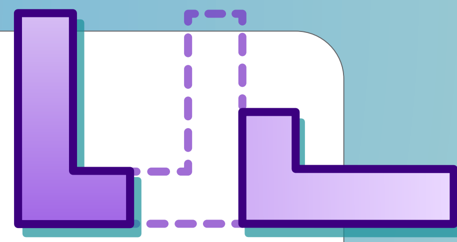
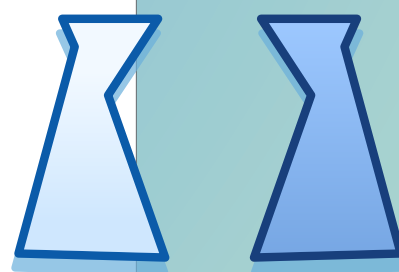


Unit 1

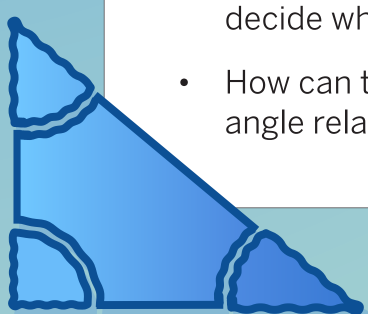
Rigid Transformations and Congruence

If you look around, you'll notice shapes in art, architecture, and everyday objects. Shapes have parts that can be measured, like sides and angles. Will anything happen to these side lengths and angle measures when you *slide*, *flip*, or *turn* these shapes?



Essential Questions

- What are different ways to transform a figure?
- How can we use rigid transformations to decide whether two figures are congruent?
- How can transformations help make sense of angle relationships?



Summary | Lesson 1

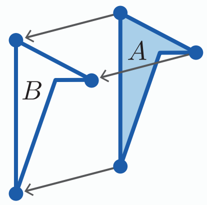
You can describe how a figure moves to get from one position to another in different ways.

- A figure can slide or move up, down, left, or right.
- A figure can spin around the middle or around another point.
- A figure can flip vertically, horizontally, or diagonally.

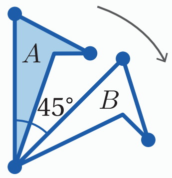
Try This

Describe how figure A moved to figure B.

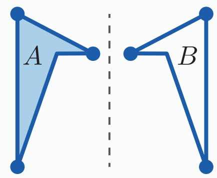
a



b



c



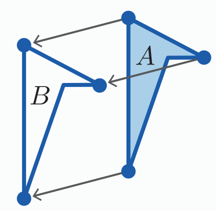
Summary | Lesson 2

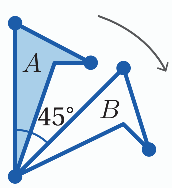
Transformations are actions that you can perform to change a figure. They are applied to every point on the figure. Here are some examples:

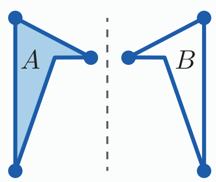
- A **rotation** turns or spins a figure.
- A **reflection** flips or mirrors a figure over a line by moving every point to a point directly on the opposite side of the line.
- A **translation** slides a figure without turning it.

Try This

Identify each transformation as a rotation, reflection, or translation.

a


b


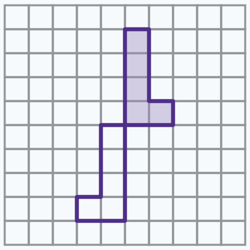
c


A **sequence of transformations** is a set of translations, rotations, and/or reflections that you can perform on a figure. Each sequence of transformations has a meaningful order.

Here are important details to consider when choosing your sequence of transformations:

- For translations, every point on the figure moves the same distance in the same direction.
- For reflections, the new point will be the same distance from the line as it was in the original figure.
- Rotations are performed around a point by a given angle and in a specific direction. The direction of a rotation can be **clockwise**, traveling in the same direction as the hands on a clock, or **counterclockwise**, traveling in the opposite direction as the hands on a clock.

You can use different sequences of transformations on a figure and get the same result.

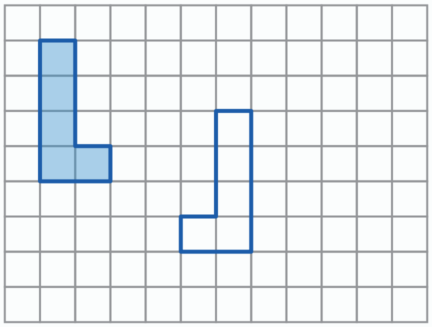


For example, here are two sequences of transformations that both move the shaded figure onto the unshaded figure:

- Sequence #1: Reflect over the longest edge and then translate the figure 4 units down.
- Sequence #2: Rotate $180°$ clockwise around the lower left corner of the shaded figure, then reflect over the horizontal line that divides the height into two halves.

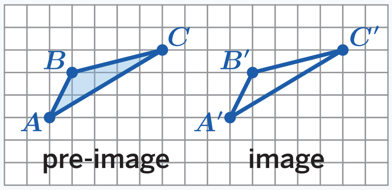
Try This

Describe a sequence of transformations that moves the shaded figure onto the unshaded figure.



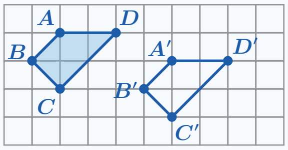
Summary | Lesson 4

When you transform a figure, the original figure is called the **pre-image** and the new figure is called the **image**. All of the points in the image **correspond** to the points in the pre-image. The points in the image are named after the pre-image point they correspond to. For example, point A' corresponds to point A.



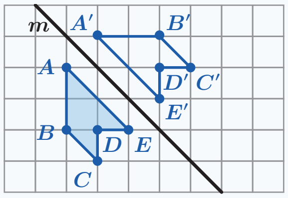
Here are important details to help you describe transformations:

Translations



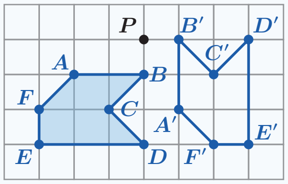
Describe the direction (up or down, left or right) and the number of units.
E.g., Figure $ABCD$ is translated 4 units right and 1 unit down.

Reflections



Describe the line of reflection.
E.g., Figure $ABCDE$ is reflected over line m.

Rotations

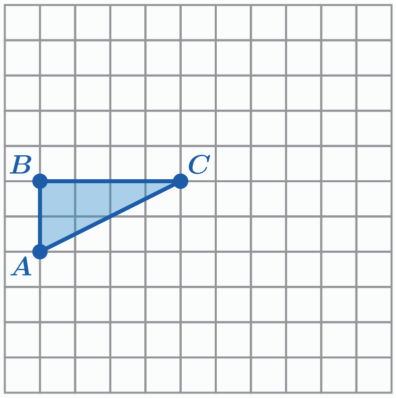


Describe the center of rotation, angle of rotation, and direction (clockwise or counterclockwise).
E.g., Figure $ABCDEF$ is rotated $90°$ counterclockwise around point P.

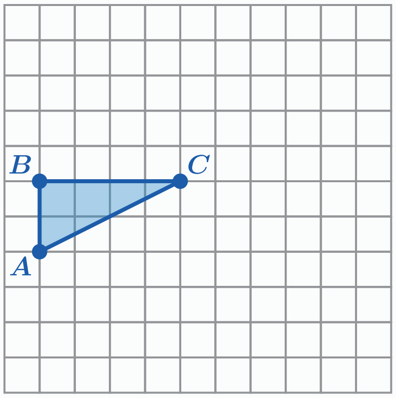
Try This

Perform each translation, then label the points in the image with A', B', and C' to correspond with points A, B, and C in the pre-image.

a Translate triangle ABC 4 units up and 2 units to the right.



b Rotate triangle ABC $90°$ clockwise around point C.

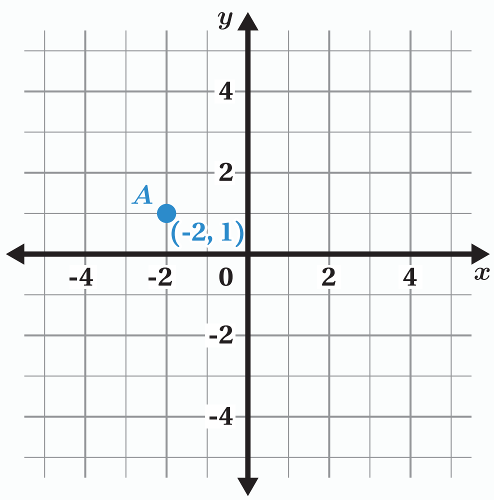


When you compare the coordinates of corresponding points in the image and pre-image, you might notice patterns in their values.

- When you translate a point to the left or right, it changes the value of the x-coordinate.
- When you translate a point up or down, it changes the value of the y-coordinate.
- When you reflect a point over the x-axis, it changes the sign of the y-coordinate. The x-coordinate remains the same.
- When you reflect a point over the y-axis, it changes the sign of the x-coordinate. The y-coordinate remains the same.

Try This

Write the coordinates of point A after each transformation.



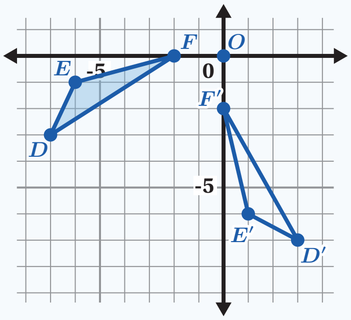
a A reflection of point A over the x-axis:

b A reflection of point A over the y-axis:

c A translation of point A 3 units to the right and 2 units down:

When you compare the coordinates of corresponding points in an image and pre-image, you might notice patterns in their values.

For a $90°$ or $270°$ rotation, the x- and y-coordinates switch and one changes sign. For a $180°$ rotation, both coordinates change signs. For a $360°$ rotation, both coordinates stay the same.



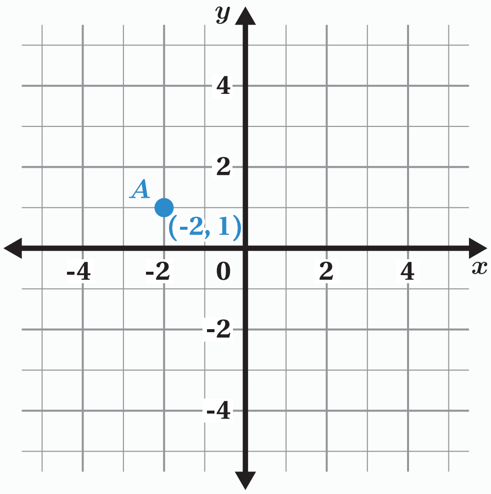
For example, triangle DEF was rotated $90°$ counterclockwise.

For triangles DEF and $D'E'F'$, the x- and y-coordinates of each point switch places, and some of the signs change.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	(0, -2)
(-6, -1)	(1, -6)
(-7, -3)	(3, -7)

Try This

Write the coordinates of point A after each transformation.

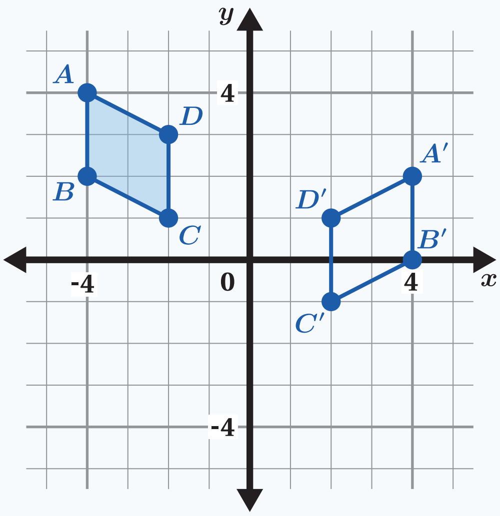


a A rotation of $180°$ clockwise around center $(0, 0)$:

b A rotation of $90°$ counterclockwise around center $(0, 0)$:

Translations, rotations, and reflections are all examples of **rigid transformations**.

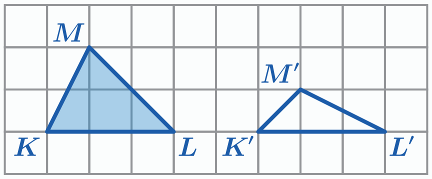
When a pre-image is transformed using a rigid transformation, corresponding sides will have the same length and corresponding angles will have the same measure. For example, figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection and a translation. Side AB has the same length as side $A'B'$ and angle C is the same measurement as angle C'.



Try This

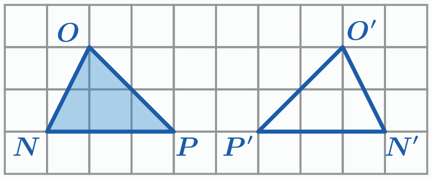
a. Explain how you know that Pair E does *not* show a rigid transformation.

Pair E (not rigid)



b. Explain how you know that Pair F *does* show a rigid transformation.

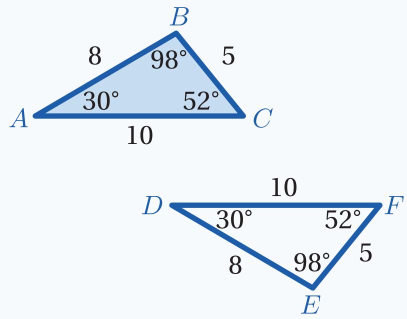
Pair F (rigid)



Two figures are **congruent** if you can use a sequence of rigid transformations to move one exactly on top of the other.

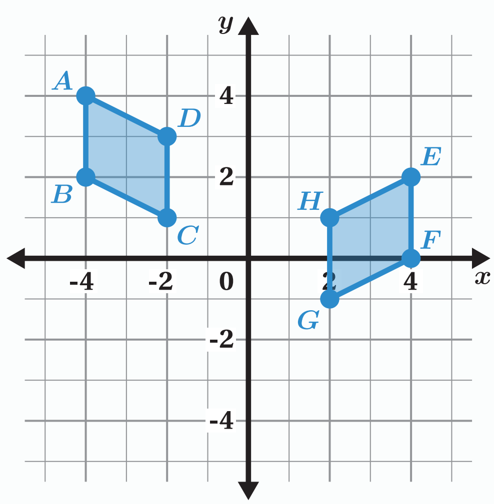
You don't need to check that all corresponding angle measures and side lengths are equal if you can show a sequence of rigid transformations.

For example, figure DEF is congruent to figure ABC because you can reflect ABC over a horizontal line and translate it to fit exactly on top of figure DEF.



Try This

Are figures $ABCD$ and $EFGH$ congruent? Explain your thinking.

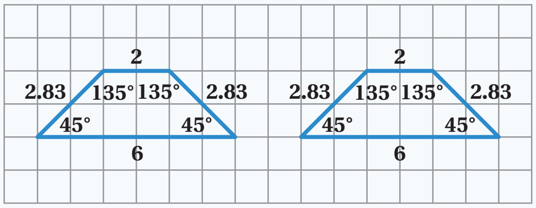


Here are two ways to determine whether two figures are congruent:

- You can determine a sequence of transformations to move one exactly onto the other.
- You can determine that *all* the corresponding sides have the same length and *all* the corresponding angles have the same measure.

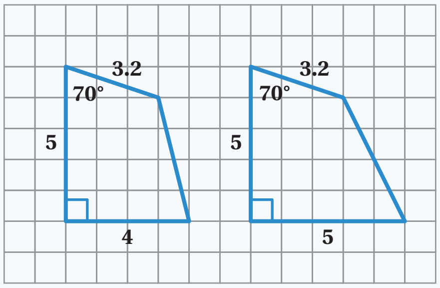
Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas.

Congruent



These figures are congruent because all the corresponding side lengths and angle measures are the same.

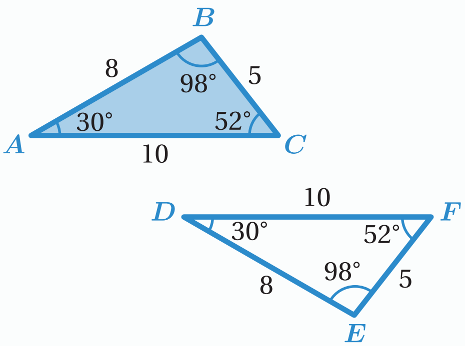
Not Congruent



These figures are not congruent because *some* of the side lengths and measures are the same, but some are different.

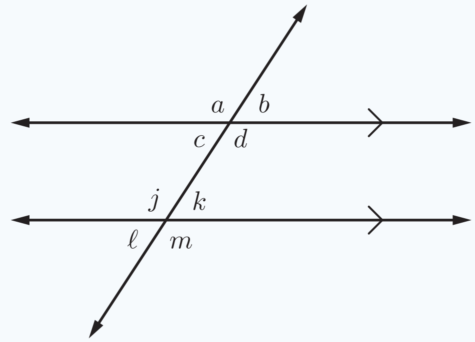
Try This

Are triangles ABC and DEF congruent? Explain your thinking.



Summary | Lesson 10

Rigid transformations of a single line result in a line. Rigid transformations of parallel lines result in parallel lines. We can use these properties to help show which angles in a diagram are congruent.

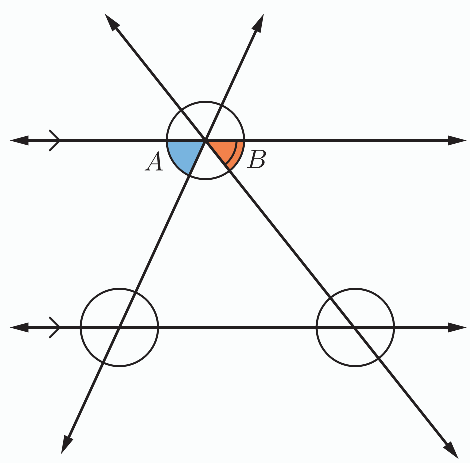


We can use transformations to show that *vertical angles* (angles opposite each other when two lines cross) are congruent. For example, vertical angles a and d are congruent because you can rotate angle a 180 degrees around where the lines intersect and it will move onto angle d.

Transformations can also help show which angles are congruent when a **transversal** intersects parallel lines. Angle b is congruent to angle k because you can translate angle b along the transversal until it moves exactly onto angle k.

Try This

Here is a pair of parallel lines with two transversals.



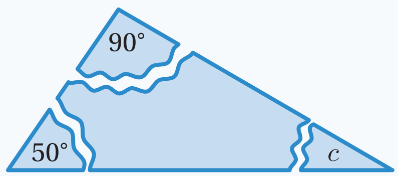
a Identify each angle that is congruent to angle A. Label each one A'.

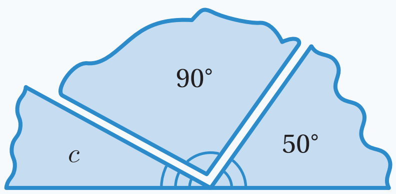
b Identify each angle that is congruent to angle B. Label each one B'.

Summary | Lesson 11

The interior angle measures of any triangle always sum to 180 degrees. We can demonstrate this by rearranging the angles of any triangle to form a straight line, which has a measure of $180°$.

If you know the measures of two angles in a triangle, you can determine the third angle by subtracting the sum of the two known angle measures from $180°$. Here is an example.

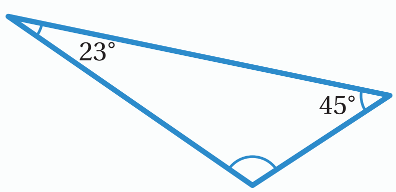




$$c = 180 - 90 - 50$$
$$c = 40$$

Try This

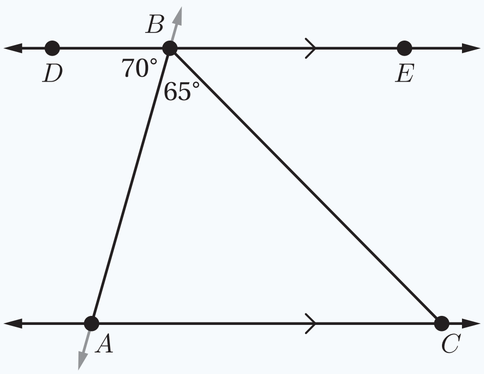
Determine the unknown angle measure in this triangle.



There are several strategies you can use to determine unknown angle measures in a diagram.

- Use translations to show that corresponding angles on parallel lines are congruent.
- Use rotations to show that vertical angles are always congruent.
- Use the fact that the sum of three interior angles in any triangle is 180 degrees.
- Use the fact that any straight line has an angle measure of $180°$.

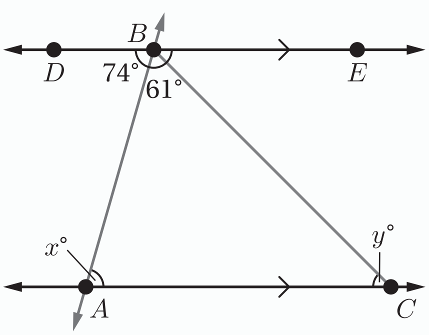
Here is an example. The measure of angle DBA is $70°$ and the measure of angle ABC is $65°$. Let's determine the remaining angle measures.



- The measure of $\angle BAC$ must also be $70°$ because you can translate and then rotate $\angle DBA$ $180°$ to fit exactly on top of $\angle BAC$.
- $\angle EBC$ must be $45°$ because it makes a straight line with $\angle DBA$ and $\angle ABC$, and $70 + 65 + 45 = 180$.
- $\angle BCA$ must also be $45°$ because you can translate and rotate $\angle EBC$ exactly on top of it and also because it is part of a triangle with the $70°$ and $65°$ angles.

Try This

Determine the values for x and y.



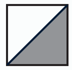
Summary | Lesson 13

You can use rigid transformations to create interesting repeating patterns of figures, such as **tessellations**. A tessellation is any repeating pattern that can fill an entire plane without gaps.

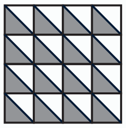
We see tessellations in many creations from around the world, but they are especially common in Islamic art and architecture. You can also see these repeating geometric patterns in floors, ceilings, wall art, and even on clothing designs.

Try This

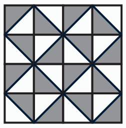
Here is a design that can be used to make a tessellation, along with two examples.



Tessellation #1

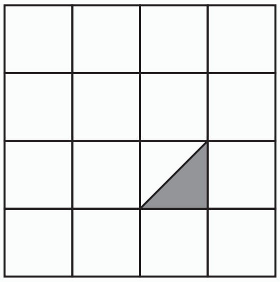


Tessellation #2



Make a tessellation by adding copies of the design to the grid.

Your Tessellation



Try This | Answer Key

Lesson 1

a *Responses vary*. It slid to the left and down.

b *Responses vary*. It spun around a point on the figure.

c *Responses vary*. It flipped horizontally over the line.

Lesson 2

a Translation

b Rotation

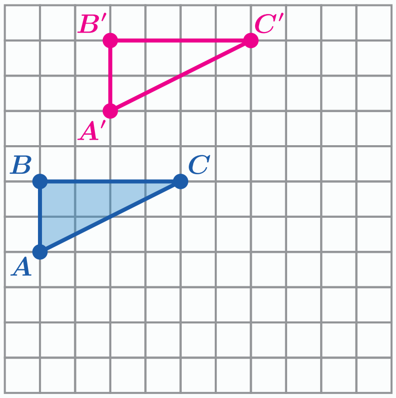
c Reflection

Lesson 3

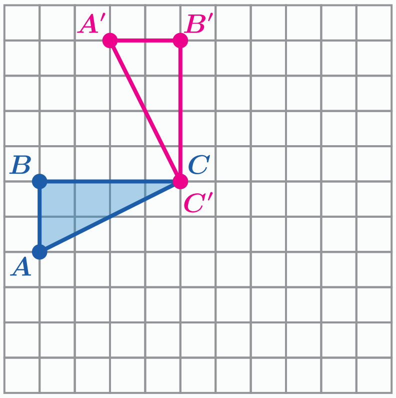
Responses vary. Reflect the shaded figure over the rightmost edge of the figure, then translate it 2 units right and 2 units down.

Lesson 4

a



b



Lesson 5

a (-2, -1)

b (2, 1)

c (1, -1)

Lesson 6

a (2, -1)

b (-1, -2)

Lesson 7

a *Responses vary.* The corresponding angles M and M' do not have the same measure.

b *Responses vary.* All of the corresponding measurements between NOP and $N'O'P'$ are the same.

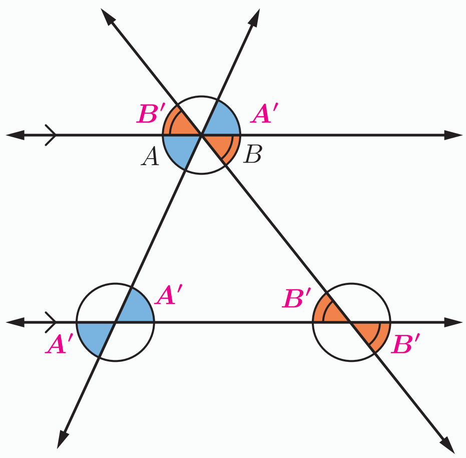
Lesson 8

Yes. *Explanations vary*. If you reflect figure $ABCD$ over the y-axis and then translate 2 units down, it will move exactly on top of figure $EFGH$.

Lesson 9

Yes. *Explanations vary*. You can reflect triangle ABC over a horizontal line and then translate to move it exactly onto triangle DEF. Also, triangles ABC and DEF both have the same side lengths and angle measurements.

Lesson 10



Lesson 11

$180 - 23 - 45 = 112°$

Lesson 12

$x = 74$ and $y = 45$

Lesson 13

Sample tessellation shown.