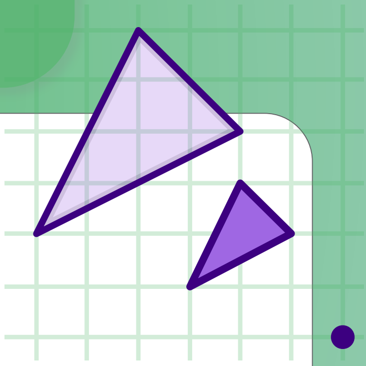
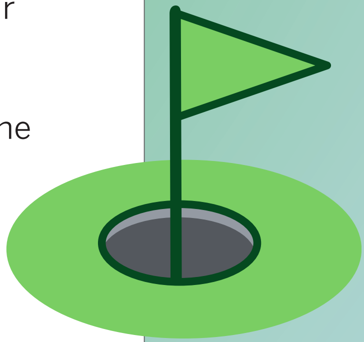


Unit 2

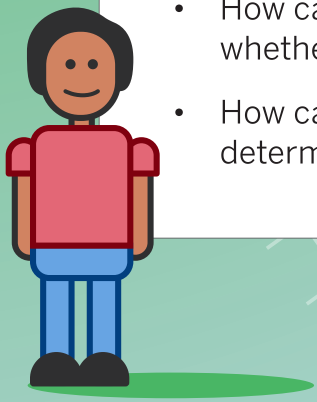
Dilations, Similarity, and Slope

We use dilations in everyday life to make objects smaller or bigger, like printing pictures in different sizes or zooming in and out on our phone screens. Unlike rigid transformations, dilations change the dimensions of a shape. We'll learn how dilations can help us determine similarity and how similarity can help us understand slope.



Essential Questions

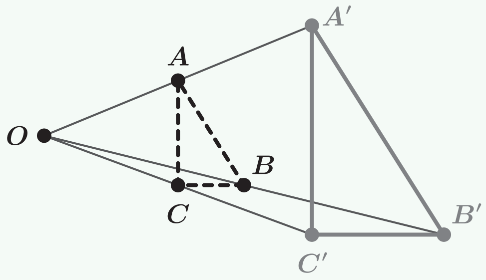
- What does it mean to dilate a figure?
- How can transformations be used to decide whether two figures are similar?
- How can similar triangles be used to determine the slope of a line?



Summary | Lesson 1

A **dilation** is a type of transformation that creates *scaled copies*. Dilating a figure means moving each of its vertices along a line that's extended from a given point. The original distance from the given point to each vertex on the *pre-image* is multiplied by the same number to create the dilated image.

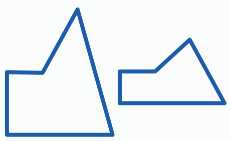
For example, Triangle ABC was dilated from the point O to create triangle $A'B'C'$.



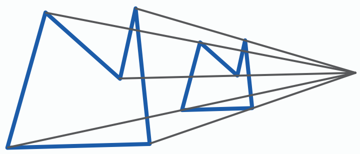
Try This

Which of these is an example of a dilation? Explain your thinking.

A.



B.



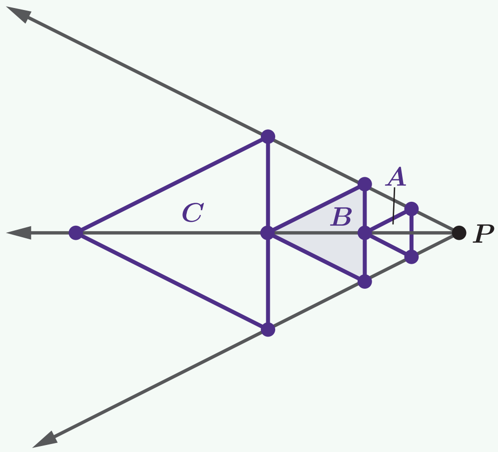
Summary | Lesson 2

A *dilation* is a transformation that involves a **center of dilation** and a **scale factor**.

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.

In this example, triangle B is the pre-image.

- Triangle A is a dilation of triangle B using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
- Triangle C is a dilation of triangle B using point P as the center of dilation and a scale factor of 2.

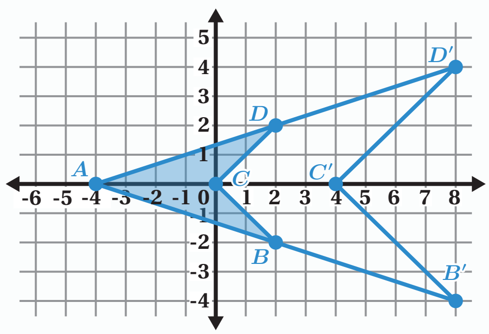


Try This

Figure $ABCD$ is dilated to create figure $A'B'C'D'$.

a What is the center of dilation?

b What is the scale factor?



Summary | Lesson 3

Dilations can be combined with other *sequences of transformations*.

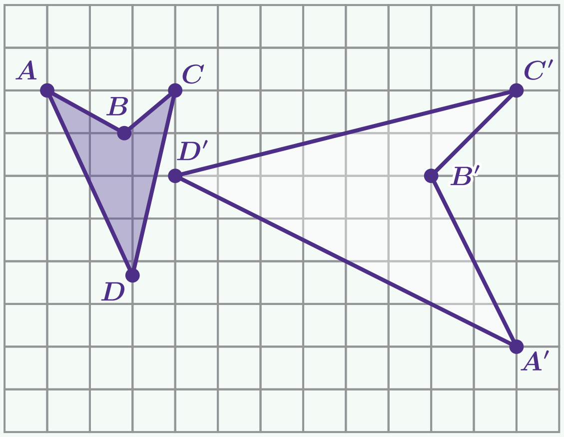
Here is a sequence of transformations that moves figure $ABCD$ onto figure $A'B'C'D'$

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2: Translate the image after Step 1 so that point D moves onto point D'.

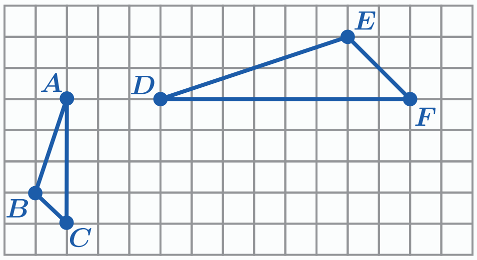
Step 3: Rotate the new image 90° clockwise around point D'.

Step 4: Reflect the new image across a horizontal line that contains points D' and B'.



Try This

Describe a sequence of transformations that moves triangle ABC onto triangle DEF.



Summary | Lesson 4

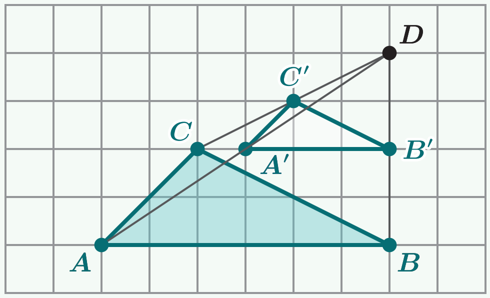
Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

If the scale factor of the dilation is:

- Greater than 1, the image will be *larger* than the pre-image and further from the center of dilation.
- Equal to 1, the image will be *the same size* as the pre-image and just as far from the center.
- Between 0 and 1, the image will be *smaller* than the pre-image and closer to the center.

For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

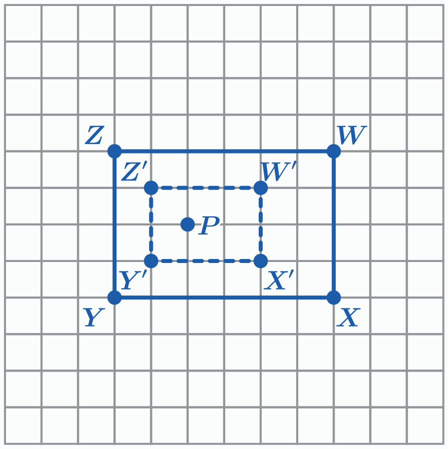
Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.



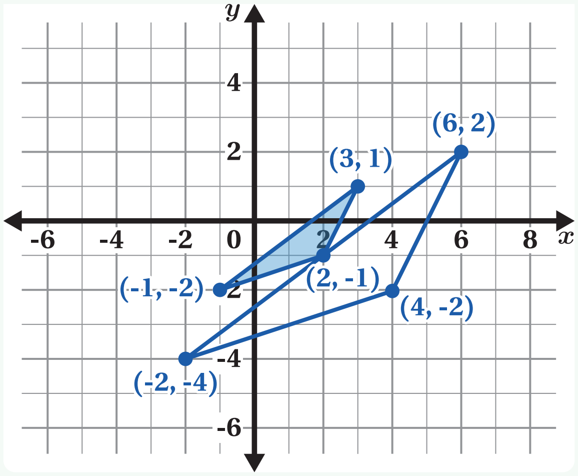
Try This

Rectangle $WXYZ$ is dilated using point P as the center of dilation to create rectangle $W'X'Y'Z'$.

Is the scale factor greater than 1, equal to 1, or between 0 and 1? Explain your thinking.



You can use coordinates to perform dilations precisely.



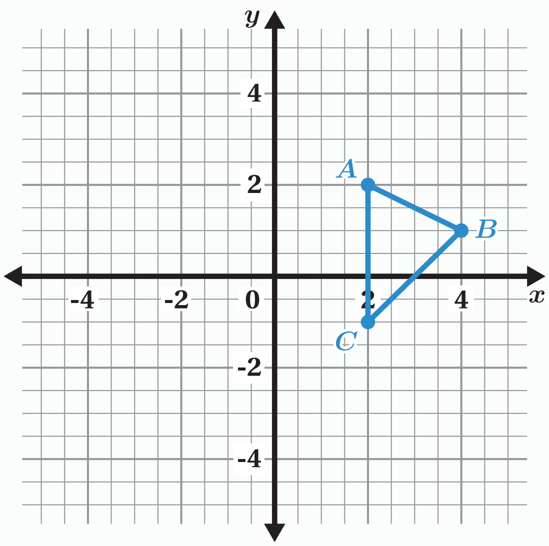
Let's say you're dilating this shaded pre-image using the center of dilation $(0, 0)$ and a scale factor of 2.

You can use the coordinate plane to measure the horizontal and vertical distances of each point from the center of dilation. Then you can multiply those distances by the scale factor to get the distances between the center of dilation and the points on the image.

In this example, the center of dilation is $(0, 0)$, so you can multiply the pre-image coordinates by the scale factor to get the image coordinates.

Try This

Dilate triangle ABC using point $(-4, 0)$ as the center of dilation and a scale factor of $\frac{1}{2}$.



Summary | Lesson 6

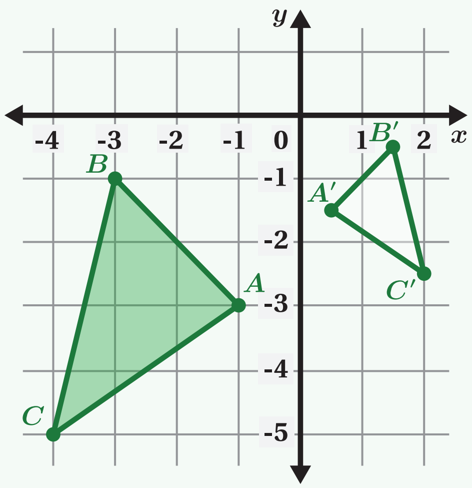
Two figures are *congruent* if one figure can be moved exactly onto the other using a sequence of rigid transformations. Two figures are **similar** if one figure can be moved exactly onto the other using a sequence of dilations and rigid transformations.

We use the symbol ~ to say that two figures are similar. For example, triangle ABC ~ triangle $A'B'C'$.

In congruent figures, the *corresponding* angles and corresponding sides are congruent.

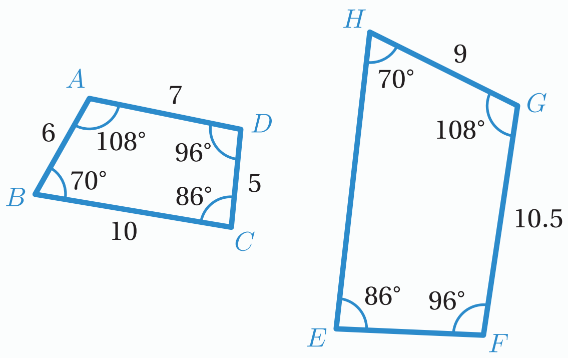
In similar figures, the corresponding angles are congruent and the corresponding side lengths are proportional but *not* always congruent.

For example, $\angle A$ is congruent to $\angle A'$, but segment AB is not the same length as segment $A'B'$. Segment $A'B'$ is twice as long as segment AB because the scale factor from triangle ABC to triangle $A'B'C'$ is 2.



Try This

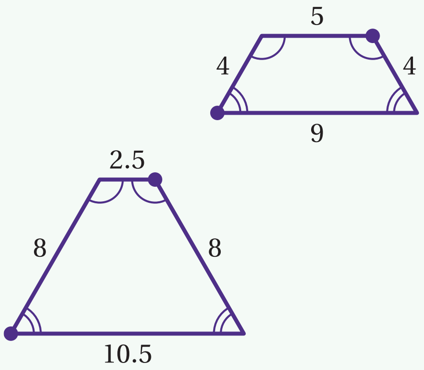
Is figure $ABCD$ similar to figure $GHEF$? Explain your thinking.



Summary | Lesson 7

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

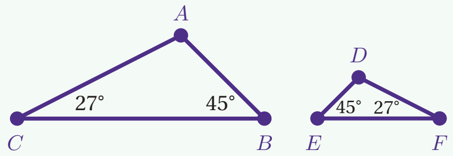
Here are two figures that have congruent corresponding angles but are not similar figures.



For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.

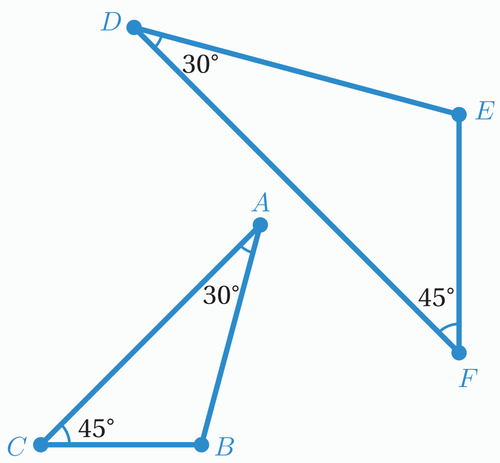
This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is $180°$ to figure out the third angle measure.

For example, the unknown angles in triangles CAB and FDE are each $108°$. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.



Try This

Is triangle ABC similar to triangle DEF? Explain your thinking.



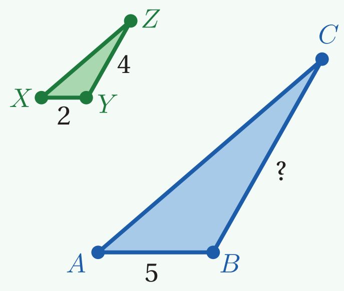
Summary | Lesson 8

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

You can also determine the ratio of two side lengths in the same triangle, then apply that ratio to the corresponding sides in a similar triangle.

Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.



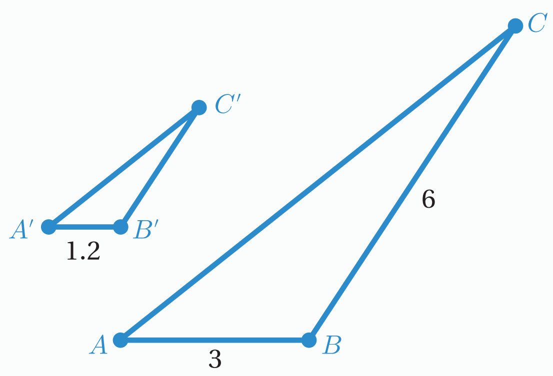
Use the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5. To calculate the length of side BC, you can multiply the length of side YZ by the scale factor. $4 \cdot 2.5 = 10$, so side BC is 10 units long.

Use ratios of side lengths within one triangle.
The ratio of side YZ to side XY is $4 : 2$, or 2. That means the length of side BC is twice the length of side AB. $5 \cdot 2 = 10$, so side BC is 10 units long.

Try This

Triangles ABC and $A'B'C'$ are similar.

Determine the length of side $B'C'$.



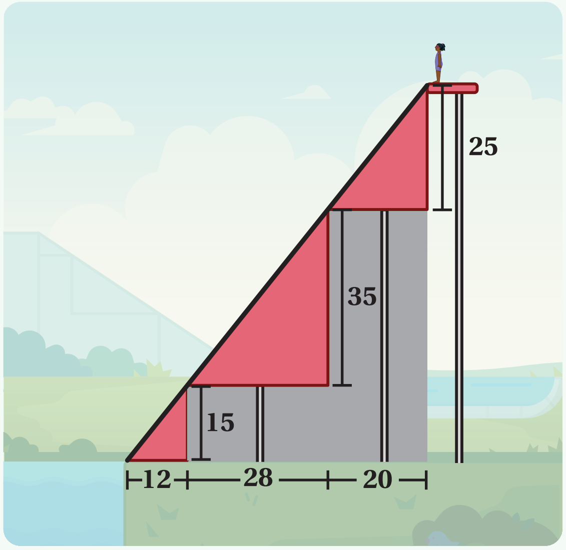
Summary | Lesson 9

Here are three ramps made up of similar triangles with proportional corresponding side lengths. Because of this, they line up perfectly to create a smooth slide.

To line up the longest side of each triangle, the **slope** of each triangle must be the same. Slope is the height-to-base *ratio* of a triangle, which describes the steepness of its longest side.

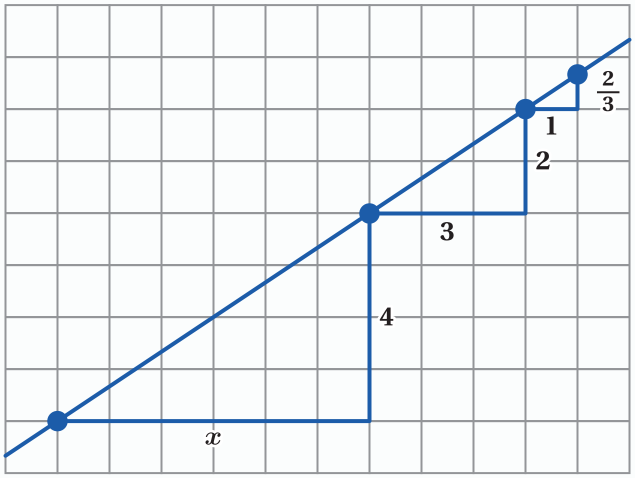
For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.

You could sketch infinite triangles on the same line that all have the same height-to-base ratio. Any of those triangles can be used to determine the slope.



Try This

What is the value of x? Explain your thinking.



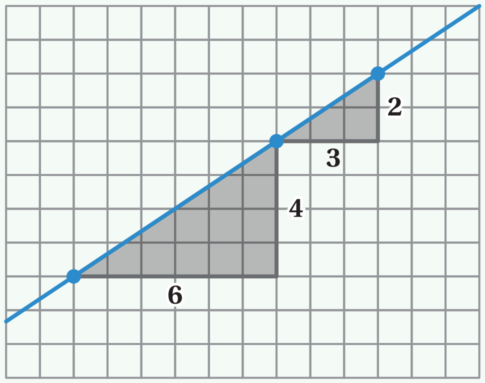
Summary | Lesson 10

You can determine the slope of a line by drawing similar right triangles, called **slope triangles**, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height of a slope triangle to its base.

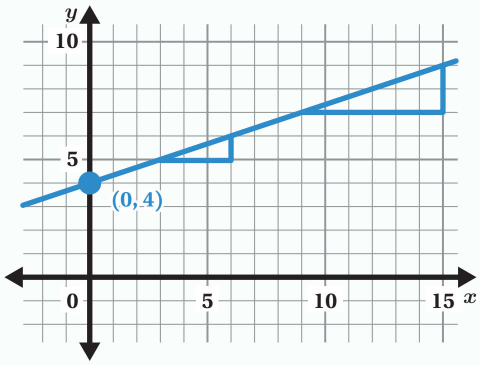
Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.



Try This

Determine the slope of this line.



Lesson 1

B. *Explanations vary.* Choice B is a dilation because the vertices are extended from a point, which results in scaled copies. The figures in choice A are not scaled copies because the side lengths have different ratios.

Lesson 2

a Point A.

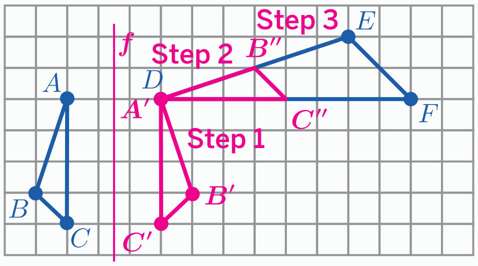
Caregiver Note: The center of dilation is point A because the points of figure $A'B'C'D'$ can be determined by drawing a straight line from point A.

b 2

Caregiver Note: The scale factor is 2 because the points in figure $A'B'C'D'$ are twice as far from A as the points in figure $ABCD$. For example, point C is 4 units from point A, while point C' is 8 units from point A.

Lesson 3

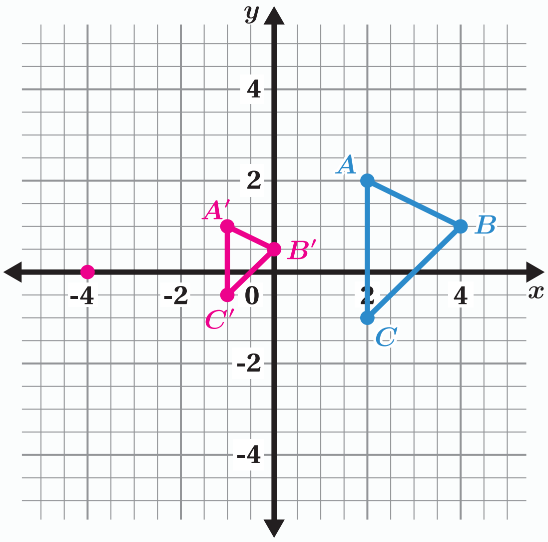
Responses vary.
Step 1: Reflect triangle ABC over a vertical line directly in between triangles ABC and DEF.
Step 2: Rotate triangle $A'B'C'$ $90°$ counterclockwise around point A'.
Step 3: Dilate triangle $A'B''C''$ using point A' as the center of dilation and a scale factor of 2.



Lesson 4

The scale factor is between 0 and 1 because the image is smaller than the pre-image and closer to the center of dilation.

Lesson 5



Lesson 6

Yes, these figures are similar. *Explanations vary.* The corresponding angle measures are congruent and the corresponding side lengths are proportional. For example, $\angle ADC$ and $\angle GFE$ are corresponding angles and both have measures of $96°$. Sides AB and GH are corresponding sides with a ratio of $6:9$ or $\frac{2}{3}$. Sides AD and GF are also corresponding sides with a ratio of $\frac{7}{10.5} = \frac{2}{3}$.

Lesson 7

Yes, triangles ABC and DEF are similar. *Explanations vary.* Since two angles of each triangle measure $45°$ and $30°$, the third angle must be $105°$, because $180 - 45 - 30 = 105$. When two triangles have three congruent angles, they are similar triangles.

Lesson 8

$B'C' = 2.4$ units.

Caregiver Note: The ratio of side AB to side BC is $1:2$ or $\frac{1}{2}$. That means side BC is two times the length of side AB, so side $B'C'$ is two times the length of side $A'B'$. Another way to determine the measure of side $B'C'$ is to notice that the ratio of side AB to side $A'B'$ is $3:1.2$. $\frac{3}{1.2} = 2.5$, so side BC is 2.5 times the length of side $B'C'$. That means $B'C'$ measures $6 \div 2.5 = 2.4$ units.

Lesson 9

$x = 6$. *Explanations vary*. The triangles all have the same slope, so their height-to-base ratios are equivalent. For example, $\frac{2}{3} = \frac{4}{6}$.

Lesson 10

The slope of the line is $\frac{1}{3}$.

Caregiver Note: There is a slope triangle on the graph with a height of 1 unit and a base of 3 units, which is a ratio of $\frac{1}{3}$.