their thinking in a table.

1 Launch

Display the tables and equations for $a(x)$ and $b(x)$ without revealing the answers.

Encourage students to use calculators to support their work evaluating both functions throughout the lesson.

Invite students to discuss in their small groups what x-values they will each evaluate on their own to make sure all the values in the table will be filled in. Then, invite each student to work individually to determine each value, checking in with their group to fill out the rest of the values in the tables for each function collectively.

A **Accessibility: Executive Functioning** Consider chunking this activity by having students focus on one function at a time. If students feel overwhelmed or need support getting started, consider strategically pairing students with the inputs 0 or 1.

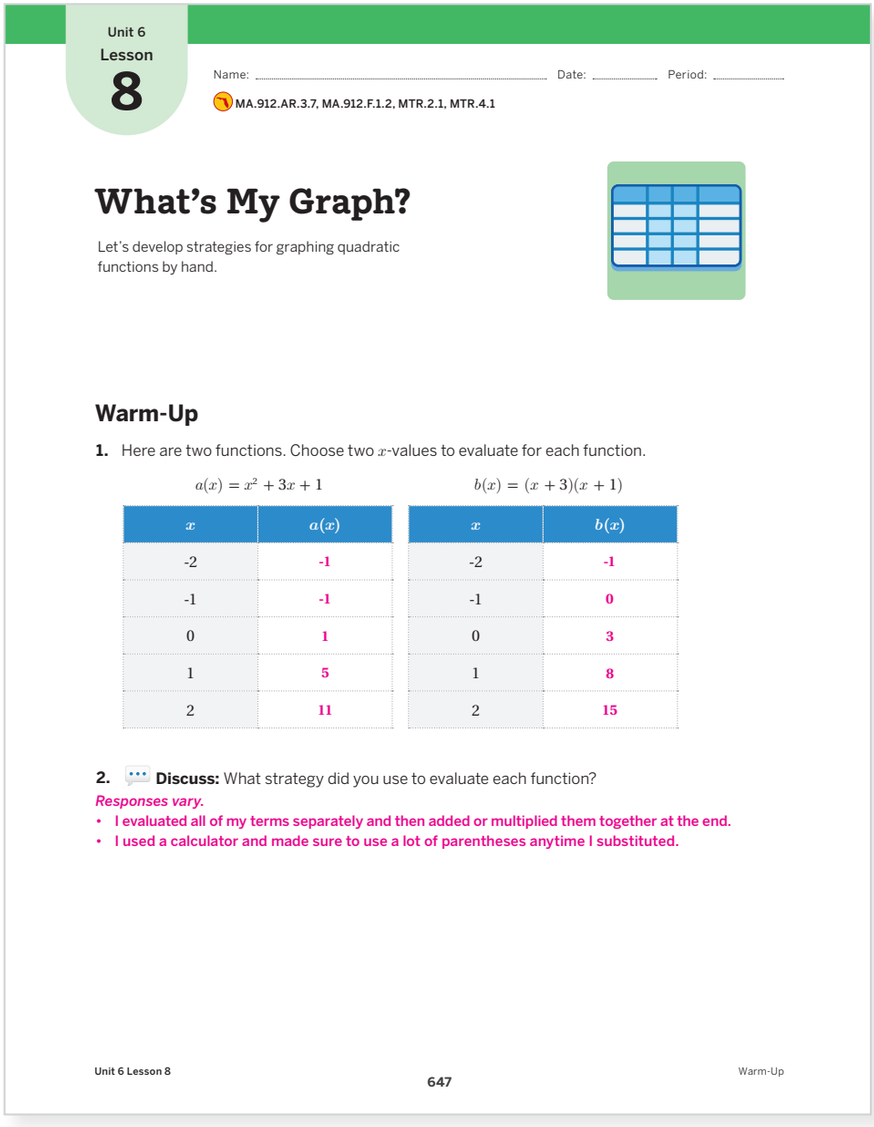
2 Connect

Display the function values for both tables. Consider asking, "How did your values match up with the table?"

Encourage students to take a moment to check in with their group about how their values matched with the table. Invite them to revise their tables if needed before sharing strategies.

Invite students to share their strategies for evaluating each function.

Consider asking, "What does each row of the table represent for the graph of each function?" Each row represents a point on the graph. For example, the first row of the table means the point (-2, -1) is on the graph of the function.



Unit 6
Lesson
8

Name: _____ Date: _____ Period: _____

MA.912.AR.3.7, MA.912.F.1.2, MTR.2.1, MTR.4.1

What's My Graph?

Let's develop strategies for graphing quadratic functions by hand.

Warm-Up

1. Here are two functions. Choose two x-values to evaluate for each function.

$a(x) = x^2 + 3x + 1$ $\quad$ $b(x) = (x + 3)(x + 1)$

x	$a(x)$
-2	-1
-1	-1
0	1
1	5
2	11

x	$b(x)$
-2	-1
-1	0
0	3
1	8
2	15

2. **Discuss:** What strategy did you use to evaluate each function?

Responses vary.

- I evaluated all of my terms separately and then added or multiplied them together at the end.
- I used a calculator and made sure to use a lot of parentheses anytime I substituted.

Unit 6 Lesson 8 647 Warm-Up

Small Groups | 15 min

Activity 1 Coordinate Co-Op

Purpose: Students use tables to graph quadratic functions collaboratively and identify strategies for evaluating quadratics written in standard and factored form.

1 Launch

Display Ethan's tables and consider asking:

- "Where does the 11 come from? The 15?" **(MTR.2.1)**
- "What was Ethan's last step in the table for $a(x)$? For $b(x)$? Why do those steps make sense?"
- "Why might tables like Ethan's be helpful for quadratics in standard form? In factored form?"

A **Accessibility: Executive Functioning** Consider chunking this part by asking students to go one by one through each number in Ethan's tables and explain where it came from.

Distribute the Activity 1 Cards and Sheets to each group.

Display the directions for a student to read aloud.

2 Monitor

Consider sharing that students can use grouping symbols on the calculator to help with squaring (e.g., $(-2)^2$).

Look for how students graph $f(x)$ and $g(x)$, including tables like Ethan's or other strategies like substitution.

D **Differentiation** *(Problems 4–5)*

Look for students who:	Respond to Student Thinking
Combine the columns in each table the same way.	**Support** Consider asking, "How did Ethan combine the terms to make $a(x)$? What about $b(x)$?"
Would enjoy an additional challenge.	**Stretch** Consider asking, "$(8, 42)$ is on the graph of $f(x)$ or $g(x)$. Which one is it? Why?" $f(x)$

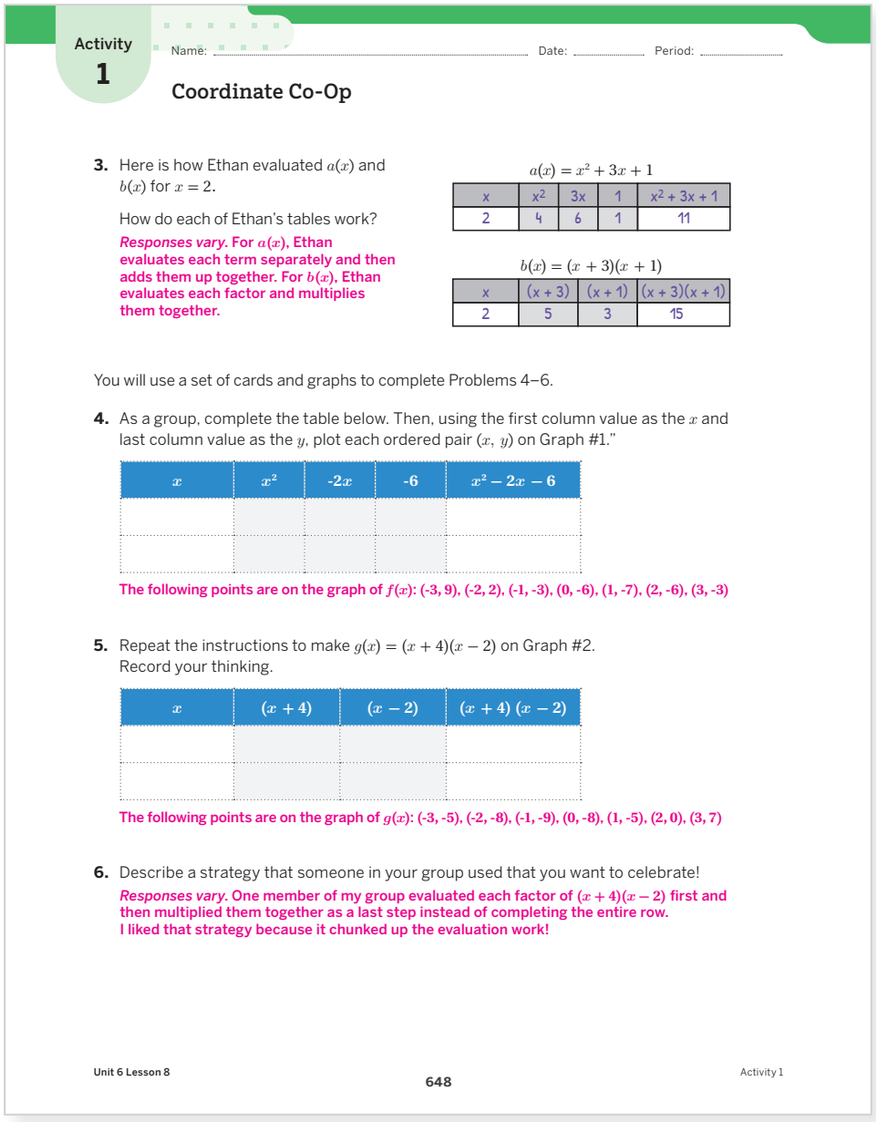
3 Connect

Invite students to share their strategies from Problem 6.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider asking:

- "How were tables like Ethan's helpful? Not helpful?"
- "What other strategies did your group use?"



Activity 1

Name: _____ Date: _____ Period: _____

Coordinate Co-Op

3. Here is how Ethan evaluated $a(x)$ and $b(x)$ for $x = 2$.

How do each of Ethan's tables work?

Responses vary. For $a(x)$, Ethan evaluates each term separately and then adds them up together. For $b(x)$, Ethan evaluates each factor and multiplies them together.

$a(x) = x^2 + 3x + 1$

x	x^2	3x	1	$x^2 + 3x + 1$
2	4	6	1	11

$b(x) = (x + 3)(x + 1)$

x	$(x + 3)$	$(x + 1)$	$(x + 3)(x + 1)$
2	5	3	15

You will use a set of cards and graphs to complete Problems 4–6.

4. As a group, complete the table below. Then, using the first column value as the x and last column value as the y, plot each ordered pair (x, y) on Graph #1."

x	x^2	$-2x$	-6	$x^2 - 2x - 6$

The following points are on the graph of $f(x)$: (-3, 9), (-2, 2), (-1, -3), (0, -6), (1, -7), (2, -6), (3, -3)

5. Repeat the instructions to make $g(x) = (x + 4)(x - 2)$ on Graph #2. Record your thinking.

x	$(x + 4)$	$(x - 2)$	$(x + 4)(x - 2)$

The following points are on the graph of $g(x)$: (-3, -5), (-2, -8), (-1, -9), (0, -8), (1, -5), (2, 0), (3, 7)

6. Describe a strategy that someone in your group used that you want to celebrate!

Responses vary. One member of my group evaluated each factor of $(x + 4)(x - 2)$ first and then multiplied them together as a last step instead of completing the entire row. I liked that strategy because it chunked up the evaluation work!

Unit 6 Lesson 8 648 Activity 1

Key Takeaway: Extended tables are a helpful tool for organizing the steps of evaluating quadratic functions written in different ways. You can create a graph of a quadratic function by evaluating the function at several inputs.

Small Groups | 10 min

Activity 2 Two Truths and a Lie

Purpose: Students attend to precision in using evaluation strategies to determine which statements are true and false about the graphs of functions. **(MTR.4.1)**

Short on time? Consider omitting Problems 8 and 10.

1 Launch

Display the functions and the directions.

ELL **English Language Learners** Invite students to ensure that everyone in the group can explain their reasoning for why statements are false. **(Writing, Speaking, and Listening)**

2 Monitor

Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider inviting them to choose either the input $x = 0$ or $x = 2$ and evaluate the function using the table.
Recognize that either B or C can be true for a general quadratic function, but not both at the same time because then the function would not show symmetry. *(Problem 7)*	**Strengthen** Celebrate their noticing and invite them to share during the Connect.
Think the point (-1, 6) is on the graph of $f(x)$ or are not sure which statement is false. *(Problem 8)*	**Support** Consider writing out $-x^2$ as $-(\ )^2$ and asking, "Which operations do you see? Which comes first?"
Think the point (0, -25) is on the graph of $g(x)$ or are not sure which statement is false. *(Problem 9)*	**Support** Consider writing out $(0-5)(0-5)$ and asking, "Which operations do you see? Which comes first?"
Add the columns of the extended table to find the output for $g(x)$. *(Problem 10)*	**Support** Consider asking, "What operation is happening between the two factors of $g(x)$?"

Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore graphing quadratic functions further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share which statements they determined were false and justify their thinking. **(MTR.4.1)**

Consider asking:

- "In Problem 7, why can't the points (2, -8) and (2, 0) both be on the graph of $f(x)$?"
- "What strategy did you use to determine that (0, -25) is not on the graph of $g(x)$?"
- "Did you have to evaluate all three points to determine the false statement? Why or why not?"

Activity 2

Name: _____ Date: _____ Period: _____

Two Truths and a Lie

Each problem has three statements about the graph of $f(x) = -x^2 - 3x + 2$.
Two are true and one is false. Use the table if it helps with your thinking. *Tables vary.*

7. Circle the statement that is false.

A. The point (0, 2) is on the graph.
B. The point (2, -8) is on the graph.
C. The point (2, 0) is on the graph.

8. Circle the statement that is false.

A. The point (-2, 4) is on the graph.
B. The point (-1, 6) is on the graph.
C. The point (1, -2) is on the graph.

x	$-x^2$	$-3x$	2	$-x^2 - 3x + 2$
-2	-4	6	2	4
-1	-1	3	2	4
0	0	0	2	2
1	-1	-3	2	-2
2	-4	-6	2	-8

Each problem has three statements about the graph of $g(x) = (x-5)(x-5)$.
Two are true and one is false. Use the table if it helps with your thinking.

9. Circle the statement that is false.

A. The point (-1, 36) is on the graph.
B. The point (0, -25) is on the graph.
C. The point (1, 16) is on the graph.

10. Circle the statement that is false.

A. The point (4, 1) is on the graph.
B. The point (5, 0) is on the graph.
C. The point (6, 2) is on the graph.

x	$(x-5)$	$(x-5)$	$(x-5)(x-5)$
-1	-6	-6	36
0	-5	-5	25
1	-4	-4	16
4	-1	-1	1
5	0	0	0
6	1	1	1

You're invited to explore more.

11. Here is another function: $h(x) = 2x^2 - 4$.

a. Write two truths and a lie about $h(x)$. *Responses vary.*
b. Share your statements with a classmate. Have them identify the statement that is false.

Statement 1: The point (0, -4) is on the graph.
Statement 2: The point (2, 0) is on the graph.
Statement 3: The point (-1, -6) is on the graph.
Note: Statement 3 is false.

Unit 6 Lesson 8 649 Activity 2

Key Takeaway: Whenever we evaluate a function f at an input x to determine an output $f(x)$, we also find a point (x, y) on the function's graph, where $y = f(x)$. We can determine if a point is on the graph of a function by substituting the x-value of an ordered pair into the function and determining if the output $f(x)$ matches the y-value of the ordered pair.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how tables can support evaluating quadratic functions.

Invite students to respond independently, and then share their thinking with a partner.

Consider asking:

- "When might a table be helpful? When might it not?"
- "What do you think is important to remember when creating graphs of quadratic functions?"

Share a variety of ideas, such as:

- The table helps to organize each step for evaluating quadratic functions.
- Each row of the table tells us a point on the graph.
- For quadratics written in standard form, you can evaluate each term individually then combine them to determine the output.
- For quadratics written in factored form, you can evaluate each factor individually, and then multiply them to determine the output.

Note: If it does not come up naturally, consider inviting students to compare the table in this problem with Ethan's table and consider asking, "Why is the middle column header in this table $-2x$ and not positive like Ethan's table? Could you use positive $2x$ instead? What would you have to do differently?" I would have to subtract this term instead of add to get the output.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and examples about how to use extended tables to evaluate quadratic functions written in standard and factored forms as they share during the whole-class discussion. Display for students to reference and add to for the rest of the unit.



Synthesis

12. How can using a table help you graph quadratic functions?

Responses vary. A table can help me organize my thinking and calculate each part of a quadratic function separately. Once I break it down into terms or factors, I can combine the parts together with addition or multiplication.

x	$-x^2$	$-2x$	3	$-x^2-2x+3$
-3	-9	6	3	0

x	$(2x+4)$	$(x-3)$	$(2x+4)(x-3)$
-3	-2	-6	12

Lesson Practice 6.08

Lesson Summary

You can graph a quadratic function by evaluating its equation at different x-values. A table can be helpful to organize your thinking for quadratics written in standard or factored form.

Here are two examples. You can use the table to determine the value of each function when $x = -3$ and $x = 1$, and determine the corresponding points that are on the graph.

$f(x) = 2x^2 + 8x - 10$

x	$2x^2$	$8x$	-10	$2x^2+8x-10$	Point on Graph
-3	$2(-3)^2 = 18$	$8(-3) = -24$	-10	$18-24-10 = -16$	(-3, -16)
1	$2(1)^2 = 2$	$8(1) = 8$	-10	$2+8-10 = 0$	(1, 0)

$g(x) = (3x-1)(x+5)$

x	$(3x-1)$	$(x+5)$	$(3x-1)(x+5)$	Point on Graph
-3	$3(-3)-1 = -10$	$-3+5 = 2$	$(-10)(2) = -20$	(-3, -20)
1	$3(1)-1 = 2$	$1+5 = 6$	$(2)(6) = 12$	(1, 12)

Unit 6 Lesson 8 650 Synthesis | Lesson Practice

Lesson Takeaway: You can graph a quadratic function by choosing several inputs, evaluating the function at those inputs, and graphing the corresponding input-output pairs as points. Tables can be helpful for organizing information while evaluating. To use a table to evaluate a function written in standard form, create a column for every term and combine the terms. To use a table to evaluate a function written in factored form, create a column for every factor and multiply the factors.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which statement about the graph of a function is false.

Today's Goals

1. **Goal:** Graph parabolas by hand by evaluating functions at different x-values.
2. **Language Goal:** Describe a strategy to determine if a point is on the graph of a quadratic function. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.08

Here are three statements about the graph of $j(x) = (2x - 1)(x + 3)$.

Two are true and one is false. Circle the statement that is false.

Use the table if it helps with your thinking. *Tables vary.*

x	$(2x - 1)$	$(x + 3)$	$(2x - 1)(x + 3)$
-2	-5	1	-5
0	-1	3	-3
2	3	5	15

A. The point (-2, -5) is on the graph.

B. The point (0, -3) is on the graph.

C. The point (2, 8) is on the graph.

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use order of operations to evaluate quadratic functions:

» **Respond to Student Thinking:**

- Revisit **Activity 1** as a class and discuss the similarities and differences in Ethan's tables.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the Sub-Unit 2 Extension Activities.
- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.

Professional Learning

Today students used extended tables and other strategies to support them in evaluating functions and making graphs. How did these tables and strategies build on students' prior knowledge of order of operations and the graphs of functions? How can you continue to use extended tables and other supports to help students as they deepen their understanding of quadratic functions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can using a table help you graph quadratic functions?

Responses vary. A table can help me organize my thinking and calculate each part of a quadratic function separately. Once I break it down into terms or factors, I can combine the parts together with addition or multiplication.

Lesson Practice 6.08

Lesson Summary

You can graph a quadratic function by evaluating its equation at different x-values. A table can be helpful to organize your thinking for quadratics written in standard or factored form.

Here are two examples. You can use the table to determine the value of each function when $x = -3$ and $x = 1$, and determine the corresponding points that are on the graph.

$f(x) = 2x^2 + 8x - 10$

x	$2x^2$	$8x$	-10	$2x^2 + 8x - 10$	Point on Graph
-3	$2(-3)^2 = 18$	$8(-3) = -24$	-10	$18 - 24 - 10 = -16$	(-3, -16)
1	$2(1)^2 = 2$	$8(1) = 8$	-10	$2 + 8 - 10 = 0$	(1, 0)

$g(x) = (3x - 1)(x + 5)$

x	$(3x - 1)$	$(x + 5)$	$(3x - 1)(x + 5)$	Point on Graph
-3	$3(-3) - 1 = -10$	$-3 + 5 = 2$	$(-10)(2) = -20$	(-3, -20)
1	$3(1) - 1 = 2$	$1 + 5 = 6$	$(2)(6) = 12$	(1, 12)

Lesson Practice 6.08

Name: _____ Date: _____ Period: _____

1. Here is a function: $k(x) = (x + 4)(x - 2)$.

Amoli says that $k(-2) = -12$.
Yasmine says that $k(-2) = -8$.

Whose thinking is correct? Explain your thinking.

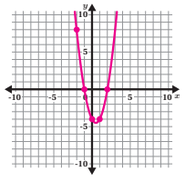
Yasmine's. *Explanations vary.* $k(-2)$ means that -2 is the value of x. $-2 + 4 = 2$ and $-2 - 2 = -4$ and $2 \cdot -4 = -8$.

Problems 2–3: Here is a function: $g(x) = 2x^2 - 2x - 4$.

2. Complete the table for $g(x)$.

x	$2x^2$	$-2x$	-4	$2x^2 - 2x - 4$
-2	8	4	-4	8
-1	2	2	-4	0
0	0	0	-4	-4
1	2	-2	-4	-4
2	8	-4	-4	0

3. Create a graph of $g(x)$.

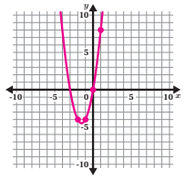


Problems 4–5: Here is a function: $h(x) = 2x(x + 3)$.

4. Complete the table for $h(x)$.

x	$2x$	$x + 3$	$2x(x + 3)$
-3	-6	0	0
-2	-4	1	-4
-1	-2	2	-4
0	0	3	0
1	2	4	8

5. Create a graph of $h(x)$.

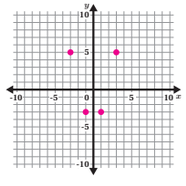


Lesson Practice 6.08

Name: _____ Date: _____ Period: _____

6. Plot a point in each quadrant that is on the graph of $g(x) = x^2 - 4$.

Responses vary.



Test Practice

7. Here are three statements about the graph of $r(x) = (x - 1)(x + 1)$. Two are true and one is false. Circle the statement that is false.

A. The point $(-1, 0)$ is on the graph.
B. The point $(0, 1)$ is on the graph.
C. The point $(1, 0)$ is on the graph.

Spiral Review

Problems 8–10: Evaluate each expression for $x = 3$.

8. $2(x - 2)$
2

9. $\frac{1}{9}x + 3$
$3\frac{1}{3}$ (or equivalent)

10. $3(x - 1) + 4$
10

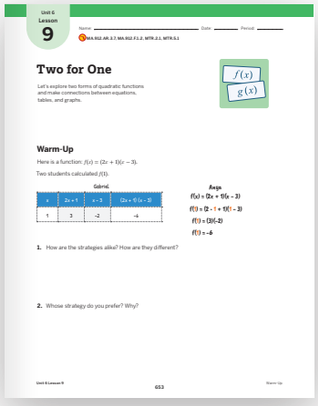
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4	2	MA.912.F.1.2
	3, 5	2	MA.912.AR.3.7
	6	3	MA.912.AR.3.7
Test Practice	7	2	MA.912.AR.3.7
Spiral Review			
Fluency	8–10	1	MA.6.AR.1.3

Unit 6

Lesson

9


Unit 6
Lesson
9
Two for One
Let's explore two forms of quadratic functions and make connections between equations, tables, and graphs.
Warm-Up
Here is a function: $f(x) = (2x + 1)(x - 3)$.
Two students calculated $f(1)$.
Gabriel
Avya
1. How are the strategies alike? How are they different?
2. Whose strategy do you prefer? Why?
653

Two for One

Standard Form and Factored Form

Let's explore two forms of quadratic equations and make connections between equations, tables, and graphs.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether a quadratic function is written in standard form, factored form, or neither.
2. **Goal:** Create tables and graphs from quadratic functions written in standard or factored form.
3. **Language Goal:** Compare and contrast the equations, tables, and graphs of quadratic functions written in standard or factored form. **(Writing, Speaking, and Listening)**

Students deepen their understanding of two forms of quadratic equations: **standard form** and **factored form**. They graph both forms of quadratic equations using tables and see that equations in standard form and factored form sometimes produce the same graph (and therefore describe the same function).

Coherence

Prior Learning

In Lesson 8, students evaluated quadratic functions at various inputs to determine corresponding points on their graphs.

Future Learning

In Lesson 10, students will use the structure of a quadratic equation written in standard form and factored form to determine key features of its graph.

Connections

- In a partner sorting activity, students explore different representations of quadratic functions (equations, tables, and graphs) and discuss the connections between them. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.F.1.5 MA.6.AR.1.3 MA.7.AR.1.1	MA.912.AR.3.1

Vocabulary

New Vocabulary

factored form

standard form (of a quadratic equation)

Lesson at a Glance

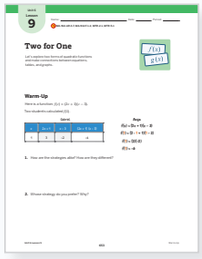
~ 45 min

MA.912.AR.3.7, MA.912.F.1.2

Warm-Up

Pairs | 5 min

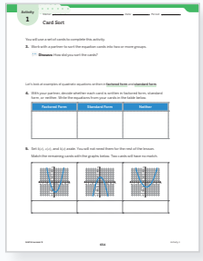
Students review two strategies for evaluating a quadratic function. MLR7: Compare and Connect



Activity 1

Pairs | 20 min

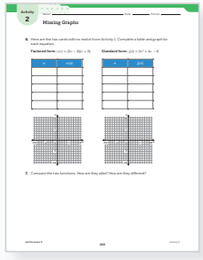
Students are introduced to quadratic equations written in **standard form** and **factored form** and their graphs.



Activity 2

Pairs | 10 min

Students graph quadratic equations written in standard and factored form by hand and see that quadratics in different forms can create the same parabola (and are equivalent). MLR7: Compare and Connect



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using tables to graph quadratic equations in standard form or factored form. MLR2: Collect and Display (MTR.5.1)



Show What You Know

Independent | 5 min

Students demonstrate their understanding by choosing their preferred form of a quadratic equation and graphing the parabola by using a table. (MTR.5.1)



Prep Checklist

Prepare the Activity 1 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students review two strategies for evaluating a quadratic function.

1 Launch

Display the two strategies.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to how they would have calculated $f(1)$.

Invite students to compare and contrast each strategy.

Note: It's okay to lack consensus about which strategy students prefer. The goal of this activity is to recognize that there are multiple strategies for evaluating quadratic functions.

ELL **English Language Learners** To support students in sharing their thinking, consider displaying sentence frames such as:

- First, they _____. Next, they _____. Finally, _____.
- Their strategy was to _____.
- I see _____ in both strategies.
- What is clear to me is _____. What is unclear to me is _____.

(Writing and Speaking)

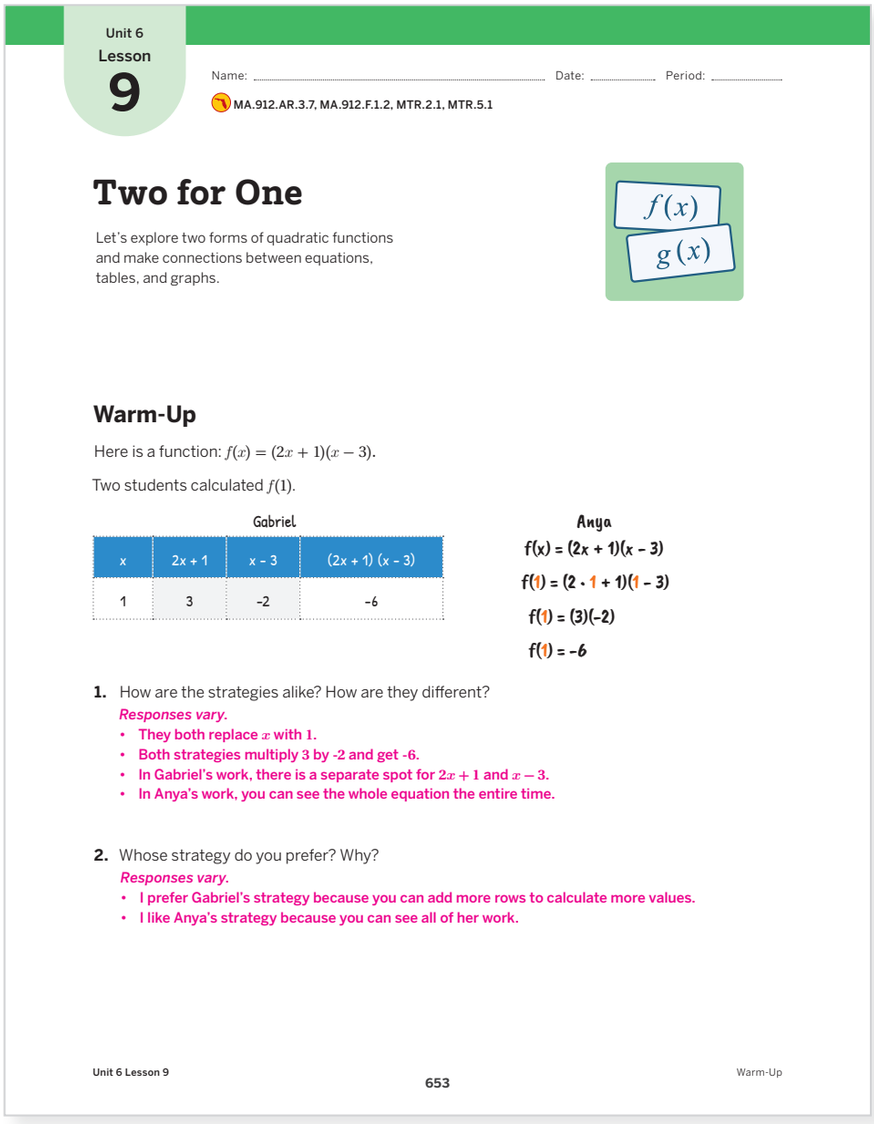
A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one step of student work at a time. To promote student focus, consider covering up steps until students are ready to address them.

2 Connect

To access connections, consider asking,

- "What did each student do?"
- "Where do you see the same information in Gabriel and Anya's strategy?"
- "How did using an extended table support Gabriel in evaluating the function at $x = 1$?"
- "Based on each student's strategy, what do you know about the graph of this quadratic function?"
- "When might you use each strategy?"
- "Why did the different strategies lead to the same outcome?"

A **Accessibility: Executive Functioning** Throughout this lesson, students use tables to make sense of points on the graph of quadratic functions. Consider inviting students to use extended tables whenever possible to support their evaluation process.



Unit 6
Lesson
9

Name: _____ Date: _____ Period: _____

MA.912.AR.3.7, MA.912.F.1.2, MTR.2.1, MTR.5.1

Two for One

Let's explore two forms of quadratic functions and make connections between equations, tables, and graphs.

$f(x)$
$g(x)$

Warm-Up

Here is a function: $f(x) = (2x + 1)(x - 3)$.

Two students calculated $f(1)$.

Gabriel

x	2x + 1	x - 3	(2x + 1) (x - 3)
1	3	-2	-6

Anya

f(x) = (2x + 1)(x - 3)
f(1) = (2 · 1 + 1)(1 - 3)
f(1) = (3)(-2)
f(1) = -6

1. How are the strategies alike? How are they different?
Responses vary.
- They both replace x with 1.
- Both strategies multiply 3 by -2 and get -6.
- In Gabriel's work, there is a separate spot for $2x + 1$ and $x - 3$.
- In Anya's work, you can see the whole equation the entire time.

2. Whose strategy do you prefer? Why?
Responses vary.
- I prefer Gabriel's strategy because you can add more rows to calculate more values.
- I like Anya's strategy because you can see all of her work.

Unit 6 Lesson 9 653 Warm-Up

Pairs | 20 min

Activity 1 Card Sort

Purpose: Students are introduced to quadratic equations written in standard form and factored form and their graphs.

Short on time? Consider omitting cards $b(x)$, $e(x)$, and $h(x)$ from the initial card sort in Problem 3.

1 Launch

Display the cards and the activity directions.

Distribute one set of cards to each pair. Invite students to work together to sort the cards.

To encourage creativity, consider sharing with students there is no one correct way to sort the cards.

A **Accessibility: Executive Functioning** Invite students to choose two cards at a time to compare

2 Monitor

Pause and display the card sort. Pair the *Standard Form* card with one equation in standard form, e.g., $a(x)$, and pair the *Factored Form* card with one equation in factored form, e.g., $c(x)$. Invite students to suggest other cards to include in each form and create a class definition for *factored form* and *standard form* using students' language and the glossary.

ELL **English Language Learners** Use the **Collect and Display** routine to create an anchor chart about the different forms of quadratic equations. Add student language to the chart throughout the lesson. **(Speaking and Listening)**

Look for strategies for matching the cards in Problem 5 to share during the Connect, including evaluating the function at many input values, using symmetry, or evaluating the function at $x = 0$.

Listen for and celebrate students who notice the relationship between equations written in factored form and the x-intercepts and equations written in standard form and the y-intercept on the graph. **Note:** It is okay if students are unsure at this stage. Students will explore these concepts in Lesson 10.

3 Connect

Display the matched cards from Problem 5. Invite several students to share their strategies for matching.

To reveal the Key Takeaway, consider asking:

- "Where do you see the factors in the factored form equation? On the graph?" **(MTR.5.1)**
- "Where do you see the values in the standard form equation on the graph?" **(MTR.5.1)**
- "What do you notice about the numbers in each equation and its graph?" **(MTR.2.1, MTR.5.1)**
- "Which graphs and equations were easy to match? Which were more challenging?"

Activity 1

Name: _____ Date: _____ Period: _____

Card Sort

You will use a set of cards to complete this activity.

3. Work with a partner to sort the equation cards into two or more groups.

Discuss: How did you sort the cards?

Responses vary.

- We sorted the cards into equations with x^2, equations with just x, and equations with x as the exponent.
- $c(x)$, $d(x)$, and $f(x)$ have two sets of parentheses. $h(x)$ has one set of parentheses. The other equations have none.

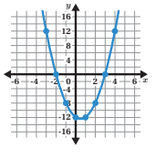
Let's look at examples of quadratic equations written in **factored form** and **standard form**.

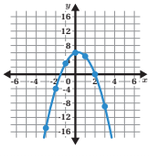
4. With your partner, decide whether each card is written in factored form, standard form, or neither. Write the equations from your cards in the table below.

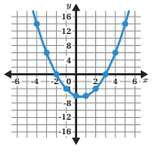
Factored Form	Standard Form	Neither
$c(x) = (2x - 2)(x + 3)$	$a(x) = x^2 - x - 6$	$b(x) = x + 3$
$d(x) = (x - 3)(x + 2)$	$g(x) = -2x^2 + x + 6$	$e(x) = 2^x$
$f(x) = (x + 2)(2x - 6)$	$j(x) = 2x^2 + 4x - 6$	$h(x) = (x - 2)^2 + 6$

5. Set $b(x)$, $e(x)$, and $h(x)$ aside. You will not need them for the rest of the lesson.

Match the remaining cards with the graphs below. Two cards will have no match.







$f(x) = (x + 2)(2x - 6)$	$g(x) = -2x^2 + x + 6$	$a(x) = x^2 - x - 6$ $d(x) = (x - 3)(x + 2)$

Cards with no match: $c(x)$, $j(x)$

Unit 6 Lesson 9 654 Activity 1

Key Takeaway: Quadratic equations can be written in **factored form** and **standard form**. The different forms that a quadratic equation is written can help reveal information, such as the key features on its graph.

Pairs | 10 min

Activity 2 Missing Graphs

Purpose: Students graph quadratic equations written in standard and factored form by hand and see that quadratics in different forms can create the same parabola (and are equivalent).

1 Launch

To access prior knowledge, consider inviting students to revisit the Warm-Up to remember two strategies for evaluating quadratic functions.

Consider asking, "What x-values would you choose to evaluate at for each function? Why?"

A **Accessibility: Executive Functioning**
To support organization in problem solving, consider writing out steps for creating a table and its graph (e.g., 1. Choose x-values to input into the equation. 2. Evaluate each x-value. 3. Graph the ordered pairs created in the table. 4. Look for patterns.).

2 Monitor

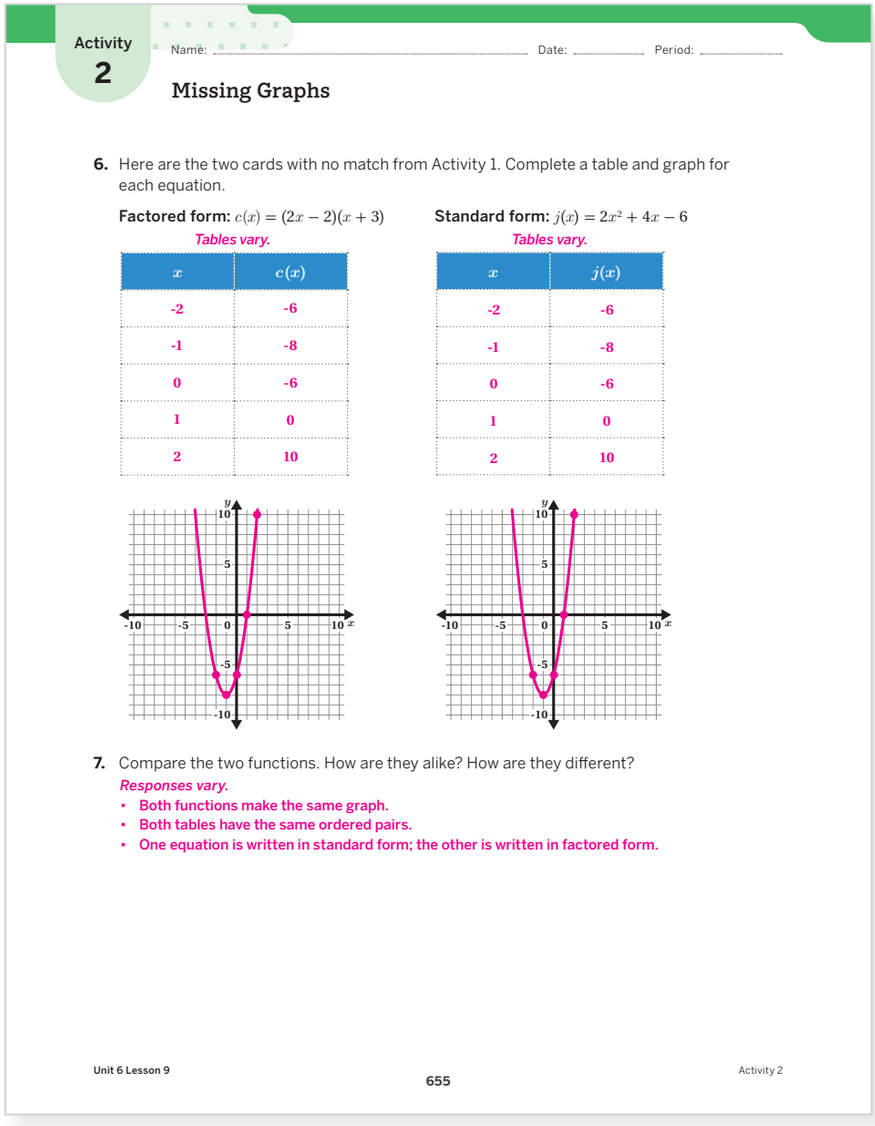
Look for strategies to share during the Connect.

D **Differentiation** *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support choosing which x-values to evaluate at.	**Support** Encourage them to use the scale of the graph (e.g., $-10 \le x \le 10$) to support their choosing, and use the evaluation to support their next choice (e.g., if the ordered pair does not fit on the graph, choose an x-value that will).
Need support with evaluating quadratic equations at specific x-values.	**Support** Encourage students to use a calculator to check their evaluations or use an extended table to support their evaluation process.
Graph incomplete parabolas.	**Strengthen** Consider asking, "What does the graph of a quadratic function look like?" Encourage them to use symmetry or evaluate at additional table values to complete the parabola.

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze each table and its graph, then find ways in which the representations are different (compare) and alike (connect). Consider asking:
- "What strategies did you use to graph each function?"
- "Which equation was easier to graph? Why?"
- "Are these equations equivalent? How do you know?"



Activity 2

Name: Date: Period:

Missing Graphs

6. Here are the two cards with no match from Activity 1. Complete a table and graph for each equation.

Factored form: $c(x) = (2x - 2)(x + 3)$

Tables vary.

x	$c(x)$
-2	-6
-1	-8
0	-6
1	0
2	10

Standard form: $j(x) = 2x^2 + 4x - 6$

Tables vary.

x	$j(x)$
-2	-6
-1	-8
0	-6
1	0
2	10

7. Compare the two functions. How are they alike? How are they different?

Responses vary.
- Both functions make the same graph.
- Both tables have the same ordered pairs.
- One equation is written in standard form; the other is written in factored form.

Unit 6 Lesson 9 655 Activity 2

Key Takeaway: There are many strategies for graphing a quadratic function. You can input x-values into the equation to find its matching output and graph the (x, y) ordered pair. You can use the equation and the form it is written in to help you choose which x-values to evaluate at (e.g., choosing 0 to determine the y-intercept). You can also use symmetry to help you graph, knowing if there is a point on one side of the line of symmetry, there is also a point on the other side as well.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using tables to graph quadratic equations in standard form or factored form.

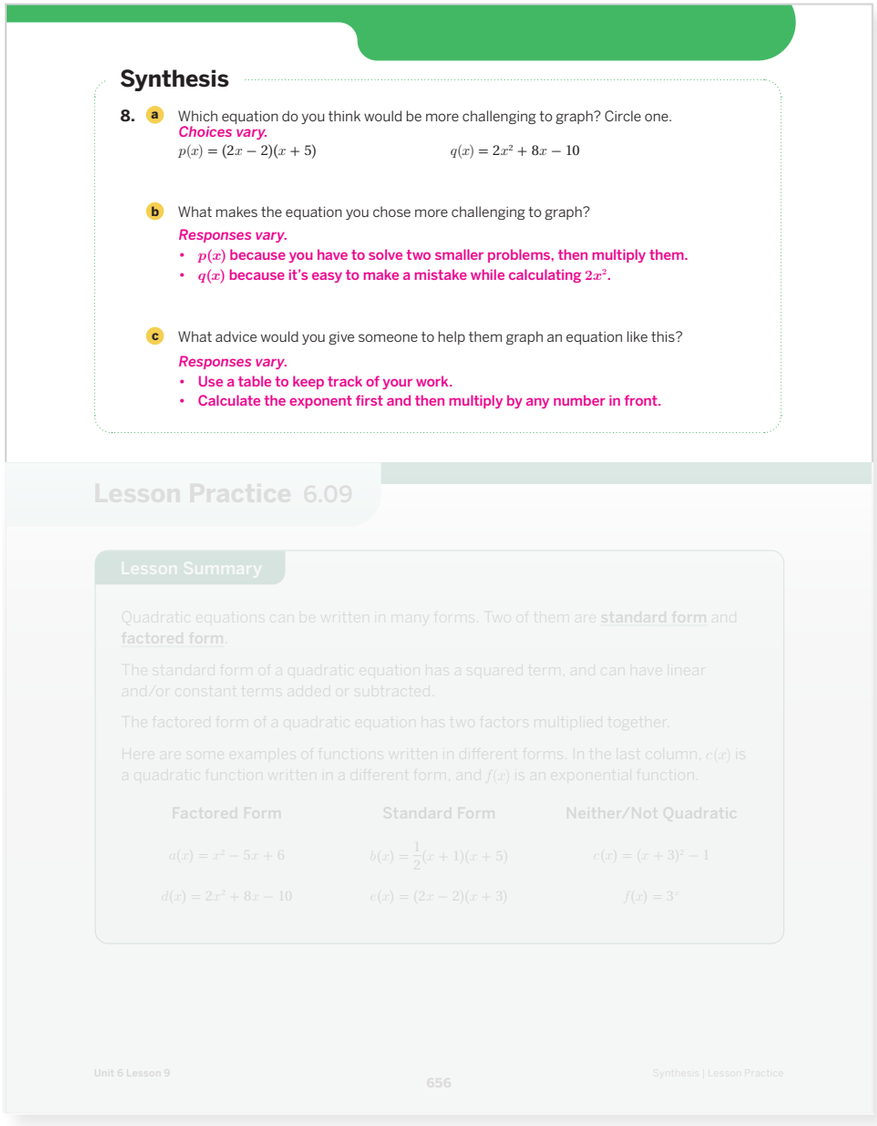
Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:

- Standard form equations are more difficult to graph because of the squared term.
- Factored form equations are more difficult to graph because you have to solve two smaller equations first.
- Using a table to evaluate the equation at different x-values helps to look for patterns.
- Evaluating the equation at strategic x-values to find key points (e.g., evaluating at $x = 0$ will reveal the y-intercept.)
- Using symmetry to graph the parabola once the parabola changes from increasing to decreasing or vice versa.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *standard form* and *factored form* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. a. Which equation do you think would be more challenging to graph? Circle one.
Choices vary.
$p(x) = (2x - 2)(x + 5)$ $q(x) = 2x^2 + 8x - 10$

b. What makes the equation you chose more challenging to graph?
Responses vary.
- $p(x)$ because you have to solve two smaller problems, then multiply them.
- $q(x)$ because it's easy to make a mistake while calculating $2x^2$.

c. What advice would you give someone to help them graph an equation like this?
Responses vary.
- Use a table to keep track of your work.
- Calculate the exponent first and then multiply by any number in front.

Lesson Practice 6.09

Lesson Summary

Quadratic equations can be written in many forms. Two of them are standard form and factored form.

The standard form of a quadratic equation has a squared term, and can have linear and/or constant terms added or subtracted.

The factored form of a quadratic equation has two factors multiplied together.

Here are some examples of functions written in different forms. In the last column, $c(x)$ is a quadratic function written in a different form, and $f(x)$ is an exponential function.

Factored Form	Standard Form	Neither/Not Quadratic
$a(x) = x^2 - 5x + 6$	$b(x) = \frac{1}{2}(x + 1)(x + 5)$	$c(x) = (x + 3)^2 - 1$
$d(x) = 2x^2 + 8x - 10$	$e(x) = (2x - 2)(x + 3)$	$f(x) = 3^x$

Unit 6 Lesson 9 656 Synthesis | Lesson Practice

Lesson Takeaway: Quadratic equations can be written in **standard form** and **factored form**. Tables can be helpful to graph quadratic equations by hand for either form.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by choosing their preferred form of a quadratic equation and graphing the parabola by using a table. **(MTR.5.1)**

Today's Goals

1. **Goal:** Determine whether a quadratic function is written in standard form, factored form, or neither.
2. **Goal:** Create tables and graphs from quadratic functions written in standard or factored form.
3. **Language Goal:** Compare and contrast the equations, tables, and graphs of quadratic functions written in standard or factored form. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

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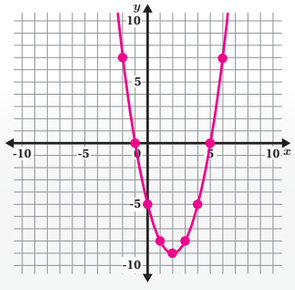
Show What You Know 6.09

Choose one equation. Graph it on the coordinate plane and show your thinking.

$f(x) = (x - 5)(x + 1)$

$g(x) = x^2 - 4x - 5$

Choices vary. The graph of both functions is the same.



Differentiation

Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use tables to graph quadratic functions:

» **Respond to Student Thinking:**

- Revisit **Lesson 8, Activity 1** (What's My Graph?).

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Encouraging students to complete the DOK3 Practice Problems.
- Invite students to explore the Sub-Unit 2 Extension Activities.

Professional Learning

Consider the next several lessons you will teach. What connections did students make between the equation, table, and graph of a quadratic function in this lesson? How will students' work today prepare them for using the structure of quadratic equations to make graphs in Lessons 10?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. a. Which equation do you think would be more challenging to graph? Circle one.
Choices vary.
$p(x) = (2x - 2)(x + 5)$ $q(x) = 2x^2 + 8x - 10$

b. What makes the equation you chose more challenging to graph?
Responses vary.
- $p(x)$ because you have to solve two smaller problems, then multiply them.
- $q(x)$ because it's easy to make a mistake while calculating $2x^2$.

c. What advice would you give someone to help them graph an equation like this?
Responses vary.
- Use a table to keep track of your work.
- Calculate the exponent first and then multiply by any number in front.

Lesson Practice 6.09

Lesson Summary

Quadratic equations can be written in many forms. Two of them are **standard form** and **factored form**.

The standard form of a quadratic equation has a squared term, and can have linear and/or constant terms added or subtracted.

The factored form of a quadratic equation has two factors multiplied together.

Here are some examples of functions written in different forms. In the last column, $c(x)$ is a quadratic function written in a different form, and $f(x)$ is an exponential function.

Factored Form	Standard Form	Neither/Not Quadratic
$a(x) = x^2 - 5x + 6$	$b(x) = \frac{1}{2}(x + 1)(x + 5)$	$c(x) = (x + 3)^2 - 1$
$d(x) = 2x^2 + 8x - 10$	$e(x) = (2x - 2)(x + 3)$	$f(x) = 3^x$

Lesson Practice 6.09

Name: _____ Date: _____ Period: _____

1. Determine whether each equation is written in standard form, factored form, or neither.

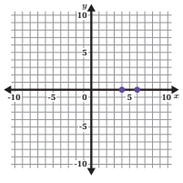
$a(x) = 3x^2 + 4$ $b(x) = 3x^2 + 4x + 2$ $c(x) = (x + 3)^2 + 4$

$f(x) = 2x(x + 4)$ $g(x) = 4x^2 - 3x$ $(4x - 4)(x - 3) = h(x)$

Factored Form	Standard Form	Neither
$f(x) = 2x(x + 4)$	$b(x) = 3x^2 + 4x + 2$	$c(x) = (x + 3)^2 + 4$
$(4x - 4)(x - 3) = h(x)$	$g(x) = 4x^2 - 3x$	
	$a(x) = 3x^2 + 4$	

2. Kala is graphing the equation $r(x) = (x - 6)(x - 4)$. Which x-value should she evaluate to best complete her graph? Circle one.

$x = 1$ $x = -1$ $x = 5$



Explain your thinking.

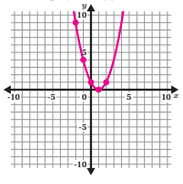
Explanations vary. Since she already has the two x-intercepts, evaluating at $x = 5$ would also give her the vertex, making it easiest to see the key features of the graph.

Problems 3–4: Here is a function: $a(x) = x^2 - 2x + 1$.

3. Complete the table for $a(x)$.

x	$a(x)$
-2	9
-1	4
0	1
1	0
2	1

4. Create a graph of $a(x)$.



Lesson Practice 6.09

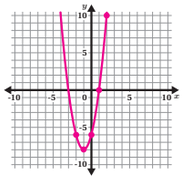
Name: _____ Date: _____ Period: _____

Problems 5–6: Here is a function: $b(x) = (2x - 2)(x + 3)$.

5. Complete the table for $b(x)$.

x	$b(x)$
-2	-6
-1	-8
0	-6
1	0
2	10

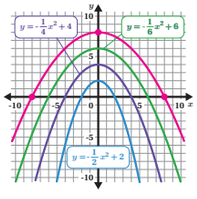
6. Create a graph of $b(x)$.



Test Practice

Problems 7–8: Here are some quadratic functions.

7. Graph the next parabola that follows this pattern.



8. Complete the table for the next parabola that follows this pattern.

x	y
-8	0
0	8
8	0

Spiral Review

9. Write the equation of a line that has a y-intercept of -2.
Responses vary.
$y = 3x - 2$

10. Write the equation of a line that has an x-intercept of 3.
Responses vary.
$9 = 3x + y$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.3.7
	2–6	2	MA.912.AR.3.7
Test Practice	7–8	3	MA.912.AR.3.7
Spiral Review			
Fluency	9–10	2	MA.8.AR.3.3

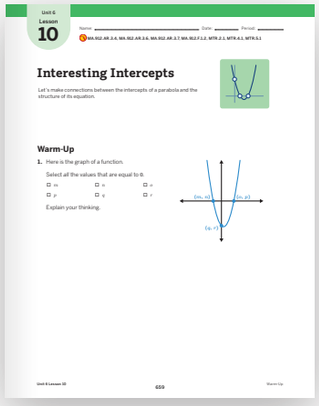
Unit 6

Lesson 10

Interesting Intercepts

Intercepts in Factored and Standard Forms

Let's make connections between the intercepts of a parabola and the structure of its equation.


Unit 6
Lesson 10
Interesting Intercepts
Let's make connections between the intercepts of a parabola and the structure of its equation.
Warm-Up
1. Here is the graph of a function.
Select all the values that are equal to 0.
Explain your thinking.
659

Learning Outcomes

Today's Goals

1. **Goal:** Identify the x- and y-intercepts of quadratic functions from their equations written in factored or standard form.
2. **Language Goal:** Explain how to determine the x- and y-intercepts of a quadratic function written in factored or standard form. **(Writing, Speaking, and Listening)**

Students make connections between the intercepts of a parabola and the structure of its equation written in factored or standard form. They explain the connection between the *factors* of an equation in factored form and the *x-intercept(s)* on its graph, and between the constant term of an equation written in standard form and the *y-intercept* on its graph. Students then use the structure of each type of equation to determine intercepts. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 8, students graphed quadratic functions written in either standard or factored form, although these terms were not introduced. In Lesson 9, students determined whether a function was written in standard form or factored form.

Future Learning

In Lesson 12, students will use quadratic equations to make sense of revenue models. In Lessons 13–14, students will be introduced to quadratic functions written in vertex form.

Connections

- Students relate the intercepts of the graph of a quadratic function to a specific form of its equation (factored or standard). (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Also Addressing: **MA.912.AR.3.6**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.F.1.2	MA.912.AR.3.8 MA.912.AR.1.3 MA.912.AR.1.4

Vocabulary

Review Vocabulary

factor
x-intercept
y-intercept

Lesson at a Glance

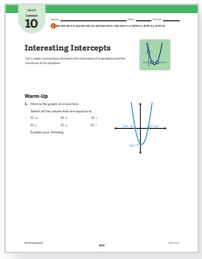
~ 45 min

MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2

Warm-Up

Pairs | 5 min

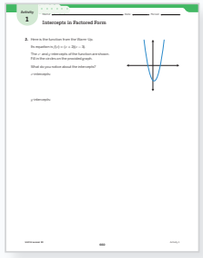
Students uncover an important feature of all intercepts: that one of their coordinates must be equal to 0. **(MTR.5.1)**



Activity 1

Pairs | 10 min

Students explore the connection between an equation written in factored form and the x-*intercepts* of its graph, using tables to understand why the y-value of an x-intercept is 0.

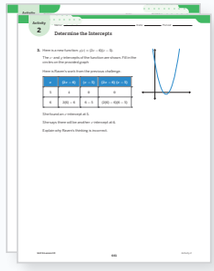


Activity 2

Pairs | 10 min

Students continue to explore the connection between an equation written in factored form and the x-intercepts of its graph, specifically when *factors* do not have a coefficient of 1.

Routines: Decide and Defend, MLR3: Critique, Correct, Clarify

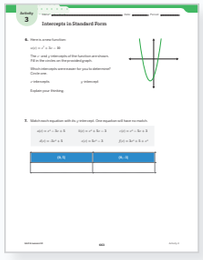


Activity 3

Pairs | 10 min

Students explore the connection between an equation written in standard form and the y-*intercept* of its graph, using tables to understand why the x-value of a y-intercept is 0.

MLR2: Collect and Display



Synthesis

Whole Class | 5 min

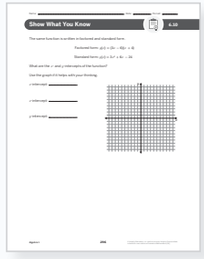
Students synthesize their understanding of what quadratic equations written in factored form and standard form reveal about the x- and y-intercepts of their graphs. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the x- and y-intercepts of a function written in factored and standard form.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students uncover an important feature of all intercepts: that one of their coordinates must be equal to 0. **(MTR.5.1)**

1 Launch

Select a and consider asking, "Does a have a value of 0? How do you know?"

Invite students to discuss the remaining variables with a partner, deciding which are equal to 0.

A **Accessibility: Visual-Spatial Processing** Invite students to sketch a similar parabola on graph paper and determine the coordinates of the x- and y-intercepts.

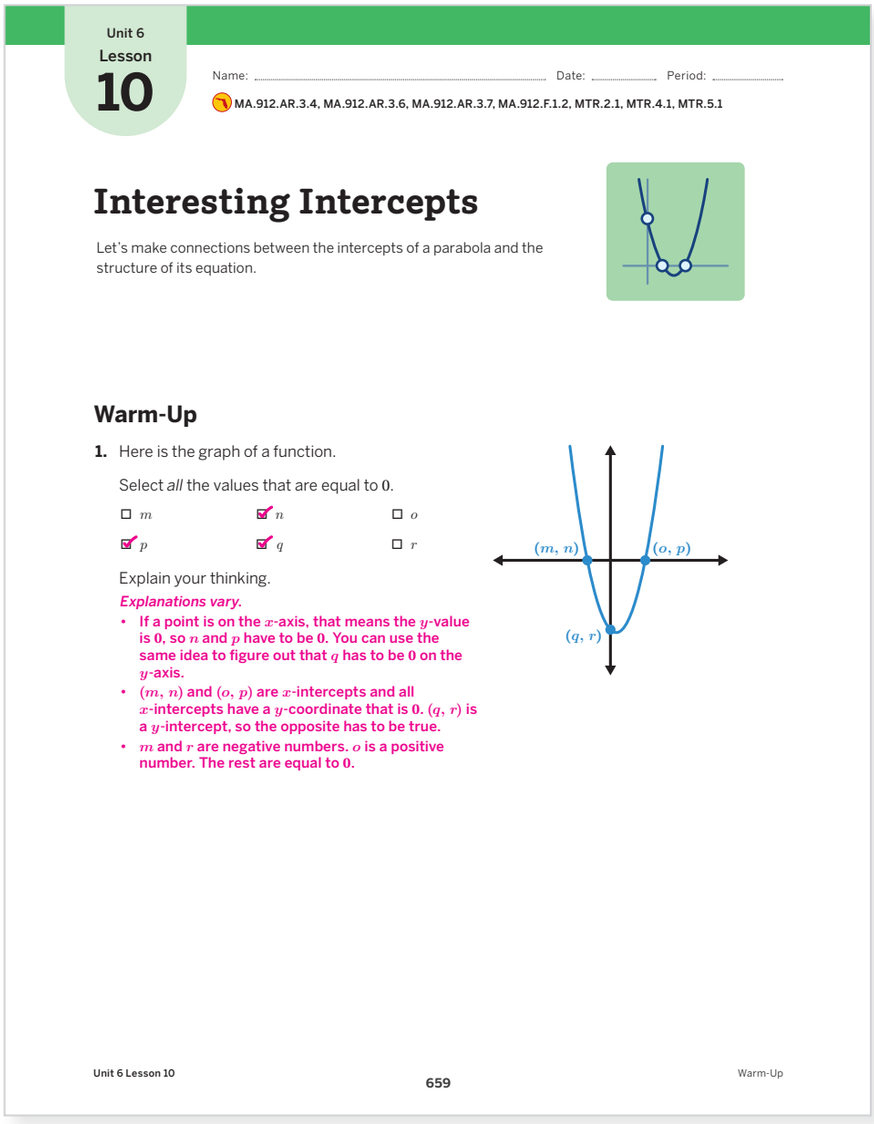
2 Connect

Discuss each value as a class, inviting students to share their reasoning for how to determine which values are equal to 0.

To reveal the Key Takeaway, consider asking:

- "How did you decide which values had to be equal to 0?"
- "What do we call the points $(a, 0)$ and $(c, 0)$? What about $(0, f)$?"

ELL **English Language Learners** Consider reviewing an anchor chart that displays the key features of a parabola. Consider asking, "Which points are x-intercepts? Which are y-intercepts? How do you know?" Display the anchor chart where students can reference it throughout the lesson. **(Reading, Writing, Speaking, and Listening)**



Unit 6
Lesson
10

Name: Date: Period:

MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Interesting Intercepts

Let's make connections between the intercepts of a parabola and the structure of its equation.

Warm-Up

1. Here is the graph of a function.

Select *all* the values that are equal to 0.

☐ m ☑ n ☐ o
☑ p ☑ q ☐ r

Explain your thinking.

Explanations vary.

- If a point is on the x-axis, that means the y-value is 0, so n and p have to be 0. You can use the same idea to figure out that q has to be 0 on the y-axis.
- (m, n) and (o, p) are x-intercepts and all x-intercepts have a y-coordinate that is 0. (q, r) is a y-intercept, so the opposite has to be true.
- m and r are negative numbers. o is a positive number. The rest are equal to 0.

Unit 6 Lesson 10 659 Warm-Up

Key Takeaway: The *x-intercepts* of a function can be written as $(a, 0)$ because they are the locations where the graph of the function crosses the x-axis and the y-value is 0. The *y-intercept* of a function can be written as $(0, b)$ because it is the location where the graph of the function crosses the y-axis and the x-value is 0.

Pairs | 10 min

Activity 1 Intercepts in Factored Form

Purpose: Students explore the connection between an equation written in factored form and the *x-intercepts* of its graph, using tables to understand why the y-value of an x-intercept is 0.

1 Launch

Consider asking, "is $x = -3$ an x-intercept or a y-intercept? How do you know?"

2 Monitor

To access connections, consider asking pairs of students, "What do you notice about the points that are intercepts? How does that connect to the function?"

A **Accessibility: Visual-Spatial Processing** Invite students to explain what happens to an input as the function evaluates from left to right in the table in question 3.

Select and sequence several student responses to share during the Connect.

D **Differentiation**

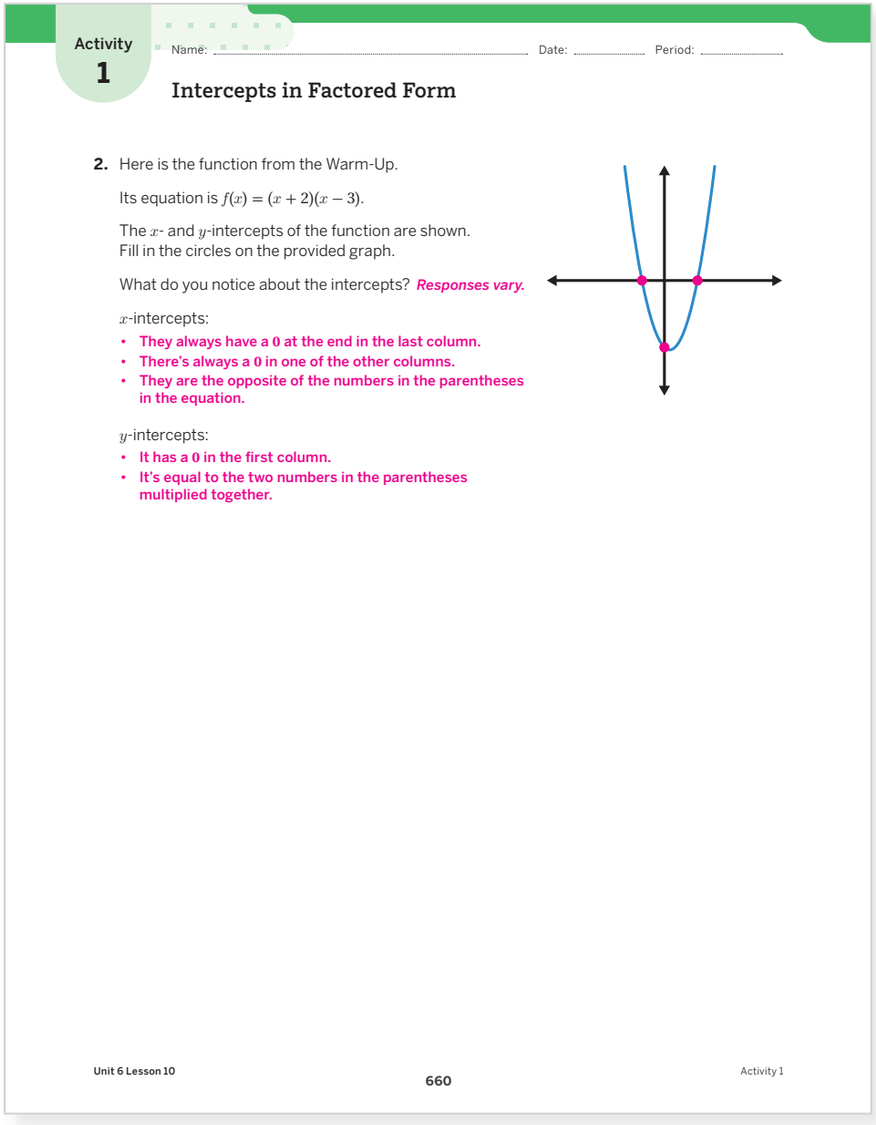
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to review the anchor chart. Consider asking, "Which rows show x-intercepts? The y-intercept? How do you know?"
Notice that the x-intercepts make each factor equal to 0.	**Strengthen** Celebrate their noticing. Invite them to share during the Connect.
Would like an additional challenge.	**Stretch** Invite these students to predict the intercepts of $g(x) = (x - 5)(x + 7)$.

3 Connect

To reveal the Key Takeaway, consider asking:

- "What did you notice about the x-intercepts in the table? In the function?" **(MTR.5.1)**
- "What did you notice about the y-intercepts in the table? In the function?" **(MTR.5.1)**

Consider sharing that $(x + 2)$ and $(x - 3)$ are called *factors* because they are multiplied together.



Activity 1

Name: _____ Date: _____ Period: _____

Intercepts in Factored Form

2. Here is the function from the Warm-Up.

Its equation is $f(x) = (x + 2)(x - 3)$.

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph.

What do you notice about the intercepts? *Responses vary.*

x-intercepts:

- They always have a 0 at the end in the last column.
- There's always a 0 in one of the other columns.
- They are the opposite of the numbers in the parentheses in the equation.

y-intercepts:

- It has a 0 in the first column.
- It's equal to the two numbers in the parentheses multiplied together.

Unit 6 Lesson 10 660 Activity 1

Key Takeaway: The factored form of a quadratic equation reveals its x-intercepts by analyzing tables to understand that the y-value at each x-intercept is 0, connecting algebraic reasoning with graphical representation.

Pairs | 10 min

Activity 2 Determine the Intercepts

Purpose: Students continue to explore the connection between an equation written in factored form and the x-intercepts of its graph, specifically when *factors* do not have a coefficient of 1.

Short on time? Consider omitting Problem 4.

1 Launch

To access connections, consider asking, "How is this equation similar to the one on the previous question? How is it different?"

A **Accessibility: Conceptual Processing** Invite students to use a calculator to support them with performing calculations.

2 Monitor

Listen for ideas from earlier activities to help determine the intercepts.

Support students getting started by asking, "How did we figure out the x-intercepts earlier in the lesson? How is this situation similar? How is it different?"

D **Differentiation: (Support)** If students try $x = 6$ because it is the opposite of -6 in the factor $(2x - 6)$, consider asking, "Did $x = 6$ make one of the factors 0? What number might you try next?"

Pause to invite students to share how they know that 6 is not an x-intercept of this parabola, what the correct x-intercept is, and why.

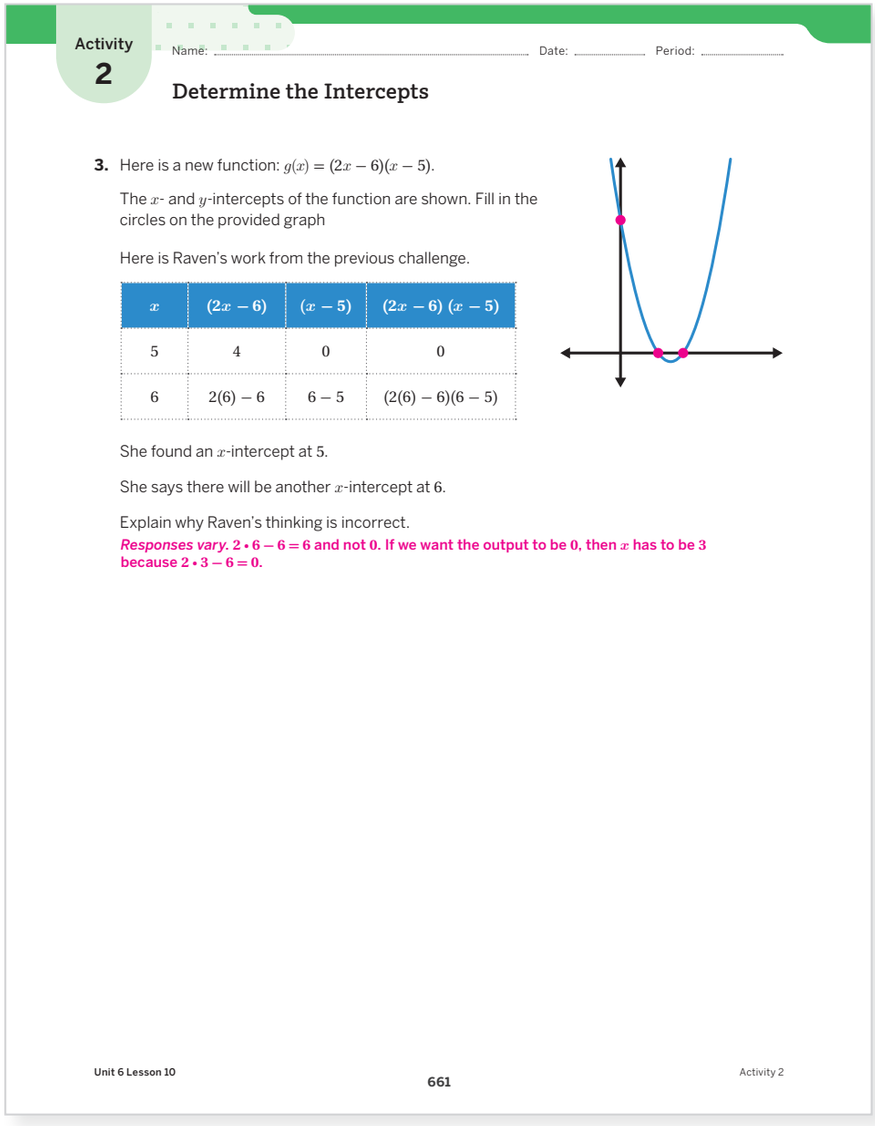
MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Raven's work. Then invite students to:

- Discuss what is *correct* about the response. Consider asking, "Why might Raven have thought the x-intercept is at 6?"
- Discuss what is *incorrect* or *unclear*. Consider asking, "How can you tell that 6 is not an x-intercept?"
- Write a question to help the student see how their work can be improved, or create a revised response. **(MTR.4.1)**

Consider asking, "What is the other x-intercept? How do you know?"

Note: If it does not come up naturally, consider sharing that $(2x + 6)$ and $(x - 5)$ are called *factors* because you multiply them together.

Activity 2 continued >



Activity 2

Name: Date: Period:

Determine the Intercepts

3. Here is a new function: $g(x) = (2x - 6)(x - 5)$.

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph

Here is Raven's work from the previous challenge.

x	$(2x - 6)$	$(x - 5)$	$(2x - 6)(x - 5)$
5	4	0	0
6	$2(6) - 6$	$6 - 5$	$(2(6) - 6)(6 - 5)$

She found an x-intercept at 5.

She says there will be another x-intercept at 6.

Explain why Raven's thinking is incorrect.

Responses vary. $2 \cdot 6 - 6 = 6$ and not 0. If we want the output to be 0, then x has to be 3 because $2 \cdot 3 - 6 = 0$.

Unit 6 Lesson 10 661 Activity 2

Pairs | 10 min

Activity 2 Determine the Intercepts (continued)

Purpose: Students continue to explore the connection between an equation written in factored form and the x-intercepts of its graph, specifically when factors do not have a coefficient of 1.

2 Monitor

Check in with pairs of students as necessary.

A **Accessibility: Conceptual Processing**
To assist students in recognizing the connections between new problems and prior work, consider asking, "How can Raven's thinking help us with this problem?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because _____.). **(Writing and Speaking)**

D **Differentiation**

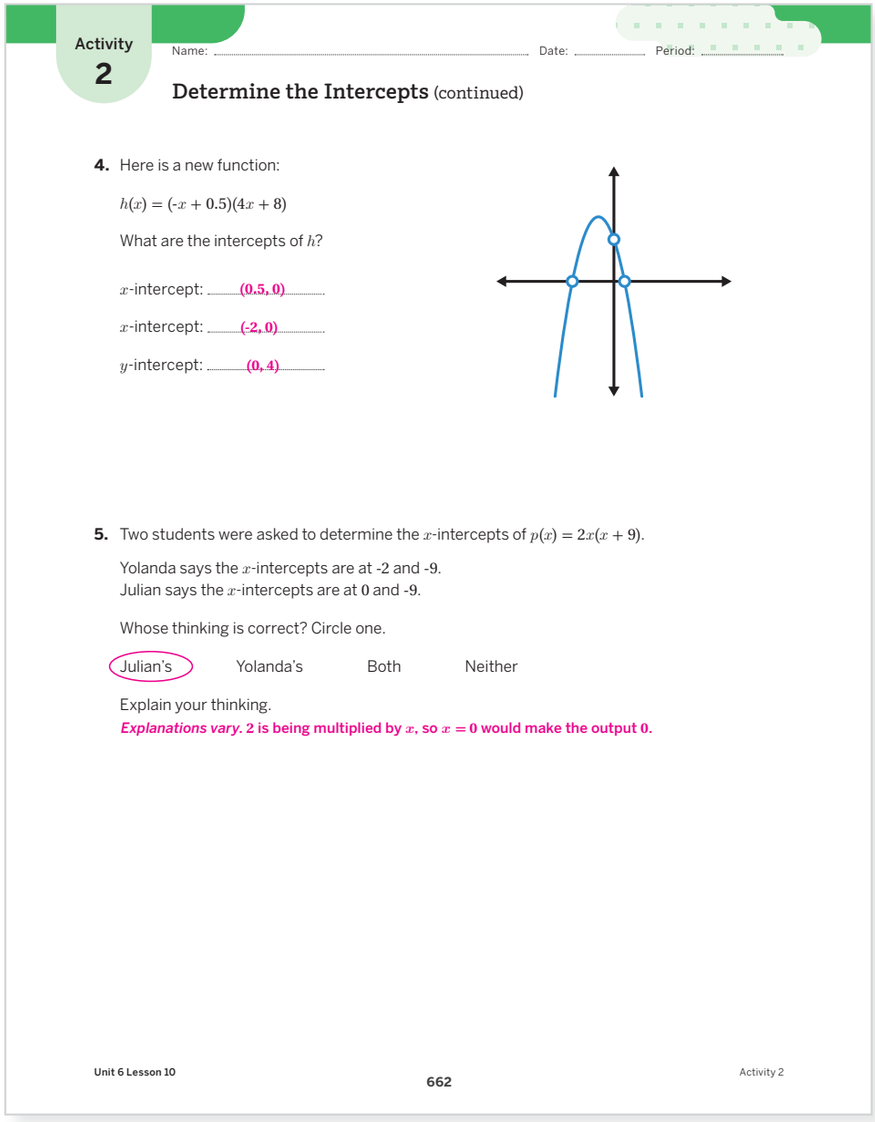
Look for students who:	Respond to Student Thinking
Notice a 2 in the factor $2x$ and select Yolanda *(Problem 5)*.	**Support** Invite them to make a table or to substitute -2 for x to help them determine the output.
Would like to explore determining the x-intercepts of a function in factored form further.	**Stretch** Encourage them to write a different equation that has the same x-intercepts as $p(x)$ and explain how they know their equation has those intercepts.

3 Connect

Display the distribution of responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Spend adequate time here to ensure that students understand why Julian's thinking is correct. **(MTR.4.1)**

To reveal the Key Takeaway, invite students to share how to determine the x- and y-intercepts from a quadratic equation in factored form.



Activity 2

Name: _____ Date: _____ Period: _____

Determine the Intercepts (continued)

4. Here is a new function:

$h(x) = (-x + 0.5)(4x + 8)$

What are the intercepts of h?

x-intercept: (0.5, 0)

x-intercept: (-2, 0)

y-intercept: (0, 4)

5. Two students were asked to determine the x-intercepts of $p(x) = 2x(x + 9)$.

Yolanda says the x-intercepts are at -2 and -9.
Julian says the x-intercepts are at 0 and -9.

Whose thinking is correct? Circle one.

Julian's Yolanda's Both Neither

Explain your thinking.

Explanations vary. 2 is being multiplied by x, so $x = 0$ would make the output 0.

Unit 6 Lesson 10 662 Activity 2

Key Takeaway: The x-value that makes one of the *factors* equal 0 will be an x-intercept of the graph of that function. When the x-value is 0, the output will be the y-intercept.

Pairs | 10 min

Activity 3 Intercepts in Standard Form

Purpose: Students explore the connection between an equation written in standard form and the y-*intercept* of its graph, using tables to understand why the x-value of a y-intercept is 0.

Short on time? Consider removing cards $d(x)$, $e(x)$, and $f(x)$ or eliminating the Card Sort routine and focusing on $a(x)$ and $b(x)$ as a whole class during the Connect.

1 Launch

Discuss the form of $w(x)$ and how students know.

To access connections, Consider asking, "Does $x = 4$ lead to an x- or y-intercept? How do you know?"

2 Monitor

Listen for strategies to determine the intercepts and noticings about the form of the equation and the intercepts.

D **Differentiation: (Support)** Encourage students to use guess-and-check and the approximate locations on the graph to determine the intercepts.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., Determining the _____ was/were easier for me because _____). **(Writing and Speaking)**

3 Connect

Display the card sort solutions, focusing on $a(x)$ and $b(x)$.

To reveal the Key Takeaway, consider asking:

- "What strategies were helpful to determine the y-intercept?"
- "Where do you see the y-intercept in $a(x)$ or $b(x)$? Why does that make sense?"
- "Do you think that the y-intercept will always be the constant term? Why or why not?"

Invite students to share how to determine the y-intercept from a quadratic equation written in standard form.

MLR **MLR2: Collect and Display** Use this routine to add to the previous class description or anchor chart using students' language. Display any new language, examples, or diagrams for students to reference later.

Activity 3

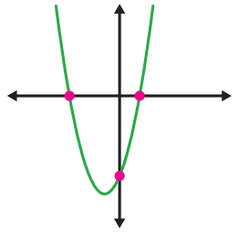
Name: _____ Date: _____ Period: _____

Intercepts in Standard Form

6. Here is a new function:

$w(x) = x^2 + 3x - 10$

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph.



Which intercepts were easier for you to determine? Circle one. *Choices vary.*

x-intercepts y-intercept

Explain your thinking.

Responses vary.

- y-intercept. When you plug in 0 for x, everything multiplies to 0 except for the -10.
- x-intercepts. I tried different numbers for x and knew I was getting close when the output got close to 0.

7. Match each equation with its y-intercept. One equation will have no match.

$a(x) = x^2 - 3x + 5$	$b(x) = x^2 + 5x - 3$	$c(x) = x^2 - 5x + 3$
$d(x) = -3x^2 + 5$	$e(x) = 5x^2 - 3$	$f(x) = 3x^2 + 5 + x^2$

(0, 5)	(0, -3)
$a(x)$, $d(x)$, $f(x)$	$b(x)$, $e(x)$

Equation with no match: $c(x)$

Unit 6 Lesson 10 663 Activity 3

Key Takeaway: In a quadratic equation written in standard form, the y-intercept is the constant term in the equation.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what quadratic equations written in factored form and standard form reveal about the x- and y-intercepts of their graphs.

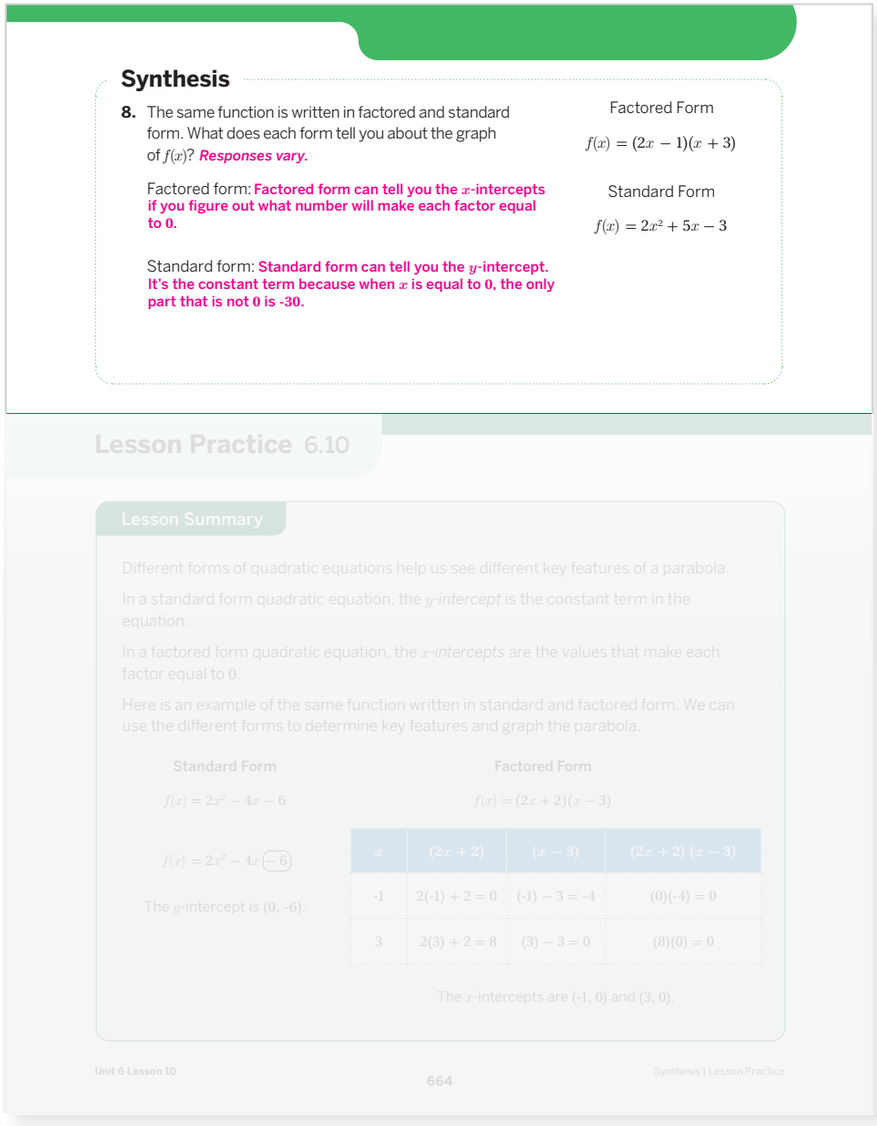
Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:
- Equations written in factored form highlight the x-intercepts.
- The x-value that makes one of the factors equal 0 will be an x-intercept of the graph of that function.
- Equations written in standard form highlight the y-intercept.
- The y-intercept is the constant term in the equation.

Consider asking:
- "Which form is more helpful for determining x-intercepts? Why? What about y-intercepts?"
- "What do you think is important to take away from the lesson today?"

MLR **MLR2 Collect and Display** Use this routine to collect students' ideas, language, examples, or strategies for determining the x- and y-intercepts. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. The same function is written in factored and standard form. What does each form tell you about the graph of $f(x)$? ***Responses vary.***

Factored form: **Factored form can tell you the x-intercepts if you figure out what number will make each factor equal to 0.**

Standard form: **Standard form can tell you the y-intercept. It's the constant term because when x is equal to 0, the only part that is not 0 is -30.**

Factored Form

$f(x) = (2x - 1)(x + 3)$

Standard Form

$f(x) = 2x^2 + 5x - 3$

Lesson Practice 6.10

Lesson Summary

Different forms of quadratic equations help us see different key features of a parabola.

In a standard form quadratic equation, the *y-intercept* is the constant term in the equation.

In a factored form quadratic equation, the *x-intercepts* are the values that make each factor equal to 0.

Here is an example of the same function written in standard and factored form. We can use the different forms to determine key features and graph the parabola.

Standard Form

$f(x) = 2x^2 - 4x - 6$

$f(x) = 2x^2 - 4x + (-6)$

The y-intercept is (0, -6).

Factored Form

$f(x) = (2x + 2)(x - 3)$

x	$(2x + 2)$	$(x - 3)$	$(2x + 2)(x - 3)$
-1	$2(-1) + 2 = 0$	$(-1) - 3 = -4$	$(0)(-4) = 0$
3	$2(3) + 2 = 8$	$(3) - 3 = 0$	$(8)(0) = 0$

The x-intercepts are (-1, 0) and (3, 0).

Unit 6 Lesson 10 664 Synthesis | Lesson Practice

Lesson Takeaway: Quadratic equations written in factored form and standard form reveal information about their graphs, specifically the x- and y-intercepts.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the x- and y-intercepts of a function written in factored and standard form.

Today's Goals

1. **Goal:** Identify the x- and y-intercepts of quadratic functions from their equations written in factored or standard form.
2. **Language Goal:** Explain how to determine the x- and y-intercepts of a quadratic function written in factored or standard form. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.10

The same function is written in factored and standard form.

Factored form: $g(x) = (3x - 6)(x + 4)$

Standard form: $g(x) = 3x^2 + 6x - 24$

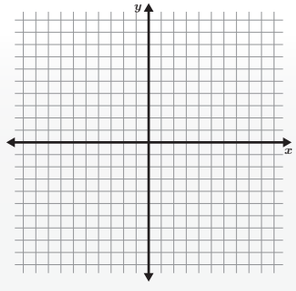
What are the x- and y-intercepts of the function?

Use the graph if it helps with your thinking.

x-intercept: (2, 0)

x-intercept: (-4, 0)

y-intercept: (0, -24)



Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the x-intercept from an equation written in factored form:

» **Respond to Student Thinking:**

- Use the *Graphing Quadratic Functions in Factored Form* **Mini-lesson**.
- Review the relationship between factors and x-intercepts.

If students need support understanding how to determine the y-intercept from an equation written in standard form:

- Review the relationship between the constant term and y-intercepts.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Encouraging students to complete the DOK3 Practice Problems.
- Invite students to explore the Sub-Unit 2 Extension Activities.

Professional Learning

Reflect on **MLR 3: Critique, Correct, Clarify** used in this lesson. Students used this routine to describe the strengths and errors in Raven's work. In what ways did this routine support students in describing Raven's thinking? How did your facilitation of the routine support students in that there is correct mathematical thinking in Raven's incorrect response? In what ways did students engage in all three steps of the routine by critiquing Raven's work, offering a correct strategy, and clarifying their thinking about the solution strategy in general?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. The same function is written in factored and standard form. What does each form tell you about the graph of $f(x)$? *Responses vary.*

Factored form: **Factored form can tell you the x-intercepts if you figure out what number will make each factor equal to 0.**

Standard form: **Standard form can tell you the y-intercept. It's the constant term because when x is equal to 0, the only part that is not 0 is -30.**

Factored Form

$f(x) = (2x - 1)(x + 3)$

Standard Form

$f(x) = 2x^2 + 5x - 3$

Lesson Practice 6.10

Lesson Summary

Different forms of quadratic equations help us see different key features of a parabola.

In a standard form quadratic equation, the *y-intercept* is the constant term in the equation.

In a factored form quadratic equation, the *x-intercepts* are the values that make each factor equal to 0.

Here is an example of the same function written in standard and factored form. We can use the different forms to determine key features and graph the parabola.

Standard Form

$f(x) = 2x^2 - 4x - 6$

$f(x) = 2x^2 - 4x(-6)$

The y-intercept is (0, -6).

Factored Form

$f(x) = (2x + 2)(x - 3)$

x	$(2x + 2)$	$(x - 3)$	$(2x + 2)(x - 3)$
-1	$2(-1) + 2 = 0$	$(-1) - 3 = -4$	$(0)(-4) = 0$
3	$2(3) + 2 = 8$	$(3) - 3 = 0$	$(8)(0) = 0$

The x-intercepts are (-1, 0) and (3, 0).

Lesson Practice 6.10

Name: _____ Date: _____ Period: _____

Problems 1–2: Determine an x-intercept(s) from each equation.

1. $a(x) = (x - 2)(x + 1)$
(2, 0) or (-1, 0)

2. $b(x) = (x + 3)^2$
(-3, 0)

3. Here is the same function written in two forms.

Factored form: $g(x) = (x + 5)(x - 2)$ **Standard form:** $g(x) = x^2 + 3x - 10$

What are the x- and y-intercepts of the function?

x-intercept: **(-5, 0)**

x-intercept: **(2, 0)**

y-intercept: **(0, -10)**

4. Evan and Ariel were working on a problem together.

Evan said: *The y-intercept of $y = (x - 3)^2$ is (0, 3).*

Ariel said: *The x-intercept of $y = (x - 3)^2$ is (3, 0).*

Whose thinking is correct? Circle one.

Evan's (Ariel's)

Explain your thinking

***Explanations vary.* If you make $x = 0$, then $(0 - 3)^2 = 9$, not 3. If you make $x = 3$, then $(3 - 3)^2 = 0$, so Ariel is correct.**

Problems 5–6: Here is a function: $h(x) = x(x + 6)$.

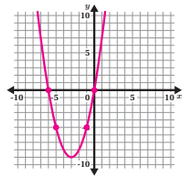
5. Determine the intercepts of $h(x)$.

x-intercept: **(-6, 0)**

x-intercept: **(0, 0)**

y-intercept: **(0, 0)**

6. Draw the graph of $h(x)$.



Lesson Practice 6.10

Name: _____ Date: _____ Period: _____

Problems 7–9: Use a graphing calculator to graph $f(x) = (x + 3)(x + 1)(x - 2)$. Change the numbers in the equation to determine an equation whose graph . . . *Responses vary.*

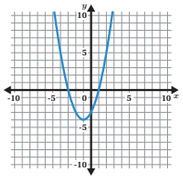
7. Has two x-intercepts. $f(x) = (x - 1)(x - 2)(x - 2)$

8. Has one x-intercept. $f(x) = (x - 2)(x - 2)(x - 2)$

9. Has an x-intercept at (7, 0). $f(x) = (x - 1)(x - 3)(x - 7)$

Test Practice

10. Here is a graph of a quadratic function.

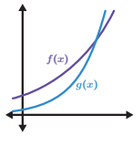


Which function could this be?

A. $y = (x - 3)(x + 1)$
B. $y = (x + 3)(x - 1)$
C. $y = (x - 3)(x - 1)$
D. $y = (x + 3)(x + 1)$

Spiral Review

11. Here are the graphs of two functions, $f(x)$ and $g(x)$.



$f(x) = 100 \cdot 2^x$

Which equation could represent $g(x)$?

A. $g(x) = 25 \cdot 4^x$
B. $g(x) = 50 \cdot 1.5^x$
C. $g(x) = 100 \cdot 4^x$
D. $g(x) = 200 \cdot 1.5^x$

Problems 12–13: Complete each equation with a number that makes the equation true.

12. $2 \cdot \underline{-2} + 4 = 0$

13. $3(3 \cdot \underline{-\frac{4}{3}} + 4) = 0$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.AR.3.7
	3–6	2	MA.912.AR.3.7
	7–9	3	MA.912.AR.3.7
Test Practice	10	1	MA.912.AR.3.7
Spiral Review			
	11	2	MA.912.AR.5.4
Fluency	12–13	1	MA.8.AR.2.1

Unit 6

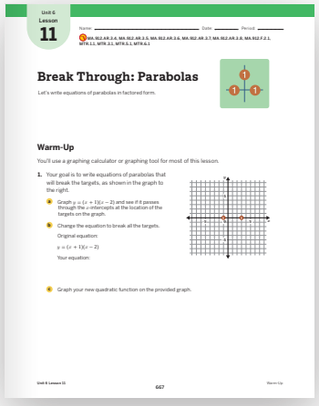
Lesson

11

Break Through: Parabolas

Building Quadratics in Factored Form

Let's write equations of parabolas in factored form.


Unit 6
Lesson
11
Name _____ Date _____ Period _____
MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1
Break Through: Parabolas
Let's write equations of parabolas in factored form.
Warm-Up
You'll use a graphing calculator or graphing tool for most of this lesson.
1. Your goal is to write equations of parabolas that will break the targets, as shown in the graph to the right.
Graph $y = (x + 1)(x - 2)$ and see if it passes through the x-intercepts at the location of the targets on the graph.
Change the equation to break all the targets.
Original equation:
$y = (x + 1)(x - 2)$
Your equation:
Graph your new quadratic function on the provided graph.
Unit 6 Lesson 11
667
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Write a quadratic function in factored form given its graph.
2. **Language Goal:** Describe how the value of a in the equation $y = a(x - m)(x - n)$ affects the graph of its corresponding parabola. **(Writing)**

Students write equations of parabolas in factored form from given key graphical features. They explore what can change in an equation written in factored form to keep the x-intercepts the same while changing other key features like the concavity, the vertical position of the vertex, or the y-intercept.

Coherence

Prior Learning

In Lessons 8–9, students graphed parabolas by hand by evaluating the function at different x-values. In Lesson 10, students described how to determine key features of parabolas (like x-intercepts, y-intercepts, and the vertex) from equations with an emphasis on those written in factored form.

Future Learning

In Lesson 12, students will create multiple representations of quadratic situations and interpret their key features in context.

Connections

- Within the context of a video game scenario, students write quadratic equations in factored form to "break the targets" at the intercepts of a parabola. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.4

Write a quadratic function to represent the relationship between two quantities from a graph, a written description, or a table of values within a mathematical or real-world context.

MA.912.AR.3.5

Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function.

Also Addressing: **MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.3.7	MA.912.F.2.1

Vocabulary

Review Vocabulary

factor

Lesson at a Glance

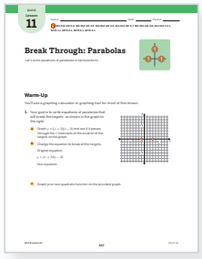
~ 45 min

MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1

Warm-Up

Pairs | 5 min

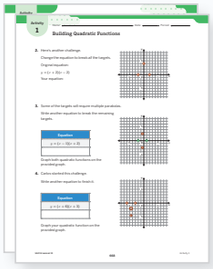
Students revisit the connection between the x-intercepts of a parabola and its equation in factored form by playing a round of *Break Through: Parabolas*. **(MTR.5.1)**



Activity 1

Pairs | 15 min

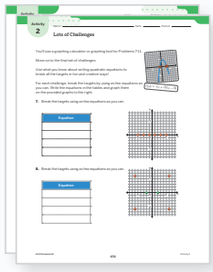
Students surface strategies for adjusting equations in factored form to keep the same x-intercepts, but change key features like the concavity or the vertical position of the vertex.



Activity 2

Pairs | 15 min

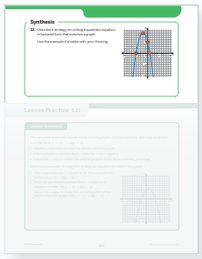
Students use their creativity and what they learned in Activity 1 to persevere in solving several different challenges. **(MTR.1.1)**



Synthesis

Whole Class | 5 min

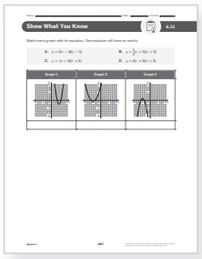
Students synthesize their strategies for writing equations in factored form to match a graph.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by matching graphs of parabolas to their equations in factored form.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students revisit the connection between the x-intercepts of a parabola and its equation in factored form by playing a round of *Break Through: Parabolas*. **(MTR.5.1)**

1 Launch

Display a blank graph and the equation $y = (x - 1)(x - 2)$ and ask:

- "What do you think the graph of $y = (x - 1)(x - 2)$ will look like?"
- "Which target(s) do you think it will break? How do you know?"

To encourage creativity, consider sharing that there are many ways to solve each challenge in this lesson. Encourage students to try to write an equation they think no one else in their class will write.

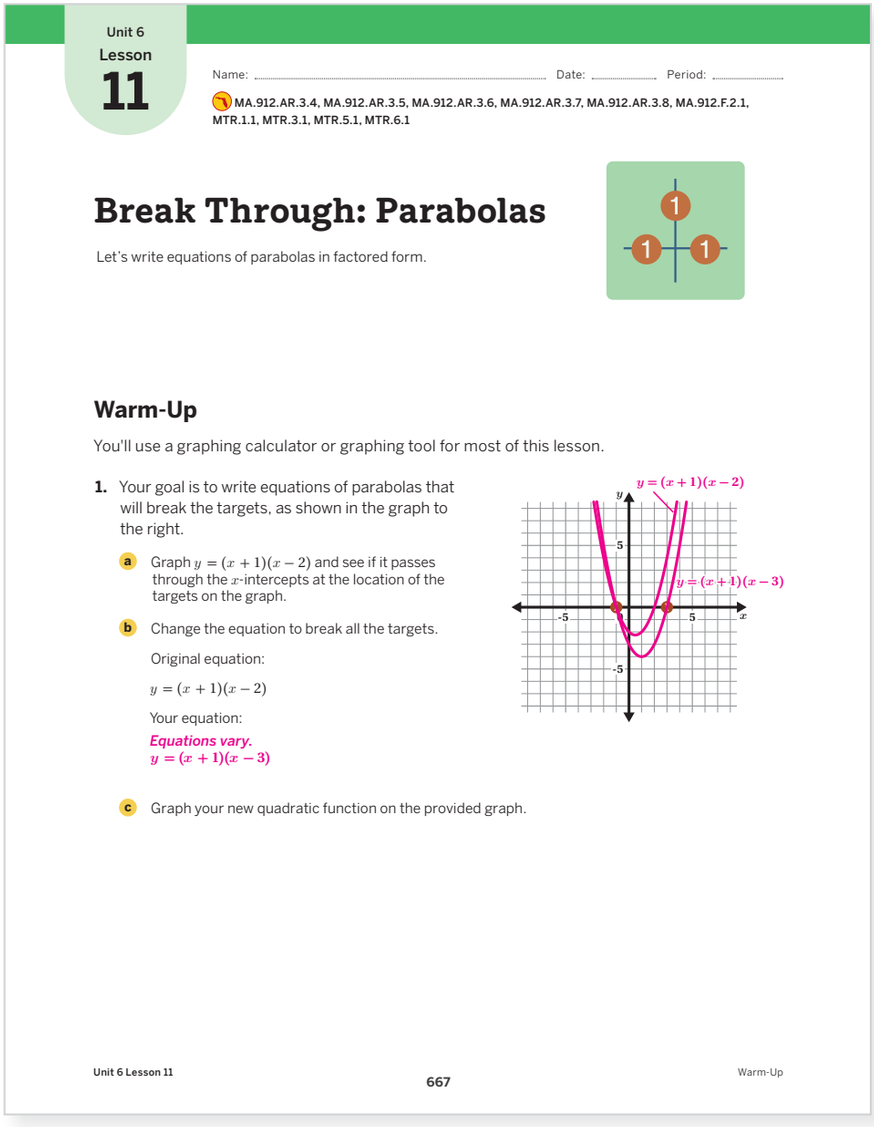
ELL **English Language Learners** Invite students to share what the word *target* means, where they've heard it in and out of school, and where they can see them on the graph. **(Reading)**

2 Connect

To access connections between quadratic functions in factored form and the graph, consider asking:

- "What form is the equation in?" **(MTR.5.1)**
- "Which key feature do these targets represent?"
- "In what ways can you change the equation while keeping the x-intercepts the same?"

Note: Students may notice that the equation is written with $y =$ instead of using function notation. Consider sharing with students that $y =$ and $f(x) =$ are equivalent ways and that they can write equations using either option. The graph of f is the graph of the equation $y = f(x)$.



Unit 6
Lesson
11

Name: ..... Date: ..... Period: .....

MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1

Break Through: Parabolas

Let's write equations of parabolas in factored form.

Warm-Up

You'll use a graphing calculator or graphing tool for most of this lesson.

1. Your goal is to write equations of parabolas that will break the targets, as shown in the graph to the right.

a. Graph $y = (x + 1)(x - 2)$ and see if it passes through the x-intercepts at the location of the targets on the graph.

b. Change the equation to break all the targets.

Original equation:

$y = (x + 1)(x - 2)$

Your equation:

Equations vary.
$y = (x + 1)(x - 3)$

c. Graph your new quadratic function on the provided graph.

Unit 6 Lesson 11 667 Warm-Up

Pairs | 15 min

Activity 1 Building Quadratic Functions

Purpose: Students surface strategies for adjusting equations in factored form to keep the same x-intercepts, but change key features like the concavity or the vertical position of the vertex.

1 Launch

Consider asking, "Which key features do the targets represent?"

To encourage playfulness, invite students to change the equation in different ways and use the feedback to revise their thinking.

2 Monitor

Observe and organize a variety of different strategies (successful and unsuccessful) as students try to create a concave down parabola.

D **Differentiation: (Support)** Students may add a negative sign into one or more of the factors, such as $y = (\text{-}x + 2)(x - 2)$. Consider asking, "Which key features changed? Why?"

A **Accessibility: Memory and Attention** Consider inviting them to record sketches of their graphs and the equations that match to recall the connections between them.

Pause briefly and invite students to share what strategies work and don't work for creating a concave down parabola. Encourage students to share what they learn from strategies that did not work.

Encourage students to predict which targets the parabola will break.

Consider sharing that students can copy and paste earlier equations to adjust if they find it helpful.

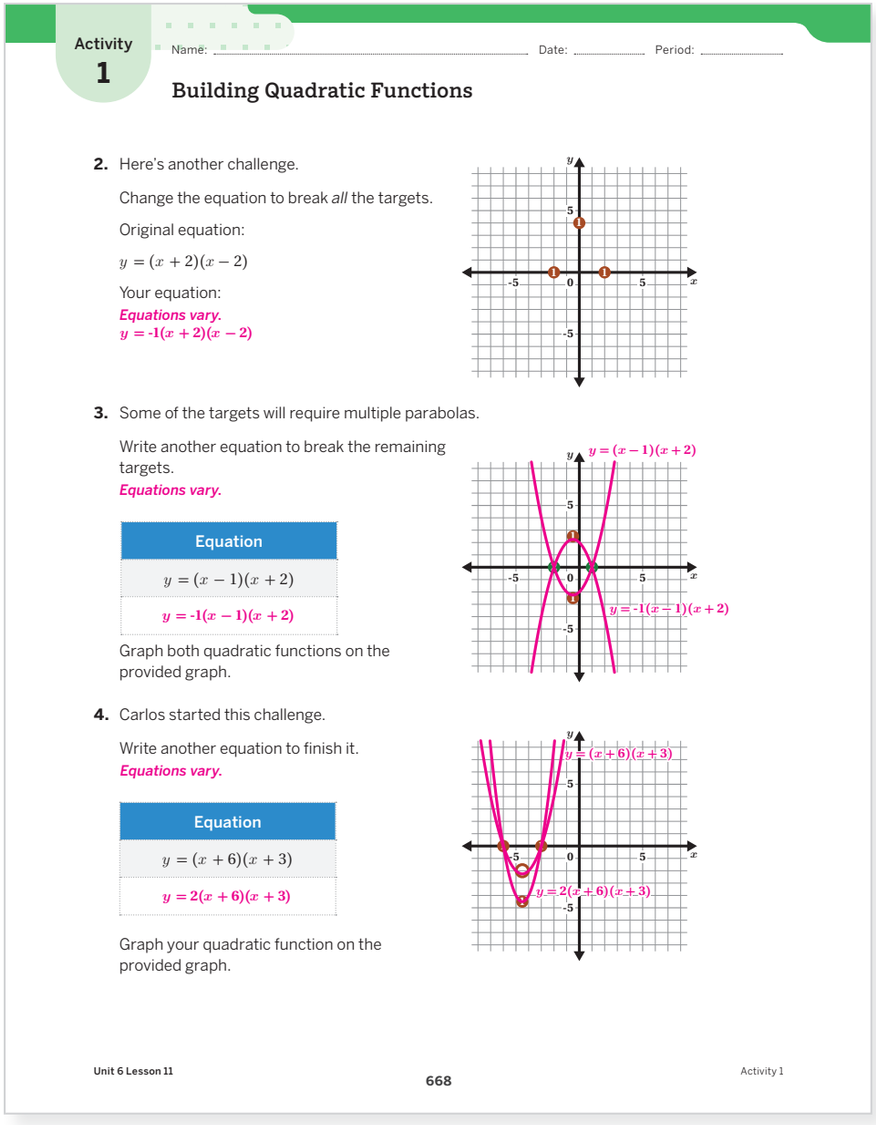
D **Differentiation: (Support)** To support students with writing a second equation, encourage them to trace or describe what they want the new parabola to look like. Consider asking, "How is it similar to and different from the first parabola?"

Note: The number on the target represents the number of times an equation needs to break that target.

Look for different strategies (successful and unsuccessful) students try to adjust the vertical position of the vertex.

Pause briefly and invite students to share what strategies work and don't work for adjusting the vertical position of the vertex without changing the x-intercepts.

Activity 1 continued >



Activity 1

Name: Date: Period:

Building Quadratic Functions

2. Here's another challenge.

Change the equation to break *all* the targets.

Original equation:

$y = (x + 2)(x - 2)$

Your equation:

Equations vary.

$y = \text{-}1(x + 2)(x - 2)$

3. Some of the targets will require multiple parabolas.

Write another equation to break the remaining targets.

Equations vary.

Equation
$y = (x - 1)(x + 2)$
$y = \text{-}1(x - 1)(x + 2)$

Graph both quadratic functions on the provided graph.

4. Carlos started this challenge.

Write another equation to finish it.

Equations vary.

Equation
$y = (x + 6)(x + 3)$
$y = 2(x + 6)(x + 3)$

Graph your quadratic function on the provided graph.

Unit 6 Lesson 11 668 Activity 1

Pairs | 15 min

Activity 1 Building Quadratic Functions (continued)

Purpose: Students surface strategies for adjusting equations in factored form to keep the same x-intercepts, but change key features like the concavity or the vertical position of the vertex.

2 Monitor

Encourage students to experiment and use the feedback to revise their thinking.

To support students getting started, consider asking:

- "What x-intercepts do you want your function to have? How do you see those in an equation?"
- "How can you adjust the height of the vertex?"

A **Accessibility: Executive Functioning** Support students in initiating complex tasks by inviting them to focus on the targets with highest numbers first.

Select and sequence a variety of sketches and explanations to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to mark the key features on the graph of $y = (x + 3)(x - 2)$, then discuss how each one of those would or would not change.
Describe which key features stay the same, which change, and how.	**Strengthen** Celebrate these ideas. Invite students to generalize their thinking by making rules about different multipliers. **(MTR.3.1)**
Sketch/describe the parabola as *wider* or a *vertical compression*.	

3 Connect

Consider asking, "Which key features did not change? Why does multiplying the equation not change the x-intercepts?" The same values of x will make each factor 0 and 0 multiplied by any number still equals 0.

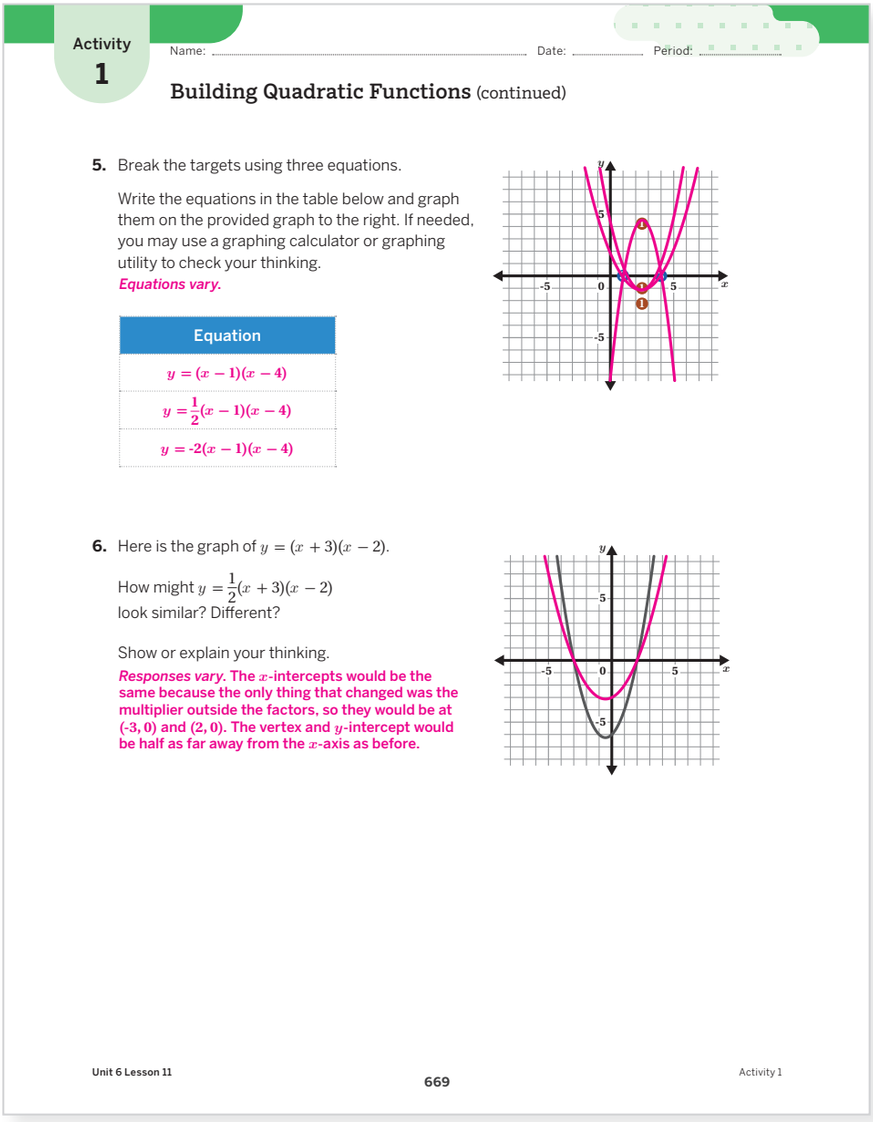
Consider asking:

- "Which key features do the factors reveal?"
- "What does the $\frac{1}{2}$ reveal about the parabola?"

To reveal the Key Takeaway, display the Desmos Graphing Calculator, typing in the equation $y = a(x + 3)(x - 2)$ with a slider for a. Adjust the a-value and invite students to make generalizations. **(MTR.3.1)**

Consider asking:

- "Which a-values make concave down parabolas?"
- "Which a-values moves the vertex farther from the x-axis? Closer?"
- "What else do you notice and wonder?"



Activity 1

Name: Date: Period:

Building Quadratic Functions (continued)

5. Break the targets using three equations.

Write the equations in the table below and graph them on the provided graph to the right. If needed, you may use a graphing calculator or graphing utility to check your thinking.

Equations vary.

Equation
$y = (x - 1)(x - 4)$
$y = \frac{1}{2}(x - 1)(x - 4)$
$y = -2(x - 1)(x - 4)$

6. Here is the graph of $y = (x + 3)(x - 2)$.

How might $y = \frac{1}{2}(x + 3)(x - 2)$ look similar? Different?

Show or explain your thinking.

Responses vary. The x-intercepts would be the same because the only thing that changed was the multiplier outside the factors, so they would be at (-3, 0) and (2, 0). The vertex and y-intercept would be half as far away from the x-axis as before.

Unit 6 Lesson 11 669 Activity 1

Key Takeaway: Multiplying an equation in factored form changes the shape of a parabola without changing the x-intercepts because the same values of x will make the function equal 0. For example, in the equation $y = a(x - m)(x - n)$:

- If the a-value is positive, the parabola will be concave up.
- If it is negative, the parabola will be concave down.
- Adjusting the value of a can also change the vertical position of the vertex.

Note: Students will be introduced to the term *stretch* as a transformation in Lesson 13.

Pairs | 15 min

Activity 2 Lots of Challenges

Purpose: Students use their creativity and what they learned in Activity 1 to persevere in solving several different challenges. **(MTR.1.1)**

Short on time? Consider inviting students to select the challenges to complete that are most interesting to them.

1 Launch

To access connections, consider asking:

- "What does the negative sign in front of the factors tell us about this parabola's shape?" **(MTR.5.1)**
- "What about $(x + 1)$? $(x - 3)$?" **(MTR.5.1)**

To encourage creativity, remind students that there are many ways to complete every challenge. Encourage them to move freely between the five challenges and work on ones that are most interesting to them.

A **Accessibility: Memory and Attention** Consider showing the graph and equation, so students can refer to the example equation and graph as they work.

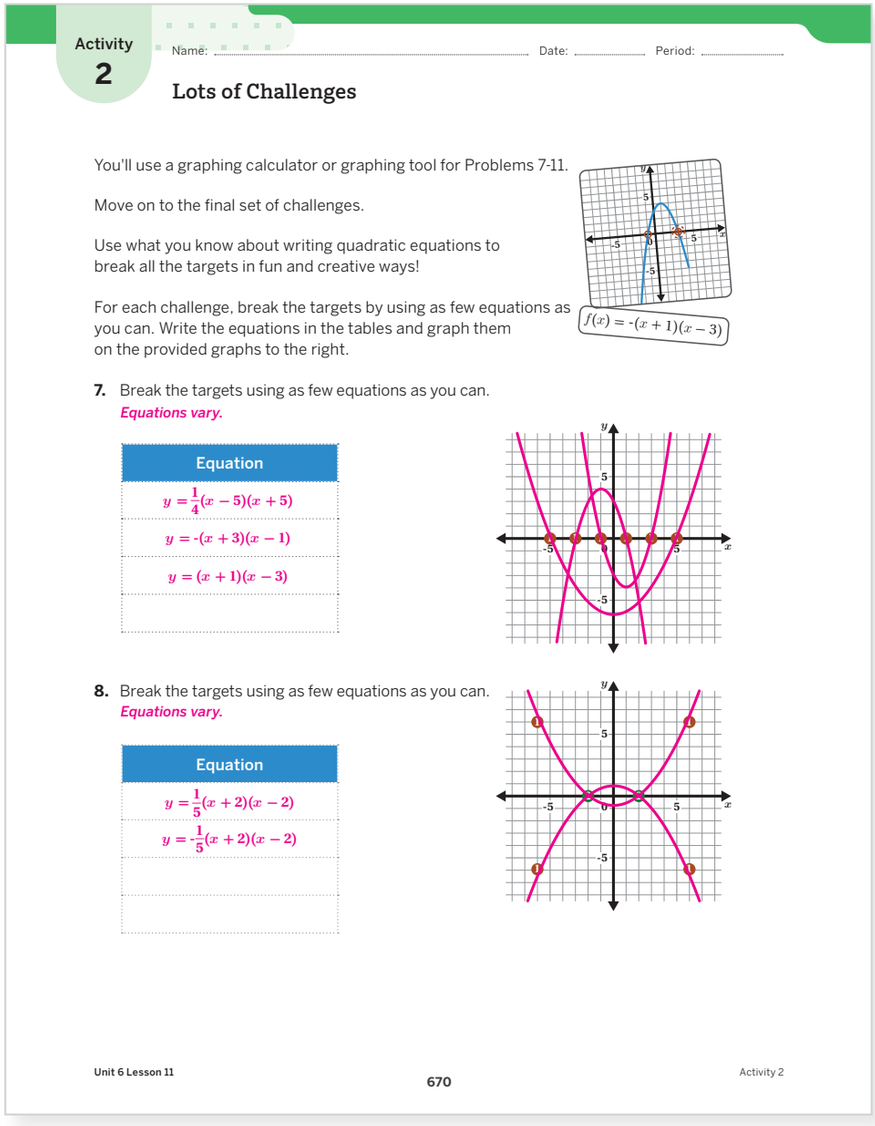
Note: This activity is intended for pairs to work at their own pace. Students do not need to complete all of the challenges to get the intended learning from this activity.

2 Monitor

Consider sharing that students can recopy earlier equations to adjust if they find it helpful.

D **Differentiation: (Support)** Encourage students to trace or describe what they want their next parabola to look like. Consider asking, "What are the x-intercepts of that parabola? Is it concave up or concave down? What does that mean about the equation?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Lots of Challenges

You'll use a graphing calculator or graphing tool for Problems 7-11.

Move on to the final set of challenges.

Use what you know about writing quadratic equations to break all the targets in fun and creative ways!

For each challenge, break the targets by using as few equations as you can. Write the equations in the tables and graph them on the provided graphs to the right.

7. Break the targets using as few equations as you can.
Equations vary.

Equation
$y = \frac{1}{4}(x - 5)(x + 5)$
$y = -(x + 3)(x - 1)$
$y = (x + 1)(x - 3)$

8. Break the targets using as few equations as you can.
Equations vary.

Equation
$y = \frac{1}{5}(x + 2)(x - 2)$
$y = -\frac{1}{5}(x + 2)(x - 2)$

Unit 6 Lesson 11 670 Activity 2

Pairs | 15 min

Activity 2 Lots of Challenges (continued)

Purpose: Students use their creativity and what they learned in Activity 1 to persevere in solving several different challenges. **(MTR.1.1)**

Short on time? Consider inviting students to select the challenges to complete that are most interesting to them.

2 Monitor

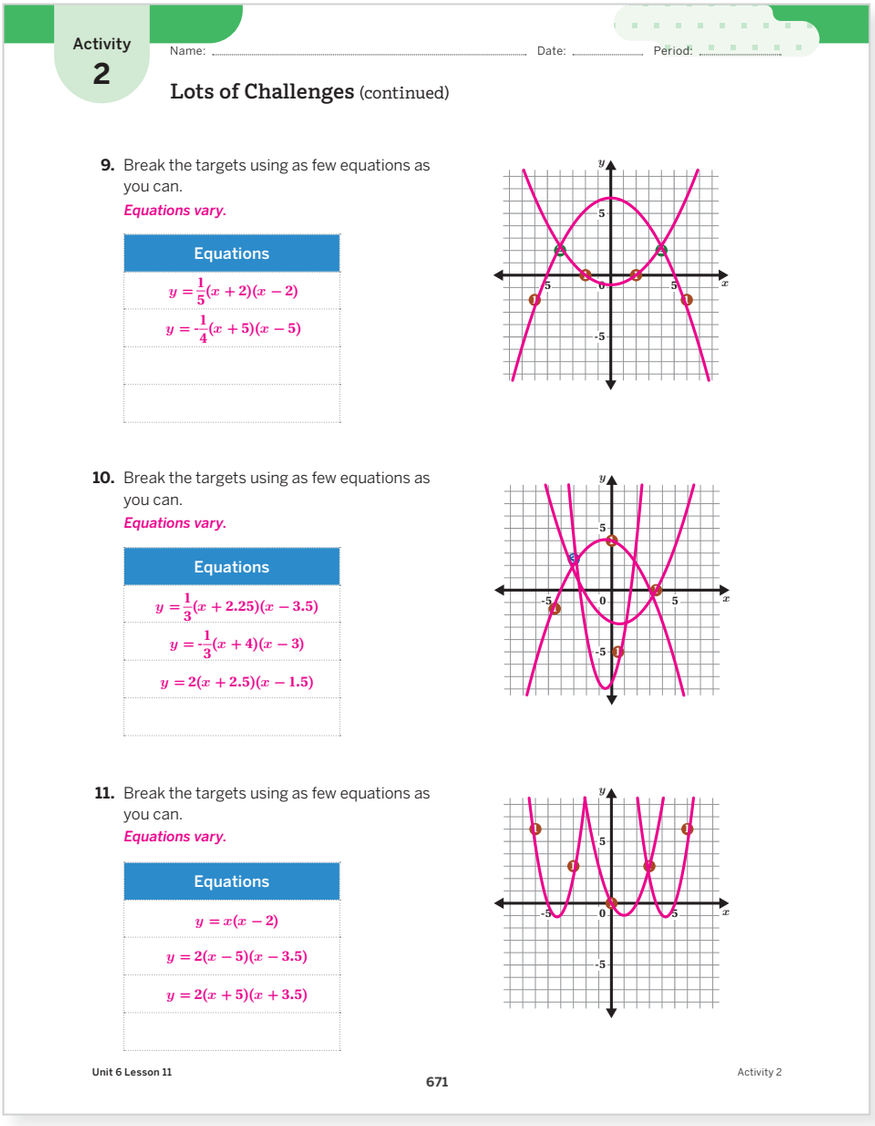
A **Accessibility: Executive Functioning** Invite students to start with one or two targets at a time, especially targets with higher numbers.

D **Differentiation: (Stretch)** Encourage students to return to a challenge and see if they can complete it using fewer parabolas.

D **Differentiation: (Stretch)** Consider asking, "Do you think it is possible to break all of these targets using one parabola? Why or why not? What other type of function could be helpful here?"

3 Connect

Note: If time allows, invite students to select one challenge they found particularly hard or exciting and consider creating a solution to it as a class. Encourage students to describe how they would start it, what equations they would write, and why.



Activity 2

Name: Date: Period:

Lots of Challenges (continued)

9. Break the targets using as few equations as you can.

Equations vary.

Equations
$y = \frac{1}{5}(x+2)(x-2)$
$y = -\frac{1}{4}(x+5)(x-5)$

10. Break the targets using as few equations as you can.

Equations vary.

Equations
$y = \frac{1}{3}(x+2.25)(x-3.5)$
$y = -\frac{1}{3}(x+4)(x-3)$
$y = 2(x+2.5)(x-1.5)$

11. Break the targets using as few equations as you can.

Equations vary.

Equations
$y = x(x-2)$
$y = 2(x-5)(x-3.5)$
$y = 2(x+5)(x+3.5)$

Unit 6 Lesson 11 671 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for writing equations in factored form to match a graph.

Invite students to respond independently, and then share their thinking with a partner.

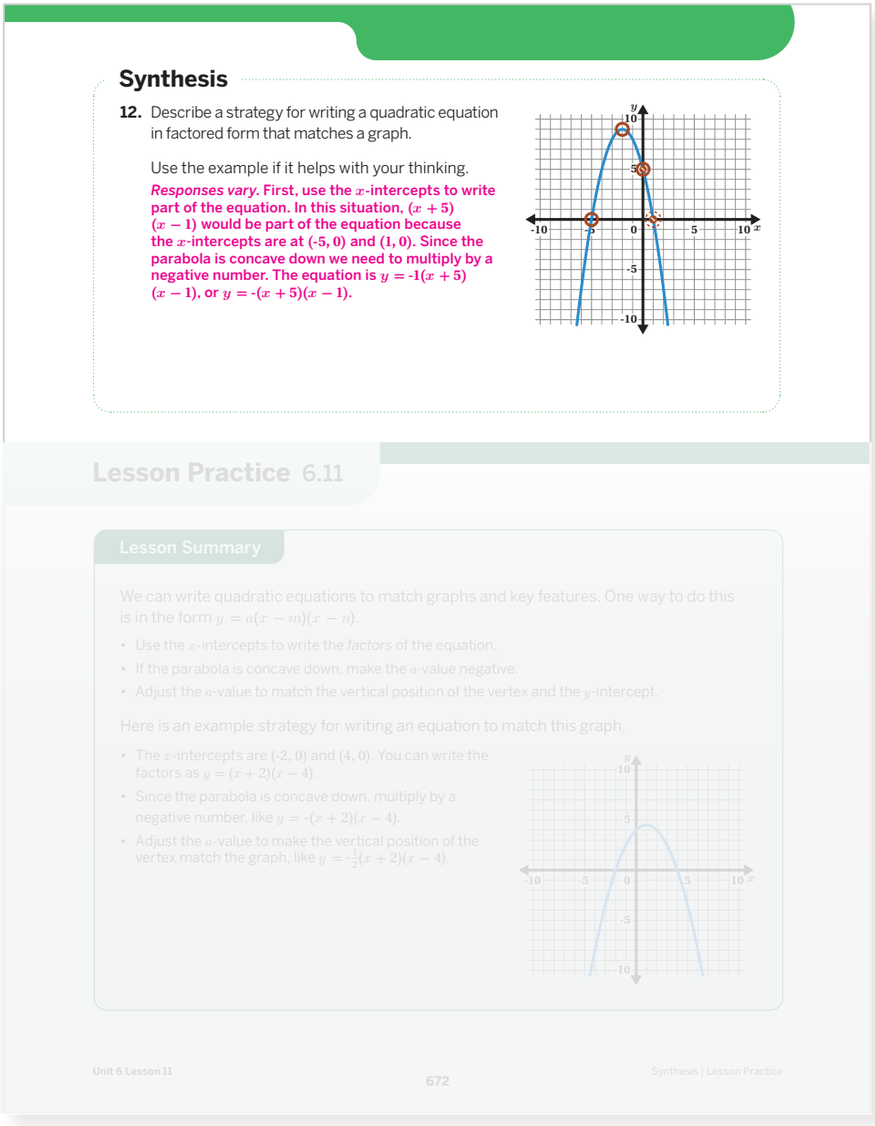
Listen for and share a variety of ideas, such as:

- Use the x-intercepts to write the factors.
- Multiply the factors by a negative if the parabola is concave down.
- Adjust the a-value so the equation matches the vertical position of the vertex.
- a-values further from 0 create narrower parabolas (vertically stretch the parabola).
- a-values closer to 0 create wider parabolas (vertically compress the parabola).

Note: Students will be introduced to the term *stretch* as a transformation in Lesson 13.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., First, _____. Then, _____.). **(Writing)**

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for writing quadratic functions in factored form from a graph along with examples. Display for students to reference and add to for the rest of the unit.



Synthesis

12. Describe a strategy for writing a quadratic equation in factored form that matches a graph.

Use the example if it helps with your thinking.

Responses vary. First, use the x-intercepts to write part of the equation. In this situation, $(x + 5)$ $(x - 1)$ would be part of the equation because the x-intercepts are at $(-5, 0)$ and $(1, 0)$. Since the parabola is concave down we need to multiply by a negative number. The equation is $y = -1(x + 5)$ $(x - 1)$, or $y = -(x + 5)(x - 1)$.

Lesson Practice 6.11

Lesson Summary

We can write quadratic equations to match graphs and key features. One way to do this is in the form $y = a(x - m)(x - n)$.

- Use the x-intercepts to write the *factors* of the equation.
- If the parabola is concave down, make the a-value negative.
- Adjust the a-value to match the vertical position of the vertex and the y-intercept.

Here is an example strategy for writing an equation to match this graph.

- The x-intercepts are $(-2, 0)$ and $(4, 0)$. You can write the factors as $y = (x + 2)(x - 4)$.
- Since the parabola is concave down, multiply by a negative number, like $y = -(x + 2)(x - 4)$.
- Adjust the a-value to make the vertical position of the vertex match the graph, like $y = -\frac{1}{2}(x + 2)(x - 4)$.

Unit 6 Lesson 11 672 Synthesis | Lesson Practice

Lesson Takeaway: One strategy for writing a quadratic equation in factored form to match a graph is to:

1. Write two *factors* to match the x-intercepts.
2. Try different a-values to match if the parabola is concave up or down, the vertical position of the vertex, and the y-intercept.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by matching graphs of parabolas to their equations in factored form.

Today's Goals

1. **Goal:** Write a quadratic function in factored form given its graph.
2. **Language Goal:** Describe how the value of a in the equation $y = a(x - m)(x - n)$ affects the graph of its corresponding parabola. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.11

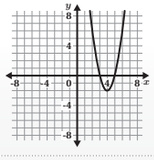
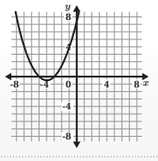
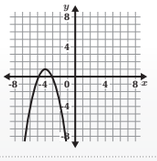
Match each graph with its equation. One equation will have no match.

A. $y = 2(x - 3)(x - 5)$

B. $y = \frac{1}{2}(x + 3)(x + 5)$

C. $y = -(x + 3)(x + 5)$

D. $y = 2(x + 3)(x + 5)$

Graph 1	Graph 2	Graph 3
		
A	B	C

Equation with no match: D

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how of writing quadratic equations:

» **Respond to Student Thinking:**

- Select a few **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Revisit the challenge from Activity 2 and invite students to try to complete it using fewer equations.
- Invite students to explore the Sub-Unit 2 Extension Activities.

Professional Learning

In this lesson, students explored how to write quadratic equations in factored form by playing a game of *Break Through*. How did playing a game support students in seeing mathematics as delightful? When in the lesson did you see evidence of students experiencing math as joy? What strategies did students use to persevere in problem solving? **(MTR.1.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe a strategy for writing a quadratic equation in factored form that matches a graph.

Use the example if it helps with your thinking.

Responses vary. First, use the x-intercepts to write part of the equation. In this situation, $(x + 5)$ $(x - 1)$ would be part of the equation because the x-intercepts are at $(-5, 0)$ and $(1, 0)$. Since the parabola is concave down we need to multiply by a negative number. The equation is $y = -1(x + 5)$ $(x - 1)$, or $y = -(x + 5)(x - 1)$.

Lesson Practice 6.11

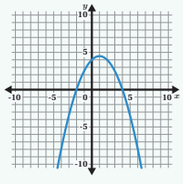
Lesson Summary

We can write quadratic equations to match graphs and key features. One way to do this is in the form $y = a(x - m)(x - n)$.

- Use the x-intercepts to write the *factors* of the equation.
- If the parabola is concave down, make the a-value negative.
- Adjust the a-value to match the vertical position of the vertex and the y-intercept.

Here is an example strategy for writing an equation to match this graph.

- The x-intercepts are $(-2, 0)$ and $(4, 0)$. You can write the factors as $y = (x + 2)(x - 4)$.
- Since the parabola is concave down, multiply by a negative number, like $y = -(x + 2)(x - 4)$.
- Adjust the a-value to make the vertical position of the vertex match the graph, like $y = -\frac{1}{2}(x + 2)(x - 4)$.



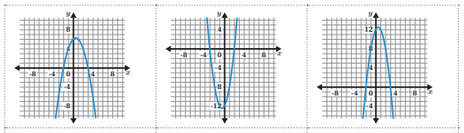
Lesson Practice 6.11

Name: _____ Date: _____ Period: _____

1. Match each equation to the graph it represents. One equation will have no match.

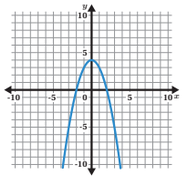
$y = -(x + 2)(x - 3)$ $y = 2(x + 3)(x - 2)$

$y = -2(x + 2)(x - 3)$ $y = (x + 2)(x - 3)$



$y = -(x + 2)(x - 3)$ | $y = 2(x + 3)(x - 2)$ | $y = -2(x + 2)(x - 3)$

Problems 2–3: Here is the graph of $y = -1(x + 2)(x - 2)$. Change the equation so the vertex goes through:



2. $(0, 8)$

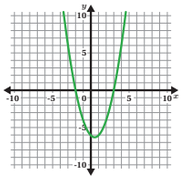
$y = -2(x + 2)(x - 2)$

3. $(0, -2)$

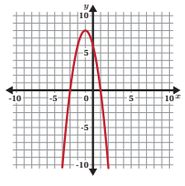
$y = \frac{1}{2}(x + 2)(x - 2)$

Problems 4–5: Write an equation to match each graph.

4. Equation: $y = (x + 2)(x - 3)$



5. Equation: $y = -2(x - 1)(x + 3)$



Lesson Practice 6.11

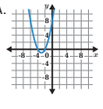
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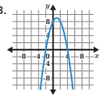
6. Write the equations of three different parabolas that have the same vertex but different x-intercepts. *Responses vary.*

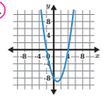
Equation 1	Equation 2	Equation 3
$y = (x + 2)(x - 2)$	$y = \frac{1}{4}(x + 4)(x - 4)$	$y = 4(x + 1)(x - 1)$

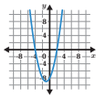
Test Practice

7. Which graph shows the function $y = (x - 4)(x + 2)$?

A. 

B. 

C. 

D. 

Spiral Review

Problems 8–9: Here are two linear functions: $f(x)$ and $h(x)$.

$f(x) = 3x + 2$

x	$h(x)$
1	5
2	9
3	13

8. Which function has a greater y-intercept? Circle one.

$f(x)$ $h(x)$

9. Which function has a greater slope? Circle one.

$f(x)$ $h(x)$

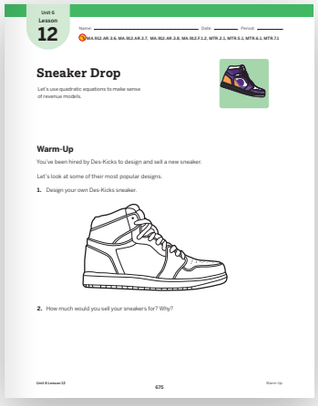
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.912.AR.3.7
	2–3	2	MA.912.AR.3.7 MA.912.F.2.1
	4–5	2	MA.912.AR.3.4 MA.912.AR.3.7
	6	2	MA.912.AR.3.4 MA.912.AR.3.7
Test Practice	7	2	MA.912.AR.3.7
Spiral Review			
Fluency	8–9	1	MA.912.F.1.5

Unit 6

Lesson

12


Unit 6
Lesson
12
Name: Date: Period:
MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1
Sneaker Drop
Let's use quadratic equations to make sense of revenue models.
Warm-Up
You've been hired by Des-Kicks to design and sell a new sneaker.
Let's look at some of their most popular designs.
1. Design your own Des-Kicks sneaker.
2. How much would you sell your sneakers for? Why?
Unit 6 Lesson 12
675
Warm-Up

Sneaker Drop

Exploring Revenue

Let's use quadratic equations to make sense of revenue models.

Learning Outcomes

Today's Goals

1. **Goal:** Compare key features of quadratic functions represented in tables, equations, and graphs.
2. **Language Goal:** Interpret key features of quadratic functions in context. **(Writing, Speaking, and Listening)**

Students use what they have learned about quadratic functions to make sense of revenue models. They consider how changing the price of an item affects the number of customers willing to buy it, and thus the maximum revenue a product can generate. In this lesson, we define *revenue* as the amount of money generated from selling an item (or the price times the number of items sold).

Coherence

Prior Learning

In Lesson 7, students used quadratic functions to model the flight of a ball launched by different robots. Students compared quadratic models by their key features.

Future Learning

In Lesson 15, students will build on their knowledge of about quadratic functions to predict diseases.

Connections

- Within the context of designing and selling a new style of sneaker, students apply their understanding of quadratic equations to compare different companies' revenue models. (real world, mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.6
Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

MA.912.AR.3.8
Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.

MA.912.F.1.2
Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

ELA Expectation: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AF.1.3	MA.912.AR.5.3

Vocabulary

Additional Vocabulary

revenue

Lesson at a Glance

~ 45 min

MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2

Warm-Up

Independent | 5 min

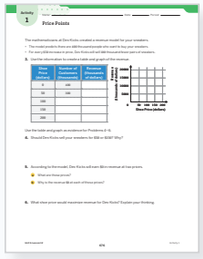
Students design a sneaker and think about pricing it to make sense of the context for this lesson.



Activity 1

Pairs | 15 min

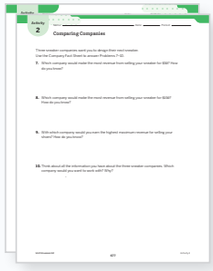
Students create representations of a revenue model and use the key features of a quadratic function to answer questions about revenue. **(MTR.2.1)**



Activity 2

Pairs | 15 min

Students analyze properties of tables, graphs, and equations of quadratic functions to compare the revenue models for three sneaker companies.
MLR7: Compare and Connect **(MTR.2.1)**



Synthesis

Whole Class | 5 min

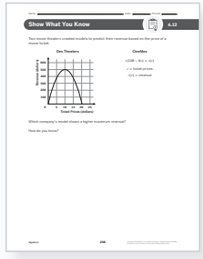
Students synthesize their understanding of how to determine the maximum revenue from an equation, a graph, and a table.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which of two models has the higher maximum revenue.



Prep Checklist

Prepare the Activity 2 Sheet.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet
  - Calculator

Independent | 5 min

Warm-Up

Purpose: Students design a sneaker and think about pricing it to make sense of the context for this lesson.

1 Launch

Discuss: The sneaker design in the student edition.

ELL **English Language Learners** Invite students to share what they know about sneakers. Consider sharing that *sneakers* may be a regional term in some areas, and that students may know other ways of referring to shoes like these (e.g., "tennis shoes" or "gym shoes"). Consider inviting students to share the word(s) for these shoes in their primary language and record these words publicly for the class to reference. **(Reading and Listening)**

To encourage creativity, invite students to design and draw a sneaker that matches their personality or that they would purchase, or one they think no one else will design.

Invite students to discuss Problem 2 with a partner after they take some time to design their sneaker.

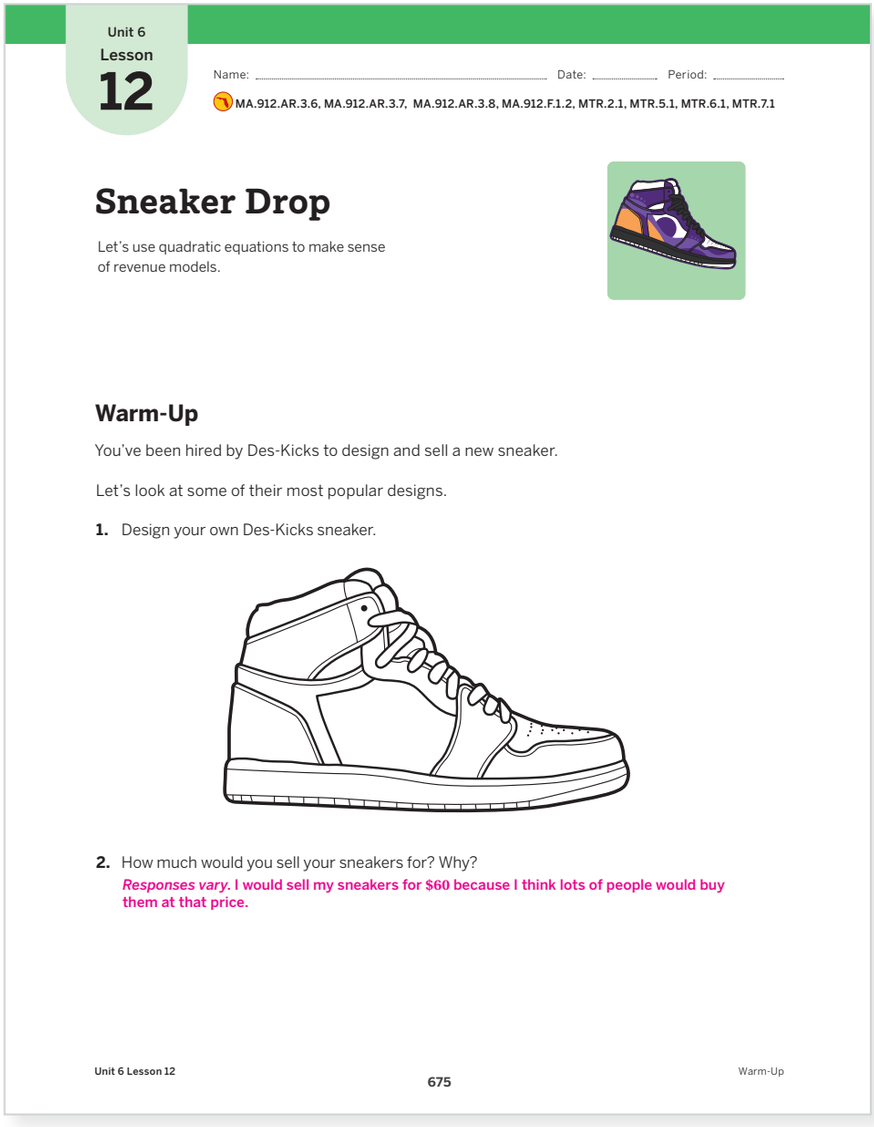
A **Accessibility: Affective Functioning** Consider setting a timer or playing music while students look for a partner to support students with collaboration.

2 Connect

Invite students to share their responses to Problem 2.

Note: If it does not come up naturally, consider asking:

- "Why might you not want to charge too low a price for your sneaker? Too high a price?" If the price is too low, it might not cover the cost of the materials, shipping, paying workers fairly, etc. If the price is too high, people might not buy the sneakers at all because they are too expensive.
- "How do you know when an item is priced too high? Too low?" I can decide based on how important the item is to me. If I think it's really important, I might pay more for it. If it's not as important to me, I might only buy it at a lower price.



Unit 6
Lesson
12

Name: _____ Date: _____ Period: _____

MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

Sneaker Drop

Let's use quadratic equations to make sense of revenue models.

Warm-Up

You've been hired by Des-Kicks to design and sell a new sneaker.

Let's look at some of their most popular designs.

1. Design your own Des-Kicks sneaker.

2. How much would you sell your sneakers for? Why?

Responses vary. **I would sell my sneakers for $60 because I think lots of people would buy them at that price.**

Unit 6 Lesson 12 675 Warm-Up

Pairs | 15 min

Activity 1 Price Points

Purpose: Students create representations of a revenue model and use the key features of a quadratic function to answer questions about revenue. **(MTR.2.1)**

1 Launch

Consider recreating the table from the student page and share that the class will be going through a process to simulate what a company might do to understand how much *revenue* they will generate if they charge different prices for a product.

Invite students to raise their hands and to keep their hands raised if they think an average customer would buy their favorite sneakers for $5. Then:

- Count the number of hands raised and write it in the "Number of Customers" column.
- Repeat this process for each price.

Consider sharing a definition for the term *revenue*, the amount of money generated from selling an item (price times the number of items sold). Encourage students to use calculators to help them determine the revenue based on the shoe price and number of customers.

Note: If it does not come up naturally, consider asking, "Why do you think the revenue does not consistently go up as the price increases?"

2 Monitor

Encourage students to complete Problems 3–6 with a partner, discussing each question as they go.

Look for students who make connections between the questions and key features of the parabola. **(MTR.2.1)**

A **Accessibility: Conceptual Processing** Consider inviting students to think about how this situation is similar to and different from the class's simulation during the Launch.

D **Differentiation: (Stretch)** Consider asking, "What is the domain of this revenue model? The range? Why are the domain and range limited to these values?"

3 Connect

Display the table and the graph.

Invite students from several different pairs to share their thinking for Problems 4–6.

To reveal the Key Takeaway, invite students to share strategies for identifying the key features of the quadratic revenue model. Consider asking:

- "How did you use the table or graph to identify the x-intercepts? The vertex?" **(MTR.2.1)**
- "How did you use the table or graph to determine the maximum revenue?" **(MTR.2.1)**

Activity 1

Name: _____ Date: _____ Period: _____

Price Points

The mathematicians at Des-Kicks created a revenue model for your sneakers.

- The model predicts there are 400 thousand people who want to buy your sneakers.
- For every $50 increase in price, Des-Kicks will sell 100 thousand fewer pairs of sneakers.

3. Use the information to create a table and graph of the revenue.

Shoe Price (dollars)	Number of Customers (thousands)	Revenue (thousands of dollars)
0	400	0
50	300	15,000
100	200	20,000
150	100	15,000
200	0	0



Use the table and graph as evidence for Problems 4–6.

4. Should Des-Kicks sell your sneakers for $50 or $150? Why?

Responses vary.

- I would sell my sneakers for $50 because you make the same amount of money and more people could afford the sneakers.
- I would rather sell my sneakers for $150 because that's how much most luxury sneakers cost.

5. According to the model, Des-Kicks will earn $0 in revenue at two prices.

a. What are those prices? $0 and $200

b. Why is the revenue $0 at each of those prices?

Explanations vary. Selling the shoes for $0 means that no matter how many shoes you sell, you will make $0 in revenue. If the shoes cost $200, then no one is buying the shoes.

6. What shoe price would maximize revenue for Des-Kicks? Explain your thinking.

Explanations vary. The graph is a parabola with a vertex at (100, 20000). According to the model, the most revenue you can make is $20,000,000 when you sell the shoes for $100.

Unit 6 Lesson 12 676 Activity 1

Key Takeaway: A revenue model represents the amount of money generated from selling goods or services at various prices, and is often modeled by a quadratic function. For revenue models, the vertex represents the maximum revenue at a specific price. The x-intercepts represent the prices for the item that will earn the company $0 in revenue.

Pairs | 15 min

Activity 2 Comparing Companies

Purpose: Students analyze properties of tables, graphs, and equations of quadratic functions to compare the revenue models for three sneaker companies. **(MTR.2.1)**

Short on time? Consider omitting Problems 7–8 and having students only calculate based on their chosen price.

1 Launch

Display the information about each company.

Distribute the Activity 2 Sheet to each pair.

To access connections, invite students to look at the company fact sheet in their pairs and consider asking:

- "Which shoe do you like the best?"
- "Which do you think will earn the most revenue?"
- "What does $(400 - 2x)$ represent in Des-Kicks's revenue model?" The number of pairs of shoes sold
- "Why does Des-Kicks's revenue model multiply x and $(400 - 2x)$?" Revenue is the price, x, times number of pairs of shoes sold, $(400 - 2x)$.

2 Monitor

Listen for students who consider revenue and other factors when making their decision.

A **Accessibility: Executive Functioning** Invite students to select one company at a time and determine that company's revenue for a $50 sneaker, a $150 sneaker, and the maximum revenue the company could make.

ELL **English Language Learners** As students share in partner discussions, provide sentence frames such as:

- The company with the most revenue is _____ because _____.
- I would work with _____ because _____. **(Writing and Speaking)**

Note: Some students may notice that Des-Kicks uses the same model in Activities 1 and 2. Encourage these students to use the equation to make connections to what they saw in the graph and table in Activity 1.

Encourage students to make decisions about rounding and reporting revenue values at a level of precision that makes sense for the model (e.g., rounding to the nearest thousand or million). Consider sharing that there can be a lot of uncertainty in calculating revenue in real-world situations, so rounding can support making sense of quantities without being exact.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Comparing Companies

Three sneaker companies want you to design their next sneaker.
Use the Company Fact Sheet to answer Problems 7–10.

7. Which company would make the most revenue from selling your sneaker for $50? How do you know?

ShoeMos. *Explanations vary.* Des-Kicks will make $15,000,000, ShoeMos will make around $17,500,000, and Green Feet will make $8,000,000 in revenue for a sneaker priced at $50.

8. Which company would make the most revenue from selling your sneaker for $150? How do you know?

Des-Kicks. *Explanations vary.* Des-Kicks will make $15,000,000, ShoeMos will make around $7,500,000, and Green Feet will make $12,000,000 in revenue for a sneaker priced at $150.

9. With which company would you earn the highest maximum revenue for selling your shoes? How do you know?

ShoeMos. *Explanations vary.* ShoeMos has the highest maximum revenue, which is more than $20,000,000. Des-Kicks's maximum revenue is exactly $20,000,000 and Green Feet's maximum revenue is closer to $12,000,000.

10. Think about all the information you have about the three sneaker companies. Which company would you want to work with? Why?

Responses vary.

- I would work with Des-Kicks because I like their style of shoes the most, and I could make $20,000,000, which is close to the maximum from ShoeMos.
- I would work with ShoeMos because you make the most revenue for the lowest shoe price.
- I would work with Green Feet because they have the best sustainability rating. I can still make $12,000,000 in revenue if I sell the shoe for $100, which is a lot of money.

Unit 6 Lesson 12 677 Activity 2

Pairs | 15 min

Activity 2 Comparing Companies (continued)

Purpose: Students analyze properties of tables, graphs, and equations of quadratic functions to compare the revenue models for three sneaker companies. **(MTR.2.1)**

Short on time? Consider omitting Problems 7–8 and having students only calculate based on their chosen price.

2 Monitor

Note: Refer to the guidance on the previous page to support students as they work on this activity.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore further to complete this optional task. Encourage them to discuss their thinking with a partner.

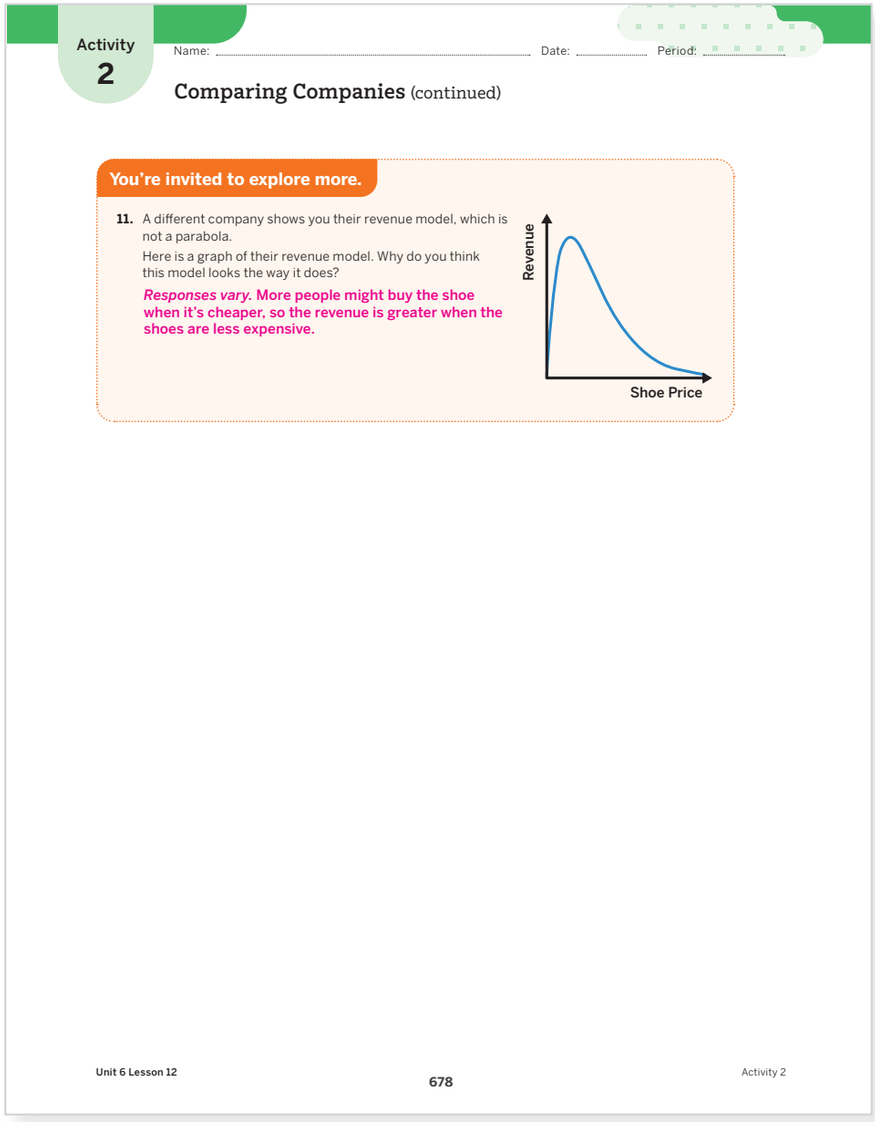
3 Connect

Discuss each company's revenue and the company students chose to work with.

MLR **MLR7: Compare and Connect** Invite students to analyze each revenue model, then find ways in which the models are different (compare) and alike (connect) between the models.

To reveal the Key Takeaway, consider asking:

- "How did you decide which company would make the most revenue from a sneaker priced at $50?"
- "How did you use the equation to figure out the most revenue DesKicks could make? What about the graph and ShoeMos? The table and Green Feet?" **(MTR.2.1)**



Activity 2

Name: Date: Period:

Comparing Companies (continued)

You're invited to explore more.

11. A different company shows you their revenue model, which is not a parabola.

Here is a graph of their revenue model. Why do you think this model looks the way it does?

Responses vary. More people might buy the shoe when it's cheaper, so the revenue is greater when the shoes are less expensive.

Unit 6 Lesson 12 678 Activity 2

Key Takeaway: You can use tables, graphs, or equations to determine the maximum revenue for a quadratic model. In a graph, the y-value of the vertex is the maximum revenue. In a table, the maximum revenue might appear as the y-value of the row that changes from increasing to decreasing (or it may be between two values). In an equation, you can use symmetry to determine the x-value in the middle of the two x-intercepts and evaluate for the y-value at that point, which represents the maximum revenue. Each representation can be used to determine the same key feature in different ways.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to determine the maximum revenue from an equation, a graph, and a table.

Invite students to respond independently, and then share their thinking with a partner.

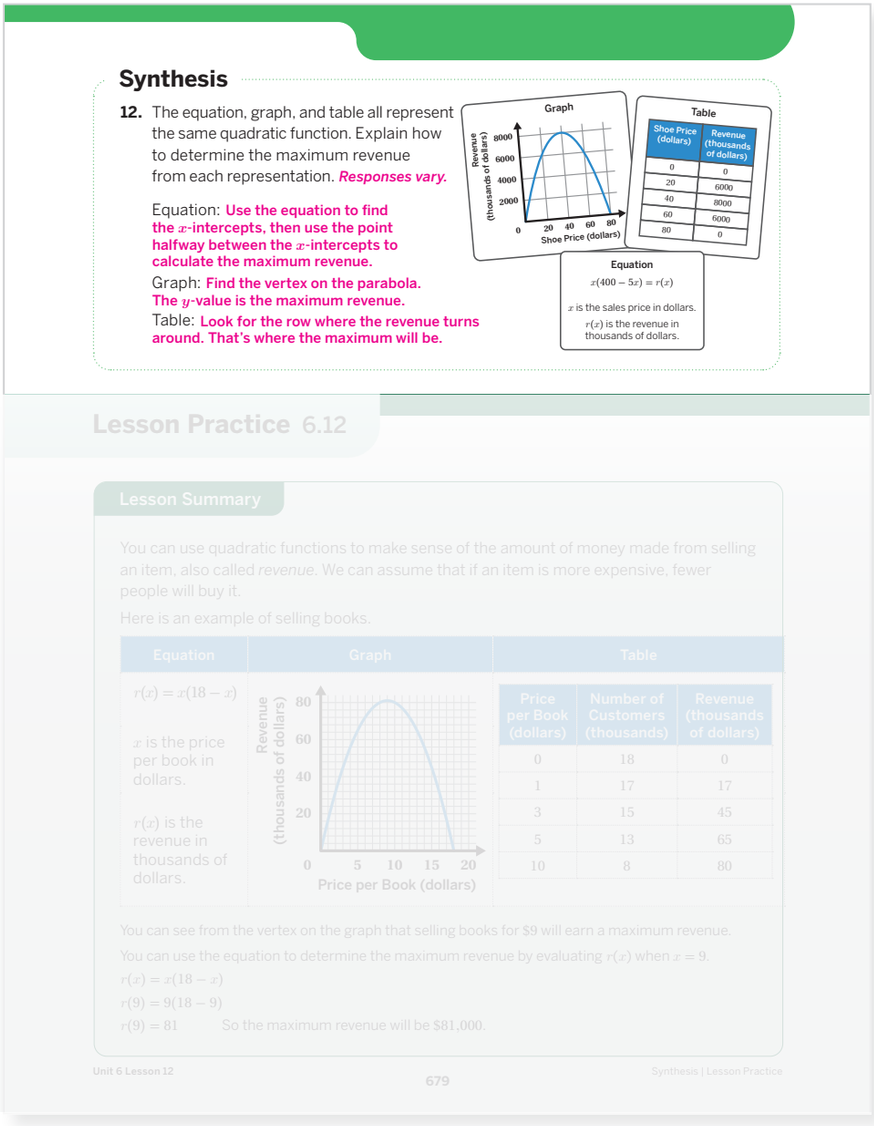
Listen for and share a variety of ideas, such as:

- Determine the vertex of the parabola from its graph. The y-coordinate of the vertex is the maximum revenue.
- Find the row in the table where the revenue values change from increasing to decreasing.
- Use the equation to determine the x-intercepts, then use the x-value halfway between the two x-intercepts to calculate the maximum revenue.

Create an anchor chart using student thinking and examples for each of the different representations.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas, language, and examples for determining and interpreting the maximum from a revenue model from tables, graphs, and equations as they share during the discussion. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. The equation, graph, and table all represent the same quadratic function. Explain how to determine the maximum revenue from each representation. *Responses vary.*

Equation: Use the equation to find the x-intercepts, then use the point halfway between the x-intercepts to calculate the maximum revenue.

Graph: Find the vertex on the parabola. The y-value is the maximum revenue.

Table: Look for the row where the revenue turns around. That's where the maximum will be.

Graph

Revenue (thousands of dollars): 8000, 6000, 4000, 2000, 0

Shoe Price (dollars): 20, 40, 60, 80

Table

Shoe Price (dollars)	Revenue (thousands of dollars)
0	0
20	6000
40	8000
60	6000
80	0

Equation

$x(400 - 5x) = r(x)$

x is the sales price in dollars.

$r(x)$ is the revenue in thousands of dollars.

Lesson Practice 6.12

Lesson Summary

You can use quadratic functions to make sense of the amount of money made from selling an item, also called *revenue*. We can assume that if an item is more expensive, fewer people will buy it.

Here is an example of selling books.

Equation	Graph	Table
$r(x) = x(18 - x)$ x is the price per book in dollars. $r(x)$ is the revenue in thousands of dollars.	Revenue (thousands of dollars): 80, 60, 40, 20, 0 Price per Book (dollars): 0, 5, 10, 15, 20	Price per Book (dollars) / Number of Customers (thousands) / Revenue (thousands of dollars) 0 / 18 / 0 1 / 17 / 17 3 / 15 / 45 5 / 13 / 65 10 / 8 / 80

You can see from the vertex on the graph that selling books for \$9 will earn a maximum revenue.

You can use the equation to determine the maximum revenue by evaluating $r(x)$ when $x = 9$.

$r(x) = x(18 - x)$

$r(9) = 9(18 - 9)$

$r(9) = 81$ So the maximum revenue will be \$81,000.

Unit 6 Lesson 12 679 Synthesis | Lesson Practice

Lesson Takeaway: The revenue that companies make from selling goods or services can be represented with revenue models. Quadratic revenue models allow us to analyze and compare these outcomes using the key features of quadratic functions represented in a graph, equation, or table.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which of two models has the higher maximum revenue.

Today's Goals

1. **Goal:** Compare Goal: Compare key features of quadratic functions represented in tables, equations, and graphs.
2. **Language Goal:** Interpret key features of quadratic functions in context. **(Writing, Speaking, and Listening)**

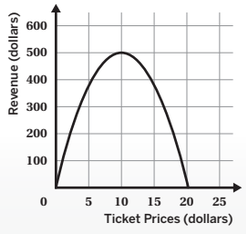
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.12

Two movie theaters created models to predict their revenue based on the price of a movie ticket.

Des Theaters



CineMos

$x(120 - 6x) = r(x)$

x = ticket prices

$r(x)$ = revenue

Which company's model shows a higher maximum revenue? CineMos

How do you know?

Explanations vary. The graph of Des Theaters shows that the maximum revenue is $500. I can use the equation for CineMos to find the x-intercepts at 0 and 20, which means the maximum revenue is when ticket prices are $10. The maximum revenue is $600.

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify key features of quadratic functions:

» **Respond to Student Thinking:**

- Revisit **Lesson 10, Activity 1** as a class and invite students to share strategies for determining x-intercepts.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.
- Invite students to explore the Sub-Unit 2 Extension Activities.

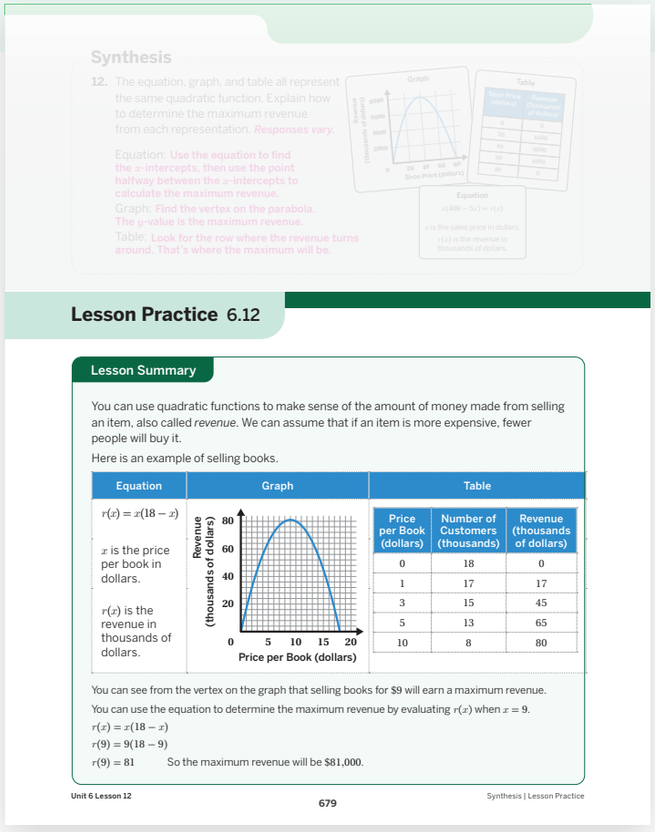
Professional Learning

Today students explored the topic of revenue models. What aspects of revenue models were students able to leverage to make decisions? How can you continue to support students in recognizing the uses and limitations of mathematical models for a situation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

12. The equation, graph, and table all represent the same quadratic function. Explain how to determine the maximum revenue from each representation. *Responses vary.*

Equation: Use the equation to find the x-intercepts, then use the point halfway between the x-intercepts to calculate the maximum revenue.

Graph: Find the vertex on the parabola. The y-value is the maximum revenue.

Table: Look for the row where the revenue turns around. That's where the maximum will be.

Lesson Practice 6.12

Lesson Summary

You can use quadratic functions to make sense of the amount of money made from selling an item, also called *revenue*. We can assume that if an item is more expensive, fewer people will buy it.

Here is an example of selling books.

Equation	Graph	Table
$r(x) = x(18 - x)$ x is the price per book in dollars. $r(x)$ is the revenue in thousands of dollars.		Price per Book (dollars) / Number of Customers (thousands) / Revenue (thousands of dollars) 0 / 18 / 0 1 / 17 / 17 3 / 15 / 45 5 / 13 / 65 10 / 8 / 80

You can see from the vertex on the graph that selling books for \$9 will earn a maximum revenue.

You can use the equation to determine the maximum revenue by evaluating $r(x)$ when $x = 9$.

$r(x) = x(18 - x)$

$r(9) = 9(18 - 9)$

$r(9) = 81$ So the maximum revenue will be \$81,000.

Lesson Practice 6.12

Name: ..... Date: ..... Period: .....

1. Match each function with its y-intercept. There will be one function with no match.

$f(x) = 4 + 2.5x - 2x^2$ $h(x) = 4x^2 - 2.5x + 2$

$g(x) = 2.5 + 4x - 2x^2$ $j(x) = 2.5x^2 + 4x - 2$

(0, 2.5)	(0, -2)	(0, 4)
$g(x) = 2.5 + 4x - 2x^2$	$j(x) = 2.5x^2 + 4x - 2$	$f(x) = 4 + 2.5x - 2x^2$

2. Here is a function: $g(x) = (x - 3)^2$. Determine if each statement is true or false, or if there is not enough information.

The y-intercept of g is at $(0, 3)$. True (False) Not enough information

An x-intercept of g is at $(3, 0)$. (True) False Not enough information

The vertex of g is at $(3, 0)$. (True) False Not enough information

Problems 3–6: Store A sells a video game called DesMan. The company asked customers how much they would pay for the video game.

3. Complete the table.

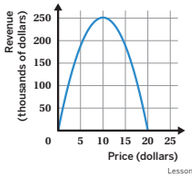
Price (\$)	Number of Customers	Revenue (\$)
10	15,000	150,000
25	7,500	187,500

Store A used your table to create a revenue model: $r(p) = p(20000 - 500p)$. p represents the price of the video game.

4. At what prices will Store A make \$0 in revenue?
\$0 and \$40

5. At what price will Store A make the most money?
\$20

6. Store B also sells DesMan. This graph models their revenue.



Which store has a higher maximum revenue? Explain your thinking.

Store B. *Explanations vary.* The vertex of the Store B graph is (10, 250). This means that Store B can make a maximum revenue of \$250,000. I found the maximum revenue of Store A by finding the revenue when the price of the game is \$20. The maximum revenue of Store A is \$200,000.

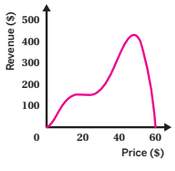
Lesson Practice 6.12

Name: ..... Date: ..... Period: .....

7. Create a revenue graph for a mystery product that:

- Makes no revenue when priced at \$0 or more than \$60.
- Has a maximum profit that is *not* halfway between the x-intercepts.

Graphs vary.



Test Practice

8. One possible revenue model for the mysterious product is $r(p) = -60p^2 + 3600p$.

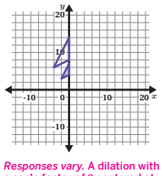
What is the revenue if the price is \$10?

A. \$35,400
B. \$72,000
(C.) \$30,000
D. \$36,000

Spiral Review

Problems 9–10: Describe a sequence of transformations that moves the shaded figure onto the unshaded figure.

9.



Responses vary. **A dilation with a scale factor of 2 centered at the origin.**

10.



Responses vary. **Reflect across the vertical line through the center of the grid.**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.AR.3.7, MA.912.F.1.2
	4–6	2	MA.912.AR.3.7, MA.912.F.1.2
	7	3	MA.912.AR.3.7, MA.912.F.1.2
Test Practice	8	3	MA.912.F.1.2
Spiral Review			
Fluency	9–10	2	MA.8.GR.2.2

Assess and Respond

Sub-Unit Quiz

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determine if relationships are quadratic or not	Lessons 2–3	2	MA.912.F.1.8 MTR.5.1
2	Make connections between quadratic equations in factored form and their graphs	Lesson 8–11	2	MA.912.AR.3.7
3a	Sketch parabolas when given key features	Lesson 6	1	MA.912.AR.3.7 MTR.5.1
3b	Write quadratic equations given a graph	Lesson 11	2	MA.912.AR.3.7 MTR.3.1
4a–b	Compare key features of different quadratic functions in context	Lessons 5, 7	2	MA.912.AR.3.6–3.8 MTR.6.1
5a–c	Use quadratic models to solve problems in context	Lessons 4,12	3	MA.912.AR.3.6 MTR.5.1

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 6

1. Select *all* the relationships that are quadratic.

☐ A.

x	y
0	1
1	4
2	7
3	10
4	13

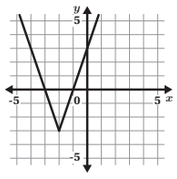
☑ B.

x	y
0	1
1	4
2	9
3	16
4	25

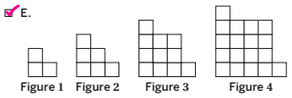
☑ C.

x	y
0	1
1	4
2	10
3	19
4	31

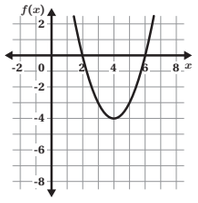
☐ D.



☑ E.



2. Here is the graph of $f(x)$.



Which could be its equation?

A. $f(x) = (x - 2)(x - 6)$

B. $f(x) = (-x - 2)(x + 6)$

C. $f(x) = (-x + 2)(x - 6)$

D. $f(x) = (x + 2)(x + 6)$

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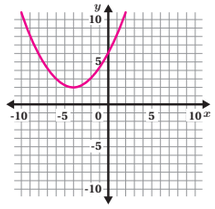
Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 6

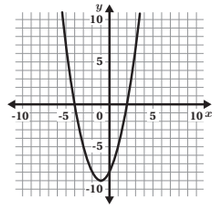
3. a Sketch a parabola that has both:

- a line of symmetry at $x = -4$.
- a y-intercept at $(0, 6)$. *Sketches vary.*



b Write a quadratic function to match this graph.

$y =$ $(x + 4)(x - 2)$ (or equivalent)



4. Yolanda and Jordan launched stomp rockets at a playground.

The heights of the rockets can be modeled by quadratic functions, $f(x)$ and $g(x)$, where x is the time in seconds since the rocket was launched.

a Whose rocket traveled higher? Circle one.

Yolanda's (Jordan's) Both rockets traveled to the same height

Explain your thinking.

Explanations vary. The highest point the rocket travels to is the maximum of the parabola. The maximum of $f(x)$ is less than 90 m, and the maximum of $g(x)$ is more than 90 m.

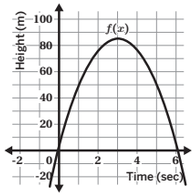
b Whose rocket was in the air for *less* time? Circle one.

(Yolanda's) Jordan's Both rockets were in the air for the same amount of time

Explain your thinking.

Explanations vary. The rocket's time in the air is represented by the second x-intercept. The second x-intercept of $f(x)$ is between 6 and 7, and the second x-intercept of $g(x)$ is 7.

Yolanda's Rocket



Jordan's Rocket

Time (sec), x	Height (m), $g(x)$
0	0
1	45
2	75

D Differentiation (Sub-Unit Quiz)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Justify whether a function is linear, quadratic, exponential, or none of these. **(Lessons 1–3)**	1	**Support** • **Teacher Move:** Revisit Lesson 3, Problems 5–9 (*Sorting Relationships*) to discuss strategies for classifying functions in multiple representations. **Strengthen** • **Practice Day 1:** Cards 8, 10 **Stretch** • **DOK3 Practice:** Lesson 3, 7
Identify and interpret key features of quadratic functions in graphs and tables. **(Lessons 4–7, 12)**	3a, 4	**Strengthen** • **Practice Day 1:** Cards 7, 11 • Practice Day 2: Hexagon Sheet **Stretch** • **You're invited to explore more:** Lesson 5 (*Stomp Rockets*)
Determine key features and graph quadratic functions from standard and factored form equations. **(Lessons 7–10, 12)**	3, 5	**Support** • **Teacher Move:** Review strategies for graphing such as creating a table or using the structure of the equation to identify key features. **Strengthen** • **Repeated Challenges:** Lesson 11 (*Parabola Zapper*) • **Practice Day 1:** Cards 2, 4, 6, 9 **Stretch** • **DOK3 Practice:** Lessons 8–10
Write quadratic equations from a graph, description, or criteria. **(Lesson 11)**	2, 3b	**Support** • **Teacher Move:** During Lesson 16, revisit students strategies for writing an equation in factored form. **Strengthen** • **Practice Day 1:** Cards 4, 12 **Stretch** • **DOK3 Practice:** Lessons 10–12

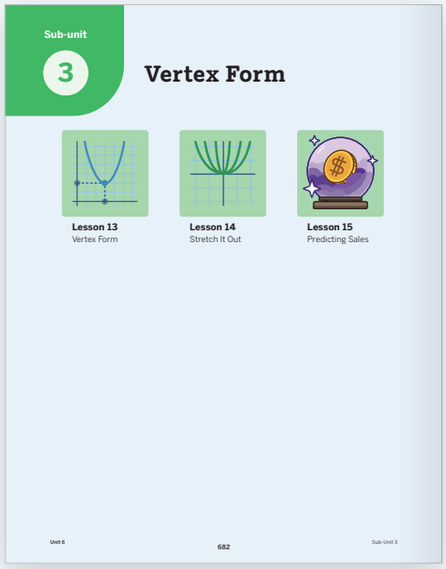
Sub-Unit 3

Vertex Form

Sub-Unit 1 Goals

- **Goal:** Create quadratic models to solve problems in context.
- **Language Goal:** Describe the effect of transformations on key features of a parabola and its equation in vertex form.

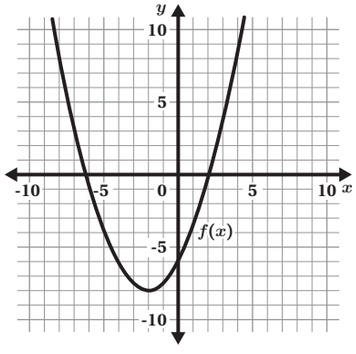
In this sub-unit, students transform parabolas with translations and vertical **stretches** to write quadratic functions in **vertex form**. They use quadratic functions in multiple forms to model real-world situations, make predictions and decisions, and solve problems.



Math That Matters Most

Concept or Skill	For example . . .
Describe the effect of vertical and horizontal translations and vertical stretches on the graph of a quadratic function	How has the equation $f(x) = -2(x - 1)^2 - 4$ been transformed from $y = x^2$? • The -2 represents a vertical stretch by a factor of 2 that also makes the parabola concave down. • The -1 represents a translation right 1 unit. • The -4 represents a translation down 4 units.
Create quadratic models and interpret their parts in context	Write an expression to represent the revenue City Roots Collective could make at any price of rent based on the data in the table.

Price of Rent ($)	Number of Rented Units	Revenue ($)
1,000	8,000	8,000,000
1,100	7,500	8,250,000
1,200	7,000	8,400,000
1,300	6,500	8,450,000
1,400	6,000	8,400,000
x	$13000 - \frac{500}{100}x$	$x\left(13000 - \frac{500}{100}x\right)$

Concept or Skill	For example . . .
Write quadratic functions in vertex and factored forms given their vertex and/or intercepts	Write an equation in vertex form and an equation in factored form for $f(x)$. Explain your thinking.  I can write the equation in factored form using the x-intercepts. $f(x) = \frac{1}{2}(x + 6)(x - 2)$ I can write the equation in vertex form using the coordinates of the vertex. $f(x) = \frac{1}{2}(x + 2)^2 - 8$

Unit 6

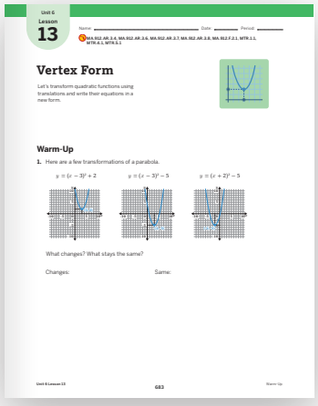
Lesson

13

Vertex Form

Translating Quadratic Functions

Let's transform quadratic functions using translations and write their equations in a new form.


Unit 6
Lesson
13
Vertex Form
Let's transform quadratic functions using translations and write their equations in a new form.
Warm-Up
1. Here are a few transformations of a parabola.
What changes? What stays the same?
Changes:
Same:
683

Learning Outcomes

Today's Goals

1. **Goal:** Identify the vertex of a quadratic function written in *vertex form*.
2. **Goal:** Write equations of the form $f(x) = (x - h)^2 + k$ for quadratic functions.
3. **Language Goal:** Describe the effect of vertical and horizontal translations on the graph of a quadratic function. **(Writing, Speaking, and Listening)**

Students make connections between translations of a parabola, the location of its vertex, and the structure of its equation written in **vertex form**. They describe what the values of h and k in $f(x) = (x - h)^2 + k$ tell us about a parabola, and use that understanding to write equations for quadratic functions with vertical and horizontal translations. **(MTR.5.1)**

Coherence

Prior Learning

In Sub-Unit 2, students determined key features of parabolas from standard and factored form and wrote standard and factored form equations given graphical key features.

Future Learning

In Lesson 15, students will build on their knowledge of quadratic functions to predict diseases.

Connections

- Students relate the vertex form of a quadratic equation to transformations of the graph of quadratic function. (mathematical)
- Students critique the thinking of another student when writing the equation of a parabola with a given vertex and explain their reasoning. (mathematical, cross-curricular)"

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.4
Write a quadratic function to represent the relationship between two quantities from a graph, a written description, or a table of values within a mathematical or real-world context.

MA.912.F.2.1
Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.

Also Addressing: **MA.912.AR.3.6, MA.912.AR.3.7**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.GR.2.2	**MA.912.F.2.1**
MA.912.AR.3.7	

Vocabulary

New Vocabulary

vertex form

Review Vocabulary

translation

Lesson at a Glance

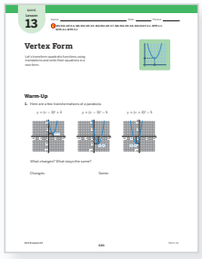
~ 45 min

MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.2.1

Warm-Up

Independent | 5 min

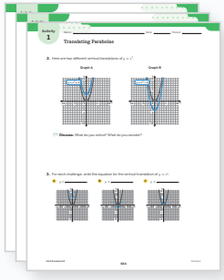
Students describe what changes and what stays the same about parabolas as they translate them.



Activity 1

Pairs | 15 min

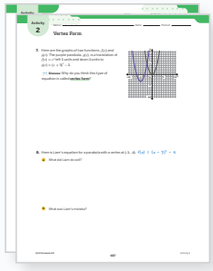
Students make generalizations about vertical and horizontal translations of parabolas to describe connections between their equations and graphs. **Routines:** Notice and Wonder, MLR2: Collect and Display (MTR.2.1)



Activity 2

Pairs | 15 min

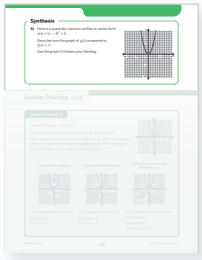
Students determine the vertex of a parabola from its equation in **vertex form** and write an equation in vertex form to match a given vertex. MLR3: Critique, Correct, Clarify (MTR.2.1)



Synthesis

Whole Class | 5 min

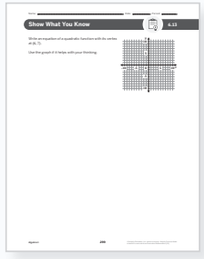
Students synthesize their understanding of what equations in vertex form reveal about the graph. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equation for a parabola with a given vertex.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students describe what changes and what stays the same about parabolas as they translate them.

1 Launch

Consider asking, "Where have you heard the word *transform* outside of math class? Inside of math class? What does it mean to you?"

ELL **English Language Learners** Invite students to research how to say *transform/transformation* in other languages. Create an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**

Invite students to suggest ways to move the points.

Look for different ways that students describe what happens when the points move.

2 Connect

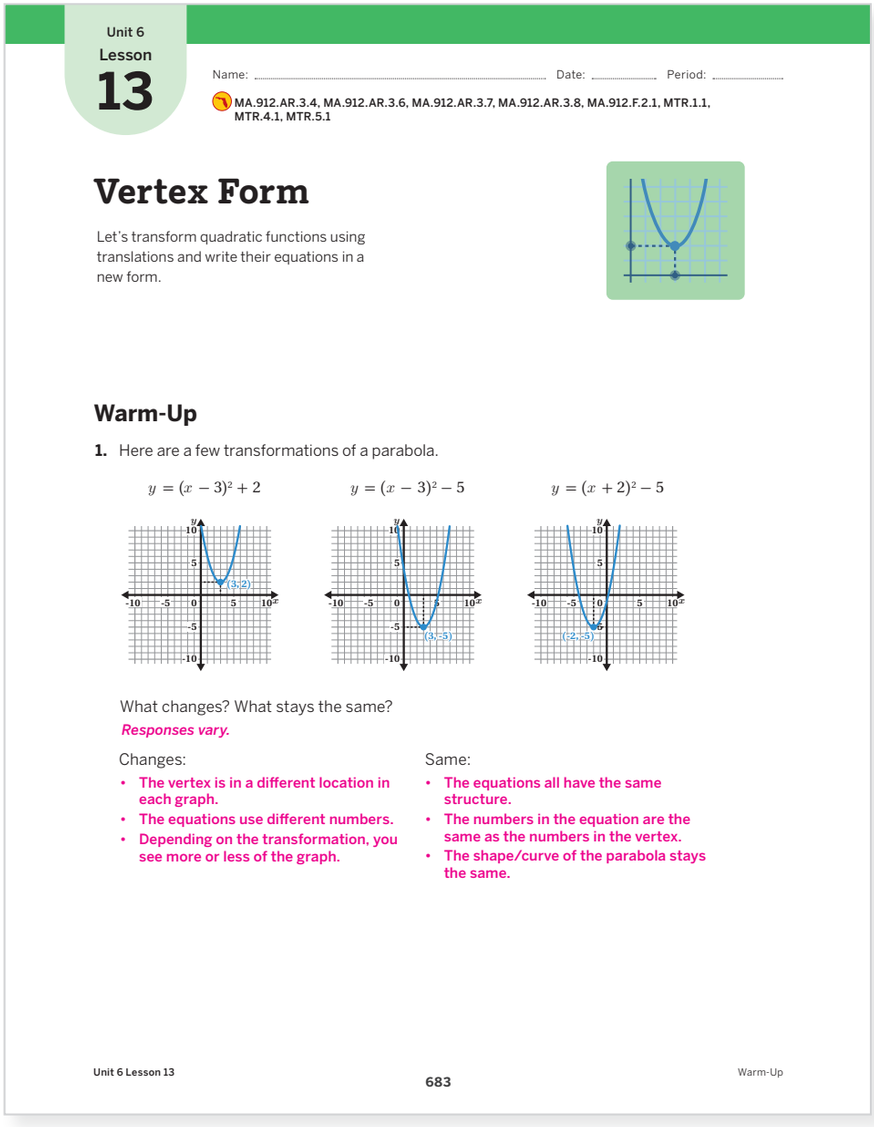
Invite students to share their noticings and further explain their thinking.

Consider asking:

- "What stays the same about the parabola as you move the points? The equation?"
- "What changes about the parabola? The equation?"

D **Differentiation: (Strengthen)** Students may think that translations can change the shape of a graph, since when the graph shifts up less of it is visible, so it appears more narrow (and for a shift down, more is visible so it appears wider). Celebrate students' noticing.

Note: If it does not come up naturally, consider asking, "What do you notice about the coordinates of the vertex and the numbers in the equation?" It is okay if there is a lack of consensus at this point. Students will explicitly connect the vertex to the equation in Activity 2 of this lesson.



Unit 6
Lesson
13

Name: ..... Date: ..... Period: .....

MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Vertex Form

Let's transform quadratic functions using translations and write their equations in a new form.

Warm-Up

1. Here are a few transformations of a parabola.

$y = (x - 3)^2 + 2$ $y = (x - 3)^2 - 5$ $y = (x + 2)^2 - 5$

What changes? What stays the same?

Responses vary.

Changes:

- The vertex is in a different location in each graph.
- The equations use different numbers.
- Depending on the transformation, you see more or less of the graph.

Same:

- The equations all have the same structure.
- The numbers in the equation are the same as the numbers in the vertex.
- The shape/curve of the parabola stays the same.

Unit 6 Lesson 13 683 Warm-Up

Pairs | 15 min

Activity 1 Translating Parabolas

Purpose: Students make generalizations about vertical and horizontal translations of parabolas to describe connections between their equations and graphs. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the two vertical translations.

ELL **English Language Learners** Invite students to share different meanings of the term *translation* inside and outside of math class (e.g., a translation between languages and a mathematical transformation). Clarify which meaning this word will have for this lesson. **(Reading and Listening)**

Invite students to share what they and their partner noticed and wondered.

Consider asking:

- "Where do you see the +3 in Graph A? The -4 in Graph B?"
- "What would $y = x^2 - 5$ look like? What about $y = x^2 + 100$?"

2 Monitor

Encourage students to discuss in their pairs before they write an equation, or alternate who writes equations.

Listen for students who notice patterns as they reason repeatedly with similar transformations. **(MTR.5.1)**

Students will complete each challenge in their Student Edition. Encourage students to check their work using a graphing calculator and consider inviting students to share their graphs with a partner.

A **Accessibility: Conceptual Processing** Invite students to revisit the transformations on the previous question to help apply concepts to new situations.

Activity 1 continued >

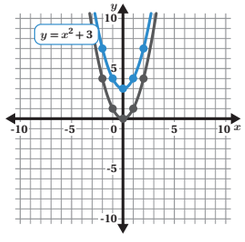
Activity 1

Name: _____ Date: _____ Period: _____

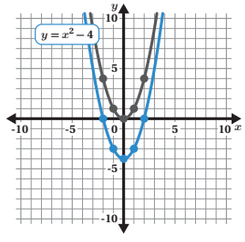
Translating Parabolas

2. Here are two different vertical *translations* of $y = x^2$.

Graph A



Graph B



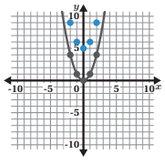
Discuss: What do you notice? What do you wonder?

Responses vary.

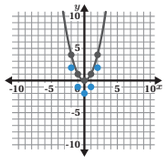
- I notice the vertex of $y = x^2 + 3$ is at $(0, 3)$.
- I notice that $y = x^2 - 4$ is shifted 4 units down from $y = x^2$.
- I wonder if $y = x^2 + 10$ will be shifted 10 units up.
- I wonder if the number after x^2 in the equation will always tell you the vertical shift.
- I wonder what the equation is like for a parabola that has a vertex to the left or right.

3. For each challenge, write the equation for the vertical translation of $y = x^2$.

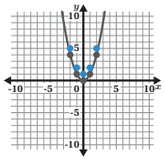
a $y =$ $x^2 + 5$



b $y =$ $x^2 - 2$



c $y =$ $x^2 + 1$



Unit 6 Lesson 13 684 Activity 1

Pairs | 15 min

Activity 1 Translating Parabolas (continued)

Purpose: Students make generalizations about vertical and horizontal translations of parabolas to describe connections between their equations and graphs. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges.

2 Monitor

To access connections, consider asking pairs, "How are these horizontal translations and the vertical translations from the previous activity alike? Different?"

Pause briefly and invite students to share their thinking. If it does not come up naturally, consider sharing that horizontal transformations use parentheses. Students may expect $(x - 3)^2$ to represent a translation *left* 3 units, because of the negative sign; it is okay for students to be unsure now, since they will have more opportunities to make sense of the differences between vertical and horizontal translations in the card sort in the next problem.

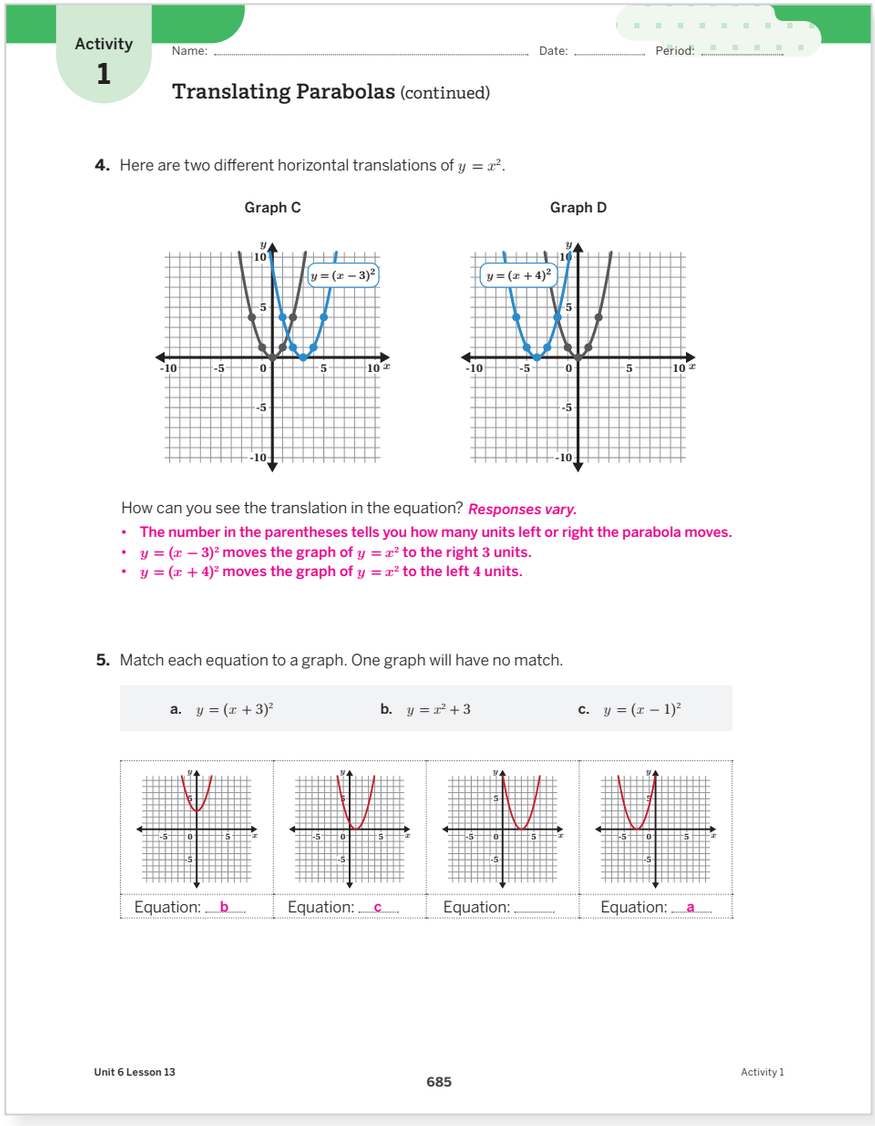
A **Accessibility: Executive Functioning** Chunk this activity by inviting students to separate the equation cards from the graphs. Invite students to choose one equation or graph at a time to match.

Look for a variety of ways that students decide how to match equations and graphs, including looking for specific points on the graph (like the vertex) or reasoning about the direction of the translation based on the equation.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Pair an equation with a non-matching graph, e.g. pairing $y = (x + 3)^2$ with the graph that has its vertex at $(3, 0)$.	**Support** Consider asking, "How could you check to see if these match?" If it does not come up naturally, invite students to use the vertex by substituting the x-coordinate into the equation and checking against the y-coordinate.
Use creative strategies like: • looking for specific points on the graph (like the vertex). • reasoning about the direction of the translation based on the equation. • the process of elimination.	**Strengthen** Celebrate their strategy and/or reasoning and invite them to check in with other pairs to support them if they're stuck.
Complete the matching correctly.	**Stretch** Invite them to write an equation for the one graph they have left over. $y = (x - 3)^2$

Activity 1 continued >



Activity 1

Name: Date: Period:

Translating Parabolas (continued)

4. Here are two different horizontal translations of $y = x^2$.

Graph C

Graph D

How can you see the translation in the equation? *Responses vary.*

- The number in the parentheses tells you how many units left or right the parabola moves.
- $y = (x - 3)^2$ moves the graph of $y = x^2$ to the right 3 units.
- $y = (x + 4)^2$ moves the graph of $y = x^2$ to the left 4 units.

5. Match each equation to a graph. One graph will have no match.

a. $y = (x + 3)^2$ **b.** $y = x^2 + 3$ **c.** $y = (x - 1)^2$

Equation: b | Equation: c | Equation: | Equation: a

Unit 6 Lesson 13 685 Activity 1

Pairs | 15 min

Activity 1 Translating Parabolas (continued)

Purpose: Students make generalizations about vertical and horizontal translations of parabolas to describe connections between their equations and graphs. **(MTR.5.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges.

2 Monitor

Invite students to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly.

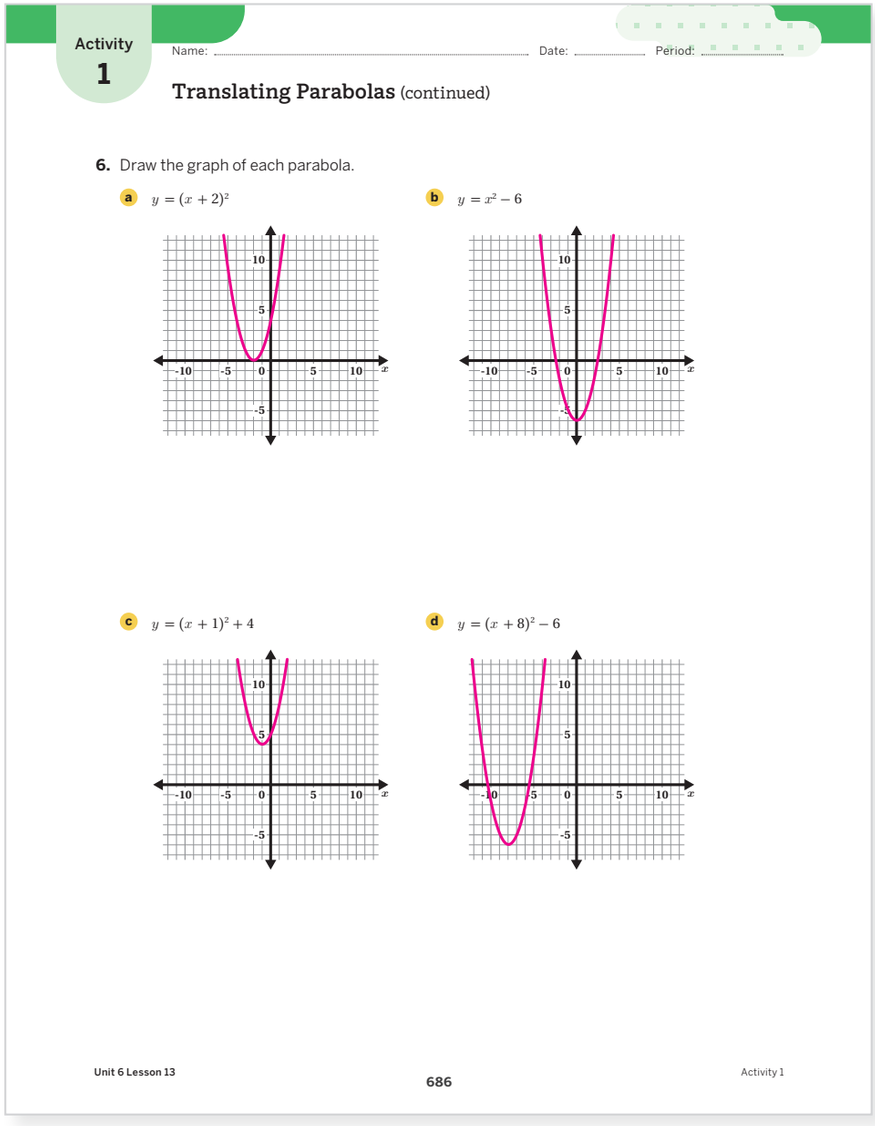
Students will complete each challenge in their Student Edition. Encourage students to check their work using a graphing calculator and consider inviting students to share their graphs with a partner.

3 Connect

To reveal the Key Takeaway, consider asking:

- "How do you tell from an equation if a parabola has been translated vertically? Horizontally?"
- "What do you think are the most common mistakes someone could make as they are learning about horizontal and vertical translations?" For horizontal translations, they might think $y = (x - 1)^2$ means translating left one unit, because of the minus sign.
- "What other patterns or generalizations did you notes about equations and graphs of transformed parabolas that haven't come up yet?"

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and strategies about how to vertical and horizontal translations affect equations and graphs of parabolas as they share during the Connect. Display for students to reference and add to for the rest of the unit.



Activity 1

Name: Date: Period:

Translating Parabolas (continued)

6. Draw the graph of each parabola.

a. $y = (x + 2)^2$

b. $y = x^2 - 6$

c. $y = (x + 1)^2 + 4$

d. $y = (x + 8)^2 - 6$

Unit 6 Lesson 13 686 Activity 1

Key Takeaway: A translation is one way to transform a parabola. Vertical translations shift each point of the parabola up or down and horizontal translations shift each point left or right. For example:

- $y = x^2 + 2$ translates $y = x^2$ up 2 units.
- $y = (x - 1)^2$ translates $y = x^2$ to the right 1 unit.

Pairs | 15 min

Activity 2 Vertex Form

Purpose: Students determine the vertex of a parabola from its equation in **vertex form** and write an equation in vertex form to match a given vertex. **(MTR.5.1)**

1 Launch

Display the graph. Consider asking:

- "Where do you see *left* 5 *units and down* 3 *units* in the equation for $f(x)$?" **(MTR.5.1)**
- "How is this equation different than the equations from the previous activity? How are they alike?" This equation has vertical and horizontal translations, but it uses function notation $f(x)$ instead of y.

ELL **English Language Learners** Invite students to demonstrate or explain how *vertex form* is related to review terms like *vertex, standard form,* and *factored form*. **(Reading and Listening)**

Invite students to share their thinking about why this type of equation is called vertex form, and consider adding their ideas to the class display from Activity 1.

2 Monitor

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Liam's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

D **Differentiation: (Support)** To support students getting started, invite students to use graph paper to put their finger on $(0, 0)$. Consider asking, "How do you get from $(0, 0)$ to the vertex (-3, -4)? What translations need to happen in the equation for $f(x)$?"

Note: In this Activity and in future lessons in this unit, students may write equations using y or $f(x)$. Both are perfectly acceptable ways of writing a quadratic function.

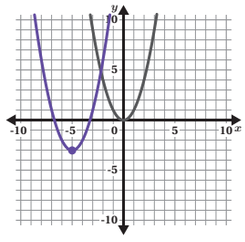
Activity 2 continued >

Activity 2

Name: Date: Period:

Vertex Form

7. Here are the graphs of two functions, $f(x)$ and $g(x)$. The purple parabola, $g(x)$, is a translation of $f(x) = x^2$ left 5 units and down 3 units to $g(x) = (x + 5)^2 - 3$.



Discuss: Why do you think this type of equation is called **vertex form**?

Responses vary. I can see the vertex at (-5, -3) in the equation. The x-value of the vertex is the number that makes the value in the parentheses equal to 0. The y-value of the vertex is the number outside of the parentheses.

8. Here is Liam's equation for a parabola with a vertex at (-3, -4). $f(x) = (x - 3)^2 - 4$

a. What did Liam do well?

Responses vary. Liam used the structure of an equation for a parabola. He also correctly wrote the vertical shift of -4.

b. What was Liam's mistake?

Responses vary. Liam wrote the equation for a parabola whose vertex is at (3, -4). He used the incorrect sign in the parentheses.

Unit 6 Lesson 13 687 Activity 2

Pairs | 15 min

Activity 2 Vertex Form (continued)

Purpose: Students determine the vertex of a parabola from its equation in vertex form and write an equation in vertex form to match a given vertex. **(MTR.5.1)**

2 Monitor

Students will complete this task in their Student Edition. Encourage students to check their work using a graphing calculator and consider inviting students to share their equation with a partner.

A **Accessibility: Conceptual Processing** Invite students to revisit the class display and/or other transformations to help apply concepts to new situations. **(MTR.5.1)**

Listen for student strategies for writing their equation to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to find the point (-6, -2) on the graph and consider asking, "What transformations would you need to get to this vertex from $(0, 0)$?"
Write an equation that includes a mistake, like $f(x) = (x - 6)^2 - 2$.	**Strengthen** Celebrate the correct part(s) of their equation and encourage them to check the vertex by substituting the x-coordinate into the equation and checking against the y-coordinate.

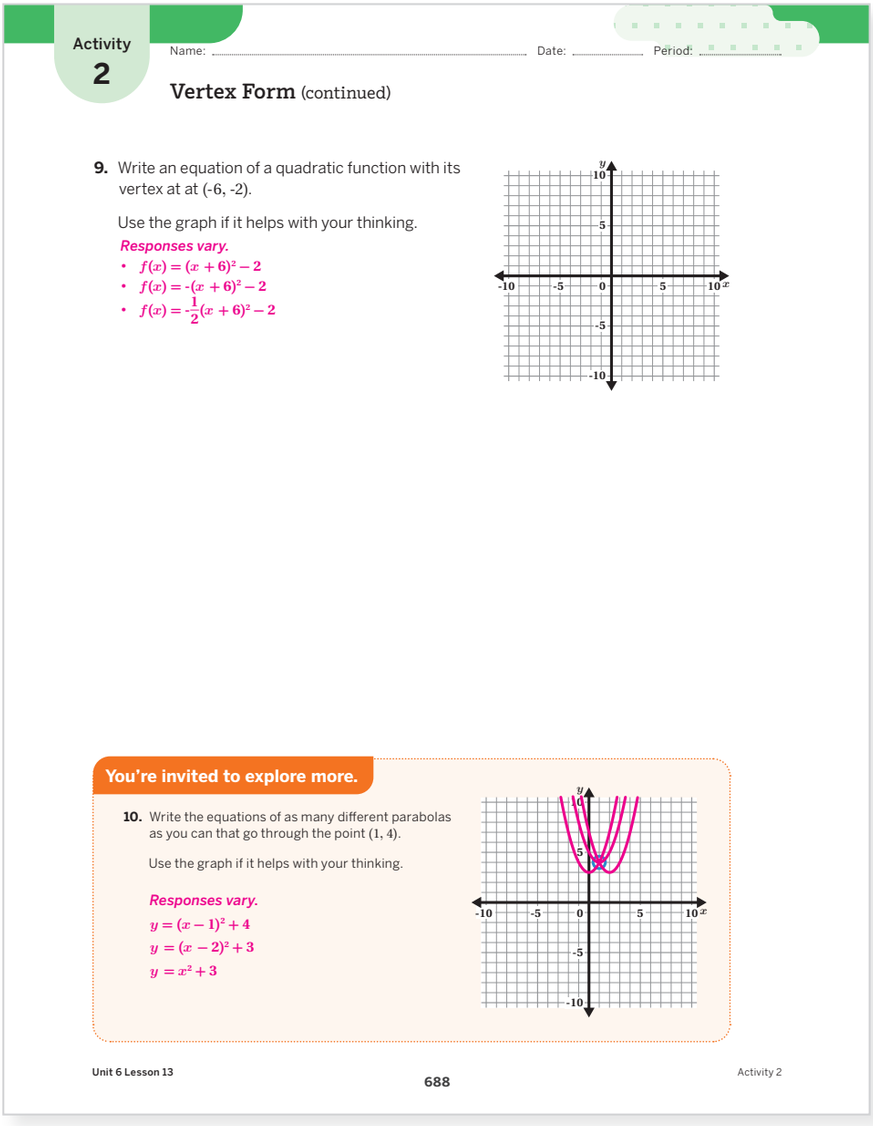
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore equations of quadratic functions further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their equation and explain their strategy for writing it.

To reveal the Key Takeaway, consider asking:

- "How would the equation have been different if the x-coordinate of the vertex had been positive? The y-coordinate?"
- "Could you still write an equation if one of the coordinates was zero? How?"
- "Can you write an equation for *any* given vertex? Why or why not?"



Activity 2

Name: Date: Period:

Vertex Form (continued)

9. Write an equation of a quadratic function with its vertex at at (-6, -2).

Use the graph if it helps with your thinking.

Responses vary.

- $f(x) = (x + 6)^2 - 2$
- $f(x) = -(x + 6)^2 - 2$
- $f(x) = -\frac{1}{2}(x + 6)^2 - 2$

You're invited to explore more.

10. Write the equations of as many different parabolas as you can that go through the point $(1, 4)$.

Use the graph if it helps with your thinking.

Responses vary.

$y = (x - 1)^2 + 4$

$y = (x - 2)^2 + 3$

$y = x^2 + 3$

Unit 6 Lesson 13 688 Activity 2

Key Takeaway: One way to write the equations of quadratic functions is in **vertex form**. This form highlights the vertex of the parabola because both coordinates of the vertex appear in the equation, like the vertex (-6, -2) in the equation $f(x) = (x + 6)^2 - 2$.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what equations in vertex form reveal about the graph.

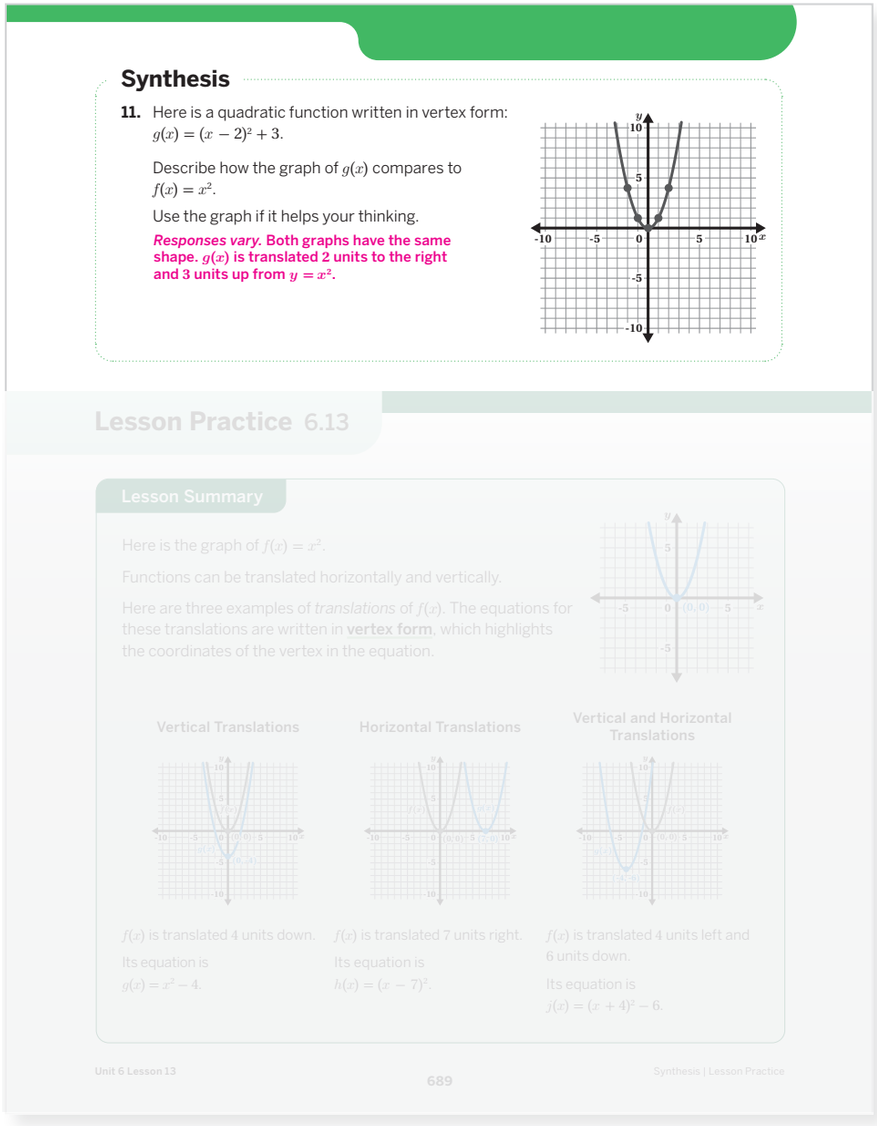
Invite students to respond independently after discussing each question with their partner.

Listen for and share a variety of ideas, such as:

- The 2 in the parentheses represents a horizontal translation that shifts every point on the parabola to the right 2 units.
- The 3 added at the end of the equation makes a vertical translation of the parabola up 3 units.
- You can get points on the transformed parabola by taking points on $y = x^2$ and shifting them to the right 2 units and up 3 units.
- The equation is in vertex form because it highlights the 2 and 3 as coordinates of the vertex, which is located at (2, 3).

MLR **MLR2: Collect and Display** Use this routine to revisit the class display about vertical and horizontal translations and vertex form adding any new student ideas as they share with the class. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each kind of transformation and what questions they still have.



Synthesis

11. Here is a quadratic function written in vertex form: $g(x) = (x - 2)^2 + 3$.

Describe how the graph of $g(x)$ compares to $f(x) = x^2$.

Use the graph if it helps your thinking.

Responses vary. Both graphs have the same shape. $g(x)$ is translated 2 units to the right and 3 units up from $y = x^2$.

Lesson Practice 6.13

Lesson Summary

Here is the graph of $f(x) = x^2$.

Functions can be translated horizontally and vertically.

Here are three examples of *translations* of $f(x)$. The equations for these translations are written in **vertex form**, which highlights the coordinates of the vertex in the equation.

Vertical Translations

$f(x)$ is translated 4 units down.

Its equation is $g(x) = x^2 - 4$.

Horizontal Translations

$f(x)$ is translated 7 units right.

Its equation is $h(x) = (x - 7)^2$.

Vertical and Horizontal Translations

$f(x)$ is translated 4 units left and 6 units down.

Its equation is $j(x) = (x + 4)^2 - 6$.

Unit 6 Lesson 13 689 Synthesis | Lesson Practice

Lesson Takeaway: Writing the equation of a parabola in **vertex form** helps to show how the parabola has been translated horizontally and vertically and reveals the coordinates of the vertex. A parabola with a vertex at (h, k) can be written in vertex form as $f(x) = (x - h)^2 + k$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing an equation for a parabola with a given vertex.

Today's Goals

1. **Goal:** Identify the vertex of a quadratic function written in vertex form.
2. **Goal:** Write equations of the form $f(x) = (x - h)^2 + k$ for quadratic functions.
3. **Language Goal:** Describe the effect of vertical and horizontal translations on the graph of a quadratic function. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

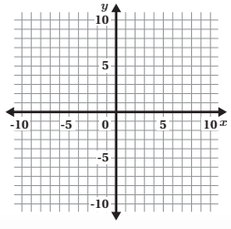
Name: _____ Date: _____ Period: _____

Show What You Know 6.13

Write an equation of a quadratic function with its vertex at (6, 7).

Use the graph if it helps with your thinking.

$y = (x - 6)^2 + 7$ (or equivalent)



D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding vertical and horizontal translations of quadratic functions:

» **Respond to Student Thinking:**

- Use the *Writing Quadratic Functions in Vertex Form* **Mini-lesson**
- Select a few **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to complete the **You're invited to explore more** task and discuss their thinking with a partner.
- Invite students to explore the Sub-Unit 3 Extension Activities.

Professional Learning

Take a moment to reflect on how students generalized their thinking by finding patterns in many graphs of translated parabolas in this lesson. Did students notice any shortcuts for determining key features of the graph of a function that had been transformed? How did you encourage students to continually check their hypotheses about the effects of horizontal and vertical translations on the graphs and equations of quadratic functions? In what ways can you continue to support this form of reasoning as students include vertical stretches to understand **vertex form** more completely in the next lesson? **(MTR.3.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here is a quadratic function written in vertex form: $g(x) = (x - 2)^2 + 3$.

Describe how the graph of $g(x)$ compares to $f(x) = x^2$.

Use the graph if it helps your thinking.

Responses vary. Both graphs have the same shape. $g(x)$ is translated 2 units to the right and 3 units up from $y = x^2$.

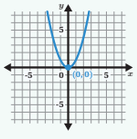
Lesson Practice 6.13

Lesson Summary

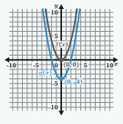
Here is the graph of $f(x) = x^2$.

Functions can be translated horizontally and vertically.

Here are three examples of *translations* of $f(x)$. The equations for these translations are written in **vertex form**, which highlights the coordinates of the vertex in the equation.



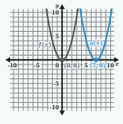
Vertical Translations



$f(x)$ is translated 4 units down.

Its equation is $g(x) = x^2 - 4$.

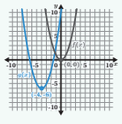
Horizontal Translations



$f(x)$ is translated 7 units right.

Its equation is $h(x) = (x - 7)^2$.

Vertical and Horizontal Translations



$f(x)$ is translated 4 units left and 6 units down.

Its equation is $j(x) = (x + 4)^2 - 6$.

Lesson Practice 6.13

Name: ..... Date: ..... Period: .....

Problems 1–2: Circle if the transformation of $y = x^2$ is a *horizontal translation*, *vertical translation*, or *neither*.

1. $y = (x - 3)^2$

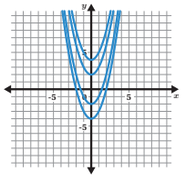
(Horizontal Translation) Vertical Translation Neither

2. $y = x^2 - 5$

Horizontal Translation (Vertical Translation) Neither

3. These parabolas are translations of $y = x^2$.

Select *all* of the equations shown in the graph.

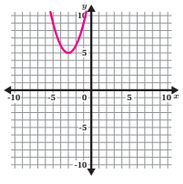


- ☑ A. $y = x^2 + 2$
- ☐ B. $y = (x - 2)^2$
- ☑ C. $y = x^2 - 2$
- ☑ D. $y = x^2 - 4$
- ☐ E. $y = (x + 4)^2$

4. Match each vertex of a parabola to its equation.

a. $y = (x - 3)^2 + 5$	e. (0, -4)
b. $y = (x + 7)^2 + 3$	b. (-7, 3)
c. $y = (x - 4)^2$	d. (-3, 5)
d. $y = (x + 3)^2 + 5$	a. (3, 5)
e. $y = x^2 - 4$	c. (4, 0)

5. Draw the graph of $y = (x + 3)^2 + 5$.

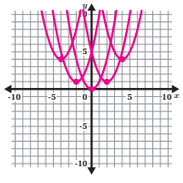


Lesson Practice 6.13

Name: ..... Date: ..... Period: .....

6. Write and graph the equations of five parabolas so that when you connect their vertices, they form a sixth parabola. *Responses vary.*

Equations
$y = x^2$
$y = (x + 2)^2 + 1$
$y = (x - 2)^2 + 1$
$y = (x + 4)^2 + 4$
$y = (x - 4)^2 + 4$



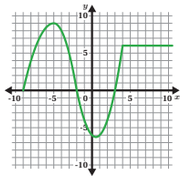
Test Practice

7. Here are four equations in vertex form. Which function has a graph with a vertex at $(1, 3)$?

- A. $y = (x + 1)^2 + 3$
- (B.) $y = (x - 1)^2 + 3$
- C. $y = (x - 3)^2 + 1$
- D. $y = (x + 3)^2 + 1$

Spiral Review

Problems 8–10: Here is a graph of $g(x)$.



8. At what point does the maximum occur?

(-5, 9)

9. At what points does $g(x) = 0$?

(-9, 0), (-2, 0), and (3, 0)

10. What is the y-intercept?

(0, -6)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.F.2.1
	4–5	2	MA.912.F.2.1 MA.912.AR.3.4
	6	3	MA.912.F.2.1
Test Practice	7	2	MA.912.F.2.1
Spiral Review			
Fluency	8–10	1	MA.912.AR.2.4

Unit 6

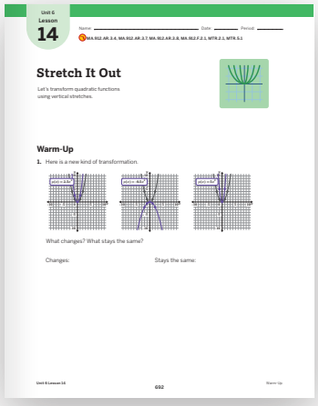
Lesson

14

Stretch It Out

Vertical Stretches and Vertex Form

Let's transform quadratic functions using vertical stretches.


Unit 6
Lesson
14
Name: Date: Period:
Stretch It Out
Let's transform quadratic functions using vertical stretches.
Warm-Up
1. Here is a new kind of transformation.
What changes? What stays the same?
Changes:
Stays the same:
692

Learning Outcomes

Today's Goals

1. **Goal:** Describe the effects of *vertical stretches* on the graph of a quadratic function.
2. **Language Goal:** Describe how the value of a in the equation $f(x) = a(x - h)^2 + k$ affects the graph of its corresponding parabola. **(Writing, Speaking, and Listening)**

Students make connections between **vertical stretches** of a parabola and the structure of its equation written in vertex form. They describe what the value of a in $f(x) = a(x - h)^2 + k$ tells us about a parabola, and use that understanding to write equations for quadratic functions with vertical stretches, as well as a combination of stretches and translations. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 13, students wrote equations of quadratic functions that had been vertically and horizontally translated. They also described the vertical and horizontal translations of a quadratic function given its equation and graph.

Future Learning

In Lesson 15 students, will generate quadratic models to fit data.

Connections

- Students transform quadratic functions using vertical stretches. (mathematical)
- Students relate the leading coefficient of a quadratic function to the behavior of its graph in a matching activity and explain their reasoning. (mathematical, cross-curriculum)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.7
Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

MA.912.F.2.1
Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.

Also Addressing: **MA.912.AR.3.4**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.GR.2.2	**MA.912.F.2.1**
MA.912.AR.3.7	

Vocabulary

New Vocabulary

vertical stretch

Lesson at a Glance

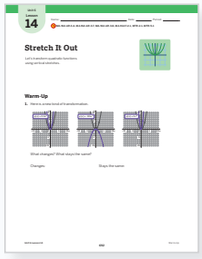
~ 45 min

MA.912.AR.3.4, MA.912.AR.3.7, MA.912.F.2.1

Warm-Up

Pairs | 5 min

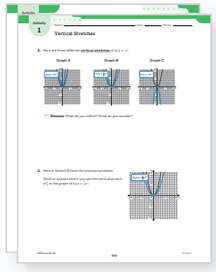
Students describe what changes and what stays the same about parabolas as they are vertically stretched.



Activity 1

Pairs | 10 min

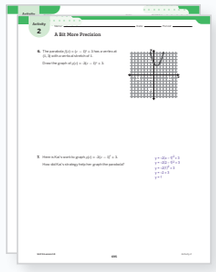
Students reason repeatedly with graphs to make sense of how **vertical stretches** affect the equations and graphs of quadratic functions.
MLR2: Collect and Display



Activity 2

Pairs | 10 min

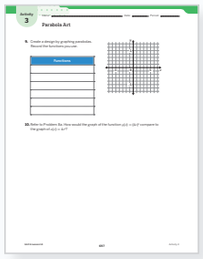
Students attend to precision in making graphs of translated and stretched parabolas.



Activity 3

Independent | 10 min

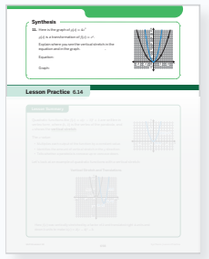
Students use their creativity to make parabola art and practice writing equations for stretched parabolas.



Synthesis

Whole Class | 5 min

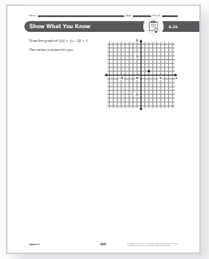
Students synthesize their understanding of how to identify a vertical stretch from a parabola and an equation in vertex form.
MLR2: Collect and Display (MTR.5.1)



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating the graph of a translated and stretched parabola.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students describe what changes and what stays the same about parabolas as they are vertically stretched.

1 Launch

Display the graphs and ask students to list what changes and what stays the same.

Consider asking, "How is this transformation similar to the translations you saw in the last lesson? How is it different?"

2 Connect

Invite students to share their noticings and further explain their thinking.

Consider asking:

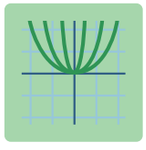
- "What happens to the graph when the coefficient in the equation is farther from 0, like 5? What happens when it is closer to 0, like 0.5?"
- "What happens to the graph when the coefficient in the equation is negative?"
- "Are the x-values or y-values changing? How can you tell?"

Unit 6
Lesson
14

Name: ..... Date: ..... Period: .....

MA.912.AR.3.4, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.2.1, MTR.5.1

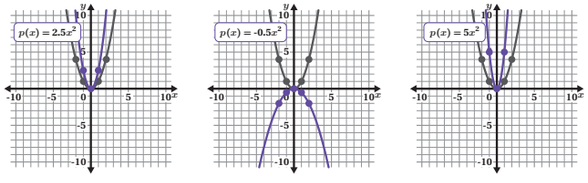
Stretch It Out



Let's transform quadratic functions using vertical stretches.

Warm-Up

1. Here is a new kind of transformation.



What changes? What stays the same?

Responses vary.

Changes:

- The number in front of the x^2.
- The width of the parabola.
- The concavity.

Stays the same:

- They are all parabolas.
- They all have a line of symmetry at $x = 0$.
- They all have a vertex at $(0, 0)$.

Unit 6 Lesson 14 692 Warm-Up

Pairs | 10 min

Activity 1 Vertical Stretches

Purpose: Students reason repeatedly with graphs to make sense of how **vertical stretches** affect the equations and graphs of quadratic functions.

1 Launch

To access connections, consider asking, "Why do you think this transformation is called a *vertical stretch*?"

ELL **English Language Learners** Invite students to share different meanings of the term *stretch*, in words and using gestures if they prefer. Clarify that, for this lesson, *stretch* will refer to the transformation of a function by multiplying each output by a number. **(Reading and Speaking)**

Use the Notice and Wonder routine to promote curiosity and help students make sense of the three vertical stretches.

Invite students to share what they and their partner noticed and wondered.

Consider asking, "Where do you see the 3.0 in Graph A? The 0.5 in Graph B? The -2.0 in Graph C?"

2 Monitor

To support students getting started, consider inviting them to label the coordinates of one point from the gray parabola $f(x) = x^2$, and compare with the point on the transformed parabola at the same x-coordinate. Consider asking, "How are the y-coordinates related to the $\frac{1}{2}$ in the equation?"

Listen for students who use specific points or calculations to support their thinking, or make generalizations about the patterns they are noticing with vertical stretches (e.g., about which values of a cause the parabola to be wider or narrower or change from concave up to concave down). Consider pausing briefly to invite students who used different points or made generalizations to explain their thinking.

D **Differentiation: (Strengthen)** Some students might think the y-coordinate of a point on the graph has to match exactly with the number in the equation, i.e. the point $(1, \frac{1}{2})$ on the transformed parabola. Consider sharing that we are looking for how the vertical distance between the vertex and the point changes with the transformation, since there will be examples in Activity 2 where the vertex is not $(0, 0)$.

Activity 1 continued >

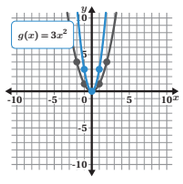
Activity 1

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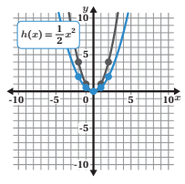
Vertical Stretches

2. Here are three different **vertical stretches** of $f(x) = x^2$.

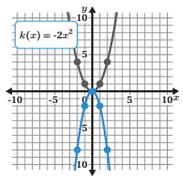
Graph A



Graph B



Graph C



Discuss: What do you notice? What do you wonder?

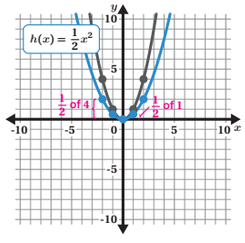
Responses vary.

- I notice that the plotted points on $f(x) = x^2$ and the stretched graphs all have the same set of x-values. Only the y-values change.
- I notice that when the number in front of the x^2 is greater than 1, the parabola looks more narrow. When the number in front of the x^2 is less than 1, the parabola looks wider.
- I wonder why multiplying by 2 makes the graph look narrower.
- I wonder if there is a way to determine the number in front of x^2 by looking for a pattern in the graph.

3. Here is Stretch B from the previous problem.

Show or explain where you see the vertical stretch of $\frac{1}{2}$ in the graph of $h(x) = \frac{1}{2}x^2$.

Responses vary. The vertical distance from the vertex to the next point went from 1 unit in the $f(x) = x^2$ graph to $\frac{1}{2}$ of a unit in the $h(x) = \frac{1}{2}x^2$ graph. Each of the outputs on $h(x) = \frac{1}{2}x^2$ are half of the value of the outputs on $f(x) = x^2$.



Unit 6 Lesson 14 693 Activity 1

Pairs | 10 min

Activity 1 Vertical Stretches (continued)

Purpose: Students reason repeatedly with graphs to make sense of how **vertical stretches** affect the equations and graphs of quadratic functions.

2 Monitor

Listen for strategies that students use to match the equations and graphs, including looking at specific points on the graph.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to start by looking at which parabolas are concave up and concave down.
Make the matches correctly, with one equation left over.	**Stretch** Invite these students to sketch a graph for the left over equation $b(x) = -2x^2$.

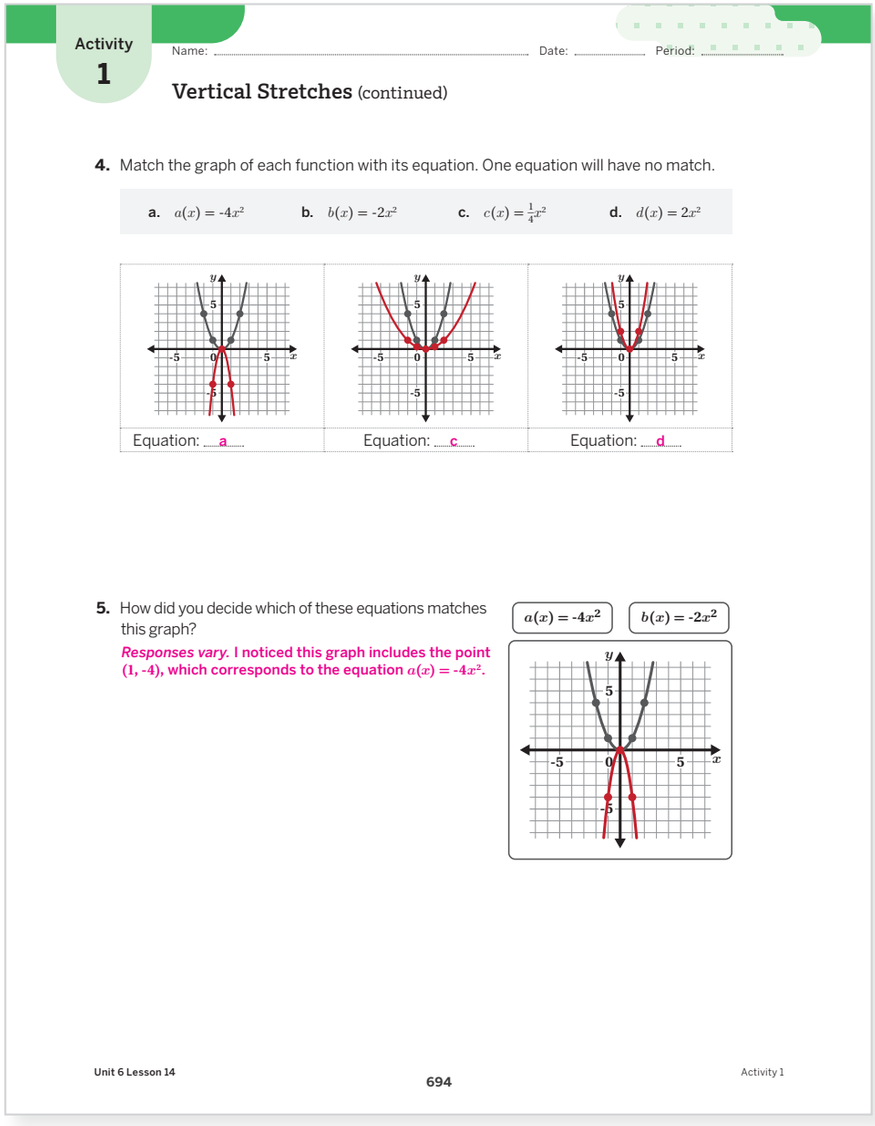
3 Connect

Invite students to share their strategies for deciding which equation to pair with each graph.

To reveal the Key Takeaway, consider asking:

- "What do these two equations have in common? What is different about them?"
- "What part(s) of the graph helped you decide which equation to pair with this graph?"
- "How would the graph for the other equation look different? Would anything stay the same?"

MLR2: Collect and Display Use this routine to collect students' ideas and generalizations about vertical stretches as they share during the Connect. Display for students to reference and add to for the rest of the unit.



Activity 1

Name: ..... Date: ..... Period: .....

Vertical Stretches (continued)

4. Match the graph of each function with its equation. One equation will have no match.

a. $a(x) = -4x^2$ **b.** $b(x) = -2x^2$ **c.** $c(x) = \frac{1}{4}x^2$ **d.** $d(x) = 2x^2$

Equation: a | Equation: c | Equation: d

5. How did you decide which of these equations matches this graph?

Responses vary. I noticed this graph includes the point (1, -4), which corresponds to the equation $a(x) = -4x^2$.

$a(x) = -4x^2$ $b(x) = -2x^2$

Unit 6 Lesson 14 694 Activity 1

Key Takeaway: A **vertical stretch** is one way to transform a parabola. Vertical stretches multiply the y-coordinate of each point of the parabola by the same number. They can change the shape of a parabola, making it narrower or wider, and they can determine if the parabola is concave up or concave down. For example:

- $b(x) = -2x^2$ vertically stretches the parabola $f(x) = x^2$ by a factor of -2 to make it narrower and concave down.
- $h(x) = \frac{1}{2}x^2$ vertically stretches the parabola $f(x) = x^2$ by a factor of $\frac{1}{2}$, compressing the parabola and making it wider.

Pairs | 10 min

Activity 2 A Bit More Precision

Purpose: Students attend to precision in making graphs of translated and stretched parabolas. **(MTR.4.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges.

1 Launch

Consider sharing with students that the equation of the graph is $f(x) = 1(x - 1)^2 + 3$.

Consider asking, "How did you determine where to drag the point to graph $g(x)$?"

Invite students to check their answer using a graphing calculator.

2 Monitor

Display Kai's graph.

Collect explanations that make sense of Kai's work by connecting it to the point $(2, 1)$ on the graph.

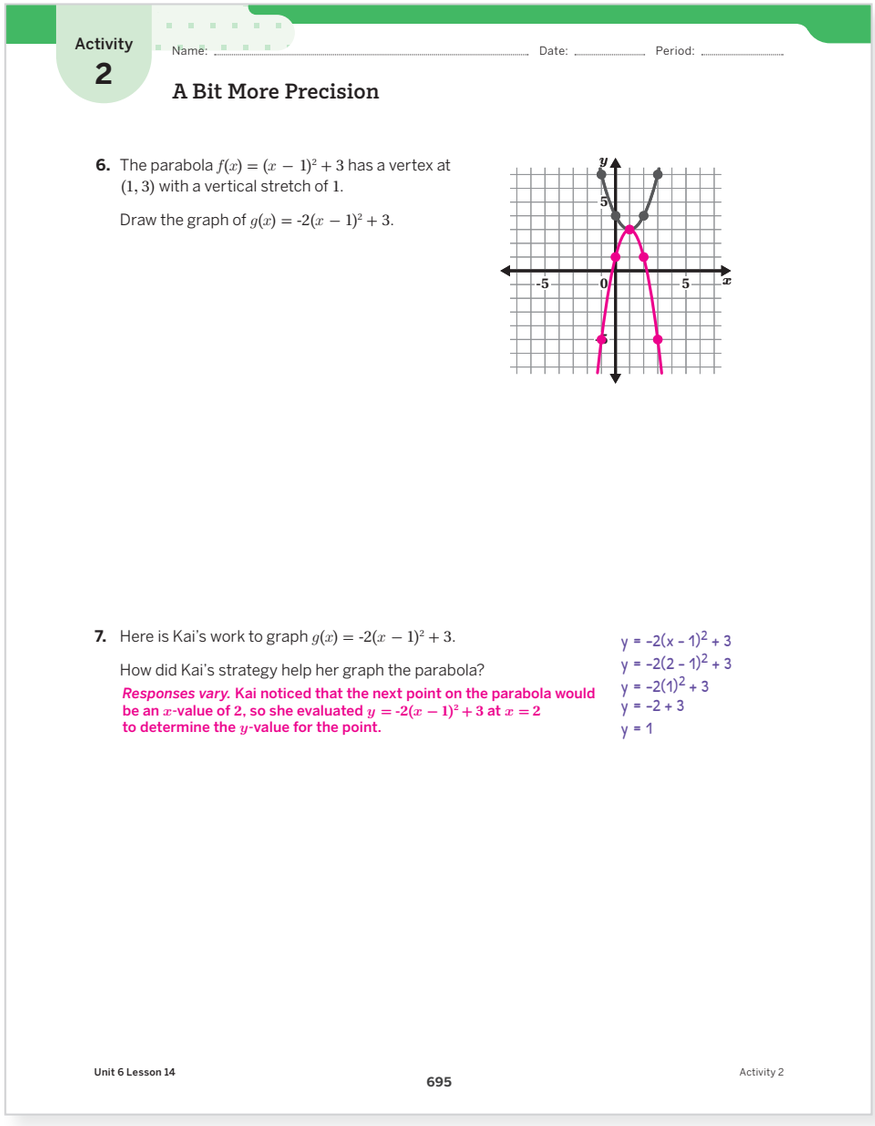
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Why do you think Kai substituted 2 in for x? Why did she choose that x-value to start with? Is there another x-value she could have started with?"
Are unsure why the -2 in the equation does not appear as a coordinate of any point in the graph.	**Support** Consider supporting these students in using their finger to show that the vertical distance is -2 between the vertex and the point $(2, 1)$ from Kai's work.
Explain Kai's strategy and why it works.	**Stretch** Consider asking, "Would this strategy work for any parabola? Why or why not?"

Pause briefly to invite students to share their explanations.

Invite students to continue.

Activity 2 continued >



Activity 2

Name: Date: Period:

A Bit More Precision

6. The parabola $f(x) = (x - 1)^2 + 3$ has a vertex at $(1, 3)$ with a vertical stretch of 1.

Draw the graph of $g(x) = -2(x - 1)^2 + 3$.

7. Here is Kai's work to graph $g(x) = -2(x - 1)^2 + 3$.

How did Kai's strategy help her graph the parabola?

Responses vary. Kai noticed that the next point on the parabola would be an x-value of 2, so she evaluated $y = -2(x - 1)^2 + 3$ at $x = 2$ to determine the y-value for the point.

$y = -2(x - 1)^2 + 3$
$y = -2(2 - 1)^2 + 3$
$y = -2(1)^2 + 3$
$y = -2 + 3$
$y = 1$

Unit 6 Lesson 14 695 Activity 2

Pairs | 10 min

Activity 2 A Bit More Precision (continued)

Purpose: Students attend to precision in making graphs of translated and stretched parabolas. **(MTR.4.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges.

2 Monitor

Students will draw the graph of eqch function in their Student Edition. Encourage students to check their work using a graphing calculator and consider inviting students to share their graphs with a partner.

A **Accessibility: Conceptual Processing**
Consider creating a display about vertex form, $f(x) = a(x - h)^2 + k$, labeled with the transformations associated with the parameters a, h, and k. Invite students to refer to it as they draw each graph.

D **Differentiation**

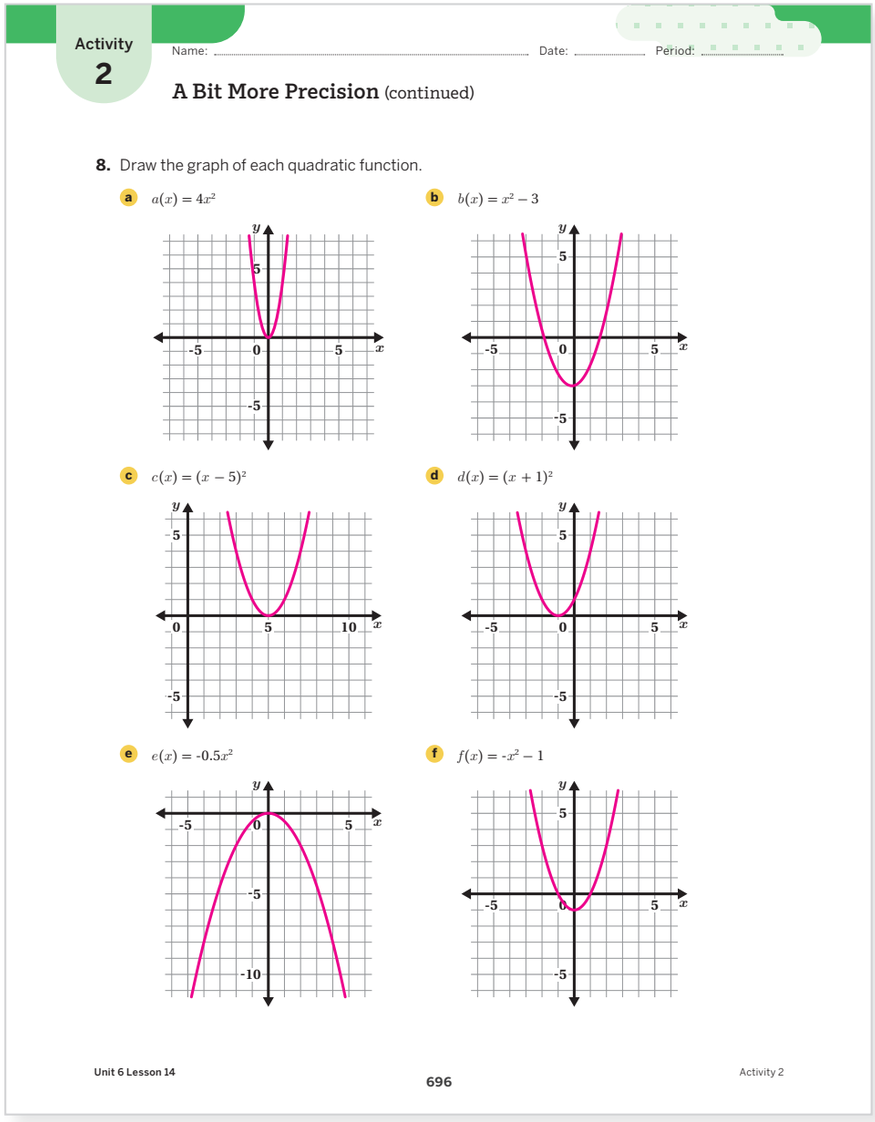
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider inviting students to list or name aloud the transformations for each function, including writing out the vertical stretch of 1 for the function $d(x)$. **Support** Consider sharing that they can determine some key features from the equation alone. For example, if the value of a is negative, the function will be concave down and have a maximum, and if a is positive, the function will be concave up and have a minimum.
Are not sure how to proceed when a is 1 or h or k is zero.	**Support** Consider supporting students in rewriting expressions with missing parameters by writing them in as zeroes or ones. For example, $p(x) = x^2$ can be rewritten as $p(x) = 1(x - 0)^2 + 0$.
Successfully complete several graphs.	**Stretch** Consider inviting students to identify and describe key features of each parabola as precisely as possible. **(MTR.4.1)**

3 Connect

Invite students to share their strategies for creating graphs for the transformed parabolas. Poll the class to see if there are any functions they would like to discuss further.

To reveal the Key Takeaway, consider asking:

- "How can you tell from a vertex form equation how the parabola has been vertically stretched?"
- "What else can you determine from a vertex form equation to help you make the graph?" The vertex
- "How can the equation help you to create a precise graph for a given vertical stretch?" The coefficient in front of $(x - h)^2$ can help to place an additional point that matches the vertical stretch exactly. **(MTR.5.1)**



Activity 2

Name: Date: Period:

A Bit More Precision (continued)

8. Draw the graph of each quadratic function.

a $a(x) = 4x^2$

b $b(x) = x^2 - 3$

c $c(x) = (x - 5)^2$

d $d(x) = (x + 1)^2$

e $e(x) = -0.5x^2$

f $f(x) = -x^2 - 1$

Unit 6 Lesson 14 696 Activity 2

Key Takeaway: Quadratic functions with the structure $f(x) = a(x - h)^2 + k$ are written in vertex form. The a-value of the equation determines the vertical stretch, and the values of h and k determine the horizontal and vertical translations of the graph. The vertex of $f(x)$ has the coordinates (h, k).

Independent | 10 min

Activity 3 Parabola Art

Purpose: Students use their creativity to make parabola art and practice writing equations for stretched parabolas.

Short on time? Consider spending less time on this activity.

1 Launch

To encourage creativity, invite students to make a design that is personal to them utilizing different colors. Consider generating excitement by creating your own parabola art or asking, "What kinds of things can you make using parabolas?"

Invite students to use coloring tools to create their parabola art. To encourage creativity, they can even add shading and domain or range restrictions to their designs.

2 Monitor

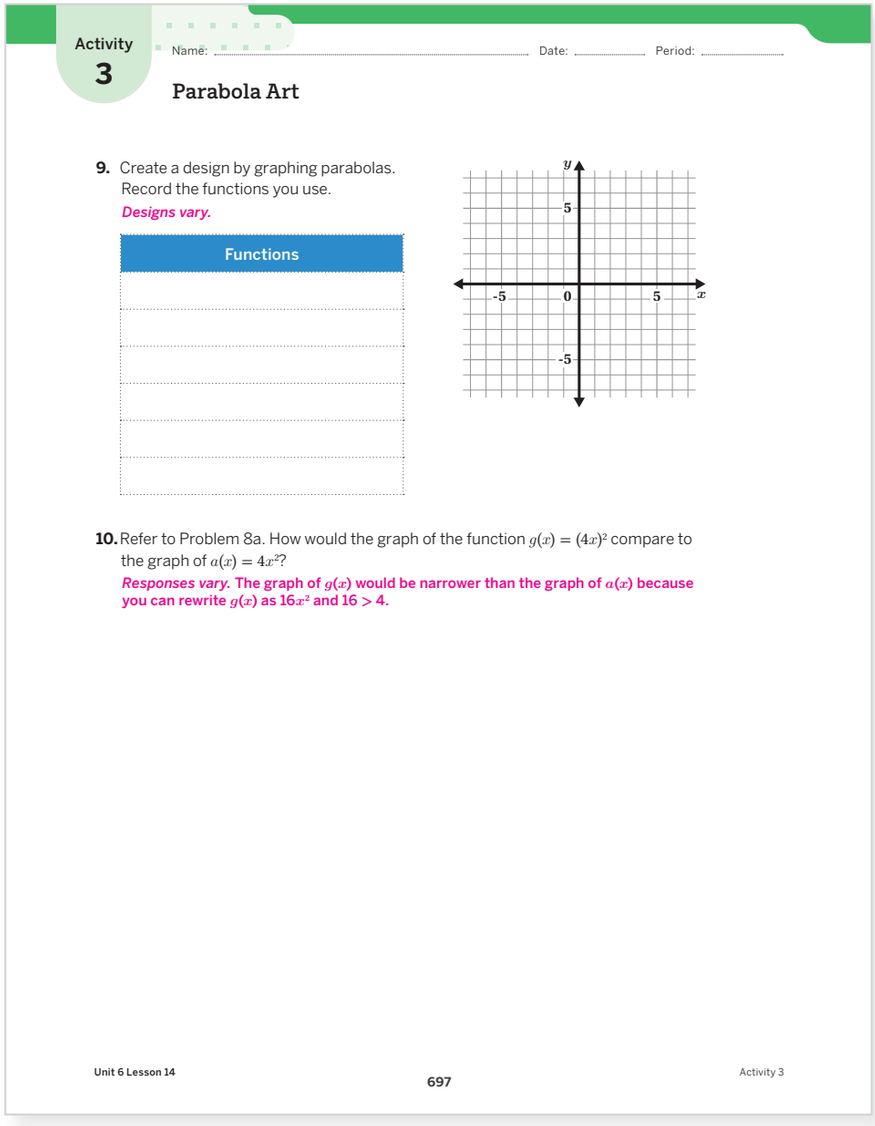
Note: Encourage students to use their strategies from earlier in this lesson to make their parabolas as precise as possible. At this point in the unit students have several strategies for thinking about making precise quadratic graphs by hand, but it can be a lot of work for students to graph many parabolas accurately. Students may draw functions that aren't quadratic or aren't perfectly precise (e.g., graphs that look like a V or a sideways U, or graphs that are not completely symmetric). Celebrate their design and creativity. Consider asking, "Which of these curves you drew are quadratic functions? Which ones are not?"

Listen for a variety of student responses to share during the Connect.

3 Connect

Share student parabola art one at a time, inviting students to describe the design and how they came up with their equations.

D **Differentiation: (Stretch)** Consider sharing that there are artists who use lots of math, like functions and geometry, in their work. Encourage students to research and explore some mathematical works of art and how they were created.



Activity 3

Name: Date: Period:

Parabola Art

9. Create a design by graphing parabolas. Record the functions you use.

Designs vary.

Functions

10. Refer to Problem 8a. How would the graph of the function $g(x) = (4x)^2$ compare to the graph of $a(x) = 4x^2$?

Responses vary. The graph of $g(x)$ would be narrower than the graph of $a(x)$ because you can rewrite $g(x)$ as $16x^2$ and $16 > 4$.

Unit 6 Lesson 14 697 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to identify a vertical stretch from a parabola and an equation in vertex form.

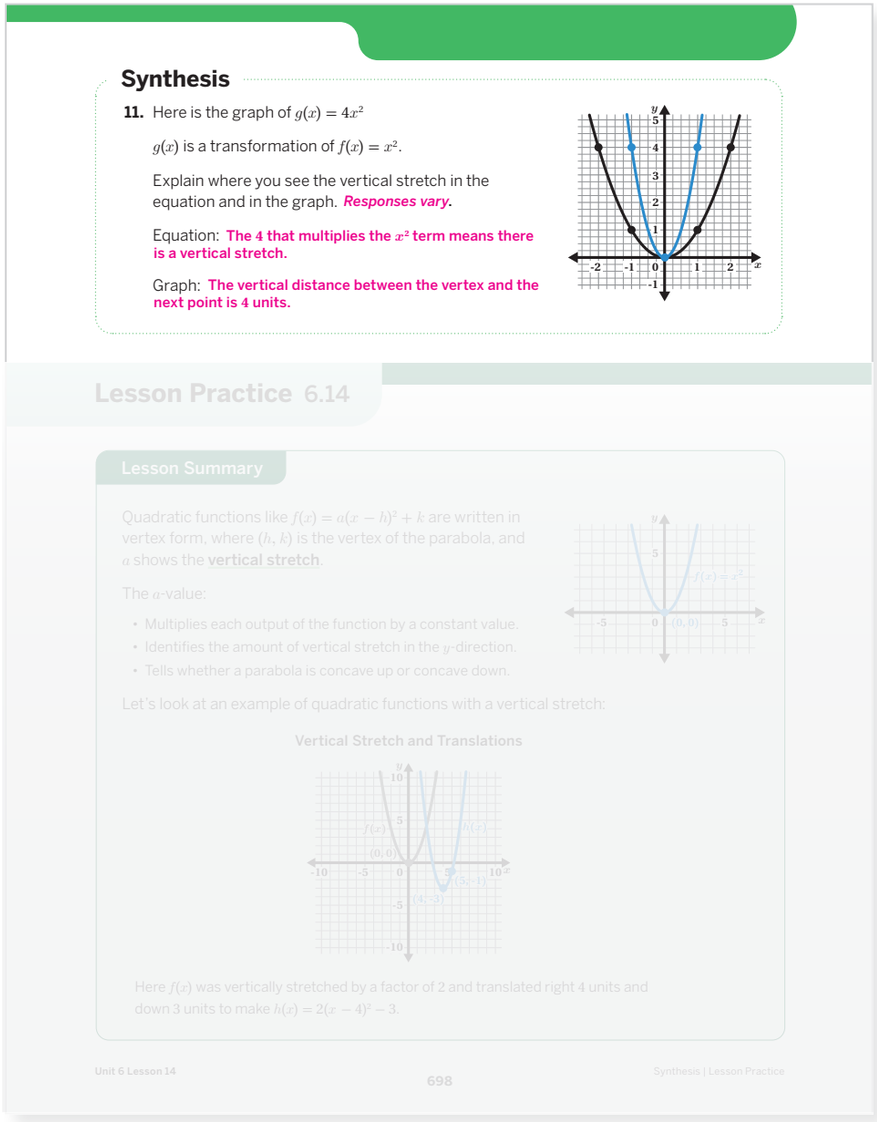
Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:
- The vertical stretch is represented in the equation by the number 4.
- The graph shows a vertical stretch because it is narrower than $f(x) = x^2$.
- The graph shows a vertical stretch of 4 because the points on the graph that are 1 unit in the x-direction away from the vertex are 4 units up from the vertex.
- The function $f(x)$ goes over one up one from the vertex, but $g(x)$ goes over one up two.
- The positive 4 means the graph is concave up.

Consider asking:
- "How would the graph be different if the 4 was negative? If it was replaced by $\frac{1}{2}$?"
- "What else can you determine from the graph besides the vertical stretch?"

MLR **MLR2: Collect and Display** Use this routine to revisit the anchor charts on vertical stretches, translations, and vertex form. Add students' ideas to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here is the graph of $g(x) = 4x^2$

$g(x)$ is a transformation of $f(x) = x^2$.

Explain where you see the vertical stretch in the equation and in the graph. ***Responses vary.***

Equation: **The 4 that multiplies the x^2 term means there is a vertical stretch.**

Graph: **The vertical distance between the vertex and the next point is 4 units.**

Lesson Practice 6.14

Lesson Summary

Quadratic functions like $f(x) = a(x - h)^2 + k$ are written in vertex form, where (h, k) is the vertex of the parabola, and a shows the **vertical stretch**.

The a-value:
- Multiplies each output of the function by a constant value.
- Identifies the amount of vertical stretch in the y-direction.
- Tells whether a parabola is concave up or concave down.

Let's look at an example of quadratic functions with a vertical stretch:

Vertical Stretch and Translations

Here $f(x)$ was vertically stretched by a factor of 2 and translated right 4 units and down 3 units to make $h(x) = 2(x - 4)^2 - 3$.

Unit 6 Lesson 14 698 Synthesis | Lesson Practice

Lesson Takeaway: Vertex form of a quadratic is $f(x) = a(x - h)^2 + k$. The value of a in the equation determines the **vertical stretch**, and the values of h and k determine the horizontal and vertical translations of the graph. The vertex of $f(x)$ has the coordinates (h, k). Different values of a can transform the graph in different ways:
- When a is positive, the parabola is concave up.
- When a is negative, the parabola is concave down.
- When a has an absolute value greater than 1, the parabola is narrower than $p(x) = x^2$.
- When a has an absolute value between 0 and 1, the parabola is wider than $p(x) = x^2$. This is sometimes called a *vertical compression*.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by creating the graph of a translated and stretched parabola.

Today's Goals

1. **Goal:** Describe the effects of vertical stretches on the graph of a quadratic function.
2. **Language Goal:** Describe how the value of a in the equation $f(x) = a(x - h)^2 + k$ affects the graph of its corresponding parabola. **(Writing, Speaking, and Listening)**

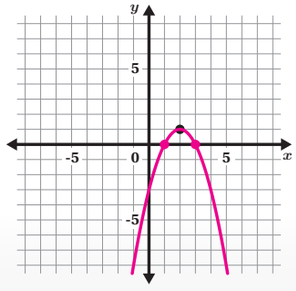
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.14

Draw the graph of $f(x) = -(x - 2)^2 + 1$.

The vertex is drawn for you.



D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support understanding understanding vertical stretches of a quadratic function written in vertex form:

» **Respond to Student Thinking:**

- Revisit the **Show What You Know** and give students individualized feedback.
- Revisit **Lesson 13, Activity 2** to give students some support with combining vertical and horizontal translations with vertical stretches.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore translations of functions of their choice by entering the function $f(x) = a(x - h)^2 + k$ in the Desmos Graphing Calculator and creating sliders for a, h, and k. If time allows, consider demonstrating this for the class.
- Invite students to explore the Sub-Unit 3 Extension Activities.

Professional Learning

Take a moment to reflect on student conceptual development of transformations of functions in the coordinate plane across this unit and course. Across the last few lessons, students transformed quadratic functions by translating them vertically and horizontally, and changed their shape by stretching them vertically. How were students able to build on their prior knowledge from transforming absolute value and exponential functions earlier in this course? How has their development in this unit extended their thinking?

In Algebra 2, students will transform many more types of functions. How will students' conceptual understanding of function transformations grow as they explore more types of functions in Algebra 2?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here is the graph of $g(x) = 4x^2$

$g(x)$ is a transformation of $f(x) = x^2$.

Explain where you see the vertical stretch in the equation and in the graph. *Responses vary.*

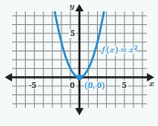
Equation: *The 4 that multiplies the x^2 term means there is a vertical stretch.*

Graph: *The vertical distance between the vertex and the next point is 4 units.*

Lesson Practice 6.14

Lesson Summary

Quadratic functions like $f(x) = a(x - h)^2 + k$ are written in vertex form, where (h, k) is the vertex of the parabola, and a shows the **vertical stretch**.

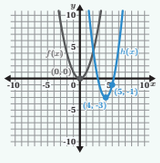


The a-value:

- Multiplies each output of the function by a constant value.
- Identifies the amount of vertical stretch in the y-direction.
- Tells whether a parabola is concave up or concave down.

Let's look at an example of quadratic functions with a vertical stretch:

Vertical Stretch and Translations



Here $f(x)$ was vertically stretched by a factor of 2 and translated right 4 units and down 3 units to make $h(x) = 2(x - 4)^2 - 3$.

Lesson Practice 6.14

Name: ..... Date: ..... Period: .....

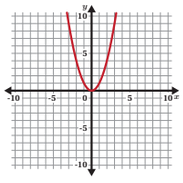
Problems 1–3: Here's the graph of $y = x^2$. Change one number to make the graph:

Responses vary.

1. Wider: $y = \frac{1}{2}x^2$ *(or equations with $0 < a < 1$)*

2. Narrower: $y = 2x^2$ *(or equations with $a > 1$)*

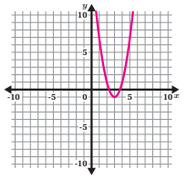
3. Open down:
$y = -1x^2$ *(or equations with $a < 0$)*



4. Describe how the graph of $y = x^2$ compares to $y = -4x^2$

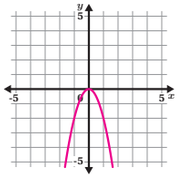
Responses vary. The vertex would stay at (0,0) and the parabola would open downward, with a vertical stretch of 4, and reflects across the x-axis.

5. Draw a graph of a parabola that has a vertex at (3, -1) and a vertical stretch of 2.

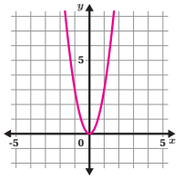


Problems 6–7: Draw a graph for each equation.

6. $a(x) = -2x^2$



7. $c(x) = 3x^2$

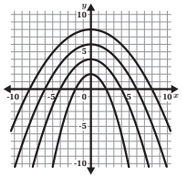


Lesson Practice 6.14

Name: ..... Date: ..... Period: .....

Test Practice

8. The 4 parabolas remind you of a rainbow. Use what you know about vertex form and choose the function that has the largest vertical stretch.



A. $y = \frac{1}{8}x^2 + 8$

B. $y = \frac{1}{6}x^2 + 6$

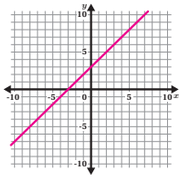
C. $y = \frac{1}{4}x^2 + 4$

D. $y = \frac{1}{2}x^2 + 2$

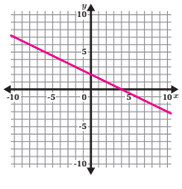
Spiral Review

Problems 9–10: Draw a graph of each equation.

9. $-3x + 3y = 9$



10. $2x + 4y = 8$



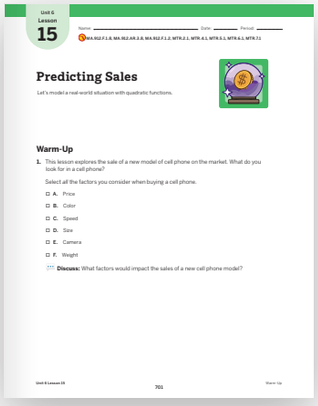
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.F.2.1
	4–7	2	MA.912.AR.3.7 MA.912.F.2.1
Test Practice	8	3	MA.912.AR.3.4 MA.912.AR.3.7
Spiral Review			
Fluency	9–10	2	MA.912.AR.3.7

Unit 6

Lesson

15


Unit 6
Lesson
15
Predicting Sales
Let's model a real-world situation with quadratic functions.
Warm-Up
1. This lesson explores the sale of a new model of cell phone on the market. What do you look for in a cell phone?
Select all the factors you consider when buying a cell phone.
A. Price
B. Color
C. Speed
D. Size
E. Camera
F. Weight
Discuss: What factors would impact the sales of a new cell phone model?
701

Predicting Sales

Linear, Exponential, and Quadratic Modeling

Let's model a real-world situation with quadratic functions.

Learning Outcomes

Today's Goals

1. **Goal:** Model situations with linear, exponential, and quadratic functions.
2. **Goal:** Use models to make predictions in context.
3. **Language Goal:** Explain when models are useful for predicting missing values. **(Writing, Speaking, and Listening)**

Students use linear, exponential, and quadratic models to make sense of real-world data about cell phone sales. They use these models to make predictions about missing data points within a set of data about the number of cell phones sold in a period of time. They consider how well different models fit the data and choose domains where the function fits the data well. They also explain the limitations of using a model outside of its useful domain.

Coherence

Prior Learning

In Units 2 and 5, students fit linear and exponential models to a data set.

Future Learning

In high school, students will represent relationships using exponential, quadratic, and polynomial functions, along with other families of functions. They will also learn about distributions and other ways to analyze data.

Connections

- Within the context of selling a new cell phone model, students use linear, quadratic, and exponential functions to model real world data and make predictions of future sales. (real world, mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.F.1.8

Determine whether a linear, quadratic or exponential function best models a given real-world situation.

MA.912.AR.3.8

Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.

Also Addressing: **MA.912.F.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.F.1.1	**MA.912.DP.2.8**

Vocabulary

Review Vocabulary

domain

Lesson at a Glance

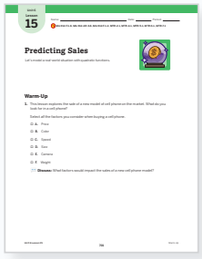
~ 45 min

MA.912.F.1.8, MA.912.F.1.2, MA.912.AR.3.8

Warm-Up

Pairs | 5 min

Students surface their prior knowledge about how cell phones are sold.

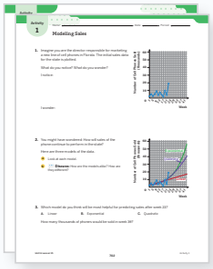


Activity 1

Pairs | 15 min

Students explore the power of mathematical models for making sense of real, messy data. **(MTR.7.1)**

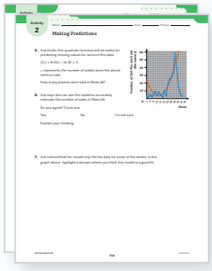
Routine(s): Notice and Wonder



Activity 2

Pairs | 15 min

Students restrict the domain of a quadratic model to determine when the model is useful for making predictions.



Synthesis

Whole Class | 5 min

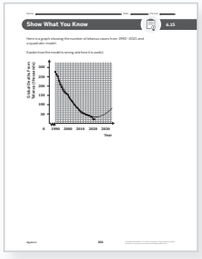
Students synthesize their understanding of making and asking questions about math models **Routine(s):** MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing a quadratic model of a real-world data set and explaining how the model is both wrong and useful. **(MTR.7.1)**



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students access their prior knowledge about how cell phones are sold.

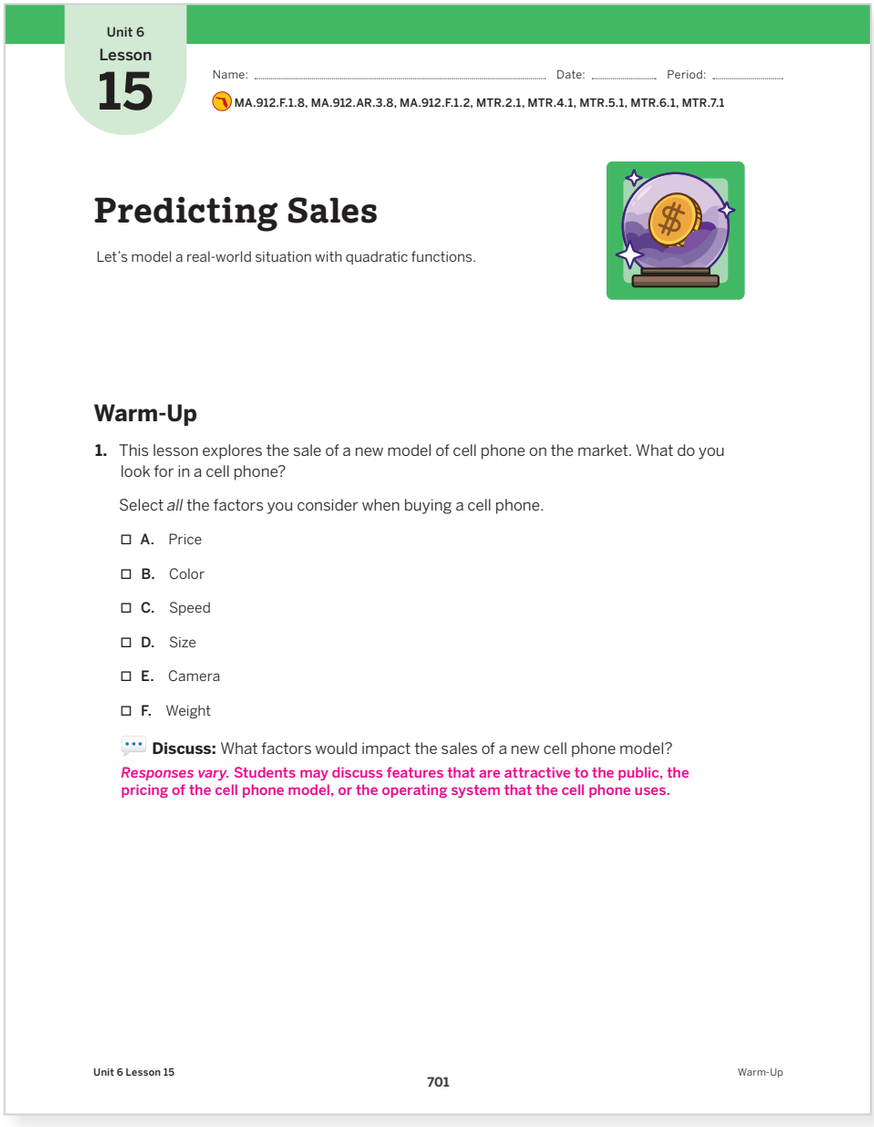
1 Launch

Consider sharing that in this lesson, students will be looking at data about cell phone sales and using the data to make predictions.

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means. Clarify any unfamiliar words. **(Reading)**

2 Connect

Consider asking, "Have you ever needed to sell a product or a good? What did you consider when making those sales?"



Unit 6
Lesson
15

Name: ..... Date: ..... Period: .....

MA.912.F.1.8, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Predicting Sales

Let's model a real-world situation with quadratic functions.

Warm-Up

1. This lesson explores the sale of a new model of cell phone on the market. What do you look for in a cell phone?

   Select *all* the factors you consider when buying a cell phone.

   - ☐ A. Price
   - ☐ B. Color
   - ☐ C. Speed
   - ☐ D. Size
   - ☐ E. Camera
   - ☐ F. Weight

   Discuss: What factors would impact the sales of a new cell phone model?

   Responses vary. Students may discuss features that are attractive to the public, the pricing of the cell phone model, or the operating system that the cell phone uses.

Unit 6 Lesson 15 701 Warm-Up

Pairs | 15 min

Activity 1 Modeling Sales

Purpose: Students explore the power of mathematical models for making sense of real, messy data. **(MTR.7.1)**

1 Launch

Invite students to choose one point from the available data and clarify what the point means as a class (e.g., $(9, 10)$ means that $10{,}000$ phones were sold in the ninth week.

Encourage students to use their fingers to find values between the axis labels on the graph.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the data. Invite students to share their noticings and wonderings.

Consider asking:

- "What do you think will happen to sales in Week 25? In Week 39?"
- "What do you think might explain what you noticed?"

2 Monitor

To support students getting started, consider asking, "Why might someone make a model? Why do you think the models don't exactly match the data?"

Listen for students' conversations about the models and what they notice as similarities and differences.

Consider asking, "Why are these models useful? What kinds of relationships do these models show?"

Encourage students to share with a partner or the class how they used the model they chose to make their prediction.

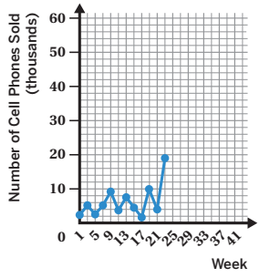
Activity 1 continued >

Activity 1

Name: Date: Period:

Modeling Sales

1. Imagine you are the director responsible for marketing a new line of cell phones in Florida. The initial sales data for the state is plotted.



What do you notice? What do you wonder?
Responses vary.
I notice:

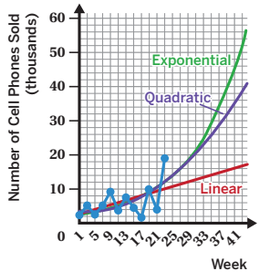
- I notice there are days with sharp increases in the number of sales.
- I notice that sales can increase or decrease from week to week.
- I notice there is a general upward trend in sales during the first thirteen weeks.

I wonder:

- I wonder what caused the spike in sales between the last two weeks.
- I wonder if the sales continued to increase after week 23 or if they declined sharply.
- I wonder why there was such a difference in sales from one week to the next.

2. You might have wondered: How will sales of the phone continue to perform in the state?

Here are three models of the data.



a. Look at each model.

b. **Discuss:** How are the models alike? How are they different?
Responses vary.

- All the models predict that the number of sales will go up over time.
- The exponential model grows much faster than the linear or quadratic models.
- All three models look about the same around week 21, and then they increase at different rates.

3. Which model do you think will be most helpful for predicting sales after week 23?

A. Linear **B.** Exponential **C.** Quadratic

How many thousands of phones would be sold in week 39?
Choices and responses vary.

Unit 6 Lesson 15 702 Activity 1

Pairs | 10 min

Activity 1 Modeling Sales (continued)

Purpose: Students explore the power of mathematical models for making sense of real, messy data. **(MTR.7.1)**

2 Monitor

Look for student responses to share during the Connect.

A **Accessibility: Visual-Spatial Processing** Invite students to pick specific points on the graph and describe what that point means to support interpreting visual representations.

ELL **English Language Learners** Provide sentence frames (e.g., "One thing the model gets right about phone sales is _____." and "One thing the model gets wrong about phone sales is _____."). **(Writing and Speaking)**

Encourage students to read others' responses and decide if others' thoughts are similar to or different from their own.

Note: If students ask how the models were created:

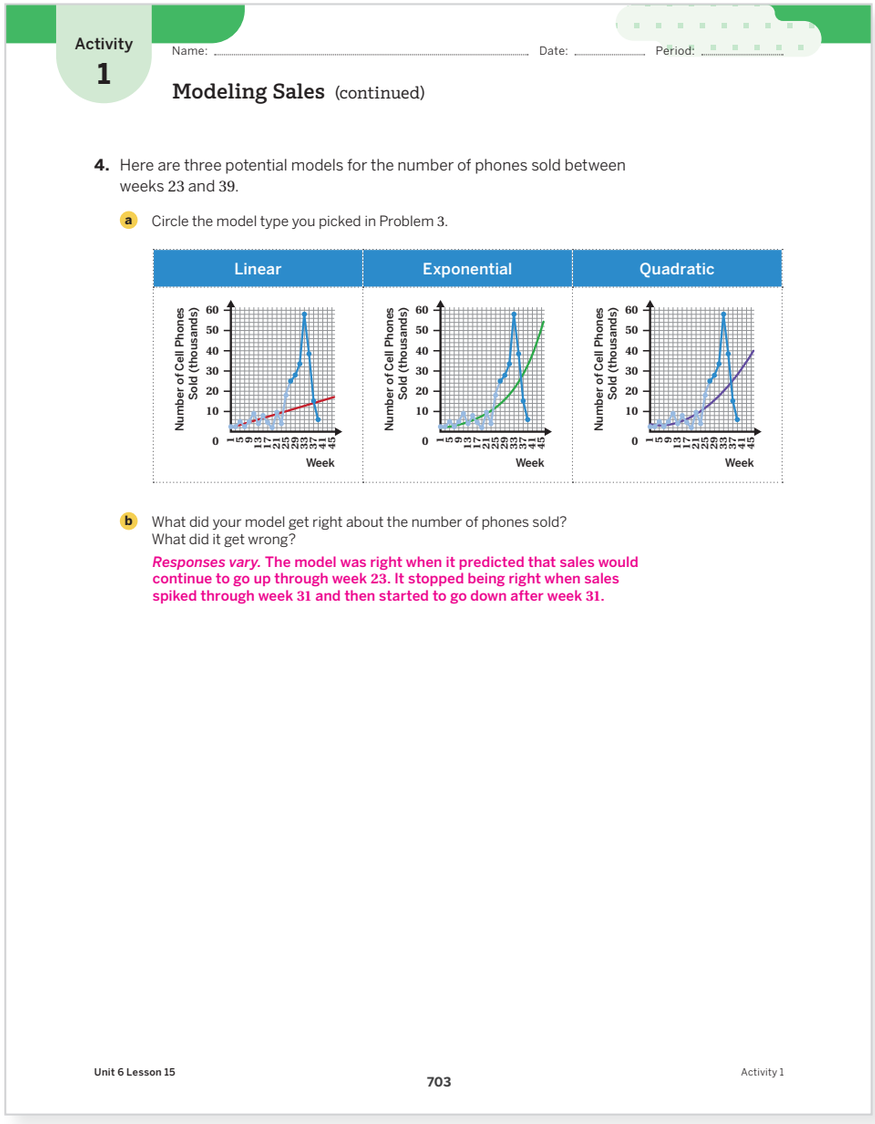
- The Linear model is a best-fit line for the data.
- The Exponential model is a best-fit exponential regression for the data.
- The Quadratic model is a best-fit quadratic regression for the data.

3 Connect

Share student responses and invite students to share more about their thinking.

To reveal the Key Takeaway, consider asking:

- "Why do you think none of these models perfectly predict cell phone sales?"
- "What might the consequences be — positive or negative — of wrong predictions? For whom?"



Activity 1

Name: _____ Date: _____ Period: _____

Modeling Sales (continued)

4. Here are three potential models for the number of phones sold between weeks 23 and 39.

a Circle the model type you picked in Problem 3.

b What did your model get right about the number of phones sold? What did it get wrong?

Responses vary. The model was right when it predicted that sales would continue to go up through week 23. It stopped being right when sales spiked through week 31 and then started to go down after week 31.

Unit 6 Lesson 15 703 Activity 1

Key Takeaway: Models of real-world data are often incorrect when made on a limited amount of data.

Pairs | 15 min

Activity 2 Making Predictions

Purpose: Students restrict the domain of a quadratic model to determine when the model is useful for making predictions.

Short on time? Consider having a class discussion on Problem 8 instead of having students create their own models.

1 Launch

Consider asking:

- "Why do you think Ava says this function will be useful for predicting missing values for some of the data?"
- "Does the model have to be a perfect fit for the data to be useful in making predictions? Why or why not?"

2 Monitor

Listen for pairs of students discussing how to use the different representations of Ava's model.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to decide whether they would prefer to make their prediction using the function or the graph. Invite students who choose the function to use the calculator to help them evaluate.
Evaluate the equation for Ava's model with $x = 1947$	**Strengthen** Celebrate the idea to use substitution to evaluate and remind students that, in the equation, x represents the number of years since 1920.

Look for student responses for all three options to share during the Connect.

ELL English Language Learners Consider inviting students to share what they remember about the terms *good fit* and *domain*. Consider creating a display or defining these terms publicly for students to refer back to in this activity. **(Reading and Listening)**

Collect and share the domains chosen by different pairs, highlighting any consensus that emerges and inviting students to share more about their thinking.

Activity 2 continued >

Activity 2

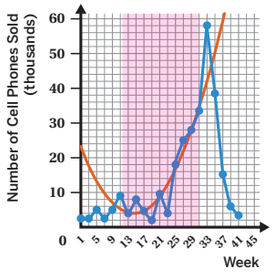
Name: Date: Period:

Making Predictions

5. Ava thinks this quadratic function will be useful for predicting missing values for some of the data:

$f(x) = 0.11(x - 14.3)^2 + 4$

x represents the number of weeks since the phone went on sale.



How many phones were sold in Week 28?

Approximately 24.65 thousand cases.

6. Ava says she can use the model to accurately estimate the number of sales in Week 28.

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

Responses and explanations vary.

- **Yes. Ava can use the model to predict sales in week 28 because her model looks like it intersects the line between the data points at week 27 and week 29.**
- **No. Ava cannot use her model because it was just lucky that her quadratic function intersected the data. After week 27, the sales were declining, but Ava's model still predicts an increase in sales.**

7. Ava noticed that her model only fits the data for some of the weeks. In the graph above, highlight a domain where you think this model is a good fit.

***Graphs vary.* The sample is shown on the graph above.**

Unit 6 Lesson 15 704 Activity 2

Pairs | 15 min

Activity 2 Making Predictions (continued)

Purpose: Students restrict the domain of a quadratic model to determine when the model is useful for making predictions.

Short on time? Consider having a class discussion on Problem 8 instead of having students create their own models.

2 Monitor

Consider asking, "Which part of the data are you planning to model? How do you know your model is a good fit for that domain?"

Look for student models that are good fits for different parts of the data to share during the Connect.

2 Connect

Invite students with each possible response to share, and consider using the sample responses in the Student Edition to share the reasoning for any options that might not be well-represented in the class distribution.

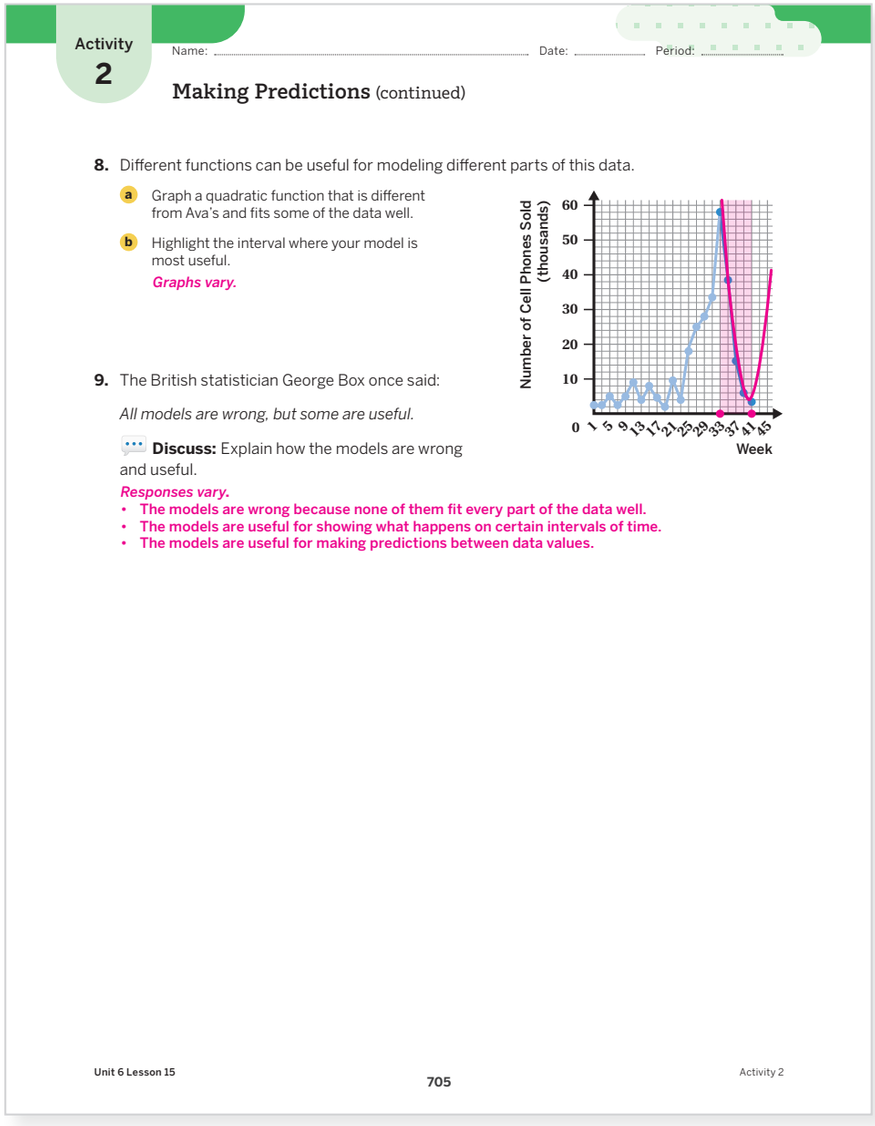
Consider sharing that when we use models to predict missing values within a domain of good fit, it is called *interpolation*. When using a model to predict outside of the highlighted domain, it is called *extrapolation*. Consider asking, "What issues might arise when extrapolating with a model?"

Share student models and invite students to share their reasoning for why they adjusted the model to fit that specific domain.

Invite students to explain their thinking about how each model is wrong and useful.

To reveal the Key Takeaway, consider asking:

- "Does the usefulness of the model depend on whether the model fits the data well in a specific domain? Why or why not?"
- "Can these models still be useful if they are not a good fit for part of the data set?"



Activity 2

Name: Date: Period:

Making Predictions (continued)

8. Different functions can be useful for modeling different parts of this data.

a. Graph a quadratic function that is different from Ava's and fits some of the data well.

b. Highlight the interval where your model is most useful.

Graphs vary.

9. The British statistician George Box once said:

All models are wrong, but some are useful.

Discuss: Explain how the models are wrong and useful.

Responses vary.

- The models are wrong because none of them fit every part of the data well.
- The models are useful for showing what happens on certain intervals of time.
- The models are useful for making predictions between data values.

Unit 6 Lesson 15 705 Activity 2

Key Takeaway: Models for real - world data sets can be helpful for predicting missing data points, even if it is only a good fit for a specific domain.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of making and asking questions about math models.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- You can make math models with many different kinds of functions, like linear, quadratic, and exponential functions.
- Math models usually fit some part of the data well and may have parts of the data they do not fit well.
- Math models can be used to interpolate missing values in a data set or extrapolate to make predictions about future values. Both kinds of predictions can be supported by models but are always uncertain.
- What other real-world issues can be modeled mathematically?

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about models as they share their thinking. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Select one of the questions to answer.

- ☐ What is something you learned about making math models?
- ☐ What is a question you have about making math models?

Responses vary.

- I learned that math models can help make predictions about the future.
- I learned that math models are sometimes very, very wrong, especially when the trend changes.
- I am wondering how people use math models in the world.
- Has anyone ever made a math model that turned out to be perfectly accurate?

Lesson Practice 6.15

Lesson Summary

Different types of functions can be used to model data and help us predict unknown data values. While models can be useful, they also have limitations. Some models may only be useful for predicting unknown values within a specific domain. Let's look at two quadratic functions that could model the data of a company's sheet pan sales, measured in thousands, since 1990.

Model A

$f(x) = -2.802(x-28)^2 + 954$

This model is a good fit for the data on the interval from 2000–2016. Beyond 2016, this model may not be useful because it suggests that sales will collapse and eventually go negative, which is impossible.

Model B

$f(x) = 4.253(x-15)^2 + 954$

This model is a good fit for the data on the interval from 2010–2020. Beyond 2020, this model may not be useful because it suggests that sales will increase to infinity, which is also impossible.

Unit 6 Lesson 15 706 Synthesis | Lesson Practice

Lesson Takeaway: Mathematical models can be used to fit real-world data. Imperfect models can still make predictions over the domain where the model is a good fit.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing a quadratic model of a real-world data set and explaining how the model is both wrong and useful. **(MTR.7.1)**

Today's Goals

1. **Goal:** Model situations with linear, exponential, and quadratic functions.
2. **Goal:** Use models to make predictions in context.
3. **Language Goal:** Explain when models are useful for predicting missing values. **(Writing, Speaking, and Listening)**

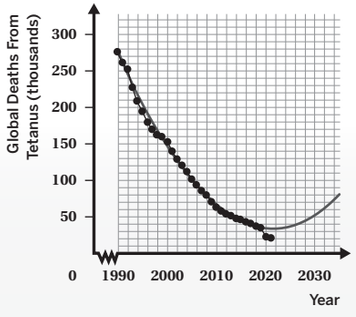
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.15

Here is a graph showing the number of tetanus cases from 1990–2021 and a quadratic model.

Explain how the model is wrong and how it is useful. ***Responses vary.***



- The model is wrong because I know there is a vaccine for tetanus, so I

D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how quadratic models are wrong and useful:

» **Respond to Student Thinking:**

- Revisit problem 10 as a class and invite students to share ways that models can be wrong and useful.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to collect data from their community and model it with a mathematical function.
- Invite students to explore the Sub-Unit 3 Extension Activities.

Professional Learning

Today students explored the effectiveness and the limitations of mathematical models. How can you continue to support students in being critical consumers and producers of data, specifically when deciding if mathematical models should be used to predict missing values? What other real-world contexts or situations that might be of interest to your students could you invite them to think about in terms of mathematical modeling?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Select one of the questions to answer.

- ☐ What is something you learned about making math models?
- ☐ What is a question you have about making math models?

Responses vary.

- *I learned that math models can help make predictions about the future.*
- *I learned that math models are sometimes very, very wrong, especially when the trend changes.*
- *I am wondering how people use math models in the world.*
- *Has anyone ever made a math model that turned out to be perfectly accurate?*

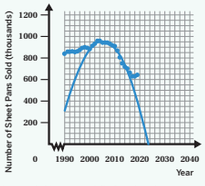
Lesson Practice 6.15

Lesson Summary

Different types of functions can be used to model data and help us predict unknown data values. While models can be useful, they also have limitations. Some models may only be useful for predicting unknown values within a specific domain. Let's look at two quadratic functions that could model the data of a company's sheet pan sales, measured in thousands, since 1990.

Model A

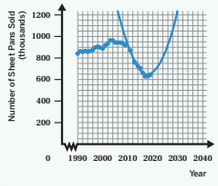
$f(x) = -2.802(x - 28)^2 + 954$



This model is a good fit for the data on the interval from 2000–2016. Beyond 2016, this model may not be useful because it suggests that sales will collapse and eventually go negative, which is impossible.

Model B

$f(x) = 4.253(x - 15)^2 + 954$

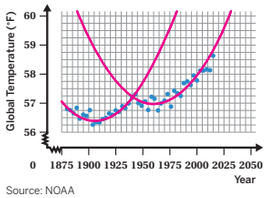


This model is a good fit for the data on the interval from 2010–2020. Beyond 2020, this model may not be useful because it suggests that sales will increase to infinity, which is also impossible.

Lesson Practice 6.15

Name: _____ Date: _____ Period: _____

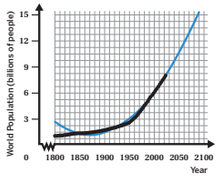
Problems 1–3: The graph shows the average global temperature, in degrees Fahrenheit, from 1875 to 2020.



Source: NOAA

1. Graph two different quadratic functions that could be useful for modeling different domains of this data. *Graphs vary.*
2. Use your model to predict the average global temperature, in degrees Fahrenheit, in 2025. *Responses vary.*
3. Do you think your model is useful for predicting average global temperature, in degrees Fahrenheit, after 2025? Explain your thinking. *Responses vary.*

Problems 4–6: The graph shows the world population in billions of people. Jacy modeled the data with a quadratic function.



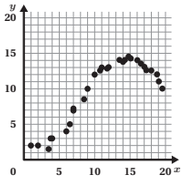
4. Describe an advantage of using Jacy's model.
   Responses vary. Jacy's model seems to follow the data very closely after 1980, so it may be a good predictor of future world population.
5. Describe a disadvantage of using Jacy's model.
   Responses vary. It shows that there was a minimum population around 1875, which doesn't make sense, as the world population has always generally been increasing.
6. How might Jacy improve this model?
   Responses vary. Jacy should specify the domain being used with the model.

Lesson Practice 6.15

Name: _____ Date: _____ Period: _____

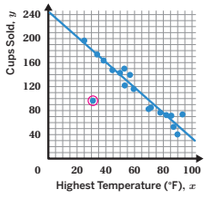
Test Practice

7. Which type of model could be used to best fit the data?
   - A. A linear function
   - B. An exponential function
   - C. A quadratic function
   - D. A combination of several different functions



Spiral Review

Problems 8–10: This scatter plot shows the number of cups of hot chocolate sold each day for different highest daily temperatures.



8. Circle the data point(s) that appear to be an outlier(s).
9. Use the linear model to estimate the number of cups sold when the highest temperature is 64°F.
   Responses between 105 and 115 cups are considered correct.
10. The equation $y = -2.11x + 245$ gives the number of cups sold, y, when the highest temperature is x °F. What does the number -2.11 mean in this situation?
    Responses vary. There are 2.11 fewer cups sold for each additional degree Fahrenheit of temperature increase.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3, 4–6	2	MA.912.F.1.8
	2	1	MA.912.F.1.8
Test Practice	7	2	MA.912.F.1.8
Spiral Review			
	8–9	1	MA.8.DP.1.2
Fluency	10	2	MA.8.AR.3.5

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Match different equation forms to a graph	Lesson 11	2	MA.912.AR.3.4, MA.912.AR.3.5 MTR.5.1
2a–b	Determine key features from equations in vertex form	Lesson 14	2	MA.912.AR.3.6, MA.912.AR.3.7 MTR.5.1
3	Graph quadratic functions written in factored form	Lesson 11	2	MA.912.AR.3.6, MA.912.AR.3.7
4a–b	Write quadratic equations given key features	Lesson 11	2	MA.912.AR.3.4, MA.912.AR.3.5 MTR.5.1
5a	Determine if relationships are quadratic or not	Lesson 3	1	MA.912.F.1.1, MA.912.F.1.8 MTR.6.1
5b–c	Identify and interpret key features from graphs and tables	Lessons 5, 7	2	MA.912.AR.3.7, MA.912.F.1.2 MTR.6.1
6	Match different equation forms to a graph	Lessons 8–11	2	MA.912.AR.3.4 MTR.4.1
7a	Use quadratic models to solve problems in context	Lessons 4, 12	2	MA.912.AR.3.6 MTR.6.1
7b	Use quadratic models to solve problems in context	Lessons 4, 12	2	MA.912.AR.3.6 MTR.6.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



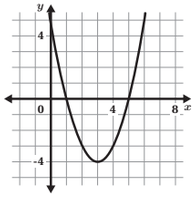
- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

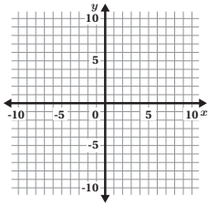
Unit 6

1. Select *all* the equations that represent this graph.



- ☐ A. $y = x^2 - 6x - 5$
- ☐ B. $y = (x + 1)(x + 5)$
- ☐ C. $y = (x - 3)^2 + 4$
- ☑ D. $y = (x - 3)^2 - 4$
- ☑ E. $y = (x - 1)(x - 5)$

2. Here is a function: $g(x) = (x + 4)^2 + 7$.
Use the graph if it helps with your thinking.

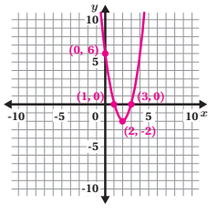


a. What is the vertex of the graph of the function g?
(-4, 7)

b. Does the vertex represent the *minimum* or the *maximum* value of the function?
Minimum

Explain or show how you know.
Explanations vary. **The graph of g is concave up.**

3. Sketch the graph of $p(x) = (x - 1)(2x - 6)$.
Include the x-intercepts, y-intercept, and vertex.



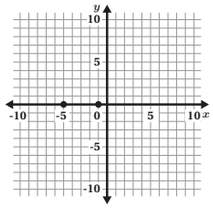
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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

4. Here are the points (-1, 0) and (-5, 0).



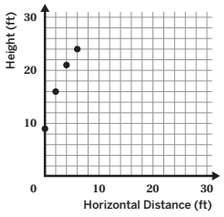
a. Write a quadratic equation whose graph passes through the points (-1, 0) and (-5, 0).
Equations vary.
- $y = (x + 1)(x + 5)$
- $y = (x + 3)^2 - 4$
- $y = -2(x + 1)(x + 5)$

b. Write a different quadratic equation whose graph passes through the points (-1, 0) and (-5, 0).
See equations in 4a.

How are the parabolas of the two equations you wrote alike? How are they different?
Responses vary.
- **Alike: The parabolas have the same x-intercepts.**
- **Different: One parabola is concave up; the other is concave down.**

5. A robot launches a ball into the air. The table and graph show some of the ball's horizontal distances from the robot and its heights above the ground.

Horizontal Distance (ft)	Height (ft)
0	9
2	16
4	21
6	24



a. What type of relationship is represented?

A. Linear B. Exponential (C.) Quadratic D. Something else

b. What is the greatest height the ball reaches?
25 feet

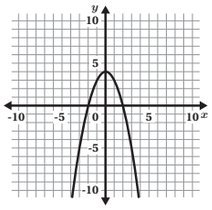
c. At what distance from the robot does the ball land?
18 feet

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

6. Esteban says the equation of this graph is $y = -(x + 2)(x - 2)$.



Josiah says the equation is $y = x^2 + 4$.

Whose thinking is correct? Circle one.

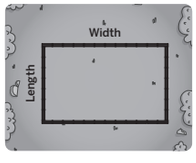
(Esteban's) Josiah's Both Neither

Explain your thinking.
Explanations vary. **Esteban's thinking is correct because his equation has the correct x-intercepts and is concave down. Josiah's equation can't be right because its graph will be concave up.**

7. A garden club is building a rectangular garden made from 60 feet of fencing.

a. Determine the length, width, and area of three different gardens that will use exactly 60 feet of fencing.

	Length (ft)	Width (ft)	Area (sq. ft)
Garden 1	20	10	200
Garden 2	15	15	225
Garden 3	25	5	125



The function $f(x) = x(30 - x)$ models the area of the garden, where x is the garden's length.

b. According to the model, what is the largest possible area for the garden?
225 square feet

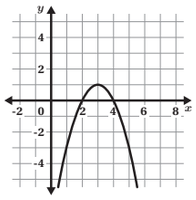
Explain your thinking.
Explanations vary. **The x-intercepts of the function are at (0, 0) and (30, 0). This means the vertex has an x-value of 15. If I plug in 15, I get the maximum value, which is 225 square feet.**

Name: Date: Period:

End-of-Unit Assessment

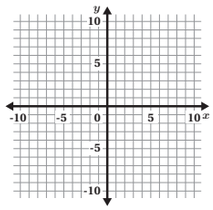
Unit 6

1. Select *all* the equations that represent this graph.



- [x] **A.** $y = -(x - 2)(x - 4)$
- [ ] **B.** $y = -(x + 2)(x + 4)$
- [x] **C.** $y = -(x - 3)^2 + 1$
- [ ] **D.** $y = -(x + 3)^2 - 1$
- [x] **E.** $y = -x^2 + 6x - 8$

2. Here is a function: $g(x) = -(x + 3)^2 + 4$.
Use the graph if it helps with your thinking.

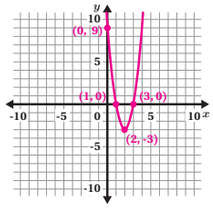


a What is the vertex of the graph of the function g?
(-3, 4)

b Does the vertex represent the *minimum* or the *maximum* value of the function?
Maximum

Explain or show how you know.
Explanations vary. **The graph of g is concave down.**

3. Sketch the graph of $p(x) = (3x - 9)(x - 1)$.
Include the x-intercepts, y-intercept, and vertex.



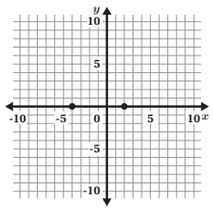
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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

4. Here are the points (2, 0) and (-4, 0).



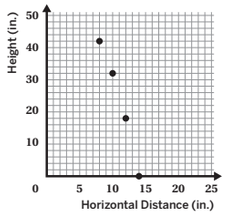
a Write a quadratic equation whose graph passes through the points (2, 0) and (-4, 0).
Equations vary.
- $y = (x - 2)(x + 4)$
- $y = (x + 1)^2 - 9$
- $y = -(x - 2)(x + 4)$

b Write a different quadratic equation whose graph passes through the points (2, 0) and (-4, 0).
See equations in 4a.

How are the parabolas of the two equations you wrote alike? How are they different?
Responses vary.
- **Alike: The parabolas have the same x-intercepts.**
- **Different: One parabola is concave up; the other is concave down.**

5. A robot launches a ball into the air. The table and graph show the ball's horizontal distances from the robot and its heights above the ground.

Horizontal Distance (in.)	Height (in.)
8	42
10	32
12	18
14	0



a What type of relationship is represented?

A. Linear **B.** Exponential **(C.)** Quadratic **D.** Something else

b What is the greatest height the ball reaches?
50 inches

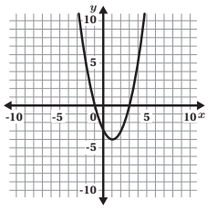
c What is the ball's height above the ground when its horizontal distance from the robot is 0?
42 inches

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

6. Emiliano says the equation of this graph is $y = (x + 1)(x - 3)$.



Dhruv says the equation is $y = x^2 - 2x - 3$.

Whose thinking is correct? Circle one.

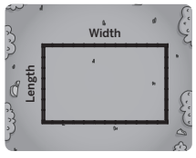
Esteban's Dhruv's (Both) Neither

Explain your thinking.
Explanations vary. **Both are correct because both equations have the same intercepts and are concave up.**

7. A farmer is building a rectangular sheep pen made from 48 feet of fencing.

a Determine the length, width, and area of three different sheep pens that will use exactly 48 feet of fencing.

	Length (ft)	Width (ft)	Area (sq. ft)
Pen 1	4	20	80
Pen 2	9	15	135
Pen 3	10	14	140



The function $f(x) = x(24 - x)$ models the area of the sheep pen, where x is the pen's width.

b According to the model, what is the largest possible area for the pen?
144 square feet

Explain your thinking
Explanations vary. **The x-intercepts of the function are at (0, 0) and (24, 0). This means the vertex has an x-value of 12. If I plug in 12, I get the maximum value, which is 144 square feet.**

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Justify whether a function is linear, quadratic, exponential, or none of these. **(Lessons 1–3)**	5a	**Support** **Strengthen** **Stretch** • **Teacher Move:** Revisit Lesson 3, Problems 5–9 (*Sorting Relationships*) and discuss student strategies for classifying functions in multiple representations. • **Practice Day 1:** Cards 8, 10 • **Practice:** Lessons 1–3 • **DOK3 Practice:** Lesson 3, 7
Identify and interpret key features of quadratic functions in graphs and tables. **(Lessons 4–7)**	5b–c	**Support** **Strengthen** **Stretch** • **Lesson:** Lesson 5 (*Stomp Rockets*) • **Practice Day 1:** Cards 7, 11 • **Practice Day 2:** Hexagon Sheet • **Practice:** Lessons 4–7 • **You're invited to explore more:** Lesson 5 (*Stomp Rockets*) • **You're invited to explore more:** Lesson 6 (*Plenty of Parabolas*) • **DOK3 Practice:** Lesson 6
Determine key features and graph quadratic functions from standard and factored form equations. **(Lessons 8–10)**	3	**Support** **Strengthen** **Stretch** • **Teacher Move:** Review strategies for graphing such as creating a table or using the structure of the equation to identify key features. • **Practice Day 1:** Cards 2, 4, 6, 9 • **Practice:** Lessons 8–10 • **DOK3 Practice:** Lessons 8–10
Write quadratic equations from a graph, description, or criteria. **(Lesson 11)**	1, 4, 6	**Support** **Strengthen** **Stretch** • **Teacher Move:** Consider revisiting a class display about the three equation forms and how each part is connected to key features. • **Practice Day 1:** Cards 4, 12 • **Practice:** Lessons 12–16 • **Teacher Move:** Invite students to revisit Lesson 11, Activity 2 and complete the challenges in a different way or with fewer parabolas. • **DOK3 Practice:** Lessons 10–11 • **You're invited to explore more:** Practice Day 2
Create quadratic models to solve problems in context. **(Lessons 12, 15)**	7	**Support** **Strengthen** • **Teacher Move:** Consider discussing strategies for determining the maximum value of a function using the Synthesis and Summary from Lesson 12 for context. • **Practice Day 1:** Cards 2, 3, 9, 11 • **Practice Day 2:** Scavenger Hunt Sheets (Crescent and Octagon)
Describe the effect of transformations on key features of a parabola and its equation in vertex form. **(Lessons 13–14)**	2	**Strengthen** **Stretch** • **Repeated Challenges:** Lesson 13 (*Vertex Form*) • **Repeated Challenges:** Lesson 14 (*Stretch It Out*) • **Practice:** Lessons 13–14 • **You're invited to explore more:** Lesson 13 (*Vertex Form*) • **DOK3 Practice:** Lesson 13 • **Teacher Move:** Invite students to transform functions of their choice by entering any function $f(x)$ in the Desmos Graphing Calculator, and then entering $af(x-h)+k$ with sliders. Encourage them to explore and discuss with a partner.



UNIT 7

Quadratic Equations

Students solve quadratic equations and systems of equations using reasoning, factoring, the zero-product property, completing the square, and the quadratic formula. They convert expressions between factored, standard, and vertex form to support them in solving equations and identifying key features of functions.

Essential Questions

- How can you solve quadratic equations symbolically and graphically?
- How can you use the structure of the equation, available tools, and your personal mathematical preferences to select a solving method strategically?
- How can rewriting expressions in different forms support both solving equations and identifying key features of functions?

Sub-Unit Goals

Sub-Unit 1

- Convert quadratic expressions between factored and standard form by multiplying and factoring.
- Use factoring and the zero-product property to solve equations and determine the zeros of quadratic functions.
- Divide a polynomial by a monomial and rewrite polynomial expressions by factoring.

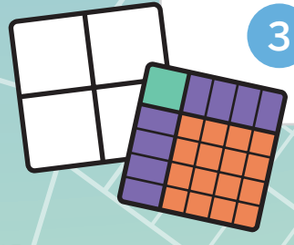
Sub-Unit 2

- Solve quadratic equations with no solutions, one solution, and two solutions using reasoning, graphs, and completing the square.
- Complete the square in order to reveal the vertex of a quadratic function.

Sub-Unit 3

- Derive the quadratic formula and use it to solve quadratic equations.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Solve quadratic equations in one variable. **(MA.912.AR.3.1)**

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **(MA.912.AR.1.7)**

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **(MA.912.AR.3.6)**

Other Standards Addressed in This Unit: MA.912.AR.1.2 MA.912.AR.1.3 MA.912.AR.1.4 MA.912.AR.3.8 MA.912.NSO.1.1 MA.912.NSO.1.2 MA.912.NSO.1.4

Mathematical Thinking and Reasoning Standards

- Students use the structure of equations, available tools, and their personal mathematical preferences in order to help them strategically choose solving methods. **(MTR.2.1)**
- Students use the structure of equations in order to determine the number of solutions, select a solving strategy, and rewrite equations in a different form. **(MTR.5.1)**
- Students communicate precisely about mathematical processes and express numerical solutions approximately and exactly. **(MTR.4.1)**

Also addressed: **MTR.1.1, MTR.3.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students use various strategies, such as factoring, completing the square, graphing, and the quadratic formula to solve quadratic equations.

Prior Learning

- **Grade 8:** Approximates square roots **(MA.8.NSO.1.1, MA.8.AR.2.3)**
- **Algebra 1:** Equivalent expressions **(MA.912.AR.1.7)**
- **Algebra 1:** Solve equations **(MA.912.AR.2.6)**
- **Grade 6:** Evaluate expressions **(MA.6.AR.1.4)**
- **Algebra 1:** Key function features **(MA.912.AR.3.7)**

Future Learning

- **Algebra 2:** Quadratic Equations **(MA.912.AR.3.1)**
- **Algebra 2:** Polynomial Functions **(MA.912.AR.1.7)**
- **Algebra 2:** Polynomial Functions **(MA.912.AR.1.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 11,** students develop conceptual understanding as they connect the structure of quadratic equations and their number of solutions to their graphs.

Procedural Fluency

In **Lesson 2,** students develop procedural fluency as they practice multiplying to rewrite factored-form expressions in standard form.

Application

In **Lesson 18,** students apply quadratic solving strategies to quadratic equations and explain what the solutions mean in context.

Strategic Competence

In **Lesson 15,** students develop strategic competence as they compare and use different strategies to complete the square.

Adaptive Reasoning

In **Lesson 5,** students develop adaptive reasoning as they use and reflect on multiple strategies to factor quadratic expressions.

Productive Disposition

In **Lesson 14,** students have opportunities to develop a productive disposition as they discover that there are many ways to solve quadratic equations.

Unit at a Glance

Assess and Respond

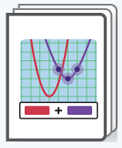


A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.NSO.1.1 MA.6.NSO.3.2 MA.6.NSO.3.3 MA.6.AR.1.3 MA.7.AR.1.2 MA.8.NSO.1.1 MA.8.AR.1.3 MA.8.AR.2.1 MA.8.AR.2.3
MTR.4.1 MTR.5.1

Sub-Unit 1

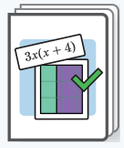


1 Sums and Differences

Adding and Subtracting Linear and Quadratic Expressions

Explore functions and expressions created by combining linear and quadratic expressions using addition and subtraction.

MA.912.AR.1.3 MA.912.NSO.1.1
MTR.1.1 MTR.4.1 MTR.5.1

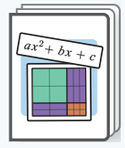


2 Two-Factor Multiplication

Rewriting Factored-Form Expressions in Standard Form

Use area models and diagrams to multiply expressions written in factored form and rewrite them in standard form.

MA.912.AR.1.3 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.1.1 MTR.3.1 MTR.4.1 MTR.5.1



3 Standard Feature

Patterns in Factored-Form and Standard-Form Expressions

Practice multiplying and use the structure of factored-form expressions to make predictions about equivalent expressions in standard form.

MA.912.AR.1.3 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.1.1 MTR.3.1 MTR.5.1

Assess and Respond



8 Shooting Stars

Determining the x-Intercepts of Quadratic Functions

Determine the x-intercepts of quadratic functions written in factored form and standard form.

MA.912.AR.1.7 MA.912.AR.3.4 MA.912.AR.3.6
MTR.2.1 MTR.4.1 MTR.5.1

Sub-Unit 2



9 Make It Zero

Solving Quadratic Equations Using the Zero-Product Property

Use the zero-product property to solve quadratic equations.

MA.912.AR.1.2 MA.912.AR.1.7 MA.912.AR.3.1 MA.912.AR.3.6 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.4.1 MTR.5.1



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.1 MA.912.AR.3.4 MA.912.AR.3.6 MA.912.AR.3.7 MA.912.AR.3.8
MTR.3.1 MTR.5.1



10 Zero, One, or Two?

Solving Equations by Reasoning

Use structure to determine whether a quadratic equation has zero, one, or two solutions.

MA.912.AR.3.1
MTR.4.1 MTR.5.1 MTR.6.1

Sub-Unit 3

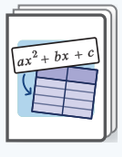


15 Back and Forth

Rewriting Quadratic Expressions in Vertex Form

Complete the square in order to reveal the vertex of a quadratic function.

MA.912.AR.1.2 MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.1
MTR.2.1 MTR.4.1 MTR.5.1 MTR.6.1

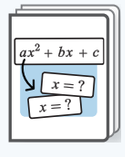


16 Formula Foundations

Introducing the Quadratic Formula

Explore how the quadratic formula can be derived from the process of completing the square.

MA.912.AR.1.2 MA.912.AR.3.1 MA.912.NSO.1.4
MTR.3.1 MTR.4.1 MTR.5.1



17 Formula Fluency

Solving Quadratic Equations Using the Quadratic Formula

Use the quadratic formula to solve quadratic equations.

MA.912.AR.3.1 MA.912.NSO.1.4
MTR.4.1 MTR.5.1 MTR.6.1



18 Stomp Rockets in Space

Solving Quadratic Equations in Context

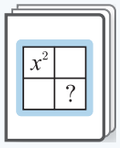
Solve quadratic equations and interpret their solutions in context.

MA.912.AR.3.1 MA.912.AR.3.8 MA.912.NSO.1.4
MTR.3.1 MTR.6.1 MTR.7.1

Pacing: 25 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 18 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

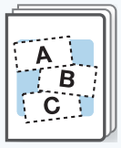


4 X-Factor

Factoring Quadratic Expressions

Rewrite standard-form quadratic expressions in factored form.

MA.912.AR.1.7 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.3.1 MTR.5.1 MTR.6.1

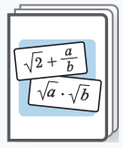


5 Form Up

More Factoring Quadratic Expressions

Identify patterns that can support them in factoring quadratic expressions with special structures, and then practice factoring.

MA.912.AR.1.4 MA.912.AR.1.7 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.4.1 MTR.5.1



6 Divide and Conquer

Dividing Polynomial Expressions

Dividing polynomials by monomials.

MA.912.AR.1.4 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.3.1 MTR.4.1 MTR.5.1



7 Consider the Factors

Factoring Polynomial Expressions

Rewriting polynomial expressions by factoring.

MA.912.AR.1.4 MA.912.AR.1.7 MA.912.NSO.1.1 MA.912.NSO.1.2
MTR.2.1 MTR.5.1



11 Graph to Solve

Solving Quadratic Equations by Graphing

Use the Graphing Calculator to solve quadratic equations with zero, one, or two solutions.

MA.912.AR.3.1 MA.912.AR.3.6
MTR.2.1 MTR.4.1 MTR.5.1



12 Couldn't Square Less

Solving by Taking the Square Root

Solve quadratic equations by taking the square root of each side.

MA.912.AR.1.2 MA.912.AR.1.7 MA.912.AR.3.1 MA.912.NSO.1.4
MTR.2.1 MTR.4.1 MTR.5.1 MTR.6.1

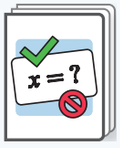


13 Square Dance

Perfect Square Expressions

Explore and create perfect square quadratic expressions in standard form.

MA.912.AR.1.2 MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.1
MTR.2.1 MTR.4.1 MTR.5.1



14 Square Tactic

Solving by Completing the Square

Develop a new strategy for solving quadratic equations called "completing the square."

MA.912.AR.1.2 MA.912.AR.1.7 MA.912.AR.3.1 MA.912.NSO.1.4
MTR.5.1 MTR.6.1

Assess and Respond



M Math in Action

Connecting Equations to History

Explore the connection between equations and history, and the contributions of Po-Shen Loh Cameron to quadratic equations.

MA.912.AR.3.6
MTR.3.1

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.1 MA.912.AR.3.4 MA.912.AR.3.6 MA.912.AR.3.7 MA.912.AR.3.8
MTR.3.1 MTR.4.1 MTR.5.1 MTR.7.1

Pacing Considerations

Lesson 1: This lesson can be omitted because the standards covered will also be covered in Algebra 2.

Lessons 2–3: These lessons support students in multiplying quadratic expressions in factored form to rewrite them in standard form. If most students demonstrate a strong understanding of multiplication in Problems 1 and 2 of the Pre-Unit Check, these lessons may be combined. One way to combine is to complete Activity 2 of Lesson 2 and Activity 1 of Lesson 3.

Lessons 8–9: In Lesson 8, students factor equations to determine their zeros. Lesson 9 supports students in solving equations using the zero-product property. If students demonstrate a strong understanding of identifying x-intercepts in Unit 6, Lesson 10 (Interesting Intercepts), Lessons 8 and 9 may be combined.

Lessons 16–17: Lesson 15 supports students in deriving the quadratic formula and understanding how it connects to completing the square. In Lesson 16, students use the quadratic formula to solve quadratic equations. If time runs short, consider omitting Lesson 15, and treat Lesson 16 as the introduction to the quadratic formula.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

Copy: Show What You Know

Materials: graphing calculators *(optional)*, coloring tools *(as needed)*

Lesson 2

Copy: Show What You Know

Materials: scientific calculators *(as needed)*, graph paper *(optional)*

Lesson 3

Copy: Show What You Know

Materials: scientific calculators *(optional)*, coloring tools *(optional)*

Lesson 4

Copy: Show What You Know

Materials: scientific calculators *(as needed)*, graph paper *(optional)*

Lesson 5

Copy: Show What You Know, Activity 2 Cards, one set per 32 students, pre-cut

Materials: scientific calculators *(as needed)*, multiplication chart *(as needed)*

Lesson 6

Copy: Show What You Know

Materials: *no required materials*

Lesson 7

- Activity 1 Sheet (**Lesson 7**)
- Activity 3 Cards (optional) (**Lesson 7**)
- ruler (**Lesson 7**)

Lesson 8

Copy: Show What You Know

Materials: scientific calculators *(as needed)*, graph paper *(optional)*

Lesson 9

Copy: Show What You Know

Materials: scientific calculators *(as needed)*

Lesson 10

Copy: Show What You Know

Materials: *no required materials*

Lesson 11

Copy: Show What You Know

Materials: graphing calculators

Lesson 12

Copy: Show What You Know

Materials: scientific calculators, highlighters *(optional)*

Lesson 13

Copy: Show What You Know, You're Invited to Explore More Sheet *(optional)*

Materials: algebra tiles, calculators (*as needed*)

Lesson 14

Copy: Show What You Know

Materials: algebra tiles *(as needed)*

Lesson 15

Copy: Show What You Know, Warm Up and Activity 2 Cards

Materials: scientific calculators *(optional)*

Lesson 16

Copy: Show What You Know

Materials: *no required materials*

Lesson 17

Copy: Show What You Know, Activity 2 Cards, one per pair

Materials: coloring tools *(as needed)*, scientific calculators

Lesson 18

Copy: Show What You Know, Activity 3 Sheet

Materials: scientific calculators

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Forms A and B
- Assessment Rubrics
- Show What You Knows



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



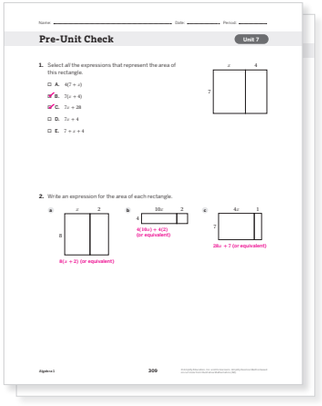
Assessments

Every unit includes a variety of formative and summative assessments, designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.NSO.3.1, MA.6.NSO.3.2, MA.6.NSO.3.3, MA.6.AR.1.3, MA.7.AR.1.2, MA.8.NSO.1.1, MA.8.AR.2.1, MA.8.AR.2.3


Pre-Unit Check

Unit 7

1. Select all the expressions that represent the area of this rectangle.

A. $4(7 + x)$

B. $7(x + 4)$

C. $7x + 28$

D. $7x + 4$

E. $7 + x + 4$

2. Write an expression for the area of each rectangle.

a. $8(x + 2)$ (or equivalent)

b. $6(18x) + 6(2)$ (or equivalent)

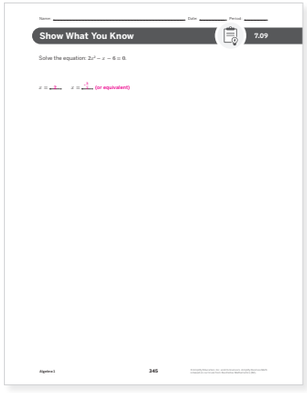
c. $28x + 7$ (or equivalent)

Algebra 1 309

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmark assessed on Lesson 9 Show What You Know: MA.912.AR.3.1


Show What You Know

7.09

Solve the equation: $2x^2 - x - 6 = 0$.

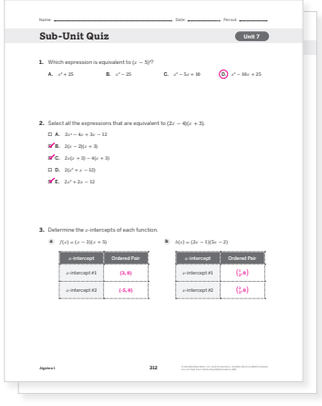
$x = \frac{3}{2}$ $x = -2$ (or equivalent)

Algebra 1 345

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed on the Sub-Unit Quiz: MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.5, MA.912.AR.3.6


Sub-Unit Quiz

Unit 7

1. Which expression is equivalent to $(x - 5)^2$?

A. $x^2 + 25$ B. $x^2 - 25$ C. $x^2 - 5x + 10$ D. $x^2 - 10x + 25$

2. Select all the expressions that are equivalent to $(2x - 4)(x + 3)$.

A. $2x^2 - 4x + 3x - 12$

B. $2(x - 2)(x + 3)$

C. $2x(x + 3) - 4(x + 3)$

D. $2(x^2 + x - 12)$

E. $2x^2 + 2x - 12$

3. Determine the x-intercepts of each function.

a. $f(x) = (x - 3)(x + 5)$

x-intercept	Ordered Pair
x-intercept #1	$(3, 0)$
x-intercept #2	$(-5, 0)$

b. $h(x) = (2x - 1)(5x - 2)$

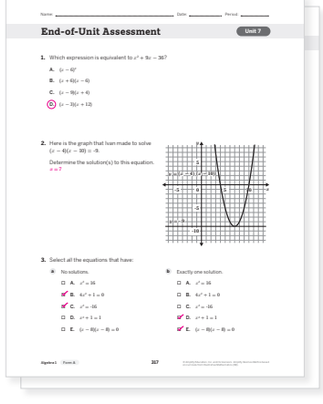
x-intercept	Ordered Pair
x-intercept #1	$(\frac{1}{2}, 0)$
x-intercept #2	$(\frac{2}{5}, 0)$

Algebra 1 312

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8


End-of-Unit Assessment

Unit 7

1. Which expression is equivalent to $x^2 + 9x - 36$?

A. $(x - 6)^2$

B. $(x + 6)(x - 6)$

C. $(x - 9)(x + 6)$

D. $(x - 3)(x + 12)$

2. Here is the graph that Ivan made to solve $(x - 6)(x - 10) = -8$.

Determine the solution(s) to this equation.

$x = 7$

3. Select all the equations that have:

a. No solutions.

A. $x^2 = 36$

B. $4x^2 + 1 = 0$

C. $x^2 = -16$

D. $x^2 + 1 = 1$

E. $(x - 8)(x - 8) = 0$

b. Exactly one solution.

A. $x^2 = 16$

B. $4x^2 + 1 = 0$

C. $x^2 = -16$

D. $x^2 + 1 = 1$

E. $(x - 8)(x - 8) = 0$

Algebra 1 Form A 317

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

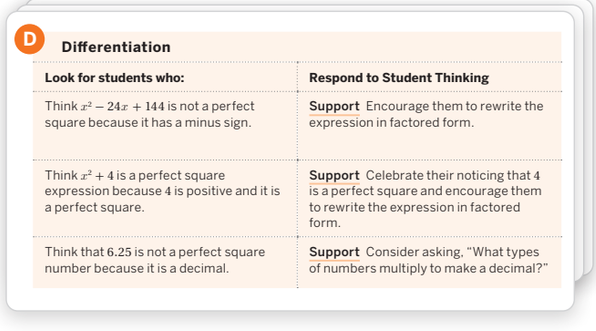
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–9) • Convert quadratic expressions between factored and standard form by multiplying and factoring. • Use factoring and the zero-product property to solve equations and determine the zeros of quadratic functions. • Divide a polynomial by a monomial and rewrite polynomial expressions by factoring.	**Mini-Lesson:** Rewriting Factored-Form Expressions in Standard Form **Previous Lessons** • Grade 6, Unit 2, Lesson 7 (*Common Factors*) • Grade 6, Unit 5, Lesson 2 (*Garden Arrangements*) • Unit 6, Lesson 9 (*Two For One*) • Unit 6, Lesson 10 (*Interesting Intercepts*)	**Practice** • Lessons 1–9	**You're invited to explore more.** • Lesson 3 (*Standard Feature*) • Lesson 8 (*Shooting Stars*) • Lesson 9 (*Make It Zero*) **Teacher Moves** Invite students to create their own quadratic expression challenges in factored and/or standard form.
Sub-Unit 2 (Lessons 10–15) • Solve quadratic equations with no solutions, one solution, and two solutions using reasoning, graphs, and completing the square. • Complete the square in order to reveal the vertex of a quadratic function.	**Mini-Lessons** • Solving Quadratic Equations by Graphing • Solving Quadratic Equations by Completing the Square **Previous Lessons** • Grade 8, Unit 4, Lesson 2 (*All, Some, or None? Part 1*) • Unit 1, Lesson 3 (*Same Position*) • Unit 4, Lesson 4 (*Lizard Lines*) **Teacher Moves** Consider creating a class display of each solving strategy and their advantages and disadvantages throughout the unit.	**Repeated Challenges** • Lesson 10 (Zero, One, or Two?) • Lesson 11 (*Graph to Solve*) • Lesson 13 (*Square Dance*) **Practice** • Lessons 10–15	**You're invited to explore more.** • Lesson 12 (*Couldn't Square Less*) • Lesson 13 (*Square Dance*) • Lesson 14 (*Square Tactic*) • Lesson 15 (*Back and Forth*)
Sub-Unit 3 (Lessons 16–18) • Derive the quadratic formula and use it to solve quadratic equations.	**Mini-Lesson:** Solving Quadratic Equations by Using the Quadratic Formula **Previous Lessons** • Grade 6, Unit 6, Lesson 11 (*Exponent Expressions*) • Grade 8, Unit 8, Lesson 4 (*Root Down*)	**Challenge Creator** • Lesson 18 (*Stomp Rockets in Space*) **Practice** • Lessons 16–18	**Teacher Moves** Invite students to solve quadratic equations using multiple methods (completing the square, factoring, graphing, quadratic formula) to check their solutions and identify the strengths of each method.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 13**, where students determine whether each expression is a perfect square or not.



Differentiation

Look for students who:	Respond to Student Thinking
Think $x^2 - 24x + 144$ is not a perfect square because it has a minus sign.	**Support** Encourage them to rewrite the expression in factored form.
Think $x^2 + 4$ is a perfect square expression because 4 is positive and it is a perfect square.	**Support** Celebrate their noticing that 4 is a perfect square and encourage them to rewrite the expression in factored form.
Think that 6.25 is not a perfect square number because it is a decimal.	**Support** Consider asking, "What types of numbers multiply to make a decimal?"

A Accessibility

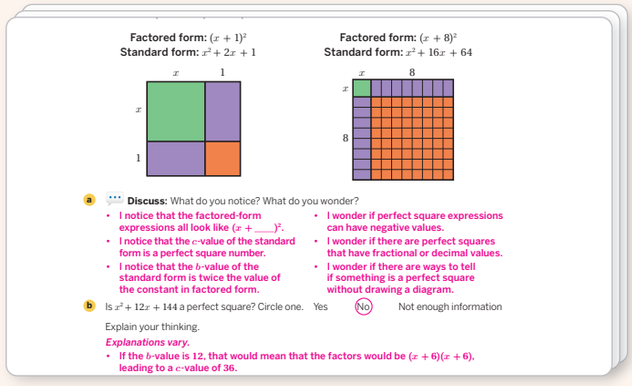
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Perception . . .

Lesson 13: Students make connections between area models and quadratic expressions in their factored form.



Factored form: $(x + 1)^2$
Standard form: $x^2 + 2x + 1$

Factored form: $(x + 8)^2$
Standard form: $x^2 + 16x + 64$

a. **Discuss:** What do you notice? What do you wonder?

- I notice that the factored-form expressions all look like $(x + ____)^2$.
- I notice that the c-value of the standard form is a perfect square number.
- I notice that the b-value of the standard form is twice the value of the constant in factored form.
- I wonder if perfect square expressions can have negative values.
- I wonder if there are perfect squares that have fractional or decimal values.
- I wonder if there are ways to tell if something is a perfect square without drawing a diagram.

b. Is $x^2 + 12x + 144$ a perfect square? Circle one. Yes (No) Not enough information

Explain your thinking.

Explanations vary.

- If the b-value is 12, that would mean that the factors would be $(x + 6)(x + 6)$, leading to a c-value of 36.

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Memory and Attention . . .

Here are two suggestions in this unit for supporting students with reducing the cognitive demand as they develop conceptual understanding of factoring quadratic equations.

Lesson 3, Activity 1

A **Accessibility: Memory and Attention** Consider displaying a labeled example of factored form and standard form for students to reference throughout the lesson.

Lesson 5, Warm-Up

A **Accessibility: Memory and Attention** IInvite students to use a calculator to support them with performing calculations and/or draw diagrams to support them with their thinking. Consider providing multiplication charts or reviewing integer operation rules for students to reference throughout this sub-unit.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
zeros	8
zero-product property	9
± (plus/minus symbol)	12
perfect square	13
completing the square	14
quadratic formula	17
closure	6
polynomials	6
remainders	6
difference of squares	7
factoring by grouping	7
perfect square trinomials	7

Review Vocabulary

coefficient
elimination
domain
irrational number
factor (of a number or expression)
factored form
point of intersection
range (of a function)
rational number
solution
solution to a system of equations
square root
standard form (of a quadratic equation)
substitution
system of equations
vertex form
x-intercept

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	1, 3–5, 7, 9–11, 13–16
MLR3: Critique, Correct, Clarify	2, 4, 6, 9–11, 17–18
MLR7: Compare and Connect	3, 8, 11–12, 14–15

Sub-Unit Language Goals

- Explain how to use factoring and the zero-product property to solve quadratic equations and determine key features of parabolas. (**Writing, Speaking, and Listening**)
- Compare and contrast different strategies for solving quadratic equations. (**Speaking and Listening**)
- Interpret the solutions to quadratic equations in context. (**Writing, Speaking, and Listening**)

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 12**, where students described what the words *exact* and *approximate* remind them of to surface multiple meanings of these terms.

ML/EL **Multilingual/English Language Learners**
Invite several students to share what *exact* and *approximate* reminds them of (e.g., When I hear exact, I think . . .). It may also be helpful to research how to say *exact solutions* and *approximate solutions* in other languages. This may help surface multiple meanings and invite students to connect to their prior knowledge.

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Solve Quadratic Equations With Complex Solutions

In this unit, students solve quadratic equations with no real solutions, one real solution, or two real solutions. **In Algebra 2**, students will learn about the complex number system and expand their understanding of solving equations to see that all quadratic equations have two complex solutions.

FL B.E.S.T. Math Benchmark:
MA.912.AR.3.2

Example:

The equation $x^2 - 4x + 13 = 0$ has no real solution, but it has two complex solutions: $x = 2 + 3i$ and $x = 2 - 3i$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-36}}{2}$$

$$x = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

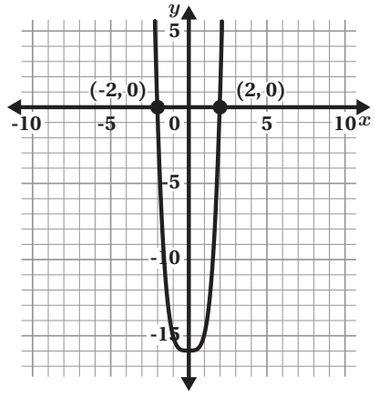
Use Structure to Rewrite Polynomial Functions

In this unit, students use structure to rewrite equations of quadratic functions to determine key features like x-intercepts. **In Algebra 2**, students will use structure to rewrite polynomial functions of higher degree to determine key features.

FL B.E.S.T. Math Benchmarks:
MA.912.AR.1.7, MA.912.AR.3.7

Example:

The polynomial function $f(x) = x^4 - 16$ can be rewritten as $f(x) = (x^2 + 4)(x^2 - 4)$ or $f(x) = (x^2 + 4)(x + 2)(x - 2)$. $f(x)$ has x-intercepts at (-2, 0) and (2, 0).



Perform Arithmetic Operations on Polynomials

In this unit, students combine linear and quadratic functions by adding and subtracting. **In Algebra 2**, students will combine general polynomial functions using addition, subtraction, multiplication, and division.

FL B.E.S.T. Math Benchmark:
MA.912.AR.1.3

Example:

Diagrams can also be used to multiply and divide polynomial functions. Consider $l(x) = 3x - 1$ and $q(x) = 3x^2 + 5x - 2$.

$q(x) \cdot l(x)$

$(3x^2 + 5x - 2)(3x - 1)$

$9x^3 + 12x^2 - 11x + 2$

	$3x^2$	$5x$	-2
$3x$	$9x^3$	$15x^2$	$-6x$
-1	$-3x^2$	$-5x$	2

$q(x) \div l(x)$

$\frac{3x^2 + 5x - 2}{3x - 1}$

$x + 2$

where $x \neq \frac{1}{3}$ because $\frac{1}{3}$ would make the denominator 0

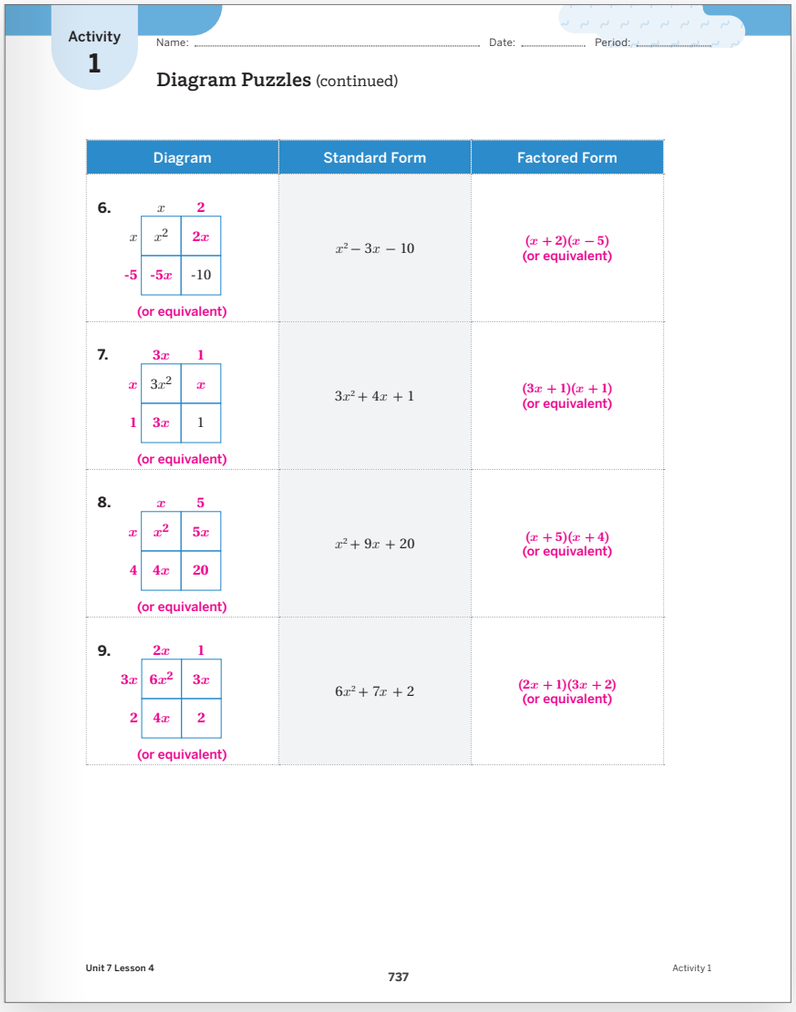
	x	2
$3x$	$3x^2$	$6x$
-1	$-x$	-2

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Application

Lesson 4, Activity 1



Activity 1

Name: Date: Period:

Diagram Puzzles (continued)

Diagram	Standard Form	Factored Form
6. x | 2; x | x^2 | $2x$; -5 | $-5x$ | -10 (or equivalent)	$x^2 - 3x - 10$	$(x + 2)(x - 5)$ (or equivalent)
7. $3x$ | 1; x | $3x^2$ | x; 1 | $3x$ | 1 (or equivalent)	$3x^2 + 4x + 1$	$(3x + 1)(x + 1)$ (or equivalent)
8. x | 5; x | x^2 | $5x$; 4 | $4x$ | 20 (or equivalent)	$x^2 + 9x + 20$	$(x + 5)(x + 4)$ (or equivalent)
9. $2x$ | 1; $3x$ | $6x^2$ | $3x$; 2 | $4x$ | 2 (or equivalent)	$6x^2 + 7x + 2$	$(2x + 1)(3x + 2)$ (or equivalent)

Unit 7 Lesson 4 737 Activity 1

Context

In this activity, students make multiple factoring attempts and surface a variety of strategies to support them in developing procedural fluency in factoring quadratic expressions with a variety of structures.

Try This

Put on your student hat and complete Problems 6–9.

Points to Ponder

- What do students need to understand about factoring quadratics to support their development of procedural fluency?
- How does procedural fluency develop across the different problems?
- Some students may think they need different strategies for different types of quadratics (e.g., where $a = 1$ and $a \neq 1$). How might you support their development of procedural fluency by sharing a strategy that works for all types of quadratics?
- How will factoring quadratics support their understanding of factoring higher degree polynomials and dividing polynomial expressions in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Identify equivalent expressions that represent area	Lesson 2	2	MA.7.AR.1.2 MTR.5.1
2	Write expressions to represent area	Lesson 2	2	MA.7.AR.1.2 MTR.5.1
3	Determine the value of expressions that involve distribution and analyze the operations involved	Lesson 2	2	MA.6.NSO.3.2, MA.6.NSO.3.3 MTR.4.1
4	List common factors of two integers	Lesson 2	1	MA.6.NSO.3.1
5	Solve linear equations	Lesson 9	2	MA.8.AR.2.1
6	Order values that include integers, square roots, and decimals	Lesson 12	1	MA.8.NSO.1.1
7	Estimate the value of irrational solutions and locate them approximately on a number line	Lesson 12	2	MA.8.NSO.1.1, MA.8.AR.2.3
8	Evaluate expressions using order of operations, with a special emphasis on squaring values	Lesson 16	1	MA.6.AR.1.3
9	Evaluate expressions	Lesson 16	1	MA.6.AR.1.3, MA.8.NSO.1.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



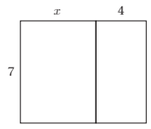
- Mini-Lessons
- Extensions

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Pre-Unit Check

Unit 7

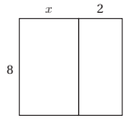
1. Select *all* the expressions that represent the area of this rectangle.



- ☐ **A.** $4(7 + x)$
- ☑ **B.** $7(x + 4)$
- ☑ **C.** $7x + 28$
- ☐ **D.** $7x + 4$
- ☐ **E.** $7 + x + 4$

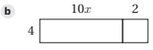
2. Write an expression for the area of each rectangle.

a



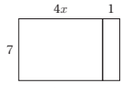
$8(x + 2)$ (or equivalent)

b



$4(10x) + 4(2)$ (or equivalent)

c



$28x + 7$ (or equivalent)

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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 7

3. Make an expression by filling each blank with one of the numbers below.

$\square(\square + \square) = ?$

-1	2	3	4
-5	6	7	8

a Determine the value of your expression.

Responses depend on the student's expression.

b Can you fill in the blanks with the listed numbers to create an expression with a greater value? Explain your thinking.

Responses vary. Yes, the value of my expression was a negative number, so any expression that has a positive value would be greater.

4. 36 and 48 have six common factors. 4 is one of them. List as many other common factors as you can.

Responses vary. 1, 2, 3, 6, and 12

5. Solve each equation.

a $5x - 20 + 3x = 0$

$x = 2.5$ (or equivalent)

b $-3(x - 11) = 0$

$x = 11$

6. Order these numbers by value:

$\sqrt{35}$	$\sqrt{24}$	6	5.5	5

$\sqrt{24}$	5	5.5	$\sqrt{35}$	6

Least Greatest

D Differentiation (Pre-Unit Check)

Note: To **Strengthen** and **Stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Prior Knowledge	Problem(s)	Respond to Student Thinking
Write equivalent expressions that involve distribution **(Preparation for Lessons 2–5)**	1–2	**Support** • **Teacher Move:** Consider revisiting Grade 6, Unit 6, Lesson 8 (*Product and Sums*) • **Mini-Lesson:** *Writing Equivalent Expressions Using the Area Model* • **Teacher Move:** During Lesson 2, consider inviting students to write each area as a product and as a sum, and describe how each can be seen in the diagram.
Operate with integers and common factors **(Preparation for Lessons 2–5)**	3–4	**Support** • **Teacher Move:** Consider revisiting Grade 6, Unit 2, Lesson 7 (*Common Factors*) • **Teacher Move:** Consider inviting students to reference a multiplication chart to help their thinking with factors.

Solve linear equations **(Preparation for Lessons 9–10)**	5	**Support**
		• **Teacher Move:** Consider solving each equation as a class and creating an anchor chart with any strategies students share.
		• **Teacher Move:** Consider revisiting:
		  • Grade 8, Unit 4, Lesson 1 (*Equation Roundtable*)
		  • Unit 1, Lesson 1 (*working Backwards*)

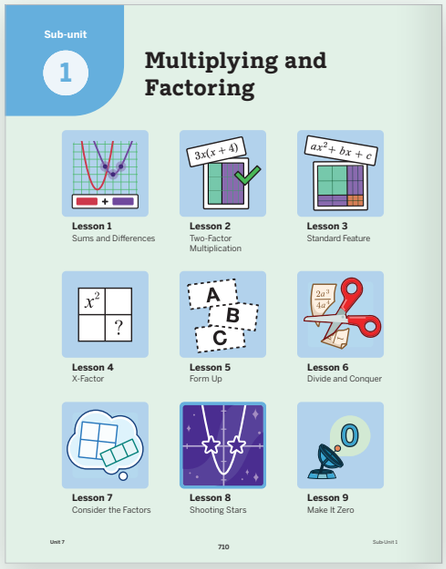
Use and approximate square roots **(Preparation for Lesson 12)**	6–7	**Support** • **Teacher Move:** Before Lesson 12, consider inviting students to describe the meaning of the square root symbol and share approximating strategies. • **Lesson:** Grade 8, Unit 8, Lesson 4 (*Root Down*) • **Mini-Lesson:** *Approximating Square Roots*
Evaluate expressions that include multiple variables and squaring values **(Preparation for Lessons 16, 18)**	8–9	**Support** • **Teacher Move:** Consider reviewing this problem before Lesson 16 as a class emphasizing order of operations and squaring values. • **Teacher Move:** Consider revisiting Grade 6, Unit 6, Lesson 11 (*Exponent Expressions*) • **Mini-Lesson:** *Evaluating Expressions With Exponents*

Multiplying and Factoring

Sub-Unit 1 Goals

- **Goal:** Convert quadratic expressions between factored and standard form by multiplying and factoring.
- **Goal:** Use factoring and the zero-product property to solve equations and determine the zeros of quadratic functions.
- **Goal:** Divide a polynomial by a monomial and rewrite polynomial expressions by factoring.
- **Language Goal:** Explain how to use factoring and the zero-product property to solve quadratic equations and determine key features of parabolas. **(Writing, Speaking, and Listening)**

Students multiply and factor to convert expressions between standard and factored form. They use factoring and the **zero-product property** to solve quadratic equations and determine the **zeros** of quadratic functions.



Math That Matters Most

Concept or Skill	Example
Add and subtract linear and quadratic expressions	Write the equation of $s(x) = (3x^2 - 2x - 1) + (-3x^2 + 8x + 12)$ using the fewest number of terms. $s(x) = 6x + 11$
Multiply to rewrite factored-form quadratic expressions in standard form	Rewrite $(3x - 4)(x + 2)$ in standard form. $3x^2 + 2x - 8$
Factor to rewrite standard-form quadratic expressions in factored form	Rewrite $3x^2 + 14x + 15$ in factored form. $(3x + 5)(x + 3)$

Concept or Skill	Example
Determine zeros of quadratic functions given in standard form	Determine the zeros of $k(x) = x^2 - 6x - 40$. $k(x) = (x + 4)(x - 10)$ so the zeros are at $x = -4$ and $x = 10$.
Solve quadratic equations using factoring and the zero-product property	Determine the solutions to the equation $2x^2 - x = 21$. $2x^2 - x - 21 = 0$ $(2x - 7)(x + 3) = 0$ $2x - 7 = 0$ and $x + 3 = 0$ $x = \frac{7}{2}$ and $x = -3$
Dividing a polynomial by a monomial	Simplify the expression $\frac{65x^6y^3 - 20x^2y^7 + 5x^4y^4}{5x^2y} = 13x^4y^2 - 4y^6 + x^2y^3$
Rewriting polynomial expressions by factoring	Factor by GCF $12x^4y^2 - 18x^3y^3 + 24x^2y^4$ $6x^2y^2(2x^2 - 3xy + 4y^2)$ Difference of Squares $49x^2 - 36$ $(7x - 6)(7x + 6)$ Perfect Square Trinomial $9z^2 + 24z + 16$ $(3z + 4)^2$ Factor by Grouping $x^3 + 3x^2 + 2x + 6$ $x^2(x + 3) + 2(x + 3)$ $(x^2 + 2)(x + 3)$

Unit 7

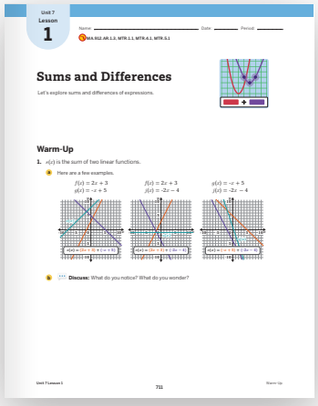
Lesson

1

Sums and Differences

Adding and Subtracting Linear and Quadratic Expressions

Let's explore sums and differences of expressions.


Unit 7
Lesson
1
Sums and Differences
Let's explore sums and differences of expressions.
Warm-Up
Unit 7 Lesson 1
711
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Add and subtract quadratic and linear expressions.
2. **Language Goal:** Explain what type of expressions can be created when adding and subtracting quadratic and linear expressions. **(Writing, Speaking, and Listening)**

Students explore functions and expressions created by combining linear and quadratic expressions using addition and subtraction. They make sense of the graphs of these functions and then analyze the structure of the expressions when combining them without a graph. They repeatedly practice writing linear and quadratic expressions using the fewest number of terms to see what kinds of expressions they can create. **(MTR.1.1, MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In Unit 1, students combined linear expressions to solve equations. In Unit 4, students added and subtracted linear expressions as part of the process of solving a system of linear equations.

Future Learning

In Lesson 2, students will multiply two linear expressions to create a quadratic expression.

Connections

- Students relate their understanding of graphs of systems of equations to combining algebraic expressions. (mathematical)
- Students compare and contrast different strategies for adding and subtracting algebraic expressions. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.3
Add, subtract and multiply polynomial expressions with rational number coefficients.

Also Addressing: **MA.912.NSO.1.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.2 **MA.7.AR.2.2** **MA.7.AR.1.1** **MA.7.NSO.1.1**	**MA.912.AR.1.7**

Vocabulary

Additional Vocabulary

polynomial

Lesson at a Glance

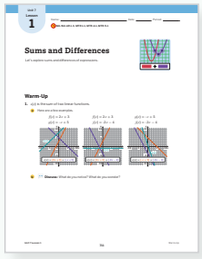
~ 45 min

MA.912.AR.1.3, MA.912.NSO.1.1

Warm-Up

Independent | 5 min

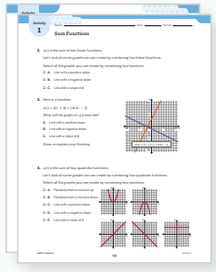
Students use the Notice and Wonder routine to combine two linear functions and make sense of the resulting graph. **(MTR.1.1)**



Activity 1

Pairs | 10 min

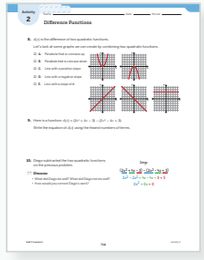
Students use the structure of expressions to add linear and quadratic functions and develop strategies for writing sums. MLR7: Compare and Connect **(MTR.5.1)**



Activity 2

Pairs | 10 min

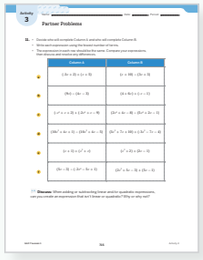
Students use the structure of expressions to subtract quadratic functions and correct student work for writing a difference. **(MTR.5.1)**



Activity 3

Pairs | 10 min

Students repeatedly practice combining linear and quadratic functions using addition and subtraction and see what kinds of expressions that process can create. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of adding and subtracting linear and quadratic functions by recording helpful things to remember. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing the equation for a function that is the sum of a quadratic and a linear expression using the fewest terms possible.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Independent | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to combine two linear functions and make sense of the resulting graph. **(MTR.1.1)**

1 Launch

Consider asking, "What does it mean for $s(x)$ to represent the *sum* of two linear functions?"

ELL **English Language Learners** Invite students to describe what *combine* means in their own words. Students may describe combine with language like *add, together, simplify, eliminate, like terms*, or words from their primary language. Consider making a display to honor this variety of language for students to reference. **(Reading and Listening)**

Use the Notice and Wonder routine to promote curiosity and help students make sense of the combined graphs. **(MTR.1.1)**

Note: In this lesson, students add and subtract quadratic and linear expressions to generalize about polynomials being closed under those operations. Students do not need to know the term *polynomial* to make sense of the ideas in this lesson and will not be assessed on their ability to classify polynomials. You may choose to introduce the term to students if it helps with their thinking.

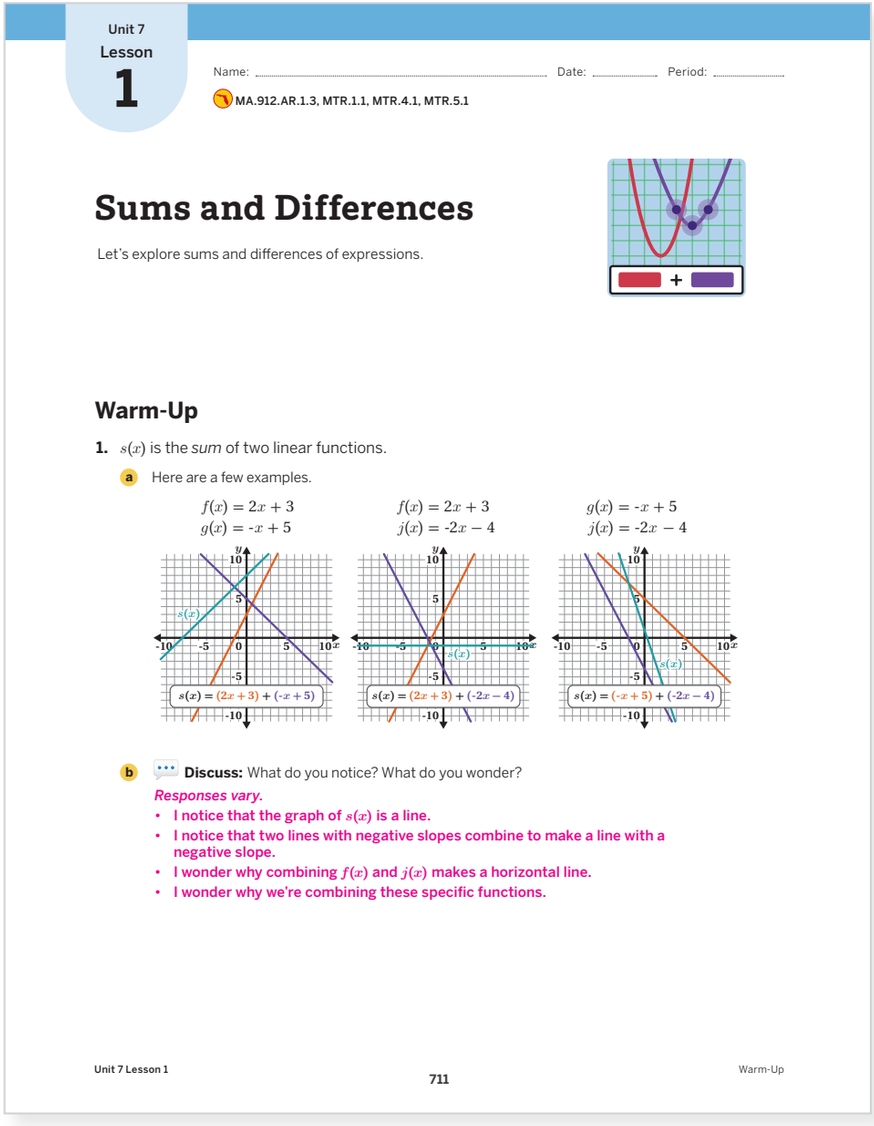
2 Connect

Invite students to share what they notice and wonder.

Create a record of the things students noticed and wondered, along with the names of the students who shared them.

Consider asking, "Did anyone notice anything interesting? Does this make you wonder about how the graph is made?"

Note: If it doesn't come up naturally, consider asking, "Did anybody notice what happens when $f(x)$ and $j(x)$ are combined? Did it surprise you? Why or why not?"



Unit 7
Lesson
1

Name: _____ Date: _____ Period: _____

MA.912.AR.1.3, MTR.1.1, MTR.4.1, MTR.5.1

Sums and Differences

Let's explore sums and differences of expressions.

Warm-Up

1. $s(x)$ is the *sum* of two linear functions.

a. Here are a few examples.

$f(x) = 2x + 3$, $g(x) = -x + 5$, $s(x) = (2x + 3) + (-x + 5)$

$f(x) = 2x + 3$, $j(x) = -2x - 4$, $s(x) = (2x + 3) + (-2x - 4)$

$g(x) = -x + 5$, $j(x) = -2x - 4$, $s(x) = (-x + 5) + (-2x - 4)$

b. **Discuss:** What do you notice? What do you wonder?

Responses vary.

- I notice that the graph of $s(x)$ is a line.
- I notice that two lines with negative slopes combine to make a line with a negative slope.
- I wonder why combining $f(x)$ and $j(x)$ makes a horizontal line.
- I wonder why we're combining these specific functions.

Unit 7 Lesson 1 711 Warm-Up

Pairs | 10 min

Activity 1 Sum Functions

Purpose: Students use the structure of expressions to add linear and quadratic functions and develop strategies for writing sums. **(MTR.5.1)**

1 Launch

Encourage students to identify which expression is represented by which line in the graph.

Consider asking, What do you notice about the graphs? The equations?"

Consider asking, "Can $s(x)$ ever be anything other than a linear function? Why or why not?"

2 Monitor

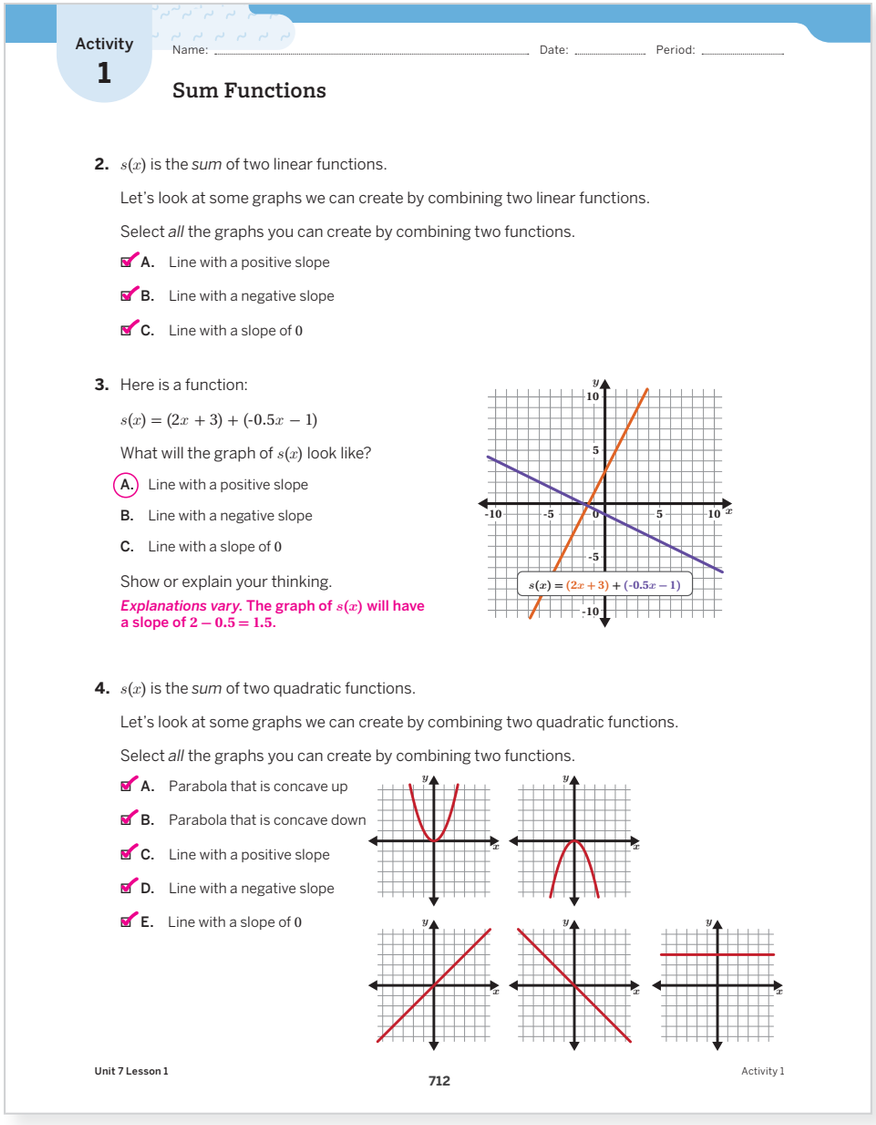
Look for different student strategies, including comparing steepness and adding the two slopes.

A **Accessibility: Conceptual Processing** Consider asking, "What is the slope of the orange line? The blue line? How do they compare?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Think you cannot make a line with two parabolas.	**Support** Encourage them to consider different parabolas that are concave down, including an example where $a = 2$. Consider asking, "What do you notice?"
Want to explore adding quadratic expressions further.	**Stretch** Invite them to consider what would happen if the two parabolas were the same. Consider asking, "What do you think the graph of $s(x)$ will look like?

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Sum Functions

2. $s(x)$ is the *sum* of two linear functions.

Let's look at some graphs we can create by combining two linear functions.

Select *all* the graphs you can create by combining two functions.

- ☑ **A.** Line with a positive slope
- ☑ **B.** Line with a negative slope
- ☑ **C.** Line with a slope of 0

3. Here is a function:

$s(x) = (2x + 3) + (-0.5x - 1)$

What will the graph of $s(x)$ look like?

- (A.) Line with a positive slope
- **B.** Line with a negative slope
- **C.** Line with a slope of 0

Show or explain your thinking.

Explanations vary. The graph of $s(x)$ will have a slope of $2 - 0.5 = 1.5$.

4. $s(x)$ is the *sum* of two quadratic functions.

Let's look at some graphs we can create by combining two quadratic functions.

Select *all* the graphs you can create by combining two functions.

- ☑ **A.** Parabola that is concave up
- ☑ **B.** Parabola that is concave down
- ☑ **C.** Line with a positive slope
- ☑ **D.** Line with a negative slope
- ☑ **E.** Line with a slope of 0

Unit 7 Lesson 1 712 Activity 1

Pairs | 10 min

Activity 1 Sum Functions (continued)

Purpose: Students use the structure of expressions to add linear and quadratic functions and develop strategies for writing sums. **(MTR.5.1)**

2 Monitor

Pause briefly to ask, "What types of functions can we create when we combine two quadratic functions? What types of functions can we *not* create?"

To support students getting started, consider asking, "What do you notice about the parts of the equation? How did they combine to make a line with slope 0?"

D **Differentiation:** (Support) Consider helping students color code each part of the equation and asking, "How would these different parts combine? Where do you see those combinations in the graph?"

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy.

Consider asking pairs of students:

- "What do the colors mean in the first strategy? How does that help you know what to combine?" **(MTR.5.1)**
- "What's important to remember about both strategies?" You can only add like terms together.

Look for student strategies for writing the equation of $s(x)$. Invite students to share during the Connect.

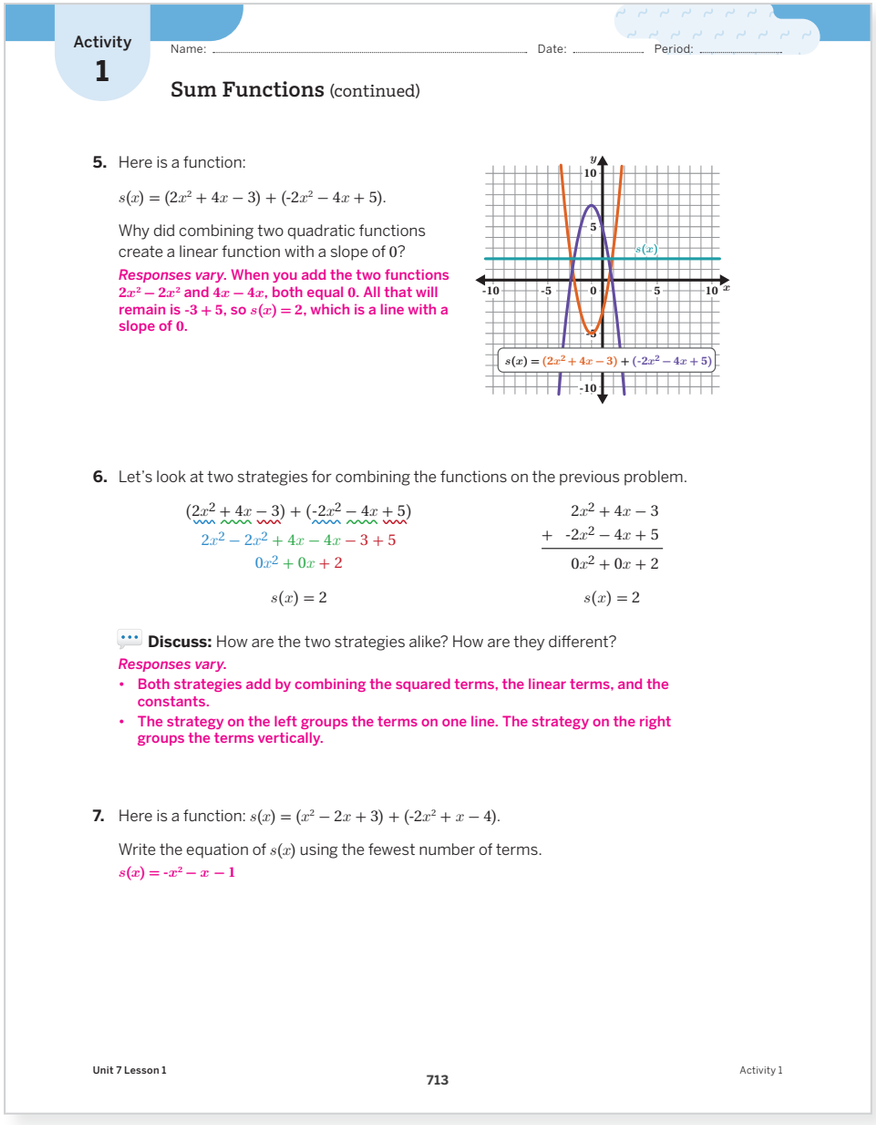
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit Problem 6 and consider asking, "How could one of these strategies help you get started?"
Write a correct equation for $s(x)$.	**Strengthen** Invite them to revisit Problem 6 and reflect on how their strategy compares to the strategies on that problem.

3 Connect

Consider asking, "Where do you see the same information in each strategy? Why did the different strategies lead to the same outcome?" **(MTR.5.1)**

To surface the Key Takeaway, invite students to share how they wrote the equation of $s(x)$. Consider asking, "How does knowing the types of functions that it is possible to make help you in writing the equation?"



Activity 1

Name: Date: Period:

Sum Functions (continued)

5. Here is a function:

$s(x) = (2x^2 + 4x - 3) + (-2x^2 - 4x + 5)$.

Why did combining two quadratic functions create a linear function with a slope of 0?

Responses vary. When you add the two functions $2x^2 - 2x^2$ and $4x - 4x$, both equal 0. All that will remain is $-3 + 5$, so $s(x) = 2$, which is a line with a slope of 0.

6. Let's look at two strategies for combining the functions on the previous problem.

$(2x^2 + 4x - 3) + (-2x^2 - 4x + 5)$

$2x^2 - 2x^2 + 4x - 4x - 3 + 5$

$0x^2 + 0x + 2$

$s(x) = 2$

$$\begin{array}{r} 2x^2 + 4x - 3 \\ +\ -2x^2 - 4x + 5 \\ \hline 0x^2 + 0x + 2 \end{array}$$

$s(x) = 2$

Discuss: How are the two strategies alike? How are they different?

Responses vary.

- Both strategies add by combining the squared terms, the linear terms, and the constants.
- The strategy on the left groups the terms on one line. The strategy on the right groups the terms vertically.

7. Here is a function: $s(x) = (x^2 - 2x + 3) + (-2x^2 + x - 4)$.

Write the equation of $s(x)$ using the fewest number of terms.

$s(x) = -x^2 - x - 1$

Unit 7 Lesson 1 713 Activity 1

Key Takeaway: Linear and quadratic expressions belong to a family called *polynomials*. Adding polynomials together can only make other polynomials. To write a polynomial sum in the fewest possible terms, we can combine the terms with similar structures (e.g., a quadratic term plus another quadratic term, a linear term plus a linear term, etc.).

Pairs | 10 min

Activity 2 Difference Functions

Purpose: Students use the structure of expressions to subtract quadratic functions and correct student work for writing a difference. **(MTR.5.1)**

1 Launch

Consider sharing that, in this activity, the focus will be on *subtracting* functions.

Consider asking, "What kinds of graphs are possible when subtracting two quadratic functions?"

ELL **English Language Learners** Invite students to describe what they think of when they hear the word *difference*. Consider sharing that in this lesson, difference will mean an expression involving subtraction. **(Reading, Writing, Speaking, and Listening)**

2 Monitor

Consider asking, "How is this like the addition you did in Activity 1? How is it different?"

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit Problem 6 and consider asking, "How could one of these strategies help you get started?"
Subtract $2x^2 - 2x^2$ to make 0 but also think that the linear and/or constant terms combine to 0 too.	**Strengthen** Celebrate their noticing that the quadratic terms combine to 0. Help them rewrite the expression using -1 and then invite them to distribute the -1: $(2x^2 + 4x - 3) + \text{-}1(2x^2 - 4x + 3)$.
Want to explore subtracting quadratic expressions further.	**Stretch** Invite students to write their own quadratic expressions that subtract to make different graphs (e.g., a concave up parabola, a line with a slope of -3, etc.).

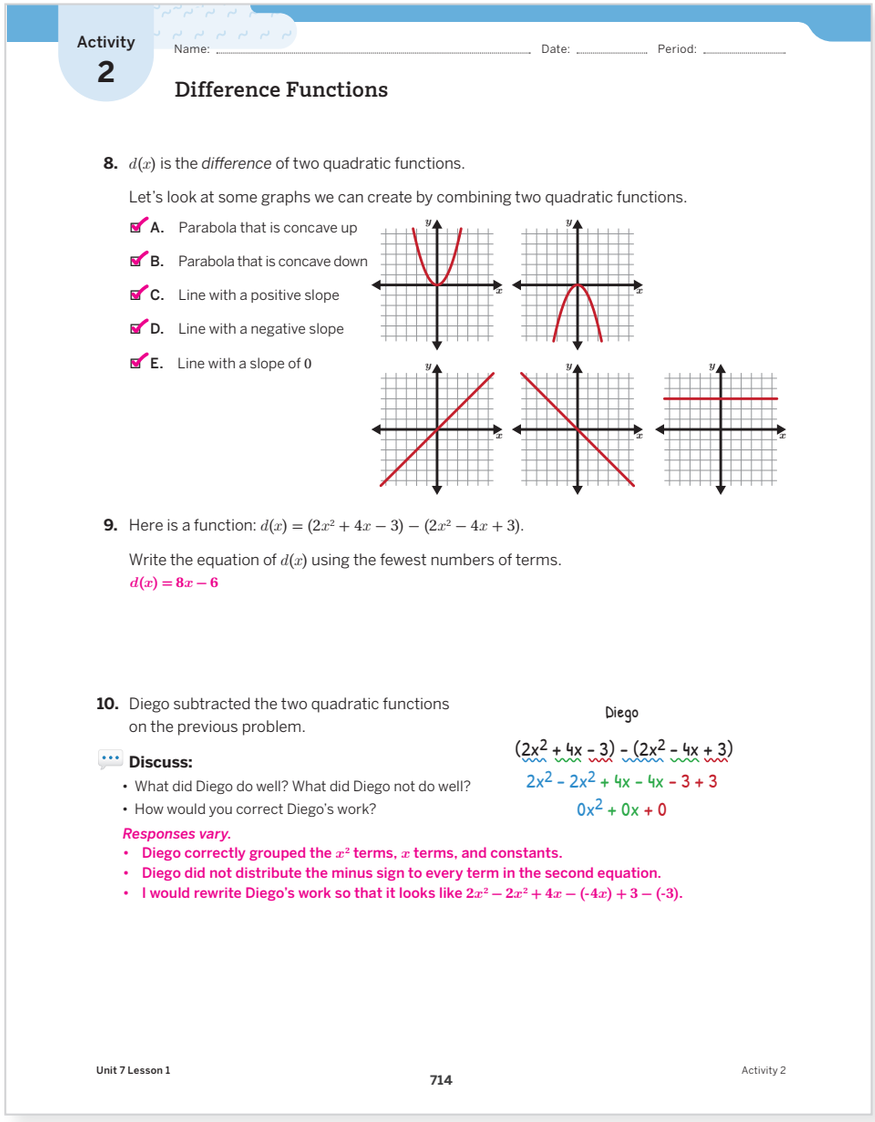
Select explanations of how to correct Diego's work to share during the Connect.

3 Connect

Invite students to share their thinking.

To surface the Key Takeaway, consider asking:

- "How did you use the different parts of the expression when you subtracted?" **(MTR.5.1)**
- "What might you suggest to help Diego subtract quadratic expressions?"
- "What types of functions can we create when subtracting two quadratic functions? Why?"



Activity 2

Name: _____ Date: _____ Period: _____

Difference Functions

8. $d(x)$ is the *difference* of two quadratic functions.

Let's look at some graphs we can create by combining two quadratic functions.

- ☑ **A.** Parabola that is concave up
- ☑ **B.** Parabola that is concave down
- ☑ **C.** Line with a positive slope
- ☑ **D.** Line with a negative slope
- ☑ **E.** Line with a slope of 0

9. Here is a function: $d(x) = (2x^2 + 4x - 3) - (2x^2 - 4x + 3)$.

Write the equation of $d(x)$ using the fewest numbers of terms.

$d(x) = 8x - 6$

10. Diego subtracted the two quadratic functions on the previous problem.

Diego

$(2x^2 + 4x - 3) - (2x^2 - 4x + 3)$

$2x^2 - 2x^2 + 4x - 4x - 3 + 3$

$0x^2 + 0x + 0$

Discuss:

- What did Diego do well? What did Diego not do well?
- How would you correct Diego's work?

Responses vary.

- Diego correctly grouped the x^2 terms, x terms, and constants.
- Diego did not distribute the minus sign to every term in the second equation.
- I would rewrite Diego's work so that it looks like $2x^2 - 2x^2 + 4x - (\text{-}4x) + 3 - (\text{-}3)$.

Unit 7 Lesson 1 714 Activity 2

Key Takeaway: Polynomials, such as quadratic and linear expressions, are also closed under subtraction. To write a polynomial difference in the fewest possible terms, it's important to subtract terms with similar structures and to pay careful attention to how the minus sign affects each term of the second expression.

Pairs | 10 min

Activity 3 Partner Problems

Purpose: Students repeatedly practice combining linear and quadratic functions using addition and subtraction and see what kinds of expressions that process can create. **(MTR.4.1)**

Short on time? Consider inviting students to complete fewer partner problems.

1 Launch

Encourage students to discuss what supporting each other without giving answers could look or sound like.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support their calculations. Encourage them to use grouping symbols to help with subtraction and negative numbers.

2 Monitor

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit their strategies from previous problems and write out the addition expressions vertically or horizontally to support their thinking.
Are not sure if their expression is equivalent to their partner's because the terms are in a different order.	**Strengthen** Celebrate that they found the sum or difference by term. Invite them to use a graphing calculator to graph both of their answers and see if the graphs are the same.
Would like an additional challenge.	**Stretch** Invite students to write their own quadratic/linear sum and difference problems and trade with a partner to solve.

3 Connect

Invite students to share which of the expressions is linear and which is quadratic.

Consider asking:

- "How did seeing many examples help you to generalize about *all* linear and quadratic expressions?"**(MTR.4.1)**
- "When adding or subtracting linear and/or quadratic expressions, is it possible to create an exponential expression? Why or why not?" No, because x is never in the exponent for linear or quadratic expressions.

Activity 3

Name: _____ Date: _____ Period: _____

Partner Problems

11.
- Decide who will complete Column A and who will complete Column B.
- Write each expression using the fewest number of terms.
- The expression in each row should be the same. Compare your expressions, then discuss and resolve any differences.

	Column A	Column B
a	$(-3x + 2) + (x + 5)$ $-2x + 7$	$(x + 10) - (3x + 3)$ $-2x + 7$
b	$(9x) - (4x - 3)$ $5x + 3$	$(4 + 6x) + (-x - 1)$ $5x + 3$
c	$(-x^2 + x + 2) + (-2x^2 + x - 9)$ $-3x^2 + 2x - 7$	$(2x^2 + 4x - 8) - (5x^2 + 2x - 1)$ $-3x^2 + 2x - 7$
d	$(10x^2 + 4x + 1) - (10x^2 + 4x - 5)$ 6	$(3x^2 + 7x + 10) + (-3x^2 - 7x - 4)$ 6
e	$(x + 1) + (x^2 + x)$ $x^2 + 2x + 1$	$(x^2 + 2) + (2x - 1)$ $x^2 + 2x + 1$
f	$(5x - 3) - (-2x^2 - 5x + 1)$ $2x^2 + 10x - 4$	$(2x^2 + 5x - 3) + (5x - 1)$ $2x^2 + 10x - 4$

Discuss: When adding or subtracting linear and/or quadratic expressions, can you create an expression that isn't linear or quadratic? Why or why not?

No. *Explanations vary.* The exponent does not change when you're adding or subtracting. So an x^2 term can either have a coefficient of 0 or some other number, but the x^2 will still be an x^2.

Unit 7 Lesson 1 715 Activity 3

Whole Class | 5 min

Synthesis

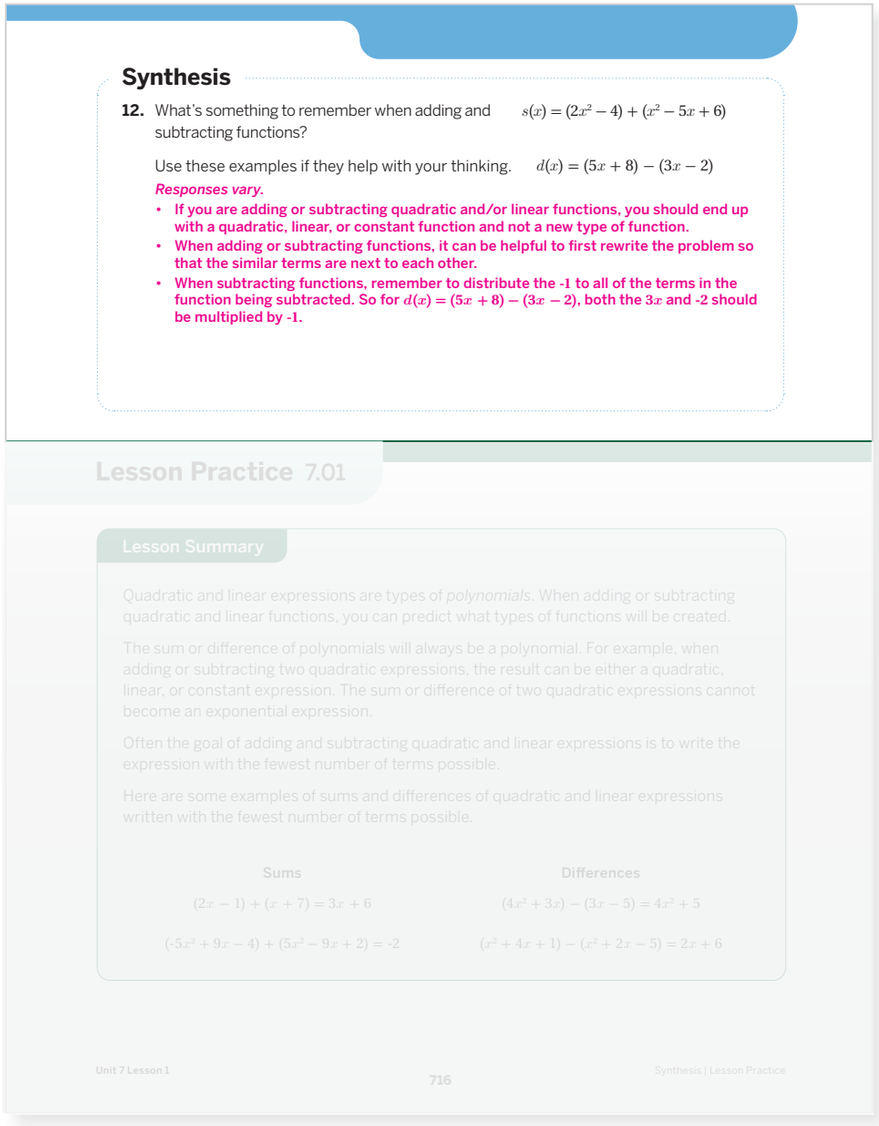
Purpose: Students synthesize their understanding of adding and subtracting linear and quadratic functions by recording helpful things to remember.

Invite students to respond independently, after discussing the question and the examples with their partner.

Share a variety of ideas, such as:

- You can add two functions by combining terms that have the same structure; for example, $2x^2$ and $-x^2$.
- When you subtract functions, the minus sign affects all the terms of the function you're subtracting, not just the first term.
- When you combine two linear and/or quadratic functions, it is only possible to create a quadratic or linear function.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the discussion. Display for students to reference and add to it for the rest of the unit.



Synthesis

12. What's something to remember when adding and subtracting functions? $s(x) = (2x^2 - 4) + (x^2 - 5x + 6)$

Use these examples if they help with your thinking. $d(x) = (5x + 8) - (3x - 2)$

Responses vary.

- If you are adding or subtracting quadratic and/or linear functions, you should end up with a quadratic, linear, or constant function and not a new type of function.
- When adding or subtracting functions, it can be helpful to first rewrite the problem so that the similar terms are next to each other.
- When subtracting functions, remember to distribute the -1 to all of the terms in the function being subtracted. So for $d(x) = (5x + 8) - (3x - 2)$, both the $3x$ and -2 should be multiplied by -1.

Lesson Practice 7.01

Lesson Summary

Quadratic and linear expressions are types of *polynomials*. When adding or subtracting quadratic and linear functions, you can predict what types of functions will be created.

The sum or difference of polynomials will always be a polynomial. For example, when adding or subtracting two quadratic expressions, the result can be either a quadratic, linear, or constant expression. The sum or difference of two quadratic expressions cannot become an exponential expression.

Often the goal of adding and subtracting quadratic and linear expressions is to write the expression with the fewest number of terms possible.

Here are some examples of sums and differences of quadratic and linear expressions written with the fewest number of terms possible.

Sums	Differences
$(2x - 1) + (x + 7) = 3x + 6$	$(4x^2 + 3x) - (3x - 5) = 4x^2 + 5$
$(-5x^2 + 9x - 4) + (5x^2 - 9x + 2) = -2$	$(x^2 + 4x + 1) - (x^2 + 2x - 5) = 2x + 6$

Unit 7 Lesson 1 716 Synthesis | Lesson Practice

Lesson Takeaway: Polynomials are closed under addition and subtraction. When adding or subtracting linear and/or quadratic expressions, the resulting expression can only be linear or quadratic.

- Linear $\pm$ Linear = Linear or Constant
- Quadratic $\pm$ Quadratic = Quadratic, Linear, or Constant
  - Linear example:
    $(2x^2 - 1) + (-2x^2 + x) = x - 1$
  - Quadratic example:
    $(3x^2 + 4x) - (x^2 + 4x) = 2x^2$

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing the equation for a function that is the sum of a quadratic and a linear expression using the fewest terms possible.

Today's Goals

1. **Goal:** Add and subtract quadratic and linear expressions.
2. **Language Goal:** Explain what type of expressions can be created when adding and subtracting quadratic and linear expressions. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.01

Here is a function: $b(x) = (-6x^2 - x + 8) + (3x + 2)$.

Write $b(x)$ using the fewest number of terms.

$b(x) = -6x^2 + 2x + 10$

D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support adding and subtracting linear and quadratic expressions:

» **Respond to Student Thinking:**

- Revisit the partner problems in **Activity 3** as a class (especially **Problems 11c and 11e**) and invite students to share strategies for adding and subtracting quadratic expressions.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write their own quadratic/linear sum and difference problems and then trade with a partner to simplify.

Professional Learning

In this lesson, students analyzed the structure of quadratic and linear expressions to support them in understanding that polynomials are closed under addition and subtraction. How did limiting the types of functions to the ones students are familiar with from prior units support them in developing conceptual understanding? When did students have opportunities to generalize their thinking to the system of polynomials? In this unit, students will have more opportunities to generalize their thinking by analyzing quadratic structures. How will students' work today prepare them for generalizing their thinking throughout the unit? **(MTR.4.1, MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What's something to remember when adding and subtracting functions?

$s(x) = (2x^2 - 4) + (x^2 - 5x + 6)$

Use these examples if they help with your thinking. $d(x) = (5x + 8) - (3x - 2)$

Responses vary.

- If you are adding or subtracting quadratic and/or linear functions, you should end up with a quadratic, linear, or constant function and not a new type of function.
- When adding or subtracting functions, it can be helpful to first rewrite the problem so that the similar terms are next to each other.
- When subtracting functions, remember to distribute the -1 to all of the terms in the function being subtracted. So for $d(x) = (5x + 8) - (3x - 2)$, both the $3x$ and -2 should be multiplied by -1.

Lesson Practice 7.01

Lesson Summary

Quadratic and linear expressions are types of *polynomials*. When adding or subtracting quadratic and linear functions, you can predict what types of functions will be created.

The sum or difference of polynomials will always be a polynomial. For example, when adding or subtracting two quadratic expressions, the result can be either a quadratic, linear, or constant expression. The sum or difference of two quadratic expressions cannot become an exponential expression.

Often the goal of adding and subtracting quadratic and linear expressions is to write the expression with the fewest number of terms possible.

Here are some examples of sums and differences of quadratic and linear expressions written with the fewest number of terms possible.

Sums	Differences
$(2x - 1) + (x + 7) = 3x + 6$	$(4x^2 + 3x) - (3x - 5) = 4x^2 + 5$
$(-5x^2 + 9x - 4) + (5x^2 - 9x + 2) = -2$	$(x^2 + 4x + 1) - (x^2 + 2x - 5) = 2x + 6$

Lesson Practice 7.01

Name: _____ Date: _____ Period: _____

Problems 1–3: Fill in the blanks to make each equation true.

1. $(6x^2 + 4x - 6) + (\underline{-3}x^2 + \underline{4}x + \underline{10}) = 3x^2 + 8x + 4$

2. $(-x^2 + 5x - 4) + (\underline{4}x^2 + \underline{-5}x + \underline{2}) = 3x^2 - 2$

3. $(8x^2 - x + 10) - (\underline{3}x^2 + \underline{5}x + \underline{10}) = 5x^2 - 6x$

Problems 4–7: Write an equivalent function using the least number of terms with $f(x) = 10x^2 - 8x - 5$ and $g(x) = -3x^2 - x + 8$.

4. $f(x) + g(x)$
$7x^2 - 9x + 3$

5. $f(x) - g(x)$
$13x^2 - 7x - 13$

6. $g(x) + f(x)$
$7x^2 - 9x + 3$

7. $g(x) - f(x)$
$-13x^2 + 7x + 13$

Problems 8–9: Neo made a mistake when subtracting $(-x^2 + 5x - 3) - (7x - 2)$.

8. Describe the error that Neo made.
Responses vary. Neo combined an x^2 term with an x term and an x term with a constant.

Neo

$$\begin{array}{r} -x^2 + 5x - 3 \\ +\ \ -7x + 2 \\ \hline -8x^2 + 7x - 3 \end{array}$$

9. Write an expression for the difference using the fewest number of terms.
$-x^2 - 2x - 1$

Lesson Practice 7.01

Name: _____ Date: _____ Period: _____

Test Practice

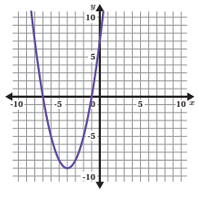
10. Here is a function: $s(x) = (2x^2 + x + 3) + (-2x^2 + 1)$.

What will the graph of $s(x)$ look like?

A. Parabola that is concave up
B. Parabola that is concave down
C. Line with a positive slope
D. Line with a negative slope
E. Line with a slope of 0

Spiral Review

11. Write two quadratic equations that represent this graph.



Responses vary.
$y = (x + 1)(x + 7)$
$y = (x + 4)^2 - 9$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.912.AR.1.3
	4–7	2	MA.912.AR.1.3
	8–9	3	MA.912.AR.1.3
Test Practice	10	2	MA.912.AR.1.3
Spiral Review			
Fluency	11	2	MA.912.AR.3.4

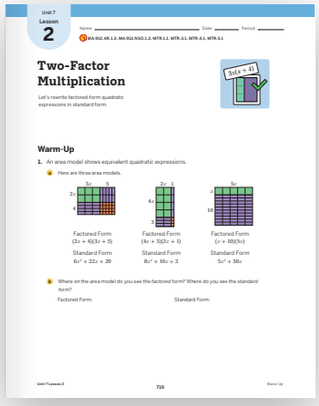
Unit 7

Lesson 2

Two-Factor Multiplication

Rewriting Factored-Form Expressions in Standard Form

Let's rewrite factored-form quadratic expressions in standard form.



Learning Outcomes

Today's Goals

1. **Goal:** Rewrite a factored-form quadratic expression in standard form.
2. **Goal:** Multiply two linear factors.
3. **Language Goal:** Describe connections between factored form, standard form, and area models representing quadratic expressions. **(Writing, Speaking, and Listening)**

Students use area models and diagrams to multiply expressions written in *factored form* and rewrite them in *standard form*. They describe connections between factored-form expressions, standard-form expressions, and area models.

Coherence

Prior Learning

In Grades 6 and 7, students added, subtracted, factored, and expanded linear expressions with rational coefficients using area models. In Unit 6, students analyzed factored-form and standard-form equations to identify key features of parabolas. In Lesson 1, students combined linear and quadratic expressions using addition and subtraction.

Future Learning

In Lesson 3, students will continue multiplying to rewrite factored-form expressions in standard form. They will work backwards using similar diagrams to factor expressions in Lessons 4 and 5.

Connections

- Using the area model, students extend their understanding of multiplying algebraic expressions to converting quadratic expressions from factored form to standard form. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.3
Add, subtract and multiply polynomial expressions with rational number coefficients.

MA.912.NSO.1.2
Generate equivalent algebraic expressions using the properties of exponents.

Also Addressing: **MA.912.NSO.1.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	**MA.912.AR.1.7**
MA.7.NSO.1.1	**MA.912.F.1.6**

Vocabulary

Review Vocabulary

factored form

standard form (of a quadratic equation)

Lesson at a Glance

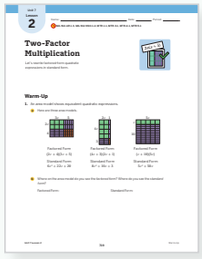
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MA.912.AR.1.3, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Whole Class | 5 min

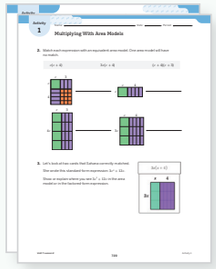
Students make sense of area models and describe how to see *factored form* and *standard form* represented in the model. **Routine:** Notice and Wonder **(MTR.1.1)**



Activity 1

Pairs | 15 min

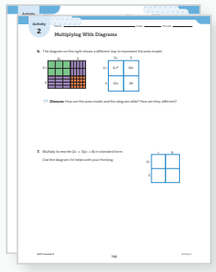
Students match factored-form expressions to area models and use area models to rewrite factored-form expressions in equivalent forms.



Activity 2

Pairs | 15 min

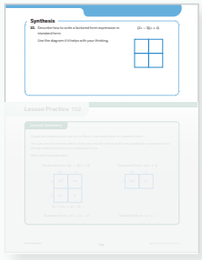
Students transition from using area models to more abstract diagrams to rewrite factored expressions in standard form. **Routines:** Think-Pair-Share; MLR3: Critique, Correct, Clarify



Synthesis

Whole Class | 5 min

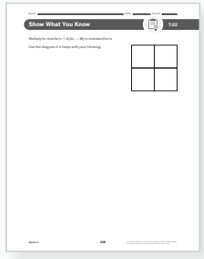
Students describe how to multiply factored-form quadratic expressions to rewrite them in standard form.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by rewriting a factored expression in standard form.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students make sense of the area model and describe how to see *factored form* and *standard form* represented in the model. **(MTR.1.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the area model. **(MTR.1.1)**

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *factored form, standard form, factors, outside, inside, squares, rectangles, big, small*, etc. **(Writing and Speaking)**

A **Accessibility: Conceptual Processing** As students think abstractly, invite them to answer more concrete questions first. For example, draw a basic rectangle and consider asking, "How can you determine the area of this 2-by-5 rectangle?"

A **Accessibility: Visual-Spatial Processing** To support students who are colorblind, consider encouraging students to refer to the tiles as *larger squares, rectangles*, and *smaller squares* instead of color descriptions like *green* or *purple* throughout the lesson.

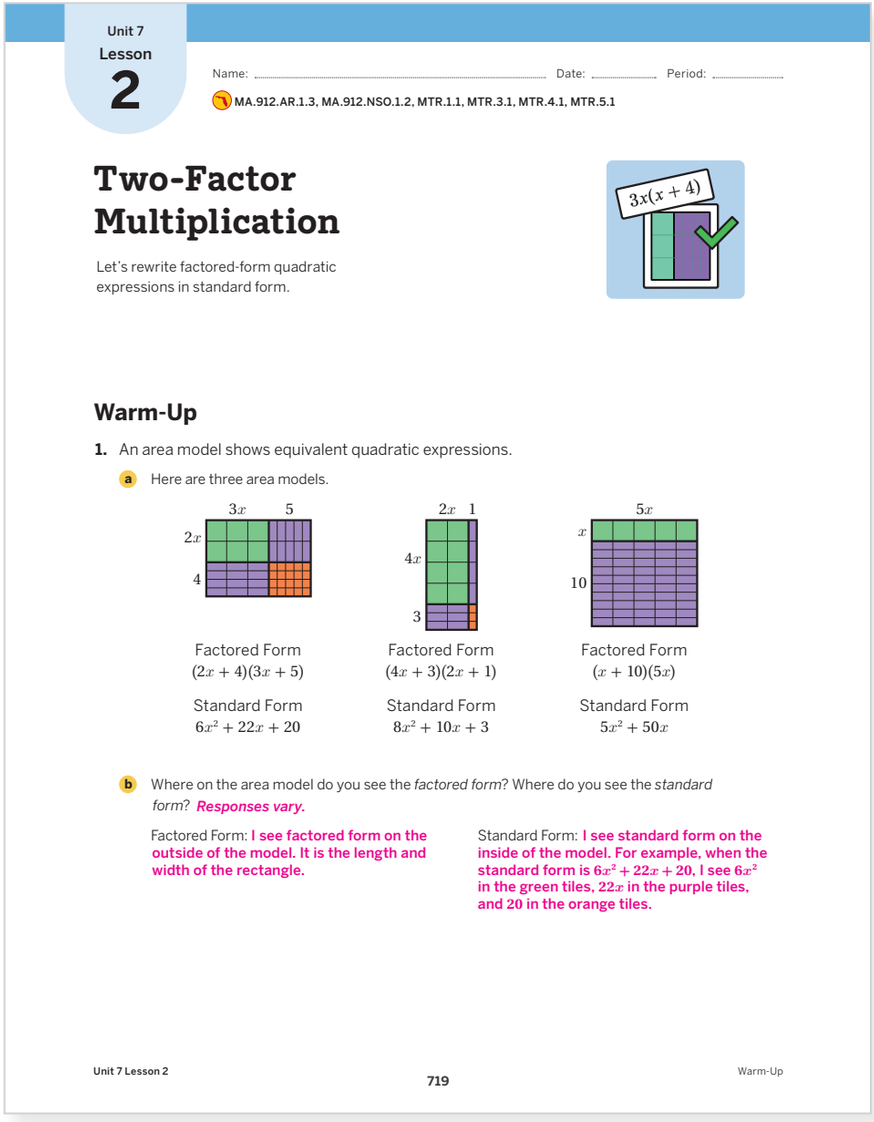
2 Connect

Display a new area model example, and invite several students to share where they see factored and standard form in the area model.

To surface the Key Takeaway, consider asking:

- "Where do you see the first factor? Second factor?"
- "What do the green tiles represent? Purple tiles? Orange tiles?"
- "Why do you think this is called an area model?"

Note: Students will continue to explore how to see standard form in area models throughout Lessons 1–4.



Unit 7
Lesson
2

Name: Date: Period:

MA.912.AR.1.3, MA.912.NSO.1.2, MTR.1.1, MTR.3.1, MTR.4.1, MTR.5.1

Two-Factor Multiplication

Let's rewrite factored-form quadratic expressions in standard form.

Warm-Up

1. An area model shows equivalent quadratic expressions.

a. Here are three area models.

Factored Form
$(2x + 4)(3x + 5)$

Standard Form
$6x^2 + 22x + 20$

Factored Form
$(4x + 3)(2x + 1)$

Standard Form
$8x^2 + 10x + 3$

Factored Form
$(x + 10)(5x)$

Standard Form
$5x^2 + 50x$

b. Where on the area model do you see the *factored form*? Where do you see the *standard form*? *Responses vary.*

Factored Form: I see factored form on the outside of the model. It is the length and width of the rectangle.

Standard Form: I see standard form on the inside of the model. For example, when the standard form is $6x^2 + 22x + 20$, I see $6x^2$ in the green tiles, $22x$ in the purple tiles, and 20 in the orange tiles.

Unit 7 Lesson 2 719 Warm-Up

Key Takeaway: Area models can be a helpful tool to show multiplication. The large green squares have an area of x^2, the purple rectangles have an area of x, and the small orange squares have an area of 1.

Pairs | 15 min

Activity 1 Multiplying With Area Models

Purpose: Students match factored-form expressions to area models and use area models to rewrite factored-form expressions in equivalent forms.

Short on time? Consider omitting the discussion on Problem 3 if the class shows a strong understanding of how standard form is connected to the area model.

1 Launch

Consider asking, "How are the three expressions alike? How are they different?"

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match a model to an expression.

2 Monitor

To surface connections, consider asking pairs:
- "What form are these expressions in?"
- "How can you see this expression represented in the area model?"

Note: If time allows, consider asking pairs to compare and justify their answers, and make revisions based on their conversation.

Encourage students to share their answers and explanations with the class.

Accessibility: Visual-Spacial Processing
Throughout this lesson, invite students to label the dimensions or areas of each of the smaller shapes inside the area model.

Pause briefly to discuss student responses and ensure all students understand that $x \cdot x = x^2$.

Consider asking:
- "Where do you see multiplication represented in the factored-form expression? In the area model?"
- "What does each green tile represent? Why?"
- "What does each purple tile represent? Why?"
- "Why can't we combine $3x^2$ and $12x$?"
- "What type of functions are $3x$ and $x + 4$?"
- "What type of function is $3x^2 + 12x$?"

Activity 1 continued >

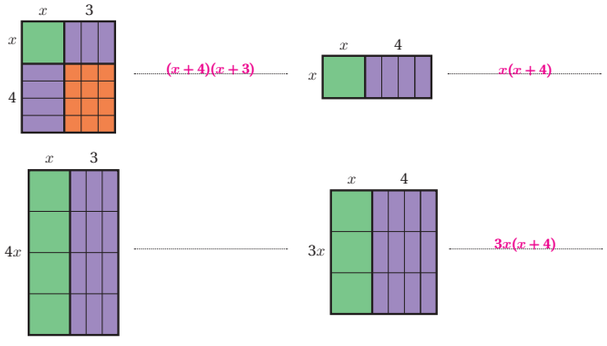
Activity 1

Name: _____ Date: _____ Period: _____

Multiplying With Area Models

2. Match each expression with an equivalent area model. One area model will have no match.

$x(x + 4)$ $\quad 3x(x + 4)$ $\quad (x + 4)(x + 3)$

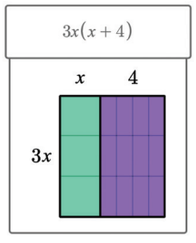


3. Let's look at two cards that Sahana correctly matched.

She wrote this standard-form expression: $3x^2 + 12x$.

Show or explain where you see $3x^2 + 12x$ in the area model or in the factored-form expression.

Responses vary.
- I see $3x^2 + 12x$ in the tiles because I counted 3 green tiles and 12 purple tiles. Each green tile represents an x^2 and each purple tile represents $1x$. Together, this makes $3x^2 + 12x$.
- I see $3x^2 + 12x$ in the expression because if you distribute $3x$ to $(x + 4)$, you end up with $3x^2 + 12x$.



Unit 7 Lesson 2 720 Activity 1

Pairs | 15 min

Activity 1 Multiplying With Area Models (continued)

Purpose: Students match factored-form expressions to area models and use area models to rewrite factored-form expressions in equivalent forms.

Short on time? Consider omitting the discussion on Problem 3 if the class shows a strong understanding of how standard form is connected to the area model.

2 Monitor

To surface connections, consider asking pairs:
- "Which expression is in standard form? Why?
- "What is the equivalent expression in factored form? Where can you see it?"

Note: If time allows, consider inviting pairs to share their responses with a pair who selected a different expression.

D **Differentiation: (Stretch)** Encourage students to justify why these three expressions are equivalent and create other equivalent expressions that they can see in the area model.

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, chunk this activity into more manageable parts by inviting students to choose one term at a time from the expression. Then ask them where they might see it in the area model.

Share models that could represent $(x + 7)(x + 3)$.

Note: Consider providing graph paper for students to support sketching models.

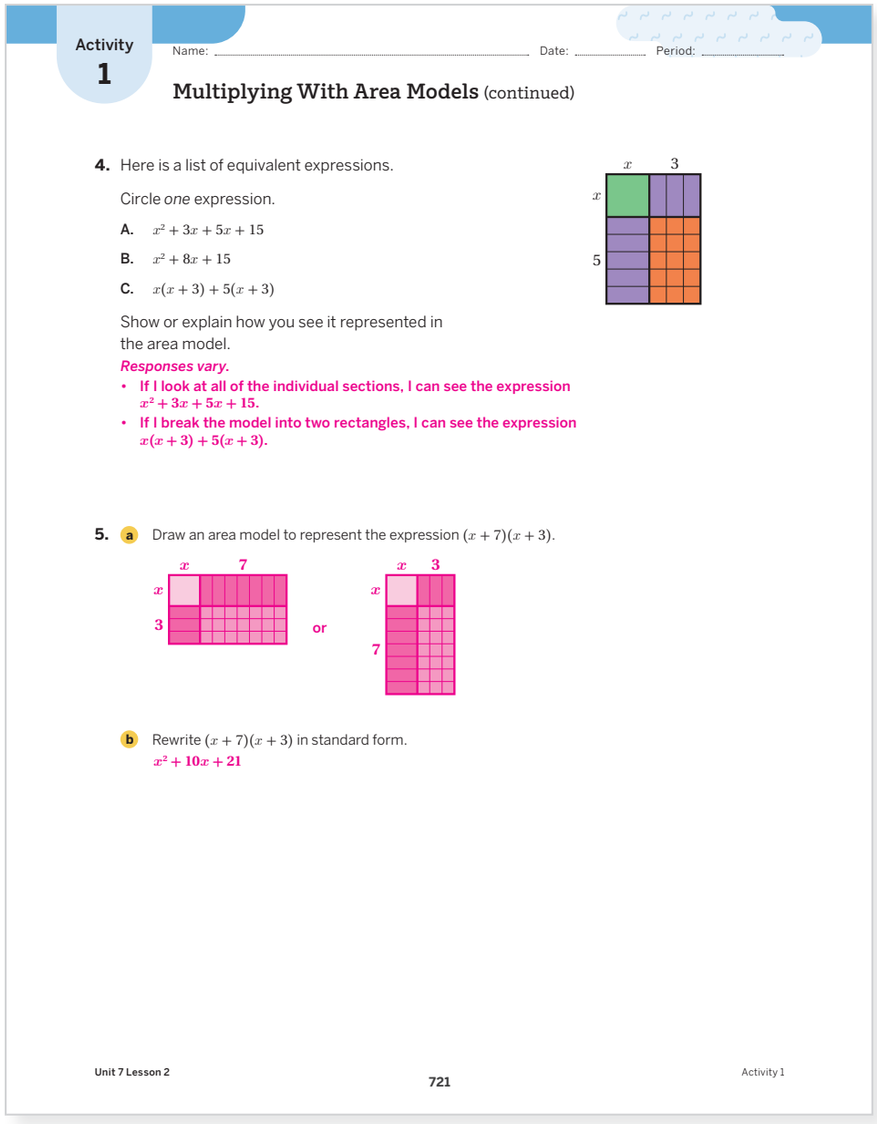
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which terms can you place along the diagram? Where can you place them?"
Write an expression with four terms, like $x^2 + 3x + 7x + 21$.	**Strengthen** Celebrate that they multiplied correctly. Consider sharing that in standard form all like-terms are combined.
Would like an additional challenge.	**Stretch** Invite students to write more expressions that are equivalent to $(x + 7)(x + 3)$.

3 Connect

Display the two area models that both represent $(x + 7)(x + 3)$. Consider asking, "Are these models equivalent? Why or why not?"

To surface the Key Takeaway, consider asking:
- "How did you create the area model?"
- "How can you determine standard form?"
- "Where do you see factored form in the area model? Standard form?"



Activity 1

Name: _____ Date: _____ Period: _____

Multiplying With Area Models (continued)

4. Here is a list of equivalent expressions.

Circle *one* expression.

A. $x^2 + 3x + 5x + 15$

B. $x^2 + 8x + 15$

C. $x(x + 3) + 5(x + 3)$

Show or explain how you see it represented in the area model.

Responses vary.
- If I look at all of the individual sections, I can see the expression $x^2 + 3x + 5x + 15$.
- If I break the model into two rectangles, I can see the expression $x(x + 3) + 5(x + 3)$.

5. a. Draw an area model to represent the expression $(x + 7)(x + 3)$.

b. Rewrite $(x + 7)(x + 3)$ in standard form.

$x^2 + 10x + 21$

Unit 7 Lesson 2 721 Activity 1

Key Takeaway: An area model is a helpful tool for rewriting a factored-form expression in *standard form*. *Factored form* can be seen on the length and width of the area model, while the area inside the model represents standard form.

Pairs | 15 min

Activity 2 Multiplying With Diagrams

Purpose: Students transition from using area models to more abstract diagrams to rewrite factored expressions in standard form.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Consider sharing that these are two representations of the same quadratic expression and ask, "Why can having multiple representations be helpful?"

Use the Think-Pair-Share routine to help students shape their ideas about the similarities and differences. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

Consider asking:

- "How are the numbers in the boxes calculated?" **(MTR.1.1)**
- "Why might the diagram on the right be useful?" **(MTR.1.1)**
- "What are other representations you could use to show a quadratic relationship?"

Note: Area models are to scale and can only use positive values. The diagram on the right is more flexible and can accommodate for more types of values.

A **Accessibility: Visual-Spatial Processing** Invite students to pick a section of the diagram and identify where to find the dimensions for that section. Consider modeling how to use your fingers to trace to the corresponding column and row label throughout the lesson.

2 Monitor

A **Accessibility: Memory and Attention** To support students with performing calculations, invite them to use a calculator. Consider displaying an anchor chart of integer operation rules for students to refer to throughout Sub-Unit 1.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support making sense of the diagram.	**Support** Consider asking, "Where did the numbers on the outside of the diagram come from? Is this the only way they could have been placed?"
Need support determining the product.	**Support** Consider reminding students that they can start in any section and ask: • "What operation can be used to calculate area?" • "What section would you like to find the area of first?"
Persist through struggle.	**Strengthen** Celebrate these students and invite them to share strategies they used to persevere. **(MTR.1.1)**

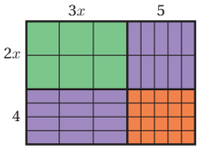
Activity 2 continued >

Activity 2

Name: Date: Period:

Multiplying With Diagrams

6. The diagram on the right shows a different way to represent the area model.



	$3x$	5
$2x$	$6x^2$	$10x$
4	$12x$	20

Discuss: How are the area model and the diagram alike? How are they different?

Responses vary.

- The area model and the diagram are similar because they both represent two forms of the same quadratic expression. In both representations, the x^2 term is in the top-left corner.
- The area model and the diagram are different because the area model shows each individual piece, while the diagram just shows terms. The area model is a scale representation, while the size of the diagram doesn't change.

7. Multiply to rewrite $(2x + 5)(x + 8)$ in standard form.

Use the diagram if it helps with your thinking.

$2x^2 + 21x + 40$

	x	8
$2x$		
5		

Unit 7 Lesson 2 722 Activity 2

Pairs | 15 min

Activity 2 Multiplying With Diagrams (continued)

Purpose: Students transition from using area models to more abstract diagrams to rewrite factored expressions in standard form.

Short on time? Consider inviting students to complete fewer challenges.

2 Monitor

To support students getting started, consider asking:

- "Did she make the diagram correctly?"
- "Did she multiply correctly?"
- "Did she write her answer in standard form?"

Pause and invite students to share their responses. Consider asking, "What is a hint or question for Karima that would help her improve her work?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Karima's work. Invite students to share their responses from the activity. **(MTR.4.1)**

Invite students to complete as many problems as they can in Problem 9.

Students will need a partner for this activity. They will solve and compare their solutions to problems instead of receiving responsive feedback.

Encourage students to use blank paper, or their Student Edition to show their thinking.

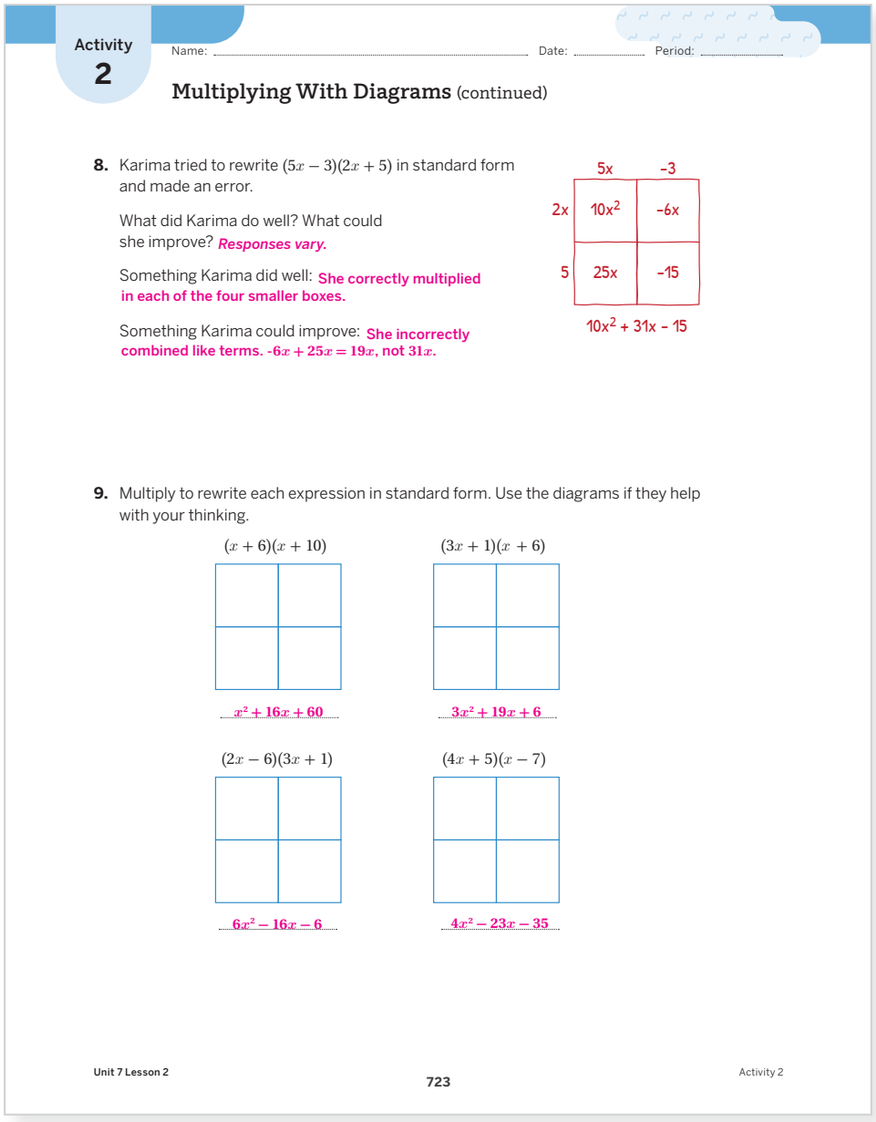
A **Accessibility: Memory and Attention** Invite students to reflect on how the expressions are similar and different before responding.

Listen for patterns and strategies students develop as they progress through the challenges such as:

- Recognizing where quadratic, linear, and constant terms show up in the diagram. **(MTR.5.1)**
- Noticing the like terms show up on a diagonal. **(MTR.4.1)**
- Noticing c will always be the product of the constants in factored form. **(MTR.4.1)**
- Noticing that when you multiply two linear functions, the result is a quadratic function.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 2

Name: _____ Date: _____ Period: _____

Multiplying With Diagrams (continued)

8. Karima tried to rewrite $(5x - 3)(2x + 5)$ in standard form and made an error.

What did Karima do well? What could she improve? *Responses vary.*

Something Karima did well: She correctly multiplied in each of the four smaller boxes.

Something Karima could improve: She incorrectly combined like terms. $-6x + 25x = 19x$, not $31x$.

	5x	-3
2x	$10x^2$	-6x
5	25x	-15

$10x^2 + 31x - 15$

9. Multiply to rewrite each expression in standard form. Use the diagrams if they help with your thinking.

$(x + 6)(x + 10)$

$x^2 + 16x + 60$

$(3x + 1)(x + 6)$

$3x^2 + 19x + 6$

$(2x - 6)(3x + 1)$

$6x^2 - 16x - 6$

$(4x + 5)(x - 7)$

$4x^2 - 23x - 35$

Unit 7 Lesson 2 723 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students describe how to multiply factored-form quadratic expressions to rewrite them in standard form.

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *factored form, standard form, factors, outside, inside, multiply, add, like terms*, etc. **(Writing and Speaking)**

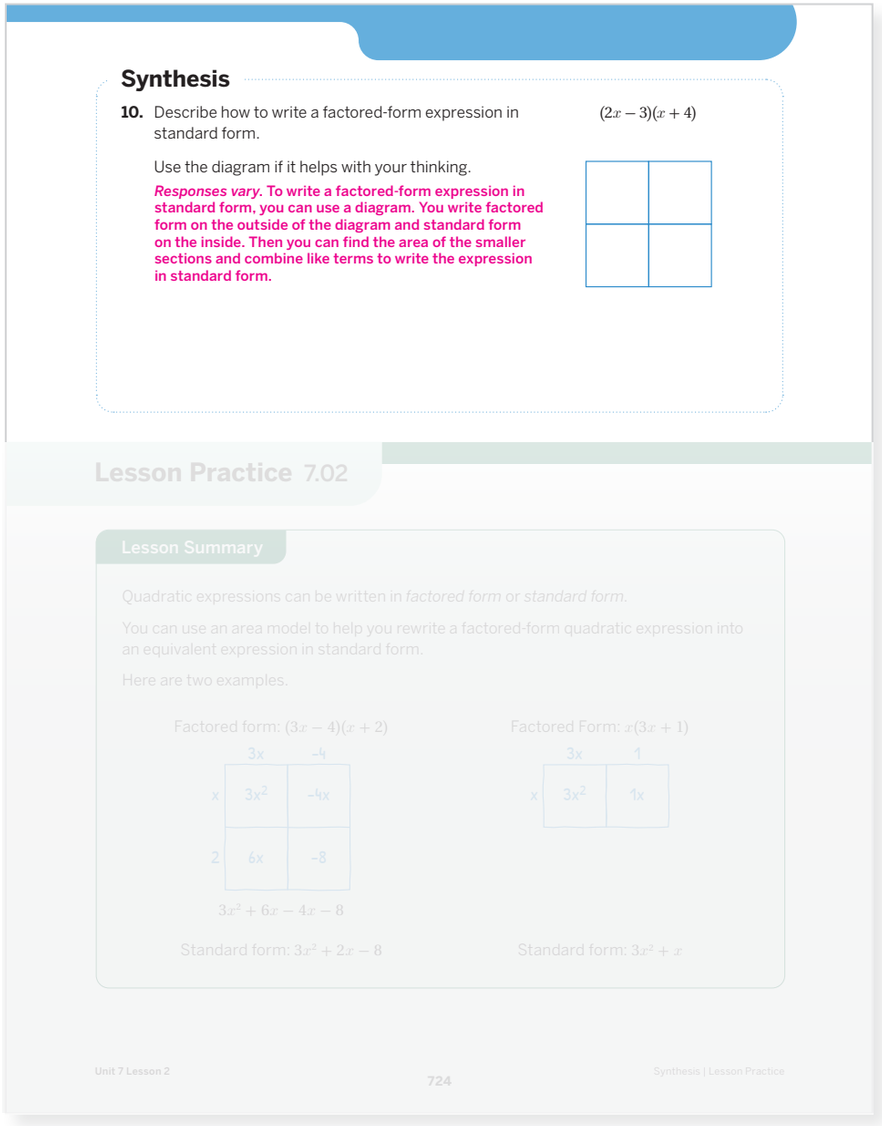
Share a variety of ideas, such as:

- Multiply the factors to get an expression in standard form.
- In standard form, all like terms are combined.
- A diagram can be a helpful tool. The factors go on the outside (length and width) of the diagram. Standard form can be found by adding together the terms on the inside of the diagram.

To surface the Lesson Takeaway, consider asking:

- "What are important things to keep in mind when rewriting a factored-form expression in standard form?"
- "How can an area model or diagram be helpful for rewriting a factored expression in standard form?"
- "What type of functions can you create when you multiply two linear factors together? Why does that make sense?"

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. Describe how to write a factored-form expression in standard form. $(2x-3)(x+4)$

Use the diagram if it helps with your thinking.

Responses vary. **To write a factored-form expression in standard form, you can use a diagram. You write factored form on the outside of the diagram and standard form on the inside. Then you can find the area of the smaller sections and combine like terms to write the expression in standard form.**

Lesson Practice 7.02

Lesson Summary

Quadratic expressions can be written in *factored form* or *standard form*.

You can use an area model to help you rewrite a factored-form quadratic expression into an equivalent expression in standard form.

Here are two examples.

Factored form: $(3x-4)(x+2)$

	3x	-4
x	$3x^2$	$-4x$
2	$6x$	-8

$3x^2+6x-4x-8$

Standard form: $3x^2+2x-8$

Factored Form: $x(3x+1)$

	3x	1
x	$3x^2$	$1x$

Standard form: $3x^2+x$

Unit 7 Lesson 2 724 Synthesis | Lesson Practice

Lesson Takeaway: Area models can help represent multiplication of expressions. Expressions in factored form can be rewritten in standard form by multiplying the factors and combining like terms. Similar to addition and subtraction, multiplication is closed for linear and quadratic functions. When you multiply two linear factors together, it creates a quadratic expression.

Independent | 5 min

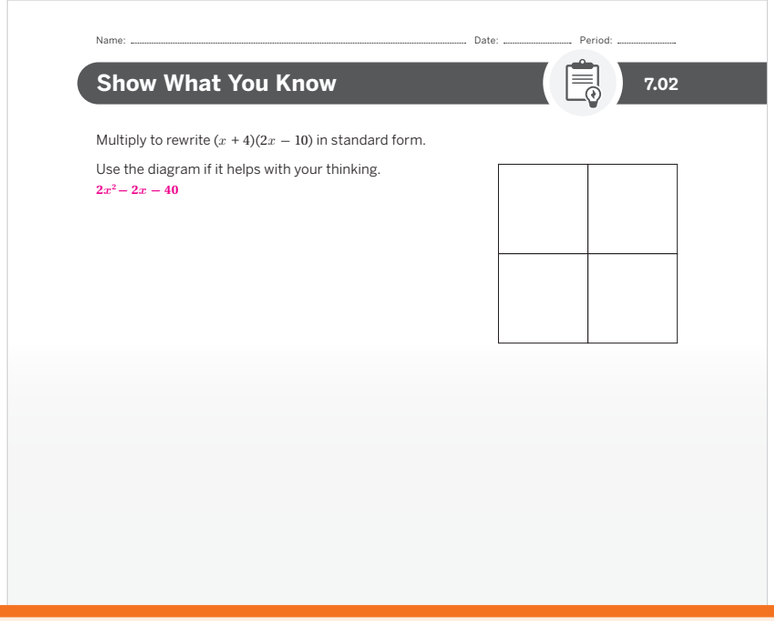
Show What You Know

Purpose: Students demonstrate their understanding by rewriting a factored expression in standard form.

Today's Goals

1. **Goal:** Rewrite a factored-form quadratic expression in standard form.
2. **Goal:** Multiply two linear factors.
3. **Language Goal:** Describe connections between factored form, standard form, and area models representing quadratic expressions. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.02

Multiply to rewrite $(x + 4)(2x - 10)$ in standard form.

Use the diagram if it helps with your thinking.

$2x^2 - 2x - 40$

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support rewriting a factored quadratic expression in standard form:

» **Respond to Student Thinking:**

- Use the ***Rewriting Factored-Form Expressions in Standard Form* Mini-Lesson.**

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write expressions in factored form that would only have two terms in standard form.

Professional Learning

Lessons 2–5 are grounded in using diagrams to multiply and factor. What knowledge about area models did your students bring to this lesson? In Lesson 3, students will look for patterns as they practice multiplying to help build fluency with multiplication and prepare them for factoring. What structures and patterns did students notice while multiplying during this lesson? How does looking for and making use of structure support students in understanding the broader concepts of multiplying and factoring, rather than viewing each quadratic as an individual case to memorize?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how to write a factored-form expression in standard form.

$(2x - 3)(x + 4)$

Use the diagram if it helps with your thinking.

Responses vary. To write a factored-form expression in standard form, you can use a diagram. You write factored form on the outside of the diagram and standard form on the inside. Then you can find the area of the smaller sections and combine like terms to write the expression in standard form.

Lesson Practice 7.02

Lesson Summary

Quadratic expressions can be written in *factored form* or *standard form*.

You can use an area model to help you rewrite a factored-form quadratic expression into an equivalent expression in standard form.

Here are two examples.

Factored form: $(3x - 4)(x + 2)$

	3x	-4
x	$3x^2$	-4x
2	6x	-8

$3x^2 + 6x - 4x - 8$

Standard form: $3x^2 + 2x - 8$

Factored Form: $x(3x + 1)$

	3x	1
x	$3x^2$	1x

Standard form: $3x^2 + x$

Lesson Practice 7.02

Name: ..... Date: ..... Period: .....

1. Write an expression that represents each area model.

	x	3
4		

$4x + 12$ (or equivalent)

	x	2
x		

$x^2 + 2x$ (or equivalent)

	x	9
$3x$		

$3x^2 + 27x$ (or equivalent)

	$3x$	5
$-2x$		

$-6x^2 - 10x$ (or equivalent)

2. Write two expressions that match the area model.

	x	6
x		
3		

Factored Form: $(x + 3)(x + 6)$

Standard Form: $x^2 + 9x + 18$

3. Multiply to rewrite $(x + 5)(x - 2)$ in standard form. Use the diagram if it helps with your thinking.

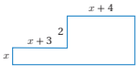
	x	5
x	x^2	$5x$
-2	$-2x$	-10

$x^2 + 3x - 10$

4. Complete the table by writing each expression in standard form.

Factored Form	Standard Form
$(x + 7)(x + 3)$	$x^2 + 10x + 21$
$(x - 2)(x - 12)$	$x^2 - 14x + 24$

5. Write an expression that represents the area and an expression for the perimeter of this figure:



Area: $2x^2 + 9x + 8$ (or equivalent)

Perimeter: $6x + 18$ (or equivalent)

Lesson Practice 7.02

Name: ..... Date: ..... Period: .....

Test Practice

6. Match each expression to its equivalent expression in standard form.

a. $(x + 2)(x + 6)$	c. $x^2 + 12x + 32$
b. $(2x + 8)(x + 2)$	d. $2x^2 + 18x + 16$
c. $(x + 8)(x + 4)$	b. $2x^2 + 12x + 16$
d. $(x + 8)(2x + 2)$	a. $x^2 + 8x + 12$

Spiral Review

Problems 7–9: Complete each equivalent expression:

7. $14a + 21 =$ 7 (2 $a +$ 3)

8. $3b + 2 + 3b + 10 =$ 6 (1 $b +$ 2)

9. $-3(3c + 3) =$ -9 $c +$ -9

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.1.3
	2–3	2	MA.912.AR.1.3
	4	2	MA.912.AR.1.3
	5	3	MA.912.AR.1.3
Test Practice	6	2	MA.912.AR.1.3
Spiral Review			
Fluency	7–9	2	MA.7.AR.1.1

Unit 7

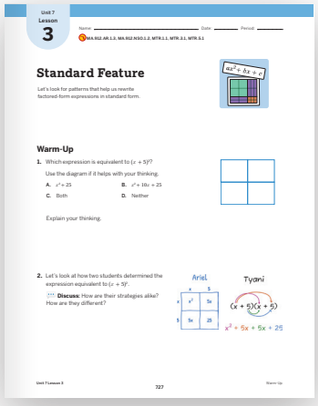
Lesson

3

Standard Feature

Patterns in Factored-Form and Standard-Form Expressions

Let's look for patterns that help us rewrite factored-form expressions in standard form.



Learning Outcomes

Today's Goals

1. **Goal:** Identify the a-, b-, and c-values in a standard-form quadratic expression.
2. **Language Goal:** Analyze the structure of a factored-form expression to make predictions about its equivalent expression in standard form. **(Writing, Speaking, and Listening)**

Students practice multiplying and use the structure of *factored-form* expressions to make predictions about equivalent expressions in *standard form*. Students match equivalent expressions that have similar constants and *coefficients*, but different structures and signs. Students describe patterns about how coefficients and constants in factored-form expressions affect the a-, b-, and c-values of standard-form expressions. These patterns will support students with factoring in Lessons 4 and 5. **(MTR.3.1, MTR.5.1)**

Coherence

Prior Learning

In Grades 6 and 7, students applied properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. In Lesson 2 of this unit, students used area models and diagrams to multiply factored-form expressions and rewrite them in standard form.

Future Learning

In Lessons 4–5, students will develop strategies for factoring quadratic expressions using the structures and patterns they notice today.

Connections

- Students explore different strategies to rewrite the factored form of quadratic expressions in standard form. (mathematical)
- In a matching activity, students identify different forms of the same quadratic expression (factored, standard, binomial squared). (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.3
Add, subtract and multiply polynomial expressions with rational number coefficients.

MA.912.NSO.1.2
Generate equivalent algebraic expressions using the properties of exponents.

Also Addressing: **MA.912.NSO.1.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	**MA.912.AR.1.7**
MA.7.NSO.1.1	**MA.912.F.1.6**

Vocabulary

Review Vocabulary

coefficient

factored form

standard form (of a quadratic equation)

Additional Vocabulary

difference of squares

Lesson at a Glance

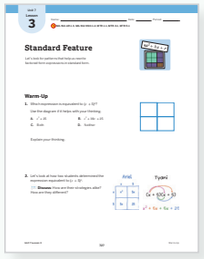
~ 45 min

MA.912.AR.1.3, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 8 min

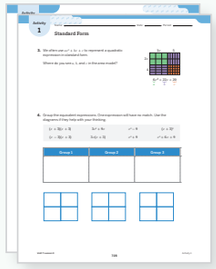
Students make sense of the expression $(x + 5)^2$ and analyze another way to visualize multiplication. **Routines:** Decide and Defend; MLR7: Compare and Connect **(MTR.1.1)**



Activity 1

Pairs | 15 min

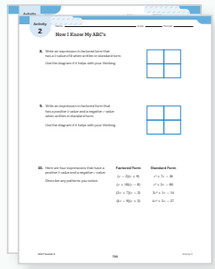
Students explore the structures of factored-form expressions to make generalizations about why some standard form expressions only have two terms. MLR2: Collect and Display **(MTR.3.1, MTR.5.1)**



Activity 2

Pairs | 12 min

Students make and test predictions about how *coefficients* and constants in factored form affect the equivalent expression in standard form.



Synthesis

Whole Class | 5 min

Students describe patterns between the structure of factored-form and standard-form expressions. MLR2: Collect and Display **(MTR.3.1, MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing equivalent expressions in factored and standard forms that meet given criteria.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 8 min

Warm-Up

Purpose: Students make sense of the expression $(x + 5)^2$ and analyze another way to visualize multiplication. **(MTR.1.1)**

Short on time? Consider omitting the Decide and Defend whole-class discussion for Problem 1.

1 Launch

Consider asking, "How would you describe *equivalent* in your own words?"

A **Accessibility: Memory and Attention** To support students with performing calculations, invite them to use a calculator or their Student Edition to record throughout this lesson.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Invite students who made different choices to use reasoning to convince their classmates.

D **Differentiation: (Stretch)** Students may think that $(x + 5)^2$ is equivalent to $x^2 + 25$ because they distributed the exponent of 2. Celebrate that students recognize that parentheses can sometimes mean distribute. Invite them to rewrite 6^2 as multiplication and then do the same for $(x + 5)^2$.

Invite students to explore the two strategies.

Encourage students to color code or label the connections they see between the two strategies.

A **Accessibility: Visual-Spatial Processing** Invite students to choose one term from standard form and describe how they see that term in each strategy (e.g., "The x^2 is in the top-left corner of Ariel's diagram and the first arrow of Tyani's work.").

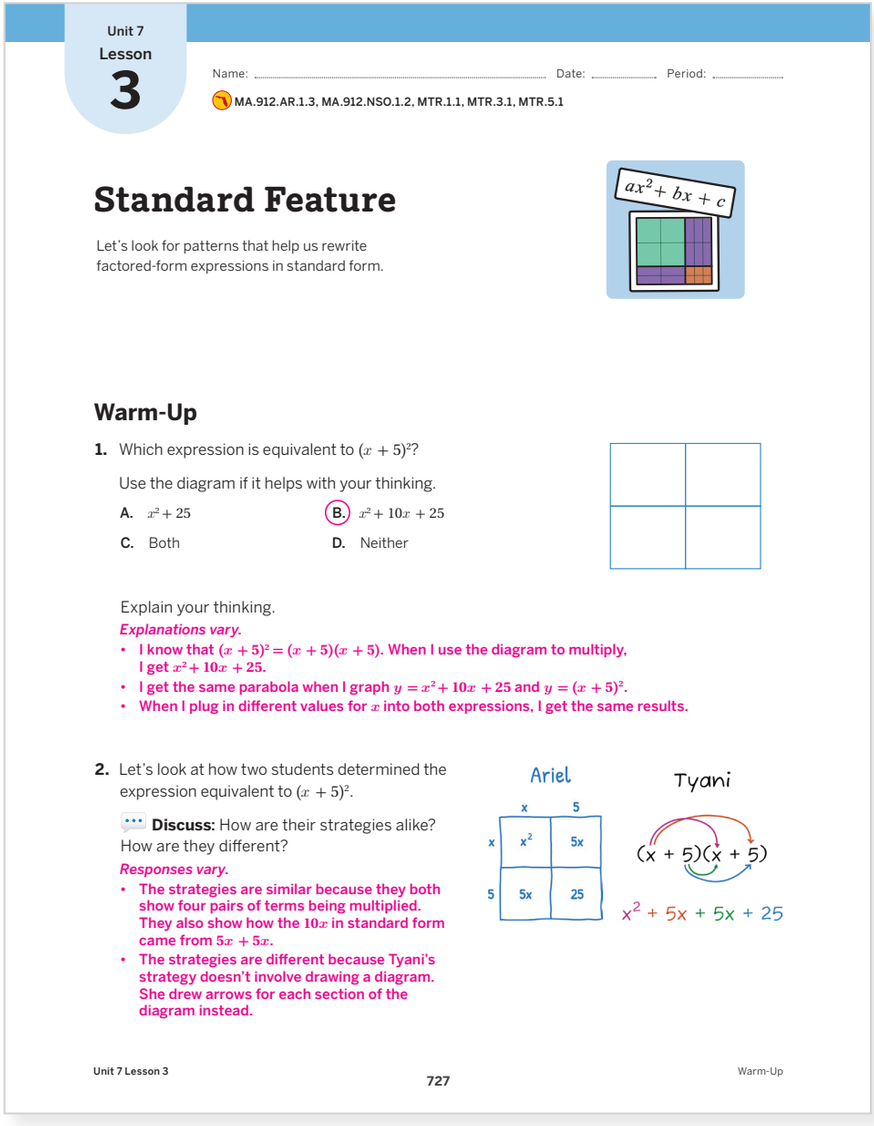
2 Connect

MLR **MLR7: Compare and Connect** With the two strategies displayed, invite students to analyze and compare each strategy, and then connect to their strategy.

Consider asking:

- "Which strategy do you like better? Why?"
- "What is helpful about each strategy?"
- "What might be difficult or tricky about each strategy?"
- "Why might having more than one strategy be useful?"

ELL **English Language Learners** If the words *coefficient* and *constant* don't come up naturally, consider reviewing their meanings to help with future discussions. **(Speaking and Listening)**



Unit 7
Lesson
3

Name: Date: Period:

MA.912.AR.1.3, MA.912.NSO.1.2, MTR.1.1, MTR.3.1, MTR.5.1

Standard Feature

Let's look for patterns that help us rewrite factored-form expressions in standard form.

Warm-Up

1. Which expression is equivalent to $(x + 5)^2$?

Use the diagram if it helps with your thinking.

A. $x^2 + 25$ B. $x^2 + 10x + 25$

C. Both D. Neither

Explain your thinking.

Explanations vary.

- I know that $(x + 5)^2 = (x + 5)(x + 5)$. When I use the diagram to multiply, I get $x^2 + 10x + 25$.
- I get the same parabola when I graph $y = x^2 + 10x + 25$ and $y = (x + 5)^2$.
- When I plug in different values for x into both expressions, I get the same results.

2. Let's look at how two students determined the expression equivalent to $(x + 5)^2$.

Discuss: How are their strategies alike? How are they different?

Responses vary.

- The strategies are similar because they both show four pairs of terms being multiplied. They also show how the $10x$ in standard form came from $5x + 5x$.
- The strategies are different because Tyani's strategy doesn't involve drawing a diagram. She drew arrows for each section of the diagram instead.

Unit 7 Lesson 3 727 Warm-Up

Pairs | 15 min

Activity 1 Standard Form

Purpose: Students explore the structures of factored-form expressions to make generalizations about why some standard-form expressions only have two terms. **(MTR.3.1, MTR.5.1)**

1 Launch

To surface prior knowledge, consider asking, "What do you remember about area models?"

A **Accessibility: Visual-Spatial Processing** To support students who are colorblind, consider encouraging students to refer to the tiles as *larger squares, rectangles*, and *smaller squares* instead of color descriptions like *green* or *purple* throughout the lesson.

2 Monitor

To support students getting started, encourage them to make an area model with smaller numbers so that the tiles are easier to count.

A **Accessibility: Memory and Attention** Consider displaying a labeled example of factored form and standard form for students to reference throughout the lesson.

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the expressions.

Encourage students to use a calculator, their Student Edition, or paper to help with their thinking throughout this lesson.

A **Accessibility: Executive Functioning** To support students in initiating complex tasks, consider asking, "What do you think a first step could be?"

D **Differentiation: (Support)** Students may think that $(x + 3)^2 = x^2 + 9$ because they distributed the exponent of 2. Encourage them to rewrite the expression as two factors and revisit Problem 2.

ELL **English Language Learners** Consider providing a word bank with words such as: *coefficients, constants, factored form, standard form, opposite, same, sign, value, terms*, a*-value*, b*-value*, c*-value*, etc. **(Speaking and Listening)**

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

Activity 1 continued >

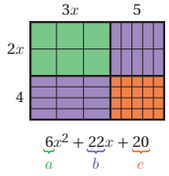
Activity 1

Name: _____ Date: _____ Period: _____

Standard Form

3. We often use $ax^2 + bx + c$ to represent a quadratic expression in *standard form*.

Where do you see a, b, and c in the area model?



Responses vary.

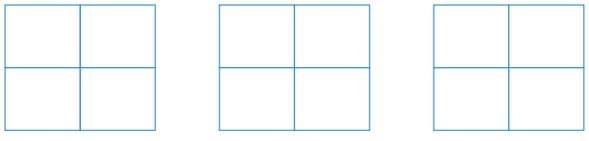
- I see the a-value in the green tiles, the b-value in the purple tiles, and the c-value in the orange tiles.
- The a-value is the number of large square tiles in the top left; the b-value is the number of rectangular tiles in the top right and bottom left; and the c-value is the number of small square tiles in the bottom right.

4. Group the equivalent expressions. One expression will have no match. Use the diagrams if they help with your thinking.

$(x + 3)(x + 3)$	$3x^2 + 9x$	$x^2 - 9$	$(x + 3)^2$
$(x - 3)(x + 3)$	$3x(x + 3)$	$x^2 + 9$	$x^2 + 6x + 9$

Group 1	Group 2	Group 3
$x^2 + 6x + 9$ $(x + 3)^2$ $(x + 3)(x + 3)$	$3x^2 + 9x$ $3x(x + 3)$	$x^2 - 9$ $(x - 3)(x + 3)$

Expression with no match: $x^2 + 9$



Unit 7 Lesson 3 728 Activity 1

Pairs | 15 min

Activity 1 Standard Form (continued)

Purpose: Students explore the structures of factored-form expressions to make generalizations about why some standard-form expressions only have two terms. **(MTR.3.1, MTR.5.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to show how to multiply $(x-3)(x+3)$.
Notice patterns such as: the linear term in the diagram is 0, or the constants in the factors are opposites.	**Strengthen** Invite them to share during the Pause. Consider asking, "What is another expression in factored form with a b-value of 0?"
Would like to explore how this connects to Unit 6.	**Stretch** Consider asking, "What would this parabola look like?"

Pause to discuss and record student generalizations. **Invite students** to share other factored-form expressions with $b = 0$. **(MTR.3.1)**

D Differentiation: **(Support)** Students might need support using a diagram since one of the factors has only one term.

D Differentiation

Look for students who:	Respond to Student Thinking
Select $(3x+1)(x-1)$ because of the $+1$ and -1.	**Support** Celebrate how they noticed the opposite constants. Invite them to multiply the expression and see what they notice.
Multiply each expression.	**Strengthen** Celebrate their work and ask, "What patterns do you notice?"
Would enjoy an additional challenge.	**Stretch** Consider asking, "Can a quadratic expression have an a-value of 0?"

3 Connect

To surface the Key Takeaway, consider asking, "How can you tell from factored form if standard form will have two terms?"

MLR MLR2: Collect and Display Use this routine to collect students' ideas and examples as they discuss each expression. Display for students to reference and add to throughout the unit.

Note: Students may notice the the structure of a *difference of squares* expression: $(rx+s)(rx-s) = r^2x^2 - s^2$. Consider sharing this with students, but students can be successful without knowing or using this term or formula.

Activity 1

Name: _____ Date: _____ Period: _____

Standard Form (continued)

5. Here are two equivalent expressions:

$(x-3)(x+3)$ and x^2-9

Discuss: Why does $(x-3)(x+3)$ have a b-value of 0 when written in standard form?

Use the diagram if it helps with your thinking.



Responses vary.

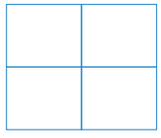
- The b-value is 0 because when you multiply you get $x^2-3x+3x-9$. Adding $-3x$ and $3x$ makes 0.
- $-3x$ and $3x$ are opposites, so they cancel out.

6. Here are two equivalent expressions:

$3x(x+3)$ and $3x^2+9x$

Why is the c-value 0?

Use the diagram if it helps with your thinking.



Responses vary.

- The c-value has no variable with it. Since I distributed an x to both terms, there is no term without a variable.
- The c-value is 0 because the first expression only has one term, so I can think of it as $3x+0$. Then when I multiply, I get $0 \cdot 3$, and that is 0.

7. Select *all* the expressions that have a b- or c-value of 0 when written in standard form.

- ☑ A. $(3x-1)(3x+1)$
- ☐ B. $(x-4)(x-4)$
- ☑ C. $x(x+4)$
- ☐ D. $(3x+1)(x-1)$
- ☑ E. $(x+10)(x-10)$

Unit 7 Lesson 3 729 Activity 1

Key Takeaway: Some expressions in *standard form* only have two terms.

- If one factor has a constant of 0, then $c = 0$ in standard form.
- If the factors have the same coefficients but opposite constants, then $b = 0$ in standard form.

Pairs | 12 min

Activity 2 Now I Know My ABC's

Purpose: Students make and test predictions about how *coefficients* and constants in factored form affect the equivalent expression in standard form. **(MTR.3.1, MTR.5.1)**

Short on time? Consider inviting students to focus on Problems 8–10. Problems 11–13 provide challenges which further prepare students for factoring.

1 Launch

Consider sharing that the goal of this activity is to support them both with multiplication and with the next lesson: rewriting standard-form expressions in factored form.

To encourage creativity, consider sharing that there are many ways to complete each challenge.

Encourage students to discuss their ideas and test them before entering expressions.

ELL **English Language Learners** Invite students to read each challenge aloud and paraphrase what it means with a partner before responding. **(Reading)**

2 Monitor

D **Differentiation: (Support)** Encourage students to use their creativity and come up with a different expression. Consider asking, "What could you change about the expression? What would have to stay the same?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to write an expression in factored form that meets just one criteria. Then ask, "Does your expression meet both requirements? Why or why not?"
Guess and check.	**Strengthen** Celebrate their persistence. Build metacognition with questions like: • "What did you try first? Why?" • "What did you try that didn't work?" • "What could you try next?"
Look for and make use of the structure in their expressions and/or make generalizations. **(MTR.3.1, MTR.5.1)**	**Strengthen** Celebrate these mathematical noticings and invite them to share these ideas during the Connect.

Share various patterns to share during the Connect.

D **Differentiation: (Stretch)** Some students may notice patterns with the constants in factored form when both coefficients are 1. Celebrate these generalizations. Then invite them to explore the fourth row and ask, "What is different about this case?"

Activity 2 continued >

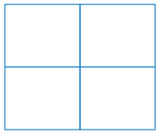
Activity 2

Name: Date: Period:

Now I Know My ABC's

8. Write an expression in *factored form* that has a b-*value of* 0 when written in standard form.

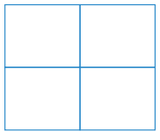
Use the diagram if it helps with your thinking.



Responses vary.
- $(x-6)(x+6)$
- $(5x-1)(5x+1)$

9. Write an expression in factored form that has a *positive* b-*value* and a *negative* c-*value* when written in standard form.

Use the diagram if it helps with your thinking.



Responses vary.
- $(x+9)(x-7)$
- $(3x-1)(x+4)$

10. Here are four expressions that have a *positive* b-*value* and a *negative* c-*value*.

Describe any patterns you notice.

Responses vary. In order to have a negative c-value, my two constants in factored form need to have opposite signs. To have a positive b-value, usually the larger constant needs to be the positive number.

Factored Form	Standard Form
$(x-2)(x+9)$	$x^2+7x-18$
$(x+10)(x-8)$	$x^2+2x-80$
$(3x+7)(x-2)$	$3x^2+1x-14$
$(4x-9)(x+3)$	$4x^2+3x-27$

Unit 7 Lesson 3 730 Activity 2

Pairs | 12 min

Activity 2 Now I Know My ABC's (continued)

Purpose: Students make and test predictions about how coefficients and constants in factored form affect the equivalent expression in standard form. **(MTR.3.1, MTR.5.1)**

Short on time? Consider inviting students to focus on Problems 8–10. Problems 11–13 provide challenges which further prepare students for factoring.

2 Monitor

To support students getting started, consider asking, "Where would 1 go in the diagram? What other parts of the diagram could this help you fill out?"

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to to write an expression in factored form that meets just one criteria. Then ask, "Does your expression meet both requirements? Why or why not?"
Guess and check.	**Strengthen** Celebrate their persistence. Encourage students to look for and make use of structure with questions like: • "What did you try first? Why?" • "What did you try that didn't work?" • "What could you try next?"
Look for and make use of the structure in their expressions and/or make generalizations. **(MTR.3.1, MTR.5.1)**	**Strengthen** Celebrate these mathematical noticings and invite them to share these ideas during the Connect.

D Differentiation: (Stretch) You're invited to explore more. Invite students who would like to explore quadratic expressions further to complete this optional task. Encourage them to discuss their thinking with a partner. Students may think that a b-value of 0 requires opposite constants in factored form, which means that the c-value will be negative. Celebrate this reasoning and invite students to explore different coefficients in factored form.

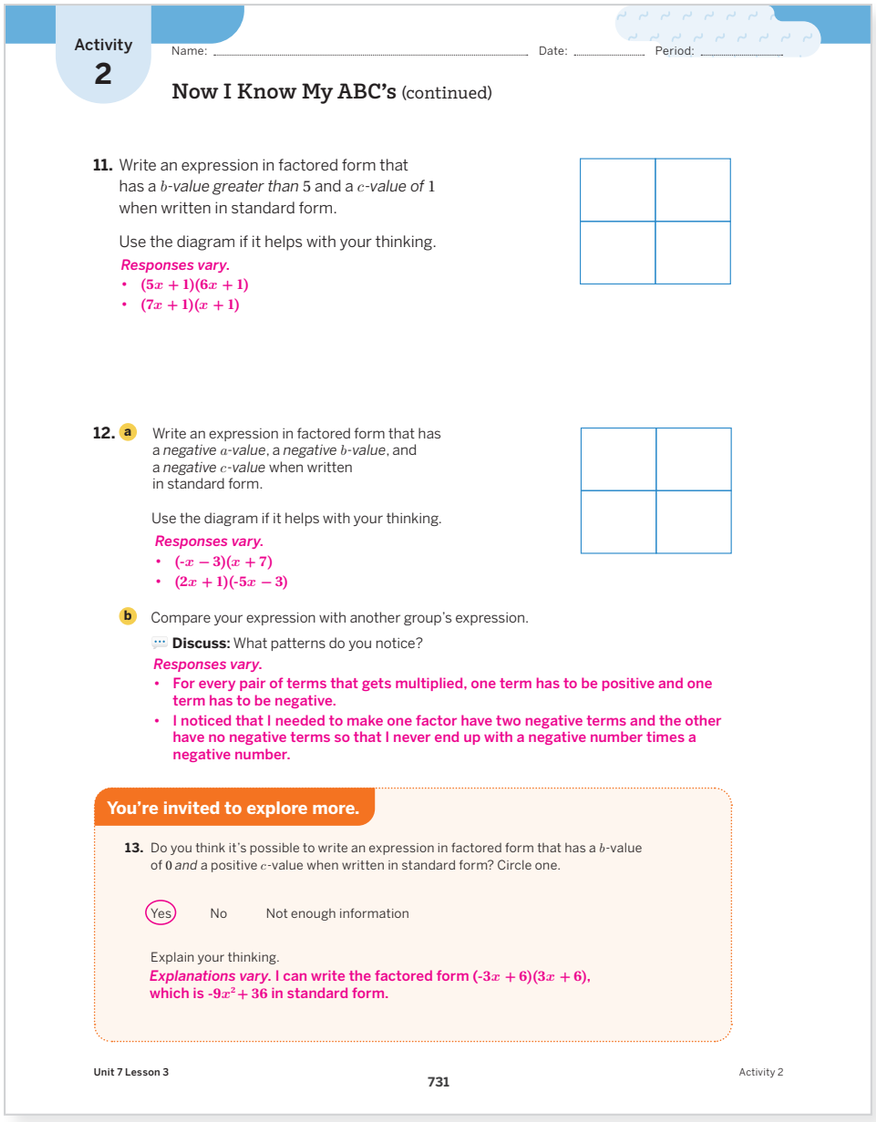
3 Connect

Invite students to share their patterns and reasoning.

Consider asking:

- "What must be true about a factored-form expression to have a negative c-value in standard form?" **(MTR.3.1, MTR.5.1)**
- "How can you tell a factored expression will have a positive b-value in standard form?" **(MTR.3.1, MTR.5.1)**

Invite students to share other new patterns they generalized during the challenges and add them to the class display. **(MTR.3.1, MTR.5.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Now I Know My ABC's (continued)

11. Write an expression in factored form that has a *b-value greater than* 5 and a *c-value of* 1 when written in standard form.

Use the diagram if it helps with your thinking.

Responses vary.
- $(5x + 1)(6x + 1)$
- $(7x + 1)(x + 1)$

12. a Write an expression in factored form that has a *negative* a*-value*, a *negative* b*-value*, and a *negative* c*-value* when written in standard form.

Use the diagram if it helps with your thinking.

Responses vary.
- $(-x - 3)(x + 7)$
- $(2x + 1)(-5x - 3)$

b Compare your expression with another group's expression.

Discuss: What patterns do you notice?

Responses vary.
- For every pair of terms that gets multiplied, one term has to be positive and one term has to be negative.
- I noticed that I needed to make one factor have two negative terms and the other have no negative terms so that I never end up with a negative number times a negative number.

You're invited to explore more.

13. Do you think it's possible to write an expression in factored form that has a b-value of 0 *and* a positive c-value when written in standard form? Circle one.

(Yes) No Not enough information

Explain your thinking.

Explanations vary. I can write the factored form $(-3x + 6)(3x + 6)$, which is $-9x^2 + 36$ in standard form.

Unit 7 Lesson 3 731 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students describe patterns between the structure of factored-form and standard-form expressions. **(MTR.3.1, MTR.5.1)**

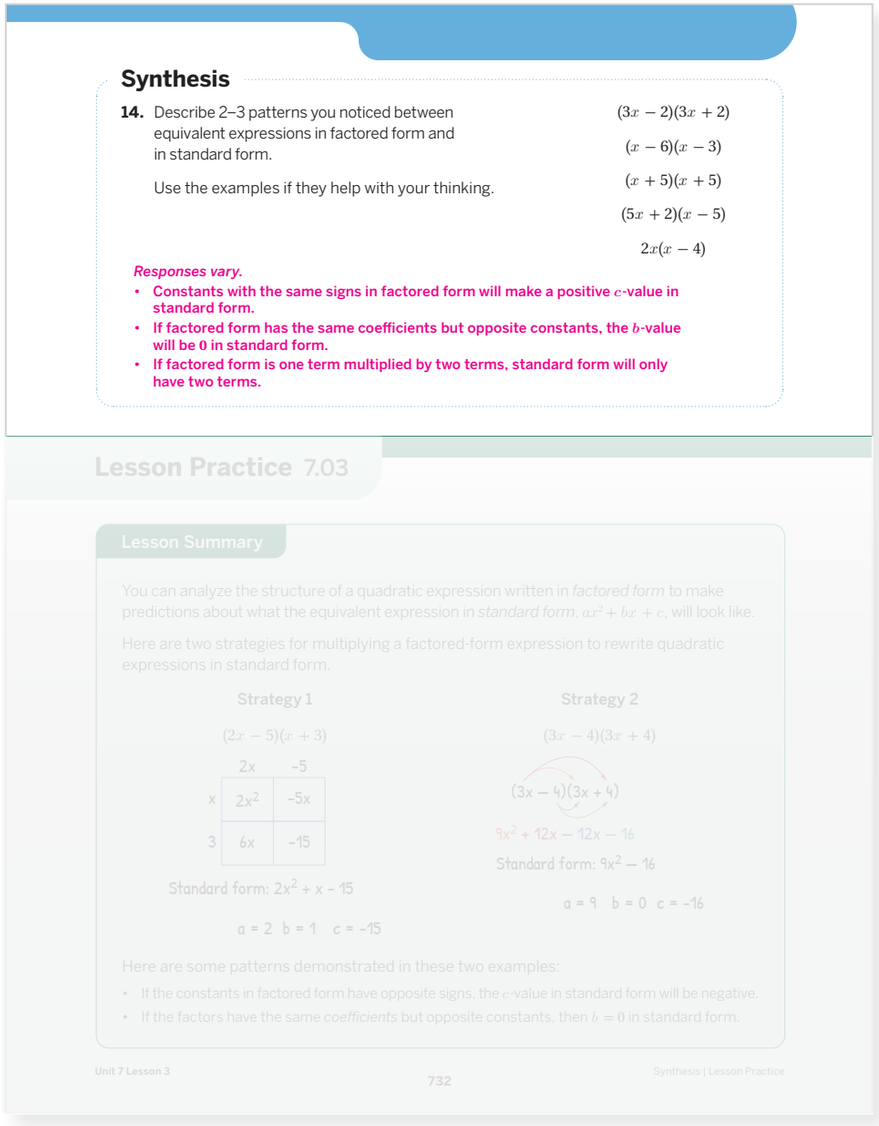
Invite students to respond independently, and then share their thinking with a partner.

Have students share a variety of ideas, such as:

- Constants with the same signs in factored form will make a positive c-value in standard form.
- If factored form has the same coefficients but opposite constants, the b-value will be 0 in standard form.
- If factored form is one term multiplied by two terms, standard form will only have two terms.

MLR **MLR2: Collect and Display** Use this routine to revisit student patterns and add any new patterns or examples as they share. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each pattern and what questions they still have.



Synthesis

14. Describe 2–3 patterns you noticed between equivalent expressions in factored form and in standard form.

Use the examples if they help with your thinking.

$(3x - 2)(3x + 2)$

$(x - 6)(x - 3)$

$(x + 5)(x + 5)$

$(5x + 2)(x - 5)$

$2x(x - 4)$

Responses vary.

- Constants with the same signs in factored form will make a positive c-value in standard form.
- If factored form has the same coefficients but opposite constants, the b-value will be 0 in standard form.
- If factored form is one term multiplied by two terms, standard form will only have two terms.

Lesson Practice 7.03

Lesson Summary

You can analyze the structure of a quadratic expression written in *factored form* to make predictions about what the equivalent expression in *standard form*, $ax^2 + bx + c$, will look like.

Here are two strategies for multiplying a factored-form expression to rewrite quadratic expressions in standard form.

Strategy 1

$(2x - 5)(x + 3)$

	2x	-5
x	$2x^2$	-5x
3	6x	-15

Standard form: $2x^2 + x - 15$

$a = 2$ $b = 1$ $c = -15$

Strategy 2

$(3x - 4)(3x + 4)$

$(3x - 4)(3x + 4)$

$9x^2 + 12x - 12x - 16$

Standard form: $9x^2 - 16$

$a = 9$ $b = 0$ $c = -16$

Here are some patterns demonstrated in these two examples:

- If the constants in factored form have opposite signs, the c-value in standard form will be negative.
- If the factors have the same *coefficients* but opposite constants, then $b = 0$ in standard form.

Unit 7 Lesson 3 732 Synthesis | Lesson Practice

Lesson Takeaway: The *coefficients* and constants in *factored form* directly impact the signs, values, and structure of *standard form* in predictable ways. Understanding some of these patterns can support us with converting between factored and standard form.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing equivalent expressions in factored and standard forms that meet given criteria.

Today's Goals

1. **Goal:** Identify the a-, b-, and c-values in a standard-form quadratic expression.
2. **Language Goal:** Analyze the structure of a factored-form expression to make predictions about its equivalent expression in standard form. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 7.03

Write an expression in factored form that has a *negative b-value* and a *negative c-value* when written in standard form.

Factored Form:

Standard Form:

Responses vary.

- Factored form: $(x-6)(x+3)$; Standard form: $x^2-3x-18$
- Factored form: $(5x-4)(2x+1)$; Standard form: $10x^2-3x-4$

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support writing equivalent expressions in factored and standard forms that meet given criteria:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on the strategies students have learned in Lessons 2–3. In Lesson 2, students learned about a strategy that will support them in factoring any quadratic expression. In this lesson, students identified patterns in the structure from Lesson 2 to support them in developing conceptual understanding, fluency, and efficiency with multiplication. How will the strategies they learned across these two lessons support them with factoring quadratic expressions? How does looking for and making use of structure support students in understanding the broader concept of factoring, rather than viewing each quadratic as an individual case to memorize? **(MTR.3.1, MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

14. Describe 2–3 patterns you noticed between equivalent expressions in factored form and in standard form.

Use the examples if they help with your thinking.

$(3x-2)(3x+2)$

$(x-6)(x-3)$

$(x+5)(x+5)$

$(5x+2)(x-5)$

$2x(x-4)$

Responses vary.

- Constants with the same signs in factored form will make a positive c-value in standard form.
- If factored form has the same coefficients but opposite constants, the b-value will be 0 in standard form.
- If factored form is one term multiplied by two terms, standard form will only have two terms.

Lesson Practice 7.03

Lesson Summary

You can analyze the structure of a quadratic expression written in *factored form* to make predictions about what the equivalent expression in *standard form*, ax^2+bx+c, will look like.

Here are two strategies for multiplying a factored-form expression to rewrite quadratic expressions in standard form.

Strategy 1

$(2x-5)(x+3)$

	2x	-5
x	$2x^2$	-5x
3	6x	-15

Standard form: $2x^2+x-15$

$a=2$ $b=1$ $c=-15$

Strategy 2

$(3x-4)(3x+4)$

$(3x-4)(3x+4)$

$9x^2+12x-12x-16$

Standard form: $9x^2-16$

$a=9$ $b=0$ $c=-16$

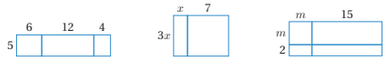
Here are some patterns demonstrated in these two examples:

- If the constants in factored form have opposite signs, the c-value in standard form will be negative.
- If the factors have the same *coefficients* but opposite constants, then $b=0$ in standard form.

Lesson Practice 7.03

Name: _____ Date: _____ Period: _____

1. Write an expression that represents each area model.



$5(6+12+14)$ (or equivalent)

$3x^2+21x$ (or equivalent)

$m^2+17m+30$ (or equivalent)

2. Complete the diagram to show that $(x-10)(x-3)$ is equivalent to $x^2-13x+30$.

	x	-10
x	x^2	-10x
-3	-3x	30

Problems 3–4: Write an expression in factored form that has:

3. A b-*value* less than 4 and a c-*value* greater than 1 when written in standard form.

Responses vary. $(x+1)(x+2)$

4. A negative a-*value*, negative b-*value*, and a positive c-*value* when written in standard form.

Responses vary. $(-x+1)(x+3)$

Lesson Practice 7.03

Name: _____ Date: _____ Period: _____

5. Select any number from the inner square.

- Multiply the number to the left of your selection by the number to the right.
- Multiply the number above your selection by the number below.
- Here's an example with the number 12 selected:



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The difference between the numbers will be 99 no matter your selection. Explain why.

Responses vary. If my selection is x, then the numbers on both sides are $x-1$ and $x+1$. Similarly, the numbers above and below are $x-10$ and $x+10$. Using algebra to multiply each pair and subtract:

$(x-1)(x+1)-(x-10)(x+10)=(x^2-1)-(x^2-100)$

$=x^2-x^2-1+100$

$=99$

Test Practice

Problems 6–8: For each expression in factored form, write an equivalent expression in standard form.

6. $(x-2)^2$

x^2-4x+4

7. $(x+1)(x-1)$

x^2-1

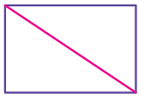
8. $(2x+4)(x-3)$

$2x^2-2x-12$

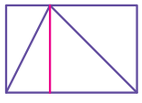
Spiral Review

9. Draw a line to show that this rectangle is made only from 2 congruent triangles.

Responses vary.



10. Draw a line to show that this rectangle is made only from 2 *pairs* of congruent triangles.



Lesson Practice Item Analysis

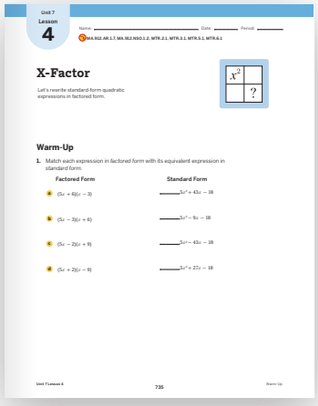
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.AR.1.3
	3–4	2	MA.912.AR.1.3
	5	3	MA.912.AR.1.3
Test Practice	6–8	2	MA.912.AR.1.3
Spiral Review			
Fluency	9–10	2	MA.6.GR.2.2

Unit 7
Lesson
4

X-Factor

Factoring Quadratic Expressions

Let's rewrite standard-form quadratic expressions in factored form.


Unit 7
Lesson
4

Name _____ Date _____ Period _____

MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.3.1, MTR.5.1, MTR.6.1

X-Factor

Let's rewrite standard-form quadratic expressions in factored form.

Warm-Up

1. Match each expression in factored form with its equivalent expression in standard form.

Factored Form	Standard Form
$(5x + 6)(x - 3)$	_____
$(5x - 3)(x + 6)$	_____
$(5x - 2)(x + 9)$	_____
$(5x + 2)(x - 9)$	_____

Learning Outcomes

Today's Goals

1. **Goal:** Factor to rewrite standard-form quadratic expressions in factored form.
2. **Language Goal:** Describe strategies for factoring and explain how to determine if a factoring attempt is correct. **(Writing, Speaking, and Listening)**

Students rewrite quadratic expressions written in *standard form* to *factored form*. They use diagrams to visualize how to factor quadratic expressions in standard form and then develop strategies to factor quadratic expressions.

Note: In this unit, students are simultaneously introduced to factoring quadratic expressions in standard form where $a = 1$ and where $a \neq 1$. This is intentional so students generate strategies that apply to factoring all quadratic expressions and are less likely to overgeneralize.

Coherence

Prior Learning

In Lessons 2 and 3, students used diagrams to multiply to rewrite factored-form expressions in standard form. They described how the coefficients and constants in factored form affect the a-, b-, and c-values of standard form, to support them with both multiplying and factoring.

Future Learning

In Lesson 5, students will factor expressions with special structures, such as differences of squares and expressions with a common factor.

Connections

- Students express quadratic expressions in different forms (area model diagram, standard form, factored form) in a puzzle activity. (mathematical)
- Students extend their understanding of finding factors of linear expressions using an area model/diagram to factor quadratic expressions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.7
Rewrite a polynomial expression as a product of polynomials over the real number system.

MA.912.NSO.1.2
Generate equivalent algebraic expressions using the properties of exponents.

Also Addressing: **MA.912.NSO.1.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.2	**MA.912.NSO.1.3**
MA.7.NSO.1.1	

Vocabulary

Review Vocabulary

factor (of a number or expression)

factored form

standard form (of a quadratic equation)

Lesson at a Glance

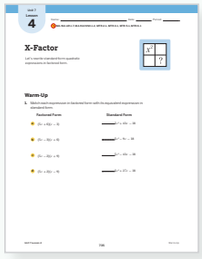
~ 45 min

MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

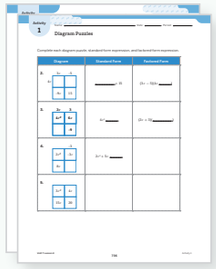
Students revisit multiplying and begin to consider structure by matching quadratic expressions written in *factored form* and *standard form*.
Routine: Notice and Wonder **(MTR.5.1)**



Activity 1

Pairs | 15 min

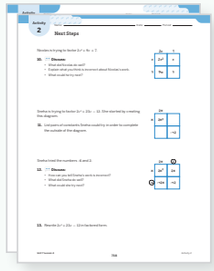
Students use multiplication and diagrams to write equivalent quadratic expressions and begin to develop strategies for factoring expressions in standard form.



Activity 2

Pairs | 15 min

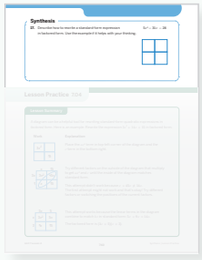
Students critique factoring attempts and surface a variety of strategies to develop fluency in factoring quadratic expressions.
MLR2: Collect and Display; MLR3: Critique, Correct, Clarify **(MTR.3.1)**



Synthesis

Whole Class | 5 min

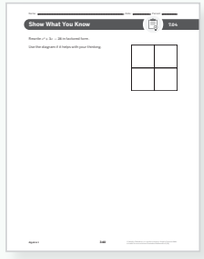
Students synthesize strategies for rewriting quadratic expressions written in standard form to factored form.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by rewriting a quadratic expression written in standard form to factored form.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit multiplying and begin to consider structure by matching quadratic expressions written in *factored form* and *standard form*. **(MTR.5.1)**

1 Launch

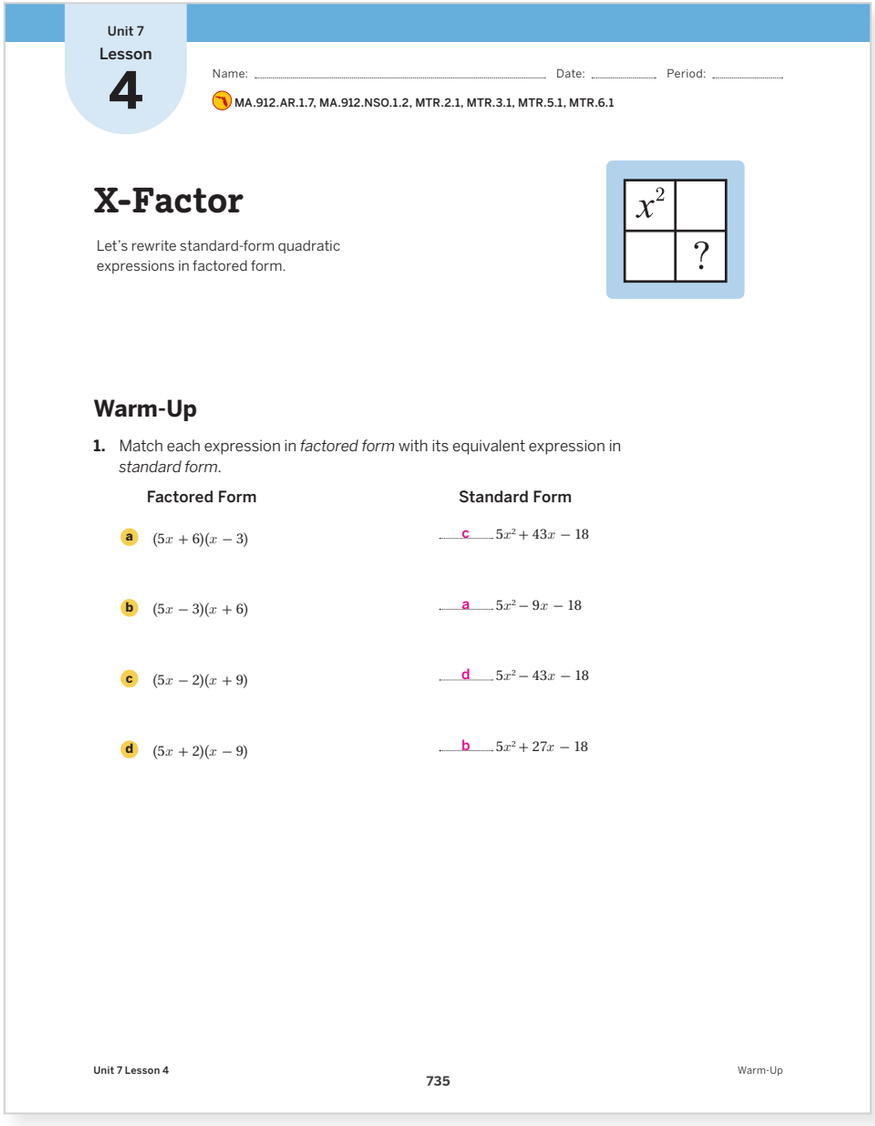
Consider asking, "How are each of these expressions alike? How are they different?"

A **Accessibility: Memory and Attention**
Throughout this lesson, invite students to use a calculator to support them with performing calculations and to draw a diagram if it helps them with their thinking.

2 Connect

Invite students to share their strategies for matching.

Use the Notice and Wonder routine to promote curiosity and help students make connections between standard form and factored form. See the Routine Facilitation Guide for more information. **(MTR.5.1)**



Unit 7
Lesson
4

Name: _____ Date: _____ Period: _____

MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.3.1, MTR.5.1, MTR.6.1

X-Factor

Let's rewrite standard-form quadratic expressions in factored form.

Warm-Up

1. Match each expression in *factored form* with its equivalent expression in *standard form*.

Factored Form	Standard Form
a $(5x + 6)(x - 3)$	c $5x^2 + 43x - 18$
b $(5x - 3)(x + 6)$	a $5x^2 - 9x - 18$
c $(5x - 2)(x + 9)$	d $5x^2 - 43x - 18$
d $(5x + 2)(x - 9)$	b $5x^2 + 27x - 18$

Unit 7 Lesson 4 735 Warm-Up

Pairs | 15 min

Activity 1 Diagram Puzzles

Purpose: Students use multiplication and diagrams to write equivalent quadratic expressions and begin to develop strategies for factoring expressions in standard form. **(MTR.5.1)**

Short on time? Consider omitting Problems 5, 7, and 9.

1 Launch

Display Problem 2 and support students in making sense of the task by asking:

- "Where do you see 15 in each representation? Where do you see $(3x - 5)$?"
- "Where are three different places we could start?" **(MTR.1.1)**

2 Monitor

To surface connections, invite students to color code or use shapes to show parts of the expressions in standard form and factored form in the diagram.

Look for strategies students used for their first steps and patterns they are noticing and utilizing.

A **Accessibility: Memory and Attention** Consider providing a multiplication table or factor chart for students to reference.

D **Differentiation: (Support)** Students may forget a plus sign in factored form (e.g., $(2x\ 3)(2x - 3)$). Invite them to read their expression aloud to catch what they are missing.

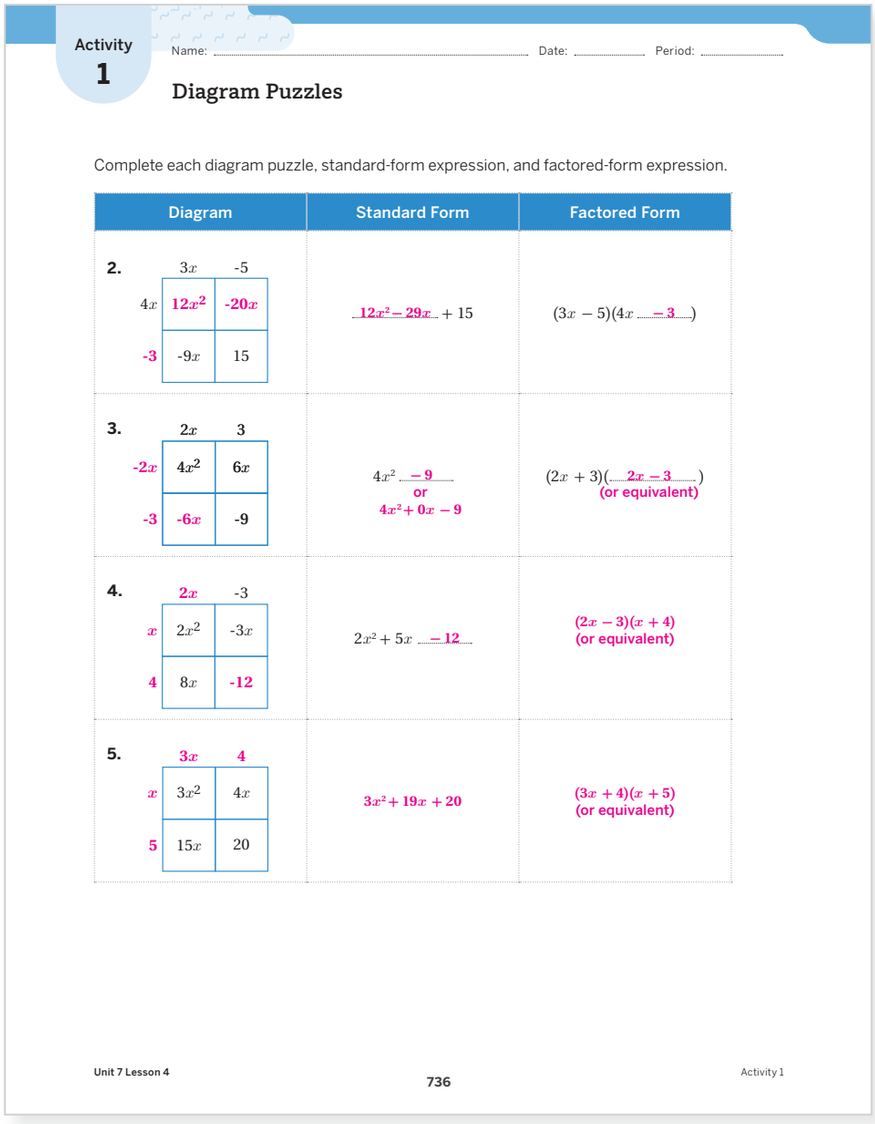
Pause briefly and reflect on Problem 4. Consider asking:

- "How did you decide where to start?"
- "How did you determine the c-value in standard form?"
- "How did you determine the factored form?"

Pause briefly and reflect on Problem 6. Consider asking:

- "How did you decide where to start?"
- "What strategies did you use to complete the puzzle?"
- "How are the terms in the factored form related to the a-, b-, and c-values in standard form?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Diagram Puzzles

Complete each diagram puzzle, standard-form expression, and factored-form expression.

Diagram	Standard Form	Factored Form
2. $3x$ / -5; $4x$: $12x^2$, $-20x$; -3: $-9x$, 15	$12x^2 - 29x + 15$	$(3x - 5)(4x - 3)$
3. $2x$ / 3; $-2x$: $4x^2$, $6x$; -3: $-6x$, -9	$4x^2 - 9$ or $4x^2 + 0x - 9$	$(2x + 3)(2x - 3)$ (or equivalent)
4. $2x$ / -3; x: $2x^2$, $-3x$; 4: $8x$, -12	$2x^2 + 5x - 12$	$(2x - 3)(x + 4)$ (or equivalent)
5. $3x$ / 4; x: $3x^2$, $4x$; 5: $15x$, 20	$3x^2 + 19x + 20$	$(3x + 4)(x + 5)$ (or equivalent)

Unit 7 Lesson 4 736 Activity 1

Pairs | 15 min

Activity 1 Diagram Puzzles (continued)

Purpose: Students use multiplication and diagrams to write equivalent quadratic expressions and begin to develop strategies for factoring expressions in standard form. **(MTR.5.1)**

Short on time? Consider omitting Problems 5, 7, and 9.

2 Monitor

Listen for student strategies to highlight during the Connect such as:

- Starting with prime terms or terms that have fewer pairs of factors.
- Determining a value on the outside of the diagram by determining a common factor or the greatest common factor of that row or column.
- Checking to make sure the diagram matches both factored and standard forms after completing it.

D Differentiation *(Problems 6–9)*

Look for students who:	Respond to Student Thinking
Complete the entire inside of a diagram first and determine the expression cannot be factored.	**Support** Consider asking, "Is that the only way to complete the inside of the diagram?" Consider sharing that there are infinitely many pairs of numbers that add to get bx, but a limited amount that multiplies to c.
Notice a classmate's diagram is different from theirs.	**Strengthen** Consider asking, "Are both diagrams equivalent? How do you know?"
Think that ax^2 will always be broken up into ax and x as factors. *(Problem 9)*	**Support** Consider asking, "What other terms multiply to $6x^2$ besides $6x$ and x?"

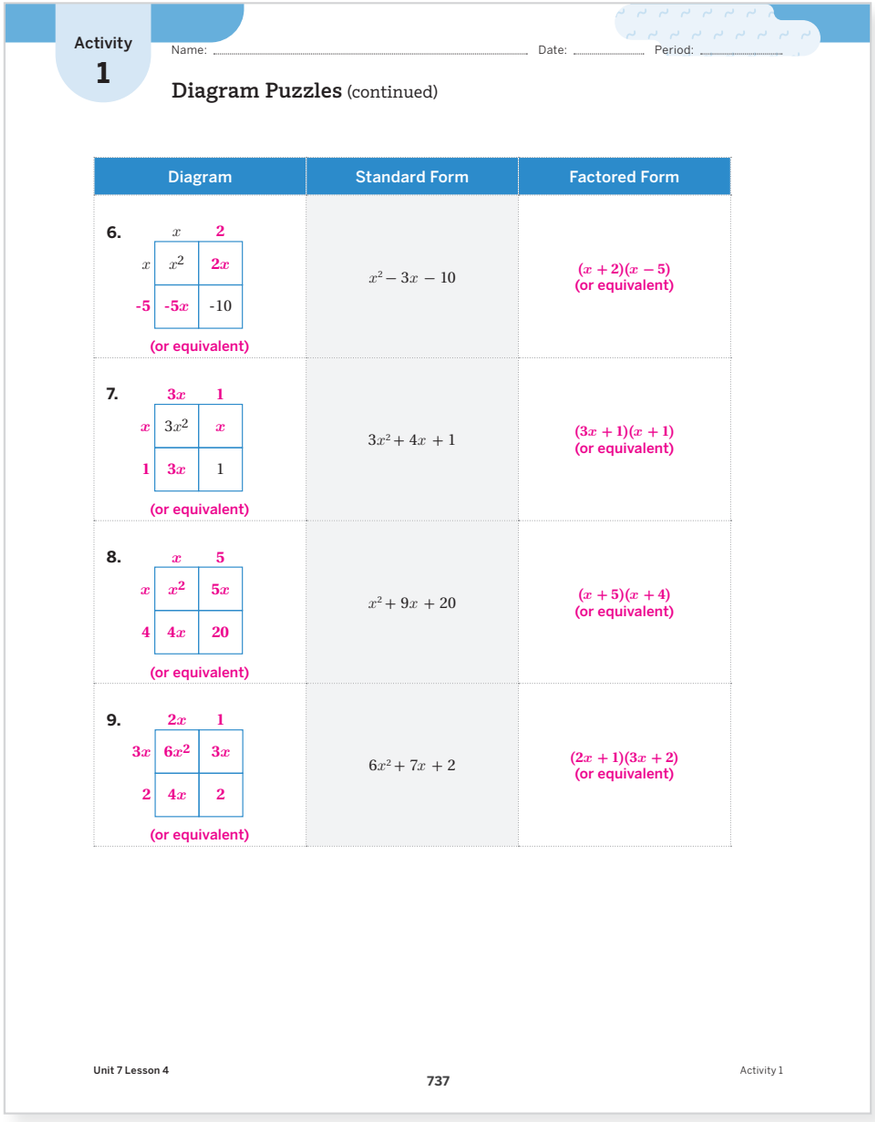
3 Connect

Display Problem 8 and consider asking:

- "When given standard form, how can you start a blank diagram puzzle?"
- "What patterns did you notice to help you complete the diagram and write the factored form?" **(MTR.5.1)**
- "How are the factored form and standard form expressions related?" **(MTR.5.1)**

Note: If time allows, consider comparing Problems 7 and 8 to surface that students may need to try different coefficients in factored form.

Consider sharing the Key Takeaway.



Activity 1

Name: Date: Period:

Diagram Puzzles (continued)

Diagram	Standard Form	Factored Form
6. x 2 / x: x^2, $2x$ / -5: $-5x$, -10 (or equivalent)	$x^2 - 3x - 10$	$(x + 2)(x - 5)$ (or equivalent)
7. $3x$ 1 / x: $3x^2$, x / 1: $3x$, 1 (or equivalent)	$3x^2 + 4x + 1$	$(3x + 1)(x + 1)$ (or equivalent)
8. x 5 / x: x^2, $5x$ / 4: $4x$, 20 (or equivalent)	$x^2 + 9x + 20$	$(x + 5)(x + 4)$ (or equivalent)
9. $2x$ 1 / $3x$: $6x^2$, $3x$ / 2: $4x$, 2 (or equivalent)	$6x^2 + 7x + 2$	$(2x + 1)(3x + 2)$ (or equivalent)

Unit 7 Lesson 4 737 Activity 1

Key Takeaway: The process of rewriting quadratic expressions from *standard form* to *factored form* is the reverse of multiplication and is called *factoring*.

Pairs | 15 min

Activity 2 Next Steps

Purpose: Students critique factoring attempts and surface a variety of strategies to develop fluency in factoring quadratic expressions. **(MTR.3.1)**

Short on time? Consider omitting Problems 14, 15, and/or 16c.

1 Launch

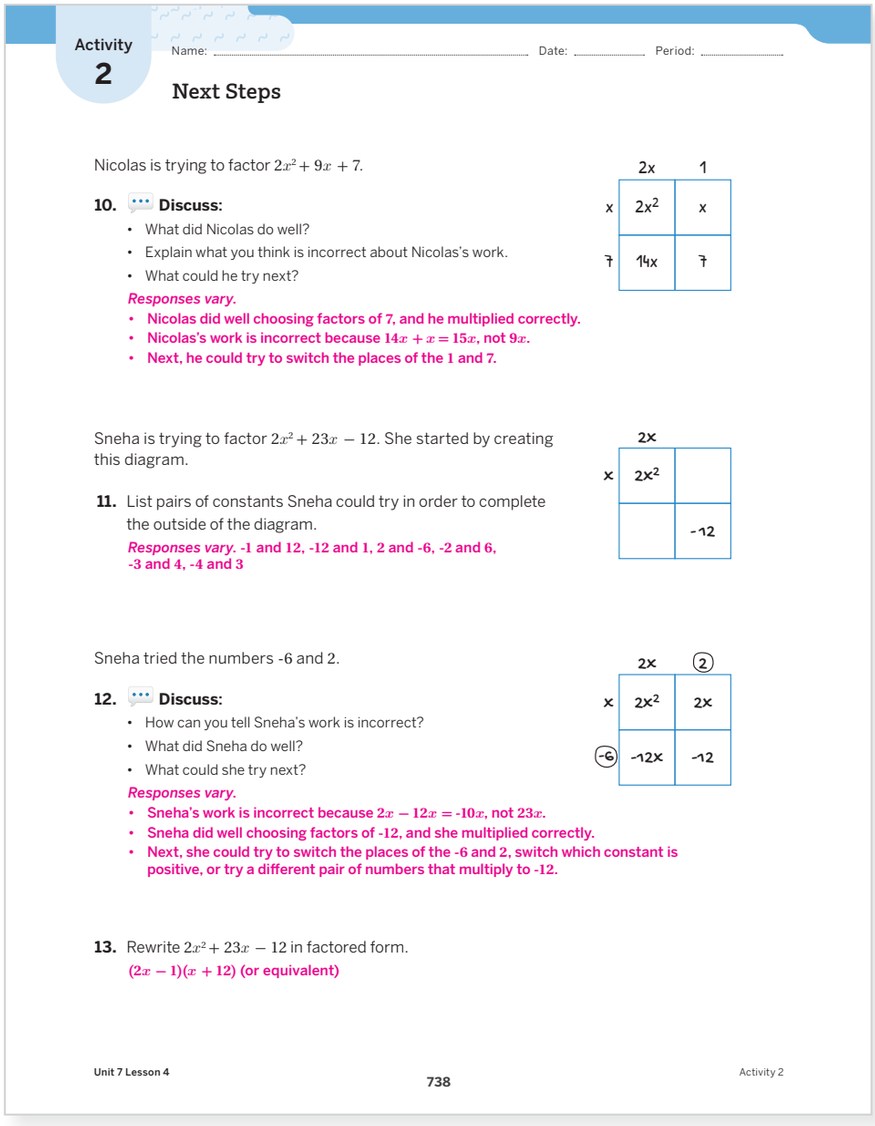
MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Nicolas's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.3.1)**

2 Monitor

Look for a variety of next steps, such as:
- Trying the same constants in different positions.
- Trying different constants that multiply to the c-value.
- Trying different coefficients that multiply to the a-value.

Look for how students factor Problem 13. Check in with pairs who need support.

Activity 2 continued >



Activity 2

Name: Date: Period:

Next Steps

Nicolas is trying to factor $2x^2 + 9x + 7$.

	2x	1
x	$2x^2$	x
7	14x	7

10. **Discuss:**
- What did Nicolas do well?
- Explain what you think is incorrect about Nicolas's work.
- What could he try next?

Responses vary.
- Nicolas did well choosing factors of 7, and he multiplied correctly.
- Nicolas's work is incorrect because $14x + x = 15x$, not $9x$.
- Next, he could try to switch the places of the 1 and 7.

Sneha is trying to factor $2x^2 + 23x - 12$. She started by creating this diagram.

	2x	
x	$2x^2$	
		-12

11. List pairs of constants Sneha could try in order to complete the outside of the diagram.

Responses vary. -1 and 12, -12 and 1, 2 and -6, -2 and 6, -3 and 4, -4 and 3

Sneha tried the numbers -6 and 2.

	2x	(2)
x	$2x^2$	2x
(-6)	-12x	-12

12. **Discuss:**
- How can you tell Sneha's work is incorrect?
- What did Sneha do well?
- What could she try next?

Responses vary.
- Sneha's work is incorrect because $2x - 12x = -10x$, not $23x$.
- Sneha did well choosing factors of -12, and she multiplied correctly.
- Next, she could try to switch the places of the -6 and 2, switch which constant is positive, or try a different pair of numbers that multiply to -12.

13. Rewrite $2x^2 + 23x - 12$ in factored form.

$(2x - 1)(x + 12)$ (or equivalent)

Unit 7 Lesson 4 738 Activity 2

Pairs | 15 min

Activity 2 Next Steps (continued)

Purpose: Students critique factoring attempts and surface a variety of strategies to develop fluency in factoring quadratic expressions. **(MTR.3.1)**

Short on time? Consider omitting Problems 14, 15, and/or 16c.

2 Monitor

Differentiation: (Support) In Problem 14, students may list factors, but be unsure about how to choose which ones. Invite them to place the factors outside of the diagram and use guess and check to see what happens. Consider asking, "What do the terms in the blank spaces need to add to? How do you know?"

English Language Learners Provide a word bank to support students in expressing their ideas, including but not limited to: *first, next, coefficient, constant, top, bottom, left, right, factors, terms, inside, outside*, etc.

Differentiation *(Problem 16)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which terms can you place in the diagram? Where can you place them?"
Persevere when their first attempt does not work.	**Support** Celebrate their persistence. Encourage them to share how they picked next steps to take during the Connect.
Would like to explore factoring quadratic expressions further.	**Stretch** Invite students to write an expression in standard form with a c-value of -12 that cannot be written in factored form using only integer constants and coefficients.

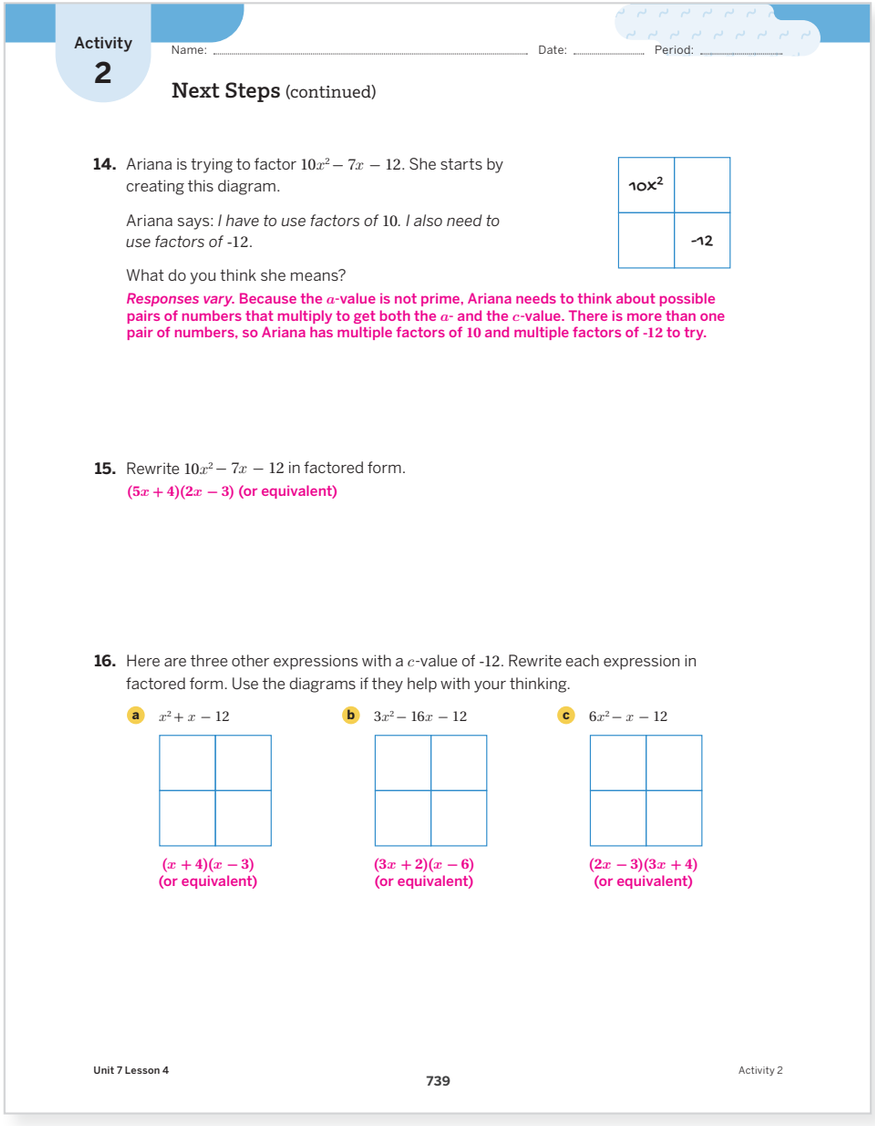
3 Connect

Consider asking:

- "How can you tell if a factoring attempt is correct?" **(MTR.3.1)**
- "What are possible next steps to try when a first factoring attempt doesn't work?"

MLR2: Collect and Display Use this routine to collect students' strategies for factoring quadratic expressions. Display for students to reference and add to for the rest of the unit.

Note: If it does not come up naturally, consider asking, "How can you check if your expressions are equivalent?"



Activity 2

Name: Date: Period:

Next Steps (continued)

14. Ariana is trying to factor $10x^2 - 7x - 12$. She starts by creating this diagram.

Ariana says: *I have to use factors of 10. I also need to use factors of -12.*

What do you think she means?

Responses vary. Because the a-value is not prime, Ariana needs to think about possible pairs of numbers that multiply to get both the a- and the c-value. There is more than one pair of numbers, so Ariana has multiple factors of 10 and multiple factors of -12 to try.

15. Rewrite $10x^2 - 7x - 12$ in factored form.

$(5x + 4)(2x - 3)$ (or equivalent)

16. Here are three other expressions with a c-value of -12. Rewrite each expression in factored form. Use the diagrams if they help with your thinking.

a. $x^2 + x - 12$

$(x + 4)(x - 3)$ (or equivalent)

b. $3x^2 - 16x - 12$

$(3x + 2)(x - 6)$ (or equivalent)

c. $6x^2 - x - 12$

$(2x - 3)(3x + 4)$ (or equivalent)

Unit 7 Lesson 4 739 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for rewriting quadratic expressions written in standard form to factored form.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

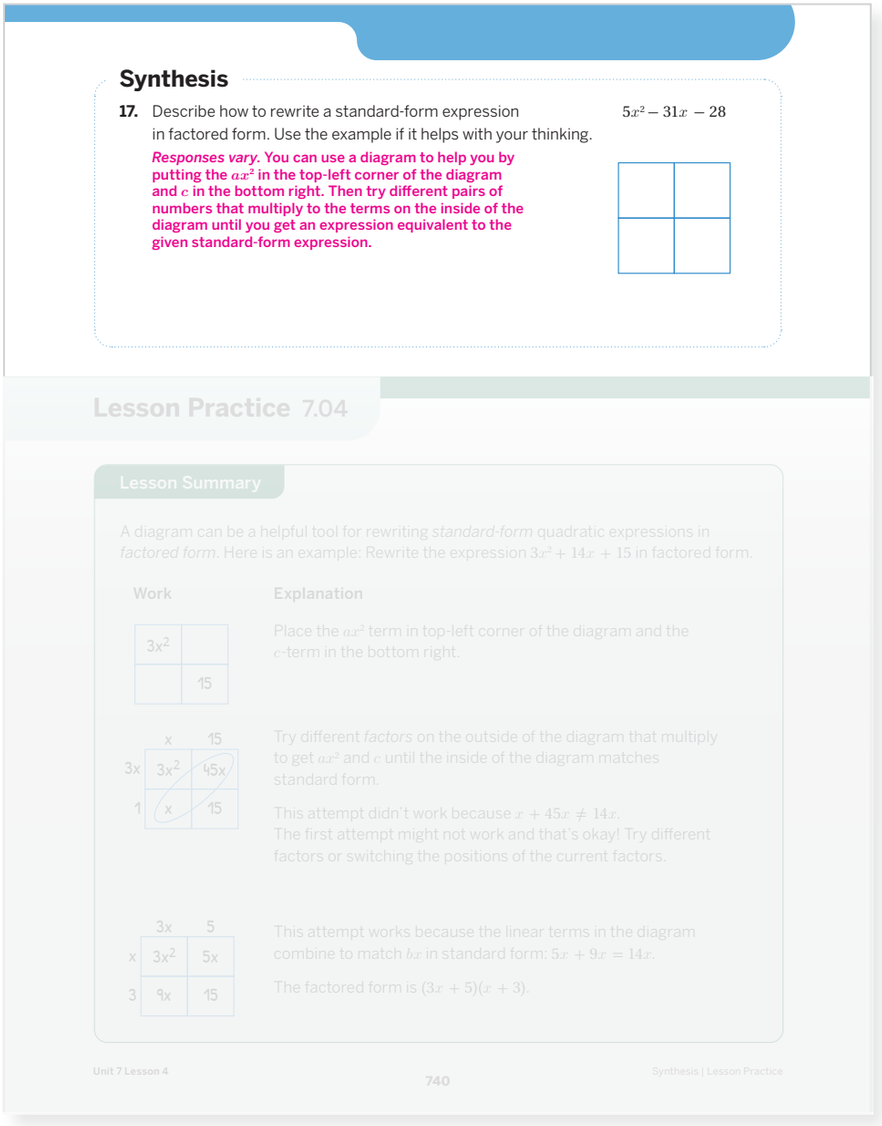
- Where to place specific terms from the standard-form expression into the diagram.
- How to pick pairs of numbers to try.
- How to determine if an attempt is correct and what to try next if it is not.

ELL **English Language Learners** To support students with processing their ideas in words, consider giving them the option to verbally share their response and point to terms and locations in the example instead of writing.

A **Accessibility: Executive Functioning** To support students in making sense of a complex process, consider asking:

- "How do you start?"
- "What do you try in the middle?"
- "How do you know you are done?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

17. Describe how to rewrite a standard-form expression in factored form. Use the example if it helps with your thinking. $5x^2 - 31x - 28$

Responses vary. You can use a diagram to help you by putting the ax^2 in the top-left corner of the diagram and c in the bottom right. Then try different pairs of numbers that multiply to the terms on the inside of the diagram until you get an expression equivalent to the given standard-form expression.

Lesson Practice 7.04

Lesson Summary

A diagram can be a helpful tool for rewriting *standard-form* quadratic expressions in *factored form*. Here is an example: Rewrite the expression $3x^2 + 14x + 15$ in factored form.

Work	Explanation
$3x^2$ | | 15	Place the ax^2 term in top-left corner of the diagram and the c-term in the bottom right.
x 15 / 3x $3x^2$ $45x$ / 1 x 15	Try different *factors* on the outside of the diagram that multiply to get ax^2 and c until the inside of the diagram matches standard form. This attempt didn't work because $x + 45x \neq 14x$. The first attempt might not work and that's okay! Try different factors or switching the positions of the current factors.
3x 5 / x $3x^2$ $5x$ / 3 $9x$ 15	This attempt works because the linear terms in the diagram combine to match bx in standard form: $5x + 9x = 14x$. The factored form is $(3x + 5)(x + 3)$.

Unit 7 Lesson 4 740 Synthesis | Lesson Practice

Lesson Takeaway: A diagram can be a helpful tool to factor quadratic expressions in *standard form*. Place the ax^2 term in the top-left corner of the diagram and the c-value in the bottom right. Try different pairs of numbers that multiply to get ax^2 and c until the inside of the diagram matches standard form. If the first attempt doesn't work, try different *factors* or different positions.

Independent | 5 min

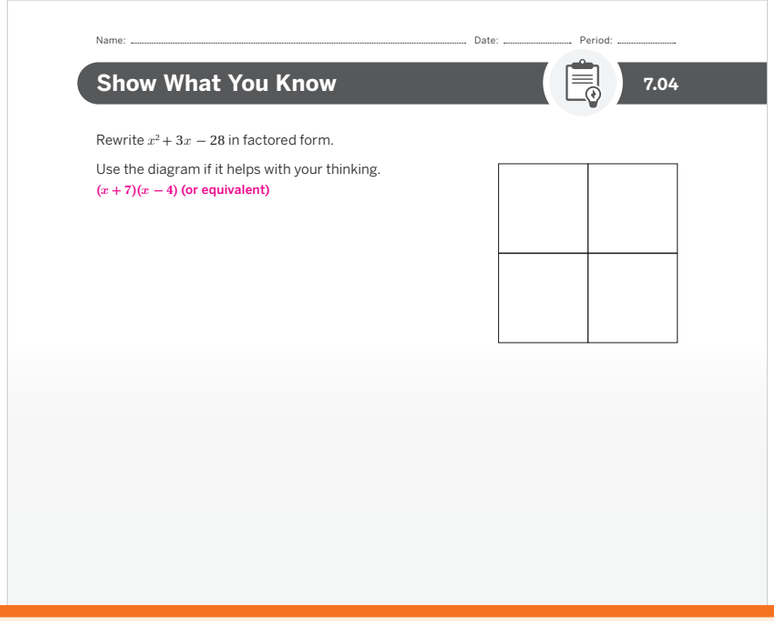
Show What You Know

Purpose: Students demonstrate their understanding by rewriting a quadratic expression written in standard form to factored form.

Today's Goals

1. **Goal:** Factor to rewrite standard form quadratic expressions in factored form.
2. **Language Goal:** Describe strategies for factoring and explain how to determine if a factoring attempt is correct. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.04

Rewrite $x^2 + 3x - 28$ in factored form.

Use the diagram if it helps with your thinking.

$(x + 7)(x - 4)$ (or equivalent)

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support factoring quadratic expressions in standard form:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students developed strategies for factoring quadratic expressions in *standard form*. How did students using diagrams to multiply in previous lessons support their introduction to factoring? How did students approach factoring quadratic expressions today, where $a = 1$ and $a \neq 1$? What structures and patterns did students notice while factoring? How does looking for and making use of structure support students in understanding the broader concepts of multiplying and factoring, rather than viewing each type of quadratic as an individual case to memorize? **(MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

17. Describe how to rewrite a standard-form expression in factored form. Use the example if it helps with your thinking.

$5x^2 - 31x - 28$

Responses vary. You can use a diagram to help you by putting the ax^2 in the top-left corner of the diagram and c in the bottom right. Then try different pairs of numbers that multiply to the terms on the inside of the given standard-form expression.

Lesson Practice 7.04

Lesson Summary

A diagram can be a helpful tool for rewriting *standard-form* quadratic expressions in *factored form*. Here is an example: Rewrite the expression $3x^2 + 14x + 15$ in factored form.

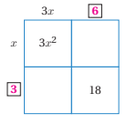
Work	Explanation
	Place the ax^2 term in top-left corner of the diagram and the c-term in the bottom right.
	Try different *factors* on the outside of the diagram that multiply to get ax^2 and c until the inside of the diagram matches standard form. This attempt didn't work because $x + 45x \neq 14x$. The first attempt might not work and that's okay! Try different factors or switching the positions of the current factors.
	This attempt works because the linear terms in the diagram combine to match bx in standard form: $5x + 9x = 14x$. The factored form is $(3x + 5)(x + 3)$.

Lesson Practice 7.04

Name: ..... Date: ..... Period: .....

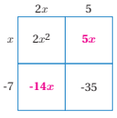
1. Write two possible constants that could complete the outside of the diagram.

Responses vary.



Problems 2–3: Complete the diagram puzzles and expressions.

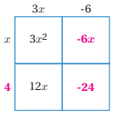
2.



Factored Form: $(2x + 5)(x - 7)$

Standard Form: $2x^2 - 9x - 35$

3.



Factored Form: $(3x - 6)(x + 4)$

Standard Form: $3x^2 + 6x - 24$

4. Rewrite each expression in factored or standard form.

Factored Form	Standard Form
$(x + 3)(x + 6)$	$x^2 + 9x + 18$
$(2x - 3)(2x - 7)$	$4x^2 - 20x + 21$
$(3x - 2)(x + 4)$	$3x^2 + 10x - 8$

5. Determine values that make the equation true.

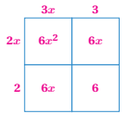
$7x^2 - 10x - _8_ = (_1_x - 2)(7x + _4_)$

Lesson Practice 7.04

Name: ..... Date: ..... Period: .....

6. Create and complete a diagram puzzle that only uses factors of 12 as coefficients and constants.

Responses vary.



Test Practice

7. This quadratic expression in standard form has an unknown b-value. If we know the expression can be factored, select the possibilties for the unknown value. Select all that apply.

$3x^2 + \boxed{?}x - 4$

- ☐ A. 12
- ☑ B. -11
- ☐ C. -8
- ☑ D. -1
- ☑ E. 4

Spiral Review

Problems 8–10: Fill in the missing values to make each equation true. *Responses vary.*

8. $3(_2_ + _4_) = 18$

9. $_5_(_1_ + _3_) = 20$

10. $4(_1_ + _3_) = 3(_4_) + 2(_2_)$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.1.7*
	2–3, 5	2	MA.912.AR.1.7
	4	1	MA.912.AR.1.7
	6	3	MA.912.AR.1.7
Test Practice	**7**	2	MA.912.AR.1.7
Spiral Review			
Fluency	**8–10**	2	MA.6.NSO.3.2

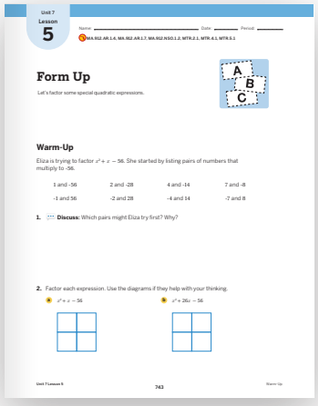
*This problem builds toward the standard shown.

Unit 7
Lesson
5

Form Up

More Factoring Quadratic Expressions

Let's factor some special quadratic expressions.


Unit 7
Lesson
5
Form Up
Let's factor some special quadratic expressions.
Warm-Up
1 and -56
2 and -28
4 and -14
7 and -8
-1 and 56
-2 and 28
-4 and 14
-7 and 8
1. Discuss: Which pairs might Eliza try first? Why?
2. Factor each expression. Use the diagrams if they help with your thinking.
743

Learning Outcomes

Today's Goals

1. **Goal:** Factor quadratic expressions in standard form.
2. **Language Goal:** Analyze the structure of a quadratic expression in standard form to make predictions about equivalent expressions in factored form. **(Writing, Speaking, and Listening)**

Students analyze quadratic expressions with specific structures: *differences of squares* (e.g., $4x^2 - 9$), expressions with a common factor (e.g., $2x^2 - 6x - 8$), and expressions with an a-value of 1. They identify patterns that can support them in factoring quadratic expressions with special structures and then apply these strategies to develop fluency with factoring. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 4, students used diagrams to factor quadratic expressions written in standard form.

Future Learning

In Lesson 6, students will use factoring to determine the x-intercepts of quadratic functions written in standard form. In Lesson 7, they will use factoring and the zero-product property to solve quadratic equations.

Connections

- Students continue to build their understanding of factoring quadratic expressions to factor special quadratic expressions, including difference of squares. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.4
Divide a polynomial expression by a monomial expression with rational number coefficients.

MA.912.AR.1.7
Rewrite a polynomial expression as a product of polynomials over the real number system.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.2 **MA.7.NSO.1.1**	**MA.912.NSO.1.3**

Vocabulary

Review Vocabulary

factor (of a number or expression)

factored form

standard form (of a quadratic equation)

Additional Vocabulary

difference of squares

Lesson at a Glance

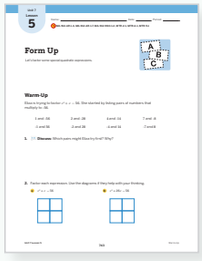
~ 45 min

MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

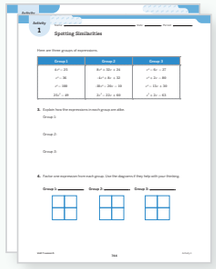
Students surface strategies for selecting pairs of numbers to try as the constants in *factored form* when given *standard form*.
Routine: Think-Pair-Share



Activity 1

Pairs | 20 min

Students surface patterns and strategies to help them factor expressions with special structures. **Routines:** Think-Pair-Share; MLR2: Collect and Display **(MTR.5.1)**



Activity 2

Pairs | 10 min

Students apply their strategies to efficiently factor quadratic expressions with various structures.



Synthesis

Whole Class | 5 min

Students synthesize strategies for factoring quadratic expressions. MLR2: Collect and Display **(MTR.2.1, MTR.4.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by factoring a quadratic expression.



Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 2 Cards, one card per student, pre-cut
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students surface strategies for selecting pairs of numbers to try as the constants in *factored form* when given *standard form*.

1 Launch

Display Eliza's work and consider asking:

- "Why do you think Eliza started by listing pairs of numbers that multiply to -56?"
- "Which pairs might be least likely to work as constants in factored form? Why?" **(MTR.5.1)**

Use the Think-Pair-Share routine to support student-to-student discourse for Problem 1. See the Routine Facilitation Guide for more information.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations and/or draw diagrams to support them with their thinking. Consider providing multiplication charts or reviewing integer operation rules for students to reference throughout this sub-unit.

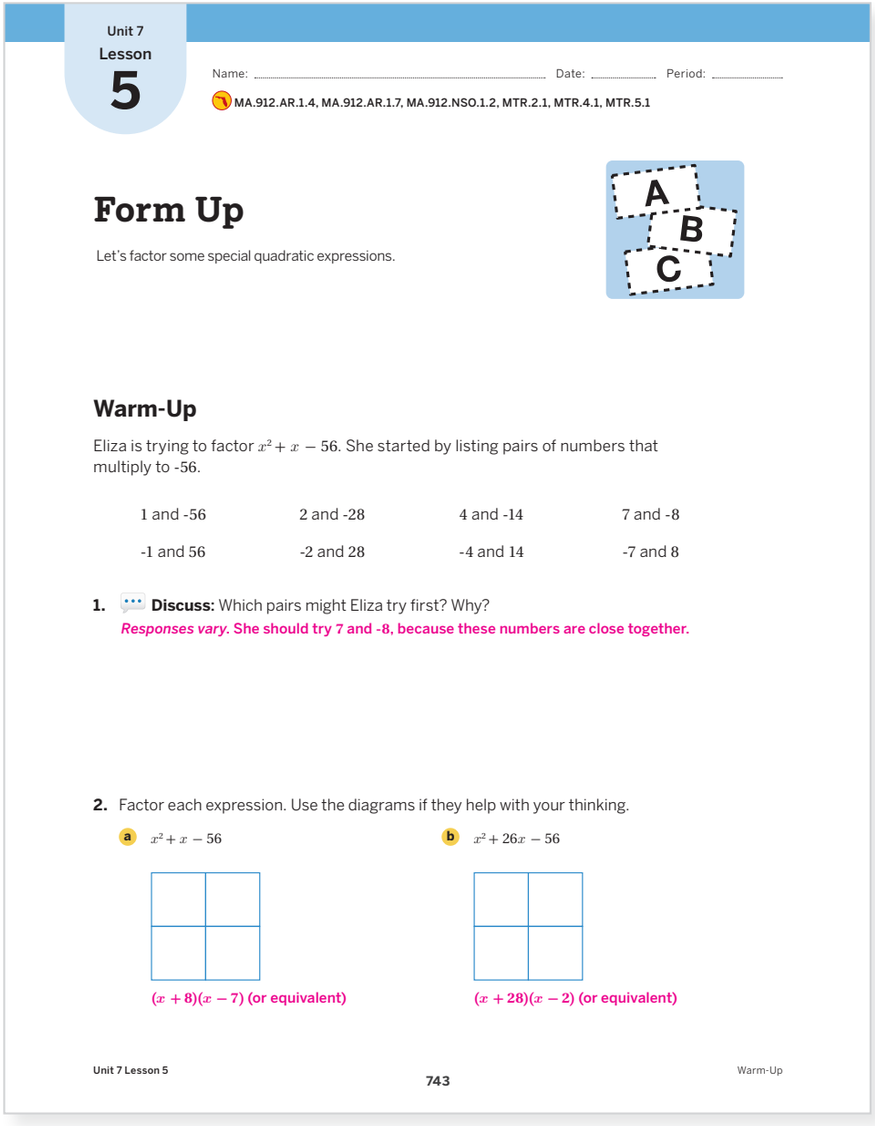
D **Differentiation:** (Stretch) Invite students to write another quadratic expression in standard form that Eliza's list could help them rewrite in factored form.

2 Connect

Invite students to share their expressions written in factored form.

To surface the Key Takeaway, consider asking:

- "What strategies did you use to select the constants for factored form?"
- "What patterns do you notice? Do you think that these will work for all quadratic expressions?"



Unit 7
Lesson
5

Name: _____ Date: _____ Period: _____

MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Form Up

Let's factor some special quadratic expressions.

Warm-Up

Eliza is trying to factor $x^2 + x - 56$. She started by listing pairs of numbers that multiply to -56.

1 and -56	2 and -28	4 and -14	7 and -8
-1 and 56	-2 and 28	-4 and 14	-7 and 8

1. **Discuss:** Which pairs might Eliza try first? Why?
Responses vary. She should try 7 and -8, because these numbers are close together.

2. Factor each expression. Use the diagrams if they help with your thinking.

a. $x^2 + x - 56$

b. $x^2 + 26x - 56$

$(x + 8)(x - 7)$ (or equivalent)

$(x + 28)(x - 2)$ (or equivalent)

Unit 7 Lesson 5 743 Warm-Up

Pairs | 20 min

Activity 1 Spotting Similarities

Purpose: Students surface patterns and strategies to help them factor expressions with special structures. **(MTR.5.1)**

Short on time? Consider omitting Problem 4 if students demonstrated a strong understanding of factoring in the previous lesson.

1 Launch

Use the Think-Pair-Share routine to support students in making sense of the structural similarities in each group. Encourage students to think independently and then take turns sharing with a partner. **(MTR.5.1)**

Invite students to share their or their partner's thinking with the class.

2 Monitor

Encourage students to use a diagram to help to factor each expression.

To support students getting started, consider asking:

- "What terms can you place in the top-left and bottom-right position?"
- "What expressions multiply to get ax^2? What pairs of numbers multiply to get c?"

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations or to use a reference sheet, like a multiplication chart.

ELL **English Language Learners** Invite one partner to read each prompt out loud before the pair discusses it. **(Reading and Listening)**

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Spotting Similarities

Here are three groups of expressions.

Group 1	Group 2	Group 3
$4x^2 - 25$	$8x^2 + 32x + 24$	$x^2 - 6x - 27$
$x^2 - 36$	$-4x^2 + 8x + 32$	$x^2 + 2x - 80$
$x^2 - 100$	$-10x^2 - 20x - 10$	$x^2 - 13x + 30$
$25x^2 - 49$	$2x^2 - 22x + 60$	$x^2 + 2x - 63$

3. Explain how the expressions in each group are alike. *Responses vary.*

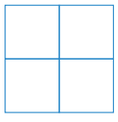
Group 1: Each of these expressions only has two terms. Both terms are perfect squares.

Group 2: Each of these expressions has an a-value that is not 1. They all have three terms. All of the terms share a common factor.

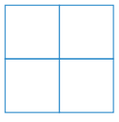
Group 3: Each of these expressions has three terms. All of the a-values are 1.

4. Factor one expression from each group. Use the diagrams if they help with your thinking. *Choices vary.*

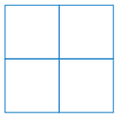
Group 1: _____ Group 2: _____ Group 3: _____



$(2x + 5)(2x - 5)$
$(x - 6)(x + 6)$
$(x + 10)(x - 10)$
$(5x - 7)(5x + 7)$



$8(x + 3)(x + 1)$
$-4(x - 4)(x + 2)$
$-10(x + 1)(x + 1)$
$2(x - 6)(x - 5)$
(or equivalent)



$(x - 9)(x + 3)$
$(x + 10)(x - 8)$
$(x - 3)(x - 10)$
$(x + 9)(x - 7)$

Unit 7 Lesson 5 744 Activity 1

Pairs | 20 min

Activity 1 Spotting Similarities (continued)

Purpose: Students surface patterns and strategies to help them factor expressions with special structures. **(MTR.5.1)**

Short on time? Consider omitting Problem 4 if students demonstrated a strong understanding of factoring in the previous lesson.

2 Monitor

Look for student strategies to highlight during the Connect such as:

- Dividing out a common factor to make the numbers in standard form smaller.
- Noticing a difference of squares structure.
- Determining factors of c that add to b when the a-value is 1 in standard form.

D Differentiation *(Problems 5–10)*

Look for students who:	Respond to Student Thinking
Are unsure how to distribute a value to three terms. *(Problem 5)*	**Support** Invite them to explain how they would multiply $7(x + 3)$ and then apply that strategy to this new situation.
Do not notice that both the a- and c-terms are perfect squares. *(Problem 7–9)*	**Strengthen** Invite students to multiply the factored expression and consider asking, "Can you write a factored expression similar to Yasmine's result? What do you notice when you multiply the two factors?"
Would like to explore factoring further.	**Stretch** Invite students to factor the expressions in Problem 10a and 10c in at least two different ways.

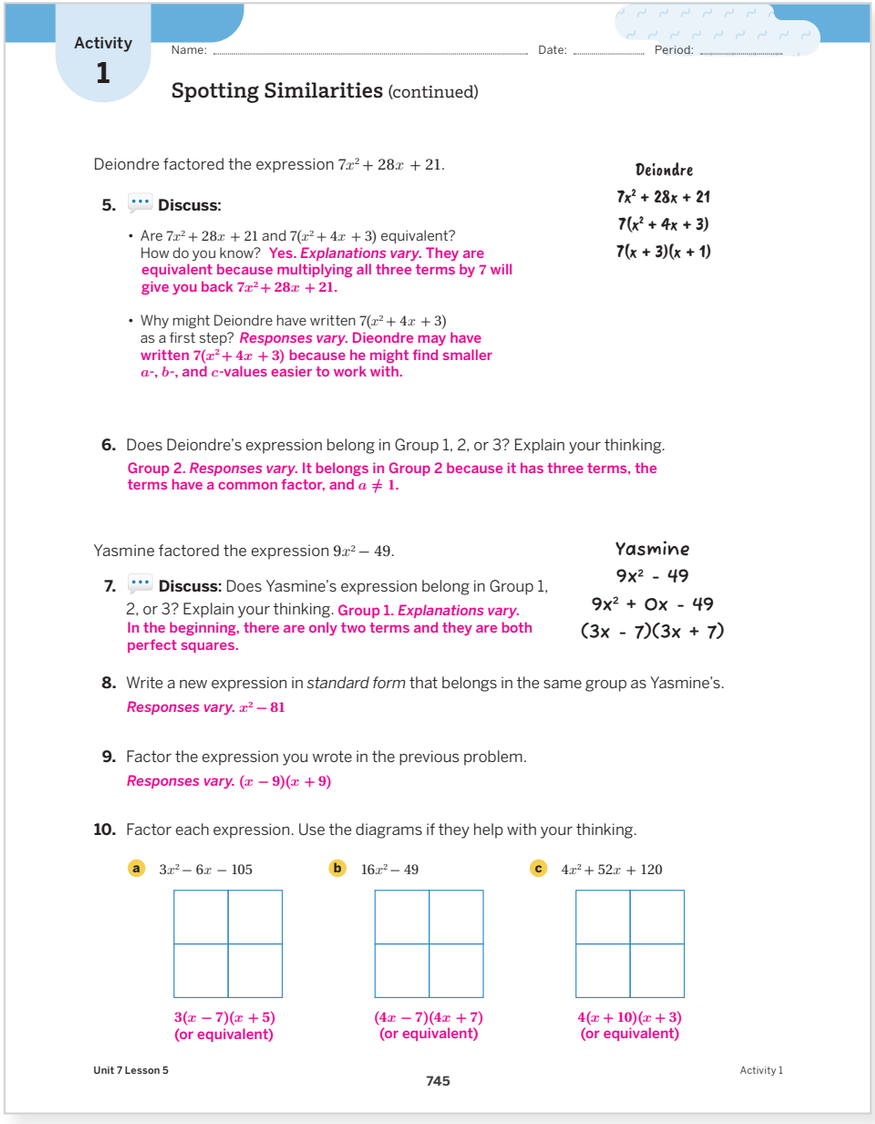
3 Connect

Invite students to discuss Deiondre and Yasmine's expressions one at a time. Consider asking:

- "Which group does this expression belong to? Why?"
- "What strategies were helpful to factor expressions in this group?"
- "Which groups would the expressions in Problem 10 belong to? Why?" **(MTR.5.1)**
- "What other strategies and patterns did you find that helped you with factoring?"

MLR MLR2: Collect and Display Use this routine to collect students' ideas and strategies for factoring quadratic expressions with special structures. Display for students to reference and add to for the rest of the unit.

Note: Students may notice the patterns that create the *difference of squares* expressions in group 1: $(rx + s)(rx - s) = r^2x^2 - s^2$. You may consider sharing this language with students, but students can be successful without knowing or using this term or formula. **(MTR.5.1)**



Activity 1

Name: Date: Period:

Spotting Similarities (continued)

Deiondre factored the expression $7x^2 + 28x + 21$.

Deiondre
$7x^2 + 28x + 21$
$7(x^2 + 4x + 3)$
$7(x + 3)(x + 1)$

5. Discuss:

- Are $7x^2 + 28x + 21$ and $7(x^2 + 4x + 3)$ equivalent? How do you know? Yes. *Explanations vary.* They are equivalent because multiplying all three terms by 7 will give you back $7x^2 + 28x + 21$.
- Why might Deiondre have written $7(x^2 + 4x + 3)$ as a first step? *Responses vary.* Dieondre may have written $7(x^2 + 4x + 3)$ because he might find smaller a-, b-, and c-values easier to work with.

6. Does Deiondre's expression belong in Group 1, 2, or 3? Explain your thinking.
Group 2. *Responses vary.* It belongs in Group 2 because it has three terms, the terms have a common factor, and $a \neq 1$.

Yasmine factored the expression $9x^2 - 49$.

Yasmine
$9x^2 - 49$
$9x^2 + 0x - 49$
$(3x - 7)(3x + 7)$

7. Discuss: Does Yasmine's expression belong in Group 1, 2, or 3? Explain your thinking. Group 1. *Explanations vary.* In the beginning, there are only two terms and they are both perfect squares.

8. Write a new expression in *standard form* that belongs in the same group as Yasmine's.
Responses vary. $x^2 - 81$

9. Factor the expression you wrote in the previous problem.
Responses vary. $(x - 9)(x + 9)$

10. Factor each expression. Use the diagrams if they help with your thinking.

a. $3x^2 - 6x - 105$ — $3(x - 7)(x + 5)$ (or equivalent)

b. $16x^2 - 49$ — $(4x - 7)(4x + 7)$ (or equivalent)

c. $4x^2 + 52x + 120$ — $4(x + 10)(x + 3)$ (or equivalent)

Unit 7 Lesson 5 745 Activity 1

Key Takeaway: There are many strategies that support us in factoring quadratic expressions such as using a diagram, dividing out a common factor first, rewriting the expression with 0 for a missing term, or looking for special structures.

Pairs | 10 min

Activity 2 Solve and Swap

Purpose: Students apply their strategies to efficiently factor quadratic expressions with various structures.

Short on time? Consider inviting students to factor fewer cards.

1 Launch

Consider asking, "What questions can we ask to help each other factor expressions?" Consider recording and displaying the responses, such as:

- "Is there a common factor?"
- "What do we know about the c-value?"

Distribute one card to each student or invite students to select a card that interests them.

A **Accessibility: Executive Functioning** To support multi-step activities, invite students to play an example round of the activity.

Invite students to factor as many expressions as they have time for. Share that it's more important to make sense of the problems than it is to work quickly.

2 Monitor

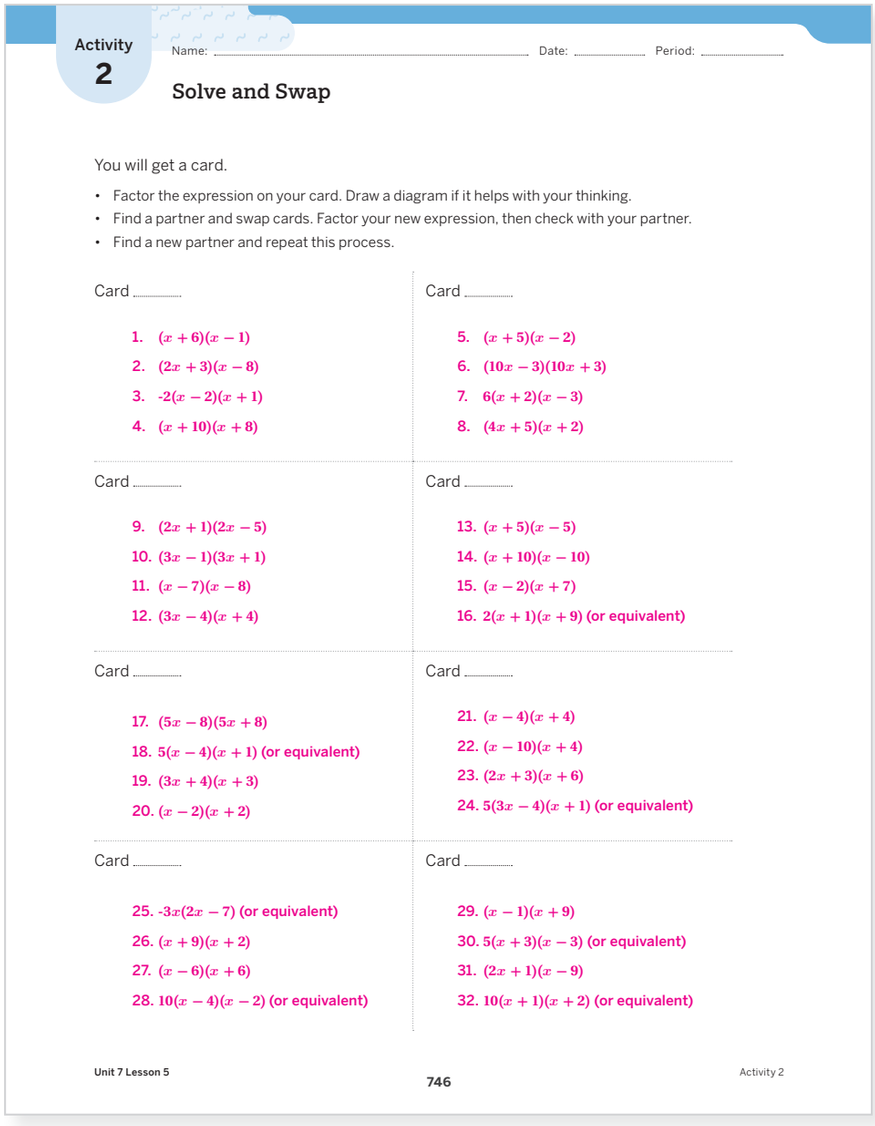
To support students getting started, encourage students to ask each other the questions brainstormed in the launch.

To surface connections, consider asking pairs:

- "Would this expression belong in one of the groups from Activity 1? How can you tell?" **(MTR.5.1)**
- "How did you start?"
- "What did you try that didn't work? How did you know it didn't work?"
- "What might you try next?"

3 Connect

To surface connections, encourage students to share any patterns and strategies they noticed for factoring the quadratic expressions, highlighting the strategies that were most efficient for certain structures. **(MTR.5.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Solve and Swap

You will get a card.

- Factor the expression on your card. Draw a diagram if it helps with your thinking.
- Find a partner and swap cards. Factor your new expression, then check with your partner.
- Find a new partner and repeat this process.

Card _____

1. $(x+6)(x-1)$
2. $(2x+3)(x-8)$
3. $-2(x-2)(x+1)$
4. $(x+10)(x+8)$

Card _____

5. $(x+5)(x-2)$
6. $(10x-3)(10x+3)$
7. $6(x+2)(x-3)$
8. $(4x+5)(x+2)$

Card _____

9. $(2x+1)(2x-5)$
10. $(3x-1)(3x+1)$
11. $(x-7)(x-8)$
12. $(3x-4)(x+4)$

Card _____

13. $(x+5)(x-5)$
14. $(x+10)(x-10)$
15. $(x-2)(x+7)$
16. $2(x+1)(x+9)$ (or equivalent)

Card _____

17. $(5x-8)(5x+8)$
18. $5(x-4)(x+1)$ (or equivalent)
19. $(3x+4)(x+3)$
20. $(x-2)(x+2)$

Card _____

21. $(x-4)(x+4)$
22. $(x-10)(x+4)$
23. $(2x+3)(x+6)$
24. $5(3x-4)(x+1)$ (or equivalent)

Card _____

25. $-3x(2x-7)$ (or equivalent)
26. $(x+9)(x+2)$
27. $(x-6)(x+6)$
28. $10(x-4)(x-2)$ (or equivalent)

Card _____

29. $(x-1)(x+9)$
30. $5(x+3)(x-3)$ (or equivalent)
31. $(2x+1)(x-9)$
32. $10(x+1)(x+2)$ (or equivalent)

Unit 7 Lesson 5 746 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for factoring quadratic expressions. **(MTR.2.1, MTR.4.1)**

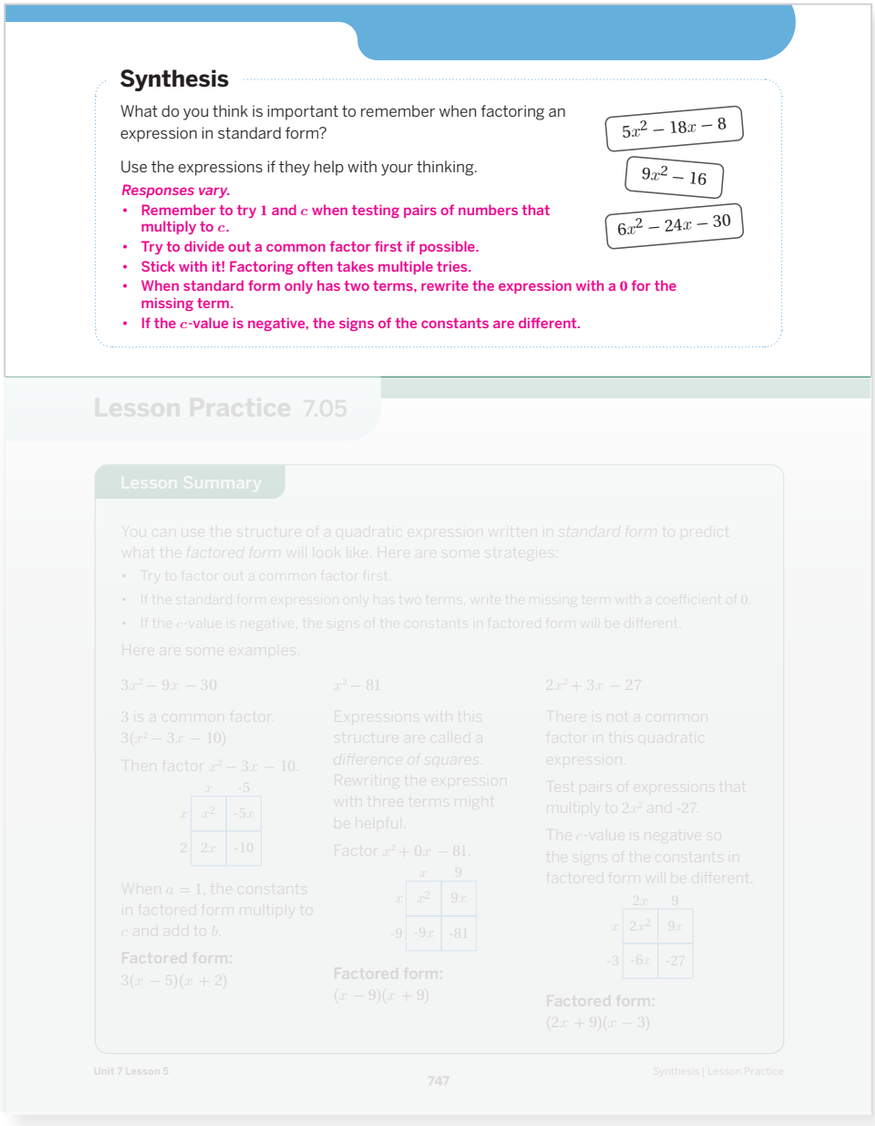
Invite students to respond independently, and then share their thinking with a partner. **(MTR.2.1, MTR.4.1)**

Collect and share a variety of ideas, such as:

- Remember to try 1 and c when testing pairs of numbers that multiply to c.
- Try to divide out a common factor first if possible.
- Check factored form by multiplying to see if I get the correct standard form.
- Be persistent. Factoring often takes multiple attempts.
- When standard form only has two terms, rewrite it with a 0 for the missing term.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *first, last, coefficient, constant, top, bottom, left, right, common, factor, terms, inside, outside*, etc.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas, language, examples, diagrams, or strategies as they share. Display for students to reference and add to for the rest of the unit.



Synthesis

What do you think is important to remember when factoring an expression in standard form?

Use the expressions if they help with your thinking.

$5x^2 - 18x - 8$

$9x^2 - 16$

$6x^2 - 24x - 30$

Responses vary.

- Remember to try 1 and c when testing pairs of numbers that multiply to c.
- Try to divide out a common factor first if possible.
- Stick with it! Factoring often takes multiple tries.
- When standard form only has two terms, rewrite the expression with a 0 for the missing term.
- If the c-value is negative, the signs of the constants are different.

Lesson Practice 7.05

Lesson Summary

You can use the structure of a quadratic expression written in *standard form* to predict what the *factored form* will look like. Here are some strategies:

- Try to factor out a common factor first.
- If the standard form expression only has two terms, write the missing term with a coefficient of 0.
- If the c-value is negative, the signs of the constants in factored form will be different.

Here are some examples.

$3x^2 - 9x - 30$

3 is a common factor.
$3(x^2 - 3x - 10)$
Then factor $x^2 - 3x - 10$.

	x	-5
x	x^2	$-5x$
2	$2x$	-10

When $a = 1$, the constants in factored form multiply to c and add to b.

Factored form:
$3(x - 5)(x + 2)$

$x^2 - 81$

Expressions with this structure are called a *difference of squares*. Rewriting the expression with three terms might be helpful.

Factor $x^2 + 0x - 81$.

	x	9
x	x^2	$9x$
-9	$-9x$	-81

Factored form:
$(x - 9)(x + 9)$

$2x^2 + 3x - 27$

There is not a common factor in this quadratic expression.

Test pairs of expressions that multiply to $2x^2$ and -27.

The c-value is negative so the signs of the constants in factored form will be different.

	$2x$	9
x	$2x^2$	$9x$
-3	$-6x$	-27

Factored form:
$(2x + 9)(x - 3)$

Unit 7 Lesson 5 747 Synthesis | Lesson Practice

Lesson Takeaway: There are many strategies to factor a quadratic expression and certain strategies may be more efficient depending on the specific structure of the expression.

Independent | 5 min

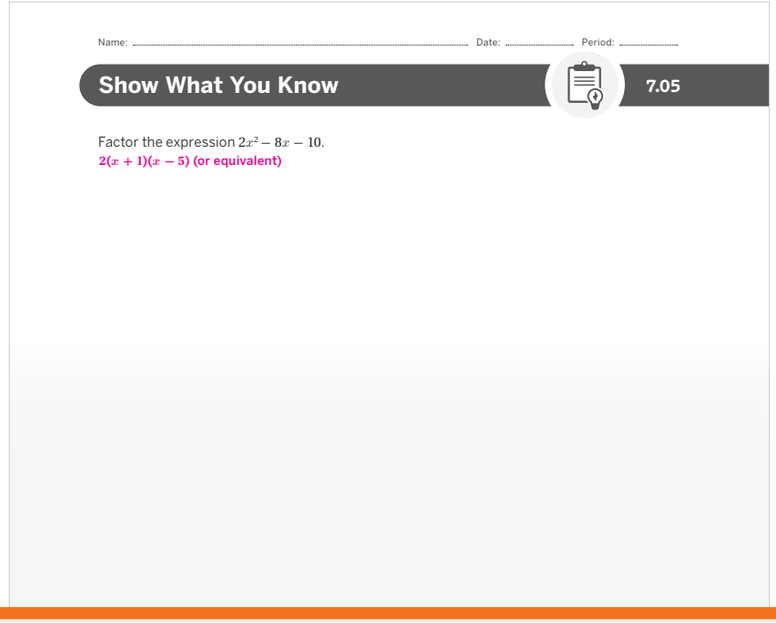
Show What You Know

Purpose: Students demonstrate their understanding by factoring a quadratic expression.

Today's Goals

1. **Goal:** Factor quadratic expressions in standard form.
2. **Language Goal:** Analyze the structure of a quadratic expression in standard form to make predictions about equivalent expressions in factored form. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.05

Factor the expression $2x^2 - 8x - 10$.

$2(x + 1)(x - 5)$ (or equivalent)

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support factoring quadratic expressions in standard form:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and give students individualized feedback as they factor more expressions from each group.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to factor expressions like: $x^4 - 16$, $25x^6 - 100$, $x^4 - 4x^2 - 5$, or $2x^3 + 5x^2 - 3x$.

Professional Learning

In Lesson 4, students learned about a strategy that will support them in factoring any quadratic expression. In this lesson, students identified patterns that can support them in factoring quadratic expressions with special structures. How will the strategies they learned across these two lessons support them with factoring quadratic expressions? How has looking for and making use of structure supported students in understanding the broader concept of factoring, rather than viewing each quadratic as an individual case to memorize? As students apply factoring to solving equations and identifying key features in future lessons, how can you continue to support students with this?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

What do you think is important to remember when factoring an expression in standard form?

Use the expressions if they help with your thinking.

$5x^2 - 18x - 8$

$9x^2 - 16$

$6x^2 - 24x - 30$

Responses vary.

- Remember to try 1 and c when testing pairs of numbers that multiply to c.
- Try to divide out a common factor first if possible.
- Stick with it! Factoring often takes multiple tries.
- When standard form only has two terms, rewrite the expression with a 0 for the missing term.
- If the c-value is negative, the signs of the constants are different.

Lesson Practice 7.05

Lesson Summary

You can use the structure of a quadratic expression written in *standard form* to predict what the *factored form* will look like. Here are some strategies:

- Try to factor out a common factor first.
- If the standard form expression only has two terms, write the missing term with a coefficient of 0.
- If the c-value is negative, the signs of the constants in factored form will be different.

Here are some examples.

$3x^2 - 9x - 30$

3 is a common factor.
$3(x^2 - 3x - 10)$

Then factor $x^2 - 3x - 10$.

	x	-5
x	x^2	$-5x$
2	$2x$	-10

When $a = 1$, the constants in factored form multiply to c and add to b.

Factored form:
$3(x - 5)(x + 2)$

$x^2 - 81$

Expressions with this structure are called a *difference of squares*. Rewriting the expression with three terms might be helpful.

Factor $x^2 + 0x - 81$.

	x	9
x	x^2	$9x$
-9	$-9x$	-81

Factored form:
$(x - 9)(x + 9)$

$2x^2 + 3x - 27$

There is not a common factor in this quadratic expression.

Test pairs of expressions that multiply to $2x^2$ and -27.

The c-value is negative so the signs of the constants in factored form will be different.

	$2x$	9
x	$2x^2$	$9x$
-3	$-6x$	-27

Factored form:
$(2x + 9)(x - 3)$

Lesson Practice 7.05

Name: ..... Date: ..... Period: .....

1. Write a + or − sign in each box to make true equations.

$(x \boxed{+} 18)(x \boxed{-} 3) = x^2 + 15x - 54$ $(x \boxed{-} 18)(x \boxed{-} 3) = x^2 - 21x + 54$

Problems 2–5: Fill in the blanks to make each equation true. Use the diagrams if they help your thinking.

2. x^2 ...−12... x ...+27... $= (x - 9)(x - 3)$

3. $x^2 + 12x$ ...+32... $= (x + 4)(x$ ...+8...$)$





4. $2x^2 + 11x + 15 = (2x$ ...+5...$)(x$ ...+3...$)$

5. $3x^2 - 11x$ ...−42... $= (x - 6)(3x$ ...+7...$)$





Problems 6–8: Factor each expression.

6. $x^2 - x - 30$
$(x - 6)(x + 5)$

7. $4x^2 + 20x + 25$
$(2x + 5)^2$ (or equivalent)

8. $2x^2 + x - 15$
$(2x - 5)(x + 3)$

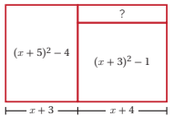
Lesson Practice 7.05

Name: ..... Date: ..... Period: .....

Test Practice

9. The diagram shows the expressions for areas and lengths. Choose the expression for the unknown area.

A. $5(x + 4)$
B. $4(x + 5)$
C. x-2
D. $-2(x + 1)$



Spiral Review

Problems 10–12: Solve each equation.

10. $6 + 2x = 0$
$x = -3$

11. $2x - 5 = 0$
$x = \frac{5}{2}$

12. $\frac{1}{2}(x - 87) = 0$
$x = 87$

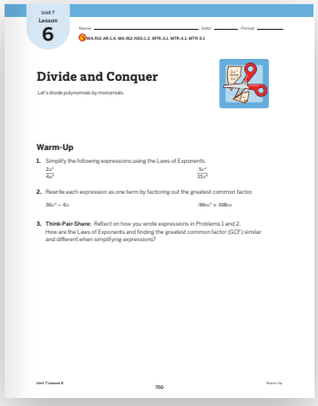
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.1.7
	2–5	2	MA.912.AR.1.7
	6–8	2	MA.912.AR.1.7
Test Practice	9	3	MA.912.AR.1.7
Spiral Review			
Fluency	10–12	1	MA.7.AR.2.2

Unit 7

Lesson

6


Unit 7
Lesson
6
Name: Date: Period:
MA.912.AR.1.4, MA.912.NSO.1.2, MTR.3.1, MTR.4.1, MTR.5.1
Divide and Conquer
Let's divide polynomials by monomials.
Warm-Up
1. Simplify the following expressions using the Laws of Exponents.
2. Rewrite each expression as one term by factoring out the greatest common factor.
3. Think-Pair-Share: Reflect on how you wrote expressions in Problems 1 and 2. How are the Laws of Exponents and finding the greatest common factor (GCF) similar and different when simplifying expressions?
Unit 7 Lesson 6
750
Warm-Up

Divide and Conquer

Dividing Polynomial Expressions

Let's divide polynomials by monomials.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether the division of a polynomial by a monomial results in a polynomial.
2. **Goal:** Analyze the meaning of closure of polynomials
3. **Language Goal:** Explain different strategies used to divide polynomials and understand closure. **(Reading, Speaking, and Listening)**

Students surface prior knowledge of the laws of exponents to develop conceptual understanding of how to divide polynomials by monomials. Students make use of different strategies to explain their thinking. **(MTR.5.1)**

Coherence

Prior Learning

In 7th grade, students learned about the Laws of Exponents. In Lesson 1, students practiced addition and subtraction with polynomials. In Unit 7, students have practiced multiplying and factoring polynomials.

Future Learning

In future grades, students will learn about polynomial long division and synthetic division.

Connections

- In a partner activity, students explore examples and non-examples of polynomials to derive a definition. (mathematical, cross-curricular)
- Students extend their understanding of the laws of exponents when dividing polynomials by monomials. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.4

Divide a polynomial expression by a monomial expression with rational number coefficients.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.1.1	**MA.912.AR.1.5**
MA.8.AR.1.2	**MA.912.AR.1.6**
MA.8.AR.1.3	**MA.912.AR.1.7**
	MA.912.AR.6.3

Vocabulary

Review Vocabulary

rational exponent	*polynomial*
radical	*remainder*

Lesson at a Glance

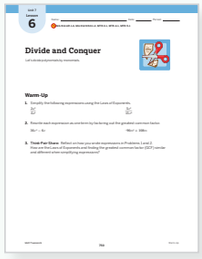
~ 45 min

MA.912.AR.1.4, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

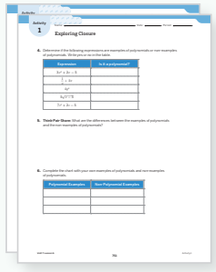
Students engage in discussion and look for patterns when applying knowledge of factoring and the Laws of Exponents. **Routine:** Think-Pair-Share **(MTR.4.1, MTR.5.1)**



Activity 1

Pairs | 10 min

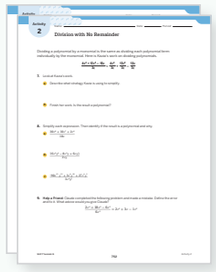
Students identify and discuss if the result of a polynomial divided by a monomial results in a polynomial. **Routine:** Think-Pair-Share **(MTR.4.1)**



Activity 2

Pairs | 10 min

Students learn to simplify accurately and recognize the result of their quotient is a polynomial. **Routine:** Critique, Correct, Clarify **(MTR.3.1)**



Activity 3

Pairs | 10 min

Students learn to simplify and recognize the result of their quotient is not a polynomial.



Synthesis

Whole Class | 5 min

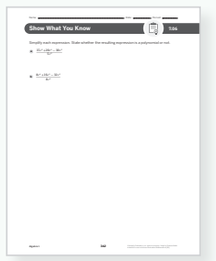
Students synthesize their understanding of the Laws of Exponents to simplify polynomials divided by monomials. **Routine:** Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by finding quotients of polynomials.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students engage in discussion and look for patterns when applying knowledge of factoring and the Laws of Exponents. **(MTR.4.1, MTR.5.1)**

Short on time? Consider doing one Laws of Exponents problem and one greatest common factor problem.

1 Launch

Invite students to answer the first two problems independently and then compare them with their partner. Invite students to share and discuss their reasoning and answers, revising their work as needed. **(MTR.4.1)**

Give students 30 seconds to one minute to read Problem 3 and think individually.

2 Connect

Invite students to turn to a neighbor and share their ideas, asking clarifying questions as needed.

Consider asking:

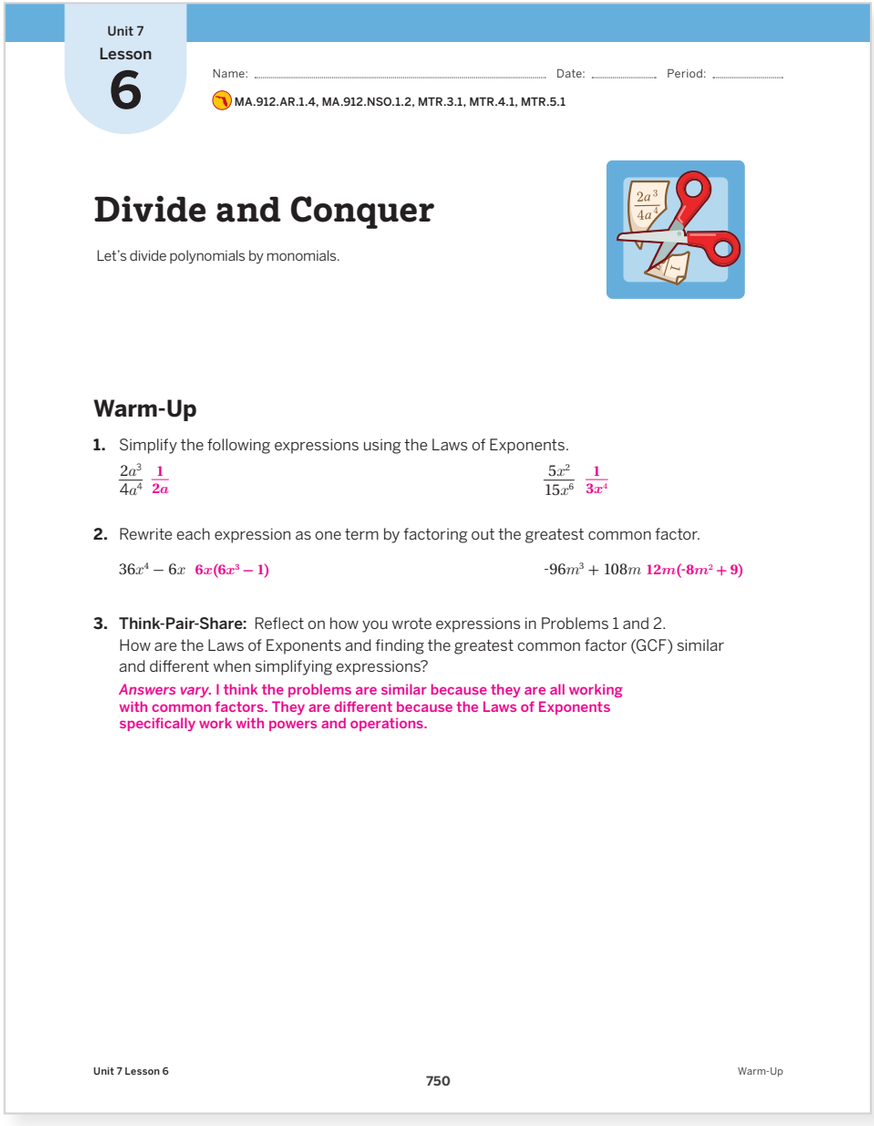
- "What do you remember about dividing with exponents?"
- "What should you do if you can't find the greatest factor right away?"
- "What happens when there is a negative exponent?"
- "What is similar and different about Problems 1 and 2?" **(MTR.5.1)**
- "For Problem 3, how does your answer compare to your partner's answer? **(MTR.4.1)**

A **Accessibility: Conceptual Processing** Invite students to talk aloud about their thinking to support self-monitoring of understanding.

Use the **Think-Pair-Share** routine to support student-to-student discourse for Problem 3 when determining the similarities and differences between the Laws of Exponents and the greatest common factor. See the Routine Facilitation Guide for more information.

Encourage students to share their ideas with the whole class.

Consider recording student's ideas on a display in the classroom to track responses. Some students may have similar responses.



Unit 7
Lesson
6

Name: ..... Date: ..... Period: .....

MA.912.AR.1.4, MA.912.NSO.1.2, MTR.3.1, MTR.4.1, MTR.5.1

Divide and Conquer

Let's divide polynomials by monomials.

Warm-Up

1. Simplify the following expressions using the Laws of Exponents.

   $\frac{2a^3}{4a^4}$ $\frac{1}{2a}$ $\frac{5x^2}{15x^6}$ $\frac{1}{3x^4}$

2. Rewrite each expression as one term by factoring out the greatest common factor.

   $36x^4 - 6x$ $6x(6x^3 - 1)$ $-96m^3 + 108m$ $12m(-8m^2 + 9)$

3. **Think-Pair-Share:** Reflect on how you wrote expressions in Problems 1 and 2. How are the Laws of Exponents and finding the greatest common factor (GCF) similar and different when simplifying expressions?

   Answers vary. I think the problems are similar because they are all working with common factors. They are different because the Laws of Exponents specifically work with powers and operations.

Unit 7 Lesson 6 750 Warm-Up

Pairs | 10 min

Activity 1 Exploring Closure

Purpose: Students identify and discuss if the result of a polynomial divided by a monomial results in a polynomial. **(MTR.4.1)**

1 Launch

Invite students to read the information about polynomials. Encourage students to annotate, mark, or highlight information they find important or new to them.

To support students in getting started, have students take turns reading aloud different parts of the text.

ELL **English Language Learners** Invite students to read the definitions aloud and paraphrase what they mean. Clarify any words they don't know. **(Reading)**

2 Monitor

Look for ideas to share during the Connect.

D **Differentiation** *(Problems 4-5)*

Look for students who:	Respond to Student Thinking
Need support getting started. (*Problem 4*)	**Support** Encourage them to answer the questions they are sure about and skip the ones they aren't.
Think the rational function is a polynomial. (*Problem 4*)	**Strengthen** Invite them to reread the definition in the box above.
Think polynomials cannot have more than one variable. (*Problem 4*)	**Strengthen** Consider reminding them that terms can have an infinite number of variables.
See many differences between the examples and non-examples. (*Problem 5*)	**Stretch** Invite the students to create their examples and non-examples of polynomials.

Use the Think-Pair-Share routine to support student-to-student discourse for Problem 5 when determining the similarities and differences between all of the examples. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

3 Connect

Listen for student reasoning that showcases the key differences in the examples and non-examples of polynomials.

To surface the Key Takeaway, consider asking:

- "What are the key differences between the examples and non-examples?"
- "What other types of functions have closure similar to polynomials?"

Activity 1

Name: _____ Date: _____ Period: _____

Exploring Closure

4. Determine if the following expressions are examples of polynomials or non-examples of polynomials. Write yes or *no* in the table.

Expression	Is it a polynomial?
$3x^2 + 2x - 5$	yes
$\frac{1}{x} + 2x$	no
$4y^4$	yes
$5\sqrt{x-3}$	no
$7z^3 + 2z - 5$	yes

5. **Think-Pair-Share:** What are the differences between the examples of polynomials and the non-examples of polynomials?

Responses vary.
For the polynomials, all the variables have positive integer exponents. In the examples, the operations used are addition, subtraction, and multiplication. In the non-examples, there are operations like division by a variable or taking the square root of a variable, which are not allowed in polynomials.

6. Complete the chart with your own examples of polynomials and non-examples of polynomials. *Responses vary.*

Polynomial Examples	Non-Polynomial Examples
$-3x^2 + x - 15$	$\frac{5}{x} - 4x$
$4y^4 + 6y - 40$	$\frac{8}{x^2} - 7x$
$2z - 5$	$-18\sqrt{x} - 9$

Unit 7 Lesson 6 751 Activity 1

Key Takeaway: Polynomials are closed under addition, subtraction, and multiplication, but not division. Polynomials must have variables with non-negative integer exponents.

Pairs | 10 min

Activity 2 Division with No Remainder

Purpose: Students learn to simplify accurately and recognize the result of their quotient is a polynomial. **(MTR.3.1)**

1 Launch

Invite students to consider how to rewrite the expression as a sum of individual terms. Then each term in the expression can be simplified. **(MTR.3.1)**

Note: It is okay if students are unsure at this stage. Students will see different strategies for approaching problems like this in the next activity.

2 Monitor

Invite students to share their answers with their partners and look for errors, revising as needed. **(MTR.4.1)**

Gather students' strategies or explanations for Problem 7 to share during the Connect.

D Differentiation *(Problems 8–9)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look over the example problem and start Problem 7 by breaking up the terms in a similar manner. **(MTR.3.1)**
Find the correct answer, but cannot explain their reasoning with accuracy. (*Problem 8a-c*)	**Strengthen** Invite students to review the definition of polynomials from Activity 1.
Are confused where there are multiple variables (*Problem 8b–c*)	**Strengthen** Consider showing an example without coefficients and focus on the variables.
Believe Claude is correct. (*Problem 9*)	**Support** Encourage students to break up the problem and compare their result to Claude's result.

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students identify errors.

To surface the Key Takeaway, consider asking:

- "What is something Claude did well? What is a mistake Claude made?"
- "What would you tell Claude to do to correct his answer?"
- "What makes a polynomial a polynomial?"

Activity 2

Name: ..... Date: ..... Period: .....

Division with No Remainder

Dividing a polynomial by a monomial is the same as dividing each polynomial term individually by the monomial. Here is Kazia's work on dividing polynomials.

$$\frac{6x^3+12x^2-18x}{3x} = \frac{6x^3}{3x} + \frac{12x^2}{3x} - \frac{18x}{3x}$$

7. Look at Kazia's work.

a. Describe what strategy Kazia is using to simplify. **Kazia separated the terms.**

b. Finish her work. Is the result a polynomial? $2x^2+4x-6$ **Yes, the result is a polynomial.**

8. Simplify each expression. Then identify if the result is a polynomial and why.

a. $\frac{30x^8+10x^7+2x^6}{10x}$ $3x^7+x^6+\frac{1}{5}x^5$

Yes it is a polynomial because all the coefficients are rational and there are no variables in the denominator of the result.

b. $\frac{16x^2y^4-8x^2y+6xy)}{2xy}$ $8xy^3-4x+3$

Yes it is a polynomial because all the coefficients are rational and there are no variables in the denominator of the result.

c. $\frac{-90x^{11}y^{15}+3x^6y^{10}+27x^4y^5}{3x^4y^5}$ $-30x^7y^{10}+x^2y^5+9$

Yes it is a polynomial because all the coefficients are rational and there are no variables in the denominator of the result.

9. Help a Friend: Claude completed the following problem and made a mistake. Define the error and fix it. What advice would you give Claude?

$$\frac{2x^4+18x^3-6x^2}{6x^2} = 2x^2+3x-1x^2$$

The correct answer is $2x^2+3x-1$. *Explanations vary.* Claude's mistake is in the third term. For the third term, $\frac{6x^2}{6x^2}=1$ because any term divided by itself is 1.

Advice for Claude: Carefully divide each term of the numerator by the denominator using the Laws of Exponents. Always double-check your work by simplifying each term individually.

Unit 7 Lesson 6 752 Activity 2

Key Takeaway: Dividing polynomials by monomials yields a polynomial if exponents are non-negative and no variables are in the denominator.

Pairs | 10 min

Activity 3 Polynomial No More!

Purpose: Students learn to simplify and recognize the result of their quotient is not a polynomial. **(MTR.3.1, MTR.4.1, MTR.5.1)**

1 Launch

Look for students who may be struggling to begin. Encourage them to approach these problems the same way they did in the Warm-Up and Activity 1. **(MTR.3.1)**

2 Monitor

Look for students noticing that these problems are different from the problems in Activity 1.

Note: If it does not come up naturally, consider asking, "Do you notice a difference between these problems and the other ones we have seen?" **(MTR.5.1)**

Pause briefly to listen to students' conversations regarding their answers to Problem 10. **(MTR.4.1)**

Invite students to share their reasoning. Discuss each possible response.

A **Accessibility: Conceptual Processing** Invite students to describe their thinking on Problem 10a–b and apply this same strategy to Problem 11a–c to encourage abstract thinking.

3 Connect

To surface the Key Takeaway, consider asking:

- "What were the key differences between Activities 2 and 3?"
- "Why aren't these problems considered polynomials?"
- "How does this relate to our discussion on closure?"

Activity 3

Name: ..... Date: ..... Period: .....

Polynomial No More!

10. Simplify the following expressions and identify if the result is a polynomial.

a. $\frac{40r^3+8r^2+2r}{8r^3}$ $5+\frac{1}{r}+\frac{1}{4r^2}$ not a polynomial

b. $\frac{20x^6y^4-2x^3y}{4x^3y^2}$ $5x^3y^2-\frac{1}{2y}$ not a polynomial

11. Determine the quotient of $\frac{1}{3}x^4-3x^3+\frac{1}{2}x^2$ and $3x^3$.

$\frac{1}{9}x-1+\frac{1}{6x}$

a. **Discuss** the strategy you used with a partner. How do your quotients compare to one another? *Responses vary.*

I broke the numerator into individual terms and divided each term by the denominator and simplified coefficients and exponents separately. I like creating separate terms because it helps me focus on one term at a time.

I focused on simplifying the coefficients of the terms first, treating the variables with exponents as a second step. Picking one area to focus on allows me to avoid mistakes.

b. Is the quotient a polynomial? Explain.

The quotient is not a polynomial. *Explanations vary.* The last term has $\frac{1}{x}$ which has a variable in the denominator. This can be written as x^{-1} breaking the rule of polynomials having exponents that are whole numbers, not negatives.

c. Is this allowed to happen? Shouldn't all polynomials turn into polynomials?

Yes. *Explanations vary.* Earlier we learned about closure. Addition, subtraction, and multiplication of polynomials will always result in another polynomial. Division does not always result in a polynomial because it is not closed.

Unit 7 Lesson 6 753 Activity 3

Key Takeaway: A remainder in polynomial division can create non-polynomials if negative exponents or variables in the denominator appear.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the Laws of Exponents to simplify polynomials divided by monomials.

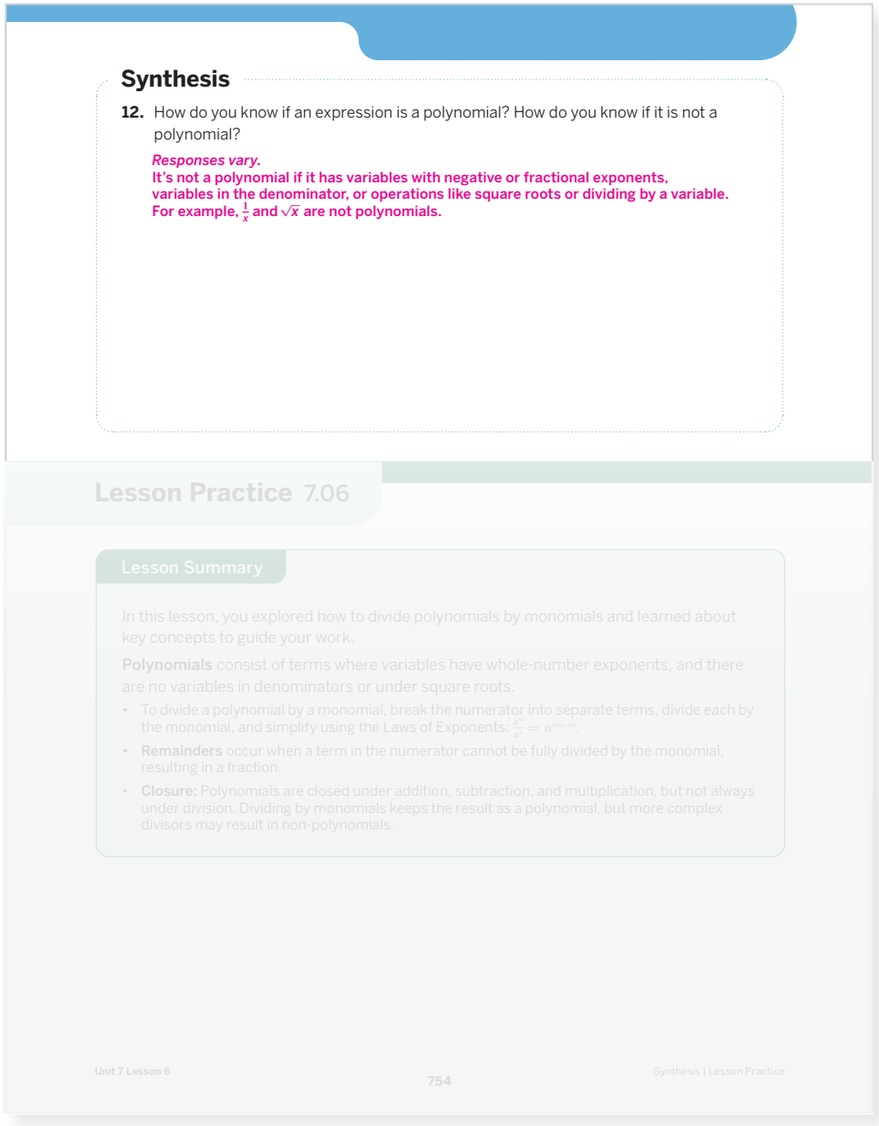
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- "An expression is a polynomial if it only has terms with variables raised to whole number exponents and the coefficients are real numbers".
- "There can't be variables in the denominator, under a square root, or with negative or fractional exponents."

Create an anchor chart of examples of polynomials and non-examples to keep up in the classroom.

MLR **MLR2: Collect and Display** Use this routine to collect students' examples as they discuss and share during the synthesis. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. How do you know if an expression is a polynomial? How do you know if it is not a polynomial?

Responses vary.
It's not a polynomial if it has variables with negative or fractional exponents, variables in the denominator, or operations like square roots or dividing by a variable. For example, $\frac{1}{x}$ and $\sqrt{x}$ are not polynomials.

Lesson Practice 7.06

Lesson Summary

In this lesson, you explored how to divide polynomials by monomials and learned about key concepts to guide your work.

Polynomials consist of terms where variables have whole-number exponents, and there are no variables in denominators or under square roots.

- To divide a polynomial by a monomial, break the numerator into separate terms, divide each by the monomial, and simplify using the Laws of Exponents: $\frac{a^m}{a^n} = a^{m-n}$.
- **Remainders** occur when a term in the numerator cannot be fully divided by the monomial, resulting in a fraction.
- **Closure:** Polynomials are closed under addition, subtraction, and multiplication, but not always under division. Dividing by monomials keeps the result as a polynomial, but more complex divisors may result in non-polynomials.

Unit 7 Lesson 6 754 Synthesis | Lesson Practice

Lesson Takeaway: Polynomials are closed under division if no variables appear in the denominator or have negative exponents.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by finding quotients of polynomials. **(MTR.3.1)**

Today's Goals

1. **Goal:** Determine whether the division of a polynomial by a monomial results in a polynomial.
2. **Goal:** Analyze the meaning of closure of polynomials.
3. **Language Goal:** Explain different strategies used to divide polynomials and understand closure. **(Reading, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 7.06

Simplify each expression. State whether the resulting expression is a polynomial or not.

a. $\frac{15x^5+20x^4-30x^3}{5x^3}$

$3x^2+4x-6$ is a polynomial

b. $\frac{8x^4+16x^3-32x^2}{8x^4}$

$1+\frac{2}{x}-\frac{4}{x^2}$ is not a polynomial

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support dividing a polynomial expression by a monomial:

» **Respond to Student Thinking:**

- Revisit **Activity 2**, specifically Katzia's strategy, as a class and provide targeted feedback on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on how students incorporated prior knowledge from previous grades into the lesson.

- Were their skills strong?
- Where do they need more support?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How do you know if an expression is a polynomial? How do you know if it is not a polynomial?

Responses vary.
It's not a polynomial if it has variables with negative or fractional exponents, variables in the denominator, or operations like square roots or dividing by a variable. For example, $\frac{1}{x}$ and $\sqrt{x}$ are not polynomials.

Lesson Practice 7.06

Lesson Summary

In this lesson, you explored how to divide polynomials by monomials and learned about key concepts to guide your work.

Polynomials consist of terms where variables have whole-number exponents, and there are no variables in denominators or under square roots.

- To divide a polynomial by a monomial, break the numerator into separate terms, divide each by the monomial, and simplify using the Laws of Exponents: $\frac{a^m}{a^n} = a^{(m-n)}$.
- **Remainders** occur when a term in the numerator cannot be fully divided by the monomial, resulting in a fraction.
- **Closure:** Polynomials are closed under addition, subtraction, and multiplication, but not always under division. Dividing by monomials keeps the result as a polynomial, but more complex divisors may result in non-polynomials.

Lesson Practice 7.06

Name: _____ Date: _____ Period: _____

Problems 1–3: James was finding the quotient of $\frac{12x^4 + 18x^3 - 24x^2}{6x^2}$

$$\frac{12x^4 + 18x^3 - 24x^2}{6x^2}$$

$$\frac{12x^4}{6x^2} + \frac{18x^3}{6x^2} - \frac{24x^2}{6x^2}$$

$$2x^2 + 3 - 4$$

1. Identify his error. James forgot to correctly apply the Laws of Exponents to the second term. He simplified $\frac{18x^3}{6x^2} = 3$ instead of $3x$.

2. What is the correct answer? $2x^2 + 3x - 4$.

3. What advice can you give James to avoid this error in the future? James needs to remember that dividing exponents means subtracting the powers.

Problems 4–5: Simplify each expression. State whether the resulting expression is a polynomial or not a polynomial.

4. $\frac{12x^4 + 18x^3 - 24x^2}{6x^2}$ $2x^2 + 3x - 4$ polynomial

5. $\frac{7x^3 + 9x^2}{2x^3}$ $\frac{7}{2} + \frac{9}{x}$ not a polynomial

Lesson Practice 7.06

Name: _____ Date: _____ Period: _____

Test Practice

6. Which two expressions when simplified have the same value?

- ☑ A. $\frac{8x^2 + 4x - 10}{2}$
- ☐ B. $\frac{8x^2 + 4x + 10x}{2}$
- ☑ C. $\frac{12x^3 + 6x - 15x}{3x}$
- ☐ D. $\frac{20x^4 + 10x^3 + 25}{5x^2}$

Spiral Review

Problems 7–10: Identify the x- and y-intercepts.

7. $2x + 3y = 12$
x-intercept (6,0)
y-intercept (0,4)

8. $5x - 4y = 20$
x-intercept (4,0)
y-intercept (0,-5)

9. $y = \frac{2}{3}x - 4$
x-intercept (6,0)
y-intercept (0,-4)

10. $y = -5x + 10$
x-intercept (2,0)
y-intercept (0,10)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3	2	MA.912.AR.1.4
	2, 4, 5	1	MA.912.AR.1.4
Test Practice	6	2	MA.912.AR.1.4
Spiral Review			
Fluency	7–10	2	MA.912.AR.3.7, MA.8.AR.3.5

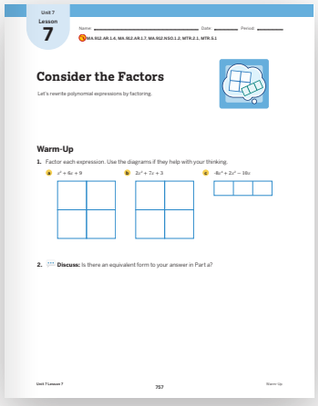
Unit 7

Lesson 7

Consider the Factors

Factoring Polynomial Expressions.

Let's write equivalent polynomial expressions by factoring.


Unit 7 Lesson 7

Consider the Factors

Let's rewrite polynomial expressions by factoring.

Warm-Up

1. Factor each expression. Use the diagrams if they help with your thinking.

2. Discuss: Is there an equivalent form to your answer in Part a?

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Learning Outcomes

Today's Goals

1. **Goal:** Identify different factoring strategies.
2. **Language Goal:** Describe and understand patterns of factoring. **(Speaking and Listening)**

In today's lesson, students rewrite polynomial expressions as products of polynomials over the real number system. They factor polynomials using a variety of methods, including factoring out the greatest common factor (GCF), recognizing and factoring perfect square trinomials, applying the difference of squares formula, and factoring by grouping for four-term polynomials. Students compare and contrast methods to identify patterns, share their reasoning, and develop flexibility in choosing appropriate strategies to simplify expressions.

Coherence

Prior Learning

In Algebra 1 Unit 7 Lessons 2–5, students practiced factoring different types of expressions.

Future Learning

In future math classes, students will extend their factoring skills to solve quadratic equations, graph polynomial functions, and analyze their key features. These skills will also support solving real-world problems, such as modeling scenarios with quadratic or higher-degree polynomial functions and finding solutions that represent meaningful outcomes.

Connections

- Students extend their understanding of factoring quadratic expressions to factoring polynomial expressions. (mathematical)
- Students explore patterns when factoring special-case polynomials, such as difference of squares, perfect square trinomials, and four-term polynomials, and discuss their findings. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.7
Rewrite a polynomial expression as a product of polynomials over the real number system.

MA.912.NSO.1.1
Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

Also Addressing: **MA.912.AR.1.4, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.1.3	**MA.912.AR.1.8**

Vocabulary

Review Vocabulary

expression *polynomial*

Lesson at a Glance

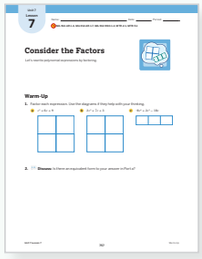
~ 45 min

MA.912.AR.1.7, MA.912.AR.1.4, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Whole Class | 5 min

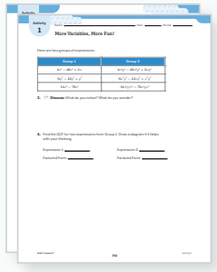
Students will demonstrate understanding by reviewing different types of factoring problems utilizing a diagram.



Activity 1

Pairs | 10 min

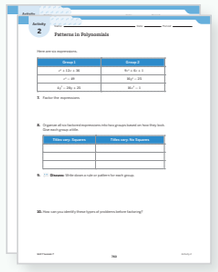
Students use the Notice and Wonder routine to make connections about how GCF can be used to factor polynomials.
Routine(s): Notice and Wonder **(MTR.2.1, MTR.5.1)**



Activity 2

Pairs | 10 min

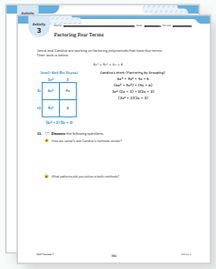
Students discover patterns in perfect square trinomials and differences of squares.
Routine(s): Think-Pair-Share **(MTR.2.1, MTR.5.1)**



Activity 3

Pairs | 10 min

Students compare two factoring strategies to factor polynomials with four terms.
Routine(s): Think-Pair-Share (MTR.5.1)



Synthesis

Whole Class | 5 min

Students synthesize their understanding of strategies for factoring polynomials.
Routine: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by factoring polynomials.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students will demonstrate understanding by reviewing different types of factoring problems utilizing a diagram. **(MTR.2.1)**

1 Launch

Encourage students to recall information from previous lessons. To help with recall, consider asking,

- "What is a trinomial? Binomial? Polynomial?"
- "What is a factor?"
- "What is a term?"
- "How many terms are in a binomial? Trinomial? Monomial?"
- "How many factors can there be in a term?"
- "How can you see all the factors in a term?"

A **Accessibility: Visual-Spatial Processing**
Consider giving students a larger diagram if needed.

Note: It is okay if students are unsure at this stage. Students will see different strategies for approaching problems like this in the rest of the activities.

To surface connections, encourage students to utilize the diagrams and recall information they learned about different factoring techniques thus far.

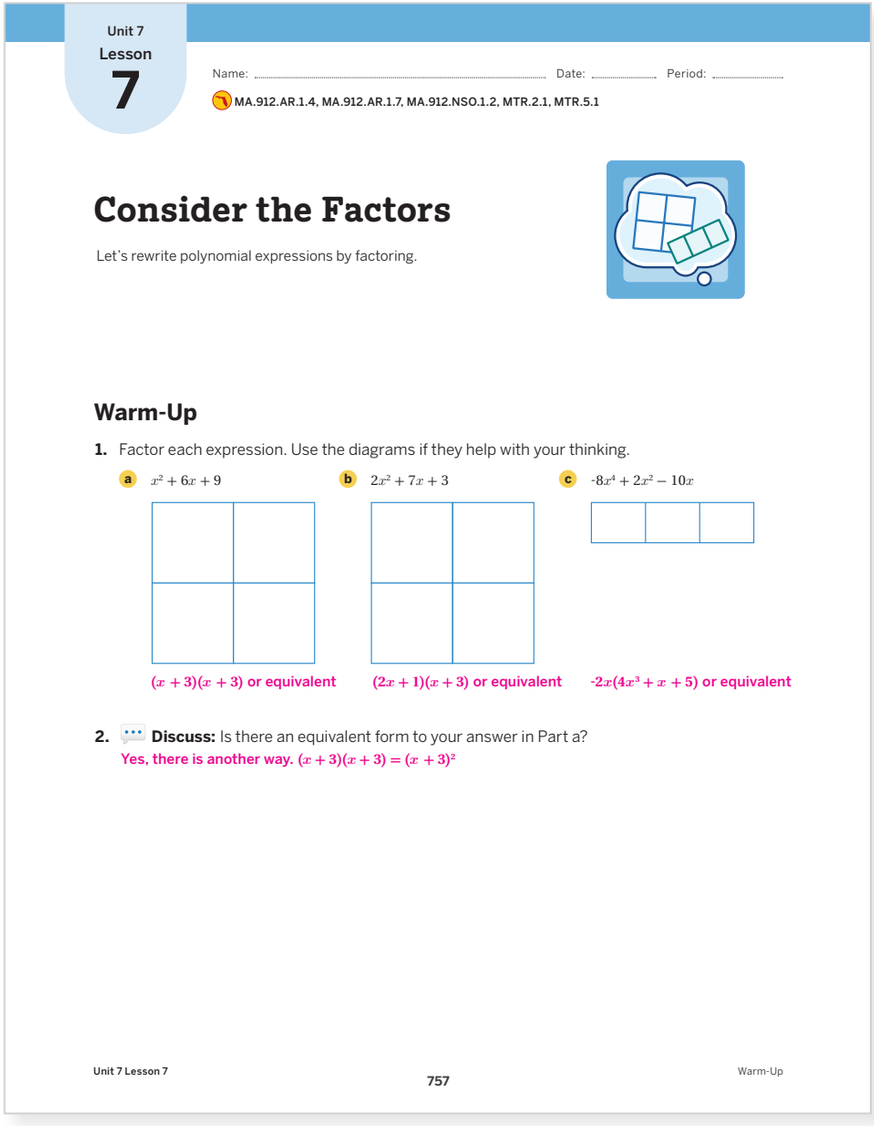
2 Connect

Share samples of student responses and write them somewhere everyone can see.

Consider asking:

- "What strategies are your favorite? Why?"
- "Did your partner pick a different strategy?"
- "Did you use multiple strategies or just stick to one?"

Invite students to show their work in front of the class.



Unit 7
Lesson
7

Name: ..... Date: ..... Period: .....

MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.5.1

Consider the Factors

Let's rewrite polynomial expressions by factoring.

Warm-Up

1. Factor each expression. Use the diagrams if they help with your thinking.

a. $x^2 + 6x + 9$

b. $2x^2 + 7x + 3$

c. $-8x^4 + 2x^2 - 10x$

$(x + 3)(x + 3)$ or equivalent

$(2x + 1)(x + 3)$ or equivalent

$-2x(4x^3 + x + 5)$ or equivalent

2. **Discuss:** Is there an equivalent form to your answer in Part a?
Yes, there is another way. $(x + 3)(x + 3) = (x + 3)^2$

Unit 7 Lesson 7 757 Warm-Up

Pairs | 10 min

Activity 1 More Variables, More Fun!

Purpose: Students use the Notice and Wonder routine to make connections about how GCF can be used to factor polynomials. **(MTR.5.1)**

Short on time? Consider having students only factor one expression instead of two. Or Save time by assigning different rows to different groups.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students identify similarities and differences between Group 1 and Group 2. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

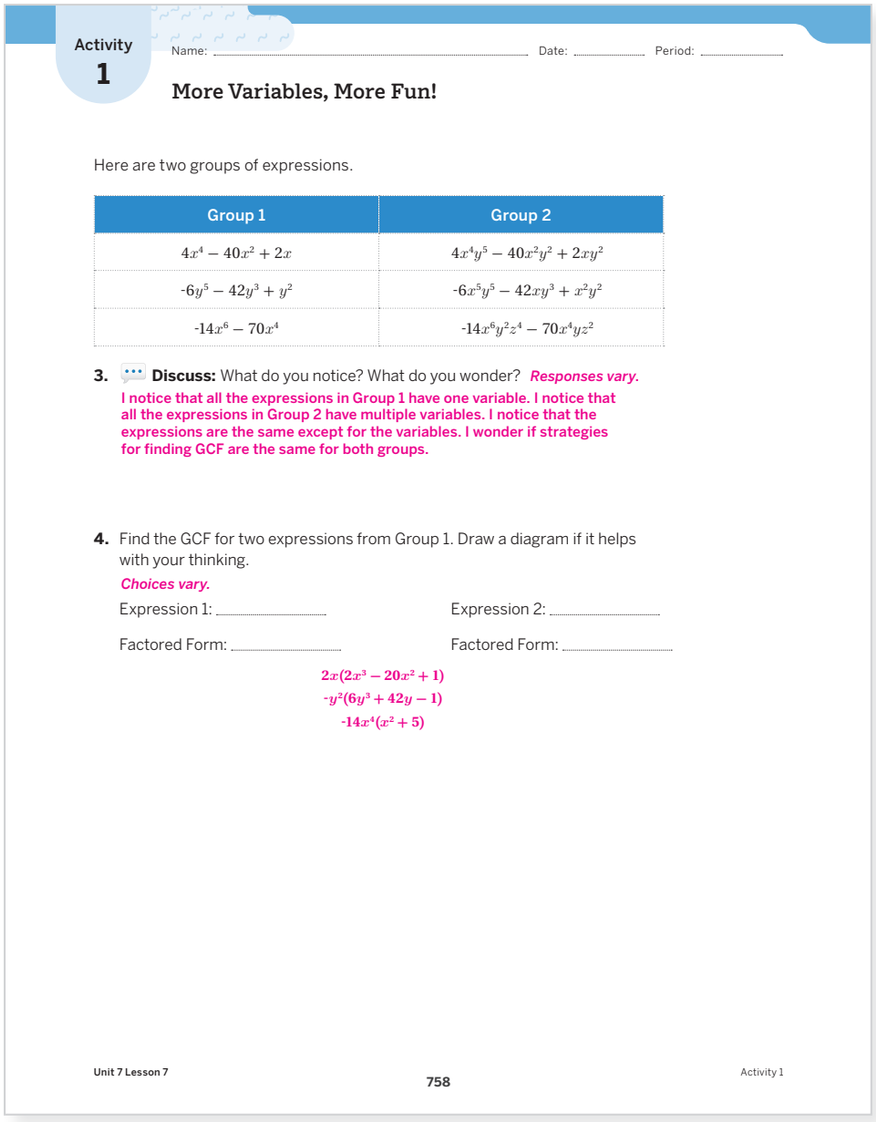
Encourage pairs to work together and check each other's work. Make sure students are engaging in discussion as they work through the problems and make connections.

2 Monitor

To support students getting started, suggest that students use highlighters to mark up commonalities between the expressions.

Look for students making connections between Group 1 and Group 2.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

More Variables, More Fun!

Here are two groups of expressions.

Group 1	Group 2
$4x^4 - 40x^2 + 2x$	$4x^4y^5 - 40x^2y^2 + 2xy^2$
$-6y^5 - 42y^3 + y^2$	$-6x^5y^5 - 42xy^3 + x^2y^2$
$-14x^6 - 70x^4$	$-14x^6y^2z^4 - 70x^4yz^2$

3. **Discuss:** What do you notice? What do you wonder? *Responses vary.*
I notice that all the expressions in Group 1 have one variable. I notice that all the expressions in Group 2 have multiple variables. I notice that the expressions are the same except for the variables. I wonder if strategies for finding GCF are the same for both groups.

4. Find the GCF for two expressions from Group 1. Draw a diagram if it helps with your thinking.
Choices vary.
Expression 1: _____ Expression 2: _____
Factored Form: _____ Factored Form: _____
$2x(2x^3 - 20x^2 + 1)$
$-y^2(6y^3 + 42y - 1)$
$-14x^4(x^2 + 5)$

Unit 7 Lesson 7 758 Activity 1

Pairs | 15 min

Activity 1 More Variables, More Fun! (continued)

Purpose: Students use the Notice and Wonder **routine** to make connections about how GCF can be used to factor polynomials. **(MTR.5.1)**

Short on time? Consider having students only factor one expression instead of two. Or save time by assigning different rows to different groups.

2 Monitor

D Differentiation *(Problems 4–5)*

Look for students who:	Respond to Student Thinking
Incorrectly factor Group 1.	**Support** Invite them to check their work with a partner.
Don't make the connection between Groups 1 and 2.	**Strengthen** Consider asking, "Is there a pattern here that we can recognize? Can we use the strategy from Group 1 here too?" **(MTR.5.1)**
Finish early.	**Stretch** Encourage students to factor all six expressions.

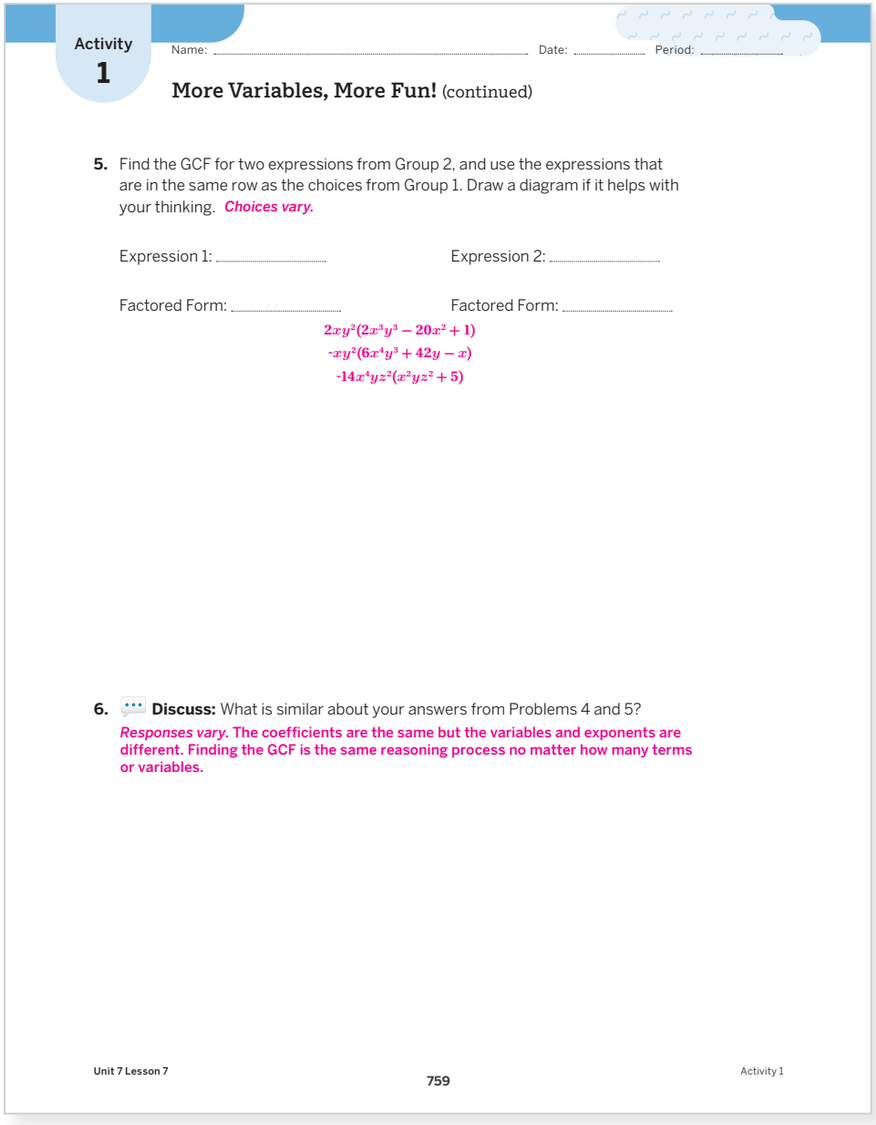
Collect and share student responses to Problem 6 by creating a list of ideas.

ELL English Language Learners Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

3 Connect

To surface the Key Takeaway, consider asking:

- "Was Group 2 really that different from Group 1?"
- "What are the steps for finding the GCF?"
- "What patterns did you notice when factoring?" **(MTR.5.1)**



Activity 1

Name: _____ Date: _____ Period: _____

More Variables, More Fun! (continued)

5. Find the GCF for two expressions from Group 2, and use the expressions that are in the same row as the choices from Group 1. Draw a diagram if it helps with your thinking. *Choices vary.*

Expression 1: _____ Expression 2: _____

Factored Form: _____ Factored Form: _____

$2xy^2(2x^3y^3 - 20x^2 + 1)$

$-xy^2(6x^4y^3 + 42y - x)$

$-14x^4yz^2(x^2yz^2 + 5)$

6. **Discuss:** What is similar about your answers from Problems 4 and 5?

Responses vary. The coefficients are the same but the variables and exponents are different. Finding the GCF is the same reasoning process no matter how many terms or variables.

Unit 7 Lesson 7 759 Activity 1

Key Takeaway: When finding the greatest common factor the strategy stays the same no matter how many terms or variables.

Pairs | 10 min

Activity 2 Patterns in Polynomials

Purpose: Students discover patterns in perfect square trinomials and differences of squares. **(MTR.5.1)**

Short on time? Consider completing these activities as a whole class.

1 Launch

Display the expressions where everyone can see.

Look for groups that are working well together and encourage them to keep going.

Encourage students to make sure their answers are fully factored by looking for factoring patterns in their results.

2 Monitor

Look for a variety of connections to perfect squares, factoring, and their patterns.

Consider asking,

- "How are you planning to group your answers?"
- "What patterns do you see?"
- "Do you think these patterns work every time?" **(MTR.5.1)**
- "What is different about the problem $16x^4 - 1$ compared to the others?"
- "How do you know when you are done factoring $16x^4 - 1$?"

Consider including a counterexample of a sum of squares and ask students to attempt to factor and discuss how factoring the sum of squares is not possible (over the real number system).

3 Connect

Introduce the formal names of the groupings: Difference of Squares and Perfect Square Trinomials.

Use the Think-Pair-Share routine to support student-to-student discourse about factoring patterns. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

To surface the Key Takeaway, consider asking:

- "Did all the groups come up with similar patterns?" **(MTR.5.1)**
- "What would the mathematical rule be?"
- "Can you create more examples quickly now that you see a pattern?" **(MTR.5.1)**

Activity 2

Name: Date: Period:

Patterns in Polynomials

Here are six expressions.

Group 1	Group 2
$x^2 + 12x + 36$	$9z^2 + 6z + 1$
$x^2 - 49$	$16y^2 - 25$
$4y^2 - 20y + 25$	$16x^4 - 1$

7. Factor the expressions $x^2 + 12x + 36 = (x + 6)^2$, $x^2 - 49 = (x - 7)(x + 7)$, $4y^2 - 20y + 25 = (2y - 5)^2$, $9z^2 + 6z + 1 = (3z + 1)^2$, $16y^2 - 25 = (4y - 5)(4y + 5)$, $16x^4 - 1 = (4x^2 + 1)(4x^2 - 1)$ or $(4x^2 + 1)(2x + 1)(2x - 1)$

8. Organize all six factored expressions into two groups based on how they look. Give each group a title. *Groupings vary.*

Titles vary. Squares	*Titles vary.* No Squares
$(x + 6)^2$	$(x - 7)(x + 7)$
$(2y - 5)^2$	$(4y - 5)(4y + 5)$
$(3z + 1)^2$	$(4x^2 + 1)(4x^2 - 1)$ or $(4x^2 + 1)(2x + 1)(2x - 1)$

9. **Discuss:** Write down a rule or pattern for each group. *Responses vary.* In one of my groups the answers were able to be written as one factored squared, I noticed that it was the same factor twice. In the other group, I notice all my numbers are the same but the only difference is the +/- in each factor.

10. How can you identify these types of problems before factoring? *Responses vary.* I can tell it's a perfect square trinomial if the first and last numbers are perfect squares, like x^2 or 36, and the middle number is twice the product of their square roots. I know it's a difference of squares if there's only two terms, they're both perfect squares, and there's a minus sign between them.

Unit 7 Lesson 7 760 Activity 2

Key Takeaway: Perfect square trinomials factor into the form $(a + b)^2$ or $(a - b)^2$. Differences of squares factor into the form $(a - b)(a + b)$.

Pairs | 10 min

Activity 3 Factoring Four Terms

Purpose: Students compare two factoring strategies to factor polynomials with four terms. **(MTR.5.1)**

1 Launch

To surface connections, consider discussing how this problem and the one from the Warm-Up are alike. Also, connect it back to previous lessons.

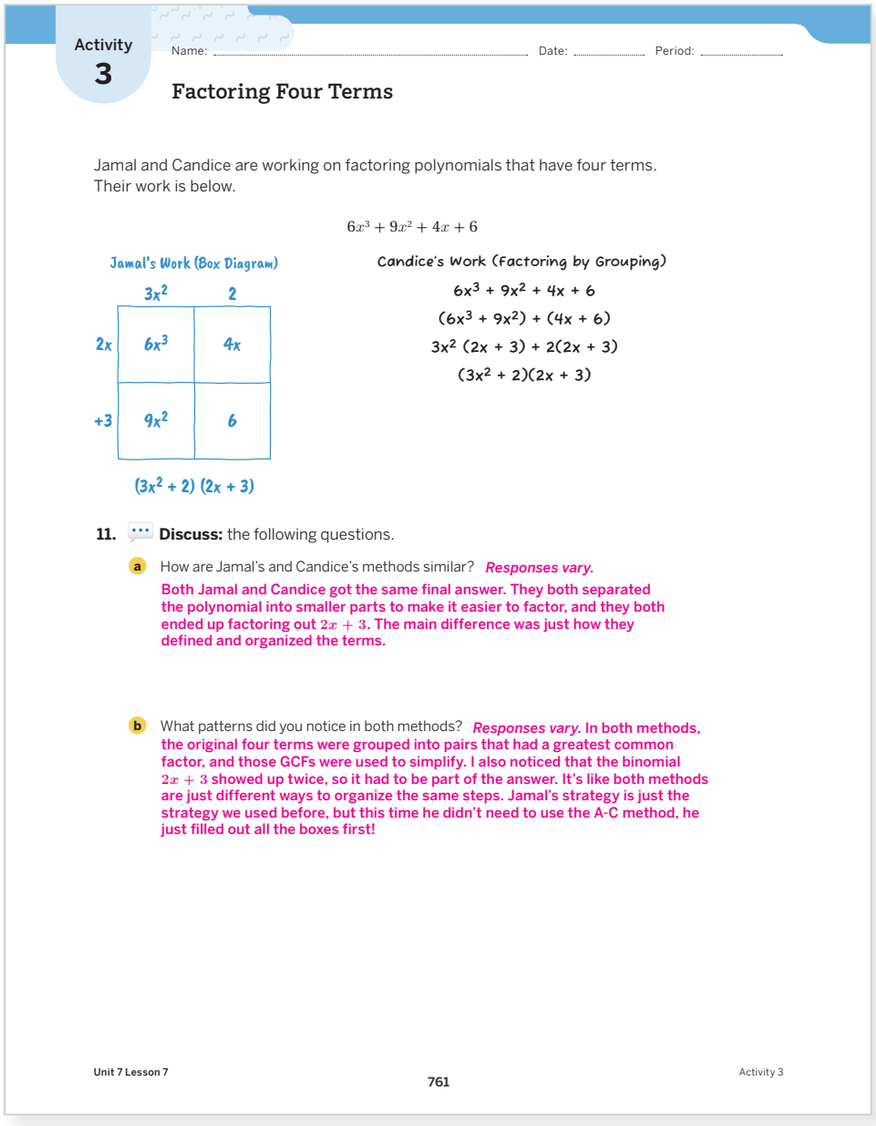
2 Monitor

Select and sequence key points you are hearing from the whole class discussion about Jamal and Candice's work. Listen for connections to the box diagram.

Consider asking the pairs,

- "What strategy do you like best?"
- "How could you check your work to ensure you are correct?"
- "Did your partner pick the same strategy?"

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Factoring Four Terms

Jamal and Candice are working on factoring polynomials that have four terms. Their work is below.

$$6x^3 + 9x^2 + 4x + 6$$

Jamal's Work (Box Diagram)

	$3x^2$	2
$2x$	$6x^3$	$4x$
$+3$	$9x^2$	6

$(3x^2 + 2)(2x + 3)$

Candice's Work (Factoring by Grouping)

$6x^3 + 9x^2 + 4x + 6$

$(6x^3 + 9x^2) + (4x + 6)$

$3x^2(2x + 3) + 2(2x + 3)$

$(3x^2 + 2)(2x + 3)$

11. **Discuss:** the following questions.

a. How are Jamal's and Candice's methods similar? *Responses vary.*
Both Jamal and Candice got the same final answer. They both separated the polynomial into smaller parts to make it easier to factor, and they both ended up factoring out $2x + 3$. The main difference was just how they defined and organized the terms.

b. What patterns did you notice in both methods? *Responses vary.* In both methods, the original four terms were grouped into pairs that had a greatest common factor, and those GCFs were used to simplify. I also noticed that the binomial $2x + 3$ showed up twice, so it had to be part of the answer. It's like both methods are just different ways to organize the same steps. Jamal's strategy is just the strategy we used before, but this time he didn't need to use the A-C method, he just filled out all the boxes first!

Unit 7 Lesson 7 761 Activity 3

Pairs | 10 min

Activity 3 Factoring Four Terms (continued)

Purpose: Students compare two factoring strategies to factor polynomials with four terms. **(MTR.5.1)**

2 Monitor

A **Accessibility: Visual-Spatial Processing** Consider giving students a larger diagram if needed.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I noticed that _____, so I _____." or "First, I _____ because _____."). **(Writing and Speaking)**

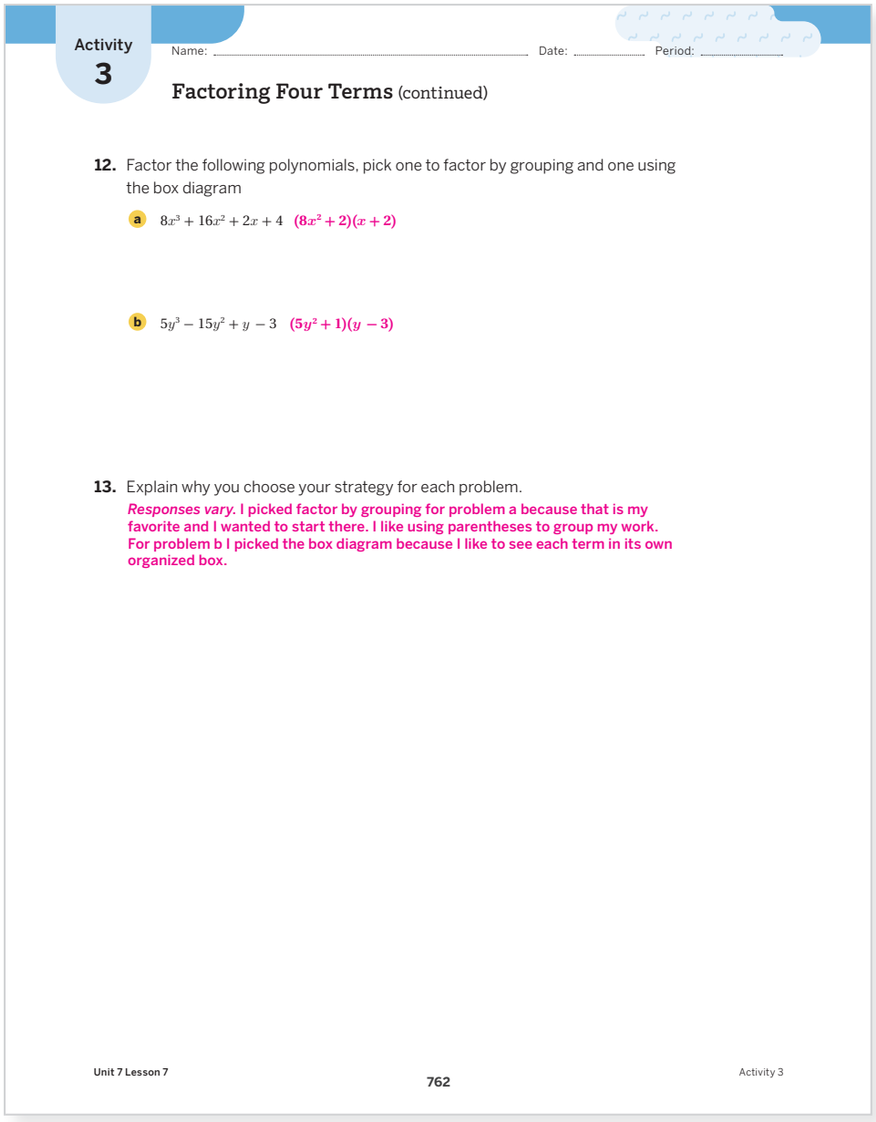
D **Differentiation** *(Problems 12–13)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to annotate both strategies and understand them before trying Problem 12 on their own.
Cannot find similarities between strategies.	**Strengthen** Ask them if they notice similar numbers or factors in both sets of work.
Struggle to set up a box diagram.	**Strengthen** Consider asking, "Is the box set up similarly to the ones in the warm-up? Can we extend that thinking here?" **(MTR.5.1)**
Do not want to try both methods.	**Stretch** Encourage them to show both methods for these examples and assure them in the future they can use the method they like best.

3 Connect

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider asking:
- "Which is your favorite strategy and why?"
- "Is one strategy faster?"
- "Will both strategies always work for polynomials of this form?"



Activity 3

Name: _____ Date: _____ Period: _____

Factoring Four Terms (continued)

12. Factor the following polynomials, pick one to factor by grouping and one using the box diagram

a. $8x^3 + 16x^2 + 2x + 4$ $(8x^2 + 2)(x + 2)$

b. $5y^3 - 15y^2 + y - 3$ $(5y^2 + 1)(y - 3)$

13. Explain why you choose your strategy for each problem.

Responses vary. I picked factor by grouping for problem a because that is my favorite and I wanted to start there. I like using parentheses to group my work. For problem b I picked the box diagram because I like to see each term in its own organized box.

Unit 7 Lesson 7 762 Activity 3

Key Takeaway: Factoring by grouping simplifies four-term polynomials by pairing terms, factoring out GCFs, and finding a common binomial factor.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for factoring polynomials.

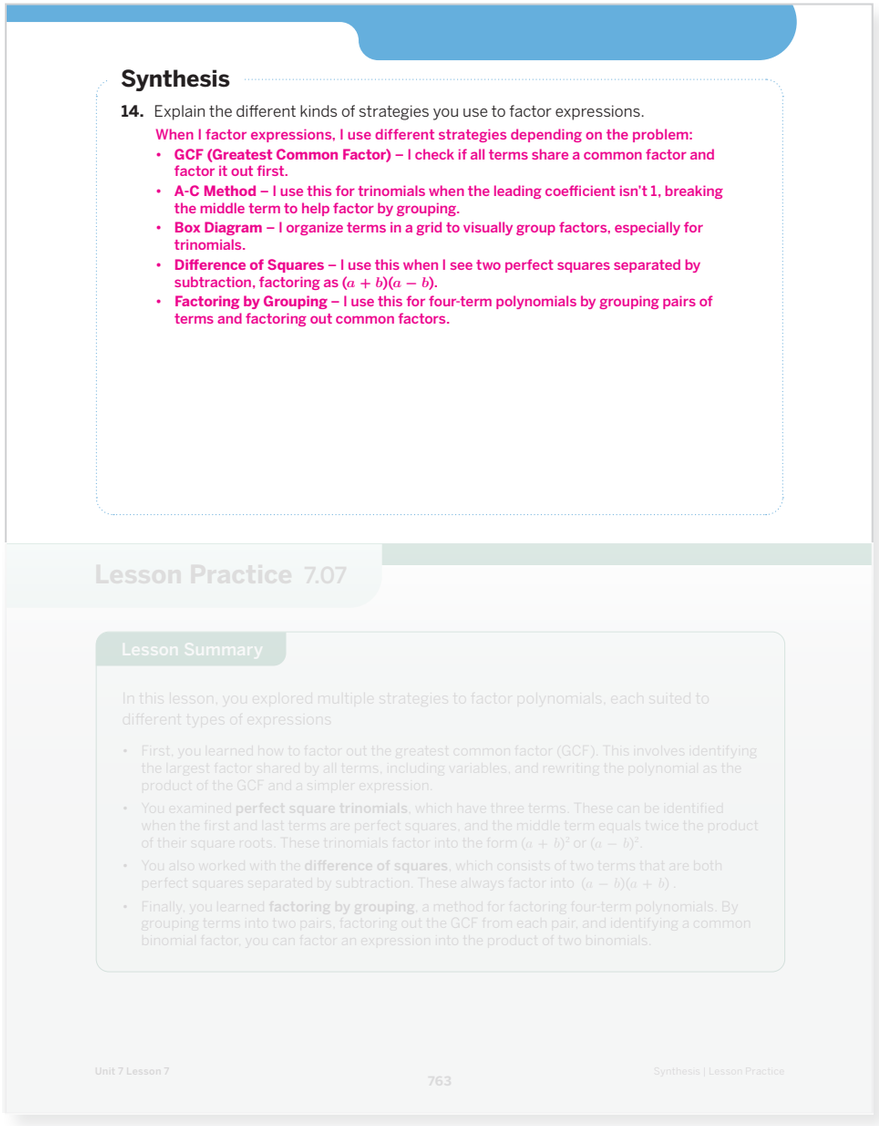
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Box diagrams are helpful in a lot of expressions.
- There are many ways to think about factoring.
- It's important to remember your factors.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

14. Explain the different kinds of strategies you use to factor expressions.

When I factor expressions, I use different strategies depending on the problem:
- **GCF (Greatest Common Factor) –** I check if all terms share a common factor and factor it out first.
- **A-C Method –** I use this for trinomials when the leading coefficient isn't 1, breaking the middle term to help factor by grouping.
- **Box Diagram –** I organize terms in a grid to visually group factors, especially for trinomials.
- **Difference of Squares –** I use this when I see two perfect squares separated by subtraction, factoring as $(a + b)(a - b)$.
- **Factoring by Grouping –** I use this for four-term polynomials by grouping pairs of terms and factoring out common factors.

Lesson Practice 7.07

Lesson Summary

In this lesson, you explored multiple strategies to factor polynomials, each suited to different types of expressions

- First, you learned how to factor out the greatest common factor (GCF). This involves identifying the largest factor shared by all terms, including variables, and rewriting the polynomial as the product of the GCF and a simpler expression.
- You examined **perfect square trinomials**, which have three terms. These can be identified when the first and last terms are perfect squares, and the middle term equals twice the product of their square roots. These trinomials factor into the form $(a + b)^2$ or $(a - b)^2$.
- You also worked with the **difference of squares**, which consists of two terms that are both perfect squares separated by subtraction. These always factor into $(a - b)(a + b)$.
- Finally, you learned **factoring by grouping**, a method for factoring four-term polynomials. By grouping terms into two pairs, factoring out the GCF from each pair, and identifying a common binomial factor, you can factor an expression into the product of two binomials.

Unit 7 Lesson 7 763 Synthesis | Lesson Practice

Lesson Takeaway: Factoring simplifies polynomials by recognizing patterns and applying methods like GCF, trinomials, or grouping.

Independent | 5 min

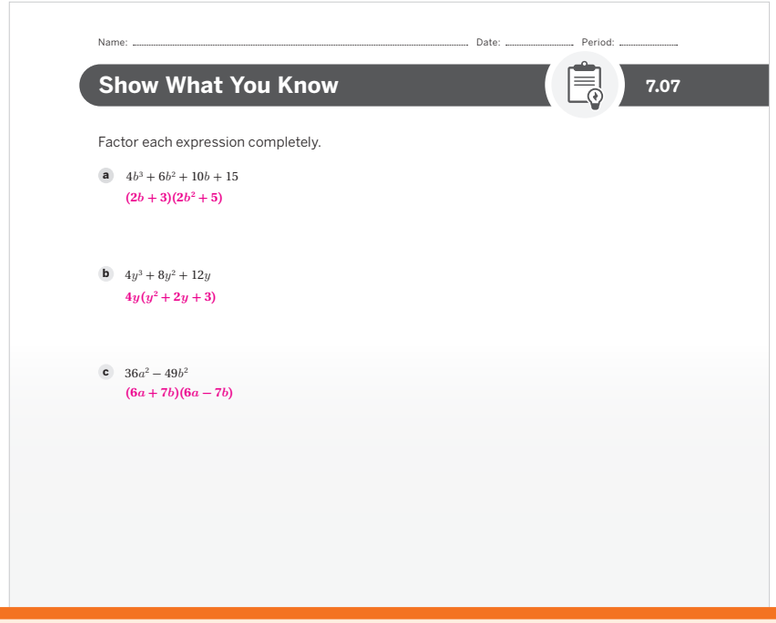
Show What You Know

Purpose: Students demonstrate their understanding by factoring polynomials.

Today's Goals

1. **Goal:** Identify different factoring strategies.
2. **Language Goal:** Describe and understand patterns of factoring. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.07

Factor each expression completely.

a. $4b^3 + 6b^2 + 10b + 15$
$(2b + 3)(2b^2 + 5)$

b. $4y^3 + 8y^2 + 12y$
$4y(y^2 + 2y + 3)$

c. $36a^2 - 49b^2$
$(6a + 7b)(6a - 7b)$

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support factoring polynomials:

» **Respond to Student Thinking:**

- Review **Activity 3**, specifically Jamal's and Candace's strategies, and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

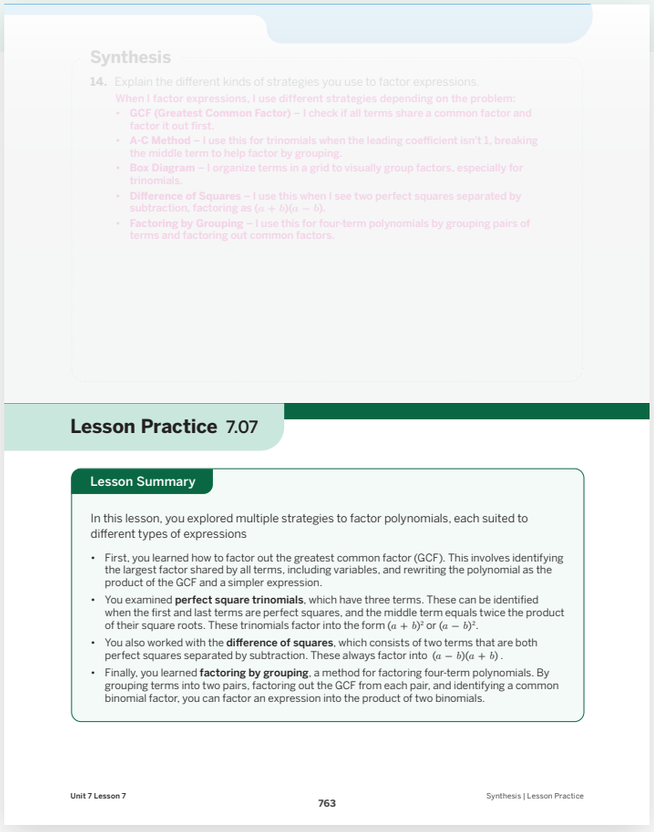
Professional Learning

Take a moment to reflect on this lesson. In this lesson, students factored many different types of expressions. How will that work support their understanding when identifying the x-intercepts of quadratic functions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

14. Explain the different kinds of strategies you use to factor expressions.

When I factor expressions, I use different strategies depending on the problem:

- GCF (Greatest Common Factor) – I check if all terms share a common factor and factor it out first.
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- Box Diagram – I organize terms in a grid to visually group factors, especially for trinomials.
- Difference of Squares – I use this when I see two perfect squares separated by subtraction, factoring as $(a+b)(a-b)$.
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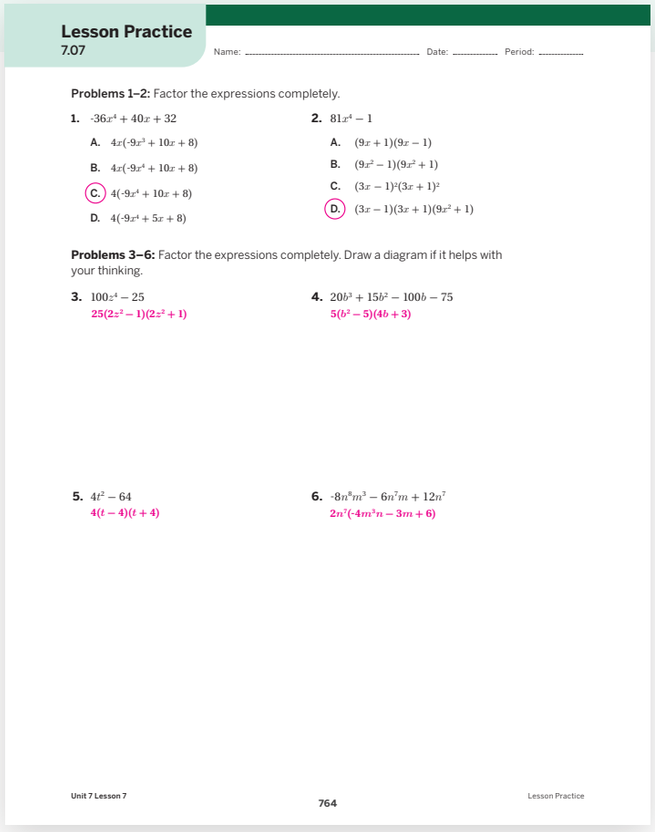
Lesson Practice 7.07

Lesson Summary

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Unit 7 Lesson 7 763 Synthesis | Lesson Practice



Lesson Practice 7.07

Name: ..... Date: ..... Period: .....

Problems 1–2: Factor the expressions completely.

1. $-36x^4+40x+32$

   A. $4x(-9x^3+10x+8)$

   B. $4x(-9x^4+10x+8)$

   C. $4(-9x^4+10x+8)$

   D. $4(-9x^4+5x+8)$

2. $81x^4-1$

   A. $(9x+1)(9x-1)$

   B. $(9x^2-1)(9x^2+1)$

   C. $(3x-1)^2(3x+1)^2$

   D. $(3x-1)(3x+1)(9x^2+1)$

Problems 3–6: Factor the expressions completely. Draw a diagram if it helps with your thinking.

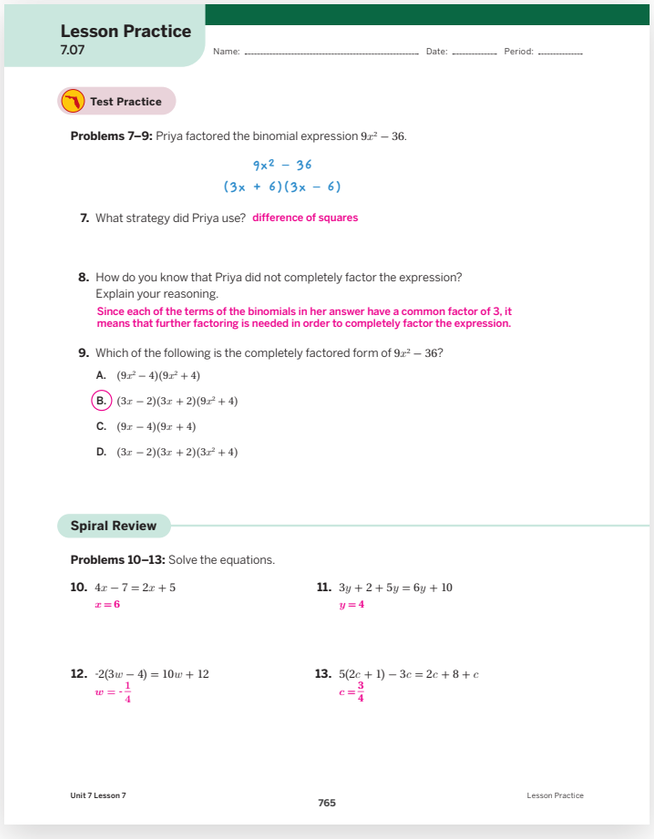
3. $100z^4-25$
   $25(2z^2-1)(2z^2+1)$

4. $20b^3+15b^2-100b-75$
   $5(b^2-5)(4b+3)$

5. $4t^2-64$
   $4(t-4)(t+4)$

6. $-8n^8m^3-6n^7m+12n^7$
   $2n^7(-4m^3n-3m+6)$

Unit 7 Lesson 7 764 Lesson Practice



Lesson Practice 7.07

Name: ..... Date: ..... Period: .....

Test Practice

Problems 7–9: Priya factored the binomial expression $9x^2-36$.

$9x^2 - 36$

$(3x + 6)(3x - 6)$

7. What strategy did Priya use? difference of squares

8. How do you know that Priya did not completely factor the expression? Explain your reasoning.
   Since each of the terms of the binomials in her answer have a common factor of 3, it means that further factoring is needed in order to completely factor the expression.

9. Which of the following is the completely factored form of $9x^2-36$?

   A. $(9x^2-4)(9x^2+4)$

   B. $(3x-2)(3x+2)(9x^2+4)$

   C. $(9x-4)(9x+4)$

   D. $(3x-2)(3x+2)(3x^2+4)$

Spiral Review

Problems 10–13: Solve the equations.

10. $4x-7=2x+5$
    $x=6$

11. $3y+2+5y=6y+10$
    $y=4$

12. $-2(3w-4)=10w+12$
    $w=-\frac{1}{4}$

13. $5(2c+1)-3c=2c+8+c$
    $c=\frac{3}{4}$

Unit 7 Lesson 7 765 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.AR.1.7
	4–6	2	MA.912.AR.1.7
Test Practice	7–9	3	MA.912.AR.1.7
Spiral Review			
Fluency	10–13	1	MA.8.AR.2.1

Unit 7

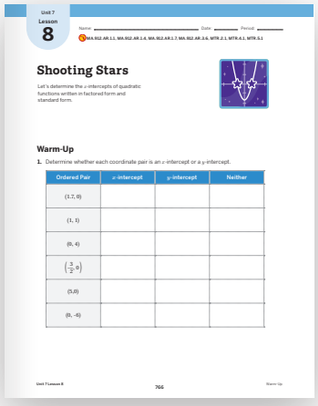
Lesson

8

Shooting Stars

Determining the x-Intercepts of Quadratic Functions

Let's determine the x-intercepts of quadratic functions written in factored form and standard form.


Unit 7
Lesson
8
Name _____ Date _____ Period _____
MA.912.AR.1.1, MA.912.AR.1.4, MA.912.AR.2.7, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1
Shooting Stars
Let's determine the x-intercepts of quadratic functions written in factored form and standard form.
Warm-Up
1. Determine whether each coordinate pair is an x-intercept or a y-intercept.
Ordered Pair | x-intercept | y-intercept | Neither
(1.7, 0)
(1, 1)
(0, 4)
(3/2, 0)
(5,0)
(0, -6)
Unit 7 Lesson 8
766
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Determine the x-intercepts of quadratic functions written in factored form and standard form.
2. **Language Goal:** Explain why the x-intercepts of a quadratic function are the same when it is written in different, but equivalent, factored forms. **(Writing and Listening)**

Students determine the x-intercepts of quadratic functions written in factored form and standard form. They compare equivalent quadratic functions factored in different ways and practice writing quadratic functions when given the x-intercepts.

Note: Students will be formally introduced to the zero-product property in the next lesson. **(MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In Unit 7, students determined the x-intercepts of quadratic functions by checking for x-values that would produce a y-value of 0. In Lessons 4 and 5, students learned to factor quadratic expressions written in standard form.

Future Learning

In Lesson 7, students will be introduced to the zero-product property and use it to solve quadratic equations written in various forms. In Lesson 16, students will solve for x-intercepts of quadratic functions and interpret them in context.

Connections

- Students extend their understanding of the relationship between the factored form of a quadratic function and the intercepts (or zeros) of a parabola as they explore equivalent forms of a factored quadratic function with an a-term greater than 1. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.7.1

Rewrite a polynomial expression as a product of polynomials over the real number system.

MA.912.AR.3.6

Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

Also Addressing: **MA.912.NSO.1.1, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2,1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.1.2	**MA.912.AR.6.5**
MA.8.AR.3.3	**MA.912.AR.3.9**

Vocabulary

New Vocabulary

zeros

Review Vocabulary

x-intercept

Lesson at a Glance

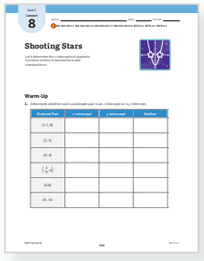
~ 45 min

MA.912.AR.1.7, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

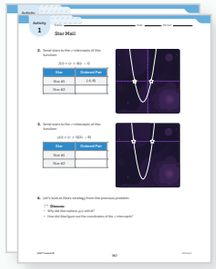
Students revisit the meaning of x- and y-intercepts by sorting ordered pairs.



Activity 1

Pairs | 10 min

Students surface strategies for determining x-intercepts from equations in factored form.

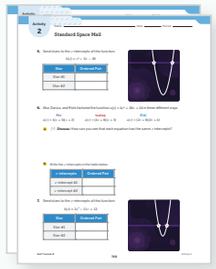


Activity 2

Pairs | 10 min

Students surface strategies for determining x-intercepts from equations in standard form and different, but equivalent, equations in factored form. **(MTR.2.1)**

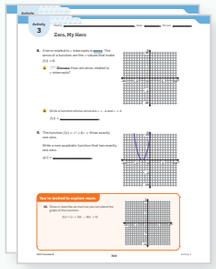
MLR7: Compare and Connect



Activity 3

Pairs | 10 min

Students are introduced to the term **zeros** and write equations for quadratic functions from given information about their zeros.



Synthesis

Whole Class | 5 min

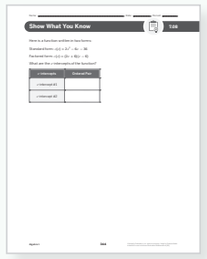
Students describe how to determine the zeros of a quadratic function given an equation in standard form or factored form. **(MTR.2.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the x-intercepts of a quadratic function.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit the meaning of x- and y-intercepts by sorting ordered pairs.

Short on time? Consider shortening or omitting the class discussion if the class shows a strong understanding of x- and y-intercepts during the Warm-Up.

1 Launch

To surface prior knowledge, consider asking:

- "What does it mean to *intercept* something?"
- "Where can you find the x-intercepts on a graph? y-intercepts?"

To support students getting started, invite them to use graph paper to graph the points to help with their thinking.

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.

ELL **English Language Learners** Revisit a class display of the key features of a parabola. Consider asking, "Which point(s) on the graph are x-intercepts? Which are y-intercepts?" **(Reading)**

2 Connect

Consider asking:

- "How are x-intercepts and y-intercepts alike? How are they different?" **(MTR.5.1)**
- "What do you notice about all of the ordered pairs that are x-intercepts?"
- "Where do you think $(0, 0)$ would go?"

Unit 7
Lesson
8

Name: _____ Date: _____ Period: _____

MA.912.AR.1.1, MA.912.AR.1.4, MA.912.AR.1.7, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1

Shooting Stars

Let's determine the x-intercepts of quadratic functions written in factored form and standard form.



Warm-Up

1. Determine whether each coordinate pair is an x-intercept or a y-intercept.

Ordered Pair	x-intercept	y-intercept	Neither
$(1.7, 0)$	✓		
$(1, 1)$			✓
$(0, 4)$		✓	
$\left(-\frac{3}{2}, 0\right)$	✓		
$(5,0)$	✓		
$(0, -6)$		✓	

Unit 7 Lesson 8 766 Warm-Up

Pairs | 10 min

Activity 1 Star Mail

Purpose: Students surface strategies for determining x-intercepts from equations in factored form.

Short on time? Consider omitting the launch.

1 Launch

Encourage students to predict what the other x-intercept of $f(x)$ will be. Invite them to describe how their predictions are connected to the equation and the graph.

2 Monitor

To surface connections, consider asking pairs:

- "What happens when you substitute -4 into $f(x)$? What other value could you substitute into $f(x)$ to make the function equal to 0?"
- "How are the numbers in the ordered pair related to the numbers in the function equation?"

D Differentiation *(Problems 2 and 3)*

Look for students who:	Respond to Student Thinking
Reverse the signs of their x-intercepts.	**Support** Celebrate their use of structure. Encourage them to substitute the x-values into $g(x)$ and consider asking: • "Where would that point be on a graph?" • "What is different about the second factor?" **(MTR.5.1)**
Use the constant in factored form to determine $(6, 0)$.	
Determine the x-intercepts.	**Strengthen** Consider asking: • "Why does $g(x)$ have two x-intercepts?" • "How can you check to see if the x-value makes $f(x) = 0$?"

Discuss Aba's strategy.

Listen for what students say about Aba's strategy to highlight in the Connect, such as:

- x-intercepts occur at x-values that make the function equal to 0.
- Aba set each factor equal to 0 to solve for x.

A Accessibility: Conceptual Processing Consider showing a step in between line 2 and line 3 of Aba's method (e.g., $x + 1 = 0$ and $2x - 6 = 0$) to support students in making sense of Aba's strategy.

3 Connect

To surface the Key Takeaway, consider asking:

- "What did Aba do to determine $x = -1$ and $x = 3$?"
- "Where do you see the x-intercepts in each line of Aba's solving process?" **(MTR.5.1)**
- "How could we check to see if these x-intercepts are correct?" **(MTR.5.1)**

Activity 1

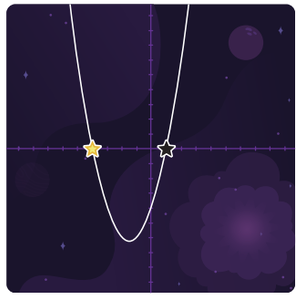
Name: _____ Date: _____ Period: _____

Star Mail

2. Send stars to the x-intercepts of this function:

$$f(x) = (x + 4)(x - 1)$$

Star	Ordered Pair
Star #1	(-4, 0)
Star #2	(1, 0)



3. Send stars to the x-intercepts of this function:

$$g(x) = (x + 1)(2x - 6)$$

Star	Ordered Pair
Star #1	(-1, 0)
Star #2	(3, 0)



4. Let's look at Aba's strategy from the previous problem.

Discuss:

- Why did Aba replace $g(x)$ with 0?
- How did Aba figure out the coordinates of the x-intercepts?

Explanations vary.

- Aba replaced $g(x)$ with 0 because an x-intercept of a function is a point where the output is 0.
- Once Aba found the inputs that make $g(x) = 0$, she wrote them as ordered pairs in the form $(x, 0)$ since x-intercepts are where the graph of the function crosses the x-axis.

Unit 7 Lesson 8 767 Activity 1

Key Takeaway: One strategy to determine x-intercepts from a quadratic equation in factored form is to set each factor equal to 0 and solve.

Note: The zero-product property will be introduced in the next lesson.

Pairs | 10 min

Activity 2 Standard Space Mail

Purpose: Students surface strategies for determining x-intercepts from equations in standard form and different, but equivalent, equations in factored form. **(MTR.2.1)**

1 Launch

Consider asking, "How is $h(x)$ similar to $g(x)$ in Problem 3? How is it different?"

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What form is this equation in? Which form reveals x-intercepts?" **(MTR.5.1)**
Guess and check.	**Strengthen** Celebrate their persistence. Consider asking: • "What's a value you know is not an x-intercept? How do you know?" • "How can you use the equation to determine the x-intercept?" **(MTR.5.1)**

A Accessibility: Memory and Attention Invite students to use a calculator and draw diagrams to support them with calculations and factoring.

Listen for descriptions to share during the Connect.

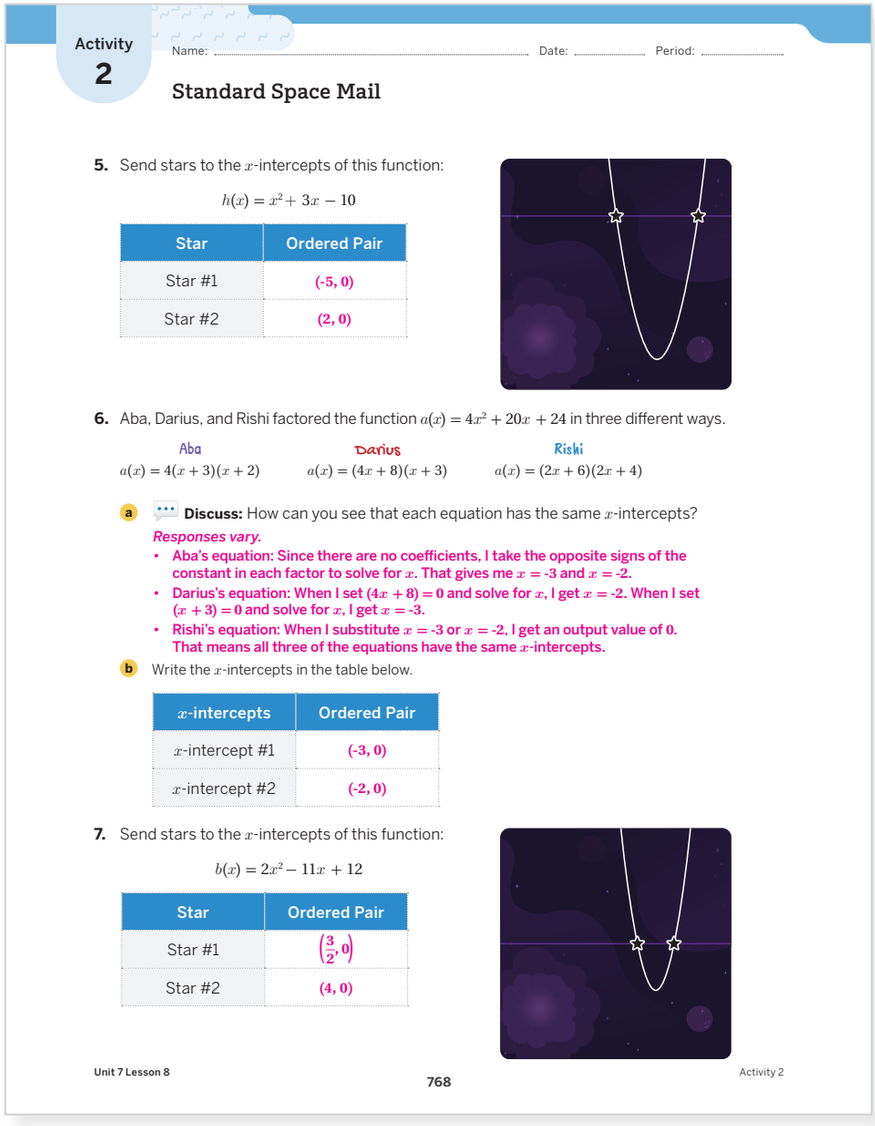
ELL English Language Learners Consider providing a word bank with words such as: *factor*, *coefficient*, *constant*, *sign*, etc. **(Speaking, and Listening)**

D Differentiation: (Strengthen) Consider asking:
- "How can you tell that one of the x-intercepts of $b(x)$ is not a whole number?"
- "Is there more than one way to factor $b(x)$? Why or why not?"

3 Connect

MLR MLR7: Compare and Connect Invite students to analyze and compare the three equations.

To surface the Key Takeaway, consider asking:
- "How are Aba's, Darius's, and Rishi's functions alike? How are they different?" They are all in factored form, but the factors are different.
- "How do you know that Aba's, Darius's, and Rishi's functions are all equivalent?" **(MTR.2.1, MTR.4.1)**
- "When might it be useful to factor the function like Aba, Darius, or Rishi did?" **(MTR.5.1)**
- "What's important to remember from Problem 6?"



Activity 2

Name: Date: Period:

Standard Space Mail

5. Send stars to the x-intercepts of this function:

$h(x) = x^2 + 3x - 10$

Star	Ordered Pair
Star #1	(-5, 0)
Star #2	(2, 0)

6. Aba, Darius, and Rishi factored the function $a(x) = 4x^2 + 20x + 24$ in three different ways.

Aba: $a(x) = 4(x + 3)(x + 2)$ Darius: $a(x) = (4x + 8)(x + 3)$ Rishi: $a(x) = (2x + 6)(2x + 4)$

a. **Discuss:** How can you see that each equation has the same x-intercepts?

Responses vary.
- Aba's equation: Since there are no coefficients, I take the opposite signs of the constant in each factor to solve for x. That gives me $x = -3$ and $x = -2$.
- Darius's equation: When I set $(4x + 8) = 0$ and solve for x, I get $x = -2$. When I set $(x + 3) = 0$ and solve for x, I get $x = -3$.
- Rishi's equation: When I substitute $x = -3$ or $x = -2$, I get an output value of 0. That means all three of the equations have the same x-intercepts.

b. Write the x-intercepts in the table below.

x-intercepts	Ordered Pair
x-intercept #1	(-3, 0)
x-intercept #2	(-2, 0)

7. Send stars to the x-intercepts of this function:

$b(x) = 2x^2 - 11x + 12$

Star	Ordered Pair
Star #1	$\left(\frac{3}{2}, 0\right)$
Star #2	(4, 0)

Unit 7 Lesson 8 768 Activity 2

Key Takeaway: Rewriting a standard-form equation in factored form can be a helpful step for determining x-intercepts. There are multiple equivalent ways to factor equations.

Pairs | 10 min

Activity 3 Zero, My Hero

Purpose: Students are introduced to the term **zeros** and write equations for quadratic functions from given information about their zeros.

1 Launch

Encourage students to describe what **zero** means in their own words.

To encourage creativity, challenge students to come up with a function that is different from ones their classmates come up with.

2 Monitor

D **Differentiation:** (Strengthen) Consider asking, "How do you know your factors will create zeros at $x = -4$ and $x = 2$?" **(MTR.2.1)**

To encourage playfulness, consider asking pairs:
- "How many answers are there?"
- "Can you make a function that is concave down?"

Look for creative responses to celebrate during the Connect (e.g., $f(x) = (.01x + .04)(180x - 360)$).

A **Accessibility: Executive Functioning** To support students in initiating complex tasks, encourage students to write a quadratic function in factored form that does not work, and use the feedback to make small revisions until the function meets the criteria.

To surface connections, consider asking, "How can you tell that $f(x) = x^2 + 6x + 9$ has exactly one zero from the equation?"

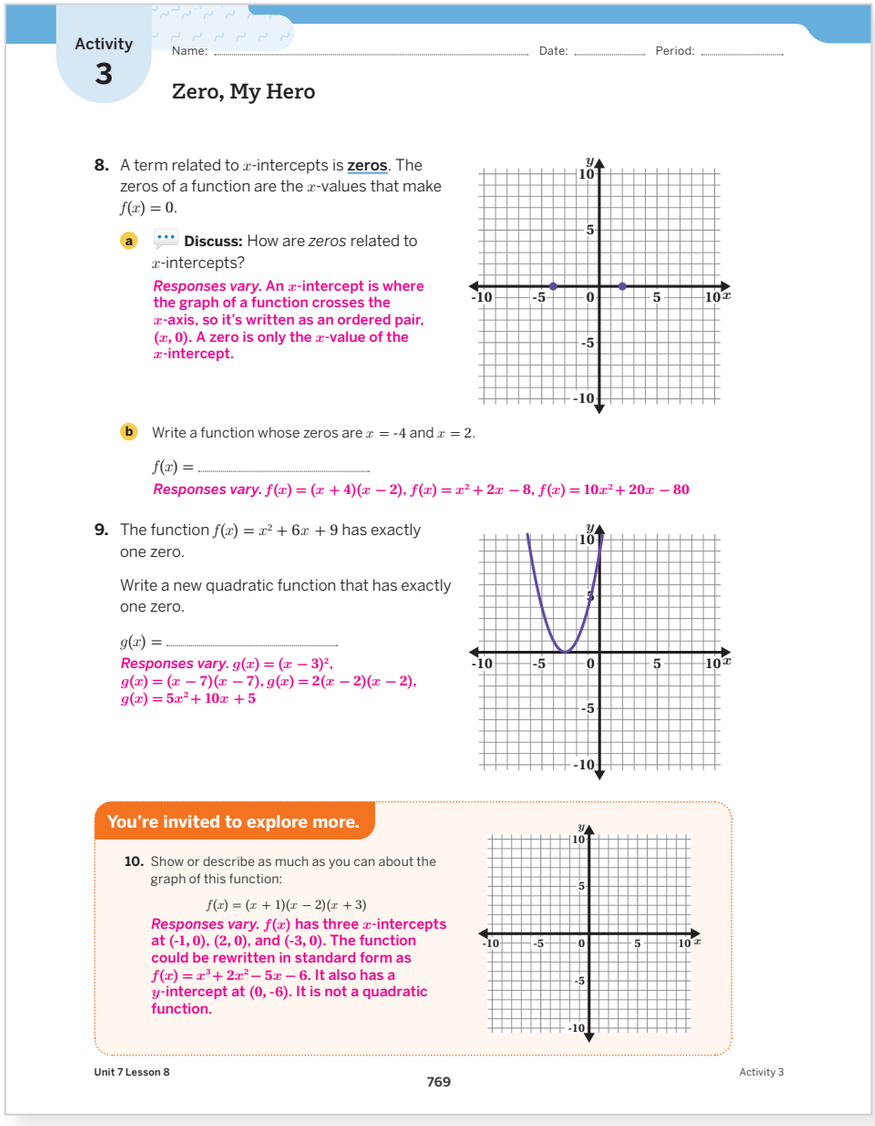
Look for creative responses to celebrate during the Connect. Students may write the function:
- As one factor: $g(x) = (x - 3)^2$.
- As two factors: $g(x) = (x - 7)(x - 7)$.
- With a greatest common factor: $g(x) = 2(x - 2)(x - 2)$.
- In standard form: $g(x) = 5x^2 + 10x + 5$.

D **Differentiation: You're invited to explore more.** (Stretch) Invite students who would like to explore graphs of equations in factored form further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share what is similar and different about the functions in Problem 9.

Consider asking, "How can you tell that $f(x) = x^2 + 6x + 9$ has one zero from the equation?"



Activity 3

Name: ..... Date: ..... Period: .....

Zero, My Hero

8. A term related to x-intercepts is **zeros**. The zeros of a function are the x-values that make $f(x) = 0$.

a. **Discuss:** How are *zeros* related to x-intercepts?

Responses vary. An x-intercept is where the graph of a function crosses the x-axis, so it's written as an ordered pair, $(x, 0)$. A zero is only the x-value of the x-intercept.

b. Write a function whose zeros are $x = -4$ and $x = 2$.

$f(x) =$ .....

Responses vary. $f(x) = (x + 4)(x - 2)$, $f(x) = x^2 + 2x - 8$, $f(x) = 10x^2 + 20x - 80$

9. The function $f(x) = x^2 + 6x + 9$ has exactly one zero.

Write a new quadratic function that has exactly one zero.

$g(x) =$ .....

Responses vary. $g(x) = (x - 3)^2$, $g(x) = (x - 7)(x - 7)$, $g(x) = 2(x - 2)(x - 2)$, $g(x) = 5x^2 + 10x + 5$

You're invited to explore more.

10. Show or describe as much as you can about the graph of this function:

$f(x) = (x + 1)(x - 2)(x + 3)$

Responses vary. $f(x)$ has three x-intercepts at (-1, 0), (2, 0), and (-3, 0). The function could be rewritten in standard form as $f(x) = x^3 + 2x^2 - 5x - 6$. It also has a y-intercept at (0, -6). It is not a quadratic function.

Unit 7 Lesson 8 769 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students describe how to determine the zeros of a quadratic function given an equation in standard form or factored form. **(MTR.2.1)**

Invite students to respond independently, and then share their thinking with a partner. **(MTR.2.1, MTR.4.1)**

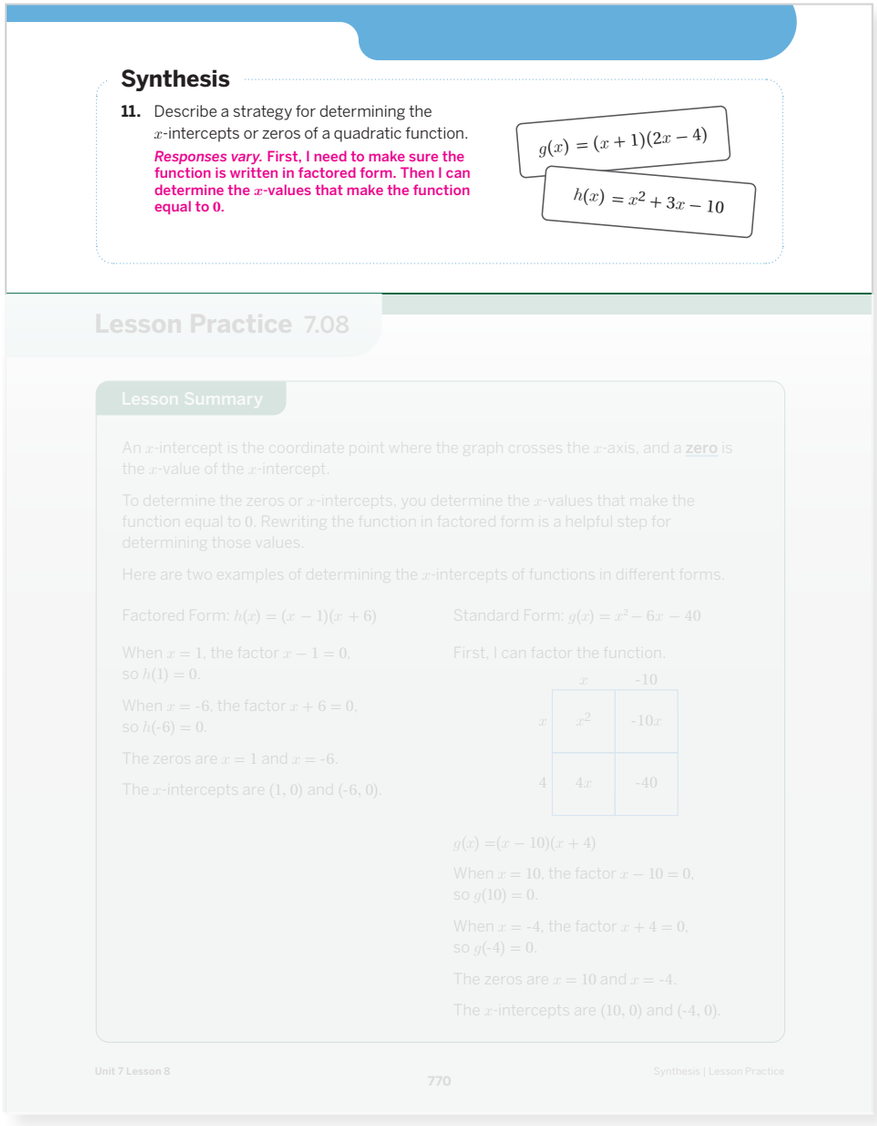
Collect and share a variety of ideas, such as:

- Write the function in factored form if it's not already written that way.
- Determine what values will make each factor equal to 0.
- To check, substitute the x-values into the function to see if the output is 0.
- Write the zeros as ordered pairs in the form $(x, 0)$ for the x-intercepts.

Note: If it doesn't come up naturally, consider asking:

- "Which form of a quadratic function reveals x-intercepts and zeros of a function? Why?"
- "What are some important things to remember about determining x-intercepts?"

Note: If time allows, invite students to revise their previous thinking based on the discussion. **(MTR.4.1)**



Synthesis

11. Describe a strategy for determining the x-intercepts or zeros of a quadratic function.

Responses vary. First, I need to make sure the function is written in factored form. Then I can determine the x-values that make the function equal to 0.

$g(x) = (x + 1)(2x - 4)$

$h(x) = x^2 + 3x - 10$

Lesson Practice 7.08

Lesson Summary

An x-intercept is the coordinate point where the graph crosses the x-axis, and a zero is the x-value of the x-intercept.

To determine the zeros or x-intercepts, you determine the x-values that make the function equal to 0. Rewriting the function in factored form is a helpful step for determining those values.

Here are two examples of determining the x-intercepts of functions in different forms.

Factored Form: $h(x) = (x - 1)(x + 6)$

When $x = 1$, the factor $x - 1 = 0$, so $h(1) = 0$.

When $x = -6$, the factor $x + 6 = 0$, so $h(-6) = 0$.

The zeros are $x = 1$ and $x = -6$.

The x-intercepts are $(1, 0)$ and $(-6, 0)$.

Standard Form: $g(x) = x^2 - 6x - 40$

First, I can factor the function.

	x	-10
x	x^2	$-10x$
4	$4x$	-40

$g(x) = (x - 10)(x + 4)$

When $x = 10$, the factor $x - 10 = 0$, so $g(10) = 0$.

When $x = -4$, the factor $x + 4 = 0$, so $g(-4) = 0$.

The zeros are $x = 10$ and $x = -4$.

The x-intercepts are $(10, 0)$ and $(-4, 0)$.

Unit 7 Lesson 8 770 Synthesis | Lesson Practice

Lesson Takeaway: In order to determine zeros from an equation in standard form, it can be helpful to rewrite the equation in standard form. Once an equation is in factored form, each factor can be set equal to 0 and solved to determine the zeros or x-intercepts.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the x-intercepts of a quadratic function.

Today's Goals

1. **Goal:** Determine the x-intercepts of quadratic functions written in factored form and standard form.
2. **Language Goal:** Explain why the x-intercepts of a quadratic function are the same when it is written in different, but equivalent, factored forms. **(Writing and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.08

Here is a function written in two forms:

Standard form: $c(x) = 2x^2 - 6x - 36$

Factored form: $c(x) = (2x + 6)(x - 6)$

What are the x-intercepts of the function?

x-intercepts	Ordered Pair
x-intercept #1	(6, 0)
x-intercept #2	(-3, 0)

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support determining the x-intercepts of a quadratic function written in standard and factored form:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **You're invited to explore more** tasks and discuss their thinking with a partner.

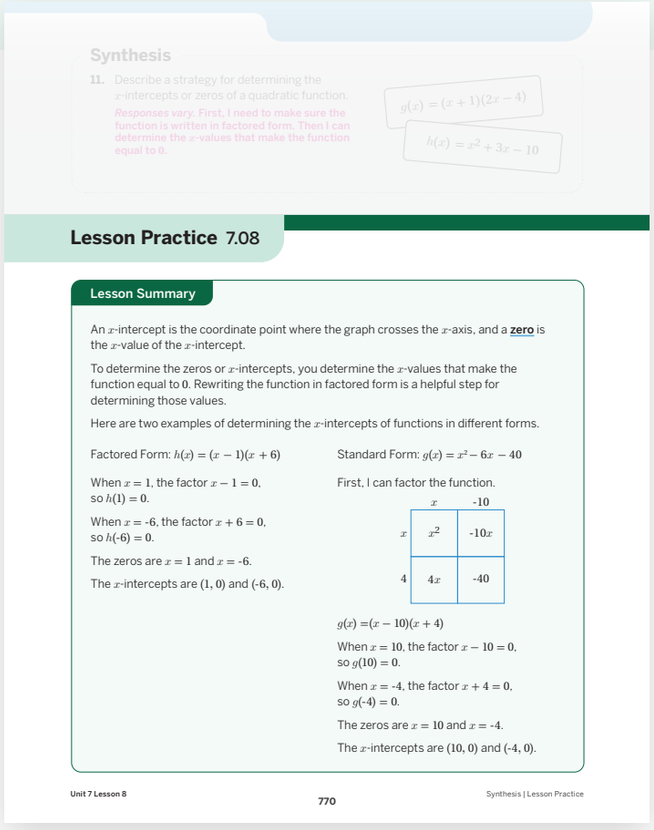
Professional Learning

Take a moment to reflect on how students determined x-intercepts in this lesson. What strategies and skills did they use? In Unit 6, students tested x-values and used the structure of equations to determine x-intercepts. In the next lesson, students will be introduced to the zero-product property to solve quadratic equations. How have the strategies and skills of substituting values, using structure, factoring, and using the zero-product property connect to each other? How can they each support students with solving quadratic equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. Describe a strategy for determining the x-intercepts or zeros of a quadratic function.

Responses vary. First, I need to make sure the function is written in factored form. Then I can determine the x-values that make the function equal to 0.

$g(x) = (x + 1)(2x - 4)$

$h(x) = x^2 + 3x - 10$

Lesson Practice 7.08

Lesson Summary

An x-intercept is the coordinate point where the graph crosses the x-axis, and a **zero** is the x-value of the x-intercept.

To determine the zeros or x-intercepts, you determine the x-values that make the function equal to 0. Rewriting the function in factored form is a helpful step for determining those values.

Here are two examples of determining the x-intercepts of functions in different forms.

Factored Form: $h(x) = (x - 1)(x + 6)$

When $x = 1$, the factor $x - 1 = 0$, so $h(1) = 0$.

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The zeros are $x = 1$ and $x = -6$.

The x-intercepts are $(1, 0)$ and $(-6, 0)$.

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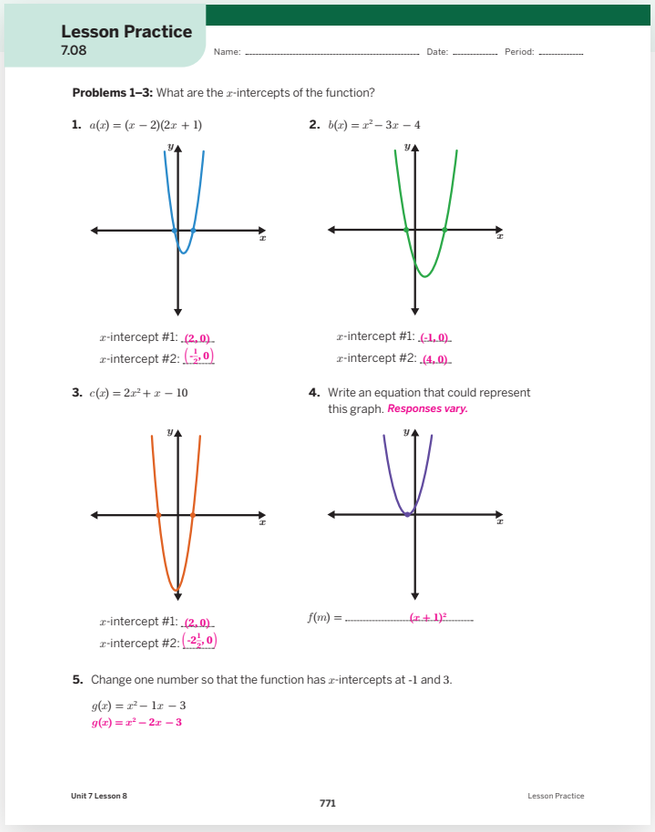
$g(x) = (x - 10)(x + 4)$

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When $x = -4$, the factor $x + 4 = 0$, so $g(-4) = 0$.

The zeros are $x = 10$ and $x = -4$.

The x-intercepts are $(10, 0)$ and $(-4, 0)$.



Lesson Practice 7.08

Name: ..... Date: ..... Period: .....

Problems 1–3: What are the x-intercepts of the function?

1. $a(x) = (x - 2)(2x + 1)$

x-intercept #1: (2, 0)

x-intercept #2: $\left(-\frac{1}{2}, 0\right)$

2. $b(x) = x^2 - 3x - 4$

x-intercept #1: (-1, 0)

x-intercept #2: (4, 0)

3. $c(x) = 2x^2 + x - 10$

x-intercept #1: (2, 0)

x-intercept #2: $\left(-2\frac{1}{2}, 0\right)$

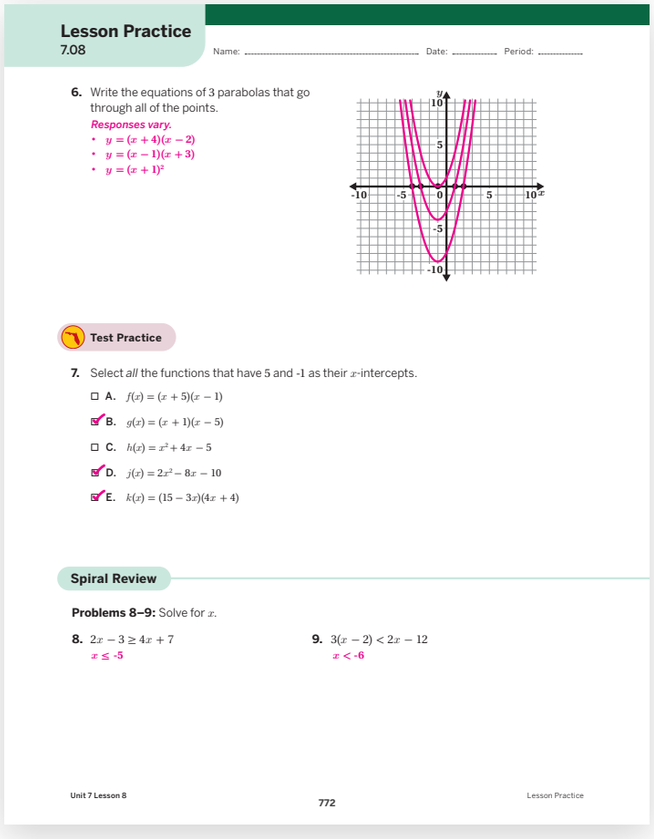
4. Write an equation that could represent this graph. *Responses vary.*

$f(m) =$ $(x + 1)^2$

5. Change one number so that the function has x-intercepts at -1 and 3.

$g(x) = x^2 - 1x - 3$

$g(x) = x^2 - 2x - 3$



Lesson Practice 7.08

Name: ..... Date: ..... Period: .....

6. Write the equations of 3 parabolas that go through all of the points.

Responses vary.

- $y = (x + 4)(x - 2)$
- $y = (x - 1)(x + 3)$
- $y = (x + 1)^2$

Test Practice

7. Select *all* the functions that have 5 and -1 as their x-intercepts.

- ☐ **A.** $f(x) = (x + 5)(x - 1)$
- ☑ **B.** $g(x) = (x + 1)(x - 5)$
- ☐ **C.** $h(x) = x^2 + 4x - 5$
- ☑ **D.** $j(x) = 2x^2 - 8x - 10$
- ☑ **E.** $k(x) = (15 - 3x)(4x + 4)$

Spiral Review

Problems 8–9: Solve for x.

8. $2x - 3 \geq 4x + 7$

$x \leq -5$

9. $3(x - 2) < 2x - 12$

$x < -6$

Lesson Practice Item Analysis

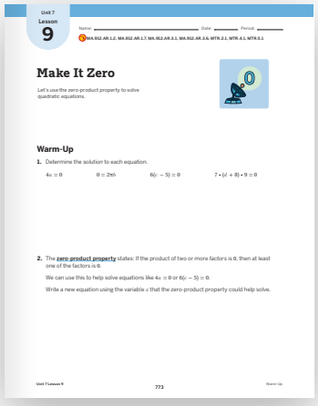
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.AR.3.6
	4, 6	2	MA.912.AR.3.4
	5	2	MA.912.AR.3.5
Test Practice	7	3	MA.912.AR.3.6
Spiral Review			
Fluency	8–9	1	MA.8.AR.2.2

Unit 7
Lesson
9

Make It Zero

Solving Quadratic Equations Using the Zero-Product Property

Let's use the zero-product property to solve quadratic equations.


Unit 7
Lesson
9
Make It Zero
Let's use the zero-product property to solve quadratic equations.
Warm-Up
1. Determine the solution to each equation.
2. The zero-product property states: If the product of two or more factors is 0, then at least one of the factors is 0.
We can use this to help solve equations like $4x = 0$ or $6(c - 5) = 0$.
Write a new equation using the variable x that the zero-product property could help solve.
773

Learning Outcomes

Today's Goals

1. **Goal:** Use the *zero-product property*: if the product of two or more factors is 0, then at least one of its factors is 0.
2. **Goal:** Solve quadratic equations by factoring and using the zero-product property.
3. **Language Goal:** Describe how to rewrite a quadratic equation so that the zero-product property can be used. **(Writing)**

Students use the zero-product property to solve quadratic equations. They surface ideas about what has to be true about the structure of an equation in order for this property to be used. **(MTR.5.1)**

Coherence

Prior Learning

Students solved linear equations in Unit 2 and exponential equations in Unit 6. In Lessons 4 and 5, students factored quadratic expressions, and in Lesson 6, they factored quadratic functions in order to determine x-intercepts.

Future Learning

In Lesson 8, students will use reasoning to dive deeper into solving quadratic equations and determine the number of *solutions* an equation has. In Lesson 9, they're introduced to graphical strategies for solving equations. Students will learn how to complete the square and use the quadratic formula to solve in the next two sub-units. In Algebra 2, students will use the zero-product property to solve polynomial equations of higher degrees.

Connections

- Students relate the factored form of a quadratic expression to solving a quadratic equation using the zero-product property. (mathematical)
- Students critique the thinking of how a quadratic equation is solved and explain their reasoning. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.7
Rewrite a polynomial expression as a product of polynomials over the real number system.

MA.912.AR.1.2
Rearrange equations or formulas to isolate a quantity of interest.

Also Addressing: **MA.912.AR.3.1, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2,1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.1.3	**MA.912.AR.1.8**
MA.8.AR.2	**MA.912.AR.3.2**

Vocabulary

New Vocabulary

zero-product property

Review Vocabulary

factor (of a number or expression)

solution

Lesson at a Glance

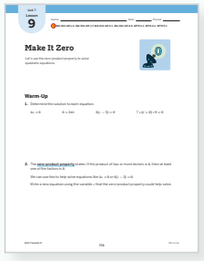
~ 45 min

MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6, MA.912.NSO.1.1, MA.912.NSO.1.2

Warm-Up

Pairs | 5 min

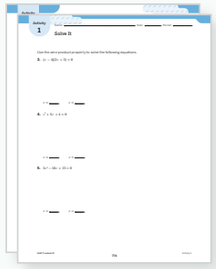
Students are introduced to the **zero-product property** through solving linear equations with a product equal to 0.



Activity 1

Pairs | 15 min

Students use the zero-product property to solve equations in both factored form and standard form that are set equal to 0. MLR3: Critique, Correct, Clarify



Activity 2

Pairs | 15 min

Students utilize structure to surface that equations must be set equal to zero and factored before the zero-product property can be used to solve them. MLR3: Critique, Correct, Clarify **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students describe how to use the zero-product property to solve quadratic equations. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a quadratic equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the **zero-product property** through solving linear equations with a product equal to 0.

1 Launch

Consider asking, "How are these equations similar? How are they different?"

Encourage students to check their solutions using substitution to help them revise their responses.

A **Accessibility: Conceptual Processing** As students solve abstract problems, invite them to say aloud what solving each equation means (e.g., "4 times what number equals 0").

2 Connect

Encourage students to paraphrase the zero-product property using their own words.

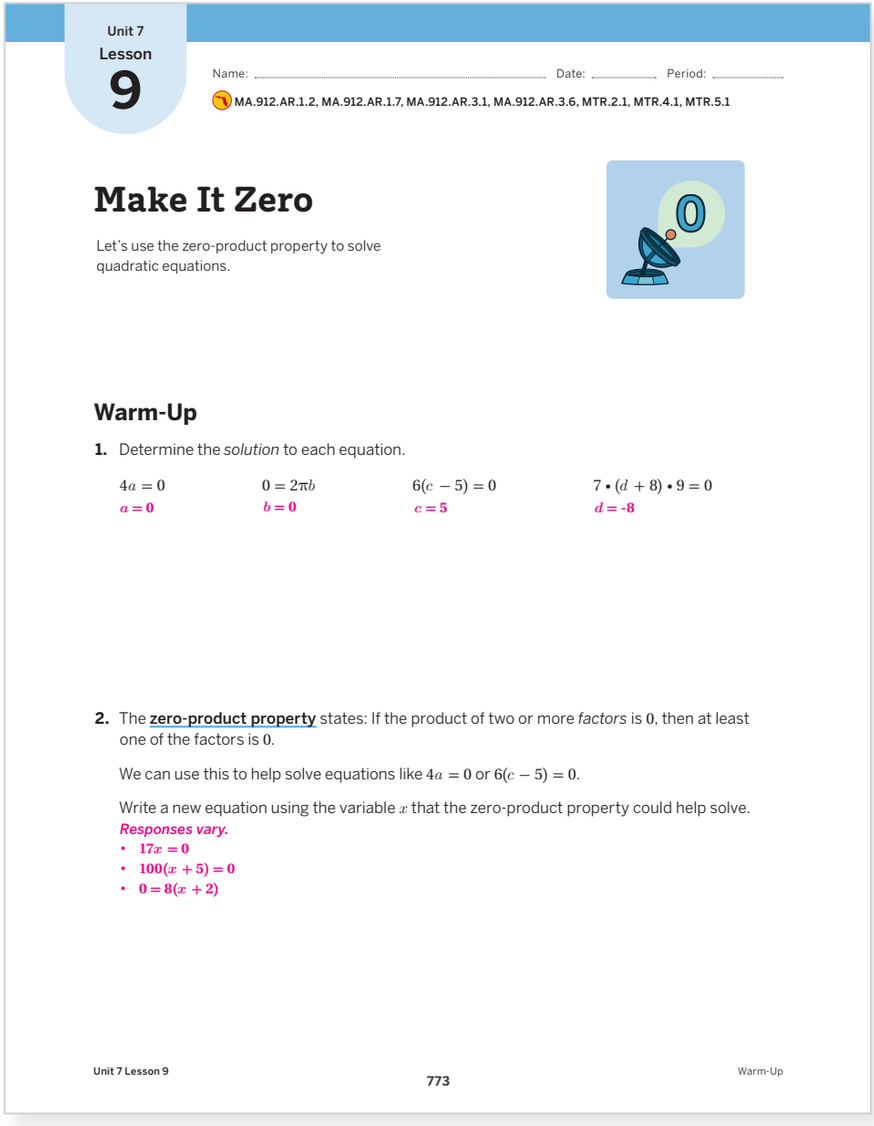
ELL **English Language Learners** Consider reviewing the meanings of *product* and *factor*. **(Reading and Listening)**

To encourage creativity, challenge students to write an equation they think will be different from their classmates'.

To surface the Key Takeaway, consider asking:

- "What do all of the equations have in common?"
- "Where do you see 0 in each of these equations?"
- "How do you see factors in each of these equations?" **(MTR.5.1)**

(MTR.5.1)



Unit 7
Lesson
9

Name: Date: Period:

MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1

Make It Zero

Let's use the zero-product property to solve quadratic equations.

Warm-Up

1. Determine the *solution* to each equation.

$4a = 0$	$0 = 2\pi b$	$6(c - 5) = 0$	$7 \cdot (d + 8) \cdot 9 = 0$
$a = 0$	$b = 0$	$c = 5$	$d = -8$

2. The **zero-product property** states: If the product of two or more *factors* is 0, then at least one of the factors is 0.

We can use this to help solve equations like $4a = 0$ or $6(c - 5) = 0$.

Write a new equation using the variable x that the zero-product property could help solve.

Responses vary.

- $17x = 0$
- $100(x + 5) = 0$
- $0 = 8(x + 2)$

Unit 7 Lesson 9 773 Warm-Up

Key Takeaway: The **zero-product property** can be a helpful tool for solving equations with a specific structure. It states that if the product of two or more *factors* is 0, then at least one of the factors is 0.

Pairs | 15 min

Activity 1 Solve It

Purpose: Students use the zero-product property to solve equations in both factored form and standard form that are set equal to 0.

Short on time? Consider omitting Problem 4. Engaging with Problem 5 will support students in accessing the discussion on Problem 6.

1 Launch

Consider asking, "How is this equation similar to the equations you solved in the Warm-Up? How is it different?" **(MTR.2.1, MTR.5.1)**

A **Accessibility: Memory and Attention** To support students with performing calculations throughout this lesson, invite them to use a calculator and/or paper, to record their work.

2 Monitor

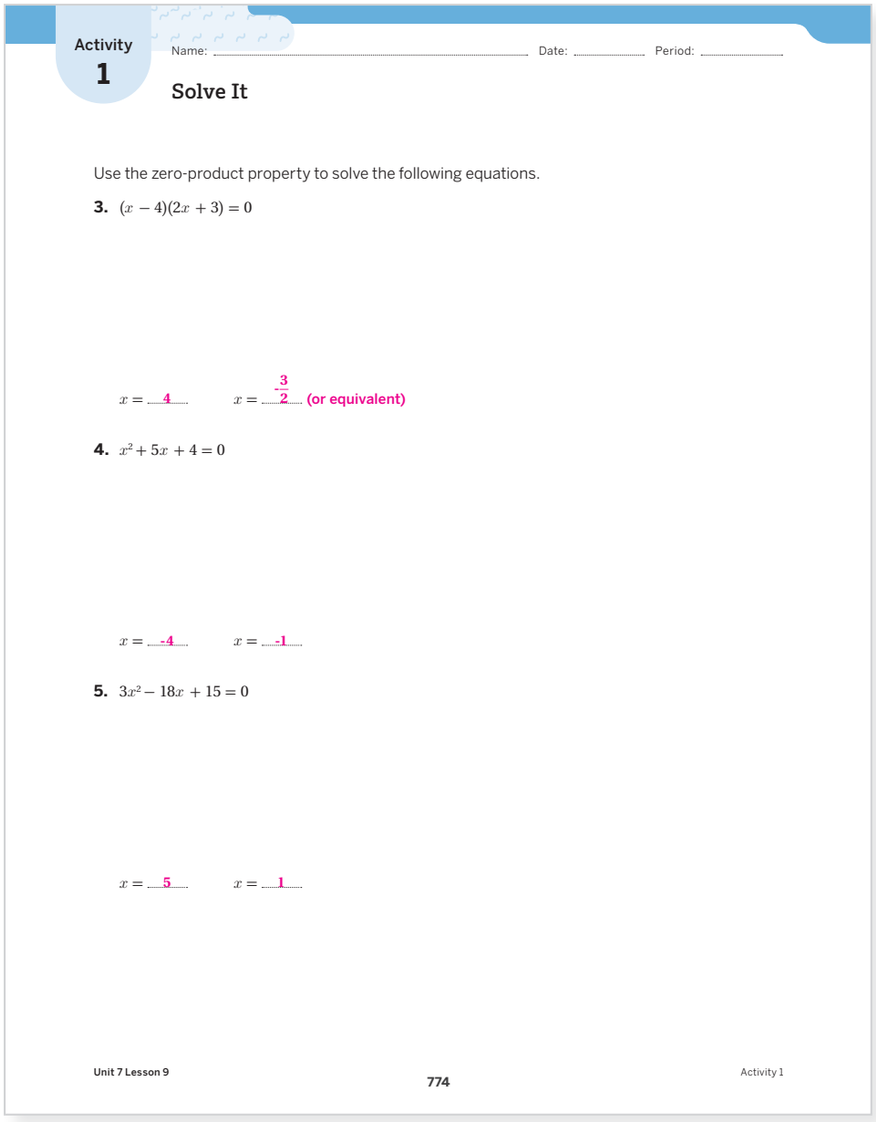
To surface connections, consider asking pairs:

- "The equations in the Warm-Up each had one *solution*. Why do these equations have two?"
- "How did you check your solutions?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking: • "How is this equation similar to the prior one? How is it different?" **(MTR.2.1, MTR.5.1)** • "Where do you see factors?" **(MTR.5.1)** • "Can you rewrite the equation in factored form?"
Think $x = -3$ is a solution because of the constant in the second factor. (*Problem 3*)	**Support** Celebrate their use of structure. Invite students to reflect on the feedback and ask, "What about the factors makes 4 a solution, but not -3?" **(MTR.5.1)**
Guess and check.	**Strengthen** Celebrate their persistence. Consider asking: • "What form is this equation in?" **(MTR.5.1)** • "Could you rewrite it in a different form to help you solve it?" **(MTR.2.1, MTR.5.1)** • Encourage them to draw a diagram if it supports them with factoring.
Solve the equation correctly.	**Strengthen** Consider asking: • "How is the structure of this equation different from the equation in Problem 3?" **(MTR.5.1)** • "How can factoring help you to solve quadratic equations?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Solve It

Use the zero-product property to solve the following equations.

3. $(x - 4)(2x + 3) = 0$

$x =$ 4 $x =$ $-\frac{3}{2}$ (or equivalent)

4. $x^2 + 5x + 4 = 0$

$x =$ -4 $x =$ -1

5. $3x^2 - 18x + 15 = 0$

$x =$ 5 $x =$ 1

Unit 7 Lesson 9 774 Activity 1

Pairs | 15 min

Activity 1 Solve It (continued)

Purpose: Students use the zero-product property to solve equations in both factored form and standard form that are set equal to 0.

Short on time? Consider omitting Problem 4. Engaging with Problem 5 will support students in accessing the discussion on Problem 6.

2 Monitor

Select and sequence a variety of student responses to share during the Connect. Some improvement ideas to highlight might include:

- Setting the factor equal to zero and solving.
- Factoring out the greatest common factor to more easily see the solutions in factored form.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *standard form, factored form, factor, constant, coefficient, solution, zero-product property*, etc. **(Writing and Speaking)**

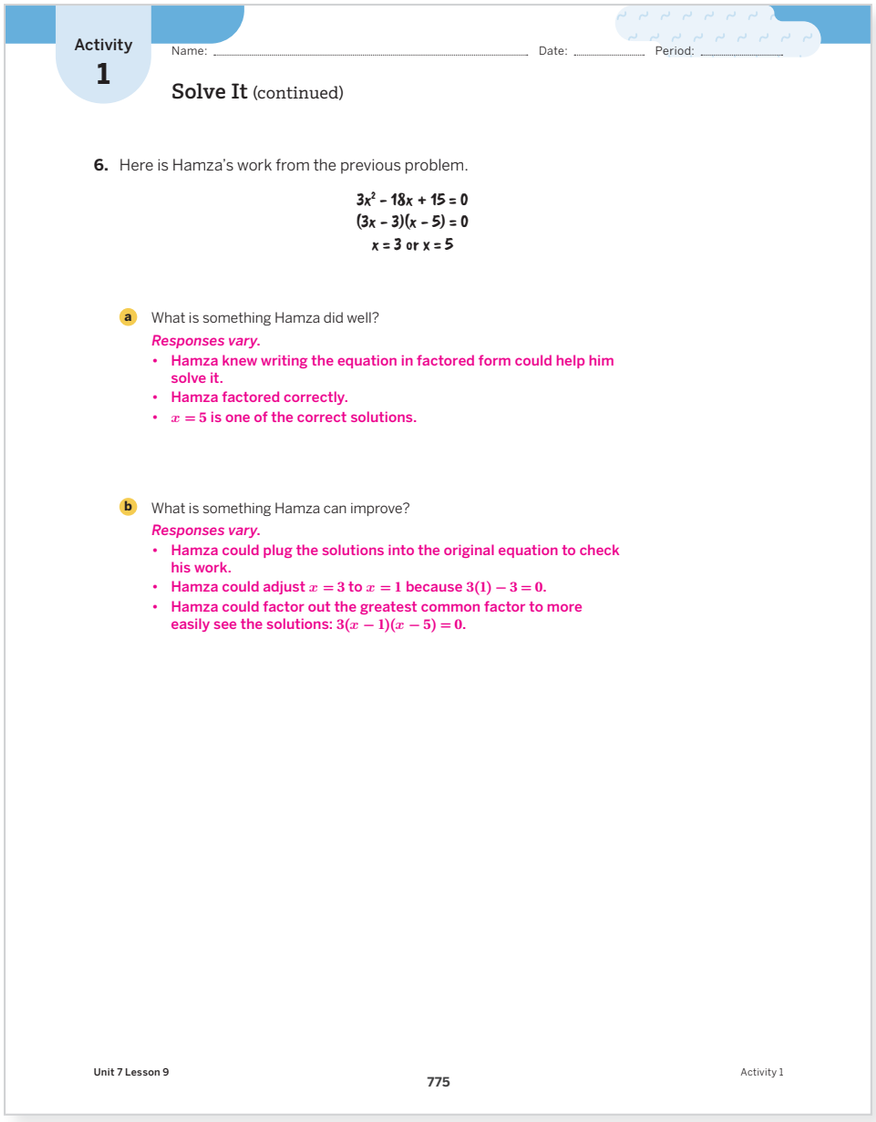
D **Differentiation: (Stretch)** Invite students to write an equation with the solutions $\frac{1}{2}$ and 6 using only integer coefficients.

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths and errors in Hamza's work. **(MTR.4.1)**
Share student responses and discuss:

a. What Hamza did well.
b. What Hamza can improve.
c. A question to help Hamza see how their work can be improved.

Consider asking, "How is Hamza's work similar to what you did on Problem 5? How is it different?"



Activity 1

Name: _____ Date: _____ Period: _____

Solve It (continued)

6. Here is Hamza's work from the previous problem.

$3x^2 - 18x + 15 = 0$
$(3x - 3)(x - 5) = 0$
$x = 3$ or $x = 5$

a. What is something Hamza did well?

Responses vary.

- Hamza knew writing the equation in factored form could help him solve it.
- Hamza factored correctly.
- $x = 5$ is one of the correct solutions.

b. What is something Hamza can improve?

Responses vary.

- Hamza could plug the solutions into the original equation to check his work.
- Hamza could adjust $x = 3$ to $x = 1$ because $3(1) - 3 = 0$.
- Hamza could factor out the greatest common factor to more easily see the solutions: $3(x - 1)(x - 5) = 0$.

Unit 7 Lesson 9 775 Activity 1

Pairs | 15 min

Activity 2 Zeroing In

Purpose: Students utilize structure to surface that equations must be set equal to zero and factored before the zero-product property can be used to solve it. **(MTR.5.1)**

1 Launch

Consider asking, "How is this equation similar to the equations you solved in the Warm-Up? How is it different?" **(MTR.2.1)**

2 Monitor

Select and sequence a variety of student responses to share such as:

- Inola is correct that she can't use the zero-product property as it looks now, but she could rewrite it.
- Setting the equation equal to zero.
- Factoring the equation.

A **Accessibility: Conceptual Processing** As students apply concepts to new situations, invite them to answer a more concrete question first, such as, "Was there an earlier equation that was easy to solve using the zero-product property? What about the structure of that equation made it easy to solve?" **(MTR.5.1)**

D **Differentiation: (Support)** Students may think that rewriting the equation as $(x + 2)(x - 2) = 3x$ could help solve it. Celebrate that they knew to factor and factored correctly. Consider asking, "Where is the zero in the equation? Why is zero important in the zero-product property?"

MLR **MLR3: Critique, Correct, Clarify** Pause, and use this routine to help students describe strengths, errors, and ambiguities in Inola's work. **(MTR.4.1)**

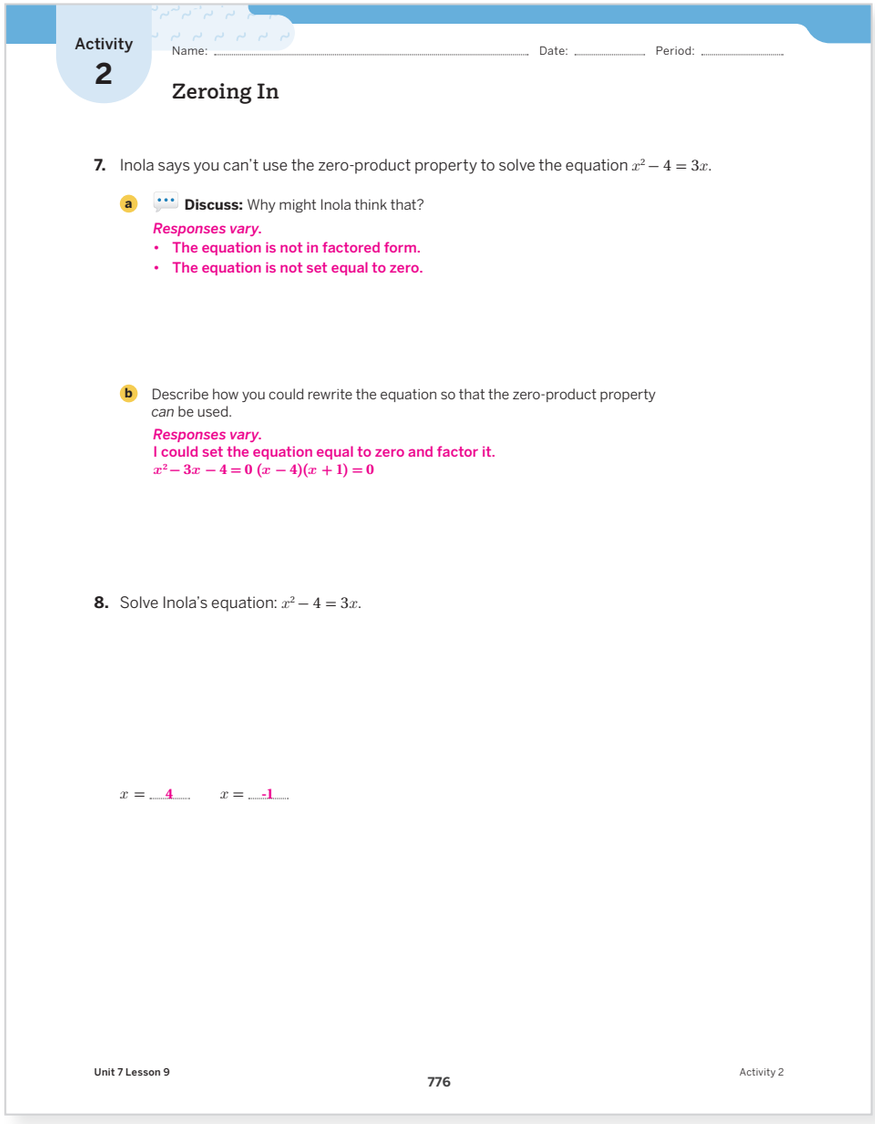
Invite students to share their responses and discuss as a class about what has to be true about the structure of an equation before the zero-product property can be used. **(MTR.5.1)**

A **Accessibility: Executive Functioning** To support students in carrying out multiple steps, consider collaboratively creating a display that demonstrates how to solve equations using the zero-product property one step at a time. For example:

1. Set the equation equal to zero.
2. Factor the equation.
3. Use the zero-product property.

Look for student progress and check in with pairs and offer support where needed.

Activity 2 continued >



Activity 2

Name: ..... Date: ..... Period: .....

Zeroing In

7. Inola says you can't use the zero-product property to solve the equation $x^2 - 4 = 3x$.

a. **Discuss:** Why might Inola think that?

Responses vary.

- The equation is not in factored form.
- The equation is not set equal to zero.

b. Describe how you could rewrite the equation so that the zero-product property *can* be used.

Responses vary.
I could set the equation equal to zero and factor it.
$x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$

8. Solve Inola's equation: $x^2 - 4 = 3x$.

$x =$ 4 $x =$ -1

Unit 7 Lesson 9 776 Activity 2

Pairs | 15 min

Activity 2 Zeroing In (continued)

Purpose: Students utilize structure to surface that equations must be set equal to zero and factored before the zero-product property can be used to solve it. **(MTR.5.1)**

2 Monitor

To support students getting started, consider asking, "What does an equation have to look like before the zero-product property can be used to solve it?" **(MTR.5.1)**

Look for a variety of solutions and explanations to share during the Connect, such as:

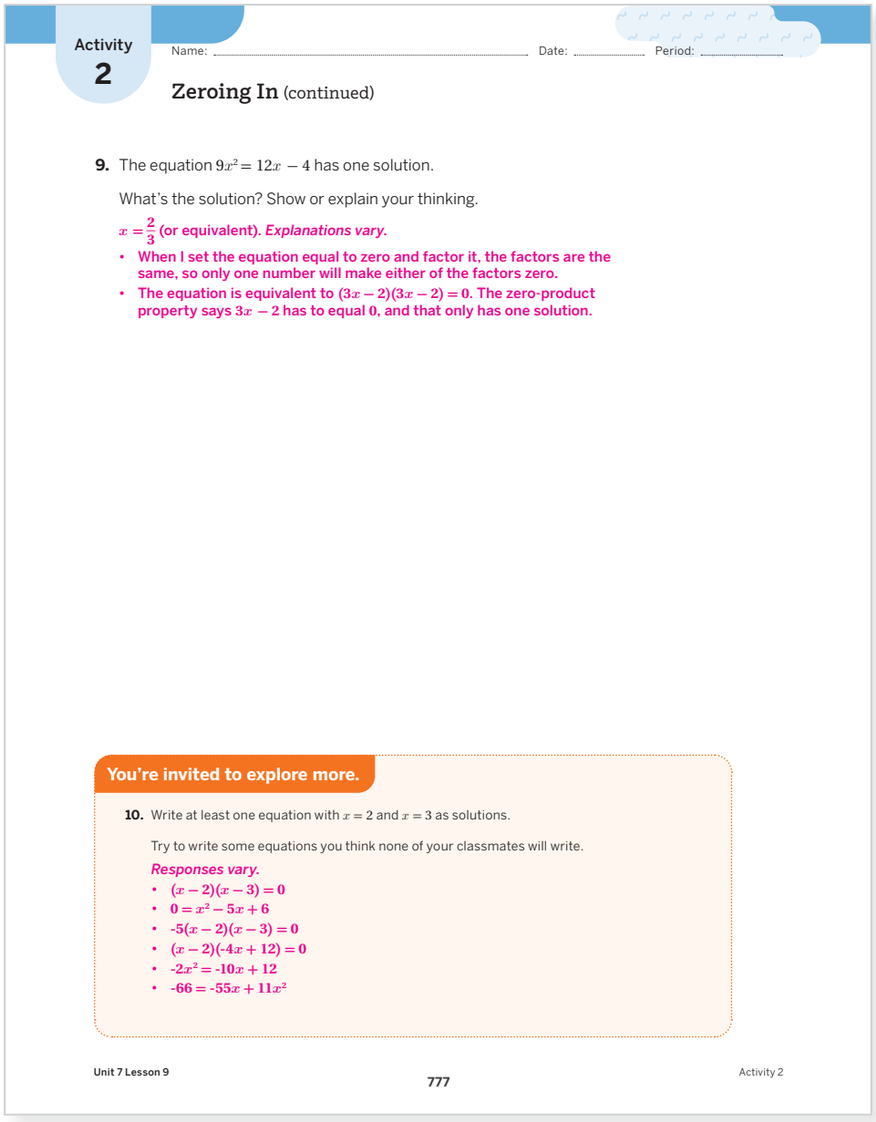
- Solutions written in different ways $\left(\text{e.g., } \frac{2}{3}, 0.66\right)$.
- Strategies that rewrite the equation as $(3x + 2)^2 = 0$ or $(3x + 2)(3x + 2) = 0$.
- Responses explaining that there is only one value that will make the factors 0 since both factors are the same.

D **Differentiation: You're invited to explore more.** **(Stretch)** Invite students who would like to think about the zero-product property in a different way to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their responses and consider asking:

- "What first steps did you take to solve this equation?"
- "How can you tell that this equation has exactly one solution?"
- "Which of these solutions are correct? What is the most precise way to write the solution?"



Activity 2

Name: _____ Date: _____ Period: _____

Zeroing In (continued)

9. The equation $9x^2 = 12x - 4$ has one solution.

What's the solution? Show or explain your thinking.

$x = \frac{2}{3}$ (or equivalent). *Explanations vary.*

- When I set the equation equal to zero and factor it, the factors are the same, so only one number will make either of the factors zero.
- The equation is equivalent to $(3x - 2)(3x - 2) = 0$. The zero-product property says $3x - 2$ has to equal 0, and that only has one solution.

You're invited to explore more.

10. Write at least one equation with $x = 2$ and $x = 3$ as solutions.

Try to write some equations you think none of your classmates will write.

Responses vary.

- $(x - 2)(x - 3) = 0$
- $0 = x^2 - 5x + 6$
- $-5(x - 2)(x - 3) = 0$
- $(x - 2)(-4x + 12) = 0$
- $-2x^2 = -10x + 12$
- $-66 = -55x + 11x^2$

Unit 7 Lesson 9 777 Activity 2

Key Takeaway: When an equation is set equal to zero and *factored*, it can be solved using the **zero-product property**.

Whole Class | 5 min

Synthesis

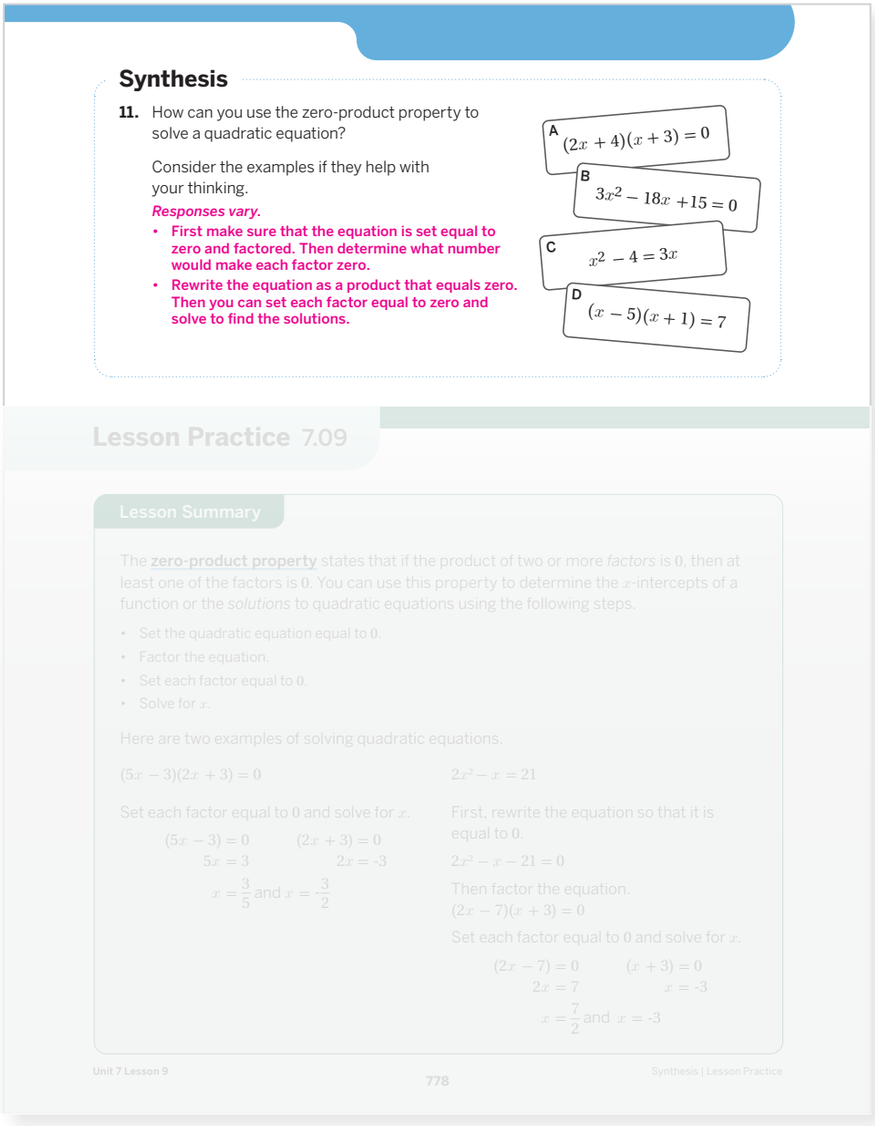
Purpose: Students describe how to use the zero-product property to solve quadratic equations.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Setting the equation equal to zero.
- Factoring the equation.
- Determining what value would make each factor equal 0.
- Checking solutions by substituting them into the original equation.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas, language, and examples of how to use the zero-product property to solve quadratic equations. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. How can you use the zero-product property to solve a quadratic equation?

Consider the examples if they help with your thinking.

Responses vary.
- First make sure that the equation is set equal to zero and factored. Then determine what number would make each factor zero.
- Rewrite the equation as a product that equals zero. Then you can set each factor equal to zero and solve to find the solutions.

A $(2x + 4)(x + 3) = 0$

B $3x^2 - 18x + 15 = 0$

C $x^2 - 4 = 3x$

D $(x - 5)(x + 1) = 7$

Lesson Practice 7.09

Lesson Summary

The **zero-product property** states that if the product of two or more *factors* is 0, then at least one of the factors is 0. You can use this property to determine the x-intercepts of a function or the *solutions* to quadratic equations using the following steps.
- Set the quadratic equation equal to 0.
- Factor the equation.
- Set each factor equal to 0.
- Solve for x.

Here are two examples of solving quadratic equations.

$(5x - 3)(2x + 3) = 0$

Set each factor equal to 0 and solve for x.

$(5x - 3) = 0$ $\quad$ $(2x + 3) = 0$

$5x = 3$ $\quad$ $2x = -3$

$x = \frac{3}{5}$ and $x = -\frac{3}{2}$

$2x^2 - x = 21$

First, rewrite the equation so that it is equal to 0.

$2x^2 - x - 21 = 0$

Then factor the equation.

$(2x - 7)(x + 3) = 0$

Set each factor equal to 0 and solve for x.

$(2x - 7) = 0$ $\quad$ $(x + 3) = 0$

$2x = 7$ $\quad$ $x = -3$

$x = \frac{7}{2}$ and $x = -3$

Unit 7 Lesson 9 778 Synthesis | Lesson Practice

Lesson Takeaway: The zero-product property is a helpful tool for solving equations that are in *factored* form and set equal to 0.

Independent | 5 min

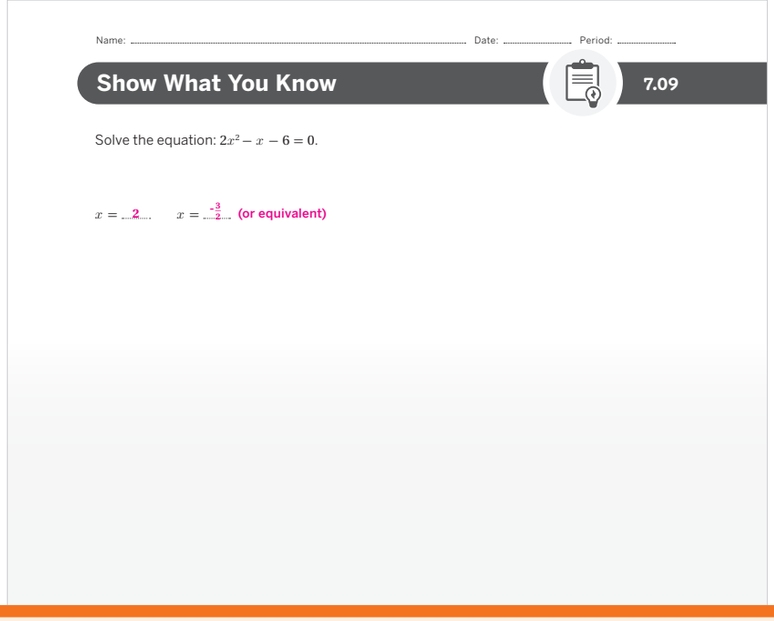
Show What You Know

Purpose: Students demonstrate their understanding by solving a quadratic equation.

Today's Goals

1. **Goal:** Use the *zero-product property*: if the product of two or more factors is 0, then at least one of its factors is 0.
2. **Goal:** Solve quadratic equations by factoring and using the zero-product property.
3. **Language Goal:** Describe how to rewrite a quadratic equation so that the zero-product property can be used. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.09

Solve the equation: $2x^2 - x - 6 = 0$.

$x =$ 2 $x =$ $-\frac{3}{2}$ (or equivalent)

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support solving a quadratic equation by factoring:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the You're invited to explore more tasks and discuss their thinking with a partner.

Professional Learning

Reflect on your facilitation of the **MLR3: Critique, Correct, Clarify** in this lesson.

- What types of strengths, errors, and ambiguities were students able to name?
- How can this routine support students in critiquing the reasoning of others?
- How can this routine support in building a positive classroom culture around learning from mistakes?
- This routine is used again in Lesson 10. What might you keep the same about your facilitation? What might you change about your facilitation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you use the zero-product property to solve a quadratic equation?

Consider the examples if they help with your thinking.

Responses vary.

- First make sure that the equation is set equal to zero and factored. Then determine what number would make each factor zero.
- Rewrite the equation as a product that equals zero. Then you can set each factor equal to zero and solve to find the solutions.

A $(2x+4)(x+3)=0$

B $3x^2-18x+15=0$

C $x^2-4=3x$

D $(x-5)(x+1)=7$

Lesson Practice 7.09

Lesson Summary

The **zero-product property** states that if the product of two or more *factors* is 0, then at least one of the factors is 0. You can use this property to determine the x-intercepts of a function or the *solutions* to quadratic equations using the following steps.

- Set the quadratic equation equal to 0.
- Factor the equation.
- Set each factor equal to 0.
- Solve for x.

Here are two examples of solving quadratic equations.

$(5x-3)(2x+3)=0$

Set each factor equal to 0 and solve for x.

$5x-3=0$ $\quad$ $2x+3=0$

$5x=3$ $\quad$ $2x=-3$

$x=\frac{3}{5}$ and $x=-\frac{3}{2}$

$2x^2-x=21$

First, rewrite the equation so that it is equal to 0.

$2x^2-x-21=0$

Then factor the equation.

$(2x-7)(x+3)=0$

Set each factor equal to 0 and solve for x.

$2x-7=0$ $\quad$ $x+3=0$

$2x=7$ $\quad$ $x=-3$

$x=\frac{7}{2}$ and $x=-3$

Lesson Practice 7.09

Name: ..... Date: ..... Period: .....

1. Rewrite each standard-form quadratic equation in factored form.

$x^2+7x+6=0$ $\quad$ $(x+6)(x+1)=0$

$x^2-5x+6=0$ $\quad$ $(x-3)(x-2)=0$

$x^2+5x-6=0$ $\quad$ $(x+6)(x-1)=0$

2. Rewrite the equation $x^2-x=6$ so that one side is equal to 0. Then solve the equation.

$x^2-x-6=0$ (or equivalent)

$x=3$ and $x=-2$

Problems 3–5: Solve each equation.

3. $(4-5x)(x+4)=0$

$x=\frac{4}{5}$ and $x=-4$

4. $x^2+14x+33=0$

$x=-3$ and $x=-11$

5. $5x+12=x^2-5x-12$

$x=-2$ and $x=12$

6. Solve the equation $3x^2-18x+24=0$. Use the diagram if it helps with your thinking.

$x=4$ and $x=2$

	x	-4
$3x$	$3x^2$	$-12x$
-6	$-6x$	24

(or equivalent)

7. Write two equations with $x=\frac{2}{3}$ and $x=-9$ as solutions.

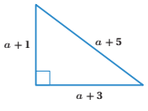
Responses vary. $(3x-2)(x+9)=0$ and $\left(x-\frac{2}{3}\right)(x+9)=0$

Lesson Practice 7.09

Name: ..... Date: ..... Period: .....

8. Determine the value of a in the right triangle. Explain your thinking.

$a=5$. *Explanations vary.* I used the Pythagorean theorem to set up the equation $(a+1)^2+(a+3)^2=(a+5)^2$. The solutions to this equation are 5 and -3, but -3 isn't a valid solution since it would cause some of the triangle's side lengths to be negative.



Test Practice

Problems 9–10: Solve each equation.

9. $x^2-7x=0$

$x=0$ and $x=7$

10. $(x+2)(x+4)=3$

$x=-5$ and $x=-1$

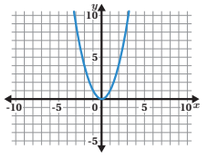
Spiral Review

11. Decide whether this equation has one solution, no solution, or infinitely many solutions: $-3(x+2)=-x+6-2x$.

One solution $\quad$ (No solution) $\quad$ Infinitely many solutions

12. Select *all* the true statements about the function $f(x)=x^2$.

- ☐ **A.** The domain has no negative values.
- ☑ **B.** The range has no negative values.
- ☐ **C.** The function has no minimum.
- ☑ **D.** The function has no maximum.



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
Fluency	1	1	MA.912.AR.1.7
	2–5	2	MA.912.AR.3.1
	6, 8	3	MA.912.AR.3.1
	7	2	MA.912.AR.3.4
Test Practice	9–10	2	MA.912.AR.3.1
Spiral Review			
	11	1	MA.8.AR.2.1
	12	2	MA.912.F.1.6

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify equivalent expressions	Lessons 2–3	1	MA.912.AR.1.3 MTR.2.1
2	Identify equivalent forms of a quadratic expression	Lessons 2–3	2	MA.912.AR.1.3 MTR.5.1
3	Determine x-intercepts of quadratic functions in factored form	Lesson 8	2	MA.912.AR.3.6
4	Factor quadratic expressions in the form of $ax^2 + bx + c$	Lessons 4–5	2	MA.912.AR.1.7
5a	Solve quadratic equations	Lesson 9	2	MA.912.AR.3.1
5b	Write quadratic equations with given solutions	Lesson 9	3	MA.912.AR.3.5 MTR.5.1

Assessment Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 7

1. Which expression is equivalent to $(x-5)^2$?

A. x^2+25 B. x^2-25 C. $x^2-5x+10$ D. $x^2-10x+25$

2. Select *all* the expressions that are equivalent to $(2x-4)(x+3)$.

- ☐ A. $2x^2-4x+3x-12$
- ☑ B. $2(x-2)(x+3)$
- ☑ C. $2x(x+3)-4(x+3)$
- ☐ D. $2(x^2+x-12)$
- ☑ E. $2x^2+2x-12$

3. Determine the x-intercepts of each function.

a $f(x)=(x-3)(x+5)$

x-intercept	Ordered Pair
x-intercept #1	$(3, 0)$
x-intercept #2	$(-5, 0)$

b $h(x)=(2x-1)(5x-2)$

x-intercept	Ordered Pair
x-intercept #1	$\left(\frac{1}{2}, 0\right)$
x-intercept #2	$\left(\frac{2}{5}, 0\right)$

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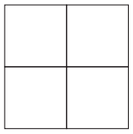
Sub-Unit Quiz (continued)

Unit 7

4. Rewrite each expression in factored form. Use the diagrams if they help with your thinking.

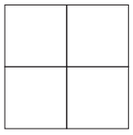
a $x^2-3x-28$

$(x-7)(x+4)$ (or equivalent)



b $3x^2+11x-4$

$(3x-1)(x+4)$ (or equivalent)



5. a Determine the solutions to $x^2-10x=24$.

$x=12$ and $x=-2$

b Write a quadratic equation with solutions $x=-2$ and $x=6$. Explain your thinking.

Responses and explanations vary. $(x+2)(x-6)=0$. When I use the zero-product property, I know that $x+2=0$ or $x-6=0$. When I solve these equations, I get $x=-2$ and $x=6$.

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
• Convert quadratic expressions between factored and standard form by multiplying and factoring. **(Lessons 1–5)**	1	Support	• **Teacher Move:** Consider revisiting Lesson 3 (*Standard Feature*)
		Stretch	• **You're invited to explore more:** • Lesson 3 (*Standard Feature*)
	2	Support	• **Teacher Move:** Consider revisiting Lesson 2 (*Two Factor Multiplication*)
	4	Support	• **Teacher Move:** Consider revisiting Problems 6 - 9 in Lesson 4 (*X-Factor*) to discuss how to use an area model to factor.
• Use factoring and the zero-product property to solve equations and determine the zeros of quadratic functions. **(Lessons 6–9)**	3, 5	Support	• **Teacher Move:** Consider revisiting Aba's Strategy in Lesson 8 (*Shooting Stars*) to discuss how to use the zero-product property to determine zeros and solve quadratic equations.
		Stretch	• **You're invited to explore more:** • Lesson 8 (*Shooting Stars*) • Lesson 9 (*Make It Zero*)

Solving Equations and Completing the Square

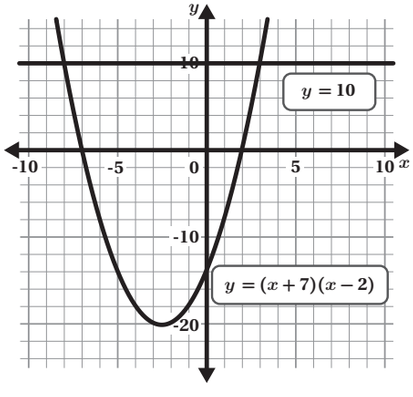
Sub-Unit 2 Goals

- **Goal:** Solve quadratic equations with no solutions, one solution, and two solutions using reasoning, graphs, and completing the square.
- **Goal:** Complete the square in order to reveal the vertex of a quadratic function.
- **Language Goal:** Compare and contrast different strategies for solving quadratic equations. **(Speaking and Listening)**

Students solve quadratic equations using reasoning, graphs, and completing the square. They analyze the structure of equations in order to determine if a quadratic equation has zero, one, or two solutions. They convert quadratic functions between different forms to identify key features, including completing the square in order to determine the vertex.



Math That Matters Most

Concept or Skill	For example . . .
Determine whether a quadratic equation has zero, one, or two solutions	Which of the following equations has no solution? Explain your thinking. **A.** $(x-3)^2 = -1$ **B.** $(x+2)^2 = 0$ **C.** $(x+1)^2 = 4$ *Equation A has no solution because no number squared will result in a negative value.*
Solve quadratic equations by graphing	Use the graph to determine the solutions to $(x+7)(x-2) = 10$.  *x = -8 and x = 3*

Concept or Skill	For example . . .
Solve quadratic equations by taking square roots	Solve the equation: $2(x+4)^2 = 12$. $2(x+4)^2 = 12$ $(x+4)^2 = 6$ $x+4 = \pm\sqrt{6}$ $x = -4 \pm \sqrt{6}$
Solve quadratic equations by completing the square	Solve the equation: $x^2 + 10x = 2$. $x^2 + 10x + 25 = 2 + 25$ $(x+5)^2 = 27$ $x+5 = \pm\sqrt{27}$ $x = -5 \pm \sqrt{27}$
Complete the square to rewrite a function in vertex form	Rewrite $f(x) = x^2 - 6x + 17$ in vertex form. $f(x) = (x^2 - 6x + 9) - 9 + 17$ $f(x) = (x-3)^2 - 9 + 17$ $f(x) = (x-3)^2 + 8$

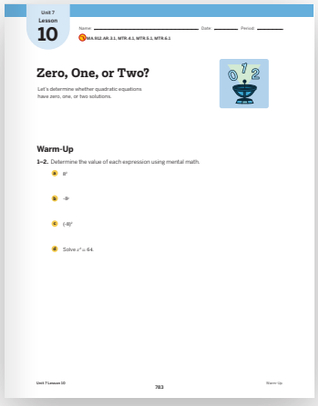
Unit 7

Lesson 10

Zero, One, or Two?

Solving Equations by Reasoning

Let's determine whether quadratic equations have zero, one, or two solutions.


Unit 7
Lesson
10
Name _____ Date _____ Period _____
MA.912.AR.3.1, MTR.4.1, MTR.5.1, MTR.6.1
Zero, One, or Two?
Let's determine whether quadratic equations have zero, one, or two solutions.
0 1 2
Warm-Up
1–2. Determine the value of each expression using mental math.
a. 8^2
b. -9^2
c. $(-8)^2$
d. Solve $x^2 = 64$.
783

Learning Outcomes

Today's Goals

1. **Goal:** Solve quadratic equations by reasoning.
2. **Language Goal:** Analyze the structure of a quadratic equation to determine if it has zero, one, or two solutions. **(Writing and Listening)**

Students determine whether a quadratic equation has zero, one, or two solutions by analyzing its structure, and then solve equations using reasoning. They analyze equations in a variety of structures and sort them by their number of solutions. After determining the solutions, students receive feedback by substituting their solutions back into the original equation. **(MTR.6.1)**

Coherence

Prior Learning

In Unit 1, students solved linear equations and inequalities using reasoning. In Lesson 9, students solved quadratic equations by factoring and using the zero-product property.

Future Learning

Later in the unit, students will solve quadratic equations by graphing, using square roots, completing the square, and using the quadratic formula. In Algebra 2, students will learn about complex solutions to quadratic equations.

Note: In this lesson, the number of solutions is referring to the number of real solutions.

Connections

- Students relate the number of solutions of a quadratic equation to the type of solutions that result from squaring a number. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

Building Toward: **MA.912.AR.3.1**
Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.3	**MA.912.AR.3.2**

Lesson at a Glance

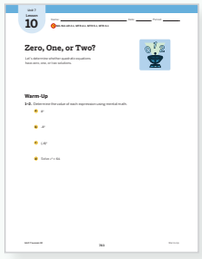
~ 45 min

MA.912.AR.3.1

Warm-Up

Whole Class | 5 min

Students discuss how they make sense of squaring positive and negative numbers, and use those ideas to solve a simple quadratic equation. **Routine:** Number Talk **(MTR.6.1)**



Activity 1

Pairs | 15 min

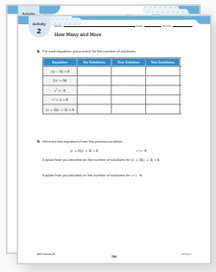
Students determine whether a quadratic equation has zero, one, or two solutions by analyzing its structure, and then solve using reasoning. MLR2: Collect and Display; MLR3: Critique, Correct, Clarify **(MTR.5.1)**



Activity 2

Pairs | 15 min

Students continue to solve a variety of quadratic equations and construct arguments about how the structure of the equation can help in determining the number of solutions. **(MTR.4.1, MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize strategies for determining how many solutions a quadratic equation has and the value of the solution(s). MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the number of solutions and the corresponding value(s) to a given quadratic equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Whole Class | 5 min

Warm-Up

Purpose: Students discuss how they make sense of squaring positive and negative numbers, and use those ideas to solve a simple quadratic equation. **(MTR.6.1)**

1 Launch

Use the Number Talk routine to surface prior knowledge about squaring positive and negative numbers. See the Routine Facilitation Guide for more information.

Invite students to evaluate the first expression independently. Then record a variety of strategies, along with the names of the students who shared them.

ELL **English Language Learners** Invite students to record their strategy publicly and have other students describe it verbally. **(Writing and Speaking)**

Consider asking, "Can someone explain ___'s strategy in their own words?"

Note: This structure is repeated for each expression in the **Number Talk**.

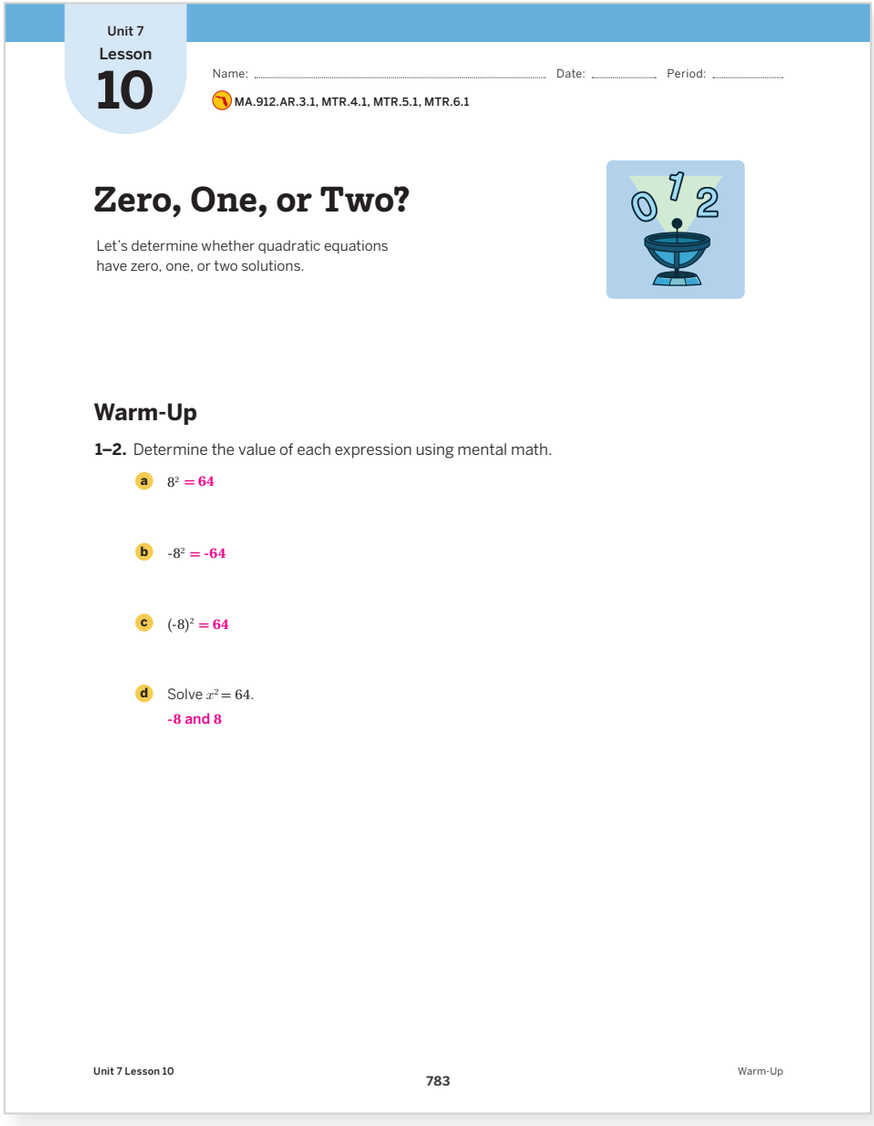
2 Connect

Invite students to compare and contrast the different expressions. If it doesn't come up naturally, consider asking:

- "Which two are equivalent? Why?"
- "Why are the expressions -8^2 and $(-8)^2$ not equivalent?" *Responses vary.* -8^2 only squares the 8, not the negative, and $(-8)^2$ squares both.

Display the equation and continue the **Number Talk routine** by inviting students to think independently and then surfacing some strategies.

To surface the Key Takeaway, consider asking "How many solutions does the equation $x^2 = 64$ have? Why?"



Unit 7
Lesson
10

Name: ..... Date: ..... Period: .....

MA.912.AR.3.1, MTR.4.1, MTR.5.1, MTR.6.1

Zero, One, or Two?

Let's determine whether quadratic equations have zero, one, or two solutions.

Warm-Up

1–2. Determine the value of each expression using mental math.

a. $8^2 = 64$

b. $-8^2 = -64$

c. $(-8)^2 = 64$

d. Solve $x^2 = 64$.
-8 and 8

Unit 7 Lesson 10 783 Warm-Up

Key Takeaway: The square root of a positive number results in two values, one positive and one negative.

Pairs | 15 min

Activity 1 How Many?

Purpose: Students determine whether a quadratic equation has zero, one, or two solutions by analyzing its structure, and then solve using reasoning. **(MTR.5.1)**

1 Launch

Display Activity 1 and consider asking, "How are the equations alike? How are they different?" **(MTR.5.1)**

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the quadratic equations. **(MTR.4.1)**

Note: It's okay — even desirable — to lack consensus at this stage. The purpose is for students to start making sense of the different structures.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to identify an equation where 4 is a solution. Consider asking: • "Why is 4 a solution?" • "Is there any other value that could be a solution to this equation?"
Sort $(x-3)^2=1$ and $(x-3)(x-3)=1$ as having a different number of solutions.	**Support** Consider asking "How else could you write $(x-3)^2=1$? What does that tell us about the number of potential solutions?"
Use strategies such as: • Substituting values. • Using the idea that $-1 \cdot -1 = 1$. • Use structure (e.g., No quantity squared will equal a negative number).	**Strengthen** Invite them to share during the Connect. Consider asking: • "Which equations were easier to sort? Which were more difficult? Why?" • "Why do $(x-3)^2=1$ and $(x-3)^2=0$ have different numbers of solutions?"

Gather a variety of hints, especially ones that allow Diya to think deeper without giving away the answer.

MLR **MLR3: Critique, Correct, Clarify** Pause briefly and use this routine to help students describe strengths, errors, and ambiguities in Diya's work. Share the gathered hints. Then consider asking, "What should we keep in mind when solving quadratic equations that was different when solving linear equations?" **(MTR.4.1, MTR.5.1)**

To support students getting started, invite other pairs to share their strategies, such as:
- Guessing and checking by substituting values into the equation.
- Reasoning like "x is bigger than 4 because . . ."
- Adding 16 to both sides of the equation before using reasoning.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

How Many?

3. For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$(x-3)^2=1$			✓
$(x-3)^2=0$		✓	
$(x-3)^2=-1$	✓		
$(x-3)=1$		✓	
$(x-3)(x-3)=1$			✓

4. Diya says that $x=4$ is the only solution to $(x-3)^2=1$.

a. **Discuss:** How do you know that $x=4$ is a solution to $(x-3)^2=1$?

Responses vary.
- When I substitute 4 for x, I get $(4-3)^2$, which is equal to 1.
- $(x-3)^2$ is the same as $(x-3)(x-3)$. Plugging in 4 for x, I get $(4-3)(4-3)=1$, which is true.

b. Write a hint to help Diya determine *another* solution.

Responses vary.
- There are two numbers that you could square to get 1. What are they?
- Since $x^2=(x)(x)$, it's important to remember that you can multiply two positives or two negatives to get a positive. So you need to check $(x-3)=1$ and $(x-3)=-1$.

5. Here is a new equation: $x^2-16=9$.

How many solutions does this equation have? Circle one.

No solutions One solution (Two solutions)

Record any solution(s) here:

$x=$ 5

$x=$ -5

Unit 7 Lesson 10 784 Activity 1

Pairs | 15 min

Activity 1 How Many? (continued)

Purpose: Students determine whether a quadratic equation has zero, one, or two solutions by analyzing its structure, and then solve using reasoning. **(MTR.5.1)**

2 Monitor

Look for opportunities to recognize the brilliance from students who have not been heard from recently.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "When would a quadratic equation have no solutions?"
Replace 9 with any negative number, convinced it will result in no solutions.	**Support** Invite students to input their answer and use what they learn from the feedback or encourage them to rewrite their equation in the structure $x^2 = ___$.
Input values that give decimal solutions, convinced that an equation whose solutions are not whole numbers have no solutions.	**Strengthen** Consider asking students questions to remind them that equations can have decimal solutions (e.g., How many solutions does $x^2 = 16$ have? $x^2 = 25$? What about $x^2 = 20$?).

Invite students to continue to the next problem after they've analyzed their classmates' equations. Consider asking pairs, "What unique ways did you see your classmates make an equation that has no solution?"

Listen for student ideas to highlight during the Connect.

3 Connect

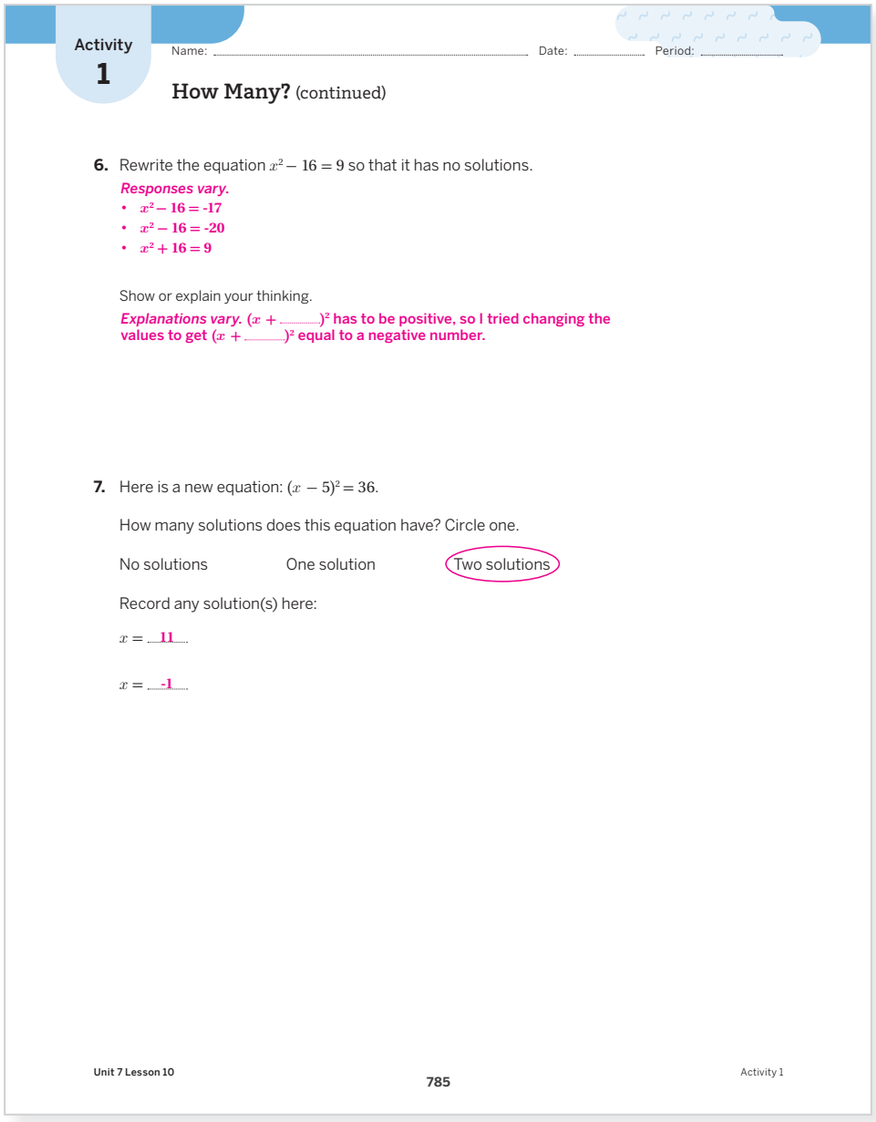
Discuss student strategies for determining the number and values of solutions for this quadratic equation Consider asking:

- "How do you know this equation has two solutions?"
- "How did you determine the solutions to this equation?"

MLR MLR2: Collect and Display Use this routine to collect students' ideas about equations that have no solutions, one solution, and two solutions as they share during the Connect. Display for students to reference and add to for the rest of the unit.

To surface the Key Takeaway, consider asking:

- "What do you notice about the equations that we have seen that have no solutions? Two solutions?" **(MTR.5.1)**
- "How can we confirm whether a value is a solution to a quadratic equation?"



Activity 1

Name: _____ Date: _____ Period: _____

How Many? (continued)

6. Rewrite the equation $x^2 - 16 = 9$ so that it has no solutions.

Responses vary.

- $x^2 - 16 = -17$
- $x^2 - 16 = -20$
- $x^2 + 16 = 9$

Show or explain your thinking.

Explanations vary. $(x + ___)^2$ has to be positive, so I tried changing the values to get $(x + ___)^2$ equal to a negative number.

7. Here is a new equation: $(x - 5)^2 = 36$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ 11

$x =$ -1

Unit 7 Lesson 10 785 Activity 1

Key Takeaway: Quadratic equations can have zero, one, or two solutions depending on the structure of the equation. A solution can be verified by substituting and determining if the equation is true.

Pairs | 15 min

Activity 2 How Many and More

Purpose: Students continue to solve a variety of quadratic equations and construct arguments about how the structure of the equation can help in determining the number of solutions. **(MTR.4.1, MTR.5.1)**

Short on time? Consider inviting students to complete fewer challenges on Problem 10.

1 Launch

Display Problem 8 and consider asking,

- "Are there any cards that feel easier to sort? Why?"
- "What strategies can you use to determine the number of solutions?"

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the quadratic equations. **(MTR.4.1)**

2 Monitor

Listen for student strategies to add the class record from Activity 1 especially those that make sense of structure, such as:

- If the squared expression is equal to a negative number there is no solution.
- If the squared expression is equal to 0 there is only one solution.
- If the squared expression is equal to a positive number there are two solutions.
- Rewriting the equation to isolate the squared expression.
- If the equation is in factored form and set equal to 0, analyze the factors to determine how many solutions it has. **(MTR.4.1, MTR.5.1)**

Encourage students to continue to the next problem after they've discussed with a partner. **(MTR.4.1)**

Invite students to read others' responses and decide if others' strategies were similar to or different from their own.

Activity 2 continued >

Activity 2

Name: Date: Period:

How Many and More

8. For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$x(x-6)=0$			✓
$2x^2=50$			✓
$x^2=-9$	✓		
$x^2+4=0$	✓		
$(x+2)(x+2)=0$		✓	

9. Here are two equations from the previous problem.

$(x+2)(x+2)=0$ $x^2=-9$

Explain how you decided on the number of solutions for $(x+2)(x+2)=0$.

Responses vary. To be true, $(x+2)$ must be 0, which means there is only one value for x.

Explain how you decided on the number of solutions for $x^2=-9$.

Responses vary. There are no values for x that would lead to a negative value when squared. Therefore, there are no solutions.

Unit 7 Lesson 10 786 Activity 2

Pairs | 15 min

Activity 2 How Many and More (continued)

Purpose: Students continue to solve a variety of quadratic equations and construct arguments about how the structure of the equation can help in determining the number of solutions. **(MTR.4.1, MTR.5.1)**

Short on time? Consider inviting students to complete fewer challenges in Problem 10.

2 Monitor

Invite students to solve as many challenges as they have time for.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

Look for students who use the structure of the equations or describe general methods for the problems as they progress through the challenges. **(MTR.5.1)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to guess and use the feedback to think of a strategy.
Guess-and-check values and allow the scanner to confirm.	**Support** Celebrate their persistence. Invite these students to reflect on the feedback to strategically choose a next guess.
Use the structure of the equations consistently to identify the number of solutions.	**Strengthen** Consider renaming the strategies in the class record with the names of students in the class who used them. Invite these students to share during the Connect.
Would like to explore identifying solutions to quadratic equations further.	**Stretch** Invite the students to write a rule to determine the number of solutions to any quadratic equation.

3 Connect

Discuss strategies students used to complete the challenges, consider asking:

- "How many solutions can a quadratic equation have?"
- "How can you tell if a quadratic equation has two solutions? One solution? No solutions?"
- "What makes sense to you about the solving strategies we have heard? What does not make sense?"
- "What connections do you see between your classmates' strategies?"



Activity 2

Name: _____ Date: _____ Period: _____

How Many and More (continued)

10. Solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$30 = x^2 - 6$	No solutions	One solution	(Two solutions)	$x =$ -6 $x =$ 6
b	$7x^2 + 1 = 1$	No solutions	(One solution)	Two solutions	$x =$ 0 $x =$
c	$(x - 4)^2 = -12$	(No solutions)	One solution	Two solutions	$x =$ $x =$
d	$x(x + 2) = 15$	No solutions	One solution	(Two solutions)	$x =$ -5 $x =$ 3
e	$(x - 4)(x - 4) = 16$	No solutions	One solution	(Two solutions)	$x =$ 8 $x =$ 0
f	$3x^2 - 3 = -3$	No solutions	(One solution)	Two solutions	$x =$ 0 $x =$

Unit 7 Lesson 10 787 Activity 2

Whole Class | 5 min

Synthesis

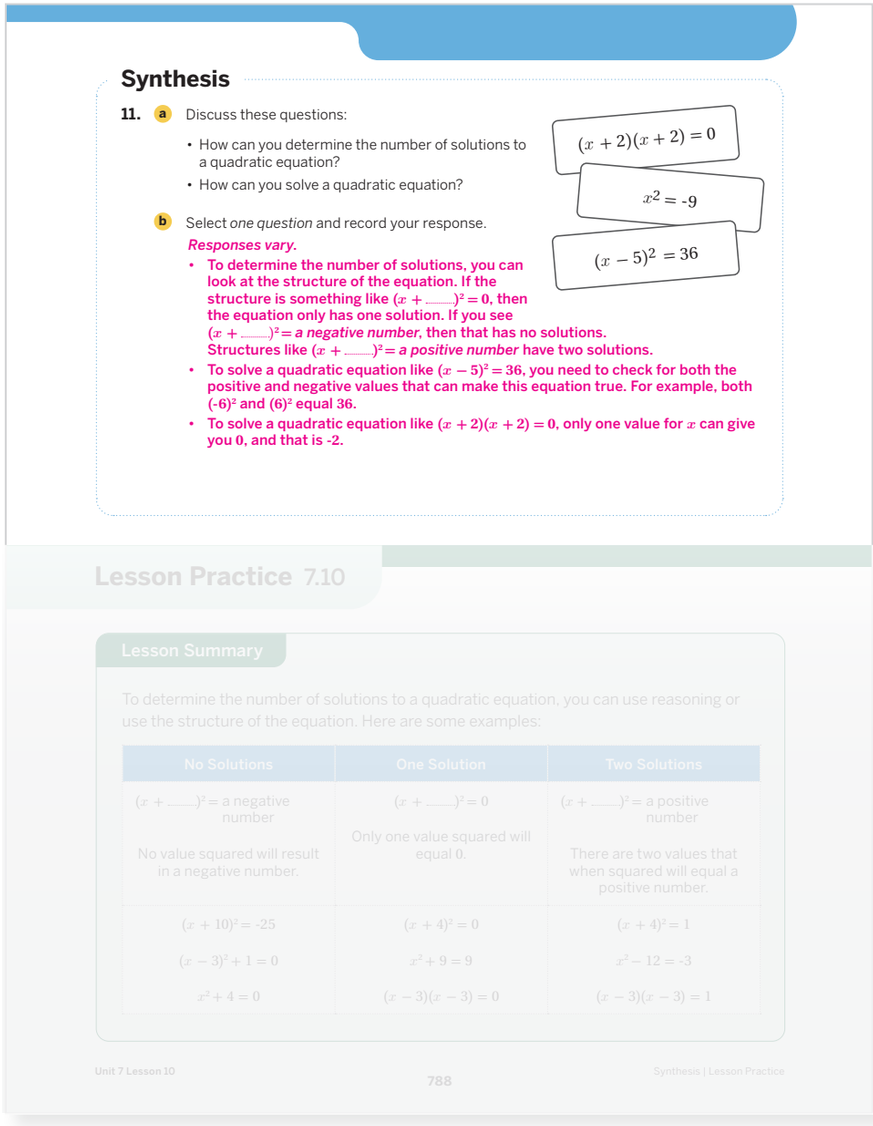
Purpose: Students synthesize strategies for determining how many solutions a quadratic equation has and the value of the solution(s).

Invite students to respond independently, after discussing each question with their partner. **(MTR.4.1, MTR.5.1)**

Listen for and share a variety of ideas, such as:

- Explaining how the structure can be used to identify the number of solutions in generic terms (e.g., if the squared expression is equal to a negative number there is no solution).
- Rewriting the equation to isolate the squared expression to make sense of the solutions.
- Using the zero-product property if the equation is factored and set equal to zero.

MLR **MLR2: Collect and Display** Use this routine to add to the class record organizing student ideas about equations that have no solutions, one solution, and two solutions.



Synthesis

11. a. Discuss these questions:

- How can you determine the number of solutions to a quadratic equation?
- How can you solve a quadratic equation?

$(x+2)(x+2)=0$

$x^2=-9$

$(x-5)^2=36$

b. Select *one question* and record your response.

Responses vary.

- **To determine the number of solutions, you can look at the structure of the equation. If the structure is something like $(x+_____)^2=0$, then the equation only has one solution. If you see $(x+_____)^2=$ *a negative number*, then that has no solutions. Structures like $(x+_____)^2=$ *a positive number* have two solutions.**
- **To solve a quadratic equation like $(x-5)^2=36$, you need to check for both the positive and negative values that can make this equation true. For example, both $(-6)^2$ and $(6)^2$ equal 36.**
- **To solve a quadratic equation like $(x+2)(x+2)=0$, only one value for x can give you 0, and that is -2.**

Lesson Practice 7.10

Lesson Summary

To determine the number of solutions to a quadratic equation, you can use reasoning or use the structure of the equation. Here are some examples:

No Solutions	One Solution	Two Solutions
$(x+_____)^2=$ a negative number No value squared will result in a negative number.	$(x+_____)^2=0$ Only one value squared will equal 0.	$(x+_____)^2=$ a positive number There are two values that when squared will equal a positive number.
$(x+10)^2=-25$	$(x+4)^2=0$	$(x+4)^2=1$
$(x-3)^2+1=0$	$x^2+9=9$	$x^2-12=-3$
$x^2+4=0$	$(x-3)(x-3)=0$	$(x-3)(x-3)=1$

Unit 7 Lesson 10 788 Synthesis | Lesson Practice

Lesson Takeaway: Quadratic equations have zero, one, or two solutions depending on the structure of the equation. Some structural examples include:

- When the squared expression can be rewritten to equal a negative number, there is no solution.
- When the squared expression can be rewritten to equal zero, there is one solution.
- When the squared expression is equal to a positive number, both the positive and negative square roots of that value need to be considered.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the number of solutions and the corresponding value(s) to a given quadratic equation.

Today's Goals

1. **Goal:** Solve quadratic equations by reasoning.
2. **Language Goal:** Analyze the structure of a quadratic equation to determine if it has zero, one, or two solutions. **(Writing and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.10

Here is a new equation: $(x + 2)(x + 2) = 9$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ -5

$x =$ 1

If you circled *No solutions*, complete the statement:
The equation has no solutions because . . .

Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support identifying the number of solutions of a quadratic equation:

» **Respond to Student Thinking:**

- Spend extra time on the class display of strategies during the next lesson's **Warm-Up**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

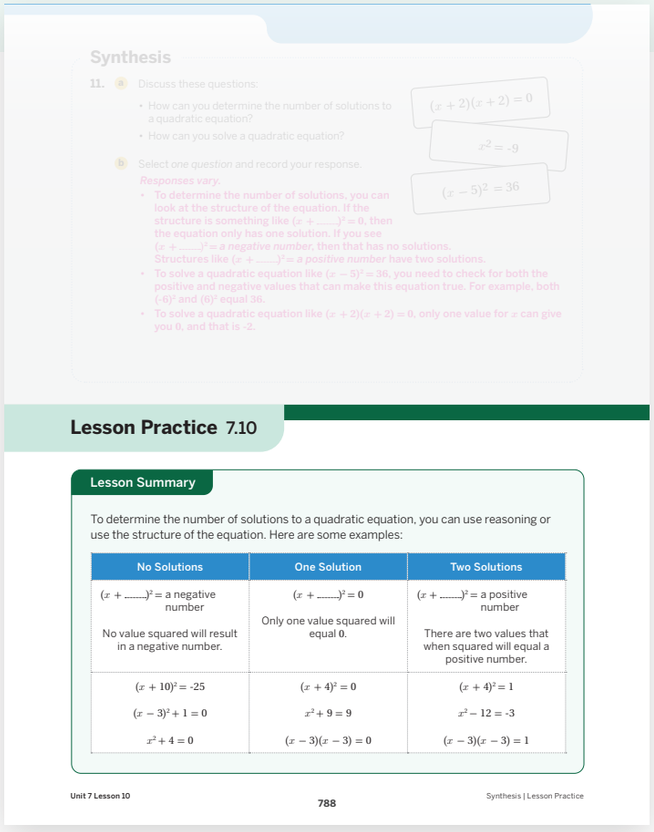
Take a moment to reflect on how students looked for and made use of structure in today's lesson. **(MTR.7.1)**

- How did the **Number Talk** routine support students in noticing and describing differences in structures?
- How did students connect the structure of a quadratic equation to the number of solutions?
- How can you continue to support students in looking for and making use of structure as they continue to explore quadratic equations throughout the unit?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. a. Discuss these questions:

- How can you determine the number of solutions to a quadratic equation?
- How can you solve a quadratic equation?

b. Select one *question* and record your response.

Responses vary.

- To determine the number of solutions, you can look at the structure of the equation. If the structure is something like $(x + _____)^2 = 0$, then the equation only has one solution. If you see $(x + _____)^2 =$ *a negative number*, then that has no solutions. Structures like $(x + _____)^2 =$ *a positive number* have two solutions.
- To solve a quadratic equation like $(x - 5)^2 = 36$, you need to check for both the positive and negative values that can make this equation true. For example, both $(-6)^2$ and $(6)^2$ equal 36.
- To solve a quadratic equation like $(x + 2)(x + 2) = 0$, only one value for x can give you 0, and that is -2.

$(x + 2)(x + 2) = 0$

$x^2 = -9$

$(x - 5)^2 = 36$

Lesson Practice 7.10

Lesson Summary

To determine the number of solutions to a quadratic equation, you can use reasoning or use the structure of the equation. Here are some examples:

No Solutions	One Solution	Two Solutions
$(x + _____)^2 =$ a negative number No value squared will result in a negative number.	$(x + _____)^2 = 0$ Only one value squared will equal 0.	$(x + _____)^2 =$ a positive number There are two values that when squared will equal a positive number.
$(x + 10)^2 = -25$ $(x - 3)^2 + 1 = 0$ $x^2 + 4 = 0$	$(x + 4)^2 = 0$ $x^2 + 9 = 9$ $(x - 3)(x - 3) = 0$	$(x + 4)^2 = 1$ $x^2 - 12 = -3$ $(x - 3)(x - 3) = 1$

Lesson Practice 7.10

Name: _____ Date: _____ Period: _____

1. Write a quadratic equation that has . . . *Responses vary.*

Two solutions: $x^2 = 16$

One solution: $x^2 + 5 = 5$

No solutions: $x^2 + 10 = 9$

2. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x(x + 3) = 0$	Two solutions
$(x + 3)(x + 1) = 0$	Two solutions
$(x + 1)(x + 1) = 0$	One solution
$x^2 - 10x = -9$	Two solutions
$(x - 5)(x - 5) = -14$	No solutions
$x^2 - 3 = -3$	One solution
$x^2 + 6 = 2$	No solutions

Problems 3–4: Determine the solution for each equation.

3. $149 + (x - 2)^2 = 149$ — $x = 2$

4. $x^2 + 3x = x - 1$ — $x = -1$

Problems 5–6: Determine the two solutions for each equation.

5. $100 + (x - 2)^2 = 149$ — $x = 9, x = -5$

6. $x^2 + 4x = x + 18$ — $x = -6, x = 3$

Test Practice

7. Which value for x is a solution to the equation $x^2 + 10 = 9$?

A. $x = 1$ B. $x = -1$ C. $x = \sqrt{19}$ D. There is no solution to the equation.

Lesson Practice 7.10

Name: _____ Date: _____ Period: _____

Spiral Review

8. Order the expressions by value from *least* to *greatest*: $5^2, \sqrt{90}, 8, 3^3, \sqrt{27}$.

$\sqrt{27}$	8	$\sqrt{90}$	5^2	3^3

Least Greatest

Problems 9–10: Complete each diagram. Then write the corresponding quadratic expression in factored form and standard form.

9.

	-3x	3
6x	$-18x^2$	$18x$
3	$-9x$	9

Factored form: $(6x + 3)(-3x + 3)$

Standard form: $-18x^2 + 9x + 9$

10.

	6x	-8
5x	$30x^2$	$-40x$
2	$12x$	-16

Factored form: $(5x + 2)(6x - 8)$

Standard form: $30x^2 - 28x - 16$

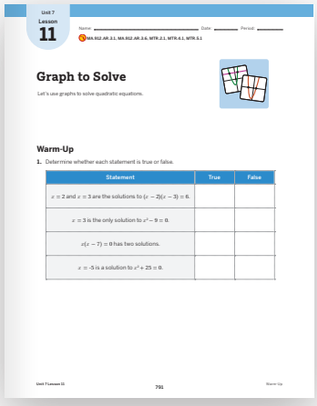
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	3	MA.912.AR.3.1
	2–6	2	MA.912.AR.3.1
Test Practice	7	2	MA.912.AR.3.1
Spiral Review			
Fluency	8	1	MA.8.NSO.1.2
Fluency	9–10	1	MA.912.AR.1.3 MA.912.AR.1.7

Unit 7

Lesson

11



Graph to Solve

Solving Quadratic Equations by Graphing

Let's use graphs to solve quadratic equations.

Learning Outcomes

Today's Goals

1. **Goal:** Solve quadratic equations by graphing.
2. **Language Goal:** Identify the number of solutions to a quadratic equation using its graph. **(Writing, Speaking, and Listening)**

Students use graphs to solve quadratic equations with zero, one, or two solutions. They compare and contrast different graphical solving strategies and describe how to see the solutions in each. **(MTR.2.1)**

Coherence

Prior Learning

In Lessons 8–9, students used factoring and the zero-product property to solve quadratic equations. In Lesson 10, they analyzed the structure of a quadratic equation and used reasoning to determine whether it had zero, one, or two solutions. In Unit 4, students solve systems of equations by graphing.

Future Learning

In Lesson 13, students will solve quadratic equations by completing the square. In Lessons 15–16, they'll derive the quadratic formula and use it as a strategy to solve quadratic equations. In Algebra 2, they'll apply what they learned about solving quadratic equations to solving polynomial equations.

Connections

- In an exploration activity, students relate the x-intercepts of the graph of a quadratic function to the solutions of a quadratic equation. (mathematical)
- Students compare, contrast, and discuss different strategies for solving a quadratic equation using graphs. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.1
Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

MA.912.AR.3.6
Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.AR.2.5	**MA.912.AR.9.2**
MA.912.AR.3.7	

Vocabulary

Review Vocabulary

point of intersection

system of equations

Lesson at a Glance

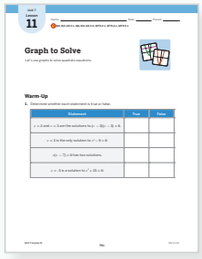
~ 45 min

MA.912.AR.3.1, MA.912.AR.3.6

Warm-Up

Pairs | 8 min

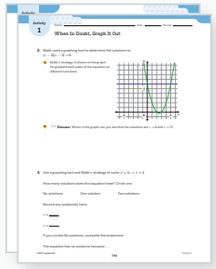
Students use the structure of different quadratic equations to determine the number of solutions.



Activity 1

Pairs | 15 min

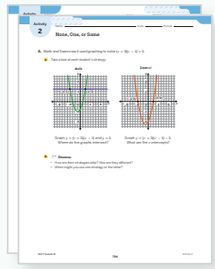
Students explore two strategies for solving quadratic equations with a graphing calculator. **(MTR.2.1)**



Activity 2

Pairs | 12 min

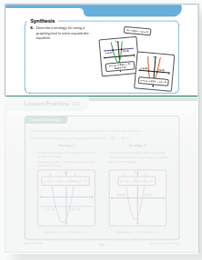
Students compare different graphical solving strategies and use appropriate tools to solve quadratic equations. MLR7: Compare and Connect **(MTR.2.1)**



Synthesis

Whole Class | 5 min

Students synthesize a strategy for solving a quadratic equation using a graphing calculator. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a quadratic equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 8 min

Warm-Up

Purpose: Students reason using the structure of different quadratic equations to determine the number of solutions. **(MTR.2.1)**

1 Launch

To surface prior knowledge, consider asking, "What strategies might we use to check if these statements about equations and their solutions are true or false?"

Invite students to think independently before discussing their ideas with a partner to reach an agreement about how to sort the statements about solutions to quadratic expressions.

Students will need access to a graphing tool throughout the lesson.

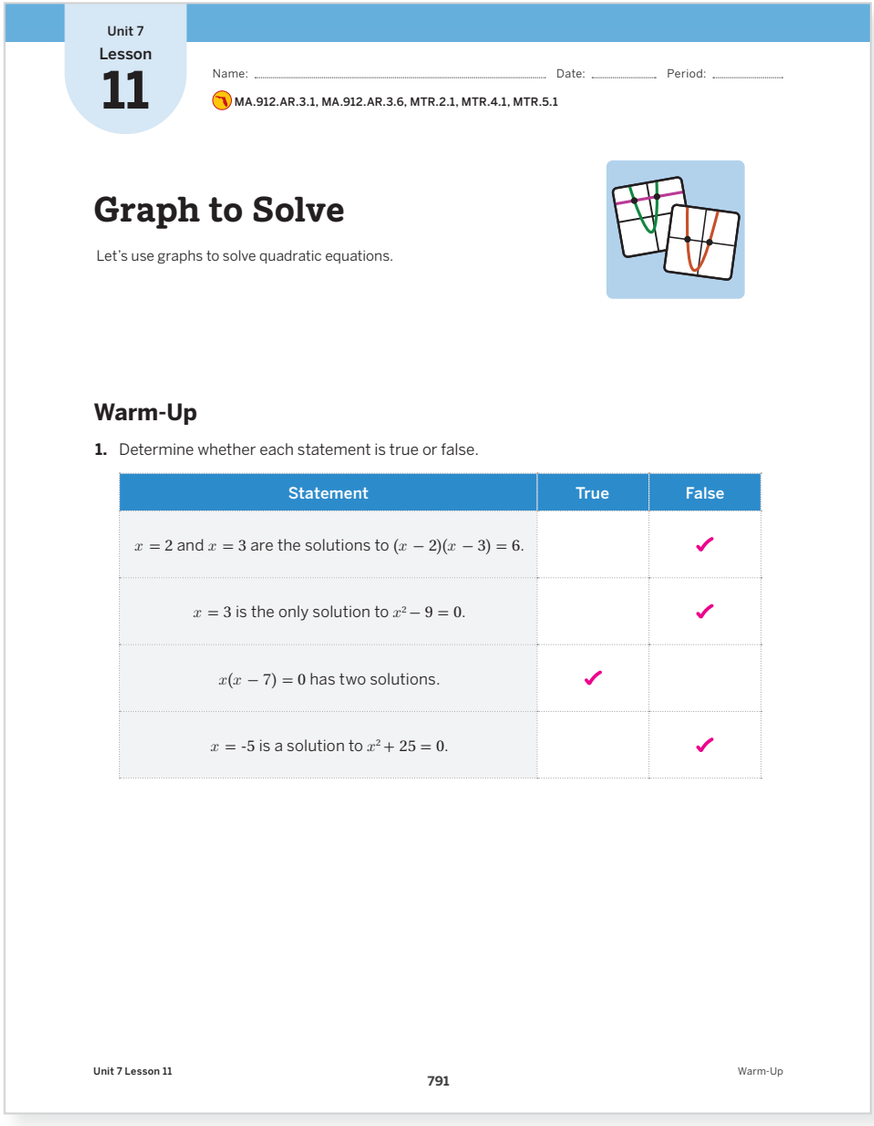
ELL **English Language Learners** Encourage students to discuss how to group the statements in their primary language. Consider discussing the meaning of words like *true*, *false*, and *only*. **(Speaking and Reading)**

A **Accessibility: Executive Function** Invite students to focus on one statement at a time to support students in carrying out multiple steps.

2 Connect

Display the Warm-Up and invite students to share the variety of tools and strategies they used, such as:

- Using a calculator.
- Using the structure of the quadratic equation to help them reason about solutions. **(MTR.5.1)**
- Using the zero-product property.
- Using their knowledge about the signs of squared numbers.



Unit 7
Lesson
11

Name: _____ Date: _____ Period: _____

MA.912.AR.3.1, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1

Graph to Solve

Let's use graphs to solve quadratic equations.

Warm-Up

1. Determine whether each statement is true or false.

Statement	True	False
$x = 2$ and $x = 3$ are the solutions to $(x - 2)(x - 3) = 6$.		✓
$x = 3$ is the only solution to $x^2 - 9 = 0$.		✓
$x(x - 7) = 0$ has two solutions.	✓	
$x = -5$ is a solution to $x^2 + 25 = 0$.		✓

Unit 7 Lesson 11 791 Warm-Up

Pairs | 15 min

Activity 1 When In Doubt, Graph It Out

Purpose: Students explore two strategies for solving quadratic equations with a graphing calculator. **(MTR.2.1)**

1 Launch

Encourage students to look closely at Malik's strategy.

To surface connections, consider asking:

- "How can we use a graphing calculator to see when two equations are equal?"
- "What happens when you substitute 0 in the equation $(x-2)(x-3)=6$? What about 5?"
- "What does each ordered pair tell you about the equation?"
- "What is a solution? An x-intercept? A zero? How are they all related?"
- "How is this similar to solving a *system of equations*?"

A **Accessibility: Visual-Spatial Processing**
Invite students to share with a partner what is happening at each step to support interpreting visual representations.

2 Monitor

Encourage students to use a graphing tool throughout the lesson, whenever they find it helpful.

Look for students who use a variety of strategies, such as solving graphically or symbolically. **(MTR.2.1)**

D **Differentiation: (Support)** Encourage students who guess and check values to revisit Malik's strategy from the previous poblem and consider asking, "How can you use Malik's strategy to help you determine which values to try?"

Note: Students may graph the equation $x^2+2x+1=4$ on one line to produce two vertical lines. The x-intercepts of the graph represent the input values that make the equation true.

Activity 1 continued >

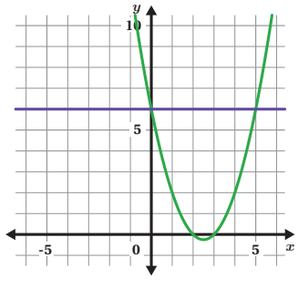
Activity 1

Name: _____ Date: _____ Period: _____

When In Doubt, Graph It Out

2. Malik used a graphing tool to determine the solutions to $(x-2)(x-3)=6$.

a Malik's strategy is shown on the graph. He graphed both sides of the equation as different functions.



b **Discuss:** Where in the graph can you see that the solutions are $x=0$ and $x=5$?

Responses vary.

- $x=0$ and $x=5$ are the input values that give an output of $y=6$ for the parabola $y=(x-2)(x-3)$.
- The x-coordinates of the points $(0,0)$ and $(5,0)$ are where the graphs of $y=(x-2)(x-3)$ and $y=6$ intersect.

3. Use a graphing tool and Malik's strategy to solve $x^2+2x+1=4$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x=$ -3

$x=$ 1

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

Unit 7 Lesson 11 792 Activity 1

Pairs | 15 min

Activity 1 When In Doubt, Graph It Out (continued)

Purpose: Students explore two strategies for solving quadratic equations with a graphing calculator. **(MTR.2.1)**

2 Monitor

Look for and celebrate the different ways that students interpret Saanvi's graph.

ELL **English Language Learners** Invite students to read Saanvi's work aloud and paraphrase what it means. Clarify any unfamiliar words. **(Reading)**

Share a variety of solving tools and strategies to share during the Connect, such as:

- Use a graphing tool to analyze where $(x-3)^2 = -1$.
- Use a graphing tool to analyze where the graph of $y = (x-3)^2 - 1$ intersects with the x-axis.
- Use structure to understand that no number can be squared to get a negative number. **(MTR.5.1)**
- Guess and check with different x-values to see what input makes the equation true. **(MTR.2.1)**

D **Differentiation: (Support)** Some students might select one or two solutions. Consider asking these students:

- "What number times itself is 1? What number times itself is -1?"
- "How does that help you know the number of solutions for this equation?"

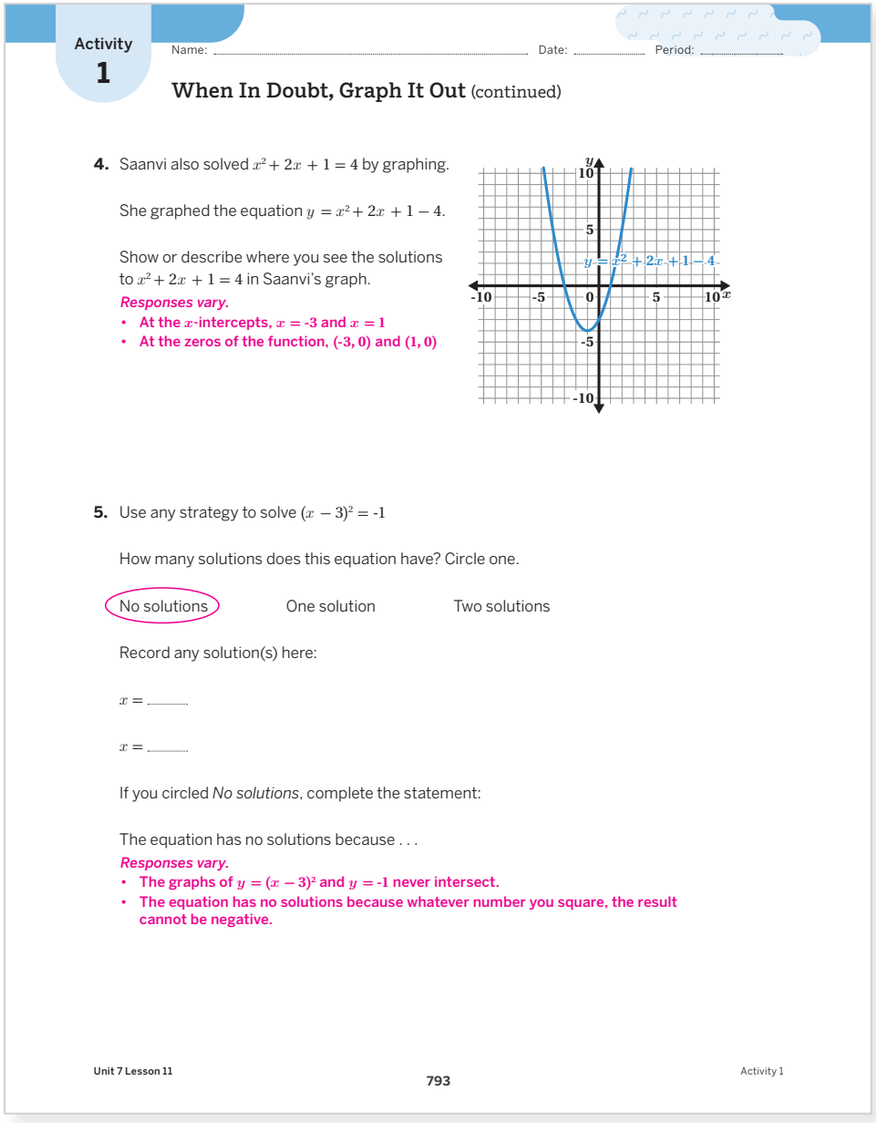
3 Connect

Share the students' strategies, highlighting at least one student who solved it graphically.

To surface the Key Takeaway, consider asking

- "How can we use a graph to identify the solutions to a quadratic equation?"
- "What might the graph look like if there are no solutions? One solution?"

Note: If it does not come up naturally, consider sharing that the intersection points are how solutions are determined and if there is no intersection, there are no solutions.



Activity 1

Name: _____ Date: _____ Period: _____

When In Doubt, Graph It Out (continued)

4. Saanvi also solved $x^2 + 2x + 1 = 4$ by graphing.

She graphed the equation $y = x^2 + 2x + 1 - 4$.

Show or describe where you see the solutions to $x^2 + 2x + 1 = 4$ in Saanvi's graph.

Responses vary.

- At the x-intercepts, $x = -3$ and $x = 1$
- At the zeros of the function, (-3, 0) and (1, 0)

5. Use any strategy to solve $(x-3)^2 = -1$

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ _____

$x =$ _____

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

Responses vary.

- The graphs of $y = (x-3)^2$ and $y = -1$ never intersect.
- The equation has no solutions because whatever number you square, the result cannot be negative.

Unit 7 Lesson 11 793 Activity 1

Key Takeaway: Graphs can be a helpful tool for solving quadratic equations. Two strategies for determining the solutions using graphs are:

- Graph each side of the equation separately and determine the x-coordinates of the *points of intersection*.
- Set the equation equal to 0. Graph the equation and determine the zeros.

Pairs | 12 min

Activity 2 None, One, or Some

Purpose: Students compare different graphical solving strategies and use appropriate tools to solve quadratic equations. **(MTR.2.1)**

Short on time? Consider inviting students to complete fewer Repeated Challenges in Problem 8.

1 Launch

Display Malik's and Saanvi's graphs.

Invite students to think independently before discussing their ideas with a partner.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategies from the previous activity.

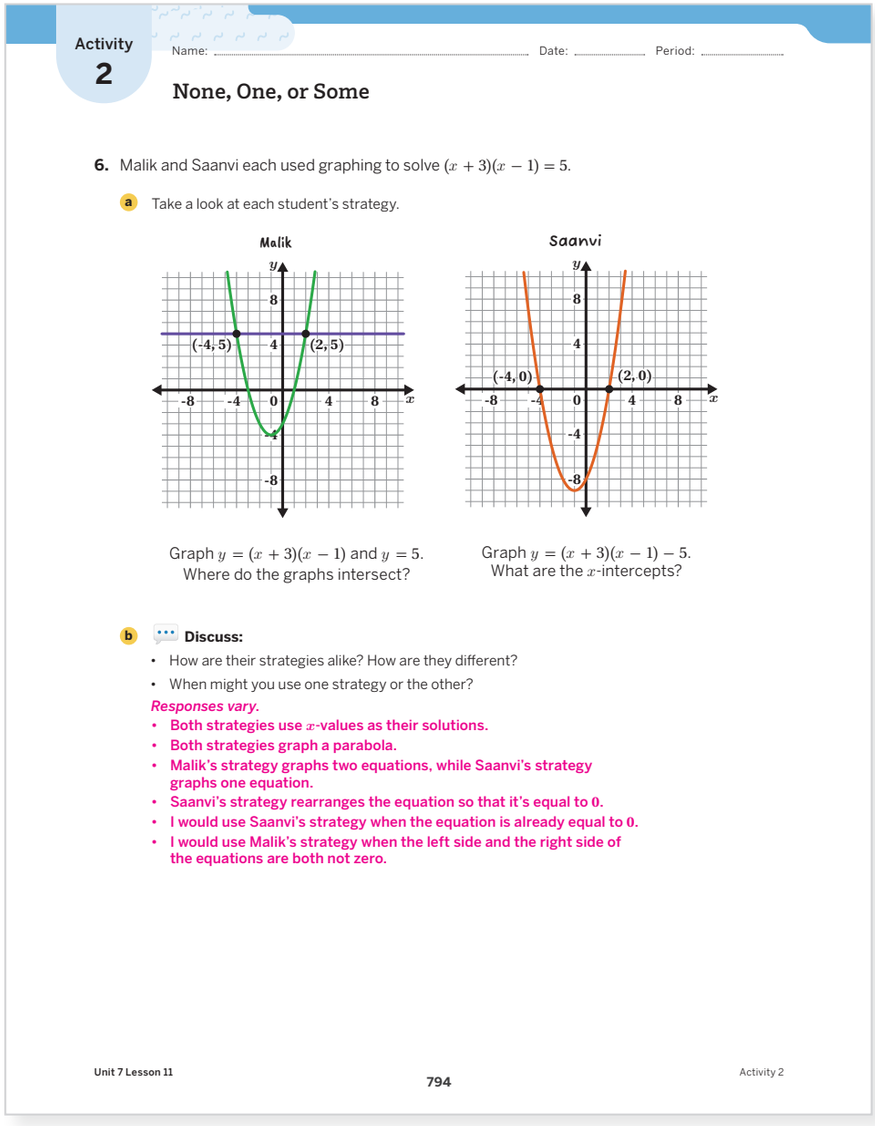
2 Monitor

A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific point on the graph and describe what that point means to support interpreting visual representations.

Pause briefly to discuss the two strategies. Consider asking:

- "What do Malik's solutions to the equation represent?" **(MTR.5.1)**
- "What do Saanvi's solutions to the equation represent?" **(MTR.5.1)**
- "How do Malik and Saanvi's strategies compare with your own?"
- "What do you think is important to remember from each student's work?"

Activity 2 continued >



Activity 2

Name: Date: Period:

None, One, or Some

6. Malik and Saanvi each used graphing to solve $(x + 3)(x - 1) = 5$.

a. Take a look at each student's strategy.

Malik

Graph $y = (x + 3)(x - 1)$ and $y = 5$. Where do the graphs intersect?

Saanvi

Graph $y = (x + 3)(x - 1) - 5$. What are the x-intercepts?

b. **Discuss:**

- How are their strategies alike? How are they different?
- When might you use one strategy or the other?

Responses vary.

- Both strategies use x-values as their solutions.
- Both strategies graph a parabola.
- Malik's strategy graphs two equations, while Saanvi's strategy graphs one equation.
- Saanvi's strategy rearranges the equation so that it's equal to 0.
- I would use Saanvi's strategy when the equation is already equal to 0.
- I would use Malik's strategy when the left side and the right side of the equations are both not zero.

Unit 7 Lesson 11 794 Activity 2

Pairs | 12 min

Activity 2 None, One, or Some (continued)

Purpose: Students compare different graphical solving strategies and use appropriate tools to solve quadratic equations. **(MTR.2.1)**

Short on time? Consider inviting students to complete fewer challenges in Problem 8.

2 Monitor

Invite students to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly.

Students will need a partner for this activity. They will solve and compare their solutions instead of receiving responsive feedback.

A **Accessibility: Conceptual Processing** Invite students to share how each problem is different from the previous one to help apply concepts to new situations.

D **Differentiation**

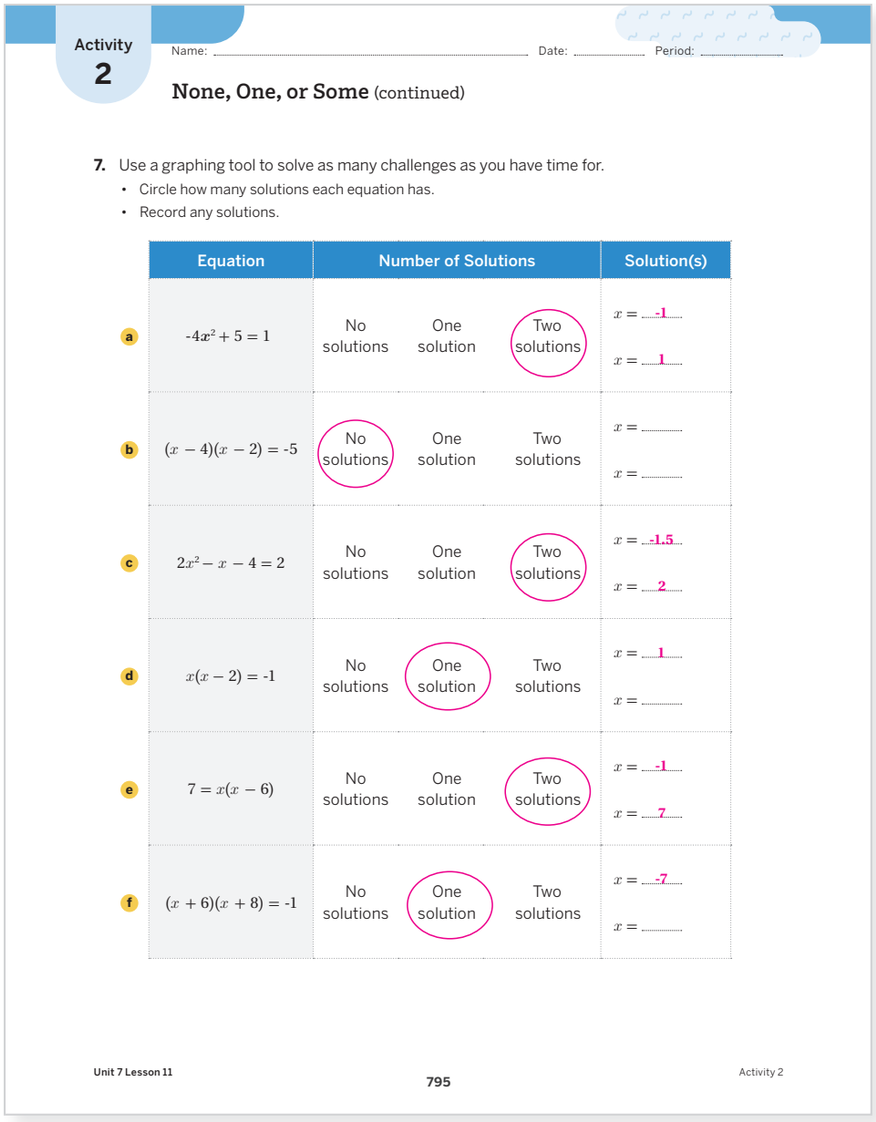
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How might we use Malik's or Saanvi's strategies to answer this using a graph?"
Want to explore using graphs to solve equations further.	**Stretch** Invite students to explore how they can solve any equation using these graphical strategies (e.g., $5\left(\frac{1}{2}\right)^x = 8x - 1$ or $4(x-2)^3 = x^2 = 5x$).

Listen for strategies students use in discussing how to solve the problems, such as:

- Using a graphing tool to plot the quadratic expression and a vertical line for the value and identify the intersection points.
- Algebraically manipulating the equation to be equal to zero and using a graphing tool to identify the x-intercepts.
- Substituting x-values into the equation to see what makes it true. **(MTR.2.1)**

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 2

Name: _____ Date: _____ Period: _____

None, One, or Some (continued)

7. Use a graphing tool to solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$-4x^2 + 5 = 1$	No solutions	One solution	(Two solutions)	$x =$ -1 $x =$ 1
b	$(x-4)(x-2) = -5$	(No solutions)	One solution	Two solutions	$x =$ _____ $x =$ _____
c	$2x^2 - x - 4 = 2$	No solutions	One solution	(Two solutions)	$x =$ -1.5 $x =$ 2
d	$x(x-2) = -1$	No solutions	(One solution)	Two solutions	$x =$ 1 $x =$ _____
e	$7 = x(x-6)$	No solutions	One solution	(Two solutions)	$x =$ -1 $x =$ 7
f	$(x+6)(x+8) = -1$	No solutions	(One solution)	Two solutions	$x =$ -7 $x =$ _____

Unit 7 Lesson 11 795 Activity 2

Whole Class | 5 min

Synthesis

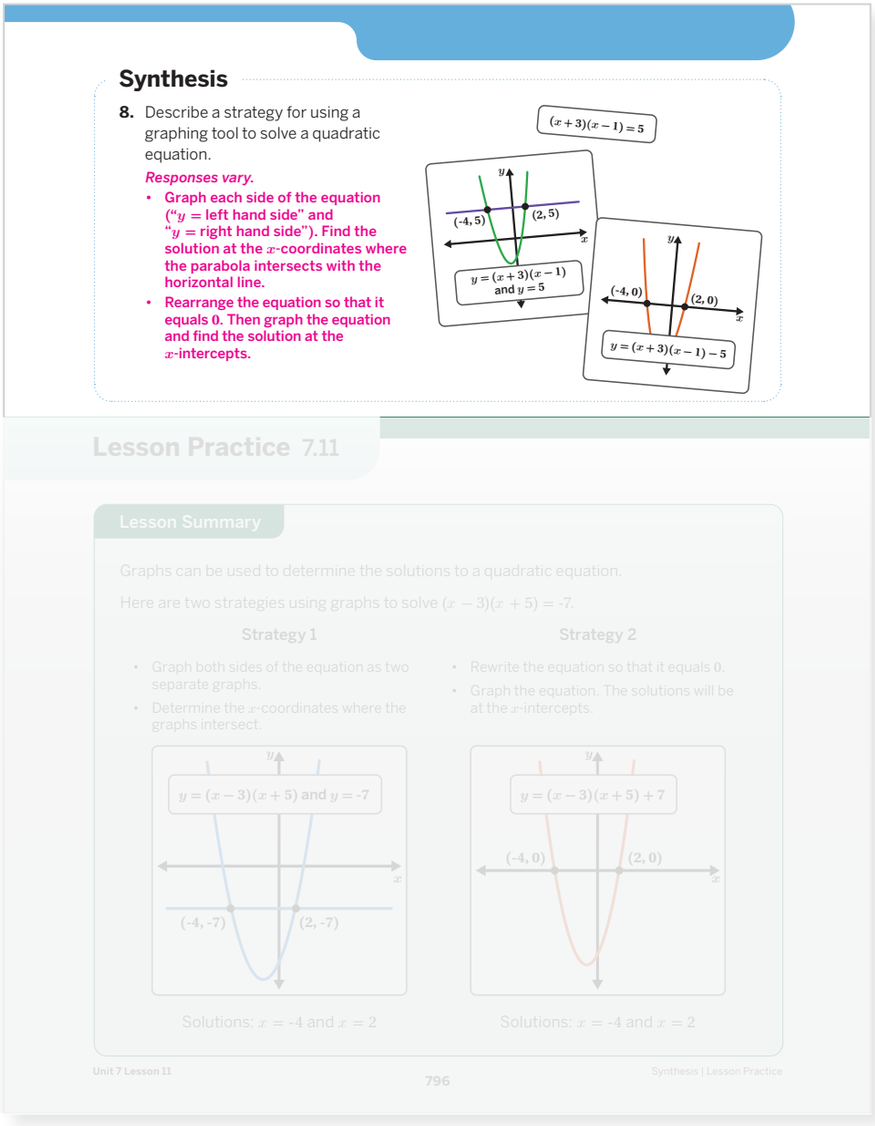
Purpose: Students synthesize a strategy for solving a quadratic equation using a graphing calculator.

Invite students to respond independently, and then share their thinking with a partner.

Share a variety of ideas, such as:
- Analyzing where the parabola and the horizontal line intersect.
- Rearranging the equation so it equals 0, then determining the x-intercepts.
- Substituting x-values into the equation to see what makes it true.

MLR **MLR7: Collect and Display** Use this routine to formalize the strategy for solving quadratic equations graphically using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

8. Describe a strategy for using a graphing tool to solve a quadratic equation.

Responses vary.
- Graph each side of the equation ("y = left hand side" and "y = right hand side"). Find the solution at the x-coordinates where the parabola intersects with the horizontal line.
- Rearrange the equation so that it equals 0. Then graph the equation and find the solution at the x-intercepts.

Lesson Practice 7.11

Lesson Summary

Graphs can be used to determine the solutions to a quadratic equation.

Here are two strategies using graphs to solve $(x-3)(x+5)=-7$.

Strategy 1
- Graph both sides of the equation as two separate graphs.
- Determine the x-coordinates where the graphs intersect.

Solutions: $x=-4$ and $x=2$

Strategy 2
- Rewrite the equation so that it equals 0.
- Graph the equation. The solutions will be at the x-intercepts.

Solutions: $x=-4$ and $x=2$

Unit 7 Lesson 11 796 Synthesis | Lesson Practice

Lesson Takeaway: Two strategies for solving quadratic equations graphically are:
- Graph each side of the equation separately and determine the x-coordinates of the intersection points.
- Set the equation equal to 0. Graph the equation and determine the zeros.

If there are no intersection points or no zeros, then there are no solutions.

Independent | 5 min

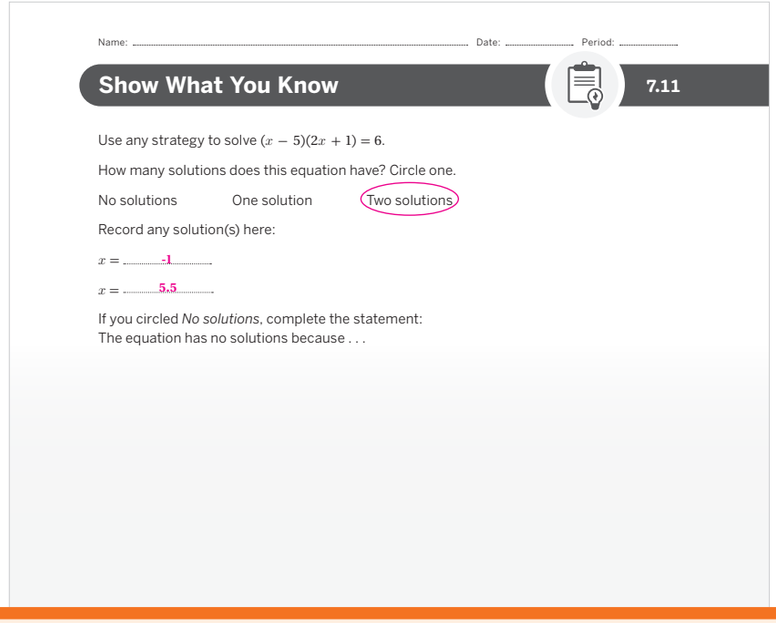
Show What You Know

Purpose: Students demonstrate their understanding by solving a quadratic expression.

Today's Goals

1. **Goal:** Solve quadratic equations by graphing.
2. **Language Goal:** Identify the number of solutions to a quadratic equation using its graph. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.11

Use any strategy to solve $(x - 5)(2x + 1) = 6$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ -1

$x =$ 5.5

If you circled *No solutions*, complete the statement:
The equation has no solutions because . . .

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support solving a quadratic equation by graphing:

» **Respond to Student Thinking:**

- Use the ***Solving Quadratic Equations by Graphing*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to explore how they can solve any equation using these graphical strategies. Some examples that can be used are:
  - $5\left(\frac{1}{2}\right)^{x} = 8x - 1$
  - $4(x - 2)^{3} = x^{3} - 5x$

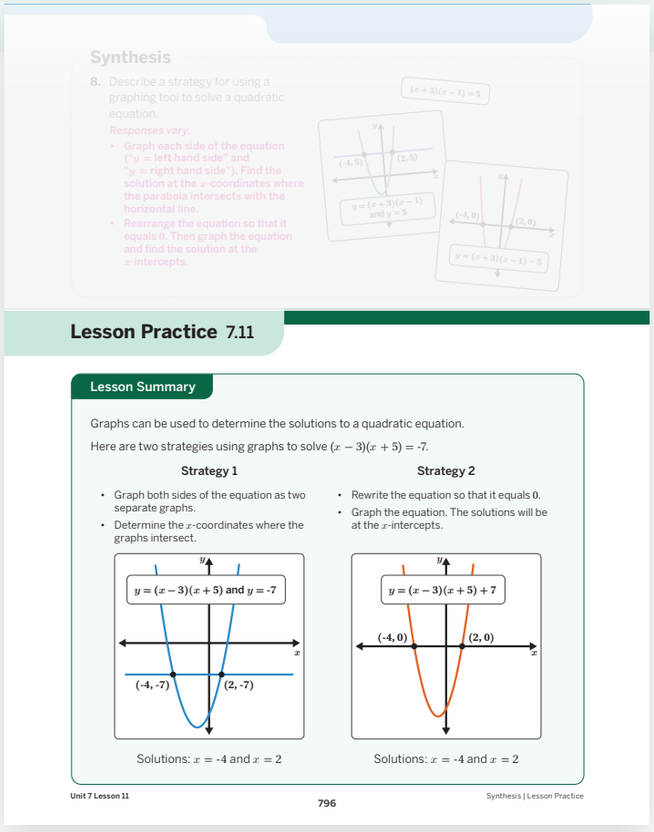
Professional Learning

Reflect on students' use of a graphing tool throughout this lesson. How did a graphing tool support students with communicating their ideas or participating in the lesson? What support might students need to use this tool effectively in the future? What connections were they able to make between the equations and the graphs?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. Describe a strategy for using a graphing tool to solve a quadratic equation.

Responses vary.

- Graph each side of the equation ("$y =$ left hand side" and "$y =$ right hand side"). Find the solution at the x-coordinates where the parabola intersects with the horizontal line.
- Rearrange the equation so that it equals 0. Then graph the equation and find the solution at the x-intercepts.

Lesson Practice 7.11

Lesson Summary

Graphs can be used to determine the solutions to a quadratic equation.

Here are two strategies using graphs to solve $(x - 3)(x + 5) = -7$.

Strategy 1

- Graph both sides of the equation as two separate graphs.
- Determine the x-coordinates where the graphs intersect.

Solutions: $x = -4$ and $x = 2$

Strategy 2

- Rewrite the equation so that it equals 0.
- Graph the equation. The solutions will be at the x-intercepts.

Solutions: $x = -4$ and $x = 2$

Lesson Practice 7.11

Name: ..... Date: ..... Period: .....

Problems 1–4: Circle how many solutions each equation has. Record any solutions.

1.	$-2x^2 + 2 = -6$	No solutions	One solution	**Two solutions**	$x =$ -2 $x =$ 2
2.	$x(x + 3) = 4$	No solutions	One solution	**Two solutions**	$x =$ -4 $x =$ 1
3.	$0 = (x - 2)(x + 4) + 9$	No solutions	**One solution**	Two solutions	$x =$ -1 $x =$ .....
4.	$2x(x + 1) = -1$	**No solutions**	One solution	Two solutions	$x =$ ..... $x =$ .....

Problems 5–6: Fill in the blank so the equation has:

5. One solution

   $3x(x + 2) =$ ...-3...

6. No solutions *Responses vary.*

   $(x - 2)(x +$ ...1...$) = -4$

7. The graphs of $y = -x^2 + 25$, $y = 24$, $y = 21$, and $y = 16$ all intersect at integer values.

   Write the equations of three more lines that follow this property. *Responses vary.*

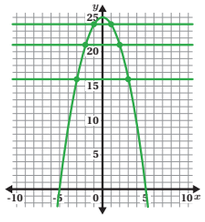
   $y =$ ...9...

   $y =$ ...0...

   $y =$ ...-11...

   What pattern do you notice?

   Responses vary. If you subtract each y-value from 25, it equals a perfect square.



Lesson Practice 7.11

Name: ..... Date: ..... Period: .....

Test Practice

8. Here is a graph of $y = x^2 - 4x - 5$ and $y = 4$. Select *all* the points that are solutions to $x^2 - 4x - 5 = 4$.

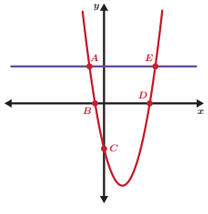
   ☑ **A.** Point A

   ☐ **B.** Point B

   ☐ **C.** Point C

   ☐ **D.** Point D

   ☑ **E.** Point E



Spiral Review

9. Fill in the blanks to make each equation true. *Responses vary.*

   $6^2 =$ ...36... (7 or -7)$^2 = 49$ $2^2 =$ ...$\sqrt{16}$... (...$\sqrt{11}$...)$^2 = 11$

10. Match each expression to an equivalent expression. You will have one left over.

    A. $1 - x^2$ ...C... $(y + x)(y - x)$

    B. $x^2 - y^2$ ...A... $(1 + x)(1 - x)$

    C. $y^2 - x^2$ ...E... $(1 + x)(x - 1)$

    D. $x^2 - 2xy + y^2$ ...D... $(x - y)(x - y)$

    E. $x^2 - 1$

Practice Problem Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.912.AR.3.1
	5–6	2	MA.912.AR.3.5
	7	3	MA.912.AR.3.1
Test Practice	8	1	MA.912.AR.3.1
Spiral Review			
Fluency	9	1	MA.8.AR.2.3
	10	2	MA.912.AR.1.7

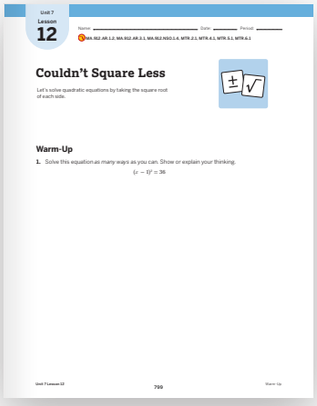
Unit 7

Lesson 12

Couldn't Square Less

Solving by Taking the Square Root

Let's solve quadratic equations by taking the square root of each side.


Unit 7
Lesson 12
Couldn't Square Less
Let's solve quadratic equations by taking the square root of each side.
Warm-Up
1. Solve this equation as many ways as you can. Show or explain your thinking.
$(x-1)^2 = 36$
799

Learning Outcomes

Today's Goals

1. **Goal:** Use square roots to determine exact solutions to quadratic equations.
2. **Goal:** Express irrational solutions to quadratic equations in the form $a \pm \sqrt{b}$.
3. **Language Goal:** Compare and contrast different strategies for solving quadratic equations. **(Speaking and Listening)**

Students solve quadratic equations that involve taking the square root. They also learn to express solutions using the ± symbol and exact irrational solutions in the form $a \pm \sqrt{b}$. Students consider the advantages and disadvantages of solving graphically versus symbolically and of writing solutions exactly versus approximately. **(MTR.6.1)**

Coherence

Prior Learning

In Lessons 7–9, students solved quadratic equations by graphing and factoring using the zero-product property. They also learned to determine whether a quadratic equation has zero, one, or two solutions.

Future Learning

In Lesson 12, students will learn to "complete the square" so that they can solve quadratic equations by taking square roots.

Connections

- Students continue to build their understanding of how to solve quadratic equations by taking the square root of each side of the equation. (mathematical)
- In a partner activity, students solve equivalent forms of a quadratic equation and share their thinking with each other. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.1
Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Also Addressing: **MA.912.AR.1.2, MA.912.AR.1.7, MA.912.NSO.1.4**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.912.NSO.1.1	**MA.912.AR.9.1**
MA.912.AR.2.1	**MA.912.NSO.1.2**

Vocabulary

New Vocabulary

± (plus/minus symbol)

Review Vocabulary

irrational number
rational number
square root

Lesson at a Glance

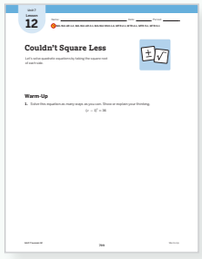
~ 45 min

MA.912.AR.3.1, MA.912.AR.1.2, MA.912.AR.1.7, MA.912.NSO.1.4

Warm-Up

Pairs | 10 min

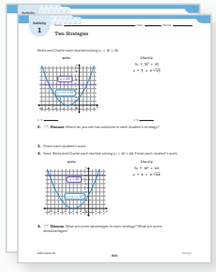
Students are introduced to solving quadratic equations by taking the *square root* of both sides and expressing the solutions using the ± symbol. **Routine:** Think-Pair-Share



Activity 1

Pairs | 5 min

Students solve quadratic equations with *rational* and *irrational* solutions by taking the square root and graphing. **MLR7: Compare and Connect**



Activity 2

Pairs | 10 min

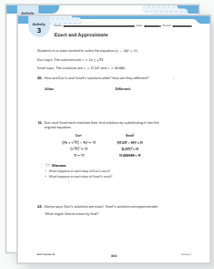
Students develop fluency in solving quadratic equations by taking the square root.



Activity 3

Pairs | 10 min

Students compare and contrast exact and approximate solutions of quadratic equations. **(MTR.6.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of expressing exact and approximate solutions of quadratic equations.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the exact solutions of a quadratic equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator

Pairs | 10 min

Warm-Up

Purpose: Students are introduced to solving quadratic equations by taking the square root of both sides and expressing the solutions using the ± symbol.

1 Launch

To surface prior knowledge, consider asking, "How many solutions can a quadratic equation have?"

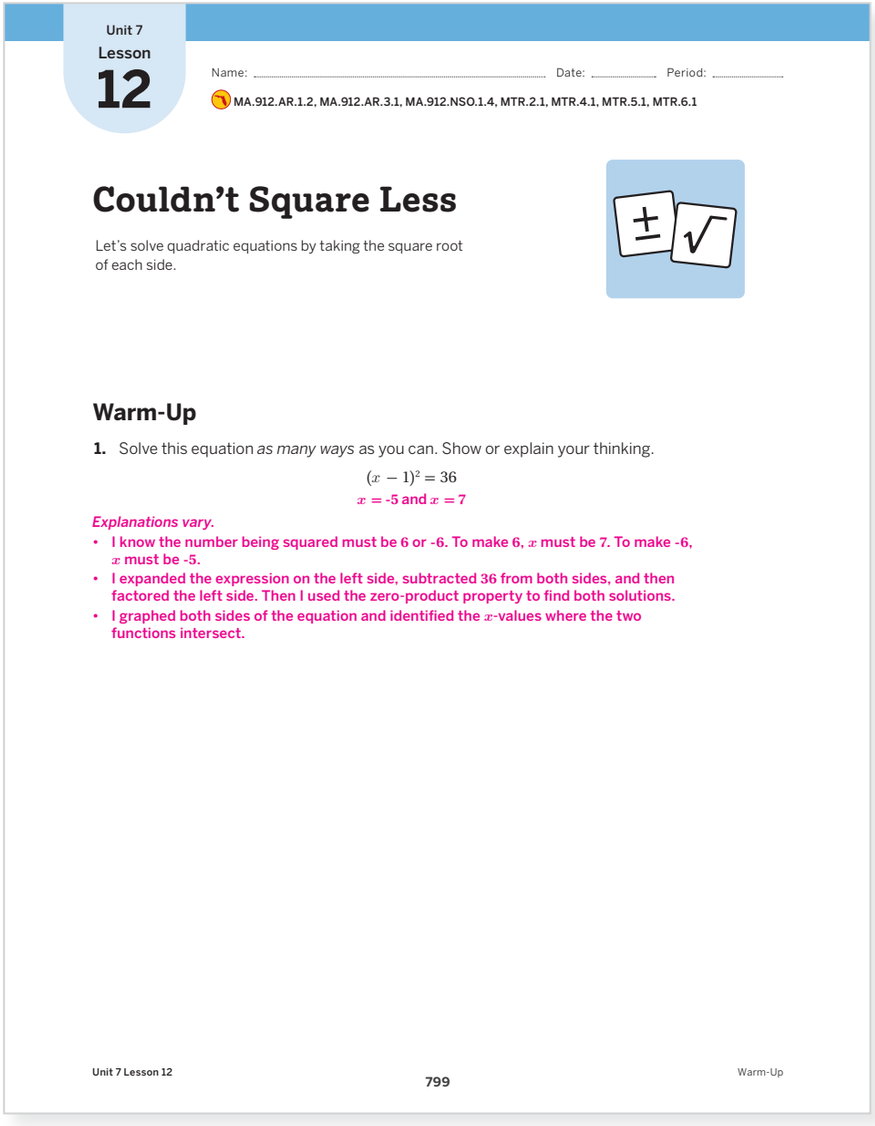
Invite students to use scientific calculators throughout the lesson.

Use the Think-Pair-Share routine to help students shape their ideas about multiple strategies for solving quadratic equations, including graphically and symbolically. See the Routine Facilitation Guide for more information.

2 Connect

Intentionally select a few students who are using different strategies to share their starting steps. Discuss as a class what their next steps could possibly be.

A **Accessibility: Executive Functioning** To support students in making sense of multiple steps, invite students to show or explain each step of their thinking.



Unit 7
Lesson
12

Name: _____ Date: _____ Period: _____

MA.912.AR.1.2, MA.912.AR.3.1, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

Couldn't Square Less

Let's solve quadratic equations by taking the square root of each side.

Warm-Up

1. Solve this equation *as many ways* as you can. Show or explain your thinking.

$$(x - 1)^2 = 36$$

$x = -5$ and $x = 7$

Explanations vary.

- I know the number being squared must be 6 or -6. To make 6, x must be 7. To make -6, x must be -5.
- I expanded the expression on the left side, subtracted 36 from both sides, and then factored the left side. Then I used the zero-product property to find both solutions.
- I graphed both sides of the equation and identified the x-values where the two functions intersect.

Unit 7 Lesson 12 799 Warm-Up

Key Takeaway: One strategy to solve a quadratic equation is to take the *square root* of both sides and express the solution using the **± (plus/minus symbol)**.

Pairs | 5 min

Activity 1 Two Strategies

Purpose: Students solve quadratic equations with *rational* and *irrational* solutions by taking the square root and graphing.

Short on time? Consider omitting the Launch.

1 Launch

To surface connections, consider asking, "How many solutions do you think $(x + 3)^2 = 25$ has? How do you know?"

2 Monitor

Look for a variety of strategies:

- Estimating the x-values of the points of intersection.
- Writing two equations, one equal to the positive *square root* and one to the negative.
- Using the $\pm$ symbol when writing the square root.

D Differentiation *(Problems 2–4)*

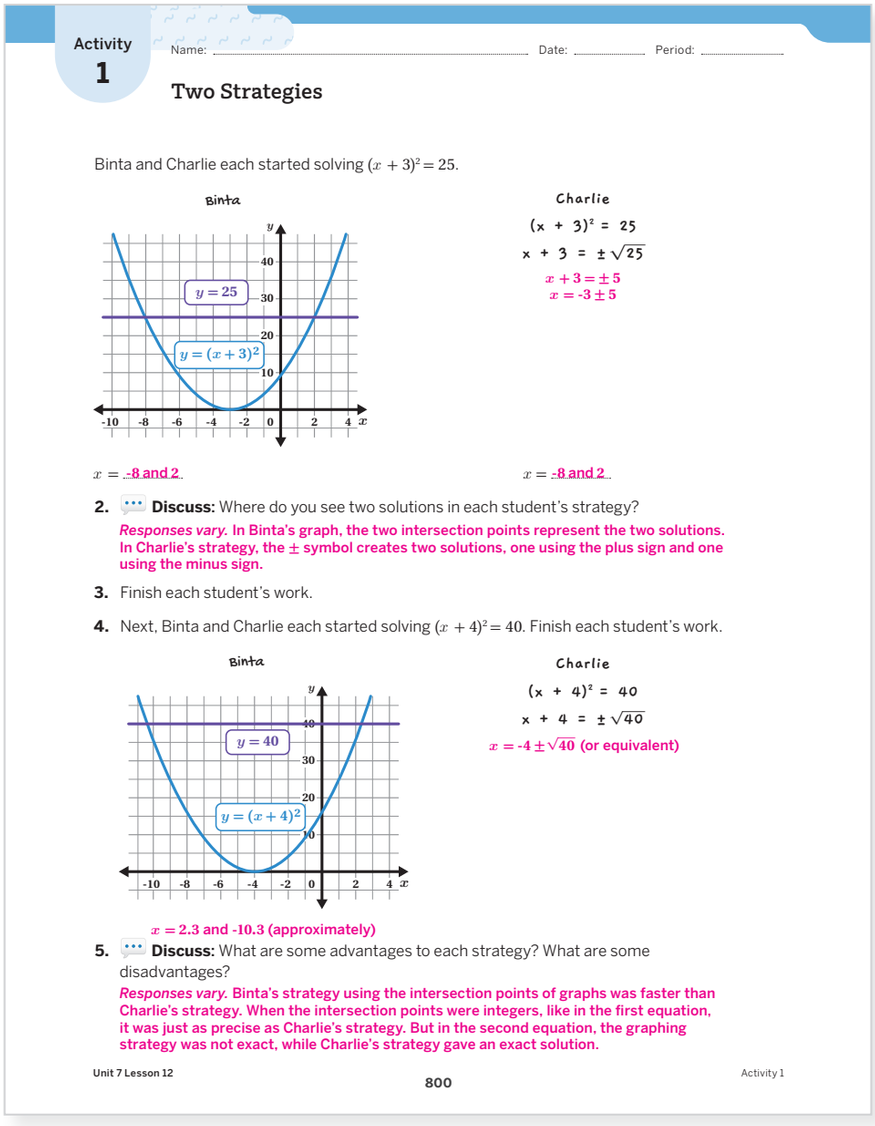
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Display Problem 2 and invite students to refer back to those strategies. Consider asking, "Which strategy would you like to try to finish first?"
Write the solutions as ± 2, because they subtract from the ± 5. *(Problem 2)*	**Support** Invite them to substitute 2 and -2 back into the original equation to check. Demonstrate how they can also write two separate equations instead of using the $\pm$ symbol (e.g., $x = 5 - 3$ and $x = -5 - 3$).
Describe interesting advantages and disadvantages.	**Strengthen** Invite them to share these during the Connect.
Would like to explore solving equations by taking the square root further.	**Stretch** Invite them to write an equation that has one solution in the form $(x + a)^2 = b$.

3 Connect

MLR MLR7: Compare and Connect Invite students to analyze and compare each strategy, and then connect to previous strategies for solving quadratic equations.

Consider asking:

- "Where do you see the same information in Binta and Charlie's strategy?"
- "What is helpful about each strategy?"
- "When might you use each strategy?" **(MTR.5.1)**
- "Is one strategy more precise? Why or why not?" **(MTR.6.1)**
- "Why is it helpful to have more than one strategy?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Two Strategies

Binta and Charlie each started solving $(x + 3)^2 = 25$.

Binta

$x =$ -8 and 2

Charlie

$(x + 3)^2 = 25$

$x + 3 = \pm\sqrt{25}$

$x + 3 = \pm 5$

$x =$ -3 $\pm$ 5

$x =$ -8 and 2

2. **Discuss:** Where do you see two solutions in each student's strategy?

Responses vary. In Binta's graph, the two intersection points represent the two solutions. In Charlie's strategy, the $\pm$ symbol creates two solutions, one using the plus sign and one using the minus sign.

3. Finish each student's work.

4. Next, Binta and Charlie each started solving $(x + 4)^2 = 40$. Finish each student's work.

Binta

Charlie

$(x + 4)^2 = 40$

$x + 4 = \pm\sqrt{40}$

$x = -4 \pm \sqrt{40}$ (or equivalent)

$x =$ 2.3 and -10.3 (approximately)

5. **Discuss:** What are some advantages to each strategy? What are some disadvantages?

Responses vary. Binta's strategy using the intersection points of graphs was faster than Charlie's strategy. When the intersection points were integers, like in the first equation, it was just as precise as Charlie's strategy. But in the second equation, the graphing strategy was not exact, while Charlie's strategy gave an exact solution.

Unit 7 Lesson 12 800 Activity 1

Pairs | 10 min

Activity 2 Pair Solving

Purpose: Students develop fluency in solving quadratic equations by taking the square root.

Short on time? Consider inviting students to complete fewer parts in Problems 6 and 7.

1 Launch

Display Charlie's claim and discuss the prompt. Consider asking, "What form does this look closer to: standard, factored, or vertex form?" **(MTR.5.1)**

2 Monitor

Encourage students to discuss their reasoning with their partner as they work on each problem.

Look for a variety of strategies:

- Using the structure of equations to help them determine the number of solutions or take first steps toward solving the equations. **(MTR.5.1)**
- Attending to precision as they express *irrational* solutions exactly. **(MTR.6.1)**

D **Differentiation: (Support)** Students may add the values inside and outside of the square root together. Consider inviting them to put each expression into their calculator to show that they are not equivalent.

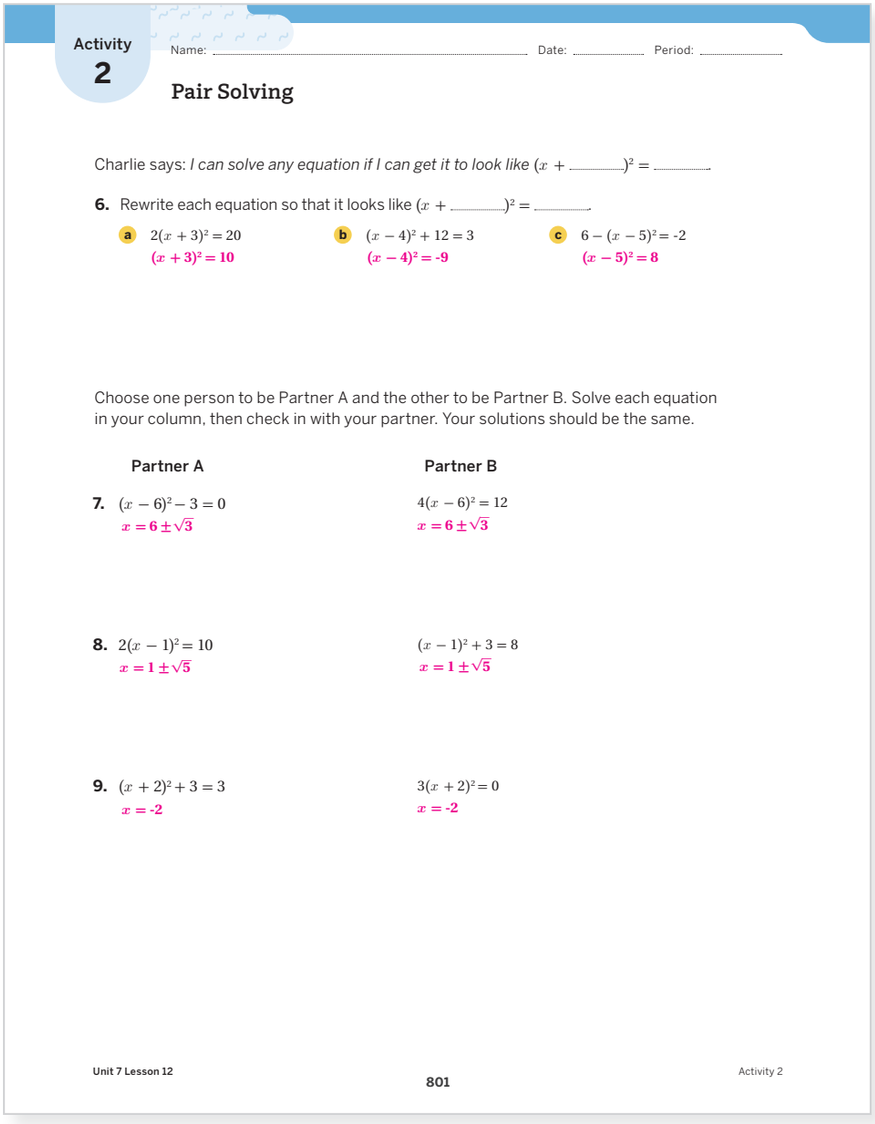
3 Connect

Share student work to highlight the process of using the square root to solve quadratic equations.

To surface the Key Takeaway, consider asking:

- "What strategies were most helpful for solving these equations?"
- "What are some other examples of equations we c ould solve by taking the square root of both sides?" **(MTR.5.1)**

Note: If it does not come up naturally, share that it is okay for solutions to include the $\pm$ and $\sqrt{\ }$ symbols, like $-4 \pm \sqrt{40}$. Representing solutions is the focus of Activity 3.



Activity 2

Name: _____ Date: _____ Period: _____

Pair Solving

Charlie says: *I can solve any equation if I can get it to look like* $(x + ____)^2 = ____$.

6. Rewrite each equation so that it looks like $(x + ____)^2 = ____$.

a. $2(x+3)^2 = 20$ → $(x+3)^2 = 10$

b. $(x-4)^2 + 12 = 3$ → $(x-4)^2 = -9$

c. $6 - (x-5)^2 = -2$ → $(x-5)^2 = 8$

Choose one person to be Partner A and the other to be Partner B. Solve each equation in your column, then check in with your partner. Your solutions should be the same.

	Partner A	Partner B
7.	$(x-6)^2 - 3 = 0$ $x = 6 \pm \sqrt{3}$	$4(x-6)^2 = 12$ $x = 6 \pm \sqrt{3}$
8.	$2(x-1)^2 = 10$ $x = 1 \pm \sqrt{5}$	$(x-1)^2 + 3 = 8$ $x = 1 \pm \sqrt{5}$
9.	$(x+2)^2 + 3 = 3$ $x = -2$	$3(x+2)^2 = 0$ $x = -2$

Unit 7 Lesson 12 801 Activity 2

Key Takeaway: Quadratic equations written in the form $(x + ___)^2 = ___$ can be solved by taking the square root of both sides before solving for the value of x. **(MTR.5.1)**

Pairs | 10 min

Activity 3 Exact and Approximate

Purpose: Students compare and contrast exact and approximate solutions of quadratic equations. **(MTR.6.1)**

1 Launch

Display Duri's and Yosef's solutions and consider asking, "What would your first step in solving $(x - 14)^2 = 11$ be? Why?"

A **Accessibility: Conceptual Processing** Invite students to take a more concrete step first to support abstract thinking, such as, "Calculate $14 + \sqrt{11}$ and $14 - \sqrt{11}$ using a calculator."

2 Monitor

To support students getting started, consider asking, "What are Duri's two solutions? Yosef's?" and invite students to use a highlighter to identify where the solution is in the first step of each student's work.

Listen for students who notice and wonder about differences between the approximate and exact solutions, as they consider Abena's claim. **(MTR.6.1)** Students may:

- Notice that the exact solutions work out perfectly when checking.
- Notice that Duri's solutions are written as one expression and Yosef's are separate.
- Wonder if Yosef rounded his solutions or if they only have three decimal places.
- Notice that it is more precise to leave the square root in the expression when the solutions are irrational numbers.

To surface connections, consider asking, "When might it be more helpful to have exact solutions? Approximate solutions?" See the Connect for a sample response.

Activity 3 continued >

Activity 3

Name: _____ Date: _____ Period: _____

Exact and Approximate

Students in a class worked to solve the equation $(x - 14)^2 = 11$.

Duri says: *The solutions are* $x = 14 \pm \sqrt{11}$.

Yosef says: *The solutions are* $x = 17.317$ *and* $x = 10.683$.

10. How are Duri's and Yosef's solutions alike? How are they different? ***Responses vary.***

Alike:
- Both students list two solutions.
- The sets of solutions are approximately equivalent.

Different:
- Duri's two solutions are represented with a single expression.
- Yosef rounded his solutions. Duri didn't do any rounding.

11. Duri and Yosef each checked their first solution by substituting it into the original equation.

Duri	Yosef
$((14 + \sqrt{11}) - 14)^2 = 11$	$(17.317 - 14)^2 = 11$
$(\sqrt{11})^2 = 11$	$(3.317)^2 = 11$
$11 = 11$	$11.002489 = 11$

Discuss:
- What happens in each step of Duri's work?
- What happens in each step of Yosef's work?

Responses vary.
- Duri substituted $14 + \sqrt{11}$ into the equation. $14 - 14 = 0$, so all that remained on the left side of the equation was $(\sqrt{11})^2$. Duri's solution is confirmed because $\sqrt{11} \cdot \sqrt{11} = 11$.
- Yosef substituted 17.317 into the equation, getting $(3.317)^2$ after subtracting. After squaring the value, probably on a calculator, Yosef got 11.002489.

12. Abena says: *Duri's solutions are exact. Yosef's solutions are approximate.*

What might Abena mean by that?

Responses vary. Abena might mean that when you substitute an "exact" solution, the left side equals the right side exactly. In this example, substituting $14 + \sqrt{11}$ leads to $11 = 11$. Meanwhile, when you substitute an "approximate" solution, the left side *almost* equals the right side, but not quite. A solution that is "approximate" might involve rounding.

Unit 7 Lesson 12 802 Activity 3

Pairs | 10 min

Activity 3 Exact and Approximate (continued)

Purpose: Students compare and contrast exact and approximate solutions of quadratic equations. **(MTR.6.1)**

2 Monitor

Look for opportunities to recognize the brilliance from students who have not been heard from recently. Some responses to highlight might include:

- Helping Riku approximate $\sqrt{11}$.
- Asking another square root question (e.g., Is $1+\sqrt{9}$ equivalent to $\sqrt{10}$?) to show that a number cannot be brought under the square root.

D **Differentiation: You're invited to explore more.** (Stretch) Invite students who would like to explore solving quadratic equations using square roots further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

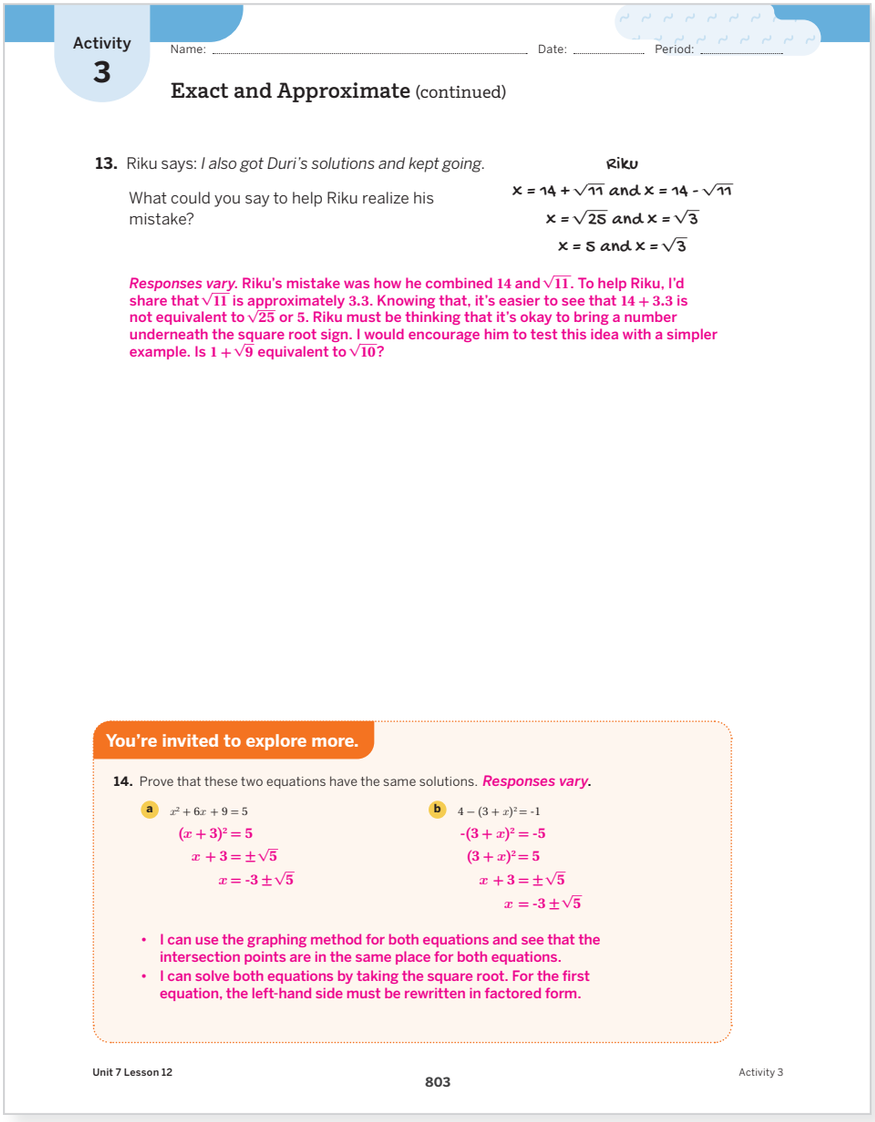
Display Problem 13 and encourage students to share what they might say to help Riku and to consider what he might have been thinking.

Consider asking:

- "What does *exact solutions* mean in your own words? *Approximate solutions*?"
- "When might it be more helpful to have exact solutions? Approximate solutions?" Exact solutions are helpful when you need to be very precise. Approximate solutions might be helpful if you're answering a question about a situation and sharing it with people. For example, "approximately 10 and 17 seconds" is more clear to most people than "$14 \pm \sqrt{11}$ seconds."

Note: Students may notice that when equations have *irrational* solutions, it is more precise to leave the radical in the expression, but they can evaluate the radical and still have precise answers when the solutions are *rational*. However, students do not need to use this language to reach the learning goal.

ELL **English Language Learners** Invite several students to share what the words *exact* and *approximate* remind them of (e.g., When I hear exact, I think . . .). It may also be helpful to research how to say *exact solutions* and *approximate solutions* in other languages. This may help surface multiple meanings and invite students to connect to their prior knowledge.



Activity 3

Name: _____ Date: _____ Period: _____

Exact and Approximate (continued)

13. Riku says: *I also got Duri's solutions and kept going.*

What could you say to help Riku realize his mistake?

Riku

$x = 14 + \sqrt{11}$ and $x = 14 - \sqrt{11}$

$x = \sqrt{25}$ and $x = \sqrt{3}$

$x = 5$ and $x = \sqrt{3}$

Responses vary. Riku's mistake was how he combined 14 and $\sqrt{11}$. To help Riku, I'd share that $\sqrt{11}$ is approximately 3.3. Knowing that, it's easier to see that $14 + 3.3$ is not equivalent to $\sqrt{25}$ or 5. Riku must be thinking that it's okay to bring a number underneath the square root sign. I would encourage him to test this idea with a simpler example. Is $1+\sqrt{9}$ equivalent to $\sqrt{10}$?

You're invited to explore more.

14. Prove that these two equations have the same solutions. *Responses vary.*

a $x^2 + 6x + 9 = 5$

$(x+3)^2 = 5$

$x + 3 = \pm\sqrt{5}$

$x = -3 \pm \sqrt{5}$

b $4 - (3+x)^2 = -1$

$-(3+x)^2 = -5$

$(3+x)^2 = 5$

$x + 3 = \pm\sqrt{5}$

$x = -3 \pm \sqrt{5}$

- I can use the graphing method for both equations and see that the intersection points are in the same place for both equations.
- I can solve both equations by taking the square root. For the first equation, the left-hand side must be rewritten in factored form.

Unit 7 Lesson 12 803 Activity 3

Whole Class | 5 min

Synthesis

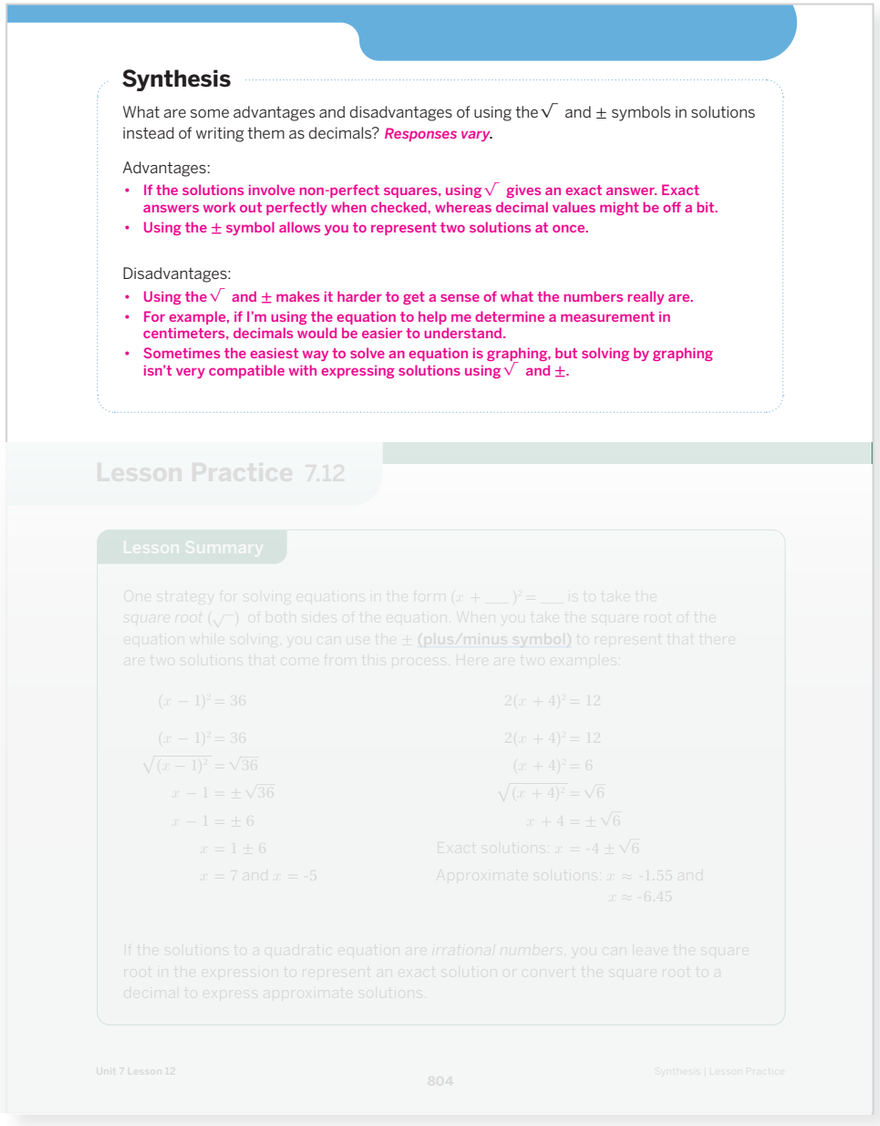
Purpose: Students synthesize their understanding of expressing exact and approximate solutions of quadratic equations.

Invite students to respond independently, and then share their thinking with a partner.

Gather and share a variety of ideas, such as:
- The $\sqrt{\ }$ symbol helps write exact solutions.
- The $\pm$ symbol can be used to write two solutions in one expression.
- Using the symbols wouldn't make sense for answering a question in real life, like how long something should be.

Consider asking:
- "Why might someone write the solutions as $x = 2 \pm \sqrt{7}$? Why might they write them as $x = -0.646$ and $x = 4.646$?" **(MTR.6.1)**
- "What do you think is important to remember when solving quadratic equations using square roots?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

What are some advantages and disadvantages of using the $\sqrt{\ }$ and $\pm$ symbols in solutions instead of writing them as decimals? ***Responses vary.***

Advantages:
- **If the solutions involve non-perfect squares, using $\sqrt{\ }$ gives an exact answer. Exact answers work out perfectly when checked, whereas decimal values might be off a bit.**
- **Using the $\pm$ symbol allows you to represent two solutions at once.**

Disadvantages:
- **Using the $\sqrt{\ }$ and $\pm$ makes it harder to get a sense of what the numbers really are.**
- **For example, if I'm using the equation to help me determine a measurement in centimeters, decimals would be easier to understand.**
- **Sometimes the easiest way to solve an equation is graphing, but solving by graphing isn't very compatible with expressing solutions using $\sqrt{\ }$ and $\pm$.**

Lesson Practice 7.12

Lesson Summary

One strategy for solving equations in the form $(x + ___)^2 = ___$ is to take the *square root* ($\sqrt{\ }$) of both sides of the equation. When you take the square root of the equation while solving, you can use the $\pm$ **(plus/minus symbol)** to represent that there are two solutions that come from this process. Here are two examples:

$(x - 1)^2 = 36$

$(x - 1)^2 = 36$

$\sqrt{(x - 1)^2} = \sqrt{36}$

$x - 1 = \pm\sqrt{36}$

$x - 1 = \pm 6$

$x = 1 \pm 6$

$x = 7$ and $x = -5$

$2(x + 4)^2 = 12$

$2(x + 4)^2 = 12$

$(x + 4)^2 = 6$

$\sqrt{(x + 4)^2} = \sqrt{6}$

$x + 4 = \pm\sqrt{6}$

Exact solutions: $x = -4 \pm \sqrt{6}$

Approximate solutions: $x \approx -1.55$ and $x \approx -6.45$

If the solutions to a quadratic equation are *irrational numbers*, you can leave the square root in the expression to represent an exact solution or convert the square root to a decimal to express approximate solutions.

Unit 7 Lesson 12 804 Synthesis | Lesson Practice

Lesson Takeaway: Quadratic equations written in the form $(x + ___)^2 = ___$ can be solved by taking the square root of both sides before solving for the value of x. If solutions are *irrational*, they can be expressed exactly, with a **± (plus/minus symbol)**, or approximately with a rounded decimal.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the exact solutions of a quadratic equation.

Today's Goals

1. **Goal:** Use square roots to determine exact solutions to quadratic equations.
2. **Goal:** Express irrational solutions to quadratic equations in the form $a \pm \sqrt{b}$.
3. **Language Goal:** Compare and contrast different strategies for solving quadratic equations. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.12

Determine the *exact* solution(s) to this equation:

$$2(x + 8)^2 = 70$$

$x = -8 \pm \sqrt{35}$ (or equivalent)

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support solving quadratic equations by taking the square root:

» **Respond to Student Thinking:**

- Use the ***Approximating Square Roots*** **Mini-Lesson**.
- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **You're invited to explore more** tasks and discuss their thinking with a partner.

Professional Learning

In this lesson, students learned a new strategy to solve quadratic equations: taking the *square root*. How will that work support the next strategy: solving quadratic equations by completing the square? How can you continue to support students in using the structure of equations and their personal preferences to help them select solving methods strategically? **(MTR.2.1, MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

What are some advantages and disadvantages of using the $\sqrt{\ }$ and $\pm$ symbols in solutions instead of writing them as decimals? *Responses vary.*

Advantages:

- If the solutions involve non-perfect squares, using $\sqrt{\ }$ gives an exact answer. Exact answers work out perfectly when checked, whereas decimal values might be off a bit.
- Using the $\pm$ symbol allows you to represent two solutions at once.

Disadvantages:

- Using the $\sqrt{\ }$ and $\pm$ makes it harder to get a sense of what the numbers really are.
- For example, if I'm using the equation to help me determine a measurement in centimeters, decimals would be easier to understand.
- Sometimes the easiest way to solve an equation is graphing, but solving by graphing isn't very compatible with expressing solutions using $\sqrt{\ }$ and $\pm$.

Lesson Practice 7.12

Lesson Summary

One strategy for solving equations in the form $(x + ___)^2 = ___$ is to take the *square root* $(\sqrt{\ })$ of both sides of the equation. When you take the square root of the equation while solving, you can use the $\pm$ **(plus/minus symbol)** to represent that there are two solutions that come from this process. Here are two examples:

$(x-1)^2 = 36$

$(x-1)^2 = 36$
$\sqrt{(x-1)^2} = \sqrt{36}$
$x - 1 = \pm\sqrt{36}$
$x - 1 = \pm 6$
$x = 1 \pm 6$
$x = 7$ and $x = -5$

$2(x+4)^2 = 12$

$2(x+4)^2 = 12$
$(x+4)^2 = 6$
$\sqrt{(x+4)^2} = \sqrt{6}$
$x + 4 = \pm\sqrt{6}$
Exact solutions: $x = -4 \pm \sqrt{6}$
Approximate solutions: $x \approx -1.55$ and $x \approx -6.45$

If the solutions to a quadratic equation are *irrational numbers*, you can leave the square root in the expression to represent an exact solution or convert the square root to a decimal to express approximate solutions.

Lesson Practice 7.12

Name: ..... Date: ..... Period: .....

1. Is the equation $\sqrt{x^2} = x$ always true? Explain your thinking.

No. *Explanations vary.* If x is positive or equal to zero, the equation is true, because squaring and finding square roots are inverse operations. $\sqrt{5^2} = 5$. But if x is negative, the equation is false: $\sqrt{(-5)^2} \neq -5$.

Problems 2–5: Determine the exact solutions to each equation. Write the solutions using $\pm$ notation.

2. $x^2 = 144$
$x = \pm 12$

3. $x^2 = 5$
$x = \pm\sqrt{5}$

4. $6x^2 = 42$
$x = \pm\sqrt{7}$

5. $4x^2 - 25 = 0$
$x = \pm\frac{5}{2}$

Problems 6–8: Determine the exact solutions to each equation. Write the solutions using $\pm$ notation.

6. $(x+4)^2 = 7$
$x = -4 \pm \sqrt{7}$

7. $5(x-4)^2 = 30$
$x = 4 \pm \sqrt{6}$

8. $4 + (3+x)^2 = 9$
$x = -3 \pm \sqrt{5}$

9. Adrian says that the two solutions to $(x-3)^2 = 2$ can be represented by $3 \pm \sqrt{2}$. Substitute each solution into the original equation and show that it makes the equation true.

$(x-3)^2 = 2$
$((3+\sqrt{2})-3)^2 = 2$
$(\sqrt{2})^2 = 2$
$2 = 2$

$(x-3)^2 = 2$
$((3-\sqrt{2})-3)^2 = 2$
$(-\sqrt{2})^2 = 2$
$2 = 2$

Lesson Practice 7.12

Name: ..... Date: ..... Period: .....

Test Practice

10. Match each expression to an equivalent expression.

a. -4 ± 1	c. -17 and 5
b. $10 \pm \sqrt{4}$	e. $4 + \sqrt{2}$ and $4 - \sqrt{2}$
c. -6 ± 11	b. 12 and 8
d. $4 \pm \sqrt{10}$	a. -3 and -5
e. $\sqrt{16} \pm \sqrt{2}$	d. $4 + \sqrt{10}$ and $4 - \sqrt{10}$

Spiral Review

11. Complete the table by writing each expression in the missing form.

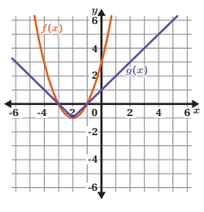
Factored Form	Standard Form
$(x-7)(x-7)$	$x^2 - 14x + 49$
$(4x+3)(4x+3)$	$16x^2 + 24x + 9$
$(2x+5)(2x+5)$	$4x^2 + 20x + 25$
$\left(x - \frac{1}{8}\right)\left(x - \frac{1}{8}\right)$	$x^2 - \frac{1}{4}x + \frac{1}{64}$

12. Here are the graphs of a quadratic function and an absolute value function.

Write the equation for each function:

$f(x) = (x+2)^2 - 1$

$g(x) = |x+2| - 1$



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	3	MA.912.NSO.1.1
	2–5	1	MA.912.AR.3.1
	6–9	2	MA.912.AR.3.1
Test Practice	10	1	MA.912.AR.3.1
Spiral Review			
Fluency	11	2	MA.912.AR.1.3, MA.912.AR.1.4
	12	2	MA.912.AR.3.4, MA.912.AR.4.3

Unit 7

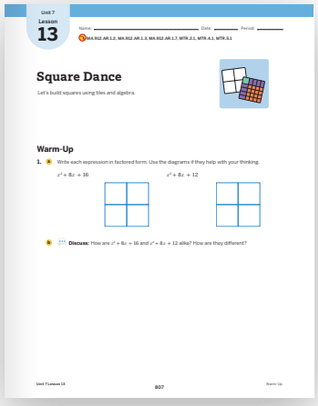
Lesson

13

Square Dance

Perfect Square Expressions

Let's build squares using tiles and algebra.


Unit 7
Lesson
13
Square Dance
Let's build squares using tiles and algebra.
Warm-Up
1. Write each expression in factored form. Use the diagrams if they help with your thinking.
$x^2 + 8x + 16$
$x^2 + 8x + 12$
Discuss: How are $x^2 + 8x + 16$ and $x^2 + 8x + 12$ alike? How are they different?
807

Learning Outcomes

Today's Goals

1. **Goal:** Determine unknown values of b or c that make the expression $x^2 + bx + c$ a *perfect square*.
2. **Language Goal:** Justify whether a quadratic expression is a perfect square. **(Writing, Speaking, and Listening)**

Students explore perfect square quadratic expressions in standard form $ax^2 + bx + c$, and in this lesson students will work with $a = 1$. They analyze the structure of examples and non-examples of perfect square expressions in order to create a class definition. Students determine if expressions are perfect squares, justify their thinking, and determine missing b- and c-values for perfect square expressions. **(MTR.5.1)**

Coherence

Prior Learning

In Lessons 1–4, students used area models to convert expressions between standard and factored form. They identified how to see the a-, b-, and c-values of standard form in the area model, including with expressions that were perfect squares.

Future Learning

In Lesson 12, students will complete the square in order to solve equations. In Lesson 13, students will complete the square to rewrite standard-form equations in vertex form to reveal the maximum or minimum value.

Connections

- Students identify patterns and use tiles to create perfect square trinomials. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.2
Rearrange equations or formulas to isolate a quantity of interest.

Also Addressing: **MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	MA.912.AR.3.6
MA.7.AR.1.1	MA.912.AR.3.1

Vocabulary

New Vocabulary

perfect square

Lesson at a Glance

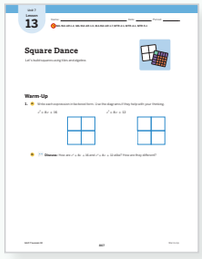
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MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1

Warm-Up

Pairs | 5 min

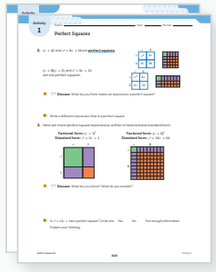
Students compare two expressions and revisit factoring.



Activity 1

Pairs | 10 min

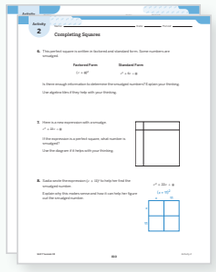
Students create a class definition of **perfect square** expressions by analyzing examples and non-examples and justifying their classifications. **Routines:** Notice and Wonder; MLR2: Collect and Display **(MTR.4.1)**



Activity 2

Pairs | 20 min

Students create perfect square expressions by determining a missing b- or c-value.



Synthesis

Whole Class | 5 min

Students synthesize how to determine whether an expression is a perfect square. MLR2: Collect and Display **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining missing b- and c-values that make a perfect square expression.



Prep Checklist

Prepare Activity 2 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 2 sheets, one per student.
- Calculator
- Activity Sheet
- Algebra Tiles

Pairs | 5 min

Warm-Up

Purpose: Students compare two expressions and revisit factoring.

1 Launch

Consider asking, "What do you remember about factoring?"

Encourage students to show their thinking.

To support students getting started, consider displaying an anchor chart from Lessons 3 or 4 and consider asking, "Where can the x^2 go in the diagram? the 16?"

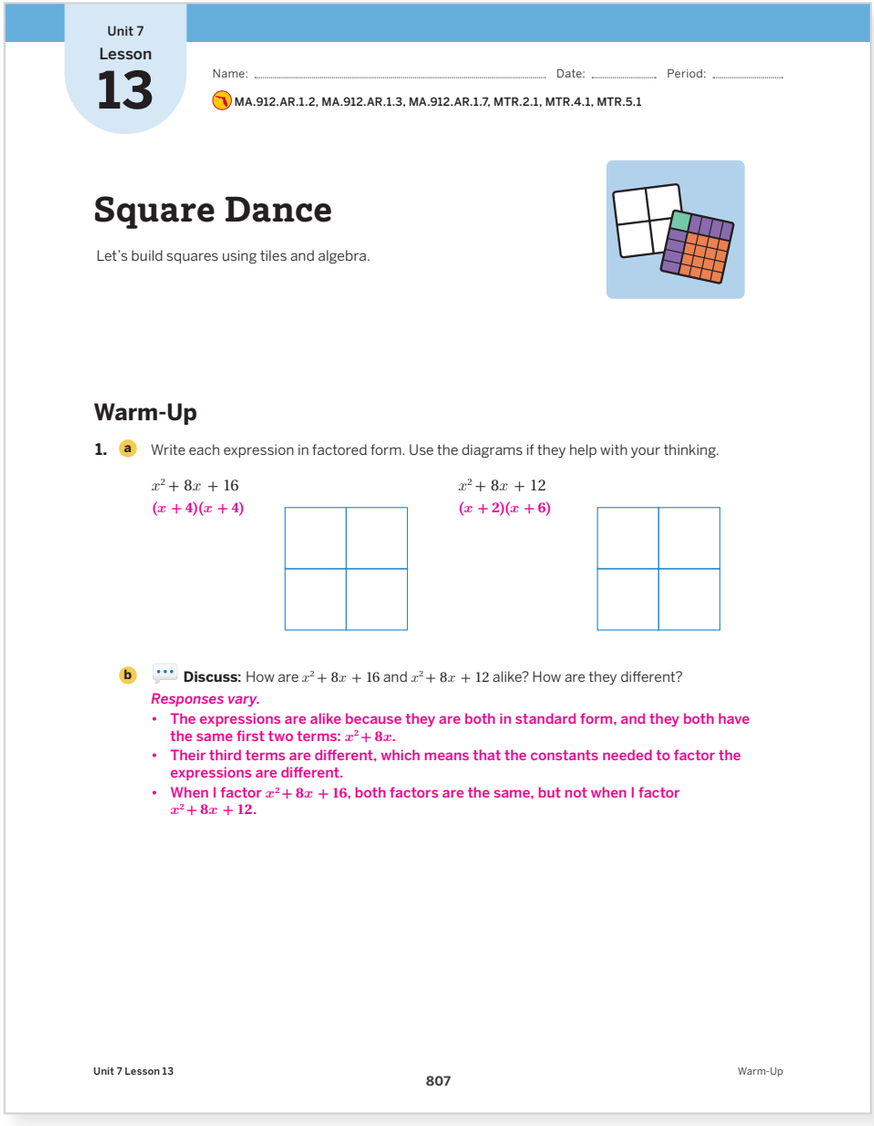
A **Accessibility: Memory and Attention**
To support students with performing calculations, invite them to use a calculator. Consider providing a multiplication table or factor chart for students to reference for the rest of the lesson.

ELL **English Language Learners** Consider providing sentence frames for the discussion prompt: "The expressions are alike because they both have _____. The expressions are different because they both have _____." **(Writing and Speaking)**

2 Connect

Invite students to share their factored-form expressions and their process for factoring.

Consider asking, "What do you notice about the factored-form expressions for $x^2 + 8x + 16$ and $x^2 + 8x + 12$?" The factored form expression for $x^2 + 8x + 16$ has the same expression for both factors: $(x + 4)$.



Unit 7
Lesson
13

Name: _____ Date: _____ Period: _____

MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MTR.2.1, MTR.4.1, MTR.5.1

Square Dance

Let's build squares using tiles and algebra.

Warm-Up

1. a Write each expression in factored form. Use the diagrams if they help with your thinking.

$x^2 + 8x + 16$
$(x + 4)(x + 4)$

$x^2 + 8x + 12$
$(x + 2)(x + 6)$

b **Discuss:** How are $x^2 + 8x + 16$ and $x^2 + 8x + 12$ alike? How are they different?

Responses vary.

- The expressions are alike because they are both in standard form, and they both have the same first two terms: $x^2 + 8x$.
- Their third terms are different, which means that the constants needed to factor the expressions are different.
- When I factor $x^2 + 8x + 16$, both factors are the same, but not when I factor $x^2 + 8x + 12$.

Unit 7 Lesson 13 807 Warm-Up

Pairs | 10 min

Activity 1 Perfect Squares

Purpose: Students create a class definition of **perfect square** expressions by analyzing examples and non-examples of perfect square expressions, justifying their classifications. **(MTR.4.1)**

1 Launch

Display Problem 2.

To surface connections, consider asking:

- "What do you notice about $x^2 + 8x + 16$ when it has been factored? What made it look like that?"
- "Why does it make sense to call this a perfect square?"
- "What else do you think of when you hear *perfect square*?"

Invite students to create a class definition of **perfect square**.

MLR **MLR2: Collect and Display** Use this routine to collect students' language and examples as they share during the class discussion. Display for students to reference and add to for the rest of the unit.

ELL **English Language Learners** Invite students to research how to say **perfect square** in other languages. Add the new terms to the anchor chart, along with examples. **(Reading and Listening)**

A **Accessibility: Visual-Spatial Processing** Invite students to describe what each type of tile in the area model represents.

2 Monitor

D **Differentiation**

Look for students who:	Respond to Student Thinking
Think that each term of standard form needs to be a perfect square (e.g., $x^2 + 9x + 25$).	**Support** Consider asking, "Were all of the values in $x^2 + 8x + 16$ perfect squares? How can we write a perfect square in factored form? In standard form?"
Would like an additional challenge.	**Stretch** Invite them to write a unique perfect square expression they don't believe anyone else submitted.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the perfect square expressions in both forms. Consider inviting students to share their noticings during the pause.

To support students getting started, consider asking, "What would happen if we could drag the point further?"

Pause if many pairs are struggling. Consider having students share their responses with a partner before sharing with the class. If there is no consensus, invite students to justify their reasoning using the expressions and the area model. **(MTR.4.1)**

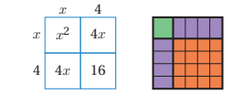
Activity 1 continued >

Activity 1

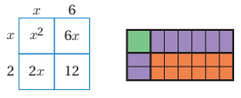
Name: _____ Date: _____ Period: _____

Perfect Squares

2. $(x + 4)^2$ and $x^2 + 8x + 16$ are **perfect squares**.



$(x + 6)(x + 2)$ and $x^2 + 8x + 12$ are not perfect squares.



a. **Discuss:** What do you think makes an expression a perfect square? *Responses vary.*

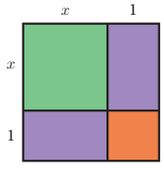
- An expression is a perfect square if you can represent it as a square using an area model.
- An expression is a perfect square if the factors as the same, like $(x + 4)(x + 4)$.
- An expression is a perfect square if it can be written like $(x + ____)^2$.

b. Write a different expression that is a perfect square.

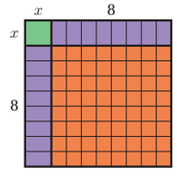
Responses vary. $(x + 6)(x + 6)$; $(x + 10)^2$; $x^2 + 6x + 9$; $x^2 + 10x + 25$

3. Here are more perfect square expressions written in factored and standard form.

Factored form: $(x + 1)^2$
Standard form: $x^2 + 2x + 1$



Factored form: $(x + 8)^2$
Standard form: $x^2 + 16x + 64$



a. **Discuss:** What do you notice? What do you wonder?

- I notice that the factored-form expressions all look like $(x + ____)^2$.
- I notice that the c-value of the standard form is a perfect square number.
- I notice that the b-value of the standard form is twice the value of the constant in factored form.
- I wonder if perfect square expressions can have negative values.
- I wonder if there are perfect squares that have fractional or decimal values.
- I wonder if there are ways to tell if something is a perfect square without drawing a diagram.

b. Is $x^2 + 12x + 144$ a perfect square? Circle one. Yes (No) Not enough information

Explain your thinking.

Explanations vary.

- If the b-value is 12, that would mean that the factors would be $(x + 6)(x + 6)$, leading to a c-value of 36.
- The c-value is 144. Based on that, the factors for a perfect square expression would be $(x + 12)(x + 12)$, but that would have a b-value of 24.

Unit 7 Lesson 13 808 Activity 1

Pairs | 10 min

Activity 1 Perfect Squares (continued)

Purpose: Students create a class definition of perfect square expressions by analyzing examples and non-examples of perfect square expressions, justifying their classifications. **(MTR.4.1)**

2 Monitor

To support students getting started, invite them to identify a pair of expressions that look alike and ask, "How are the expressions alike? Different?"

A **Accessibility: Executive Functioning** Consider chunking this activity into more manageable parts by inviting students to sort one expression at a time.

Look for the most common incorrect student responses. Note that students will discuss $x^2 + 5x + 6.25$ in Problem 5.

D **Differentiation**

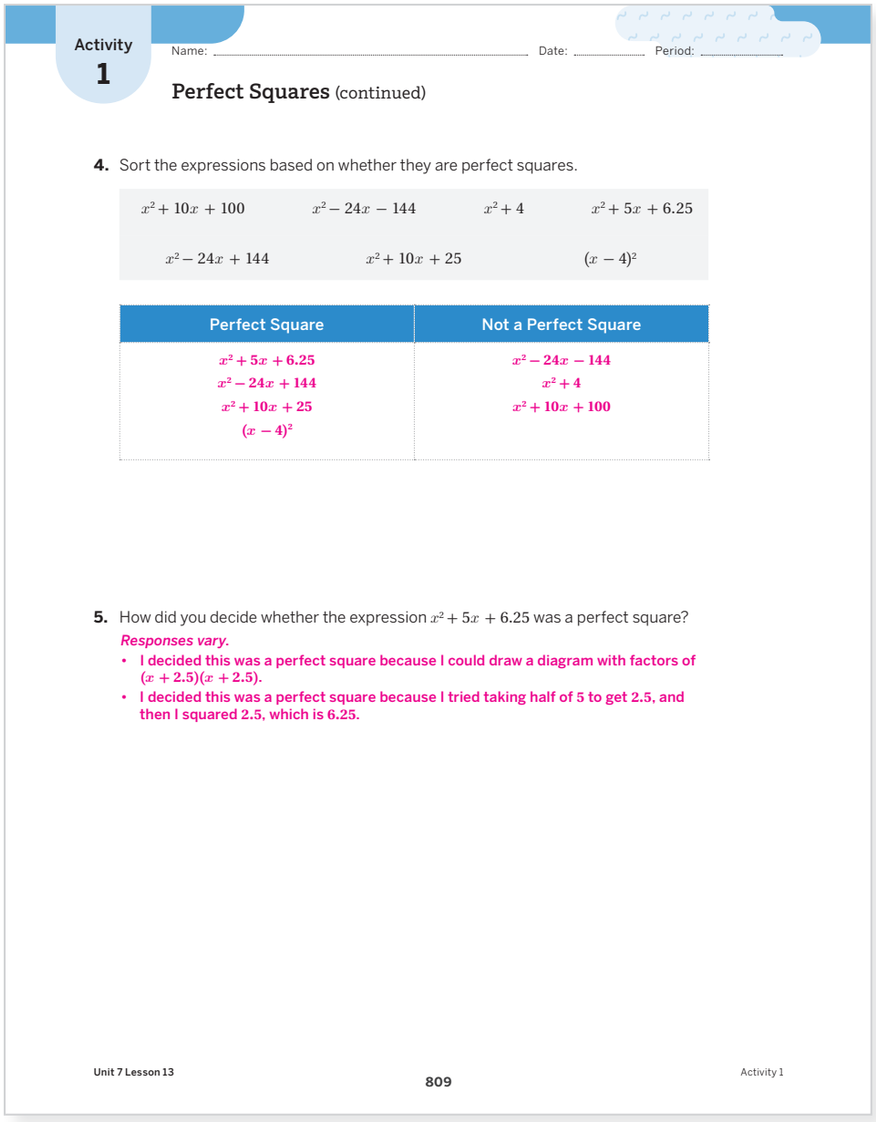
Look for students who:	Respond to Student Thinking
Think $x^2 - 24x + 144$ is not a perfect square because it has a minus sign.	**Support** Encourage them to rewrite the expression in factored form.
Think $x^2 + 4$ is a perfect square expression because 4 is positive and it is a perfect square.	**Support** Celebrate their noticing that 4 is a perfect square and encourage them to rewrite the expression in factored form.
Think that 6.25 is not a perfect square number because it is a decimal.	**Support** Consider asking, "What types of numbers multiply to make a decimal?"

3 Connect

To surface the Key Takeaway, consider asking:

- "What do the perfect square expressions have in common?"
- "Why does the c-value need to be positive in a perfect square expression?"
- "Can a perfect square expression have two terms in standard form? Why or why not?"

Invite students to share whether they think this expression is a perfect square by polling the class. Encourage them to make arguments and justify their reasoning for each response. **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Perfect Squares (continued)

4. Sort the expressions based on whether they are perfect squares.

$x^2 + 10x + 100$ $x^2 - 24x - 144$ $x^2 + 4$ $x^2 + 5x + 6.25$

$x^2 - 24x + 144$ $x^2 + 10x + 25$ $(x - 4)^2$

Perfect Square	Not a Perfect Square
$x^2 + 5x + 6.25$ $x^2 - 24x + 144$ $x^2 + 10x + 25$ $(x - 4)^2$	$x^2 - 24x - 144$ $x^2 + 4$ $x^2 + 10x + 100$

5. How did you decide whether the expression $x^2 + 5x + 6.25$ was a perfect square?

Responses vary.

- I decided this was a perfect square because I could draw a diagram with factors of $(x + 2.5)(x + 2.5)$.
- I decided this was a perfect square because I tried taking half of 5 to get 2.5, and then I squared 2.5, which is 6.25.

Unit 7 Lesson 13 809 Activity 1

Key Takeaway: A **perfect square** is an expression that can be represented as something multiplied by itself, e.g., $(x + 1)^2$. A quadratic expression written in standard form is a perfect square when it can be rewritten as a perfect square in factored form.

Pairs | 20 min

Activity 2 Completing Squares

Purpose: Students create perfect square expressions by determining a missing b- or c-value.

Short on time? Consider inviting students to complete fewer Repeated Challenges.

1 Launch

Demonstrate how to work with the algebra tiles.

Invite students to use algebra tiles to support their thinking.

Consider asking, "What does each tile represent?"

A **Accessibility: Fine Motor Skills** Encourage students to work in pairs and discuss together how to arrange the tiles.

2 Monitor

To support students getting started, consider asking pairs of students:

- "How could you arrange the purple rectangle tiles to make the area model a perfect square?"
- "What is each smudged number? Is there only one possibility for each?"
- "How is the area model related to the smudges?"
- "How are the values in factored form connected to the values in standard form?" **(MTR.5.1)**

Look for and celebrate the variety of correct answers, e.g., 121, 11^2, or $\left(\frac{22}{2}\right)^2$.

D **Differentiation**

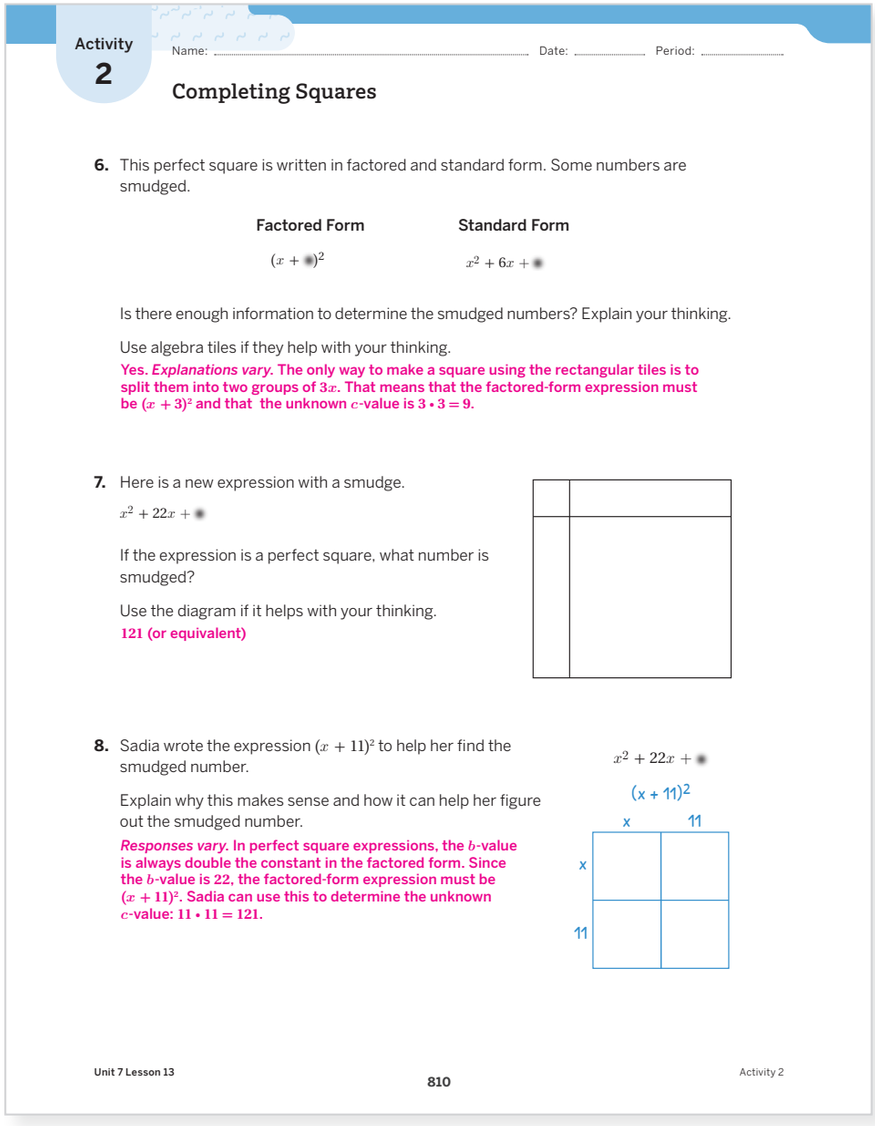
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How is this expression similar to the expression on the last problem? How is it different?"
Think that the number is 22^2 or 484.	**Support** Celebrate that they thought to square 22 and consider asking, "Why is this too many tiles for this area model?"
Guess and check.	**Strengthen** Encourage them to reflect before trying a new number. Consider asking, "Where do you see the 22 in the area model?"

Note: Consider pausing for a class discussion if enough pairs are struggling to find the c-value.

Consider asking pairs of students:

- "How is this diagram like the area model? How is it different?"
- "How can you tell this expression is a square?"
- "Where do you see the $22x$ in this diagram?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Completing Squares

6. This perfect square is written in factored and standard form. Some numbers are smudged.

Factored Form	Standard Form
$(x + \blacksquare)^2$	$x^2 + 6x + \blacksquare$

Is there enough information to determine the smudged numbers? Explain your thinking.

Use algebra tiles if they help with your thinking.

Yes. *Explanations vary.* The only way to make a square using the rectangular tiles is to split them into two groups of $3x$. That means that the factored-form expression must be $(x + 3)^2$ and that the unknown c-value is $3 \cdot 3 = 9$.

7. Here is a new expression with a smudge.

$x^2 + 22x + \blacksquare$

If the expression is a perfect square, what number is smudged?

Use the diagram if it helps with your thinking.

121 (or equivalent)

8. Sadia wrote the expression $(x + 11)^2$ to help her find the smudged number.

Explain why this makes sense and how it can help her figure out the smudged number.

Responses vary. In perfect square expressions, the b-value is always double the constant in the factored form. Since the b-value is 22, the factored-form expression must be $(x + 11)^2$. Sadia can use this to determine the unknown c-value: $11 \cdot 11 = 121$.

$x^2 + 22x + \blacksquare$

$(x + 11)^2$

x 11

x

11

Unit 7 Lesson 13 810 Activity 2

Pairs | 20 min

Activity 2 Completing Squares (continued)

Purpose: Students create perfect square expressions by determining a missing b- or c-value.

Short on time? Consider inviting students to complete fewer Repeated Challenges.

2 Monitor

Invite students to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly.

Look for a variety of interesting solutions to a variety of challenges to share during the Connect.

Students will need a partner for this activity. They will solve and compare their solutions to problems instead of receiving responsive feedback.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

To support students getting started, consider asking, "How can you use the area model to help you? Where would the x^2 go? The $-10x$?"

Listen for student strategies in their pair discussions and offer help or encouragement.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore perfect squares further to complete this optional task. Encourage them to discuss their thinking with a partner.

Note: Students will need copies of the You're invited to explore more Sheet.

3 Connect

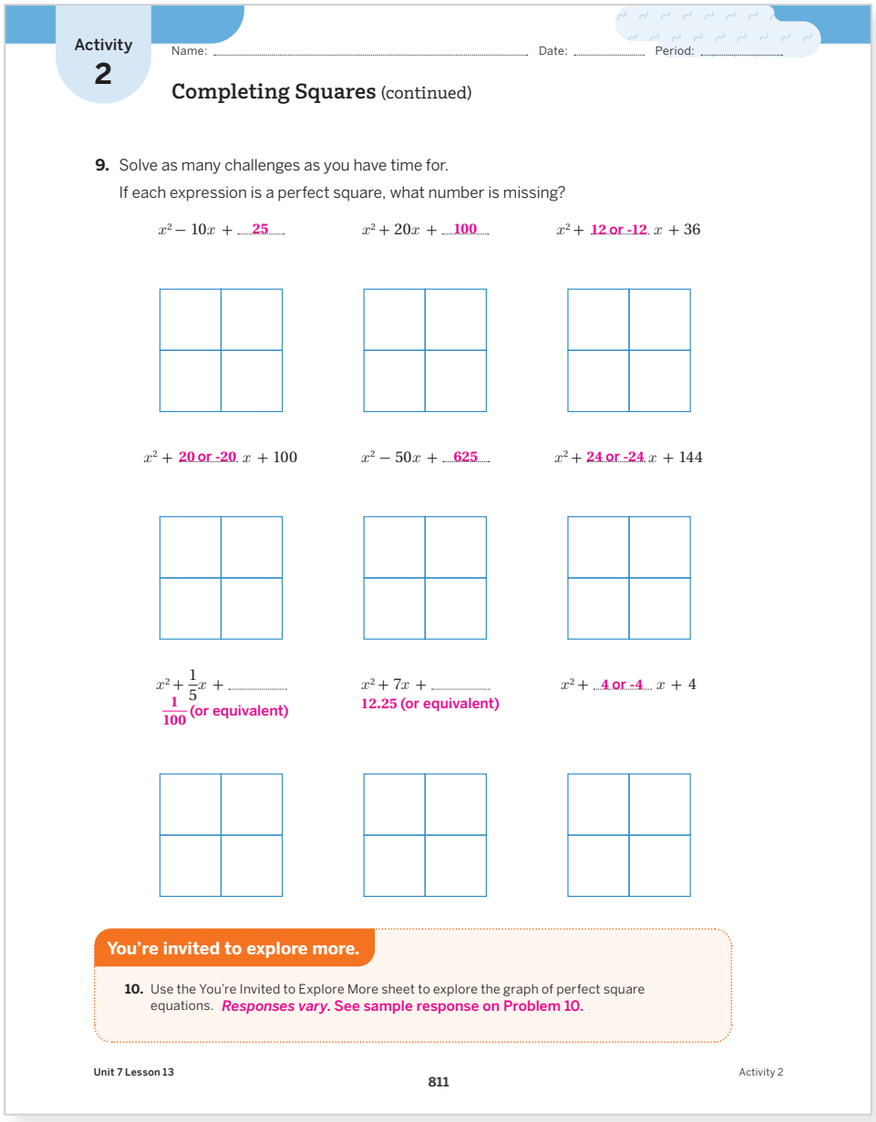
Invite students to share their explanations and sketches.

To surface the Key Takeaway, consider asking:

- "How can the diagram help us determine the c-value?"
- "How can I determine the c-value to make a perfect square expression if I was given $x^2 + 30x +$ ____? $x^2 + bx +$ ____?"

Invite students to share by polling the class to see which challenges they would like to discuss.

Note: Consider adding key ideas about perfect square expressions to the anchor chart.



Activity 2

Name: _____ Date: _____ Period: _____

Completing Squares (continued)

9. Solve as many challenges as you have time for.
If each expression is a perfect square, what number is missing?

$x^2 - 10x +$ 25 | $x^2 + 20x +$ 100 | $x^2 +$ 12 or -12 $x + 36$

$x^2 +$ 20 or -20 $x + 100$ | $x^2 - 50x +$ 625 | $x^2 +$ 24 or -24 $x + 144$

$x^2 + \frac{1}{5}x +$ $\frac{1}{100}$ (or equivalent) | $x^2 + 7x +$ 12.25 (or equivalent) | $x^2 +$ 4 or -4 $x + 4$

You're invited to explore more.

10. Use the You're Invited to Explore More sheet to explore the graph of perfect square equations. *Responses vary.* See sample response on Problem 10.

Unit 7 Lesson 13 811 Activity 2

Key Takeaway: A perfect square expression $x^2 + bx + c$ can be written in factored form as $(x + p)^2$, where p is half of b and $p^2 = c$.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize how to determine whether an expression is a perfect square. **(MTR.5.1)**

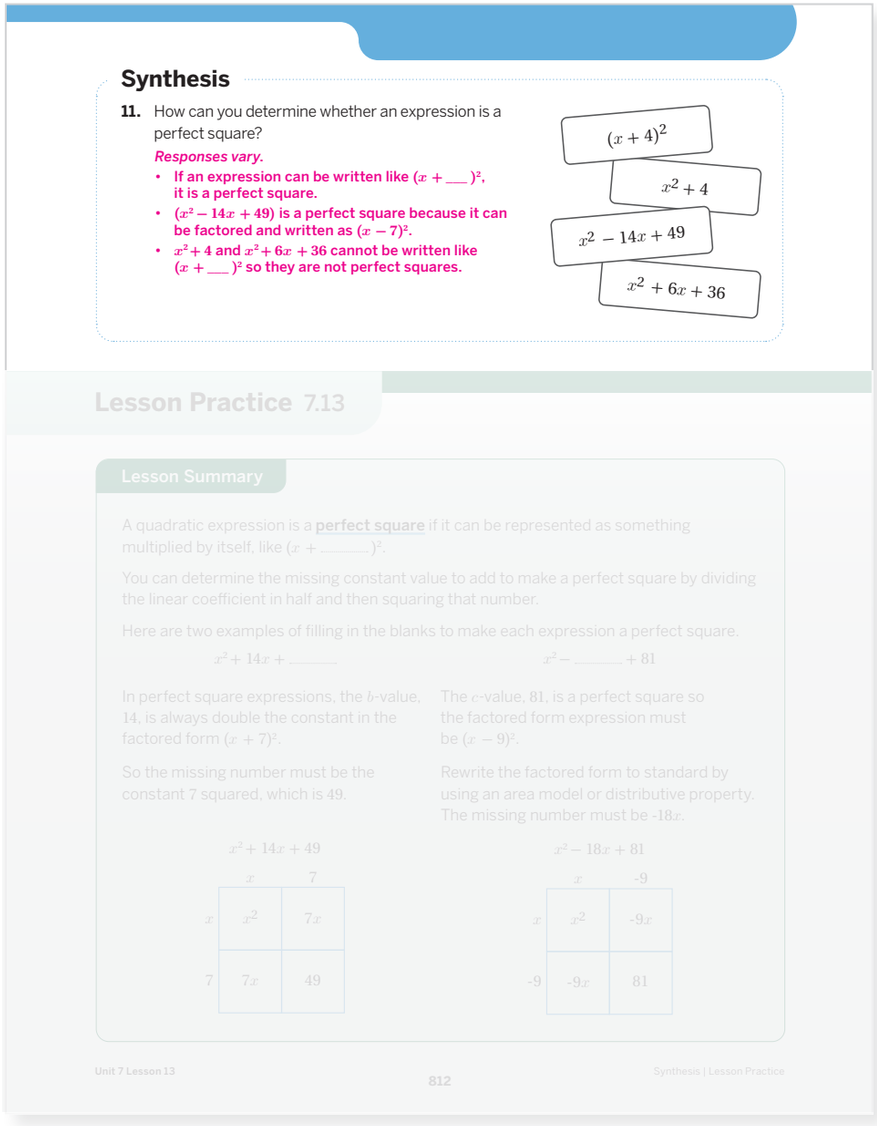
Invite students to respond independently, and then share their thinking with a partner.

A **Accessibility: Conceptual Processing** As students think abstractly, invite them to answer more concrete questions first, like, "Which of the examples is a perfect square expression? How do you know?"

Collect and share a variety of ideas, such as:
- It can be represented as a square area model.
- When factored, both factors are the same.
- It can be written as $(x + __)^2$.
- In standard form, the c-value is a positive perfect square and b is twice the number that you square to get c.

Consider asking:
- "What connections do you see between your classmates' strategies?"
- "If you know an expression is a perfect square, how can you determine an unknown c-value? b-value?"

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they share during the class discussion. Display for students to reference and add to for the rest of the unit.



Synthesis

11. How can you determine whether an expression is a perfect square?

Responses vary.
- If an expression can be written like $(x + __)^2$, it is a perfect square.
- $(x^2 - 14x + 49)$ is a perfect square because it can be factored and written as $(x - 7)^2$.
- $x^2 + 4$ and $x^2 + 6x + 36$ cannot be written like $(x + __)^2$ so they are not perfect squares.

$(x + 4)^2$

$x^2 + 4$

$x^2 - 14x + 49$

$x^2 + 6x + 36$

Lesson Practice 7.13

Lesson Summary

A quadratic expression is a **perfect square** if it can be represented as something multiplied by itself, like $(x + _____)^2$.

You can determine the missing constant value to add to make a perfect square by dividing the linear coefficient in half and then squaring that number.

Here are two examples of filling in the blanks to make each expression a perfect square.

$x^2 + 14x + _____$

In perfect square expressions, the b-value, 14, is always double the constant in the factored form $(x + 7)^2$.

So the missing number must be the constant 7 squared, which is 49.

$x^2 + 14x + 49$

	x	7
x	x^2	$7x$
7	$7x$	49

$x^2 - _____ + 81$

The c-value, 81, is a perfect square so the factored form expression must be $(x - 9)^2$.

Rewrite the factored form to standard by using an area model or distributive property. The missing number must be $-18x$.

$x^2 - 18x + 81$

	x	-9
x	x^2	$-9x$
-9	$-9x$	81

Unit 7 Lesson 13 812 Synthesis | Lesson Practice

Lesson Takeaway: A **perfect square** is an expression that can be represented as something multiplied by itself. A quadratic expression written in factored form is a perfect square when it is one factor multiplied by itself. A perfect square expression in standard form $x^2 + bx + c$ can be written in factored form as $(x + p)^2$, where p is half of b and $p^2 = c$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining missing b- and c-values that make a perfect square expression.

Today's Goals

1. **Goal:** Determine unknown values of b or c that make the expression $x^2 + bx + c$ a *perfect square*.
2. **Language Goal:** Justify whether a quadratic expression is a perfect square. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.13

Complete each expression so that it is a perfect square.

$x^2 + 8x +$ 16

$x^2 -$ $10x$ $+ 25$

$x^2 - 20x +$ 100

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support determining unknown values of b or c that make the expression $x^2 + bx + c$ a perfect square:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problem 9** as a class and invite students to share strategies for finding b- and c-values.
- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **You're invited to explore more** tasks and discuss their thinking with a partner.

Professional Learning

In this lesson, students extended their understanding of the term **perfect square** to quadratic expressions written in standard and factored form. How can you continue to support students in extending their algebraic thinking as they continue their study of quadratics in this course and more general polynomials in future courses? What other algebraic and numerical ideas can you support students in generalizing more broadly beyond the numerical setting where they may have first learned them (e.g., divisibility, factors/factoring)?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you determine whether an expression is a perfect square?

Responses vary.

- If an expression can be written like $(x + ____)^2$, it is a perfect square.
- $(x^2 - 14x + 49)$ is a perfect square because it can be factored and written as $(x - 7)^2$.
- $x^2 + 4$ and $x^2 + 6x + 36$ cannot be written like $(x + ____)^2$ so they are not perfect squares.

Lesson Practice 7.13

Lesson Summary

A quadratic expression is a **perfect square** if it can be represented as something multiplied by itself, like $(x + ____)^2$.

You can determine the missing constant value to add to make a perfect square by dividing the linear coefficient in half and then squaring that number.

Here are two examples of filling in the blanks to make each expression a perfect square.

$x^2 + 14x + ____$

In perfect square expressions, the b-value, 14, is always double the constant in the factored form $(x + 7)^2$.

So the missing number must be the constant 7 squared, which is 49.

$x^2 + 14x + 49$

	x	7
x	x^2	$7x$
7	$7x$	49

$x^2 - ____ + 81$

The c-value, 81, is a perfect square so the factored form expression must be $(x - 9)^2$.

Rewrite the factored form to standard by using an area model or distributive property. The missing number must be $-18x$.

$x^2 - 18x + 81$

	x	-9
x	x^2	$-9x$
-9	$-9x$	81

Lesson Practice 7.13

Name: ..... Date: ..... Period: .....

1. Which expression is equivalent to $(x - 6)^2$?

A. $(x + 6)^2$ B. $x^2 - 12x - 36$ C. $x^2 - 36$ D. $x^2 - 12x + 36$

2. Write each perfect square expression in factored form.

$x^2 + 6x + 9$ → $(x + 3)^2$

$x^2 - 16x + 64$ → $(x - 8)^2$

$x^2 - 12x + 36$ → $(x - 6)^2$

$x^2 + 5x + \frac{25}{4}$ → $\left(x + \frac{5}{2}\right)^2$ (or equivalent)

3. LaShawn claims that $x^2 + bx + \frac{b^2}{4}$ is a perfect square. Do you agree? Explain your thinking.

Yes. *Explanations vary.* To be a perfect square, the c-value needs to be equal to the b-value divided by 2 and squared. $\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$.

4. Daniela says that if a perfect square expression is written in the form $x^2 + bx + c$, the value of c cannot be negative. Why is this true?

Responses vary. If the quadratic expression is a perfect square, it can be written like $(x + h)^2$ or $(x - h)^2$. The c-value in $x^2 + bx + c$ is equal to h^2. Any number squared is either positive or 0.

Problems 5–8: Fill in the blanks to complete each perfect square.

5. $x^2 + 24x + \underline{144}$

6. $x^2 - 2x + \underline{1}$

7. $x^2 - \underline{16x} + 64$

8. $x^2 + \frac{2}{3}x + \underline{\frac{1}{9}}$

Lesson Practice 7.13

Name: ..... Date: ..... Period: .....

9. The expressions $(x - 4)^2$ and $(4 - x)^2$ are both perfect squares. Are they equivalent to one another? Explain your thinking.

Yes, they are equivalent. *Explanations vary.*

- I converted both expressions to standard form, and they both resulted in $x^2 - 8x + 16$.
- I graphed both expressions and noticed they were the same parabola.

Test Practice

10. Select *all* the expressions that are perfect squares.

- ☑ A. $x^2 + 10x + 25$
- ☑ B. $x^2 - 10x + 25$
- ☐ C. $x^2 + 4$
- ☑ D. $(x + 3.5)(3.5 + x)$
- ☐ E. $(x - 10)(10 - x)$
- ☑ F. $x^2 + \frac{1}{2}x + \frac{1}{16}$

Spiral Review

Problems 11–12: The equations $y = x^2 + 5x + 6$ and $y = (x + 2)(x + 3)$ are equivalent.

11. Which equation would you use to determine the x-intercepts? Explain your thinking.

Responses and explanations vary. I would use $y = (x + 2)(x + 3)$. To find the x-intercepts, I would set $y = 0$. Factored-form equations equal to 0 are easy for me to solve.

12. Which equation would you use to determine the y-intercept? Explain your thinking.

Responses and explanations vary. I would use $y = x^2 + 5x + 6$. To find the y-intercept, I would set $x = 0$. When I do that in a standard form equation like this one, the c-term is the y-intercept.

13. Without using a graphing calculator, select *all* the equations with a positive y-intercept.

- ☐ A. $y = x^2 + 3x - 2$
- ☑ B. $y = (x + 1)(x + 5)$
- ☐ C. $y = x^2 - 10x$
- ☑ D. $y = (x - 3)^2$
- ☐ E. $y = -5x^2 + 3x - 12$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.912.AR.1.7
	2	2	MA.912.AR.1.7
	3	3	MA.912.AR.1.7
	4, 9	2	MA.912.AR.1.7
	5–8	1	MA.912.AR.1.7
Test Practice	10	2	MA.912.AR.1.7
Spiral Review			
	11–13	2	MA.912.AR.1.7

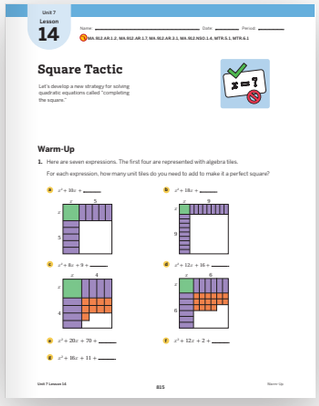
Unit 7

Lesson 14

Square Tactic

Solving by Completing the Square

Let's develop a new strategy for solving quadratic equations called "completing the square."


Unit 7
Lesson 14
Square Tactic
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Solve quadratic equations by *completing the square*.
2. **Language Goal:** Compare and contrast different strategies for solving quadratic equations. **(Speaking and Listening)**

Students develop a new strategy for solving quadratic equations, **completing the square**, which involves adding a constant to a quadratic expression to make it a perfect square and then taking the square root. Students use their understanding of the underlying structure of quadratic expressions to determine the most effective strategy for solving them. **(MTR.5.1)**

Coherence

Prior Learning

In Grade 8, students learned about square roots and solved problems using square roots. In Lessons 10–12, students identified the number of solutions for a quadratic equation and solved quadratics using square roots when applicable. In Lesson 12, students identified perfect square expressions and wrote equivalent standard-form quadratic expressions as perfect squares.

Future Learning

In Lesson 15, students will use the strategy of completing the square to rewrite quadratic expressions to reveal a function's vertex. In Lesson 16, students will derive the quadratic formula by completing the square.

Connections

- Students apply their understanding of perfect trinomial squares to solve quadratic equations by completing the square. (mathematical)
- Students compare, contrast, and discuss two different strategies for solving a quadratic equation. (mathematical, cross-curricular)
- Students explore how the method of completing the square to solve quadratic equations was shared in an ancient text. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.7
Rewrite a polynomial expression as a product of polynomials over the real number system.

MA.912.AR.3.1
Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Also Addressing: **MA.912.AR.1.2, MA.912.NSO.1.4**

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.1	**MA.912.AR.3.2**
MA.8.AR.2.3	
MA.8.NSO.1.1	

Vocabulary

New Vocabulary

completing the square

Lesson at a Glance

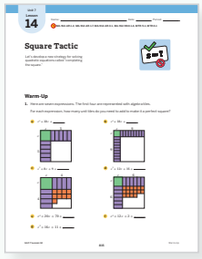
~ 45 min

MA.912.AR.1.2 , MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4

Warm-Up

Independent | 5 min

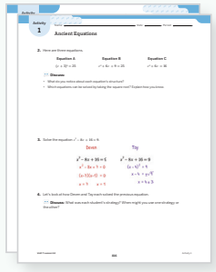
Students analyze the structure of a quadratic expression to determine what constant could be added to make it a perfect square. **(MTR.5.1)**



Activity 1

Pairs | 20 min

Students compare the strategies of factoring and solving using square roots, and learn about mathematician Muḥammad ibn Mūsā al-Khwarizmi. **Routines:** Think-Pair-Share; MLR7: Compare and Connect



Activity 2

Pairs | 10 min

Students repeatedly practice using a new strategy, called **completing the square**, to solve a variety of quadratic equations. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the advantages and disadvantages of different strategies for solving quadratic equations. MLR2: Collect and Display **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a quadratic equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculator
- Activity Sheet
- Algebra Tiles

Independent | 5 min

Warm-Up

Purpose: Students analyze the structure of a quadratic expression to determine what constant could be added to make it a perfect square. **(MTR.5.1)**

1 Launch

Consider asking:

- "What makes a quadratic expression a *perfect square*?"
- "Is $x^2 + 10x$ a perfect square? How would you explain your reasoning?"

ELL **English Language Learners** Consider spending time clarifying the phrase *unit tiles* by labeling each algebra tile with what it represents (e.g., labeling the larger square tile x^2, the rectangular tile $1x$, and the smaller square unit tile 1). Consider showing a few example constructions with labeled algebra tiles. **(Reading, Writing, Speaking, and Listening)**

Note: Consider having students work together on the first challenge before trying the remaining challenges on their own.

A **Accessibility: Visual-Spatial Processing** Invite pairs to think about key features of the figure and discuss them aloud before responding to support their reasoning.

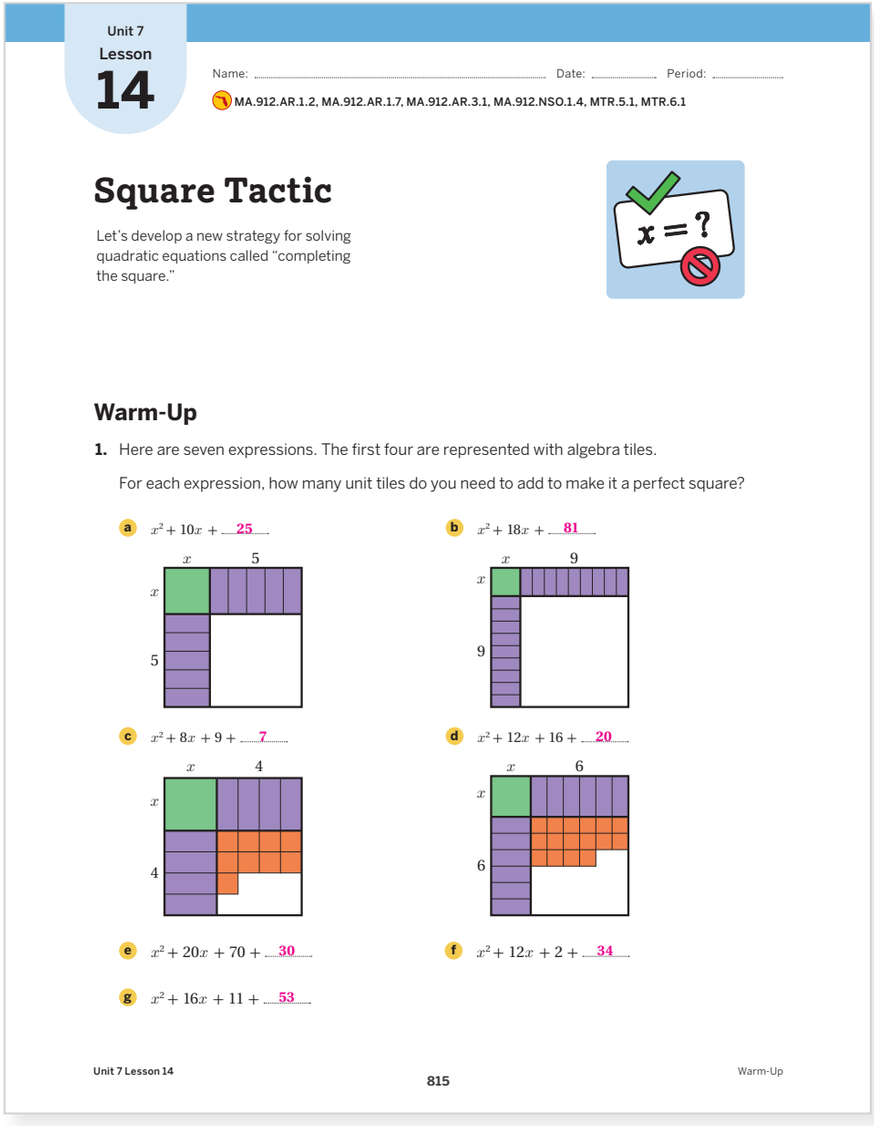
Invite students to use algebra tiles to construct each expression and check their work.

Encourage students to share the strategies they develop as they progress through the challenges.

2 Connect

Consider asking:

- "What strategies did you use to figure out how many tiles you needed?"
- "When we see a quadratic expression like $x^2 + 16x + 11$, what clues help us decide the number of tiles needed to create a perfect square?"
- "How does your strategy change if there is a constant in the quadratic expression?"



Unit 7
Lesson
14

Name: _____ Date: _____ Period: _____

MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4, MTR.5.1, MTR.6.1

Square Tactic

Let's develop a new strategy for solving quadratic equations called "completing the square."

Warm-Up

1. Here are seven expressions. The first four are represented with algebra tiles.

For each expression, how many unit tiles do you need to add to make it a perfect square?

a. $x^2 + 10x +$ 25

b. $x^2 + 18x +$ 81

c. $x^2 + 8x + 9 +$ 7

d. $x^2 + 12x + 16 +$ 20

e. $x^2 + 20x + 70 +$ 30

f. $x^2 + 12x + 2 +$ 34

g. $x^2 + 16x + 11 +$ 53

Unit 7 Lesson 14 815 Warm-Up

Pairs | 20 min

Activity 1 Ancient Equations

Purpose: Students compare the strategies of factoring and solving using square roots, and learn about mathematician Muḥammad ibn Mūsā al-Khwarizmi.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

A **Accessibility: Conceptual Processing** Invite students to consider two equations at a time. Consider asking, "How is Equation A like Equation B? How is B like C? How is A like C?"

Consider asking:

- "Could Equation B be rewritten as a perfect square? Equation C? How do you know?"**(MTR.5.1)**
- "What solving strategy would you use to find the solutions to each equation?"**(MTR.5.1)**

2 Monitor

To support students getting started, consider asking, "Is $x^2 - 8x + 16$ a perfect square? How do you know? What value when squared is equal to 16?"

Look for students who use strategies similar to Deven's and Tay's on the next problem or other creative strategies. Invite them to share during the pause.

Display Deven's and Tay's strategies.

Pause after pairs of students have had enough time to respond and discuss the prompts on this problem.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

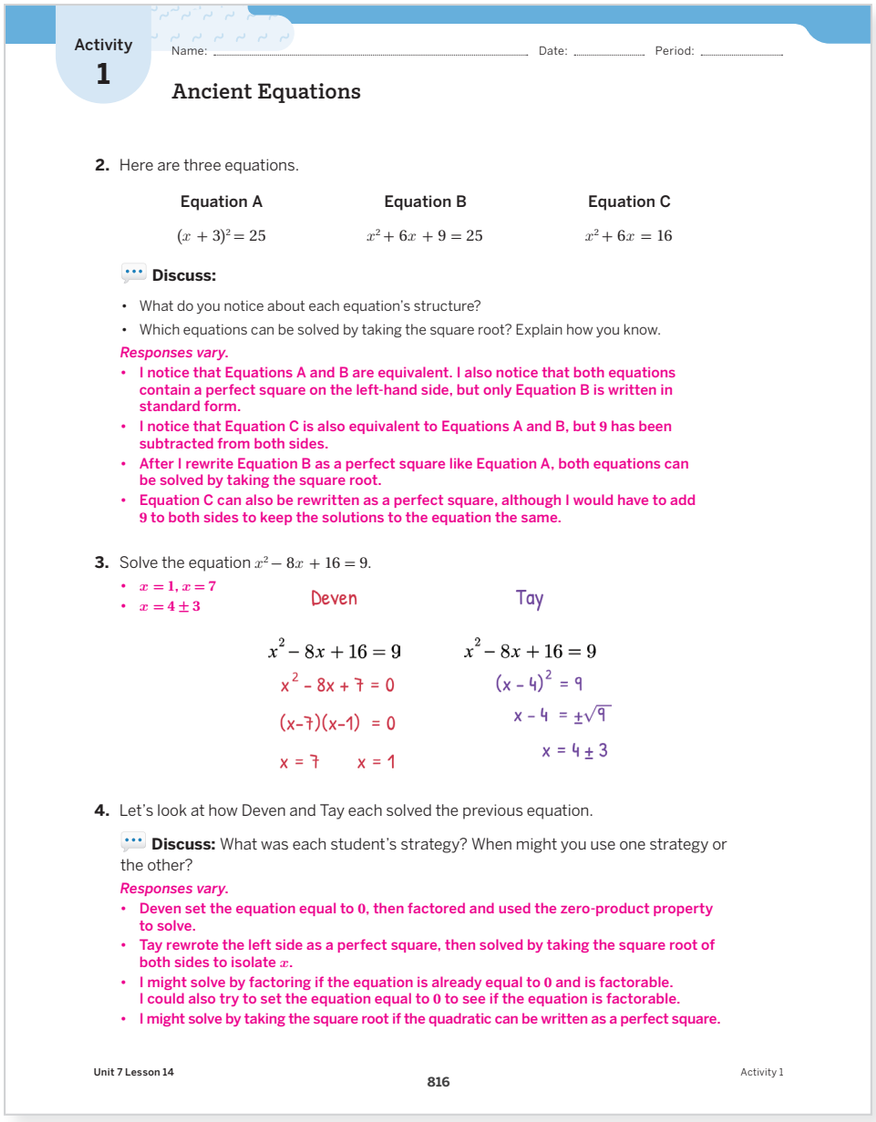
- "I know _____'s strategy works because . . ."
- "_____'s idea makes sense to me because . . ."
- "Another strategy would be _____ because . . ."

(Writing, Speaking, and Listening)

Consider asking:

- "How did Tay arrive at their solution?"
- "How did Deven and Tay write their two solutions?"
- "What might be important to remember from each student's work?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Ancient Equations

2. Here are three equations.

Equation A	Equation B	Equation C
$(x+3)^2 = 25$	$x^2 + 6x + 9 = 25$	$x^2 + 6x = 16$

Discuss:

- What do you notice about each equation's structure?
- Which equations can be solved by taking the square root? Explain how you know.

Responses vary.

- I notice that Equations A and B are equivalent. I also notice that both equations contain a perfect square on the left-hand side, but only Equation B is written in standard form.
- I notice that Equation C is also equivalent to Equations A and B, but 9 has been subtracted from both sides.
- After I rewrite Equation B as a perfect square like Equation A, both equations can be solved by taking the square root.
- Equation C can also be rewritten as a perfect square, although I would have to add 9 to both sides to keep the solutions to the equation the same.

3. Solve the equation $x^2 - 8x + 16 = 9$.

- $x = 1, x = 7$
- $x = 4 \pm 3$

Deven

$x^2 - 8x + 16 = 9$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 7 \quad x = 1$

Tay

$x^2 - 8x + 16 = 9$

$(x-4)^2 = 9$

$x - 4 = \pm\sqrt{9}$

$x = 4 \pm 3$

4. Let's look at how Deven and Tay each solved the previous equation.

Discuss: What was each student's strategy? When might you use one strategy or the other?

Responses vary.

- Deven set the equation equal to 0, then factored and used the zero-product property to solve.
- Tay rewrote the left side as a perfect square, then solved by taking the square root of both sides to isolate x.
- I might solve by factoring if the equation is already equal to 0 and is factorable. I could also try to set the equation equal to 0 to see if the equation is factorable.
- I might solve by taking the square root if the quadratic can be written as a perfect square.

Unit 7 Lesson 14 816 Activity 1

Activity 1 Ancient Equations (continued)

Purpose: Students compare the strategies of factoring and solving using square roots, and learn about mathematician Muḥammad ibn Mūsā al-Khwarizmi.

2 Monitor

Consider sharing more information about Muḥammad ibn Mūsā al-Khwarizmi, such as the pronunciation of his name and that he improved world maps, accurately predicted planetary movement, and was the namesake of the word *algorithm*.

Consider asking, "Are there any symbols on the document that look familiar?"

Listen for student ideas such as solving by factoring, graphing, using square roots, and trying different values.

Encourage students to share their ideas during Connect. Encourage students to show their thinking.

D **Differentiation: (Support)** To support students getting started, consider asking:
- "How would you describe his first step?"
- "Why did he choose to add 25 to the left? Would 35 or 36 have been as helpful? Why or why not?"
- "Does this new equation have the same or different answers as the first one? How do you know?" **(MTR.5.1)**

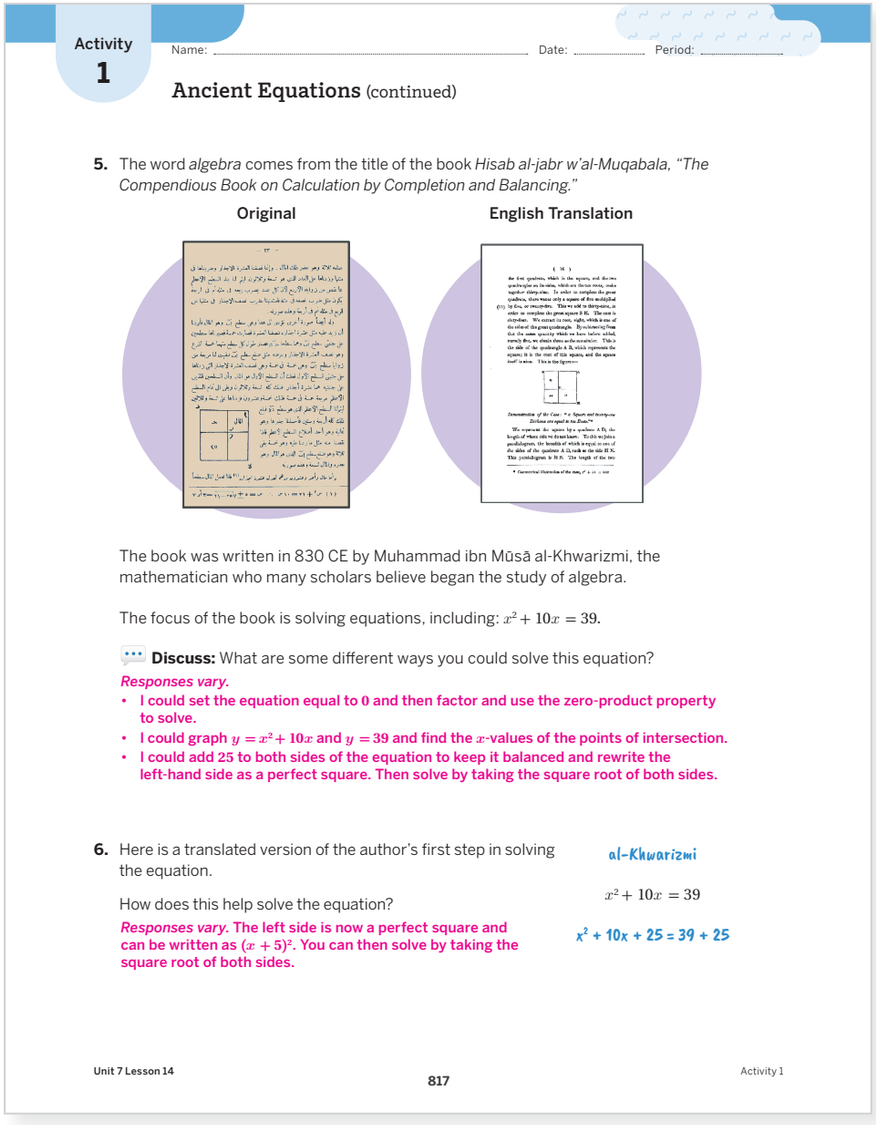
A **Accessibility: Conceptual Processing** As students make generalizations, invite them to revisit Deven's and Tay's strategies. Consider asking, "How are these strategies and al-Khwarizmi's strategies alike? How are they different?"

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze al-Khwarizmi's strategy, and then connect to their preferred strategy.

Share the student responses. Invite students to discuss their reasoning.

To surface the Key Takeaway, consider asking:
- "How is al-Khwarizmi's strategy connected to what we've already learned?"
- "What do you think his next steps would be?"
- "How can we determine what number to add?"
- "Why must we add that number to both sides?"



Activity 1

Name: _____ Date: _____ Period: _____

Ancient Equations (continued)

5. The word *algebra* comes from the title of the book *Hisab al-jabr w'al-Muqabala, "The Compendious Book on Calculation by Completion and Balancing."*

Original | English Translation

The book was written in 830 CE by Muhammad ibn Mūsā al-Khwarizmi, the mathematician who many scholars believe began the study of algebra.

The focus of the book is solving equations, including: $x^2 + 10x = 39$.

Discuss: What are some different ways you could solve this equation?

Responses vary.
- I could set the equation equal to 0 and then factor and use the zero-product property to solve.
- I could graph $y = x^2 + 10x$ and $y = 39$ and find the x-values of the points of intersection.
- I could add 25 to both sides of the equation to keep it balanced and rewrite the left-hand side as a perfect square. Then solve by taking the square root of both sides.

6. Here is a translated version of the author's first step in solving the equation.

al-Khwarizmi

$x^2 + 10x = 39$

$x^2 + 10x + 25 = 39 + 25$

How does this help solve the equation?

Responses vary. The left side is now a perfect square and can be written as $(x + 5)^2$. You can then solve by taking the square root of both sides.

Unit 7 Lesson 14 817 Activity 1

Key Takeaway: There are many strategies for solving quadratic equations, including factoring and solving using square roots. One other possible strategy related to solving using square roots is to add a number to both sides of an equation so that one side can be rewritten as a perfect square.

Pairs | 10 min

Activity 2 Completing the Square

Purpose: Students repeatedly practice using a new strategy, called **completing the square**, to solve a variety of quadratic equations. **(MTR.5.1)**

Short on time? Consider facilitating Problem 9 as a whole-class discussion instead of inviting pairs to work on it first.

1 Launch

Consider asking:

- "Why do you think people called al-Khwarizmi's strategy **completing the square**?"
- "What do you think a good first step could be?"

Invite students to check their answers with a partner or with other pairs of students.

2 Monitor

Consider asking pairs of students:

- "What would happen if we tried solving this using factoring instead?"
- "Why might we prefer to write our solutions to this equation using the plus/minus symbol (±)?"

A **Accessibility: Memory and Attention** To support students with keeping track of information, invite them to record their work for each problem on paper or in their Student Edition as they work.

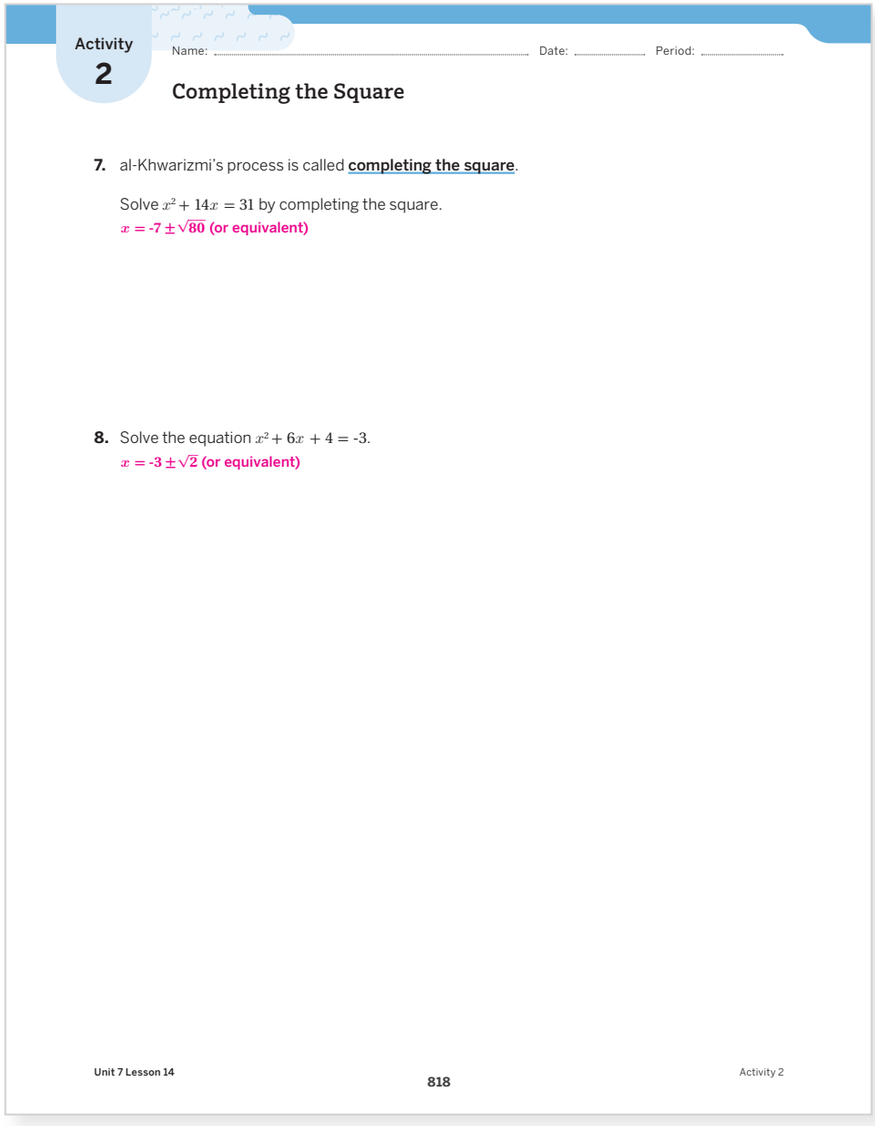
Consider asking pairs of students:

- "What does this equation have in common with the previous one? How is it different?"
- "Why won't factoring work as a solving strategy for this equation?" **(MTR.5.1)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started or use trial and error.	**Support** Invite students to revisit their work from the Warm-Up. Consider asking, "What number might make sense to add in this equation?"
Claim there is no right answer since it is not factorable or because there's a negative on the right side.	**Support** Invite students to solve graphically. Consider asking, "How many solutions are there? How can we use completing the square to determine them?"
Add 9 to both sides of the equation and do not obtain a correct answer.	**Strengthen** Celebrate their idea to add 9. Consider asking, "How might we write the expression $x^2 + 6x + 9$ in factored form? How can we use this to solve our new equation?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Completing the Square

7. al-Khwarizmi's process is called **completing the square**.

Solve $x^2 + 14x = 31$ by completing the square.

$x = -7 \pm \sqrt{80}$ (or equivalent)

8. Solve the equation $x^2 + 6x + 4 = -3$.

$x = -3 \pm \sqrt{2}$ (or equivalent)

Unit 7 Lesson 14 818 Activity 2

Pairs | 10 min

Activity 2 Completing the Square (continued)

Purpose: Students repeatedly practice using a new strategy, called **completing the square**, to solve a variety of quadratic equations. **(MTR.5.1)**

Short on time? Consider facilitating Problem 9 as a whole-class discussion instead of inviting pairs to work on it first.

2 Monitor

Encourage students to use paper and pencil to solve the problem on their own first and then compare their work to Roberto's work.

ELL **English Language Learners** As students process their ideas in words, invite them to show their thinking on paper before expressing their ideas in words. **(Writing and Speaking)**

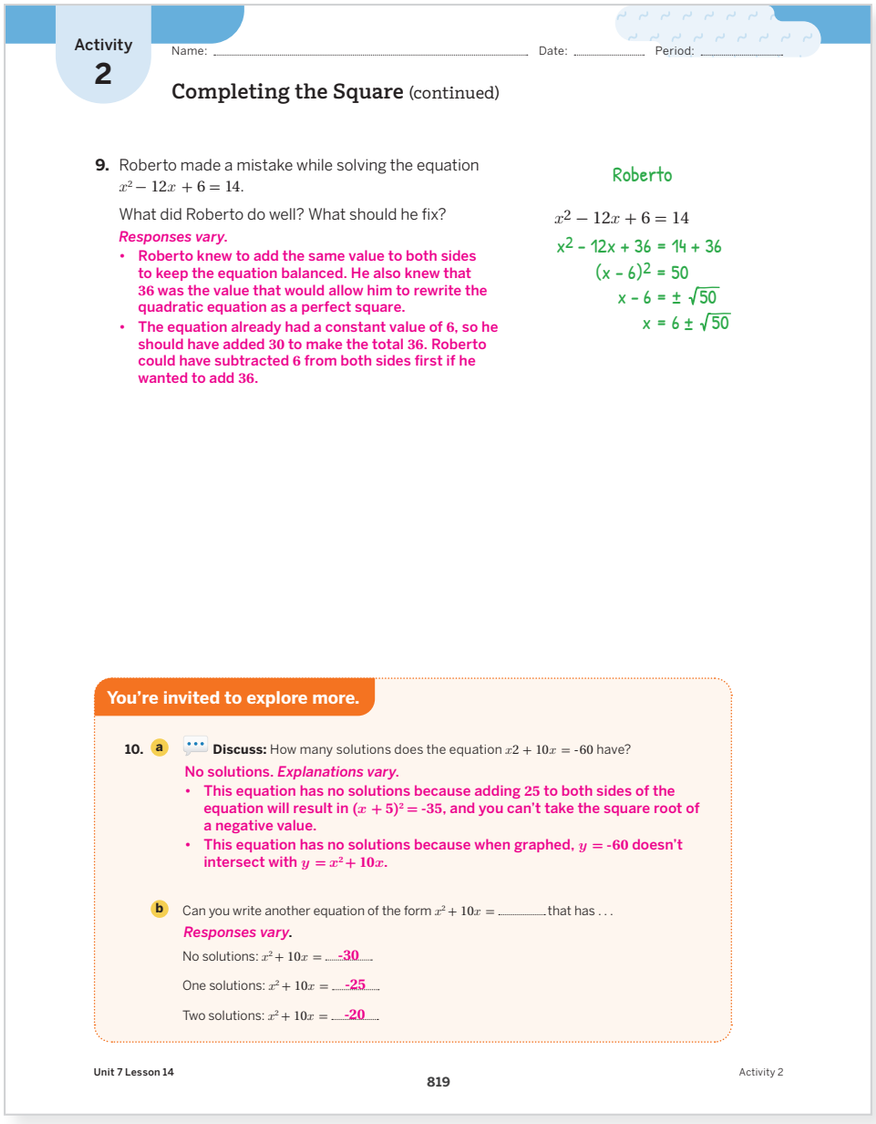
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore completing the square further to complete this optional task. Encourage them to discuss their thinking with a partner. Consider asking, "What is the constant that allows you to complete the square for this quadratic equation? Does knowing that number give you clues about how to create an equation with one solution? Two solutions? No solutions?"

3 Connect

To surface the Key Takeaway, consider asking:

- "Why can't we add any number to both sides of the equation to make this strategy work?"
- "Why might someone prefer using the strategy of completing the square to solve quadratic equations compared to other solving strategies?" Factoring doesn't work for every quadratic equation, but completing the square does. **(MTR.5.1)**

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Activity 2

Name: _____ Date: _____ Period: _____

Completing the Square (continued)

9. Roberto made a mistake while solving the equation $x^2 - 12x + 6 = 14$.

What did Roberto do well? What should he fix?

Responses vary.

- Roberto knew to add the same value to both sides to keep the equation balanced. He also knew that 36 was the value that would allow him to rewrite the quadratic equation as a perfect square.
- The equation already had a constant value of 6, so he should have added 30 to make the total 36. Roberto could have subtracted 6 from both sides first if he wanted to add 36.

Roberto

$x^2 - 12x + 6 = 14$

$x^2 - 12x + 36 = 14 + 36$

$(x - 6)^2 = 50$

$x - 6 = \pm\sqrt{50}$

$x = 6 \pm \sqrt{50}$

You're invited to explore more.

10. a **Discuss:** How many solutions does the equation $x2 + 10x = -60$ have?

No solutions. *Explanations vary.*

- This equation has no solutions because adding 25 to both sides of the equation will result in $(x + 5)^2 = -35$, and you can't take the square root of a negative value.
- This equation has no solutions because when graphed, $y = -60$ doesn't intersect with $y = x^2 + 10x$.

b Can you write another equation of the form $x^2 + 10x =$ _____ that has . . .

Responses vary.

No solutions: $x^2 + 10x =$ -30

One solutions: $x^2 + 10x =$ -25

Two solutions: $x^2 + 10x =$ -20

Unit 7 Lesson 14 819 Activity 2

Key Takeaway: **Completing the square** is the process of rewriting quadratic equations to include perfect squares, often by adding the same number to both sides. Once the perfect square is isolated, it is possible to take the square root of both sides to find solutions to any quadratic equation.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the advantages and disadvantages of different strategies for solving quadratic equations. **(MTR.5.1)**

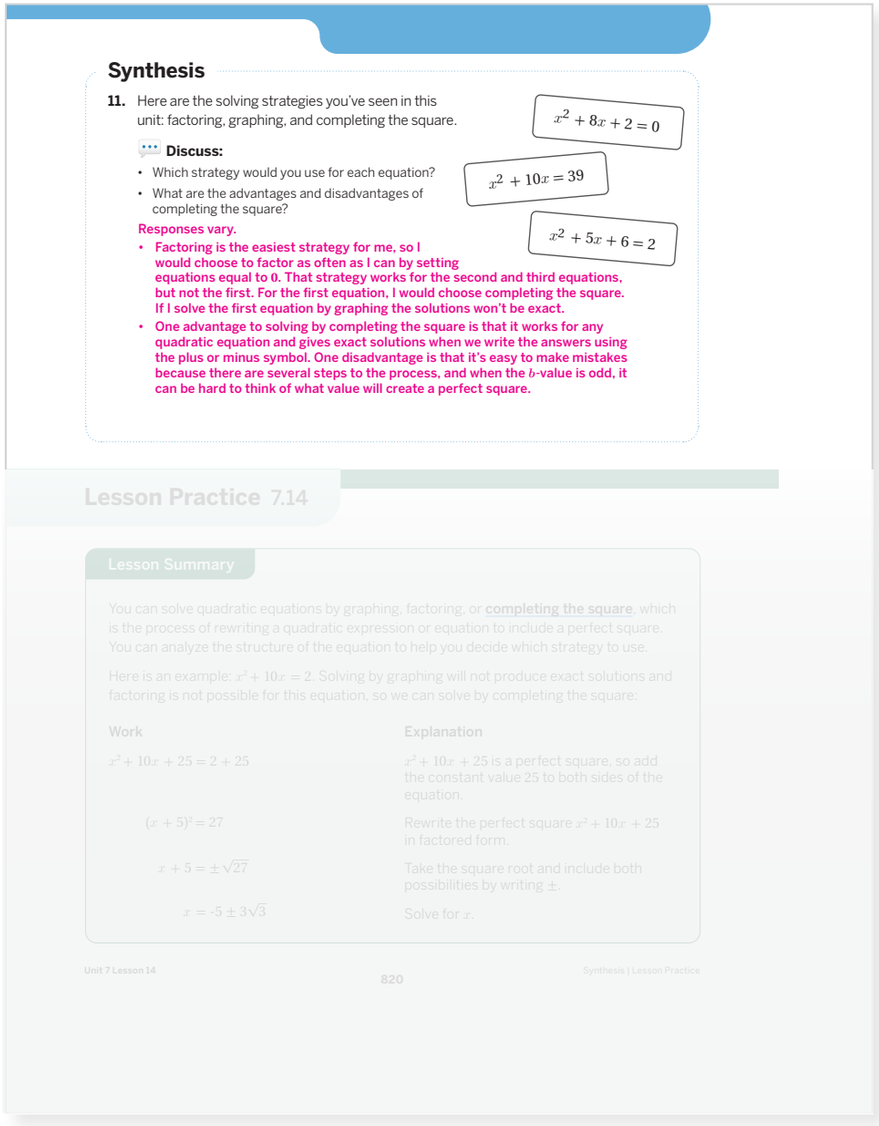
Invite students to discuss their thinking with a partner before responding.

Share a variety of ideas, such as:
- Factoring is a helpful strategy but doesn't allow us to solve all problems.
- Graphing shows us the number of solutions, allows us to see when we have no solutions, and gives us approximations, but doesn't always give exact solutions.
- Completing the square can help us solve any quadratic equation.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about different strategies for solving quadratic equations as they share with the class. Display for students to reference and add to for the rest of the unit.

Consider asking:
- "What makes sense to you about each strategy? What does not make sense?"
- "Which strategies can we use for the top equation? Middle equation? Bottom equation?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here are the solving strategies you've seen in this unit: factoring, graphing, and completing the square.

$x^2 + 8x + 2 = 0$

$x^2 + 10x = 39$

$x^2 + 5x + 6 = 2$

Discuss:
- Which strategy would you use for each equation?
- What are the advantages and disadvantages of completing the square?

Responses vary.
- Factoring is the easiest strategy for me, so I would choose to factor as often as I can by setting equations equal to 0. That strategy works for the second and third equations, but not the first. For the first equation, I would choose completing the square. If I solve the first equation by graphing the solutions won't be exact.
- One advantage to solving by completing the square is that it works for any quadratic equation and gives exact solutions when we write the answers using the plus or minus symbol. One disadvantage is that it's easy to make mistakes because there are several steps to the process, and when the b-value is odd, it can be hard to think of what value will create a perfect square.

Lesson Practice 7.14

Lesson Summary

You can solve quadratic equations by graphing, factoring, or completing the square, which is the process of rewriting a quadratic expression or equation to include a perfect square. You can analyze the structure of the equation to help you decide which strategy to use.

Here is an example: $x^2 + 10x = 2$. Solving by graphing will not produce exact solutions and factoring is not possible for this equation, so we can solve by completing the square:

Work	Explanation
$x^2 + 10x + 25 = 2 + 25$	$x^2 + 10x + 25$ is a perfect square, so add the constant value 25 to both sides of the equation.
$(x + 5)^2 = 27$	Rewrite the perfect square $x^2 + 10x + 25$ in factored form.
$x + 5 = \pm\sqrt{27}$	Take the square root and include both possibilities by writing $\pm$.
$x = -5 \pm 3\sqrt{3}$	Solve for x.

Unit 7 Lesson 14 820 Synthesis | Lesson Practice

Lesson Takeaway: There are many strategies for solving quadratic equations, including **completing the square**. It involves adding the same number to both sides to create a perfect square in factored form and then taking the square root. Although it requires careful algebra, this strategy works for any quadratic equation and gives exact solutions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving a quadratic equation.

Today's Goals

1. **Goal:** Solve quadratic equations by *completing the square*.
2. **Language Goal:** Compare and contrast different strategies for solving quadratic equations. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.14

Solve the equation $x^2 - 16x = -5$.

$x = 8 \pm \sqrt{59}$ (or equivalent)

D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support solving a quadratic equation using the completing the square method:

» **Respond to Student Thinking:**

- Use the ***Solving Quadratic Equations by Completing the Square*** **Mini-Lesson**.
- Review the **Lesson Summary** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **You're invited to explore more** tasks and discuss their thinking with a partner.

Professional Learning

Completing the square may be the first strategy students have seen for solving quadratic equations that will work for solving any quadratic equation. How can you support students in recognizing the value of mathematical tools and strategies that work across all possible instances as well as tools that may be easier or more efficient in special cases? How can you help students to build trust in themselves as mathematicians to make choices about what tools to use? **(MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are the solving strategies you've seen in this unit: factoring, graphing, and completing the square.

$x^2 + 8x + 2 = 0$

$x^2 + 10x = 39$

$x^2 + 5x + 6 = 2$

Discuss:

- Which strategy would you use for each equation?
- What are the advantages and disadvantages of completing the square?

Responses vary.

- Factoring is the easiest strategy for me, so I would choose to factor as often as I can by setting equations equal to 0. That strategy works for the second and third equations, but not the first. For the first equation, I would choose completing the square. If I solve the first equation by graphing the solutions won't be exact.
- One advantage to solving by completing the square is that it works for any quadratic equation and gives exact solutions when we write the answers using the plus or minus symbol. One disadvantage is that it's easy to make mistakes because there are several steps to the process, and when the b-value is odd, it can be hard to think of what value will create a perfect square.

Lesson Practice 7.14

Lesson Summary

You can solve quadratic equations by graphing, factoring, or **completing the square**, which is the process of rewriting a quadratic expression or equation to include a perfect square. You can analyze the structure of the equation to help you decide which strategy to use.

Here is an example: $x^2 + 10x = 2$. Solving by graphing will not produce exact solutions and factoring is not possible for this equation, so we can solve by completing the square:

Work	Explanation
$x^2 + 10x + 25 = 2 + 25$	$x^2 + 10x + 25$ is a perfect square, so add the constant value 25 to both sides of the equation.
$(x + 5)^2 = 27$	Rewrite the perfect square $x^2 + 10x + 25$ in factored form.
$x + 5 = \pm\sqrt{27}$	Take the square root and include both possibilities by writing $\pm$.
$x = -5 \pm 3\sqrt{3}$	Solve for x.

Lesson Practice 7.14

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are perfect squares.

- ☑ A. $(x + 5)(5 + x)$
- ☑ B. $(x - 3)^2$
- ☐ C. $x^2 - 3^2$
- ☐ D. $x^2 + 8x + 64$
- ☑ E. $x^2 + 10x + 25$

2. Add the number that would make the expression a perfect square. Then write an equivalent expression in factored form.

$x^2 - 6x +$ 9 ~~$(x - 3)^2$~~

$x^2 + 2x +$ 1 ~~$(x + 1)^2$~~

$x^2 - 14x +$ 49 ~~$(x - 7)^2$~~

3. Match each equation to an equivalent equation.

- a. $x^2 - 12x = 6$ — b. $(x - 6)^2 = 30$
- b. $x^2 - 12x + 6 = 0$ — d. $(x - 3)^2 = 42$
- c. $x^2 - 6x = 6$ — a. $(x - 6)^2 = 42$
- d. $x^2 - 6x = 33$ — c. $(x - 3)^2 = 15$

4. Alexis solved the equation $x^2 + 12x = 13$ by completing the square, but some parts are blank. Fill in the blanks.

$x^2 + 12x = 13$

$x^2 + 12x + 36 = 13 + 36$ (or equivalent)

$(x + 6)^2 = 49$

$x + 6 = \pm 7$

$x =$ -13 and $x =$ 1

Lesson Practice 7.14

Name: ..... Date: ..... Period: .....

Problems 5–7: Solve each equation by completing the square.

5. $x^2 - 2x = 8$
$x = -2$ and $x = 4$

6. $7 = x^2 + 4x - 1$
$x = -2 \pm 2\sqrt{3}$

7. $x^2 - 18x + 60 = -11$
$x = 9 \pm \sqrt{10}$

Test Practice

8. Write a quadratic equation of the form $x^2 + bx + c = 0$ with solutions that are $x = 5 - \sqrt{2}$ and $x = 5 + \sqrt{2}$.
$x^2 - 10x + 23 = 0$

Spiral Review

9. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x^2 + 144 = 0$	No solutions
$x^2 - 144 = 0$	Two solutions
$(x - 7)^2 = 0$	One solution

10. The graph of $y = (x - 1)^2 + 4$ is the same as the graph of $y = x^2$, but:

- (A.) It is shifted 1 unit to the right and 4 units up.
- B. It is shifted 1 unit to the left and 4 units up.
- C. It is shifted 1 unit to the right and 4 units down.
- D. It is shifted 1 unit to the left and 4 units down.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.AR.1.7
	3	2	MA.912.AR.3.1
	4	2	MA.912.AR.3.1
Fluency	5	3	MA.912.AR.3.1
	6–7	2	MA.912.AR.3.1
Test Practice	8	2	MA.912.AR.3.5
Spiral Review			
	9	2	MA.912.AR.3.1
	10	2	MA.912.F.1

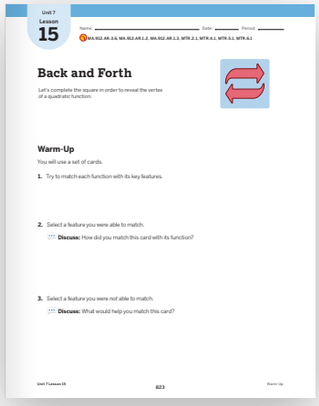
Unit 7

Lesson 15

Back and Forth

Rewriting Quadratic Expressions in Vertex Form

Let's complete the square in order to reveal the vertex of a quadratic function.


Unit 7
Lesson 15
Back and Forth
Let's complete the square in order to reveal the vertex of a quadratic function.
Warm-Up
You will use a set of cards.
1. Try to match each function with its key features.
2. Select a feature you were able to match.
Discuss: How did you match this card with its function?
3. Select a feature you were not able to match.
Discuss: What would help you match this card?
823

Learning Outcomes

Today's Goals

1. **Goal:** Determine key features of graphs of quadratic functions using their equations.
2. **Goal:** Complete the square in order to reveal the vertex of a quadratic function.
3. **Language Goal:** Describe a strategy for rewriting quadratic equations in different forms. **(Writing, Speaking, and Listening)**

Students complete the square in order to reveal the vertex of a quadratic function. They make sense of different strategies to complete the square and discuss how to maintain equivalence throughout the process. Students match functions in standard, factored, and vertex forms to their corresponding key features (y-intercept, x-intercepts, and vertex) Strategies. **(MTR.2.1, MTR.5.1)**

Coherence

Prior Learning

In Unit 6, students explored the standard, factored, and vertex forms of quadratic functions and determined key features from each form. In Lesson 14, students solved quadratic equations by completing the square.

Future Learning

In Lesson 14, students will complete the square to derive the quadratic formula.

Connections

- Students relate the process of completing the square to the vertex form of a quadratic function. (mathematical)
- Students analyze and discuss the thinking of others to make sense of a strategy for rewriting the standard form of a quadratic function in vertex form. (mathematical, cross-curriculum)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.6

Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

Also Addressing: **MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.1.2	**MA.912.AR.3.2**
MA.8.AR.2.3	**MA.912.AR.6.4**
	MA.912.AR.6.5

Vocabulary

Review Vocabulary

vertex form

Lesson at a Glance

~ 45 min

MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6

Warm-Up

Pairs | 5 min

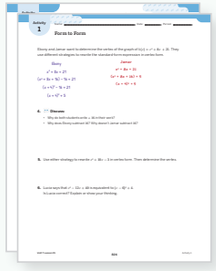
Students revisit how the different structures of a quadratic equation highlight different key features of its graph. MLR2: Collect and Display (MTR.2.1)



Activity 1

Pairs | 15 min

Students explore how completing the square can be used to rewrite a standard-form function in vertex form by making sense of two different strategies. MLR7: Compare and Connect (MTR.2.1, MTR.5.1)



Activity 2

Pairs | 15 min

Students rewrite quadratic functions in different structures in order to identify their x-intercepts, y-intercept, and vertex. (MTR.2.1, MTR.5.1)



Synthesis

Whole Class | 5 min

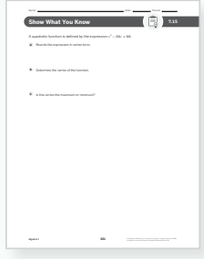
Students synthesize their understanding of rewriting standard-form expressions in vertex form.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by rewriting a standard-form expression in vertex form and identifying the vertex of the function.



Prep Checklist

Prepare Warm-Up and Activity 2 Cards

This lesson includes:

- Student Edition
- Show What You Know
- Copy Warm-Up and Activity 2 Cards, 1 set per pair, pre-cut
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students revisit how the different structures of a quadratic equation highlight different key features of its graph. **(MTR.2.1, MTR.5.1)**

Short on time? Consider omitting functions $f(x)$ and $g(x)$.

1 Launch

Distribute the pre-cut cards to each pair of students.

Invite pairs to match key features to functions without graphing or performing any calculations.

Note: Ensure that students know they're not expected to match all of the cards during the Warm-Up.

A **Accessibility: Executive Functioning** Consider chunking this activity into more manageable parts by inviting students to choose one function at a time. Consider asking, "Which key feature is revealed from this equation form? Where can you see it in the equation?" **(MTR.2.1, MTR.5.1)**

Pause briefly for students to share their thinking for Problems 2 and 3.

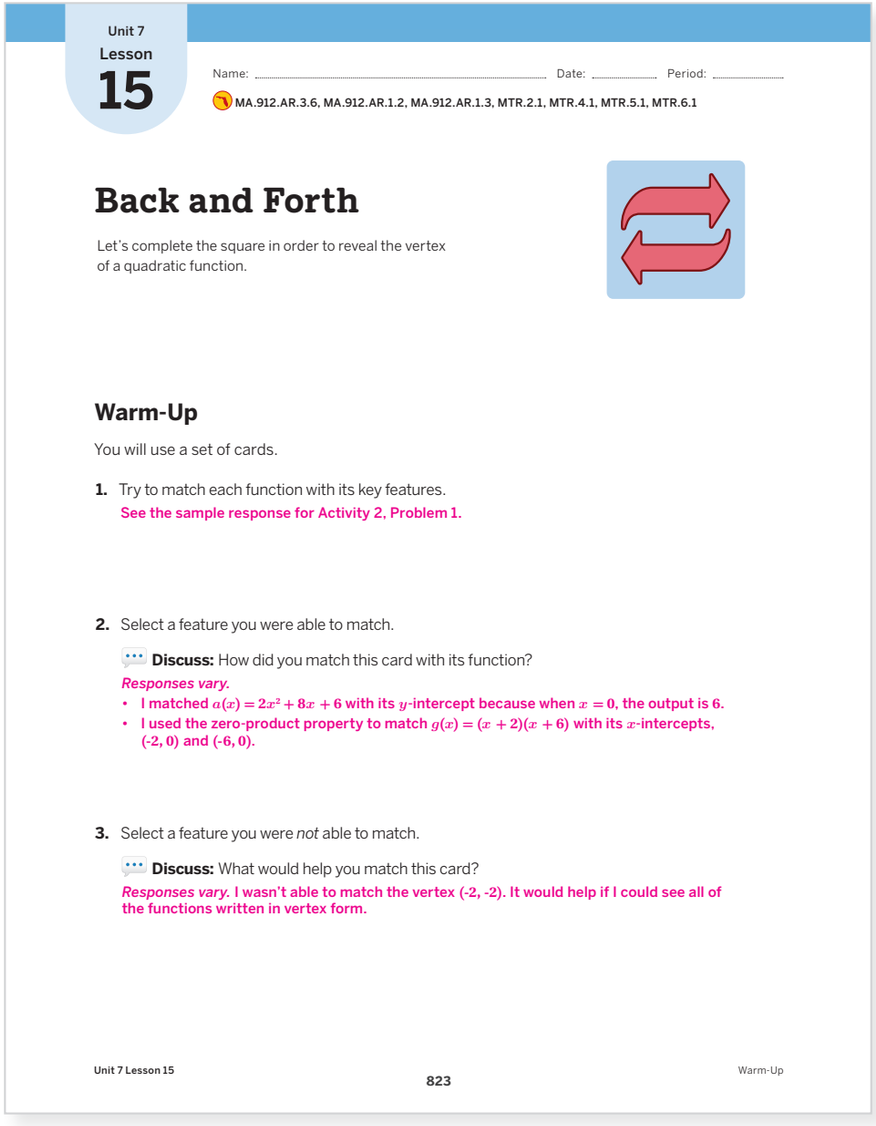
2 Connect

To surface connections, consider asking:

- "What form is each equation in?" **(MTR.5.1)**
- "Which key feature is revealed in each form? How can you see it?" **(MTR.2.1, MTR.5.1)**
- "How can you determine the x-intercepts from standard form?"
- "What ideas do you have about how we can determine the vertex from standard form?"

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about which key features are revealed in standard form, factored form, and vertex form. Display for students to reference throughout the lesson. **(MTR.2.1, MTR.5.1)**

Consider sharing that students will finish the card sort during Activity 2.



Unit 7
Lesson
15

Name: _____ Date: _____ Period: _____

MA.912.AR.3.6, MA.912.AR.1.2, MA.912.AR.1.3, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

Back and Forth

Let's complete the square in order to reveal the vertex of a quadratic function.

Warm-Up

You will use a set of cards.

1. Try to match each function with its key features.
   See the sample response for Activity 2, Problem 1.

2. Select a feature you were able to match.

   Discuss: How did you match this card with its function?

   Responses vary.
   - I matched $a(x) = 2x^2 + 8x + 6$ with its y-intercept because when $x = 0$, the output is 6.
   - I used the zero-product property to match $g(x) = (x + 2)(x + 6)$ with its x-intercepts, (-2, 0) and (-6, 0).

3. Select a feature you were *not* able to match.

   Discuss: What would help you match this card?

   Responses vary. I wasn't able to match the vertex (-2, -2). It would help if I could see all of the functions written in vertex form.

Unit 7 Lesson 15 823 Warm-Up

Pairs | 15 min

Activity 1 Form to Form

Purpose: Students explore how completing the square can be used to rewrite a standard-form function in vertex form by making sense of two different strategies. **(MTR.2.1, MTR.5.1)**

1 Launch

Draw students' attention to Ebony and Jamar's strategies.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy.

Consider asking:
- "How are their strategies alike? How are they different?"
- "Where do you see the same information in Ebony's and Jamar's strategy?"

A **Accessibility: Executive Functioning** Throughout this activity, support students in making sense of multiple steps by inviting them to cover student work and reveal/analyze one line at a time.

2 Monitor

To support students getting started, consider asking, "What number would make $x^2 + 10x$ into a perfect square quadratic expression?" Invite students to draw a diagram if it helps with their thinking.

D **Differentiation: (Support)** Students might add 25 in a way that does not keep the expression equivalent. Consider celebrating their strategic choice in picking the value 25. After they engage with Problem 6, invite them to use their strategy from Problem 6 to check their answer for Problem 5.

Look for a variety of strategies students use to justify their thinking for Problem 6. **(MTR.4.1)**
- Rewrite the vertex form to see if it matches the standard form.
- Compare the graphs of the functions defined by each expression.
- Complete the square in standard form to see if it matches the vertex form.

Pause briefly for students to share their strategies for Problems 5 and 6.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Form to Form

Ebony and Jamar want to determine the vertex of the graph of $h(x) = x^2 + 8x + 21$. They use different strategies to rewrite the standard-form expression in *vertex form*.

Ebony

$x^2 + 8x + 21$

$(x^2 + 8x + 16) - 16 + 21$

$(x + 4)^2 - 16 + 21$

$(x + 4)^2 + 5$

Jamar

$x^2 + 8x + 21$

$(x^2 + 8x + 16) + 5$

$(x + 4)^2 + 5$

4. Discuss:
- Why do both students write $+ 16$ in their work?
- Why does Ebony subtract 16? Why doesn't Jamar subtract 16?

Responses vary.
- Adding 16 to $x^2 + 8x$ completes the square in order to rewrite the expression as $(x + 4)^2$.
- Ebony introduces a 16 to the expression to complete the square, so then she has to subtract a 16 to keep the value of the expression equivalent. Jamar doesn't have to subtract 16 because he just splits the 21 that was already there into 16 and 5.

5. Use either strategy to rewrite $x^2 + 10x - 3$ in vertex form. Then determine the vertex.

Vertex form: $(x + 5)^2 - 28$
Vertex: (-5, -28)

6. Lucia says that $x^2 - 12x + 40$ is equivalent to $(x - 6)^2 + 4$. Is Lucia correct? Explain or show your thinking.

Yes. *Explanations vary.*
- You can complete the square to rewrite the expression as $(x^2 - 12x + 36) + 4$, which is equivalent to $(x - 6)^2 + 4$.
- I multiplied $(x - 6)^2$ to get $x^2 - 12x + 36$. If you add 4, you get the original expression.

Unit 7 Lesson 15 824 Activity 1

Pairs | 15 min

Activity 1 Form to Form (continued)

Purpose: Students explore how completing the square can be used to rewrite a standard-form function in vertex form by making sense of two different strategies. **(MTR.2.1, MTR.5.1)**

2 Monitor

A **Accessibility: Conceptual Processing** Invite students to share how each expression is different from the previous one to help apply concepts to new situations.

D **Differentiation** *(Problems 7–8)*

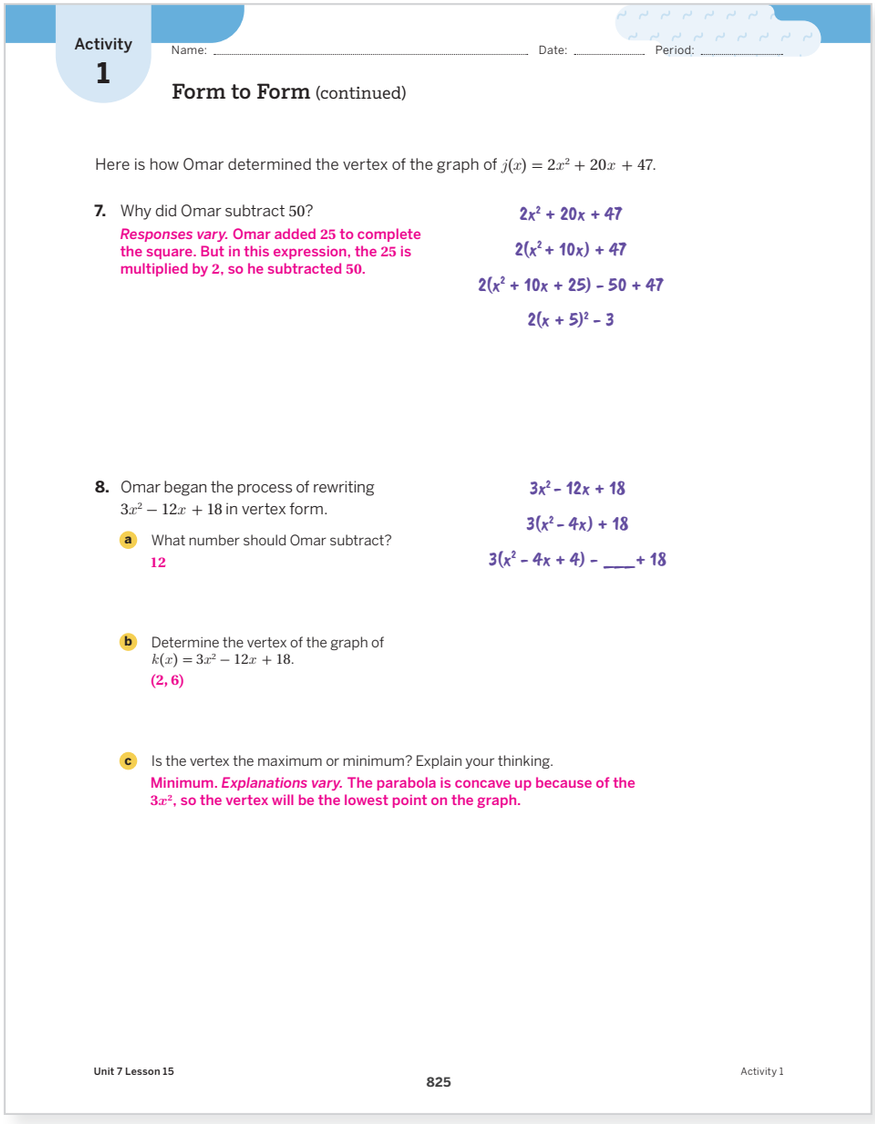
Look for students who:	Respond to Student Thinking
Need support getting started. (*Problem 7*)	**Support** Consider asking, "How is Omar's expression different from the ones in Problems 5 and 6?"
Subtract 4 instead of 12. (*Problem 8*)	**Support** Consider asking, "Why did Omar subtract 50 and not 25 in Problem 7?"
Would like to explore the connections between quadratic equations and their graphs further.	**Stretch** Consider inviting students to sketch a graph of one of the functions from this activity and describe how the function compares to $y = x^2$.

3 Connect

To surface the Key Takeaway, consider asking:

- "In Problems 4–6, where do you see the strategy of completing the square?"
- "Why is it helpful to complete the square in these cases?"
- "How does the strategy change in Problems 7–8 when the a-value is not equal to 1?" **(MTR.2.1, MTR.5.1)**

ELL **English Language Learners** Invite students to discuss and respond in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Writing, Speaking, and Listening)**



Activity 1

Name: _____ Date: _____ Period: _____

Form to Form (continued)

Here is how Omar determined the vertex of the graph of $j(x) = 2x^2 + 20x + 47$.

7. Why did Omar subtract 50?

Responses vary. Omar added 25 to complete the square. But in this expression, the 25 is multiplied by 2, so he subtracted 50.

$2x^2 + 20x + 47$

$2(x^2 + 10x) + 47$

$2(x^2 + 10x + 25) - 50 + 47$

$2(x + 5)^2 - 3$

8. Omar began the process of rewriting $3x^2 - 12x + 18$ in vertex form.

a What number should Omar subtract?

12

$3x^2 - 12x + 18$

$3(x^2 - 4x) + 18$

$3(x^2 - 4x + 4) - ___ + 18$

b Determine the vertex of the graph of $k(x) = 3x^2 - 12x + 18$.

(2, 6)

c Is the vertex the maximum or minimum? Explain your thinking.

Minimum. *Explanations vary.* The parabola is concave up because of the $3x^2$, so the vertex will be the lowest point on the graph.

Unit 7 Lesson 15 825 Activity 1

Key Takeaway: Completing the square to rewrite a standard-form quadratic expression in vertex form is a useful strategy to determine the vertex.

Pairs | 15 min

Activity 2 Card Sort

Purpose: Students rewrite quadratic functions in different structures in order to identify their x-intercepts, y-intercept, and vertex. **(MTR.2.1, MTR.5.1)**

Short on time? IConsider omitting functions $f(x)$ and $g(x)$.

1 Launch

To surface prior knowledge, consider asking, "How can rewriting expressions in different forms help us determine key features?"

A **Accessibility: Memory and Attention** Invite students to refer to the chart created in the Warm-Up to activate prior knowledge.

2 Monitor

Encourage students to use scientific calculators and to draw diagrams if it helps them with their thinking.

Look for students who are using a variety of strategies to justify their thinking: **(MTR.4.1)**

- Completing the square to determine the vertex.
- Factoring to determine the x-intercepts.
- Rewriting the expression in standard form or substituting 0 for x to determine the y-intercept.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore relating the structure of an equation and key features of its graph further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their strategies for matching each function to its key features.

To surface the Key Takeaway, consider asking:

- "How do you see each key feature in the equation?" **(MTR.2.1, MTR.5.1)**
- "Which key features were the most challenging to match? Why?"
- "Which key features were you able to match by completing the square?"

Activity 2

Name: _____ Date: _____ Period: _____

Card Sort

Look at the set of cards that you used during the Warm-Up.

9. Continue to match each function with *two* key features of its graph. Record your thinking in the table.

$a(x) = 2x^2 + 8x + 6$	y-intercept: (0, 6)	x-intercepts: (-3, 0) and (-1, 0)*	vertex: (-2, -2)
$b(x) = (2x + 6)(x - 3)$	y-intercept: (0, -18)	x-intercepts: (-3, 0) and (3, 0)	vertex: (0, -18)*
$c(x) = 3(x - 2)^2 + 6$	y-intercept: (0, 18)	x-intercepts: None*	vertex: (2, 6)
$d(x) = x^2 - 5x - 6$	y-intercept: (0, -6)	x-intercepts: (-1, 0) and (6, 0)	vertex: (2.5, -12.25)*
$f(x) = x^2 + 4x + 10$	y-intercept: (0, 10)	x-intercepts: None*	vertex: (-2, 6)
$g(x) = (x + 2)(x + 6)$	y-intercept: (0, 12)*	x-intercepts: (-2, 0) and (-6, 0)	vertex: (-4, -4)

Responses with an asterisk (*) are the solutions to Problem 11.

10. **Discuss:** What strategies did you use to match the cards this time?

Responses vary. To match the vertex, I rewrote the expression in vertex form. To match the x-intercepts, I substituted the x-values into the equations to see if they resulted in an output of 0.

You're invited to explore more.

11. In Activity 2, each function matches with two key features. For each function, determine the missing key feature: its x-intercept, y-intercept, or vertex.

See responses with an asterisk (*) in Problem 9.

Unit 7 Lesson 15 826 Activity 2

Key Takeaway: A quadratic function can be written in equivalent forms to reveal key features. Factored form reveals the zeros, standard form reveals the y-intercept, and vertex form reveals the vertex.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of rewriting standard-form expressions in vertex form.

Invite students to respond independently, and then share their thinking with a partner. **(MTR.4.1)**

Gather and share a variety of ideas, such as:

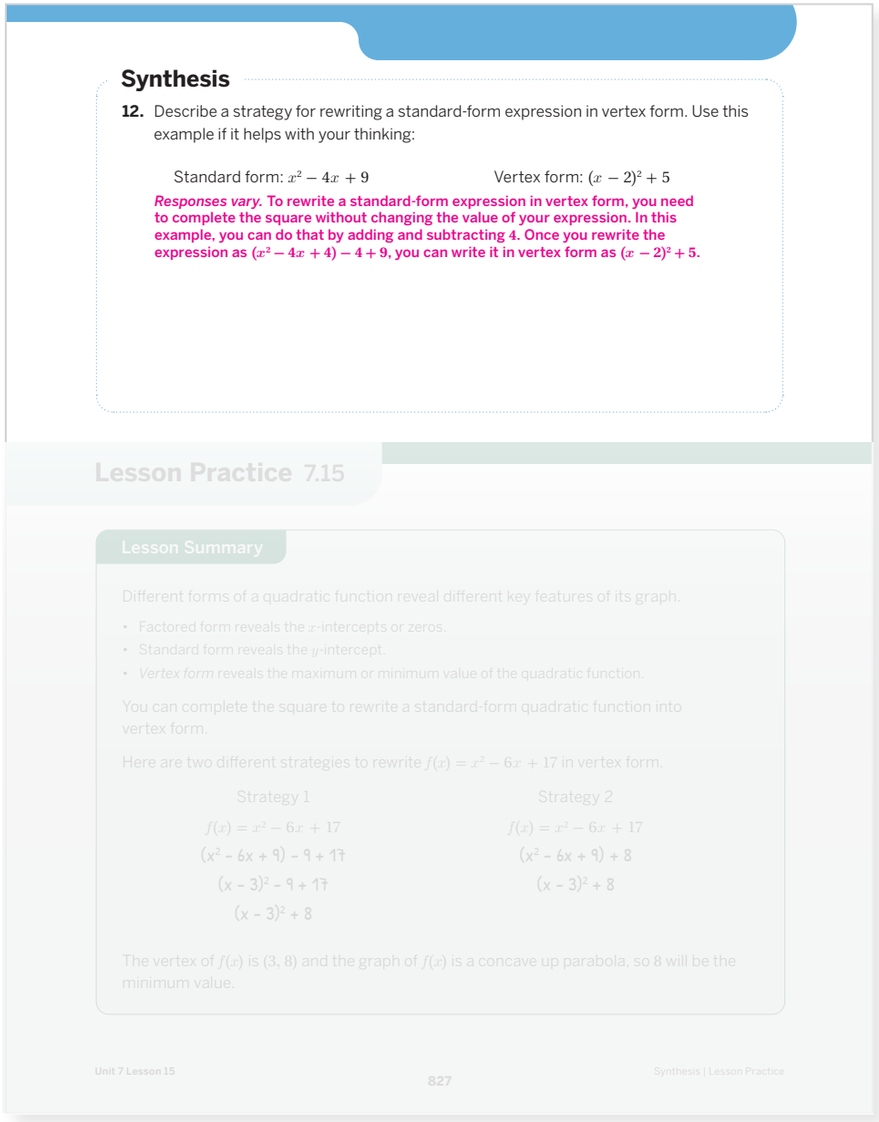
- To rewrite a standard-form expression in vertex form, you can complete the square without changing the value of your expression.
- In this example, you can add and subtract 4, $(x^2 - 4x + 4) - 4 + 9$, then complete the square.
- In this example, you can rewrite the expression as $(x^2 - 4x + 4) + 5$, then complete the square.

Consider asking,

- "How can you make sure the standard form and vertex form are equivalent?"
- "What makes sense to you about each strategy? What doesn't make sense?"
- "What connections do you see between your classmates' strategies?"

ELL **English Language Learners** Consider offering students the option to use a scribe or to verbally share their response instead of writing it. **(Writing and Speaking)**

Note: If time allows, invite students to revise their previous thinking based on the discussion. **(MTR.4.1, MTR.6.1)**



Synthesis

12. Describe a strategy for rewriting a standard-form expression in vertex form. Use this example if it helps with your thinking:

Standard form: $x^2 - 4x + 9$ Vertex form: $(x - 2)^2 + 5$

Responses vary. To rewrite a standard-form expression in vertex form, you need to complete the square without changing the value of your expression. In this example, you can do that by adding and subtracting 4. Once you rewrite the expression as $(x^2 - 4x + 4) - 4 + 9$, you can write it in vertex form as $(x - 2)^2 + 5$.

Lesson Practice 7.15

Lesson Summary

Different forms of a quadratic function reveal different key features of its graph.

- Factored form reveals the x-intercepts or zeros.
- Standard form reveals the y-intercept.
- *Vertex form* reveals the maximum or minimum value of the quadratic function.

You can complete the square to rewrite a standard-form quadratic function into vertex form.

Here are two different strategies to rewrite $f(x) = x^2 - 6x + 17$ in vertex form.

Strategy 1

$f(x) = x^2 - 6x + 17$
$(x^2 - 6x + 9) - 9 + 17$
$(x - 3)^2 - 9 + 17$
$(x - 3)^2 + 8$

Strategy 2

$f(x) = x^2 - 6x + 17$
$(x^2 - 6x + 9) + 8$
$(x - 3)^2 + 8$

The vertex of $f(x)$ is (3, 8) and the graph of $f(x)$ is a concave up parabola, so 8 will be the minimum value.

Unit 7 Lesson 15 827 Synthesis | Lesson Practice

Lesson Takeaway: Rewriting functions in different forms reveals different key features of the graph. Completing the square to rewrite a standard-form quadratic expression in vertex form is a useful strategy to determine the vertex.

Independent | 5 min

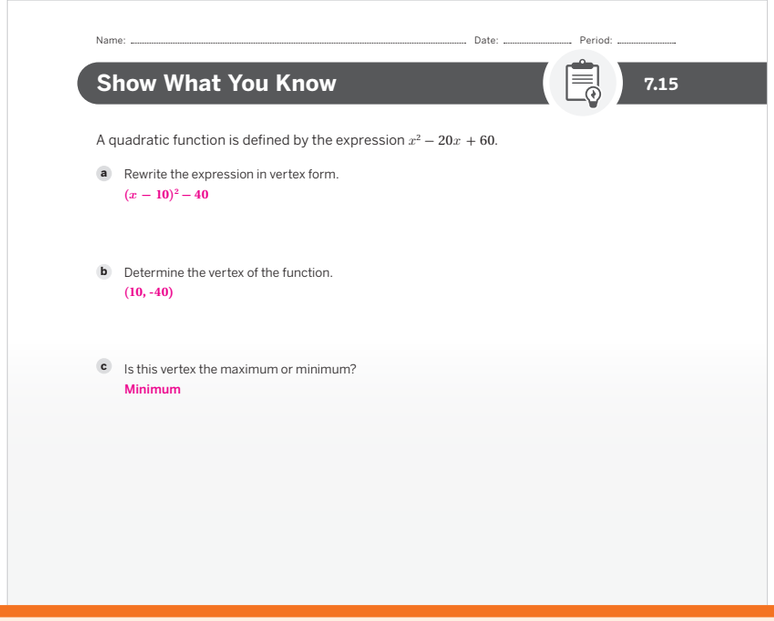
Show What You Know

Purpose: Students demonstrate their understanding by rewriting a standard-form expression in vertex form and identifying the vertex of the function.

Today's Goals

1. **Goal:** Determine key features of graphs of quadratic functions using their equations.
2. **Goal:** Complete the square in order to reveal the vertex of a quadratic function.
3. **Language Goal:** Describe a strategy for rewriting quadratic equations in different forms. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.15

A quadratic function is defined by the expression $x^2 - 20x + 60$.

a. Rewrite the expression in vertex form.
$(x - 10)^2 - 40$

b. Determine the vertex of the function.
$(10, -40)$

c. Is this vertex the maximum or minimum?
Minimum

D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support rewriting a quadratic expression from standard form to vertex form:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and provide feedback on targeted **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **You're invited to explore more** tasks and discuss their thinking with a partner.

Professional Learning

In this lesson, students expanded on their knowledge of identifying key features from quadratic equations in different forms. What strategies and strengths did your students exhibit today? How were they able to build from those strengths to support them in rewriting standard-form expressions in vertex form? What connections were students able to make about the symbolic process of completing the square and what it means graphically? **(MTR.2.1, MTR.5.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe a strategy for rewriting a standard-form expression in vertex form. Use this example if it helps with your thinking.

Standard form: $x^2 - 4x + 9$ Vertex form: $(x - 2)^2 + 5$

Responses vary. To rewrite a standard-form expression in vertex form, you need to complete the square without changing the value of your expression. In this example, you can do that by adding and subtracting 4. Once you rewrite the expression as $(x^2 - 4x + 4) - 4 + 9$, you can write it in vertex form as $(x - 2)^2 + 5$.

Lesson Practice 7.15

Lesson Summary

Different forms of a quadratic function reveal different key features of its graph.

- Factored form reveals the x-intercepts or zeros.
- Standard form reveals the y-intercept.
- *Vertex form* reveals the maximum or minimum value of the quadratic function.

You can complete the square to rewrite a standard-form quadratic function into vertex form.

Here are two different strategies to rewrite $f(x) = x^2 - 6x + 17$ in vertex form.

Strategy 1	Strategy 2
$f(x) = x^2 - 6x + 17$	$f(x) = x^2 - 6x + 17$
$(x^2 - 6x + 9) - 9 + 17$	$(x^2 - 6x + 9) + 8$
$(x - 3)^2 - 9 + 17$	$(x - 3)^2 + 8$
$(x - 3)^2 + 8$	

The vertex of $f(x)$ is (3, 8) and the graph of $f(x)$ is a concave up parabola, so 8 will be the minimum value.

Lesson Practice 7.15

Name: _____ Date: _____ Period: _____

Problems 1–3: Each of these equations represents the same function in an equivalent form. Complete the table with which feature of the function's graph (vertex, y-intercept, x-intercepts) can easily be determined and the coordinates.

	Function form	Feature of function's graph	Coordinates
1.	$f(x) = x^2 - 20x + 21$	y-intercept	(0, 21)
2.	$f(x) = (x - 7)(x - 3)$	x-intercepts	(3, 0) and (7, 0)
3.	$f(x) = (x - 5)^2 - 4$	vertex	(5, -4)

Problems 4–5: Gabriela made a mistake while rewriting the expression $x^2 - 12x + 10$ in vertex form.

$x^2 - 12x + 10$
$x^2 - 12x + 36 + 10$
$(x - 6)^2 + 10$

4. Describe Gabriela's mistake.

Responses vary. Gabriela added 36 to the expression, which changed the expression's value. She can only add 36 if she also subtracts 36.

5. Rewrite $x^2 - 12x + 10$ in vertex form. Show your work.

$(x - 6)^2 - 26$.

Work varies.
$x^2 - 12x + 10$
$x^2 - 12x + 36 - 36 + 10$

Problems 6–9: Complete the table by writing each expression in the missing form.

	Vertex Form	Standard Form	Factored Form
6.	$(x + 3)^2 - 16$	$x^2 + 6x - 7$	$(x + 7)(x - 1)$
7.	$(x - 3)^2 - 9$	$x^2 - 6x$	$x(x - 6)$
8.	$(x + 7)^2 - 9$	$x^2 + 14x + 40$	$(x + 10)(x + 4)$
9.	$2(x - 4)^2 - 50$	$2x^2 - 16x - 18$	$2(x - 9)(x + 1)$

Lesson Practice 7.15

Name: _____ Date: _____ Period: _____

10. Using the digits 0 to 9, without repeating, fill in the boxes to create three equations that produce the exact same parabola.

Responses vary.

$y = (x + \boxed{3})^2 - \boxed{1}$
$y = (x + \boxed{2})(x + \boxed{4})$
$y = x^2 + \boxed{6}x + \boxed{8}$

Test Practice

11. Select *all* the expressions that are equivalent to $x^2 - 16x$.

- ☑ A. $x(x - 16)$
- ☐ B. $(x - 4)(x + 4)$
- ☑ C. $x^2 - 16x + 64 - 64$
- ☐ D. $(x - 4)^2$
- ☑ E. $(x - 8)^2 - 64$

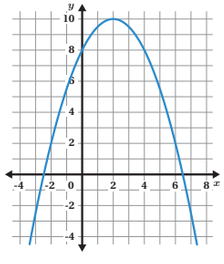
Spiral Review

12. Fill in the blanks to complete each perfect square.

$x^2 - 18x + \boxed{81}$
$x^2 + \frac{4}{3}x + \boxed{\frac{4}{9}}$

13. Determine the following values of the quadratic function $g(x) = -\frac{1}{2}x^2 + 2x + 8$.

$g(4) =$ 8
$g(-2) =$ -2
$g(x) = -2.5$; $x =$ -3, or 7



Lesson Practice Item Analysis

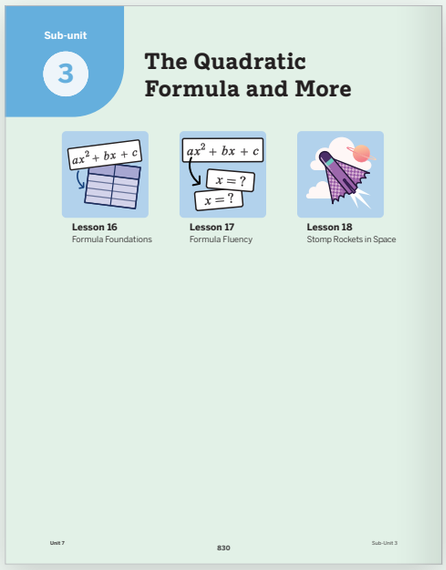
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.912.AR.3.6
	4–5	2	MA.912.AR.1.2 MA.912.AR.3.4
	6–9	2	MA.912.AR.1.2 MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.4
	10	3	MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.4
Test Practice	11	2	MA.912.AR.1.3 MA.912.AR.1.7 MA.912.AR.3.4
Spiral Review			
Fluency	12	1	MA.912.AR.1.7
	13	1	MA.912.F.1.2

The Quadratic Formula and More

Sub-Unit 3 Goals

- **Goal:** Derive the quadratic formula and use it to solve quadratic equations.
- **Language Goal:** Interpret the solutions to quadratic equations in context. **(Writing, Speaking and Listening)**

Students solve equations using the **quadratic formula**, explore sums and products of irrational and rational numbers, and solve systems of equations.



Math That Matters Most

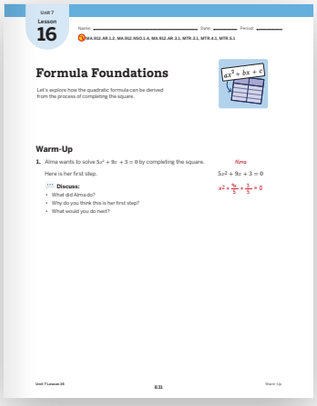
Concept or Skill	Example
Solve quadratic equations using the quadratic formula	Solve the equation: $0 = 3x^2 - 5x + 2$. $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$ $x = \frac{5 \pm \sqrt{1}}{6}$ $x = \frac{2}{3}$ and $x = 1$
Write equations to solve problems in a quadratic context	The function $h(t) = -1.5t^2 + 12t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched. Write an equation that can be solved to determine when the rocket will touch the ground. $0 = -1.5t^2 + 12t + 8$ When will the rocket touch the ground?
Interpret solutions in context	When I solve the equation I get $t \approx -0.619$ and $t \approx 8.619$. Since going back in time doesn't make sense in this situation, the rocket touches the ground at approximately 8.619 seconds.

Concept or Skill	Example
Make arguments about why the sums and products of rational and irrational numbers are always, sometimes, or never rational	Determine if the following statement is true or false and explain why: *The sum of a rational number and a rational number is always rational.* True. Rational numbers can be written as fractions of integers. To add two fractions, I can create a common denominator and then add the numerators, making a new fraction.
Solve systems of linear and quadratic equations	Solve the system of equations: $y = x^2 - 5x + 3$ $y = 2x - 7$ $y = \boxed{2x - 7}$ $y = x^2 - 5x + 3$ $2x - 7 = x^2 - 5x + 3$ $0 = x^2 - 7x + 10$ $0 = (x - 5)(x - 2)$ $\boxed{x = 5}$ $\boxed{x = 2}$ $y = 2(5) - 7$ $y = 2(2) - 7$ $\boxed{y = 3}$ $\boxed{y = -3}$ Points of intersection: $\boxed{(5, 3)}$ $\boxed{(2, -3)}$

Unit 7

Lesson

16


Unit 7
Lesson
16
Formula Foundations
Let's explore how the quadratic formula can be derived from the process of completing the square.
Warm-Up
1. Alma wants to solve $5x^2 + 9x + 3 = 0$ by completing the square.
Here is her first step.
Alma
$5x^2 + 9x + 3 = 0$
$x^2 + \frac{9}{5}x + \frac{3}{5} = 0$
Discuss:
• What did Alma do?
• Why do you think this is her first step?
• What would you do next?
8.31

Formula Foundations

Introducing the Quadratic Formula

Let's explore how the quadratic formula can be derived from the process of completing the square.

Learning Outcomes

Today's Goals

1. **Goal:** Use the *quadratic formula* to solve quadratic equations written in standard form.
2. **Goal:** Understand that the quadratic formula can be derived by generalizing the process of completing the square.
3. **Language Goal:** Describe how to use the quadratic formula and what parts of the formula represent. **(Speaking and Listening)**

Students explore how the **quadratic formula** can be derived from the process of completing the square. They also reason abstractly and quantitatively to make sense of the structure of the quadratic formula and determine whether equations have zero, one, or two solutions. **(MTR.5.1)**

Coherence

Prior Learning

In Lessons 9–14, students solved quadratic equations by factoring, graphing, and completing the square.

Future Learning

In Lessons 17 and 18, students will use the quadratic formula to solve quadratic equations and interpret the solutions in context. In Algebra 2, students will solve problems by completing the square for quadratic equations that have real and complex solutions.

Connections

- Students apply their understanding of completing the square to derive the quadratic formula through a series of guided activities. (mathematical)
- Students build an understanding and appreciation for formulas as a tool for efficient problem solving. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.1.2
Rearrange equations or formulas to isolate a quantity of interest.

MA.912.AR.3.1
Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Also Addressing: **MA.912.NSO.1.4**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.8.AR.2.1	**MA.912.AR.3.2**
MA.8.AR.2.3	

Vocabulary

New Vocabulary

quadratic formula

Lesson at a Glance

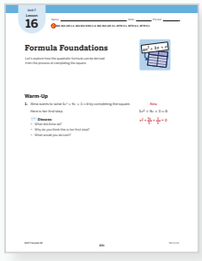
~ 45 min

MA.912.AR.1.2, MA.912.AR.3.1, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

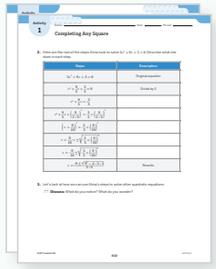
Students make sense of the first step of completing the square to solve a quadratic equation.



Activity 1

Pairs | 10 min

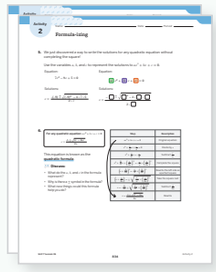
Students describe the steps of solving a quadratic equation by completing the square and notice those steps can be used to solve any quadratic equation. **Routine:** Notice and Wonder **(MTR.5.1)**



Activity 2

Pairs | 8 min

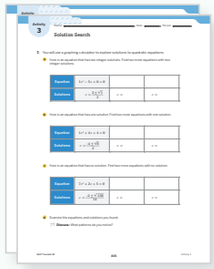
Students are introduced to the **quadratic formula** as a way to solve any quadratic equation without going through the process of completing the square. MLR2: Collect and Display



Activity 3

Pairs | 12 min

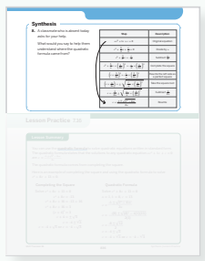
Students explore solutions to different quadratic equations and describe patterns in quadratic equations that have zero, one, or two solutions. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize the connection between completing the square and the quadratic formula. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of the quadratic formula's relationship with completing the square by explaining why the solutions to both approaches are equivalent.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students make sense of the first step of completing the square to solve a quadratic equation.

1 Launch

Consider asking, "What are some different ways we can solve quadratic equations? How would you solve this equation?"

To support students getting started, consider displaying an anchor chart of the perfect squares from Lesson 13 and asking questions like, "Where might the parts of Alma's second equation go in the perfect square diagram?"

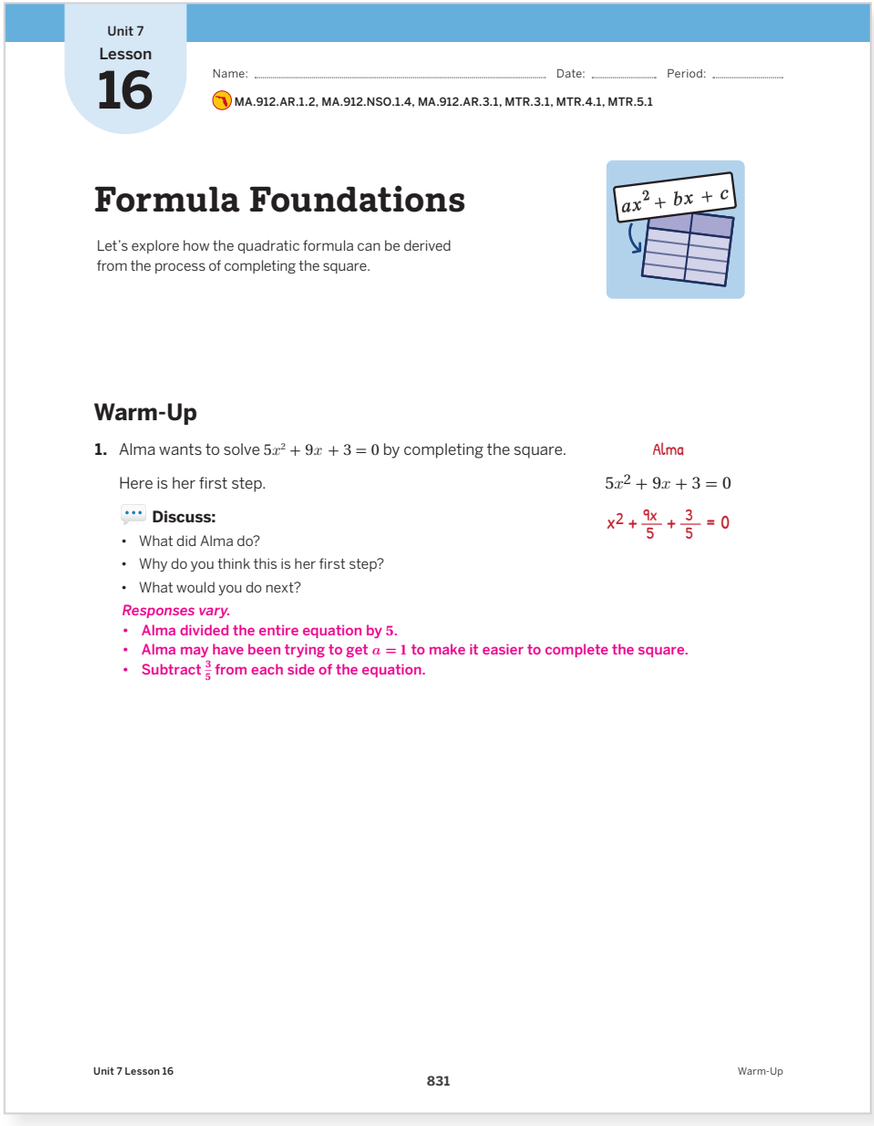
D **Differentiation: (Support)** Look for students who are not sure why Alma divided by 5. Invite them to try completing the square using $a = 5$ in an area model and reflect on what is challenging about it. Invite them to then to fill in the area model with Alma's second equation and ask, "Why would making $a = 1$ be helpful?"

ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share their ideas with others. **(Writing and Speaking)**

2 Connect

Invite students to share their responses.

Note: Spend adequate time here to ensure that students understand how Alma's first step and the next step are related to the process of completing the square.



Unit 7
Lesson
16

Name: ..... Date: ..... Period: .....

MA.912.AR.1.2, MA.912.NSO.1.4, MA.912.AR.3.1, MTR.3.1, MTR.4.1, MTR.5.1

Formula Foundations

Let's explore how the quadratic formula can be derived from the process of completing the square.

Warm-Up

1. Alma wants to solve $5x^2 + 9x + 3 = 0$ by completing the square.

Here is her first step.

Alma

$5x^2 + 9x + 3 = 0$

$x^2 + \frac{9x}{5} + \frac{3}{5} = 0$

Discuss:

- What did Alma do?
- Why do you think this is her first step?
- What would you do next?

Responses vary.

- Alma divided the entire equation by 5.
- Alma may have been trying to get $a = 1$ to make it easier to complete the square.
- Subtract $\frac{3}{5}$ from each side of the equation.

Unit 7 Lesson 16 831 Warm-Up

Pairs | 10 min

Activity 1 Completing Any Square

Purpose: Students describe the steps of solving a quadratic equation by completing the square and notice those steps can be used to solve any quadratic equation. **(MTR.2.1, MTR.5.1)**

Short on time? Consider not pausing on Problem 3 and have the discussion on Problem 4 instead.

1 Launch

Draw students attention to Problem 2. Consider asking,"What did Alma do to get from the second step to the third step?"

Invite students to describe Alma's work using whatever words make sense to them.

Note: A description of each of Alma's steps will be given in Problem 3. The purpose of this problem is to give students a chance to describe Alma's steps in their own words before seeing the full description. Consider waiting until Problem 3 to discuss Alma's steps.

2 Monitor

To support students getting started, invite them to point at the parts of the equation that have changed with each step and ask, "What has changed? How is this step related to completing the square?"

Note: Some students might wonder how they can know that the last two rows are the same. While students will not be expected to rewrite such expressions in this unit, you may choose to take the time to show students how to rewrite it.

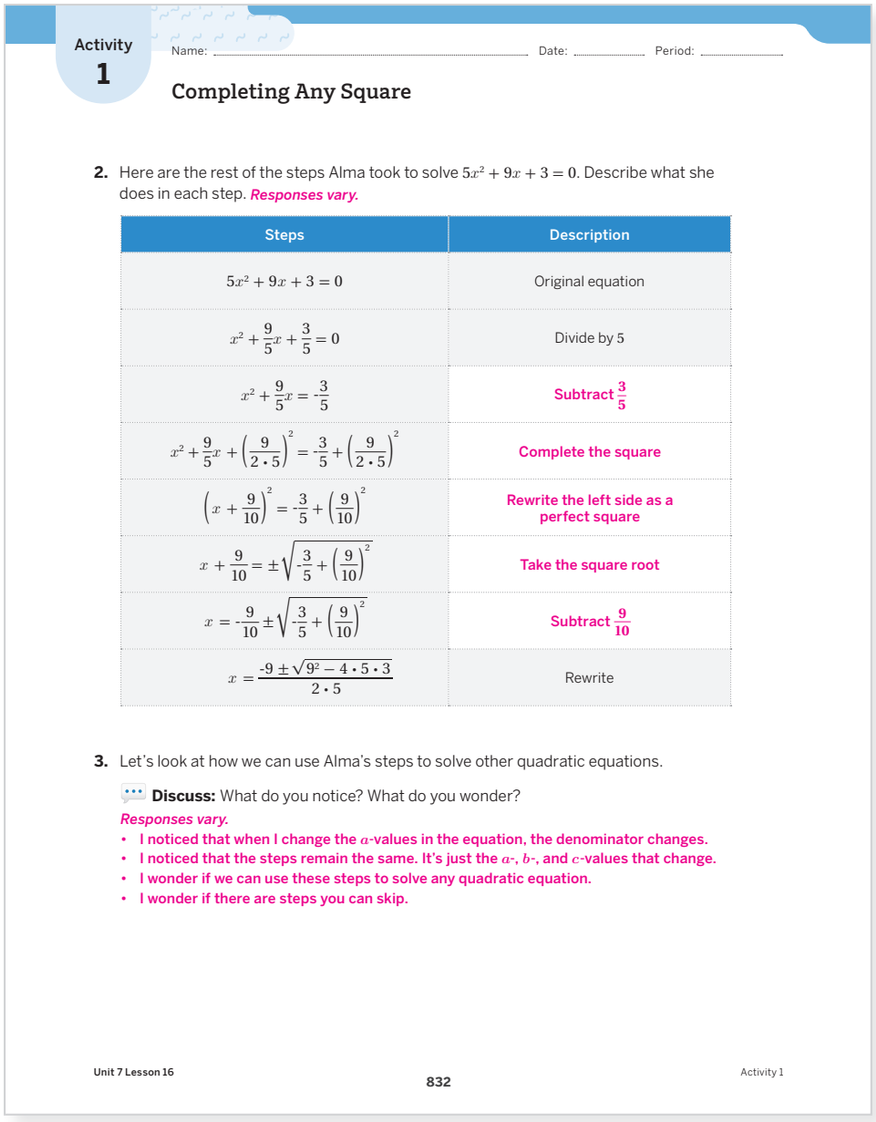
Encourage students to explore both positive and negative numbers.

To support students getting started, invite them to change one number at a time and describe what is changing in each row.

Pause briefly and **use the Notice and Wonder routine** to support students in making sense of patterns. **(MTR.5.1)**

Consider asking questions like:
- "When c is -7, where do you see -7 in each step?"
- "How will the last row change if you change the a-value to 5?"
- "Can you predict what the last row would look like if a is 1, b is 2, and c is 3?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Completing Any Square

2. Here are the rest of the steps Alma took to solve $5x^2 + 9x + 3 = 0$. Describe what she does in each step. *Responses vary.*

Steps	Description
$5x^2 + 9x + 3 = 0$	Original equation
$x^2 + \frac{9}{5}x + \frac{3}{5} = 0$	Divide by 5
$x^2 + \frac{9}{5}x = -\frac{3}{5}$	Subtract $\frac{3}{5}$
$x^2 + \frac{9}{5}x + \left(\frac{9}{2 \cdot 5}\right)^2 = -\frac{3}{5} + \left(\frac{9}{2 \cdot 5}\right)^2$	Complete the square
$\left(x + \frac{9}{10}\right)^2 = -\frac{3}{5} + \left(\frac{9}{10}\right)^2$	Rewrite the left side as a perfect square
$x + \frac{9}{10} = \pm\sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	Take the square root
$x = -\frac{9}{10} \pm \sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	Subtract $\frac{9}{10}$
$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5}$	Rewrite

3. Let's look at how we can use Alma's steps to solve other quadratic equations.

Discuss: What do you notice? What do you wonder?

Responses vary.
- I noticed that when I change the a-values in the equation, the denominator changes.
- I noticed that the steps remain the same. It's just the a-, b-, and c-values that change.
- I wonder if we can use these steps to solve any quadratic equation.
- I wonder if there are steps you can skip.

Unit 7 Lesson 16 832 Activity 1

Pairs | 10 min

Activity 1 Completing Any Square (continued)

Purpose: Students describe the steps of solving a quadratic equation by completing the square and notice those steps can be used to solve any quadratic equation. **(MTR.5.1)**

Short on time? Consider not pausing on Problem 3 and have the discussion on Problem 4 instead.

2 Monitor

Look and listen for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on one value at a time.
Think they need to replace each instance of 10, 7, and 1 in each row with 3, 8, and -15.	**Strengthen** Invite them to return to the previous problem and pay attention to the first and last rows as they change one value at a time.
Show or explain why Felipe can skip all these steps.	**Strengthen** Invite students to share their thinking during the Connect.
	Stretch Invite students to use Felipe's shortcut to determine the solutions to $-5x^2 - 9 = 0$.

A

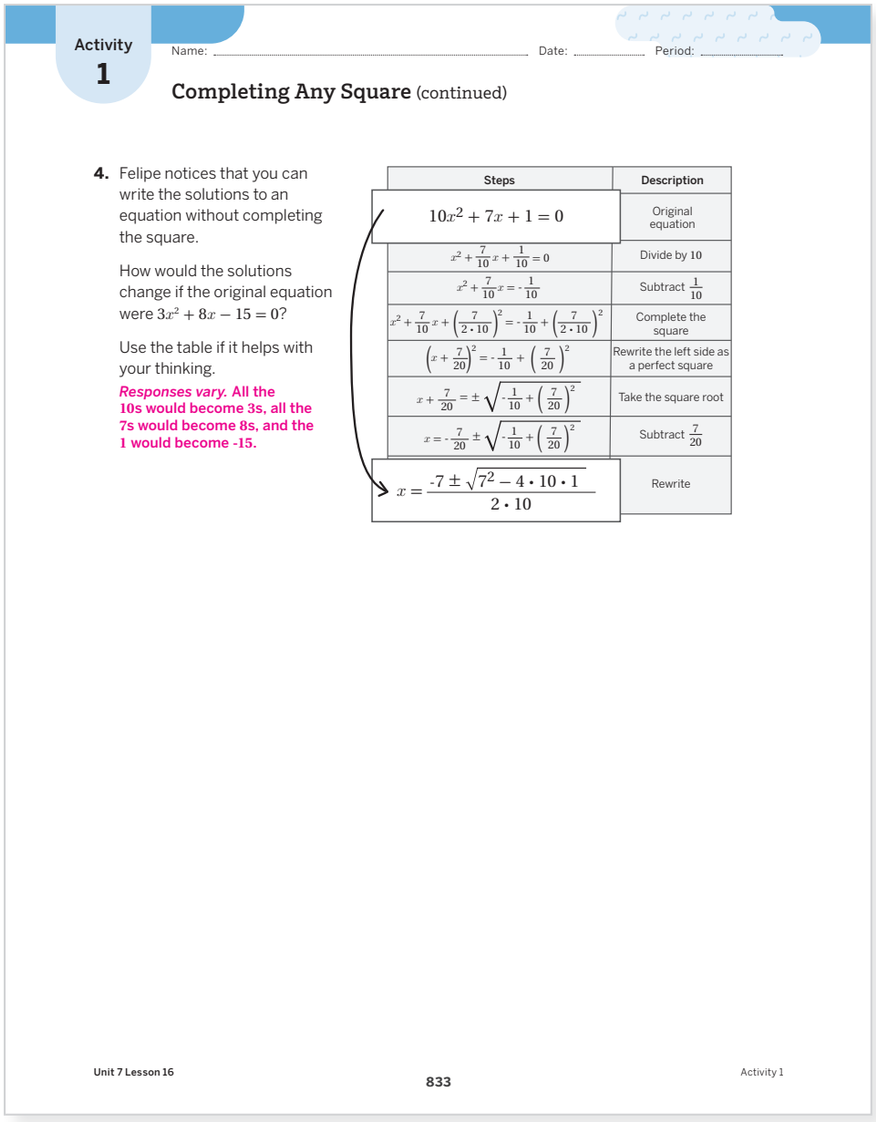
Accessibility: Executive Functioning Consider chunking this problem into more manageable parts by inviting students to consider how the solutions would change if the original equation were $3x^2 + 7x + 1 = 0$.

3 Connect

Invite students to share their responses and, if it is helpful, use the previous problem to help make connections between the original equation and its solution for several different examples.

To surface the Key Takeaway, consider asking:

- "What allows Felipe to skip all of those steps?"
- "Will this always work? How do you know?"



Activity 1

Name: _____ Date: _____ Period: _____

Completing Any Square (continued)

4. Felipe notices that you can write the solutions to an equation without completing the square.

How would the solutions change if the original equation were $3x^2 + 8x - 15 = 0$?

Use the table if it helps with your thinking.

Responses vary. All the 10s would become 3s, all the 7s would become 8s, and the 1 would become -15.

Steps	Description
$10x^2 + 7x + 1 = 0$	Original equation
$x^2 + \frac{7}{10}x + \frac{1}{10} = 0$	Divide by 10
$x^2 + \frac{7}{10}x = -\frac{1}{10}$	Subtract $\frac{1}{10}$
$x^2 + \frac{7}{10}x + \left(\frac{7}{2 \cdot 10}\right)^2 = -\frac{1}{10} + \left(\frac{7}{2 \cdot 10}\right)^2$	Complete the square
$\left(x + \frac{7}{20}\right)^2 = -\frac{1}{10} + \left(\frac{7}{20}\right)^2$	Rewrite the left side as a perfect square
$x + \frac{7}{20} = \pm\sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Take the square root
$x = -\frac{7}{20} \pm \sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Subtract $\frac{7}{20}$
$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 10 \cdot 1}}{2 \cdot 10}$	Rewrite

Unit 7 Lesson 16 833 Activity 1

Key Takeaway: The steps to solve a quadratic equation using completing the square are generally the same and can be used to solve any quadratic equation.

Pairs | 8 min

Activity 2 Formula-izing

Purpose: Students are introduced to the **quadratic formula** as a way to solve any quadratic equation without going through the process of completing the square.

1 Launch

Consider asking, "What are the values of a, b, and c in the equation on the left?"

2 Monitor

Encourage students to use different colors as they make connections between the a-, b-, and c-values in the equation and the solution and the solution.

To support students getting started, invite them to try an answer they know is wrong and to compare each part of their solution to the given solution by asking:

- "What number is supposed to be in that position?"
- "What variable represents that number?"

Listen for students who notice that the **quadratic formula** can be used to solve any quadratic equation by factoring or completing the square. **(MTR.5.1)**

D **Differentiation: (Stretch)** Invite students to write one quadratic equation for which they would not use the quadratic equation to determine the solutions, and one for which they would. Have them share their reasoning for each equation during the Connect.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- The a in the formula represents . . .
- There is a $\pm$ symbol in the quadratic formula because . . .
- The quadratic formula could help you . . .

(Reading, Writing, Speaking, and Listening)

3 Connect

Invite students to share their responses.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the Connect. Create a color-coded anchor chart about the quadratic formula and what it can help with to reference and add to for the rest of the unit.

Consider asking, "When might someone want to write out all of Alma's steps to solve a quadratic equation? When might they want to skip these steps?"

Activity 2

Name: Date: Period:

Formula-izing

5. We just discovered a way to write the solutions for any quadratic equation without completing the square!

Use the variables a, b, and c to represent the solutions to $ax^2 + bx + c = 0$.

Equation: $7x^2 - 9x + 5 = 0$

Solutions: $x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 7 \cdot 5}}{2 \cdot 7}$

Equation: $a\,x^2 + b\,x + c = 0$

Solutions: $x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$

(or equivalent)

6. For any quadratic equation: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation is known as the **quadratic formula**.

Discuss:

- What do the a, b, and c in the formula represent?
- Why is there a $\pm$ symbol in the formula?
- What new things could this formula help you do?

Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Responses vary.

- a, b, and c represent the coefficients and the constant in the standard-form quadratic equation.
- There is a $\pm$ symbol because when you take the square root, there is a positive and a negative value.
- I can use this formula to solve quadratic equations that may be hard to solve by factoring or completing the square.

Unit 7 Lesson 16 834 Activity 2

Pairs | 12 min

Activity 3 Solution Search

Purpose: Students explore solutions to different quadratic equations and describe patterns in quadratic equations that have zero, one, or two solutions. **(MTR.5.1)**

Short on time? Consider inviting students to find two of each kind of equation instead of three.

1 Launch

Direct students' attention to Problem 7 and invite them to solve $x^2 - 5x + 6 = 0$ using whatever method they prefer.

Invite students to solve the equation independently, then regroup students into cooperative pairs to share and compare their solutions to the given solutions.

Note: To help students notice patterns, encourage them to record the step before final simplification $\left(\text{e.g., } \frac{5 \pm \sqrt{1}}{1}\right)$.

2 Monitor

D **Differentiation: (Stretch)** Invite students to find an equation that has interesting solutions and share that equation with a partner.

A **Accessibility: Memory and Attention** To support students with keeping track of information, invite them to record each equation they try, along with the number of solutions.

3 Connect

To surface the Key Takeaway, consider asking:

- "What patterns did you notice?"
- "What strategy did you use to find equations that had no solutions? One solution?" **(MTR.5.1)**
- "What is true about the a-, b-, and c-values of a quadratic equation that has one solution? No solutions?"

Note: Consider adding patterns that students noticed to the anchor chart from Problem 6.

Activity 3

Name: Date: Period:

Solution Search

7. You will use a graphing calculator to explore solutions to quadratic equations.

a Here is an equation that has *two integer solutions*. Find two more equations with two integer solutions.

Responses vary.

Equation	$1x^2 - 5x + 6 = 0$	$2x^2 + 8x + 6 = 0$	$1x^2 + 0x - 9 = 0$
Solutions	$x = \frac{5 \pm \sqrt{1}}{2}$	$x = \frac{-8 \pm \sqrt{16}}{4}$	$x = \frac{0 \pm \sqrt{36}}{2}$

b Here is an equation that has *one solution*. Find two more equations with one solution.

Responses vary.

Equation	$1x^2 + 4x + 4 = 0$	$9x^2 + 0x + 0 = 0$	$1x^2 + 6x + 9 = 0$
Solutions	$x = \frac{-4 \pm \sqrt{0}}{2}$	$x = \frac{0 \pm \sqrt{0}}{18}$	$x = \frac{-6 \pm \sqrt{0}}{2}$

c Here is an equation that has *no solution*. Find two more equations with no solution.

Responses vary.

Equation	$7x^2 + 2x + 5 = 0$	$-2x^2 + 5x - 4 = 0$	$1x^2 + 0x + 9 = 0$
Solutions	$x = \frac{-2 \pm \sqrt{-136}}{14}$	$x = \frac{-5 \pm \sqrt{-7}}{-4}$	$x = \frac{0 \pm \sqrt{-36}}{2}$

d Examine the equations and solutions you found.

Discuss: What patterns do you notice?

Responses vary.

- I notice that the number under the square root tells me how many solutions the equation has.
- I notice that when there is a square root of a negative number, there are no solutions.

Unit 7 Lesson 16 835 Activity 3

Key Takeaway: When solving using the quadratic formula, the expression under the square root can tell us how many solutions the equation has.

Whole Class | 5 min

Synthesis

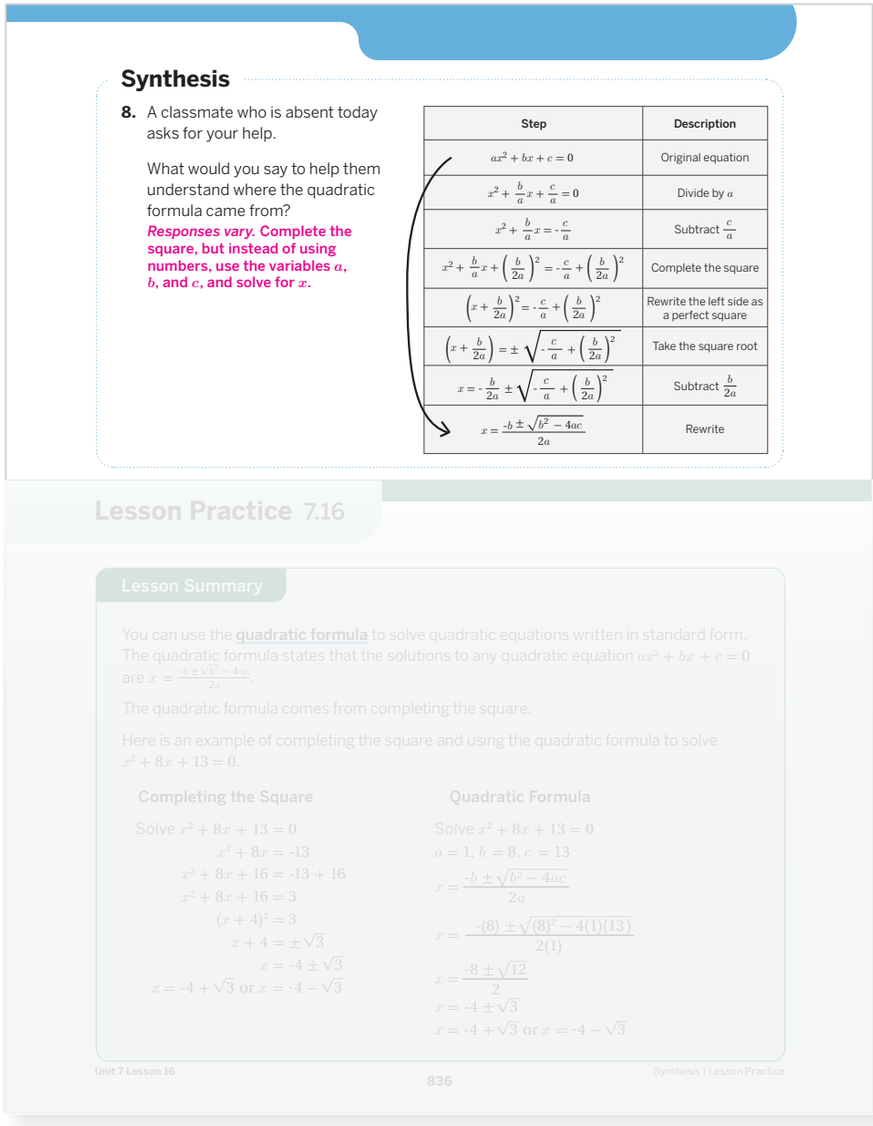
Purpose: Students synthesize the connection between completing the square and the quadratic formula.

Invite students to respond independently, and then share their thinking with a partner.

Consider asking:
- "Why do we use a, b, and c in the quadratic formula?"
- "Why are we able to use the quadratic formula to determine the solution to a quadratic equation?"
- "Why might someone want to solve an equation by using the quadratic formula instead of completing the square?"

Collect and share a variety of ideas, such as:
- The quadratic formula helps us "skip" the process of completing the square.
- The quadratic formula comes from solving an equation by completing the square with variables instead of numbers.

MLR **MLR2: Collect and Display** Use this routine to revisit students' ideas about the **quadratic formula**. Add any new ideas, including adding patterns that students noticed during the Solution Search, to the display for students to reference for the rest of the unit.



Synthesis

8. A classmate who is absent today asks for your help.

What would you say to help them understand where the quadratic formula came from?

Responses vary. Complete the square, but instead of using numbers, use the variables a, b, and c, and solve for x.

Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Lesson Practice 7.16

Lesson Summary

You can use the **quadratic formula** to solve quadratic equations written in standard form. The quadratic formula states that the solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The quadratic formula comes from completing the square.

Here is an example of completing the square and using the quadratic formula to solve $x^2 + 8x + 13 = 0$.

Completing the Square

Solve $x^2 + 8x + 13 = 0$

$x^2 + 8x = -13$

$x^2 + 8x + 16 = -13 + 16$

$x^2 + 8x + 16 = 3$

$(x + 4)^2 = 3$

$x + 4 = \pm\sqrt{3}$

$x = -4 \pm \sqrt{3}$

$x = -4 + \sqrt{3}$ or $x = -4 - \sqrt{3}$

Quadratic Formula

Solve $x^2 + 8x + 13 = 0$

$a = 1, b = 8, c = 13$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(13)}}{2(1)}$

$x = \frac{-8 \pm \sqrt{12}}{2}$

$x = -4 \pm \sqrt{3}$

$x = -4 + \sqrt{3}$ or $x = -4 - \sqrt{3}$

Unit 7 Lesson 16 836 Synthesis | Lesson Practice

Lesson Takeaway: The process of completing the square can be generalized and rewritten as the **quadratic formula**, which can be used to solve any quadratic equation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of the quadratic formula's relationship with completing the square by explaining why the solutions to both approaches are equivalent.

Today's Goals

1. **Goal:** Use the *quadratic formula* to solve quadratic equations written in standard form.
2. **Goal:** Understand that the quadratic formula can be derived by generalizing the process of completing the square.
3. **Language Goal:** Describe how to use the quadratic formula and what parts of the formula represent. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

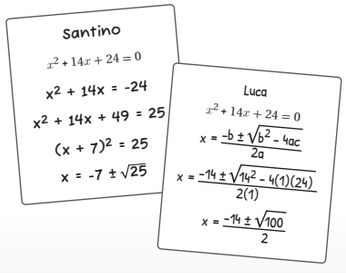
Show What You Know 7.16

Santino and Luca used two different strategies to solve $x^2 + 14x + 24 = 0$.

Santino started to solve the equation by completing the square.

Luca started to solve the equation using the quadratic formula.

Explain why the two last equations of each strategy are equivalent and will lead to the same solutions.



Responses vary.

- When Santino simplifies his last step to $x = -7 \pm 5$ he will arrive at $x = -12$ or $x = -2$.
- When Luca simplifies his last step to $x = \frac{(-14 \pm 10)}{2}$, he will arrive at $x = \frac{-24}{2}$ or -12 and $x = \frac{-4}{2}$ or -2. They will both arrive at the same solutions for x.

Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support simplifying radical expressions:

» **Respond to Student Thinking:**

- Revisit **Activity 3** as a class and invite students to share strategies for simplifying radical expressions.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

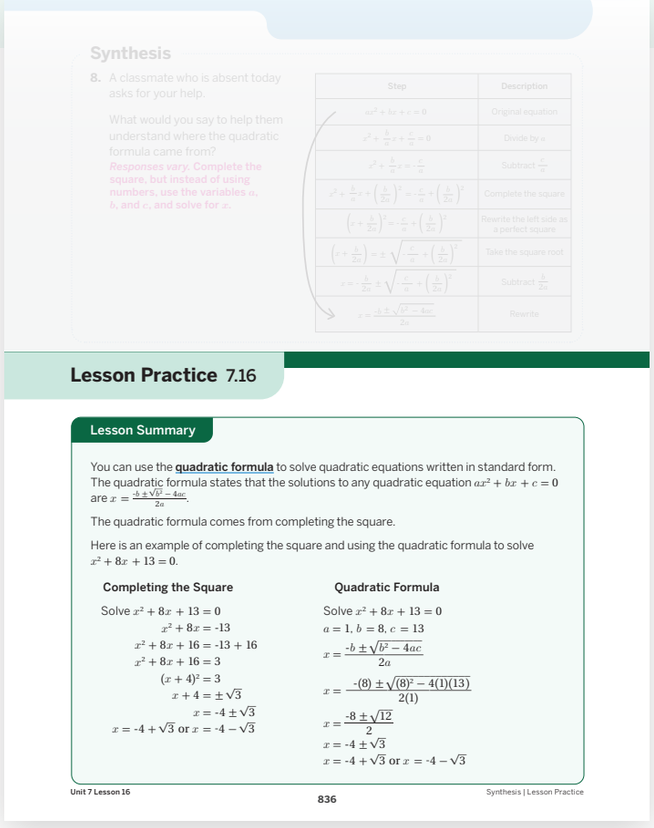
Professional Learning

In this lesson, students are introduced to the quadratic formula. How will that work support them in solving quadratic equations? How can you encourage and motivate students to be strategic about using certain solving methods in different situations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. A classmate who is absent today asks for your help.

What would you say to help them understand where the quadratic formula came from?

Responses vary. Complete the square, but instead of using numbers, use the variables a, b, and c, and solve for x.

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Lesson Practice 7.16

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$x = -4 \pm \sqrt{3}$

$x = -4 + \sqrt{3}$ or $x = -4 - \sqrt{3}$

Quadratic Formula

Solve $x^2 + 8x + 13 = 0$

$a = 1, b = 8, c = 13$

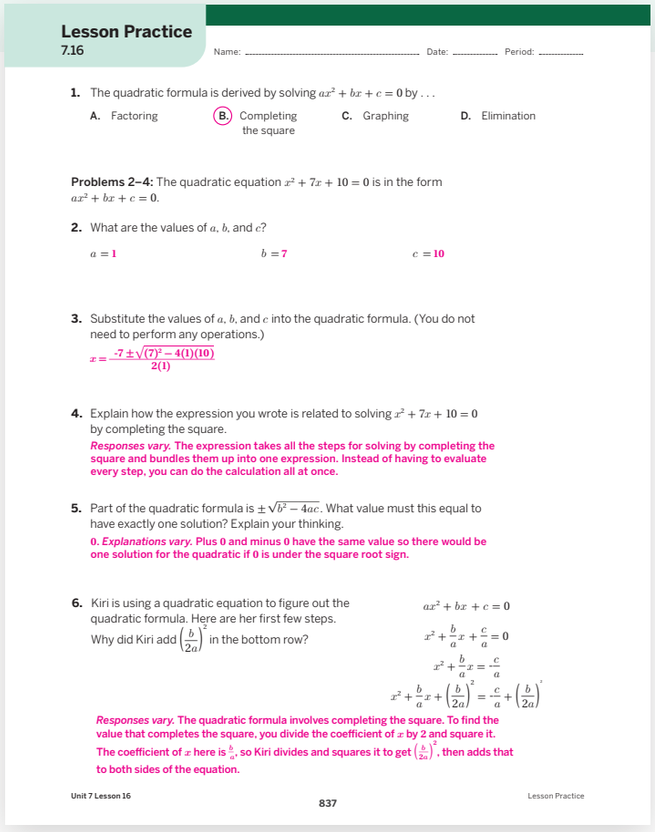
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(13)}}{2(1)}$

$x = \frac{-8 \pm \sqrt{12}}{2}$

$x = -4 \pm \sqrt{3}$

$x = -4 + \sqrt{3}$ or $x = -4 - \sqrt{3}$



Lesson Practice 7.16

Name: ..... Date: ..... Period: .....

1. The quadratic formula is derived by solving $ax^2 + bx + c = 0$ by . . .

A. Factoring (B.) Completing the square C. Graphing D. Elimination

Problems 2–4: The quadratic equation $x^2 + 7x + 10 = 0$ is in the form $ax^2 + bx + c = 0$.

2. What are the values of a, b, and c?

$a = 1$ $b = 7$ $c = 10$

3. Substitute the values of a, b, and c into the quadratic formula. (You do not need to perform any operations.)

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(10)}}{2(1)}$

4. Explain how the expression you wrote is related to solving $x^2 + 7x + 10 = 0$ by completing the square.

Responses vary. The expression takes all the steps for solving by completing the square and bundles them up into one expression. Instead of having to evaluate every step, you can do the calculation all at once.

5. Part of the quadratic formula is $\pm\sqrt{b^2 - 4ac}$. What value must this equal to have exactly one solution? Explain your thinking.

0. *Explanations vary.* Plus 0 and minus 0 have the same value so there would be one solution for the quadratic if 0 is under the square root sign.

6. Kiri is using a quadratic equation to figure out the quadratic formula. Here are her first few steps. Why did Kiri add $\left(\frac{b}{2a}\right)^2$ in the bottom row?

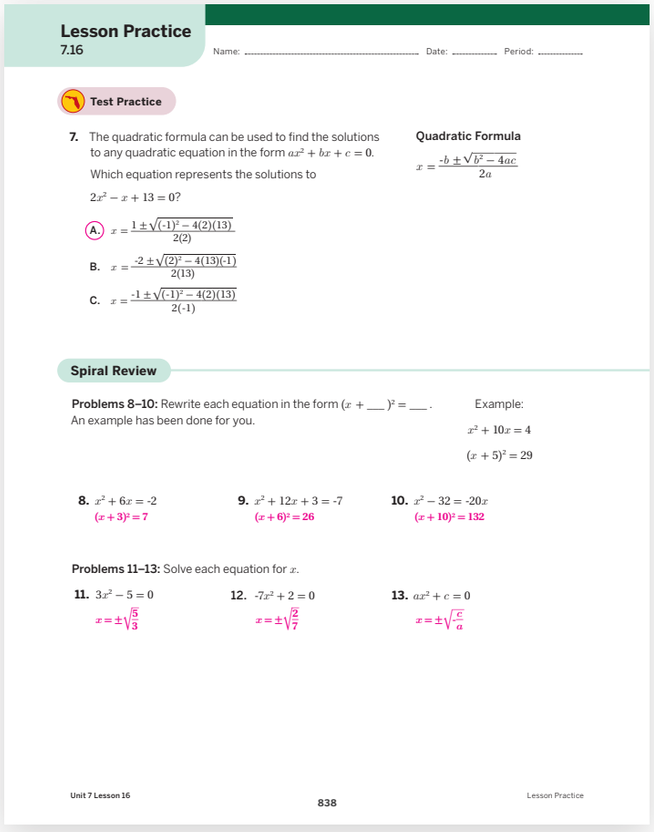
$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

Responses vary. The quadratic formula involves completing the square. To find the value that completes the square, you divide the coefficient of x by 2 and square it. The coefficient of x here is $\frac{b}{a}$, so Kiri divides and squares it to get $\left(\frac{b}{2a}\right)^2$, then adds that to both sides of the equation.



Lesson Practice 7.16

Name: ..... Date: ..... Period: .....

Test Practice

7. The quadratic formula can be used to find the solutions to any quadratic equation in the form $ax^2 + bx + c = 0$.

Which equation represents the solutions to $2x^2 - x + 13 = 0$?

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(A.) $x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(2)}$

B. $x = \frac{-2 \pm \sqrt{(2)^2 - 4(13)(-1)}}{2(13)}$

C. $x = \frac{-1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(-1)}$

Spiral Review

Problems 8–10: Rewrite each equation in the form $(x + ___)^2 = ___$. An example has been done for you.

Example:

$x^2 + 10x = 4$

$(x + 5)^2 = 29$

8. $x^2 + 6x = -2$ $(x + 3)^2 = 7$

9. $x^2 + 12x + 3 = -7$ $(x + 6)^2 = 26$

10. $x^2 - 32 = -20x$ $(x + 10)^2 = 132$

Problems 11–13: Solve each equation for x.

11. $3x^2 - 5 = 0$ $x = \pm\sqrt{\frac{5}{3}}$

12. $-7x^2 + 2 = 0$ $x = \pm\sqrt{\frac{2}{7}}$

13. $ax^2 + c = 0$ $x = \pm\sqrt{\frac{c}{a}}$

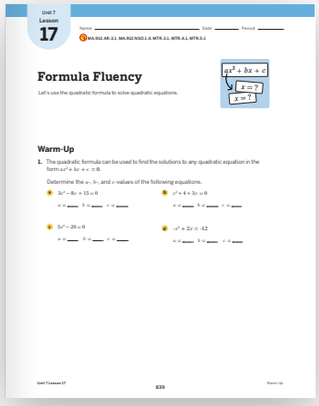
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.912.AR.1.2
	2–3	1	MA.912.AR.1.2 MA.912.AR.3.1
	4–5	2	MA.912.AR.1.2 MA.912.AR.3.1
	6	2	MA.912.AR.1.2 MA.912.AR.3.1 MA.912.NSO.1.4
Test Practice	7	2	MA.912.AR.1.2 MA.912.AR.3.1 MA.912.NSO.1.4
Spiral Review			
	8–10	1	MA.912.AR.3.4
Fluency	11–13	1	MA.912.AR.3.1

Unit 7

Lesson

17



Formula Fluency

Solving Quadratic Equations Using the Quadratic Formula

Let's use the quadratic formula to solve quadratic equations.

Learning Outcomes

Today's Goals

1. **Goal:** Use the quadratic formula to solve quadratic equations.
2. **Language Goal:** Describe the advantages and disadvantages of using the quadratic formula to solve equations with different structures. **(Speaking and Listening)**

Students develop fluency in applying the quadratic formula to solve equations, including those not yet in the form $ax^2 + bx + c = 0$. They make sense of common errors that can occur when solving quadratic equations using the quadratic formula.

Coherence

Prior Learning

In Lessons 7–12, students solved quadratic equations using various methods/strategies, including factoring, graphing, and completing the square. In Lesson 14, students derived the quadratic formula and used it to solve quadratic equations in the form $ax^2 + bx + c = 0$.

Future Learning

In Lesson 16, students will use the quadratic formula to solve and interpret quadratic equations in context.

Connections

- Students build their fluency with the quadratic formula by solving problems given various equations, including error analysis and predicting error spots in using the formula accurately. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.1

Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Also Addressing: **MA.912.NSO.1.4**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.3	**MA.912.AR.9.1**

Lesson at a Glance

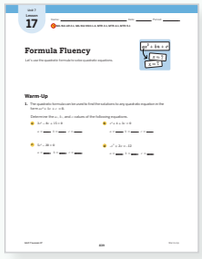
~ 45 min

MA.912.AR.3.1, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

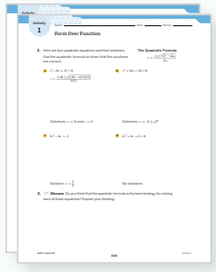
Students determine the a-, b-, and c-values of quadratic equations written in different forms.
Routine: Think-Pair-Share



Activity 1

Pairs | 15 min

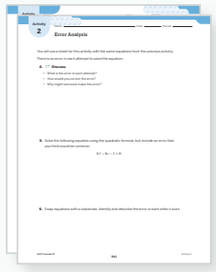
Students use the quadratic formula to verify the solutions of four quadratic equations written in various forms.



Activity 2

Pairs | 15 min

Students critique fictional work to make sense of common errors made when solving quadratic equations using the quadratic formula.
MLR3: Critique, Correct, Clarify **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize the advantages and disadvantages of using the quadratic formula to solve quadratic equations.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using the quadratic formula to solve a quadratic equation.



Prep Checklist

Prepare the Activity 2 Sheet.

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 2 Sheets, one per pair
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students determine the a-, b-, and c-values of quadratic equations written in different forms.

1 Launch

To surface prior knowledge, consider asking, "What does it mean to find the solutions to a quadratic equation?"

Use the Think-Pair-Share routine to help support student discourse about identifying the a-, b-, and c-values of quadratic equations in different forms. Encourage students to think independently and then take turns sharing with a partner.

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools or highlighters to underline or annotate each a-, b-, and c-value to support interpreting visual representations.

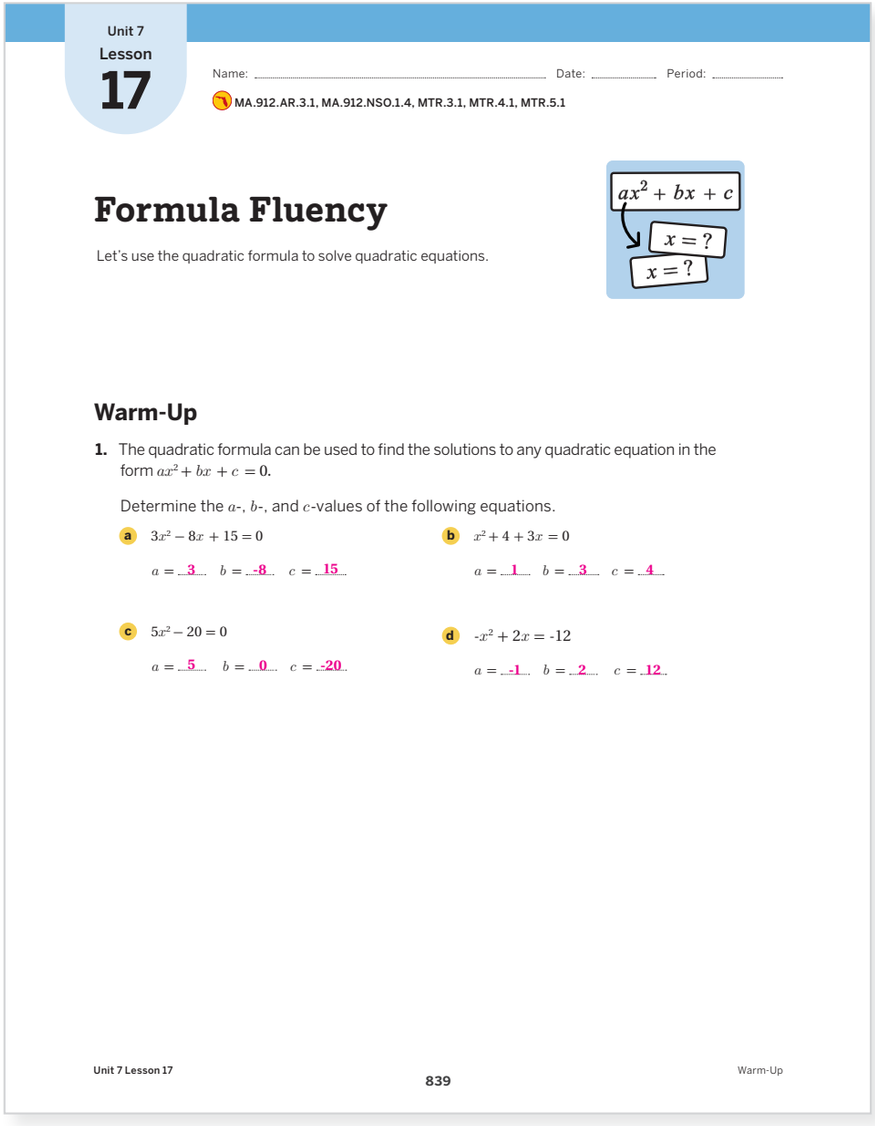
D **Differentiation: (Support)** Students may think the b-value in $5x^2 - 20 = 0$ is -20. Consider asking, "How is $5x^2 - 20 = 0$ similar to $3x^2 - 8x + 15 = 0$? How is it different? How can that help you determine the correct b-value?"

2 Connect

Invite students to share their strategies for determining the values.

Consider asking:

- "Why do we set quadratic equations equal to 0 when we solve them?"
- "How can you decide if the a-, b-, and c-values are positive or negative?"
- "Why might someone think the b-value of $x^2 + 4 + 3x = 0$ is 4 instead of 3?" **(MTR.5.1)**
- "How did you determine the c-value for $-x^2 + 2x = -12$?" **(MTR.5.1)**
- "Why might someone think the b-value of $5x^2 - 20 = 0$ is -20? What would you say to someone to help them with equations where there are missing terms?"



Unit 7
Lesson
17

Name: _____ Date: _____ Period: _____

MA.912.AR.3.1, MA.912.NSO.1.4, MTR.3.1, MTR.4.1, MTR.5.1

Formula Fluency

Let's use the quadratic formula to solve quadratic equations.

Warm-Up

1. The quadratic formula can be used to find the solutions to any quadratic equation in the form $ax^2 + bx + c = 0$.

   Determine the a-, b-, and c-values of the following equations.

   a. $3x^2 - 8x + 15 = 0$
   $a =$ 3 $b =$ -8 $c =$ 15

   b. $x^2 + 4 + 3x = 0$
   $a =$ 1 $b =$ 3 $c =$ 4

   c. $5x^2 - 20 = 0$
   $a =$ 5 $b =$ 0 $c =$ -20

   d. $-x^2 + 2x = -12$
   $a =$ -1 $b =$ 2 $c =$ 12

Unit 7 Lesson 17 839 Warm-Up

Pairs | 15 min

Activity 1 Form Over Function

Purpose: Students use the quadratic formula to verify the solutions of four quadratic equations written in various forms.

1 Launch

Display Troy's work and consider discussing, "Did Troy make any mistakes so far? How do you know? What advice would you give Troy as he continues his work?"

Encourage students to use a scientific calculator to support their calculations.

2 Monitor

Look for students who notice that they can use other strategies to solve these problems, such as reasoning, factoring, or completing the square. Invite these students to share during the Connect.

D Differentiation *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite other students to share strategies they use themselves, such as: • Using colored pencils or highlighters to identify the a-, b-, and c-values. • Calculating one part at a time, such as simplifying the expression under the square root symbol or squaring a negative b-value. **(MTR.6.1)**
Did not need to finish all of the calculations in part d.	**Strengthen** Consider asking, "At what point in the calculation did you know there would be no solutions? Why?" **(MTR.5.1)**
Would like to explore solving quadratic equations further.	**Stretch** Invite students to solve one of the equations using a different strategy. Consider asking, "What were the advantages and disadvantages of the strategy you chose?"

3 Connect

To surface the Key Takeaway, consider asking:

- "For which problems do you think using the quadratic formula was the best strategy? The worst strategy?"
- "For which problems would you prefer to factor? Why?"

Activity 1

Name: _____ Date: _____ Period: _____

Form Over Function

2. Here are four quadratic equations and their solutions. Use the quadratic formula to show that the solutions are correct. ***Work varies.***

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a $x^2 - 8x + 15 = 0$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$

$x = \frac{8 \pm \sqrt{64 - 60}}{2}$

$x = \frac{8 \pm \sqrt{4}}{2}$

$x = \frac{8 \pm 2}{2}$

$x = \frac{10}{2}$ and $x = \frac{6}{2}$

Solutions: $x = 5$ and $x = 3$

b $x^2 + 10x + 18 = 0$

$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(18)}}{2(1)}$

$x = \frac{-10 \pm \sqrt{100 - 72}}{2}$

$x = \frac{-10 \pm \sqrt{28}}{2}$

Solutions: $x = -5 \pm \sqrt{7}$

c $9x^2 - 6x = -1$

$9x^2 - 6x + 1 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$

$x = \frac{6 \pm \sqrt{36 - 36}}{18}$

$x = \frac{6 \pm \sqrt{0}}{18}$

$x = \frac{6 \pm 0}{18}$

Solution: $x = \frac{1}{3}$

d $2x^2 + 6x + 5 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)}$

$x = \frac{-6 \pm \sqrt{36 - 40}}{4}$

$x = \frac{-6 \pm \sqrt{-4}}{4}$

No solutions

3. **Discuss**: Do you think that the quadratic formula is the best strategy for solving each of these equations? Explain your thinking.

Responses vary. **The quadratic formula is the best strategy for solving $9x^2 - 6x = -1$ and $2x^2 + 6x + 5 = 0$ because I didn't know how to factor either equation. I would rather solve $x^2 - 8x + 15 = 0$ by factoring because I can rewrite it as $(x - 5)(x - 3) = 0$.**

Unit 7 Lesson 17 840 Activity 1

Key Takeaway: There are many strategies for solving quadratic equations. The quadratic formula can be used to solve any quadratic equation, although it may not always be the best strategy.

Pairs | 15 min

Activity 2 Error Analysis

Purpose: Students critique fictional work to make sense of common errors made when solving quadratic equations using the quadratic formula. **(MTR.4.1)**

Short on time? Consider omitting Problems 5 and 6.

1 Launch

To surface connections, consider asking, "Which problem from Activity 1 did you think was the most challenging? Why might someone make an error while solving that problem?"

Distribute the Activity 2 Sheet to each pair of students.

A **Accessibility: Executive Functioning** Invite students to use their fingers or blank paper to cover up most of the work so they can focus on one row of the work at a time.

2 Monitor

To support students getting started, encourage them to look back at their work from Activity 1, then compare their work to that of the fictional student.

ELL **English Language Learners**

- Invite one partner to read each prompt out loud before the pair discusses it. **(Reading and Listening)**
- Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**
- Invite students to circle the error and write the corrections instead of verbally explaining the error. **(Reading and Writing)**

D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Think there is no error in part a.	**Support** Invite them to use a scientific calculator to check each calculation in the work.
Think there is no error in part b.	**Support** Consider sharing that students can rewrite $\frac{-10 \pm \sqrt{28}}{2}$ as $\frac{-10}{2} \pm \frac{\sqrt{28}}{2}$ and asking, "How else could you rewrite these terms?"
Think there is no error in part c.	**Support** Invite them to revisit part d of the Warm-Up.
Think there is no error in part d.	**Support** Consider asking, "How is this work similar to the other parts? How is it different? Can any of the other parts help you to find the error?"

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Error Analysis

You will use a sheet for this activity with the same equations from the previous activity.

There is an error in each attempt to solve the equation.

4. **Discuss:**

- What is the error in each attempt?
- How would you correct the error?
- Why might someone make this error?

Responses vary.

a. $(-8)^2$ is equal to 64, not -64.

b. $\frac{-10 \pm \sqrt{28}}{2}$ is equivalent to $\frac{-10}{2} \pm \frac{\sqrt{28}}{2}$. $\frac{-10}{2}$ can be reduced to -5 but $\frac{\sqrt{28}}{2}$ cannot be reduced to $\sqrt{14}$.

c. The c-value should be 1 because the equation is $9x^2 - 6x + 1 = 0$ when written in standard form.

d. There are no numbers that would give you $\sqrt{-4}$ when squared, so $2x^2 + 6x + 5 = 0$ has no solutions.

5. Solve the following equation using the quadratic formula, but include an error that you think would be common. *Errors vary.*

$3x^2 - 6x - 1 = 0$

$3x^2 - 6x - 1 = 0$

$a = 3,\ b = -6,\ c = -1$

$x = \frac{-6 \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{36 + 12}}{6}$

$x = \frac{-6 \pm \sqrt{48}}{6}$

$x = -1 \pm \frac{\sqrt{48}}{6}$

6. Swap equations with a classmate. Identify and describe the error in each other's work.

Responses vary. $b = -6$, so when you substitute b into the quadratic formula, $-b$ is $-(-6)$, which is equal to 6.

Unit 7 Lesson 17 841 Activity 2

Pairs | 15 min

Activity 2 Error Analysis (continued)

Purpose: Students critique fictional work to make sense of common errors made when solving quadratic equations using the quadratic formula. **(MTR.4.1)**

Short on time? Consider omitting Problems 5 and 6.

2 Monitor

Look for a variety of errors such as:

- Not rewriting the equation equal to 0.
- Mislabeling the a-, b-, or c-values.
- Incorrectly substituting the values into the quadratic formula.
- Incorrectly squaring the b-value.
- Incorrectly simplifying the fraction.
- Incorrectly simplifying the square root.

Invite students to record their favorite mistake to share during the Connect.

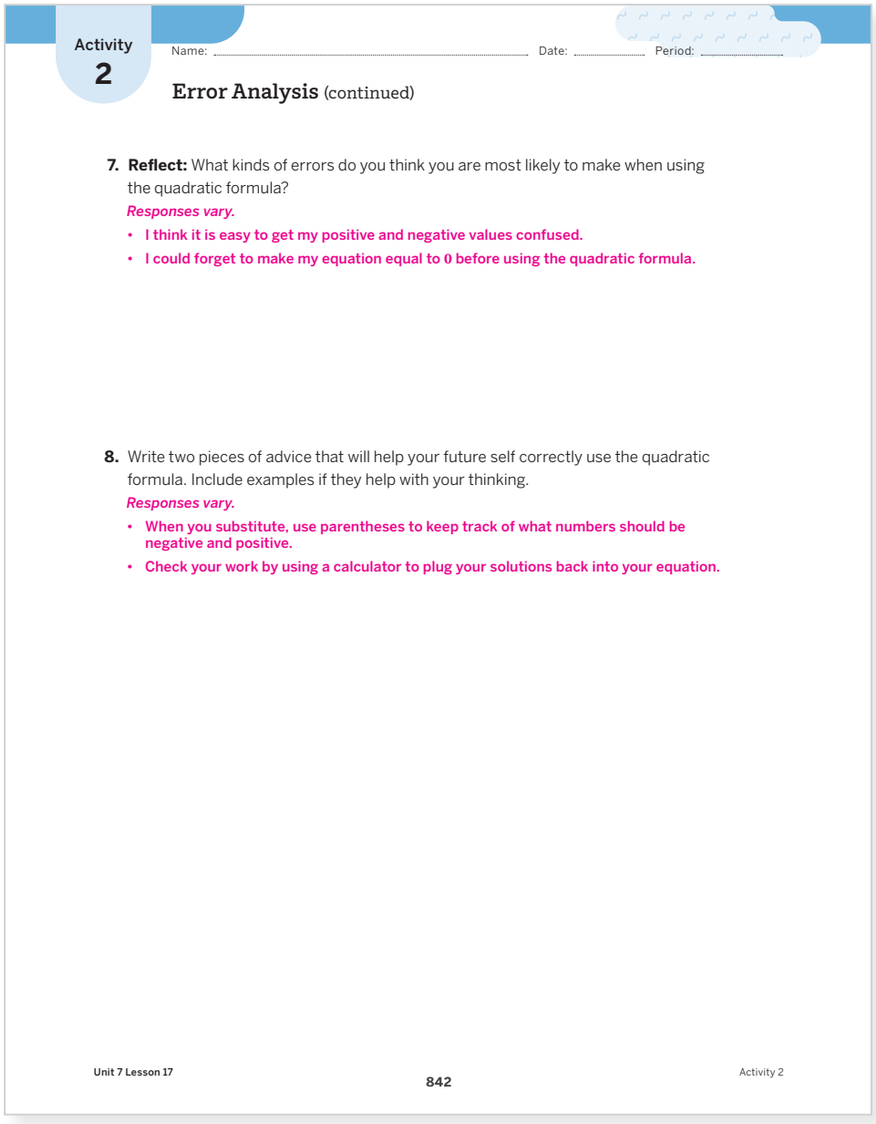
ELL **English Language Learners** Invite students to write first in their primary language and then translate it to English during the second draft. **(Writing)**

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in the fictional work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.5.1, MTR.6.1)**

Consider asking, "What is your favorite error?"

Invite students to share why someone might make these errors and how to correct them. **(MTR.4.1)**



Activity 2

Name: Date: Period:

Error Analysis (continued)

7. Reflect: What kinds of errors do you think you are most likely to make when using the quadratic formula?

Responses vary.

- I think it is easy to get my positive and negative values confused.
- I could forget to make my equation equal to 0 before using the quadratic formula.

8. Write two pieces of advice that will help your future self correctly use the quadratic formula. Include examples if they help with your thinking.

Responses vary.

- When you substitute, use parentheses to keep track of what numbers should be negative and positive.
- Check your work by using a calculator to plug your solutions back into your equation.

Unit 7 Lesson 17 842 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize the advantages and disadvantages of using the quadratic formula to solve quadratic equations.

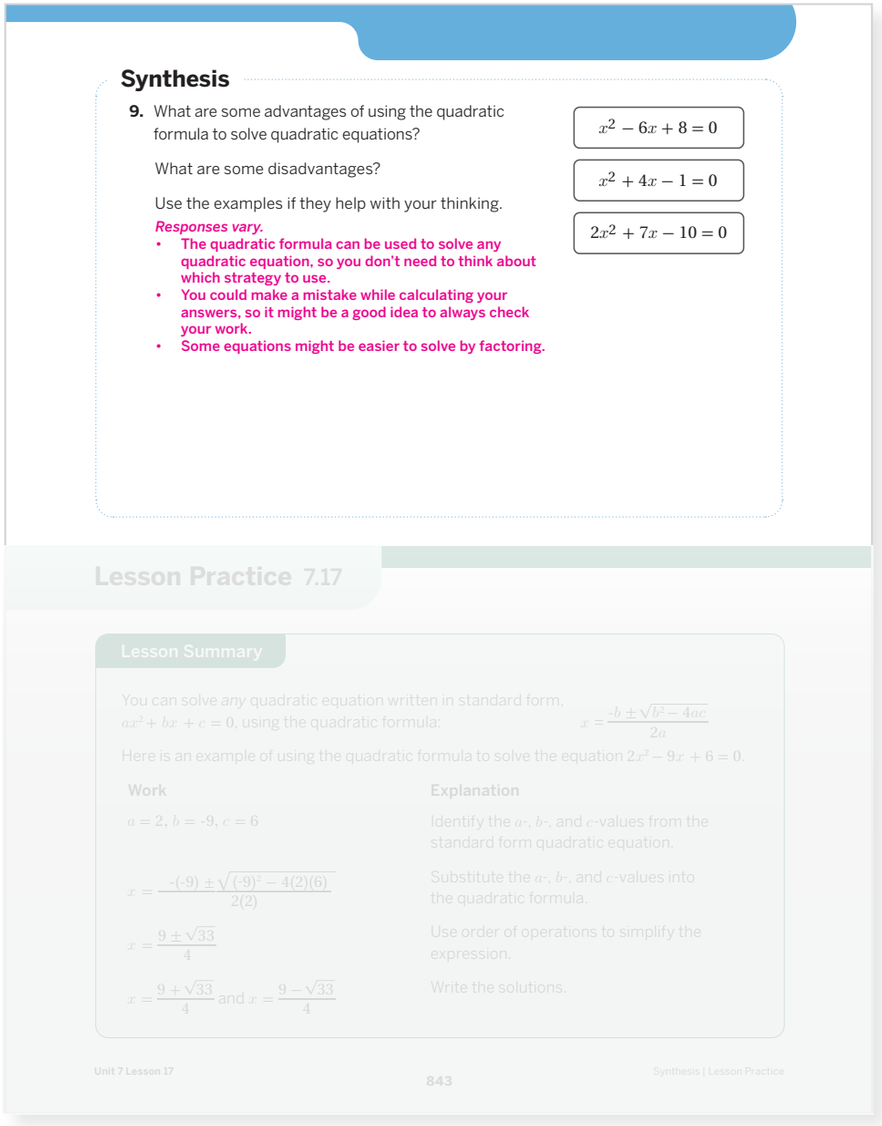
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- If the equation can be factored, using the quadratic formula can feel like a lot of work.
- Using the quadratic formula requires a lot of steps, so there are more chances to make an error when calculating by hand.
- The quadratic formula is not always the most efficient method for solving a quadratic equation, but it can be used to solve any quadratic equation.

Consider asking:
- "For which equation would the quadratic formula be the most efficient to use? Why?"
- "For which equation would the quadratic formula be the most challenging to use? Why?"
- "For which equation would the quadratic formula be most helpful to use? Least helpful? Why?"

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *quadratic equation, quadratic formula, solutions, factoring, completing the square, coefficients*, etc. **(Writing and Speaking)**

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. What are some advantages of using the quadratic formula to solve quadratic equations?

What are some disadvantages?

Use the examples if they help with your thinking.

$x^2 - 6x + 8 = 0$

$x^2 + 4x - 1 = 0$

$2x^2 + 7x - 10 = 0$

Responses vary.
- The quadratic formula can be used to solve any quadratic equation, so you don't need to think about which strategy to use.
- You could make a mistake while calculating your answers, so it might be a good idea to always check your work.
- Some equations might be easier to solve by factoring.

Lesson Practice 7.17

Lesson Summary

You can solve *any* quadratic equation written in standard form, $ax^2 + bx + c = 0$, using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here is an example of using the quadratic formula to solve the equation $2x^2 - 9x + 6 = 0$.

Work	Explanation
$a = 2, b = -9, c = 6$	Identify the a-, b-, and c-values from the standard form quadratic equation.
$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$	Substitute the a-, b-, and c-values into the quadratic formula.
$x = \frac{9 \pm \sqrt{33}}{4}$	Use order of operations to simplify the expression.
$x = \frac{9 + \sqrt{33}}{4}$ and $x = \frac{9 - \sqrt{33}}{4}$	Write the solutions.

Unit 7 Lesson 17 843 Synthesis | Lesson Practice

Lesson Takeaway: The quadratic formula can be used to solve any quadratic equation. It is a strategy that is always useful, although sometimes it can involve more calculations than other strategies like factoring.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using the quadratic formula to solve a quadratic equation.

Today's Goals

1. **Goal:** Use the quadratic formula to solve quadratic equations.
2. **Language Goal:** Describe the advantages and disadvantages of using the quadratic formula to solve equations with different structures. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.17

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula to solve the equation $2x^2 + 5x - 12 = 0$. Show your thinking.

$x = -4$ and $x = 1.5$

D Differentiation Use after Lesson 17

Support

Provide targeted intervention for students by using these resources.

If students need support solving quadratic equations using the quadratic formula:

» **Respond to Student Thinking:**

- Use the ***Solving Quadratic Equations Using the Quadratic Formula*** **Mini-Lesson**.
- Revisit **Activity 1** and provide students with individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to try and solve each quadratic equation from the lesson using a different strategy than the quadratic formula.

Professional Learning

Reflect on how students analyzed student work and solved quadratic equations today. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of each student's work? How can you support students in valuing learning from both correct and incorrect thinking?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What are some advantages of using the quadratic formula to solve quadratic equations?

What are some disadvantages?

Use the examples if they help with your thinking.

$x^2 - 6x + 8 = 0$

$x^2 + 4x - 1 = 0$

$2x^2 + 7x - 10 = 0$

Responses vary.

- The quadratic formula can be used to solve any quadratic equation, so you don't need to think about which strategy to use.
- You could make a mistake while calculating your answers, so it might be a good idea to always check your work.
- Some equations might be easier to solve by factoring.

Lesson Practice 7.17

Lesson Summary

You can solve *any* quadratic equation written in standard form, $ax^2 + bx + c = 0$, using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here is an example of using the quadratic formula to solve the equation $2x^2 - 9x + 6 = 0$.

Work	Explanation
$a = 2, b = -9, c = 6$	Identify the a-, b-, and c-values from the standard form quadratic equation.
$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$	Substitute the a-, b-, and c-values into the quadratic formula.
$x = \frac{9 \pm \sqrt{33}}{4}$	Use order of operations to simplify the expression.
$x = \frac{9 + \sqrt{33}}{4}$ and $x = \frac{9 - \sqrt{33}}{4}$	Write the solutions.

Lesson Practice 7.17

Name: ..... Date: ..... Period: .....

Problems 1–2: Each equation is in the form $ax^2 + bx + c = 0$. Determine the values of a, b, and c.

1. $x^2 - 5x + 9 = 0$

$a =$ 1 $b =$ -5 $c =$ 9

2. $-x^2 + 8 = 0$

$a =$ -1 $b =$ 0 $c =$ 8

Problems 3–5: Solve each equation using any method.

3. $2x^2 - 7x = 15$

$x = 5$ and $x = -1.5$ (or equivalent)

4. $2x^2 + 5x - 1 = 0$

$x = \frac{-5 \pm \sqrt{33}}{4}$

5. $4x^2 = 5$

$x = \pm\frac{\sqrt{5}}{2}$

6. Santiago determined that the solutions to $3x^2 - 6x - 9 = 0$ are $x = 3$ and $x = -1$. Is he correct? Explain your thinking.

Yes. *Explanations vary.*

- I graphed $y = 3x^2 - 6x - 9$ and saw that the parabola's x-intercepts were $(3, 0)$ and $(-1, 0)$.
- I substituted each solution into the original equation to see if it made the equation true.

7. Write a quadratic equation that would be simpler to solve *without* using the quadratic equation.

Responses vary.

$(x + 2)(x - 1) = 0$

Lesson Practice 7.17

Name: ..... Date: ..... Period: .....

Test Practice

8. Choose a quadratic function with the following x-intercepts.

$\left(\frac{2 \pm \sqrt{40}}{6}, 0\right)$

A. $2x^2 + 3x - 2$

B. $3x^2 - 2x - 3$

C. $2x^2 + 2x + 3$

D. $3x^2 - 3x - 2$

Spiral Review

Problems 9–10: Here is the function $f(x) = (x + 1)(x + 5)$.

9. What are the coordinates of the x-intercepts?

(-5, 0) and (-1, 0)

10. What are the coordinates of the vertex? Explain your thinking.

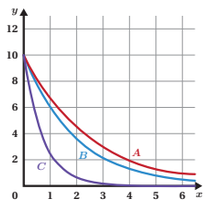
(-3, -4). *Explanations vary.* The x-intercepts tell me that the line of symmetry is at $x = -3$. When I input -3 into the function, the output is -4.

11. Here are the graphs of three equations. Match each graph with its equation.

A $y = 10\left(\frac{2}{3}\right)^x$

C $y = 10\left(\frac{1}{4}\right)^x$

B $y = 10\left(\frac{3}{5}\right)^x$



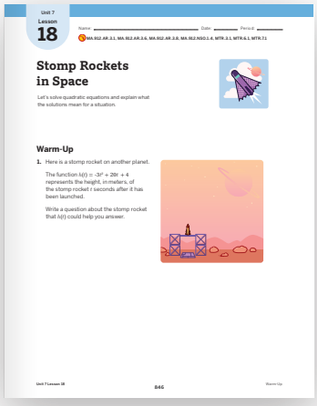
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.912.AR.3.1
	3–5	2	MA.912.AR.3.1
	6	2	MA.912.AR.3.1
	7	3	MA.912.AR.3.4
Test Practice	8	3	MA.912.AR.3.1
Spiral Review			
Fluency	9–10	1	MA.912.AR.3.7
	11	2	MA.912.AR.5.4

Unit 7

Lesson

18


Unit 7
Lesson
18
Name: Date: Period:
MA.912.AR.3.1, MA.912.AR.3.8, MA.912.AR.3.9, MA.912.NSO.1.4, MTR.3.1, MTR.6.1, MTR.7.1
Stomp Rockets in Space
Let's solve quadratic equations and explain what the solutions mean for a situation.
Warm-Up
1. Here is a stomp rocket on another planet.
The function $h(t) = -3t^2 + 28t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.
Write a question about the stomp rocket that $h(t)$ could help you answer.
Unit 7 Lesson 18
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Warm-Up

Stomp Rockets in Space

Solving Quadratic Equations in Context

Let's solve quadratic equations and explain what the solutions mean for a situation.

Learning Outcomes

Today's Goals

1. **Goal:** Write quadratic equations to represent a situation.
2. **Goal:** Solve quadratic equations in context.
3. **Language Goal:** Interpret the solutions to quadratic equations in context. **(Speaking and Listening)**

Students solve quadratic equations and interpret their solutions in context, reasoning abstractly and quantitatively. They also explore how to write equations to help them answer questions and interpret solutions in context to determine which one(s) make sense. Then they use their creativity to build their own rocket launch and answer questions about rocket launches created by their classmates. **(MTR.3.1)**

Coherence

Prior Learning

In Units 2, 5, and 6, students solved problems by writing linear equations, linear inequalities, systems of linear equations and inequalities, and exponential equations. In Unit 7, students used graphs and tables to represent a similar stomp rocket context. In Unit 8, students explored several strategies for solving quadratic equations, including factoring, completing the square, and using the quadratic formula.

Future Learning

Students will continue to write equations and use them to solve problems in context in Algebra 2.

Connections

- Students continue to build their fluency with solving quadratic equations using the quadratic formula and make sense of the solutions within each problem context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.912.AR.3.1
Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Also Addressing: **MA.912.AR.3.8, MA.912.NSO.1.4**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.NSO.2.3	**MA.912.F.1.6**

Vocabulary

Review Vocabulary

domain

range (of a function)

Lesson at a Glance

~ 45 min

MA.912.AR.3.1, MA.912.AR.3.8, MA.912.NSO.1.4

Warm-Up

Pairs | 5 min

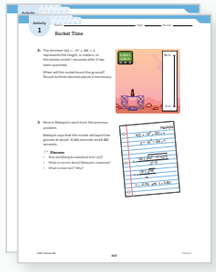
Students write questions that can be answered using an equation that models a stomp rocket's height over time. **Routine:** Notice and Wonder **(MTR.7.1)**



Activity 1

Pairs | 8 min

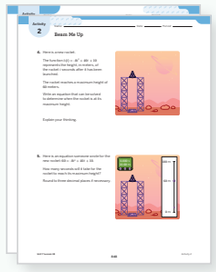
Students surface strategies for writing quadratic equations to help answer questions and interpreting their solutions in context. MLR3: Critique, Correct, Clarify **(MTR.3.1)**



Activity 2

Pairs | 12 min

Students write and solve quadratic equations and practice interpreting their solutions in context. **Routine:** Decide and Defend **(MTR.3.1)**



Activity 3

Pairs | 10 min

Students create their own rocket launch and answer questions authored by their classmates. **Routine:** Notice and Wonder



Synthesis

Whole Class | 5 min

Students synthesize their strategies for using quadratic equations to model contexts and answer questions. **(MTR.7.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using an equation to answer a question about a quadratic context.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Copy Activity 3 Sheets, 1 per pair or small group
- Calculator

Pairs | 5 min

Warm-Up

Purpose: Students write questions that can be answered using an equation that models a stomp rocket's height over time. **(MTR.7.1)**

1 Launch

To surface prior knowledge, consider asking:
- "Have you ever seen a stomp rocket before?"
- "How do they work?"
- "How long do you think it was in the air?"

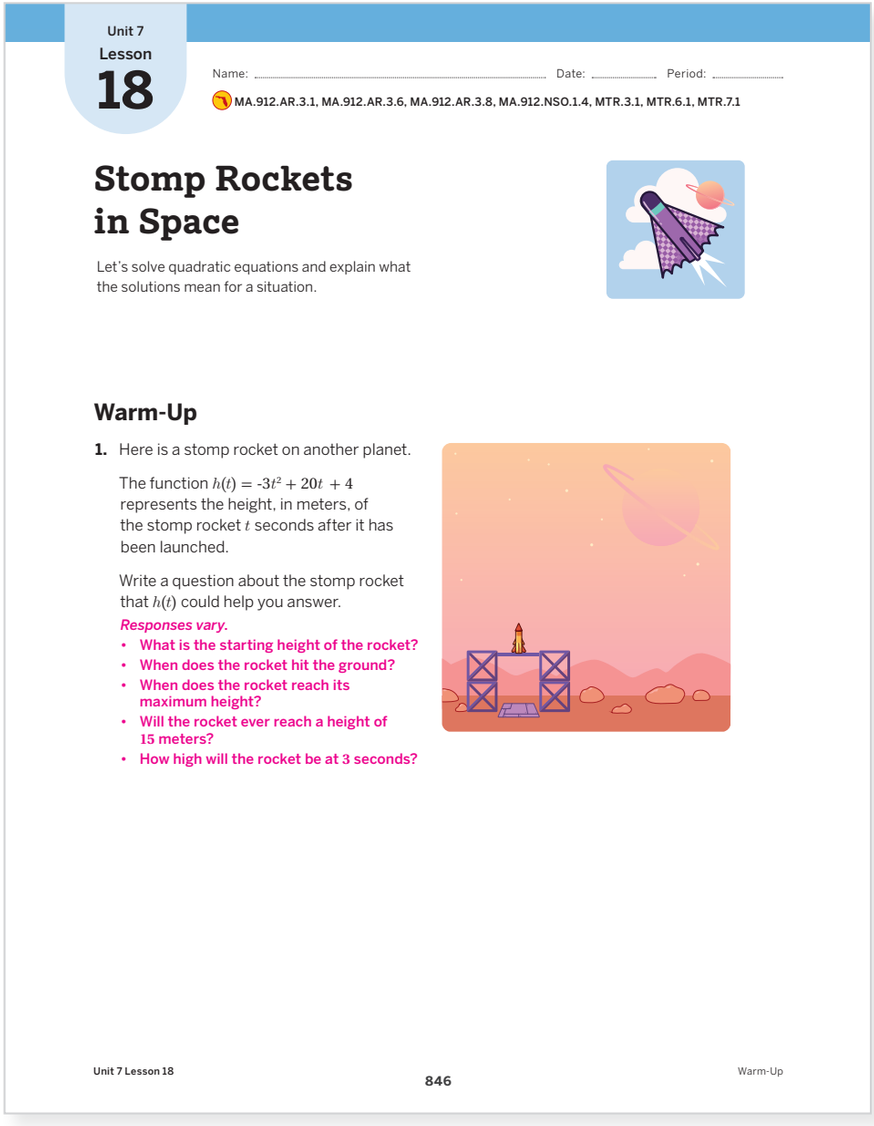
To encourage creativity, invite students to write a question that none of their classmates will write.

ELL **English Language Learners** Throughout this lesson, invite them to read the question aloud and paraphrase what it means. Clarify any unfamiliar words or concepts. **(Reading, Writing, Speaking, and Listening)**

2 Connect

Use the Notice and Wonder routine to promote curiosity and help students make sense of what types of questions can and can't be answered with a function. **(MTR.7.1)**

To surface the Key Takeaway, consider asking:
- "What do these questions have in common?"
- "What is a question about the stomp rocket that $h(x)$ cannot help us answer?"
- "Why are function equations helpful?"



Unit 7
Lesson
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Name: Date: Period:

MA.912.AR.3.1, MA.912.AR.3.6, MA.912.AR.3.8, MA.912.NSO.1.4, MTR.3.1, MTR.6.1, MTR.7.1

Stomp Rockets in Space

Let's solve quadratic equations and explain what the solutions mean for a situation.

Warm-Up

1. Here is a stomp rocket on another planet.

   The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

   Write a question about the stomp rocket that $h(t)$ could help you answer.

   Responses vary.
   - What is the starting height of the rocket?
   - When does the rocket hit the ground?
   - When does the rocket reach its maximum height?
   - Will the rocket ever reach a height of 15 meters?
   - How high will the rocket be at 3 seconds?

Unit 7 Lesson 18 846 Warm-Up

Key Takeaway: A situation may involve many variables. Functions can mathematically model some of the variables in the situation and can be used to answer questions about one or two of the variables involved in the function. **(MTR.7.1)**

Pairs | 8 min

Activity 1 Rocket Time

Purpose: Students surface strategies for writing quadratic equations to help answer questions and interpreting their solutions in context. **(MTR.3.1)**

1 Launch

Consider sharing this is the same function from the previous problem and ask, "What is a prediction that you think would be too short? Too long?"

2 Monitor

Encourage students to use a scientific calculator or paper as they find it helpful throughout the lesson.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What's the rocket's height when it's on the ground? Where can you substitute that into the equation?"
Guess and check.	**Strengthen** Celebrate their persistence. Consider asking: • "What information does the problem give you about the time or the rocket's height?" • "Is there a variable in the equation you could replace with a number?"

A Accessibility: Conceptual Processing Invite them to answer more concrete questions like: "What are the units for $t \approx 6.861$? What does it mean about the rocket?"

A Accessibility: Executive Functioning Invite them to cover up Makayla's work, then show one line at a time and describe one thing that she did correctly or incorrectly.

D Differentiation: (Stretch) Invite students to determine the *domain* of this function within this context.

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Makayla's work. Invite students to share their responses. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider asking:
- "Will negative t-values ever make sense in this situation? Why or why not?"
- "Are there other situations where negative inputs could make sense?"

Activity 1

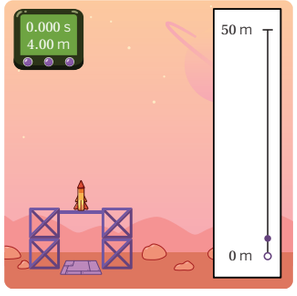
Name: _____ Date: _____ Period: _____

Rocket Time

2. The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

When will the rocket touch the ground? Round to three decimal places if necessary.

$\frac{-20 - \sqrt{448}}{-6}$ or 6.861 seconds

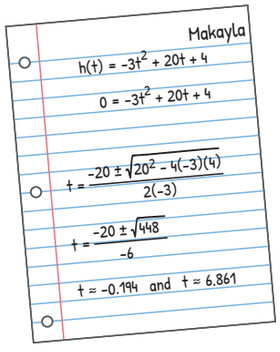


3 Here is Makayla's work from the previous problem.

Makayla says that the rocket will touch the ground at about -0.194 seconds and 6.861 seconds.

Discuss:
- Why did Makayla substitute 0 for $h(t)$?
- What is correct about Makayla's response?
- What is incorrect? Why?

Responses vary.
- In this function, $h(t)$ represents height. To determine when the rocket will touch the ground, Makayla set $h(t)$ equal to 0.
- Makayla correctly used the quadratic formula to solve $0 = -3t^2 + 20t + 4$.
- Makayla incorrectly interpreted -0.194 as a time when the rocket hits the ground. In this situation, it doesn't make sense for time to be negative.



Unit 7 Lesson 18 847 Activity 1

Key Takeaway: When using a mathematical model to solve problems in context, it is important to interpret if the solutions are reasonable for the context. Sometimes solutions that make sense mathematically do not make sense in the situation. **(MTR.6.1)**

Pairs | 12 min

Activity 2 Beam Me Up

Purpose: Students write and solve quadratic equations and practice interpreting their solutions in context. **(MTR.3.1, MTR.7.1)**

Short on time? Consider only discussing Problem 6 during the Connect.

1 Launch

Invite students to read the text aloud and ask, "Why might someone want to know when the rocket reaches its maximum height?"

Encourage students to discuss with their partner before entering an equation.

2 Monitor

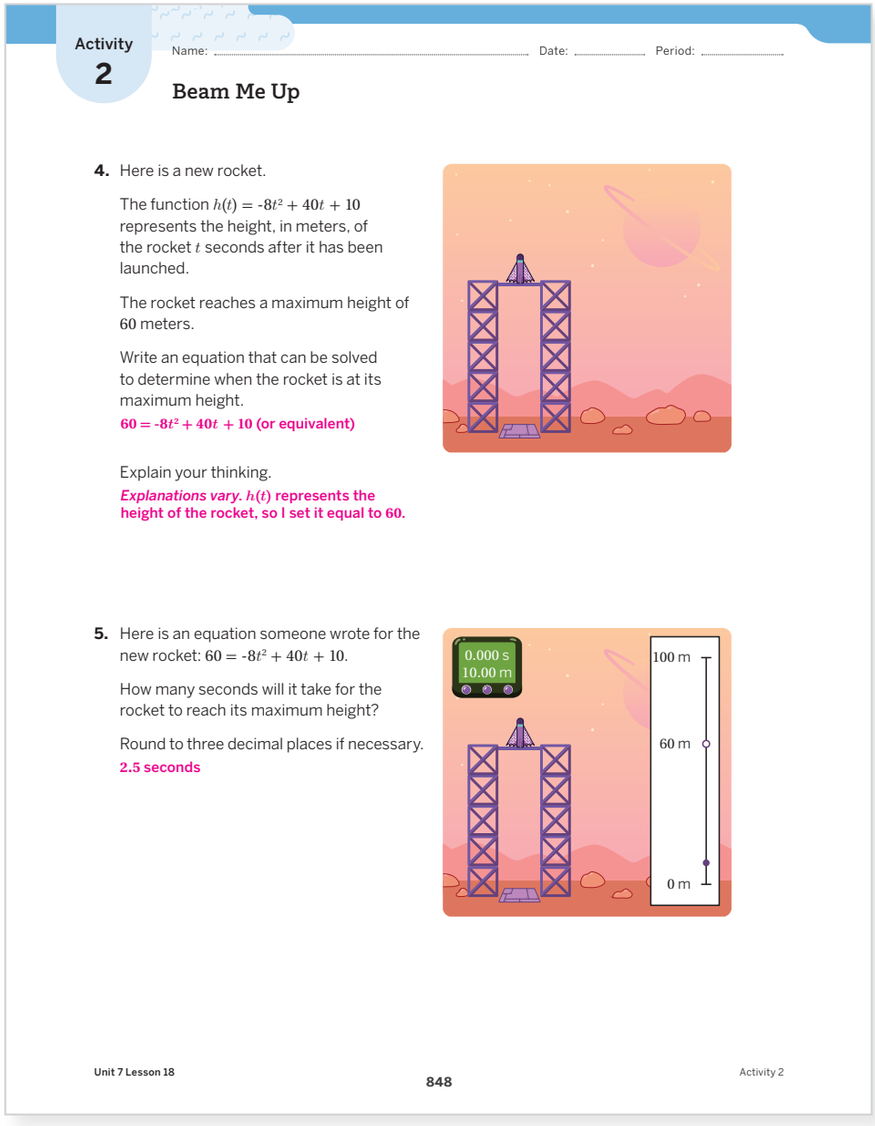
To support students getting started, consider asking, "What information were you given about the rocket launch? Where can you substitute that information into the equation?"

D **Differentiation: (Support)** Students might try to solve $60 = -8t^2 + 40t + 10$ using factoring or the quadratic formula without first setting it equal to 0. Consider asking students, "How is this equation different from the one Makayla solved on Problem 3? What has to be true before we can use the quadratic formula or zero-product property?"

To surface connections, consider asking pairs:

- "Does your answer make sense in this situation?"
- "Can there be more than one answer to this question? How do you know?"
- "Is there another way we could have solved this equation?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Beam Me Up

4. Here is a new rocket.

The function $h(t) = -8t^2 + 40t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.

The rocket reaches a maximum height of 60 meters.

Write an equation that can be solved to determine when the rocket is at its maximum height.

$60 = -8t^2 + 40t + 10$ (or equivalent)

Explain your thinking.

Explanations vary. $h(t)$ represents the height of the rocket, so I set it equal to 60.

5. Here is an equation someone wrote for the new rocket: $60 = -8t^2 + 40t + 10$.

How many seconds will it take for the rocket to reach its maximum height?

Round to three decimal places if necessary.

2.5 seconds

Unit 7 Lesson 18 848 Activity 2

Pairs | 12 min

Activity 2 Beam Me Up (continued)

Purpose: Students write and solve quadratic equations and practice interpreting their solutions in context. **(MTR.3.1, MTR.7.1)**

Short on time? Consider only discussing Problem 6 during the Connect.

2 Monitor

Select and sequence a variety of student explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What equation could you solve to help us answer this question?"
Evaluate the square root of a negative number as if it were a positive number.	**Strengthen** Consider asking, "Is it possible to multiply a number by itself and get a negative product? Why or why not?"
Would like an additional challenge.	**Stretch** Invite students to determine the range in this context.

Look for a variety of questions to share during the Connect.

D Differentiation: (Stretch) Invite students to answer one of their classmate's questions from the Warm-Up.

3 Connect

Gather and display the distribution of responses.

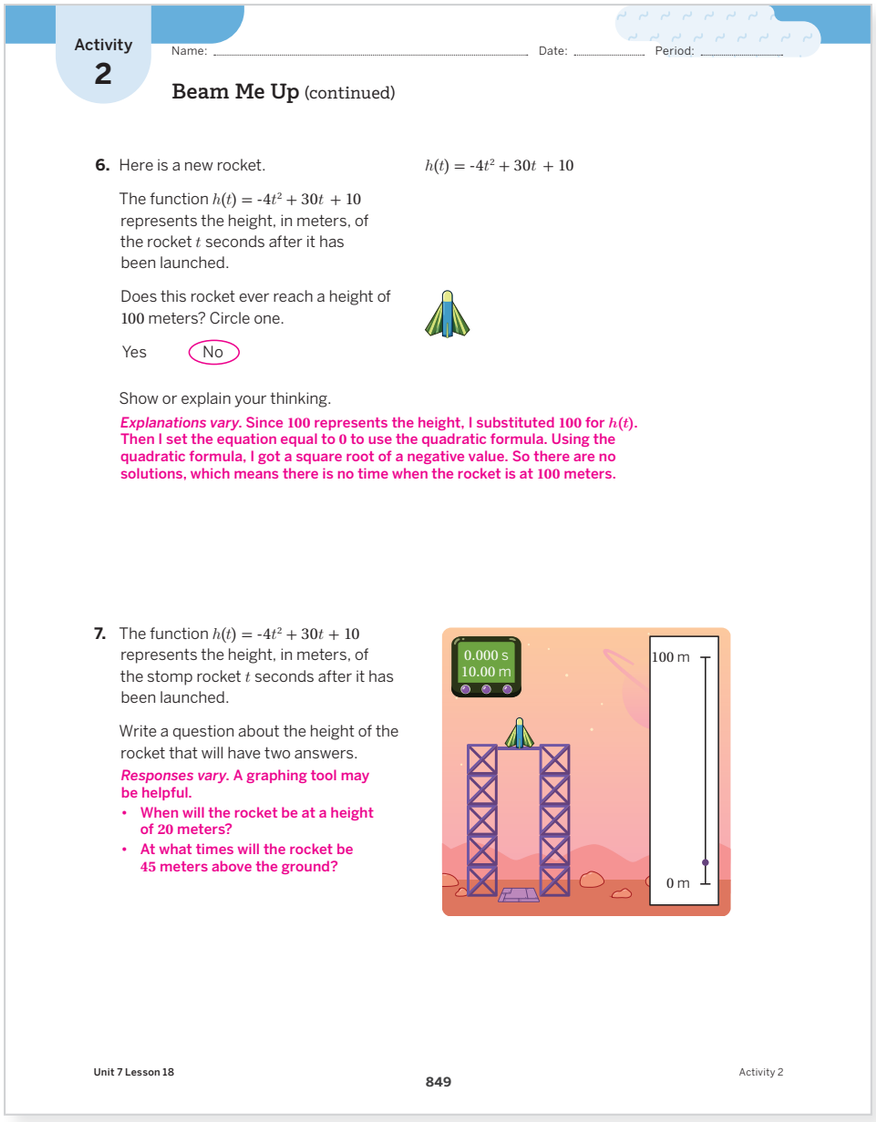
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

ELL English Language Learners To support discussions, provide sentence frames such as:
- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How do you know that . . . ?

(Speaking and Listening)

Note: If it doesn't come up naturally, consider asking:
- "Do you have to use the entire quadratic formula to answer this question?"
- "How could you answer this question by completing the square or graphing?"

Note: If time allows, display several student questions and ask:
- "How are these questions alike? How are they different?"
- "Which heights will the rocket be at twice? Once? Which heights will the rocket never reach?" **(MTR.3.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Beam Me Up (continued)

6. Here is a new rocket. $h(t) = -4t^2 + 30t + 10$

The function $h(t) = -4t^2 + 30t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.

Does this rocket ever reach a height of 100 meters? Circle one.

Yes (No)

Show or explain your thinking.

Explanations vary. Since 100 represents the height, I substituted 100 for $h(t)$. Then I set the equation equal to 0 to use the quadratic formula. Using the quadratic formula, I got a square root of a negative value. So there are no solutions, which means there is no time when the rocket is at 100 meters.

7. The function $h(t) = -4t^2 + 30t + 10$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

Write a question about the height of the rocket that will have two answers.

Responses vary. A graphing tool may be helpful.
- When will the rocket be at a height of 20 meters?
- At what times will the rocket be 45 meters above the ground?

Unit 7 Lesson 18 849 Activity 2

Pairs | 10 min

Activity 3 Rocket Scientist

Purpose: Students create their own rocket launch and answer questions authored by their classmates.

Short on time? Consider inviting students to complete fewer of their classmates' challenges.

1 Launch

Use the Notice and Wonder routine to support students make observations about how adjusting the gravity, velocity, and height affect the function. **(MTR.3.1)**

Consider sharing that students will create their own challenge and then create another challenge for others to solve.

To encourage creativity, invite students to create rockets they think none of their classmates will make.

Students will need one or more partners for this activity. Distribute the Activity 3 Sheet so they can choose their stomp rocket challenge. These students will not receive responsive feedback on their challenge before sharing it with others.

2 Monitor

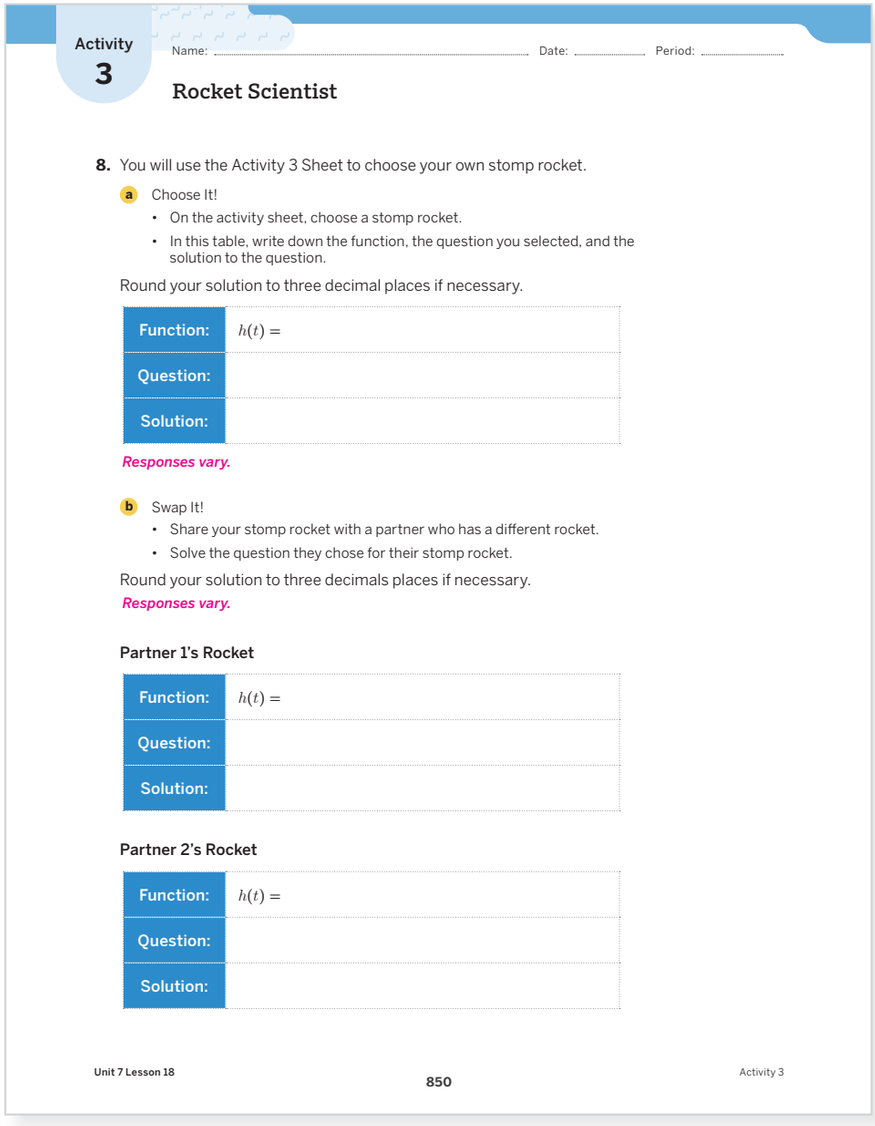
To surface connections, consider asking pairs:

- "How did you pick the challenges you wanted to solve?"
- "How did you know which solutions made sense in the situation and which did not?" **(MTR.3.1)**
- "What made some challenges more or less complicated than others?"

A **Accessibility: Conceptual Processing** Invite students to share how each problem is different from the previous one to help apply concepts to new situations.

ELL **English Language Learners** Invite students to read the questions aloud and paraphrase what it means. Clarify any unfamiliar words. **(Reading)**

D **Differentiation: (Stretch)** Invite students to answer one of their classmate's questions from the Warm-Up.



Activity 3

Name: Date: Period:

Rocket Scientist

8. You will use the Activity 3 Sheet to choose your own stomp rocket.

a. Choose It!
- On the activity sheet, choose a stomp rocket.
- In this table, write down the function, the question you selected, and the solution to the question.

Round your solution to three decimal places if necessary.

Function:	$h(t) =$
Question:	
Solution:	

Responses vary.

b. Swap It!
- Share your stomp rocket with a partner who has a different rocket.
- Solve the question they chose for their stomp rocket.

Round your solution to three decimals places if necessary.

Responses vary.

Partner 1's Rocket

Function:	$h(t) =$
Question:	
Solution:	

Partner 2's Rocket

Function:	$h(t) =$
Question:	
Solution:	

Unit 7 Lesson 18 850 Activity 3

Whole Class | 5 min

Synthesis

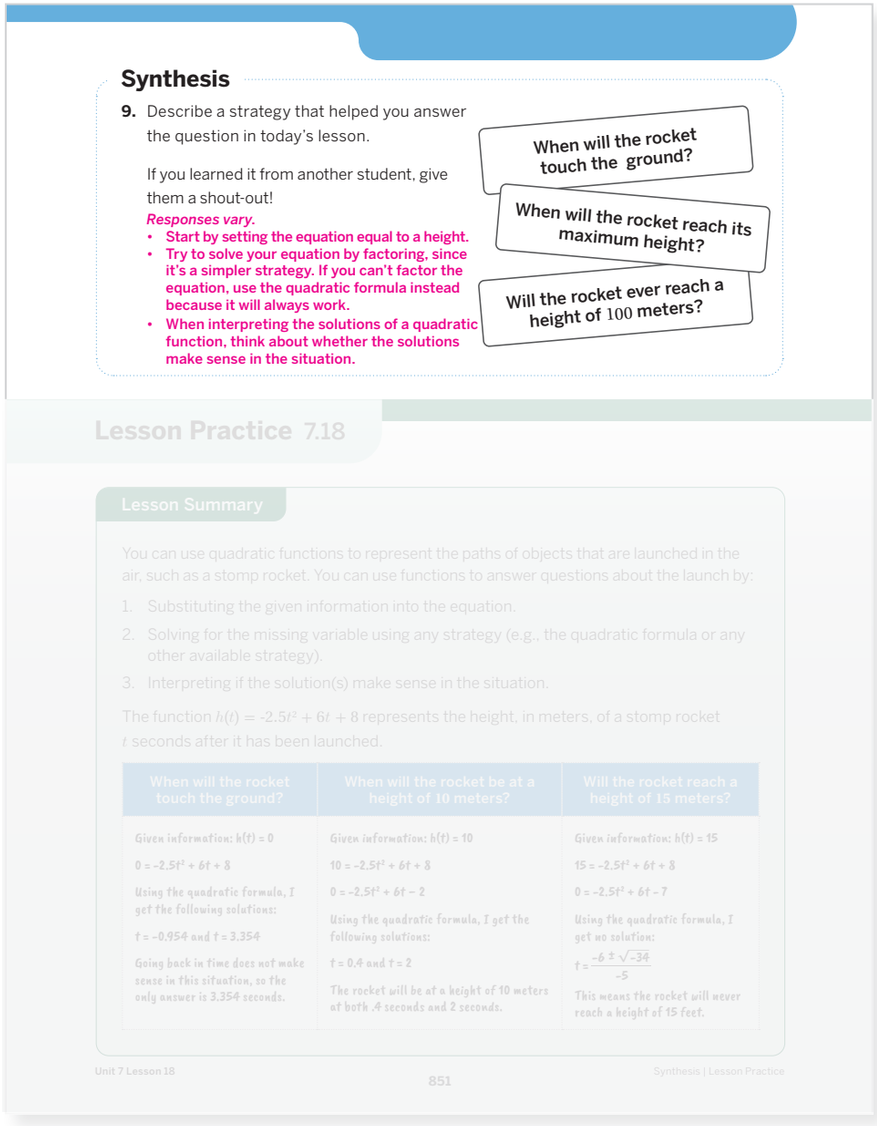
Purpose: Students synthesize their strategies for using quadratic equations to model contexts and answer questions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Identify the quantities given in the question.
- Substitute given quantities into the correct variables of the equation.
- Use the quadratic formula, factoring, completing the square, or other strategies to solve the equation.
- Determine if each solution makes sense in the situation.

Consider asking:
- "What makes sense to you about each strategy? What doesn't make sense?
- "How did you know which solutions made sense in the situation and which did not?" **(MTR.3.1)**

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. Describe a strategy that helped you answer the question in today's lesson.

If you learned it from another student, give them a shout-out!

Responses vary.
- Start by setting the equation equal to a height.
- Try to solve your equation by factoring, since it's a simpler strategy. If you can't factor the equation, use the quadratic formula instead because it will always work.
- When interpreting the solutions of a quadratic function, think about whether the solutions make sense in the situation.

When will the rocket touch the ground?

When will the rocket reach its maximum height?

Will the rocket ever reach a height of 100 meters?

Lesson Practice 7.18

Lesson Summary

You can use quadratic functions to represent the paths of objects that are launched in the air, such as a stomp rocket. You can use functions to answer questions about the launch by:

1. Substituting the given information into the equation.
2. Solving for the missing variable using any strategy (e.g., the quadratic formula or any other available strategy).
3. Interpreting if the solution(s) make sense in the situation.

The function $h(t) = -2.5t^2 + 6t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

When will the rocket touch the ground?	When will the rocket be at a height of 10 meters?	Will the rocket reach a height of 15 meters?
Given information: $h(t) = 0$	Given information: $h(t) = 10$	Given information: $h(t) = 15$
$0 = -2.5t^2 + 6t + 8$	$10 = -2.5t^2 + 6t + 8$	$15 = -2.5t^2 + 6t + 8$
Using the quadratic formula, I get the following solutions:	$0 = -2.5t^2 + 6t - 2$	$0 = -2.5t^2 + 6t - 7$
$t = -0.954$ and $t = 3.354$	Using the quadratic formula, I get the following solutions:	Using the quadratic formula, I get no solution:
Going back in time does not make sense in this situation, so the only answer is 3.354 seconds.	$t = 0.4$ and $t = 2$	$t = \frac{-6 \pm \sqrt{-34}}{-5}$
	The rocket will be at a height of 10 meters at both .4 seconds and 2 seconds.	This means the rocket will never reach a height of 15 feet.

Unit 7 Lesson 18 851 Synthesis | Lesson Practice

Lesson Takeaway: Equations that model contexts can be used to answer questions by substituting known values into the equation and solving. Since quadratic equations can mathematically have zero, one, or two solutions, it is important to interpret which one(s) make sense in context.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using an equation to answer a question about a quadratic context.

Today's Goals

1. **Goal:** Write quadratic equations to represent a situation.
2. **Goal:** Solve quadratic equations in context.
3. **Language Goal:** Interpret the solutions to quadratic equations in context. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.18

The function $h(t) = -5t^2 + 10t + 1$ represents the height of an apple, in meters, t seconds after it is tossed straight up in the air.

How many seconds will it take for the apple to reach the ground?

Round to three decimal places if necessary.

$t = \frac{(-10 \pm \sqrt{(120)})}{-10}$ or 2.095 seconds

D Differentiation Use after Lesson 18

Support

Provide targeted intervention for students by using these resources.

If students need support solving quadratic equations to answer questions in context:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and invite students to share strategies for writing an equation and interpreting the solutions in context.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Take a moment to reflect on how students interacted with mathematical models of quadratic contexts. How did writing questions support students in making sense of what mathematical models could be used for? How did students interpret solutions in context? What reasoning did students use to determine if solutions were reasonable in a context or not? **(MTR.3.1, MTR.7.1)**

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe a strategy that helped you answer the question in today's lesson.

If you learned it from another student, give them a shout-out!

Responses vary.

- Start by setting the equation equal to a height.
- Try to solve your equation by factoring, since it's a simpler strategy. If you can't factor the equation, use the quadratic formula instead because it will always work.
- When interpreting the solutions of a quadratic function, think about whether the solutions make sense in the situation.

When will the rocket touch the ground?

When will the rocket reach its maximum height?

Will the rocket ever reach a height of 100 meters?

Lesson Practice 7.18

Lesson Summary

You can use quadratic functions to represent the paths of objects that are launched in the air, such as a stomp rocket. You can use functions to answer questions about the launch by:

1. Substituting the given information into the equation.
2. Solving for the missing variable using any strategy (e.g., the quadratic formula or any other available strategy).
3. Interpreting if the solution(s) make sense in the situation.

The function $h(t) = -2.5t^2 + 6t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

When will the rocket touch the ground?	When will the rocket be at a height of 10 meters?	Will the rocket reach a height of 15 meters?
Given information: $h(t) = 0$ $0 = -2.5t^2 + 6t + 8$ Using the quadratic formula, I get the following solutions: $t = -0.954$ and $t = 3.354$ Going back in time does not make sense in this situation, so the only answer is 3.354 seconds.	Given information: $h(t) = 10$ $10 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 2$ Using the quadratic formula, I get the following solutions: $t = 0.4$ and $t = 2$ The rocket will be at a height of 10 meters at both .4 seconds and 2 seconds.	Given information: $h(t) = 15$ $15 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 7$ Using the quadratic formula, I get no solution: $t = \frac{-6 \pm \sqrt{-34}}{-5}$ This means the rocket will never reach a height of 15 feet.

Lesson Practice 7.18

Name: ..... Date: ..... Period: .....

Problems 1–2: The function $f(t) = 4 + 12t - 16t^2$ models the height of a tennis ball, in feet, t seconds after it was hit.

1. Select *all* of the solutions to the equation $0 = 4 + 12t - 16t^2$.

☑ A. $-\frac{1}{4}$ ☐ B. $\frac{1}{4}$ ☐ C. 4 ☑ D. 1 ☐ E. -1

2. How many seconds until the tennis ball hits the ground? Explain how you know.

1 second. *Explanations vary.* $f(1) = 0$, which means the ball is on the ground at 1 second. The other solution to the equation is a negative value of t, which doesn't make sense for this situation.

Problems 3–5: Katie is planning to go skydiving. She writes the function $h(t) = -16t^2 + 13500$ to represent her height, in feet, t seconds after jumping out of the airplane.

3. According to Katie's function, how high is the airplane when she jumps?

13,500 feet

4. It's recommended that skydivers open their parachutes at 5000 feet. Use $h(t)$ to approximate how many seconds after jumping Katie should open her parachute.

About 23 seconds

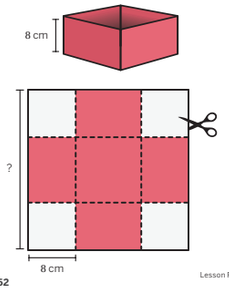
5. When Katie actually jumps, do you think she will reach 5000 feet in less time, more time, or exactly the amount of time you approximated? Explain your thinking.

***Responses and explanations vary.* I think it will take Katie more than 23 seconds because her body is working against a lot of wind, which might cause her to slow down.**

6. A company wants to make a square box with no top. The requirements are:

- It must be 8 cm tall.
- Its volume must be 1,000 cubic cm.

The boxes are made by cutting four corners from a square piece of cardboard and folding the flaps up.



What should the length of the starting square be? Show or explain your thinking.

About 27.2 cm. *Explanations vary.* If x is the length and the width of the square box, I can write an equation for the volume: $8x^2 = 1000$. The positive solution to this equation is $\sqrt{125}$ cm, or about 11.2 cm. So the length of the square is $8 + 8 + 11.2 = 27.2$ cm.

Lesson Practice 7.18

Name: ..... Date: ..... Period: .....

Test Practice

Problems 7–9: The function $h(t) = -75t^2 + 60t$ models the height, in inches, of a jumping frog, where t is the number of seconds after it jumped.

7. Solve the equation $-75t^2 + 60t = 0$.

$t = 0$ and $t = 0.8$

8. What do the solutions tell us about the jumping frog?

***Responses vary.* Each solution corresponds to a moment when the frog was on the ground. It was on the ground when it started jumping, and it returned to the ground after 0.8 seconds.**

9. When does the frog reach its maximum height of 12 inches? **0.4 seconds**

Spiral Review

10. Select all the irrational numbers.

☑ A. $\sqrt{13}$ ☐ B. $-\frac{53}{11}$ ☑ C. $-\sqrt{17}$
☐ D. $\sqrt{\frac{28}{7}}$ ☐ E. $\sqrt{81}$ ☐ D. $(-\sqrt{8})^2$

11. Write an equation of each type that starts with $x^2 + 8x =$ .....

No Solutions	$x^2 + 8x =$ .....
One Solution	$x^2 + 8x =$ .....
Two Solutions	$x^2 + 8x =$ .....

Equations vary.

No solutions
- $x^2 + 8x = -50$
- $x^2 + 8x = -17$

One solution
- $x^2 + 8x = -16$
- $x^2 + 8x = 8x$

Two solutions
- $x^2 + 8x = 50$
- $x^2 + 8x = 0$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.912.AR.3.1
	3–5	2	MA.912.AR.3.1
	6	3	MA.912.AR.3.1
	7	1	MA.912.AR.3.1
	8	2	MA.912.AR.3.1
Test Practice	9	2	MA.912.AR.3.1
Spiral Review			
Fluency	10	1	MA.8.NSO.1.1
	11	3	MA.912.AR.3.1

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students solved quadratic equations using reasoning, factoring, properties, completing the square, and the quadratic formula.

Career Connection | Meet Po-Shen Loh

Invite students to read about how mathematics professor Po-Shen Loh made math history in 2019 when he discovered a new method for solving quadratic equations.

Consider asking:

- "Po-Shen Loh's method was based on observations by other mathematicians. How might studying the work of others help people make new discoveries, both in mathematics and in other fields of study?"

B.E.S.T. Benchmark Connection

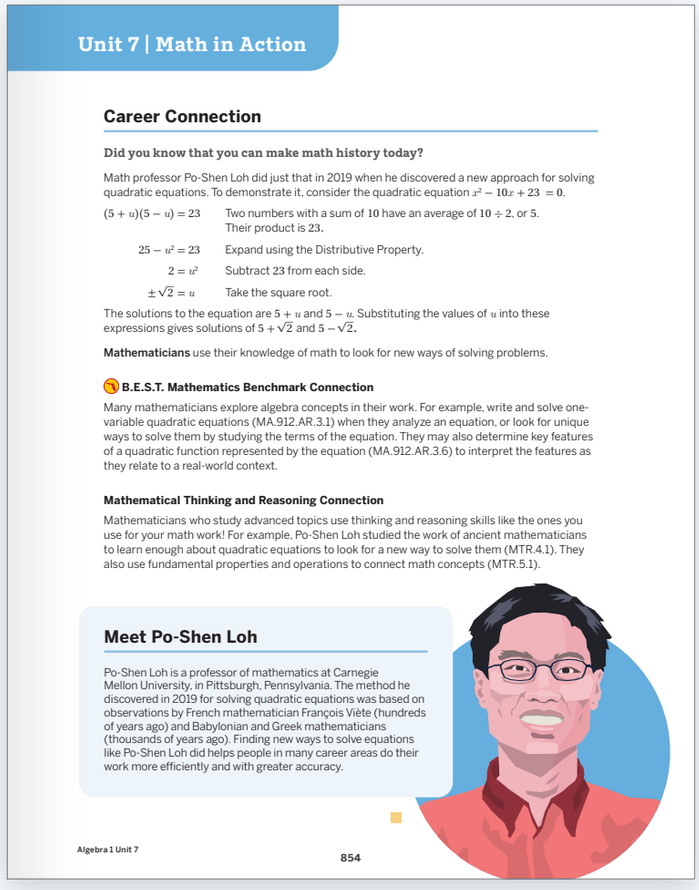
Consider asking:

- "What other math concepts have you studied in Units 6 and 7 that you could apply to quadratic equations shown with Po-Shen's solution method?" Sample response: You can graph the quadratic function and compare the points of intersection of the graph with the x-axis to the solutions found by his method to see if they are the same. (MA.912.AR.3.6)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "Why would you expect mathematicians like Po-Shen Loh to apply Mathematical Thinking and Reasoning standard MTR.3.1 in their work?" Sample response: They would have to show mathematical fluency in order to be confident that their methods are accurate.



Unit 7 | Math in Action

Career Connection

Did you know that you can make math history today?

Math professor Po-Shen Loh did just that in 2019 when he discovered a new approach for solving quadratic equations. To demonstrate it, consider the quadratic equation $x^2 - 10x + 23 = 0$.

$(5 + u)(5 - u) = 23$ Two numbers with a sum of 10 have an average of $10 \div 2$, or 5. Their product is 23.

$25 - u^2 = 23$ Expand using the Distributive Property.

$2 = u^2$ Subtract 23 from each side.

$\pm\sqrt{2} = u$ Take the square root.

The solutions to the equation are $5 + u$ and $5 - u$. Substituting the values of u into these expressions gives solutions of $5 + \sqrt{2}$ and $5 - \sqrt{2}$.

Mathematicians use their knowledge of math to look for new ways of solving problems.

B.E.S.T. Mathematics Benchmark Connection

Many mathematicians explore algebra concepts in their work. For example, write and solve one-variable quadratic equations (MA.912.AR.3.1) when they analyze an equation, or look for unique ways to solve them by studying the terms of the equation. They may also determine key features of a quadratic function represented by the equation (MA.912.AR.3.6) to interpret the features as they relate to a real-world context.

Mathematical Thinking and Reasoning Connection

Mathematicians who study advanced topics use thinking and reasoning skills like the ones you use for your math work! For example, Po-Shen Loh studied the work of ancient mathematicians to learn enough about quadratic equations to look for a new way to solve them (MTR.4.1). They also use fundamental properties and operations to connect math concepts (MTR.5.1).

Meet Po-Shen Loh

Po-Shen Loh is a professor of mathematics at Carnegie Mellon University, in Pittsburgh, Pennsylvania. The method he discovered in 2019 for solving quadratic equations was based on observations by French mathematician François Viète (hundreds of years ago) and Babylonian and Greek mathematicians (thousands of years ago). Finding new ways to solve equations like Po-Shen Loh did helps people in many career areas do their work more efficiently and with greater accuracy.

Algebra 1 Unit 7 854

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify the factored form of a quadratic expression in standard form	Lessons 4–5	1	MA.912.AR.1.7
2	Determine solutions to a quadratic equation using a graph	Lesson 11	1	MA.912.AR.3.6, MA.912.AR.3.7
3	Determine the number of solutions to a quadratic equation	Lesson 10	2	MA.912.AR.3.1 MTR.5.1
4a–b	Critique the use of completing the square to solve a quadratic equation	Lesson 14	2	MA.912.AR.3.1 MTR.4.1
4c	Solve quadratic equations	Lesson 14	2	MA.912.AR.3.1
5	Rewrite quadratic expressions in different forms	Lessons 2–5, 15	2	MA.912.AR.1.3, MA.912.AR.1.7, MA.912.AR.3.1 MTR.3.1
6	Solve quadratic equations	Lessons 9, 12, 14, 17	2	MA.912.AR.3.1
7a	Write a quadratic equation that models a situation	Lesson 18	2	MA.912.AR.3.4 MTR.7.1
7b	Determine the solution to a quadratic equation in context	Lesson 18	2	MA.912.AR.3.8 MTR.7.1
7c	Identify and interpret the vertex in context	Lessons 15, 18	3	MA.912.AR.3.6 MTR.7.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 7

1. Which expression is equivalent to $x^2 + 9x - 36$?

A. $(x - 6)^2$

B. $(x + 6)(x - 6)$

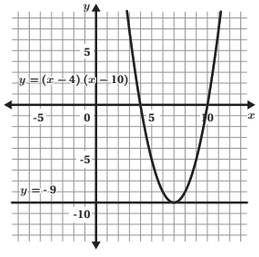
C. $(x - 9)(x + 4)$

D. $(x - 3)(x + 12)$

2. Here is the graph that Ivan made to solve $(x - 4)(x - 10) = -9$.

Determine the solution(s) to this equation.

$x = 7$



3. Select *all* the equations that have:

a No solutions.

- ☐ A. $x^2 = 16$
- ☑ B. $4x^2 + 1 = 0$
- ☑ C. $x^2 = -16$
- ☐ D. $x^2 + 1 = 1$
- ☐ E. $(x - 8)(x - 8) = 0$

b Exactly one solution.

- ☐ A. $x^2 = 16$
- ☐ B. $4x^2 + 1 = 0$
- ☐ C. $x^2 = -16$
- ☑ D. $x^2 + 1 = 1$
- ☑ E. $(x - 8)(x - 8) = 0$

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 7

4. Jalen made a mistake when solving $x^2 - 10x + 9 = 0$ by completing the square.

$x^2 - 10x + 9 = 0$
$x^2 - 10x = -9$
$x^2 - 10x + 25 = -9$
$(x - 5)^2 = -9$
$x - 5 = \pm\sqrt{-9}$
no solution

a Describe one thing that Jalen did correctly.

Responses vary. Jalen correctly added 25 to make $x^2 - 10x$ into a perfect square.

b Show or explain why Jalen's work is incorrect.

Explanations vary. Jalen did not add 25 to both sides of the equation.

c Determine the solution(s) to the equation.

$x = 1$ and $x = 9$

5. Complete the table with equivalent forms of each expression.

	Vertex Form	Standard Form	Factored Form
Expression A	$(x + 3)^2 - 4$	$x^2 + 6x + 5$	$(x + 5)(x + 1)$ (or equivalent)
Expression B	$(x - 4)^2 - 1$	$x^2 - 8x + 15$	$(x - 5)(x - 3)$
Expression C	$(x + 1)^2 - 9$	$x^2 + 2x - 8$	$(x - 2)(x + 4)$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 7

6. Determine the exact solution(s) for each equation.

a $x^2 + 10x - 24 = 0$
$x = -12$ and $x = 2$

b $5x^2 + 13x = 6$
$x = -3$ and $x = 0.4$ (or equivalent)

c $(x - 3)^2 = 5$
$x = 3 \pm \sqrt{5}$

7. $h(x) = -5x^2 + 10x + 2$ represents the height of a football, in meters, x seconds after it was thrown.



a Write an equation that can be solved to determine when the football will touch the ground at a height of 0 meters.

$0 = -5x^2 + 10x + 2$

b Determine the number of seconds it will take for the football to touch the ground.

2.183 seconds (or equivalent, e.g., $1 + \sqrt{\frac{7}{5}}$, $\frac{-10 - \sqrt{140}}{-10}$)

c The vertex form of this function is $h(x) = -5(x - 1)^2 + 7$.
Determine the coordinates of the vertex and explain what it means about this situation.

The coordinates of the vertex are $(1, 7)$. *Explanations vary.* After 1 second, the football reaches its maximum height of 7 meters.

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 7

1. Which expression is equivalent to $x^2 - 10x + 16$?

A. $(x - 4)^2$

B. $(x - 8)(x - 2)$

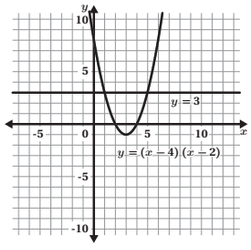
C. $(x + 10)(x + 6)$

D. $(x - 16)(x + 1)$

2. Here is the graph that Ivan made to solve $(x - 4)(x - 2) = 3$.

Determine the solution(s) to this equation.

$x = 1$ and $x = 5$



3. Select *all* the equations that have:

a. One solution.

- ☐ A. $x^2 = 64$
- ☑ B. $(x - 9)^2 = 0$
- ☐ C. $x^2 + 6 = 2$
- ☑ D. $x^2 + 4 = 4$
- ☐ E. $(x + 3)(x - 6) = 0$

b. Two solutions.

- ☑ A. $x^2 = 64$
- ☐ B. $(x - 9)^2 = 0$
- ☐ C. $x^2 + 6 = 2$
- ☐ D. $x^2 + 4 = 4$
- ☑ E. $(x + 3)(x - 6) = 0$

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Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 7

4. Felipe made a mistake when solving $x^2 - 8x + 3 = 0$ by completing the square.

$x^2 - 8x + 3 = 0$
$x^2 - 8x = -3$
$x^2 - 8x + 64 = -3 + 64$
$(x - 8)^2 = 61$
$x - 8 = \pm\sqrt{61}$
$x = 8 \pm \sqrt{61}$

a. Describe one thing that Felipe did correctly.

Responses vary. Felipe correctly took the square root of both sides after factoring, including adding a $\pm$ on the right side.

b. Show or explain why Felipe's work is incorrect.

Explanations vary. Felipe did not divide by 2 before squaring the second coefficient to find the perfect square.

c. Determine the solution(s) to the equation.

$x = 4 \pm \sqrt{13}$ (or equivalent)

5. Complete the table with equivalent forms of each expression.

	Factored Form	Standard Form	Vertex Form
Expression A	$(x - 3)(x + 1)$ (or equivalent)	$x^2 - 2x - 3$	$(x - 1)^2 - 4$
Expression B	$(x + 12)(x + 6)$	$x^2 + 18x + 72$	$(x + 9)^2 - 9$
Expression C	$(x - 6)(x - 4)$	$x^2 - 10x + 24$	$(x - 5)^2 - 1$

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 7

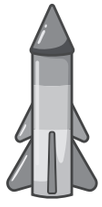
6. Determine the exact solution(s) for each equation.

a. $x^2 - 6x = 16$
$x = -2$ and $x = 8$

b. $2x^2 + 13x + 15 = 0$
$x = -5$ and $x = -1.5$ (or equivalent)

c. $(x - 2)^2 = 10$
$x = 2 \pm \sqrt{10}$

7. $h(x) = -3x^2 + 18x + 10$ represents the height of a stomp rocket, in meters, x seconds after it was thrown on another planet.



a. Write an equation that can be solved to determine when the rocket will touch the ground at a height of 0 meters.

$0 = -3x^2 + 18x + 10$

b. Determine the number of seconds it will take for the rocket to touch the ground.

6.512 seconds (or equivalent, e.g., $3 + \sqrt{\frac{37}{3}}$, $\frac{-18 - \sqrt{444}}{-6}$)

c. The vertex form of this function is $h(x) = -3(x - 3)^2 + 37$.
Determine the coordinates of the vertex and explain what it means about this situation.

The coordinates of the vertex are (3, 37). *Explanations vary.* After 3 seconds, the rocket reaches its maximum height of 37 meters.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1 • Convert quadratic expressions between factored and standard form by multiplying and factoring. **(Lessons 1–5)** • Use factoring and the zero-product property to solve equations and determine the zeros of quadratic functions. **(Lessons 6–9)**	1	**Support**	• **Mini-Lesson:** Rewriting Factored-Form Expressions in Standard Form • **Teacher Move:** Consider revisiting Lesson 4 (*X-Factor*)
		Stretch	• **You're invited to explore more:** • Lesson 3 (*Standard Feature*)
	6a–b	**Support**	• **Teacher Move:** Consider revisiting Lesson 9 (*Make It Zero*) to discuss how to use the zero-product property to determine zeros and solve quadratic equations.

		Stretch	• **You're invited to explore more:**
			  • Lesson 8 (*Shooting Stars*)
			  • Lesson 9 (*Make It Zero*)

Sub-Unit 2 • Solve quadratic equations with no solutions, one solution, and two solutions using reasoning, graphs, and completing the square. **(Lessons 10–14)** • Complete the square in order to reveal the vertex of a quadratic function. **(Lesson 15)**	2	**Support**	• **Teacher Move:** Consider revisiting Lesson 11 (*Graph to Solve*) and discuss how graphing can be used to solve equations.
		Strengthen	• **Repeated Challenges:** • Lesson 11 (*Graph to Solve*)
	3	**Support**	• **Teacher Move:** Consider reviewing the class display of ideas about equations that have no solutions, one solution, and two solutions.
		Strengthen	• **Repeated Challenges:** • Lesson 10 (*Zero, One, or Two*)
	4	**Support**	• **Teacher Move:** Consider revisiting Lesson 14 (*Square Tactic*) and discuss common mistakes in completing the square.
		Strengthen	• **Repeated Challenges:** • Lesson 13 (*Square Dance*)

		Stretch	• **You're invited to explore more:**
			  • Lesson 13 (*Square Dance*)
			  • Lesson 14 (*Square Tactic*)

	6c	**Support**	• **Teacher Move:** Consider revisiting Lesson 12 (*Couldn't Square Less*) to discuss how to solve quadratic equations by taking the square root of each side.
	5	**Support**	• **Teacher Move:** Consider revisiting Lesson 15 (*Back and Forth*) and discuss student strategies for writing an equation and interpreting the solutions in context.
Sub-Unit 3 • Derive the quadratic formula and use it to solve quadratic equations. **(Lessons 16–18)**	7	**Support**	• **Teacher Move:** Consider revisiting Lesson 18 (*Stomp Rockets in Space*) and invite students to share strategies for writing an equation and interpreting the solutions in context.
		Strengthen	• **Repeated Challenges:** • Lesson 18 (*Stomp Rockets in Space*)

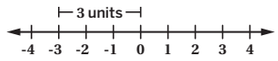
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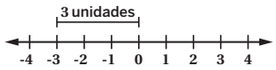
absolute value The absolute value of a number is its distance from 0 on the number line.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

$|4| = 4$ and $|-4| = 4$. They are both 4 units away from 0.

valor absoluto El valor absoluto de un número es su distancia al 0 en la recta numérica.

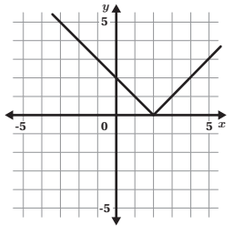


Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

$|4| = 4$ y $|-4| = 4$. Ambos están a 4 unidades del 0.

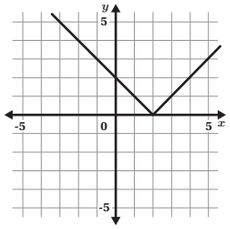
absolute value function A function that is defined using absolute value symbols. When written in the form $f(x) = |x - h|$, its output is the distance of its input from a given value, h.

For example, $f(x) = |x - 2|$ outputs the distance from 2 for every input value.



función de valor absoluto Una función que se define utilizando signos de valor absoluto. Cuando se escribe en la forma $f(x) = |x - h|$, su salida es la distancia de su entrada a un valor determinado, h.

Por ejemplo, $f(x) = |x - 2|$ arroja la distancia de cada valor de entrada al 2.

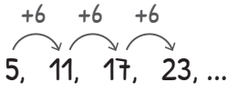


area The amount of space inside a two-dimensional shape. It is measured in square units, such as square inches or square centimeters.

área La cantidad de espacio dentro de una figura bidimensional. Se mide en unidades cuadradas, como pulgadas cuadradas o centímetros cuadrados.

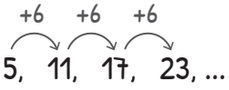
arithmetic sequence A sequence that changes by a constant difference.

In this arithmetic sequence, the first term is 5 and the constant difference is 6.



secuencia aritmética Una secuencia que cambia con una diferencia constante.

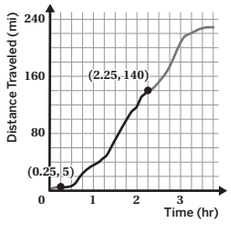
En esta secuencia aritmética, el primer término es 5 y la diferencia constante es 6.



association If two variables are statistically related to one another, we say that there is an association between the two variables. A *correlation* is a type of association.

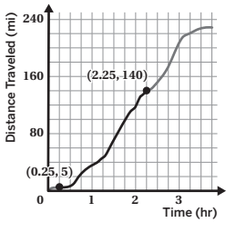
asociación Si dos variables están relacionadas estadísticamente, decimos que existe una asociación entre ambas variables. Una *correlación* es un tipo de asociación.

average rate of change A measure of how much a function changes, on average, over an interval. To calculate the average rate of change over an interval, find the slope between the point on the graph of the function where the interval begins and the point where it ends.



To calculate the average rate of change over the interval from 0.25 hours to 2.25 hours, calculate the change in y-values $(140 - 5)$ and divide by the change in x-values $(2.25 - 0.25)$. The average rate of change from 0.25 to 2.25 is 67.5, which means the average speed on that trip was 67.5 miles per hour in that interval.

tasa de cambio promedio Una medida de cuánto cambia una función, en promedio, en un intervalo. Para calcular la tasa de cambio promedio a lo largo de un intervalo, se halla la pendiente entre el punto donde empieza el intervalo y el punto donde termina en la gráfica de la función.



Para calcular la tasa de cambio promedio en el intervalo de 0.25 horas a 2.25 horas, se calcula el cambio en los valores de y $(140 - 5)$ y se divide por el cambio en los valores de x $(2.25 - 0.25)$. La tasa de cambio promedio de 0.25 a 2.25 es 67.5, lo que significa que la velocidad media en ese trayecto fue de 67.5 millas por hora en ese intervalo.

B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.

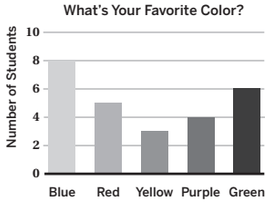
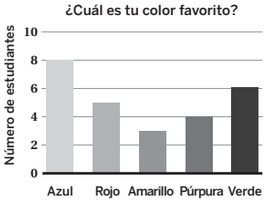


gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.



base A number or expression that is raised to an exponent. The exponent describes the number of times to multiply the base by itself.

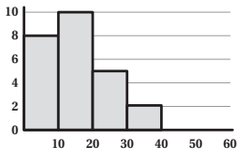
For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

base Un número o una expresión que se eleva a un exponente. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

English

bin (of a histogram) The intervals used to group data values in a histogram are called *bins*. The distance of each interval is called the *bin width*.

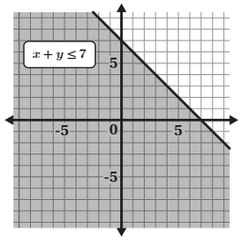


For example, this histogram shows 4 bins, each with a bin width of 10 units.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

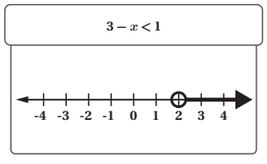
For example, a collection of students' heights and shoe sizes would be a bivariate data set.

boundary line The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \leq x$). A dashed boundary line indicates that they are not (e.g., $y < x$).



The solution region to $x + y \leq 7$ has a boundary line at $x + y = 7$. The line is solid because the points on the line $x + y = 7$ are included in the solution region.

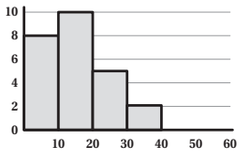
boundary point The value that separates the solution set of an inequality from non-solutions. A solid boundary point indicates that the point is included in the solution set. An empty boundary point indicates the point is not included in the solution set.



The solution set to $3 - x < 1$ has a boundary point at $x = 2$. The point at 2 is empty because 2 is not included in the solution set.

Español

intervalo (de un histograma) Se denominan intervalos a los espacios que se usan para agrupar los valores de datos en un histograma. La distancia de cada intervalo se denomina *ancho del intervalo*.

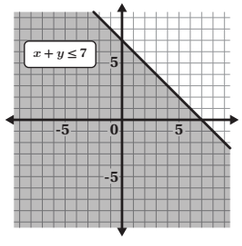


Por ejemplo, este histograma muestra 4 intervalos, cada uno con un ancho de intervalo de 10 unidades.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

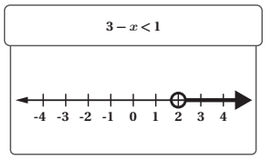
Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

recta límite La línea que separa la región solución de una desigualdad lineal de todos los valores que no son soluciones. Una desigualdad lineal (p. ej., $y < 2x + 5$) tiene una recta límite representada simbólicamente por la ecuación correspondiente (p. ej., $y = 2x + 5$). Una recta límite continua indica que esos puntos están incluidos en el conjunto de soluciones (p. ej., $y \leq x$). Una recta límite discontinua indica que no lo están (p. ej., $y < x$).



La región solución de $x + y \leq 7$ tiene una recta límite en $x + y = 7$. La recta es continua porque los puntos en la recta $x + y = 7$ se encuentran dentro de la región solución.

punto límite El valor que separa el conjunto de soluciones de una desigualdad de todos los valores que no son soluciones. Un punto límite sólido (relleno) indica que el punto está incluido en el conjunto de soluciones. Un punto límite vacío indica que el punto no está incluido en el conjunto de soluciones.



El conjunto de soluciones de $3 - x < 1$ tiene un punto límite en $x = 2$. El punto en 2 no está relleno porque 2 no está incluido en el conjunto de soluciones.

C

categorical data A type of data that is divided into groups or categories. Categorical data describes qualities or characteristics, such as colors, names, or zip codes, rather than numerical values.

For example, the breeds of 10 dogs are categorical data. Another example is the colors of 100 flowers.

causation Describes a relationship between two variables in which a change in one variable causes a change in the other variable. A special type of *correlation*.

datos categóricos Un tipo de datos que se dividen en grupos o categorías. Los datos categóricos describen cualidades o características, como colores, nombres o códigos postales, en vez de valores numéricos.

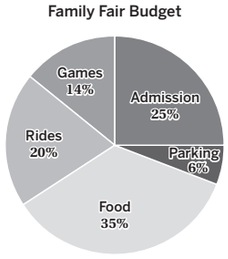
Por ejemplo, las razas de 10 perros diferentes son datos categóricos. Otro ejemplo son los colores de 100 flores diferentes.

causalidad Describe una relación entre dos variables en la que un cambio en una variable provoca un cambio en la otra variable. Un tipo especial de *correlación*.

Glossary/Glosario

English

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a *pie chart*.



coefficient The number that multiplies a variable in an algebraic expression. If no number is shown, the coefficient is understood to be 1.

For example, in the expression $5x$, the coefficient is 5. In y, the coefficient is 1.

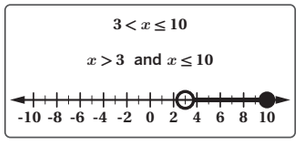
completing the square The process of rewriting a quadratic expression or equation to include a perfect square.

$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

compound To increase in value by applying an operation repeatedly. In mathematics, compounding can involve repeated multiplication or growth over time.

For example, when bacteria multiply by splitting in half every hour, the number of bacteria compounds, doubling each hour.

compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or."

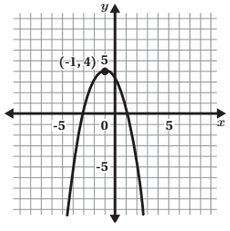


The numbers greater than 3 and less than or equal to 10 can be written as: $x > 3$ and $x \leq 10$ or $3 < x \leq 10$.

compound interest Interest calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.). The balance in an account that earns compound interest can be modeled by an exponential function.

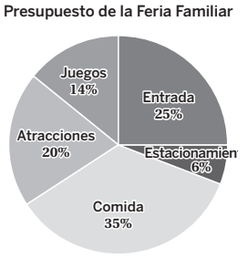
concave down A parabola that opens downward is described as *concave down*. A function with a negative a-value will produce a concave down parabola.

This parabola is concave down. Two ways to write the equation are $f(x) = -1(x + 1)^2 + 4$ and $f(x) = -x^2 - 2x + 3$.



Español

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado *gráfico de torta*.



coeficiente El número que multiplica una variable en una expresión algebraica. Si no se muestra ningún número, se entiende que el coeficiente es 1.

Por ejemplo, en la expresión $5x$, el coeficiente es 5. En y, el coeficiente es 1.

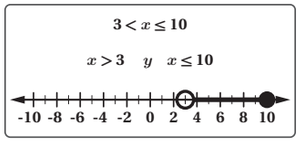
completación del cuadrado El proceso de reescribir una expresión o ecuación cuadrática para incluir un cuadrado perfecto.

$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

crecimiento compuesto Aumentar en valor aplicando una operación repetidamente. En matemáticas, el crecimiento compuesto puede implicar la multiplicación repetida o el crecimiento a lo largo del tiempo.

Por ejemplo, cuando las bacterias se multiplican dividiéndose por la mitad cada hora, el número de bacterias crece de forma compuesta y se duplica cada hora.

desigualdad compuesta Dos o más desigualdades juntas. Una desigualdad compuesta puede escribirse con signos o las palabras "y" u "o".

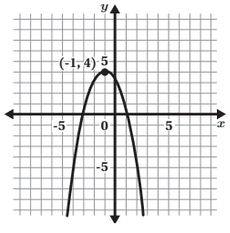


Los números mayores que 3 y menores o iguales que 10 pueden escribirse de la siguiente forma:
$x > 3$ y $x \leq 10$ o $3 < x \leq 10$.

interés compuesto Interés que se calcula según una cantidad inicial y el interés de los períodos anteriores. Se calcula en intervalos regulares (diariamente, mensualmente, anualmente, etc.). El saldo de una cuenta que produce interés compuesto puede modelarse con una función exponencial.

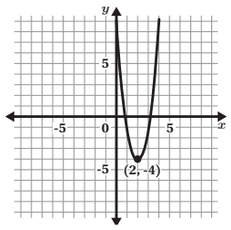
cóncava hacia abajo Una parábola que se abre hacia abajo se describe como cóncava hacia abajo. Una función con un valor a negativo producirá una parábola cóncava hacia abajo.

Esta parábola es cóncava hacia abajo. Dos formas de escribir la ecuación son $f(x) = -1(x + 1)^2 + 4$ y $f(x) = -x^2 - 2x + 3$.



English

concave up A parabola that opens upward is described as *concave up*. A function with a positive a-value will produce a concave up parabola.



This parabola is concave up. Two ways to write the equation are $f(x) = 4(x-2)^2 - 4$ and $f(x) = 4x^2 - 16x + 12$.

confidence interval A range of values used to estimate an unknown population value. It shows how likely it is that the true value is within the range.

A survey estimates that 60% of students prefer online learning, with a confidence interval of 55% to 65%. This means the true percentage is likely between 55% and 65%.

constant difference When the difference between every two consecutive values in a pattern is the same, there is a *constant difference*.

x	y	
0	5	
1	7	+2
2	9	+2
3	11	+2

The pattern in the table has a constant difference of 2.

constant ratio When the ratio between every two consecutive values in a pattern is the same, there is a *constant ratio*.

x	y	
0	1	
1	3	·3
2	9	·3
3	27	·3

The pattern in the table has a constant ratio of 3.

constraint A limitation on the possible values of variables in a model. Equations and inequalities are often used to represent constraints.

The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality $h \geq 36$.

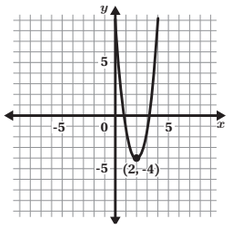
correlation A statistical relationship between two or more variables. Also called an *association*.

cube root The cube root of a number n (written as $\sqrt[3]{n}$) is the number that can be cubed to get n. The cube root is also the edge length of a cube with a volume of n.

For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

Español

cóncava hacia arriba Una parábola que se abre hacia arriba se describe como cóncava hacia arriba. Una función con un valor a positivo producirá una parábola cóncava hacia arriba.



Esta parábola es cóncava hacia arriba. Dos formas de escribir la ecuación son $f(x) = 4(x-2)^2 - 4$ y $f(x) = 4x^2 - 16x + 12$.

intervalo de confianza Un rango de valores utilizado para estimar un valor poblacional desconocido. Muestra la probabilidad de que el valor real esté dentro del rango.

Una encuesta estima que el $60\ \%$ de los estudiantes prefieren el aprendizaje en línea, con un intervalo de confianza del $55\ \%$ al $65\ \%$. Esto significa que el porcentaje real probablemente esté entre el $55\ \%$ y el $65\ \%$.

diferencia constante Cuando la diferencia entre dos valores consecutivos en un patrón permanece igual, hay una diferencia constante.

x	y	
0	5	
1	7	+2
2	9	+2
3	11	+2

El patrón en la tabla tiene una diferencia constante de 2.

razón constante Cuando la razón entre dos valores consecutivos en un patrón permanece igual, hay una razón constante.

x	y	
0	1	
1	3	·3
2	9	·3
3	27	·3

El patrón en la tabla tiene una razón constante de 3.

restricción Una limitación de los posibles valores de las variables en un modelo. Suelen usarse ecuaciones o desigualdades para representar restricciones.

La restricción "debes medir 36 pulgadas o más para subirte a la rueda de la fortuna" puede representarse con la desigualdad $h \geq 36$.

correlación Una relación estadística entre dos o más variables. También se denomina *asociación*.

raíz cúbica La raíz cúbica de un número n (se escribe $\sqrt[3]{n}$) es el número que puede elevarse al cubo para obtener n. La raíz cúbica también es la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de arista de un cubo que tiene un volumen de 64 unidades cúbicas.

D

data A collection of information, usually in the form of numbers, words, or measurements, that can be analyzed.

A list of students' test scores is an example of numerical data.

data sets A collection of values gathered for reference, analysis, or interpretation.

A list of test scores collected from a class is an example of data.

datos Una recopilación de información, generalmente en forma de números, palabras o medidas, que se puede analizar.

Una lista de las calificaciones de los exámenes de unos estudiantes es un ejemplo de datos numéricos.

conjuntos de datos Una recopilación de valores reunidos para referencia, análisis o interpretación.

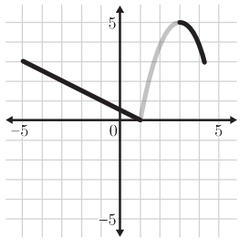
Una lista de las calificaciones de exámenes recopiladas de una clase es un ejemplo de datos.

Glossary/Glosario

English

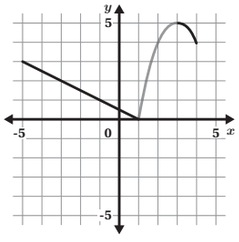
decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.



decreasing (interval or function) A function is decreasing when its outputs decrease as its inputs increase. A function can be decreasing for its entire domain or over an interval.

For example, the function $f(x)$ is decreasing when $-5 < x < 1$ and when $3 < x < 4$.



dependent variable The value of a dependent variable is based on the value of another variable or set of variables. In a function, the value of the dependent variable represents the output. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

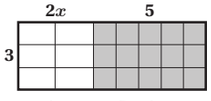
difference of squares An expression that can be written as one perfect square subtracted from another perfect square. It has the structure $r^2 - s^2$ in standard form and $(r - s)(r + s)$ in factored form.

These expressions are differences of squares: $x^2 - 25$, $9x^2 - 100$, and $16x^2 - 1$.

discrete A set of values is discrete if there is separation between the values. A graph can be described as discrete when it consists of unconnected points or intervals.

The set of numbers $1, 2, 3$ is discrete, while the set of all values between 1 and 3 is not discrete.

distributive property Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

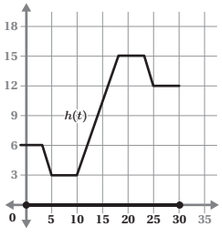


Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.

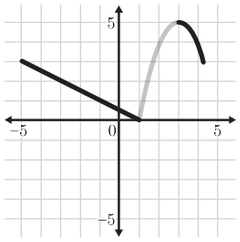
The domain of this graph can be described as: All numbers from 0 to 30 or $0 \le t \le 30$.



Español

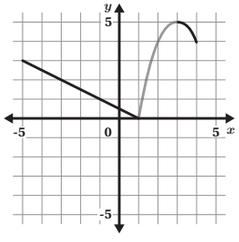
decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

Las partes resaltadas de esta función son decrecientes.



decreciente (intervalo o función) Una función es decreciente cuando sus valores de salida decrecen a medida que crecen sus valores de entrada. Una función puede ser decreciente en todo su dominio o en un intervalo.

Por ejemplo, la función $f(x)$ es decreciente cuando $-5 < x < 1$ y cuando $3 < x < 4$.



variable dependiente El valor de una variable dependiente se basa en el valor de otra variable o un conjunto de variables. En una función, el valor de la variable dependiente representa la salida. La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

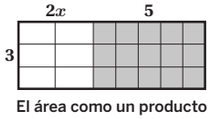
diferencia de cuadrados Una expresión que puede escribirse como un cuadrado perfecto que se resta a otro cuadrado perfecto. Tiene la estructura $r^2 - s^2$ en forma estándar y $(r - s)(r + s)$ en forma factorizada.

Estas expresiones son diferencias de cuadrados: $x^2 - 25$, $9x^2 - 100$ y $16x^2 - 1$.

discreto Un conjunto de valores es discreto si hay separación entre los valores. Una gráfica puede describirse como discreta cuando consta de puntos o intervalos que no se conectan.

El conjunto de números $1, 2, 3$ es discreto, mientras que el conjunto de todos los valores entre 1 y 3 no es discreto.

propiedad distributiva Multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente antes de sumarlos.

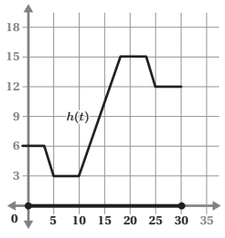


El área como un producto
$3(2x + 5)$

Ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.

El dominio de esta gráfica puede describirse de la siguiente manera:
Todos los números del 0 al 30 o $0 \le t \le 30$.



English

E

Español

elimination A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

In the example, subtraction is used to eliminate y and create an equation that can be solved for x.

eliminación Un método para resolver sistemas de ecuaciones en el que se suman o restan las ecuaciones para producir una nueva ecuación con menos variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

En el ejemplo, se usa la resta para eliminar y y producir una ecuación que puede resolverse para determinar el valor de x.

equation Two expressions with an equal sign between them.

$-3(2x + 5) = 5 + x$ is an example of an equation.

ecuación Dos expresiones con un signo igual entre ambas.

$-3(2x + 5) = 5 + x$ es un ejemplo de una ecuación.

equivalent equations Equations that have the same solution(s).

$3x + 4 = 10$ and $9x + 12 = 30$ are equivalent equations because if you multiply the first equation by 3, you create the second. The solution to each equation is $x = 2$.

ecuaciones equivalentes Ecuaciones que tienen exactamente la misma o las mismas soluciones.

$3x + 4 = 10$ y $9x + 12 = 30$ son ecuaciones equivalentes porque si se multiplica la primera por 3, se forma la segunda. La solución de cada ecuación es $x = 2$.

equivalent systems Two or more systems of equations that have the same set of solutions.

sistemas equivalentes Dos o más sistemas de ecuaciones que tienen el mismo conjunto de soluciones.

exponent Exponents describe repeated multiplication. The exponent describes the number of times to multiply the base by itself.

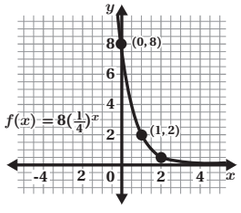
For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

exponente Los exponentes describen una multiplicación repetida. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

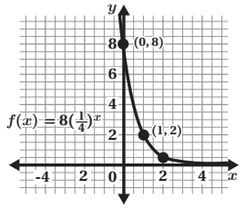
exponential decay An exponential relationship that decreases. An exponential decay relationship has a growth factor between 0 and 1.

For example, $f(x) = 8\left(\frac{1}{4}\right)^x$ is an example of exponential decay because the growth factor, $\frac{1}{4}$, is between 0 and 1.



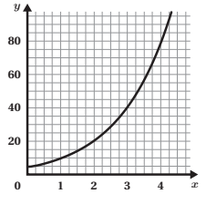
decaimiento exponencial Una relación exponencial que decrece. Una relación de decaimiento exponencial tiene un factor de crecimiento entre 0 y 1.

Por ejemplo, $f(x) = 8\left(\frac{1}{4}\right)^x$ es un ejemplo de decaimiento exponencial porque el factor de crecimiento, $\frac{1}{4}$, está entre 0 y 1.



exponential function A function that increases or decreases by a constant percent rate of change. A function that changes by equal factors over equal intervals.

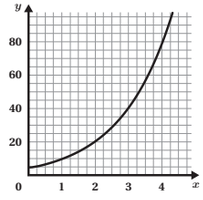
x	$f(x)$
0	5
1	10
2	20
3	40



For example, the table and graph show the exponential function $f(x) = 5 \cdot (2)^x$, which has an initial value of 5 and has a growth factor of 2.

función exponencial Una función que crece o decrece con una tasa porcentual de cambio constante. Una función que cambia por factores iguales en intervalos iguales.

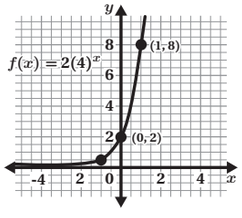
x	$f(x)$
0	5
1	10
2	20
3	40



Por ejemplo, la tabla y la gráfica muestran la función exponencial $f(x) = 5 \cdot (2)^x$, que tiene un valor inicial de 5 y un factor de crecimiento de 2.

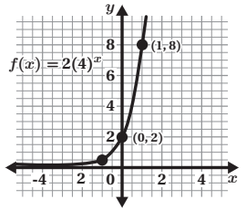
exponential growth An exponential relationship that increases. An exponential growth relationship has a growth factor greater than 1.

For example, $f(x) = 2(4)^x$ is an example of exponential growth because the growth rate, 4, is greater than 1.



crecimiento exponencial Una relación exponencial que crece. Una relación con crecimiento exponencial tiene un factor de crecimiento mayor que 1.

Por ejemplo, $f(x) = 2(4)^x$ es un ejemplo de crecimiento exponencial porque la tasa de crecimiento, 4, es mayor que 1.



exponential relationship See *exponential function*.

relación exponencial Ver *función exponencial*.

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

5, $2x$, $7 - 3^p$, and $4(2 - 5x^2) + 8$ are all examples of expressions.

expresión Un conjunto de números, variables, operaciones y signos de agrupación que representan una cantidad.

5, $2x$, $7 - 3^p$ y $4(2 - 5x^2) + 8$ son ejemplos de expresiones.

Glossary/Glosario

English

Español

F

factor (of a number or expression) A number or expression multiplied with other numbers or expressions to make a product.

For example, 1, 2, 4, and 8 are all factors of the number 8 because $1 \cdot 8 = 8$ and $2 \cdot 4 = 8$. Also, $(x + 3)$ and $(x - 5)$ are factors of $x^2 - 2x - 15$ because $(x + 3)(x - 5) = x^2 - 2x - 15$.

factor (de un número o una expresión) Un número o una expresión que se multiplica por otros números o expresiones para dar como resultado un producto.

Por ejemplo, 1, 2, 4 y 8 son factores del número 8 porque $1 \cdot 8 = 8$ y $2 \cdot 4 = 8$. Además, $(x + 3)$ y $(x - 5)$ son factores de $x^2 - 2x - 15$ porque $(x + 3)(x - 5) = x^2 - 2x - 15$.

factored form One of three common forms of a quadratic equation. A quadratic equation in factored form looks like $f(x) = a(x - m)(x - n)$.

These equations are in factored form:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

forma factorizada Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma factorizada tiene el siguiente orden: $f(x) = a(x - m)(x - n)$.

Estas ecuaciones están en forma factorizada:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

frequency table A table that shows the number of times each value or category occurs in a data set.

For example, this frequency table shows that 5 students selected red as their favorite color.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

function A rule, or relation, that assigns exactly one output to each possible input. Every function is a relation, but not every relation is a function. In a function, the value of the output variable depends on the value of the input variable.

función Una regla, o relación, que asigna exactamente una salida a cada entrada posible. Toda función es una relación, pero no toda relación es una función. En una función, el valor de la variable de salida depende del valor de la variable de entrada.

function notation A notation used to represent the inputs and outputs of a function. For a function f, when x is an input (or domain value), the symbol $f(x)$ shows the corresponding output (or range value).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the independent variable is 4, the value of the dependent variable is 9.

notación de función Una notación utilizada para representar las entradas y las salidas de una función. Para una función (x), cuando x es una entrada (o valor de dominio), el símbolo $f(x)$ muestra la salida correspondiente (o valor de rango).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

Por ejemplo, $f(4) = 9$ es una expresión escrita en notación de funciones. Indica que cuando la entrada de la función f es 4, la salida es 9. En otras palabras, cuando el valor de la variable independiente es 4, el valor de la variable dependiente es 9.

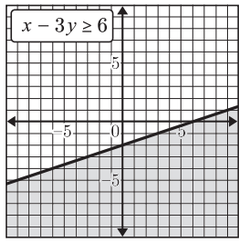
G

growth factor The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.

factor de crecimiento La razón constante (o el factor común) que multiplica a cada término para generar un patrón exponencial.

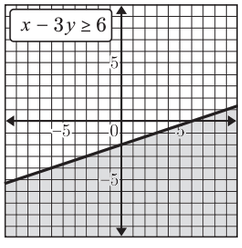
H

half-plane A region that defines the solution set of a single two-variable linear inequality. The boundary line of the inequality splits the coordinate plane into two equal half-planes.



The graph shows the solution region to $x - 3y \geq 6$. The boundary line $x - 3y = 6$ divides the coordinate plane into two equal half-planes.

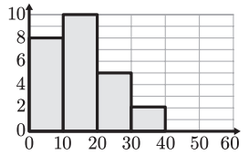
semiplano Una región que define el conjunto de soluciones de una desigualdad lineal con dos variables. La recta límite de la desigualdad divide el plano de coordenadas en dos semiplanos iguales.



La gráfica muestra la región solución de $x - 3y \geq 6$. La recta límite $x - 3y = 6$ divide el plano de coordenadas en dos semiplanos iguales.

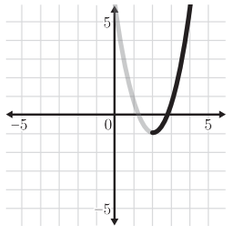
English

histogram A visual display of quantitative data that groups values into intervals, called *bins*. Each bin is represented by a rectangle, where the height shows the frequency or relative frequency of the data in that bin.



For example, this histogram shows that there are 8 values between 0 and 10.

increasing (interval or function) A function is increasing when its outputs increase as its inputs increase. A function can be increasing for its entire domain or over an interval.



For example, the function $f(x)$, whose graph is shown, is increasing when $x > 2$.

independent variable The value of an independent variable is not based on the value of any other variable. In a function, the value of the independent variable represents the input. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

index In a radical $\sqrt[n]{x}$ the quantity n is called the *index*. (See *radical*.)

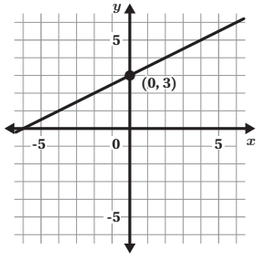
inequality A comparison statement that uses the symbols $<$, $>$, $\leq$, or $\geq$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y + 2 \geq 30$ means that the value of the expression $y + 2$ will be greater than or equal to 30.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. In a system of equations with infinitely many solutions, every point on the graph is a solution to the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

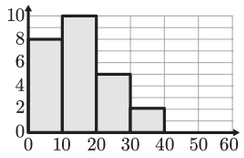
initial value A point where the graph of an equation or function crosses the y-axis or when $x = 0$.



The initial value of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.

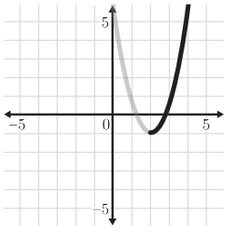
Español

histograma Una representación visual de datos cuantitativos que agrupa los valores en intervalos, llamados *contenedores*. Cada contenedor está representado por un rectángulo, donde la altura muestra la frecuencia o la frecuencia relativa de los datos en ese contenedor.



Por ejemplo, este histograma muestra que hay 8 valores entre 0 y 10.

creciente (intervalo o función) Una función es creciente cuando sus valores de salida crecen a medida que crecen sus valores de entrada. Una función puede ser creciente en todo su dominio o en un intervalo.



Por ejemplo, la función $f(x)$, cuya gráfica se muestra, es creciente cuando $x > 2$.

variable independiente El valor de una variable independiente no depende del valor de ninguna otra variable. En una función, el valor de la variable independiente representa la entrada. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

índice En un radical $\sqrt[n]{x}$, la cantidad n se llama índice. (Ver *radical*).

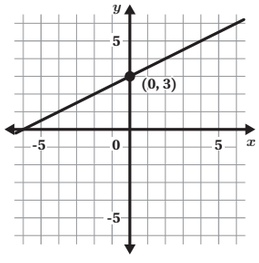
desigualdad Un enunciado de comparación que utiliza los signos $<$, $>$, $\leq$ o $\geq$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y + 2 \geq 30$ significa que el valor de la expresión $y + 2$ será mayor o igual que 30.

infinitas soluciones Una ecuación tiene infinitas soluciones si es verdadera sea cual sea el valor de la variable. Un sistema de ecuaciones tiene infinitas soluciones si las ecuaciones del sistema son equivalentes. En un sistema de ecuaciones con infinitas soluciones, cada punto de la gráfica es una solución del sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene infinitas soluciones porque la ecuación es verdadera con cualquier valor de x.

valor inicial Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.



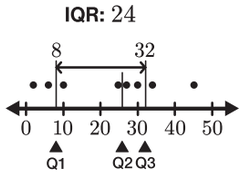
El valor inicial de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.

Glossary/Glosario

English

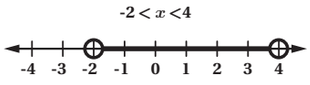
input In a function, any value that you substitute for x is called an input. The input is sometimes called the *independent variable*. The input typically appears on the horizontal axis of a graph and in the left-hand column of a table.

interquartile range (IQR) A measure of variation in a set of numerical data. The interquartile range is the distance between the first quartile (Q1) and the third quartile (Q3) of the data set.



For example, the IQR of this data set is $32 - 8 = 24$.

interval A set of values between two points.



In words: All numbers between -2 and 4.
Inequality: $-2 < x < 4$.

interval notation A way to represent a range of values on a number line using brackets and parentheses.

$(2, 6]$ represents all values between 2 and 6, not including 2 but including 6. $[-3, 5)$ represents all values between -3 and 5, including -3 but not including 5.

inverse operations Two operations that undo each other are called *inverse operations*.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

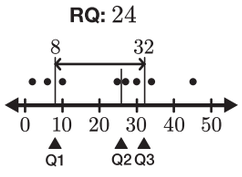
irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

2 is a rational number because it can be written as $\frac{2}{1}$, whereas $\sqrt{3}$ is irrational because it cannot be written as a fraction made up of two integers.

Español

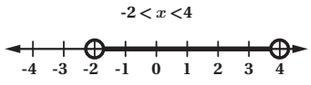
entrada En una función, todo valor que sustituya a x se denomina entrada. La entrada a veces se denomina variable independiente. La entrada suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

rango intercuartil (RIC) Una medida de variación en un conjunto de datos numéricos. El rango intercuartil es la distancia entre el primer cuartil (Q1) y el tercer cuartil (Q3) del conjunto de datos.



Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.

intervalo Un conjunto de valores entre dos puntos.



Con palabras: Todos los números entre el -2 y el 4. Desigualdad: $-2 < x < 4$.

notación de intervalo Una forma de representar un rango de valores en una recta numérica usando corchetes y paréntesis.

$(2, 6]$ representa todos los valores entre 2 y 6, excluyendo 2 pero incluyendo 6. $[-3, 5)$ representa todos los valores entre -3 y 5, incluyendo -3 pero excluyendo 5.

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

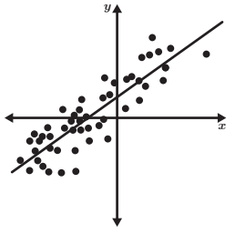
La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

número irracional Un número que no se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

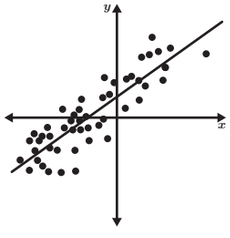
2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que $\sqrt{3}$ es irracional porque no se puede escribir como una fracción de dos números enteros.

L

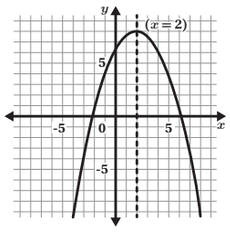
line of fit The line on a scatter plot that best represents the trend created by the points in a data set. There are many lines of fit for a single data set, but there is only one line of best fit.



línea de (mejor) ajuste La línea de un diagrama de dispersión que mejor representa la tendencia que definen los puntos de un conjunto de datos. Existen muchas líneas de ajuste para un mismo conjunto de datos, pero solo hay una línea de mejor ajuste.

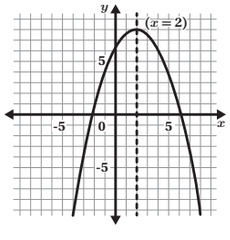


line of symmetry A line that divides a figure or the graph of a function into two halves. For every point (except the vertex), there is a corresponding point on the other side of the line that is the same distance from the line.



The equation of this line of symmetry is $x = 2$.

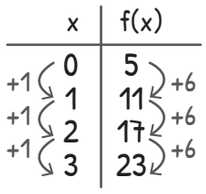
eje de simetría Una línea que divide una figura o la gráfica de una función en dos mitades. Cada punto (excepto el vértice) tiene un punto correspondiente en el otro lado de la línea, el cual está a la misma distancia de la línea.



La ecuación de este eje de simetría es $x = 2$.

English

linear function A function that increases or decreases by a constant rate of change, or shows a linear relationship. A function that changes by equal differences over equal intervals. The graph of a linear function is a line.



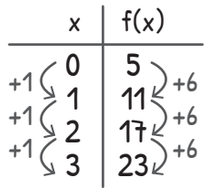
For example, $f(x) = 5 + 6x$ represents a linear function that has an initial value of 5 and increases by a constant difference of 6.

linear relationship See *linear function*.

lurking variable An additional variable that has an effect on the other variables being analyzed. Lurking variables can lead us to make conclusions about data and relationships that are inaccurate.

Español

función lineal Función que crece o decrece con una tasa de cambio constante o muestra una relación lineal. Una función que cambia con diferencias iguales en intervalos iguales. La gráfica de una función lineal es una recta.



Por ejemplo, $f(x) = 5 + 6x$ representa una función lineal que tiene un valor inicial de 5 y crece con una diferencia constante de 6.

relación lineal Ver *función lineal*.

variable de confusión Una variable adicional que tiene un efecto sobre las demás variables que se analizan. Las variables de confusión pueden producir conclusiones imprecisas en torno a los datos y las relaciones.

M

margin of error A measure that shows how much the results of a survey or study might differ from the true population value.

If a survey shows that 60% of people like a product with a margin of error of ±3%, the true percentage is likely between 57% and 63%.

margen de error Una medida que muestra cuánto pueden diferir los resultados de una encuesta o un estudio del valor real de la población.

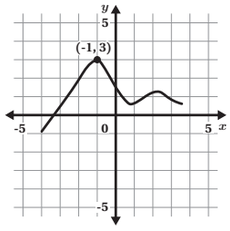
Si una encuesta muestra que al 60 % de las personas les gusta un producto con un margen de error de ±3 %, el porcentaje real probablemente esté entre el 57 % y el 63 %.

maximum The greatest value in a data set.

máximo El mayor valor en un conjunto de datos.

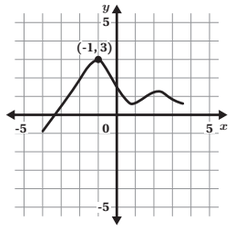
maximum (of a function) The highest point on the graph.

The maximum of this function is $(-1, 3)$.

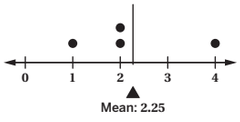


máximo (de una función) El punto más alto en una gráfica.

El máximo de esta función está en $(-1, 3)$.

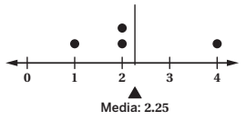


mean A statistic used to describe the typical value of a data set. To calculate the mean, you can add up all the data values and divide by the number of data points. The mean is a measure of center and is also called the *average*.



In this data set, the mean is 2.25.
$1 + 2 + 2 + 4 = 9$
$\frac{9}{4} = 2.25$

media Una estadística que se usa para describir el valor típico de un conjunto de datos. Para calcular la media, se suman todos los valores y el resultado se divide por el número de puntos de datos. La media es una medida de tendencia central y también se denomina promedio.



En este conjunto de datos, la media es 2.25.
$1 + 2 + 2 + 4 = 9$
$\frac{9}{4} = 2.25$

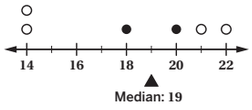
measure of center A numerical value that describes the overall center or clustering of data in a set. The three most common measures of central tendency are the mean, median, and mode.

medida de centro Un valor numérico que describe el centro general o la agrupación de datos en un conjunto. Las tres medidas de tendencia central más comunes son la media, la mediana y la moda.

Glossary/Glosario

English

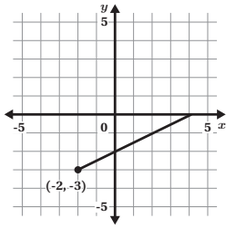
median A statistic used to describe the typical value of a data set. It is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the mean of those two values. The median is a measure of center.



maximum The greatest value in a data set.

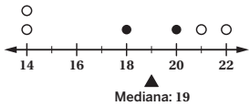
minimum (of a function) The lowest point on the graph.

The minimum of this function is (-2, -3).



Español

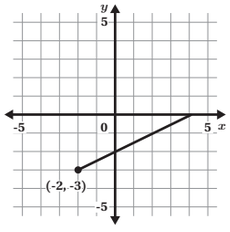
mediana Una estadística que se usa para describir el valor típico de un conjunto de datos. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay dos valores en el medio del conjunto de datos, la mediana es la media de esos dos valores. La mediana es una medida de tendencia central.



máximo El mayor valor en un conjunto de datos.

mínimo (de una función) El punto más bajo en una gráfica.

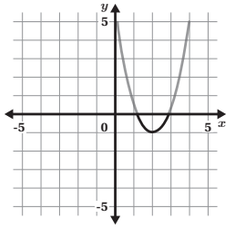
El mínimo de esta función está en (-2, -3).



N

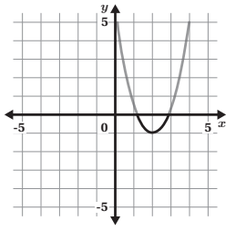
negative (interval or function) A function is negative when its outputs are negative and its graph is below the x-axis. A function can be negative for its entire domain or over an interval.

This function is negative when $1 < x < 3$.

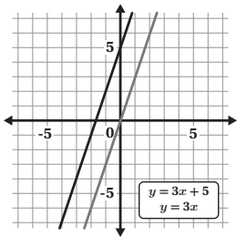


negativo (intervalo o función) Una función es negativa cuando sus salidas son negativas y su gráfica está por debajo del eje x. Una función puede ser negativa en todo su dominio o en un intervalo.

Esta función es negativa cuando $1 < x < 3$.

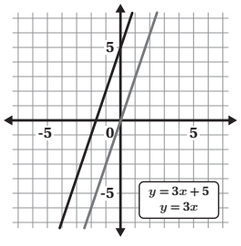


no solution An equation has no solution if there is no value of the variable that will make the equation true. A system of equations has no solution if there is no set of values that makes all the equations in the system true. In a system of equations with no solution, there is no point that is on the graph of every equation in the system.



For example, the system of equations containing $y = 3x + 5$ and $y = 3x$ has no solution because the graphs are parallel and never intersect.

sin solución Una ecuación no tiene solución si no hay ningún valor de la variable que haga que la ecuación sea verdadera. Un sistema de ecuaciones no tiene solución si no hay ningún conjunto de valores que haga que todas las ecuaciones de ese sistema sean verdaderas. En un sistema de ecuaciones sin solución, no hay ningún punto que esté en la gráfica de cada una de las ecuaciones del sistema.



Por ejemplo, el sistema de ecuaciones que contiene $y = 3x + 5$ y $y = 3x$ no tiene solución porque las gráficas son paralelas y nunca se intersecan.

non-linear relationship A pattern, table, scenario, graph, or equation that does not have a constant rate of change. A non-linear relationship is called non-linear because its graph is not a line.

Day	Teal Globs
0	1
1	2
2	5
3	10

+1, +1, +3, +5

The equation $y = x^2 + 1$ represents a non-linear relationship because its rate of change is not constant and its graph is not a line.

relación no lineal Patrón, tabla, contexto, gráfica o ecuación que no tiene una tasa de cambio constante. Una relación no lineal se denomina no lineal porque su gráfica no es una recta.

Día	Manchitas turquesa
0	1
1	2
2	5
3	10

+1, +1, +3, +5

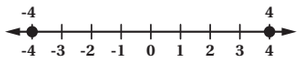
La ecuación $y = x^2 + 1$ representa una relación no lineal porque su tasa de cambio no es constante y su gráfica no es una recta.

numerical data See *quantitative data*.

datos numéricos Ver *datos cuantitativos*.

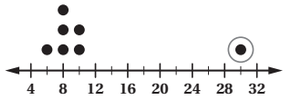
O

opposite Two numbers that are the same distance from 0 and on different sides of the number line. Two terms that are opposites are also referred to as zero pairs and additive inverses.



For example, 4 and -4 are opposites.

outlier A data value that is far from the other values in the data set. Values that are 1.5 times the IQR from either quartlile 1 or quartile 3 are outliers.

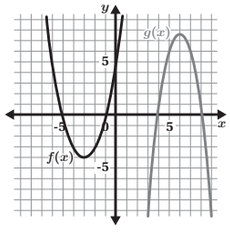


The circled point is an outlier.

output The value of a function after it has been evaluated for an input value of x is called the *output*. The output is sometimes called the *dependent variable*. The output typically appears on the vertical axis of a graph and in the right-hand column of a table.

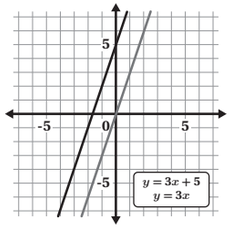
P

parabola The graph of a quadratic function, which is a U-shaped curve.



parallel lines Lines that never cross or intersect. On a graph, two lines with the same slope and different y-intercepts are parallel.

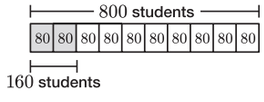
For example, the lines $y = 3x + 5$ and $y = 3x$ are parallel and never intersect.



percent decrease Describes how much a quantity goes down, expressed as a percent of the starting amount.

For example, the function $f(x) = 1000(.78)^x$ represents the value of a phone with an initial value of $1,000 with a percent decrease in value of 22% each year, x.

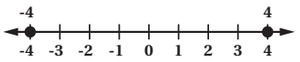
percent (percentage) Percent means for every 100. It is represented by the percent symbol: %. We use percentages to represent ratios and fractions.



For example, 20% means 20 : 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number.
Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

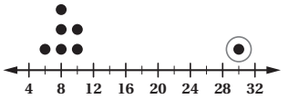
Español

opuestos Dos números que están a la misma distancia del 0 y en diferentes lados de la recta numérica. Dos términos que son opuestos también se conocen como inversos aditivos.



Por ejemplo, 4 y -4 son opuestos.

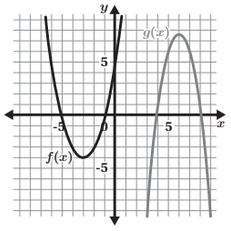
valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos. Son valores atípicos aquellos que están a 1.5 veces el RQ del cuartil 1 o del cuartil 3.



El punto encerrado en un círculo es un valor atípico.

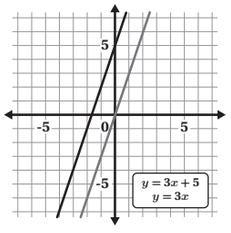
salida El valor de una función tras haber sido evaluada con un valor de entrada x se denomina salida. La salida a veces se denomina variable dependiente. La salida suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

parábola La gráfica de una función cuadrática, que es una curva en forma de U.



líneas (o rectas) paralelas Líneas que nunca se cruzan o intersecan. En una gráfica, dos rectas con la misma pendiente e intersecciones diferentes con el eje y son paralelas.

Por ejemplo, las rectas $y = 3x + 5$ y $y = 3x$ son paralelas y nunca se intersecan.



disminución porcentual Describe cuánto disminuye una cantidad y se expresa como un porcentaje de la cantidad inicial.

Por ejemplo, la función $f(x) = 1000(.78)^x$ representa el valor de un teléfono con un valor inicial de $1,000 y una disminución porcentual del 22% de su valor cada año, x.

por ciento (porcentaje) Por ciento significa por cada 100. Se representa con el símbolo de porcentaje: %. Usamos porcentajes para representar razones y fracciones.

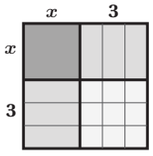


Por ejemplo, 20% significa 20 : 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número.
Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos están en una excursión, entonces eso es 160 estudiantes porque 20 están de viaje por cada 100 estudiantes en total.

Glossary/Glosario

English

perfect square An expression that can be written as something multiplied by itself.



These expressions are perfect squares:
3^2, 9, $(x + 3)^2$, and $x^2 + 6x + 9$.

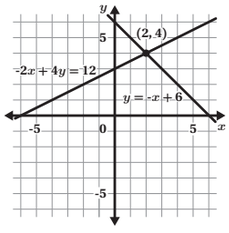
perimeter The sum of the lengths of all the sides of a polygon.

perpendicular Describes a line that crosses or meets another line at a 90° angle.

± (plus/minus symbol) A symbol used to represent both the positive and negative values of a number. It also can be used to represent two expressions.

± 9 represents -9 and +9.
2 ± 3 represents $2 - 3$ and $2 + 3$.

point of intersection A point where two lines or curves meet.



For example, $(2, 4)$ is the point of intersection for the lines $-2x + 4y = 12$ and $y = -x + 6$.

polynomial The sum or difference of terms that include variables raised to non-negative integer powers, with coefficients that can be real or complex numbers.

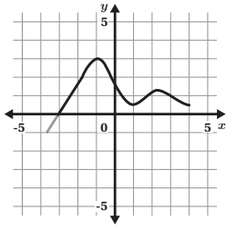
$2x - 7$ and $3x^2 - x + 4$ are polynomial expressions.
$f(x) = 2x - 7$ and $g(x) = 3x^2 - x + 4$ are polynomial functions.

population A set of people or things that are being studied.

For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

population mean The average of all the values in a population. It is calculated by adding all the values and dividing by the total number of values.

positive (interval or function) A function is positive when its outputs are positive and its graph is above the x-axis. A function can be positive for its entire domain or over an interval.

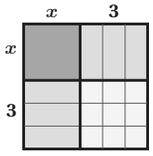


This function is positive when $-3 < x < 4$.

principal The total amount (initial value) of money borrowed or invested, not including any interest.

Español

cuadrado perfecto Una expresión que puede escribirse como algo multiplicado por sí mismo.



Estas expresiones son cuadrados perfectos:
3^2, 9, $(x + 3)^2$ y $x^2 + 6x + 9$.

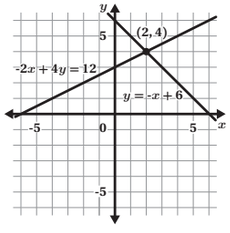
perímetro La suma de las longitudes de todos los lados de un polígono.

perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

± (signo más menos) Un símbolo que se usa para representar los valores positivos y negativos de un número. También puede usarse para representar dos expresiones.

± 9 representa -9 and +9.
2 ± 3 representa $2 - 3$ y $2 + 3$.

punto de intersección Un punto donde se cruzan dos rectas o curvas.



Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.

polinomio La suma o la diferencia de términos que incluyen variables elevadas a potencias enteras no negativas, con coeficientes que pueden ser números reales o complejos.

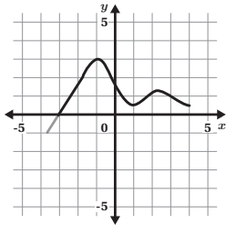
$2x - 7$ y $3x^2 - x + 4$ son expresiones polinómicas.
$f(x) = 2x - 7$ y $g(x) = 3x^2 - x + 4$ son funciones polinómicas.

población Conjunto de personas o cosas que se estudian.

Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

media poblacional El promedio de todos los valores en una población. Se calcula sumando todos los valores y dividiendo por el número total de valores.

positivo (intervalo o función) Una función es positiva cuando sus salidas son positivas y su gráfica está por encima del eje x. Una función puede ser positiva en todo su dominio o en un intervalo.



Esta función es positiva cuando $-3 < x < 4$.

principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, sin incluir los intereses.

English

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Carpet (sq. ft.)	Cost (dollars)
10 $\xrightarrow{\times 1.5}$	15.00
20 $\xrightarrow{\times 1.5}$	30.00
50 $\xrightarrow{\times 1.5}$	75.00

Q

quadratic formula A formula that can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function A function with output values that change by a constant second difference. Equations of quadratic functions have a squared term when written in standard form. The graph of a quadratic function is a parabola.

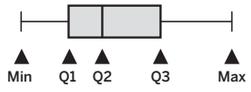
x	f(x)		
1	5		
2	8	+3	
3	14	+6	+3
4	23	+9	+3
5	35	+12	+3

quadratic relationship See *quadratic function*.

quantitative data Data represented as numbers, quantities, or measurements that can be meaningfully compared is called *quantitative data*, or *numerical data*.

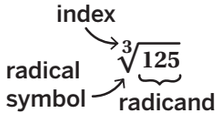
How many pets do you have? is a question that produces quantitative data. This is different from categorical data.

quartile Quartiles divide a data set into four sections.
Quartile 1 is the median of the lower half of the data.
Q2 is the median.
Q3 is the median of the upper half of the data.
Q4 is the maximum.



R

radical A square root, cube root, fourth root, etc. The radical symbol is $\sqrt{\ }$.



radicand The expression under a radical symbol. In the radical $\sqrt[n]{x}$ the quantity x is called the *radicand*. (See *radical*).

random Happening without a predictable pattern or order. In mathematics, a random event has outcomes that occur by chance.

Rolling a die is a random event because any number from 1 to 6 can appear with equal probability.

Español

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

Alfombra (pies cuadrados)	Costo (dólares)
10 $\xrightarrow{\times 1.5}$	15.00
20 $\xrightarrow{\times 1.5}$	30.00
50 $\xrightarrow{\times 1.5}$	75.00

Q

fórmula cuadrática Una fórmula que puede usarse para determinar las soluciones de una ecuación cuadrática $ax^2 + bx + c = 0$, donde $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

función cuadrática Una función con valores de salida que cambian de acuerdo con una segunda diferencia constante. Las ecuaciones de las funciones cuadráticas tienen un término elevado al cuadrado cuando se escriben en forma estándar. La gráfica de una función cuadrática es una parábola.

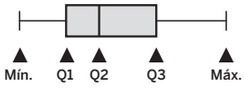
x	f(x)		
1	5		
2	8	+3	
3	14	+6	+3
4	23	+9	+3
5	35	+12	+3

relación cuadrática Ver *función cuadrática*.

datos cuantitativos Los datos que se representan como números, cantidades o medidas y que pueden compararse de forma significativa se denominan datos cuantitativos o datos numéricos.

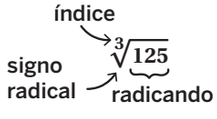
¿Cuántas mascotas tienes? es una pregunta que produce datos cuantitativos, que son diferentes de los datos categóricos.

cuartil Los cuartiles dividen un conjunto de datos en cuatro secciones.
El cuartil 1 es la mediana de la mitad inferior de los datos.
Q2 es la mediana.
Q3 es la mediana de la mitad superior de los datos.
Q4 es el máximo.



R

radical Una raíz cuadrada, raíz cúbica, raíz cuarta, etc. El signo radical es $\sqrt{\ }$.



radicando La expresión bajo un signo radical. En un radical $\sqrt[n]{x}$, la cantidad x se llama radicando. (Ver *radical*).

aleatorio Algo que ocurre sin un patrón o un orden predecibles. En matemáticas, un suceso aleatorio tiene resultados que ocurren por casualidad.

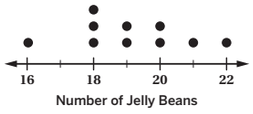
Lanzar un dado es un suceso aleatorio porque cualquier número del 1 al 6 puede aparecer con la misma probabilidad.

Glossary/Glosario

English

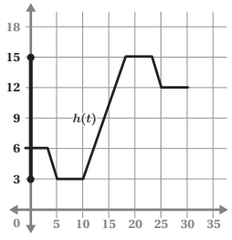
range The difference between the maximum and minimum values in a data set. Range is a measure of spread.

For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.



range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.

The range of this graph can be described as: All numbers from 3 to 15 or $3 \leq h(t) \leq 15$.



rate of change The ratio of the change in one quantity to the corresponding change in another quantity. In a linear relationship, the rate of change is the change in y divided by the change in x, which is also the slope of the graph.

rational exponent A real number that can be expressed as the ratio of two integers, where the denominator is not zero. Rational numbers can also appear as fractional exponents in mathematical expressions.

In the expression $5^{\frac{1}{2}}$, $\frac{1}{2}$ is a rational exponent.

rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5, and $\sqrt{9}$ are rational numbers.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

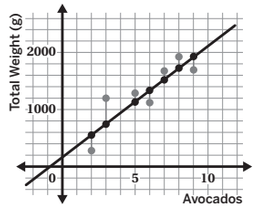
For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a *function*.

relative frequency table A type of two-way table used to compare data across two categorical variables. Relative frequency tables present the fraction or percent of the data that is in that category, instead of the actual number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

residual The difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.

The short lines connecting each point to the line of fit show the residual value for that point.



Español

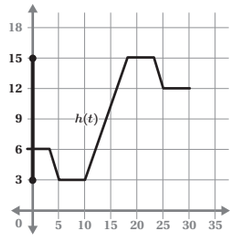
rango La diferencia entre los valores máximo y mínimo de un conjunto de datos. El rango es una medida de dispersión.

Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.



fluctuación (de una función) El conjunto de todos los posibles valores de salida de una función o relación. La fluctuación puede describirse con palabras o como una desigualdad.

La fluctuación de esta gráfica puede describirse de la siguiente manera: Todos los números del 3 al 15 o $3 \leq h(t) \leq 15$.



tasa de cambio La razón entre el cambio en una cantidad y el cambio correspondiente en otra cantidad. En una relación lineal, la tasa de cambio es el cambio en (x) dividido por el cambio en x, que también es la pendiente del gráfico.

exponente racional Un número real que puede expresarse como la razón de dos números enteros, donde el denominador no es cero. Los números racionales también pueden aparecer como exponentes fraccionarios en expresiones matemáticas.

En la expresión $5^{\frac{1}{2}}$, $\frac{1}{2}$ es un exponente racional.

número racional Un número que se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5 y $\sqrt{9}$ son números racionales.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

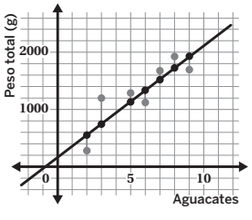
Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

tabla de frecuencia relativa Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa presentan la fracción o el porcentaje de los datos que están en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

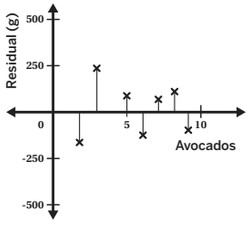
residuo La diferencia entre el valor y de un punto en un diagrama de dispersión y el valor que predice la línea de mejor ajuste.

Las líneas cortas que conectan cada punto con la línea de ajuste muestran el valor del residuo del punto en cuestión.



English

residual plot A scatter plot of residual values for a data set. The x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.



The residual plot shows how the total weight of different numbers of avocados vary from their predicted values.

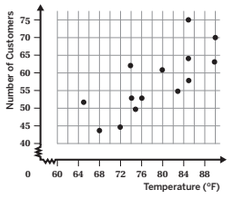
revenue The amount of money generated by selling a product or service.

S

sample A part of a population.

For example, a population could be all the seventh-grade students at one school. One sample of that population is all the seventh-grade students who are in band.

scatter plot A graph in the coordinate plane that shows the relationship between two variables by plotting individual data points. Scatter plots are used to observe patterns in bivariate numerical data.



second difference The differences between consecutive output values in the table of a function are called *first differences*. The differences between those values are called *second differences*. Quadratic functions have constant second differences.

x	f(x)		
1	5		
		+3	
2	8		+3
		+6	
3	14		+3
		+9	
4	23		+3
		+12	
5	35		

In this example, the first differences are 3, 6, 9, and 12. The second differences are constant at 3, so $f(x)$ is a quadratic function.

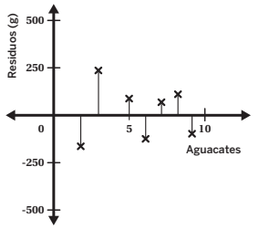
segmented bar graph A bar graph where each bar is divided into segments that represent different categories, showing how the total is split into parts.

set-builder notation A shorthand used to describe sets, often those with an infinite number of elements. It is written in the form $\{x : x > 5\}$, which is read as "the set of all x such that x is greater than 5." The colon (:) can also be replaced by a vertical line (|), as in $\{x \mid x > 5\}$, and is read the same way.

The set of all numbers greater than 8 can be written in set-builder notation as: $\{x \mid x>8\}$
This is read as "the set of all x such that x is greater than 8."

Español

diagrama de residuos Un diagrama de dispersión de los valores de los residuos de un conjunto de datos. El eje x representa el valor que predice la línea de mejor ajuste, y el valor y de cada punto representa el valor del residuo.



El diagrama de residuos muestra que el peso total de diferentes cantidades de aguacates varía con respecto a sus valores esperados.

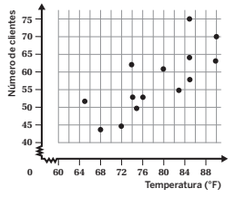
ingresos La cantidad de dinero que genera la venta de un producto o servicio.

S

muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

diagrama de dispersión Un gráfico en el plano de coordenadas que muestra la relación entre dos variables mediante el trazado de puntos de datos individuales. Los diagramas de dispersión se utilizan para observar patrones en datos numéricos bivariados.



segunda diferencia Las diferencias entre valores de salida consecutivos en la tabla de una función se llaman primeras diferencias. Las diferencias entre dichos valores se llaman segundas diferencias. Las funciones cuadráticas tienen segundas diferencias constantes.

x	f(x)		
1	5		
		+3	
2	8		+3
		+6	
3	14		+3
		+9	
4	23		+3
		+12	
5	35		

En este ejemplo, las primeras diferencias son 3, 6, 9 y 12. Las segundas diferencias son constantes, de 3, por lo que $f(x)$ es una función cuadrática.

gráfico de barras segmentadas Un gráfico de barras donde cada barra está dividida en segmentos que representan diferentes categorías y muestran cómo el total se divide en partes.

notación de construcción de conjuntos Una abreviatura utilizada para describir conjuntos, a menudo aquellos con un número infinito de elementos. Se escribe en la forma $\{x : x > 5\}$, que se lee como "el conjunto de todas las x, de modo que x es mayor que 5". Los dos puntos (:) también se pueden reemplazar por una línea vertical (|), como en $\{x \mid x > 5\}$, y se leen de la misma manera.

El conjunto de todos los números mayores que 8 se puede escribir en notación de construcción de conjuntos como: $\{x \mid x > 8\}$
Esto se lee como "el conjunto de todas las x, de modo que x es mayor que 8".

Glossary/Glosario

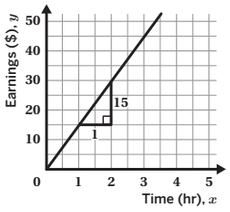
English

simple interest Interest that is calculated solely based on the initial amount. The balance in an account with simple interest is modeled by a linear function.

simulation An approximate imitation of a statistical experiment, often performed using a computer program to analyze the results of a large number of trials.

A simulation can be used to predict the likelihood of getting heads or tails by flipping a virtual coin many times.

slope (rate of change) The ratio of the change in the vertical direction (y-axis) to the change in the horizontal direction (x-axis).



In this graph, y increases by 15 dollars when x increases by 1 hour. The slope of the line is 15, and the rate of change is 15 dollars per hour.

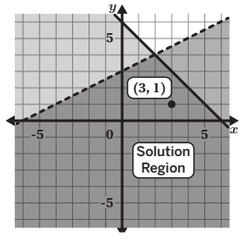
slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where m represents the slope, b represents the y-intercept of the line, and x and y are variables.

The equations $y = 2x + 4$ and $y = -5x - 10$ are in slope-intercept form. The equation $2x + 5y = 20$ is not in slope-intercept form.

solution A value or set of values that makes an equation or inequality true.

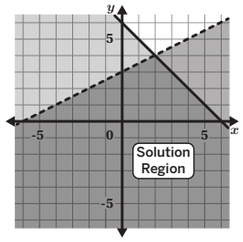
For example, $x = 2$ is a solution to the equation $3x + 4 = 10$.
$x > 2$ are the solutions to the inequality $3x + 4 > 10$.
The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.

solution (to a system of inequalities) An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the solution region where the graphs overlap.



For system $y \leq -x + 6$ and $-2x + 4y < 12$, (3,1) is a solution to this system of inequalities because it makes both inequalities true and falls in the region where the inequalities overlap.

solution region The set of all ordered pairs that make an inequality or each inequality in a system true. For a two-variable linear inequality, the solution region is a half-plane. For a system of inequalities, the solution region is located where the graphs overlap.



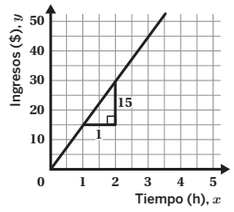
Español

interés simple Interés que se calcula únicamente en función de la cantidad inicial. El saldo de una cuenta con interés simple se modela con una función lineal.

simulación Una imitación aproximada de un experimento estadístico, a menudo realizada mediante un programa informático para analizar los resultados de una gran cantidad de ensayos.

Se puede utilizar una simulación para predecir la probabilidad de obtener cara o cruz lanzando una moneda virtual muchas veces.

pendiente (tasa de cambio) La razón entre el cambio en la dirección vertical (eje y) y el cambio en la dirección horizontal (eje x).



En esta gráfica, y incrementa en 15 dólares cuando x incrementa en 1 hora. La pendiente de la recta es 15 y la tasa de cambio es de 15 dólares por hora.

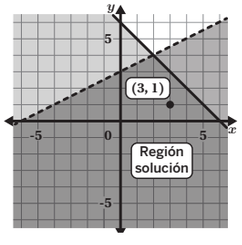
forma pendiente-intersección, forma pendiente-ordenada al origen Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje y (o la ordena al origen) de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben $y = mx + b$, donde m representa la pendiente, b representa la intersección con el eje y de la recta, y tanto x como y son variables.

Las ecuaciones $y = 2x + 4$ y $y = -5x - 10$ están en la forma pendiente-intersección. La ecuación $2x + 5y = 20$ no está en la forma pendiente-intersección.

solución Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

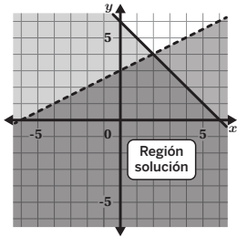
Por ejemplo, $x = 2$ es una solución de la ecuación $3x + 4 = 10$.
$x > 2$ son las soluciónes de la desigualdad $3x + 4 > 10$.
El par ordenado $(1, 2)$ es una solución de la ecuación $3x + 4y = 11$.

solución (de un sistema de desigualdades) Un par ordenado que hace que cada desigualdad de un sistema sea verdadera. Cada par ordenado que sea una solución de un sistema se encuentra en la región solución donde se superponen las gráficas.



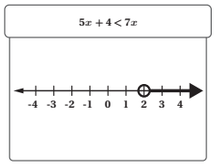
Por ejemplo, en el sistema de desigualdades $y \leq -x + 6$ y $-2x + 4y < 12$, la solución es (3,1) porque hace que ambas desigualdades sean verdaderas y se ubica en la región donde las desigualdades se suporponen.

región solución El conjunto de todos los pares ordenados que hacen que una desigualdad, o cada desigualdad de un sistema, sean verdaderas. En una desigualdad lineal de dos variables, la región solución es un semiplano. En un sistema de desigualdades, la región solución se encuentra donde se superponen las gráficas.



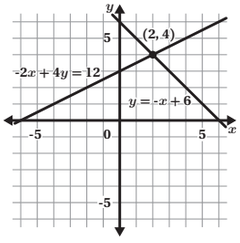
English

solution set The set of all values that makes an inequality true. To describe a solution set symbolically, we often use inequalities. To describe a solution set graphically, we often shade a portion of a number line or a region of the coordinate plane.



The solution set for the inequality $5x + 4 < 7x$ includes all of the values that are larger than 2. 2 is not included in the solution set.

solution to a system of equations A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the point of intersection.



For the system $y = -x + 6$ and $-2x + 4y = 12$, $(2, 4)$ is the solution to this system of equations and the point of intersection on the graph.

square root The square root of a number n (written as $\sqrt{n}$) is the positive number that can be squared to get n. The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

standard form (of a linear equation) Linear equations that are written in the form $ax + by = c$, where a, b, and c are constants and x and y are variables.

The equations $2x + 5y = 20$ and $3x - 4y = -10$ are in standard form. The equation $y = 2x + 4$ is not in standard form.

standard form (of a quadratic equation) One of three common forms of a quadratic equation. A quadratic equation in standard form looks like $f(x) = ax^2 + bx + c$.

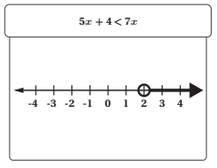
These equations are in standard form:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

substitute Replace a variable or expression with a value or another expression. $4x = 4(5)$

In this example, 5 is substituted for x in the expression $4x$.

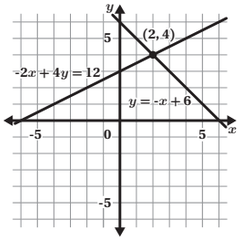
Español

conjunto de soluciones El conjunto de todos los valores que hacen verdadera una desigualdad. Para describir con signos un conjunto de soluciones suelen emplearse desigualdades. Para describir con gráficas un conjunto de soluciones suele colorearse o sombrearse una parte de una recta numérica o una región del plano de coordenadas.



El conjunto de soluciones de la desigualdad $5x + 4 \geq 7x$ incluye todos los valores mayores que 2. El 2 no está incluido en el conjunto de soluciones.

solución de un sistema de ecuaciones Una solución de un sistema de ecuaciones es un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.



Por ejemplo, en el sistema de ecuaciones $y = -x + 6$ y $-2x + 4y = 12$, la solución y el punto de intersección de la gráfica es $(2, 4)$.

raíz cuadrada La raíz cuadrada de un número n (se escribe $\sqrt{n}$) es el número positivo que puede elevarse al cuadrado para obtener n. La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

forma estándar (de una ecuación lineal) Las ecuaciones lineales que se escriben en la forma $ax + by = c$, donde a, b y c son constantes, y tanto x como y son variables, se conocen como ecuaciones en forma estándar.

Las ecuaciones $2x + 5y = 20$ y $3x - 4y = -10$ están en forma estándar. La ecuación $y = 2x + 4$ no está en forma estándar.

forma estándar (de una ecuación cuadrática) Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma estándar tiene el siguiente orden: $f(x) = ax^2 + bx + c$.

Estas ecuaciones están en forma estándar:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

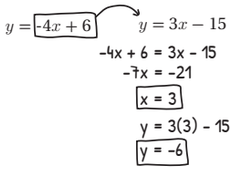
sustituir Reemplazar una variable o expresión por un valor u otra expresión. $4x = 4(5)$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

Glossary/Glosario

English

substitution A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.



For example, we can substitute $-4x + 6$ in for y in $y = 3x - 15$ because they are equivalent.

survey A method of collecting data by asking people to answer a given set of questions.

system of equations Two or more equations that represent the constraints on a shared set of variables form a system of equations.

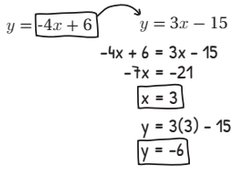
These equations make a system:
$3b + c = -2$
$b - 5c = 12$

system of inequalities Two or more inequalities that represent the constraints on a shared set of variables form a system of inequalities.

These inequalities make a system:
$10m + 5n > -2$
$m - 5n \leq 12$

Español

sustitución Un método para resolver sistemas de ecuaciones donde una variable se reemplaza con una expresión equivalente para producir una nueva ecuación con menos variables.



Por ejemplo, podemos introducir $-4x + 6$ en lugar de y en $y = 3x - 15$ porque son equivalentes.

encuesta Un método de recopilación de datos que consiste en pedir a personas que respondan una serie de preguntas.

sistema de ecuaciones Dos o más ecuaciones que representan las restricciones de un conjunto compartido de variables forman un sistema de ecuaciones.

Estas ecuaciones forman un sistema:
$3b + c = -2$
$b - 5c = 12$

sistema de desigualdades Dos o más desigualdades que representan las restricciones de un conjunto compartido de variables forman un sistema de desigualdades.

Estas desigualdades forman un sistema:
$10m + 5n > -2$
$m - 5n \leq 12$

T

total relative frequency table A type of two-way table used to compare data across two categorical variables. Total relative frequency tables present the fraction or percentage of the data that is in that category, instead of the number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

	Meditated	Did Not Meditate	Total
Calm	46%	8%	54%
Agitated	24%	22%	46%
Total	70%	30%	100%

translation A transformation that moves every point in a function a given distance in a given direction. A translation changes the location of a function, but does not change its shape.

two-way table A way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

	Meditated	Did Not Meditate	Total
Calm	45	8	53
Agitated	23	21	44
Total	68	29	97

tabla de frecuencia relativa total Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa total presentan la fracción o el porcentaje de los datos que se encuentran en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

	Meditaron	No meditaron	Total
Calmados	46%	8%	54%
Agitados	24%	22%	46%
Total	70%	30%	100%

traslación Una transformación que mueve cada punto de una función una determinada distancia en una determinada dirección. Una traslación cambia la ubicación de una función, pero no su forma.

tabla de doble entrada Una manera de comparar dos variables categóricas. Muestra una de las variables en la fila superior y la otra variable en la columna de un costado. Cada entrada de la tabla es la frecuencia o frecuencia relativa de la categoría que describen los títulos de la columna y la fila.

	Meditaron	No meditaron	Total
Calmados	45	8	53
Agitados	23	21	44
Total	68	29	97

English

Español

U

univariate data A data set that involves one variable. Each data point contains one piece of information.

A collection of students' heights is a univariate data set.
Tables, dot plots, and bar graphs are useful for displaying univariate data.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

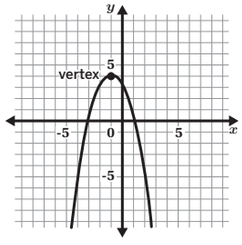
variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

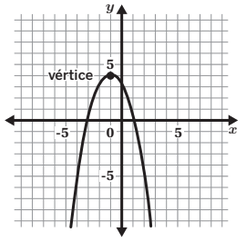
variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

vertex On the graph of a quadratic or absolute value function, the vertex is the maximum or minimum point. The vertex is also where the function changes from increasing to decreasing, or vice versa.



vértice En la gráfica de una función cuadrática o una función de valor absoluto, el vértice es el punto máximo o mínimo. El vértice también es donde la función cambia de creciente a decreciente, o viceversa.



vertex form One of three common forms of a quadratic equation. A quadratic equation in vertex form looks like $f(x) = a(x - h)^2 + k$.

These equations are in vertex form:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

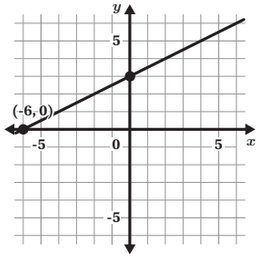
forma de vértice Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma de vértice tiene el siguiente orden: $f(x) = a(x - h)^2 + k$.

Estas ecuaciones están en forma de vértice:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

X

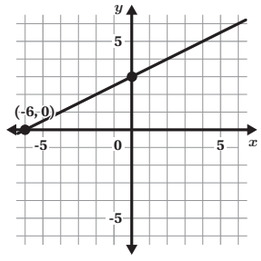
x-intercept A point where the graph of an equation or function crosses the x-axis or when $y = 0$.

The x-intercept of the graph of $-2x + 4y = 12$ is $(-6, 0)$, or just -6.



intersección con el eje x, abscisa al origen Un punto donde la gráfica de una ecuación o función cruza el eje x, o cuando $y = 0$.

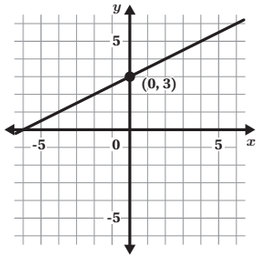
La intersección con el eje x de la gráfica de $-2x + 4y = 12$ es $(-6, 0)$, o simplemente -6.



Y

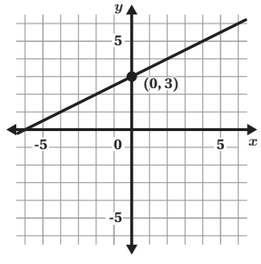
y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

The y-intercept of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.



intersección con el eje y, ordenada al origen Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.

La intersección con el eje y de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.



Glossary/Glosario

Z

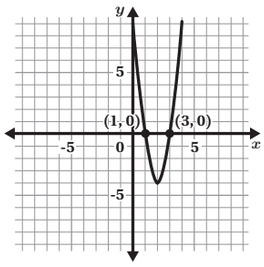
English

zero-product property A property which states that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to help solve equations.

If $(2x - 3)(x + 1) = 0$, then either $2x - 3 = 0$ or $x + 1 = 0$.

zeros The x-values that make a function equal zero, or $f(x) = 0$.

The zeros of $f(x) = 4(x - 1)(x - 3)$ are 1 and 3.



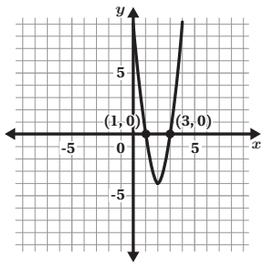
Español

propiedad del producto cero Una propiedad que establece que si el producto de dos o más factores es 0, entonces al menos uno de los factores es 0. Esta propiedad puede usarse como ayuda para resolver ecuaciones.

Si $(2x - 3)(x + 1) = 0$, entonces $2x - 3 = 0$ o $x + 1 = 0$.

ceros Los valores x que hacen que una función sea igual a cero, o $f(x) = 0$.

Los ceros de $f(x) = 4(x - 1)(x - 3)$ son 1 y 3.



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