Amplify Desmos Math FLORIDA

Student Edition

ACC 7

Volume 2

Amplify Desmos Math FLORIDA

Accelerated 7

Volume 2: Units 6–11

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

Cover illustration by Caroline Hadilaksono and J Yang.

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55 Washington Street, Suite 800
Brooklyn, NY 11201
www.amplify.com

ISBN: 9798895804605
Printed in [e.g., the United States of America]
[# of print run] [print vendor] [year of printing]

Dear Student,

Welcome to Amplify Desmos Math Florida! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math Florida is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- Lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important ideas in mathematics.

We hope you enjoy exploring math this year as you work with friends to solve problems and learn about different concepts.

–The Amplify Desmos Math Florida Team

Unit 1 Rigid Transformations and Congruence

In this unit, you will investigate translations, reflections, and rotations, and use these transformations to make arguments about congruence. You will also explore angle relationships and the triangle sum theorem.

Sub-Unit 1 Rigid Transformations 2

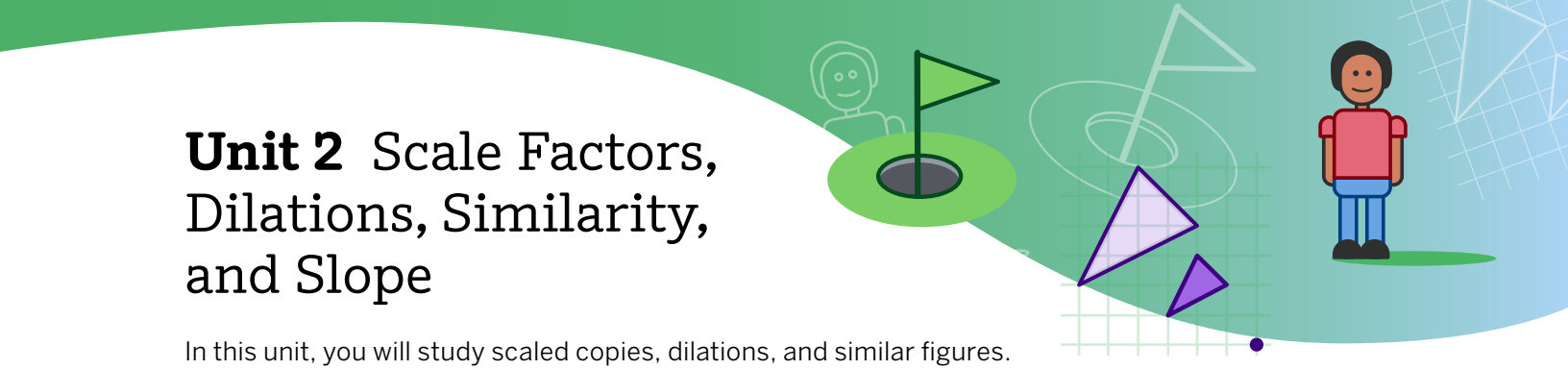
1.01 Spinning, Flipping, Sliding | Naming Transformations 3
1.02 Moving Day | Describing Transformations Precisely 10
1.03 Getting Coordinated, Part 1 | Coordinates of Translations and Reflections 17
1.04 Getting Coordinated, Part 2 | Coordinates of Transformations 24

Sub-Unit 2 Congruence 32

1.05 No Bending, No Stretching | Rigid Transformations 33
1.06 Are They the Same? | Defining Congruence 39
1.07 Are They Congruent? | Rigid Transformations and Congruent Figures 47

Sub-Unit 3 Angle Relationships 54

1.08 Tearing It Up | Angle Sums in Triangles 55
1.09 Friendly Angles | Complementary, Supplementary, Vertical, and Adjacent Angles 62
1.10 Sum It Up | Exploring Angle Sums in Polygons 72



Unit 2 Scale Factors, Dilations, Similarity, and Slope

In this unit, you will study scaled copies, dilations, and similar figures. You will use similar triangles to explain, understand, and apply the concept of slope.

Sub-Unit 1 Scaled Copies 80

2.01 Scaling Machines | What Are Scaled Copies? 81
2.02 Scaling Robots | Lengths and Scaled Copies 90
2.03 Make It Scale | Drawing Scaled Copies 97
2.04 Scale Factor Challenges | Effects of Scale Factors 104
2.05 Tiles | Scaling and Area 112
2.06 Scale Drawings | Comparing Scale Factor and Scale 120

Sub-Unit 2 Dilations 126

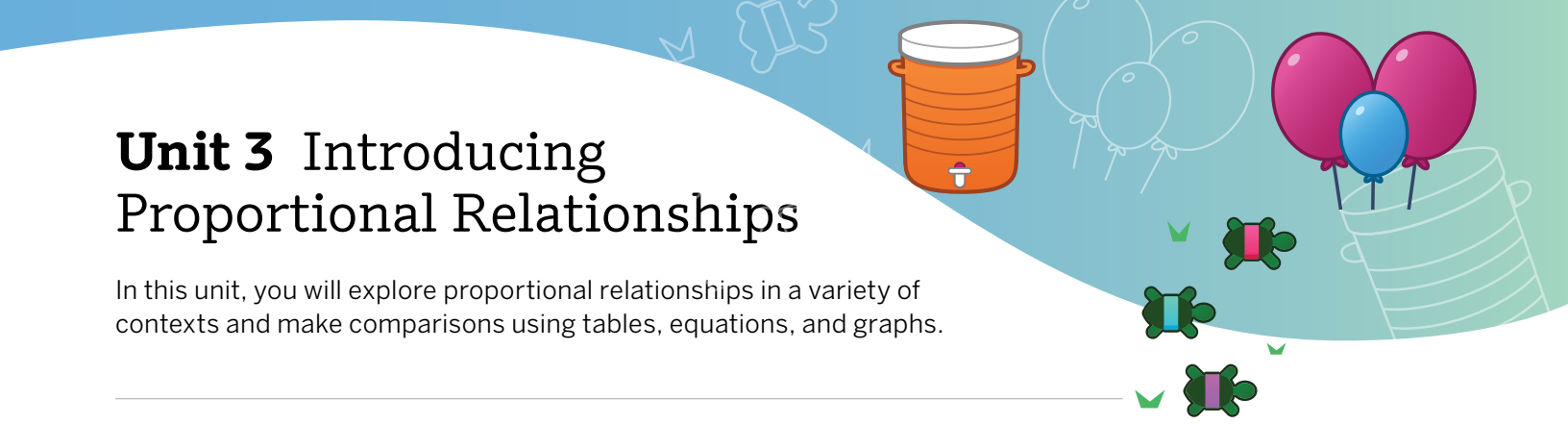
2.07 Sketchy Dilations | Introducing Dilations 127
2.08 Dilation Mini Golf | Dilations With No Grid 133
2.09 Make It a Combo! | Combining a Rigid Transformation and a Dilation 140
2.10 Match My Dilation | Dilations on a Square Grid 146
2.11 Dilations on a Plane | Dilations With Coordinates 153

Sub-Unit 3 Similarity 160

2.12 Scavenger Hunt | Similar Figures, Sides, and Angles 161
2.13 Are Angles Enough? | Similar Triangles 168
2.14 Shadows | Side Length Relationships in Similar Triangles 175

Sub-Unit 4 Slope 182

2.15 Water Slide | Similarity and Slope 183
2.16 Slope Challenges | Developing Fluency with Slope 191



Unit 3 Introducing Proportional Relationships

In this unit, you will explore proportional relationships in a variety of contexts and make comparisons using tables, equations, and graphs.

Sub-Unit 1 Proportional Relationships in Tables 200

3.01 Balloon Float | Introducing Proportional Relationships With Tables 201

3.02 Sugar, Spice, and Everything Rice | Constant of Proportionality 209

3.03 Pen Pals | Converting Units 215

Sub-Unit 2 Proportional Relationships in Equations 222

3.04 Robot Factory | Proportional Relationships and Equations 223

3.05 Snapshots | More Equations of Proportional Relationships 229

3.06 Two and Two | Two Equations for Each Relationship 236

3.07 All Kinds of Equations | Equations of Proportional Relationships 243

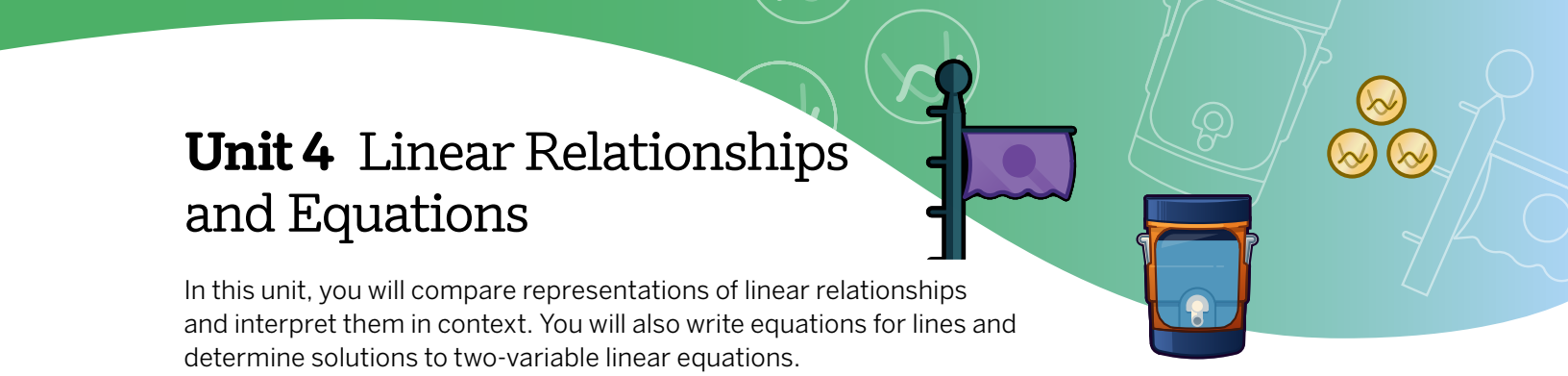
Sub-Unit 3 Proportional Relationships in Graphs 250

3.08 DinoPops | Introducing Graphs of Proportional Relationships 251

3.09 Gallon Challenge | Interpreting Graphs of Proportional Relationships 259

3.10 Three Turtles | Proportional Relationships and Graphs 266

3.11 Four Representations | Connecting Descriptions, Tables, Graphs, and Equations 274



Unit 4 Linear Relationships and Equations

In this unit, you will compare representations of linear relationships and interpret them in context. You will also write equations for lines and determine solutions to two-variable linear equations.

Sub-Unit 1 Linear Relationships 282

4.01 Flags | Introduction to Linear Relationships 283

4.02 Water Cooler | Slopes Don't Have to Be Positive 291

4.03 Ups and Downs | Analyzing Linear Relationships 299

4.04 Stacking Cups | More With Linear Relationships 306

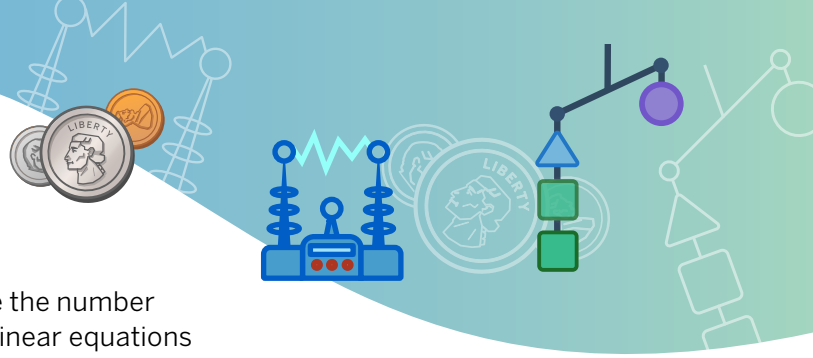
Sub-Unit 2 Linear Equations and Points 314

4.05 Landing Planes | Calculating Slope 315

4.06 Coin Capture | Horizontal and Vertical Lines 322

4.07 Why Intercepts? | Equations of All Kinds of Lines 328

4.08 Pennies and Quarters | Using Linear Relationships to Solve Problems 336



Unit 5 Linear Equations and Linear Systems

In this unit, you will solve linear equations and determine the number of possible solutions. You will also solve systems of two linear equations algebraically and graphically.

Sub-Unit 1 Two-Step Equations 346

5.01 Balancing Equations | Solving Equations With Balanced Hangers 347
5.02 Keeping It True | Solving Equations 354
5.03 Factoring and Expanding | Options for Solving One Equation 360
5.04 Pass the Equation | Solving Equations Multiple Ways 367
5.05 Community Day | Using Equations to Solve Problems 373

Sub-Unit 2 Multi-Step Equations 380

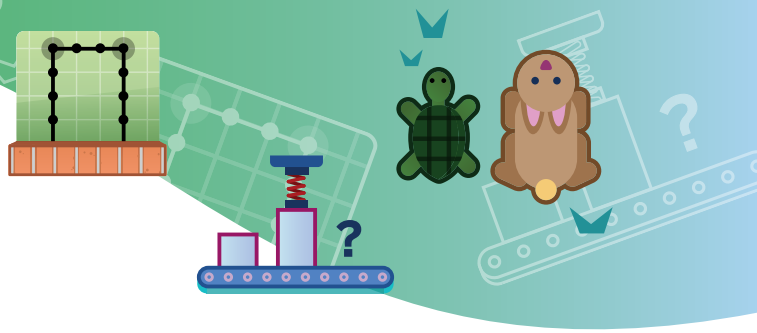
5.06 Equation Roundtable | Analyzing Unbalanced Equations 381
5.07 All, Some, or None? Part 1 | How Many Solutions? 387
5.08 Strategic Solving | Solving Many Types of Equations 395
5.09 When Will They Meet? | Solving Linear Equations in Context 402
5.10 Do the Two-Step | Solving Two-Step Linear Inequalities 410

Sub-Unit 3 Systems of Linear Equations 418

5.11 On or Off the Line? | Interpreting Points On or Off the Line 419
5.12 On Both Lines | Representing Systems of Linear Equations 426
5.13 Make Them Balance | Graphing Systems of Linear Equations 432
5.14 Line Zapper | Solving Systems of Linear Equations 440
5.15 All, Some, or None? Part 2 | Systems of Equations with No, One, or Many Solutions 447

Unit 6 Functions

In this unit, you will learn about functions, analyze representations of functions, and examine functions in different contexts.



Sub-Unit 1 Introduction to Functions 456

6.01 Guess My Rule | Introduction to Functions 457

6.02 The Ins and Outs | Determining Domain and Range 465

6.03 Function or Not? | Graphs of Functions and Non-Functions 474

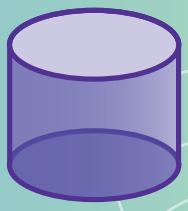
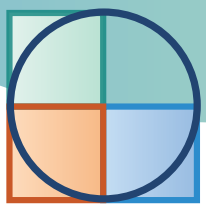
6.04 Window Frames | Functions and Equations 482

Sub-Unit 2 Representing and Interpreting Functions 490

6.05 The Tortoise and the Hare | Interpreting Graphs of Functions 491

6.06 Elevation Examination | Analyzing Function Representations That Are Linear or Nonlinear 498

6.07 Comparing Linear Functions | Comparing Representations of Functions 505

Unit 7 Circles and Cylinders

In this unit, you will explore the relationships between the radius, diameter, circumference, and area of a circle. You will also explore the volume and surface area of cylinders.

Sub-Unit 1 Circles 514

7.01 Is It a Circle? | Parts of a Circle 515

7.02 Measuring Around | Estimating and Calculating Circumference 522

7.03 Radius Squares | Exploring Circle Area 529

7.04 Circle Area | Calculating Circle Area 536

7.05 Area Challenges | Calculating Areas of Complex Shapes 542

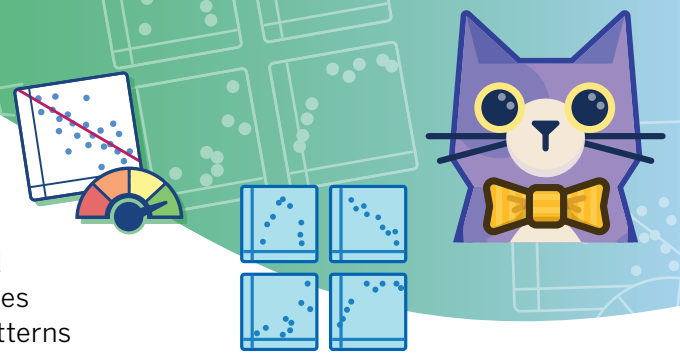
7.06 Circle vs. Square | Calculating Area Given Perimeter 548

Sub-Unit 2 Solid Geometry 554

7.07 Cylinders | Volumes of Cylinders 555

7.08 Scaling Cylinders | Applying Volumes of Cylinders 563

7.09 From Flat to Round | Surface Area of Cylinders 570



Unit 8 Associations in Data

In this unit, you will decide the best display for numerical and categorical data. Then you will use scatter plots and fitted lines to analyze numerical data with two variables and observe patterns in the data.

Sub-Unit 1 Displaying Data 580

8.01 The County Fair | Creating Circle Graphs 581

8.02 Choices, Choices | Choosing an Appropriate Representation for Data 589

8.03 On Display | Creating an Appropriate Representation for Data 596

Sub-Unit 2 Organizing Numerical Data 604

8.04 Click Battle | Organizing Data 605

8.05 Birds, Birds, Birds | Plotting Data 613

8.06 Robots | What a Point on a Scatter Plot Means 619

Sub-Unit 3 Analyzing Numerical Data 626

8.07 Dapper Cats | Linear Models and Outliers 627

8.08 Interpreting Scatter Plots | Showing Patterns in Data 634

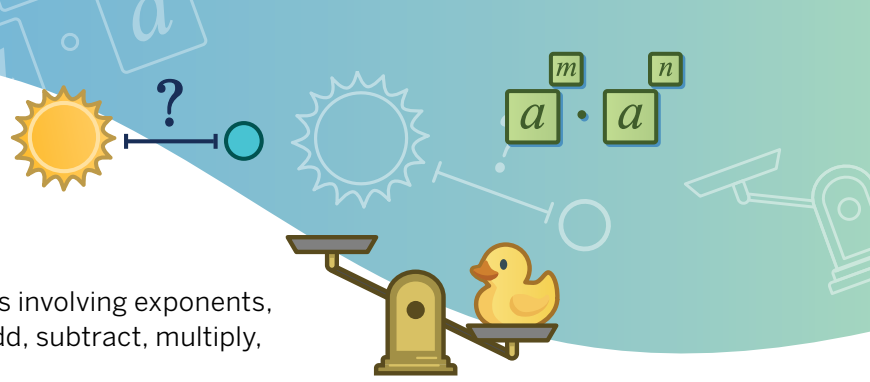
8.09 Find the Fit | Fitting a Line to Data 641

8.10 Interpreting Slopes | The Slope of a Linear Model 649

8.11 Scatter Plot City | Observing More Patterns in Scatter Plots 658

8.12 Animal Brains | Analyzing Bivariate Data 664

Unit 9 Exponents and Scientific Notation



In this unit, you will develop fluency with expressions involving exponents, powers of 10, and scientific notation. You will also add, subtract, multiply, and divide numbers written in scientific notation.

Sub-Unit 1 Exponent Properties 674

9.01 Circles | Exponent Notation 675

9.02 Combining Exponents | Equivalent Expressions With Exponents 682

9.03 Power Pairs | Multiplying Powers and Powers of Powers 689

9.04 Rewriting Powers | Rewriting Exponential Expressions as a Single Power 695

9.05 Exponent Zero | Using Patterns to Understand Zero Exponents 702

9.06 Negative Bases | Powers With Negative Bases 708

Sub-Unit 2 Algebraic Expressions 716

9.07 Negative Exponents | Understanding Negative Exponents and Expressions With Integer Exponents 717

9.08 Write a Rule | Generalizing Exponent Properties 725

9.09 Algebraic Expressions with Exponents | Using Rules of Exponents to Evaluate and Simplify Expressions 731

9.10 Multiplying Linear Expressions | Multiplying Monomials and Binomials 739

9.11 Fabulous Factors | Factoring Algebraic Expressions 746

Sub-Unit 3 Scientific Notation 754

9.12 Scales and Weights, Part 1 | Describing Large Numbers Using Powers of 10 755

9.13 Scales and Weights, Part 2 | Describing Small Numbers Using Powers of 10 762

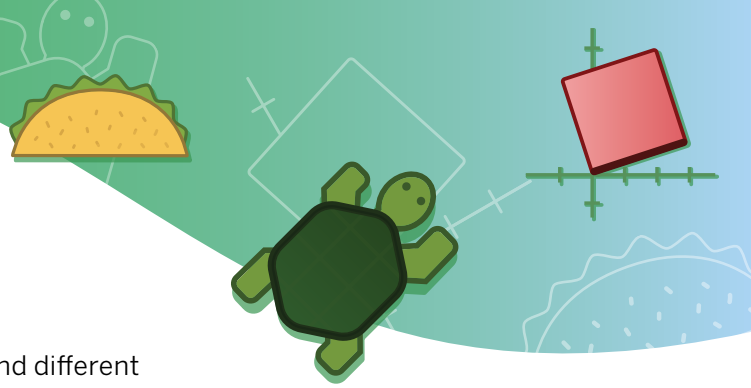
9.14 Specific and Scientific | Definition of Scientific Notation 771

9.15 Multiplying and Dividing | Multiplying and Dividing With Scientific Notation 777

9.16 Balance the Scale | Multiplying, Dividing, and Estimating with Scientific Notation 784

9.17 Use Your Powers | Applications of Arithmetic With Scientific Notation 791

9.18 City Lights | Adding and Subtracting With Scientific Notation 797



Unit 10 The Pythagorean Theorem and Irrational Numbers

In this unit, you will explore the Pythagorean theorem and different types of numbers (square roots and cube roots, rational and irrational numbers).

Sub-Unit 1 Square Roots and Cube Roots 806

10.01 Tilted Squares | The Areas of Tilted Squares 807

10.02 From Squares to Roots | Side Lengths and Areas 814

10.03 Between Squares | Approximating Square Roots 821

10.04 Root Down | Reasoning About Square Roots 828

10.05 Filling Cubes | Edge Lengths, Volumes, and Cube Roots 835

Sub-Unit 2 The Pythagorean Theorem 844

10.06 The Pythagorean Theorem | Squares of Side Lengths 845

10.07 Pictures to Prove It | A Proof of the Pythagorean Theorem 851

10.08 Triangle-Tracing Turtle | Finding Unknown Side Lengths 858

10.09 Make It Right | The Converse of the Pythagorean Theorem 865

10.10 Can You Build It? | The Triangle Inequality 873

10.11 Taco Truck | Applications of the Pythagorean Theorem 881

10.12 Pond Hopper | Finding Distances on the Coordinate Plane 887

Sub-Unit 3 Rational and Irrational Numbers 894

10.13 Code Breaking | Equivalent Forms 895

10.14 Hit the Target | Rational and Irrational Numbers 902

10.15 Road Trip! | Solving Problems Using Order of Operations 909

10.16 Simplify It! | Expressions With Exponents and Radicals 917

Unit 11 Probability

In this unit, you will learn ways to display all of the possible outcomes for a repeated experiment. You will learn the difference between theoretical and experimental probability and calculate them.

Sub-Unit 1 Theoretical Probability 926

11.01 What Can Happen? | Understanding Sample Spaces 927

11.02 Representing Sample Spaces | Tables, Tree Diagrams, and Descriptions 934

11.03 A Likely Story | Theoretical Probability 941

11.04 We're on a Roll! | Understanding Experimental Probability 949

11.05 What Could Happen? | Using Probability to Make Predictions 957

Florida's B.E.S.T. Standards for Mathematics

Benchmark	B.E.S.T Mathematics Benchmark
Number Sense and Operations	
MA.7.NSO.1.1	Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.
MA.7.NSO.1.2	Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.
MA.8.NSO.1.1	Extend previous understanding of rational numbers to define irrational numbers within the real number system. Locate an approximate value of a numerical expression involving irrational numbers on a number line.
MA.8.NSO.1.2	Plot, order and compare rational and irrational numbers, represented in various forms.
MA.8.NSO.1.3	Extend previous understanding of the Laws of Exponents to include integer exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to integer exponents and rational number bases, with procedural fluency.
MA.8.NSO.1.4	Express numbers in scientific notation to represent and approximate very large or very small quantities. Determine how many times larger or smaller one number is compared to a second number.
MA.8.NSO.1.5	Add, subtract, multiply and divide numbers expressed in scientific notation with procedural fluency.
MA.8.NSO.1.6	Solve real-world problems involving operations with numbers expressed in scientific notation.
MA.8.NSO.1.7	Solve multi-step mathematical and real-world problems involving the order of operations with rational numbers including exponents and radicals.
Algebraic Reasoning	
MA.7.AR.2.2	Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.
MA.7.AR.3.3	Solve mathematical and real-world problems involving the conversion of units across different measurement systems.
MA.7.AR.4.1	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.
MA.7.AR.4.2	Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.
MA.7.AR.4.3	Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.
MA.7.AR.4.4	Given any representation of a proportional relationship, translate the representation to a written description, table or equation.
MA.7.AR.4.5	Solve real-world problems involving proportional relationships.
MA.8.AR.1.1	Apply the Laws of Exponents to generate equivalent algebraic expressions, limited to integer exponents and monomial bases.
MA.8.AR.1.2	Apply properties of operations to multiply two linear expressions with rational coefficients.

Florida's B.E.S.T. Standards for Mathematics

MA.8.AR.1.3	Rewrite the sum of two algebraic expressions having a common monomial factor as a common factor multiplied by the sum of two algebraic expressions.
MA.8.AR.2.1	Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.
MA.8.AR.2.2	Solve two-step linear inequalities in one variable and represent solutions algebraically and graphically.
MA.8.AR.2.3	Given an equation in the form of $x^2 = p$ and $x^3 = q$, where p is a whole number and q is an integer, determine the real solutions.
MA.8.AR.3.1	Determine if a linear relationship is also a proportional relationship.
MA.8.AR.3.2	Given a table, graph or written description of a linear relationship, determine the slope.
MA.8.AR.3.3	Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.
MA.8.AR.3.4	Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form.
MA.8.AR.3.5	Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.
MA.8.AR.4.1	Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.
MA.8.AR.4.2	Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions.
MA.8.AR.4.3	Given a mathematical or real-world context, solve systems of two linear equations by graphing.
Functions	
MA.8.F.1.1	Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.
MA.8.F.1.2	Given a function defined by a graph or an equation, determine whether the function is a linear function. Given an input-output table, determine whether it could represent a linear function.
MA.8.F.1.3	Analyze a real-world written description or graphical representation of a functional relationship between two quantities and identify where the function is increasing, decreasing or constant.
Geometric Reasoning	
MA.7.GR.1.3	Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.
MA.7.GR.1.4	Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.
MA.7.GR.1.5	Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.
MA.7.GR.2.1	Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure's net.

MA.7.GR.2.2	Solve real-world problems involving surface area of right circular cylinders.
MA.7.GR.2.3	Solve mathematical and real-world problems involving volume of right circular cylinders.
MA.8.GR.1.1	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving unknown side lengths in right triangles.
MA.8.GR.1.2	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving the distance between two points in a coordinate plane.
MA.8.GR.1.3	Use the Triangle Inequality Theorem to determine if a triangle can be formed from a given set of sides. Use the converse of the Pythagorean Theorem to determine if a right triangle can be formed from a given set of sides.
MA.8.GR.1.4	Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.
MA.8.GR.1.5	Solve problems involving the relationships of interior and exterior angles of a triangle.
MA.8.GR.1.6	Develop and use formulas for the sums of the interior angles of regular polygons by decomposing them into triangles.
MA.8.GR.2.1	Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.
MA.8.GR.2.2	Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.
MA.8.GR.2.3	Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.
MA.8.GR.2.4	Solve mathematical and real-world problems involving proportional relationships between similar triangles.
Data Analysis and Probability	
MA.7.DP.1.4	Use proportional reasoning to construct, display and interpret data in circle graphs.
MA.7.DP.1.5	Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.
MA.8.DP.1.1	Given a set of real-world bivariate numerical data, construct a scatter plot or a line graph as appropriate for the context.
MA.8.DP.1.2	Given a scatter plot within a real-world context, describe patterns of association.
MA.8.DP.1.3	Given a scatter plot with a linear association, informally fit a straight line.
MA.8.DP.2.1	Determine the sample space for a repeated experiment.
MA.8.DP.2.2	Find the theoretical probability of an event related to a repeated experiment.
MA.8.DP.2.3	Solve real-world problems involving probabilities related to single or repeated experiments, including making predictions based on theoretical probability.

Mathematical Thinking and Reasoning Standards

MTR 1.1	Actively participate in effortful learning both individually and collectively.
MTR 2.1	Demonstrate understanding by representing problems in multiple ways.
MTR 3.1	Complete tasks with mathematical fluency.
MTR 4.1	Engage in discussions that reflect on the mathematical thinking of self and others.
MTR 5.1	Use patterns and structure to help understand and connect mathematical concepts.
MTR 6.1	Assess the reasonableness of solutions.
MTR 7.1	Apply mathematics to real-world contexts.

Florida's B.E.S.T. Standards for English Language Arts

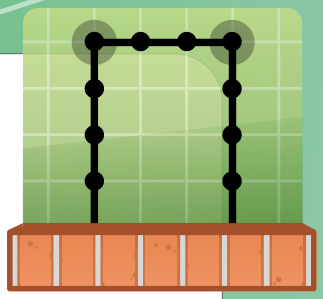
ELA.K12.EE.1.1	Cite evidence to explain and justify reasoning.
ELA.K12.EE.2.1	Read and comprehend grade-level complex texts proficiently.
ELA.K12.EE.3.1	Make inferences to support comprehension.
ELA.K12.EE.4.1	Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.
ELA.K12.EE.5.1	Use the accepted rules governing a specific format to create quality work.
ELA.K12.EE.6.1	Use appropriate voice and tone when speaking or writing.

English Language Development Standards

ELD.K12.ELL.MA.1	Communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
ELD.K12.ELL.SI.1	Read and comprehend grade-level complex texts proficiently.

Unit 6

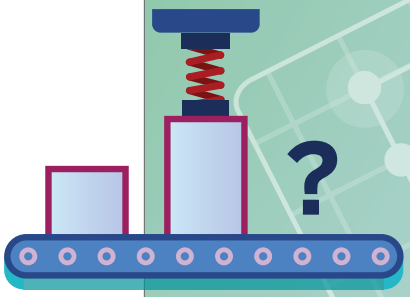
Functions

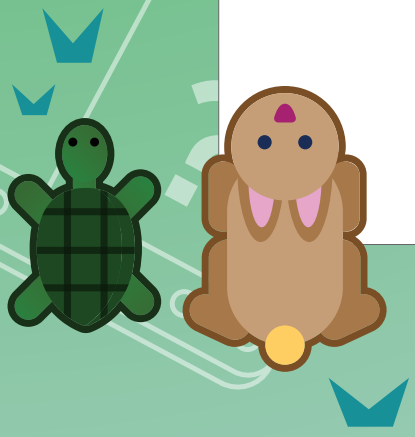


You will learn about functions for the first time. You will analyze representations of functions, and examine functions in different contexts.

Essential Questions

- What makes a relationship a function?
- How are functions useful in representing situations?
- How can I tell if a function is linear or nonlinear?



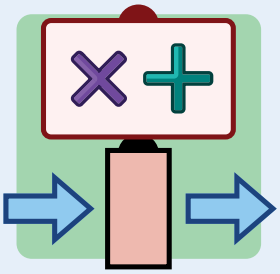


Sub-unit

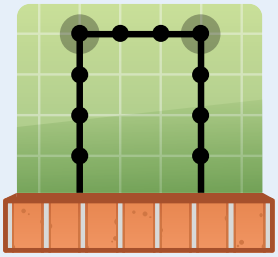
1

Introduction to Functions

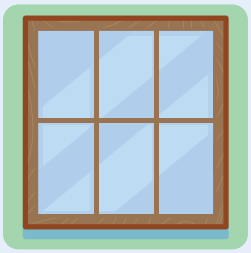
Lesson 1
Guess My Rule



Lesson 2
The Ins and Outs



Lesson 3
Function or Not?



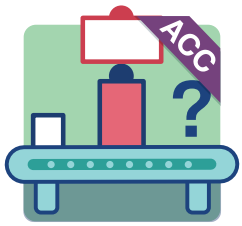
Lesson 4
Window Frames

Name: ..... Date: ..... Period: .....

MA.8.F.1.1, MTR.4.1

Guess My Rule

Let's explore rules to develop the concept of a function.

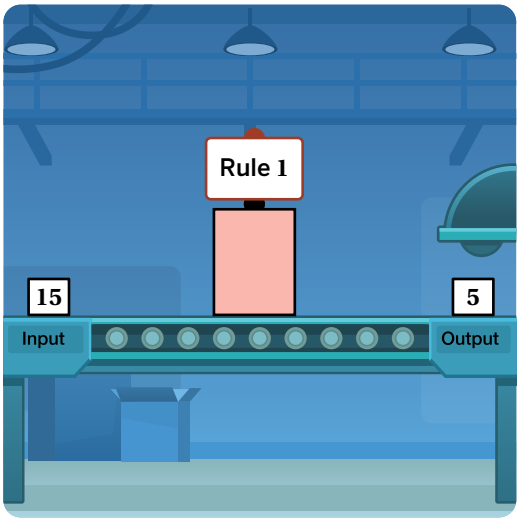


Warm-Up

1. This machine uses a secret rule, Rule #1, to turn inputs into outputs.

Rule #1 allows *all integers* as inputs.

What could Rule #1 be? Select *all* that apply.



(A.) Divide by 2, then add 5.

(B.) Divide by 3.

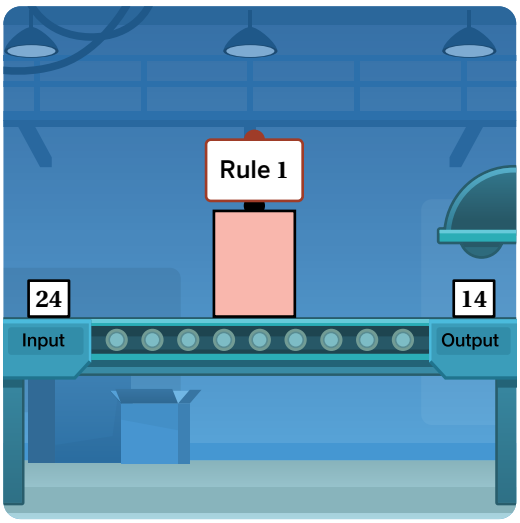
(C.) Take the ones digit.

(D.) Multiply by 3.

(E.) Subtract 10.

2. Let's enter a different integer to help you decide what Rule #1 is.

What could Rule #1 be now? Select *all* that apply.



(A.) Divide by 2, then add 5.

(B.) Divide by 3.

(C.) Take the ones digit.

(D.) Multiply by 3.

(E.) Subtract 10.

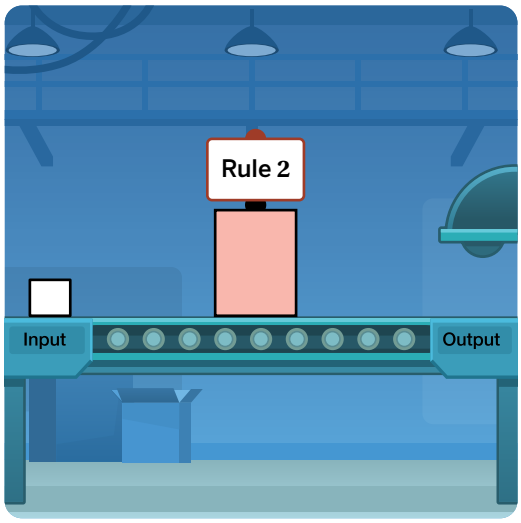
Name: ..... Date: ..... Period: .....

Guess My Rule

3. This machine uses a new rule called Rule #2.

Rule #2 allows *all numbers* as inputs.

Here are some inputs and outputs.



Rule #2

Input	Output
-13	7
-61	7
81	7
-100	7
60	7

If you input 6 into Rule #2, what do you think the output will be?

Explain your thinking.

Name: ..... Date: ..... Period: .....

Guess My Rule (continued)

4. Rule #3 allows *single words* as inputs.

Here are some inputs and outputs.

Rule #3

Input	Output
mint	u
hen	o
clear	s
friend	
wallet	
friend	
party	

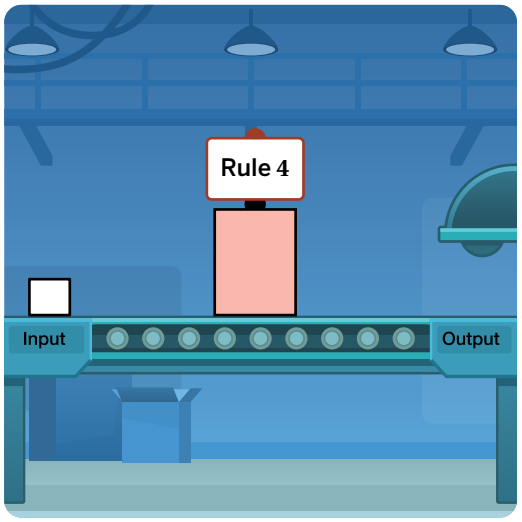
a **Discuss** Using the first three input-output pairs in the table, what do you think the rule might be?

b Test your understanding of Rule #3 by completing the remaining outputs in the table.

5. Rule #4 allows *single letters* (like "A") as inputs.

Here are some inputs and outputs.

Input	Output
W	William
A	Anand
A	Adam



What do you think the output will be if you input "A" into Rule #4? Explain your thinking.

Then compare your response with your classmates' responses.

Name: ..... Date: ..... Period: .....

What Is a Function?

6. Rules #1, #2, and #3 all represent **functions**.

Rule #4 does *not* represent a function.

a What do you think makes a rule a function?

b Compare your response with a classmate's. Then revise your response to make it stronger and clearer.

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

7. Determine whether each table represents a function.

a

Button Selected (Input)	Drink Received (Output)
A	Water
B	Seltzer
C	Juice
D	Water

b

Money Spent (Input)	Number of Items (Output)
$1	2
$8	12
$7	1
$1	3

c

Height in Feet (Input)	Height in Inches (Output)
5	60
4.5	54
6	72
5.4	64.8

d

Time in Seconds (Input)	Musical Note (Output)
1.5	D
1.25	D
1.5	E
2	D

8. This table does *not* represent a function.

a Change at least one number in the table so that it could represent a function.

b Explain your thinking.

Input	Output
1	5
2	10
3	15
2	20
1	24

Activity 3

Name: Date: Period:

Functions and Mapping Diagrams

A **mapping diagram** is a visual representation of a relation. It shows a list of inputs and outputs and uses arrows to show how they are related.

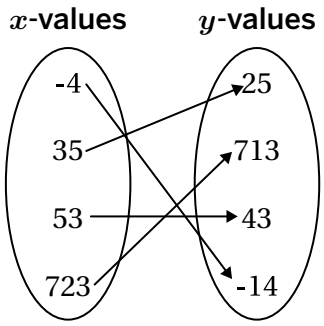
9. Here are two relations from the previous activities and their mapping diagrams.

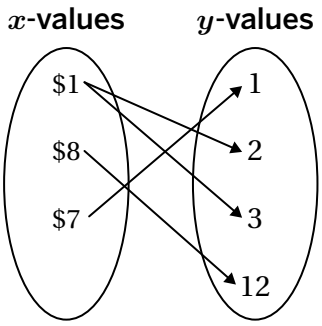
Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Not a Function

Money Spent (Input)	Number of Items (Output)
$1	2
$8	12
$7	1
$1	3





a Explain how a mapping diagram shows the input and output values of a relation.

b **Discuss** How can you tell a relation is not a function from its mapping diagram?

Synthesis

10. How can you determine if a rule represents a function when given a table or a mapping diagram?

Use the examples if they help with your thinking.

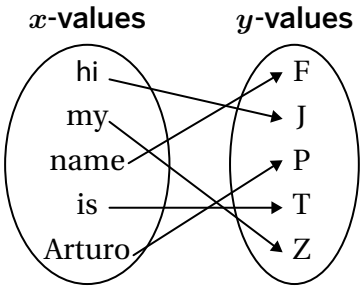
Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

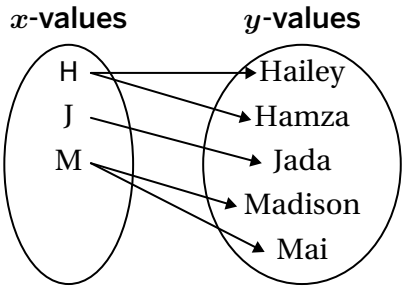
Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3



Rule #4



Lesson Practice ACC7.6.01

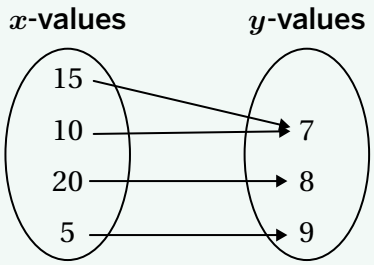
Lesson Summary

A **function** is a rule that assigns exactly one output for each possible input. Another way to say this is that the output is a function of the input.

Input/Output Table	Mapping Diagram

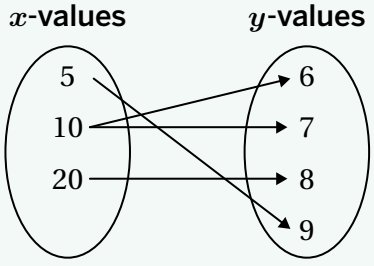
Function

Input	Output
15	7
10	7
20	8
5	9



Not a function

Input	Output
10	6
10	7
20	8
5	9



Lesson Practice
ACC7.6.01

Name: _____ Date: _____ Period: _____

Problems 1–2: Complete each table based on its rule.

1. Rule: Divide by 4 and then add 2.

Input	Output
0	
2	
4	
6	
8	

2. Rule: If odd, write 1. If even, write 0.

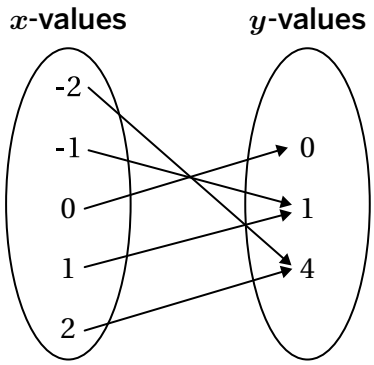
Input	Output
1	
2	
3	
7	
12	

Problems 3–4: Determine whether each table or mapping diagram could represent a function. Explain your thinking.

3.

Input	Output
4	-2
1	-1
0	0
1	1
4	2

4.



5. Complete the table so that it could represent a function.

Input	Output
-2	0
0	
	10
4	

Name: _____ Date: _____ Period: _____

6. Many people consider mathematician Ada Lovelace to be the world's first computer programmer. Lovelace used functions to write programs for an early computing machine. Certain inputs (such as pressing a certain key on a keyboard) caused certain outputs.

Here's a table that shows how pressing different keys causes different movements in a video game character. Could this table represent a function? Explain your thinking.

Key Pressed	Movement
Right arrow	Walk right
Left arrow	Walk left
Up arrow	Walk right
Down arrow	Jump

FAST Practice

7. Which of the following tables could represent a function?

A.

Input	Output
1	0
2	5
3	2.5
4	5
5	8

B.

Input	Output
0	1
5	2
2.5	3
5	4
8	5

C.

Input	Output
3	-8
3	-2
3	-1
3	6
3	12

D.

Input	Output
-1	0
-2	8
2	2
-1	-1
-2	9

Spiral Review

Problems 8–10: Determine whether each ordered pair is a solution for the equation $2x + 4y = 16$.

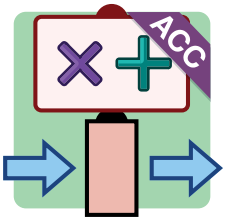
8. $(1, 3)$ **9.** $(6, 1)$ **10.** $(0, 4)$

Name: ..... Date: ..... Period: .....

MA.8.F.1.1, MTR.2.1

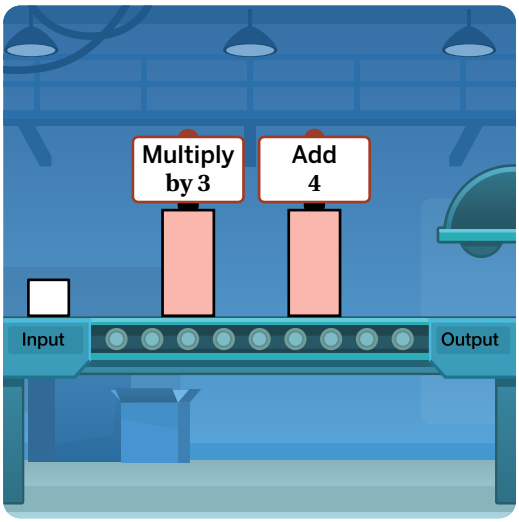
The Ins and Outs

Let's explore the inputs and outputs of relations.



Warm-Up

The rule for this input-output machine is "Multiply by 3, then add 4."



1. Pick a few inputs and determine their outputs.

2. Now, list all of your inputs together and all of your outputs together.

Inputs:

Outputs:

Activity 1

Name: ..... Date: ..... Period: .....

What Are Domain and Range?

The table of values shows a **relation**.

x	y
2	5
6	9
8	14
15	20

3. Describe what you notice about the table.

The domain and range of the relation can be determined from the table.

The **domain** is $\{2, 6, 8, 15\}$. The **range** is $\{5, 9, 14, 20\}$.

4. Describe how domain and range are shown in a table.

Name: ..... Date: ..... Period: .....

What Are Domain and Range? (continued)

5. Daeja and Ivan are asked to determine the domain and range of the relation shown in the table.

x	y
1	-1
4	8
6	11
4	17
10	15

Daeja

Domain: {-1, 8, 11, 15, 17}

Range: {1, 4, 6, 10}

Ivan

Domain: {1, 4, 6, 10}

Range: {-1, 8, 11, 15, 17}

Whom do you agree with? Why?

6. Complete the table to show a relation with a domain of $\{1, 7, 9, 13\}$ and a range of $\{2, 3\}$.

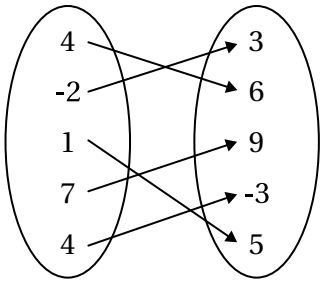
x	y

Activity 2

Name: ..... Date: ..... Period: .....

Determining Domain and Range

7. The mapping diagram shows a relation.



a Describe how the domain and range are shown in the diagram.

b Identify the domain and range.

8. The list of ordered pairs shows a relation.

(2, 3), (4, 9), (6, 1), (4, 5), (3, 9), (7, 3)

a Describe how the domain and range are shown in the list of ordered pairs.

b Identify the domain and range.

9. Create a table of values to show a relation.

x	y

Trade with a classmate

.....

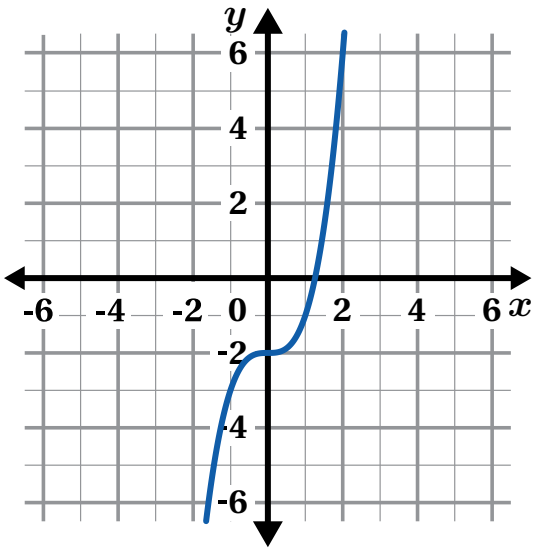
Name ..... Domain and range .....

Activity 3

Name: ..... Date: ..... Period: .....

Domain and Range in Graphs

The graph shows a relation.



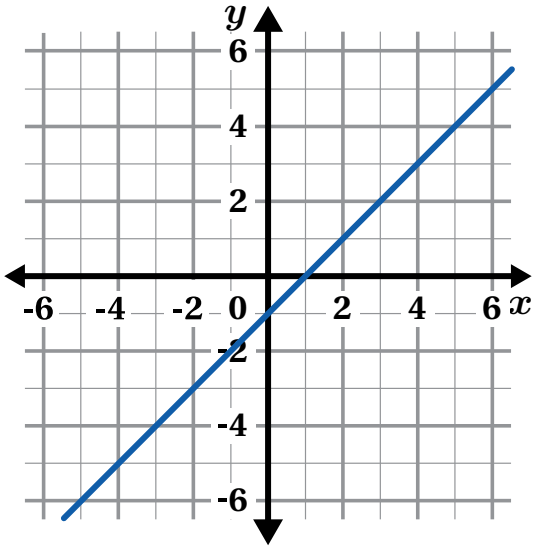
The domain is all real numbers. This can be written as D: $-\infty < x < \infty$.

The range is all real numbers. This can be written as R: $-\infty < y < \infty$.

10. Why might you want to describe the domain and range of this relation with words or inequalities instead of a list?

11. Write the domain and range for each graph.

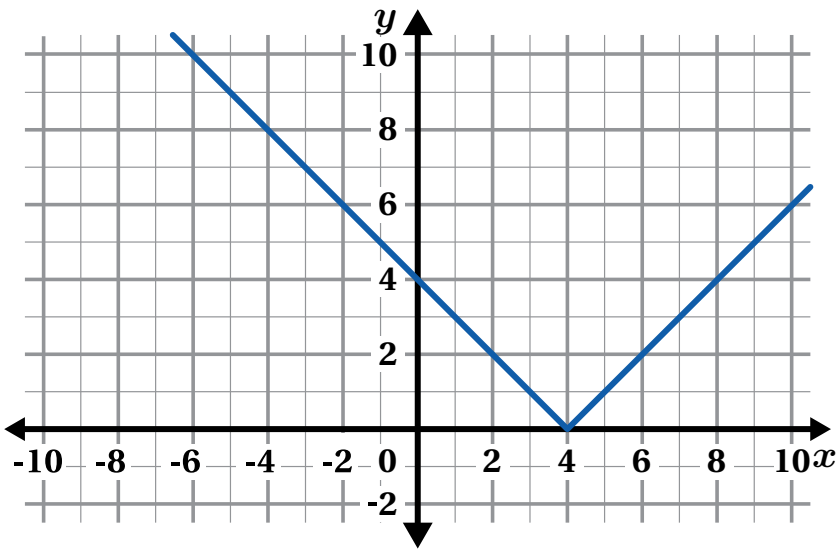
a



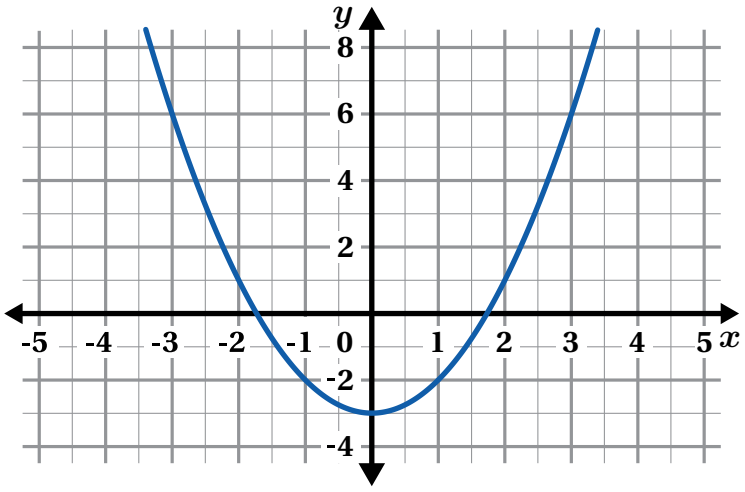
Name: ..... Date: ..... Period: .....

Domain and Range in Graphs (continued)

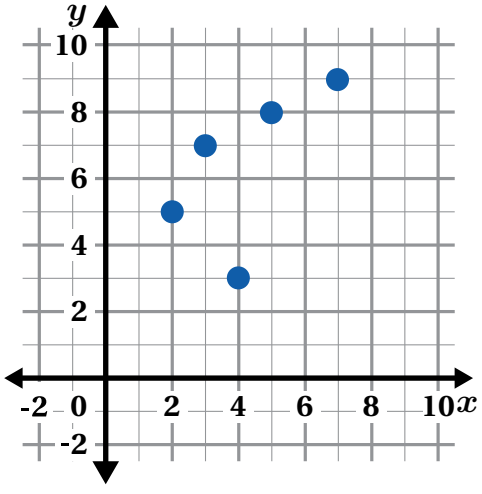
b



c

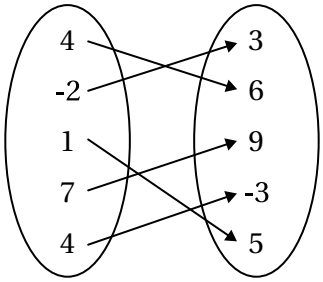


d



Synthesis

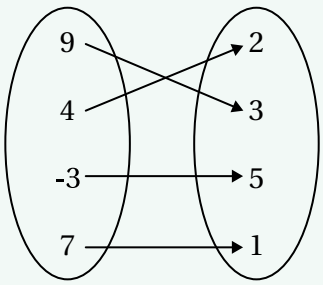
12. Define *domain* and *range* in your own words. Use the example if it helps with your thinking.



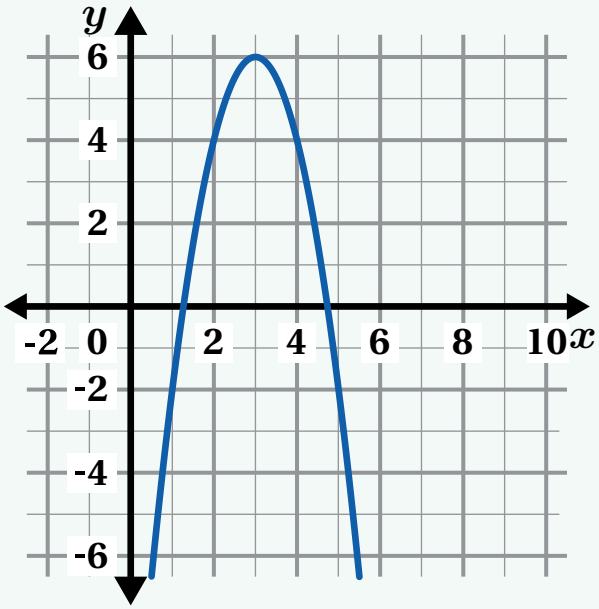
Lesson Practice ACC7.6.02

Lesson Summary

- You can determine the **domain** and **range** of **relations** from mapping diagrams, lists of ordered pairs, tables, and graphs.
- The domain is the set of all possible inputs of a relation. The range is the set of all possible outputs of a relation.
- Domain and range can be shown as a list of elements in brackets or with inequalities.



D: {-3, 4, 7, 9}; R: {1, 2, 3, 5}

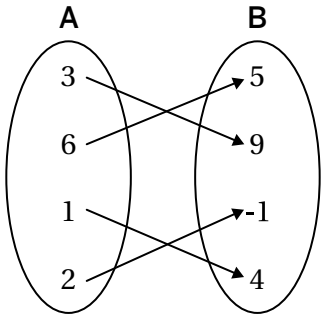


D: $-\infty < x < \infty$; R: $y \leq 6$

Lesson Practice
ACC7.6.02

Name: ..... Date: ..... Period: .....

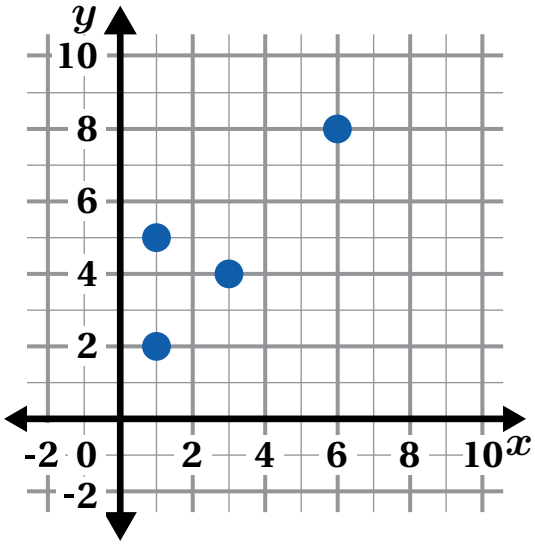
1. What are the domain and range of the relation shown in the mapping diagram?



2. What is the domain of the relation shown in the list of ordered pairs?

{(4, 7), (2, 5), (3, 10), (8, -1), (2, 6)}

3. What is the domain of the relation shown in the graph?



(A.) {1, 3, 6}

(B.) {3, 6, 8}

(C.) {1, 4, 8}

(D.) {2, 4, 5, 8}

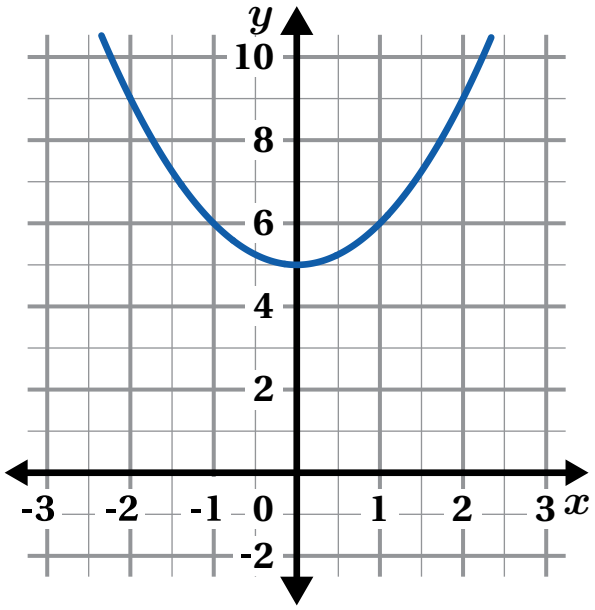
Name: _____ Date: _____ Period: _____

4. Complete the table of values so that it is a relation that has a domain of $\{2, 6, 8\}$ and a range of $\{3, 4, 10\}$.

x	y

FAST Practice

5. What is the range of the relation shown by the graph?



(A.) $y > 5$

(B.) $-\infty < y < \infty$

(C.) $y \leq 5$

(D.) $y \geq 5$

Spiral Review

Problems 6–9: Determine if each ordered pair is a solution for the equation $2x + 8y = 40$.

6. $(4, 4)$

7. $(0, 5)$

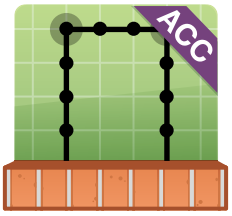
8. $(20, 0)$

9. $(10, 3)$

Name: ..... Date: ..... Period: .....

MA.8.F.1.1, MTR.3.1, MTR.4.1

Function or Not?



Let's determine whether a graph represents a function.

Warm-Up

1. We learned that a function is a rule that assigns exactly one output to each possible input.

That means each input determines a single output.

Complete the table so that y is *not* a function of x.

x	y
3	
5	4
	6

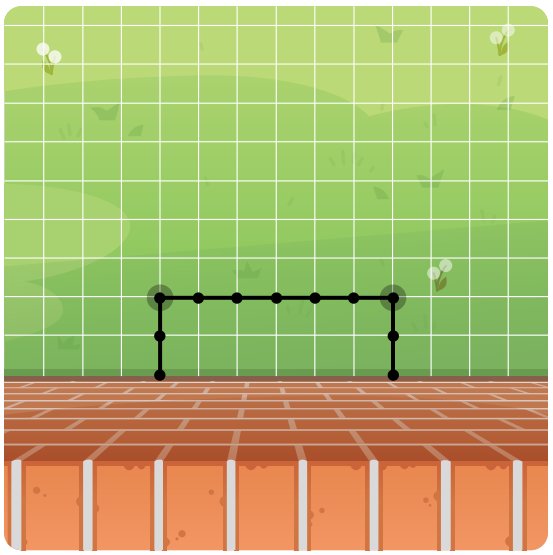
Activity 1

Name: ..... Date: ..... Period: .....

Rectangular Garden

2. Emma is hiring builders to create a fence around her rectangular garden to keep the vegetables in her garden from being eaten by animals. She wants to build three sides of fencing against a brick wall.

Enter the amount of fencing and the area for the garden in the plan shown.



Amount of Fencing (m)	Area (sq. m)

3. Here is a table that shows the amount of fencing and the area for several gardens.

Amount of Fencing (m)	Area (sq. m)
8	8
12	10
14	24
12	16
16	24
20	32

Emma gave her builders 12 meters of fencing to build around her garden.

Is it possible for the builders to determine the area of Emma's garden?

Yes No I'm not sure

Explain your thinking.

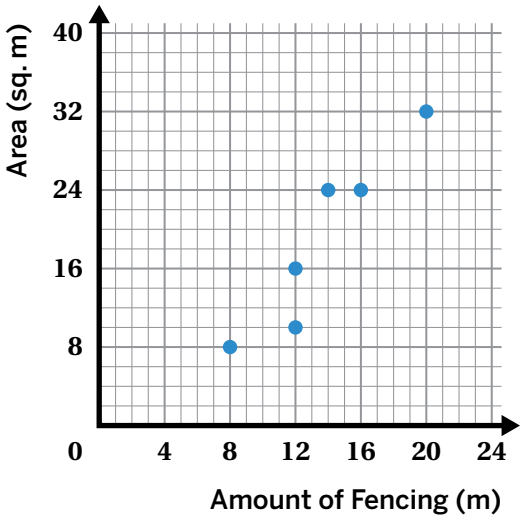
Name: ..... Date: ..... Period: .....

Rectangular Garden (continued)

4. **a** Here is a graph that shows the relationship between area and amount of fencing for several gardens.

How can you use the graph to quickly determine that area is not a function of the amount of fencing?

Area is not a function of the amount of fencing because . . .



b Create a mapping diagram to justify your reasoning.

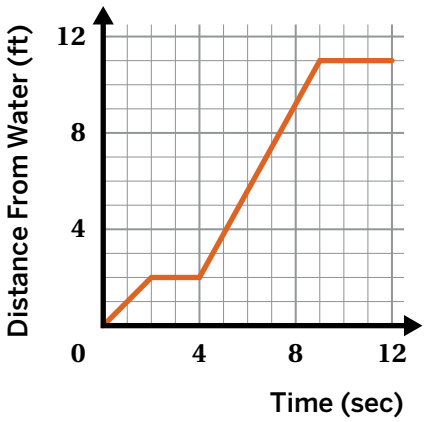
Activity 2

Name: Date: Period:

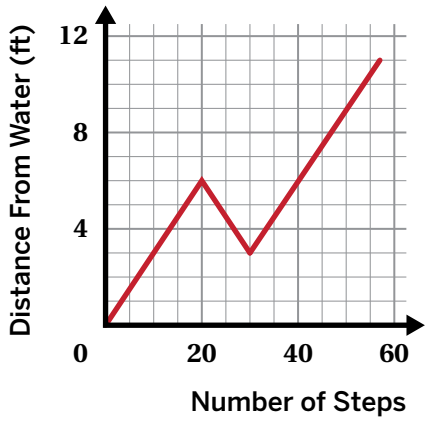
Turtle Crossing

5. The graph represents a turtle's journey across the sand.

Does the graph show that the distance from the water is a function of time? Explain your thinking.



6. How can you use this graph to decide whether distance from the water is a function of the number of steps?

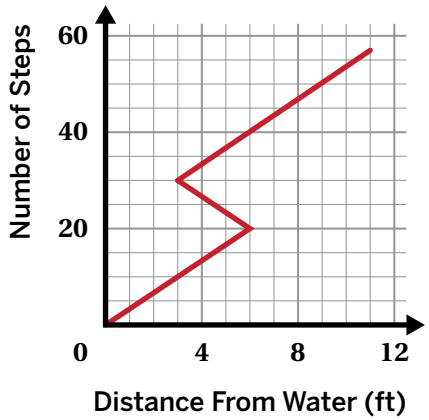


7. Here is the relationship in reverse.

Is the number of steps a function of distance? Circle one.

Yes No

Explain your thinking.

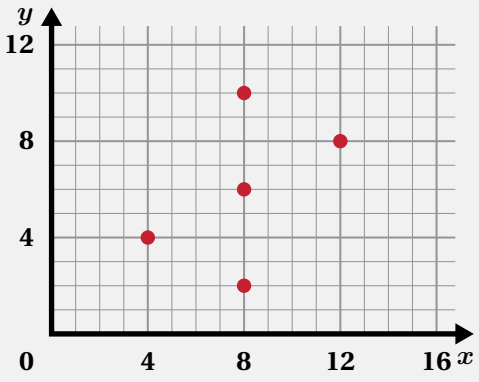


Name: ..... Date: ..... Period: .....

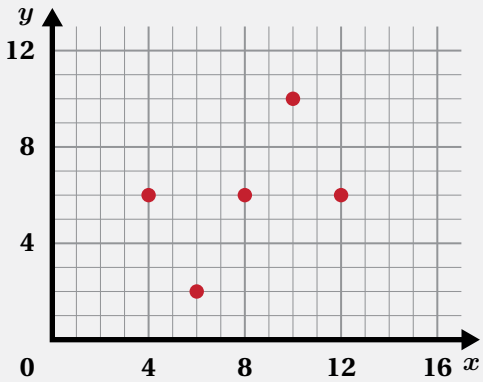
Turtle Crossing (continued)

8. Determine whether each graph represents a function or not.

Graph A

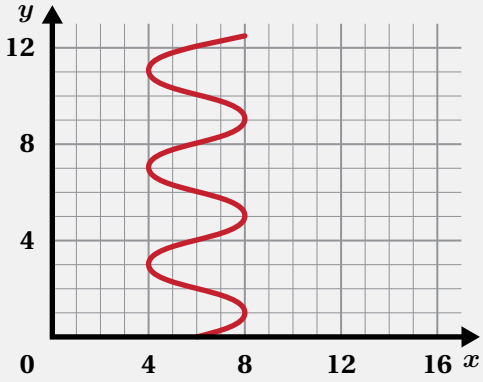


Graph B



Graph C

Graph D

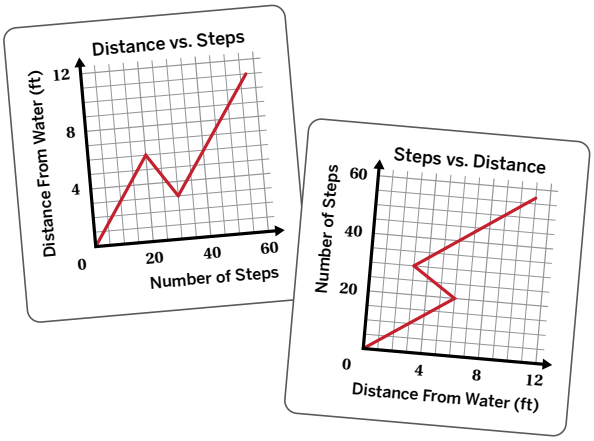


Function	Not a function

Synthesis

9. How can you determine whether a graph represents a function?

Use the examples if it helps with your thinking.



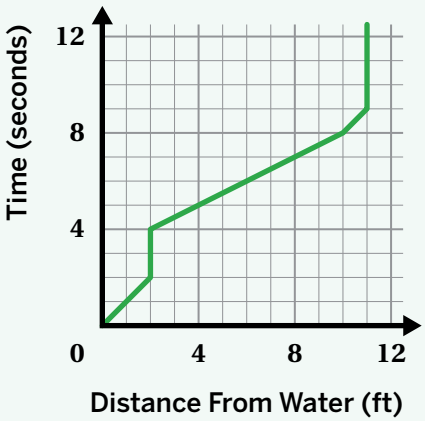
Lesson Practice ACC7.6.03

Lesson Summary

A graph represents a function when each x-value, or input, only has one corresponding y-value, or output. If a graph has multiple y-values for the same x-value, it does not represent a function.

Here are two graphs of the same turtle's journey.

This graph represents a function because for every second, x, the turtle is at only one corresponding distance, y.



This graph does not represent a function because at both 2 feet and 11 feet, the turtle has multiple corresponding times.

Lesson Practice
ACC7.6.03

Name: _____ Date: _____ Period: _____

Problems 1–3: A group of students are timed while sprinting 100 meters.

Time (seconds)	Speed (meters per second)
13.8	7.246
15.9	6.289
16.3	6.135
17.1	5.848
18.2	5.495

1. Is speed a function of time? Explain your thinking.

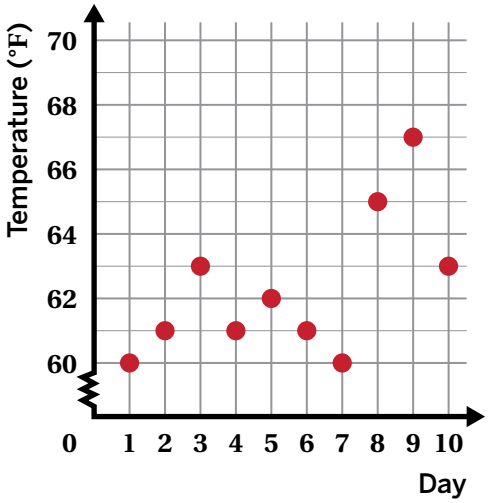
2. Is distance a function of time? Explain your thinking.

Time (seconds)	Distance (meters)
13.8	100
15.9	100
16.3	100
17.1	100
18.2	100

3. Is time a function of distance? Explain your thinking.

Distance (meters)	Time (seconds)
100	13.8
100	15.9
100	16.3
100	17.1
100	18.2

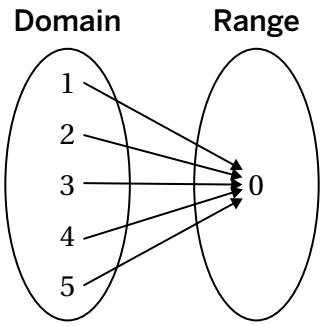
4. This graph represents a city's high temperatures over a 10-day period. Determine whether this statement is true or false: *The high temperature is a function of the day.* Explain your thinking.



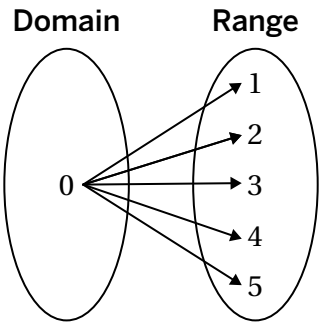
Name: ..... Date: ..... Period: .....

Problems 5–6: Determine whether each mapping diagram could represent a function. Explain your thinking.

5.



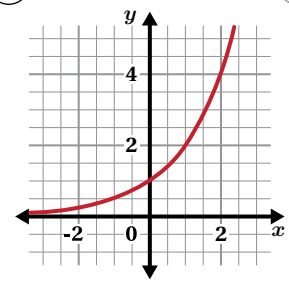
6.



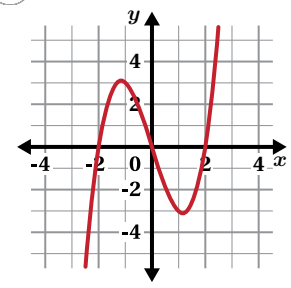
FAST Practice

7. Which graph does *not* represent y as a function of x?

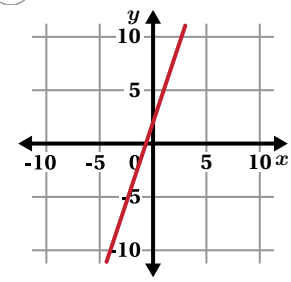
A.



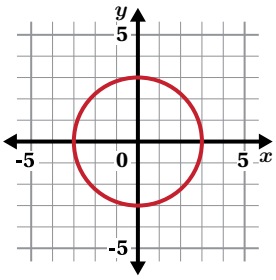
B.



C.



D.



Spiral Review

Problems 8–10: Determine whether each system of equations has one solution, no solution, or infinitely many solutions.

8. $\begin{cases} y = x + 4 \\ y = x + 4 \end{cases}$

9. $\begin{cases} y = -\frac{4}{5}x + 7 \\ y = \frac{4}{5}x - 2 \end{cases}$

10. $\begin{cases} y = 2x + \frac{1}{5} \\ y = 2x + 42 \end{cases}$

Name: ..... Date: ..... Period: .....

MA.8.F.1.1, MA.8.AR.2.1, MA.8.AR.3.3, MA.8.AR.3.5, MTR.4.1, MTR.7.1

Window Frames

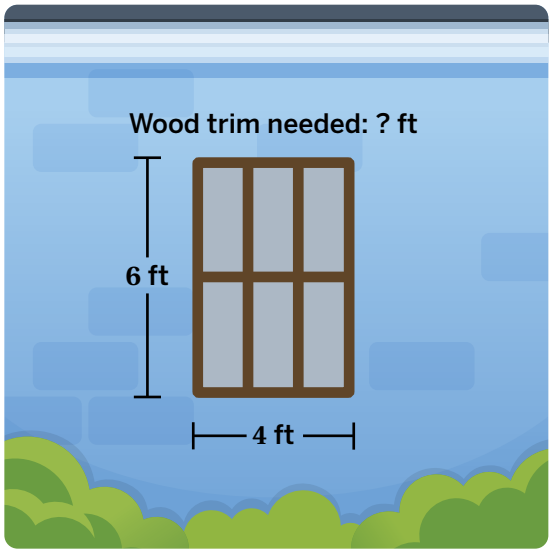
Let's explore the relationship between equations and functions.

Warm-Up

1. Ricardo owns a window framing company.

How much wood trim would he need to build this frame?

Explain your thinking.



Activity 1

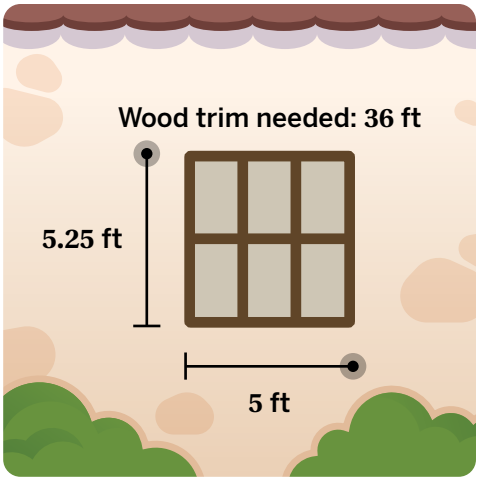
Name: ..... Date: ..... Period: .....

Window Frames

Ricardo's company will cut 36 feet of wood trim into pieces of any size.

2. Let's look at a window that uses exactly 36 feet of wood trim.

Which of the following is true?



A. Width is a function of height.

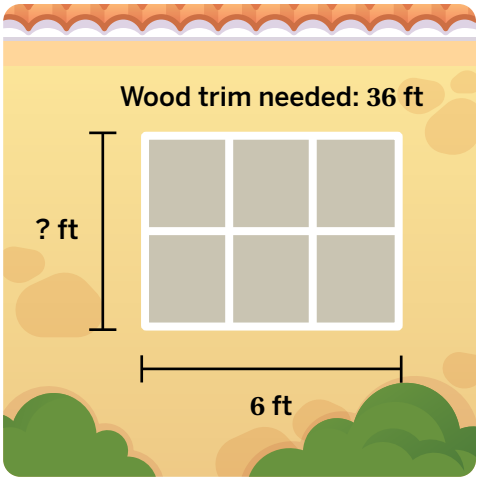
B. Height is a function of width.

C. Both

D. Neither

3. A customer wants a window that is 6 feet wide.

How tall should the window frame be?



4. Ricardo was determining the window frame's height from the previous problem. Here is his table.

Frame Width (ft)	Frame Height (ft)
6	$\frac{(36 - 3 \cdot 6)}{4}$

a Does Ricardo's expression correctly calculate the window frame's height? Circle one.

Yes No

b If yes, explain what 3, 6, and 4 represent about the frame. If no, how would you change his expression to make it correct?

Name: Date: Period:

Window Frames (continued)

5. This table shows Ricardo's expression for frame height given a frame with a width of 6 feet.

Three other windows with different frame widths also use exactly 36 feet of window trim. Calculate how tall each frame should be.

Frame Width (ft)	Frame Height (ft)
6	$\frac{(36 - 3 \cdot 6)}{4}$
8	
5	
7.5	

6. Ricardo wants an equation to help him more efficiently calculate the height, h, given any frame width, w.

a Which equation would be most helpful for Ricardo? Circle one.

Equation 1: $h = \frac{36 - 3w}{4}$ Equation 2: $w = \frac{36 - 4h}{3}$

b Use the equation you selected to find the height of the window frame if the width is 9 feet.

Name: ..... Date: ..... Period: .....

Independent and Dependent Variables

7. Zwena also works at Ricardo's company.

Zwena is using the equation $w = \frac{36 - 4h}{3}$.

Which question can be answered most efficiently using her equation?

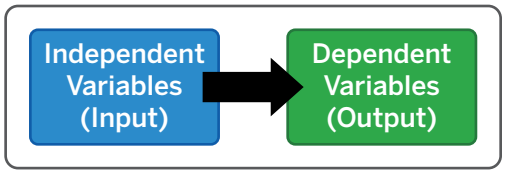
A. If the width of a window frame is 5.5 feet and the height is 3 feet, how much wood trim would you need to build the frame?

B. What is the width of a window frame if the height is 5.5 feet?

C. What is the height of a window frame that is 5.5 feet wide?

Explain your thinking.

8. We can describe the same situation using what we call **independent** and **dependent variables**.



The independent variable is an input that we can choose. The dependent variable, or output, depends on the input.

For situations where Zwena uses the equation $w = \frac{36 - 4h}{3}$, which variable do you think is the dependent variable? Circle one.

Frame width, w Frame height, h I'm not sure

Explain your thinking.

Name: ..... Date: ..... Period: .....

Independent and Dependent Variables (continued)

9. Zwena wants to know the width of a window frame if the height is 5.5 feet.

Circle *all* the statements that are true.

(A.) The frame height, h, is the independent variable.

(B.) The frame width, w, is the independent variable.

(C.) The frame height, h, is the dependent variable.

(D.) The frame width, w, is the dependent variable.

(E.) The frame width, w, is a function of the height, h.

10. Match the equations and sets of variables to each question.

A. Equation: $h = 2c + 10$

B. Equation: $c = \frac{h - 10}{2}$

C. Independent variable: height of stack
Dependent variable: number of cups

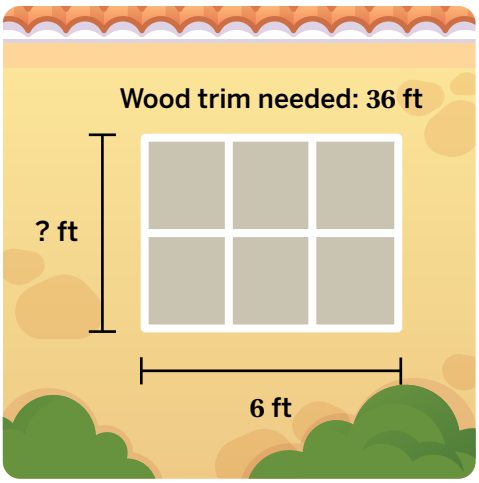
D. Independent variable: number of cups
Dependent variable: height of stack

How many cups, c, do you need to stack to reach a basketball player with height h?	What is the height, h, of a stack of c cups?

Synthesis

11. How can you determine when a variable is independent or dependent?

Use the example if it helps with your thinking.



Lesson Practice ACC7.6.04

Lesson Summary

In a situation represented by a function, the input is often called the **independent variable** and the output is called the **dependent variable**. The independent variable and dependent variable can change depending on the problem you are trying to solve.

The value of an independent variable is not based on the value of any other variable. The value of a dependent variable is based on the value of another variable or set of variables.

Lesson Practice
ACC7.6.04

Name: ..... Date: ..... Period: .....

Problems 1–2: Write an equation that expresses the output as a function of the input. Then determine the independent and dependent variables.

1. The perimeter, p, of a square with side length s:

Equation: .....

Independent variable: .....

Dependent variable: .....

2. The total cost, c, after a sales tax of 7% is applied to the cost of a purchase, p.

Equation: .....

Independent variable: .....

Dependent variable: .....

Problems 3–6: Rafael earns $\$10.50$ per hour helping his neighbor with chores.

3. Is the amount he earns a function of the number of hours he works? Explain your thinking.

4. Is the number of hours he works a function of the amount he earns? Explain your thinking.

5. Write an equation that describes the situation. Use x to represent the independent variable and y to represent the dependent variable.

6. How much will Rafael earn if he works 3 hours each weekday next week? Show or explain your thinking.

Name: _____ Date: _____ Period: _____

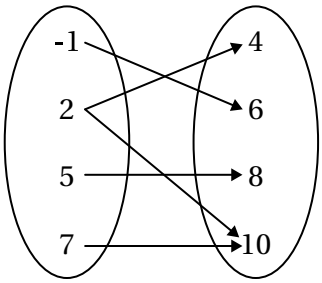
FAST Practice

7. Which of these could represent a function? Select *all* that apply.

☐ **A.** $y = \frac{2}{3}x - 5$

☐ **B.** $\{(1, 8), (2, 6), (4, 4), (2, 7), (5, 9)\}$

☐ **C.**



☐ **D.**

x	y
-1	7
-3	7
-2	7
-1	7

Spiral Review

Problems 8–9: Solve each system of equations. Show your thinking.

8. $\begin{cases} y = 7x + 10 \\ y = -4x - 23 \end{cases}$

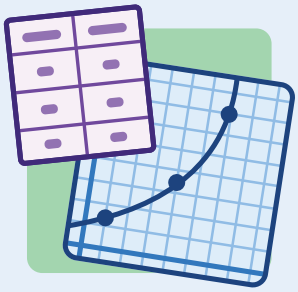
9. $\begin{cases} y = 3x - 6 \\ y = -2x - 1 \end{cases}$

Sub-unit

2

Representing and Interpreting Functions

Lesson 5
The Tortoise and the Hare



Lesson 6
Elevation Examination

Lesson 7
Comparing Linear Functions

Name: ..... Date: ..... Period: .....

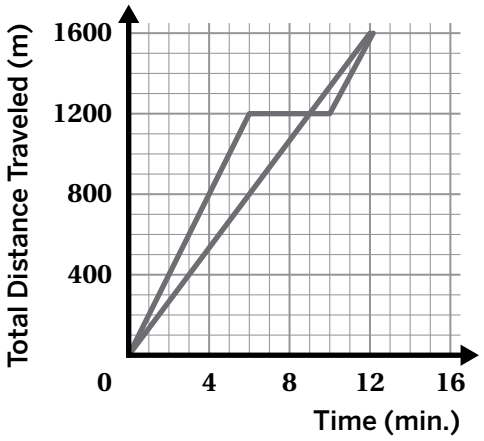
MA.8.F.1.3, MA.8.AR.3.4, MA.8.AR.3.5, MTR.4.1

The Tortoise and the Hare

Let's interpret the graph of a function in context.

Warm-Up

1. Think about the different ways the race between the tortoise and the hare could turn out based on changes in their speeds. This graph shows two functions representing the relationship between distance and time: one for the tortoise and one for the hare.



a Which animal does each function represent? Label the graph with "Tortoise" and "Hare".

b Explain your thinking.

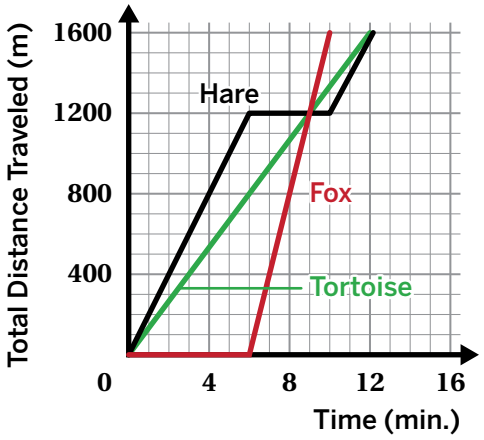
Name: ..... Date: ..... Period: .....

The Tortoise, the Hare, and the Fox

2. This graph shows the relationship between distance and time for a third animal, the fox.

Tell a story about the fox's journey during the race.

Include specific details about time and distance.



3. Share your story with a partner. How are your two stories alike? How are they different?

4. Here are five statements about the race. Select *all* the true statements.

(A.) The fox's distance was always increasing.

(B.) Between 6 and 10 minutes, the fox was traveling faster than the other animals.

(C.) When the hare reached 800 meters, the fox was still at the starting line.

(D.) The graph of the tortoise represents a function, but the other two graphs do not represent functions.

(E.) All three graphs represent functions.

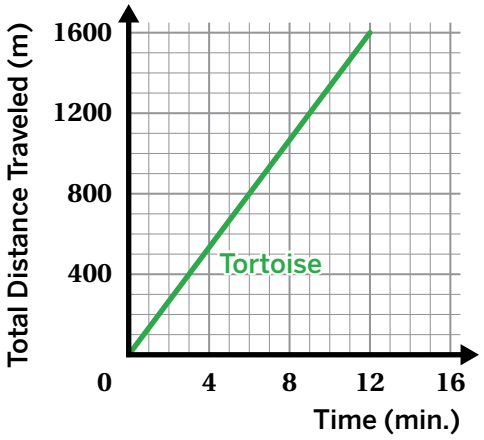
Name: ..... Date: ..... Period: .....

The Tortoise and the Dog

5. Next, the tortoise raced a dog.

Draw a graph representing the relationship between distance and time for a dog that makes *all* of these statements true:

- The dog got a head start but lost the race.
- The dog and tortoise were tied at 800 meters.
- The dog's distance was decreasing for 3 minutes.



6. Share your graph with a different partner. How are your two graphs alike? How are they different?

7. The blue graph shows what Brielle drew to represent the dog's race.

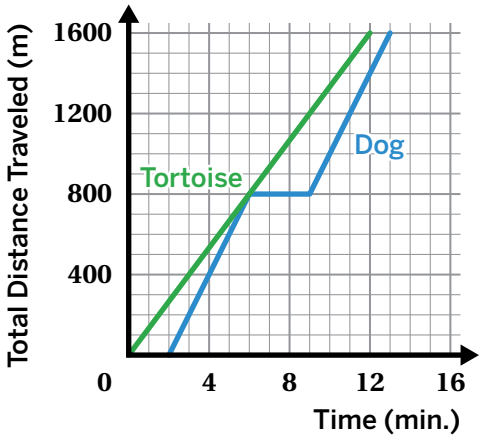
At least one of the following statements is false. Circle a false statement.

(A.) The dog got a head start but lost the race.

(B.) The dog and tortoise were tied at 800 meters.

(C.) The dog's distance was decreasing for 3 minutes.

Explain your thinking.

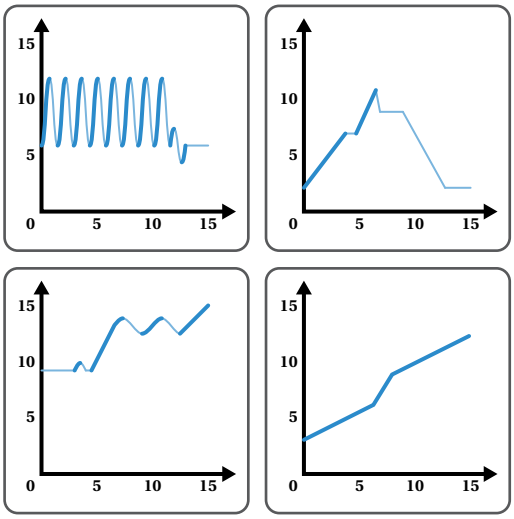


Name: ..... Date: ..... Period: .....

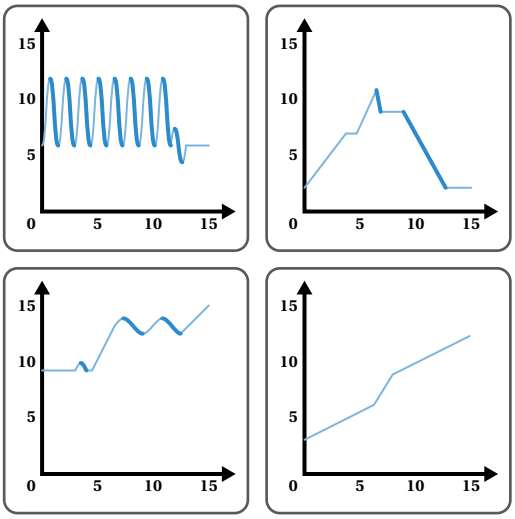
Describing Graphs

8. Here are some graphs. Parts of the graphs are bolded to show where they are either *increasing*, *decreasing*, or *constant*.

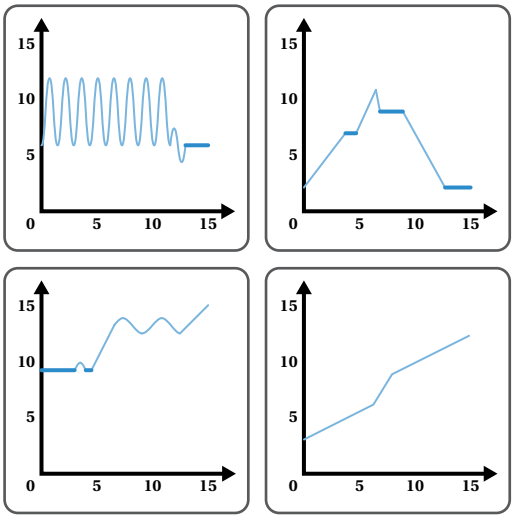
Increasing



Decreasing



Constant

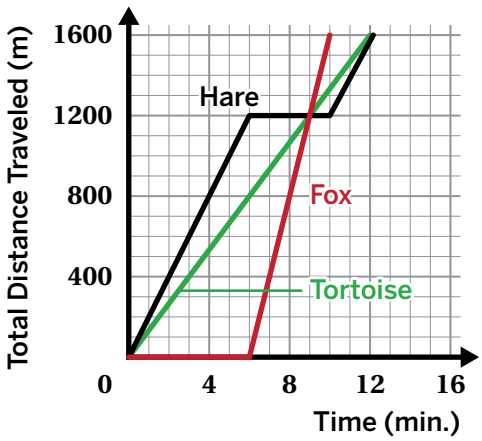


Discuss: What do you think each term means?

Synthesis

9. What can a function's graph tell us about a situation?

Use the example if it helps with your thinking.

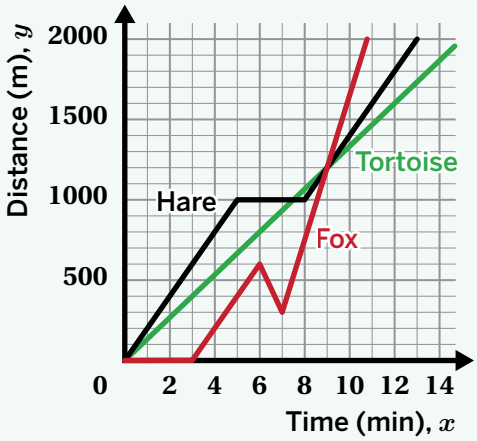


Lesson Practice ACC7.6.05

Lesson Summary

A graph can be helpful when comparing multiple functions in a situation, such as by comparing the initial value, slope, and points of intersection.

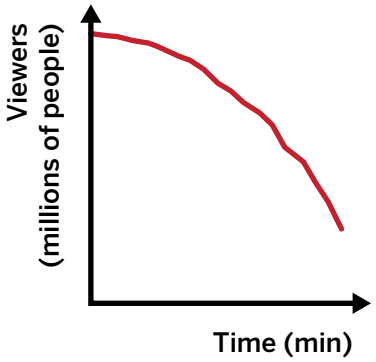
For example, this graph represents a race between a hare, a tortoise, and a fox. From 0 to 5 minutes, the hare is moving at a steady pace of 200 meters per minute and is in first place. The graph is *increasing* on this interval. From 5 to 8 minutes, the Hare does not move at all and the graph is *constant*. At 9 minutes, the race is tied. The fox does not begin the race until three minutes have passed, so its function is constant from 0 to 3 minutes. From 6 to 7 minutes, the function is *decreasing*, which indicates that the fox is moving in the wrong direction! Then, at 7 minutes, it speeds up to a pace of 450 meters per minute. The fox wins the race at about 11 minutes.



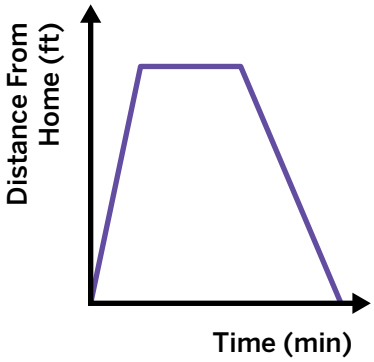
Name: ..... Date: ..... Period: .....

Problems 1–2: For each graph, write a story that the graph tells about each situation.

1. The relationship between number of viewers of a short video and time.



2. The distance of a cat from its home as a function of time.



Problems 3–8: Elena filled up the tub and gave her dog a bath. Then she let the water in the tub drain. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes.

3. Label the axes.

4. When did Elena turn off the water faucet?

5. How much water was in the tub when she bathed her dog?

6. On what interval is the function decreasing?

7. At what rate did the faucet fill the tub?

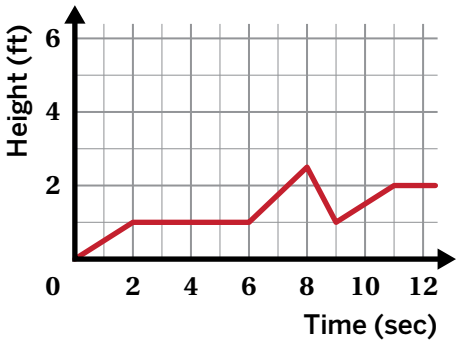
8. At what rate did the water drain from the tub?

Lesson Practice
ACC7.6.05

Name: ..... Date: ..... Period: .....

FAST Practice

9. This graph shows the height, in feet, that a butterfly is flying above the ground over time. At what interval is the function constant? Select *all* that apply.



- ☐ **A.** Between 0 and 2 seconds
- ☐ **B.** Between 2 and 6 seconds
- ☐ **C.** Between 6 and 8 seconds
- ☐ **D.** Between 9 and 11 seconds
- ☐ **E.** Between 11 and 12 seconds

Spiral Review

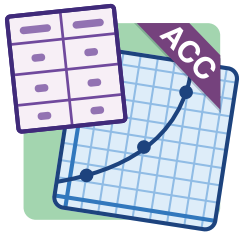
10. A car is traveling at a speed of either 55 miles per hour or 35 miles per hour, depending on the speed limits, until it reaches its destination 200 miles away. Let x represent the amount of time, in hours, that the car is traveling at 55 miles per hour. Let y represent the amount of time, in hours, that the car is traveling at 35 miles per hour. The equation $55x + 35y = 200$ represents this situation.

If the car spends 2.5 hours traveling at 35 miles per hour on the trip, how long does it spend traveling at 55 miles per hour? Show or explain your thinking.

Name: ..... Date: ..... Period: .....

MA.8.F.1.1, MA.8.F.1.2, MA.8.AR.3.1, MTR.2.1, MTR.4.1

Elevation Examination



Let's analyze whether a function is linear or nonlinear.

Warm-Up

1. The tables show the time, in seconds, that it takes for each elevator to go from the first floor to another floor without stopping.

Elevator A

Floor	Time (sec)
1	0
2	8
3	16
4	24

Elevator B

Floor	Time (sec)
1	0
2	8
3	12
4	14

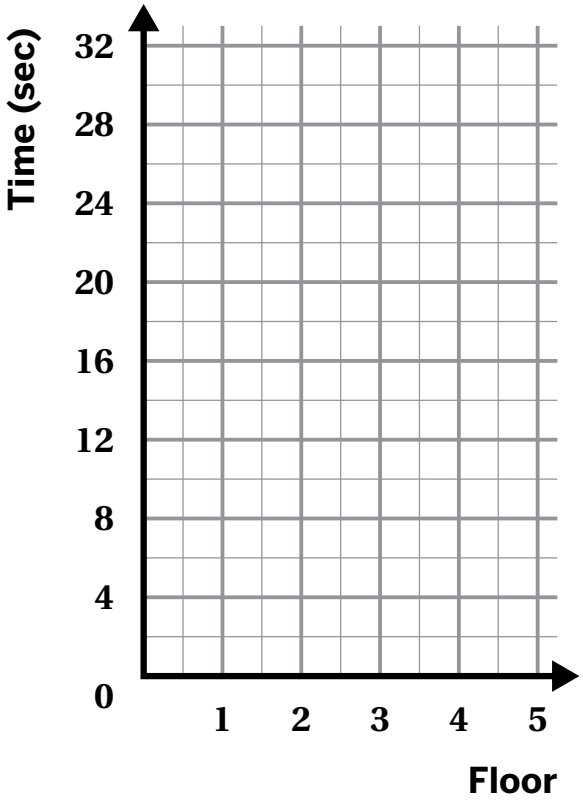
Examine the data and write a story about these elevators.

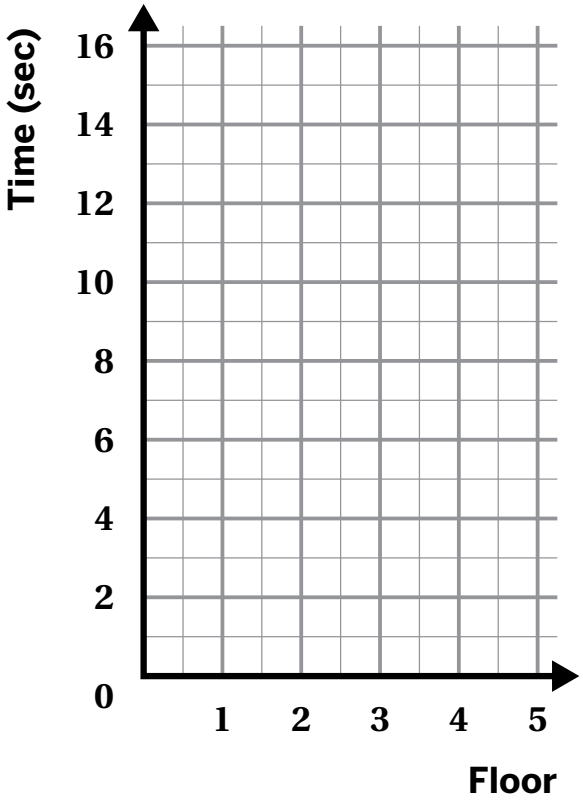
Name: Date: Period:

More Elevator Analysis

A **linear function** has a constant rate of change, and its graph is a straight line or points that form a line. A **nonlinear function** has a graph that is not a straight line.

2. Graph the input-output pairs for each elevator from the tables in the Warm-Up.





3. Analyze each graph and determine if the relationship between floor and time is a function. If so, determine what type of function. Explain your thinking.

4. Juan believes that the number of seconds, y, it takes for Elevator A to get to a floor, x, can be modeled by the equation $y = 8x - 8$, and Keya believes this equation should be $y = x + 8$. Whom do you agree with? Explain your thinking.

Activity 2

Name: Date: Period:

Linear and Nonlinear Functions

The Activity 2 Cards contain 12 relations shown in tables, equations, and graphs.

5. **a** Sort each relation.

Linear Function	Nonlinear Function	Not a Function

b Choose one relation from each category and justify how you sorted it.

c **Discuss:** How can you tell if a table represents a linear function? How can you tell if an equation represents a linear function?

6. **a** Which function(s) would be classified as a proportion?

b Raven says that all linear relationships are proportional. Nikhil says all proportional relationships are linear. Whom do you agree with? Circle one.

Raven Nikhil Both Neither

Show or explain your thinking.

Name: ..... Date: ..... Period: .....

Linear and Nonlinear Functions (continued)

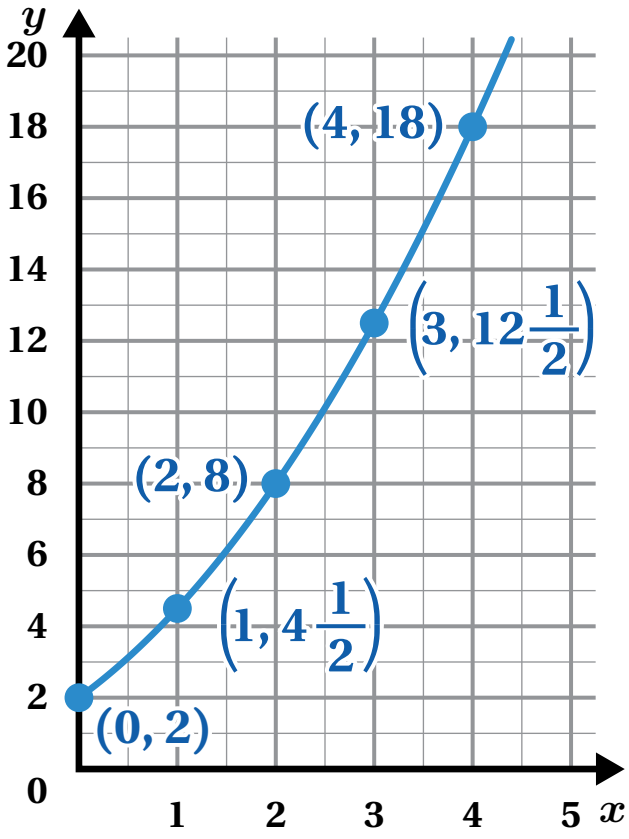
7. Santino and Tyrone are analyzing the function defined by the equation $y = \frac{1}{2}(x + 2)^2$ to determine whether it is linear or nonlinear. Here is each student's method.

Santino

x	y	Rate of Change
0	2	$m = \frac{8 - 2}{2 - 0} = \frac{6}{2}$
2	8	$m = \frac{3}{1}$
4	18	$m = \frac{32 - 18}{6 - 4} = \frac{14}{2}$
6	32	$m = \frac{7}{1}$

Since the rate of change is not constant, this equation must be a nonlinear function.

Tyrone



Since the graph is not a line, it is a nonlinear function.

Discuss: How are Santino's and Tyrone's methods the same? How are they different?

Synthesis

8. How can you determine if an equation, table, or graph represents a linear function? Use the examples if it helps with your thinking.

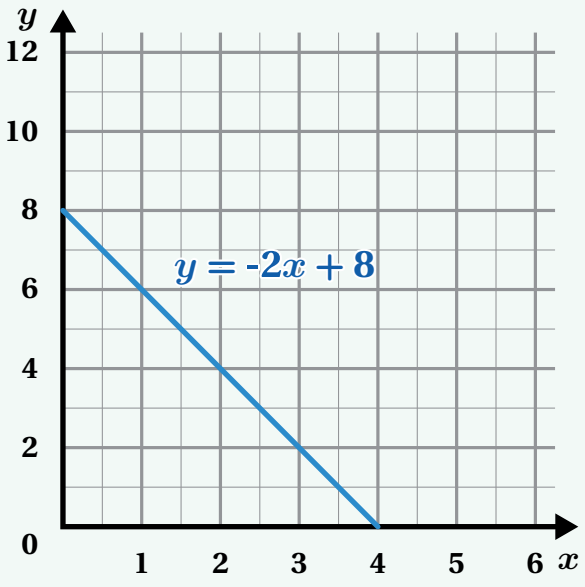
$y = \frac{1}{2}x - 2$

x	0	4	8
y	-2	0	2

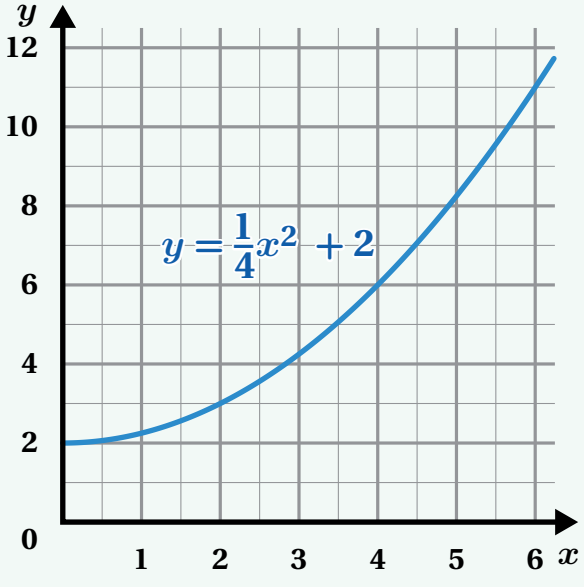
Lesson Practice ACC7.6.06

Lesson Summary

A **linear function** has a constant rate of change, and its graph is a straight line. Its equation can be written in the form $y = mx + b$.



A **nonlinear function** does not have a constant rate of change, and its graph is not a straight line.



Lesson Practice
ACC7.6.06

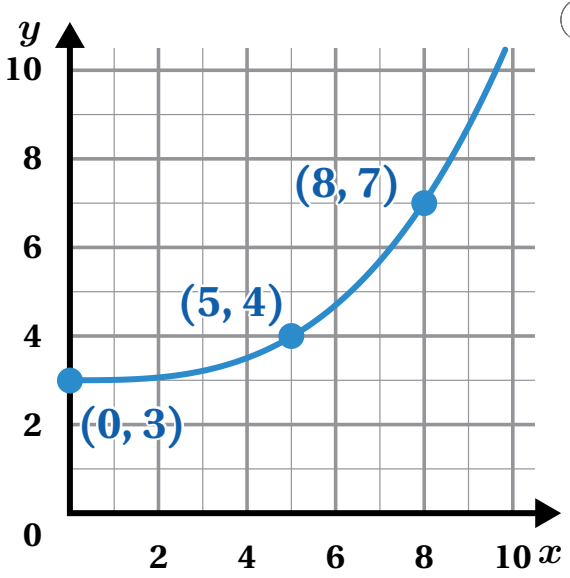
Name: _____ Date: _____ Period: _____

1. Which represents a linear function?

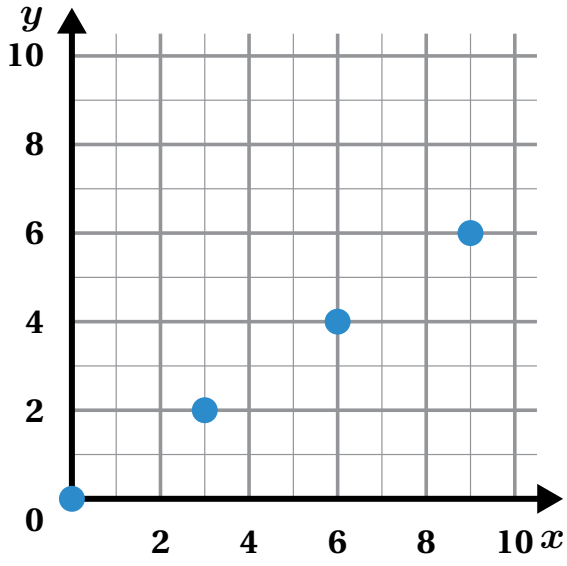
A. $y = x(x + 10)$

B. $y = 2x^2 - 1$

C.



D.



2. Determine whether each table of values could represent a linear function or a nonlinear function. Circle the correct answer for each table.

Linear Nonlinear

x	y
5	3
7	9
12	15
15	23

Linear Nonlinear

x	y
2	16
6	12
8	10
9	9

Linear Nonlinear

x	y
1	0
3	0
9	0
12	0

Name: _____ Date: _____ Period: _____

3. Which statements about the relation $4x = y$ are true? Select *all* that apply.

- ☐ **A.** It is a linear function.
- ☐ **B.** It is a nonlinear function.
- ☐ **C.** It is not a function.
- ☐ **D.** It has a constant rate of change of 4.
- ☐ **E.** Its graph is a line.

Problems 4–7: Determine whether each statement is *always*, *sometimes*, or *never* true.

4. Proportional relationships are linear functions.

5. The graph of a vertical line is a linear function.

6. Nonlinear functions have graphs that are curves.

7. Linear functions have the same rate of change between any pair of input-output values.

FAST Practice

8. A function includes the points shown in the table. Which of the following statements about the function are true? Select *all* that apply.

x	y
1	2
2	7
6	27
7	32

- ☐ **A.** The function is linear.
- ☐ **B.** The function is nonlinear.
- ☐ **C.** The rate of change shown in the table is $\frac{1}{5}$.
- ☐ **D.** The rate of change shown in the table is $\frac{5}{1}$.
- ☐ **E.** There is not a constant rate of change.

Spiral Review

Problems 9–10: Solve each equation. Show your thinking.

9. $-(-2x + 1) = 9 - 14x$

10. $3x + \frac{3}{5} = \frac{1}{3}(5x + 5)$

Name: ..... Date: ..... Period: .....

MA.8.F.1.2, MA.8.F.1.3, MA.8.AR.3.5, MTR.2.1, MTR.4.1

Comparing Linear Functions

Let's compare linear functions represented in different ways.

Warm-Up

Jada has $50 in her savings account and saves $7 per week.

1. What could the independent variable and dependent variable be in this situation? Explain your thinking.

2. Write an equation representing this situation. Be sure to define the variables that you use.

3. Is your relationship from Problem 2 a function? Explain your thinking.

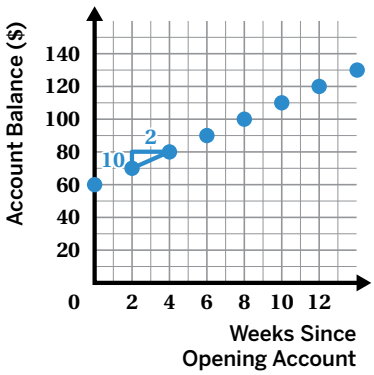
Activity 1

Name: Date: Period:

Which Is Growing Faster?

Noah and Elena both opened a savings account on the same day. Each person's account balance is a function of the weeks since the account was opened. Here is some information about each account.

Noah's Account



Elena's Account

$a = 8w + 60$, where w is the number of weeks since the account was opened, and a is the account balance

4. Who started with more money in their account? Circle one.

Noah Elena They started with the same amount.

Explain your thinking.

5. Who is saving money at a faster rate? Circle one.

Noah Elena They are saving at the same rate.

Explain your thinking.

6. How much will Noah save over the course of a year if he does not make any withdrawals?

(Note: There are 52 weeks in a year.) Show or explain your thinking.

7. How long will it take Elena to save the same amount? Show or explain your thinking.

Activity 2

Name: Date: Period:

Making Deposits vs. Withdrawals

Take a look at the accounts of four customers. They each have an account balance, a, measured over w weeks.

Account A

The account balance, a, is represented by the function $a = 65 + 10w$, where w represents the number of weeks since the account was opened.

Account B

The account balance, a, starts at $\$40$ and decreases by $\$8.50$ per week.

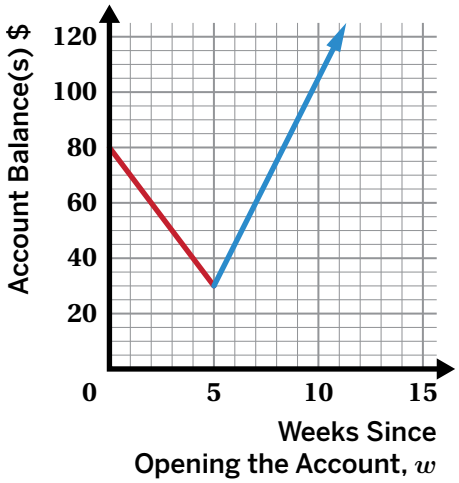
Account C

The account balance is represented by this table:

Number of Weeks	Account Balance ($)
1	71
3	48
7	2

Account D

The account balance is represented by this graph:



8. Discuss:

a. Which account(s) show customers making deposits? Explain your thinking.

b. Which account(s) show customers making withdrawals? Explain your thinking.

Name: Date: Period:

Making Deposits vs. Withdrawals (continued)

9. Kiana says that all four relationships are linear functions and that each situation can be represented with one equation in the form $y = mx + b$. Is Kiana's claim correct? Explain your thinking.

10. Which account will have the most money at the end of a year? Explain your thinking.

11. **a** How much money did Account A have after one year?

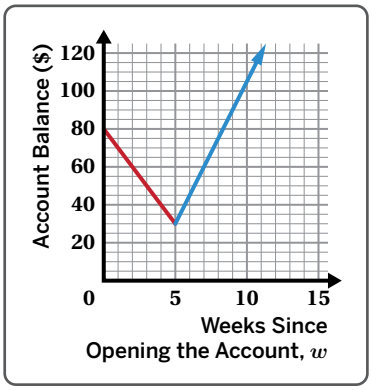
b How long did it take Account D to have the same amount?

Synthesis

12. How can you compare linear functions shown using different representations?

Number of weeks	Account balance ($)
1	71
3	48
7	2

$a = 65 + 10w$



Lesson Practice ACC7.6.07

Lesson Summary

Linear relationships that have exactly one output for every possible input are called linear functions. All linear functions can be represented with an equation in the form $y = mx + b$, where m is the rate of change and b is the initial value.

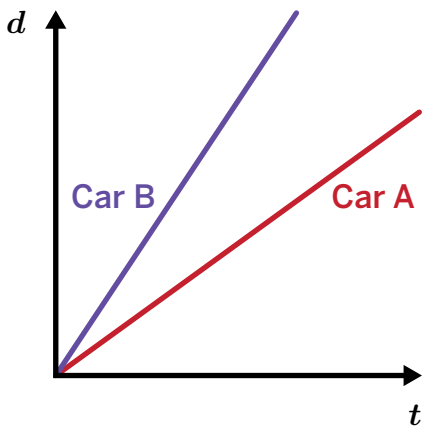
In a graph, you can find the rate of change using slope triangles to calculate the vertical and horizontal change between two points on the coordinate plane. You can find the initial value by locating the y-intercept.

In a table, you can find the rate of change by determining the ratio of the difference between the y-values and the difference between the x-values. You can find the initial value by determining the dependent value when the independent value is 0.

In an equation, you can find the rate of change by determining the coefficient of the independent variable. You can find the initial value by determining the constant, b, in the equation $y = mx + b$.

Name: ..... Date: ..... Period: .....

1. Two cars are traveling on the same highway in the same direction. The graphs show the distance, d, of each car from the starting point as a function of time, t. Which car is traveling faster? Explain your thinking.



Problems 2–4: Kiri and Chloe race each other home from school. They run at the same speed, but Kiri's house is slightly closer to the school than Chloe's house. Suppose there is a graph that shows their distances from home, in meters, as a function of the time, in seconds, from when they began the race.

2. If you were to read the graphs from left to right, would you expect the lines to increase or decrease? Explain your thinking.

3. How would you expect the lines representing Kiri's run and Chloe's run to be different? Explain your thinking.

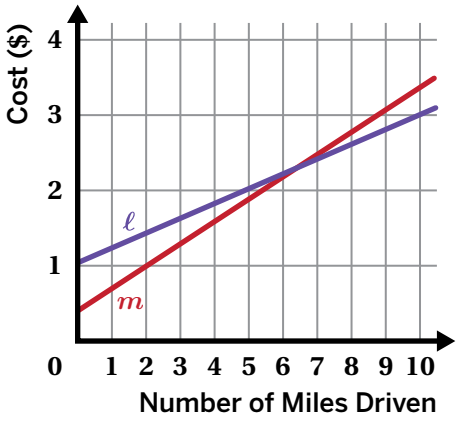
4. How would you expect the lines representing Kiri's run and Chloe's run to be alike? Explain your thinking.

Lesson Practice
ACC7.6.07

Name: _____ Date: _____ Period: _____

FAST Practice

5. Two car services offer to pick up a customer and take them to their destination. Service A charges a flat fee of $\$0.40$ plus $\$0.30$ for each mile of the trip. Service B charges a flat fee of $\$1.10$ plus c dollars for each mile of their trip.

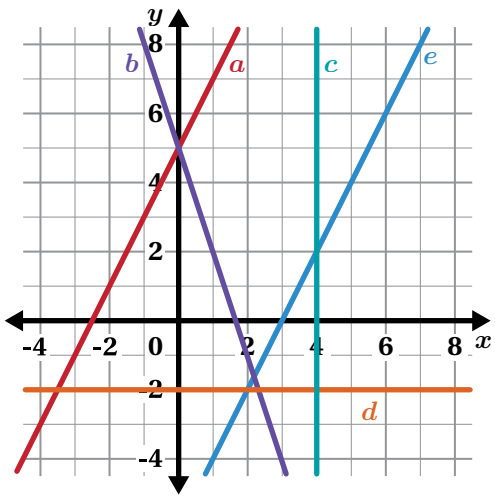


Match the services with the lines l and m. Record your answers in the spaces provided.

Line l represents Service ____. Line m represents Service ____.

Spiral Review

6. Write an equation for each line shown on the graph.



Line a:

Line b:

Line c:

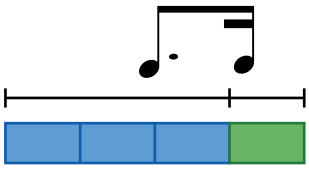
Line d:

Line e:

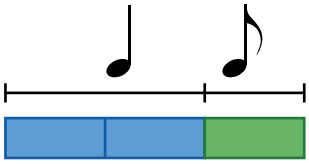
Career Connection

How might ratios alter the mood of a song?

Musicians perform and create musical pieces for people to listen to and enjoy. They use ratios and proportional relationships to create sounds that when played together, are pleasing to the ear.



Ratio of 3 : 4



Ratio of 2 : 3

B.E.S.T. Mathematics Benchmark Connection

Believe it or not, musicians apply math extensively in their work. For example, the combination of notes that are played depend on proportional relationships (MA.7.AR.3.2, MA.7.AR.4.1) to create a certain sound or to choose how fast or slow to play a part of a musical piece. Musicians who write and play music also apply their understanding of ratios and proportions (MA.7.GR.4.2, MA.7.GR.4.4) as they determine which instruments are played during certain parts of a piece of music. How the music is played often sets the tone for a piece of music, even when there are no words being sung.

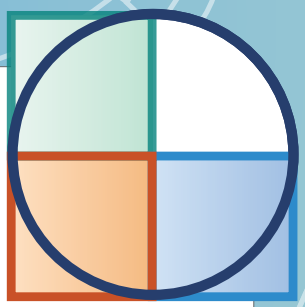
Mathematical Thinking and Reasoning Connection

People who are involved with music use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work with people they collaborate with, such as the music arranger, the conductor of the orchestra, or fellow musicians in a band (MTR.1.1). When musicians study arrangements for performances such as a concert, they learn to complete the tasks of organizing and displaying the sheet music, attachments for their instruments, and other details with fluency (MTR.3.1).

Meet Eugenia Cheng

Eugenia Cheng, a mathematician and concert pianist, has helped to explain how math and music are connected. Both can involve patterns and ratios. The duration of the notes in a sound creates a rhythmic pattern based on ratios. Different rhythmic patterns can set the mood for a song. According to Eugenia Cheng's research, a rhythmic pattern where the first note takes up $\frac{3}{4}$ of the beat tends to sound more lively, while a rhythmic pattern where the first note takes up $\frac{2}{3}$ of the beat might sound more relaxed.

Paul Crisanti



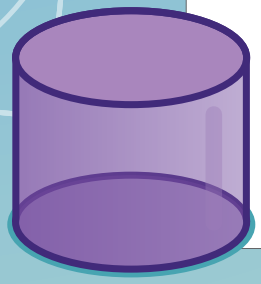
Unit 7

Circles and Cylinders

In this unit, you will determine formulas for the circumference and area of a circle. Then you will explore volume and surface area of a cylinder.

Essential Questions

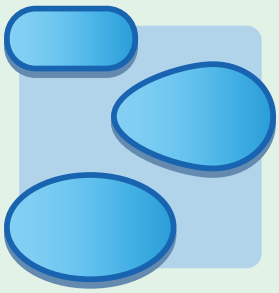
- How do we measure circles when all of our tools are straight?
- What is π and what does it have to do with circles?
- Where do you see volume and surface area of cylinders used in the real world?



Sub-unit

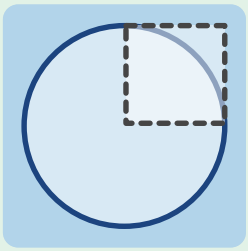
1

Circles

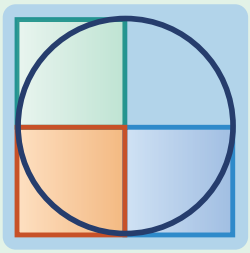


Lesson 1
Is It a Circle?

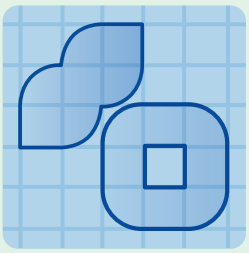
Lesson 2
Measuring Around



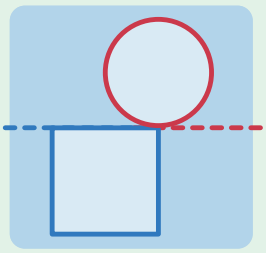
Lesson 3
Radius Squares



Lesson 4
Circle Area



Lesson 5
Area Challenges



Lesson 6
Circle vs. Square

Name: ..... Date: ..... Period: .....

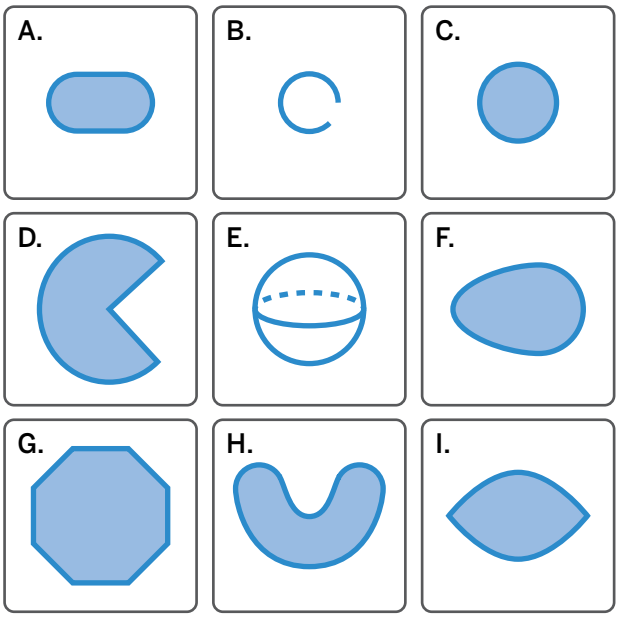
MA.7.GR.1.3, MTR.4.1

Is It a Circle?

Let's explore what makes a circle a circle.

Warm-Up

1. Circle the shapes you would describe as round.

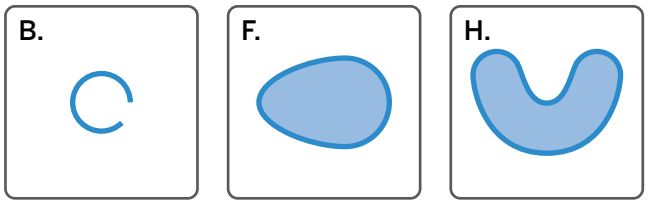


Discuss: Which shape(s) would you call a *circle*? Why?

Name: ..... Date: ..... Period: .....

Circle Sizes

2. Here are some shapes students said were round.

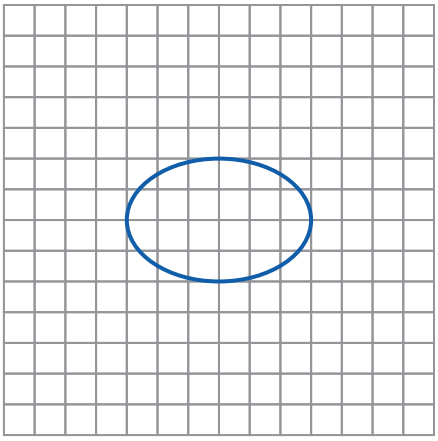


Choose one that isn't a circle.

Explain your thinking.

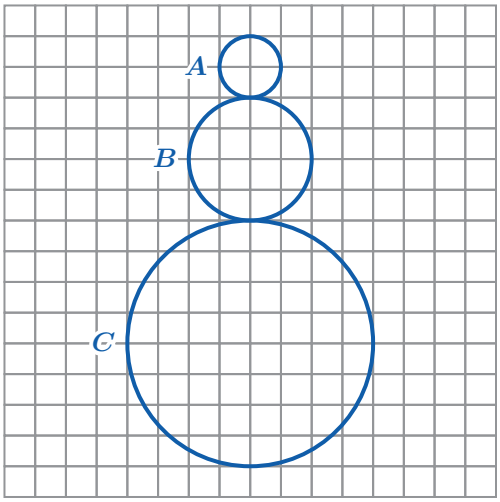
Shape _____ is not a circle because . . .

3. What is one reason someone might say this figure is not a circle?



Draw if it helps to show your thinking.

4. Adhira made a drawing using circles.



She said: *One of the circles measures 4 units.*

Which circle do you think Adhira is describing? Circle one.

Circle A Circle B Circle C

Explain your thinking.

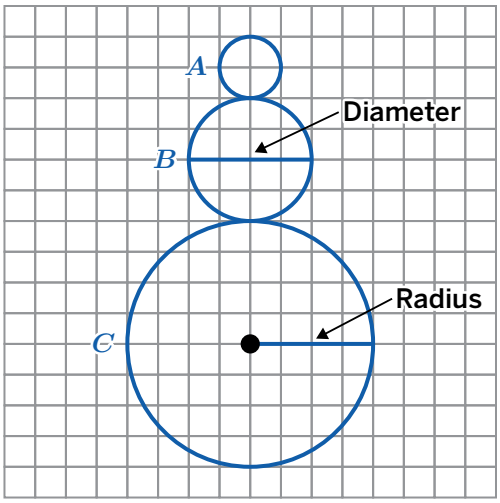
Name: Date: Period:

Circle Sizes (continued)

5. We can measure circles using **radius** or **diameter**.

a Record the radius and diameter of each circle in the table. Some are done for you.

Circle	Radius (units)	Diameter (units)
A		
B		4
C	4	

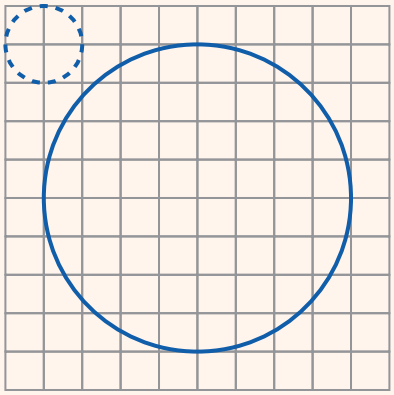


b **Discuss:** How is the diameter of a circle related to its radius?

You're invited to explore more.

6. How many full circles with a radius of 1 unit can you fit inside a circle of radius 4 units so that they don't overlap?

Sketch on the diagram if that helps with your thinking.

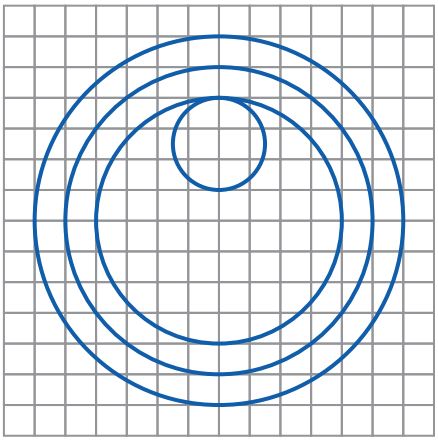


Name: ..... Date: ..... Period: .....

Circle Centers

7. Madison made a drawing using circles. Describe her drawing as precisely as you can so that someone who can't see her drawing could recreate it.

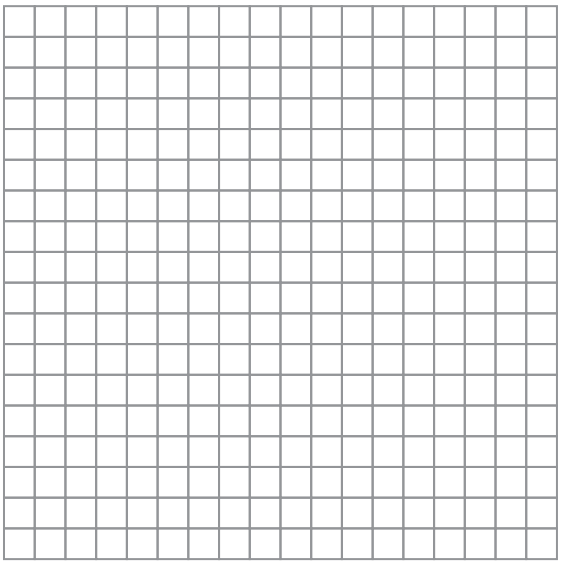
Draw if it helps with your thinking.



8. Callen also made a drawing using circles. He wrote the following description:

- My drawing has 4 circles.
- All the circles touch at a single point.
- Each circle is a different size.
- The largest circle has a radius of 5 units.
- The smallest circle has a diameter of 2 units.

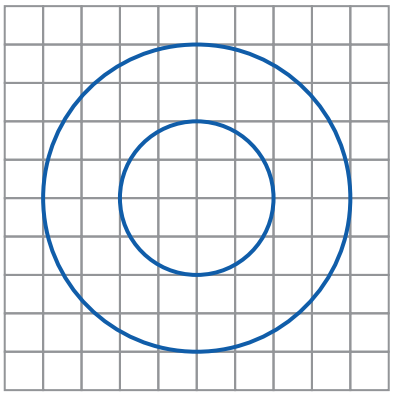
Draw circles on the grid or use the circles from the Activity 2 Sheet to recreate Callen's design.



Synthesis

9. Describe the drawing as precisely as you can so that someone who can't see it could recreate it.

Include the terms you used today in your description.



Lesson Practice ACC7.7.01

Lesson Summary

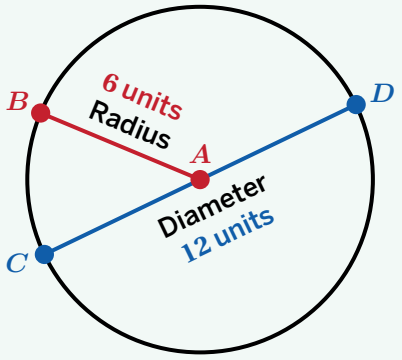
A *circle* is a shape made out of all the points that are the same distance from a center point.

You can measure a circle with its **radius**. A radius is a line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

You can also measure a circle with its **diameter**, which is the distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

For any circle, the length of the diameter is two times the length of the radius.

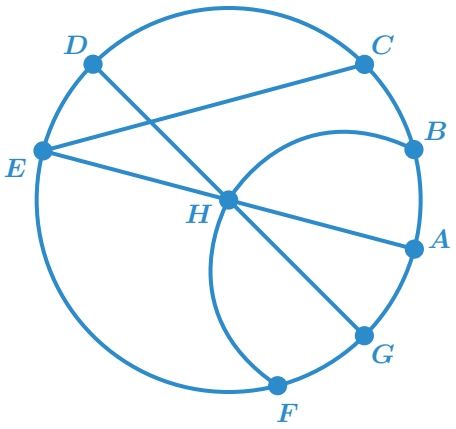
For example, here is a circle. The radius AB is 6 units. The diameter CD is 12 units.



Lesson Practice
ACC7.7.01

Name: ..... Date: ..... Period: .....

Problems 1–3: Here is a circle. Point H is the circle's center.



1. Trace a segment that is a radius.

2. Trace a segment that is a diameter.

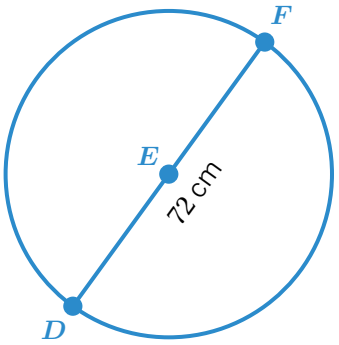
3. Trace a segment that is neither a radius nor a diameter.

4. A circle has a radius of 4.5 units. What is its diameter?

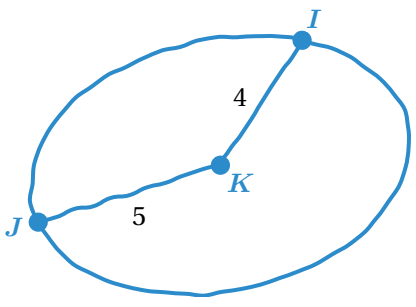
5. Can a circle with a radius of 5.2 inches have a diameter of 13 inches?

Explain your thinking.

6. What is the length of the radius in the circle?



7. Kala's teacher asked her to create a circle and draw a diameter. Here is her drawing. Identify and describe her mistakes.



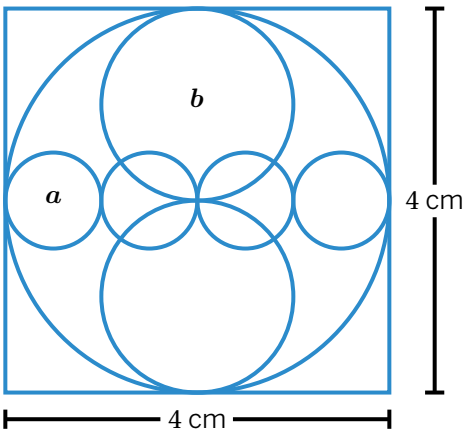
Lesson Practice
ACC7.7.01

Name: _____ Date: _____ Period: _____

FAST Practice

8. This square contains several circles. Determine the radius and diameter of circles a and b.

Circle	Radius (cm)	Diameter (cm)
a		
b		



Spiral Review

9. A lemonade recipe calls for $\frac{1}{4}$ cups of lemon juice, $\frac{1}{2}$ cups of sugar, and 1 cup of water. Complete the table to show the amount of lemon juice for different-sized batches.

Sugar (cups)	Lemon Juice (cups)
2	
10	
5	

10. The graph of a proportional relationship includes the point $(3, 12)$.

What is one constant of proportionality for this relationship? Show or explain your thinking.

Name: ..... Date: ..... Period: .....

MA.7.GR.1.3, MTR.1.1, MTR.4.1, MTR.6.1

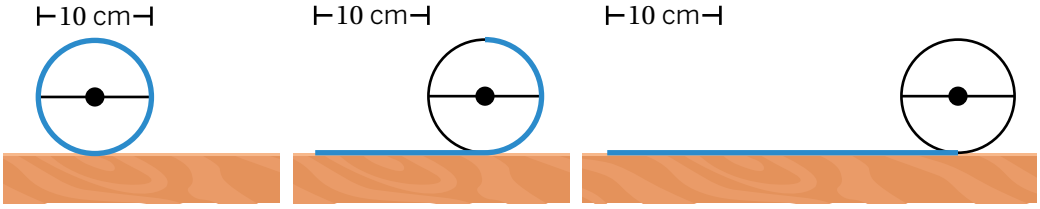
Measuring Around

Let's explore the relationship between the diameter of a circle and its circumference.

Warm-Up

1. The **circumference** of a circle is the distance all the way around it.

a Imagine unrolling the circumference to show it as a line segment.



b Estimate the length of the circumference.

Activity 1

Name: Date: Period:

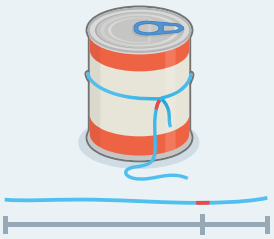
So Many Circles

2. Measure the diameter and the circumference of at least three circular objects. Record your results in the table.

Object	Diameter (cm)	Circumference (cm)
Toilet paper tube	4.1	12.9

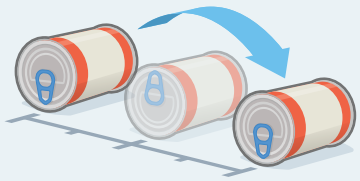
Three Ways to Measure the Circumference

String

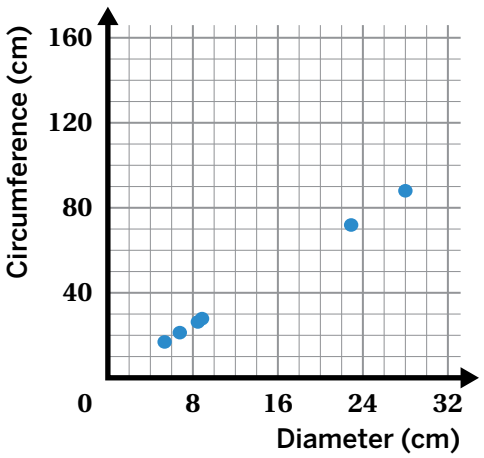


Tape Measure

Roll



3. This graph shows the diameter and circumference of several circular objects.



Do you think the relationship between circumference and diameter is proportional? Circle one.

Yes No I'm not sure

Explain your thinking.

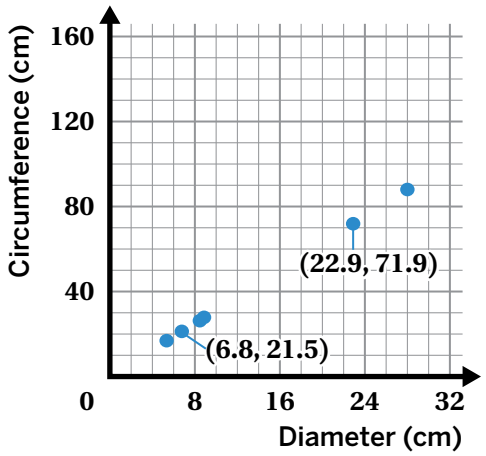
Name: _____ Date: _____ Period: _____

Introducing π

4. The relationship between circumference, C, and diameter, d, is proportional.

Estimate the *constant of proportionality* that completes the equation relating circumference and diameter.

$C =$ _____ d

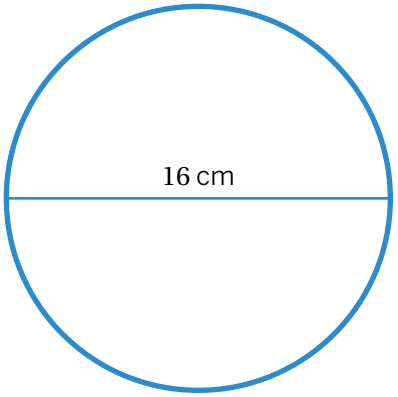


5. The constant of proportionality between a circle's diameter, d, and its circumference, C, is π (written as **pi** and pronounced "pie").

In other words, $C = \pi d$.

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. None of these are exactly π.

Calculate the circumference of a circle with a diameter of 16 centimeters.

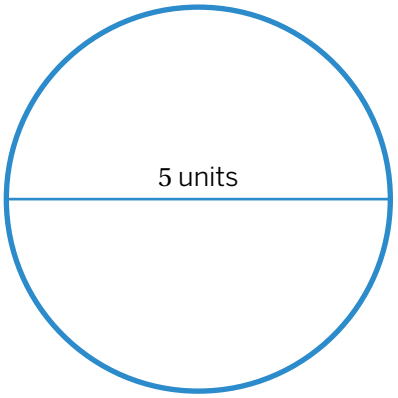


6. This circle has a diameter of 5 units.

Four students tried to calculate its circumference.

Order their answers starting with the farthest from the exact circumference to the closest to the exact circumference.

15.7 units	15 units	5π units	15π units



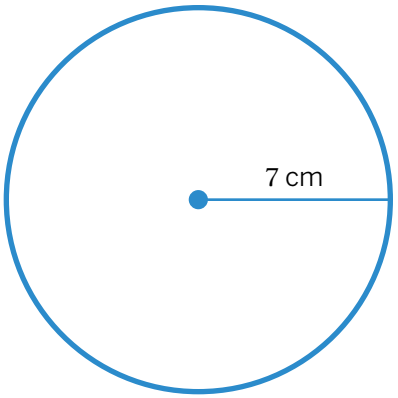
Farthest from exact circumference **Closest to exact circumference**

Name: Date: Period:

Proportionality and π

7. The radius of this circle is 7 centimeters.

What is its circumference?



8. The constant of proportionality in the relationship between diameter and circumference is π.

a Complete the table.

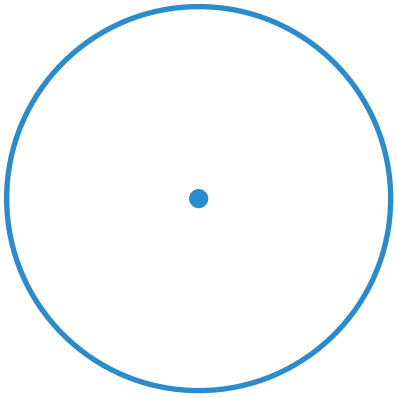
Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Cookie	3	6	
Small plate	9		
Quarter		2.4	
Frisbee			21π
Tennis ball can	$\frac{7}{2}$		
Vinyl record			100

b When you finish, compare your answers with a classmate.

Discuss: How are your answers alike? How are they different?

Synthesis

9. Describe the relationship between radius, diameter, and circumference.



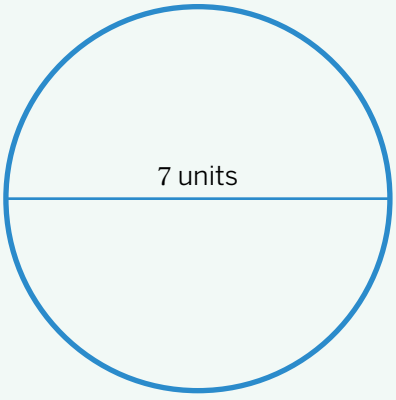
Lesson Practice ACC7.7.02

Lesson Summary

The distance around a circle is called the **circumference**. There is a proportional relationship between the diameter of a circle and its circumference. The *constant of proportionality* for that relationship is π (**pi**). π is often approximated as 3.14 or $\frac{22}{7}$.

For any circle, we can calculate the circumference, C, using the equation $C = \pi d$.

For example, if the diameter of a circle is 7 units, the circumference of the circle can be calculated approximately as $7 \cdot 3.14 = 21.98$. More accurately, $C = 7\pi$ units.



If we know the *radius* of a circle, we can calculate the circumference by first determining the diameter, then using the equation $C = \pi d$.

Lesson Practice
ACC7.7.02

Name: _____ Date: _____ Period: _____

1. Lucy measured the diameter and circumference of several circular objects and recorded her measurements in a table.

Which object's circumference measurement do you think is the least accurate?

Explain your thinking.

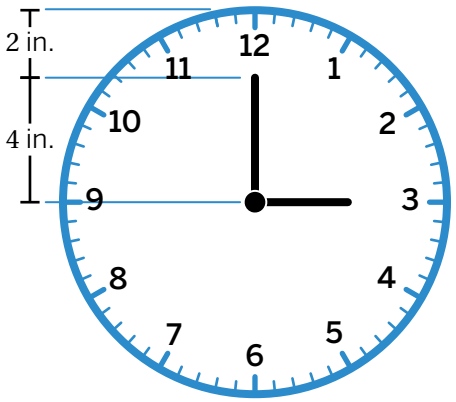
Object	Diameter (cm)	Circumference (cm)
Half dollar coin	3	10
Flying disc	23	50
Jar lid	8	25
Flower pot	15	48

2. Complete the table to determine the missing measurements for each object.

Object	Radius	Diameter	Circumference
Hula hoop		36 inches	
Circular pond			556 feet
Magnifying glass		5.2 centimeters	
Car tire			71.6 inches

3. The circumference of a circle is approximately 50.27 inches. Determine the length of the radius of the circle in inches. Round your answer to the nearest whole number.

Problems 4–5: The minute hand of a circular clock measures 4 inches. The distance from the end of the minute hand to the outer edge of the clock is 2 inches.



4. What is the length of the radius of the clock?

5. What is the circumference of the clock?

Name: ..... Date: ..... Period: .....

Problems 6–7: The size of a drum affects its pitch. When comparing drums, the one with a larger diameter has a lower pitch because the frequency of its sound waves is slower.

6. If a drummer wants a drum with a lower pitch, should she choose a drum with a circumference of 18π inches or 22.5π inches?

7. Determine the diameter of the drum she should choose.

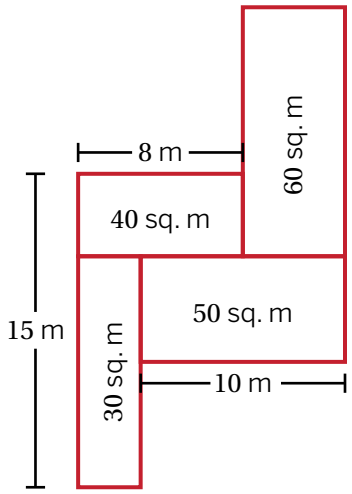
FAST Practice

8. Determine whether the measurement described is the diameter, radius, or circumference.

	Diameter	Radius	Circumference
The tires on a tractor are 4.5 feet tall.	☐	☐	☐
The distance from the tip of a slice of pizza to the crust is 6 inches.	☐	☐	☐
The length of the metal rim around a glass lens is 190 millimeters.	☐	☐	☐

Spiral Review

9. This shape is made up of four rectangles. Determine the total perimeter of the shape.

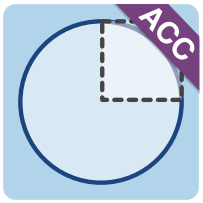


Name: ..... Date: ..... Period: .....

MA.7.GR.1.4, MTR.4.1, MTR.5.1, MTR.6.1

Radius Squares

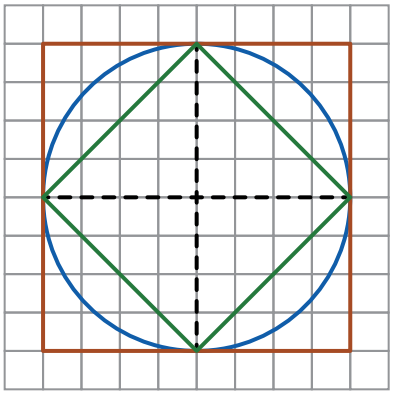
Let's start to explore the relationship between radius squares and circles.



Warm-Up

1. Ayaan used this drawing to estimate the area of a circle. He said the area is between 32 and 64 square units.

Where do you think the 32 comes from? What about the 64?



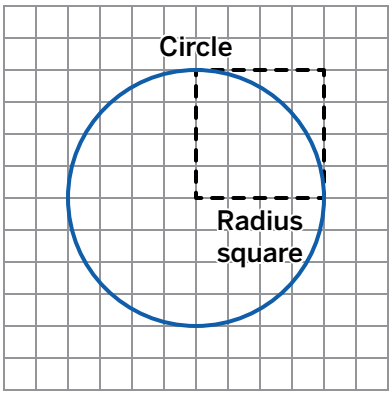
Activity 1

Name: ..... Date: ..... Period: .....

Estimating Circle Area

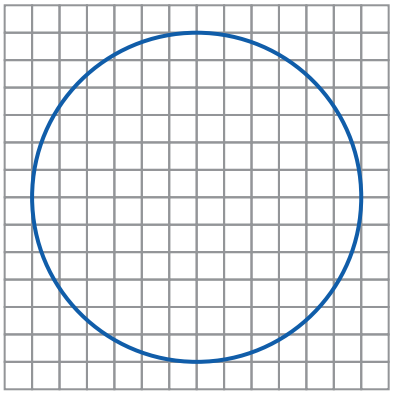
Every circle has a matching radius square.

Here is the radius square for Ayaan's circle.

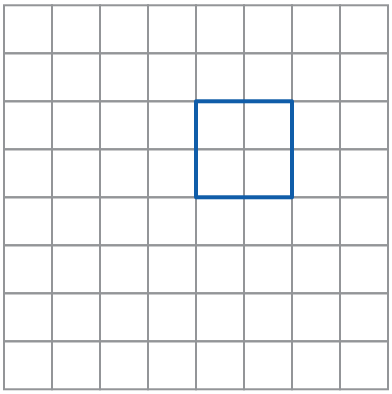


2. Why do you think this is called a radius square?

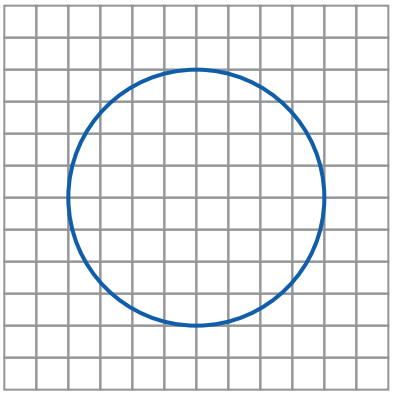
3. Draw a radius square for this circle.



4. Draw a circle for this radius square.



5. Estimate the area of Ayaan's circle.



a Record your own estimate and strategy.

b Find a person who got a different estimate or used a different strategy, and record their estimate and strategy.

My estimate	..... 's estimate
My strategy	..... 's strategy

Name: Date: Period:

Squares Cover Circle

Let's find out how many radius squares are needed to cover a circle.

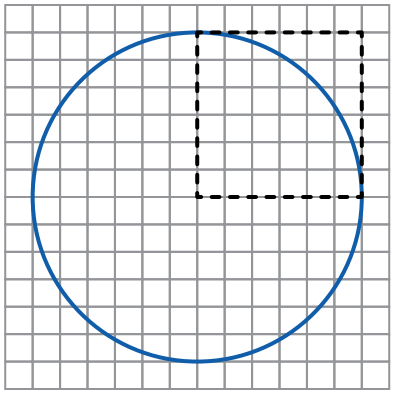
You will get four circles and four sets of radius squares. For each circle, cut up the radius squares and rearrange the pieces to cover the circle.

6. Use your work to complete this table.

	Radius of the Circle (units)	Number of Radius Squares Needed to Cover the Circle
Circle A		
Circle B		
Circle C		
Circle D		

7. In general, how many radius squares do you think can cover a circle?

8. Here is a circle.



a Ayaan says the area of the radius square is 6^2.
Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

b Estimate the area of the circle.

Explain your thinking.

9. **Discuss:** Why might it be helpful to know about how many radius squares are needed to cover a circle?

Activity 3

Name: Date: Period:

Piecing It Together

Let's explore how to derive the formula for the area of a circle.

10. Use the Activity 3 Sheet to rearrange equal-size wedges of a circle into a rectangle. Paste or sketch the rectangle you created from the wedges.

11. Describe what you notice about the rectangle formed by the wedges.

a How is the area of the rectangle related to the area of the original circle?

b What do you notice about the highlighted bases of the wedges?

c How does the length of the rectangle relate to the circumference?

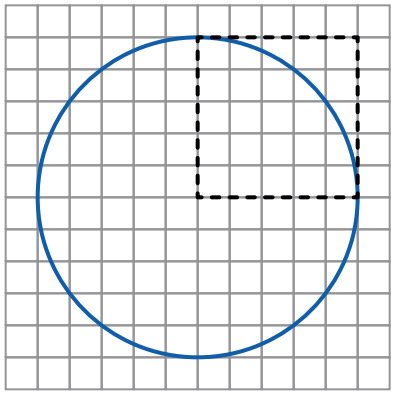
d How does the height of the rectangle relate to the circle?

12. How can you use the radius and the formula for the circumference of a circle to write a formula for the area of a circle?

Synthesis

13. How can you use radius squares to estimate the area of a circle?

Use the example if it helps with your thinking.



Lesson Practice ACC7.7.03

Lesson Summary

You can use radius squares to estimate the area of a circle.

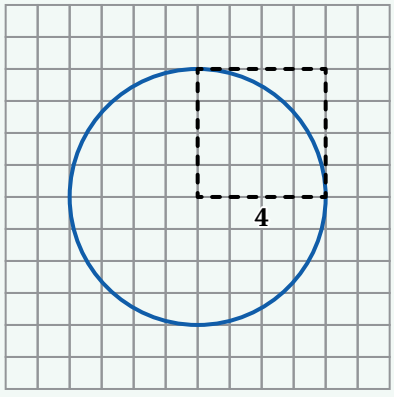
A radius square is a square whose side length is the same as the radius of the circle.

When you break apart and rearrange the radius squares, it takes a little more than 3 radius squares to cover a circle. This means that the area of the circle is a little more than three times the area of the radius square.

In the example, the radius is 4 units.

The area of the radius square is $4 \cdot 4$ or 4^2, which is equal to 16 square units.

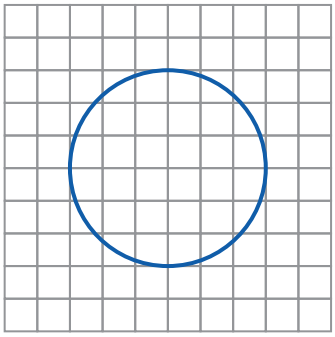
Since it takes a little more than 3 radius squares to cover a circle, the area of the circle is a little more than $3 \cdot 16$, or a little more than 48 square units.



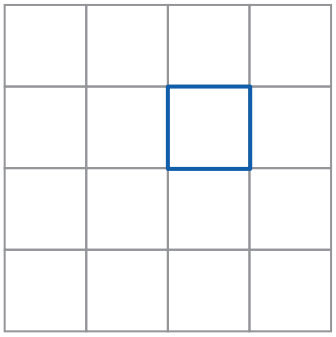
Lesson Practice
ACC7.7.03

Name: ..... Date: ..... Period: .....

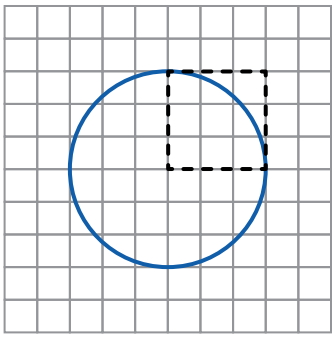
1. Draw a radius square for this circle.



2. Draw a circle for this radius square.

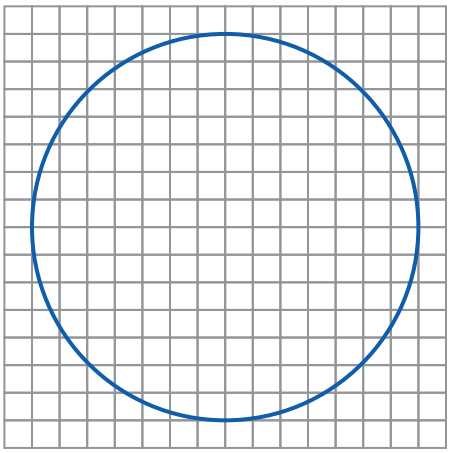


Problems 3–4: Prisha says the area of this circle is a little more than 27 square units.

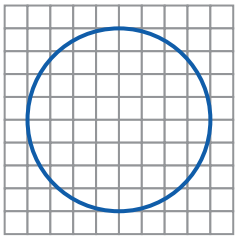


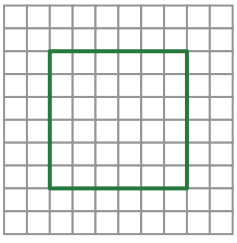
3. What is one way Prisha might have determined her estimate?

4. Estimate the area of this new circle. Show or explain your thinking.



FAST Practice





5. Compare the areas of the circle and the square.

Select **ONE** correct answer in each box to complete the sentence.

The area of the [A. circle / B. square] is larger because the circle's area is a little over [A. 24 / B. 36 / C. 48] square units and the square's area is [A. 24 / B. 36 / C. 48] square units.

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 6–8: When the High Roller observation wheel opened in Las Vegas, NV, in 2014, it became the tallest observation wheel in the world, breaking the record set by the Singapore Flyer in Downtown Core, Singapore, in 2008.

Singapore Flyer

Sergey Peterman/Shutterstock.com

High Roller

Kobay Dagan/Shutterstock.com

6. The Singapore Flyer has a diameter of 150 meters. If you rode all the way around the observation wheel, how far would you travel?

7. The High Roller has a diameter of 158.5 meters. If you rode all the way around the observation wheel, how far would you travel?

8. If you rode once all the way around each observation wheel, how much farther would you travel on the High Roller than on the Singapore Flyer?

9. The cost for 16 ounces of strawberries is $\$4.00$.

Fill in the blank to write an equation that represents the relationship between the number of ounces of strawberries, s, and the cost, c.

$c =$ ..... s

Name: ..... Date: ..... Period: .....

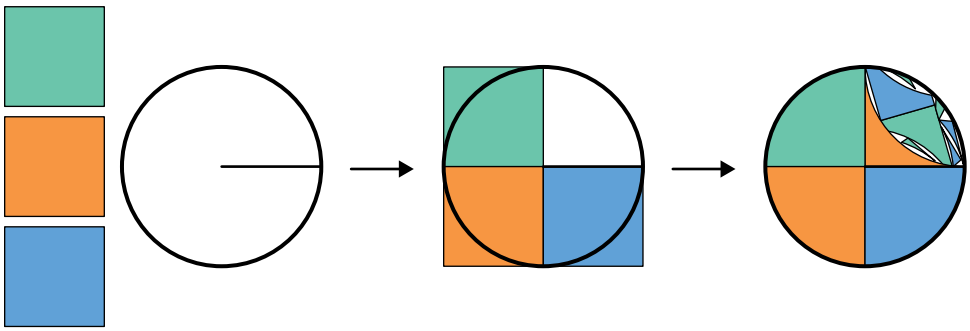
MA.7.GR.1.4, MTR.1.1, MTR.3.1, MTR.4.1

Circle Area

Let's determine a formula for the relationship between the radius of a circle and its area.

Warm-Up

Study the three radius squares and how they fit into the circle.



1. What do you notice? What do you wonder?

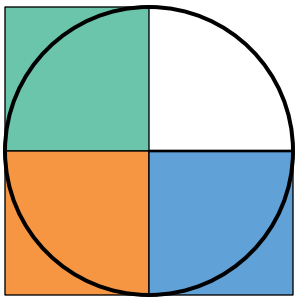
I notice: I wonder:

Name: ..... Date: ..... Period: .....

Finding a Formula

2. Precious says that you can determine the approximate area of a circle by calculating $3 \cdot r^2$.

What do you think each part of her expression means? Draw on the diagram if it helps with your thinking.



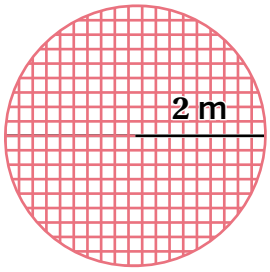
3. Jaleel says you can determine the approximate area of a circle by calculating $r \cdot r \cdot 3$.

Do you agree with Jaleel? Circle one.

Yes No I'm not sure

Explain your thinking.

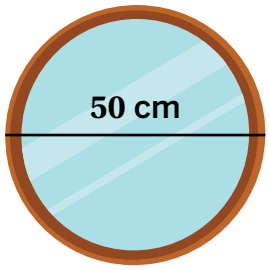
4. Here are four circles, their radius or diameter lengths, and their areas



$A \approx 12.57$ square meters

$A \approx 50.27$ square inches

$A \approx 3848.45$ square centimeters



$A \approx 1963.50$ square centimeters

Write a formula to calculate the exact area of a circle, A, with a radius, r. Use these examples and your work from Problems 2 and 3 if it is helpful.

$A =$ .....

Name: ..... Date: ..... Period: .....

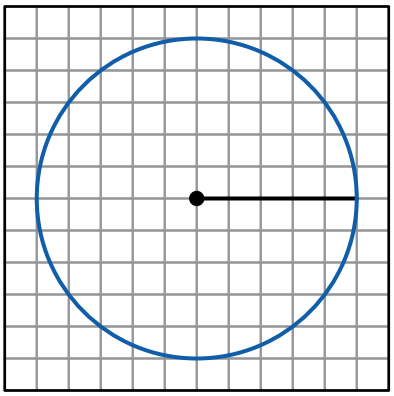
Circle Area

5. Rewrite the formula for the exact area of a circle you developed in Activity 1. Use A for the area of the circle and r for the length of the radius.

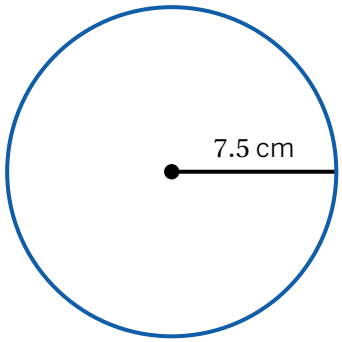
$A =$ .....

6. Calculate the exact area of each circle.

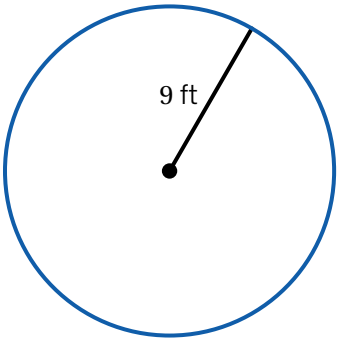
a Area .....



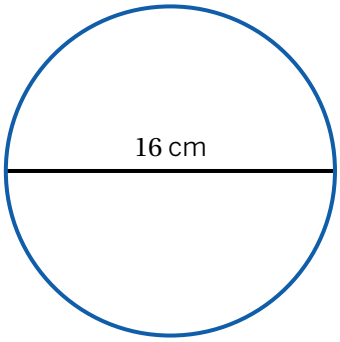
b Area .....



c Area .....

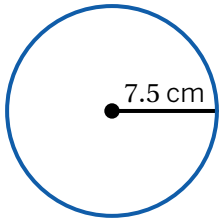


d Area .....

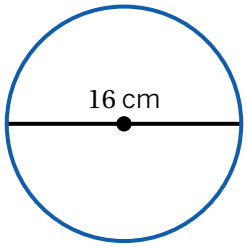


Synthesis

7. Describe a strategy to calculate the area of a circle if you know its radius.



How does your strategy change if you know a circle's diameter?

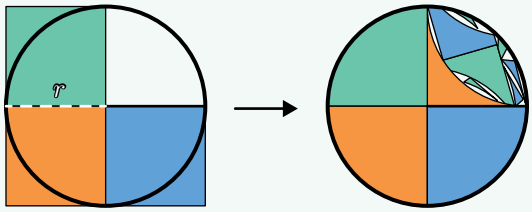


Lesson Practice ACC7.7.04

Lesson Summary

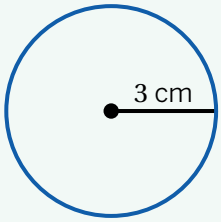
You can calculate the area of a circle if you know the length of its radius, r.

The approximate area of a circle is equal to the area of a little more than 3 radius squares. Each radius square has an area of r^2.



The exact area of a circle is equal to the area of π radius squares. You can express this with the formula $A = \pi \cdot r^2$.

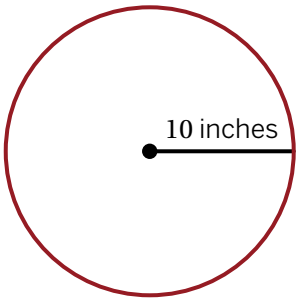
For example, to calculate the area of a circle with a radius of 3 centimeters, you can calculate 3^2, then multiply the result by π. The area of the circle is 9π square centimeters.



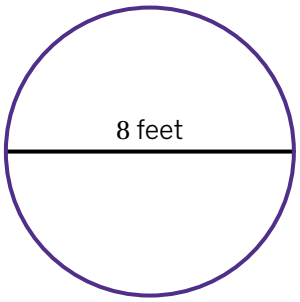
Name: _____ Date: _____ Period: _____

Problems 1–2: Calculate the exact area of each circle.

1.



2.



Problems 3–4: The table shows the diameters of 4 different coins.

3. To determine how much metal is on one face of a coin, it is more useful to use the area rather than the circumference. Explain why this is the case.

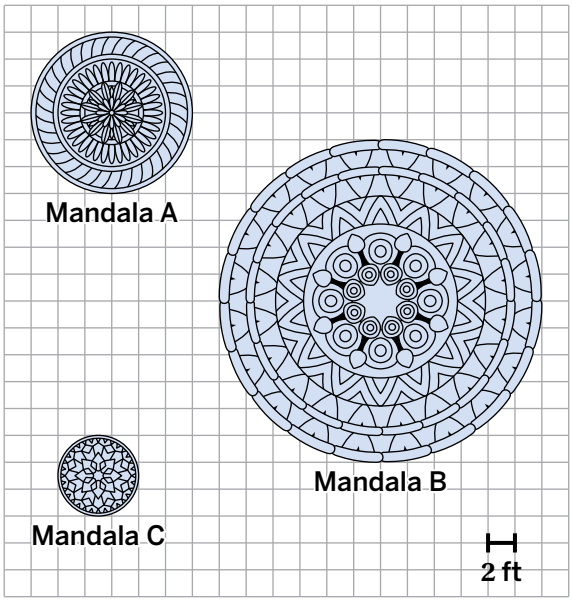
Coin	Diameter (cm)	Area (sq. cm)
Penny	1.9	
Nickel	2.1	
Dime	1.8	
Quarter	2.4	

4. Complete the table by calculating the exact area of each coin.

5. A mandala is a geometric figure. The word *mandala* is Sanskrit for *circle*.

Saanvi designed three mandalas. Determine the exact circumference and area for each mandala.

	Circumference (ft)	Area (sq. ft)
A		
B		
C		



Name: ..... Date: ..... Period: .....

FAST Practice

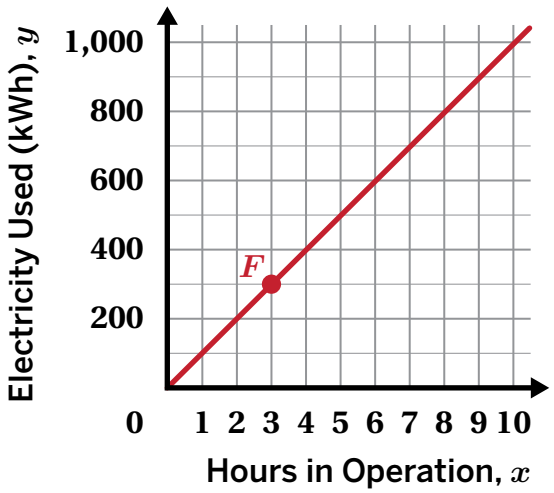
6. The radius of Earth is approximately $6{,}400$ kilometers. The equator is the circle around Earth that divides it into the northern and southern hemispheres.

What is the length of the equator?

_____ kilometers

Spiral Review

Problems 7–9: This graph shows a proportional relationship between the number of hours a manufacturing factory is in operation and the number of kilowatt-hours (kWh) of electricity used. Use the graph to determine whether each statement is true or false.



7. Point F represents the number of kilowatt-hours of electricity used when the factory is in operation for 3 hours.

8. The factory uses 6 kilowatt-hours of electricity when it is in operation for 600 hours.

9. The factory uses 700 kilowatt-hours of electricity when it is in operation for 7 hours.

Name: ..... Date: ..... Period: .....

MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.4.1, MTR.5.1

Area Challenges

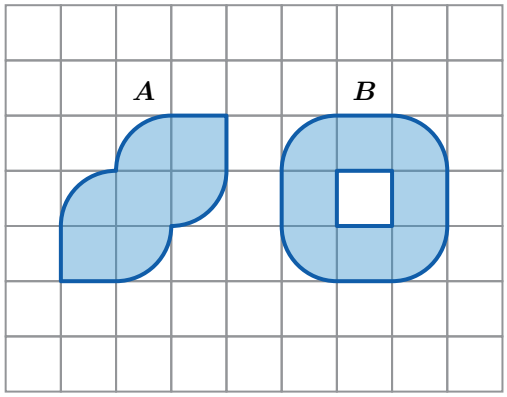
Let's calculate the areas of complex shapes.

Warm-Up

1. Which shape has the greater area? Circle one.

Shape A Shape B They're the same

Explain your thinking.

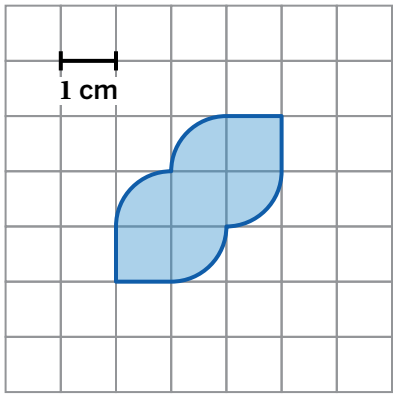


Name: Date: Period:

Calculating Areas

2. Here is shape A from the Warm-Up.

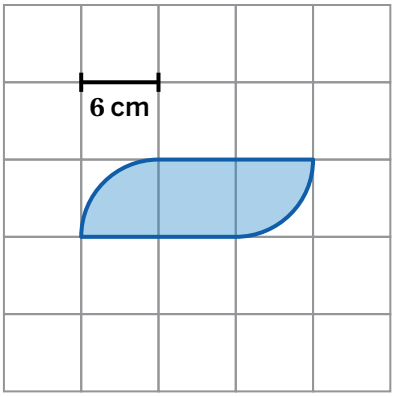
What is its area?



Explain your thinking.

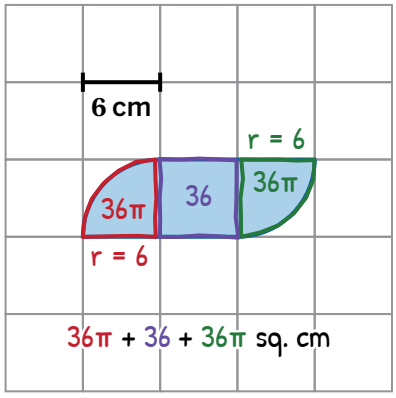
3. Here is a new shape with a new scale.

What is its area?



4. Taylor calculated the area of the shape using the expression $36\pi + 36 + 36\pi$, but made a mistake.

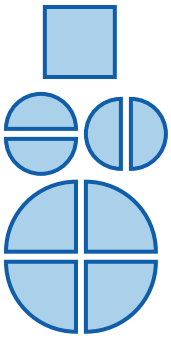
Identify the mistake and explain why it is incorrect.

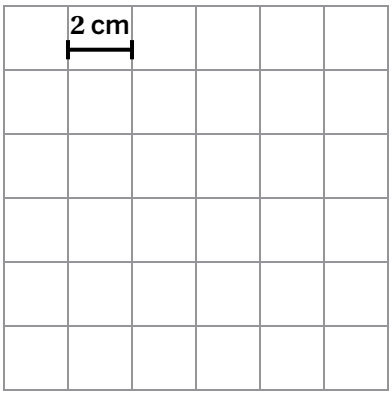


Name: ..... Date: ..... Period: .....

Create a Shape

5. Create a shape made up of squares and parts of circles that has an area of $8 + 4\pi$ square centimeters.





6. Create your own challenge!

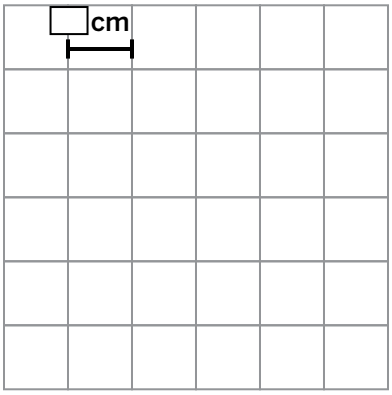
a Circle the scale you'd like to use, then label the diagram with your choice.

2 cm 4 cm 6 cm 8 cm

b Draw a shape made up of squares and parts of circles.

c Determine the exact area of your shape.

My Challenge



Now it's time to complete your partner's challenge!

d Ask your partner what scale they used, then label the diagram with what they chose.

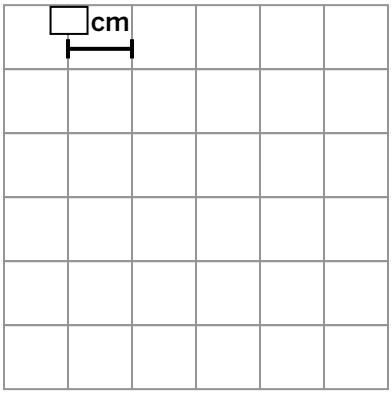
e Ask your partner the area of their shape and record it:

..... square centimeters

f Draw a shape made up of squares and parts of circles that has the same area as your partner's shape.

g When you're done, compare your shape with your partner's shape. How are they alike? How are they different?

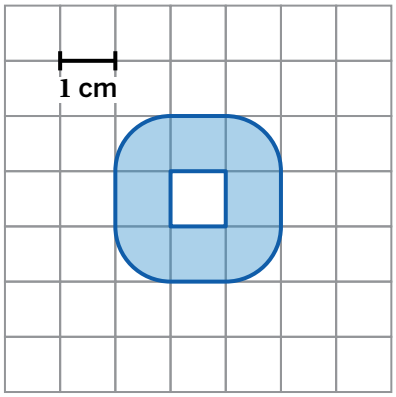
My Partner's Challenge



Synthesis

7. Describe a strategy for determining the area of a shape that's made up of squares and parts of circles.

Use the example if it helps to show your thinking.

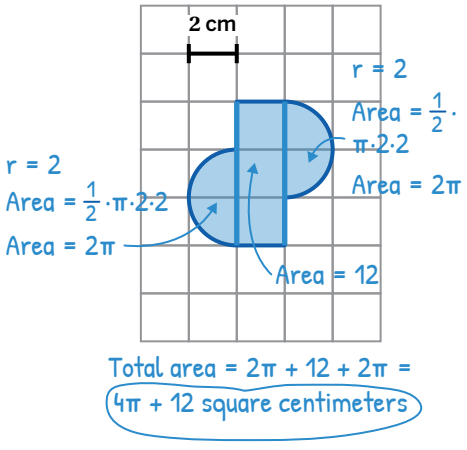


Lesson Practice ACC7.7.05

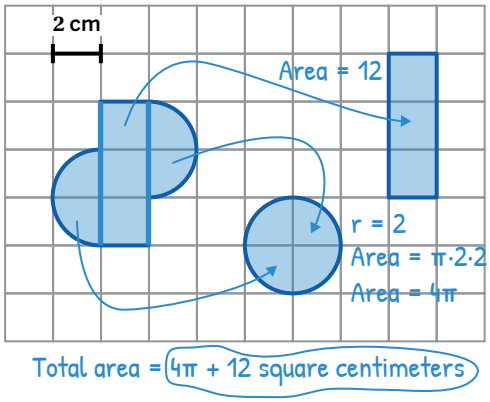
Lesson Summary

We can use different strategies and different ways of thinking to calculate the area of complex figures. Here are two ways we can determine the area of this shape:

Strategy 1



Strategy 2

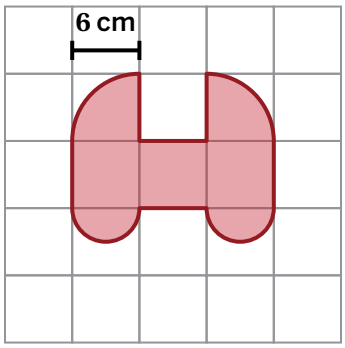


The path to the solution may not be obvious at first, but by breaking the shape down into squares, circles, and parts of circles, we can figure things out!

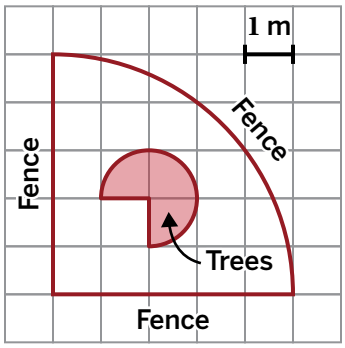
Lesson Practice
ACC7.7.05

Name: ..... Date: ..... Period: .....

1. Calculate the area of this shape.



Problems 2–4: Here is the architectural plan for a fenced-in garden. The shaded region represents an area for trees.



2. What is the area of the tree region?

3. The rest of the fenced-in garden is for flowers. How much area will be covered in flowers? Show your thinking.

4. How long is the fence around the edge of the garden? (Note: The edge around the tree region is not a part of the fence.)

5. Each of these squares has a side length of 12 units. Compare the areas of the shaded regions in these three figures. Which figure has the largest shaded region? Show or explain your thinking.

Figure A

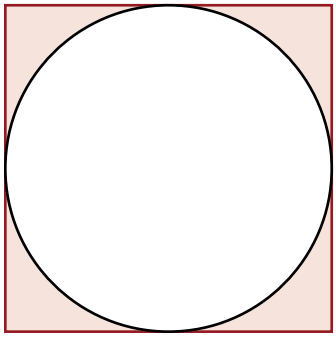


Figure B

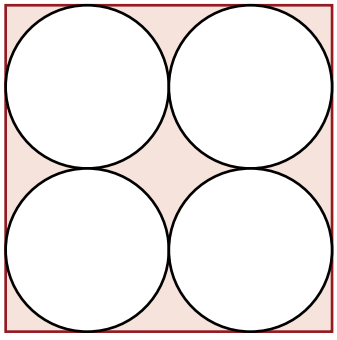
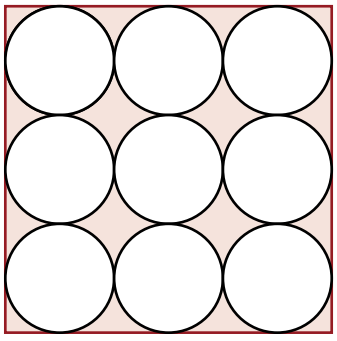


Figure C



Name: ..... Date: ..... Period: .....

FAST Practice

6. A circle with a 12-inch long diameter is folded in half and then folded in half again. What is the exact area of the new shape? Write your answer in terms of π, and show your thinking.

☐ square inches

Spiral Review

7. A graffiti artist is spray painting a circular mural onto a wall. The mural will have a diameter that is 10 feet long. Each can of spray paint covers about 24 square feet. How many cans of spray paint will the artist need to create the mural? Explain your thinking.

Problems 8–9: Determine each value.

8. What is the length of the radius of a circle with a diameter that measures 10 units?

9. What is the length of the diameter of a circle with a radius that measures 10 units?

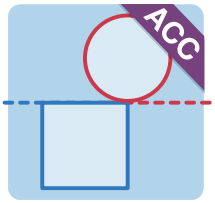
10. Abena used wire fencing to form a border around a circular garden in her backyard. If the radius of the circular garden was 10 yards long, what was the total length of the border, rounded to the nearest tenth of a yard?

(A.) 31.4 (B.) 62.8 (C.) 314.2 (D.) 628.3

Name: ..... Date: ..... Period: .....

MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.3.1

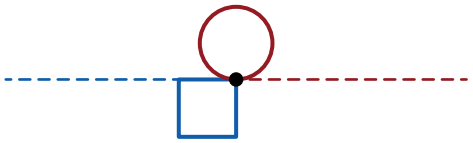
Circle vs. Square



Let's solve challenges with circles and squares.

Warm-Up

1. Let's look at this diagram.



What do you notice? What do you wonder?

I notice:

I wonder:

Name: Date: Period:

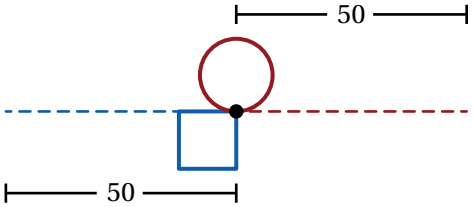
Which is Greater?

2. The length of the original segment is 100 units.

We can cut this segment in half and use 50 units to form each shape.

Which shape do you think has the greater area? Circle one.

Square Circle They're the same



3. What information would you need to determine which shape has the larger area?

4. Calculate the area of each shape.

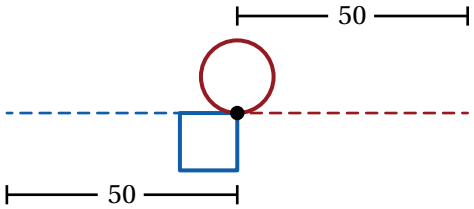
Square Area (square units)	Circle Area (square units)

Name: Date: Period:

Greatest Total Area

5. When we cut the 100-unit segment in half and use 50 units to form each shape, the total area of both shapes is about 355.19 square units.

Let's explore if the total area stays the same if the 100 units is split between the shapes in different ways.



a You and your partner will use a card set for this activity. Pick one of the cards.

b Calculate the new total area.

c **Discuss:** How does the new total area compare to the 355.19 square units from before?

6. **a** Record the areas you calculated in the previous problem.

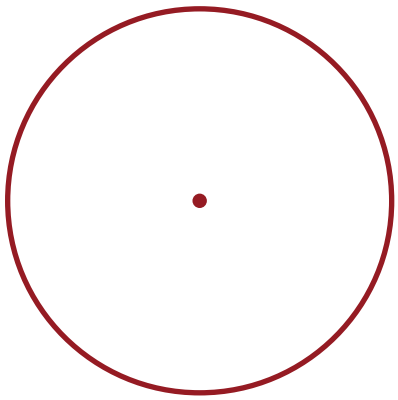
b Complete the table to determine the greatest total area you can make from the circle and square.

Card	Left Segment Length (units)	Square Area (sq. units)	Circle Area (sq. units)	Total Area (sq. units)
1	100			
2	75			
3	60			
4	40			
5	25			
6	0			

Synthesis

7. How can you determine the area of a circle if you only know its circumference?

Make a drawing if it helps to show your thinking.

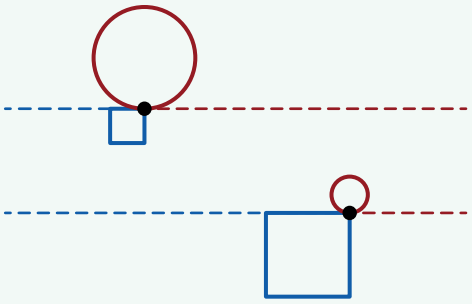


Lesson Practice ACC7.7.06

Lesson Summary

One of the fun things about math is how you can test predictions using calculations. You can make a guess about something, like when the total area of the square and circle will be greatest, and then determine how close your prediction is to the actual answer.

Sometimes your guess will turn out to be correct. Other times, you get to experience the surprise and wonder of discovering something unexpected!



Name: ..... Date: ..... Period: .....

Problems 1–3: A circle's circumference is approximately 76 centimeters. Determine each of these measurements and show your thinking:

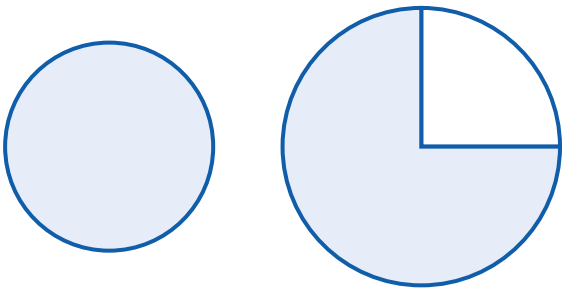
1. The length of the diameter of the circle.

2. The length of the radius of the circle.

3. The area of the circle.

4. The metal rim of a magnifying glass measures 10 inches around. What is the area of the surface of the glass inside? Show your thinking.

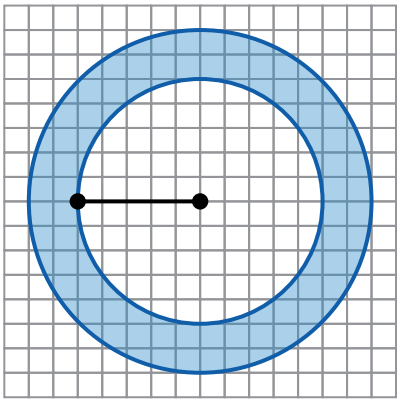
5. The diameter of the smaller circle is 4.5 centimeters long. The diameter of the larger circle is 6 centimeters long. Calculate the difference in the areas of the shaded regions of the two circles. Show your thinking.



The figures may not be drawn to scale.

Name: _____ Date: _____ Period: _____

6. Determine the area of the shaded region. Show or explain your thinking.



FAST Practice

7. The circumference of a circle is 30π centimeters. What is the area of the circle in terms of π?

A. 15π square centimeters

B. 30π square centimeters

C. 225π square centimeters

D. 900π square centimeters

Spiral Review

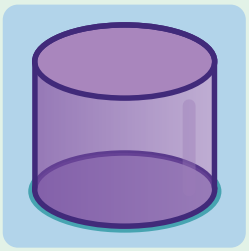
Problems 8–10: Complete the table by determining the unknown measurements of each circle.

	Radius (in.)	Diameter (in.)	Circumference (in.)	Area (sq. in.)
8.	12			
9.		12		
10.			12	

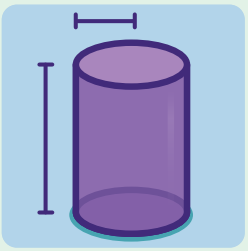
Sub-unit

2

Solid Geometry



Lesson 7
Cylinders



Lesson 8
Scaling Cylinders

Lesson 9
From Flat to Round

Name: ..... Date: ..... Period: .....

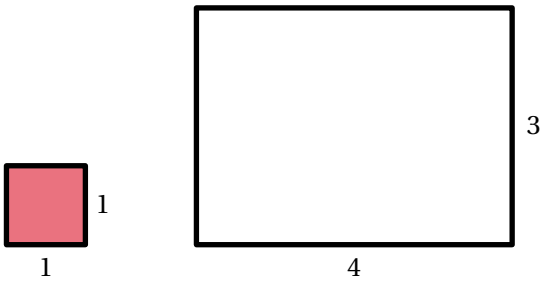
MA.7.GR.2.3, MTR.2.1, MTR.3.1, MTR.4.1

Cylinders

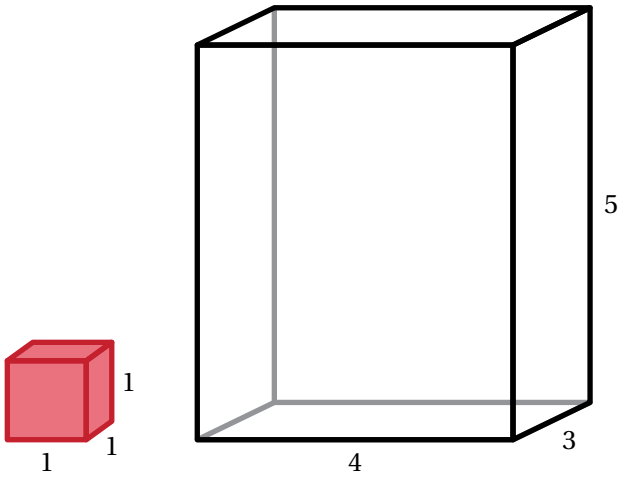
Let's calculate the volume of cylinders.

Warm-Up

1. How many unit squares are needed to cover this rectangle?



2. How many unit cubes are needed to fill this rectangular prism?



Name: Date: Period:

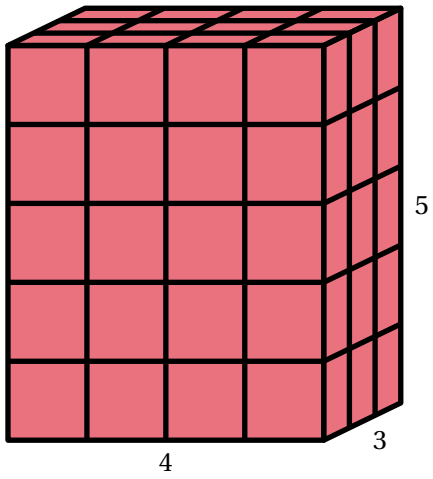
Volume Strategies

3. DeAndre calculated the *volume* in three ways:

- $V = 12 \cdot 5$
- $V = 15 \cdot 4$
- $V = 20 \cdot 3$

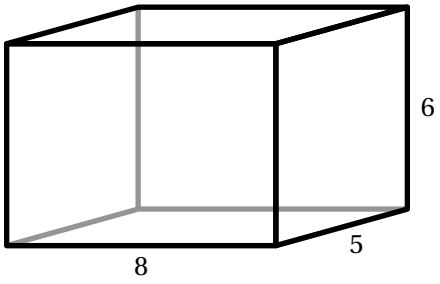
Let's focus on DeAndre's first equation.

Explain what 12 and 5 represent in the diagram.



4. Here is a new rectangular prism.

Use DeAndre's strategy to write an expression for its volume.

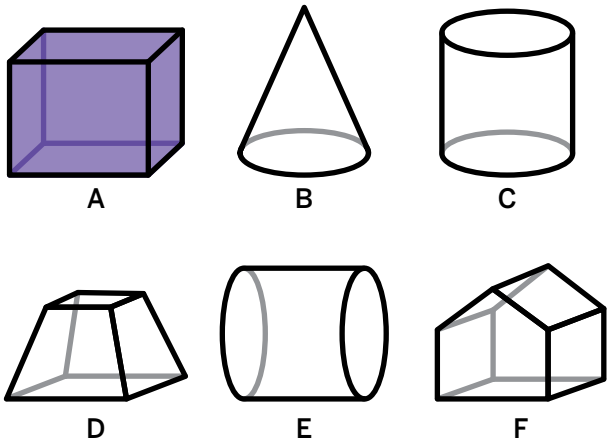


Name: ..... Date: ..... Period: .....

Volume Strategies (continued)

5. DeAndre described his strategy for calculating the volume of a solid.

- First, calculate the area of the base.
- Then multiply that area by the height of the object.

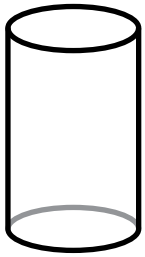


a Circle *all* of the objects for which DeAndre's strategy will work.

b **Discuss:** How did you decide which objects to choose?

6. DeAndre's strategy for calculating volume works for **cylinders**.

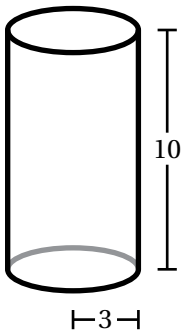
What information would you need to calculate the volume of this cylinder?



Name: Date: Period:

Cylinder Volumes

7. This cylinder has a height of 10 units and a radius of 3 units.



Calculate the volume of the cylinder.

8. Caasi incorrectly determined the volume of the cylinder with this calculation:

$V = 3 \cdot 3 \cdot 10 = 90$

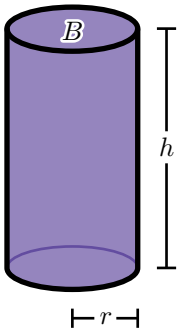
What did Caasi do well and what mistake did she make?

9. Here are two formulas for the volume of a cylinder:

$V = \pi r^2 \cdot h$

$V = B \cdot h$

Describe how the formulas are related.



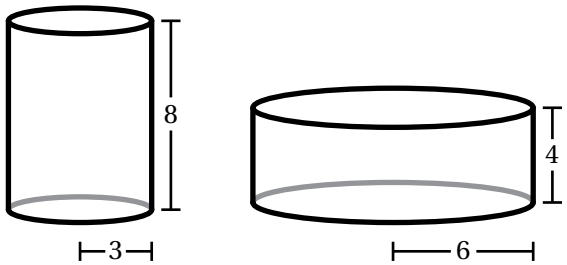
Name: Date: Period:

Calculating Cylindrical Volume

10. Calculate the volume of each cylinder.

Tall cylinder:

Short cylinder:



11. Sketch two different cylinders that have the same volume. What is the volume of each cylinder?

Cylinder 1 **Cylinder 2**

You're invited to explore more.

12. **a** Write dimensions for a rectangular prism and a cylinder with volumes that are close to equal.

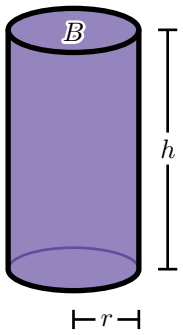
Rectangular prism:

Cylinder:

b Prove that they have close to equal volumes.

Synthesis

13. Describe a strategy for determining the volume of a cylinder given its radius and height.

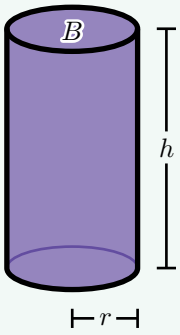


Lesson Practice ACC7.7.07

Lesson Summary

A prism has two congruent bases connected by perpendicular lines. Its *volume* can be determined by multiplying the area of its base by its height. A **cylinder** has two congruent circles for its base and the sides are perpendicular to the bases. This means you can also determine the volume of a cylinder by using the area of its base multiplied by its height.

If you know the radius and height of a cylinder, then you can determine the volume of the cylinder. The base area, B, is determined using the expression $\pi \cdot r^2$. The volume, in cubic units, can be determined by multiplying the base area by the height, h. The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$.



$V = \pi r^2 \cdot h$

Lesson Practice
ACC7.7.07

Name: _____ Date: _____ Period: _____

Problems 1–3: Draw a cylinder.

1. Label the radius 3 units and the height 10 units.

2. Determine the area of the base. Write your response in terms of π.

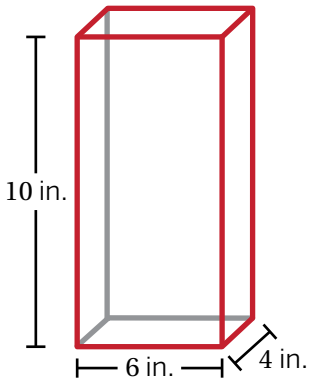
3. Determine the volume of the cylinder. Write your response in terms of π.

Problems 4–6: Containers A and B hold oatmeal. Container A is a rectangular prism and container B is a cylinder.

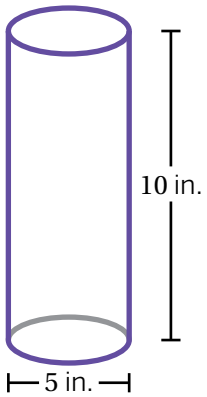
4. The diameter of container B is 5 inches. What is the radius of the container?

5. Which container's base has a larger area? Explain your thinking.

Container *A*



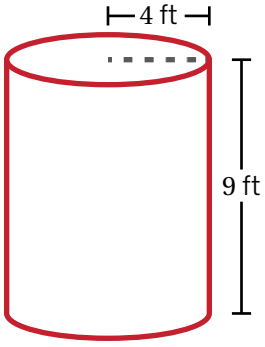
Container *B*



6. Which has a larger volume: container A or B? Explain your thinking.

FAST Practice

7. Here is a cylinder with a radius of 4 feet and a height of 9 feet. What is the volume of the cylinder in cubic feet? Round your answer to the nearest hundredth.



_____ cubic feet

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 8–9: A gardener is designing a circular flower bed. The distance around the flower bed is 31.4 feet.

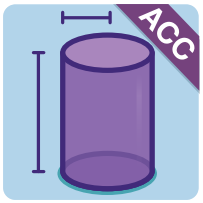
8. The gardener wants to create a straight path through the center of the flower bed from one end to the other. What will be the length of the path? Use 3.14 for π.

9. What is the area of the flower bed?

Name: ..... Date: ..... Period: .....

MA.7.GR.2.3, MTR.1.1, MTR.5.1, MTR.7.1

Scaling Cylinders

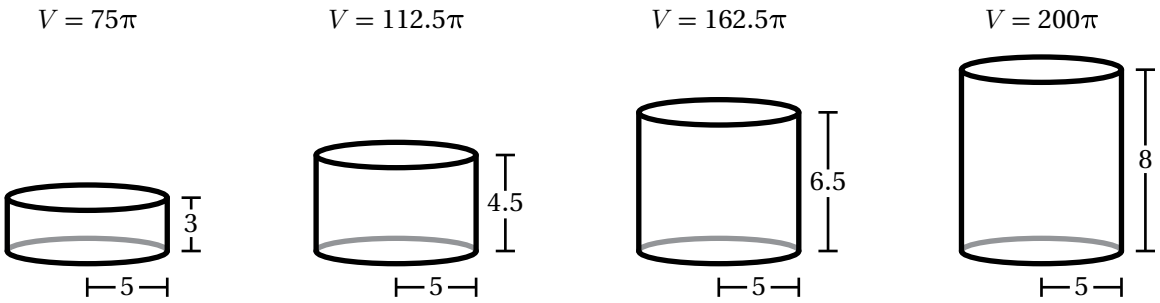


Let's see how changing a cylinder's radius or height impacts its volume.

Warm-Up

1. Think about objects that you have seen that are shaped like the cylinders shown. Tell a story about why a company would want to consider using cylinders of varying sizes for its products.

Here are several cylinders:



Name: Date: Period:

Changing the Dimensions

2. Let's see what happens if we keep the radius of a cylinder constant but change the height.

a Complete the table for different cylinders that each have a radius of 5 centimeters. Express each volume in terms of π.

b What do you notice about the height and volume of the cylinders?

Object	Height	Volume (cu. cm)
Cylinder A	2	
Cylinder B	4	100π
Cylinder C	8	
Cylinder D	16	

3. Let's see what happens if we keep the height of a cylinder constant but change the radius.

a Complete the table for different cylinders that each have a height of 10 centimeters. Express each volume in terms of π.

b What do you notice about the radius and volume of the cylinders?

	Radius (cm)	Volume (cu. cm)
Cylinder 1	2	
Cylinder 2	4	160π
Cylinder 3	8	
Cylinder 4	16	

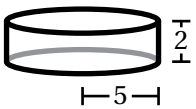
4. Make a prediction about how tripling the radius of a cylinder will affect its volume.

Name: Date: Period:

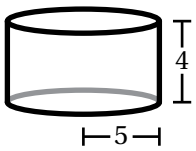
Choosing Cylinders

5. Ashley has several sizes of jars that each have a radius of 5 centimeters. She needs jars that will each hold 300 cubic centimeters of modeling clay. Which cylinder should she use? Explain your thinking.

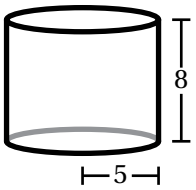
Cylinder 1



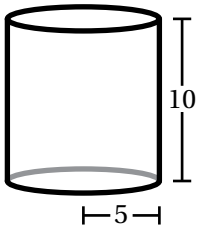
Cylinder 2



Cylinder 3



Cylinder 4



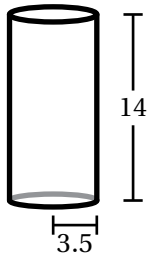
6. Ramon needs a water bottle that holds 1,000 cubic centimeters of water. An orange bottle has a height of 20 centimeters and a diameter of 12 centimeters. A blue bottle also has a height of 20 centimeters, but its diameter is 8 centimeters. Which water bottle should Ramon choose? Explain your thinking.

Name: Date: Period:

Choosing Cylinders (continued)

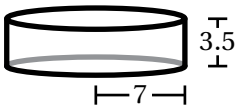
7. A nut company has four different sizes of gift containers with measures shown in inches. Which two containers would be appropriate choices to hold 540 cubic inches of nuts? Show or explain your thinking.

Cylinder A

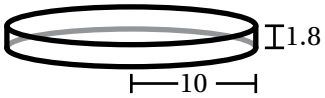


Cylinder B

Cylinder C

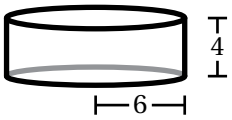


Cylinder D



8. Two cylinders have the same volume. One cylinder has a radius of 4 inches and a height of 3 inches. If the other cylinder has a radius of 2 inches, what is its height?

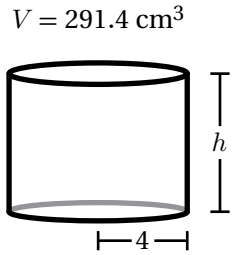
9. The cylinder shown has a radius of 6 centimeters and a height of 4 centimeters. Draw a cylinder that has a smaller radius but a greater volume.



Synthesis

10. Describe a strategy for determining the height of a cylinder when you know the radius and volume.

Use the example if it helps with your thinking.



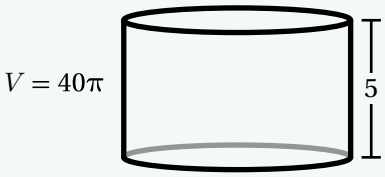
Lesson Practice ACC7.7.08

Lesson Summary

The volume of a cylinder depends on its radius and height. If you know the radius and the height, you can determine the volume using the formula $V = \pi r^2 h$ or $V = Bh$.

If you know the volume and either the height or the radius, you can determine the other dimensions, too.

For example, if a cylinder has a height of 5 inches and a volume of 40π, you can calculate the area of the base by dividing the volume by the height. The area of the base of the cylinder is 8π.



$40\pi = B \cdot 5$

$8\pi = B$

Name: ..... Date: ..... Period: .....

1. Each row of this table lists information about a specific cylinder. Complete the table.

Diameter (units)	Area of Base (sq. units)	Height (units)	Volume (cu. units)
4		10	
6			63π
	25π	6	

Problems 2–3: A gas company's delivery truck has a cylindrical tank with a diameter of 14 feet and a height of 40 feet.

2. Draw the tank, then label its radius and height.

3. How much gas can fit in the tank? Show or explain your thinking.

Problems 4–6: Imagine several cylinders that all have a height of 18 meters. Let r represent the radiuses of the cylinders, in meters, and V represent the volume of the cylinders, in cubic meters.

4. Write an equation that represents the relationship between the volume, V, and the radius, r, for all cylinders with a height of 18 meters.

5. Complete this table:

r (m)	1	2	3
V (cu. m)			

6. A village water treatment container must be purchased to hold at least 300 cubic meters of water. The village can choose from the three cylinders in Problem 5. Which one of these three cylinders is best suited to meet the village's needs? Explain your thinking.

Name: _____ Date: _____ Period: _____

7. A cylinder has a volume of 48π cubic centimeters and a height represented by h.

Complete this table with the volumes of other cylinders that have the same radius but different heights.

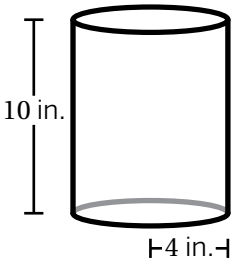
Height (cm)	Volume (cu. cm)
h	48π
$2h$	
$5h$	
$\frac{h}{2}$	
$\frac{h}{5}$	

FAST Practice

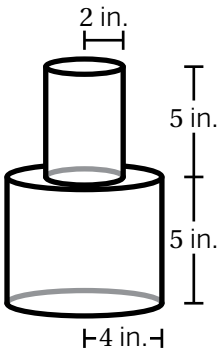
8. A bakery offers two types of celebration cakes, as shown. Select *all* the true statements.

- ☐ **A.** Cake B has more volume than cake A.
- ☐ **B.** Cake B has a volume of about 314 cubic inches.
- ☐ **C.** If cake A costs \$130, that is about \$0.26 per cubic inch.
- ☐ **D.** Cake A has a volume of 438 cubic inches.
- ☐ **E.** If cake B costs \$150, that is about \$0.48 per cubic inch.
- ☐ **F.** The difference in the cakes' volumes is 63 cubic inches.

Cake A



Cake B



Spiral Review

9. The area of a circle is approximately 201.06 square inches. What is its radius in inches?

10. Match each circle with its area.

- Circle A has a radius of 4 units.
- Circle B has a radius of 10 units.
- Circle C has a radius of 8 units.

Area of the Circle	Circle
About 314 square units	
64π square units	
16π square units	

Name: ..... Date: ..... Period: .....

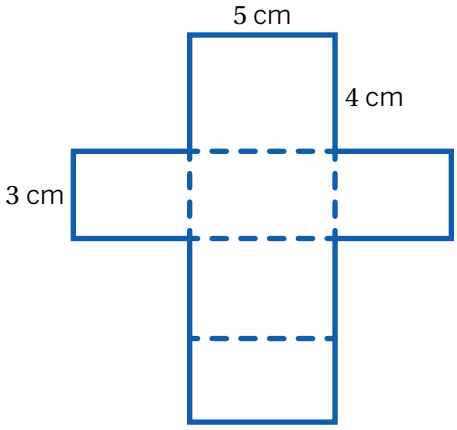
MA.7.GR.2.1, MA.7.GR.2.2, MTR.1.1, MTR.2.1, MTR.7.1

From Flat to Round

Let's calculate the surface area of cylinders.

Warm-Up

1. A *net* of a rectangular prism is shown.



a Describe the shapes that make up the net.

b How would you use the net to determine the total surface area of the rectangular prism?

Name: ..... Date: ..... Period: .....

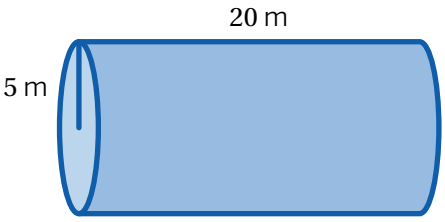
What's in the Net?

2. A can of tomato soup is shaped like a cylinder. The can has a label that wraps around it.

a Describe the shape of the label if you unwrap it from the can.

b How can you use the dimensions of the can to determine the dimensions of the label?

3. Compare this right circular cylinder to the can of tomato soup from Problem 2.

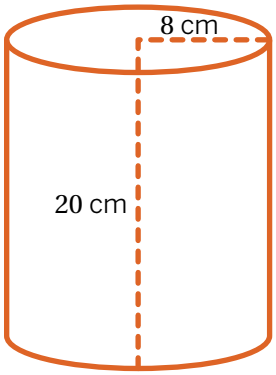


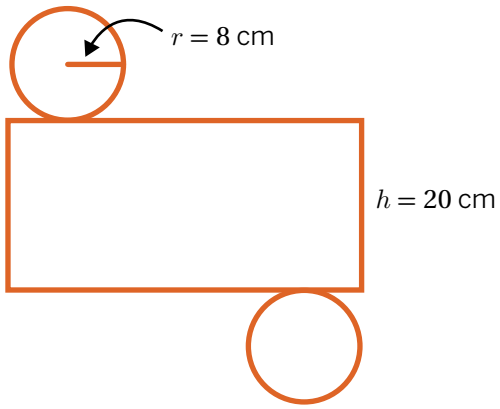
Describe how the nets for the two cylinders would look similar.

Name: Date: Period:

What's in the Net? (continued)

4. You can use a net to help you determine the **surface area** of a cylinder. Complete the table to determine the exact surface area of the right circular cylinder shown.



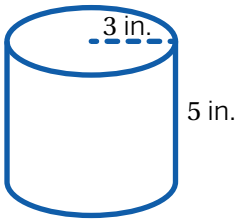


	Formula for Area	Expression	Area
Circle 1	πr^2	$\pi(8)^2$	64π sq. cm
Circle 2	πr^2		
Rectangle	$2\pi r \cdot h$		
Total Surface Area			

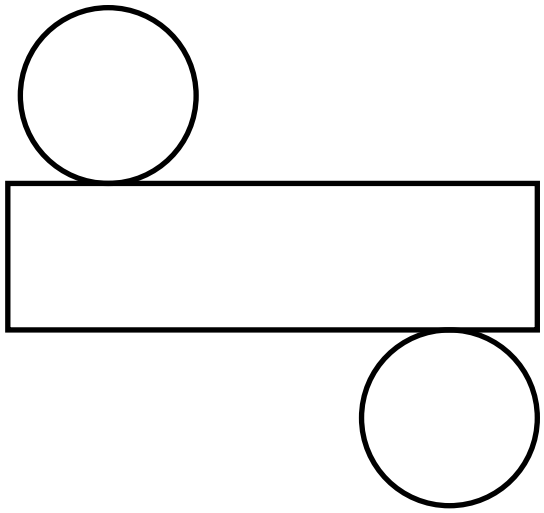
Name: Date: Period:

Without a Net

The cylinder has a height of 5 inches and a radius of 3 inches.



5. Here is a net for the cylinder. Label the measurements for radius of the circle and the base and height of the rectangle.



6. What information do you need to determine the surface area of the cylinder?

7. Write a formula you can use to determine the surface area. Use r to represent the radius and h to represent the height of the cylinder.

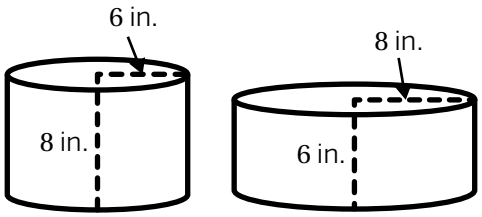
Surface area = area of curved surface + area of circular bases

Use your formula to calculate the surface area of the cylinder. Write your response in terms of π.

Name: Date: Period:

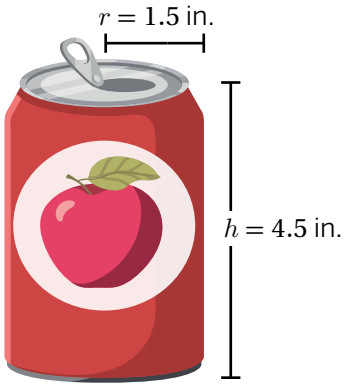
Without a Net (continued)

8. Arjun says that the two cylinders have the same surface area.



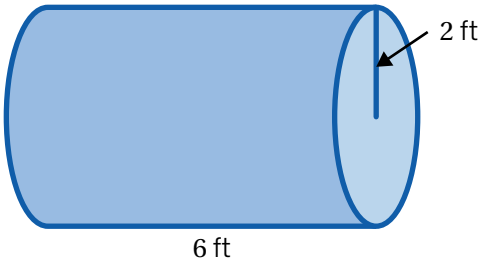
Is Arjun correct? Explain your thinking.

9. A can of apple juice like the one shown is cut from a large sheet of metal. Determine the amount of metal needed to make a similar can. Show your thinking. Use. 3.14 for π and round to the nearest tenth.



Synthesis

10. Describe the steps you would take to determine the surface area of a right circular cylinder. Use this cylinder if it helps you with your thinking.

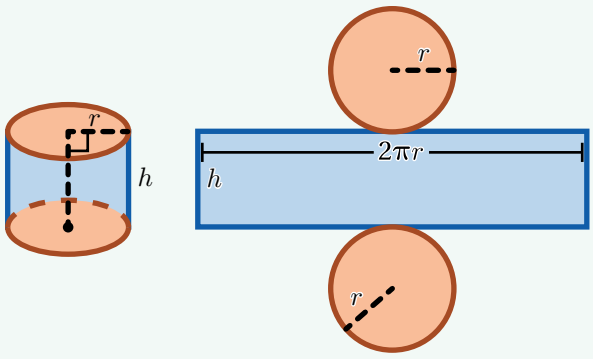


Lesson Practice ACC7.7.09

Lesson Summary

To determine the **surface area** of a right circular cylinder, it's helpful to visualize what the net of the cylinder looks like. You can add the areas of the parts without having to rely on a formula.

The net for all right circular cylinders is composed of two identical circular bases and a rectangle that forms the curved surface.



To calculate the area of the circular bases, use πr^2 and multiply by 2.

To calculate the area of the curved surface, use the circumference of the base, $2\pi r$, and multiply by the height, h.

Add the areas of the two bases to the area of the curved surface to calculate the total surface area of the cylinder.

Name: ..... Date: ..... Period: .....

Problems 1–3: Determine if each statement is true or false.

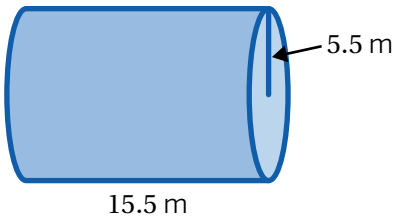
1. The net of a right circular cylinder is composed of two circular bases and one rectangle.

2. The two circular bases in the net of a cylinder have different areas.

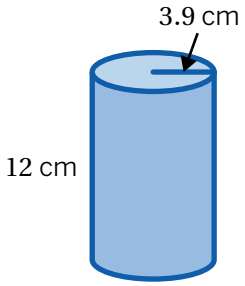
3. The area of the curved surface of a cylinder can be calculated by multiplying the circumference of the base by the height of the cylinder.

Problems 4–5: Calculate the surface area of each cylinder. Use 3.14 for π. Round to the nearest tenth.

4.



5.



6. A city used a large cylindrical water tank to store water. The tank has a radius of 10 meters and a height of 20 meters. The city painted the outside of the tank, including the top, bottom, and sides. Determine the exact area that was painted.

Name: ..... Date: ..... Period: .....

7. A baker made a round cake in the shape of a cylinder with a radius of 6 inches and a height of 3 inches. The baker covered the entire outside of the cake and the top of the cake with frosting. How much frosting did the baker use to cover these areas? Use 3.14 for π.

FAST Practice

8. The surface area of a cylinder is 154π sq. cm. The radius of the cylinder is 7 cm. What is the height of the cylinder?

(A.) 2 cm

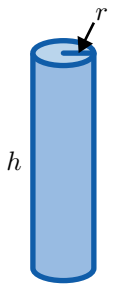
(B.) 4 cm

(C.) 6 cm

(D.) 7 cm

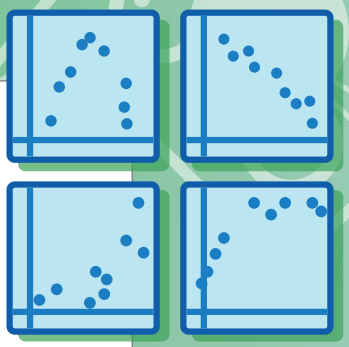
Spiral Review

9. The cylinder shown has a height of $10\frac{1}{2}$ inches and a radius measuring $1\frac{1}{2}$ inches. What is the volume of the cylinder in cubic inches? Use 3.14 for π. Round to the nearest tenth.



Unit 8

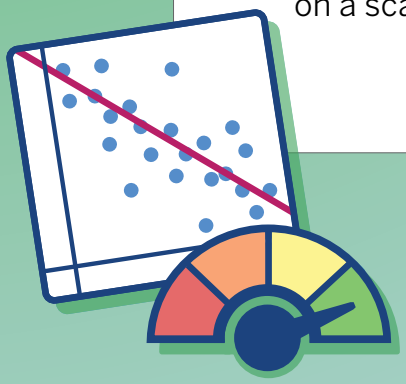
Associations in Data



Data is one way we make sense of the world around us. Organizing and displaying data can allow us to describe trends and make predictions. In this unit, you will choose the best representation to display data and investigate data points that represent two pieces of information.

Essential Questions

- How do we choose the most appropriate display to represent data?
- What is a scatter plot and what can it tell you?
- How can lines help you model data on a scatter plot?

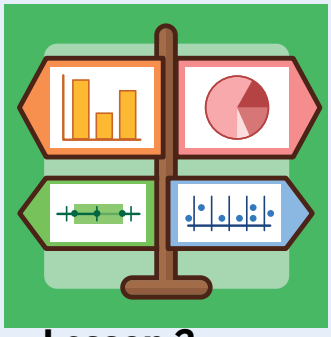


Sub-unit

1

Displaying Data

Lesson 1
The County Fair



Lesson 2
Choices, Choices

Lesson 3
On Display

Name: ..... Date: ..... Period: .....

MA.7.DP.1.4, MTR.4.1, MTR.5.1

The County Fair

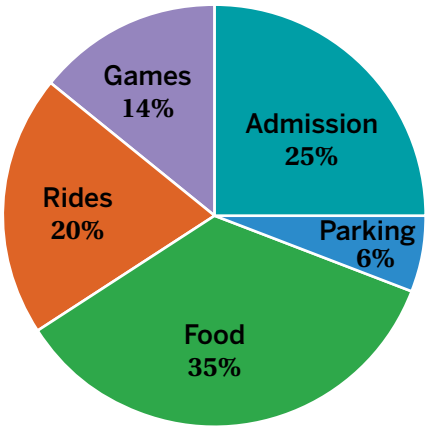
Let's use proportional reasoning to make sense of circle graphs.

1. Here is a circle graph showing a family's budget at a county fair. What do you notice? What do you wonder?

I notice:

I wonder:

Family Fair Budget

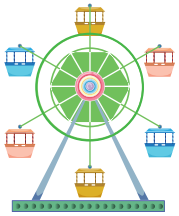


Name: _____ Date: _____ Period: _____

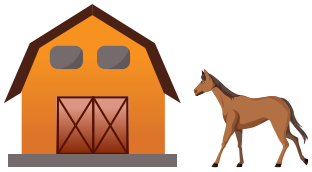
Percents and Fractions

The diagram shows the percent of space occupied by different activities at the fair.

Rides
30%



Animal barns
25%



Concessions
20%

Games
10%

Exhibits
10%

Entertainment
5%

2. Represent each category with a ratio that represents the space occupied when compared to the whole.

Activity	Percent	Ratio
Rides	30%	
Animal Barns	25%	
Concessions	20%	
Games	10%	
Exhibits	10%	
Entertainment	5%	

3. Mia wrote the fraction $\frac{30}{70}$ for the space needed for rides as a fraction. Describe the error she might have made and show the correct response.

Name: ..... Date: ..... Period: .....

Setting Up

Circle graphs visually display data as parts of a whole. In a circle graph, the parts add up to 100 percent. There are 360° in a circle.

4. The data in this table matches the diagram from Activity 1. Use a proportional relationship to determine what part of a circle will represent each percent in the table.

Activity	Percent	Angle Measure
Rides	30%	108°
Animal Barns	25%	
Concessions	20%	
Games	10%	
Exhibits	10%	
Entertainment	5%	

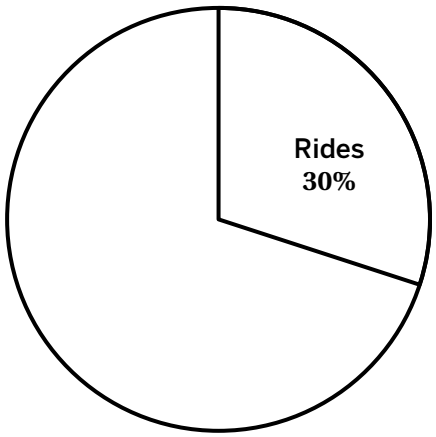
5. Select one activity from the table and explain how you calculated the angle measure.

Name: ..... Date: ..... Period: .....

Setting Up (continued)

6. Use a protractor to complete the circle graph. Label each section with the category and percent.

Space Occupied by Fair Activities



7. How can you check your angle measures before creating a circle graph?

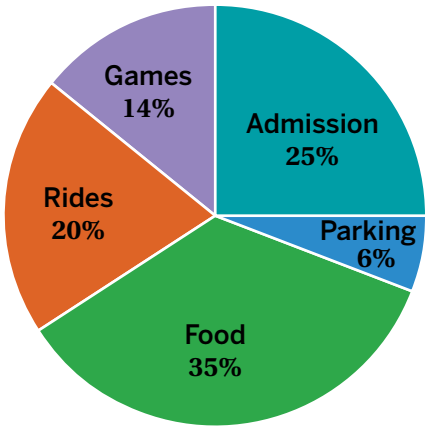
Activity 3

Name: Date: Period:

Have Some Fun

Here is the circle graph from the Warm-Up again.

Family Fair Budget



8. Suppose the family spends $160 at the county fair. Complete the table with the amount of money spent on each category. Round to the nearest dollar.

Category	Money Spent ($)
Admission	
Parking	
Food	
Rides	
Games	

9. Evan found that the total spent on parking was $96. Is he correct? Explain your thinking.

Synthesis

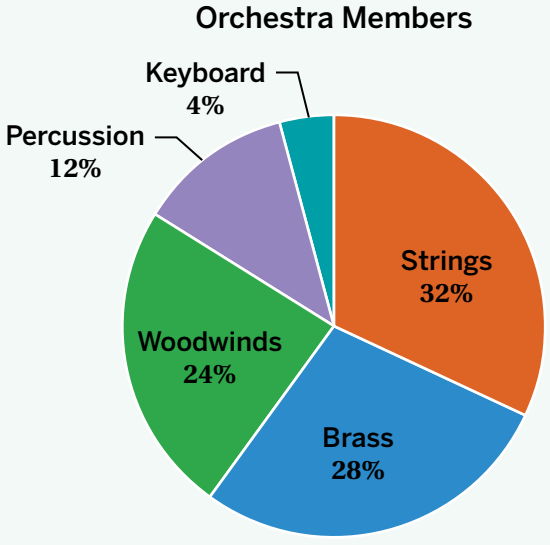
10. Explain how you would display this data using a circle graph.

Exhibit	Number of Vendors
Arts and Crafts	20
Baked Goods	32
Flowers	10
Photography	14
Produce	24

Lesson Practice ACC7.8.01

Lesson Summary

- **Circle graphs** show categories as percents. The whole set of data is represented by the circle and its interior. The categories are represented by fractional parts of the circle. Use proportions to construct a circle graph. The equation $\frac{\text{percent}}{100} = \frac{x}{360}$ gives the angle measure, x, for each category. You can use a protractor to create an accurate circle graph.
- You can apply information from a circle graph to solve problems. If 50 members are in an orchestra, the number of people playing brass instruments, x, is $\frac{28}{100} = \frac{x}{50}$. Solving the proportion gives $x = 14$, so 14 members of the orchestra play brass instruments.



Lesson Practice
ACC7.8.01

Name: ..... Date: ..... Period: .....

Problems 1–3: The table shows the number of pigs of different breeds shown at the county fair. Use the table to answer the problems.

Breed	Number	Ratio	Percent	Number of Degrees
Berkshire	45			
Landrace	11			
Duroc	33			
Chester White	42			
Hampshire	19			

1. What is the total number of pigs shown at the fair? Why do you need this total to complete the table?

2. For each breed, complete the table to show how it compares to the whole. For the Ratio column, do not simplify the fraction. For the Percent column and Number of Degrees column, round to the nearest whole.

3. Explain how you determined the number of degrees for Duroc pigs.

Lesson Practice
ACC7.8.01

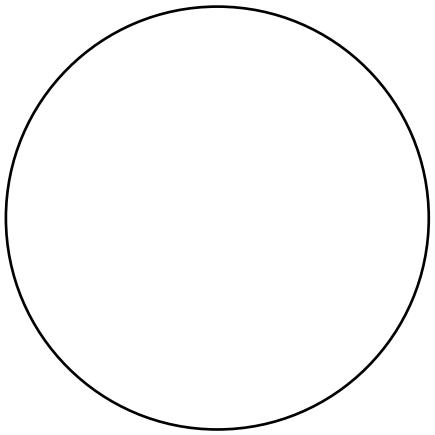
Name: ..... Date: ..... Period: .....

FAST Practice

Problems 4–5: Use the completed table from the previous page.

4. Draw the circle graph that shows the number of pigs of different breeds shown at the county fair. Label each section with the breed and percent.

Pig Breeds



5. Which is an accurate comparison of the breeds of pigs at the county fair?

A. The angle for each breed of pig is $\frac{12}{5}$ times the number of pigs of each breed.

B. The difference in the angle measure for the Duroc and Landrace pigs is exactly $53°$.

C. The ratio of Landrace pigs to Hampshire pigs is greater than the ratio of Duroc pigs to Berkshire pigs.

D. The percent of pigs that are Berkshire is greater than the sum of the percents for any two other breeds.

Spiral Review

6. Calculate the sum of $-\frac{3}{8} + \frac{4}{5}$.

7. Calculate the product of $-\frac{3}{8} \times \frac{4}{5}$.

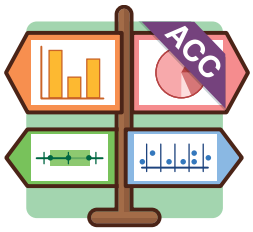
8. Calculate the difference of $-\frac{3}{8} - \frac{4}{5}$.

9. Calculate the quotient of $-\frac{3}{8} \div \frac{4}{5}$.

Name: ..... Date: ..... Period: .....

MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.4.1, MTR.7.1

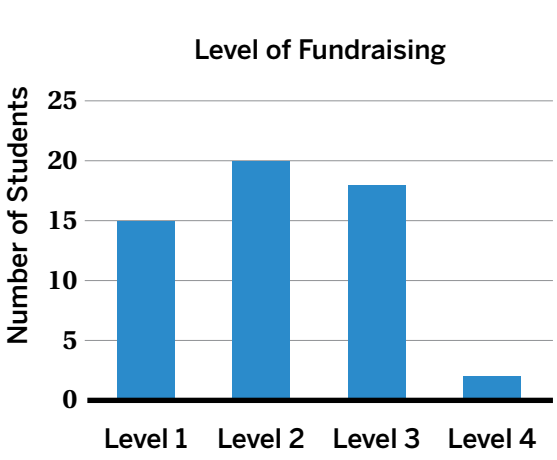
Choices, Choices

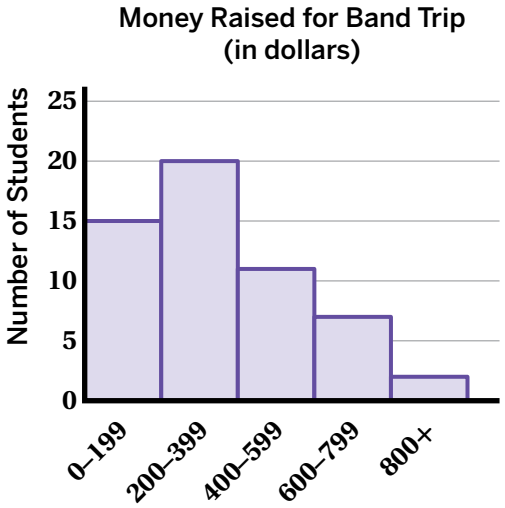


Let's explore how to choose an appropriate representation for data.

Warm-Up

Look at the *bar chart* and *histogram*.





1. How are the graphical displays similar? How are they different?

Activity 1

Name: Date: Period:

Different Ways to Display Data

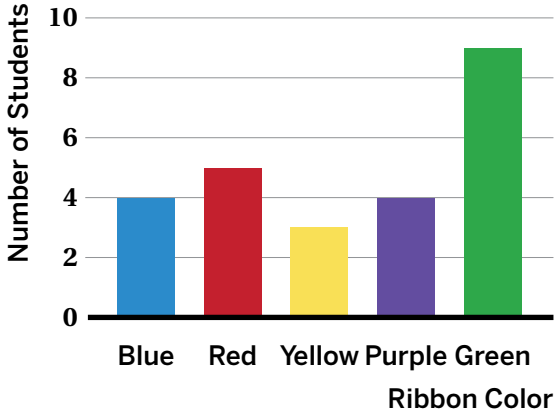
Students in Mr. Bach's class participated in a reading contest. The students earned points based on the number of books they read and the difficulty level of the book. Students received ribbons based on their total number of points.

Each graphical display shows different information about the contest, but they all represent the same set of data.

Here is a *bar chart* and a circle graph.

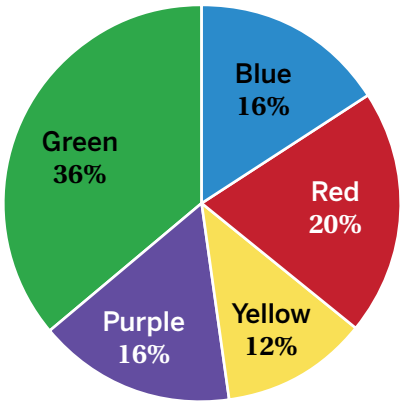
Bar Chart

Reading Contest Ribbons



Circle Graph

Reading Contest Ribbons



2. Do these displays show numerical data or *categorical data*?

3. Which display shows how many students received each color of ribbon? Explain your thinking.

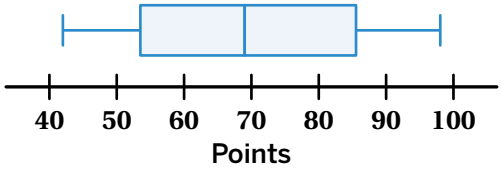
Name: ..... Date: ..... Period: .....

Different Ways to Display Data (continued)

Here is a *box plot* and a *dot plot*.

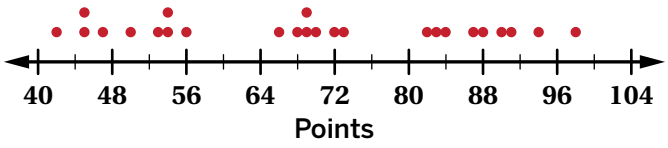
Box Plot

Reading Contest



Dot Plot

Reading Contest



4. Which display would you use to answer each question?

a What is the median?

b What is the least number of points a student earned?

c How many students earned more than 90 points?

Here is a *stem-and-leaf plot* and a *histogram*.

Stem-and-Leaf Plot

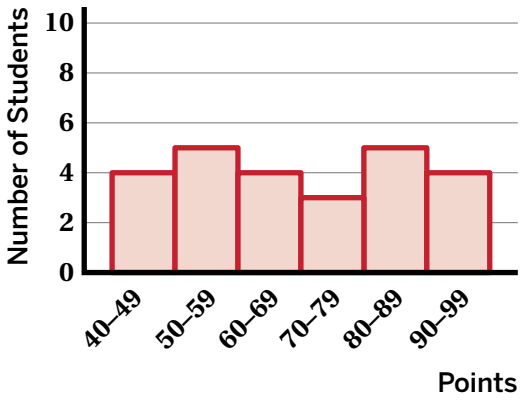
Points Earned in Reading Contest

Stem	Leaf
4	2 5 5 7
5	0 3 4 4 6
6	6 8 9 9
7	0 2 3
8	2 3 4 7 8
9	0 1 4 8

Key: 4 | 2 = 42

Histogram

Reading Contest



5. When would it be better to use a stem-and-leaf plot? When would it be better to use a histogram? Explain your thinking.

Activity 2

Name: Date: Period:

Choose an Appropriate Display

6. The data shows the number of laps swum per day by a random sample of patrons.

{2, 3, 4, 1, 4, 2, 1, 3, 3, 1, 1, 2, 1, 3, 1, 4, 4, 2, 3, 1, 1, 3, 1, 2, 1}

Choose the most appropriate display(s) to show the number of people who swam each number of laps.

Bar chart Box plot Dot plot Histogram Stem-and-leaf plot Circle graph

Explain your thinking.

7. The data shows the most commonly bought dog treat for a sample of dog owners.

Dog Treat	Biscuit	Dental Chew	Jerky Strip	Soft and Chewy
Number of Orders	9	6	4	6

Choose the most appropriate display(s) to compare the number of orders for each treat.

Bar chart Box plot Dot plot Histogram Stem-and-leaf plot Circle graph

Explain your thinking.

8. A researcher wants to analyze the college entrance exam scores for a sample of 500 students.

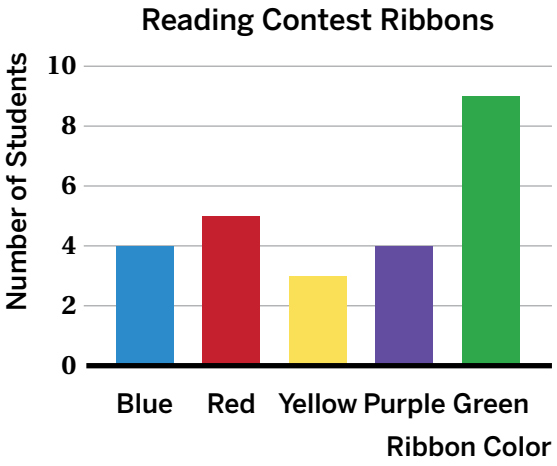
Rafael says that the researcher should use a histogram to show the data, and Kanna says the researcher should use a box plot. Whose thinking is correct? Circle one.

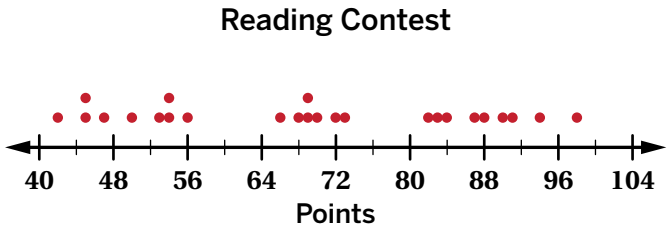
Rafael's Kanna's Both Neither

Explain your thinking.

Synthesis

9. Explain how these displays represent data differently.





Lesson Practice ACC7.8.02

Lesson Summary

We can use bar charts and circle graphs to show *categorical data*.

- *Bar charts* show the number of items in categories.
- Circle graphs show how categories represent parts of a whole.

We can use box plots, dot plots, histograms, and stem-and-leaf plots to show numerical data.

- *Box plots* separate data into quartiles and show the minimum and maximum values.
- *Dot plots* show how many times each data value occurs.
- *Stem-and-leaf plots* show all numerical data values in a condensed list.
- *Histograms* show numerical data that is continuous in equal intervals.

Histograms and box plots are most effective for representing large amounts of data. Dot plots and stem-and leaf plots are useful when individual data points are needed for calculations such as determining the mean.

Name: _____ Date: _____ Period: _____

Problems 1–4: Choose the appropriate graphical display.

1. The data shows the systolic blood pressure number for a random sample of patients at a hospital.

Systolic Blood Pressure									
149	147	109	117	147	93	125	115	123	129
123	137	115	120	133	137	123	96	126	114

Which type of display makes it easiest to see the mode?

(A.) Bar chart

(B.) Dot plot

(C.) Stem-and-leaf plot

(D.) Histogram

2. The data shows the field trip preferences for a random sample of students.

Which type of display would you use to compare the percentages of students who selected each field trip location?

Destination	Number of Students
Anaheim	3
Palm Springs	5
San Diego	13
San Francisco	4

(A.) Bar chart

(B.) Box plot

(C.) Histogram

(D.) Circle graph

3. The data shows the weights, in pounds, of a random sample of Chihuahuas.

{3.7, 4.7, 5.2, 3.9, 6.1, 5.8, 5.7, 4.5, 4.3, 5.6, 3.4, 6.9, 5.2, 3.3, 5.1, 4.2, 3.2, 4.4, 3.6, 6.1}

Which type of display should be used to represent the continuous data?

(A.) Bar chart

(B.) Dot plot

(C.) Stem-and-leaf plot

(D.) Histogram

Name: _____ Date: _____ Period: _____

Problems 4–6: Choose an appropriate display for each situation. Explain your reasoning.

4. The favorite scent of customers for a new line of body lotions

5. The median salary of teachers in a school district

6. The test scores for each student on a quiz

FAST Practice

7. Choose *all* the situations that a dot plot would be a good choice to display the data.

- ☐ **A.** Movie genre preferences
- ☐ **B.** Heights of cactus plants in a nursery
- ☐ **C.** Ages of students in a chess club
- ☐ **D.** Number of students in each class
- ☐ **E.** Percent of students in favor of, against, or neutral regarding a new lunch item

Spiral Review

8. Tariq buys a notebook for $\$3.78$ and 2 pens that cost $\$1.15$ each. He pays with a $\$20$ bill. How much change will he get back?

9. The walking path around a pond is $\frac{5}{6}$ of a mile. Kayleen walks around the pond 3 times. How many miles does Kayleen walk in all?

Name: ..... Date: ..... Period: .....

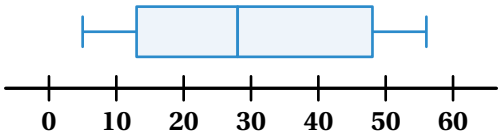
MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.7.1

On Display

Let's explore how to create an appropriate representation for data.

Warm-Up

The box plot and stem-and-leaf plot both show the number of boxes of protein bars sold by each student in a class.



Stem	Leaf
0	5 5 6 7
1	2 2 4 8 9
2	0 2 5 8
3	0 5 7
4	3 6 8 8
5	1 1 3 4 6

Key: $1 \mid 5 = 15$

1. What information do you need to know to create each display?

Activity 1

Name: Date: Period:

Choose a Display

2. The data shows the cracker flavor preferences for a random sample of students.

garlic	avocado	sour cream	avocado	avocado
avocado	sour cream	avocado	avocado	sour cream
garlic	avocado	avocado	original	garlic
avocado	garlic	avocado	sour cream	sour cream
original	garlic	garlic	garlic	avocado

a Complete the table to show the frequency of each flavor preference.

Flavor	Number of Students
Avacado	
Garlic	
Original	
Sour Cream	

b Select an appropriate display to represent the data and justify your choice.

c Create the display.

d Create a question that can be answered by using the display. Then provide the answer.

Name: ..... Date: ..... Period: .....

Choose a Display (continued)

3. The data shows the distances traveled, in feet, by small cars designed and built by an engineering class.

8.09	0.5	5.61	8.36	33.68	16.71	12.4	3.19	4.72	29.13
33.92	36.16	23.44	28.61	36.13	33.22	1.18	34.15	12.93	22.04

a Select an appropriate display to represent the data and justify your choice.

b Create the display.

c Create a question that can be answered by using the display. Then provide the answer.

Name: Date: Period:

Same Data, Different Display

4. The stem-and-leaf plot shows the quiz scores for students in an English class. Create a different display that shows the same data.

Stem-and-Leaf Plot

Stem	Leaf
3	0 1 5
4	3 4 5
5	5 8
6	3 4
7	1 6
8	1 1 4 5 6 6
9	0 0 0 7 7 9 9

Key: $3 \mid 0 = 30$

My Display

5. A new pet store owner gives away 25 small gifts worth varying amounts on her first day of business as part of a promotional effort. The data shows how much each gift is worth (in dollars).

$$15, 20, 25, 5, 10, 10, 5, 20, 5, 5, 25, 5, 20, 20, 25, 10, 5, 20, 5, 25, 5, 15, 15, 15, 10$$

Create two different displays that can be used to represent the data.

First Display

Second Display

Synthesis

6. What graphical displays can you create to represent categorical data and numerical data? Use the data shown if it helps with your thinking.

Wall Paint Color Preferences

blue, green, gray, taupe, taupe, green, blue, blue, blue, gray, gray, green, taupe, green, blue, gray, taupe

Weekly Amount Spent on Lunch ($)

30, 50, 40, 40, 25, 60, 30, 20, 50, 30, 20, 10, 20, 25, 20, 40, 50, 35, 40, 20

Lesson Practice ACC7.8.03

Lesson Summary

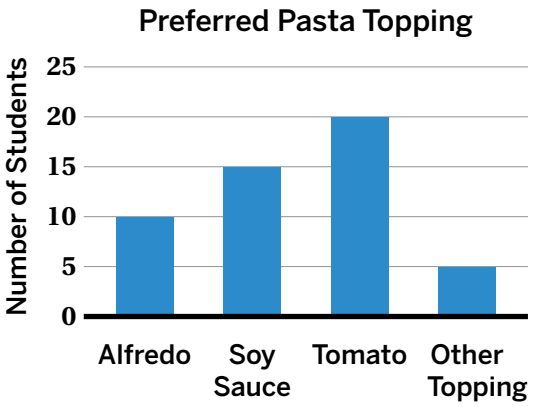
There are different ways to display data.

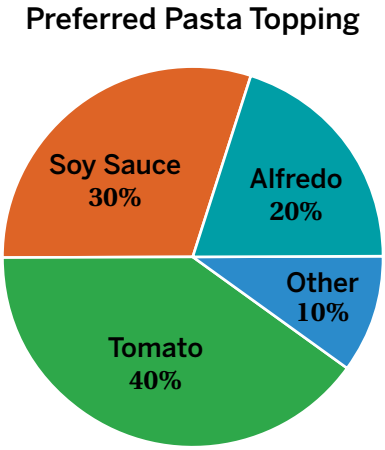
- When creating a bar chart or a histogram, it is helpful to create a frequency table first to track how many times a data value appears individually or within an interval. The heights of the bars or bins show the frequency of data.
- To create a circle graph when percentages are not given, create a relative frequency table to show what part of the whole each category represents.
- For box plots, you must first identify the minimum, maximum, and quartiles of the data.
- To create a line plot, place a dot above a number on the number line each time that data value occurs.
- To create a stem-and-leaf plot, organize the data from least to greatest, then list the data using the digits of the numbers as the stems and leaves.

Lesson Practice
ACC7.8.03

Name: ..... Date: ..... Period: .....

1. Look at the bar chart and circle graph.





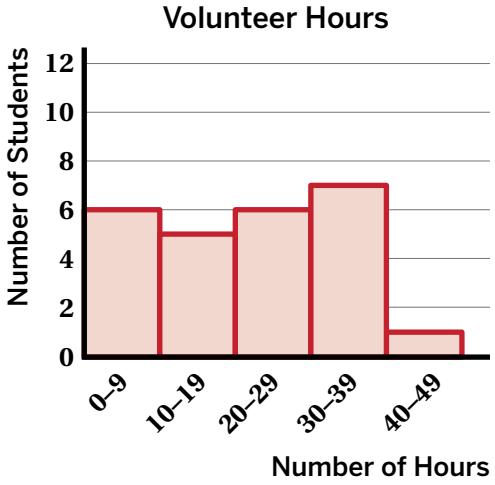
Do the displays show the same data? Explain your thinking.

2. The data shows the number of hours students in a random sample volunteer per month at different organizations.

4, 28, 6, 8, 34, 21, 14, 34, 17, 2, 6, 35, 37, 40, 6, 31, 14, 24, 25, 13, 25, 37, 14, 28, 32

LaShawn represents the data using the histogram shown.

Is LaShawn's histogram correct? Explain your thinking.



3. The data shows the scores on a band playing test.

30, 34, 35, 42, 43, 43, 47, 48, 52, 53, 55, 65, 65, 67, 68, 71, 73, 74, 78, 80, 82, 82, 83, 87, 91

Create a display that you could use to show the interquartile range of the scores.

Name: ..... Date: ..... Period: .....

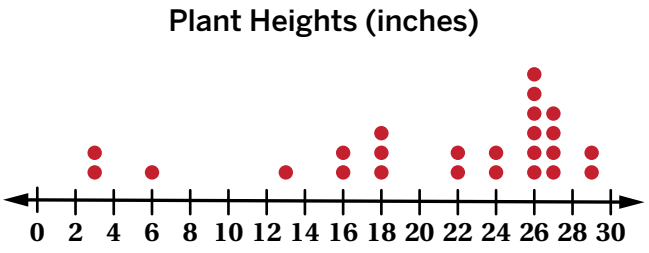
4. The data shows the grade levels of students who are participating in a community cleanup effort.

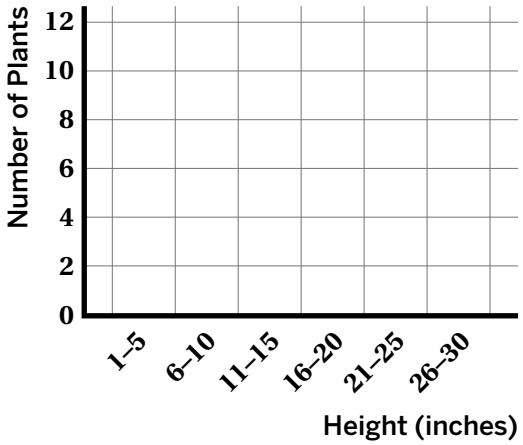
4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 8

Create an appropriate display for the data. Explain how your display might be used to answer questions.

FAST Practice

5. The line plot shows the plant heights in a science experiment. Create a histogram to represent the same data.





Spiral Review

Problems 6–8: Determine the value of the expression.

6. $\frac{1}{2}\left|3-\frac{1}{4}\right|+3^3$

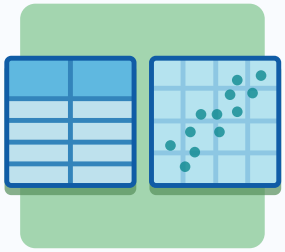
7. $12 \div (6-4)^2 - 8$

8. $(14.85 \div 3 + 6) \times 2$

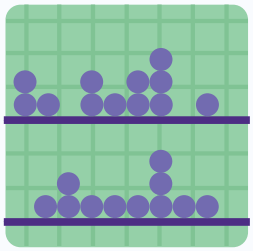
Sub-unit

2

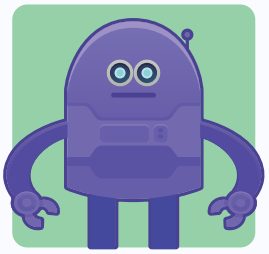
Organizing Numerical Data



Lesson 4
Click Battle



Lesson 5
Birds, Birds, Birds



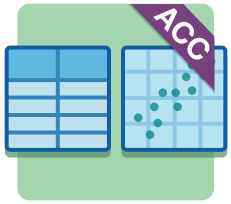
Lesson 6
Robots

Name: ..... Date: ..... Period: .....

MA.8.DP.1.1, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

Click Battle

Let's find ways to show patterns in data.



Warm-Up

1. Tap your pencil on your desk as many times as you can for 2 seconds. Record your number of taps.

2. Tap your pencil on your desk as many times as you can for 6 seconds. Record your number of taps.

Name: ..... Date: ..... Period: .....

Organizing Data

3. Let's look at some class data about button clicks, which are similar to pencil taps.

List

14 clicks in 2.5 seconds	24 clicks in 6.5 seconds
17 clicks in 3 seconds	30 clicks in 5.5 seconds
16 clicks in 4 seconds	24 clicks in 4.5 seconds
42 clicks in 8 seconds	36 clicks in 7 seconds
22 clicks in 5.5 seconds	23 clicks in 4 seconds

Discuss: What patterns do you see in the data?

4. Let's look at one way to represent the data. What do you notice?

Table

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

Name: ..... Date: ..... Period: .....

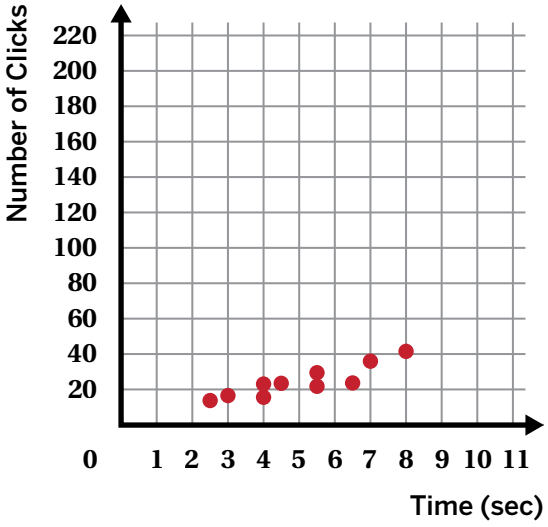
Organizing Data (continued)

5. Let's look at another way to represent the data.

Table

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

Scatter Plot



Discuss: What connections do you see between the **scatter plot** and the table?

Name: ..... Date: ..... Period: .....

Make a Prediction

Here is click data organized as a list, table, and scatter plot.

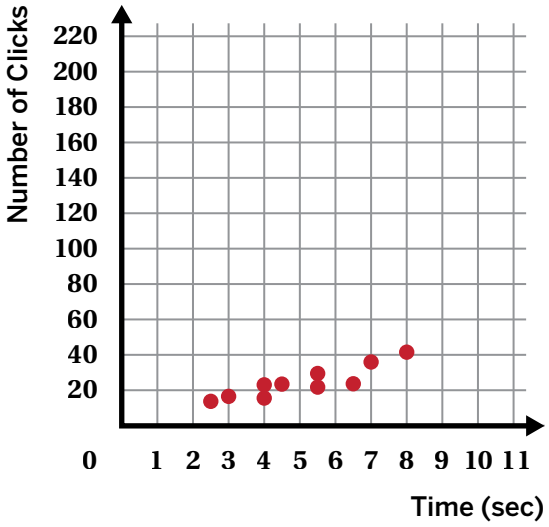
List

14 clicks in 2.5 seconds	24 clicks in 6.5 seconds
17 clicks in 3 seconds	30 clicks in 5.5 seconds
16 clicks in 4 seconds	24 clicks in 4.5 seconds
42 clicks in 8 seconds	36 clicks in 7 seconds
22 clicks in 5.5 seconds	23 clicks in 4 seconds

Table

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

Scatter Plot



6. Select a representation and use it to answer this question:

How many clicks do you think a typical student in your class would make in 10 seconds?

Explain your thinking.

7. Test your prediction by counting the number of pencil taps you can make in 10 seconds.

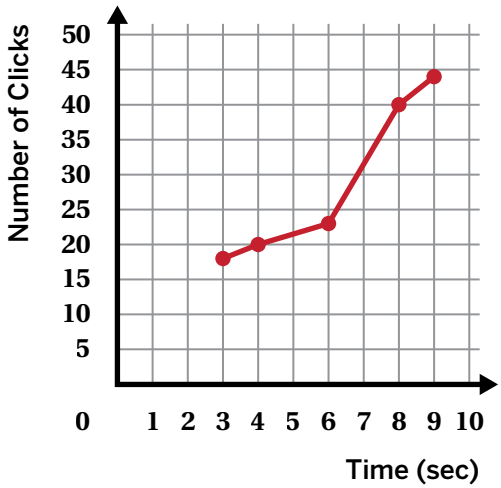
Activity 3

Name: ..... Date: ..... Period: .....

Line Graphs

Here is a set of click data for Yolanda, shown in a table and a **line graph**.

Time (sec)	Number of Clicks
3	18
4	20
6	23
8	40
9	44



8. **a** Determine the rate of change between 3 and 4 seconds and between 6 and 8 seconds.

b **Discuss:** How does the line graph compare to the scatter plot of similar data?

c Write a story about Yolanda's clicks.

Synthesis

9. What are some advantages of using a list, a table, a scatter plot, or a line graph to organize data?

Lesson Practice ACC7.8.04

Lesson Summary

You can organize and display data that includes numbers in different ways, including in a table and in a scatter plot. A **scatter plot** is a set of disconnected data points plotted on a coordinate plane.

A table and a scatter plot both display the same data, but can be helpful in different ways. For example, you can use a scatter plot to investigate connections between two variables, while a table is helpful for looking for the exact values of specific data points.

Another way to show data is with a line graph. Like a scatter plot, a **line graph** displays data points on a coordinate grid, but the points are connected. This can help to show rates of change between pairs of points or within sections of the graph. Line graphs show trends and are useful for showing how data changes over time.

Lesson Practice
ACC7.8.04

Name: ..... Date: ..... Period: .....

Problems 1–3: Here is data on the number of number of music records produced by a certain company from 1944 to 1955.

Year	Number of Records
1944	109,873
1945	133,792
1946	109,860
1947	156,517
1948	74,715
1949	64,479
1950	120,718
1951	68,687
1952	45,030
1953	37,129
1954	60,866
1955	62,786

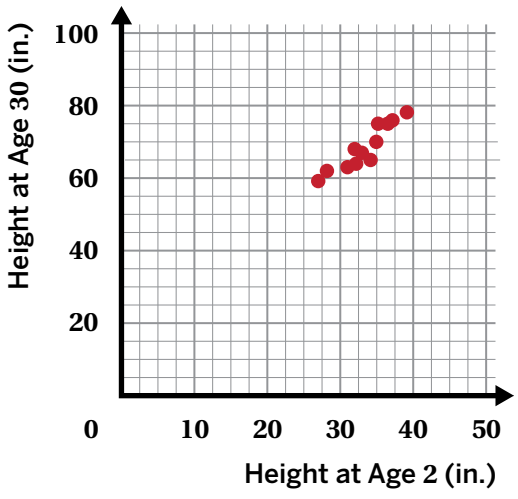
1. Describe another way to sort this table. What is a question that can be answered when the table is sorted this way?

2. Which years in this period of time had more than 100,000 records produced?

3. Based on this data, would you expect 1956 to have closer to 50,000 records or 100,000 records? Explain your thinking.

Problems 4–5: A research study measured the heights of twelve people on their birthday at age 2 and at age 30.

4. What patterns do you notice in the data?

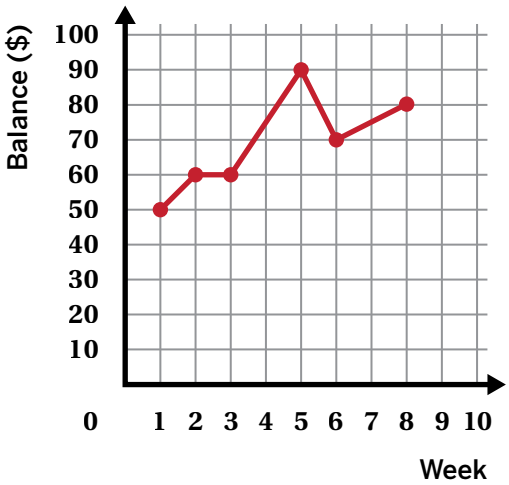


5. A two-year-old has a height of 38 inches. Based on this data, predict their height at age 30.

Name: ..... Date: ..... Period: .....

FAST Practice

6. The line graph shows the balance in a bank account over time.



Select *all* the true statements about the bank account.

- ☐ **A.** The balance in the account went up each week at a constant rate.
- ☐ **B.** The rate of change from week 3 to week 5 was 30 dollars per week.
- ☐ **C.** The rate of change from week 5 to week 6 was -20 dollars per week.
- ☐ **D.** The maximum account balance occurred in week 5.
- ☐ **E.** The point $(1, 50)$ indicates that the account had $\$50$ in week 1.

Spiral Review

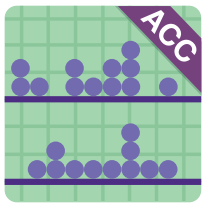
7. Solve this system of linear equations. Show your thinking.

$$\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$$

Name: ..... Date: ..... Period: .....

MA.8.DP.1.1, MTR.2.1, MTR.4.1, MTR.7.1

Birds, Birds, Birds



Let's compare dot plots and scatter plots.

Warm-Up

1. The table shows the average height and weight of seven different bird species.

Height (in.)	Weight (oz.)
12	3.5
9	1.5
14	6.4
16	13
19	24
25	56
34	64

Discuss:

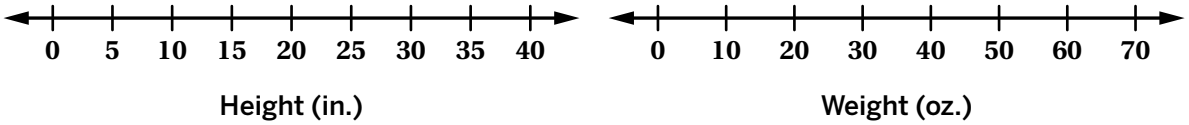
- How could we reorganize the data to make it more useful for analyzing?
- What are some questions this data could help answer?

Activity 1

Name: Date: Period:

Visualizing Data

2. Plot points on the *dot plots* to represent the birds' heights and weights.



3. Let's look at dot plots of height and weight data.

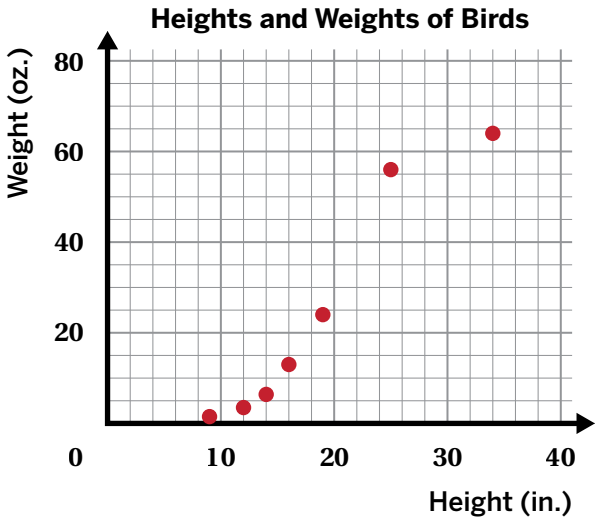
a What is a question you *can* answer based on the dot plots?

b What is a question you *cannot* answer based on the dot plots?

Name: ..... Date: ..... Period: .....

Visualizing Data (continued)

4. Let's look at a scatter plot that represents height and weight data.



a What is a question you *can* answer based on the scatter plot?

b What is a question you *cannot* answer based on the scatter plot?

Synthesis

5. What are some advantages of using a dot plot or a scatter plot to represent data? Use the examples in the Summary if they help with your thinking.

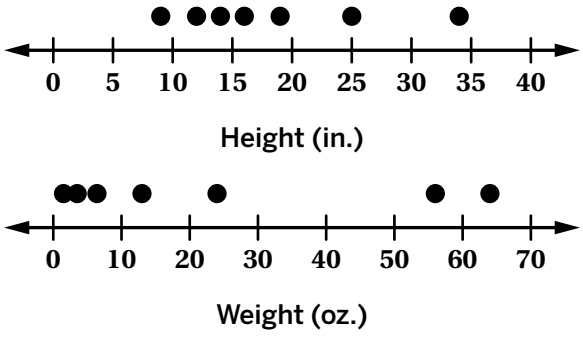
Lesson Practice ACC7.8.05

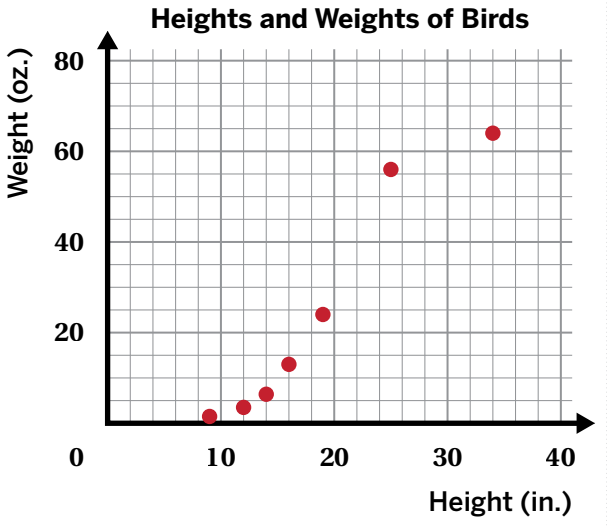
Lesson Summary

Data presented as numbers, quantities, or measurements that can be compared in a meaningful way is called *numerical data*, or *quantitative data*. You can investigate *univariate data*, which involves one variable, and *bivariate data*, which involves two variables.

There are different ways to represent numerical data. A *dot plot*, also known as a line plot, shows data for one variable and a scatter plot shows data for two variables at the same time. Seeing two numerical variables at the same time allows us to notice trends and connections.

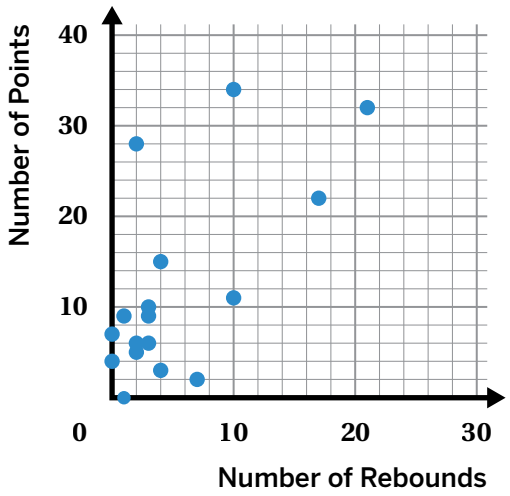
Dot Plot	Scatter Plot





Name: _____ Date: _____ Period: _____

Problems 1–5: This scatter plot shows the number of rebounds and points for each player in a recent professional basketball game.



1. Circle the point on the graph that represents the player with the most rebounds. How many rebounds did that player have?

2. How many players had 0 rebounds?

3. What is another question you *can* answer based on this scatter plot?

4. What is a question you *cannot* answer based on this scatter plot?

5. The table shows the data for another basketball player. Plot the point for the player on the graph.

Number of Rebounds	Number of Points
20	15

6. Which representation(s) are appropriate for comparing the heights of students on a volleyball team to the heights of students on a soccer team?

A. Scatter plot

B. Dot plots

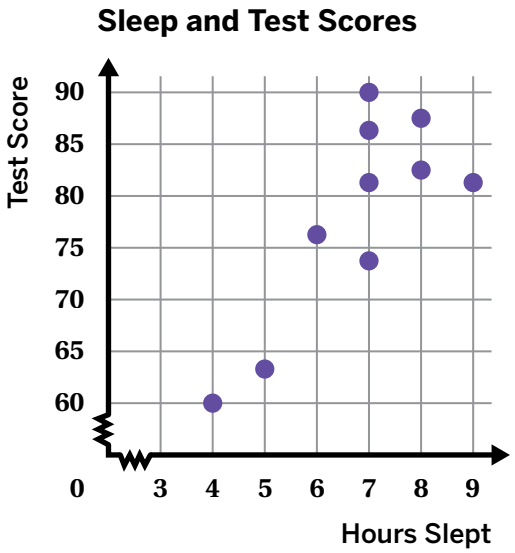
C. Both

Explain your thinking.

Name: ..... Date: ..... Period: .....

FAST Practice

7. A teacher collected data about her students' test scores and how many hours they slept the night before a test. She represented the data with a scatter plot.



Select *all* of the questions that you can answer from the scatter plot.

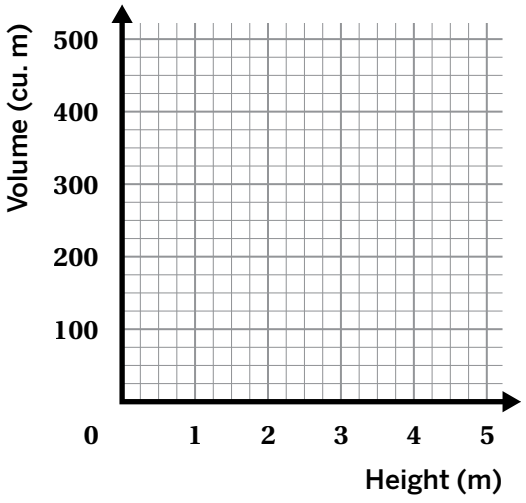
- ☐ **A.** How many students slept for 7 hours?
- ☐ **B.** How many students scored higher than an 80 on the test?
- ☐ **C.** Who got the highest score in the class?
- ☐ **D.** Is there a relationship between hours slept and test scores?
- ☐ **E.** What other factors influenced test scores?

Spiral Review

Problems 8–10: There are many cylinders with a radius of 6 meters. Let h represent the height in meters and V represent the volume in cubic meters.

8. Write an equation that represents the volume, V, as a function of the height, h.

9. Sketch the graph of the equation, using 3.14 as an approximation for π.



10. If you double the height of a cylinder, what happens to the volume? Use the equation to help you explain your thinking.

Name: ..... Date: ..... Period: .....

MA.8.DP.1.1, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

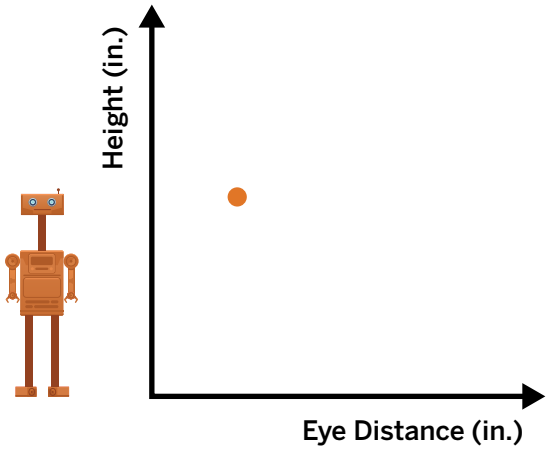
Robots

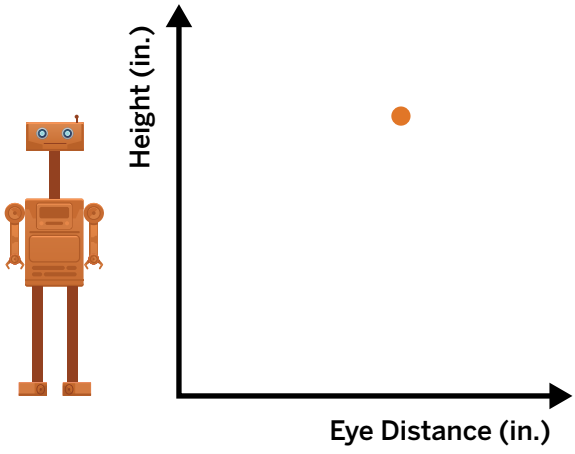
Let's investigate points on a scatter plot.

Warm-Up

1. Here are two graphs and images of a robot.

Discuss: What do you notice?

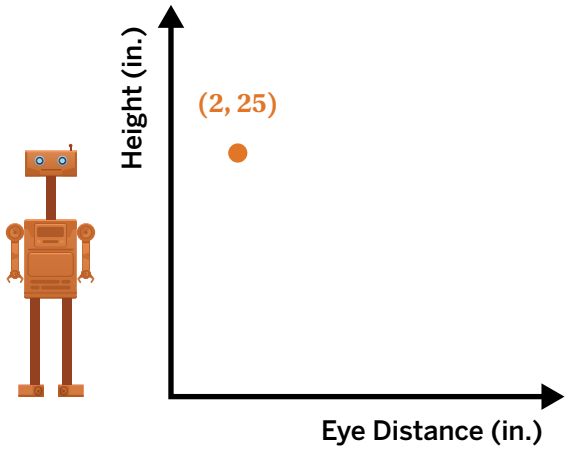




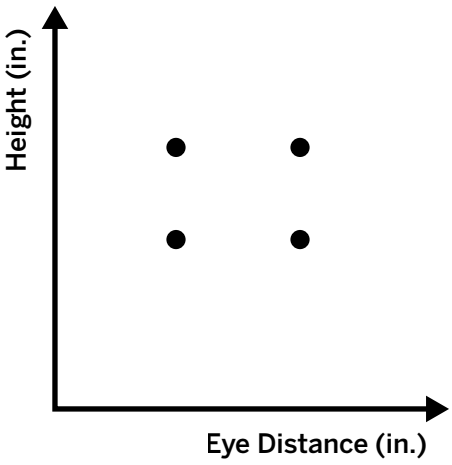
Name: ..... Date: ..... Period: .....

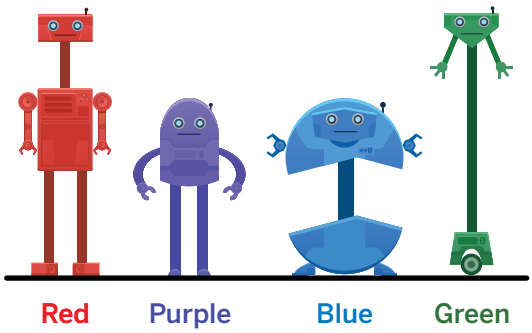
Robots

2. Describe something you know about the robot based on the graph.



3. Here are four different robots. Label each point on the graph with the color robot it represents.





Name: ..... Date: ..... Period: .....

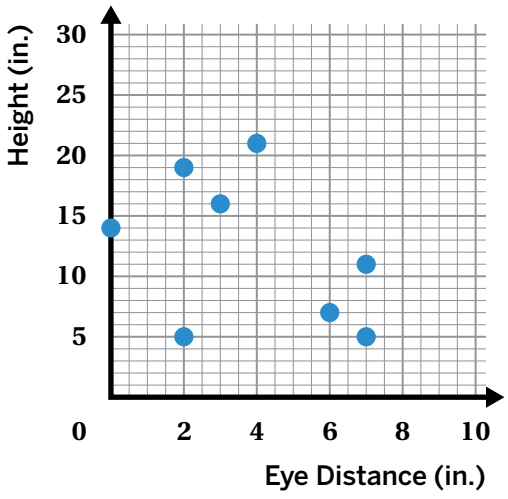
Robots (continued)

4. The table shows the heights and eye distances for five different robots. Plot a point to represent the pink robot.

Robot Color	Eye Distance (in.)	Height (in.)
Teal	2	30
Black	4	10
Gray	8	10
Orange	6	20
Pink	8	20

5. The graph shows the heights and eye distances for eight blue robots.

Plot a point for a red robot to make this statement true: *The red robot is taller than all the blue robots, and its eye distance is 5 inches.*



Activity 2

Name: Date: Period:

Challenge Creator

6. You will use a set of cards with scatter plots to create your own challenge.

a Choose one card that interests you and plot a point somewhere you think is interesting.

b On this page, write the point as an ordered pair. Then tell a story about this point.

My Point	My Story

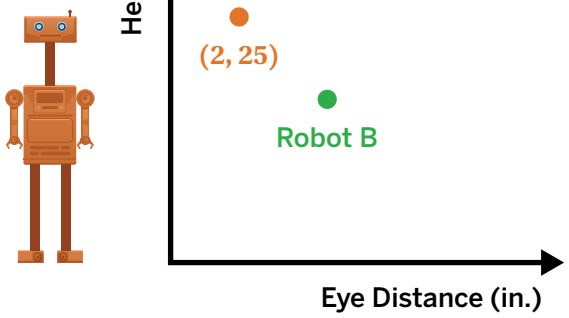
c Swap your challenge with one or more partners. Write the point they plotted as an ordered pair and tell a story about it.

	Point	Story
Partner 1		
Partner 2		
Partner 3		

Synthesis

7. This graph shows the height and eye distance for two robots.

Describe some things you know about Robot B given the information about Robot A.

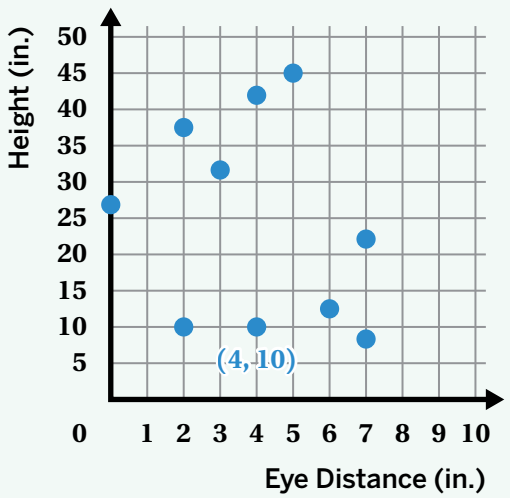


Lesson Practice ACC7.8.06

Lesson Summary

A point on a scatter plot represents two pieces of information. The axes labels tell you how to interpret the coordinates of each point.

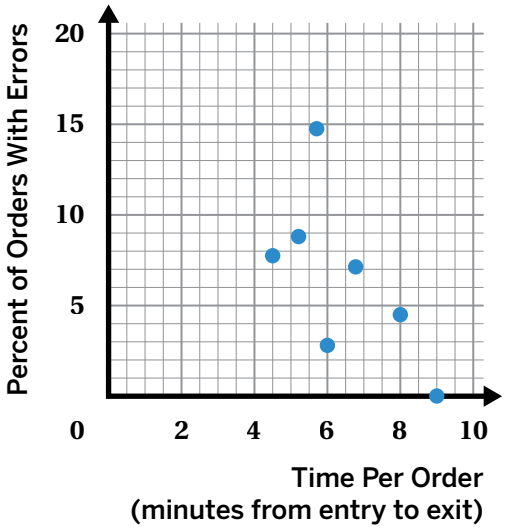
In this example, the point $(4, 10)$ represents a robot with an eye distance of 4 inches and a height of 10 inches.



Name: _____ Date: _____ Period: _____

Problems 1–3: A study gathered data about different drive-thru restaurants. The table and scatter plot show the average time per order and the percent of orders with errors for each restaurant.

Restaurant	Time Per Order (min)	Percent of Orders With Errors
CraveBite	8	4.5
Taco Tango	6	2.8
Bite Master	9	0
Noodle Nest	4.5	7.7
Burger Whiz	5.2	8.8
Pajaro	6.8	7.1
NachoLoco	5.7	14.7

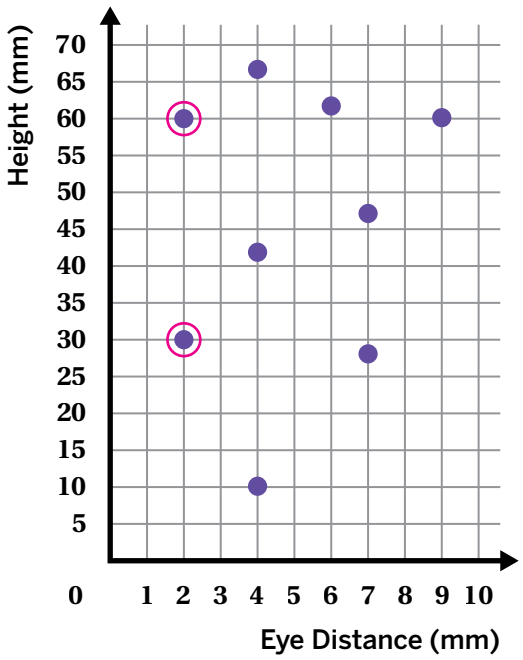


1. Circle the point on the scatter plot that represents the data for NachoLoco.

2. What does the point $(6, 2.8)$ represent?

3. In the same study, the data showed that Dumpling Delight takes 4.5 minutes per order and 13% of their orders had errors. Add a point to the scatter plot to represent Dumpling Delight.

4. Circle the point(s) for the robot(s) with the shortest eye distance. Write the height and eye distance of each point you circled.

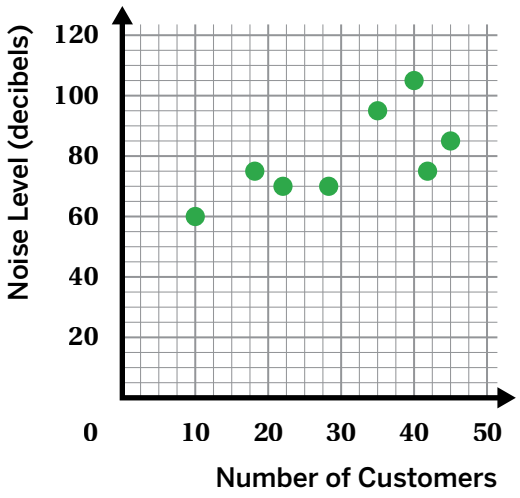


Lesson Practice
ACC7.8.06

Name: ..... Date: ..... Period: .....

FAST Practice

Problems 5–7: This scatter plot shows the noise level and the number of customers for eight restaurants.



5. What is the noise level at the loudest restaurant?

6. What is the noise level at the restaurant with the most customers?

7. The noise level at a restaurant with 35 customers is 80 decibels. What are the coordinates of the point that represents this restaurant on a graph? Record your answer in the space provided.

Spiral Review

8. Select *all* the representations that are appropriate for comparing exam score to hours of sleep the night before an exam.

- ☐ **A.** Histogram
- ☐ **B.** Scatter plot
- ☐ **C.** Dot plot
- ☐ **D.** Table
- ☐ **E.** Box plot

Problems 9–12: Evaluate each expression.

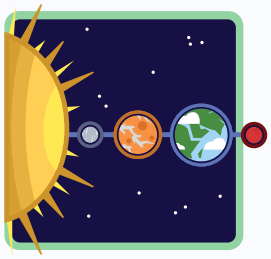
9. $-2 \cdot (-4)$ **10.** $-7 \cdot 2$ **11.** $9 \cdot (-10)$ **12.** $-2 \cdot (-6) \cdot (5)$

Sub-unit

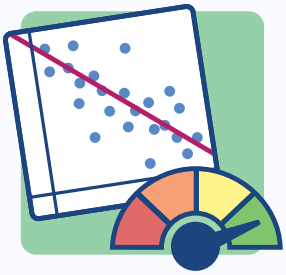
3

Analyzing Numerical Data

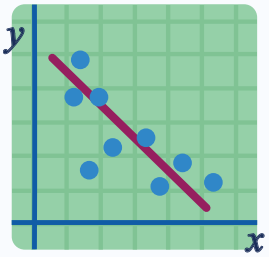
Lesson 7
Dapper Cats



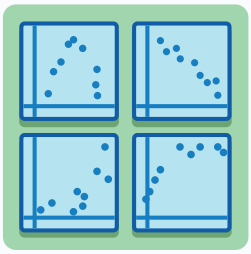
Lesson 8
Interpreting Scatter Plots



Lesson 9
Find the Fit



Lesson 10
Interpreting Slopes



Lesson 11
Scatter Plot City

Lesson 12
Animal Brains

Name: ..... Date: ..... Period: .....

MA.8.DP.1.2, MA.8.DP.1.3, MTR.4.1, MTR.6.1, MTR.7.1

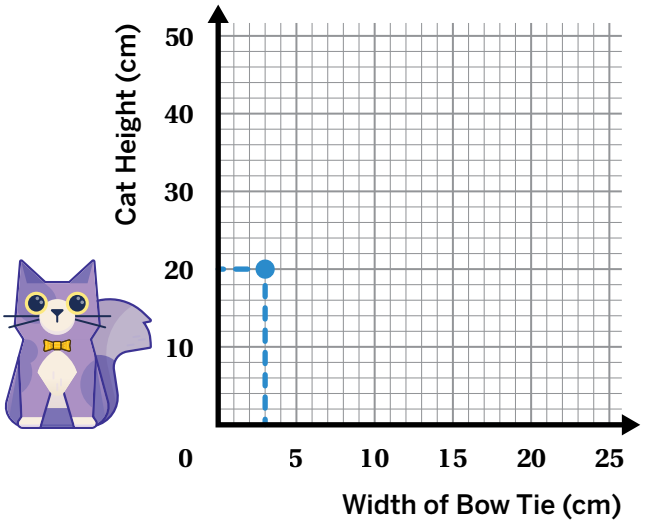
Dapper Cats

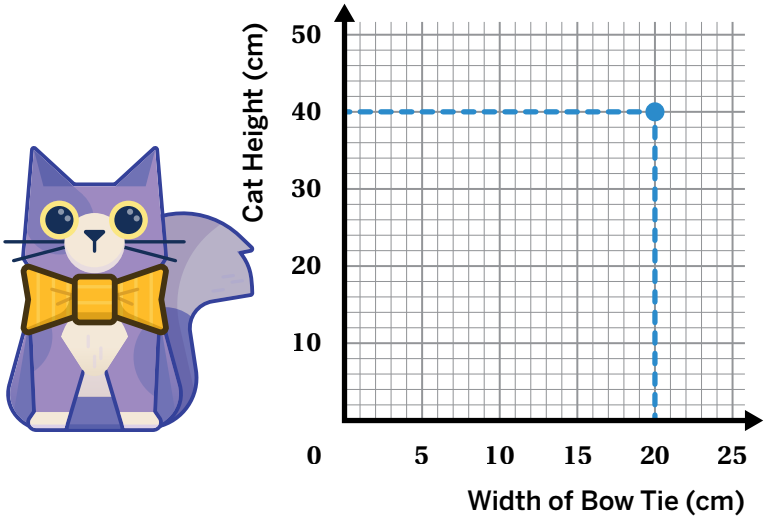
Let's identify potential outliers and use a linear model to predict values.

Warm-Up

1. Here are two toy cats built at the Build-a-Cat workshop.

Discuss: What do you notice? What do you wonder?





Name: ..... Date: ..... Period: .....

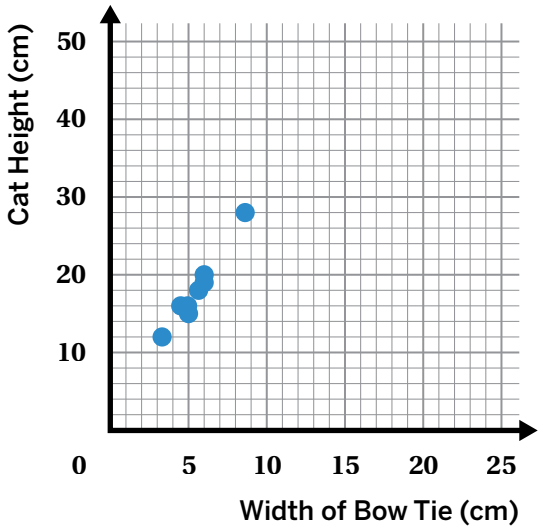
Dapper Cats

2. Some of these toy cats are dapper and some are not.

What do you think makes a toy cat "dapper"? What makes a toy cat "not dapper"?

3. This scatter plot shows many "dapper cat" orders.

A customer just ordered a dapper cat that is 24 centimeters tall. Plot a point on the scatter plot to represent this cat.

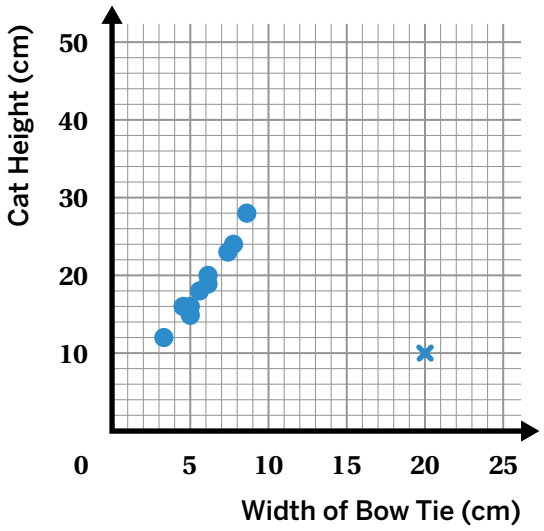


4. Describe what you notice about the scatter plot.

Name: ..... Date: ..... Period: .....

Dapper Cats (continued)

5. Plot a point to represent a dapper cat that is taller than any of the other cats in the scatter plot.



6. Plot a point to represent a not-dapper cat that is very short with a very large bow tie.

7. A student plotted the point $(20, 10)$. It is shown on the scatter plot with an X. This point is an **outlier** because it is far from the rest of the data.

a Plot another outlier on the scatter plot.

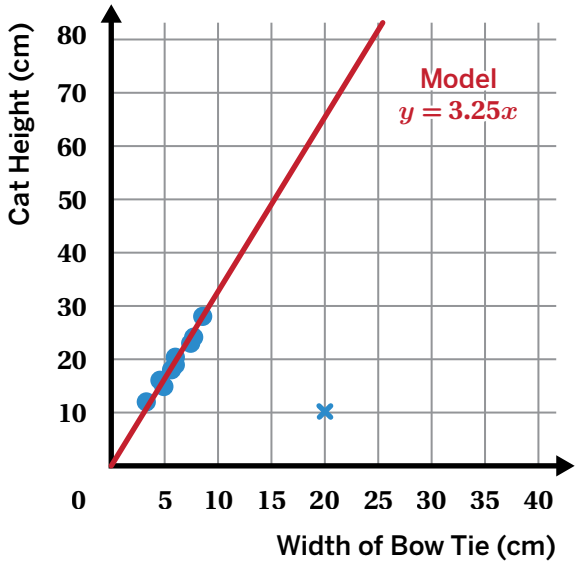
b Describe the cat your point represents.

Name: ..... Date: ..... Period: .....

Using a Linear Model to Predict Data

A **linear model** (also called a *line of fit*) can be used to help identify trends in data and to make predictions. The line and equation model the relationship between bow tie width, x, and cat height, y.

8. Use the linear model to predict the height of a dapper cat with a 12-centimeter bow tie.



9. Another dapper cat is 65 centimeters tall. Use the linear model to predict the width of its bow tie.

10. Lucy's cat has a 60-centimeter bow tie width. What does the linear model predict for its height?

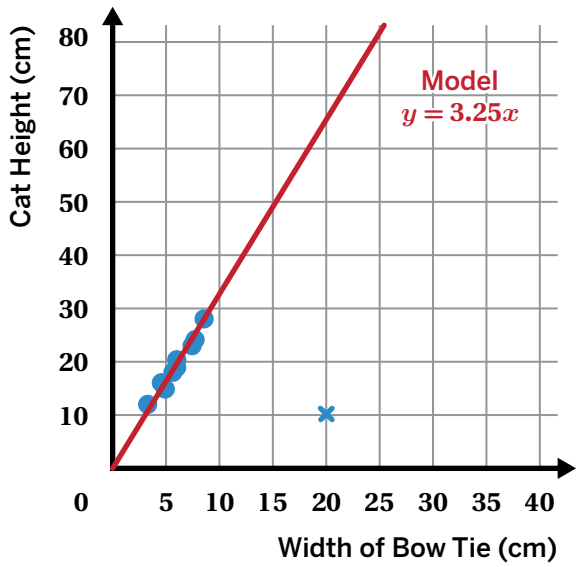
Explain your thinking.

Synthesis

11. A line of fit and an equation are two ways to represent a linear model.

a How can a linear model be helpful?

b How can you identify an outlier on a scatter plot?



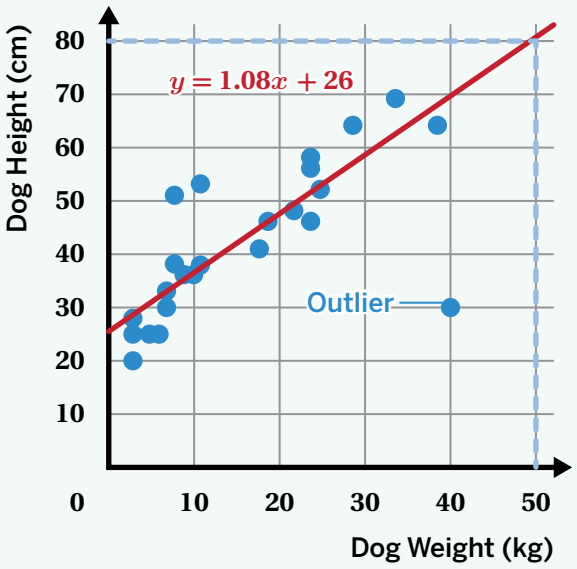
Lesson Practice ACC7.8.07

Lesson Summary

A **linear model** is a line on a scatter plot that helps identify trends in data more clearly. You can also use a linear model to make a prediction.

For example, there are two ways you can use this linear model to predict a dog's height when it weighs 50 kilograms.

- Use the graph to locate 50 on the x-axis and follow it up to meet the linear model, which shows a y-value of 80. This means when the dog's weight is 50 kilograms, its height is 80 centimeters.
- Use the equation for the linear model, $y = 1.08x + 26$, by replacing x with 50 and evaluating for y, which is approximately 80 centimeters.

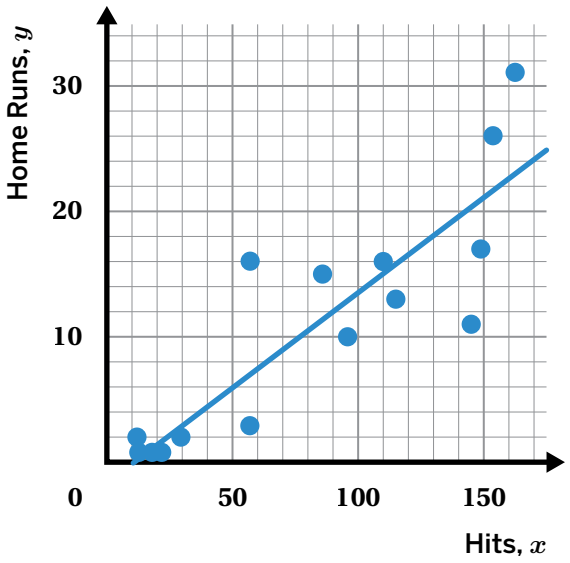


You can identify an **outlier** by looking for points that are far away from the other points and from the predicted values. The point (40, 30) is an outlier on the graph of dog weights and heights.

Lesson Practice
ACC7.8.07

Name: ..... Date: ..... Period: .....

Problems 1–3: This scatter plot shows the number of hits and home runs for 15 baseball players last season and the linear model $y = 0.15x - 1.5$.

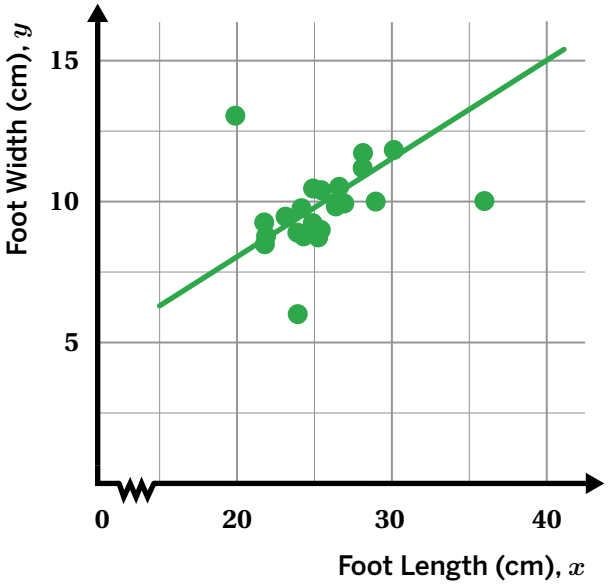


1. How many home runs did the player with 154 hits have?

2. If a player has 20 home runs, how many hits does the linear model predict they will have?

3. How many home runs does the linear model predict a player with 250 hits will have?

Problems 4–5: This scatter plot shows several foot lengths and widths, along with a linear model represented by the equation $y = 0.35x + 1$.



4. Use the linear model's equation to predict the width of a foot that is 50 centimeters long.

5. Does the scatter plot appear to have any outliers? If so, circle them and describe what they represent about foot length and foot width.

6. In your own words, what does an outlier represent?

Lesson Practice
ACC7.8.07

Name: _____ Date: _____ Period: _____

FAST Practice

7. This scatter plot shows the weight and age for many dogs of a certain breed.

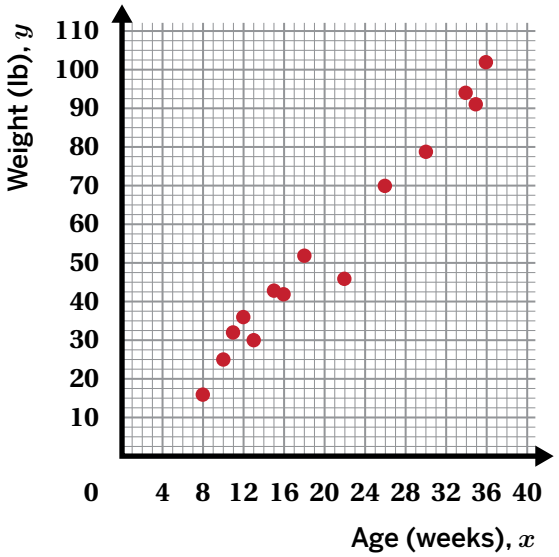
Which is the best prediction for the weight of a dog that is 24 weeks old?

(A.) 10 pounds

(B.) 45 pounds

(C.) 60 pounds

(D.) 90 pounds

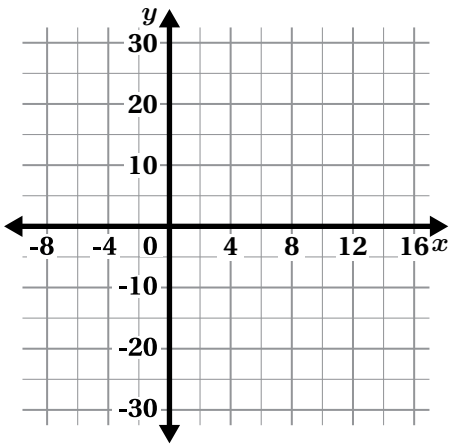


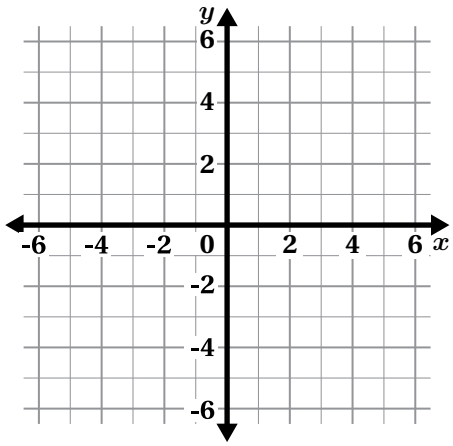
Spiral Review

Problems 8–9: Solve each system of equations by graphing and using an algebraic method of your choice. Write the solution as an ordered pair.

8. $\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$

9. $\begin{cases} y = x + 1 \\ y = -x + 5 \end{cases}$





Name: ..... Date: ..... Period: .....

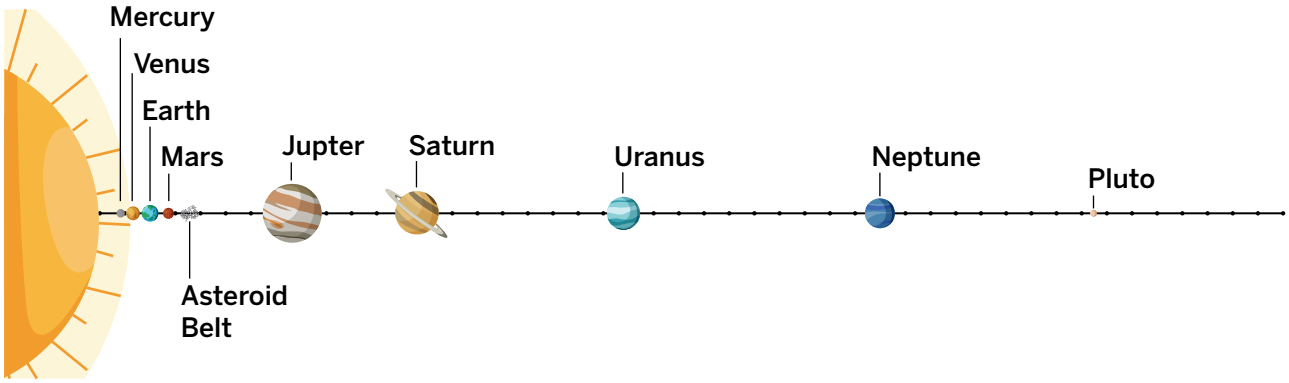
MA.8.DP.1.1, MA.8.DP1.3, MTR.4.1, MTR.6.1, MTR.7.1

Interpreting Scatter Plots

Let's find ways to show and identify patterns in data.

Warm-Up

1. Each planet in our solar system is in a unique orbit around the Sun.



The table shows the distance of each planet from the Sun and its mean temperature. What do you notice? What do you wonder?

Planet	Distance from Sun (106 km)	Mean Temperature (°C)
Mercury	57.9	167
Venus	108.2	464
Earth	149.6	15
Mars	228.0	-65
Jupiter	778.5	-110
Saturn	1432.0	-140
Uranus	2867.0	-195
Neptune	4515.0	-200

I notice:

I wonder:

Activity 1

Name: ..... Date: ..... Period: .....

Creating a Scatter Plot

An industrial engineer created an experiment to test the burner settings on a stove. For each setting 1–6, the engineer placed a pot with 2.0 L of water at 15°C onto the burner and turned it on to that setting. After 5 minutes, the temperature of the water was recorded.

The table shows the recorded data.

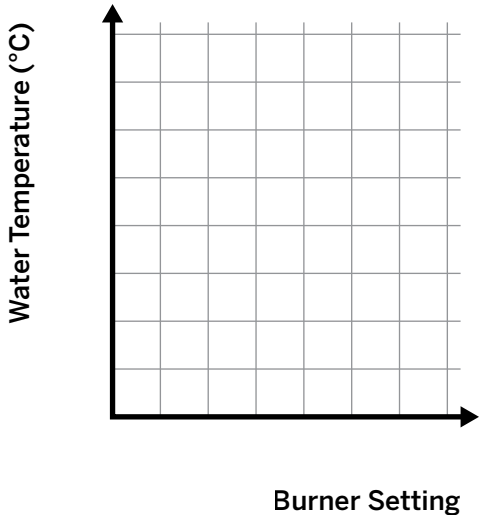
Burner Setting	Water Temperature After 5 Minutes (°C)
1	22
2	31
3	43
4	49
5	57
6	70

2. **Discuss:** What are some questions you can ask about this situation?

3. Let's make a scatter plot of this data.

a Create a scale for the graph so it fits all of the data.

b Create a scatter plot of the data.



4. Use the graph to estimate the water temperature if a burner setting of 7 were to be used.

5. A linear model for this data is $y = 9x + 14$. Use the model to predict the temperature for a burner setting of 7. Show your thinking and compare this result to your response to Problem 4.

Activity 2

Name: ..... Date: ..... Period: .....

Scavenger Hunt

Start at any of the Scavenger Hunt Sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: .....

Answer →

Sheet: .....

↙ **Answer**

Sheet: .....

Answer →

Sheet: .....

Answer ↗

continued on next page...

Name: ..... Date: ..... Period: .....

Scavenger Hunt (continued)

Sheet: ..... **Answer** →	**Sheet:** ..... ↙ **Answer**
Sheet: ..... **Answer** →	**Sheet:** ..... ↙ **Answer**
Sheet: ..... **Answer** →	**Sheet:** ..... **Answer**

Synthesis

6. How can a scatter plot help make sense of the relationship between two variables? Use the scatter plot you created in Activity 1 if it helps with your thinking.

Lesson Practice ACC7.8.08

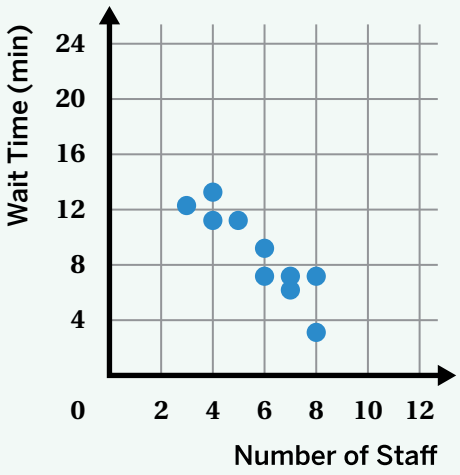
Lesson Summary

You can use a scatter plot to help identify patterns in data points and relationships between two variables.

For example, this scatter plot shows data about how long customers waited at a drive-thru restaurant and the number of staff working at that time.

The scatter plot shows both specific information and general trends, including:

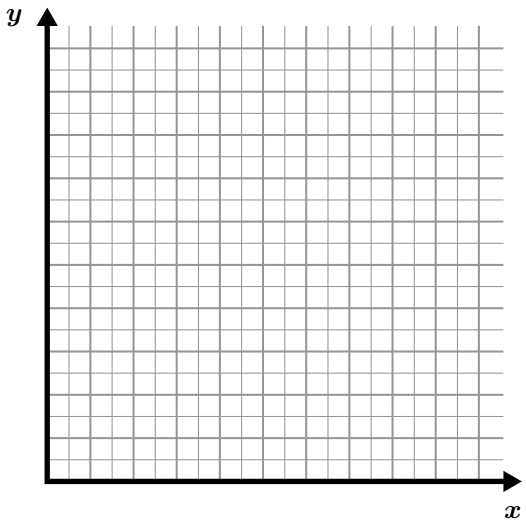
- When 3 staff were working, the wait time was about 12 minutes.
- The more staff there are, the shorter the wait time seems to be.



Lesson Practice
ACC7.8.08

Name: _____ Date: _____ Period: _____

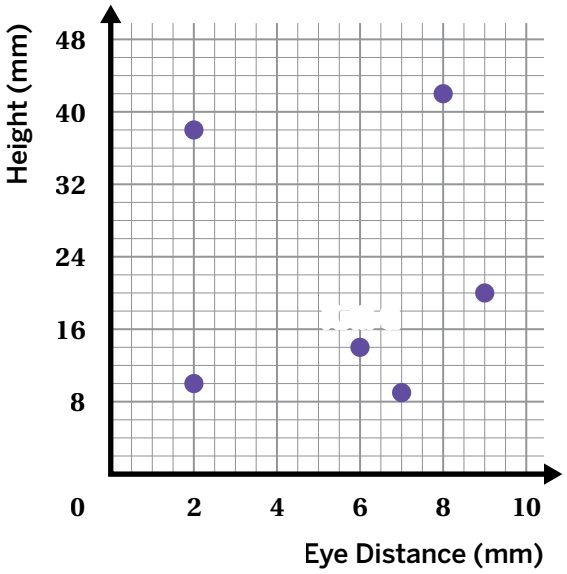
1. Create a scale for the graph so it fits all the data. Then create a scatter plot of the data.



x	y
2	3
5	5
4	2
8	3
6	3
3	5
1	6
4	4

Problems 2–4: The table and scatter plot show the heights and eye distances of seven different krill (small shrimp-like crustaceans).

Krill	Eye Distance (mm)	Height (mm)
A	7	9
B	2	38
C	2	10
D	8	42
E	7	22
F	9	20
G		



2. On the scatter plot, circle the point that represents the tallest krill.

3. Complete the table with the values that represent Krill G.

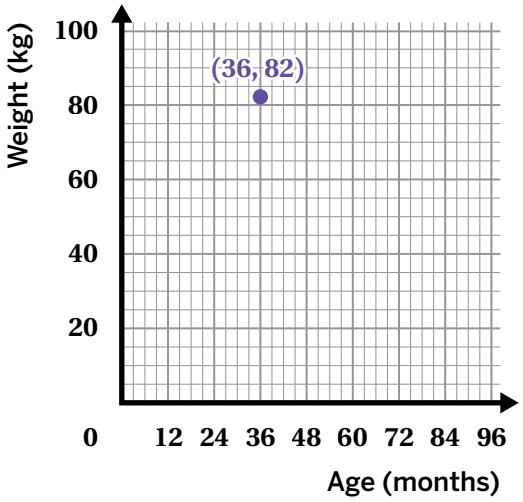
4. On the scatter plot, plot a point that represents Krill E.

Name: ..... Date: ..... Period: .....

5. When do you think it is better to use a table to represent data? When do you think it is better to use a scatter plot?

FAST Practice

6. The graph shows the age and weight of a giant panda. Complete the statement so that it correctly describes the point shown on the graph.



Select *one* correct answer in each box to complete the sentence.

The point represents a giant panda that is

[A. 36 B. 82] months old and weighs [A. 36 B. 82] kilograms.

Spiral Review

Problems 7–8: Solve each equation. Show your thinking.

7. $2(3 - 2c) = 30$

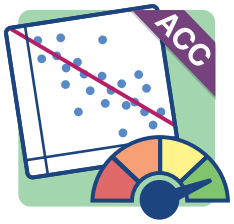
8. $3x - 2 = 7 - 6x$

Name: ..... Date: ..... Period: .....

MA.8.DP.1.3, MTR.4.1, MTR.6.1

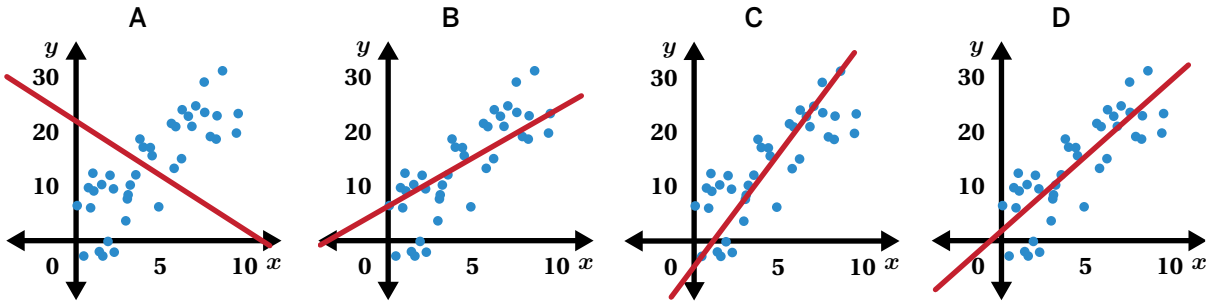
Find the Fit

Let's fit a line to data on a scatter plot.



Warm-Up

1. Which one doesn't belong? Explain your thinking.

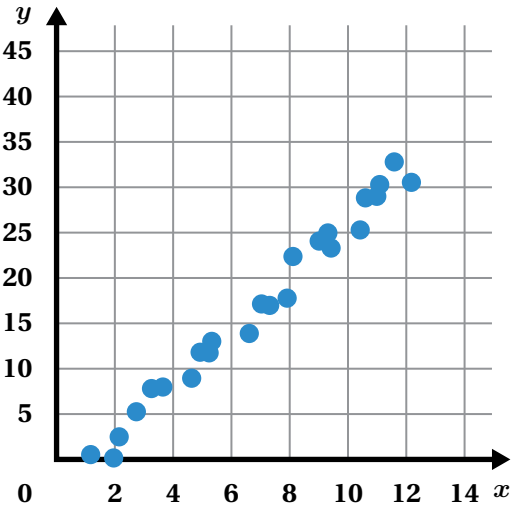


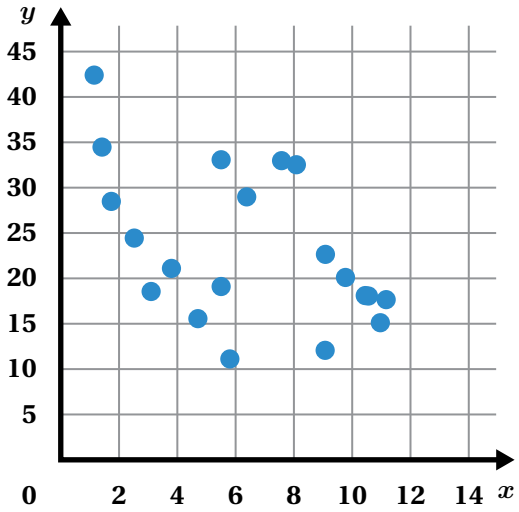
Activity 1

Name: ..... Date: ..... Period: .....

Lines of Fit

2. **a** Create a line that is a good fit for each data set.





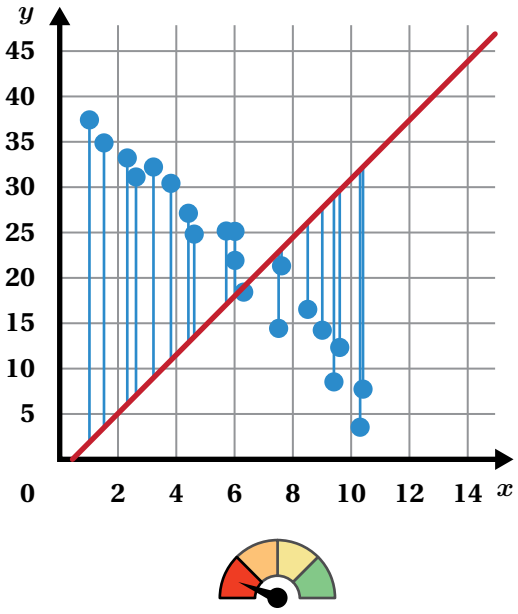
b **Discuss:** How could you decide if a line is a good fit for the data?

Name: ..... Date: ..... Period: .....

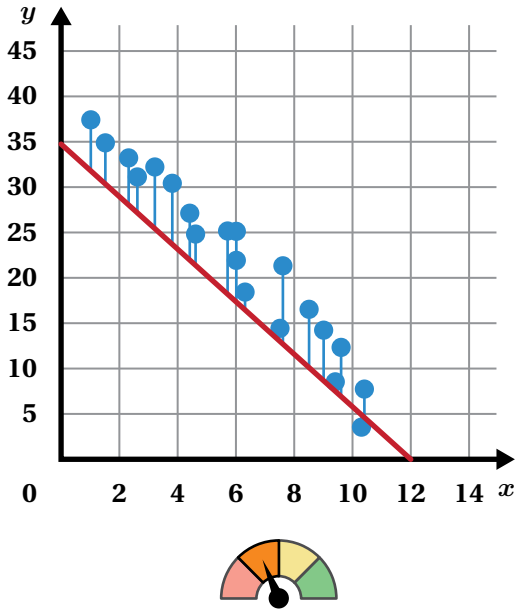
Lines of Fit (continued)

Here are four different lines for the same scatter plot. The meter shows a score for each line.

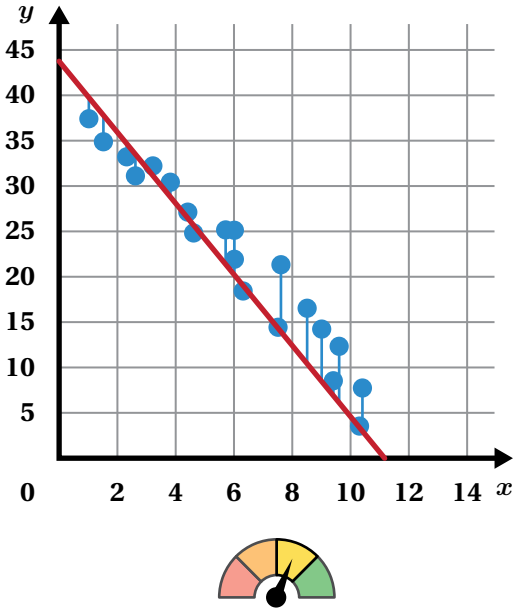
Graph A



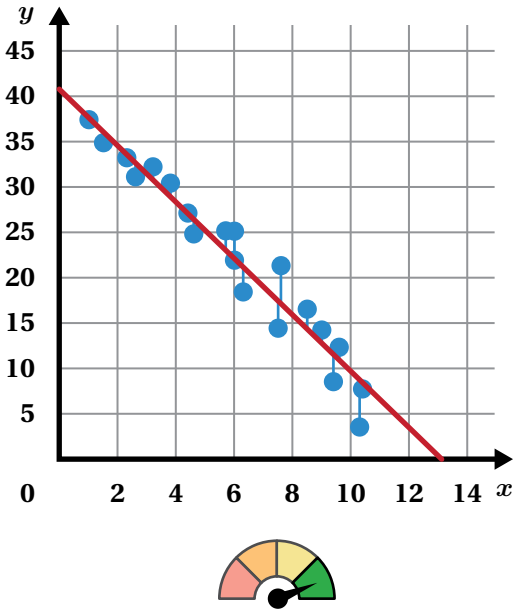
Graph B



Graph C



Graph D



3. Describe how to get a high score (green) on the meter.

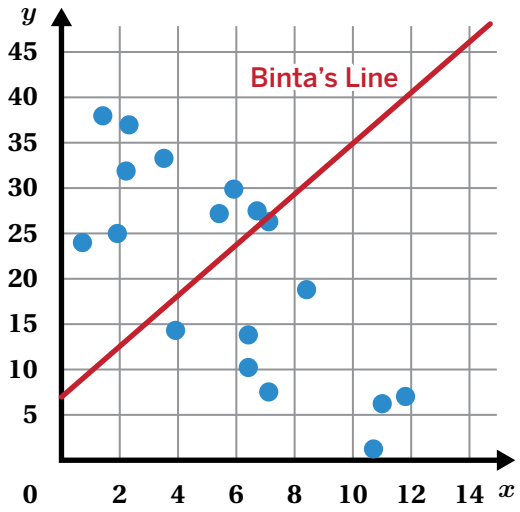
Name: Date: Period:

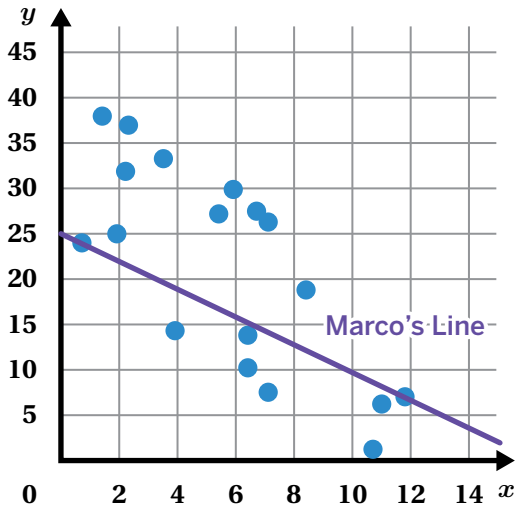
Find the Fit

4. Binta and Marco each sketched a *line of fit* on this scatter plot.

Binta says: *My line is a good fit because half of the points are on each side of the line.*

Marco says: *My line is a good fit because it passes through the leftmost and rightmost points.*



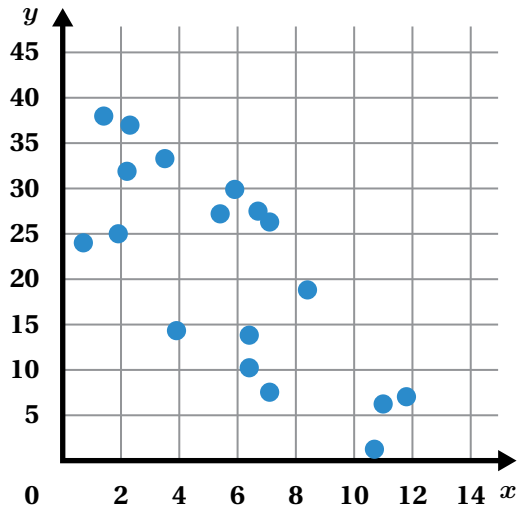


Whose line is a good fit for the data? Circle one.

Binta's Marco's Both Neither

Explain your thinking.

5. Sketch a line that fits the data from the previous problem.



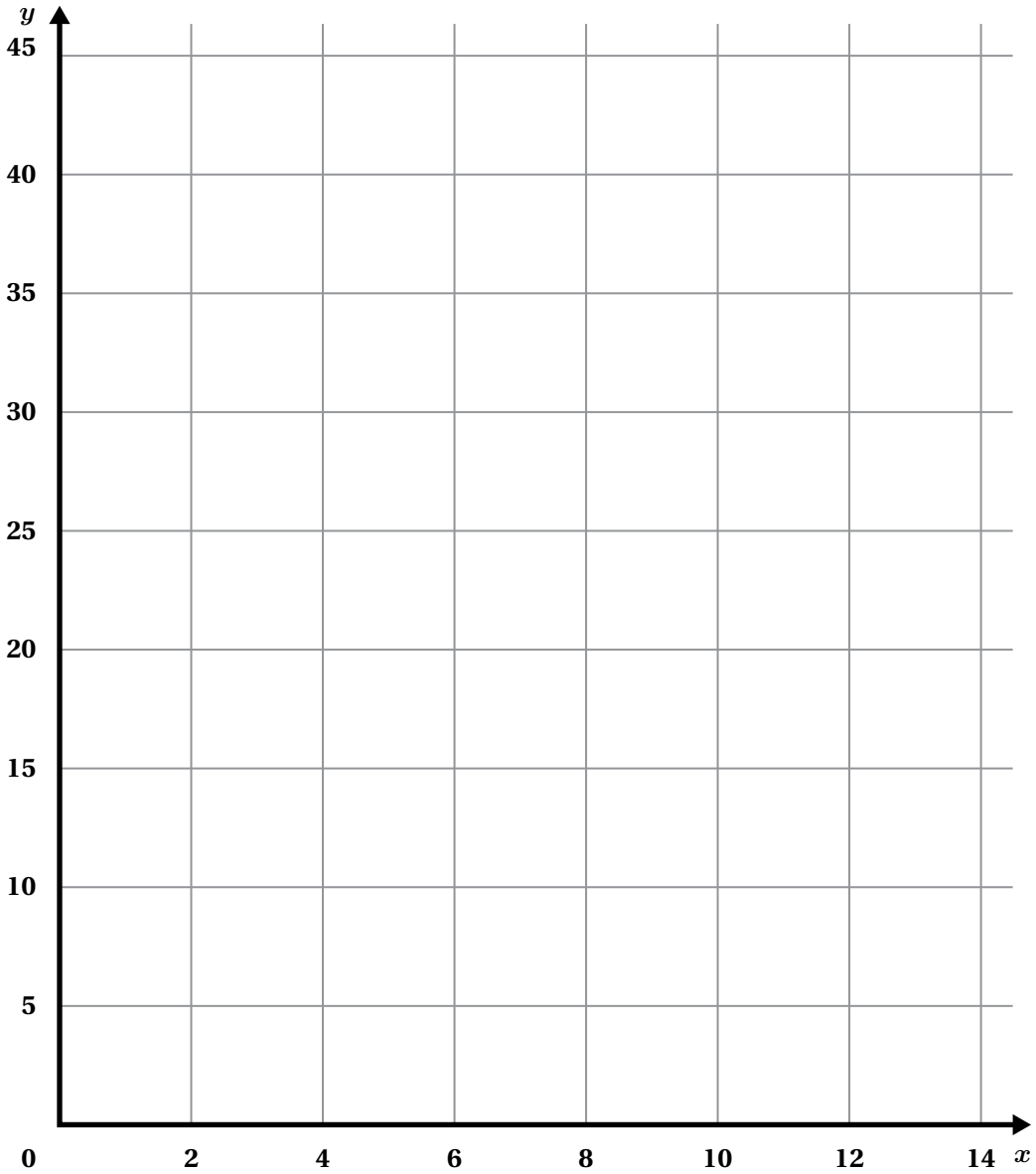
Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

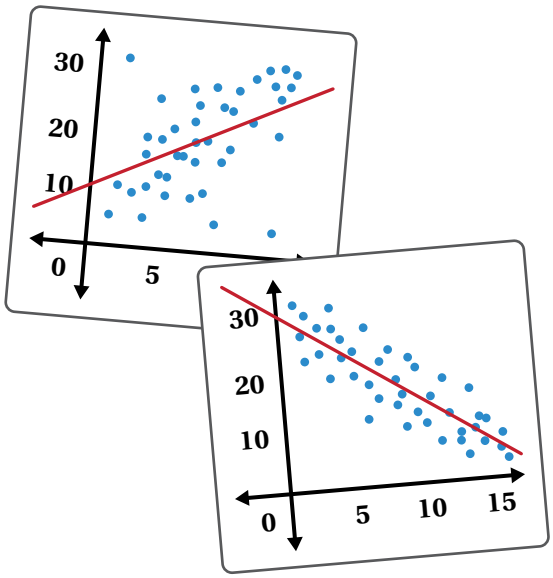
6. You will use the Activity 3 Sheet to complete this activity.

- **a** **Make It!** On the Activity 3 Sheet, create a scatter plot with at least ten points.
- **b** Pass the Activity 3 Sheet with your scatter plot to another student.
- **c** **Solve It!** Draw a line of fit for the data you received.
- **d** **Make It!** Create a new scatter plot in the next available space.
- **e** Repeat steps b–d until everyone has drawn lines for four scatter plots.
- **f** Choose one of the scatter plots you created and copy the plot and line of fit onto the grid below.



Synthesis

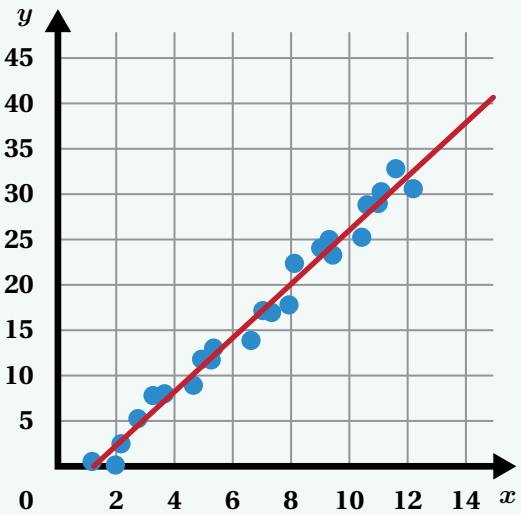
7. What are some things to consider when creating a line of fit? Use the examples if they help with your thinking.



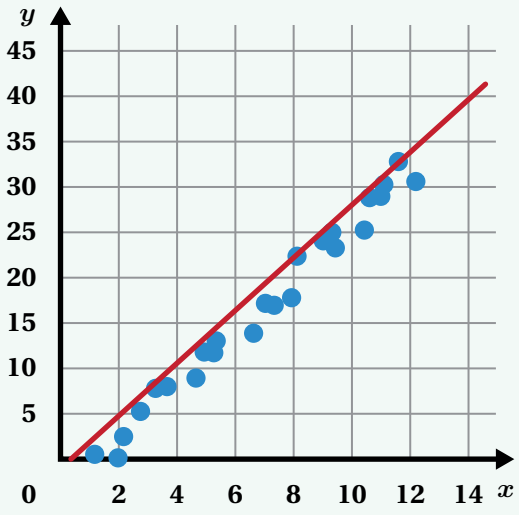
Lesson Practice ACC7.8.09

Lesson Summary

When creating a line of fit for a scatter plot, it's important to determine how well the line fits the data. A good line of fit follows the trend of the data, is as close to the plotted points as possible, and has about the same number of points above and below the line. The line may pass through some, all, or none of the points.



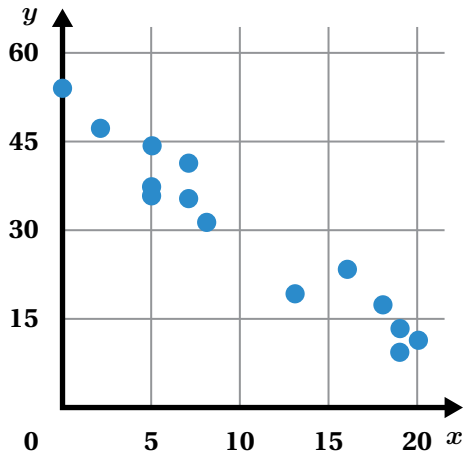
This line is a good fit for the data.



This line is not a good fit for the data.

Name: _____ Date: _____ Period: _____

Problems 1–2: Use this scatter plot.

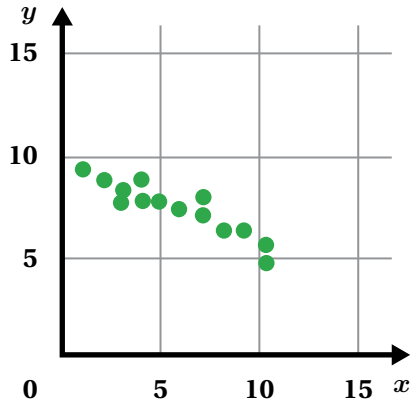


1. Sketch a line of fit for the data.

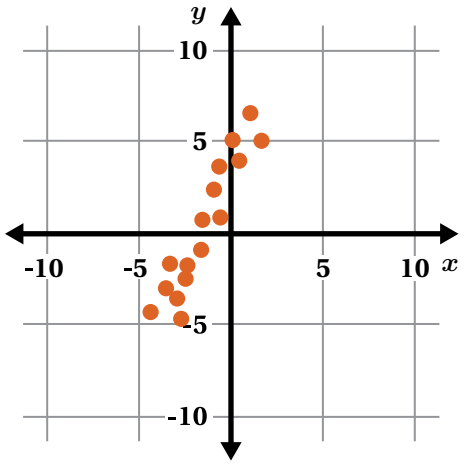
2. If a new data point has an x-value of 10, what does your line of fit predict for the value of y?

Problems 3–4: Sketch a line that fits the data.

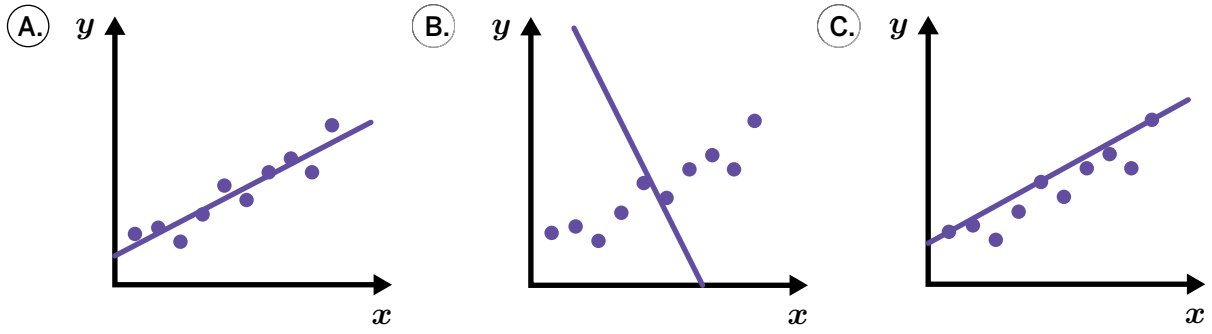
3.



4.



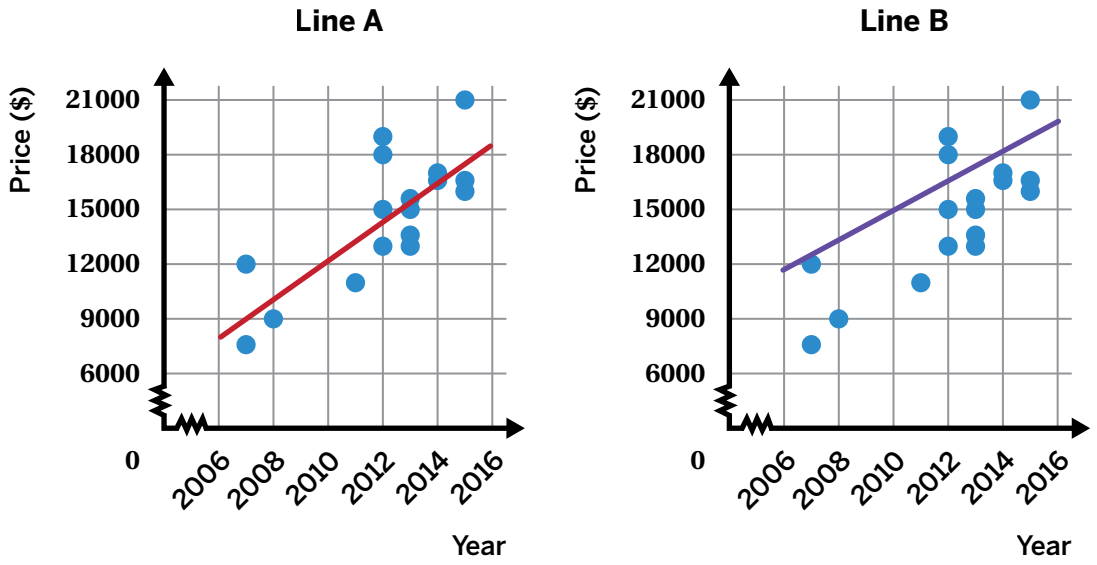
5. Which line best fits the data? Explain your thinking.



Name: ..... Date: ..... Period: .....

FAST Practice

6. Each line of fit applies to the same data.



Which line is a better fit for the data?

Select **ONE** correct answer in each box to complete the sentence.

Line (A.) A (B.) B is a better fit because it

(A.) has a good balance of points above and below it (B.) does not represent the trend of the data.

Spiral Review

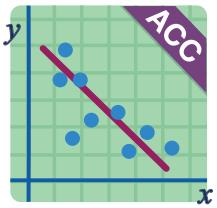
7. Match each equation with the scenario it represents.

Equation		Scenario
a $y = 3x$	.....	A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 cubic feet of dirt.
b $\frac{1}{2}x = y$	.....	You are making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
c $y = 3.5x$	.....	10 blueberries weigh 4 grams.
d $y = \frac{2}{5}x$	.....	For every 48 cookies I bake, my students receive 24 cookies.

Name: ..... Date: ..... Period: .....

MA.8.DP.1.2, MA.8.DP.1.3, MA.8.AR.3.5, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Interpreting Slopes

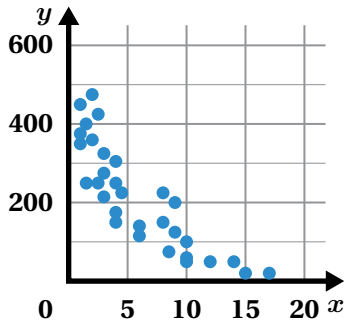


Let's identify different types of associations.

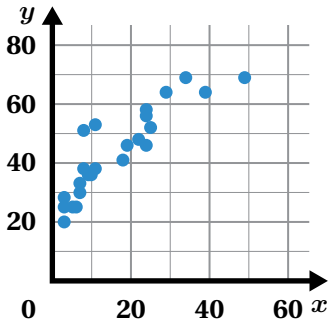
Warm-Up

1. Which one doesn't belong? Explain your thinking.

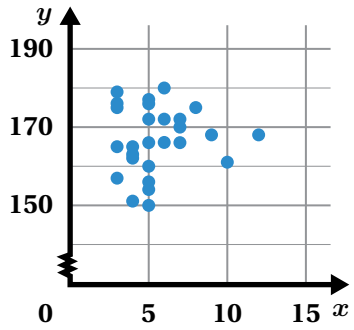
A.



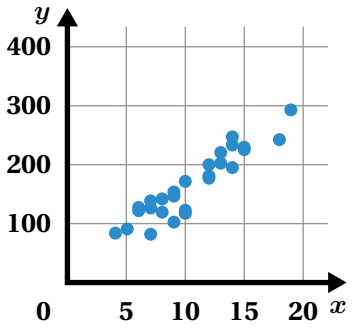
B.



C.



D.

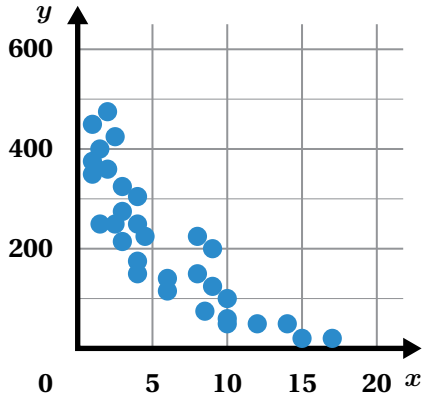


Name: ..... Date: ..... Period: .....

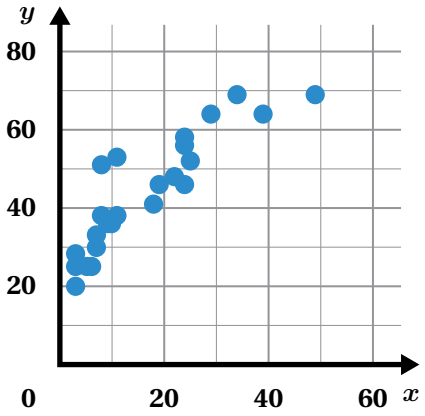
Positive and Negative Associations

2. Match each scatter plot with the variables that it most likely represents.

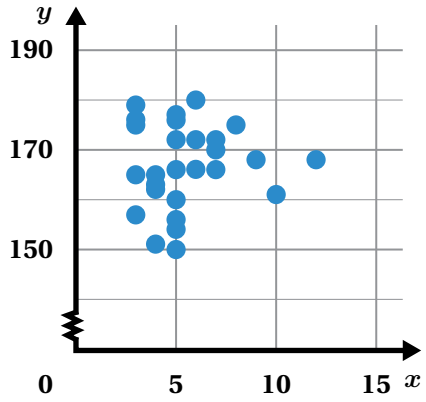
A.



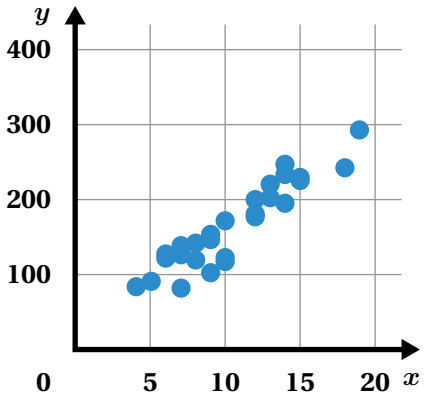
B.



C.



D.



x: Number of Floors y: Building Height (ft)	x: Age of Bike (years) y: Bike Price ($)	x: Dog Weight (kg) y: Dog Height (cm)	x: Letters in Name y: Height (cm)
.....	.....	.....	.....

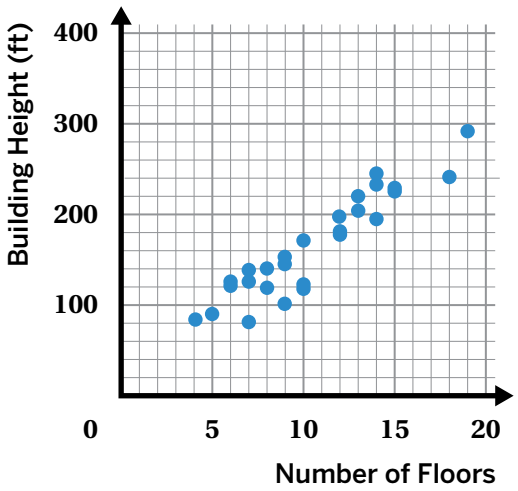
3. How did you decide which scatter plot matches these variables?

x: Number of Floors
y: Building Height (ft)

Name: ..... Date: ..... Period: .....

Positive and Negative Associations (continued)

4. Here is the scatter plot that shows data from some buildings.



a Sketch a linear model to fit the data.

b Esi sketched a linear model whose equation is $y = 13x + 30$.

Discuss:

- What is the *slope* of her line?
- What does the slope represent in this situation?

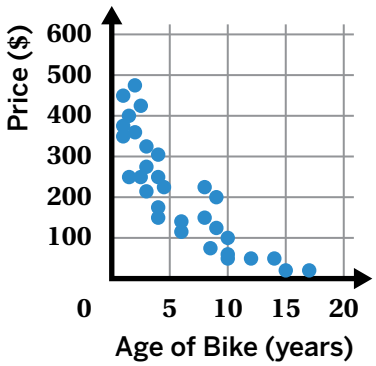
5. An **association** is a relationship between two variables. There is a positive association if both variables increase together and a negative association if one variable decreases as the other increases.

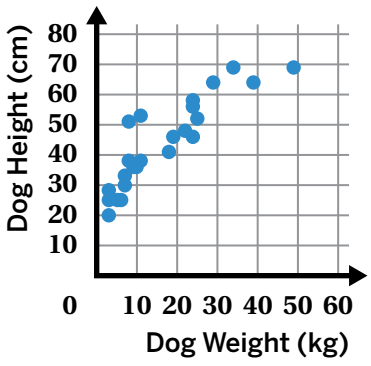
What type of association is there between building height and number of floors? Circle one.

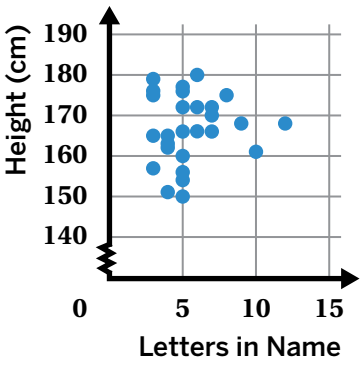
Positive association Negative association No association

Explain your thinking.

6. Determine what type of association each scatter plot shows. Discuss your thinking.



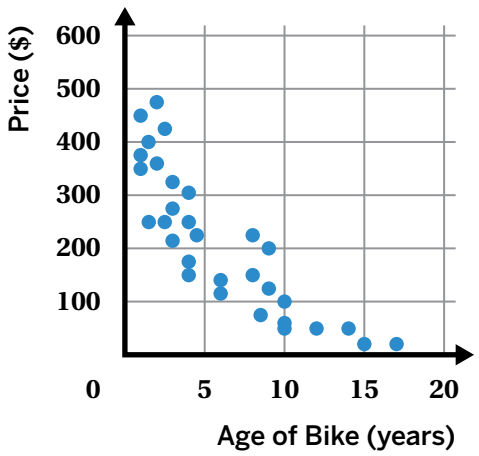




Name: Date: Period:

Interpretations

7. Here is the scatter plot that shows the prices and ages of some used bikes.



a Sketch a linear model that fits the data.

b **Discuss:** How can you tell from the linear model that there is a negative association between bike age and price?

8. Troy drew a linear model for the bike data using the equation $y = -25x + 375$. Use Troy's model to finish this sentence:

The model predicts that as the age of a bike increases by 1 year:

A. The price will increase by $25.

B. The price will decrease by $25.

C. The price will increase by $375.

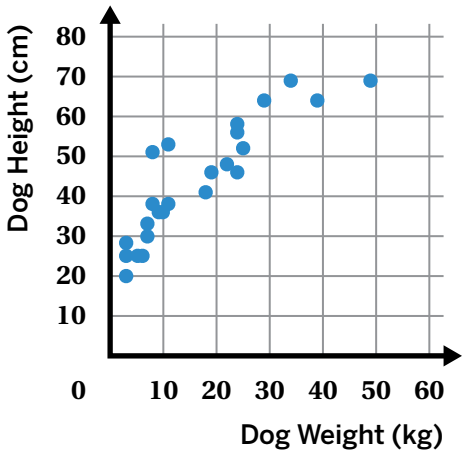
D. The price will decrease by $375.

Explain your thinking.

Name: ..... Date: ..... Period: .....

Interpretations (continued)

9. Here is the scatter plot that shows the heights and weights of some dogs.



a Sketch a linear model that fits the data.

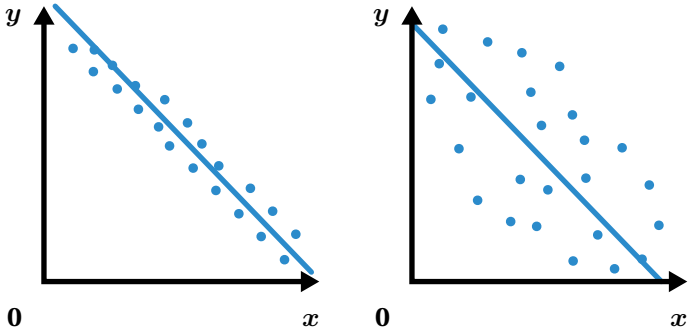
b Nikolai sketched a linear model whose equation is $y = 1.2x + 28$.

Identify the slope of Nikolai's model and describe what it means in this situation.

Name: ..... Date: ..... Period: .....

Strong and Weak Associations

10. Both of these graphs show a negative association, but one shows a strong negative association and the other shows a weak negative association. The graphs have the same linear model.



Strong Association | Weak Association

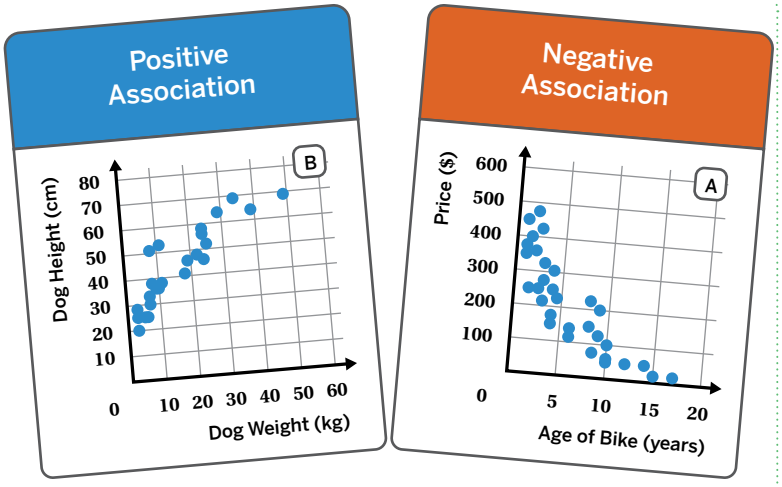
a. **Discuss:** What do you think it means to have a strong or weak association?

b. How accurate do you think a prediction made using the linear model will be for each graph? Explain your thinking.

Synthesis

11. Use the example scatterplots if they help to answer these questions:

a What are some clues that a scatter plot might have a positive or negative association?

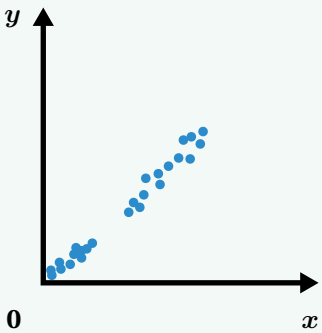


b What does the slope of a linear model tell you about the data?

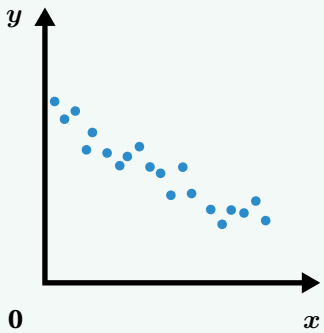
Lesson Practice ACC7.8.10

Lesson Summary

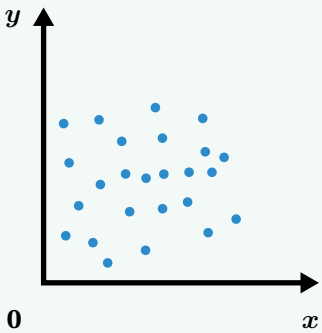
If two variables on a scatter plot are related, there is an **association**. The *slope* of a linear model can help determine the type of association. A positive association means that when one variable increases, the other also increases. A negative association means that when one variable increases, the other decreases. These associations may be weak or strong. A strong association shows a clear positive or negative association, with the points fitted closely along a line. In a weak association, there is still an upward or downward trend, but the points do not closely follow the same line. If the scatter plot shows no clear trend between the two variables, then the variables have no association.



Positive association



Negative association

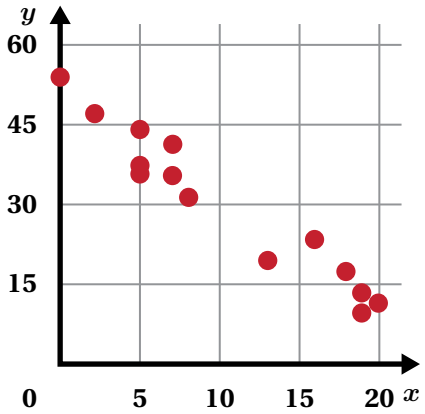


No association

Lesson Practice
ACC7.8.10

Name: _____ Date: _____ Period: _____

1. Which type of association does the scatter plot show?



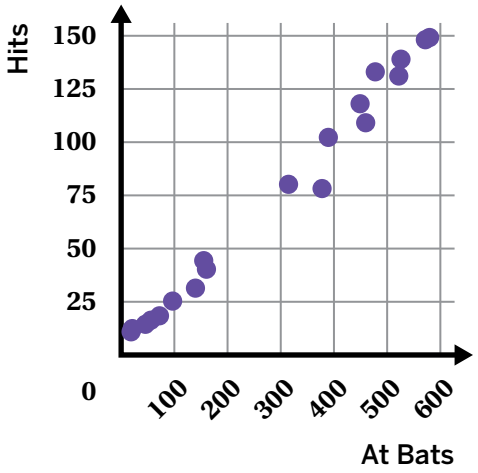
(A.) Positive association

(B.) Negative association

(C.) No association

Explain your thinking.

2. The scatter plot shows the number of hits and at bats for players on a baseball team. Which conclusion is best supported by the scatterplot?



(A.) As the number of at bats increases, the number of hits also increases.

(B.) As the number of at bats increases, the number of hits decreases.

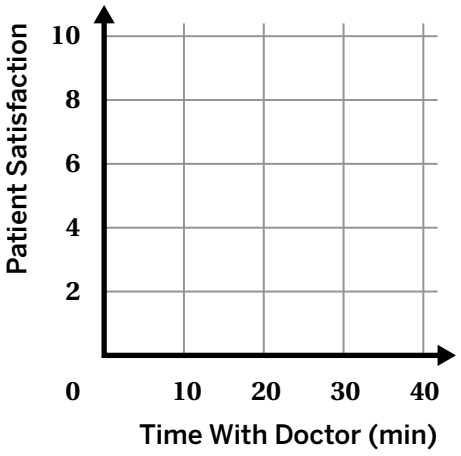
(C.) As the number of hits increases, the number of at bats remains the same.

(D.) There is no relationship between the number of at bats and the number of hits.

Problems 3–5: The doctors in a medical clinic looked at the relationship between patient satisfaction on a 0–10 scale and the number of minutes spent with a doctor. They found the variables had a positive association.

3. What does this positive association mean about the relationship between patient satisfaction and time with a doctor?

4. Create a scatter plot that represents this situation.



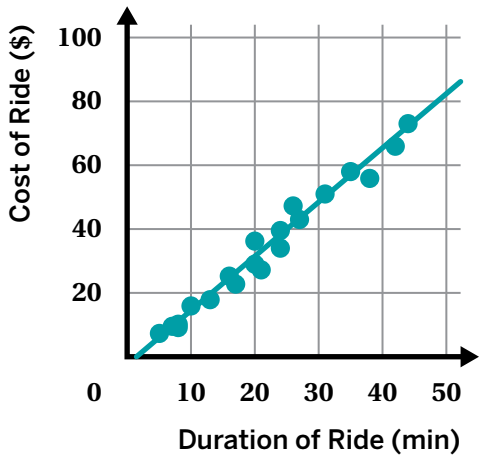
5. Does this graph show a strong or weak association? Explain your thinking.

Lesson Practice
ACC7.8.10

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 6–8: The scatter plot shows the data from 20 taxi rides in Austin, Texas, along with a linear model whose equation is $y = 1.7x - 2.5$.



6. What is the slope of the linear model?

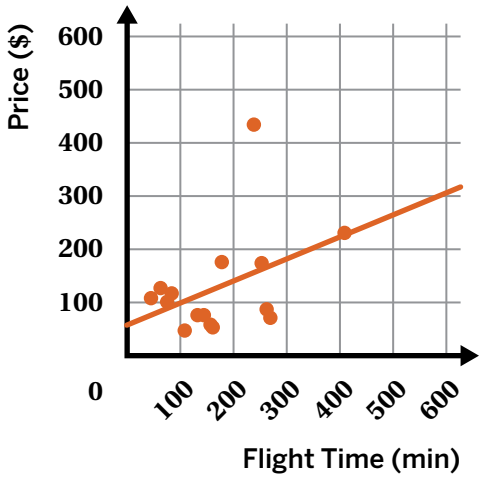
7. What does the slope represent in this situation?

8. Select *all* of the associations that describe the relationship between duration and cost of a taxi ride.

- ☐ **A.** Positive association
- ☐ **B.** Negative association
- ☐ **C.** No association
- ☐ **D.** Strong association
- ☐ **E.** Weak association

Spiral Review

Problems 9–10: The scatter plot shows the flight time and price for several different flights from O'Hare Airport in Chicago.



9. Circle any data point(s) that appear to be outliers.

10. Use the linear model to estimate the price of a 10-hour flight from O'Hare Airport.

11. The equation $y = 5280x$ gives the number of feet, y, in x miles. What does the number 5,280 represent in this relationship?

Name: ..... Date: ..... Period: .....

MA.8.DP.1.2, MTR.4.1, MTR.5.1

Scatter Plot City

Let's use precise language to describe the trends in a scatter plot.

Warm-Up

1. Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with scatter plots for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

Record helpful questions from each round in the space below.

Activity 1

Name: Date: Period:

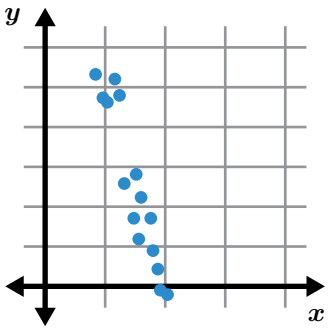
Scatter Plot City

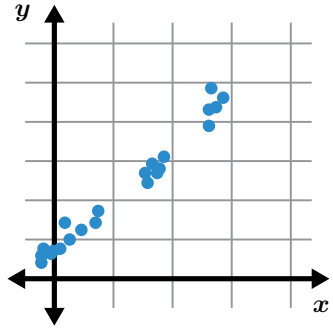
2. You will use the Activity 1 Sheet to see some scatter plots from the Polygraph and some terms that describe them.

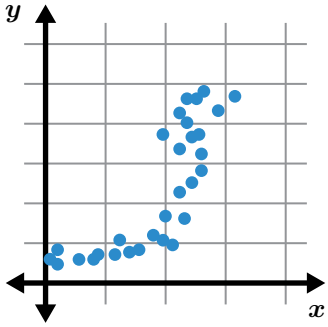
Discuss: What does each term mean?

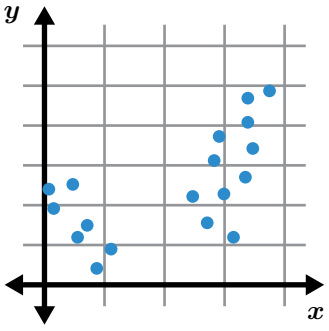
- Linear association
- Nonlinear association
- With **clusters**
- Without clusters

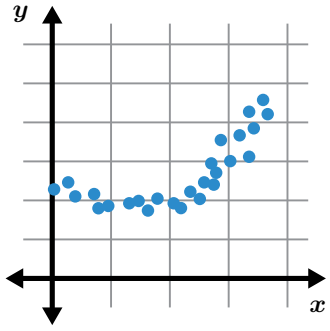
Here are six new scatter plots.

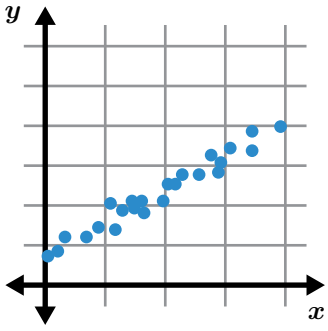
A. 

B. 

C. 

D. 

E. 

F. 

3. Sort them according to their type of association.

Linear Association	Nonlinear Association

4. Sort them in a different way: those with clusters and those without.

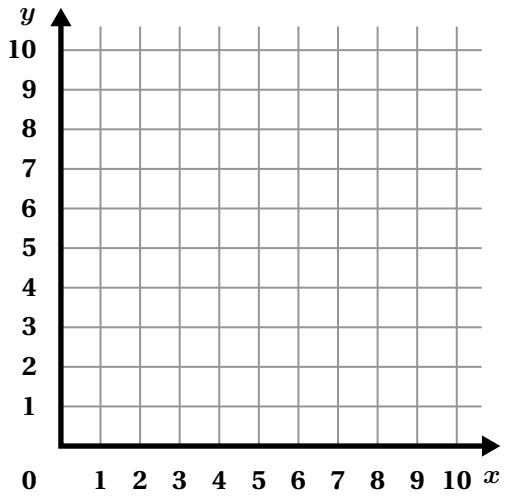
With Clusters	Without Clusters

Activity 2

Name: ..... Date: ..... Period: .....

Putting It All Together

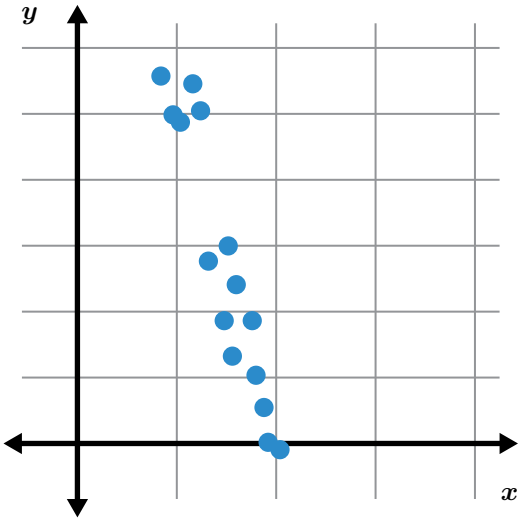
5. **a** Create a scatter plot that has a negative nonlinear association, without clusters.



b Compare your scatter plot with your partner's.

6. Here is one of the scatter plots from before.

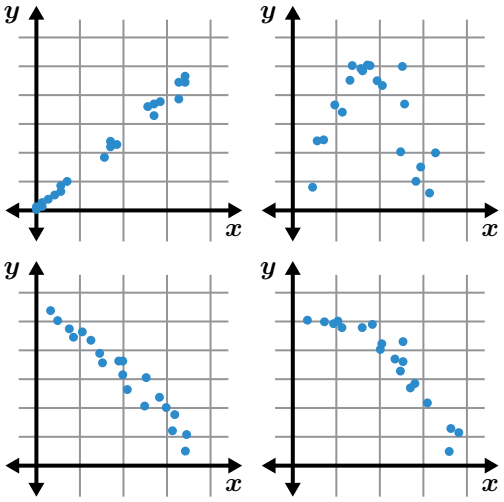
Describe the scatter plot using vocabulary from this unit.



positive association	negative association	clusters
linear association	nonlinear association	outlier
strong association	weak association	

Synthesis

7. How can you identify a nonlinear association or clusters in a scatter plot? Use the examples if they help with your thinking.

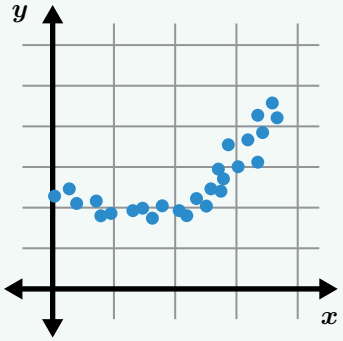


Lesson Practice ACC7.8.11

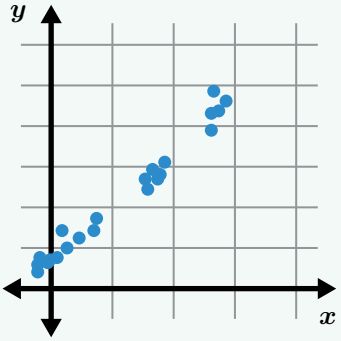
Lesson Summary

When you can model data on a scatter plot with a straight line, we say it has a linear association. Data that can't be modeled by a straight line has a nonlinear association. Sometimes groups of data points appear close together, which are called **clusters**.

This scatter plot is an example of a nonlinear association, without clusters.



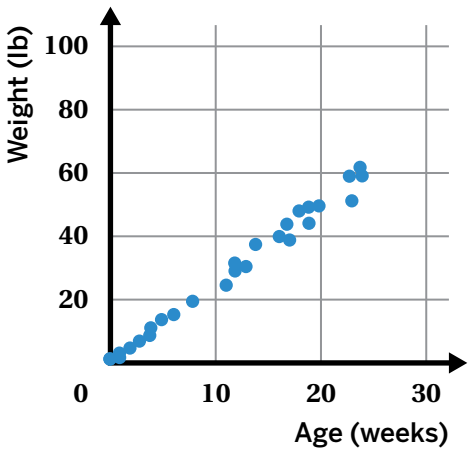
This scatter plot is an example of a linear association, with clusters.



Lesson Practice
ACC7.8.11

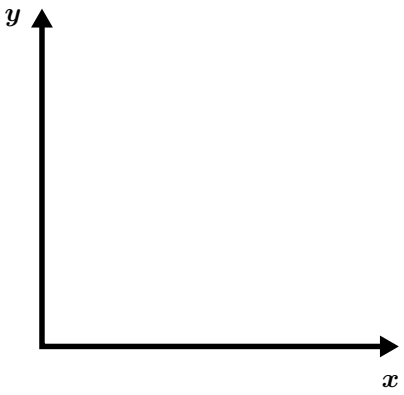
Name: _____ Date: _____ Period: _____

1. The graph shows the age and weight of babies in a nursery. Select *all* the terms that describe the association on the scatter plot.



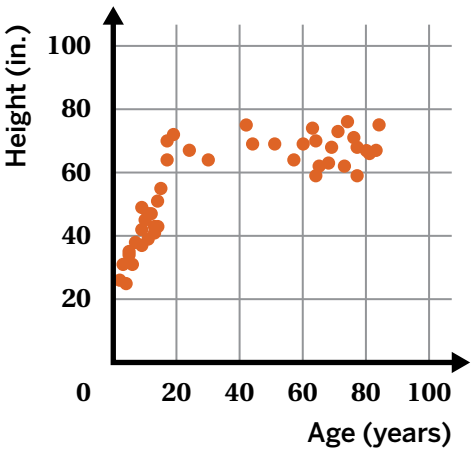
- ☐ **A.** Linear association
- ☐ **B.** Nonlinear association
- ☐ **C.** Positive association
- ☐ **D.** Negative association
- ☐ **E.** No association
- ☐ **F.** Strong association
- ☐ **G.** Weak association

2. Create a scatter plot that has a weak positive linear association, with clusters.



FAST Practice

3. A doctor in a small town sees patients of all ages. The scatter plot shows her patients' ages and heights. Select **ONE** correct answer in each box to complete each sentence.



There [(A.) is (B.) is not] a nonlinear association between the ages and heights of the doctor's patients.

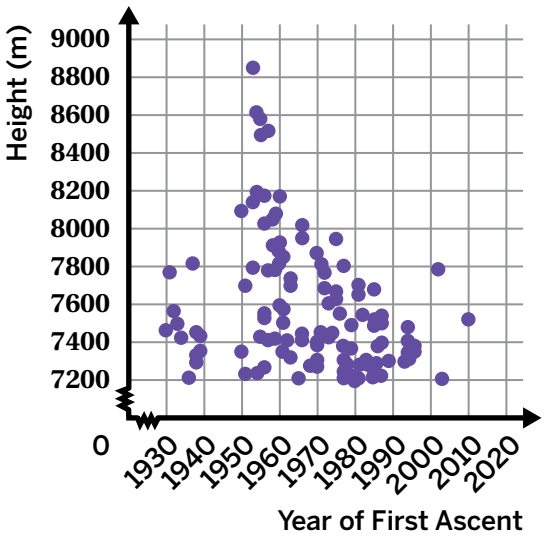
As patients' ages increase, their heights [(A.) tend (B.) do not tend] to decrease.

Lesson Practice
ACC7.8.11

Name: ..... Date: ..... Period: .....

Spiral Review

4. The scatter plot shows data for some of the tallest mountains on Earth. Which of the following terms best describes the association between the heights of the mountains and years of first recorded ascent?



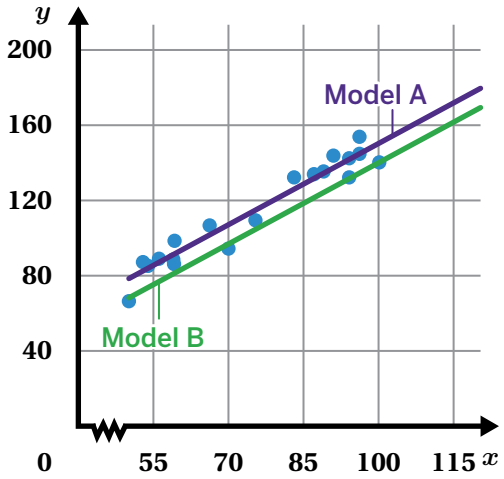
(A.) Linear association

(B.) Positive association

(C.) Negative association

(D.) No association

5. Here are two different linear models for the same data. Which model is a better fit for the data? Explain your thinking.



6. The points $(2, 4)$ and $(6, 7)$ fall on a line. What is the slope of the line?

(A.) 1

(B.) 2

(C.) $\frac{4}{3}$

(D.) $\frac{3}{4}$

Name: ..... Date: ..... Period: .....

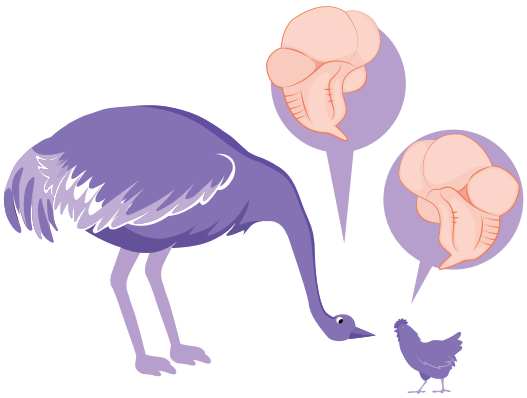
MA.8.AR.3.5, MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3, MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

Animal Brains

Let's analyze bivariate data.

Warm-Up

1. Do you think heavier animals have heavier brains? Explain your thinking.



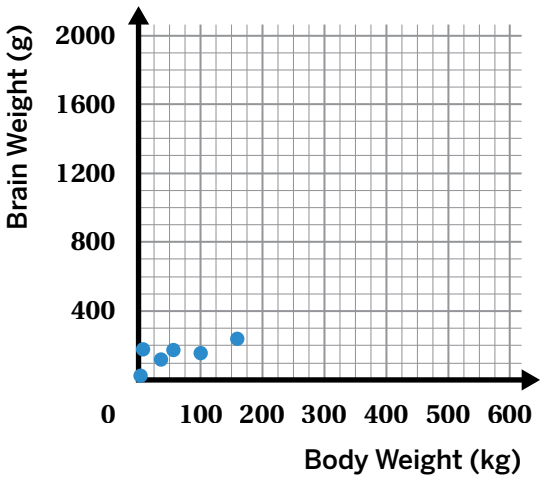
2. What information would help you determine whether heavier animals have heavier brains?

Activity 1

Name: Date: Period:

Creating a Scatter Plot

3. The scatter plot shows the body weight and brain weight of 6 different animals. The table shows the weights of 4 more animals. Plot and label a point on the graph to represent each animal's data from the table.



Animal	Body Weight (kg)	Brain Weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423

4. Look back at your prediction from the Warm-Up. Based on the scatter plot, what types of association do there appear to be between brain weight and body weight? Circle all that apply.

Positive association Negative association No association

Linear association Nonlinear association Strong association Weak association

Explain your thinking.

5. The table shows the body weight of three more animals.

Plot and label points on the graph to predict the brain weight of each animal.

Complete the table with your predictions.

Animal	Body Weight (kg)	Brain Weight (g)
Dog	10	
Pig	192	
Horse	521	

Activity 2

Name: ..... Date: ..... Period: .....

Line of Fit

Fitting a line to data can help make predictions more accurate.

6. Draw a line that fits the data on the scatter plot on the previous page.

7. The equation for Inola's line of fit is $y = 0.9x + 79$.

a What is the slope of the line? What is the y-*intercept*?

Slope: ..... y-intercept: .....

b **Discuss:**

- What does each number mean in this situation?
- Do these values make sense in this situation?

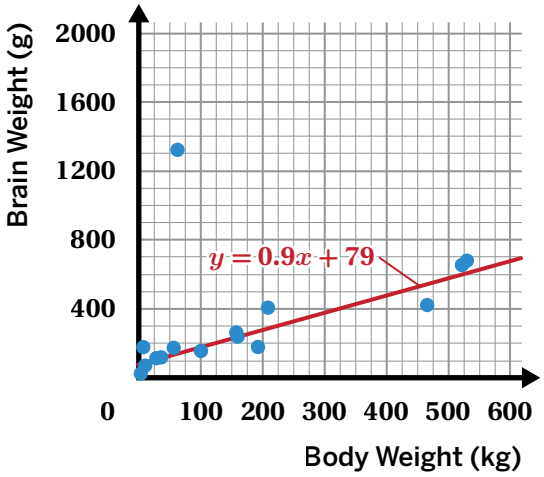
8. Use your line of fit to predict the brain weight for a gorilla and a human. Plot and label points on the previous page to show your predictions.

Animal	Body Weight (kg)	Brain Weight (g)
Gorilla	207	
Human	62	

Name: Date: Period:

Line of Fit (continued)

9. Let's look at a scatter plot that shows body and brain weight data, including the data for the gorilla and human.



What do you notice? What do you wonder?

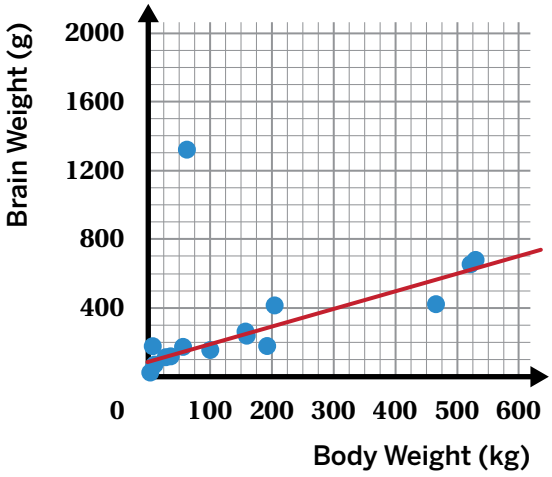
I notice:

I wonder:

Synthesis

10. Describe an advantage and disadvantage of using a line of fit to make predictions.

Use the graph if it helps with your thinking.



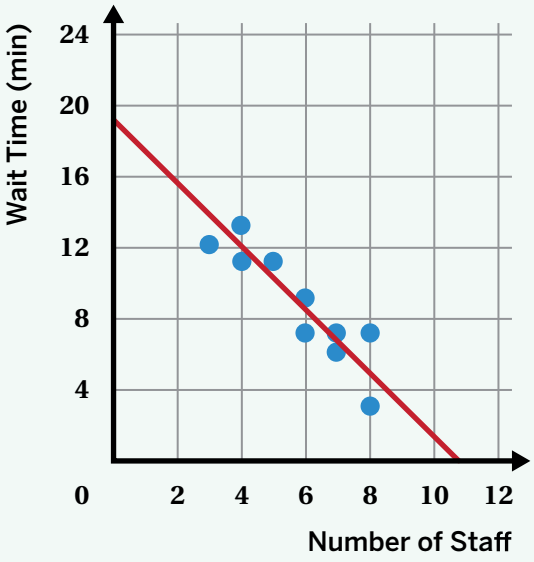
Lesson Practice ACC7.8.12

Lesson Summary

By understanding the association between two variables, you can make predictions about unknown values. When there's a linear association, using a linear model can often make predictions more accurate.

For example, this scatter plot shows data about how many minutes customers waited at a drive-through restaurant and the number of staff working at that time. This data can be modeled by the equation $y = -1.75x + 19$.

- The *slope* of the linear model is -1.75, which means that if the number of staff increases by 1 person, the wait time decreases by 1.75 minutes.
- The linear model predicts that if there are 2 staff working, the wait time will be approximately 15.5 minutes.
- But the linear model also predicts that when there are 0 staff working, the wait time will be 19 minutes, which is impossible!

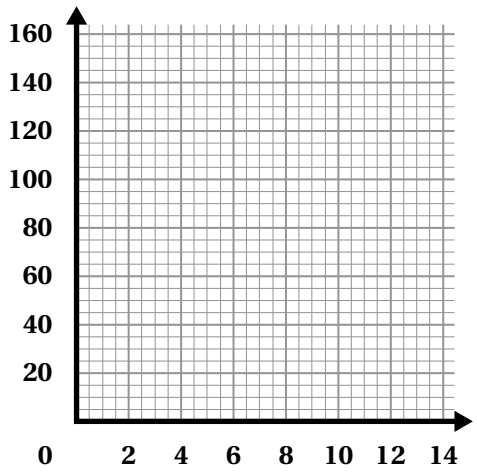


Name: ..... Date: ..... Period: .....

Problems 1–5: Stores across the country sell a particular book at different prices. The table shows the price of the book and the number of books sold at that price.

Price ($)	Number of Books Sold
10.50	60
12.10	30
8.45	81
9.25	70
9.75	80
7.25	120
12	37
9.99	130
7.99	100

1. Create a scatter plot for this data. Include labels for the horizontal and vertical axes.



2. Are there any outliers? Explain your thinking.

3. What type of association does there appear to be between the price of the book and the number of books sold? Explain your thinking.

4. Draw a line on the graph that you think is a good fit for the data.

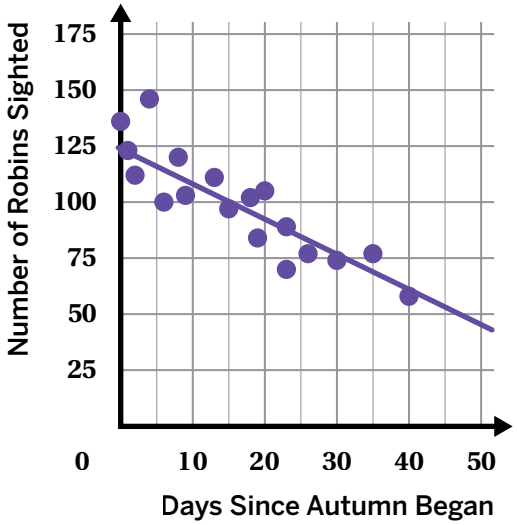
5. A bookstore plans to sell the book for $6. Use your line to predict the number of books the store will sell. Explain your thinking.

Lesson Practice
ACC7.8.12

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 6–7: This scatter plot shows data about the number of robins sighted at a local park and the number of days since autumn began.



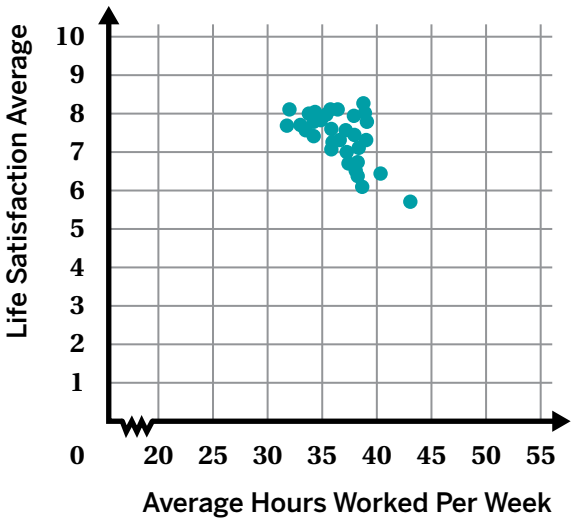
6. Approximately how many robins were sighted 15 days since autumn began?

7. Use the line of fit to choose the best prediction for the number of robins that will be sighted 50 days since autumn began.

(A.) 10 robins (B.) 25 robins (C.) 45 robins (D.) 75 robins

Spiral Review

Problems 8–9: Martina is interested in the quality of life in different countries. She created this scatter plot to show the average number of hours people worked in a week and their average life satisfaction (on a 0–10 scale). Each point on the graph represents a different country.



8. Circle the point that represents the country where average life satisfaction is lowest. How many hours are in the average work week in that country?

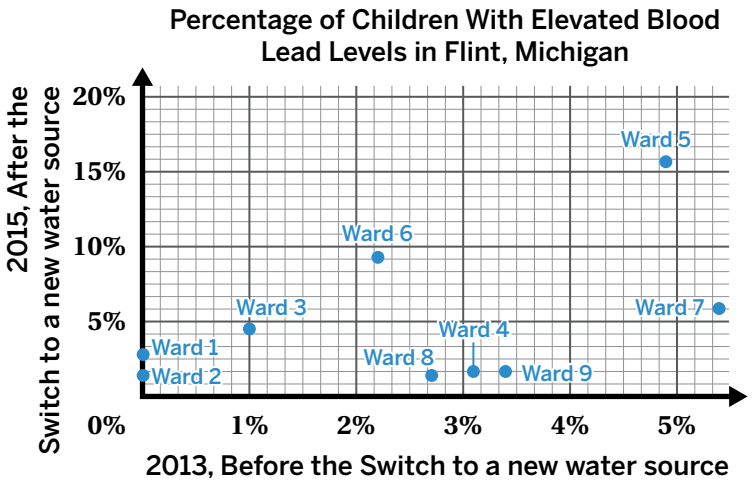
9. Select *all* the true statements about the data in the scatter plot.

- ☐ **A.** The variables have a positive association.
- ☐ **B.** The variables have a negative association.
- ☐ **C.** A linear model that fits the data would have a positive slope.
- ☐ **D.** A linear model that fits the data would have a negative slope.
- ☐ **E.** More than half of the countries have an average life satisfaction above 7.

Career Connection

Inventors identify areas in everyday life that can be improved and then design and create new products or processes. Many inventions today use STEM (Science, Technology, Engineering, and Math) to solve challenging problems in our world.

12-year-old Gitanjali Rao, invented a device and an app to detect water contaminants and display the results. In 2020, she was named TIME magazine's first Kid of the Year.



B.E.S.T. Mathematics Benchmark Connection

Scientists and statisticians rely on graphing and statistics concepts in their work. For example, they use a scatter plot (MA.8.DP.1.1) when they analyze data collected over shorter (daily) and longer (yearly) periods of time. They may also look for linear relationships (MA.8.DP.1.3) to describe patterns they see in a scatter plot to make predictions for future water quality.

Mathematical Thinking and Reasoning Connection

Scientists who collect data to develop models use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work with their team members as well as others who may be studying a similar situation (MTR.1.1). When they study data collected for the same locations at different times, they learn to complete the tasks of organizing and displaying the data with mathematical fluency (MTR.3.1).

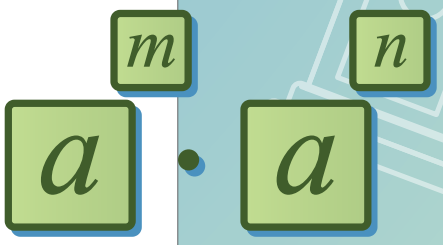
Meet Kimberly F. Sellers

Kimberly F. Sellers is the Head of the Department of Statistics at North Carolina State University. She is a specialist on count data, which is a data type that describes quantities that can be counted. She has applied her statistics expertise in several areas, including prote omics, the study of proteins.

Unit 9

Exponents and Scientific Notation

Numbers and operations with numbers can be represented in many different ways. How can you represent very large or very small numbers? How can you work with these numbers? In this unit, you will see how exponents, properties of exponents, and powers of 10 will be used to solve problems involving scales, city lights, and the distance to the Moon.



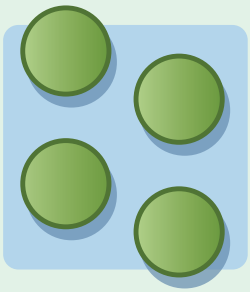
Essential Questions

- How can you use the properties of exponents to make connections between expressions?
- What does it mean when an exponent is negative?
- What is scientific notation and how can it be used to represent small and large numbers?

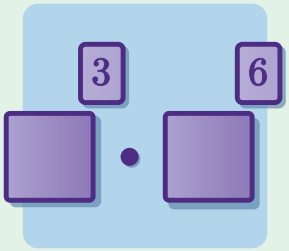
Sub-unit

1

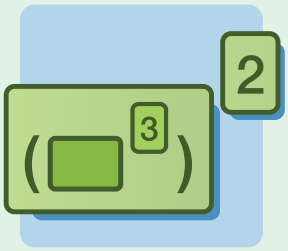
Exponent Properties



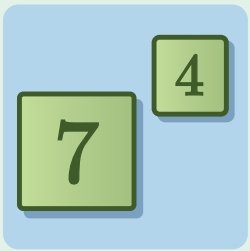
Lesson 1
Circles



Lesson 2
Combining Exponents



Lesson 3
Power Pairs



Lesson 4
Rewriting Powers

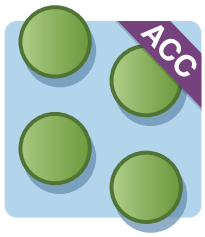
Lesson 5
Exponent Zero

Lesson 6
Negative Bases

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MTR.2.1, MTR.3.1, MTR.5.1

Circles



Let's revisit exponents.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. $5 \cdot 2$

2. $5 \cdot 2 \cdot 2$

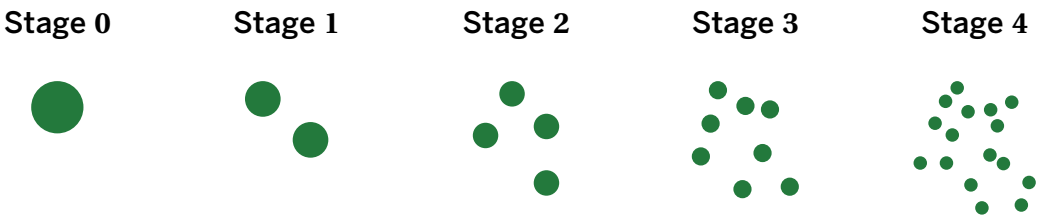
3. $5 \cdot 2 \cdot 2 \cdot 2$

4. $5 \cdot 2^4$

Name: Date: Period:

Lots of Circles

5. Let's look at a pattern.



Discuss: What do you notice? What do you wonder?

6. How many circles will there be in Stage 5?

Explain your thinking.

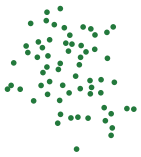
Name: Date: Period:

Lots of Circles (continued)

7. Here are Stages 5–12 of the same pattern.

Stage 5

Stage 6

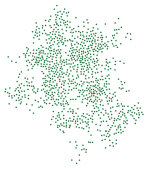


Stage 7

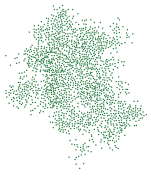
Stage 8

Stage 9

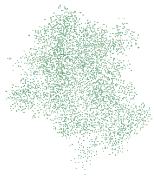
Stage 10



Stage 11



Stage 12



How many circles are there in Stage 12?

8. Adah and Jamal were calculating the number of circles in Stage 12.

Adah wrote: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Jamal wrote: 2^{12}

Whose expression is correct? Circle one.

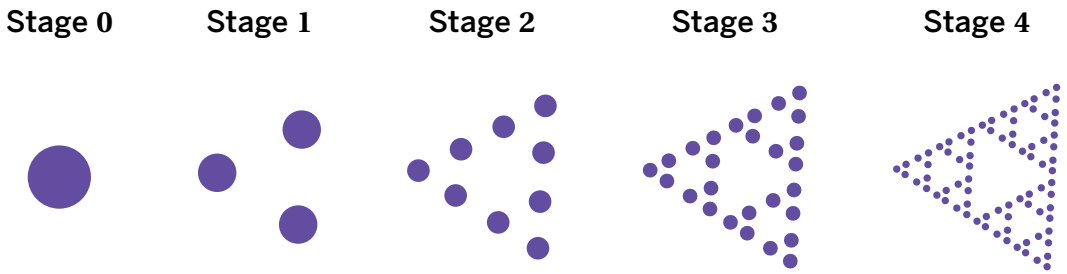
Adah's Jamal's Both Neither

Explain your thinking.

Name: Date: Period:

A New Pattern

9. Here is a new pattern.



How many circles are there in Stage 4?

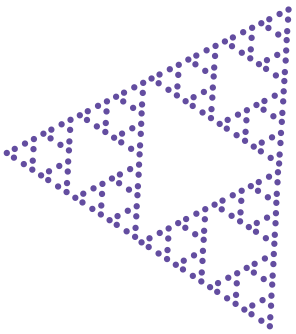
Explain your thinking.

10. There are 243 circles in Stage 5.

Select *all* the expressions that represent the number of circles in Stage 7.

- ☐ **A.** 3^7
- ☐ **B.** $243 \cdot 3^2$
- ☐ **C.** $243 \cdot (3 \cdot 2)$
- ☐ **D.** $243 + 243 + 243$
- ☐ **E.** $243 \cdot 3 \cdot 3$

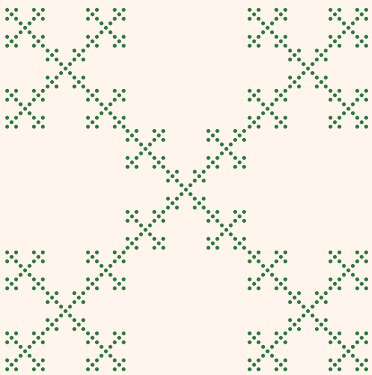
Stage 5



You're invited to explore more.

11. How many circles are in this image?

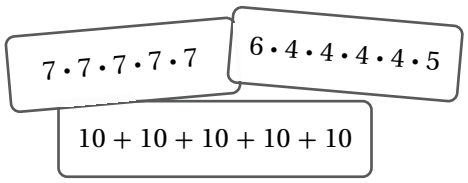
Explain your thinking.



Synthesis

12. When might it be helpful to write values or expressions using *exponents*?

Use the examples if they help with your thinking.



Lesson Practice ACC7.9.01

Lesson Summary

Expressions with *exponents* are useful for representing repeated multiplication. In the expression 3^5, 5 is the exponent. When the exponent is a positive integer, it says how many times the number or expression is multiplied.

For example, $3^5 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{5 \text{ times}}$. Imagine writing 3^{100} using multiplication!

Here are a few more examples:

- $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^5$
- $5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 = 5^4 \cdot 8^3$
- $10 \cdot 10 \cdot 10 + 10 \cdot 10 = 10^3 + 10^2$

Lesson Practice
ACC7.9.01

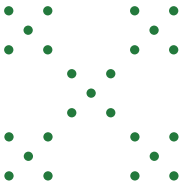
Name: _____ Date: _____ Period: _____

1. Notice how the number of circles changes at each stage.

Stage 0

Stage 1

Stage 2



How many circles will there be in Stage 4?

2. Complete the table.

Expanded Expression	Exponent Expression	Value
$3 \cdot 3 \cdot 3 \cdot 3$		
$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$		
	2^5	
	4^3	

Problems 3–4: Write an equivalent expression that uses exponents.

3. $2 \cdot 9 \cdot 2 \cdot 9 \cdot 2 \cdot 2$

4. $7 \cdot 7 \cdot 7 + 7 \cdot 7 \cdot 7 \cdot 7$

5. Each day, the number of grains of rice you have triples. On Day 1, you have 3 grains of rice. On Day 2, you have 9 grains of rice. On what day will you have 243 grains of rice?

Show or explain your thinking.

FAST Practice

6. Adnan starts with two coins on Day 1. The number of coins doubles every day. How many coins will he have on Day 8? Write your answer as an expression with an exponent. Record your answer in the space provided.

Name: _____ Date: _____ Period: _____

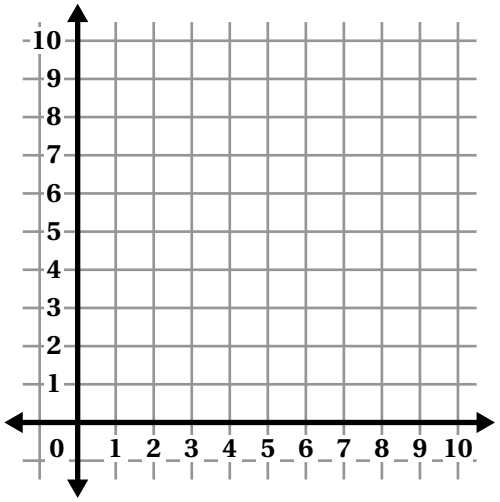
Spiral Review

7. Which expression is equivalent to $10000 + 225$?

(A.) $10^3 + 9 \cdot 25$ (B.) $10^4 + 15^2$ (C.) $100^3 + 9 \cdot 25$ (D.) $1000^2 + 15^2$

8. The points $(2, 4)$ and $(6, 7)$ lie on a line. What is the slope of the line?

Sketch on the grid if it helps with your thinking.



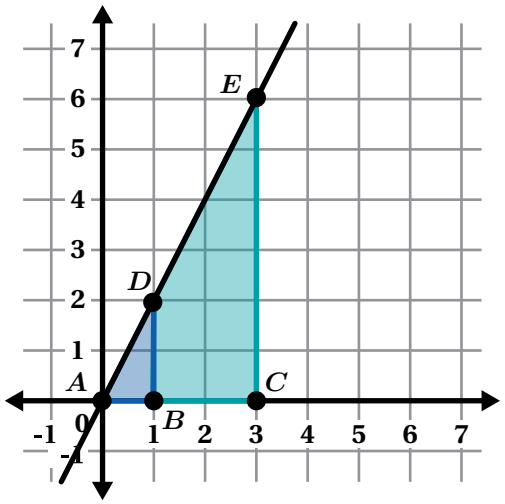
(A.) 2

(B.) 1

(C.) $\frac{4}{3}$

(D.) $\frac{3}{4}$

Problems 9–10: Here is a diagram that shows a pair of similar figures.



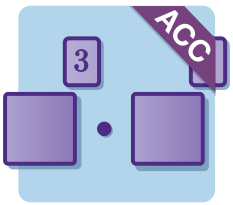
9. What does the center of dilation need to be to dilate triangle ACE onto triangle ABD?

10. What does the scale factor need to be to dilate triangle ACE onto triangle ABD?

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MTR.2.1, MTR.5.1, MTR.6.1

Combining Exponents



Let's explore equivalent expressions with exponents.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. $(2^2)^3$

B. $2^3 \cdot 3 \cdot 2^2$

C. $2 \cdot 32$

D. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Activity 1

Name: Date: Period:

Combining Exponents

2. Here are some different ways to build a billion, 10^9, by multiplying two **powers of ten**.

$(10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$
$10^2 \cdot 10^7 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$
$10^4 \cdot 10^5 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10)$
$10^6 \cdot 10^3 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10)$
$10^8 \cdot 10^1 = 10^9$

Discuss: What do you notice? What do you wonder?

3. Group the expressions by whether they are equivalent to 8^5, 8^6, or neither. Some expressions will have no match.

$8 + 8 + 8 + 8 + 8 + 8$	40	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8 \cdot 8)$	$8^3 \cdot 8^3$
$8^3 \cdot 8^2$	$(8^3)^2$	$8^4 + 8$	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8)$

Equivalent to 8^5	Equivalent to 8^6	Neither

4. Here is a new expression: $4^5 \cdot 2^5$. Is this expression equivalent to 8^5? Show or explain your thinking.

Name: Date: Period:

Odd One Out

5. **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$3 + 3 + 3 + 3 + 3$ $3^2 \cdot 3 \cdot 3 \cdot 3$ 3^5

b How could you change this expression so that it has the same value as the others?

6. **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ 20^3 $(5 \cdot 3) \cdot (4 \cdot 3)$

b How could you change this expression so that it has the same value as the others?

7. **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$(2 \cdot 2 \cdot 2)\ (2 \cdot 2 \cdot 2)$ 6^2 $(2^3)^2$

b How could you change this expression so that it has the same value as the others?

Name: Date: Period:

Odd One Out (continued)

8. Here are some more pairs of equivalent expressions.

For each row, write one more equivalent expression.

Expression 1	Expression 2	Expression 3
$4 \cdot 4 \cdot 4^3$	4^5	
6^4	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	
$(7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$	$(7^2)^3$	
5^{18}	$(5^3)^6$	

You're invited to explore more.

9. Using whole numbers 0 through 9 without repeating, fill in the blanks so that all expressions have the same value.

$$\left(5^{\square}\right)^{\square}$$

$$5^{\square} \cdot 5^{\square}$$

$$5^{\square} \cdot 5^{\square} \cdot 5^{\square}$$

$$5^{\square}$$

Synthesis

10. **a** Describe some strategies for writing equivalent expressions involving exponents.

Use the examples if they help with your thinking.

$3^4 \cdot 3^2 = 3^6$

$3^5 \cdot 4^5 = 12^5$

$(3^2)^4 = 3^8$

b Fill in the blanks to write a general pattern that can be used to create an expression equivalent to each of the expressions below.

$(a^m)(a^n) = a^{\square}$

$(ab)^m = (a^{\square})(b^{\square})$

Lesson Practice ACC7.9.02

Lesson Summary

Expanding is one strategy for determining if expressions with exponents are equivalent.

Here are two **powers of ten** that are equivalent to 10^8. Each expression can be expanded to "$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$."

- $10^5 \cdot 10^3 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^8$
- $(10^2)^4 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^8$

The product of powers law states that when powers with the same *base* are multiplied, you can add the exponents to get an equivalent expression.

For example, $10^4 \cdot 10^3 = 10^{(4+3)} = 10^7$.

The power of a product law states that when a product is raised to a power, you can distribute the exponent to each term inside the parentheses.

For example, $(5 \cdot 2)^6 = 5^6 \cdot 2^6$.

Lesson Practice
ACC7.9.02

Name: _____ Date: _____ Period: _____

Problems 1–2: For each expression, write an equivalent expression with a single power. Two examples are shown.

1.

Expression	Single Power
$6^3 \cdot 6^9$	6^{12}
$2 \cdot 2^4$	
$12^5 \cdot 12^{12}$	
$7^6 \cdot 7^6 \cdot 7^6$	

2.

Expression	Single Power
$(3^7)^2$	3^{14}
$(2^9)^3$	
$(7^6)^3$	
$(11^2)^3$	

3. Which expression is equivalent to $6^4 \cdot 2^4$?

A. 8^4 B. 8^8 C. 12^4 D. 12^{16}

Problems 4–6: Here is a large rectangular swimming pool. The pool is filled to the top with water.

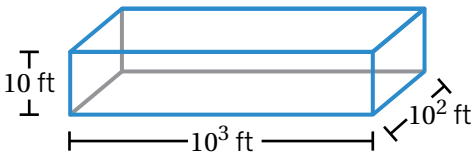


Figure may not be drawn to scale.

4. Write an expression with exponents to represent how much water the pool holds.

_____ cubic feet

5. Write an equivalent expression with a single power.

_____ cubic feet

6. You want to buy a cover to put on top of the pool. Write an expression with exponents to represent the area of the cover you need to buy.

_____ square feet

Name: ..... Date: ..... Period: .....

7. Fill in each blank using the whole numbers 1 to 9 only once to make a true statement.

$$6^{\square} \cdot 6^{\square} = \left(6^{\square}\right)^{\square}$$

FAST Practice

8. Select *all* the expressions that are equivalent to 6^8.

- ☐ **A.** $2^8 \cdot 3^8$
- ☐ **B.** $(6^4)^4$
- ☐ **C.** $(6^2)^4$
- ☐ **D.** $6^4 + 6^4$
- ☐ **E.** $6^4 \cdot 6^2$

Spiral Review

9. The expression $0.7 \cdot 0.52$ is equivalent to what number?

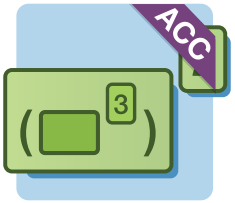
10. Determine the quotient of 1.44 and 0.06.

11. Diego has a length of ribbon 2.6 meters long. For a crafts project, he needs to cut the ribbon into pieces that are 0.4 meters long. How many pieces of ribbon can he make to be used for the project? Explain your thinking.

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MTR.2.1, MTR.5.1, MTR.6.1

Power Pairs



Let's determine if expressions with exponents are equivalent.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. 2^2

2. 2^3

3. $2^2 \cdot 2^3$

4. 2^5

Name: Date: Period:

Power Pairs

5. Here are two expressions.

$(3^4)^2$ $3^5 \cdot 3^3$

Discuss Are these expressions equivalent? How do you know?

6. You will use a set of cards to complete this activity. Follow the instructions on the presentation screen to play the game *Power Pairs* with your group. Record the pairs of equivalent expressions in the table.

Player's Name	Expression 1	Expression 2

Workspace:

Activity 2

Name: Date: Period:

Are They Equivalent?

7. Decide if the expressions in each pair are equivalent. Show your thinking.

	Expression 1	Expression 2	Equivalent? Circle one
a	$(5^5)^2$	$5^4 \cdot 5^3$	Yes No
b	$4^3 \cdot 2^5$	8^8	Yes No
c	$15^3 \cdot 2^3$	$(5 \cdot 2)^3 \cdot 3^3$	Yes No

8. Decide whether each expression is equivalent to 10^6. Rewrite any expressions that are *not* equivalent so that they are equivalent to 10^6.

a. $\frac{10 \cdot 10^4 \cdot 10^2}{10}$

b. 100^5

c. $10^3 + 10^3$

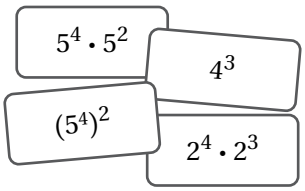
d. $(10^2)^3$

You're invited to explore more.

9. Write at least four different expressions that are equivalent to $(3^4)^2$ using only numbers, multiplication, and exponents.

Synthesis

10. **a** What are some important things to remember when determining whether expressions with exponents are equivalent?



b Fill in the blanks to write a general pattern that can be used to create an expression equivalent to the expression below.

$(a^m)^n = a$ ☐

Lesson Practice ACC7.9.03

Lesson Summary

Rewriting powers can help you make sense of different bases with the same exponent. Here is one example.

If you expand $4^6 \cdot 3^6$, it is equal to $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)\ (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$. You can rearrange the factors to get $(4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)$, which is equal to 12^6. This means that $4^6 \cdot 3^6$ is equivalent to 12^6.

You may not need to expand an expression completely to determine whether it is equivalent to another expression. For example, $(12^4)^2$ is not equivalent to 12^6 because it is $(12^4) \cdot (12^4) = 12^8$.

The power of a power law states that to determine the power of a power, multiply the exponents.

For example, $(5^4)^3 = 5^{4 \cdot 3} = 5^{12}$.

Lesson Practice
ACC7.9.03

Name: ..... Date: ..... Period: .....

1. Write four different expressions that have the same value as $(5^2)^3$ using only numbers, multiplication, and exponents.

....

2. Show or explain why $6^5 \cdot 6^3$ is equivalent to $(6^4)^2$.

3. Write another expression that is equivalent to both $6^5 \cdot 6^3$ and $(6^4)^2$.

4. Select *all* the expressions that are equivalent to $2^3 \cdot 2^3$.

- ☐ **A.** $2^2 \cdot 2^4$
- ☐ **B.** 2^9
- ☐ **C.** 4^6
- ☐ **D.** $(2^3)^2$
- ☐ **E.** $2^5 \cdot 2$

5. If $(3^2)^n = 3^8$, what value of n will make this a true statement? Explain your thinking

FAST Practice

6. For each pair, determine whether the expressions are equivalent or not equivalent.

	Equivalent	Not Equivalent
Expression 1: $(12^2)^3$ Expression 2: $12^4 \cdot 12^2$	☐	☐
Expression 1: $(7 \cdot 2)^3$ Expression 2: $7^3 \cdot 2^3$	☐	☐
Expression 1: $10^2 + 10^4 + 10$ Expression 2: 10^7	☐	☐
Expression 1: 15^6 Expression 2: $(5 \cdot 3 \cdot 3 \cdot 5)^4$	☐	☐

Name: ..... Date: ..... Period: .....

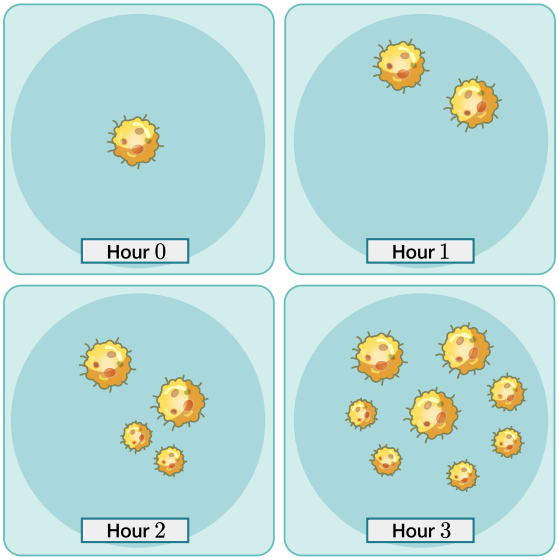
Spiral Review

Problems 7–10: An amoeba (a single-celled animal) is being observed in a dish. Every hour, each amoeba divides to form two amoebas.

7. How many amoebas will there be after 5 hours?

8. Write an expression for the number of amoebas after 8 hours.

9. Write an expression for the number of amoebas after 24 hours.



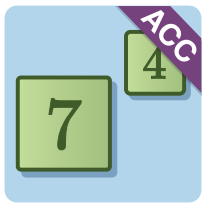
10. Why might an expression with an exponent, like 2^6, be useful for determining the number of amoebas?

Name: Date: Period:

MA.7.NSO.1.1, MTR.2.1, MTR.5.1, MTR.6.1

Rewriting Powers

Let's rewrite expressions with exponents as a single power.



Warm-Up

1. Order the expressions from what you think is *least complicated* to *most complicated*.

$\frac{7^7}{7^3}$ $\quad \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\quad \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

$\frac{7^3 \cdot 7^3}{7^2}$ $\quad 7^4$

Least Complicated

Most Complicated

Activity 1

Name: _____ Date: _____ Period: _____

Single Powers

2. Circle one expression.

$\frac{7^3 \cdot 7^3}{7^2}$ $\quad$ $\frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\quad$ $\frac{7^7}{7^3}$ $\quad$ $\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

Show or explain how to rewrite this expression as the single power 7^4.

3. Here is how Jayla rewrote $\frac{7^5 \cdot 7^2}{7^3}$ as a single power.

Discuss: How could you rewrite $\frac{4^9}{4^2 \cdot 4^4}$ as a single power?

Jayla

$$\frac{7^5 \cdot 7^2}{7^3} = \frac{(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7 \cdot 7}$$
$$= \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$$
$$= 1 \cdot 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7$$
$$= 7^4$$

7^4 ← single power of 7

4. Rewrite each expression as a single power.

Expression	Single Power
$\frac{7^5 \cdot 7^2}{7^3}$	7^4
$\frac{4^9}{4^2 \cdot 4^4}$	
$\frac{2^3 \cdot 2^3 \cdot 2^3}{2 \cdot 2 \cdot 2}$	

Expression	Single Power
$2^4 \cdot 3^4$	
$\frac{6^7}{2^7}$	
$\frac{(8^4)^2}{8}$	

Name: Date: Period:

Single Powers (continued)

5. Sort the expressions based on whether they are equivalent to 6^8.

$2^8 \cdot 3^8$ $\quad 2^3 \cdot 3^5$ $\quad \frac{12^8}{2^8}$ $\quad \frac{6^4 \cdot 6^4}{6^1}$ $\quad \frac{6^7 \cdot 6^7}{(6^3)^2}$

Equivalent to 6^8	Not Equivalent to 6^8

6. Diamond and Luca are working together to rewrite an expression the expression $\frac{6^9}{6^4}$ as a single power. Diamond uses the quotient of powers law, but Luca prefers to write the powers in expanded form.

Diamond

$$\frac{6^9}{6^4} = 6^{9-4}$$

$$= 6^5$$

Luca

$$\frac{6^9}{6^4} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6 \cdot 6}$$

$$= \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$$

$$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$$

$$= 6^5$$

a Why does each strategy result in the same value?

b What is an advantage of knowing both strategies?

7. Rewrite the expression $\frac{3^5 \cdot 3^2}{3^3}$ as a single power using each strategy.

Power Laws	Expanded Form

Activity 2

Name: Date: Period:

Challenge Creator

8. You will use a separate sheet of paper to create your own single power challenge.

a. **Make It!** On your sheet of paper, write down a single power, using a positive integer for the base and any positive integer for the exponent. (For example, 4^3 or 5^{16} or 2^{100}.)

b. **Solve It!** On this page, record your single power, then create an expression that is equivalent to it. (For example, $5^{10} \cdot 5^6$ is equivalent to 5^{16}.)

My Single Power	Equivalent Expression

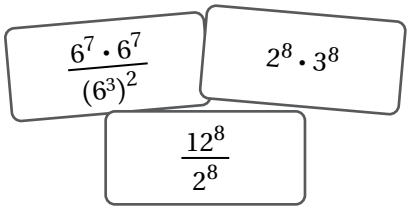
c. **Swap It!** Swap your challenge with one or more partners. Record your partner's single power, then create an expression that is equivalent to it.

	Single Power	Equivalent Expression
Partner 1		
Partner 2		
Partner 3		
Partner 4		

Synthesis

9. **a** Describe a strategy for rewriting an expression as a single power.

Use these expressions if they help with your thinking.



b Fill in the blank to write a general pattern that can be used to create an expression equivalent to the expression below.

$$\frac{a^m}{a^n} = a^{\square}$$

Lesson Practice ACC7.9.04

Lesson Summary

You can rewrite expressions as a single power like 7^3 to help you make sense of more complex expressions, especially ones that involve division. Expanding is one strategy for rewriting expressions with exponents as single powers.

Here are two examples.

$$\begin{aligned}\frac{(3^3)^2}{3^4} &= \frac{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3} \\ &= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \\ &= 3^2\end{aligned}$$

$$\begin{aligned}\frac{9^2 \cdot 3^5}{3^3} &= \frac{(9 \cdot 9) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} \\ &= \frac{(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} \\ &= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 3^6\end{aligned}$$

Knowing that $\frac{(3^3)^2}{3^4} = 3^2$ and $\frac{9^2 \cdot 3^5}{3^3} = 3^6$ can help you compare the two: $\frac{9^2 \cdot 3^5}{3^3}$ is greater than $\frac{(3^3)^2}{3^4}$ because 3^6 is greater than 3^2.

The quotient of powers law states that when you divide two powers with the same base, you can subtract their exponents to get an equivalent expression.

$$\frac{a^m}{a^n} = a^{m-n}$$

For example, $\frac{5^{10}}{5^4} = 5^{10-4} = 5^6$.

Lesson Practice
ACC7.9.04

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are equivalent to 10^6.

- ☐ **A.** $5^6 \cdot 2^6$
- ☐ **B.** $10^8 - 10^2$
- ☐ **C.** $10^3 \cdot 10 \cdot 10^2$
- ☐ **D.** $\frac{10^9}{10^3}$
- ☐ **E.** $(10^3)^3$

Problems 2–9: Rewrite each expression as a single power. Show your thinking.

2. $4^4 \cdot 5^4$

3. $\frac{5^6}{5^3}$

4. $(14^3)^6$

5. $\frac{21^3 \cdot 21^5}{21^2}$

6. $8^3 \cdot 8^6$

7. $\frac{3^{10}}{3}$

8. $(12^2)^7 \cdot 12$

9. $\frac{16^6}{2^6}$

Name: ..... Date: ..... Period: .....

10. Diamond and Luca are rewriting another expression. Luca prefers to write the problem in expanded form, but Diamond wants to help her by using the quotient of powers law. Explain how Diamond can use the quotient of powers law to help Luca finish the problem.

$$\frac{9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9}{9 \cdot 9 \cdot 9}$$

11. Rewrite the expression $\frac{10^8 \cdot 10^6}{10^7}$ as a single power using your preferred strategy. Explain your reasoning.

FAST Practice

12. What is the value of n in the equation $\frac{7^{12}}{7^n} = 7^8$?

$n =$ ☐

Spiral Review

Neena is selling her famous lemonade at the town fair. The following table shows the number of lemons used compared to the number of pitchers made.

Pitchers Made (x)	Lemons Used (y)
5	40
10	80
15	120

13. Write an equation to represent the number of lemons used (y) based on the number of pitchers made (x).

14. How many lemons she would need to make 25 pitchers of lemonade.

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MTR.4.1, MTR.5.1

Exponent Zero

Let's explore the exponent zero.

Warm-Up

1. Order the expressions by value from *least* to *greatest*.

$\frac{2^6}{2^3}$	$\frac{2^5}{2^5}$	2^6	0	$\frac{2^3}{2^4}$

Least

Greatest

Name: Date: Period:

Zero Exponents

2. Complete as much of the table as you can.

Exponent Form	Expanded Form	Value
10^4	$10 \cdot 10 \cdot 10 \cdot 10$	10,000
10^3	$10 \cdot 10 \cdot 10$	
10^2		100
	10	10
10^0		

3. What patterns do you see in the table? Describe as many as you can.

4. Cameron wanted to investigate more about zero exponents. He decided to write some expressions and apply exponent properties.

Discuss: What patterns do you notice in Cameron's expressions?

Cameron

$10^5 \cdot 10^4 = 10^9$

$10^5 \cdot 10^3 = 10^8$

$10^5 \cdot 10^2 = 10^7$

$10^5 \cdot 10^1 = 10^6$

$10^5 \cdot 10^0 = 10^5$

Name: Date: Period:

Beyond 10

5. Complete this new table about powers of 3.

Exponent Form	Expanded Form	Value
3^3	$3 \cdot 3 \cdot 3$	27
3^2	$3 \cdot 3$	
3^1		3
3^0		

6. Match each expression to an equivalent expression.

$\left(\frac{1}{3}\right)^5$	$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$
$3^5 \cdot 3^5 \cdot 3^0$	$\frac{1}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$
$\frac{1}{5^3}$	3^{10}
1	15^0

7. Look at the two expressions shown.

9^0 99^0

Are these equivalent? Circle one.

Yes No I'm not sure.

Explain your thinking.

Synthesis

8. How could you use the table to convince someone that $6^0 = 1$?

Exponent Form	Value
6^3	216
6^2	36
6^1	6
6^0	

Lesson Practice ACC7.9.05

Lesson Summary

We can use patterns to make generalizations about powers with zero exponents. For example, this table shows that each time the exponent decreases by 1, the value gets divided by 4. Based on this pattern, we can determine that $4^0 = 1$.

Exponent Form	Expanded Form	Value
4^3	$4 \cdot 4 \cdot 4$	64
4^2	$4 \cdot 4$	16
4^1	4	4
4^0	1	1

Any power with an exponent of 0 is equal to 1.

Examples:

$$\left(\frac{1}{5}\right)^0 = 1 \qquad (3)^0 = 1$$

Name: _____ Date: _____ Period: _____

Problems 1–2: Prisha says, "I can determine the value of 5^0 by looking at other powers of 5." She organizes her work in the table shown.

1. What pattern do you notice in Prisha's table?

2. If this pattern continues, what could the value of 5^0 be?

Exponent Form	Value
5^3	125
5^2	25
5^1	5
5^0	

3. Select *all* the statements that are true about zero exponents.

- ☐ **A.** $10^0 = 10$
- ☐ **B.** $8^0 = 1$
- ☐ **C.** $12^0 = 0$
- ☐ **D.** $x^0 = 1$ for any nonzero value of x.
- ☐ **E.** Any whole number raised to the power of zero is equal to 0.

4. Determine if each equation is true or false. Then change one side of each false equation to make it true.

Equation	True	False	Changed Equation
$\frac{4^3}{4^3} = 4^0$			
$4^0 \cdot 4^4 = 4^5$			
$\left(\frac{1}{4}\right)^0 = 1$			

Name: ..... Date: ..... Period: .....

Problems 5–7: Rewrite each expression as a single power.

5. $2^0 \cdot 2^4 \cdot 2^3$

6. $\frac{x^3}{x^0} \cdot x^2$

7. $7^0 \cdot 7^0 \cdot 7^3 \cdot 7^5$

FAST Practice

8. In the expressions, a and b represent integers. The value of Expression 2 is 100 times the value of Expression 1. What is the value of b?

Expression 1: 10^{a+b} Expression 2: 10^a

(A.) 2 (B.) 0 (C.) -2 (D.) -4

Spiral Review

Problems 9–10: Determine the value of each expression.

9. $42 - 8 \cdot 2^3$

10. $\frac{4 + (3 + 1)^2}{4}$

11. A student in your class showed his work while determining the value of an expression. Circle any line that contains an error and correct the error beside it.

$$(4 + 4)^2 + 10 \cdot 2 - 3$$

$$(8)^2 + 10 \cdot 2 - 3$$

$$64 + 10 \cdot 2 - 3$$

$$74 \cdot 2 - 3$$

$$148 - 3$$

$$145$$

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MTR.4.1, MTR.5.1

Negative Bases

Let's explore powers with negative bases.

Warm-Up

1. Write each expression using a single exponent.

a $(-3) \cdot (-3) \cdot (-3)$

b $\left(-\frac{4}{5}\right) \cdot \left(-\frac{4}{5}\right)$

c $(-0.8) \cdot (-0.8) \cdot (-0.8) \cdot (-0.8)$

Name: Date: Period:

Negative or Positive Values

2. Complete the table.

Exponent Form	Expanded Form	Value
$(-5)^5$	$(-5) \cdot (-5) \cdot (-5) \cdot (-5) \cdot (-5)$	-3,125
$(-5)^4$	$(-5) \cdot (-5) \cdot (-5) \cdot (-5)$	
$(-5)^3$		
$(-5)^2$		
$(-5)^1$		
$(-5)^0$		

3. Describe any patterns you notice in the table.

4. Oscar simplified the expression $\frac{(-2)^5}{(-2)^3}$. He said that the expression is equivalent to a positive value. Is Oscar correct? Explain your thinking.

Name: ..... Date: ..... Period: .....

Even or Odd Exponents

5. Complete the table.

Expression	Expression With a Single Exponent	Value	Is the Value Positive or Negative?	Is the Exponent Even or Odd?
$(-4)^4$	$(-4)^4$	256	Positive	Even
$(-0.5)^3$	$(-0.5)^3$			
$\left(-\frac{1}{4}\right)^2$				
$\frac{(-2)^2}{(-2)^1}$				
$(-3)^1 \cdot (-3)^3$				

6. Describe any patterns you notice in the table.

Name: Date: Period:

Even or Odd Exponents (continued)

7. Here are two expressions.

$(-2)^3 \cdot (-2)^4$ $-2^3 \cdot -2^4$

Imani says that the two expressions are equivalent.

Nekeisha claims the two expressions are not equivalent.

Whose claim do you agree with? Circle one.

Imani Nekeisha

Explain your thinking.

Synthesis

8. Describe the importance of parentheses when simplifying exponential expressions with negative bases. Use the expressions if it helps with your thinking.

-4^2

$(-4)^2$

Lesson Practice ACC7.9.06

Lesson Summary

You can simplify and write exponential expressions with negative bases by understanding the importance of parentheses. Here are two scenarios to think about:

- When parentheses are around the entire base, including the negative sign, the negative sign is included when you evaluate the exponent. This means that the negative number is used as a factor n times and the result will depend on whether the exponent n is **even** or **odd**.
- When there are no parentheses around the base or the negative sign is outside the parentheses, you apply the negative sign after you evaluate the power of the base.

Examples:

- $(-5)^2 = 25$
- $-5^2 = -25$
- $\left(-\frac{1}{3}\right)^2 = \frac{1}{9}$
- $-\left(\frac{1}{3}\right)^2 = -\frac{1}{9}$

Lesson Practice
ACC7.9.06

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value for each expression.

1. $(-4)^2$

2. $\left(-\frac{1}{5}\right)^3$

3. -2^4

4. $(-3)^5$

5. Order the expressions from *least* to *greatest*.

-2	$-2^2 \cdot (-2)^2$	$(-2)^3$	$(-2)^2$

Least **Greatest**

6. What is the value of p in the equation $\frac{(-3)^p}{(-3)} = (-3)^2$?

7. Show or explain why $-6^2 \cdot (-6)^2$ is *not* equal to $(-6)^4$.

Name: ..... Date: ..... Period: .....

FAST Practice

8. Select *all* the expressions that are equivalent to a *positive* whole number.

- ☐ **A.** $(-9)^5$
- ☐ **B.** $(-9)^4$
- ☐ **C.** $(-9)^3$
- ☐ **D.** $(-9)^2$
- ☐ **E.** $(-9)^1$

Spiral Review

9. What is the value of $5^0 \cdot 5^1 \cdot 5^2$?

10. Look at the two expressions shown.

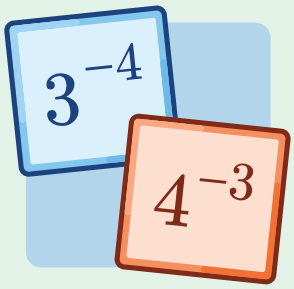
$$\frac{9^3}{9^0} \qquad 729$$

Are these equivalent? Explain your thinking.

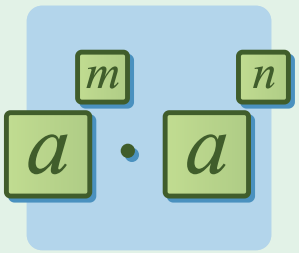
Sub-unit

2

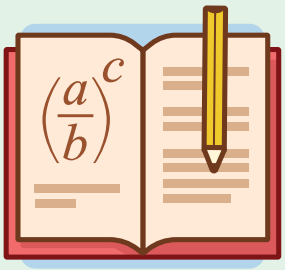
Algebraic Expressions



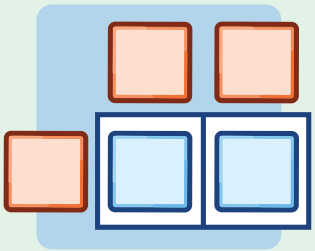
Lesson 7
Negative Exponents



Lesson 8
Write a Rule



Lesson 9
Algebraic Expressions With Exponents



Lesson 10
Multiplying Linear Expressions

Lesson 11
Fabulous Factors

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.AR.1.1, MA.8.AR.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Negative Exponents

Let's explore negative exponents.

Warm-Up

1. Order the expressions by value from *least* to *greatest*.

$\frac{3^4}{3^3}$	$\frac{3^0}{3^2}$	3^0	$\frac{3^2}{3^3}$	$\frac{3^5}{3^2}$

Least

Greatest

Name: Date: Period:

Understanding Negative Exponents

2. Complete as much of this table as you can about the powers of 4.

Exponent Form	Expanded Form	Value
4^3	$4 \cdot 4 \cdot 4$	64
4^2	$4 \cdot 4$	
4^1	4	4
4^0		
	$\frac{1}{4}$	$\frac{1}{4}$
4^{-2}		
4^{-3}		

3. The value of $4^5 = 1024$. Predict the value of 4^{-5}.

Name: ..... Date: ..... Period: .....

Understanding Negative Exponents (continued)

4. Here are some expressions.

$\left(\frac{1}{4}\right)^3$	$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$	4^{-3}	
$\frac{1}{81}$		3^{-4}	
-12	$\frac{1}{4^3}$	$\frac{1}{12}$	

a Write down two different groups of equivalent expressions. Some expressions may be left over.

Group 1:

Group 2:

b Here are two expressions from earlier.

4^{-3} -12

Are these equivalent? Circle one.

Yes No I'm not sure.

Explain your thinking.

Name: Date: Period:

Evaluating Expressions With Negative Exponents

5. Complete the table and determine which other expressions are equivalent to $\frac{1}{2^3}$. Rewrite each using a single exponent.

Expression	With a Single Exponent	Equivalent to $\frac{1}{2^3}$?
$2^{-3} \cdot 2^{-1}$	2^{-4}	No
$2^{-2} \cdot 2^{-1}$		
$2^3 \cdot 2^1$		
$2^2 \cdot 2^{-5}$		
$2^3 \cdot 2^{-4}$		

6. Determine the value of $\left(\frac{2^2}{2^3}\right)^2$. Show your work.

7. Emika and Latifa used different strategies to rewrite $\frac{1}{3^{-5}}$ with a positive exponent.

Emika: $\frac{1}{3^{-5}} = \frac{1}{\frac{1}{3^5}} = 1 \div \frac{1}{3^5} = 1 \cdot \frac{3^5}{1} = 3^5$

Latifa: $\frac{1}{3^{-5}} = \frac{3^0}{3^{-5}} = 3^{0-(-5)} = 3^5$

Discuss: How are Emika's and Latifa's strategies alike? How are they different?

Name: Date: Period:

Algebraic Expressions With Integer Exponents

8. Complete this new table.

Exponent Form	Expanded Form
	$x \cdot x \cdot x$
x^2	
x^1	x
x^0	
	$\frac{1}{x}$
x^{-2}	
x^{-3}	

9. Use the table to justify why $x^0 = 1$.

10. Rewrite each expression using only positive exponents.

a $m^{-4}\, s^{-5}$

b $\frac{x^{-2}}{y^{-5}}$

Synthesis

11. Explain the difference between $-y$ and y^{-1}. Use the examples if it helps you explain your thinking.

-4 4^{-1}

Lesson Practice ACC7.9.07

Lesson Summary

Positive, negative, and zero exponents are all related.

For example, this table shows that each time the exponent decreases by 1, the value gets divided by 5. Based on the pattern, you can determine that

$5^{-1} = \frac{1}{5}$.

You can also use these patterns to make generalizations about algebraic expressions with integer exponents.

Examples:

- $x^2 = x \cdot x$
- $x^{-2} = \frac{1}{x} \cdot \frac{1}{x} = \frac{1}{x^2}$
- $\frac{1}{x^{-3}} = \frac{x^0}{x^{-3}} = x^{0-(-3)} = x^3$

Exponent Form	Expanded Form	Value
5^2	$5 \cdot 5$	25
5^1	5	5
5^0	1	1
5^{-1}	$\frac{1}{5}$	$\frac{1}{5}$
5^{-2}	$\frac{1}{5} \cdot \frac{1}{5}$	$\frac{1}{25}$

Lesson Practice
ACC7.9.07

Name: _____ Date: _____ Period: _____

Problems 1–2: Josiah says, "I can determine the value of 7^{-2} by looking at other powers of 7." Josiah organizes his work in a table.

Exponent Form	Expanded Form
7^2	49
7^1	7
7^0	1
7^{-1}	$\frac{1}{7}$
7^{-2}	

1. What pattern do you notice in Josiah's table?

2. If the pattern continues, what would be the value of 7^{-2}?

Problems 3–5: Determine if each equation is true or false. If it is false, change one side of the equation to make it true.

	Equation	True	False	Changed Equation
3.	$5^{-4} = -(5^4)$			
4.	$5^3 \cdot 5^{-5} \cdot 5^2 = 1$			
5.	$\frac{1}{5^{-2}} = \frac{1}{25}$			

Name: ..... Date: ..... Period: .....

Problems 6–8: Rewrite each expression with only positive exponents.

6. $x^{-6} \cdot z^{-2}$

7. $\frac{x^{-4}}{y}$

8. $\left(\frac{y^{-1}}{z^{-3}}\right)^2$

9. What is the value of 9^{-2}?

FAST Practice

10. Select *all* the expressions that are equivalent to $\frac{1}{2^3}$.

- ☐ **A.** 2^{-3}
- ☐ **B.** $2^2 \cdot 2^1$
- ☐ **C.** $2^{-5} \cdot 2^2$
- ☐ **D.** $\left(\frac{1}{2}\right)^3$
- ☐ **E.** -8
- ☐ **F.** $\frac{1}{8}$

Spiral Review

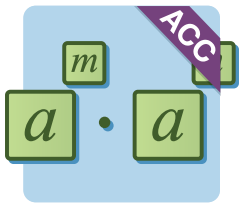
11. What is the value of h in the equation $7^{11} = 7^7 \cdot 7^h$?

12. Write two different expressions that have the same value as $(3^2)^0$.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.AR.1.1, MTR.4.1, MTR.5.1

Write a Rule



Let's describe the rules for exponents.

Warm-Up

1. You and your partner will use a set of cards. Sort the expressions into groups in a way that makes sense to you.

Name: Date: Period:

Write Your Own Rules

2. You and your partner will use the cards from the Warm-Up and the instructions from your teacher to create your own rules for three exponent situations.

Multiplying Powers With the Same Base

Examples	Rule (First Draft)	Rule (Second Draft)
$6^4 \cdot 6^{11} = 6^{15}$		

Dividing Powers With the Same Base

Examples	Rule (First Draft)	Rule (Second Draft)
$\frac{3^9}{3^1} = 3^8$		

Powers of Powers

Examples	Rule (First Draft)	Rule (Second Draft)
$\left(\left(\frac{7}{8}\right)^5\right)^2 = \left(\frac{7}{8}\right)^{10}$		

3. Choose any one of the three rules. Show or explain how you know the rule always works.

Name: Date: Period:

Three More Rules

4. You will use the cards from the Warm-Up and the instructions from your teacher to create your own rules for three more exponent situations.

Negative Exponents

Examples	Rule (First Draft)	Rule (Second Draft)
$3^{-6} = \frac{1}{3^6}$		

Powers With Different Bases

Examples	Rule (First Draft)	Rule (Second Draft)
$7^3 \cdot 3^3 = 21^3$		

Zero Exponents

Examples	Rule (First Draft)	Rule (Second Draft)
$10^0 = 1$		

5. Choose any one of the three rules. Show or explain how you know the rule always works.

Synthesis

6. Describe how exponent rules can help you rewrite exponential expressions.

Lesson Practice ACC7.9.08

Lesson Summary

There are several rules about powers that can be helpful when rewriting or comparing expressions with exponents.

Types of Powers	Rule With Variables	Example
Multiplying Powers With the Same Base	$a^n \cdot a^m = a^{n+m}$	$6^2 \cdot 6^7 = 6^{2+7} = 6^9$
Dividing Powers With the Same Base	$\frac{a^n}{a^m} = a^{n-m}$	$\frac{3^{11}}{3^4} = 3^{11-4} = 3^7$
Powers of Powers	$(a^n)^m = a^{n \cdot m}$	$(1.7^5)^3 = 1.7^{(5 \cdot 3)} = 1.7^{15}$
Negative Exponents	$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$14^{-2} = \frac{1}{14^2} = \left(\frac{1}{14}\right)^2$
Powers With Different Bases	$a^n \cdot b^n = (ab)^n$ $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$2^3 \cdot 5^3 = 10^3$ $\frac{2^3}{5^3} = \left(\frac{2}{5}\right)^3$
Zero Exponents	$a^0 = 1$	$\left(\frac{5}{9}\right)^0 = 1$

Note: The variables a and b are not equal to 0, and n and m are integers.

Lesson Practice
ACC7.9.08

Name: ..... Date: ..... Period: .....

Problems 1–3: Evaluate each expression.

1. 5^0

2. $\frac{6^3}{6^3}$

3. $2^2 + 2^1 + 2^0$

Problems 4–9: Rewrite each expression as a single power.

4. $\frac{5^3 \cdot 5^4}{5^5}$

5. $(7^4) \cdot \frac{7^{12}}{7^7}$

6. $\left(\frac{3^5}{3^3}\right)^4$

7. $\frac{2^4 \cdot 2^5 \cdot 2^6}{2^3 \cdot 2^7}$

8. $\frac{(10^5)^2}{(10^2)^3}$

9. $\left(\left(\frac{2}{3}\right)^4\right)^6$

Problems 10–12: Rewrite each expression as a single power with a negative exponent.

	Expression	Single Power With Negative Exponent
	$\frac{1}{6} \cdot \frac{1}{6}$	6^{-2}
10.	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$	
11.	$\frac{1}{5^7}$	
12.	$\left(\frac{1}{10^3}\right)^3$	

Name: ..... Date: ..... Period: .....

13. Kai tried to write $\left(\frac{5^8}{5^{10}}\right)^3$ as a single power of 5:

$$\left(\frac{5^8}{5^{10}}\right)^3 = (5^2)^3 = 5^{2 \cdot 3} = 5^6$$

Is Kai's work correct?

Explain your thinking.

14. Fill in each blank using the digits 0 to 9 only once to create a result with the greatest exponent.

$$\frac{3^{\square}}{3^{\square}} \times \left(3^{\square}\right)^{\square} = 3^{\square\square}$$

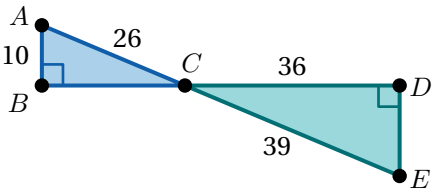
FAST Practice

15. Determine the value of n for the equation $6^n = \frac{1}{6^4}$.

Record your answer in the space provided.

Spiral Review

Problems 16–18: Here is a diagram.



16. Explain why triangle ABC is similar to triangle EDC.

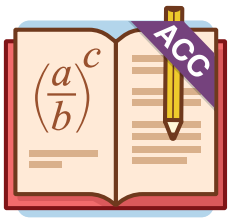
17. Determine the length of side BC.

18. Determine the length of side DE.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.AR.1.1, MTR.3.1, MTR.4.1, MTR.5.1

Algebraic Expressions With Exponents



Let's explore algebraic expressions with exponents.

Warm-Up

1. Which one doesn't belong?

A. 3^{-2}

B. $\frac{1}{2^4}$

C. 2^3

D. $\frac{1}{8}$

Explain your thinking.

Name: Date: Period:

Evaluating Exponents With Fractional Bases

2. Here are two students' strategies for evaluating the expression $\left(\frac{3}{5}\right)^3$.

Aba's work

$$\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$$

Dylan's work

$$\left(\frac{3}{5}\right)^3 = \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{3 \cdot 3 \cdot 3}{5 \cdot 5 \cdot 5} = \frac{27}{125}$$

Discuss: How are their strategies alike? How are they different?

3. Maneli started to evaluate the expression $\left(\frac{2}{3}\right)^{-3}$ as shown.

$$\left(\frac{2}{3}\right)^{-3} = \left(\left(\frac{2}{3}\right)^{-1}\right)^3 = \left(\frac{1}{\frac{2}{3}}\right)^3$$

a What do you think Maneli should do next? Explain.

b What is the value of the expression? Show your work.

c Could you use Aba's strategy from the previous problem to evaluate $\left(\frac{2}{3}\right)^{-3}$? Show or explain your thinking.

Name: ..... Date: ..... Period: .....

Evaluating Exponents With Fractional Bases (continued)

4. Complete as much of the table as you can. The first few rows have been completed for you.

Expression	Rewrite With a Positive Exponent	Evaluate or SImplify
$\left(\frac{1}{2}\right)^{-2}$	$\frac{1}{\left(\frac{1}{2}\right)^2}$ or $\left(\frac{2}{1}\right)^2$	4
$\left(-\frac{1}{2}\right)^{-2}$	$\frac{1}{\left(-\frac{1}{2}\right)^2}$ or $\left(-\frac{2}{1}\right)^2$	4
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{1}\right)^3$	
$\left(-\frac{1}{2}\right)^{-3}$		
$\left(\frac{a}{b}\right)^{-4}$		

5. Determine the value of $\left(-\frac{2}{5}\right)^{-2} \cdot (0.5)^3$. Show your work.

Name: Date: Period:

Putting the Rules Together

6. Use the laws of exponents to rewrite each expression using the variable y only once.

a $y^2 \cdot y^3$

b $\frac{y^2}{y^3}$

c $\frac{y^2 \cdot y^3 \cdot y^2}{y^2 \cdot y^3}$

7. Consider the expression $(4ab^5)^3$.

Is the expression $64a^3b^8$ an equivalent expression? Circle one.

Yes No Not sure

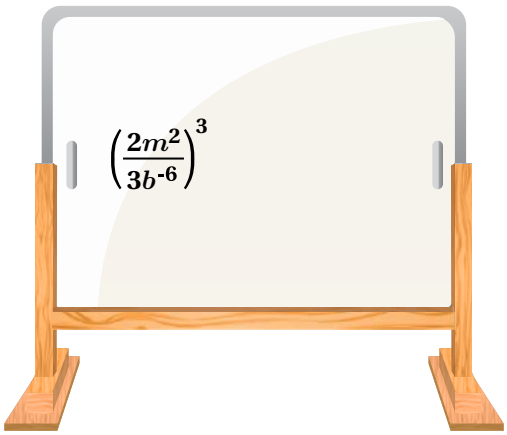
Show or explain your thinking.

Name: ..... Date: ..... Period: .....

Putting the Rules Together (continued)

8. The expression $(3x^4y^{-3})^2$ is equivalent to $9x^ny^m$. What are the values of n and m?

9. Your teacher writes an expression on the whiteboard and gives you two challenges to complete.



a **Challenge 1**: Write an equivalent expression with only two exponents.

b **Challenge 2:** Write a different equivalent expression with only two exponents and no negative exponents.

Synthesis

10. Explain how you can use the rules of exponents to simplify an expression. Use the example if it helps with your thinking.

$$\left(\frac{x^3y^6z^0}{x^2y}\right)^{-2}$$

Lesson Practice ACC7.9.09

Lesson Summary

When simplifying expressions with exponents, different laws of exponents can be applied to rewrite the expression in a different form.

Example: $(a^3b^{-2}c^0)^5$

1. Apply the rule for powers with different bases: $(a^3)^5 \cdot (b^{-2})^5 \cdot (c^0)^5$.
2. Apply the power of powers rule: $a^{3 \cdot 5}b^{-2 \cdot 5}c^{0 \cdot 5} = a^{15}b^{-10}c^0$.
3. Apply the rule for zero exponents: $a^{15}b^{-10} \cdot 1 = a^{15}b^{-10}$.
4. Apply the rule for negative exponents: b^{-10} can be written as $\frac{1}{b^{10}}$.

The expression is equivalent to $\frac{a^{15}}{b^{10}}$.

Lesson Practice
ACC7.9.09

Name: ..... Date: ..... Period: .....

Problems 1–2: Evaluate each expression. Show your thinking.

1. $\left(\frac{1}{5}\right)^{-2}$

2. $\left(-\frac{1}{4}\right)^{-3}$

Problems 3–6: Rewrite each expression using the variable only once.

3. a^4a^3

4. $a^2a^2a^{-2}$

5. $\frac{4x^5}{2x^3}$

6. $-\frac{6x^3}{3x^4x^2}$

Problems 7–8: Simplify each expression using only two exponents. Show your thinking.

7. $(5x^2y^4y^2)^{-2}$

8. $-\frac{6m^2n}{3m^4n^{-2}}$

Lesson Practice
ACC7.9.09

Name: _____ Date: _____ Period: _____

FAST Practice

9. Select *all* the expressions that are equivalent to $\left(\frac{3x^3}{12xy^5}\right)^{-1}$.

- ☐ **A.** $\frac{4y^2}{x^2}$
- ☐ **B.** $\frac{12xy^5}{3x^3}$
- ☐ **C.** $\frac{4y^5}{x^2}$
- ☐ **D.** $\left(\frac{x^2}{4y^5}\right)^{-1}$
- ☐ **E.** $\frac{12x^3}{3xy^5}$

Spiral Review

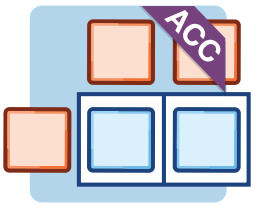
10. What is the value of 3^{-2}?

11. Rewrite the expression $5^{-5} \cdot 5^2$ with a single exponent. Then, determine its value.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.AR.1.2, MTR.3.1, MTR.4.1

Multiplying Linear Expressions



Let's multiply expressions with numbers and variables.

Warm-Up

1. Describe how you can use the *commutative property* to simplify $\frac{1}{2} \cdot 5 \cdot 20$.

2. Brianna and Riku simplify $\frac{1}{2}(10 + 8)$ using two different strategies.

Brianna

$$\frac{1}{2}(10 + 8)$$
$$= \frac{1}{2}(18)$$
$$= 9$$

Riku

$$\frac{1}{2}(10 + 8)$$
$$\left(\frac{1}{2} \cdot 10\right) + \left(\frac{1}{2} \cdot 8\right)$$
$$(5) + (4)$$
$$9$$

Compare their strategies.

Name: Date: Period:

One by One

3. Group the expressions by whether or not they are equivalent to $8x^8$.

$0.5x^6 \cdot 16x^2$	$\frac{1}{2}x^4 \cdot 16x^2$	$\frac{4}{2}x \cdot 4x^7$
$4x^2 \cdot 4x^6$	$1.25x^2 \cdot 6.4x^4$	$\frac{2}{3}x^3 \cdot 12x^5$

Equivalent to $8x^8$	Not Equivalent to $8x^8$

4. **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$0.25x^2 \cdot 16x^4$ $\frac{1}{2}x \cdot 8x^5$ $2x^3 \cdot 2x^2$

b Justify why the two uncircled expressions are equivalent.

5. Write a multiplication expression that is equivalent to $1.5x^{12}$.

Activity 2

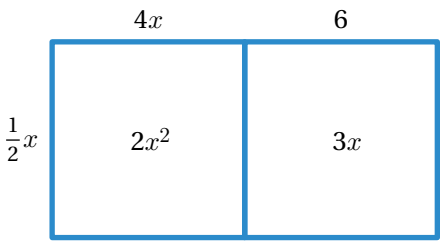
Name: ..... Date: ..... Period: .....

One by Two

6. DeShawn and Mayra each wrote an expression to represent the area model.

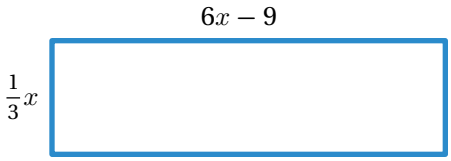
DeShawn: $\frac{1}{2}x(4x + 6)$

Mayra: $2x^2 + 3x$



Discuss: How are the expressions alike? How are they different?

7. Consider this rectangle with side lengths represented by algebraic expressions.



a Write a multiplication expression that represents the area of the rectangle.

b Explain how you can use the *distributive property* to simplify the expression.

8. Julian multiplied the expression $-0.25x(8x + 20)$. His work is shown below.

$$-0.25x(8x + 20) = -2x^2 + 20$$

Is Julian's answer correct? Explain your thinking. If it is incorrect, determine the correct product.

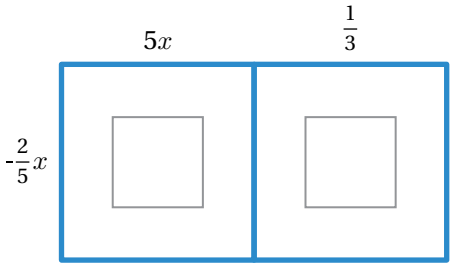
Activity 3

Name: Date: Period:

Multiplication Challenges

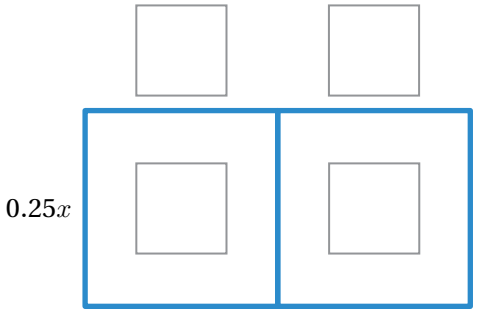
Problems 9–12: Complete the missing information in each area model and table.

9.



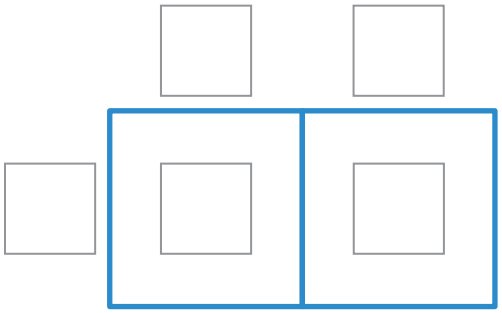
Factors	Product
$-\frac{2}{5}x\left(5x+\frac{1}{3}\right)$	

10.



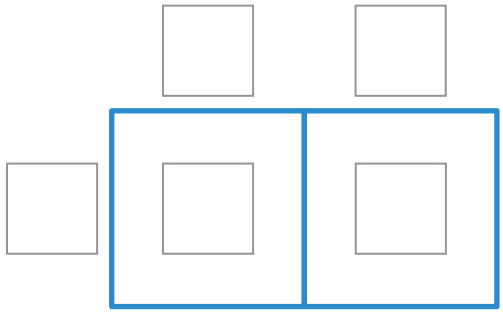
Factors	Product
$0.25x(8x-2)$	

11.



Factors	Product
$2x(-2.5x+4)$	

12.



Factors	Product
$-\frac{1}{4}x(12x-3y)$	

Synthesis

13. Describe some strategies for multiplying algebraic expressions.

Use the examples if they help you with your thinking.

$-1.2x^2 \cdot 5x^3 = -6x^5$

$\frac{3}{4}x\left(\frac{1}{2}x + 8\right) = \frac{3}{8}x^2 + 6x$

Lesson Practice ACC7.9.10

Lesson Summary

You can use properties you already know to multiply algebraic expressions.

To multiply $0.25x^2 \cdot 24x^5$, you can expand the expressions and use the *commutative property* to multiply the coefficients first.

Then you can use the product of powers property to multiply the variables.

$0.25x^2 \cdot 24x^5 =$

$0.25 \cdot x^2 \cdot 24 \cdot x^5 =$

$0.25 \cdot 24 \cdot x^2 \cdot x^5 =$

$6 \cdot x^{2+5} = 6x^7$

To multiply one term by the sum or difference of two terms, you can apply the *distributive property* or use an area model to organize your thinking.

Distributive Property

$-\frac{3}{4}x\left(\frac{16}{3}x + 4\right)$

$\left(-\frac{3}{4}x \cdot \frac{16}{3}x\right) + \left(-\frac{3}{4}x \cdot 4\right)$

$(-4x^2) + (-3x)$

Area Model

	$\frac{16}{3}x$	4
$-\frac{3}{4}x$	$-4x^2$	$-3x$

Lesson Practice
ACC7.9.10

Name: ..... Date: ..... Period: .....

Problems 1–2: Multiply each set of factors and write the product. Two examples are shown.

1.

Factors	Product
$\frac{3}{4}x \cdot \frac{5}{6}x^2$	$\frac{5}{8}x^3$
$-2.5x^2 \cdot -3x^3$	
$-\frac{1}{3}x^6 \cdot 9x^{-2}$	

2.

Factors	Product
$-1.5x(3x + 2.2)$	$-4.5x^2 - 3.3x$
$\frac{6}{7}x\left(7 + \frac{1}{3}x\right)$	
$-4x\left(x + \frac{1}{2}\right)$	

3. Which expression is the product of $-0.5x^3 \cdot 1.2x^2$?

(A.) $-6x^6$ (B.) $-0.6x^6$ (C.) $-6x^5$ (D.) $-0.6x^5$

4. Fill in each blank using the whole numbers 1 to 9 only once to make a true statement.

$$\frac{\square}{\square}x^{\square} \cdot 8x^{\square} = 6x^{\square}$$

5. Select *all* expressions equivalent to $3x^2 - 6x$.

☐ A. $\frac{3}{4}x(4 - 8x)$ ☐ B. $-0.75x(-4x + 8)$ ☐ C. $\frac{2}{5}x\left(\frac{15}{2}x - 15\right)$

☐ D. $1.25x(2.4x - 4.8)$ ☐ E. $-0.4x(7.5x + 1.5)$ ☐ F. $\frac{1}{2}x(6 - 12x)$

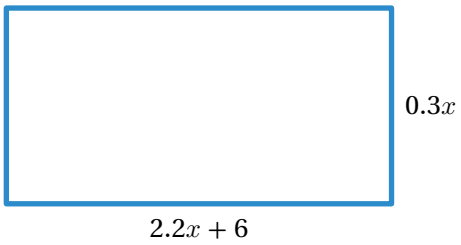
6. Determine the product: $\frac{1}{2}a(10b + 12)$.

Lesson Practice
ACC7.9.10

Name: ..... Date: ..... Period: .....

Problems 7–8: Here is a rectangle with side lengths given as algebraic expressions.

7. Determine the area in terms of x.



8. If the value of x is 5 cm, what is the area of the rectangle in square centimeters?

FAST Practice

9. Match the equivalent expressions.

	$6x^2 - 3x$	$4x^2 - 6x$
$\frac{3}{4}x\left(\frac{16}{3}x - 8\right)$	☐	☐
$-0.4x(15 - 10x)$	☐	☐
$1.2x(5x - 2.5)$	☐	☐

Spiral Review

Problems 10–11: Decide whether the given equation has *one solution*, *no solution*, or *infinitely many solutions*. Explain your thinking

10. $\frac{1}{3}(12x - 9) - 2x = 2\left(x - \frac{3}{2}\right)$

11. $1.2(4x - 5) = 0.8(3x + 2) + 2$

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.AR.1.3, MTR.4.1

Fabulous Factors

Let's use factors to rewrite expressions.

1. Identify the *greatest common factor* for each pair of numbers.

a 12 and 30

b 15 and 45

c 42 and 56

2. Write each expression as a product of numbers and variables.

a $20x$

b $100xy$

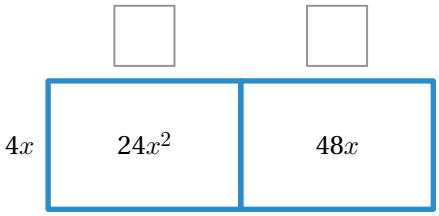
c $7x^2 z$

Name: Date: Period:

Factoring Expressions

To **factor** is to use the distributive property to rewrite an expression as the product of two or more factors.

3. The expression $24x^2 + 48x$ is shown in the area model.



a Fill in the missing values along the top of the model.

b Use the model to factor the expression.

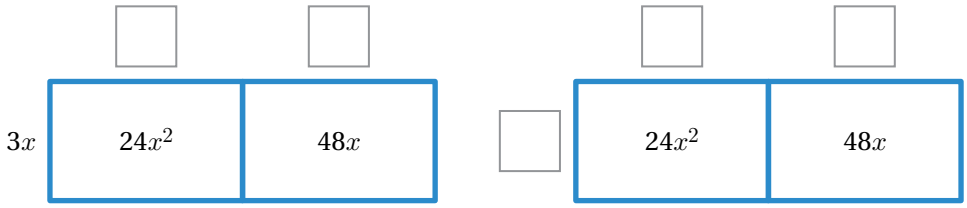
4. Emmanuel thinks that $2(12x^2 + 24x)$ is also equivalent to $24x^2 + 48x$. Do you agree? Circle one.

Agree Disagree There's not enough information

Show or explain your thinking.

5. Use the area models to factor $24x^2 + 48x$ in different ways from the ones above.

Model:



Factored Expression:

6. **Discuss:** Explain how multiplication and factoring are related.

Activity 2

Name: Date: Period:

Greatest Common Factor

One way to rewrite an expression is to factor out the greatest common factor, or GCF.

7. Here are Miriam's and Jalen's strategies for determining the *greatest common factor* of $30x^3y$ and $45x^2y$.

Miriam

$30x^3y = 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot y$

$45x^2y = 3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y$

$GCF = 3 \cdot 5 \cdot x \cdot x$

Jalen

	coefficients	variables
factors of $30x^3y$:	1, 2, 3, 5, 6, 10, 15, 30	x^2, x, y
factors of $45x^2y$:	1, 3, 5, 9, 15, 45	x^2, y

Discuss: What are the advantages and disadvantages of each strategy?

8. Look back at the factored expressions you wrote for $24x^2 + 48x$ in Activity 1.

a What is the greatest common factor of $24x^2$ and $48x$? Explain your thinking.

b Rewrite the expression $24x^2 + 48x$ by factoring out the GCF.

9. Find the GCF of each pair of expressions.

a $24x^2y^2$ and $36xy^3$

b $\frac{1}{3}x^3y$ and $\frac{2}{3}xy^2$

Activity 3

Name: ..... Date: ..... Period: .....

Factoring Challenges

10. Rewrite as many expressions using the greatest common factor as you have time for. Sensemaking is more important than speed.

a $8y^3 - 4y^2$

b $-9x^2 y - 12xy^2$

c $1.5a^3 + 0.75ab^2$

d $32m^3n - 16m^2n^2$

e $\frac{5}{2}xz - \frac{3}{2}x^2z^3$

f $\frac{7}{3}x^3 + \frac{14}{3}y^2$

Synthesis

11. How can you rewrite an expression as the product of a common factor and a sum or difference of two expressions? Use the expression shown if it helps with your thinking.

$12x^3y + 18x^2$

Lesson Practice ACC7.9.11

Lesson Summary

The distributive property is a rule that says $a(b + c) = ab + ac$. This means if you have an expression with terms that share a common factor, you can **factor** that expression. So, you can use the distributive property in reverse to rewrite the expression with the common factor outside parentheses.

For example, in the expression $12x^2\,y + 16x$, the two terms have a common factor of $2x$. Using the distributive property, you can rewrite it as $2x(6xy + 8)$.

Sometimes, there are several ways to factor an expression. One way is by using the *greatest common factor* (GCF). The GCF is the biggest number and variable that divides both terms.

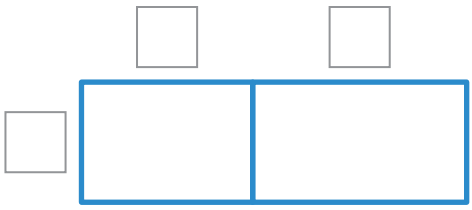
$$\begin{aligned} 12x^2y &= 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \\ 16x &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \\ \hline \text{GCF} &= 2 \cdot 2 \cdot x \end{aligned}$$

The GCF of $12x^2y$ and $16x$ is $4x$ because the numbers 12 and 16 have a GCF of 4 and both terms have at least one x. So you can factor the expression as $4x(3xy + 4)$.

Lesson Practice
ACC7.9.11

Name: _____ Date: _____ Period: _____

1. Use the area model to write $24xy + 30x^2y$ in factored form.



2. Select *all* expressions equivalent to $40x^3y + 32x^2y^2$.

- ☐ **A.** $16x^2y\,(5x + 2y)$
- ☐ **B.** $8x^2y\,(5x + 4y)$
- ☐ **C.** $2x^2y\,(10x + 16y)$
- ☐ **D.** $4x^2y\,(20x + 9y)$
- ☐ **E.** $4x^2y\,(10x + 8y)$
- ☐ **F.** $2x^2\,(20xy + 16y^2)$

3. Match each pair of monomials to their greatest common factor.

Monomial Pairs	GCF
a. $16x^3y,\ 8x^2$	_____ $6x^2y^2$
b. $12x^2y,\ 30xy^2$	_____ $8y^2$
c. $12x^2y^2,\ 18x^3y^7$	_____ $6xy$
d. $16x^3y^2,\ 24y^2$	_____ $8x^2$

4. Determine the GCF of $35x^2y^2$ and $28xy^2$. Explain your thinking.

Name: ..... Date: ..... Period: .....

Problems 5–8: Factor each expression using the greatest common factor.

5. $-14xy - 21xy^2$

6. $2.1x^3 + 0.7xy^2$

7. $12y^3 + 96xy^2$

8. $\frac{3}{4}xy - \frac{1}{4}x^2y$

FAST Practice

9. Pilar is building a sandbox for her neighborhood playground. The expression $16 + 24x$ represents how much material she has to build the perimeter of the sandbox. If she wants the sandbox to be a regular octagon, what expression would represent the length of each side? Record your answer in the space provided.

If the sandbox were a regular octagon, the expression ☐ would represent the length of each side.

Spiral Review

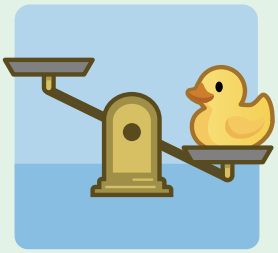
10. Determine the number of degrees for the sum of the interior angles of an octagon.

Sub-unit

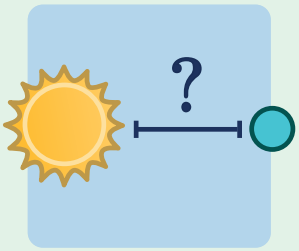
3

Scientific Notation

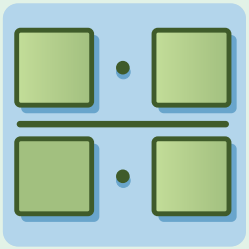
Lesson 12
Scales and Weights, Part 1



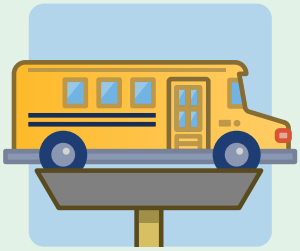
Lesson 13
Scales and Weights, Part 2



Lesson 14
Specific and Scientific



Lesson 15
Multiplying and Dividing



Lesson 16
Balance the Scale

Lesson 17
Use Your Powers

Lesson 18
City Lights

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.4, MA.8.NSO.1.6, MTR.2.1, MTR.5.1, MTR.7.1

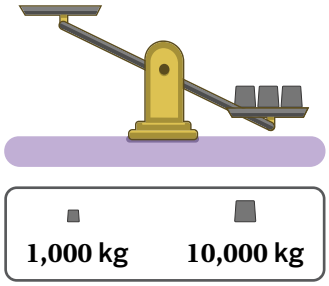
Scales and Weights, Part 1

Let's explore ways to represent large numbers.

Warm-Up

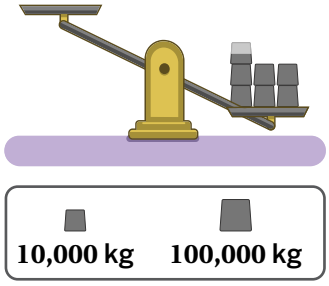
1.

a.



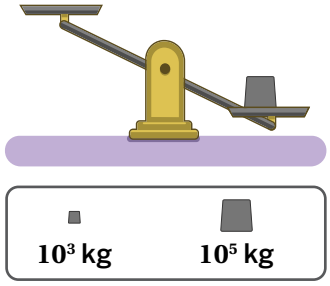
How many 1,000 kg weights are needed to balance with three 10,000 kg weights?

b.



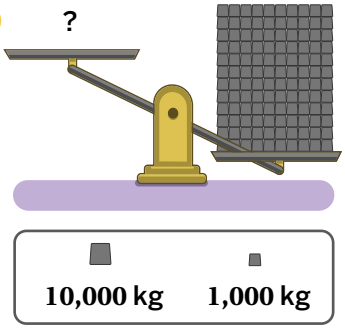
How many 10,000 kg weights are needed to balance with 6.5 100,000 kg weights?

c.



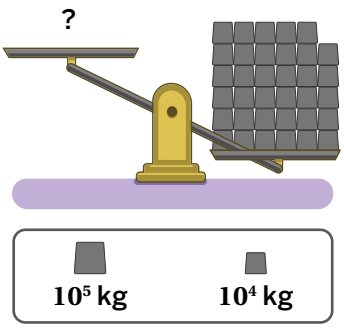
How many 10^3 kg weights are needed to balance with one 10^5 kg weight?

d.



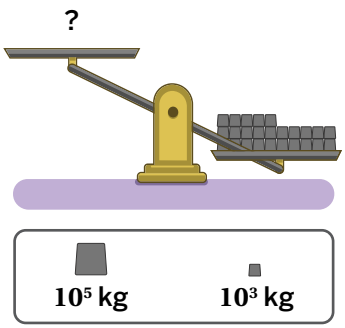
How many 10,000 kg weights are needed to balance with 120 1,000 kg weights?

e.



How many 10^5 kg weights are needed to balance with 35 10^4 kg weights?

f.



How many 10^5 kg weights are needed to balance with 25 10^3 kg weights?

Name: Date: Period:

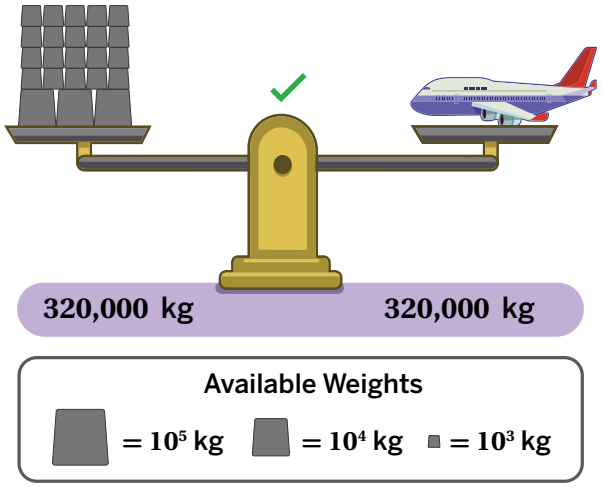
Scales and Weights

2. A plane weighs 320,000 kilograms.

The table shows how a student balanced the scale using the weights provided.

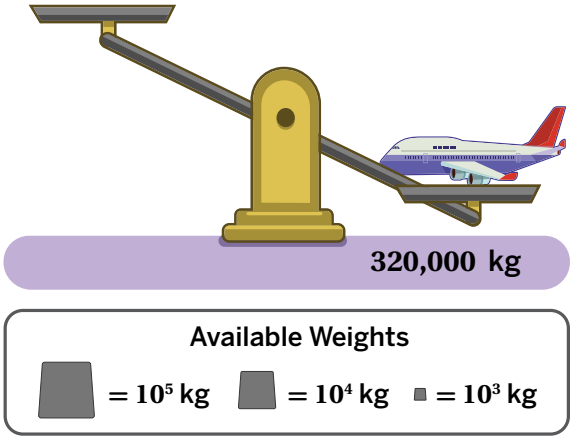
Write two other combinations of weights that will balance the scale.

10^5 kg Weights	10^4 kg Weights	10^3 kg Weights
3	0	20



3. Another way to write 320,000 kilograms is to use a combination of powers of 10. For example: $3 \cdot 10^5 + 2 \cdot 10^4$ kilograms.

a. **Discuss:** What does each part of the expression represent in terms of weights?



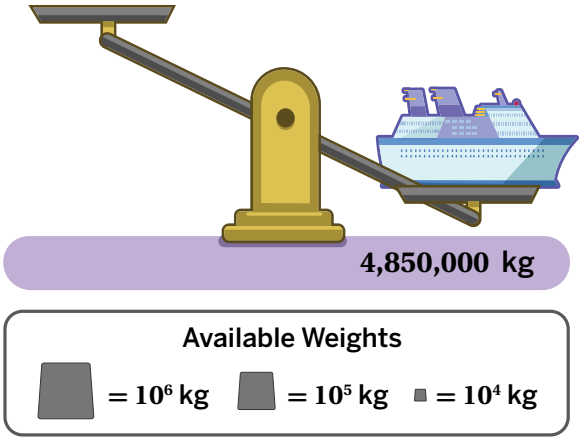
b. Write an expression to represent a *different* combination of available weights that will balance the scale.

Name: Date: Period:

Ships and Shuttles

4. A ship weighs $4{,}850{,}000$ kilograms.

Write an expression to represent a combination of available weights that will balance the scale.



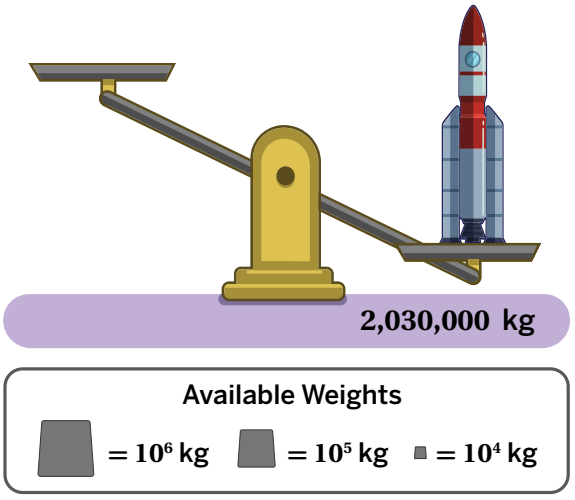
5. Rishi and Parv tried to balance the scale with a $2{,}030{,}000$-kilogram space shuttle.

- Rishi wrote: $2.03 \cdot 10^6$ kilograms
- Parv wrote: $20 \cdot 10^5 + 3 \cdot 10^4$ kilograms

Whose expression represents a combination of weights that will balance the scale? Circle one.

Rishi's Parv's Both Neither

Explain your thinking.



6. Write an expression to represent a *different* combination of available weights that will balance the scale.

Name: Date: Period:

Ships and Shuttles (continued)

7. Match each expression with the value it is equivalent to. One expression will have no match.

$4.7 \cdot 10^5$	$4.7 \cdot 10^4$	$7 \cdot 10^4 + 400 \cdot 10^3$
$4 \cdot 10^4 + 7 \cdot 10^2$		$4 \cdot 10^5 + 7 \cdot 10^3$
$40 \cdot 10^3 + 70 \cdot 10^2$		$4 \cdot 10^5 + 5 \cdot 10^4 + 20 \cdot 10^3$

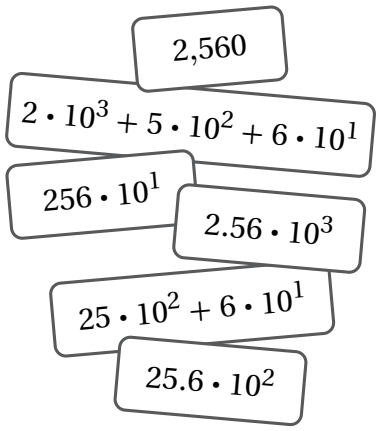
407,000	47,000	470,000

8. In the previous problem, how did you decide which value this expression is equivalent to?

$7 \cdot 10^4 + 400 \cdot 10^3$

Synthesis

9. What are some strategies for writing a number as a combination of powers of 10? Use the examples if they help with your thinking.



Lesson Practice ACC7.9.12

Lesson Summary

You can write large numbers as a combination of powers of 10 to make them less awkward to work with and to prevent having to count 0s.

The number 90,700,000 can be written many different ways using powers of 10.

For example:

- $90700000 = 9 \cdot 10^7 + 7 \cdot 10^5$
- $90700000 = 90 \cdot 10^6 + 7 \cdot 10^5$
- $90700000 = 9.07 \cdot 10^7$

Lesson Practice
ACC7.9.12

Name: ..... Date: ..... Period: .....

Problems 1–3: Write each value as a number times 10^3.

1. 42,300

2. 2,000

3. 301,000

Problems 4–6: Rewrite each expression as a combination of powers of 10.

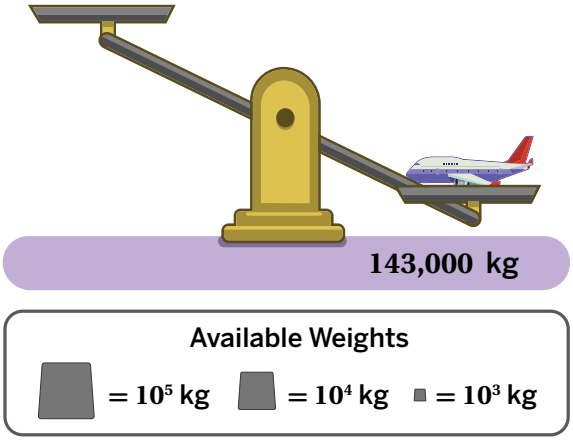
4. 4,200,000

5. 40,700

6. 999

Problems 7–8: Three students tried to balance the scale using weights measuring 10^5 kg, 10^4 kg, and 10^3 kg.

- Lucy wrote the weight of the plane as $14.3 \cdot 10^4$ kilograms.
- Parv wrote the weight of the plane as $143 \cdot 10^2$ kilograms.
- Kiri wrote the weight of the plane as $1 \cdot 10^5 + 4 \cdot 10^4 + 3 \cdot 10^3$ kilograms.



7. Which of these expressions are accurate?

8. Write an expression to represent a different combination of available weights that will balance the scale.

Lesson Practice
ACC7.9.12

Name: _____ Date: _____ Period: _____

FAST Practice

9. Which of the following numbers has the greatest value?

A. $48.34 \cdot 10^3$

B. $4.834 \cdot 10^5$

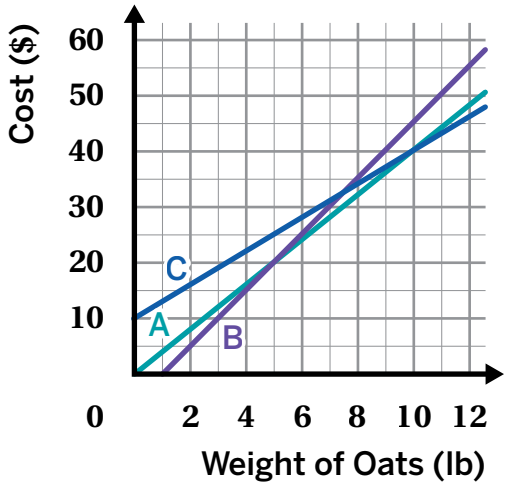
C. $4.834 \cdot 10^4$

D. $4.83 \cdot 10^5$

Spiral Review

Problems 10–12: Three stores are selling oats by the pound.

- Store A sells oats for a dollars per pound.
- Store B sells oats for b dollars per pound, with a coupon for \$5 off the total purchase.
- Store C is an online store. They sell oats for c dollars per pound, with a \$10 delivery fee.



10. Which store has the lowest price for two pounds of oats?

11. If a customer wants to buy 6 pounds of oats to make granola, which store has the lowest price?

12. How many pounds of oats would a customer need to buy to make Store C a good option? Explain your thinking.

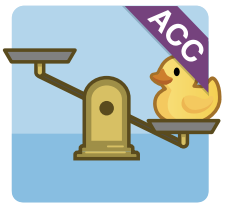
13. Select *all* expressions that are equivalent to 10^{-3}.

☐ A. $(5 \cdot 2)^{-3}$ ☐ B. $\frac{10^6}{10^3}$ ☐ C. $\frac{1}{10^3}$ ☐ D. $(10^{-3})^{-1}$ ☐ E. $\frac{1}{1000}$

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.6, MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

Scales and Weights, Part 2



Let's explore ways to represent small numbers with powers of 10.

Warm-Up

1. Match each number to a verbal description. One number will have no match.

1,000	0.000001	10^{-6}
10^{-3}	0.001	
0.00001	10^{3}	

One thousand	One thousandth	One millionth

Activity 1

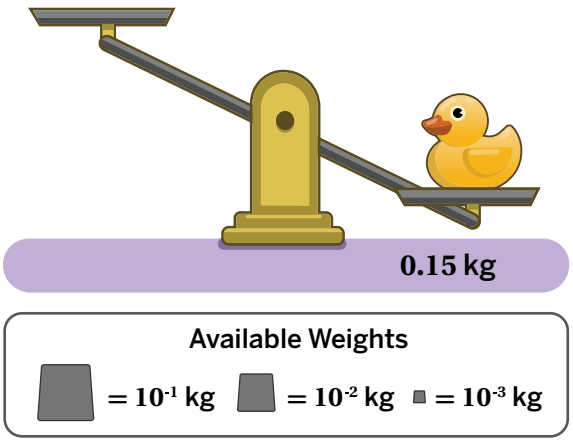
Name: Date: Period:

Light Weights

2. A rubber duck weighs 0.15 kilograms. A student balanced the scale using the weights shown in the top row of the table.

Write two other combinations of weights that will balance the scale.

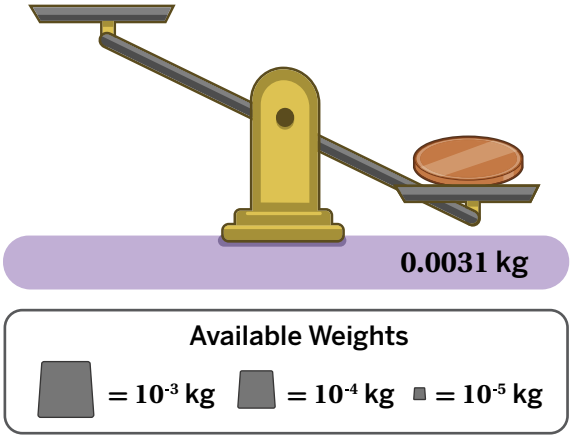
10^{-1} kg	10^{-2} kg	10^{-3} kg
1	5	0



3. A penny weighs 0.0031 kilograms. Here's one way to write its weight using a combination of powers of 10:

$3 \cdot 10^{-3} + 1 \cdot 10^{-4}$

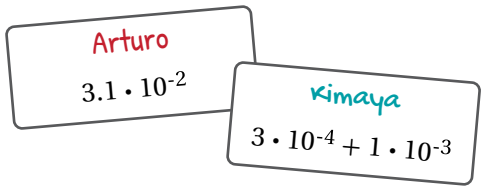
Write an expression to represent a *different* combination of available weights that will balance the scale.



Name: Date: Period:

Light Weights (continued)

4. Two students made mistakes writing 0.0031 using combinations of powers of 10.



Circle your favorite mistake.

Arturo's mistake Kimaya's mistake

a What is correct about this student's work?

b What could you add or change to make *all* of their work correct?

Name: ..... Date: ..... Period: .....

Getting Smaller and Smaller

5. The weight of a raisin is 0.000572 kilograms. Here is how two students rewrote 0.000572.

0.000572 kg

Alina

$5 \cdot 10^{-4} + 7 \cdot 10^{-5} + 2 \cdot 10^{-6}$

Lukas

$57.2 \cdot 10^{-5}$

Discuss:

- What are some advantages of Alina's strategy?
- What are some advantages of Lukas's strategy?

6. Lukas's strategy was to rewrite the raisin's weight, 0.000572 kilograms, as a number times a single power of 10:

$57.2 \cdot 10^{-5}$

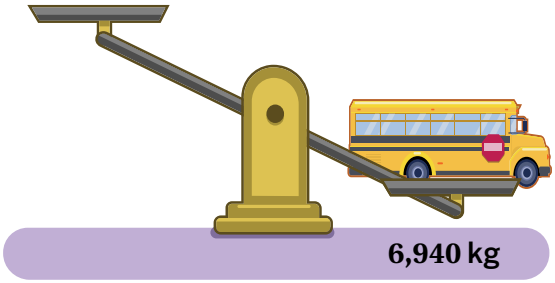
Write the same weight as:

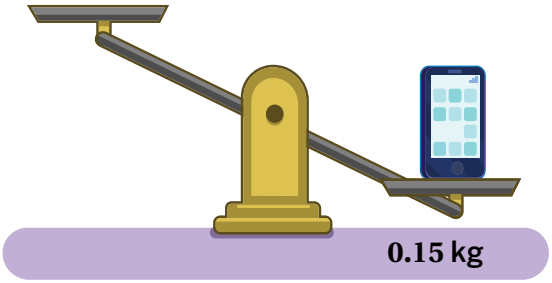
- A number times 10^{-6}: .....
- A number times 10^{-4}: .....

Name: ..... Date: ..... Period: .....

Repeated Challenges

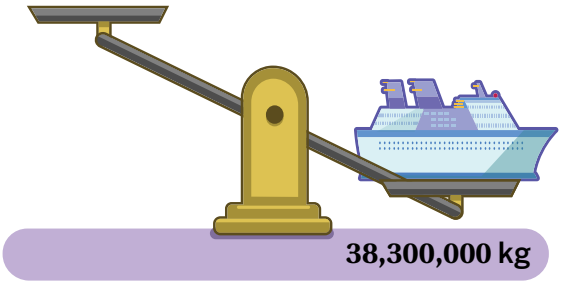
7. Write the weight of each object using a number times a single power of 10.

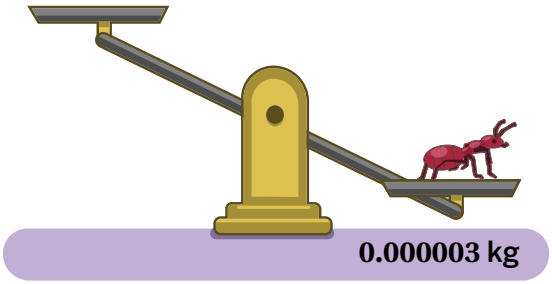




a Write the weight of the bus (6,940 kg) as a number times 10^3.

b Write the weight of the cell phone (0.15 kg) as a number times 10^{-2}.



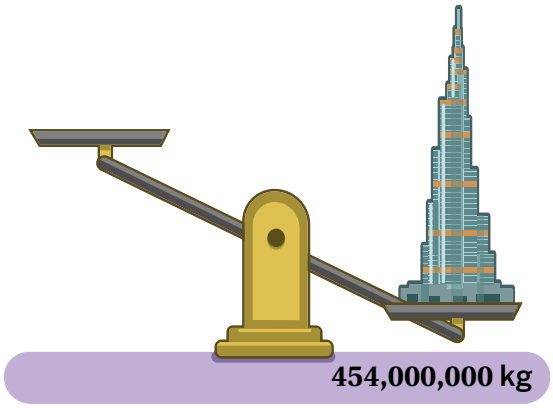


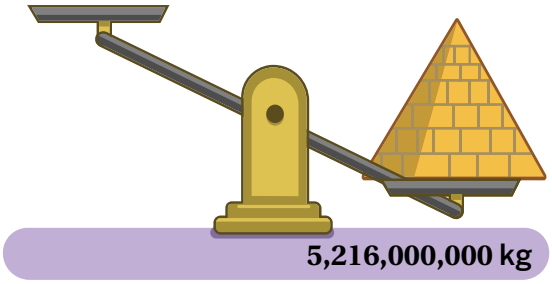
c Write the weight of the cruise ship (38,300,000 kg) as a number times 10^7.

d Write the weight of the ant (0.000003 kg) as a number times 10^{-5}.

Name: ..... Date: ..... Period: .....

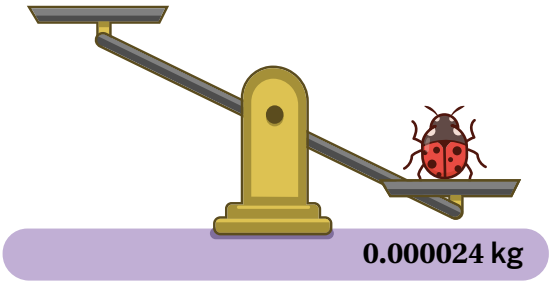
Repeated Challenges (continued)

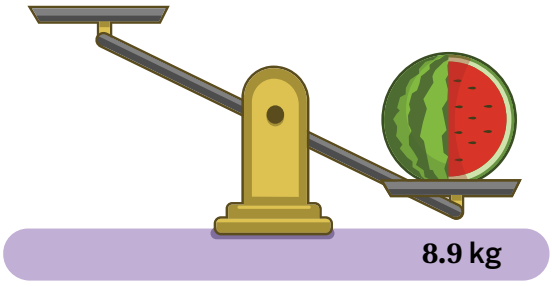




e Write the weight of the Burj Khalifa (454,000,000 kg) as a number times 10^7.

f Write the weight of the pyramid (5,216,000,000 kg) as a number times 10^9.



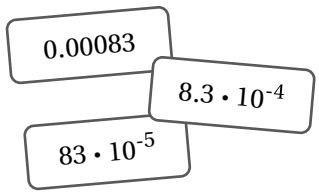


g Write the weight of the ladybug (0.000024 kg) as a number times 10^{-6}.

h Write the weight of the watermelon (8.9 kg) as a number times 10^1.

Synthesis

8. What are some strategies for writing very small values as a number times a single power of 10? Use the examples if they help with your thinking.



Lesson Practice ACC7.9.13

Lesson Summary

Like large numbers, you can write small numbers using combinations of powers of 10. Numbers less than 1 will use negative powers of 10.

For example:

- $0.000000877 = 8 \cdot 10^{-7} + 7 \cdot 10^{-8} + 7 \cdot 10^{-9}$
- $0.00000000034 = 3 \cdot 10^{-10} + 4 \cdot 10^{-11}$
- $0.00000049 = 4 \cdot 10^{-7} + 9 \cdot 10^{-8}$

You can write large and small values as a number times a single power of 10 to help compare those values and get a sense of their scale.

For example:

- $42000000000 = 4.2 \cdot 10^{10}$
- $2500000000 = 25 \cdot 10^{8}$
- $0.00000000034 = 3.4 \cdot 10^{-10}$
- $0.00000049 = 49 \cdot 10^{-8}$

Lesson Practice
ACC7.9.13

Name: ..... Date: ..... Period: .....

1. Select *all* expressions that are equivalent to $\frac{1}{1000}$.

- ☐ **A.** $1 \cdot 10^{-3}$
- ☐ **B.** $-1 \cdot 10^{3}$
- ☐ **C.** $1 \cdot 10^{\frac{1}{3}}$
- ☐ **D.** $10 \cdot 10^{-4}$
- ☐ **E.** $10 \cdot 10^{-3}$

2. Order the expressions from *least* to *greatest*.

$2 \cdot 10^{-3}$	$3 \cdot 10^{-2}$	$-3 \cdot 10^{-2}$	$-2 \cdot 10^{-3}$	$3 \cdot 10^{2}$

Least **Greatest**

Problems 3–4: Write each sum as a decimal.

3. $3 \cdot 10^{-4} + 2 \cdot 10^{-5} + 3 \cdot 10^{-6}$

4. $2 \cdot 10^{-7} + 3 \cdot 10^{-5} + 5 \cdot 10^{-3}$

Problems 5–6: Write each value as a number times a single power of 10.

5. $\frac{7}{10000}$

6. 0.0013

7. Write 0.00573 in three different ways, using a single multiple of 10.

....

Name: _____ Date: _____ Period: _____

8. Fill in each blank using the digits 0 to 9 to make a sum that's as close to 10 as possible.

$\square\square \cdot 10^{-\square} + \square\square \cdot 10^{-\square}$

FAST Practice

9. Which shows 0.004 as a number times a single power of 10?

A. 0.4×10^{-1}

B. 4×10^{-2}

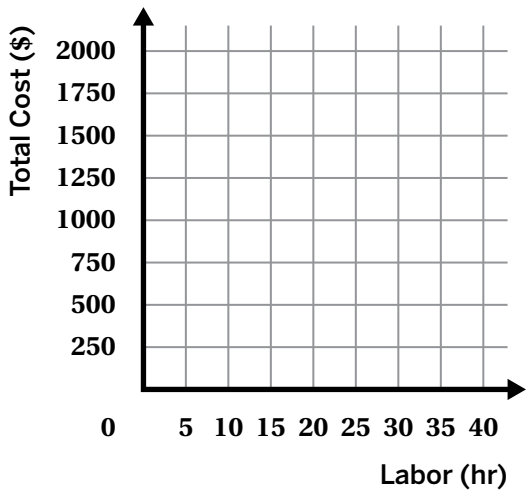
C. 4×10^{-3}

D. 0.4×10^{2}

Spiral Review

Problems 10–12: An electrician charges a flat rate of $250, plus $25 for each hour of labor.

10. Graph a line representing the relationship between the number of hours of labor and the total cost.



11. What is the total cost for 20 hours of labor?

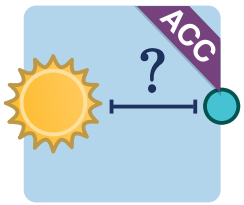
12. What is the slope of this line?

Explain its meaning in context.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.4, MA.8.NSO.1.6, MTR.2.1, MTR.3.1, MTR.5.1

Specific and Scientific



Let's explore scientific notation.

Warm-Up

1. Order these numbers from *least* to *greatest*.

$75 \cdot 10^5$	4,000,000	$0.6 \cdot 10^7$	$5 \cdot 10^5$

Least **Greatest**

2. Order these numbers from *least* to *greatest*.

$4 \cdot 10^6$	$6 \cdot 10^6$	$5 \cdot 10^5$	$7.5 \cdot 10^6$

Least **Greatest**

3. Which list was easier to sort? Explain your thinking.

Activity 1

Name: Date: Period:

Scientific Notation

Scientific notation is a specific way of writing very large or very small numbers that can help us compare numbers.

4. Some of these numbers are written in scientific notation and some are not.

What do you think it means for a number to be written in scientific notation?

In Scientific Notation	Not in Scientific Notation
$3 \cdot 10^9$	3,000,000,000
$1.257 \cdot 10^5$	$125.7 \cdot 10^3$
$2 \cdot 10^{-1}$	0.2
$5.1 \cdot 10^{-4}$	$0.51 \cdot 10^{-3}$

5. Sort the numbers based on whether they are written in scientific notation.

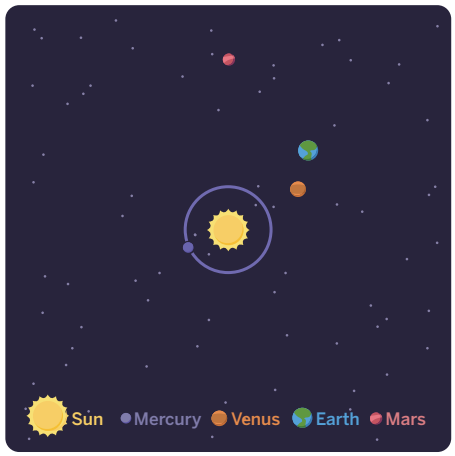
0.00099	48,200	$0.78 \cdot 10^{-3}$
$5.23 \cdot 10^8$	$8.7 \cdot 10^{-12}$	$36 \cdot 10^5$

In Scientific Notation	Not in Scientific Notation

Name: Date: Period:

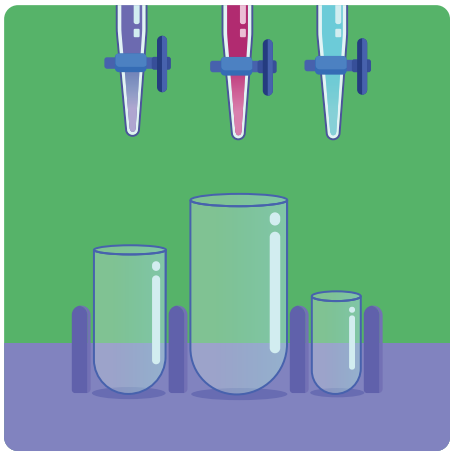
Solar System and Test Tubes

6. Here are the distances of four planets from the Sun. Write each distance in scientific notation. Mercury has been done for you.



Planet	Distance From Sun (mi)	Scientific Notation (mi)
Mercury	36,000,000	$3.6 \cdot 10^7$
Venus	67,000,000	
Earth	92,960,000	
Mars	$1417 \cdot 10^5$	

7. We can use scientific notation to represent small numbers, too! Write each volume in scientific notation.

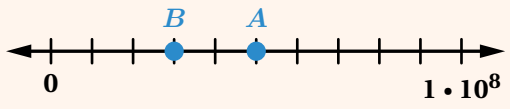


Liquid Color	Volume (L)	Scientific Notation (L)
Purple	0.000125	
Red	0.0002	
Blue	$325 \cdot 10^{-8}$	

You're invited to explore more.

8. What are the values of points A and B? Write your answers in scientific notation.

Point	Value
A	
B	



Explain your thinking.

Synthesis

9. Describe a strategy for writing a number in scientific notation. Use the examples if they help with your thinking.

Not in Scientific Notation	In Scientific Notation
$36{,}000{,}000$	$3.6 \cdot 10^7$
$6{,}700{,}000$	
0.00024	
$417 \cdot 10^3$	

Lesson Practice ACC7.9.14

Lesson Summary

There are many ways to express a number using a power of 10. One specific way is called **scientific notation**, which can be helpful for comparing very large or very small numbers. When a number is written in scientific notation, the first part is a number greater than or equal to 1, but less than 10. The second part is an integer power of 10.

For example:

- $425{,}000{,}000$ is $4.25 \cdot 10^8$ in scientific notation
- 0.0000000000783 is $7.83 \cdot 10^{-11}$ in scientific notation

Lesson Practice
ACC7.9.14

Name: ..... Date: ..... Period: .....

Problems 1–4: Write each value as a number times a power of 10.

1. 0.04 **2.** 0.072 **3.** 0.0000325 **4.** 0.003

Problems 5–10: Write each value in scientific notation.

5. 0.00083 **6.** $760{,}000{,}000$ **7.** $147 \cdot 10^{6}$

8. 0.038 **9.** 3.8 **10.** $38 \cdot 10^{-4}$

11. Angela says: *Any positive number multiplied by* 10^{5} *will be greater than any other number multiplied by* 10^{3}. Is this statement *always*, *sometimes*, or *never* true? Explain your thinking.

12. Luis wrote 0.0000683 as $0.683 \cdot 10^{-4}$. Is this number written in scientific notation? Explain your thinking.

FAST Practice

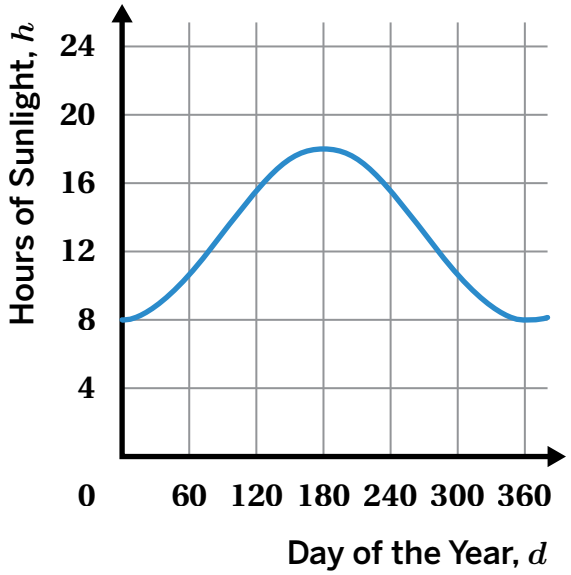
13. There are a total of $367{,}400$ books in a library. Express the number of books in scientific notation.
Record your answer in the space provided.

Lesson Practice
ACC7.9.14

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 14–17: Here is the graph representing the predicted number of hours of sunlight, h, on a given day of the year, d, in Metropolis.



14. Is hours of sunlight a function of days of the year? Explain your thinking.

15. For what days of the year do the hours of sunlight increase?

16. For what days of the year do the hours of sunlight decrease?

17. Which day of the year has the most hours of sunlight?

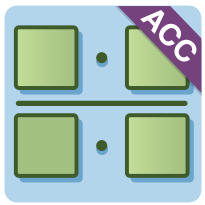
18. Select *all* the expressions that are equivalent to $4 \cdot 10^{-3}$.

- ☐ **A.** $4 \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right)$
- ☐ **B.** $4 \cdot (-10) \cdot (-10) \cdot (-10)$
- ☐ **C.** $4 \cdot 0.001$
- ☐ **D.** $4 \cdot 0.0001$
- ☐ **E.** 0.004
- ☐ **F.** 0.0004

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6, MTR.2.1, MTR.4.1, MTR.5.1

Multiplying and Dividing



Let's explore how to multiply and divide with scientific notation.

Warm-Up

1. What is the value of $(2 \cdot 10^3) \cdot (4 \cdot 10^6)$?

A. $6 \cdot 10^9$

B. $8 \cdot 10^{18}$

C. $8 \cdot 10^9$

D. $6 \cdot 10^{18}$

Explain your thinking.

Activity 1

Name: Date: Period:

Multiplication Strategies

Anya and Rudra are trying to multiply $(4 \cdot 10^5) \cdot (3 \cdot 10^6)$.
Here is each student's work.

Anya

$(4 \cdot 10^5) \cdot (3 \cdot 10^6)$

$400{,}000 \cdot 3{,}000{,}000$

$1{,}200{,}000{,}000{,}000$

$1.2 \cdot 10^{12}$

Rudra

$(4 \cdot 10^5) \cdot (3 \cdot 10^6)$

$4 \cdot 3 \cdot 10^5 \cdot 10^6$

$12 \cdot 10^{5+6}$

$12 \cdot 10^{11}$

$1.2 \cdot 10^{12}$

2. **Discuss:** How are Anya's and Rudra's strategies alike? How are they different?

3. Here are four new expressions.

- For each expression, circle whether you would use Anya's strategy, Rudra's strategy, or another strategy.
- Choose two expressions to multiply. Write your answers in scientific notation.
- Compare your strategies and responses with a partner.

a $(3 \cdot 10^5) \cdot (6 \cdot 10^2)$

Anya Rudra Other

b $(4 \cdot 10^2) \cdot (2 \cdot 10^{-3})$

Anya Rudra Other

c $(6 \cdot 10^3) \cdot 400$

Anya Rudra Other

d $(5 \cdot 10^{22}) \cdot (7 \cdot 10^{17})$

Anya Rudra Other

Activity 2

Name: Date: Period:

Division Strategies

Anya and Rudra are trying to divide $\frac{7 \cdot 10^6}{2 \cdot 10^4}$. Here is each student's work.

Anya

$$\frac{7 \cdot 10^6}{2 \cdot 10^4}$$

$$\frac{7{,}000{,}000}{20{,}000}$$

$$350$$

$$3.5 \cdot 10^2$$

Rudra

$$\frac{7 \cdot 10^6}{2 \cdot 10^4}$$

$$\frac{7}{2} \cdot \frac{10^6}{10^4}$$

$$3.5 \cdot 10^{6-4}$$

$$3.5 \cdot 10^2$$

4. **Discuss:** How are Anya's and Rudra's strategies alike? How are they different?

5. Here are four new expressions.

- For each expression, circle whether you would use Anya's strategy, Rudra's strategy, or another strategy.
- Choose two expressions to divide. Write your answers in scientific notation.
- Compare your strategies and responses with a partner.

a $\frac{8 \cdot 10^3}{2 \cdot 10^2}$

Anya Rudra Other

b $\frac{9 \cdot 10^{-4}}{3}$

Anya Rudra Other

c $\frac{3 \cdot 10^5}{6 \cdot 10^2}$

Anya Rudra Other

d $\frac{8 \cdot 10^9}{4 \cdot 10^{-4}}$

Anya Rudra Other

Name: Date: Period:

Two Truths and a Lie

Problems 6–9: For each problem, two statements are true and one is false. Circle the false statement.

6.
- (A.) $1.5 \cdot 10^7 = (5 \cdot 10^2) \cdot (3 \cdot 10^4)$
- B. $1.5 \cdot 10^7 = (5 \cdot 10^2) \cdot (3 \cdot 10^5)$
- C. $1.5 \cdot 10^7 = (3 \cdot 10^3) \cdot (5 \cdot 10^3)$

7.
- (A.) $(7 \cdot 10^{-4}) \cdot (4 \cdot 10^{15}) = (7 \cdot 4) \cdot 10^{(-4+15)}$
- B. $(7 \cdot 10^{-4}) \cdot (4 \cdot 10^{15}) = 2.8 \cdot 10^{10}$
- C. $(7 \cdot 10^{-4}) \cdot (4 \cdot 10^{15}) = 2.8 \cdot 10^{12}$

8.
- (A.) $(6 \cdot 10^3) \cdot 200 = (6 \cdot 200) \cdot 10^3$
- B. $(6 \cdot 10^3) \cdot 200 = 6 \cdot 10^3 \cdot 2 \cdot 10^2$
- C. $(6 \cdot 10^3) \cdot 200 = (6 \cdot 200) \cdot 10^{(3+2)}$

9.
- (A.) $\frac{9 \cdot 10^7}{2 \cdot 10^3} = (9 \div 2) \cdot 10^{(7-3)}$
- B. $\frac{9 \cdot 10^7}{2 \cdot 10^3} = 7 \cdot 10^4$
- C. $\frac{9 \cdot 10^7}{2 \cdot 10^3} = 4.5 \cdot 10^4$

10. Yosuf is buying apples at the store. A 5-pound bag containing 12 apples costs \$6.00. He wants to compare the cost of buying the same number of apples if he buys them separately. He puts 12 apples on the scale, and it shows a weight of 5.00 pounds.

Discuss: Which measure is more precise: 5 pounds or 5.00 pounds?

11. Anya is measuring the dimensions of a large wall in her home to estimate how much paint to buy. She measured the length of the base of the wall using a tape measure and found it to be 32.5 feet. She measured the height of the room using a laser measuring tool and got a reading of $2.20625 \cdot 10^1$ feet. She calculates the area as follows:

$$(3.25 \cdot 10^1) \cdot (2.20625 \cdot 10^1) = 7.1703125 \cdot 10^2 = 717.03125 \text{ square feet}$$

She wants her final answer to be as precise as her least precise measurement. How should Anya round her final answer for the area of the wall? Explain your thinking.

Synthesis

12. Describe a strategy for multiplying or dividing numbers written in scientific notation. Use the examples if they help with your thinking.

$(4 \cdot 10^{-2}) \cdot (5 \cdot 10^{8})$

$\frac{5 \cdot 10^{9}}{2 \cdot 10^{4}}$

Lesson Practice ACC7.9.15

Lesson Summary

Multiplying numbers written in scientific notation is an extension of multiplying decimals.

To multiply two numbers written in scientific notation:

- Multiply the first parts of each number.
- Multiply the powers of 10 using exponent properties.

For example: $(2 \cdot 10^{3}) \cdot (4 \cdot 10^{6}) = (2 \cdot 4) \cdot 10^{(3+6)} = 8 \cdot 10^{9}$.

To divide two numbers written in scientific notation, it can be helpful to rewrite the expression as a fraction.

- Divide the first part in the numerator by the first part in the denominator.
- Divide the powers of 10 using exponent properties.

For example: $\frac{8 \cdot 10^{7}}{4 \cdot 10^{2}} = \frac{8}{4} \cdot 10^{(7-2)} = 2 \cdot 10^{5}$.

Significant digits are the important digits in a measurement that show how precise it is.

- Zeros in a number are only considered *significant* when they are between two non-zero numbers or when they are trailing zeros in a number that has a decimal point. For example, 450 has only two significant digits, but 450.0 has four.
- When multiplying or dividing numbers, the number of significant digits in the solution will be the same as the least number of significant digits of the numbers being multiplied or divided. This is to make sure the final answer is as precise as the least precise number in the computation.

Lesson Practice
ACC7.9.15

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are equivalent to $4 \cdot 10^3$.

- ☐ **A.** $(2 \cdot 10^{-5}) \cdot (2 \cdot 10^8)$
- ☐ **B.** $\frac{8 \cdot 10^5}{2 \cdot 10^8}$
- ☐ **C.** $\frac{8 \cdot 10^{-5}}{2 \cdot 10^{-8}}$
- ☐ **D.** $(4 \cdot 10^3) \cdot (1 \cdot 10^3)$
- ☐ **E.** $(4 \cdot 10) \cdot (100)$

Problems 2–5: Determine the value of a that makes each equation true.

2. $(5 \cdot 10^a) \cdot (3 \cdot 10^3) = 1.5 \cdot 10^8$

$a =$ .....

3. $(6 \cdot 10^{-5}) \cdot (2 \cdot 10^{-4}) = 1.2 \cdot 10^a$

$a =$ .....

4. $\frac{3.2 \cdot 10^7}{1.6 \cdot 10^a} = 2 \cdot 10^2$

$a =$ .....

5. $\frac{6.2 \cdot 10^a}{2 \cdot 10^{-3}} = 3.1 \cdot 10^{-4}$

$a =$ .....

6. Fill in each blank using the digits 1 to 9 only once to create the largest product possible.

$\square\square \cdot 10^{-\square} \cdot \square\square \cdot 10^{-\square}$

7. Changing the sign of which part of the expression would make the value of $\frac{-3 \cdot 10^{-4}}{-2.4 \cdot 10^{-2}}$ greater than 1?

(A.) -3 (B.) -4 (C.) -2.4 (D.) -2

8. Select *all* the numbers that have 4 significant digits.

- ☐ **A.** 6,070
- ☐ **B.** 672.80
- ☐ **C.** 087.34
- ☐ **D.** 657.09
- ☐ **E.** 32.45

Name: ..... Date: ..... Period: .....

9. An experienced cyclist travels $2.1 \cdot 10^2$ miles in 8.0125 hours. What is the cyclist's speed in miles per hour? Express your answer using the correct number of significant digits. Show your thinking.

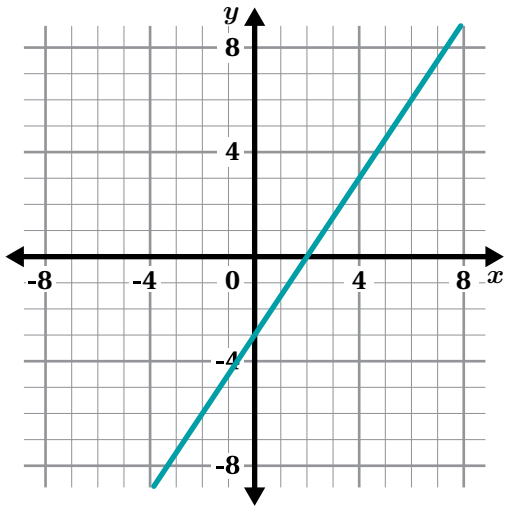
FAST Practice

10. What is the value of $\frac{4.6 \cdot 10^{-5}}{2.3 \cdot 10^{-2}}$? Express your answer in scientific notation.

Record your answer in the space provided.

Spiral Review

Problems 11–13: Here is a graph of the equation $y = \frac{3}{2}x - 3$ in a system of equations.



11. Write a second equation so that the system has infinitely many solutions.

Equation:

12. Write a second equation so that the system has no solutions.

Equation:

13. Write a second equation so that the system has one solution at $(4, 3)$.

Equation:

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6, MTR.5.1, MTR.6.1, MTR.7.1

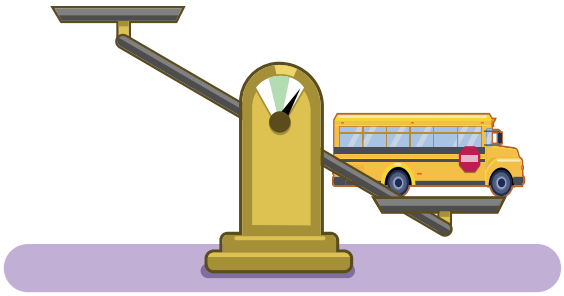
Balance the Scale

Let's use multiplication and division to compare large and small numbers in scientific notation.

Warm-Up

1. A school bus sits on one side of the scale.

How many jelly beans do you think it would take to balance the scale? Use scientific notation.

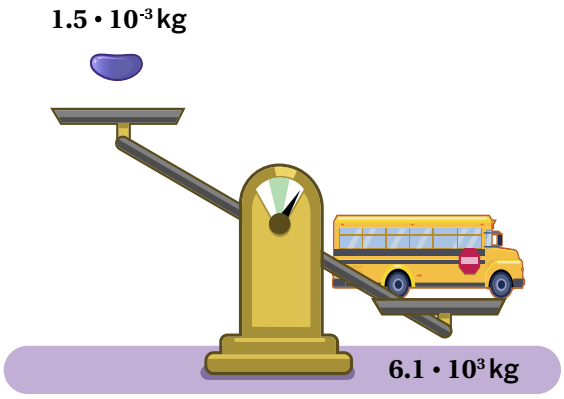


Name: Date: Period:

Balance the Scale, Part 1

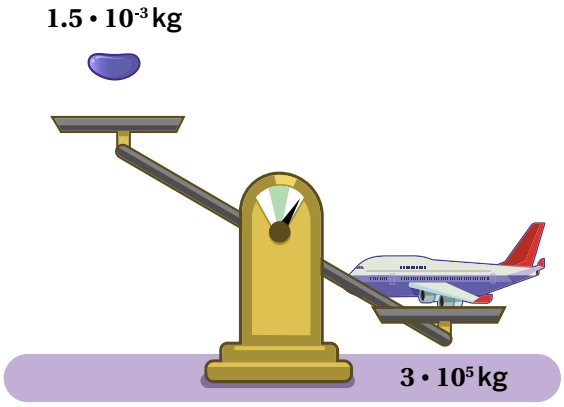
2. One jelly bean weighs $1.5 \cdot 10^{-3}$ kilograms. A school bus weighs $6.1 \cdot 10^{3}$ kilograms.

Describe a strategy for determining about how many jelly beans weigh as much as a bus.



3. One jelly bean weighs $1.5 \cdot 10^{-3}$ kilograms. A jumbo jet weighs $3 \cdot 10^{5}$ kilograms.

How many jelly beans will it take to balance the scale?

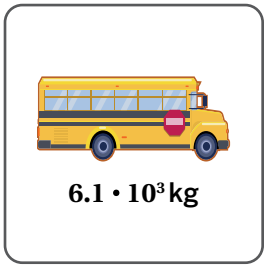


4. Basheera says the jumbo jet weighs about 200 times as much as the school bus. Elena says it weighs about 50 times as much as the school bus.

Whose claim is correct? Circle one.

Basheera's Elena's Neither

Explain your thinking.

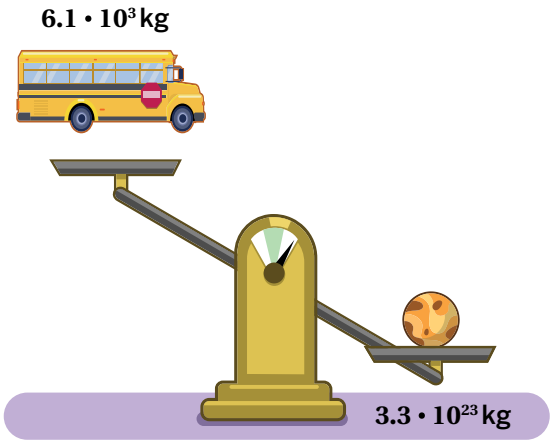


Name: Date: Period:

Balance the Scale, Part 2

5. A school bus weighs $6.1 \cdot 10^3$ kilograms. Mercury weighs $3.3 \cdot 10^{23}$ kilograms.

About how many school buses will it take to balance the scale? Write your answer in scientific notation.

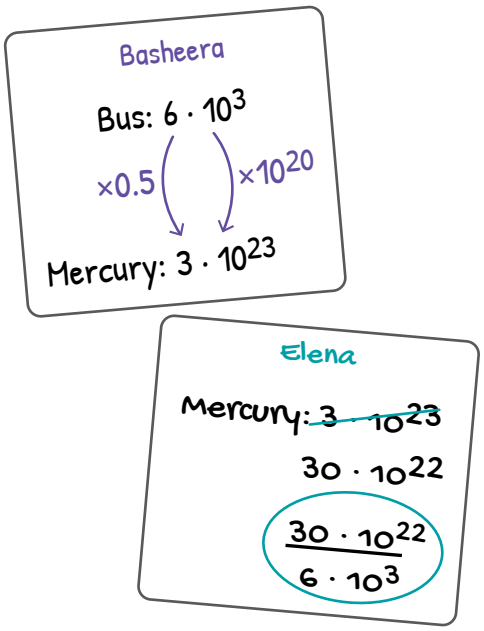


6. Here are two students' strategies for the previous problem. Examine their work.

Explain how Basheera and Elena each arrived at the answer $5 \cdot 10^{19}$ buses.

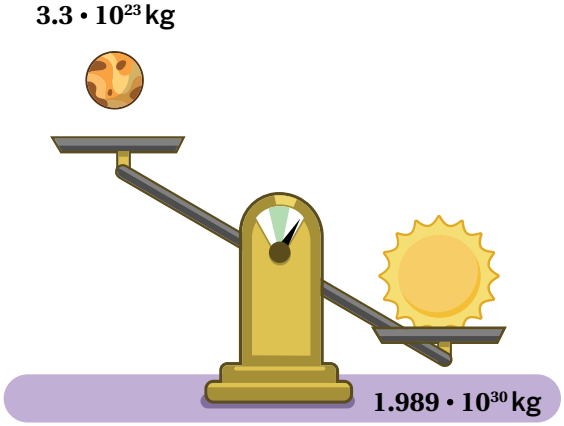
Basheera:

Elena:



7. Mercury weighs $3.3 \cdot 10^{23}$ kilograms. The Sun weighs $1.989 \cdot 10^{30}$ kilograms.

About how many Mercurys will it take to balance the scale? Write your answer in scientific notation.



Name: ..... Date: ..... Period: .....

Balance the Scale, Part 2 (continued)

8. Choose an object from each row and then compare them.

- Use these weights to determine how many of your first object weighs as much as your second object.
- Complete as many comparisons as you have time for.

Watermelon $8.9 \cdot 10^0$ kilograms	**Horse** $7.1 \cdot 10^2$ kilograms	**Ant** $3 \cdot 10^{-6}$ kilograms	**Cell Phone** $1.5 \cdot 10^{-1}$ kilograms	**Penny** $3.1 \cdot 10^{-3}$ kilograms
Bus $7.81 \cdot 10^3$ kilograms	**Moon** $7.348 \cdot 10^{22}$ kilograms	**Rocket** $2.03 \cdot 10^6$ kilograms	**Cruise Ship** $3.83 \cdot 10^7$ kilograms	**Pyramid** $5.216 \cdot 10^9$ kilograms

Comparison 1

How many _____ weigh about as much as the _____?

Write your answer in scientific notation.

Comparison 2

How many _____ weigh about as much as the _____?

Write your answer in scientific notation.

Comparison 3

How many _____ weigh about as much as the _____?

Write your answer in scientific notation.

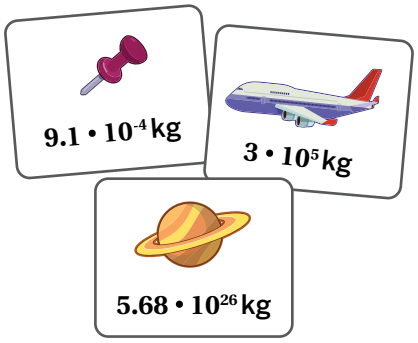
Comparison 4

How many _____ weigh about as much as the _____?

Write your answer in scientific notation.

Synthesis

9. Describe a strategy for determining how many times as large one number is compared to another.



Lesson Practice ACC7.9.16

Lesson Summary

You can use scientific notation when comparing quantities.

Here is one example. How many jelly beans weigh as much as one Egyptian pyramid?

- Jelly bean weight: $1.5 \cdot 10^{-3}$ kilograms
- Egyptian pyramid weight: $5.216 \cdot 10^{9}$ kilograms

It can be helpful to round the first parts of both numbers before calculating.

- $1.5 \cdot 10^{-3}$ is about $2 \cdot 10^{-3}$.
- $5.216 \cdot 10^{9}$ is about $5 \cdot 10^{9}$.

There are many strategies you can use when comparing quantities in scientific notation. Here are two:

Divide the larger number by the smaller number.

$$\frac{5 \cdot 10^{9}}{2 \cdot 10^{-3}} = 2.5 \cdot 10^{12}$$

$2.5 \cdot 10^{12}$ jelly beans weigh about the same as the pyramid.

Multiply the smaller number by the number needed to equal the larger number.

$$2 \cdot 10^{-3} \cdot ? = 5 \cdot 10^{9}$$

$2.5 \cdot 10^{12}$ jelly beans weigh about the same as the pyramid.

Lesson Practice
ACC7.9.16

Name: _____ Date: _____ Period: _____

1. Which number is greater? Circle one.

$17 \cdot 10^8$ $4 \cdot 10^8$

About how many times greater is one than the other?

2. Which number is greater? Circle one.

$2 \cdot 10^6$ $7.839 \cdot 10^6$

About how many times greater is one than the other?

3. Which number is greater? Circle one.

$42 \cdot 10^7$ $8.5 \cdot 10^8$

About how many times greater is one than the other?

Problems 4–5: Complete each sentence by writing a number in scientific notation.

4. $10.3 \cdot 10^9$ is about _____ times as large as $5.2 \cdot 10^3$.

5. $12.5 \cdot 10^{11}$ is about $4 \cdot 10^8$ times as large as _____.

6. A number is $3 \cdot 10^5$ times as large as another number. Determine two numbers that make this relationship true and complete the statement below.

_____ is $3 \cdot 10^5$ times as large as _____.

Lesson Practice
ACC7.9.16

Name: ..... Date: ..... Period: .....

FAST Practice

7. The mass of a penny is $3.1 \cdot 10^{-3}$ kilograms and the mass of an Egyptian pyramid is $5.216 \cdot 10^{9}$ kilograms. Based on this information, how many pennies weigh about as much as one pyramid?

(A.) 1.7×10^{10}

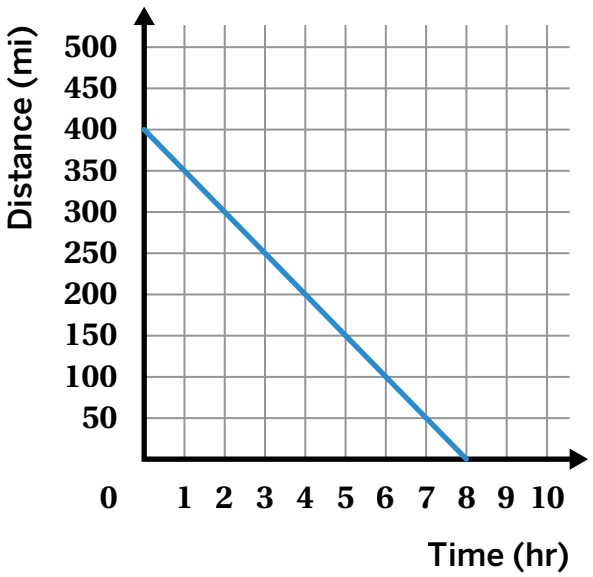
(B.) 1.675×10^{11}

(C.) 1.675×10^{12}

(D.) 1.7×10^{13}

Spiral Review

Problems 8–11: A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance to their cousins' house each hour of the trip.



8. How far is the trip?

9. How long did the trip take?

10. How fast are they traveling?

Explain your thinking

11. Is the slope positive or negative?

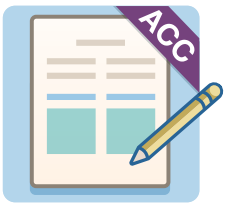
Explain how you know and why that fits the situation.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6, MTR.5.1, MTR.6.1, MTR.7.1

Use Your Powers

Let's use scientific notation to help us make calculations with large numbers.



Warm-Up

Complete each sentence by writing a number in scientific notation.

1. $6.1 \cdot 10^{13}$ is about ..... times as large as $2.1 \cdot 10^{2}$.

2. $2.9 \cdot 10^{7}$ is about ..... times as large as $1.5 \cdot 10^{-3}$.

3. $1.4 \cdot 10^{9}$ is about ..... times as large as $5.8 \cdot 10^{5}$.

Activity 1

Name: Date: Period:

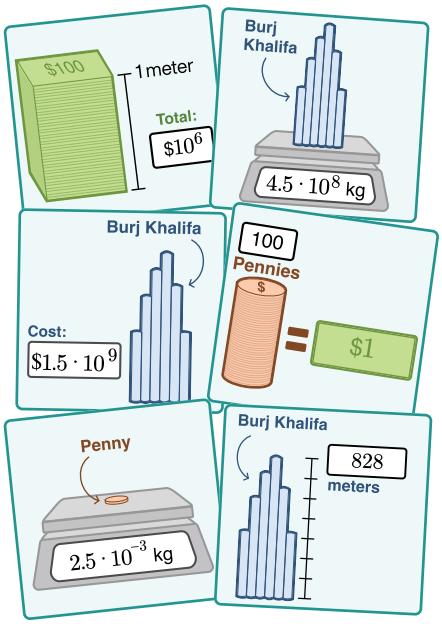
Make a Poster

Read each situation. Then circle one situation to explore.

Situation 1	Situation 2
The Burj Khalifa	Distance to the Moon

Situation 1

The Burj Khalifa



a Which is taller: the Burj Khalifa or the stack of 50 dollar bills it cost to build the Burj Khalifa?

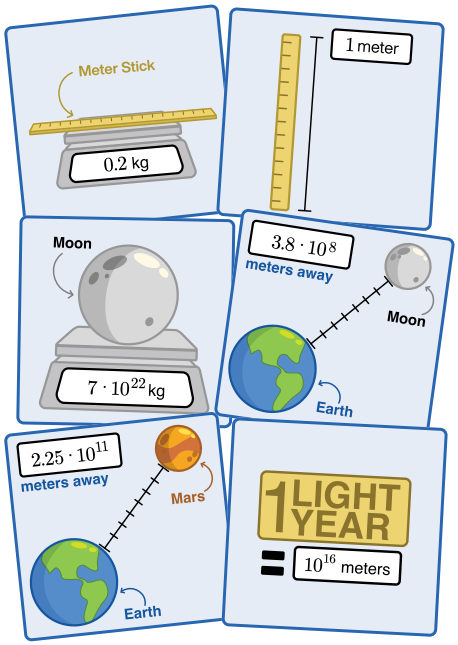
b How many times as tall?

Name: ..... Date: ..... Period: .....

Make a Poster (continued)

Situation 2

Distance to the Moon



a How many meter sticks does it take to equal the mass of the Moon?

b If you took all those meter sticks and lined them up end to end, how many times would they reach from Earth to the Moon?

4. Create a poster answering the questions for the situation you chose. Your poster should include:

- ☐ A summary of the situation.
- ☐ All of the measurements you used to answer each question (with units).
- ☐ Your calculations.
- ☐ Your answers (with units).

Synthesis

5. Describe something you learned about scientific notation while making your poster.

Lesson Practice ACC7.9.17

Lesson Summary

You can use scientific notation and exponent rules to solve real-world problems that include very large or very small numbers. For example, you can use the rules to calculate how many dollars worth of food are wasted in the United States each year or the total amount of student debt in the United States.

When solving a real-world problem, it is important to look at the information you know, determine what information is needed to solve the problem, and think about appropriate units of measurement. You can also use rounding to make some quantities simpler to work with.

Lesson Practice
ACC7.9.17

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are equal to $9 \cdot 10^8$.

- ☐ **A.** $(9 \cdot 10^{10}) \cdot (1 \cdot 10^{-2})$
- ☐ **B.** $\frac{18 \cdot 10^4}{2 \cdot 10^4}$
- ☐ **C.** $3 \cdot 3 \cdot 10^6 \cdot 10^2$
- ☐ **D.** 90^8
- ☐ **E.** $\frac{18 \cdot 10^{12}}{2 \cdot 10^4}$

2. The Sun is roughly $1 \cdot 10^2$ times as wide as Earth. The star KW Sagittarii is roughly $1 \cdot 10^5$ times as wide as Earth. About how many times as wide is KW Sagittarii as the Sun?

Show or explain your thinking.

3. The mass of Saturn is about $5.68 \cdot 10^{26}$ kilograms. If the average mass of an apple is about 0.085 kilograms, how many apples does it take to equal the mass of Saturn?

Show or explain your thinking.

Problems 4–6: Here are some interesting facts.

- The average distance from Earth to the Sun is about $1.5 \cdot 10^{11}$ meters.
- The *Helios* 2 spacecraft traveled at a speed of $2.53 \cdot 10^8$ meters per hour.
- There are about $3{,}000$ blades of grass in a square foot.
- A soccer field has an area of $81{,}000$ square feet.

4. The rings of Saturn are about $2.82 \cdot 10^8$ meters across. How long would it take for *Helios* 2 to travel across the rings of Saturn? Show or explain your thinking.

5. How long would it take for *Helios* 2 to travel from Earth to the Sun? Show or explain your thinking.

6. Which is greater: the time it would take for *Helios* 2 to travel from Earth to the Sun or the number of blades of grass on a soccer field? Show or explain your thinking.

Name: ..... Date: ..... Period: .....

7. The Burj Khalifa in Dubai is the tallest structure in the world at approximately $8.3 \cdot 10^2$ meters tall. The Big Ben clock tower in London is 96 meters tall. About how many times taller is the height of the Burj Khalifa than Big Ben? Show or explain your thinking.

FAST Practice

8. The body of a 154-pound person contains approximately $1.4 \cdot 10^2$ grams of potassium and $2 \cdot 10^{-3}$ grams of silver. Based on this information, how many times greater is the number of grams of potassium in the body than the number of grams of silver in the body? Record your answer in the space provided.

Spiral Review

9. Select *all* expressions that are equivalent to 6^{-3}.

- ☐ **A.** -18
- ☐ **B.** $\frac{6}{6^4}$
- ☐ **C.** $\frac{1}{6^3}$
- ☐ **D.** $\left(\frac{1}{6}\right) \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{1}{6}\right)$
- ☐ **E.** $\frac{12^6}{2^9}$

10. Ira has a marble and a penny in his pocket. The mass of the marble is $6.937 \cdot 10^0$ grams. The mass of the penny is $2.51 \cdot 10^0$ grams. What is the total mass, in grams, of the marble and penny in Ira's pocket? Express your answer to the appropriate number of significant digits.

- (A.) 4.427
- (B.) 4.43
- (C.) 9.45
- (D.) 9.447

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6, MTR.5.1, MTR.6.1, MTR.7.1

City Lights

Let's apply our understanding of place value to add and subtract with scientific notation.

Warm-Up

1. Ariel says: $2 \cdot 10^2 + 3 \cdot 10^3 = 5 \cdot 10^5$.

Is Ariel's claim correct? Circle one.

Yes No I'm not sure

Explain your thinking.

Name: Date: Period:

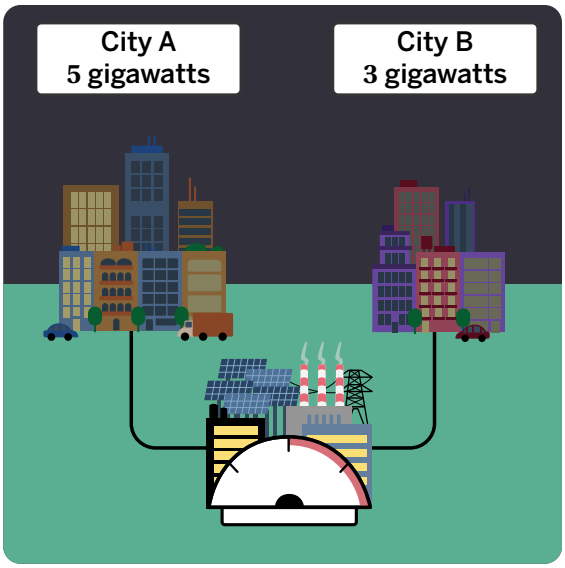
City Lights, Part 1

2. City A and City B get electricity from the same source.

City A needs 5 gigawatts of electricity.

City B needs 3 gigawatts.

How many gigawatts are needed to power both cities?

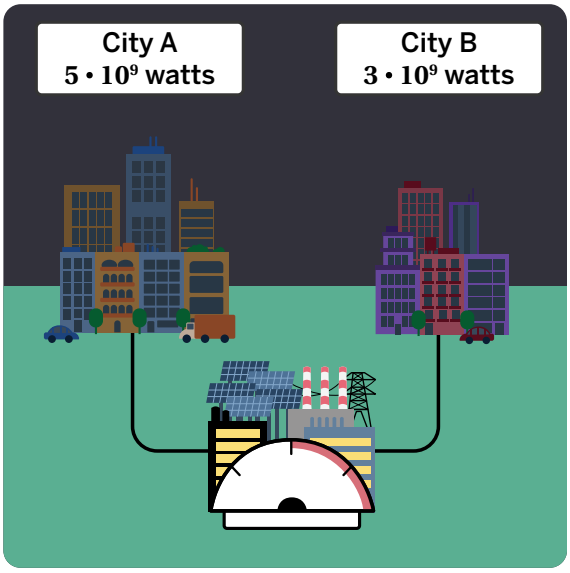


3. 1 gigawatt is equal to 10^9 watts.

City A needs $5 \cdot 10^9$ watts of electricity.

City B needs $3 \cdot 10^9$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.



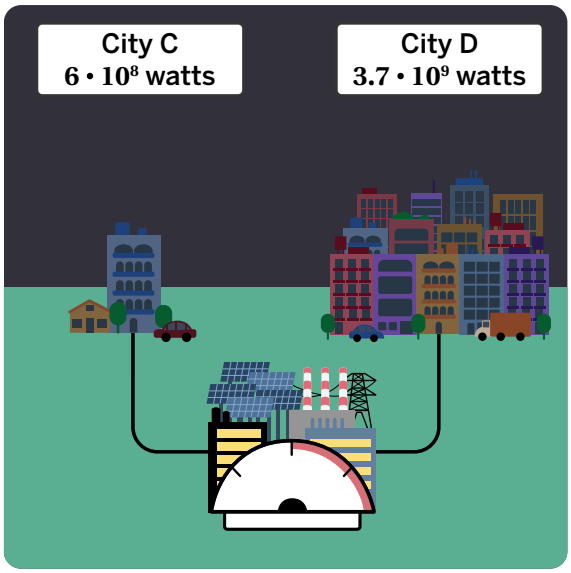
Name: Date: Period:

City Lights, Part 1 (continued)

4. Here are two new cities: City C and City D.

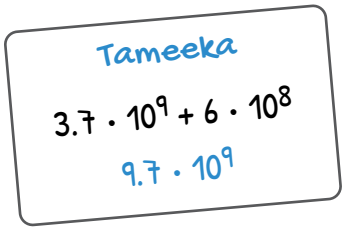
City C needs $6 \cdot 10^8$ watts of electricity.
City D needs $3.7 \cdot 10^9$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.



5. Tameeka made a mistake on the previous problem.

a What do you think Tameeka did well?



b What would you recommend Tameeka change in her work?

Name: Date: Period:

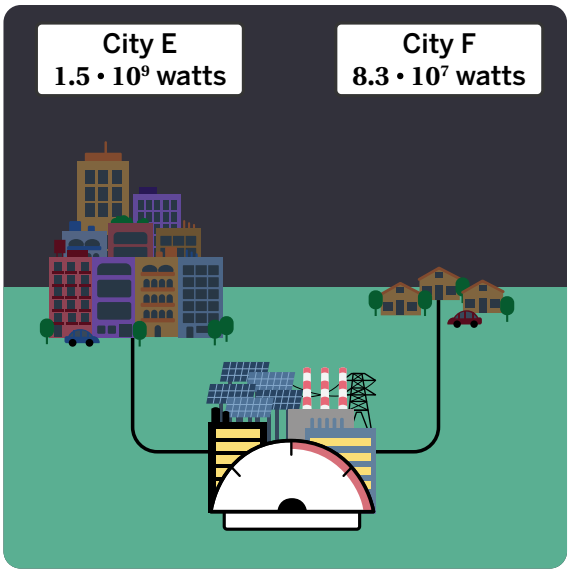
City Lights, Part 2

6. Here are two new cities: City E and City F.

City E needs $1.5 \cdot 10^9$ watts of electricity.

City F needs $8.3 \cdot 10^7$ watts.

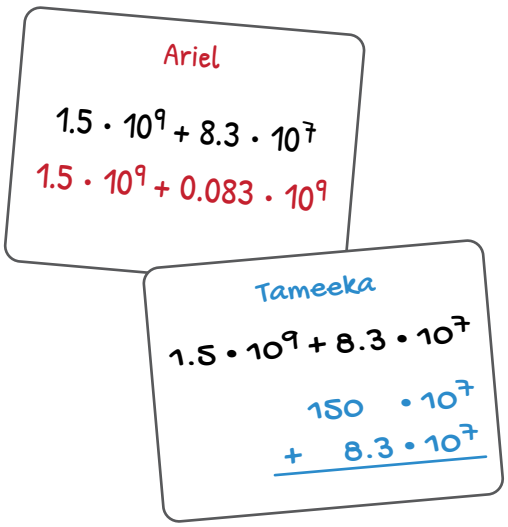
How many watts are needed to power both cities? Write your answer in scientific notation.



7. Here are Ariel's and Tameeka's strategies for the previous problem.

Discuss:

- How might each student finish the problem?
- After seeing both strategies, how would you add $3.6 \cdot 10^6 + 2.5 \cdot 10^5$?



Name: _____ Date: _____ Period: _____

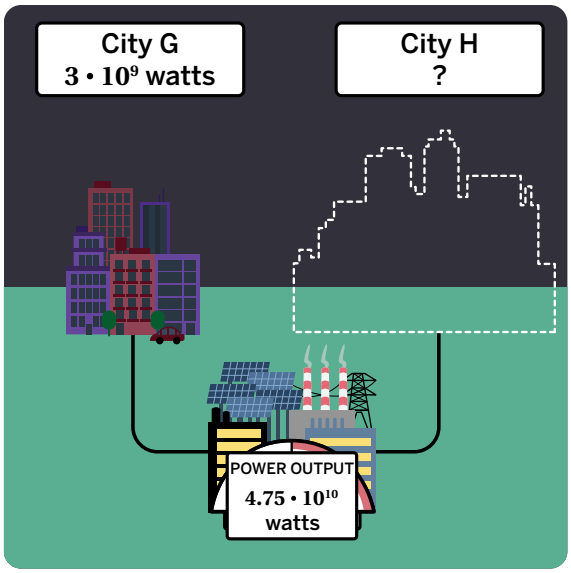
City Lights, Part 2 (continued)

8. Here are two new cities: City G and City H.

The power plant provides exactly enough electricity for both cities: $4.75 \cdot 10^{10}$ watts.

City G uses $3 \cdot 10^9$ watts.

How many watts does City H use? Write your answer in scientific notation.



9. Match each value with the correct situation. Note: All units are in watts.

$3.76 \cdot 10^{10}$	$1.03 \cdot 10^{12}$	$1.6 \cdot 10^{10}$	$1.03 \cdot 10^{11}$

Situation A	Situation B	Situation C	Situation D
City 1: $8 \cdot 10^{10}$	City 1: $4.5 \cdot 10^{10}$	City 1: $9.6 \cdot 10^{10}$	City 1: _____
City 2: $9.5 \cdot 10^{11}$	City 2: _____	City 2: $7 \cdot 10^9$	City 2: $2.4 \cdot 10^9$
Total: _____	Total: $6.1 \cdot 10^{10}$	Total: _____	Total: $4 \cdot 10^{10}$

Synthesis

10. What are some important things to remember when adding or subtracting numbers written in scientific notation?

Use the examples if they help with your thinking.

$4.6 \cdot 10^7 + 3.2 \cdot 10^6$

$1.57 \cdot 10^8 - 4 \cdot 10^6$

Lesson Practice ACC7.9.18

Lesson Summary

Scientific notation can be useful for adding or subtracting very large or very small numbers. It is important to pay attention to place value when adding and subtracting numbers written in scientific notation.

For example: Let's add $3.4 \cdot 10^5 + 2.1 \cdot 10^6$.

It may appear that you can add the first parts: 3.4 and 2.1. However, these numbers *do not* have the same place value because they are multiplied by different powers of 10.

If you rewrite one number so that both numbers have the same power of 10, then you can add their first parts. In this case, let's rewrite $2.1 \cdot 10^6$ as $21 \cdot 10^5$.

$$\begin{aligned} 3.4 \cdot 10^5 + 2.1 \cdot 10^6 &= 3.4 \cdot 10^5 + 21 \cdot 10^5 \\ &= 24.4 \cdot 10^5 \\ &= 2.44 \cdot 10^6 \end{aligned}$$

Now that the power of 10 is the same, you can add 3.4 and 21. The sum is $24.4 \cdot 10^5$, or $2.44 \cdot 10^6$ when rewritten in scientific notation.

Lesson Practice
ACC7.9.18

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value of each expression. Write your answers in scientific notation.

1. $5.3 \cdot 10^4 + 4.7 \cdot 10^4$

2. $3.7 \cdot 10^6 - 3.3 \cdot 10^6$

3. $4.8 \cdot 10^{-3} + 6.3 \cdot 10^{-3}$

4. $6.6 \cdot 10^{-5} - 6.1 \cdot 10^{-5}$

Problems 5–7: Decide whether each statement is true or false. Show or explain your thinking.

5. $3 \cdot 10^2 + 4 \cdot 10^3 = 7 \cdot 10^5$

6. $8 \cdot 10^2 - 5.1 \cdot 10^3 = 2.9 \cdot 10^2$

7. $7 \cdot 10^{-4} + 9 \cdot 10^{-3} = 9.7 \cdot 10^{-3}$

Name: ..... Date: ..... Period: .....

8. Fill in each blank. Try to make an expression none of your classmates will.

$\square \cdot 10^{\square} + \square \cdot 10^{\square} = 3.6 \cdot 10^4$

FAST Practice

9. Write the value of $2.3 \cdot 10^4 + 4.1 \cdot 10^5$ in scientific notation. Record your answer in the space provided.

Spiral Review

Problems 10–13: Multiply the numbers in each expression. Write your answers in scientific notation.

10. $4.1 \cdot 10^7 \cdot 2$

11. $3 \cdot (1.5 \cdot 10^{11})$

12. $(3 \cdot 10^3)^2$

13. $(9 \cdot 10^6) \cdot (3 \cdot 10^6)$

Problems 14–15: Diego was trying to solve an equation. But when he checked his answer, he saw his solution was incorrect.

14. What would you recommend Diego change in his work?

Diego

$-4(7 - 2x) = 3(x + 4)$

$-28 - 8x = 3x + 12$

$-28 = 11x + 12$

$-40 = 11x$

$-\frac{40}{11} = x$

15. What is the correct solution to the equation?

16. Philipe made a candle in the shape of a cylinder. The base of the candle is $1.256 \cdot 10^1$ square inches. The height of the candle is 6 inches. The formula for the volume, V, of a cylinder with a base area of B and height of h, is $V = Bh$. Does the number of significant digits change from four significant digits when finding the volume of the candle? Explain your thinking.

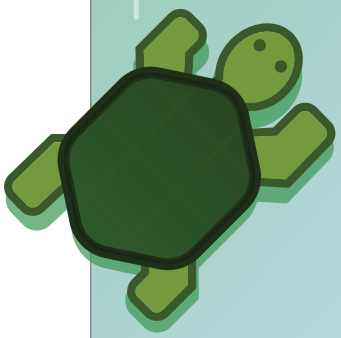
Unit 10

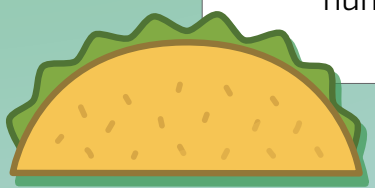
The Pythagorean Theorem and Irrational Numbers

There are lots of different types of triangles. What is the relationship between the sides of a right triangle? To answer this, a representation for the edge length of a square is needed. In this unit, you'll explore how side lengths of right triangles are related, and learn about new types of numbers.

Essential Questions

- How can you estimate the square root of a number? What does the square root of a number represent?
- Is it true that $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ for all right triangles? If so, can you prove it?
- What is the difference between a rational number and an irrational number?

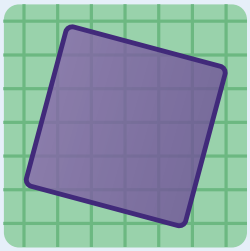




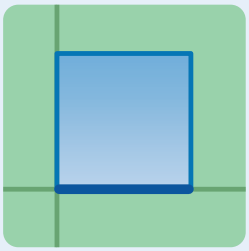
Sub-unit

1

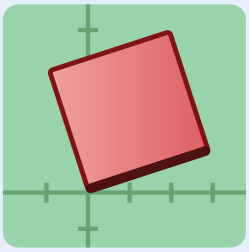
Square Roots and Cube Roots



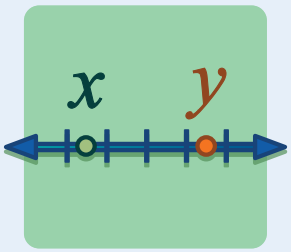
Lesson 1
Tilted Squares



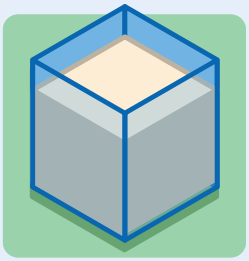
Lesson 2
From Squares to Roots



Lesson 3
Between Squares



Lesson 4
Root Down



Lesson 5
Filling Cubes

Name: ..... Date: ..... Period: .....

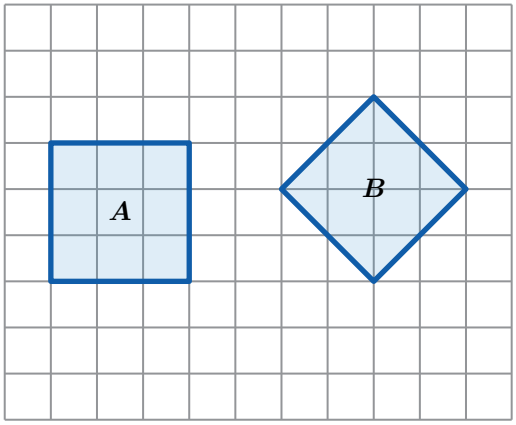
Building Toward MA.8.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Tilted Squares

Let's explore finding the areas of tilted squares.

Warm-Up

1. Which shaded region is larger?

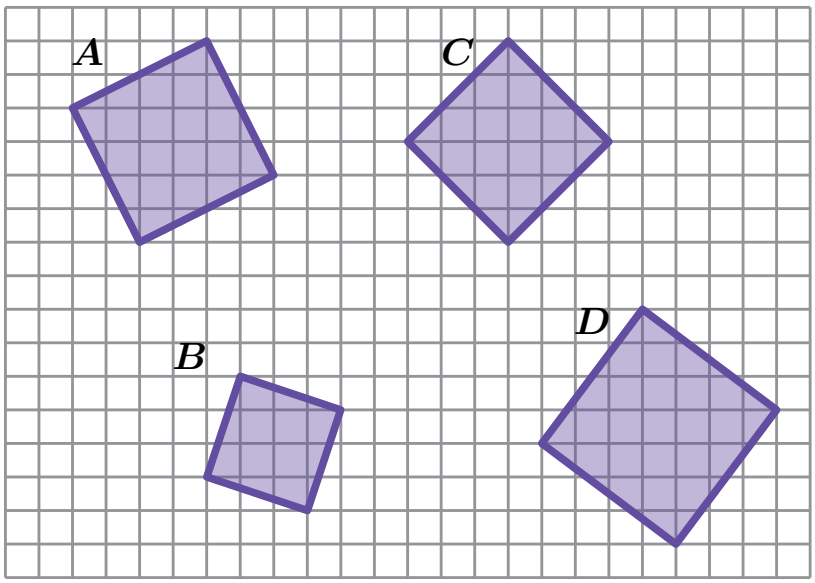


Explain your thinking.

Name: Date: Period:

Area of Tilted Squares

2. Determine the area of each tilted square (in square units). Record the areas in the table.



Square	A	B	C	D
Area (sq. units)				

3. What strategies did you use to determine the areas of the tilted squares?

4. What is the side length of square D?

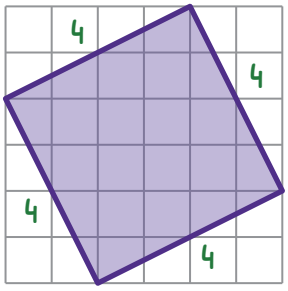
Explain your thinking.

Name: Date: Period:

Different Strategies

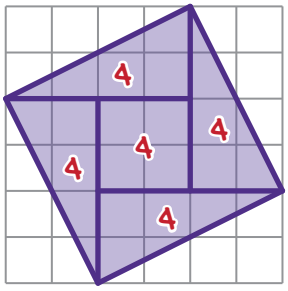
Here are Trevon's and Zahra's strategies for finding the area of tilted square A.

Trevon



$6 \cdot 6 - 4 \cdot 4 = 20$ square units

Zahra



$4 \cdot 4 + 4 = 20$ square units

5. How are Trevon's and Zahra's strategies alike? How are they different?

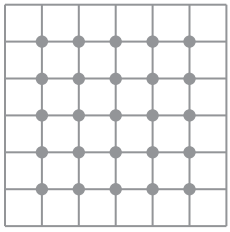
6. How does each strategy compare to your own?

Name: ..... Date: ..... Period: .....

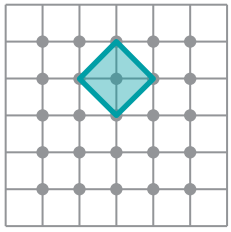
Building Squares With Different Areas

7. Here are squares with areas of 2 square units and 9 square units. On each dot grid, try to draw a square with the given area. Then circle "P" for any area that is possible to draw and "N" for any area that's not possible to draw.

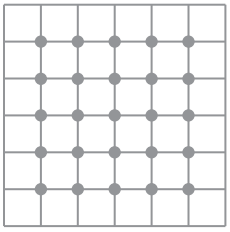
Area:
1 square unit
P / N



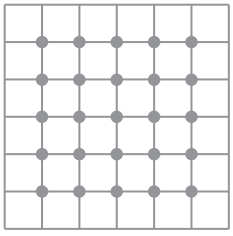
Area:
2 square units
(P) / N



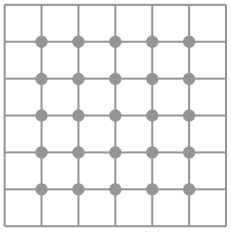
Area:
3 square units
P / N



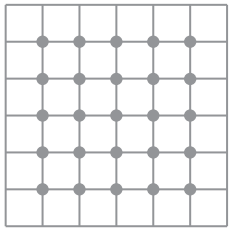
Area:
4 square units
P / N



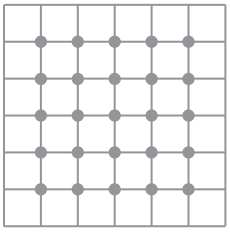
Area:
5 square units
P / N



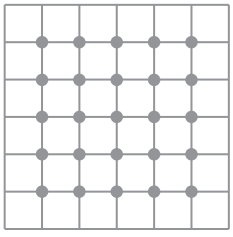
Area:
6 square units
P / N



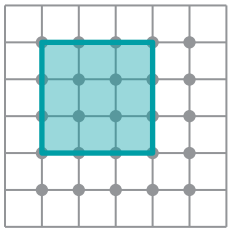
Area:
7 square units
P / N



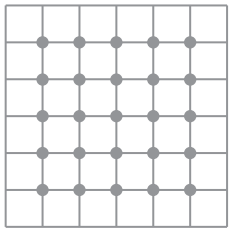
Area:
8 square units
P / N



Area:
9 square units
(P) / N



Area:
10 square units
P / N

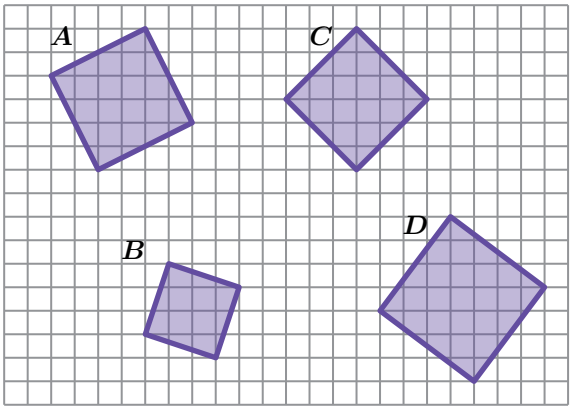


8. Choose one of the squares and determine its side length.

Synthesis

9. Describe a strategy for determining the area of a tilted square.

Use the examples if they help with your thinking.

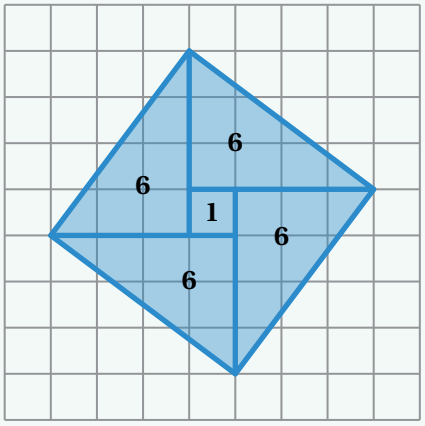


Lesson Practice ACC7.10.01

Lesson Summary

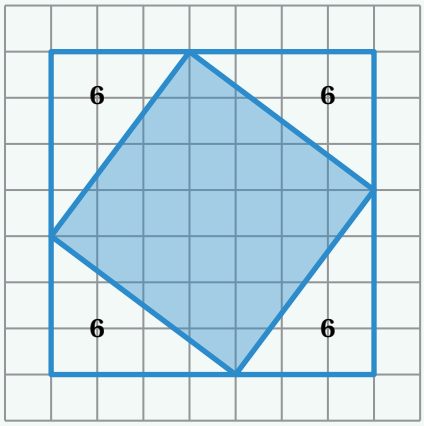
There are many strategies for determining the area of a tilted square. Here are two strategies called "decompose and rearrange" and "surround and subtract."

Decompose and Rearrange
Area is calculated by adding the areas of the four triangles and one center square.



$4 \cdot 6 + 1 = 25$ square units

Surround and Subtract
Area is calculated by finding the area of the large square minus the area of the four triangles.

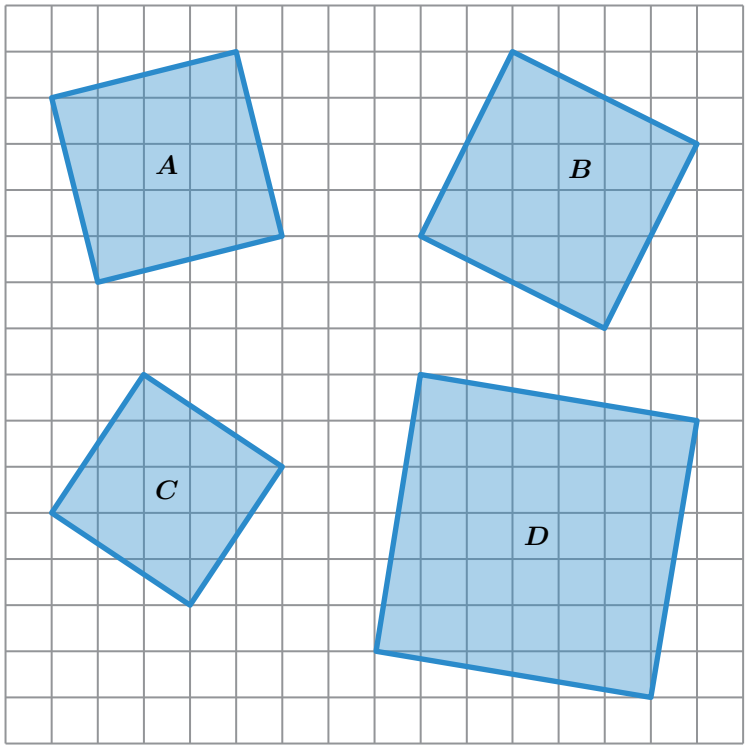


$7 \cdot 7 - 4 \cdot 6 = 25$ square units

Lesson Practice
ACC7.10.01

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the area of each tilted square. Each square grid represents 1 square unit.



1. Square A

2. Square B

3. Square C

4. Square D

Problems 5–7: Determine the area of each square given its side length.

5. Side length: 3 inches

6. Side length: 100 centimeters

7. Side length: x units

Problems 8–10: Here are the areas of three squares. Determine the side length of each square.

8. Area: 81 square inches

9. Area: $\frac{4}{25}$ square centimeters

10. Area: m^2 units

Lesson Practice
ACC7.10.01

Name: ..... Date: ..... Period: .....

FAST Practice

11. A square has a side length of 6 cm. Each side of the square is increased by 3 cm. How much more is the area of the new square than the original square?

Record your answer in the space provided.

☐ square centimeters

Spiral Review

12. Select *all* the expressions that are equivalent to 3^8.

☐ **A.** $3^6 \cdot 10^2$ ☐ **B.** 8^3 ☐ **C.** $\frac{3^6}{3^{-2}}$ ☐ **D.** $(3^4)^2$ ☐ **E.** $(3^2)^4$

13. In July 2023, the population of Arlington, Texas, was 392,786. What is the value of the exponent when this number is written in scientific notation?

14. Which expression is equal to $(3.1 \cdot 10^4) \cdot (2 \cdot 10^6)$?

(A.) $5.1 \cdot 10^{10}$ (B.) $5.1 \cdot 10^{24}$ (C.) $6.2 \cdot 10^{10}$ (D.) $6.2 \cdot 10^{24}$

15. Here is Oliver's work for solving this problem:

Evaluate $5.4 \cdot 10^5 + 2.3 \cdot 10^4$ and write the answer in scientific notation.

Oliver

$5.4 \cdot 10^5$ can be rewritten as $54 \cdot 10^4$

$54 \cdot 10^4 + 2.3 \cdot 10^4 = 56.3 \cdot 10^4$

Is Oliver's solution correct?

Explain your thinking.

Name: ..... Date: ..... Period: .....

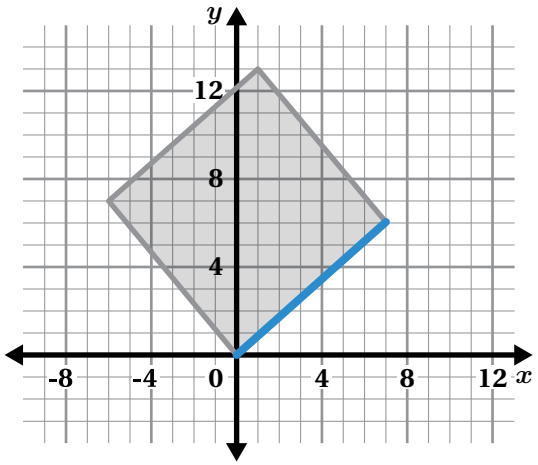
MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

From Squares to Roots

Let's explore the connection between the area and side length of a square.

Warm-Up

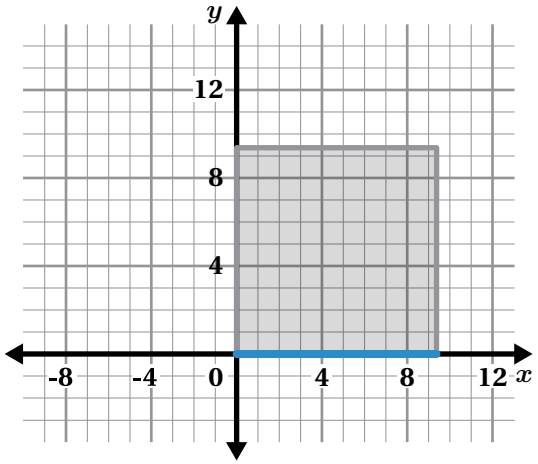
1. Estimate the side length of the square.



2. You can *approximate* the side length of a tilted square by rotating it onto an axis.

a Here is the square from the previous problem rotated so that the highlighted side length is along the x-axis.

b Write a new estimate for the side length of the square.

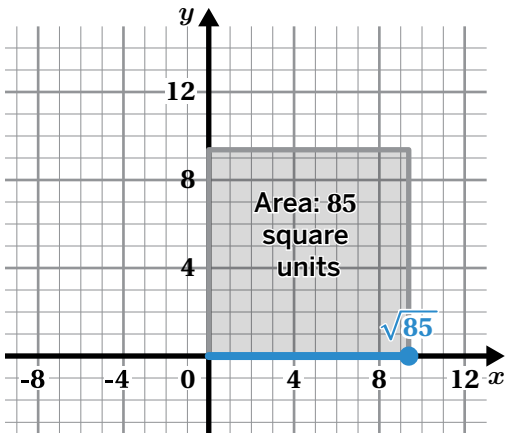


Activity 1

Name: Date: Period:

Square Roots

3. The *exact* side length of this square is the **square root** of 85, written as $\sqrt{85}$.

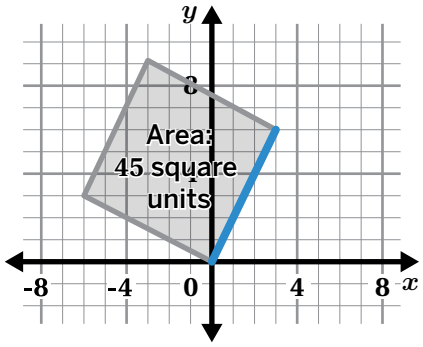


a. Take a look at the square and how the side length is written.

b. **Discuss:** Explain what you think a *square root* is in your own words.

4. The area of this square is 45 square units.

Write the exact value of the side length.



5. Determine the unknown side lengths and areas for each square.

Square	Side Length (units)	Area (sq. units)
A	$\sqrt{55}$	55
B		81
C	2.5	
D		14
E	$\sqrt{44}$	
F		32

Name: ..... Date: ..... Period: .....

Square Roots (continued)

6. Order the squares from *smallest* to *largest* area.

A	*B*	*C*	*D*
Area of 50 square units	Side length of $\sqrt{81}$ units	Side length of $\sqrt{55}$ units	Side length of 8 units

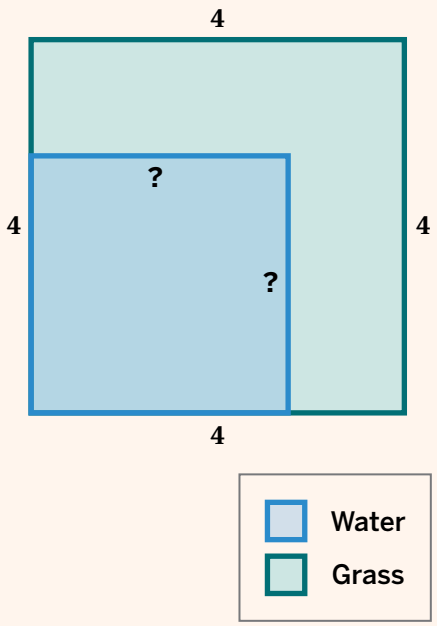
Smallest **Largest**

You're invited to explore more.

7. Metropolis has a park surrounded by a square fence with 4-meter side lengths.

The city would like to build a square pool as shown in the figure.

What should the side length of the pool be so that half of the area is grass and half is water? Explain your thinking.

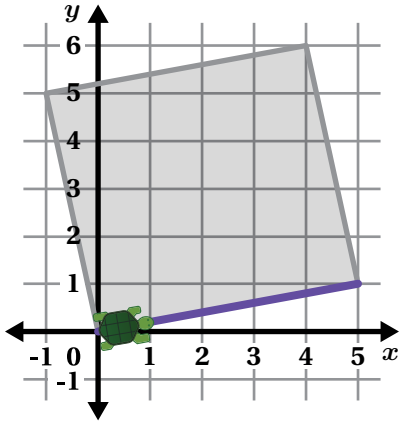


Activity 2

Name: ..... Date: ..... Period: .....

Turtle Tracing

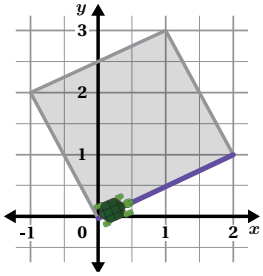
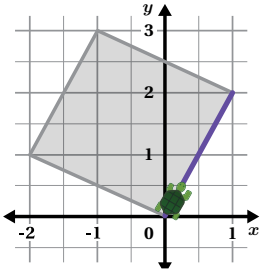
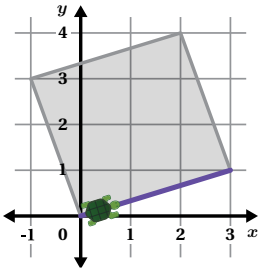
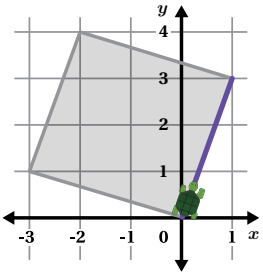
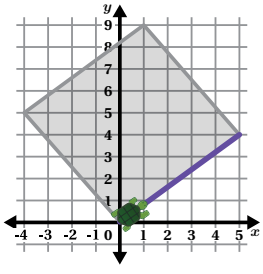
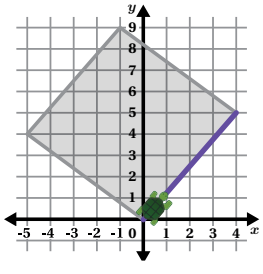
Tiam the turtle is walking on one side of a square.



8. Exactly how far does Tiam need to travel?

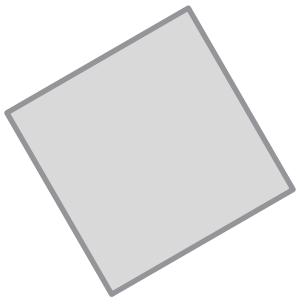
9. Complete this chart with a partner.

- Decide with your partner who will complete Column A and who will complete Column B.
- Determine how far Tiam the turtle needs to travel, and then compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.
- Determine the side lengths of as many squares as you have time for.

	Column A	Column B
a	..... units 	..... units 
b	..... units 	..... units 
c	..... units 	..... units 

Synthesis

10. Describe the relationship between the side length and the area of a square using the term *square root*.

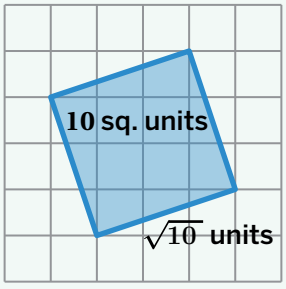


Lesson Practice ACC7.10.02

Lesson Summary

There is a known relationship between the area of any square and its side length. The exact value of the side length of a square can be written as the **square root** of its area.

For example, $\sqrt{10}$ is the exact value for the side length of a square with an area of 10 square units.



Name: ..... Date: ..... Period: .....

Problems 1–4: Here are the areas of different squares. Determine the side length for each square.

1. Area: 37 square units

2. Area: $\frac{100}{9}$ square units

3. Area: $\frac{2}{5}$ square units

4. Area: 0.0001 square units

Problems 5–7: Here is some information about three squares. Determine which side length matches each square.

- Square A is smaller than square B.
- Square B is smaller than square C.
- The three squares' side lengths are $\sqrt{26}$, 4.2, and $\sqrt{11}$ units.

5. Square A

6. Square B

7. Square C

Problems 8–10: Here are the side lengths of different squares. Determine the area of each square.

8. Side length: $\frac{1}{5}$ centimeters

9. Side length: $\frac{3}{7}$ units

10. Side length: 0.1 meter

Lesson Practice
ACC7.10.02

Name: _____ Date: _____ Period: _____

FAST Practice

11. Square A has an area of 81 square feet. Select *all* the expressions that are equal to the side length of this square (in feet).

☐ **A.** 3 ☐ **B.** $\frac{81}{2}$ ☐ **C.** $\sqrt{81}$ ☐ **D.** $\sqrt{9}$ ☐ **E.** 9

Spiral Review

12. Which expression is equivalent to $(12^3)(12^{-8})$?

A. 12^{-24} **B.** -12^5 **C.** $\frac{1}{12^5}$ **D.** $\frac{1}{12^{-5}}$

Problems 13–14: Here is a table showing the areas of six large countries.

Country	Area (sq. km)
Russia	$1.71 \cdot 10^7$
Canada	$9.98 \cdot 10^6$
China	$9.60 \cdot 10^6$
United States	$9.53 \cdot 10^6$
Brazil	$8.52 \cdot 10^6$
India	$3.29 \cdot 10^6$

13. About how many times greater is the area of Russia than the area of Canada?

14. The Eastern Hemisphere countries on this list are Russia, China, and India. The Western Hemisphere countries are Canada, the United States, and Brazil. Which has the greater total area?

A. The three Eastern Hemisphere countries

B. The three Western Hemisphere countries

Explain your thinking.

15. Select *all* the expressions that are equivalent to 10^{-6}.

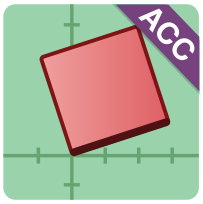
☐ **A.** $-\frac{1}{1000000}$ ☐ **B.** $\left(\frac{1}{10}\right)^6$ ☐ **C.** $10^8 \cdot 10^{-2}$

☐ **D.** $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$ ☐ **E.** $\frac{1}{10^6}$

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

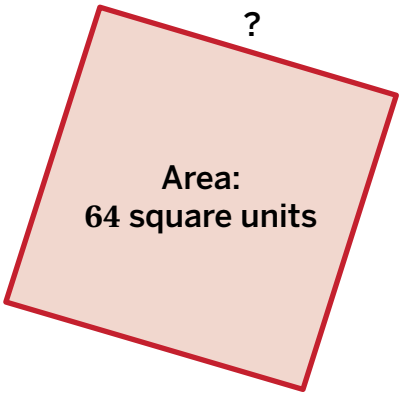
Between Squares



Let's approximate the value of square roots.

Warm-Up

1. Select *one* correct expression for the side length of this square.



- ☐ **A.** $\frac{64}{2}$
- ☐ **B.** $\sqrt{64}$
- ☐ **C.** 8
- ☐ **D.** $\sqrt{8}$
- ☐ **E.** 4
- ☐ **F.** $\frac{64}{4}$

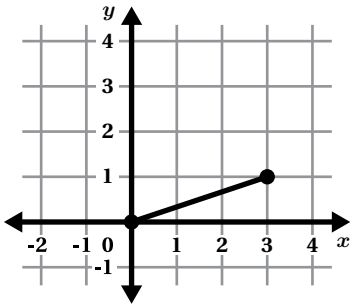
Explain your thinking.

Name: Date: Period:

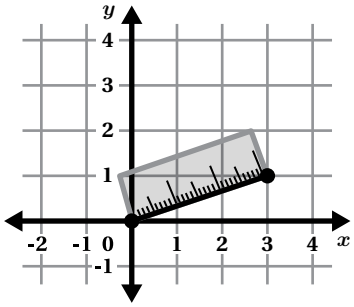
Squaring Lines

2. What do you think is the length of this segment?

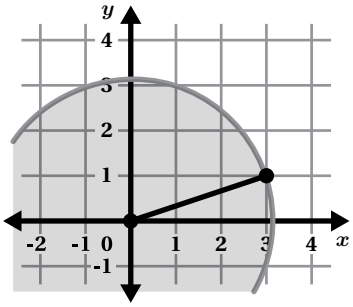
Use the ruler, circle, or square to help with your thinking.



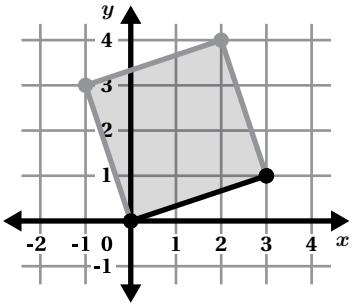
Ruler



Circle



Square



3. Ava says that the segment length is $\sqrt{10}$ units because the area of the square is 10 square units.

Rebecca says that the segment length is about 3.2 units because that's the approximate length of the circle's radius.

Discuss:

- How are Ava's and Rebecca's strategies alike? How are they different?
- What is helpful about each strategy?

Activity 2

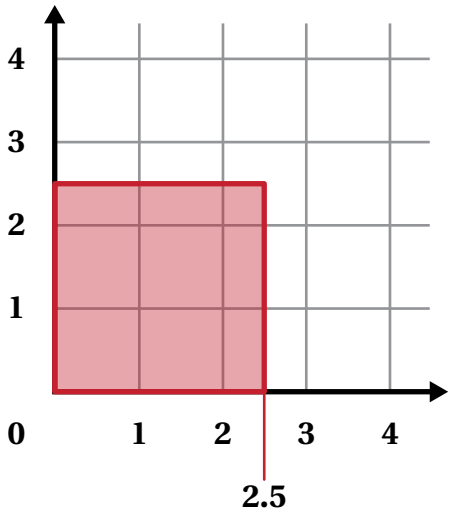
Name: ..... Date: ..... Period: .....

Using Squares to Estimate

4. How does $\sqrt{5}$ compare to 2.5? Circle one.
Use the square to help with your thinking.

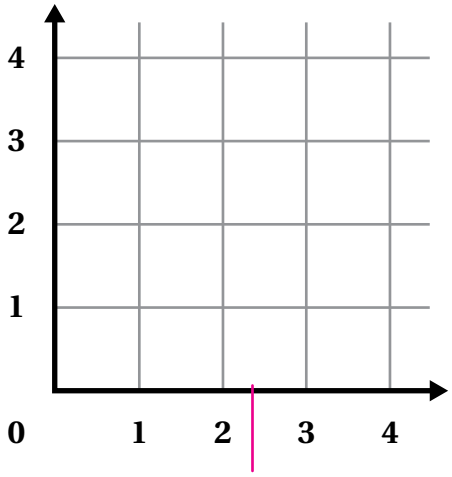
$\sqrt{5}$ is less than 2.5 | $\sqrt{5}$ is greater than 2.5 | $\sqrt{5}$ is equal to 2.5

Explain your thinking.



5. **a** Sketch a square to help you estimate $\sqrt{5}$.

b Write your estimate for $\sqrt{5}$.



Name: Date: Period:

Closest Decimal Approximation

6. $\sqrt{5}$ is a number that equals 5 when squared.

Use a calculator to approximate the value of $\sqrt{5}$ as closely as you can. Record each guess, n, and its square, n^2, in the table.

n	n^2
2.0	$(2.0)^2 = 4.00$

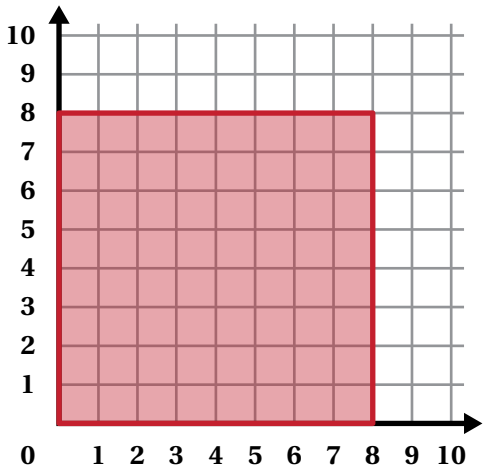
7. Describe your strategy for finding a decimal approximation that is as close as possible to $\sqrt{5}$.

8. Use a calculator to approximate the value of $\sqrt{30}$ as closely as you can. Record each guess, n, and its square, n^2, in the table.

n	n^2
5.2	$(5.2)^2 = 27.04$

Synthesis

9. What are some strategies for approximating a square root, such as $\sqrt{75}$?



n	n^2
8.0	64

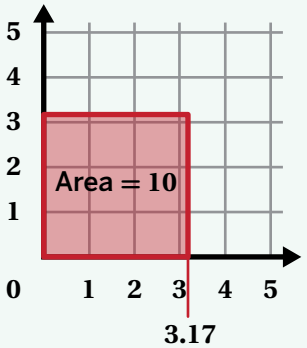
Lesson Practice ACC7.10.03

Lesson Summary

You can use several strategies to approximate the values of square roots. One strategy is to use the areas of squares. The side length of a square is equal to the square root of its area. Another strategy is to create a table of values for n and determine n^2. Squaring a number and taking the square root are inverse operations: remember that $(\sqrt{n})^2 = n$. Below is a description of how each strategy can be used to approximate $\sqrt{10}$.

Using Squares

- Create a square that has an area equal to about 10 square units.
- Approximate the side length of the square created.



Using a Table

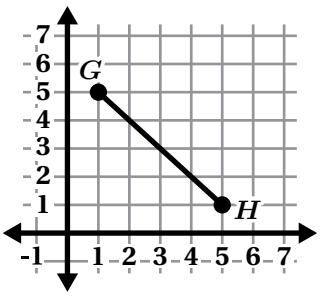
- Create a table of decimal value guesses for n.
- Calculate n^2 for each guess of n.
- The closer n^2 is to 10, the better that value of n is as an approximation for $\sqrt{10}$.

n	n^2
3.1	9.61
3.16	9.9856
3.17	10.0489
3.165	10.017225

Lesson Practice
ACC7.10.03

Name: ..... Date: ..... Period: .....

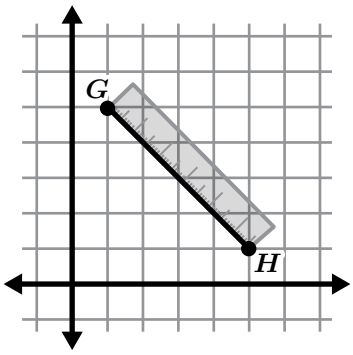
Problems 1–2: Here is the line segment GH. Each grid square represents 1 square unit. Use the ruler, circle, or square if they help with your thinking.



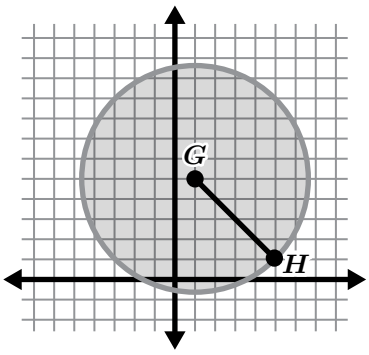
1. Determine the approximate length of GH.

2. Determine the exact length of GH.

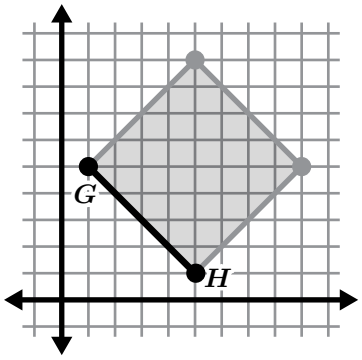
Ruler



Circle



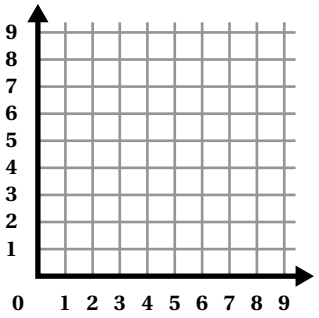
Square



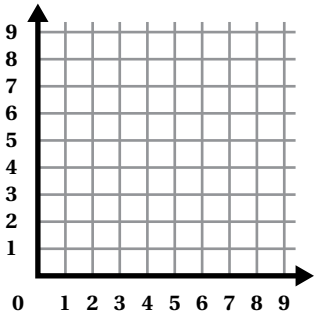
3. Determine the value of $\sqrt{16}$.

Problems 4–5: Estimate each square root. Draw a square if it helps with your thinking.

4. $\sqrt{35}$



5. $\sqrt{66}$



Problems 6–7: Determine which two whole numbers each square root is between.

6. $\sqrt{7}$

7. $\sqrt{31}$

Lesson Practice
ACC7.10.03

Name: _____ Date: _____ Period: _____

FAST Practice

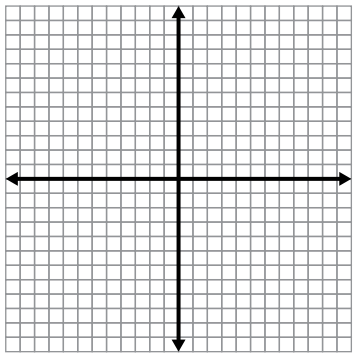
8. Here is a list of values ordered from least to greatest. One value is unknown. Which could be the unknown value?

$2.5, \frac{19}{3}, \sqrt{51}, _____$

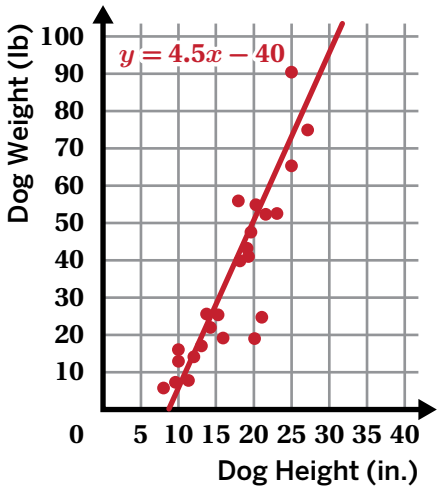
(A) $(3.1)^2$ (B) $\frac{15}{4}$ (C) 6.89 (D) 2.1

Spiral Review

9. Identify two points on a line that would create a slope of 6. Use the coordinate plane if it helps with your thinking.



Problems 10–12: Here is a scatter plot that shows the heights and weights of 25 dogs, as well as a linear model for the same situation and its equation.



10. What does the slope of the linear model mean in this situation?

11. Based on the model, what will be the weight of a dog that is 25 inches tall?

12. Does the data show a positive or negative association?

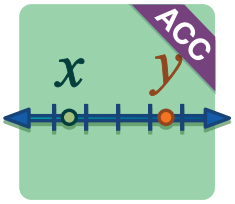
Explain your thinking.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3, MTR.5.1, MTR.6.1

Root Down

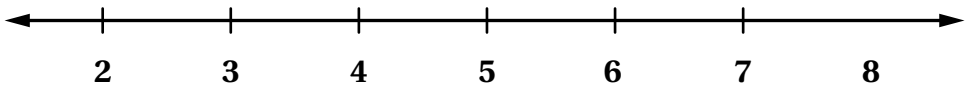
Let's estimate the value of square roots and represent them on a number line.



Warm-Up

1. Plot these values on the number line.

$\sqrt{49}$ $\sqrt{64}$ $\sqrt{9}$ $\sqrt{25}$ $\sqrt{55}$



2. 9, 25, 49, and 64 are **perfect squares**. 55 is not.

What do you think a perfect square is?

Activity 1

Name: ..... Date: ..... Period: .....

Between Whole Numbers

3. Match each value to the whole numbers it is between. Two values will not have a match. Note: The numbers x, y, and z are positive numbers.

The value of x when $x^2 = 50$	The value of y when $y^2 = 20$	The value of z when $z^2 = 80$	$\sqrt{24}$
$\sqrt{62}$	$\sqrt{60}$	$\sqrt{17}$	$\sqrt{15}$

Between 4 and 5	Between 7 and 8	No Match

4. Esi thinks that the description "The value of z when $z^2 = 80$" doesn't belong in either category.

Between 7 and 8 | Between 4 and 5

The value of z when $z^2 = 80$

a Which two whole numbers is the value of z between?

(A.) 4 and 5

(B.) 6 and 7

(C.) 7 and 8

(D.) 8 and 9

b Of those two numbers, which would z be closer to?

Explain your thinking.

Name: Date: Period:

Between Whole Numbers (continued)

5. Order the numbers from *least* to *greatest*.

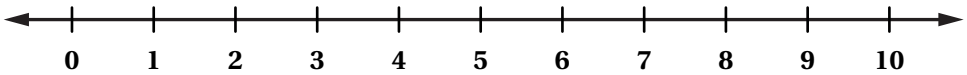
$\sqrt{99}$	$\sqrt{75}$	9	9.5	10

Least **Greatest**

6. The numbers x and y are positive. $x^2 = 3$ and $y^2 = 35$.

a Solve each equation for the variable.

b Plot x and y on the number line.



c **Discuss:** How did you decide where to plot each point?

Activity 2

Name: ..... Date: ..... Period: .....

How Many Roots?

7. Duri says the equation $x^2 = 36$ has one solution: $x = 6$. Taylor says the equation has two solutions: $x = 6$ and $x = -6$.

Whom do you agree with? Circle one.

Duri Taylor Neither

Explain your thinking.

8. Give all possible solutions for each equation.

a $a^2 = \sqrt{100}$

b $y^2 = 20$

9. Write $<$ or $>$ to complete each inequality. Then explain your thinking.

a 8 ..... $\sqrt{62}$

b 11 ..... $\sqrt{130}$

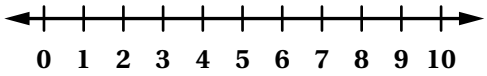
c $\sqrt{51}$ 7.5

Synthesis

10. What are some strategies for plotting square roots on a number line?

Use the number line and examples if they help to show your thinking.

$\sqrt{25}$ $\sqrt{36}$ $\sqrt{31}$ $\sqrt{40}$



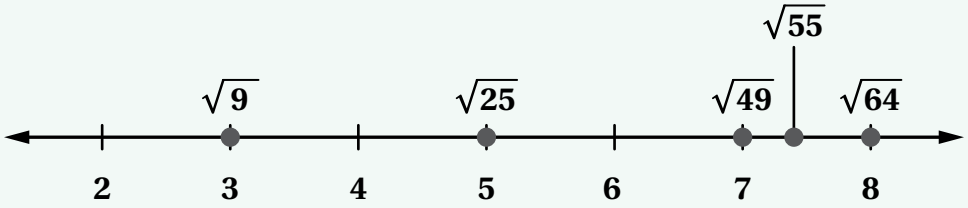
Lesson Practice ACC7.10.04

Lesson Summary

You can represent a square root on a number line. We write a solution to an equation, such as $x^2 = 3$, using square root notation. The positive solution to this equation is $x = \sqrt{3}$. This equation also has a negative solution, $-\sqrt{3}$.

You can approximate a square root on a number line by observing the whole numbers around it.

For example, you can determine that $\sqrt{55}$ is between 7 and 8 because $7^2 = 49$ and $8^2 = 64$, and 55 is between 49 and 64. More precisely, $\sqrt{55}$ should be plotted slightly left of 7.5 since it is closer to 7 than 8.



A **perfect square** is a number that is the square of an integer. For example, 49 is a perfect square because $7 \cdot 7 = 7^2 = 49$, but 55 is not a perfect square because no integer can be squared to equal 55.

Name: ..... Date: ..... Period: .....

1. The number z is positive. Determine the exact value of z if $z^2 = 60$.

(A.) 30 (B.) $\sqrt{7.75}$ (C.) 7.75 (D.) $\sqrt{60}$

2. Determine *all* possible values of n if $n^2 = 150$.

☐ **A.** 12 ☐ **B.** $\sqrt{150}$ ☐ **C.** $-\sqrt{150}$ ☐ **D.** 75 ☐ **E.** -75

3. Which statement *best* describes the value of $\sqrt{41}$?

(A.) The value of $\sqrt{41}$ is between 6 and 6.5.

(B.) The value of $\sqrt{41}$ is between 6.5 and 7.

(C.) The value of $\sqrt{41}$ is between 7 and 7.5.

(D.) The value of $\sqrt{41}$ is between 7.5 and 8.

4. Write two square roots that are between 7 and 8.

5. Explain how you know that $\sqrt{30}$ is between 5 and 6.

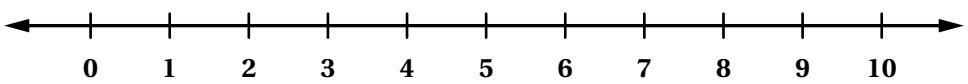
Problems 6–7: Compare the numbers by writing <, >, or =. Then, explain your thinking.

6. $\sqrt{37}$ 6

7. $\sqrt{95}$ 10

8. Estimate each number and plot it on the number line.

$\sqrt{83}$ $\sqrt{40}$ $\sqrt{64}$



Name: ..... Date: ..... Period: .....

9. Fill in each blank using the digits 1 to 9 only once to make the inequality true.

$$\square < \sqrt{\square\square} < \square$$

FAST Practice

10. Select *all the* numbers that are greater than 10 and less than 11.

- ☐ **A.** $\sqrt{120}$
- ☐ **B.** $\sqrt{122}$
- ☐ **C.** $\sqrt{130}$
- ☐ **D.** $\sqrt{95}$
- ☐ **E.** $\sqrt{105}$

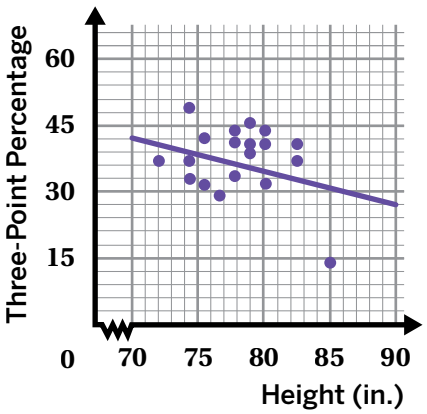
Spiral Review

Problems 11–12: Evaluate each expression. Write your answer in scientific notation.

11. $(2 \cdot 10^3)(3.4 \cdot 10^{11})$

12. $\dfrac{4.6 \cdot 10^3}{2 \cdot 10^5}$

Problems 13–14: This scatter plot shows the heights (in inches) and the three-point percentages for different basketball players last season.



13. Predict the three-point percentage for a player who is 70 inches tall.

14. Identify a data point that appears to be an outlier.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3, MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

Filling Cubes

Let's explore the relationship between the edge length and the volume of a cube.

Warm-Up

1. Order each value from *least* to *greatest*. Note: Let a, b, c, and d be positive numbers.

a when $a^2 = 9$	b when $b^3 = 8$	c when $c^2 = 8$	d when $d^3 = 9$

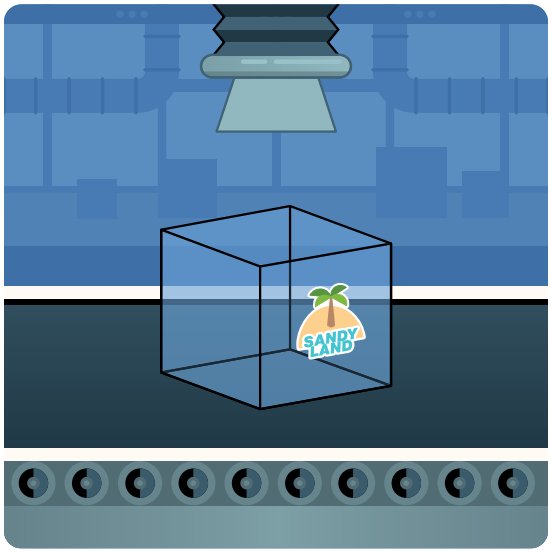
Least **Greatest**

Activity 1

Name: Date: Period:

Filling Cubes

Your job is to make sure the right amount of sand ends up in each cube. Use a calculator if it helps with your thinking.



2. This cube has an edge length of 6 inches.

How much sand is needed to fill it?

Edge Length (in.)	Amount of Sand (cu. in.)
6	

3. Four new orders just came in. Complete the table for each order.

Edge Length (in.)	Amount of Sand (cu. in.)
3	
2.1	
	64
	125

4. Describe a strategy you used to find the unknown edge lengths.

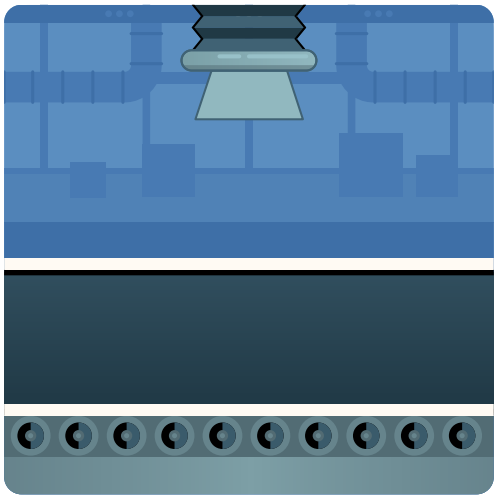
Name: ..... Date: ..... Period: .....

Filling Cubes (continued)

5. A customer wants a cube filled with 100 cubic inches of sand.

Let's try to find the *exact* edge length of this cube.

Choose an edge length and use your calculator to determine how much sand will fill that cube. Keep revising your estimate to get as close to the target as possible.



Edge Length (in.)	Amount of Sand (cu. in.)
4.5	$(4.5)^3 = 91.125$

6. The equation $x^3 = 100$ can help you determine the edge length of a cube that holds 100 cubic inches of sand. The exact solution to this equation is a **cube root**: $x = \sqrt[3]{100}$.

a Enter $\sqrt[3]{100}$ on your calculator to see its approximate value.

b **Discuss:** What is the relationship between the edge length and the volume of a cube?

7. Determine the *exact* unknown value for each cube.

Edge Length (in.)	Amount of Sand (cu. in.)
	200
	150
	91.125
$\sqrt[3]{42}$	

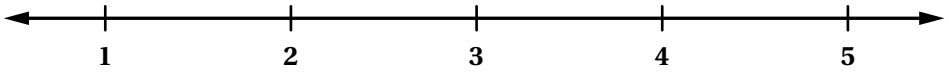
Name: ..... Date: ..... Period: .....

The Number Line

8. Here are three cube roots.

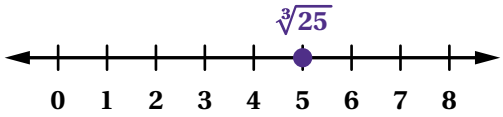
$\sqrt[3]{27}$ $\sqrt[3]{64}$ $\sqrt[3]{15}$

a Plot the cube roots on the number line.



b Describe your strategy for plotting $\sqrt[3]{15}$.

9. Nia incorrectly plotted $\sqrt[3]{25}$.



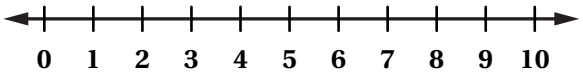
Discuss:

- What mistake could Nia have made?
- What question could you ask Nia to help her correct her work?

10. Here is an equation: $x^3 = 30$.

a Write the exact solution to the equation.

b Plot the solution on the number line.

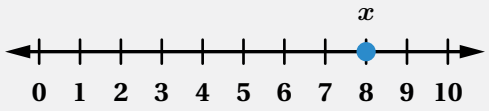
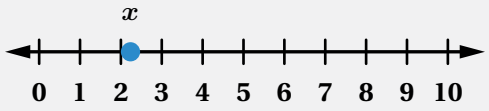
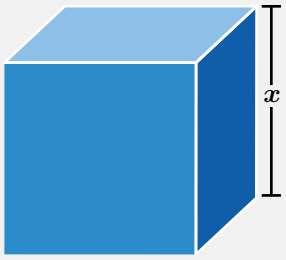
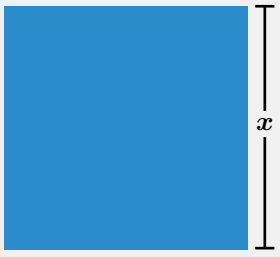


Name: ..... Date: ..... Period: .....

The Number Line (continued)

11. Match each equation to the visual that represents the same value of x.

$x = \sqrt{10}$ $x = \sqrt{64}$ $x = \sqrt[3]{10}$ $x = \sqrt[3]{64}$

Visual	Equation
	
	
Volume: 64 cu. in. 	
Area: 10 sq. in. 	

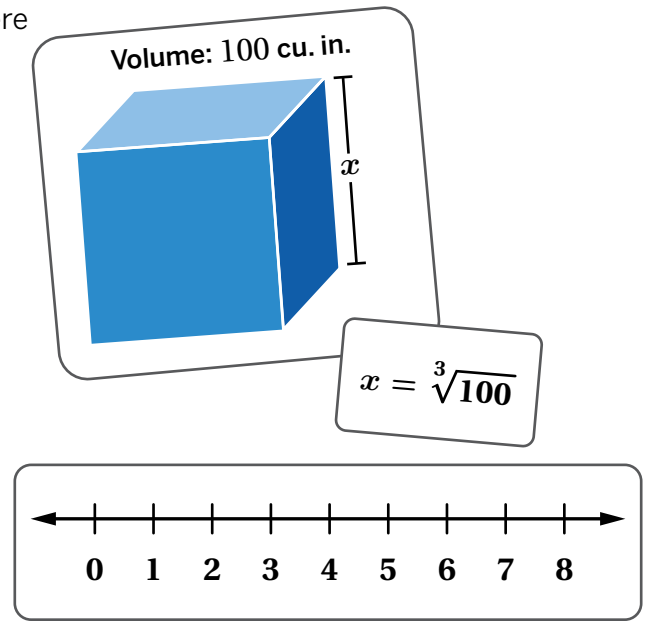
You're invited to explore more.

12. **a** If you double the edge length of a cube, what happens to the volume?

b If you double the volume of a cube, what happens to the edge length?

Synthesis

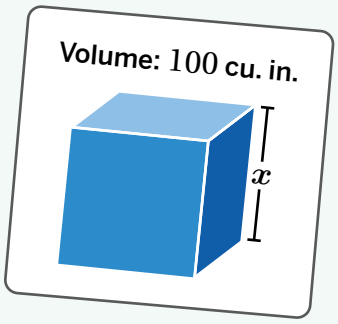
13. Explain a strategy for determining where to plot a cube root on the number line.



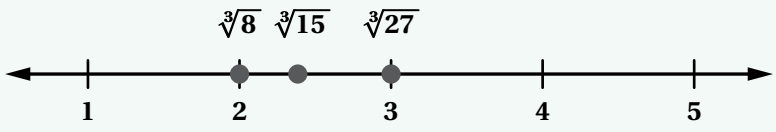
Lesson Practice ACC7.10.05

Lesson Summary

A **cube root** describes the edge length of a cube given its volume. For the cube shown with a volume of 100 cubic inches, the equation $x^3 = 100$ can help you find its edge length. Its exact solution would be represented as $x = \sqrt[3]{100}$. Cubing a number and taking the cube root are inverse operations.



We can approximate a cube root on a number line by observing the whole numbers around it. For example, you can determine that $\sqrt[3]{15}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$, and 15 is between 8 and 27.



8 and 27 are perfect cubes because they are both the cube of an integer: $2 \cdot 2 \cdot 2 = 2^3 = 8$ and $3 \cdot 3 \cdot 3 = 3^3 = 27$.

Name: _____ Date: _____ Period: _____

1. What is the volume of each cube based on its edge length?

	Edge Length	Volume
Cube A	4 cm	
Cube B	$\sqrt[3]{11}$ ft	
Cube C	s units	

2. What is the exact edge length of each cube based on its volume?

	Edge Length	Volume
Cube D		1,000 cu. cm
Cube E		23 cu. in.
Cube F		v cu. units

Problems 3–6: Write an equivalent expression that doesn't use a cube root symbol.

3. $\sqrt[3]{1}$

4. $\sqrt[3]{216}$

5. $\sqrt[3]{\frac{27}{125}}$

6. $\sqrt[3]{\frac{1}{64}}$

Problems 7–10: Write an exact solution to each equation.

7. $t^2 = 64$

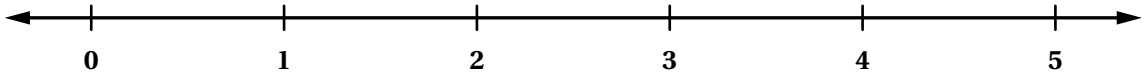
8. $f^3 = 181$

9. $m^2 = 8$

10. $c^3 = 343$

11. Plot each cube root on the number line.

$\sqrt[3]{5}$ $\sqrt[3]{11}$ $\sqrt[3]{80}$ $\sqrt[3]{120}$



Name: _____ Date: _____ Period: _____

12. Order the following from *least* to *greatest*. Use the number line if it helps with your thinking.

$\sqrt[3]{27}$	$\sqrt[3]{530}$	$\sqrt{48}$	$\sqrt{121}$	π	$\frac{19}{2}$

Least **Greatest**

FAST Practice

13. Compare the numbers by writing $<$, $>$, or $=$.

Record your answer in the space provided.

$\sqrt{50}$ ☐ 3.5

Spiral Review

14. What two consecutive whole numbers does $\sqrt{40}$ lie between?

(A.) 4 and 5 (B.) 5 and 6 (C.) 6 and 7 (D.) 7 and 8

15. Which set of ordered pairs represents a function?

(A.) $\{(-5, 10), (-20, 0), (-20, 25)\}$

(B.) $\{(-10, 0), (-10, 5), (-10, 25)\}$

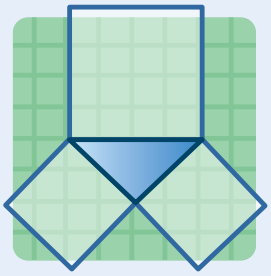
(C.) $\{(-25, 0), (5, -10), (-25, 30)\}$

(D.) $\{(-20, 0), (5, -10), (30, -25)\}$

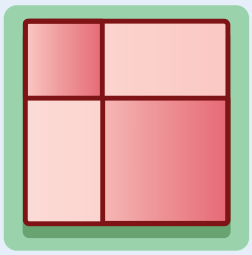
Sub-unit

2

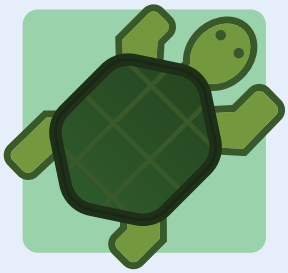
The Pythagorean Theorem



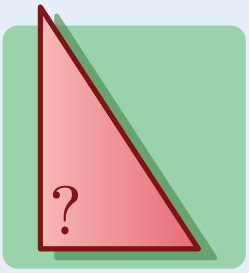
Lesson 6
The Pythagorean Theorem



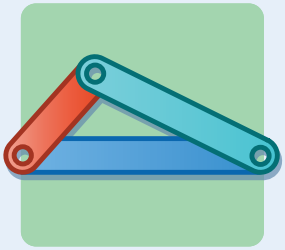
Lesson 7
Pictures to Prove It



Lesson 8
Triangle-Tracing Turtle

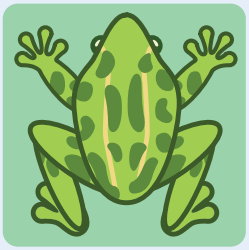


Lesson 9
Make It Right



Lesson 10
Can You Build It?

Lesson 11
Taco Truck



Lesson 12
Pond Hopper

Name: ..... Date: ..... Period: .....

MA.8.GR.1.1, MA.8.NSO.1.1, MTR.2.1, MTR.5.1

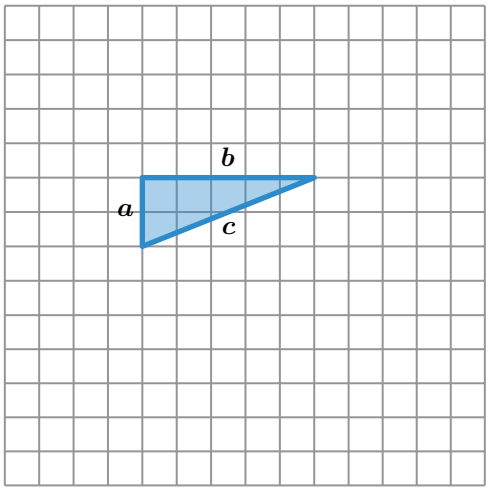
The Pythagorean Theorem

Let's explore the relationship between the squares of side lengths in triangles.

Warm-Up

1. Use any strategy to determine the value of c^2.

Explain your thinking.

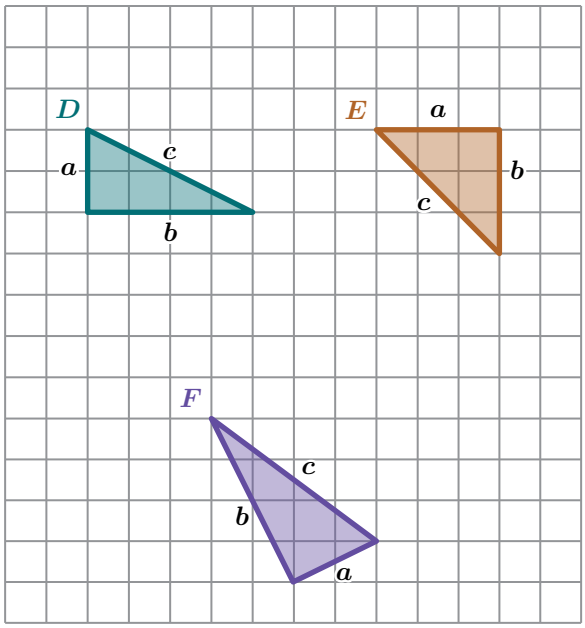


Activity 1

Name: ..... Date: ..... Period: .....

Squares of Side Lengths

2. Use these triangles to complete the table.



Triangle	a^2	b^2	c^2
D			
E			
F			

3. What do you notice? What do you wonder?

I notice . . .

I wonder . . .

Activity 2

Name: ..... Date: ..... Period: .....

True for Every Triangle?

4. You will use a set of cards for this problem.

a Work with a partner to create groups where $a^2 + b^2 = c^2$ and $a^2 + b^2 \neq c^2$. Complete the table with the card numbers.

$a^2 + b^2 = c^2$	$a^2 + b^2 \neq c^2$

b Revisit your noticings and wonderings from Activity 1. What do you notice and wonder now?

I notice . . .

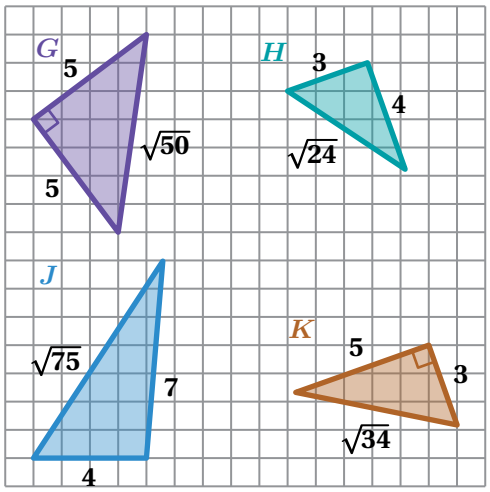
I wonder . . .

For each triangle, let a and b represent the two shorter sides and c represent the longest side.

5. Circle one triangle where the equation $a^2 + b^2 = c^2$ is true.

Triangle G Triangle H Triangle J Triangle K

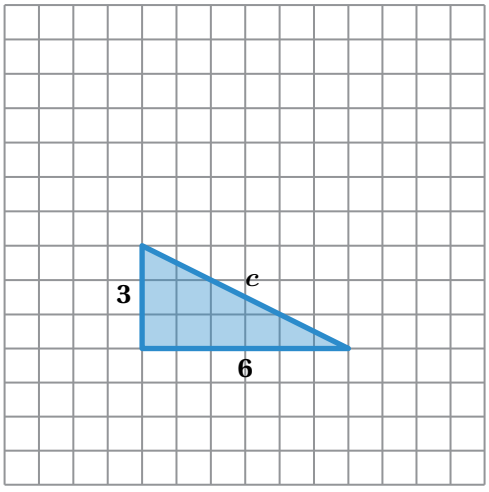
Show or explain your thinking.



6. Sai says that the value of c is $\sqrt{45}$.
Do you agree? Circle one.

Agree Disagree There's not enough information

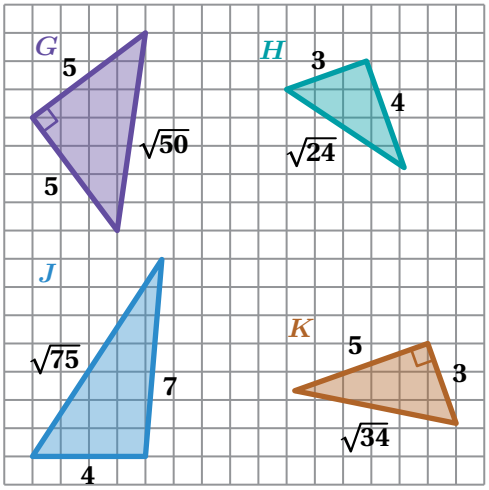
Show or explain your thinking.



Synthesis

7. The **Pythagorean theorem** says that for right triangles, $a^2 + b^2 = c^2$. The date of the first discovery is unknown, but the Babylonians used the Pythagorean theorem over 3,500 years ago (1,000 years before Pythagoras was born).

Explain the Pythagorean theorem in your own words. Use the triangles if they help with your thinking.

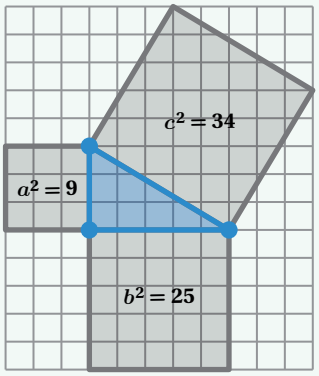


Lesson Practice ACC7.10.06

Lesson Summary

The **Pythagorean theorem** says that for right triangles, $a^2 + b^2 = c^2$, where a and b represent the lengths of the two shorter sides and c represents the length of the longest side.

The relationship $a^2 + b^2 = c^2$ is only true for right triangles.



For triangle H:

$3^2 + 4^2 = 25$

$c^2 = (\sqrt{24})^2 = 24$

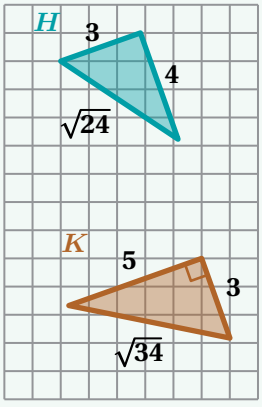
$25 \neq 24$ so $a^2 + b^2 = c^2$ is not true.

For triangle K:

$5^2 + 3^2 = 34$

$c^2 = (\sqrt{34})^2 = 34$

$34 = 34$ so $a^2 + b^2 = c^2$ is true.

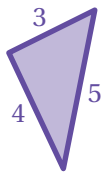


Lesson Practice
ACC7.10.06

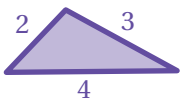
Name: ..... Date: ..... Period: .....

1. For which of the following triangles is $a^2 + b^2 = c^2$ true?

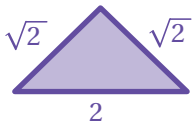
Triangle A



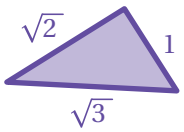
Triangle B



Triangle C



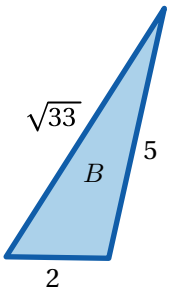
Triangle D



Show or explain your thinking.

2. Felipe claims the Pythagorean theorem is true for any triangle. Is his claim correct?

Explain your thinking using triangle B as an example.



3. Draw a triangle where $a^2 + b^2 = c^2$. What are its side lengths?

$a =$ ....., $b =$ ....., $c =$ .....

4. Draw a triangle where $a^2 + b^2 \neq c^2$. What are its side lengths?

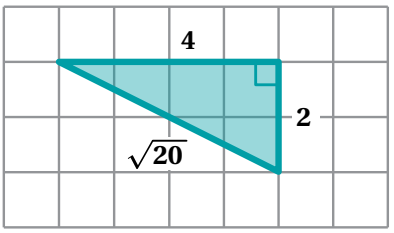
$a =$ ....., $b =$ ....., $c =$ .....

Lesson Practice
ACC7.10.06

Name: _____ Date: _____ Period: _____

FAST Practice

5. Select *all* the equations that represent the relationship between the side lengths.



- ☐ **A.** $(\sqrt{20})^2 + 4^2 = 2^2$
- ☐ **B.** $(\sqrt{20})^2 = 4^2 + 2^2$
- ☐ **C.** $2^2 + 4^2 = (\sqrt{20})^2$
- ☐ **D.** $2^2 + (\sqrt{20})^2 = 4^2$
- ☐ **E.** $(\sqrt{20})^2 = 2^2 + 4^2$

Spiral Review

6. Order the following expressions from *least* to *greatest*.

$25 \div 10$	$250000 \div 1000$	$2.5 \div 1000$	$0.025 \div 1$

Least **Greatest**

Problems 7–8: A teacher tells her students she is just over 1.5 billion seconds old.

7. Write her age in seconds using scientific notation.

What is a more reasonable unit of measurement for this situation?

8. Convert the teacher's age to a new unit. Show or explain your thinking.

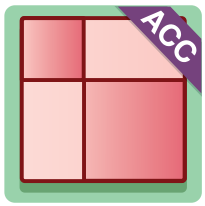
9. Kwasi borrows money from his older brother to buy a new game and begins to pay him back. The amount of money he owes his brother can be represented by the equation $y = -6x + 30$ where y is the amount of remaining money, in dollars, to be paid after x months. Based on the equation, what amount of money, in dollars, will Kwasi still owe after 3.5 months?

- (A.) 6
- (B.) 9
- (C.) 21
- (D.) 30

Name: ..... Date: ..... Period: .....

MA.8.GR.1.1, MA.8.NSO.1.1, MTR.2.1, MTR.4.1, MTR.5.1

Pictures to Prove It



Let's prove the Pythagorean theorem.

Warm-Up

1. Here are two figures.

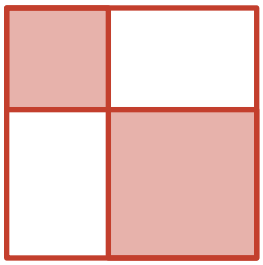


Figure A

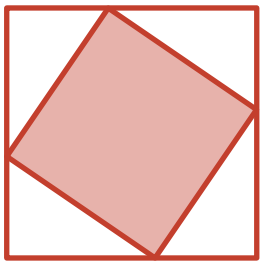


Figure B

Which one do you think has a larger *shaded* area? Circle one.

Figure A Figure B The areas are equal I'm not sure

Explain your thinking.

Activity 1

Name: Date: Period:

Pictures to Prove It

Note: For this lesson, assume figures that look like squares are squares.

2. Determine the total area of the shaded region in each figure. Mark the diagram if it helps with your thinking.

	Figure C	Figure D
Total Shaded Area (sq. units)		

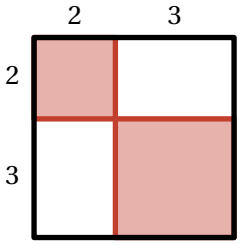


Figure C

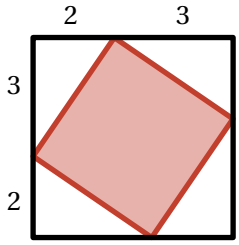


Figure D

3. Here are three different dimensions for a pair of new figures.

a Choose one of the options and circle your choice.

Option 1

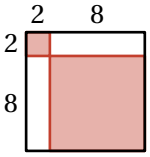


Figure E

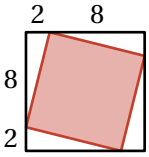


Figure F

Option 2

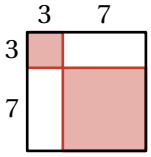


Figure E

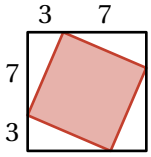


Figure F

Option 3

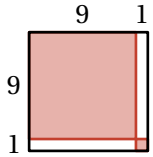


Figure E

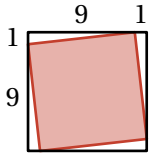


Figure F

b Determine the total area of the shaded region in each figure.

	Figure E	Figure F
Total Shaded Area (sq. units)		

4. Do you think the total shaded area will *always* be equal between figures like E and F, even if their outer dimensions change?

Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Thinking More Generally

Let's generalize using variables instead of numbers.

5. Determine the area of the shaded region in each figure using the variables a, b, or c.

	Figure G	Figure H
Total Shaded Area (sq. units)		

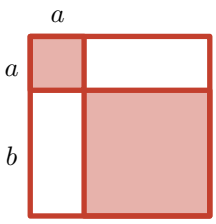


Figure G

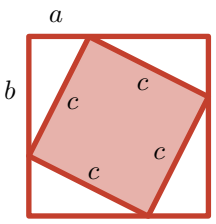


Figure H

6. The Pythagorean theorem states that for any *right* triangle, $a^2 + b^2 = c^2$.

How can we use these figures to prove that the Pythagorean theorem is true?

Mark the diagram if it helps to show your thinking.

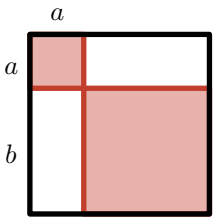


Figure G

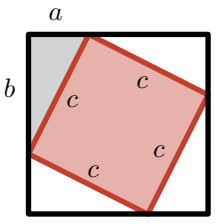


Figure H

Activity 3

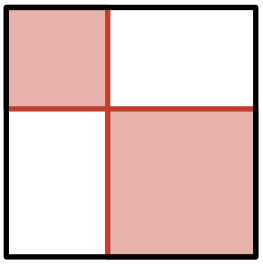
Name: ..... Date: ..... Period: .....

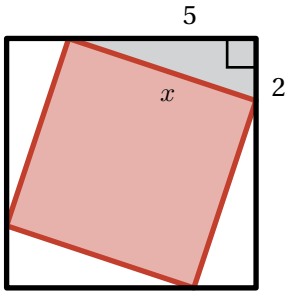
Let's Put It to Work

Let's put the Pythagorean theorem to work.

7. Calculate the value of x.

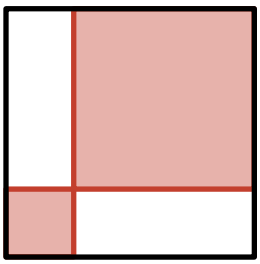
Draw on the diagram if it helps with your thinking.

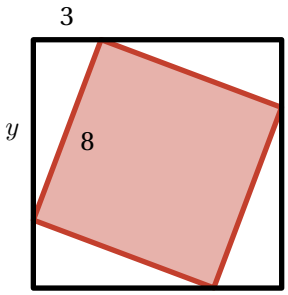




8. Calculate the value of y.

Draw on the diagram if it helps with your thinking.





You're invited to explore more.

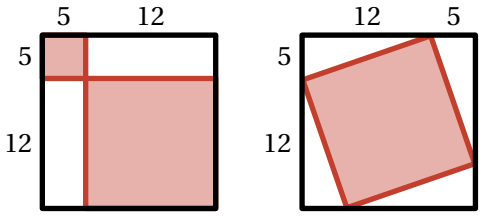
9. You will need the Activity 3 Sheet. Use scissors to cut along the dashed lines.

a Arrange the pieces in the smaller squares to fit in the large square.

b Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.

Synthesis

10. Explain how the equation $5^2 + 12^2 = 13^2$ is related to the figures on the right and to the Pythagorean theorem.



Lesson Practice ACC7.10.07

Lesson Summary

The Pythagorean theorem states that for any right triangle, $a^2 + b^2 = c^2$. There are many proofs for the Pythagorean theorem.

For example, you can draw squares on the sides of a right triangle, as shown in Figure 1.

In Figure 2 and Figure 3, each total area is equal to $(a + b)^2$.

Since the area of the right triangle is equal to $\frac{1}{2}ab$, the unshaded areas of Figures 2 and 3 are equal and each sum to $2ab$.

Since the total areas and the unshaded areas of Figures 2 and 3 are equal, then their shaded areas must be equal. This shows that $a^2 + b^2 = c^2$.

Figure 1

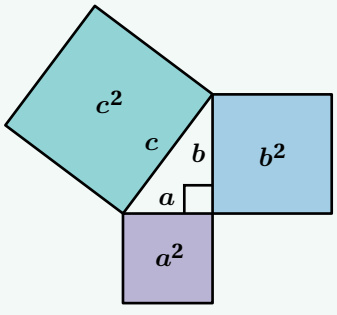


Figure 2

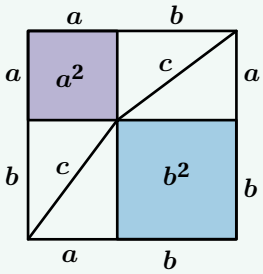
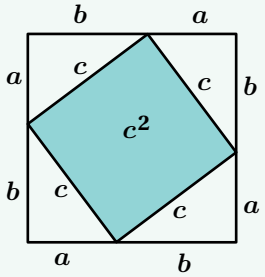


Figure 3

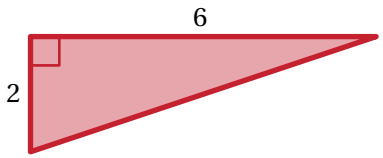


Lesson Practice
ACC7.10.07

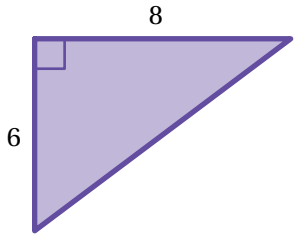
Name: ..... Date: ..... Period: .....

Problems 1–2: Determine the length of each unlabeled side.

1.

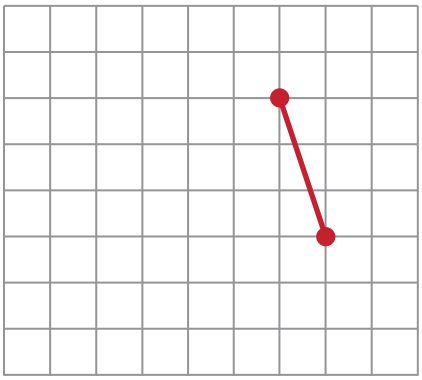


2.

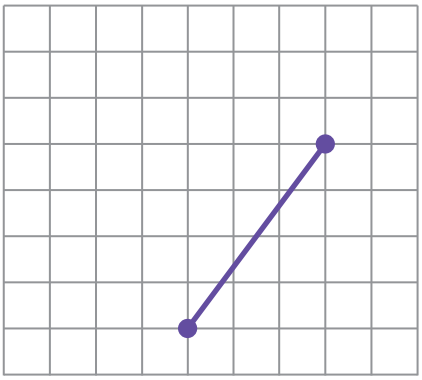


Problems 3–4: Determine the exact length of each segment. Each grid line represents one unit.

3.



4.



5. Determine a set of values for a, b, and c that meet the following criteria:

- Make the shaded areas of figure G and figure H equal.
- Make $a^2 + b^2 = c^2$ true.

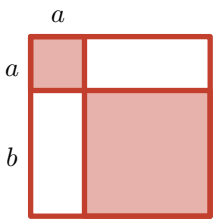


Figure G

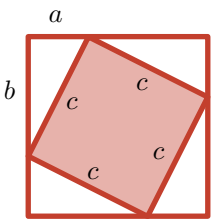


Figure H

Lesson Practice
ACC7.10.07

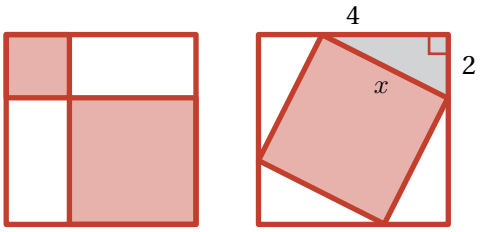
Name: _____ Date: _____ Period: _____

FAST Practice

6. Determine the exact value of x.

Record your answer in the space provided.

☐ units



Spiral Review

Problems 7–10: For each square root, write which two consecutive whole numbers the value is between. For example, $\sqrt{2}$ is between 1 and 2.

7. $\sqrt{10}$

8. $\sqrt{54}$

9. $\sqrt{18}$

10. $\sqrt{99}$

Problems 11–13: Rewrite each expression as a single power of 10. For example, $(10^5)^0$ can be rewritten as 10^0.

11. $10^5 \cdot 10^0$

12. $\frac{10^9}{10^0}$

13. Which expression is equivalent to $3^{-4} \cdot 3^9$?

A. $\frac{3^{-4}}{3^{-1}}$ B. $(3^5)^{-1}$ C. $\frac{3^4}{3^{-1}}$ D. $(3^{-1})^5$

Name: ..... Date: ..... Period: .....

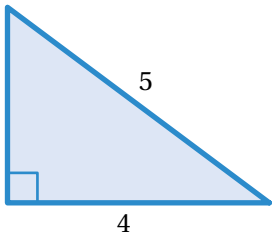
MA.8.GR.1.1, MA.8.NSO.1.1, MTR.4.1, MTR.6.1, MTR.7.1

Triangle-Tracing Turtle

Let's calculate unknown side lengths in right triangles.

Warm-Up

1. Select *all* the equations that could help you calculate the unknown side length of this triangle.



- ☐ **A.** $a^2 + 4^2 = 5^2$
- ☐ **B.** $a = \sqrt{4^2 + 5^2}$
- ☐ **C.** $b^2 = 5^2 + 4^2$
- ☐ **D.** $4^2 + b^2 = 5^2$
- ☐ **E.** $b = \sqrt{5^2 - 4^2}$

Activity 1

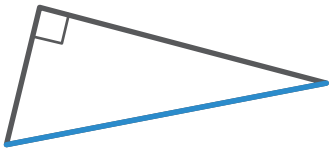
Name: ..... Date: ..... Period: .....

Hypotenuse

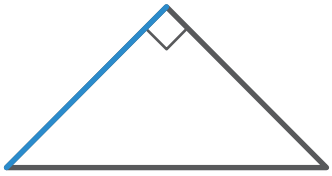
2. The **hypotenuse** is the side of a right triangle that is opposite the right angle. The **legs** of a right triangle are the sides that make the right angle.

Select *all* the triangles where a hypotenuse is highlighted.

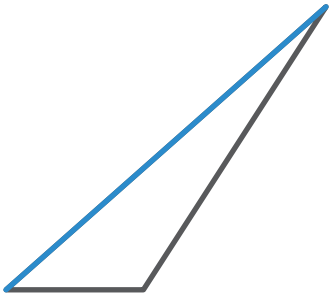
☐ **A.**



☐ **B.**



☐ **C.**



☐ **D.**

3. Melissa incorrectly thinks that triangles A, C, and D from the previous problem have a highlighted hypotenuse.

a What do you think Melissa did well?

b What mistake might she have made?

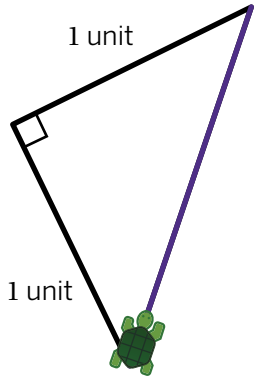
Activity 2

Name: Date: Period:

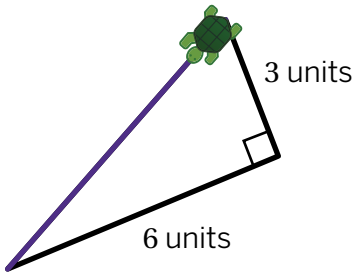
Turtle Tracing

Tiam the turtle is walking on one side of a triangle.

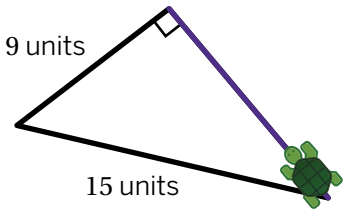
4. Exactly how far does Tiam need to travel?



5. Exactly how far does Tiam need to travel?



6. Exactly how far does Tiam need to travel?



Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

7. You will use a separate sheet of paper to create your own triangle challenge.

a. **Make It!** On the paper, sketch a right triangle. Label two of the sides with their approximate lengths.

b. **Solve It!** On this page, write the two side lengths you labeled on your triangle. Then calculate the length of the third side.

My Sides	My Lengths (units)
Leg 1	
Leg 2	
Hypotenuse	

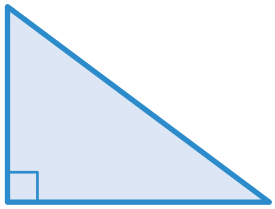
c. **Swap It!** Swap your challenge with one or more partners. Calculate the unknown side length for each partner's triangle.

	Side	Length (units)
Partner 1	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 2	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 3	Leg 1	
	Leg 2	
	Hypotenuse	

Synthesis

8. If you know two side lengths of a right triangle, how can you calculate the third side length?

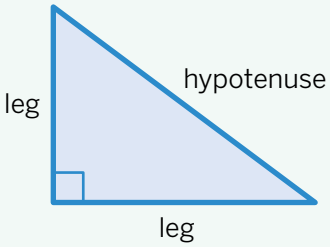
Use the image if it helps to show your thinking.



Lesson Practice ACC7.10.08

Lesson Summary

The **hypotenuse** is the side of a right triangle that is opposite the right angle, and is the longest side. Only right triangles have a *hypotenuse*. The **legs** of a right triangle are the sides that make the right angle.



The Pythagorean theorem says that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. This can be represented by the equation $a^2 + b^2 = c^2$, where a and b represent the lengths of the legs and c represents the length of the hypotenuse.

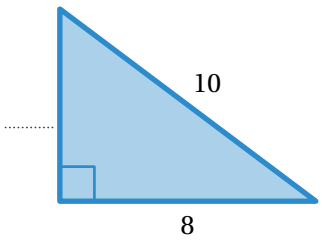
When any two side lengths of a right triangle are known, the Pythagorean theorem can be used to calculate the length of the third side, whether it is the hypotenuse or a leg. You can substitute the lengths you know into the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ or $a^2 + b^2 = c^2$, and then solve for the unknown value.

Lesson Practice
ACC7.10.08

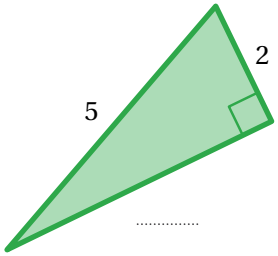
Name: ..... Date: ..... Period: .....

Problems 1–3: Calculate the exact value of the unknown side length in each right triangle.

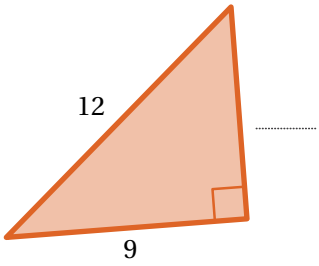
1.



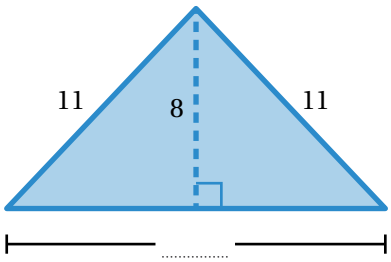
2.



3.



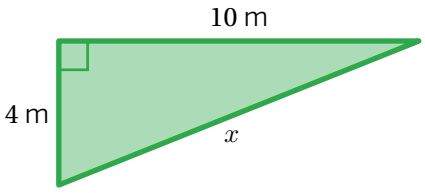
4. Calculate the value of the unknown side length to the nearest tenth.



5. Ivory is walking home from her friend's house. She starts by walking 2 kilometers west and then 3 kilometers south. How many kilometers, to the nearest tenth, does she live from her friend's house?

FAST Practice

6. This diagram shows a right triangle.



Which measurement is closest to the value of x in meters?

(A.) 3.8 meters

(B.) 8.7 meters

(C.) 10.8 meters

(D.) 11.2 meters

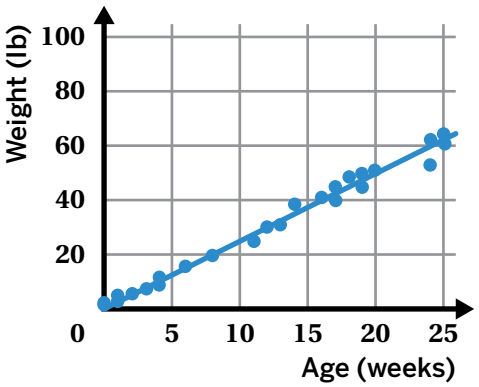
Name: ..... Date: ..... Period: .....

Spiral Review

7. In 2015, there were roughly $1 \cdot 10^6$ high school football players and $2 \cdot 10^3$ professional football players in the United States. About how many times more high school football players were there?

Show or explain your thinking.

Problems 8–9: The scatter plot shows some ages and weights for a large dog breed. The scatter plot shows the model of $y = 2.45x + 1.22$.



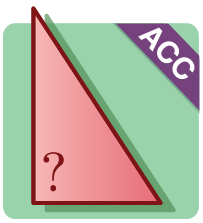
8. What does the slope mean in this situation?

9. Based on this model, how heavy would you expect a newborn puppy to be?

Name: ..... Date: ..... Period: .....

MA.8.GR.1.3, MA.8.NSO.1.1, MTR.2.1, MTR.5.1, MTR.6.1

Make It Right

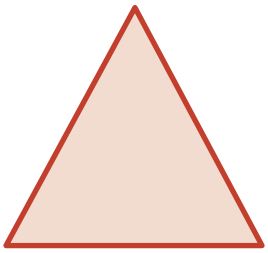


Let's determine if a triangle is a right triangle.

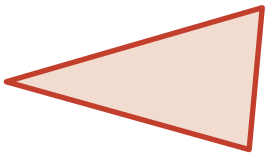
Warm-Up

1. Which one doesn't belong?

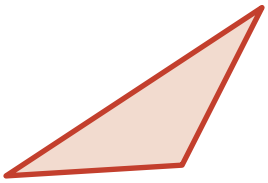
A.



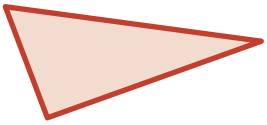
B.



C.



D.



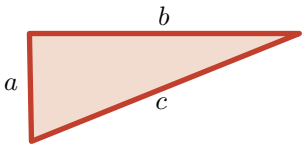
Explain your thinking.

Activity 1

Name: Date: Period:

Is the Converse True?

2. Mathematicians sometimes think about a statement's *converse*, which is a statement in the opposite direction.



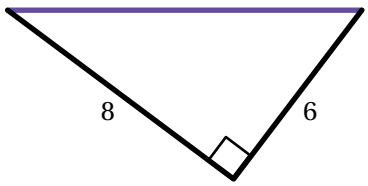
The converse of the Pythagorean theorem says: *If a triangle has side lengths such that* $a^2 + b^2 = c^2$*, it is a right triangle.*

Do you think this statement is always, sometimes, or never true? Circle one.

Always Sometimes Never

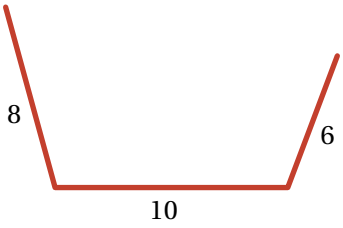
Explain your thinking.

3. Let's explore the converse of the Pythagorean theorem by focusing on a specific example. Here is a right triangle with legs that are 6 and 8 units long.



What is the length of its hypotenuse?

4. You will use the cutouts from the Activity 1 Card to create several triangles.



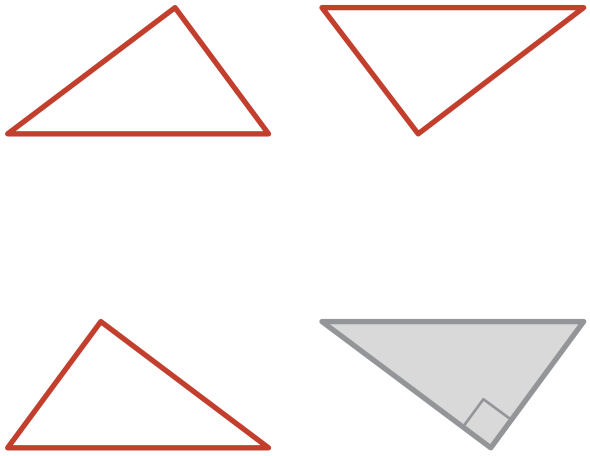
a Experiment with making two other triangles with sides lengths measuring 6, 8, and 10 units.

b **Discuss:** What do you notice? What do you wonder?

Name: Date: Period:

Is the Converse True? (continued)

5. Yosef says that every triangle with sides lengths measuring 6, 8, and 10 units *must* be a right triangle, because the triangles with these lengths are all *congruent*.



Do you agree? Circle one. Use tracing paper to help with your thinking.

Yes No I'm not sure

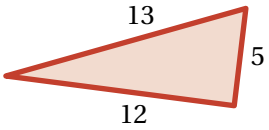
Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Make It Right

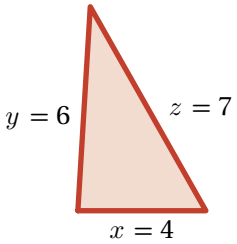
6. What type of triangle is this? Circle one.



A right triangle Not a right triangle

Explain your thinking.

7. Change *one* of the values to make this triangle a right triangle.



There are many different solutions. Try to find *at least* four.

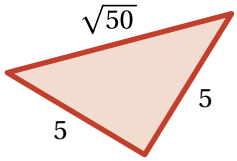
x	y	z

Name: ..... Date: ..... Period: .....

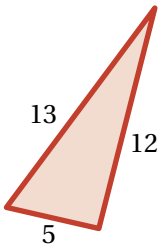
Make It Right (continued)

8. Circle whether each triangle is a right triangle.

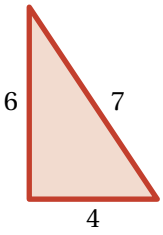
a Right Triangle Not a Right Triangle



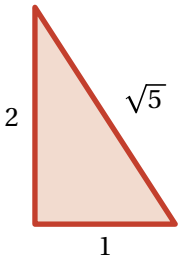
b Right Triangle Not a Right Triangle



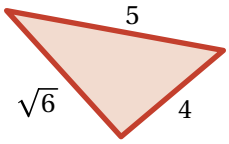
c Right Triangle Not a Right Triangle



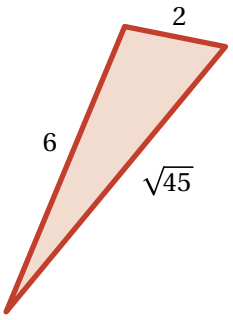
d Right Triangle Not a Right Triangle



e Right Triangle Not a Right Triangle



f Right Triangle Not a Right Triangle

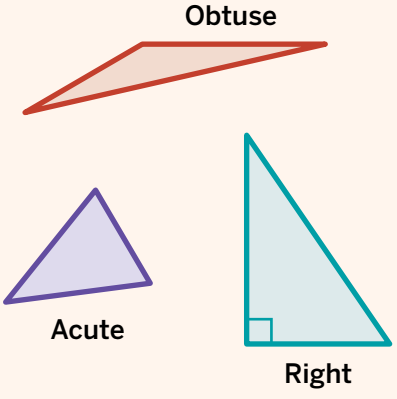


You're invited to explore more.

9. Here is an obtuse triangle, an acute triangle, and a right triangle. *All* triangles are one of these three types.

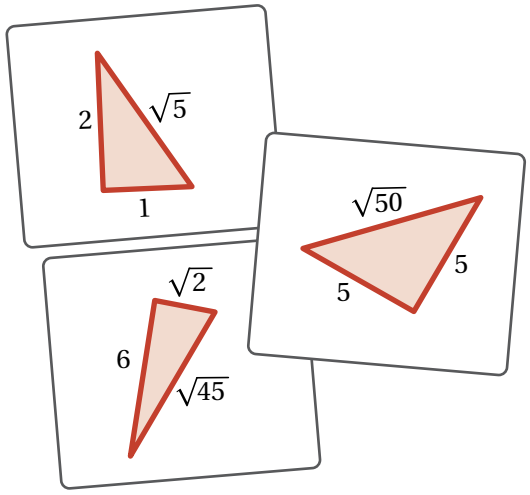
Decide whether triangles X, Y, and Z are acute, right, or obtuse based on their side lengths.

Triangle X, side lengths: 15, 20, 8
Triangle Y, side lengths: 8, 15, 13
Triangle Z, side lengths: 17, 8, 15



Synthesis

10. How can you tell from just the side lengths if a triangle is a right triangle?

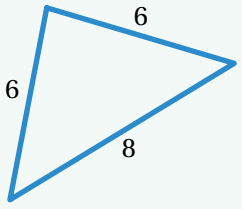


Lesson Practice ACC7.10.09

Lesson Summary

If a triangle has side lengths a, b, and c, where c is the longest side and $a^2 + b^2 = c^2$, then the converse of the Pythagorean theorem says that you must have a right triangle. We can use this to determine whether any triangle is a right triangle. If the sides of a triangle *do not* make the equation $a^2 + b^2 = c^2$ true, then you know it is *not* a right triangle.

In the triangle shown, let $a = 6$, $b = 6$, and $c = 8$. You can use substitution to determine whether the triangle is a right triangle.



$$a^2 + b^2 = 36 + 36$$
$$= 72$$

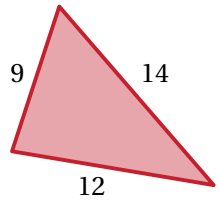
Because $c^2 = 8^2$, or 64, the triangle cannot be a right triangle because $a^2 + b^2 \neq c^2$.

Lesson Practice
ACC7.10.09

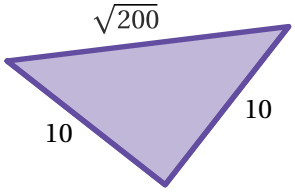
Name: ..... Date: ..... Period: .....

Problems 1–3: Determine whether each triangle is a right triangle.

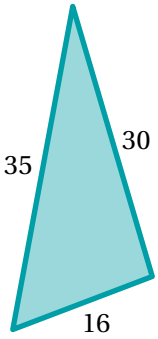
1.



2.

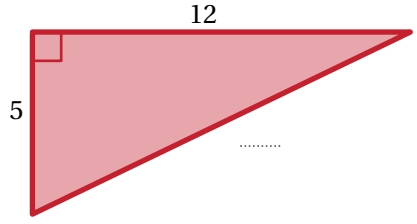


3.

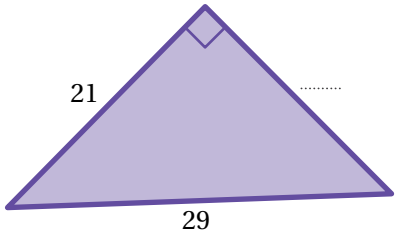


Problems 4–5: Calculate the value of the unknown side length so that the triangle is a right triangle.

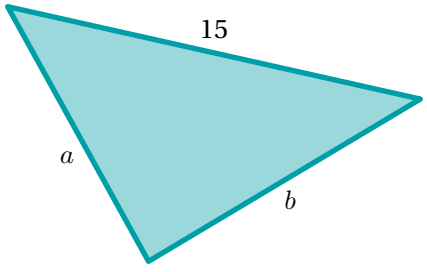
4.



5.



6. The longest side of this triangle has a length of 15 centimeters. What could the lengths of the other two sides be so that this is a right triangle?



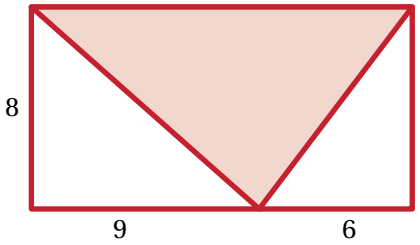
Name: _____ Date: _____ Period: _____

7. The side lengths of a triangle are 5, 7, and 9 units. Does the length of the longest side need to *increase*, *decrease*, or *stay the same* to make the triangle a right triangle? Circle one.

Increase Decrease Stay the same

Explain your thinking.

8. Here is a 15-by-8 rectangle divided into triangles. Is the shaded triangle a right triangle? Circle one.



Yes No

Explain your thinking.

FAST Practice

9. The lengths of two sides of a triangle are 5 and 6. Which side length would make the triangle a right triangle?

(A.) $\sqrt{8}$ (B.) $\sqrt{11}$ (C.) 9 (D.) 10

Spiral Review

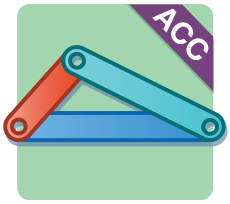
10. Determine which two whole numbers $\sqrt{53}$ is between.

11. Write an expression equivalent to $\sqrt[3]{\frac{64}{125}}$ that does not use a cube root symbol.

Name: ..... Date: ..... Period: .....

MA.8.GR.1.3, MA.8.NSO.1.1, MTR.3.1, MTR.5.1

Can You Build It?

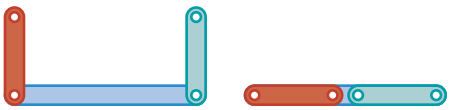


Let's explore what combinations of three line segments form a triangle.

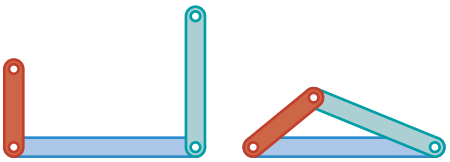
Warm-Up

1. Here are two sets of line segments that were used to try to form a triangle.

Set 1



Set 2



What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

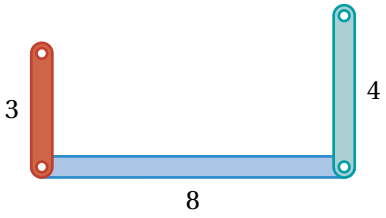
Name: Date: Period:

Length of the Third Side

You will use a set of line segments to complete this activity.

2. Will these three line segments form a triangle? Circle one.

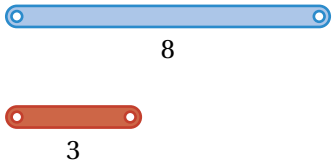
Yes No I'm not sure



Explain your thinking.

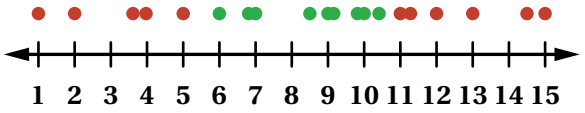
3. Set 1 and Set 2 from the Warm-Up each have one line segment that is 8 units long and one that is 3 units long. Set 1 does not form a triangle. Set 2 does.

Try other lengths for the third segment. Try to find several that do and do not form a triangle.



Forms a Triangle	Does Not Form a Triangle
• 10 units	• 2 units

4. Here is a graph of other lengths that students tried. The lengths that form a triangle are represented by green dots.



Describe what you notice about those lengths.

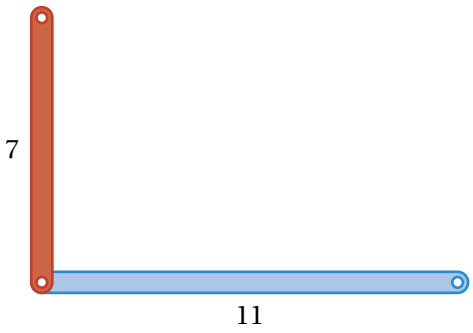
Activity 2

Name: ..... Date: ..... Period: .....

Not Too Long, Not Too Short

You will use a set of line segments to complete this activity.

5. Diamond is convinced that a third segment that is 19 units long will form a triangle.



Mohamed thinks that 19 units is too long.

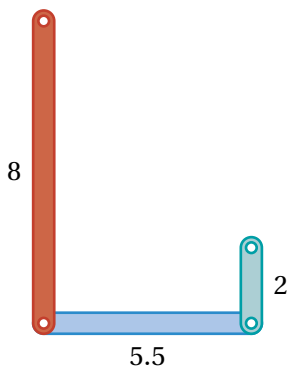
Whose claim is correct? Circle one.

Diamond Mohamed

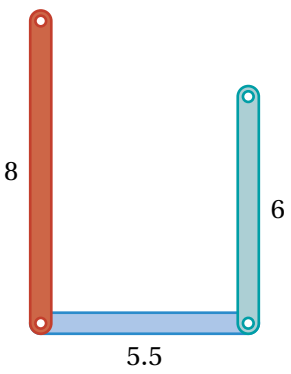
Explain your thinking.

6. Which of these lengths will form a triangle with side lengths 5.5 and 8 units? Circle one.

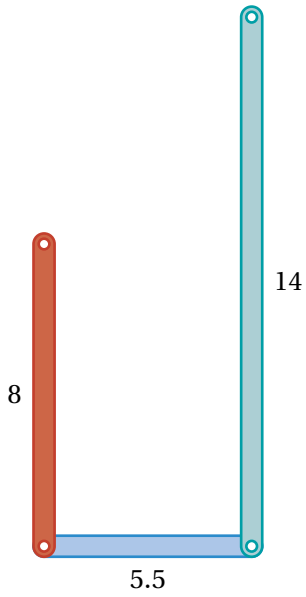
2 units



6 units



14 units



Name: ..... Date: ..... Period: .....

Not Too Long, Not Too Short (continued)

7. You will use a challenge card to create your own triangle challenge.

- Choose two line segments of any length.
- Determine three multiple choice options. One option should form a triangle with your other line segments, and two options should not.
- Swap your triangle challenge with one or more partners.
- Determine the lengths that will form a triangle for each of your partners' challenges.
- Complete as many challenges as you have time for.

	Given Length	Given Length	Correct Third Length
..... 's Challenge			
..... 's Challenge			
..... 's Challenge			
..... 's Challenge			
..... 's Challenge			

Activity 3

Name: Date: Period:

Does It Add Up?

You will use a set of line segments to complete this activity.

8. A triangle has a perimeter of 24 units.

Perimeter = 24 units

What are three possible lengths for the sides of this triangle?

Length 1	Length 2	Length 3

9. Abena made her first side 12 units long.

12

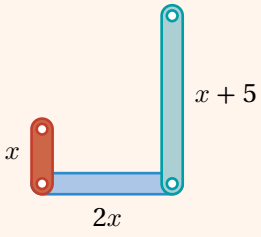
Perimeter = 24 units

Will she be able to form a triangle with a perimeter of 24 units? Explain your thinking.

You're invited to explore more.

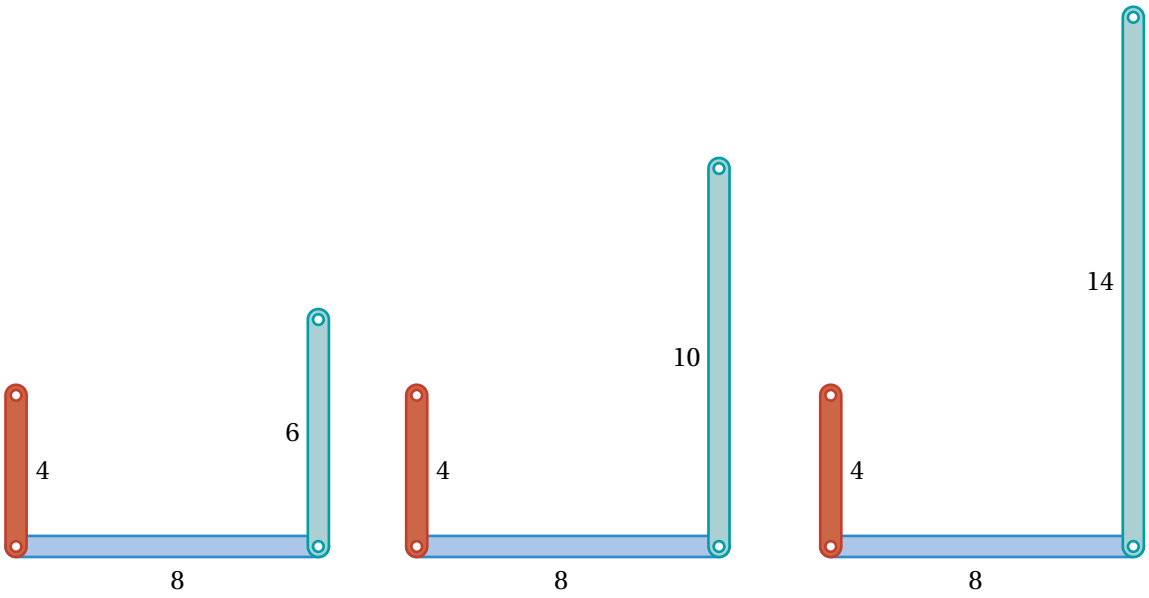
10. A triangle has sides that are x, $2x$, and $x + 5$ units.

What are some values of x that would form a triangle?



Synthesis

11. Explain how you can determine whether three line segments will form a triangle.



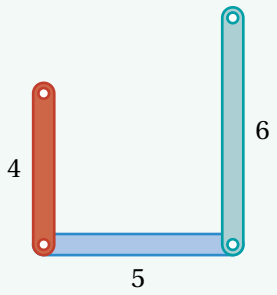
Lesson Practice ACC7.10.10

Lesson Summary

Three line segments do not always make a triangle.

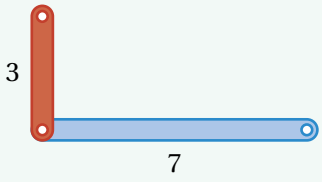
The **triangle inequality theorem** states that in order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

For example, $4 + 5 > 6$, so these three line segments would make a triangle.



You can use this relationship to determine what possible lengths would create a triangle.

For example, if two side lengths of a triangle are 3 units and 7 units, then the third side must be greater than 4 and less than 10 units.



Name: _____ Date: _____ Period: _____

1. Select *all* the sets of line segments that will form a triangle.

- ☐ **A.** 3, 4, and 8 units
- ☐ **B.** 7, 6, and 12 units
- ☐ **C.** 5, 13, and 11 units
- ☐ **D.** 12, 6, and 4 units
- ☐ **E.** 4, 6, and 10 units

2. Choose one of the other sets of segments from Problem 1 and explain why it will *not* form a triangle.

3. One side of a triangle is 5.5 inches long and another side is 10.5 inches long. Select *all* possible side lengths for the third side.

- ☐ **A.** 3 inches
- ☐ **B.** 5 inches
- ☐ **C.** 7 inches
- ☐ **D.** 10 inches
- ☐ **E.** 12 inches
- ☐ **F.** 20 inches

Problems 4–5: A triangle has one side that is 4 centimeters long and another side that is 9 centimeters long. The third side is a whole number of centimeters.

4. What is the shortest possible length of the third side? _____ centimeters

5. What is the longest possible length of the third side? _____ centimeters

6. The direct distance between Airport A and Airport B is 120 miles. The direct distance between Airport B and Airport C is 240 miles. Which of these could be the direct distance between Airport A and Airport C?

- (A.) 370 miles
- (B.) 300 miles
- (C.) 120 miles
- (D.) 110 miles

7. Create a set of four segment lengths so that:

- Each length is different.
- Each length is a whole number (in centimeters).
- No matter which of the three lengths you choose, you will *always* be able to form a triangle.

Explain your thinking.

Segment	Length (cm)
A	
B	
C	
D	

Lesson Practice
ACC7.10.10

Name: _____ Date: _____ Period: _____

FAST Practice

8. You have one straw that is 3 inches long and one that is 8 inches long. Which of these could be the length of a third straw so that the three straws form a triangle?

(A.) 12 inches

(B.) 6 inches

(C.) 5 inches

(D.) 3 inches

Spiral Review

9. Write a number and power of 10 to show the value of $(6.2 \cdot 10^5) \cdot (3.4 \cdot 10^2)$ in scientific notation.

_____ • _____

10. Select *all* the expressions that are equivalent to 3^8.

☐ A. $(3^2)^3 \cdot 3^2$

☐ B. $\frac{1}{3^4 \cdot 3^4}$

☐ C. $\frac{(3^4)^3 \cdot 3^0 \cdot 3^3}{3^2 \cdot 3^5}$

☐ D. $(3^2 \cdot 3^2)^4$

☐ E. $\frac{3^3 \cdot 3^3 \cdot 3^4}{3^2}$

Name: ..... Date: ..... Period: .....

MA.8.GR.1.1, MA.8.AR.3.2, MA.8.NSO.1.1, MTR.4.1, MTR.6.1, MTR.7.1

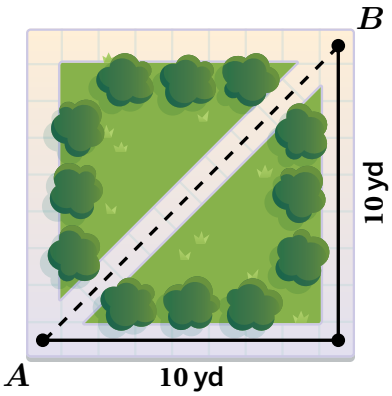
Taco Truck

Let's solve problems with the Pythagorean theorem.

Warm-Up

1. Alma is going to walk through the garden from point A to point B.

What distance will she walk?



2. If Alma walks at a speed of 1.3 yards per second, how long will it take for her to walk across the garden?

Activity 1

Name: ..... Date: ..... Period: .....

Taco Truck

3. Imagine you're on the beach, and you're getting hungry.

Sketch the route you would take to the taco truck.

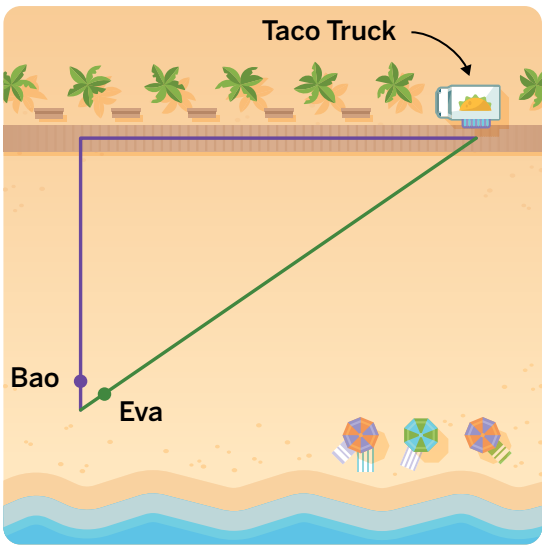
Explain your thinking.

4. Bao and Eva choose different routes to get to the taco truck.

a Who do you think will reach the taco truck first?

Bao Eva They'll arrive at the same time

b What information would help you know for sure?



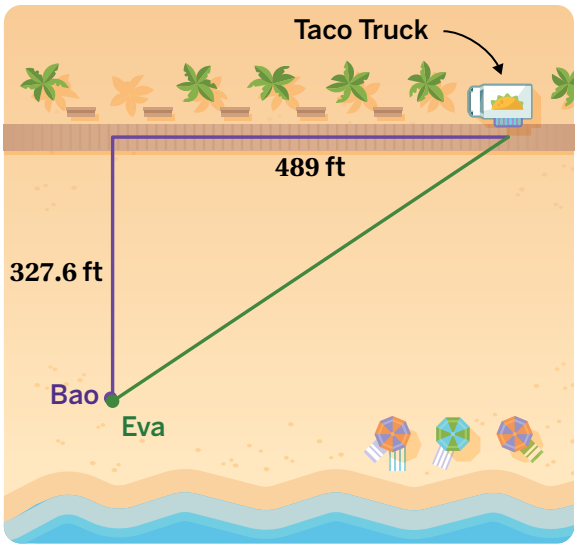
Name: ..... Date: ..... Period: .....

Let's Eat

5. Let's look at some additional information about Bao's and Eva's routes:

- Some distances are shown in the image.
- The speed on the boardwalk is 5 feet per second.
- The speed on the sand is 3 feet per second.

Use the information to calculate how long it will take for each person to get to the taco truck. Show your thinking.



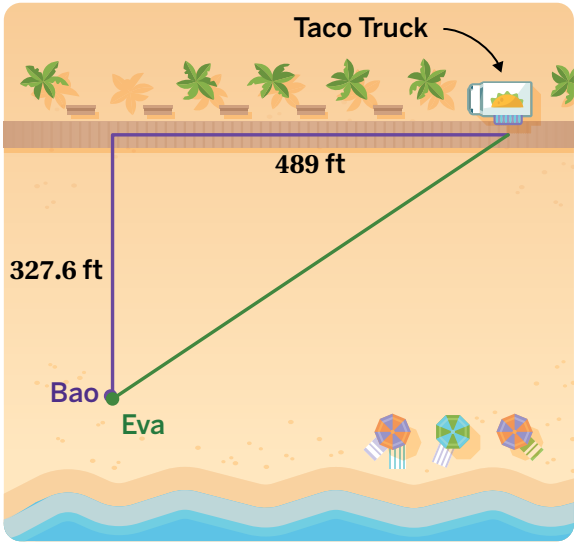
You're invited to explore more.

6. Determine the speed on the boardwalk that would make Eva and Bao arrive at the same time.

Speed on Sand	Speed on Boardwalk
3 feet per second	

Synthesis

7. What are some important things to remember when using the Pythagorean theorem to solve problems?



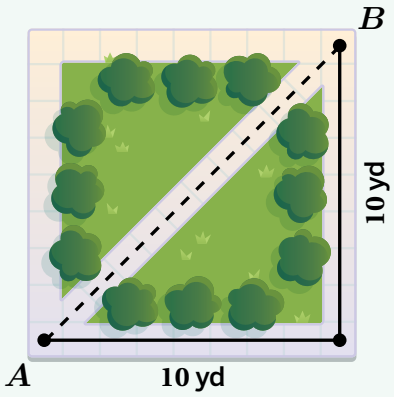
Lesson Practice ACC7.10.11

Lesson Summary

The Pythagorean theorem can be used to solve problems that can be modeled with right triangles. The sides of a triangle might represent units such as the length of an object or the distance between two objects.

To apply the Pythagorean theorem, the lengths of two sides must be known so the length of the third side can be determined.

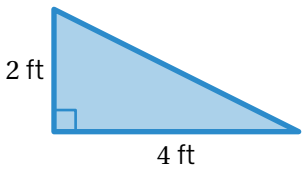
For example, you can use the Pythagorean theorem to calculate the distance to walk through the park from point A to point B.



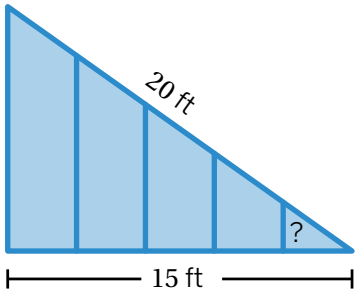
Lesson Practice
ACC7.10.11

Name: ..... Date: ..... Period: .....

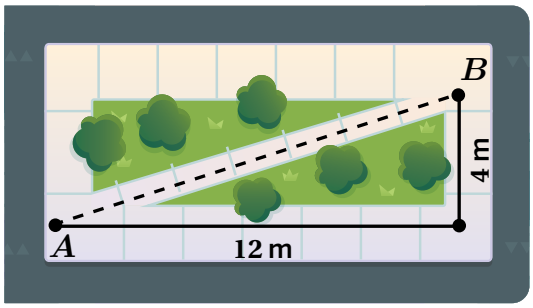
1. Here is a skateboarding ramp. The length of the base is 4 feet, and the height of the ramp is 2 feet. What is the approximate length of the ramp in feet?



2. A 20-foot roof needs 5 support beams placed equidistant along the floor. Each support is a different length. Determine the exact length of the shortest support beam.



Problems 3–4: Sol is thinking about cutting diagonally through the park to get from point A to point B.

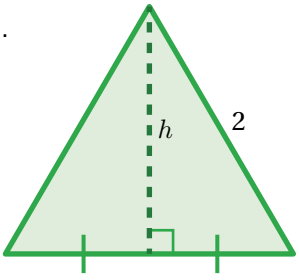


3. Determine the distance Sol would walk if he cut through the park.

4. If Sol walks an average of 1.42 meters per second, how much time will he save by cutting through the park instead of walking around it? Round your answer to the nearest second.

FAST Practice

5. Here is an equilateral triangle. The length of each side is 2 units. A height, h, is drawn. Determine the exact height.



Record your answer in the space provided.

☐ units

Name: ..... Date: ..... Period: .....

Spiral Review

6. Select *all* the sets of side lengths that form a right triangle.

- ☐ **A.** $7, 8, 13$
- ☐ **B.** $4, 10, \sqrt{84}$
- ☐ **C.** $\sqrt{8}, 11, \sqrt{129}$
- ☐ **D.** $\sqrt{1}, 2, \sqrt{3}$
- ☐ **E.** $\sqrt{2}, 3, \sqrt{13}$

Problems 7–9: Circle the number that is greater.

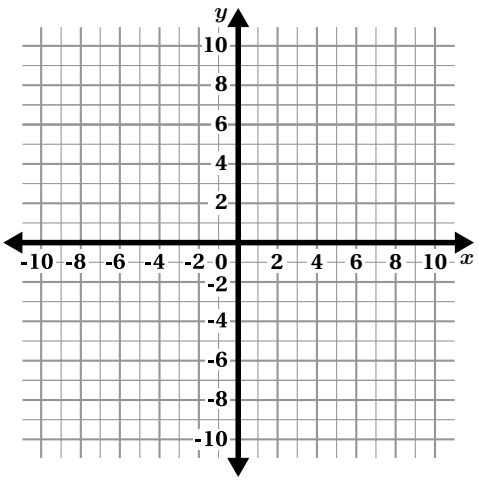
7. $12 \cdot 10^9$ or $4 \cdot 10^9$

8. $1.5 \cdot 10^{-12}$ or $3 \cdot 10^{-12}$

9. $20 \cdot 10^4$ or $6 \cdot 10^5$

10. A line contains the point $(3, 5)$. If the line has a negative slope, which of these points could also be on the line?

Use the graph if it helps with your thinking.



(A) $(6, 5)$ (B) $(4, 7)$

(C) $(5, 4)$ (D) $(2, 0)$

Name: ..... Date: ..... Period: .....

MA.8.GR.1.2, MA.8.NSO.1.1, MTR.5.1, MTR.6.1

Pond Hopper

Let's calculate distances between points on the coordinate plane.

Warm-Up

1. Order the pairs of points from closest together to farthest apart.

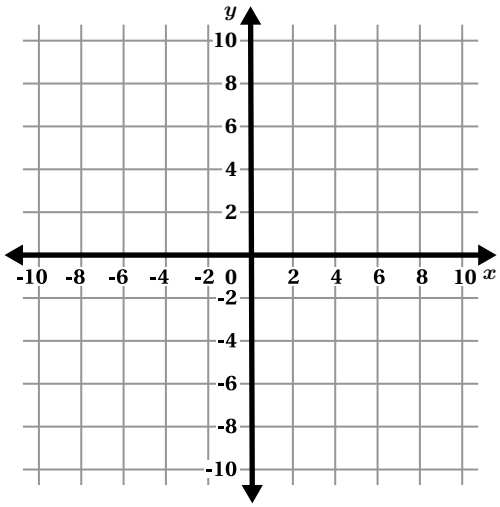
Use the graph if it helps with your thinking.

Pair A: $(-8, 1)$ and $(-8, 8)$

Pair B: $(7, 0)$ and $(7, -9)$

Pair C: $(2, 3)$ and $(2, 9)$

Pair D: $(-3, 6)$ and $(5, 6)$



Closest Together **Farthest Apart**

Activity 1

Name: Date: Period:

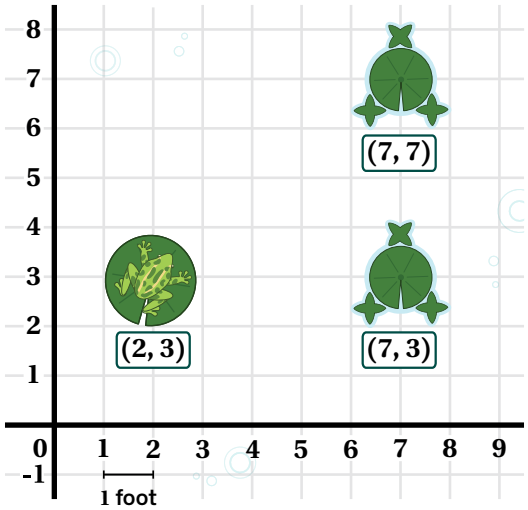
Pond Hopper

2. Let's help the frog hop to all the lily pads.

Pick a lily pad. Write its coordinates in the table.

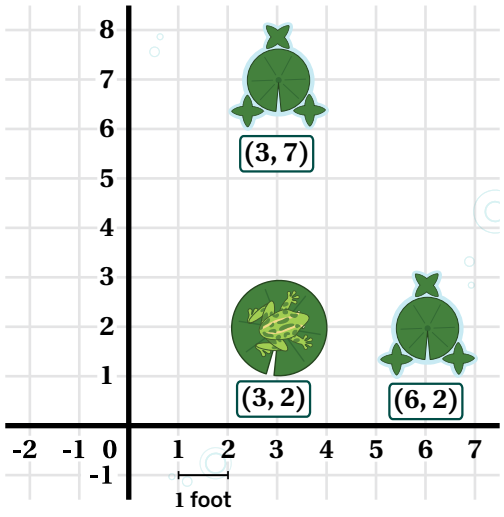
Then write the distance between the frog and the lily pad.

Repeat until you have hopped to all the lily pads.



	Lily Pad Coordinates	Distance (ft)
Hop 1		
Hop 2		
Hop 3		
Hop 4		

3. Help the frog hop to all the lily pads in *as few hops as possible*.



	Lily Pad Coordinates	Distance (ft)
Hop 1		
Hop 2		
Hop 3		
Hop 4		

Name: ..... Date: ..... Period: .....

Pond Hopper (continued)

4. Which expression represents the distance between the frog and the lily pad?

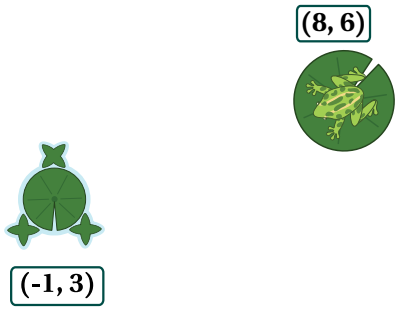
A. $\sqrt{7^2 - 3^2}$

B. $\sqrt{7^2 + 3^2}$

C. $\sqrt{9^2 - 3^2}$

D. $\sqrt{9^2 + 3^2}$

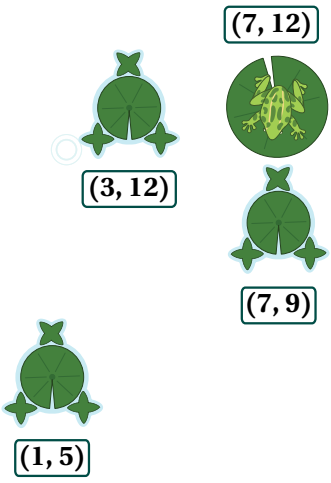
Show or explain your thinking.



All measurements in feet

5. Help the frog hop to all the lily pads in *as few hops as possible*.

	Lily Pad Coordinates	Distance (ft)
Hop 1		
Hop 2		
Hop 3		
Hop 4		
Hop 5		



All measurements in feet

Activity 2

Name: ..... Date: ..... Period: .....

Challenge Creator

6. You will use the Activity 2 Sheet to complete this activity.

a **Make It!** On the Activity 2 Sheet, create a lily pad challenge by sketching lily pads and a rock.

b **Solve It!** On this page, help the frog hop to all the lily pads in your challenge in *as few hops as possible*.

My Challenge

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

c **Swap It!** Swap your challenge with one or more partners. Help the frog hop to all the lily pads in each of your partners' challenges in *as few hops as possible*.

Partner 1

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

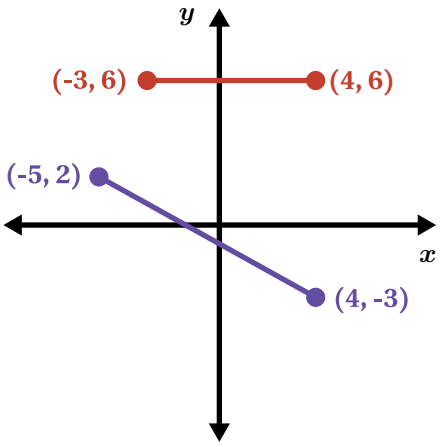
Partner 2

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

Synthesis

7. What are some strategies to calculate the distance between two points on the coordinate plane?

Use the examples in the graph if they help to show your thinking.



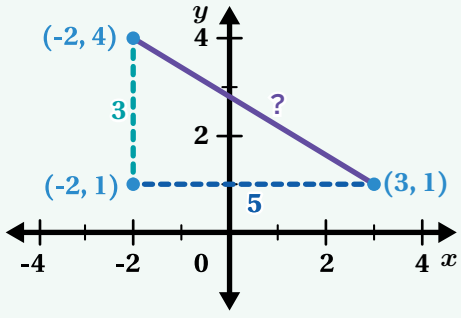
Lesson Practice ACC7.10.12

Lesson Summary

You can use the Pythagorean theorem to calculate the distance between two points that are on a diagonal line segment. To do this, start by drawing horizontal and vertical legs to form a right triangle. Then use the Pythagorean theorem to calculate the length of the hypotenuse, which will be the distance between the two points.

When two points are on a horizontal line segment, you can calculate the distance between them by determining the absolute value of the difference between their x-coordinates. For the points (-2, 1) and (3, 1) the distance is $|-2 - 3| = 5$ units.

Similarly, when two points are on a vertical line segment, you can calculate the distance between them by determining the absolute value of the distance between their y-coordinates. For the points (-2, 4) and (-2, 1) the distance is $|4 - 1| = 3$ units.



$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

$3^2 + 5^2 = x^2$

$9 + 25 = x^2$

$34 = x^2$

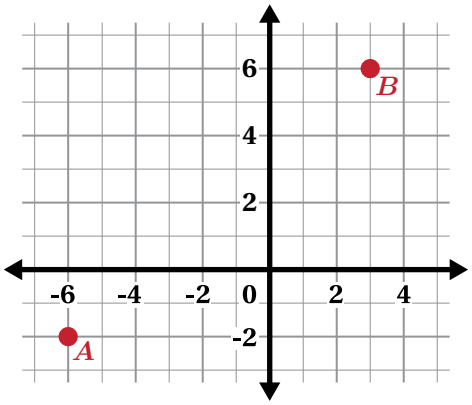
$\sqrt{34} = x$

Lesson Practice
ACC7.10.12

Name: _____ Date: _____ Period: _____

1. Here is a coordinate plane with labeled points.

What is the distance between point A and point B?



(A) $\sqrt{17}$

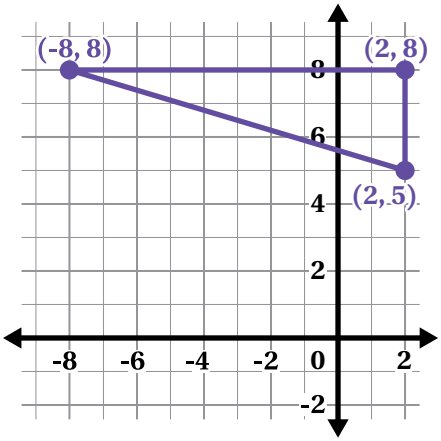
(B) $\sqrt{128}$

(C) $\sqrt{145}$

(D) 17

Problems 2–3: Calculate the length of each side of these right triangles.

2.

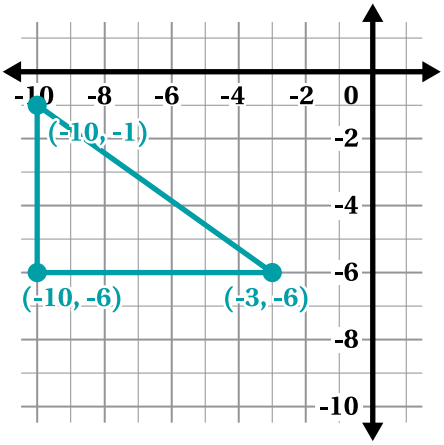


Shorter leg: _____

Longer leg: _____

Hypotenuse: _____

3.



Shorter leg: _____

Longer leg: _____

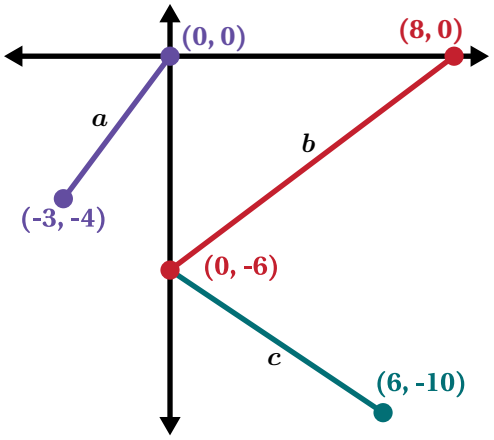
Hypotenuse: _____

Problems 4–6: Calculate the length of each segment.

4. Segment a

5. Segment b

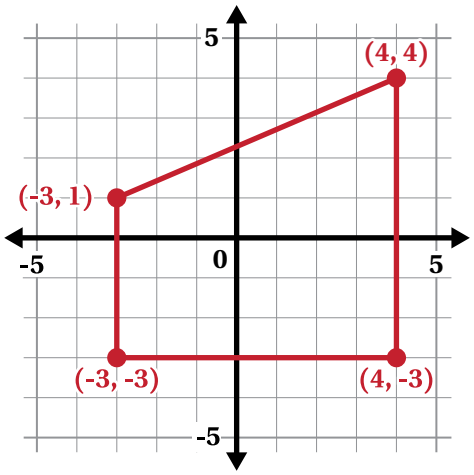
6. Segment c



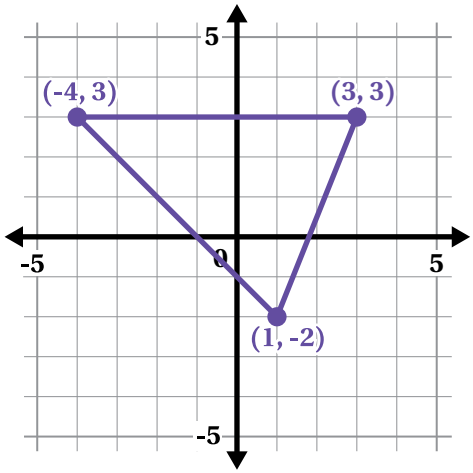
Name: ..... Date: ..... Period: .....

Problems 7–8: Calculate the perimeter of each polygon.

7.



8.



FAST Practice

9. The distance between $(0, -8)$ and another point is 5 units. Which could be the coordinates for the second point if the two points do not lie on a horizontal or vertical line?

(A.) $(-2, -5)$

(B.) $(-3, -6)$

(C.) $(4, -5)$

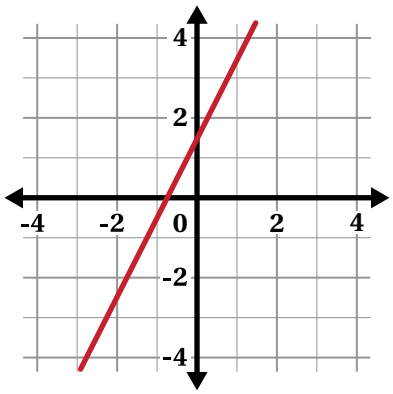
(D.) $(5, -6)$

Spiral Review

10. Determine the value of the expression. Write your answer in scientific notation.

$(5.6 \cdot 10^8) + (7.3 \cdot 10^8)$

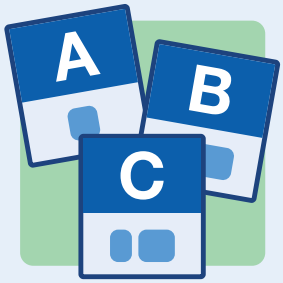
11. Write an equation for the line.



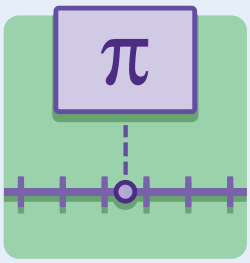
Sub-unit

3

Rational and Irrational Numbers



Lesson 13
Code Breaking



Lesson 14
Hit the Target

Lesson 15
Road Trip!

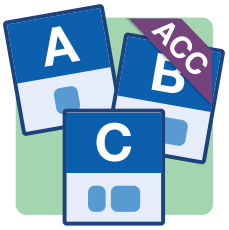
Lesson 16
Simplify It!

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.2, MTR.3.1, MTR.4.1

Code Breaking

Let's break a code by representing rational numbers in different ways.



Warm-Up

1. Which one doesn't belong? Circle one.

$\frac{75}{100}$ $\frac{3}{4}$ -0.75 75%

Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

Breaking the Code

2. Ada leaves a note for Zack stating where she plan to go for the day. When Zack looks at the note, he realizes he needs to decode the message. What does the note say?

$1\frac{2}{15}$	$\frac{11}{9}$	$-\frac{7}{9}$	$-0.\overline{7}$	$122.\overline{2}\%$	$1.08\overline{}$	$1.\overline{2}$	$-\frac{23}{9}$	$-2\frac{5}{9}$	$\frac{7}{3}$	$2.\overline{3}$

Key:

A	B	C	D	E	F
$1\frac{2}{9}$	2.3%	2.05	$\frac{5}{4}$	$2\frac{1}{3}$	$-166.\overline{6}\%$

G	H	I	J	K	L
1.3	$\frac{49}{45}$	$\frac{109}{99}$	-1.1	1.14	$-77.\overline{7}\%$

M	N	O	P	Q	R
$10.\overline{2}$	2.1	$-\frac{16}{15}$	$-2\frac{1}{2}$	$1\frac{1}{3}$	-103.5%

S	T	U	V	W	X
$-2.\overline{5}$	$113.\overline{3}\%$	102%	$-\frac{4}{3}$	$\frac{113}{99}$	$3\frac{1}{3}$

Y	Z
2	$\frac{5}{2}$

Discuss: What strategies did you use to break the code?

Name: ..... Date: ..... Period: .....

Breaking the Code (continued)

3. The key says that E is $2\frac{1}{3}$. What other ways did you represent this number to break the code? How did you determine the other representations?

4. When Zoe decodes the message, she uses her calculator to help her. She notices that $-\frac{7}{9}$ on her calculator is -0.7777777778. She doesn't see that number in the key Ada left for her. What happened?

Name: Date: Period:

Writing a Code

5. Write your own code about your weekend plans. Include positive and negative **rational numbers** written as fractions, mixed numbers, decimals, and percents.

6. Trade your code with a partner and decode their message.

Synthesis

7. Describe strategies you can use to write a rational number in different but equivalent forms. Use the number $\frac{13}{9}$ if it helps you explain your thinking.

Lesson Practice ACC7.10.13

Lesson Summary

- **Rational numbers** can be represented in equivalent forms: fraction, decimal, mixed number, and percent.
- If a rational number is written as a fraction, you can use long division to write an equivalent decimal.
  - The fraction is a decimal that is either *terminating* or *repeating*.
  - For a repeating decimal, you can use *bar notation* to indicate the part of the decimal that repeats.
- If a rational number is written as a decimal, you can multiply by 100 to write an equivalent percent.
  - Use a percent sign, %, to indicate that the number is a percent.
- If a rational number is written as a percent, you can write a fraction with a denominator of 100 or you can divide by 100 to write the number as a decimal.

Examples

$1\frac{3}{8} = \frac{11}{8} = 1.375 = 137.5\%$

$\frac{5}{9} = 0.\underline{5} = 55.\overline{5}\%$

$0.12 = \frac{12}{100} = 12\%$

$0.165 = \frac{165}{1000} = \frac{33}{200}$

$7.5\% = 0.075 = \frac{75}{1000} = \frac{3}{40}$

$150\% = \frac{150}{100} = 1\frac{50}{100} = 1\frac{1}{2} = 1.5$

Lesson Practice
ACC7.10.13

Name: _____ Date: _____ Period: _____

1. Match the equivalent rational numbers.

a. $2.\overline{3}$ _____ -1.025

b. -102.5% _____ $183.\overline{3}\%$

c. $1\frac{1}{9}$ _____ $1.\overline{1}$

d. $\frac{11}{6}$ _____ $\frac{7}{3}$

2. Complete the table.

Fraction	Decimal	Mixed Number	Percent
	1.35		
			$235.\overline{5}\%$
$-\frac{11}{9}$			
		$2\frac{3}{5}$	

3. What is the same about each decimal representation in the table and what is different?

Name: ..... Date: ..... Period: .....

4. How do you know if two rational numbers are equivalent?

FAST Practice

5. Select *all* the rational numbers that are equivalent to $\frac{4}{3}$.

- ☐ **A.** $1.\overline{3}$
- ☐ **B.** $1\frac{1}{3}$
- ☐ **C.** 133%
- ☐ **D.** $\frac{8}{6}$
- ☐ **E.** $13.\overline{3}\%$

Spiral Review

Problems 6–9: Determine the value of the expression.

6. $\left(-\frac{1}{2}\right)(3)$

7. $7 - 12$

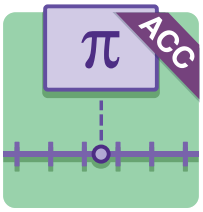
8. $-\frac{3}{5} + \left(-\frac{1}{3}\right)$

9. $-1.85 \div 5$

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.1, MA.8.NSO.1.2, MTR.5.1, MTR.6.1

Hit the Target



Let's build an understanding of two new types of numbers.

Warm-Up

1. Which number is greater? Circle one.

$\sqrt{13}$ π I'm not sure

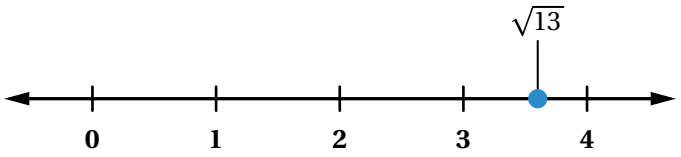
Explain your thinking.

Activity 1

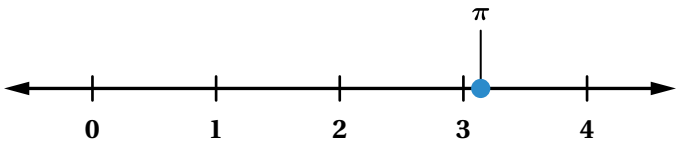
Name: ..... Date: ..... Period: .....

Hit the Target

2. Write a fraction as close to $\sqrt{13}$ as you can (without using the square root symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



3. Write a fraction as close to π as you can (without using the π symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



Name: Date: Period:

Irrational Numbers

4. A rational number is a number that can be written as a fraction using non-zero integers for the numerator and denominator.

It is impossible to write $\sqrt{13}$ as a fraction with non-zero integers, which makes it an **irrational number**.

Discuss: What are some other numbers you think might be irrational?

5. Is $\sqrt{\frac{9}{4}}$ rational or irrational? Circle one.

Rational Irrational I'm not sure

Explain your thinking.

6. Sort the numbers into groups based on whether they are rational or irrational.

$\frac{8}{4}$	$2\frac{3}{20}$	$1.\overline{73}$	$\sqrt{2}$
$2 \cdot \sqrt{13}$	$\sqrt{10}$	2π	$\sqrt{\frac{1}{4}}$
$\sqrt[3]{9}$	$\sqrt[3]{8}$	$1.73205080757...$	1.73

Rational	Irrational	I'm Not Sure

Name: Date: Period:

Irrational Number Challenges

7. Abdel and Irene each estimated the value of $2 + \sqrt{12}$. Their work is shown.

Abdel

12 is between the perfect squares 9 and 16 but closer to 9. So a reasonable guess for $\sqrt{12}$ is about 3.25.

$2 + 3.25 = 5.25$

$2 + \sqrt{12}$ is approximately equal to 5.25.

Irene

$\sqrt{9} < \sqrt{12} < \sqrt{16}$, so $\sqrt{12}$ is between 3 and 4 but closer to 3.

$3.3^2 = 10.89$

$3.4^2 = 11.56$

$3.5^2 = 12.25 \rightarrow$ This is closest to 12!

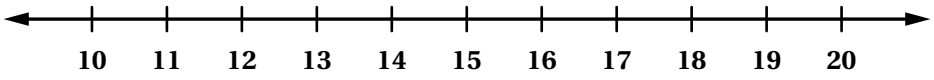
$2 + 3.5 = 5.5$, so that is my estimate.

How are their strategies alike? How are they different?

8. For each expression, estimate the value of the irrational number and then use it to determine the value of the expression. The first row has been done for you.

Expression	Work	Estimated Value
$3\sqrt{20}$	$3\sqrt{20} \approx 3 \cdot 4.5$	13.5
$\sqrt{40} + 10$		
$5\sqrt{5}$		
15π		
$\frac{1}{2}\pi$		

9. Plot $10\sqrt{2}$ on the number line.



Show or explain your reasoning.

Synthesis

10. What is an irrational number? Give at least one example.

Lesson Practice ACC7.10.14

Lesson Summary

A rational number is a number that can be written as a fraction of two non-zero integers. An **irrational number** is a number that cannot be written as a fraction of two non-zero integers.

Here are some examples of rational and irrational numbers.

Examples of Rational Numbers

- Fractions: $\frac{10}{5}$, $3\frac{11}{20} = \frac{71}{20}$
- Terminating decimals: $1.5 = \frac{3}{2}$, $1.73 = \frac{173}{100}$
- Repeating decimals: $1.\overline{73} = \frac{172}{99}$, $0.1212... = \frac{12}{99}$
- Square roots of perfect squares and cube roots of perfect cubes: $\sqrt[3]{8} = \frac{2}{1}, \sqrt{64} = \frac{8}{1}, \sqrt{\frac{1}{9}} = \frac{1}{3}$

Examples of Irrational Numbers

- Non-terminating, non-repeating decimals: π, $0.743...$, $2.742050...$
- Square roots of non-perfect squares and cube roots of non-perfect cubes: $\sqrt{2}, 3 \cdot \sqrt{5}, \sqrt[3]{9}$

Lesson Practice
ACC7.10.14

Name: _____ Date: _____ Period: _____

1. Sort the numbers based on whether they are rational or irrational.

0.1234	$-\sqrt{12}$	$\frac{-13}{3}$
$-\sqrt{100}$	$\sqrt{37}$	-77

Rational	Irrational

2. Select *all* the rational numbers.

- ☐ **A.** π^2
- ☐ **B.** $\sqrt{14}$
- ☐ **C.** $-\sqrt{99}$
- ☐ **D.** $-\sqrt{100}$
- ☐ **E.** $-\frac{123}{45}$
- ☐ **F.** $\sqrt{64}$

3. Determine whether each number is rational or irrational.

Number	Rational	Irrational
$\frac{1}{\sqrt{16}}$		
$\sqrt{10}$		
$-2\frac{1}{3}$		
0.532		

4. Eliza claims that $\sqrt{\frac{7}{9}}$ is rational. Is her claim correct?

Explain your thinking.

Lesson Practice
ACC7.10.14

Name: ..... Date: ..... Period: .....

FAST Practice

5. Which is the best estimate for the value of $\sqrt{30} + 7$?

(A.) 12.3

(B.) 12.5

(C.) 13.1

(D.) 22

Spiral Review

Problems 6–7: Rewrite each expression as a single power.

6. $3^{-5} \cdot 4^{-5}$

7. $(3^{-3})^2$

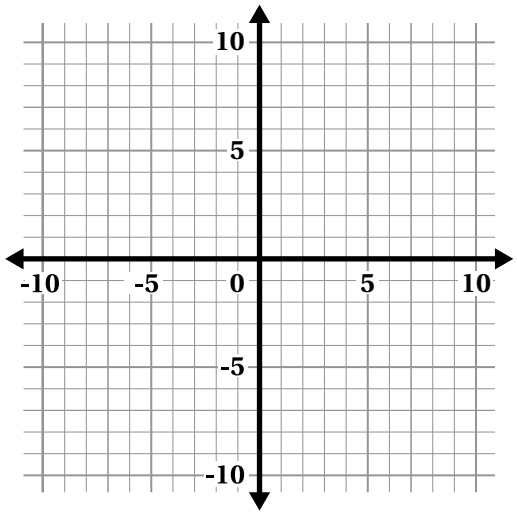
Problems 8–9: Write an equation that expresses the relationship between a right triangle's side lengths if they measure:

8. 10, 6, and 8 units

9. 2, 2, and $\sqrt{8}$ units

10. A square has vertices $(0, 0)$, $(5, 2)$, $(3, 7)$, and $(-2, 5)$. Which of these statements is true?

Use the graph if it helps with your thinking.



(A.) The square's side length is 5.

(B.) The square's side length is between 6 and 7.

(C.) The square's side length is between 5 and 6.

(D.) The square's side length is 7.

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.7, MTR.5.1, MTR.7.1

Road Trip!

Let's solve multistep problems using order of operations with rational numbers.

Warm-Up

1. Simplify the following expressions:

a $\frac{2}{3} + (2 \times 4) - \frac{1}{3}$

b $\frac{2}{3} + 2 \times \left(4 - \frac{1}{3}\right)$

Discuss: How are these expressions alike? How are they different?

Alike:

Different:

Activity 1

Name: Date: Period:

Family Road Trip

Adriana and her family took a road trip.

Here are the details of the trip:

- The family traveled a total of 150 miles.
- The car they drove uses $\frac{3}{4}$ gallon of gas for every 25 miles driven.
- The price of a gallon of gas was $\$3.50$.
- The family took three rest stops during the trip.
- The family paid two tolls along the way. Each toll was $\$2.50$.

2. You can use the expression $\left[(150 \div 25) \times \frac{3}{4} \times 3.50\right] + 2 \times 2.50$ to calculate how much the family spent on transportation during their trip. Identify what each part of the expression means and why the grouping symbols are needed.

3. When simplifying your expression from Problem 2, what step in the calculation should you do first? Explain your reasoning.

Name: ..... Date: ..... Period: .....

Family Road Trip (continued)

4. What was the total cost of transportation for the trip? Show each step as you simplify the expression.

$$\left[(150 \div 25) \times \frac{3}{4} \times 3.50\right] + 2 \times 2.50$$

5. Look at how you simplified the expression to find its value. Would you get a different answer if you removed the brackets? The parentheses? Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Group Road Trip

A group of five friends took a road trip. Two students, Irelle and Sora, calculated the cost per person for the trip. The group will share the transportation and snack costs equally.

Here are the details of their trip:

- The group traveled 240 miles, and the price of a gallon of gas was $3.25.
- The car they drove can travel 30 miles per gallon of gas.
- The group spent $12.50 on snacks and $5.00 in tolls.

6. Irelle and Sora each entered an expression into their calculator to determine the amount that **each** person owes, but they got different answers. Their calculator screens and conclusions are shown.

Irelle

$240 \div 30 \times 3.25 + 12.5 + 5 \div 5 \quad = 39.5$

Each friend owes $39.50.

Sora

$(240 \div 30 \times 3.25 + 12.5 + 5) \div 5 \quad = 8.7$

Each friend owes $8.70

Whose work is correct? Circle one.

Irelle's Sora's

Justify your choice.

7. Why is there such a difference between the two answers that Irelle and Sora calculated?

Activity 3

Name: Date: Period:

Work Trip

A group of six coworkers are driving to a conference, and they have a combined budget of $\$100$ to spend on the trip. They travel a total of 171 miles using a car that can travel 28.5 miles per gallon. Gas costs $\$3.15$ per gallon. They decide to buy lunch, which costs $\$12.75$ per person.

8. Write an expression for the total amount of money spent on the trip.

9. Did they stay within their budget? Show your work and explain.

10. The coworkers also have a $\$75$ budget for breakfast. Each person orders a coffee for $\$2.75$ and a bagel for $\$9.50$. There is an 8% sales tax. Did they stay within their budget for breakfast? Show your work and explain.

Synthesis

11. Terrance and his three friends eat lunch at a restaurant. The bill is $\$44.89$. The group applies a $\$5$ coupon to the bill and then leaves a 20% tip. Write and simplify an expression for the amount that Terrance pays if the total cost is split evenly among the group. Show each step as you simplify.

Lesson Practice ACC7.10.15

Lesson Summary

Using order of operations can help you solve real-world problems. This involves using grouping symbols, ( ) and [ ], correctly given the context of a problem.

Consider this problem: "Mr. Tyson bought tickets to a concert. Each ticket cost $\$12.50$. He decides to buy 4 tickets. There's also a $\$5$ service fee added to the total ticket price. What is the total cost of the tickets?"

You *might* come up with different expressions to solve this problem.

Expression 1: $(12.50 \times 4) + 5$ **Expression 2:** $(12.50 + 5) \times 4$

Expression 1 adds the service fee **after** multiplying the ticket price by the number of tickets, which gives a total cost of $\$55$.

Expression 2 adds the service fee **before** multiplying, which gives a total cost of $\$70$. This is incorrect given the context of the problem.

It is also important to remember that, in a fraction, the numerator and denominator are each a group that must be simplified before the division happens. This is especially important to remember when using technology to simplify. Sometimes you have to add parentheses around a numerator or denominator to get the correct answer.

Lesson Practice
ACC7.10.15

Name: ..... Date: ..... Period: .....

Problems 1–2: Each equation is missing some grouping symbols. Draw grouping symbols that would make each equation true.

1. $\frac{1}{8} \times \quad 3 + 2 \quad = \frac{5}{8}$

2. $\frac{1}{4} + 4 \quad - 10 \times 5 = -45\frac{3}{4}$

Problems 3–5: Write and simplify an expression to solve each problem.

3. Mr. Tyler is making smoothies for a party. The recipe calls for $\frac{1}{2}$ cup of yogurt for every 2 servings and $\frac{1}{8}$ cup of honey per serving. How much yogurt will Mr. Tyler need for 8 servings?

4. Three friends go to breakfast and spend a total of $\$55.60$, including tax. They will leave a 15% tip for the server and split the cost equally. Write and simplify an expression to find the cost for each person. Round to the nearest cent.

5. Omari bought 3.5 pounds of apples for $\$1.35$ per pound and 1.2 pounds of grapes for $\$3.99$ per pound. He has a coupon for $\$1.50$ off. What is the total cost of the fruit to the nearest cent?

Name: _____ Date: _____ Period: _____

FAST Practice

6. An art teacher is buying supplies for a class project.

- She buys 3.5 yards of fabric, and each yard costs $\$4.50$.
- She buys 2.5 yards of ribbon, and each yard costs $\$2.50$.
- The total cost includes a 7% sales tax.

Select *all* the statements about the situation that are true.

- ☐ **A.** The expression $[(3.5 \times 4.50) + (2.5 \times 2.50)] \times 1.07$ models the situation.
- ☐ **B.** The expression $3.5 \times 4.50 + (2.5 \times 2.50) \times 1.07$ models the situation.
- ☐ **C.** The total cost is $\$23.54$.
- ☐ **D.** The total cost is $\$22.44$.
- ☐ **E.** The total cost is $\$23.10$.

Spiral Review

7. Which number is greater? Circle one.

$\sqrt{23}$ $\quad$ 4

Explain your thinking.

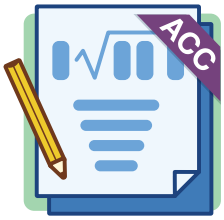
8. Select *all* the irrational numbers.

- ☐ **A.** -56
- ☐ **B.** $\sqrt{3}$
- ☐ **C.** 1.12222
- ☐ **D.** 5.55
- ☐ **E.** $\frac{1}{999}$
- ☐ **F.** $-\sqrt{725}$

Name: ..... Date: ..... Period: .....

MA.8.NSO.1.7, MTR.4.1, MTR.5.1

Simplify It!



Let's apply order of operations with exponents and radicals.

Warm-Up

1. Look at the expression shown.

$$\left(\frac{1}{2}\right)^2 + \sqrt{9+7} - (4^2 \times 2)$$

Discuss: What do you think you should do first to simplify this expression?

Name: ..... Date: ..... Period: .....

Step-by-Step

2. The table shows the steps you could take to simplify $\left(\frac{1}{2}\right)^2 + \sqrt{9+7} - (4^2 \times 2)$.

a Fill in a reason for each step.

	Math Work	Reason
Step 1	$\left(\frac{1}{2}\right)^2 + \sqrt{9+7} - (16 \times 2)$	Simplify exponent in parentheses
Step 2	$\left(\frac{1}{2}\right)^2 + \sqrt{9+7} - 32$	Multiply within parentheses
Step 3	$\frac{1}{4} + \sqrt{9+7} - 32$	
Step 4	$\frac{1}{4} + \sqrt{16} - 32$	
Step 5	$\frac{1}{4} + 4 - 32$	
Step 6	$-27\frac{3}{4}$	

b Explain why a radical is like a grouping symbol.

c Imagine that you want to use a calculator to determine the value of $\sqrt{125-25}$. What would you do to make sure you get the correct answer?

Name: ..... Date: ..... Period: .....

Step-by-Step (continued)

3. Consider the expression shown.

$$\frac{6+\sqrt{2\times 32}}{5+2}$$

a What step would you do first to simplify the expression?

b Use the table below to simplify the expression. You may not need all of the rows, or you may want to add rows.

	Math Work	Reason
Step 1	$\frac{6+\sqrt{2\times 32}}{5+2}$	Original expression
Step 2		
Step 3		
Step 4		
Step 5		
Step 6		

Name: Date: Period:

Simplifying in Groups

4. Look at the expression shown. Notice that there are no grouping symbols. Circle parts of the expression that you need to simplify further **before** you add, multiply, or divide.

$$\frac{3}{\sqrt[3]{8}+1}+\sqrt{9^2+19}\times 2^3$$

5. Zola simplified the expression in Problem 4. Her work is shown.

$$\frac{3}{\sqrt[3]{8}+1}+\sqrt{9^2+19}\times 2^3$$

$$\frac{3}{3}+\sqrt{9^2+19}\times 2^3$$

$$1+\sqrt{100}\times 2^3$$

$$1+10\times 8$$

$$11\times 8$$

$$88$$

What did Zola do well? What mistake did Zola make? Explain your answers with words and numbers.

Name: ..... Date: ..... Period: .....

Simplifying in Groups (continued)

6. Simplify the expression in the table by following the steps that are given.

	Math Work	Reason
Step 1	$\frac{3^2+1}{6} \times \sqrt{40+9} + 10$	Original expression
Step 2		Simplify numerator
Step 3		Simplify radicand
Step 4		Take square root
Step 5		Multiply
Step 6		Add

7. **Discuss:** When simplifying expressions using order of operations, what are some strategies you can use?

Synthesis

8. Calculate the value of the expression. Show each step.

$$\frac{3^2}{2^4} \times \sqrt{20 - 4} - \frac{1}{2}$$

Lesson Practice ACC7.10.16

Lesson Summary

When simplifying expressions involving radicals and exponents, apply the order of operations. Look for grouping symbols like parentheses and brackets to simplify parts of the expression first.

You can also identify other groups or terms that need to be simplified further before performing the order of operations. These include radicands and numerators and denominators of fractions.

In the expression shown, the different colors show parts of the expression that you must simplify first before moving on.

$$2^4 \times \frac{5 + 1}{\sqrt{20 + 5}}$$

$$16 \times \frac{6}{5}$$

$$\frac{96}{5}$$

Name: ..... Date: ..... Period: .....

Problems 1–4: Calculate the value of each expression.

1. $3^2 \times \sqrt{16} + 7.15$

2. $1.2(2^4 + 1) - \sqrt[3]{8}$

3. $\dfrac{\sqrt{25} + 3^2}{3\sqrt{9}}$

4. $\dfrac{1}{2 + \sqrt{25}} \times 5^2$

Problems 5–6: Each expression was simplified two different ways. Circle the correct work.

5. Sample Work A

$(6 + \sqrt[3]{60 + 4}) - 8^2$

$(6 + 4) - 8^2$

$10 - 64$

-54

Sample Work B

$(6 + \sqrt[3]{60 + 4}) - 8^2$

$(6 + 8) - 8^2$

$14 - 64$

-50

6. Sample Work A

$\dfrac{2^2}{\sqrt{100}} + 4 \times 3$

$0.4 + 4 \times 3$

4.4×3

13.2

Sample Work B

$\dfrac{2^2}{\sqrt{100}} + 4 \times 3$

$0.4 + 4 \times 3$

$0.4 + 12$

12.4

Lesson Practice
ACC7.10.16

Name: ..... Date: ..... Period: .....

FAST Practice

7. What is the value of the expression?

$$\frac{2\sqrt{100}+3^2}{\sqrt{36}-2^3}$$

A. $\frac{29}{2}$

B. $\frac{25}{28}$

C. $-\frac{25}{28}$

D. $-\frac{29}{2}$

Spiral Review

Problems 9–10: Circle the part of each expression that you need to simplify first.

8. $9+3\times 1.5$

9. $10\times\left[8+\frac{1}{2}\right]+\frac{2}{3}$

10. Santiago built a birdhouse. The expression represents the total cost, in dollars, of the materials for the birdhouse he built.

$$4.50+2-8\times 0.5$$

What is the total cost of the materials Santiago bought? Show your work.

Unit 11

Probability

When an experiment is repeated, how can you know every possible outcome? You can use structures like organized lists, tree diagrams, and tables to make sure every possibility is included. In this unit, you will determine all possible outcomes for repeated experiments and calculate and compare theoretical and experimental probabilities.

Essential Questions

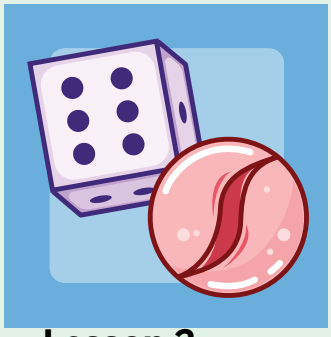
- How can you determine all of the possible outcomes when an experiment is performed more than once?
- What is experimental probability and how is it different from theoretical probability?

Sub-unit

1

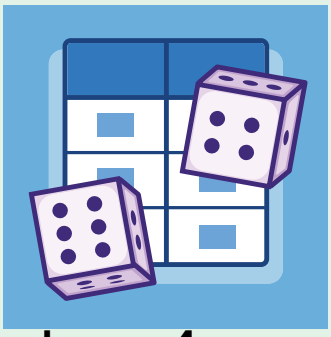
Theoretical Probability

Lesson 1
What Can Happen?



Lesson 2
Representing Sample Spaces

Lesson 3
A Likely Story



Lesson 4
We're On a Roll!

Lesson 5
What Could Happen?

Name: ..... Date: ..... Period: .....

MA.8.DP.2.1, MTR.5.1

What Can Happen?

Let's determine the possible outcomes of repeated experiments.

Warm-Up

1. Consider the following **experiments**. How are they similar? How are they different?

A. flipping a coin

B. rolling a number cube labeled 1 through 6

C. picking a card from a standard deck of cards

D. spinning a spinner with 10 equal sections, each labeled with a different name

Activity 1

Name: Date: Period:

Coin Flip

You will do a repeated experiment by flipping a coin two times. Use "H" for heads and "T" for tails.

2. Perform the repeated experiment four times and record your results in the table.

	Flip 1	Flip 2
Experiment 1		
Experiment 2		
Experiment 3		
Experiment 4		

3. The outcome of an experiment or a repeated experiment is an **event**. List all of your events. For example, if you had heads for both flips, you would list HH.

4. Did you get all of the possible events? Explain.

5. List all of the possible events for the repeated experiment.

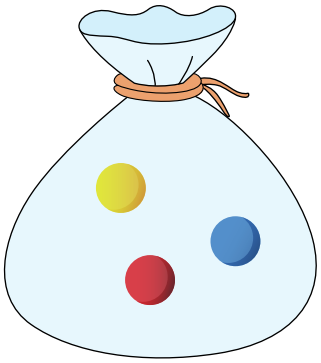
6. For a new repeated experiment, the coin is flipped four times. What is a possible event?

Activity 2

Name: ..... Date: ..... Period: .....

Sample Spaces

The set of all the possible outcomes for a repeated experiment is called the **sample space**.



7. A bag contains 3 marbles. There is 1 blue marble (B), 1 red marble (R), and 1 yellow marble (Y). A marble is drawn and replaced. Then a second marble is drawn.

a Why is it important whether or not the marble is replaced?

b Is the outcome BB possible? What does it mean? Explain your thinking.

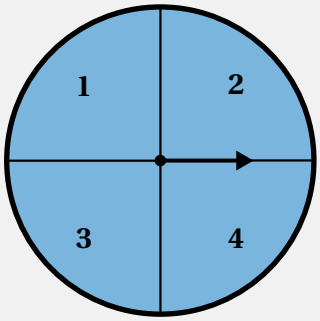
c What is the sample space for this repeated experiment?

d How does the sample space compare to the sample space for flipping a coin twice in Activity 1? Why do you think that is?

Name: ..... Date: ..... Period: .....

Sample Spaces (continued)

8. Fill in the sample space for each repeated experiment. Use an organized list.

Repeated Experiment	Sample Space
A dime is flipped three times.	
A bag contains tiles with the letters P, Q, R, and S. A tile is picked and replaced, and a second tile is picked.	
The spinner is spun twice. 	

9. **Discuss:** What do you notice about the bag of letters and the spinner and their sample spaces?

Synthesis

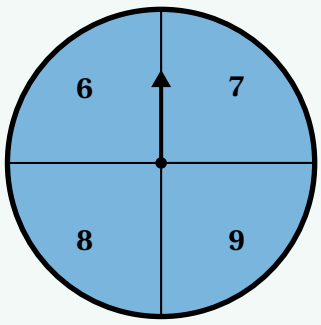
10. Consider the repeated experiment of rolling a standard 6-sided number cube twice. List the sample space.

Lesson Practice ACC7.11.01

Lesson Summary

- An **experiment** is an action or activity that you can do over and over again without knowing what the outcome will be each time. When the same experiment is done more than once, it is a repeated experiment.
- The result of an experiment or a repeated experiment is an **event**, and a **sample space** is the set of every possible event.
- If the repeated experiment is two spins of the spinner shown, then the sample space is:

6, 6	6, 7	6, 8	6, 9
7, 6	7, 7	7, 8	7, 9
8, 6	8, 7	8, 8	8, 9
9, 6	9, 7	9, 8	9, 9

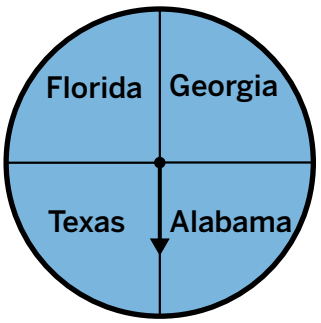


Name: ..... Date: ..... Period: .....

1. Use a list to show the sample space for flipping a fair coin three times.

2. A set of four cards has a red 4, a blue 4, a green 2, and a yellow 3. What is the sample space for the repeated experiment of choosing one card at random from the set, replacing it, then choosing another card?

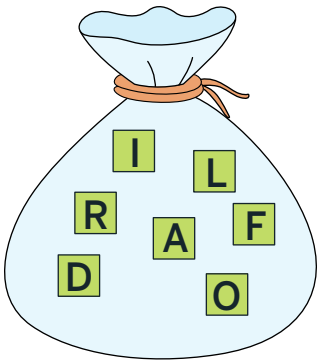
3. List the sample space when spinning this spinner twice.



4. Lucia says that when she rolls a standard 6-sided number cube twice, there are 12 events in the sample space. Do you agree with Lucia? Explain your thinking.

Name: ..... Date: ..... Period: .....

5. A bag has a set of tiles. Each tile has one of the letters of the word *FLORIDA*. You pick a tile, replace it in the bag, and pick a second tile. What is the sample space?



FAST Practice

6. A spinner has three equally divided sections labeled with the letters A, B, and C. What is the sample space if you spin this spinner twice?

(A.) {A, B, C}

(B.) {A, B, C, A, B, C}

(C.) {AA, BB, CC}

(D.) {AA, AB, AC, BA, BB, BC, CA, CB, CC}

Spiral Review

Problems 7–8: Determine the value(s) of x in each equation.

7. $81 = v^2$

8. $z^3 = -125$

9. During a lunar eclipse, the Moon is on the opposite side of Earth from the Sun, and the three are in a straight line. The distance between the Moon and Earth is about 3.85×10^5 kilometers and the distance between Earth and the Sun is about 1.5×10^8 kilometers. What is the distance between the Sun and the Moon during a lunar eclipse?

Name: ..... Date: ..... Period: .....

MA.8.DP.2.1, MTR.2.1

Representing Sample Spaces

Let's explore different ways to show a sample space.

Warm-Up

1. Consider the sample spaces for the following repeated experiments. How are they similar? How are they different?

A. flipping a fair coin four times

B. choosing a marble from a bag that has a white marble, a blue marble, and a red marble, replacing the marble, choosing a second marble, then replacing that marble and choosing a third marble

C. picking a card from a standard deck of cards, replacing it, then picking another card

D. spinning a spinner twice that has 10 equal sections, each labeled with a different name

Activity 1

Name: Date: Period:

We're Rolling

You will do a repeated experiment by rolling a number cube two times. Write the digit that shows on top for each roll.

2. Perform the repeated experiment four times and record your results in the table.

	Roll 1	Roll 2
Experiment 1		
Experiment 2		
Experiment 3		
Experiment 4		

3. Describe the sample space for the experiment in words.

4. **a** Fill in the rest of the table to show the sample space.

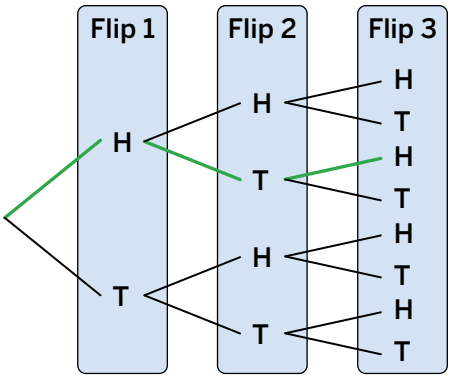
	1	2	3	4	5	6
1	11	12	13	14	15	16
2						
3						
4						
5						
6						

b **Discuss:** Do you think a table is a good way to represent this sample space? Explain.

Name: ..... Date: ..... Period: .....

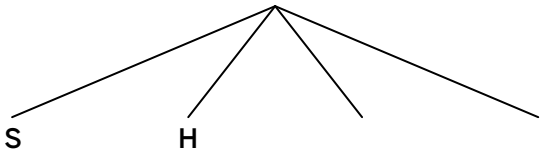
Tree Diagrams

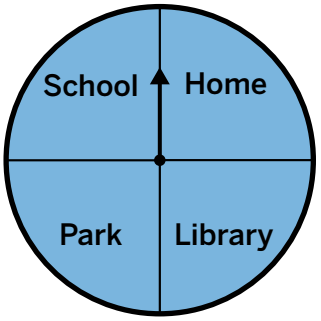
A **tree diagram** is one way to represent the sample space for a repeated experiment. It uses branches to show possible outcomes for each repeat of the experiment.



5. This tree diagram shows the sample space for flipping a coin three times. What outcome do you think the green highlighted path represents?

6. Complete the tree diagram that shows the sample space for spinning this spinner twice.





7. A bag of marbles has one blue marble, one red marble, one purple marble, one orange marble, and one white marble. In a repeated experiment, you choose one marble and replace it, choose a second marble and replace it, and then choose a third marble.

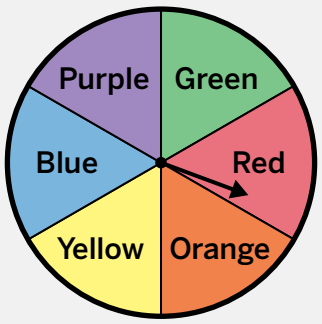
a Describe what a tree diagram for the experiment would look like.

b Could you use a table like the one in Activity 1 to represent this sample space? Explain.

Name: Date: Period:

You Choose!

8. Represent the sample space for each repeated experiment. You can use a list, write a description, create a table, or create a tree diagram.

Repeated Experiment	Sample Space
Flip a two-color counter that is red on one side and yellow on the other three times.	
A bag contains tiles with the letters M, N, O, and P. You pick a tile, replace it, then pick a second tile.	
Spin this spinner twice. 	

Synthesis

9. The table shows the sample space for a repeated experiment. Describe a possible repeated experiment that could be represented by the table.

	A	B	C
A	AA	AB	AC
B	BA	BB	BC
C	CA	CB	CC

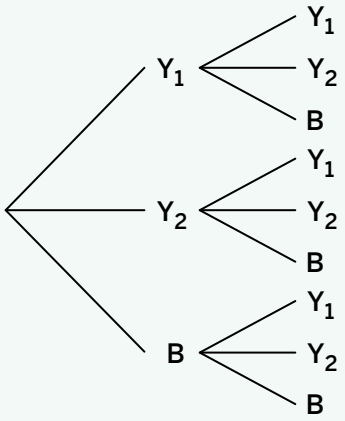
Lesson Practice ACC7.11.02

Lesson Summary

To show the sample space for a repeated experiment, you can use an ordered list, a written description, a table, or a **tree diagram**. A table works only for an experiment that is performed twice. A tree diagram uses branches to represent the possible outcomes of each experiment.

This table and tree diagram show the sample space for spinning a spinner with three equal sections twice. The spinner has two yellow sections and one black section.

	Y_1	Y_2	B
Y_1	Y_1Y_1	Y_1Y_2	Y_1B
Y_2	Y_2Y_1	Y_2Y_2	Y_2B
B	BY_1	BY_2	BB



Lesson Practice
ACC7.11.02

Name: ..... Date: ..... Period: .....

1. Draw a tree diagram to show the sample space for flipping a coin four times.

2. A box contains four tiles labeled 6, 7, 8, and 9. A tile is drawn from the box and replaced. Then a second tile is drawn. Use a table to represent the sample space of the repeated experiment.

3. A bag has two red marbles and one blue marble. Represent the sample space for selecting a marble from the bag, replacing it, and then selecting another marble.

4. How could you represent the sample space for choosing a card from a standard deck of playing cards, replacing the card, then choosing a second card?

Lesson Practice
ACC7.11.02

Name: _____ Date: _____ Period: _____

FAST Practice

5. A spinner is divided equally into five sections labeled A, B, C, D, and E. It is spun twice. Which shows the sample space for the repeated experiment?

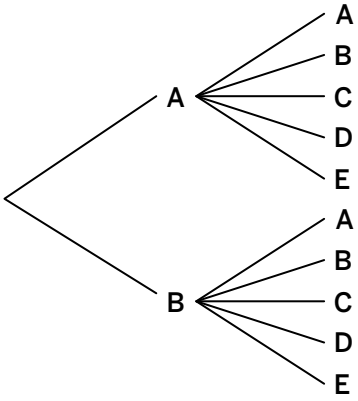
A. {A, B, C, D, E}

B.

	A	B	C	D	E
A	AA	AB	AC	AD	AE
B	BA	BB	BC	BD	BE
C	CA	CB	CC	CD	CE
D	DA	DB	DC	DD	DE
E	EA	EB	EC	ED	EE

C. {AA, BB, CC, DD, EE}

D.



Spiral Review

6. Write the expression $\frac{8^{-5}}{8^{-2}}$ without exponents.

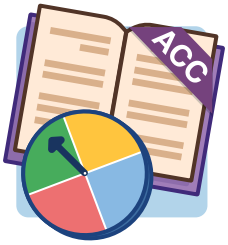
7. A right triangle has a base length of 7 centimeters and a hypotenuse of 14 centimeters. What is the height of the triangle?

Name: ..... Date: ..... Period: .....

MA.8.DP.2.1, MA.8.DP.2.2, MTR.5.1

A Likely Story

Let's determine the theoretical probability of events in repeated experiments.



Warm-Up

1. Which event is most likely? Show or explain your thinking.

A. Choosing a white T-shirt randomly from a bag that has 5 white T-shirts, 3 blue T-shirts, 4 gray T-shirts, and 2 black T-shirts

B. Rolling a 5 or 6 on a standard number cube

C. Flipping heads on a fair coin

D. Spinning a spinner labeled 1 to 10 and having it land on a number less than 4

Activity 1

Name: Date: Period:

Theoretical Probability

Consider the repeated experiment of flipping a coin 3 times. Use H for heads and T for tails.

2. **a** Give the sample space for the repeated experiment.

b How many events are in the sample space for this experiment?

c Which events in the sample space have exactly 2 heads?

d How would you describe the likelihood of flipping exactly 2 heads when flipping a coin 3 times?

3. Imani says the **theoretical probability** of all three flips landing on heads is $\frac{1}{8}$, or 12.5%.

Do you agree? Circle one.

Yes No I don't know

4. **Discuss:** How is calculating probabilities for a single coin flip different from calculating probabilities for a repeated experiment of several coin flips?

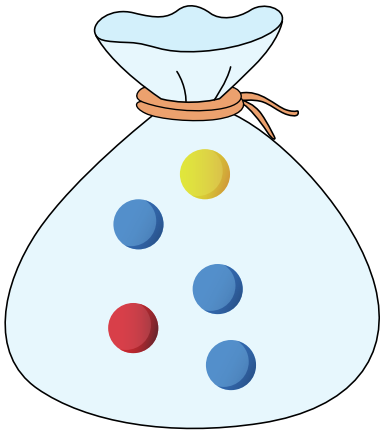
Activity 2

Name: ..... Date: ..... Period: .....

Don't Lose Your Marbles!

The probability of a certain event can be written as P(event). For example, in a coin flip experiment, you could use P(H) to indicate the probability of flipping heads or P(at least 2 tails) to indicate the probability of getting at least 2 tails in a repeated experiment.

A bag contains 5 marbles. There are 3 blue marbles, 1 red marble, and 1 yellow marble. A marble is drawn and replaced. Then, a second marble is drawn.



5. **a** What is the sample space for this repeated experiment? You can use numbers to represent the different blue marbles, such as B1, B2, and B3.

b What is P(blue, yellow), the theoretical probability of drawing a blue marble and then a yellow marble? Explain your thinking.

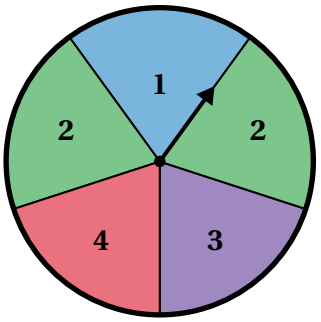
c What does P(at least 1 blue) mean, and what is its value?

d **Discuss:** Are B1, Y and B2, Y the same event? Explain.

Name: Date: Period:

More Probability Situations

6. The spinner shown is spun twice.



a Determine $P(\text{at least one } 2)$.

b Determine $P(\text{same number})$.

7. A standard number cube is rolled twice.

a Determine $P(\text{sum of } 8)$.

b Determine $P(2, 3)$.

c Determine $P(3, 2)$.

d **Discuss:** What do you notice about your answers for parts b and c? What does this tell you about each outcome in a repeated experiment?

Name: ..... Date: ..... Period: .....

More Probability Situations (continued)

8. A pop quiz has 2 multiple-choice questions. Each question has 4 answer choices, and only one of the answer choices is correct. Mariam wants to determine the probabilities associated with guessing the answers at random.

Mariam makes the following table to represent the sample space of correct (C) and wrong (W) answer choices.

	C	W1	W2	W3
C	C, C	C, W1	C, W2	C, W3
W1	W1, C	W1, W1	W1, W2	W1, W3
W2	W2, C	W2, W1	W2, W2	W2, W3
W3	W3, C	W3, W1	W3, W2	W3, W3

a Explain how the table represents the sample space.

b Use the table to determine P(both answers correct).

c Haruto used the table to determine P(at least 1 answer correct). He thinks he made a mistake.

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

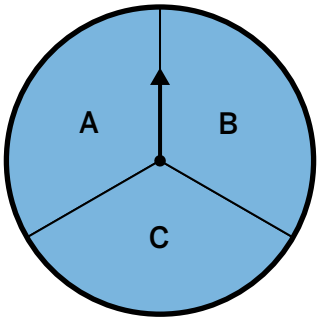
There are 6 events with at least 1 answer correct: C, W1; C, W2; C, W3; W1, C; W2, C; and W3, C.

There are a total of 16 possible outcomes in the sample space. The probability is $\frac{6}{16}$, or 37.5%.

Synthesis

9. The spinner on the right is spun twice. The sample space for the repeated experiment is shown in the table.

	A	B	C
A	A, A	A, B	A, C
B	B, A	B, B	B, C
C	C, A	C, B	C, C



a Explain what P(both letters the same) means and how you would calculate it.

b How is the sample space for an experiment related to the probabilities of the events in that sample space?

Lesson Practice ACC7.11.03

Lesson Summary

- The **theoretical probability** of an event can be written as P(event) and expressed as:
  - a fraction
  - a decimal between 0 and 1
  - a percent between 0% and 100%
- The theoretical probability of an event is equal to $\frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$.

Name: ..... Date: ..... Period: .....

Problems 1–3: A fair coin is tossed 4 times.

1. What is the sample space? Use H for heads and T for tails.

2. What is P(at least 2 heads)?

3. What is P(all tails)?

Problems 4–6: A standard number cube is rolled twice.

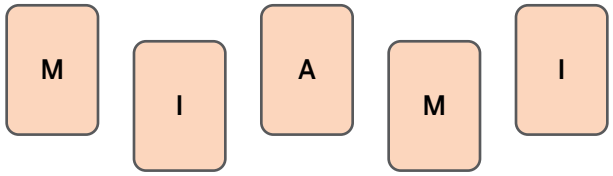
4. What is P(sum of at least 10)?

5. What is P(sum of 9 or less)?

6. What do you notice about your answers to Problems 4 and 5? Why do you think this is?

Name: ..... Date: ..... Period: .....

Problems 7–8: A deck has a set of 5 cards. Each card has one of the letters of the word MIAMI. You pick a card, replace it in the deck, and pick a second card.



7. What is the theoretical probability of choosing an A first and an M second?

8. What is the theoretical probability of not choosing an I either time?

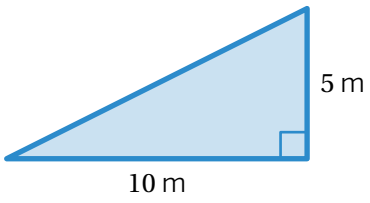
FAST Practice

9. A spinner has 4 equal-sized sections labeled with the letters A, B, C, and D. What is P(exactly one D) expressed as a fraction?

Record your answer in the space provided.

Spiral Review

10. Determine the unknown side length of the triangle.



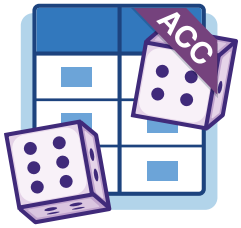
11. There are about 3.2×10^7 seconds in a year, and light travels at a speed of about 3×10^5 kilometers per second. Using these values, what is the distance in kilometers that light travels in a year (a light-year)?

Name: ..... Date: ..... Period: .....

MA.8.DP.2.1, MA.8.DP.2.2, MA.8.DP.2.3, MTR.4.1

We're on a Roll!

Let's explore experimental probability.



Warm-Up

1. For each experiment, describe the sample space and determine the theoretical probability of the given outcome.

a A pile has 6 white socks, 3 black socks, and 1 blue sock. You choose a sock at random.

Sample space:

Probability of choosing a blue sock, P(blue):

b You flip a fair coin twice.

Sample space:

Probability of getting the same result on both flips, P(same):

Activity 1

Name: Date: Period:

Let's Roll!

2. Roll a standard number cube 20 times and complete the table with your results.

Roll	Result	Roll	Result	Roll	Result	Roll	Result
1		6		11		16	
2		7		12		17	
3		8		13		18	
4		9		14		19	
5		10		15		20	

3. Use your results to complete this table, showing the frequency of each possible outcome.

Result	1	2	3	4	5	6
Frequency						

4. Analyze your results. What do you notice? What do you wonder?

Activity 2

Name: Date: Period:

Experimental or Theoretical?

The **experimental probability** of an outcome is the ratio of the number of times an event occurs to the total number of trials conducted.

5. What is the theoretical probability for rolling each number on a standard number cube?

6. Calculate the experimental probability for rolling each number based on your results in Activity 1.

	$P(1)$	$P(2)$	$P(3)$	$P(4)$	$P(5)$	$P(6)$
Experimental Probability						

7. Compare the theoretical probabilities with the experimental probabilities you calculated.

8. Darius did not roll a 6 during his repeated experiment, so he thinks that rolling a 6 is impossible. Do you agree? Circle one.

Agree Disagree I don't know

Show or explain your thinking.

Name: Date: Period:

Experimental or Theoretical? (continued)

A bag contains a blue marble, a red marble, a white marble, and a yellow marble. A marble is drawn and replaced in the bag, and then a second marble is drawn from the bag. The results of 14 repetitions of this experiment are shown in the table.

Trial	Result
1	blue, red
2	white, blue
3	white, yellow
4	blue, yellow
5	red, red
6	yellow, white
7	blue, white

Trial	Result
8	white, red
9	yellow, yellow
10	blue, yellow
11	white, red
12	red, blue
13	blue, blue
14	white, yellow

9. Calculate the experimental probability of drawing the same color twice. Show or explain your thinking.

10. Calculate the theoretical probability of drawing the same color twice. Show or explain your thinking.

11. Compare the experimental probability with the theoretical probability of drawing the same color both times.

Name: Date: Period:

How Many Heads?

Number of Heads	0	1	2	3
Frequency (first 5 trials)				
Experimental Probability (out of 5)				
Frequency (next 10 trials)				
Experimental Probability (out of 15)				

12. Flip a coin 3 times. Repeat this 5 times. Use tallies to record the number of heads for each trial. Then calculate the probability of flipping each number of heads out of 5 trials. Complete the first two rows of the table.

13. Repeat the experiment for another 10 trials and record your results. Then calculate the new experimental probabilities out of 15 total trials. Complete the last two rows of the table.

14. A computer simulated this same experiment for 100 trials and for 1,000 trials. The results are shown in the table along with the theoretical probability for each outcome.

Number of Heads	0	1	2	3
Frequency out of 100 Trials	15	39	34	12
Frequency out of 1,000 Trials	118	382	373	127
Theoretical Probability	12.5%	37.5%	37.5%	12.5%

Discuss: How do the results change as the number of trials changes?

Synthesis

15. The table shows the results for 14 trials of rolling a standard number cube twice. Use the results to determine the theoretical and experimental probabilities of each outcome.

Trial	Result	Trial	Result
1	4, 2	8	1, 2
2	2, 6	9	4, 4
3	4, 3	10	3, 5
4	2, 2	11	6, 2
5	6, 3	12	2, 2
6	4, 3	13	4, 2
7	6, 3	14	4, 4

a The same number is rolled both times.

b The sum of the two rolls is even.

c At least one roll is a 1.

Lesson Practice ACC7.11.04

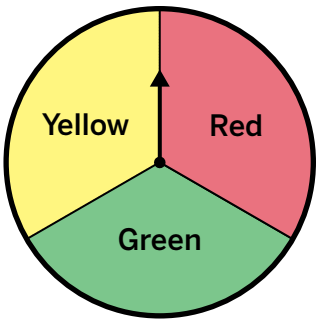
Lesson Summary

- The **experimental probability** for an outcome in an experiment is the ratio of the number of times the event occurs in the experiment to the total number of trials conducted.
- While theoretical probability is based on what is expected to occur given the sample space, experimental probability is calculated based on what actually happens in an experiment or simulation.
- The theoretical and experimental probabilities for an event are often different. However, as the number of trials increases, the experimental probability gets closer to the theoretical probability.

Name: ..... Date: ..... Period: .....

Problems 1–2: The repeated experiment of spinning the spinner twice was done 200 times. The results are shown in the table.

Result	Frequency
yellow, yellow	18
yellow, red	21
yellow, green	25
red, yellow	19
red, red	22
red, green	21
green, yellow	24
green, red	21
green, green	29



1. Based on the table, what is the experimental probability that the first spin landed on red? Show or explain your thinking.

2. How does the experimental probability of landing on red first compare to the theoretical probability?

3. Mariam rolled a standard number cube 30 times and got three 5s. She says that that the experimental probability for rolling a 5 was $\frac{1}{6}$. Do you agree with Mariam? Circle one.

Yes No

Show or explain your reasoning.

Name: ..... Date: ..... Period: .....

Problems 4–6: The table shows the results of an experiment where a coin was flipped 20 times.

Result	Heads	Tails
Frequency	7	13

4. Based on the table, what is the experimental probability $P(\text{heads})$?

5. How does the experimental probability of the coin landing on heads compare to the theoretical probability?

6. How would you expect the experimental probability to change if the experiment were repeated another 150 times?

FAST Practice

7. The repeated experiment of rolling a fair 6-sided number cube twice was done 50 times. There were 10 trials in which both of the numbers rolled were even numbers. Select the true statement.

A. The experimental probability is 20%, which is less than the theoretical probability.

B. The experimental probability is 25%, which is greater than the theoretical probability.

C. The experimental probability is 17%, which is less than the theoretical probability.

D. The experimental probability and the theoretical probability are both 20%.

Spiral Review

8. Write the expression $\frac{2^3}{2^{-4}}$ as a whole number or a fraction without exponents or decimals.

Problems 9–10: Determine whether each set of line segments can form a right triangle. Explain your thinking.

9. 8 in., 10 in., 12 in.

10. 5 cm, 12 cm, 13 cm

Name: ..... Date: ..... Period: .....

MA.8.DP.2.3, MTR.5.1, MTR.7.1

What Could Happen?

Let's predict the results of experiments.

Warm-Up

1. For each experiment, give the theoretical probability and experimental probability of the given outcome.

a A bag of marbles has 3 white marbles, 4 black marbles, 6 green marbles, and 7 blue marbles. You choose a marble at random.

Probability of choosing a blue marble

Out of 25 draws, a blue marble is drawn 8 times.

b You roll a fair number cube twice.

Probability of rolling two even numbers

Out of 60 trials, two even numbers are rolled 18 times.

Activity 1

Name: Date: Period:

Flipping Out

Use the table for Problems 2–4.

	# of Heads	# of Tails
Guess		
Actual		
Difference		

2. Imagine you will flip a fair coin 50 times. How many times would you expect to get heads? How many times would you expect to get tails?

a Record your guesses in the first row of the table.

b **Discuss:** Explain how you came up with your guesses.

3. With your partner, flip a coin 50 times or simulate this experiment with a computer program. Record the actual number of heads and tails in the second row of the table. You can use the space below to keep track of your flips.

Heads	Tails

4. Compare the actual numbers from your experiment with your guesses.

a Record the difference for each in the third row of the table.

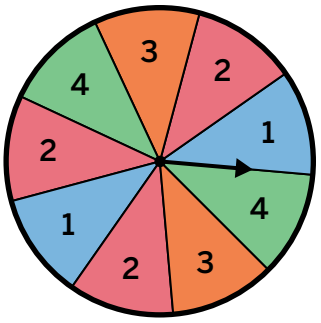
b How do the actual results compare to your predictions?

c **Discuss:** Why do you think the guesses and results were different?

Name: ..... Date: ..... Period: .....

Put Some Spin on It

Use the spinner shown for Problems 5–7.



5. Determine the theoretical probability of the spinner landing on each number.

6. Based on your probability calculations, which number would you expect the spinner to land on most often in an experiment? Explain your thinking.

7. Here are two students' strategies for predicting how many times the spinner will land on 3 if it is spun 180 times.

Riya's work

$$P(3) = \frac{2}{9}$$

$$\frac{2}{9} = \frac{x \text{ 3s predicted}}{180}$$

$$\frac{2}{9} \times \frac{20}{20} = \frac{40}{180}$$

$$x = 40$$

I expect the spinner to land on 3 about 40 times.

Zion's work

$$P(3) = \frac{2}{9}$$

$$180 \times \frac{2}{9} = 40$$

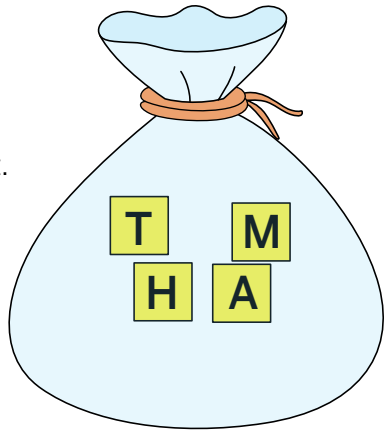
I predict the spinner will land on 3 forty times.

Discuss: How are their strategies alike? How are they different?

Name: Date: Period:

Put Some Spin on It (continued)

A bag of tiles contains the letters for the word *math*, with one tile for each letter. In a repeated experiment, a tile is drawn at random, the tile is replaced, and then a second tile is drawn.



8. Represent the sample space for the repeated experiment.

9. The experiment is performed 100 times. Predict the number of times that the A tile is drawn both times. Show or explain your thinking.

10. The experiment is performed 250 times. Predict the number of times that the M tile will be drawn at least once. Show or explain your thinking.

11. **Discuss:** Consider the predictions you made for Problems 9 and 10. Which do you think would be more accurate if tested?

Synthesis

12. Consider the repeated experiment of flipping a fair coin 3 times.

a Calculate the theoretical probability of getting 0, 1, 2, or 3 heads. Record your results in the first row of the table.

b Predict how many times you would expect to get 0, 1, 2, or 3 heads in 50 trials. Record your results in the second row of the table.

# of Heads	0	1	2	3
Probability				
Prediction				

c If you were to conduct the repeated experiment, would you expect your predictions to be exactly the same as the results? Explain your thinking.

Lesson Practice ACC7.11.05

Lesson Summary

To calculate the predicted number of outcomes for an experiment, start by finding the theoretical probability of the event, P(event). Then you can:

- Set up and solve a proportion: $\frac{\text{number of outcomes of the event}}{\text{number of outcomes in the sample space}} = \frac{\text{predicted number of outcomes}}{\text{number of trials}}$

or

- Multiply the theoretical probability by the number of trials: P(event) $\times$ number of trials

Note that the results of an experiment will likely not match the predicted values but are usually close, especially as the number of trials in the experiment increases.

Name: ..... Date: ..... Period: .....

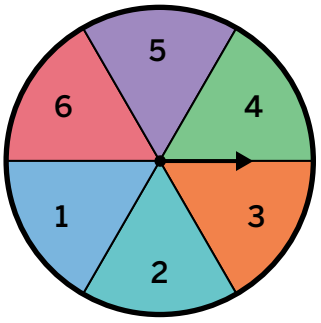
Problems 1–2: A math class used a computer to perform an experiment about flipping coins. The experiment they wanted to simulate was flipping a single coin 4 times. They performed this simulation 700 times, and the results are recorded in the table.

# of Heads	0	1	2	3	4
Frequency	53	162	252	194	39
Prediction					

1. For each number of heads, calculate the predicted number of results you would expect out of 700 trials. Round your answers to the nearest whole and record them in the table. Explain how you found your answers.

2. Compare your predictions with the actual frequencies in the simulation.

Problems 3–4: Use the spinner at the right.

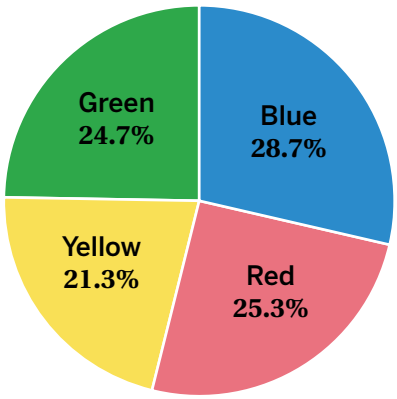


3. If you spun this wheel 250 times, how many times would you expect it to land on 2? Explain your answer.

4. Marquis predicted that in 135 spins, the spinner would land 29 times on the number 5. Is this a reasonable prediction? If not, how would you revise the prediction?

Name: _____ Date: _____ Period: _____

5. The results of choosing a marble from a bag and then replacing it 40 times are shown in the circle graph. How many times would you expect to choose a blue marble from the same bag in 500 trials?



6. Renata chooses a card from a standard deck of 52 cards and replaces the card each time. She does this a total of 260 times. What is a reasonable prediction of how many times she will choose an ace? Show or explain your thinking.

FAST Practice

7. A fair number cube labeled 1–6 is rolled 500 times. Which is the most reasonable prediction of how many times a 3 will be rolled?

A. 83

B. 100

C. 125

D. 167

Spiral Review

8. The areas of two squares are 121 square feet and 36 square feet. What is the difference between the length of a side of the larger square and the length of the side of the smaller square?

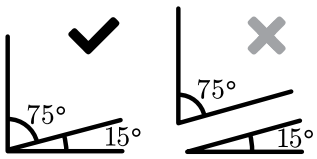
9. Write the sum $5.68 \times 10^{26} + 1.89 \times 10^{27}$ in scientific notation.

Glossary/Glosario

English

A

adjacent angles Angles that share a side and a vertex.

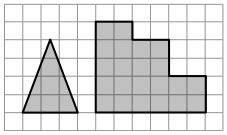


The image with the check mark shows a 75° angle and a 15° angle when they are adjacent.

approximation A rounded value that you can use to represent a number that may be difficult to work with, such as an irrational number or a repeating decimal.

For example, the exact value of pi (π) is an irrational number, so we often use the approximate value of 3.14 in calculations involving pi.

area The space inside a two-dimensional figure. It is expressed in square units.



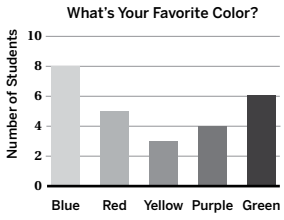
The area of the triangle is 6 square units. The area of the other shape is 22 square units.

association If two variables are related to one another, we say that there is an association between them. Associations can be described as positive or negative, linear or nonlinear, and strong or weak.

Some examples of associations are:
Positive: One variable increases as the other also increases.
Negative: One variable decreases as the other increases.
Linear: Can be modeled by a straight line.
Nonlinear: Cannot be modeled by a straight line.

B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.



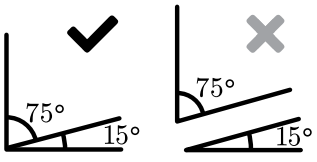
bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

For example, the repeating decimal 0.090909... can be written in bar notation as $0.\overline{09}$.

Español

A

ángulos adyacentes Ángulos que comparten un lado y un vértice.

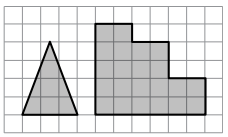


La imagen con la marca de verificación muestra un ángulo de 75° y un ángulo de 15° cuando son adyacentes.

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un número irracional o un decimal periódico.

Por ejemplo, el valor exacto de pi (π) es un número irracional, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

área El espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.



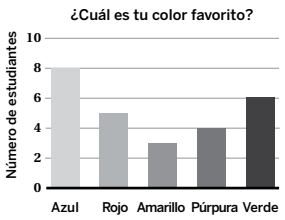
El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.

asociación Si dos variables están relacionadas entre sí, decimos que existe una asociación entre ambas. Las asociaciones pueden describirse como positivas o negativas, lineales o no lineales lineales y fuertes o débiles.

Algunos ejemplos de asociaciones son los siguientes:
Positiva: El valor de una variable aumenta a medida que el valor de la otra también aumenta. Negativa: El valor de una variable disminuye a medida que el valor de la otra aumenta.
Lineal: Puede modelarse con una línea recta.
No lineal: No puede modelarse con una línea recta.

B

gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.

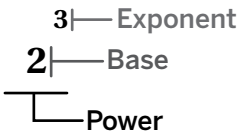


raya indicadora de decimales periódicos (vinculum) Una forma de representar los dígitos que se repiten en un número decimal. Sobre los dígitos que se repiten se traza una pequeña línea.

Por ejemplo, el decimal periódico 0.090909... puede escribirse con la raya indicadora de decimales periódicos (vinculum) como $0.\overline{09}$.

English

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.

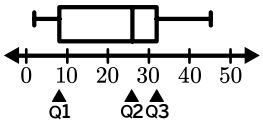


In this example, 2 is the base.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

For example, a collection of students' heights and shoe sizes would be a bivariate data set.

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



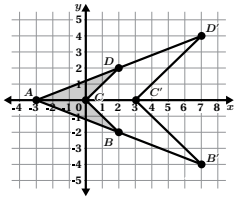
C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

"What kind of pet do you have?" is a question that would result in categorical data.

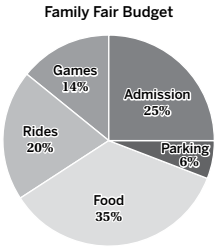
center of a dilation The point from which we measure distances in a dilation.

The center of dilation in this example is point A.



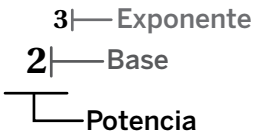
circle A shape made out of all the points that are the same distance from a center point.

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a pie chart.



Español

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.

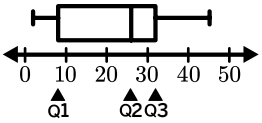


En este ejemplo, 2 es la base.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.

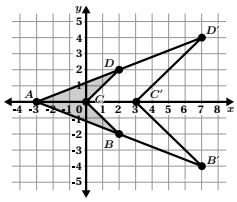


datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

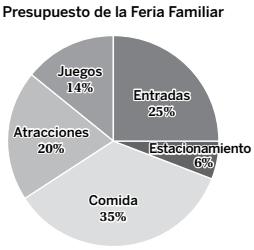
centro de una dilatación El punto desde el cual medimos las distancias en una dilatación.

El centro de dilatación en este ejemplo es el punto A.



círculo Una figura formada por todos los puntos que están a la misma distancia de un punto central.

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado gráfico de torta.

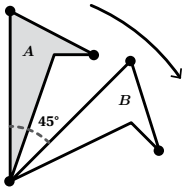


Glossary/Glosario

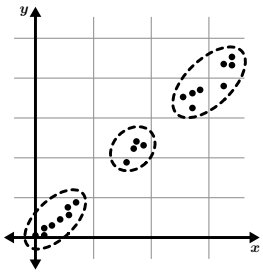
English

circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string. The circumference of a circle, C, can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

clockwise In the same direction as the hands of a clock; to the right (of a turn).



clusters Groups of data values that are close together.



coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

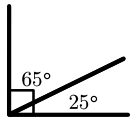
In the expression $5x + 8$, 5 is the coefficient of x.

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

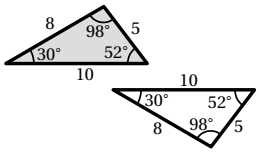
For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

complementary angles Two angles whose measures add up to $90°$.

For example, a $65°$ angle and a $25°$ angle are complementary.



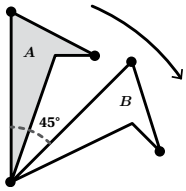
congruent One figure is congruent to another if they have exactly the same size and shape, or if one can be mapped to the other with a translation, rotation, or reflection.



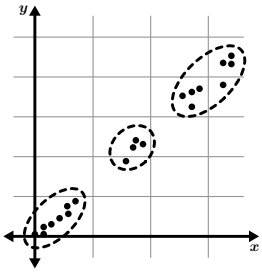
Español

circunferencia La distancia alrededor de un círculo. Si imaginamos el círculo como un trozo de cuerda, es la longitud de la cuerda. La circunferencia de un círculo, C, puede calcularse mediante la fórmula $C = \pi d$, donde d es el diámetro del círculo, o $C = 2\pi r$, donde r es el radio.

en el sentido de las manecillas del reloj Girar en la misma dirección que las manecillas de un reloj. En otras palabras, un giro a la derecha.



agrupaciones (clusters) Grupos de valores de datos que están próximos entre sí.



coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

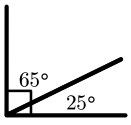
En la expresión $5x + 8$, el coeficiente de x es 5.

propiedad conmutativa La propiedad indica que $a + b = b + a$ and $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente del orden en el que estén los números.

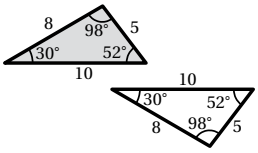
Por ejemplo, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

ángulos complementarios Dos ángulos cuyas medidas suman $90°$.

Por ejemplo, un ángulo de $65°$ y otro ángulo de $25°$ son complementarios.



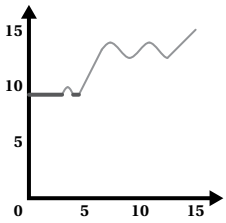
congruente Una figura es congruente con otra si tienen exactamente el mismo tamaño y forma, o si una puede ser transformada en la otra mediante una traslación, rotación o reflexión.



English

constant (function) A function or interval of a function is constant if the y-values stay the same when the x-values go up.

The bolded parts of this function are constant.



constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

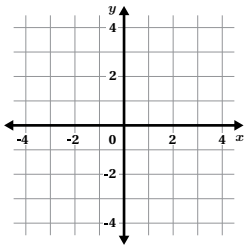
Carpet (sq. ft.)	Cost (dollars)
10	$\times\frac{2}{3}$ 15.00
20	$\times\frac{2}{3}$ 30.00
50	$\times\frac{2}{3}$ 75.00

In this table, one constant of proportionality is $\frac{2}{3}$.

converse A statement written in the opposite direction.

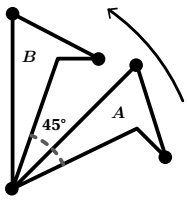
For example, the Pythagorean theorem states: If a triangle is right, it has side lengths such that $a^2 + b^2 = c^2$. The converse of the Pythagorean theorem is: If a triangle has side lengths such that $a^2 + b^2 = c^2$, it is a right triangle.

coordinate plane The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

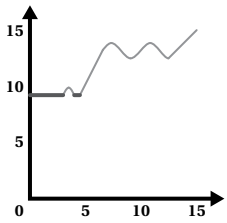
counterclockwise In the opposite direction as the hands of a clock; to the left (of a turn).



Español

constante (función) Una función o el intervalo de una función es constante si los valores de y se mantienen iguales cuando los valores de x aumentan.

Las partes resaltadas de esta función son constantes.



constante de proporcionalidad En una relación proporcional, una constante de proporcionalidad es el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.

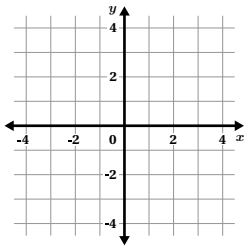
Alfombra (pies cuadrados)	Costo (dólares)
10	$\times\frac{2}{3}$ 15.00
20	$\times\frac{2}{3}$ 30.00
50	$\times\frac{2}{3}$ 75.00

En esta tabla, una constante de proporcionalidad es $\frac{2}{3}$.

recíproco Una expresión escrita en sentido contrario.

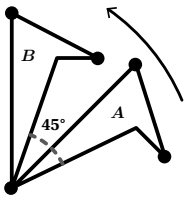
Por ejemplo, el teorema de Pitágoras indica lo siguiente: Si un triángulo es rectángulo, tiene longitudes de lado tales que $a^2 + b^2 = c^2$. El recíproco del teorema de Pitágoras sería el siguiente: Todo triángulo que tiene longitudes de lado tales que $a^2 + b^2 = c^2$, es un triángulo rectángulo.

plano de coordenadas, plano cartesiano El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).



corresponder (partes correspondientes) Corresponder es coincidir. Cuando una parte de una figura original corresponde con una parte de una copia, se llaman partes correspondientes. Pueden ser puntos, segmentos, ángulos o distancias.

en sentido contrario a las manecillas del reloj Girar en la dirección opuesta a las manecillas de un reloj.



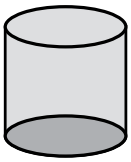
Glossary/Glosario

English

cube root The number that can be cubed to get n, written as $\sqrt[3]{n}$.
The cube root also describes the edge length of a cube with a volume of n.

For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

cylinder A 3-D figure that has two parallel circular bases connected by a curved surface.

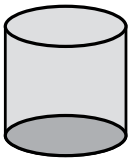


Español

raíz cúbica El número que se puede elevar al cubo para obtener n. Se escribe $\sqrt[3]{n}$.
La raíz cúbica también describe la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de lado de un cubo que tiene un volumen de 64 unidades cúbicas.

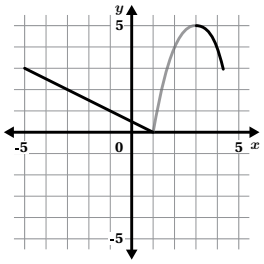
cilindro Una figura tridimensional que tiene dos bases circulares paralelas conectadas por una superficie curva.



D

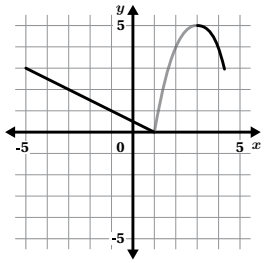
decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.



decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

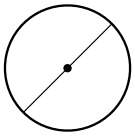
Las partes resaltadas de esta función son decrecientes.



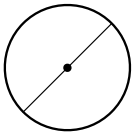
dependent variable (output) A variable whose value is based on the value of another variable or set of variables. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.
In a function, the dependent variable is sometimes called the output.

variable dependiente (salida) Una variable cuyo valor se basa en el valor de otra variable o un conjunto de variables. La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.
En una función, la variable dependiente a veces se denomina la salida.

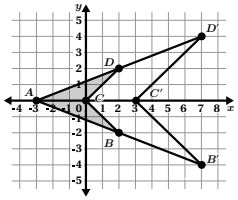
diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.



diámetro La distancia entre un punto y otro en un círculo, pasando por el centro. También es la distancia mayor entre un punto y otro en un círculo.

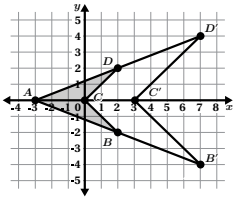


dilation A type of transformation that creates scaled copies. A dilation moves every point in a figure away from a center of dilation by the distance of a scale factor.



In this example, polygon $ABCD$ is dilated using point A as the center of dilation and a scale factor of 2. Point D' is twice as far away from point A as point D along the same ray.

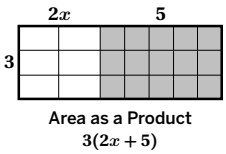
dilatación Un tipo de transformación que produce copias a escala. Una dilatación aleja cada punto de una figura del centro de dilatación de acuerdo con un factor de escala.



Aquí, el polígono $ABCD$ se dilata usando punto A como centro de dilatación y un factor de escala de 2. El punto D' está al doble de la distancia del punto A que el punto D en la misma semirrecta.

English

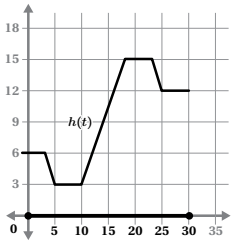
distributive property The distributive property says that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.



Area as a Product
$3(2x + 5)$

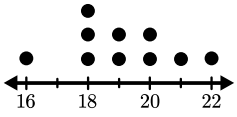
For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as: All numbers from 0 to 30 or $0 \le t \le 30$.

dot plot One way to visualize data. Every data point is shown as a dot above its value on a number line. Data sets with repeating values will have multiple dots aligned with the same value. A dot plot is sometimes called a line plot.

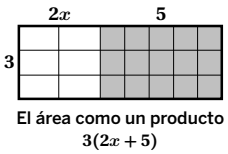


Number of Jelly Beans

For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

Español

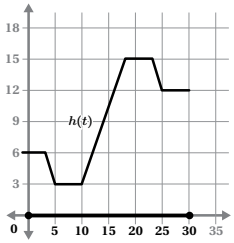
propiedad distributiva La propiedad distributiva indica que la multiplicación de un número por la suma de dos o más términos es igual a la suma de las multiplicaciones de dicho número por cada término individualmente.



El área como un producto
$3(2x + 5)$

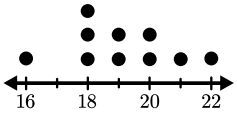
Por ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera: Todos los números del 0 al 30 o $0 \le t \le 30$.

diagrama de puntos Una forma de visualizar datos. Cada punto de datos se muestra como un punto encima de su valor en una recta numérica. Los conjuntos de datos con valores repetidos tienen múltiples puntos alineados en un mismo valor. Un diagrama de puntos a veces se llama gráfico de líneas.



Cantidad de frijolitos

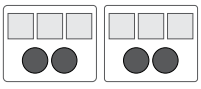
Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

E

equivalent expressions Expressions that are equal for every value of a variable.

For example, $6x + 2x$ is equivalent to $5x + 3x$. No matter what value x is, the two expressions are always equal.

equivalent ratios Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



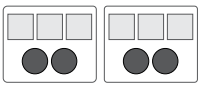
3 : 2 is equivalent to 6 : 4 because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. Both recipes will taste the same.

expresiones equivalentes Expresiones que son iguales para todos los valores de una variable.

Por ejemplo, $6x + 2x$ es equivalente a $5x + 3x$. Independientemente del valor de x, las dos expresiones son siempre iguales.

razónes equivalentes Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



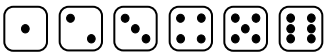
3 : 2 es equivalente a 6 : 4 porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. Ambas recetas tendrán el mismo sabor.

Glossary/Glosario

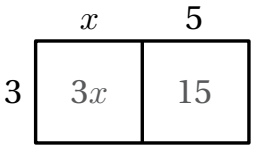
English

event A set of one or more outcomes in a chance experiment.



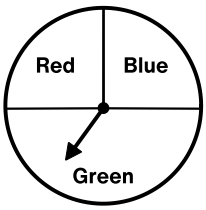
For example, if we roll a number cube, there are six possible outcomes.

expand To expand is to use the distributive property to multiply the factors in an expression and rewrite the expression as a sum. The new expression is equivalent to the original expression.



For example, we can expand $3(x + 5)$ to get the equivalent expression $3x + 15$.
The sum of the two boxes in the diagram show the expression in expanded form.

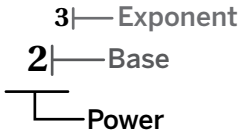
experiment An action or activity that you can do over and over again without knowing what the outcome will be each time.



For example, each time you spin the spinner, it could land on red, blue, or green.

experimental probability Experimental probability is the ratio of the number of times an event occurs to the total number of trials conducted. It is determined through actual experiments rather than theoretical calculations.

exponent A number used to describe repeated multiplication.

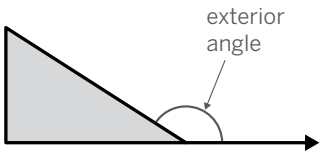


In this example, 3 is the exponent, which means $2^3 = 2 \cdot 2 \cdot 2$.

expression A set of numbers, symbols, and operators (such as + and •) grouped together to represent a value. Unlike equations, expressions do not include an equal sign (=).

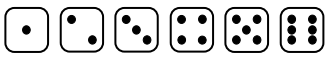
For example, $5x$, $m - 4$, $3y + 0.5$, and $2(n + 8)$ are all expressions.

exterior angle An exterior angle of a polygon is the angle that is formed between a side of the polygon and a line extended from the next side.



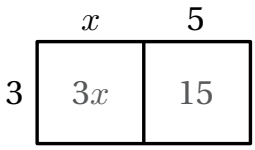
Español

evento Un conjunto de uno o más resultados en un experimento de azar.



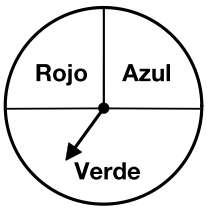
Por ejemplo, si tiramos un dado, hay seis resultados posibles.

expandir Expandir significa usar la propiedad distributiva para multiplicar los factores en una expresión y reescribir la expresión como una suma. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos expandir $3(x + 5)$ para obtener la expresión equivalente $3x + 15$.
La suma de las dos casillas en el diagrama muestra la expresión en forma expandida.

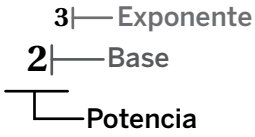
experimento Una acción o actividad que se puede llevar a cabo una y otra vez sin saber cuál será el resultado en cada ocasión.



Por ejemplo, cada vez que se gira la ruleta, podría caer en rojo, azul o verde.

probabilidad experimental La probabilidad experimental es la razón entre el número de veces que ocurre un suceso y el número total de ensayos realizados. Se determina mediante experimentos reales en vez de cálculos teóricos.

exponente Un número que se usa para describir multiplicaciones repetidas.

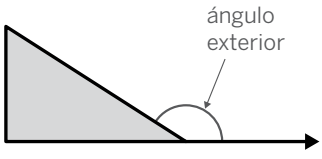


En este ejemplo, 3 es el exponente, lo cual significa que $2^3 = 2 \cdot 2 \cdot 2$.

expresión Un conjunto de números, símbolos y operadores (como + y •) agrupados para representar un valor. A diferencia de las ecuaciones, las expresiones no incluyen un signo igual (=).

Por ejemplo, $5x$, $m - 4$, $3y + 0.5$, y $2(n + 8)$ son expresiones.

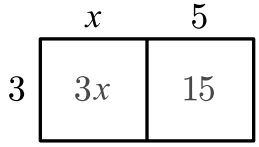
ángulo exterior Un ángulo exterior de un polígono es el ángulo que se forma entre un lado del polígono y una línea extendida desde el siguiente lado.



English

F

factor To factor is to use the distributive property to rewrite an expression as the product of two or more factors. The new expression is equivalent to the original expression.



For example, we can factor $3x + 15$ to get the equivalent expression $3(x + 5)$.
The product of the two sides in the diagram show the expression in factored form.

function A relationship that assigns exactly one output to each possible input.

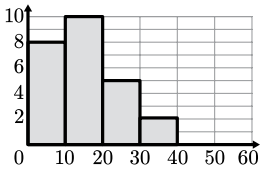
G

greatest common factor (GCF) The largest number that is a common factor of two numbers.

The common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is 4.

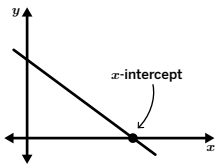
H

histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.

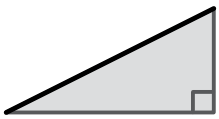


For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.

horizontal intercept The point where the graph of a line crosses the horizontal axis or when $y = 0$. The horizontal intercept is sometimes called the x-intercept.



hypotenuse The side of a right triangle that is opposite the right angle. The hypotenuse is always the longest side of a right triangle.



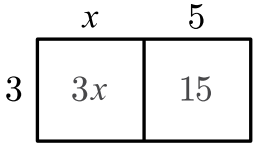
I

image A new figure that is created after a transformation of an original figure (called the pre-image). Every part of the pre-image moves in the same way to match up with a part of the image.

Español

F

factorizar Factorizar significa usar la propiedad distributiva para reescribir una expresión como el producto de dos o más factores. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos factorizar $3x + 15$ para obtener la expresión equivalente $3(x + 5)$.
El producto de los dos lados del diagrama muestra la expresión en forma factorizada.

función Una relación que asigna exactamente una salida a cada entrada posible.

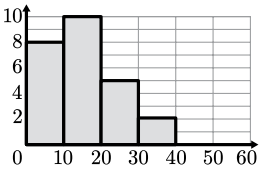
G

máximo común divisor (MCD) El número mayor que es factor común de dos números.

Los factores comunes de 8 y 12 son 1, 2 y 4. El máximo común divisor es 4.

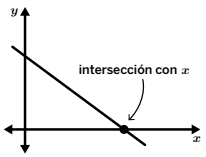
H

histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.

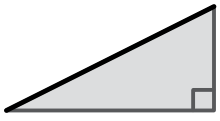


Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

intersección horizontal El punto donde la gráfica de una recta se cruza con el eje horizontal o cuando $y = 0$. La intersección horizontal a veces se denomina intersección con el eje x.



hipotenusa El lado del triángulo rectángulo que está opuesto al ángulo recto. La hipotenusa siempre es el lado más largo de un triángulo rectángulo.



I

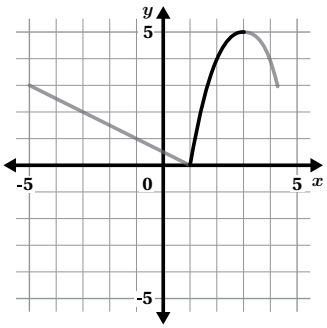
imagen Una nueva figura que se produce después de la transformación de una figura original (denominada preimagen). Todas las partes de la preimagen se mueven de la misma forma para coincidir con cada parte de la imagen.

Glossary/Glosario

English

increasing A function, or interval of a function, is increasing if the y-values go up when the x-values go up.

The bolded part of this function is increasing.



independent variable (input) A variable whose value is not based on the value of any other variable. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table. In a function, the independent variable is sometimes called the input.

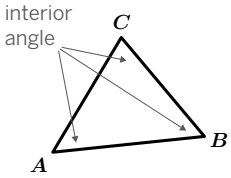
inequality A comparison statement that uses the symbols $<$ or $>$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y > 30$ means that the value of the expression y is any number greater than 30.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. Equivalent equations will create the same line on a graph, so every point on the line is a solution to each equation in the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

interior angle An angle between adjacent sides of a polygon, inside the polygon.



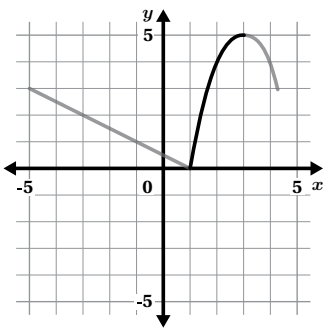
inverse operations Two operations that undo each other are called inverse operations.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

Español

creciente Una función, o un intervalo de una función, es creciente si los valores de y aumentan cuando los valores de x aumentan.

La parte resaltada de esta función es creciente.



variable independiente (entrada) Una variable cuyo valor no depende del valor de ninguna otra variable. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla. En una función, la variable independiente a veces se denomina la entrada.

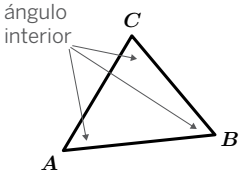
desigualdad Un enunciado de comparación que utiliza los símbolos $<$ o $>$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y > 30$ significa que el valor de la expresión y es cualquier número mayor que 30.

infinitas soluciones Una ecuación tiene un número infinito de soluciones si es verdadera independientemente del valor de la variable. Un sistema de ecuaciones tiene un número infinito de soluciones si las ecuaciones en el sistema son equivalentes. Las ecuaciones equivalentes producen la misma recta en una gráfica, por lo que cada punto en la recta es una solución de cada ecuación en el sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene un número infinito de soluciones porque la ecuación es verdadera para cualquier valor de x.

ángulo interior Un ángulo entre lados adyacentes de un polígono, dentro del polígono.



operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

English

irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas pi (π) is irrational because it cannot be written as a fraction of two non-zero integers.

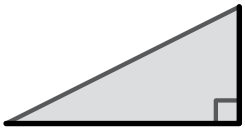
Español

número irracional Un número que no se puede escribir como una fracción de números enteros, donde el denominador es diferente de cero.

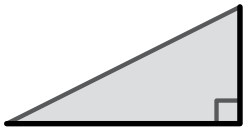
Por ejemplo, 2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que pi (π) es irracional porque no se puede escribir como una fracción de dos números enteros distintos de cero.

L

legs The two sides of a right triangle that are not the hypotenuse. The legs are the sides that form the right angle.



catetos Los dos lados de un triángulo rectángulo que no son la hipotenusa. Los catetos son los lados que forman el ángulo recto.



like terms Two or more terms that have the same variables and exponent values. Numbers, decimals, and fractions are all like terms.

For example, $8x$ and $12x$ are like terms because they both have a variable of x. $8x$ and 12 are not like terms. $8x$ and $12x^2$ are also not like terms because they have different exponents.

términos semejantes Dos o más términos que tienen variables y valores de exponentes iguales. Los números enteros, los decimales y las fracciones son términos semejantes.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable x. $8x$ y 12 no son términos semejantes. $8x$ y $12x^2$ tampoco son términos semejantes porque tienen exponentes diferentes.

line graph A set of data points plotted on a coordinate plane and connected by lines. Line graphs allow us to investigate connections between two variables, including rates of change between consecutive points or groups of points.

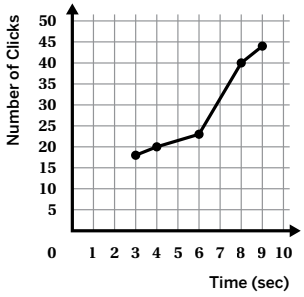
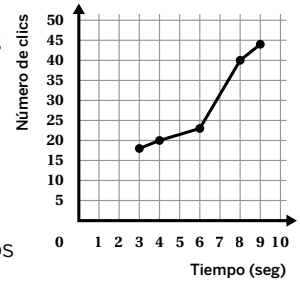


gráfico lineal Un conjunto de puntos de datos trazados en un plano de coordenadas y conectados por líneas. Los gráficos de líneas nos permiten investigar las conexiones entre dos variables, incluyendo las tasas de cambio entre puntos consecutivos o grupos de puntos.

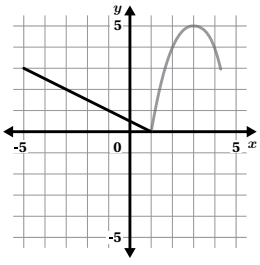


line of fit See *linear model*.

línea de ajuste Ver *modelo lineal*.

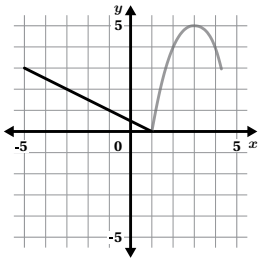
linear A function is linear when its graph is a straight, non-vertical line.

The bolded part of this function is linear.



lineal Una función es lineal cuando su gráfica es una línea recta que no es vertical.

La parte resaltada de esta función es lineal.



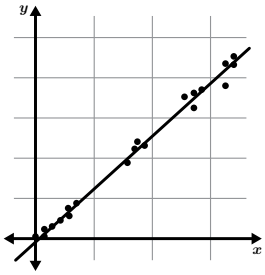
linear function A function that can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept. A vertical line is not a linear function because it has an input with different outputs.

función lineal Una función que puede definirse mediante una ecuación de la forma $y = mx + b$, donde m representa la pendiente y b representa la intersección vertical.
Una línea vertical no es una función lineal porque tiene una entrada con diferentes salidas.

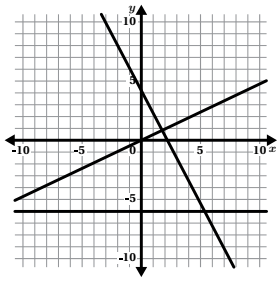
Glossary/Glosario

English

linear model A line that shows, or models, the general direction or trend of a group of points in a data set.
We can use linear models to make predictions about values related to a given data set.

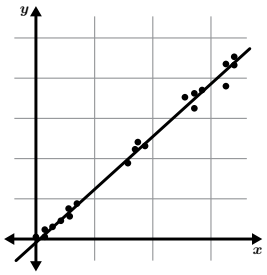


linear relationship A relationship is called linear when its graph is a line.

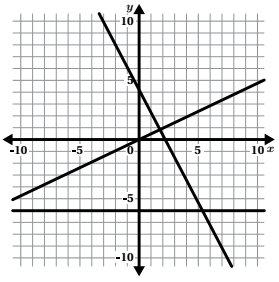


Español

modelo lineal Una línea que muestra, o modela, la dirección o tendencia general de un grupo de puntos en un conjunto de datos.
Podemos emplear modelos lineales para hacer predicciones sobre valores relativos a un conjunto de datos determinado.



relación lineal Una relación se llama lineal cuando su gráfica es una recta.



M

mapping diagram A visual representation of a relation that shows a list of inputs and outputs and uses arrows to show how they are related.

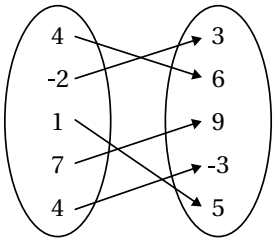
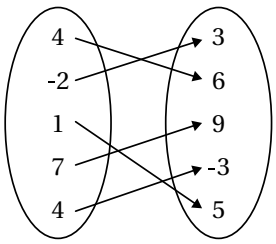


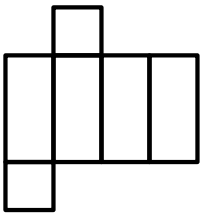
diagrama de mapeo Una representación visual de una relación que muestra una lista de entradas y salidas y utiliza flechas para mostrar cómo están relacionadas.



N

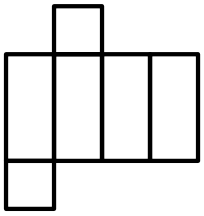
net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

Here is a net for a rectangular prism.



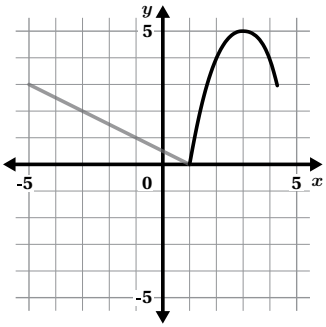
red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.

Esta es una red de un prisma rectangular.



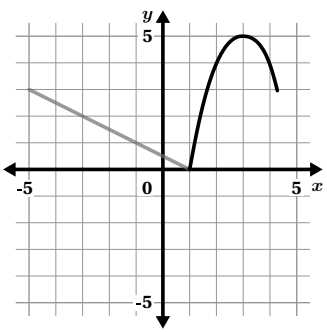
nonlinear A function is nonlinear when its graph is not a straight line.

The bolded part of this function is nonlinear.



no lineal Una función es no lineal cuando su gráfica no es una línea recta.

La parte resaltada de esta función es no lineal.



English

numerical data Data presented as numbers, quantities, or measurements that can be meaningfully compared. It is sometimes called quantitative data.

For example, data on a group of students' heights or arm spans.

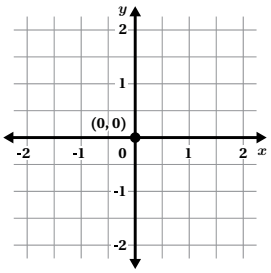
Español

datos numéricos Datos que se presentan como números, cantidades o medidas que pueden compararse de forma significativa. A veces se denominan datos cuantitativos.

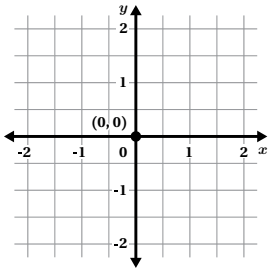
Por ejemplo, los datos de las estaturas o las longitudes de los brazos de un grupo de estudiantes.

O

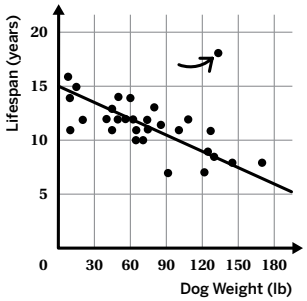
origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



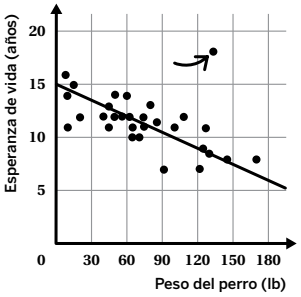
origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan el eje x y el eje y.



outlier A data value that is far from the other values in the data set.



valor atípico Un valor que está lejos de los demás valores del conjunto de datos.



P

perfect cubes The cube of an integer is called a perfect cube.

For example, 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 3^3$ and $3^3 = 27$.

cubo perfecto El cubo de un número entero se denomina cubo perfecto.

Por ejemplo, 27 es un cubo perfecto porque $3 \cdot 3 \cdot 3 = 3^3$ y $3^3 = 27$.

perfect square The square of an integer is called a perfect square.

For example, 49 is a perfect square because $7 \cdot 7 = 7^2$ and $7^2 = 49$.

cuadrado perfecto El cuadrado de un número entero se denomina cuadrado perfecto.

Por ejemplo, 49 es un cuadrado perfecto porque $7 \cdot 7 = 7^2$ y $7^2 = 49$.

pi A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π.

Some common approximations for π are 3.14 and $\frac{22}{7}$.

pi Un número que representa la constante de proporcionalidad entre el diámetro y la circunferencia de cualquier círculo. El símbolo de pi es π.

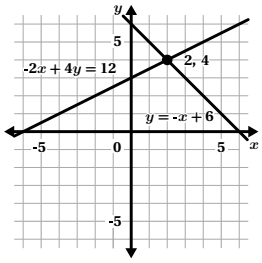
Algunas aproximaciones comunes de π son 3.14 y $\frac{22}{7}$.

Glossary/Glosario

English

point of intersection A point where two lines meet.

For example, $(2, 4)$ is the point of intersection for the lines $y = -x + 6$ and $-2x + 4y = 12$.



power of ten A number written in the form 10^n, where n represents the number of times 10 is multiplied.

pre-image The name of a figure before any transformations are performed.

prime notation Using the prime symbol (′) to name points in an image that correspond to points in the pre-image.

For example, if point A is transformed, the corresponding point would be named A'.

proportional Quantities are proportional if they form equivalent ratios.

For example, the heights and bases of these triangles are proportional because the ratio $6 : 12$ is equivalent to $4 : 8$.

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

Carpet (sq. ft.)	Cost (dollars)
10	15.00
20	30.00
50	75.00

×1.5 →

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Pythagorean theorem The theorem that describes the relationship between the side lengths of a right triangle. The Pythagorean theorem says that the square of the hypotenuse is equal to the sum of the squares of the legs. We can write this as $a^2 + b^2 = c^2$.

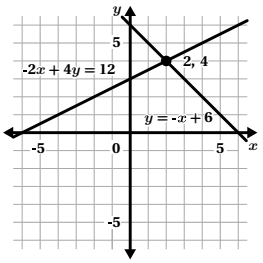
Q

quantitative data See *numerical data*.

Español

punto de intersección Un punto donde se cruzan dos rectas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



potencia de diez Un número escrito de la forma 10^n, donde n representa el número de veces que se multiplica el 10.

preimagen El nombre de una figura antes de realizar una transformación.

notación prima Uso del signo prima (′) para denominar los puntos de una imagen que corresponden a puntos de la preimagen.

Por ejemplo, si el punto A se transforma, el punto correspondiente se denominaría A'.

proporcional Diferentes cantidades son proporcionales si forman razones equivalentes.

Por ejemplo, las alturas y las bases de estos triángulos son proporcionales porque la relación $6 : 12$ es equivalente a $4 : 8$.

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
10	15.00
20	30.00
50	75.00

×1.5 →

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

teorema de Pitágoras El teorema que describe la relación entre las longitudes de lado de los triángulos rectángulos. El teorema de Pitágoras dice que el cuadrado de la hipotenusa es igual a la suma de los cuadrados de los catetos. Podemos escribirlo como $a^2 + b^2 = c^2$.

Q

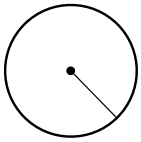
datos cuantitativos Ver *datos numéricos*.

English

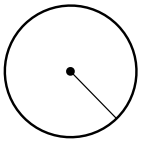
Español

R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

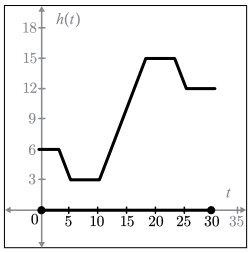


radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.



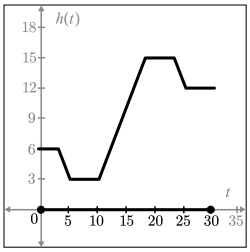
range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.

The range of this graph can be described as:
All numbers from 3 to 15.
$3 \leq h(t) \leq 15$.



rango (de una función) El conjunto de todos los posibles valores de salida de una función o relación. El rango puede describirse con palabras o como una desigualdad.

El rango de esta gráfica puede describirse de la siguiente manera:
Todos los números del 3 al 15.
$3 \leq h(t) \leq 15$



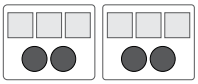
rate A comparison, or ratio, that describes how two quantities change together.

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

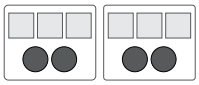
rate of change See *slope*.

tasa de cambio Véase *pendiente*.

ratio A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.



razón Una razón $a : b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.



If the ratio of apples to oranges in a fruit bowl is $2 : 3$, then for every 2 apples, there are 3 oranges.
There are several ways to describe ratios.
- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is $3 : 2$.

Si la razón de manzanas a naranjas en un frutero es $2 : 3$, entonces por cada 2 manzanas hay 3 naranjas.
Hay varias formas de describir razones.
- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es $3 : 2$.

rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

Examples of rational numbers include 13, -74, 0, 0.2, and $\sqrt{9}$.

número racional Un número que se puede escribir como fracción de números enteros, donde el denominador es diferente de cero.

Algunos ejemplos de números racionales son 13, -74, 0, 0.2 y $\sqrt{9}$.

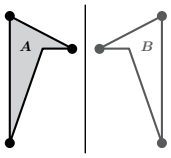
reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \times \frac{2}{3} = 1$.

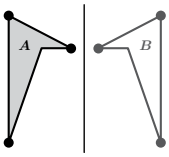
recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \times \frac{2}{3} = 1$.

reflection A reflection across a line moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.



reflexión Una reflexión con respecto a una línea mueve cada punto de una figura a un punto directamente en el lado opuesto de la línea. El nuevo punto está a la misma distancia de la línea que estaba en la figura original.

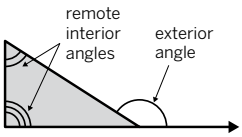


Glossary/Glosario

English

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a function.

remote interior angles The two angles that are inside the triangle and opposite from the exterior angle are the remote interior angles.



repeating decimal A decimal with one or more digits (not all zeros) that repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (...) at the end.

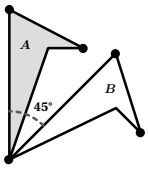
If the repeating digits are all zeros, it would be called a terminating decimal.

For example, the decimal representation of $\frac{1}{3}$ is $0.\overline{3}$, which means $0.33333...$

The decimal representation of $\frac{25}{22}$ is $1.1\overline{36}$, which means $1.1363636...$

rigid transformation A move that does not change any measurements of a figure. Translations, rotations, and reflections (or any sequence of these) are rigid transformations.

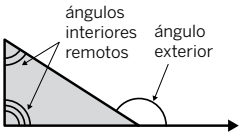
rotation A rotation moves every point on a figure around a center by a given angle in a specific direction.



Español

relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

ángulos interiores remotos Los dos ángulos que están dentro del triángulo y opuestos al ángulo exterior.



decimal periódico Un decimal con uno o más dígitos (no todos son ceros) que se repiten infinitamente. Un decimal periódico puede escribirse usando la raya indicadora de decimales periódicos (vinculum) encima de los dígitos que se repiten o con puntos suspensivos (...) al final.

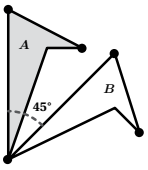
Si todos los dígitos periódicos son ceros, entonces se denomina decimal exacto.

Por ejemplo, la representación decimal de $\frac{1}{3}$ es $0.\overline{3}$, lo que significa $0.33333...$

La representación decimal de $\frac{25}{22}$ es $1.1\overline{36}$, o sea $1.1363636...$

transformación rígida Un movimiento que no cambia ninguna de las medidas de una figura. Las traslaciones, rotaciones y reflexiones son transformaciones rígidas, así como lo es cualquier secuencia de ellas.

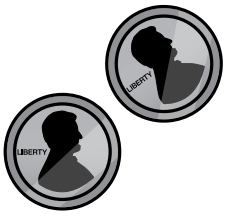
rotación Una rotación mueve cada punto en una figura alrededor de un centro hacia una dirección específica y con un ángulo determinado.



S

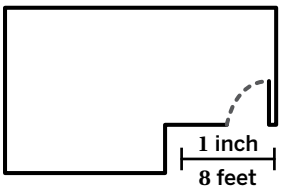
sample space The list of every possible outcome for a chance experiment.

For example, the sample space for tossing two coins is: heads-heads, tails-heads, heads-tails, tails-tails.



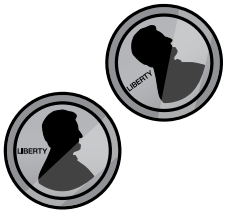
scale A scale tells us how the actual measurements of an object are represented in a drawing.

The scale of this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room.



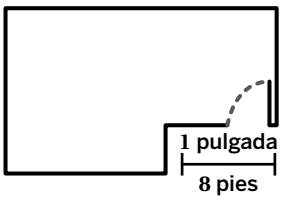
espacio muestral La lista de todos los posibles resultados de un experimento de azar.

Por ejemplo, el espacio muestral del lanzamiento de dos monedas es: cara-cara, sello-cara, cara-sello, sello-sello.



escala Una escala nos indica cómo están representadas en un dibujo o diagrama las medidas reales de un objeto.

La escala de este plano nos indica que 1 pulgada en el dibujo representa 8 pies en la habitación real.

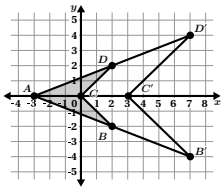


English

scale drawing A two-dimensional representation of an actual object or place. All the measurements in the drawing correspond to the measurements of the actual object by the same scale. Floor plans and maps are examples of scale drawings.

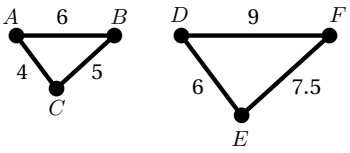
A scale may show that 1 centimeter on a map represents 30 miles on land. The distance marked on this map is 8 centimeters, or 240 miles.

scale factor The number used to create a dilation. All distances from the pre-image to the center of dilation are multiplied by the same number, called the *scale factor*.



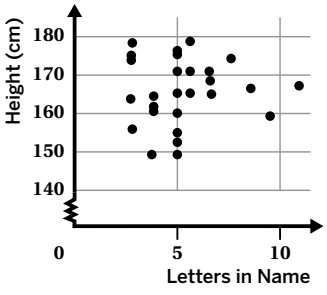
For example, the scale factor from polygon $ABCD$ to polygon $A'B'C'D'$ is 2.

scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a figure looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.



For example, triangle DEF is a scaled copy of triangle ABC.

scatter plot A set of disconnected data points plotted on a coordinate plane. Scatter plots allow us to investigate connections between two variables.



scientific notation A way to write very large or very small numbers. In scientific notation, a number between 1 and 10 is multiplied by a power of 10.

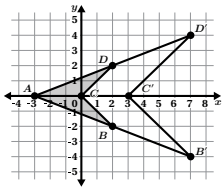
For example, the number $425{,}000{,}000$ in scientific notation is $4.25 \cdot 10^8$. The number 0.0000000783 in scientific notation is $7.83 \cdot 10^{-8}$.

Español

dibujo a escala Una representación bidimensional de un objeto o lugar reales. Todas las medidas del dibujo corresponden a las medidas del objeto real usando la misma escala. Los planos y los mapas son ejemplos de dibujos a escala.

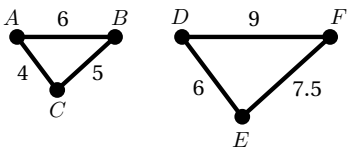
Una escala puede mostrar que 1 centímetro en un mapa representa 30 millas en tierra. La distancia mostrada en este mapa es 8 centímetros o 240 millas.

factor de escala El número que se usa para hacer una dilatación. Todas las distancias de la preimagen al centro de dilatación se multiplican por el mismo número, que se denomina *factor de escala*.



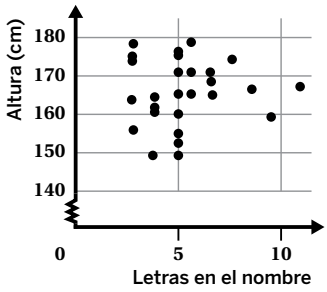
Por ejemplo, el factor de escala del polígono $ABCD$ al polígono $A'B'C'D'$ es 2.

copia a escala Una copia de una imagen que puede cambiar de tamaño, pero que siempre conserva la forma y las medidas de los ángulos de la imagen original. Si una figura luce aplastada o estirada en comparación con la original, no es una copia a escala. Para crear una copia a escala, multiplicamos todas las longitudes de la figura original por el mismo número.



Por ejemplo, el triángulo DEF es una copia a escala del triángulo ABC.

diagrama de dispersión Un conjunto de puntos de datos que no están conectados trazados en un plano de coordenadas. Los diagramas de dispersión nos permiten analizar las conexiones entre dos variables.



notación científica Una forma de escribir números muy grandes o muy pequeños. Cuando un número entre 1 y 10 está multiplicado por una potencia de 10, significa que está escrito en notación científica.

Por ejemplo, el número $425{,}000{,}000$ en notación científica es $4.25 \cdot 10^8$. El número 0.0000000783 en notación científica es $7.83 \cdot 10^{-8}$.

Glossary/Glosario

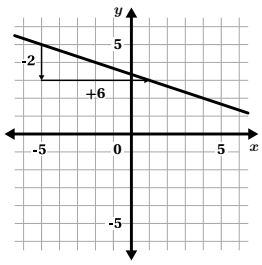
English

significant digits The important digits in a measurement that show how precise it is. Zeros in a number are only considered significant when they are between two non-zero numbers or when they are trailing zeros in a number that has a decimal point.

For example, 450 has only two significant digits, but 450.0 has four.

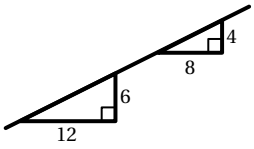
similar One figure is similar to another if translations, rotations, reflections, and dilations can be used to fit one exactly over the other. Two figures are similar if one is a scaled copy of the other.

slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.



The slope of this line is $-\frac{2}{6} = -\frac{1}{3}$.

slope triangle A triangle drawn to include two points on a line in order to determine the slope of that line. The longest side lies on the line and the other two sides are vertical and horizontal. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.



Both of these triangles are slope triangles.

solution to an equation The value or set of values that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y).

The solution to the equation $x + 15 = 8$ is $x = -7$ because $(-7) + 15 = 8$.
One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.

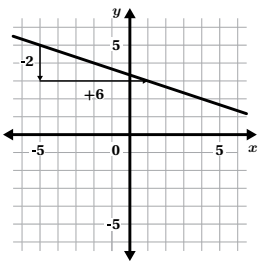
Español

dígitos significativos Los dígitos importantes en una medición que muestran cuán precisa es. Los ceros en un número solo se consideran significativos cuando están entre dos números distintos de cero o cuando son ceros finales en un número que tiene un punto decimal.

Por ejemplo, 450 tiene solo dos dígitos significativos, pero 450.0 tiene cuatro.

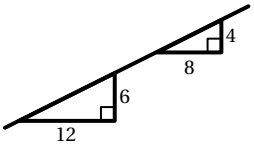
semejante Una figura es semejante a otra si se pueden emplear traslaciones, rotaciones, reflexiones y dilataciones para que coincida exactamente con la otra. Dos figuras son semejantes si una es una copia a escala de la otra.

pendiente Un número que describe la dirección e inclinación de una línea. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Es por eso que la pendiente de una recta a veces se denomina tasa de cambio. Para calcular la pendiente, la distancia vertical entre dos puntos cualesquiera en la recta se divide entre la distancia horizontal entre dichos puntos.



La pendiente de esta recta es $-\frac{2}{6} = -\frac{1}{3}$.

triángulo de pendiente Un triángulo que se traza para incorporar dos puntos de una recta y así determinar la pendiente de dicha recta. El lado más largo se sitúa sobre la recta y los dos lados restantes son uno vertical y otro horizontal. La altura del triángulo de pendiente representa la distancia vertical entre los puntos, mientras que la base del triángulo representa la distancia horizontal entre los puntos.



Estos dos triángulos son triángulos de pendiente.

solución de una ecuación El valor o conjunto de valores que hacen que la ecuación sea verdadera. Una solución de una ecuación con dos variables es un par de valores que hacen que la ecuación sea verdadera, y a menudo se escribe como un par ordenado, (x, y).

La solución de la ecuación $x + 15 = 8$ es $x = -7$ porque $(-7) + 15 = 8$.
Una solución de la ecuación $4x + 3y = 24$ es $(6, 0)$ porque $4(6) + 3(0) = 24$.

English

solution to a system of equations A set of values that makes all equations in that system true.
When the equations are graphed, the solution to the system is the point of intersection.

For example, $(2, 4)$ is the solution to this system of equations, and the point of intersection on the graph.
$y = -x + 6$
$-2x + 4y = 12$

square root A positive number that can be squared to get n. Written as $\sqrt{n}$.
The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

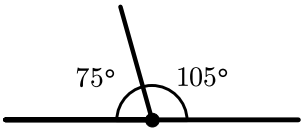
stem-and-leaf plot A table that organizes data by place value to compare data frequencies.

The stem-and-leaf plot shows the data set $(5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64)$.

Stem	Leaf
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

Key: $1 \mid 0 = 10$

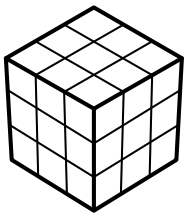
supplementary angles Two angles whose measures add up to $180°$.



For example, a $75°$ angle and a $105°$ angle are supplementary.

surface area The sum of the areas of the surfaces of a solid.

For example, the six faces of this cube each have an area of 9 square centimeters, so the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



system of equations Two or more equations that represent the constraints on a shared set of variables.

For example, these equations make up a system of equations:
$x + y = -2$
$x - y = 12$

Español

solución de un sistema de ecuaciones Un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.

Por ejemplo, $(2, 4)$ es la solución de este sistema de ecuaciones y el punto de intersección en la gráfica.
$y = -x + 6$
$-2x + 4y = 12$

raíz cuadrada Un número positivo que se puede elevar al cuadrado para obtener n. Se escribe $\sqrt{n}$.
La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

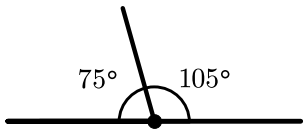
diagrama de tallo y hojas Una tabla que organiza los datos por valor posicional para comparar frecuencias de datos.

El diagrama de tallo y hojas muestra el conjunto de datos $(5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64)$.

Tallo	Hoja
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

Leyenda: $1 \mid 0 = 10$

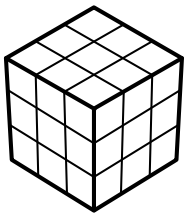
ángulos suplementarios Dos ángulos cuyas medidas suman $180°$.



Por ejemplo, un ángulo de $75°$ y otro ángulo de $105°$ son suplementarios.

área de superficie La suma de las áreas de las superficies de un sólido.

Por ejemplo, cada una de las seis caras de este cubo tiene un área de 9 centímetros cuadrados, por lo tanto, el área de superficie del cubo mide $6 \cdot 9$ o 54 centímetros cuadrados.



sistema de ecuaciones Dos o más ecuaciones que representan las restricciones en un conjunto compartido de variables.

Por ejemplo, estas ecuaciones forman un sistema de ecuaciones:
$x + y = -2$
$x - y = 12$

Glossary/Glosario

T

terminating decimal A decimal with a finite number of non-zero digits after the decimal point.

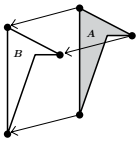
For example, 0.08, 1.5, and 0.2563 are all terminating decimals.

theoretical probability The probability that a certain outcome will occur, as determined through reasoning or calculation. It is equal to the ratio

$$\frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

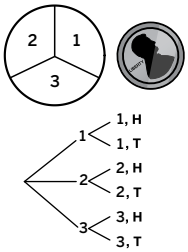
transformation An action or rule for moving or changing figures on a plane. Transformations include translations, reflections, rotations, and dilations.

translation A translation moves every point in a figure a given distance in a given direction.



tree diagram A diagram that represents all the possible outcomes in an experiment. It is often used to represent compound events.

For example, this tree diagram shows the outcomes of spinning a spinner and then flipping a coin.

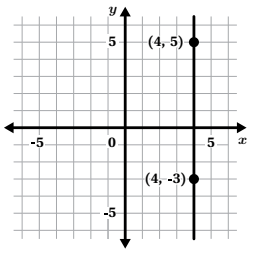


triangle inequality theorem A theorem which states that in order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

For example, a triangle can have side lengths of 4, 5, and 6, because $4 + 5 > 6$.

U

undefined An expression or term that has no understandable value. Dividing any (non-zero) value by 0 creates an undefined expression because we cannot divide by 0.



The slope of a vertical line is undefined because there is 0 horizonal change between any two points on the line. This means any vertical change would be divided by 0, which creates an undefined value.

Español

T

decimal exacto Un decimal con un número finito de dígitos distintos de cero después del punto decimal.

Por ejemplo: 0.08, 1.5 y 0.2563 son decimales exactos.

probabilidad teórica La probabilidad de que ocurra un determinado resultado, determinada mediante razonamiento o cálculo. Es igual a la razón

$$\frac{\text{número de resultados en el suceso}}{\text{número de resultados en el espacio de muestra}}$$

transformación Una acción o regla para mover o cambiar figuras en un plano. Las transformaciones incluyen traslaciones, reflexiones, rotaciones y dilataciones.

traslación Una traslación mueve cada punto de una figura una determinada distancia en una determinada dirección.

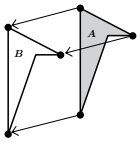
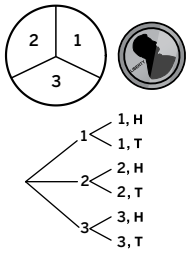


diagrama de árbol Diagrama que representa todos los resultados posibles de un experimento. A menudo se utiliza para representar eventos compuestos.

Por ejemplo, este diagrama de árbol muestra los resultados de girar una ruleta y luego lanzar una moneda.

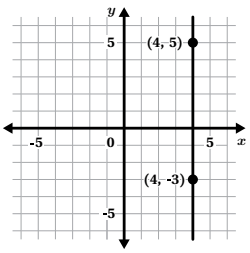


teorema de la desigualdad triangular Un teorema que establece que, para que tres segmentos de recta formen un triángulo, la suma de las longitudes de los dos segmentos más cortos debe ser mayor que la longitud del tercer segmento.

Por ejemplo, un triángulo puede tener lados de 4, 5 y 6, porque $4 + 5 > 6$.

U

indefinido Una expresión o un término que no tiene ningún valor comprensible. La división de un valor (diferente de cero) entre 0 produce una expresión indefinida porque no es posible dividir por 0.



La pendiente de una recta vertical es indefinida porque hay un cambio horizontal de 0 entre dos puntos cualesquiera en la recta. Esto significa que cualquier cambio vertical se dividiría por 0, lo cual produce un valor indefinido.

English

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

univariate data A data set that involves one variable. Each data point contains one piece of information.

A collection of students' heights is a univariate data set. Tables, dot plots, and bar graphs are useful for displaying univariate data.

Español

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

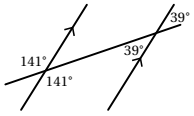
Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

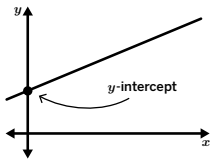
Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

vertical angles Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal.

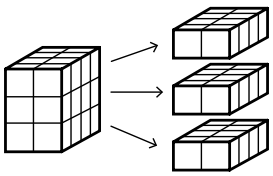


vertical intercept The point where the graph of a line crosses the vertical axis or when $x = 0$. The vertical intercept is sometimes called the y-intercept.

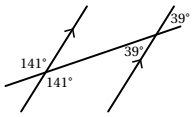


volume The number of cubic units that fill a 3-D region without any gaps or overlaps.

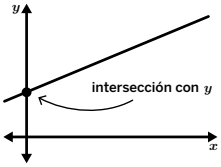
For example, the volume of this rectangular prism is 24 cubic units because it is made of 3 layers that are each 8 cubic units.



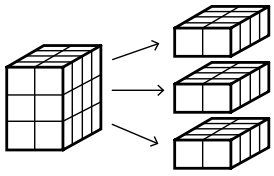
ángulos verticales Los ángulos verticales son ángulos opuestos que comparten el mismo vértice. Se forman con un par de rectas que se intersecan. Las medidas de sus ángulos son iguales.



intersección vertical El punto donde la gráfica de una recta se cruza con el eje vertical o cuando $x = 0$. La intersección vertical a veces se denomina intersección con el eje y.



volumen La cantidad de unidades cúbicas que llenan un espacio tridimensional, sin vacíos ni superposiciones.



Por ejemplo, el volumen de este prisma rectangular mide 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.

X

x-intercept See *horizontal intercept*.

intersección con el eje *x* Véase *intersección horizontal*.

Y

y-intercept See *vertical intercept*.

intersección con el eje *y* Véase *intersección vertical*.

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VOL. 1

Unit 1: Rigid Transformations and Congruence
Unit 2: Scale Factors, Dilations, Similarity, and Slope
Unit 3: Introducing Proportional Relationships
Unit 4: Linear Relationships and Equations
Unit 5: Linear Equations and Linear Systems

VOL. 2

Unit 6: Functions
Unit 7: Circles and Cylinders
Unit 8: Associations in Data
Unit 9: Exponents and Scientific Notation
Unit 10: The Pythagorean Theorem and Irrational Numbers
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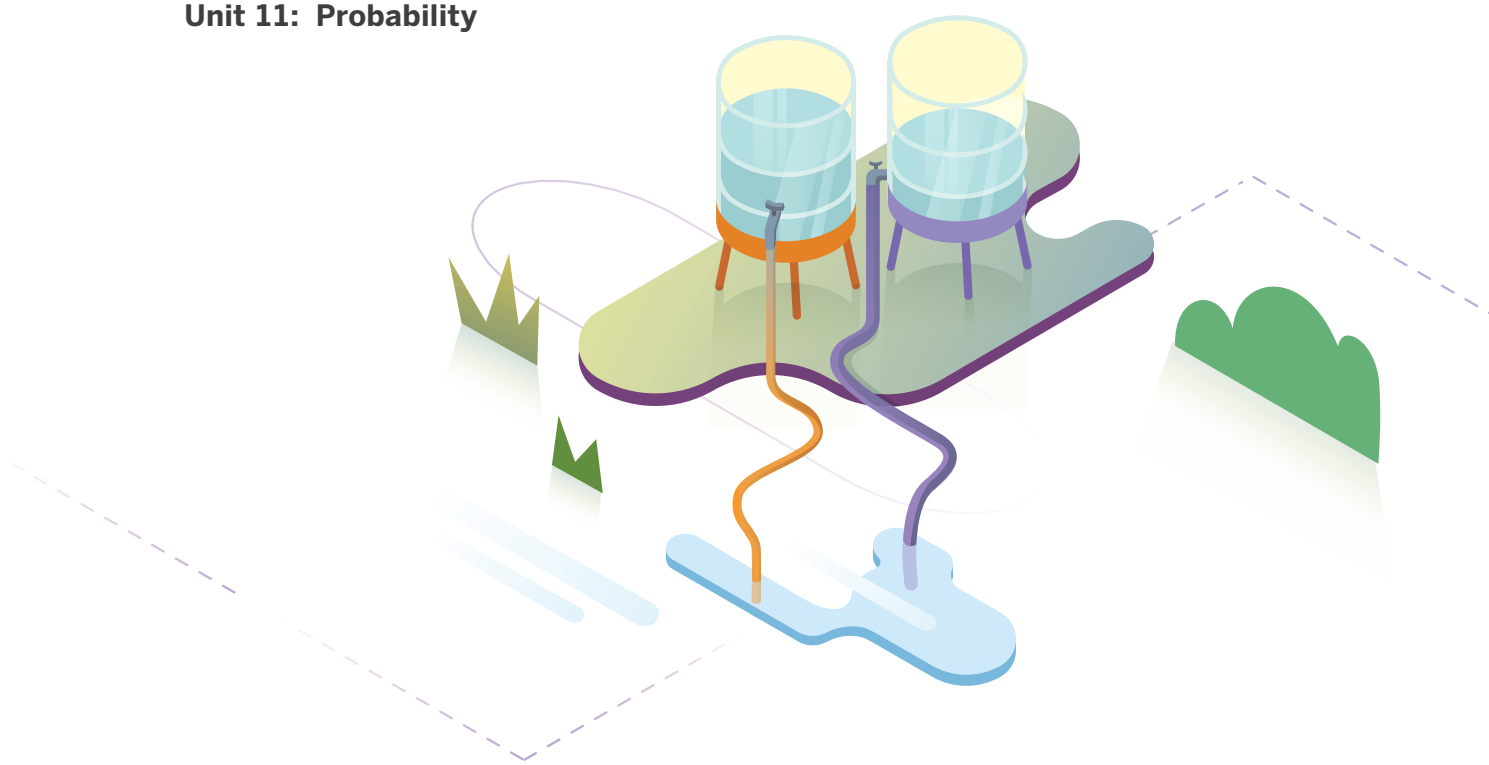

ISBN 9798895804605

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