Unit 1

Patterns and Sequences

What is a sequence? In this unit, you'll use patterns, tables, and graphs to compare and contrast arithmetic and geometric sequences. You will describe sequences recursively using the relationship from one output to the next, or explicitly using the relationship between an input and its output.

Essential Questions

- What strategies can be used to make predictions about sequences?
- How can sequences be defined recursively and explicitly?
- How can sequences be used to model and interpret situations in context?

Summary | Lesson 1

When you're trying to visualize how a pattern will grow, it helps to have a toolbox of different strategies.

Here are the first three figures in a pattern and a table showing the number of tiles at each stage. You could determine how many tiles there will be in Figure 7 by:

- Drawing the next four figures and adding a row of 3 each time.
- Continuing the table, increasing the number of tiles by 3 in each new row.
- Noticing that the number of tiles is 3 times the figure number, plus 2, and then calculating the tiles for Figure 7.

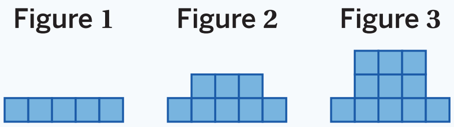
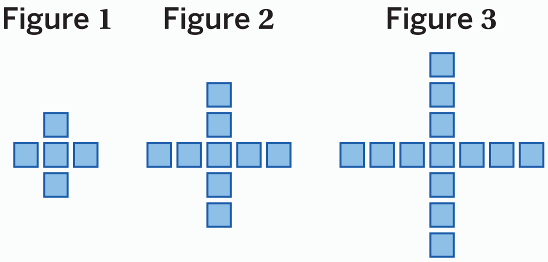


Figure	Number of Tiles
1	5
2	8
3	11

Try This

Here is a visual pattern.



a Sketch Figure 4.

b How many tiles will there be in Figure 10?

Explain your thinking.

Summary | Lesson 2

A **sequence** is a list of numbers in a particular order.

Sequences can change in predictable ways. Two examples of predictable change are constant differences and constant ratios.

A sequence has a **constant difference** when the difference between any two consecutive terms is the same.

$$5, \overset{+6}{\curvearrowright} 11, \overset{+6}{\curvearrowright} 17, \overset{+6}{\curvearrowright} 23$$

A sequence has a **constant ratio** when the ratio between any two consecutive terms is the same.

$$2, \overset{\times 4}{\curvearrowright} 8, \overset{\times 4}{\curvearrowright} 32, \overset{\times 4}{\curvearrowright} 128$$

Sequences can also change in predictable ways that are neither a constant difference or ratio.

$$13, \overset{-1}{\curvearrowright} 12, \overset{-2}{\curvearrowright} 10, \overset{-3}{\curvearrowright} 7$$

Once you know how a sequence changes, you can use that change to determine unknown terms.

Here is an example:

- The known terms in this sequence have a constant ratio of 0.5.
- The value of the unknown term should be 0.5 times the term before, 40, or 2 times the term after, 10.
- That means the unknown term is 20.

$$80, \overset{\times 0.5}{\curvearrowright} 40, \ ?, \ 10, \overset{\times 0.5}{\curvearrowright} 5$$

Try This

Here is a sequence.

$$5, 15, 45, \ldots$$

a Does this sequence have a constant ratio or a constant difference?

b What is the next term in the sequence? Explain your thinking.

Summary | Lesson 3

There are several ways to define, or describe, a sequence. When you define a sequence recursively, you're determining each term using the previous term.

A **recursive definition** of a sequence includes a first term and a rule for finding every term that follows.

Here are some examples of recursive definitions for this sequence: $32, 16, 8, 4, 2, 1, 0.5$.

First term: 32

Rule: Half of the previous term

First term: 32

Rule: Constant ratio of $\frac{1}{2}$

First term: 32

Rule: Multiply the previous term by 0.5

Try This

Create three different recursive definitions of sequences where the second term is 10.

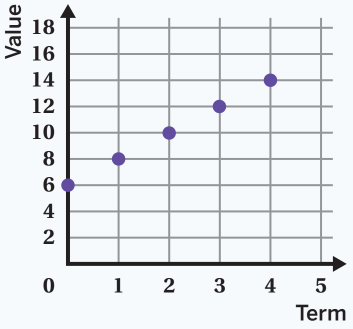
Sequence	First Term	Rule
______, 10, ______, ______, ______		
______, 10, ______, ______, ______		
______, 10, ______, ______, ______		

Summary | Lesson 4

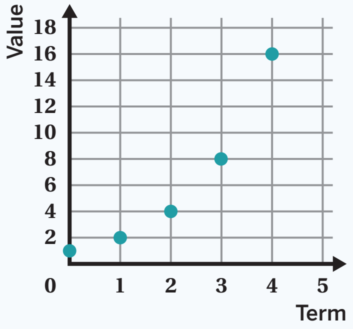
Sequences that change by a constant difference are called **arithmetic sequences**, while sequences that change by a constant ratio are called **geometric sequences**.

Sequences can be represented in multiple ways: as a list of numbers, in a table, on a graph, or with a recursive definition. In each representation, there are ways to identify if the sequence is arithmetic, geometric, or neither. Here are some examples of graphs of different kinds of sequences.

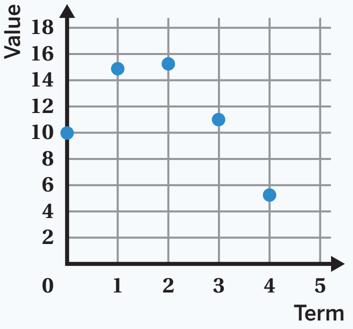
Arithmetic



Geometric



Neither



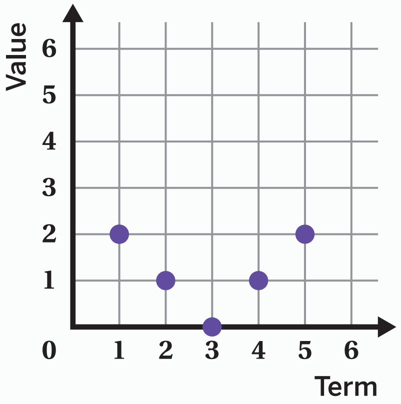
Try This

Determine whether each sequence is arithmetic, geometric, or neither. Explain your thinking.

a

Term	Value
1	8
2	16
3	24
4	32
5	40

b



c **First term:** 5

Rule: Divide the previous term by 3.

Summary | Lesson 5

An **explicit definition** is a formula that determines the value of any term in a sequence using the term number. An explicit definition of a sequence can be written as an *expression* or in words.

Here are two situations and the explicit expressions that define them.

Situation: Julian's family is 250 miles away from home. They drive 50 miles toward their home for every n hours.

Expression: $250 - 50n$
The expression represents the number of miles Julian's family is from home after n hours.

- 250 represents the starting distance.
- -50 represents the distance traveled each hour.
- $-50n$ represents the total distance traveled.
- n represents the number of hours driven.

Situation: A scientist tracks a group of 250 bugs in a mound. Each month, the number of bugs increases by 1.5 times, for n months.

Expression: $250 \cdot 1.5^n$
The expression represents the number of termites in the mound after n months.

- 250 represents the initial number of bugs.
- 1.5 represents the ratio that the bugs grow by each month.
- n represents the number of months.

Try This

Saanvi woke up to 8 inches of snow outside. The weather forecast predicts that there will be about 5 inches of snow every hour for the rest of the day.

a Write an explicit expression that represents the number of inches of snow after n hours.

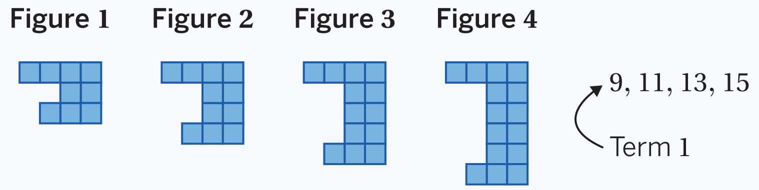
Make a table if it helps with your thinking.

b What does each part of your expression represent in this situation?

Summary | Lesson 6

You can write an explicit expression for a pattern or sequence in multiple equivalent ways, just by referencing different term numbers.

Here's an example of a pattern and its matching sequence.



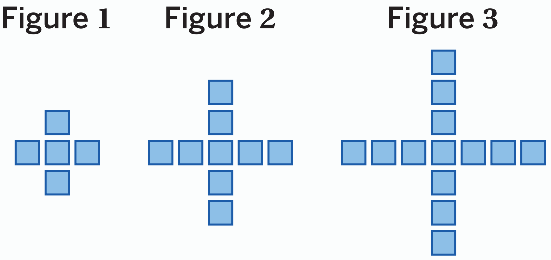
$7 + 2(1)$ $7 + 2(2)$ $7 + 2(3)$ $7 + 2(4)$ $9 + 2(n - 1)$

One expression that represents this sequence is $9 + 2(n - 1)$; this expression uses Term 1 as the starting value and $n - 1$ to calculate the change.

The expression $7 + 2n$ also represents this sequence, using Term 0 as the starting value, and n to calculate the change.

Try This

Select *all* the expressions that could represent the number of tiles in Figure n of this pattern.



☐ **A.** $n + 4$

☐ **B.** $4n + 1$

☐ **C.** $5 + 4(n - 1)$

☐ **D.** $9n - 4$

☐ **E.** $9 + 4(n - 2)$

Summary | Lesson 7

A **model** is a mathematical representation (such as a graph, equation, or relationship) of a situation. You can use a model to make predictions or decisions.

For example, let's say a group of scientists track the amount of algae on the surface of a lake, in square feet, over n weeks.

Weeks (n)	0	1	2	3	4
Amount of Algae (sq. ft)	2	2.8	3.92	5.49	7.68

The scientists model this information with the expression $2 \cdot 1.4^n$, which can be used to predict how much algae there will be in the upcoming weeks. Predictions might not be exactly accurate because factors like weather or human action can affect algae growth. The model might also be limited by the size of the lake.

No mathematical model applies perfectly to real life, but a model can still be a powerful and useful tool. We can even improve models when we take into account more information.

Try This

An RV park has 20 rented plots. The management company is hoping the number of rented plots doubles every year until the park is full.

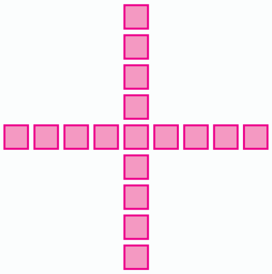
a Would a model for the number of rented plots each year be arithmetic, geometric, or neither? Explain your thinking.

b Create an explicit expression to model the number of rented plots after n years until the park is full.

Try This | Answer Key

Lesson 1

a



b 41 tiles. *Explanations vary*. There are 4 groups of 10 tiles and 1 tile in the middle.

Lesson 2

a Constant ratio

b 135. *Explanations vary*. The sequence has a constant ratio of 3, so to get the next term, multiply by 3. $45 \cdot 3 = 135$.

Lesson 3

Responses vary.

- First term: 4. Rule: Add 6 to the previous term.
- First term: 5. Rule: Constant ratio of 2.
- First term: 20. Rule: Constant difference of -10.

Lesson 4

a Arithmetic sequence. *Explanations vary*. There is a constant difference of 8, and graphing the points forms a line.

b Neither. *Explanations vary*. There is neither a constant difference nor a constant ratio. The difference between the second and third terms is -1, but the difference between the third and fourth terms is 1.

c Geometric sequence. *Explanations vary*. Another way to think of dividing the previous term by 3 is multiplying by a constant ratio of $\frac{1}{3}$.

Try This | Answer Key

Lesson 5

ⓐ $8 + 5n$ (or equivalent)

ⓑ *Responses vary*. The 8 represents the starting amount of snow. There is a constant difference of 5 inches, so there will be 5 inches of snow every hour for n hours since Saanvi woke up.

Lesson 6

B. $4n + 1$

C. $5 + 4(n - 1)$

E. $9 + 4(n - 2)$

Lesson 7

ⓐ *Responses and explanations vary*.

- Geometric. Since the management company is hoping that the number of rented plots doubles each year, there is a constant ratio of 2.
- Neither. Although the number of plots will double for some period of time, eventually the park will be full, and the number of plots will become constant.

ⓑ $20 \cdot 2^n$ (or equivalent)