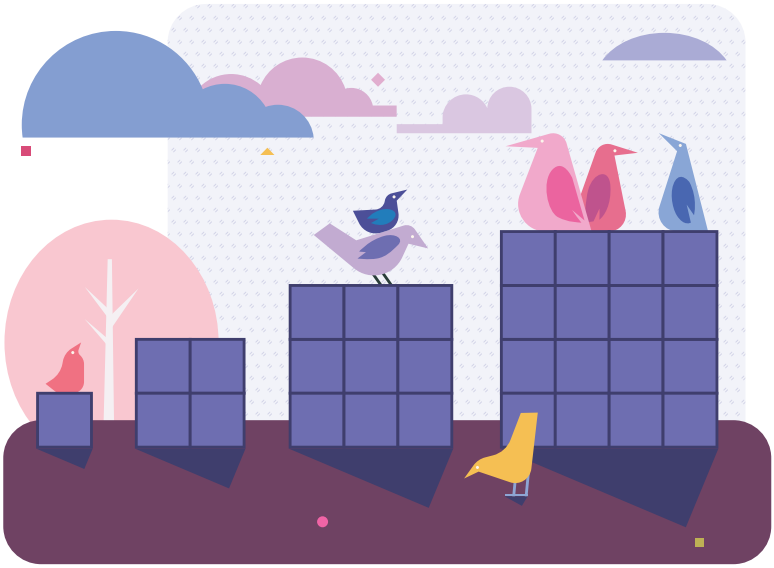
How Does It Change?

Let's describe some patterns of change.



Warm-up Notice and Wonder

Study the figures. What do you notice? What do you wonder?

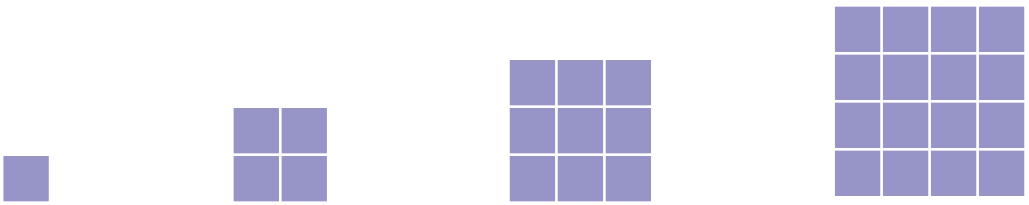


Figure 1 Figure 2 Figure 3 Figure 4

1. I notice . . .

2. I wonder . . .

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Activity 1 Growing Squares

Study each pattern.

Pattern A

Figure 1 **Figure 2** **Figure 3** **Figure 4**

Pattern B

Figure 1 **Figure 2** **Figure 3** **Figure 4**

1. How does each pattern change? Explain your thinking.

2. How would you determine the number of dots in Figure 5 for each pattern? Sketch Figure 5 for each pattern.

3. How would you describe the shape of the figures in Pattern B?

4. In Pattern B, how does the figure number relate to the number of dots in the figure?

5. Complete the table with the number of dots used for each figure in the pattern.

Figure number	Number of dots in Pattern A	Number of dots in Pattern B
1		
2		
3		
4		
5		
10		
n		

6. Describe the relationship in Pattern B between the shapes of the figures and the number of dots in Figure n.

A squared variable, by itself or in an expression, is called a ***quadratic*** or ***quadratic expression***. It comes from the Latin *quadrare*, which means "to make square." The expression n^2 is quadratic.

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Activity 2 First and Second Differences

1. Study the figures in each pattern. Do you notice a pattern? Explain your thinking.

Pattern A

Figure 1 Figure 2 Figure 3 Figure 4

Pattern B

Figure 1 Figure 2 Figure 3 Figure 4

What do you notice about each pattern? Complete the table to help with your thinking.

Pattern A

Figure number	1	2	3	4
Number of squares				

Pattern B

Figure number	1	2	3	4
Number of squares				

Activity 2 First and Second Differences (continued)

2. Study the relationship between the figure number and number of squares.

a Calculate the difference between the number of squares in each figure.

Pattern A

Figure number	1	2	3	4
Number of squares	1	3	5	7

Difference: ☐ ☐ ☐

Pattern B

Figure number	1	2	3	4
Number of squares	1	4	9	16

Difference: ☐ ☐ ☐

b What do you notice about the difference(s) in the number of squares between Pattern A and Pattern B ?

c For Pattern A, calculate the differences of the differences. That is, subtract the preceding difference in the table from the next difference.

d For Pattern B, calculate the differences of the differences.

3. What do you notice about the differences of the differences, also known as the *second differences*?

4. Form your own hypothesis about first and second differences for linear and quadratic relationships. Explain your thinking.

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Activity 3 Patterns of Dots

Compare Patterns X and Y.

Pattern X

Figure 0 Figure 1 Figure 2 Figure 3

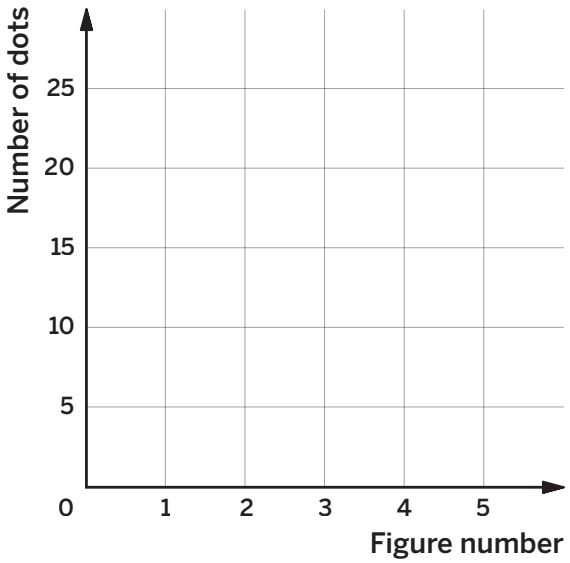
Pattern Y

Figure 0 Figure 1 Figure 2 Figure 3

1. Complete the table with the number of dots in each pattern. Then compare and contrast Patterns X and Y.

Figure	Number of dots in Pattern X	Number of dots in Pattern Y
0		
1		
2		
3		
4		
5		

2. In the graph, plot the number of dots in each figure number in Pattern X and Pattern Y. Use different colors or symbols for each pattern.



3. Does the graph of each pattern confirm your comparison in Problem 1? Explain your thinking.

STOP

Summary

In today's lesson . . .

You observed some patterns that do not change linearly or exponentially. Instead, the change is ***quadratic***, meaning the pattern grows by raising a number or term to the second power, or squaring it. For example, the area of a square with side length n is the ***quadratic expression*** n^2. (The prefix "quad-" means four. While the exponent in quadratic expressions is 2, quadratics are closely related to squares, which have 4 sides.)

You can determine if a pattern is linear or quadratic by analyzing its first and second differences. In a linear relationship, the first differences are equal while the second differences are all 0. In a quadratic relationship, the first differences are not equal, but the second differences are equal.

Reflect:

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Practice

1. Pattern A grows by three dots in each successive figure. In Pattern B, the number of dots in each figure is expressed by n^2, where n is the figure number. Sketch Figures 1–3 for each pattern.

Pattern A

Figure 0 Figure 1 Figure 2 Figure 3

Pattern B

Figure 0 Figure 1 Figure 2 Figure 3

2. Examine each pattern.

Pattern Y

Figure 0 Figure 1 Figure 2 Figure 3

Pattern Z

Figure 0 Figure 1 Figure 2 Figure 3

a How many dots will there be in Figure 4 of each pattern?

b Which pattern shows a quadratic relationship between the figure number and the number of dots? Explain your thinking.

Practice

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3. Select *all* the expressions for the number of dots in a pattern that represent a quadratic relationship with the figure number n.

A. n^2 **C.** $n \cdot n$ **E.** $n + 2$

B. $2n$ **D.** $n + 1$ **F.** $n \div 2$

4. A garden has a perimeter of 40 ft. Some of the possible measurements are shown in the table.

a Complete the missing measurements in the table.

b What lengths and widths produce the greatest area?

Length (ft)	Width (ft)	Area (ft^2)
4	16	64
8	12	
10		
12		96
14		
		64

5. The function $C(x)$ gives the percentage of homes using only cell phone service x years after 2004. Explain the meaning of each statement.

a $C(10) = 35$

b $C(x) = 10$

c How is $C(10)$ different from $C(x) = 10$?

6. How many small squares will there be in Figure 10?

Figure 1

Figure 2

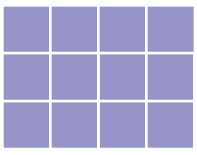

Figure 3