

The Pythagorean Theorem and Irrational Numbers

There are lots of different types of triangles. What is the relationship between the sides of a right triangle? To answer this, a representation for the edge length of a square is needed. In this unit, you'll explore how side lengths of right triangles are related, and learn about new types of numbers.

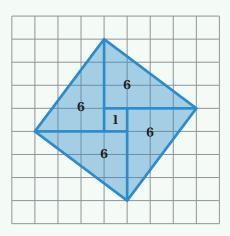
Essential Questions

- How can you estimate the square root of a number? What does the square root of a number represent?
- Is it true that leg² + leg² = hypotenuse² for all right triangles? If so, can you prove it?
- What is the difference between a rational number and an irrational number?

There are many strategies for determining the area of a tilted square. Here are two strategies called "decompose and rearrange" and "surround and subtract."

Decompose and Rearrange

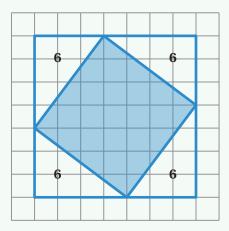
Area is calculated by adding the areas of the four triangles and one center square.



 $4 \cdot 6 + 1 = 25$ square units

Surround and Subtract

Area is calculated by finding the area of the large square minus the area of the four triangles.

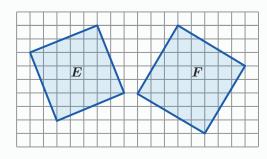


 $7 \cdot 7 - 4 \cdot 6 = 25$ square units

Try This

Use any strategy to calculate the area of each square.

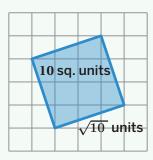
- **a** Square *E*
- **b** Square *F*



Summary | Lesson 2

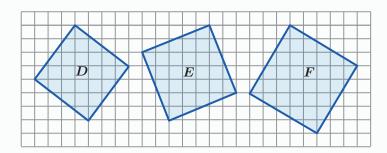
There is a known relationship between the area of any square and its side length. The exact value of the side length of a square can be written as the **square root** of its area.

For example, $\sqrt{10}$ is the exact value for the side length of a square with an area of 10 square units.



Try This

Complete the table with the missing values.

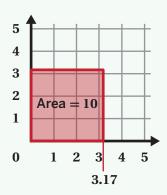


Square	Area (sq. units)	Side Length (sq. units)
D	25	
E		$\sqrt{29}$
F		

You can use several strategies to approximate the values of square roots. One strategy is to use the areas of squares. The side length of a square is equal to the square root of its area. Another strategy is to create a table of values for n and determine n^2 . Remember that $(\sqrt{n})^2 = n$. Below is a description of how each strategy can be used to approximate $\sqrt{10}$.

Using Squares

- Create a square that has an area equal to about 10 square units.
- Approximate the side length of the square created.



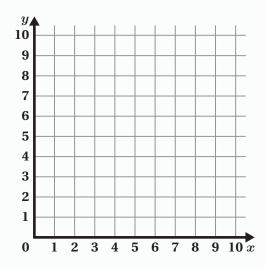
Using a Table

- Create a table of decimal value guesses for n.
- Calculate n^2 for each guess of n.
- The closer n^2 is to 10, the better that value of n is as an approximation for $\sqrt{10}$.

n	n^2
3.1	9.61
3.16	9.9856
3.17	10.0489
3.165	10.017225

Try This

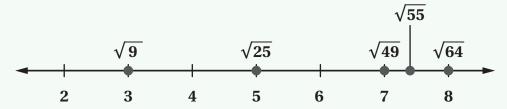
Square B has an area of 17 square units. Estimate the side length of square B. Explain your thinking. Use the graph if it helps with your thinking.



You can represent a square root on a number line. We write a solution to an equation, such as $x^2 = 3$, using square root notation. The positive solution to this equation is $x = \sqrt{3}$.

You can approximate a square root on a number line by observing the whole numbers around it.

For example, you can determine that $\sqrt{55}$ is between 7 and 8 because $7^2 = 49$ and $8^2 = 64$, and 55 is between 49 and 64. More precisely, $\sqrt{55}$ should be plotted slightly left of 7.5 since it is closer to 7 than 8.



A <u>perfect square</u> is a number that is the square of an integer. For example, 49 is a perfect square because $7 \cdot 7 = 7^2 = 49$, but 55 is not a perfect square because no integer can be squared to equal 55.

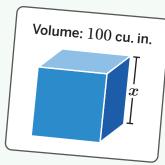
Try This

Sort each of the values into the correct category. Then plot each value on the number line.

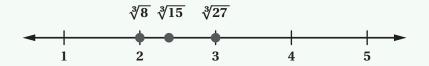


	Betwee	en 2 and 3	;	Between	4 and 5
	-		+	+	
2	2.5	3	3.5	4	4.5

A <u>cube root</u> describes the edge length of a cube given its volume. For the cube shown with a volume of 100 cubic inches, the equation $x^3 = 100$ can help you find its edge length. Its exact solution would be represented as $x = \sqrt[3]{100}$.



We can approximate a cube root on a number line by observing the whole numbers around it. For example, you can determine that $\sqrt[3]{15}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$, and 15 is between 8 and 27.



8 and 27 are perfect cubes because they are both the cube of an integer: $2 \cdot 2 \cdot 2 = 2^3 = 8$ and $3 \cdot 3 \cdot 3 = 3^3 = 27$.

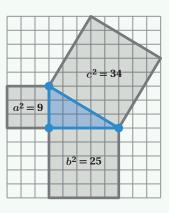
Try This

Complete the table without using a calculator.

Exact Cube Edge Length (units)	Approximate Cube Edge Length (units)	Volume of Cube (cu. units)
	Betweenand	60
$\sqrt[3]{4}$	Betweenand	
	Between and	25

The **Pythagorean theorem** says that for right triangles, $a^2 + b^2 = c^2$, where a and b represent the lengths of the two shorter sides and c represents the length of the longest side.

The relationship $a^2 + b^2 = c^2$ is only true for right triangles.



For triangle *H*:

$$(\sqrt{10})^2 + (\sqrt{17})^2 = 27$$

$$(\sqrt{10})^2 + (\sqrt{40})^2 = 50$$

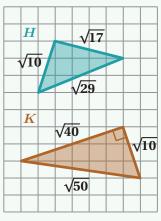
$$c^2 = (\sqrt{29})^2 = 29$$

$$c^2 = (\sqrt{50})^2 = 50$$

For triangle *K*:

$$27 \neq 29$$
 so $a^2 + b^2 = c^2$ is not true. $50 = 50$ so $a^2 + b^2 = c^2$ is true.

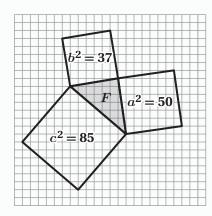
$$50 = 50$$
 so $a^2 + b^2 = c^2$ is true



Try This

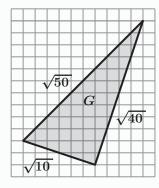
Use the Pythagorean theorem to determine if each triangle is a right triangle.





Is triangle F a right triangle? Explain your thinking.





Is triangle G a right triangle? Explain your thinking.

The Pythagorean theorem states that for any right triangle, $a^2 + b^2 = c^2$. There are many proofs for the Pythagorean theorem.

For example, you can draw squares on the sides of a right triangle, as shown in Figure 1.

In Figure 2 and Figure 3, each total area is equal to $(a + b)^2$.

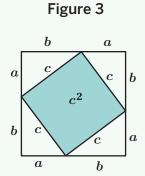
Since the area of the right triangle is equal to $\frac{1}{2}ab$, the unshaded areas of Figures 2 and 3 are equal and each sum to 2ab.

Since the total areas and the unshaded areas of Figures 2 and 3 are equal, then their shaded areas must be equal. This shows that $a^2 + b^2 = c^2$.

Figure 1 c^2 b b^2

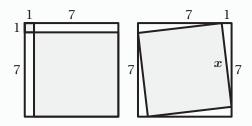
 a^2

Figure 2



Try This

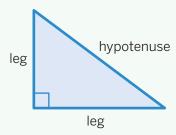
Determine the exact value of x.



Summary | Lesson 8

The **hypotenuse** is the side of a right triangle that is opposite the right angle, and is the longest side. Only right triangles have a *hypotenuse*. The **legs** of a right triangle are the sides that make the right angle.

The Pythagorean theorem says that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. This can be represented by the equation $a^2 + b^2 = c^2$, where a and b represent the lengths of the legs and c represents the length of the hypotenuse.



When any two side lengths of a right triangle are known, the Pythagorean theorem can be used to calculate the length of the third side, whether it is the hypotenuse or a leg. You can substitute the lengths you know into the equation $\log^2 + \log^2 = \text{hypotenuse}^2$ or $a^2 + b^2 = c^2$, and then solve for the unknown value.

Try This

For each triangle in the table, determine whether the missing side is a leg or a hypotenuse. Then calculate the exact length of the missing side.

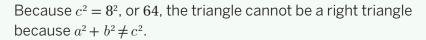
Triangle	Leg or Hypotenuse?		Missing Side Length
5	Leg	Hypotenuse	
2 6	Leg	Hypotenuse	

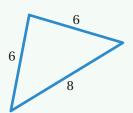
If a triangle has side lengths a, b, and c, where c is the longest side and $a^2 + b^2 = c^2$, then the converse of the Pythagorean theorem says that you must have a right triangle, according to the converse of the Pythagorean theorem. We can use this to determine whether any triangle is a right triangle. If the sides of a triangle do not make the equation $a^2 + b^2 = c^2$ true, then you know it is not a right triangle.

In the triangle shown, let a=6, b=6, and c=8. You can use substitution to determine whether the triangle is a right triangle.

$$a^2 + b^2 = 36 + 36$$

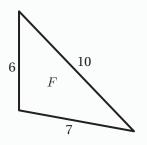
= 72





Try This

Change one of the side lengths to make triangle ${\cal F}$ a right triangle.



The Pythagorean theorem can be used to solve problems that can be modeled with right triangles. The sides of a triangle might represent units such as the length of an object or the distance between two objects.

To apply the Pythagorean theorem, the lengths of two sides must be known so the length of the third side can be determined.

For example, you can use the Pythagorean theorem to calculate the distance to walk through the park from point A to point B.



Try This

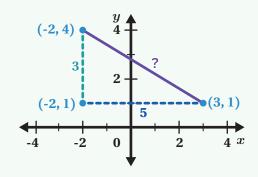
A 17-foot ladder is leaning against a wall. The ladder can reach a window 15 feet up the wall.

a Draw a picture of the situation.

b How far should the base of the ladder be from the wall so that it reaches the window? Show your thinking. You can use the Pythagorean theorem to calculate the distance between two points that are on a diagonal line segment. To do this, start by drawing horizontal and vertical legs to form a right triangle. Then use the Pythagorean theorem to calculate the length of the hypotenuse, which will be the distance between the two points.

When two points are on a horizontal line segment, you can calculate the distance between them by determining the absolute value of the difference between their x-coordinates. For the points (-2, 1) and (3, 1) the distance is |-2 - 3| = 5 units.

Similarly, when two points are on a vertical line segment, you can calculate the distance between them by determining the absolute value of the distance between their y-coordinates. For the points (-2, 4) and (-2, 1) the distance is |4 - 1| = 3 units.



$$leg^2 + leg^2 = hypotenuse^2$$

$$3^2 + 5^2 = x^2$$

$$9 + 25 = x^2$$

$$34 = x^2$$

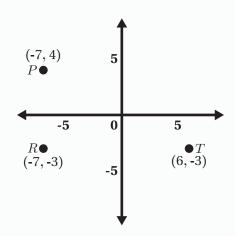
$$\sqrt{34} = x$$

Try This

Calculate the length of each segment.

a Segment
$$PR =$$
____units

b Segment
$$RT =$$
____units



You can write every number as a decimal. Some fractions can be written as terminating decimals, while others can be written as repeating decimals. To write a fraction as a decimal, you can use long division.

For example, here is how you can use long division to rewrite $\frac{1}{15}$ as a decimal.

To avoid writing the repeating part of a decimal over and over, you can use **bar notation**, which shows a line over the part of the decimal that repeats. For example, when writing $\frac{1}{15}$ as a decimal, you would write 0.06666... as $0.0\overline{6}.$

0.0 15)1.0 -9	
1	.00
-	90
	100
	-90
	100
	-90
	10

Try This

a Sort the fractions based on whether they are terminating or repeating.

 $\frac{3}{8}$

 $\frac{3}{11}$

 $\frac{98}{6}$

Terminating	Repeating

b Describe the strategy you used to determine whether each fraction represents a terminating or repeating decimal.

You can express all repeating decimals as fractions. One way to do this is to multiply equations by factors of 10 until the repeating decimals can subtract to 0. Once the repetition is removed, the resulting equation can be solved and left in fraction form.

For example, see these steps to represent $0.\overline{57} = 0.575757575...$ as a fraction.

If a decimal expansion of a number is a repeating or terminating decimal, the number can be written as a fraction. If the digits in the decimal expansion do not repeat (non-repeating) and do not terminate (non-terminating), then the number cannot be written as a fraction.

$x = 0.\overline{57}$
$10x = 5.\overline{75}$
$100x = 57.\overline{57}$
$100x = 57.\overline{57}$
$-(x=0.\overline{57})$
99x = 57
$x = \frac{57}{99}$

 $0.\overline{57} = \frac{57}{99}$

Try This

a Match each repeating decimal to its equivalent fraction. One decimal does not have a match.

Repeating Decimal	Fraction
a. 5.37	<u>48</u>
b. 5.3	34
c. $0.\overline{53}$	<u>532</u> 99
d. $0.3\overline{7}$	

b Use any strategy to convert the remaining repeating decimal into a fraction.

A <u>rational number</u> is a number that can be written as a fraction of two non-zero integers. An <u>irrational number</u> is a number that cannot be written as a fraction of two non-zero integers.

Here are some examples of rational and irrational numbers.

Examples of Rational Numbers

- Fractions: $\frac{10}{5}$, $3\frac{11}{20} = \frac{71}{20}$
- Terminating decimals: $1.5 = \frac{3}{2}$, $1.73 = \frac{173}{100}$
- Repeating decimals: $1.\overline{73} = \frac{172}{99}$, $0.1212... = \frac{12}{99}$
- Square roots of perfect squares and cube roots of perfect cubes: $\sqrt[3]{8} = \frac{2}{1}, \sqrt{64} = \frac{8}{1}, \sqrt{\frac{1}{9}} = \frac{1}{3}$

Examples of Irrational Numbers

- Non-terminating, non-repeating decimals: π , 0.743..., 2.742050...
- Square roots of non-perfect squares and cube roots of non-perfect cubes: $\sqrt{2}$, $3 \cdot \sqrt{5}$, $\sqrt[3]{9}$

Try This

- a Select all the irrational numbers.
 - \Box A. $\frac{5}{9}$

□ **B.** $\sqrt{3}$

□ **C**. $-\sqrt{25}$

 \Box **D.** $0.2\overline{4}$

□ **E.** $\sqrt[3]{25}$

- □ **F**. -π
- **b** Remy says $\sqrt{27}$ is a rational number because $3^3 = 27$. Mio states that $\sqrt{27}$ is not rational because 27 is not a perfect square. Who do you agree with? Explain your thinking.

- a 29 square units
- **b** 34 square units

Lesson 2

Square	Area (sq. units)	Side Length (sq. units)
D	25	5
E	29	$\sqrt{29}$
F	34	$\sqrt{34}$

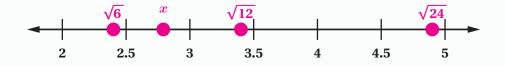
Lesson 3

Responses vary. Square B has an exact side length of $\sqrt{17}$, which lies between 4 and 5. Since $\sqrt{17}$ is closer to $\sqrt{16}$ than it is to $\sqrt{25}$, we can assume that $\sqrt{17}$ is closer to 4 than it is to 5. Using a table and calculator technology, $\sqrt{17}\approx 4.12$.

n	n^2
4	16
4.1	16.81
4.14	17.1396
4.12	16.9744
4.13	17.0569

Lesson 4

Between 2 and 3	Between 4 and 5
$\sqrt{6}$ x when $x^2 = 8$	$\sqrt{24}$



Exact Cube Edge Length (units)	Approximate Cube Edge Length (units)	Volume of Cube (cu. units)
∛60	Between <u>3</u> and <u>4</u>	60
$\sqrt[3]{4}$	Between <u>1</u> and <u>2</u>	4
∛25	Between <u>2</u> and <u>3</u>	25

Lesson 6

- a No. *Explanations vary*. Because $37 + 50 \neq 85$, the Pythagorean theorem $(a^2 + b^2 = c^2)$ is not true for this triangle.
- **b** Yes. *Explanations vary*. The Pythagorean theorem is true for this triangle. $(\sqrt{10})^2 + (\sqrt{40})^2 = (\sqrt{50})^2$ 10 + 40 = 50

Lesson 7

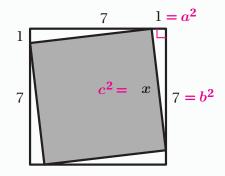
$$a^{2} + b^{2} = c^{2}$$

$$1^{2} + 7^{2} = x^{2}$$

$$1 + 49 = x^{2}$$

$$50 = x^{2}$$

$$x = \sqrt{50}$$



Triangle	Leg or Hypotenuse?	Missing Side Length
5/12	Leg Hypotenuse	$5^{2} + 12^{2} = c^{2}$ $25 + 144 = c^{2}$ $169 = c^{2}$ $13 = c$
2 6	(Leg) Hypotenuse	$2^{2} + b^{2} = 6^{2}$ $4 + b^{2} = 36$ $-4 \qquad -4$ $b^{2} = 32$ $b = \sqrt{32}$

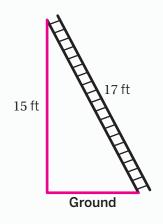
Lesson 9

Responses vary.

- Change 7 to 8, then $6^2 + 8^2 = 10^2$.
- Change 6 to $\sqrt{51}$, then $(\sqrt{51})^2 + 7^2 = 10^2$.
- Change 10 to $\sqrt{85}$, then $6^2 + 7^2 = (\sqrt{85})^2$.

Lesson 10





b 8 feet
$$d^{2} + 15^{2} = 17^{2}$$
$$d^{2} + 225 = 289$$
$$d^{2} = 64$$
$$d = \sqrt{64}$$
$$d = 8$$

a Segment PR = 4 - (-3) = 7 units

b Segment RT = 6 - (-7) = 13 units

c Segment $PT = \sqrt{7^2 + 13^2} = \sqrt{49 + 169} = \sqrt{218}$ units

Lesson 12

а	Terminating	Repeating
	$\frac{3}{8} = 0.375$	$\frac{3}{11} = 0.\overline{27} \\ \frac{98}{6} = 16.\overline{3}$

b Responses vary. I used long division to determine whether each decimal was terminating or repeating. When I used long division to divide 3 into 8, there were no remainders left after a certain point, which makes the decimal representation of $\frac{3}{8}$ a terminating decimal. But when I divided 3 by 11 and when I divided 98 by 6, I eventually got to the point where I continued to get the same remainder, creating a pattern of repeating decimals.



d
$$\frac{34}{90}$$

$$\frac{532}{99}$$

b Responses vary.

$$x = 0.\overline{53}$$

$$x = 52.\overline{52}$$

$$100x = 53.\overline{53}$$

$$100x = 53.\overline{53}$$

$$-(x = 0.\overline{53})$$

$$99x = 53$$

$$x = \frac{53}{99}$$
 (or equivalent)

Lesson 14

a B.
$$\sqrt{3}$$

E.
$$\sqrt[3]{25}$$

b Mio. *Explanations vary*. 27 is a perfect cube and not a perfect square. If the number had been 3√27, then Remy would be correct.

Grade 8 Unit 8 Glossary/8.º grado Unidad 8 Glosario

English

A

Español

approximation A rounded value that you can use to represent a number that may be difficult to work with, such as an irrational number or a repeating decimal.

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un número irracional o un decimal periódico.

For example, the exact value of pi (π) is an irrational number, so we often use the approximate value of 3.14 in calculations involving pi.

Por ejemplo, el valor exacto de pi (π) es un número irracional, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

raya indicadora de decimales periódicos (vinculum) Una forma de representar los dígitos que se repiten en un número decimal. Sobre los dígitos que se repiten se traza una pequeña línea.

For example, the repeating decimal 0.090909... can be written in bar notation as $0.\overline{09}$.

Por ejemplo, el decimal periódico 0.090909... puede escribirse con la raya indicadora de decimales periódicos (vinculum) como $0.\overline{09}$.

congruent One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

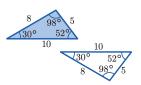
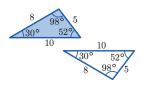


figura es congruente con otra si se puede mover por medio de traslaciones,

congruente Una



rotaciones y reflexiones de forma tal que coincida exactamente con la otra.

converse A mathematical statement written in the opposite direction.

converso Una expresión escrita en sentido contrario.

For example, the Pythagorean theorem states: If a triangle is right, it has side lengths such that $a^2+b^2=c^2$. The converse of the Pythagorean theorem is: If a triangle has side lengths such that $a^2+b^2=c^2$, it is a right triangle.

Por ejemplo, el teorema de Pitágoras indica lo siguiente: Si un triángulo es rectángulo, tiene longitudes de lado tales que $a^2 + b^2 = c^2$. El coverso del teorema de Pitágoras sería lo siguiente: Todo triángulo que tiene longitudes de lado tales que $a^2 + b^2 = c^2$, es un triángulo rectángulo.

English

cube root The number that can be cubed to get n, written as $\sqrt[3]{n}$.

The cube root also describes the edge length of a cube with a volume of n.

For example, the cube root of $64 \left(\sqrt[3]{64}\right)$ is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

hypotenuse The side of a right triangle that is opposite the right angle. The

hypotenuse is always the longest side of a right triangle.

irrational number A number that cannot be written as a fraction with integers as the numerator and denominator.

For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas pi (π) is irrational because it cannot be written as a fraction of two integers.

legs The two sides of a right triangle that are not the hypotenuse. The legs are the sides that form the right angle.

perfect cubes The cube of an integer is called a perfect cube.

For example, 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 3^3$ and $3^3 = 27$.

Español

raíz cúbical El número que se puede elevar al cubo para obtener n. Se escribe $\sqrt[3]{n}$. La raíz cúbica también describe la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de $64 \left(\sqrt[3]{64}\right)$ es 4 porque 4^3 es 64. 4 también es la longitud de lado de un cubo que tiene un volumen de 64 unidades cúbicas.

hipotenusa El lado del triángulo rectángulo que está opuesto al ángulo recto. La

hipotenusa siempre es el lado más largo de un triángulo rectángulo.

número irracional Un número que no se puede escribir como una fracción de dos enteros diferentes de cero.

Por ejemplo, 2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que pi (π) es irracional porque no se pueden escribir como una fracción de dos números enteros distintos de cero.

catetos Los dos lados de un triángulo rectángulo que no son la hipotenusa.

Los catetos son los lados que forman el ángulo recto.

cubo perfecto El cubo de un número entero se denomina cubo perfecto.

Por ejemplo, 27 es un cubo perfecto porque $3 \cdot 3 \cdot 3 = 3^3$ y $3^3 = 27$.

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English

perfect square The square of an integer is called a perfect square.

For example, 49 is a perfect square because $7 \cdot 7 = 7^2$ and $7^2 = 49$.

Pythagorean theorem The theorem that describes the relationship between the side lengths of a right triangle.

The Pythagorean theorem says that the square of the hypotenuse is equal to the sum of the squares of the legs. We can write this as $a^2 + b^2 = c^2$.

rational number A number that can be written as a fraction with a non-zero denominator.

Examples of rational numbers include 13, -74, 0, and 0.2.

repeating decimal A decimal with one or more digits (not all zeros) that repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (...) at the end. If the repeating digits are all zeros, this would be called a terminating decimal.

For example, the decimal representation of $\frac{1}{3}$ is $0.\overline{3}$, which means 0.33333...

The decimal representation of $\frac{25}{22}$ is $1.1\overline{36}$, which means 1.1363636...

Español

cuadrado perfecto El cuadrado de un número entero se denomina cuadrado perfecto.

Por ejemplo, 49 es un cuadrado perfecto porque 7 • $7 = 7^2 \vee 7^2 = 49$.

teorema de Pitágoras El teorema que describe la relación entre las longitudes de lado de los triángulos rectángulos. El teorema de Pitágoras dice que el cuadrado de la hipotenusa es igual a la suma de los cuadrados de los catetos. Podemos escribirlo como $a^2 + b^2 = c^2$.

número racional Un número que se puede escribir como fracción de dos

enteros distintos de cero.

Algunos ejemplos de números racionales son 13, -74, 0 y 0.2.

decimal periódico Un decimal con uno o más dígitos (no todos son cero) que se repiten infinitamente. Un decimal periódico puede escribirse usando la raya indicadora de decimales periódicos (vinculum) encima de los dígitos que se repiten o con puntos suspensivos (...) al final.

Si todos los dígitos que se repiten son cero, entonces se denomina decimal exacto.

Por ejemplo, la representación decimal de $\frac{1}{3}$ es $0.\overline{3}$, lo que significa 0.33333... La representación decimal de $\frac{25}{22}$ es $1.1\overline{36}$, lo que

significa 1.1363636...

English

square root A positive number that can be squared to get n. Written as \sqrt{n} .

The square root is also the side length of a square with an area of n.

The square root of $16 (\sqrt{16})$ is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

terminating decimal A decimal with a finite number of digits after the decimal point (not all zeros).

For example, 0.08, 1.5, and 0.2563 are all terminating decimals.

unit fraction A fraction with a numerator of 1 and a denominator that is a non-zero integer.

raíz cuadrada Un número positivo que se puede elevar al cuadrado para obtener n. Se escribe \sqrt{n} .

La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de $16 \left(\sqrt{16} \right)$ es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

decimal exacto Un decimal con un número finito de dígitos distintos de cero después del punto decimal.

Por ejemplo, 0.08, 1.5 y 0.2563 son decimales exactos.

fracción unitaria Una fracción con numerador 1 y denominador entero distinto de cero.