Amplify Desmos Math FLORIDA

Teacher Edition

GRADE 7

Volume 2



Amplify Desmos Math FLORIDA

Grade 7

Volume 2: Units 5–8

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from the English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Teacher,

Welcome to Amplify Desmos Math Florida! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

- A program designed around the **Florida B.E.S.T. Standards for Mathematics**, grounded in the strands, standards, and benchmarks, and the importance of student exploration and investigation.
- **Deep and intentional alignment** to the Florida B.E.S.T. standards, Florida English Language Arts standards for K-12 Mathematics, and Florida English Language Development Standards related to the Language of Mathematics.
- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Manipulatives, visual models, discussions, and drawings to encourage perseverance and revision.
  - » Easy-to-follow lesson plans, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency, help students review previous topics, and gain experience with FAST Assessment problem types.
- **Recommended differentiation moves** that meet the needs of all learners.
- **Diagnostic, formative, and summative assessments** along with lesson-level checks for understanding.
- **A caregiver resource** for each unit that includes explanations of key math concepts and problems to try with your student.

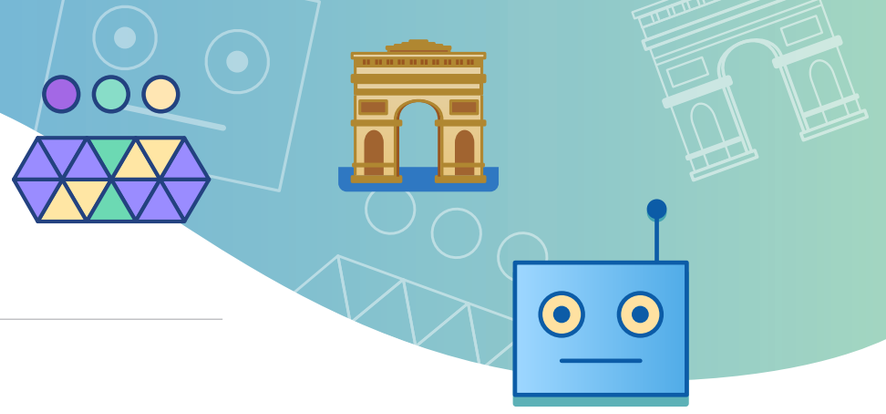
We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math Florida team



Unit 1 Scale Drawings

Students examine and create scaled copies of figures, then explore scale drawings of real-life objects.



Pre-Unit Check 1L

Sub-Unit 1 Scaled Copies 2

		B.E.S.T. Benchmark Connection
1.01	Scaling Machines | What Are Scaled Copies? 3A	1.01: MA.7.GR.1.5, MTR.4.1, MTR.5.1
1.02	Scaling Robots | Lengths and Scaled Copies 12A	1.02: MA.7.GR.1.5, MTR.4.1, MTR.5.1
1.03	Make It Scale | Drawing Scaled Copies 19A	1.03: MA.7.GR.1.5, MTR.4.1
1.04	Scale Factor Challenges | Effects of Scale Factors 26A	1.04: MA.7.GR.1.5, MTR.4.1, MTR.5.1
1.05	Tiles | Scaling and Area 34A	1.05: MA.7.GR.1.5, MTR.1.1, MTR.3.1, MTR.4.1

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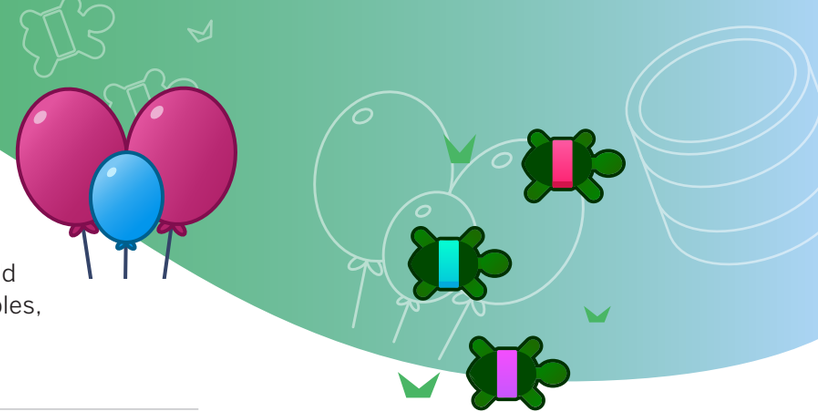
Sub-Unit 2 Scale Drawings 42

		B.E.S.T. Benchmark Connection
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1.07	Will It Fit? | Scale Drawings 49A	1.07: MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1
1.08	Scaling States, Part 1 | Creating Scale Drawings 56A	1.08: MA.7.GR.1.5, MTR.4.1, MTR.7.1
1.09	Scaling States, Part 2 | Creating Scale Drawings 62A	1.09: MA.7.GR.1.5, MTR.5.1, MTR.7.1
1.10	Scaling Buildings | Same Object, Different Scales 68A	1:10: MA.7.GR.1.5, MTR.2.1, MTR.4.1, MTR.5.1

End-of-Unit Assessment 75A

Unit 2 Introducing Proportional Relationships

Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.



Pre-Unit Check 77L

Sub-Unit 1 Proportional Relationships in Tables 78

B.E.S.T. Benchmark Connection
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2.02 Balloon Float | Introducing Proportional Relationships With Tables ..... 86A — **2.02: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2,** MTR.4.1, MTR.5.1
2.03 Sugar, Spice, and Everything Rice | Constant of Proportionality ..... 94A — **2.03: MA.7.AR.3.2, MA.7.AR.4.1, MA.7.AR.4.2,** MTR.2.1, MTR.6.1

Sub-Unit 2 Proportional Relationships in Equations 100

B.E.S.T. Benchmark Connection
2.04 Robot Factory | Proportional Relationships and Equations ..... 101A — **2.04: MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4,** MTR.2.1, MTR.3.1
2.05 Snapshots | More Equations of Proportional Relationships ..... 107A — **2.05: MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.2.1, MTR.7.1
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2.09 Gallon Challenge | Interpreting Graphs of Proportional Relationships ..... 137A — **2.09: MA.7.AR.3.2, MA.7.AR.4.2,** MTR.3.1, MTR.7.1
2.10 Three Turtles | Proportional Relationships and Graphs ..... 144A — **2.10: MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.1.1, MTR.2.1, MTR.4.1

Sub-Unit 4 Using Proportional Relationships 152

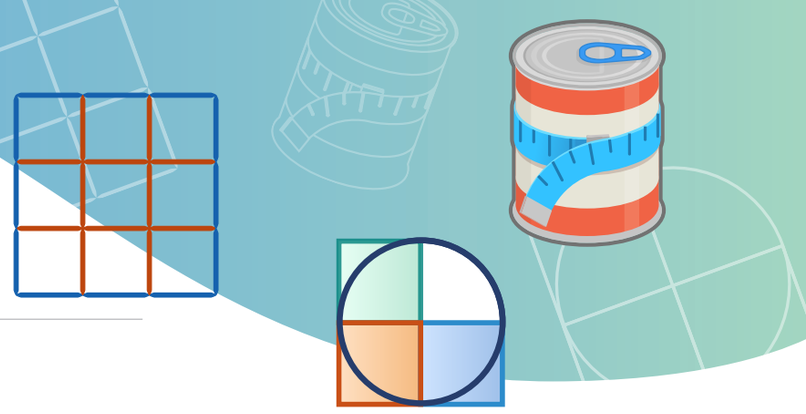
B.E.S.T. Benchmark Connection
2.11 Four Representations | Connecting Descriptions, Tables, Graphs, and Equations ..... 153A — **2.11: MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4,** MTR.2.1, MTR.7.1
2.12 Water Efficiency, Part 1 | Let's Put It to Work ..... 159A — **2.12: MA.7.AR.3.2, MA.7.AR.4.2 MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.1.1, MTR.7.1
2.13 Water Efficiency, Part 2 | Let's Put It to Work ..... 165A — **2.13: MA.7.AR.3.2, MA.7.AR.4.3, MA.7.AR.4.4, MA.7.AR.4.5,** MTR.2.1

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End-of-Unit Assessment 171A

Unit 3 Measuring Circles

Students explore the relationships between the radius, diameter, circumference, and area of a circle.



Pre-Unit Check 173L

Sub-Unit 1 Circumference of a Circle 174

B.E.S.T. Benchmark Connection
3.01: Building Toward MA.7.GR.1.3, MTR.5.1, MTR.6.1
3.02: MA.7.GR.1.3, MTR.4.1
3.03: MA.7.GR.1.3, MTR.1.1, MTR.4.1, MTR.6.1

3.01 Toothpicks | Perimeter and Proportional Relationships 175A

3.02 Is It a Circle? | Parts of a Circle 182A

3.03 Measuring Around | Estimating and Calculating Circumference 189A

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Sub-Unit 2 Area of a Circle 196

B.E.S.T. Benchmark Connection
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3.05: MA.7.GR.1.4, MTR.4.1, MTR.5.1, MTR.6.1
3.06: MA.7.GR.1.4, MTR.1.1, MTR.3.1, MTR.4.1

3.04 Area Strategies | Estimating and Calculating Area 197A

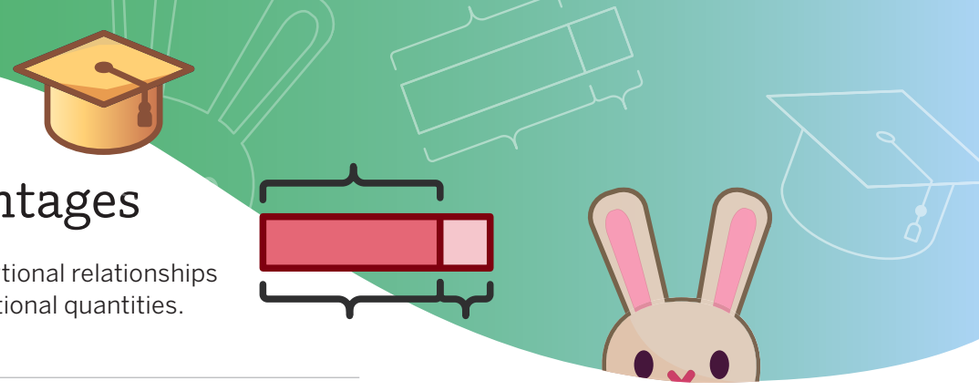
3.05 Radius Squares | Exploring Circle Area 203A

3.06 Circle Area | Calculating Circle Area 210A

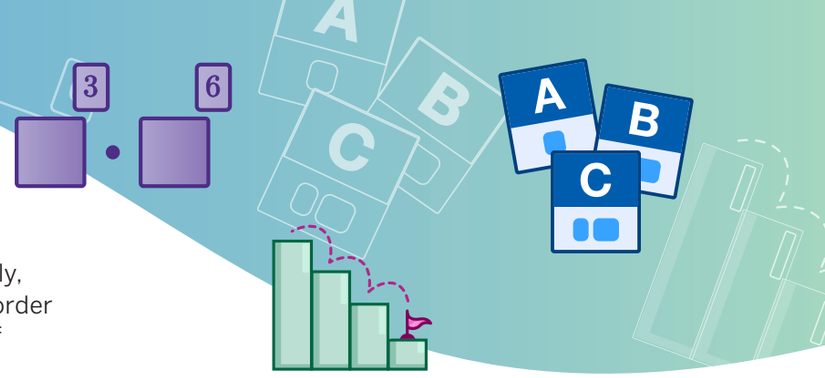
End-of-Unit Assessment 215A

Unit 4 Proportional Relationships and Percentages

Students apply what they have learned about proportional relationships to solve problems involving percent change and fractional quantities.



		B.E.S.T. Benchmark Connection
	Pre-Unit Check 217L	
Sub-Unit 1	Percentages as Proportional Relationships 218	B.E.S.T. Benchmark Connection
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4.03	100% | Percent Increase and Decrease With Double Number Lines 234A	**4.03: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.1.1, MTR.2.1
4.04	Percent Machines | Calculating Unknowns With Percentages 241A	**4.04: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.2.1, MTR.4.1
4.05	Back in My Day | Real-World Situations Involving Percent Increase 248A	**4.05: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.6.1, MTR.7.1
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4.06	Tax, Tips, and Fees | Multistep Percent Problems 257A	**4.06: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.3.1, MTR.4.1
4.07	How Simple, How Interesting | Calculating Simple Interest Using a Formula 265A	**4.07: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.3.1, MTR.7.1
4.08	Bookcase Builder | Percent Error 271A	**4.08: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.3.1, MTR.4.1
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4.11	Pen Pals | Converting Units 295A	**4.11: MA.7.AR.3.3,** MTR.2.1, MTR.4.1
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4.13	Population and Fast Fashion | Posing Problems Involving Percents 309A	**4.13: MA.7.NSO.1.2, MA.7.AR.3.1,** MTR.4.1, MTR.7.1
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	End-of-Unit Assessment 316A	



Unit 5 Positive and Negative Rational Numbers

Students extend what they learned in Grade 6 to add, subtract, multiply, and divide positive and negative rational numbers. Students also use order of operations to evaluate numerical expressions and apply the Laws of Exponents to generate equivalent expressions.

Pre-Unit Check 317L

Sub-Unit 1 Operations with Rational Numbers 318

5.01	Code Breaking | Equivalent Forms	319A
5.02	Bumpers | Adding and Subtracting on the Number Line	326A
5.03	Back in Time | Position, Rate, and Time	333A
5.04	Speeding Turtles | Dividing Rational Numbers	341A
5.05	Order Matters | Evaluating Numerical Expressions	348A

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B.E.S.T. Benchmark Connection
5.01: MA.7.NSO.1.2, MTR.3.1, MTR.4.1
5.02: MA.7.NSO.2.2, MTR.3.1, MTR.4.1, MTR.5.1
5.03: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1
5.04: MA.7.NSO.2.2, MA.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1
5.05: MA.7.NSO.2.1, MA.7.NSO.2.2, MTR.1.1, MTR.5.1

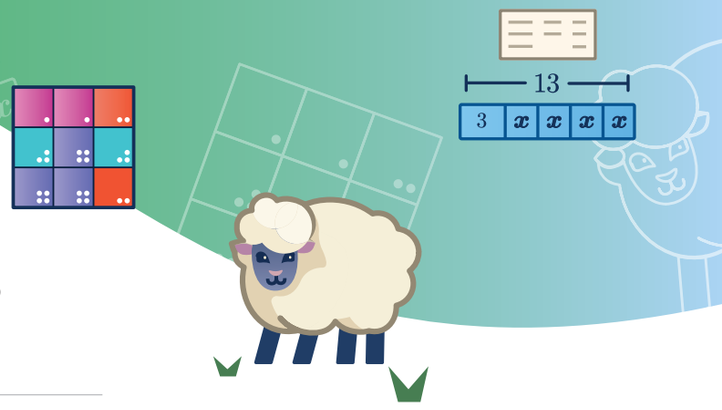
Sub-Unit 2 Laws of Exponents 356

5.06	Combining Exponents | Equivalent Expressions with Exponents	357A
5.07	Power Pairs | Multiplying Powers and Powers of Powers	364A
5.08	Rewriting Powers | Rewriting Exponential Expressions as a Single Power	370A
5.09	Exponent Zero | Using Patterns to Understand Zero Exponents	377A
5.10	Negative Bases | Powers With Negative Bases	383A

End-of-Unit Assessment 389A

B.E.S.T. Benchmark Connection
5.06: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
5.07: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
5.08: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
5.09: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1
5.10: MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1

Unit 6 Expressions, Equations, and Inequalities



Students extend what they learned in Grade 6 about solving one-step equations to solve equations of the form $px + q = r$ and $p(x + q) = r$, and equations that include expanding, factoring, or adding terms. Students also solve inequalities and graph their solutions on a number line.

Pre-Unit Check 391L

Sub-Unit 1 Equations and Tape Diagrams 392

B.E.S.T. Benchmark Connection
6.01 Smudged Receipts | Connecting Contexts and Tape Diagrams ..... 393A — **6.01: MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.5.1, MTR.7.1
6.02 Equations | Representing Contexts With Tape Diagrams and Equations ..... 400A — **6.02: MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.03 Seeing Structure | Practice With Tape Diagrams and Equations ..... 408A — **6.03: MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.5.1, MTR.7.1

Sub-Unit 2 Solving Equations 414

B.E.S.T. Benchmark Connection
6.04 Balancing Moves | Introduction to Balanced Hangers ..... 415A — **6.04: MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.05 Balancing Equations | Solving Equations With Balanced Hangers ..... 422A — **6.05: MA.7.NSO.2.2, MA.7.AR.1.2, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.06 Keeping It True | Solving Equations ..... 429A — **6.06: MA.7.NSO.2.2, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.07 Factoring and Expanding | Options for Solving One Equation ..... 435A — **6.07: MA.7.NSO.2.2, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.08 Always-Equal Machines | Equivalent Expressions ..... 442A — **6.08: MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.09 The Same but Different | Determining Whether Expressions are Equivalent ..... 450A — **6.09: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1
6.10 Collect the Squares | Adding and Subtracting Expressions ..... 457A — **6.10: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1,** MTR.3.1, MTR.4.1, MTR.5.1
6.11 Pass the Equation | Solving Equations Multiple Ways ..... 465A — **6.11: MA.7.NSO.2.2, MA.7.AR.2.2,** MTR.2.1, MTR.3.1, MTR.5.1
6.12 Community Day | Using Equations to Solve Problems ..... 471A — **6.12: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.2.2,** MTR.2.1, MTR.4.1, MTR.7.1

Sub-Unit Quiz 476B

Sub-Unit 3 Inequalities 477–478

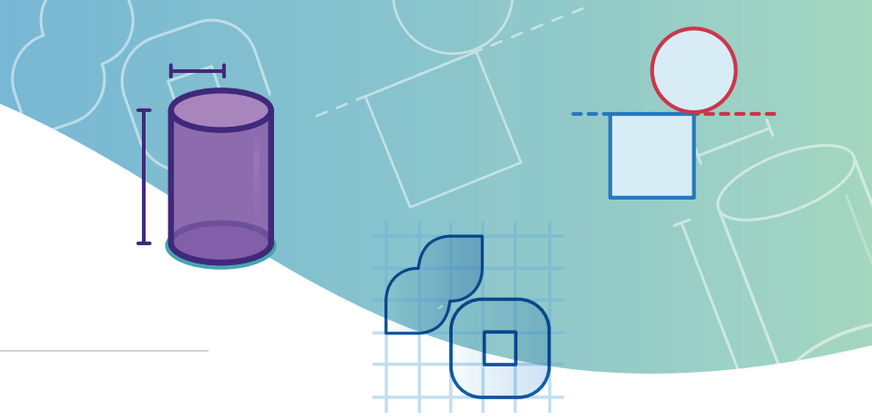
B.E.S.T. Benchmark Connection
6.13 I Saw the Signs | Inequalities on the Number Line ..... 479A — **6.13: Building Toward MA.7.AR.2.1,** MTR.2.1, MTR.4.1, MTR.5.1
6.14 Unbalanced Hangers | Solutions to Inequalities ..... 487A — **6.14: MA.7.NSO.2.2, MA.7.AR.2.1,** MTR.2.1, MTR.3.1, MTR.4.1
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6.16 Write Them and Solve Them | Modeling With Inequalities ..... 502A — **6.16: MA.7.NSO.2.2, MA.7.AR.2.1,** MTR.3.1, MTR.6.1, MTR.7.1

Math in Action 508

End-of-Unit Assessment 508A

Unit 7 Polygons and Cylinders

Students calculate the areas of quadrilaterals, polygons, and composite figures. Students also explore the volume and surface area of cylinders.



Pre-Unit Check 509L

Sub-Unit 1 Area 510

B.E.S.T. Benchmark Connection
7.01: MA.7.GR.1.1, MTR.1.1, MTR.2.1, MTR.5.1
7.02: MA.7.GR.1.2, MTR.1.1, MTR.4.1, MTR.5.1
7.03: MA.7.GR.1.1, MA.7.GR.1.2, MTR.2.1, MTR.4.1, MTR.7.1
7.04: MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.4.1, MTR.5.1
7.05: MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.3.1

7.01 Access to All Areas | Investigating Areas of Special Quadrilaterals 511A

7.02 Piles of Polygons | Investigating Polygons and Their Areas 518A

7.03 Shapes Inside Shapes | Area of Composite Figures 525A

7.04 Area Challenges | Calculating Areas of Complex Shapes 532A

7.05 Circle vs. Square | Calculating Area Given Perimeter 538A

Sub-Unit Quiz 543A

Sub-Unit 2 Solid Geometry 544

B.E.S.T. Benchmark Connection
7.06: MA.7.GR.2.3, MTR.2.1, MTR.3.1, MTR.4.1
7.07: MA.7.GR.2.3, MTR.1.1, MTR.5.1, MTR.7.1
7.08: MA.7.GR.2.1, MA.7.GR.2.2, MTR.1.1, MTR.2.1, MTR.7.1

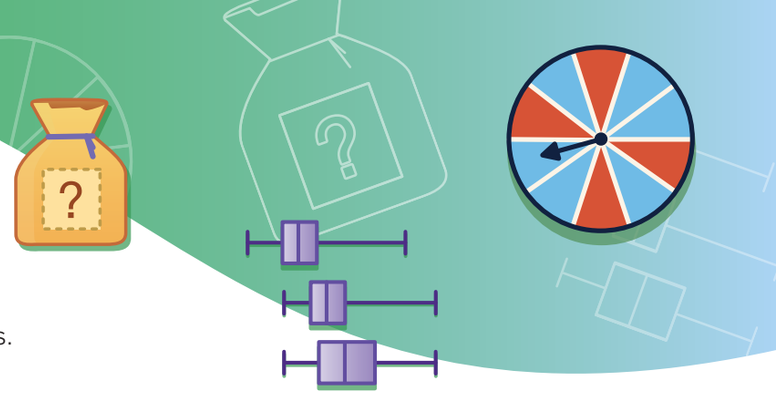
7.06 Cylinders | Volumes of Cylinders 545A

7.07 Scaling Cylinders | Applying Volumes of Cylinders 553A

7.08 From Flat to Round | Surface Area of Cylinders 560A

End-of-Unit Assessment 567A

Unit 8 Probability and Sampling



Students learn about probability as a way to describe the likelihood of unknown events and estimate the probability of real-world situations. They use samples to draw conclusions about and compare populations.

Pre-Unit Check 569L

Sub-Unit 1 Probability 570

8.01 How Likely? | Chance Experiments 571A
8.02 Prob-bear-bilities | Intro to Probability 577A
8.03 Mystery Bag | Predicting Sample Spaces 586A
8.04 Spin Class | Repeated Experiments 594A
8.05 Is It Fair? | Comparing Probabilities and Experiments 602A

Sub-Unit Quiz 609A

Sub-Unit 2 Sampling 610

8.06 The County Fair | Creating Circle Graphs 611A
8.07 Choices, Choices | Choosing an Appropriate Representation for Data 619A
8.08 On Display | Creating an Appropriate Representation for Data 626A
8.09 Car, Bike, or Train? | Using Mean, Median, and Range to Compare Groups 633A
8.10 Crab Island | Sampling From Large Populations 640A
8.11 Flower Power | Using Percentages to Predict Populations 648A
8.12 Plots and Samples | Using Statistical Measures to Make Predictions 655A
8.13 School Newspaper | Comparing Populations 663A

Math in Action 670

End-of-Unit Assessment 670A

B.E.S.T. Benchmark Connection
8.01: MA.7.DP.2.1, MTR.2.1, MTR.3.1
8.02: MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1
8.03: MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.3.1, MTR.6.1
8.04: MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.1.1, MTR.4.1
8.05: MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.5.1, MTR.6.1

B.E.S.T. Benchmark Connection
8.06: MA.7.DP.1.4, MTR.4.1, MTR.5.1
8.07: MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.4.1, MTR.7.1
8.08: MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.7.1
8.09: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1
8.10: MA.7.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1
8.11: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3, MTR.4.1, MTR.5.1, MTR.6.1
8.12: MA.7.DP.1.1, MA.7.DP.1.2, MTR.5.1, MTR.6.1
8.13: MA.7.DP.1.2, MTR.4.1, MTR.7.1

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math Florida, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

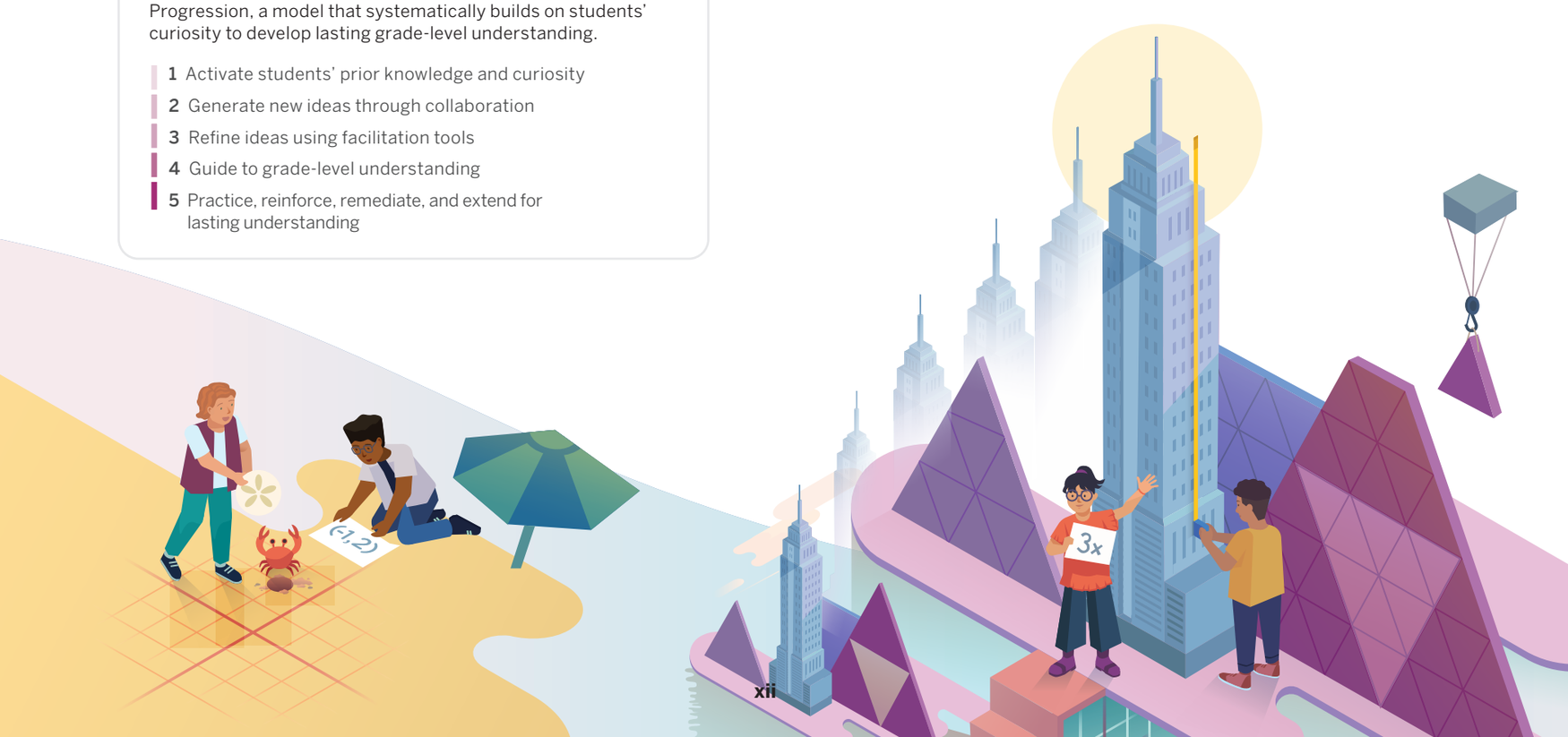
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.





Student thinking is valuable and can be made evident

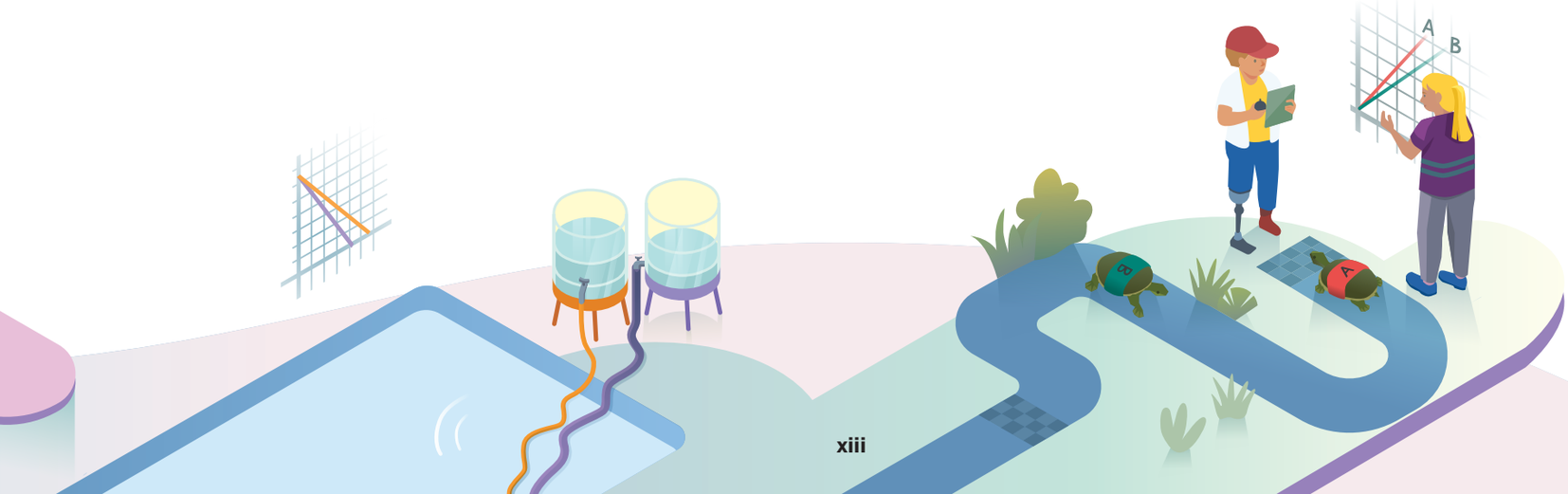
Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Math that motivates students

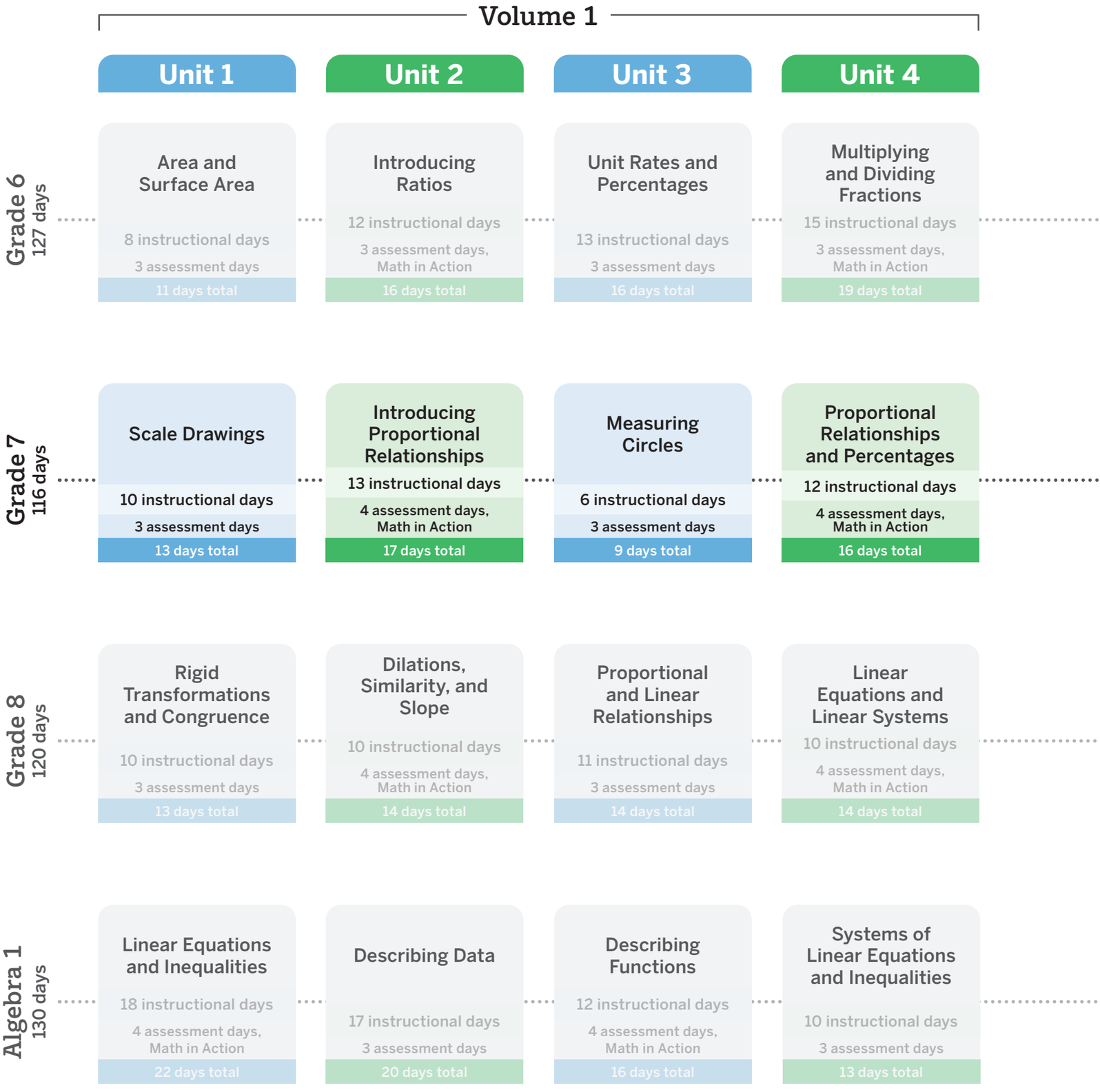
Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity.

Our curriculum supports collaborative classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.



Scope and Sequence | Grade 6–Algebra 1



Volume 2

Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Decimal Arithmetic 10 instructional days 3 assessment Days 13 days total	**Expressions and Equations** 11 instructional days 4 assessment Days, Math in Action 15 days total	**Positive and Negative Numbers** 18 instructional days 3 assessment Days 21 days total	**Describing Data** 12 instructional days 4 assessment Days, Math in Action 16 days total	
Positive and Negative Rational Numbers 10 instructional days 3 assessment days 13 days total	**Equations, Expressions, and Inequalities** 16 instructional days 4 assessment days, Math in Action 20 days total	**Polygons and Cylinders** 8 instructional days 3 assessment days 11 days total	**Probability and Sampling** 13 instructional days 4 assessment days, Math in Action 17 days total	
Functions 7 instructional days 3 assessment days 10 days total	**Associations in Data** 9 instructional days 4 assessment days, Math in Action 13 days total	**Exponents and Scientific Notation** 13 instructional days 3 assessment days 16 days total	**The Pythagorean Theorem and Irrational Numbers** 15 instructional days 4 assessment days, Math in Action 19 days total	**Probability** 5 instructional days 2 assessment days 7 days total
Exponential Functions 15 instructional days 4 assessment days, Math in Action 19 days total	**Quadratic Functions** 15 instructional days 3 assessment days 18 days total	**Quadratic Equations** 18 instructional days 4 assessment days, Math in Action 22 days total		

Program Resources

For Students


Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1

Make It Scale

Let's draw scaled copies.

Warm-Up

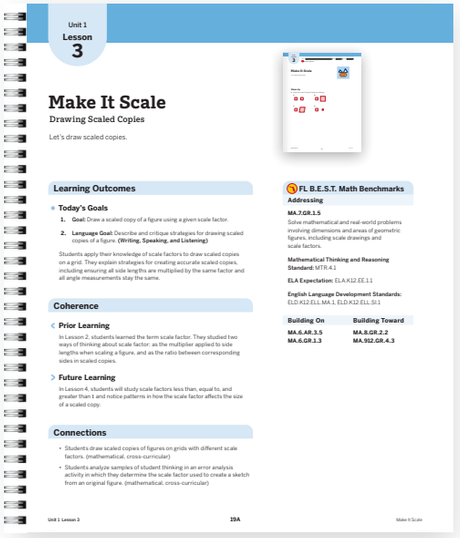
1. Which pair doesn't belong? Explain your thinking.

A. B. C. D.

Unit 1 Lesson 3 19 Warm-Up

- Student Edition (two-volume)

For Teachers


Unit 1
Lesson
3

Make It Scale

Drawing Scaled Copies

Let's draw scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Draw a scaled copy of a figure using a given scale factor.
2. **Language Goal:** Describe and critique strategies for drawing scaled copies of a figure. **(Writing, Speaking, and Listening)**

Students apply their knowledge of scale factors to draw scaled copies on a grid. They explain strategies for creating accurate scaled copies, including ensuring all side lengths are multiplied by the same factor and all angle measurements stay the same.

Coherence

Prior Learning

In Lesson 2, students learned the term scale factor. They studied two ways of thinking about scale factor: as the multiplier applied to side lengths when scaling a figure, and as the ratio between corresponding sides in scaled copies.

Future Learning

In Lesson 4, students will study scale factors less than, equal to, and greater than 1 and notice patterns in how the scale factor affects the size of a scaled copy.

Connections

- Students draw scaled copies of figures on grids with different scale factors. (mathematical, cross-curricular)
- Students analyze samples of student thinking in an error analysis activity in which they determine the scale factor used to create a sketch from an original figure. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Unit 1 Lesson 3 19A Make It Scale

- Teacher Edition (two-volume)

Additional Resources



Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior grade Mini-Lessons
- Extensions

Optional: Manipulative Kits



hand2mind

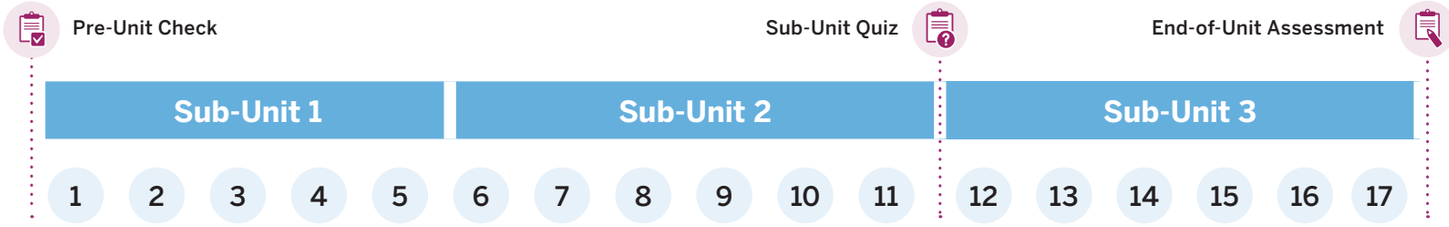
Program Architecture

Course

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
13 days	17 days	9 days	16 days	13 days	20 days	11 days	17 days

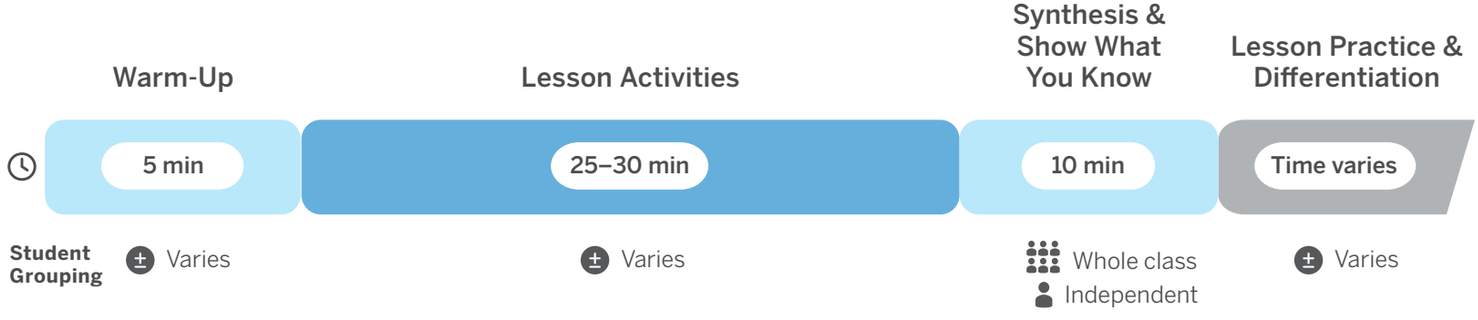
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

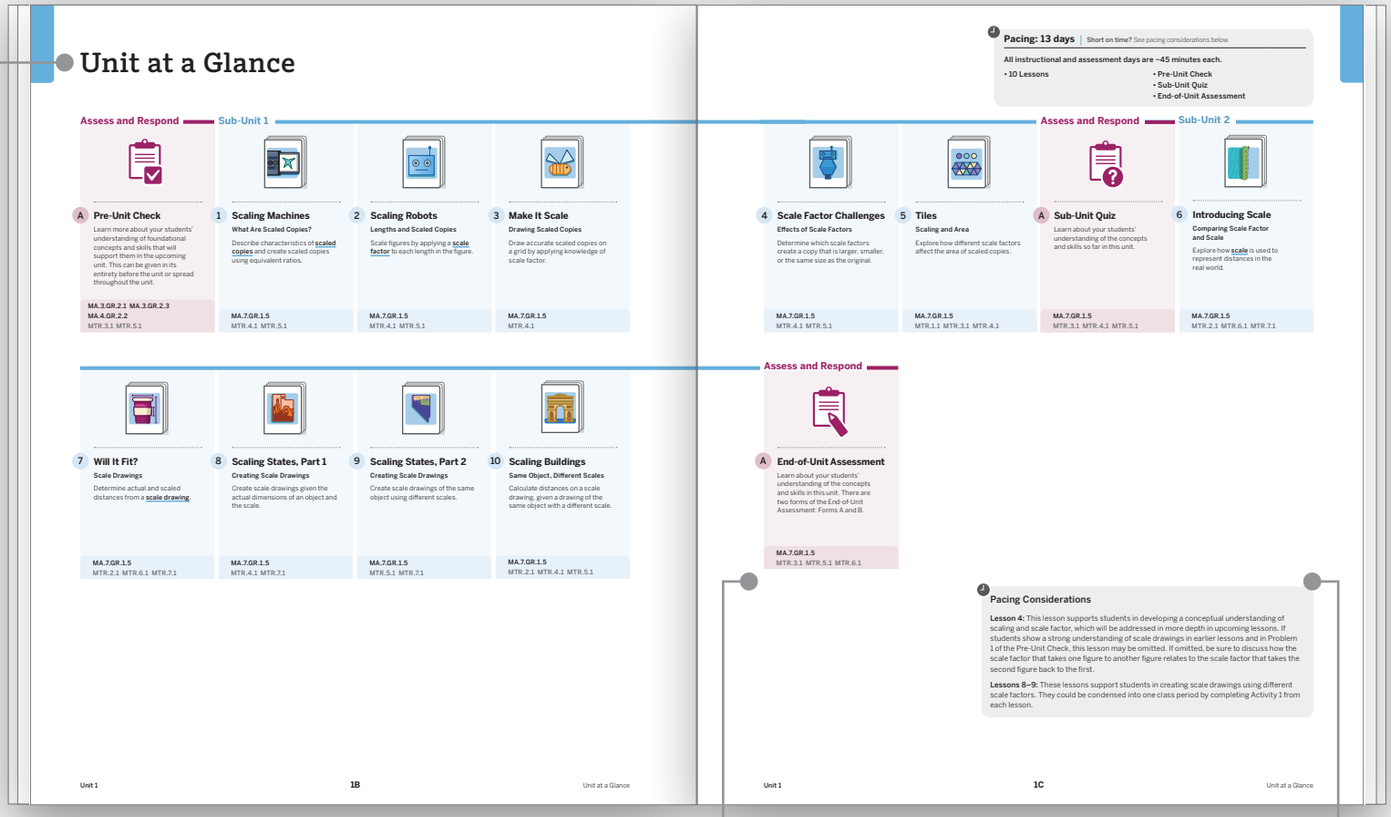
Lesson



Navigating This Program

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Unit at a Glance

Assess and Respond — Sub-Unit 1

A Pre-Unit Check
Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.
MA.3.GR.2.1 MA.3.GR.2.3 MA.4.GR.2.2 MTR.3.1 MTR.5.1

1 Scaling Machines
What Are Scaled Copies?
Describe characteristics of scaled copies and create scaled copies using equivalent ratios.
MA.7.GR.1.5 MTR.4.1 MTR.5.1

2 Scaling Robots
Lengths and Scaled Copies
Scale figures by applying a scale factor to each length in the figure.
MA.7.GR.1.5 MTR.4.1 MTR.5.1

3 Make It Scale
Drawing Scaled Copies
Draw accurate scaled copies on a grid by applying knowledge of scale factor.
MA.7.GR.1.5 MTR.4.1

7 Will It Fit?
Scale Drawings
Determine actual and scaled distances from a scale drawing.
MA.7.GR.1.5 MTR.2.1 MTR.6.1 MTR.7.1

8 Scaling States, Part 1
Creating Scale Drawings
Create scale drawings given the actual dimensions of an object and the scale.
MA.7.GR.1.5 MTR.4.1 MTR.7.1

9 Scaling States, Part 2
Creating Scale Drawings
Create scale drawings of the same object using different scales.
MA.7.GR.1.5 MTR.5.1 MTR.7.1

10 Scaling Buildings
Same Object, Different Scales
Calculate distances on a scale drawing, given a drawing of the same object with a different scale.
MA.7.GR.1.5 MTR.2.1 MTR.4.1 MTR.5.1

Unit 1 1B Unit at a Glance

Pacing: 13 days | Short on time? See pacing considerations below.
All instructional and assessment days are ~45 minutes each.
• 10 Lessons
• Pre-Unit Check
• Sub-Unit Quiz
• End-of-Unit Assessment

Assess and Respond — Sub-Unit 2

4 Scale Factor Challenges
Effects of Scale Factors
Determine which scale factors create a copy that is larger, smaller, or the same size as the original.
MA.7.GR.1.5 MTR.4.1 MTR.5.1

5 Tiles
Scaling and Area
Explore how different scale factors affect the area of scaled copies.
MA.7.GR.1.5 MTR.1.1 MTR.3.1 MTR.4.1

A Sub-Unit Quiz
Learn about your students' understanding of the concepts and skills so far in this unit.
MA.7.GR.1.5 MTR.3.1 MTR.4.1 MTR.5.1

6 Introducing Scale
Comparing Scale Factor and Scale
Explore how scale is used to represent distances in the real world.
MA.7.GR.1.5 MTR.2.1 MTR.6.1 MTR.7.1

Assess and Respond

A End-of-Unit Assessment
Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.
MA.7.GR.1.5 MTR.3.1 MTR.5.1 MTR.6.1

Pacing Considerations

Lesson 4: This lesson supports students in developing a conceptual understanding of scaling and scale factor, which will be addressed in more depth in upcoming lessons. If students show a strong understanding of scale drawings in earlier lessons and in Problem 1 of the Pre-Unit Check, this lesson may be omitted. If omitted, be sure to discuss how the scale factor that takes one figure to another figure relates to the scale factor that takes the second figure back to the first.

Lessons 8–9: These lessons support students in creating scale drawings using different scale factors. They could be condensed into one class period by completing Activity 1 from each lesson.

Unit 1 1C Unit at a Glance

Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

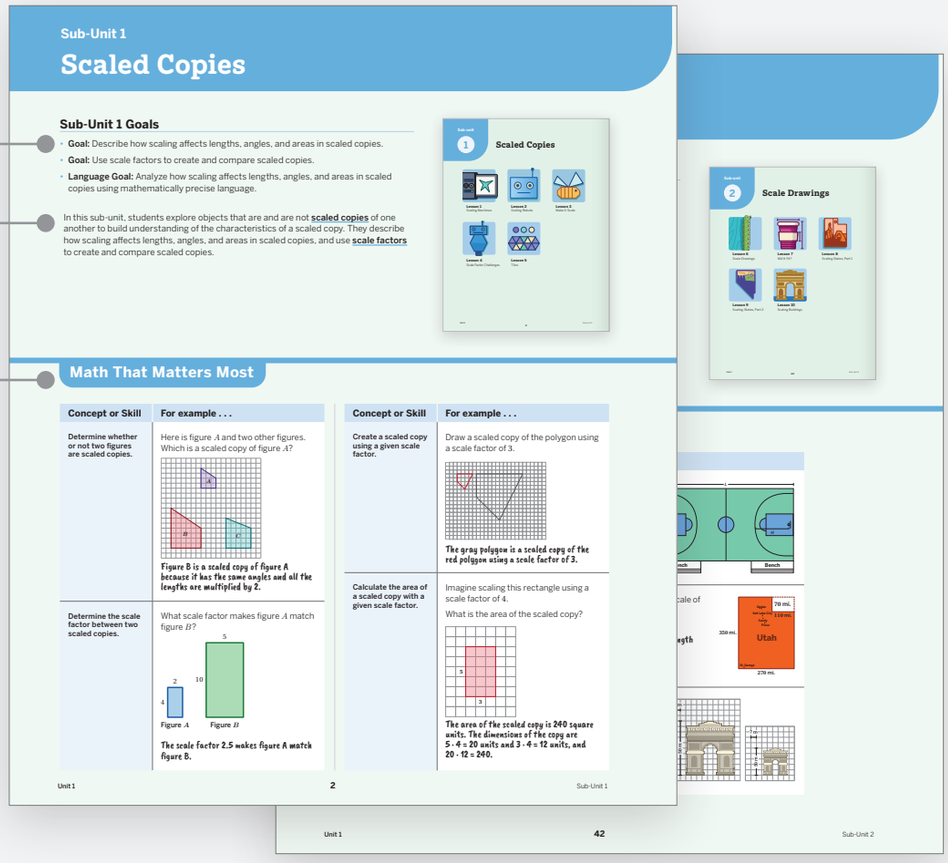
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, a preview of each of the unit assessments, and guidance about differentiation, accessibility, language development, materials, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



Sub-Unit 1

Scaled Copies

Sub-Unit 1 Goals

- **Goal:** Describe how scaling affects lengths, angles, and areas in scaled copies.
- **Goal:** Use scale factors to create and compare scaled copies.
- **Language Goal:** Analyze how scaling affects lengths, angles, and areas in scaled copies using mathematically precise language.

In this sub-unit, students explore objects that are and are not **scaled copies** of one another to build understanding of the characteristics of a scaled copy. They describe how scaling affects lengths, angles, and areas in scaled copies, and use **scale factors** to create and compare scaled copies.

Math That Matters Most

Concept or Skill	For example . . .
Determine whether or not two figures are scaled copies.	Here is figure *A* and two other figures. Which is a scaled copy of figure *A*? Figure B is a scaled copy of figure A because it has the same angles and all the lengths are multiplied by 2.
Determine the scale factor between two scaled copies.	What scale factor makes figure *A* match figure *B*? Figure *A* Figure *B* The scale factor 2.5 makes figure A match figure B.
Create a scaled copy using a given scale factor.	Draw a scaled copy of the polygon using a scale factor of 3. The gray polygon is a scaled copy of the red polygon using a scale factor of 3.
Calculate the area of a scaled copy with a given scale factor.	Imagine scaling this rectangle using a scale factor of 4. What is the area of the scaled copy? The area of the scaled copy is 240 square units. The dimensions of the copy are 5 · 4 = 20 units and 3 · 4 = 12 units, and 20 · 12 = 240.

Unit 1 2 Sub-Unit 1

Unit 1 42 Sub-Unit 2

The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

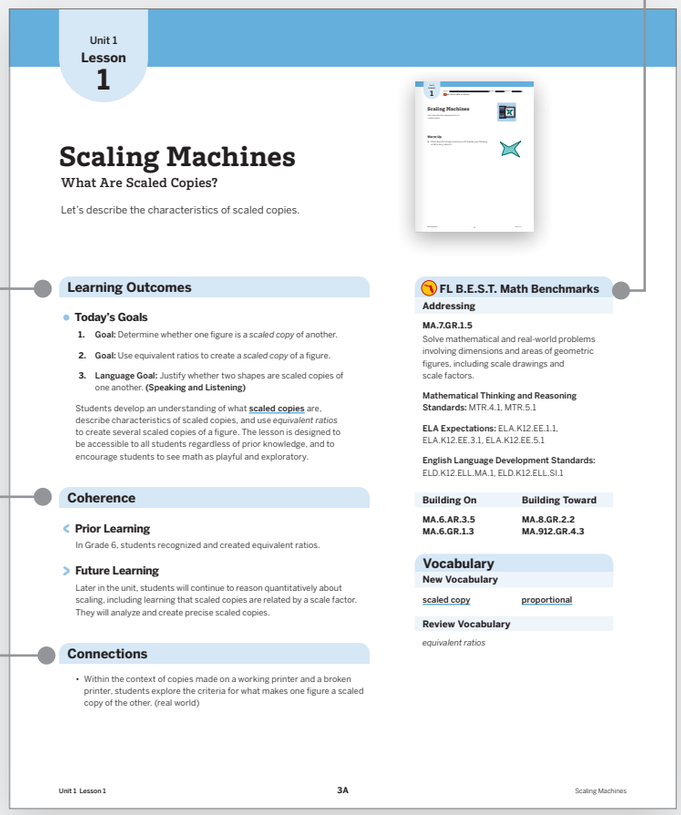
The lessons within each unit are grouped into sub-units that address a related group of concepts. All sub-units include lesson practice and Show What You Know formative assessments, and targeted sub-units include a quiz.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

The **Florida B.E.S.T. Math Benchmarks** section will list all standards addressed in this lesson, including the **Mathematical Thinking and Reasoning Standards,** the **ELA Expectations,** and the **English Language Development Standards.** This section shows the standards the lesson builds on and the standards the lesson builds toward.



The **Learning Outcomes** section is where teachers will find the goals of the lesson, including language goals

In the **Coherence** section, teachers will find information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

In the **Connections** section, teachers will find opportunities to help their students make connections cross-curricularly, to the real world, or to other mathematical learning.

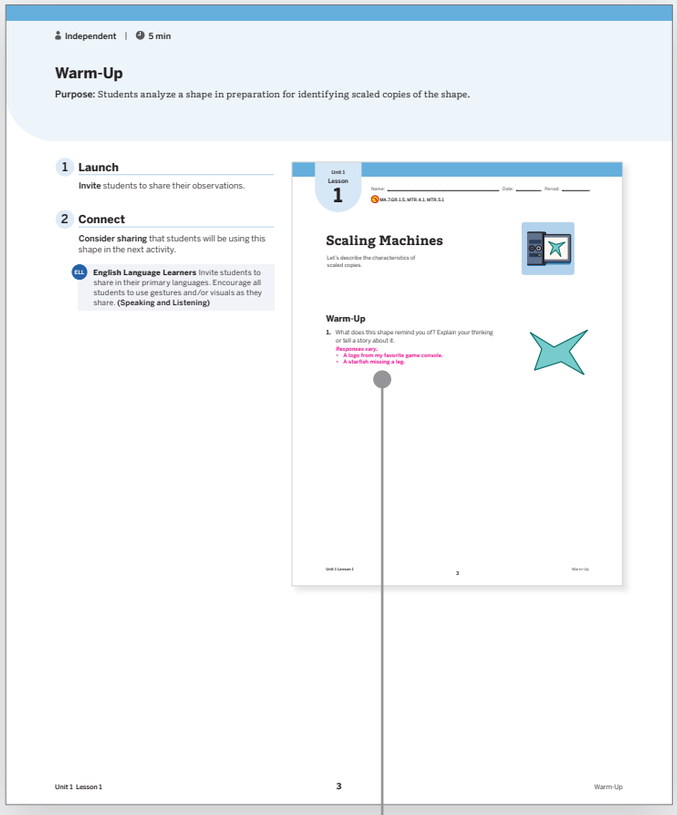
Lesson Overview

This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.





The **Prep Checklist** displays all materials students will need for the lesson.

Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class. The page also lists any hands-on materials that may be needed.

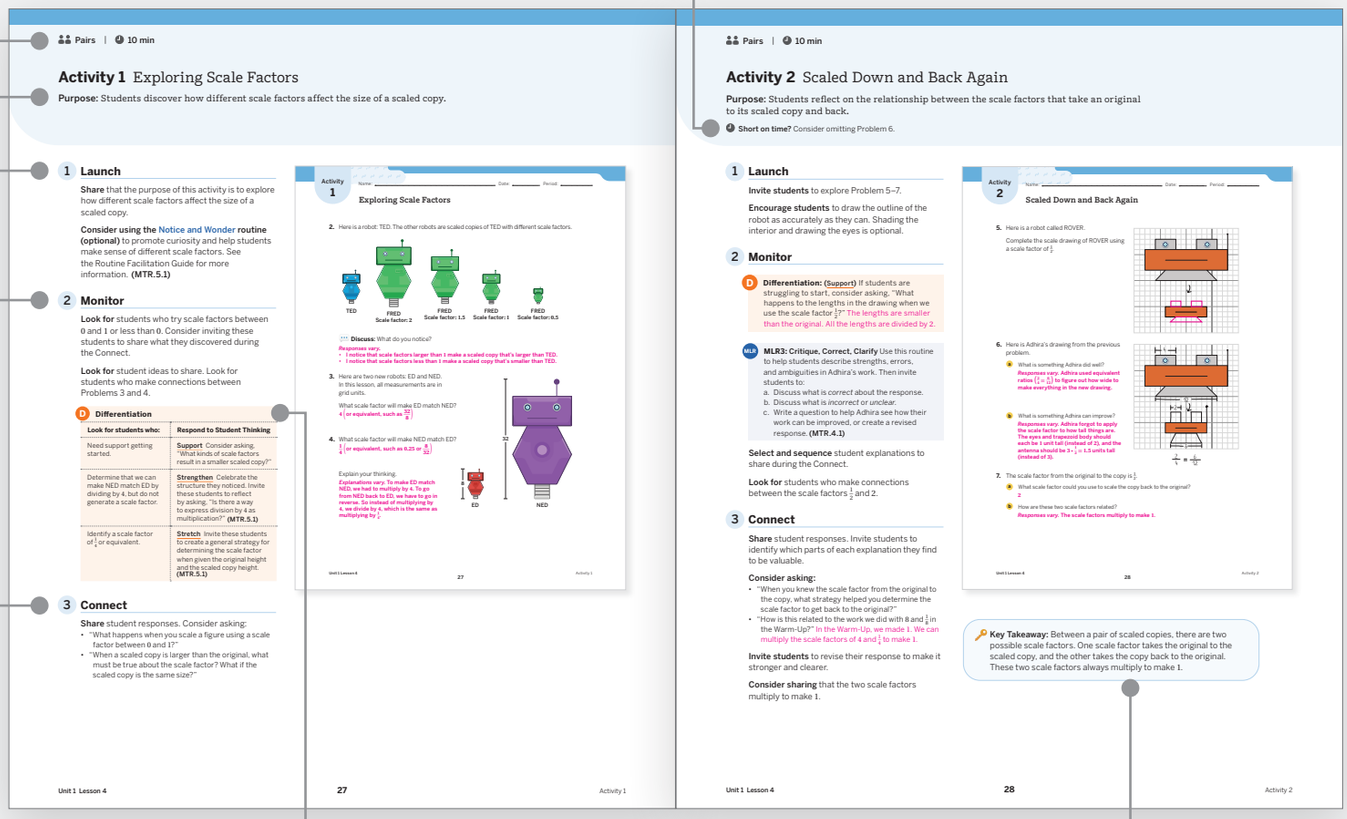
Warm-Up

Every Amplify Desmos Math Florida lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping** and **time frame** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.



Pairs | 10 min

Activity 1 Exploring Scale Factors

Purpose: Students discover how different scale factors affect the size of a scaled copy.

1 Launch

Share that the purpose of this activity is to explore how different scale factors affect the size of a scaled copy.

Consider using the Notice and Wonder routine (optional) to promote curiosity and help students make sense of different scale factors. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

2 Monitor

Look for students who try scale factors between 0 and 1 or less than 0. Consider inviting these students to share what they discovered during the Connect.

Look for student ideas to share. Look for students who make connections between Problems 3 and 4.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What kinds of scale factors result in a smaller scaled copy?"
Determine that we can make NED match ED by dividing by 4, but do not generate a scale factor.	**Strengthen** Celebrate the structure they noticed. Invite these students to reflect by asking, "Is there a way to express division by 4 as multiplication?" **(MTR.5.1)**
Identify a scale factor of $\frac{1}{4}$ or equivalent.	**Stretch** Invite these students to create a general strategy for determining the scale factor when given the original height and the scaled copy height. **(MTR.5.1)**

3 Connect

Share student responses. Consider asking:

- "What happens when you scale a figure using a scale factor between 0 and 1?"
- "When a scaled copy is larger than the original, what must be true about the scale factor? What if the scaled copy is the same size?"

Activity 1 — Exploring Scale Factors

2. Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.

TED; FRED Scale factor: 2; FRED Scale factor: 1.5; FRED Scale factor: 1; FRED Scale factor: 0.5

Discuss: What do you notice?

Responses vary.
- I notice that scale factors larger than 1 make a scaled copy that's larger than TED.
- I notice that scale factors less than 1 make a scaled copy that's smaller than TED.

3. Here are two new robots: ED and NED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED? 4 (or equivalent, such as $\frac{32}{8}$)

4. What scale factor will make NED match ED? $\frac{1}{4}$ (or equivalent, such as 0.25 or $\frac{8}{32}$)

Explain your thinking.

Explanations vary. To make ED match NED, we had to multiply by 4. To go from NED back to ED, we have to go in the opposite direction of multiplying by 4, we divide by 4, which is the same as multiplying by $\frac{1}{4}$.

ED; NED; 32

Unit 1 Lesson 4 — 27 — Activity 1

Pairs | 10 min

Activity 2 Scaled Down and Back Again

Purpose: Students reflect on the relationship between the scale factors that take an original to its scaled copy and back.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to explore Problem 5–7.

Encourage students to draw the outline of the robot as accurately as they can. Shading the interior and drawing the eyes is optional.

2 Monitor

D Differentiation: (Support) If students are struggling to start, consider asking, "What happens to the lengths in the drawing when we use the scale factor $\frac{1}{2}$?" The lengths are smaller than the original. All the lengths are divided by 2.

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Adhira's work. Then invite students to:
a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Adhira see how their work can be improved, or create a revised response. **(MTR.4.1)**

Select and sequence student explanations to share during the Connect.

Look for students who make connections between the scale factors $\frac{1}{2}$ and 2.

3 Connect

Share student responses. Invite students to identify which parts of each explanation they find to be valuable.

Consider asking:
- "When you knew the scale factor from the original to the copy, what strategy helped you determine the scale factor to get back to the original?"
- "How is this related to the work we did with 8 and $\frac{1}{8}$ in the Warm-Up?" In the Warm-Up, we made 1. We can multiply the scale factors of 4 and $\frac{1}{4}$ to make 1.

Invite students to revise their response to make it stronger and clearer.

Consider sharing that the two scale factors multiply to make 1.

Activity 2 — Scaled Down and Back Again

5. Here is a robot called ROVER.

Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.

6. Here is Adhira's drawing from the previous problem.

What is something Adhira did well?

Responses vary. Adhira used equivalent ratios ($\frac{1}{2} = \frac{2}{4}$) to figure out how wide to make everything in the new drawing.

What is something Adhira can improve?

Responses vary. Adhira forgot to apply the scale factor to how tall things are. The eyes and trapezoid body should each be 1 unit tall (instead of 2), and the antenna should be $3 \times \frac{1}{2} = 1.5$ units tall (instead of 3).

7. The scale factor from the original to the copy is $\frac{1}{2}$.

What scale factor could you use to scale the copy back to the original?

2

How are these two scale factors related?

Responses vary. The scale factors multiply to make 1.

Unit 1 Lesson 4 — 28 — Activity 2

Key Takeaway: Between a pair of scaled copies, there are two possible scale factors. One scale factor takes the original to the scaled copy, and the other takes the copy back to the original. These two scale factors always multiply to make 1.

In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a **D Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

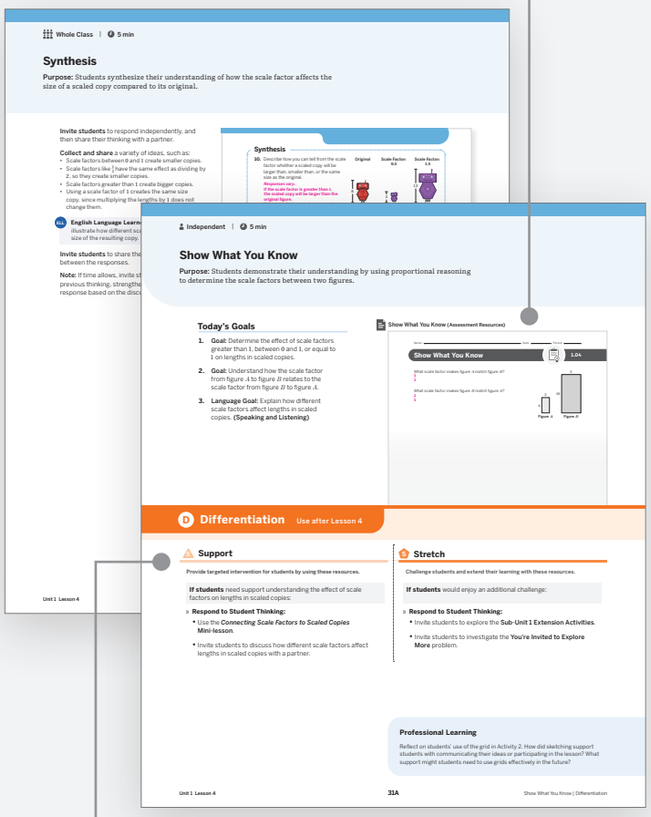
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

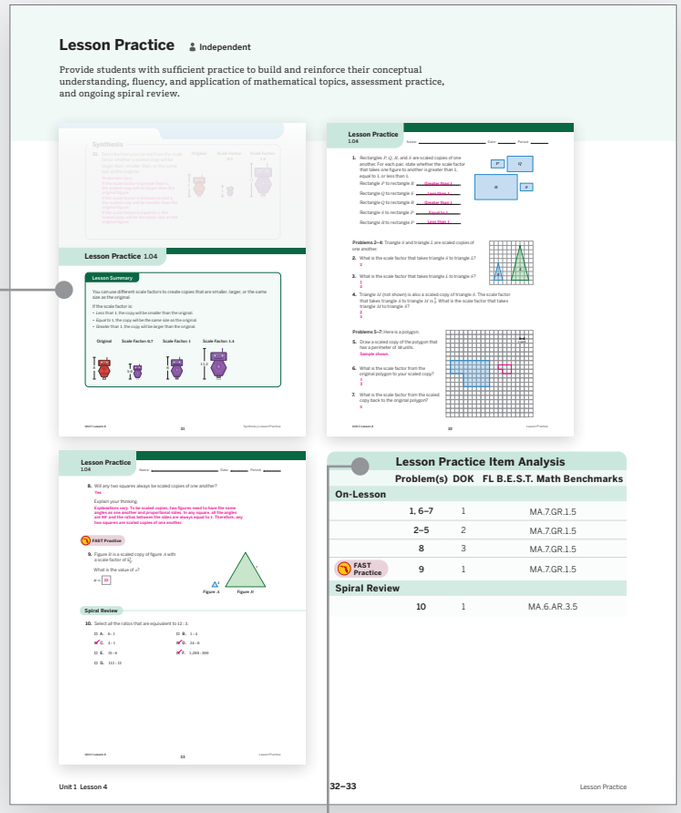
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, and ways to increase access to the language and content. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** begins with a **Summary** of the big ideas in the lesson, often including a worked example. Students can highlight parts of the summary or share it with a caregiver or classmate.



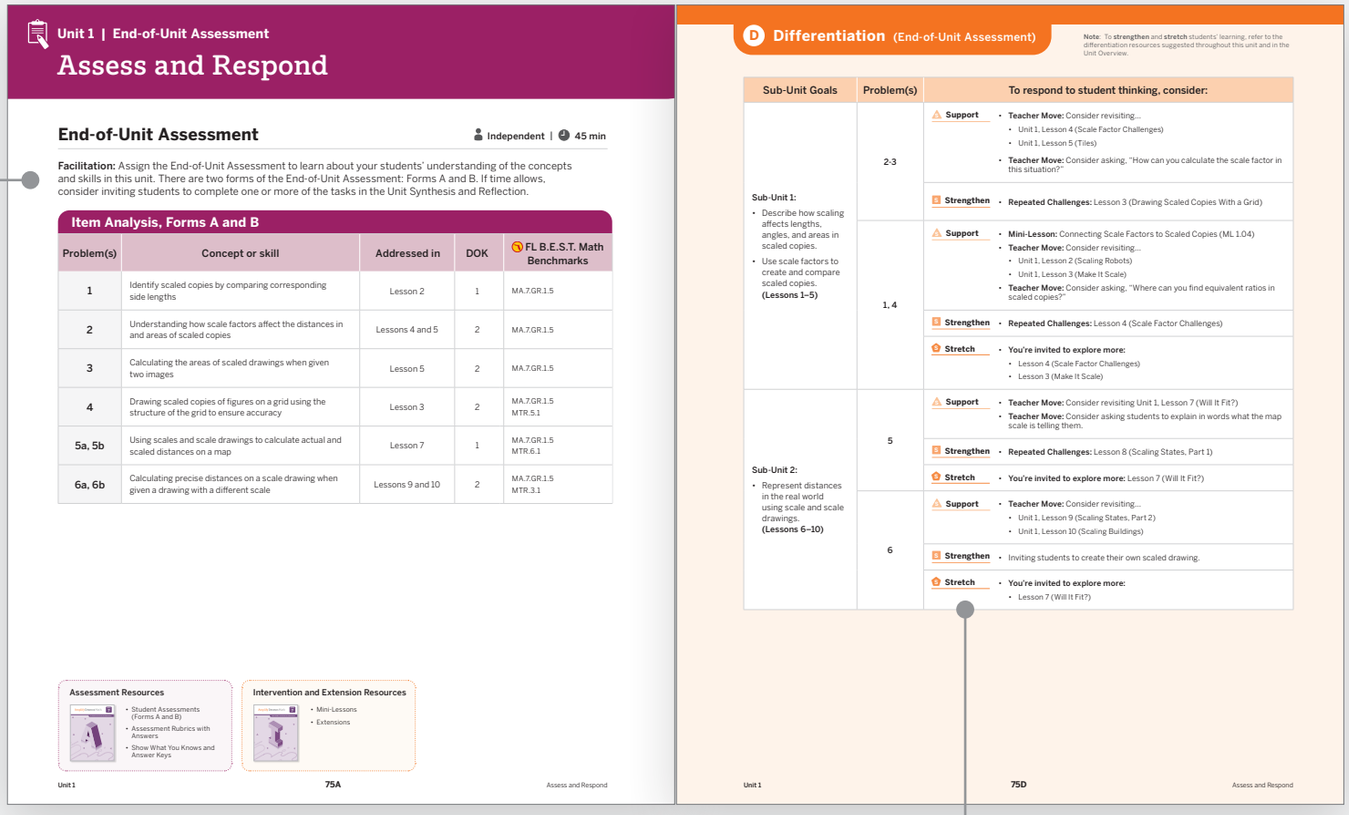
A **Practice Problem Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding B.E.S.T. Math Benchmarks.

Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

Lesson Practice

Daily practice problems for the day's lesson are included both online and in the print Student Edition, including Fluency, FAST Practice, and Spiral Review.



Unit 1 | End-of-Unit Assessment

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identify scaled copies by comparing corresponding side lengths	Lesson 2	1	MA.7.GR.1.5
2	Understanding how scale factors affect the distances in and areas of scaled copies	Lessons 4 and 5	2	MA.7.GR.1.5
3	Calculating the areas of scaled drawings when given two images	Lesson 5	2	MA.7.GR.1.5
4	Drawing scaled copies of figures on a grid using the structure of the grid to ensure accuracy	Lesson 3	2	MA.7.GR.1.5 MTR.5.1
5a, 5b	Using scales and scale drawings to calculate actual and scaled distances on a map	Lesson 7	1	MA.7.GR.1.5 MTR.6.1
6a, 6b	Calculating precise distances on a scale drawing when given a drawing with a different scale	Lessons 9 and 10	2	MA.7.GR.1.5 MTR.3.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Unit 1 75A Assess and Respond

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Describe how scaling affects lengths, angles, and areas in scaled copies.
• Use scale factors to create and compare scaled copies.
(Lessons 1–5)
	2-3	**Support**
		• **Teacher Move:** Consider revisiting...
		  • Unit 1, Lesson 4 (Scale Factor Challenges)
		  • Unit 1, Lesson 5 (Tiles)
		• **Teacher Move:** Consider asking, "How can you calculate the scale factor in this situation?"

		Strengthen • **Repeated Challenges:** Lesson 3 (Drawing Scaled Copies With a Grid)

	1, 4	**Support**
		• **Mini-Lesson:** Connecting Scale Factors to Scaled Copies (ML 1.04)
		• **Teacher Move:** Consider revisiting...
		  • Unit 1, Lesson 2 (Scaling Robots)
		  • Unit 1, Lesson 3 (Make It Scale)
		• **Teacher Move:** Consider asking, "Where can you find equivalent ratios in scaled copies?"

		Strengthen • **Repeated Challenges:** Lesson 4 (Scale Factor Challenges)

		Stretch
		• **You're invited to explore more:**
		  • Lesson 4 (Scale Factor Challenges)
		  • Lesson 3 (Make It Scale)

Sub-Unit 2: • Represent distances in the real world using scale and scale drawings. **(Lessons 6–10)**	5	**Support** • **Teacher Move:** Consider revisiting Unit 1, Lesson 7 (Will It Fit?) • **Teacher Move:** Consider asking students to explain in words what the map scale is telling them.
		Strengthen • **Repeated Challenges:** Lesson 8 (Scaling States, Part 1)
		Stretch • **You're invited to explore more:** Lesson 7 (Will It Fit?)

	6	**Support**
		• **Teacher Move:** Consider revisiting...
		  • Unit 1, Lesson 9 (Scaling States, Part 2)
		  • Unit 1, Lesson 10 (Scaling Buildings)

		Strengthen • Inviting students to create their own scaled drawing.
		Stretch • **You're invited to explore more:** • Lesson 7 (Will It Fit?)

Unit 1 75D Assess and Respond

An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math Florida is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions*.

1 Launch



The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor



As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect



Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

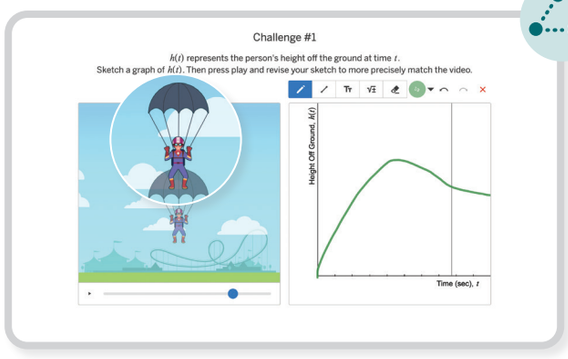
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

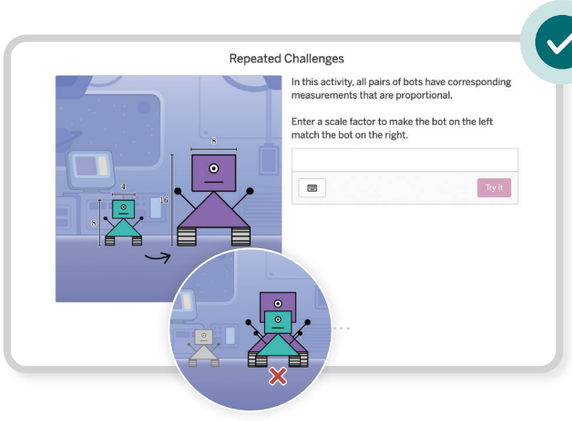
The Power of Digital

Amplify Desmos Math Florida provides the option for teachers to deliver lessons using the power of digital technology. This includes the use of visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. These interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



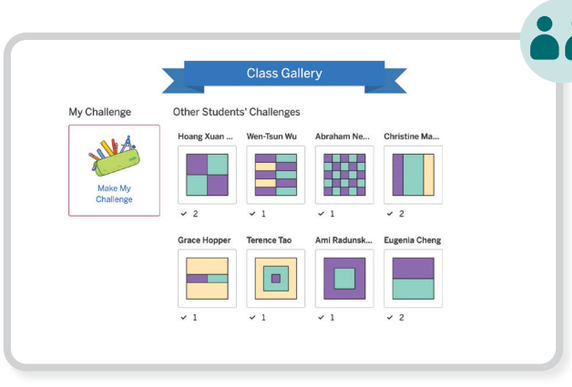
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math Florida, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

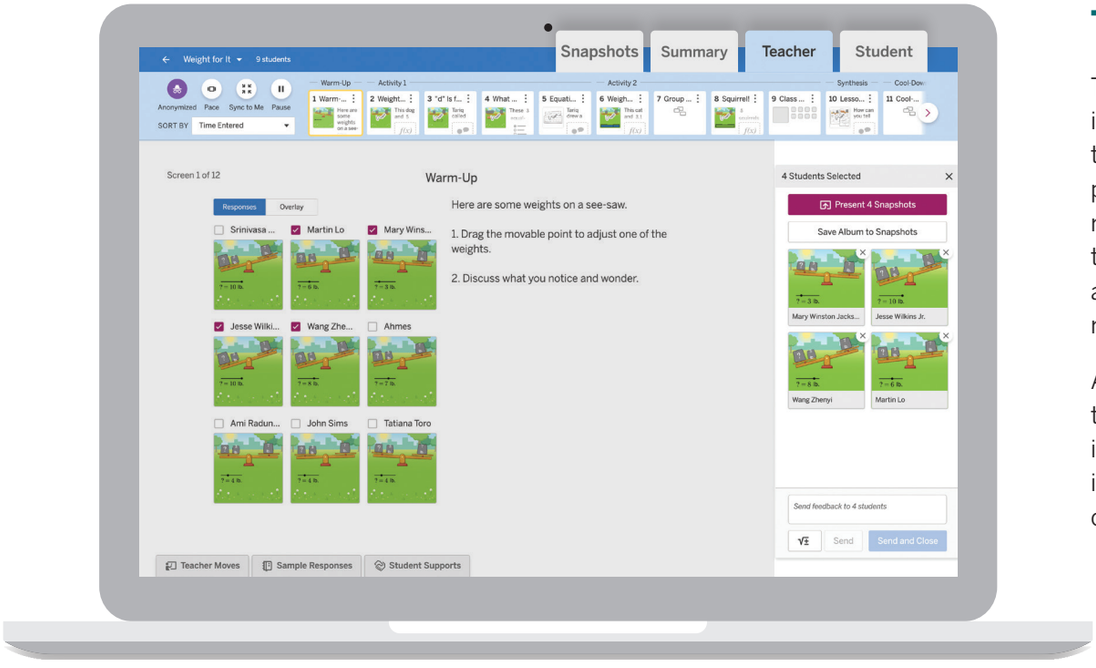


Collaborative experiences

Digital tools allow students to interact with each other's mathematical ideas in a variety of ways. Students can use the "Share With Class" button to exchange mathematical thinking with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Facilitation Tools

Amplify Desmos Math Florida includes a suite of digital facilitation tools. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

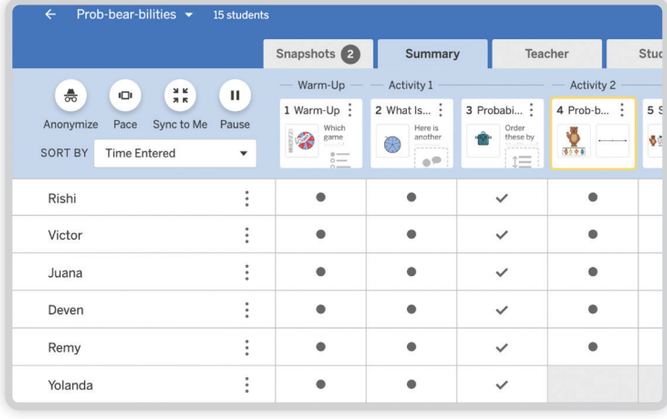
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math in Action

Each Math in Action feature highlights an individual whose work in the field of mathematics, or a connected field, have made an impact on American society. These features can be used to help make connections between the mathematical content of the grade and how math shows up in a wide variety of careers.

Career Connections

Amplify Desmos Math Florida intentionally celebrates traditional mathematics fields as well as elevates work that engages mathematics in unique ways.

Representation of Mathematicians

Students can read about the contributions and work of a variety of individuals.



Unit 6 | Math in Action

Career Connection

Did you know that there are over 400 different types of oranges in the world? And that oranges make up about 70% of all the citrus produced worldwide? The state of Florida is the second largest producer of citrus in the United States.

How many types of oranges can you name? A few are shown in the picture.

Horticulturalists are among the many careers that involve the citrus industry. They are people who study and whose work includes growing fruits, vegetables, flowers, and other plants. They use expressions and equations to determine measurements, temperatures and to describe the characteristics of the plants they are studying. Data collection and graphing will allow a horticulturalist to create new varieties of plants through cross-pollination and other techniques.

B.E.S.T. Mathematics Benchmark Connection

Horticulturalists and others involved in producing citrus use many math concepts in their work. For example, they apply inequality concepts and use number lines to track temperature and growth rates (MA.7.AR.2.1) so that plants grow in the best conditions possible. IT Specialists will use diagrams and equations (MA.7.NSO.2.3, MA.7.AR.2.2) as they use technology tools to monitor weather conditions, and control watering systems.

Mathematical Thinking and Reasoning Connection

Horticulturalists use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as diagrams or equations (MTR.2.1).

Meet Lue Gim Gong

Lue Gim Gong was a horticulturist who is credited for his work in growing the American citrus industry. He developed his interest in plants while he was working in orange groves in China and his mother taught him about cross-pollination. Even when he moved to Massachusetts to work in a shoe factory, he continued to study plants. When he moved to Florida, he focused his studies on the cold frosts that damaged orange crops and is credited for developing a cold-resistant orange!

Nature and Science/Alamy Stock Photo

Grade 7 Unit 6

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Eugenia Cheng

Mathematics and music? Eugenia Cheng found a way to combine her interest in both, studying how patterns and ratios are applied to make unique sounds.



Ebonya Washington

Mathematics and money? Ebonya Washington found a way to combine her interest in both, studying how percents are applied to make good banking decisions.



Lue Gim Gong

Mathematics and horticulture? Lue Lim Gong found a way to combine his interest in both, studying how temperatures impacted the growth of citrus to invent a cold-resistant orange.



Franklin R. Chang-Diaz

Mathematics and space? Franklin R. Chang-Diaz found a way to combine his interest in both, studying how geometry and engineering are applied to construct space vehicles.

Facilitating Meaningful Conversations

Amplify Desmos Math Florida is designed to support interactive and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the ideas of others, and appreciate different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the real-world and mathematical experiences that students bring to the classroom.

Use the Tell a Story routine to help students make sense of the turtle race animation. Invite students to write a story about the animation using their experiences, perspectives, and creativity. **(MP1)**



In the lesson **Turtle Time Trials**, students are invited to use the **Tell a Story** routine to create their own narratives about what the turtles in the animation are doing and why. This is a chance for students to use their creativity while engaging deeply in the math.

1 Launch

Direct students' attention to the images of the rice dishes.

To surface connections, consider asking, "What is your favorite rice dish? When or where do you eat it?"



In the lesson **Rice Ratios**, students are introduced to the concept of equivalent ratios in the context of making rice. The Teacher Edition provides teachers with prompts they can use to help students connect the rice dishes in the activities to their own lives.

Supporting Student Collaboration

Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.

How will you support student collaboration?

- Discuss what supporting each other without giving answers could look or sound like.
- Invite a student from each group to read the task aloud.

The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

In the lesson **Subway Seats**, students use the **Which One Doesn't Belong?** routine to begin exploring two-variable equations. Classroom conversations about activities like this are a way for teachers to elicit different kinds of student brilliance. The Teacher Edition provides suggestions for how teachers can engage students in conversation and reinforce the idea that each student has something uniquely valuable to contribute.

Accessibility

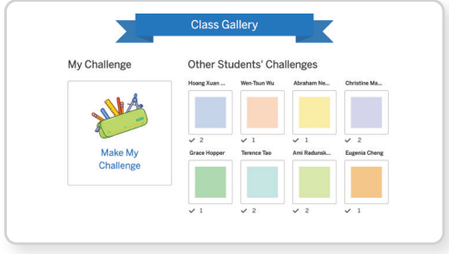
Amplify Desmos Math Florida is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

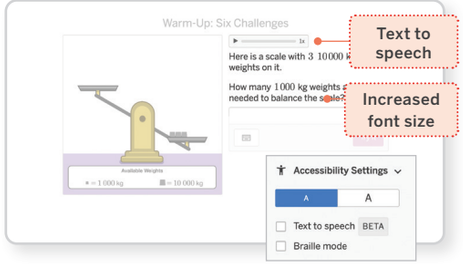
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome student interests, support sustaining effort and persistence, and develop an awareness of others.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

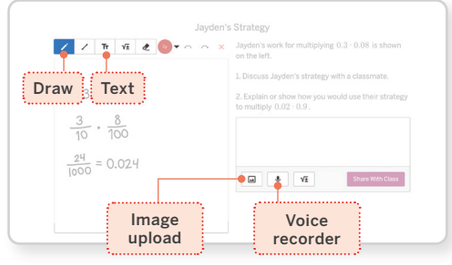
Learners make sense of information differently. Amplify Desmos Math Florida includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math Florida ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

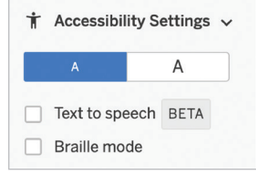
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math Florida enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.

Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Differentiation: Beyond the Lesson

In each lesson, students' understanding of the learning goals is broken down for teachers into three categories: students who need support to understand the learning goals, students who need to continue strengthening their understanding of the learning goals, and students who are ready to stretch their learning.

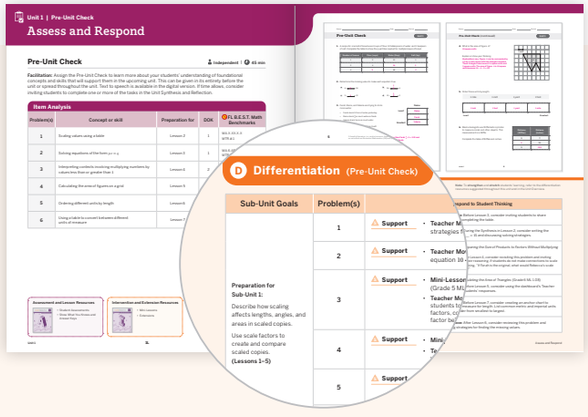
Each **Support**, **Strengthen**, and **Stretch** resource is designed to take 15 minutes:

- **Mini-Lessons:** Targeted intervention for students who need additional support or need more time.
- **Extensions:** Small group or independent challenges.



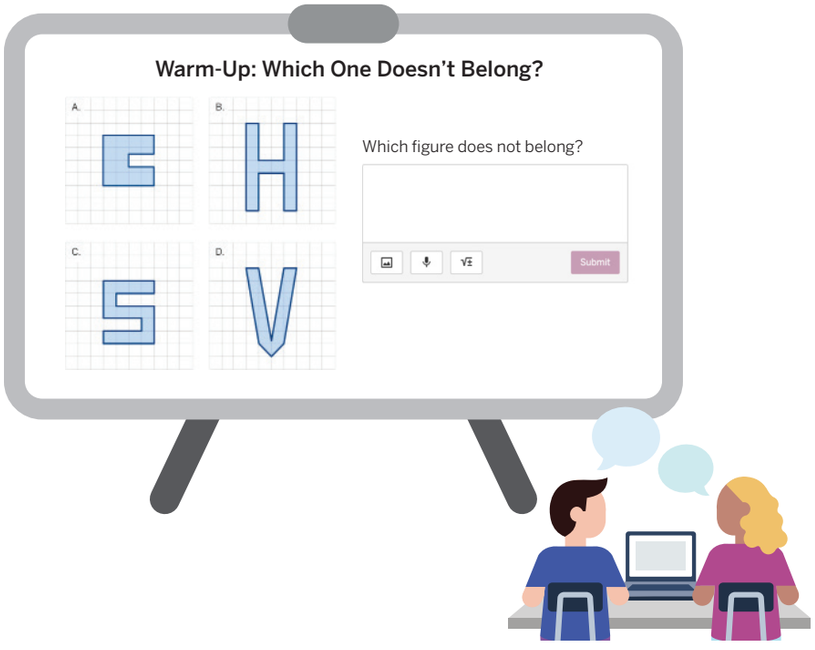
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

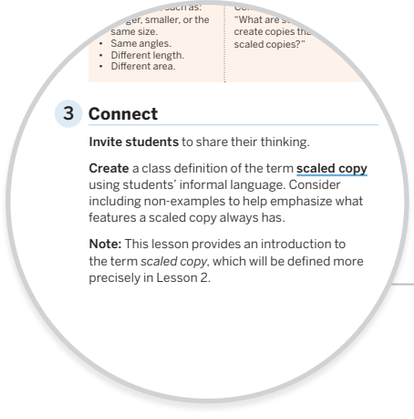
- The **Which One Doesn't Belong?** routine communicates to students that their ideas have value, that there are many ways to be correct in math, and that they can learn math by sharing their math thinking with each other.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

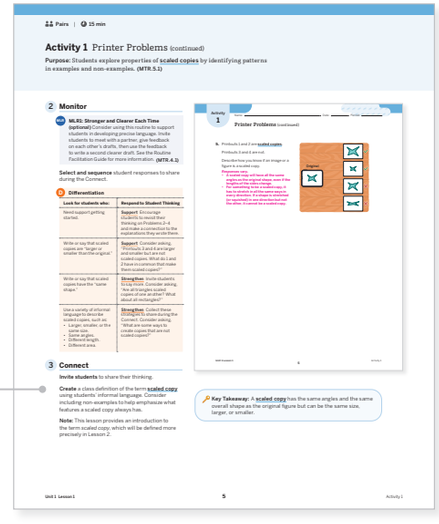
Each instructional routine included in an Amplify Desmos Math Florida lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' mathematical thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math Florida curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?





For more information about the Instructional Routines embedded in the Amplify Desmos Math Florida curriculum, see the Routine Facilitation Guide in the Professional Development Library.

Math Language Development

At Amplify Desmos Math Florida, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of English Language Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math Florida includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for English Language Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* (′). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does *pre* mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Calculate the area of a parallelogram on a grid using a variety of strategies.
  2. **Language Goal:** Compare and contrast strategies for determining the area of a parallelogram. **(Writing, Speaking, and Listening)**
  3. **Language Goal:** Describe parallelograms using precise mathematical language. **(Speaking and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

English Language Learners Support

Supports for ELL **English Language Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for English Language Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. English Language Learner supports may also be attached to MLRs.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Developing Fluency

Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another*.

In Grade 7, Florida students are expected to perform all four operations with rational numbers with procedural fluency.

Expectation

Exploration

To develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.

Procedural Reliability

To utilize skills to develop an accurate, reliable method that aligns with the student's understanding and learning style.

Procedural Fluency

To utilize skills to become fluent with an efficient, generalizable and accurate procedure, including a standard algorithm.

How Amplify Desmos Math Florida Supports the Expectation

- Manipulatives and visual models and diagrams are used extensively to support student's exploration of new concepts.
- Discussions are strongly encouraged in each lesson, including 💬 Discuss prompts in the student edition as well as suggested discussion prompts in the teacher's edition.
- Students are encouraged to estimate and compare estimations to actual answers to support reasonableness of problem solving solutions.
- Students are often asked to analyze a drawing or to make a drawing to explain their reasoning and strengthen their understanding of the math concepts being studied.

- Frequently in each course, students are asked to analyze and evaluate work presented in their student edition that represents work done by other students for accuracy, preference to learning style, and for efficiency.

- Program features such as practice, spiral reviews, mini lessons, and pre-unit assessments are all designed to assist students with achieving procedural fluency.

*National Research Council [2001, 2005, 2012]; Star [2005]

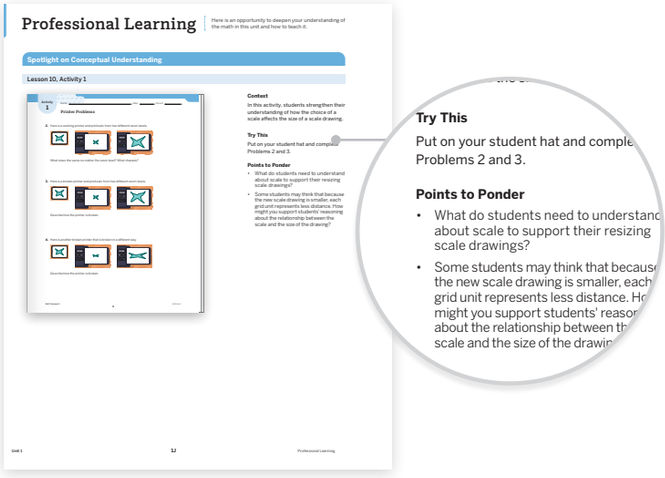
Professional Learning

The Amplify Desmos Math Florida program includes regular opportunities for teachers to deepen their understanding while they teach.

Embedded Professional Development

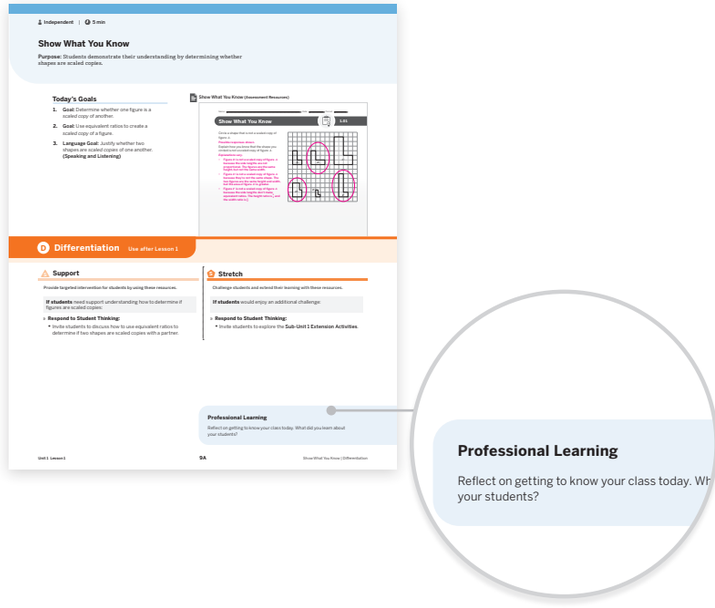
Unit-Level Professional Development

The Unit Overview for each unit includes a Professional Development page, which lists opportunities for teachers to deepen their understanding of the math in the unit and how to teach it.



Lesson-Level Professional Development

Each lesson provides a moment of professional learning as an opportunity for teachers to reflect on how well the lesson went. These professional learning moments can relate to principles, instruction, pedagogy, or program content.



Professional Development Sessions

Amplify's professional development team supports both teachers and administrators throughout their entire implementation journey. These sessions are available as on-site or virtual options.

Launch

Launch sessions introduce Amplify programs and support strong implementation. After learning about the program's foundational principles, you'll practice implementing the program in a collaborative environment.

Strengthen

Strengthen sessions deepen understanding of the program. These offerings are targeted and are meant to take your practice – and your students' learning to the next level.

Coach

These sessions are tailored to elevate instructional practices and meet the unique needs of teachers and leaders. These sessions can be customized leveraging our menu of supports.

Professional learning on your schedule

The PD Library serves as a centralized location where educators can access an ever-growing collection of professional learning resources and courses designed to support the implementation of Amplify Desmos Math Florida. Educators will find bite-sized how to videos, unit and lesson planning guides and templates, resources for leaders, and so much more!

Program Assessments

A variety of performance data in Amplify Desmos Math Florida provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math Florida, understanding is made continually visible.

Lesson-Level Assessments

Amplify Desmos Math lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.



Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.



Pre-Unit Check

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.



Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.



End-of-Unit Assessment

Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight. Two forms are included to help facilitate assessment administration in the classroom.

Florida's B.E.S.T. Standards for Mathematics

The following correlations show the alignment of Amplify Desmos Math Florida to the FL B.E.S.T. Mathematics Standards for Grade 7.

Benchmark	B.E.S.T Mathematics Benchmark	Amplify Desmos Math Lesson(s)
Number Sense and Operations		
MA.7.NSO.1.1	Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.	**Unit 5** Lessons 6, 7, 8, 9, 10
MA.7.NSO.1.2	Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.	**Unit 5** Lessons 1, 2, 6, 7, 8, 9 ,10 **Unit 8** Lessons 3, 4, 5
MA.7.NSO.2.1	Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.	**Unit 5** Lessons 5, 6, 7 **Unit 6** Lessons 9, 10
MA.7.NSO.2.2	Add, subtract, multiply and divide rational numbers with procedural fluency.	**Unit 5** Lessons 2, 3, 4, 5 **Unit 6** Lessons 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16 **Unit 8** Lessons 9, 11
MA.7.NSO.2.3	Solve real-world problems involving any of the four operations with rational numbers.	**Unit 5** Lessons 3, 4 **Unit 6** Lessons 1, 2, 3, 12 **Unit 7** Lessons 6, 7, 8 **Unit 8** Lessons 9, 11
Algebraic Reasoning		
MA.7.AR.1.1	Apply properties of operations to add and subtract linear expressions with rational coefficients.	**Unit 6** Lesson 10
MA.7.AR.1.2	Determine whether two linear expressions are equivalent.	**Unit 6** Lessons 4, 5, 8, 9
MA.7.AR.2.1	Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.	**Unit 6** Lessons 13, 14, 15, 16
MA.7.AR.2.2	Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.	**Unit 6** Lessons 1, 2, 3, 5, 6, 7, 11, 12
MA.7.AR.3.1	Apply previous understanding of percentages and ratios to solve multi-step realworld percent problems.	**Unit 4** Lessons 1, 2, 3, 4, 5, 6, 7, 8, 13
MA.7.AR.3.2	Apply previous understanding of ratios to solve real-world problems involving proportions.	**Unit 2** Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13 **Unit 4** Lessons 9, 10 **Unit 8** Lesson 11
MA.7.AR.3.3	Solve mathematical and real-world problems involving the conversion of units across different measurement systems.	**Unit 4** Lesson 11
MA.7.AR.4.1	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.	**Unit 2** Lessons 2, 3, 7, 8, 10
MA.7.AR.4.2	Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.	**Unit 2** Lessons 2, 3, 4, 5, 6, 7, 9, 10, 11, 12 **Unit 4** Lessons 9, 10 **Unit 8** Lessons 3, 4, 5

Florida's B.E.S.T. Standards for Mathematics

MA.7.AR.4.3	Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.	**Unit 2** Lessons 11, 23
MA.7.AR.4.4	Given any representation of a proportional relationship, translate the representation to a written description, table or equation.	**Unit 2** Lessons 4, 5, 6, 7, 9, 10, 11, 13 **Unit 4** Lesson 9
MA.7.AR.4.5	Solve real-world problems involving proportional relationships.	**Unit 2** Lessons 5, 6, 7, 10, 12, 13 **Unit 4** Lessons 9, 10 **Unit 8** Lesson 11
	Geometric Reasoning	
MA.7.GR.1.1	Apply formulas to find the areas of trapezoids, parallelograms and rhombi.	**Unit 7** Lessons 1, 3
MA.7.GR.1.2	Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.	**Unit 3** Lesson 4 **Unit 7** Lessons 2, 3
MA.7.GR.1.3	Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.	**Unit 3** Lessons 1, 2, 3
MA.7.GR.1.4	Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.	**Unit 3** Lessons 5, 6 **Unit 7** Lessons 4, 5
MA.7.GR.1.5	Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.	**Unit 1** Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **Unit 3** Lesson 2 **Unit 4** Lesson 9 **Unit 7** Lessons 4, 5
MA.7.GR.2.1	Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure's net.	**Unit 7** Lesson 8
MA.7.GR.2.2	Solve real-world problems involving surface area of right circular cylinders.	**Unit 7** Lesson 8
MA.7.GR.2.3	Solve mathematical and real-world problems involving volume of right circular cylinders.	**Unit 7** Lessons 6, 7
	Data Analysis and Probability	
MA.7.DP.1.1	Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.	**Unit 8** Lessons 9, 12
MA.7.DP.1.2	Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.	**Unit 8** Lessons 9, 12, 13
MA.7.DP.1.3	Given categorical data from a random sample, use proportional relationships to make predictions about a population.	**Unit 8** Lessons 10, 11
MA.7.DP.1.4	Use proportional reasoning to construct, display and interpret data in circle graphs.	**Unit 8** Lessons 6, 7, 8

Florida's B.E.S.T. Standards for Mathematics

MA.7.DP.1.5	Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.	**Unit 8** Lessons 7, 8
MA.7.DP.2.1	Determine the sample space for a simple experiment.	**Unit 8** Lesson 2
MA.7.DP.2.2	Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.	**Unit 8** Lessons 1, 2, 3, 4
MA.7.DP.2.3	Find the theoretical probability of an event related to a simple experiment.	**Unit 8** Lessons 2, 3, 4, 5
MA.7.DP.2.4	Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.	**Unit 8** Lessons 3, 4, 5

Mathematical Thinking and Reasoning Standards

The following correlations show the alignment of Amplify Desmos Math Florida to the Mathematical Thinking and Reasoning Standards.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Unit 1 Lesson 5
Unit 2 Lessons 7, 10, 12
Unit 3 Lessons 3, 4, 6
Unit 4 Lessons 2, 3
Unit 5 Lesson 5
Unit 7 Lessons 1, 2, 4, 5, 7, 8
Unit 8 Lessons 4, 10

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Unit 1 Lessons 6, 7, 10
Unit 2 Lessons 3, 4, 5, 6, 8, 10, 11, 12
Unit 4 Lessons 1, 3, 4, 9, 11
Unit 5 Lessons 6, 7, 8
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
Unit 7 Lessons 1, 3, 6, 8
Unit 8 Lessons 1, 2, 7, 8

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Unit 1 Lesson 5
Unit 2 Lessons 4, 9
Unit 3 Lesson 6
Unit 4 Lessons 6, 7, 8
Unit 5 Lessons 1, 2, 3, 4
Unit 6 Lessons 11, 14, 15, 16
Unit 7 Lessons 5, 6
Unit 8 Lessons 1, 2, 3

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Unit 1 Lessons 1, 2, 3, 4, 5, 8, 10
Unit 2 Lessons 1, 2, 7, 10
Unit 3 Lessons 2, 3, 4, 5, 6
Unit 4 Lessons 4, 6, 8, 9, 10, 11, 13
Unit 5 Lessons 1, 2, 9, 10
Unit 6 Lessons 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15
Unit 7 Lessons 2, 3, 4, 6
Unit 8 Lessons 4, 6, 7, 9, 10, 11, 13

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Unit 1 Lessons 1, 2, 4, 9, 10
Unit 2 Lessons 1, 2, 6, 7, 8
Unit 3 Lessons 1, 5
Unit 4 Lesson 12
Unit 5 Lessons 2, 3, 4, 5, 6, 7, 8, 9, 10
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13
Unit 7 Lessons 1, 2, 4, 7
Unit 8 Lessons 5, 6, 11, 12

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Unit 1 Lessons 6, 7
Unit 2 Lesson 3
Unit 3 Lessons 1, 3, 4, 5
Unit 4 Lesson 5
Unit 5 Lessons 6, 7, 8
Unit 6 Lessons 15, 16
Unit 8 Lessons 3, 5, 11, 12

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Unit 1 Lessons 6, 7, 8, 9
Unit 2 Lessons 5, 9, 11, 12
Unit 4 Lessons 5, 7, 13
Unit 5 Lessons 3, 4
Unit 6 Lessons 1, 3, 12, 16
Unit 7 Lessons 3, 7, 8
Unit 8 Lessons 7, 8, 9, 10, 13

Florida's B.E.S.T. Standards for English Language Arts

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

Unit 1 Lessons 2, 3, 5
Unit 2 Lessons 5, 6, 7, 10, 11
Unit 3 Lessons 3
Unit 4 Lessons 6, 8
Unit 5 Lessons 1, 2, 4, 5, 6, 10
Unit 6 Lessons 1, 2, 4, 5, 6, 7, 10, 14, 16
Unit 7 Lessons 3, 5, 8
Unit 8 Lessons 3, 4, 6, 7, 10, 11, 12, 13

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

Unit 1 Lessons 1, 5
Unit 2 Lesson 7
Unit 3 Lesson 3
Unit 4 Lessons
Unit 5 Lessons
Unit 6 Lesson 12
Unit 7 Lessons 3, 7
Unit 8 Lessons 2, 4, 9

ELA.K12.EE.3.1 Make inferences to support comprehension.

Unit 1 Lessons 1, 2, 4, 6, 7, 8, 10
Unit 2 Lessons 2, 3, 5, 6, 7, 9, 10, 13
Unit 3 Lessons 1, 3, 5, 6
Unit 4 Lessons 3, 4, 5, 10, 11, 12
Unit 5 Lessons 6, 7, 9
Unit 6 Lessons 1, 3, 6
Unit 7 Lesson 7
Unit 8 Lesson 11

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

Unit 1 Lessons 2, 5, 7, 10
Unit 2 Lessons 9, 12
Unit 3 Lessons 2, 4, 6
Unit 4 Lessons 1, 2, 3, 6, 7, 10, 11
Unit 5 Lessons 3, 8, 9, 10
Unit 6 Lessons 3, 6, 9, 11, 14
Unit 7 Lessons 1, 7, 8
Unit 8 Lessons 7, 8, 9, 10

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

Unit 1 Lessons 1, 4, 5, 8, 9, 10
Unit 2 Lessons 1, 2, 4, 9, 12, 13
Unit 3 Lesson 4
Unit 4 Lessons 1, 5, 6, 9, 11, 12, 13
Unit 5 Lessons 1, 3, 4, 7
Unit 6 Lessons 2, 5, 7, 8, 9, 10, 11, 13, 15
Unit 7 Lessons 1, 2, 3, 4, 6, 7
Unit 8 Lessons 1, 2, 3, 5, 8, 9, 10, 12

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

Unit 1 Lesson 6
Unit 2 Lessons 1, 5, 11
Unit 3 Lesson 2
Unit 4 Lesson 2, 4, 7, 8, 13
Unit 5 Lessons 4, 5
Unit 6 Lessons 1, 2
Unit 7 Lesson 3
Unit 8 Lessons 6, 7, 13

English Language Development Standards

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

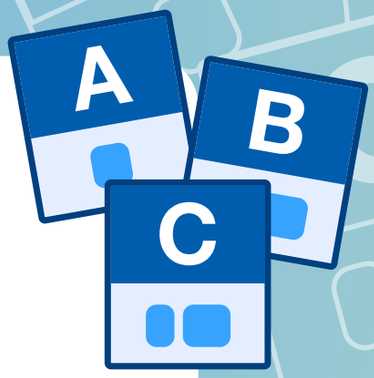
Unit 1 All Lessons
Unit 2 All Lessons
Unit 3 All Lessons
Unit 4 All Lessons
Unit 5 All Lessons
Unit 6 All Lessons
Unit 7 All Lessons
Unit 8 All Lessons

ELD.K12.ELL.SI.1 English language learners communicate for social and instructional purposes within the school setting.

Unit 1 All Lessons
Unit 2 All Lessons
Unit 3 All Lessons
Unit 4 All Lessons
Unit 5 All Lessons
Unit 6 All Lessons
Unit 7 All Lessons
Unit 8 All Lessons

UNIT 5

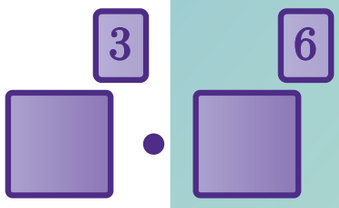
Positive and Negative Rational Numbers



Students extend what they learned in Grade 6 to add, subtract, multiply, and divide positive and negative rational numbers. Students also use order of operations to evaluate numerical expressions and apply the Laws of Exponents to generate equivalent expressions.

Essential Questions

- How is solving problems with positive and negative fractions and decimals the same as or different from solving problems with integers?
- How do you know what operation to do first when there are multiple grouping symbols?
- How can you use properties of exponents to simplify powers with the same base?



Sub-Unit Goals

Sub-Unit 1

- Perform all four operations with positive and negative rational numbers using a variety of strategies.
- Use order of operations to evaluate numerical expressions.

Sub-Unit 2

- Apply the Laws of Exponents to generate equivalent expressions involving whole-number exponents.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Rewrite numbers in equivalent forms. **(MA.7.NSO.1)**

Add, subtract, multiply and divide rational numbers. **(MA.7.NSO.2)**

Benchmarks Addressed in This Unit: MA.7.NSO.1.1 MA.7.NSO.1.2 MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.NSO.2.3

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Assess the reasonableness of solutions. **(MTR.6.1)**

Also addressed: **MTR.1.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students extend what they learned in Grade 6 to add, subtract, multiply, and divide positive and negative rational numbers. Students also use order of operations to evaluate numerical expressions and apply the Laws of Exponents to generate equivalent expressions.

Prior Learning

- **Grade 6,** Units 4 and 5: Operations with positive rational numbers **(MA.6.NSO.2.1, MA.6.NSO.2.2, MA.6.NSO.2.3)**
- **Grade 6,** Unit 7: Add, subtract, multiply, and divide integers **(MA.6.NSO.4.1, MA.6.NSO.4.2)**
- **Grade 6,** Unit 6: Evaluate exponents **(MA.6.NSO.3.3)**
- **Grade 6,** Unit 6: Generate equivalent expressions **(MA.6.AR.1.3, MA.6.AR.1.4)**

Future Learning

- **Grade 8,** Unit 8: Rational and irrational numbers **(MA.8.NSO.1.1, MA.8.NSO.1.2)**
- **Grade 8,** Unit 7 Negative exponents **(MA.NSO.1.3, MA.8.AR.1.1)**
- **Grade 8**, Unit 7 Operations with scientific notation **(MA.8.NSO.1.5)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 2, 3 and 4**, students develop conceptual understanding of the effects of adding, subtracting, multiplying, and dividing positive and negative numbers on a number line.

Procedural Fluency

In **Lessons 2, 3, and 4**, students develop procedural fluency in performing operations with positive and negative numbers by solving number line problems represented by bumpers and the position, rate, and time of turtles.

Application

In **Lesson 3**, students apply their understanding of unit rate to determine the position and time of a turtle along its path.

Strategic Competence

In **Lesson 4**, students justify why their expressions and equations represent positive or negative values as it relates to the positions, rates, and times of turtles along the number line.

Adaptive Reasoning

In **Lessons 1**, students apply their understanding of equivalent forms of numbers to solve a code puzzle.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.5.AR.2.2 MA.5.AR.2.3 MA.6.AR.2.1 MA.6.NSO.1.1 MA.6.NSO.1.1 MA.6.NSO.4.1
MTR.2.1

Sub-Unit 1



1 Code Breaking

Equivalent Forms

Decode a message by rewriting **rational numbers** in equivalent forms.

MA.7.NSO.1.2
MTR.3.1 MTR.4.1

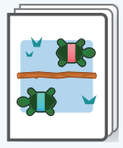


2 Bumpers

Adding and Subtracting on the Number Line

Add and subtract positive and negative decimals and fractions on a number line.

MA.7.NSO.2.2
MTR.3.1 MTR.4.1 MTR.5.1



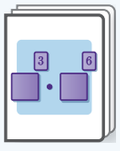
3 Back in Time

Position, Rate, and Time

Make sense of a different model for multiplying positive and negative numbers involving position, rate, and time.

MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.NSO.2.3
MTR.3.1 MTR.5.1 MTR.7.1

Sub-Unit 2

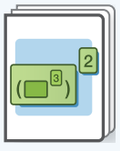


6 Combining Exponents

Equivalent Expressions with Exponents

Make use of structure to discover ways to write equivalent expressions involving products of powers and powers of powers.

MA.7.NSO.1.1 MA.7.NSO.2.1
MTR.2.1 MTR.5.1 MTR.6.1



7 Power Pairs

Multiplying Powers and Powers of Powers

Justify strategies for determining whether two expressions with exponents are equivalent.

MA.7.NSO.1.1 MA.7.NSO.2.1
MTR.2.1 MTR.5.1 MTR.6.1



8 Rewriting Powers

Rewriting Exponential Expressions as a Single Power

Develop strategies for rewriting exponential expressions as a single power and work with division of powers.

MA.7.NSO.1.1 MA.7.NSO.2.1
MTR.2.1 MTR.5.1 MTR.6.1



9 Exponent Zero

Using Patterns to Understand Zero Exponents

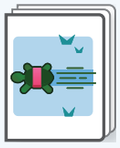
Identify and use patterns to reason and generalize about zero exponents.

MA.7.NSO.1.1 MA.7.NSO.2.1
MTR.4.1 MTR.5.1

Pacing: 13 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 10 Lessons
- Pre-Unit Check
- Sub-Unit Quiz(zes)
- End-of-Unit Assessment



4 **Speeding Turtles**

Dividing Rational Numbers

Make sense of division of positive and negative numbers using position, rate, and time.

MA.7.NSO.2.2 MA.7.NSO.2.3
MTR.3.1 MTR.5.1 MTR.7.1



5 **Order Matters**

Evaluating Expressions with Exponents

Use order of operations to evaluate numerical expressions involving grouping symbols, exponents, and absolute value.

MA.7.NSO.2.1 MA.7.NSO.2.2
MTR.1.1 MTR.5.1

Assess and Respond



A **Sub-Unit Quiz**

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.NSO.1.2 MA.7.NSO.2.1

Assess and Respond



10 **Negative Bases**

Powers with Negative Bases

Evaluate powers with negative rational bases.

MA.7.NSO.1.1 MA.7.NSO.2.1
MTR.4.1 MTR.5.1



A **End-of-Unit Assessment**

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.NSO.1.1 MA.7.NSO.1.2
MA.7.NSO.2.1

Pacing Considerations

Lessons 5-7: These lessons give students an opportunity to apply what they've learned about all four operations involving positive and negative numbers. There is no new content in these lessons, so one or two of these lessons may be omitted.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know

Lesson 2

- **Copy:** Show What You Know, Horizontal Number Lines Sheet *(one per student, optional)*

Lesson 3

- **Copy:** Show What You Know, Activity 3 You're Invited to Explore More Sheet *(one per student, as needed)*

Lesson 4

- **Copy:** Show What You Know

Lesson 5

- **Copy:** Show What You Know
- **Cut:** Activity 2 Sheet *(one per group)*

Lesson 6

- **Copy:** Show What You Know

Lesson 7

- **Copy:** Show What You Know
- **Cut:** Activity 1 Cards *(one set per group)*

Lesson 8

- **Copy:** Show What You Know
- **Materials:** blank paper, index cards, or sticky notes

Lesson 9

- **Copy:** Show What You Know

Lesson 10

- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

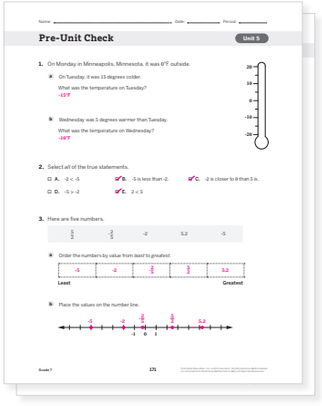
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.5.AR.2.2, MA.5.AR.2.3, MA.6.AR.2.1, MA.6.NSO.1.1, MA.6.NSO.1.1, MA.6.NSO.4.1


Pre-Unit Check

Unit 5

1. On Monday in Minneapolis, Minnesota, it was 6°F outside.

a. On Tuesday, it was 15 degrees colder. What was the temperature on Tuesday? -15°F

b. Wednesday was 5 degrees warmer than Tuesday. What was the temperature on Wednesday? -10°F

2. Select all of the true statements.

A. $-2 < -5$ B. -5 is less than -2 C. -2 is closer to 0 than 5 is.

D. $-5 > -2$ E. $2 < 5$

3. Here are five numbers.

$\frac{1}{2}$ $\frac{3}{5}$ -2 5.2 -5

a. Order the numbers by value from least to greatest.

-5 -2 $-\frac{1}{2}$ $\frac{3}{5}$ 5.2

Least Greatest

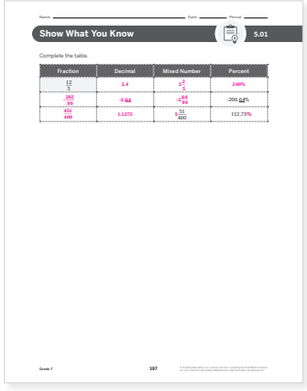
b. Place the values on the number line.

Grade 7 171

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 1 Show What You Know: MA.7.NSO.1.2


Show What You Know

5.01

Complete the table.

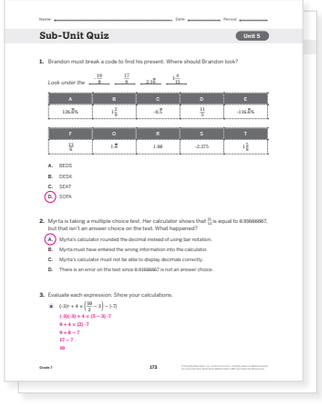
Fraction	Decimal	Mixed Number	Percent
$\frac{12}{5}$	2.4	$2\frac{2}{5}$	240%
$-\frac{369}{400}$	$-0.\overline{44}$	$-\frac{4}{9}$	$-200.\overline{44}\%$
$\frac{451}{400}$	1.1275	$1\frac{51}{400}$	112.75%

Grade 7 197

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: MA.7.NSO.1.2, MA.7.NSO.2.1


Sub-Unit Quiz

Unit 5

1. Brandon must break a code to find his present. Where should Brandon look?

Look under the $\frac{10}{3}$ $\frac{17}{5}$ $\frac{4}{\sqrt{25}}$ $\frac{4.5}{1}$

A	B	C	D	E
$136.\overline{36}$	$1\frac{2}{5}$	$-6.\overline{3}$	$\frac{11}{5}$	$-136.\overline{36}$

F	G	H	S	T
$\frac{10}{3}$	$1.\overline{3}$	1.08	-2.375	$4\frac{1}{2}$

A. BEDS

B. DESK

C. SEAT

D. SOFA

2. Myrta is taking a multiple choice test. Her calculator shows that $\frac{11}{12}$ is equal to 0.916666667, but that isn't an answer choice on the test. What happened?

A. Myrta's calculator rounded the decimal instead of using bar notation.

B. Myrta must have entered the wrong information into the calculator.

C. Myrta's calculator must not be able to display decimals correctly.

D. There is an error on the test since 0.916666667 is not an answer choice.

3. Evaluate each expression. Show your calculations.

a. $(-3)^2 + 4 \times \left[\frac{10}{2} - 3\right] - |-7|$

$(-3)(-3) + 4 \times (5 - 3) - 7$

$9 + 4 \times (2) - 7$

$9 + 8 - 7$

$17 - 7$

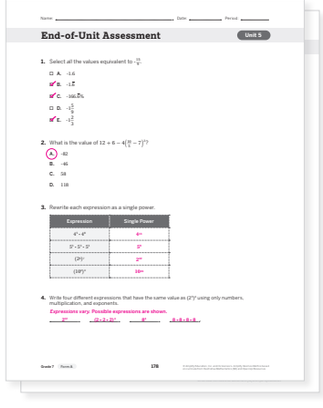
10

Grade 7 173

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.NSO.1.1, MA.7.NSO.1.2, MA.7.NSO.2.1


End-of-Unit Assessment

Unit 5

1. Select all the values equivalent to $-\frac{25}{4}$.

A. $-6.\overline{6}$

B. $-1.\overline{6}$

C. $-\frac{100}{4}$

D. $-\frac{1.5}{0.6}$

E. $-\frac{5^2}{-2^2}$

2. What is the value of $12 \div 6 - 4\left[\frac{10}{2} - 7\right]^2$?

A. 40

B. −46

C. 58

D. 148

3. Rewrite each expression as a single power.

Expression	Single Power
$4^3 \cdot 4^4$	4^{7}
$5^4 \cdot 5^{-2} \cdot 5^3$	5^{5}
$(2^3)^2$	2^{6}
$(10^2)^4$	10^{8}

4. Write four different expressions that have the same value as $(2^3)^2$ using only numbers, multiplication, and exponents.

Expressions vary. Possible expressions are shown.

2^6 $2^3 \cdot 2^3$ 4^3 $8 \cdot 8$

Grade 7 Form A 178

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

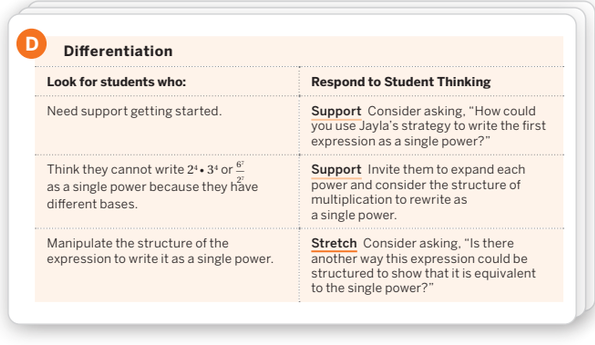
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Perform all four operations with positive and negative rational numbers using a variety of strategies. • Use order of operations to evaluate numerical expressions.	**Mini-Lesson** • *Dividing Integers*	**Repeated Challenges** • Lesson 2 *(Bumpers)* • Lesson 3 *(Back in Time)* • Lesson 4 *(Speeding Turtles)* **Practice** • Lessons 1–5	**You're invited to explore more.** • Lesson 3 *(Back in Time)*
Sub-Unit 2 (Lessons 6–10) • Apply the Laws of Exponents to generate equivalent expressions involving whole-number exponents.	**Mini-Lesson** • *Multiplying Powers*	**Challenge Creator** • Lesson 8 *(Rewriting Powers)* **Practice** • Lessons 6–10	**You're invited to explore more.** • Lesson 6 *(Combining Exponents)* • Lesson 7 *(Power Pairs)*

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 8**, where students sort expressions based on whether they are equivalent to a given power.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How could you use Jayla's strategy to write the first expression as a single power?"
Think they cannot write $2^4 \cdot 3^4$ or $\frac{6^7}{2^2}$ as a single power because they have different bases.	**Support** Invite them to expand each power and consider the structure of multiplication to rewrite as a single power.
Manipulate the structure of the expression to write it as a single power.	**Stretch** Consider asking, "Is there another way this expression could be structured to show that it is equivalent to the single power?"

Accessibility

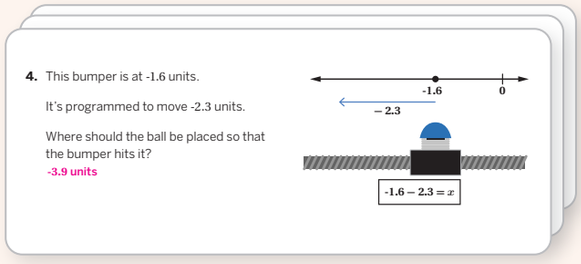
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Building Knowledge

Lesson 1: Students use models to make sense of operations with negative and positive rational numbers, which guides information processing and visualization.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing

Here are two suggestions in this unit for supporting students with making sense of complex tasks.

Lesson 3, Activity 3

Accessibility: Conceptual Processing Invite students to make a table of values or write out where Tommy was at shorter intervals to make sense of going back in time with a negative rate. (For example, 1 minute ago Tommy was at 62.2 feet.)

Lesson 6, Activity 2

Accessibility: Conceptual Processing To support abstract thinking, invite students to expand expressions with a single exponent and use parentheses to re-group and write a new expression.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
rational number	1
power of ten	6

Review Vocabulary

absolute value
bar notation
base (of a power)
exponent
integer
order of operations
repeating decimal
terminating decimal

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	4
MLR2: Collect and Display	4 and 7
MLR3: Critique, Correct, Clarify	3

Sub-Unit Language Goals

- Describe strategies for determining the sum, difference, product, or quotient of two rational numbers.
- Explain and interpret expressions involving exponents and their properties.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 7**, where students play a card game in small groups to identify pairs of equivalent expressions and justify their thinking. Students are encouraged to share in their primary language and to use gestures or visuals as they share to make sense of the mathematical strategies.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Rational and Irrational Numbers

In this unit, students perform operations with rational numbers. In **Grade 8, Unit 8**, students will consider what makes a number rational as they compare them to irrational numbers.

FL B.E.S.T. Math Benchmarks:
MA.8.NSO.1.1, MA.8.NSO.1.2

Example:

Rational numbers are numbers that can be written as a fraction of two integers. Some examples of rational numbers are 5, 2.4, 2, $\frac{1}{2}$, $\sqrt{16}$, and $\frac{17}{3}$.

We call numbers that cannot be written this way irrational numbers. Some examples of irrational numbers are $\sqrt{3}$, $3\sqrt{2}$, 2π, and $1.73204082\ldots$ (neither repeats nor ends).

Negative Exponents

In this unit, students evaluate numerical expressions with whole-number exponents. In **Grade 8, Unit 7**, students extend their understanding to include integer exponents.

FL B.E.S.T. Math Benchmarks:
MA.8.NSO.1.3, MA.8.AR.1.1

Example:

Rewrite $\frac{1}{x^{-4}}$ with a positive exponent.

$\frac{1}{x^{-4}} = \frac{x^0}{x^{-4}} = x^{0-(-4)} = x^4$

Operations with Scientific Notation

In this unit, students apply Laws of Exponents to generate equivalent expressions . In **Grade 8, Unit 7**, students solve problems with number expressed in scientific notation.

FL B.E.S.T. Math Benchmark:
MA.8.NSO.1.5

Example:

What is the value of $(7 \cdot 10^4) \cdot (3 \cdot 10^2)$?

$(7 \cdot 10^4) \cdot (3 \cdot 10^2)$

$(7 \cdot 3) \cdot (10^4 \cdot 10^2)$

$21 \cdot 10^{4+2}$

$21 \cdot 10^6$

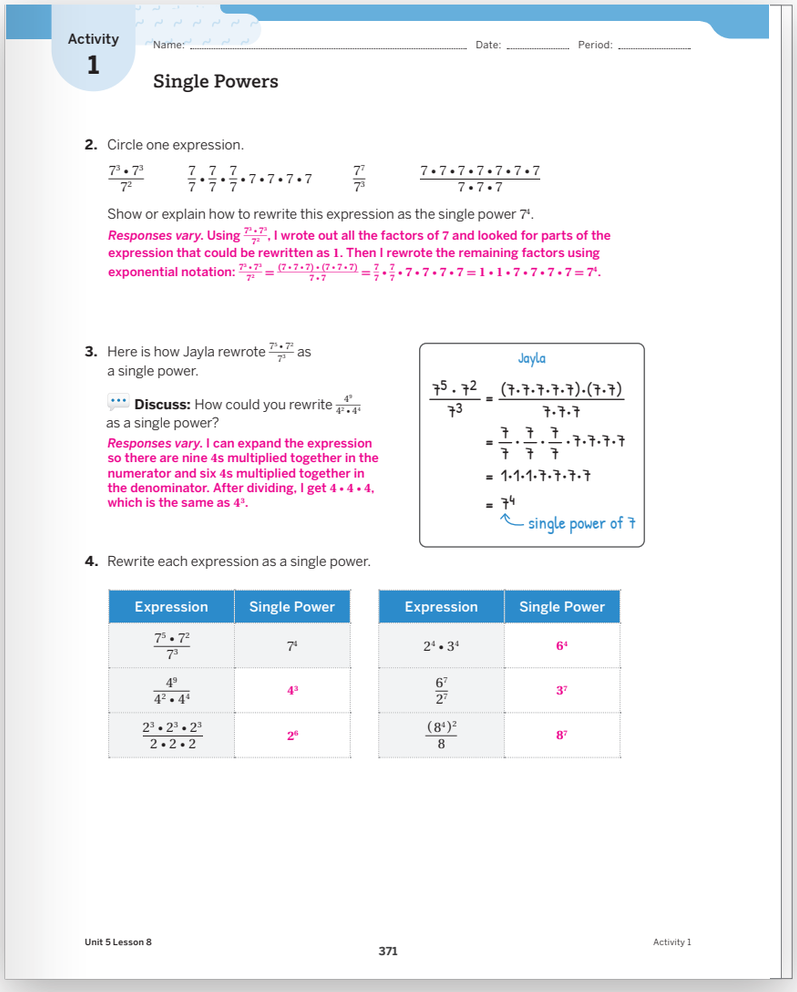
$2.1 \cdot 10^7$

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 8, Activity 1



Activity 1

Name: _____ Date: _____ Period: _____

Single Powers

2. Circle one expression.

$\frac{7^3 \cdot 7^3}{7^2}$ $\quad \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\quad \frac{7^7}{7^3}$ $\quad \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

Show or explain how to rewrite this expression as the single power 7^4.

Responses vary. Using $\frac{7^3 \cdot 7^2}{7^2}$, I wrote out all the factors of 7 and looked for parts of the expression that could be rewritten as 1. Then I rewrote the remaining factors using exponential notation: $\frac{7^3 \cdot 7^2}{7^2} = \frac{(7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7} = \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^4$.

3. Here is how Jayla rewrote $\frac{7^3 \cdot 7^2}{7^3}$ as a single power.

Jayla

$\frac{7^5 \cdot 7^2}{7^3} = \frac{(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7 \cdot 7}$

$= \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$= 1 \cdot 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$= 7^4$ ← single power of 7

Discuss: How could you rewrite $\frac{4^9}{4^2 \cdot 4^4}$ as a single power?

Responses vary. I can expand the expression so there are nine 4s multiplied together in the numerator and six 4s multiplied together in the denominator. After dividing, I get $4 \cdot 4 \cdot 4$, which is the same as 4^3.

4. Rewrite each expression as a single power.

Expression	Single Power	Expression	Single Power
$\frac{7^5 \cdot 7^2}{7^3}$	7^4	$2^4 \cdot 3^4$	6^4
$\frac{4^9}{4^2 \cdot 4^4}$	4^3	$\frac{6^7}{2^7}$	3^7
$\frac{2^3 \cdot 2^3 \cdot 2^3}{2 \cdot 2 \cdot 2}$	2^6	$\frac{(8^4)^2}{8}$	8^7

Unit 5 Lesson 8 371 Activity 1

Context

In this activity, students extend their understanding of Laws of Exponents to write exponential expressions as a single power.

Try This

Put on your student hat and complete Problems 2–4.

Points to Ponder

- What do students need to understand about Laws of Exponents to identify ways exponential expressions can be written in equivalent forms?
- How does writing powers as repeated multiplication support their understanding of dividing powers.

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Reasoning with negative numbers	Lesson X	1	MA.6.NSO.1.1, MTR.2.1
2	Comparing positive and negative numbers	Lesson X	1	MA.6.NSO.1.1
3a	Comparing and ordering positive and negative numbers	Lesson 2	2	MA.6.NSO.1.2
3b	Plotting positive and negative numbers on a number line	Lesson 2	1	MA.6.NSO.1.2, MTR.2.1
4	Adding and subtracting fractions and decimals	Lesson 2	1	MA.6.NSO.4.1
5	Determining if equations are equivalent	Lesson 2, 4	2	MA.5.AR.2.2, MA.5.AR.2.3
6	Determining solutions to one-step equations	Lesson 2, 4	1	MA.6.AR.2.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



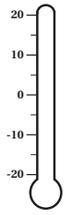
- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 5

1. On Monday in Minneapolis, Minnesota, it was 0°F outside.



a. On Tuesday, it was 15 degrees colder.

What was the temperature on Tuesday?

-15°F

b. Wednesday was 5 degrees warmer than Tuesday.

What was the temperature on Wednesday?

-10°F

2. Select *all* of the true statements.

- ☐ A. $-2 < -5$
- ☑ B. -5 is less than -2.
- ☑ C. -2 is closer to 0 than 5 is.
- ☐ D. $-5 > -2$
- ☑ E. $2 < 5$

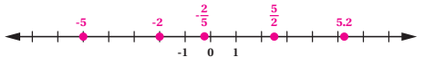
3. Here are five numbers.

$\frac{5}{2}$	$-\frac{2}{5}$	-2	5.2	-5

a. Order the numbers by value from *least* to *greatest*.

-5	-2	$-\frac{2}{5}$	$\frac{5}{2}$	5.2
Least				Greatest

b. Place the values on the number line.



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Name: Date: Period:

Pre-Unit Check (continued)

Unit 5

4. Determine the value of each expression.

Expression	Value
$\frac{11}{4} + \frac{9}{4}$	**5 (or equivalent)**
$24.14 - 12.16$	**11.98 (or equivalent)**
$\frac{5}{8} + \frac{3}{2}$	**$\frac{17}{8}$ (or equivalent)**
$102.82 + 13.43$	**116.25 (or equivalent)**

5. a. If the equation $2 + a = 5$ is true, which other equation(s) must be true?
Select *all* that apply.

- ☑ A. $5 - a = 2$
- ☐ B. $2 - a = 5$
- ☑ C. $a + 2 = 5$
- ☑ D. $5 - 2 = a$
- ☐ E. $2 - 5 = a$

b. If the equation $\frac{8}{2} = b$ is true, which other equation(s) must be true?
Select *all* that apply.

- ☑ A. $2 \cdot b = 8$
- ☑ B. $\frac{8}{b} = 2$
- ☐ C. $2 \cdot 8 = b$
- ☐ D. $\frac{1}{2} \cdot b = 8$
- ☑ E. $8 \cdot \frac{1}{2} = b$

6. Determine the value of the variable that makes each equation true.

a. $x - 2.6 = 5$
$x = 7.6$

b. $2.6 \cdot 5 = y$
$y = 13$

c. $\frac{1}{4}z = 8$
$z = 32$

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

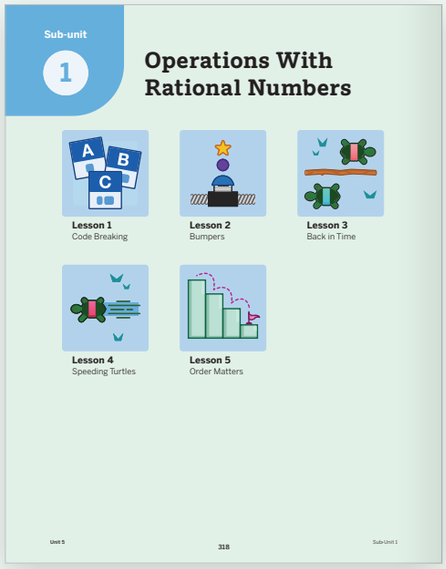
Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Perform all four operations with positive and negative rational numbers using a variety of strategies. • Use order of operations to evaluate numerical expressions. **(Lessons 1–5)**	1	**Support** • **Teacher Move:** Consider revisiting Grade 6, Unit 7, Lesson 3 (Sub-Zero) • **Teacher Move:** Invite students to share how they used the number line on the thermometer to help them determine temperatures in Problem 1.
	2, 3	**Support** • **Mini-Lesson:** *Ordering and Comparing Positive and Negative Numbers* • **Teacher Move:** Consider revisiting Grade 6, Unit 7, Lesson 2 (Digging Deeper) and Grade 6, Unit 7, Lesson 4 (Order in the Class). • **Teacher Move:** Consider reviewing Problem 6 and then inviting students to generate numbers that would fall between two of the plotted numbers.
	5	**Support** • **Teacher Move:** Invite students to share how they know that a does not have the same value in $2 + a = 5$ and $2 - a = 5$. • **Teacher Move:** Consider revisiting Grade 6, Unit 6, Lesson 2 (Five Equations) and Grade 6, Unit 6, Lesson 6 (Swap and Solve)
	4, 6	**Support** • **Teacher Move:** During Lessons 2 consider continually inviting students to share strategies for determining the value of a variable in an equation.

Operations With Rational Numbers

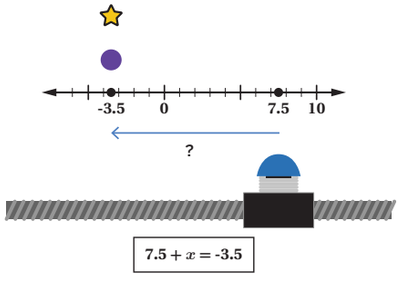
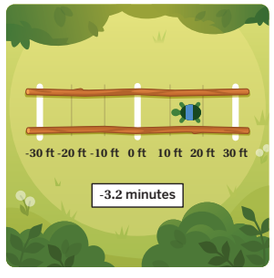
Sub-Unit 1 Goals

- **Goal:** Perform all four operations with positive and negative rational numbers using a variety of strategies.
- **Goal:** Use order of operations to evaluate numerical expressions.
- **Language Goal:** Describe strategies for determining the sum, difference, product, or quotient of two rational numbers.

In this sub-unit, students develop models to add, subtract, multiply, and divide rational numbers. They extend their learning by using order of operations to evaluate numerical expressions that include grouping symbols, absolute value, and exponents.



Math That Matters Most

Concept or Skill	For example . . .
A number line can help to identify the start, change, and end to determine an unknown value in equations.	The bumper is at 7.5 units and the ball is at -3.5 units. Write the number that would move the bumper and hit the ball. **-11 units** 
We can determine the position of an object by multiplying the rate it is traveling by the time.	One student wrote the expression $(-5)(-3.2) = 16$ to determine Tam's position. Explain what each number represents in the situation. **-5 represents that Tam is walking left 5 feet every minute.** **-3.2 represents 3.2 minutes back in time.** **16 represents where Tam was at that time.** 
Use order of operations to evaluate numerical expressions.	What is the value of the expression $18 - 3^2 + 10 \times 2$? **First, evaluate the term with the exponent.** **18 – 9 + 10 × 2** **Next, multiply 10 times 2.** **18 – 9 + 20** **Finally, subtract and add from left to right.** **9 + 20** **The value of the expression is 29.**

Unit 5

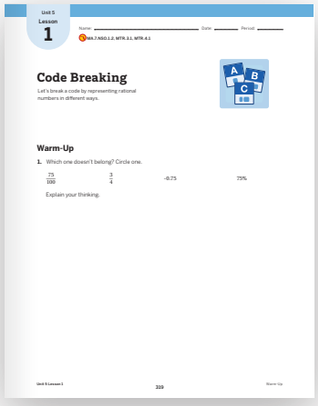
Lesson

1

Code Breaking

Equivalent Forms

Let's break a code by representing rational numbers in different ways.



Learning Outcomes

Today's Goals

1. **Goal:** Represent *rational numbers* as fractions, mixed numbers, decimals, and percents.
2. **Language Goal:** Describe strategies for representing *rational numbers* in different ways. **(Speaking and Listening)**

Students build on their knowledge of **rational numbers**, fractions, decimals, and percents to decode a message by rewriting rational numbers in equivalent forms. They describe the strategies they use to write numbers in equivalent forms.

Coherence

Prior Learning

In Unit 4, students converted fractions to repeating and terminating decimals using long division and converted percents to decimals to solve a variety of problems.

Future Learning

In Grade 8, students will use decimals as they explore irrational numbers.

Connections

- Students demonstrate the understanding of rational numbers by analyzing the frequency of letters in a ciphered message that can be represented as the ratio of a letter's occurrence to the total number of letters. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.2
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.1.1	**MA.8.NSO.1.1**
MA.6.NSO.3.5	**MA.8.NSO.1.2**

Vocabulary

New Vocabulary

rational number

Review Vocabulary

bar notation
repeating decimal
terminating decimal

Lesson at a Glance

~ 45 min

MA.7.NSO.1.2

Warm-Up

Whole Class | 5 min

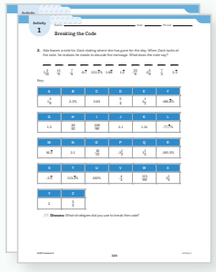
Students make sense of the different ways a number can be written to be ready to write **rational numbers** in different forms.
Routine: Which One Doesn't Belong?



Activity 1

Pairs | 15 min

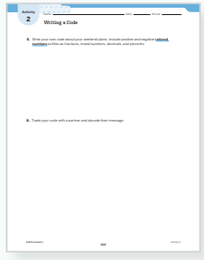
Students identify rational numbers in different forms to decode a message.
Routine: Think-Pair-Share



Activity 2

Pairs | 15 min

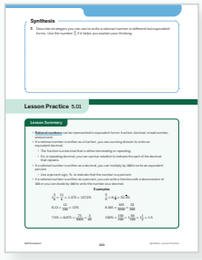
Students write rational numbers in different forms to create their own code.



Synthesis

Whole Class | 5 min

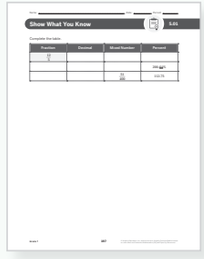
Students synthesize strategies for writing rational numbers in equivalent forms.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by [insert what students are doing here].
(MP#, MP#)



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students make sense of the different ways a number can be written to be ready to write **rational numbers** in different forms.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing different forms of rational numbers.

Consider sharing that each number does not belong in some way. Encourage students to look for more than one possibility.

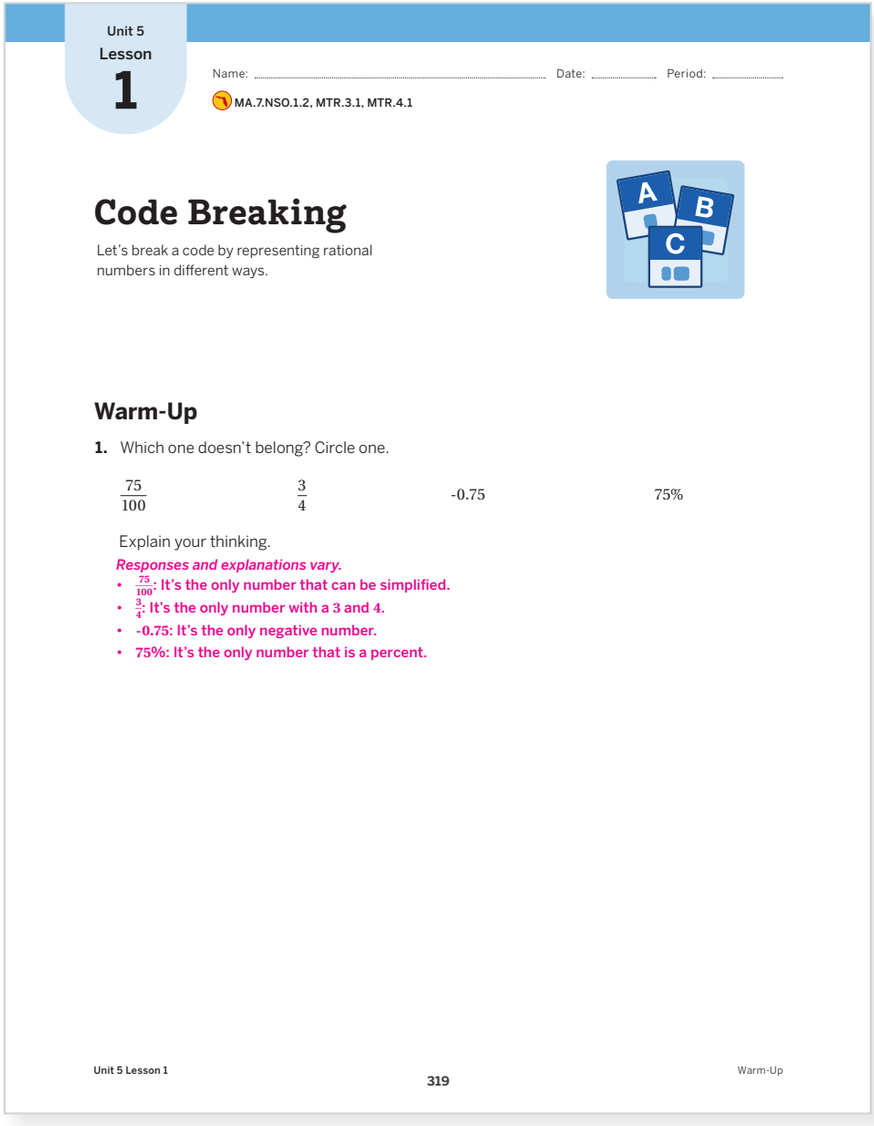
ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share their ideas with others. **(Writing and Speaking)**

2 Connect

Invite students to share at least one reason why each rational number doesn't belong.

Consider asking:

- "What is another way to say why the number ____ doesn't belong?"
- "Is there another reason why the number ____ doesn't belong?"



Unit 5
Lesson
1

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.2, MTR.3.1, MTR.4.1

Code Breaking

Let's break a code by representing rational numbers in different ways.

Warm-Up

1. Which one doesn't belong? Circle one.

$\frac{75}{100}$ $\frac{3}{4}$ -0.75 75%

Explain your thinking.

Responses and explanations vary.

- $\frac{75}{100}$: It's the only number that can be simplified.
- $\frac{3}{4}$: It's the only number with a 3 and 4.
- -0.75: It's the only negative number.
- 75%: It's the only number that is a percent.

Unit 5 Lesson 1 319 Warm-Up

Pairs | 15 min

Activity 1 Breaking the Code

Purpose: Students identify rational numbers in different forms to decode a message.

Short on time? Consider having pairs of students work on different sections of the message and then as a class combine the information to decode the note.

1 Launch

Direct students' attention to the note that Ada leaves for Zoe. Consider asking:

- "What do you need to do to decode the message?"
- "How does the key help you decode the message?"
- "What are the different forms the numbers are written in?"
- "What do you do if you don't see the given number in the key?"

2 Monitor

Look for strategies students use to write the numbers in different formats to highlight during the Connect. **(MTR.3.1)**

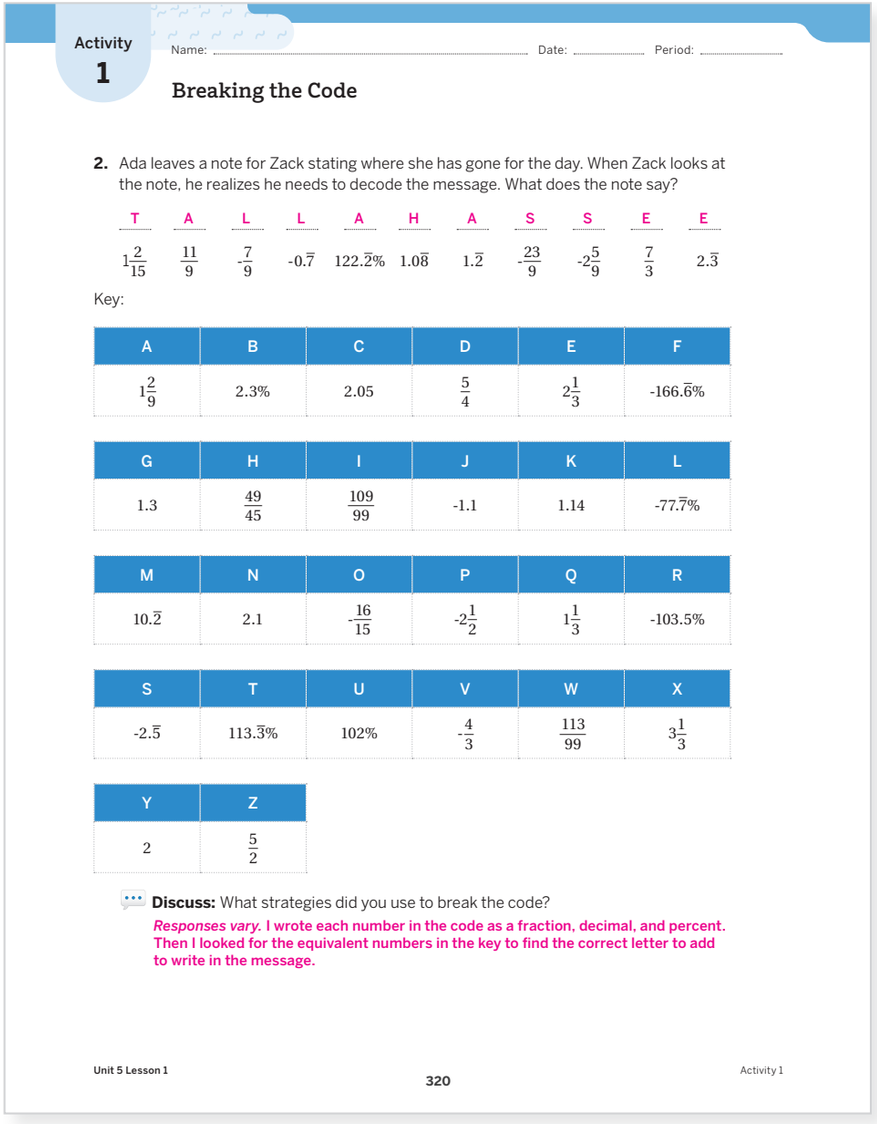
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try writing the given number in a different form.
Need support writing a fraction as a decimal.	**Support** Consider asking, "What strategies do you know for writing a fraction as a decimal?"
Need support writing a decimal as a percent.	**Support** Consider asking, "What happens when you try rewriting a simpler number as a percent? Can that help you decode that message?"
Write a rational number in a different form from what is given.	**Strengthen** Celebrate the strategy. Consider asking, "How do you know these two numbers are equivalent?"

A Accessibility: Executive Functioning Consider providing a graphic organizer to support students in representing rational numbers in multiple forms.

Use the Think-Pair-Share routine to support student-to-student discourse about the strategies they used to break the code.

Activity 1 continued >



Activity 1

Name: Date: Period:

Breaking the Code

2. Ada leaves a note for Zack stating where she has gone for the day. When Zack looks at the note, he realizes he needs to decode the message. What does the note say?

T	A	L	L	A	H	A	S	S	E	E
$1\frac{2}{15}$	$\frac{11}{9}$	$-\frac{7}{9}$	$-0.\overline{7}$	$122.\overline{2}\%$	$1.0\overline{8}$	$1.\overline{2}$	$-\frac{23}{9}$	$-2\frac{5}{9}$	$\frac{7}{3}$	$2.\overline{3}$

Key:

A	B	C	D	E	F
$1\frac{2}{9}$	2.3%	2.05	$\frac{5}{4}$	$2\frac{1}{3}$	$-166.\overline{6}\%$

G	H	I	J	K	L
1.3	$\frac{49}{45}$	$\frac{109}{99}$	-1.1	1.14	$-77.\overline{7}\%$

M	N	O	P	Q	R
$10.\overline{2}$	2.1	$-\frac{16}{15}$	$-2\frac{1}{2}$	$1\frac{1}{3}$	-103.5%

S	T	U	V	W	X
$-2.\overline{5}$	$113.\overline{3}\%$	102%	$-\frac{4}{3}$	$\frac{113}{99}$	$3\frac{1}{3}$

Y	Z
2	$\frac{5}{2}$

Discuss: What strategies did you use to break the code?

Responses vary. I wrote each number in the code as a fraction, decimal, and percent. Then I looked for the equivalent numbers in the key to find the correct letter to add to write in the message.

Unit 5 Lesson 1 320 Activity 1

Pairs | 15 min

Activity 1 Breaking the Code (continued)

Purpose: Students identify rational numbers in different forms to decode a message.

Short on time? Consider having pairs of students work on different sections of the message and then as a class combine the information to decode the note.

2 Monitor

Invite students to share their reasoning. Discuss each possible response.

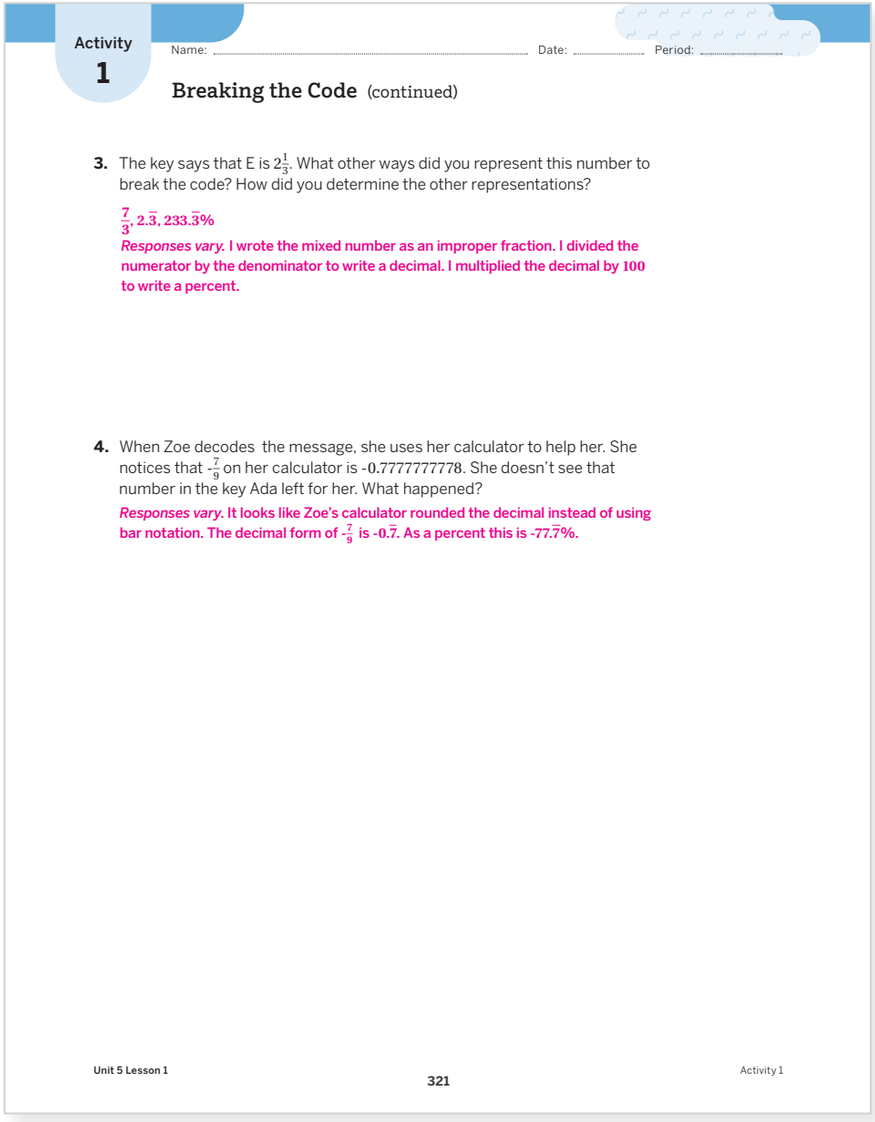
Encourage students to justify their strategies for writing the numbers in equivalent forms. **(MTR.4.1)**

Collect and share a variety of explanations about what to consider when using calculators in Problem 4.

3 Connect

To surface the Key Takeaway, consider asking:

- "What are different ways you can write rational numbers?"
- "What strategies can you use to write a rational number in a different form?"



Activity 1

Name: Date: Period:

Breaking the Code (continued)

3. The key says that E is $2\frac{1}{3}$. What other ways did you represent this number to break the code? How did you determine the other representations?

$\frac{7}{3}$, $2.\overline{3}$, $233.\overline{3}\%$

Responses vary. I wrote the mixed number as an improper fraction. I divided the numerator by the denominator to write a decimal. I multiplied the decimal by 100 to write a percent.

4. When Zoe decodes the message, she uses her calculator to help her. She notices that $-\frac{7}{9}$ on her calculator is -0.7777777778. She doesn't see that number in the key Ada left for her. What happened?

Responses vary. It looks like Zoe's calculator rounded the decimal instead of using bar notation. The decimal form of $-\frac{7}{9}$ is $-0.\overline{7}$. As a percent this is $-77.\overline{7}\%$.

Unit 5 Lesson 1 321 Activity 1

Key Takeaway: **Rational numbers** can be written in different equivalent forms. There are many different strategies you can use to write a number in a different form.

Pairs | 15 min

Activity 2 Writing a Code

Purpose: Students write rational numbers in different forms to create their own code.

Short on time? Consider having pairs of students write one code and trade with another pair of students.

1 Launch

Direct students' attention to the key from Activity 1. Consider asking:

- "How can you use the key to create a code?"
- "What steps should you take to write your own code?"
- "What equivalent form seems the most challenging?"

Create a class definition for the term **rational number** using students' language and the glossary. Consider including non-examples to help emphasize what features a rational number must have.

2 Monitor

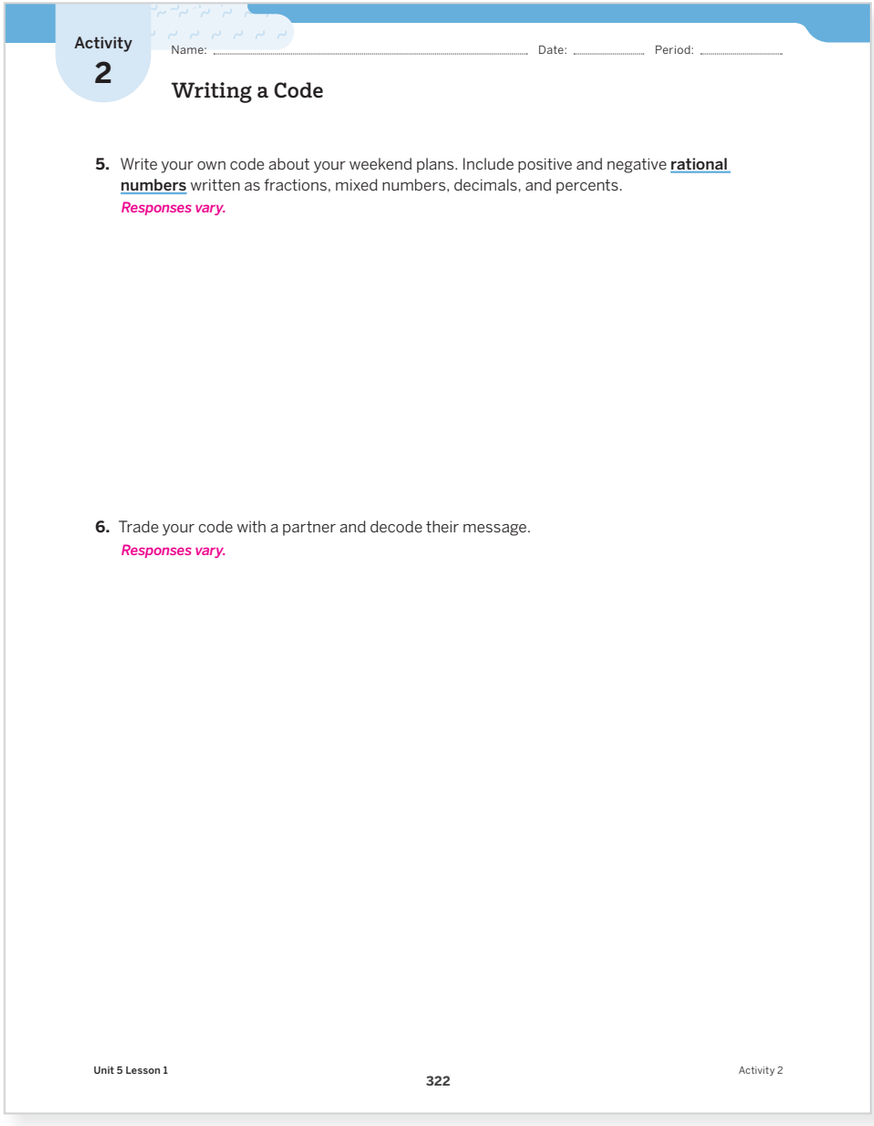
Look for strategies students use to write rational numbers in equivalent forms while creating their code to highlight during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to determine the message they want to encode.
Need support writing rational numbers in multiple equivalent forms.	**Support** Consider asking, "What different types of equivalent forms are there? How do you write rational numbers in those forms?"
Write rational numbers in an equivalent form.	**Strengthen** Celebrate students' achievements. Encourage them to write the number in another form to strengthen their understanding.
Finish early.	**Stretch** Invite these students to write a longer message and use more equivalent forms of each number.

3 Connect

Invite students to share the strategies they used to encode and decode messages. Consider recording the strategies students have for writing rational numbers in equivalent forms for students to reference later. **(MTR.4.1)**



Activity 2

Name: Date: Period:

Writing a Code

5. Write your own code about your weekend plans. Include positive and negative **rational numbers** written as fractions, mixed numbers, decimals, and percents.
Responses vary.

6. Trade your code with a partner and decode their message.
Responses vary.

Unit 5 Lesson 1 322 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for writing rational numbers in equivalent forms.

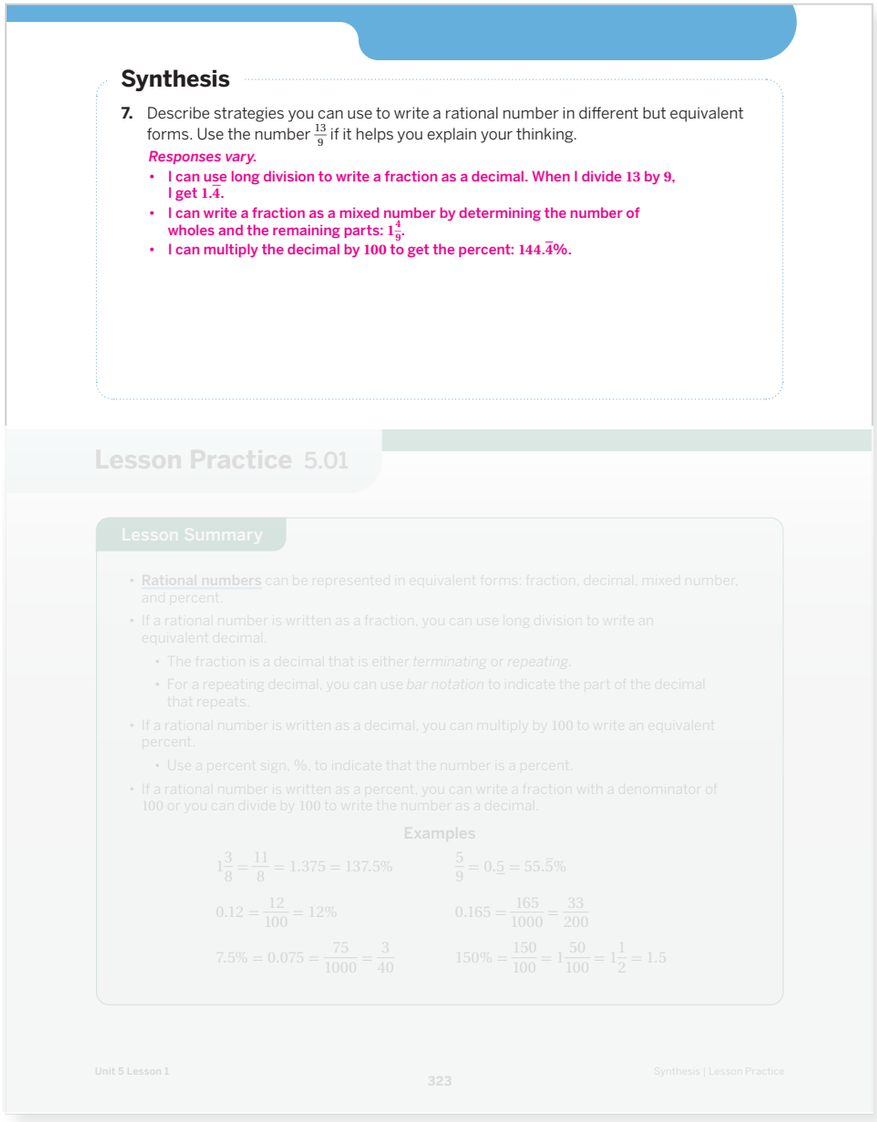
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- To write a fraction as a decimal, use long division.
- To write a fraction as a mixed number, determine the number of wholes and the remaining parts.
- To write a decimal as a percent, multiply by 100.
- To write a percent as a decimal, divide by 100.
- To write a percent as a fraction, use a denominator of 100.

Consider asking, "How does writing a fraction with a denominator of 100 or $1{,}000$ help you to more easily convert the fraction to a decimal or a percent?"

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for writing rational numbers in equivalent forms as they share during the Synthesis. Display for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

7. Describe strategies you can use to write a rational number in different but equivalent forms. Use the number $\frac{13}{9}$ if it helps you explain your thinking.

Responses vary.
- I can use long division to write a fraction as a decimal. When I divide 13 by 9, I get $1.\overline{4}$.
- I can write a fraction as a mixed number by determining the number of wholes and the remaining parts: $1\frac{4}{9}$.
- I can multiply the decimal by 100 to get the percent: $144.\overline{4}\%$.

Lesson Practice 5.01

Lesson Summary

- Rational numbers can be represented in equivalent forms: fraction, decimal, mixed number, and percent.
- If a rational number is written as a fraction, you can use long division to write an equivalent decimal.
  - The fraction is a decimal that is either *terminating* or *repeating*.
  - For a repeating decimal, you can use *bar notation* to indicate the part of the decimal that repeats.
- If a rational number is written as a decimal, you can multiply by 100 to write an equivalent percent.
  - Use a percent sign, %, to indicate that the number is a percent.
- If a rational number is written as a percent, you can write a fraction with a denominator of 100 or you can divide by 100 to write the number as a decimal.

Examples

$1\frac{3}{8} = \frac{11}{8} = 1.375 = 137.5\%$ $\quad \frac{5}{9} = 0.\underline{5} = 55.\overline{5}\%$

$0.12 = \frac{12}{100} = 12\%$ $\quad 0.165 = \frac{165}{1000} = \frac{33}{200}$

$7.5\% = 0.075 = \frac{75}{1000} = \frac{3}{40}$ $\quad 150\% = \frac{150}{100} = 1\frac{50}{100} = 1\frac{1}{2} = 1.5$

Unit 5 Lesson 1 323 Synthesis | Lesson Practice

Lesson Takeaway: Rational numbers can be written in many equivalent forms, including fractions, decimals, and percents.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by [insert what students are doing here]. **(MP#)**

Today's Goals

1. **Goal:** Represent *rational numbers* as fractions, mixed numbers, decimals, and percents.
2. **Language Goal:** Describe strategies for representing *rational numbers* in different ways. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.01

Complete the table.

Fraction	Decimal	Mixed Number	Percent
$\frac{12}{5}$	2.4	$2\frac{2}{5}$	240%
$-\frac{262}{99}$	$-2.\overline{64}$	$-2\frac{64}{99}$	$-200.\overline{64}\%$
$\frac{451}{400}$	1.1275	$1\frac{51}{400}$	112.75%

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to represent rational numbers in different ways:

» **Respond to Student Thinking:**

- Use the *Insert title here* Mini-lesson.
- Use the *Insert title here of Prerequisite* Mini-Lesson.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on students' progress with writing rational numbers in equivalent forms. In what ways have students' skills grown? What strategies do students need additional support with? How can you use students' strengths to help everyone become proficient with the different strategies?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Describe strategies you can use to write a rational number in different but equivalent forms. Use the number $\frac{13}{9}$ if it helps you explain your thinking.

Responses vary.

- I can use long division to write a fraction as a decimal. When I divide 13 by 9, I get $1.\overline{4}$.
- I can write a fraction as a mixed number by determining the number of wholes and the remaining parts: $1\frac{4}{9}$.
- I can multiply the decimal by 100 to get the percent: $144.\overline{4}\%$.

Lesson Practice 5.01

Lesson Summary

- **Rational numbers** can be represented in equivalent forms: fraction, decimal, mixed number, and percent.
- If a rational number is written as a fraction, you can use long division to write an equivalent decimal.
  - The fraction is a decimal that is either *terminating* or *repeating*.
  - For a repeating decimal, you can use *bar notation* to indicate the part of the decimal that repeats.
- If a rational number is written as a decimal, you can multiply by 100 to write an equivalent percent.
  - Use a percent sign, %, to indicate that the number is a percent.
- If a rational number is written as a percent, you can write a fraction with a denominator of 100 or you can divide by 100 to write the number as a decimal.

Examples

$1\frac{3}{8} = \frac{11}{8} = 1.375 = 137.5\%$

$\frac{5}{9} = 0.\overline{5} = 55.\overline{5}\%$

$0.12 = \frac{12}{100} = 12\%$

$0.165 = \frac{165}{1000} = \frac{33}{200}$

$7.5\% = 0.075 = \frac{75}{1000} = \frac{3}{40}$

$150\% = \frac{150}{100} = 1\frac{50}{100} = 1\frac{1}{2} = 1.5$

Lesson Practice 5.01

Name: _____ Date: _____ Period: _____

1. Match the equivalent rational numbers.

a. $2.\overline{3}$ — b

b. -102.5% — d

c. $1\frac{1}{9}$ — c

d. $\frac{11}{6}$ — a

b. -1.025

d. $183.\overline{3}\%$

c. $1.\overline{1}$

a. $\frac{7}{3}$

2. Complete the table.

Fraction	Decimal	Mixed Number	Percent
$\frac{27}{20}$	1.35	$1\frac{7}{20}$	135%
$\frac{106}{45}$	$2.3\overline{5}$	$2\frac{16}{45}$	$235.\overline{5}\%$
$-\frac{11}{9}$	$-1.\overline{2}$	$-1\frac{2}{9}$	$-122.\overline{2}\%$
$\frac{13}{5}$	2.6	$2\frac{3}{5}$	206%

3. What is the same about each decimal representation in the table and what is different?

Responses vary. Each decimal has a decimal point. Some of the decimals are repeating and some are terminating. The repeating decimals only have one digit that repeats. Some of the decimals are positive and some are negative.

Lesson Practice 5.01

Name: _____ Date: _____ Period: _____

4. How do you know if two rational numbers are equivalent?

Responses vary. To determine if two representations are equivalent, write the rational numbers in the same form. For example, if one number is written as a decimal and the other is written as a fraction, write both numbers as fractions or both numbers as decimals and then compare the numbers.

FAST Practice

5. Select *all* the rational numbers that are equivalent to $\frac{4}{3}$.

- ☑ A. $1.\overline{3}$
- ☑ B. $1\frac{1}{3}$
- ☐ C. 133%
- ☑ D. $\frac{8}{6}$
- ☐ E. $13.\overline{3}\%$

Spiral Review

Problems 6–9: Determine the value of the expression.

6. $\left(-\frac{1}{2}\right)(3)$ — $-\frac{3}{2}$

7. $7 - 12$ — -5

8. $-\frac{3}{5} + \left(-\frac{1}{3}\right)$ — $-\frac{14}{15}$

9. $-1.85 \div 5$ — -0.37

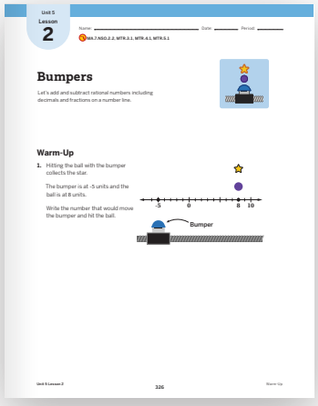
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.NSO.1.2
	2–3	2	MA.7.NSO.1.2
	4	3	MA.7.NSO.1.2
FAST Practice	5	2	MA.7.NSO.1.2
Spiral Review			
	6–9	1	MA.7.NSO.2.2

Unit 5

Lesson

2



Bumpers

Adding and Subtracting on the Number Line

Let's add and subtract rational numbers including decimals and fractions on a number line.

Learning Outcomes

Today's Goals

1. **Goal:** Add and subtract positive and negative numbers, including decimals and fractions, on a number line.
2. **Language Goal:** Describe a strategy for adding and subtracting positive and negative decimals and fractions. **(Reading, Writing, Speaking, and Listening)**

Students use number lines as a model for adding and subtracting rational numbers (e.g., positive and negative *integers*, decimals, and fractions). The lesson begins with the context of a bumper moving to hit a ball and progresses toward performing operations without a number line to develop fluency in adding and subtracting rational numbers. **(MTR.3.1)**

Coherence

Prior Learning

In Grade 6, students added and subtracted integers.

Future Learning

In Lesson 3, students will extend their knowledge of multiplication of integers to multiply positive and negative rational numbers.

Connections

- Students explore how to represent real world scenarios in the context of a bumper and ball on a number line. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2

Add, subtract, multiply, and divide rational numbers with procedural fluency.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.4.1	**MA.8.NSO.1.5** **MA.8.NSO.1.7**

Vocabulary

Review Vocabulary

integer

Lesson at a Glance

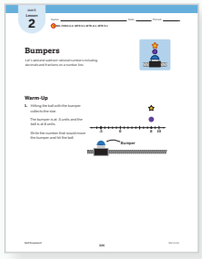
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MA.7.NSO.2.2

Warm-Up

Pairs | 5 min

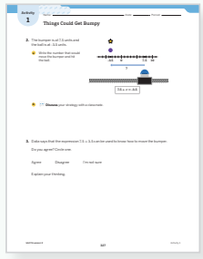
Students are introduced to the context of moving a bumper along a number line to model addition of positive and negative numbers.



Activity 1

Pairs | 10 min

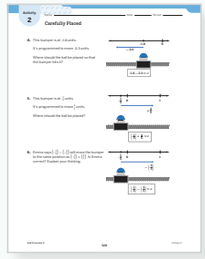
Students determine an unknown value in a bumper challenge involving positive and negative decimals.



Activity 2

Pairs | 10 min

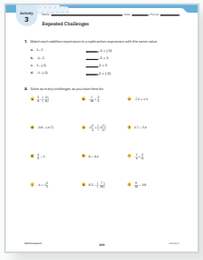
Students use number lines to determine an unknown value in an equation by adding or subtracting positive and negative decimals and fractions. **Routine:** Decide and Defend



Activity 3

Pairs | 10 min

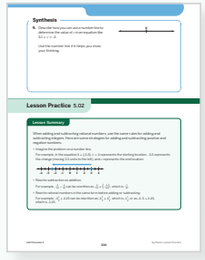
Students complete repeated challenges to practice adding and subtracting rational numbers. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

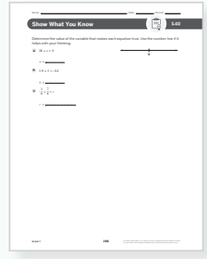
Students synthesize their understanding of how to model adding positive and negative decimals using a number line.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining an unknown amount in a one-step addition equation with positive and negative fractions and decimals.



Prep Checklist

Prepare Horizontal Number Lines Sheets

This lesson includes:

Student Edition
Show What You Know

- Horizontal Number Lines Sheet, one per student (optional)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of moving a bumper along a number line to model addition of positive and negative numbers.

1 Launch

Consider sharing that the bumper can move to the left and to the right along the number line.

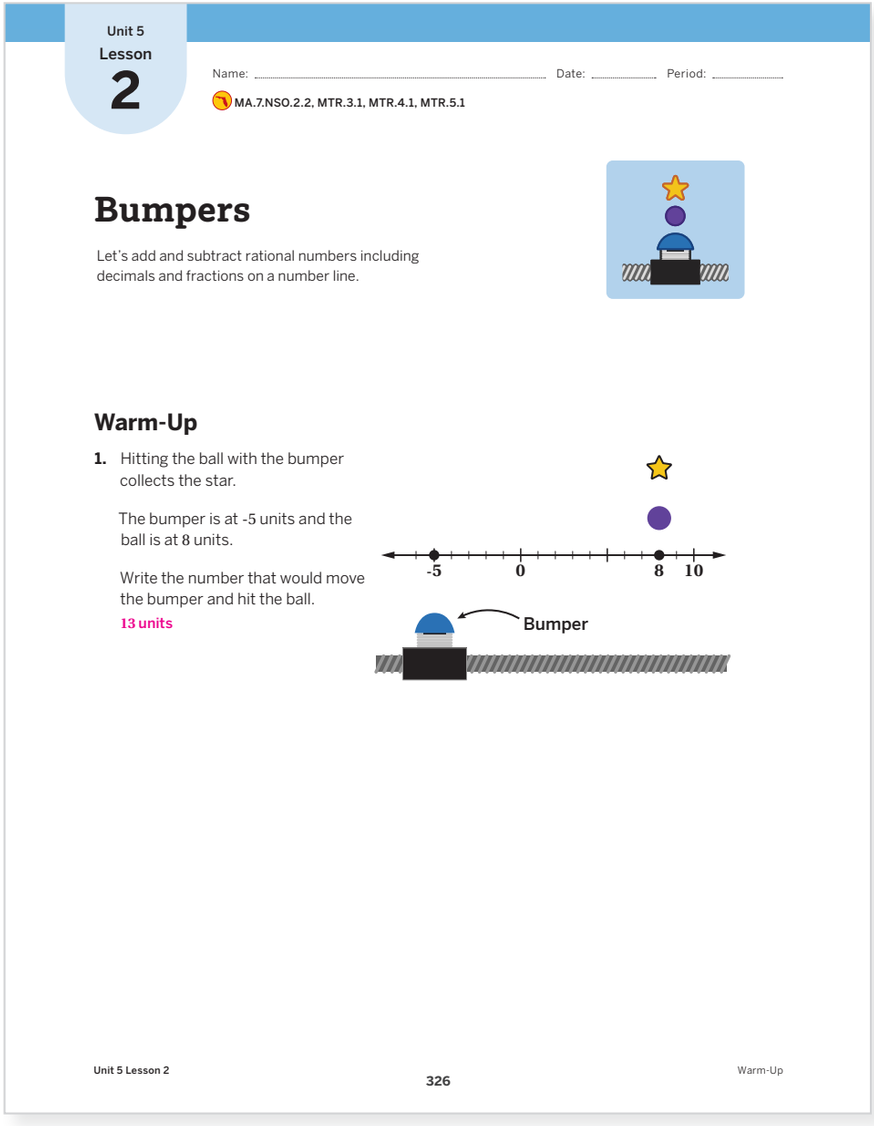
A **Accessibility: Visual-Spatial Processing**
Consider inviting students to use their fingers or pencils to count the spaces on the number line.

2 Connect

Invite students to share their responses.

Consider asking:

- "Why does it make sense that the change from -5 to 8 is 13?"
- "How would you explain this to someone else?"
- "What equation could you write to represent the number line?"



Unit 5
Lesson
2

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.2, MTR.3.1, MTR.4.1, MTR.5.1

Bumpers

Let's add and subtract rational numbers including decimals and fractions on a number line.

Warm-Up

1. Hitting the ball with the bumper collects the star.

The bumper is at -5 units and the ball is at 8 units.

Write the number that would move the bumper and hit the ball.

13 units

Unit 5 Lesson 2 326 Warm-Up

Pairs | 10 min

Activity 1 Things Could Get Bumpy

Purpose: Students determine an unknown value in a bumper challenge involving positive and negative decimals.

1 Launch

To surface connections, consider discussing how it is possible to move the bumper to the left by adding a number. **(MTR.5.1)**

Consider distributing the Horizontal Number Lines Sheet to each student as a space to write their thinking throughout the lesson.

2 Monitor

Look for a variety of ways students make sense of the challenge.

D Differentiation

Look for students who:	Respond to Student Thinking
Estimate the distance and use the equation to revise or check their work.	**Strengthen** Consider asking, "How is the answer you determined related to the values -3.5 and 7.5?"
Determine the distances on each side of 0 and add them.	**Strengthen** Celebrate this strategy. Consider asking, "Which direction does the bumper need to move? How will you communicate that?"

A Accessibility: Visual-Spatial Processing Invite pairs to think about key features of the visual, such as the start, change, and end points. Encourage pairs to discuss them aloud and make connections with the equation representation.

Look for a variety of reasoning students use to evaluate Dalia's claim.

ELL English Language Learners Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

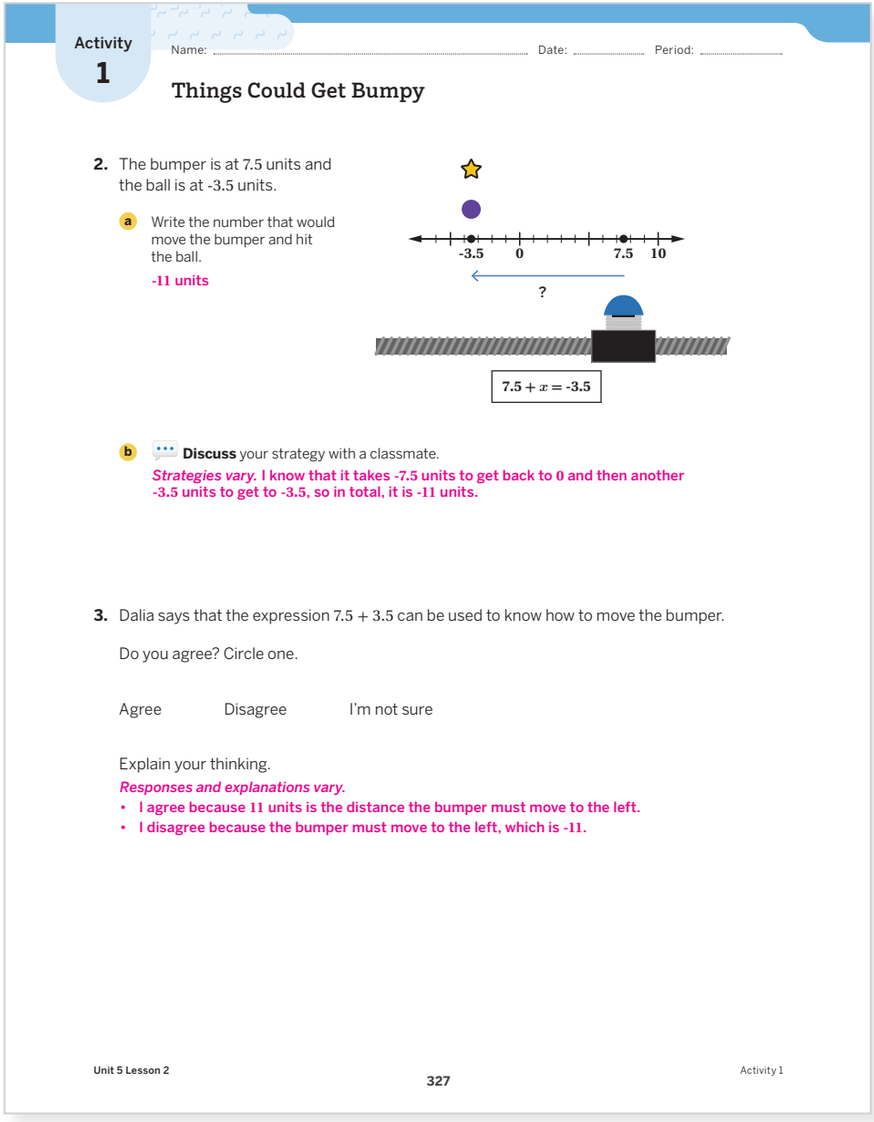
3 Connect

Record the distribution of responses by polling the class, calling attention to any conflict or consensus.

Invite students who made different choices to share their reasoning.

To surface the Key Takeaway, consider asking:

- "How does what you see on the number line connect with what you see in Dalia's work?"
- "What is correct about Dalia's work?"
- "What could you add or change to make Dalia's claim stronger and clearer?"



Activity 1

Name: Date: Period:

Things Could Get Bumpy

2. The bumper is at 7.5 units and the ball is at -3.5 units.

a. Write the number that would move the bumper and hit the ball.

-11 units

b. **Discuss** your strategy with a classmate.

Strategies vary. I know that it takes -7.5 units to get back to 0 and then another -3.5 units to get to -3.5, so in total, it is -11 units.

3. Dalia says that the expression $7.5 + 3.5$ can be used to know how to move the bumper.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Responses and explanations vary.

- I agree because 11 units is the distance the bumper must move to the left.
- I disagree because the bumper must move to the left, which is -11.

Unit 5 Lesson 2 327 Activity 1

Key Takeaway: When solving a bumper challenge, it is important to consider both the distance and the direction the bumper needs to travel. If the bumper is crossing zero, you can determine the distance on either side of zero and add them together. Adding a negative number moves the bumper to the left, and adding a positive number moves the bumper to the right.

Pairs | 10 min

Activity 2 Carefully Placed

Purpose: Students use number lines to determine an unknown value in an equation by adding or subtracting positive and negative decimals and fractions.

1 Launch

Direct students' attention to the bumper and equation in Problem 4. Consider asking, "How is this like the previous challenge? How is it different?" They both have a bumper on a number line, but this time we are deciding where to put the ball instead of how to move the bumper. The previous challenge involved addition, but this challenge involves subtraction.

2 Monitor

To support students getting started, consider inviting them to model $-1 - 2 = x$ on the Horizontal Number Lines Sheet. Then ask, "How is this challenge similar to and different from that problem?"

Look for a variety of strategies students use to decide where the ball should be placed.

D Differentiation

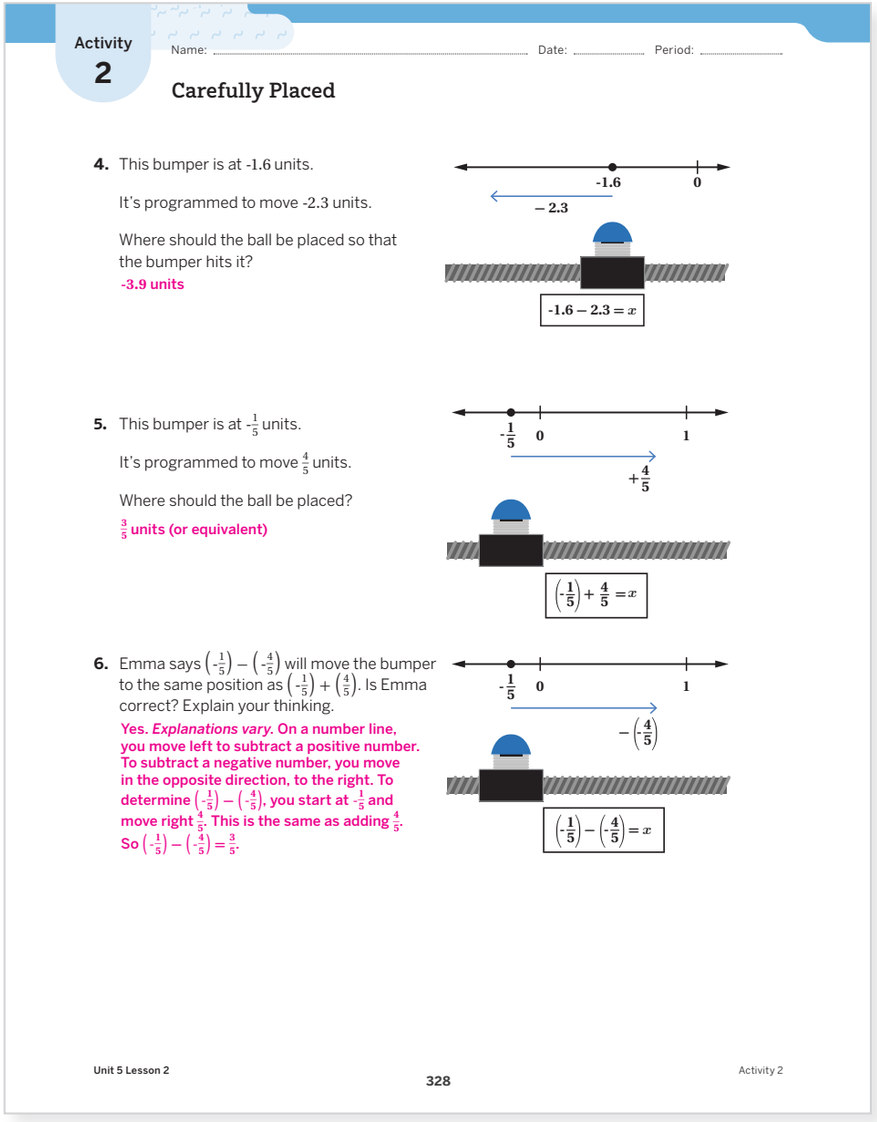
Look for students who:	Respond to Student Thinking
Think the ball should be placed at $\frac{5}{5}$ because $\frac{1}{5}+\frac{4}{5}=\frac{5}{5}$.	**Support** Consider asking, "How many fifths do you need to move the ball to 0? How many more fifths past 0?"
Add the denominators of the fractions, e.g., write $\frac{3}{10}$ or $\frac{5}{10}$.	**Support** Consider asking, "How big is $\frac{1}{10}$ compared to $\frac{1}{5}$? Where would your answer be on the number line?"
Use a strategy such as: • Counting 4 fifths forward from $-\frac{1}{5}$. • Subtracting $\frac{1}{5}$ from $\frac{4}{5}$.	**Strengthen** Invite students to compare their strategy with a classmate.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"

To surface the Key Takeaway, consider asking:
- "Why is subtracting a negative value like adding the opposite of the value?"
- "What direction would we move on the number line if we subtracted -2.3 from a number?"



Activity 2

Name: Date: Period:

Carefully Placed

4. This bumper is at -1.6 units.

It's programmed to move -2.3 units.

Where should the ball be placed so that the bumper hits it?

-3.9 units

5. This bumper is at $-\frac{1}{5}$ units.

It's programmed to move $\frac{4}{5}$ units.

Where should the ball be placed?

$\frac{3}{5}$ units (or equivalent)

6. Emma says $\left(-\frac{1}{5}\right)-\left(-\frac{4}{5}\right)$ will move the bumper to the same position as $\left(-\frac{1}{5}\right)+\left(\frac{4}{5}\right)$. Is Emma correct? Explain your thinking.

Yes. *Explanations vary.* On a number line, you move left to subtract a positive number. To subtract a negative number, you move in the opposite direction, to the right. To determine $\left(-\frac{1}{5}\right)-\left(-\frac{4}{5}\right)$, you start at $-\frac{1}{5}$ and move right $\frac{4}{5}$. This is the same as adding $\frac{4}{5}$. So $\left(-\frac{1}{5}\right)-\left(-\frac{4}{5}\right)=\frac{3}{5}$.

Unit 5 Lesson 2 328 Activity 2

Key Takeaway: Subtraction is the opposite of addition. To subtract a negative number, move to the right on a number line because subtracting a negative value has the same change as adding its positive value.

Pairs | 10 min

Activity 3 Bumper Challenge

Purpose: Students complete repeated challenges to practice adding and subtracting rational numbers. **(MTR.3.1)**

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Consider asking, "Will you always be able to represent a subtraction problem as addition?"

Share that it's more important to make sense of the challenges than it is to work quickly.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

Listen for strategies students use in discussing how to solve the problems, such as:

- Drawing each challenge on a number line.
- Determining the start, change, and end.
- Using 0 as a benchmark, e.g., determining the movement necessary to get to 0 and adding the movement past 0.
- Generalizing about when to use subtraction or addition to calculate the difference. **(MTR.3.1)**

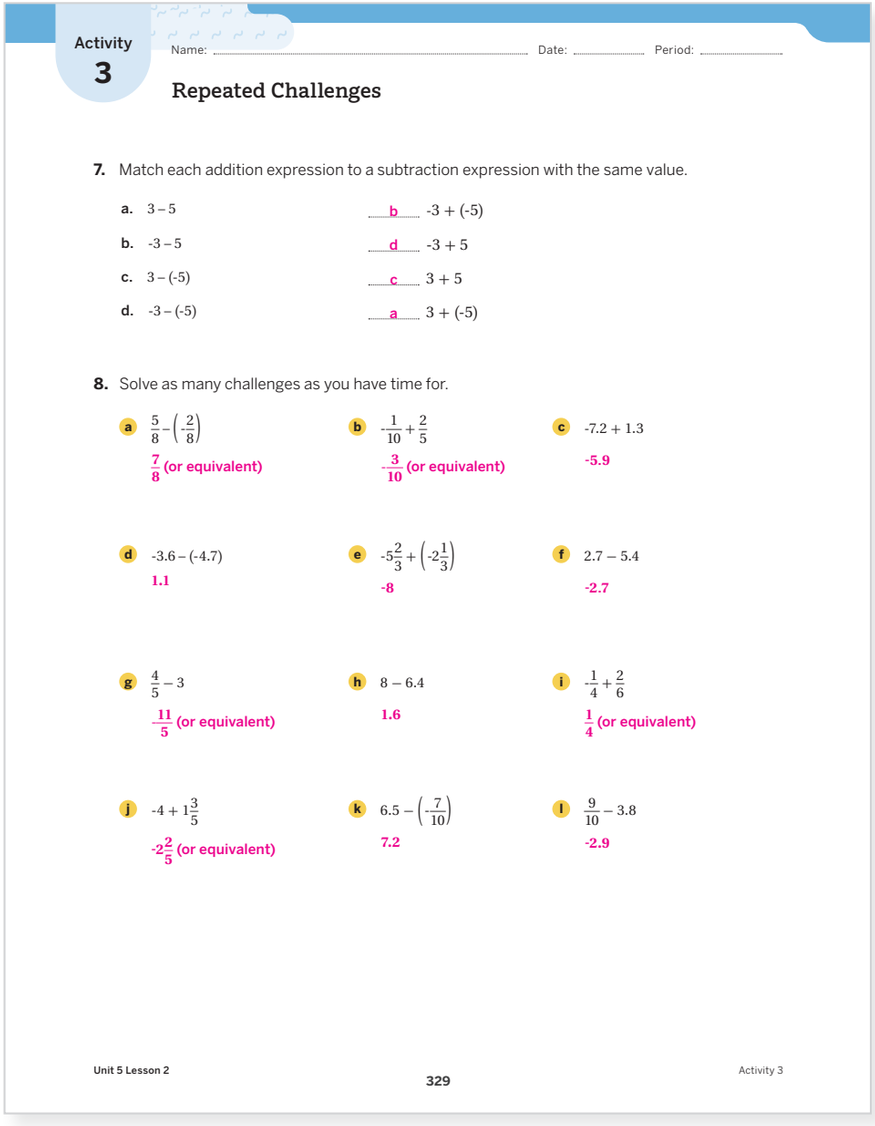
D **Differentiation: (Support)** Students may need support performing calculations involving fractions. Encourage them to choose a unit fraction to scale their number line before labeling the values from the problem.

3 Connect

Discuss the strategies students used to approach the repeated challenges.

Consider asking:

- "What strategies were useful for more than one challenge?"
- "What patterns did you notice?" **(MTR.5.1)**



Activity 3

Name: _____ Date: _____ Period: _____

Repeated Challenges

7. Match each addition expression to a subtraction expression with the same value.

a. $3-5$	b. $-3+(-5)$
b. $-3-5$	d. $-3+5$
c. $3-(-5)$	c. $3+5$
d. $-3-(-5)$	a. $3+(-5)$

8. Solve as many challenges as you have time for.

a. $\frac{5}{8}-\left(\frac{2}{8}\right)$ — $\frac{7}{8}$ (or equivalent)

b. $-\frac{1}{10}+\frac{2}{5}$ — $\frac{3}{10}$ (or equivalent)

c. $-7.2+1.3$ — -5.9

d. $-3.6-(-4.7)$ — 1.1

e. $-5\frac{2}{3}+\left(-2\frac{1}{3}\right)$ — -8

f. $2.7-5.4$ — -2.7

g. $\frac{4}{5}-3$ — $-\frac{11}{5}$ (or equivalent)

h. $8-6.4$ — 1.6

i. $-\frac{1}{4}+\frac{2}{6}$ — $\frac{1}{4}$ (or equivalent)

j. $-4+1\frac{3}{5}$ — $-2\frac{2}{5}$ (or equivalent)

k. $6.5-\left(-\frac{7}{10}\right)$ — 7.2

l. $\frac{9}{10}-3.8$ — -2.9

Unit 5 Lesson 2 329 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to model adding positive and negative decimals using a number line.

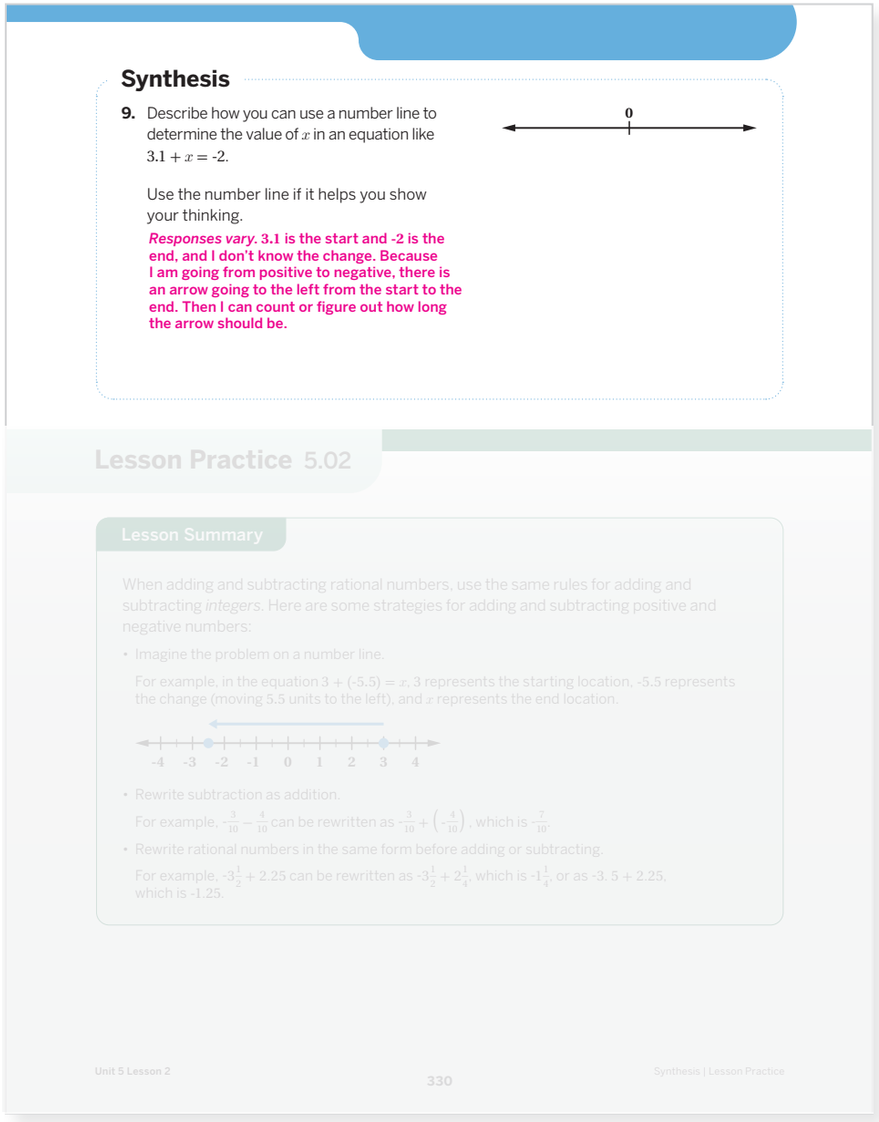
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using 3.1 as a starting point and -2 as the end, then determining the distance between them.
- Noticing that the value of x must be negative because the number line shows moving from a positive to a negative number.
- Determining the distance from 3.1 to 0 and then adding the distance from 0 to -2.

Consider asking, "How could the number line visually show each part of the equation? Why does that make the number line an effective tool for these problems?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = -2$.

Use the number line if it helps you show your thinking.

Responses vary. **3.1 is the start and -2 is the end, and I don't know the change. Because I am going from positive to negative, there is an arrow going to the left from the start to the end. Then I can count or figure out how long the arrow should be.**

Lesson Practice 5.02

Lesson Summary

When adding and subtracting rational numbers, use the same rules for adding and subtracting *integers*. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem on a number line.

  For example, in the equation $3 + (-5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving 5.5 units to the left), and x represents the end location.

- Rewrite subtraction as addition.

  For example, $-\frac{3}{10} - \frac{4}{10}$ can be rewritten as $-\frac{3}{10} + \left(-\frac{4}{10}\right)$, which is $-\frac{7}{10}$.

- Rewrite rational numbers in the same form before adding or subtracting.

  For example, $-3\frac{1}{2} + 2.25$ can be rewritten as $-3\frac{1}{2} + 2\frac{1}{4}$, which is $-1\frac{1}{4}$, or as $-3.5 + 2.25$, which is -1.25.

Unit 5 Lesson 2 330 Synthesis | Lesson Practice

Lesson Takeaway: Using a number line to identify the start, change, and end can help you determine an unknown value in equations involving adding and subtracting positive and negative numbers, including equations with fractions and decimals.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining an unknown amount in a one-step addition equation with positive and negative fractions and decimals.

Today's Goals

1. **Goal:** Add and subtract positive and negative numbers, including decimals and fractions, on a number line.
2. **Language Goal:** Describe a strategy for adding and subtracting positive and negative decimals and fractions. **(Reading, Writing, Speaking, and Listening)**

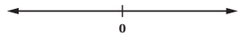
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.02

Determine the value of the variable that makes each equation true. Use the number line if it helps with your thinking.

a. $10 + a = 3$



$a =$ -7

b. $1.9 + b = -3.2$

$b =$ -5.1

c. $-\frac{1}{6} + \frac{7}{6} = c$

$c =$ $\frac{6}{6}$ (or equivalent)

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support adding and subtracting rational numbers using a number line:

» **Respond to Student Thinking:**

- Revisit Activity 2 and invite students to share strategies for adding and subtracting fractions and decimals.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to create their own number line challenges, trade them with a partner, and discuss strategies for solving.

Professional Learning

In this lesson, students are introduced to a model that supports their understanding of adding and subtracting rational numbers. In Lesson 3, they will continue to use number line models as they solve multiplication problems. How does this model integrate with other representations of adding and subtracting positive and negative numbers, such as algebra tiles?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = -2$.

Use the number line if it helps you show your thinking.

Responses vary. 3.1 is the start and -2 is the end, and I don't know the change. Because I am going from positive to negative, there is an arrow going to the left from the start to the end. Then I can count or figure out how long the arrow should be.

Lesson Practice 5.02

Lesson Summary

When adding and subtracting rational numbers, use the same rules for adding and subtracting *integers*. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem on a number line.

  For example, in the equation $3 + (-5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving 5.5 units to the left), and x represents the end location.



- Rewrite subtraction as addition.

  For example, $-\frac{3}{10} - \frac{4}{10}$ can be rewritten as $-\frac{3}{10} + \left(-\frac{4}{10}\right)$, which is $-\frac{7}{10}$.

- Rewrite rational numbers in the same form before adding or subtracting.

  For example, $-3\frac{1}{2} + 2.25$ can be rewritten as $-3\frac{1}{2} + 2\frac{1}{4}$, which is $-1\frac{1}{4}$, or as -3. 5 + 2.25, which is -1.25.

Lesson Practice 5.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $40 + a = 30$
$a = -10$

2. $-3.5 + c = 4.5$
$c = 8$

3. $d + 2.4 = 0.9$
$d = -1.5$

Problems 4–9: Evaluate each expression.

4. $\frac{1}{10} + \frac{2}{5}$
$\frac{3}{10}$ (or equivalent)

5. $-7.2 + 1.3$
-5.9

6. $-5\frac{7}{12} + \left(-2\frac{5}{12}\right)$
-8

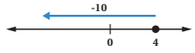
7. $3.7 - 5.4$
-1.7

8. $8 - 6.09$
1.91

9. $\frac{1}{4} + \frac{2}{6}$
$\frac{1}{4}$ (or equivalent)

FAST Practice

10. Select *all* the equations that represent the number line.



- ☑ A. $4 + (-10) = x$
- ☐ B. $4 - (-10) = x$
- ☐ C. $4 + 10 = x$
- ☑ D. $4 - 10 = x$
- ☐ E. $-4 + (-10) = x$

Lesson Practice 5.02

Name: _____ Date: _____ Period: _____

Spiral Review

11. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select *all* that apply.

- ☐ A. $g - 0.05$
- ☑ B. $g - 0.05g$
- ☑ C. $0.95g$
- ☐ D. $0.05g$
- ☑ E. $(1 - 0.05)g$

Problems 12–13: Determine whether each equation represents a proportional relationship. Explain your thinking.

12. Volume measured in cups, c, and the same volume measured in ounces, z. $c = \frac{1}{8}z$
Yes. *Explanations vary.* The equation is in the form $y = kx$ and the constant of proportionality is $\frac{1}{8}$.

13. Length, l, and width, w, for a rectangle whose area is 60 square units. $l = \frac{60}{w}$
No. *Explanations vary.* The ratio of w to l will not remain constant. When w is 1, l is 60, but when w is 2, l is 30.

14. Tiam bought a train ticket online. The original price of the train ticket was $83.00. He used a coupon to receive a 20% discount. A sales tax of 9% was applied after the discount. How much did Tiam end up paying?

A. $18.09 B. $56.44 (C.) $72.38 D. $75.00

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–9	1	MA.7.NSO.2.2
FAST Practice	10	2	MA.7.NSO.2.2
Spiral Review			
	11	2	MA.6.AR.1.1
	12–13	2	MA.7.AR.4.1
	14	2	MA.7.AR.3.1

Unit 5

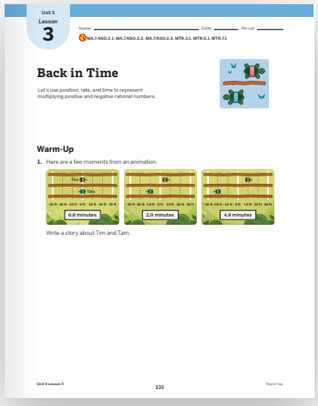
Lesson

3

Back in Time

Position, Rate, and Time

Let's use position, rate, and time to represent multiplying positive and negative rational numbers.


Unit 5
Lesson 3
Back in Time
Let's use position, rate, and time to represent multiplying positive and negative rational numbers.
Warm-Up
1. Here are a few moments from an animation.
Write a story about Tim and Tam.
333

Learning Outcomes

Today's Goals

1. **Goal:** Represent rate, time, and position using the product of positive and negative rational numbers.
2. **Language Goal:** Explain why the product of two negative numbers is positive in the context of rate, time, and position. **(Writing)**

Students make sense of a different model for multiplying positive and negative rational numbers (rate • time = position).

Coherence

Prior Learning

In Grade 6, students multiplied positive rational numbers and multiplied with integers. In Lesson 2, students made sense of adding and subtracting positive and negative rational numbers.

Future Learning

In Lesson 4, students will extend the model in this lesson to make sense of division with positive and negative rational numbers.

Connections

- Students explore annimation to represent multiplying postive and negative rational numbers. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2

Add, subtract, **multiply** and divide **rational numbers with procedural fluency.**

MA.7.NSO.2.3

Solve real-world problems involving any of the four operations with rational numbers.

Also Addressing: **MA.7.NSO.2.1**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12. EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.2.1	MA.8.NSO.1.5
MA.6.NSO.2.2	MA.8.NSO.1.6
MA.6.NSO.2.3	MA.8.NSO.1.7
MA.6.NSO.4.2	

Lesson at a Glance

~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.NSO.2.3

Warm-Up

Pairs | 5 min

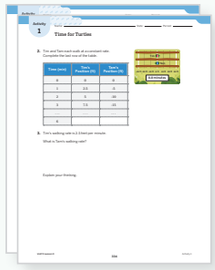
Students are introduced to the context of turtles walking at different rates in opposite directions.
Routine: Tell a Story



Activity 1

Pairs | 15 min

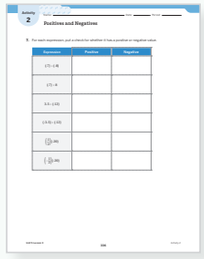
Students use rate, time, and position to make sense of multiplying rational numbers.
MLR3: Critique, Correct, Clarify **(MTR.5.1)**



Activity 2

Pairs | 5 min

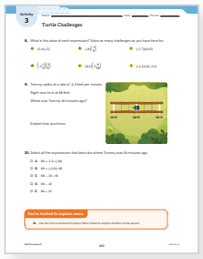
Students practice determining when multiplication expressions are positive or negative.



Activity 3

Pairs | 10 min

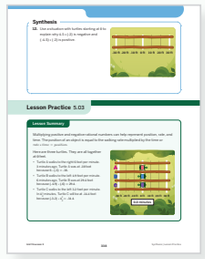
Students extend what they have learned to consider expressions that involve both addition and multiplication and gain fluency with multiplying positive and negative rational numbers. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

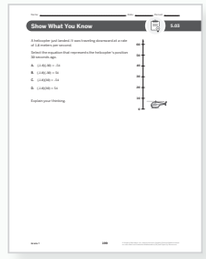
Students synthesize their understanding of why the product of two numbers is positive or negative using the turtle context.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which equation represents a situation in context using distance, time, and rate.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More Sheet, one per student (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of turtles walking at different rates in opposite directions.

1 Launch

Use the Tell a Story routine to help students make sense of the images of the two turtles. Invite students to write a story about the images using their unique experiences, perspectives, and creativity. **(MTR.7.1)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

Select several student graphs and stories. Consider capturing stories that include:

- A mention that both turtles start from the same location.
- Tim and Tam traveling in opposite directions.
- Tam traveling a further distance or faster than Tim in the same amount of time.
- A creative way to explain Tim's and Tam's journeys.

A **Accessibility: Conceptual Processing** As students connect visual representations to the context of different rates in different directions, consider asking:

- "What do you notice about the rate at which Tim and Tam are walking?"
- "What do you notice about the direction that Tim and Tam are traveling?"

ELL **English Language Learners** Give students time to rehearse and formulate their stories before sharing them with the class. **(Speaking and Listening)**

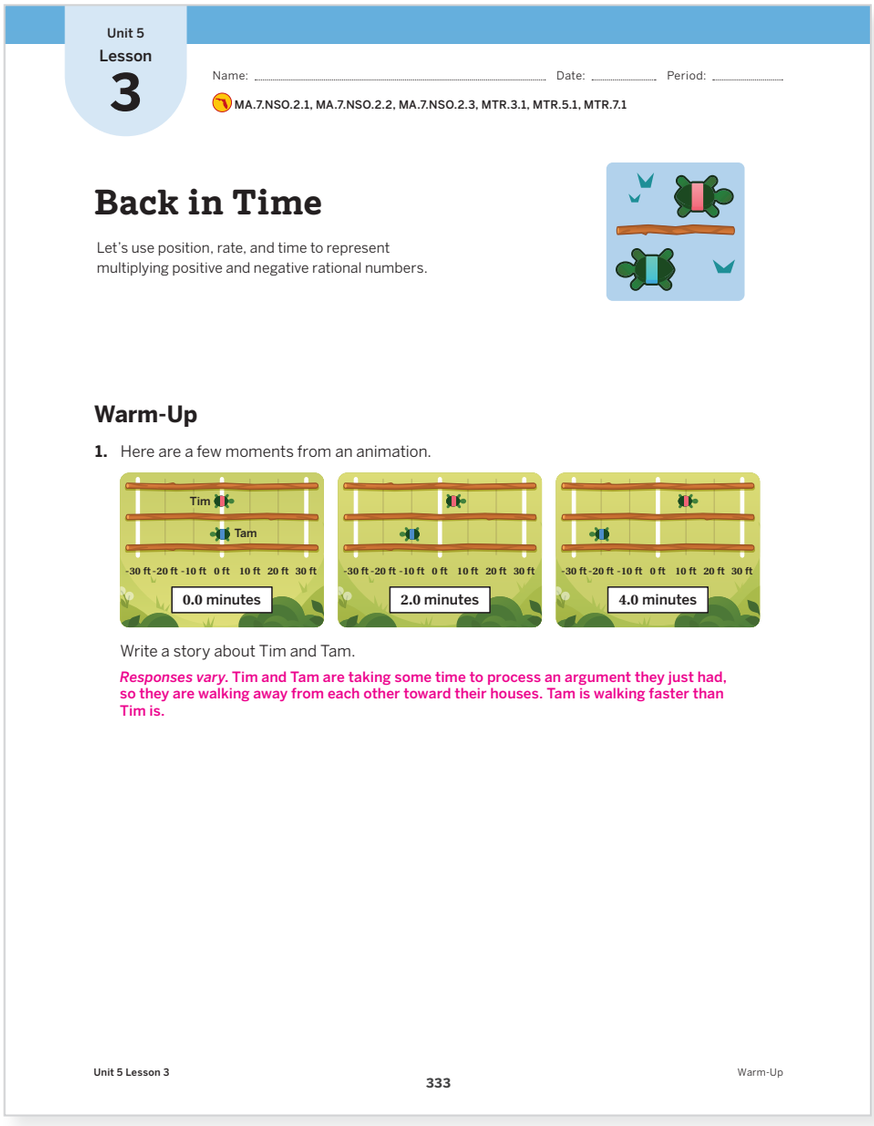
2 Connect

Invite students to share their stories.

Invite students to share about their stories and what inspired them in their story writing.

Encourage students to ask questions about each other's stories. Consider asking:

- "How do you see the parts of ___'s story in the images?"
- "What parts of each of our stories are alike? How are they different?"



Unit 5
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1

Back in Time

Let's use position, rate, and time to represent multiplying positive and negative rational numbers.

Warm-Up

1. Here are a few moments from an animation.

Write a story about Tim and Tam.

Responses vary. Tim and Tam are taking some time to process an argument they just had, so they are walking away from each other toward their houses. Tam is walking faster than Tim is.

Unit 5 Lesson 3 333 Warm-Up

Pairs | 15 min

Activity 1 Time for Turtles

Purpose: Students use rate, time, and position to make sense of multiplying rational numbers. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "Is the information in the table consistent with the stories you told in the Warm-Up?"

Invite students to predict both positions after 6 seconds.

A **Accessibility: Conceptual Processing** Invite students to look at the relationship between the turtle's starting position, the values in the table, and the turtle's ending position to support identifying patterns. **(MTR.5.1)**

Note: The position of each turtle is measured from the shell stripe.

2 Monitor

Encourage students to make connections between the walking rate and their work with proportional relationships. **(MTR.5.1)**

Listen for different ways students explain Tam's walking rate, including those that use a positive value for the rate.

D **Differentiation**

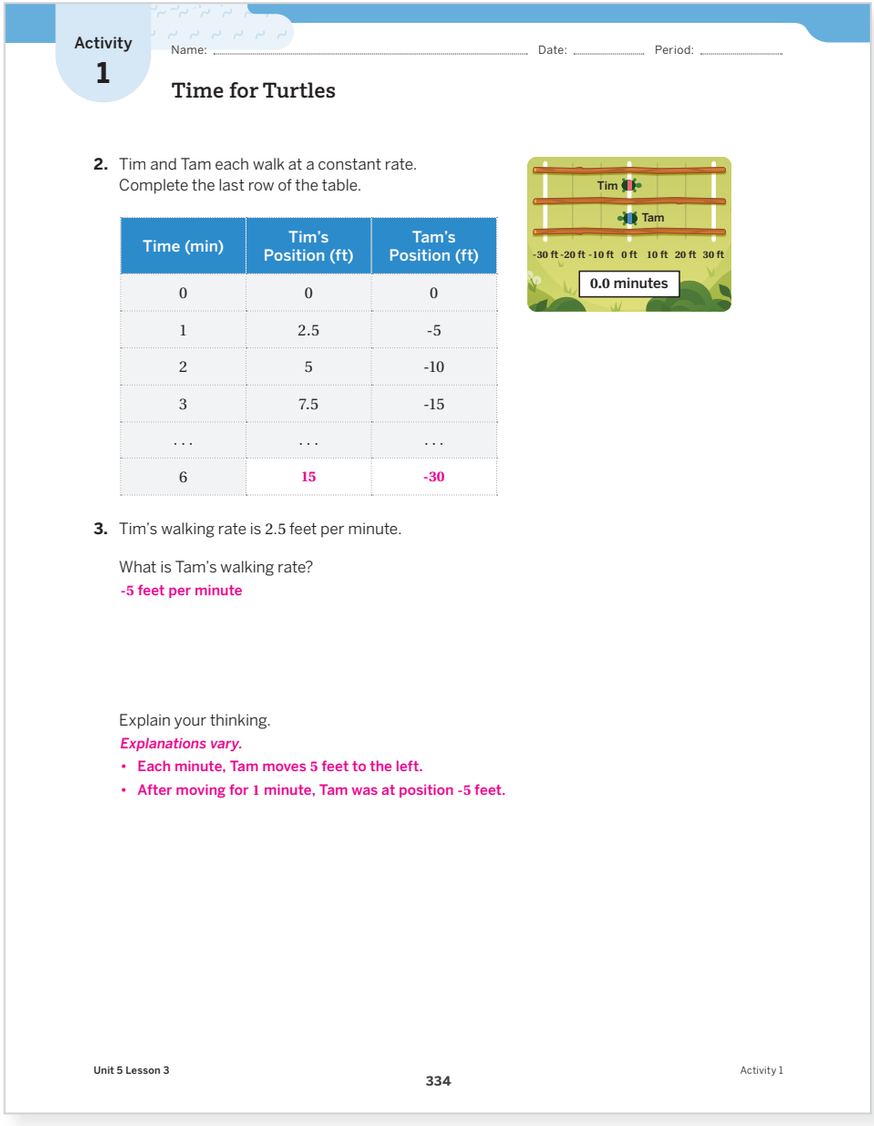
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on one row to determine the number of feet per second each turtle walks.
Use a positive value to represent Tam's walking rate.	**Support** Consider asking, "What direction on the number line would a positive walking rate represent? What direction did Tam go in the image?"
Determine the walking rate of -5 feet per second quickly.	**Stretch** Consider asking, "When will Tim and Tam be 150 feet apart?"

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in each other's responses. As a class:

- Discuss what is *correct* about each other's responses.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct and clear* solution.

Activity 1 continued >



Activity 1

Name: Date: Period:

Time for Turtles

2. Tim and Tam each walk at a constant rate. Complete the last row of the table.

Time (min)	Tim's Position (ft)	Tam's Position (ft)
0	0	0
1	2.5	-5
2	5	-10
3	7.5	-15
...	...	...
6	15	-30

3. Tim's walking rate is 2.5 feet per minute.

What is Tam's walking rate?

-5 feet per minute

Explain your thinking.

Explanations vary.

- Each minute, Tam moves 5 feet to the left.
- After moving for 1 minute, Tam was at position -5 feet.

Unit 5 Lesson 3 334 Activity 1

Pairs | 15 min

Activity 1 Time for Turtles (continued)

Purpose: Students use rate, time, and position to make sense of multiplying rational numbers. **(MTR.5.1)**

2 Monitor

Consider sharing they are going to use Tam's rate of change to make predictions about positions at various points in time.

Encourage students to look for connections between Problems 4 and 5. Consider asking, "How are the prompts similar? Different?"

Select and sequence student responses that explain the connection between the equations and the image of Tam from Problems 4 and 5. **(MTR.5.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Use Tam's position in the image to describe his position.	**Strengthen** Consider asking, "What about the equation shows that Tam will be facing to the left?"
Say that Tam moves to the right because of the positive 16, but don't mention the rate or going back in time.	**Strengthen** Invite students to write about Tam's final position using the -5 and the -3.2, and consider asking what each number represents.
Use language such as "to the right", "to the left", or "back in time" to describe Tam's movements.	**Stretch** Consider asking, "What are other combinations of positive and negative rates and times that would result in Tam moving to the right? To the left?"

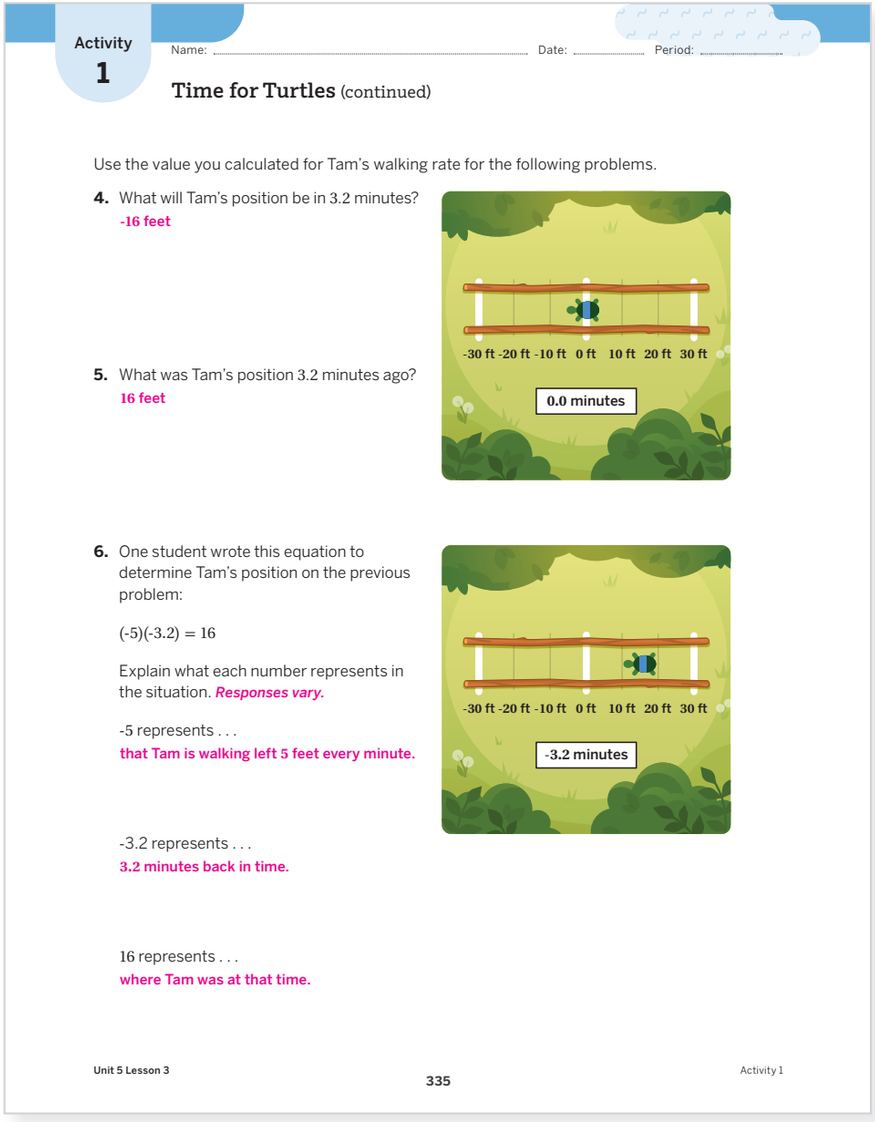
ELL **English Language Learners** Create an anchor chart that publicly displays words describing the turtle scenario to aid explanations and reasoning. **(Reading and Listening)**

3 Connect

Invite students to explain what each value in the expression means and why it makes sense that the position is positive. **(MTR.7.1)**

To surface the Key Takeaway, consider asking:

- "Why does it make sense that Tam's position is positive in this situation?"
- "Will Tam's position always be positive if both the rate and time are negative? Why or why not?"



Activity 1

Name: Date: Period:

Time for Turtles (continued)

Use the value you calculated for Tam's walking rate for the following problems.

4. What will Tam's position be in 3.2 minutes?
-16 feet

5. What was Tam's position 3.2 minutes ago?
16 feet

6. One student wrote this equation to determine Tam's position on the previous problem:

$(-5)(-3.2) = 16$

Explain what each number represents in the situation. *Responses vary.*

-5 represents . . .
that Tam is walking left 5 feet every minute.

-3.2 represents . . .
3.2 minutes back in time.

16 represents . . .
where Tam was at that time.

Unit 5 Lesson 3 335 Activity 1

Key Takeaway: The direction and time an object travels helps us to determine the final position of the object and whether it will be positive or negative. On a number line, moving to the right represents a positive change while moving to the left represents a negative change. The rate multiplied by the time represents the position.

Pairs | 5 min

Activity 2 Positives and Negatives

Purpose: Students practice determining when multiplication expressions are positive or negative.

1 Launch

Encourage students to share their reasoning with a partner and to work together to reach an agreement about how to sort the expressions.

A **Accessibility: Visual-Spatial Processing**
To support distinguishing between different expressions with positive and negative numbers, invite pairs to read each equation aloud and discuss how they represent Tam's position from the previous problems. **(MTR.5.1)**

2 Monitor

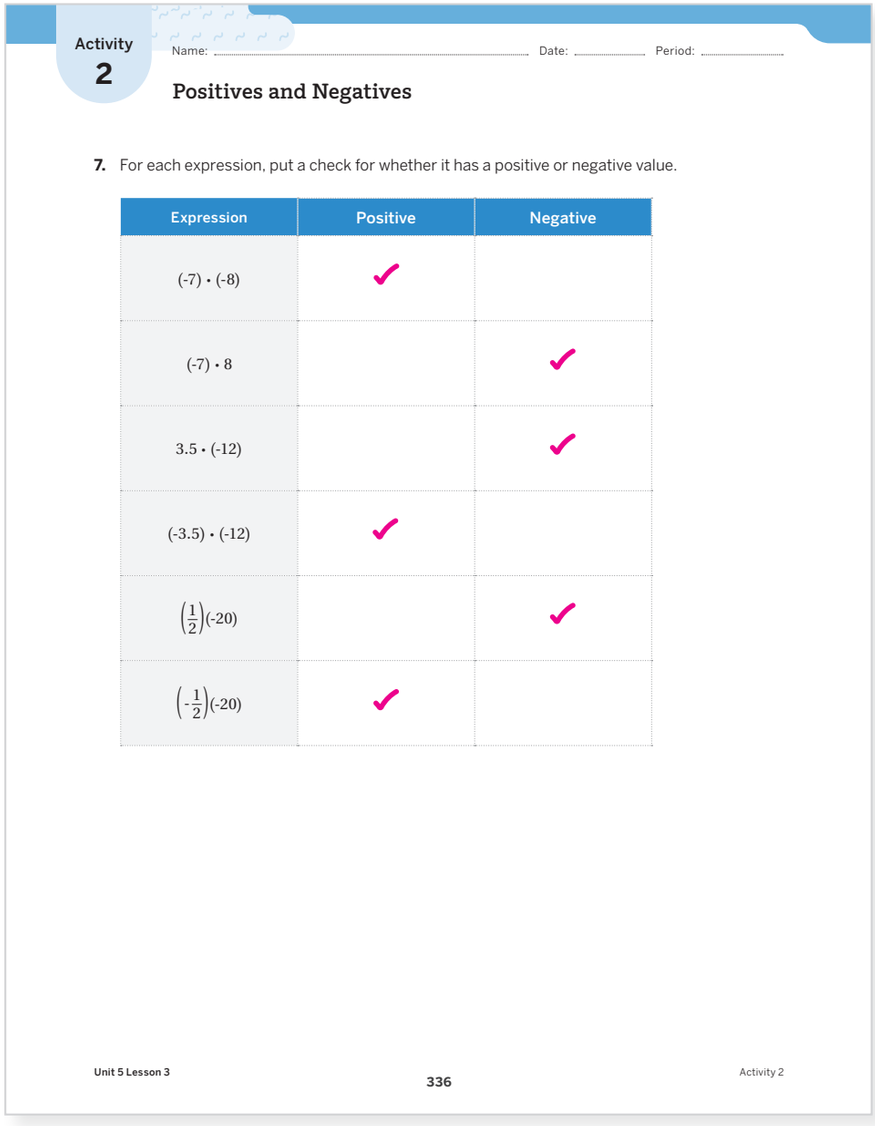
Invite students to compare their answers with another pair of students, justify their decisions, and make revisions based on their conversations.

D **Differentiation: (Stretch)** Invite students who finish early to write about how one of the expressions could represent Tam's position from the previous problems.

3 Connect

Invite students to share strategies they used to group the expressions.

Consider asking, "If we think about how these expressions represent our turtle's position, how do $3.5 \cdot (-12)$ and $(-7) \cdot 8$ both represent a turtle at a negative final position?" *Responses vary.* $3.5 \cdot (-12)$ represents a turtle that traveled 3.5 feet per minute starting 12 minutes ago so they will start in a negative position. $(-7) \cdot 8$ represents a turtle that travels 7 feet to the left every minute for 8 minutes so they will end up in a negative position.



Activity 2

Name: Date: Period:

Positives and Negatives

7. For each expression, put a check for whether it has a positive or negative value.

Expression	Positive	Negative
$(-7) \cdot (-8)$	✓	
$(-7) \cdot 8$		✓
$3.5 \cdot (-12)$		✓
$(-3.5) \cdot (-12)$	✓	
$\left(\frac{1}{2}\right)(-20)$		✓
$\left(-\frac{1}{2}\right)(-20)$	✓	

Unit 5 Lesson 3 336 Activity 2

Pairs | 10 min

Activity 3 Turtle Challenges

Purpose: Students extend what they have learned to consider expressions that involve both addition and multiplication and gain fluency with multiplying positive and negative rational numbers. **(MTR.3.1)**

Short on time? Consider omitting Problems 8–9.

1 Launch

Invite students to discuss strategies for multiplying decimals and fractions. Consider asking, "How does multiplying positive and negative rational numbers compare to multiplying integers?" **(MTR.3.1)**

A **Accessibility: Conceptual Processing** Invite students to make a table of values or write out where Tommy was at shorter intervals to make sense of going back in time with a negative rate. (For example, 1 minute ago Tommy was at 62.2 feet.)

2 Monitor

Look for how many challenges each student has completed.

Look for students using negative numbers to make sense of moving to the left and going back in time.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Since Tommy was moving -2.2 feet per minute, would he be further to the left or right 10 minutes ago?"
Write expressions to represent Tommy's position, like $60 + (-2.2)(-10)$.	**Stretch** Encourage them to write out in words what each number in their expression represents to share during the Connect.

Listen for students who notice that 60 is in each expression because it is the starting position.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore the relationship between position, rate, and time further to complete this optional task. Encourage them to discuss their thinking with a partner. **Note**: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Display the distribution of responses, calling attention to any conflicts or consensus you see.

To surface the Key Takeaway, consider asking, "How does each expression connect to a turtle scenario? How does that help determine its value?"

Activity 3

Name: Date: Period:

Turtle Challenges

8. What is the value of each expression? Solve as many challenges as you have time for.

a. $(3.4)(-5)$ — **-17**

b. $(-8)\left(-\frac{3}{4}\right)$ — **6**

c. $(-1.7)(0.25)$ — **-0.425**

d. $\left(-2\frac{1}{2}\right)\left(\frac{3}{5}\right)$ — **$-1\frac{1}{2}$ (or equivalent)**

e. $(0.2)\left(-5\frac{9}{10}\right)$ — **-1.18 (or equivalent)**

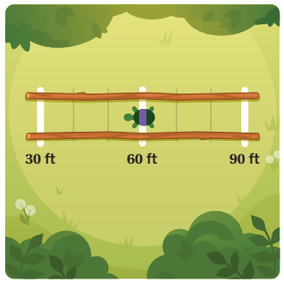
f. $(-4.1)(3)(-2.3)$ — **28.29**

9. Tommy walks at a rate of -2.2 feet per minute.

Right now he is at 60 feet.

Where was Tommy 10 minutes ago?

82 feet



Explain how you know.

Explanations vary. **Tommy was at 82 feet because he started 22 feet to the right of where he is now.**

10. Select *all* the expressions that describe where Tommy was 10 minutes ago.

- ☐ A. $60 + 2.2 \cdot (-10)$
- ☑ B. $60 + (-2.2)(-10)$
- ☐ C. $60 - 22 \cdot 10$
- ☐ D. $60 - 22$
- ☑ E. $60 + 22$

You're invited to explore more.

11. Use the You're Invited to Explore More Sheet to explore another turtle puzzle.

Unit 5 Lesson 3 337 Activity 3

Key Takeaway: When an object doesn't start at 0, the starting position is added to, or subtracted from, the value of the rate multiplied by the time to get the final position.

Whole Class | 5 min

Synthesis

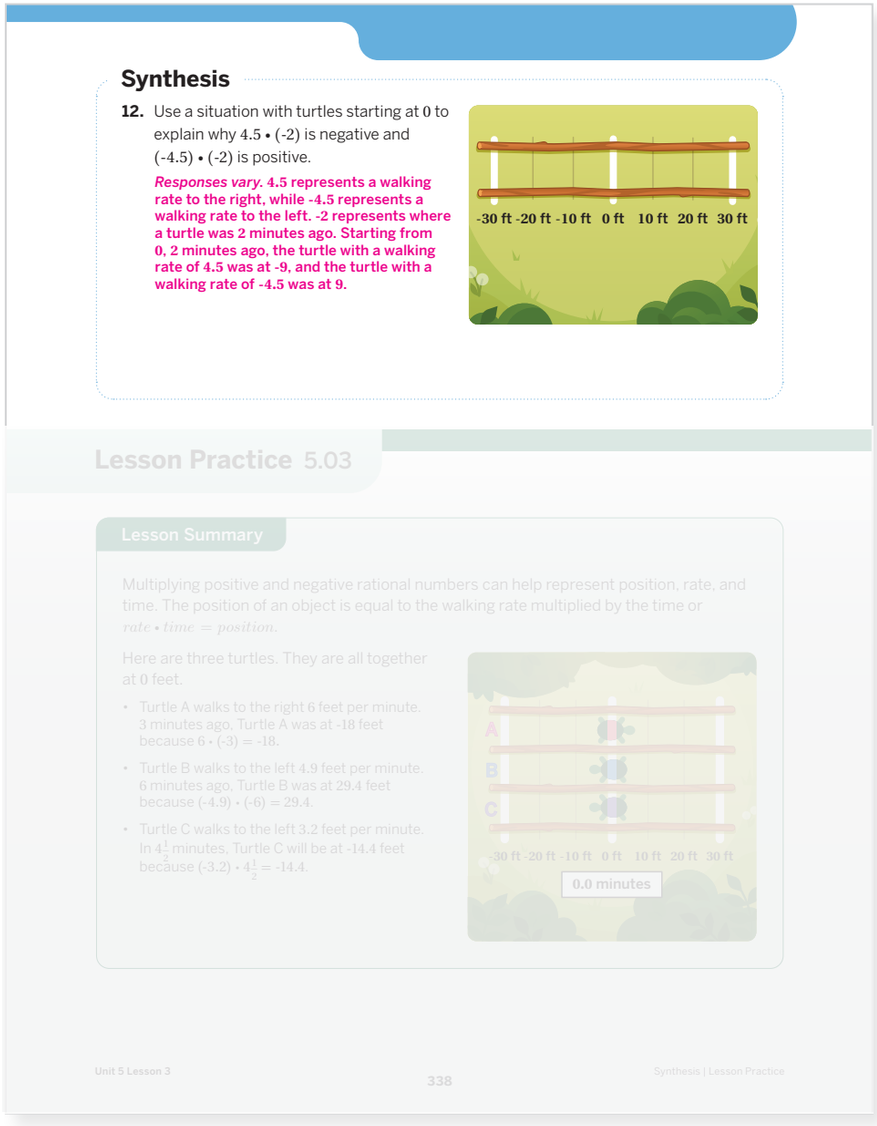
Purpose: Students synthesize their understanding of why the product of two numbers is positive or negative using the turtle context.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- A turtle that traveled 4.5 feet per minute two minutes ago would have been at -9 feet to begin with.
- A turtle that traveled -4.5 feet per minute two minutes ago would have been at 9 feet to begin with.
- If we go back in time two minutes with a turtle that is traveling to the left, it would have been in the positive position to start.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Use a situation with turtles starting at 0 to explain why $4.5 \cdot (-2)$ is negative and $(-4.5) \cdot (-2)$ is positive.

Responses vary. 4.5 represents a walking rate to the right, while -4.5 represents a walking rate to the left. -2 represents where a turtle was 2 minutes ago. Starting from 0, 2 minutes ago, the turtle with a walking rate of 4.5 was at -9, and the turtle with a walking rate of -4.5 was at 9.

Lesson Practice 5.03

Lesson Summary

Multiplying positive and negative rational numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or $rate \cdot time = position$.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left 4.9 feet per minute. 6 minutes ago, Turtle B was at 29.4 feet because $(-4.9) \cdot (-6) = 29.4$.
- Turtle C walks to the left 3.2 feet per minute. In $4\frac{1}{2}$ minutes, Turtle C will be at -14.4 feet because $(-3.2) \cdot 4\frac{1}{2} = -14.4$.

Unit 5 Lesson 3 338 Synthesis | Lesson Practice

Lesson Takeaway: We can determine the position of an object by multiplying the rate it is traveling by the time. A positive number represents a walking rate to the right, and a negative number represents a walking rate to the left. Traveling forward in time is represented by a positive number, and going back in time is represented by a negative number.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which equation represents a situation in context using distance, time, and rate.

Today's Goals

1. **Goal:** Represent rate, time, and position using the product of positive and negative rational numbers.
2. **Language Goal:** Explain why the product of two negative numbers is positive in the context of rate, time, and position. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.03

A helicopter just landed. It was traveling downward at a rate of 1.8 meters per second.

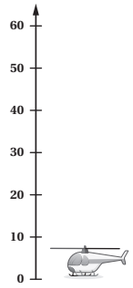
Select the equation that represents the helicopter's position 30 seconds ago.

A. $(-1.8)(-30) = -54$

B. $(-1.8)(-30) = 54$

C. $(-1.8)(30) = -54$

D. $(-1.8)(30) = 54$



Explain your thinking.

Explanations vary. The helicopter's rate can be represented using a negative number since it was moving downward to land. The time is also represented using a negative number since it refers to a time in the past. Multiplying two negative numbers has a positive value. In this case, this makes sense because 30 seconds ago, the helicopter would have still been in the air, so its position would be represented by a positive number.

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to model multiplication using time, rate, and position:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Consider the next several lessons you will teach. How will students' work today in Activity 1 prepare them for dividing rational numbers in Lesson 4? What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Use a situation with turtles starting at 0 to explain why $4.5 \cdot (-2)$ is negative and $(-4.5) \cdot (-2)$ is positive.

Responses vary. 4.5 represents a walking rate to the right, while -4.5 represents a walking rate to the left. -2 represents where a turtle was 2 minutes ago. Starting from 0, 2 minutes ago, the turtle with a walking rate of 4.5 was at -9, and the turtle with a walking rate of -4.5 was at 9.

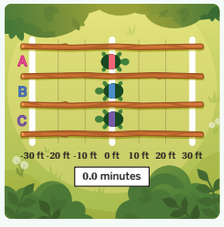
Lesson Practice 5.03

Lesson Summary

Multiplying positive and negative rational numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or $rate \cdot time = position$.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left 4.9 feet per minute. 6 minutes ago, Turtle B was at 29.4 feet because $(-4.9) \cdot (-6) = 29.4$.
- Turtle C walks to the left 3.2 feet per minute. In $4\frac{1}{2}$ minutes, Turtle C will be at -14.4 feet because $(-3.2) \cdot 4\frac{1}{2} = -14.4$.



Lesson Practice 5.03

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine the value of each expression.

1. $5(-3) = -15$

2. $-5 \cdot 3 = -15$

3. $(-5)(-3) = 15$

4. $-5 \cdot (-0.3) = 1.5$

Problems 5–8: Anton the Ant can travel up and down an anthill. Currently, his crawling rate is -6.7 centimeters per second, which means he's crawling down the anthill. Anton passes ground level at position 0.

5. Which equation represents Anton's position 8 seconds *after* he passes ground level?

A. $-6.7 \cdot 8 = 53.6$ (B.) $-6.7 \cdot 8 = -53.6$ C. $-6.7 \cdot -8 = 53.6$ D. $-6.7 \cdot -8 = -53.6$

6. Which equation represents Anton's position 8 seconds *before* he passes ground level?

A. $-6.7 \cdot 8 = 53.6$ B. $-6.7 \cdot 8 = -53.6$ (C.) $-6.7 \cdot -8 = 53.6$ D. $-6.7 \cdot -8 = -53.6$

7. Write an ant story that could be represented by the equation $5 \cdot \left(-6\frac{1}{2}\right) = -32\frac{1}{2}$.

Responses vary. An ant is crawling up an anthill at a rate of 5 centimeters per second. The ant passes ground level. The expression shows the position of the ant $6\frac{1}{2}$ seconds before it passes ground level.

8. Anton has friends in nearby hills named Anto and Anty. For each description, write an equation in the table that represents it. Assume each ant starts at position 0.

Responses vary.

Description	Equation
Anto crawls for 28 seconds at -2.4 centimeters per second.	$-2.4 \cdot 28 = -67.2$
Anty crawls for $3\frac{1}{2}$ seconds at $-8\frac{1}{4}$ centimeters per second.	$-8\frac{1}{4} \cdot 3\frac{1}{2} = -28\frac{7}{8}$

Lesson Practice 5.03

Name: _____ Date: _____ Period: _____

FAST Practice

9. What is the value of the expression $(0.8)\left(-\frac{2}{9}\right)$?

$-\frac{8}{45}$ (or equivalent)

Spiral Review

Problems 10–11: A tank of water breaks and begins draining at a constant rate.

10. Complete the table.

Time (min)	Water in the Tank (L)
0	770
1	756
2	742
. . .	. . .
5	700

11. Is there a proportional relationship between time and the amount of water left in the tank? Explain your thinking.

No. *Explanations vary.* The water in the tank at the beginning is 770 liters. For it to be a proportional relationship, it would need to begin at 0 liters.

12. Angela buys a pair of shoes. The shoes cost $40. The store is having a 20% off sale on everything in the store. How much money does Angela save from the sale?

$8.00

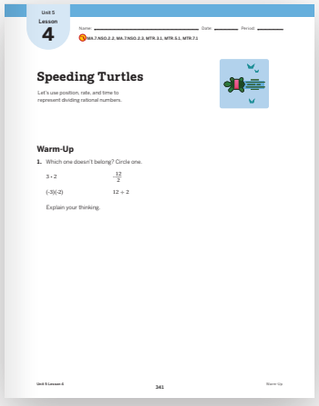
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.NSO.2.2
	5–8	2	MA.7.NSO.2.3
FAST Practice	9	2	MA.7.NSO.2.2
Spiral Review			
	10	2	MA.7.AR.3.2
	11	2	MA.7.AR.4.1
	12	2	MA.7.AR.3.1

Unit 5

Lesson

4



Speeding Turtles

Dividing Rational Numbers

Let's use position, rate, and time to represent dividing rational numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Divide positive and negative rational numbers.
2. **Language Goal:** Describe the operations used to determine position, time, and rate and when values will be positive or negative. **(Writing)**

Students make sense of division of positive and negative rational numbers using the context of position, rate, and time as a model. Students use the turtle context to determine unknown rates and times, reason about their signs, and build fluency with dividing rational numbers.

Coherence

Prior Learning

In Lesson 3, students made sense of multiplying positive and negative rational numbers using the context of position, rate, and time as a model.

Future Learning

In Lesson 5, students will use order of operations to evaluate expressions with rational numbers.

Connections

- Students use rate, time, and position to make sense of multiplying rational numbers through the context of walking turtles. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2

Add, subtract, multiply and divide rational numbers with procedural fluency.

MA.7.NSO.2.3

Solve real-world problems involving any of the four operations with rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.2.1	MA.8.NSO.1.5
MA.6.NSO.2.2	MA.8.NSO.1.6
MA.6.NSO.2.3	MA.8.NSO.1.7
MA.6.NSO.4.2	

Lesson at a Glance

~ 45 min

MA.7.NSO.2.2, MA.7.NSO.2.3

Warm-Up

Independent | 5 min

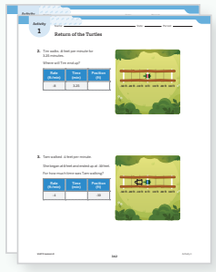
Students reason about numerical expressions involving multiplication and division. **Routine:** Which One Doesn't Belong?



Activity 1

Pairs | 15 min

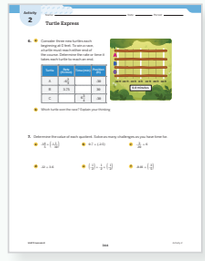
Students multiply and divide rational numbers in context to determine unknown positions, rates, and times. MLR1: Stronger and Clearer Each Time **(MTR.7.1)**



Activity 2

Pairs | 15 min

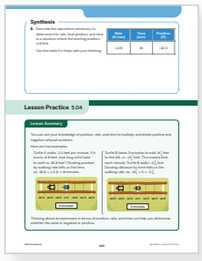
Students build fluency in determining quotients of rational numbers. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of dividing positive and negative rational numbers. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which two expressions have the same value.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students reason about numerical expressions involving multiplication and division.

1 Launch

Use the Which One Doesn't Belong? routine to support students in describing properties of expressions with positive and negative numbers. **(MTR.5.1)**

Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., "_____ doesn't belong because _____."). **(Writing and Speaking)**

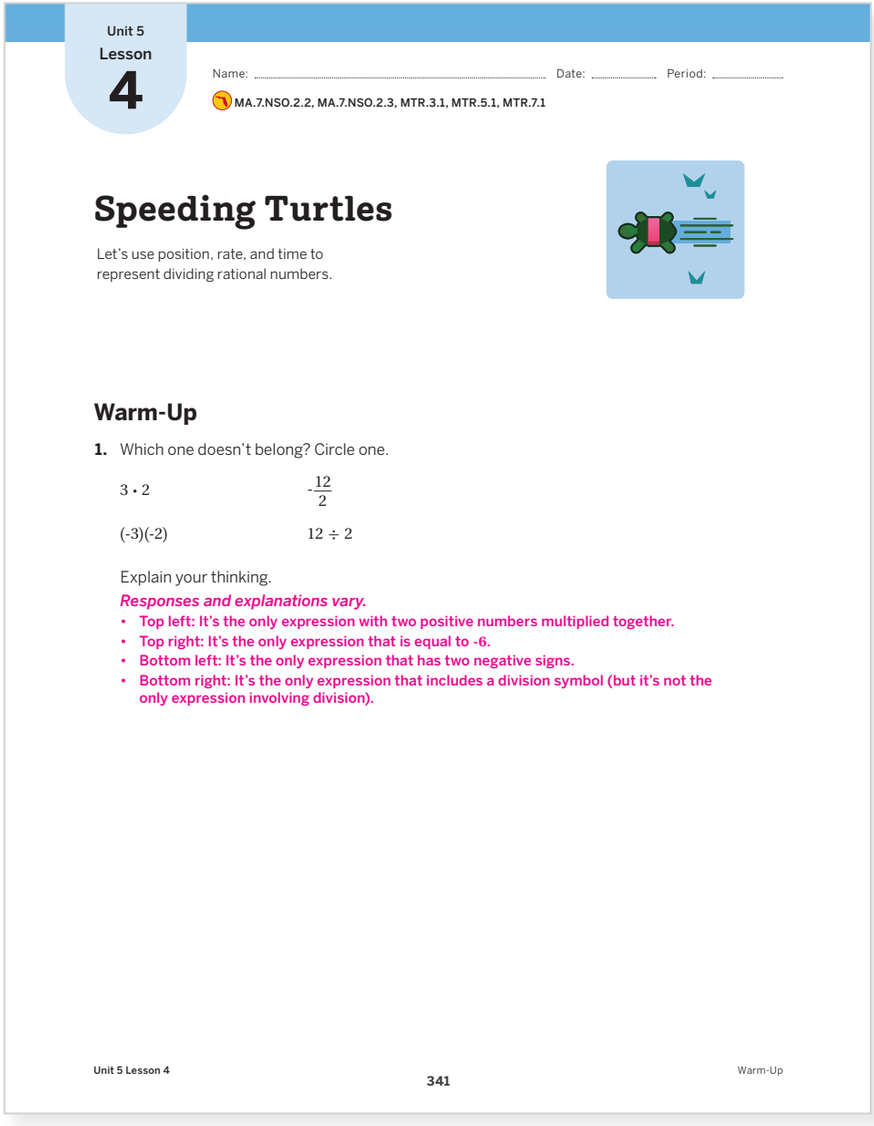
2 Connect

Invite students to share at least one reason why *each* expression doesn't belong.

Create a record of why each expression doesn't belong, along with the name of the student who shared the reasoning.

Consider asking:

- "Does the expression $-\frac{12}{2}$ represent division?"
- "Does $3 \cdot 2$ have the same value as $-3 \cdot -2$?"



Unit 5
Lesson
4

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1

Speeding Turtles

Let's use position, rate, and time to represent dividing rational numbers.

Warm-Up

1. Which one doesn't belong? Circle one.

$3 \cdot 2$ $\quad -\frac{12}{2}$

$(-3)(-2)$ $\quad 12 \div 2$

Explain your thinking.

Responses and explanations vary.

- Top left: It's the only expression with two positive numbers multiplied together.
- Top right: It's the only expression that is equal to -6.
- Bottom left: It's the only expression that has two negative signs.
- Bottom right: It's the only expression that includes a division symbol (but it's not the only expression involving division).

Unit 5 Lesson 4 341 Warm-Up

Pairs | 15 min

Activity 1 Return of the Turtles

Purpose: Students multiply and divide rational numbers in context to determine unknown positions, rates, and times. **(MTR.7.1)**

1 Launch

To surface prior knowledge, consider asking, "What do you remember about negative rates and times from the turtles in the last lesson?" **(MTR.7.1)**

Note: The position of each turtle is measured from the shell stripe.

A **Accessibility: Memory and Attention** Provide students with a graphic organizer to support them with keeping track of the relationship between the position, rate, and time.

2 Monitor

Look for a variety of student strategies for determining the time including writing expressions such as $\frac{-10}{-4}$ or setting up equations.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How much time did it take Tam to reach -8 feet? -12 feet?"
Multiply the rate and the position to get the time.	**Support** Consider asking, "What would the units be for the value you calculated? Does that match the units for time?"
Write an equation like $-10 = -4 \cdot t$.	**Strengthen** Celebrate the way they used the relationship to make sense of the situation. Encourage them to solve the equation to determine the time. **(MTR.5.1)**
Divide the position by the rate to determine the time.	**Stretch** Consider asking, "How do the units for each of your values help you to make sense of the division?"

Pause briefly to discuss students' strategies for determining the unknown time. Consider asking:

- "How do the units of position and rate help to determine the unknown time?" When I divide the units of feet by feet/minute, the result is minutes. Minutes is what time is measured in so I know that I need to divide the two values.
- "Based on the starting and ending positions, how do we know that the time was positive?" Since the rate is negative, and the turtle ends to the left of where it started, I know that time had to be positive.

Note: Consider creating an anchor chart of rate · time = position and displaying it with examples to support students' reasoning.

Activity 1 continued >

Activity 1

Name: Date: Period:

Return of the Turtles

2. Tim walks -8 feet per minute for 3.25 minutes.

Where will Tim end up?

Rate (ft/min)	Time (min)	Position (ft)
-8	3.25	-26

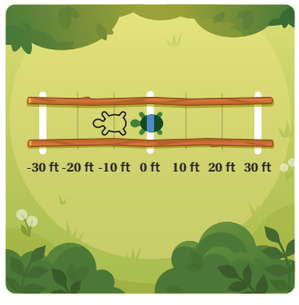


3. Tam walked -4 feet per minute.

She began at 0 feet and ended up at -10 feet.

For how much time was Tam walking?

Rate (ft/min)	Time (min)	Position (ft)
-4	2.5	-10



Unit 5 Lesson 4 342 Activity 1

Pairs | 15 min

Activity 1 Return of the Turtles (continued)

Purpose: Students multiply and divide rational numbers in context to determine unknown positions, rates, and times. **(MTR.7.1)**

2 Monitor

Invite students to continue to Problem 4, writing a first draft about why Fabiana's equation makes sense.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own. Consider asking, "Is there anything in someone else's explanation that you can use to make your writing stronger and clearer?"
Select and sequence student responses that calculate the correct unit rate.

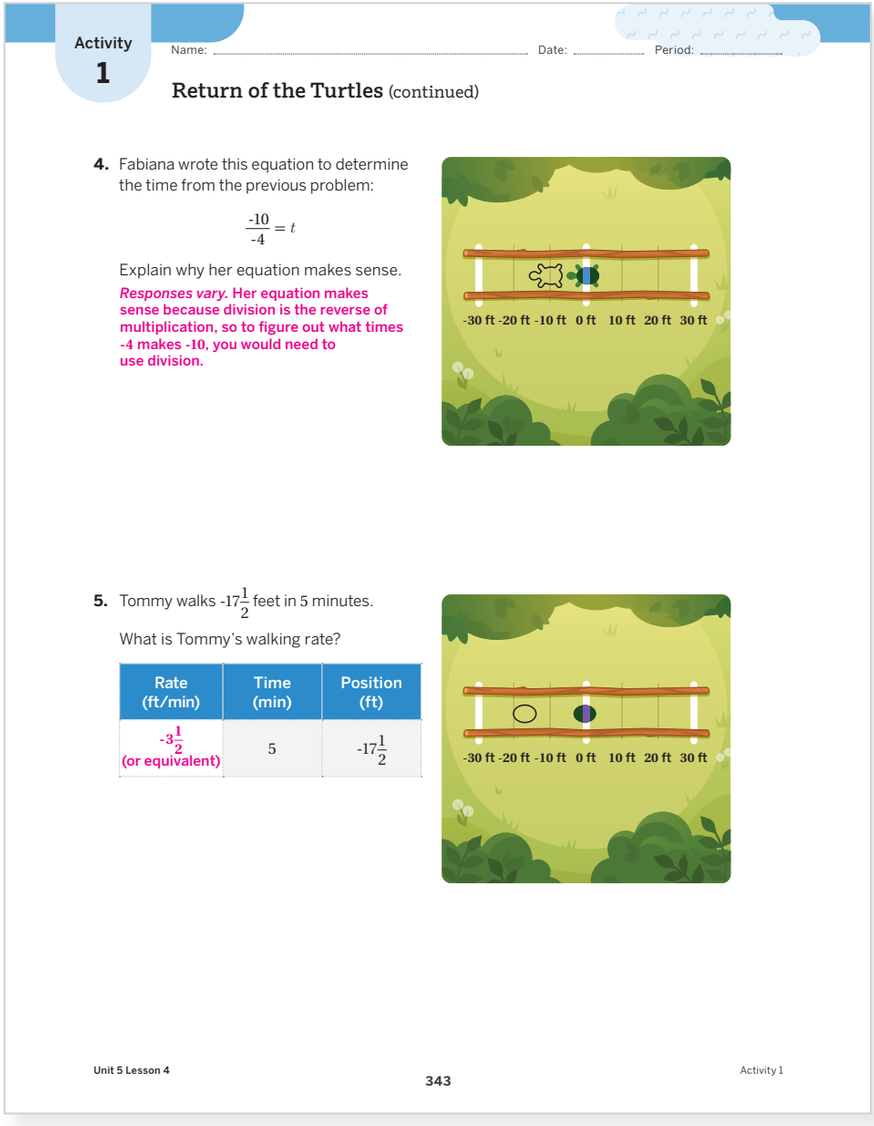
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you use Fabiana's strategy to help make sense of this problem?" **(MTR.5.1)**
Multiply the time and position to try to determine the rate.	**Support** Consider asking, "What are the units if we multiply position by time?"
Enter a division expression $\left(\frac{-17}{5}\right)$ for the rate.	**Strengthen** Consider asking, "Why is the walking rate negative here? How does your expression help to show the negative position?"

3 Connect

Invite students to share their strategies for determining the unknown rate.

To surface the Key Takeaway, consider asking, "Why do we use division to determine an unknown rate when we know the position and time?"



Activity 1

Name: Date: Period:

Return of the Turtles (continued)

4. Fabiana wrote this equation to determine the time from the previous problem:

$$\frac{-10}{-4} = t$$

Explain why her equation makes sense.

Responses vary. Her equation makes sense because division is the reverse of multiplication, so to figure out what times -4 makes -10, you would need to use division.

5. Tommy walks $-17\frac{1}{2}$ feet in 5 minutes.

What is Tommy's walking rate?

Rate (ft/min)	Time (min)	Position (ft)
$-3\frac{1}{2}$ (or equivalent)	5	$-17\frac{1}{2}$

Unit 5 Lesson 4 343 Activity 1

Key Takeaway: When using the relationship position = rate • time, we can use division to figure out an unknown rate or time.

Pairs | 15 min

Activity 2 Turtle Express

Purpose: Students build fluency in determining quotients of rational numbers. **(MTR.3.1)**

Short on time? Consider completing Problem 6 as a class.

1 Launch

Invite a student to read the problem aloud, then discuss the table.

To surface connections, invite students to share experiences they've had with foot races. Consider asking, "Have you ever run a race where people run in different directions?"

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., Turtle _____ had the _____ rate, so this turtle _____.). **(Writing and Speaking)**

2 Monitor

Look for students who make sense of rate and time using the relationship between position, time, and rate.

Encourage students to share their reasoning with a partner and revise their solutions based on feedback and collective decisions.

3 Connect

Invite students to share their responses, calling attention to any conflicts or consensus you see.

Consider asking:

- "Can you follow the same process as with dividing integers to determine the sign of the answer?" **(MTR.3.1)**
- "How are multiplying and dividing positive and negative rational numbers alike? How are they different?"

Activity 2

Name: _____ Date: _____ Period: _____

Turtle Express

6. a Consider three new turtles each beginning at 0 feet. To win a race, a turtle must reach either end of the course. Determine the rate or time it takes each turtle to reach an end.

Turtle	Rate (ft/min)	Time (min)	Position (ft)
A	$-6\frac{2}{3}$	$-4\frac{1}{2}$	-30
B	3.75	8	30
C	$-4\frac{4}{9}$	$6\frac{3}{4}$	-30



b Which turtle won the race? Explain your thinking

Turtle A won the race. *Explanations vary.* Turtle A had the fastest rate, so it went the distance in the shortest amount of time.

7. Determine the value of each quotient. Solve as many challenges as you have time for.

a $-3\frac{4}{5} \div \left(-1\frac{1}{10}\right)$

$\frac{38}{11}$ (or equivalent)

b $0.7 \div (-2.5)$

-0.28

c $-\frac{3}{20} \div 6$

$-\frac{1}{40}$ (or equivalent)

d $-12 \div 3.6$

$-3.\overline{3}$

e $\left(-\frac{1}{2}\right) \div -\frac{1}{2} \div \left(-\frac{1}{2}\right)$

-2

f $-0.01 \div \left(-\frac{3}{5}\right)$

60

Unit 5 Lesson 4 344 Activity 2

Whole Class | 5 min

Synthesis

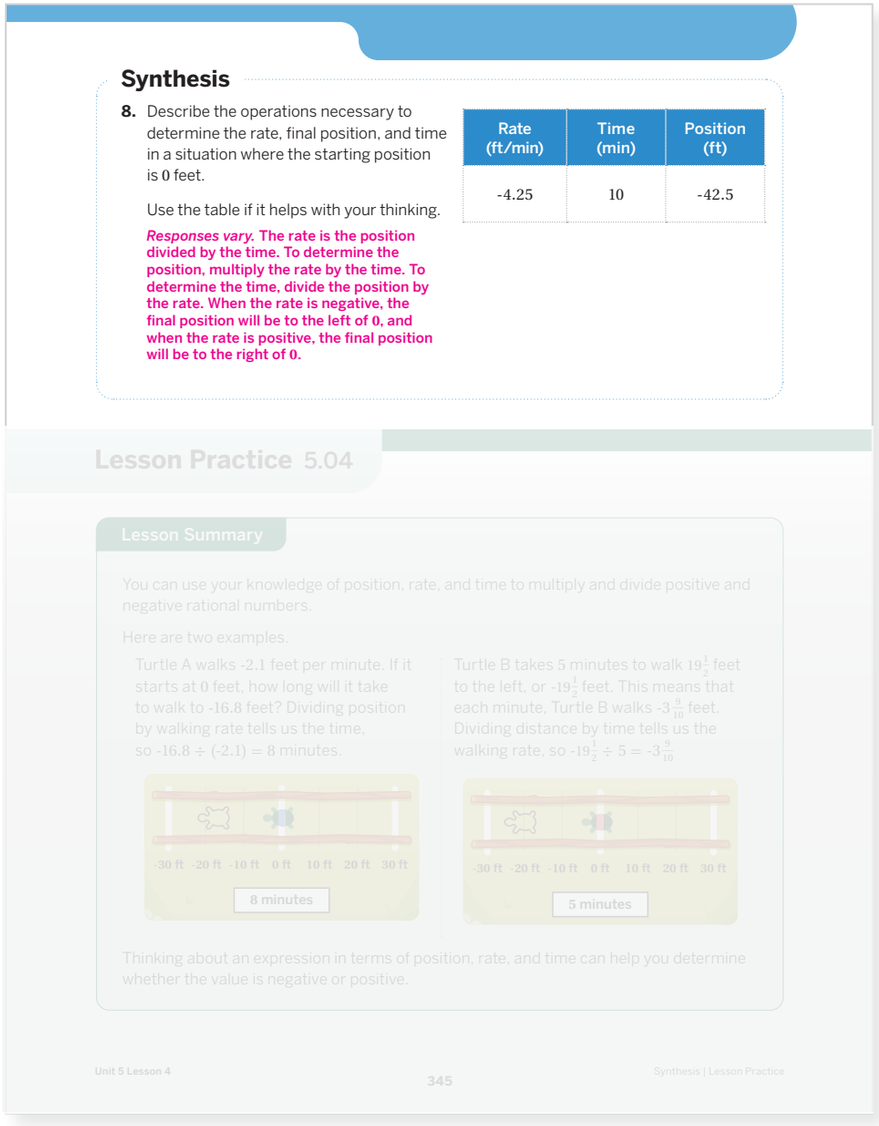
Purpose: Students synthesize their understanding of dividing positive and negative rational numbers.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- We can use the equation position = rate • time to determine the unknown quantity.
- The unit of measure for time is minutes. When you divide the position (in feet) by the rate (in feet/minute), the unit of measure that remains is minutes.
- Negative numbers are to the left of 0, and positive numbers are to the right of 0.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for dividing positive and negative rational numbers as they discuss with a partner. Consider asking, "When dividing two rational numbers, how do you know whether a quotient is positive or negative?" Display for students to reference and add to for the rest of the unit.



Synthesis

8. Describe the operations necessary to determine the rate, final position, and time in a situation where the starting position is 0 feet.

Use the table if it helps with your thinking.

Rate (ft/min)	Time (min)	Position (ft)
-4.25	10	-42.5

Responses vary. The rate is the position divided by the time. To determine the position, multiply the rate by the time. To determine the time, divide the position by the rate. When the rate is negative, the final position will be to the left of 0, and when the rate is positive, the final position will be to the right of 0.

Lesson Practice 5.04

Lesson Summary

You can use your knowledge of position, rate, and time to multiply and divide positive and negative rational numbers.

Here are two examples.

Turtle A walks -2.1 feet per minute. If it starts at 0 feet, how long will it take to walk to -16.8 feet? Dividing position by walking rate tells us the time, so $-16.8 \div (-2.1) = 8$ minutes.

Turtle B takes 5 minutes to walk $19\frac{1}{2}$ feet to the left, or $-19\frac{1}{2}$ feet. This means that each minute, Turtle B walks $-3\frac{9}{10}$ feet. Dividing distance by time tells us the walking rate, so $-19\frac{1}{2} \div 5 = -3\frac{9}{10}$

Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Unit 5 Lesson 4 345 Synthesis | Lesson Practice

Lesson Takeaway: The sign of the quotient of a positive number divided by a negative number (or a negative number divided by a positive number) is negative. The sign of the quotient of a negative number divided by a negative number is positive. We can use models, such as the relationship between position, rate, and time to help us make sense of why the signs of these division problems are positive or negative.

Independent | 5 min

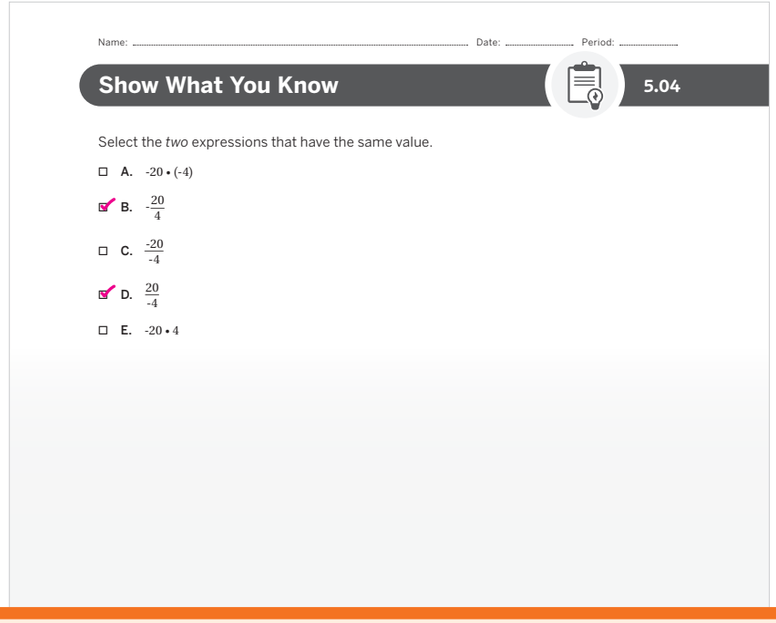
Show What You Know

Purpose: Students demonstrate their understanding by determining which two expressions have the same value.

Today's Goals

1. **Goal:** Divide positive and negative rational numbers.
2. **Language Goal:** Describe the operations used to determine position, time, and rate and when values will be positive or negative. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.04

Select the *two* expressions that have the same value.

- ☐ A. $-20 \cdot (-4)$
- ☑ B. $-\frac{20}{4}$
- ☐ C. $\frac{-20}{-4}$
- ☑ D. $\frac{20}{-4}$
- ☐ E. $-20 \cdot 4$

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of multiplying and dividing integers:

» **Respond to Student Thinking:**

- **Mini-Lesson:** Dividing Integers.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on your facilitation of the **MLR1: Stronger and Clearer Each Time** routine in this lesson. How did students engage with making their response stronger about why the equation made sense? What about how you facilitated the routine resulted in students revising their responses to make them stronger? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe the operations necessary to determine the rate, final position, and time in a situation where the starting position is 0 feet.

Use the table if it helps with your thinking.

Rate (ft/min)	Time (min)	Position (ft)
-4.25	10	-42.5

Responses vary. The rate is the position divided by the time. To determine the position, multiply the rate by the time. To determine the time, divide the position by the rate. When the rate is negative, the final position will be to the left of 0, and when the rate is positive, the final position will be to the right of 0.

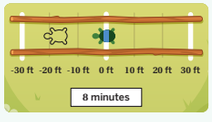
Lesson Practice 5.04

Lesson Summary

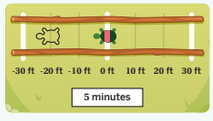
You can use your knowledge of position, rate, and time to multiply and divide positive and negative rational numbers.

Here are two examples.

Turtle A walks -2.1 feet per minute. If it starts at 0 feet, how long will it take to walk to -16.8 feet? Dividing position by walking rate tells us the time, so $-16.8 \div (-2.1) = 8$ minutes.



Turtle B takes 5 minutes to walk $19\frac{1}{2}$ feet to the left, or $-19\frac{1}{2}$ feet. This means that each minute, Turtle B walks $-3\frac{9}{10}$ feet. Dividing distance by time tells us the walking rate, so $-19\frac{1}{2} \div 5 = -3\frac{9}{10}$



Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Lesson Practice 5.04

Name: _____ Date: _____ Period: _____

1. Select *all* the expressions that have a negative value.

- ☑ A. $\frac{-15}{12}$
- ☑ B. $\frac{15}{-12}$
- ☐ C. $\frac{-15}{-12}$
- ☑ D. $-\frac{15}{12}$
- ☐ E. $\frac{15}{12}$

Problems 2–3: Determine each quotient.

2. $6.3 \div (-3)$

-2.1

3. $0.36 \div \left(-1\frac{1}{5}\right)$

-0.3

Problems 4–5: A machine that drills holes for wells drilled at a constant rate to a depth of $-76\frac{4}{5}$ feet in one day (24 hours).

4. How much did the depth change each hour? Make sure to show whether the change is positive or negative.

$-3\frac{1}{5}$ feet per hour (or equivalent)

5. What was the depth after 15 hours?

-48 feet

FAST Practice

6. Which expression has the same value as (-0.75)(-0.6)?

- (A.) $0.135 \div 0.3$
- B. $-0.135 \div 0.3$
- C. $11\frac{1}{4} \div \frac{1}{4}$
- D. $11\frac{1}{4} \div -\frac{1}{4}$

Lesson Practice 5.04

Name: _____ Date: _____ Period: _____

Spiral Review

7. The equation $30 + (-30) = 0$ is an example of two numbers whose sum is 0. Write a different equation with two numbers whose sum is 0.

Responses vary. $44 + (-44) = 0$

8. Write an equation with three numbers whose sum is 0.

Responses vary. $33 + (-22) + (-11) = 0$

9. Write an equation with *four* numbers, none of which are opposites, whose sum is 0. One example of opposites is -30 and 30.

Responses vary. $44 + (-33) + (-22) + 11 = 0$

10. Write an equation using *four* numbers that multiply to 0.

Responses vary. $33 \cdot 22 \cdot 11 \cdot 0 = 0$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.NSO.2.2
	2–3	2	MA.7.NSO.2.2
	4–5	2	MA.7.NSO.2.3
FAST Practice	**6**	2	MA.7.NSO.2.2
Spiral Review			
	7–10	2	MA.7.NSO.2.2

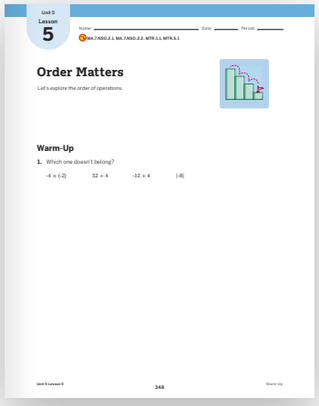
Unit 5

Lesson 5

Order Matters

Evaluating Numerical Expressions

Let's explore how to evaluate numerical expressions using order of operations.


Unit 5
Lesson 5
Order Matters
Let's explore the order of operations.
Warm-Up
1. Which one doesn't belong?
-4 × (-2)
32 ÷ 4
-12 + 4
|-8|
348

Learning Outcomes

Today's Goals

1. **Goal:** Use order of operations to evaluate numerical expressions.
2. **Language Goal:** Describe how order of operations is used to evaluate a numerical expression. **(Reading, Writing, Speaking, and Listening)**

Students use structure when using the *order of operations* to evaluate numerical expressions. They notice that they should perform operations within grouping symbols first, followed by *exponents*, then multiplication and division in order from left to right, and finally addition and subtraction in order from left to right. **(MTR.5.1)**

Coherence

Prior Learning

In Grade 6, students used order of operations to evaluate algebraic expressions involving integers.

Future Learning

In Lessons 6–8, students will use properties of exponents to write equivalent expressions involving exponents.

Connections

- Students explore evaluating expressions through analyzing student work. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

MA.7.NSO.2.2
Add, subtract, multiply, and divide rational numbers with procedural fluency.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.2 **MA.6.NSO.4** **MA.6.AR.1.3**	**MA.8.NSO.1.7**

Vocabulary

Review Vocabulary

absolute value
exponent
order of operations

Lesson at a Glance

~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2

Warm-Up

Pairs | 5 min

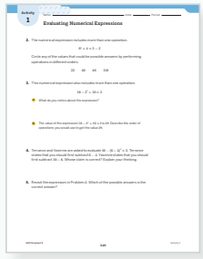
Students use the **Which One Doesn't Belong?** routine to surface their understanding of expressions and operations.



Activity 1

Pairs | 15 min

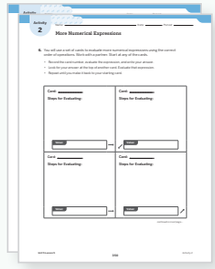
Students use *order of operations* to evaluate numerical expressions.



Activity 2

Small Groups | 10 min

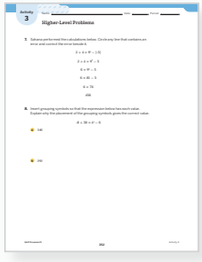
Students apply what they have learned to evaluate more numerical expressions. **(MTR.5.1)**



Activity 3

Pairs | 10 min

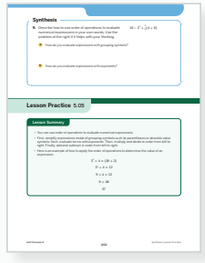
Students analyze student work for potential errors and place grouping symbols in an expression to create different values.
MLR1: Stronger and Clearer Each Time **(MTR.5.1)**



Synthesis

Whole Class | 5 min

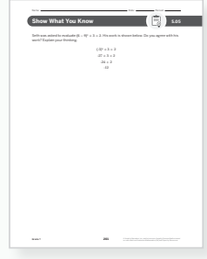
Students synthesize their understanding about using order of operations to evaluate numerical expressions.
Routine: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by critiquing the process of another student when evaluating a numerical expression and explaining their thinking.



Prep Checklist

Prepare Activity 2 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per group, pre-cut

Pairs | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to surface their understanding of expressions and operations.

1 Launch

Use the Which One Doesn't Belong routine to support students in noticing signs of numbers, absolute value, and operations.

Direct students' attention to the four expressions. Invite students to choose one they believe doesn't belong and justify their choice.

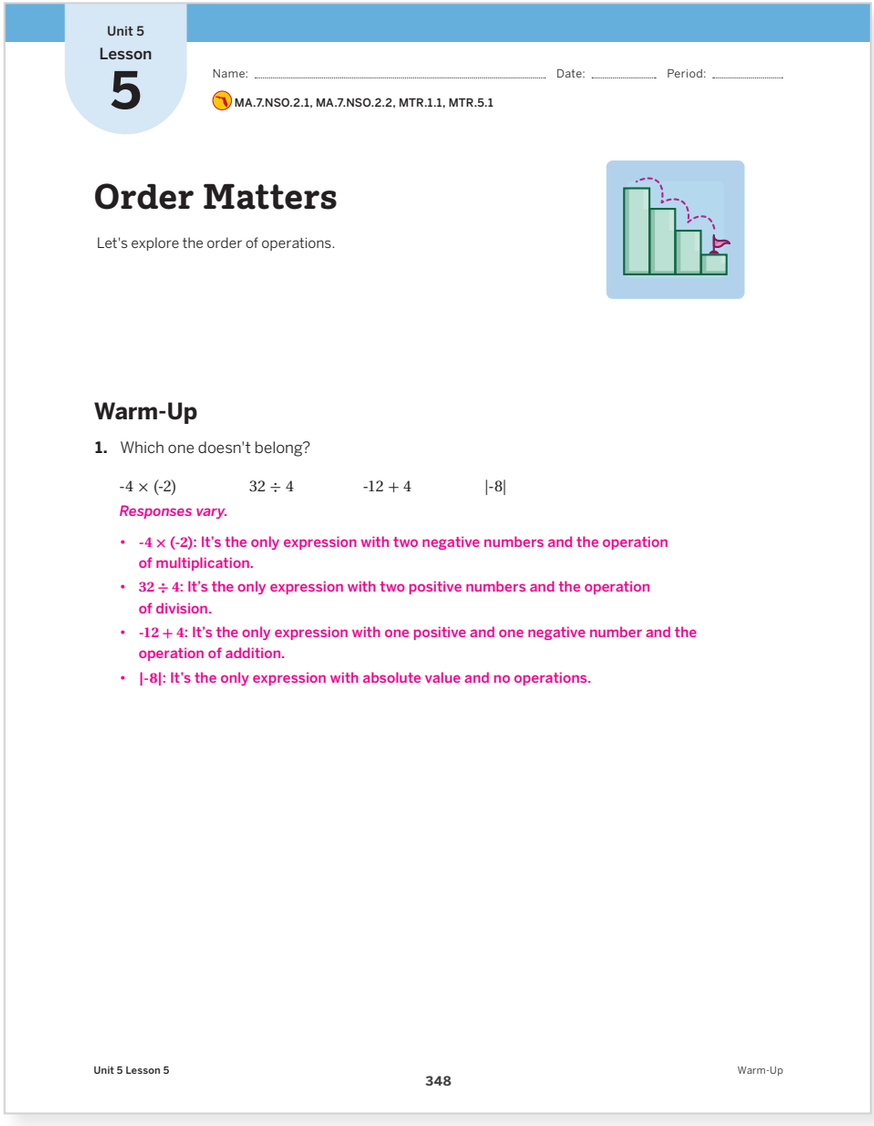
Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

A **Accessibility: Conceptual Processing** Encourage students who struggle with determining which one doesn't belong to think about what could be different about the one that doesn't belong. Ask students to discuss what they notice about each given expression.

2 Connect

Invite students to share at least one reason why each expression doesn't belong. Consider asking, "What were you thinking while you were determining which expression doesn't belong?"

ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____ is the only one that has/doesn't have _____."). **(Writing and Speaking)**



Unit 5
Lesson
5

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.1, MA.7.NSO.2.2, MTR.1.1, MTR.5.1

Order Matters

Let's explore the order of operations.

Warm-Up

1. Which one doesn't belong?

$-4 \times (-2)$ $32 \div 4$ $-12 + 4$ $|-8|$

Responses vary.

- $-4 \times (-2)$: It's the only expression with two negative numbers and the operation of multiplication.
- $32 \div 4$: It's the only expression with two positive numbers and the operation of division.
- $-12 + 4$: It's the only expression with one positive and one negative number and the operation of addition.
- $|-8|$: It's the only expression with absolute value and no operations.

Unit 5 Lesson 5 348 Warm-Up

Pairs | 10 min

Activity 1 Evaluating Numerical Expressions

Purpose: Students use *order of operations* to evaluate numerical expressions.

1 Launch

Direct students' attention to the numerical expression. Invite students to discuss how this numerical expression is similar to or different from the expressions in the Warm-up.

Consider asking:

- "Do you think you will evaluate this expression the same as your classmates?"
- "How can you come up with different possible answers?"
- "Is there only one correct answer?"

Pause briefly to have students share their descriptions.

2 Monitor

Encourage students to share their descriptions with a partner and work together to reach an agreement about the *order of operations* used to get the value of 29.

D Differentiation

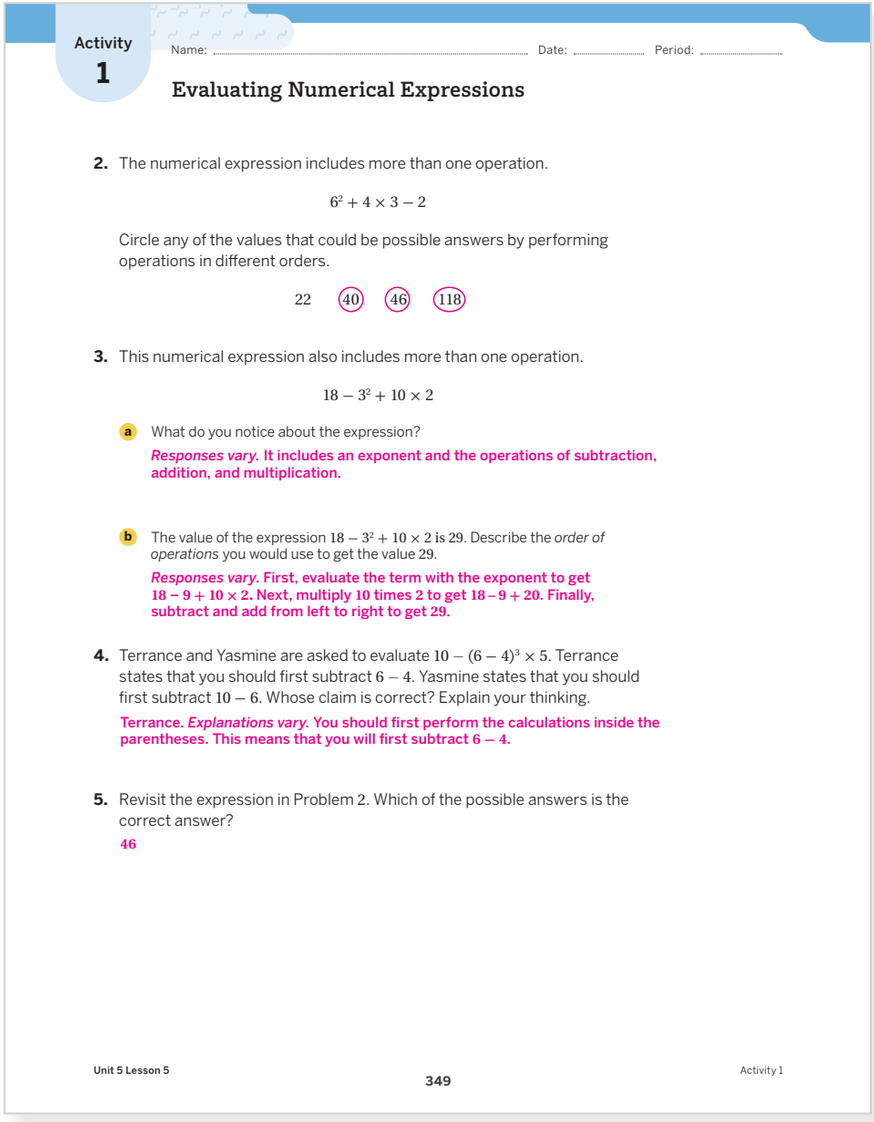
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to state aloud which operation should be completed first. Consider asking, "Can you just perform the operations from left to right as they appear? Why or why not?"
Add $-9 + 10$.	**Support** Consider asking, "Can you perform addition before multiplication in the order of operations?"
Show two different ways to calculate 29.	**Stretch** Challenge students to write a statement that says that addition and subtraction are on the same level.

3 Connect

Create an anchor chart that explains what operations to perform first when they see other numerical expressions in this form. Record it for students to refer to later in the unit.

Collect and share students' answers. Consider creating a bar graph to show the students' answers using the possible answers they got for Problem 2 as the values along the horizontal axis.

Invite students to share how they know their steps and value are correct.



Activity 1

Name: Date: Period:

Evaluating Numerical Expressions

2. The numerical expression includes more than one operation.

$$6^2 + 4 \times 3 - 2$$

Circle any of the values that could be possible answers by performing operations in different orders.

22 (40) (46) (118)

3. This numerical expression also includes more than one operation.

$$18 - 3^2 + 10 \times 2$$

a. What do you notice about the expression?

Responses vary. It includes an exponent and the operations of subtraction, addition, and multiplication.

b. The value of the expression $18 - 3^2 + 10 \times 2$ is 29. Describe the *order of operations* you would use to get the value 29.

Responses vary. First, evaluate the term with the exponent to get $18 - 9 + 10 \times 2$. Next, multiply 10 times 2 to get $18 - 9 + 20$. Finally, subtract and add from left to right to get 29.

4. Terrance and Yasmine are asked to evaluate $10 - (6 - 4)^3 \times 5$. Terrance states that you should first subtract $6 - 4$. Yasmine states that you should first subtract $10 - 6$. Whose claim is correct? Explain your thinking.

Terrance. *Explanations vary.* You should first perform the calculations inside the parentheses. This means that you will first subtract $6 - 4$.

5. Revisit the expression in Problem 2. Which of the possible answers is the correct answer?

46

Unit 5 Lesson 5 349 Activity 1

Small Groups | 10 min

Activity 2 More Numerical Expressions

Purpose: Students apply what they have learned to evaluate more numerical expressions. **(MTR5.1)**

1 Launch

Distribute the Activity 2 Sheet, one per group.

Direct students' attention to the numerical expressions on the cards. Ask students to describe what they notice about these expressions. **(MTR.5.1)**

Consider asking:

- "What operations are involved in these expressions?"
- "Which expressions require you to work with a grouping symbol inside a grouping symbol?"
- "At what point should you evaluate *absolute value* in an expression?"

Pause briefly to have students share their thoughts.

Demonstrate how to determine the next card. Consider sharing that the values for the expressions can be found at the top of a card in the set. If students do not see their answer at the top of any card, they should check their work.

2 Monitor

Listen for students' strategies for evaluating operations, particularly when there are operations on the same level when performing the order of operations.

Consider asking:

- "In this expression, is there a place where you have a choice in which operation you do next?"
- "What are some examples of operations that are on the same level?"

Encourage students to select a numerical expression to discuss with the class and share the steps they used to evaluate it.

Collect and share students' discussions.

ELL **English Language Learners Supports** Invite students to restate another group's steps for evaluating to help solidify understanding and use appropriate mathematical language and terms.

Activity 2 continued >

Activity 2

Name: ..... Date: ..... Period: .....

More Numerical Expressions

6. You will use a set of cards to evaluate more numerical expressions using the correct order of operations. Work with a partner. Start at any of the cards.

- Record the card number, evaluate the expression, and write your answer.
- Look for your answer at the top of another card. Evaluate that expression.
- Repeat until you make it back to your starting card.

The card students start with varies.

Card: 1

Steps for Evaluating:

$|-8|\left(\frac{1}{4}\times 8\right)\times 3+1$

$8\left(\frac{1}{4}\times 8\right)\times 3+1$

$8(2)\times 3+1$

$16\times 3+1$

$48+1$

Value: 49 →

Card: 5

Steps for Evaluating:

$20\div(3+|2|)-1\times 4$

$20\div(3+2)-1\times 4$

$20\div 5-1\times 4$

$4-4$

↙ **Value: 0**

Card: 3

Steps for Evaluating:

$-6+\left(-\frac{1}{4}\times(-48)\right)^2-|-7|$

$-6+(12)^2-|-7|$

$-6+(12)^2-7$

$-6+144-7$

$138-7$

Value: 131 →

Card: 2

Steps for Evaluating:

$(13-|2|)+2\div 0.4$

$(13-2)+2\div 0.4$

$11+2\div 0.4$

$11+5$

Value: 16 ↗

continued on next page...

Unit 5 Lesson 5 350 Activity 2

Activity 2 More Numerical Expressions (continued)

Purpose: Students apply what they have learned to evaluate more numerical expressions. **(MTR5.1)**

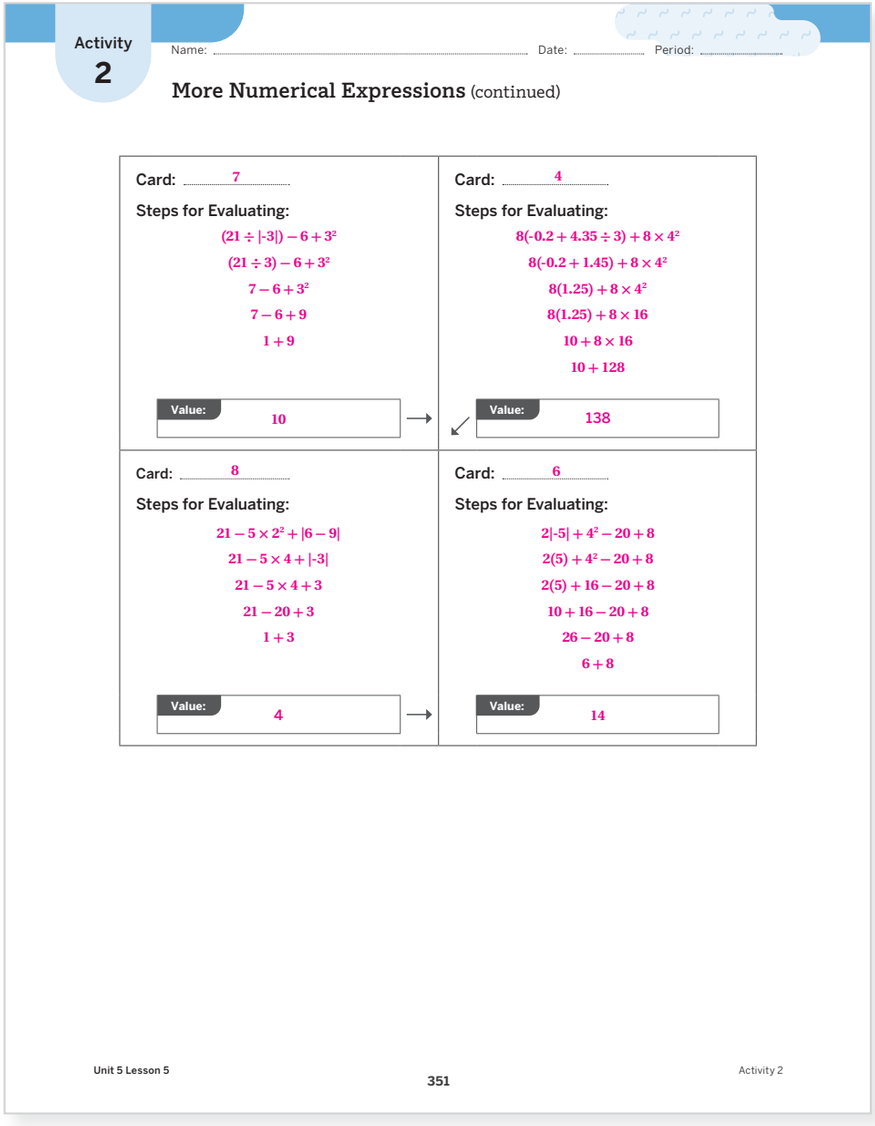
2 Monitor

Note: Refer to the guidance on the previous page to support students as they work through the remainder of the cards.

3 Connect

Invite students to share any difficulties they encountered when working through the activity. Encourage them to ask questions about anything that is still not clear. **(MTR.1.1)**

To surface the Key Takeaway, consider asking, "From what you have seen, how should you approach any numerical expression when working to determine its value? What is an example of a mistake you could make?"



Activity 2

Name: Date: Period:

More Numerical Expressions (continued)

Card: 7

Steps for Evaluating:

$(21 \div |-3|) - 6 + 3^2$

$(21 \div 3) - 6 + 3^2$

$7 - 6 + 3^2$

$7 - 6 + 9$

$1 + 9$

Value: 10

Card: 4

Steps for Evaluating:

$8(-0.2 + 4.35 \div 3) + 8 \times 4^2$

$8(-0.2 + 1.45) + 8 \times 4^2$

$8(1.25) + 8 \times 4^2$

$8(1.25) + 8 \times 16$

$10 + 8 \times 16$

$10 + 128$

Value: 138

Card: 8

Steps for Evaluating:

$21 - 5 \times 2^2 + |6 - 9|$

$21 - 5 \times 4 + |-3|$

$21 - 5 \times 4 + 3$

$21 - 20 + 3$

$1 + 3$

Value: 4

Card: 6

Steps for Evaluating:

$2|-5| + 4^2 - 20 + 8$

$2(5) + 4^2 - 20 + 8$

$2(5) + 16 - 20 + 8$

$10 + 16 - 20 + 8$

$26 - 20 + 8$

$6 + 8$

Value: 14

Unit 5 Lesson 5 351 Activity 2

Key Takeaway: The *order of operations* must be followed when evaluating a numerical expression to get the correct value.

Pairs | 10 min

Activity 3 Higher-Level Problems

Purpose: Students analyze student work for potential errors and place grouping symbols in an expression to create different values. **(MTR5.1)**

1 Launch

To support students getting started, encourage them to say the numerical expression in words. Then ask them to identify what they first notice about the student's work.

2 Monitor

Pause after most students have finished Problem 7 to discuss the errors in Sahana's work.

Consider asking:

- "What should you be thinking when deciding on where to place the grouping symbols?"
- "How can you check your work?"

3 Connect

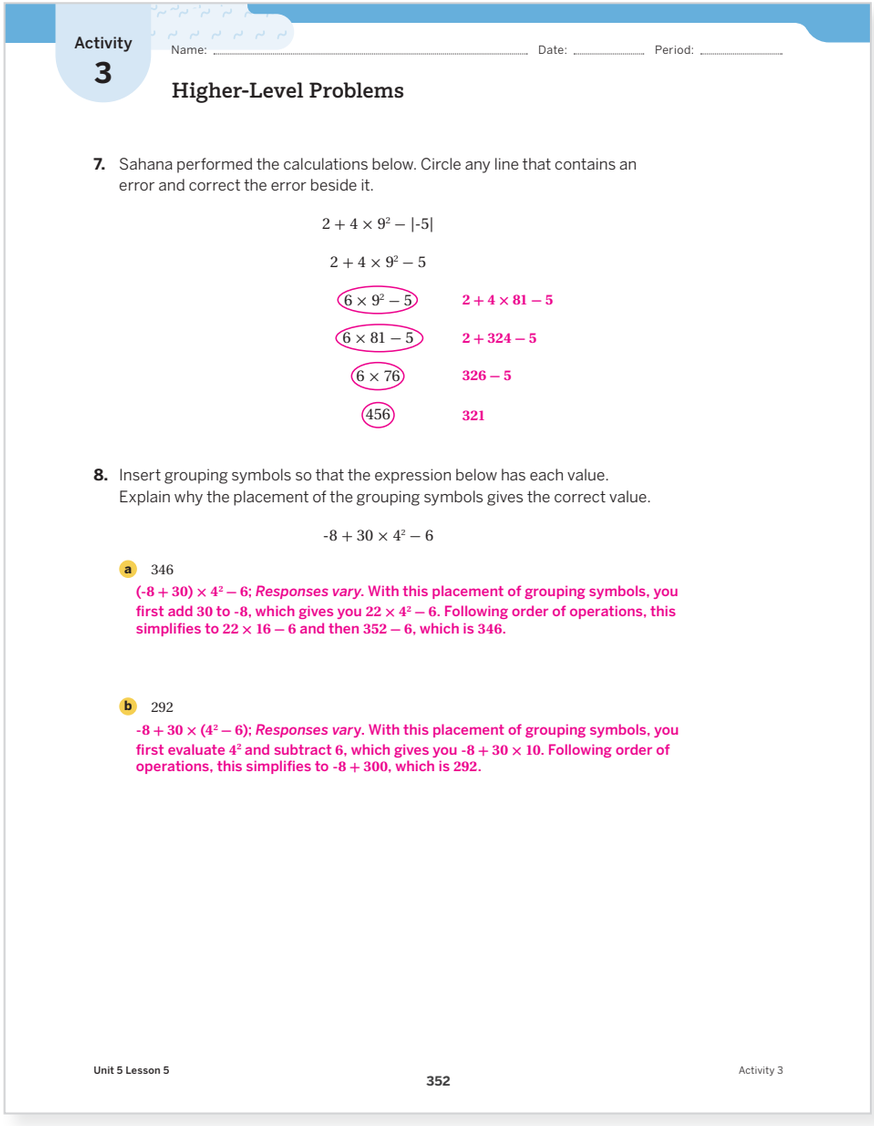
Invite students to use structure when providing the explanation for why the numerical expression has the correct value. **(MTR.5.1)**

Consider asking:

- "What was difficult about completing this problem?"
- "What strategy can be used to determine where to include grouping symbols in an expression to show a certain value?"
- "Prior to working through Problem 8, did you think about the importance of grouping symbols and how they impact the value of a numerical expression?" **(MTR.1.1)**

Invite students to share their strategies and ideas.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language when explaining why their expressions have the given values. Invite students to meet with other groups, give feedback on each other's explanations, and then use the feedback to write a second clearer explanation. See the Routine Facilitation Guide for more information.



Activity 3

Name: Date: Period:

Higher-Level Problems

7. Sahana performed the calculations below. Circle any line that contains an error and correct the error beside it.

$2 + 4 \times 9^2 - |-5|$

$2 + 4 \times 9^2 - 5$

$6 \times 9^2 - 5$ $\quad 2 + 4 \times 81 - 5$

$6 \times 81 - 5$ $\quad 2 + 324 - 5$

6×76 $\quad 326 - 5$

456 $\quad 321$

8. Insert grouping symbols so that the expression below has each value. Explain why the placement of the grouping symbols gives the correct value.

$-8 + 30 \times 4^2 - 6$

a. 346

$(-8 + 30) \times 4^2 - 6$; *Responses vary.* With this placement of grouping symbols, you first add 30 to -8, which gives you $22 \times 4^2 - 6$. Following order of operations, this simplifies to $22 \times 16 - 6$ and then $352 - 6$, which is 346.

b. 292

$-8 + 30 \times (4^2 - 6)$; *Responses vary.* With this placement of grouping symbols, you first evaluate 4^2 and subtract 6, which gives you $-8 + 30 \times 10$. Following order of operations, this simplifies to $-8 + 300$, which is 292.

Unit 5 Lesson 5 352 Activity 3

Whole Class | 5 min

Synthesis

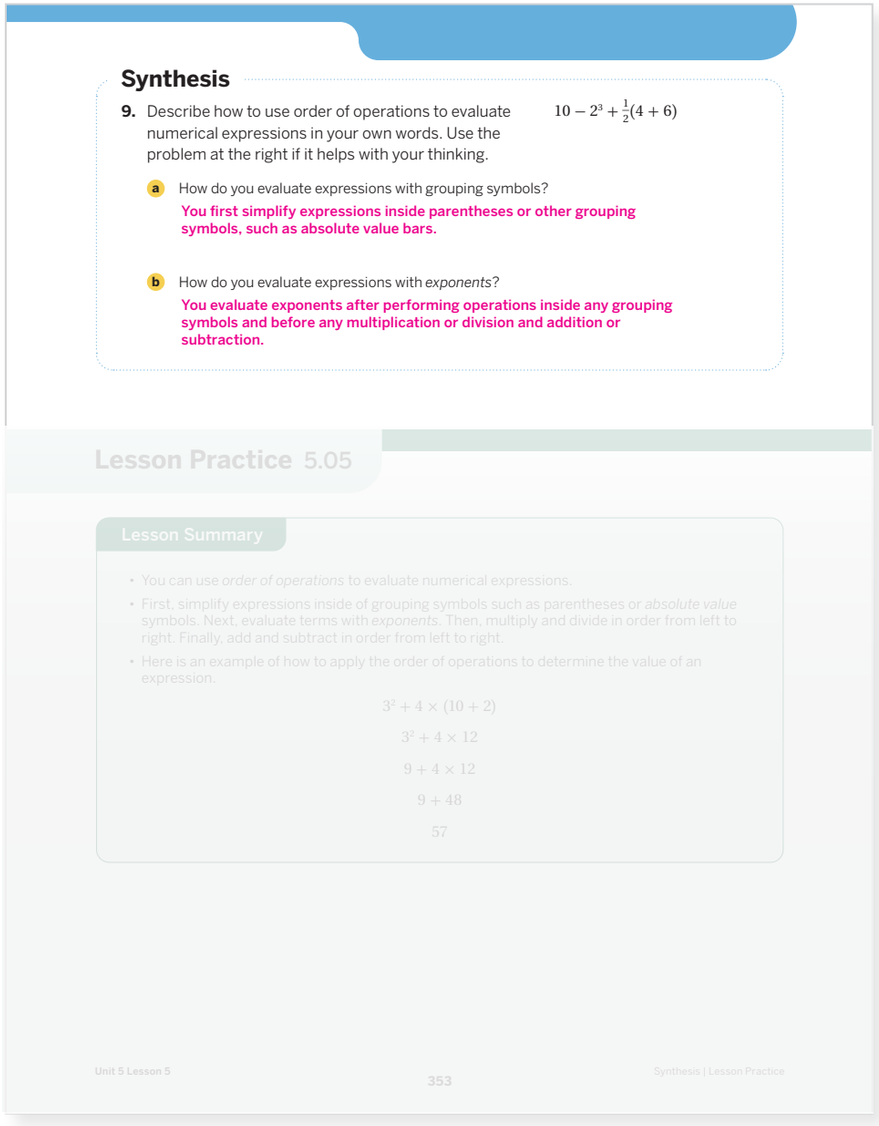
Purpose: Students synthesize their understanding about using order of operations to evaluate numerical expressions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share students' ideas about evaluating numerical expressions using the order of operations.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about using the order of operations to evaluate numerical expressions. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to discuss examples of numerical expressions that will require even more care in order to evaluate.



Synthesis

9. Describe how to use order of operations to evaluate numerical expressions in your own words. Use the problem at the right if it helps with your thinking. $10 - 2^3 + \frac{1}{2}(4 + 6)$

a. How do you evaluate expressions with grouping symbols?

You first simplify expressions inside parentheses or other grouping symbols, such as absolute value bars.

b. How do you evaluate expressions with *exponents*?

You evaluate exponents after performing operations inside any grouping symbols and before any multiplication or division and addition or subtraction.

Lesson Practice 5.05

Lesson Summary

- You can use *order of operations* to evaluate numerical expressions.
- First, simplify expressions inside of grouping symbols such as parentheses or *absolute value* symbols. Next, evaluate terms with *exponents*. Then, multiply and divide in order from left to right. Finally, add and subtract in order from left to right.
- Here is an example of how to apply the order of operations to determine the value of an expression.

$3^2 + 4 \times (10 + 2)$

$3^2 + 4 \times 12$

$9 + 4 \times 12$

$9 + 48$

57

Unit 5 Lesson 5 353 Synthesis | Lesson Practice

Lesson Takeaway: When using the *order of operations*, first simplify expressions inside grouping symbols, next evaluate *exponents*, then perform multiplication and division in order from left to right, and finally perform addition and subtraction from left to right.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by critiquing the process of another student when evaluating a numerical expression and explaining their thinking.

Today's Goals

1. **Goal:** Use order of operations to evaluate numerical expressions.
2. **Language Goal:** Describe how order of operations is used to evaluate a numerical expression. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.05

Seth was asked to evaluate $(6 - 9)^3 + 3 \div 2$. His work is shown below. Do you agree with his work? Explain your thinking.

$$(-3)^3 + 3 \div 2$$
$$-27 + 3 \div 2$$
$$-24 \div 2$$
$$-12$$

Responses vary. No, Seth should have divided 3 by 2 and added the quotient to -27 to get a value of -25.5.

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to evaluate numerical expressions:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

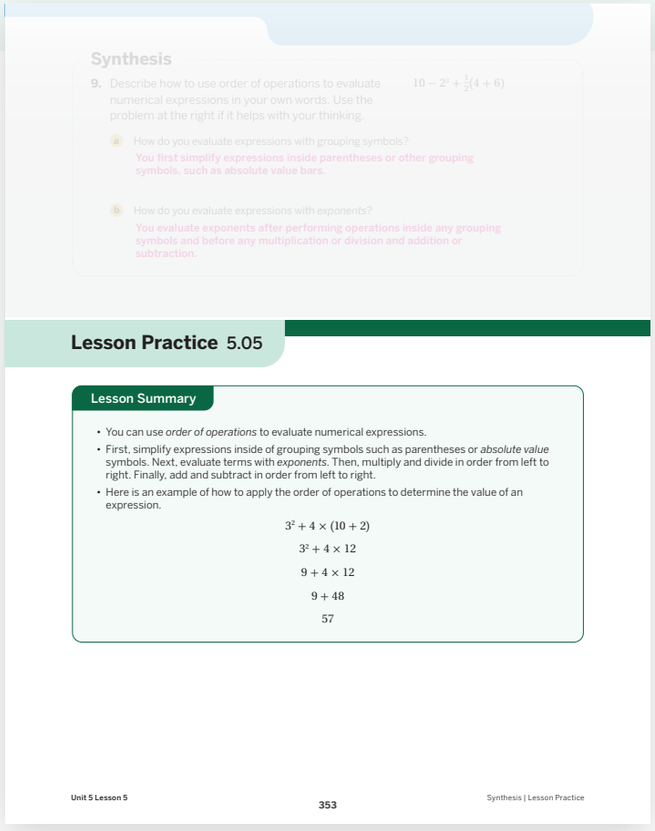
Professional Learning

Take a moment to reflect on this lesson. How will students' work today prepare them for writing equivalent expressions with exponents in Lessons 6–8?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

9. Describe how to use order of operations to evaluate $10 - 2^3 + \frac{1}{2}(4 + 6)$ numerical expressions in your own words. Use the problem at the right if it helps with your thinking.

a. How do you evaluate expressions with grouping symbols?

You first simplify expressions inside parentheses or other grouping symbols, such as absolute value bars.

b. How do you evaluate expressions with *exponents*?

You evaluate exponents after performing operations inside any grouping symbols and before any multiplication or division and addition or subtraction.

Lesson Practice 5.05

Lesson Summary

- You can use *order of operations* to evaluate numerical expressions.
- First, simplify expressions inside of grouping symbols such as parentheses or *absolute value* symbols. Next, evaluate terms with *exponents*. Then, multiply and divide in order from left to right. Finally, add and subtract in order from left to right.
- Here is an example of how to apply the order of operations to determine the value of an expression.

$3^2 + 4 \times (10 + 2)$

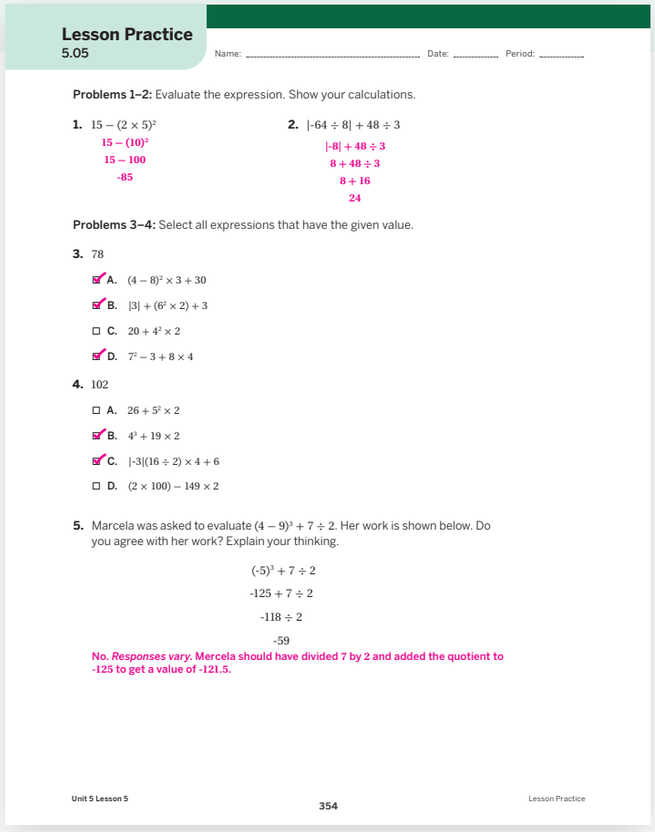
$3^2 + 4 \times 12$

$9 + 4 \times 12$

$9 + 48$

57

Unit 5 Lesson 5 353 Synthesis | Lesson Practice



Lesson Practice 5.05

Name: Date: Period:

Problems 1–2: Evaluate the expression. Show your calculations.

1. $15 - (2 \times 5)^2$

$15 - (10)^2$

$15 - 100$

-85

2. $|-64 \div 8| + 48 \div 3$

$|-8| + 48 \div 3$

$8 + 48 \div 3$

$8 + 16$

24

Problems 3–4: Select all expressions that have the given value.

3. 78

- [x] A. $(4 - 8)^2 \times 3 + 30$
- [x] B. $|3| + (6^2 \times 2) + 3$
- [ ] C. $20 + 4^2 \times 2$
- [x] D. $7^2 - 3 + 8 \times 4$

4. 102

- [ ] A. $26 + 5^2 \times 2$
- [x] B. $4^3 + 19 \times 2$
- [x] C. $|-3|(16 \div 2) \times 4 + 6$
- [ ] D. $(2 \times 100) - 149 \times 2$

5. Marcela was asked to evaluate $(4 - 9)^3 + 7 \div 2$. Her work is shown below. Do you agree with her work? Explain your thinking.

$(-5)^3 + 7 \div 2$

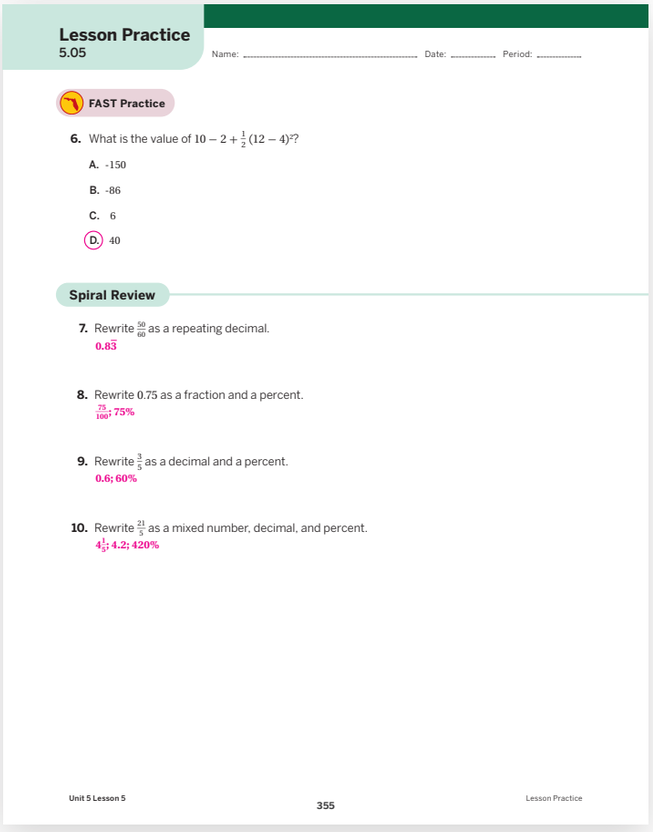
$-125 + 7 \div 2$

$-118 \div 2$

-59

No. *Responses vary.* Mercela should have divided 7 by 2 and added the quotient to -125 to get a value of -121.5.

Unit 5 Lesson 5 354 Lesson Practice



Lesson Practice 5.05

Name: Date: Period:

FAST Practice

6. What is the value of $10 - 2 + \frac{1}{2}(12 - 4)^2$?

A. -150

B. -86

C. 6

D. 40

Spiral Review

7. Rewrite $\frac{50}{60}$ as a repeating decimal.

$0.8\overline{3}$

8. Rewrite 0.75 as a fraction and a percent.

$\frac{75}{100}$; 75%

9. Rewrite $\frac{3}{5}$ as a decimal and a percent.

0.6; 60%

10. Rewrite $\frac{21}{5}$ as a mixed number, decimal, and percent.

$4\frac{1}{5}$; 4.2; 420%

Unit 5 Lesson 5 355 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.NSO.2.1
	5	3	MA.7.NSO.2.1
FAST Practice	6	2	MA.7.NSO.2.1
Spiral Review			
	7–10	1	MA.7.NSO.1.2

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1, 2	Comparing expressions containing the same positive and negative numbers	Lesson 1	2	MA.7.NSO.1.2, MTR.2.1
3a, 3b, 3c	Using the structure of order of operations to evaluate numerical expressions	Lesson 5	2	MA.7.NSO.2.1, MTR.2.1
4a, 4b, 4c	Comparing expressions containing the same positive and negative numbers	Lesson 1	1	MA.7.NSO.1.2
5	Using the structure of exponential expressions to justify why given exponential expressions are equivalent	Lesson 5	3	MA.7.NSO.1.2, MTR.4.1

Assessment and Lesson Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 5

1. Brandon must break a code to find his present. Where should Brandon look?

Look under the $-\frac{19}{8}$ $\frac{17}{9}$ $2.1\overline{6}$ $1\frac{4}{15}$

A	B	C	D	E
$126.\overline{6}\%$	$1\frac{2}{9}$	$-0.\overline{5}$	$\frac{11}{5}$	$-116.\overline{6}\%$

F	O	R	S	T
$\frac{13}{6}$	$1.\overline{8}$	1.08	-2.375	$1\frac{5}{9}$

A. BEDS
B. DESK
C. SEAT
D. SOFA

2. Myrta is taking a multiple choice test. Her calculator shows that $\frac{11}{12}$ is equal to 0.91666667, but that isn't an answer choice on the test. What happened?

A. Myrta's calculator rounded the decimal instead of using bar notation.
B. Myrta must have entered the wrong information into the calculator.
C. Myrta's calculator must not be able to display decimals correctly.
D. There is an error on the test since 0.91666667 is not an answer choice.

3. Evaluate each expression. Show your calculations.

a. $(-3)^2 + 4 \times \left(\frac{10}{2} - 3\right) - |-7|$

$(-3)(-3) + 4 \times (5 - 3)$ -7
$9 + 4 \times (2)$ -7
$9 + 8 - 7$
$17 - 7$
10

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 5

b. $\frac{12}{4} + 4^2 - |5 - 9| \times 2$

$3 + (4 \cdot 4) - |-4| \times 2$
$3 + 16 - 4 \times 2$
$3 + 16 - 8$
$19 - 8$
11

c. $4(-0.12 + 9.36 \div 3) + \frac{16}{2} - 5^2$

$4(-0.12 + 3.12) + 8 - (5 \cdot 5)$
$4(3) + 8 - 25$
$12 + 8 - 25$
$20 - 25$
-5

4. Circle all of the equivalent values.

A. Equal to $\frac{5}{3}$	1.6	$1.\overline{6}$	$1\frac{2}{5}$	$1\frac{2}{3}$	$166.\overline{6}\%$
B. Equal to $1.8\underline{3}$	$1\frac{5}{6}$	1.83	$\frac{11}{6}$	$1\frac{83}{100}$	1.83%
C. Equal to $155.\underline{5}\%$	$\frac{14}{9}$	$1\frac{5}{9}$	$1.\overline{5}$	1.5555	$\frac{155}{100}$

5. Cecelia was asked to evaluate $(3 - 7)^2 + |\frac{12}{6} - 8| \div 2$. Her work is shown below. Do you agree with her work? Explain your thinking.

$(-4)^2 + |2 - 8| \div 2$
$-16 - 6 \div 2$
$-16 - 3$
-19

Responses vary.
I do not agree with Cecelia. She forgot to square the negative in $(-4)^2$ and forgot to take the absolute value of -6. The final answer should have been 19, not -19.

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: Students perform all four operations with positive and negative numbers using a variety of strategies. Students evaluate numerical expressions. **(Lessons 1–5)**	1, 2, 4a, 4b, 4c	**Support** • **Mini-Lesson:** *Dividing Integers* • **Teacher Move:** Consider revisiting Lesson 1 (*Code Breaking*)
		Strengthen • **Writing a Code Challenge:** • Lesson 1 (*Code Breaking*)
	3a, 3b, 3c, 5	**Support** • **Teacher Move:** Consider revisiting: • Lesson 5 (*Order Matters*)

		Strengthen
		• **Repeated Challenges:**
		  • Lesson 2 (*Bumpers*)
		  • Lesson 3 (*Back in Time*)
		  • Lesson 4 (*Speeding Turtles*)

		Stretch • **You're invited to explore more:** • Lesson 3 (*Back in Time*)

Sub-Unit 2

Laws of Exponents

Sub-Unit 2 Goals

- **Goal:** Apply the Laws of Exponents to generate equivalent expressions involving whole-number exponents.
- **Language Goal:** Explain and interpret expressions involving exponents and their properties.

In this sub-unit, students identify and create equivalent expressions involving whole-number exponents and rational number bases. In Grade 8, students will investigate properties of negative exponents.



Math that Matters Most

Concept or Skill	For example . . .
Identify and write equivalent expressions.	Two of these expressions are equivalent. $5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ $\quad 20^3 \quad (5 \cdot 3)(4 \cdot 3)$ a. Which expression is not equivalent to the others? b. How could you change this expression so that it has the same value as the others? $(5 \cdot 3)(4 \cdot 3)$ is not equivalent to the others. If I change the expression to $(5^3)(4^3)$, the expressions would all have the same value.
Rewrite exponential expressions as single powers.	How could you rewrite $\frac{4^9}{4^2 \cdot 4^4}$ as a single power? I can expand the expression so there are nine 4s multiplied together in the numerator and six 4s multiplied together in the denominator. $\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$ After dividing, I get $4 \cdot 4 \cdot 4$, which is the same as 4^3.
Determine if expressions involving negative bases are equivalent.	Determine whether the expressions are equivalent. $(-2)^3 \cdot (-2)^4$ and $-2^3 \cdot -2^4$ I can use the product of powers to evaluate $(-2)^3 \cdot (-2)^4$. $(-2)^3 \cdot (-2)^4 = (-2)^{(3+4)}$ $= (-2)^7$ $= -128$ I can use order of operations to evaluate $-2^3 \cdot -2^4$. $-2^3 \cdot -2^4 = -8 \cdot -16$ $= 128$ The expressions are not equivalent.

Unit 5

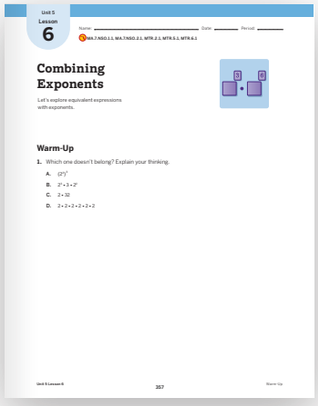
Lesson

6

Combining Exponents

Equivalent Expressions with Exponents

Let's explore equivalent expressions with exponents.


Unit 5
Lesson
6
Name _____ Date _____ Period _____
MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1
Combining Exponents
Let's explore equivalent expressions with exponents.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
A. $(2^3)^2$
B. $2^3 \cdot 2^2$
C. $2 \cdot 32$
D. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Unit 5 Lesson 6
357
Warm-Up

Learning Outcomes

Today's Goal

1. **Goal:** Identify and write equivalent expressions, such as $6^3 \cdot 6^5$, $(6^4)^2$, and $2^8 \cdot 3^8$.

Students use patterns and structure to discover ways to write equivalent expressions involving products of powers and powers of powers. At this time, they only work with positive exponents. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 5, students reviewed order of operations and how it can be used to solve multi-step problems.

Future Learning

In Lesson 7, students will further explore the Laws of Exponents by examining the Power of a Power Law.

Connections

- Students explore more compact and efficient way to express mathematical relationships through equivalent expressions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.1
Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.3	MA.8.NSO.1.3
MA.6.NSO.3.4	MA.8.AR.1.1

Vocabulary

New Vocabulary

power of ten

Review Vocabulary

base (of a power)

Lesson at a Glance

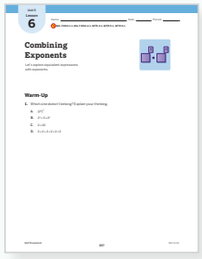
~ 45 min

MA.7.NSO.1.1, MA.7.NSO.2.1

Warm-Up

Whole Class | 5 min

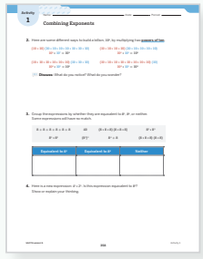
Students use the **Which One Doesn't Belong?** routine to compare equivalent expressions written in different forms.



Activity 1

Pairs | 15 min

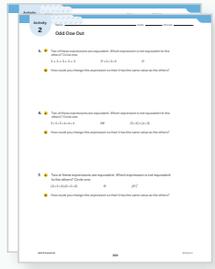
Students consider different ways to represent expressions with exponents to identify when two expressions are equivalent. **Routine: Notice and Wonder** **(MTR.2.1)**



Activity 2

Pairs | 15 min

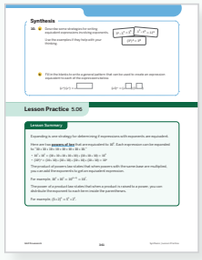
Students make use of structure to compare expressions with exponents and write equivalent expressions involving products of powers and powers of powers. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

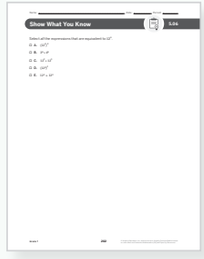
Students synthesize their understanding of strategies for writing equivalent expressions with exponents.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting all expressions that are equivalent to a given expression with exponents.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong routine to compare equivalent expressions written in different forms.

1 Launch

Use the Which One Doesn't Belong? routine to support students in looking for structure in expressions with exponents and justifying their thinking precisely. **(MTR.5.1)**

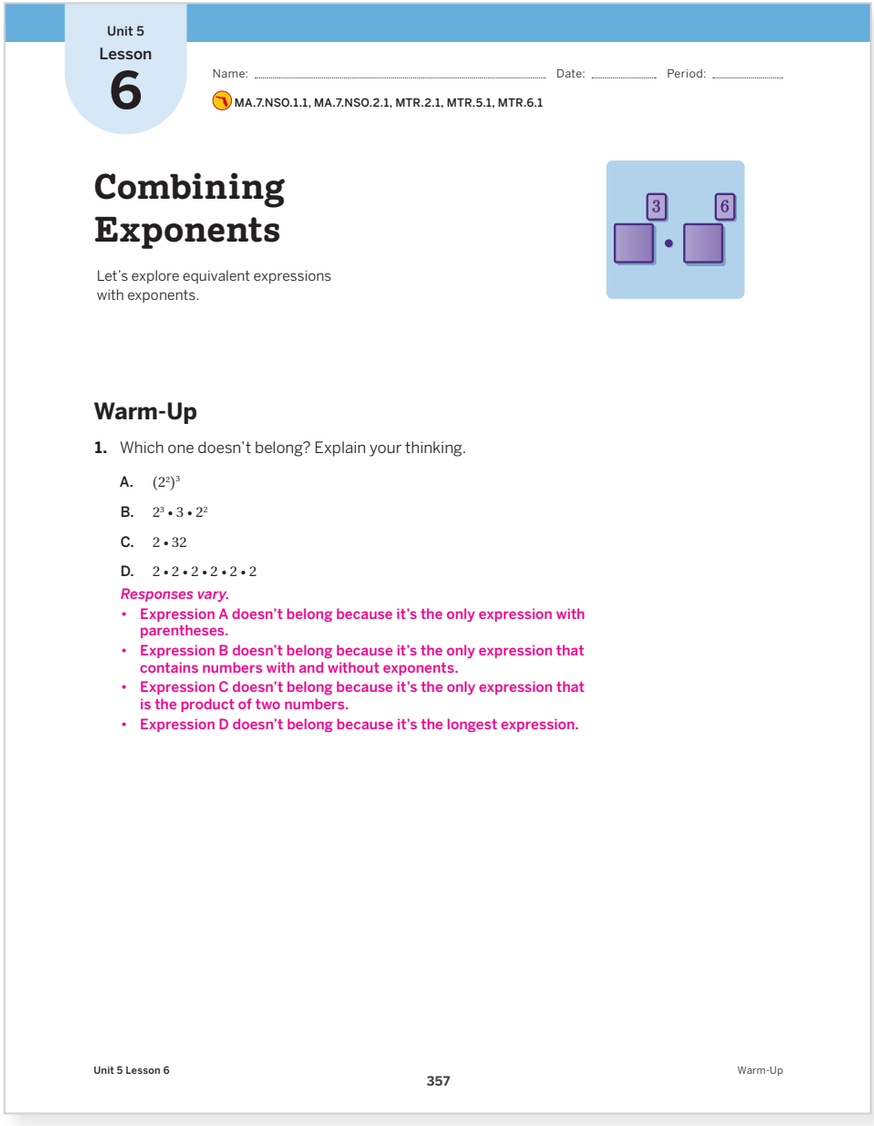
Encourage students to look for more than one possibility.

2 Connect

Discuss why *each* representation doesn't belong, asking students to justify their choice.

Consider asking, "What do you notice about the value of each expression? What does this tell you?"

Note: If it doesn't come up naturally, consider reviewing the terms *power*, *exponent*, and *base* with students.



Unit 5
Lesson
6

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1

Combining Exponents

Let's explore equivalent expressions with exponents.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. $(2^2)^3$

B. $2^3 \cdot 3 \cdot 2^2$

C. $2 \cdot 32$

D. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Responses vary.

- Expression A doesn't belong because it's the only expression with parentheses.
- Expression B doesn't belong because it's the only expression that contains numbers with and without exponents.
- Expression C doesn't belong because it's the only expression that is the product of two numbers.
- Expression D doesn't belong because it's the longest expression.

Unit 5 Lesson 6 357 Warm-Up

Pairs | 15 min

Activity 1 Combining Exponents

Purpose: Students consider different ways to represent expressions with exponents to identify when two expressions are equivalent. **(MTR.2.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the product of powers. See the Routine Facilitation Guide for more information. **(MTR.2.1)**

Note: If it doesn't come up naturally, consider asking, "What do you notice about the exponents for the **powers of ten**?" The vocabulary term *power of ten* will be important for future lessons on scientific notation in Grade 8.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the expressions.

A Accessibility: Executive Functioning Invite students to focus on one card at a time to support students in carrying out multiple steps.

D Differentiation

Look for students who:	Respond to Student Thinking
Match: • $8+8+8+8+8+8$ to 8^6 • 8^4+8 to 8^5 • 40 to 8^5	**Support** Invite them to check the value of each expression with a calculator. Consider asking, "Why don't these expressions have the same value?"
Match: • $(8^3)^2$ to 8^5 • $8^3 \cdot 8^2$ to 8^6	**Support** Encourage them to expand the expressions using the structure of exponents.

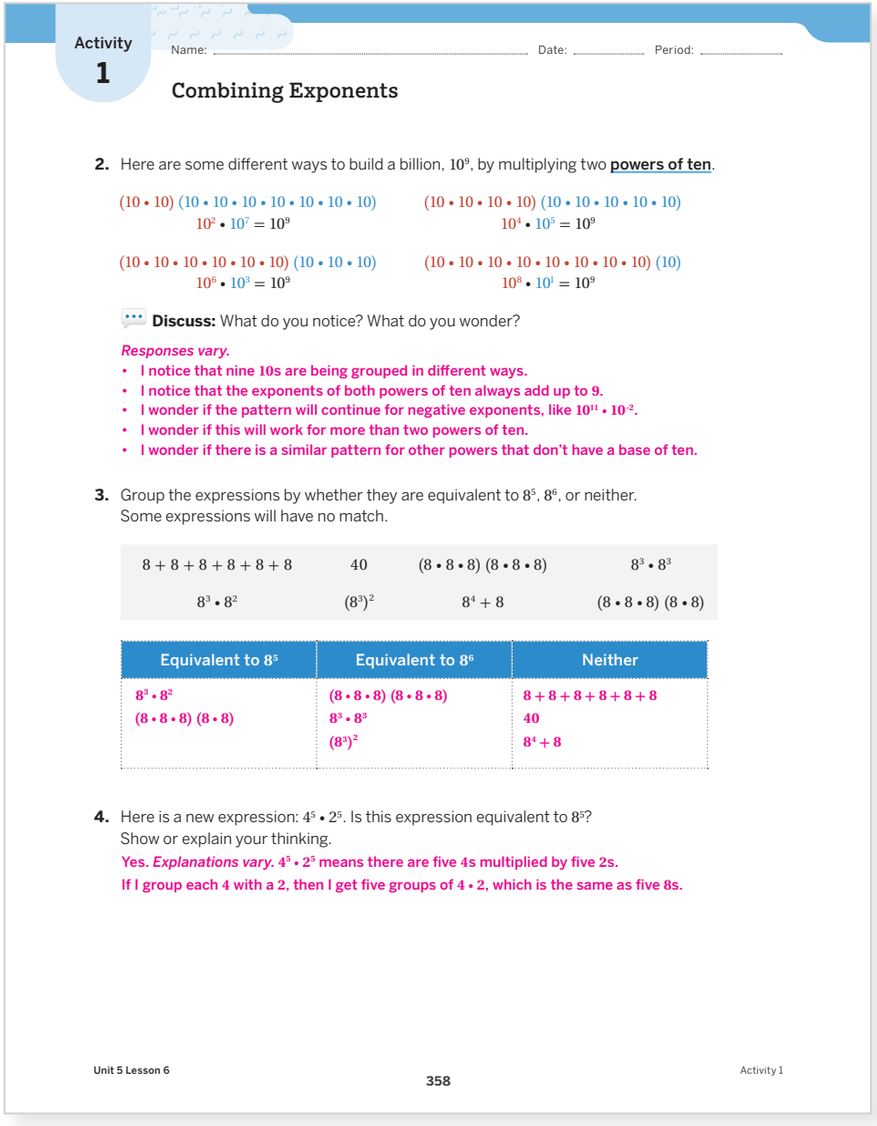
Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

Look for students who use the structure of exponents to explain why $4^5 \cdot 2^5$ can be matched with 8^5. Invite these students to share during the Connect. **(MTR.5.1)**

3 Connect

Collect and display a variety of student responses. Invite pre-selected students to share their reasoning.

To surface the Key Takeaway, consider asking, "How would you explain that these expressions are equivalent?"



Activity 1

Name: Date: Period:

Combining Exponents

2. Here are some different ways to build a billion, 10^9, by multiplying two **powers of ten**.

$(10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$
$10^2 \cdot 10^7 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$
$10^4 \cdot 10^5 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10)$
$10^6 \cdot 10^3 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10)$
$10^8 \cdot 10^1 = 10^9$

Discuss: What do you notice? What do you wonder?

Responses vary.
- I notice that nine 10s are being grouped in different ways.
- I notice that the exponents of both powers of ten always add up to 9.
- I wonder if the pattern will continue for negative exponents, like $10^{11} \cdot 10^{-2}$.
- I wonder if this will work for more than two powers of ten.
- I wonder if there is a similar pattern for other powers that don't have a base of ten.

3. Group the expressions by whether they are equivalent to 8^5, 8^6, or neither. Some expressions will have no match.

$8+8+8+8+8+8$	40	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8 \cdot 8)$	$8^3 \cdot 8^3$
$8^3 \cdot 8^2$	$(8^3)^2$	8^4+8	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8)$

Equivalent to 8^5	Equivalent to 8^6	Neither
$8^3 \cdot 8^2$ $(8 \cdot 8 \cdot 8)\ (8 \cdot 8)$	$(8 \cdot 8 \cdot 8)\ (8 \cdot 8 \cdot 8)$ $8^3 \cdot 8^3$ $(8^3)^2$	$8+8+8+8+8+8$ 40 8^4+8

4. Here is a new expression: $4^5 \cdot 2^5$. Is this expression equivalent to 8^5? Show or explain your thinking.

Yes. *Explanations vary.* $4^5 \cdot 2^5$ means there are five 4s multiplied by five 2s.
If I group each 4 with a 2, then I get five groups of $4 \cdot 2$, which is the same as five 8s.

Unit 5 Lesson 6 358 Activity 1

Key Takeaway: Look for structure in expressions with exponents to determine whether two expressions are equivalent.

Pairs | 15 min

Activity 2 Odd One Out

Purpose: Students make use of structure to compare expressions with exponents and write equivalent expressions involving products of powers and powers of powers. **(MTR.5.1)**

Short on time? Invite students to focus on writing two equivalent expressions for Problem 8.

1 Launch

Consider asking, "What does it mean for two expressions to be equivalent?"

Encourage students to discuss their strategy with a partner for the next three problems and to show their thinking on paper.

2 Monitor

Look and listen for a variety of responses.

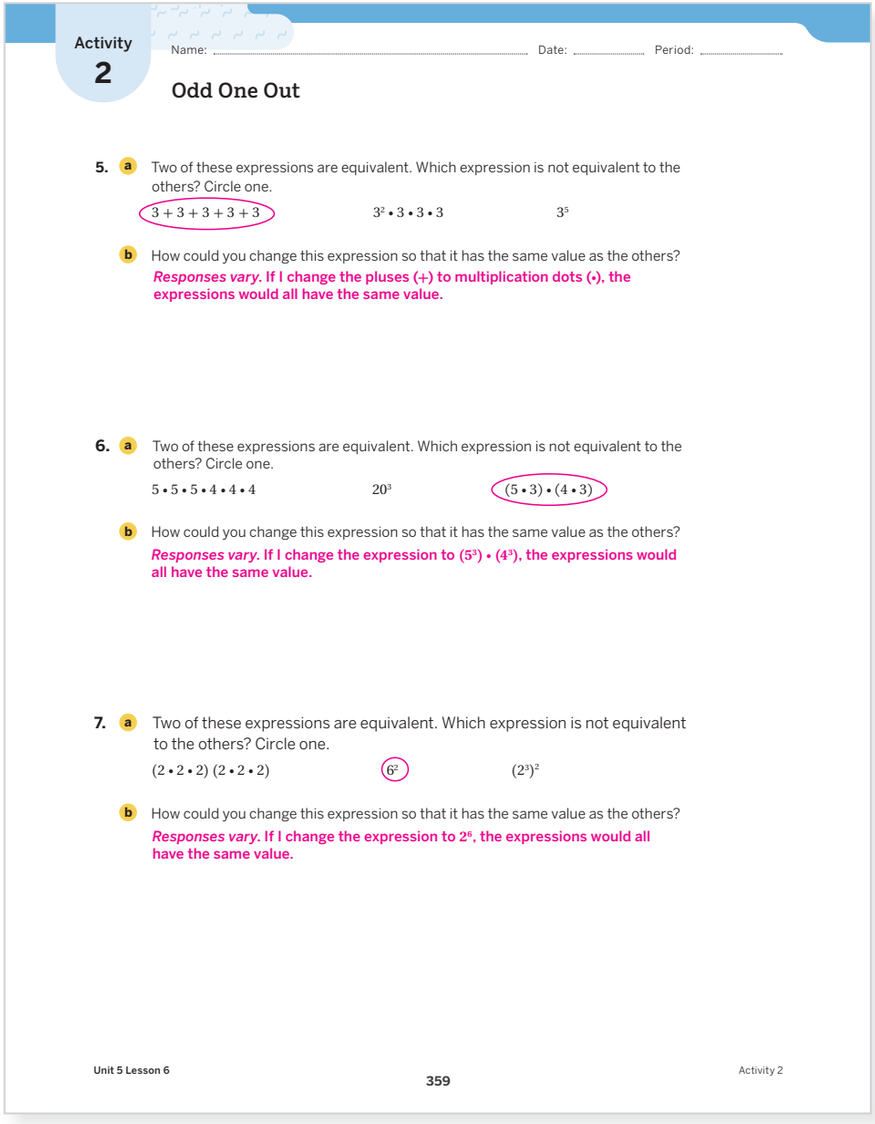
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to expand expressions using the structure of exponents.
Think these pairs of expressions are equivalent: • $3+3+3+3+3$ and 3^5 • $5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ and $(5 \cdot 3) \cdot (4 \cdot 3)$	**Support** Consider asking, • "What does an exponent represent?" • "What is different about repeated addition versus repeated multiplication?" **(MTR.6.1)**
Think 20^3 is not equivalent to $5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ because it has a different base.	**Support** Consider inviting students to write 20 as a product of 5 and 4. **(MTR.6.1)**
Use strategies like: • Change addition to multiplication • Change multipliers to exponents • Switch the base and exponent	**Stretch** Consider asking, "How could you use the changed expression to write another expression that is equivalent to all three?"

A **Accessibility: Conceptual Processing** Invite students to revisit their work on identifying equivalent expressions with exponents to help apply concepts to new situations.

ELL **English Language Learners** Consider offering students the option to use a scribe or to verbally share their response instead of writing it. **(Writing and Speaking)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Odd One Out

5. a. Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$3+3+3+3+3$ $3^2 \cdot 3 \cdot 3 \cdot 3$ 3^5

b. How could you change this expression so that it has the same value as the others?

Responses vary. If I change the pluses (+) to multiplication dots (•), the expressions would all have the same value.

6. a. Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ 20^3 $(5 \cdot 3) \cdot (4 \cdot 3)$

b. How could you change this expression so that it has the same value as the others?

Responses vary. If I change the expression to $(5^3) \cdot (4^3)$, the expressions would all have the same value.

7. a. Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$(2 \cdot 2 \cdot 2)\,(2 \cdot 2 \cdot 2)$ 6^2 $(2^3)^2$

b. How could you change this expression so that it has the same value as the others?

Responses vary. If I change the expression to 2^6, the expressions would all have the same value.

Unit 5 Lesson 6 359 Activity 2

Pairs | 15 min

Activity 2 Odd One Out (continued)

Purpose: Students make use of structure to compare expressions with exponents and write equivalent expressions involving products of powers and powers of powers. **(MTR.5.1)**

Short on time? Invite students to focus on writing two equivalent expressions for Problem 8.

2 Monitor

To support students getting started, encourage them to choose one pair of expressions to focus on. Consider asking, "Why are the two expressions in the pair you chose equivalent?"

A **Accessibility: Conceptual Processing** To support abstract thinking, invite students to expand expressions with a single exponent and use parentheses to re-group and write a new expression.

Invite students to check their expressions with a partner.

Look for sample student expressions to share during the Connect.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore the structure of expressions with exponents further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Discuss each pair of expressions. Consider asking, "What strategies did you use to write a third expression that is equivalent to the other two?" **(MTR.5.1)**

Invite students to share the expressions they wrote that are equivalent to each pair of expressions. Consider recording student responses in a place where the whole class can view.

Note: If time allows, invite students to consider one or more sets of expressions from the previous problems in this activity and share how they determined which two expressions are equivalent.

Activity 2

Name: _____ Date: _____ Period: _____

Odd One Out (continued)

8. Here are some more pairs of equivalent expressions.

For each row, write one more equivalent expression. *Responses vary.*

Expression 1	Expression 2	Expression 3
$4 \cdot 4 \cdot 4^3$	4^5	$4^2 \cdot 4^3$
6^4	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	$2^4 \cdot 3^4$
$(7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$	$(7^2)^3$	7^6
5^{18}	$(5^3)^6$	$(5^2)^9$

You're invited to explore more.

9. Using whole numbers 0 through 9 without repeating, fill in the blanks so that all expressions have the same value.

Responses vary.

- $(5^2)^4$
- $5^1 \cdot 5^7$
- $5^3 \cdot 5^5 \cdot 5^0$
- 5^8

$(5^{\square})^{\square}$ $\quad 5^{\square} \cdot 5^{\square}$ $\quad 5^{\square} \cdot 5^{\square} \cdot 5^{\square}$ $\quad 5^{\square}$

Unit 5 Lesson 6 360 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for writing equivalent expressions with exponents.

Invite students to respond independently, and then share their thinking with a partner.

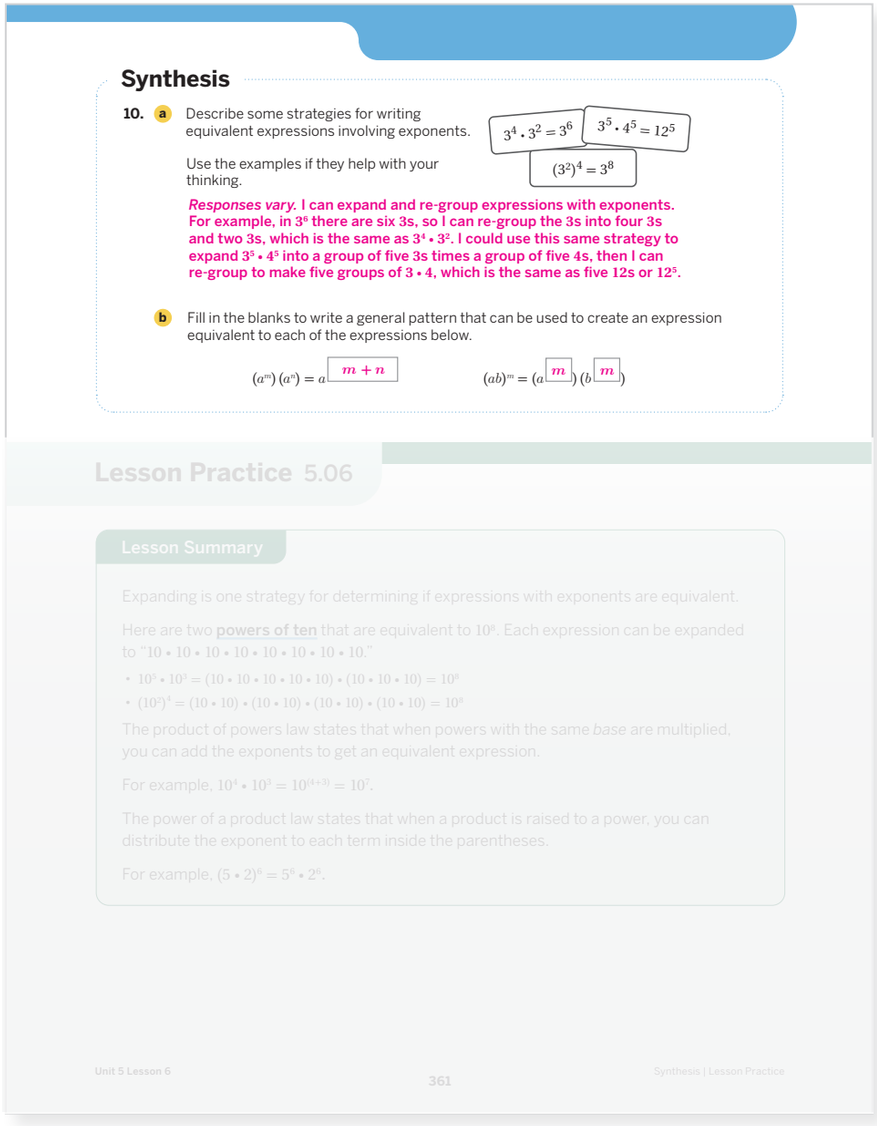
Collect and share a variety of ideas, such as:

- Expand and re-group expressions with exponents to write equivalent expressions.
- Write the base as a product of whole numbers, then expand and re-group to write equivalent expressions.
- Generalize by looking for patterns.

A **Accessibility: Conceptual Processing** Invite students to revisit their work from Activity 2 to support students in generalizing.

ELL **English Language Learners** Invite students to share an example for one of their strategies, and display it for reference. **(Writing and Speaking)**

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. **a** Describe some strategies for writing equivalent expressions involving exponents.

Use the examples if they help with your thinking.

$3^4 \cdot 3^2 = 3^6$ $3^5 \cdot 4^5 = 12^5$ $(3^2)^4 = 3^8$

Responses vary. I can expand and re-group expressions with exponents. For example, in 3^6 there are six 3s, so I can re-group the 3s into four 3s and two 3s, which is the same as $3^4 \cdot 3^2$. I could use this same strategy to expand $3^5 \cdot 4^5$ into a group of five 3s times a group of five 4s, then I can re-group to make five groups of $3 \cdot 4$, which is the same as five 12s or 12^5.

b Fill in the blanks to write a general pattern that can be used to create an expression equivalent to each of the expressions below.

$(a^m)(a^n) = a^{\boxed{m+n}}$ $(ab)^m = (a^{\boxed{m}})(b^{\boxed{m}})$

Lesson Practice 5.06

Lesson Summary

Expanding is one strategy for determining if expressions with exponents are equivalent.

Here are two powers of ten that are equivalent to 10^8. Each expression can be expanded to "$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$."

- $10^5 \cdot 10^3 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^8$
- $(10^2)^4 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^8$

The product of powers law states that when powers with the same *base* are multiplied, you can add the exponents to get an equivalent expression.

For example, $10^4 \cdot 10^3 = 10^{(4+3)} = 10^7$.

The power of a product law states that when a product is raised to a power, you can distribute the exponent to each term inside the parentheses.

For example, $(5 \cdot 2)^6 = 5^6 \cdot 2^6$.

Unit 5 Lesson 6 361 Synthesis | Lesson Practice

Lesson Takeaway: Use the structure of expressions with exponents to identify and write equivalent expressions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting all expressions that are equivalent to a given expression with exponents.

Today's Goal

1. **Goal:** Identify and write equivalent expressions, such as $6^3 \cdot 6^5$, $(6^4)^2$, and $2^8 \cdot 3^8$.

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.06

Select *all* the expressions that are equivalent to 12^6.

- ☐ A. $(12^3)^4$
- ☑ B. $3^6 \cdot 4^6$
- ☐ C. $12^2 \cdot 12^3$
- ☑ D. $(12^3)^2$
- ☐ E. $12^3 + 12^3$

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how write equivalent exponential expressions:

» **Respond to Student Thinking:**

- Revisit Activity 2 as a class and invite students to share strategies for determining whether two exponential expressions are equivalent.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the You're invited to **Explore More** task from this lesson.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson.

- What opportunities were there to celebrate specific students' strategies for writing equivalent exponential expressions?
- What opportunities might there be in future lessons to connect back to the strategies students used today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. a. Describe some strategies for writing equivalent expressions involving exponents.

$3^4 \cdot 3^2 = 3^6$ $3^3 \cdot 4^3 = 12^3$ $(3^2)^4 = 3^8$

Use the examples if they help with your thinking.

Responses vary. I can expand and re-group expressions with exponents. For example, in 3^6 there are six 3s, so I can re-group the 3s into four 3s and two 3s, which is the same as $3^4 \cdot 3^2$. I could use this same strategy to expand $3^3 \cdot 4^3$ into a group of five 3s times a group of five 4s, then I can re-group to make five groups of $3 \cdot 4$, which is the same as five 12s or 12^5.

b. Fill in the blanks to write a general pattern that can be used to create an expression equivalent to each of the expressions below.

$(a^m)(a^n) = a^{(\boxed{m+n})}$ $(ab)^m = (a^{\boxed{m}})(b^{\boxed{m}})$

Lesson Practice 5.06

Lesson Summary

Expanding is one strategy for determining if expressions with exponents are equivalent.

Here are two **powers of ten** that are equivalent to 10^8. Each expression can be expanded to "$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$."

- $10^5 \cdot 10^3 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^8$
- $(10^2)^4 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^8$

The product of powers law states that when powers with the same *base* are multiplied, you can add the exponents to get an equivalent expression.

For example, $10^4 \cdot 10^3 = 10^{(4+3)} = 10^7$.

The power of a product law states that when a product is raised to a power, you can distribute the exponent to each term inside the parentheses.

For example, $(5 \cdot 2)^6 = 5^6 \cdot 2^6$.

Lesson Practice 5.06

Name: _____ Date: _____ Period: _____

Problems 1–2: For each expression, write an equivalent expression with a single power. Two examples are shown.

1.

Expression	Single Power
$6^3 \cdot 6^9$	6^{12}
$2 \cdot 2^4$	2^5
$12^5 \cdot 12^{12}$	12^{17}
$7^6 \cdot 7^5 \cdot 7^6$	7^{18}

2.

Expression	Single Power
$(3^7)^2$	3^{14}
$(2^9)^3$	2^{27}
$(7^6)^3$	7^{18}
$(11^2)^3$	11^6

3. Which expression is equivalent to $6^4 \cdot 2^4$?

A. 8^4 B. 8^8 (C.) 12^4 D. 12^{16}

Problems 4–6: Here is a large rectangular swimming pool. The pool is filled to the top with water.

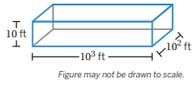


Figure may not be drawn to scale.

4. Write an expression with exponents to represent how much water the pool holds.

$10^3 \cdot 10^2 \cdot 10^1$ cubic feet
(or equivalent)

5. Write an equivalent expression with a single power.

10^6 cubic feet

6. You want to buy a cover to put on top of the pool. Write an expression with exponents to represent the area of the cover you need to buy.

$10^3 \cdot 10^2$ square feet
(or equivalent)

Lesson Practice 5.06

Name: _____ Date: _____ Period: _____

7. Fill in each blank using the whole numbers 1 to 9 only once to make a true statement.

$6^{\square} \cdot 6^{\square} = (6^{\square})^{\square}$

Responses vary. $6^1 \cdot 6^3 = (6^2)^2$

FAST Practice

8. Select *all* the expressions that are equivalent to 6^8.

☑ A. $2^8 \cdot 3^8$
☐ B. $(6^4)^4$
☑ C. $(6^2)^4$
☐ D. $6^4 + 6^4$
☐ E. $6^4 \cdot 6^2$

Spiral Review

9. The expression $0.7 \cdot 0.52$ is equivalent to what number?

0.364

10. Determine the quotient of 1.44 and 0.06.

24

11. Diego has a length of ribbon 2.6 meters long. For a crafts project, he needs to cut the ribbon into pieces that are 0.4 meters long. How many pieces of ribbon can he make to be used for the project? Explain your thinking.

6. *Explanations vary.* Diego is cutting one long ribbon into smaller equal parts, so I can divide 2.6 by 0.4 to determine how many equal-sized ribbons he can make. When I divide 2.6 by 0.4, I get 6.5, which means he can cut 6 whole ribbons of the same length with some ribbon left over that is not long enough for the project.

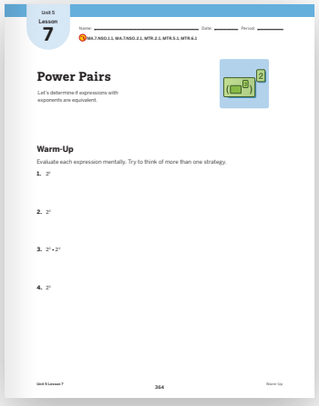
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4–5	1	MA.7.NSO.1.1
	3, 6	2	MA.7.NSO.1.1
	7	3	MA.7.NSO.1.1
FAST Practice	8	2	MA.7.NSO.1.1
Spiral Review			
	9–10	1	MA.6.NSO.2.1
	11	2	MA.6.NSO.2.1

Unit 5

Lesson

7


Power Pairs

Let's determine if expressions with exponents are equivalent.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. 2^5
2. 2^3
3. $2^3 \cdot 2^3$
4. 2^6

364

Power Pairs

Multiplying Powers and Powers of Powers

Let's determine if expressions with exponents are equivalent.

Learning Outcomes

- H┼❋ź=’é◘Ç┼źˆ

1. **Language Goal:** Justify whether expressions with exponents are equivalent. **(Writing, Speaking, and Listening)**

Students make use of patterns and structure to relate equivalent exponential expressions that use the powers of powers and products of powers. They justify their strategies for determining whether two expressions are equivalent. Students continue to work solely with positive exponents. **(MTR.5.1)**

Coherence

< P$┼$ĮÊåź$¸ˇ{

In Lesson 6, students wrote equivalent exponential expressions involving the products of powers and powers of powers.

> F⁸ è⁸ $åąÊåź$¸ˇ{

In Lesson 8, students will begin to work with division of powers and use the structure of exponential expressions to rewrite as a single power.

Connections

- Students recognize that different expressions can represent the same numerical value as they determine if expressions with exponents are equivalent. (mathematical)

FL B.E.S.T. Math Benchmarks

A❋❋$åéé¸ˇ{

MA.7.NSO.1.1

Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

MA.7.NSO.2.1

Solve mathematical problems using multi-step operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12. EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

B⁸ ˇ❋¸ˇ{ąN¸	B⁸ ˇ❋¸ˇ{ąH┼– ź$❋
MA.6.NSO.3.3	**MA.8.NSO.1.3**
MA.6.NSO.3.4	**MA.8.AR.1.1**

Lesson at a Glance

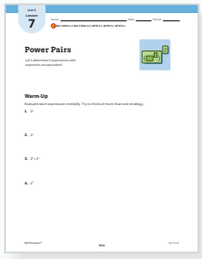
~ 45 min

MA.7.NSO.1.1, MA.7.NSO.2.1

Warm-Up

Whole Class | 5 min

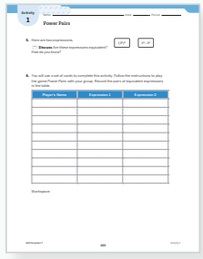
Students use the **Number Talk** routine to recall what they have learned about the properties of expressions with exponents.



Activity 1

Small Groups | 20 min

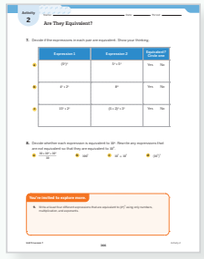
Students play a card game where they identify and justify equivalence for pairs of expressions to build fluency working with expressions with exponents. **(MTR.2.1)**



Activity 2

Pairs | 10 min

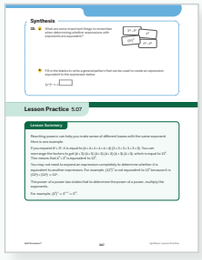
Students determine whether given expressions are equivalent to build fluency making use of the structure of expressions with exponents. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

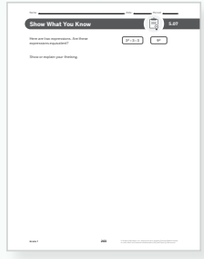
Students synthesize their understanding of how to determine whether expressions with exponents are equivalent. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by justifying whether two given expressions are equivalent.



Prep Checklist

Prepare the Activity 1 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards, one set per group, pre-cut

Whole Class | 5 min

Warm-Up

Purpose: Students use the Number Talk routine to recall what they have learned about the properties of expressions with exponents.

1 Launch

Use the Number Talk routine to support students' ability to think flexibly about evaluating expressions with exponents.

Invite students to evaluate the first expression independently and then share their strategies aloud.

Create a record of each strategy that students share, along with the name of the student who shared it.

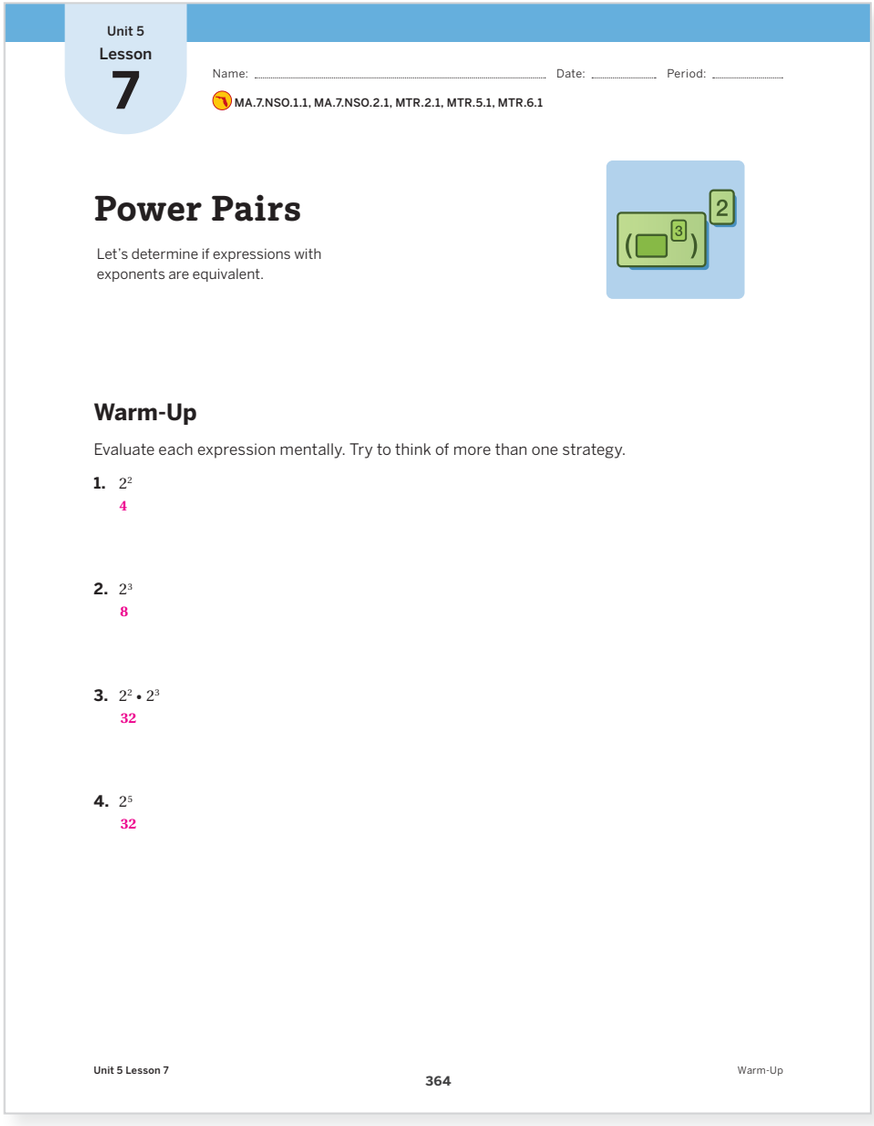
Consider asking:
- "Who thought about it in a different way?"
- "Can someone explain ___'s strategy in their own words?"

Note: Repeat this structure for each expression in the Number Talk.

2 Connect

Invite students to compare and contrast the different expressions.

Consider asking:
- "What connections do you notice among the different expressions?"
- "Why do the last two expressions have the same value?"
- "Is there a way to tell just by looking at the expressions that they are equal? How?"



Unit 5
Lesson
7

Name: ..... Date: ..... Period: .....

MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1

Power Pairs

Let's determine if expressions with exponents are equivalent.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. 2^2
4

2. 2^3
8

3. $2^2 \cdot 2^3$
32

4. 2^5
32

Unit 5 Lesson 7 364 Warm-Up

Small Groups | 20 min

Activity 1 Power Pairs

Purpose: Students play a card game where they identify and justify equivalence for pairs of expressions to build fluency working with expressions with exponents. **(MTR.2.1)**

Short on time? Consider inviting students to focus on playing a few rounds with their group.

1 Launch

Direct students' attention to the two expressions with exponents. Invite students to think independently before discussing their ideas with a partner. **(MTR.2.1)**

Discuss students' strategies for determining whether the expressions are equivalent. Consider asking, "Who has a different strategy?"

Consider sharing that students are going to play a card game where they match up equivalent expressions and justify their reasoning like they did with these expressions.

Invite students to read the game instructions aloud then restate it in their own words.

A **Accessibility: Conceptual Processing** Consider demonstrating the steps for the game by having a group of students and staff play an example round while the rest of the class observes.

Distribute one set of cards to each group of students.

2 Monitor

Encourage students to get consensus from the group that their selected pair does contain equivalent expressions before continuing to the next player.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Consider asking:

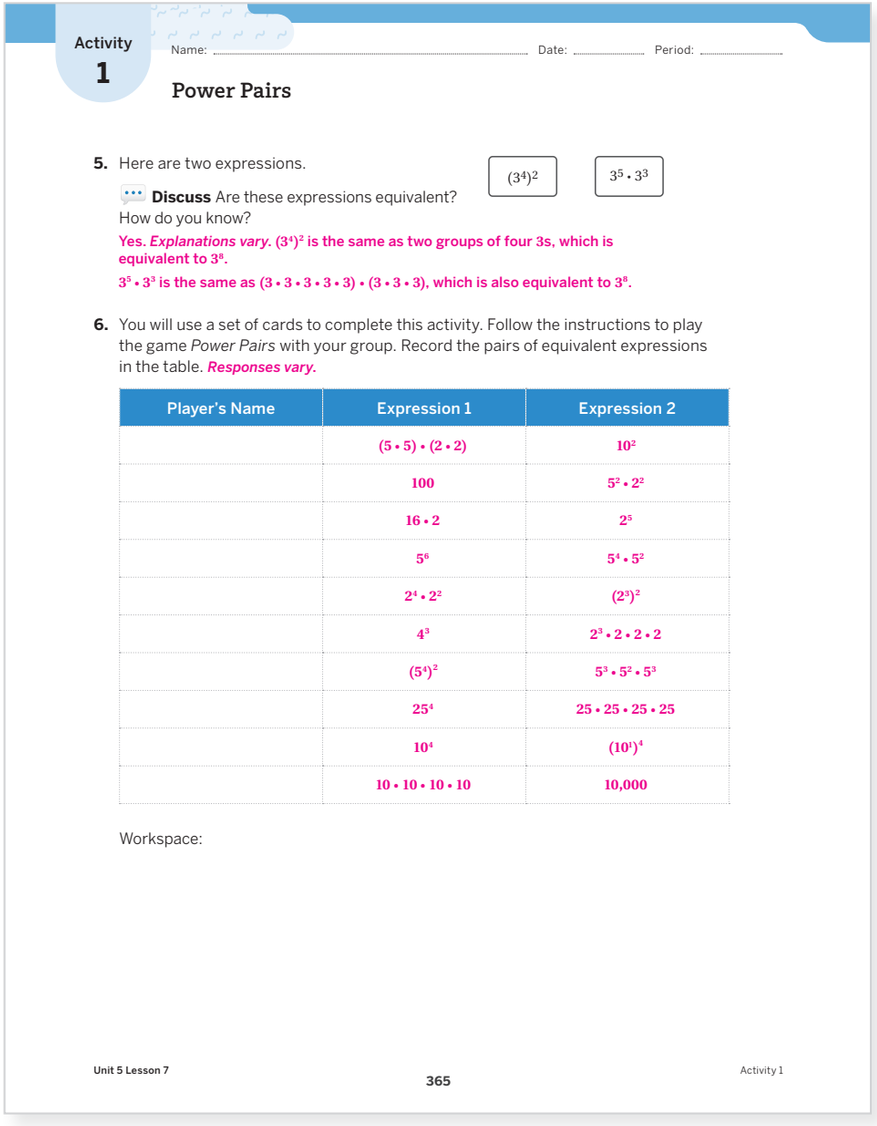
- "Can you explain your strategy for determining matching cards?"
- "Did anyone have a different strategy for determining matching cards?"
- "Do you agree or disagree? Why?"

3 Connect

Invite a few students to share sample expressions from the Power Pairs game.

Consider asking:

- "What equivalent expressions surprised you?"
- "What pairs of expressions were difficult to match?"
- "What is an explanation someone in your group gave that you found helpful?"



Activity 1

Name: _____ Date: _____ Period: _____

Power Pairs

5. Here are two expressions.

$(3^4)^2$ $3^5 \cdot 3^3$

Discuss Are these expressions equivalent? How do you know?

Yes. *Explanations vary.* $(3^4)^2$ is the same as two groups of four 3s, which is equivalent to 3^8.

$3^5 \cdot 3^3$ is the same as $(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)$, which is also equivalent to 3^8.

6. You will use a set of cards to complete this activity. Follow the instructions to play the game *Power Pairs* with your group. Record the pairs of equivalent expressions in the table. *Responses vary.*

Player's Name	Expression 1	Expression 2
	$(5 \cdot 5) \cdot (2 \cdot 2)$	10^2
	100	$5^2 \cdot 2^2$
	$16 \cdot 2$	2^5
	5^6	$5^4 \cdot 5^2$
	$2^4 \cdot 2^2$	$(2^3)^2$
	4^3	$2^3 \cdot 2 \cdot 2 \cdot 2$
	$(5^4)^2$	$5^3 \cdot 5^2 \cdot 5^3$
	25^4	$25 \cdot 25 \cdot 25 \cdot 25$
	10^4	$(10^1)^4$
	$10 \cdot 10 \cdot 10 \cdot 10$	$10{,}000$

Workspace:

Unit 5 Lesson 7 365 Activity 1

Pairs | 10 min

Activity 2 Are They Equivalent?

Purpose: Students determine whether given expressions are equivalent to build fluency making use of the structure of expressions with exponents. **(MTR.5.1)**

Short on time? Consider inviting students to focus on 1–2 pairs of expressions in Problem 7.

1 Launch

Direct students' attention to the expressions and discuss the directions.

Listen for and record the strategies students share so that they can reference them during the activity.

2 Monitor

Listen to partner discussions for student ideas to highlight in the Connect.

D Differentiation *(Problems 7–8)*

Look for students who:	Respond to Student Thinking
Think that $(5^5)^2$ and $5^4 \cdot 5^3$ are equivalent because $5 + 2 = 3 + 4$. *(Problem 7a)*	**Support** Invite them to expand each expression using what they know about exponents. **(MTR.6.1)**
Are uncertain what to do with powers that have different bases.	**Support** Consider asking, "What are the common factors between the bases?"
Use a variety of strategies for Problem 8: • Evaluate to check for equivalence. • Write out all factors. • Make groupings. • Use properties of exponents.	**Strengthen** Encourage these students to think about how they would share their strategy. Consider inviting them to record their work in a place where the whole class can view it.

Note: Problem 8a contains an expression with division. Students will have opportunities in the next lesson to work with dividing powers.

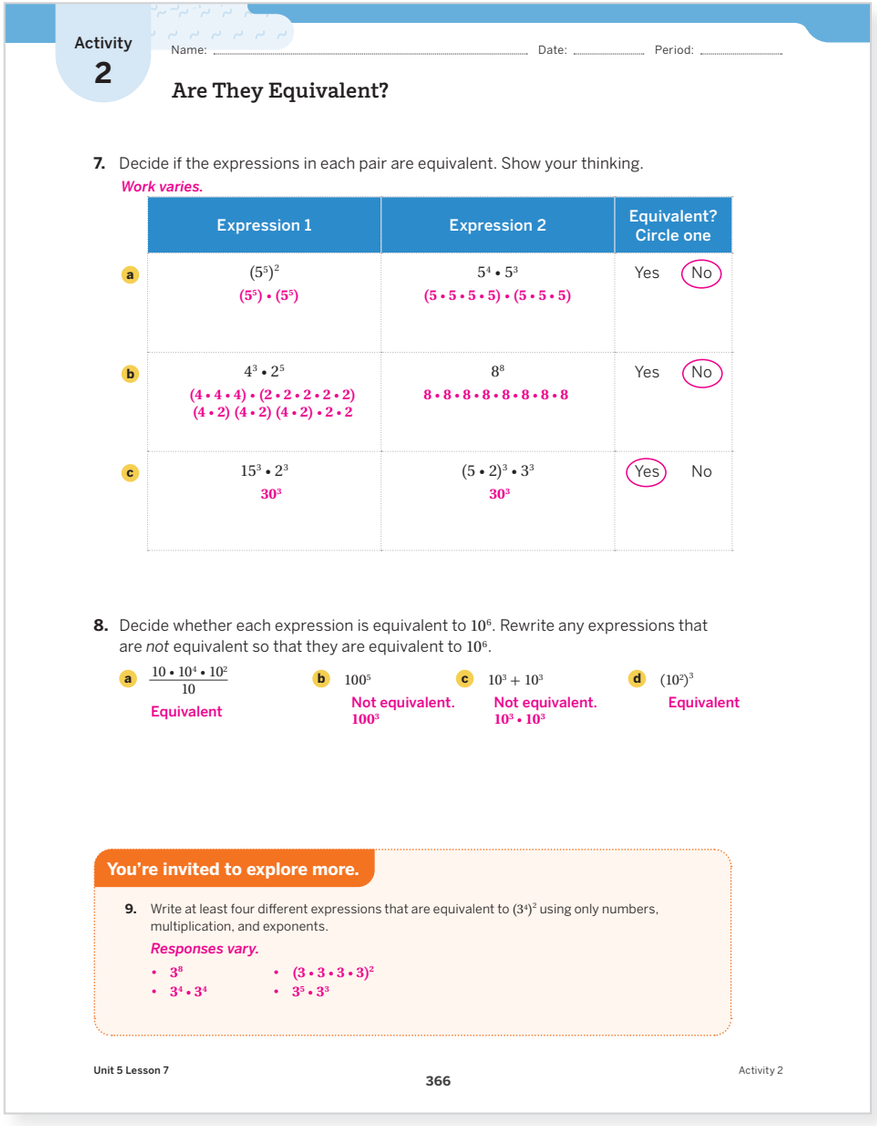
D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore equivalent expressions involving exponents further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share strategies they used for each part in Problem 8.

To surface the Key Takeaway, highlight a variety of strategies students used to determine equivalence. Consider asking, "Is there a way to tell just by looking at the expressions which ones are equivalent to 10^6? How?" **(MTR.5.1)**

Note: If time allows, invite students to share how they changed the expressions in Problems 8b and 8c to be equivalent to 10^6.



Activity 2

Name: Date: Period:

Are They Equivalent?

7. Decide if the expressions in each pair are equivalent. Show your thinking.

Work varies.

	Expression 1	Expression 2	Equivalent? Circle one
a	$(5^5)^2$ $(5^5) \cdot (5^5)$	$5^4 \cdot 5^3$ $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	Yes (No)
b	$4^3 \cdot 2^5$ $(4 \cdot 4 \cdot 4) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ $(4 \cdot 2)\ (4 \cdot 2)\ (4 \cdot 2) \cdot 2 \cdot 2$	8^8 $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$	Yes (No)
c	$15^3 \cdot 2^3$ 30^3	$(5 \cdot 2)^3 \cdot 3^3$ 30^3	(Yes) No

8. Decide whether each expression is equivalent to 10^6. Rewrite any expressions that are *not* equivalent so that they are equivalent to 10^6.

a. $\frac{10 \cdot 10^4 \cdot 10^2}{10}$ Equivalent

b. 100^5 Not equivalent. 100^3

c. $10^3 + 10^3$ Not equivalent. $10^3 \cdot 10^3$

d. $(10^2)^3$ Equivalent

You're invited to explore more.

9. Write at least four different expressions that are equivalent to $(3^4)^2$ using only numbers, multiplication, and exponents.

Responses vary.

- 3^8
- $3^4 \cdot 3^4$
- $(3 \cdot 3 \cdot 3 \cdot 3)^2$
- $3^5 \cdot 3^3$

Unit 5 Lesson 7 366 Activity 2

Key Takeaway: When determining if two expressions are equivalent, expanding an expression with exponents can be helpful, but not always efficient. Recognizing the structure of an expression and applying the products of powers or powers of powers can help when looking for equivalent expressions.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to determine whether expressions with exponents are equivalent.

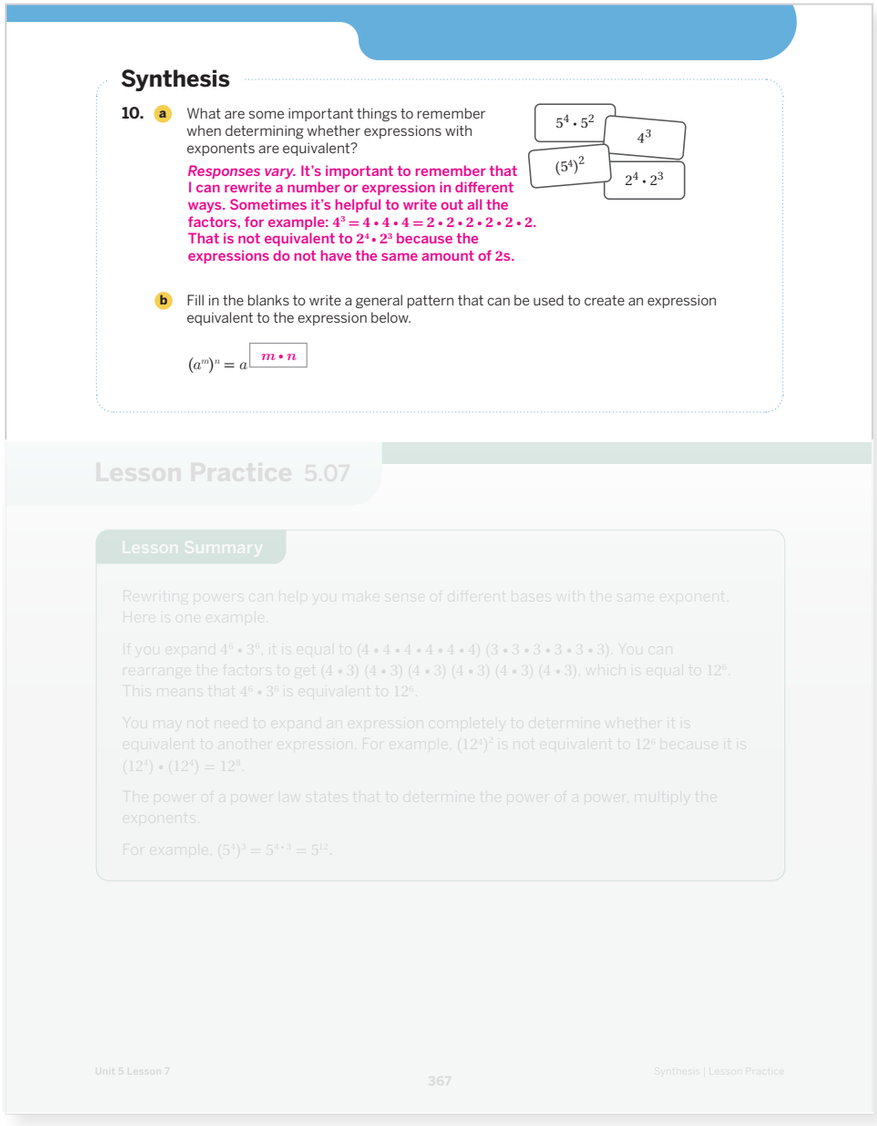
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Manipulating the structure of expressions with exponents to rewrite and compare.
- Write out all factors of an expression to see how it compares to a different expression.
- Exponential expressions with different bases can still be equivalent.
- Raising a power to a power is not the same as the product of two powers.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for identifying equivalent expressions. Display these publicly for students to reference or add to for the rest of the unit.

Note: If time allows, invite students to write equivalent expressions for the expressions shown.



Synthesis

10. **a** What are some important things to remember when determining whether expressions with exponents are equivalent?

$5^4 \cdot 5^2$ 4^3 $(5^4)^2$ $2^4 \cdot 2^3$

Responses vary. It's important to remember that I can rewrite a number or expression in different ways. Sometimes it's helpful to write out all the factors, for example: $4^3 = 4 \cdot 4 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. That is not equivalent to $2^4 \cdot 2^3$ because the expressions do not have the same amount of 2s.

b Fill in the blanks to write a general pattern that can be used to create an expression equivalent to the expression below.

$(a^m)^n = a^{\boxed{m \cdot n}}$

Lesson Practice 5.07

Lesson Summary

Rewriting powers can help you make sense of different bases with the same exponent. Here is one example.

If you expand $4^6 \cdot 3^6$, it is equal to $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) \, (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$. You can rearrange the factors to get $(4 \cdot 3) \, (4 \cdot 3) \, (4 \cdot 3) \, (4 \cdot 3) \, (4 \cdot 3) \, (4 \cdot 3)$, which is equal to 12^6. This means that $4^6 \cdot 3^6$ is equivalent to 12^6.

You may not need to expand an expression completely to determine whether it is equivalent to another expression. For example, $(12^4)^2$ is not equivalent to 12^8 because it is $(12^4) \cdot (12^4) = 12^8$.

The power of a power law states that to determine the power of a power, multiply the exponents.

For example, $(5^4)^3 = 5^{4 \cdot 3} = 5^{12}$.

Unit 5 Lesson 7 367 Synthesis | Lesson Practice

Lesson Takeaway: Using the structure of exponents is one way to compare expressions with exponents and determine equivalence. When comparing expressions with different bases, it is helpful to expand and regroup factors in the expressions.

Independent | 5 min

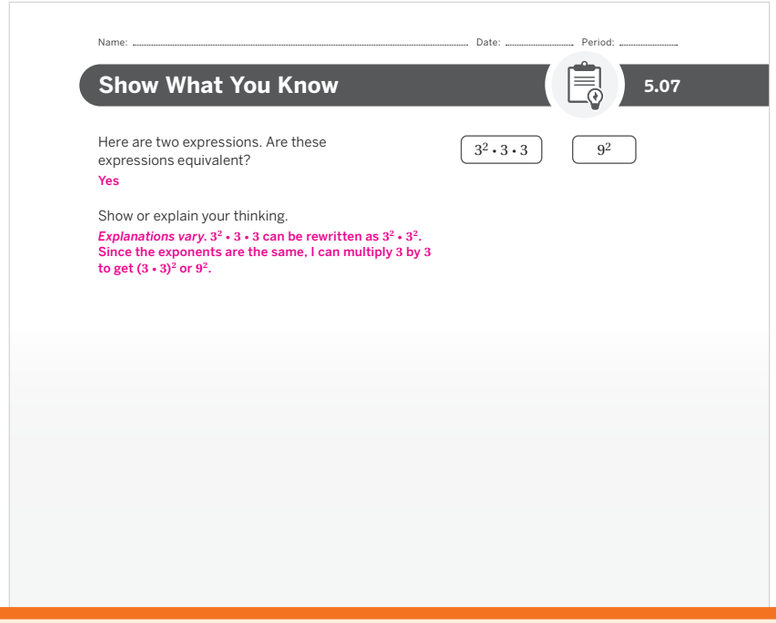
Show What You Know

Purpose: Students demonstrate their understanding by justifying whether two given expressions are equivalent.

Today's Goal

1. **Language Goal:** Justify whether expressions with exponents are equivalent. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.07

Here are two expressions. Are these expressions equivalent?

$3^2 \cdot 3 \cdot 3$ 9^2

Yes

Show or explain your thinking.

Explanations vary. $3^2 \cdot 3 \cdot 3$ can be rewritten as $3^2 \cdot 3^2$. Since the exponents are the same, I can multiply 3 by 3 to get $(3 \cdot 3)^2$ or 9^2.

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine whether two exponential expressions are equivalent:

» **Respond to Student Thinking:**

- Using the ***Multiplying Powers*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on how students made use of patterns and structure in today's lesson.

- How did the card game in Activity 1 Power Pairs support students in noticing patterns and determining equivalent expressions?
- How can you continue to support students in making use of patterns and structure as they explore how to rewrite more complex expressions as a single power in the next lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. a. What are some important things to remember when determining whether expressions with exponents are equivalent?

Responses vary. It's important to remember that I can rewrite a number or expression in different ways. Sometimes it's helpful to write out all the factors, for example: $4^3 = 4 \cdot 4 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. That is not equivalent to $2^4 \cdot 2^3$ because the expressions do not have the same amount of 2s.

b. Fill in the blanks to write a general pattern that can be used to create an expression equivalent to the expression below.

$(a^m)^n = a^{\boxed{mn} \cdot n}$

Lesson Practice 5.07

Lesson Summary

Rewriting powers can help you make sense of different bases with the same exponent. Here is one example.

If you expand $4^6 \cdot 3^6$, it is equal to $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)\ (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$. You can rearrange the factors to get $(4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)$, which is equal to 12^6. This means that $4^6 \cdot 3^6$ is equivalent to 12^6.

You may not need to expand an expression completely to determine whether it is equivalent to another expression. For example, $(12^4)^2$ is not equivalent to 12^6 because it is $(12^4) \cdot (12^4) = 12^8$.

The power of a power law states that to determine the power of a power, multiply the exponents.

For example, $(5^4)^3 = 5^{4 \cdot 3} = 5^{12}$.

Lesson Practice 5.07

Name: _____ Date: _____ Period: _____

1. Write four different expressions that have the same value as $(5^2)^3$ using only numbers, multiplication, and exponents.

Responses vary.

$(25)^3$ $(5 \cdot 5)^3$ $5^2 \cdot 5^2 \cdot 5^2$ $25 \cdot 25 \cdot 25$

2. Show or explain why $6^5 \cdot 6^3$ is equivalent to $(6^4)^2$.

Responses vary.

$6^5 \cdot 6^3 = (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6) = (6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6 \cdot 6) = 6^4 \cdot 6^4 = (6^4)^2$

3. Write another expression that is equivalent to both $6^5 \cdot 6^3$ and $(6^4)^2$.

Responses vary. $(6^2)^4$

4. Select *all* the expressions that are equivalent to $2^3 \cdot 2^3$.

☑ A. $2^2 \cdot 2^4$ ☐ B. 2^9 ☐ C. 4^6

☑ D. $(2^3)^2$ ☑ E. $2^5 \cdot 2$

5. If $(3^2)^n = 3^8$, what value of n will make this a true statement? Explain your thinking

Explanations vary. I thought about how many groups of $(3 \cdot 3)$ would make 3^8, or $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. If I write four groups, like $(3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$, I can see that there are eight 3s. So $(3^2)^4 = 3^8$.

FAST Practice

6. For each pair, determine whether the expressions are equivalent or not equivalent.

	Equivalent	Not Equivalent
Expression 1: $(12^2)^3$ Expression 2: $12^4 \cdot 12^2$	☑	☐
Expression 1: $(7 \cdot 2)^3$ Expression 2: $7^3 \cdot 2^3$	☑	☐
Expression 1: $10^2 + 10^4 + 10$ Expression 2: 10^7	☐	☑
Expression 1: 15^6 Expression 2: $(5 \cdot 3 \cdot 3 \cdot 5)^4$	☐	☑

Lesson Practice 5.07

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 7–10: An amoeba (a single-celled animal) is being observed in a dish. Every hour, each amoeba divides to form two amoebas.

7. How many amoebas will there be after 5 hours?

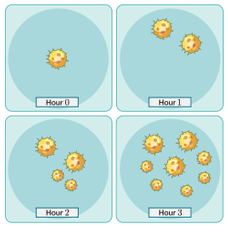
32 amoebas

8. Write an expression for the number of amoebas after 8 hours.

2^8 (or equivalent)

9. Write an expression for the number of amoebas after 24 hours.

2^{24} (or equivalent)



10. Why might an expression with an exponent, like 2^6, be useful for determining the number of amoebas?

Responses vary. Because the number of amoebas doubles every hour, it's more efficient to use an expression with exponents than expanded form.

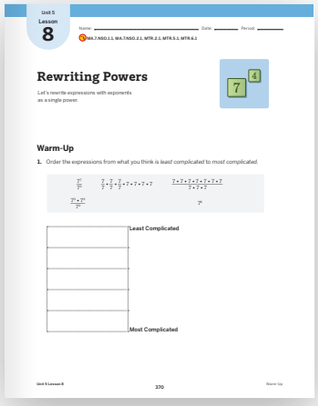
Lesson Practice Item Analysis

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Ŋ¸ -Êåéé†¸		
	1, 3–4 2	MA.7.NSO.1.1
	2 2	MA.7.NSO.1.1 MA.7.NSO.2.1
	5 3	MA.7.NSO.1.1 MA.7.NSO.2.1
FAST Practice	**6** 2	MA.7.NSO.1.1
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	7–10 2	MA.7.NSO.1.1

Unit 5

Lesson

8



Rewriting Powers

Rewriting Exponential Expressions as a Single Power

Let's rewrite expressions with exponents as a single power.

Learning Outcomes

Today's Goals

1. **Goal:** Divide expressions involving exponents that have the same base.
2. **Goal:** Rewrite expressions with positive exponents as single powers.

Students use patterns and structure to develop strategies for rewriting exponential expressions as a single power. They continue to work with expressions involving multiplication of powers, powers of powers, and now division of powers. The emphasis of this lesson is not on identifying and then memorizing rules, but on making sense of the structure of expressions and identifying parts of those expressions that can be written in an equivalent form (e.g., writing $\frac{5}{5}$ as 1). **(MTR.5.1)**

Coherence

Prior Learning

In Lessons 6 and 7, students worked with other properties of exponents. They determined if two expressions with exponents are equivalent and wrote equivalent expressions with exponents.

Future Learning

In Lesson 9, students will extend their work with exponents to develop an understanding of zero exponents.

Connections

- Students apply a set of rules that reflect the sequence of the order in which expressions with multiple operations should be solved. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.1
Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.

Also Addressing: **MA.7.NSO.2.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.3	**MA.8.NSO.1.3**
MA.6.NSO.3.4	**MA.8.AR.1.1**

Lesson at a Glance

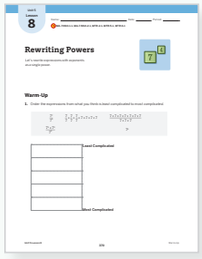
~ 45 min

MA.7.NSO.1.1, MA.7.NSO.2.1

Warm-Up

Pairs | 5 min

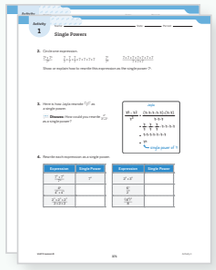
Students order expressions from least to most complicated, to surface prior knowledge of rewriting expressions using properties of exponents.



Activity 1

Pairs | 20 min

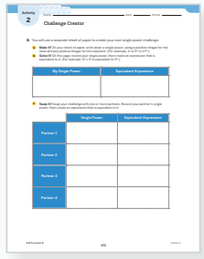
Students use patterns and structure to develop and practice strategies for rewriting expressions with exponents as a single power. **(MTR.5.1)**



Activity 2

Independent | 10 min

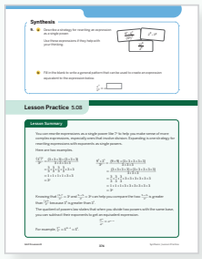
Students create a specific single power for their peers to write a more complicated expression to build fluency using exponent properties when writing equivalent expressions.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of general strategies developed and used throughout the lesson to rewrite expressions as single powers.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying all expressions that can be rewritten as a specific single power.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- blank paper, Index cards, or Sticky notes

Pairs | 5 min

Warm-Up

Purpose: Students order expressions from least to most complicated, to surface prior knowledge of rewriting expressions using properties of exponents.

1 Launch

Consider sharing that students will be working with equivalent expressions written in several different ways in this lesson. **(MTR.2.1)**

Invite students to explain to a classmate what makes an expression more or less complicated in this context.

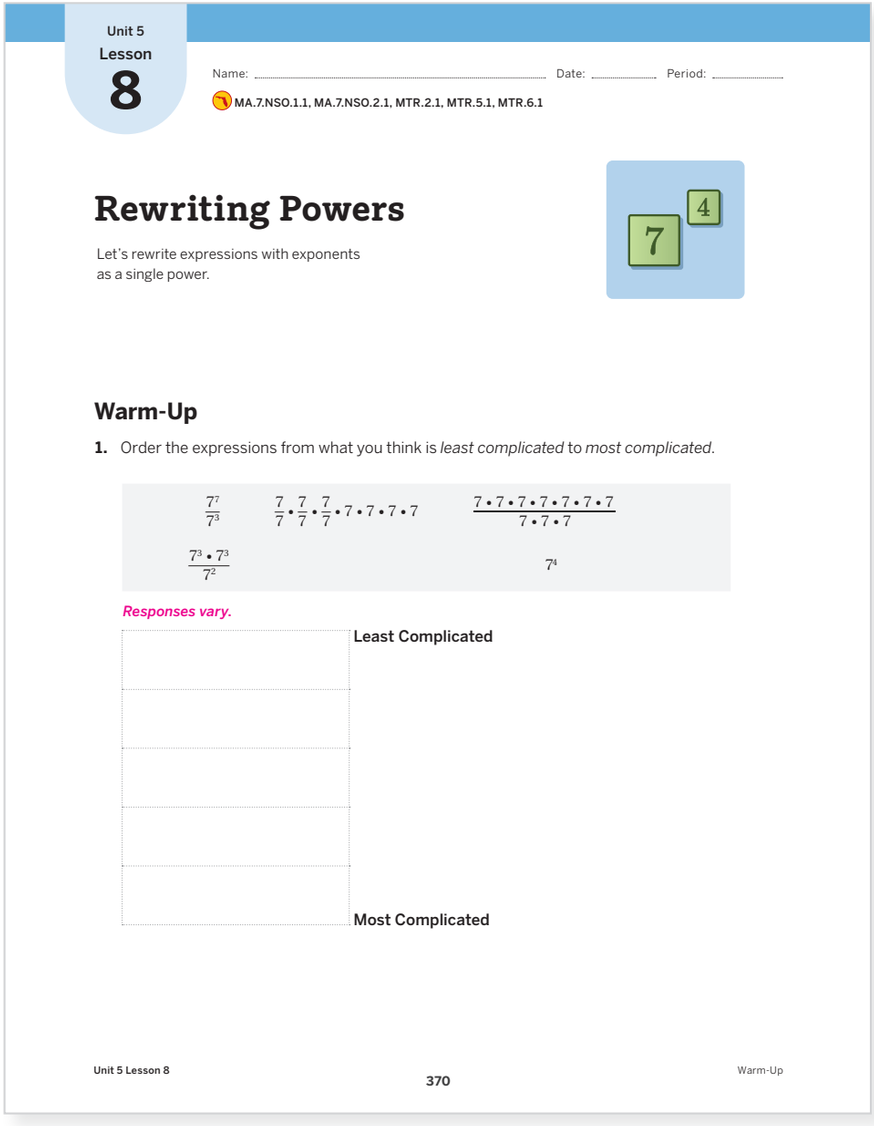
2 Connect

Invite students to share how they ordered the expressions.

Note: While students may agree that 7^4 is the "least complicated" expression, it's okay to lack consensus on the remaining order. The goal for this problem is to invite students to think about the structure of several related expressions involving exponents.

Consider asking:

- "Why might someone want a simpler expression?"
- "Why might someone want a more complicated expression?"



Unit 5
Lesson
8

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.2.1, MTR.5.1, MTR.6.1

Rewriting Powers

Let's rewrite expressions with exponents as a single power.

Warm-Up

1. Order the expressions from what you think is *least complicated* to *most complicated*.

$\frac{7^7}{7^3}$ $\quad \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\quad \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

$\frac{7^3 \cdot 7^3}{7^2}$ $\quad 7^4$

Responses vary.

Least Complicated

Most Complicated

Unit 5 Lesson 8 370 Warm-Up

Pairs | 20 min

Activity 1 Single Powers

Purpose: Students use patterns and structure to develop and practice strategies for rewriting expressions with exponents as a single power. **(MTR.5.1)**

Short on time? Consider discussing Problem 6 as a class.

1 Launch

Invite students to choose one expression and show how it is equivalent to 7^4. Encourage them to use paper and pencil support their thinking and explanation.

2 Monitor

Encourage students to share their explanation with another pair of students who chose a different expression.

Listen for students who discuss how Jayla's work can be applied to the new expression. **(MTR.5.1)**

Invite students to continue to the next problem after they've discussed with a partner.

Invite students to check their single powers with a partner. **(MTR.6.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How could you use Jayla's strategy to write the first expression as a single power?"
Think they cannot write $2^4 \cdot 3^4$ or $\frac{6^7}{2^7}$ as a single power because they have different bases.	**Support** Invite them to expand each power and consider the structure of multiplication to rewrite as a single power.
Manipulate the structure of the expression to write it as a single power.	**Stretch** Consider asking, "Is there another way this expression could be structured to show that it is equivalent to the single power?"

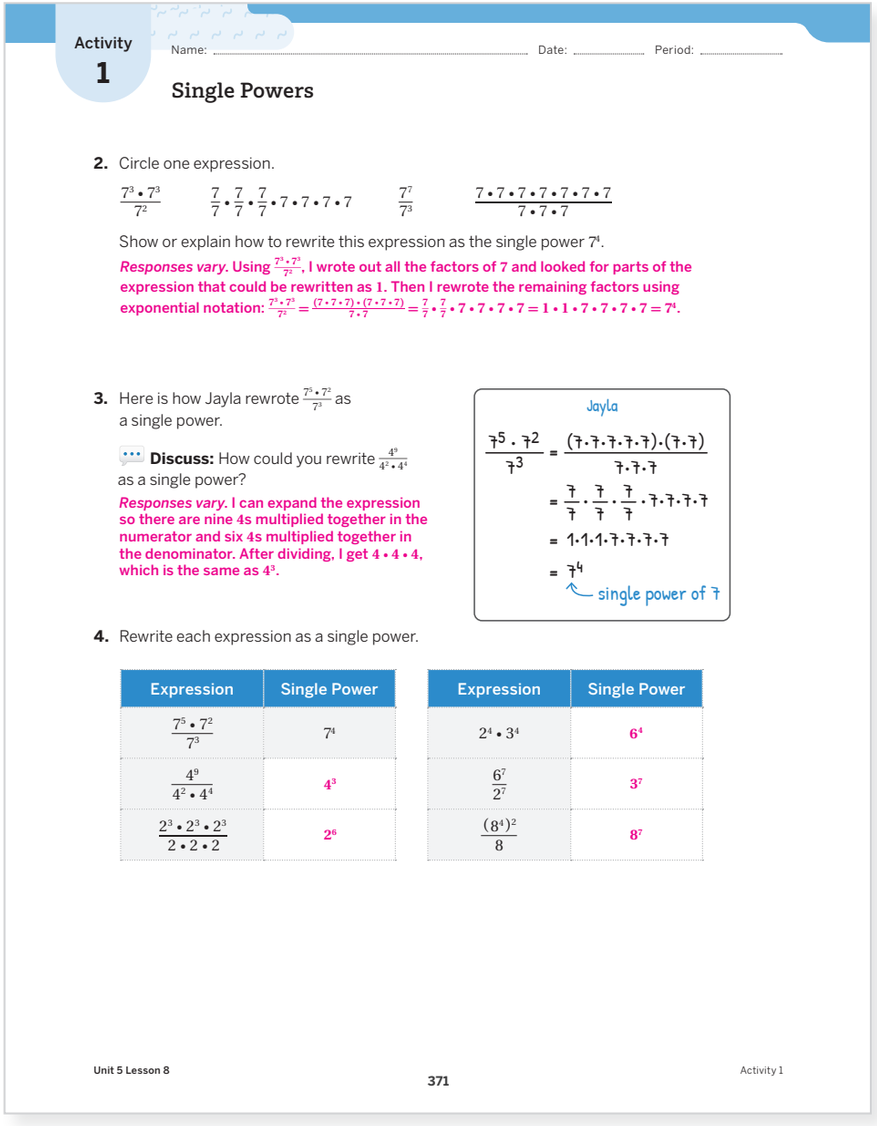
Pause to surface strategies for rewriting expressions with exponents as a single power.

Invite students to choose an expression and explain their strategy for rewriting it as a single power. Consider asking:

- "How does this strategy compare to Jayla's strategy?"
- "What is another strategy we could use to rewrite this expression as a single power?"

Note: Repeat this structure for all expressions.

Activity 1 continued >



Activity 1

Name: Date: Period:

Single Powers

2. Circle one expression.

$\frac{7^3 \cdot 7^3}{7^2}$ $\quad \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\quad \frac{7^7}{7^3}$ $\quad \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

Show or explain how to rewrite this expression as the single power 7^4.

Responses vary. Using $\frac{7^3 \cdot 7^3}{7^2}$, I wrote out all the factors of 7 and looked for parts of the expression that could be rewritten as 1. Then I rewrote the remaining factors using exponential notation: $\frac{7^3 \cdot 7^3}{7^2} = \frac{(7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7)}{7 \cdot 7} = \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^4$.

3. Here is how Jayla rewrote $\frac{7^5 \cdot 7^2}{7^3}$ as a single power.

Jayla

$\frac{7^5 \cdot 7^2}{7^3} = \frac{(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7 \cdot 7}$

$= \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$= 1 \cdot 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$= 7^4$ ← single power of 7

Discuss: How could you rewrite $\frac{4^9}{4^2 \cdot 4^4}$ as a single power?

Responses vary. I can expand the expression so there are nine 4s multiplied together in the numerator and six 4s multiplied together in the denominator. After dividing, I get $4 \cdot 4 \cdot 4$, which is the same as 4^3.

4. Rewrite each expression as a single power.

Expression	Single Power	Expression	Single Power
$\frac{7^5 \cdot 7^2}{7^3}$	7^4	$2^4 \cdot 3^4$	6^4
$\frac{4^9}{4^2 \cdot 4^4}$	4^3	$\frac{6^7}{2^7}$	3^7
$\frac{2^3 \cdot 2^3 \cdot 2^3}{2 \cdot 2 \cdot 2}$	2^6	$\frac{(8^4)^2}{8}$	8^7

Unit 5 Lesson 8 371 Activity 1

Pairs | 20 min

Activity 1 Single Powers (continued)

Purpose: Students use patterns and structure to develop and practice strategies for rewriting expressions with exponents as a single power. **(MTR.5.1)**

Short on time? Consider discussing Problem 6 as a class.

2 Monitor

Encourage students to share their reasoning with a partner and to work together to reach an agreement about how to sort the expressions.

A **Accessibility: Executive Function** Invite students to focus on one expression at a time to support students in carrying out multiple steps.

Look for sorting strategies to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Think $2^3 \cdot 3^5$ is equivalent to 6^8.	**Support** Celebrate the structure they noticed. Encourage them to expand each power to check their thinking.
Think $\frac{12^8}{2^8}$ and/or $2^8 \cdot 3^8$ are not equivalent to 6^8.	**Support** Consider asking, "How could you rewrite this expression?"
Determine which expressions are equivalent to 6^8 by manipulating the structure of the expression.	**Stretch** Consider asking, "Why might someone think that one of the expressions that is not equivalent to 6^8 is in fact equivalent to 6^8?"

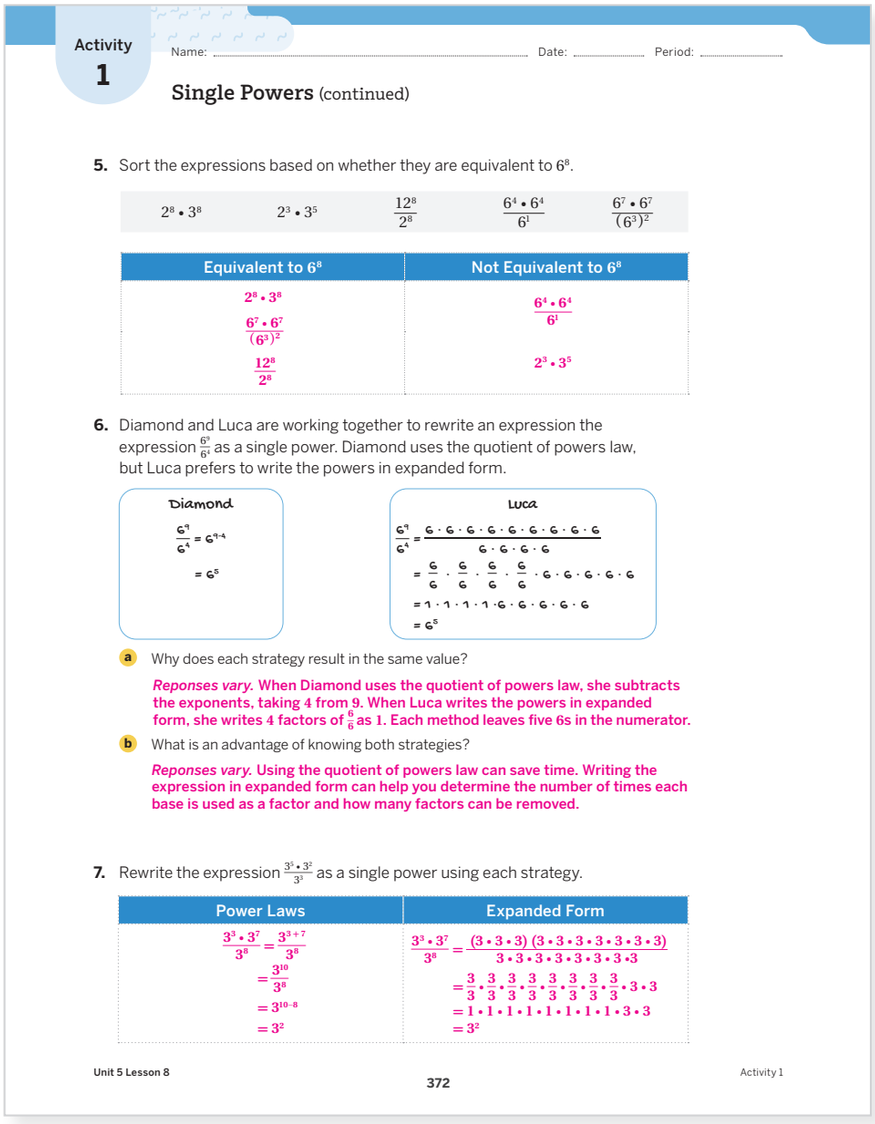
Support students getting started by asking them to paraphrase each student's strategy in their own words.

Consider asking:
- "What did each student do?"
- "Where do you see the same information in each strategy?"

3 Connect

Invite students to share their strategies for determining which expressions are equivalent to 6^8.

To surface the Key Takeaway, consider asking:
- "How did you determine $\frac{(6^7 \cdot 6^7)}{(6^3)^2}$ is equivalent to 6^8?"
- "What is another strategy we could use?"
- "What are some things you found helpful when working with expressions with division?"



Activity 1

Name: Date: Period:

Single Powers (continued)

5. Sort the expressions based on whether they are equivalent to 6^8.

$2^8 \cdot 3^8$ $\quad 2^3 \cdot 3^5$ $\quad \frac{12^8}{2^8}$ $\quad \frac{6^4 \cdot 6^4}{6^1}$ $\quad \frac{6^7 \cdot 6^7}{(6^3)^2}$

Equivalent to 6^8	Not Equivalent to 6^8
$2^8 \cdot 3^8$	$\frac{6^4 \cdot 6^4}{6^1}$
$\frac{6^7 \cdot 6^7}{(6^3)^2}$	$2^3 \cdot 3^5$
$\frac{12^8}{2^8}$	

6. Diamond and Luca are working together to rewrite an expression the expression $\frac{6^9}{6^4}$ as a single power. Diamond uses the quotient of powers law, but Luca prefers to write the powers in expanded form.

Diamond

$\frac{6^9}{6^4} = 6^{9-4}$

$= 6^5$

Luca

$\frac{6^9}{6^4} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6 \cdot 6}$

$= \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

$= 6^5$

a. Why does each strategy result in the same value?

Reponses vary. When Diamond uses the quotient of powers law, she subtracts the exponents, taking 4 from 9. When Luca writes the powers in expanded form, she writes 4 factors of $\frac{6}{6}$ as 1. Each method leaves five 6s in the numerator.

b. What is an advantage of knowing both strategies?

Reponses vary. Using the quotient of powers law can save time. Writing the expression in expanded form can help you determine the number of times each base is used as a factor and how many factors can be removed.

7. Rewrite the expression $\frac{3^5 \cdot 3^2}{3^3}$ as a single power using each strategy.

Power Laws	Expanded Form
$\frac{3^3 \cdot 3^7}{3^8} = \frac{3^{3+7}}{3^8}$ $= \frac{3^{10}}{3^8}$ $= 3^{10-8}$ $= 3^2$	$\frac{3^3 \cdot 3^7}{3^8} = \frac{(3 \cdot 3 \cdot 3)\,(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ $= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3$ $= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3$ $= 3^2$

Unit 5 Lesson 8 372 Activity 1

Key Takeaway: You can use the definition of an exponent, apply properties of exponents, and evaluate when dividing expressions with exponents.

Independent | 10 min

Activity 2 Challenge Creator

Purpose: Students create a specific single power for their peers to write a more complicated expression to build fluency using exponent properties when writing equivalent expressions.

1 Launch

Consider sharing that students will create their own challenge similar to the Problem 7.

Distribute a separate sheet of paper, index card, or sticky note to students so they can create their single power challenge.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

2 Monitor

Listen to student strategies and discussions, offering help or encouragement where needed.

Invite students to check their answers with the person who created the single power that they are creating an equivalent expression for. Encourage them to discuss and resolve any differences. **(MTR.6.1)**

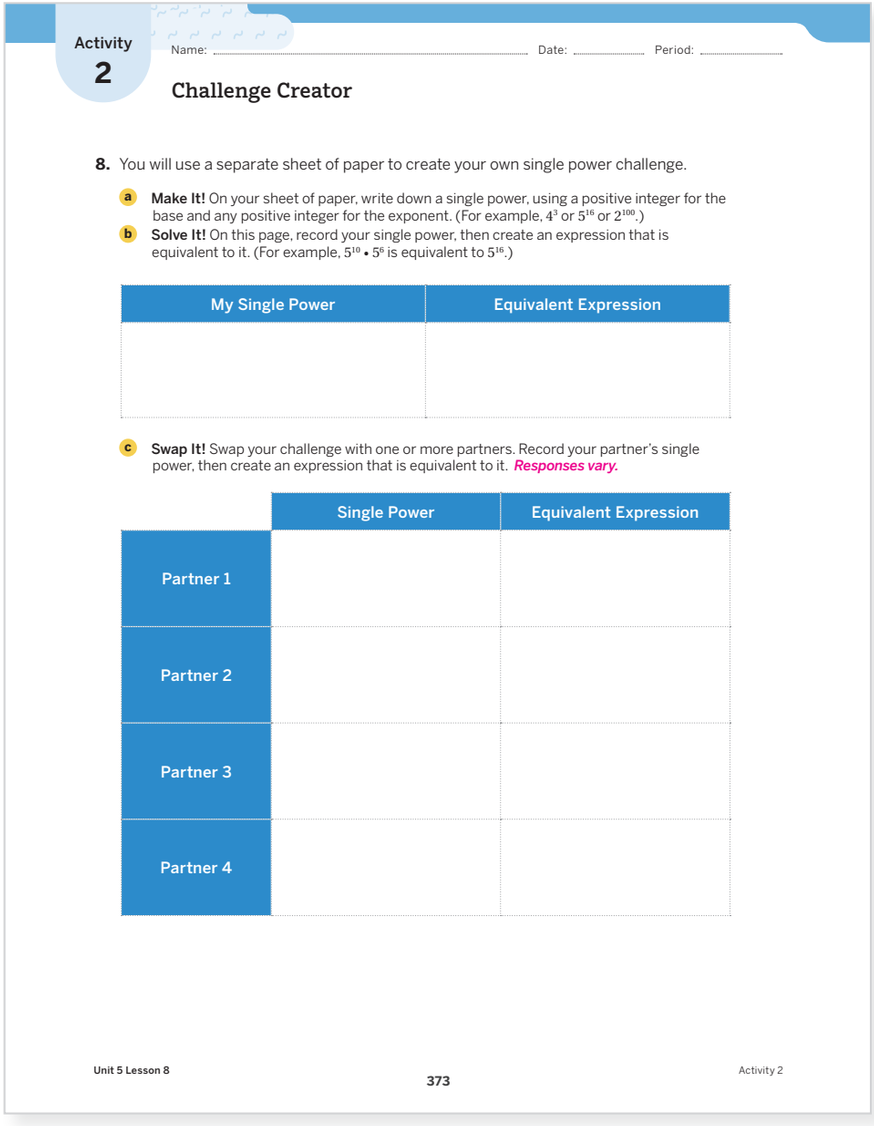
Look for students' single powers and several equivalent expressions to share during the Connect.

3 Connect

Share the students' single powers and their equivalent expressions.

Invite students to share a variety of strategies for writing an equivalent expression.

Consider asking, "Was there a strategy you heard that surprised you?"



Activity 2

Name: Date: Period:

Challenge Creator

8. You will use a separate sheet of paper to create your own single power challenge.

a. **Make It!** On your sheet of paper, write down a single power, using a positive integer for the base and any positive integer for the exponent. (For example, 4^3 or 5^{16} or 2^{100}.)

b. **Solve It!** On this page, record your single power, then create an expression that is equivalent to it. (For example, $5^{10} \cdot 5^6$ is equivalent to 5^{16}.)

My Single Power	Equivalent Expression

c. **Swap It!** Swap your challenge with one or more partners. Record your partner's single power, then create an expression that is equivalent to it. *Responses vary.*

	Single Power	Equivalent Expression
Partner 1		
Partner 2		
Partner 3		
Partner 4		

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of general strategies developed and used throughout the lesson to rewrite expressions as single powers.

Invite students to respond independently, and then share their thinking with a partner.

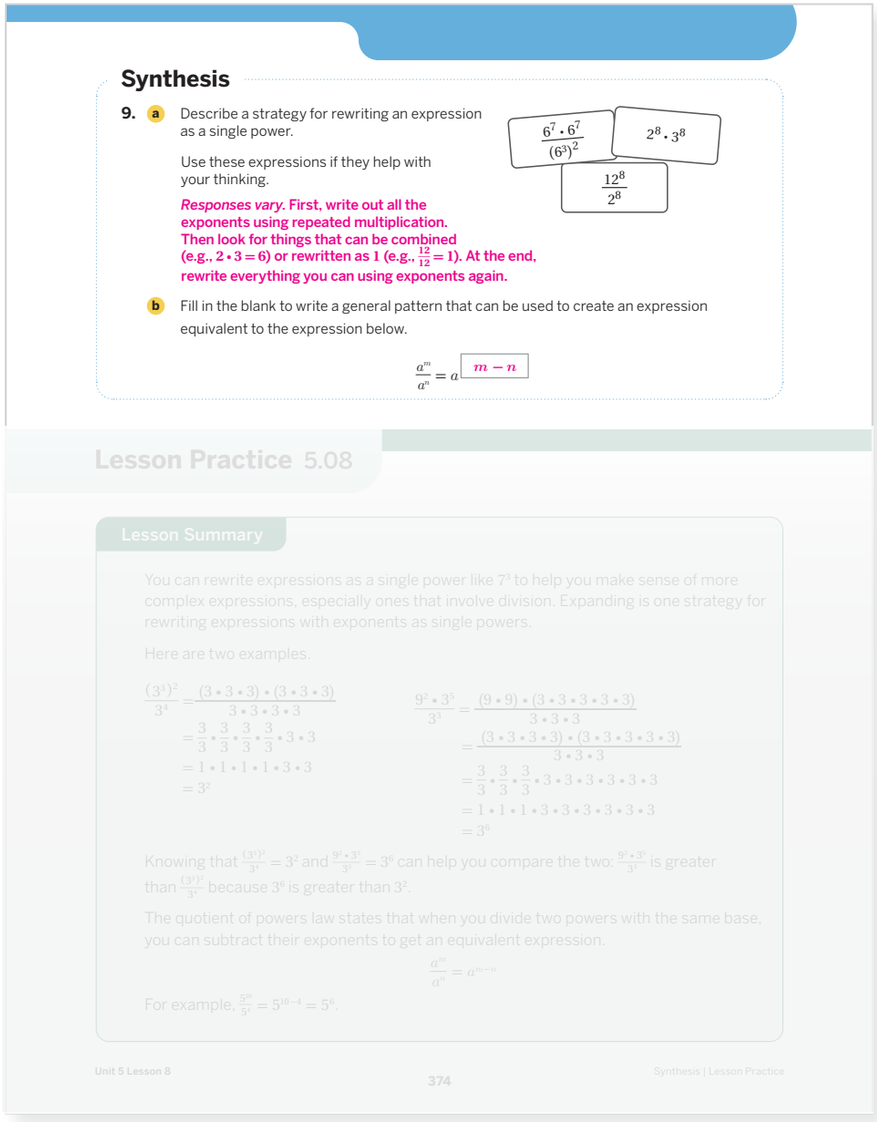
Collect and share a variety of ideas, such as:

- Expanding and simplifying the expression.
- Using the structure of powers with different bases to re-group and write as a single power.
- Focus on simplifying the expressions in the numerator and denominator separately, then looking at the expression as a whole.

A **Accessibility: Conceptual Processing** Invite students to revisit their work writing single powers to support students in generalizing.

ELL **English Language Learners** Invite students to demonstrate or explain how a single power is related to review terms like *power*, *exponent*, and *base*. **(Reading and Listening)**

Note: If time allows, invite students to consider the equivalent expressions their classmates created for their challenge and explain to a partner why each is equivalent to their single power.



Synthesis

9. **a** Describe a strategy for rewriting an expression as a single power.

Use these expressions if they help with your thinking.

$\frac{6^7 \cdot 6^7}{(6^3)^2}$ $2^8 \cdot 3^8$ $\frac{12^8}{2^8}$

Responses vary. First, write out all the exponents using repeated multiplication. Then look for things that can be combined (e.g., $2 \cdot 3 = 6$) or rewritten as 1 (e.g., $\frac{12}{12} = 1$). At the end, rewrite everything you can using exponents again.

b Fill in the blank to write a general pattern that can be used to create an expression equivalent to the expression below.

$\frac{a^m}{a^n} = a^{\boxed{m-n}}$

Lesson Practice 5.08

Lesson Summary

You can rewrite expressions as a single power like 7^3 to help you make sense of more complex expressions, especially ones that involve division. Expanding is one strategy for rewriting expressions with exponents as single powers.

Here are two examples.

$$\frac{(3^3)^2}{3^4} = \frac{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 = 3^2$$

$$\frac{9^2 \cdot 3^5}{3^3} = \frac{(9 \cdot 9) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} = \frac{(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} = \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$$

Knowing that $\frac{(3^3)^2}{3^4} = 3^2$ and $\frac{9^2 \cdot 3^5}{3^3} = 3^6$ can help you compare the two: $\frac{9^2 \cdot 3^5}{3^3}$ is greater than $\frac{(3^3)^2}{3^4}$ because 3^6 is greater than 3^2.

The quotient of powers law states that when you divide two powers with the same base, you can subtract their exponents to get an equivalent expression.

$$\frac{a^m}{a^n} = a^{m-n}$$

For example, $\frac{5^{10}}{5^4} = 5^{10-4} = 5^6$.

Unit 5 Lesson 8 374 Synthesis | Lesson Practice

Lesson Takeaway: You can use the properties of exponents to manipulate the structure of expressions with exponents and write as a single power.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying all expressions that can be rewritten as a specific single power.

Today's Goals

1. **Goal:** Divide expressions involving exponents that have the same base.
2. **Goal:** Rewrite expressions with positive exponents as single powers.

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.08

Select *all* the expressions that are equivalent to 8^8.

- ☐ A. $2^3 \cdot 4^5$
- ☑ B. $2^8 \cdot 4^8$
- ☑ C. $\frac{8^{10}}{8^2}$
- ☐ D. $8^3 + 8^5$
- ☑ E. $\frac{8^2 \cdot 8^3 \cdot 8^4}{8^1}$

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to divide exponential expressions that have the same base:

» **Respond to Student Thinking:**

- Revisit Jayla's method from Activity 1 as a class and invite students to share strategies for dividing and rewriting expressions as a single power.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Which students' ideas were you able to highlight during Activity 1? How were students able to effectively show and explain their ideas?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. a. Describe a strategy for rewriting an expression as a single power.

Use these expressions if they help with your thinking.

$\frac{5^5 \cdot 6^7}{(6^3)^2}$ $2^8 \cdot 3^8$ $\frac{12^8}{2^8}$

Responses vary. First, write out all the exponents using repeated multiplication. Then look for things that can be combined (e.g., $2 \cdot 3 = 6$) or rewritten as 1 (e.g., $\frac{12}{12} = 1$). At the end, rewrite everything you can using exponents again.

b. Fill in the blank to write a general pattern that can be used to create an expression equivalent to the expression below.

$\frac{a^m}{a^n} = a^{\boxed{m-n}}$

Lesson Practice 5.08

Lesson Summary

You can rewrite expressions as a single power like 7^3 to help you make sense of more complex expressions, especially ones that involve division. Expanding is one strategy for rewriting expressions with exponents as single powers.

Here are two examples.

$$\frac{(3^3)^2}{3^4} = \frac{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 = 3^2$$

$$\frac{9^2 \cdot 3^5}{3^3} = \frac{(9 \cdot 9) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} = \frac{(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} = \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$$

Knowing that $\frac{(3^3)^2}{3^4} = 3^2$ and $\frac{9^2 \cdot 3^5}{3^3} = 3^6$ can help you compare the two: $\frac{9^2 \cdot 3^5}{3^3}$ is greater than $\frac{(3^3)^2}{3^4}$ because 3^6 is greater than 3^2.

The quotient of powers law states that when you divide two powers with the same base, you can subtract their exponents to get an equivalent expression.

$$\frac{a^m}{a^n} = a^{m-n}$$

For example, $\frac{5^6}{5^4} = 5^{10-4} = 5^6$.

Lesson Practice 5.08

Name: _____ Date: _____ Period: _____

1. Select *all* the expressions that are equivalent to 10^6.

- [x] A. $5^6 \cdot 2^6$
- [ ] B. $10^8 - 10^2$
- [x] C. $10^3 \cdot 10 \cdot 10^2$
- [x] D. $\frac{10^8}{10^2}$
- [ ] E. $(10^2)^3$

Problems 2–9: Rewrite each expression as a single power. Show your thinking. *Work varies.*

2. $4^4 \cdot 5^4$

$4^4 \cdot 5^4 = (4 \cdot 5)^4 = 20^4$

3. $\frac{5^6}{5^3}$

$\frac{5^6}{5^3} = 5^{(6-3)} = 5^3$

4. $(14^3)^6$

$(14^3)^6 = 14^{(3 \cdot 6)} = 14^{18}$

5. $\frac{21^3 \cdot 21^5}{21^2}$

$\frac{21^3 \cdot 21^5}{21^2} = \frac{21^{(3+5)}}{21^2} = \frac{21^8}{21^2} = 21^{(8-2)} = 21^6$

6. $8^3 \cdot 8^6$

$8^3 \cdot 8^6 = 8^{(3+6)} = 8^9$

7. $\frac{3^{10}}{3}$

$\frac{3^{10}}{3} = 3^{(10-1)} = 3^9$

8. $(12^2)^7 \cdot 12$

$(12^2)^7 \cdot 12 = 12^{(2 \cdot 7)} \cdot 12 = 12^{14} \cdot 12 = 12^{(14+1)} = 12^{15}$

9. $\frac{16^6}{2^6}$

$\frac{16^6}{2^6} = \left(\frac{16}{2}\right)^6 = 8^6$

Lesson Practice 5.08

Name: _____ Date: _____ Period: _____

10. Diamond and Luca are rewriting another expression. Luca prefers to write the problem in expanded form, but Diamond wants to help her by using the quotient of powers law. Explain how Diamond can use the quotient of powers law to help Luca finish the problem.

$$\frac{9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9}{9 \cdot 9 \cdot 9}$$

Explanations vary. Diamond can begin by counting the number of 9s in the numerator and denominator. There are eight 9s in the numerator, or 9^8, and three 9s in the denominator, or 9^3. Since the powers are being divided, she can subtract the exponents, $8 - 3 = 5$, to write the expression using a single exponent as 9^5.

11. Rewrite the expression $\frac{10^4 \cdot 10^4}{10^7}$ as a single power using your preferred strategy. Explain your reasoning.

10^1. *Explanations vary.* I used exponent laws to rewrite the expression because I think I am less likely to make a mistake adding and subtracting the exponents than by writing out all the 10s in the problem.

FAST Practice

12. What is the value of n in the equation $\frac{7^{11}}{7^n} = 7^7$?

$n =$ 4

Spiral Review

Neena is selling her famous lemonade at the town fair. The following table shows the number of lemons used compared to the number of pitchers made.

Pitchers Made (x)	Lemons Used (y)
5	40
10	80
15	120

13. Write an equation to represent the number of lemons used (y) based on the number of pitchers made (x).

$y = 8x$

14. How many lemons she would need to make 25 pitchers of lemonade.

200 lemons

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 10–11	2	MA.7.NSO.1.1
	2–9	1	MA.7.NSO.1.1
FAST Practice	12	2	MA.7.NSO.1.1
Spiral Review			
	13	2	MA.7.AR.4.2 MA.7.AR.4.4
	14	1	MA.7.AR.4.5

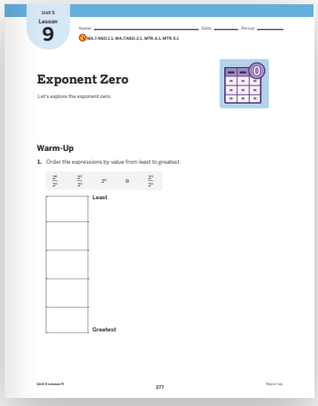
Unit 5

Lesson 9

Exponent Zero

Using Patterns to Understand Zero Exponents

Let's explore the exponent zero.



Learning Outcomes

Today's Goals

1. **Goal:** Use patterns to determine that any power with an exponent of 0 is equal to 1.
2. **Goal:** Determine if expressions involving positive and zero exponents are equivalent.
3. **Language Goal:** Describe patterns in tables with positive and zero exponents. **(Reading, Writing, Speaking, and Listening)**

Students develop an understanding of zero exponents. They use their understanding of positive exponents to rewrite exponential expressions in expanded form, then identify and use patterns to reason and generalize about zero exponents. **(MTR.5.1)**

Coherence

Prior Learning

In Lessons 6–8, students wrote equivalent expressions involving the products, quotients, and powers of powers. They practiced writing exponential expressions as single powers.

Future Learning

In Lesson 10, students will apply exponent rules for expressions with negative bases.

Connections

- Students explore the exponent zero by investigating patterns in student work samples. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.1
Know and apply the Law of Exponents to evaluate numerical expressions and generate equivalent expressions, limited to whole-number exponents and rational number bases.

Also Addressing: **MA.7.NSO.2.1**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12. EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.3	**MA.8.NSO.1.3**
MA.6.NSO.3.4	**MA.8.AR.1.1**

Lesson at a Glance

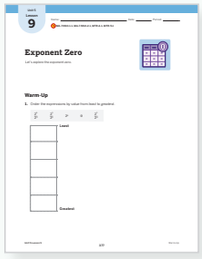
~ 35 min

MA.7.NSO.1.1, MA.7.NSO.2.1

Warm-Up

Pairs | 5 min

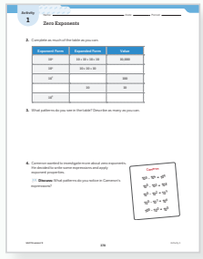
Students order expressions involving powers of 2 to recall properties of exponents.
Routine: Think-Pair-Share



Activity 1

Pairs | 15 min

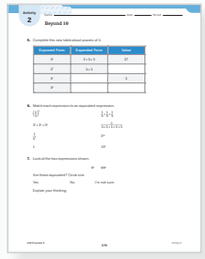
Students consider powers of 10 in exponent form, in expanded form, and by value to identify patterns with zero exponents.
Routine: Notice and Wonder **(MTR.5.1)**



Activity 2

Pairs | 15 min

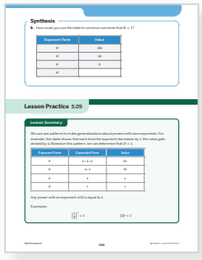
Students consider additional exponential expressions in exponent form, in expanded form, and by value to surface properties of zero exponents.



Synthesis

Whole Class | 5 min

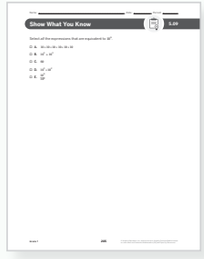
Students synthesize their understanding of zero exponents.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying all expressions that are equivalent to a power of 10.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students order expressions involving powers of 2 to recall properties of exponents.

1 Launch

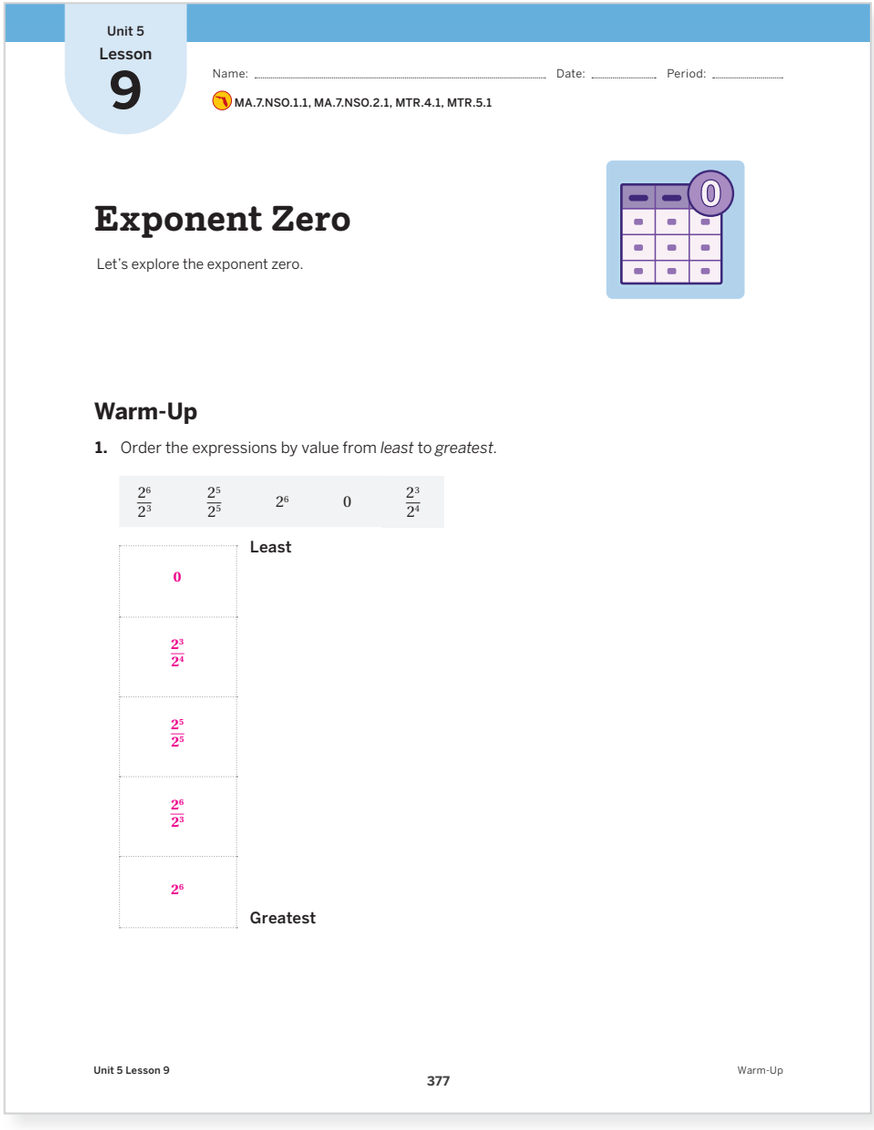
Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

Note: Students do not need to determine the value of any individual expression (though they are welcome to if they find that helpful).

2 Connect

Invite students to explain how they decided where to place each of the expressions from least to greatest.

Invite students to critique and respond to the reasons they hear from their classmates. **(MTR.4.1)**



Unit 5
Lesson
9

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1

Exponent Zero

Let's explore the exponent zero.

Warm-Up

1. Order the expressions by value from *least* to *greatest*.

$\frac{2^6}{2^3}$ $\frac{2^5}{2^5}$ 2^6 0 $\frac{2^3}{2^4}$

Least

0

$\frac{2^3}{2^4}$

$\frac{2^5}{2^5}$

$\frac{2^6}{2^3}$

2^6

Greatest

Unit 5 Lesson 9 377 Warm-Up

Pairs | 15 min

Activity 1 Zero Exponents

Purpose: Students consider powers of 10 in exponent form, in expanded form, and by value to identify patterns with zero exponents. **(MTR.5.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the table of powers of 10. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

2 Monitor

Select and sequence patterns to share during the Connect.

D Differentiation *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to describe the pattern(s) they recognize in the first column.
See symmetry in the table.	**Strengthen** Consider asking, "How can you compare the size of two powers?"
Recognize the overall pattern as approaching 0.	**Stretch** Consider asking, "Can you use $\frac{10^8}{10^9}$ along with what you know about dividing powers to explain why $10^0 = 1$?"

Direct students' attention to Cameron's work and invite students to discuss with their partner and listen for any consensus.

A Accessibility: Conceptual Processing Invite students to look at the relationship between lines 4 and 5 of Cameron's work to support identifying patterns.

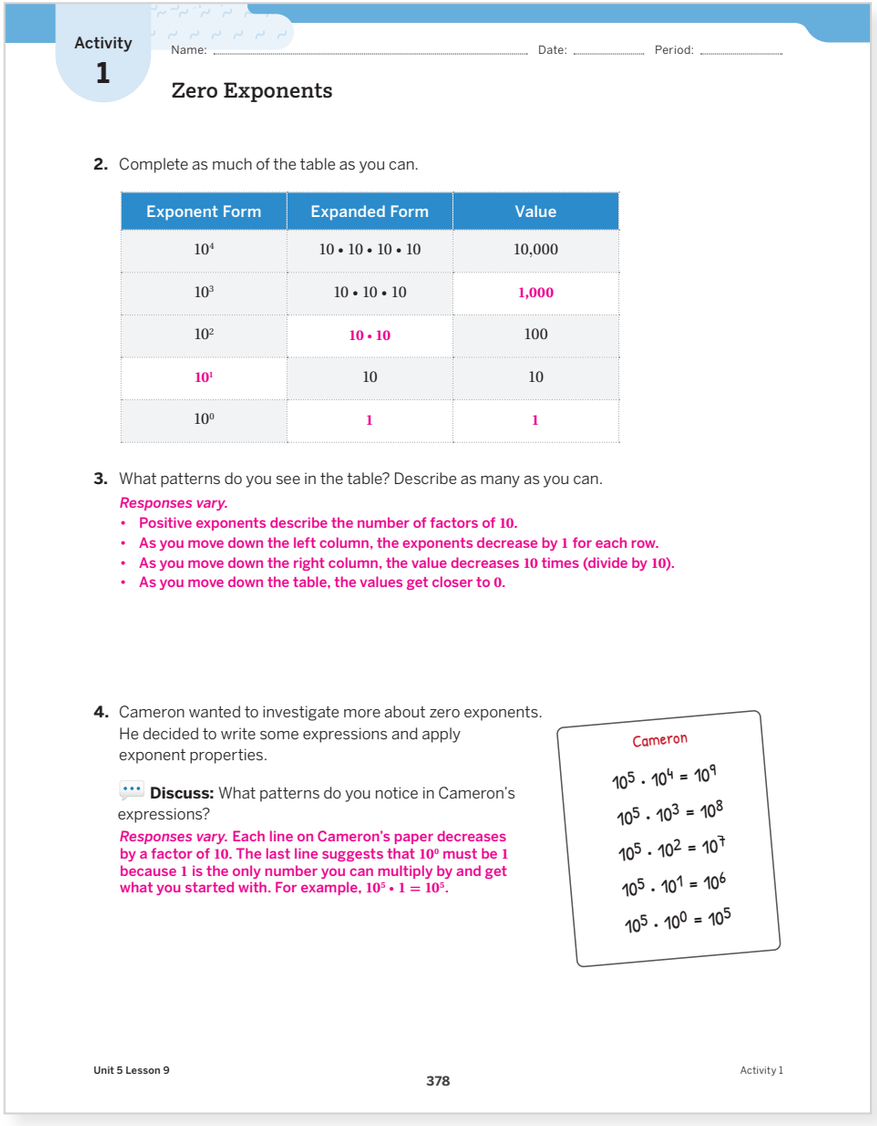
ELL English Language Learners Learners Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

3 Connect

Share student responses and invite students to share more patterns they found in the table.

To surface the Key Takeaway, consider asking:
- "What do you know about zero exponents?"
- "How can Cameron's work confirm what we know about zero exponents?"

Note: If time allows, invite students to update their table based on what they learned during the discussion.



Activity 1

Name: Date: Period:

Zero Exponents

2. Complete as much of the table as you can.

Exponent Form	Expanded Form	Value
10^4	$10 \cdot 10 \cdot 10 \cdot 10$	10,000
10^3	$10 \cdot 10 \cdot 10$	1,000
10^2	$10 \cdot 10$	100
10^1	10	10
10^0	1	1

3. What patterns do you see in the table? Describe as many as you can.

Responses vary.
- Positive exponents describe the number of factors of 10.
- As you move down the left column, the exponents decrease by 1 for each row.
- As you move down the right column, the value decreases 10 times (divide by 10).
- As you move down the table, the values get closer to 0.

4. Cameron wanted to investigate more about zero exponents. He decided to write some expressions and apply exponent properties.

Discuss: What patterns do you notice in Cameron's expressions?

Responses vary. Each line on Cameron's paper decreases by a factor of 10. The last line suggests that 10^0 must be 1 because 1 is the only number you can multiply by and get what you started with. For example, $10^5 \cdot 1 = 10^5$.

Cameron

$10^5 \cdot 10^4 = 10^9$

$10^5 \cdot 10^3 = 10^8$

$10^5 \cdot 10^2 = 10^7$

$10^5 \cdot 10^1 = 10^6$

$10^5 \cdot 10^0 = 10^5$

Unit 5 Lesson 9 378 Activity 1

Key Takeaway: $10^0 = 1$, and any non-zero whole number raised to the power of zero is 1.

Pairs | 15 min

Activity 2 Beyond 10

Purpose: Students consider additional exponential expressions in exponent form, in expanded form, and by value to surface properties of zero exponents.

Short on time? Consider inviting students in Problem 6 to determine just one pair of equivalent expressions.

1 Launch

To surface connections to the powers of 10 table, consider asking, "What do you see that is the same as Activity 1? What is different?" **(MTR.5.1)**

2 Monitor

Invite students to share their strategies and solutions with a partner. If there is disagreement, consider capturing these ideas to further facilitate a whole-class discussion. **(MTR.4.1)**

D Differentiation *(Problems 5–6)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 5)*	**Support** Consider asking, "What patterns do you notice from the table with powers of 10? How can you apply those patterns here?" **(MTR.5.1)**
Understand that 3^0 is equal to 1. *(Problem 5)*	**Strengthen** Consider asking, "Can you generalize this idea to other whole-number bases?"
Match all pairs of equivalent expressions. *(Problem 6)*	**Stretch** Invite students to write down any additional pairs of equivalent expressions they can think of.

Look for students who respond "Yes" or "I'm not sure." Consider asking them:

- "What is 9^0 equal to?"
- "What is 99^0 equal to?"
- "What patterns did you notice in the table from Problem 5 that could convince you that the expressions are equivalent?"

3 Connect

Invite students to share strategies they used to match the expressions in Problem 6.

Activity 2

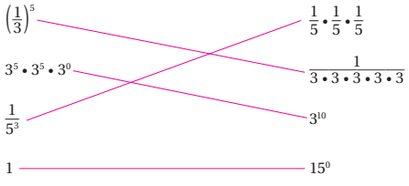
Name: Date: Period:

Beyond 10

5. Complete this new table about powers of 3.

Exponent Form	Expanded Form	Value
3^3	$3 \cdot 3 \cdot 3$	27
3^2	$3 \cdot 3$	9
3^1	3	3
3^0	1	1

6. Match each expression to an equivalent expression.



7. Look at the two expressions shown.

9^0 99^0

Are these equivalent? Circle one.

(Yes) No I'm not sure.

Explain your thinking.

Responses vary. These are equivalent because they both have zero exponents. Both expressions are equal to 1.

Unit 5 Lesson 9 379 Activity 2

Whole Class | 5 min

Synthesis

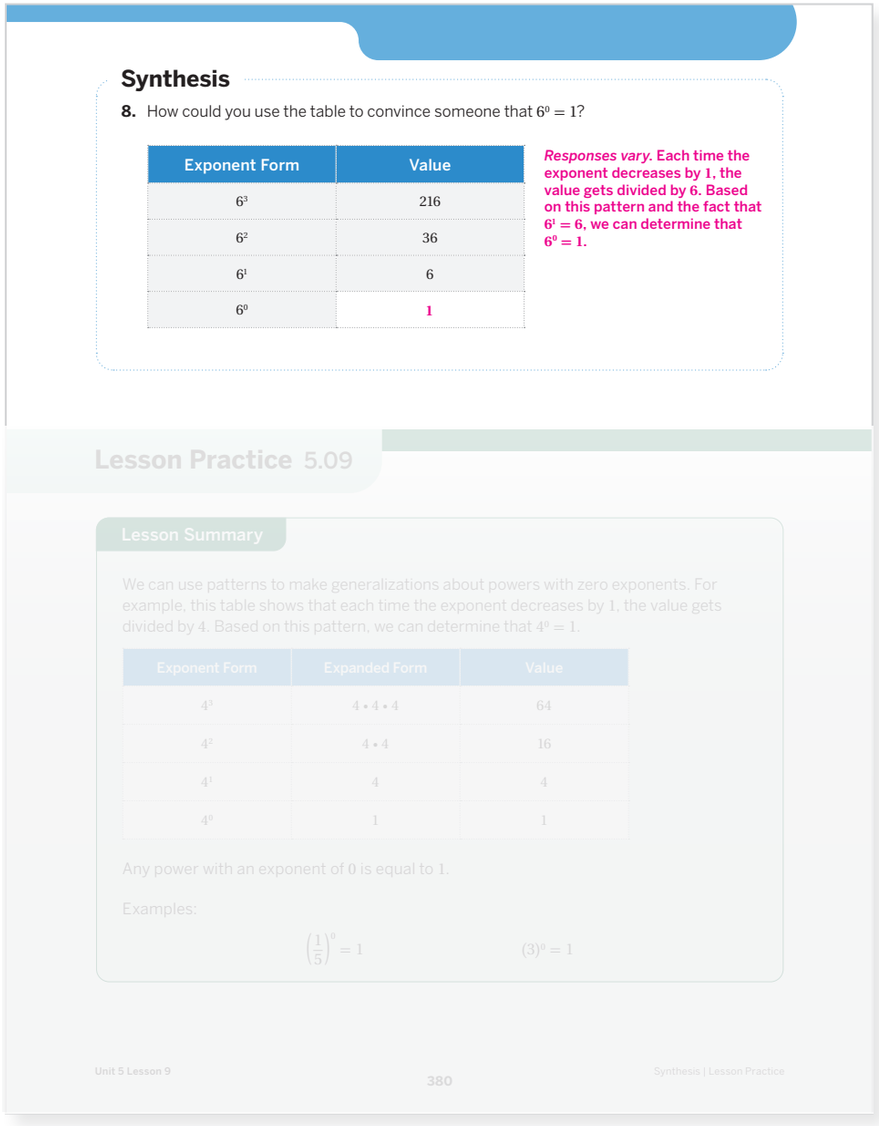
Purpose: Students synthesize their understanding of zero exponents.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using the pattern of division in the table to explain why 6^0 is equal to 1.
- Using properties of exponents such as $\frac{6^3}{6^3} = 1$ to show that $6^0 = 1$.

Create an anchor chart that generalizes the idea that any non-zero whole number that is raised to the power of 0 is equal to 1.

Note: If time allows, invite students to revise their previous thinking based on the discussion. Also, invite students to share any questions that they may still have.



Synthesis

8. How could you use the table to convince someone that $6^0 = 1$?

Exponent Form	Value
6^3	216
6^2	36
6^1	6
6^0	1

Responses vary. Each time the exponent decreases by 1, the value gets divided by 6. Based on this pattern and the fact that $6^1 = 6$, we can determine that $6^0 = 1$.

Lesson Practice 5.09

Lesson Summary

We can use patterns to make generalizations about powers with zero exponents. For example, this table shows that each time the exponent decreases by 1, the value gets divided by 4. Based on this pattern, we can determine that $4^0 = 1$.

Exponent Form	Expanded Form	Value
4^3	$4 \cdot 4 \cdot 4$	64
4^2	$4 \cdot 4$	16
4^1	4	4
4^0	1	1

Any power with an exponent of 0 is equal to 1.

Examples:

$\left(\frac{1}{5}\right)^0 = 1$ $\quad$ $(3)^0 = 1$

Unit 5 Lesson 9 380 Synthesis | Lesson Practice

Lesson Takeaway: Any non-zero base raised to the 0 power will result in 1.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying all expressions that are equivalent to a power of 10.

Today's Goals

1. **Goal:** Use patterns to determine that any power with an exponent of 0 is equal to 1.
2. **Goal:** Determine if expressions involving positive and zero exponents are equivalent.
3. **Language Goal:** Describe patterns in tables with positive and zero exponents. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.09

Select *all* the expressions that are equivalent to 10^6.

- [x] A. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
- [ ] B. $10^3 + 10^3$
- [ ] C. 60
- [ ] D. $10^3 \cdot 10^2$
- [x] E. $\frac{10^9}{10^3}$

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support applying exponent rules to identify equivalent forms of powers of 10:

» **Respond to Student Thinking:**

- Revisit the **Activity 2** table and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

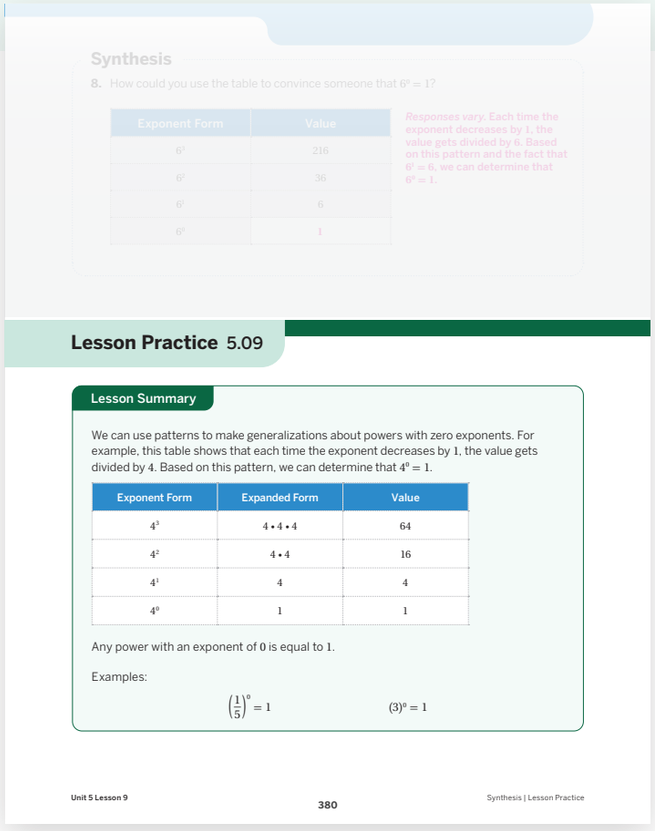
Professional Learning

Take a moment and reflect on how students engaged with Mathematical Thinking and Reasoning Standard 5.1 in today's lesson. In what ways did the students use patterns and structure of the expressions of powers of 10 to analyze the expressions of powers of 3?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. How could you use the table to convince someone that $6^0 = 1$?

Exponent Form	Value
6^3	216
6^2	36
6^1	6
6^0	1

Responses vary. Each time the exponent decreases by 1, the value gets divided by 6. Based on this pattern and the fact that $6^1 = 6$, we can determine that $6^0 = 1$.

Lesson Practice 5.09

Lesson Summary

We can use patterns to make generalizations about powers with zero exponents. For example, this table shows that each time the exponent decreases by 1, the value gets divided by 4. Based on this pattern, we can determine that $4^0 = 1$.

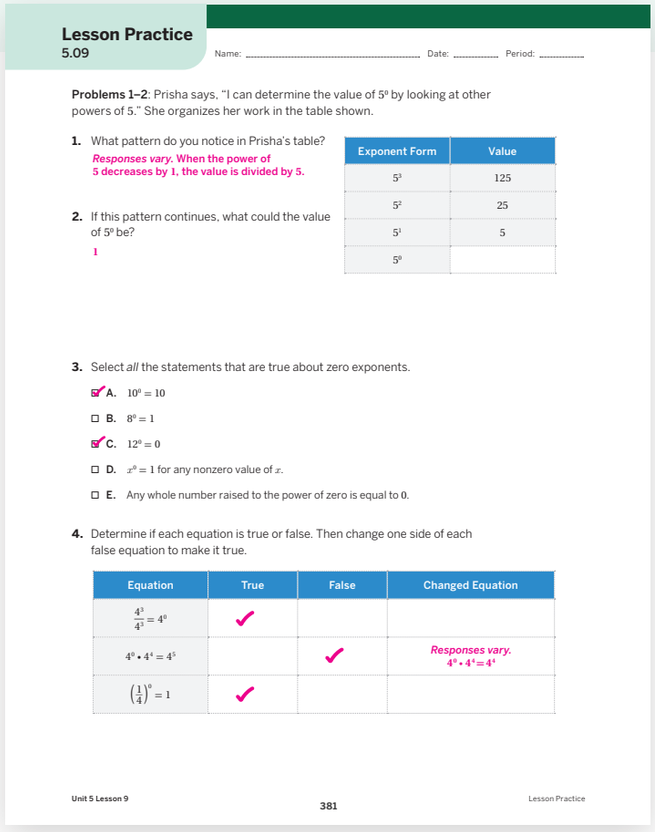
Exponent Form	Expanded Form	Value
4^3	$4 \cdot 4 \cdot 4$	64
4^2	$4 \cdot 4$	16
4^1	4	4
4^0	1	1

Any power with an exponent of 0 is equal to 1.

Examples:

$\left(\frac{1}{5}\right)^0 = 1$ $\quad$ $(3)^0 = 1$

Unit 5 Lesson 9 380 Synthesis | Lesson Practice



Lesson Practice 5.09

Name: _____ Date: _____ Period: _____

Problems 1–2: Prisha says, "I can determine the value of 5^0 by looking at other powers of 5." She organizes her work in the table shown.

1. What pattern do you notice in Prisha's table?

Responses vary. When the power of 5 decreases by 1, the value is divided by 5.

2. If this pattern continues, what could the value of 5^0 be?

1

Exponent Form	Value
5^3	125
5^2	25
5^1	5
5^0	

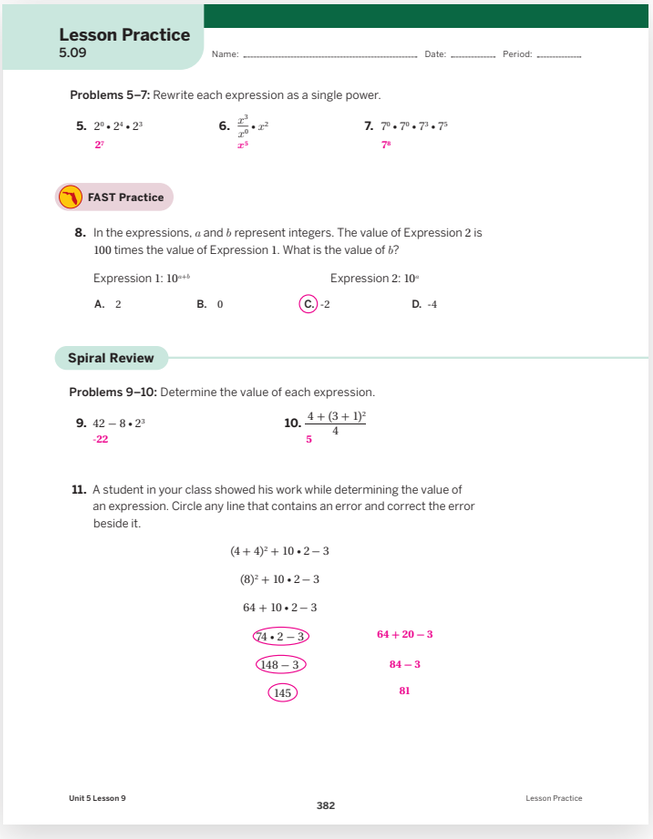
3. Select *all* the statements that are true about zero exponents.

- ☑ A. $10^0 = 10$
- ☐ B. $8^0 = 1$
- ☑ C. $12^0 = 0$
- ☐ D. $x^0 = 1$ for any nonzero value of x.
- ☐ E. Any whole number raised to the power of zero is equal to 0.

4. Determine if each equation is true or false. Then change one side of each false equation to make it true.

Equation	True	False	Changed Equation
$\frac{4^3}{4^3} = 4^0$	✓		
$4^0 \cdot 4^4 = 4^5$		✓	*Responses vary.* $4^0 \cdot 4^4 = 4^4$
$\left(\frac{1}{4}\right)^0 = 1$	✓		

Unit 5 Lesson 9 381 Lesson Practice



Lesson Practice 5.09

Name: _____ Date: _____ Period: _____

Problems 5–7: Rewrite each expression as a single power.

5. $2^0 \cdot 2^4 \cdot 2^3$ — 2^7

6. $\frac{x^3}{x^0} \cdot x^2$ — x^5

7. $7^0 \cdot 7^0 \cdot 7^3 \cdot 7^5$ — 7^8

FAST Practice

8. In the expressions, a and b represent integers. The value of Expression 2 is 100 times the value of Expression 1. What is the value of b?

Expression 1: 10^{a+b} $\quad$ Expression 2: 10^a

A. 2 $\quad$ B. 0 $\quad$ C. -2 $\quad$ D. -4

Spiral Review

Problems 9–10: Determine the value of each expression.

9. $42 - 8 \cdot 2^3$ — -22

10. $\frac{4 + (3 + 1)^2}{4}$ — 5

11. A student in your class showed his work while determining the value of an expression. Circle any line that contains an error and correct the error beside it.

$(4 + 4)^2 + 10 \cdot 2 - 3$

$(8)^2 + 10 \cdot 2 - 3$

$64 + 10 \cdot 2 - 3$

$74 \cdot 2 - 3$ $\quad$ $64 + 20 - 3$

$148 - 3$ $\quad$ $84 - 3$

145 $\quad$ 81

Unit 5 Lesson 9 382 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.NSO.1.1
	5–7	1	MA.7.NSO.1.1
FAST Practice	8	3	MA.7.NSO.1.1
Spiral Review			
	9–10	2	MA.7.NSO.2.1
	11	3	MA.7.NSO.2.1

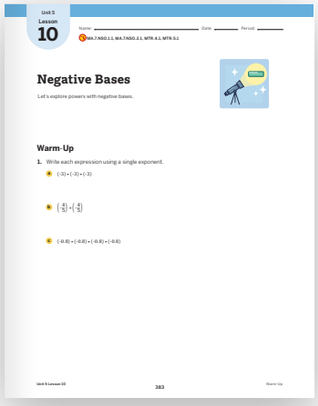
Unit 5

Lesson 10

Negative Bases

Powers With Negative Bases

Let's explore powers with negative bases.


Unit 5
Lesson
10
Name _____ Date _____ Period _____
MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1
Negative Bases
Let's explore powers with negative bases.
Warm-Up
1. Write each expression using a single exponent.
a. $(-3)\cdot(-3)\cdot(-3)$
b. $\left(-\frac{1}{4}\right)\cdot\left(-\frac{1}{4}\right)$
c. $(-0.8)\cdot(-0.8)\cdot(-0.8)\cdot(-0.8)$
383

Learning Outcomes

Today's Goals

1. **Goal:** Evaluate numerical expressions with negative rational bases.
2. **Goal:** Determine if exponential numerical expressions are equivalent to a negative or positive value.
3. **Language Goal:** Describe patterns in tables with negative rational bases and discuss the importance of parentheses. **(Reading, Writing, Speaking, and Listening)**

Students use patterns to develop an understanding of negative bases. They use laws of exponents to evaluate numerical expressions and generate equivalent numerical expressions with negative rational numbers having whole-number exponents. Students then identify patterns with odd- and even-numbered exponents and investigate the importance of parentheses when working with negative bases.

Coherence

Prior Learning

In Lesson 9, students wrote exponential expressions in expanded form and identified patterns to reason and generalize about zero exponents.

Future Learning

In Unit 6, students will represent different contextual situations with equations.

Connections

- Students investigate patterns and explore powers with negative bases. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.1.1
Know and apply the Law of Exponents to evaluate numerical expressions and generate equivalent expressions, limited to whole-number exponents and rational number bases.

Also Addressing: **MA.7.NSO.2.1**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.3	**MA.8.NSO.1.3**
MA.6.NSO.3.4	**MA.8.AR.1.1**

Lesson at a Glance

~ 45 min

MA.7.NSO.1.1, MA.7.NSO.2.1

Warm-Up

Pairs | 5 min

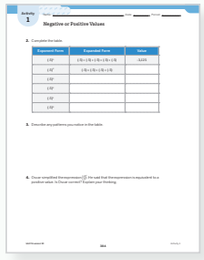
Students write repeated multiplication expressions as a power using a single exponent.



Activity 1

Pairs | 15 min

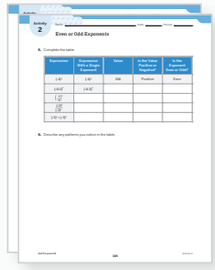
Students evaluate exponential expressions with negative bases and determine if the result is negative or positive.
Routine: Think-Pair-Share



Activity 2

Pairs | 15 min

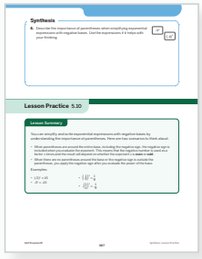
Students describe patterns in expressions with odd- and even-numbered exponents, and investigate how parentheses can affect the values of expressions that contain negative signs. **Routine:** Decide and Defend



Synthesis

Whole Class | 5 min

Students synthesize their understanding of simplifying exponential expressions with negative bases by describing the importance of parentheses.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by evaluating numerical expressions with negative rational bases.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students write repeated multiplication expressions as a power using a single exponent.

1 Launch

To surface prior knowledge about multiplying integers, consider asking:

- "Is the product of two negative numbers positive or negative?"
- "Is the product of a negative number and a positive number positive or negative?"

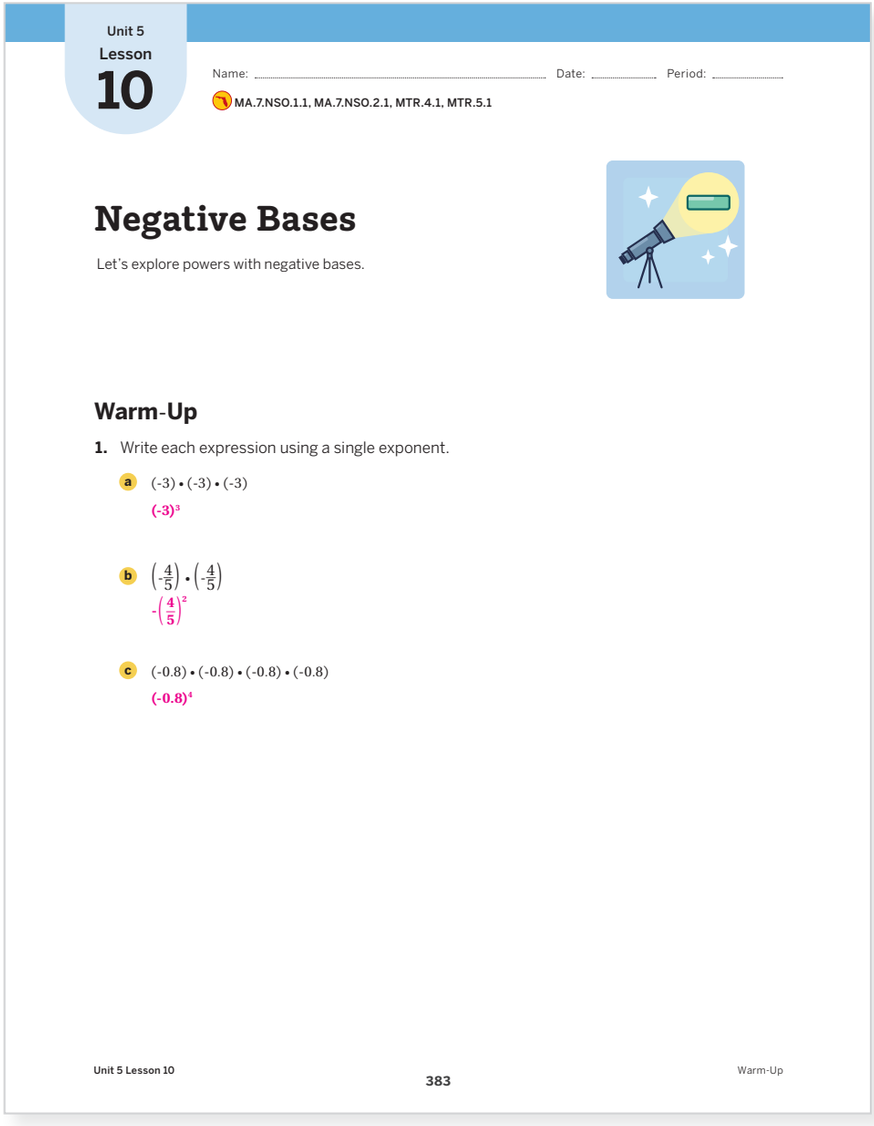
Invite students to write the first expression independently and then share their strategy or thinking aloud with a partner.

2 Connect

Invite students to compare and contrast the different expressions.

Consider asking:

- "Is the expression with a single exponent still equivalent to the original? How do you know?"
- "What patterns do you notice?"
- "Why would you want to write the expressions as a single power?"



Unit 5
Lesson
10

Name: _____ Date: _____ Period: _____

MA.7.NSO.1.1, MA.7.NSO.2.1, MTR.4.1, MTR.5.1

Negative Bases

Let's explore powers with negative bases.

Warm-Up

1. Write each expression using a single exponent.

a. $(-3) \cdot (-3) \cdot (-3)$

$(-3)^3$

b. $\left(-\frac{4}{5}\right) \cdot \left(-\frac{4}{5}\right)$

$-\left(\frac{4}{5}\right)^2$

c. $(-0.8) \cdot (-0.8) \cdot (-0.8) \cdot (-0.8)$

$(-0.8)^4$

Unit 5 Lesson 10 383 Warm-Up

Pairs | 15 min

Activity 1 Negative or Positive Values

Purpose: Students evaluate exponential expressions with negative bases and determine if the result is negative or positive.

1 Launch

To support students getting started, consider having students analyze the first row in the table and pause for any questions students may have.

A **Accessibility: Memory and Attention** Invite students to use additional paper or a calculator to support them with performing calculations.

2 Monitor

Look for a variety of patterns students notice in the table to share during the Connect.

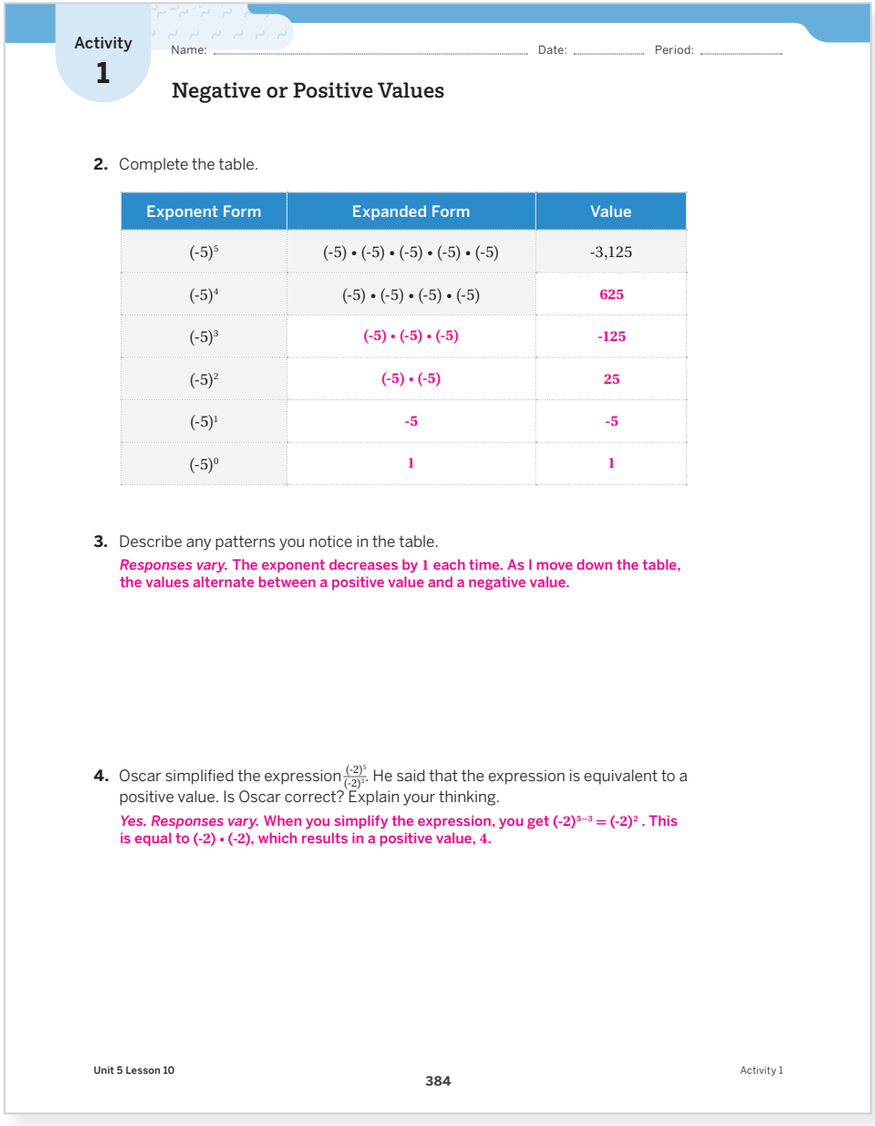
Listen for students who identify a relationship between the exponent and whether the value is positive or negative. **(MTR.5.1)**

Use the Think-Pair-Share routine to support student-to-student discourse about any patterns they notice in the table. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class. **(MTR.4.1)**

3 Connect

Invite students to share their responses. Consider asking, "What reasoning supports your claim?"

Create a class anchor chart for powers with negative bases and display it with examples to support students' reasoning.



Activity 1

Name: Date: Period:

Negative or Positive Values

2. Complete the table.

Exponent Form	Expanded Form	Value
$(-5)^5$	(-5) • (-5) • (-5) • (-5) • (-5)	-3,125
$(-5)^4$	(-5) • (-5) • (-5) • (-5)	625
$(-5)^3$	(-5) • (-5) • (-5)	-125
$(-5)^2$	(-5) • (-5)	25
$(-5)^1$	-5	-5
$(-5)^0$	1	1

3. Describe any patterns you notice in the table.

Responses vary. The exponent decreases by 1 each time. As I move down the table, the values alternate between a positive value and a negative value.

4. Oscar simplified the expression $\frac{(-2)^5}{(-2)^3}$. He said that the expression is equivalent to a positive value. Is Oscar correct? Explain your thinking.

Yes. Responses vary. When you simplify the expression, you get $(-2)^{5-3} = (-2)^2$. This is equal to (-2) • (-2), which results in a positive value, 4.

Unit 5 Lesson 10 384 Activity 1

Pairs | 15 min

Activity 2 Even or Odd Exponents

Purpose: Students describe patterns in expressions with odd- and even-numbered exponents and investigate how parentheses can affect the values of expressions that contain negative signs.

Short on time? Consider completing Problems 5–6 as a class.

1 Launch

Invite students to make a prediction about whether the value of an expression will be odd or even. **(MTR.5.1)**

Consider asking, "Which expressions will you need to rewrite with a single exponent?"

2 Monitor

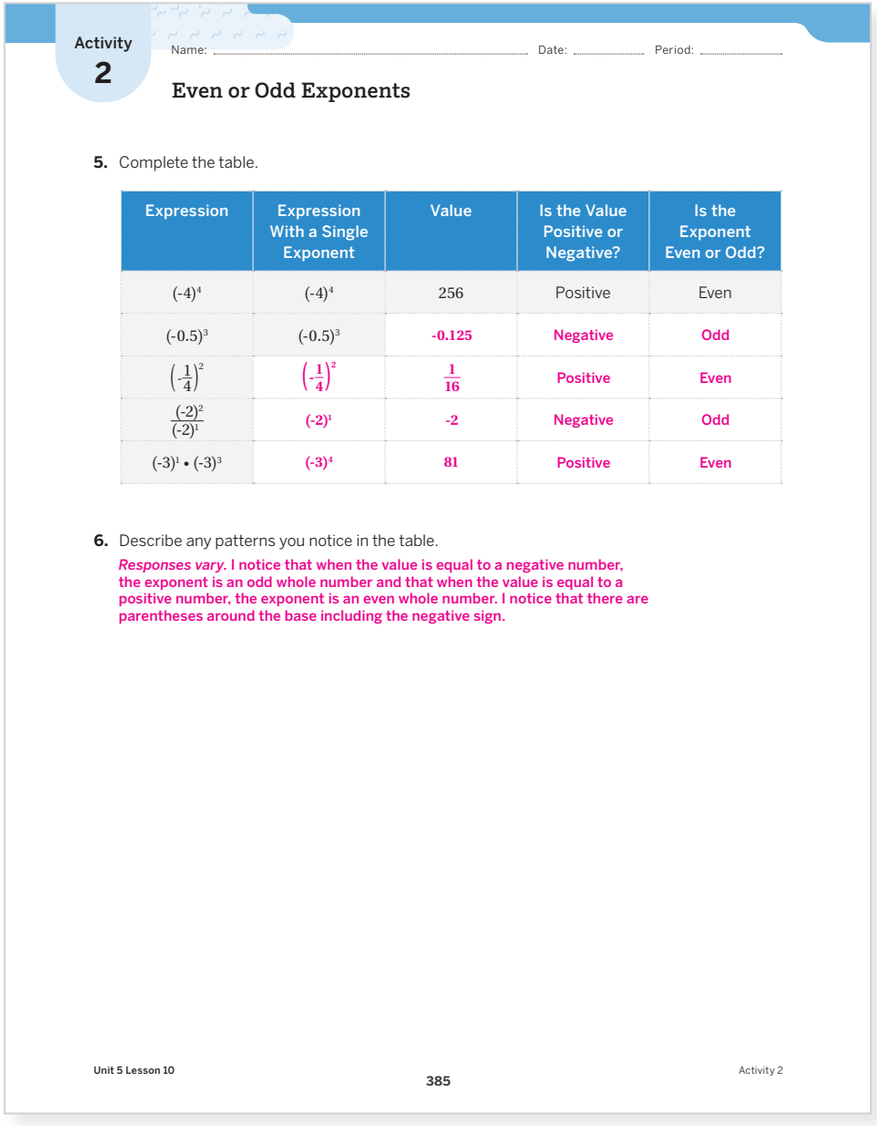
Look for student strategies, listen to small-group discussions, and offer help or encouragement.

D Differentiation *(Problems 5–6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to review the Warm-Up activity to activate prior knowledge and refer to the anchor chart showing examples of powers with negative bases.
Think the value of $(-0.5)^3$ is positive or need support with determining the value. *(Problem 5)*	**Strengthen** Invite them to write the expression using repeated multiplication and think about the rules for multiplying integers.
Recognize that each expression uses parentheses that include the negative base.	**Stretch** Invite students to write different expressions that use negative bases, but without parentheses, and evaluate them.

ELL English Language Learners Encourage students to paraphrase each other's ideas to help make connections and incorporate math vocabulary from the unit. **(Speaking and Listening)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Even or Odd Exponents

5. Complete the table.

Expression	Expression With a Single Exponent	Value	Is the Value Positive or Negative?	Is the Exponent Even or Odd?
$(-4)^4$	$(-4)^4$	256	Positive	Even
$(-0.5)^3$	$(-0.5)^3$	-0.125	Negative	Odd
$\left(-\frac{1}{4}\right)^2$	$\left(-\frac{1}{4}\right)^2$	$\frac{1}{16}$	Positive	Even
$\frac{(-2)^2}{(-2)^1}$	$(-2)^1$	-2	Negative	Odd
$(-3)^1 \cdot (-3)^3$	$(-3)^4$	81	Positive	Even

6. Describe any patterns you notice in the table.

Responses vary. I notice that when the value is equal to a negative number, the exponent is an odd whole number and that when the value is equal to a positive number, the exponent is an even whole number. I notice that there are parentheses around the base including the negative sign.

Unit 5 Lesson 10 385 Activity 2

Pairs | 15 min

Activity 2 Even or Odd Exponents (continued)

Purpose: Students describe patterns in expressions with odd- and even-numbered exponents and investigate how parentheses can affect the value of expressions that contain negative signs.

Short on time? Consider completing Problems 5–6 as a class.

2 Monitor

To support students in making connections, consider highlighting different parts of the expressions in different colors to emphasize and visualize the different parts (e.g,. base, exponents, factors).

Consider sharing that -2^3 is the opposite of 2^3.

Listen for explanations to share during the Connect.

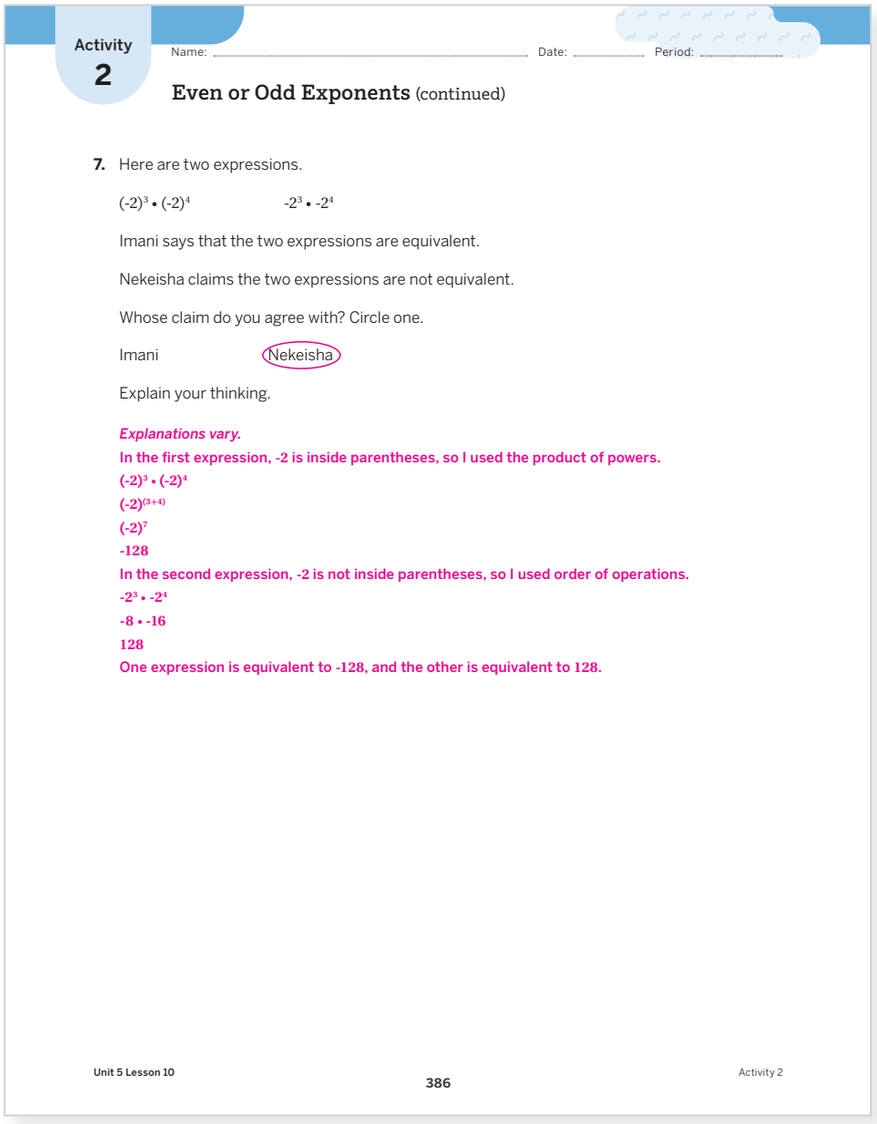
3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Display the distribution of responses by polling the class.

To surface the Key Takeaway, consider asking:

- "How does the placement of the parentheses in an exponential expression change the result?"
- "When a negative base is inside parentheses, how do you know if its value is a positive number?"
- "When a negative base is inside parentheses, how do you know if its value is a negative number?"



Activity 2

Name: Date: Period:

Even or Odd Exponents (continued)

7. Here are two expressions.

$(-2)^3 \cdot (-2)^4$ $-2^3 \cdot -2^4$

Imani says that the two expressions are equivalent.

Nekeisha claims the two expressions are not equivalent.

Whose claim do you agree with? Circle one.

Imani Nekeisha

Explain your thinking.

Explanations vary.

In the first expression, -2 is inside parentheses, so I used the product of powers.

$(-2)^3 \cdot (-2)^4$

$(-2)^{(3+4)}$

$(-2)^7$

-128

In the second expression, -2 is not inside parentheses, so I used order of operations.

$-2^3 \cdot -2^4$

$-8 \cdot -16$

128

One expression is equivalent to -128, and the other is equivalent to 128.

Unit 5 Lesson 10 386 Activity 2

Key Takeaway: When a negative base is inside parentheses, raise the negative number to the power. The sign of the result will depend on whether the exponent is even or odd.

Whole Class | 5 min

Synthesis

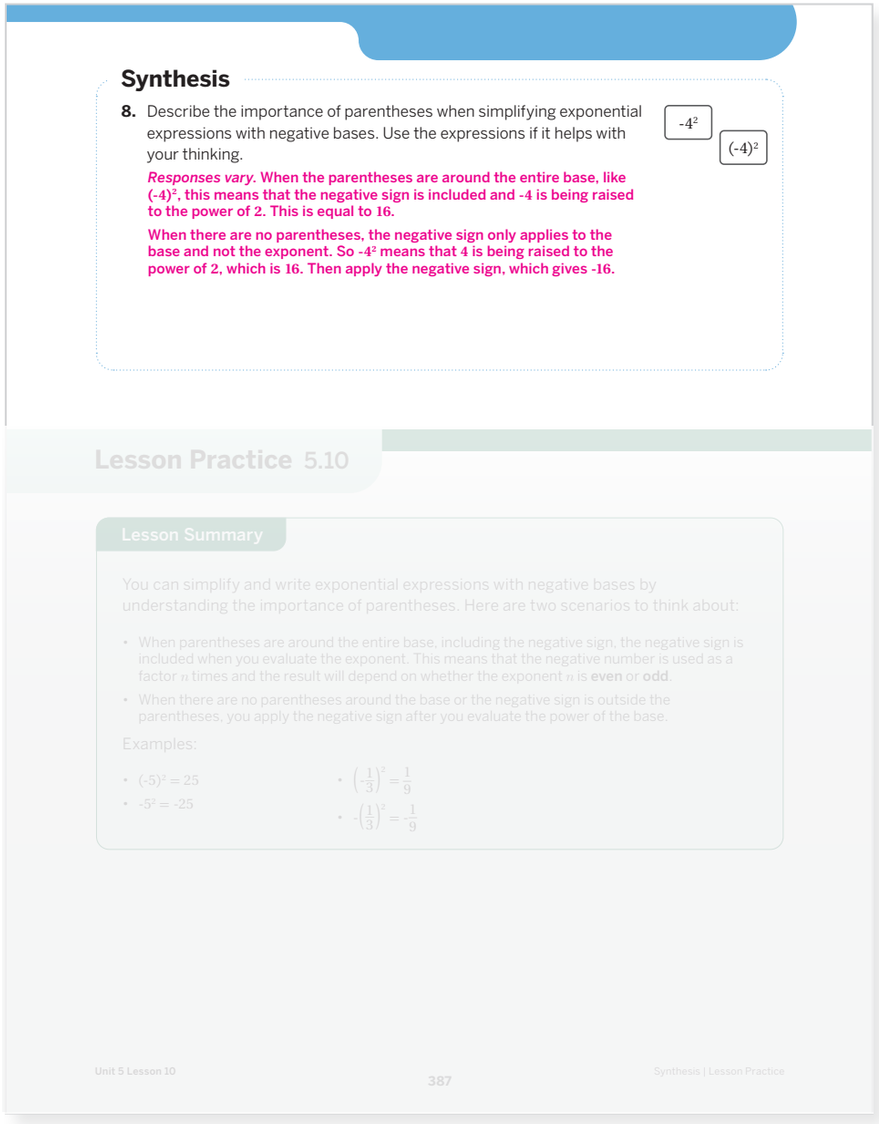
Purpose: Students synthesize their understanding of simplifying exponential expressions with negative bases by describing the importance of parentheses.

Invite students to respond independently, and then share their thinking with a partner.

Collect and Share a variety of ideas, such as:

- When parentheses are around the negative base, the negative number is raised to the power.
- When there are no parentheses, the negative sign is applied after you calculate the power.
- With parentheses, if the exponent is even, the result is positive. If the exponent is odd, the result is negative.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. Describe the importance of parentheses when simplifying exponential expressions with negative bases. Use the expressions if it helps with your thinking.

-4^2

$(-4)^2$

Responses vary. When the parentheses are around the entire base, like $(-4)^2$, this means that the negative sign is included and -4 is being raised to the power of 2. This is equal to 16.

When there are no parentheses, the negative sign only applies to the base and not the exponent. So -4^2 means that 4 is being raised to the power of 2, which is 16. Then apply the negative sign, which gives -16.

Lesson Practice 5.10

Lesson Summary

You can simplify and write exponential expressions with negative bases by understanding the importance of parentheses. Here are two scenarios to think about:

- When parentheses are around the entire base, including the negative sign, the negative sign is included when you evaluate the exponent. This means that the negative number is used as a factor n times and the result will depend on whether the exponent n is **even** or **odd**.
- When there are no parentheses around the base or the negative sign is outside the parentheses, you apply the negative sign after you evaluate the power of the base.

Examples:

- $(-5)^2 = 25$
- $-5^2 = -25$
- $\left(-\frac{1}{3}\right)^2 = \frac{1}{9}$
- $-\left(\frac{1}{3}\right)^2 = -\frac{1}{9}$

Unit 5 Lesson 10 387 Synthesis | Lesson Practice

Lesson Takeaway: The value of a power with a negative base is positive when the exponent is even and negative when the exponent is odd. When a power contains a negative sign, the placement of parentheses can affect its value.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by evaluating numerical expressions with negative rational bases.

Today's Goals

1. **Goal:** Evaluate numerical expressions with negative rational bases.
2. **Goal:** Determine if exponential numerical expressions are equivalent to a negative or positive value.
3. **Language Goal:** Describe patterns in tables with negative rational bases and discuss the importance of parentheses. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.10

Show or explain why $-4^2 \cdot (-4)^2$ is *not* equal to $(-4)^4$.

-4^2 is equal to -16, since the negative sign is applied after raising 4 to the power of 2.
$(-4)^2$ is equal to 16, since the negative sign is inside the parentheses.
When multiplying these two values together the result is -256.
Since $(-4)^4$ has an even numbered exponent, the result will be positive, not negative.

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding powers with negative bases:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

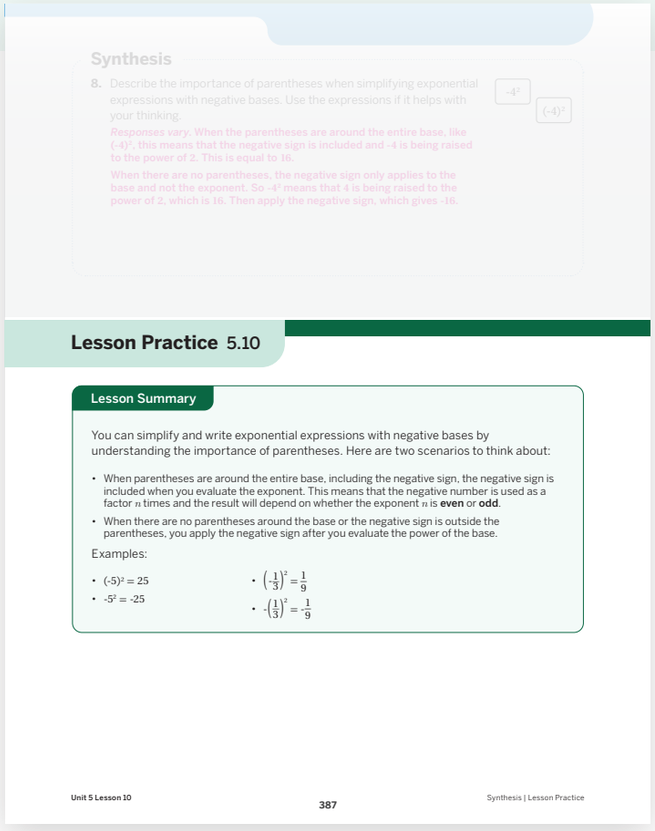
Professional Learning

Take a moment to reflect on how students' understanding of the laws of exponents has developed throughout this unit. How did their previous knowledge help them recognize patterns in this lesson? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. Describe the importance of parentheses when simplifying exponential expressions with negative bases. Use the expressions if it helps with your thinking.

-4^2 $(-4)^2$

Responses vary. When the parentheses are around the entire base, like $(-4)^2$, this means that the negative sign is included and -4 is being raised to the power of 2. This is equal to 16.

When there are no parentheses, the negative sign only applies to the base and not the exponent. So -4^2 means that 4 is being raised to the power of 2, which is 16. Then apply the negative sign, which gives -16.

Lesson Practice 5.10

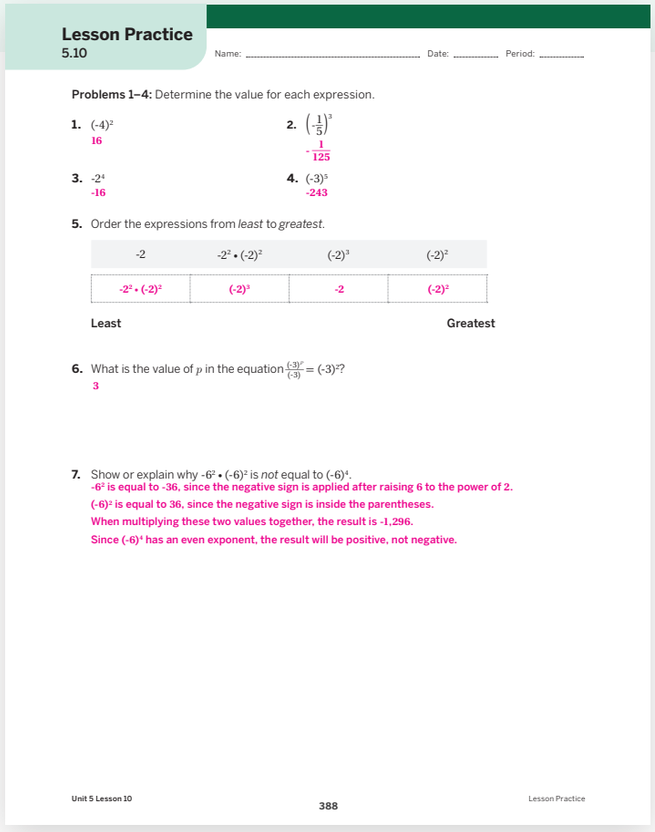
Lesson Summary

You can simplify and write exponential expressions with negative bases by understanding the importance of parentheses. Here are two scenarios to think about:

- When parentheses are around the entire base, including the negative sign, the negative sign is included when you evaluate the exponent. This means that the negative number is used as a factor n times and the result will depend on whether the exponent n is **even** or **odd**.
- When there are no parentheses around the base or the negative sign is outside the parentheses, you apply the negative sign after you evaluate the power of the base.

Examples:

- $(-5)^2 = 25$
- $-5^2 = -25$
- $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$
- $-\left(\frac{1}{3}\right)^2 = -\frac{1}{9}$



Lesson Practice 5.10

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine the value for each expression.

1. $(-4)^2$ — 16
2. $\left(\frac{1}{5}\right)^3$ — $\frac{1}{125}$
3. -2^4 — -16
4. $(-3)^5$ — -243

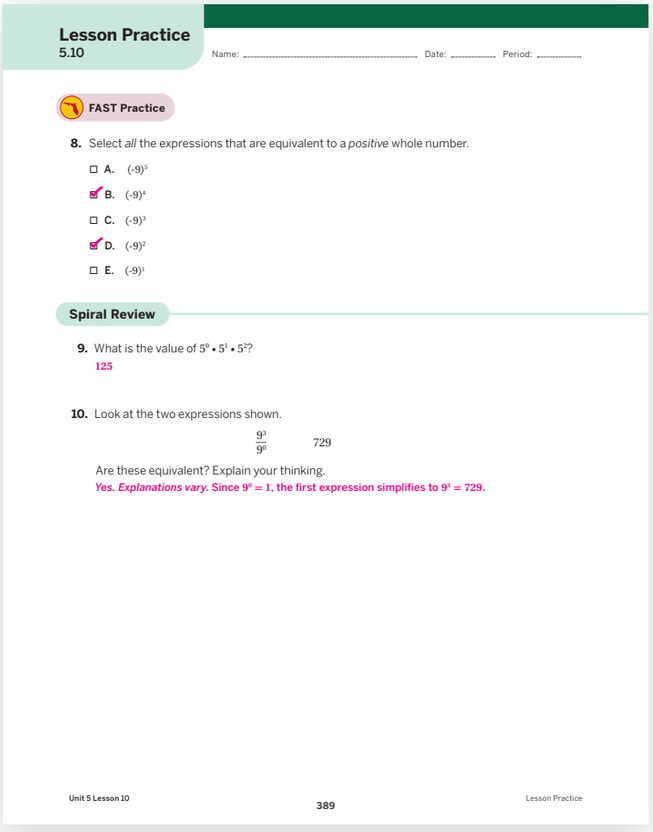
5. Order the expressions from *least* to *greatest*.

-2	$-2^2 \cdot (-2)^2$	$(-2)^3$	$(-2)^2$
$-2^2 \cdot (-2)^2$	$(-2)^3$	-2	$(-2)^2$

Least Greatest

6. What is the value of p in the equation $\frac{(-3)^p}{(-3)} = (-3)^2$?
3

7. Show or explain why $-6^2 \cdot (-6)^2$ is *not* equal to $(-6)^4$.
-6^2 is equal to -36, since the negative sign is applied after raising 6 to the power of 2.
$(-6)^2$ is equal to 36, since the negative sign is inside the parentheses.
When multiplying these two values together, the result is -1,296.
Since $(-6)^4$ has an even exponent, the result will be positive, not negative.



Lesson Practice 5.10

Name: _____ Date: _____ Period: _____

FAST Practice

8. Select *all* the expressions that are equivalent to a *positive* whole number.

- ☐ A. $(-9)^5$
- ☑ B. $(-9)^4$
- ☐ C. $(-9)^3$
- ☑ D. $(-9)^2$
- ☐ E. $(-9)^1$

Spiral Review

9. What is the value of $5^0 \cdot 5^1 \cdot 5^2$?
125

10. Look at the two expressions shown.

$\frac{9^3}{9^0}$ 729

Are these equivalent? Explain your thinking.
Yes. Explanations vary. Since $9^0 = 1$, the first expression simplifies to $9^3 = 729$.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.NSO.1.1
	5–6	2	MA.7.NSO.1.1
	7	3	MA.7.NSO.1.1
FAST Practice	8	1	MA.7.NSO.1.1
Spiral Review			
	9	1	MA.7.NSO.1.1
	10	2	MA.7.NSO.1.1

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying rational numbers that have the same value	Lesson 1	1	MA.7.NSO.1.2
2	Using the structure of order of operations to evaluate numerical expressions	Lessons 5	2	MA.7.NSO.2.1, MTR.5.1
3	Rewriting expressions with exponents as a single power	Lesson 6	2	MA.7.NSO.1.1, MTR.5.1
4	Writing expressions with exponents that are equivalent	Lesson 7	3	MA.7.NSO.1.1, MTR.2.1
5	Rewriting expressions with exponents as a single power	Lesson 8	2	MA.7.NSO.1.1, MTR.5.1
6a, 6b, 6c	Creating true statements using the properties of exponents	Lessons 8, 10	3	MA.7.NSO.1.1, MTR.5.1
7a, 7b, 7c	Explaining why any number to the 0 power is equal to 1	Lesson 9	3	MA.7.NSO.1.1, MTR.5.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 5

1. Select *all* the values equivalent to $-\frac{15}{9}$.

- ☐ **A.** -1.6
- ☑ **B.** $-1.\overline{6}$
- ☑ **C.** $-166.\overline{6}\%$
- ☐ **D.** $-1\frac{5}{9}$
- ☑ **E.** $-1\frac{2}{3}$

2. What is the value of $12 + 6 - 4\left(\frac{10}{5} - 7\right)^2$?

- (A.) -82
- **B.** -46
- **C.** 58
- **D.** 118

3. Rewrite each expression as a single power.

Expression	Single Power
$4^5 \cdot 4^8$	4^{13}
$5^3 \cdot 5^3 \cdot 5^3$	5^9
$(2^6)^7$	2^{42}
$(10^4)^4$	10^{16}

4. Write four different expressions that have the same value as $(2^3)^4$ using only numbers, multiplication, and exponents.

Expressions vary. Possible expressions are shown.

2^{12} $(2 \cdot 2 \cdot 2)^4$ 8^4 $8 \cdot 8 \cdot 8 \cdot 8$

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

5. Rewrite each expression as a single power.

Expression	Single Power
$\frac{4^9}{4^3 \cdot 4^4}$	4^2
$\frac{(5^1)^2}{5}$	5^7
$\frac{9^8 \cdot 9^3}{9^4}$	9^7
$\frac{3^4 \cdot 3^4 \cdot 3^4}{3 \cdot 3 \cdot 3}$	3^9

6. Write a number in each box so that each equation is true.

Answers vary. Possible exponents are shown.

A. $\frac{5^{\boxed{6}} \cdot 5^{\boxed{4}}}{5^{\boxed{2}}} = 5^8$

B. $(-5)^{\boxed{2}} \cdot (-5)^{\boxed{2}} = 5^{\boxed{4}}$

C. $\frac{5^{\boxed{3}} \cdot 5^{\boxed{5}}}{(5^{\square})^{\boxed{2}}} = 5^4$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

7. Complete the table below and use it to answer the following questions.

a

Expression	Value
4^3	64
4^2	16
4^1	4
4^0	1

b How could you use this table to convince someone that $4^0 = 1$?

Responses vary. Each time the exponent decreases by 1, the value is divided by 4. Based on the pattern, we can determine that $4^0 = 1$.

c Would this work with a number other than 4 as the base? Explain your reasoning.

Responses vary. Yes, it would work for any number. I tested it with a base of 5. The pattern of dividing by the base remained the same. Any nonzero number with an exponent of 0 will be equal to 1.

Name: Date: Period:

End-of-Unit Assessment

Unit 5

1. Select *all* of the values equivalent to $\frac{19}{6}$.

- ☑ **A.** $3\frac{1}{6}$
- ☐ **B.** $3\frac{1}{19}$
- ☑ **C.** $316.\overline{6}\%$
- ☑ **D.** $3.1\overline{6}$
- ☐ **E.** 3.16

2. What is the value of $15 + |3 - 8| - 4^2$?

- **A.** -6
- **B.** -3
- **C.** 4
- **D.** 10

3. Rewrite each expression as a single power.

Expression	Single Power
$5^6 \cdot 5^3$	5^9
$7^2 \cdot 7^2 \cdot 7^2$	7^6
$(3^5)^2$	3^{10}
$(6^4)^4$	6^{16}

4. Write four different expressions that have the same value as $(2^4)^3$ using only numbers, multiplication, and exponents.

Expressions vary. Possible expressions are shown.

2^{12} $(2 \cdot 2 \cdot 2 \cdot 2)^3$ 16^3 $16 \cdot 16 \cdot 16$

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

5. Rewrite each expression as a single power.

Expression	Single Power
$\frac{5^8}{5^3 \cdot 5^2}$	5^3
$\frac{(3^{11})^3}{3}$	3^{35}
$\frac{8^6 \cdot 8^2}{8^5}$	8^3
$\frac{7^5 \cdot 7^5 \cdot 7^5}{7 \cdot 7 \cdot 7}$	7^{12}

6. Write a number in each box so that each statement is true.

Answers vary. Possible exponents are shown.

A. $\frac{7^{\boxed{4}} \cdot 7^{\boxed{4}}}{7^{\boxed{2}}} = 7^{\boxed{3}}$

B. $(-6)^{\boxed{2}} \cdot (-6)^{\boxed{3}} = (-6)^{\boxed{5}}$

C. $\frac{9^{\boxed{6}} \cdot 9^{\boxed{5}}}{(9^{\boxed{2}})} = 9^{\boxed{7}}$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

7. Complete the table below and use it to answer the following questions.

a

Expression	Value
6^3	216
6^2	36
6^1	6
6^0	1

b How could you use this table to convince someone that $6^0 = 1$?

Responses vary. Each time the exponent decreases by 1, the value is divided by 6. Based on the pattern, we can determine that $6^0 = 1$.

c Would this work with a number other than 6 as the base? Explain your reasoning.

Responses vary. Yes, it would work for any number. I tested it with a base of 9. The pattern of dividing by the base remained the same. Any nonzero number with an exponent of 0 will be equal to 1.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: Use order of operations to evaluate numerical expressions. **(Lessons 1–5)**	**1**	**Support** • **Mini-Lesson:** *Dividing Integers* • **Teacher Move:** Consider revisiting Lesson 1 (*Code Breaking*)
	2	**Support** • **Teacher Move:** Consider revisiting Unit 5, Lesson 5 (*Order Matters*)

		Strengthen
		• **Repeated Challenges:**
		  • Lesson 2 (*Bumpers*)
		  • Lesson 3 (*Back in Time*)
		  • Lesson 4 (*Speeding Turtles*)

		Stretch • **You're invited to explore more:** • Lesson 3 (*Back in Time*)

Sub-Unit 2:
Apply the Laws of Exponents to generate equivalent expressions involving whole-number exponents
(Lessons 6–8)
	3, 4	**Support**
		Strengthen
		• **Mini-Lesson:** *Multiplying Powers*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 6 (*Combining Exponents*)
		  • Lesson 7 (*Power Pairs*)

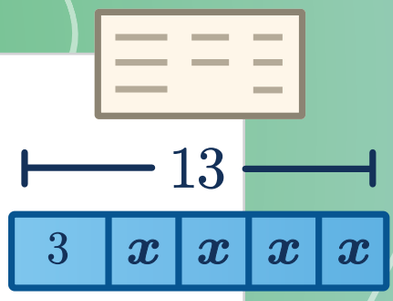
		Stretch
		• **You're invited to explore more:**
		  • Lesson 6 (*Combining Exponents*)
		  • Lesson 7 (*Power Pairs*)

	5, 6a, 6b, 6c	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 8 (*Rewriting Power*)
		  • Lesson 10 (*Negative Bases*)

		Strengthen • **Challenge Creator:** Lesson 8 (*Rewriting Powers*)
	7a, 7b, 7c	**Support** **Strengthen** • **Teacher Move:** Consider revisiting: • Lesson 9 (*Exponent Zero*)

UNIT 6

Expressions, Equations, and Inequalities

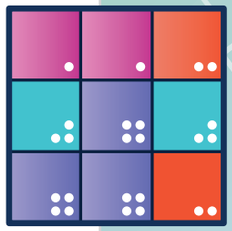


Students extend what they learned in Grade 6 about solving one-step equations to solve equations of the form $px + q = r$ and $p(x + q) = r$, and equations that include expanding, factoring, or adding terms. Students also solve inequalities and graph their solutions on a number line.

Essential Questions

- Which representations help you make sense of different mathematical situations?
- Which strategies for solving simple equations or inequalities can help you solve more complex ones?
- What are ways to increase efficiency when solving mathematical problems?

Sub-Unit Goals



Sub-Unit 1

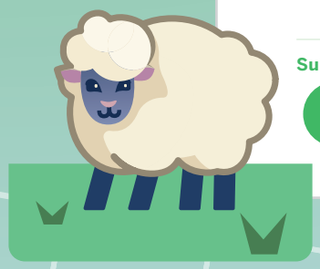
- Use tape diagrams to represent equations and situations in context and to determine unknown values.

Sub-Unit 2

- Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems.
- Write equivalent expressions by adding, subtracting, expanding, and factoring.

Sub-Unit 3

- Solve one-step inequalities with positive and negative coefficients.
- Create graphs that represent solutions to inequalities, including those with ≥ or ≤.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Add, subtract, multiply and divide rational numbers. **(MA.7.NSO.2)**

Rewrite algebraic expressions in equivalent forms. **(MA.7.AR.1)**

Write and solve equations and inequalities in one variable. **(MA.7.AR.2)**

Benchmarks Addressed in This Unit: MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.AR.1.1 MA.7.AR.1.2 MA.7.AR.2.1 MA.7.AR.2.2

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**

Also Addressed: **MTR.6.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students extend what they learned in Grade 6 about solving one-step equations to solve multistep equations, including expanding, factoring, or adding terms. Students also solve inequalities and graph their solutions on a number line.

Prior Learning

- **Grade 6,** Unit 6: Generate equivalent expressions **(MA.6.AR.1.4)**
- **Grade 6,** Unit 6: Use substitution to determine if a value is a solution to an equation or an inequality **(MA.6.AR.2.1)**
- **Grade 6,** Units 6 and 7: Write and solve one-step equations **(MA.6.AR.2.2, MA.6.AR.2.3)**
- **Grade 6,** Unit 7: Write and graph inequalities **(MA.6.AR.1.2)**

Future Learning

- **Grade 8,** Unit 4: Solve linear equations with variables on both sides **(MA.8.AR.2.1)**
- **Grade 8,** Unit 4: Solve two-step linear inequalities **(MA.8.AR.2.2)**
- **Grade 8,** Unit 4: Solve systems of linear equations **(MA.8.AR.4.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1 and 6**, students develop conceptual understanding of the effects of adding, subtracting, multiplying, and dividing positive and negative numbers on a submarine using floats and anchors.

Procedural Fluency

In **Lessons 3, 7, and 8**, students develop procedural fluency in performing operations with positive and negative numbers by solving number line problems represented by bumpers and the position, rate, and time of turtles.

Application

In **Lesson 16**, students apply their understanding of writing and solving inequalities to solve problems about situations in context.

Strategic Competence

In **Lesson 8**, students justify why their expressions and equations represent positive or negative values as it relates to the positions, rates, and times of turtles along the number line.

Adaptive Reasoning

In **Lessons 4 and 10**, students reflect on how they can apply operations with positive and negative numbers to solve number puzzles.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.AR.1.2 MA.6.AR.1.4
MA.6.AR.2.2 MA.6.AR.2.3
MA.6.AR.3.2
MTR.2.1

Sub-Unit 1

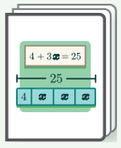


1 Smudged Receipts

Connecting Contexts and Tape Diagrams

Revisit tape diagrams and surface strategies for using them to determine unknown values in situations.

MA.7.NSO.2.3 MA.7.AR.2.2
MTR.2.1 MTR.5.1 MTR.7.1

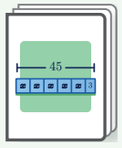


2 Equations

Representing Contexts With Tape Diagrams and Equations

Use knowledge about tape diagrams and situations to make sense of equations of non-proportional relationships.

MA.7.NSO.2.3 MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1

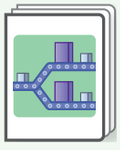


3 Seeing Structure

Practice With Tape Diagrams and Equations

Use structure to analyze and solve a question in context and interpret the meaning of its solution.

MA.7.NSO.2.3 MA.7.AR.2.2
MTR.2.1 MTR.5.1 MTR.7.1



8 Always-Equal Machines

Equivalent Expressions

Understand that equivalent expressions have the same value for any value of the variable.

MA.7.NSO.2.2 MA.7.AR.1.2
MTR.2.1 MTR.4.1 MTR.5.1



9 The Same but Different

Determining Whether Expressions Are Equivalent

Use algebra tiles, area models, and properties of operations to determine whether more complex expressions are equivalent.

MA.7.NSO.2.1 MA.7.NSO.2.2
MA.7.AR.1.2
MTR.2.1 MTR.4.1

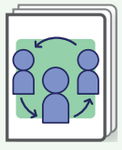


10 Collect the Squares

Adding and Subtracting Expressions

Write expressions with fewer terms by adding, subtracting, and expanding terms.

MA.7.NSO.2.1 MA.7.NSO.2.2
MA.7.AR.1.1
MTR.3.1 MTR.4.1 MTR.5.1



11 Pass the Equation

Solving Equations Multiple Ways

Solve equations using a variety of strategies.

MA.7.NSO.2.2 MA.7.AR.2.2
MTR.2.1 MTR.3.1 MTR.5.1



15 Shira the Sheep

Solving Inequalities With Positive and Negative Numbers

Solve inequalities with both positive and negative coefficients and connect the solutions of inequalities to their graphs.

MA.7.NSO.2.2 MA.7.AR.2.1
MTR.3.1 MTR.4.1 MTR.6.1



16 Write Them and Solve Them

Modeling With Inequalities

Write and solve inequalities and critically examine what the solutions to those inequalities mean in context.

MA.7.NSO.2.2 MA.7.AR.2.1
MTR.3.1 MTR.6.1 MTR.7.1

M Math in Action

Connecting Inequalities to Oranges

Explore the connection between inequalities and horticulture, as well as the contributions of Lue Gim Gong to the American citrus industry.

MA.7.AR.2.1 MA.7.AR.2.2
MA.7.NSO.2.2
MTR.2.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

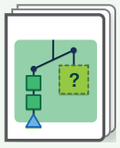
MA.7.AR.1.1 MA.7.AR.2.1
MA.7.AR.2.2 MA.7.AR.2.3
MA.7.AR.3.3
MTR.4.1 MTR.6.1 MTR.7.1

Pacing: 20 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 16 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2



4 Balancing Moves

Introduction to Balanced Hangers

Develop strategies for determining an unknown weight by creating balanced hangers with fewer objects.

MA.7.NSO.2.2 MA.7.AR.1.2
MTR.2.1 MTR.4.1 MTR.5.1

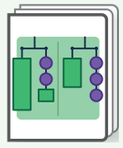


5 Balancing Equations

Solving Equations With Balanced Hangers

Understand strategies for solving an equation and connect those to balanced moves on a hanger.

MA.7.NSO.2.2 MA.7.AR.1.2
MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1

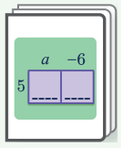


6 Keeping It True

Solving Equations

Describe and use strategies for solving equations of the forms $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers.

MA.7.NSO.2.2 MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1



7 Factoring and Expanding

Options for Solving One Equation

Rewrite expressions by **factoring** and **expanding**, and then solve equations in multiple ways and compare the results.

MA.7.NSO.2.2 MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1

Assess and Respond

Sub-Unit 3



12 Community Day

Using Equations to Solve Problems

Represent and answer questions about situations in context.

MA.7.NSO.2.2 MA.7.NSO.2.3
MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.7.1



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.AR.1.1 MA.7.AR.2.2
MA.7.AR.3.3
MTR.4.1 MTR.5.1 MTR.6.1



13 I Saw the Signs

Inequalities on the Number Line

Extend learning about graphing inequalities and using inequality symbols to describe situations.

Building Toward: MA.7.AR.2.1
MTR.2.1 MTR.4.1 MTR.5.1



14 Unbalanced Hangers

Solutions to Inequalities

Make sense of how to solve inequalities using hanger diagrams, a familiar representation.

MA.7.NSO.2.2 MA.7.AR.2.1
MTR.2.1 MTR.3.1 MTR.4.1

Pacing Considerations

Lessons 1–2: These lessons may be condensed into one lesson if students show a strong understanding of matching tape diagrams with expressions in the Pre-Unit Check.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1	Lesson 2	Lesson 3	Lesson 4
Copy: Show What You Know	**Copy:** Show What You Know **Materials:** graph paper *(as needed)*	**Copy:** Show What You Know	**Copy:** Show What You Know, Activity 2 Sheet *(one per student)* **Materials:** coloring tools *(optional)*, concrete model of a hanger *(optional)*
Lesson 5	**Lesson 6**	**Lesson 7**	**Lesson 8**
Copy: Show What You Know **Materials:** coloring tools *(as needed)*	**Copy:** Show What You Know	**Copy:** Show What You Know **Materials:** coloring tools *(as needed)*	**Copy:** Show What You Know
Lesson 9	**Lesson 10**	**Lesson 11**	**Lesson 12**
Copy: Show What You Know	**Copy:** Show What You Know	**Copy:** Show What You Know	**Copy:** Show What You Know, Activity 2 Sheet *(one per student)*
Lesson 13	**Lesson 14**	**Lesson 15**	**Lesson 16**
Copy: Show What You Know	**Copy:** Show What You Know	**Copy:** Show What You Know	**Copy:** Show What You Know **Cut:** Activity 2 Cards *(one per pair)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

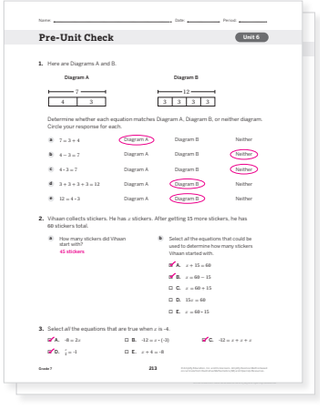
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.AR.1.2, MA.6.AR.1.4, MA.6.AR.2.2, MA.6.AR.2.3, MA.6.AR.3.2


Pre-Unit Check

Unit 6

1. Here are Diagrams A and B.

Diagram A

Diagram B

Determine whether each equation matches Diagram A, Diagram B, or neither diagram. Circle your response for each.

a. $7 = 3 + 4$	Diagram A	Diagram B	Neither
b. $4 - 3 = 7$	Diagram A	Diagram B	Neither
c. $4 \cdot 3 = 7$	Diagram A	Diagram B	Neither
d. $3 + 3 + 3 + 3 = 12$	Diagram A	Diagram B	Neither
e. $12 = 4 \cdot 3$	Diagram A	Diagram B	Neither

2. Vihaan collects stickers. He has x stickers. After getting 15 more stickers, he has 68 stickers total.

a. How many stickers did Vihaan start with?
45 stickers

b. Select all the equations that could be used to determine how many stickers Vihaan started with.
- A. $x + 15 = 68$
- B. $x = 68 - 15$
- C. $x - 68 = 15$
- D. $15x = 68$
- E. $x = 68 + 15$

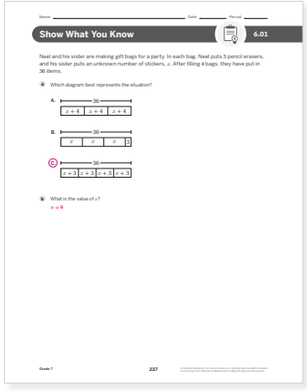
3. Select all the equations that are true when x is -4.
- A. $-8 = 2x$
- B. $-12 = x + (-3)$
- C. $-12 = x + x + x$
- D. $\frac{x}{-4} = 1$
- E. $x + 4 = -8$

Grade 7 213

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmark assessed on Lesson 1 Show What You Know: MA.7.AR.2.2


Show What You Know

6.01

Neel and his sister are making gift bags for a party. In each bag, Neel puts 3 pencil erasers, and his sister puts an unknown number of stickers, s. After filling 4 bags, they have put in 36 items.

a. Which diagram best represents the situation?

A.

B.

C.

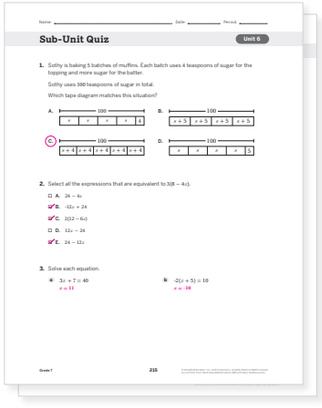
b. What is the value of s?
$s = 6$

Grade 7 237

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: MA.7.AR.1.1, MA.7.AR.2.2, MA.7.AR.3.3


Sub-Unit Quiz

Unit 6

1. Sothy is baking 5 batches of muffins. Each batch uses 4 teaspoons of sugar for the topping and more sugar for the batter.

Sothy uses 100 teaspoons of sugar in total.

Which tape diagram matches this situation?

A.

B.

C.

D.

2. Select all the expressions that are equivalent to $3(8 - 4x)$.
- A. $24 - 4x$
- B. $-12x + 24$
- C. $2(12 - 6x)$
- D. $12x - 24$
- E. $24 - 12x$

3. Solve each equation.

a. $3x + 7 = 40$
$x = 11$

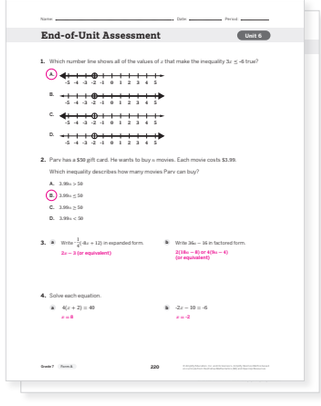
b. $-2(x + 5) = 10$
$x = -10$

Grade 7 215

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.AR.1.1, MA.7.AR.2.1, MA.7.AR.2.2, MA.7.AR.2.3, MA.7.AR.3.3


End-of-Unit Assessment

Unit 6

1. Which number line shows all of the values of x that make the inequality $3x \leq -6$ true?

A.

B.

C.

D.

2. Parv has a $50 gift card. He wants to buy x movies. Each movie costs $3.99.

Which inequality describes how many movies Parv can buy?

A. $3.99x > 50$

B. $3.99x \leq 50$

C. $3.99x \geq 50$

D. $3.99x < 50$

3. a. Write $\frac{1}{2}(8x + 12)$ in expanded form.
$2x + 3$ (or equivalent)

b. Write $36x - 16$ in factored form.
$2(18x - 8)$ or $4(9x - 4)$ (or equivalent)

4. Solve each equation.

a. $4(x + 2) = 40$
$x = 8$

b. $-2x - 10 = -6$
$x = -2$

Grade 7 Form A 220

D Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

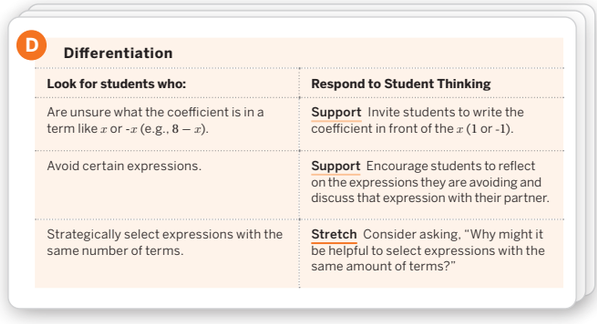
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–3) • Use tape diagrams to represent equations and situations in context and to determine unknown values.	**Mini-Lesson** • *Writing Equations from Descriptions and Tape Diagrams*	**Practice** • Lessons 1–3	**You're invited to explore more.** • Lesson 3 *(Seeing Structure)*
Sub-Unit 2 (Lessons 4–12) • Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems. • Write equivalent expressions by adding, subtracting, expanding, and factoring.	**Mini-Lesson** • *Solving Equations With Positive and Negative Numbers*	**Repeated Challenges** • Lesson 5 *(Balancing Equations)* **Challenge Creator** • Lesson 4 *(Balancing Moves)* **Practice** • Lessons 4–12	**You're invited to explore more.** • Lesson 8 *(Always-Equal Machines)* • Lesson 10 *(Collect the Squares)*
Sub-Unit 3 (Lessons 13–16) • Solve one-step inequalities with positive and negative coefficients. • Create graphs that represent solutions to inequalities, including those with ≥ or ≤.	**Mini-Lesson** • *Solving Inequalities*	**Repeated Challenges** • Lesson 14 *(Unbalanced Hangers)* **Practice** • Lessons 13–16	

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 10**, where students write equivalent expressions using the fewest number of terms.



D **Differentiation**

Look for students who:	Respond to Student Thinking
Are unsure what the coefficient is in a term like x or $-x$ (e.g., $8 - x$).	**Support** Invite students to write the coefficient in front of the x (1 or -1).
Avoid certain expressions.	**Support** Encourage students to reflect on the expressions they are avoiding and discuss that expression with their partner.
Strategically select expressions with the same number of terms.	**Stretch** Consider asking, "Why might it be helpful to select expressions with the same amount of terms?"

Accessibility

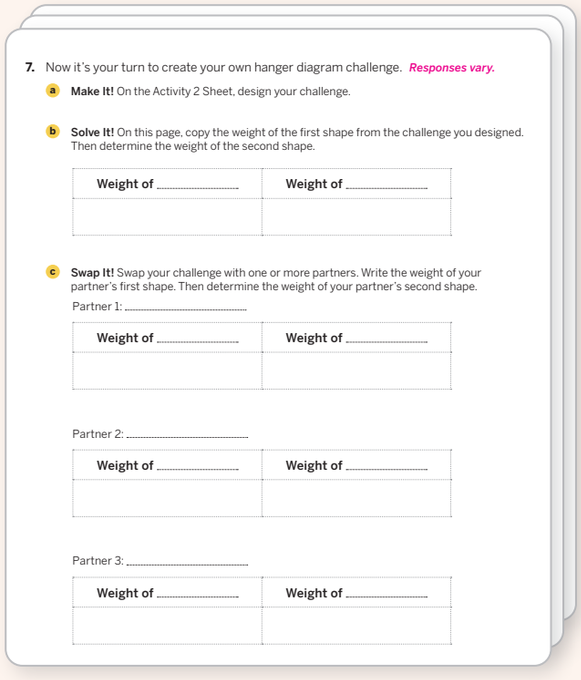
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action and Expression.

Spotlight on Sustaining Effort & Persistence

Lesson 4: Students create their own challenges and solve each other's challenges, which fosters a sense of community. Students work with their partners to check their answers, receive feedback, and resolve any differences.



7. Now it's your turn to create your own hanger diagram challenge. *Responses vary.*

a. **Make It!** On the Activity 2 Sheet, design your challenge.

b. **Solve It!** On this page, copy the weight of the first shape from the challenge you designed. Then determine the weight of the second shape.

Weight of	Weight of .....

c. **Swap It!** Swap your challenge with one or more partners. Write the weight of your partner's first shape. Then determine the weight of your partner's second shape.

Partner 1: .....

Weight of	Weight of .....

Partner 2: .....

Weight of	Weight of .....

Partner 3: .....

Weight of	Weight of .....

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing . . .

Here are two suggestions in this unit for supporting students with abstract thinking.

Lesson 5, Warm-Up

Accessibility: Conceptual Processing Invite students to make connections to familiar contexts to support abstract thinking.

Lesson 12, Activity 1

Accessibility: Conceptual Processing Invite students to answer a more concrete question first to support abstract thinking, like: *How can a tape diagram help you?*

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson	Review Vocabulary	Additional Vocabulary
expand **factor**	7	coefficient constant distributive property solution to an equation tape diagram term variable	boundary point like terms inverse operations
equivalent expressions	8		
solutions to an inequality	14		

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	**6–8, 11, and 13**
MLR3: Critique, Correct, Clarify	**2, 8–10, and 15**
MLR5: Co-Craft Questions	**3**
MLR6: Three Reads	**12**
MLR7: Compare and Connect	**5 and 7**

Sub-Unit Language Goals

- Analyze descriptions of situations in context using tape diagrams and equations to interpret the solution.
- Describe and use strategies for writing equivalent expressions that involve adding, subtracting, factoring, expanding, and reordering terms.
- Analyze descriptions of situations in context to interpret the solution of inequalities.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 13**, where students make connections between the words, symbol, and graph of each type of inequality.

ELL **English Language Learners** Consider creating an anchor chart about *greater than*, *greater-than-or-equal-to*, *less than*, and *less-than-or-equal-to*. Add student ideas to the chart throughout the lesson. **(Writing and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Solving Linear Equations with Variables on Both Sides

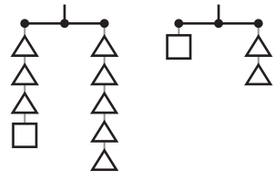
In this unit, students solve equations of the form $px + q = r$ and $p(x + q) = r$. Students focus on solving equations with a variable only on one side. **In Grade 8, Unit 4,** students will learn to solve equations with variables on both sides.

FL B.E.S.T. Math Benchmark:
MA.8.AR.2.1

Example:

We can also represent equations with two variables using hanger diagrams. One way to solve the equation $3x + 5 = 5x$ is by subtracting $3x$ from both sides (or removing three triangles from both sides).

This creates the equivalent equation $5 = 2x$.
The solution to both equations is $x = \frac{5}{2}$.



$3x + 5 = 5x$ $\quad$ $5 = 2x$

Solve Two-Step Linear Inequalities

In this unit, students solve one-step inequalities and represent solutions algebraically and graphically. **In Grade 8, Unit 4,** students will solve two-step inequalities in the forms $px \pm q > r$ and $p(x \pm q) > r$ where p, q, and r are rational numbers.

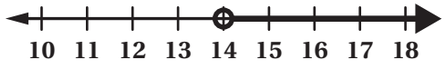
FL B.E.S.T. Math Benchmark:
MA.8.AR.2.2

Example:

You can use properties of inequalities to solve a two-step inequality. Apply the same operations to both sides of the inequality so that the inequality remains true.

The solution is $x > 14$.

$5x - 24 > 46$ Given inequality
$5x - 24 + 24 > 46 + 24$ Add 24 to both sides.
$5x > 70$ Simplify.
$\frac{5x}{5} > \frac{70}{5}$ Divide by 5 on both sides.
$x > 14$ Simplify.



Solving Systems of Linear Equations

In this unit, students solve equations of the form $px + q = r$ and $p(x + q) = r$ in and out of context. **In Grade 8, Unit 4,** students will use these solving strategies to help them solve systems of linear equations.

FL B.E.S.T. Math Benchmark:
MA.8.AR.4.3

Example:

Yona and Haruto's school is 600 meters from their home. Yona is running home from school at 4 meters per second and, at the same time, Haruto starts walking from home to school at 2 meters per second. When will they pass each other?

First, write an equation for each child's distance from home. When Yona and Haruto pass each other, their distances from home will be equal. You can set the expressions equal to each other, then solve for the time.

Yona: $d = 600 - 4t$

Haruto: $d = 2t$

$600 - 4t = 2t$

$600 = 6t$

$100 = t$

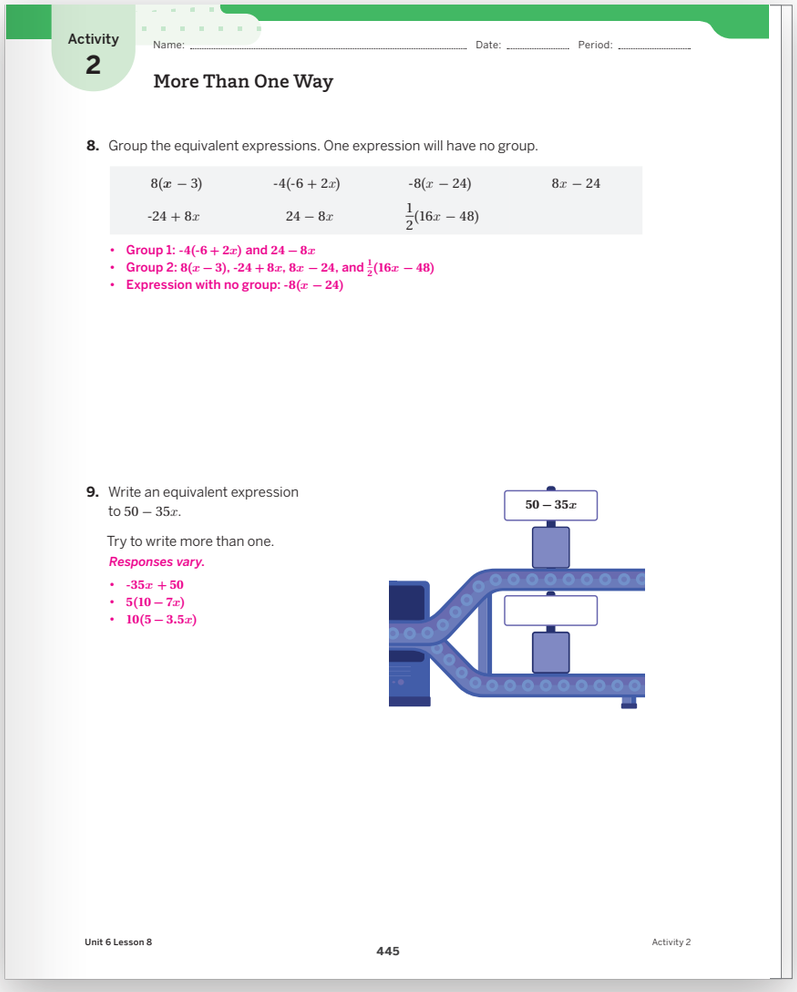
Yona and Haruto pass each other after 100 seconds.

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 8, Activity 2



Activity 2

Name: _____ Date: _____ Period: _____

More Than One Way

8. Group the equivalent expressions. One expression will have no group.

$8(x-3)$	$-4(-6+2x)$	$-8(x-24)$	$8x-24$
$-24+8x$	$24-8x$	$\frac{1}{2}(16x-48)$	

- Group 1: $-4(-6+2x)$ and $24-8x$
- Group 2: $8(x-3)$, $-24+8x$, $8x-24$, and $\frac{1}{2}(16x-48)$
- Expression with no group: $-8(x-24)$

9. Write an equivalent expression to $50-35x$.

Try to write more than one.

Responses vary.

- $-35x+50$
- $5(10-7x)$
- $10(5-3.5x)$

Unit 6 Lesson 8 445 Activity 2

Context

In this activity, students develop procedural fluency as they write and identify equivalent expressions.

Try This

Put on your student hat and complete Problems 8–9.

Points to Ponder

- What do students need to understand about factoring, expanding, or reordering terms to support their writing equivalent expressions?
- How does writing equivalent expressions build from their understanding of operations with positive and negative numbers?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Representing numerical equations with tape diagrams	Lesson 1	1	MA.6.AR.2.2 MTR.2.1
2a	Solving multi-step application problems	Lesson 2	1	MA.6.AR.2.2
2b	Representing real-world situations with equations	Lesson 2	2	MA.6.AR.2.2 MTR.2.1
3	Using the structure of expressions to select equivalent algebraic expressions	Lesson 5	1	MA.6.AR.2.2, MA.6.AR.2.3
4	Using structure to solve equations of the form $x + p = q$ and $px = q$	Lesson 5	1	MA.6.AR.2.2, MA.6.AR.2.3
5	Using the structure of expressions and the distributive property to write equivalent expressions and justifying the response	Lesson 5	1	MA.6.AR.1.4
6a	Representing the solution set to inequalities on a number line and justifying the response	Lesson 13	2	MA.6.AR.1.2 MTR.2.1
6b	Solving multi-step application problems involving unit rates and justifying the response	Lesson 13	2	MA.6.AR.3.2

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 6

1. Here are Diagrams A and B.

Diagram A

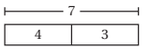
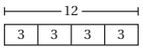


Diagram B



Determine whether each equation matches Diagram A, Diagram B, or neither diagram. Circle your response for each.

a	$7 = 3 + 4$	(Diagram A)	Diagram B	Neither
b	$4 - 3 = 7$	Diagram A	Diagram B	(Neither)
c	$4 \cdot 3 = 7$	Diagram A	Diagram B	(Neither)
d	$3 + 3 + 3 + 3 = 12$	Diagram A	(Diagram B)	Neither
e	$12 = 4 \cdot 3$	Diagram A	(Diagram B)	Neither

2. Vihaan collects stickers. He has x stickers. After getting 15 more stickers, he has 60 stickers total.

a. How many stickers did Vihaan start with?

45 stickers

b. Select *all* the equations that could be used to determine how many stickers Vihaan started with.

- ☑ **A.** $x + 15 = 60$
- ☑ **B.** $x = 60 - 15$
- ☐ **C.** $x = 60 + 15$
- ☐ **D.** $15x = 60$
- ☐ **E.** $x = 60 \cdot 15$

3. Select *all* the equations that are true when x is -4.

- ☑ **A.** $-8 = 2x$
- ☐ **B.** $-12 = x \cdot (-3)$
- ☑ **C.** $-12 = x + x + x$
- ☑ **D.** $\frac{x}{4} = -1$
- ☐ **E.** $x + 4 = -8$

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Name: Date: Period:

Pre-Unit Check (continued)

Unit 6

4. Solve each equation.

a. $p + 7 = 12$
$p = 5$

b. $90 = -20r$
$r = -4.5$
(or equivalent)

c. $\frac{1}{3}s = 7$
$s = 21$

5. Which expression is equivalent to $4(x + 2)$?

A. $12x$ B. $4x + 2$ C. $6x$ (D.) $4x + 8$

Explain your thinking.

Explanations vary. **This is like 4 groups of $x + 2$, which is like $4 \cdot x + 4 \cdot 2$ or $4x + 8$.**

6. Ishaan and Arnav are selling boxes of cookies.

a. Ishaan's goal is to sell more than 5 boxes. Which graph shows how many boxes Ishaan must sell in order to reach his goal?

A.



(B.)



C.



D.



Explain your thinking.

Explanations vary. **Ishaan wants to sell more than 5 boxes, so that's like shading to the right of 5. The number 5 doesn't get shaded because if he sells 5 boxes, he doesn't reach his goal.**

b. Arnav sells each box for $3.75. His goal is to make more than $30. How many boxes of cookies could he sell to reach his goal?

9 or more boxes

Explain your thinking.

Explanations vary. **To make exactly $30, Arnav needs to sell 8 boxes of cookies since $\frac{30}{3.75} = 8$. But he wants to make more than $30, so he needs to sell 9 or more boxes.**

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Use tape diagrams to represent equations and situations in context and to determine unknown values. **(Lessons 1–3)**	1	**Support** • **Teacher Move:** During Lesson 1, consider spending extra time during the Warm-Up discussing the structure of the tape diagram.
	2	**Support** • **Teacher Move:** Move: Prior to Lesson 2, consider reviewing this problem or spending extra time during Activity 1 discussing the relationships between the scenarios and the equations that represent them.
	3	**Support** • **Mini-Lesson:** Writing and Solving Equations • **Teacher Move:** Prior to Lesson 5, consider spending extra time discussing strategies for solving equations. • During Lesson 6, consider checking all the solutions in Activity 1 together as a class.
	5	**Support** • **Teacher Move:** Move: During Lesson 7, consider spending extra time reviewing the Warm-Up, helping students see the connections between the various representations.
	6	**Support** • **Teacher Move:** Prior to Lesson 13, consider reviewing this problem and prompting, "Test whether 8 boxes of cookies will help Arnav reach his goal." Help students make connections between the number line diagrams and the word problem.

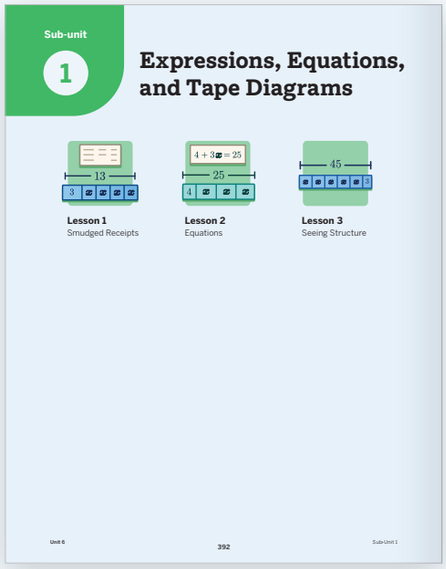
Sub-Unit 1

Equations and Tape Diagrams

Sub-Unit 1 Goals

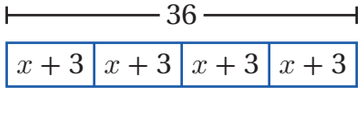
- **Goal:** Use tape diagrams to represent equations and situations in context and to determine unknown values.
- **Language Goal:** Analyze descriptions of situations in context using tape diagrams and equations to interpret the solution.

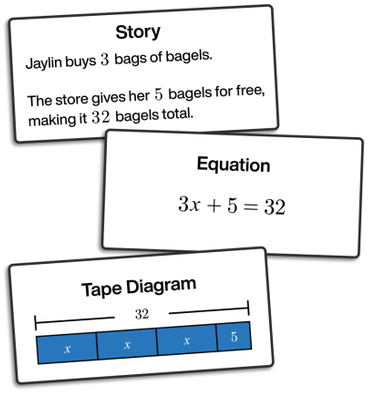
In this sub-unit, students use tape diagrams to represent equations and situations in context and determine unknown values.



Math That Matters Most

Concept or Skill	For example . . .
Use a tape diagram to solve for an unknown in context.	Neel and his sister are making gift bags for a party. In each bag, Neel puts 3 pencil erasers, and his sister puts an unknown number of stickers, x. After filling 4 bags, they have put in 36 items. The tape diagram represents this situation. What is the value of x? ***x = 6***
Make connections between tape diagrams, equations, and verbal descriptions.	Explain how the number 9 is important in each representation. ***In the story, there must be 9 bagels in each bag.*** ***In the equation, 9 makes sense as the value for x because 3(9) + 5 = 32.*** ***In the tape diagram, 9 is the length of the x segments because 9 + 9 + 9 + 5 = 32.***
Write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent a context, then solve the equation using a tape diagram.	Amir has 24 ounces of juice. He pours equal amounts for each of his 2 friends and then adds 4 more ounces for each person. Write an equation to represent this story, then solve your equation. Explain what the solution tells you about the story. ***The story is represented by the equation 2(x + 4) = 24. The value of x is 8. In the story, this means that Amir originally poured 8 ounces of juice for each of his friends.***





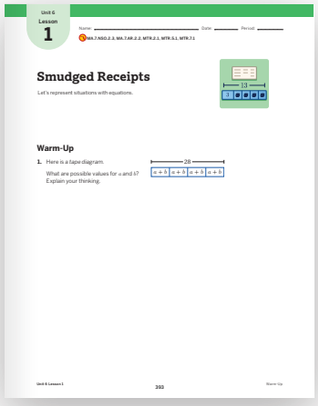
Unit 6

Lesson 1

Smudged Receipts

Connecting Contexts and Tape Diagrams

Let's represent situations with equations.


Unit 6
Lesson 1
Smudged Receipts
Let's represent situations with equations.
Warm-Up
1. Here is a tape diagram.
What are possible values for x and k?
Explain your thinking.
393

Learning Outcomes

Today's Goals

1. **Goal:** Use a tape diagram to determine an unknown value in context.
2. **Language Goal:** Interpret a tape diagram that represents a relationship in context. **(Writing)**

Students revisit *tape diagrams* and surface strategies for using tape diagrams to determine unknown values in situations. They use tape diagrams to informally represent equations of the form $px + q = r$ and $p(x + q) = r$. **(MTR.2.1)**

Coherence

Prior Learning

In Unit 4, students used tape diagrams to reason about percent increases and decreases.

Future Learning

In Lesson 2, students will use what they know about tape diagrams and situations to make sense of equations of the form $px + q = r$ and $p(x + q) = r$.

Connections

- Students use tape diagrams as a visual tool to help understand and solve equations by representing mathematical relationships in a concrete way. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.3
Solve real-world problems involving any of the four operations with rational numbers.

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2	MA.8.AR.2.1
MA.6.AR.2.3	

Vocabulary

Review Vocabulary

tape diagram

Lesson at a Glance

~ 45 min

MA.7.NSO.2.3, MA.7.AR.2.2

Warm-Up

Independent | 5 min

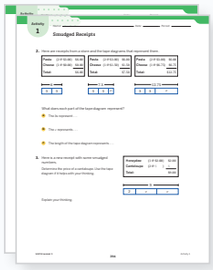
Students revisit *tape diagrams* and surface strategies for reasoning about unknowns in tape diagrams.



Activity 1

Pairs | 15 min

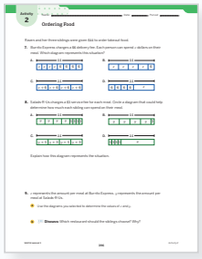
Students connect *tape diagrams* to situations involving smudged receipts and use the tape diagrams to determine unknown costs.



Activity 2

Pairs | 15 min

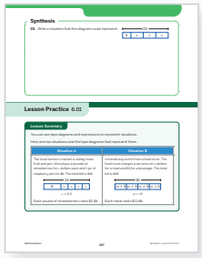
Students connect situations to tape diagrams, including comparing and contrasting situations that lead to equations like $px + q = r$ and $p(x + q) = r$. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

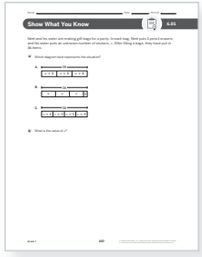
Students synthesize their understanding of how to solve for unknowns using tape diagrams. **Routine:** Tell a Story



Show What You Know

Independent | 5 min

Students demonstrate their understanding by connecting situations to tape diagrams.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students revisit *tape diagrams* and surface strategies for reasoning about unknowns in tape diagrams.

1 Launch

To surface connections, consider asking students to share if they have seen similar diagrams. Students may connect these *tape diagrams* to the diagrams that they used in Unit 4 to represent percentages.

To encourage creativity, invite students to think of several different possible values for a and b.

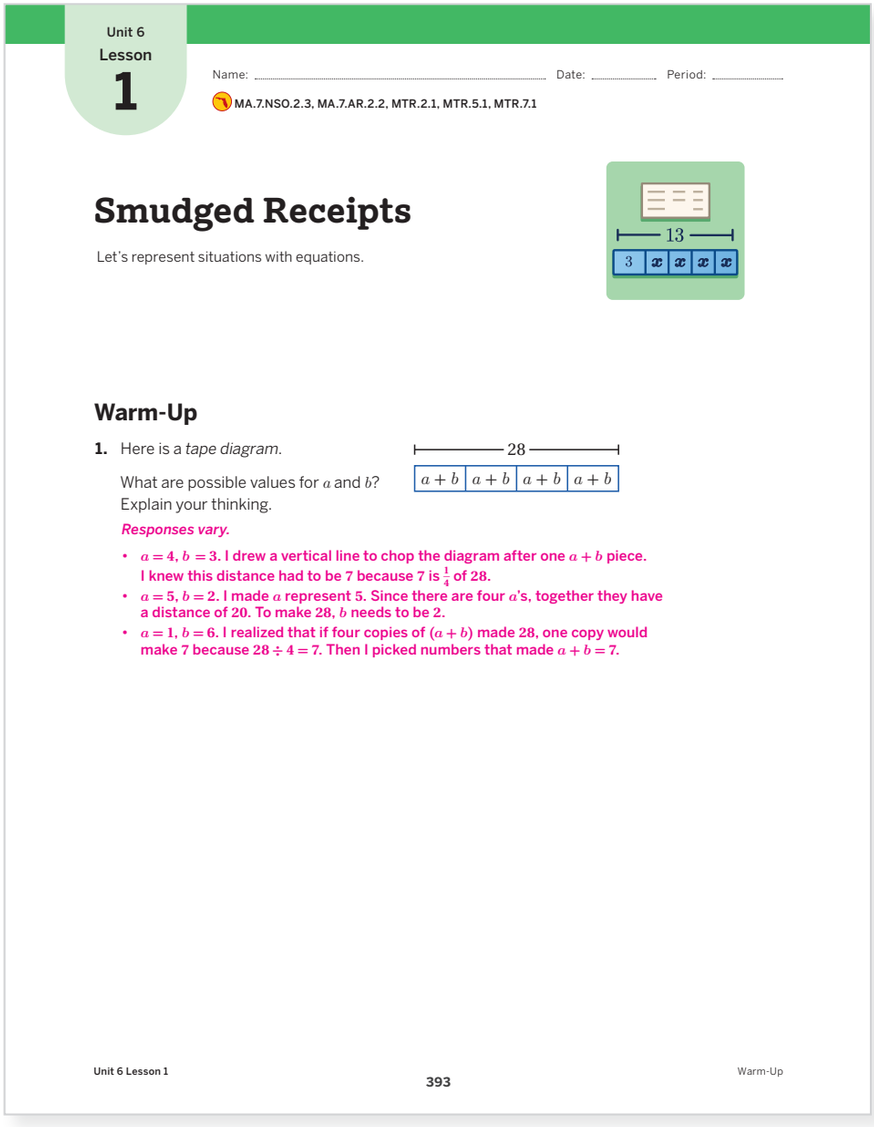
A **Accessibility: Conceptual Processing** Invite students to solve a similar question with smaller numbers to support thinking abstractly.

2 Connect

Invite students to share their responses.

Consider asking:

- "Which of these values make the tape diagram true? Why or why not?"
- "How long do you think each piece of the tape diagram is? How do you know?"



Unit 6
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.5.1, MTR.7.1

Smudged Receipts

Let's represent situations with equations.

Warm-Up

1. Here is a *tape diagram*.

What are possible values for a and b? Explain your thinking.

Responses vary.

- $a = 4$, $b = 3$. I drew a vertical line to chop the diagram after one $a + b$ piece. I knew this distance had to be 7 because 7 is $\frac{1}{4}$ of 28.
- $a = 5$, $b = 2$. I made a represent 5. Since there are four a's, together they have a distance of 20. To make 28, b needs to be 2.
- $a = 1$, $b = 6$. I realized that if four copies of $(a + b)$ made 28, one copy would make 7 because $28 \div 4 = 7$. Then I picked numbers that made $a + b = 7$.

Unit 6 Lesson 1 393 Warm-Up

Pairs | 15 min

Activity 1 Smudged Receipts

Purpose: Students connect *tape diagrams* to situations involving smudged receipts and use the tape diagrams to determine unknown costs.

1 Launch

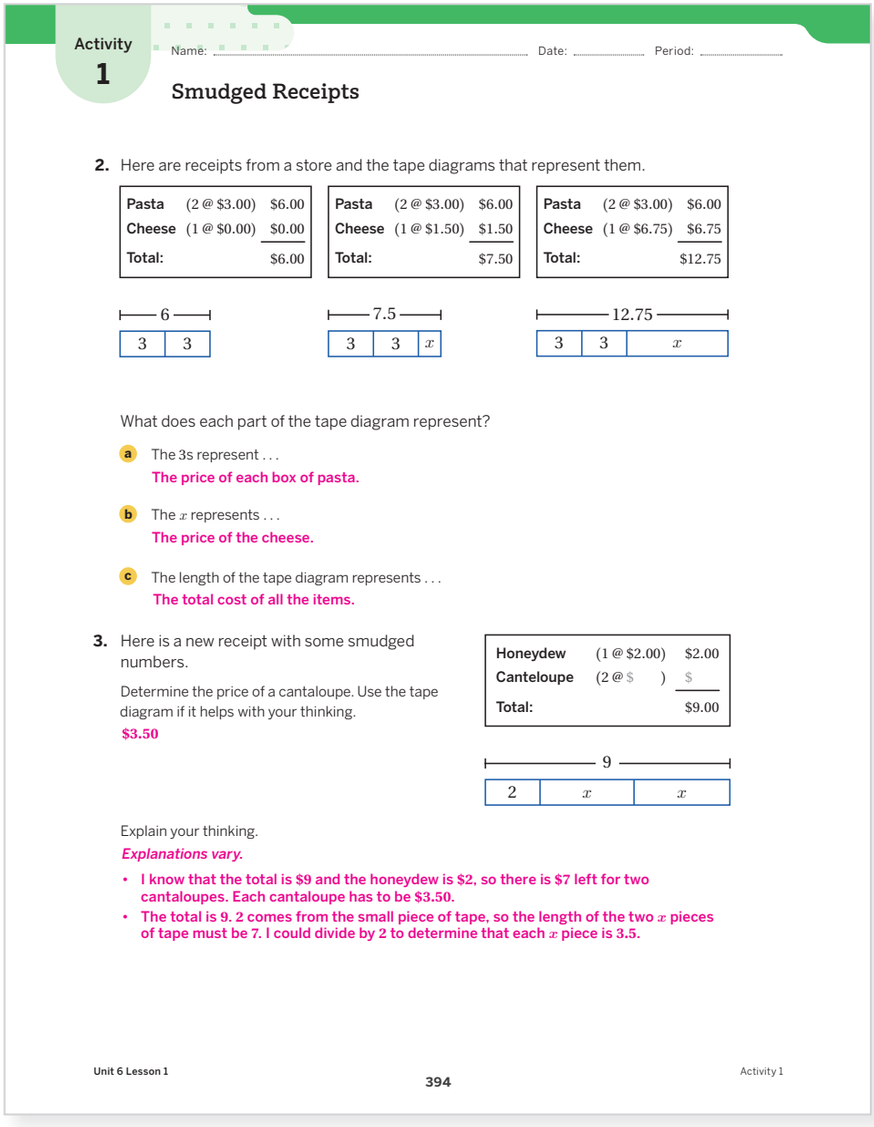
Direct students' attention to the three receipts. Consider asking, "What do you notice is the same in the receipts? What is different?" **(MTR.5.1)**

2 Monitor

Invite students to share their thinking.

Pause briefly to discuss strategies students used to determine the price of the cantaloupe.

Activity 1 continued >



Activity 1

Name: Date: Period:

Smudged Receipts

2. Here are receipts from a store and the tape diagrams that represent them.

Pasta	(2 @ $3.00)	$6.00
Cheese	(1 @ $0.00)	$0.00
Total:		$6.00

Pasta	(2 @ $3.00)	$6.00
Cheese	(1 @ $1.50)	$1.50
Total:		$7.50

Pasta	(2 @ $3.00)	$6.00
Cheese	(1 @ $6.75)	$6.75
Total:		$12.75

What does each part of the tape diagram represent?

a. The 3s represent . . .
The price of each box of pasta.

b. The x represents . . .
The price of the cheese.

c. The length of the tape diagram represents . . .
The total cost of all the items.

3. Here is a new receipt with some smudged numbers.

Honeydew	(1 @ $2.00)	$2.00
Canteloupe	(2 @ $)	$
Total:		$9.00

Determine the price of a cantaloupe. Use the tape diagram if it helps with your thinking.
$3.50

Explain your thinking.
Explanations vary.

- **I know that the total is $9 and the honeydew is $2, so there is $7 left for two cantaloupes. Each cantaloupe has to be $3.50.**
- **The total is 9. 2 comes from the small piece of tape, so the length of the two x pieces of tape must be 7. I could divide by 2 to determine that each x piece is 3.5.**

Unit 6 Lesson 1 394 Activity 1

Pairs | 15 min

Activity 1 Smudged Receipts (continued)

Purpose: Students connect *tape diagrams* to situations involving smudged receipts and use the tape diagrams to determine unknown costs.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement.

Select and sequence student responses to highlight in the Connect.

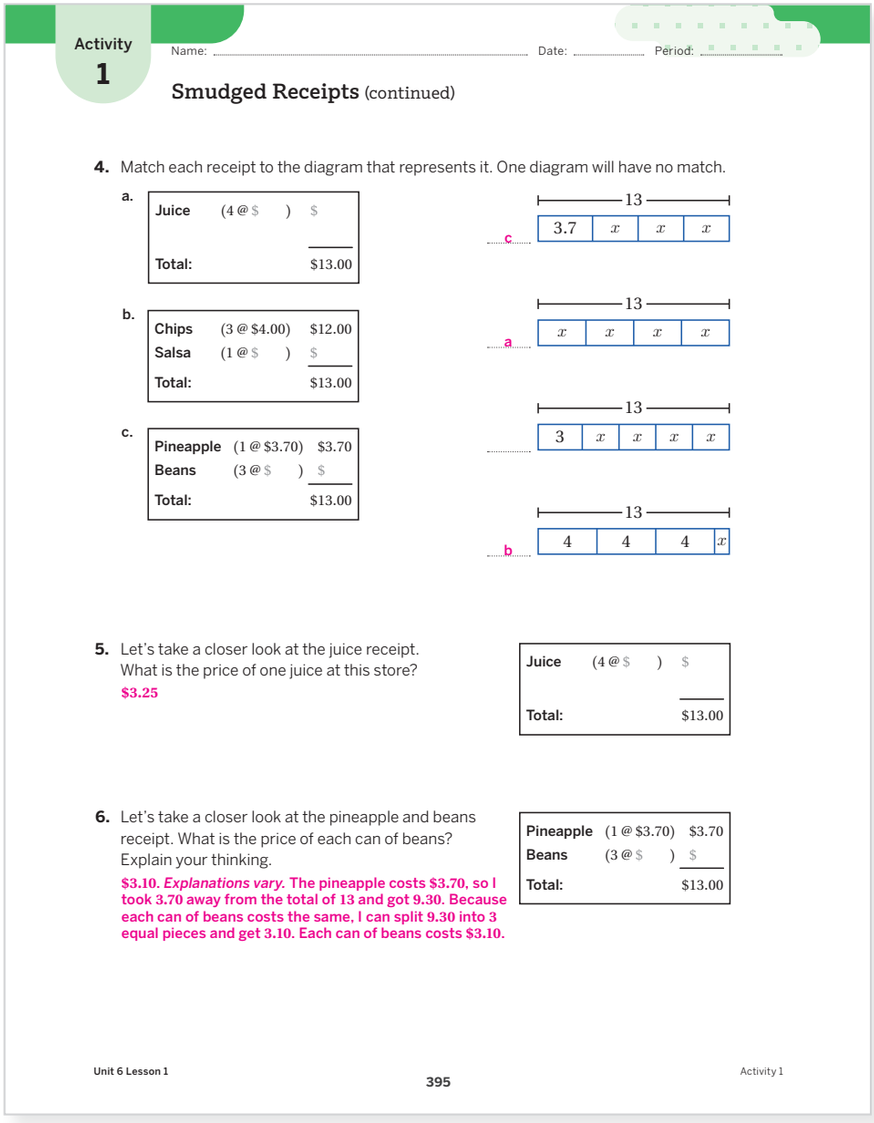
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider discussing how each problem is different from the previous one to help students apply concepts to new situations.
Have difficulty determining unknown values using a tape diagram.	**Support** Consider asking, "What is the relationship between each section of the tape diagram and the total?"
Use a variety of strategies to determine the unknown values.	**Strengthen** Consider asking, "If you substitute your solution into the variable, does your tape diagram match the total?"
Would like to explore tape diagrams and receipts further.	**Stretch** Encourage students to create their own receipt for the unpaired diagram from the matching exercise.

3 Connect

Invite students to share their responses for the price of juice and beans.

To surface the Key Takeaway, invite students to share strategies that were successful and strategies that were not successful. Record their strategies so everyone can refer to them throughout the lesson. **(MTR.2.1)**



Activity 1

Name: Date: Period:

Smudged Receipts (continued)

4. Match each receipt to the diagram that represents it. One diagram will have no match.

a.

Juice	(4 @ $)	$
Total:		$13.00

b.

Chips	(3 @ $4.00)	$12.00
Salsa	(1 @ $)	$
Total:		$13.00

c.

Pineapple	(1 @ $3.70)	$3.70
Beans	(3 @ $)	$
Total:		$13.00

5. Let's take a closer look at the juice receipt. What is the price of one juice at this store?

$3.25

Juice	(4 @ $)	$
Total:		$13.00

6. Let's take a closer look at the pineapple and beans receipt. What is the price of each can of beans? Explain your thinking.

$3.10. *Explanations vary.* The pineapple costs $3.70, so I took 3.70 away from the total of 13 and got 9.30. Because each can of beans costs the same, I can split 9.30 into 3 equal pieces and get 3.10. Each can of beans costs $3.10.

Pineapple	(1 @ $3.70)	$3.70
Beans	(3 @ $)	$
Total:		$13.00

Unit 6 Lesson 1 395 Activity 1

Key Takeaway: *Tape diagrams* are useful when determining unknown values because each piece represents one part of a situation.

Pairs | 15 min

Activity 2 Ordering Food

Purpose: Students connect situations to *tape diagrams*, including comparing and contrasting situations that lead to equations like $px + q = r$ and $p(x + q) = r$. **(MTR.2.1)**

Short on time? Consider omitting Problem 7.

1 Launch

Consider using the Which One Doesn't Belong? routine to support students in noticing the features of each representation.

A **Accessibility: Visual-Spatial Processing** Invite students to use their Student Edition to draw on their tape diagrams to support interpreting visual representations.

2 Monitor

Capture and share the distribution of responses, calling attention to any conflict or consensus.

Pause and invite students to share their reasoning for how they connected diagrams to situations. Invite students to critique each other's reasoning. **(MTR.2.1)**

Consider asking, "What words or phrases did you pay attention to as you selected each tape diagram?"

Look for student ideas to highlight during the Connect.

ELL **English Language Learners** Pair students to aid them in making sense of the discussion prompt and expressing their understanding before the whole-class discussion.

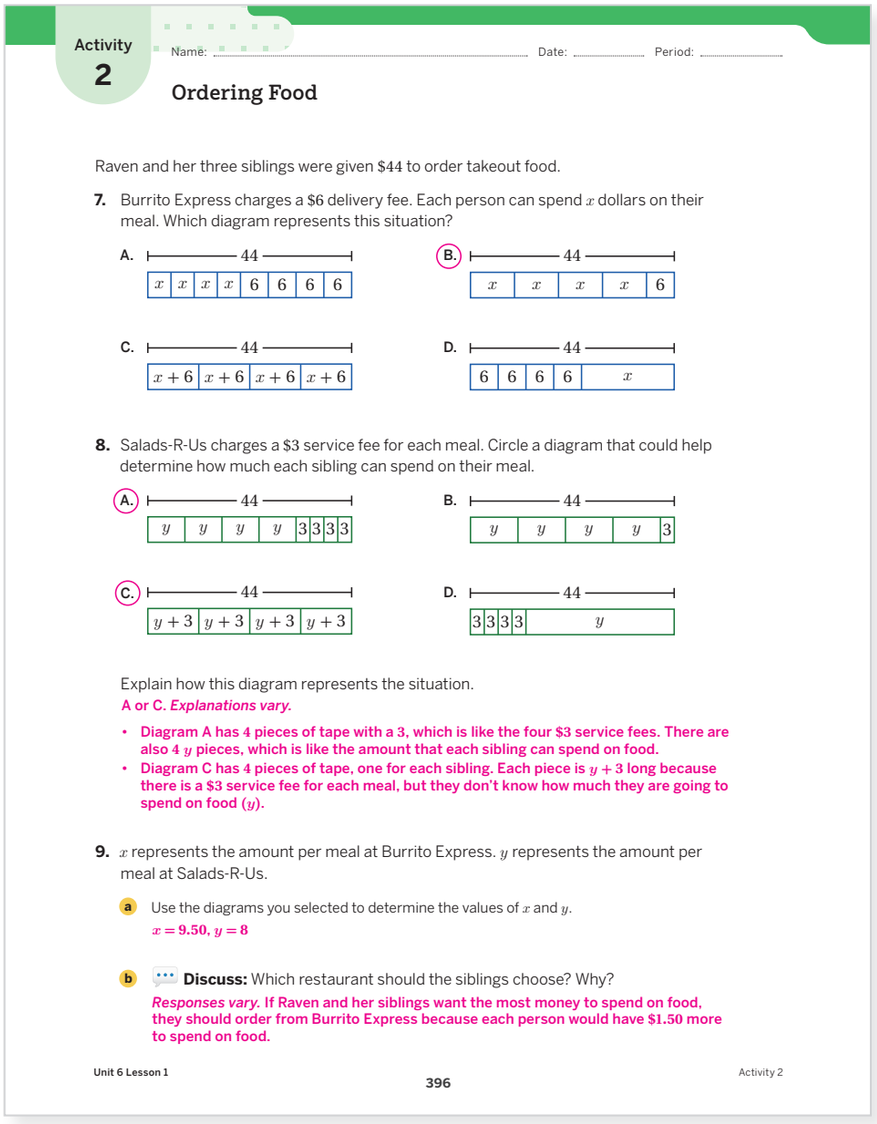
D **Differentiation: (Stretch)** Encourage students to consider what system of charging fees they think is most fair and why. If students are curious why restaurants might charge a service or delivery fee, consider asking them what costs different restaurants might have that would cause them to charge fees differently. **(MTR.7.1)**

3 Connect

Invite students to share their responses for the values of x and y.

Invite students to share their strategies for determining the unknown value and what it represents in the situation.

Consider asking, "If you were Raven and her siblings, where would you choose to eat? Why?" **(MTR.7.1)**



Activity 2

Name: Date: Period:

Ordering Food

Raven and her three siblings were given $44 to order takeout food.

7. Burrito Express charges a $6 delivery fee. Each person can spend x dollars on their meal. Which diagram represents this situation?

A. 44 | x | x | x | x | 6 | 6 | 6 | 6

B. 44 | x | x | x | x | 6

C. 44 | $x+6$ | $x+6$ | $x+6$ | $x+6$

D. 44 | 6 | 6 | 6 | 6 | x

8. Salads-R-Us charges a $3 service fee for each meal. Circle a diagram that could help determine how much each sibling can spend on their meal.

A. 44 | y | y | y | y | 3 | 3 | 3 | 3

B. 44 | y | y | y | y | 3

C. 44 | $y+3$ | $y+3$ | $y+3$ | $y+3$

D. 44 | 3 | 3 | 3 | 3 | y

Explain how this diagram represents the situation.

A or C. *Explanations vary.*

- Diagram A has 4 pieces of tape with a 3, which is like the four $3 service fees. There are also 4 y pieces, which is like the amount that each sibling can spend on food.
- Diagram C has 4 pieces of tape, one for each sibling. Each piece is $y + 3$ long because there is a $3 service fee for each meal, but they don't know how much they are going to spend on food (y).

9. x represents the amount per meal at Burrito Express. y represents the amount per meal at Salads-R-Us.

a. Use the diagrams you selected to determine the values of x and y.

$x = 9.50$, $y = 8$

b. **Discuss:** Which restaurant should the siblings choose? Why?

Responses vary. If Raven and her siblings want the most money to spend on food, they should order from Burrito Express because each person would have $1.50 more to spend on food.

Unit 6 Lesson 1 396 Activity 2

Whole Class | 5 min

Synthesis

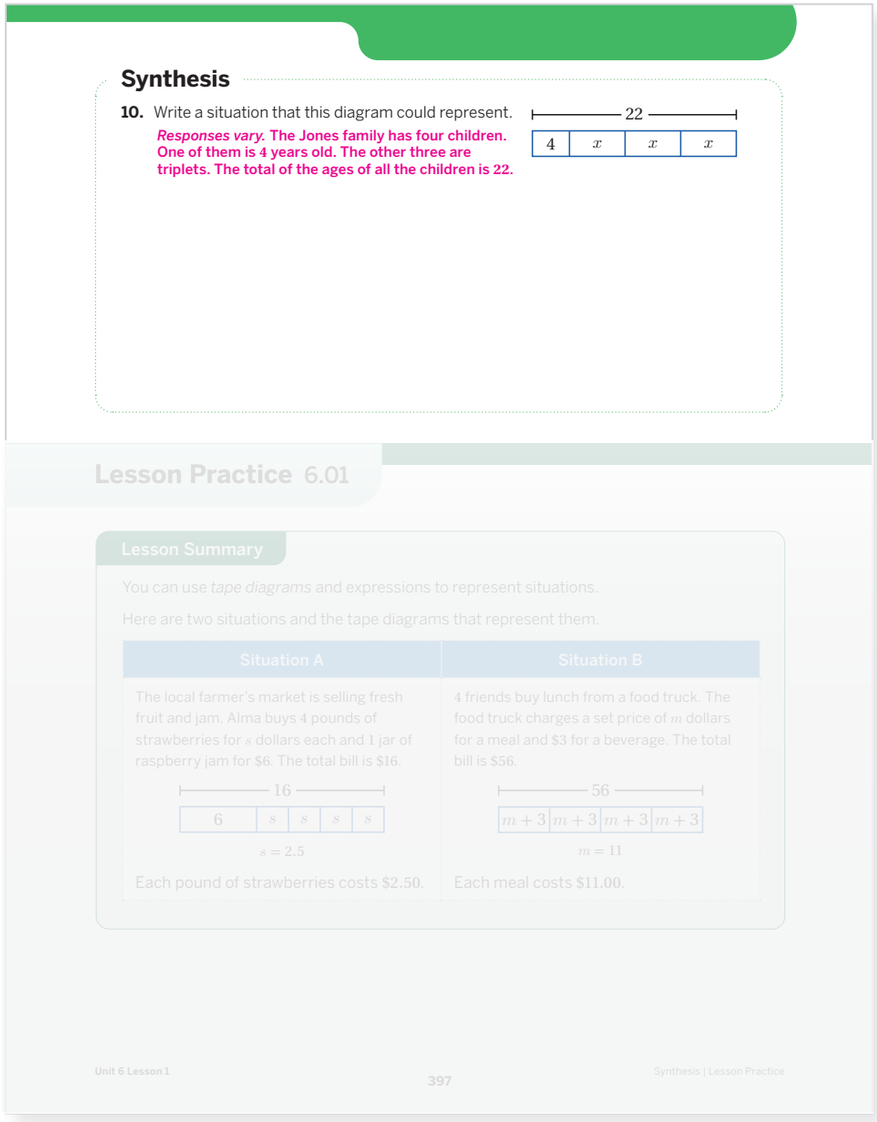
Purpose: Students synthesize their understanding of how to solve for unknowns using tape diagrams.

Invite students to respond independently, then share their thinking with a partner.

Use the Tell a Story routine. Invite students to write a story about the tape diagram using their unique experiences, perspectives, identities, and creativity. See the Routine Facilitation Guide for more information.

Capture and share a variety of ideas, such as:
- Creative and unique stories.
- Determining the value of x to enhance their story.

Note: If time allows, invite students to revise their story based on the discussion.



Synthesis

10. Write a situation that this diagram could represent.

Responses vary. The Jones family has four children. One of them is 4 years old. The other three are triplets. The total of the ages of all the children is 22.

Lesson Practice 6.01

Lesson Summary

You can use *tape diagrams* and expressions to represent situations.

Here are two situations and the tape diagrams that represent them.

Situation A	Situation B
The local farmer's market is selling fresh fruit and jam. Alma buys 4 pounds of strawberries for s dollars each and 1 jar of raspberry jam for \$6. The total bill is \$16.	4 friends buy lunch from a food truck. The food truck charges a set price of m dollars for a meal and \$3 for a beverage. The total bill is \$56.
16 6 | s | s | s | s $s = 2.5$	56 $m+3$ | $m+3$ | $m+3$ | $m+3$ $m = 11$
Each pound of strawberries costs \$2.50.	Each meal costs \$11.00.

Unit 6 Lesson 1 397 Synthesis | Lesson Practice

Lesson Takeaway: *Tape diagrams* with variables can be used to represent relationships and determine unknown values. They can also be used to determine an unknown value in context.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by connecting situations to tape diagrams.

Today's Goals

1. **Goal:** Use a tape diagram to determine an unknown value in context.
2. **Language Goal:** Interpret a tape diagram that represents a relationship in context. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.01

Neel and his sister are making gift bags for a party. In each bag, Neel puts 3 pencil erasers, and his sister puts an unknown number of stickers, x. After filling 4 bags, they have put in 36 items.

a. Which diagram best represents the situation?

A. 36 — $x+4$ | $x+4$ | $x+4$

B. 36 — x | x | x | 3

C. 36 — $x+3$ | $x+3$ | $x+3$ | $x+3$

b. What is the value of x?

$x = 6$

D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to interpret tape diagrams that represent relationships in context:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of tape diagrams in Lessons 2–3. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students used tape diagrams to make sense of relationships. How will that support students in writing and solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Write a situation that this diagram could represent.

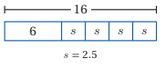
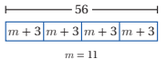
Responses vary. The Jones family has four children. One of them is 4 years old. The other three are triplets. The total of the ages of all the children is 22.

Lesson Practice 6.01

Lesson Summary

You can use *tape diagrams* and expressions to represent situations.

Here are two situations and the tape diagrams that represent them.

Situation A	Situation B
The local farmer's market is selling fresh fruit and jam. Alma buys 4 pounds of strawberries for s dollars each and 1 jar of raspberry jam for $6. The total bill is $16.	4 friends buy lunch from a food truck. The food truck charges a set price of m dollars for a meal and $3 for a beverage. The total bill is $56.
 $s = 2.5$	 $m = 11$
Each pound of strawberries costs $2.50.	Each meal costs $11.00.

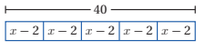
Lesson Practice 6.01

Name: _____ Date: _____ Period: _____

Problems 1–2: Axel wants to save $40 to buy a gift for his friend. Every week, his neighbor pays him to mow the lawn, and he donates $2 of what he earns to charity. Axel calculates that it will take him 5 weeks to earn enough for his friend's gift.

1. Which tape diagram represents this situation?

A. 

B. 

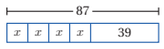
Explain how the tape diagram represents the situation.

Explanations vary. The 5 equal parts represent the 5 weeks. Each week, Axel earns x dollars for mowing his neighbor's lawn and donates $2, so he is saving $x - 2$ dollars. In 5 weeks, he will save a total of $40.

2. How much does Axel's neighbor pay him each week to mow the lawn?

$10

3. Select *all* the situations that the tape diagram can represent.



- ☑ **A.** There are 87 first graders in school. After 39 students are picked up, the teachers put the remaining students into 4 groups for an activity.
- ☑ **B.** Latifa buys a pack of 87 pencils. She gives 39 to her teacher and shares the remaining pencils between herself and 3 friends.
- ☐ **C.** Emiliano buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.
- ☑ **D.** Shanice's family buys 4 tickets to a fair and spends $39 on dinner. They spend $87 total.
- ☐ **E.** Daniela went shopping and spent $39 on 4 shirts. She then spent $87 on new shoes.

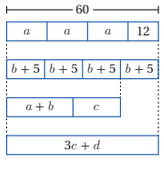
Lesson Practice 6.01

Name: _____ Date: _____ Period: _____

FAST Practice

4. Determine the values of a, b, c, and d.

Variable	Value
a	16
b	10
c	19
d	3



Spiral Review

Problems 5–12: Solve each equation.

5. $2x = 10$ — $x = 5$

6. $-3x = 21$ — $x = -7$

7. $\frac{1}{3}x = 6$ — $x = 18$

8. $\frac{1}{2}x = -7$ — $x = 14$

9. $8.5 \cdot (-3) = a$ — $a = -25.5$

10. $(-7) + b = -11$ — $b = -4$

11. $c - (-3) = 15$ — $c = 12$

12. $d \cdot (-4) = 32$ — $d = -8$

13. What is the value of the expression $-20 \div 4 + 2(-8) + 10$?

A. 11 B. -11 C. -58 D. $-\frac{13}{3}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.7.AR.2.2
FAST Practice	4	3	MA.7.AR.2.2
Spiral Review			
	5–12	1	MA.6.AR.2.2 MA.6.AR.2.3
	13	1	MA.7.NSO.2.1

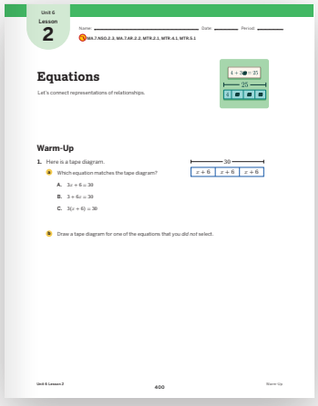
Unit 6

Lesson 2

Equations

Representing Contexts With Tape Diagrams and Equations

Let's connect representations of relationships.


Unit 6
Lesson 2
Name _____ Date _____ Period _____
MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
Equations
Let's connect representations of relationships.
Warm-Up
1. Here is a tape diagram.
Which equation matches the tape diagram?
A. $3x + 6 = 30$
B. $3 + 6x = 30$
C. $3(x + 6) = 30$
30
$x + 6$ $x + 6$ $x + 6$
Draw a tape diagram for one of the equations that you did not select.
Unit 6 Lesson 2
400
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Connect tape diagrams, equations, and verbal descriptions in context.
2. **Goal:** Write an equation to represent a tape diagram and a context.
3. **Language Goal:** Interpret the meaning of situations in a mathematical context. **(Reading)**

Students use what they know about tape diagrams and situations to make sense of equations of the form $px + q = r$ and $p(x + q) = r$. They reason abstractly and quantitatively to make sense of quantities and connect different representations of the same situation. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 1, students connected contexts and tape diagrams to informally represent equations of the form $px + q = r$ and $p(x + q) = r$.

Future Learning

In Lesson 3, students will use tape diagrams and equations to solve problems.

Connections

- Students continue use tape diagrams as a visual tool to help understand and solve equations by representing mathematical relationships in a concrete way. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.3
Solve real-world problems involving any of the four operations with rational numbers.

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2	**MA.8.AR.2.1**
MA.6.AR.2.3	

Lesson at a Glance

~ 45 min

MA.7.NSO.2.3, MA.7.AR.2.2

Warm-Up

Pairs | 5 min

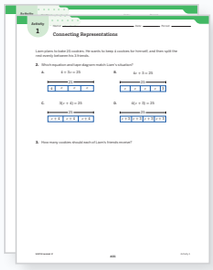
Students connect tape diagrams to equations.



Activity 1

Pairs | 15 min

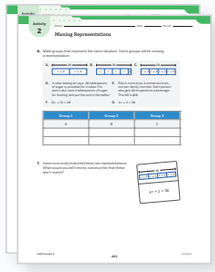
Students use equations and tape diagrams to make sense of a situation. **(MTR.2.1)**



Activity 2

Pairs | 15 min

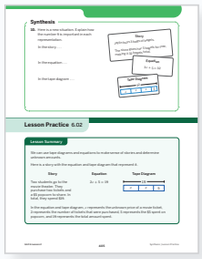
Students use the **Tell a Story** routine to practice connecting situations, tape diagrams, and equations.
MLR3: Critique, Correct, Clarify



Synthesis

Whole Class | 5 min

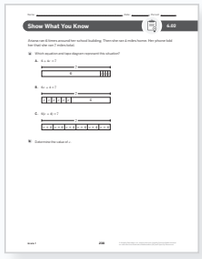
Students synthesize their understanding of the connections between stories, equations, and tape diagrams with their solution.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by connecting an equation to a situation and determining its solution.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- graph paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students connect tape diagrams to equations.

1 Launch

Consider asking students what they notice about the tape diagram.

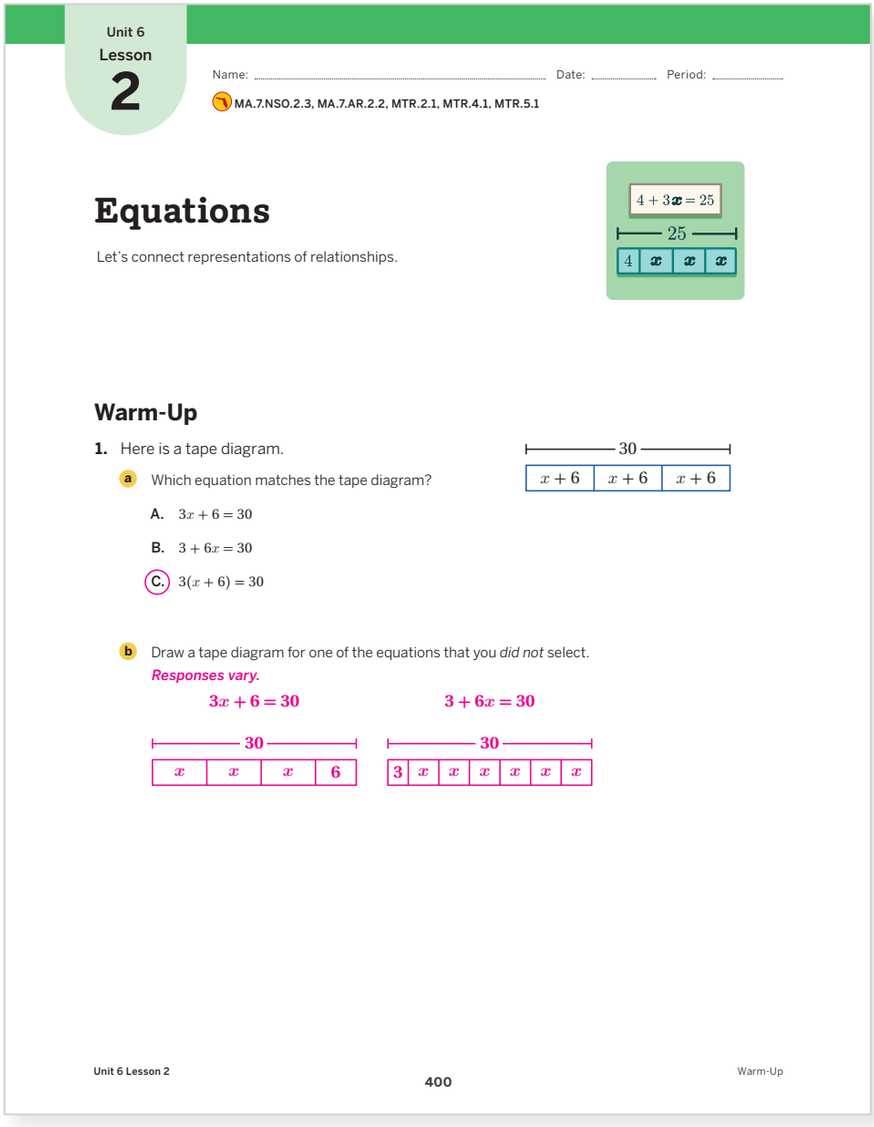
Listen for and amplify terms such as *equal segments*, *groups of*, *repeated values*, and *one-time values*.

Consider asking, "What is the value of x in the tape diagram?" Encourage students to test that value in each equation.

2 Connect

Invite students to share how they decided which equation to choose.

Consider asking, "What would the tape diagram for $3x + 6 = 30$ look like? What about $3 + 6x = 30$?"



Unit 6
Lesson
2

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Equations

Let's connect representations of relationships.

$4 + 3x = 25$

25

4	x	x	x

Warm-Up

1. Here is a tape diagram.

30

$x + 6$	$x + 6$	$x + 6$

a. Which equation matches the tape diagram?

A. $3x + 6 = 30$

B. $3 + 6x = 30$

C. $3(x + 6) = 30$

b. Draw a tape diagram for one of the equations that you *did not* select.

Responses vary.

$3x + 6 = 30$

30

x	x	x	6

$3 + 6x = 30$

30

3	x	x	x	x	x	x

Unit 6 Lesson 2 400 Warm-Up

Pairs | 15 min

Activity 1 Connecting Representations

Purpose: Students use equations and tape diagrams to make sense of a situation. **(MTR.2.1)**

Short on time? Consider omitting Problem 5.

1 Launch

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means, clarifying any unknown words.

Consider asking, "Why might someone make a tape diagram for a situation like this?"

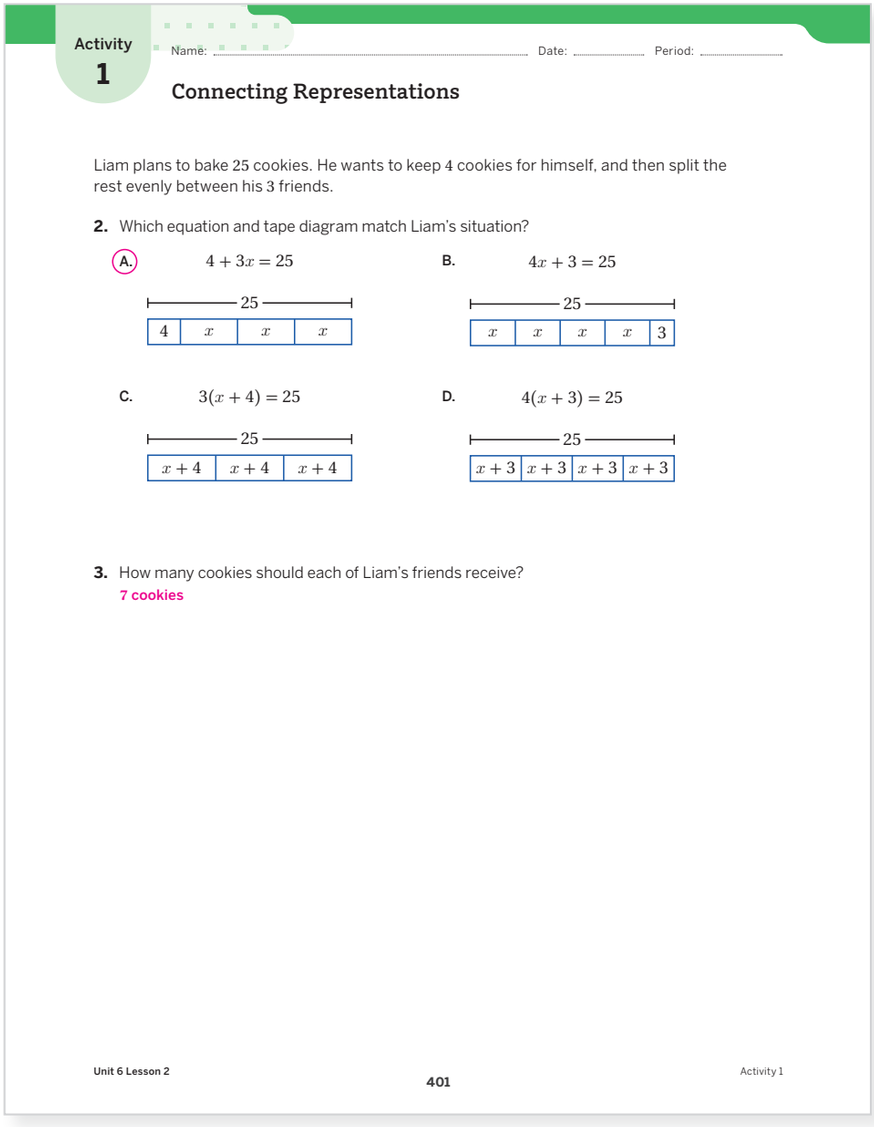
2 Monitor

Look for any consensus in student responses.

Look for strategies students use to determine the number of cookies, such as equations, tape diagrams, and direct reasoning. **(MTR.2.1)**

Encourage students to check their solution in both the tape diagram and equation.

Activity 1 continued >



Activity 1

Name: Date: Period:

Connecting Representations

Liam plans to bake 25 cookies. He wants to keep 4 cookies for himself, and then split the rest evenly between his 3 friends.

2. Which equation and tape diagram match Liam's situation?

A. $4 + 3x = 25$

B. $4x + 3 = 25$

C. $3(x + 4) = 25$

D. $4(x + 3) = 25$

3. How many cookies should each of Liam's friends receive?

7 cookies

Unit 6 Lesson 2 401 Activity 1

Pairs | 15 min

Activity 1 Connecting Representations (continued)

Purpose: Students use equations and tape diagrams to make sense of a situation. **(MTR.2.1)**

Short on time? Consider omitting Problem 5.

2 Monitor

To support students getting started, consider asking, "What do you think Liam's new tape diagram would look like? How is each part of the equation $4 + 3x = 25$ related to the original tape diagram?" **(MTR.5.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., This equation might be helpful because _____.). **(Writing and Speaking)**

Select and sequence several student responses that use different ways to describe the equation and the tape diagram and make connections between the situation and the other representations. **(MTR.2.1)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Write an equation that represents a different situation, like $7x - 5 = 33$ or $8x = 33$.	**Support** Consider asking, "How many cookies would Liam get? How many cookies would each of his friends get?"
Write $5 + 7x = 33$.	**Strengthen** Invite students to share how they determined this equation and how it will help them draw a tape diagram.
Would like to explore representations further.	**Stretch** Have students use their equations and tape diagrams to determine the solution.

A **Accessibility: Visual-Spatial Processing** Provide graph paper so students can draw their own tape diagram before responding to the prompt.

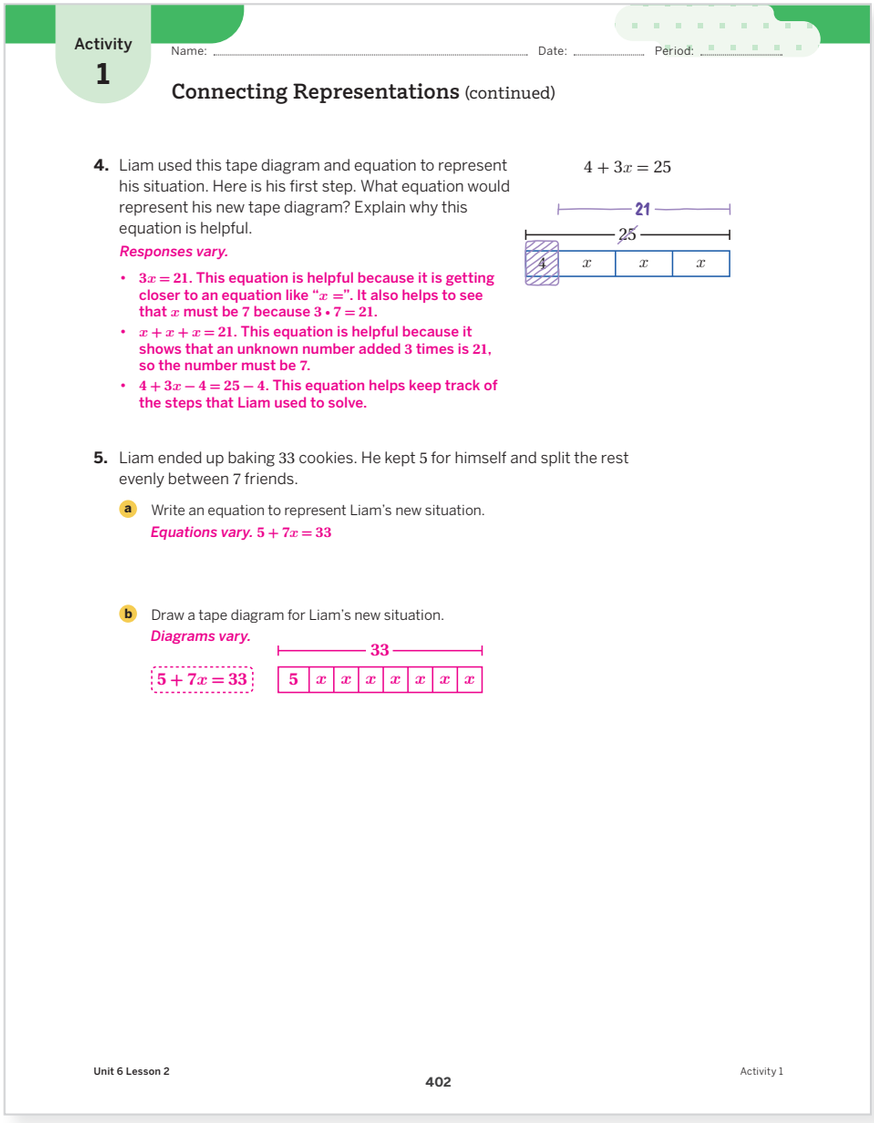
3 Connect

Share the captured equations and descriptions.

Consider asking:

- "How are the new equation and tape diagram connected?"
- "How can we see the parts of the story in each representation?"

ELL **English Language Learners** Give students time to think independently before sharing their ideas verbally. **(Writing)**



Activity 1

Name: _____ Date: _____ Period: _____

Connecting Representations (continued)

4. Liam used this tape diagram and equation to represent his situation. Here is his first step. What equation would represent his new tape diagram? Explain why this equation is helpful.

$4 + 3x = 25$

21

25

4 | x | x | x

Responses vary.

- $3x = 21$. This equation is helpful because it is getting closer to an equation like "$x =$". It also helps to see that x must be 7 because $3 \cdot 7 = 21$.
- $x + x + x = 21$. This equation is helpful because it shows that an unknown number added 3 times is 21, so the number must be 7.
- $4 + 3x - 4 = 25 - 4$. This equation helps keep track of the steps that Liam used to solve.

5. Liam ended up baking 33 cookies. He kept 5 for himself and split the rest evenly between 7 friends.

a. Write an equation to represent Liam's new situation.

Equations vary. $5 + 7x = 33$

b. Draw a tape diagram for Liam's new situation.

Diagrams vary.

$5 + 7x = 33$

33

5 | x | x | x | x | x | x

Unit 6 Lesson 2 402 Activity 1

Key Takeaway: Equations and tape diagrams can be used to represent and model situations.

Pairs | 15 min

Activity 2 Missing Representations

Purpose: Students use the **Tell a Story** routine to practice connecting situations, tape diagrams, and equations.

1 Launch

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to group the representations.

2 Monitor

Pause to give students time to compare their grouped representations with another pair of students. Invite students to discuss and make revisions based on their conversation. **(MTR.4.1)**

Consider asking, "How do your matches compare to your classmates'?"

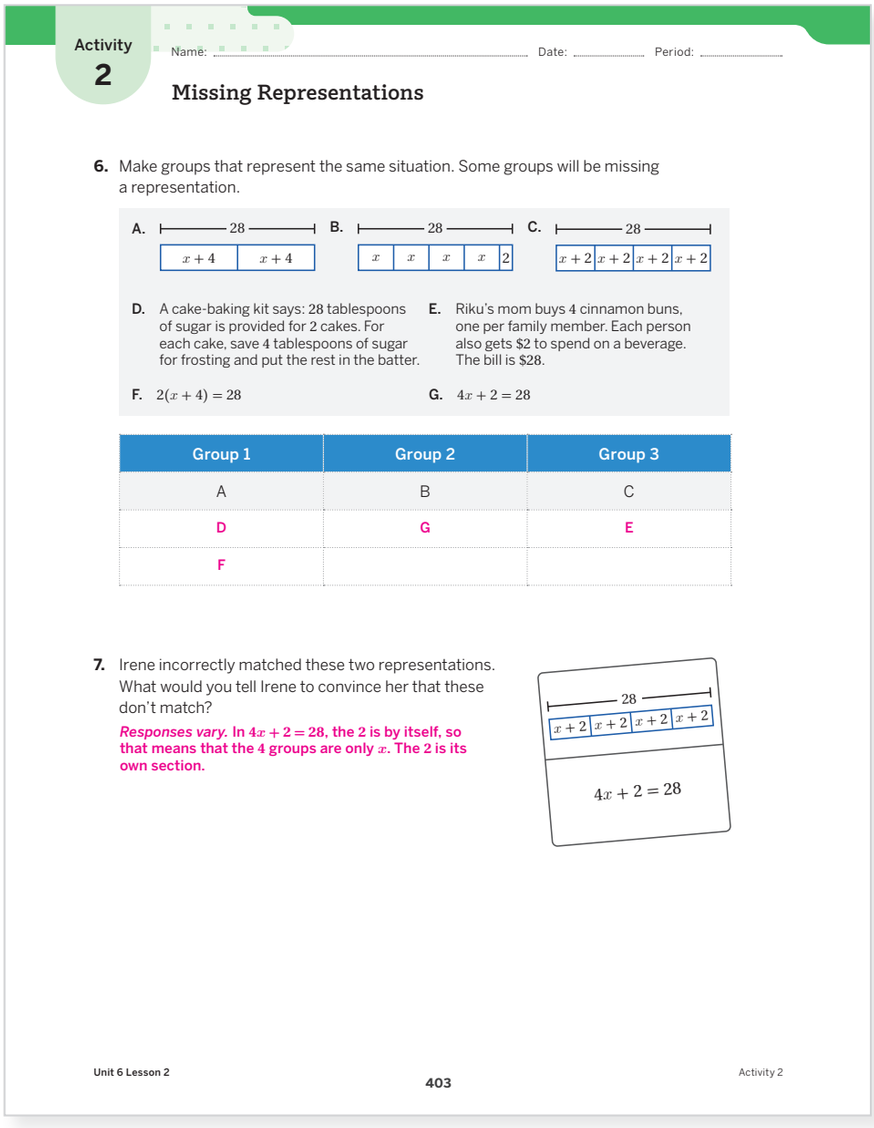
ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., These do not match because _____.). **(Writing and Speaking)**

Consider asking, "How do you know that this tape diagram and equation do not match?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Irene's card match. Then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write advice or a question to help Irene see how her work can be improved, or create a revised response. **(MTR.4.1)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Missing Representations

6. Make groups that represent the same situation. Some groups will be missing a representation.

A.

B.

C.

D. A cake-baking kit says: 28 tablespoons of sugar is provided for 2 cakes. For each cake, save 4 tablespoons of sugar for frosting and put the rest in the batter.

E. Riku's mom buys 4 cinnamon buns, one per family member. Each person also gets $2 to spend on a beverage. The bill is $28.

F. $2(x+4)=28$

G. $4x+2=28$

Group 1	Group 2	Group 3
A	B	C
D	G	E
F		

7. Irene incorrectly matched these two representations. What would you tell Irene to convince her that these don't match?

Responses vary. In $4x+2=28$, the 2 is by itself, so that means that the 4 groups are only x. The 2 is its own section.

Unit 6 Lesson 2 403 Activity 2

Pairs | 15 min

Activity 2 Missing Representations (continued)

Purpose: Students use the Tell a Story routine to practice connecting situations, tape diagrams, and equations.

2 Monitor

D **Differentiation: (Support)** Look for students who write an expression instead of an equation, such as $4(x + 2) + 28$. Encourage them to identify the total value.

Capture student responses to share during the Connect.

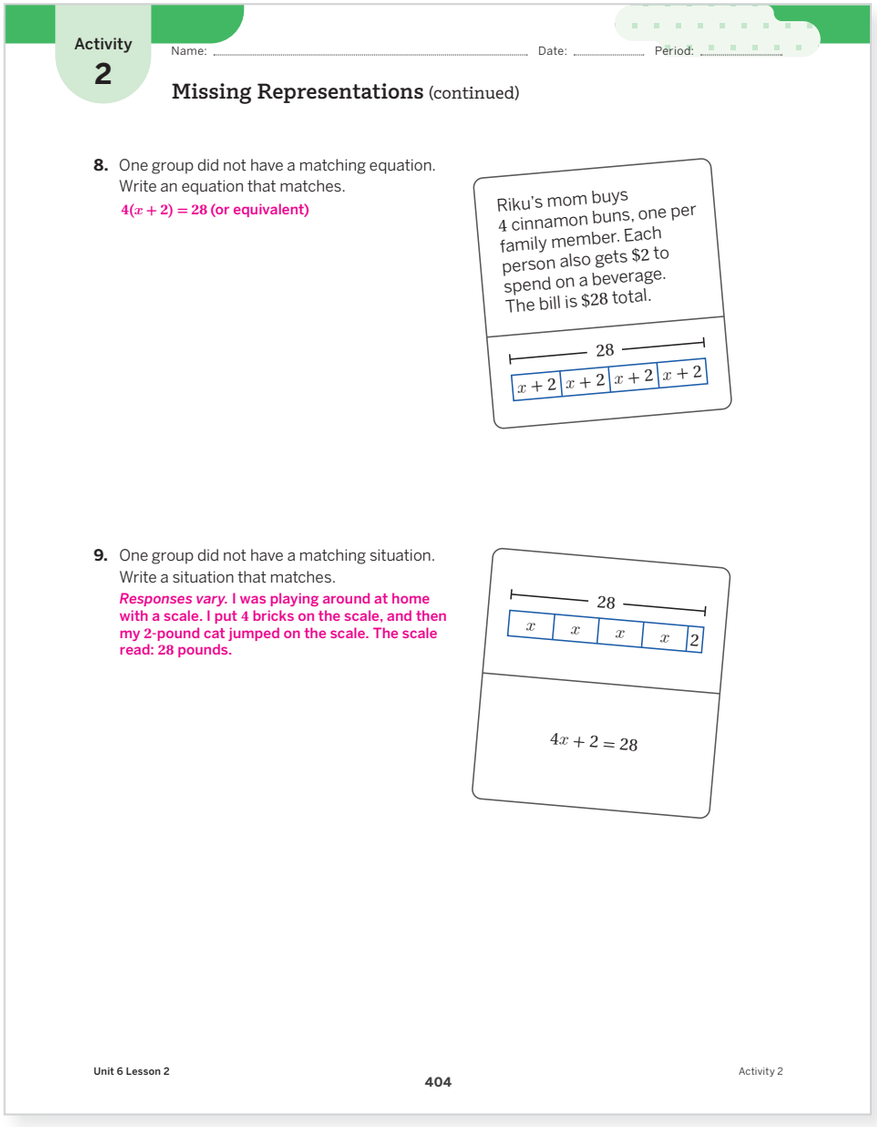
Use the Tell a Story routine. Invite students to write a story about the group using their unique experiences, perspectives, identities, and creativity. See the Routine Facilitation Guide for more information.

Look for unique or creative student stories to highlight during the Connect.

3 Connect

Direct students' attention to the group missing an equation, and invite students to share their equations. Amplify the variety of equations shared and discuss both correct and incorrect thinking.

Share the captured stories. Invite students to speak about their inspiration.



Activity 2

Name: Date: Period:

Missing Representations (continued)

8. One group did not have a matching equation. Write an equation that matches.

$4(x + 2) = 28$ (or equivalent)

Riku's mom buys 4 cinnamon buns, one per family member. Each person also gets $2 to spend on a beverage. The bill is $28 total.

9. One group did not have a matching situation. Write a situation that matches.

Responses vary. I was playing around at home with a scale. I put 4 bricks on the scale, and then my 2-pound cat jumped on the scale. The scale read: 28 pounds.

Unit 6 Lesson 2 404 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the connections between stories, equations, and tape diagrams with their solution.

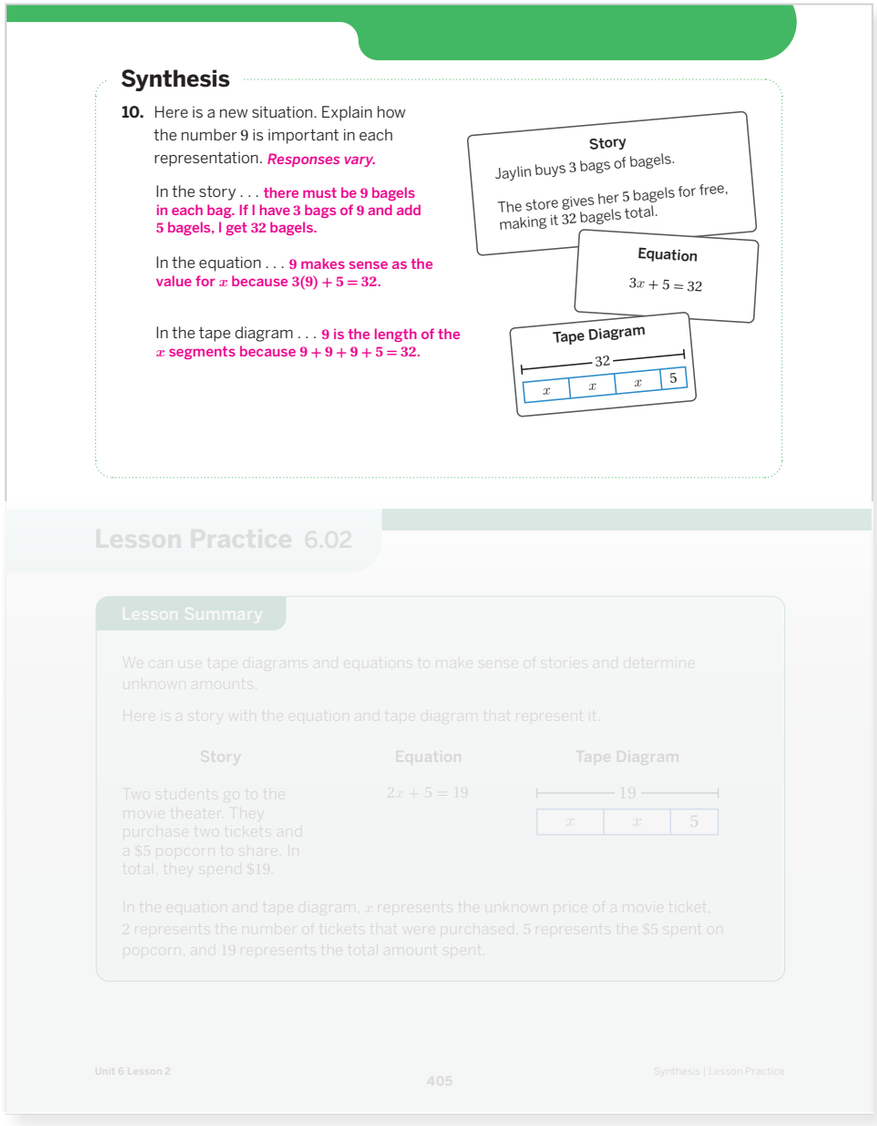
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Subtracting 5 free bagels from the 32 bagel total and then dividing this by the 3 bags of bagels to get 9.
- Substituting 9 for x in the equation and simplifying.
- Annotating the x values in the tape diagram as 9 and then adding 9, 9, 9, and 5 to get 32. **(MTR.2.1)**

Note: If time allows, invite students to revise their previous thinking based on the discussion.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**



Synthesis

10. Here is a new situation. Explain how the number 9 is important in each representation. *Responses vary.*

In the story . . . **there must be 9 bagels in each bag. If I have 3 bags of 9 and add 5 bagels, I get 32 bagels.**

In the equation . . . **9 makes sense as the value for x because $3(9) + 5 = 32$.**

In the tape diagram . . . **9 is the length of the x segments because $9 + 9 + 9 + 5 = 32$.**

Lesson Practice 6.02

Lesson Summary

We can use tape diagrams and equations to make sense of stories and determine unknown amounts.

Here is a story with the equation and tape diagram that represent it.

Story	Equation	Tape Diagram
Two students go to the movie theater. They purchase two tickets and a $5 popcorn to share. In total, they spend $19.	$2x + 5 = 19$	19; x x 5

In the equation and tape diagram, x represents the unknown price of a movie ticket, 2 represents the number of tickets that were purchased, 5 represents the $5 spent on popcorn, and 19 represents the total amount spent.

Unit 6 Lesson 2 405 Synthesis | Lesson Practice

Lesson Takeaway: Solutions represent important information connecting stories, equations, and tape diagrams.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by connecting an equation to a situation and determining its solution.

Today's Goals

1. **Goal:** Connect tape diagrams, equations, and verbal descriptions in context.
2. **Goal:** Write an equation to represent a tape diagram and a context.
3. **Language Goal:** Interpret the meaning of situations in a mathematical context. **(Reading)**

Show What You Know (Assessment Resources)

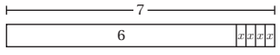
Name: _____ Date: _____ Period: _____

Show What You Know 6.02

Ariana ran 6 times around her school building. Then she ran 4 miles home. Her phone told her that she ran 7 miles total.

a. Which equation and tape diagram represent this situation?

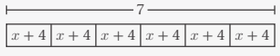
A. $6 + 4x = 7$



B. $6x + 4 = 7$



C. $6(x + 4) = 7$



b. Determine the value of x.

$x = 0.5$ (or equivalent)

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to represent contexts with equations:

» **Respond to Student Thinking:**

- Use the ***Writing Equations from Descriptions and Tape Diagrams*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

How did the Tell a Story routine support students in connecting situations to tape diagrams and equations? How did students tap into their creativity while writing their stories?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Here is a new situation. Explain how the number 9 is important in each representation. *Responses vary.*

In the story . . . there must be 9 bagels in each bag. If I have 3 bags of 9 and add 5 bagels, I get 32 bagels.

In the equation . . . 9 makes sense as the value for x because $3(9) + 5 = 32$.

In the tape diagram . . . 9 is the length of the x segments because $9 + 9 + 9 + 5 = 32$.

Lesson Practice 6.02

Lesson Summary

We can use tape diagrams and equations to make sense of stories and determine unknown amounts.

Here is a story with the equation and tape diagram that represent it.

Story	Equation	Tape Diagram
Two students go to the movie theater. They purchase two tickets and a $5 popcorn to share. In total, they spend $19.	$2x + 5 = 19$	19: x | x | 5

In the equation and tape diagram, x represents the unknown price of a movie ticket, 2 represents the number of tickets that were purchased, 5 represents the $5 spent on popcorn, and 19 represents the total amount spent.

Lesson Practice 6.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Here are two different situations.

1. Match each equation and situation with its tape diagram.

Equation/Situation	Diagram
a. $3x + 6 = 27$	a, d
b. $6x + 3 = 27$	
c. A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 total.	
d. Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.	b, c

Diagram A: 27: x \| x \| x \| 6

Diagram B: 27: x \| x \| x \| x \| x \| x \| 3

2. Using your responses from Problem 1, determine the value and meaning of x in Diagram A. Explain what x represents in context.

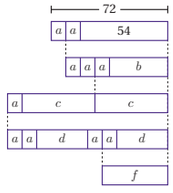
$x = 7$. x represents the number of ounces of juice he gives each friend, so Diego gives each friend 7 ounces of juice.

3. Using your responses from Problem 1, determine the value and meaning of x in Diagram B. Explain what x represents in context.

$x = 4$. x represents the cost of the ticket, so each ticket costs $4.

4. Determine the value of each variable.

Variable	Value
a	9
b	36
c	45
d	31.5
f	40.5



Lesson Practice 6.02

Name: _____ Date: _____ Period: _____

FAST Practice

5. Match each equation with a tape diagram.

Equation	Tape Diagram
a. $5x + 1 = 20$	b
b. $5(x + 1) = 20$	a
c. $5 + x + 1 = 20$	c

Tape Diagram b: 20: $x+1$ \| $x+1$ \| $x+1$ \| $x+1$ \| $x+1$

Tape Diagram a: 20: x \| x \| x \| x \| x \| 1

Tape Diagram c: 20: 5 \| x \| 1

Spiral Review

6. Determine the number of miles each car can travel in 1 hour assuming the operator drives at a constant speed.

Speed	Number of Miles in 1 Hour
135 miles in 3 hours	45
22 miles in $\frac{1}{2}$ hour	44
7.5 miles in $\frac{1}{4}$ hour	30
$97\frac{1}{2}$ miles in $\frac{3}{2}$ hours	65

Problems 7–10: Determine the value of each expression.

7. $100 \cdot (-0.09) = -9$

8. $-7 \cdot (-1.1) = 7.7$

9. $-7.3 \cdot 5 = -36.5$

10. $-0.2 \cdot (-0.3) = 0.06$

11. Select *all the* expressions that have products that are positive.

- ☑ A. $-\frac{5}{2} \cdot (-4) \cdot 1\frac{1}{2} \cdot 2$
- ☑ B. $-2\frac{1}{5} \cdot \left(-\frac{1}{6}\right) \cdot \left(\frac{3}{4}\right) \cdot \left(-9\frac{7}{8}\right)$
- ☑ C. $-4 \cdot 0.3 \cdot (-8)$
- ☐ D. $-2.1 \cdot (-5.3) \cdot 7.2 \cdot (-0.4)$
- ☐ E. $\frac{4}{3} \cdot \frac{3}{4} \cdot \left(-\frac{1}{2}\right)$

Lesson Practice Item Analysis

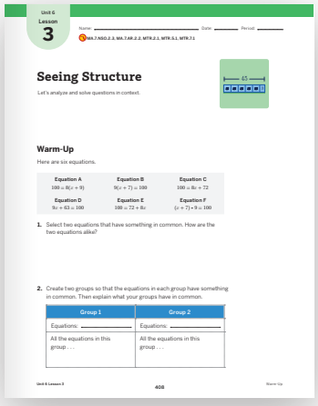
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.7.AR.2.2
	4	3	MA.7.AR.2.2
FAST Practice	5	2	MA.7.AR.2.2
Spiral Review			
	6	2	MA.6.AR.3.2
	7–11	1	MA.7.NSO.2.2

Unit 6
Lesson
3

Seeing Structure

Practice With Tape Diagrams and Equations

Let's analyze and solve questions in context.


Unit 6
Lesson 3
Name: Date: Period:
MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.5.1, MTR.7.1
Seeing Structure
Let's analyze and solve questions in context.
Lesson 3 Learning Targets
Warm-Up
Here are six equations.
Equation A
Equation B
Equation C
Equation D
Equation E
Equation F
1. Select two equations that have something in common. How are the two equations alike?
2. Create two groups so that the equations in each group have something in common. Then explain what your groups have in common.
Group 1
Group 2
Equations:
All the equations in this group . . .
408
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Connect features of a situation to the tape diagram, equation, and solution that represent it.
2. **Goal:** Write equations of the form $px + q = r$ and $p(x + q) = r$ to represent situations and determine their solutions using a tape diagram.
3. **Language Goal:** Analyze a situation in context to write an equation and interpret the meaning of its solution. **(Reading, Writing, Speaking, and Listening)**

Students use tape diagrams and equations to answer questions about situations. They compare and contrast equations of the form $px + q = r$ and $p(x + q) = r$, then use the solutions to these equations to answer questions about the situations. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 2, students wrote equations to represent tape diagrams and contexts but did not yet solve these equations or interpret their solutions. In Grade 6, students solved equations of the form $x + p = q$ and $px = q$.

Future Learning

In Sub-Unit 2, students will use a different strategy to solve equations: balancing moves using hanger diagrams. They will also rewrite expressions using fewer terms before solving.

Connections

- Students analyze visual representations to solve equations and word problems by breaking them down into smaller, more manageable parts. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.3
Solve real-world problems involving any of the four operations with rational numbers.

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2 **MA.6.AR.2.3**	**MA.8.AR.2.1**

Lesson at a Glance

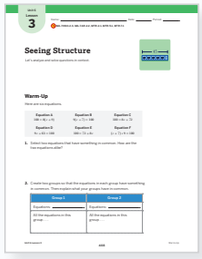
~ 45 min

MA.7.NSO.2.3, MA.7.AR.2.2

Warm-Up

Pairs | 10 min

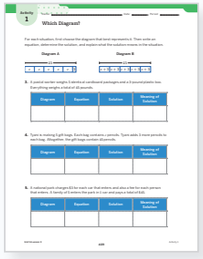
Students attend to the structure of equations of the form $px + q = r$ and $p(x + q) = r$.
Routine: Think-Pair-Share **(MTR.5.1)**



Activity 1

Pairs | 10 min

Students connect the features of a situation to a tape diagram, then write an equation, determine a solution, and interpret the meaning of the solution in the situation. **(MTR.2.1)**



Activity 2

Pairs | 15 min

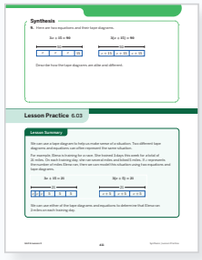
Students use the features of a situation to write their own question, then write an equation and determine its solution to answer the question.
MLR5: Co-Craft Questions **(MTR.7.1)**



Synthesis

Whole Class | 5 min

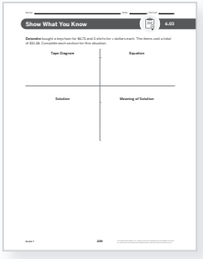
Students synthesize their understanding of comparing tape diagrams for similar equations with different structures.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by representing a situation with a tape diagram and an equation. They use these to solve and interpret the meaning of the solution in the situation.



Prep Checklist

Invite students to use their Student Edition.

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 10 min

Warm-Up

Purpose: Students attend to the structure of equations of the form $px + q = r$ and $p(x + q) = r$. **(MTR.5.1)**

1 Launch

Direct students' attention to the equations.

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

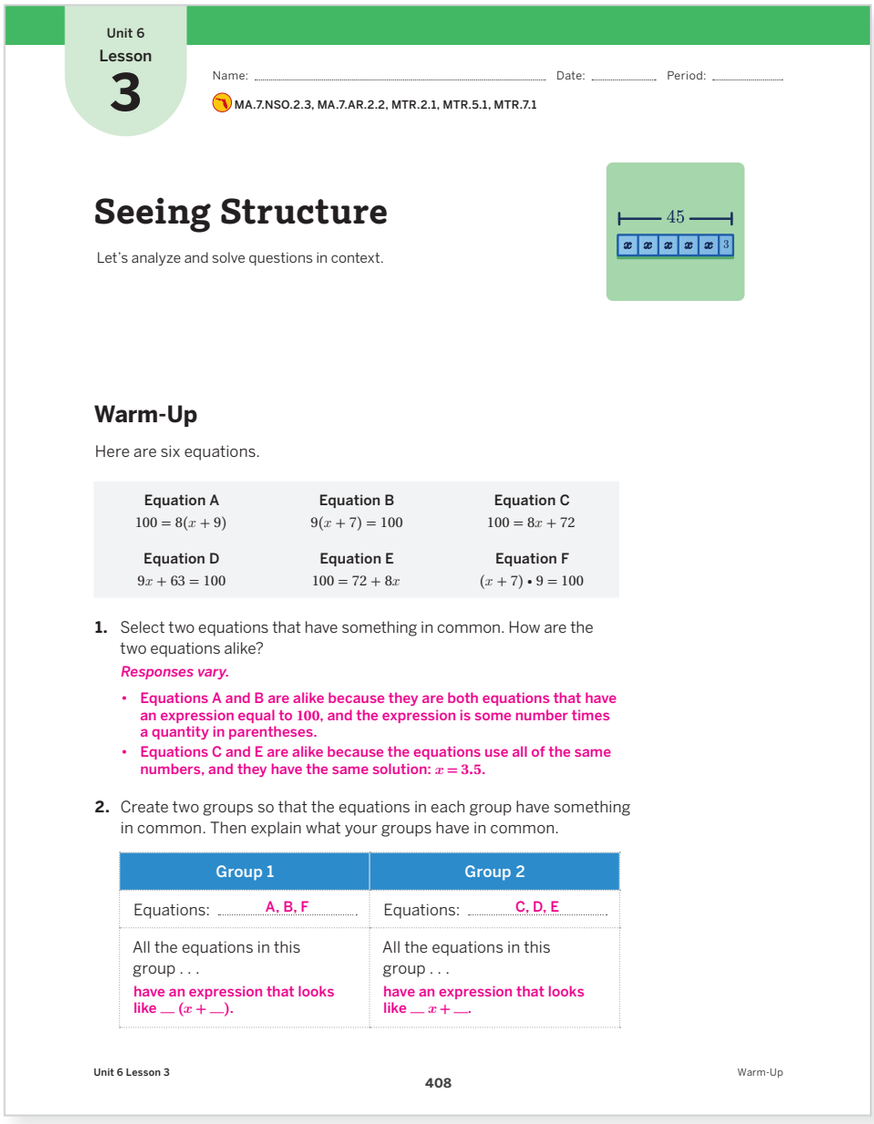
Consider asking, "What do you notice about the structure of the equations?"

A **Accessibility: Conceptual Processing** Check for understanding with individual students as needed at each step of the activity.

2 Connect

Invite students to share how they grouped the equations along with their reasoning. Emphasize groupings based on the structure of the equation. **(MTR.5.1)**

To surface the Key Takeaway, consider asking, "Did anyone group the equations based on whether there were parentheses? How would including parentheses change the tape diagram?" The tape diagrams for the equations with parentheses would be split into segments of equal length. For example, the tape diagram for the equation $9(x + 7) = 100$ would be split evenly into 9 segments each with a length of $x + 7$. The tape diagrams for the equations without parentheses would not be split into segments of equal length. For example, the equation $100 = 8x + 72$ would be represented by a tape diagram with a 72-unit segment and 8 segments of x units.



Unit 6
Lesson
3

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.5.1, MTR.7.1

Seeing Structure

Let's analyze and solve questions in context.

Warm-Up

Here are six equations.

Equation A	Equation B	Equation C
$100 = 8(x + 9)$	$9(x + 7) = 100$	$100 = 8x + 72$
Equation D	**Equation E**	**Equation F**
$9x + 63 = 100$	$100 = 72 + 8x$	$(x + 7) \cdot 9 = 100$

1. Select two equations that have something in common. How are the two equations alike?

   Responses vary.

   - Equations A and B are alike because they are both equations that have an expression equal to 100, and the expression is some number times a quantity in parentheses.
   - Equations C and E are alike because the equations use all of the same numbers, and they have the same solution: $x = 3.5$.

2. Create two groups so that the equations in each group have something in common. Then explain what your groups have in common.

Group 1	Group 2
Equations: A, B, F	Equations: C, D, E
All the equations in this group . . . have an expression that looks like __ $(x + $ __$)$.	All the equations in this group . . . have an expression that looks like __ $x + $ __.

Unit 6 Lesson 3 408 Warm-Up

Key Takeaway: Equations of the form $px + q = r$ represent p groups of x's plus an additional quantity, q. Equations of the form $p(x + q) = r$ represent p groups of the quantity $x + q$.

Pairs | 10 min

Activity 1 Which Diagram?

Purpose: Students connect the features of a situation to a tape diagram, then write an equation, determine a solution, and interpret the meaning of the solution in the situation. **(MTR.2.1)**

Short on time? Consider omitting Problem 3.

1 Launch

Direct students' attention to the tape diagrams.

To surface connections, consider asking, "How are these tape diagrams alike? How are they different?"

Consider asking, "How can you determine which diagram matches the situation?" Then invite students to use reasoning to compare their matches and work together to write the equation, solution, and meaning of the solution. **(MTR.2.1)**

2 Monitor

Select and sequence student strategies to share during the Connect.

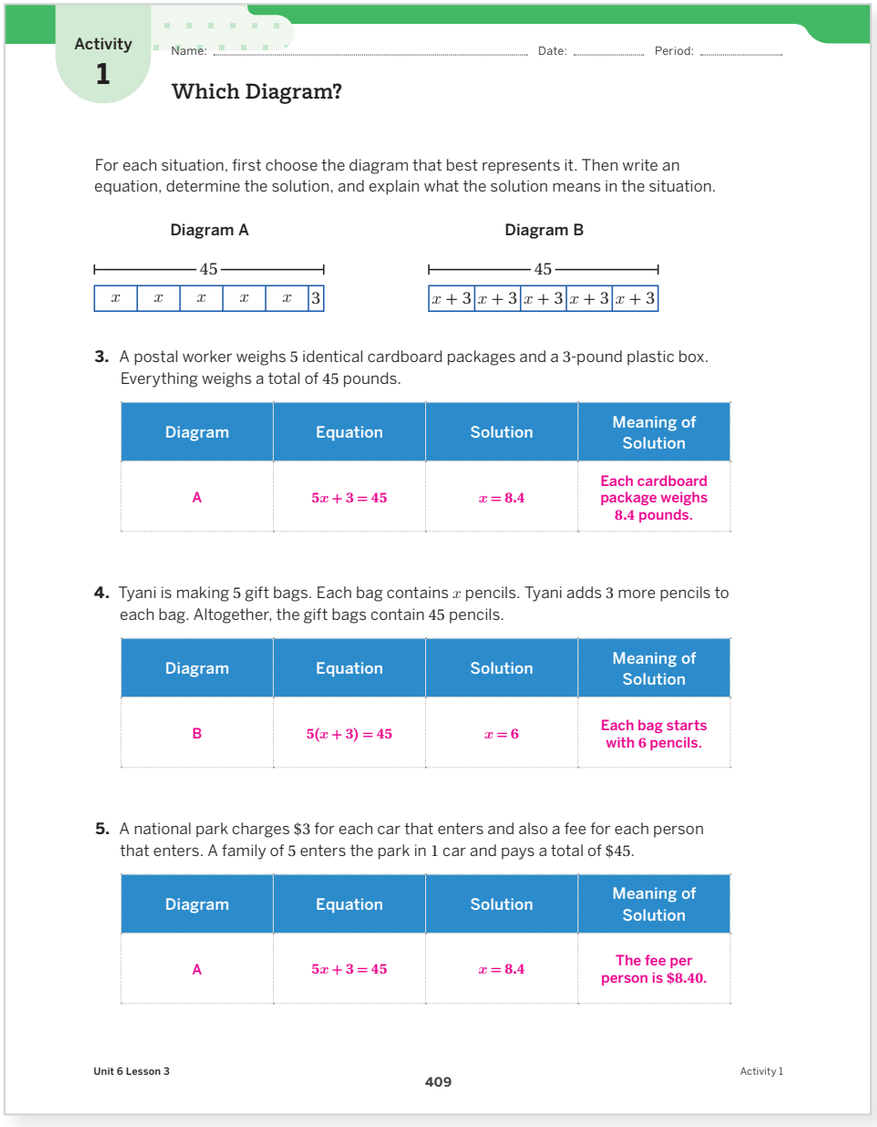
D Differentiation *(Problems 3–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to match their groups from the Warm-Up with a diagram structure.
Discuss whether 3 is a one-time or repeated value.	**Strengthen** Invite students to share during the Connect how a one-time or repeated 3 is represented in a tape diagram and in an equation.
Would like to explore real-world situations further.	**Stretch** Invite students to write their own situation that matches one of the diagrams, then trade situations with another student to determine which diagram they selected.

3 Connect

Share the correct solutions to focus the conversation on students' reasoning.

To surface the Key Takeaway, invite students to share the strategies they used to decide which diagram matched each situation, to write the equation, and to determine the meaning of the solution.



Activity 1

Name: Date: Period:

Which Diagram?

For each situation, first choose the diagram that best represents it. Then write an equation, determine the solution, and explain what the solution means in the situation.

Diagram A

Diagram B

3. A postal worker weighs 5 identical cardboard packages and a 3-pound plastic box. Everything weighs a total of 45 pounds.

Diagram	Equation	Solution	Meaning of Solution
A	$5x + 3 = 45$	$x = 8.4$	Each cardboard package weighs 8.4 pounds.

4. Tyani is making 5 gift bags. Each bag contains x pencils. Tyani adds 3 more pencils to each bag. Altogether, the gift bags contain 45 pencils.

Diagram	Equation	Solution	Meaning of Solution
B	$5(x + 3) = 45$	$x = 6$	Each bag starts with 6 pencils.

5. A national park charges $3 for each car that enters and also a fee for each person that enters. A family of 5 enters the park in 1 car and pays a total of $45.

Diagram	Equation	Solution	Meaning of Solution
A	$5x + 3 = 45$	$x = 8.4$	The fee per person is $8.40.

Unit 6 Lesson 3 409 Activity 1

Key Takeaway: The context of a situation determines the structure of both the equation and tape diagram that represent it. Equations of the form $px + q = r$ represent repeatedly adding the value p and adding the value q only once. Equations of the form $p(x + q) = r$ represent repeatedly summing the value $x + q$.

Pairs | 15 min

Activity 2 Questions and Answers

Purpose: Students use the features of a situation to write their own question, then write an equation and determine its solution to answer the question. **(MTR.7.1)**

Short on time? Consider having students choose one of the two problems.

1 Launch

Consider asking:

- "What do you remember about how to write good math questions?"
- "What do you think makes a good question?"

MLR **MLR5: Co-Craft Questions** Use this routine to support students in making sense of a context, by asking questions without the pressure to produce answers.

Consider sharing that the goal is to generate many questions before choosing one to answer.

Invite students to create questions that could be asked and answered about each situation. **(MTR.7.1)**

2 Monitor

Listen for unique and creative student questions, including ones that are related to the activity they're about to explore.

D **Differentiation** *(Problems 6–8)*

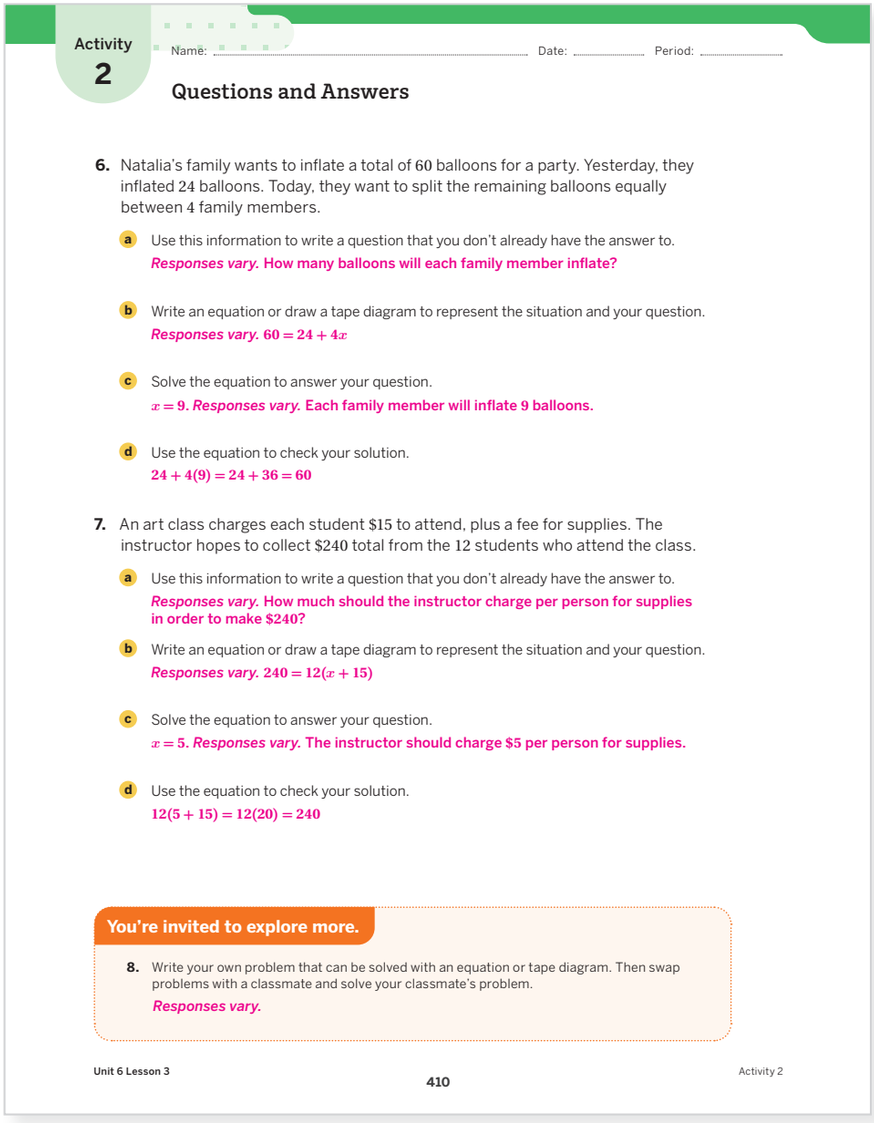
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to highlight or circle important values and information in the situation. Have students brainstorm with their partner to identify a good question.
Draw a visual (tape diagram or hanger) to represent the situation or use another tool to help solve. **(MTR.2.1)**	**Strengthen** Invite them to share their visuals with their partner. Consider asking, "How can you use this visual to create a diagram and equation?"
Would like to explore real-world problems further. *(Problem 8)*	**Stretch** **You're invited to explore more.** Encourage students to write their own problem that can be solved with an equation or a tape diagram.

3 Connect

Invite students to share the questions that they wrote.

Consider asking:

- "Which question(s) are most interesting to you?"
- "How are the diagrams and equations that you wrote for Problem 1 and Problem 2 alike? How are they different?"
- "How did you determine whether the structure of the equations you wrote should be in $px + q = r$ or $p(x + q) = r$ form?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Questions and Answers

6. Natalia's family wants to inflate a total of 60 balloons for a party. Yesterday, they inflated 24 balloons. Today, they want to split the remaining balloons equally between 4 family members.

a. Use this information to write a question that you don't already have the answer to.
Responses vary. How many balloons will each family member inflate?

b. Write an equation or draw a tape diagram to represent the situation and your question.
Responses vary. $60 = 24 + 4x$

c. Solve the equation to answer your question.
$x = 9$. *Responses vary.* Each family member will inflate 9 balloons.

d. Use the equation to check your solution.
$24 + 4(9) = 24 + 36 = 60$

7. An art class charges each student $15 to attend, plus a fee for supplies. The instructor hopes to collect $240 total from the 12 students who attend the class.

a. Use this information to write a question that you don't already have the answer to.
Responses vary. How much should the instructor charge per person for supplies in order to make $240?

b. Write an equation or draw a tape diagram to represent the situation and your question.
Responses vary. $240 = 12(x + 15)$

c. Solve the equation to answer your question.
$x = 5$. *Responses vary.* The instructor should charge $5 per person for supplies.

d. Use the equation to check your solution.
$12(5 + 15) = 12(20) = 240$

You're invited to explore more.

8. Write your own problem that can be solved with an equation or tape diagram. Then swap problems with a classmate and solve your classmate's problem.
Responses vary.

Unit 6 Lesson 3 410 Activity 2

Whole Class | 5 min

Synthesis

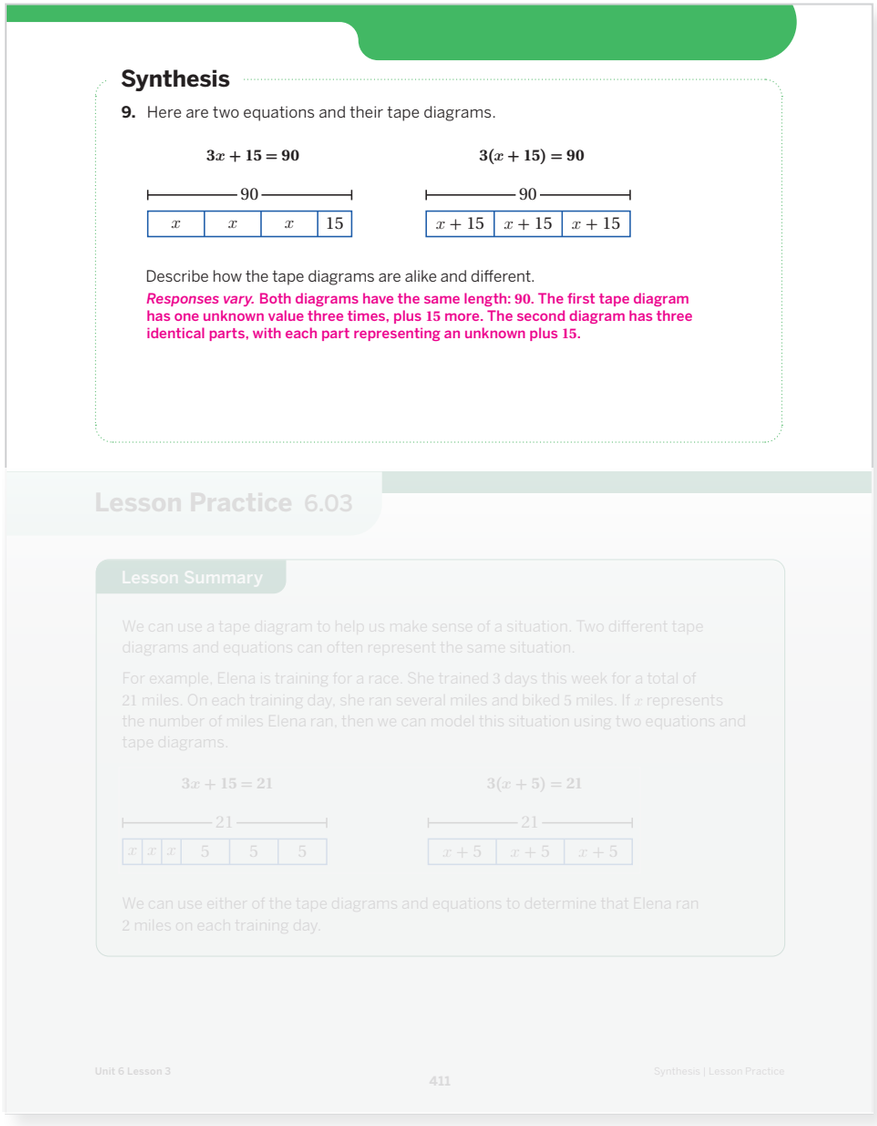
Purpose: Students synthesize their understanding of comparing tape diagrams for similar equations with different structures. **(MTR.5.1)**

Direct students' attention to the two equations.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:
- Comparing the values in the equations and tape diagrams.
- Contrasting the structure of the equations, tape diagrams, solutions, and the meaning of the solutions.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Here are two equations and their tape diagrams.

$3x + 15 = 90$

$3(x + 15) = 90$

Describe how the tape diagrams are alike and different.

Responses vary. Both diagrams have the same length: 90. The first tape diagram has one unknown value three times, plus 15 more. The second diagram has three identical parts, with each part representing an unknown plus 15.

Lesson Practice 6.03

Lesson Summary

We can use a tape diagram to help us make sense of a situation. Two different tape diagrams and equations can often represent the same situation.

For example, Elena is training for a race. She trained 3 days this week for a total of 21 miles. On each training day, she ran several miles and biked 5 miles. If x represents the number of miles Elena ran, then we can model this situation using two equations and tape diagrams.

$3x + 15 = 21$

$3(x + 5) = 21$

We can use either of the tape diagrams and equations to determine that Elena ran 2 miles on each training day.

Unit 6 Lesson 3 411 Synthesis | Lesson Practice

Lesson Takeaway: A situation can be represented by the features of its tape diagram and the structure of its equation.

Independent | 5 min

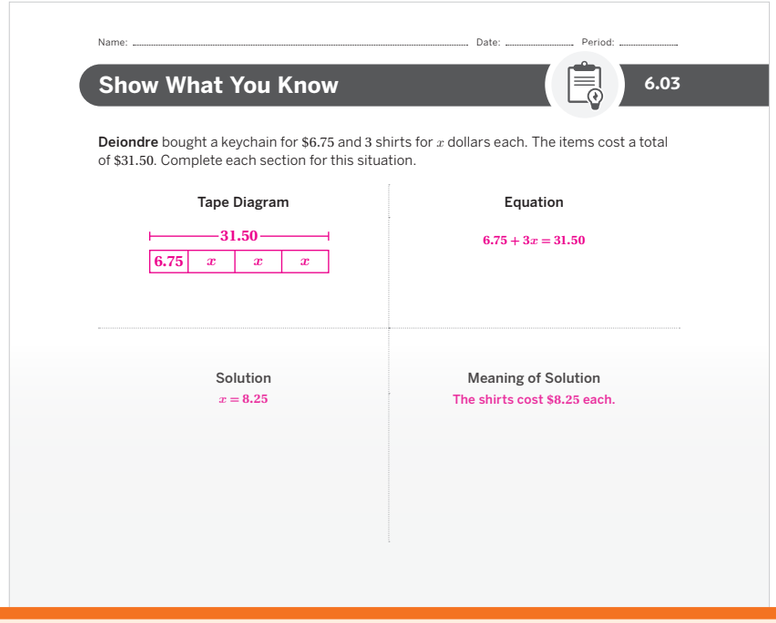
Show What You Know

Purpose: Students demonstrate their understanding by representing a situation with a tape diagram and an equation. They use these to solve and interpret the meaning of the solution in the situation.

Today's Goals

1. **Goal:** Connect features of a situation to the tape diagram, equation, and solution that represent it.
2. **Goal:** Write equations of the form $px + q = r$ and $p(x + q) = r$ to represent situations and determine their solutions using a tape diagram.
3. **Language Goal:** Analyze a situation in context to write an equation and interpret the meaning of its solution. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.03

Deiondre bought a keychain for $6.75 and 3 shirts for x dollars each. The items cost a total of $31.50. Complete each section for this situation.

Tape Diagram

Equation

$6.75 + 3x = 31.50$

Solution

$x = 8.25$

Meaning of Solution

The shirts cost $8.25 each.

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of connect situations to tape diagrams, equations, and solutions:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and give students individualized feedback.
- Review the **Lesson 2** Summary.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite them to complete the You're invited to **Explore More** task and discuss their thinking with a partner.

Professional Learning

What challenges did students encounter as they worked on the MLR5: Co-Craft Questions routine in Activity 2? How did they work through these challenges? Reflect on how you supported students during this activity. What went well? What might you change?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Here are two equations and their tape diagrams.

$3x + 15 = 90$

$3(x + 15) = 90$

Describe how the tape diagrams are alike and different.

Responses vary. Both diagrams have the same length: 90. The first tape diagram has one unknown value three times, plus 15 more. The second diagram has three identical parts, with each part representing an unknown plus 15.

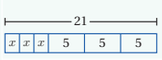
Lesson Practice 6.03

Lesson Summary

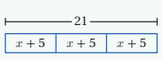
We can use a tape diagram to help us make sense of a situation. Two different tape diagrams and equations can often represent the same situation.

For example, Elena is training for a race. She trained 3 days this week for a total of 21 miles. On each training day, she ran several miles and biked 5 miles. If x represents the number of miles Elena ran, then we can model this situation using two equations and tape diagrams.

$3x + 15 = 21$



$3(x + 5) = 21$

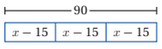


We can use either of the tape diagrams and equations to determine that Elena ran 2 miles on each training day.

Lesson Practice 6.03

Name: _____ Date: _____ Period: _____

1. A school ordered 3 large boxes of markers. 3 teachers got 15 markers each, so there were 90 markers left. This tape diagram represents the situation. How many markers were originally in each box?



45 markers

Problems 2–4: A family buys 4 tickets to a show. Each family member also spends $2 on a snack. They spend $24 total.

2. Which equation represents this situation?

A. $2(x + 4) = 24$ B. $4(x + 2) = 24$

3. What does x represent in the equation you chose?

x represents the cost of a ticket.

4. Solve the equation you chose. What does the solution tell you about this situation?

$x = 4$. Tickets to the show cost $4 each.

Problems 5–7: Amir has 24 ounces of juice. He pours equal amounts for each of his 2 friends and then adds 4 more ounces for each person.

5. Which equation represents this situation?

A. $2(x + 4) = 24$ B. $4(x + 2) = 24$

6. What does x represent in the equation you chose?

x represents the number of ounces of juice Amir originally poured for each friend.

7. Solve the equation you chose. What does the solution tell you about this situation?

$x = 8$. Amir originally poured 8 ounces of juice for each friend.

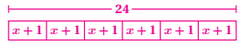
Problems 8–9: Draw a tape diagram and determine the solution for each equation.

8. $6x + 11 = 21$



$x = \frac{5}{3}$ (or equivalent)

9. $6(x + 1) = 24$



$x = 3$

Lesson Practice 6.03

Name: _____ Date: _____ Period: _____

FAST Practice

10. Each tape diagram is the same length. Write an equation for each tape diagram and determine the solution to the equation.

Diagram	Equation	Solution
56: $a+3$ | $a+3$ | $a+3$ | $a+3$	$4(a + 3) = 56$	$a = 11$
56: b | b | b | b | 3	$4b + 3 = 56$	$b = \frac{53}{4}$
56: $c+4$ | $c+4$ | $c+4$	$3(c + 4) = 56$	$c = \frac{44}{3}$
56: d | d | d | 4	$3d + 4 = 56$	$d = \frac{52}{3}$

Spiral Review

Problems 11–14: Determine the value of each expression.

11. $\frac{2}{3} \cdot \left(\frac{-4}{5}\right) = \frac{8}{15}$

12. $\left(\frac{-5}{7}\right) \cdot \left(\frac{7}{5}\right) = -1$

13. $\left(\frac{-2}{39}\right) \cdot 39 = -2$

14. $\left(\frac{2}{5}\right) \cdot \left(\frac{-3}{4}\right) = -\frac{3}{10}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–9	2	MA.7.AR.2.2
FAST Practice	10	3	MA.7.AR.2.2
Spiral Review			
	11–14	1	MA.7.NSO.2.2

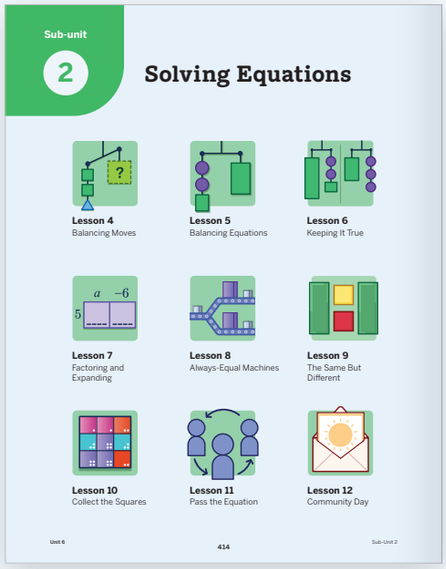
Sub-Unit 2

Solving Equations

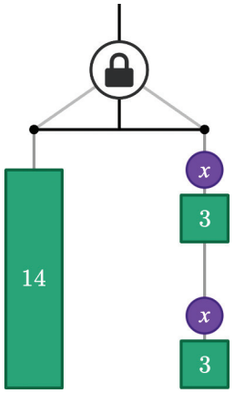
Sub-Unit 2 Goals

- **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems.
- **Goal:** Write equivalent expressions by adding, subtracting, expanding, and factoring.
- **Language Goal:** Describe and use strategies for writing equivalent expressions that involve adding, subtracting, factoring, expanding, and reordering terms.

In this sub-unit, students learn how to solve equations of the form $px + q = r$ and $p(x + q) = r$. They also write **equivalent expressions** using fewer terms by adding, **expanding**, and **factoring**, which can help make complex equations look more familiar before solving.



Math That Matters Most

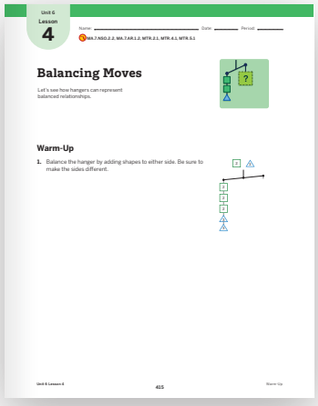
Concept or Skill	For example . . .
Use hanger diagrams to represent equations, making connections between the representations.	Write an equation to represent this hanger diagram.  **2(x + 3) = 14**
Solve equations of the forms $px + q = r$ and $p(x + q) = r$ that involve positive and negative rational numbers.	Solve the equation $-4(x + 1) = -28$. **-4(x + 1) = -28** **(x + 1) = 7** **x = 6**

Concept or Skill	For example . . .
Write equivalent expressions using a variety of strategies including factoring, expanding, reordering terms, and adding or subtracting terms.	Here is an expression. Write an equivalent expression using the fewest number of terms. $6(x + 1) + 8 - x + (-x - 12)$ **6x + 6 + 8 − x + (-x − 12)** **6x + 14 − x − x − 12** **4x + 2**
Write and solve equations to represent situations involving positive and negative numbers.	6 members of the Martinez family are going to their school's Community Day. They have a coupon for $\$4.50$ off each ticket. If they pay $\$40.50$ for all their tickets, how much does one ticket cost without the coupon? **6(x − 4.5) = 40.5** **x = 11.25** **Each ticket costs $11.25 without the coupon.**

Unit 6

Lesson

4



Balancing Moves

Introduction to Balanced Hangers

Let's see how hangers can represent balanced relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Determine an unknown value on a hanger diagram.
2. **Language Goal:** Describe strategies that will keep a hanger balanced (e.g., adding or removing equal items from each side). **(Writing, Speaking, and Listening)**

Students develop strategies for determining an unknown weight by creating balanced hangers with fewer objects. This is the first of a two lesson sequence where students connect moves on balanced hangers to the steps for solving an equation.

Coherence

Prior Learning

In Lessons 1–3, students used tape diagrams to represent equations and situations in context to determine unknown values.

Future Learning

In Lesson 5, students will connect steps for determining weights on balanced hangers to steps for solving equations.

Connections

- Students demonstrate their understanding of balanced relationships through the visual representations of the total weight (or value) on one side of a hanger to the total weight (or value) on the other side. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2
Add, subtract, multiply and divide rational numbers with procedural fluency.

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2	MA.7.AR.2.2
MA.6.AR.2.3	MA.8.AR.2.1

Lesson at a Glance

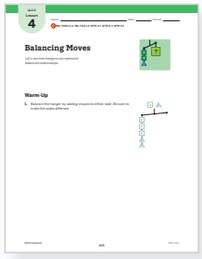
~ 45 min

MA.7.NSO.2.2, MA.7.AR.1.2

Warm-Up

Pairs | 5 min

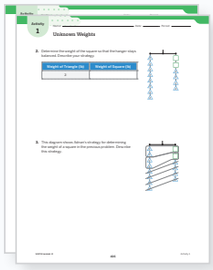
Students are introduced to balanced hangers and recognize that hangers are balanced when the total weight on each side is equal.



Activity 1

Pairs | 15 min

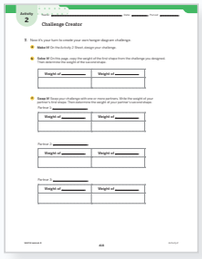
Students use and analyze different strategies to determine unknown weights on balanced hangers. **Routine:** Decide and Defend **(MTR.5.1)**



Activity 2

Independent | 15 min

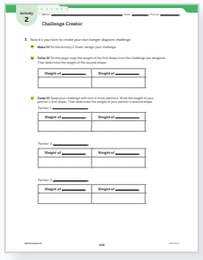
Students express their creativity by creating their own hangers and practice determining the weight of an unknown shape on a hanger.



Synthesis

Whole Class | 5 min

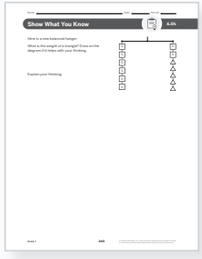
Students synthesize their understanding of strategies for creating a balanced hanger with fewer objects.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the weight of an unknown object on a hanger.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student
- coloring tools (optional)
- concrete model of a hanger (optional)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to balanced hangers and recognize that hangers are balanced when the total weight on each side is equal.

1 Launch

Demonstrate how to add shapes to each side of the hanger and how to adjust the balance. Consider using belts, socks, or ties placed on each end of a hanger as a concrete model.

Consider asking, "What do you think would happen if you added 5 triangles to the right side?"

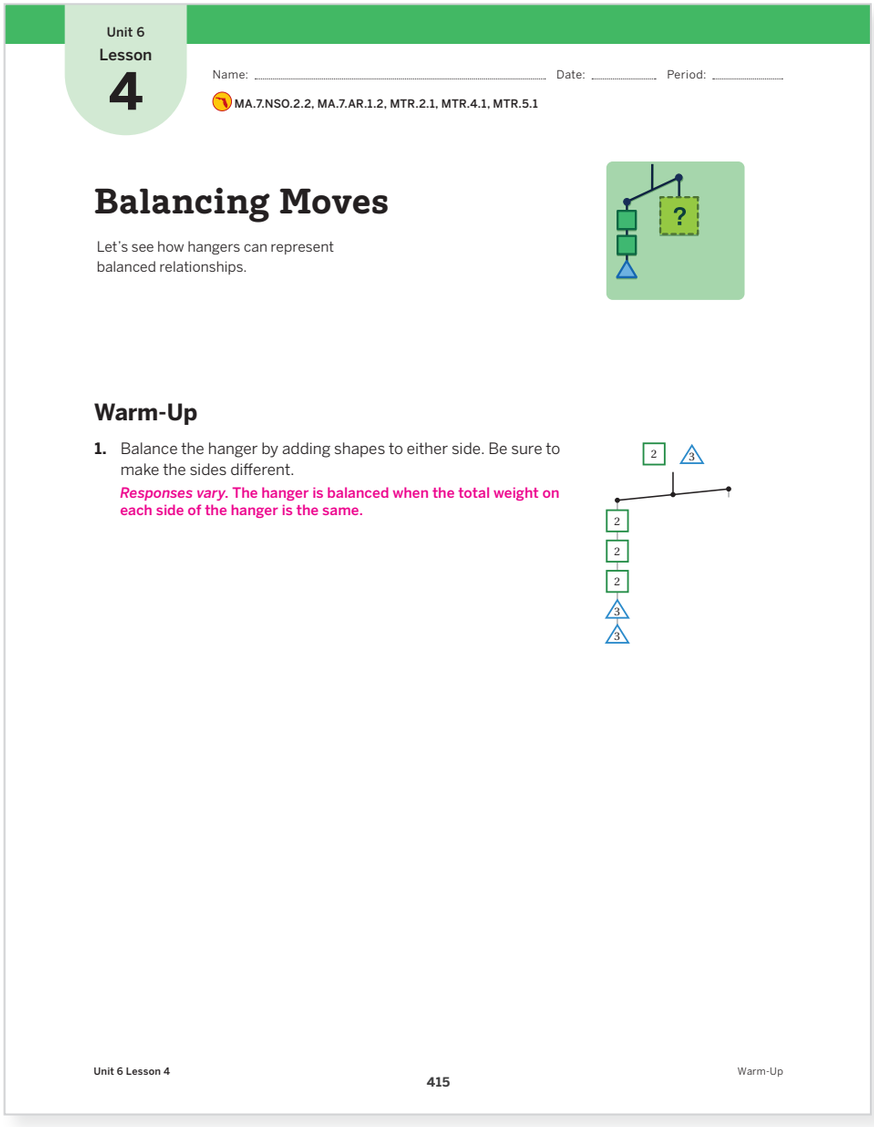
Invite students to use coloring tools to show their thinking throughout the lesson.

To encourage creativity, invite students to create several different hangers.

2 Connect

Invite students to share the shapes they added to each side, and the strategies they used to create a balanced hanger.

To surface the Key Takeaway, consider asking, "How can you predict if a hanger will be balanced?"



Unit 6
Lesson
4

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.AR.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Balancing Moves

Let's see how hangers can represent balanced relationships.

Warm-Up

1. Balance the hanger by adding shapes to either side. Be sure to make the sides different.

   Responses vary. The hanger is balanced when the total weight on each side of the hanger is the same.

Unit 6 Lesson 4 415 Warm-Up

Key Takeaway: A hanger is balanced if the total weight on each side is equal. There are several different ways of creating balanced hangers, including using the same weights in different orders or using different sets of weights that have the same total.

Pairs | 15 min

Activity 1 Unknown Weights

Purpose: Students use and analyze different strategies to determine unknown weights on balanced hangers. **(MTR.5.1)**

1 Launch

Consider asking, "What would happen if each square weighed 100 pounds?"

2 Monitor

Listen for a variety of strategies used to determine the weight of one square. **(MTR.5.1)**

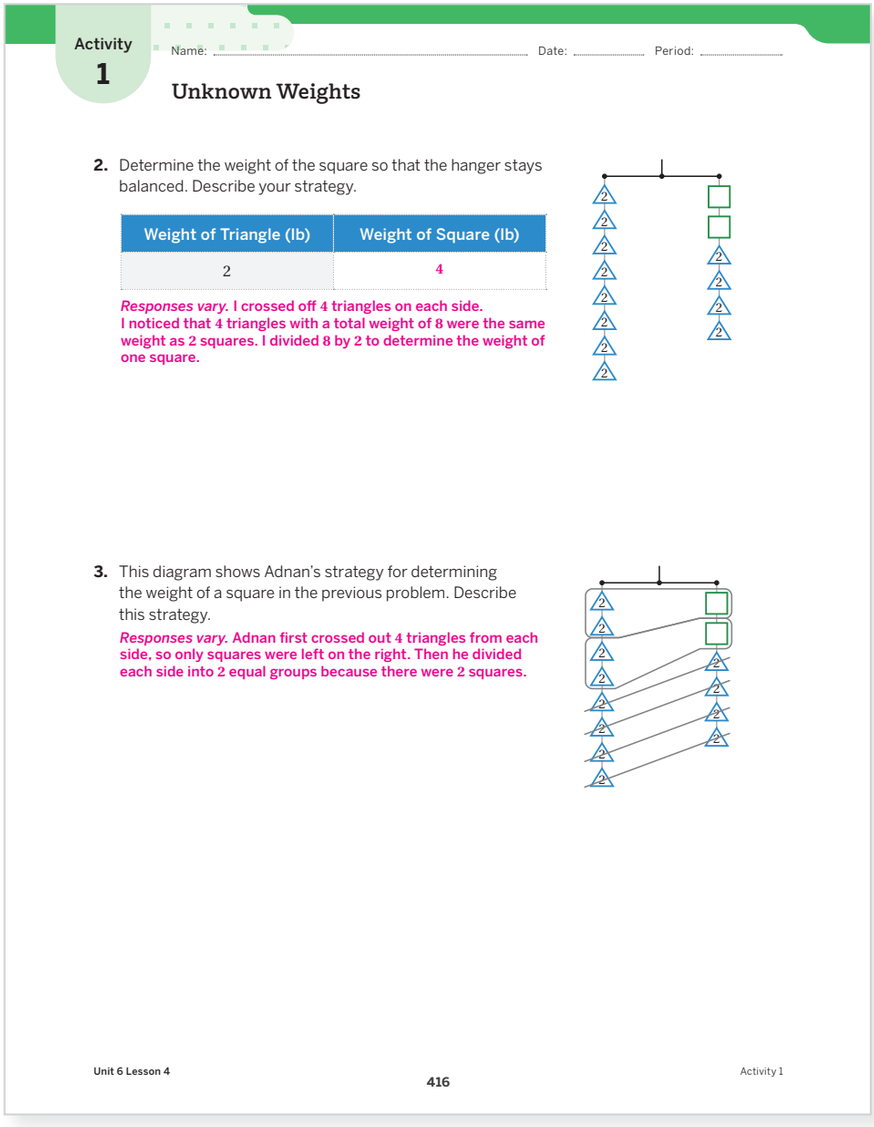
D Differentiation

Look for students who:	Respond to Student Thinking
Think that the weight of a square is the same as the weight of a triangle.	**Support** Consider asking, "If written as a ratio, how many triangles are there to one square?"
Use a strategy like Adnan's (cross off four triangles on each side of the hanger).	**Strengthen** Invite them to share how this helps to determine the weight of one square.
Rearrange the hanger diagram to show that one square and two triangles on the right has the same weight as four triangles on the left.	

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., First, Adnan _____. Then, he _____.). **(Writing and Speaking)**

Pause to discuss Adnan's strategy and other strategies students used to determine the weight of the square. **(MTR.4.1)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Unknown Weights

2. Determine the weight of the square so that the hanger stays balanced. Describe your strategy.

Weight of Triangle (lb)	Weight of Square (lb)
2	4

Responses vary. I crossed off 4 triangles on each side. I noticed that 4 triangles with a total weight of 8 were the same weight as 2 squares. I divided 8 by 2 to determine the weight of one square.

3. This diagram shows Adnan's strategy for determining the weight of a square in the previous problem. Describe this strategy.

Responses vary. Adnan first crossed out 4 triangles from each side, so only squares were left on the right. Then he divided each side into 2 equal groups because there were 2 squares.

Unit 6 Lesson 4 416 Activity 1

Pairs | 15 min

Activity 1 Unknown Weights (continued)

Purpose: Students use and analyze different strategies to determine unknown weights on balanced hangers. **(MTR.5.1)**

2 Monitor

Look for the distribution of responses in students' work. Consider selecting the hangers with the least consensus to discuss during the Connect.

A **Accessibility: Visual-Spatial Processing** Invite pairs to think about key features of the hangers and discuss them aloud before responding to support distinguishing between visual representations.

Select and sequence a variety of strategies, including ones that are similar to Zoe's.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Write that the circle weighs 80 because they only removed two squares from each side.	**Support** Consider asking, "What is remaining on each side? How can you use this to determine the weight of one circle?"
Use a strategy like Zoe's (use the structure of the hanger to divide the diagram into two equal parts).	**Strengthen** Invite students to share how this helps in determining the weight of one circle.
Use other strategies like: • Testing values. • Removing 40 from each side and then dividing.	
Would like to explore strategies for solving further.	**Stretch** Invite students to describe how they would determine the weight of a square using Zoe's strategy and the weight of a circle using Adnan's strategy.

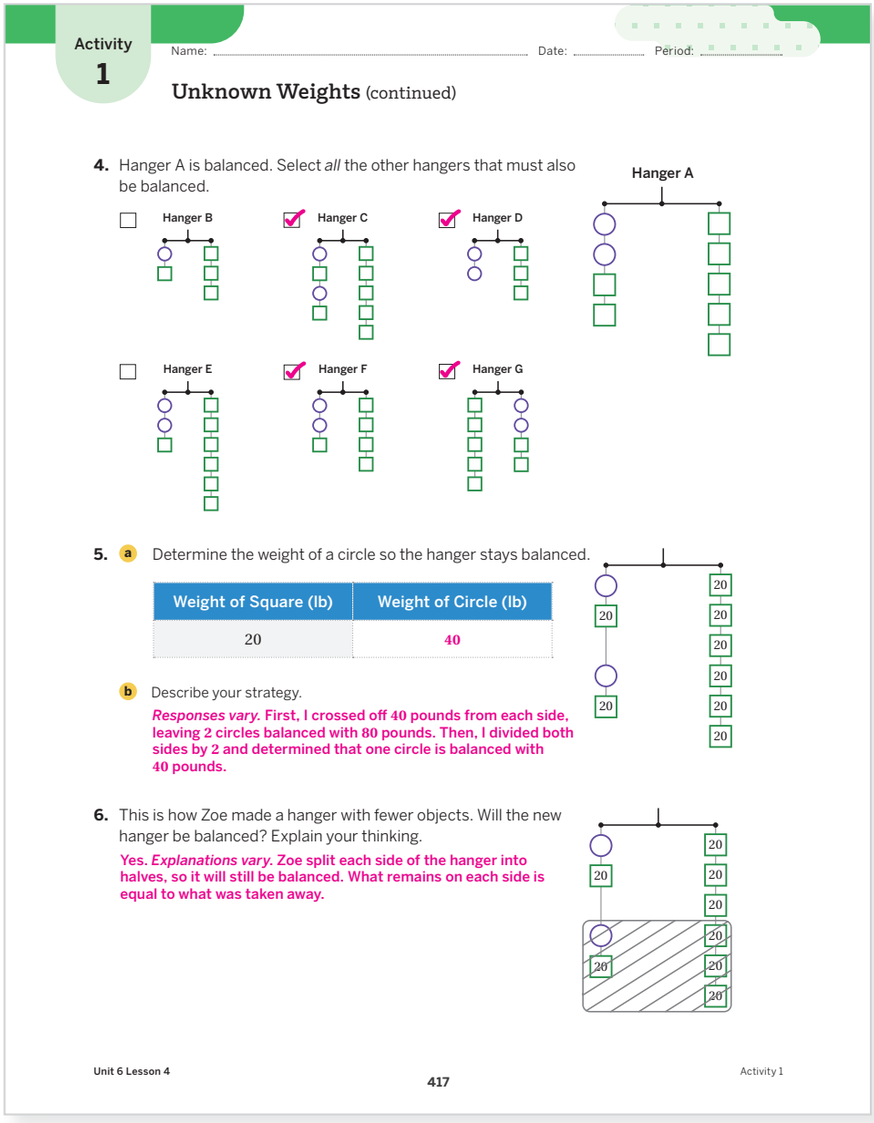
3 Connect

Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates.

To surface the Key Takeaway, share the captured strategies and explanations. Discuss Zoe's strategy and other strategies students used to determine the weight of the circle.



Activity 1

Name: Date: Period:

Unknown Weights (continued)

4. Hanger A is balanced. Select *all* the other hangers that must also be balanced.

Hanger A

☐ Hanger B ☑ Hanger C ☑ Hanger D

☐ Hanger E ☑ Hanger F ☑ Hanger G

5. a Determine the weight of a circle so the hanger stays balanced.

Weight of Square (lb)	Weight of Circle (lb)
20	40

b Describe your strategy.

Responses vary. First, I crossed off 40 pounds from each side, leaving 2 circles balanced with 80 pounds. Then, I divided both sides by 2 and determined that one circle is balanced with 40 pounds.

6. This is how Zoe made a hanger with fewer objects. Will the new hanger be balanced? Explain your thinking.

Yes. *Explanations vary.* Zoe split each side of the hanger into halves, so it will still be balanced. What remains on each side is equal to what was taken away.

Unit 6 Lesson 4 417 Activity 1

Key Takeaway: There are several strategies for determining an unknown weight on a hanger, including removing the same weight from each side and dividing into equal parts. For all strategies, it's important to make sure the weight stays balanced.

Independent | 15 min

Activity 2 Challenge Creator

Purpose: Students express their creativity by creating their own hangers and practice determining the weight of an unknown shape on a hanger.

Short on time? Consider having students complete only one partner challenge.

1 Launch

Consider sharing that students will create their own challenge similar to the one on the previous problem. Demonstrate how students can create their hanger.

Distribute the Activity 2 sheet to students so they can create their hanger challenge.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

2 Monitor

Listen to student strategies and discussions, offering help and encouragement as needed.

Note: Creating a balanced hanger may take several minutes, including several rounds of adjusting objects and weights.

Invite students to check their answers with the person who created the hanger they're solving. Encourage them to discuss and resolve any differences.

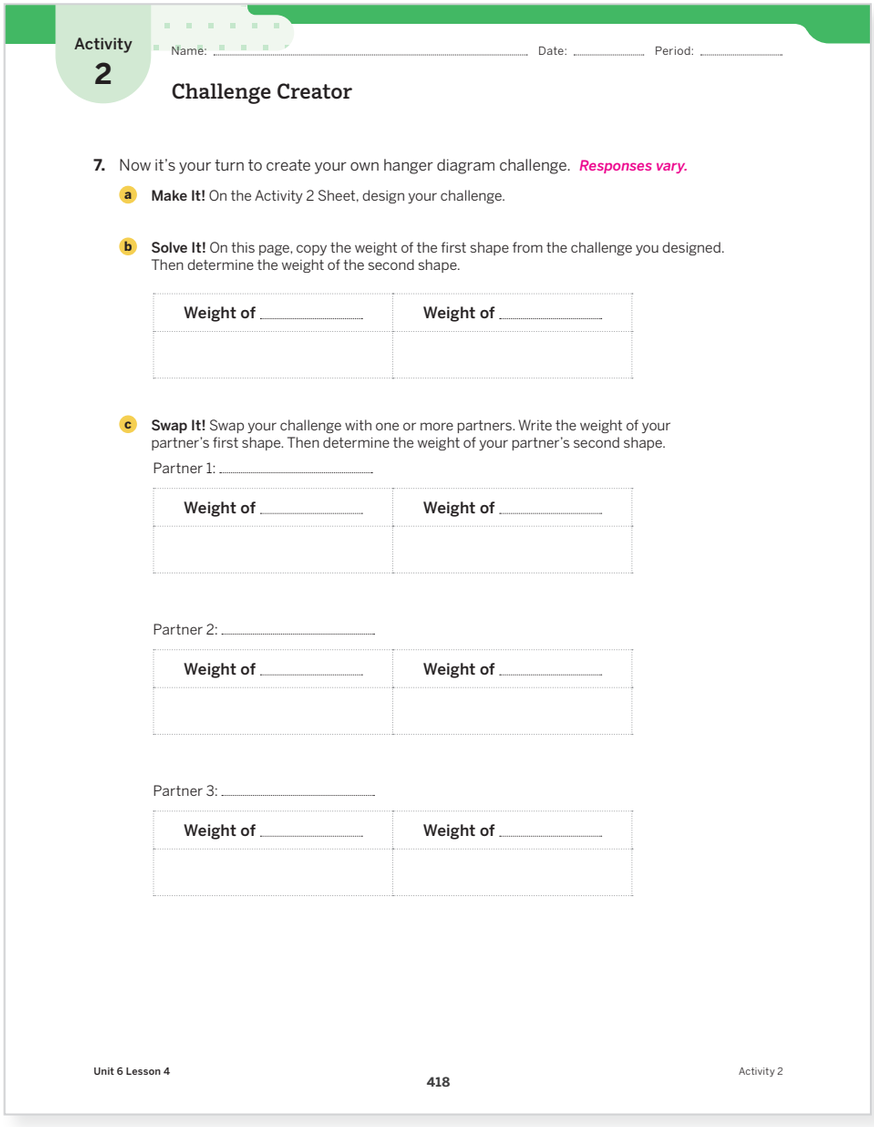
Capture several students' hangers to share during the Connect.

3 Connect

Share captured hangers. Invite students to share how they decided how many shapes and how much weight to put on each side of their hanger. **(MTR.2.1)**

Invite students to share a variety of strategies for determining the unknown weight of each hanger.

Consider asking, "Was there a strategy you heard that surprised you?"



Activity 2

Name: _____ Date: _____ Period: _____

Challenge Creator

7. Now it's your turn to create your own hanger diagram challenge. *Responses vary.*

a. **Make It!** On the Activity 2 Sheet, design your challenge.

b. **Solve It!** On this page, copy the weight of the first shape from the challenge you designed. Then determine the weight of the second shape.

Weight of _____	Weight of _____

c. **Swap It!** Swap your challenge with one or more partners. Write the weight of your partner's first shape. Then determine the weight of your partner's second shape.

Partner 1: _____

Weight of _____	Weight of _____

Partner 2: _____

Weight of _____	Weight of _____

Partner 3: _____

Weight of _____	Weight of _____

Unit 6 Lesson 4 418 Activity 2

Whole Class | 5 min

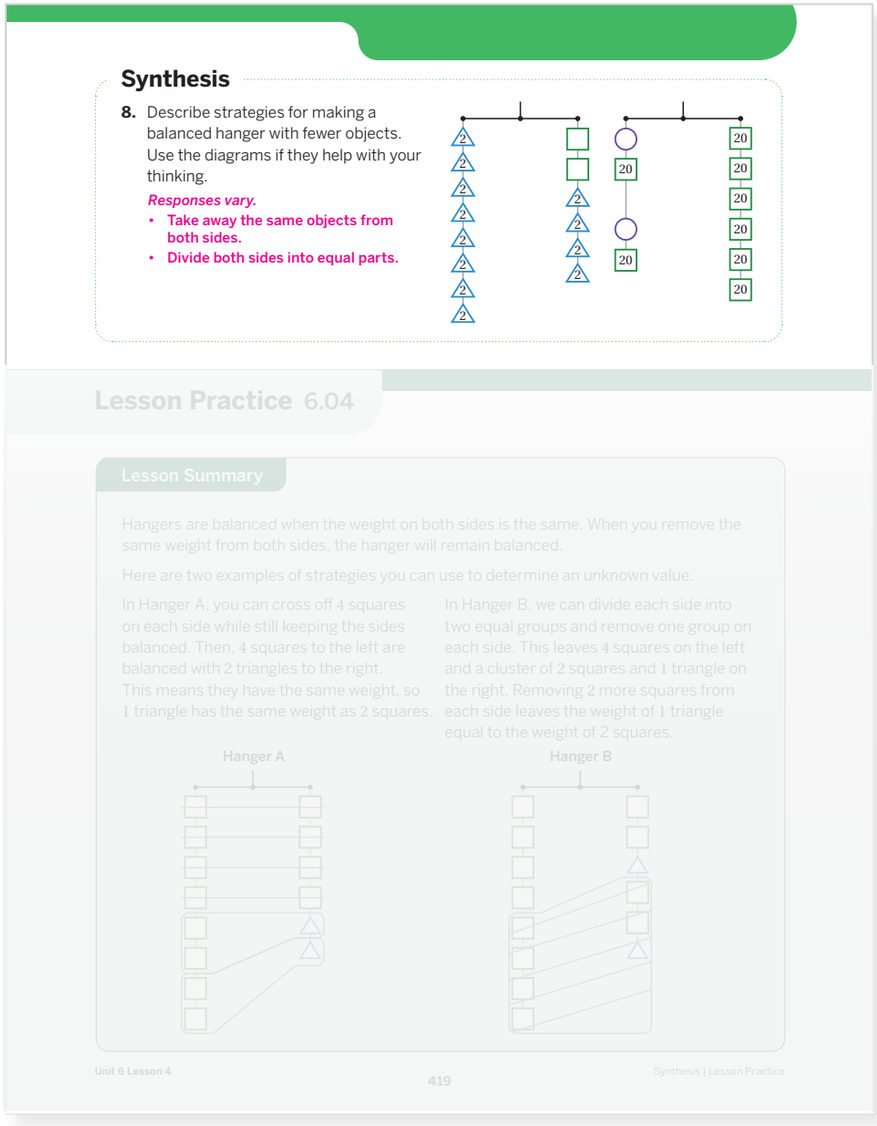
Synthesis

Purpose: Students synthesize their understanding of strategies for creating a balanced hanger with fewer objects.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:
- Taking away the same objects from both sides of the hanger.
- Dividing the hanger into equal parts.

Note: If time allows, invite students to revise their previous thinking based on the discussion and make sense of each strategy using the structure of a balanced hanger with fewer objects.



Synthesis

8. Describe strategies for making a balanced hanger with fewer objects. Use the diagrams if they help with your thinking.

Responses vary.
- Take away the same objects from both sides.
- Divide both sides into equal parts.

Lesson Practice 6.04

Lesson Summary

Hangers are balanced when the weight on both sides is the same. When you remove the same weight from both sides, the hanger will remain balanced.

Here are two examples of strategies you can use to determine an unknown value.

In Hanger A, you can cross off 4 squares on each side while still keeping the sides balanced. Then, 4 squares to the left are balanced with 2 triangles to the right. This means they have the same weight, so 1 triangle has the same weight as 2 squares.

In Hanger B, we can divide each side into two equal groups and remove one group on each side. This leaves 4 squares on the left and a cluster of 2 squares and 1 triangle on the right. Removing 2 more squares from each side leaves the weight of 1 triangle equal to the weight of 2 squares.

Hanger A

Hanger B

Unit 6 Lesson 4 419 Synthesis | Lesson Practice

Lesson Takeaway: There are different strategies for determining an unknown weight on a hanger. For each strategy, it's important to remove an equal amount of weight from each side to keep the hanger balanced.

Independent | 5 min

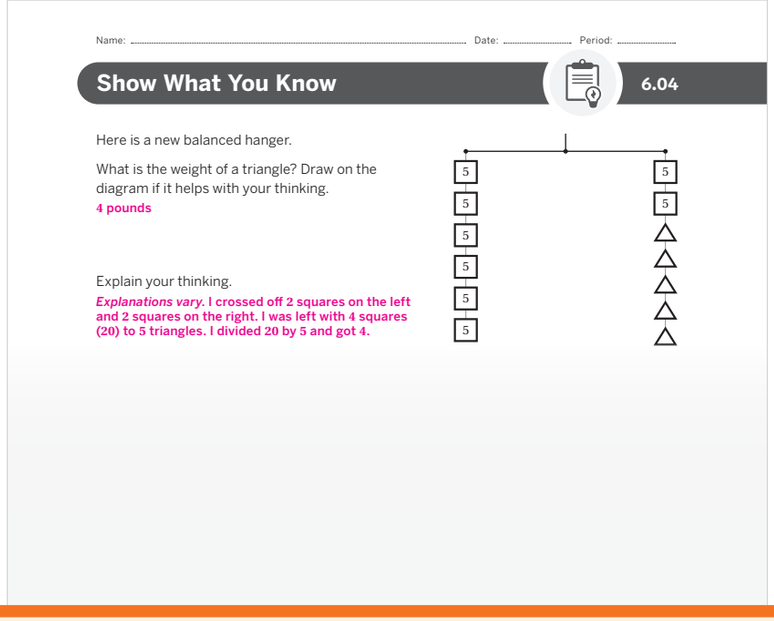
Show What You Know

Purpose: Students demonstrate their understanding by determining the weight of an unknown object on a hanger.

Today's Goals

1. **Goal:** Determine an unknown value on a hanger diagram.
2. **Language Goal:** Describe strategies that will keep a hanger balanced (e.g., adding or removing equal items from each side). **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.04

Here is a new balanced hanger.

What is the weight of a triangle? Draw on the diagram if it helps with your thinking.

4 pounds

Explain your thinking.

Explanations vary. I crossed off 2 squares on the left and 2 squares on the right. I was left with 4 squares (20) to 5 triangles. I divided 20 by 5 and got 4.

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of determine unknown values on a hanger diagram:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and discuss strategies for keeping a hanger balanced, including removing the same weight from each side and dividing into equal parts.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

In this lesson, students use hanger diagrams in a context that supports their understanding of the steps for solving an equation. How does creating balanced hangers integrate with other representations for solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe strategies for making a balanced hanger with fewer objects. Use the diagrams if they help with your thinking.

Responses vary.

- Take away the same objects from both sides.
- Divide both sides into equal parts.

Lesson Practice 6.04

Lesson Summary

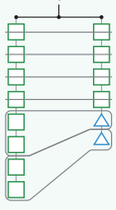
Hangers are balanced when the weight on both sides is the same. When you remove the same weight from both sides, the hanger will remain balanced.

Here are two examples of strategies you can use to determine an unknown value.

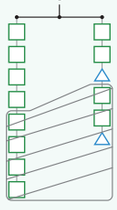
In Hanger A, you can cross off 4 squares on each side while still keeping the sides balanced. Then, 4 squares to the left are balanced with 2 triangles to the right. This means they have the same weight, so 1 triangle has the same weight as 2 squares.

In Hanger B, we can divide each side into two equal groups and remove one group on each side. This leaves 4 squares on the left and a cluster of 2 squares and 1 triangle on the right. Removing 2 more squares from each side leaves the weight of 1 triangle equal to the weight of 2 squares.

Hanger A



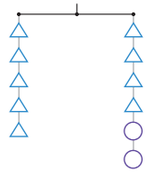
Hanger B



Lesson Practice 6.04

Name: _____ Date: _____ Period: _____

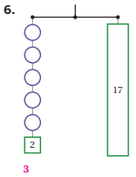
Problems 1–5: Determine the weight of a circle based on these different weights for a triangle.



1. 2 pounds
   1 pound
2. 1 pound
   0.5 pounds
3. 0.5 pounds
   0.25 pounds
4. 3 pounds
   1.5 pounds
5. 100 pounds
   50 pounds

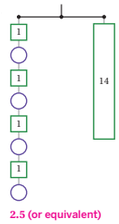
Problems 6–7: Determine the weight of a circle so the hanger stays balanced.

6.



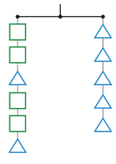
3

7.



2.5 (or equivalent)

8. Here is a balanced hanger. Darryl says the weight of four squares is equal to the weight of three triangles.



Is Darryl correct? Explain your thinking.

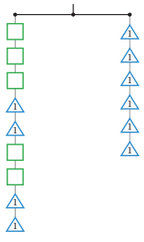
Yes. *Explanations vary.* If he removes two triangles from each side, the hanger will stay balanced. Then, he will have four squares equal to three triangles.

Lesson Practice 6.04

Name: _____ Date: _____ Period: _____

FAST Practice

9. Use this model to determine the value of a square.



- **A.** 0.4
- **B.** 0.5
- **C.** 2.5
- **D.** 4

Spiral Review

Problems 10–11: Solve each equation.

10. $8.5 \cdot 3 = a$

$25.5 = a$

11. $c - 3 = 15$

$c - 3 + 3 = 15 + 3$

$c = 18$

12. Select *all* the expressions that are equivalent to $2(x + 3)$.

- ☑ **A.** $2 \cdot (x + 3)$
- ☑ **B.** $(x + 3) \cdot 2$
- ☑ **C.** $2 \cdot x + 2 \cdot 3$
- ☐ **D.** $2x + 3$
- ☐ **E.** $(2 \cdot x) + 3$

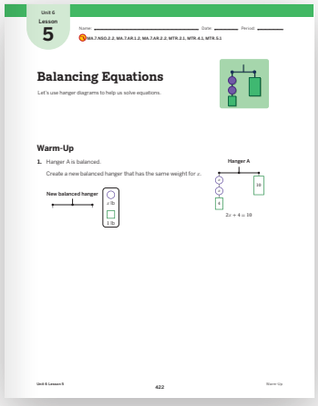
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–8	2	MA.7.NSO.2.2
FAST Practice	9	2	MA.7.NSO.2.2
Spiral Review			
	10–11	1	MA.6.AR.2.2 MA.6.AR.2.3
	12	1	MA.6.AR.1.4

Unit 6

Lesson

5


Unit 6
Lesson
5
Balancing Equations
Let's use hanger diagrams to help us solve equations.
Warm-Up
1. Hanger A is balanced.
Create a new balanced hanger that has the same weight for x.
New balanced hanger
Hanger A
$2x + 4 = 10$
422

Balancing Equations

Solving Equations With Balanced Hangers

Let's use hanger diagrams to help us solve equations.

Learning Outcomes

Today's Goals

1. **Goal:** Connect moves on hanger diagrams to moves that determine the value of a variable in an equation.
2. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive rational numbers.
3. **Language Goal:** Describe how solving equations of the form $px + q = r$ and $p(x + q) = r$ connect to a hanger diagram. **(Reading, Writing, Speaking, and Listening)**

Students understand strategies for solving an equation and connect those to balanced moves on a hanger. Students also create their own hangers to match equations, and shift from solving for unknowns on hangers to solving for unknowns in equations.

Coherence

Prior Learning

In Lesson 4, students developed strategies for determining an unknown weight by creating balanced hangers with fewer objects.

Future Learning

In Lesson 6, students will describe and use strategies for solving equations of the form $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers without a visual model.

Connections

- Students effectively connect the visual representation of a balanced hanger with the concept of an equation and its solution. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2

Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Also Addressing: **MA.7.NSO.2.2, MA.7.AR.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2 **MA.6.AR.2.3**	**MA.8.AR.2.1**

Lesson at a Glance

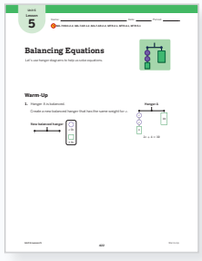
~ 45 min

MA.7.NSO.2.2, MA.7.AR.1.2, MA.7.AR.2.2

Warm-Up

Pairs | 5 min

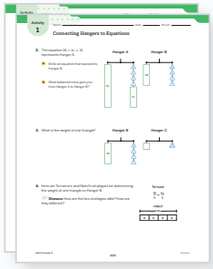
Students revisit balanced hangers and connect them to equations.



Activity 1

Pairs | 15 min

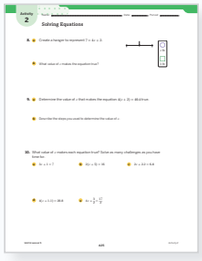
Students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation. **Routines:** Decide and Defend, MLR7: Compare and Connect



Activity 2

Pairs | 15 min

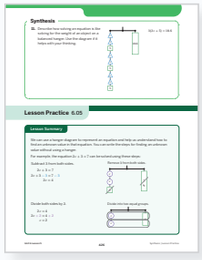
Students shift from solving for unknown weights of objects on hangers to solving for unknowns in equations.



Synthesis

Whole Class | 5 min

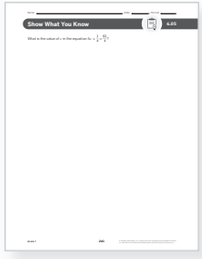
Students synthesize their understanding of the connections between moves on a hanger diagram and moves used to solve an equation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the value of x in an equation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students revisit balanced hangers and connect them to equations.

1 Launch

Consider asking, "Where do you see each part of the equation in the hanger?" **(MTR.5.1)**

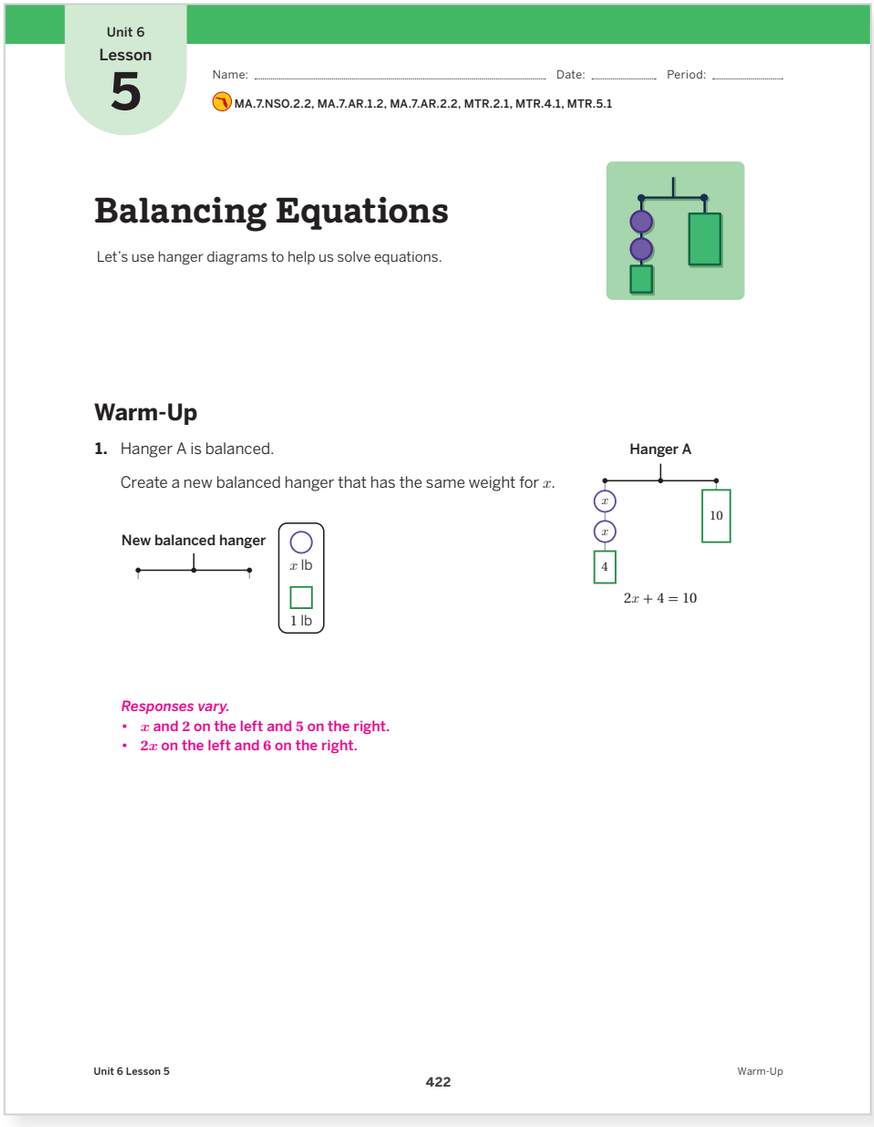
A **Accessibility: Conceptual Processing** Invite students to make connections to familiar contexts to support abstract thinking.

Invite students to share their hangers with a partner and give feedback on each other's work.

2 Connect

Invite students to share any connections they make between the hangers and their equations, or what they notice about each of the hangers that balance.

To surface the Key Takeaway, consider asking, "What value of x makes the hanger balanced?"



Unit 6
Lesson
5

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.2, MA.7.AR.1.2, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Balancing Equations

Let's use hanger diagrams to help us solve equations.

Warm-Up

1. Hanger A is balanced.

Create a new balanced hanger that has the same weight for x.

New balanced hanger

x lb

1 lb

Hanger A

x x 4 10

$2x + 4 = 10$

Responses vary.

- x and 2 on the left and 5 on the right.
- $2x$ on the left and 6 on the right.

Unit 6 Lesson 5 422 Warm-Up

Key Takeaway: Hanger diagrams can be used to represent equations where a value is unknown.

Pairs | 15 min

Activity 1 Connecting Hangers to Equations

Purpose: Students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation.

1 Launch

To surface prior knowledge, consider asking, "How does a hanger stay balanced as new moves are made?" **(MTR.2.1)**

2 Monitor

To support students getting started, consider asking, "How are the two hangers related?"

D Differentiation

Look for students who:	Respond to Student Thinking
Think that the value of x is 4 because 4 triangles balance the rectangle.	**Support** Encourage them to determine the weight of the rectangle in Hanger C.
Divide 10 by 4.	**Strengthen** Invite them to share how this move keeps the hanger balanced.

Consider asking, "How can the weight you determined for one triangle be checked for accuracy in Hanger A and Hanger B?"

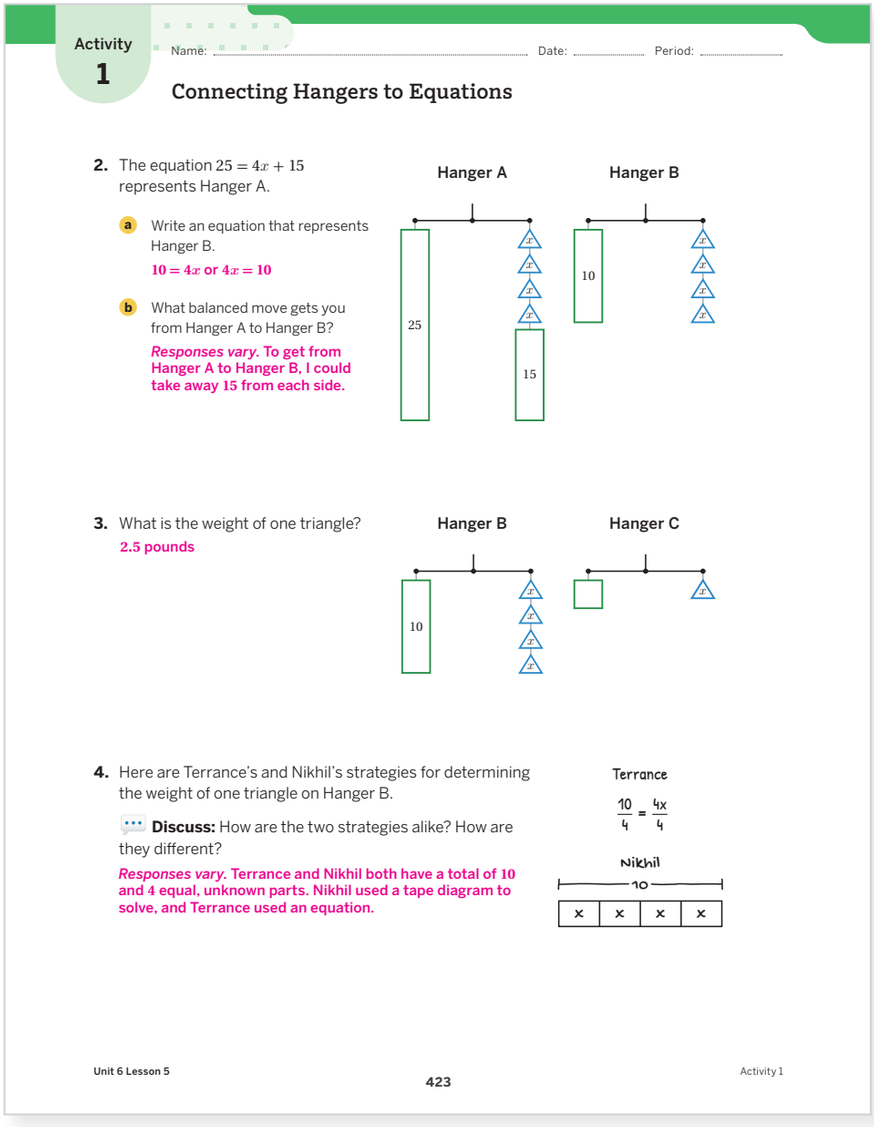
Listen for student responses mentioning the multiple representations.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy. **(MTR.4.1)**

Consider asking:

- "What did each student do?"
- "Where do you see the same information in Terrance's and Nikhil's strategies?"
- "What do you think was helpful about each strategy?
- "How does each strategy compare to your own?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Connecting Hangers to Equations

2. The equation $25 = 4x + 15$ represents Hanger A.

a. Write an equation that represents Hanger B.

$10 = 4x$ or $4x = 10$

b. What balanced move gets you from Hanger A to Hanger B?

Responses vary. To get from Hanger A to Hanger B, I could take away 15 from each side.

Hanger A

Hanger B

3. What is the weight of one triangle?

2.5 pounds

Hanger B

Hanger C

4. Here are Terrance's and Nikhil's strategies for determining the weight of one triangle on Hanger B.

Discuss: How are the two strategies alike? How are they different?

Responses vary. Terrance and Nikhil both have a total of 10 and 4 equal, unknown parts. Nikhil used a tape diagram to solve, and Terrance used an equation.

Terrance

$\frac{10}{4} = \frac{4x}{4}$

Nikhil

Unit 6 Lesson 5 423 Activity 1

Pairs | 15 min

Activity 1 Connecting Hangers to Equations (continued)

Purpose: Students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation.

2 Monitor

Support students getting started by asking, "What can you remove from the right and left side to keep this hanger balanced?" **(MTR.5.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Remove 6 from each side.	**Strengthen** Invite them to share how this move keeps the hanger balanced.
Split the hanger into two equal groups.	

Invite students to share their responses. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display during the Connect.

Look for examples of student thinking to share during the Connect.

3 Connect

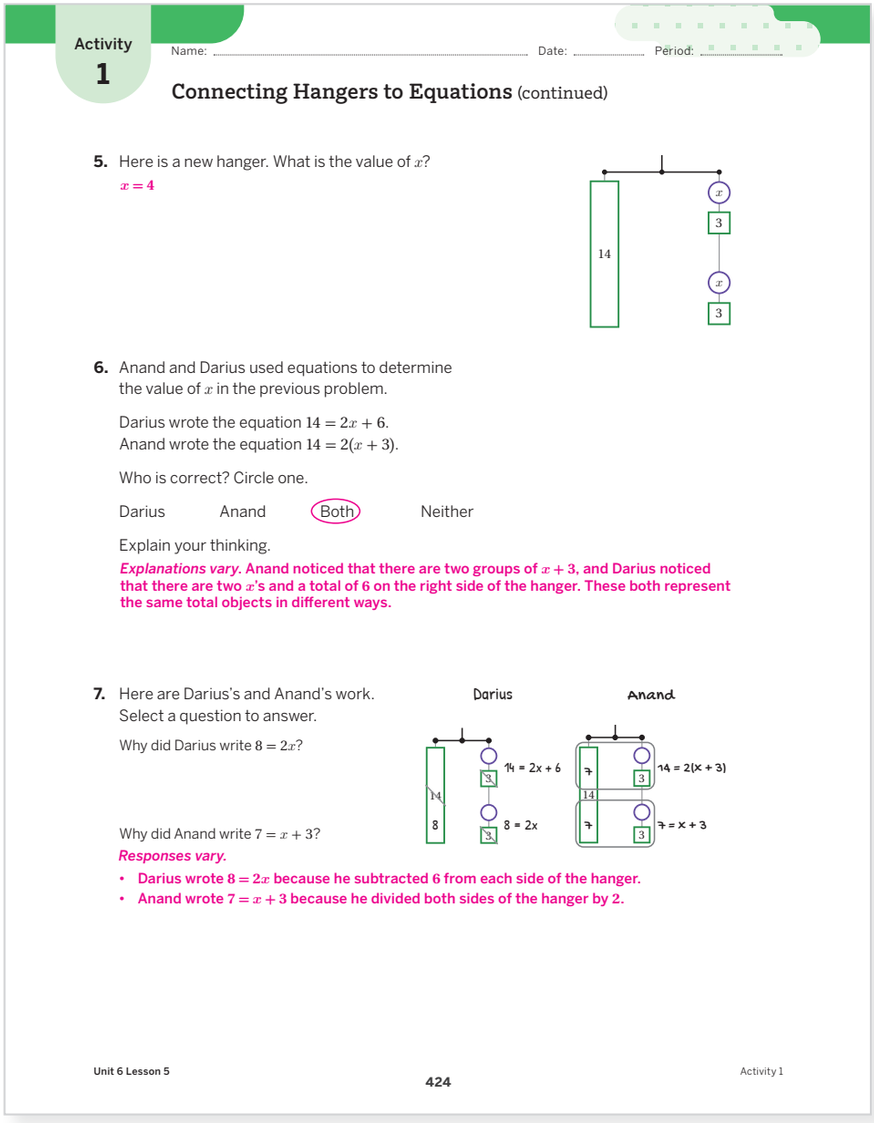
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"

Invite students to explain their reasoning for the question they chose to answer.

Consider asking, "What could each student's next step be to determine the weight of a circle? Why?"



Activity 1

Name: Date: Period:

Connecting Hangers to Equations (continued)

5. Here is a new hanger. What is the value of x?

$x = 4$

6. Anand and Darius used equations to determine the value of x in the previous problem.

Darius wrote the equation $14 = 2x + 6$.
Anand wrote the equation $14 = 2(x + 3)$.

Who is correct? Circle one.

Darius Anand (Both) Neither

Explain your thinking.

Explanations vary. Anand noticed that there are two groups of $x + 3$, and Darius noticed that there are two x's and a total of 6 on the right side of the hanger. These both represent the same total objects in different ways.

7. Here are Darius's and Anand's work. Select a question to answer.

Why did Darius write $8 = 2x$?

Why did Anand write $7 = x + 3$?

Responses vary.
- Darius wrote $8 = 2x$ because he subtracted 6 from each side of the hanger.
- Anand wrote $7 = x + 3$ because he divided both sides of the hanger by 2.

Unit 6 Lesson 5 424 Activity 1

Pairs | 15 min

Activity 2 Solving Equations

Purpose: Students shift from solving for unknown weights of objects on hangers to solving for unknowns in equations.

Short on time? Consider inviting students to complete 1 or 2 problems on the Repeated Challenges.

1 Launch

Encourage students to think about how many of each shape are needed on each side of the hanger to represent the equation. **(MTR.2.1)**

Invite students to share their responses and consider asking, "Does it matter which side of the hanger the 7 is on?"

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Use a variety of strategies to determine the value of x such as: • Using guess-and-check. • Rewriting the equation as $4x + 8 = 40.4$. • Dividing both sides by 4 first.	**Strengthen** Invite these students to share their thinking.
Connect equations to hanger diagrams.	**Strengthen** Invite them to share how each equation would look if represented on a hanger diagram.
Would like to explore using hangers to solve equations further.	**Stretch** Invite students to write their own equation, draw a hanger to represent the equation, and then solve it to determine the value of the variable.

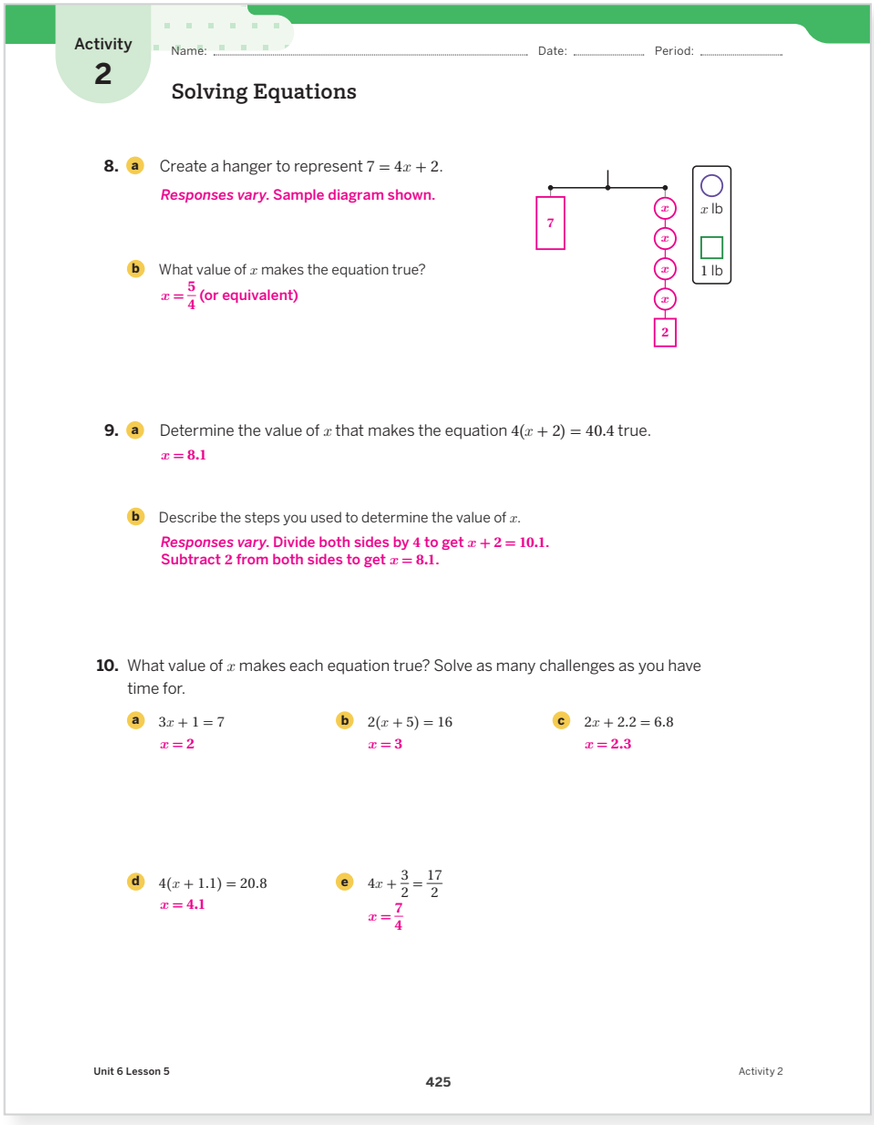
A Accessibility: Executive Functioning Invite students to draw a hanger diagram representing the equation and color-code corresponding features.

Look for how many challenges each student has completed correctly.

3 Connect

Invite students to discuss the steps they used to determine the value of x. Consider asking, "How can you check your solution to an equation?"

To surface the Key Takeaway, consider asking, "How are the equations $4x + 8 = 40.4$ and $4(x + 2) = 40.4$ related?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Solving Equations

8. a. Create a hanger to represent $7 = 4x + 2$.

Responses vary. Sample diagram shown.

b. What value of x makes the equation true?

$x = \frac{5}{4}$ (or equivalent)

9. a. Determine the value of x that makes the equation $4(x + 2) = 40.4$ true.

$x = 8.1$

b. Describe the steps you used to determine the value of x.

Responses vary. Divide both sides by 4 to get $x + 2 = 10.1$. Subtract 2 from both sides to get $x = 8.1$.

10. What value of x makes each equation true? Solve as many challenges as you have time for.

a. $3x + 1 = 7$, $x = 2$

b. $2(x + 5) = 16$, $x = 3$

c. $2x + 2.2 = 6.8$, $x = 2.3$

d. $4(x + 1.1) = 20.8$, $x = 4.1$

e. $4x + \frac{3}{2} = \frac{17}{2}$, $x = \frac{7}{4}$

Unit 6 Lesson 5 425 Activity 2

Key Takeaway: There are multiple strategies to solve an equation such as connecting the moves on a hanger diagram and removing the same weight on both sides first or dividing the hanger into equal groups first.

Whole Class | 5 min

Synthesis

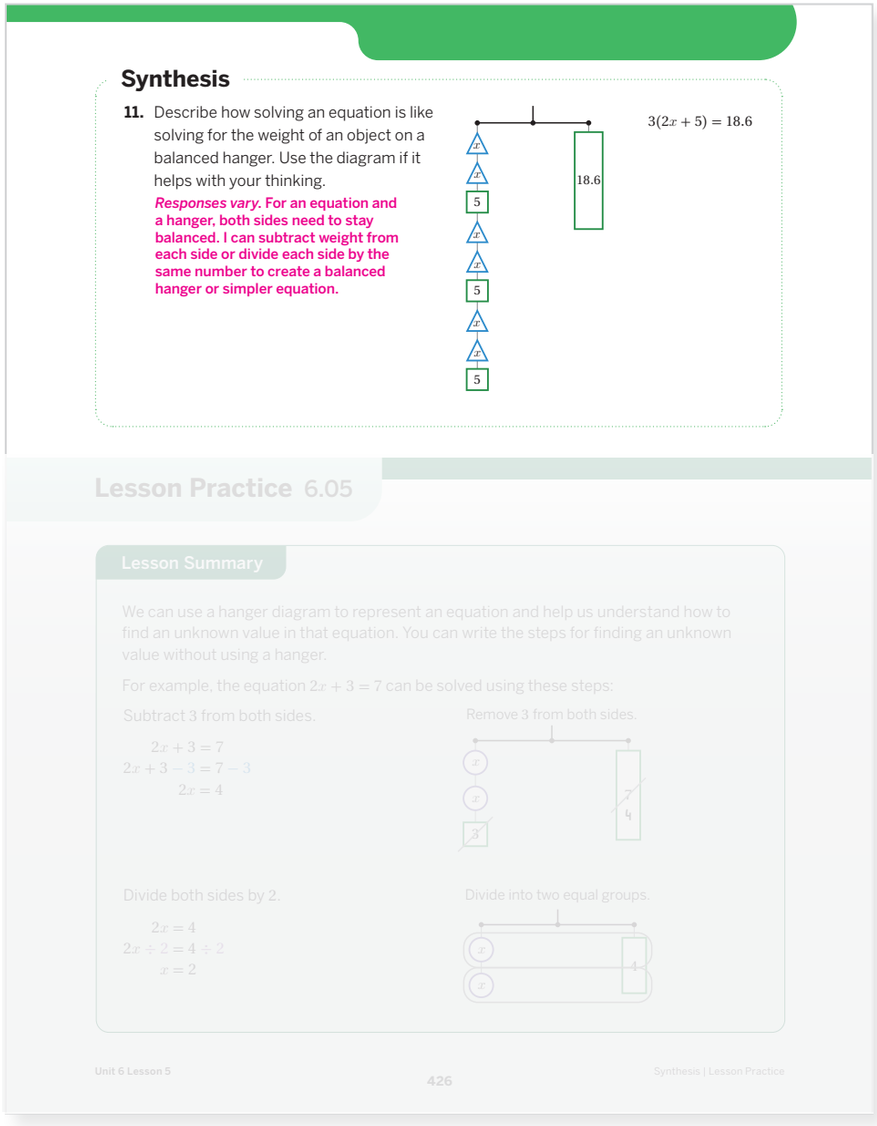
Purpose: Students synthesize their understanding of the connections between moves on a hanger diagram and moves used to solve an equation.

Invite students to respond independently, and then share their thinking with a partner.

Record and share a variety of ideas, such as:
- Applying the same move to both sides of the hanger and the equation.
- Keeping both sides of the hanger and the equation balanced.

Consider asking, "What connections do you see between these responses?"

Note: If time allows, invite students to share with a partner what makes sense about each response and what questions they still have.



Synthesis

11. Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.

Responses vary. For an equation and a hanger, both sides need to stay balanced. I can subtract weight from each side or divide each side by the same number to create a balanced hanger or simpler equation.

Lesson Practice 6.05

Lesson Summary

We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

Remove 3 from both sides.

Divide both sides by 2.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Divide into two equal groups.

Unit 6 Lesson 5 426 Synthesis | Lesson Practice

Lesson Takeaway: When solving an equation of the form $p(x + q) = r$ using a hanger diagram, different strategies can be helpful, such as removing weight on both sides first or dividing the hanger into equal groups.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the value of x in an equation.

Today's Goals

1. **Goal:** Connect moves on hanger diagrams to moves that determine the value of a variable in an equation.
2. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive rational numbers.
3. **Language Goal:** Describe how solving equations of the form $px + q = r$ and $p(x + q) = r$ connect to a hanger diagram. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.05

What is the value of x in the equation $5x + \frac{1}{4} = \frac{61}{4}$?

$x = 3$

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations in the form $px + q = r$:

» **Respond to Student Thinking:**

- Review the Lesson Summary.
- Select a few related **Practice Problems** and give individualized feedback

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

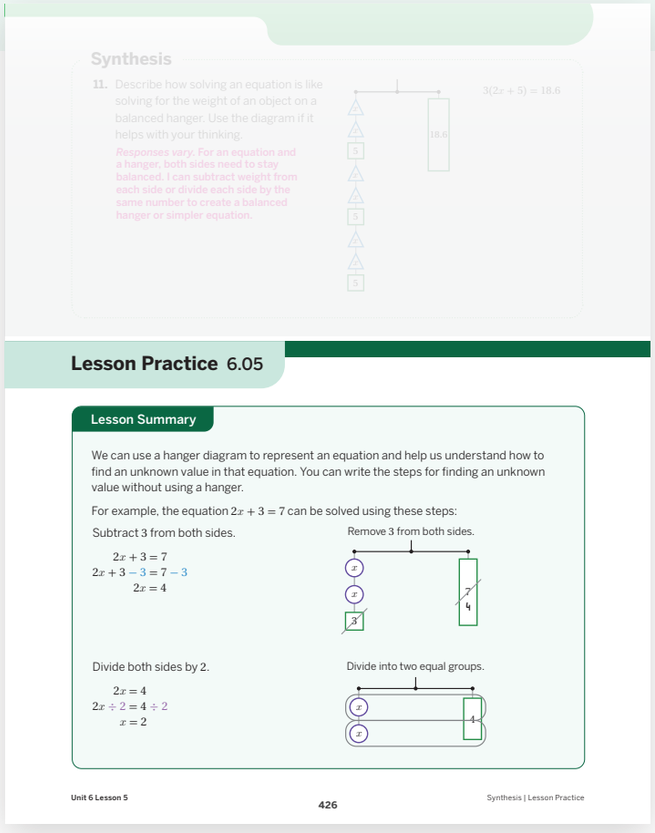
Reflect on the facilitation of the **MLR7: Compare and Connect** routine in this lesson.

- What about the routine felt successful?
- What are some areas for growth?
- What other opportunities might there be in the next few lessons for inviting students to analyze and compare strategies for solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.

Responses vary. For an equation and a hanger, both sides need to stay balanced. I can subtract weight from each side or divide each side by the same number to create a balanced hanger or simpler equation.

$3(2x + 5) = 18.6$

18.6

5

5

5

Lesson Practice 6.05

Lesson Summary

We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$2x + 3 = 7$
$2x + 3 - 3 = 7 - 3$
$2x = 4$

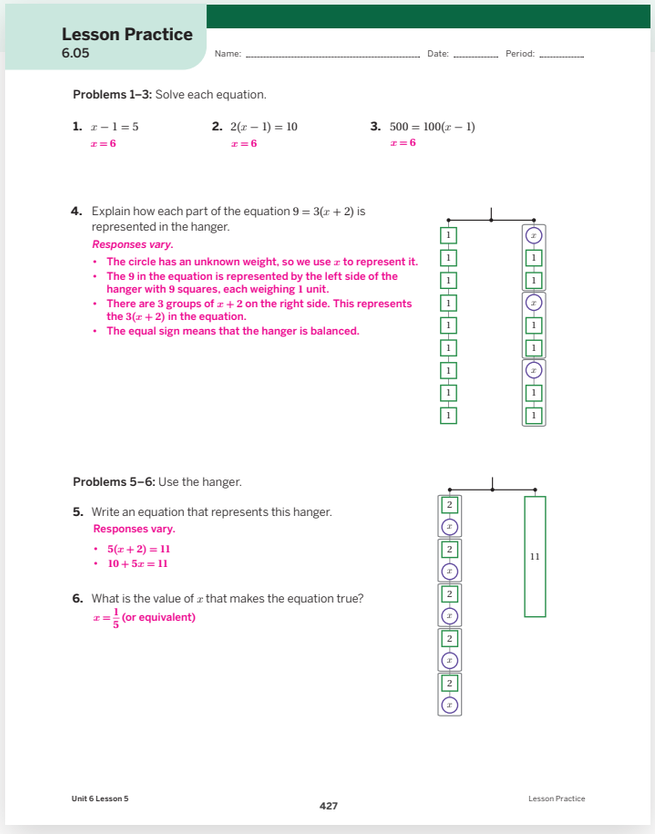
Remove 3 from both sides.

Divide both sides by 2.

$2x = 4$
$2x \div 2 = 4 \div 2$
$x = 2$

Divide into two equal groups.

Unit 6 Lesson 5 426 Synthesis | Lesson Practice



Lesson Practice 6.05

Name: Date: Period:

Problems 1–3: Solve each equation.

1. $x - 1 = 5$
   $x = 6$
2. $2(x - 1) = 10$
   $x = 6$
3. $500 = 100(x - 1)$
   $x = 6$

4. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.

Responses vary.

- The circle has an unknown weight, so we use x to represent it.
- The 9 in the equation is represented by the left side of the hanger with 9 squares, each weighing 1 unit.
- There are 3 groups of $x + 2$ on the right side. This represents the $3(x + 2)$ in the equation.
- The equal sign means that the hanger is balanced.

Problems 5–6: Use the hanger.

5. Write an equation that represents this hanger.

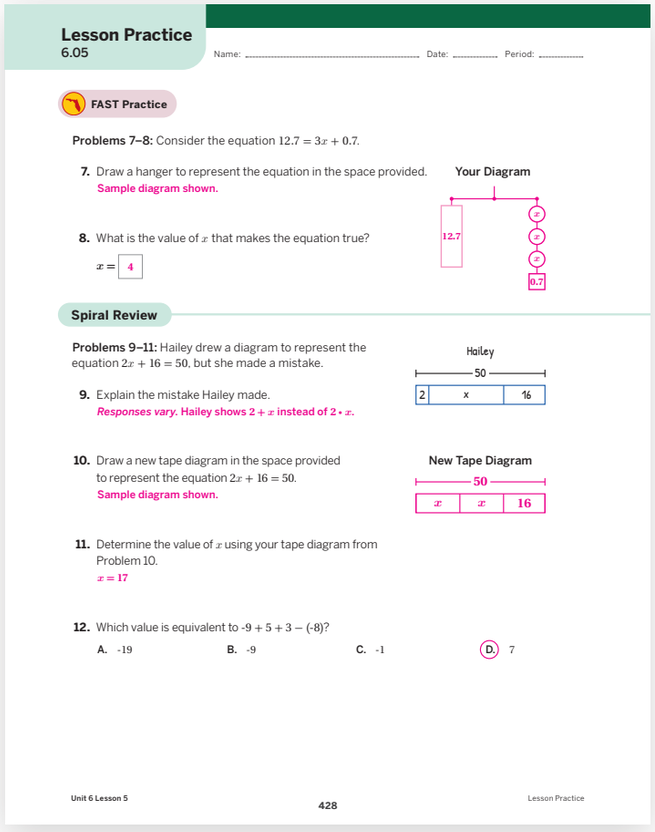
Responses vary.

- $5(x + 2) = 11$
- $10 + 5x = 11$

6. What is the value of x that makes the equation true?
   $x = \frac{1}{5}$ (or equivalent)

11

Unit 6 Lesson 5 427 Lesson Practice



Lesson Practice 6.05

Name: Date: Period:

FAST Practice

Problems 7–8: Consider the equation $12.7 = 3x + 0.7$.

7. Draw a hanger to represent the equation in the space provided.
   Sample diagram shown.

Your Diagram

12.7

0.7

8. What is the value of x that makes the equation true?
   $x =$ 4

Spiral Review

Problems 9–11: Hailey drew a diagram to represent the equation $2x + 16 = 50$, but she made a mistake.

Hailey

50

2 | x | 16

9. Explain the mistake Hailey made.
   Responses vary. Hailey shows $2 + x$ instead of $2 \cdot x$.

10. Draw a new tape diagram in the space provided to represent the equation $2x + 16 = 50$.
    Sample diagram shown.

New Tape Diagram

50

x | x | 16

11. Determine the value of x using your tape diagram from Problem 10.
    $x = 17$

12. Which value is equivalent to $-9 + 5 + 3 - (-8)$?
    A. -19 B. -9 C. -1 D. 7

Unit 6 Lesson 5 428 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.2.2
	4–6	2	MA.7.AR.2.2
FAST Practice	7–8	2	MA.7.AR.2.2
Spiral Review			
	9–11	2	MA.7.AR.2.2
	12	2	MA.6.NSO.4.1

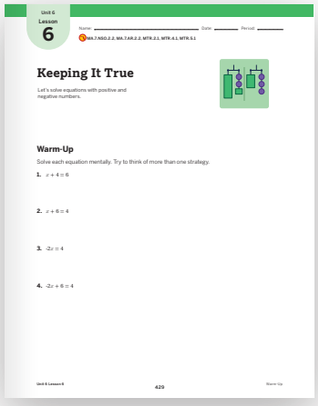
Unit 6

Lesson 6

Keeping It True

Solving Equations

Let's solve equations with positive and negative numbers.


Unit 6
Lesson 6
Name: _____ Date: _____ Period: _____
MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
Keeping It True
Let's solve equations with positive and negative numbers.
Warm-Up
Solve each equation mentally. Try to think of more than one strategy.
1. $x + 4 = 6$
2. $x + 6 = 4$
3. $2x = 4$
4. $-2x + 6 = 4$
Unit 6 Lesson 6
429
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive and negative numbers.
2. **Language Goal:** Explain why a solution to an equation is accurate. **(Reading and Writing)**

Students describe and use strategies for solving equations of the form $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers. This is the first lesson where students are not given a visual model (either a tape diagram or a hanger diagram) to support their reasoning.

Coherence

Prior Learning

In Lesson 5, students connected moves on a hanger diagram to moves that determine the unknown value in an equation.

Future Learning

In Lesson 7, students will solve equations in multiple ways, often by dividing first or using the distributive property, and then compare the results.

Connections

- Students continue to effectively connect the visual representation of a balanced hanger with the concept of an equation and its solution. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On MA.6.AR.2.2, MA.6.AR.2.3

Building Toward

MA.8.AR.2.1

Vocabulary

Review Vocabulary

solution to an equation

Lesson at a Glance

~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.2

Warm-Up

Independent | 7 min

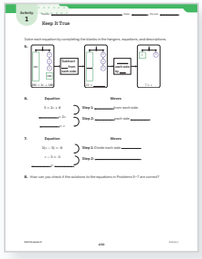
Students surface strategies for solving equations, including those with negative numbers, using the Number Talk routine.



Activity 1

Small Groups | 10 min

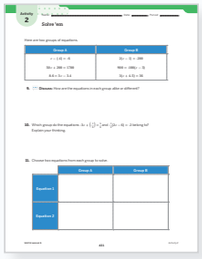
Students connect their understanding of balanced hangers from Lessons 4 and 5 to steps for solving an equation. MLR2: Collect and Display



Activity 2

Pairs | 18 min

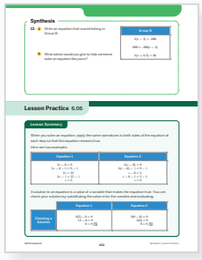
Students use the structure of equations to group them according to how they would be solved, then practice solving some of the equations. **Routine:** Think-Pair-Share **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the structure of equations by creating their own equation and providing advice on how it can be solved.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving equations that require different first steps in solving.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 7 min

Warm-Up

Purpose: Students surface strategies for solving equations, including those with negative numbers, using the **Number Talk** routine.

1 Launch

Use the Number Talk routine to support students in determining the value of x in an equation. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

Invite students to solve the first equation independently. Then record a variety of strategies, along with the name of the student who shared them.

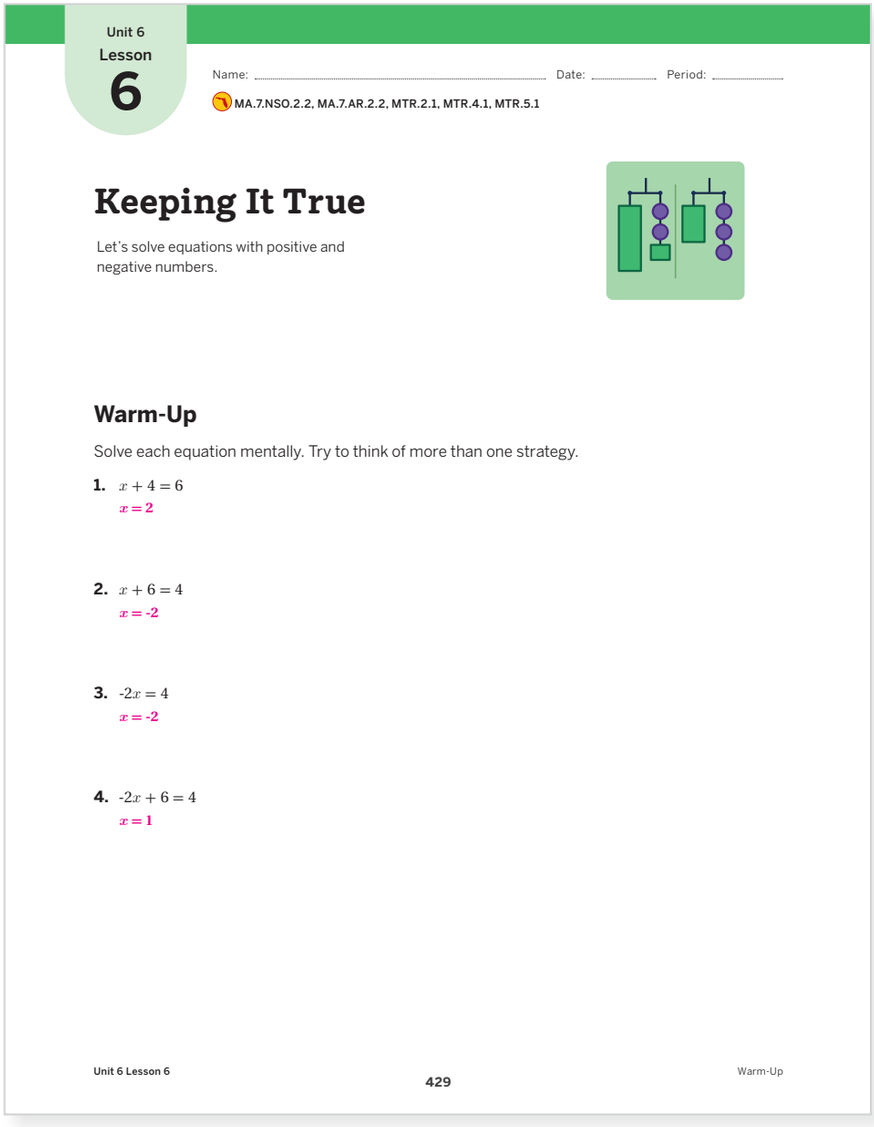
Consider asking, "Who thought about it in a different way?"

Note: This structure is repeated for each problem in the Number Talk.

2 Connect

Consider asking:

- "What does determining the *solution to an equation* mean?"
- "How can you tell whether a value for a variable in an equation makes the equation true?"



Unit 6
Lesson
6

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Keeping It True

Let's solve equations with positive and negative numbers.

Warm-Up

Solve each equation mentally. Try to think of more than one strategy.

1. $x + 4 = 6$
   $x = 2$

2. $x + 6 = 4$
   $x = -2$

3. $-2x = 4$
   $x = -2$

4. $-2x + 6 = 4$
   $x = 1$

Unit 6 Lesson 6 429 Warm-Up

Small Groups | 10 min

Activity 1 Keep It True

Purpose: Students connect their understanding of balanced hangers from Lessons 5 and 6 to steps for solving an equation.

Short on time? Consider discussing Problem 8 as a class.

1 Launch

Direct students' attention to the hanger and equation.

Invite students to share their thinking about why someone might subtract from both sides first. **(MTR.2.1)**

2 Monitor

Select and sequence student thinking to discuss during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 5)*	**Support** Reference the hanger diagram and ask, "What move was made to remove 180 from the right side of the diagram?"
Check their solution using substitution. *(Problems 6–7)*	**Strengthen** Invite students to share how this helps them.
Think about solving the equation using a different first step. *(Problem 7)*	**Stretch** Invite students to share how to solve the equation in a different way.

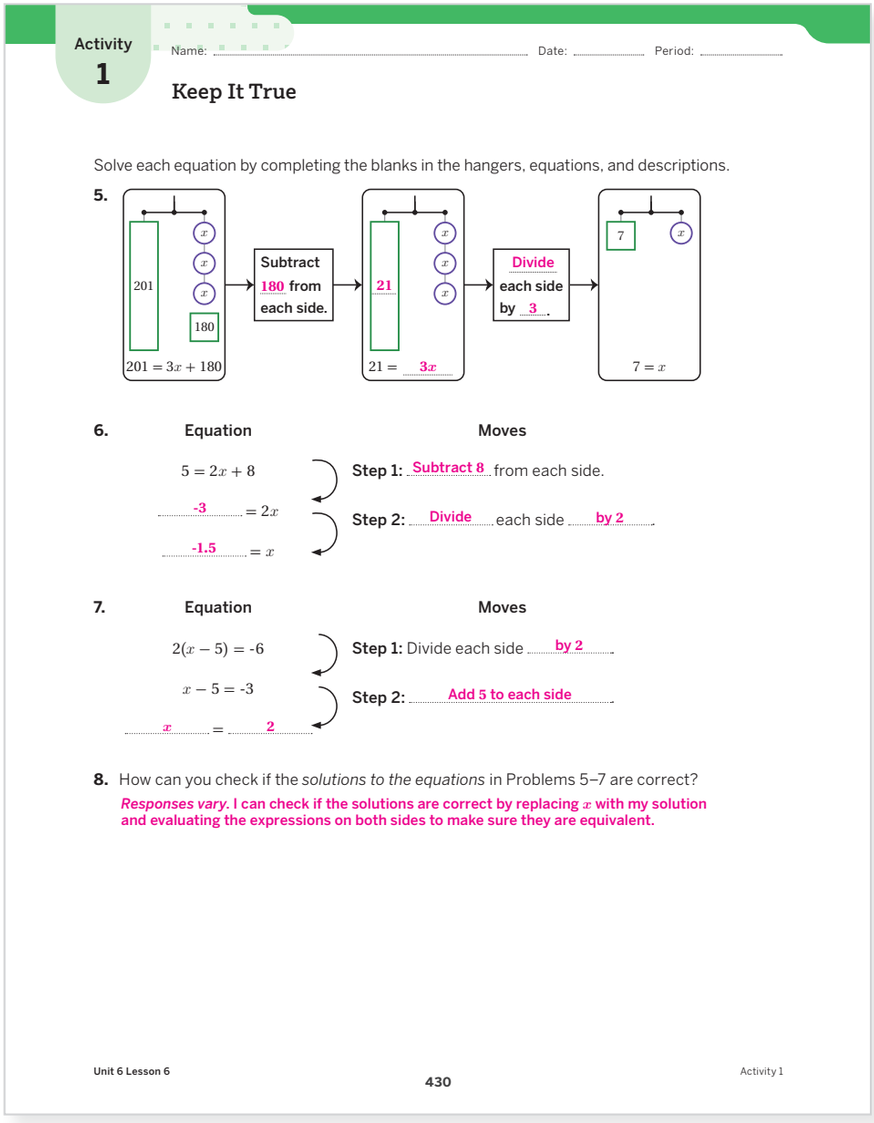
A Accessibility: Conceptual Processing Create or review an anchor chart that publicly displays possible steps for solving an equation.

3 Connect

Invite students to share their responses and strategies for each problem. Encourage students to share how to check their solution.

MLR MLR2: Collect and Display Use this routine to collect students' language, as they solve an equation. Display for students to reference and add to for the rest of the unit.

Direct students' attention to the equation and discuss the strategy of using the distributive property first to solve the equation.



Activity 1

Name: Date: Period:

Keep It True

Solve each equation by completing the blanks in the hangers, equations, and descriptions.

5.

6.

Equation	Moves
$5 = 2x + 8$	**Step 1:** Subtract 8 from each side.
-3 $= 2x$	**Step 2:** Divide each side by 2.
-1.5 $= x$	

7.

Equation	Moves
$2(x - 5) = -6$	**Step 1:** Divide each side by 2.
$x - 5 = -3$	**Step 2:** Add 5 to each side.
$x = 2$	

8. How can you check if the *solutions to the equations* in Problems 5–7 are correct?

Responses vary. I can check if the solutions are correct by replacing x with my solution and evaluating the expressions on both sides to make sure they are equivalent.

Unit 6 Lesson 6 430 Activity 1

Pairs | 18 min

Activity 2 Solve 'em

Purpose: Students use the structure of equations to group them according to how they would be solved, then practice solving some of the equations. **(MTR.5.1)**

1 Launch

To surface prior knowledge, consider asking, "What operation is represented by a number outside of a set of parentheses?"

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

Direct students' attention to the equations and invite them to share their partner's reasoning instead of their own. **(MTR.4.1)**

2 Monitor

Select and sequence student thinking to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 9)*	**Support** Consider asking, "What is a first step that you can take to solve each equation in Group A? Group B?"
Think the equation $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ belongs in Group B. *(Problem 10)*	**Support** Consider asking, "How is the use of parentheses different in the equations in Group A versus those in Group B?"
Check their solutions. *(Problem 11)*	**Strengthen** Invite students to share how they can ensure the accuracy of their solutions.

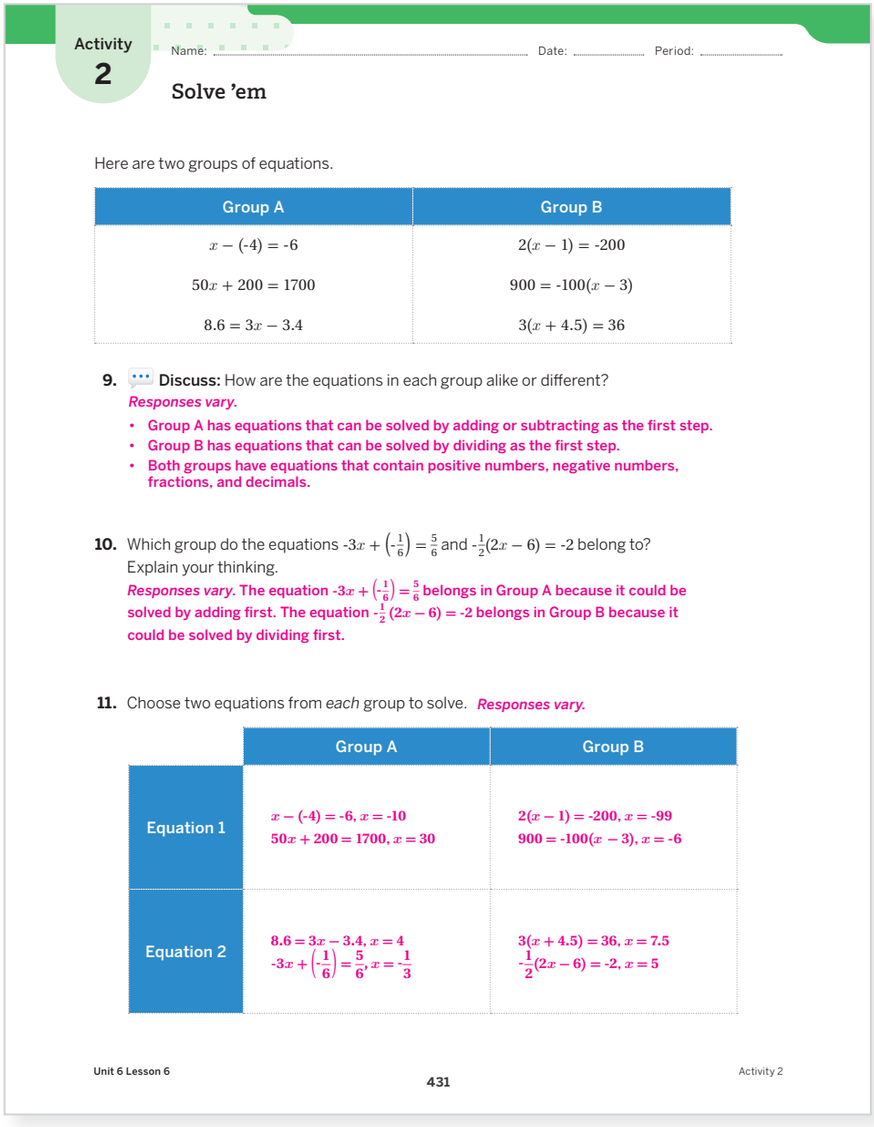
A Accessibility: Conceptual Processing Invite students to look at the relationship between the equations in each group to support identifying patterns.

3 Connect

Invite students to share how they used the structure of the equations to determine the placement of the two equations. **(MTR.5.1)**

To surface the Key Takeaway, consider asking:

- "What strategies were helpful to solve the equations in Group A?"
- "What strategies were helpful to solve the equations in Group B?"



Activity 2

Name: Date: Period:

Solve 'em

Here are two groups of equations.

Group A	Group B
$x - (-4) = -6$	$2(x - 1) = -200$
$50x + 200 = 1700$	$900 = -100(x - 3)$
$8.6 = 3x - 3.4$	$3(x + 4.5) = 36$

9. **Discuss:** How are the equations in each group alike or different?

Responses vary.

- Group A has equations that can be solved by adding or subtracting as the first step.
- Group B has equations that can be solved by dividing as the first step.
- Both groups have equations that contain positive numbers, negative numbers, fractions, and decimals.

10. Which group do the equations $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ and $-\frac{1}{2}(2x - 6) = -2$ belong to? Explain your thinking.

Responses vary. The equation $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ belongs in Group A because it could be solved by adding first. The equation $-\frac{1}{2}(2x - 6) = -2$ belongs in Group B because it could be solved by dividing first.

11. Choose two equations from *each* group to solve. *Responses vary.*

	Group A	Group B
Equation 1	$x - (-4) = -6, x = -10$ $50x + 200 = 1700, x = 30$	$2(x - 1) = -200, x = -99$ $900 = -100(x - 3), x = -6$
Equation 2	$8.6 = 3x - 3.4, x = 4$ $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}, x = -\frac{1}{3}$	$3(x + 4.5) = 36, x = 7.5$ $-\frac{1}{2}(2x - 6) = -2, x = 5$

Unit 6 Lesson 6 431 Activity 2

Key Takeaway: The structure of an equation can help you determine how to solve it. For equations like $p(x + q) = r$, you might start by dividing by p or distributing p. For equations like $px + q = r$, you might add or subtract as a first step instead.

Whole Class | 5 min

Synthesis

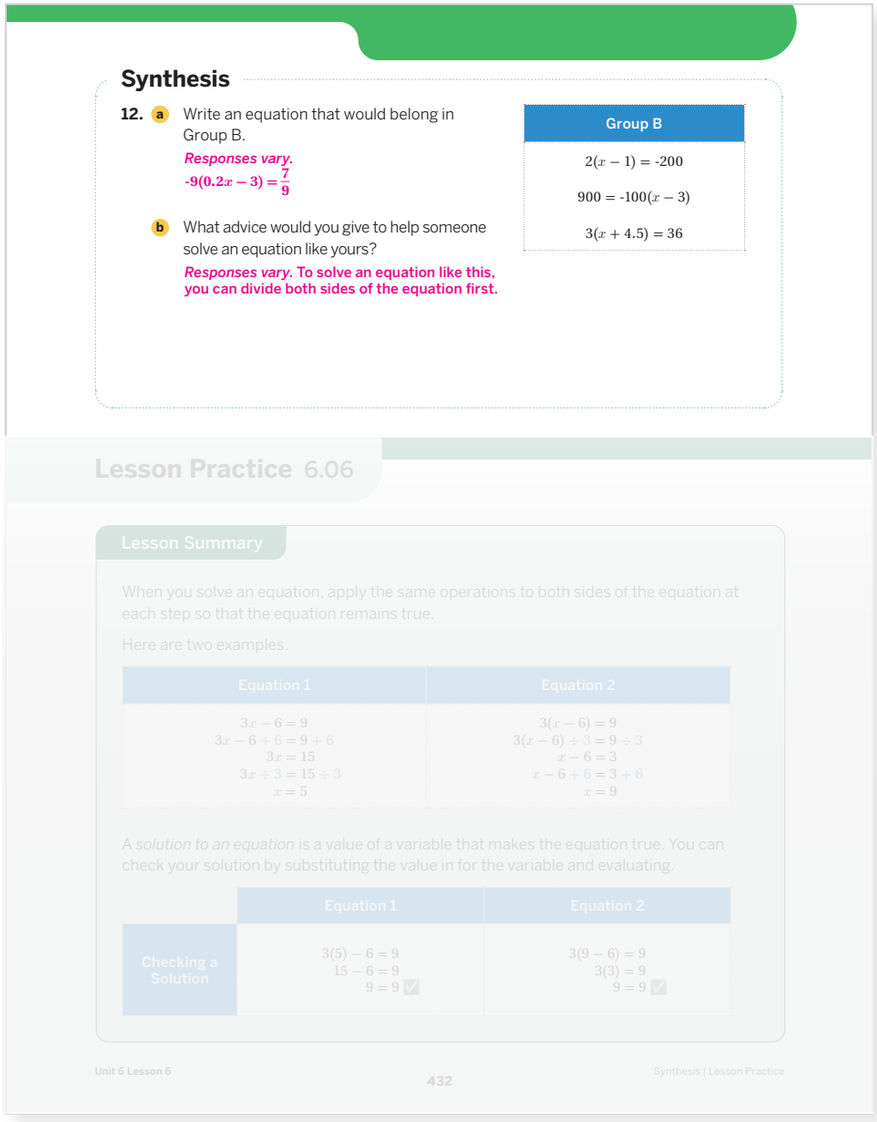
Purpose: Students synthesize their understanding of the structure of equations by creating their own equation and providing advice on how it can be solved.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Giving the advice of dividing both sides of the equation first.
- Giving the advice of distributing first.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. a. Write an equation that would belong in Group B.

Responses vary.

$-9(0.2x - 3) = \frac{7}{9}$

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

b. What advice would you give to help someone solve an equation like yours?

Responses vary. To solve an equation like this, you can divide both sides of the equation first.

Lesson Practice 6.06

Lesson Summary

When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✓	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✓

Unit 6 Lesson 6 432 Synthesis | Lesson Practice

Lesson Takeaway: The first step in solving equations may be different depending on the structure of the equation, but it is important that both sides of the equation remain balanced.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving equations that require different first steps in solving.

Today's Goals

1. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive and negative numbers.
2. **Language Goal:** Explain why a solution to an equation is accurate. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.06

Solve each equation. Show your thinking.

a. $-3x - 5 = 16$

$x = -7$

Work varies.

$-3x - 5 + 5 = 16 + 5$

$-3x = 21$

$-3x \div -3 = 21 \div -3$

$x = -7$

b. $12 = -4(x - 2)$

$x = -1$

Work varies.

$\frac{12}{-4} = \frac{-4}{-4}(x - 2)$

$-3 = x - 2$

$-3 + 2 = x - 2 + 2$

$-1 = x$

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations:

» **Respond to Student Thinking:**

- Use the ***Solving Equations With Positive and Negative Numbers*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write two equations with the same solution, one in the form $px + q = r$ and one in the form $p(x + q) = r$.

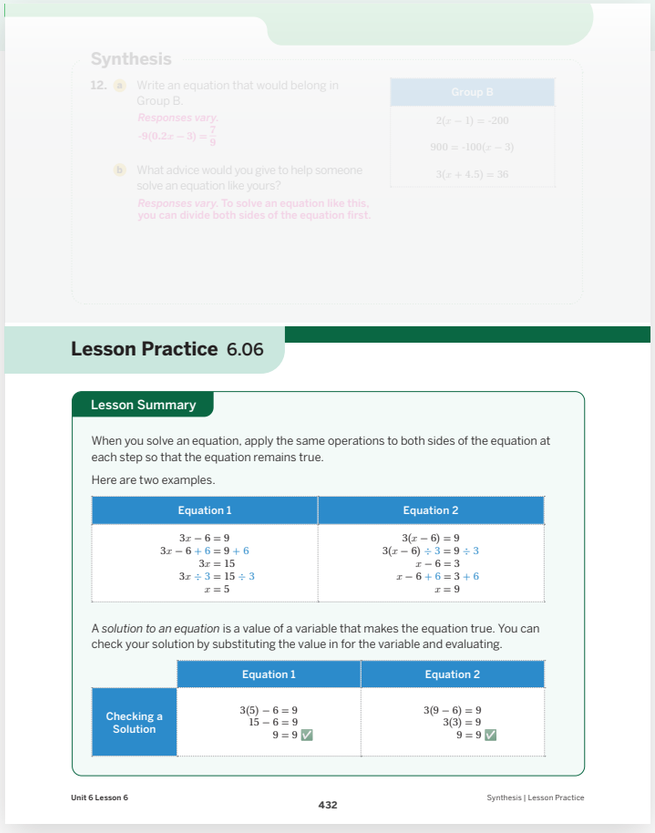
Professional Learning

What strengths did your students exhibit today? How can you build from those strengths as students continue learning about strategies for solving equations with and without visual models?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

12. a. Write an equation that would belong in Group B.

Responses vary.

$-9(0.2x - 3) = \frac{7}{9}$

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

b. What advice would you give to help someone solve an equation like yours?

Responses vary. To solve an equation like this, you can divide both sides of the equation first.

Lesson Practice 6.06

Lesson Summary

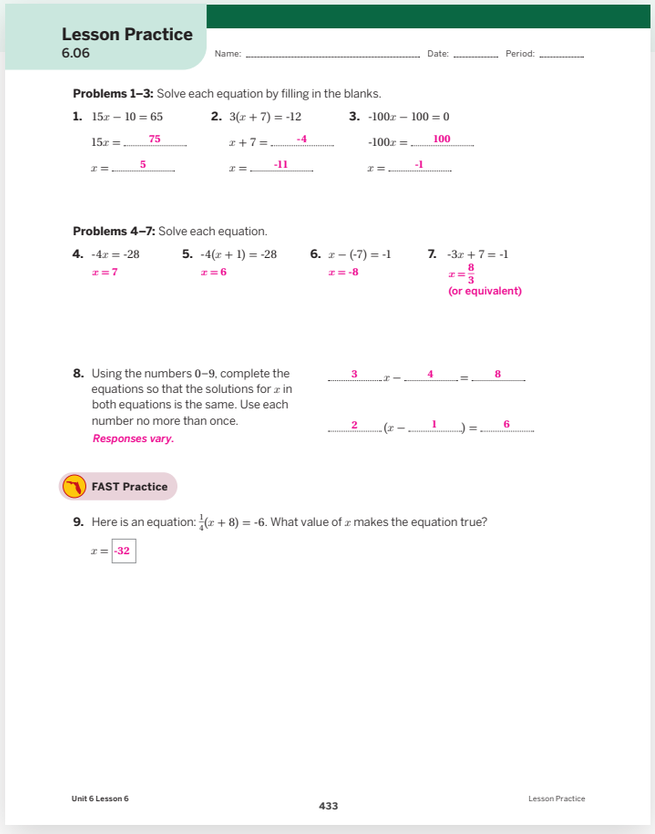
When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✅	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✅



Lesson Practice 6.06

Name: _____ Date: _____ Period: _____

Problems 1–3: Solve each equation by filling in the blanks.

1. $15x - 10 = 65$

   $15x =$ 75

   $x =$ 5

2. $3(x + 7) = -12$

   $x + 7 =$ -4

   $x =$ -11

3. $-100x - 100 = 0$

   $-100x =$ 100

   $x =$ -1

Problems 4–7: Solve each equation.

4. $-4x = -28$

   $x = 7$

5. $-4(x + 1) = -28$

   $x = 6$

6. $x - (-7) = -1$

   $x = -8$

7. $-3x + 7 = -1$

   $x = \frac{8}{3}$ (or equivalent)

8. Using the numbers 0–9, complete the equations so that the solutions for x in both equations is the same. Use each number no more than once.

   Responses vary.

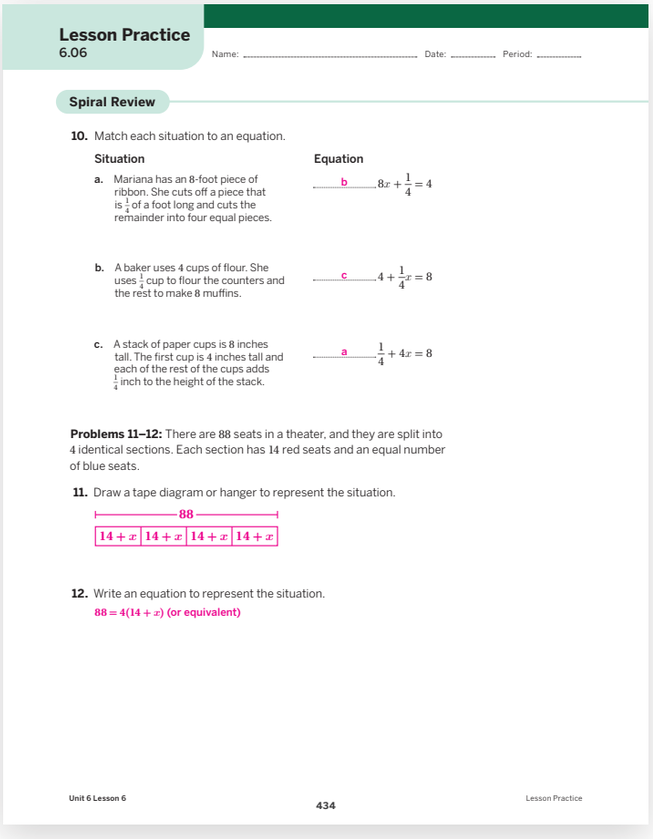
   3 $x -$ 4 $=$ 8

   2 $(x -$ 1 $) =$ 6

FAST Practice

9. Here is an equation: $\frac{1}{4}(x + 8) = -6$. What value of x makes the equation true?

   $x =$ -32



Lesson Practice 6.06

Name: _____ Date: _____ Period: _____

Spiral Review

10. Match each situation to an equation.

Situation	Equation
a. Mariana has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four equal pieces.	b. $8x + \frac{1}{4} = 4$
b. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 muffins.	c. $4 + \frac{1}{4}x = 8$
c. A stack of paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups adds $\frac{1}{4}$ inch to the height of the stack.	a. $\frac{1}{4} + 4x = 8$

Problems 11–12: There are 88 seats in a theater, and they are split into 4 identical sections. Each section has 14 red seats and an equal number of blue seats.

11. Draw a tape diagram or hanger to represent the situation.

12. Write an equation to represent the situation.

$88 = 4(14 + x)$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	2	MA.7.AR.2.2
	8	3	MA.7.AR.2.2
FAST Practice	9	2	MA.7.AR.2.2
Spiral Review			
	10–12	2	MA.7.AR.2.2

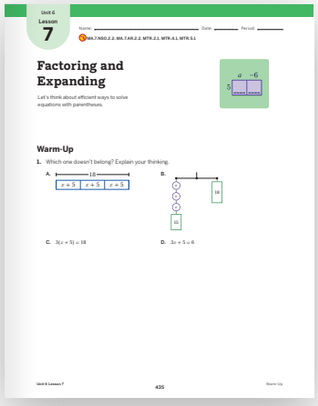
Unit 6

Lesson 7

Factoring and Expanding

Options for Solving One Equation

Let's think about efficient ways to solve equations with parentheses.


Unit 6 Lesson 7

Factoring and Expanding

Let's think about efficient ways to solve equations with parentheses.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. $x+5$ | $x+5$ | $x+5$ | 18

B. $\times 3$, $+5$, 18

C. $3(x+5)=18$

D. $3x+5=6$

435

Learning Outcomes

Today's Goals

1. **Goal:** *Expand* and *factor* expressions that include positive and negative coefficients.
2. **Goal:** Solve equations that involve *expanding* an expression.
3. **Language Goal:** Analyze different strategies for solving the same equation. **(Reading and Writing)**

Students extend what they know about distributing and solving to **factor** and **expand** expressions, and then solve equations in multiple ways and compare the results.

Coherence

Prior Learning

In Lesson 6, students described and used strategies for solving equations of the form $px + q = r$ and $p(x + q) = r$ without a visual model.

Future Learning

In Lesson 8, students will reason about equivalent expressions that involve factoring, expanding, and reordering terms.

Connections

- Students use visual hanger diagrams to help understand the distributive property and balance. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.3**
MA.6.AR.2.2	**MA.8.AR.2.1**
MA.6.AR.2.3	

Vocabulary

New Vocabulary

expand	**factor**

Review Vocabulary

coefficient	*distributive property*

Lesson at a Glance

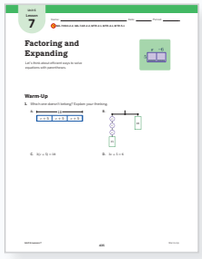
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.2

Warm-Up

Pairs | 5 min

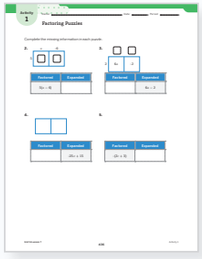
Students compare and contrast visual representations and equations in **factored** and **expanded** form using the Which One Doesn't Belong? routine.



Activity 1

Pairs | 10 min

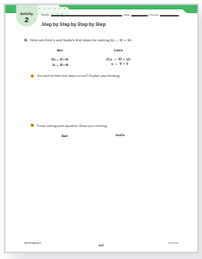
Students practice factoring and expanding expressions after analyzing diagrams using the Notice and Wonder routine.



Activity 2

Pairs | 10 min

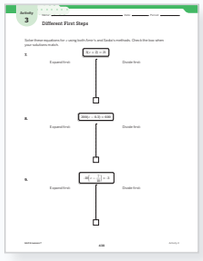
Students compare and contrast strategies to solve equations of the form $p(x + q) = r$ using the MLR7: Compare and Connect routine. **(MTR.4.1)**



Activity 3

Pairs | 10 min

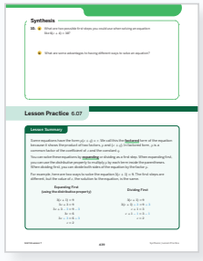
Students practice solving equations by expanding and dividing by a common factor.



Synthesis

Whole Class | 5 min

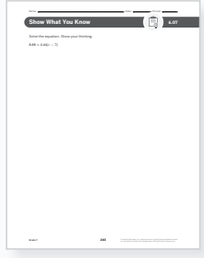
Students synthesize their understanding of solving equations of the form $p(x + q) = r$ by expanding first or by dividing first, and examine the advantages of both strategies.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving an equation of the form $p(x + q) = r$.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students compare and contrast visual representations and equations in factored and expanded form using the Which One Doesn't Belong? routine.

1 Launch

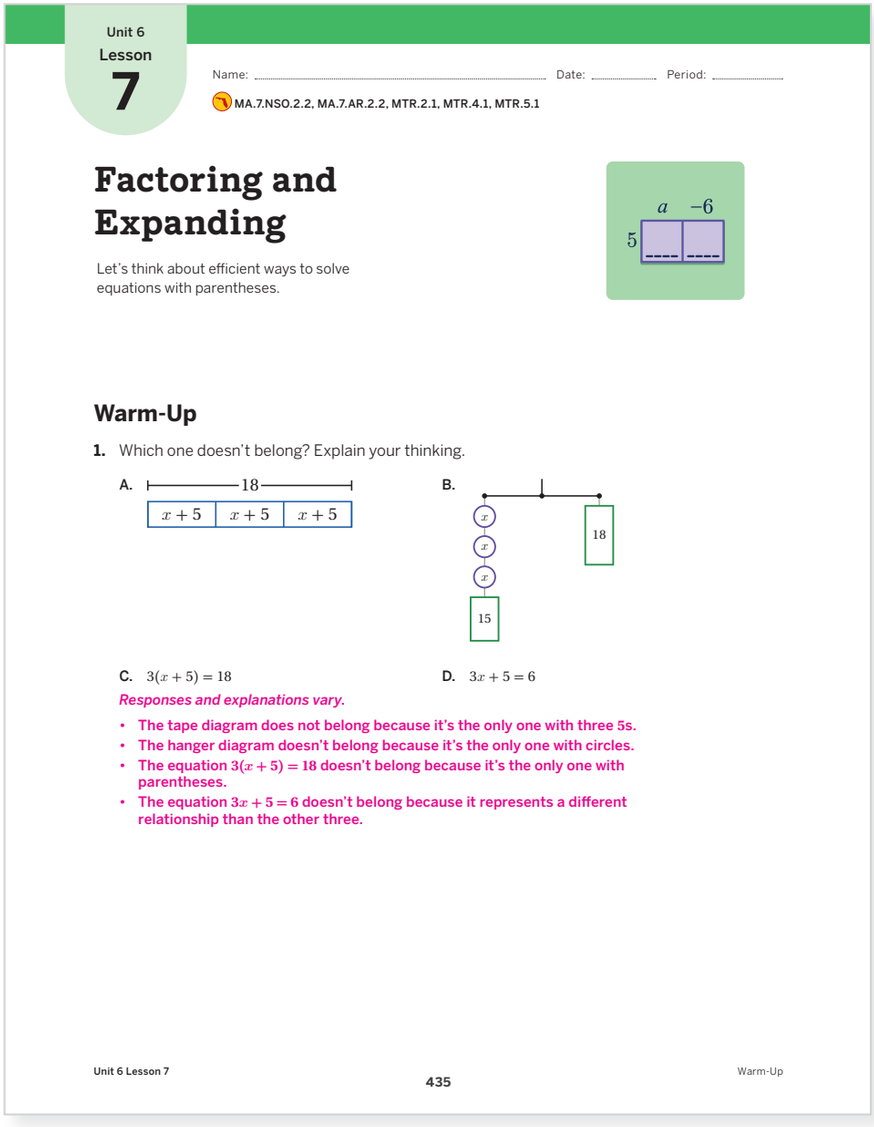
To surface prior knowledge, consider asking, "What are the diagrams in Choice A and Choice B called? Why are they used?"

Use the Which One Doesn't Belong? routine to support students in comparing and contrasting representations. See the Routine Facilitation Guide for more information. **(MTR.2.1)**

2 Connect

Invite students to share their responses.

Consider asking, "Which representation does not show the same relationship as the others? How do you know?"



Unit 6
Lesson
7

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Factoring and Expanding

Let's think about efficient ways to solve equations with parentheses.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.

B.

C. $3(x + 5) = 18$

D. $3x + 5 = 6$

Responses and explanations vary.

- The tape diagram does not belong because it's the only one with three 5s.
- The hanger diagram doesn't belong because it's the only one with circles.
- The equation $3(x + 5) = 18$ doesn't belong because it's the only one with parentheses.
- The equation $3x + 5 = 6$ doesn't belong because it represents a different relationship than the other three.

Unit 6 Lesson 7 435 Warm-Up

Pairs | 10 min

Activity 1 Factoring Puzzles

Purpose: Students practice factoring and expanding expressions after analyzing diagrams using the Notice and Wonder routine.

Short on time? Consider having students complete only two puzzles.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the two forms: factored and expanded. See the Routine Facilitation Guide for more information.

To surface connections between the diagrams and the area of a rectangle, consider asking, "How do the value(s) inside of the rectangular section relate to the area?" **(MTR.5.1)**

Create a class definition for the terms **factor** and **expand**.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation *(Problems 2–5)*

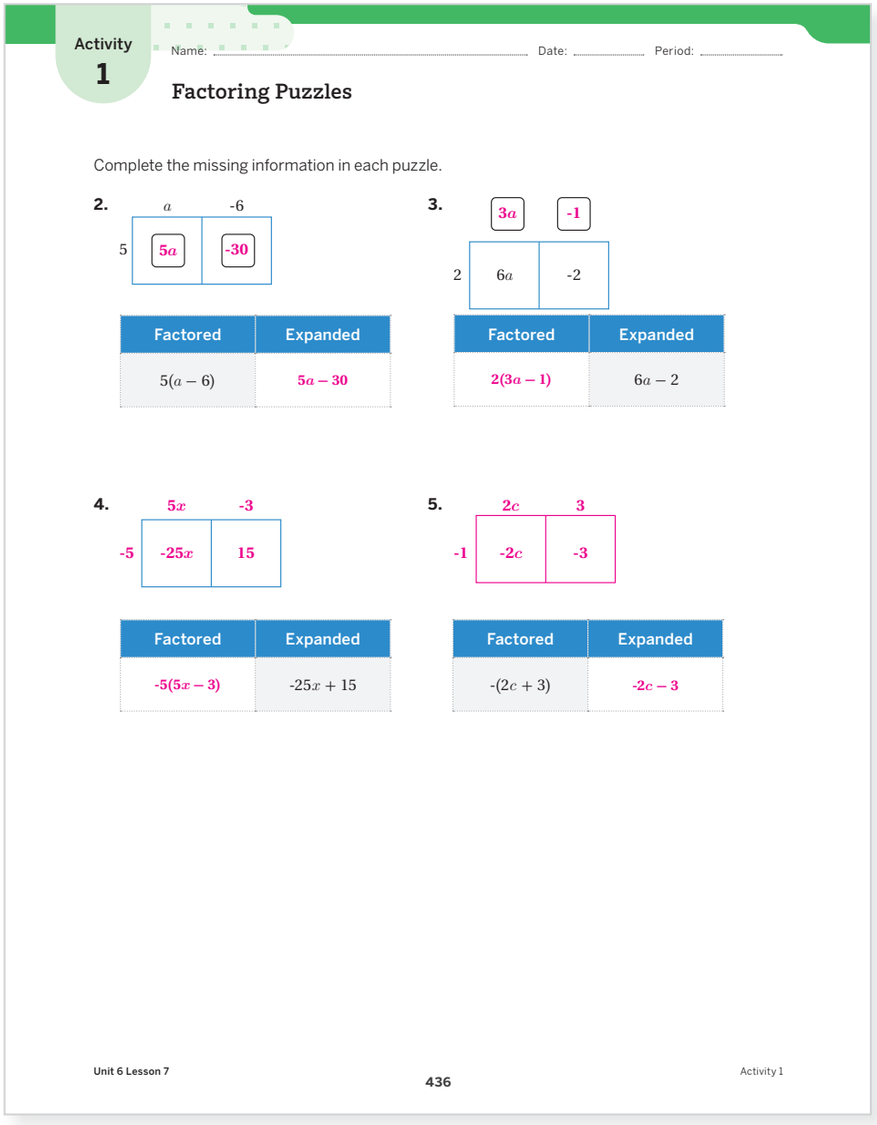
Look for students who:	Respond to Student Thinking
Combine numbers using addition or subtraction rather than multiplication.	**Support** Consider asking, "What does the operation 5(3) or 5(a) represent?"
Determine the correct values with incorrect signs.	**Support** Encourage students to review their work, looking specifically at the sign of each number to make sure that they make sense.
Form the connection that factoring is the same as "undoing" the *distributive property*.	**Strengthen** Invite students to share how factored form can be written as expanded form and vice versa.

A Accessibility: Visual-Spatial Processing To support interpreting visual representations, invite students to color-code parts of the rectangle and the corresponding parts in the expression.

3 Connect

Invite students to share their responses to the four problems and explain how the two forms are related.

To surface the Key Takeaway, consider asking, "How can you turn your factored-form expression into expanded form and vice versa?"



Activity 1

Name: Date: Period:

Factoring Puzzles

Complete the missing information in each puzzle.

2.

	a	-6
5	$5a$	-30

Factored	Expanded
$5(a-6)$	$5a-30$

3.

	$3a$	-1
2	$6a$	-2

Factored	Expanded
$2(3a-1)$	$6a-2$

4.

	$5x$	-3
-5	$-25x$	15

Factored	Expanded
$-5(5x-3)$	$-25x+15$

5.

	$2c$	3
-1	$-2c$	-3

Factored	Expanded
$-(2c+3)$	$-2c-3$

Unit 6 Lesson 7 436 Activity 1

Key Takeaway: An expression can be written in **factored** form or **expanded** form, and the new expression is equivalent to the original expression.

Pairs | 10 min

Activity 2 Step by Step by Step by Step

Purpose: Students compare and contrast strategies to solve equations of the form $p(x + q) = r$ using the **MLR7: Compare and Connect** routine. **(MTR.4.1)**

1 Launch

Direct students' attention to the equation $2(x - 9) = 10$, and invite students to share how they would solve it.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy. **(MTR.4.1)**

Consider asking:

- "What did each student do?"
- "Where do you see the same information in Amir's and Sadia's strategies?"
- "How does each strategy compare to your own?"
- "When might we use each strategy?"
- "Why did the different strategies lead to the same solution?"

2 Monitor

Select and sequence strategies to share during the Connect.

D **Differentiation** *(Problem 6)*

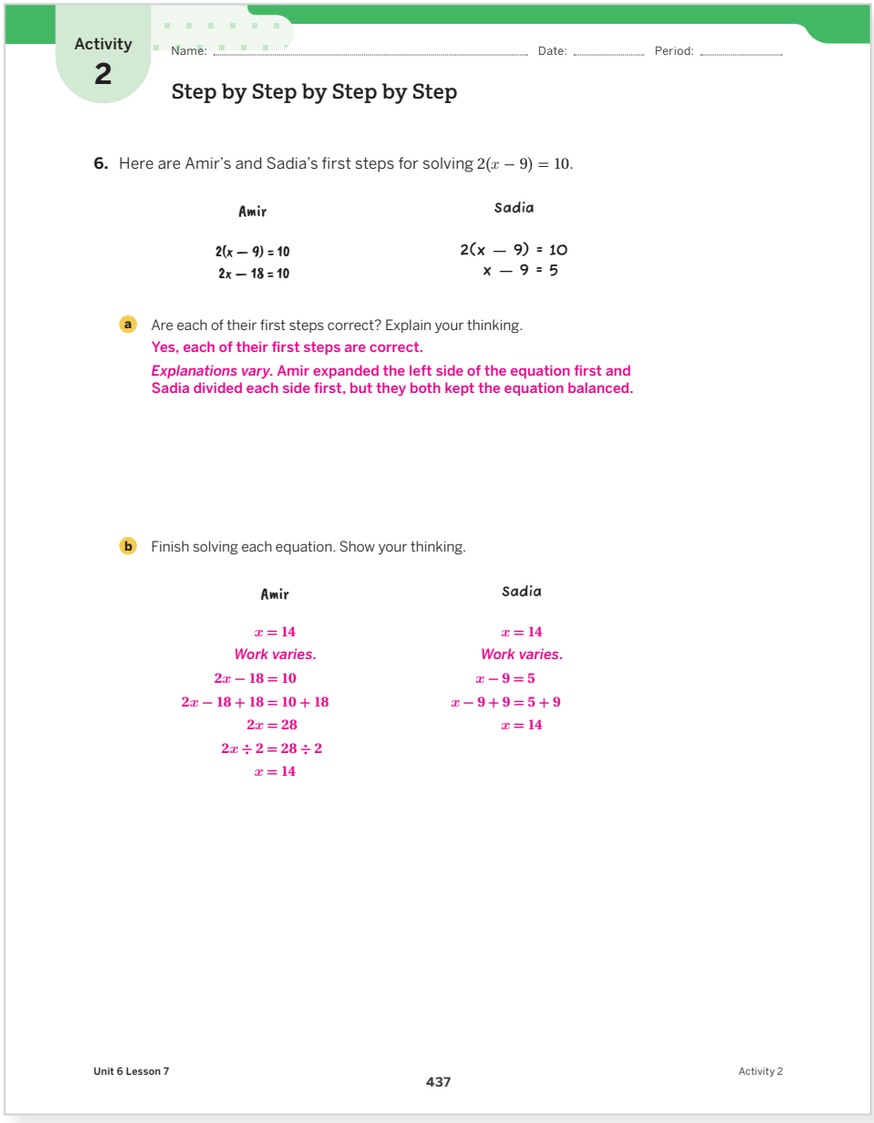
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What operation could we do to both sides of the equation to get $x - 9$ on the left and 5 on the right?"
Solve the equation on their own and substitute their response in Sadia's and Amir's first step.	**Strengthen** Invite students to share how this allows them to check the accuracy of the two equations.
Think that Sadia's strategy is the optimal method.	**Stretch** Invite students to solve the problem $5(x + 2) = 18$ using both methods and discuss what they notice.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., These are/are not both correct because _____.). **(Speaking and Listening)**

3 Connect

Invite students to share how they finished solving the equations and what they noticed about the solutions.

To surface connections between the two approaches, consider asking, "Why do both strategies lead to the same solution?"



Activity 2

Name: _____ Date: _____ Period: _____

Step by Step by Step by Step

6. Here are Amir's and Sadia's first steps for solving $2(x - 9) = 10$.

Amir	Sadia
$2(x - 9) = 10$ $2x - 18 = 10$	$2(x - 9) = 10$ $x - 9 = 5$

a. Are each of their first steps correct? Explain your thinking.

Yes, each of their first steps are correct.

Explanations vary. Amir expanded the left side of the equation first and Sadia divided each side first, but they both kept the equation balanced.

b. Finish solving each equation. Show your thinking.

Amir	Sadia
$x = 14$ *Work varies.* $2x - 18 = 10$ $2x - 18 + 18 = 10 + 18$ $2x = 28$ $2x \div 2 = 28 \div 2$ $x = 14$	$x = 14$ *Work varies.* $x - 9 = 5$ $x - 9 + 9 = 5 + 9$ $x = 14$

Unit 6 Lesson 7 437 Activity 2

Pairs | 10 min

Activity 3 Different First Steps

Purpose: Students practice solving equations by expanding and dividing by a common factor.

Short on time? Consider having students solve using one method while their partner solves using the other method. Ensure students practice each method.

1 Launch

Direct students' attention to the equation $3(x + 2) = 21$ and consider asking, "What does it mean to expand first? What does it mean to divide first?"

To surface connections to Sadia's and Amir's methods, invite students to discuss how to complete the first step of each strategy.

2 Monitor

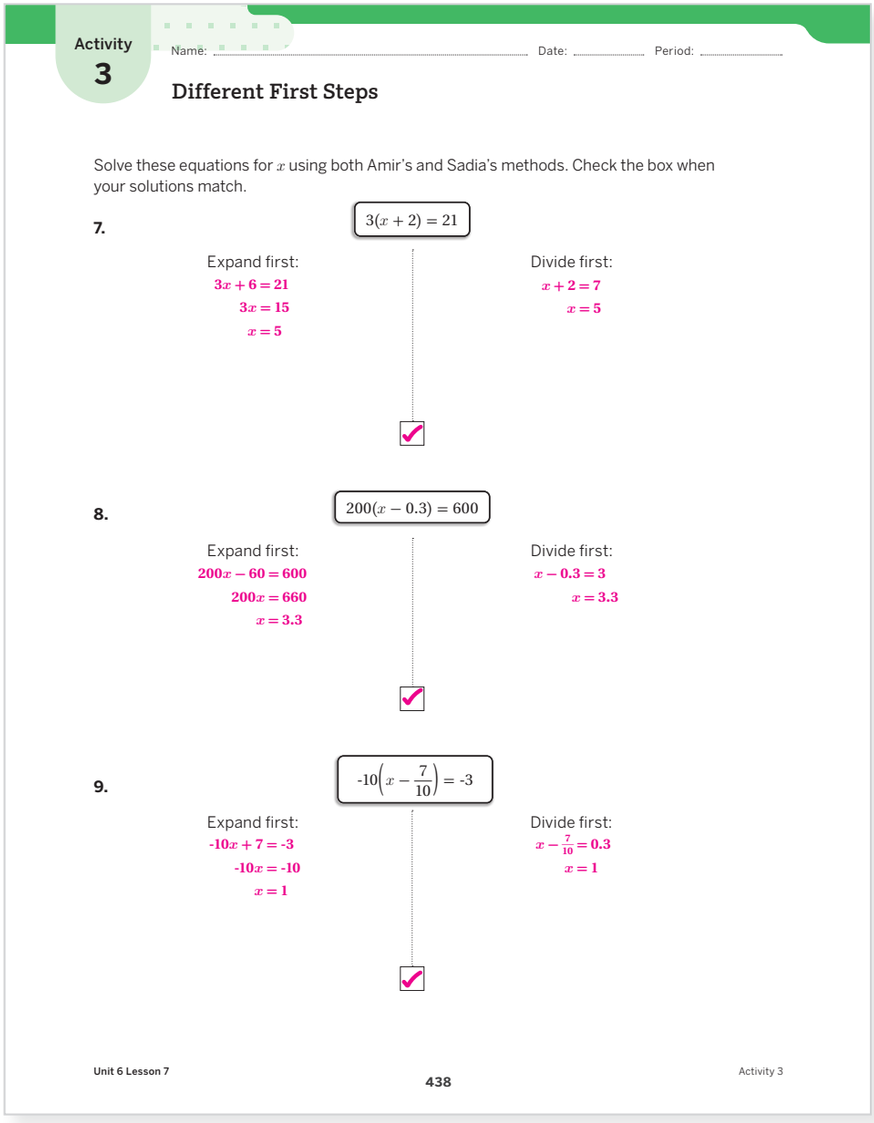
Select and sequence student strategies to share during the Connect.

D Differentiation *(Problems 7–9)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How is this problem similar to Activity 2?"
Get solutions that don't match and substitute them into the original equation to assess which solution is incorrect.	**Strengthen** Invite students to share how this allows them to discover which method needs to be reworked.
Would like to explore solving equations involving expanding first or dividing first further.	**Stretch** Invite students to create an equation that they think would be more efficient to solve by expanding first and one that is more efficient to solve by dividing first, and then convince a classmate this is true.

3 Connect

Invite students to share which strategy they prefer and explain why.



Activity 3

Name: Date: Period:

Different First Steps

Solve these equations for x using both Amir's and Sadia's methods. Check the box when your solutions match.

7. $3(x + 2) = 21$

Expand first:
$3x + 6 = 21$
$3x = 15$
$x = 5$

Divide first:
$x + 2 = 7$
$x = 5$

8. $200(x - 0.3) = 600$

Expand first:
$200x - 60 = 600$
$200x = 660$
$x = 3.3$

Divide first:
$x - 0.3 = 3$
$x = 3.3$

9. $-10\left(x - \frac{7}{10}\right) = -3$

Expand first:
$-10x + 7 = -3$
$-10x = -10$
$x = 1$

Divide first:
$x - \frac{7}{10} = 0.3$
$x = 1$

Unit 6 Lesson 7 438 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of solving equations of the form $p(x + q) = r$ by expanding first or by dividing first, and examine the advantages of both strategies.

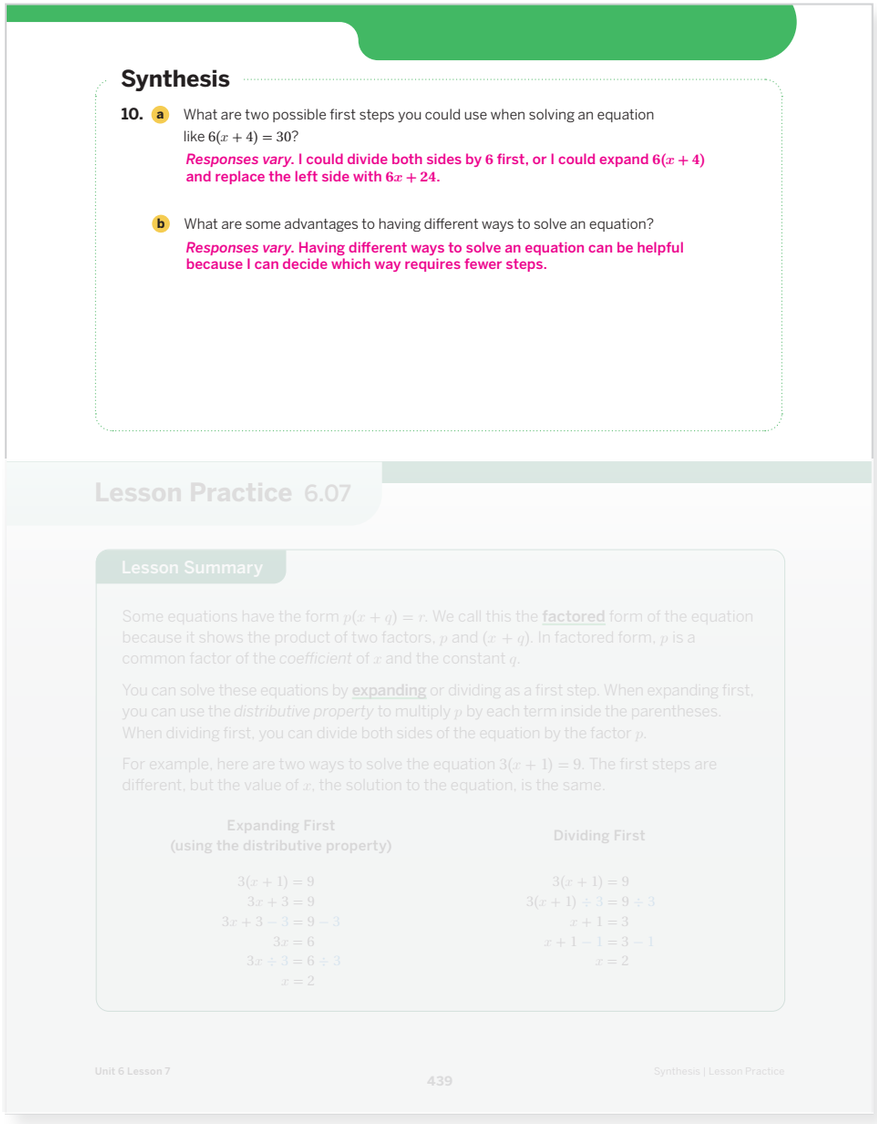
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Having fewer steps depending on the method used to solve.
- Using formal vocabulary like *expand* when describing their first step to solve.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *expand* and *factor* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. a. What are two possible first steps you could use when solving an equation like $6(x + 4) = 30$?

Responses vary. I could divide both sides by 6 first, or I could expand $6(x + 4)$ and replace the left side with $6x + 24$.

b. What are some advantages to having different ways to solve an equation?

Responses vary. Having different ways to solve an equation can be helpful because I can decide which way requires fewer steps.

Lesson Practice 6.07

Lesson Summary

Some equations have the form $p(x + q) = r$. We call this the factored form of the equation because it shows the product of two factors, p and $(x + q)$. In factored form, p is a common factor of the *coefficient* of x and the constant q.

You can solve these equations by expanding or dividing as a first step. When expanding first, you can use the *distributive property* to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the solution to the equation, is the same.

Expanding First (using the distributive property)

$$\begin{aligned} 3(x + 1) &= 9 \\ 3x + 3 &= 9 \\ 3x + 3 - 3 &= 9 - 3 \\ 3x &= 6 \\ 3x \div 3 &= 6 \div 3 \\ x &= 2 \end{aligned}$$

Dividing First

$$\begin{aligned} 3(x + 1) &= 9 \\ 3(x + 1) \div 3 &= 9 \div 3 \\ x + 1 &= 3 \\ x + 1 - 1 &= 3 - 1 \\ x &= 2 \end{aligned}$$

Unit 6 Lesson 7 439 Synthesis | Lesson Practice

Lesson Takeaway: Equations of the form $p(x + q) = r$ can be solved by **expanding** first or dividing first, and there are advantages to both depending on the equation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving an equation of the form $p(x + q) = r$.

Today's Goals

1. **Goal:** *Expand* and *factor* expressions that include positive and negative coefficients.
2. **Goal:** Solve equations that involve *expanding* an expression.
3. **Language Goal:** Analyze different strategies for solving the same equation. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.07

Solve the equation. Show your thinking.

$8.88 = 4.44(x - 7)$

$x = 9$

Work varies.

$2 = x - 7$

$2 + 7 = x - 7 + 7$

$9 = x$

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations that involve expanding an expression:

» **Respond to Student Thinking:**

- Revisit Activity 2 and invite students to reflect on how they might apply Amir's and Sadia's strategies to other equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write an equation they would prefer to solve by factoring first, and another they would prefer to solve by dividing first.

Professional Learning

Students are asked to solve using Amir's and Sadia's methods. Are they comfortable with both methods, just one, or a different method entirely? Do any students need additional support with solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. a. What are two possible first steps you could use when solving an equation like $6(x + 4) = 30$?

Responses vary. I could divide both sides by 6 first, or I could expand $6(x + 4)$ and replace the left side with $6x + 24$.

b. What are some advantages to having different ways to solve an equation?

Responses vary. Having different ways to solve an equation can be helpful because I can decide which way requires fewer steps.

Lesson Practice 6.07

Lesson Summary

Some equations have the form $p(x + q) = r$. We call this the **factored** form of the equation because it shows the product of two factors, p and $(x + q)$. In factored form, p is a common factor of the *coefficient* of x and the constant q.

You can solve these equations by **expanding** or dividing as a first step. When expanding first, you can use the *distributive property* to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the solution to the equation, is the same.

Expanding First (using the distributive property)

$3(x + 1) = 9$
$3x + 3 = 9$
$3x + 3 - 3 = 9 - 3$
$3x = 6$
$3x \div 3 = 6 \div 3$
$x = 2$

Dividing First

$3(x + 1) = 9$
$3(x + 1) \div 3 = 9 \div 3$
$x + 1 = 3$
$x + 1 - 1 = 3 - 1$
$x = 2$

Lesson Practice 6.07

Name: _____ Date: _____ Period: _____

Problems 1–3: Write each expression in expanded form.

1. $-2(-6) = 12$
2. $-2(-y) = 2y$
3. $-2(-6 - y) = 12 + 2y$

Problems 4–6: Complete the missing information in the puzzle and complete the table.

4.

	a	-5
4	$4a$	-20

Factored	Expanded
$4(a - 5)$	$4a - 20$

5.

	-4a	$\frac{1}{3}$
-3	$12a$	-1

Factored	Expanded
$-3\left(-4a + \frac{1}{3}\right)$	$12a - 1$

6.

	3x	-7
3	$9x$	-21

Factored	Expanded
$3(3x - 7)$	$9x - 21$

Responses vary.

7. Emmanuel and Mauricio started solving the equation $7(x - 2) = 91$. Finish solving each equation.

Emmanuel

$7(x - 2) = 91$
$7x - 14 = 91$
$7x = 105$
$x = 15$

Mauricio

$7(x - 2) = 91$
$x - 2 = 13$
$x = 15$

Lesson Practice 6.07

Name: _____ Date: _____ Period: _____

Problems 8–10: Solve each equation. Show your thinking.

8. $2(x - 3) = 14$
$2(x - 3) \div 2 = 14 \div 2$
$x - 3 = 7$
$x - 3 + 3 = 7 + 3$
$x = 10$

9. $-5(x - 1) = 40$
$-5(x - 1) \div (-5) = 40 \div (-5)$
$x - 1 = -8$
$x - 1 + 1 = -8 + 1$
$x = -7$

10. $\frac{5}{7}(x - 9) = 25$
$\frac{5}{7}(x - 9) \div \frac{5}{7} = 25 \div \frac{5}{7}$
$x - 9 = 35$
$x - 9 + 9 = 35 + 9$
$x = 44$

FAST Practice

11. A student incorrectly solved the equation $-5(x - 1) = 40$ as shown.

Step 1: $-5x + 5 = 40$

Step 2: $-5x = 45$

Step 3: $x = -9$

Select **ONE** correct answer in each box to complete the sentence.

The student made the first error in [A. Step 1 / (B.) Step 2]. The correct solution to the equation is [(A.) -7 / B. 9].

Spiral Review

12. Use long division to write $\frac{2}{5}$ as a decimal.

$5\overline{)2.0}$ → 0.4, -20, 0

13. Which equation is true when $x = 2$?

A. $3x + 4 = 8$ B. $5x - 3 = 12$ (C.) $6x + 4 = 16$ D. $4x + 5 = 17$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.1.2
	4–6	2	MA.7.AR.1.2
	7–10	2	MA.7.AR.2.2
FAST Practice	11	2	MA.7.AR.2.2
Spiral Review			
	12	1	MA.6.NSO.3.5
	13	2	MA.6.AR.2.1

Unit 6

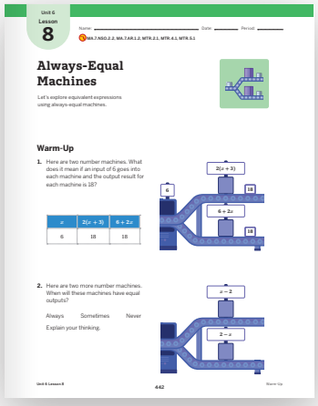
Lesson

8

Always-Equal Machines

Equivalent Expressions

Let's explore equivalent expressions using always-equal machines.


Unit 6
Lesson
8
Always-Equal Machines
Let's explore equivalent expressions using always-equal machines.
Warm-Up
1. Here are two number machines. What does it mean if an input of 6 goes into each machine and the output result for each machine is 18?
x	$2(x+3)$	$6+2x$
6	18	18
2. Here are two more number machines. When will these machines have equal outputs?
Always Sometimes Never
Explain your thinking.
442

Learning Outcomes

Today's Goals

1. **Goal:** Write *equivalent expressions* that involve factoring, expanding, and reordering terms.
2. **Language Goal:** Justify whether factored, expanded, or reordered expressions are *equivalent*. **(Speaking and Listening)**

Students learn that **equivalent expressions** have the same value for any value of the variable. They reason about expressions that involve factoring, expanding, and reordering terms, and justify whether two expressions are equivalent. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 7, students expanded and factored expressions that included positive and negative coefficients. Students also analyzed different strategies for solving the same equation.

Future Learning

In Lesson 9, students will determine whether two expressions are equivalent using models and properties of operations.

Connections

- Students use "always-equal machines" as a visual way to represent equivalent expressions demonstrating that different-looking expressions can have the same value. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.3**

Vocabulary

New Vocabulary

equivalent expressions

Lesson at a Glance

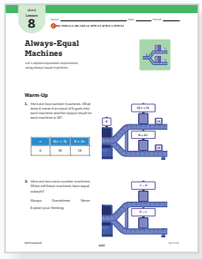
~ 45 min

MA.7.NSO.2.2, MA.7.AR.1.2

Warm-Up

Independent | 5 min

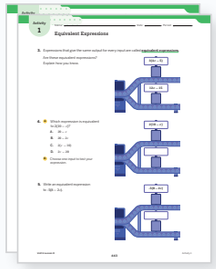
Students observe and make predictions about two number machines that use the same input.



Activity 1

Pairs | 15 min

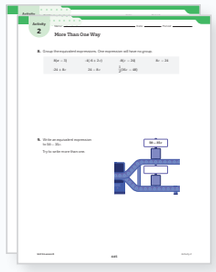
Students write **equivalent expressions** using factoring, expanding, and reordering terms. They use the MLR3: Critique, Correct, Clarify routine to critique common mistakes. **(MTR.4.1)**



Activity 2

Pairs | 15 min

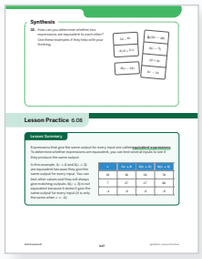
Students write multiple equivalent expressions using factoring, expanding, and reordering terms. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of strategies for determining whether two expressions are equivalent. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equivalent expression.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students observe and make predictions about two number machines that use the same input.

1 Launch

Invite students to share what they notice about the number machine.

Consider asking, "Where do you see each part of the machine in the table?" **(MTR.5.1)**

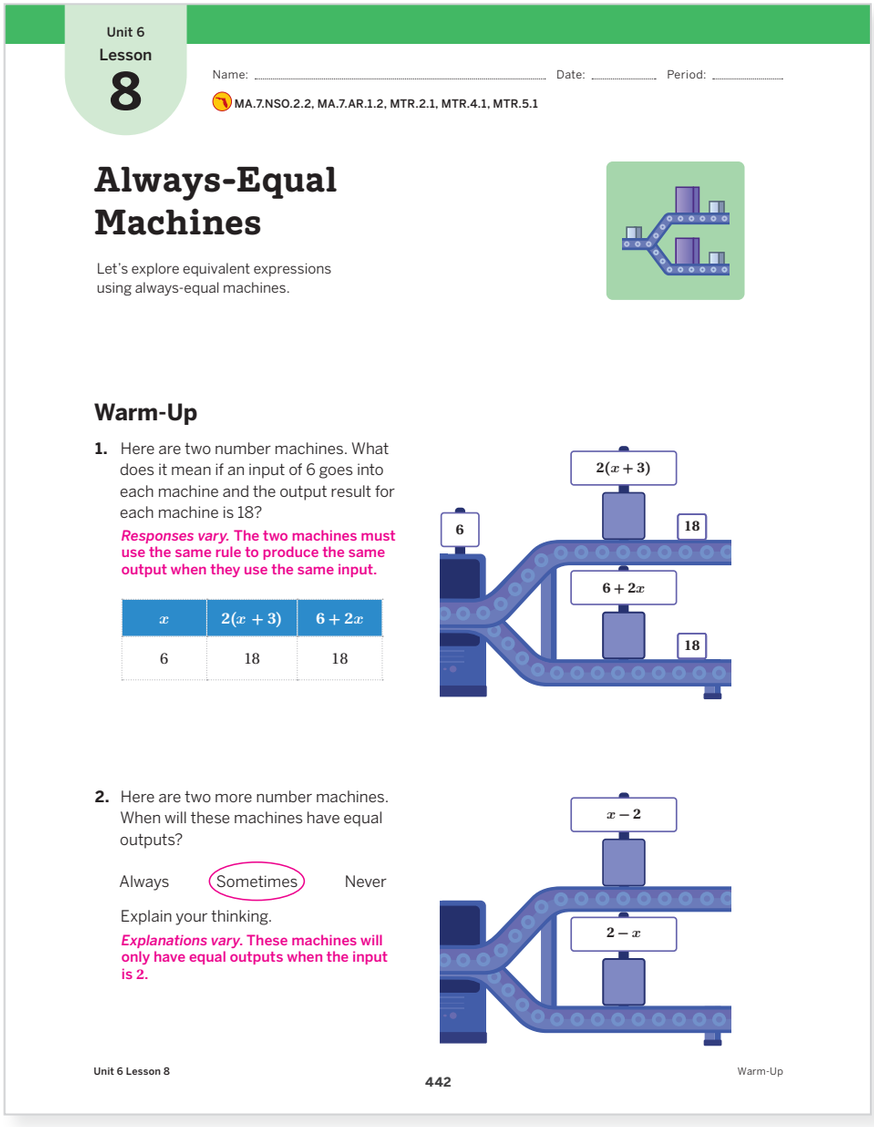
2 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Invite students to justify their response.

Consider asking, "What value(s) of x make the machines have equal outputs?"

ELL **English Language Learners** Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



Unit 6
Lesson
8

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.2, MA.7.AR.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Always-Equal Machines

Let's explore equivalent expressions using always-equal machines.

Warm-Up

1. Here are two number machines. What does it mean if an input of 6 goes into each machine and the output result for each machine is 18?

Responses vary. The two machines must use the same rule to produce the same output when they use the same input.

x	$2(x+3)$	$6+2x$
6	18	18

2. Here are two more number machines. When will these machines have equal outputs?

Always Sometimes Never

Explain your thinking.

Explanations vary. These machines will only have equal outputs when the input is 2.

Unit 6 Lesson 8 442 Warm-Up

Pairs | 15 min

Activity 1 Equivalent Expressions

Purpose: Students write **equivalent expressions** using factoring, expanding, and reordering terms. They use the **MLR3: Critique, Correct, Clarify** routine to critique common mistakes. **(MTR.4.1)**

1 Launch

Invite students to read the prompt aloud to a partner, restate it in their own words, and prepare to share with the class.

Create a class definition for the term **equivalent expressions** using students' language and the glossary.

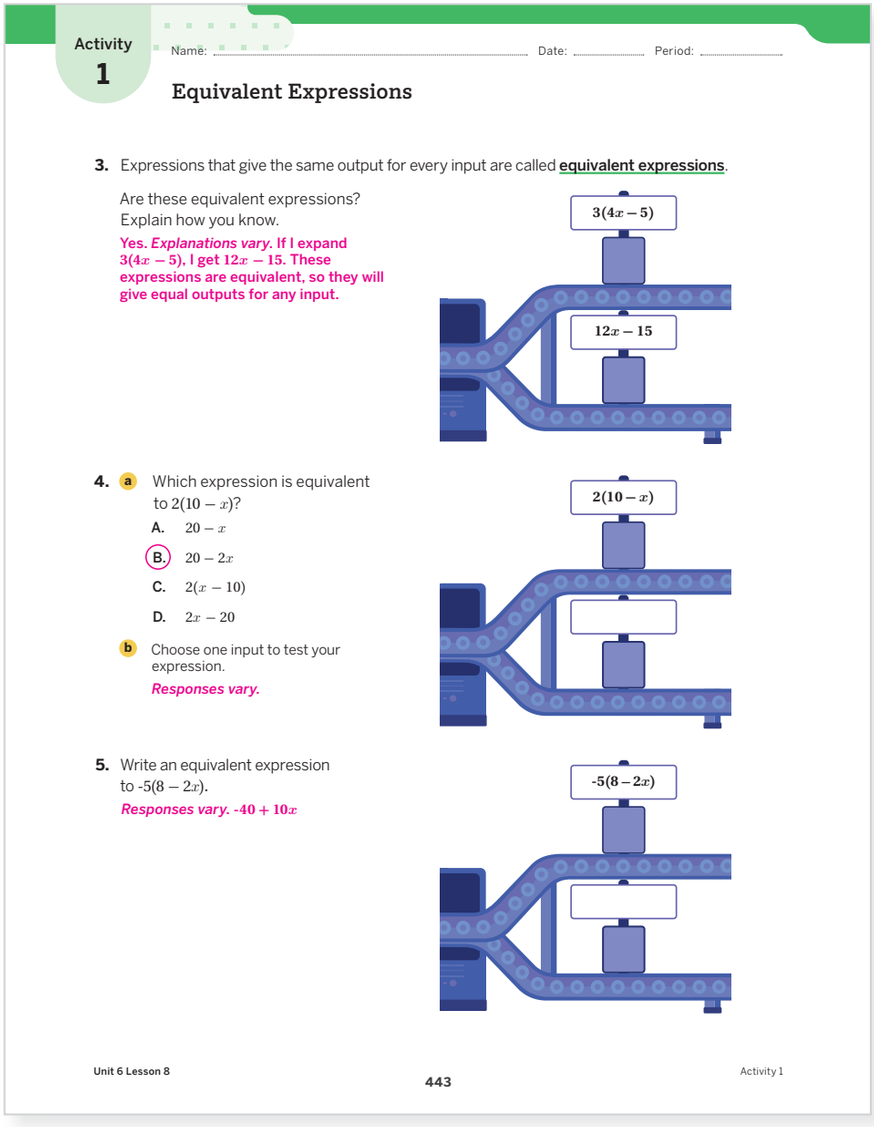
2 Monitor

Listen for different ways students explain whether expressions are equivalent.

Invite students to share different inputs that they tested. Discuss whether these inputs had the same output for both expressions. Consider asking, "Would the outputs for the equivalent expressions be the same regardless of the input chosen?"

Invite students to check their responses with a partner. Encourage them to discuss and resolve any differences.

Activity 1 continued >



Activity 1

Name: Date: Period:

Equivalent Expressions

3. Expressions that give the same output for every input are called **equivalent expressions**.

Are these equivalent expressions? Explain how you know.

Yes. *Explanations vary.* If I expand $3(4x - 5)$, I get $12x - 15$. These expressions are equivalent, so they will give equal outputs for any input.

4. a. Which expression is equivalent to $2(10 - x)$?

A. $20 - x$

B. $20 - 2x$

C. $2(x - 10)$

D. $2x - 20$

b. Choose one input to test your expression.

Responses vary.

5. Write an equivalent expression to $-5(8 - 2x)$.

Responses vary. $-40 + 10x$

Unit 6 Lesson 8 443 Activity 1

Pairs | 15 min

Activity 1 Equivalent Expressions (continued)

Purpose: Students write **equivalent expressions** using factoring, expanding, and reordering terms. They use the **MLR3: Critique, Correct, Clarify** routine to critique common mistakes. **(MTR.4.1)**

2 Monitor

Look for students describing these common mistakes in ways that might support the discussion during the Connect. **(MTR.4.1)**

Capture several correct and incorrect student explanations to highlight during the Connect.

D Differentiation

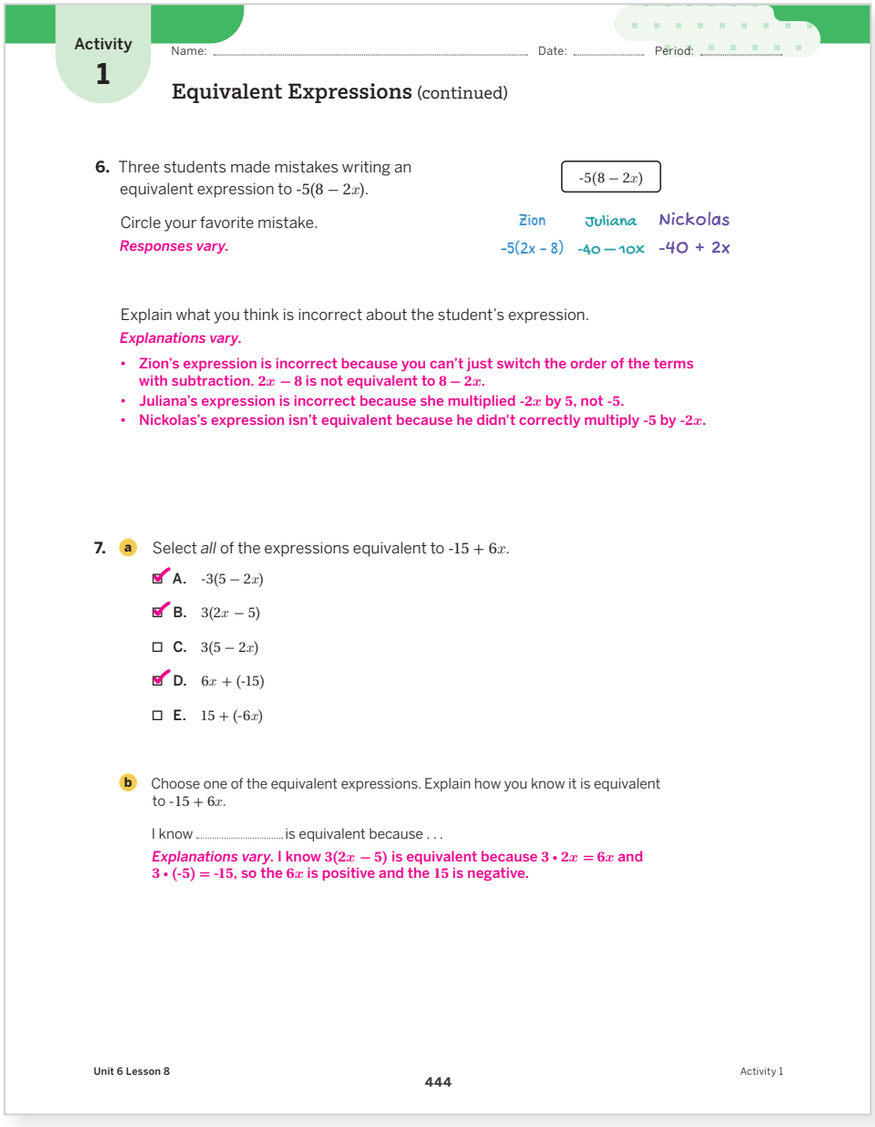
Look for students who:	Respond to Student Thinking
Have the correct values but incorrect signs (e.g., $15 + (-6x)$).	**Support** Invite students to share what they think the negative sign is connected to.
Test values for x to help them decide if expressions are equivalent.	**Strengthen** Consider asking, "How might you show that these are equivalent for all values of x?"
Create equivalent expressions.	**Stretch** Consider asking, "How many different equivalent expressions can you write? How could you determine which expression was most useful?"
Use language such as *inputs, outputs, factoring,* or *expanding* as they discuss.	

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Zion's, Juliana's, and Nickolas's work. As a class:

- Discuss what is *correct* about their responses.
- Discuss what is *incorrect* or *unclear.*
- Create *correct and clear* solutions.

To surface the Key Takeaway, consider highlighting a few student responses that use the terms *factoring*, *expanding*, and *reordering*, either formally or informally.



Activity 1

Name: Date: Period:

Equivalent Expressions (continued)

6. Three students made mistakes writing an equivalent expression to $-5(8 - 2x)$.

$-5(8 - 2x)$

Circle your favorite mistake.

Responses vary.

Zion	Juliana	Nickolas
$-5(2x - 8)$	$-40 - 10x$	$-40 + 2x$

Explain what you think is incorrect about the student's expression.

Explanations vary.

- Zion's expression is incorrect because you can't just switch the order of the terms with subtraction. $2x - 8$ is not equivalent to $8 - 2x$.
- Juliana's expression is incorrect because she multiplied $-2x$ by 5, not -5.
- Nickolas's expression isn't equivalent because he didn't correctly multiply -5 by $-2x$.

7. a. Select *all* of the expressions equivalent to $-15 + 6x$.

- ☑ A. $-3(5 - 2x)$
- ☑ B. $3(2x - 5)$
- ☐ C. $3(5 - 2x)$
- ☑ D. $6x + (-15)$
- ☐ E. $15 + (-6x)$

b. Choose one of the equivalent expressions. Explain how you know it is equivalent to $-15 + 6x$.

I know ..... is equivalent because . . .

Explanations vary. I know $3(2x - 5)$ is equivalent because $3 \cdot 2x = 6x$ and $3 \cdot (-5) = -15$, so the $6x$ is positive and the 15 is negative.

Unit 6 Lesson 8 444 Activity 1

Key Takeaway: **Equivalent expressions** can be generated by factoring, expanding, or reordering terms.

Pairs | 15 min

Activity 2 More Than One Way

Purpose: Students write multiple equivalent expressions using factoring, expanding, and reordering terms. **(MTR.2.1)**

Short on time? Consider having a short discussion about Problem 9 and omitting Problems 10 and 11.

1 Launch

Invite students to record their thinking and to attend to precision as they sort the expressions.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the expressions.

Note: If time allows, invite pairs to compare their sorted expressions, justify their placement, and make revisions based on their conversation.

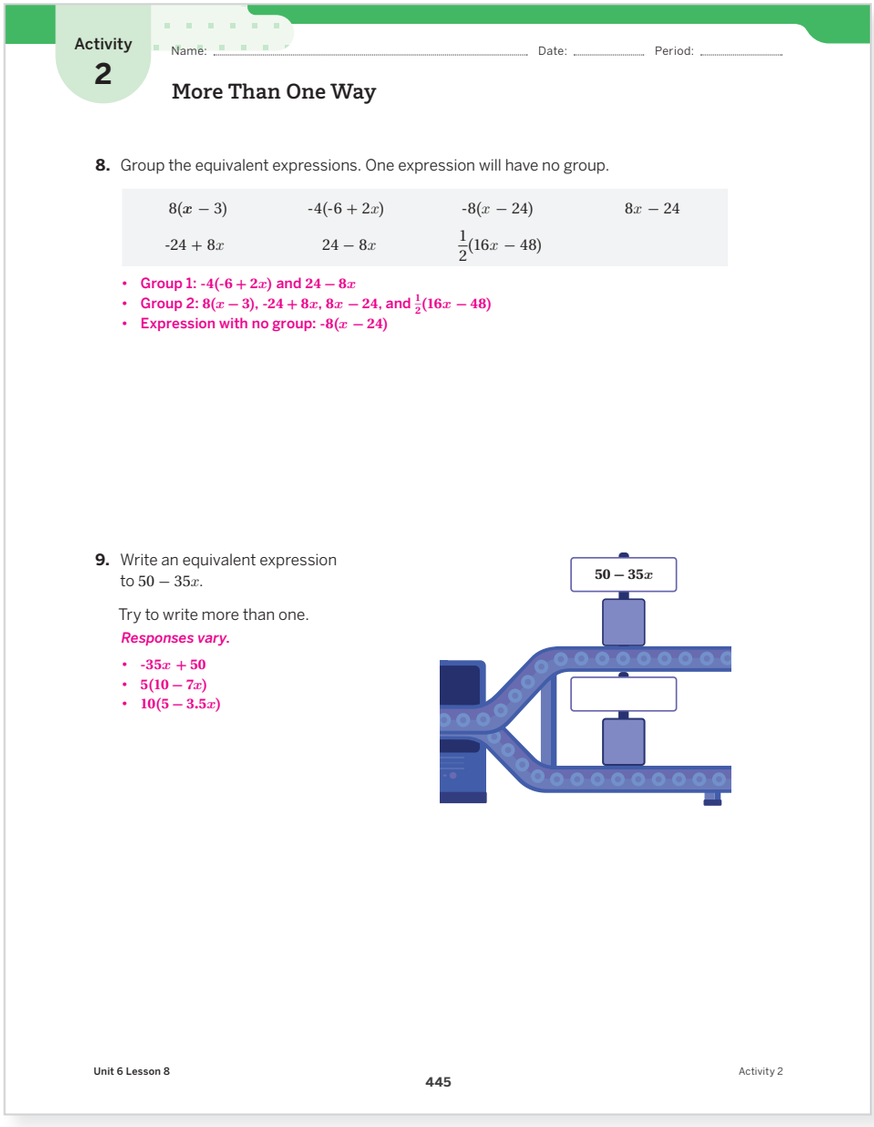
Invite students to get feedback from a partner.

Invite students to share their expressions to help the class generate many equivalent expressions. **(MTR.2.1)**

Create a record of a few equivalent expressions for students to reference throughout the remainder of the lesson.

Listen for students who mix up "infinitely many solutions" with "every solution works."

Activity 2 continued >



Activity 2

Name: Date: Period:

More Than One Way

8. Group the equivalent expressions. One expression will have no group.

$8(x-3)$	$-4(-6+2x)$	$-8(x-24)$	$8x-24$
$-24+8x$	$24-8x$	$\frac{1}{2}(16x-48)$	

- Group 1: $-4(-6+2x)$ and $24-8x$
- Group 2: $8(x-3)$, $-24+8x$, $8x-24$, and $\frac{1}{2}(16x-48)$
- Expression with no group: $-8(x-24)$

9. Write an equivalent expression to $50-35x$.

Try to write more than one.

Responses vary.

- $-35x+50$
- $5(10-7x)$
- $10(5-3.5x)$

Unit 6 Lesson 8 445 Activity 2

Pairs | 15 min

Activity 2 More Than One Way (continued)

Purpose: Students write multiple equivalent expressions using factoring, expanding, and reordering terms. **(MTR.2.1)**

Short on time? Consider having a short discussion about Problem 9 and omitting Problems 10 and 11.

2 Monitor

Select and sequence student strategies to discuss during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Rearrange without attending to signs (e.g., $16 - 64x$).	**Support** Encourage students to consider what the subtraction sign means and what term it connects with. Also consider substituting for x and testing if the expressions are equivalent.
Multiply by an integer (e.g., $128x - 32$).	**Strengthen** Invite students to multiply by different values (including non-integers).
Factor an integer (e.g., $4(16x - 4)$).	**Strengthen** Consider asking, "Are there other ways to factor this expression?"
Use a combination of strategies from above. E.g., factor and reorder $2(-8 + 32x)$.	**Stretch** Challenge students to imagine why different representations would be helpful. **(MTR.2.1)**

A Accessibility: Conceptual Processing Check in with individual students at each step of the activity to check for understanding.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore equivalent expressions further to complete this problem. Encourage them to discuss their thinking with a partner.

3 Connect

Display the captured equivalent and not equivalent expressions. Consider opening with the common mistake of rearranging without attention to signs.

Consider asking:

- "Is this expression equivalent? How do you know?"
- "What strategy might someone have used to create this expression?"

Activity 2

Name: Date: Period:

More Than One Way (continued)

10. Write at least three different equivalent expressions to $64x - 16$.

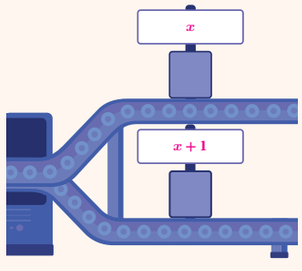
Responses vary.

- $-16 + 64x$
- $-1(-64x + 16)$
- $16(4x - 1)$

You're invited to explore more.

11. Make a never-equal machine that will never give the same output for any input. Write your expressions on the two machines.

Responses vary. Sample shown in the number machine.



Explain how you know these are never equal.

Explanations vary. These are never equal because one machine will always output a value that is one greater than the other machine.

Unit 6 Lesson 8 446 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for determining whether two expressions are equivalent.

Synthesis

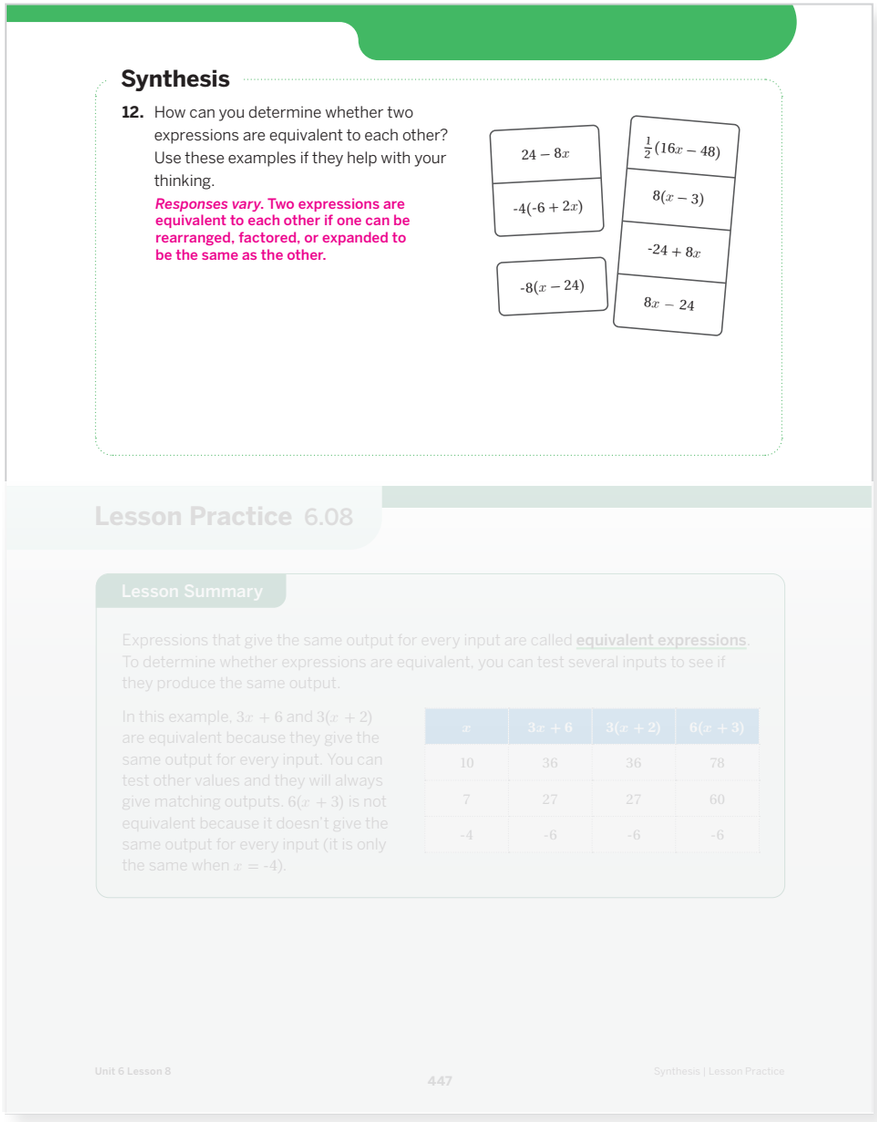
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Expressions that are factored or expanded are equivalent.
- Expressions that are rearranged are equivalent (as long as we attend to negative signs).

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *equivalent expressions* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. How can you determine whether two expressions are equivalent to each other? Use these examples if they help with your thinking.

Responses vary. Two expressions are equivalent to each other if one can be rearranged, factored, or expanded to be the same as the other.

$24 - 8x$

$-4(-6 + 2x)$

$-8(x - 24)$

$\frac{1}{2}(16x - 48)$

$8(x - 3)$

$-24 + 8x$

$8x - 24$

Lesson Practice 6.08

Lesson Summary

Expressions that give the same output for every input are called equivalent expressions. To determine whether expressions are equivalent, you can test several inputs to see if they produce the same output.

In this example, $3x + 6$ and $3(x + 2)$ are equivalent because they give the same output for every input. You can test other values and they will always give matching outputs. $6(x + 3)$ is not equivalent because it doesn't give the same output for every input (it is only the same when $x = -4$).

x	$3x + 6$	$3(x + 2)$	$6(x + 3)$
10	36	36	78
7	27	27	60
-4	-6	-6	-6

Unit 6 Lesson 8 447 Synthesis | Lesson Practice

Lesson Takeaway: Two expressions are equivalent to each other if one can be rearranged, factored, or expanded to be the same as the other.

Independent | 5 min

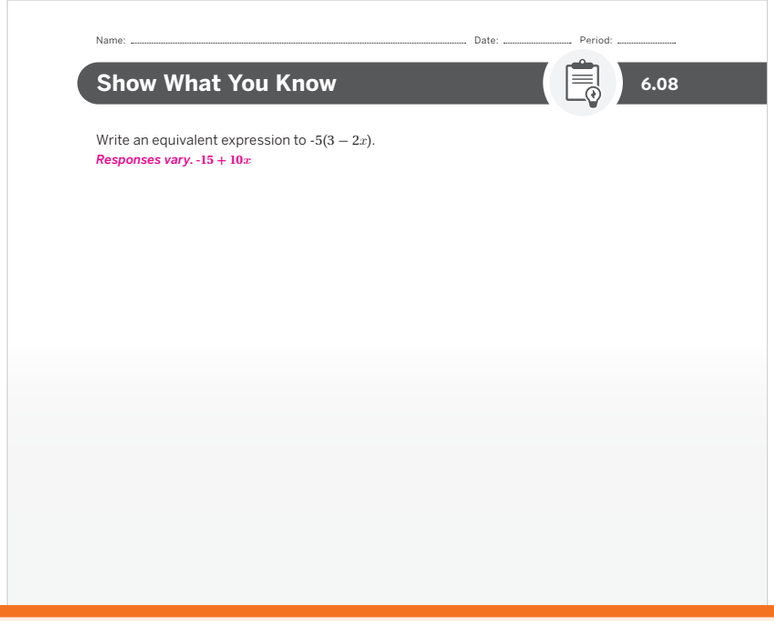
Show What You Know

Purpose: Students demonstrate their understanding by writing an equivalent expression.

Today's Goals

1. **Goal:** Write *equivalent expressions* that involve factoring, expanding, and reordering terms.
2. **Language Goal:** Justify whether factored, expanded, or reordered expressions are *equivalent*. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.08

Write an equivalent expression to $-5(3 - 2x)$.

Responses vary. $-15 + 10x$

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write equivalent expressions:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to complete the You're invited to **Explore More** task and discuss their thinking with a partner

Professional Learning

How did the **MLR3: Critique, Correct, Clarify** routine support students in justifying whether factored, expanded, and reordered expressions are equivalent?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can you determine whether two expressions are equivalent to each other? Use these examples if they help with your thinking.

Responses vary. Two expressions are equivalent to each other if one can be rearranged, factored, or expanded to be the same as the other.

Lesson Practice 6.08

Lesson Summary

Expressions that give the same output for every input are called **equivalent expressions**. To determine whether expressions are equivalent, you can test several inputs to see if they produce the same output.

In this example, $3x + 6$ and $3(x + 2)$ are equivalent because they give the same output for every input. You can test other values and they will always give matching outputs. $6(x + 3)$ is not equivalent because it doesn't give the same output for every input (it is only the same when $x = -4$).

x	$3x + 6$	$3(x + 2)$	$6(x + 3)$
10	36	36	78
7	27	27	60
-4	-6	-6	-6

Lesson Practice 6.08

Name: _____ Date: _____ Period: _____

1. Alejandro says that $10x + 6$ and $5x + 11$ are equivalent because they equal 16 when x is 1. Do you agree with Alejandro? Explain your thinking.

No. Explanations vary. Equivalent expressions are equal for any value of the variable. When x is 0, these are not equal.

2. Write at least three different expressions that are equivalent to $16a - 24$.

Responses vary.
- $-24 + 16a$
- $8(2a - 3)$
- $-2(-8a + 12)$

3. Write at least three different expressions that are equivalent to $-\frac{1}{2}(-12x + 30)$.

Responses vary.
- $6x - 15$
- $3(2x - 5)$
- $6x + (-15)$

Problems 4–7: Write an equivalent expression in expanded form.

4. $8\left(-x + \frac{1}{4}\right) = -8x + 2$ (or equivalent)

5. $-2(-6x - 1) = 12x + 2$ (or equivalent)

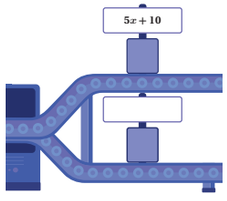
6. $\frac{1}{5}(20y - 13) = 4y - \frac{13}{5}$ (or equivalent)

7. $9\left(4x + 3y + \frac{2}{3}\right) = 36x + 27y + 6$ (or equivalent)

FAST Practice

8. Write an expression that will always be equal to $5x + 10$ and an expression that will never be equal. Use x as the variable in each expression.

Responses vary. $5(x + 2)$ will always be equal and $5(x + 10)$ will never be equal.



Lesson Practice 6.08

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 9–11: The tables show the energy output, in megawatt-hours (MWh), from a wind farm and a solar power plant.

Wind Farm

Energy (MWh)	Number of Days
1,200	2.4
1,800	3.6
4,000	8
10,000	20

9. Is the energy output of the wind farm proportional to the number of days?

Yes

10. Write an equation showing the relationship. Use E to represent energy and d to represent the number of days.

$E = 500d$ (or equivalent)

11. Is the energy output of the solar power plant proportional to the number of days? Explain your thinking.

No. Explanations vary. The relationship is not proportional because there is no constant of proportionality between the energy output and the number of days.

Solar Power Plant

Energy (MWh)	Number of Days
100	2.4
650	3.6
1,200	8
1,750	20

12. Match each expression to its value.

Expression	Value
a. $-30 \cdot (-10)$	b, c ... -40
b. $-10 + (-30)$	a ... 300
c. $-30 - 10$	d ... 40
d. $10 - (-30)$	

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	2	MA.7.AR.1.2
FAST Practice	8	2	MA.7.AR.1.2
Spiral Review			
	9, 11	2	MA.7.AR.4.1
	10	2	MA.7.AR.4.2 MA.7.AR.4.4
	12	1	MA.7.NSO.2.2

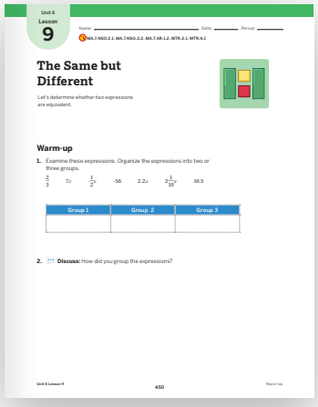
Unit 6

Lesson 9

The Same but Different

Determining Whether Expressions Are Equivalent

Let's determine whether two expressions are equivalent.



Learning Outcomes

Today's Goals

1. **Goal:** Determine whether two linear expressions in any form with rational coefficients are equivalent.
2. **Language Goal:** Describe the steps used to simplify expressions. **(Reading, Writing)**

Students extend their knowledge of simplifying expressions to determine whether two linear expressions are equivalent. They use manipulatives such as algebra tiles and area models to visualize whether expressions are the same. They also compare more complex expressions by using the properties of operations to simplify them.

Coherence

Prior Learning

In Lesson 8, students wrote equivalent expressions that involved factoring, expanding, and reordering terms.

Future Learning

In Lesson 10, students will write equivalent expressions with fewer terms.

Connections

- Students determine if two expressions are equivalent using algebra tiles. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

Also Addressing: **MA.7.NSO.2.2**

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.1** **MA.8.AR.1.3**

Vocabulary

Review Vocabulary

constant *variable*

Additional Vocabulary

like terms

Lesson at a Glance

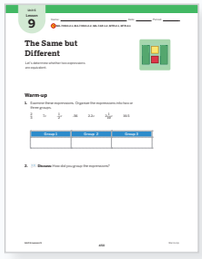
~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2

Warm-Up

Pairs | 5 min

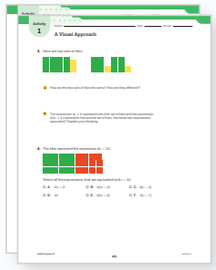
Students sort expressions into groups using any criteria they choose.



Activity 1

Pairs | 15 min

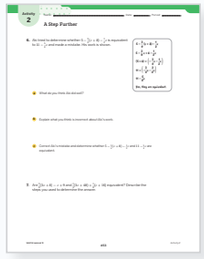
Students use algebra tiles and area models to explore equivalent expressions.
Routine: Think-Pair-Share **(MTR.2.1)**



Activity 2

Pairs | 15 min

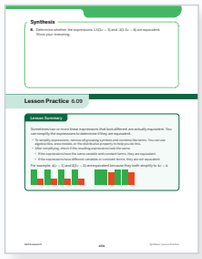
Students use the properties of operations to simplify expressions and determine equivalence. MLR3: Critique, Correct, Clarify **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of equivalent expressions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether two linear expressions are equivalent.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students sort expressions into groups using any criteria they choose.

1 Launch

Direct students' attention to the expressions. Consider sharing that there are many different ways to sort the expressions and that the purpose of this sort is to think about differences among the expressions and to notice what they have in common.

2 Connect

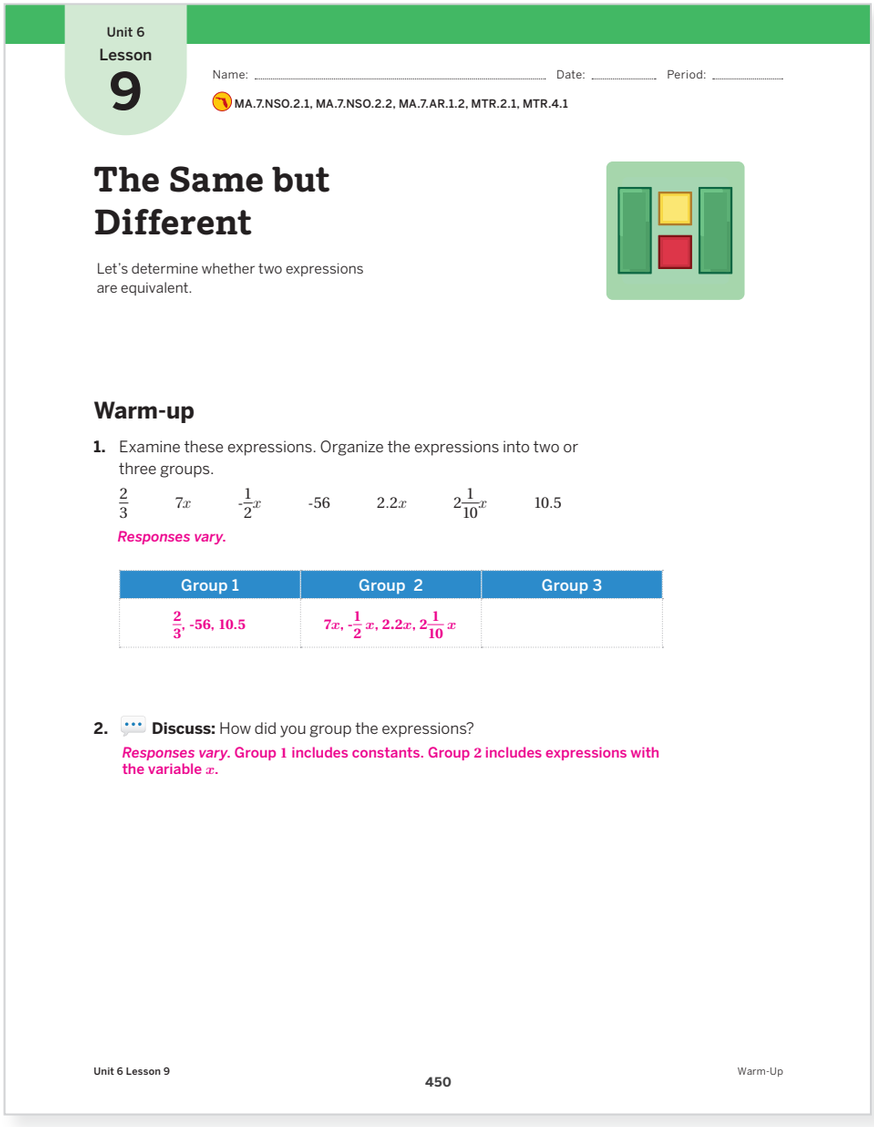
Pause and invite students to share the groups they made.

Create a record of the way students grouped the terms, along with the name of the students who shared them.

Look for and celebrate the different ways that students group the expressions, such as:

- *Constants* and expressions with *variables*.
- Expressions with whole numbers, fractions, and decimals.
- Positive and negative numbers/coefficients.
- Numbers/coefficients that are negative, between 0 and 1, and greater than 1.

ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share them with others. (Writing and Speaking)



Unit 6
Lesson
9

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2, MTR.2.1, MTR.4.1

The Same but Different

Let's determine whether two expressions are equivalent.

Warm-up

1. Examine these expressions. Organize the expressions into two or three groups.

$\frac{2}{3}$ $7x$ $-\frac{1}{2}x$ -56 $2.2x$ $2\frac{1}{10}x$ 10.5

Responses vary.

Group 1	Group 2	Group 3
$\frac{2}{3}$, -56, 10.5	$7x$, $-\frac{1}{2}x$, $2.2x$, $2\frac{1}{10}x$	

2. **Discuss:** How did you group the expressions?

Responses vary. Group 1 includes constants. Group 2 includes expressions with the variable x.

Unit 6 Lesson 9 450 Warm-Up

Pairs | 15 min

Activity 1 A Visual Approach

Purpose: Students use algebra tiles and area models to explore equivalent expressions. **(MTR.2.1)**

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about equivalent expressions. Encourage students to think independently and then take turns sharing with a partner.

To encourage movement, invite students to find a partner in a different location.

2 Monitor

Select and sequence responses to share during the Connect, such as students who:

- Use the algebra tiles to justify their answer.
- Use the properties of operations to justify their answer.

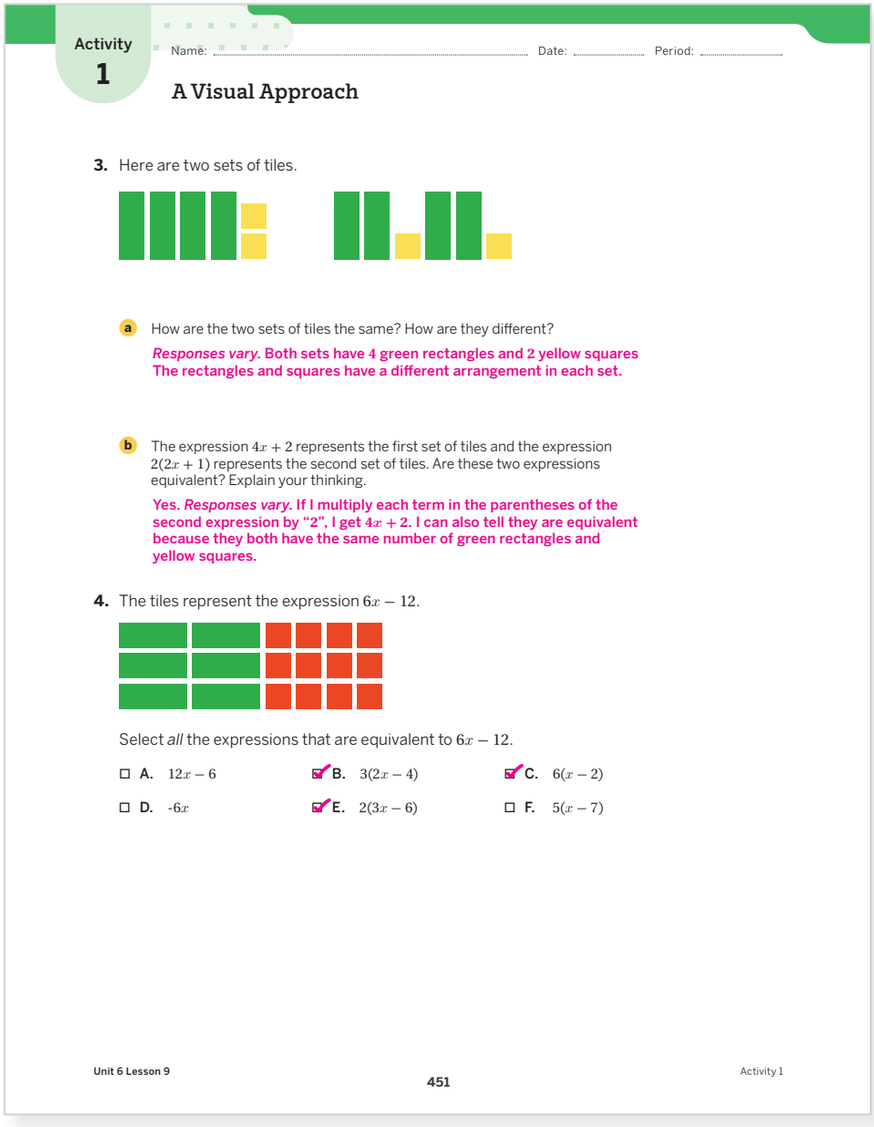
D Differentiation *(Problems 3–4)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 4)*	**Support** Encourage them to draw their own algebra tiles to model the options.
Use the properties of equality. *(Problem 3)*	**Strengthen** Invite them to name the properties of equality they are using.
Use Algebra tiles. *(Problem 3)*	**Strengthen** Consider asking, "How can you show this algebraically?"
Quickly identify the equivalent expressions. *(Problem 4)*	**Stretch** Challenge them to write a different equivalent expression. Hint: They can use fractions or decimals.

Listen for connections between visual manipulatives and the properties of operations.

A Accessibility: Visual-Spatial Processing Provide students with algebra tiles and encourage them to separate them into equal groups to identify equivalent expressions.

Activity 1 continued >



Activity 1

Name: Date: Period:

A Visual Approach

3. Here are two sets of tiles.

a. How are the two sets of tiles the same? How are they different?

Responses vary. Both sets have 4 green rectangles and 2 yellow squares The rectangles and squares have a different arrangement in each set.

b. The expression $4x + 2$ represents the first set of tiles and the expression $2(2x + 1)$ represents the second set of tiles. Are these two expressions equivalent? Explain your thinking.

Yes. *Responses vary.* If I multiply each term in the parentheses of the second expression by "2", I get $4x + 2$. I can also tell they are equivalent because they both have the same number of green rectangles and yellow squares.

4. The tiles represent the expression $6x - 12$.

Select *all* the expressions that are equivalent to $6x - 12$.

- ☐ A. $12x - 6$
- ☑ B. $3(2x - 4)$
- ☑ C. $6(x - 2)$
- ☐ D. $-6x$
- ☑ E. $2(3x - 6)$
- ☐ F. $5(x - 7)$

Unit 6 Lesson 9 451 Activity 1

Pairs | 15 min

Activity 1 A Visual Approach (continued)

Purpose: Students use algebra tiles and area models to explore equivalent expressions. **(MTR.2.1)**

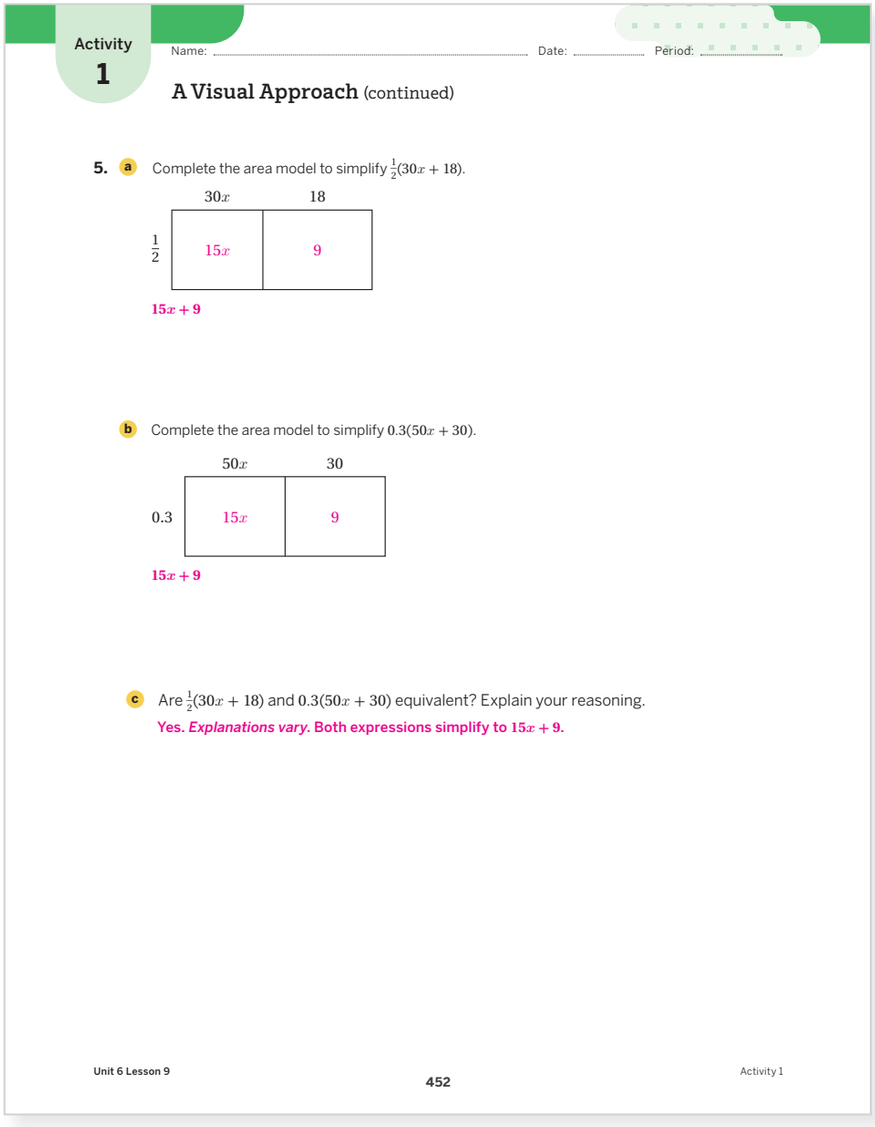
2 Monitor

To support students getting started, consider asking:

- "How do you calculate the area of a rectangle? How does that relate here?"
- "Does the fraction or decimal change the process?"

3 Connect

To surface the Key Takeaway, consider asking, "What can you say about all of the equivalent expressions in this activity?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

A Visual Approach (continued)

5. **a** Complete the area model to simplify $\frac{1}{2}(30x + 18)$.

	$30x$	18
$\frac{1}{2}$	$15x$	9

$15x + 9$

b Complete the area model to simplify $0.3(50x + 30)$.

	$50x$	30
0.3	$15x$	9

$15x + 9$

c Are $\frac{1}{2}(30x + 18)$ and $0.3(50x + 30)$ equivalent? Explain your reasoning.

Yes. *Explanations vary.* Both expressions simplify to $15x + 9$.

Unit 6 Lesson 9 452 Activity 1

Key Takeaway: Two expressions are equivalent if they simplify or expand to the same expression.

Pairs | 15 min

Activity 2 A Step Further

Purpose: Students use the properties of operations to simplify expressions and determine equivalence. **(MTR.4.1)**

Short on time? Consider discussing Problems 6a and 6b as a class.

1 Launch

To support students getting started, consider asking, "Why might Aki have used this method instead of using algebra tiles?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Aki's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., _____ is correct/incorrect. _____ did _____ correctly/incorrectly.). (Writing and Speaking)

2 Monitor

Look for students who list different things that Aki did well to have them share during the Connect.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one step of Aki's work at a time. To promote student focus, consider covering up later steps until students are ready to address them.

3 Connect

Invite students to discuss what they observed about Aki's work and how they corrected it.

To surface connections, consider asking, "What steps do you take to simplify an expression? Do fractions change how you do that?"

Invite several students to share their responses and justify their thinking.

D **Differentiation: (Support)** Provide students who need support with different-colored highlighters and encourage them to highlight constants in one color and x-terms in another color.

Note: If it does not come up naturally, consider highlighting that, for every expression in this activity, students applied the distributive property and then combined *like terms*.

Activity 2

Name: _____ Date: _____ Period: _____

A Step Further

6. Aki tried to determine whether $5 - \frac{3}{4}(x + 8) - \frac{1}{2}x$ is equivalent to $11 - \frac{5}{4}x$ and made a mistake. His work is shown.

$5 - \frac{3}{4}(x + 8) - \frac{1}{2}x$

$5 - \frac{3}{4}x + 6 - \frac{1}{2}x$

$(5 + 6) + \left(-\frac{3}{4}x - \frac{1}{2}x\right)$

$11 + \left(-\frac{3}{4}x - \frac{2}{4}x\right)$

$11 - \frac{5}{4}x$

Yes, they are equivalent.

a. What do you think Aki did well?

Responses vary. Aki correctly distributed the $-\frac{3}{4}$ to the x in the parentheses and then combined the variable terms correctly.

b. Explain what you think is incorrect about Aki's work.

Responses vary. Aki only distributed a negative to the first term in the parentheses, not the second, which made the 6 positive instead of negative. He should subtract 6 from 5 instead of adding.

c. Correct Aki's mistake and determine whether $5 - \frac{3}{4}(x + 8) - \frac{1}{2}x$ and $11 - \frac{5}{4}x$ are equivalent.

$5 - \frac{3}{4}x - 6 - \frac{1}{2}x$

$(5 - 6) + \left(-\frac{3}{4}x - \frac{1}{2}x\right)$

$-1 - \frac{5}{4}x$

No, they are not equivalent.

7. Are $\frac{5}{8}(3x + 8) - x + 9$ and $\frac{1}{4}(3x + 48) + \frac{1}{8}(x + 16)$ equivalent? Describe the steps you used to determine the answer.

Yes. *Responses vary.* I used the distributive property and combined like terms to find that they both simplify to $\frac{7}{8}x + 14$.

$\frac{5}{8}(3x + 8) - x + 9$

$\frac{15}{8}x + 5 - x + 9$

$\frac{15}{8}x - \frac{8}{8}x + 5 + 9$

$\frac{7}{8}x + 14$

$\frac{1}{4}(3x + 48) + \frac{1}{8}(x + 16)$

$\frac{3}{4}x + 12 + \frac{1}{8}x + 2$

$\frac{6}{8}x + \frac{1}{8}x + 12 + 2$

$\frac{7}{8}x + 14$

Unit 6 Lesson 9 453 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of equivalent expressions.

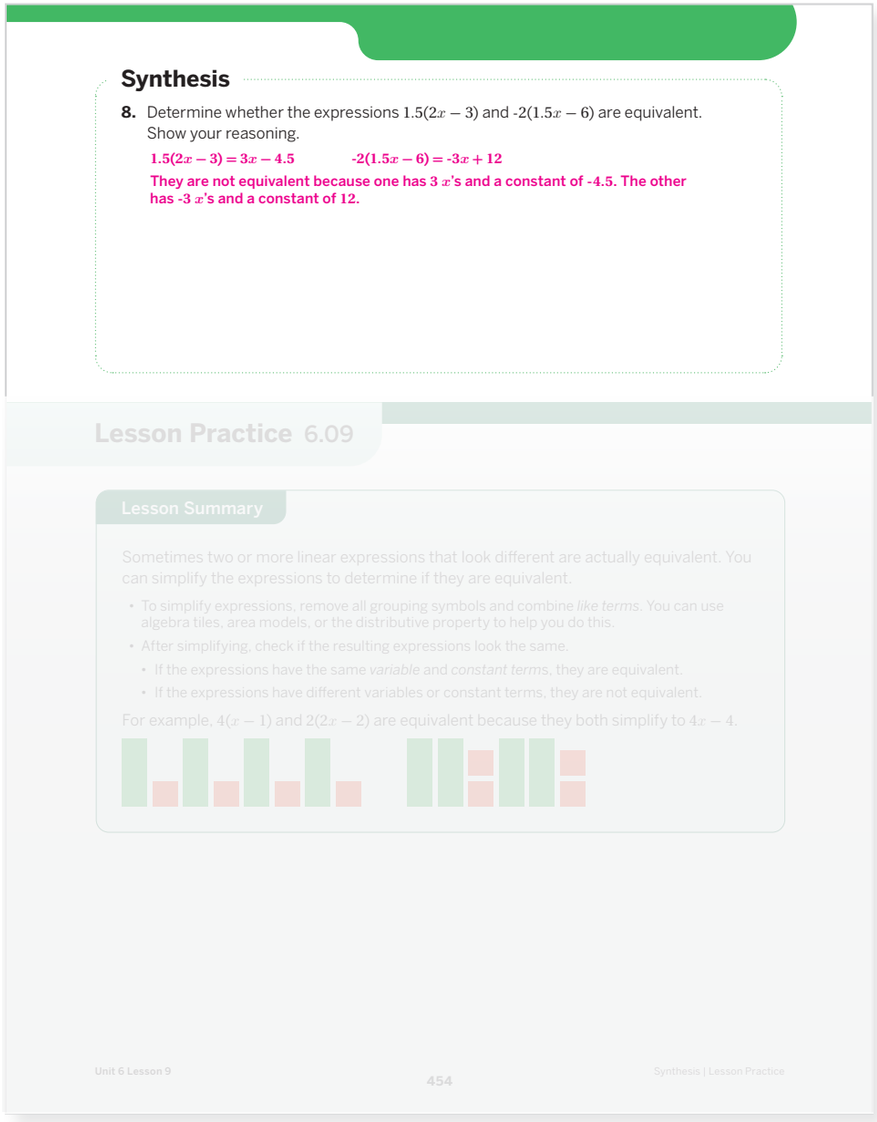
Invite students to respond independently, and then share their thinking with a partner

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with you because _____.
- I think that makes sense because _____.
- How did you know _____.

(Writing and Speaking)

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took earlier on determining whether expressions are equivalent to activate prior knowledge.



Synthesis

8. Determine whether the expressions $1.5(2x - 3)$ and $-2(1.5x - 6)$ are equivalent. Show your reasoning.

$1.5(2x - 3) = 3x - 4.5$ $\quad -2(1.5x - 6) = -3x + 12$

They are not equivalent because one has 3 x's and a constant of -4.5. The other has -3 x's and a constant of 12.

Lesson Practice 6.09

Lesson Summary

Sometimes two or more linear expressions that look different are actually equivalent. You can simplify the expressions to determine if they are equivalent.

- To simplify expressions, remove all grouping symbols and combine *like terms*. You can use algebra tiles, area models, or the distributive property to help you do this.
- After simplifying, check if the resulting expressions look the same.
  - If the expressions have the same *variable* and *constant terms*, they are equivalent.
  - If the expressions have different variables or constant terms, they are not equivalent.

For example, $4(x - 1)$ and $2(2x - 2)$ are equivalent because they both simplify to $4x - 4$.

Unit 6 Lesson 9 454 Synthesis | Lesson Practice

Lesson Takeaway: Expressions with rational numbers can be simplified using properties of operations to determine if they are equivalent.

Independent | 5 min

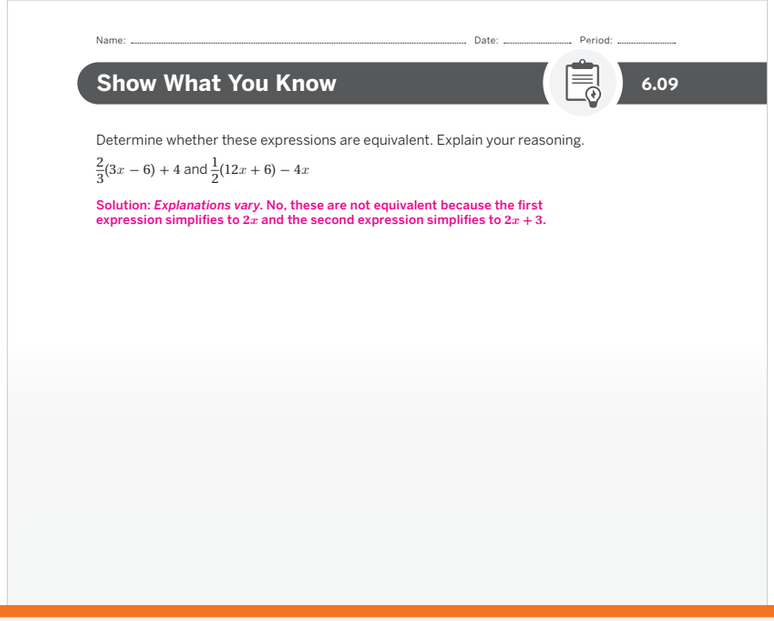
Show What You Know

Purpose: Students demonstrate their understanding by determining whether two linear expressions are equivalent.

Today's Goals

1. **Goal:** Determine whether two linear expressions in any form with rational coefficients are equivalent.
2. **Language Goal:** Describe the steps used to simplify expressions. (Reading, Writing)

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.09

Determine whether these expressions are equivalent. Explain your reasoning.

$\frac{2}{3}(3x - 6) + 4$ and $\frac{1}{2}(12x + 6) - 4x$

Solution: *Explanations vary.* No, these are not equivalent because the first expression simplifies to $2x$ and the second expression simplifies to $2x + 3$.

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine if two expressions are equivalent:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students used properties of operations to create and identify equivalent expressions. How will that work support solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Determine whether the expressions $1.5(2x - 3)$ and $-2(1.5x - 6)$ are equivalent. Show your reasoning.

$1.5(2x - 3) = 3x - 4.5$ $\quad -2(1.5x - 6) = -3x + 12$

They are not equivalent because one has 3 x's and a constant of -4.5. The other has -3 x's and a constant of 12.

Lesson Practice 6.09

Lesson Summary

Sometimes two or more linear expressions that look different are actually equivalent. You can simplify the expressions to determine if they are equivalent.

- To simplify expressions, remove all grouping symbols and combine *like terms*. You can use algebra tiles, area models, or the distributive property to help you do this.
- After simplifying, check if the resulting expressions look the same.
  - If the expressions have the same *variable* and *constant terms*, they are equivalent.
  - If the expressions have different variables or constant terms, they are not equivalent.

For example, $4(x - 1)$ and $2(2x - 2)$ are equivalent because they both simplify to $4x - 4$.



Lesson Practice 6.09

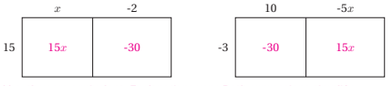
Name: _____ Date: _____ Period: _____

1. Which expression is equivalent to $13 + \frac{3}{5}(x - 10) + 4$?

A. $\frac{3}{5}x + 11$ $\quad$ B. $\frac{3}{5}x + 7$

C. $\frac{3}{5}x + 11$ $\quad$ D. $\frac{3}{5}x - 3$

2. Use area models to determine whether $15(x - 2)$ and $-3(10 - 5x)$ are equivalent. Explain your thinking.



Yes, they are equivalent. *Explanations vary.* Both expressions simplify to $15x - 30$.

3. Use the two sets of algebra tiles to write a pair of equivalent expressions.



The expressions $4(2x - 4)$ and $8(x - 2)$ are equivalent.

Lesson Practice 6.09

Name: _____ Date: _____ Period: _____

Problems 4–7: Determine whether each pair of expressions is equivalent. Explain your reasoning.

4. $2(1.4x - 3) + 3(2x + 1)$ and $4(0.7x - 1.5) + 6(x + 0.5)$

Yes, they are equivalent. They both simplify to $8.8x - 3$.

5. $2(3x + 5)$ and $6\left(x + \frac{5}{3}\right)$

Yes, they are equivalent. They both simplify to $6x + 10$.

6. $\frac{2}{3}(3x + 9) - 7$ and $\frac{3}{4}(8 + 12x) - 7x$

No, they are not equivalent. $\frac{2}{3}(3x + 9) - 7 = 2x - 1$ and $\frac{3}{4}(8 + 12x) - 7x = 6 + 2x$, which are not equal.

7. $\frac{1}{2}(6x + 4) + \frac{1}{3}(9x - 3)$ and $6x + 1$

Yes, they are equivalent. The first expression simplifies to $6x + 1$.

FAST Practice

8. Select all the expressions equivalent to $-7x + 4$.

- ☑ A. $-2(-2 + 3.5x)$
- ☐ B. $2(-3.5x - 2)$
- ☑ C. $7(1 - x) - 3$
- ☑ D. $-(2x + 5) - (5x - 9)$
- ☐ E. $-(6 - 7x) + 10$

Spiral Review

Problems 9–10: Evaluate each expression.

9. $5\frac{1}{4} \div \left(\frac{3}{8}\right)$

-14

10. $-6.75 + 4(1.8) - 8.2$

-7.75

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.AR.1.2
	3	3	MA.7.AR.1.2
	4–7	2	MA.7.AR.1.2
FAST Practice	8	2	MA.7.AR.1.2
Spiral Review			
	9–10	2	MA.7.NSO.2.2

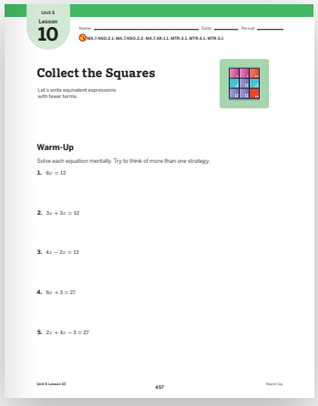
Unit 6

Lesson 10

Collect the Squares

Adding Expressions

Let's write equivalent expressions with fewer terms.


Unit 6
Lesson 10
Collect the Squares
Let's write equivalent expressions with fewer terms.
Warm-Up
Solve each equation mentally. Try to think of more than one strategy.
1. $4x = 12$
2. $3x + 3x = 12$
3. $4x - 2x = 12$
4. $6x + 3 = 27$
5. $2x + 4x - 3 = 27$
457

Learning Outcomes

Today's Goals

1. **Goal:** Write equivalent expressions with fewer terms by expanding, adding, and subtracting terms.
2. **Language Goal:** Explain how to write equivalent expressions using fewer terms. **(Speaking and Writing)**

Students explore the concept of a *term* and why writing an expression with fewer terms may be helpful for solving problems. This lesson will support students with "combining like terms" as they make use of structure to rewrite expressions with fewer terms by adding, subtracting, and expanding. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 8, students wrote multiple equivalent expressions by factoring, expanding, and reordering. In Grade 6, Unit 6, students used the distributive property to write equivalent expressions and described what it meant for two expressions to be equal.

Future Learning

In Lesson 11, students will begin solving more complex equations including adding and expanding expressions.

Connections

- Students use visual representaions to simplifying expressions by grouping together terms that have the same squared variable. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

MA.7.AR.1.1
Apply properties of operations to add and subtract linear expressions with rational coefficients.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.2**
	MA.8.AR.1.3

Vocabulary

Review Vocabulary

term

Lesson at a Glance

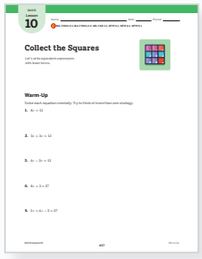
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MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1

Warm-Up

Whole Class | 10 min

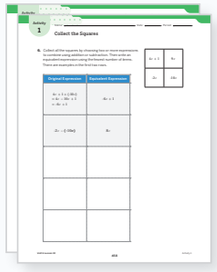
Students use the **Number Talk** routine to make use of structure, which creates a need for writing equivalent expressions using fewer *terms*. **(MTR.5.1)**



Activity 1

Pairs | 10 min

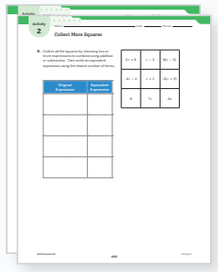
Students practice writing equivalent expressions with fewer terms by expanding, adding, and subtracting terms, addressing common errors throughout. **MLR3: Critique, Correct, Clarify**



Activity 2

Pairs | 15 min

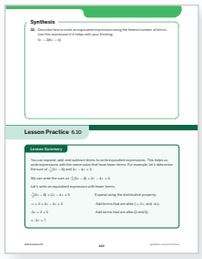
Students develop fluency by selecting expressions to combine and rewriting them with fewer terms. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to write an expression using fewer terms.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing expressions with the fewest number of terms.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 10 min

Warm-Up

Purpose: Students use the **Number Talk** routine to make use of structure, which creates a need for writing equivalent expressions using fewer *terms*. **(MTR.5.1)**

1 Launch

Use the Number Talk routine to support students in making use of structure to write equivalent expressions with fewer terms. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

Invite students to solve the first equation independently. Then record a variety of strategies, along with the name of the students who shared them.

Consider asking, "Who thought about it in a different way?"

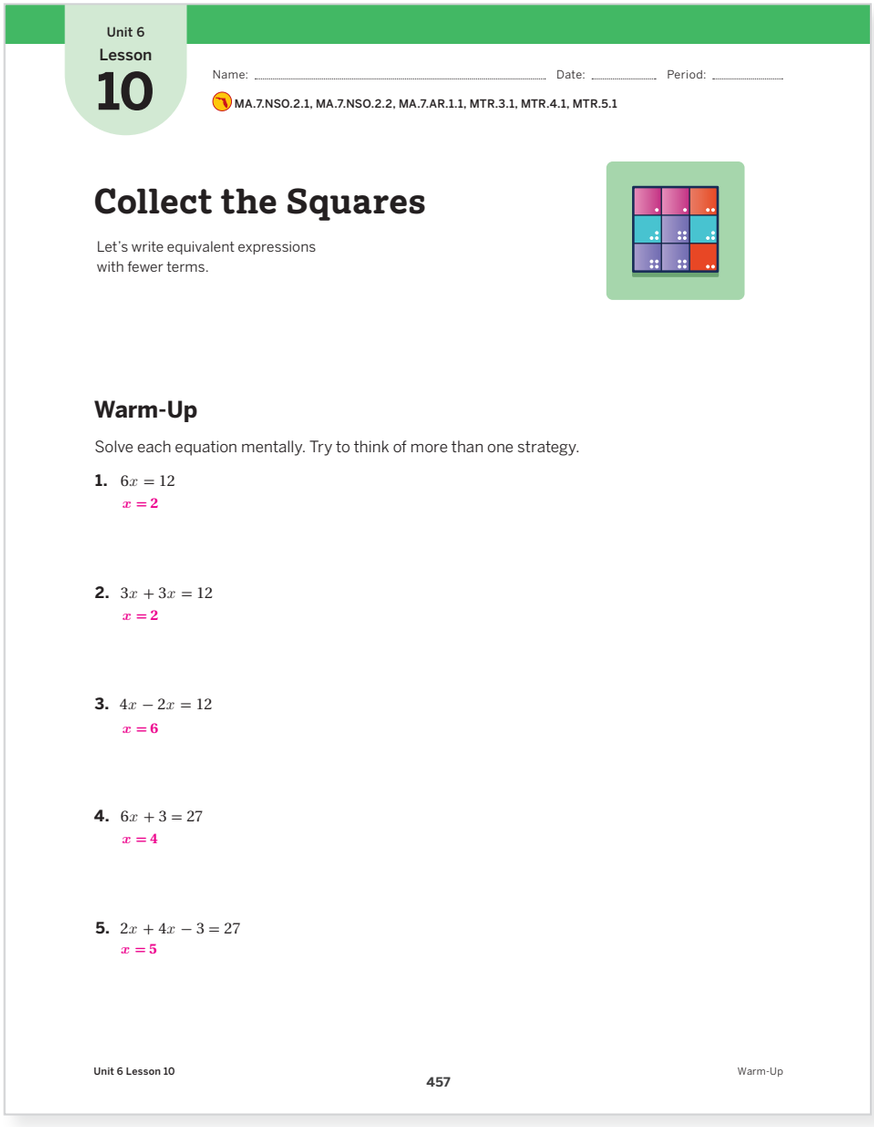
Note: This structure is repeated for each problem in the Number Talk.

2 Connect

Consider asking:

- "Why do the first three equations all have the same solution?"
- "Why might writing an expression with fewer terms help you to solve it?"

Consider sharing that the second expression has two *terms* if it does not come up naturally. Invite students to share what they think *term* means, celebrating multiple interpretations.



Unit 6
Lesson
10

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1, MTR.3.1, MTR.4.1, MTR.5.1

Collect the Squares

Let's write equivalent expressions with fewer terms.

Warm-Up

Solve each equation mentally. Try to think of more than one strategy.

1. $6x = 12$
   $x = 2$
2. $3x + 3x = 12$
   $x = 2$
3. $4x - 2x = 12$
   $x = 6$
4. $6x + 3 = 27$
   $x = 4$
5. $2x + 4x - 3 = 27$
   $x = 5$

Unit 6 Lesson 10 457 Warm-Up

Key Takeaway: Writing expressions with fewer *terms* can be helpful when solving equations.

Pairs | 10 min

Activity 1 Collect the Squares

Purpose: Students practice writing equivalent expressions with fewer terms by expanding, adding, and subtracting terms, addressing common errors throughout.

1 Launch

Direct students' attention to the examples in the table. Consider asking, "How do you combine expressions with addition and subtraction?"

Discuss the example $4x + 1 + (\text{-}10x)$, emphasizing why 1 cannot be combined with the other terms.

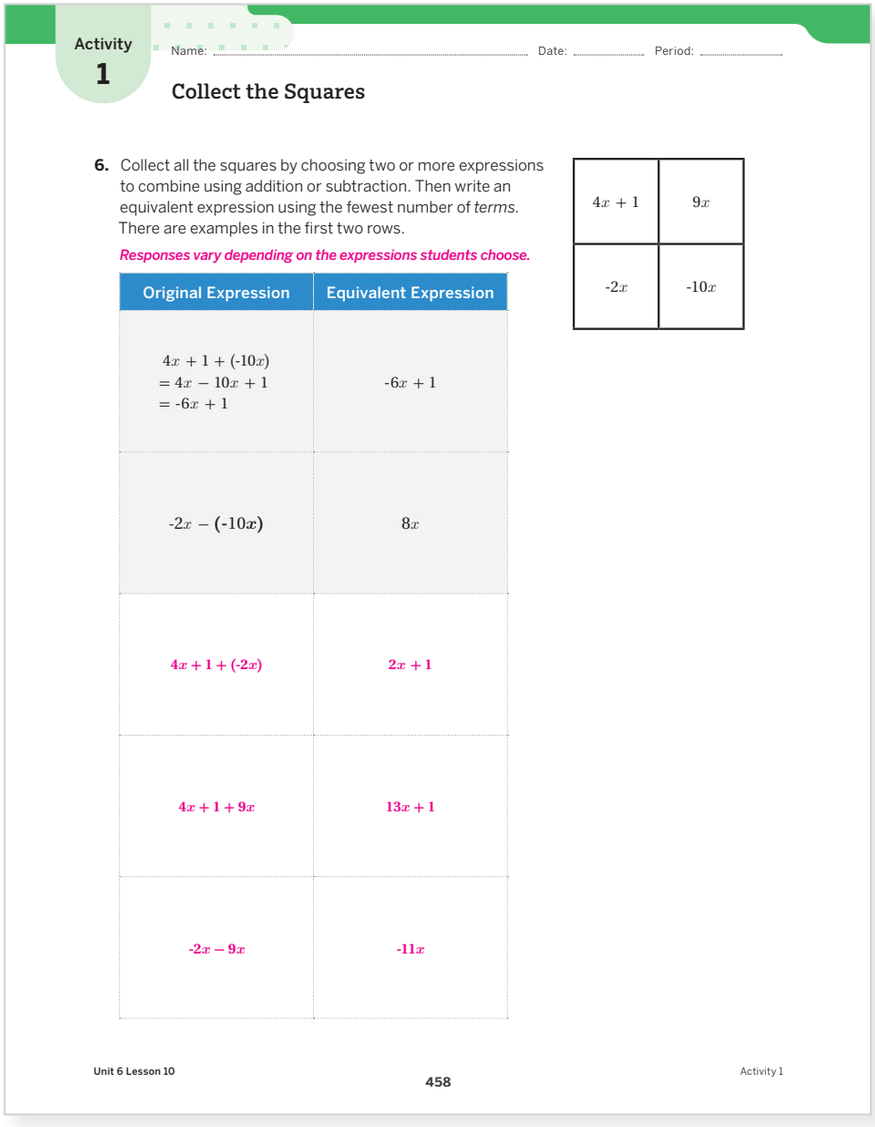
Encourage students to record the expressions they picked and make their thinking visible as they rewrite them.

2 Monitor

Invite students to share their thinking with a partner and give feedback on each other's expressions.

Capture several combinations and equivalent expressions to share during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Collect the Squares

6. Collect all the squares by choosing two or more expressions to combine using addition or subtraction. Then write an equivalent expression using the fewest number of *terms*. There are examples in the first two rows.

$4x + 1$	$9x$
$\text{-}2x$	$\text{-}10x$

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$4x + 1 + (\text{-}10x)$ $= 4x - 10x + 1$ $= \text{-}6x + 1$	$\text{-}6x + 1$
$\text{-}2x - (\text{-}10x)$	$8x$
$4x + 1 + (\text{-}2x)$	$2x + 1$
$4x + 1 + 9x$	$13x + 1$
$\text{-}2x - 9x$	$\text{-}11x$

Unit 6 Lesson 10 458 Activity 1

Pairs | 10 min

Activity 1 Collect the Squares (continued)

Purpose: Students practice writing equivalent expressions with fewer terms by expanding, adding, and subtracting terms, addressing common errors throughout.

2 Monitor

Select and sequence a variety of correct and incorrect ideas to highlight during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to share what they recognize from the expression and what feels confusing.
Add before distributing (e.g., $-3 + 3(x + 1) = 0(x + 1)$).	**Support** Consider asking, "What does it mean for a term to be on the outside of parentheses? Why might it be important to expand first?"
Add terms with a variable and terms without one (e.g., $2x + 8 = 10x$).	**Support** Invite them to share what $2x$ means and how that differs from 8 and that they cannot be combined because we don't yet know what x is. Note: This error is similar to Caleb's error. Consider capturing.

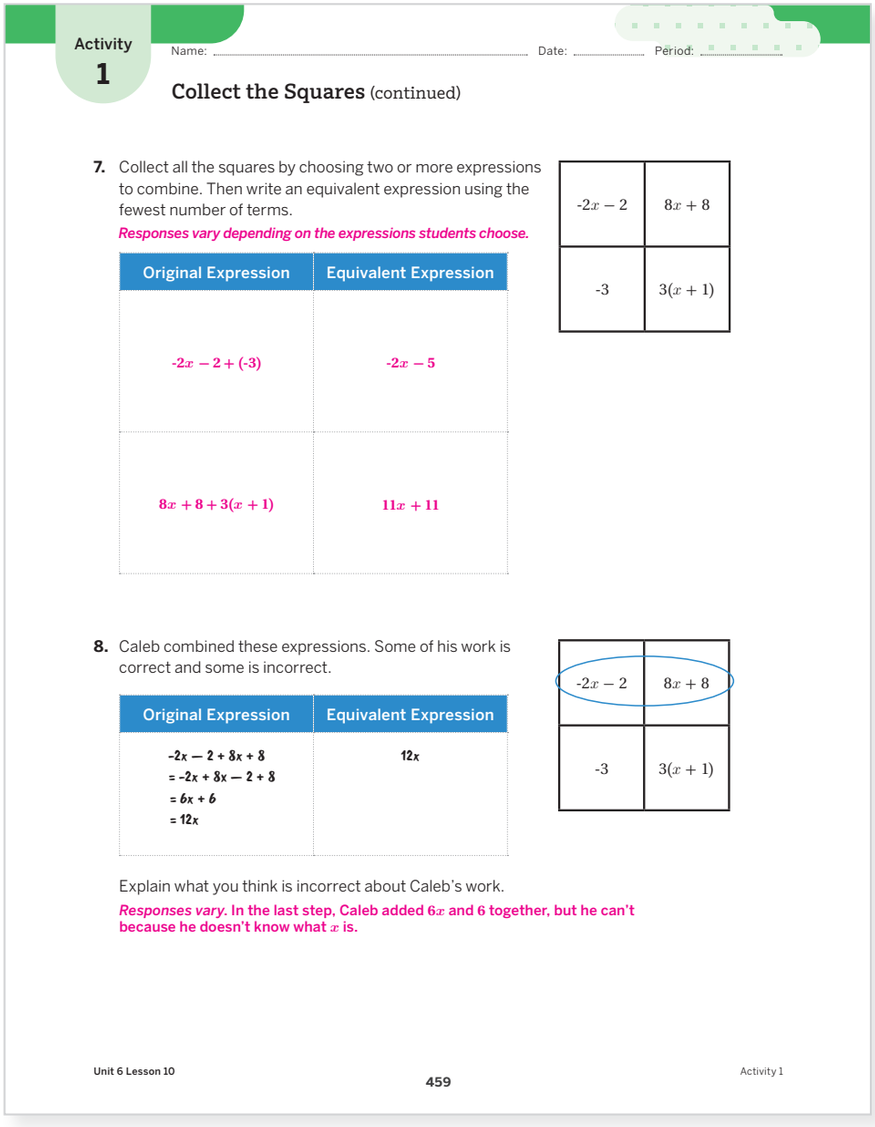
Listen for two or more explanations of Caleb's error that amplify why the sum of $6x$ and 6 is not $12x$ (e.g., without knowing the value of x, the structure of the terms is not the same).

D Differentiation: (Stretch) Invite students to add all of the squares at once. Consider asking, "What is a fifth expression that we could add to make the total sum equal to zero?"

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Caleb's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider inviting students to share how they would describe terms that can be combined.



Activity 1

Name: Date: Period:

Collect the Squares (continued)

7. Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$-2x - 2 + (-3)$	$-2x - 5$
$8x + 8 + 3(x + 1)$	$11x + 11$

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

8. Caleb combined these expressions. Some of his work is correct and some is incorrect.

Original Expression	Equivalent Expression
$-2x - 2 + 8x + 8$ $= -2x + 8x - 2 + 8$ $= 6x + 6$ $= 12x$	$12x$

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

Explain what you think is incorrect about Caleb's work.

Responses vary. In the last step, Caleb added $6x$ and 6 together, but he can't because he doesn't know what x is.

Unit 6 Lesson 10 459 Activity 1

Key Takeaway: It can be helpful to think about which *terms* can be combined and which cannot when writing equivalent expressions.

Pairs | 15 min

Activity 2 Collect More Squares

Purpose: Students develop fluency by selecting expressions to combine and rewriting them with fewer terms. **(MTR.3.1)**

Short on time? Consider inviting students to choose either Problem 9 or 10 to complete.

1 Launch

To surface connections to Activity 1, ask:

- "Which expression stands out to you? Which other expression do you think would be least difficult to add to that one? The most difficult? Why?"
- "Which strategies from earlier might help us combine some of these expressions?" **(MTR.3.1)**

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., asking students to collect one row or two squares).

2 Monitor

Look for trends in the squares that students select first. Consider asking, "Why did you choose to combine those two expressions first?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Are unsure what the coefficient is in a term like x or $-x$ (e.g., $8 - x$).	**Support** Invite students to write the coefficient in front of the x (1 or -1).
Avoid certain expressions.	**Support** Encourage students to reflect on the expressions they are avoiding and discuss that expression with their partner.
Strategically select expressions with the same number of terms.	**Strengthen** Consider asking: "Why might it be helpful to select expressions with the same amount of terms? What should we do with expressions with parentheses?" Consider capturing a few of these responses for the Connect.
Use addition or subtraction to collect terms (e.g., $7 - 8x$ or $7 + (-8x)$).	**Strengthen** Consider asking, "Do both of these produce equivalent expressions? Why?" Consider capturing one of these responses for the Connect.

Activity 2 continued >

Activity 2

Name: Date: Period:

Collect More Squares

9. Collect all the squares by choosing two or more expressions to combine using addition or subtraction. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$3x + 6 + x - 2$	$4x + 4$
$-4x - 4 + 6 + x + 1$	$-3x + 3$
$8(x - 3) + (-8x)$	-24
$-2(x + 2) - 7x$	$-9x - 4$

$3x + 6$	$x - 2$	$8(x - 3)$
$-4x - 4$	$x + 1$	$-2(x + 2)$
6	$7x$	$-8x$

Unit 6 Lesson 10 460 Activity 2

Pairs | 15 min

Activity 2 Collect More Squares (continued)

Purpose: Students develop fluency by selecting expressions to combine and rewriting them with fewer terms. **(MTR.3.1)**

Short on time? Consider inviting students to choose either Problem 9 or 10 to complete.

2 Monitor

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore expressions further to complete this optional task. Encourage them to discuss their thinking with a partner.

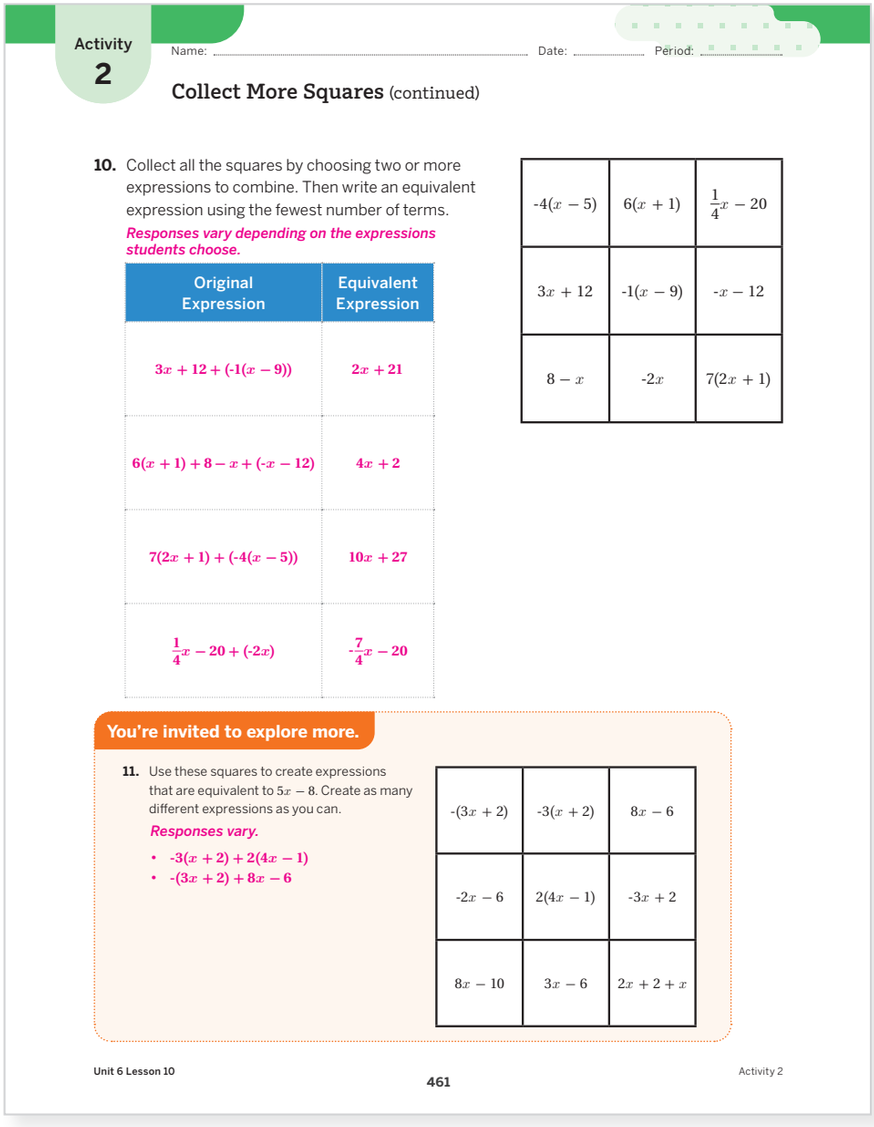
3 Connect

Display a collection of responses.

Consider asking:

- "Which expressions did you start with? Why?"
- "Which expressions did you leave until the end? Why?"
- "What happens when we select an expression with a negative sign in front?"
- "What are the strategies that you found helpful?"
- "How would we know if we have written an expression with the fewest possible terms?"

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit.



Activity 2

Name: Date: Period:

Collect More Squares (continued)

10. Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$3x + 12 + (-1(x - 9))$	$2x + 21$
$6(x + 1) + 8 - x + (-x - 12)$	$4x + 2$
$7(2x + 1) + (-4(x - 5))$	$10x + 27$
$\frac{1}{4}x - 20 + (-2x)$	$-\frac{7}{4}x - 20$

$-4(x - 5)$	$6(x + 1)$	$\frac{1}{4}x - 20$
$3x + 12$	$-1(x - 9)$	$-x - 12$
$8 - x$	$-2x$	$7(2x + 1)$

You're invited to explore more.

11. Use these squares to create expressions that are equivalent to $5x - 8$. Create as many different expressions as you can.

Responses vary.

- $-3(x + 2) + 2(4x - 1)$
- $-(3x + 2) + 8x - 6$

$-(3x + 2)$	$-3(x + 2)$	$8x - 6$
$-2x - 6$	$2(4x - 1)$	$-3x + 2$
$8x - 10$	$3x - 6$	$2x + 2 + x$

Unit 6 Lesson 10 461 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to write an expression using fewer terms.

Synthesis

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., First, _____. Then, _____.). **(Speaking and Listening)**

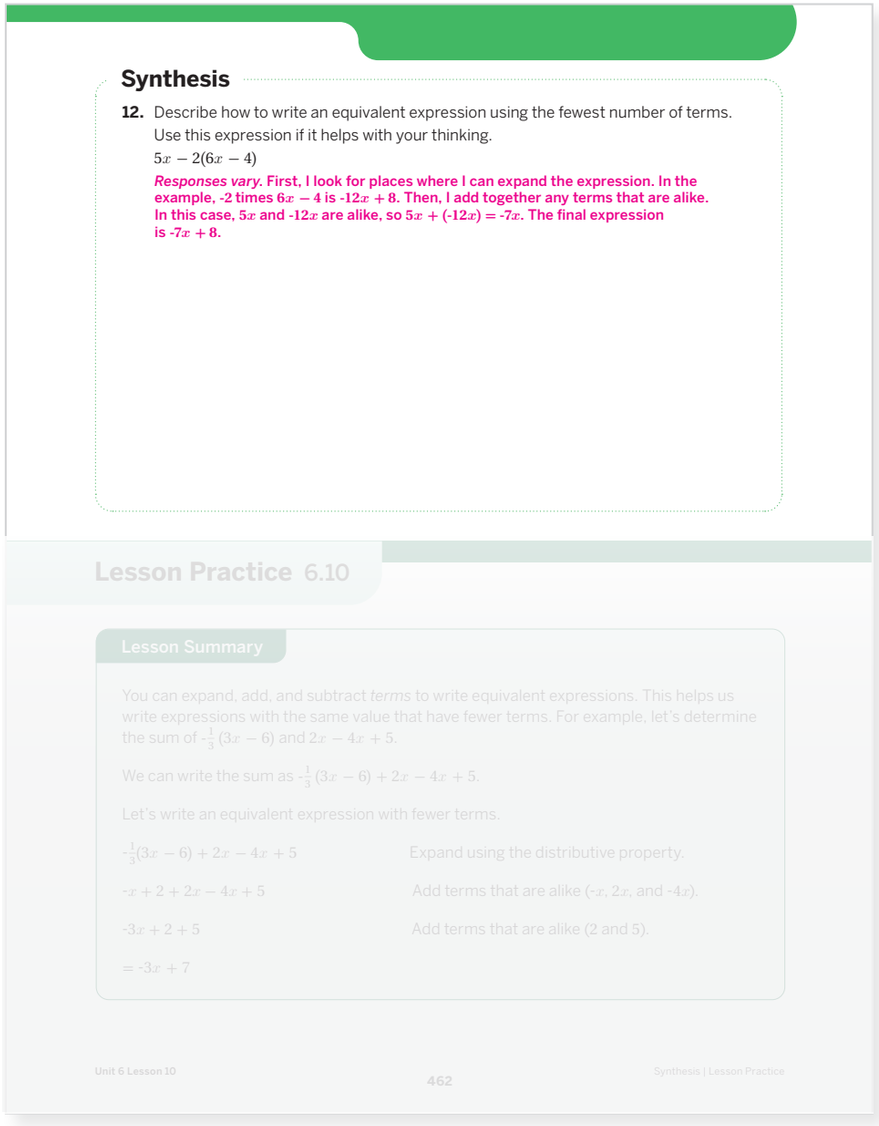
Collect and share a variety of ideas, such as:

- Expanding the expression first.
- Analyzing the structure of each of the terms to decide if they can be combined.

Consider asking:

- "What are some ways we could rewrite this expression using expanding, adding, or subtracting?"
- "What are some common mistakes someone might make on this problem?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Describe how to write an equivalent expression using the fewest number of terms. Use this expression if it helps with your thinking.
$5x - 2(6x - 4)$

Responses vary. First, I look for places where I can expand the expression. In the example, -2 times $6x - 4$ is $-12x + 8$. Then, I add together any terms that are alike. In this case, $5x$ and $-12x$ are alike, so $5x + (-12x) = -7x$. The final expression is $-7x + 8$.

Lesson Practice 6.10

Lesson Summary

You can expand, add, and subtract *terms* to write equivalent expressions. This helps us write expressions with the same value that have fewer terms. For example, let's determine the sum of $-\frac{1}{3}(3x - 6)$ and $2x - 4x + 5$.

We can write the sum as $-\frac{1}{3}(3x - 6) + 2x - 4x + 5$.

Let's write an equivalent expression with fewer terms.

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$ Expand using the distributive property.

$-x + 2 + 2x - 4x + 5$ Add terms that are alike ($-x$, $2x$, and $-4x$).

$-3x + 2 + 5$ Add terms that are alike (2 and 5).

$= -3x + 7$

Unit 6 Lesson 10 462 Synthesis | Lesson Practice

Lesson Takeaway: Equivalent expressions can be written with fewer *terms* by expanding, adding, and subtracting terms.

Independent | 5 min

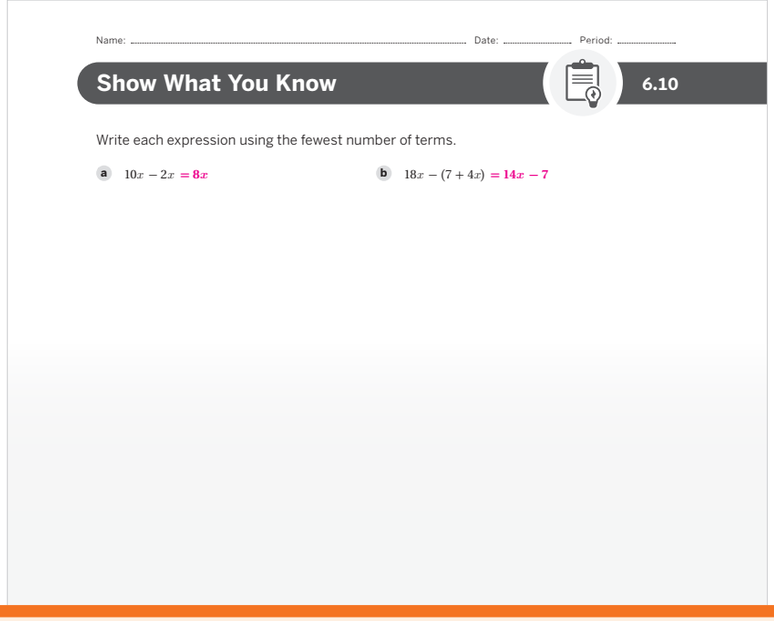
Show What You Know

Purpose: Students demonstrate their understanding by writing expressions with the fewest number of terms.

Today's Goals

1. **Goal:** Write equivalent expressions with fewer terms by expanding, adding, and subtracting terms.
2. **Language Goal:** Explain how to write equivalent expressions using fewer terms. **(Speaking and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.10

Write each expression using the fewest number of terms.

a $10x - 2x = 8x$

b $18x - (7 + 4x) = 14x - 7$

Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support with rewriting an expression with fewer terms by adding, subtracting, and expanding:

» **Respond to Student Thinking:**

- Revisit the **Synthesis** before giving individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the You're invited to **Explore More task**.

Professional Learning

Which students' ideas were you able to highlight during Activity 1? Did students prefer to combine expressions using addition rather than subtraction?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe how to write an equivalent expression using the fewest number of terms. Use this expression if it helps with your thinking.

$5x - 2(6x - 4)$

Responses vary. First, I look for places where I can expand the expression. In the example, -2 times $6x - 4$ is $-12x + 8$. Then, I add together any terms that are alike. In this case, $5x$ and $-12x$ are alike, so $5x + (-12x) = -7x$. The final expression is $-7x + 8$.

Lesson Practice 6.10

Lesson Summary

You can expand, add, and subtract *terms* to write equivalent expressions. This helps us write expressions with the same value that have fewer terms. For example, let's determine the sum of $-\frac{1}{3}(3x - 6)$ and $2x - 4x + 5$.

We can write the sum as $-\frac{1}{3}(3x - 6) + 2x - 4x + 5$.

Let's write an equivalent expression with fewer terms.

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$	Expand using the distributive property.
$-x + 2 + 2x - 4x + 5$	Add terms that are alike ($-x$, $2x$, and $-4x$).
$-3x + 2 + 5$	Add terms that are alike (2 and 5).
$= -3x + 7$	

Lesson Practice 6.10

Name: _____ Date: _____ Period: _____

1. Fill in the blanks to make each equation true.

Responses vary.

$6x +$ ___4x___ $= 10x$

$6x +$ ___-4x___ $= 2x$

$6x +$ ___-16x___ $= -10x$

$6x +$ ___4x + 5___ $= 10x + 5$

$6x -$ ___4x___ $= 2x$

$6x -$ ___5x___ $= x$

$6x +$ ___-6x + 10___ $= 10$

$6x - ($ ___2x + 10___ $) = 4x - 10$

2. Fill in each blank with a number or expression such that each row and column has the same sum.

$2x$	$x + 2$	$2 - x$
$5 - x$	x	$2x - 1$
$x - 1$	2	$x + 3$

FAST Practice

3. Select *all* of the expressions that are equivalent to $4x - 5 + 6$.

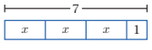
- ☑ **A.** $4x + (-5) + 6$
- ☐ **B.** $4x - 6 + 5$
- ☑ **C.** $4x + 1$
- ☐ **D.** $5x$
- ☐ **E.** $5 + 6 - 4x$

Lesson Practice 6.10

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 4–6: This diagram can be represented by the equation $7 = 3x + 1$.



4. Explain where you can see the 3 in the diagram.

Explanations vary. There are 3 parts labeled x.

5. Determine the value of x.

$x = 2$

6. Select *all* the situations that could be represented by this equation.

- ☑ **A.** Aaliyah is studying 7 hours this week for end-of-year exams. She spends 1 hour on English and an equal number of hours each on math, science, and history.
- ☐ **B.** Lan spends $3 on 7 markers and $1 on a pen.
- ☑ **C.** Sneha shares 7 grapes with 3 friends. She eats 1 grape and gives each friend the same number of grapes.
- ☑ **D.** Kiri read 1 book this week and has a goal to read 7 total books within the next 3 weeks. She will read the same number of books each week.
- ☐ **E.** Adriana runs 7 miles every day for 3 days and then runs 1 mile on the fourth day.

Problems 7–10: Solve each equation.

7. $5(n - 4) = -60$

$n = -8$

8. $-3t + (-8) = 25$

$t = -11$

9. $7p - 8 = -22$

$p = -2$

10. $\frac{2}{5}(j + 40) = -4$

$j = -50$

11. A small town had a population of 960 people last year. The population grew to 1,200 people this year. By what percent did the population grow?

25%

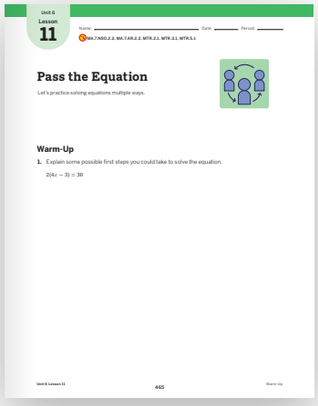
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.7.AR.1.1
	2	3	MA.7.AR.1.1
FAST Practice	3	1	MA.7.AR.1.1
Spiral Review			
	4–10	2	MA.7.AR.2.2
	11	2	MA.7.AR.3.1

Unit 6

Lesson

11


Unit 6
Lesson
11
Name: Date: Period:
MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.3.1, MTR.5.1
Pass the Equation
Let's practice solving equations multiple ways.
Warm-Up
1. Explain some possible first steps you could take to solve the equation.
$2(4x - 3) = 30$
Unit 6 Lesson 11
465
Warm-Up

Pass the Equation

Solving Equations Multiple Ways

Let's practice solving equations multiple ways.

Learning Outcomes

Today's Goals

1. **Goal:** Solve equations using a variety of strategies.
2. **Language Goal:** Compare and contrast strategies for solving the same equation. **(Reading, Writing, Speaking, and Listening)**

Students practice solving equations that involve adding terms and expanding expressions. They recognize that there are several possible sets of steps for the same equation that will lead to the same solution. This lesson is also an opportunity for students to practice converting between decimals and fractions as they solve equations.

Coherence

Prior Learning

In Lessons 2–5, students practiced solving equations. In Lessons 7–10, students factored, expanded, and added expressions.

Future Learning

In Lesson 12, students will write and solve equations. They will interpret the meaning of their solutions in context.

Connections

- Students solve equations using mutliple ways that provides different perspectives on the same problem. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2
Add, subtract, multiply and divide rational numbers with procedural fluency.

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2	**MA.8.AR.2.1**
MA.6.AR.2.3	

Vocabulary

Additional Vocabulary

inverse operations

Lesson at a Glance

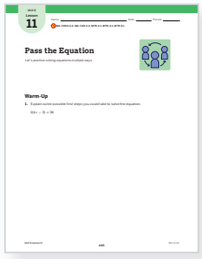
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.2

Warm-Up

Pairs | 5 min

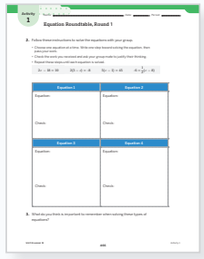
Students surface several first steps for the same equation to show that there is often more than one useful first step. **Routine:** Think-Pair-Share



Activity 1

Small Groups | 15 min

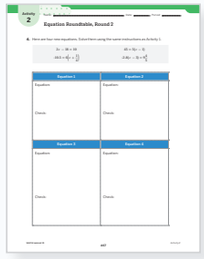
Students practice solving equations that have multiple viable solving pathways and get feedback from their peers to revise their thinking.



Activity 2

Small Groups | 15 min

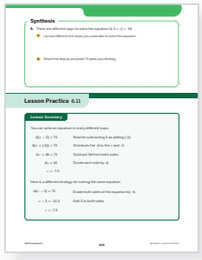
Students practice solving equations that involve both adding and expanding expressions.



Synthesis

Whole Class | 5 min

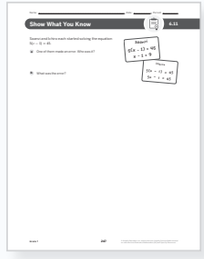
Students synthesize their understanding of a variety of first steps for solving complex equations. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by finding errors made in solving equations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students surface several first steps for the same equation to show that there is often more than one useful first step.

1 Launch

Direct students' attention to the equation and consider asking, "What do you notice about the equation?"

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's strategy with the class.

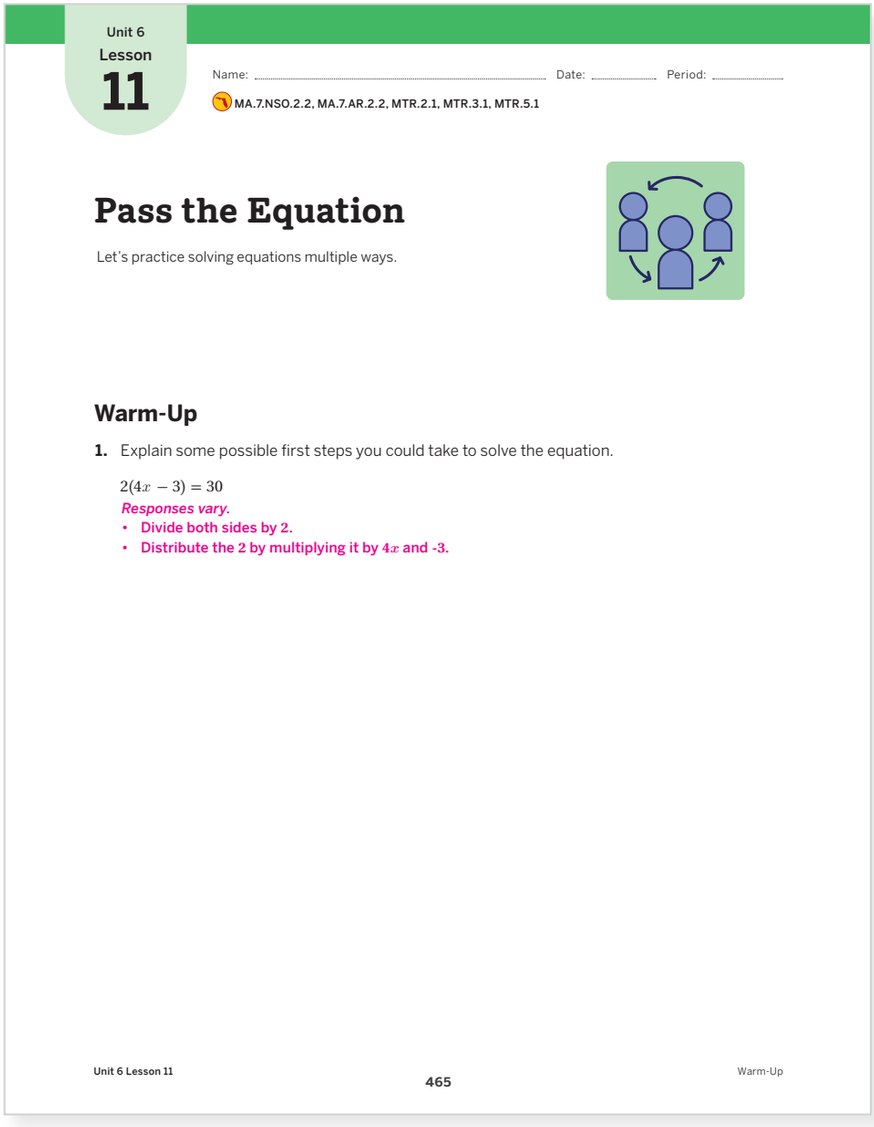
2 Connect

Invite students to share several possible first steps.

ELL **English Language Learners** Create or review an anchor chart that publicly displays common solving steps, along with vocabulary like *add*, *subtract*, *expand*, *factor*, *term*, and *equivalent expression* to aid students in explaining their reasoning.

Consider asking:

- "Which first step do you prefer? Why?"
- "Will you get the same solution regardless of which first step you choose?"



Unit 6
Lesson
11

Name: Date: Period:

MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.3.1, MTR.5.1

Pass the Equation

Let's practice solving equations multiple ways.

Warm-Up

1. Explain some possible first steps you could take to solve the equation.

$2(4x - 3) = 30$

Responses vary.

- Divide both sides by 2.
- Distribute the 2 by multiplying it by $4x$ and -3.

Unit 6 Lesson 11 465 Warm-Up

Small Groups | 15 min

Activity 1 Equation Roundtable, Round 1

Purpose: Students practice solving equations that have multiple viable solving pathways and get feedback from their peers to revise their thinking.

1 Launch

Direct students' attention to the Equation Roundtable instructions.

Invite students to read through the instructions and ask clarifying questions. It may be helpful to do a practice round as a class before students begin.

2 Monitor

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took earlier on solving equations to activate prior knowledge.

D **Differentiation** *(Problem 2)*

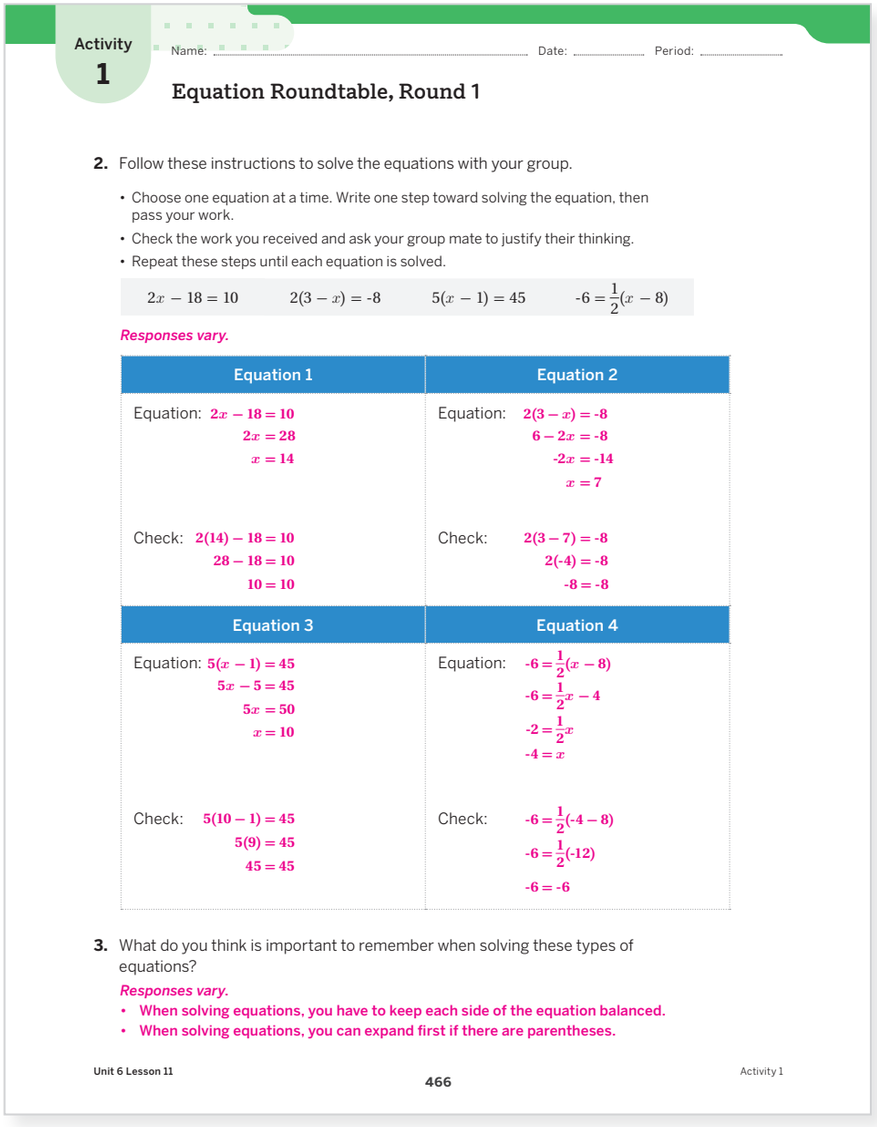
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to refer to the strategies from the Warm-Up.
Distribute to only one of the two terms in the parentheses.	**Support** Consider asking, "How could you check if this equation is equivalent to your first equation?"
Use strategies like: • Reasoning about the unknown value. • Distributing first. • Dividing both sides by the number outside the parentheses. • Using *inverse operations*. • Rewriting the equation with the variable on the left.	**Strengthen** Consider asking, "Why is the sign of a term important when distributing?" Select and sequence these strategies to share during the Connect.
Would like to explore this idea more.	**Stretch** Consider asking, "Will you get the same solution if you take a different first step?"

3 Connect

Direct students' attention to the four equations and invite the class to choose one to discuss.

Consider asking:

- "Did your partner ever make a move different than the one you expected them to? Describe it."
- "What advice would you give yourself or others when solving equations like these?"



Activity 1

Name: Date: Period:

Equation Roundtable, Round 1

2. Follow these instructions to solve the equations with your group.

- Choose one equation at a time. Write one step toward solving the equation, then pass your work.
- Check the work you received and ask your group mate to justify their thinking.
- Repeat these steps until each equation is solved.

$2x - 18 = 10$ $2(3 - x) = -8$ $5(x - 1) = 45$ $-6 = \frac{1}{2}(x - 8)$

Responses vary.

Equation 1	Equation 2
Equation: $2x - 18 = 10$ $2x = 28$ $x = 14$ Check: $2(14) - 18 = 10$ $28 - 18 = 10$ $10 = 10$	Equation: $2(3 - x) = -8$ $6 - 2x = -8$ $-2x = -14$ $x = 7$ Check: $2(3 - 7) = -8$ $2(-4) = -8$ $-8 = -8$
Equation 3	**Equation 4**
Equation: $5(x - 1) = 45$ $5x - 5 = 45$ $5x = 50$ $x = 10$ Check: $5(10 - 1) = 45$ $5(9) = 45$ $45 = 45$	Equation: $-6 = \frac{1}{2}(x - 8)$ $-6 = \frac{1}{2}x - 4$ $-2 = \frac{1}{2}x$ $-4 = x$ Check: $-6 = \frac{1}{2}(-4 - 8)$ $-6 = \frac{1}{2}(-12)$ $-6 = -6$

3. What do you think is important to remember when solving these types of equations?

Responses vary.

- When solving equations, you have to keep each side of the equation balanced.
- When solving equations, you can expand first if there are parentheses.

Unit 6 Lesson 11 466 Activity 1

Key Takeaway: There are multiple ways to solve equations, including using the distributive property or using inverse operations on both sides of the equation. No matter which strategy you choose to solve an equation, the solution should be the same.

Small Groups | 15 min

Activity 2 Equation Roundtable, Round 2

Purpose: Students practice solving equations that involve both adding and expanding expressions.

Short on time? Consider omitting this activity or invite each student in the group to choose one equation to solve, then to take turns explaining their solving strategy to the group.

1 Launch

Direct students' attention to the instructions and invite students to share helpful strategies from Activity 1.

To surface connections, encourage students to compare and contrast the equations in Activity 1 and Activity 2. Students may notice that the structure of equations in Activity 2 requires combining like terms on the same side, which they practiced with expressions in Lesson 10. **(MTR.5.1)**

2 Monitor

Encourage students to explain how they know the equation is still balanced after each step.

D Differentiation *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to review strategies from Activity 1.
Combine unlike terms.	**Support** Encourage students to circle or highlight like terms on the same side before rewriting.
Use strategies like: • Using properties of integers and equations to correctly solve. • Rewriting subtraction as addition of a negative value. • Rewriting fractions and decimals in the same form.	**Strengthen** Consider asking, "How can you check that your solution is correct?"
Would like to explore this idea more.	**Stretch** Encourage groups to determine a solution path for each equation that uses the fewest number of steps. **(MTR.3.1)**

Activity 2

Name: Date: Period:

Equation Roundtable, Round 2

4. Here are four new equations. Solve them using the same instructions as Activity 1.

$2x - 18 = 10$ $\quad$ $45 = 5(x - 1)$

$-10.5 = 6\left(x + \frac{1}{4}\right)$ $\quad$ $-2.8(x - 3) = 9\frac{4}{5}$

Responses vary.

Equation 1

Equation: $2x - 18 = 10$; $2x = 28$; $x = 14$

Check: $2(14) - 18 = 10$; $28 - 18 = 10$; $10 = 10$

Equation 2

Equation: $45 = 5(x - 1)$; $45 = 5x - 5$; $50 = 5x$; $10 = x$

Check: $45 = 5(10 - 1)$; $45 = 5(9)$; $45 = 45$

Equation 3

Equation: $-10.5 = 6\left(x + \frac{1}{4}\right)$; $-\frac{21}{2} = 6x + \frac{6}{4}$; $-\frac{42}{4} = \frac{24}{4}x + \frac{6}{4}$; $-42 = 24x + 6$; $-48 = 24x$; $-2 = x$

Check: $-10.5 = 6\left(-2 + \frac{1}{4}\right)$; $-\frac{21}{2} = 6\left(-\frac{8}{4} + \frac{1}{4}\right)$; $-\frac{21}{2} = 6\left(-\frac{7}{4}\right)$; $-\frac{42}{4} = -\frac{42}{4}$

Equation 4

Equation: $-2.8(x - 3) = 9\frac{4}{5}$; $-2.8x + 8.4 = 9\frac{4}{5}$; $-2.8x + 8.4 = 9.8$; $-2.8x = 1.4$; $x = -0.5$ (or equivalent)

Check: $-2.8(-0.5 - 3) = 9\frac{4}{5}$; $-2.8(-3.5) = 9.8$; $9.8 = 9.8$

Unit 6 Lesson 11 467 Activity 2

3 Connect

Invite students to compare and contrast the strategies their group used to solve the equations. Encourage students to share which first steps they prefer and their reasoning. **(MTR.2.1)**

To surface the Key Takeaway, consider asking, "What is helpful to remember when solving equations?"

Key Takeaway: There are multiple ways to solve equations, including using integer properties and writing equivalent expressions.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of a variety of first steps for solving complex equations.

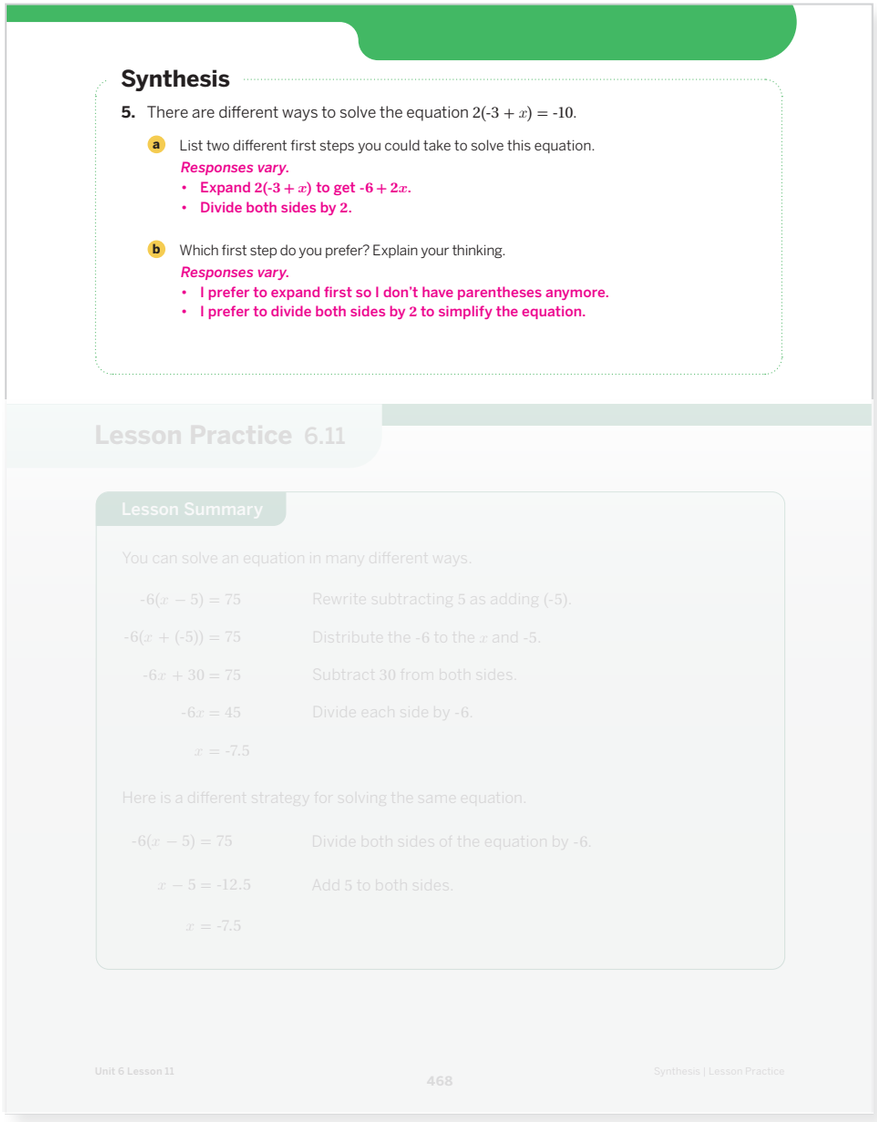
Direct students' attention to the equation and discussion questions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Dividing by 2 on both sides.
- Expanding the equation by distributing 2 to -3 and x.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas or strategies as they solve complex equations. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

5. There are different ways to solve the equation $2(-3 + x) = -10$.

a. List two different first steps you could take to solve this equation.

Responses vary.
- Expand $2(-3 + x)$ to get $-6 + 2x$.
- Divide both sides by 2.

b. Which first step do you prefer? Explain your thinking.

Responses vary.
- I prefer to expand first so I don't have parentheses anymore.
- I prefer to divide both sides by 2 to simplify the equation.

Lesson Practice 6.11

Lesson Summary

You can solve an equation in many different ways.

$-6(x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(x + (-5)) = 75$	Distribute the -6 to the x and -5.
$-6x + 30 = 75$	Subtract 30 from both sides.
$-6x = 45$	Divide each side by -6.
$x = -7.5$	

Here is a different strategy for solving the same equation.

$-6(x - 5) = 75$	Divide both sides of the equation by -6.
$x - 5 = -12.5$	Add 5 to both sides.
$x = -7.5$	

Unit 6 Lesson 11 468 Synthesis | Lesson Practice

Lesson Takeaway: There are many possible first steps for solving complex equations, including combining like terms and expanding expressions.

Independent | 5 min

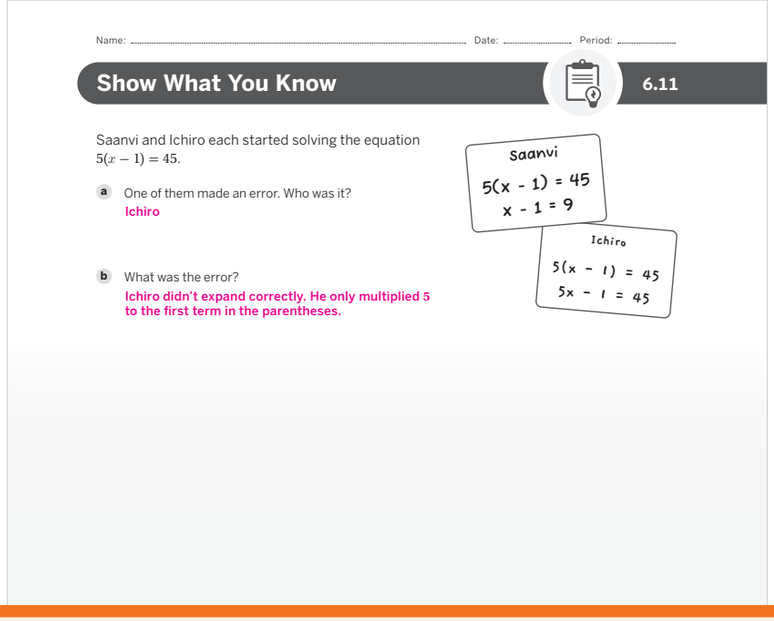
Show What You Know

Purpose: Students demonstrate their understanding by finding errors made in solving equations.

Today's Goals

1. **Goal:** Solve equations using a variety of strategies.
2. **Language Goal:** Compare and contrast strategies for solving the same equation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.11

Saanvi and Ichiro each started solving the equation $5(x - 1) = 45$.

a. One of them made an error. Who was it?
Ichiro

b. What was the error?
Ichiro didn't expand correctly. He only multiplied 5 to the first term in the parentheses.

Saanvi
5(x - 1) = 45
x - 1 = 9

Ichiro
5(x - 1) = 45
5x - 1 = 45

Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations using a variety of strategies:

» **Respond to Student Thinking:**

- Revisit **Activity 1** as a class and inviting students to share strategies for how to solve equations.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

In what ways are students taking risks when trying out new strategies for solving equations? What types of questions might support students in exploring and testing new strategies?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

5. There are different ways to solve the equation $2(-3 + x) = -10$.

a. List two different first steps you could take to solve this equation.

Responses vary.

- Expand $2(-3 + x)$ to get $-6 + 2x$.
- Divide both sides by 2.

b. Which first step do you prefer? Explain your thinking.

Responses vary.

- I prefer to expand first so I don't have parentheses anymore.
- I prefer to divide both sides by 2 to simplify the equation.

Lesson Practice 6.11

Lesson Summary

You can solve an equation in many different ways.

$-6(x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(x + (-5)) = 75$	Distribute the -6 to the x and -5.
$-6x + 30 = 75$	Subtract 30 from both sides.
$-6x = 45$	Divide each side by -6.
$x = -7.5$	

Here is a different strategy for solving the same equation.

$-6(x - 5) = 75$	Divide both sides of the equation by -6.
$x - 5 = -12.5$	Add 5 to both sides.
$x = -7.5$	

Lesson Practice 6.11

Name: _____ Date: _____ Period: _____

Problems 1–3: Kwame and Maia each solved the same equation but used different strategies to do so. Show what they might have done to get from one step to another.

Kwame	Maia
$3.3(x - 10) = 66$	$3.3(x - 10) = 66$
$3.3x - 3.3 \cdot 10 = 66$	$3.3(x - 10) \div 3.3 = 66 \div 3.3$
$3.3x - 33 = 66$	$x - 10 = 20$
$3.3x - 33 + 33 = 66 + 33$	$x - 10 + 10 = 20 + 10$
$3.3x = 99$	$x = 30$
$3.3x \div 3.3 = 99 \div 3.3$	
$x = 30$	

1. How did Kwame begin? Maia?

   Kwame began by distributing first. Maia began by dividing each side by 3.3.

2. Complete the missing steps for each student.

   Sample shown in work.

3. Which method do you prefer to use for this equation? Why?

   Responses vary.

Problems 4–7: Solve each equation. Show your thinking.

4. $4(x - 3) = 16$

   $4(x - 3) \div (4) = 16 \div (4)$
   $x - 3 = 4$
   $x - 3 + 3 = 4 + 3$
   $x = 7$

5. $-5(x - 4) = 40$

   $-5(x - 4) \div (-5) = 40 \div (-5)$
   $x - 4 = -8$
   $x - 4 + 4 = -8 + 4$
   $x = -4$

6. $\frac{3}{7}(x - 8) = 15$

   $\frac{3}{7}(x - 8) \div \frac{3}{7} = 15 \div \frac{3}{7}$
   $x - 8 = 35$
   $x - 8 + 8 = 35 + 8$
   $x = 43$

7. $\frac{1}{6}(x + 6) = 11$

   $\frac{1}{6}(x + 6) \div \frac{1}{6} = 11 \div \frac{1}{6}$
   $x + 6 = 66$
   $x + 6 - 6 = 66 - 6$
   $x = 60$

Lesson Practice 6.11

Name: _____ Date: _____ Period: _____

Problems 8–9: Alina and Naoki are each solving the equation $7(x + 2) = 91$.
Alina starts by using the distributive property.
Naoki starts by dividing each side by 7.

8. Show what Alina's and Naoki's full solution methods might look like.

Alina	Naoki
$7(x + 2) = 91$	$7(x + 2) = 91$
$7x + 14 = 91$	$7(x + 2) \div 7 = 91 \div 7$
$7x + 14 - 14 = 91 - 14$	$x + 2 = 13$
$7x = 77$	$x + 2 - 2 = 13 - 2$
$7x \div 7 = 77 \div 7$	$x = 11$
$x = 11$	

9. What is the same and what is different about their methods?

   Responses vary. **Both divided each side by 7 (at different steps) and determined the solution $x = 11$. Alina subtracted 14 from each side while Naoki subtracted 2 from each side.**

FAST Practice

10. What is the value of x in the equation $9 - \frac{x}{3} = 14$?

    $x =$ **-15**

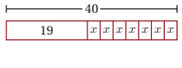
Spiral Review

11. Vicente and Zwena are trying to write $9x - 2x + 4x$ using fewer terms.
    - Vicente says that $9x - 2x + 4x = 3x$ because the subtraction sign tells us to subtract everything that comes after $9x$.
    - Zwena says that $9x - 2x + 4x = 11x$ because the subtraction only applies to $2x$.

    Who is correct? Explain your thinking.

    Zwena. *Explanations vary.* Rewriting addition as subtraction gives us $9x + (-2x) + 4x$, which shows that the subtraction symbol in front of the $2x$ applies only to the $2x$ and not to the terms that come after it.

12. Write three different equations that represent the tape diagram. Then determine the value of x.

    $x = 3$. ***Equations vary.***
    - $7x + 19 = 40$
    - $19 + x + x + x + x + x + x + x = 40$
    - $7x = 40 - 19$

13. Choose the expression that is equivalent to $(9x + 5) + (-7x + 2)$.

    A. $-2x + 10$ B. $-63x + 10$ C. $16x + 7$ D. $2x + 7$

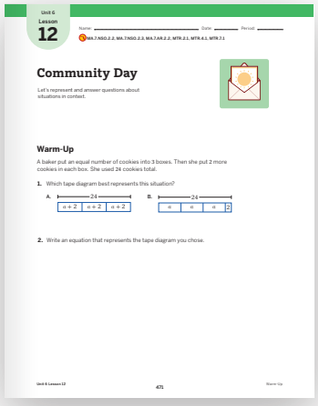
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	1	MA.7.AR.2.2
	8–9	2	MA.7.AR.2.2
FAST Practice	10	2	MA.7.AR.2.2
Spiral Review			
	11	2	MA.6.AR.1.4
	12	2	MA.7.AR.2.2
	13	1	MA.7.AR.1.1

Unit 6

Lesson

12


Unit 6
Lesson
12
Name _____ Date _____ Period _____
MA.7.NSO.2.3, MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.7.1
Community Day
Let's represent and answer questions about situations in context.
Warm-Up
A baker put an equal number of cookies into 3 boxes. Then she put 2 more cookies in each box. She used 24 cookies total.
1. Which tape diagram best represents this situation?
A. 24
n + 2 n + 2 n + 2
B. 24
n n n 2
2. Write an equation that represents the tape diagram you chose.
Unit 6 Lesson 12
471
Warm-Up

Community Day

Using Equations to Solve Problems

Let's represent and answer questions about situations in context.

Learning Outcomes

Today's Goals

1. **Goal:** Write and solve equations to represent situations involving positive and negative numbers.
2. **Goal:** Connect visual representations, equations, and verbal descriptions of the same situation.
3. **Language Goal:** Compare and contrast situations using different visual representations and equations. **(Reading and Writing)**

Students use everything they have learned so far in the unit to create visuals and equations to represent and answer questions about situations in context. They make sense of verbal descriptions, and compare and contrast situations in similar contexts. **(MTR.2.1, MTR.7.1)**

Coherence

Prior Learning

In Lessons 1–3, students used tape diagrams to represent equations and situations in context and to determine unknown values. In Lessons 4–11, students developed fluency in writing and solving equations using inverse operations.

Future Learning

In Lesson 13, students will use inequalities to represent and answer questions about situations in context.

Connections

- Students use tape diagrams as a visual tool to represent and solve equations in real world situations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.3
Solve real-world problems involving any of the four operations with rational numbers.

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectation: ELA.K12.EE.2.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2	**MA.8.AR.2.1**
MA.6.AR.2.3	

Lesson at a Glance

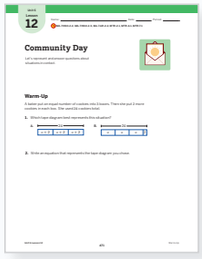
~ 45 min

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.2.2

Warm-Up

Pairs | 5 min

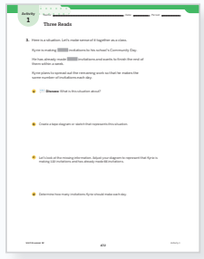
Students connect a visual representation and an equation to a description of a situation.



Activity 1

Pairs | 10 min

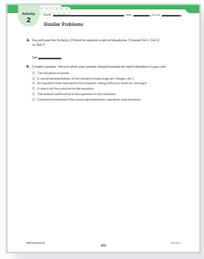
Students make sense of a situation described in words, represent the situation with a visual, and answer a question about it. MLR6: Three Reads (MTR.7.1)



Activity 2

Pairs | 20 min

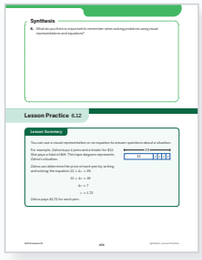
Students practice creating visuals in order to write and solve equations for situations in context. (MTR.2.1)



Synthesis

Whole Class | 5 min

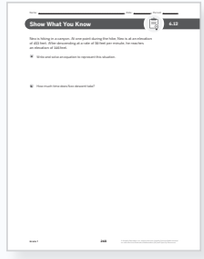
Students synthesize their understanding of what is important to remember when representing and answering questions about situations.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing and solving an equation in context.



Prep Checklist

Prepare the Activity 2 Sheet.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student

Pairs | 5 min

Warm-Up

Purpose: Students connect a visual representation and an equation to a description of a situation.

1 Launch

Direct students' attention to the situation and questions.

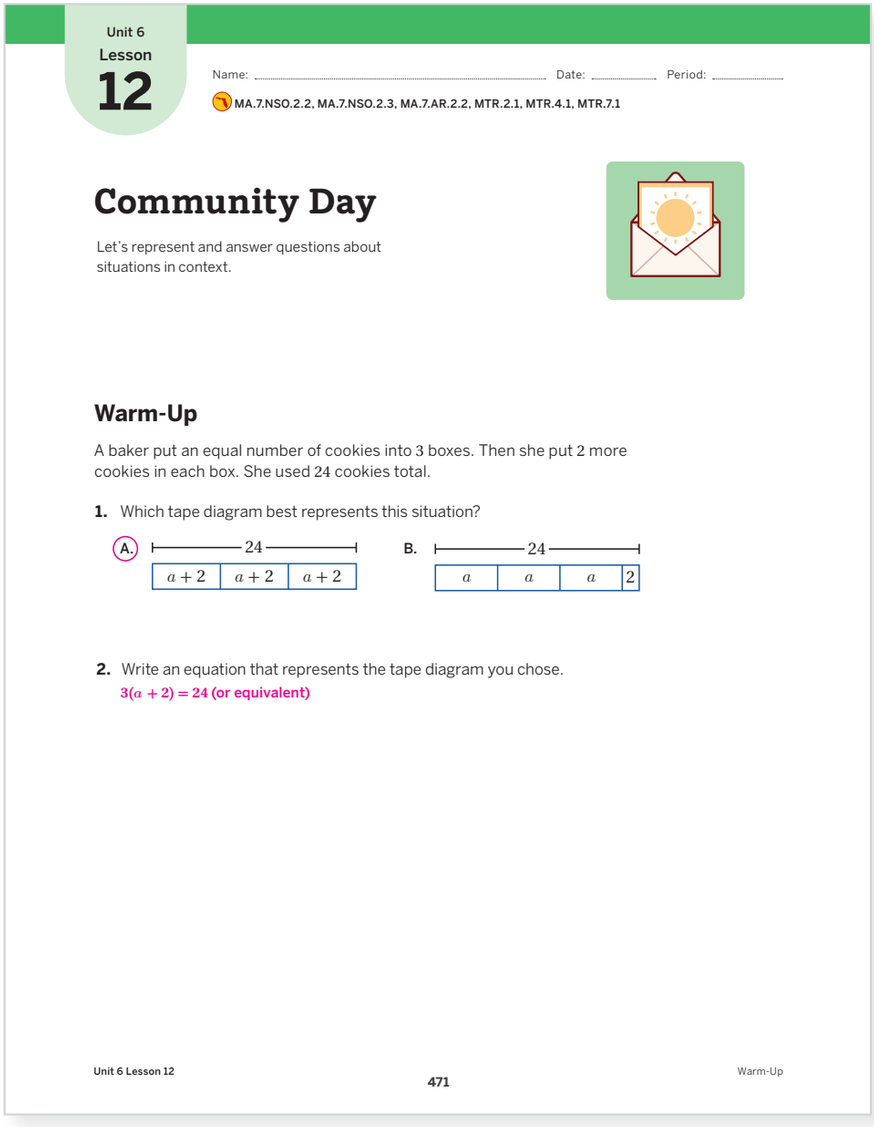
Invite a student to read the situation out loud and other students to paraphrase it in their own words.

2 Connect

To surface connections, consider asking:

- "What words in the situation helped you decide which diagram to choose?"
- "How do you know the other diagram does not represent this situation?"
- "What equation did you write? Are there any other equations that also represent the situation?"

D **Differentiation:** (Stretch) Encourage students to write a story that matches the tape diagram they did not select.



Unit 6
Lesson
12

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.7.1

Community Day

Let's represent and answer questions about situations in context.

Warm-Up

A baker put an equal number of cookies into 3 boxes. Then she put 2 more cookies in each box. She used 24 cookies total.

1. Which tape diagram best represents this situation?

A.

B.

2. Write an equation that represents the tape diagram you chose.

$3(a+2)=24$ (or equivalent)

Unit 6 Lesson 12 471 Warm-Up

Pairs | 10 min

Activity 1 Three Reads

Purpose: Students make sense of a situation described in words, represent the situation with a visual, and answer a question about it. **(MTR.7.1)**

1 Launch

MLR **MLR6: Three Reads** Invite students to read the text three times, each with a particular focus, to support students in making sense of the situation. **(MTR.7.1)**

Direct students' attention to the situation with the hidden information and invite a student to read it aloud, then discuss the prompt in part a.

To surface connections, invite students to share a personal experience they have had with a Community Day.

Direct students' attention to part b and invite a student to read it aloud, then discuss the prompt.

Invite students to share their tape diagram or sketch.

To encourage creativity, celebrate the variety of diagrams and sketches students create.

Direct students' attention to parts c and d. After the third read, invite students to discuss the question with a partner, then as a class. **(MTR.4.1)**

2 Monitor

D **Differentiation** *(Problem 3)*

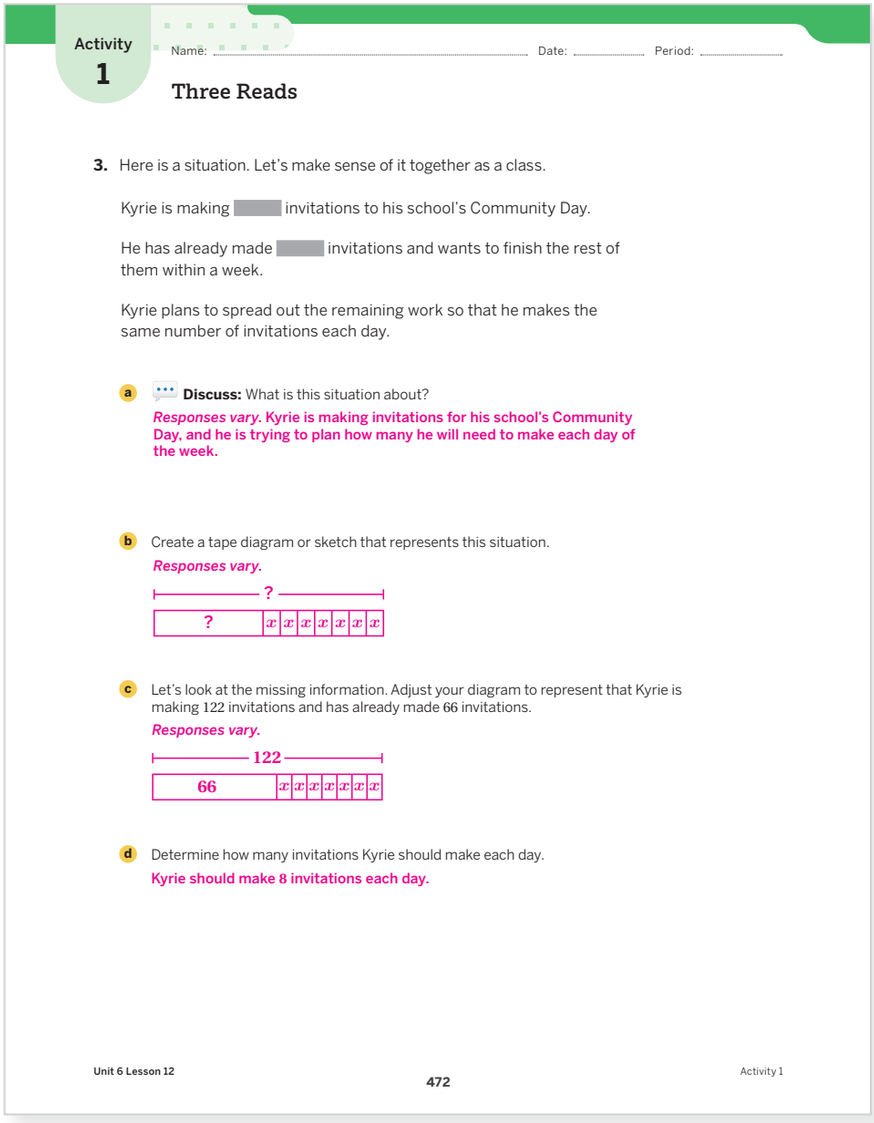
Look for students who:	Respond to Student Thinking
Need support interpreting the phrase "within a week."	**Support** Encourage students to choose a day of the week for the story and determine when Kyrie would be finished by.
Substitute the given values into the tape diagram.	**Strengthen** Consider asking, "What equation would represent this tape diagram and situation?"

A **Accessibility: Conceptual Processing** Invite students to answer a more concrete question first to support abstract thinking, like: *How can a tape diagram help you?*

3 Connect

Invite students to think of new values for this scenario, then to update their diagrams to match. **(MTR.2.1)**

To surface the Key Takeaway, consider asking, "How did you adjust your diagram to the new values?"



Activity 1

Name: _____ Date: _____ Period: _____

Three Reads

3. Here is a situation. Let's make sense of it together as a class.

Kyrie is making [redacted] invitations to his school's Community Day.

He has already made [redacted] invitations and wants to finish the rest of them within a week.

Kyrie plans to spread out the remaining work so that he makes the same number of invitations each day.

a. **Discuss:** What is this situation about?

Responses vary. Kyrie is making invitations for his school's Community Day, and he is trying to plan how many he will need to make each day of the week.

b. Create a tape diagram or sketch that represents this situation.

Responses vary.

c. Let's look at the missing information. Adjust your diagram to represent that Kyrie is making 122 invitations and has already made 66 invitations.

Responses vary.

d. Determine how many invitations Kyrie should make each day.

Kyrie should make 8 invitations each day.

Unit 6 Lesson 12 472 Activity 1

Key Takeaway: Visuals and diagrams can help you represent situations and answer questions about them.

Pairs | 20 min

Activity 2 Similar Problems

Purpose: Students practice creating visuals in order to write and solve equations for situations in context. **(MTR.2.1)**

Short on time? Invite pairs to select one set. Have one partner work on Situation A and the other on Situation B and then compare their equations and solutions.

1 Launch

Direct students' attention to the instructions and review them as a class.

Distribute the Activity 2 sheet, one per student.

Consider sharing that the situations (A and B) in each set are very similar but not exactly the same.

Note: The sets go in order of difficulty. Invite students to choose one set that they believe would present a meaningful challenge to them.

2 Monitor

Look for student strategies, listen to small group discussions, and offer help or encouragement where needed.

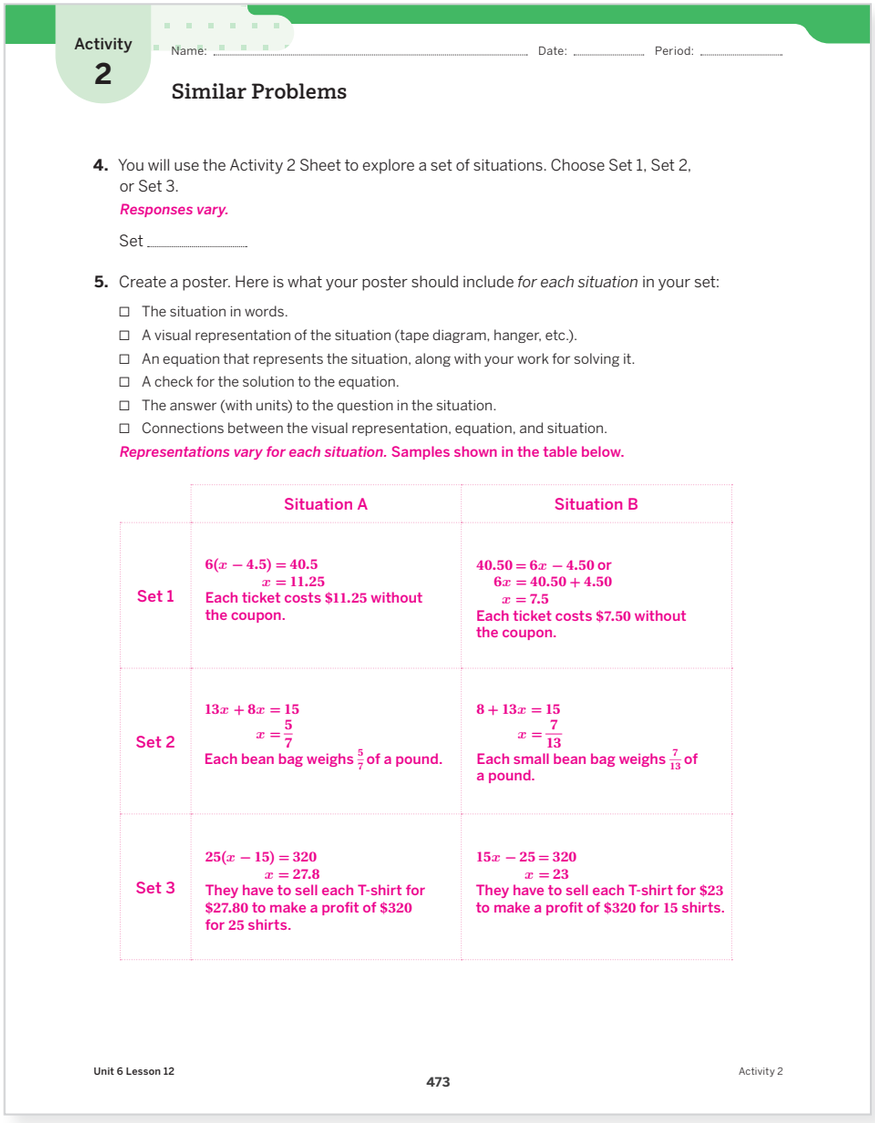
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to discuss specific words to help them determine the differences between the two situations.
Use their visual representation to solve. Use their equation to solve.	**Strengthen** Consider asking, "Would you get the same solution regardless of the representation or equation that you used to solve? Why or why not?"
Would like to explore real-world situations further.	**Stretch** Encourage students to create a new set of two related situations to represent.

3 Connect

Invite students to share their posters and explain connections between the visual representations, equations, and situations. Highlight students who created different visual representations or equations for the same situations. **(MTR.2.1)**

Consider asking, "What words did you find helpful to distinguish the two situations?" **(MTR.7.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Similar Problems

4. You will use the Activity 2 Sheet to explore a set of situations. Choose Set 1, Set 2, or Set 3.

Responses vary.

Set _____

5. Create a poster. Here is what your poster should include *for each situation* in your set:

- ☐ The situation in words.
- ☐ A visual representation of the situation (tape diagram, hanger, etc.).
- ☐ An equation that represents the situation, along with your work for solving it.
- ☐ A check for the solution to the equation.
- ☐ The answer (with units) to the question in the situation.
- ☐ Connections between the visual representation, equation, and situation.

Representations vary for each situation. Samples shown in the table below.

	Situation A	Situation B
Set 1	$6(x - 4.5) = 40.5$ $x = 11.25$ Each ticket costs \$11.25 without the coupon.	$40.50 = 6x - 4.50$ or $6x = 40.50 + 4.50$ $x = 7.5$ Each ticket costs \$7.50 without the coupon.
Set 2	$13x + 8x = 15$ $x = \frac{5}{7}$ Each bean bag weighs $\frac{5}{7}$ of a pound.	$8 + 13x = 15$ $x = \frac{7}{13}$ Each small bean bag weighs $\frac{7}{13}$ of a pound.
Set 3	$25(x - 15) = 320$ $x = 27.8$ They have to sell each T-shirt for \$27.80 to make a profit of \$320 for 25 shirts.	$15x - 25 = 320$ $x = 23$ They have to sell each T-shirt for \$23 to make a profit of \$320 for 15 shirts.

Unit 6 Lesson 12 473 Activity 2

Whole Class | 5 min

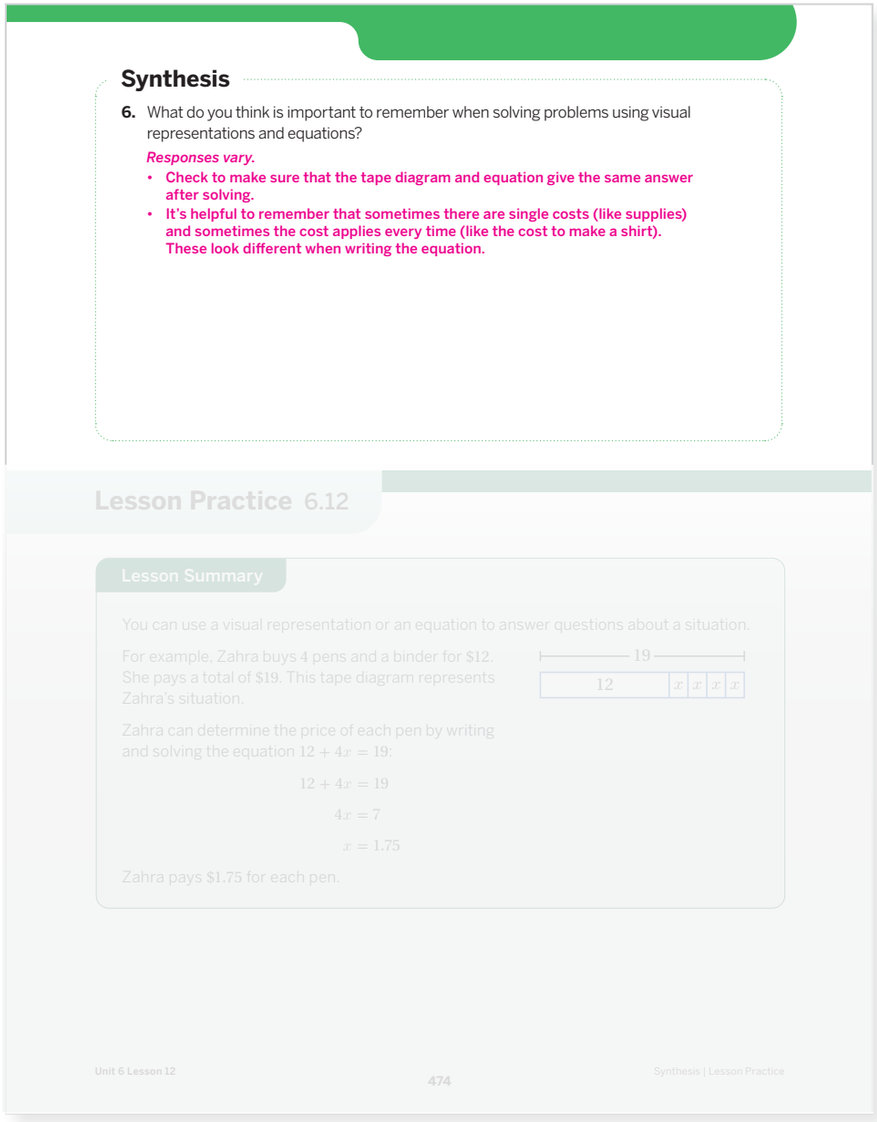
Synthesis

Purpose: Students synthesize their understanding of what is important to remember when representing and answering questions about situations.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using multiple representations to confirm their solution.
- Identifying the difference between constant values and coefficients.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

6. What do you think is important to remember when solving problems using visual representations and equations?

Responses vary.
- Check to make sure that the tape diagram and equation give the same answer after solving.
- It's helpful to remember that sometimes there are single costs (like supplies) and sometimes the cost applies every time (like the cost to make a shirt). These look different when writing the equation.

Lesson Practice 6.12

Lesson Summary

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for \$12. She pays a total of \$19. This tape diagram represents Zahra's situation.

Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$12 + 4x = 19$$
$$4x = 7$$
$$x = 1.75$$

Zahra pays \$1.75 for each pen.

Unit 6 Lesson 12 474 Synthesis | Lesson Practice

Lesson Takeaway: Creating tape diagrams and equations can help answer questions about situations. Key words like *same*, *each*, *every*, and *total* can help determine if a situation has a single value or a value that applies every time.

Independent | 5 min

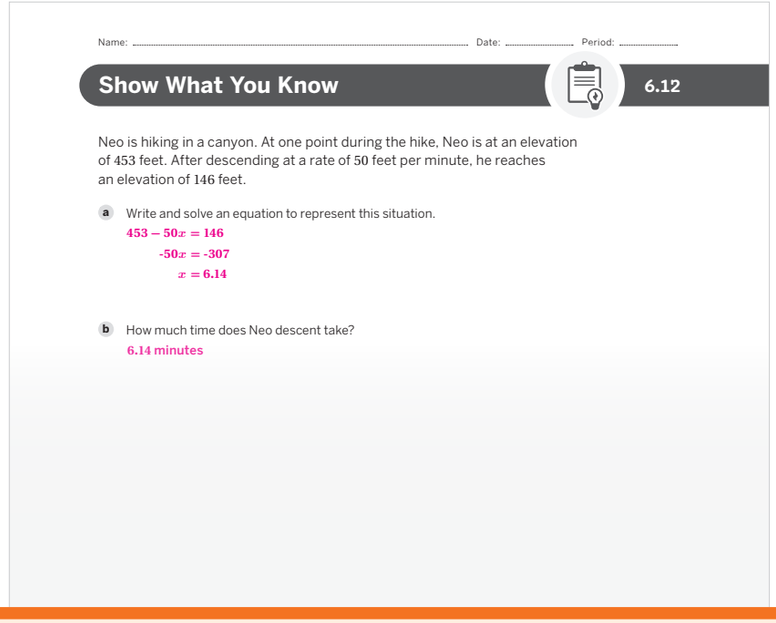
Show What You Know

Purpose: Students demonstrate their understanding by writing and solving an equation in context.

Today's Goals

1. **Goal:** Write and solve equations to represent situations involving positive and negative numbers.
2. **Goal:** Connect visual representations, equations, and verbal descriptions of the same situation.
3. **Language Goal:** Compare and contrast situations using different visual representations and equations. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.12

Neo is hiking in a canyon. At one point during the hike, Neo is at an elevation of 453 feet. After descending at a rate of 50 feet per minute, he reaches an elevation of 146 feet.

a. Write and solve an equation to represent this situation.

$453 - 50x = 146$

$-50x = -307$

$x = 6.14$

b. How much time does Neo descent take?

6.14 minutes

Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write equations to represent situations:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and invite students to use the MLR6: Three Reads routine to make sense of a few related Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to write an equation that could represent a real-world situation and determine the corresponding solutions.

Professional Learning

Why was the context of *Community Day* particularly useful in building student understanding of representing and answering questions about situations in context?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. What do you think is important to remember when solving problems using visual representations and equations?

Responses vary.

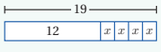
- Check to make sure that the tape diagram and equation give the same answer after solving.
- It's helpful to remember that sometimes there are single costs (like supplies) and sometimes the cost applies every time (like the cost to make a shirt). These look different when writing the equation.

Lesson Practice 6.12

Lesson Summary

You can use a visual representation or an equation to answer questions about a situation.

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$$12 + 4x = 19$$
$$4x = 7$$
$$x = 1.75$$

Zahra pays $1.75 for each pen.

Lesson Practice 6.12

Name: _____ Date: _____ Period: _____

Problems 1–4: Select an equation to represent the situation. Then solve the equation and explain the solution's meaning in context.

$5x - 7 = 3$ $\quad 7 = 3(5 - x)$ $\quad 3x + 5 = -7$ $\quad x + 7 = 3 \cdot 5$

	Situation	Equation	Solution	Solution's Meaning
1.	The temperature outside is currently -7°C. Since midnight, the temperature tripled and then rose 5 degrees. What was the temperature at midnight?	$3x + 5 = -7$	$x = -4$	The temperature at midnight was -4°C.
2.	Ama has 7 pink roses plus some white roses. She gives all of her roses away by giving 5 roses to each of her 3 favorite teachers. How many white roses does Ama give away?	$x + 7 = 3 \cdot 5$	$x = 8$	Ama gives away 8 white roses.
3.	A family of 3 goes to a fair. Tickets cost $5 each, but each person has a coupon. They pay $7 altogether. How much money does each person save on buying their ticket?	$7 = 3(5 - x)$	$x = \frac{8}{3}$ (or equivalent)	Each person saved $\frac{8}{3}$ on their ticket cost, or $2.67.
4.	A club puts its members into 5 groups for an activity. 7 students leave early, so there are only 3 students left to finish the activity. How many students were in each group?	$5x - 7 = 3$	$x = 2$	2 students were in each group.

Lesson Practice 6.12

Name: _____ Date: _____ Period: _____

FAST Practice

5. 6 soccer teams are practicing on a field. Each team has the same number of players, x. A coach asks 2 players from each team to leave the field to help move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

Equation: $6(x - 2) = 78$ or $6x - 12 = 78$

Solution: $x =$ 15 players on each team

Spiral Review

Problems 6–9: Determine the value of the variable that makes each equation true.

6. $a \cdot 3 = -30$; $a = -10$

7. $-9 \cdot b = -45$; $b = 5$

8. $-89 \cdot 12 = c$; $c = -1068$

9. $d \cdot 88 = -88000$; $d = -1000$

10. Select *all* the expressions that show x increased by 35%.

- ☑ A. $1.35x$
- ☐ B. $\frac{35}{100}x$
- ☑ C. $x + \frac{35}{100}x$
- ☑ D. $(1 + 0.35)x$
- ☐ E. $(100 + 35)x$
- ☑ F. $\frac{100 + 35}{100}x$

11. Alma has a loan of $12,589. The loan requires Alma to pay 2.1% of the loan amount in interest at the end of each year. About how much interest will Alma pay on this loan at the end of the year?

A. $2,644 (B.) $264 C. $15,233 D. $12,853

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.2.2
FAST Practice	5	2	MA.7.AR.2.2
Spiral Review			
	6–9	1	MA.6.AR.2.3
	10–11	2	MA.7.AR.3.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Representing real-world situations with tape diagrams that contain sections labeled with variable expressions	Lesson 2	1	MA.7.AR.2.2, MTR.6.1
2	Using the structure of expressions to select equivalent algebraic expressions	Lesson 8	1	MA.7.AR.1.1, MTR.5.1
3a, 3b	Using structure to solve equations of the form $px + q = r$ and $p(x + q) = r$	Lesson 6	1	MA.7.AR.2.2
4a	Critiquing the work of another student involving writing equivalent expressions using fewer terms	Lesson 10	2	MA.7.AR.1.1, MTR.4.1
4b	Using the structure of expressions and the distributive property to write equivalent expressions using fewer terms	Lesson 10	1	MA.7.AR.1.1
5a	Interpreting variables within the context of a situation	Lesson 12	1	MA.7.AR.3.3, MTR.6.1
5b	Using the structure of expressions and combining like terms to write equivalent expressions using fewer terms	Lesson 12	2	MA.7.AR.3.3, MTR.6.1
5c	Solving multi-step application problems using algebraic expressions and interpreting the solution within context	Lesson 12		MA.7.AR.3.3, MTR.6.1

Assessment and Lesson Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

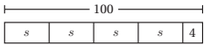
Unit 6

1. Sothy is baking 5 batches of muffins. Each batch uses 4 teaspoons of sugar for the topping and more sugar for the batter.

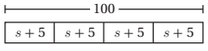
Sothy uses 100 teaspoons of sugar in total.

Which tape diagram matches this situation?

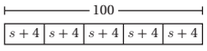
A.



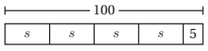
B.



C.



D.



2. Select *all* the expressions that are equivalent to $3(8 - 4x)$.

- ☐ A. $24 - 4x$
- ☑ B. $-12x + 24$
- ☑ C. $2(12 - 6x)$
- ☐ D. $12x - 24$
- ☑ E. $24 - 12x$

3. Solve each equation.

a. $3x + 7 = 40$
$x = 11$

b. $-2(x + 5) = 10$
$x = -10$

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 6

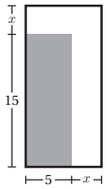
4. Here is Kwame's work writing the expression $4 - 2(x + 5)$ using fewer terms.

Kwame
$4 - 2(x + 5)$
$2(x + 5)$
$2x + 10$

a. Describe the mistake that Kwame made.
Responses vary. Kwame subtracted $4 - 2$ first, but the -2 is actually multiplied by $x + 5$, so that needs to happen before the like terms can be combined.

b. Write an expression equivalent to $4 - 2(x + 5)$ that has two terms.
$-2x - 6$ (or equivalent)

5. Isaiah's vegetable garden is 15 feet long by 5 feet wide. He plans to increase the width and length of his garden by the same amount and put a fence around it.



He writes this expression for the total amount of fencing:
$(x + 15) + (x + 5) + (x + 15) + (x + 5)$.

a. Describe what x represents in this situation.
Responses vary. x is how much longer and wider Isaiah's new garden is than his old garden.

b. Write an equivalent expression that uses fewer terms.
$4x + 40$ (or equivalent)

c. If Isaiah uses 50 feet of fencing in total, how much will the length of his garden increase by?
2.5 feet

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Use tape diagrams to represent equations and situations in context and to determine unknown values. **(Lessons 1–3)**	1	Support	• **Mini-Lesson:** Writing Equations From Descriptions and Tape Diagrams (ML 6.02)
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 2, Activity 1 and discuss Problem 5. Ask students to share with a partner their tap diagram for Liam's new situation.
		Stretch	• **You're invited to explore more:** Lesson 3 (Seeing Structure)
Sub-Unit 2: • Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems. • Write equivalent expressions by adding, subtracting, expanding, and factoring. **(Lessons 4–12)**	2, 4	Support	• **Teacher Move:** Consider revisiting Lesson 7, Activity 1 and asking, "How can you use an area model to help you expand or factor?" Then invite students to revise Problems 2 and 4.
	3	Support	• **Mini-Lesson:** Solving Equations With Positive and Negative Numbers (ML 6.06)
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 5, Activity 2 and asking, "How can you use a hanger diagram to solve the equation?" • **Challenge Creator:** Lesson 4 (Balancing Moves)
	5	Support	• **Teacher Move:** Consider asking, "How does the diagram connect with the expression and situation? What is known and unknown?" Then invite students to collaborate and revise Problem 5.

Sub-Unit 3

Inequalities

Sub-Unit 3 Goals

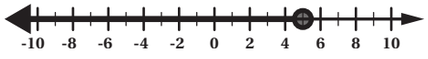
- **Goal:** Solve one-step inequalities with positive and negative coefficients.
- **Goal:** Create graphs that represent solutions to inequalities, including those with ≥ or ≤.
- **Language Goal:** Analyze descriptions of situations in context to interpret the solution of inequalities.

In this sub-unit, students use what they have learned about solving equations to solve inequalities that represent mathematical and real-world problems. They also create graphs that represent **solutions to inequalities**, including those with ≤, less-than-or-equal-to or ≥, greater-than-or-equal-to.



Math That Matters Most

Concept or Skill	For example . . .
Understand and use the symbols ≤ and ≥ to describe inequalities.	To work at an amusement park, employees must be at least 16 years old. Write an inequality to represent this situation. $x \geq 16$
Solve inequalities with positive and negative coefficients.	Solve this inequality. $-6x < 60$ $\frac{-6x}{-6} > \frac{60}{-6}$ $x > -10$

Concept or Skill	For example . . .
Draw and label a graph on the number line that represents an inequality.	Make a graph on the number line that represents the inequality $x \leq 5$. 
Write and solve an inequality to answer a question about a situation with a constraint.	Wey Wey is trying to figure out how many movies she can download to her hard drive. Each movie is 8 gigabytes. The hard drive is supposed to hold 500 gigabytes of data. Write and solve an inequality to represent Wey Wey's situation. Inequality: $8x \leq 500$ Solution: $x \leq 62.5$ Wey Wey can download at most 62 movies.

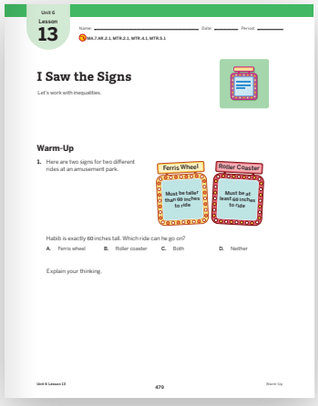
Unit 6

Lesson 13

I Saw the Signs

Inequalities on the Number Line

Let's work with inequalities.


Unit 6
Lesson 13
I Saw the Signs
Let's work with inequalities.
Warm-Up
1. Here are two signs for two different rides at an amusement park.
Ferris Wheel
Must be taller than 60 inches to ride
Roller Coaster
Must be at least 60 inches to ride
Habib is exactly 60 inches tall. Which ride can he go on?
A. Ferris wheel B. Roller coaster C. Both D. Neither
Explain your thinking.
479

Learning Outcomes

Today's Goals

1. **Goal:** Understand and use the symbols ≤ and ≥ to describe inequalities.
2. **Goal:** Draw and label a graph on the number line that represents an inequality.
3. **Language Goal:** Interpret an inequality that represents a context. **(Writing)**

Students build on their knowledge from Grade 6 about graphing inequalities. Students explore inequality symbols using words and graphs as they describe restrictions for riding amusement park rides.

Coherence

Prior Learning

In Grade 6, students learned about the symbols <, >, ≤, and ≥. They also recognized that inequalities have infinitely many solutions and represented those solutions on a number line diagram. In Lesson 12, students wrote and solved equations to answer questions about scenarios in context.

Future Learning

In Lesson 14, students will make sense of solving inequalities using unbalanced hanger diagrams. In Lesson 15, they will work with inequalities with both negative and positive coefficients.

Connections

- Students use inequalities to represent real-world height requirements in contexts like amusement park rides. (real world)

FL B.E.S.T. Math Benchmarks

Building Toward

MA.7.AR.2.1
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On

MA.6.AR.1.2
MA.6.AR.2.1

Vocabulary

Additional Vocabulary

boundary point

Lesson at a Glance

~ 45 min

MA.7.AR.2.1

Warm-Up

Pairs | 5 min

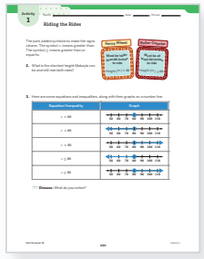
Students attend to precision as they examine common ways of describing greater than and greater than or equal to in the context of height requirements at an amusement park.



Activity 1

Pairs | 10 min

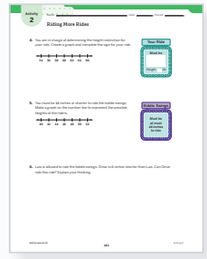
Students explore inequality symbols and connect them to the context of heights at an amusement park and graphs on a number line.



Activity 2

Pairs | 5 min

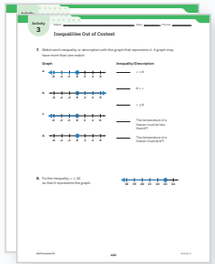
Students connect words, symbols, and graphs as ways of describing who can ride a specific amusement park ride. **(MTR.2.1)**



Activity 3

Pairs | 15 min

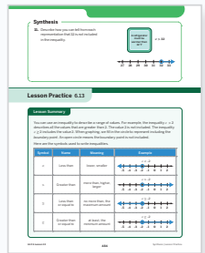
Students use the **Decide and Defend** routine to strengthen their ability to make arguments and to critique other students' reasoning about inequalities out of context. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of inequalities by analyzing multiple representations. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing and graphing an inequality to represent a given context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students attend to precision as they examine common ways of describing greater than and greater than or equal to in the context of height requirements at an amusement park.

1 Launch

To surface connections, ask students if they have ever been too tall or too short to be allowed to do something.

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means, clarifying any unknown words. **(Reading)**

Consider sharing that some amusement parks have restrictions for safety that only allow people of a certain height to ride specific rides.

A **Accessibility: Memory and Attention** To surface prior knowledge, consider asking, "What information have we used earlier that might help us now?"

2 Connect

Display the distribution of responses by polling the class calling attention to any conflict or consensus.

Invite students to share their reasoning for each of the choices. It is ok to lack consensus at this point.

Consider asking:

- "How are the two amusement park signs different?"
- "Do these signs reflect something you would actually see in an amusement park? Why or why not?"

ELL **English Language Learners** Invite students to share different meanings of the term *sign*. Clarify that for this activity, "sign" will refer to the information on the images. Inequalities such as *greater than or equal to* and *less than* or *equal to* are represented by "symbols."



Unit 6
Lesson
13

Name: _____ Date: _____ Period: _____

MA.7.AR.2.1, MTR.2.1, MTR.4.1, MTR.5.1

I Saw the Signs

Let's work with inequalities.

Warm-Up

1. Here are two signs for two different rides at an amusement park.

Ferris Wheel: Must be taller than 60 inches to ride

Roller Coaster: Must be at least 60 inches to ride

Habib is exactly 60 inches tall. Which ride can he go on?

A. Ferris wheel (B.) Roller coaster C. Both D. Neither

Explain your thinking.

Explanations vary. Habib is 60 inches tall, so he can ride the roller coaster. He is not taller than 60 inches, so he can't ride the Ferris wheel.

Unit 6 Lesson 13 479 Warm-Up

Pairs | 10 min

Activity 1 Riding the Rides

Purpose: Students explore inequality symbols and connect them to the context of heights at an amusement park and graphs on a number line.

1 Launch

Invite students to read the prompt aloud to a partner, restate it in their own words, and prepare to share with the class.

To encourage creativity, ask students why they think these symbols were chosen and if they would select different symbols, if given the chance.

2 Monitor

Look for a variety of responses about Makayla's possible height. Celebrate the precision of students who write things like 60.00001 and connect their response to the signs.

Invite students to look at the equations and inequalities alongside their graphs and share their thinking with a partner.

Encourage students to look for patterns across the inequalities and graphs. **(MTR.5.1)**

3 Connect

Invite students to share their responses, calling attention to each type of inequality.

Consider asking:

- "How do the words and symbols change when you change the graph from having a filled-in *boundary point* to an empty circle?"
- "If $\geq$ represents greater than or equal to, what do you think $\leq$ would represent?"
- "Where do you see connections between the words, the symbol, and the graph?" **(MTR.2.1)**

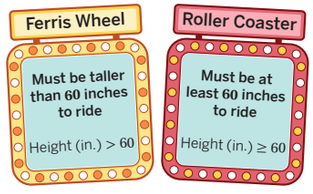
ELL **English Language Learners** Consider creating an anchor chart about *greater than, greater than or equal to, less than,* and *less than or equal to*. Add student ideas to the chart throughout the lesson. **(Writing and Speaking)**

Activity 1

Name: Date: Period:

Riding the Rides

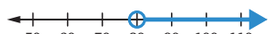
The park added symbols to make the signs clearer. The symbol $>$ means greater than. The symbol $\geq$ means greater than or equal to.



2. What is the shortest height Makayla can be and still ride both rides?

Responses vary.
- 61 inches
- 60.00001 inches

3. Here are some equations and inequalities, along with their graphs on a number line.

Equation/Inequality	Graph
$x = 80$	
$x < 80$	
$x > 80$	
$x \leq 80$	
$x \geq 80$	

Discuss: What do you notice?

Responses vary.
- I notice that when the circle is filled in, the symbol is $\leq$ or $\geq$, and when the circle is empty, the symbol is $<$ or $>$.
- I notice that when the symbol is $=$, there is no arrow and the circle is filled in.

Unit 6 Lesson 13 480 Activity 1

Pairs | 5 min

Activity 2 Riding More Rides

Purpose: Students connect words, symbols, and graphs as ways of describing who can ride a specific amusement park ride. **(MTR.2.1)**

1 Launch

To generate excitement, consider asking, "What kind of ride do you think you want to make?"

Consider asking, "How does the inequality symbol you choose affect the wording and the graph?"

2 Monitor

Consider sharing a picture of a kiddie swing to increase access to the task.

A **Accessibility: Conceptual Processing** To support students with thinking abstractly, demonstrate an incorrect solution and ask, "What information from the signs matches? What doesn't match?"

Capture student responses to share during the Connect.

D **Differentiation**

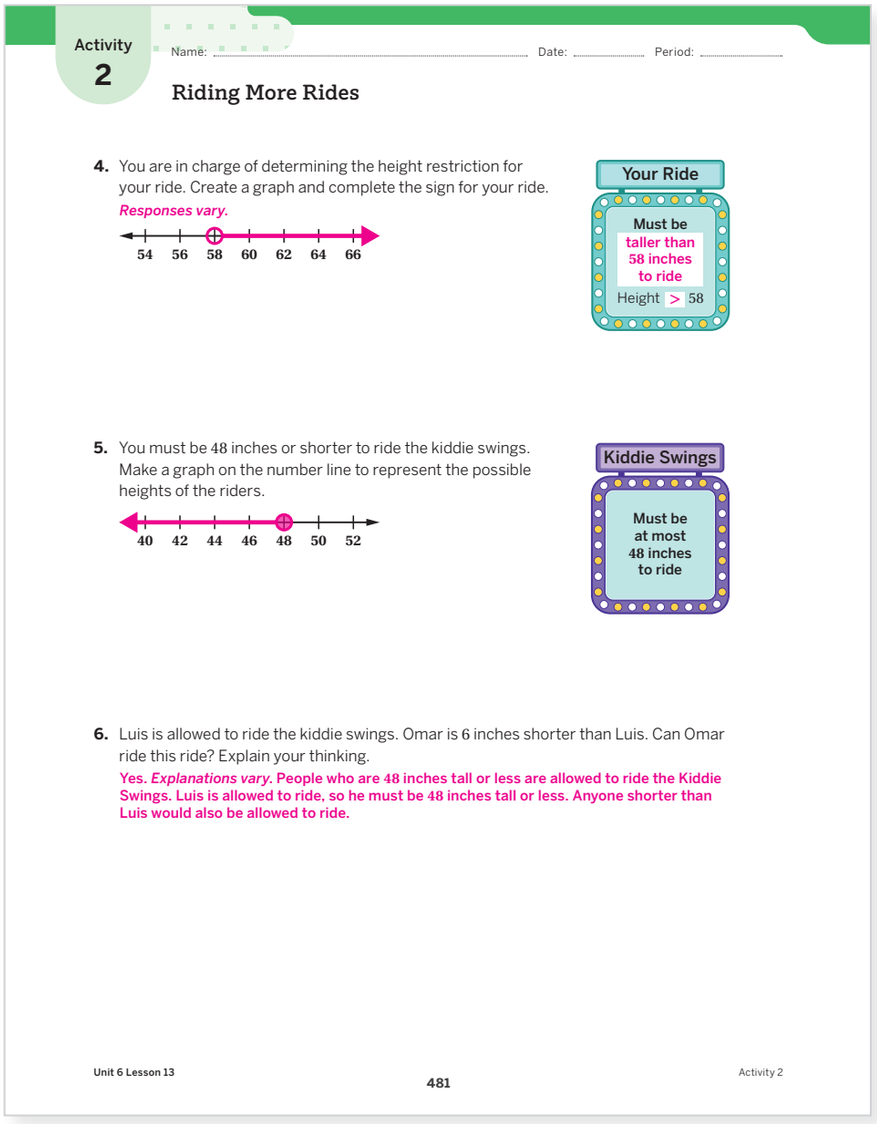
Look for students who:	Respond to Student Thinking
Graph an open boundary point on 48.	**Support** Invite students to reflect on the feedback to adjust their graph. Consider asking, "Could someone who is exactly 48 inches tall ride the kiddie swings?"
Graph an arrow pointing to the right.	**Support** Invite students to reflect on the feedback to adjust their graph.
Graph a closed boundary point on 48 and an arrow pointing to the left.	**Strengthen** Consider asking, "What is a different way to word this sign?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., Omar can/cannot ride the ride because _____.). **(Writing and Speaking)**

3 Connect

Share the distribution of responses and invite students to defend their position using mathematics and reasoning. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "Which representation (words, symbols, or graphs) feels most helpful to you? Why?" **(MTR.2.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Riding More Rides

4. You are in charge of determining the height restriction for your ride. Create a graph and complete the sign for your ride.

Responses vary.

54 56 58 60 62 64 66

Your Ride

Must be taller than 58 inches to ride

Height > 58

5. You must be 48 inches or shorter to ride the kiddie swings. Make a graph on the number line to represent the possible heights of the riders.

40 42 44 46 48 50 52

Kiddie Swings

Must be at most 48 inches to ride

6. Luis is allowed to ride the kiddie swings. Omar is 6 inches shorter than Luis. Can Omar ride this ride? Explain your thinking.

Yes. *Explanations vary.* People who are 48 inches tall or less are allowed to ride the Kiddie Swings. Luis is allowed to ride, so he must be 48 inches tall or less. Anyone shorter than Luis would also be allowed to ride.

Unit 6 Lesson 13 481 Activity 2

Key Takeaway: Words, symbols, and graphs are all different ways of expressing the same idea in an inequality.

Pairs | 15 min

Activity 3 Inequalities Out of Context

Purpose: Students use the **Decide and Defend** routine to strengthen their ability to make arguments and to critique other students' reasoning about inequalities out of context. **(MTR.4.1)**

Short on time? Consider omitting Problems 7 and 10.

1 Launch

Invite students to share what they notice about the graphs with a partner before beginning the activity.

ELL **English Language Learners** Invite students to identify unknown terms and annotate their meanings (e.g., *temperature* and *freezer*). **(Reading)**

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., asking students to focus on one number line at a time).

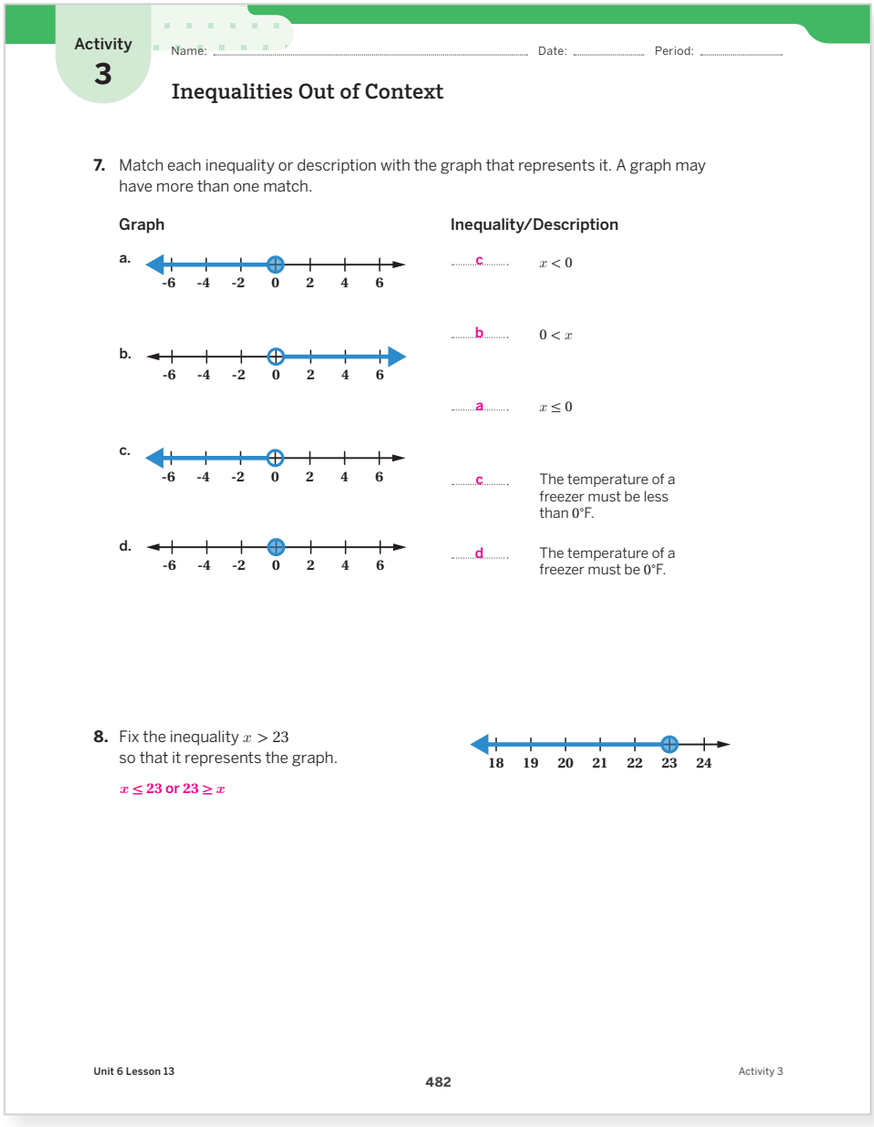
2 Monitor

Invite students to compare their thinking with a partner before moving on.

Encourage students to choose either the inequality or the graph and predict what the other representation would look like before revising the given response.

D **Differentiation: (Support)** Students may guess about which direction their inequality symbol should go. Consider encouraging them to make meaning of each symbol by asking, "What values would make this inequality true? Does that match our current graph?"

Activity 3 continued >



Activity 3

Name: Date: Period:

Inequalities Out of Context

7. Match each inequality or description with the graph that represents it. A graph may have more than one match.

Graph	Inequality/Description
a.	c $x < 0$
b.	b $0 < x$
	a $x \leq 0$
c.	c The temperature of a freezer must be less than 0°F.
d.	d The temperature of a freezer must be 0°F.

8. Fix the inequality $x > 23$ so that it represents the graph.

$x \leq 23$ or $23 \geq x$

Unit 6 Lesson 13 482 Activity 3

Pairs | 15 min

Activity 3 Inequalities Out of Context (continued)

Purpose: Students use the **Decide and Defend** routine to strengthen their ability to make arguments and to critique other students' reasoning about inequalities out of context. **(MTR.4.1)**

Short on time? Consider omitting Problems 7 and 10.

2 Monitor

Listen for students making connections between the graph and the inequality they wrote, using phrases such as "greater than" or "to the right" to justify their response.

Consider asking, "What do you notice on this graph? Where do you see a boundary point? Is it included in the solution?" **(MTR.2.1)**

Select and sequence explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Write minimal explanations.	**Support** Consider asking, "What other information can you add to the explanation to help your classmates better understand your thinking?"
Write creative explanations for each response. See sample responses for examples.	**Strengthen** Invite students to share during the Connect.

3 Connect

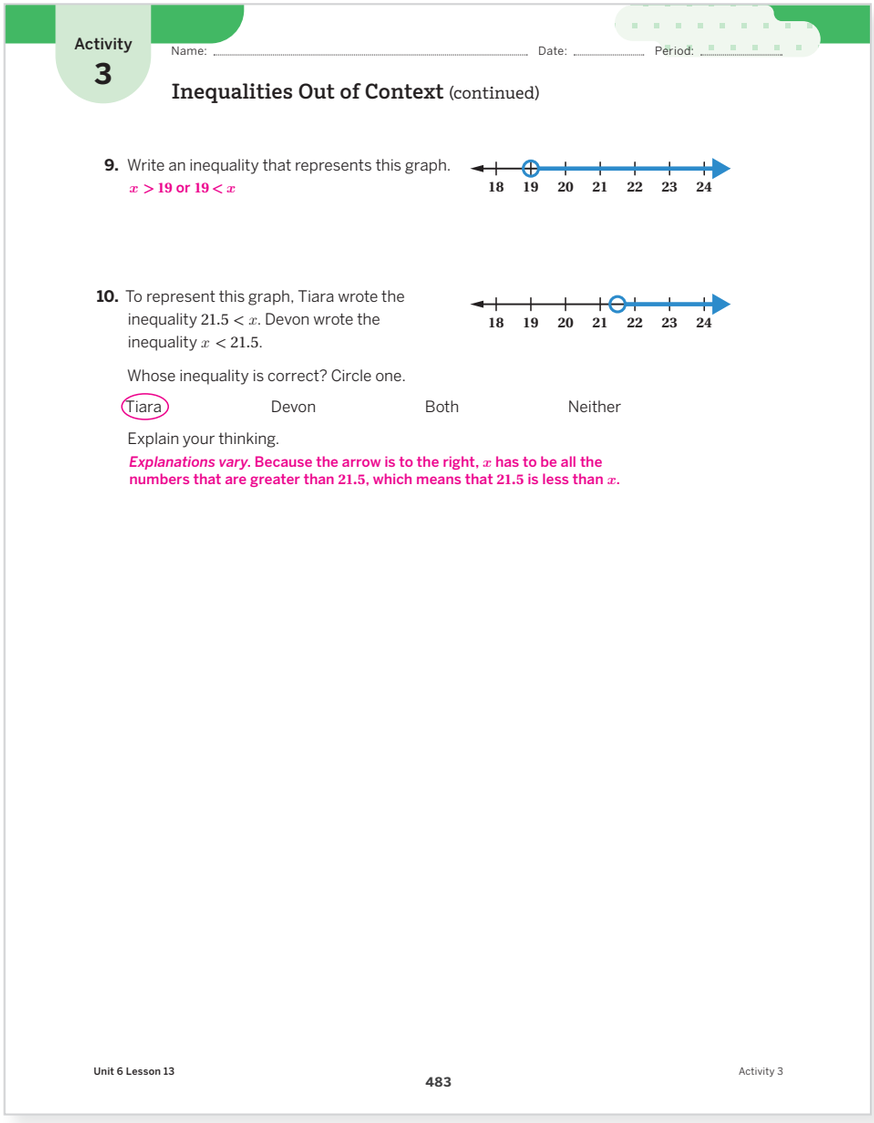
Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Note: If everyone is in agreement, consider inviting students to imagine what an argument would be for one of the other responses.

Consider asking:

- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"



Activity 3

Name: Date: Period:

Inequalities Out of Context (continued)

9. Write an inequality that represents this graph.

$x > 19$ or $19 < x$

10. To represent this graph, Tiara wrote the inequality $21.5 < x$. Devon wrote the inequality $x < 21.5$.

Whose inequality is correct? Circle one.

(Tiara) Devon Both Neither

Explain your thinking.

Explanations vary. Because the arrow is to the right, x has to be all the numbers that are greater than 21.5, which means that 21.5 is less than x.

Unit 6 Lesson 13 483 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of inequalities by analyzing multiple representations.

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., In the inequality, _____. In the number line, _____. In the sign, _____). **(Writing and Speaking)**

Capture and share a variety of ideas, such as:

- The inequality symbol does not have a line under it.
- The boundary point is open, not filled in.
- The words say "warmer than," not "at most" or "at least."

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *greater-than-or-equal-to* and *less-than-or-equal-to* using students' language. Display any new language, examples, or diagrams for students to reference later.

Consider asking, "Why are each of these three representations useful or powerful?" Consider adding these responses to the anchor chart.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Describe how you can tell from each representation that 32 is *not* included in the inequality.

Responses vary. In the inequality, the symbol is >, not ≥. In the graph, there is an open circle, so 32 is not included in the inequality. In the sign, it says "warmer than," which is like "more than." 32 is not more than 32.

Lesson Practice 6.13

Lesson Summary

You can use an inequality to describe a range of values. For example, the inequality $x > 2$ describes all the values that are greater than 2. The value 2 is not included. The inequality $x \geq 2$ includes the value 2. When graphing, we fill in the circle to represent including the *boundary point*. An open circle means the boundary point is not included.

Here are the symbols used to write inequalities.

Symbol	Name	Meaning	Example
$<$	Less than	lower, smaller	$x < -2$
$>$	Greater than	more than, higher, larger	$x > -2$
$\leq$	Less than or equal to	no more than, the maximum amount	$x \leq -2$
$\geq$	Greater than or equal to	at least, the minimum amount	$x \geq -2$

Unit 6 Lesson 13 484 Synthesis | Lesson Practice

Lesson Takeaway: There are inequality symbols that include the boundary point and they can be represented as graphs and with words.

Independent | 5 min

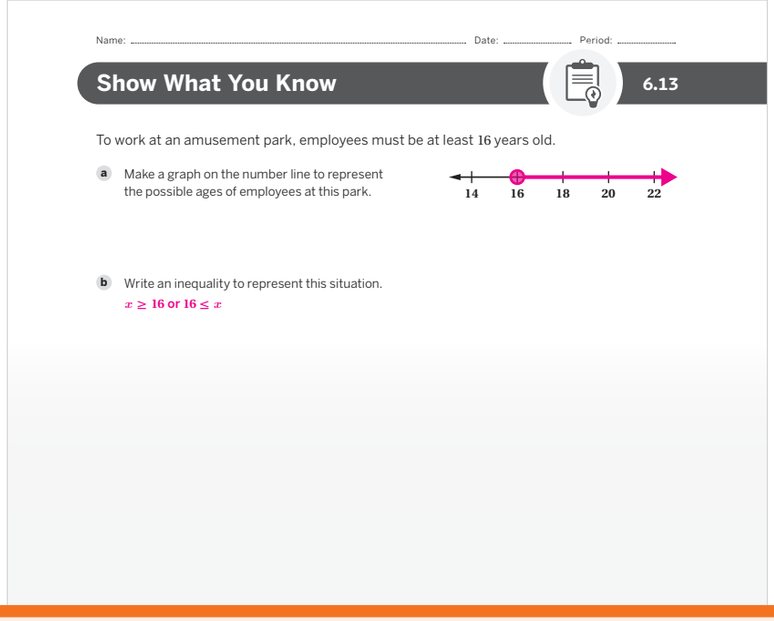
Show What You Know

Purpose: Students demonstrate their understanding by writing and graphing an inequality to represent a given context.

Today's Goals

1. **Goal:** Understand and use the symbols ≤ and ≥ to describe inequalities.
2. **Goal:** Draw and label a graph on the number line that represents an inequality.
3. **Language Goal:** Interpret an inequality that represents a context. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.13

To work at an amusement park, employees must be at least 16 years old.

a. Make a graph on the number line to represent the possible ages of employees at this park.

14 16 18 20 22

b. Write an inequality to represent this situation.

$x \geq 16$ or $16 \leq x$

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to draw and label a graph on the number line to represent an inequality:

» **Respond to Student Thinking:**

- Use the ***Identifying Solutions to Inequalities*** **Mini-Lesson** from Grade 6, Unit 7.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

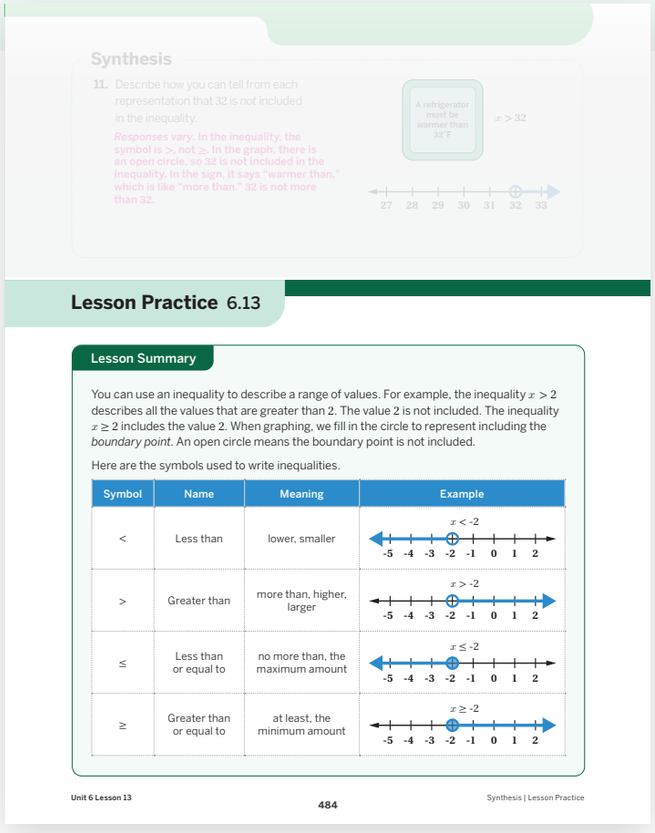
Professional Learning

Take a moment to reflect on the **Decide and Defend** routine in Activity 3 of this lesson. How did you support students in making a clear and concise claim, asking clarifying questions, and posing counterarguments? What about the routine felt successful?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. Describe how you can tell from each representation that 32 is *not* included in the inequality.

Responses vary. In the inequality, the symbol is >, not ≥. In the graph, there is an open circle, so 32 is not included in the inequality. In the sign, it says "warmer than," which is like "more than." 32 is not more than 32.

A refrigerator must be warmer than 32°F

$x > 32$

27 28 29 30 31 32 33

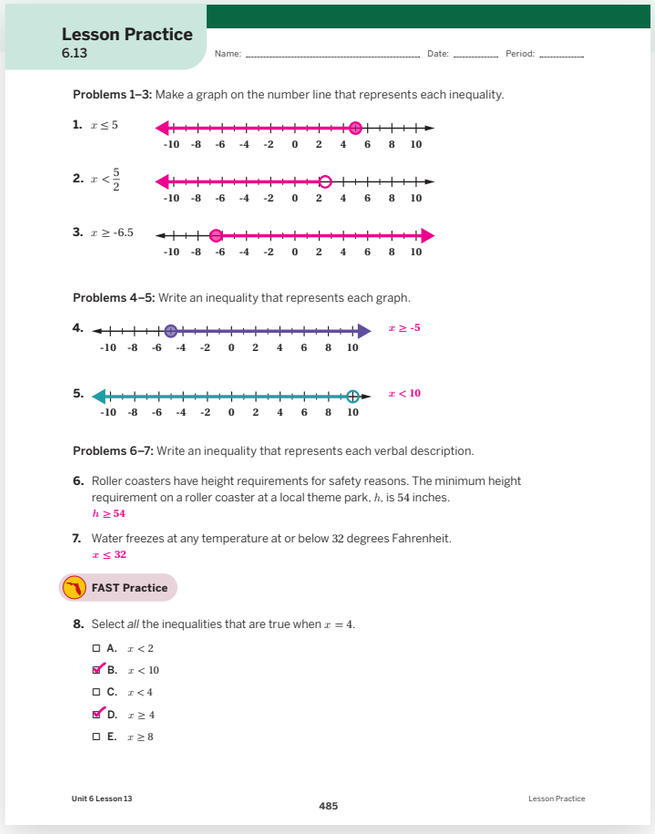
Lesson Practice 6.13

Lesson Summary

You can use an inequality to describe a range of values. For example, the inequality $x > 2$ describes all the values that are greater than 2. The value 2 is not included. The inequality $x \geq 2$ includes the value 2. When graphing, we fill in the circle to represent including the *boundary point*. An open circle means the boundary point is not included.

Here are the symbols used to write inequalities.

Symbol	Name	Meaning	Example
<	Less than	lower, smaller	$x < -2$
>	Greater than	more than, higher, larger	$x > -2$
≤	Less than or equal to	no more than, the maximum amount	$x \leq -2$
≥	Greater than or equal to	at least, the minimum amount	$x \geq -2$



Lesson Practice 6.13

Name: Date: Period:

Problems 1–3: Make a graph on the number line that represents each inequality.

1. $x \leq 5$

2. $x < \frac{5}{2}$

3. $x \geq -6.5$

-10 -8 -6 -4 -2 0 2 4 6 8 10

Problems 4–5: Write an inequality that represents each graph.

4. $x \geq -5$

5. $x < 10$

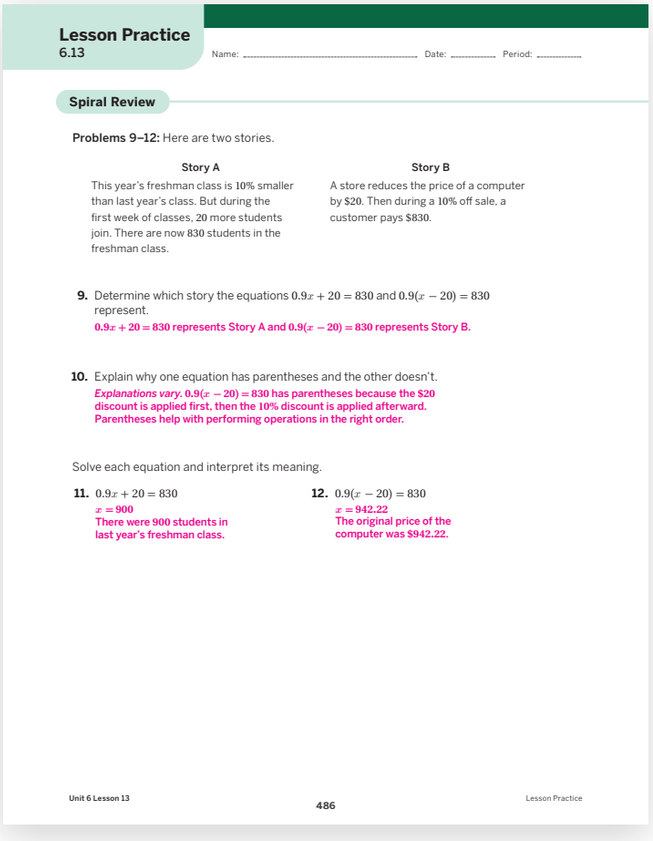
Problems 6–7: Write an inequality that represents each verbal description.

6. Roller coasters have height requirements for safety reasons. The minimum height requirement on a roller coaster at a local theme park, h, is 54 inches.
   $h \geq 54$

7. Water freezes at any temperature at or below 32 degrees Fahrenheit.
   $x \leq 32$

FAST Practice

8. Select *all* the inequalities that are true when $x = 4$.
   - ☐ A. $x < 2$
   - ☑ B. $x < 10$
   - ☐ C. $x < 4$
   - ☑ D. $x \geq 4$
   - ☐ E. $x \geq 8$



Lesson Practice 6.13

Name: Date: Period:

Spiral Review

Problems 9–12: Here are two stories.

Story A

This year's freshman class is 10% smaller than last year's class. But during the first week of classes, 20 more students join. There are now 830 students in the freshman class.

Story B

A store reduces the price of a computer by $20. Then during a 10% off sale, a customer pays $830.

9. Determine which story the equations $0.9x + 20 = 830$ and $0.9(x - 20) = 830$ represent.
   $0.9x + 20 = 830$ represents Story A and $0.9(x - 20) = 830$ represents Story B.

10. Explain why one equation has parentheses and the other doesn't.
    Explanations vary. $0.9(x - 20) = 830$ has parentheses because the $20 discount is applied first, then the 10% discount is applied afterward. Parentheses help with performing operations in the right order.

Solve each equation and interpret its meaning.

11. $0.9x + 20 = 830$
    $x = 900$
    There were 900 students in last year's freshman class.

12. $0.9(x - 20) = 830$
    $x = 942.22$
    The original price of the computer was $942.22.

Lesson Practice Item Analysis

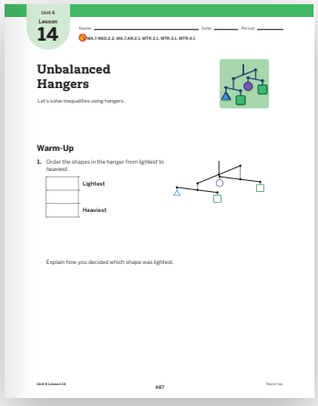
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	1	MA.7.AR.2.1*
	6–7	2	MA.7.AR.2.1*
FAST Practice	8	1	MA.7.AR.2.1*
Spiral Review			
	9–10	2	MA.7.AR.2.2
	11–12	2	MA.7.AR.2.2

*This problem builds toward the standard shown.

Unit 6

Lesson

14


Unit 6
Lesson
14
Name: Date: Period:
MA.7.NSO.2.2, MA.7.AR.2.1, MTR.2.1, MTR.3.1, MTR.4.1
Unbalanced Hangers
Let's solve inequalities using hangers.
Warm-Up
1. Order the shapes in the hanger from lightest to heaviest.
Lightest
Heaviest
Explain how you decided which shape was lightest.
487
Warm-Up

Unbalanced Hangers

Solutions to Inequalities

Let's solve inequalities using hangers.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the *solutions to an inequality* with only positive numbers.
2. **Language Goal:** Compare and contrast solutions to equations and *solutions to inequalities*. **(Speaking and Listening)**

Students determine **solutions to inequalities** using a familiar representation: hanger diagrams. Students connect solving an inequality with determining the values of x that will keep one side of a hanger heavier than the other, and they recognize that the solutions to inequalities are themselves inequalities.

Coherence

Prior Learning

In Lesson 13, students revisited inequalities and were introduced to greater-than-or-equal-to and less-than-or-equal-to symbols. They drew and labeled graphs that represent inequalities on number lines.

Future Learning

In Lesson 15, students will practice solving inequalities with both positive and negative coefficients and connect the solutions of inequalities to their graphs.

Connections

- Students use hanger diagrams as a visual way to solve inequalities by representing the relationship between variables and constants. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.1
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.2	**MA.8.AR.2.2**
MA.6.AR.2.1	

Vocabulary

New Vocabulary

solutions to an inequality

Lesson at a Glance

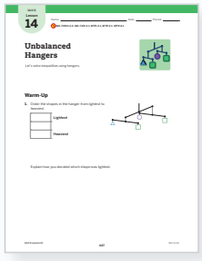
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.1

Warm-Up

Pairs | 5 min

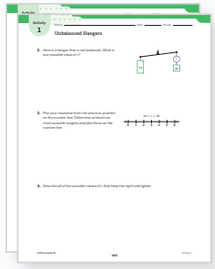
Students revisit hangers and reason about relative weights of objects on an unbalanced hanger. **Routine:** Think-Pair-Share



Activity 1

Pairs | 15 min

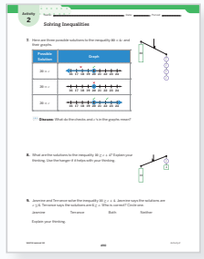
Students connect unbalanced hangers, inequalities, and their solutions. They explore why the **solutions to an inequality** are also inequalities. **Routine:** Decide and Defend **(MTR.2.1)**



Activity 2

Pairs | 10 min

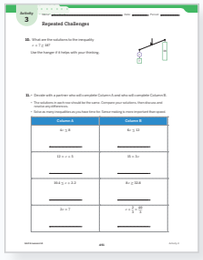
Students develop strategies for determining solutions to inequalities, making connections to equations, and using unbalanced hangers to help them revise their thinking.



Activity 3

Independent | 5 min

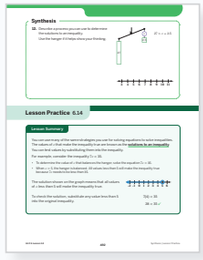
Students practice solving inequalities with positive values using a set of partner problems. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

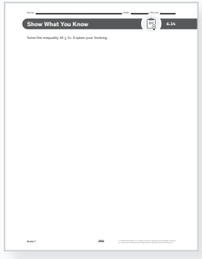
Students synthesize their understanding of how to solve an inequality.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving an inequality and explaining their thinking.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students revisit hangers and reason about relative weights of objects on an unbalanced hanger.

1 Launch

Invite students to share what they notice about the hanger.

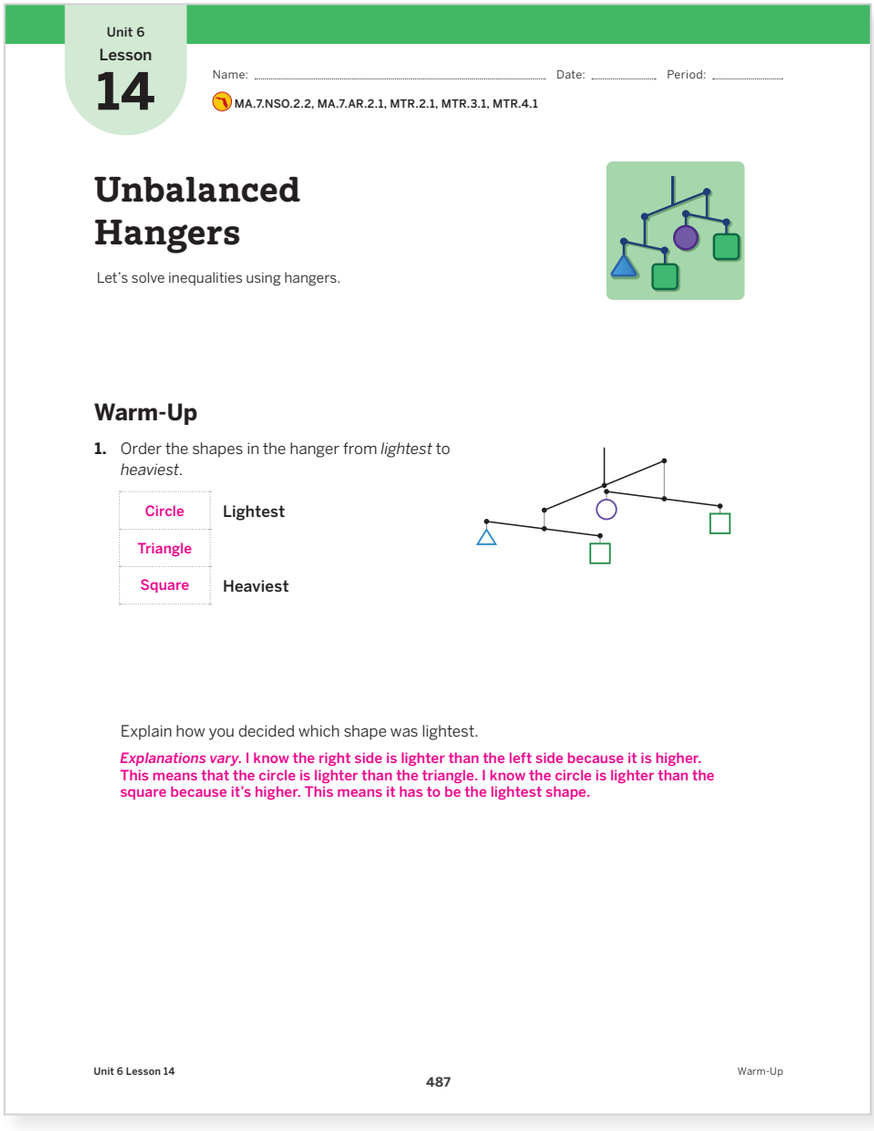
Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's strategy with the class. **(MTR.4.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., The ____ is lighter/heavier than the ____ because ____.). **(Writing and Speaking)**

2 Connect

Display a few student explanations. Invite several students to share their reasoning.

To surface the Key Takeaway, consider asking, "Which side of an unbalanced hanger is heavier? How do you know?"



Unit 6
Lesson
14

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.AR.2.1, MTR.2.1, MTR.3.1, MTR.4.1

Unbalanced Hangers

Let's solve inequalities using hangers.

Warm-Up

1. Order the shapes in the hanger from *lightest* to *heaviest*.

Circle	Lightest
Triangle	
Square	Heaviest

Explain how you decided which shape was lightest.

Explanations vary. I know the right side is lighter than the left side because it is higher. This means that the circle is lighter than the triangle. I know the circle is lighter than the square because it's higher. This means it has to be the lightest shape.

Unit 6 Lesson 14 487 Warm-Up

Key Takeaway: Unbalanced hangers are a visual representation for inequalities.

Pairs | 15 min

Activity 1 Unbalanced Hangers

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the **solutions to an inequality** are also inequalities. **(MTR.2.1)**

1 Launch

Invite students to share what they notice and wonder about the hanger. Listen for and amplify words such as *heavier*, *lighter*, *tilted*, and *unbalanced*.

Note: In previous grades and lessons, students have solved problems to make a hanger balanced. In this lesson, students will make sense of unbalanced hangers.

2 Monitor

Look for students' progress, checking in with students as needed.

Encourage students to record several values that do and do not make the left side of the hanger heavier.

Invite students to check their values with a partner.

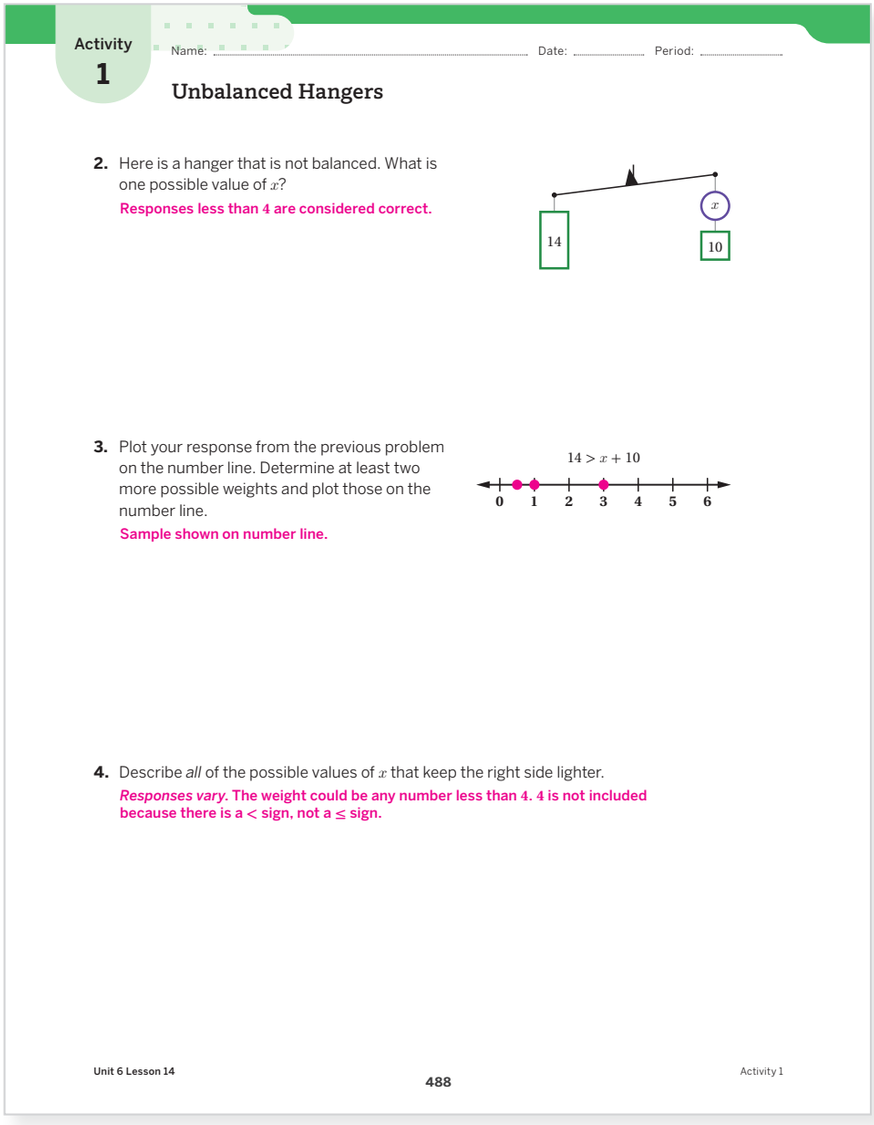
Capture several responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to make connections between the dots on the number line and the hanger.
Discuss 4 as a point that makes it equal or balanced.	**Support** Consider asking, "Does that keep the right side lighter? What is important about this point?" Consider capturing this to show during the Connect.
List specific numbers (1, 2, 5, etc.).	**Strengthen** Consider asking, "Are there other solutions that aren't listed here yet? How could we incorporate those?"
Write "any number less than 4."	**Strengthen** Consider asking, "How might we represent that using symbols?"
Write $x < 4$.	**Stretch** Challenge students to describe the largest and the smallest solutions.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., All of the possible weights are _____ because _____.). **(Writing and Speaking)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Unbalanced Hangers

2. Here is a hanger that is not balanced. What is one possible value of x?

Responses less than 4 are considered correct.

3. Plot your response from the previous problem on the number line. Determine at least two more possible weights and plot those on the number line.

Sample shown on number line.

4. Describe *all* of the possible values of x that keep the right side lighter.

Responses vary. **The weight could be any number less than 4. 4 is not included because there is a < sign, not a ≤ sign.**

Unit 6 Lesson 14 488 Activity 1

Pairs | 15 min

Activity 1 Unbalanced Hangers (continued)

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the **solutions to an inequality** are also inequalities. **(MTR.2.1)**

2 Monitor

Display the distribution of responses by polling the class.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Note: If it doesn't come up naturally, consider asking:

- "What would the hanger look like for each of the responses?"
- "What strategies can you use to solve inequalities such as $3x \leq 24$?"

Listen for students who reference the graph of $3x = 24$ as they create the graph of $3x \leq 24$.

Invite students to share their thoughts aloud during the whole-class discussion.

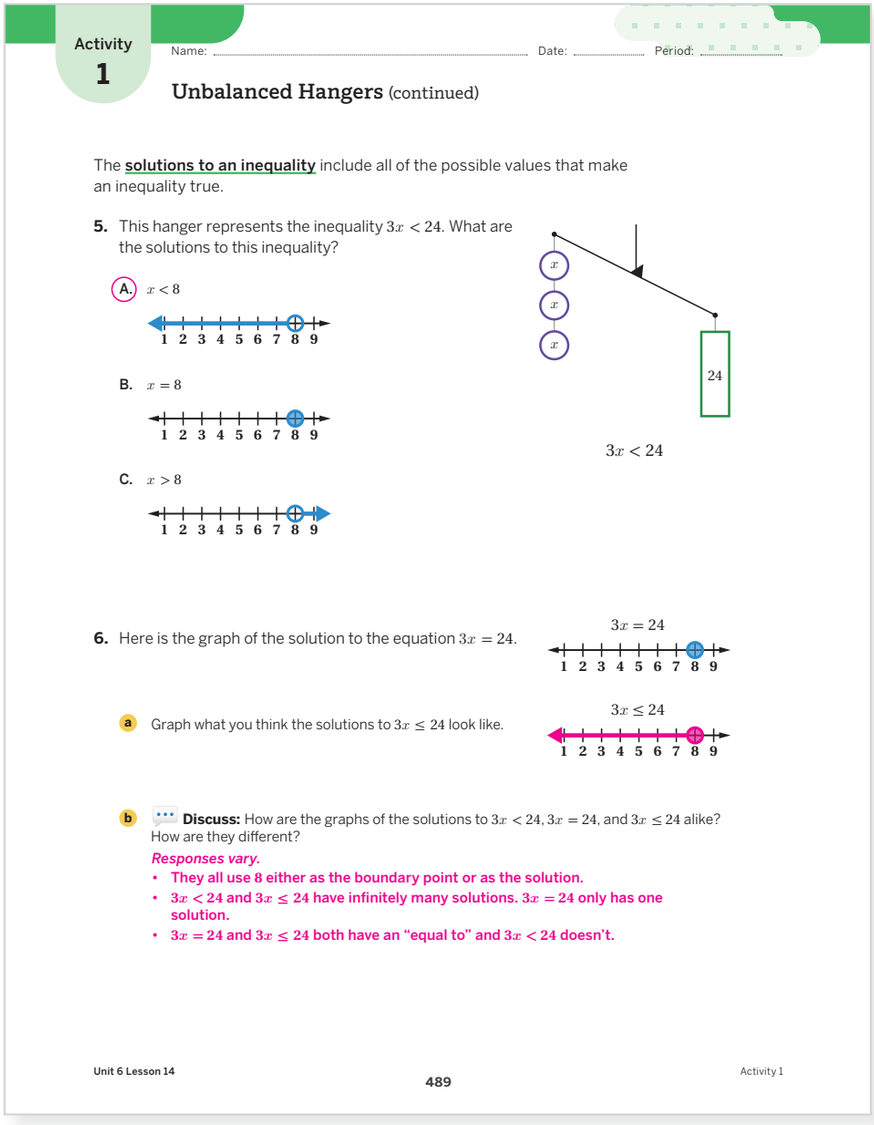
3 Connect

Share the captured strategies to connect the unbalanced hanger, the inequality, and the graph. Consider opening with a strategy where the student identified 4 as the balancing point. **(MTR.2.1)**

Consider asking, "How can you tell from each representation that the 4 isn't included in the inequality?"

To surface connections with Problem 4, consider asking, "How can you tell that the 8 is included in the inequality?"

To surface the Key Takeaway, consider asking, "What do solutions to inequalities look like graphically? Symbolically?"



Activity 1

Name: Date: Period:

Unbalanced Hangers (continued)

The **solutions to an inequality** include all of the possible values that make an inequality true.

5. This hanger represents the inequality $3x < 24$. What are the solutions to this inequality?

$3x < 24$

A. $x < 8$

B. $x = 8$

C. $x > 8$

6. Here is the graph of the solution to the equation $3x = 24$.

$3x = 24$

a. Graph what you think the solutions to $3x \leq 24$ look like.

$3x \leq 24$

b. **Discuss:** How are the graphs of the solutions to $3x < 24$, $3x = 24$, and $3x \leq 24$ alike? How are they different?

Responses vary.

- They all use 8 either as the boundary point or as the solution.
- $3x < 24$ and $3x \leq 24$ have infinitely many solutions. $3x = 24$ only has one solution.
- $3x = 24$ and $3x \leq 24$ both have an "equal to" and $3x < 24$ doesn't.

Unit 6 Lesson 14 489 Activity 1

Key Takeaway: **Solutions to inequalities** are written as inequalities because there are a range of values that make them true.

Pairs | 10 min

Activity 2 Solving Inequalities

Purpose: Students develop strategies for determining solutions to inequalities, making connections to equations, and using unbalanced hangers to help them revise their thinking.

1 Launch

To surface connections, consider asking, "How could you solve this problem if it were an equation?"

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does the green check above 20 mean? What about the red x above 20?"

2 Monitor

ELL **English Language Learners** Encourage students to use gestures or hand motions to help them predict what the hanger should look like. **(Reading and Listening)**

Look for students who justify the response "both" in creative ways, and capture several responses to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Have minimal or incomplete explanations.	**Support** Encourage students to justify their thinking because their ideas are valuable for the class discussion that is coming up.
Choose Terrance because the inequality sign is facing the same direction as the original.	**Support** Consider asking, "What is the difference between Jasmine's and Terrance's solutions?" Encourage students to test values.
Choose both.	**Strengthen** Consider asking, "Are there always multiple ways to write inequalities? How do you know?"

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

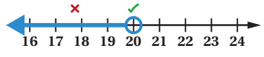
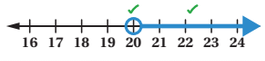
To surface the Key Takeaway, consider asking, "How is solving an inequality like solving an equation? How is it different?"

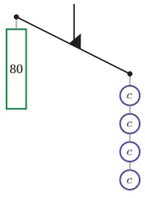
Activity 2

Name: Date: Period:

Solving Inequalities

7. Here are three possible solutions to the inequality $80 < 4c$ and their graphs.

Possible Solution	Graph
$20 > c$	
$20 = c$	
$20 < c$	

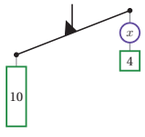


Discuss: What do the checks and x's in the graphs mean?

Responses vary. In the last row of the table, the check on the open circle on 20 means 20 is the boundary. The x in the middle row means that 20 is not included. The check on the arrow facing right means this is correct because the solution for c includes all of the values that are greater than 20.

8. What are the solutions to the inequality $10 \geq x + 4$? Explain your thinking. Use the hanger if it helps with your thinking.

$x \leq 6$ or $6 \geq x$. *Explanations vary.* First, I determined the boundary point of 6, which keeps the hanger balanced because of the "equal to" in the inequality. A value greater than 6 will make the right side of the hanger heavier. A value less than 6 will keep the hanger lighter on the right as shown in the hanger.



9. Jasmine and Terrance solve the inequality $10 \geq x + 4$. Jasmine says the solutions are $x \leq 6$. Terrance says the solutions are $6 \geq x$. Who is correct? Circle one.

Jasmine Terrance (Both) Neither

Explain your thinking.

Explanations vary. These both mean the same thing because $x \leq 6$ and $6 \geq x$ are both true when x is any number less than or equal to 6.

Unit 6 Lesson 14 490 Activity 2

Key Takeaway: One strategy for solving inequalities is to solve a related equation to determine the boundary point, then test values to determine the direction of the solutions.

Independent | 5 min

Activity 3 Repeated Challenges

Purpose: Students practice solving inequalities with positive values using a set of partner problems. **(MTR.3.1)**

Short on time? Consider omitting this activity.

1 Launch

Invite students to make a prediction about the solution before solving.

2 Monitor

Look for a variety of strategies and consider creating a record of them to support students with the repeated challenges.

Invite students to set their own goal for how many repeated challenges they would like to complete and share that sense-making is more important than speed.

Encourage students to solve and compare their solutions with a partner.

3 Connect

Invite students to share strategies that they found helpful or mistakes they made while solving these challenges. **(MTR.3.1)**

Activity 3

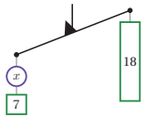
Name: Date: Period:

Repeated Challenges

10. What are the solutions to the inequality $x + 7 \geq 18$?

Use the hanger if it helps with your thinking.

$x \geq 11$ or $11 \leq x$



11. • Decide with a partner who will complete Column A and who will complete Column B.

- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many inequalities as you have time for. Sense-making is more important than speed.

Column A	Column B
$4x \leq 8$ $x \leq 2$	$6x \leq 12$ $x \leq 2$
$12 > x + 5$ $7 > x$	$15 > 3x$ $5 > x$
$10.4 \leq x + 2.2$ $8.2 \leq x$	$8x \geq 32.8$ $x \geq 4.1$
$2x > 7$ $x > \frac{7}{2}$	$x + \frac{2}{3} > \frac{40}{3}$ $x > \frac{38}{3}$

Unit 6 Lesson 14 491 Activity 3

Whole Class | 5 min

Synthesis

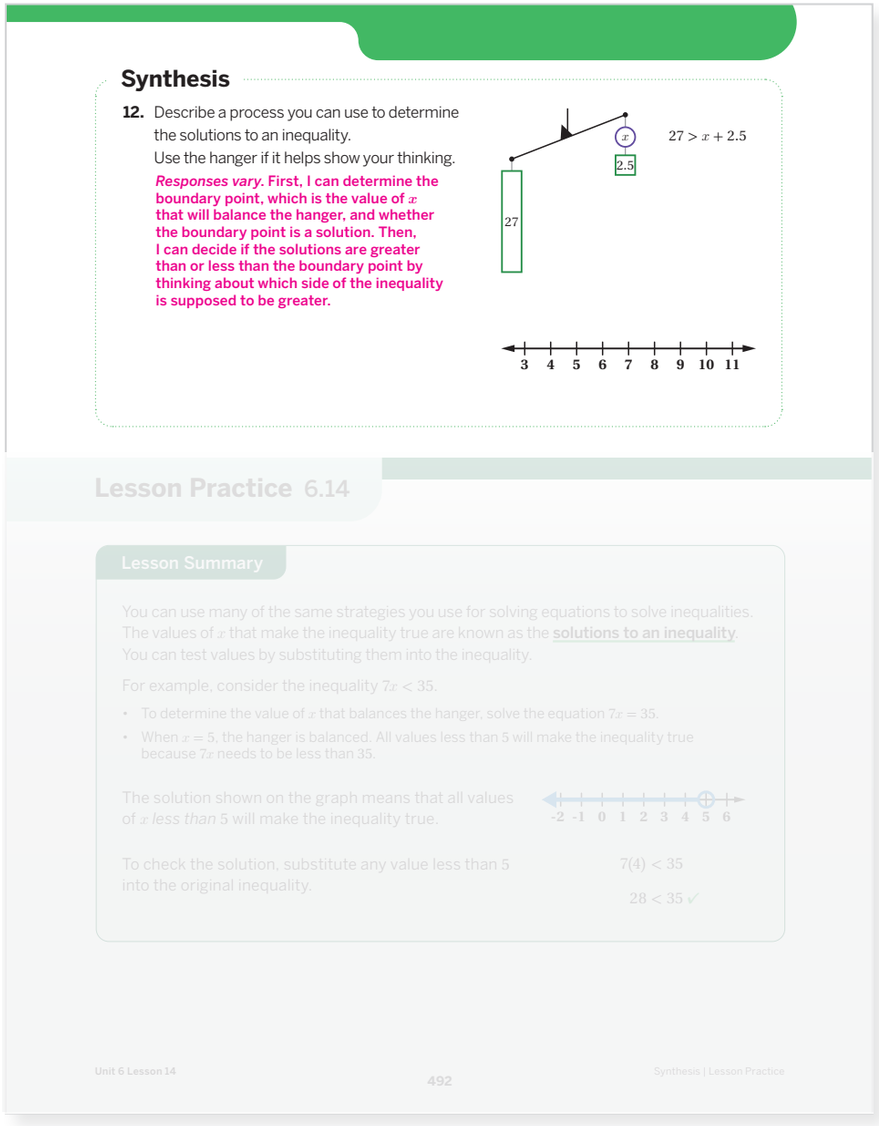
Purpose: Students synthesize their understanding of how to solve an inequality.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Connecting the inequality to the hanger.
- Solving as if this were an equation to determine the boundary point.
- Substituting values for x and testing if they make the inequality true.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.

Responses vary. First, I can determine the boundary point, which is the value of x that will balance the hanger, and whether the boundary point is a solution. Then, I can decide if the solutions are greater than or less than the boundary point by thinking about which side of the inequality is supposed to be greater.

Lesson Practice 6.14

Lesson Summary

You can use many of the same strategies you use for solving equations to solve inequalities. The values of x that make the inequality true are known as the solutions to an inequality. You can test values by substituting them into the inequality.

For example, consider the inequality $7x < 35$.

- To determine the value of x that balances the hanger, solve the equation $7x = 35$.
- When $x = 5$, the hanger is balanced. All values less than 5 will make the inequality true because $7x$ needs to be less than 35.

The solution shown on the graph means that all values of x *less than* 5 will make the inequality true.

-2 -1 0 1 2 3 4 5 6

To check the solution, substitute any value less than 5 into the original inequality.

$7(4) < 35$

$28 < 35$ ✓

Unit 6 Lesson 14 492 Synthesis | Lesson Practice

Lesson Takeaway: An inequality can be solved by determining the boundary point and then testing a value to determine the direction for the graph.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving an inequality and explaining their thinking.

Today's Goals

1. **Goal:** Determine the *solutions to an inequality* with only positive numbers.
2. **Language Goal:** Compare and contrast solutions to equations and *solutions to inequalities*. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.14

Solve the inequality $19 \geq 2x$. Explain your thinking.

$9.5 \geq x$ (or equivalent). *Explanations vary.* First, I found the boundary point, which is $x = 9.5$ because that is the solution to the equation $19 = 2x$. The solution to the inequality is $9.5 \geq x$ because 9.5 is the greatest value that makes the inequality true.

D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve an inequality:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop understanding of inequalities in Lessons 15–16. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students developed early ideas of **solutions to an inequality**. How will their understanding of *solutions to an inequality* build over the next several lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.

Responses vary. First, I can determine the boundary point, which is the value of x that will balance the hanger, and whether the boundary point is a solution. Then, I can decide if the solutions are greater than or less than the boundary point by thinking about which side of the inequality is supposed to be greater.

Lesson Practice 6.14

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Lesson Practice 6.14

Name: _____ Date: _____ Period: _____

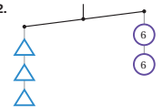
Problems 1–2: Here are two unbalanced hangers. Write an inequality to represent the relationship between the weights on each hanger. Use t to represent the weight of the triangle in grams.

1.



$t > 1$ (or equivalent)

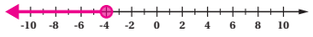
2.



$t > 4$ (or equivalent)

Problems 3–5: Use the number line diagrams to graph possible solutions for each inequality.

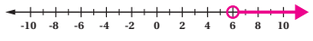
3. $5x \leq -20$



4. $30 > 6x$

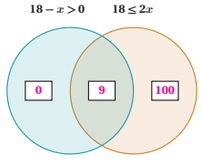


5. $x + 3 > 9$



6. List three values for x that would make $5x \leq -20$ true.
Responses less than or equal to -4 are considered correct.

7. Write a value in each region that makes the inequality or inequalities true.
Responses vary. **Sample shown on diagram.**



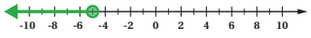
Lesson Practice 6.14

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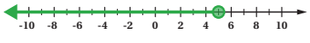
FAST Practice

8. Which number line represents the solutions to the inequality $x + 8 \leq 13$?

A.



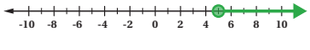
B.



C.



D.



Spiral Review

9. Select *all* the values that are solutions to $x \leq -4$.

☐ A. 4 ☑ B. -4 ☐ C. -3.99
☑ D. -4.01 ☐ E. 0

10. Finish writing $\frac{5}{8}$ as a decimal.

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
-48\downarrow \\
\hline
20 \\
-16\downarrow \\
\hline
40 \\
-40 \\
\hline
0
\end{array}
$$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.2.1
	6	1	MA.7.AR.2.1
	7	3	MA.7.AR.2.1
FAST Practice	8	2	MA.7.AR.2.1
Spiral Review			
	9	1	MA.6.AR.2.1
	10	1	MA.7.NSO.1.2

Unit 6

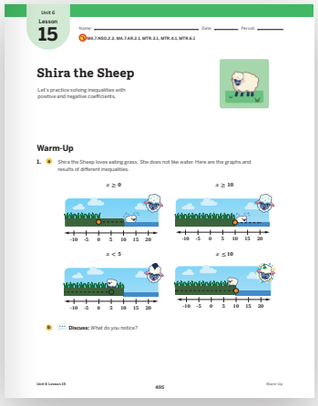
Lesson

15

Shira the Sheep

Solving Inequalities With Positive and Negative Numbers

Let's practice solving inequalities with positive and negative coefficients.


Unit 6
Lesson
15
Name _____ Date _____ Period _____
MA.7.NSO.2.2, MA.7.AR.2.1, MTR.3.1, MTR.4.1, MTR.6.1
Shira the Sheep
Let's practice solving inequalities with positive and negative coefficients.
Warm-Up
1. Shira the Sheep loves eating grass. She does not like water. Here are the graphs and results of different inequalities.
$x \geq 0$
$x \geq 10$
$x < 5$
$x \leq 10$
Discuss: What do you notice?
Unit 6 Lesson 15
495
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Solve inequalities with positive and negative coefficients, and graph their solutions.
2. **Goal:** Test values to determine which inequality symbol represents the solutions to an inequality.
3. **Language Goal:** Explain how to determine the graph of the solutions to any inequality. **(Writing, Speaking, and Listening)**

Students practice solving inequalities with positive and negative coefficients, and connect the solutions of inequalities to their graphs. They reason about the direction of an inequality when solving by asking, "What happens if I make x larger or smaller than the boundary?"

Coherence

Prior Learning

In Lesson 14, students made sense of solving inequalities using hanger diagrams.

Future Learning

In Lesson 16, students will critique solutions to an inequality, including whether fractional or negative values are reasonable.

Connections

- Students determine the range of values for the variable that a sheep can safely graze on. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.1
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

Also addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.2	**MA.8.AR.2.2**
MA.6.AR.2.1	

Lesson at a Glance

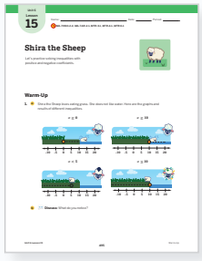
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.1

Warm-Up

Independent | 5 min

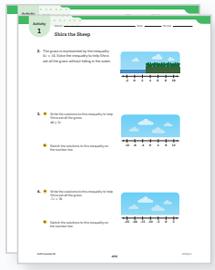
Students are oriented to the context they will explore throughout the lesson: a fictional sheep who eats grass according to an inequality.



Activity 1

Pairs | 15 min

Students solve inequalities with positive and negative coefficients to solve a variety of Shira the Sheep challenges.

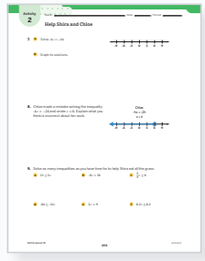


Activity 2

Pairs | 15 min

Students practice solving inequalities and critique other students' solutions.

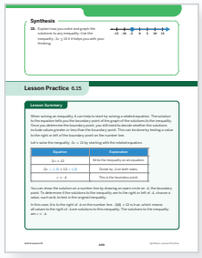
MLR3: Critique, Correct, Clarify (MTR.4.1)



Synthesis

Whole Class | 5 min

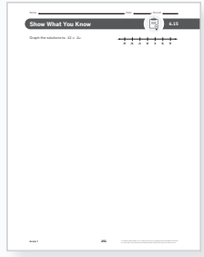
Students synthesize their understanding of solving and graphing inequalities with a negative coefficient.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by graphing the solutions to an inequality.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students are oriented to the context they will explore throughout the lesson: a fictional sheep who eats grass according to an inequality.

1 Launch

Direct students' attention to the inequalities and their graphs.

Consider sharing that Shira the Sheep starts at the circle and moves along the dotted line.

To support students getting started, consider asking, "How is the dotted line related to the inequality in each situation?"

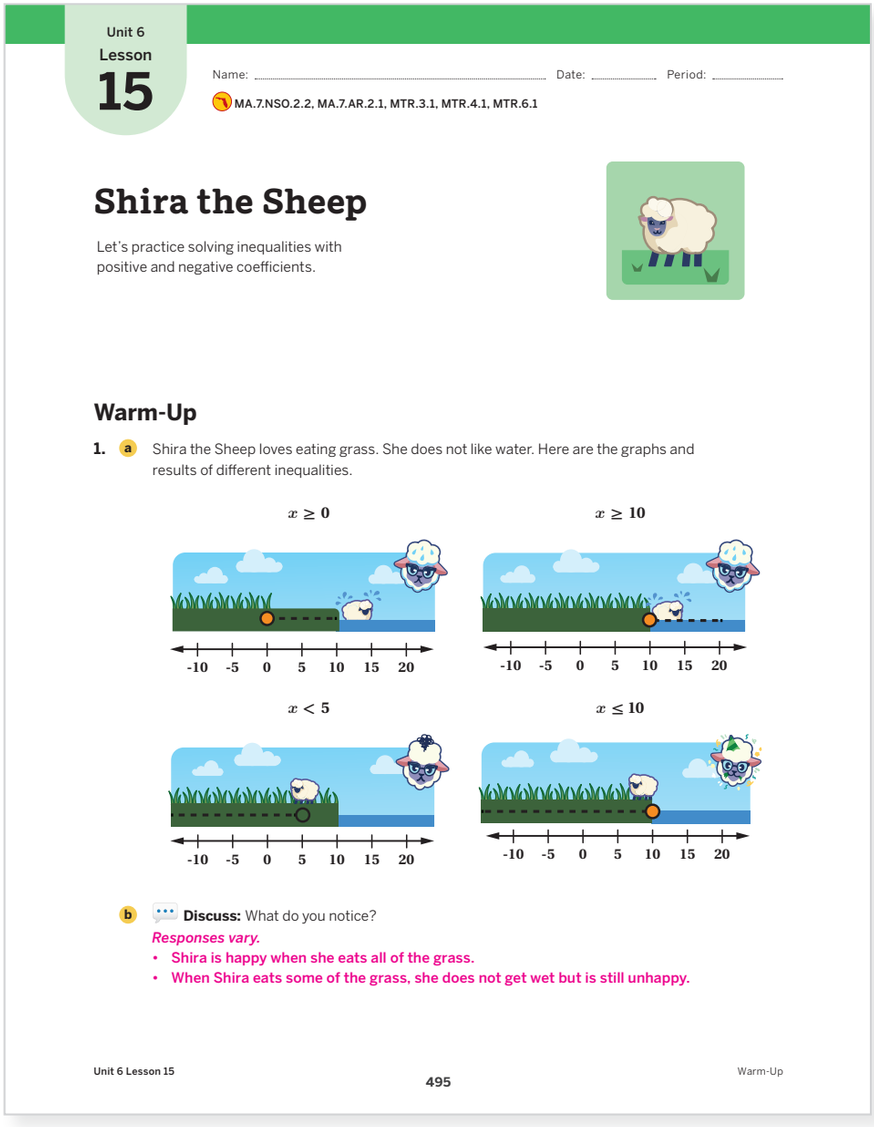
Invite students to share what they notice about Shira's reaction in each case.

2 Connect

Invite students to share what they noticed.

Note: If it does not come up naturally, consider asking:

- "What does the grass represent?"
- "Which inequalities cause Shira to get wet?"
- "Which inequality allows Shira to eat some but not all of the grass without getting wet?"
- "Which inequality allows Shira to eat all of the grass without getting wet?".



Unit 6
Lesson
15

Name: Date: Period:

MA.7.NSO.2.2, MA.7.AR.2.1, MTR.3.1, MTR.4.1, MTR.6.1

Shira the Sheep

Let's practice solving inequalities with positive and negative coefficients.

Warm-Up

1. a Shira the Sheep loves eating grass. She does not like water. Here are the graphs and results of different inequalities.

$x \geq 0$

$x \geq 10$

$x < 5$

$x \leq 10$

b **Discuss:** What do you notice?

Responses vary.

- Shira is happy when she eats all of the grass.
- When Shira eats some of the grass, she does not get wet but is still unhappy.

Unit 6 Lesson 15 495 Warm-Up

Pairs | 15 min

Activity 1 Shira the Sheep

Purpose: Students solve inequalities with positive and negative coefficients to solve a variety of Shira the Sheep challenges.

1 Launch

Consider asking, "What is the same about this image and the one from the Warm-Up? What is different?"

Invite students to check their inequalities with a partner.

2 Monitor

Encourage students to revise their solutions based on the feedback from their partner.

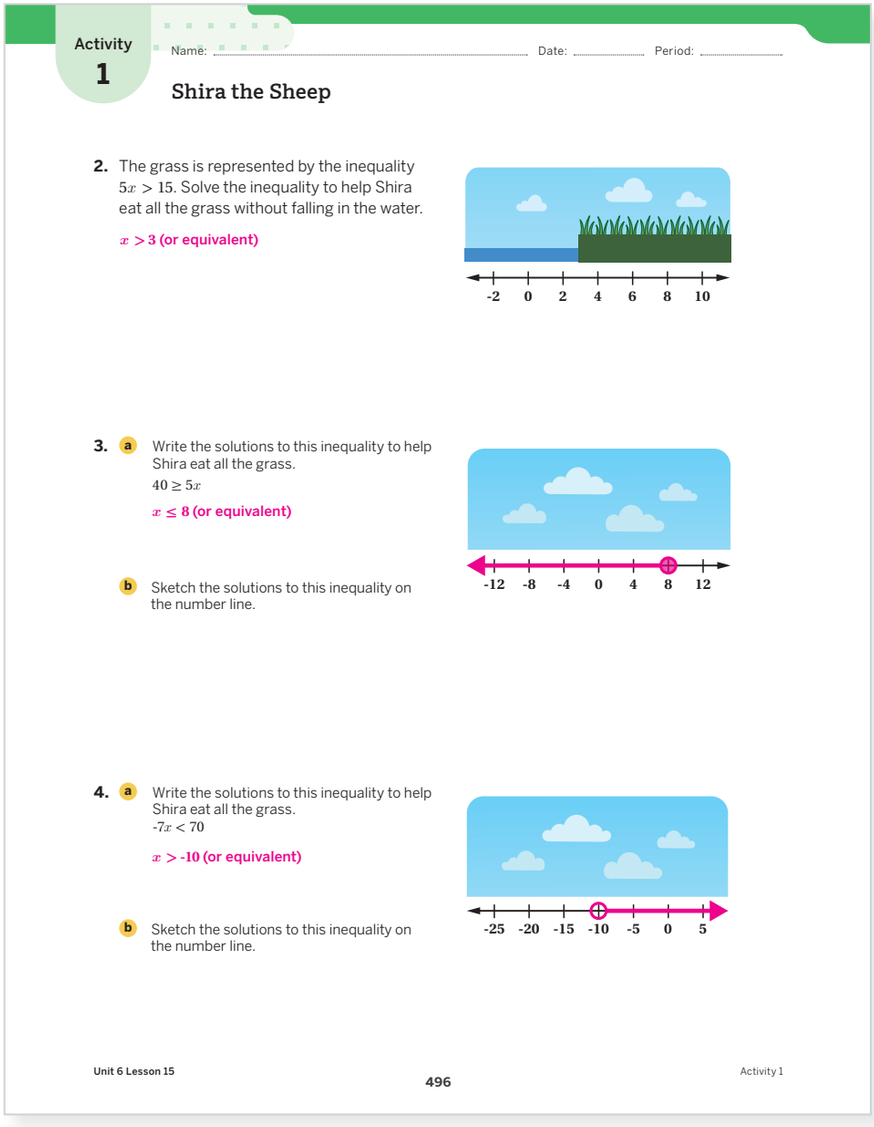
Look for students who use a variety of strategies to solve each inequality, including students who make connections to solving equations. **(MTR.3.1)**

Note: If time allows, pause to celebrate the different ways students solve the inequalities.

D **Differentiation: (Support)** Look for students who wrote $x < -10$ as the solution. Encourage them to test the value then ask, "How does the inequality need to change to prevent Shira from falling in the water?" **(MTR.6.1)**

A **Accessibility: Conceptual Processing** Invite students to talk aloud about their thinking to support self-monitoring of understanding.

Activity 1 continued >



Activity 1

Name: Date: Period:

Shira the Sheep

2. The grass is represented by the inequality $5x > 15$. Solve the inequality to help Shira eat all the grass without falling in the water.

$x > 3$ (or equivalent)

-2 0 2 4 6 8 10

3. a. Write the solutions to this inequality to help Shira eat all the grass.

$40 \geq 5x$

$x \leq 8$ (or equivalent)

b. Sketch the solutions to this inequality on the number line.

-12 -8 -4 0 4 8 12

4. a. Write the solutions to this inequality to help Shira eat all the grass.

$-7x < 70$

$x > -10$ (or equivalent)

b. Sketch the solutions to this inequality on the number line.

-25 -20 -15 -10 -5 0 5

Unit 6 Lesson 15 496 Activity 1

Pairs | 15 min

Activity 1 Shira the Sheep (continued)

Purpose: Students solve inequalities with positive and negative coefficients to solve a variety of Shira the Sheep challenges.

2 Monitor

Capture several student responses to discuss during the Connect, including ones similar to the sample responses or other creative ideas.

Select and sequence strategies for solving inequalities with negative coefficients.

D Differentiation

Look for students who:	Respond to Student Thinking
Guess and check values.	**Support** Consider asking, "How could solving an equation help determine the boundary value?"
Write the equation $-\frac{1}{2}x = 1$ before solving.	**Strengthen** Invite them to share their strategy during the Connect.
Test a value on either side of the boundary point to determine the direction of the inequality.	
Recognize that the negative coefficient is changing the inequality sign.	**Stretch** Invite these students to test their theory and explain why this might be.

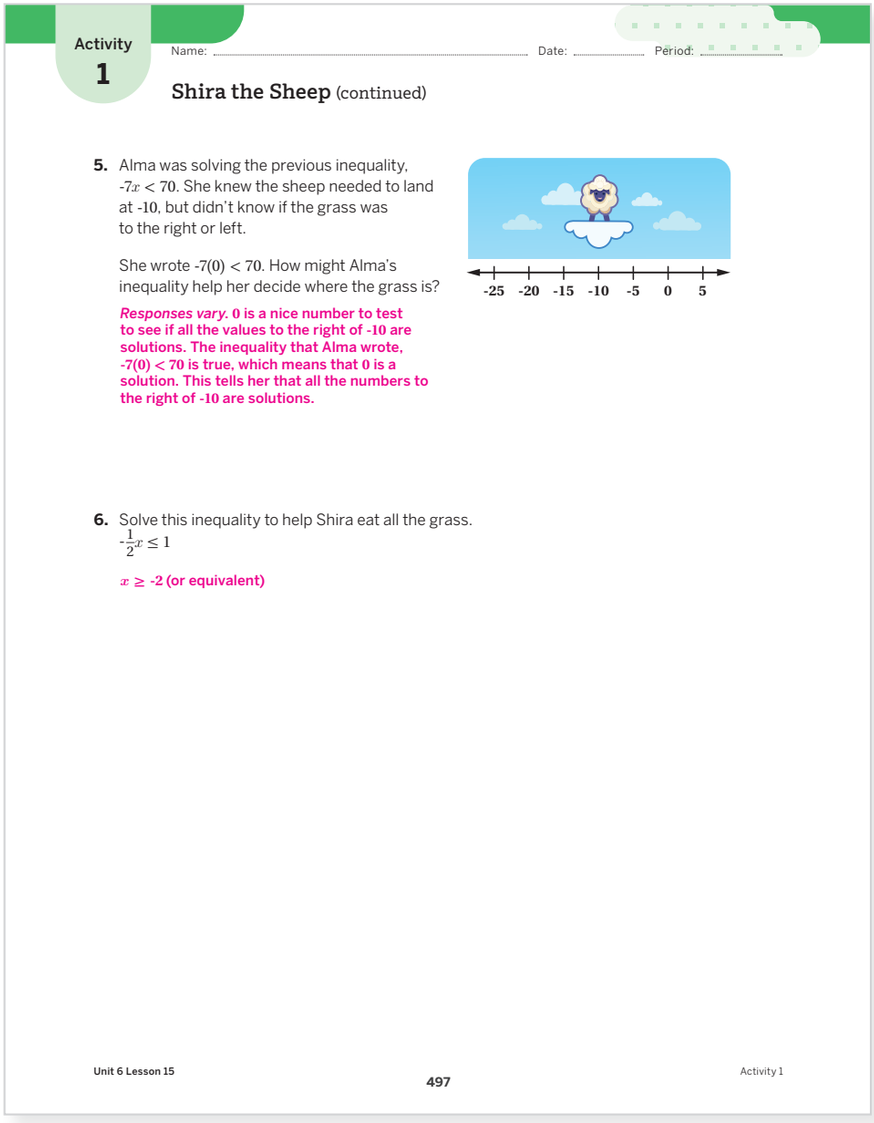
3 Connect

Display several student responses and invite students to critique Alma's reasoning. **(MTR.4.1)**

Consider asking:

- "Is 0 a solution to the original inequality? Why or why not?"
- "What do you think would happen if you tried a number less than -10, like -100 or -1,000?"
- "How does testing numbers help to determine which inequality symbol to use?"

To surface the Key Takeaway, share the captured strategies and discuss the connection between solving equations and solving inequalities.



Activity 1

Name: Date: Period:

Shira the Sheep (continued)

5. Alma was solving the previous inequality, $-7x < 70$. She knew the sheep needed to land at -10, but didn't know if the grass was to the right or left.

She wrote $-7(0) < 70$. How might Alma's inequality help her decide where the grass is?

Responses vary. 0 is a nice number to test to see if all the values to the right of -10 are solutions. The inequality that Alma wrote, $-7(0) < 70$ is true, which means that 0 is a solution. This tells her that all the numbers to the right of -10 are solutions.

6. Solve this inequality to help Shira eat all the grass.

$-\frac{1}{2}x \leq 1$

$x \geq -2$ (or equivalent)

Unit 6 Lesson 15 497 Activity 1

Key Takeaway: Solving inequalities is similar to solving equations. To determine the direction of the inequality, you can test values on either side of the boundary point. The direction of the inequality is determined by which values make the inequality true.

Pairs | 15 min

Activity 2 Help Shira and Chloe

Purpose: Students practice solving inequalities and critique other students' solutions. **(MTR.4.1)**

Short on time? Consider setting a timer for students to attempt as many challenges as they can in Problem 9.

1 Launch

To encourage connections, consider asking, "Is 0 a solution to this inequality? Why or why not?"

Invite students to check their inequalities with a partner.

Note: It is okay if students are feeling stuck here. They will revisit this inequality in the next problem.

2 Monitor

Select and sequence students' solving strategies.

D Differentiation

Look for students who:	Respond to Student Thinking
Think Chloe is correct.	**Support** Consider asking, "If you substitute 1 for x, is your inequality true?"
Test values on either side of the boundary point to determine the direction of the inequality.	**Strengthen** Invite them to share during the Connect.
Recognize that the negative coefficient is why the inequality sign is > and not <.	**Stretch** Invite them to test their theory and explain why this might be.

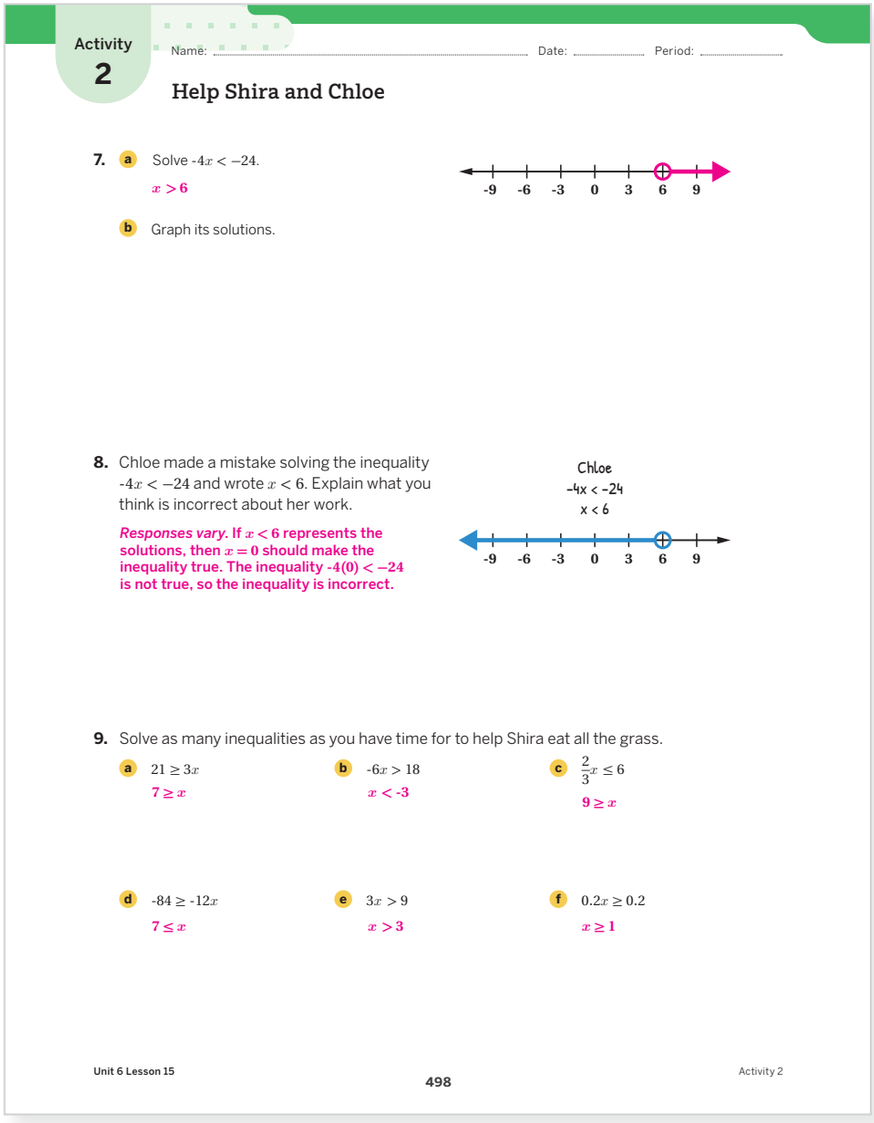
Invite students to set their own goal for how many problems they would like to complete.

To support students getting started, ask, "How might you determine the boundary point?"

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Chloe's work. **(MTR.4.1)**
As a class:
- Discuss what is *correct* about her response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct* and *clear* solution.

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these problems. **(MTR.3.1)**



Activity 2

Name: Date: Period:

Help Shira and Chloe

7. a. Solve $-4x < -24$.

$x > 6$

b. Graph its solutions.

8. Chloe made a mistake solving the inequality $-4x < -24$ and wrote $x < 6$. Explain what you think is incorrect about her work.

Responses vary. If $x < 6$ represents the solutions, then $x = 0$ should make the inequality true. The inequality $-4(0) < -24$ is not true, so the inequality is incorrect.

9. Solve as many inequalities as you have time for to help Shira eat all the grass.

a. $21 \geq 3x$
$7 \geq x$

b. $-6x > 18$
$x < -3$

c. $\frac{2}{3}x \leq 6$
$9 \geq x$

d. $-84 \geq -12x$
$7 \leq x$

e. $3x > 9$
$x > 3$

f. $0.2x \geq 0.2$
$x \geq 1$

Unit 6 Lesson 15 498 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of solving and graphing inequalities with a negative coefficient.

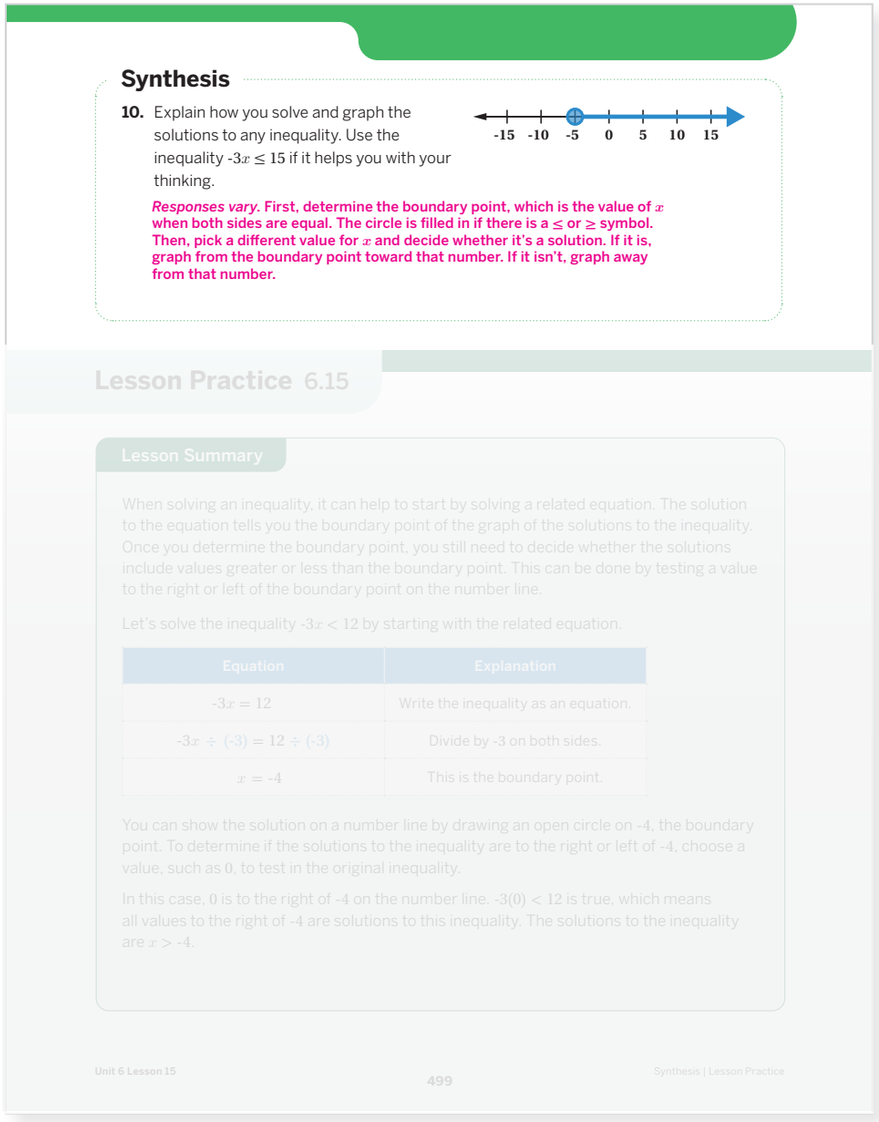
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of student responses to display, such as:

- Rewriting the inequality as an equation before solving.
- Testing values on both sides of the boundary point.
- Substituting 0 to test the solution.
- Referencing the inequality sign changing.

Discuss the connections students see between responses.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Explain how you solve and graph the solutions to any inequality. Use the inequality $-3x \leq 15$ if it helps you with your thinking.

Responses vary. First, determine the boundary point, which is the value of x when both sides are equal. The circle is filled in if there is a $\leq$ or $\geq$ symbol. Then, pick a different value for x and decide whether it's a solution. If it is, graph from the boundary point toward that number. If it isn't, graph away from that number.

Lesson Practice 6.15

Lesson Summary

When solving an inequality, it can help to start by solving a related equation. The solution to the equation tells you the boundary point of the graph of the solutions to the inequality. Once you determine the boundary point, you still need to decide whether the solutions include values greater or less than the boundary point. This can be done by testing a value to the right or left of the boundary point on the number line.

Let's solve the inequality $-3x < 12$ by starting with the related equation.

Equation	Explanation
$-3x = 12$	Write the inequality as an equation.
$-3x \div (-3) = 12 \div (-3)$	Divide by -3 on both sides.
$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle on -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) < 12$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

Unit 6 Lesson 15 499 Synthesis | Lesson Practice

Lesson Takeaway: Solving inequalities is similar to solving equations. The solutions to an inequality can be determined by first solving an equation, then testing whether the values on the left or right side of the boundary point make the inequality true. Testing a value is especially important when there are negative signs in the inequality.

Independent | 5 min

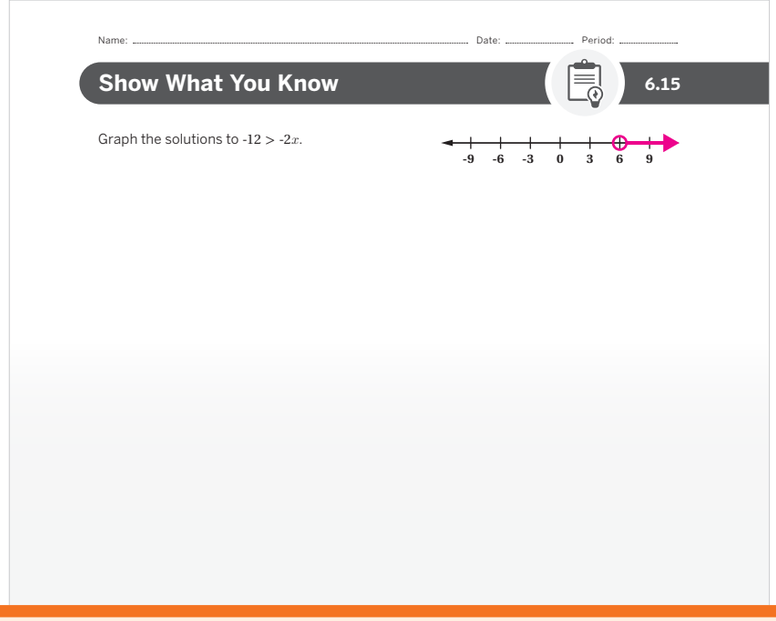
Show What You Know

Purpose: Students demonstrate their understanding by graphing the solutions to an inequality.

Today's Goals

1. **Goal:** Solve inequalities with positive and negative coefficients, and graph their solutions.
2. **Goal:** Test values to determine which inequality symbol represents the solutions to an inequality.
3. **Language Goal:** Explain how to determine the graph of the solutions to any inequality. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)


Name: _____ Date: _____ Period: _____

Show What You Know 6.15

Graph the solutions to $-12 > -2x$.

Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of solve inequalities with negative coefficients:

» **Respond to Student Thinking:**

- Use the ***Solving Inequalities*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Reflect on how students solved inequalities with positive and negative coefficients.

- How did they talk about mistakes?
- Were they able to notice the correct and incorrect parts of Alma's and Chloe's work?
- How can you support students in valuing learning from both correct and incorrect thinking?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Explain how you solve and graph the solutions to any inequality. Use the inequality $-3x \leq 15$ if it helps you with your thinking.

Responses vary. First, determine the boundary point, which is the value of x when both sides are equal. The circle is filled in if there is a $\leq$ or $\geq$ symbol. Then, pick a different value for x and decide whether it's a solution. If it is, graph from the boundary point toward that number. If it isn't, graph away from that number.

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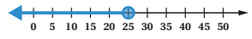
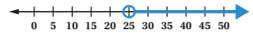
Lesson Practice 6.15

Name: _____ Date: _____ Period: _____

1. Select *all* the values of x that make the inequality $-x + 6 \geq 10$ true.

- ☐ A. -3.9
- ☐ B. 4
- ☑ C. -4.01
- ☑ D. -4
- ☐ E. 4.01
- ☐ F. 3.9
- ☐ G. 0
- ☑ H. -7

2. A library is having a party for any student who read at least 25 books over the summer. Match each situation or number line graph with an inequality.

Situation/Graph	Inequality
a. Ricardo read x books and was invited to the party.	b. $x < 25$
b. Prisha read x books but was not invited to the party.	d. $x > 25$
c. 	c. $x \leq 25$
d. 	a. $x \geq 25$

Problems 3–4: Use the inequality $-3x \geq -150$.

3. Select *all* the values of x that make the inequality $-3x \geq -150$ true.

☑ A. 0 ☑ B. 50 ☑ C. -50 ☑ D. 49.9 ☐ E. 50.1

4. In order to solve the inequality $-3x \geq -150$, Makalya solved the equation $-3x = -150$ and got $x = 50$. What is the solution to the inequality?

$x \leq 50$

5. Diego is solving the inequality $-3x \geq 45$. He solves the equation $-3x = 45$ to determine $x = -15$. What is the solution to the inequality?

A. $x < -15$

B. $x > -15$

C. $x \leq -15$

D. $x \geq -15$

Lesson Practice 6.15

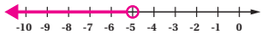
Name: _____ Date: _____ Period: _____

Problems 6–7: Complete the table to determine the solutions to each inequality. Write the solutions as an inequality and graph them on the number line.

6. $-6x > 30$

$x < -5$

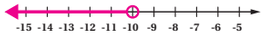
x	-7	-6	-5	-4	-3	-2	-1	0
$-6x$	42	36	30	24	18	12	6	0



7. $\frac{3}{4}x < -\frac{15}{2}$

$x < -10$

x	-16	-14	-12	-10	-8	-6	-4	-2
$\frac{3}{4}x$	-12	$-\frac{21}{2}$	-9	$-\frac{15}{2}$	-6	$-\frac{9}{2}$	-3	$-\frac{3}{2}$



FAST Practice

8. Solve the inequality $x + 2 < 5$.

$x < 3$

Spiral Review

9. Alma makes 5 cups of her favorite color of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ a cup of white paint. Alma has 2 cups of white paint. How much blue paint and red paint will Alma need to use with the 2 cups of white paint?

12 cups of blue paint and 6 cups of red paint

10. A backpack normally costs $25, but is on sale for $21. What percent is the discount? Show your thinking.

16%. *Work varies.* $25 - 21 = 4$, $\frac{4}{25} \cdot 100 = 16$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.AR.2.1
	2–7	2	MA.7.AR.2.1
FAST Practice	8	1	MA.7.AR.2.1
Spiral Review			
	9	2	MA.7.AR.3.2
	10	2	MA.7.AR.3.1

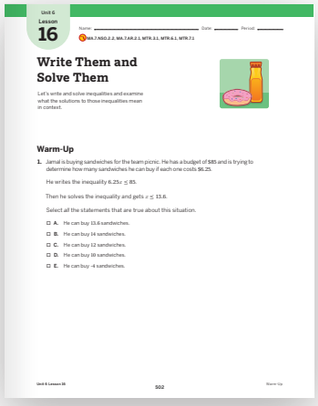
Unit 6

Lesson 16

Write Them and Solve Them

Modeling With Inequalities

Let's write and solve inequalities and examine what the solutions to those inequalities mean in context.


Unit 6
Lesson 16

Name _____ Date _____ Period _____

MA.7.NSO.2.2, MA.7.AR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Write Them and Solve Them

Let's write and solve inequalities and examine what the solutions to those inequalities mean in context.

Warm-Up

1. Jamal is buying sandwiches for the team picnic. He has a budget of $85 and is trying to determine how many sandwiches he can buy if each one costs $6.25.

   He writes the inequality $6.25x \leq 85$.

   Then he solves the inequality and gets $x \leq 13.6$.

   Select all the statements that are true about this situation.

   - ☐ A. He can buy 13.6 sandwiches.
   - ☐ B. He can buy 14 sandwiches.
   - ☐ C. He can buy 12 sandwiches.
   - ☐ D. He can buy 10 sandwiches.
   - ☐ E. He can buy -4 sandwiches.

Unit 6 Lesson 16 502 Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Write and solve an inequality to answer a question about a situation with a constraint.
2. **Language Goal:** Critique solutions to an inequality, including whether fractional or negative values are reasonable. **(Reading, Writing, Speaking, and Listening)**

Students practice writing and solving inequalities and critically examine what the solutions to those inequalities mean in context. They determine whether it is appropriate to round their calculations in each situation.

Coherence

Prior Learning

In Lessons 13–15, students solved inequalities with both positive and negative coefficients and graphed their solutions.

Future Learning

In Algebra 1, students will revisit solutions to inequalities and extend their work to solutions to inequalities with two variables.

Connections

- Students write and solve inequalities to find all possible variables that make that inequality true in the context of the cost of breakfast at a store. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.1
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.6.1, MTR.7.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.2	**MA.8.AR.2.2**
MA.6.AR.2.1	

Lesson at a Glance

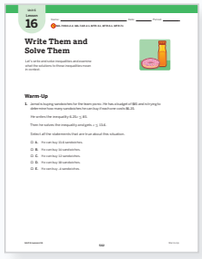
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.1

Warm-Up

Whole Class | 5 min

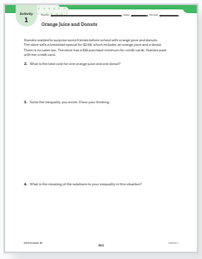
Students reason about which solutions to an inequality make sense based on its situation. **(MTR.6.1)**



Activity 1

Pairs | 10 min

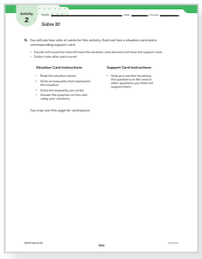
Students write and solve an inequality and then interpret their solution. **(MTR.7.1)**



Activity 2

Pairs | 20 min

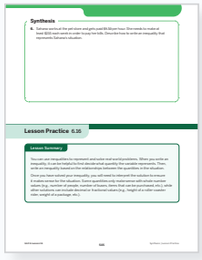
Students develop fluency writing and solving inequalities and interpreting their solutions in context. They support each other in making sense of each problem and its solutions. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

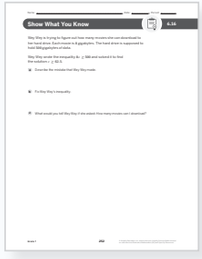
Students synthesize their understanding of how to write an inequality to represent a situation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing and interpreting the solutions to an inequality in context.



Prep Checklist

Prepare the Activity 2 Cards. Invite students to use their Student Edition.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one per student, pre-cut

Whole Class | 5 min

Warm-Up

Purpose: Students reason about which solutions to an inequality make sense based on its situation. **(MTR.6.1)**

1 Launch

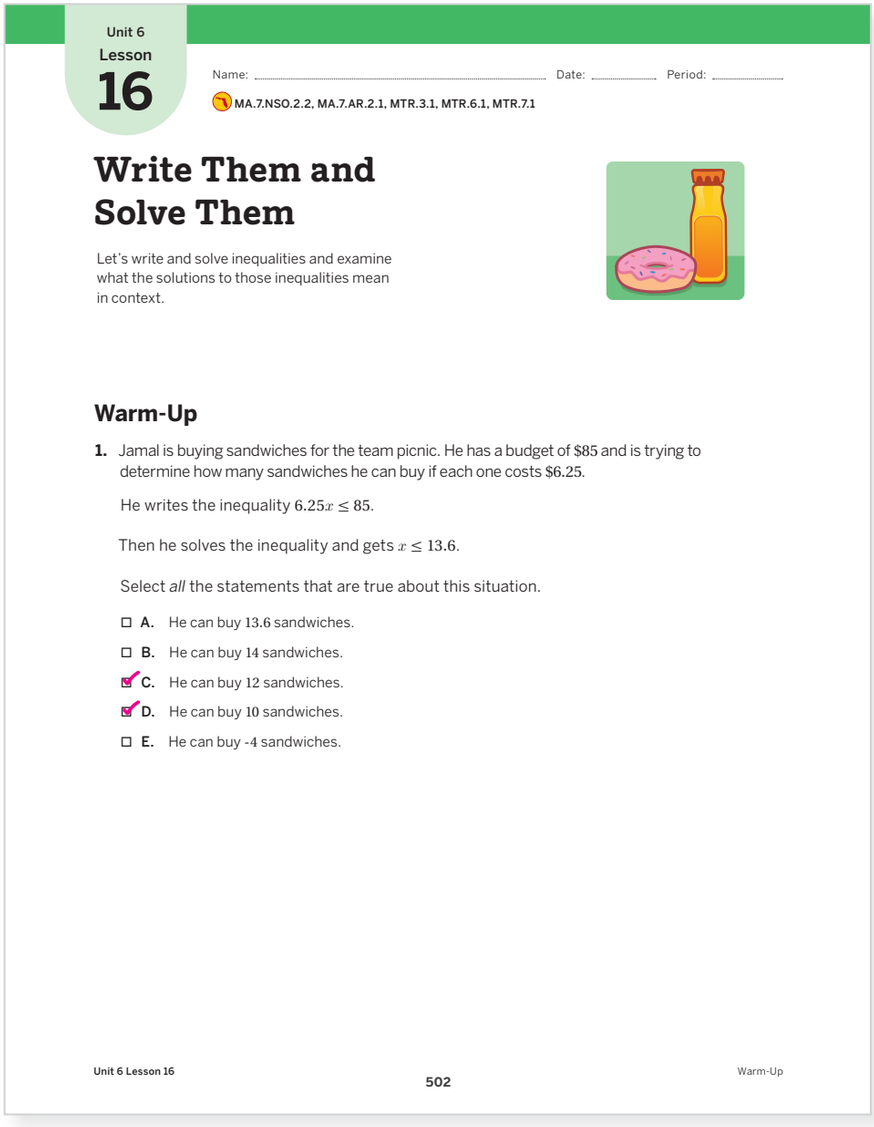
Direct students' attention to the situation. Consider reading the story as a class and asking students what connections they make to the story.

Consider asking, "What does the variable x represent in Jamal's inequality?" **(MTR.7.1)**

2 Connect

Invite students to share how $x \leq 13.6$ connects to the situation.

Consider asking, "How can we use Jamal's solution to help him determine how many sandwiches he can buy?" **(MTR.6.1)**



Unit 6
Lesson
16

Name: Date: Period:

MA.7.NSO.2.2, MA.7.AR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Write Them and Solve Them

Let's write and solve inequalities and examine what the solutions to those inequalities mean in context.

Warm-Up

1. Jamal is buying sandwiches for the team picnic. He has a budget of $85 and is trying to determine how many sandwiches he can buy if each one costs $6.25.

   He writes the inequality $6.25x \leq 85$.

   Then he solves the inequality and gets $x \leq 13.6$.

   Select *all* the statements that are true about this situation.

   - ☐ **A.** He can buy 13.6 sandwiches.
   - ☐ **B.** He can buy 14 sandwiches.
   - ☑ **C.** He can buy 12 sandwiches.
   - ☑ **D.** He can buy 10 sandwiches.
   - ☐ **E.** He can buy -4 sandwiches.

Unit 6 Lesson 16 502 Warm-Up

Pairs | 10 min

Activity 1 Orange Juice and Donuts

Purpose: Students write and solve an inequality and then interpret their solution. **(MTR.7.1)**

1 Launch

Direct students' attention to the situation and read it aloud. Discuss the first question as a class.

ELL **English Language Learners** Consider sharing or sketching a picture of the situation to increase access to the task.

2 Monitor

Invite students to work together to share their reasoning and reach an agreement about how to answer each question.

A **Accessibility: Conceptual Processing** Invite students to read the inequality aloud using a verbal description for the variable (e.g., $2.90 times the number of people is greater than or equal to $10).

Listen for student strategies and offer help or encouragement where needed.

D **Differentiation** *(Problems 2–3)*

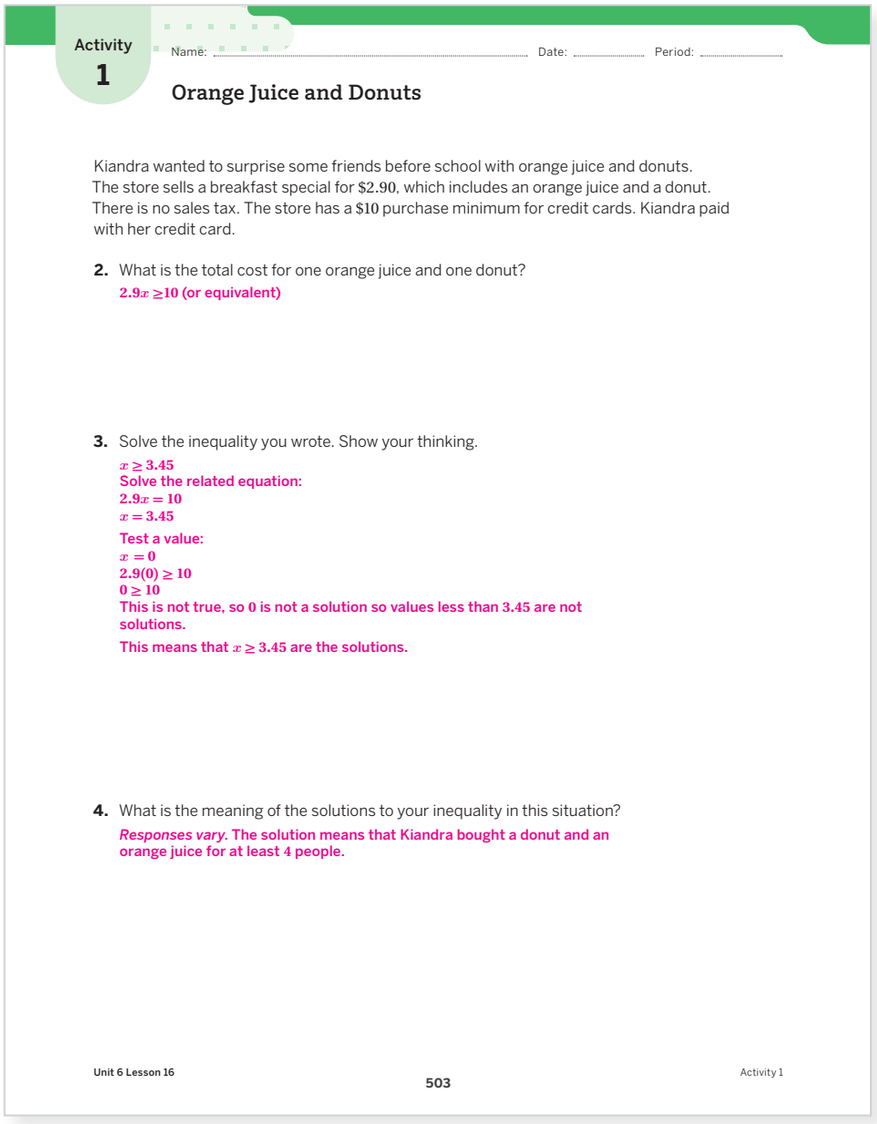
Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 2)*	**Support** Consider asking, "What quantity might be represented with a variable?"
Say that they can purchase food for at least 3.44 people. *(Problem 3)*	**Support** Consider asking, "What might 3.44 people mean?"
Round to 3 people.	**Support** Consider asking, "Do you reach the minimum amount you need to use a credit card?"
Use a variety of strategies, such as: • Guessing and checking. • Annotating the text. • Using their personal experience to reason.	**Strengthen** Invite them to share these strategies during the Connect.
Would like to explore this idea more.	**Stretch** Encourage them to research the cost of orange juice and donuts in their community and answer Problems 1–3 using the costs they find.

3 Connect

Consider asking:

- "How did you start solving the inequality?"
- "What is the meaning of the solutions to your inequality in this situation?" **(MTR.7.1)**

To surface the Key Takeaway, invite students to share their strategies for writing and solving their inequalities, including what their solutions mean in Kiandra's situation.



Activity 1

Name: Date: Period:

Orange Juice and Donuts

Kiandra wanted to surprise some friends before school with orange juice and donuts. The store sells a breakfast special for $2.90, which includes an orange juice and a donut. There is no sales tax. The store has a $10 purchase minimum for credit cards. Kiandra paid with her credit card.

2. What is the total cost for one orange juice and one donut?

$2.9x \geq 10$ (or equivalent)

3. Solve the inequality you wrote. Show your thinking.

$x \geq 3.45$
Solve the related equation:
$2.9x = 10$
$x = 3.45$
Test a value:
$x = 0$
$2.9(0) \geq 10$
$0 \geq 10$
This is not true, so 0 is not a solution so values less than 3.45 are not solutions.
This means that $x \geq 3.45$ are the solutions.

4. What is the meaning of the solutions to your inequality in this situation?

Responses vary. The solution means that Kiandra bought a donut and an orange juice for at least 4 people.

Unit 6 Lesson 16 503 Activity 1

Key Takeaway: The solution to an inequality does not always make sense in the context of the situation. To answer a question about a situation, reason about whether to round the solutions and in what way.

Pairs | 20 min

Activity 2 Solve It!

Purpose: Students develop fluency writing and solving inequalities and interpreting their solutions in context. They support each other in making sense of each problem and its solutions. **(MTR.3.1)**

Short on time? Consider reducing the number of situation cards students complete.

1 Launch

Direct students' attention to the instructions and review them as a class.

To generate excitement, consider sharing that students will take turns solving problems and supporting, and that each role is crucial to their success. **(MTR.3.1)**

Distribute the Activity 2 Cards to each pair of students.

2 Monitor

Listen to partner discussions, including conversations about using whole number or fractional solutions.

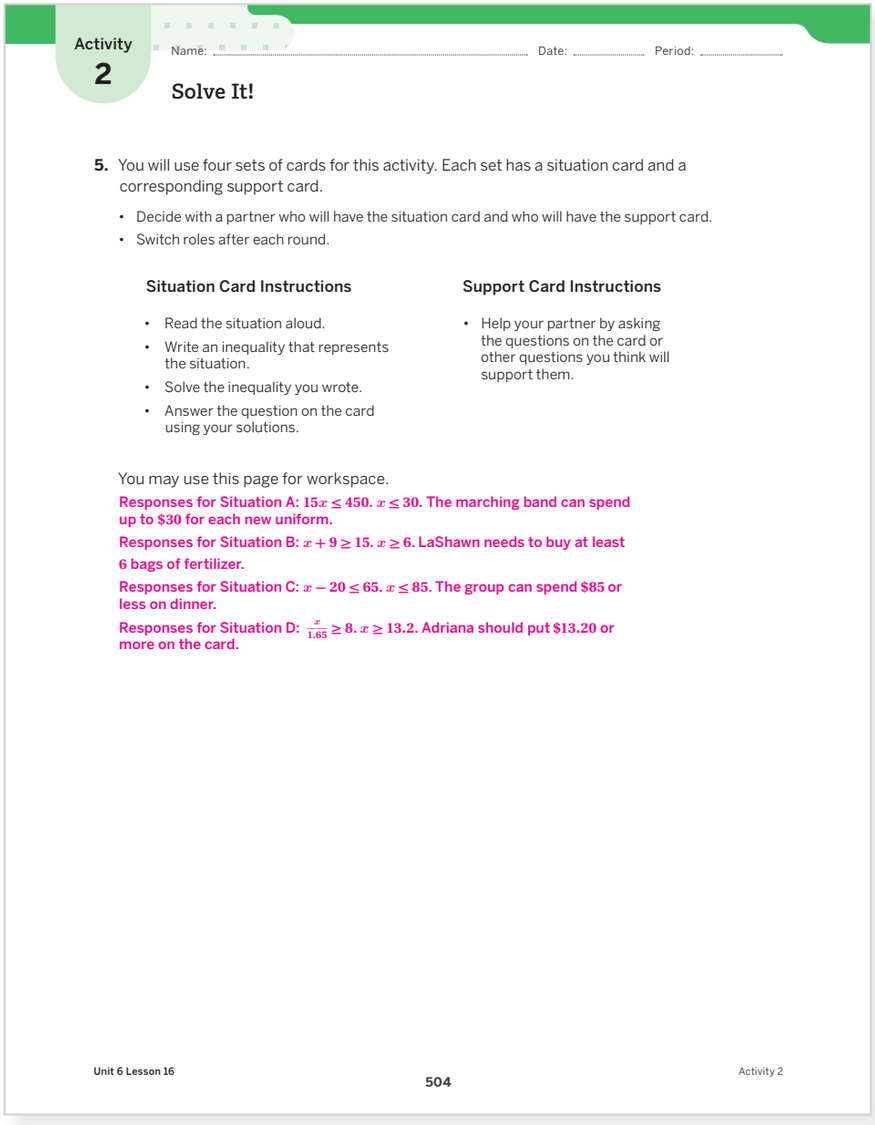
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider inviting a group to act as an example pair while the rest of the class observes.
Include only whole numbers in their answers to situations. Include fractions in situations where that does not make sense.	**Support** Consider asking, "Is it possible that your answer could also be a decimal or a fraction? Would this solution make sense in this situation?"
Use a variety of strategies to determine inequalities.	**Strengthen** Invite them to share various strategies during the Connect.

3 Connect

Invite students to share their responses and strategies.

Consider asking, "When might you come across an inequality that can only have a whole number as a solution? When might the solution to an inequality be a fraction?" **(MTR.6.1)**



Activity 2

Name: Date: Period:

Solve It!

5. You will use four sets of cards for this activity. Each set has a situation card and a corresponding support card.

- Decide with a partner who will have the situation card and who will have the support card.
- Switch roles after each round.

Situation Card Instructions

- Read the situation aloud.
- Write an inequality that represents the situation.
- Solve the inequality you wrote.
- Answer the question on the card using your solutions.

Support Card Instructions

- Help your partner by asking the questions on the card or other questions you think will support them.

You may use this page for workspace.

Responses for Situation A: $15x \leq 450$. $x \leq 30$. The marching band can spend up to $30 for each new uniform.

Responses for Situation B: $x + 9 \geq 15$. $x \geq 6$. LaShawn needs to buy at least 6 bags of fertilizer.

Responses for Situation C: $x - 20 \leq 65$. $x \leq 85$. The group can spend $85 or less on dinner.

Responses for Situation D: $\frac{x}{1.65} \geq 8$. $x \geq 13.2$. Adriana should put $13.20 or more on the card.

Unit 6 Lesson 16 504 Activity 2

Whole Class | 5 min

Synthesis

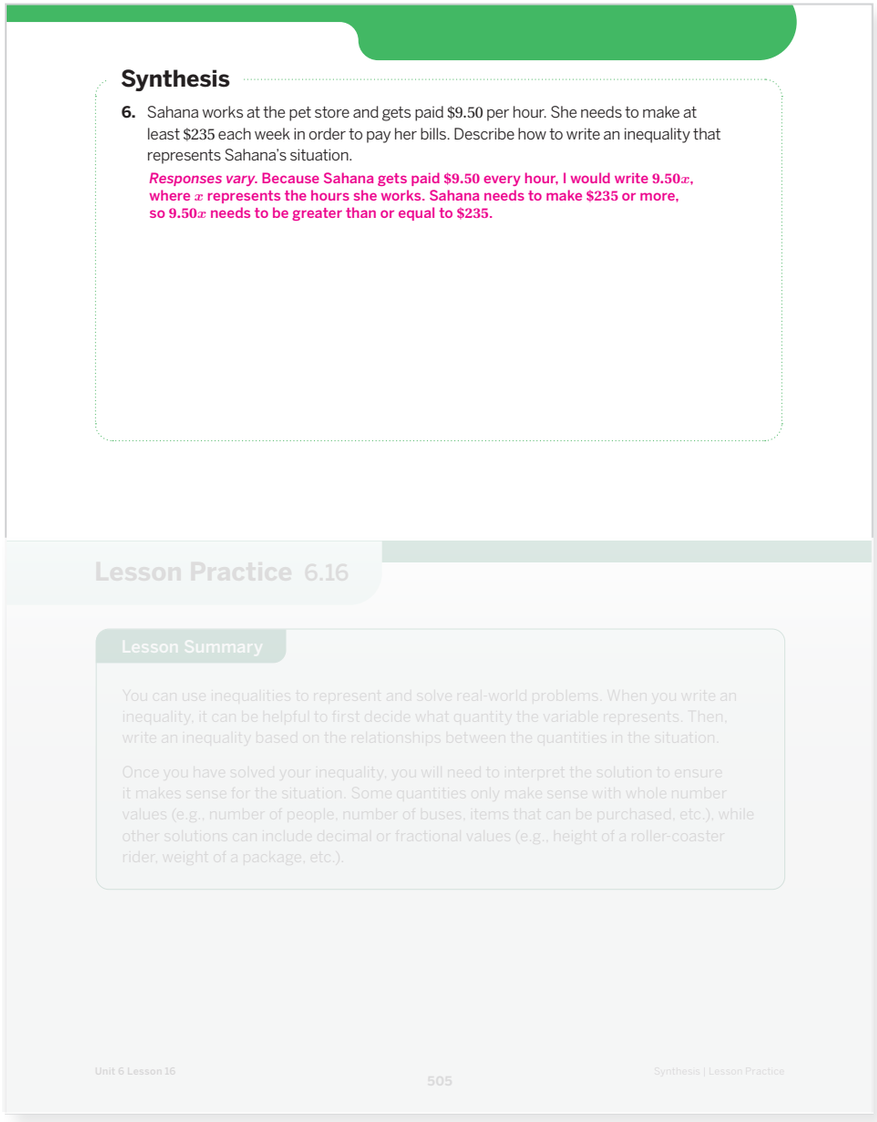
Purpose: Students synthesize their understanding of how to write an inequality to represent a situation.

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of responses, including ones similar to the sample response.

Encourage students to share how they decided which inequality symbol to use.

Note: If time allows, invite students to write their own situation and an inequality to match it.



Synthesis

6. Sahana works at the pet store and gets paid $9.50 per hour. She needs to make at least $235 each week in order to pay her bills. Describe how to write an inequality that represents Sahana's situation.

Responses vary. **Because Sahana gets paid $9.50 every hour, I would write $9.50x$, where x represents the hours she works. Sahana needs to make $235 or more, so $9.50x$ needs to be greater than or equal to $235.**

Lesson Practice 6.16

Lesson Summary

You can use inequalities to represent and solve real-world problems. When you write an inequality, it can be helpful to first decide what quantity the variable represents. Then, write an inequality based on the relationships between the quantities in the situation.

Once you have solved your inequality, you will need to interpret the solution to ensure it makes sense for the situation. Some quantities only make sense with whole number values (e.g., number of people, number of buses, items that can be purchased, etc.), while other solutions can include decimal or fractional values (e.g., height of a roller-coaster rider, weight of a package, etc.).

Unit 6 Lesson 16 505 Synthesis | Lesson Practice

Lesson Takeaway: When using an inequality to answer a question about a situation, consider what the solutions represent. Sometimes solutions can only be whole number values, while other solutions can be fractional or negative.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing and interpreting the solutions to an inequality in context.

Today's Goals

1. **Goal:** Write and solve an inequality to answer a question about a situation with a constraint.
2. **Language Goal:** Critique solutions to an inequality, including whether fractional or negative values are reasonable. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.16

Wey Wey is trying to figure out how many movies she can download to her hard drive. Each movie is 8 gigabytes. The hard drive is supposed to hold 500 gigabytes of data.

Wey Wey wrote the inequality $8x \geq 500$ and solved it to find the solution $x \geq 62.5$.

a. Describe the mistake that Wey Wey made.
Wey Wey should have used the less-than-or-equal-to symbol.

b. Fix Wey Wey's inequality.
$8x \leq 500$

c. What would you tell Wey Wey if she asked: *How many movies can I download?*
I would tell Wey Wey that she can download 62 movies or fewer.

Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write inequalities to answer a question about a situation with a constraint:

» **Respond to Student Thinking:**

- Revisit the **Warm-Up** and invite students to share strategies for writing inequalities for situations and interpreting their solutions.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to look for other real- world examples of inequalities and challenge them to write inequalities and solutions to represent those examples.

Professional Learning

Take a moment to reflect on how students' understanding of *solutions to an inequality* has developed throughout this unit. How has their definition of *solutions to an inequality* expanded over the last few lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. Sahana works at the pet store and gets paid $9.50 per hour. She needs to make at least $235 each week in order to pay her bills. Describe how to write an inequality that represents Sahana's situation.

Responses vary. Because Sahana gets paid $9.50 every hour, I would write $9.50x$, where x represents the hours she works. Sahana needs to make $235 or more, so $9.50x$ needs to be greater than or equal to $235.

Lesson Practice 6.16

Lesson Summary

You can use inequalities to represent and solve real-world problems. When you write an inequality, it can be helpful to first decide what quantity the variable represents. Then, write an inequality based on the relationships between the quantities in the situation.

Once you have solved your inequality, you will need to interpret the solution to ensure it makes sense for the situation. Some quantities only make sense with whole number values (e.g., number of people, number of buses, items that can be purchased, etc.), while other solutions can include decimal or fractional values (e.g., height of a roller-coaster rider, weight of a package, etc.).

Lesson Practice 6.16

Name: _____ Date: _____ Period: _____

Problems 1–3: A store sold $\frac{2}{5}$ of the shirts on display. The store likes to keep at least 120 shirts on display. The manager wrote the inequality $\frac{3}{5}x \geq 120$ to describe the situation.

1. Explain what $\frac{3}{5}$ means in the manager's inequality.
   Explanations vary. It represents the amount of shirts that remained in the display. After $\frac{2}{5}$ of the shirts were sold, $\frac{3}{5}$ remained.
2. Solve the inequality.
   $x \geq 200$
3. What do the solutions to the inequality mean in this situation?
   The solution means that the store initially had at least 200 shirts on display.

Problems 4–6: Camila has up to $100 to spend on her birthday party at a city swimming pool. She plans to spend $38.50 on admission to the pool and spend the rest on pizza and drinks.

4. Write an inequality to represent the situation.
   $x + 38.50 \leq 100$ (or equivalent)
5. Solve the inequality.
   $x \leq 61.50$ (or equivalent)
6. What do the solutions to your inequality mean in this situation?
   The solution to the inequality is the most she can spend on pizza and drinks.

FAST Practice

Problems 7–8: Write an inequality to represent each situation.

7. The cafeteria has several 4-seat tables. There are enough tables to fit at most 200 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.
   $4x \leq 200$ (or equivalent)
8. There are 4 barrels that catch rainwater in the schoolyard. The barrels are the same size. The Environmental Club is hoping that the barrels can catch at least 200 liters of water to use to water the school's garden. Write an inequality whose solution is the possible size of each of the 4 barrels.
   $4x \geq 200$ (or equivalent)

Lesson Practice 6.16

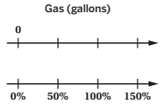
Name: _____ Date: _____ Period: _____

Spiral Review

9. Select *all* the inequalities that have the same solutions as $-4x < 20$.
   - [x] A. $-x < 5$
   - [x] B. $4x > -20$
   - [ ] C. $4x < -20$
   - [ ] D. $x < -5$
   - [x] E. $x > -5$

For Problems 10–13: Solve each equation.

10. $-1d - 4 = -3$
    $d = -1$
11. $-\frac{1}{4}m + 5 = 16$
    $m = -44$
12. $10b + (-45) = -43$
    $b = \frac{1}{5}$
13. $-8(y - 1.25) = 4$
    $y = 0.75$
14. The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does the car's tank hold? Use the double number line if it helps with your thinking.
    15 gallons



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.AR.2.1
	2–6	2	MA.7.AR.2.1
FAST Practice	7–8	2	MA.7.AR.2.1
Spiral Review			
	9	2	MA.7.AR.2.1
	10–13	2	MA.7.AR.2.2
	14	1	MA.7.AR.3.1

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students solved equations that included expanding, factoring, or combining like terms. They also solved inequalities and graphed their solutions on number lines.

Career Connection | Meet Lue Gim Gong

Invite students to read about horticulturalist Lue Gim Gong and how his work led to the development of cold-resistant oranges.

Consider asking:

- "What kinds of oranges do you like to eat? Do you know which kinds of oranges Florida is known for?" Sample response: Varieties of oranges grown in Florida include navel, Valencia, hamlin, and pineapple.

B.E.S.T. Benchmark Connections

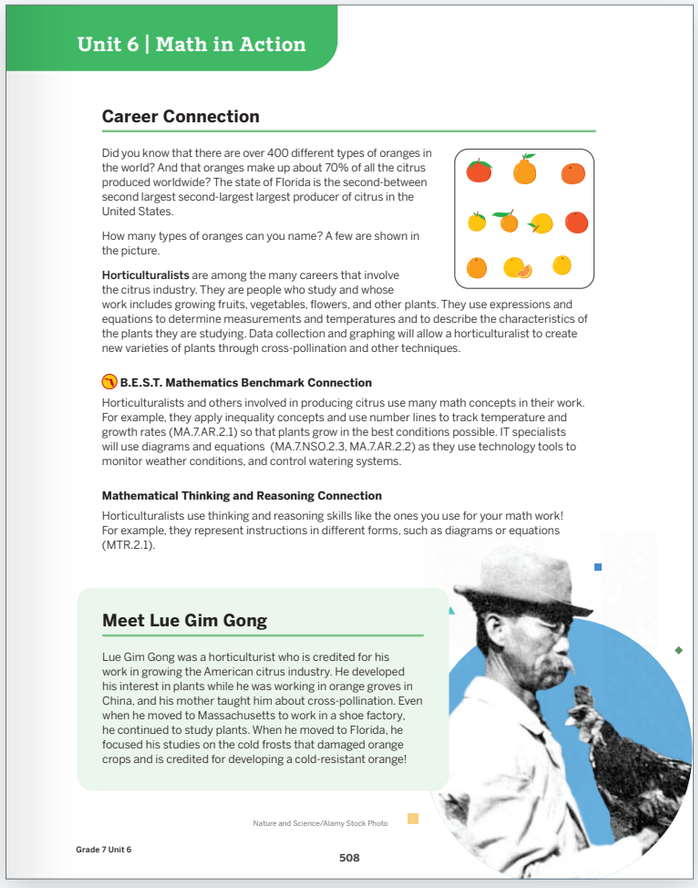
Consider asking:

- "What other math concepts have you studied in this unit that you could you apply to the citrus industry?" Sample response: You can calculate the volume of cylinders to construct containers for storing or shipping oranges, or designing a can to hold frozen orange juice. (MA.7.GR.2.3)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "Scientists often work in teams as important learning opportunities. How is this similar to how you engage with math through Mathematical Thinking and Reasoning?" Sample response: Actively participate in effortful learning both individually and collectively. (MTR.1.1) Working on activities in this unit, sometimes on your own, and other times with a partner or team, can help you better understand concepts like equations and inequalities.



Unit 6 | Math in Action

Career Connection

Did you know that there are over 400 different types of oranges in the world? And that oranges make up about 70% of all the citrus produced worldwide? The state of Florida is the second-between second largest second-largest largest producer of citrus in the United States.

How many types of oranges can you name? A few are shown in the picture.

Horticulturalists are among the many careers that involve the citrus industry. They are people who study and whose work includes growing fruits, vegetables, flowers, and other plants. They use expressions and equations to determine measurements and temperatures and to describe the characteristics of the plants they are studying. Data collection and graphing will allow a horticulturalist to create new varieties of plants through cross-pollination and other techniques.

B.E.S.T. Mathematics Benchmark Connection

Horticulturalists and others involved in producing citrus use many math concepts in their work. For example, they apply inequality concepts and use number lines to track temperature and growth rates (MA.7.AR.2.1) so that plants grow in the best conditions possible. IT specialists will use diagrams and equations (MA.7.NSO.2.3, MA.7.AR.2.2) as they use technology tools to monitor weather conditions, and control watering systems.

Mathematical Thinking and Reasoning Connection

Horticulturalists use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as diagrams or equations (MTR.2.1).

Meet Lue Gim Gong

Lue Gim Gong was a horticulturist who is credited for his work in growing the American citrus industry. He developed his interest in plants while he was working in orange groves in China, and his mother taught him about cross-pollination. Even when he moved to Massachusetts to work in a shoe factory, he continued to study plants. When he moved to Florida, he focused his studies on the cold frosts that damaged orange crops and is credited for developing a cold-resistant orange!

Nature and Science/Alamy Stock Photo

Grade 7 Unit 6

508

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Representing the solution set to inequalities on a number line	Lesson 14	1	MA.7.AR.2.3
2	Representing real-world situations with algebraic inequalities	Lesson 16	2	MA.7.AR.2.3, MTR.6.1
3a, 3b	Using the structure of expressions and the distributive property to factor and expand expressions	Lessons 7, 8	1	MA.7.AR.1.1
4a, 4b	Using structure to solve equations of the form $px + q = r$ and $p(x + q) = r$	Lesson 6	1	MA.7.AR.2.2
5a	Critiquing the work of another student involving writing equivalent expressions using fewer terms	Lessons 4, 10	2	MA.7.AR.1.1. MTR.4.1
5b	Using the structure of expressions and combining like terms to write equivalent expressions using fewer terms	Lesson 12	1	MA.7.AR.1.1
6a	Solving a multi-step application problem involving rational numbers and justifying the response	Lessons 15, 16	2	MA.7.AR.3.3, MTR6.1
6b	Representing real-world situations with algebraic expressions and inequalities	Lesson 16	3	MA.7.AR.2.1, MTR.6.1
6c	Solving algebraic inequalities and interpreting the solution within the context of the situation	Lessons 15, 16	2	MA.7.AR.3.3, MA.7.AR.2.1, MTR.7.1

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



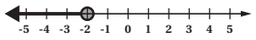
- Mini-Lessons
- Extensions

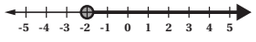
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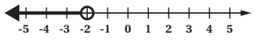
End-of-Unit Assessment

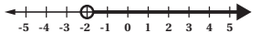
Unit 6

1. Which number line shows all of the values of x that make the inequality $3x \leq -6$ true?

A.


B.


C.


D.


2. Parv has a $50 gift card. He wants to buy n movies. Each movie costs $3.99.

Which inequality describes how many movies Parv can buy?

A. $3.99n > 50$

B. $3.99n \leq 50$

C. $3.99n \geq 50$

D. $3.99n < 50$

3. **a** Write $-\frac{1}{4}(-8x + 12)$ in expanded form.

$2x - 3$ (or equivalent)

b Write $36a - 16$ in factored form.

$2(18a - 8)$ or $4(9a - 4)$ (or equivalent)

4. Solve each equation.

a $4(x + 2) = 40$

$x = 8$

b $-2x - 10 = -6$

$x = -2$

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

5. Here is Diya's work writing the expression $6 - 2x + 5 + 4x$ with fewer terms.

Diya
$6 - 2x + 5 + 4x$
$11 + 6x$
$17x$

a Describe the mistake that Diya made.

Responses vary. Diya added all of the terms together instead of adding $6 + 5$ and $-2x + 4x$.

b Write an expression equivalent to $6 - 2x + 5 + 4x$ that has two terms.

$11 + 2x$ (or equivalent)

6. Joel's family car has a 14-gallon gas tank. The car uses about 0.5 gallons of gas each day.



a If Joel's family starts with a full tank, can they drive the car for 15 days? Circle one.

Yes No

Explain how you know.

Explanations vary. Joel's family would use 7.5 gallons of gas in 15 days, so there would still be $14 - 7.5 = 6.5$ gallons of gas left in the tank.

b Which expression describes the gallons of gas in the tank after d days?

A. $14 - 0.5d$ B. $14d - 0.5$ C. $0.5 - 14d$ D. $0.5d - 14$

Write an inequality that represents the number of days Joel's family can drive.

$0.5d < 14$ (or equivalent)

c Solve the inequality you wrote.

$d < 28$

Explain what the solutions mean in this situation.

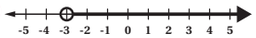
Explanations vary. The solutions to this inequality represent the number of days that Joel's family can drive.

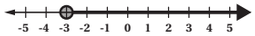
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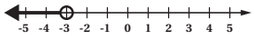
End-of-Unit Assessment

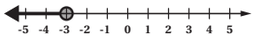
Unit 6

1. Which number line shows all of the values of x that make the inequality $2x < -4$ true?

A. 

B. 

C. 

D. 

2. Kayla has a goal to hike at least 75 miles. She plans to hike n miles each week for the next 6 weeks.

Which inequality describes the number of miles Kayla should hike each week to meet her goal?

A. $6n > 75$

B. $6n \leq 75$

C. $6n \geq 75$

D. $6n < 75$

3. a. Write $-\frac{1}{3}(9x - 15)$ in expanded form.

$-3x + 5$ (or equivalent)

b. Write $24a - 18$ in factored form.

$2(12a - 9)$ or $6(4a - 3)$ (or equivalent)

4. Solve each equation.

a. $30 = 2(x + 3)$

$x = 12$

b. $-4x - 17 = -5$

$x = -3$

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5. Here is Caleb's work writing the expression $5 - 3x + 11 - 9x$ with fewer terms.

Caleb
5 - 3x + 11 - 9x
16 - 6x
10x

a. Describe the mistake that Caleb made.

Responses vary.

- Caleb combined $16 - 6x$ to be $10x$, but they can't be combined.
- Caleb subtracted $3x - 9x$ instead of $-3x - 9x$.

b. Write an expression equivalent to $5 - 3x + 11 - 9x$ that has two terms.

$16 - 12x$ (or equivalent)

6. When fully charged, Jaylin's computer works for 20 hours. She uses her computer for about 1.5 hours each day.



a. If Jaylin's computer starts with a full charge, can she use it for 13 days? Circle one.

(Yes) No

Explain how you know.

Explanations vary. Jaylin would use the computer for 19.5 hours in 13 days, so there would only be 0.5 hours remaining in the battery.

b. Which expression describes the hours of computer charge remaining after d days?

A. $20 - 2d$ B. $1.5 - 20d$ (C.) $20 - 1.5d$ D. $1.5d - 20$

Write an inequality that represents the number of days Jaylin can use her computer.

$1.5d < 20$ (or equivalent)

c. Solve the inequality you wrote.

$d < 13\frac{1}{3}$

Explain what the solutions mean in this situation.

Explanations vary. The solutions to this inequality represent the number of days that Jaylin can use the computer.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 2: • Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical Problems. • Write equivalent expressions by adding, subtracting, expanding, and factoring. **(Lessons 4–12)**	3	**Support**	• **Teacher Move:** Consider revisiting Lesson 7, Activity 1 and asking, "How can you use an area model to help you expand or factor?" Then invite students to revise Problem 3.
	4	**Support**	• **Mini-Lesson:** *Solving Equations With Positive and Negative Numbers*
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 5, Activity 2 and asking, "How can you use a hanger diagram to solve an equation?" Then invite students to revise Problem 4 by drawing tape diagrams to help with their thinking.
	5	**Support**	• **Teacher Move:** Consider revisiting Lesson 10, Activity 1 and discussing, "What did Caleb do well? What was his error?"
		Stretch	• **You're invited to explore more:** Lesson 10 (*Collect the Squares*)
Sub-Unit 3: • Solve inequalities of the form $px + q > r$ and $px + q < r$ that represent real-world and mathematical problems. • Create graphs that represent solutions to inequalities, including those with ≥ or ≤. **(Lessons 13–16)**	1	**Support**	• **Teacher Move:** Consider revisiting Lesson 14, Synthesis and invite students to work together to solve and predict what the solutions will look like when graphed.
		Strengthen	• **Repeated Challenges:** Lesson 14 (*Unbalanced Hangers*)
	2, 6	**Support**	• **Mini-Lesson:** Solving Inequalities
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 15, Activity 1 and students share with a partner their strategies for problem 5.

UNIT 7

Polygons and Cylinders

Students calculate the areas of quadrilaterals, polygons, and composite figures. Students also explore the volume and surface area of cylinders.

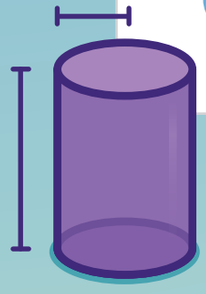
Essential Questions

- How are the area formulas for rectangles, triangles, parallelograms, rhombi, and trapezoids related?
- When is it more efficient to subtract the area of a composite figure from a larger figure instead of decomposing the figure?
- Where do you see volume and surface area of cylinders used in the real world?



Sub-Unit Goals

Sub-Unit 1	• Calculate the areas of parallelograms, rhombi, trapezoids, polygons, and composite shapes.
Sub-Unit 2	• Calculate the volume and surface area of cylinders.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Solve problems involving two-dimensional figures, including circles. **(MA.7.GR.1)**

Solve problems involving three-dimensional figures, including right circular cylinders. **(MA.7.GR.2)**

Benchmarks Addressed in This Unit: MA.7.GR.1.1 MA.7.GR.1.2 MA.7.GR.1.4 MA.7.GR.1.5 MA.7.GR.2.1 MA.7.GR.2.2 MA.7.GR.2.3

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**

Also Addressed: **MTR.5.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students calculate the areas of quadrilaterals, polygons, and composite figures. Students also explore the volume and surface area of cylinders.

Prior Learning

- **Grade 6, Unit 1:** Calculate the area of triangles **(MA.6.GR.2.1)**
- **Grade 6, Unit 1:** Calculate the area of composite figures composed of rectangles and triangles **(MA.6.GR.2.2)**
- **Grade 6, Unit 1:** Calculate the surface area of rectangular prisms **(MA.6.GR.2.4)**
- **Grade 6, Unit 4:** Calculate the volume of rectangular prisms **(MA.6.GR.2.3)**

Future Learning

- **Grade 8, Unit 1:** Sum of interior angles of a regular polygon **(MA.8.GR.1.6)**
- **High School:** Volumes of pyramids, cones, and spheres **(MA.912.GR.4.5)**
- **High School:** Surface area of pyramids, cones, and spheres **(MA.912.GR.4.6)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 4**, students use decomposing and rearranging to find the area of complex shapes. They generate a formula for the area of circles by making sense of several visual representations.

Strategic Competence

In **Lesson 5**, students generate methods for solving multistep problems involving circumference and area.

Procedural Fluency

In **Lessons 3–4**, students build fluency in calculating the circumference of circles and the perimeter of complex shapes that include parts of circles.

Adaptive Reasoning

In **Lesson 5**, students reflect on the precision of their estimates for the area of a circle, determining whether they are greater than or less than the exact area.

Application

In **Lesson 7**, students apply their understanding of finding the volume of cylinders in context.

Productive Disposition

In **Lesson 4**, students use their creativity to create challenges for their classmates and develop their own strategies for calculating circumference.

Unit at a Glance

Assess and Respond

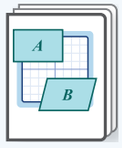


A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.5.GR,3,1 MA.5.GR.3.3
MA.6.GR.2.1 MA.6.GR.2.2
MA.6.GR.2.3 MA.6.GR.2.4
MA.6.AR.1.3 MA.6.AR.2.4
MA.7.GR.1.3
MTR.1.1

Sub-Unit 1



1 Access to All Areas

Investigating Areas of Special Quadrilaterals

Derive the area formulas for a parallelogram, rhombus, and trapezoid and determine the areas of these quadrilaterals.

MA.7.GR.1.1
MTR.1.1 MTR.2.1 MTR.5.1



2 Pile of Polygons

Investigating Polygons and Their Areas

Apply knowledge of area to a new category of shapes: **polygons**.

MA.7.GR.1.2
MTR.1.1 MTR.4.1 MTR.5.1

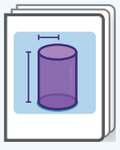


3 Shapes Inside Shapes

Area of Composite Figures

Use different strategies to determine the area of composite figures.

MA.7.GR.1.1 MA.7.GR.1.2
MTR.2.1 MTR.4.1 MTR.7.1



7 Scaling Cylinders

Applying Volumes of Cylinders

Explore how changing a cylinder's radius or height impacts its volume.

MA.7.GR.2.3
MTR.1.1 MTR.5.1 MTR.7.1



8 From Flat to Round

Surface Area of Cylinders

Explore and use strategies to calculate the surface area of a cylinder.

MA.7.GR.2.1 MA.7.GR.2.2
MTR.1.1 MTR.2.1 MTR.7.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.GR.1.1 MA.7.GR.1.2
MA.7.GR.1.3 MA.7.GR.1.4
MA.7.GR.2.2 MA.7.GR.2.3

Pacing: 11 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- **8 Lessons**
- **Pre-Unit Check**
- **Sub-Unit Quiz**
- **End-of-Unit Assessment**

Sub-Unit 2

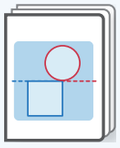


4 Area Challenges

Calculating Areas of Complex Shapes

Apply the area formula to solve problems involving the area of circles as well as shapes composed of parts of circles and squares.

MA.7.GR.1.4 MA.7.GR.1.5
MTR.1.1 MTR.4.1 MTR.5.1



5 Circle vs. Square

Calculating Area Given Perimeter

Apply knowledge of circumference and area to make sense of a challenge involving calculating the areas of two shapes based on their perimeters.

MA.7.GR.1.4 MA.7.GR.1.5
MTR.1.1 MTR.3.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.GR.1.1 MA.7.GR.1.2
MA.7.GR.1.3 MA.7.GR.1.4

Sub-Unit 2



6 Cylinders

Volumes of Cylinders

Explore and use strategies to calculate the volume of a cylinder.

MA.7.GR.2.3
MTR.2.1 MTR.3.1 MTR.4.1

Pacing Considerations

Lesson 4: This lesson supports students in developing fluency calculating the area of shapes composed of squares and parts of circles. This lesson may be omitted as it addresses standards already thoroughly covered elsewhere in the unit.

Lesson 5: This lesson supports students in developing fluency calculating area and circumference of shapes through a playful problem-solving challenge. This lesson may be omitted as it addresses standards already thoroughly covered elsewhere in the unit.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1	Lesson 2	Lesson 3	Lesson 4
Copy: Show What You Know	**Copy:** Show What You Know, Activity 2 Sheet *(one per student)* **Materials:** graph paper / straightedges *(optional)*	**Copy:** Show What You Know	**Copy:** Show What You Know **Materials:** coloring tools *(as needed)*
Lesson 5	**Lesson 6**	**Lesson 7**	**Lesson 8**
Copy: Show What You Know **Cut:** Activity 2 Cards *(one per pair)*	**Copy:** Show What You Know **Materials:** three-dimensional solids, calculators *(as needed)*	**Copy:** Show What You Know	**Copy:** Show What You Know **Materials:** calculators *(as needed)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

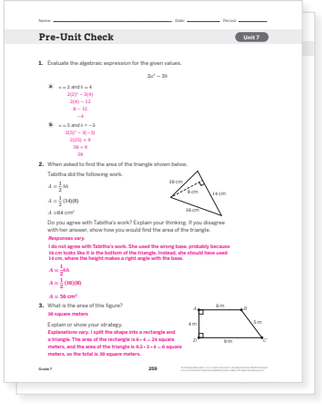
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.5.GR,3,1, MA.5.GR.3.3, MA.6.GR.2.1, MA.6.GR.2.2, MA.6.GR.2.3, MA.6.GR.2.4, MA.6.AR.1.3, MA.6.AR.2.4, MA.7.GR.1.3


Pre-Unit Check

Unit 7

1. Evaluate the algebraic expression for the given values.

$2a^2 - 3b$

a. $a = 2$ and $b = 4$

$2(2^2 - 3(4)$

$2(4) - 12$

$8 - 12$

-4

b. $a = 5$ and $b = -3$

$2(5^2 - 3(-3)$

$2(25) + 9$

$50 + 9$

59

2. When asked to find the area of the triangle shown below.

Tabitha did the following work.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(14)(8)$

$A = 64$ cm^2

Do you agree with Tabitha's work? Explain your thinking. If you disagree with her answer, show how you would find the area of the triangle.

Responses vary.

I do not agree with Tabitha's work. She used the wrong base, probably because 14 cm looks like it is the bottom of the triangle. Instead, she should have used 16 cm, where the height makes a right angle with the base.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(16)(8)$

$A = 56$ cm^2

3. What is the area of this figure?

36 square meters

Explain or show your strategy.

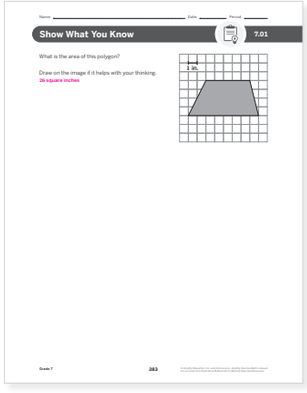
Explanations vary. I split the shape into a rectangle and a triangle. The area of the rectangle is $6 \times 4 = 24$ square meters, and the area of the triangle is $6.3 \div 4 = 6$ square meters, so the total is 36 square meters.

Grade 7 259

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmark assessed on Lesson 1 Show What You Know: MA.7.GR.1.1


Show What You Know

7.01

What is the area of this polygon?

Draw on the image if it helps with your thinking.

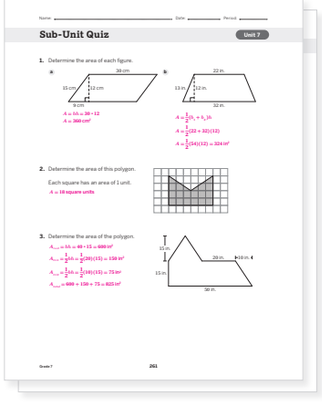
36 square inches

Grade 7 283

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: MA.7.GR.1.1, MA.7.GR.1.2, MA.7.GR.1.3, MA.7.GR.1.4


Sub-Unit Quiz

Unit 7

1. Determine the area of each figure.

a. $A = 55 + 30 + 12$

$A = 360$ cm^2

b. $A = \frac{1}{2}(h_1 + h_2)b$

$A = \frac{1}{2}(22 + 32)(12)$

$A = \frac{1}{2}(54)(12) = 324$ m^2

2. Determine the area of this polygon.

Each square has an area of 1 unit.

$A = 18$ square units

3. Determine the area of the polygon.

$A_{rect} = 55 + 60 + 15 = 600$ in^2

$A_{tri} = \frac{1}{2}bh = \frac{1}{2}(30)(10) = 150$ in^2

$A_{tri} = \frac{1}{2}bh = \frac{1}{2}(10)(15) = 75$ in^2

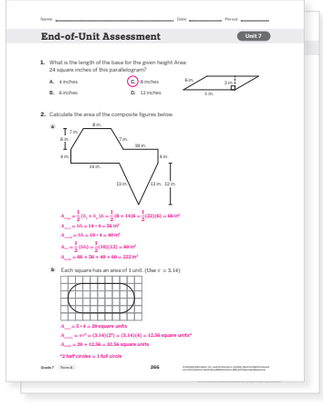
$A_{total} = 600 + 150 + 75 = 825$ in^2

Grade 7 261

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.GR.1.1, MA.7.GR.1.2, MA.7.GR.1.3, MA.7.GR.1.4, MA.7.GR.2.2, MA.7.GR.2.3


End-of-Unit Assessment

Unit 7

1. What is the length of the base for the given height Area: 24 square inches of this parallelogram?

A. 4 inches C. 8 inches

B. 6 inches D. 12 inches

2. Calculate the area of the composite figures below.

a. $A_{rect} = \frac{1}{2}(h_1 + h_2)b = \frac{1}{2}(8 + 14)8 = \frac{1}{2}(22)(16) = 44$ m^2

$A_{rect} = 10 \times 5.6 = 4 = 56$ in^2

$A_{rect} = 5.5 \times 10 = 4 = 40$ in^2

$A_{tri} = \frac{1}{2}(bh) = \frac{1}{2}(8)(12) = 48$ in^2

$A_{total} = 66 + 56 + 40 + 60 = 222$ in^2

b. Each square has an area of 1 unit. (Use $\pi \approx 3.14$)

$A_{rect} = 2 \times 4 = 24$ square units

$A_{semi} = \pi r^2 = (3.14)(2^2) = (3.14)(4) = 12.56$ square units*

$A_{total} = 20 + 12.56 = 32.56$ square units

*2 half circles = 1 full circle

Grade 7 Form A 266

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

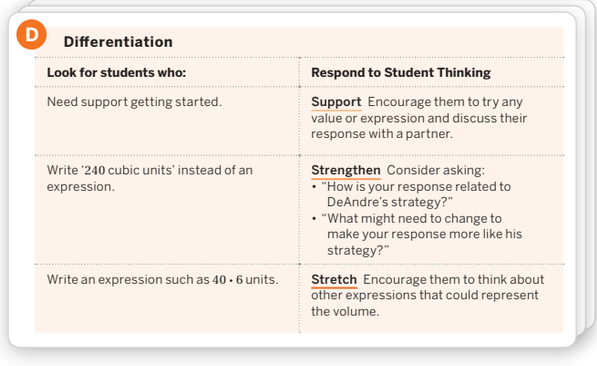
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Calculate the areas of parallelograms, rhombi, trapezoids, polygons, and composite shapes.	**Mini-Lesson** • *Calculating the Area of Parallelograms* • *Calculating Areas of Complex Shapes*	**Challenge Creator** • Lesson 2 (*Pile of Polygons*) • Lesson 4 *(Area Challenges)* **Practice** • Lessons 1–5	**Teacher Moves:** Invite students to create and find the area of famous logos which are composite shapes.
Sub-Unit 2 (Lessons 6–8) • Calculate the volume and surface area of cylinders.	**Mini-Lesson** • *Calculating Volumes of Cylinders*	**Practice** • Lessons 6–8	**You're invited to explore more.** • Lesson 6 *(Cylinders)* **Teacher Moves:** Invite students to measure and find the volume of classroom objects.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 6**, where students determine the volume of a prism using a fictional student's strategy.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try any value or expression and discuss their response with a partner.
Write '240 cubic units' instead of an expression.	**Strengthen** Consider asking: • "How is your response related to DeAndre's strategy?" • "What might need to change to make your response more like his strategy?"
Write an expression such as $40 \cdot 6$ units.	**Stretch** Encourage them to think about other expressions that could represent the volume.

Accessibility

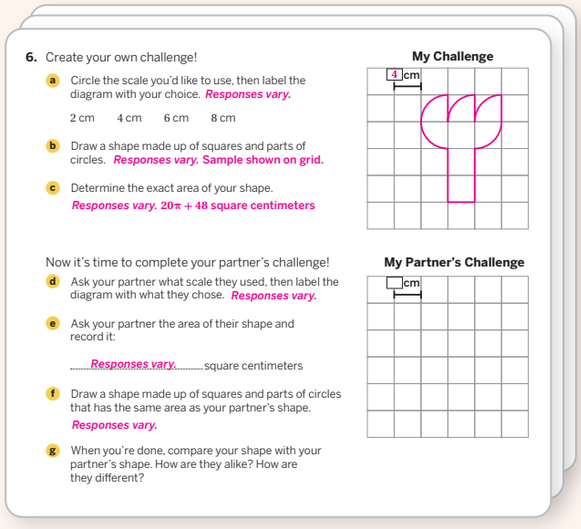
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Sustaining Effort and Persistence

Lesson 4: In this lesson, students create their own challenge to submit to the class gallery. Students then solve each other's challenges which fosters a sense of community.



6. Create your own challenge!

My Challenge

a. Circle the scale you'd like to use, then label the diagram with your choice. *Responses vary.*

2 cm 4 cm 6 cm 8 cm

b. Draw a shape made up of squares and parts of circles. *Responses vary.* **Sample shown on grid.**

c. Determine the exact area of your shape.

Responses vary. **$20\pi + 48$ square centimeters**

Now it's time to complete your partner's challenge!

My Partner's Challenge

d. Ask your partner what scale they used, then label the diagram with what they chose. *Responses vary.*

e. Ask your partner the area of their shape and record it:

Responses vary. square centimeters

f. Draw a shape made up of squares and parts of circles that has the same area as your partner's shape.

Responses vary.

g. When you're done, compare your shape with your partner's shape. How are they alike? How are they different?

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Fine Motor Skills

Here are two suggestions in this unit for supporting students with decomposing figures and creating composite figures.

Lesson 3, Activity 2

Accessibility: Fine Motor Skills Consider providing ways to decompose the figure and labeling one or more measurements in the figure so students can focus on determining the area.

Lesson 4, Activity 2

Accessibility: Fine Motor Skills To reduce anxiety about precision, share that the goal is to communicate ideas rather than to draw a perfect circle. Invite students to label their drawing to justify their area calculations.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary and Language Goals

New Vocabulary	Lesson(s)
base (of a parallelogram) **height (of a parallelogram)**	1
polygon	2
cylinder	6
surface area	8

Review Vocabulary
composite figure
net
parallelogram
perpendicular
quadrilateral
rhombus
trapezoid
volume

Math Language Routines

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	3
MLR2: Collect and Display	2–3 and 6–7
MLR3: Critique, Correct, Clarify	2, 4, and 6
MLR7: Compare and Connect	1

Sub-Unit Language Goals

- Describe the strategies for deriving the area formulas for parallelograms, rhombi, and trapezoids.
- Describe strategies for determining areas of polygons and composite figures.
- Explain how to derive the formulas for the volume and surface area of a cylinder.

Multilingual / English Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 2**, where students describe a strategy for calculating the area of a polygon. Students are encouraged to begin by sharing their strategy in their primary language to open access to the task. Students are given sentence frames to support them in expressing the steps they took to calculate the area of the shape.

ELL **English Language Learners**
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others. Provide sentence frames to help students explain their thinking (e.g., To determine the area of this polygon, first _____. Then, _____.).**(Listening and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Sum of Interior Angles of a Regular Polygon

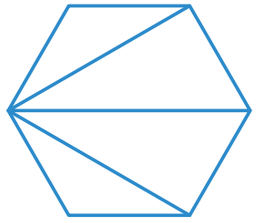
In this unit, students decompose polygons into triangles and quadrilaterals to determine the area of a given polygon. In **Grade 8, Unit 1**, students will decompose polygons into triangles to calculate the sum of the interior angles of a regular polygon.

FL B.E.S.T. Math Benchmark:
MA.8.GR.1.6

Example:

To determine the sum of the interior angle measures of a polygon, decompose the figure into triangles by drawing lines from the same vertex. Multiply the number of triangles by $180°$.

A hexagon can be decomposed into 4 triangles. The sum of the interior angle measures of a hexagon is $4 \times 180° = 720°$.



Volume of Pyramids, Cones, and Spheres

In this unit, students calculate the volume of cylinders. In **High School**, students will solve problems involving the volumes of pyramids, cones, and spheres.

FL B.E.S.T. Math Benchmark:
MA.912.GR.4.5

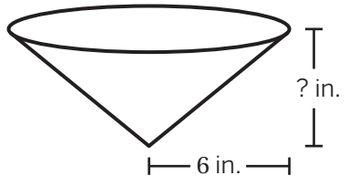
Example:

The volume of this cone is 60π cubic inches.

What is the height of the cone?

$$V = \frac{1}{3}\pi r^2 \cdot h$$
$$60\pi = \frac{1}{3}\pi(6)^2 \cdot h$$
$$60\pi = 12\pi \cdot h$$
$$5 = h$$

The height of the cone is 5 inches.



Surface Area of Pyramids, Cones, and Spheres

In this unit, students calculate the surface area of cylinders. In **High School**, students will solve problems involving the surface areas of pyramids, cones, and spheres.

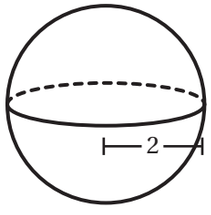
FL B.E.S.T. Math Benchmark:
MA.912.GR.4.6

Example:

What is the surface area of a sphere with a radius of 2 centimeters?

$$SA = 4\pi r^2$$
$$SA = 4\pi(2)^2$$
$$SA = 16\pi$$

The surface area of the sphere is 16π square centimeters.

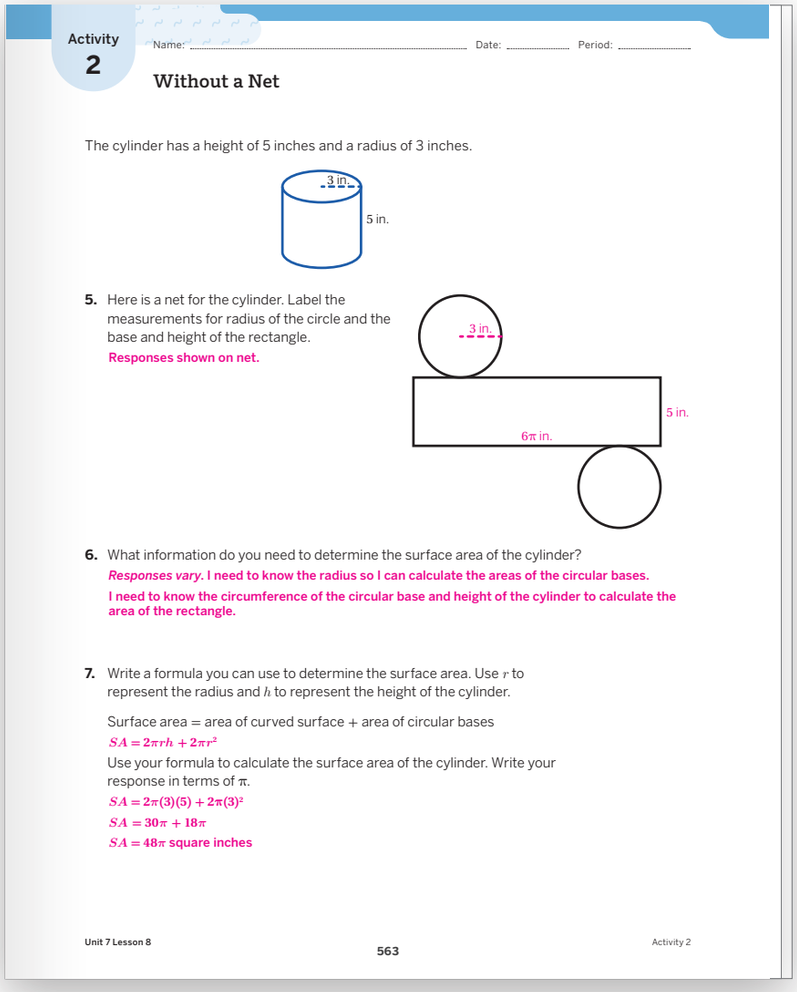


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Strategic Competence

Lesson 8, Activity 2



Activity 2

Name: _____ Date: _____ Period: _____

Without a Net

The cylinder has a height of 5 inches and a radius of 3 inches.

5. Here is a net for the cylinder. Label the measurements for radius of the circle and the base and height of the rectangle.

Responses shown on net.

6. What information do you need to determine the surface area of the cylinder?

Responses vary. **I need to know the radius so I can calculate the areas of the circular bases.**

I need to know the circumference of the circular base and height of the cylinder to calculate the area of the rectangle.

7. Write a formula you can use to determine the surface area. Use r to represent the radius and h to represent the height of the cylinder.

Surface area = area of curved surface + area of circular bases

$SA = 2\pi rh + 2\pi r^2$

Use your formula to calculate the surface area of the cylinder. Write your response in terms of π.

$SA = 2\pi(3)(5) + 2\pi(3)^2$

$SA = 30\pi + 18\pi$

$SA = 48\pi$ square inches

Unit 7 Lesson 8 563 Activity 2

Context

In this activity, students strengthen their understanding of how nets and models can be used to determine the surface area of a cylinder.

Try This

Put on your student hat and complete Problems 5–7.

Points to Ponder

- What do students need to understand about the circumference of the circular base to support their labeling of the rectangular section of the net?
- How does calculating the surface area of a cylinder build from their understanding of formulas for circles and rectangles?
- How will nets support their understanding of calculating the surface area of figures in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Evaluate expressions	Lesson 1	1	MA.6.AR.1.3
2	Calculating the area of a triangle	Lesson 1	1	MA.6.GR.2.1
3	Using structure to calculate the area of a non-rectangular quadrilateral	Lesson 2	1	MA.6.GR.2.2
4	Calculating the area and circumference of a circle	Lesson 5	2	MA.7.GR.1.3, MTR.1.1
5	Describing understanding of volume and surface area	Lesson 6	1	MA.5.GR.3.3, MA.6.GR.2.4
6	Calculating the volume and surface area of a rectangular prism and justifying the response	Lesson 6	1	MA.5.GR.3.1, MA.6.GR.2.3

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 7

1. Evaluate the algebraic expression for the given values.

$$2a^2 - 3b$$

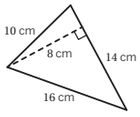
a $a = 2$ and $b = 4$

$2(2)^2 - 3(4)$
$2(4) - 12$
$8 - 12$
-4

b $a = 5$ and $b = -3$

$2(5)^2 - 3(-3)$
$2(25) + 9$
$50 + 9$
59

2. When asked to find the area of the triangle shown below,



Tabitha did the following work.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(14)(8)$

$A = 64 \text{ cm}^2$

Do you agree with Tabitha's work? Explain your thinking. If you disagree with her answer, show how you would find the area of the triangle.

Responses vary.

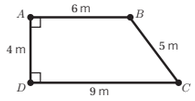
I do not agree with Tabitha's work. She used the wrong base, probably because 16 cm looks like it is the bottom of the triangle. Instead, she should have used 14 cm, where the height makes a right angle with the base.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(16)(8)$

$A = 56 \text{ cm}^2$

3. What is the area of this figure?



30 square meters

Explain or show your strategy.

Explanations vary. I split the shape into a rectangle and a triangle. The area of the rectangle is $6 \cdot 4 = 24$ square meters, and the area of the triangle is $0.5 \cdot 3 \cdot 4 = 6$ square meters, so the total is 30 square meters.

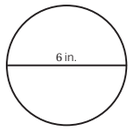
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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 7

4. Use the circle to answer the questions.



a Calculate the circumference of the circle. Use 3.14 for π.

$A = \pi d$
$A = (3.14)(6)$
$A = 18.84$ in.

b Calculate the area of the circle. Use 3.14 for π.

$A = \pi r^2$
$A = (3.14)(3)^2$
$A = (3.14)(9)$
$A = 28.26 \text{ in}^2$

5. Consider volume and surface area.

a What are some things you know about volume and surface area?

Responses vary.

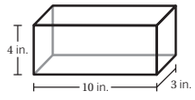
- Surface area is the amount it takes to cover the outside of an object, and volume is the amount it takes to fill up an object.
- The volume of a cube is the side length cubed.

b What are some things you still wonder about volume and surface area?

Responses vary.

- Is the volume of every object length times width times height?
- Why is surface area measured in units squared?
- How do you remember the difference between volume and surface area?

6. Here is a rectangular prism.



a How many 1-by-1-by-1-inch cubes fit inside of this rectangular prism?

120 cubes

b How many square inches of paper would you need to cover the entire prism?

164 square inches

Show or explain your thinking.

Explanations vary. There are two faces whose areas are $4 \cdot 3 = 12$ square inches, two faces whose areas are $4 \cdot 10 = 40$ square inches, and two faces whose areas are $3 \cdot 10 = 30$ square inches. So the total surface area is $12 + 12 + 40 + 40 + 30 + 30 = 164$ square inches.

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Calculate the areas of parallelograms, rhombi, trapezoids, polygons, and composite shapes. **(Lessons 1–5)**	1, 2	**Support** • **Mini-Lesson:** *Evaluating Expressions With Exponents* • **Teacher Move:** Have students share their mathematical process with a partner.
	3	**Support** • **Teacher Move:** Consider highlighting several different student strategies from Problem 3, such as total area – including surrounding and subtracting – and decomposing.
	4	**Support** • **Teacher Move:** Consider making an anchor chart for students to reference the area and circumference of a circle.
	5	**Support** • **Teacher Move:** Consider having a public place to record what students know, wonder, and learn about volume and surface area.
	6	**Support** • **Mini-Lesson:** *Determining the Surface Area of Rectangular Prisms* • **Teacher Move:** Consider spending extra time highlighting several different students' strategies from Problem 6.

Area

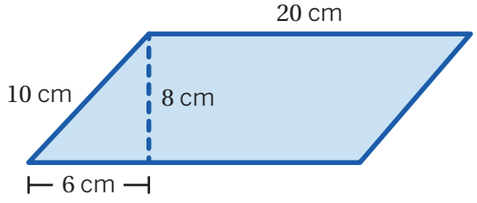
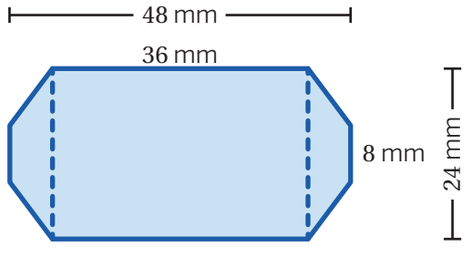
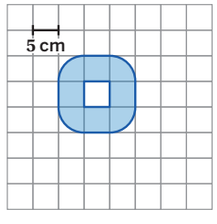
Sub-Unit 1 Goals

- **Goal:** Calculate the areas of parallelograms, rhombi, trapezoids, polygons, and composite shapes.
- **Language Goal:** Describe strategies for deriving the area formulas for parallelograms, rhombi, and trapezoids.
- **Language Goal:** Describe strategies for determining areas of polygons and composite figures.

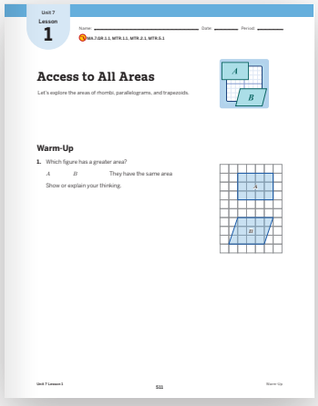
In this sub-unit, students develop strategies for calculating the areas of parallelograms, rhombi, trapezoids, and **polygons**, including generalizing and using formulas using the **base** and **height** of these figures.



Math That Matters Most

Concept or Skill	For example . . .
Calculate the area of a parallelogram.	What is the area of the parallelogram? The base of the parallelogram is 20 centimeters. The height of the parallelogram is 8 centimeters. $A = b \cdot h$ $A = 20 \cdot 8$ $A = 160$ The area of the parallelogram is 160 square centimeters. 
Explain how to calculate the area of a composite figure.	Determine the area of the composite figure. We can decompose the figure into 1 rectangle and 2 identical trapezoids. Area of rectangle: $A = 36 \cdot 24 = 864$ Area of one trapezoid: $A = \frac{1}{2}(6)(8 + 24) = 96$ Add the areas: $A = 864 + 96 + 96 = 1056$ square millimeters 
Calculate the area of complex shapes that include squares and fractions of circles.	What is the area of the shape? There are four quarter circles with a radius of 5 centimeters in the shape. Together, their area is $\pi \cdot (5)^2 = 25\pi$ square centimeters. There are also 4 squares, each with an area of $5 \cdot 5 = 25$ square centimeters. Together, their area is 100 square centimeters. The total area of the shape is $25\pi + 100$ square centimeters. 

Unit 7
Lesson
1


Unit 7
Lesson
1
Name _____ Date _____ Period _____
MA.7.GR.1.1, MTR.1.1, MTR.2.1, MTR.5.1
Access to All Areas
Let's explore the areas of rhombi, parallelograms, and trapezoids.
Warm-Up
1. Which figure has a greater area?
A B They have the same area
Show or explain your thinking.
511
Warm-Up

Access to All Areas

Investigating Areas of Special Quadrilaterals

Let's explore the areas of rhombi, parallelograms, and trapezoids.

Learning Outcomes

Today's Goals

1. **Goal:** Derive the formula for the area of a parallelogram from the formula of a rectangle and determine areas of parallelograms and rhombi.
2. **Goal:** Derive the formula for the area of a trapezoid from the formula for a parallelogram and determine the areas of trapezoids.
3. **Language Goal:** Compare and contrast strategies for determining the areas of a parallelogram, a rhombus, and a trapezoid. **(Speaking and Listening)**

Students decompose *parallelograms*, *rhombi*, and *trapezoids* into squares, rectangles, and triangles to explore the areas of these quadrilaterals. Students use the measurements of the **base** and the **height** to derive area formulas for parallelograms and trapezoids and then apply those formulas to calculate areas.

Coherence

Prior Learning

In Grade 6, Unit 1, students calculated the areas of triangles and rectangles using formulas. Then they calculated the areas of composite figures by decomposing them into triangles and rectangles.

Future Learning

In Lesson 2, students will determine areas of polygons by decomposing them into triangles and quadrilaterals.

Connections

- Students extend their understanding of the area of rectangles and decomposing them to find the area of rhombi, parallelograms, and trapezoids on a grid. (mathematical)
- Students generate formulas for the areas of parallelograms and trapezoids by relating them to the area formula of a rectangle. (mathematical)

FL B.E.S.T. Math Benchmarks

Building Toward

MA.7.GR.1.1
Apply formulas to find the areas of trapezoids, parallelograms and rhombi.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.1	**MA.7.GR.1.2**
MA.6.GR.2.2	**MA.912.GR.3.4**
	MA.912.GR.4.4

Vocabulary

New Vocabulary

base (of a parallelogram)
height (of a parallelogram)

Review Vocabulary

parallelogram
perpendicular
quadrilateral
rhombus
trapezoid

Lesson at a Glance

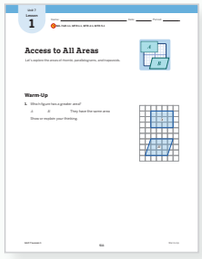
~ 45 min

MA.7.GR.1.1

Warm-Up

Pairs | 5 min

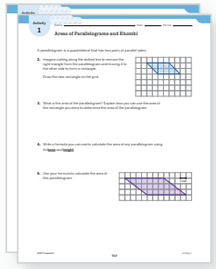
Students determine whether a rectangle or a *parallelogram* has the greater area.
Routine: Think-Pair-Share



Activity 1

Pairs | 15 min

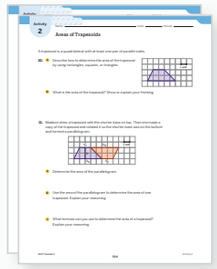
Students explore strategies for determining the areas of parallelograms and *rhombi* and then write a general formula to determine the area of any parallelogram.



Activity 2

Pairs | 15 min

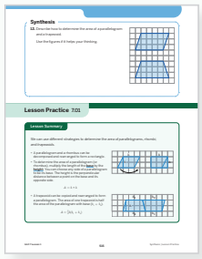
Students explore strategies for determining the area of a *trapezoid* and write a general formula to determine the area of any trapezoid. MLR7: Compare and Connect



Synthesis

Whole Class | 5 min

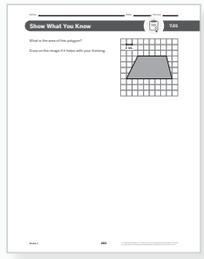
Students synthesize their understanding of how to use different strategies to determine the areas of a parallelogram and a trapezoid.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the area of the parallelograms, rhombi, and trapezoids using different approaches.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students determine whether a rectangle or a *parallelogram* has the greater area.

1 Launch

Direct students' attention to figures *A* and *B*. Consider asking, "How could you describe these shapes?"

Use the Think-Pair-Share routine to help students shape their ideas about which figure has a greater area. Invite students to think independently and then take turns sharing with a partner.

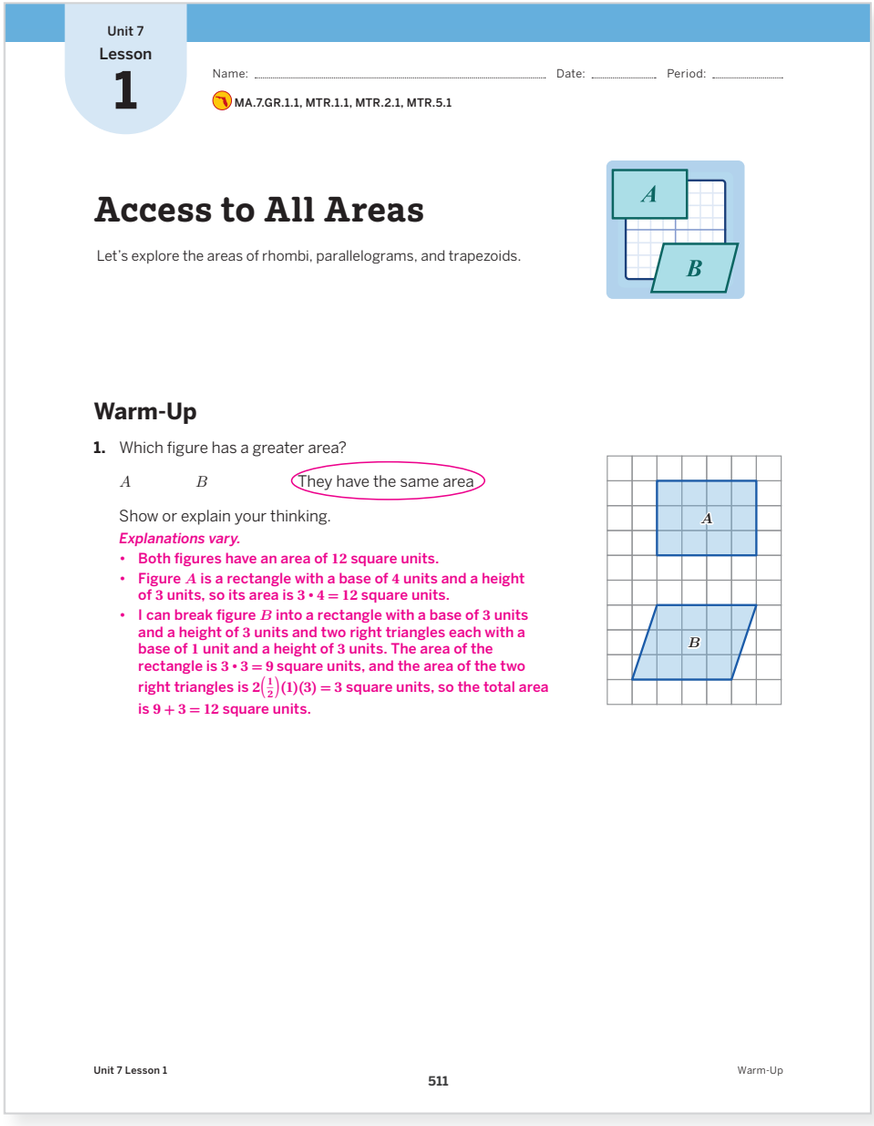
A **Accessibility: Memory and Attention** To support students in surfacing prior knowledge, create or review an anchor chart that displays area formulas for rectangles and triangles.

2 Connect

Invite students to share their responses about their strategies for determining the area of each figure.

Consider asking:

- "What is the formula for the area of a rectangle?"
- "How can you break apart figure B into simpler shapes to use area formulas that you know?"



Unit 7
Lesson
1

Name: ..... Date: ..... Period: .....

MA.7.GR.1.1, MTR.1.1, MTR.2.1, MTR.5.1

Access to All Areas

Let's explore the areas of rhombi, parallelograms, and trapezoids.

Warm-Up

1. Which figure has a greater area?

A *B* They have the same area

Show or explain your thinking.

Explanations vary.

- Both figures have an area of 12 square units.
- Figure A is a rectangle with a base of 4 units and a height of 3 units, so its area is $3 \cdot 4 = 12$ square units.
- I can break figure B into a rectangle with a base of 3 units and a height of 3 units and two right triangles each with a base of 1 unit and a height of 3 units. The area of the rectangle is $3 \cdot 3 = 9$ square units, and the area of the two right triangles is $2\left(\frac{1}{2}\right)(1)(3) = 3$ square units, so the total area is $9 + 3 = 12$ square units.

Unit 7 Lesson 1 511 Warm-Up

Pairs | 15 min

Activity 1 Areas of Parallelograms and Rhombi

Purpose: Students explore strategies for determining the areas of parallelograms and *rhombi* and then write a general formula to determine the area of any parallelogram.

1 Launch

To surface prior knowledge, consider asking, "What are the names and characteristics of the special quadrilaterals that you know?"

Note: If it does not come up naturally, revisit the characteristics of a *parallelogram*, rectangle, square, *rhombus*, and *trapezoid*.

Create an anchor chart with students' descriptions for the properties of special quadrilaterals.

ELL **English Language Learners** Provide a reference that includes descriptions and visuals of geometric shapes, including *quadrilateral*, *rectangle*, *parallelogram*, *rhombus/rhombi*, and *trapezoid*, that students can refer to during the lesson.

2 Monitor

Look for correct explanations of how students explain how to determine the area of the parallelogram using the area of a rectangle to share during the Connect. **(MTR.2.1)**

Consider asking, "How can breaking a shape into parts and rearranging it help determine its area?"

Pause briefly so students can share their formula for the area of a parallelogram.

Create a class definition for the terms **base** and **height** (**of a parallelogram**) using students' language and the glossary.

Consider sharing that the base of a parallelogram can be any one of its sides. The height is the *perpendicular* distance from the chosen base to the opposite side.

Activity 1 continued >

Activity 1

Name: Date: Period:

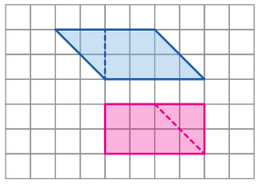
Areas of Parallelograms and Rhombi

A *parallelogram* is a *quadrilateral* that has two pairs of parallel sides.

2. Imagine cutting along the dotted line to remove the right triangle from the parallelogram and moving it to the other side to form a rectangle.

Draw the new rectangle on the grid.

Responses vary. Sample shown on grid.



3. What is the area of the parallelogram? Explain how you can use the area of the rectangle you drew to determine the area of the parallelogram.

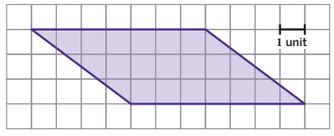
8 square units. *Explanations vary.* The parallelogram and the rectangle have the same base and height. By removing a triangle from the left side of the parallelogram and moving it to the right side of the parallelogram, it forms a rectangle. The rectangle and parallelogram have the same area, which can be found by multiplying the length of the base and the height.

4. Write a formula you can use to calculate the area of any parallelogram using its **base** and **height**.

$A = b \cdot h$

5. Use your formula to calculate the area of this parallelogram.

21 square units



Unit 7 Lesson 1 512 Activity 1

Pairs | 15 min

Activity 1 Areas of Parallelograms and Rhombi (continued)

Purpose: Students explore strategies for determining the areas of parallelograms and *rhombi* and then write a general formula to determine the area of any parallelogram.

2 Monitor

Consider asking, "How can breaking a shape into parts and rearranging it help determine its area?"

D Differentiation *(Problems 6–7)*

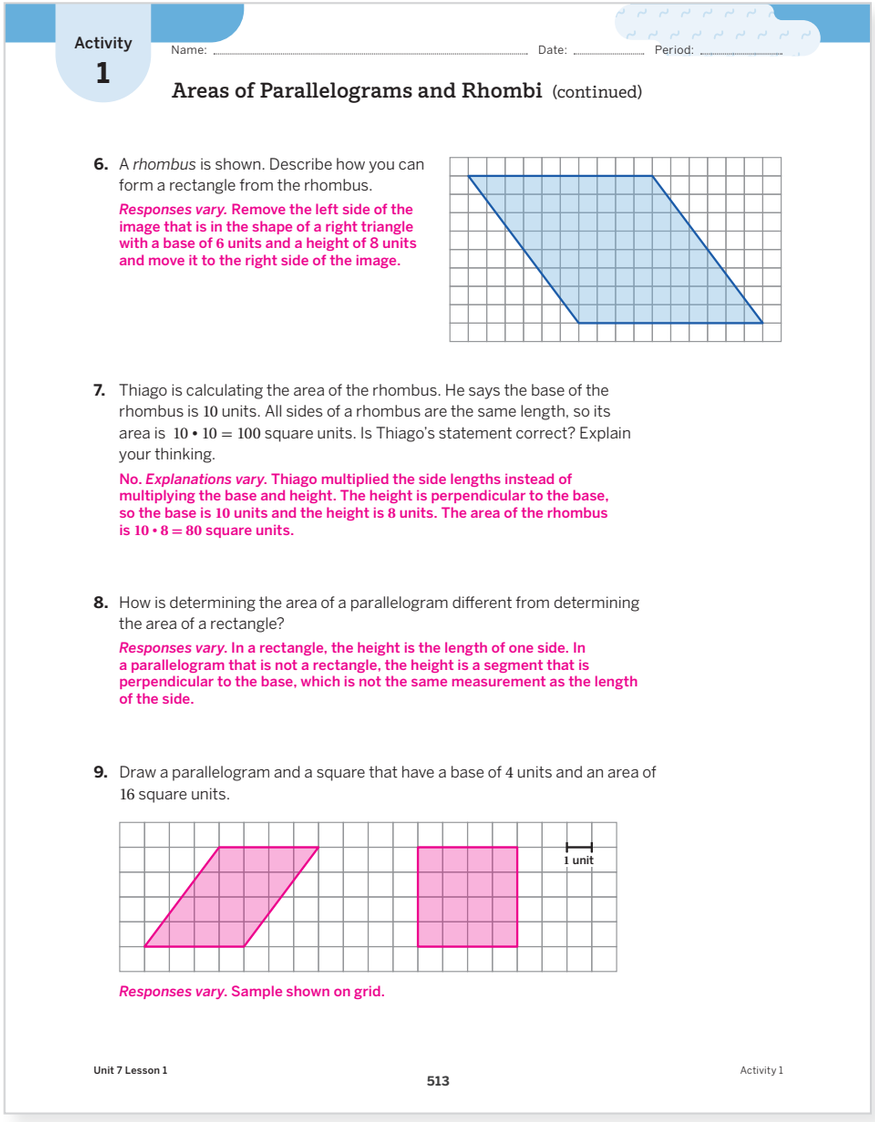
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw the height on the rhombus.
Think that the side length and height have the same measure.	**Support** Invite them to measure the side length and the height with a ruler.
Identify the error in Thiago's statement.	**Stretch** Encourage students to use a lateral side as a base, draw the height, and confirm the height is the same regardless of which side is used as the base.

3 Connect

Invite students to share their responses. Consider asking, "Why don't you need to draw a line to represent the height of a rectangle?"

Invite students to share their drawings. Consider asking, "Is the parallelogram you drew also a rhombus?"

To surface the Key Takeaway, consider asking, "What are the differences and similarities between determining the area of a rectangle and the areas of a parallelogram and a rhombus?" **(MTR.5.1)**



Activity 1

Name: Date: Period:

Areas of Parallelograms and Rhombi (continued)

6. A *rhombus* is shown. Describe how you can form a rectangle from the rhombus.

Responses vary. Remove the left side of the image that is in the shape of a right triangle with a base of 6 units and a height of 8 units and move it to the right side of the image.

7. Thiago is calculating the area of the rhombus. He says the base of the rhombus is 10 units. All sides of a rhombus are the same length, so its area is $10 \cdot 10 = 100$ square units. Is Thiago's statement correct? Explain your thinking.

No. *Explanations vary.* Thiago multiplied the side lengths instead of multiplying the base and height. The height is perpendicular to the base, so the base is 10 units and the height is 8 units. The area of the rhombus is $10 \cdot 8 = 80$ square units.

8. How is determining the area of a parallelogram different from determining the area of a rectangle?

Responses vary. In a rectangle, the height is the length of one side. In a parallelogram that is not a rectangle, the height is a segment that is perpendicular to the base, which is not the same measurement as the length of the side.

9. Draw a parallelogram and a square that have a base of 4 units and an area of 16 square units.

1 unit

Responses vary. Sample shown on grid.

Unit 7 Lesson 1 513 Activity 1

Key Takeaway: To calculate the area of a *parallelogram*, multiply the **base** by its **height**: $A = b \cdot h$.

Pairs | 15 min

Activity 2 Areas of Trapezoids

Purpose: Students explore strategies for determining the area of a *trapezoid* and write a general formula to determine the area of any trapezoid.

Short on time? Consider completing Problem 11 as a class.

1 Launch

Consider sharing that the trapezoids in Problems 10 and 11 have the same dimensions.

Consider asking, "How do you know which sides are the bases in a trapezoid?

2 Monitor

Look and listen for students making sense of each strategy.

Consider asking, "Can you think of another strategy to determine the area of the trapezoid?" **(MTR.1.1)**

Note: If it does not come up naturally, discuss the strategy of drawing a rectangle around the trapezoid and subtracting the parts that aren't in the trapezoid.

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare both strategies for determining the area of a trapezoid. Consider asking:

- "What was helpful about each strategy?"
- "Can you think of another strategy for determining the area of a trapezoid?"

To surface connections, consider discussing why the height of the parallelogram is the same as the height of the trapezoid and the base of the parallelogram is the same as the sum of the two bases of the trapezoid. **(MTR.5.1)**

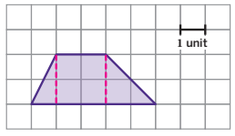
Activity 2

Name: Date: Period:

Areas of Trapezoids

A *trapezoid* is a quadrilateral with at least one pair of parallel sides.

10. a. Describe how to determine the area of the trapezoid by using rectangles, squares, or triangles.

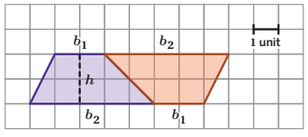


Responses vary. I can draw two vertical lines to form a square and two right triangles and then add the areas of each part.

b. What is the area of the trapezoid? Show or explain your thinking.

7 square units. *Responses vary.* The area of the rectangle is $2 \cdot 2 = 4$ square units, the area of the right triangle on the left is $\left(\frac{1}{2}\right)(2)(1) = 1$ square unit, and the area of the right triangle on the right is $\left(\frac{1}{2}\right)(2)(2) = 2$ square units, so the total area of the trapezoid is $4 + 1 + 2 = 7$ square units.

11. Madison drew a trapezoid with the shorter base on top. Then she made a copy of the trapezoid and rotated it so the shorter base was on the bottom and formed a parallelogram.



a. Determine the area of the parallelogram.

$A = b \cdot h$

$A = 7 \cdot 2$

$A = 14$ square units

b. Use the area of the parallelogram to determine the area of one trapezoid. Explain your reasoning.

7 square units, *Explanations vary.* The area of one trapezoid is equal to half the area of the parallelogram. $\frac{1}{2} \cdot 14 = 7$ square units

c. What formula can you use to determine the area of a trapezoid? Explain your reasoning.

$A = \frac{1}{2}h(b_1 + b_2)$. *Explanations vary.* The height of the trapezoid is the same as the height of the parallelogram. The base of the parallelogram is the sum of b_1 and b_2. The area of one trapezoid is half the area of the parallelogram.

Unit 7 Lesson 1 514 Activity 2

Key Takeaway: The area of a *trapezoid* is half the product of the sum of its bases and its height: $A = \frac{1}{2}h(b_1 \cdot b_2)$.

Whole Class | 5 min

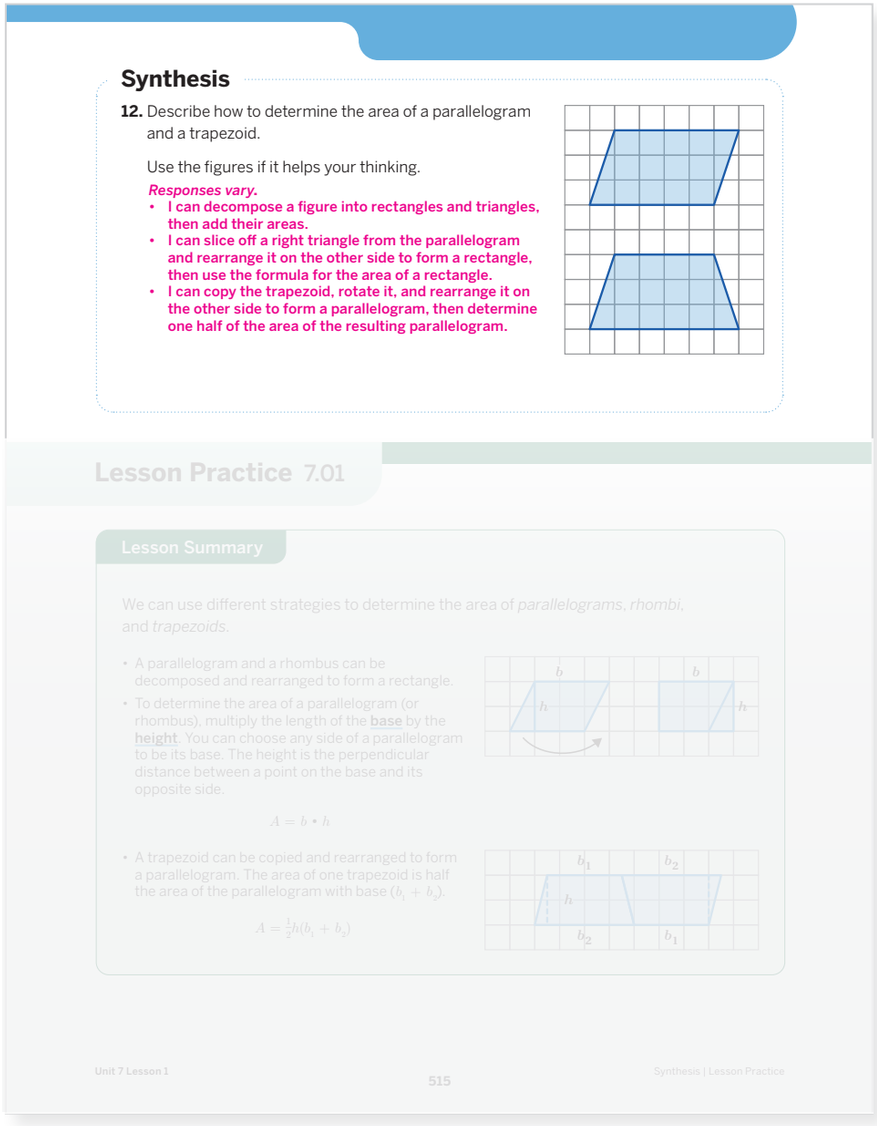
Synthesis

Purpose: Students synthesize their understanding of how to use different strategies to determine the areas of a parallelogram and a trapezoid.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Decomposing and rearranging is a helpful strategy for determining areas of quadrilaterals.
- A parallelogram and a rhombus have the same area as a rectangle with the same base and height.
- Two copies of a trapezoid can form a parallelogram, so you can halve the area of the parallelogram to get the area of the trapezoid.

Create an anchor chart with images and formulas determining the areas of special quadrilaterals. Use different colors to connect the dimensions in the figures to the variables in the formulas.



Synthesis

12. Describe how to determine the area of a parallelogram and a trapezoid.

Use the figures if it helps your thinking.

Responses vary.
- I can decompose a figure into rectangles and triangles, then add their areas.
- I can slice off a right triangle from the parallelogram and rearrange it on the other side to form a rectangle, then use the formula for the area of a rectangle.
- I can copy the trapezoid, rotate it, and rearrange it on the other side to form a parallelogram, then determine one half of the area of the resulting parallelogram.

Lesson Practice 7.01

Lesson Summary

We can use different strategies to determine the area of *parallelograms*, *rhombi*, and *trapezoids*.

- A parallelogram and a rhombus can be decomposed and rearranged to form a rectangle.
- To determine the area of a parallelogram (or rhombus), multiply the length of the base by the height. You can choose any side of a parallelogram to be its base. The height is the perpendicular distance between a point on the base and its opposite side.

$A = b \cdot h$

- A trapezoid can be copied and rearranged to form a parallelogram. The area of one trapezoid is half the area of the parallelogram with base $(b_1 + b_2)$.

$A = \frac{1}{2}h(b_1 + b_2)$

Unit 7 Lesson 1 515 Synthesis | Lesson Practice

Lesson Takeaway: To calculate the area A of a parallelogram, use the formula $A = b \cdot h$, where b represents the length of the **base** and h represents the length of the corresponding **height**. To calculate the area A of a trapezoid, use the formula $A = \frac{1}{2}h(b_1 \cdot b_2)$, where b_1 and b_2 are its bases and h is its height.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the area of the parallelograms, rhombi, and trapezoids using different approaches.

Today's Goals

1. **Goal:** Derive the formula for the area of a parallelogram from the formula of a rectangle and determine areas of parallelograms and rhombi.
2. **Goal:** Derive the formula for the area of a trapezoid from the formula for a parallelogram and determine the areas of trapezoids.
3. **Language Goal:** Compare and contrast strategies for determining the areas of a parallelogram, a rhombus, and a trapezoid. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

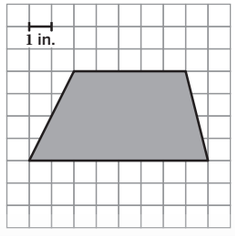
Name: _____ Date: _____ Period: _____

Show What You Know 7.01

What is the area of this polygon?

Draw on the image if it helps with your thinking.

26 square inches



Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the area of parallelograms, rhombi and trapezoids:

» **Respond to Student Thinking:**

- Use ***Calculating the Area of Parallelograms Mini-lesson***.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on the facilitation of the Compare and Connect routine in this lesson. Who shared in the whole class discussion? Who did not? What follow up questions did you ask to help students deepen their thinking? How did you push students to think about multiple strategies?"

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe how to determine the area of a parallelogram and a trapezoid.

Use the figures if it helps your thinking.

Responses vary.

- I can decompose a figure into rectangles and triangles, then add their areas.
- I can slice off a right triangle from the parallelogram and rearrange it on the other side to form a rectangle, then use the formula for the area of a rectangle.
- I can copy the trapezoid, rotate it, and rearrange it on the other side to form a parallelogram, then determine one half of the area of the resulting parallelogram.

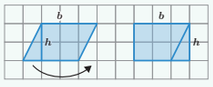
Lesson Practice 7.01

Lesson Summary

We can use different strategies to determine the area of *parallelograms*, *rhombi*, and *trapezoids*.

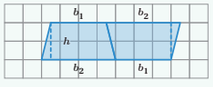
- A parallelogram and a rhombus can be decomposed and rearranged to form a rectangle.
- To determine the area of a parallelogram (or rhombus), multiply the length of the **base** by the **height**. You can choose any side of a parallelogram to be its base. The height is the perpendicular distance between a point on the base and its opposite side.

$$A = b \cdot h$$



- A trapezoid can be copied and rearranged to form a parallelogram. The area of one trapezoid is half the area of the parallelogram with base $(b_1 + b_2)$.

$$A = \frac{1}{2}h(b_1 + b_2)$$

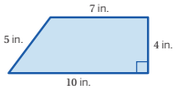


Lesson Practice 7.01

Name: _____ Date: _____ Period: _____

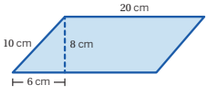
Problems 1–4: Determine the area of each figure.

1.



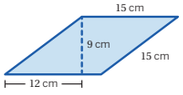
34 square inches

2.



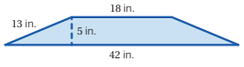
160 square centimeters

3.



135 square centimeters

4.



150 square inches

5. A wall tile in the shape of a parallelogram has horizontal side lengths of 4 inches and diagonal side lengths of 2.5 inches. The height of the tile is 1.5 inches. What is the area of the wall tile in square inches?

6 square inches

6. A sign is in the shape of a rhombus. Each side length is 28 centimeters with a height of 12 centimeters. What is the area of the sign in square centimeters?

336 square centimeters

Lesson Practice 7.01

Name: _____ Date: _____ Period: _____

7. The shape of a park resembles a trapezoid with bases 15 miles and 20 miles and a height of 10 miles. What is the area of the park in square miles?

175 square miles

FAST Practice

8. The parallel sides of an isosceles trapezoid are 20 millimeters and 56 millimeters. The congruent sides are 30 millimeters. The height of the trapezoid is 24 millimeters. What is the area in square millimeters?

A. 912 square millimeters

B. 1,140 square millimeters

C. 1,824 square millimeters

D. 2,280 square millimeters

Spiral Review

Problems 9–12: Evaluate each expression.

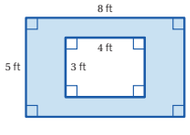
9. $0.5 \cdot 16 \cdot 3 = 24$

10. $\frac{3}{5} \cdot 4 \cdot 15 = 36$

11. $2\frac{3}{4} \div \frac{5}{8} = 4\frac{2}{5}$

12. $36.9 \div 1.8 = 20.5$

13. Calculate the area of the shaded region. Show or explain your thinking.



28 square feet. *Explanations vary.* I subtracted the area of the small rectangle from the area of the large rectangle. $(5 \cdot 8) - (3 \cdot 4) = 40 - 12 = 28$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.GR.1.1
	5–7	2	MA.7.GR.1.1
FAST Practice	8	2	MA.7.GR.1.1
Spiral Review			
	9–12	2	MA.7.NSO.2.2
	13	2	MA.6.GR.2.2

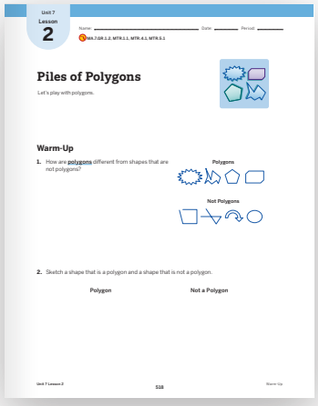
Unit 7

Lesson 2

Pile of Polygons

Investigating Polygons and Their Areas

Let's play with polygons.


Unit 7
Lesson 2

Name _____ Date _____ Period _____

MA.7.GR.1.2, MTR.1.1, MTR.4.1, MTR.5.1

Piles of Polygons

Let's play with polygons.

Warm-Up

1. How are polygons different from shapes that are not polygons?

Polygons

Not Polygons

2. Sketch a shape that is a polygon and a shape that is not a polygon.

Polygon

Not a Polygon

Unit 7 Lesson 2

518

Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Determine the area of a *polygon* with and without a grid using the areas of rectangles and triangles.
2. **Language Goal:** Describe strategies for determining the area of polygons using precise mathematical language. **(Writing, Speaking, and Listening)**
3. **Language Goal:** Describe polygons using their characteristics. **(Writing, Speaking, and Listening)**

Students apply what they've learned about calculating the areas of quadrilaterals and triangles to a new category of shapes: **polygons**. They first define the characteristics of polygons, then develop strategies for calculating their areas with and without a grid. They analyze the features of shapes to inform the way they choose to decompose a figure.

Coherence

Prior Learning

In Lesson 1, students explored strategies for determining the areas of parallelograms, trapezoids, and rhombi.

Future Learning

In Lesson 3, students will solve problems involving areas of composite figures by decomposing them into triangles and quadrilaterals.

Connections

- Students explore visuals of the areas of polygons given it's vertices on a coordinate plane. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.2

Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	MA.912.GR.3.4, MA.912.GR.4.4

Vocabulary

New Vocabulary

polygon

Lesson at a Glance

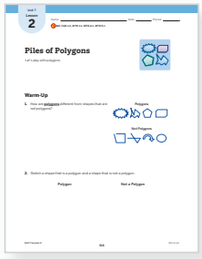
~ 45 min

MA.7.GR.1.2

Warm-Up

Pairs | 5 min

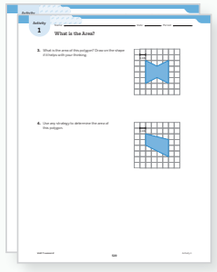
Students identify features of **polygons** by looking at examples and non-examples before drawing one of each. MLR2: Collect and Display



Activity 1

Pairs | 15 min

Students use MLR3: Critique, Correct, Clarify to make sense of multiple strategies for precisely calculating the area of a polygon with and without a grid. **(MTR.4.1)**



Activity 2

Pairs | 15 min

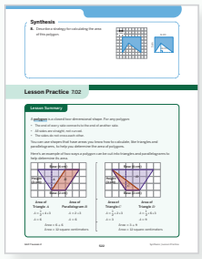
Students create their own polygon and calculate its area. They then challenge themselves to calculate the area of their classmates' polygons.



Synthesis

Whole Class | 5 min

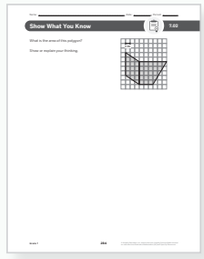
Students synthesize their understanding of how to calculate the area of a polygon with and without a grid. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the area of a polygon.



Prep Checklist

Prepare copies of the Warm-Up Sheet and the Activity 2

This lesson includes:

- Student Edition
- Show What You Know
- Graph paper / straightedges (optional)

Pairs | 5 min

Warm-Up

Purpose: Students identify features of **polygons** by looking at examples and non-examples before drawing one of each.

1 Launch

Direct students' attention to the shapes labeled "Polygons" and "Not Polygons." Consider sharing that *polygon* is a word that describes some types of shapes but not others, and that students' task is to figure out what makes a shape a polygon.

Listen for students who mention the traditional characteristics of polygons (straight sides, closed shapes, no intersections) as well as creative or unique characteristics, and display these ideas during the Connect.

2 Connect

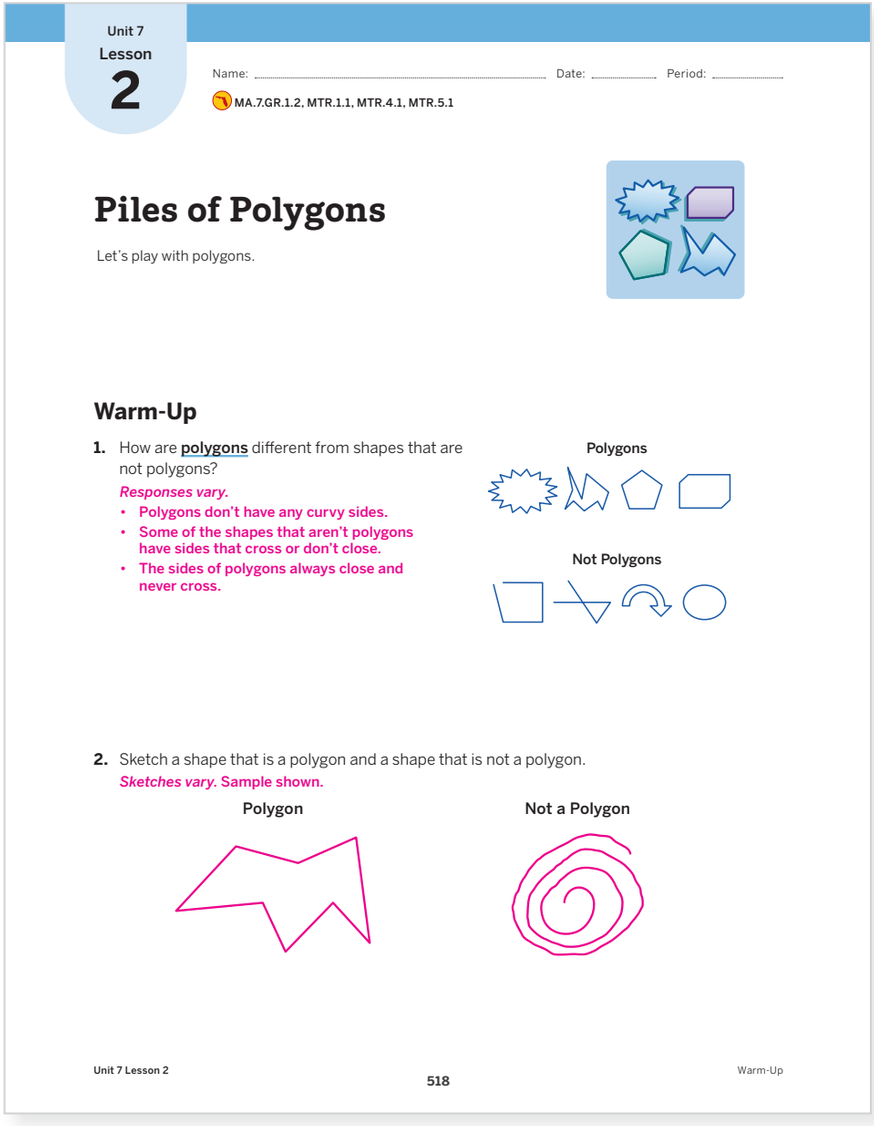
Invite students to share their sketches. Look for a variety of polygons and non-polygons.

MLR **MLR2: Collect and Display** Use this routine to collect students' language as they share words to describe polygons and their polygon examples. Display for students to reference and add to for the rest of the lesson.

Pause briefly to draw a cube and consider asking, "Would a 3-D shape like a cube be considered a polygon?" No, polygons are 2-D shapes.

Create a class definition for the term **polygon** using students' language and the glossary. Consider asking, "What words are most important to include in our definition?"

Display sketched polygons and invite students to decide if they meet the class-created definition of polygons.



Unit 7
Lesson
2

Name: Date: Period:

MA.7.GR.1.2, MTR.1.1, MTR.4.1, MTR.5.1

Piles of Polygons

Let's play with polygons.

Warm-Up

1. How are **polygons** different from shapes that are not polygons?

Responses vary.

- Polygons don't have any curvy sides.
- Some of the shapes that aren't polygons have sides that cross or don't close.
- The sides of polygons always close and never cross.

Polygons

Not Polygons

2. Sketch a shape that is a polygon and a shape that is not a polygon.

Sketches vary. Sample shown.

Polygon

Not a Polygon

Unit 7 Lesson 2 518 Warm-Up

Key Takeaway: A **polygon** is a closed 2-D shape with straight sides that do not cross each other.

Pairs | 15 min

Activity 1 What is the Area?

Purpose: Students use MLR3: Critique, Correct, Clarify to make sense of multiple strategies for precisely calculating the area of a polygon with and without a grid. **(MTR.4.1)**

Short on time? Consider omitting Problem 5, taking note of common mistakes in Problems 4 and 6 instead.

1 Launch

Direct students' attention to the image and consider inviting students to discuss the following with their peers:

- "How would you describe the shape?"
- "How does the structure of the shape influence what strategy you might use to calculate the area?" **(MTR.5.1)**

2 Monitor

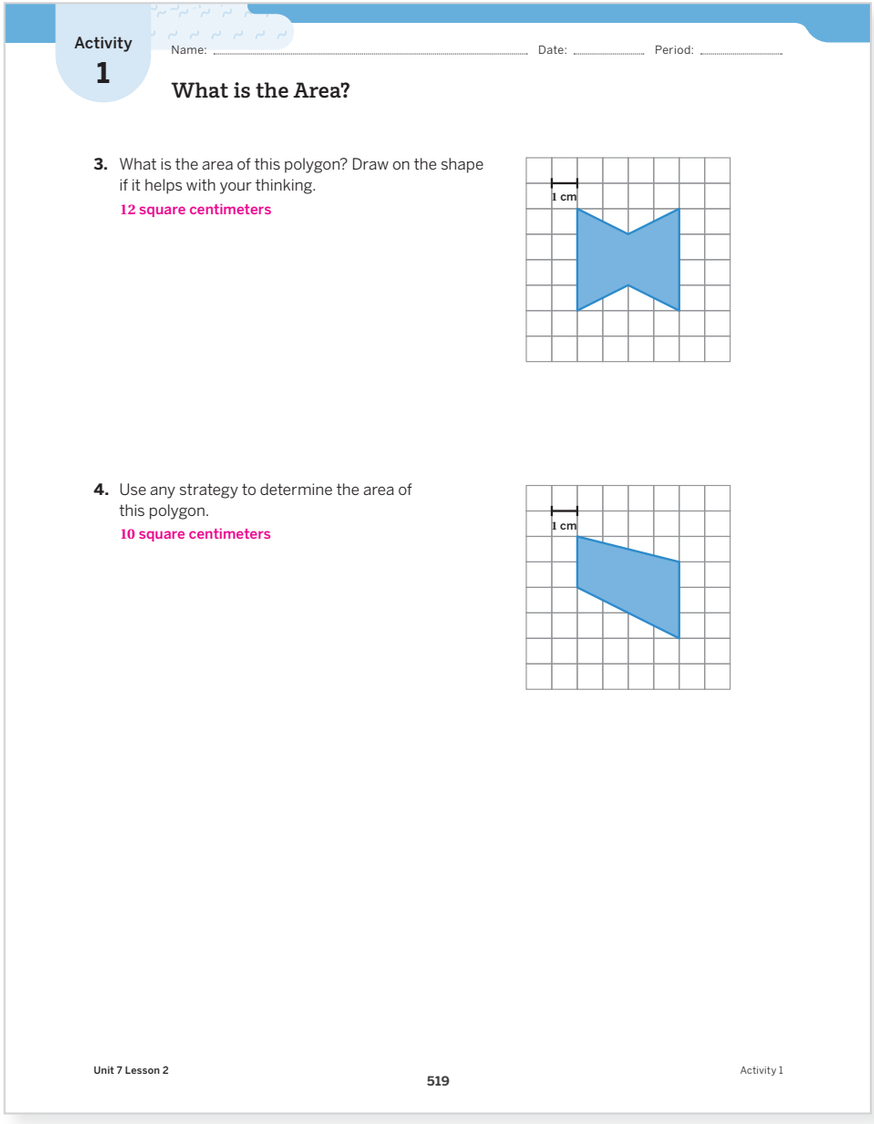
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to use the grid to support their thinking. Consider asking: • "How is this polygon similar to one you know how to determine the area of?" • "Can you cut the polygon into pieces that would be helpful?"
Calculate the area by counting one by one.	**Strengthen** Consider asking, "What would a more efficient strategy look like to determine the area?"
Calculate the area by breaking the shape into triangles and parallelograms.	**Stretch** Invite students to share their strategy with a peer and consider asking, "Are there other ways to decompose this shape? Can you prove it would give you the same area?" **(MTR.1.1)**
Calculate the area by enclosing the shape in a rectangle.	

A Accessibility: Visual-Spatial Processing

Pair students and invite them to record any known measurements (like the vertical side length) before calculating the area of the polygon in order to support students who struggle with visual-spatial processing.

Activity 1 continued >



Activity 1

Name: Date: Period:

What is the Area?

3. What is the area of this polygon? Draw on the shape if it helps with your thinking.

12 square centimeters

4. Use any strategy to determine the area of this polygon.

10 square centimeters

Unit 7 Lesson 2 519 Activity 1

Pairs | 15 min

Activity 1 What is the Area? (continued)

Purpose: Students use MLR3: Critique, Correct, Clarify to make sense of multiple strategies for precisely calculating the area of a polygon with and without a grid. **(MTR.4.1)**

Short on time? Consider omitting Problem 5, taking note of common mistakes in Problems 4 and 6 instead.

2 Monitor

Invite students to compare the strategy they used in the previous problem to Jasmine's and Callen's strategies.

Encourage students to share their responses with a partner and decide if their reasoning was similar to or different from their own. **(MTR.4.1)**

To support students getting started, consider asking:

- "How might we divide this polygon into more recognizable shapes like parallelograms and triangles?"
- "How do the side lengths 5 and 2 give us helpful information about the height of the triangle?" **(MTR.1.1)**

  The height of the triangle is 3 because $5 - 2 = 3$.

A **Accessibility: Visual-Spatial Processing** Provide graph paper for students to draw the polygon on if desired.

Select and sequence responses that include different methods of decomposing areas, including:

- parallelogram + triangle
- triangle + triangle
- enclosed rectangle − triangle

D **Differentiation: (Stretch)** Draw a different polygon with the same area.

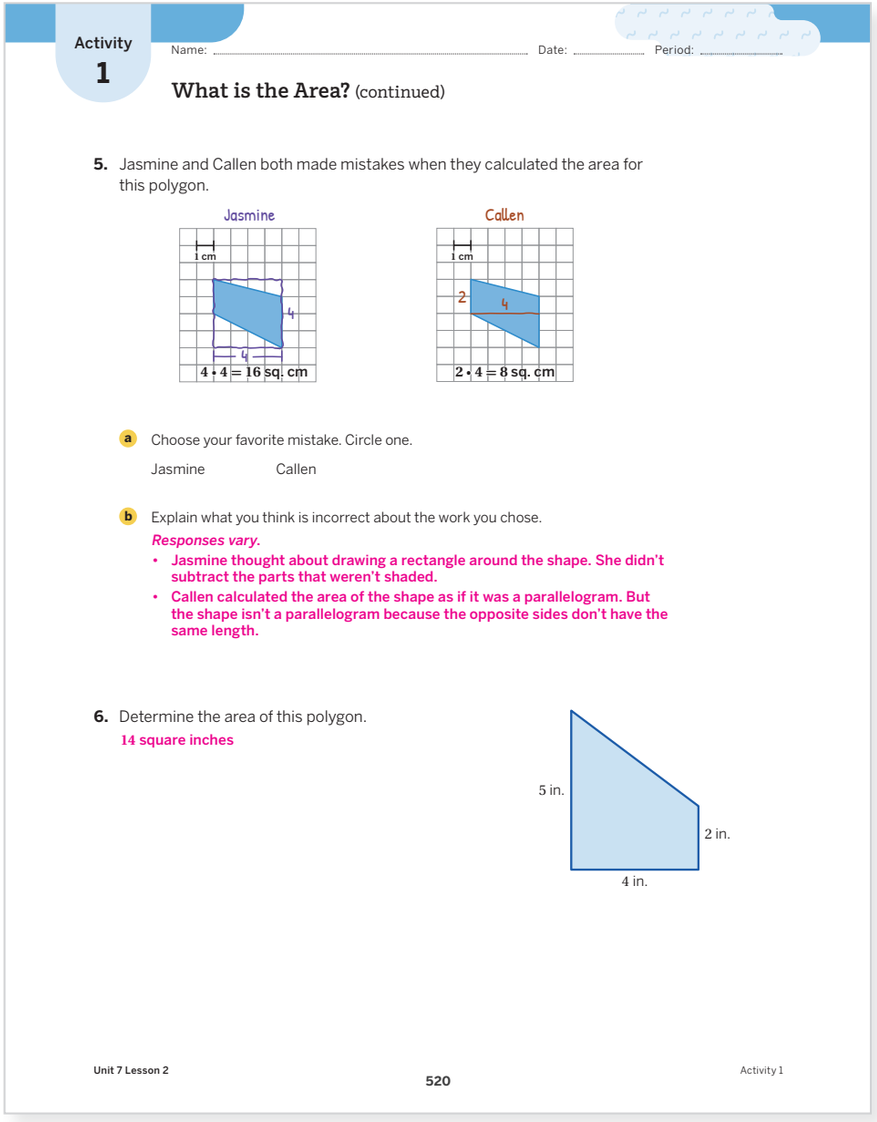
3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Jasmine and Callen's work. Then invite students to:

a. Discuss what is *correct* about the responses.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Jasmine see how their work can be improved, or create a revised response. **(MTR.4.1)**

Invite students to share how they determined the area of the given polygon.

To surface the Key Takeaway, consider asking, "Which strategy for determining the area of a polygon makes the most sense to you? Which do you have questions about?"



Activity 1

Name: Date: Period:

What is the Area? (continued)

5. Jasmine and Callen both made mistakes when they calculated the area for this polygon.

Jasmine

Callen

a. Choose your favorite mistake. Circle one.

Jasmine Callen

b. Explain what you think is incorrect about the work you chose.

Responses vary.

- Jasmine thought about drawing a rectangle around the shape. She didn't subtract the parts that weren't shaded.
- Callen calculated the area of the shape as if it was a parallelogram. But the shape isn't a parallelogram because the opposite sides don't have the same length.

6. Determine the area of this polygon.

14 square inches

Unit 7 Lesson 2 520 Activity 1

Key Takeaway: There are many strategies to determine the area of a **polygon**, including breaking it into familiar shapes, enclosing and subtracting, or using a grid or given measurements.

Pairs | 15 min

Activity 2 Challenge Creator

Purpose: Students create their own polygon and calculate its area. They then challenge themselves to calculate the area of their classmates' polygons.

1 Launch

Invite students to create and calculate the area of their own polygon and then create a challenge for others to solve.

Distribute the Activity 2 Sheet so students can create their polygon challenge. Students will need one or more partners for this activity.

Invite students to try as many challenges as they can with the given time and to use multiple strategies.

2 Monitor

A **Accessibility: Executive Functioning** Invite students to work together on a challenge before trying one independently.

Listen for students discussing how to solve for the areas, including decomposing and rearranging or enclosing and subtracting.

Encourage students to annotate their shapes to show their area-calculating strategies so that they can be shared and celebrated with the class.

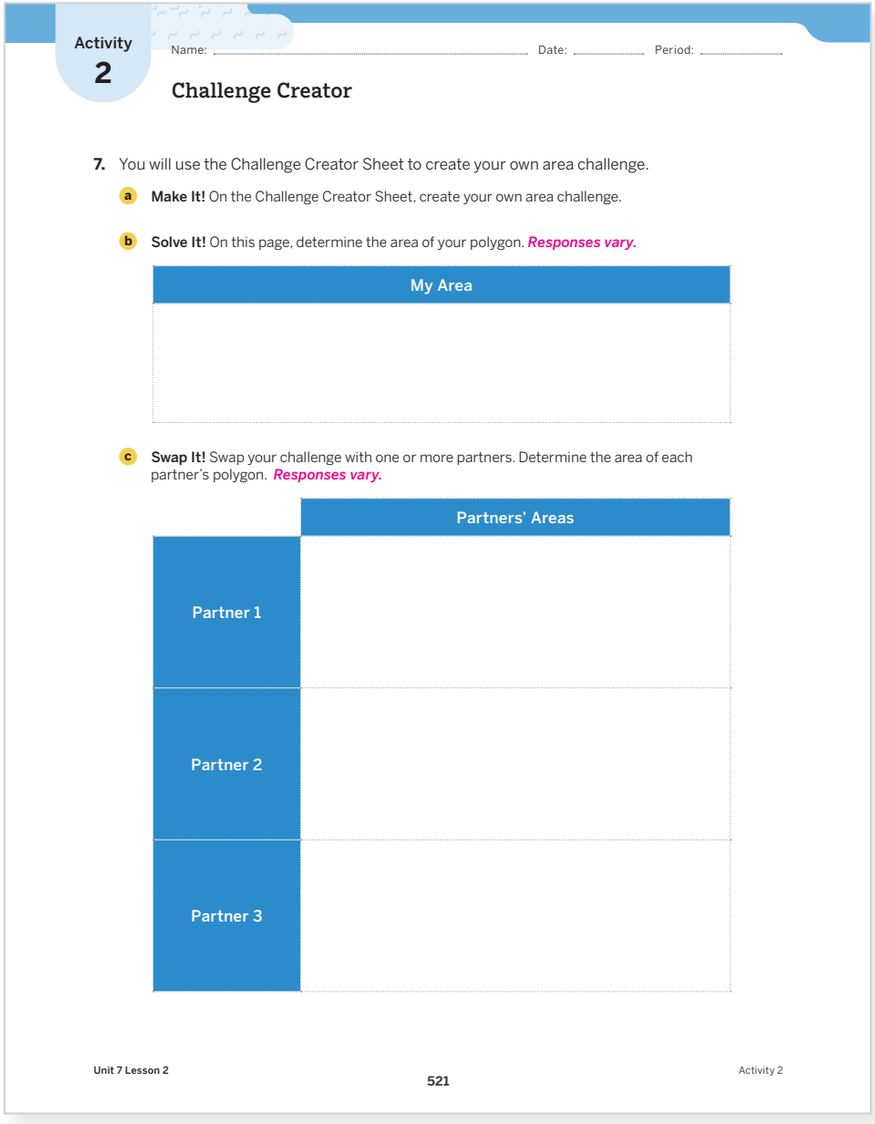
Look for several creative challenges and strategies to share during the Connect.

3 Connect

Share the polygons. Invite students to discuss their shape and what was most challenging about calculating its area.

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "What made some challenges more complex than others?"
- "How do you decide which strategy to choose?"
- "Would your strategy change if there was no grid? If so, how?"



Activity 2

Name: _____ Date: _____ Period: _____

Challenge Creator

7. You will use the Challenge Creator Sheet to create your own area challenge.

a. **Make It!** On the Challenge Creator Sheet, create your own area challenge.

b. **Solve It!** On this page, determine the area of your polygon. ***Responses vary.***

My Area

c. **Swap It!** Swap your challenge with one or more partners. Determine the area of each partner's polygon. ***Responses vary.***

	Partners' Areas
Partner 1	
Partner 2	
Partner 3	

Unit 7 Lesson 2 521 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to calculate the area of a polygon with and without a grid.

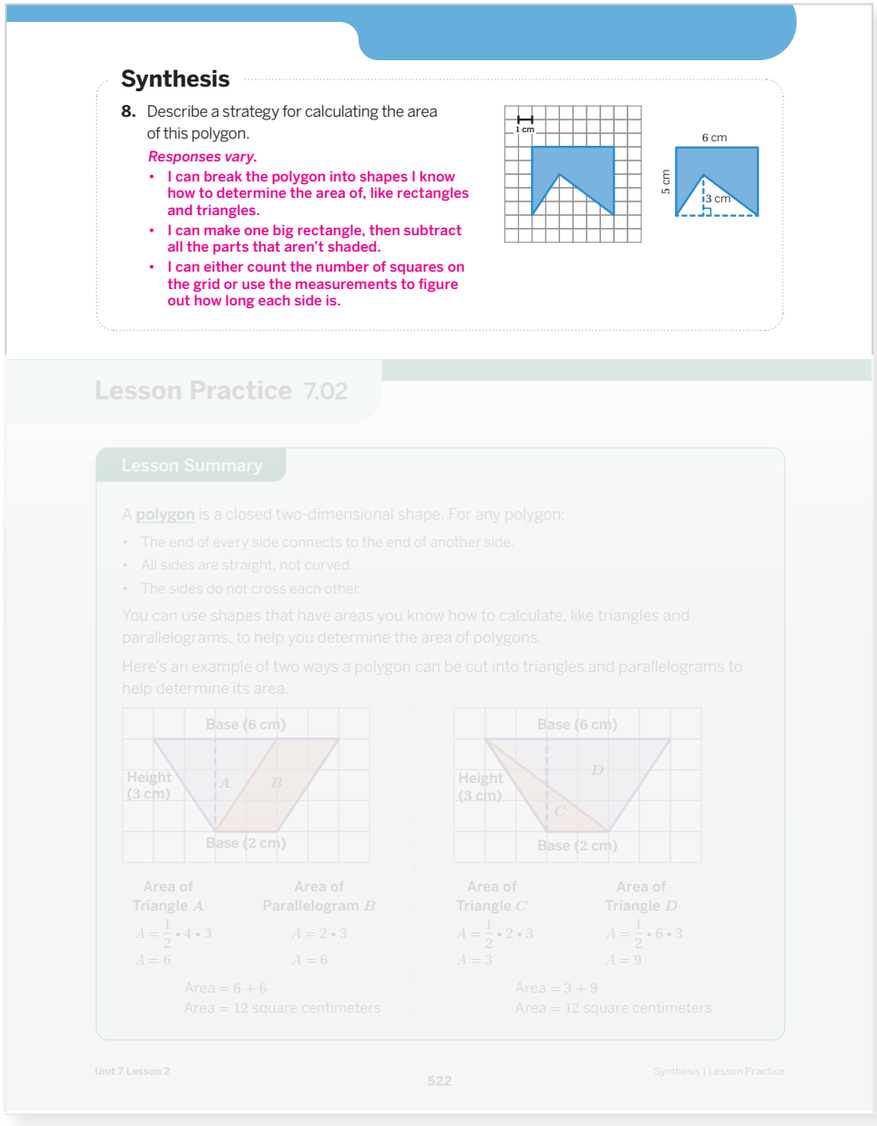
Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others. Provide sentence frames to help students explain their thinking (e.g., To determine the area of this polygon, first _____. Then, _____.).

Look for and share a variety of ideas, such as:

- I can break the polygon into familiar shapes like rectangles and triangles.
- I can make one big rectangle, then subtract all the parts that are not shaded.
- I can either count the number of squares on the grid or use measurements to figure out how long each side is.

MLR **MLR2: Collect and Display** Use this routine to add strategies for calculating the area of a polygon to the anchor chart created during the Warm-Up.



Synthesis

8. Describe a strategy for calculating the area of this polygon.

Responses vary.

- I can break the polygon into shapes I know how to determine the area of, like rectangles and triangles.
- I can make one big rectangle, then subtract all the parts that aren't shaded.
- I can either count the number of squares on the grid or use the measurements to figure out how long each side is.

Lesson Practice 7.02

Lesson Summary

A polygon is a closed two-dimensional shape. For any polygon:

- The end of every side connects to the end of another side.
- All sides are straight, not curved.
- The sides do not cross each other.

You can use shapes that have areas you know how to calculate, like triangles and parallelograms, to help you determine the area of polygons.

Here's an example of two ways a polygon can be cut into triangles and parallelograms to help determine its area.

Area of Triangle A: $A = \frac{1}{2} \cdot 4 \cdot 3$, $A = 6$

Area of Parallelogram B: $A = 2 \cdot 3$, $A = 6$

Area $= 6 + 6$

Area $= 12$ square centimeters

Area of Triangle C: $A = \frac{1}{2} \cdot 2 \cdot 3$, $A = 3$

Area of Triangle D: $A = \frac{1}{2} \cdot 6 \cdot 3$, $A = 9$

Area $= 3 + 9$

Area $= 12$ square centimeters

Unit 7 Lesson 2 522 Synthesis | Lesson Practice

Key Takeaway: A **polygon** is a closed 2-D shape with straight sides that do not cross each other. There are many strategies to determine the area of a polygon, including breaking it into familiar shapes, enclosing and subtracting, or using a grid or given measurements.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by calculating the area of a polygon.

Today's Goals

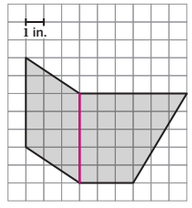
1. **Goal:** Determine the area of a *polygon* with and without a grid using the areas of rectangles and triangles.
2. **Language Goal:** Describe strategies for determining the area of polygons using precise mathematical language. **(Writing, Speaking, and Listening)**
3. **Language Goal:** Describe polygons using their characteristics. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.02

What is the area of this polygon?



Show or explain your thinking.

37.5 square inches

Explanations may vary. I divided the figure into a parallelogram and a trapezoid. The parallelogram had a base of 5 inches and a height of 3 inches. The trapezoid had bases of 6 inches and 3 inches and a height of 5 inches. The parallelogram has an area of $A = 5 \times 3 = 15$ square inches. The trapezoid has an area of $A = \frac{(6+3)}{2} \times 5 = 22.5$ square inches. $15 + 22.5 = 37.5$ square inches.

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to make sense of different strategies:

» **Respond to Student Thinking:**

- Revisit the Challenge Creator in Activity 2 and invite students to solve additional problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

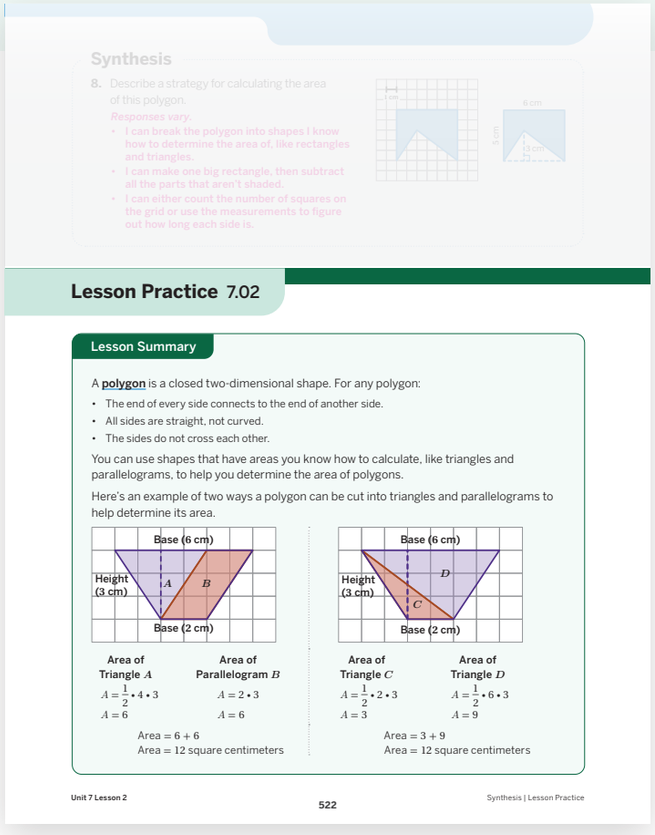
Professional Learning

Reflect on your facilitation of the Polygraph in this lesson. How did students engage in the activity? What about how you facilitated the routine enhanced student engagement?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. Describe a strategy for calculating the area of this polygon.

Responses vary.

- I can break the polygon into shapes I know how to determine the area of, like rectangles and triangles.
- I can make one big rectangle, then subtract all the parts that aren't shaded.
- I can either count the number of squares on the grid or use the measurements to figure out how long each side is.

Lesson Practice 7.02

Lesson Summary

A **polygon** is a closed two-dimensional shape. For any polygon:

- The end of every side connects to the end of another side.
- All sides are straight, not curved.
- The sides do not cross each other.

You can use shapes that have areas you know how to calculate, like triangles and parallelograms, to help you determine the area of polygons.

Here's an example of two ways a polygon can be cut into triangles and parallelograms to help determine its area.

Base (6 cm), Height (3 cm), Base (2 cm), A, B

Area of Triangle A	Area of Parallelogram B
$A = \frac{1}{2} \cdot 4 \cdot 3$	$A = 2 \cdot 3$
$A = 6$	$A = 6$

Area = 6 + 6
Area = 12 square centimeters

Base (6 cm), Height (3 cm), Base (2 cm), C, D

Area of Triangle C	Area of Triangle D
$A = \frac{1}{2} \cdot 2 \cdot 3$	$A = \frac{1}{2} \cdot 6 \cdot 3$
$A = 3$	$A = 9$

Area = 3 + 9
Area = 12 square centimeters

Lesson Practice 7.02

Name: _____ Date: _____ Period: _____

1. Select *all* the polygons.

☑ A. ☐ B. ☑ C. ☐ D. ☐ E. ☐ F.

2. Determine the area of this polygon. Each square has an area of 1 square unit.

18 square units

1 unit

Problems 3–4: Determine the area of each polygon. Show your thinking.

3. 1 unit; A = 4, A = 2, A = 6, A = 2, A = 4

18 square units

4. 1 unit; A = 7.5, A = 7.5

15 square units

5. Clare is reviewing the blueprint for a new addition to her home. What is the area for the addition if each unit represents 1 square foot?

4, 2, 5, A = 20, A = 3, 5, A = 9, 7

32 square feet

Lesson Practice 7.02

Name: _____ Date: _____ Period: _____

FAST Practice

6. What is the total area of the shaded figure in square inches?

A. 16
B. 64
C. 82
D. 100

3 in., 4 in., 3 in., 3 in., 4 in., 3 in.

Spiral Review

7. Select *all* the ratios that are equivalent to 24 : 8.

☑ A. 6:2
☐ B. 1:3
☑ C. 12:4
☑ D. 3:1
☑ E. 48:16
☐ F. 120:48

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.GR.1.2
	2–5	2	MA.7.GR.1.2
FAST Practice	6	2	MA.7.GR.1.2
Spiral Review			
	7	1	MA.6.AR.3.5

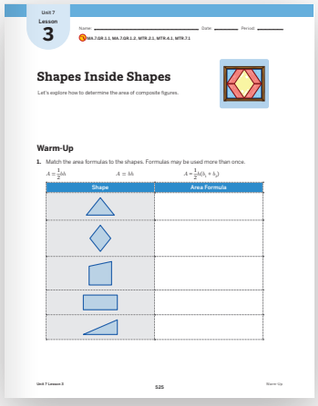
Unit 7

Lesson 3

Shapes Inside Shapes

Area of Composite Figures

Let's explore how to determine the area of composite figures.


Unit 7
Lesson 3
Shapes Inside Shapes
Let's explore how to determine the area of composite figures.
Warm-Up
1. Match the area formulas to the shapes. Formulas may be used more than once.
Shape
Area Formula
525

Learning Outcomes

Today's Goals

1. **Goal:** Decompose *composite figures* to determine the total area.
2. **Goal:** Decompose *composite figures* to determine the area of shaded parts.
3. **Language Goal:** Describe how decomposition can be used to calculate area of composite figures and area of shaded parts in composite figures. **(Reading, Writing, and Speaking)**

Students use their knowledge of determining the area of triangles and quadrilaterals to determine the total area of *composite figures* and the area of shaded parts of composite figures. They apply this knowledge to solve real-world problems involving composite figures. **(MTR.2.1, MTR.7.1)**

Coherence

Prior Learning

In Lesson 1, students used formulas to determine the areas of trapezoids, parallelograms, and rhombi. In Lesson 2, students used decomposition to determine the area of quadrilaterals.

Future Learning

In Lesson 4, students will work through area challenges and calculate the area of complex shapes composed of parts of circles and squares.

Connections

- Students connect the concept of real-world scenarios involving area calculations, such as finding the area of a deck by breaking them down into simpler shapes like rectangles and triangles. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.2
Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.

Also Addressing: **MA.7.GR.1.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	**MA.912.GR.3.4**
	MA.912.GR.4.4

Vocabulary

Review Vocabulary

composite figure

Lesson at a Glance

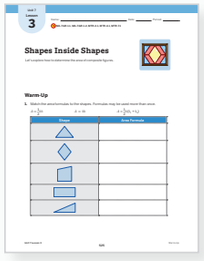
~ 45 min

MA.7.GR.1.1, MA.7.GR.1.2

Warm-Up

Pairs | 5 min

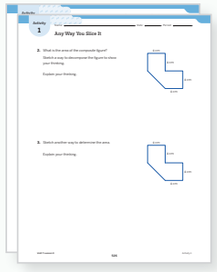
Students match area formulas to different figures.



Activity 1

Pairs | 15 min

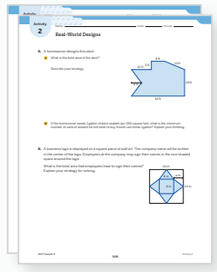
Students decompose figures in more than one way and analyze different strategies to determine the area of *composite figures*.
Routine: Decide and Defend **(MTR.2.1)**



Activity 2

Pairs | 15 min

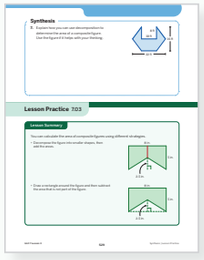
Students determine the total area of a composite figure and the area of a shaded region to solve real-world problems. MLR1: Stronger and Clearer Each Time **(MTR.7.1)**



Synthesis

Whole Class | 5 min

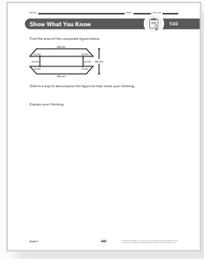
Students synthesize their understanding of how to calculate the area of composite figures.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by of finding area by decomposing composite figures.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students match area formulas to different figures.

Short on time? Consider omitting the Warm-Up

1 Launch

To support students getting started, consider asking them to describe and categorize the figures named.

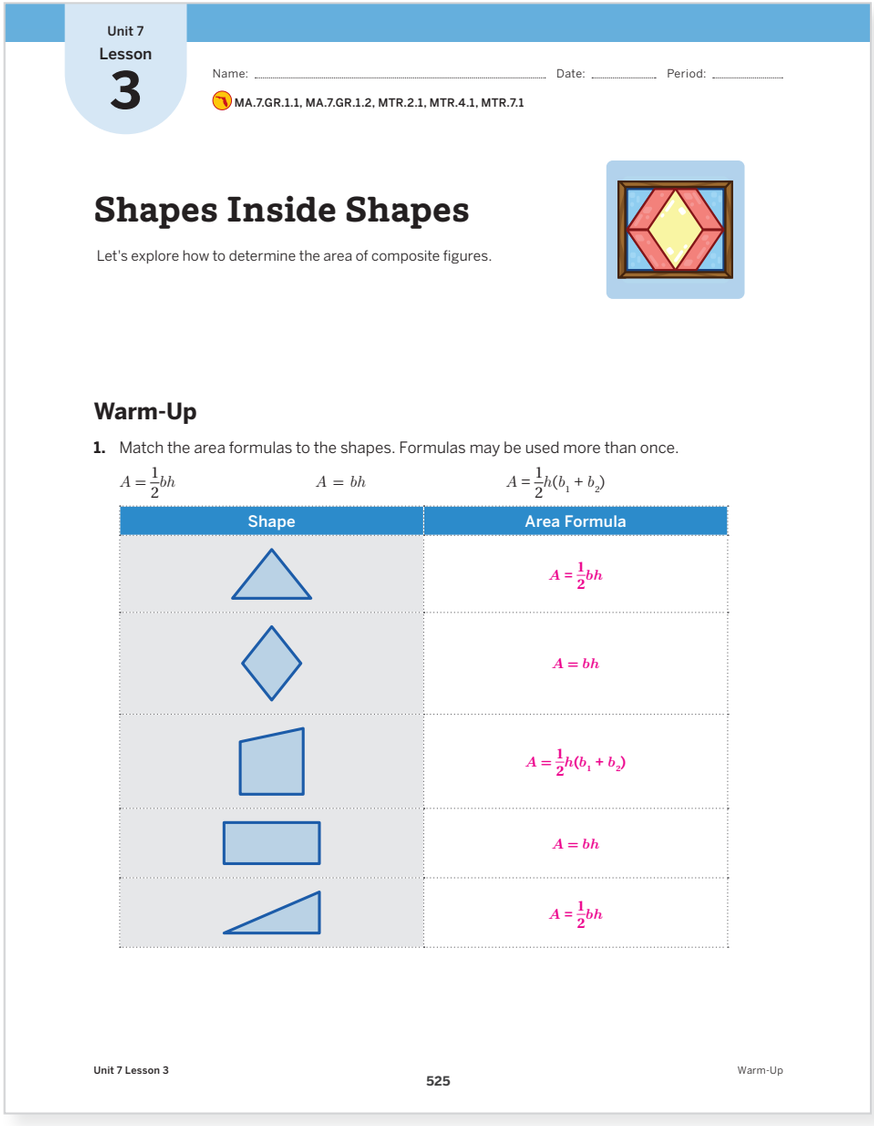
A **Accessibility: Memory and Attention** Encourage students who struggle with understanding what the variables represent in the formulas to write out the words next to the variables. For example, students would write "height" for h and "base 1" for b_1.

2 Connect

Look for and share students' matches, inviting each pair to share their method for determining the matches.

Consider asking:

- "What do the variables represent in the formulas?"
- "How are the formulas similar and different?"



Unit 7
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.GR.1.1, MA.7.GR.1.2, MTR.2.1, MTR.4.1, MTR.7.1

Shapes Inside Shapes

Let's explore how to determine the area of composite figures.

Warm-Up

1. Match the area formulas to the shapes. Formulas may be used more than once.

$A = \frac{1}{2}bh$ $A = bh$ $A = \frac{1}{2}h(b_1 + b_2)$

Shape	Area Formula
	$A = \frac{1}{2}bh$
	$A = bh$
	$A = \frac{1}{2}h(b_1 + b_2)$
	$A = bh$
	$A = \frac{1}{2}bh$

Unit 7 Lesson 3 525 Warm-Up

Pairs | 15 min

Activity 1 Any Way You Slice It

Purpose: Students decompose figures in more than one way and analyze different strategies to determine the area of *composite figures*. **(MTR.2.1)**

1 Launch

To support students getting started, consider asking:

- "What are some shapes for which you know their area formulas?"
- "Do you see a way to decompose the figure into these shapes?"

2 Monitor

Listen for students discussing how to decompose the figure using different strategies and shapes.

D Differentiation *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw lines to determine the different shapes included in the figure.
Use a wrong formula when solving.	**Strengthen** Encourage students to write down the possible area formulas they may need to solve.
Get a different area when solving this second way.	**Strengthen** Consider asking, "Does the area in the figure change when you change how you break it up?"
Would like to explore more ways to decompose the figure.	**Stretch** Challenge students to decompose the figure into different types of shapes such as a parallelogram or an isosceles triangle.

ELL English Language Learners Invite students to share what the word decompose means to them in their own words. Encourage them to create different drawings to illustrate the meaning of the word.

3 Connect

Invite students to share their strategies for decomposing the *composite figure*. Consider sharing different ways to decompose the figure if they do not come up during the discussion. **(MTR.2.1)**

Consider asking, "How does performing this second decomposition validate your original answer?"

Activity 1 continued >

Activity 1

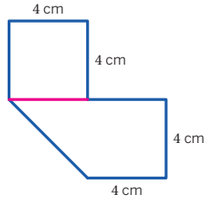
Name: Date: Period:

Any Way You Slice It

2. What is the area of the *composite figure*?

Sketch a way to decompose the figure to show your thinking.

40 sq. cm. *Responses vary.* Sample response shown on figure.



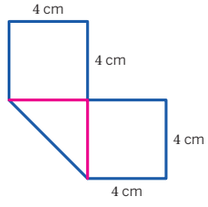
Explain your thinking.

***Explanations vary.* I divided the figure into a square with a side length of 4 cm and a trapezoid with bases 4 cm and 8 cm and a height of 4 cm.**

The area of the trapezoid is $A = \frac{1}{2}(4)(4 + 8) = 24$ sq. cm.
The area of the square is $A = 4 \cdot 4 = 16$ sq. cm. The total area of the figure is $24 + 16 = 40$ sq. cm.

3. Sketch another way to determine the area.

***Responses vary.* Sample response shown on figure.**



Explain your thinking.

***Explanations vary.* I divided the figure into a right triangle with a base of 4 cm and height of 4 cm and two squares each with a side length of 4 cm.**

The area of the right triangle is $A = \frac{1}{2}(4)\ (4) = 8$ sq. cm.
The area of the each square is $A = (4)(4) = 16$ sq. cm.
The total area of the figure is $8 + 16 + 16 = 40$ sq. cm.

Unit 7 Lesson 3 526 Activity 1

Activity 1 Any Way You Slice It (continued)

Purpose: Students decompose figures in more than one way and analyze different strategies to determine the area of *composite figures*. **(MTR.2.1)**

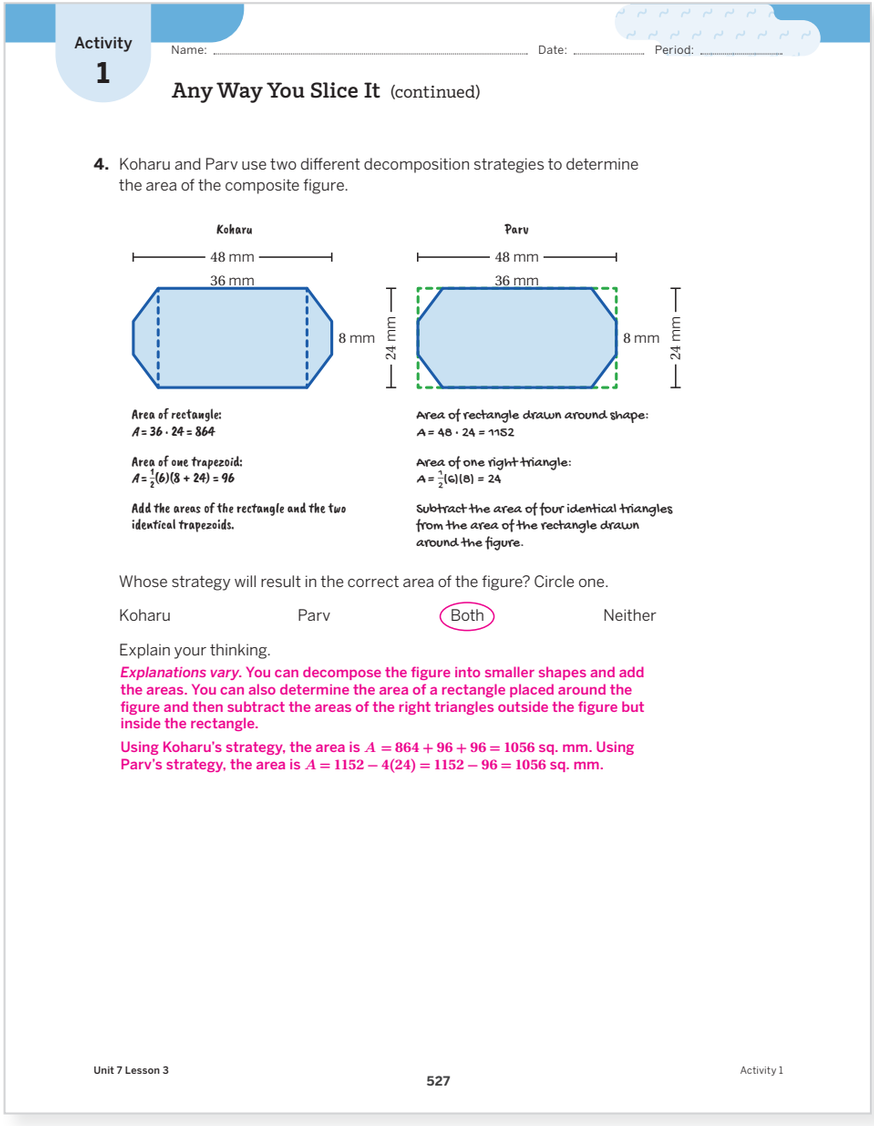
3 Connect

Display the distribution of responses by polling the class.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates.

To surface the Key Takeaway, consider asking, "Which shapes are involved in the two strategies for solving?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

Any Way You Slice It (continued)

4. Koharu and Parv use two different decomposition strategies to determine the area of the composite figure.

Koharu

Area of rectangle:
$A = 36 \cdot 24 = 864$

Area of one trapezoid:
$A = \frac{1}{2}(6)(8 + 24) = 96$

Add the areas of the rectangle and the two identical trapezoids.

Parv

Area of rectangle drawn around shape:
$A = 48 \cdot 24 = 1152$

Area of one right triangle:
$A = \frac{1}{2}(6)(8) = 24$

Subtract the area of four identical triangles from the area of the rectangle drawn around the figure.

Whose strategy will result in the correct area of the figure? Circle one.

Koharu Parv Both Neither

Explain your thinking.

Explanations vary. You can decompose the figure into smaller shapes and add the areas. You can also determine the area of a rectangle placed around the figure and then subtract the areas of the right triangles outside the figure but inside the rectangle.

Using Koharu's strategy, the area is $A = 864 + 96 + 96 = 1056$ sq. mm. Using Parv's strategy, the area is $A = 1152 - 4(24) = 1152 - 96 = 1056$ sq. mm.

Unit 7 Lesson 3 527 Activity 1

Key Takeaway: The area of a *composite figure* can be determined by using decomposition into smaller shapes and adding or by surrounding the figure and subtracting.

Pairs | 15 min

Activity 2 Real-World Designs

Purpose: Students determine the total area of a composite figure and the area of a shaded region to solve real-world problems. **(MTR.7.1)**

1 Launch

Direct students' attention to the design of the deck. Consider asking, "How can you draw lines on the figure to break it into smaller shapes?"

2 Monitor

Look for students who are able to show multiple ways of decomposing the figure to solve.

A **Accessibility: Fine Motor Skills** Consider providing ways to decompose the figure and labeling one or more measurements in the figure so students can focus on determining the area.

Pause to have students discuss their method for decomposing the figure, calculating the area, and determining how many cans of sealant are needed. **(MTR.7.1)**

Listen for students who notice that they need to determine the area of the larger square first, then determine the area of the shaded parts, and finally subtract those areas to determine the area of the non-shaded region.

MLR **MLR1: Stronger and Clearer Each Time** Consider using this routine to support students in developing precise language when explaining how they determined the area that could be used for writing employees' names. Invite students to meet with other groups, give feedback on each other's explanations, and then use the feedback to write a second clearer explanation. See the Routine Facilitation Guide for more information.

3 Connect

Invite students to share how they determined the minimum number of cans of sealant needed to cover the deck. **(MTR.7.1)**

Create a class summary statement that describes how to use decomposition of composite figures to solve mathematical and real-world problems, including those that ask for total area and those that ask for partial area. Record the statement for students to refer back to later in the unit.

Activity 2

Name: Date: Period:

Real-World Designs

5. A homeowner designs this deck.

a. What is the total area of the deck?

534 sq. ft

Describe your strategy.

Responses vary. I drew a line to separate the trapezoid from the bottom portion. I drew a 32 by 16 rectangle around the bottom portion and subtracted the area of the triangle. Then I added the area of the trapezoid.

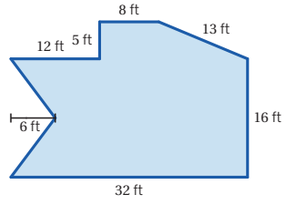
Area of rectangle: $A = 32 \cdot 16 = 512$ sq. ft.

Area of missing triangle: $A = \frac{1}{2}(16)(6) = 48$ sq. ft.

Area of bottom portion: $A = 512 - 48 = 464$ sq. ft.

Area of trapezoid: $A = \frac{1}{2}(5)(8 + 20) = 70$ sq. ft.

Total area: $A = 464 + 70 = 534$ sq. ft.



b. If the homeowner needs 1 gallon of deck sealant per 250 square feet, what is the minimum number of cans of sealant he will need to buy if each can holds 1 gallon? Explain your thinking.

3 cans. *Responses vary.* The homeowner needs $534 \div 250 = 2.136$ gallons of sealant. Since each can is 1 gallon, the homeowner will need a minimum of 3 cans to cover the deck.

6. A business logo is displayed on a square piece of wall art. The company name will be written in the center of the logo. Employees at the company may sign their names in the non-shaded space around the logo.

What is the total area that employees have to sign their names? Explain your strategy for solving.

112 sq. in. *Responses vary.* I determined the area of the larger square, then subtracted the area of the shaded region.

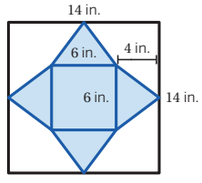
Area of the larger square: $A = 14 \cdot 14 = 196$ sq. in.

Area of the smaller shaded square: $A = 6 \cdot 6 = 36$ sq. in.

Area of one shaded triangle: $A = \frac{1}{2}(6)(4) = 12$ sq. in.

Total shaded area: $A = 36 + 4(12) = 84$ sq. in.

Area of unshaded space: $A = 196 - 84 = 112$ sq. in.



Unit 7 Lesson 3 528 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to calculate the area of composite figures.

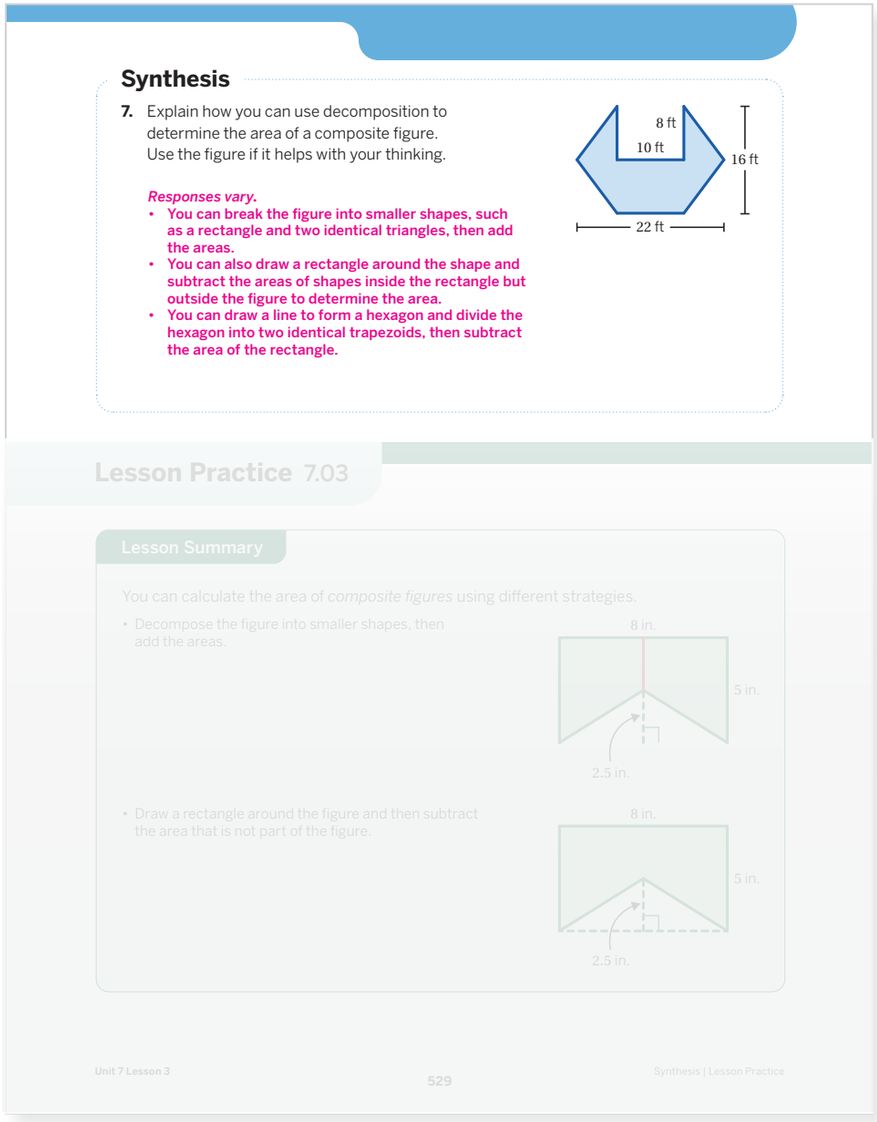
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- Decomposing the figure into smaller shapes such as a rectangle and 2 identical isosceles triangles.
- Decomposing the figure into smaller shapes such as a trapezoid and 2 identical right triangles.
- Drawing a rectangle around the shape and subtracting the parts that aren't shaded.
- Drawing a line to form a hexagon, then decomposing the hexagon into 2 identical trapezoids and subtracting the part that isn't shaded.

MLR **MLR1: Collect and Display** Use this routine to collect students' ideas about calculating the area of a composite figure. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to discuss with a partner when it might be more efficient to draw a rectangle around a portion of the figure and subtract areas, instead of adding smaller areas.



Synthesis

7. Explain how you can use decomposition to determine the area of a composite figure. Use the figure if it helps with your thinking.

Responses vary.

- You can break the figure into smaller shapes, such as a rectangle and two identical triangles, then add the areas.
- You can also draw a rectangle around the shape and subtract the areas of shapes inside the rectangle but outside the figure to determine the area.
- You can draw a line to form a hexagon and divide the hexagon into two identical trapezoids, then subtract the area of the rectangle.

Lesson Practice 7.03

Lesson Summary

You can calculate the area of *composite figures* using different strategies.

- Decompose the figure into smaller shapes, then add the areas.

- Draw a rectangle around the figure and then subtract the area that is not part of the figure.

Unit 7 Lesson 3 529 Synthesis | Lesson Practice

Lesson Takeaway: *Composite figures* can be decomposed into familiar shapes to determine the total area of the figure and to solve problems that require using the area of shaded parts to solve.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of finding area by decomposing composite figures.

Today's Goals

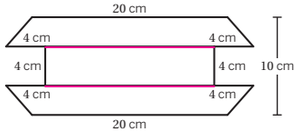
1. **Goal:** Decompose composite figures to determine the total area.
2. **Goal:** Decompose composite figures to determine the area of shaded parts.
3. **Language Goal:** Describe how decomposition can be used to calculate area of composite figures and area of shaded parts in composite figures. **(Reading, Writing, and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.03

Find the area of the composite figure below.



Sketch a way to decompose the figure to help show your thinking.

224 sq. cm. *Responses vary*. Sample response shown on figure.

Explain your thinking.

Explanations vary. I divided the figure into a rectangle with a length of 20 cm and a width of 4 cm. The other parts of the figure are two congruent trapezoids with bases 20 cm and 28 cm and a height of 3 cm. The area of the rectangle is $A = (20)(4) = 80$ sq. cm. The area of each trapezoid is $A = \frac{1}{2}(3)(20 + 28) = 72$ sq. cm. The total area of the figure is $2(72) + 80 = 224$ sq. cm.

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use decomposition to determine the area of a composite figure:

» **Respond to Student Thinking:**

- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students connected their previous understanding of calculating areas of triangles and quadrilaterals to determine areas of composite figures. How will that work support their understanding of determining areas of complex shapes in Lesson 4?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Explain how you can use decomposition to determine the area of a composite figure. Use the figure if it helps with your thinking.

Responses vary.

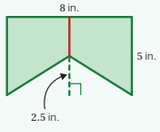
- You can break the figure into smaller shapes, such as a rectangle and two identical triangles, then add the areas.
- You can also draw a rectangle around the shape and subtract the areas of shapes inside the rectangle but outside the figure to determine the area.
- You can draw a line to form a hexagon and divide the hexagon into two identical trapezoids, then subtract the area of the rectangle.

Lesson Practice 7.03

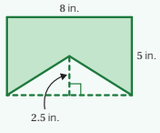
Lesson Summary

You can calculate the area of *composite figures* using different strategies.

- Decompose the figure into smaller shapes, then add the areas.



- Draw a rectangle around the figure and then subtract the area that is not part of the figure.

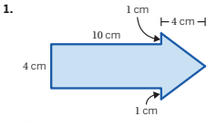


Lesson Practice 7.03

Name: _____ Date: _____ Period: _____

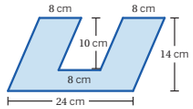
Problems 1–2: Determine the area of the composite figure.

1.



52 sq. cm

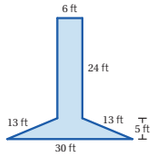
2.



256 sq. cm

3. The design of a stage and fashion runway is shown.

What is the total area of the stage and fashion runway? Explain your thinking.



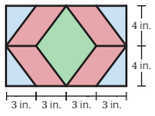
234 sq. ft. *Explanations vary.* You can decompose the figure into a rectangle with sides 6 ft and 24 ft and a trapezoid with bases 6 ft and 30 ft and a height of 5 ft.

Area of the rectangle: $A = 6 \cdot 24 = 144$ sq. ft.

Area of trapezoid: $A = \frac{1}{2}(5)(6 + 30) = 90$ sq. ft.

Total area: $A = 144 + 90 = 234$ sq. ft.

Problems 4–5: An artist drew a design for a piece of stained glass. The design is composed of 4 identical right triangles, 4 identical parallelograms, and a rhombus in the center.



4. Determine the total area for each part of the design.

Outer rectangle: 96 sq. in.

Right triangles: 24 sq. in.

Parallelograms: 48 sq. in.

5. Use two different strategies to determine the area of the rhombus. Show or explain your thinking.

Explanations vary. I can subtract the areas of the right triangles and parallelograms from the outer rectangle: $A = 96 - 24 - 48 = 24$ sq. in. I can decompose the rhombus into 2 identical triangles with a base of 6 in. and a height of 4 in.: $A = 2 \cdot \left(\frac{1}{2}\right)(6)(4) = 24$ sq. in.

Lesson Practice 7.03

Name: _____ Date: _____ Period: _____

FAST Practice

6. The shaded region in the figure represents the section of a small garden marked off for flowers. The unshaded region will be covered with rocks.

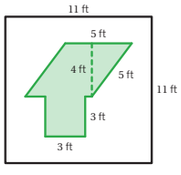
What is the area of the garden that will be covered with rocks?

A. 72 sq. ft

B. 92 sq. ft

C. 98 sq. ft

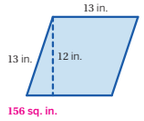
D. 101 sq. ft



Spiral Review

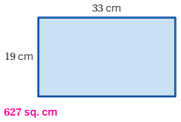
Problems 7–10: Determine the area of each figure.

7.



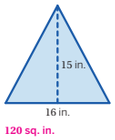
156 sq. in.

8.



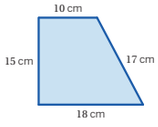
627 sq. cm

9.



120 sq. in.

10.



210 sq. cm

Lesson Practice Item Analysis

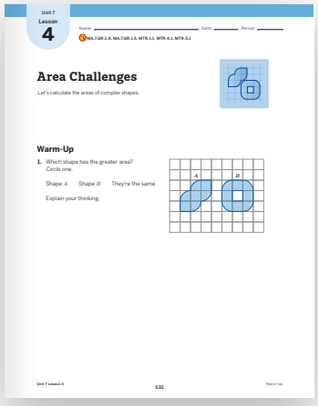
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.7.GR.1.1, MA.7.GR.1.2
	4–5	3	MA.7.GR.1.1, MA.7.GR.1.2
FAST Practice	6	2	MA.7.GR.1.1, MA.7.GR.1.2
Spiral Review			
	7, 10	1	MA.7.GR.1.1
	8–9	1	MA.6.GR.2.1

Unit 7
Lesson
4

Area Challenges

Calculating Areas of Complex Shapes

Let's calculate the areas of complex shapes.


Unit 7
Lesson
4

Name _____ Date _____ Period _____

MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.4.1, MTR.5.1

Area Challenges

Let's calculate the areas of complex shapes.

Warm-Up

1. Which shape has the greater area? Circle one.

Shape A Shape B They're the same

Explain your thinking.

A

B

532

Unit 7 Lesson 4

Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the area of a complex shape composed of squares and fractions of circles.
2. **Goal:** Write area as an expression that includes π, such as $20\pi + 50$.
3. **Language Goal:** Explain how to write an expression to represent the area of a complex shape with circular parts. **(Reading, Writing, Speaking, and Listening)**

Students apply the area formula to solve problems involving the area of circles, as well as the area of shapes composed of parts of circles and squares. They develop fluency and make sense of the many ways to decompose and rearrange shapes as they create and solve challenges.

Coherence

Prior Learning

In Unit 3 Lessons 5–6, students developed a formula for calculating the area of a circle and made connections with the formula for circumference. They determined the areas of circles when given the diameter or radius.

Future Learning

In Lesson 5, students will use what they have learned throughout the unit to solve challenges involving calculating the area of a circle when given the circumference.

Connections

- Using visual shapes on a coordinate grid student calculate the area of complex shapes by breaking them down into simpler, familiar shapes like rectangles, triangles, and circles. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.4

Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.

MA.7.GR.1.5

Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	**MA.912.GR.4.4** **MA.912.GR.6.4**

Lesson at a Glance

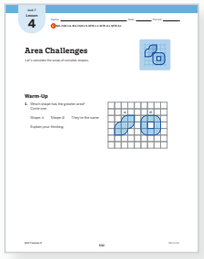
~ 45 min

MA.7.GR.1.4, MA.7.GR.1.5

Warm-Up

Pairs | 5 min

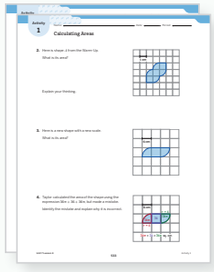
Students reason about the area of two shapes composed of squares and parts of circles.



Activity 1

Pairs | 12 min

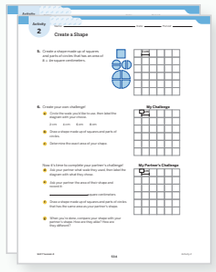
Students use the structure of complex shapes to calculate their areas. MLR3: Critique, Correct, Clarify (MTR.5.1)



Activity 2

Independent | 18 min

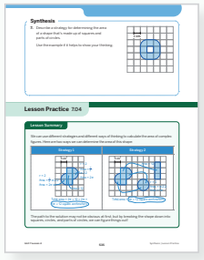
Students express their creativity by creating their own shapes composed of squares and parts of circles and calculating their areas.



Synthesis

Whole Class | 5 min

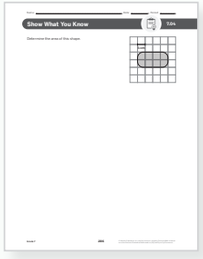
Students synthesize their strategies for calculating areas of complex shapes composed of squares and parts of circles.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the area of a shape composed of squares and parts of circles on a grid with a scale.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students reason about the area of two shapes composed of squares and parts of circles.

1 Launch

To encourage creativity, consider asking, "What do these shapes remind you of?"

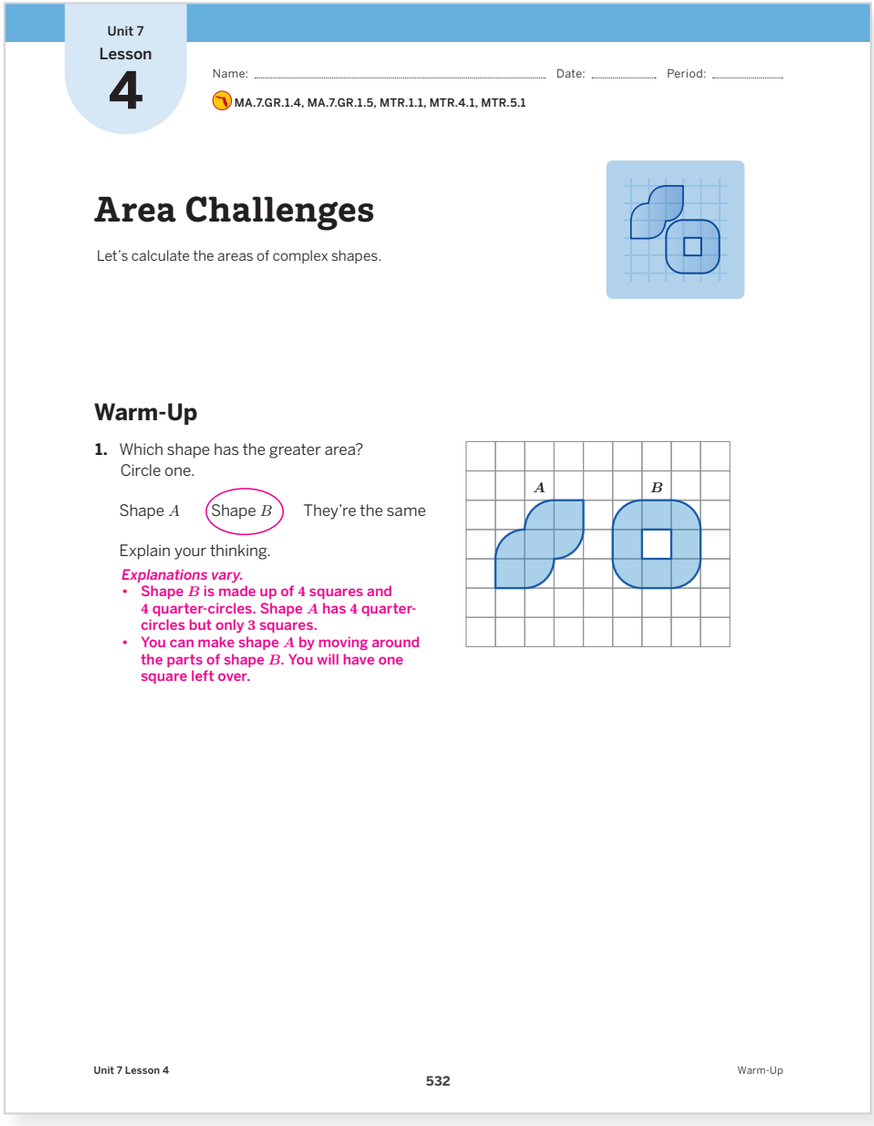
Invite students to decide which shape has a greater area and compare their thinking with a classmate.

2 Connect

Direct students' attention to the two shapes and invite students to share what strategies they used to compare their areas.

Consider asking, "What names could you give to the parts of circles that you see?" Quarter-circles, one-fourth of a circle

Note: If it does not come up naturally, consider sharing that you can rearrange or match up parts of the two shapes that have the same area. For example, each shape has four quarter circles of the same radius and some squares. **(MTR.5.1)**



Unit 7
Lesson
4

Name: _____ Date: _____ Period: _____

MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.4.1, MTR.5.1

Area Challenges

Let's calculate the areas of complex shapes.

Warm-Up

1. Which shape has the greater area? Circle one.

Shape A Shape B They're the same

Explain your thinking.

Explanations vary.
- Shape B is made up of 4 squares and 4 quarter-circles. Shape A has 4 quarter-circles but only 3 squares.
- You can make shape A by moving around the parts of shape B. You will have one square left over.

Unit 7 Lesson 4 532 Warm-Up

Pairs | 12 min

Activity 1 Calculating Areas

Purpose: Students use the structure of complex shapes to calculate their areas. **(MTR.5.1)**

1 Launch

To support students getting started, consider asking, "How many total circles do you see? How many squares?" **(MTR.5.1)**

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to support interpreting visual representations.

Encourage students to write expressions that show their thinking about the shape.

2 Monitor

Look for a variety of strategies students use to calculate each area.

D **Differentiation**

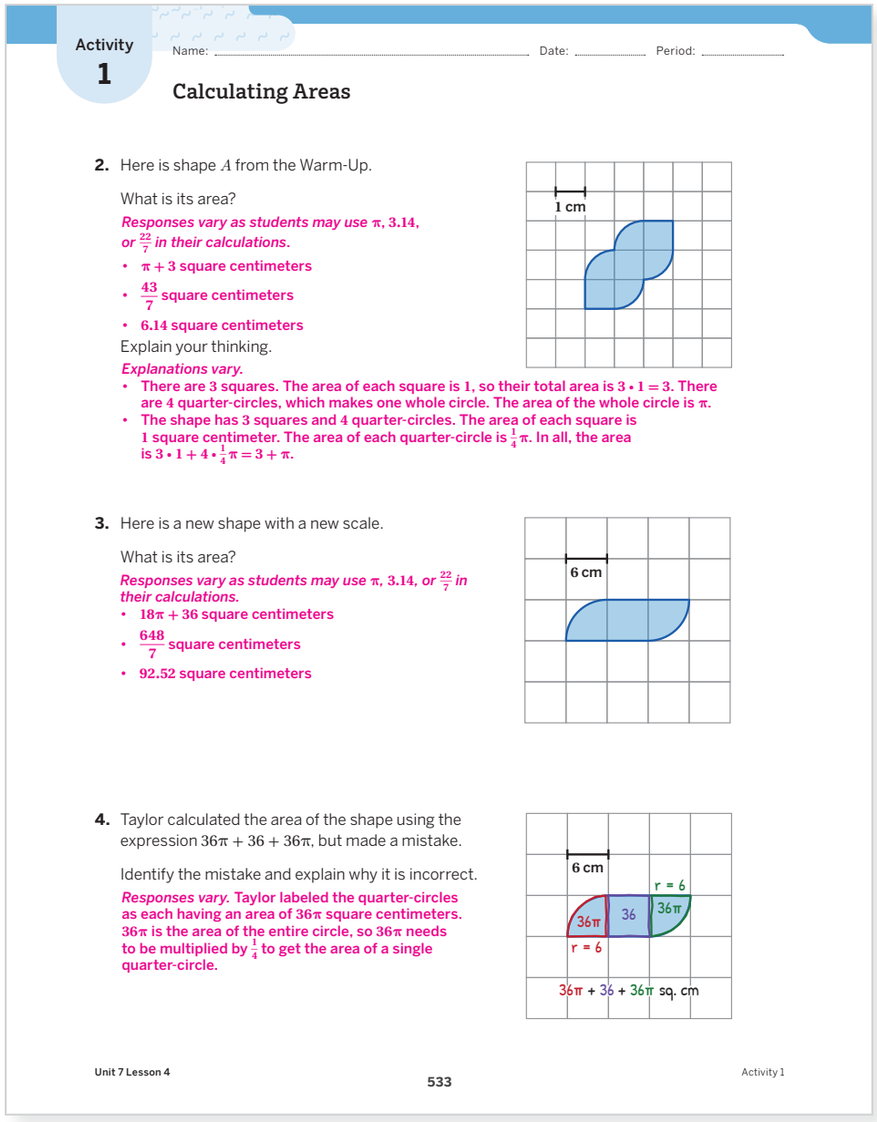
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to split the shape into parts of circles and squares and calculate the area of one part.
Think that the area of a quarter circle can be calculated as πr^2.	**Support** Consider asking, "What portion of a circle is that shape? How is it different from a whole circle?"
Calculate the area with a scale of 1 unit to 1 centimeter. (*Problem 3*)	**Support** Consider asking, "What does the 6 centimeter marker on the graph represent? How might it impact the area of the shape?"
Count all the quarter circles in the shape and calculate their total area.	**Strengthen** Celebrate this strategy. Invite students to compare their strategy with a classmate's.
Write expressions using π.	**Strengthen** Celebrate these students for attending to precision. Invite them to think about how they might share their strategy with a classmate.

Look for a variety of student thinking about Taylor's work to highlight during the Connect.

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Taylor's work. As a class:

- Discuss what is correct about her response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct* and *clear* expression for the area of the shape.
- Consider asking: "How did we calculate each part of the expression?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Calculating Areas

2. Here is shape A from the Warm-Up.

What is its area?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- $\pi + 3$ square centimeters
- $\frac{43}{7}$ square centimeters
- 6.14 square centimeters

Explain your thinking.

Explanations vary.

- There are 3 squares. The area of each square is 1, so their total area is $3 \cdot 1 = 3$. There are 4 quarter-circles, which makes one whole circle. The area of the whole circle is π.
- The shape has 3 squares and 4 quarter-circles. The area of each square is 1 square centimeter. The area of each quarter-circle is $\frac{1}{4}\pi$. In all, the area is $3 \cdot 1 + 4 \cdot \frac{1}{4}\pi = 3 + \pi$.

3. Here is a new shape with a new scale.

What is its area?

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- $18\pi + 36$ square centimeters
- $\frac{648}{7}$ square centimeters
- 92.52 square centimeters

4. Taylor calculated the area of the shape using the expression $36\pi + 36 + 36\pi$, but made a mistake.

Identify the mistake and explain why it is incorrect.

Responses vary. Taylor labeled the quarter-circles as each having an area of 36π square centimeters. 36π is the area of the entire circle, so 36π needs to be multiplied by $\frac{1}{4}$ to get the area of a single quarter-circle.

Unit 7 Lesson 4 533 Activity 1

Key Takeaway: To calculate the area of a complex shape, break the shape into squares and parts of circles, calculate the area of each piece, then add them together. To show the exact area, use the symbol π when calculating the area of parts of circles.

Independent | 18 min

Activity 2 Create a Shape

Purpose: Students express their creativity by creating their own shapes composed of squares and parts of circles and calculating their areas.

Short on time? Consider inviting students to complete fewer of their classmates' challenges.

1 Launch

Encourage students to include rotations of the quarter circle and semicircle pieces in their drawings.

2 Monitor

Look for students' shapes that use different numbers of semicircles and quarter circles.

D **Differentiation: (Support)** To support students getting started, invite them to calculate the area of each available piece.

Pause briefly to share the shapes. Consider asking:

- "Does this shape have an area of $8 + 4\pi$ square units? How do you know?"
- "Where do you see the 8 in this shape? The 4π?"

Consider sharing that students will now create their own challenge similar to the previous problem.

Invite students to choose a partner for the challenge and to get feedback from the partner.

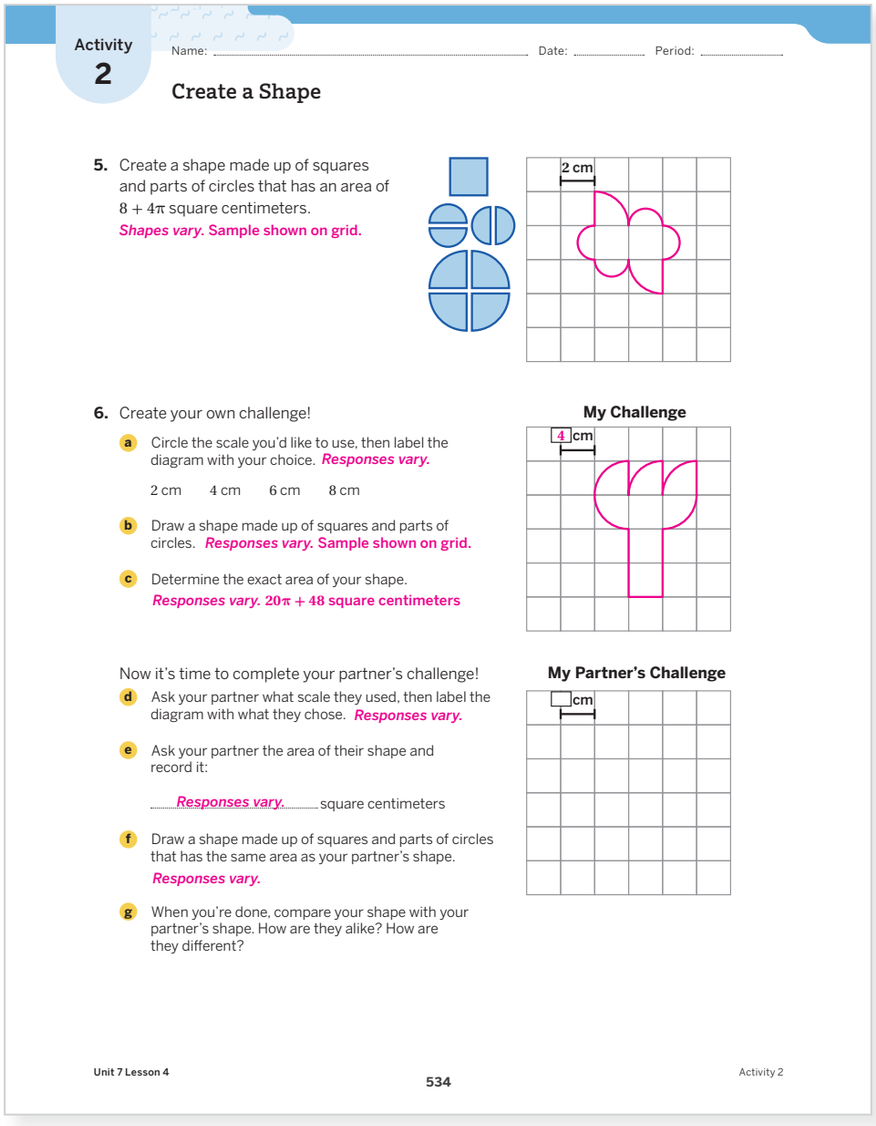
A **Accessibility: Fine Motor Skills Share** that the goal is to communicate ideas rather than to draw a perfect circle. Invite students to label their drawing to justify their area calculations.

Listen for strategies students use to create and solve challenges, such as working backwards to determine the radius that will produce a certain area. **(MTR.1.1)**

Look for several creative challenges and solutions to share during the Connect.

3 Connect

Share the challenges and solutions. Consider asking, "How do you know that these shapes have the same area?"



Activity 2

Name: _____ Date: _____ Period: _____

Create a Shape

5. Create a shape made up of squares and parts of circles that has an area of $8 + 4\pi$ square centimeters.

Shapes vary. **Sample shown on grid.**

6. Create your own challenge!

My Challenge

a. Circle the scale you'd like to use, then label the diagram with your choice. *Responses vary.*

2 cm 4 cm 6 cm 8 cm

b. Draw a shape made up of squares and parts of circles. *Responses vary.* **Sample shown on grid.**

c. Determine the exact area of your shape.

Responses vary. **$20\pi + 48$ square centimeters**

Now it's time to complete your partner's challenge!

My Partner's Challenge

d. Ask your partner what scale they used, then label the diagram with what they chose. *Responses vary.*

e. Ask your partner the area of their shape and record it:

Responses vary. square centimeters

f. Draw a shape made up of squares and parts of circles that has the same area as your partner's shape.

Responses vary.

g. When you're done, compare your shape with your partner's shape. How are they alike? How are they different?

Unit 7 Lesson 4 534 Activity 2

Whole Class | 5 min

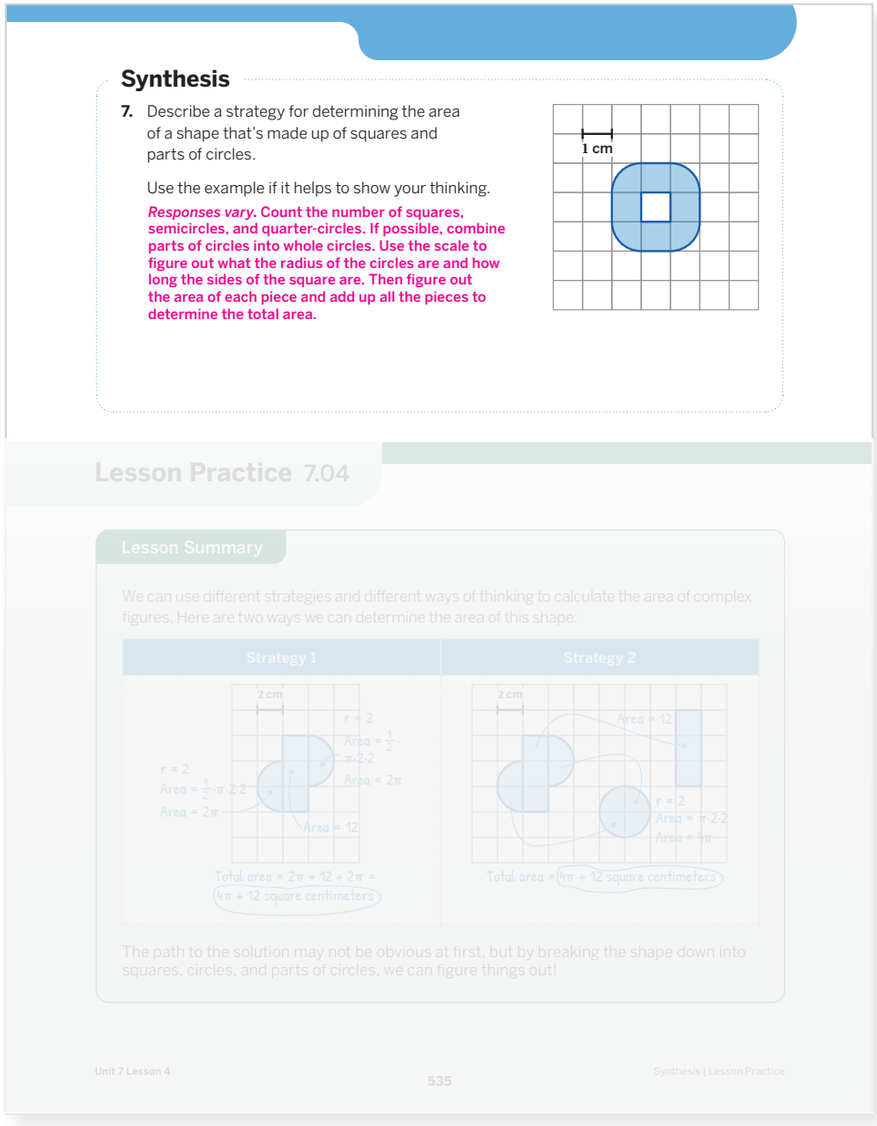
Synthesis

Purpose: Students synthesize their strategies for calculating areas of complex shapes composed of squares and parts of circles.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- Breaking the shape into squares and partial circles.
- Identifying the radius of each circle or partial circle and using it to calculate an area.
- Combining partial circles into whole circles.
- Adding together the areas of each part of the shape.



Synthesis

7. Describe a strategy for determining the area of a shape that's made up of squares and parts of circles.

Use the example if it helps to show your thinking.

Responses vary. **Count the number of squares, semicircles, and quarter-circles. If possible, combine parts of circles into whole circles. Use the scale to figure out what the radius of the circles are and how long the sides of the square are. Then figure out the area of each piece and add up all the pieces to determine the total area.**

Lesson Practice 7.04

Lesson Summary

We can use different strategies and different ways of thinking to calculate the area of complex figures. Here are two ways we can determine the area of this shape:

Strategy 1	Strategy 2
2 cm r = 2 Area = $\frac{1}{2} \cdot \pi \cdot 2 \cdot 2$ Area = 2π r = 2 Area = $\frac{1}{2} \cdot \pi \cdot 2 \cdot 2$ Area = 2π Area = 12 Total area = $2\pi + 12 + 2\pi$ = $4\pi + 12$ square centimeters	2 cm Area = 12 r = 2 Area = $\pi \cdot 2 \cdot 2$ Area = 4π Total area = $4\pi + 12$ square centimeters

The path to the solution may not be obvious at first, but by breaking the shape down into squares, circles, and parts of circles, we can figure things out!

Unit 7 Lesson 4 535 Synthesis | Lesson Practice

Key Takeaway: Decomposing shapes made of squares and parts of circles can help you determine their total area.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the area of a shape composed of squares and parts of circles on a grid with a scale.

Today's Goals

1. **Goal:** Calculate the area of a complex shape composed of squares and fractions of circles.
2. **Goal:** Write area as an expression that includes π, such as $20\pi + 50$.
3. **Language Goal:** Explain how to write an expression to represent the area of a complex shape with circular parts. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

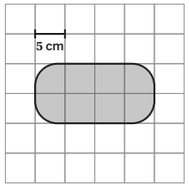
Name: _____ Date: _____ Period: _____

Show What You Know 7.04

Determine the area of this shape.

Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- $25\pi + 100$ square centimeters
- $\frac{1250}{7}$ square centimeters
- 178.5 square centimeters



Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the area of complex shapes:

» **Respond to Student Thinking:**

- Using the **Calculating Areas of Complex Shapes Mini-Lesson**.
- Revisit Activity 1 as a class and invite students to share strategies for determining the area of complex shapes.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to draw a shape on graph paper, decide on a scale, then calculate the area of their shape. Consider inviting them to draw multiple shapes with the same area.

Professional Learning

Reflect on Activity 2 of this lesson. How did students engage when creating their own shapes? How did the structure promote student sense-making around determining areas of complex shapes? What opportunities were there to recognize and encourage students' creativity?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Describe a strategy for determining the area of a shape that's made up of squares and parts of circles.

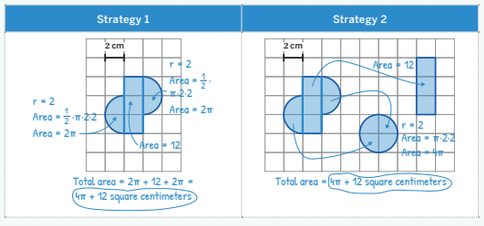
Use the example if it helps to show your thinking.

Responses vary. Count the number of squares, semicircles, and quarter-circles. If possible, combine parts of circles into whole circles. Use the scale to figure out what the radius of the circles are and how long the sides of the square are. Then figure out the area of each piece and add up all the pieces to determine the total area.

Lesson Practice 7.04

Lesson Summary

We can use different strategies and different ways of thinking to calculate the area of complex figures. Here are two ways we can determine the area of this shape:

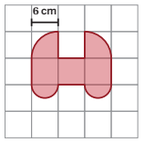


The path to the solution may not be obvious at first, but by breaking the shape down into squares, circles, and parts of circles, we can figure things out!

Lesson Practice 7.04

Name: _____ Date: _____ Period: _____

1. Calculate the area of this shape.



Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

$27\pi + 108 \approx 192.82$ square centimeters

Problems 2–4: Here is the architectural plan for a fenced-in garden. The shaded region represents an area for trees.



2. What is the area of the tree region?

$0.75\pi \approx 2.36$ square meters

3. The rest of the fenced-in garden is for flowers. How much area will be covered in flowers? Show your thinking.

About 17.28 square meters.

Work varies. $6.25\pi - 0.75\pi = 5.5\pi \approx 17.28$

4. How long is the fence around the edge of the garden? (Note: The edge around the tree region is not a part of the fence.)

$2.5\pi + 10 \approx 17.85$ meters

5. Each of these squares has a side length of 12 units. Compare the areas of the shaded regions in these three figures. Which figure has the largest shaded region? Show or explain your thinking.

Figure A

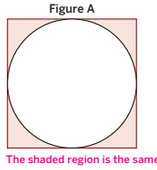


Figure B

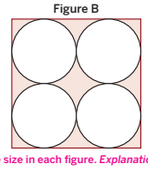
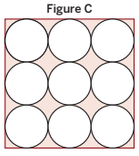


Figure C



The shaded region is the same size in each figure. *Explanations vary.*

In Figure A, the radius length of the circle is half the side length of the square, or 6 units.

$A = \pi r^2$
$A = \pi \cdot 6^2$
$A = 36\pi$
Shaded region: $144 - 36\pi$

In Figure B, the radius length of each circle is one fourth the side length of the square, or 3 units.

$A = \pi r^2$
$A = \pi \cdot 3^2$
$A = 9\pi$
4 circles: $4 \cdot 9\pi = 36\pi$
Shaded region: $144 - 36\pi$

In Figure C, the radius length of each circle is one sixth the side length of the square, or 2 units.

$A = \pi r^2$
$A = \pi \cdot 2^2$
$A = 4\pi$
9 circles: $9 \cdot 4\pi = 36\pi$
Shaded region: $144 - 36\pi$

Lesson Practice 7.04

Name: _____ Date: _____ Period: _____

FAST Practice

6. A circle with a 12-inch long diameter is folded in half and then folded in half again. What is the exact area of the new shape? Write your answer in terms of π, and show your thinking.

9π square inches

Spiral Review

7. A graffiti artist is spray painting a circular mural onto a wall. The mural will have a diameter that is 10 feet long. Each can of spray paint covers about 24 square feet. How many cans of spray paint will the artist need to create the mural? Explain your thinking.

4 cans. Explanations vary. If the diameter is 10 feet long, then the radius is 5 feet long. That means the area of the mural is $\pi \cdot 5^2 = \pi \cdot 25 \approx 79$ square feet. Because 1 can covers 24 square feet, the artist will need $79 \div 24$, or about 3.29 cans of paint. This means the artist will actually need to purchase 4 cans for the mural.

Problems 8–9: Determine each value.

8. What is the length of the radius of a circle with a diameter that measures 10 units?

5 units

9. What is the length of the diameter of a circle with a radius that measures 10 units?

20 units

10. Abena used wire fencing to form a border around a circular garden in her backyard. If the radius of the circular garden was 10 yards long, what was the total length of the border, rounded to the nearest tenth of a yard?

A. 31.4 B. 62.8 C. 314.2 D. 628.3

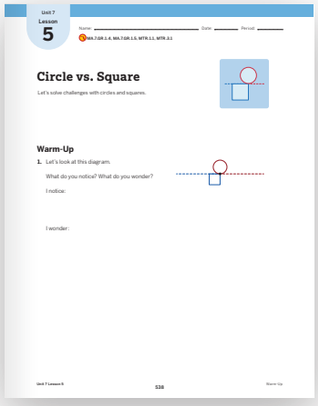
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.GR.1.4, MA.7.GR.1.5
FAST Practice	6	2	MA.7.GR.1.4
Spiral Review			
	7	3	MA.7.GR.1.4
	8–9	1	MA.7.GR.1.3
	10	2	MA.7.GR.1.3

Unit 7

Lesson

5


Unit 7
Lesson
5
Name: Date: Period:
MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.3.1
Circle vs. Square
Let's solve challenges with circles and squares.
Warm-Up
1. Let's look at this diagram.
What do you notice? What do you wonder?
I notice:
I wonder:
538
Warm-Up

Circle vs. Square

Calculating Area Given Perimeter

Let's solve challenges with circles and squares.

Learning Outcomes

Today's Goals

1. **Goal:** Explore the relationship between perimeter and area for a circle and a square.
2. **Language Goal:** Describe how to determine the area of a circle given its circumference. **(Writing, Speaking, and Listening)**

Students apply their knowledge of circumference and area to make sense of a challenge involving calculating the areas of two shapes based on their perimeters. They persevere to answer questions such as: *Which shape is larger? When will their total area be greatest? When will the areas be equal?* **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 4, students calculated the areas of shapes made up of parts of circles and squares. In Unit 3, students calculated the circumference of circles.

Future Learning

In Lessons 6–8, students will build on their knowledge of circles to calculate surface area and volume of cylinders.

Connections

- Students explore greatest total area of circles and squares using visuals and recording the results on a graphic organizer. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.4
Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	MA.7.GR.2.1
	MA.7.GR.2.2
	MA.7.GR.2.3

Lesson at a Glance

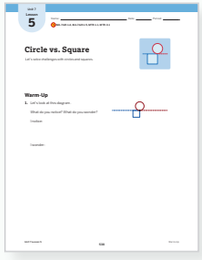
~ 45 min

MA.7.GR.1.4, MA.7.GR.1.5

Warm-Up

Independent | 5 min

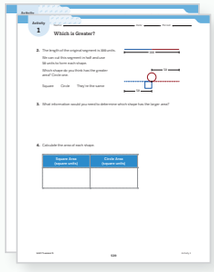
Students use the **Notice and Wonder** routine to explore a diagram of a line broken into two parts, where one part forms a square and the other forms a circle.



Activity 1

Small groups | 15 min

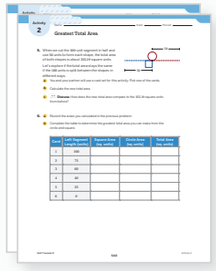
Students develop and apply a strategy for determining the areas of a circle and a square based on their perimeters. **(MTR.1.1)**



Activity 2

Small groups | 15 min

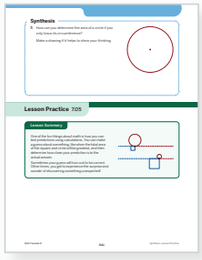
Students build fluency in calculating area given perimeter as they strategize about how to maximize the total area. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

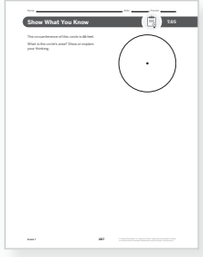
Students synthesize their understanding of the relationship between the circumference and area of circles.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the area of a circle when given its circumference.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, on per pair, pre-cut

Independent | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to explore a diagram of a line broken into two parts, where one part forms a square and the other forms a circle.

1 Launch

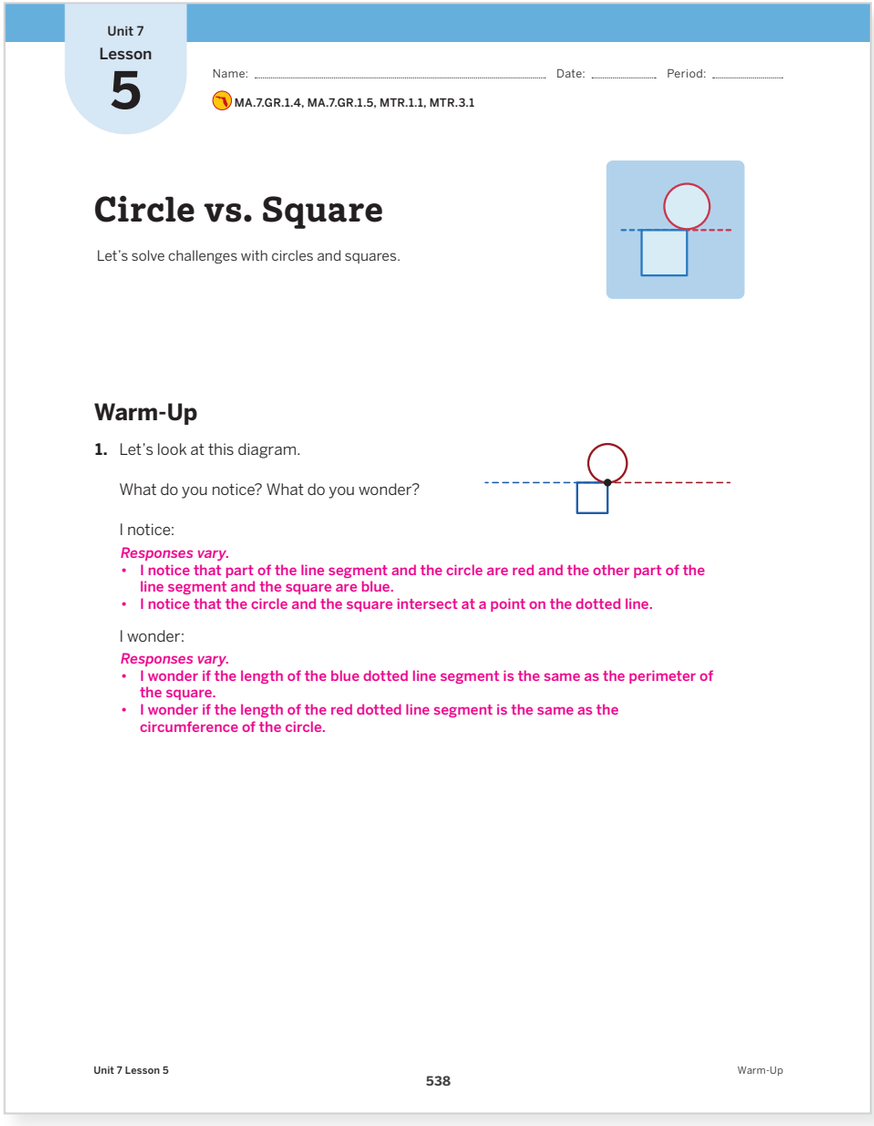
Use the Notice and Wonder routine to promote curiosity and help students make sense of the scenario.

2 Connect

Create a record of what students noticed and wondered, along with the names of the students who shared them.

Consider asking, "Did anyone notice that the length of blue part of the line segment is the same as the length of the red part of the line segment?"

Note: If it does not come up naturally, consider sharing that square is formed from the blue dotted line segment and the circle is formed from the red dotted line segment.



Unit 7
Lesson
5

Name: Date: Period:

MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.3.1

Circle vs. Square

Let's solve challenges with circles and squares.

Warm-Up

1. Let's look at this diagram.

What do you notice? What do you wonder?

I notice:

Responses vary.
- I notice that part of the line segment and the circle are red and the other part of the line segment and the square are blue.
- I notice that the circle and the square intersect at a point on the dotted line.

I wonder:

Responses vary.
- I wonder if the length of the blue dotted line segment is the same as the perimeter of the square.
- I wonder if the length of the red dotted line segment is the same as the circumference of the circle.

Unit 7 Lesson 5 538 Warm-Up

Small Groups | 15 min

Activity 1 Which is Greater?

Purpose: Students develop and apply a strategy for determining the areas of a circle and a square based on their perimeters. **(MTR.1.1)**

1 Launch

Consider asking, "When you cut the segment in half, what is the same about the two shapes? What may be different?" **(MTR.1.1)**

Invite students to make a prediction about which shape has the greater area.

Note: It's okay — even desirable — to lack consensus at this stage. The activity will build toward consensus at the end of the activity when students calculate the area of each shape.

2 Monitor

Look for students who mention side length or radius.

ELL **English Language Learners** Consider providing a word bank to support students in expressing their ideas with words such as: *radius*, *circumference*, *perimeter*, *area*, *formula*, *equation*, etc. **(Writing and Speaking)**

Pause briefly and invite students to share what information they think would help. Consider asking:

- "What does the number 50 tell us about the circle?"
- "How can you figure out the diameter and radius of a circle if you know its circumference?" **(MTR.1.1)**

Look for students who organize their work in clear or creative ways. Select and sequence these to share during the Connect.

Note: Make sure all students have access to paper.

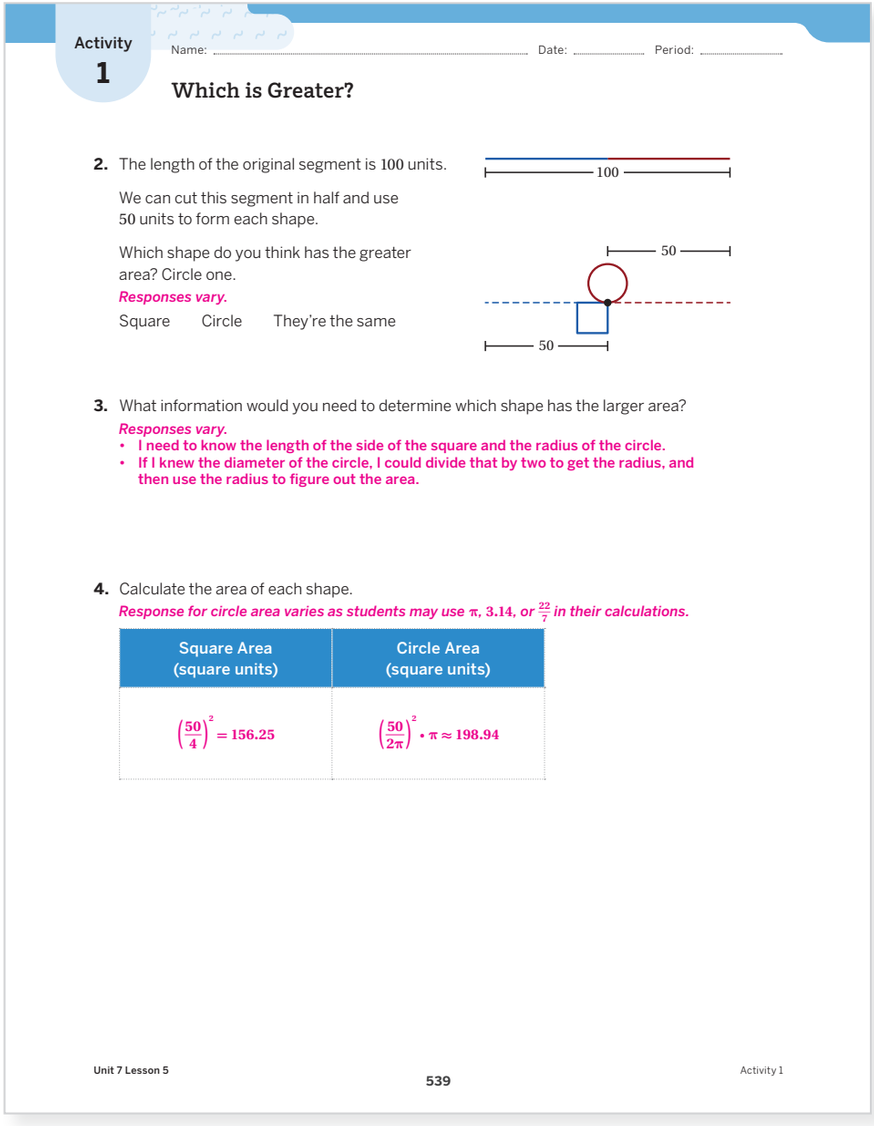
D **Differentiation: (Support)** To support students getting started, consider asking them to guess a value for the radius of the circle and see if the circumference is close to 50 units. This repeated reasoning may support students with working backwards.

A **Accessibility: Executive Functioning** Consider providing a graphic organizer to support students organizing their thinking. Encourage them to separate their work for the square and circle, and include space to record the information identified in Problem 3.

3 Connect

Display the selected student strategies and invite students to share their reasoning.

Note: Spend adequate time here to ensure that all groups have successfully used a strategy to determine the area of each shape. Students will have more practice with these calculations in Activity 2.



Activity 1

Name: Date: Period:

Which is Greater?

2. The length of the original segment is 100 units.

We can cut this segment in half and use 50 units to form each shape.

Which shape do you think has the greater area? Circle one.

Responses vary.

Square Circle They're the same

3. What information would you need to determine which shape has the larger area?

Responses vary.

- I need to know the length of the side of the square and the radius of the circle.
- If I knew the diameter of the circle, I could divide that by two to get the radius, and then use the radius to figure out the area.

4. Calculate the area of each shape.

Response for circle area varies as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

Square Area (square units)	Circle Area (square units)
$\left(\frac{50}{4}\right)^2 = 156.25$	$\left(\frac{50}{2\pi}\right)^2 \cdot \pi \approx 198.94$

Unit 7 Lesson 5 539 Activity 1

Small Groups | 15 min

Activity 2 Greatest Total Area

Purpose: Students build fluency in calculating area given perimeter as they strategize about how to maximize the total area. **(MTR.3.1)**

Short on time? Consider omitting Problem 5 and sharing that the total area changes when the 100 units are distributed differently.

1 Launch

To support students getting started, consider clarifying the instructions prior to the students working in their groups.

To generate excitement, consider polling the class by asking, "Who thinks the area will change? Who thinks the area will stay the same?"

Distribute the Activity 2 Cards.

2 Monitor

Encourage students to check their responses with the group to reach an agreement.

A **Accessibility: Executive Functioning** Consider inviting students to fold their paper into quarters or eighths and record the areas for each attempt in one section of the paper.

Look for students who discuss a general method for determining the areas, such as using division, then applying known area formulas. **(MTR.3.1)**

3 Connect

Invite students to share the greatest total area they created.

Consider asking, "Do you think you found the greatest possible area? How could you be sure?"

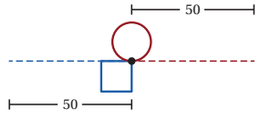
Activity 2

Name: Date: Period:

Greatest Total Area

5. When we cut the 100-unit segment in half and use 50 units to form each shape, the total area of both shapes is about 355.19 square units.

Let's explore if the total area stays the same if the 100 units is split between the shapes in different ways.



a. You and your partner will use a card set for this activity. Pick one of the cards.

b. Calculate the new total area. *Responses vary.*

c. **Discuss:** How does the new total area compare to the 355.19 square units from before? *Responses vary.*

6. a. Record the areas you calculated in the previous problem.

b. Complete the table to determine the greatest total area you can make from the circle and square. *Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.*

Card	Left Segment Length (units)	Square Area (sq. units)	Circle Area (sq. units)	Total Area (sq. units)
1	100	$\left(\frac{100}{4}\right)^2 = 625$	0	625
2	75	$\left(\frac{75}{4}\right)^2 = 351.5625$	$\left(\frac{25}{2\pi}\right)^2 \cdot \pi \approx 49.74$	401.30
3	60	$\left(\frac{60}{4}\right)^2 = 225$	$\left(\frac{40}{2\pi}\right)^2 \cdot \pi \approx 127.32$	352.32
4	40	$\left(\frac{40}{4}\right)^2 = 100$	$\left(\frac{60}{2\pi}\right)^2 \cdot \pi \approx 286.48$	386.48
5	25	$\left(\frac{25}{4}\right)^2 = 39.0625$	$\left(\frac{75}{2\pi}\right)^2 \cdot \pi \approx 447.63$	486.69
6	0	0	$\left(\frac{100}{2\pi}\right)^2 \cdot \pi \approx 795.77$	795.77

Unit 7 Lesson 5 540 Activity 2

Whole Class | 5 min

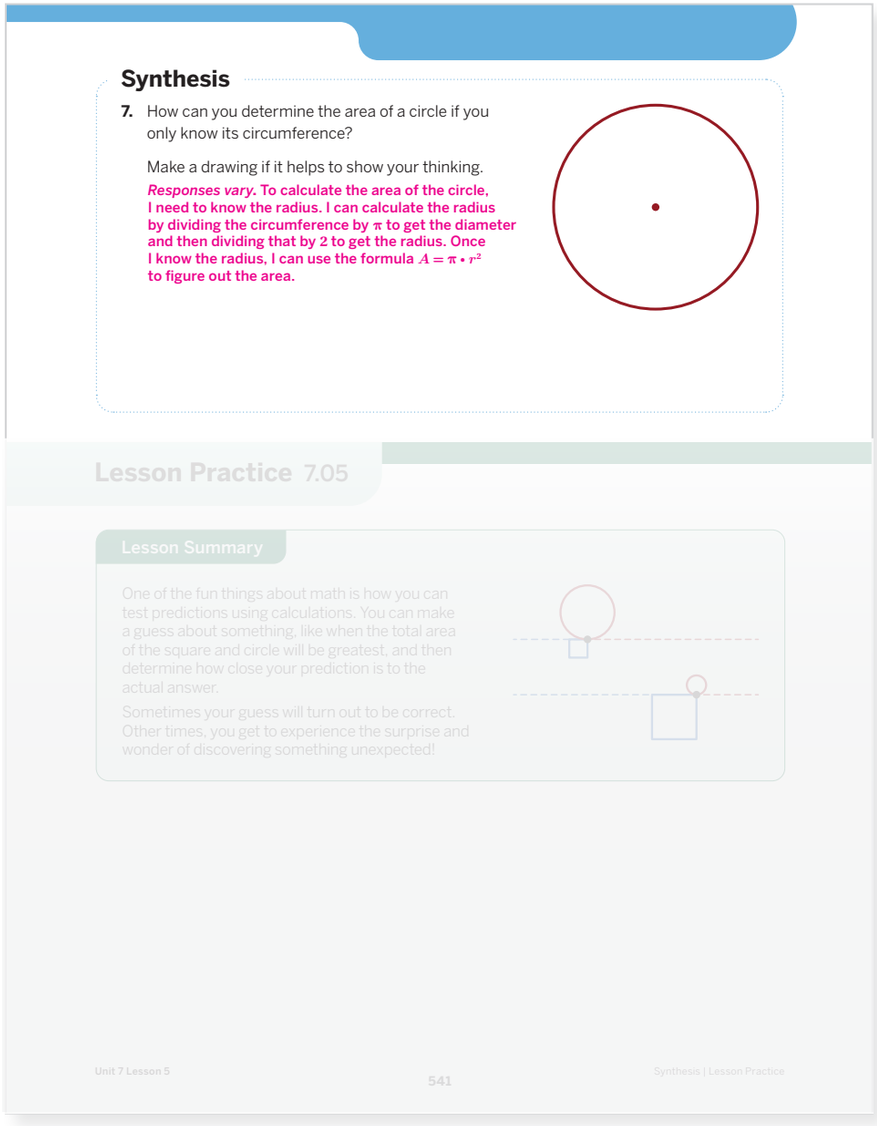
Synthesis

Purpose: Students synthesize their understanding of the relationship between the circumference and area of circles.

Invite students to respond independently, then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Working backwards from the circumference to calculate the diameter and radius of the circle.
- Using the equation $A = \pi r^2$.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Synthesis

7. How can you determine the area of a circle if you only know its circumference?

   Make a drawing if it helps to show your thinking.

   Responses vary. To calculate the area of the circle, I need to know the radius. I can calculate the radius by dividing the circumference by π to get the diameter and then dividing that by 2 to get the radius. Once I know the radius, I can use the formula $A = \pi \cdot r^2$ to figure out the area.

Key Takeaway: If you know the circumference of a circle, you can calculate the area. First, you work backwards to determine the radius. Then, you use the radius to calculate area.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the area of a circle when given its circumference.

Today's Goals

1. **Goal:** Explore the relationship between perimeter and area for a circle and a square.
2. **Language Goal:** Describe how to determine the area of a circle given its circumference. **(Writing, Speaking, and Listening)**

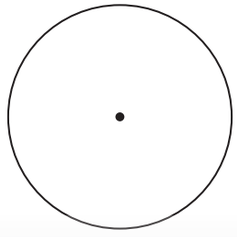
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.05

The circumference of this circle is 60 feet.

What is the circle's area? Show or explain your thinking.



Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.

- $\frac{900}{\pi}$ (or equivalent) square feet
- 286.62 square feet
- $\frac{3150}{11}$ square feet

Explanations vary. I can divide 60 by 2π to get the radius of the circle. Then, I can square the radius and multiply by π to get the area: $\left(\frac{60}{2\pi}\right)^2 \cdot \pi$

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the area of a circle, given its circumference:

» **Respond to Student Thinking:**

- Invite students to first determine the diameter and radius of a circle given its circumference.
- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on student progress with calculating area and circumference of circles from their previous learning in Unit 3. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you utilize student strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. How can you determine the area of a circle if you only know its circumference?

Make a drawing if it helps to show your thinking.

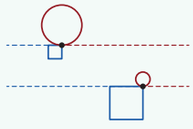
Responses vary. To calculate the area of the circle, I need to know the radius. I can calculate the radius by dividing the circumference by π to get the diameter and then dividing that by 2 to get the radius. Once I know the radius, I can use the formula $A = \pi \cdot r^2$ to figure out the area.

Lesson Practice 7.05

Lesson Summary

One of the fun things about math is how you can test predictions using calculations. You can make a guess about something, like when the total area of the square and circle will be greatest, and then determine how close your prediction is to the actual answer.

Sometimes your guess will turn out to be correct. Other times, you get to experience the surprise and wonder of discovering something unexpected!



Lesson Practice 7.05

Name: _____ Date: _____ Period: _____

Problems 1–3: A circle's circumference is approximately 76 centimeters. Determine each of these measurements and show your thinking:

1. The length of the diameter of the circle.
   About 24.2 centimeters.
   Work varies.
   $C = \pi d$
   $76 = \pi \cdot d$
   $76 \div \pi = \pi \cdot d \div \pi$
   $24.2 \approx d$

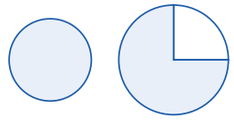
2. The length of the radius of the circle.
   About 12.1 centimeters.
   Work varies.
   $24.2 \div 2 = 12.1 = r$

3. The area of the circle.
   About 460 square centimeters.
   Work varies.
   $A = \pi r^2$
   $A = \pi \cdot 12.1^2$
   $A = \pi \cdot 146.41$
   $A \approx 460$

4. The metal rim of a magnifying glass measures 10 inches around. What is the area of the surface of the glass inside? Show your thinking.
   About 7.96 square inches.
   Work varies.
   $\left(\frac{10}{2\pi}\right)^2 \cdot \pi \approx 7.96$



5. The diameter of the smaller circle is 4.5 centimeters long. The diameter of the larger circle is 6 centimeters long. Calculate the difference in the areas of the shaded regions of the two circles. Show your thinking.
   Approximately 5.3 square centimeters. ***Work varies.***



The figures may not be drawn to scale.

Smaller circle shaded area:
$r = \frac{4.5}{2} = 2.25$
$A = \pi \cdot 2.25^2$
$A = \pi \cdot 5.0625$
$A = 5.0625\pi$

Larger circle:
$r = \frac{6}{2} = 3$
$A = \pi \cdot 3^2$
$A = \pi \cdot 9$
$A = 9\pi$

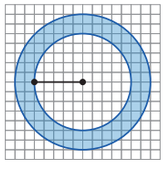
Shaded area:
$A = \frac{3}{4}(9\pi)$
$A = 6.75\pi$

Difference in shaded areas:
$A = 6.75\pi - 5.0625\pi$
$A = 1.6875\pi$
$A \approx 5.3$

Lesson Practice 7.05

Name: _____ Date: _____ Period: _____

6. Determine the area of the shaded region. Show or explain your thinking.
   24π square units. ***Work varies.***
   $7^2 \cdot \pi - 5^2 \cdot \pi = 49\pi - 25\pi = 24\pi$



FAST Practice

7. The circumference of a circle is 30π centimeters. What is the area of the circle in terms of π?

   A. 15π square centimeters
   B. 30π square centimeters
   C. 225π square centimeters
   D. 900π square centimeters

Spiral Review

Problems 8–10: Complete the table by determining the unknown measurements of each circle. ***Some responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.***

	Radius (in.)	Diameter (in.)	Circumference (in.)	Area (sq. in.)
8.	12	**24**	**24π**	**144π**
9.	**6**	12	**12π**	**36π**
10.	**$\frac{6}{\pi}$**	**$\frac{12}{\pi}$**	12	**$\frac{36}{\pi}$**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.GR.1.4 MA.7.GR.1.5
	4, 6	2	MA.7.GR.1.4
	5	3	MA.7.GR.1.4
FAST Practice	7	2	MA.7.GR.1.4
Spiral Review			
	8–10	2	MA.7.GR.1.3 MA.7.GR.1.4

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1a, 1b	Calculating the area of parallelogram	Lesson 1	1	MA.7.1.GR.1.1
2	Calculating the area of a polygon composed of rectangles and triangles on a grid	Lesson 2	1	MA.7.1.GR.1.2
3	Determining the area of polygon	Lesson 2	1	MA.7.1.GR.1.2
4	Determining the area of composite figures using decomposition strategies	Lesson 3, 4	2	MA.7.1.GR.1.2, MA.7.1.GR.1.4
5	Determining the area of a shaded region	Lesson 5	2	MA.7.1.GR.1.3, MA.7.1.GR.1.4
6	Determining the area of circle given the circumference	Lesson 5	2	MA.7.1.GR.1.3, MA.7.1.GR.1.4

Assessment and Lesson Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

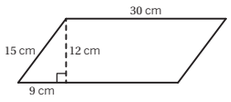
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 7

1. Determine the area of each figure.

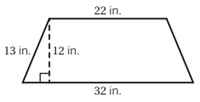
a



$A = bh = 30 \cdot 12$

$A = 360 \text{ cm}^2$

b



$A = \frac{1}{2}(b_1 + b_2)h$

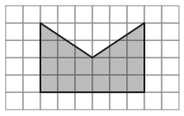
$A = \frac{1}{2}(22 + 32)(12)$

$A = \frac{1}{2}(54)(12) = 324 \text{ in}^2$

2. Determine the area of this polygon.

Each square has an area of 1 unit.

$A = 18$ square units



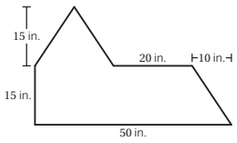
3. Determine the area of the polygon.

$A_{rect} = bh = 40 \cdot 15 = 600 \text{ in}^2$

$A_{tri1} = \frac{1}{2}bh = \frac{1}{2}(20)(15) = 150 \text{ in}^2$

$A_{tri2} = \frac{1}{2}bh = \frac{1}{2}(10)(15) = 75 \text{ in}^2$

$A_{total} = 600 + 150 + 75 = 825 \text{ in}^2$



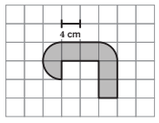
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 7

4. Determine the area of the composite figure.

(Use $\pi = 3.14$)

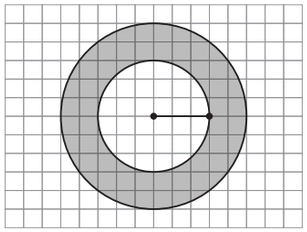


$A_{square} = 4(4 \cdot 4) = 64 \text{ cm}^2$

$A_{partial\ circles} = 3\left(\frac{1}{4}(3.14)(16)\right) = 37.68 \text{ cm}^2$

$A_{total} = 64 + 37.68 = 101.68 \text{ cm}^2$

5. Determine the exact area of the shaded region. Each square has an area of 1 unit.



$A_{large\ circle} = \pi(r^2) = \pi(5^2)$
$= 25\pi$ square units

$A_{small\ circle} = \pi(r^2) = \pi(3^2)$
$= 9\pi$ square units

$A_{shaded} = 25\pi - 9\pi$
$= 16\pi$ square units

6. The circumference of a circle is 24π cm. What is the area of the circle, in terms of π? Show or explain your thinking.

Responses vary.

First, I need to find the radius of the circle using the circumference.

$C = \pi d$

$24\pi = \pi d$

$\frac{24\pi}{\pi} = \frac{\pi d}{\pi}$

$24 = d$

The diameter is equal to $2r$, so $24 = 2r$ or $r = 12$.

$A = \pi r^2$

$A = \pi(12^2)$

$A = 144\pi \text{ cm}^2$

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

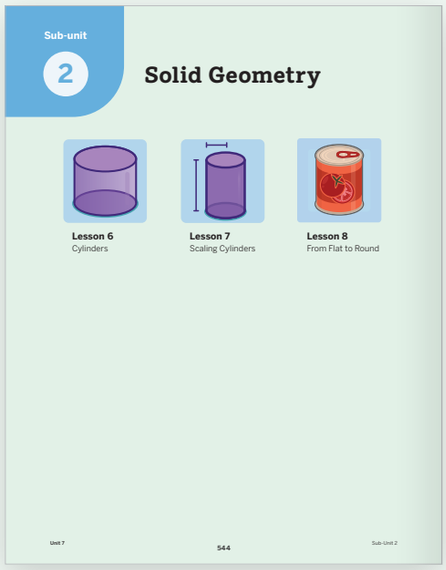
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: • Calculate the areas of parallelograms, rhombi, trapezoids, polygons, and composite shapes. **(Lessons 1–5)**	1a, 1b	**Support** **Strengthen** • **Mini-Lesson:** Calculating the Area of Parallelograms (ML 7.01) • **Teacher Move:** Consider revisiting Lesson 1, Activity 1 (Areas of Parallelograms and Rhombi)
	2	**Support** **Strengthen** • **Teacher Move:** Consider revisiting Lesson (Pile of Polygons) • **Challenge Creator:** Activity 2 (Pile of Polygons)
	3, 4	**Support** **Strengthen** • **Mini Lesson:** Calculating Areas of Complex Shapes (ML 7.04) • **Teacher Move:** Consider revisiting Lesson 3 (Shapes Inside Shapes) Synthesis
	5, 6	**Support** **Strengthen** • **Teacher Move:** Consider revisiting Lesson 5 (Circle vs. Square) • **Teacher Move:** Consider asking, "How does knowing the circumference of a circle help in finding the area of a circle?

Solid Geometry

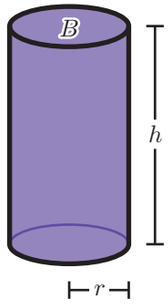
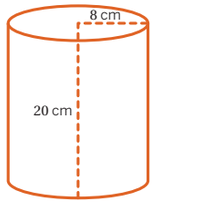
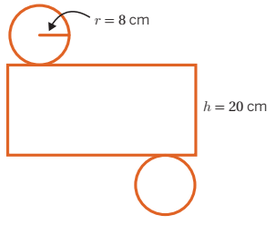
Sub-Unit 2 Goals

- **Goal:** Calculate the volume and surface area of cylinders.
- **Language Goal:** Explain how to derive the formulas for the volume and surface area of a cylinder.

In this sub-unit, students calculate the volume and surface area of cylinders. Students also determine strategies for calculating unknown dimensions and reason about how the volume of a cylinder changes as another dimension changes.



Math that Matters Most

Concept or Skill	For example . . .
Calculate the volume of a cylinder by building on how to calculate the volume of a prism.	Here are two formulas for the volume of a cylinder: $V = \pi r^2 \cdot h$ $V = B \cdot h$ In the first formula, πr^2 represents the area of the base of the cylinder, while h represents the height, so the formulas are essentially the same. They both say "find the area of the base, then multiply by the height." The first formula can only be used for cylinders. The second formula can be used for both cylinders and prisms. 
Calculate the surface area of a cylinder using its net.	The cylinder has a height of 20 centimeters and a radius of 8 centimeters. Determine the surface area of the cylinder. The cylinder has two identical circular bases with a radius of 8 centimeters. The curved surface of the cylinder is shaped like a rectangle with a height of 20 centimeters. The length of the rectangle is the same as the circumference of the circular base. Area of one circle: $A = \pi r^2$, $A = \pi(8)^2$, $A = 64\pi$ Area of rectangle: $A = 2\pi r \cdot h$, $A = 2\pi(8) \cdot 20$, $A = 320\pi$ Total surface area: $SA = 64\pi + 64\pi + 320\pi = 448\pi$ square centimeters  

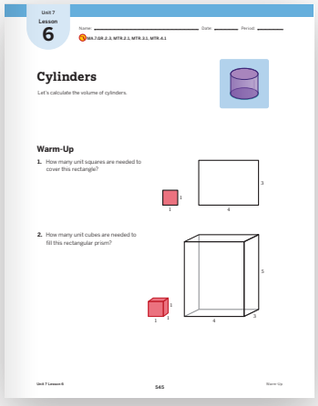
Unit 7

Lesson 6

Cylinders

Volumes of Cylinders

Let's calculate the volumes of cylinders.


Unit 7
Lesson 6
Name: Date: Period:
MA.7.GR.2.3, MTR.2.1, MTR.3.1, MTR.4.1
Cylinders
Let's calculate the volume of cylinders.
Warm-Up
1. How many unit squares are needed to cover this rectangle?
2. How many unit cubes are needed to fill this rectangular prism?
545
Warm-Up

Learning Outcomes

Today's Goal

1. **Goal:** Calculate the volume of a *cylinder* by building on how to calculate the volume of a prism.

Students develop strategies for determining the *volume* of a **cylinder** by multiplying the area of the base times the height. This builds on a strategy for determining the volume of prisms. After discovering how to calculate the volume of specific cylinders, students generalize to the formulas $V = \pi r^2 h$ and $V = Bh$.

Coherence

Prior Learning

In Grade 6, students determined volumes of right rectangular prisms. In Lessons 5–6, they solved problems related to the area and circumference of a circle.

Future Learning

In Lesson 7, students will explore how changing a cylinder's radius or height impacts its volume.

Connections

- Students extend their knowledge of volumes of prisms to discover the voulume of cylinders using visual representations. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.2.3

Solve mathematical and real-world problems involving volume of right circular cylinders.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.4.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.3	**MA.912.GR.4.5**

Vocabulary

New Vocabulary

cylinder

Review Vocabulary

volume

Lesson at a Glance

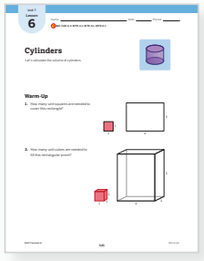
~ 45 min

MA.7.GR.2.3

Warm-Up

Independent | 5 min

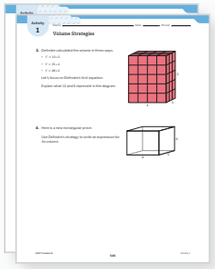
Students surface prior knowledge about area and *volume* of rectangular figures.



Activity 1

Pairs | 15 min

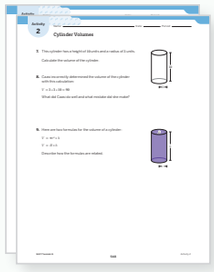
Students make connections between structures of prisms and **cylinders** to support calculating volume. **(MTR.2.1)**



Activity 2

Pairs | 10 min

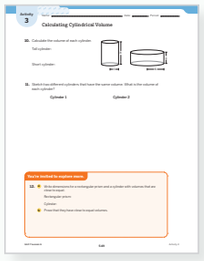
Students generalize a formula for calculating the volume of a cylinder. **MLR3: Critique, Correct, Clarify**



Activity 3

Pairs | 5 min

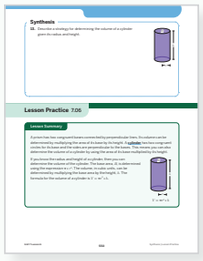
Students calculate the volume of different cylinders with precision. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

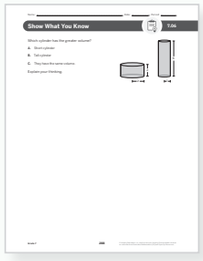
Students synthesize their understanding of how to calculate the volume of a cylinder. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by comparing the volumes of two cylinders with different dimensions.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Three-dimensional solids, calculators (as needed)

Independent | 5 min

Warm-Up

Purpose: Students surface prior knowledge about area and *volume* of rectangular figures.

1 Launch

Direct students' attention to the rectangle and invite students to think about how they can visualize the area before responding.

Consider sharing that students can write their response as an expression and provide examples (e.g., $2 + 2$).

Invite students to use any strategy to calculate the *volume*.

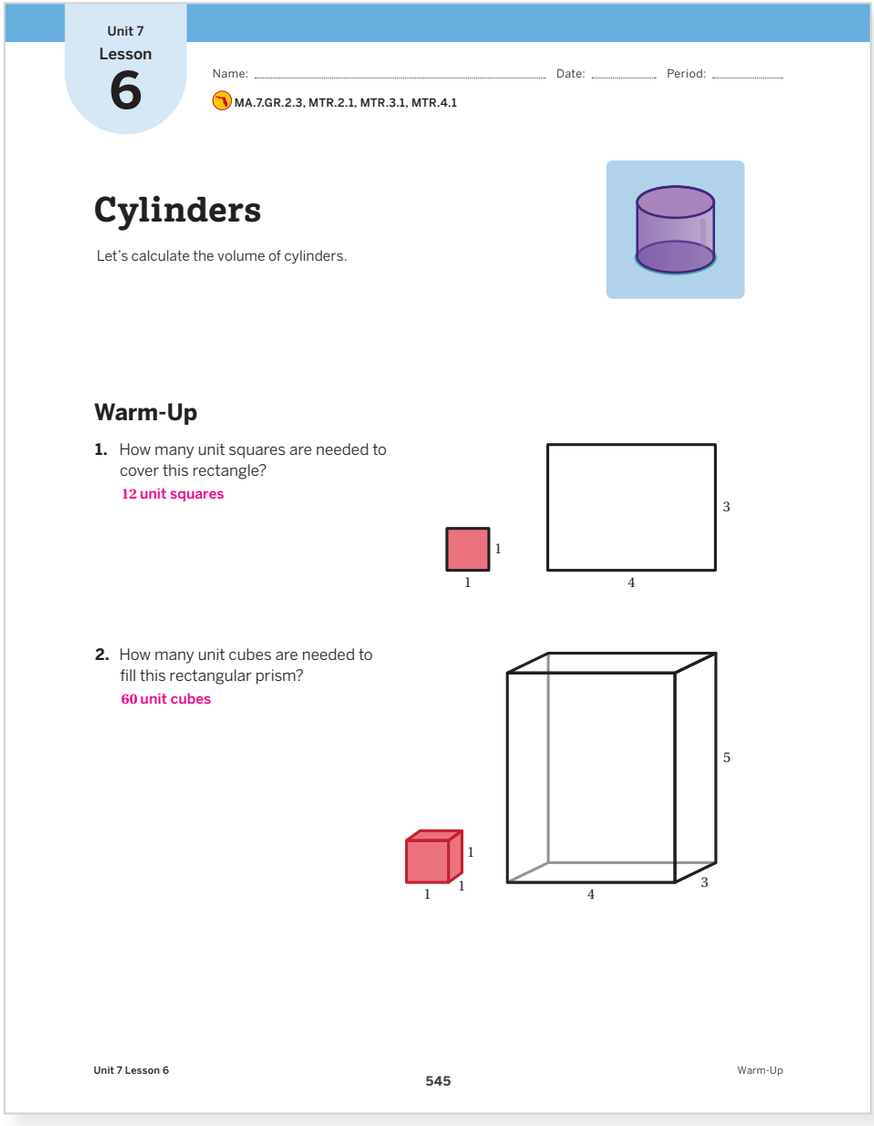
A **Accessibility: Visual-Spatial Processing**
Provide access to three-dimensional cylinders and prisms that students can reference to support processing visual information.

2 Connect

Listen for and share student responses, particularly ones that include expressions.

To surface connections, consider asking:

- "How did you determine the number of unit cubes needed to fill up the figure?"
- "How is this process related to Problem 1?"
- "What are other expressions we could write for the same volume?"



Unit 7
Lesson
6

Name: _____ Date: _____ Period: _____

MA.7.GR.2.3, MTR.2.1, MTR.3.1, MTR.4.1

Cylinders

Let's calculate the volume of cylinders.

Warm-Up

1. How many unit squares are needed to cover this rectangle?
   12 unit squares

2. How many unit cubes are needed to fill this rectangular prism?
   60 unit cubes

Unit 7 Lesson 6 545 Warm-Up

Pairs | 15 min

Activity 1 Volume Strategies

Purpose: Students make connections between structures of prisms and cylinders to support calculating volume. **(MTR.2.1)**

1 Launch

To support making connections, invite students to think independently about DeAndre's strategy before discussing their ideas with a partner.

ELL **English Language Learners** Invite students to show their thinking by annotating the Student Edition with connections between the expressions and the prism. **(Writing)**

Pause briefly to discuss how students interpreted the 12 and 5 in the diagram. Consider asking:

- "How are these equations alike? How are they different?"
- "When might an equation be useful for calculating volume?"

2 Monitor

Encourage students to move at their own pace through the rest of the activity.

Look for and respond to a variety of student expressions that represent the volume. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try any value or expression and discuss their response with a partner.
Write '240 cubic units' instead of an expression.	**Strengthen** Consider asking: • "How is your response related to DeAndre's strategy?" • "What might need to change to make your response more like his strategy?" **(MTR.4.1)**
Write an expression such as $40 \cdot 6$ units.	**Stretch** Encourage them to think about other expressions that could represent the volume.

Activity 1 continued >

Activity 1

Name: Date: Period:

Volume Strategies

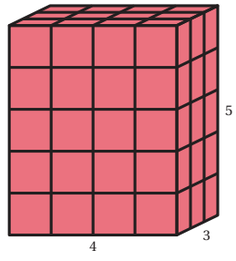
3. DeAndre calculated the *volume* in three ways:

- $V = 12 \cdot 5$
- $V = 15 \cdot 4$
- $V = 20 \cdot 3$

Let's focus on DeAndre's first equation.

Explain what 12 and 5 represent in the diagram.

Responses vary. 12 represents the area of the bottom face (3 by 4). 5 represents the height.

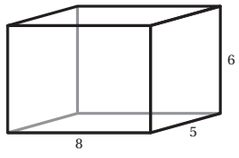


4. Here is a new rectangular prism.

Use DeAndre's strategy to write an expression for its volume.

Responses vary.

- $40 \cdot 6$ cubic units
- $30 \cdot 8$ cubic units
- $48 \cdot 5$ cubic units



Unit 7 Lesson 6 546 Activity 1

Pairs | 15 min

Activity 1 Volume Strategies (continued)

Purpose: Students make connections between structures of prisms and cylinders to support calculating volume. **(MTR.2.1)**

2 Monitor

Look for conflict and consensus in student responses.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer to DeAndre's strategy to make sense of which objects follow this structure.
Select object D or B.	**Support** Invite them to reflect on how this object is different from object A. Consider asking, "How is this object different from the prism that DeAndre investigated?"
Select some of the correct objects.	**Strengthen** Consider asking, "Are there any other objects like these?"
Select objects C, E, and F.	**Stretch** Consider asking, "How might we calculate volume for each object you chose?"

Note: If time allows, consider pausing to invite students to justify their responses if the class lacks consensus on any of the choices.

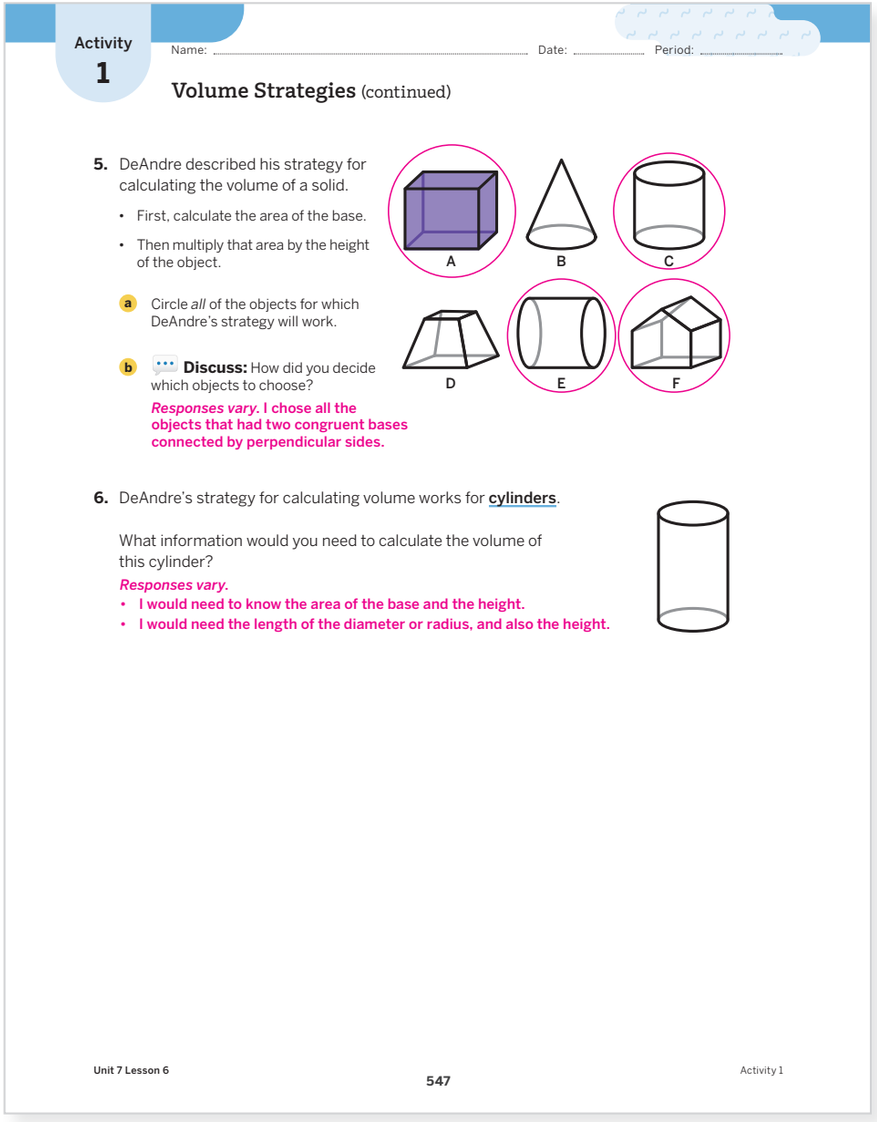
Listen for different suggestions to share with the class.

3 Connect

Share selected suggestions of information.

Create a class definition for the term **cylinder** using students' language and the glossary.

To surface the Key Takeaway, consider asking:
- "Is this enough information to calculate the volume?"
- "Is any of this information unnecessary?"
- "How would we use DeAndre's strategy to calculate the volume of a cylinder?"



Activity 1

Name: Date: Period:

Volume Strategies (continued)

5. DeAndre described his strategy for calculating the volume of a solid.
- First, calculate the area of the base.
- Then multiply that area by the height of the object.

A B C D E F

a. Circle *all* of the objects for which DeAndre's strategy will work.

b. **Discuss:** How did you decide which objects to choose?

Responses vary. I chose all the objects that had two congruent bases connected by perpendicular sides.

6. DeAndre's strategy for calculating volume works for **cylinders**.

What information would you need to calculate the volume of this cylinder?

Responses vary.
- I would need to know the area of the base and the height.
- I would need the length of the diameter or radius, and also the height.

Unit 7 Lesson 6 547 Activity 1

Key Takeaway: To determine the *volume* of a **cylinder**, multiply the area of of its circular base times its height.

Pairs | 10 min

Activity 2 Cylinder Volumes

Purpose: Students generalize a formula for the volume of a cylinder.

1 Launch

To surface connections, consider asking, "How would we calculate the area of the base of this cylinder?"

2 Monitor

Select and sequence volume calculations to share during the pause in Problem 8. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to take an educated guess and check their work.
Multiply the height by the radius and include π.	**Support** Consider asking, "How did you set up your calculations?"
Write the volume without including π.	**Support** Consider asking, "What needs to be included when calculating the area of a circle?"
Write an expression like $3 \cdot 3 \cdot 10\pi$.	**Stretch** Invite them to share their strategy during the Connect.

Pause to discuss Caasi's work. **(MTR.4.1)**

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Caasi's work. As a class:
- Discuss what is *correct* about her response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct and clear* solution.

Share the student thinking from Problem 7 if these strategies have not yet come up.

Encourage students to sketch in their Student Edition to support their explanation.

3 Connect

Create a record of ideas about the volume formula.

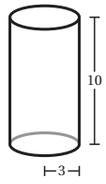
To surface the Key Takeaway, consider asking:
- "Why does the first formula use the expression πr^2? How is this connected to the B in the second formula?"
- "What does the h represent? Why is it important?"

Activity 2

Name: Date: Period:

Cylinder Volumes

7. This cylinder has a height of 10 units and a radius of 3 units.



Calculate the volume of the cylinder.

90π cubic units (or equivalent)

8. Caasi incorrectly determined the volume of the cylinder with this calculation:

$V = 3 \cdot 3 \cdot 10 = 90$

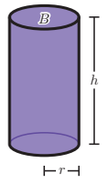
What did Caasi do well and what mistake did she make?

Responses vary. **Caasi used the dimensions given. She made a mistake in that she didn't multiply by π. She might have been thinking she could calculate the area of the base by multiplying the radius by itself. This works when the base of a prism is a square, but not for a cylinder, where the base is a circle.**

9. Here are two formulas for the volume of a cylinder:

$V = \pi r^2 \cdot h$

$V = B \cdot h$



Describe how the formulas are related.

Responses vary. **In the first formula, πr^2 represents the area of the base of the cylinder, while h represents the height, so the formulas are essentially the same. They both say "calculate the area of the base, then multiply by the height." The first formula can only be used for cylinders. The second formula can be used for both cylinders and prisms.**

Unit 7 Lesson 6 548 Activity 2

Key Takeaway: The formula for the *volume* of a **cylinder** is like the formula for the volume of a rectangular prism. In both formulas, calculate the volume by multiplying the area of the base by the height of the object.

Pairs | 5 min

Activity 3 Calculating Cylindrical Volume

Purpose: Students calculate the volume of different cylinders with precision. **(MTR.3.1)**

Short on time? Consider omitting this activity.

1 Launch

Invite students to apply the strategy for calculating the volume of cylinders from earlier in the lesson. **(MTR.3.1)**

A **Accessibility: Memory and Attention** Invite students to refer back to the anchor chart created in the previous activity on volume formulas of cylinders and prisms to activate prior knowledge.

2 Monitor

To support students getting started, consider inviting them to revisit DeAndre's strategy.

D **Differentiation: (Support)** Look for students who guess volumes for each cylinder. Encourage them to write out a volume formula and substitute what they know.

Select and sequence cylinder examples and calculations to share during the Connect.

D **Differentiation**

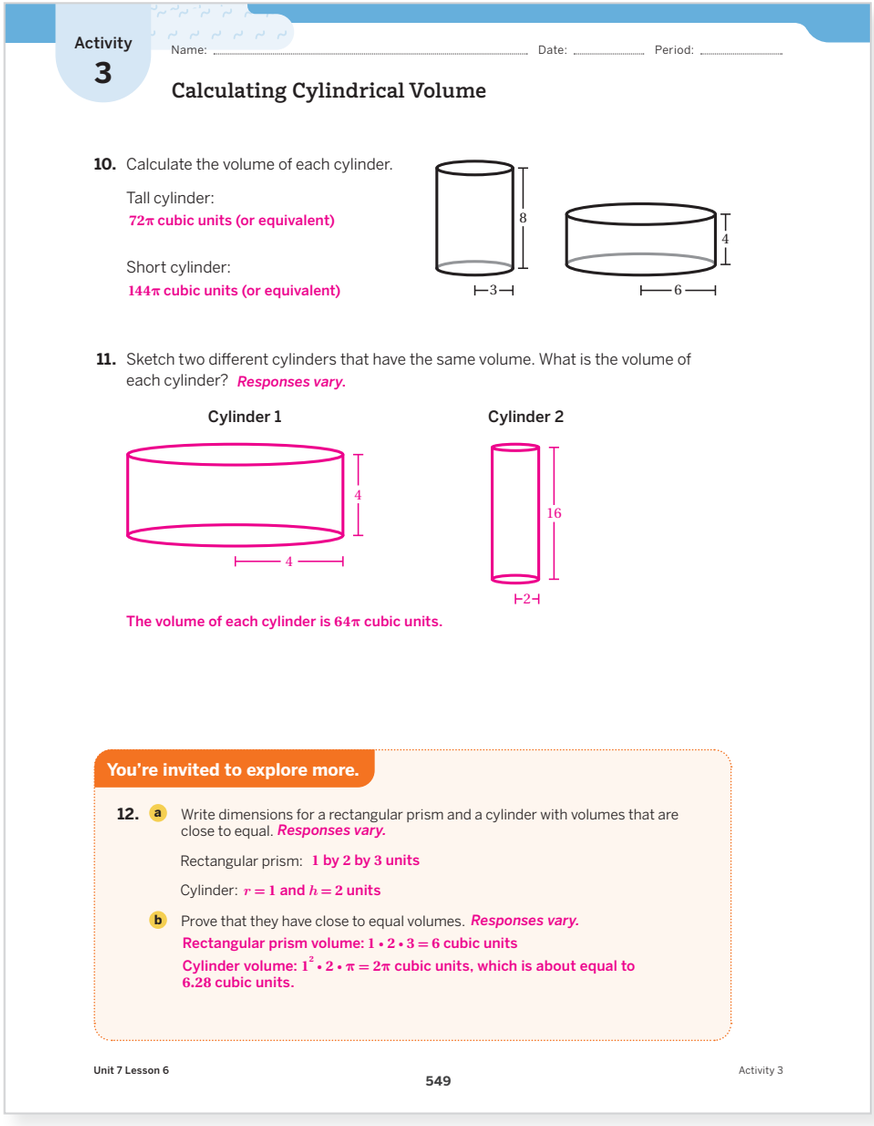
Look for students who:	Respond to Student Thinking
Create two copies of the same cylinder.	**Support** Encourage them to reread the directions and to create two different cylinders with the same volume.
Write the volumes without including π.	**Strengthen** Invite them to test their volume. Consider asking, "How do you calculate the area of the circular base?"
Test different dimensions to check if the volumes are equal.	**Stretch** Invite students to compare the relationship between the two cylinders. Consider asking, "Is there a pattern that you're noticing between the two cylinders?"
Substitute different heights and radiuses into the volume formula.	

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore the volume of solids further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share the cylinders they sketched. Consider asking, "How do you know if two cylinders have the same volume?" **(MTR.2.1)**

Note: If time allows, consider asking, "What relationships did you notice between the pairs of cylinders that have the same volume?"



Activity 3

Name: Date: Period:

Calculating Cylindrical Volume

10. Calculate the volume of each cylinder.

Tall cylinder:
72π cubic units (or equivalent)

Short cylinder:
144π cubic units (or equivalent)

11. Sketch two different cylinders that have the same volume. What is the volume of each cylinder? *Responses vary.*

Cylinder 1

Cylinder 2

The volume of each cylinder is 64π cubic units.

You're invited to explore more.

12. a. Write dimensions for a rectangular prism and a cylinder with volumes that are close to equal. *Responses vary.*

Rectangular prism: 1 by 2 by 3 units

Cylinder: $r = 1$ and $h = 2$ units

b. Prove that they have close to equal volumes. *Responses vary.*

Rectangular prism volume: $1 \cdot 2 \cdot 3 = 6$ cubic units

Cylinder volume: $1^2 \cdot 2 \cdot \pi = 2\pi$ cubic units, which is about equal to 6.28 cubic units.

Unit 7 Lesson 6 549 Activity 3

Whole Class | 5 min

Synthesis

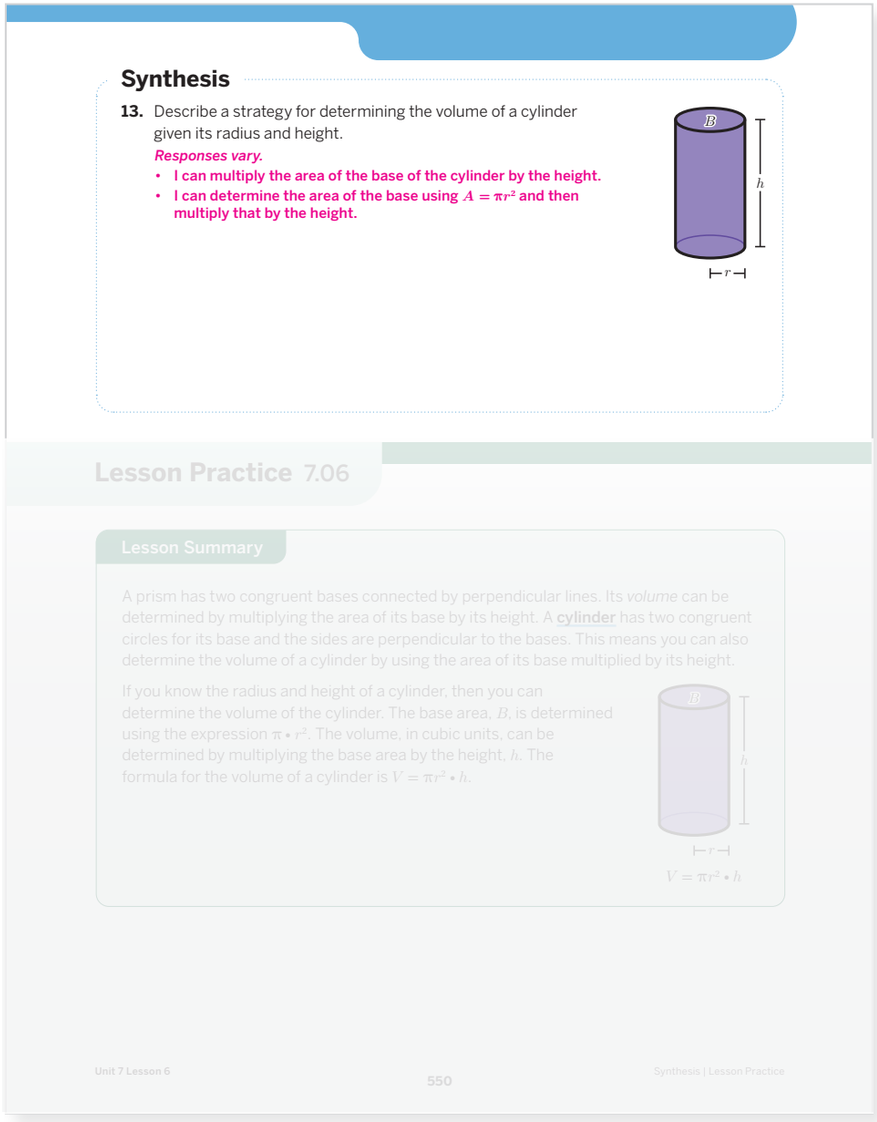
Purpose: Students synthesize their understanding of how to calculate the volume of a cylinder.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- The formula for the volume of a cylinder is connected to the volume formula for a prism.
- The area of the base in a cylinder is πr^2.
- The volume of a cylinder is the area of the base times the height.

MLR **MLR2: Collect and Display** Use this routine to collect students' formulas as they share during the Connect and Synthesis and formalize the defintion of *cylinder* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

13. Describe a strategy for determining the volume of a cylinder given its radius and height.

Responses vary.
- I can multiply the area of the base of the cylinder by the height.
- I can determine the area of the base using $A = \pi r^2$ and then multiply that by the height.

Lesson Practice 7.06

Lesson Summary

A prism has two congruent bases connected by perpendicular lines. Its *volume* can be determined by multiplying the area of its base by its height. A **cylinder** has two congruent circles for its base and the sides are perpendicular to the bases. This means you can also determine the volume of a cylinder by using the area of its base multiplied by its height.

If you know the radius and height of a cylinder, then you can determine the volume of the cylinder. The base area, B, is determined using the expression $\pi \cdot r^2$. The volume, in cubic units, can be determined by multiplying the base area by the height, h. The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$.

$V = \pi r^2 \cdot h$

Unit 7 Lesson 6 550 Synthesis | Lesson Practice

Lesson Takeaway: The *volume* of a **cylinder** is the area of its base times its height. This is represented by the formulas $V = \pi r^2 h$ and $V = Bh$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by comparing the volumes of two cylinders with different dimensions.

Today's Goal

1. **Goal:** Calculate the volume of a *cylinder* by building on how to calculate the volume of a prism.

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.06

Which cylinder has the greater volume?

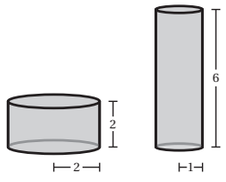
A. Short cylinder

B. Tall cylinder

C. They have the same volume.

Explain your thinking.

Explanations vary. **The volume of the short cylinder is $V = \pi \cdot (2)^2 \cdot 2 = 8\pi$. The volume of the tall cylinder is $V = \pi \cdot (1)^2 \cdot 6 = 6\pi$. So the short cylinder has the greater volume.**



Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate volume of cylinders:

» **Respond to Student Thinking:**

- Use the ***Calculating Volumes of Cylinders*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite Students to complete the You're invited to **Explore More** task.

Professional Learning

Reflect on your facilitation of the routine **MLR3: Critique, Correct, Clarify** in this lesson. How did students engage in the routine? What about how you facilitated the routine promoted student learning? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. Describe a strategy for determining the volume of a cylinder given its radius and height.

Responses vary.

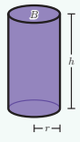
- I can multiply the area of the base of the cylinder by the height.
- I can determine the area of the base using $A = \pi r^2$ and then multiply that by the height.

Lesson Practice 7.06

Lesson Summary

A prism has two congruent bases connected by perpendicular lines. Its *volume* can be determined by multiplying the area of its base by its height. A **cylinder** has two congruent circles for its base and the sides are perpendicular to the bases. This means you can also determine the volume of a cylinder by using the area of its base multiplied by its height.

If you know the radius and height of a cylinder, then you can determine the volume of the cylinder. The base area, B, is determined using the expression $\pi \cdot r^2$. The volume, in cubic units, can be determined by multiplying the base area by the height, h. The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$.



$V = \pi r^2 \cdot h$

Lesson Practice 7.06

Name: _____ Date: _____ Period: _____

Problems 1–3: Draw a cylinder.

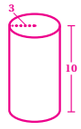
1. Label the radius 3 units and the height 10 units.

2. Determine the area of the base. Write your response in terms of π.

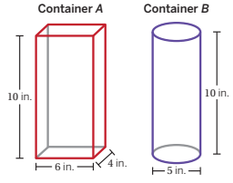
9π square units

3. Determine the volume of the cylinder. Write your response in terms of π.

90π cubic units



Problems 4–6: Containers A and B hold oatmeal. Container A is a rectangular prism and container B is a cylinder.



4. The diameter of container B is 5 inches. What is the radius of the container?

2.5 inches

5. Which container's base has a larger area? Explain your thinking.

Container A. *Explanations vary.* The area of container A's base is 24 square inches. The area of container B's base is about 19.6 square inches.

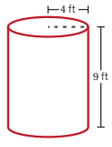
6. Which has a larger volume: container A or B? Explain your thinking.

Container A. *Explanations vary.* The two containers have the same height, but container A's base area is larger, so its volume is larger.

FAST Practice

7. Here is a cylinder with a radius of 4 feet and a height of 9 feet. What is the volume of the cylinder in cubic feet? Round your answer to the nearest hundredth.

452.39 cubic feet



Lesson Practice 7.06

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 8–9: A gardener is designing a circular flower bed. The distance around the flower bed is 31.4 feet.

8. The gardener wants to create a straight path through the center of the flower bed from one end to the other. What will be the length of the path? Use 3.14 for π.

10 feet

9. What is the area of the flower bed?

78.5 square feet

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.GR.2.3
	2–6	2	MA.7.GR.2.3
FAST Practice	7	2	MA.7.GR.2.3
Spiral Review			
	8	2	MA.7.GR.1.3
	9	2	MA.7.GR.1.4

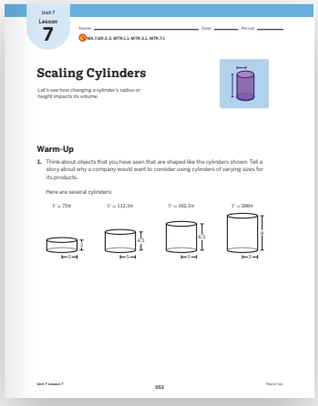
Unit 7

Lesson 7

Scaling Cylinders

Applying Volumes of Cylinders

Let's see how changing a cylinder's radius or height impacts its volume.


Unit 7
Lesson 7
Name _____ Date _____ Period _____
MA.7.GR.2.3, MTR.1.1, MTR.5.1, MTR.7.1
Scaling Cylinders
Let's see how changing a cylinder's radius or height impacts its volume.
Warm-Up
1. Think about objects that you have seen that are shaped like the cylinders shown. Tell a story about why a company would want to consider using cylinders of varying sizes for its products.
Here are several cylinders:
$V = 75\pi$
$V = 112.5\pi$
$V = 162.5\pi$
$V = 200\pi$
Unit 7 Lesson 7
553
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Analyze the relationship between the height or radius of a cylinder and its volume using tables.
2. **Goal:** Solve mathematical real-world problems involving volumes of cylinders.
3. **Language Goal:** Explain how changing the radius or height of a cylinder affects its volume and explain how to calculate a missing dimension given the volume of a cylinder and its radius. **(Speaking and Listening)**

Students explore how changing a cylinder's radius or height impacts its volume. They compute volumes of cylinders given the radius and height and determine the height given a cylinder's volume and radius to solve problems. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students developed strategies for determining the volume of a cylinder using the formulas $V = \pi r^2 h$ and $V = Bh$.

Future Learning

In Lesson 8, students will derive a formula for the surface area of cylinders.

Connections

- Studnents explore how changing a cylinder's radius or height impacts its volume in the context of real world examples such as different sizes of containers. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.2.3

Solve mathematical and real-world problems involving volume of right circular cylinders.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.3	**MA.912.GR.4.5**

Lesson at a Glance

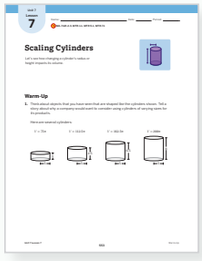
~ 45 min

MA.7.GR.2.3

Warm-Up

Independent | 5 min

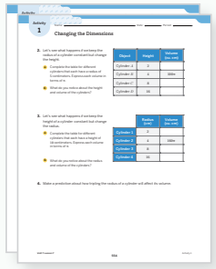
Students explore the possible reasons why and how different-sized cylinders with varying dimensions and volumes might be used.



Activity 1

Pairs | 10 min

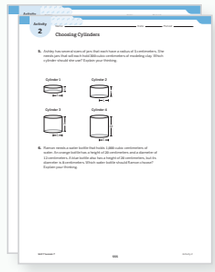
Students explore how doubling the radius or doubling the height of a cylinder affects its volume. **Routine:** Think-Pair-Share **(MTR.5.1)**



Activity 2

Pairs | 20 min

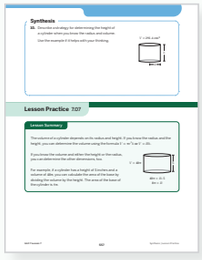
Students determine the best choice for a cylinder to use in different real-world scenarios. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

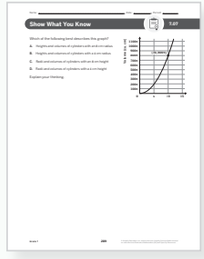
Students synthesize their understanding of the relationships among radius, height, and volume of a cylinder. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing the graph of the relationship between two dimensions of a three-dimensional solid.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students explore the possible reasons why and how different-sized cylinders with varying dimensions and volumes might be used.

1 Launch

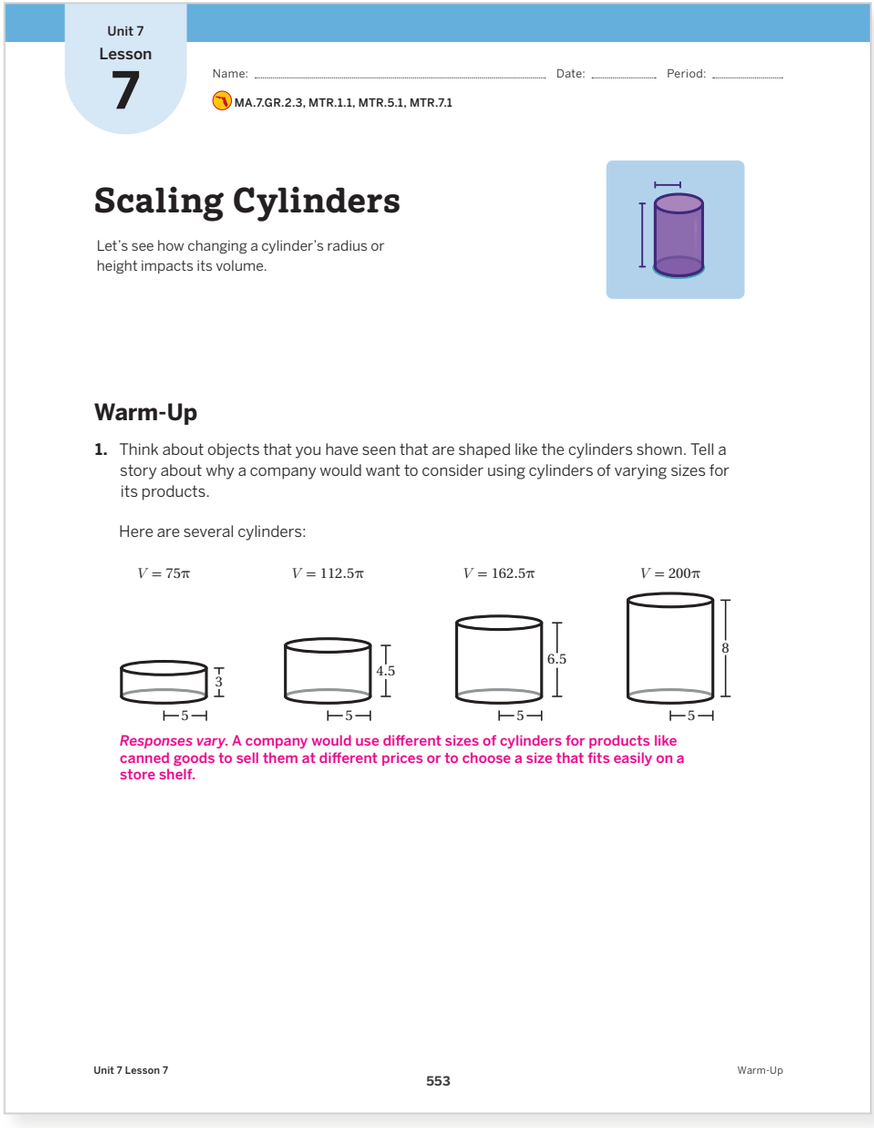
To generate excitement, invite students to share where they have seen cylinders in real life. Consider showing students examples of different-sized cans or jars.

Draw students' attention to the four cylinders. Consider asking, "What is the same in these cylinders? What is different?"

A **Accessibility: Memory and Attention** Invite students to refer back to the anchor chart created previously on cylinder volume to activate prior knowledge.

2 Connect

Invite students to share their reasoning about why a company might use various sizes for its packaging.



Unit 7
Lesson
7

Name: ..... Date: ..... Period: .....

MA.7.GR.2.3, MTR.1.1, MTR.5.1, MTR.7.1

Scaling Cylinders

Let's see how changing a cylinder's radius or height impacts its volume.

Warm-Up

1. Think about objects that you have seen that are shaped like the cylinders shown. Tell a story about why a company would want to consider using cylinders of varying sizes for its products.

   Here are several cylinders:

Responses vary. A company would use different sizes of cylinders for products like canned goods to sell them at different prices or to choose a size that fits easily on a store shelf.

Unit 7 Lesson 7 553 Warm-Up

Pairs | 10 min

Activity 1 Changing the Dimensions

Purpose: Students explore how doubling the radius or doubling the height of a cylinder affects its volume. **(MTR.5.1)**

Short on time? Consider omitting either Problem 4.

1 Launch

Direct students' attention to the tables.

To surface connections, consider asking:

- "What is changing in each cylinder in the first table? What is staying the same?"
- "What is changing in each cylinder in the second table? What is staying the same?"

2 Monitor

Encourage students to reflect on the type of relationship they notice between volume and height and volume and radius from the tables. **(MTR.5.1)**

Select and sequence ideas to share during the Connect.

D Differentiation *(Problems 2–3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to analyze the given information about each cylinder and relate those known measures to the formula for volume of a cylinder. Refer to the classroom display of the volume formula.
Complete the table by calculating the volume for cylinders A and C. *(Problem 2)*	**Strengthen** Invite them to reflect on the type of pattern they notice between the height and volume. Consider asking, "How can we use this pattern to determine the volume for cylinder D?"
Determine the volumes of all four cylinders. *(Problem 3)*	**Stretch** Consider asking, "Why does doubling the radius have a greater effect on increasing the volume than doubling the height?"

3 Connect

Invite students to share their responses about how doubling the radius or doubling the height affects the volume of a cylinder.

Invite students to check their predictions by completing a table similar to the table in Problem 3.

To surface connections, consider asking, "How does the volume formula for a cylinder help you to explain this relationship?" **(MTR.5.1)**

To surface the Key Takeaway, consider asking:

- "What measures affect the volume of a cylinder?"
- "As height doubles, what happens to volume? Why?"
- "As radius doubles, what happens to volume? Why?"

Activity 1

Name: Date: Period:

Changing the Dimensions

2. Let's see what happens if we keep the radius of a cylinder constant but change the height.

a. Complete the table for different cylinders that each have a radius of 5 centimeters. Express each volume in terms of π.

b. What do you notice about the height and volume of the cylinders?

Object	Height	Volume (cu. cm)
Cylinder A	2	50π
Cylinder B	4	100π
Cylinder C	8	200π
Cylinder D	16	400π

Responses vary. I notice that when the the height doubles, the volume also doubles.

3. Let's see what happens if we keep the height of a cylinder constant but change the radius.

a. Complete the table for different cylinders that each have a height of 10 centimeters. Express each volume in terms of π.

b. What do you notice about the radius and volume of the cylinders?

	Radius (cm)	Volume (cu. cm)
Cylinder 1	2	40π
Cylinder 2	4	160π
Cylinder 3	8	640π
Cylinder 4	16	$2{,}560\pi$

Responses vary. I notice that when the the radius doubles, the volume quadruples, or is four times the original volume.

4. Make a prediction about how tripling the radius of a cylinder will affect its volume.

Responses vary. When you triple the radius of a cylinder, the volume will be 9 times the original volume because $3 \cdot 3 = 9$.

Unit 7 Lesson 7 554 Activity 1

Key Takeaway: When you change one dimension of a cylinder, the volume changes in a predictable way.

Pairs | 20 min

Activity 2 Choosing Cylinders

Purpose: Students determine the best choice for a cylinder to use in different real-world scenarios. **(MTR.7.1)**

1 Launch

Direct students' attention to the four cylinders. Consider asking, "How are the cylinders related?"

To surface connections, consider asking, "What feels similar to the previous activity?"

Consider sharing that the diameter, not the radius, is given in the problem.

2 Monitor

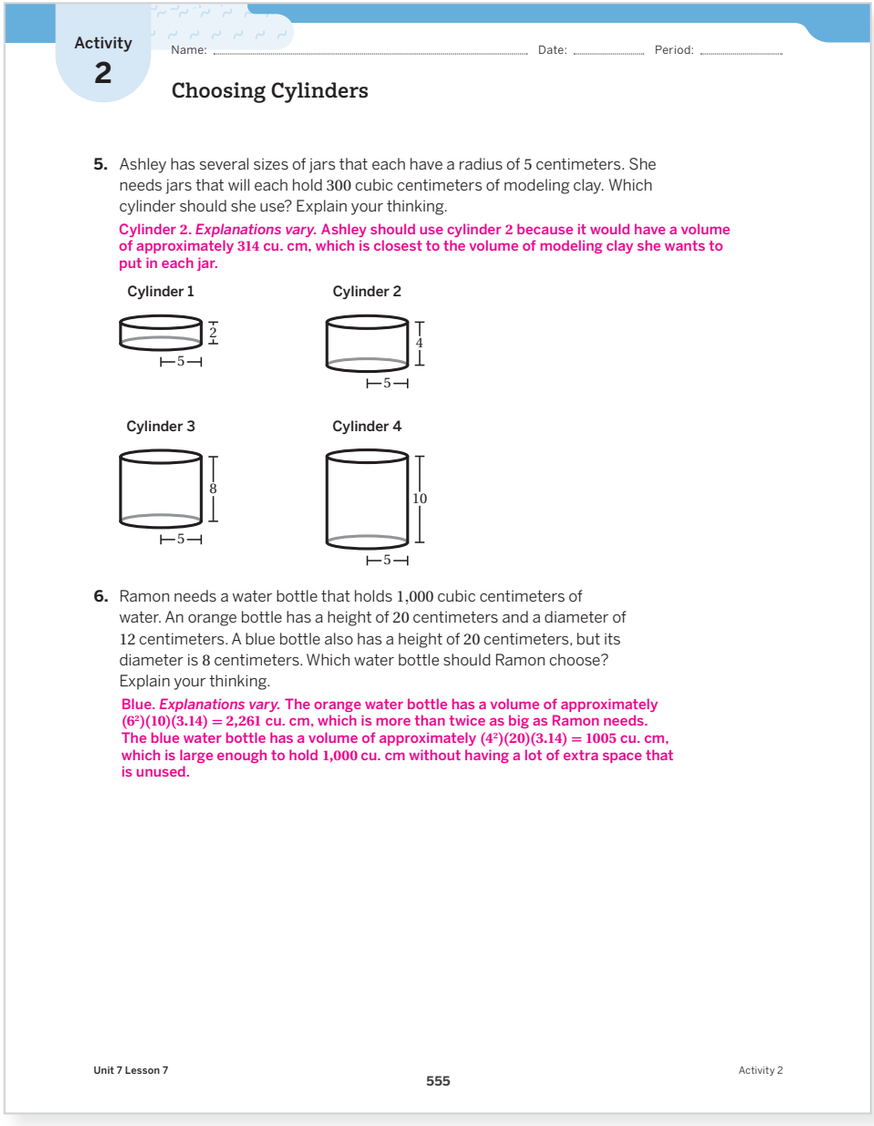
Listen for student thinking to share during the Connect.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Consider asking:

- "How did you determine the best cylinder to meet the needs of the problem?"
- "Why aren't the other containers appropriate choices?"
- "Why would a cylinder with a volume that is too large not be a good choice?" **(MTR.7.1)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Choosing Cylinders

5. Ashley has several sizes of jars that each have a radius of 5 centimeters. She needs jars that will each hold 300 cubic centimeters of modeling clay. Which cylinder should she use? Explain your thinking.

Cylinder 2. *Explanations vary.* Ashley should use cylinder 2 because it would have a volume of approximately 314 cu. cm, which is closest to the volume of modeling clay she wants to put in each jar.

Cylinder 1

Cylinder 2

Cylinder 3

Cylinder 4

6. Ramon needs a water bottle that holds 1,000 cubic centimeters of water. An orange bottle has a height of 20 centimeters and a diameter of 12 centimeters. A blue bottle also has a height of 20 centimeters, but its diameter is 8 centimeters. Which water bottle should Ramon choose? Explain your thinking.

Blue. *Explanations vary.* The orange water bottle has a volume of approximately $(6^2)(10)(3.14) = 2{,}261$ cu. cm, which is more than twice as big as Ramon needs. The blue water bottle has a volume of approximately $(4^2)(20)(3.14) = 1005$ cu. cm, which is large enough to hold 1,000 cu. cm without having a lot of extra space that is unused.

Unit 7 Lesson 7 555 Activity 2

Pairs | 20 min

Activity 2 Choosing Cylinders (continued)

Purpose: Students determine the best choice for a cylinder to use in different real-world scenarios. **(MTR.7.1)**

2 Monitor

Encourage students to individually calculate an approximate volume using 3.14 for π for each of the given cylinders.

Use the Think-Pair-Share routine to gain insight as to students' mathematical discourse and understanding as you monitor their conversations. Encourage students to reflect and revise their work as needed. **(MTR.1.1)**

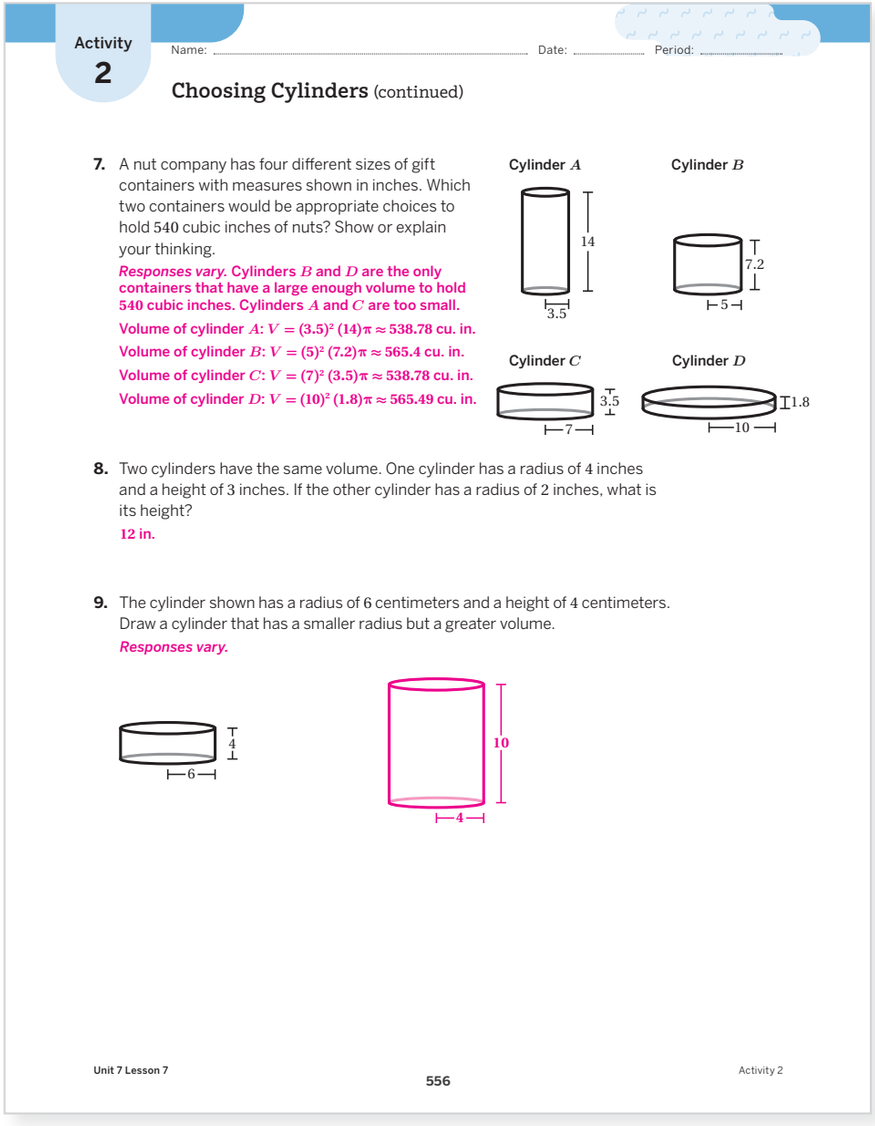
D **Differentiation:** Support For students that need support getting started, consider asking them how they can modify the formula for volume and use substitution of the known values to determine the height.

3 Connect

Display the distribution of responses by polling the class. Invite students to share their responses that support calculating precise volumes and determining which cylinders would be appropriate for the scenario.

Invite students to share the dimensions for their drawings. Check that the cylinders have a greater volume as a class.

To surface the Key Takeaway, consider asking, "How do you choose which cylinder is the better choice when given a selection of varying sizes to fit any storage scenario?" **(MTR.7.1)**



Activity 2

Name: Date: Period:

Choosing Cylinders (continued)

7. A nut company has four different sizes of gift containers with measures shown in inches. Which two containers would be appropriate choices to hold 540 cubic inches of nuts? Show or explain your thinking.

Responses vary. Cylinders B and D are the only containers that have a large enough volume to hold 540 cubic inches. Cylinders A and C are too small.

Volume of cylinder A: $V = (3.5)^2\,(14)\pi \approx 538.78$ cu. in.

Volume of cylinder B: $V = (5)^2\,(7.2)\pi \approx 565.4$ cu. in.

Volume of cylinder C: $V = (7)^2\,(3.5)\pi \approx 538.78$ cu. in.

Volume of cylinder D: $V = (10)^2\,(1.8)\pi \approx 565.49$ cu. in.

Cylinder A: 14, 3.5

Cylinder B: 7.2, 5

Cylinder C: 3.5, 7

Cylinder D: 1.8, 10

8. Two cylinders have the same volume. One cylinder has a radius of 4 inches and a height of 3 inches. If the other cylinder has a radius of 2 inches, what is its height?

12 in.

9. The cylinder shown has a radius of 6 centimeters and a height of 4 centimeters. Draw a cylinder that has a smaller radius but a greater volume.

Responses vary.

Unit 7 Lesson 7 556 Activity 2

Key Takeaway: When determining if a cylinder is fit for a real-world scenario, the cylinder must be close to but not less than the defined volume.

Whole Class | 5 min

Synthesis

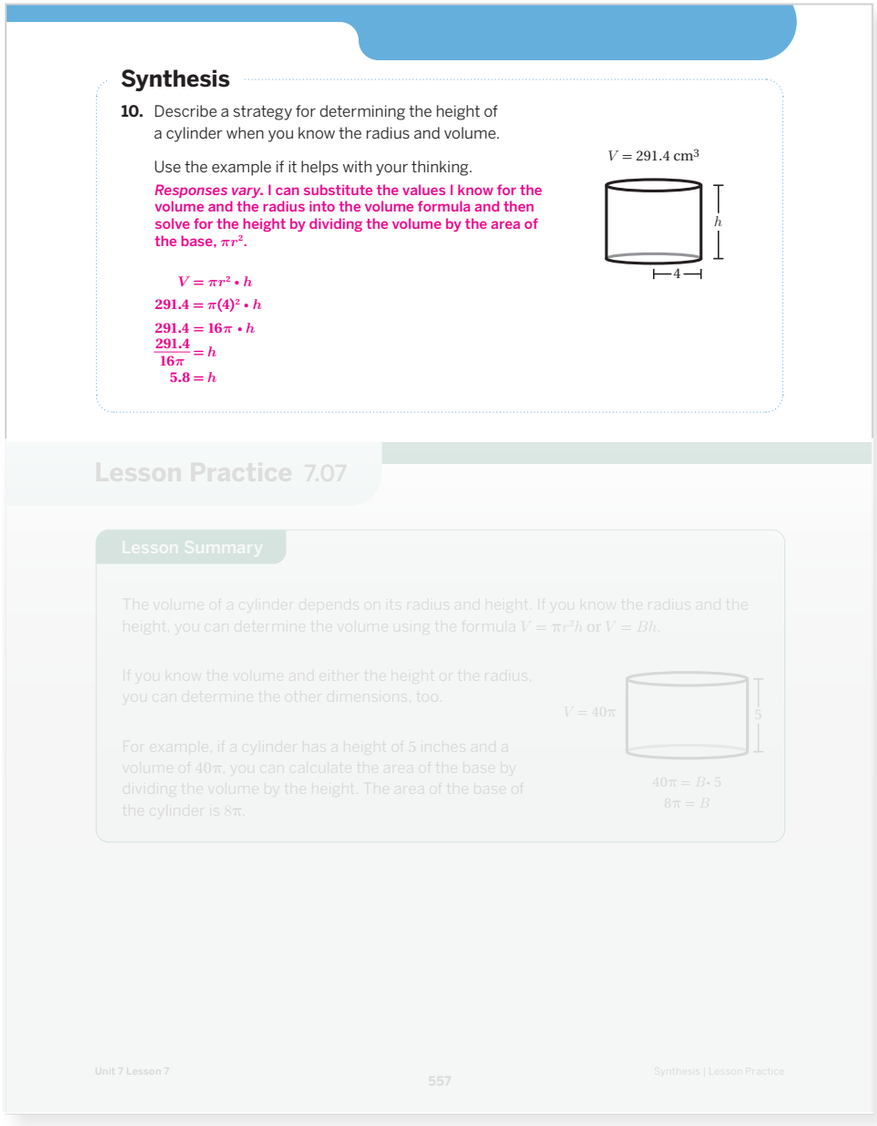
Purpose: Students synthesize their understanding of the relationships among radius, height, and volume of a cylinder.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Writing an equation and solving for the unknown.
- Dividing the volume by the area of the base to determine the height.

MLR **MLR2: Collect and Display** Use this routine to collect students' language as they share during the Connect and Synthesis. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe a strategy for determining the height of a cylinder when you know the radius and volume.

Use the example if it helps with your thinking.

Responses vary. I can substitute the values I know for the volume and the radius into the volume formula and then solve for the height by dividing the volume by the area of the base, πr^2.

$V = 291.4 \text{ cm}^3$

$V = \pi r^2 \cdot h$

$291.4 = \pi(4)^2 \cdot h$

$291.4 = 16\pi \cdot h$

$\frac{291.4}{16\pi} = h$

$5.8 = h$

Lesson Practice 7.07

Lesson Summary

The volume of a cylinder depends on its radius and height. If you know the radius and the height, you can determine the volume using the formula $V = \pi r^2 h$ or $V = Bh$.

If you know the volume and either the height or the radius, you can determine the other dimensions, too.

For example, if a cylinder has a height of 5 inches and a volume of 40π, you can calculate the area of the base by dividing the volume by the height. The area of the base of the cylinder is 8π.

$40\pi = B \cdot 5$

$8\pi = B$

Unit 7 Lesson 7 557 Synthesis | Lesson Practice

Lesson Takeaway: Volumes of cylinders change as the dimensions change. When determining a cylinder for a given scenario, the cylinder's volume should be close to but not less than the volume needed.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing the graph of the relationship between two dimensions of a three-dimensional solid.

Today's Goals

1. **Goal:** Analyze the relationship between the height or radius of a cylinder and its volume using tables.
2. **Goal:** Solve mathematical real-world problems involving volumes of cylinders.
3. **Language Goal:** Explain how changing the radius or height of a cylinder affects its volume and explain how to determine a missing dimension given the volume of a cylinder and its radius. **(Speaking and Listening)**

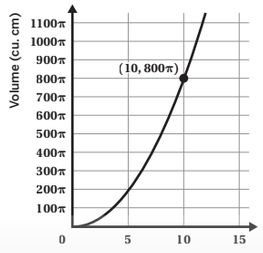
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.07

Which of the following best describes this graph?

A. Heights and volumes of cylinders with an 8 cm radius
B. Heights and volumes of cylinders with a 4 cm radius
C. Radii and volumes of cylinders with an 8 cm height
D. Radii and volumes of cylinders with a 4 cm height



Explain your thinking.

Explanations vary. The relationship between height and volume is linear, and this graph is not linear, so I knew it had to show the relationship between radius and volume. I noticed that the point $(10, 800\pi)$ was on the curve. That could represent a cylinder that has a radius of 10 and a height of 8, so I concluded that the graph was all cylinders with a height of 8 centimeters.

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of analyze relationships between dimensions and volume:

» **Respond to Student Thinking:**

- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students explored how changing dimensions affected the volume of a cylinder. What considerations did they make when choosing an appropriate container size? How did students' fluency and understanding improve as they used the formula for the volume of a cylinder to solve different types of problems?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe a strategy for determining the height of a cylinder when you know the radius and volume.

Use the example if it helps with your thinking.

Responses vary. I can substitute the values I know for the volume and the radius into the volume formula and then solve for the height by dividing the volume by the area of the base, πr^2.

$V = \pi r^2 \cdot h$

$291.4 = \pi(4)^2 \cdot h$

$291.4 = 16\pi \cdot h$

$\frac{291.4}{16\pi} = h$

$5.8 = h$

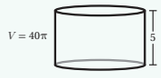
Lesson Practice 7.07

Lesson Summary

The volume of a cylinder depends on its radius and height. If you know the radius and the height, you can determine the volume using the formula $V = \pi r^2 h$ or $V = Bh$.

If you know the volume and either the height or the radius, you can determine the other dimensions, too.

For example, if a cylinder has a height of 5 inches and a volume of 40π, you can calculate the area of the base by dividing the volume by the height. The area of the base of the cylinder is 8π.



$40\pi = B \cdot 5$

$8\pi = B$

Lesson Practice 7.07

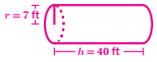
Name: _____ Date: _____ Period: _____

1. Each row of this table lists information about a specific cylinder. Complete the table.

Diameter (units)	Area of Base (sq. units)	Height (units)	Volume (cu. units)
4	4π	10	40π
6	9π	7	63π
10	25π	6	150π

Problems 2–3: A gas company's delivery truck has a cylindrical tank with a diameter of 14 feet and a height of 40 feet.

2. Draw the tank, then label its radius and height.



3. How much gas can fit in the tank? Show or explain your thinking.

$1960\pi \approx 6158$ cubic feet of gas. *Explanations vary.*

$V = \pi r^2 h$

If $r = 7$ and $h = 40$, then

$V = \pi \cdot 7^2 \cdot 40$

$V = 1960 \cdot \pi$

$V \approx 6157.52$

Problems 4–6: Imagine several cylinders that all have a height of 18 meters. Let r represent the radiuses of the cylinders, in meters, and V represent the volume of the cylinders, in cubic meters.

4. Write an equation that represents the relationship between the volume, V, and the radius, r, for all cylinders with a height of 18 meters.

$V = \pi r^2 \cdot 18$

5. Complete this table:

r (m)	1	2	3
V (cu. m)	18π	72π	162π

6. A village water treatment container must be purchased to hold at least 300 cubic meters of water. The village can choose from the three cylinders in Problem 5. Which one of these three cylinders is best suited to meet the village's needs? Explain your thinking.

The cylinder with a radius of 3 m. Explanations vary. The largest cylinder is the one with radius of 3 meters and height of 18 meters. The total volume of this tank would be 508.68 cubic meters. This is the only tank that has enough volume to meet the village's needs.

The tank with a radius of 2 meters and height of 18 meters only has a volume of about 226 cubic meters, so it is too small to meet the village's needs.

Lesson Practice 7.07

Name: _____ Date: _____ Period: _____

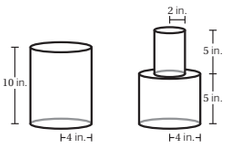
7. A cylinder has a volume of 48π cubic centimeters and a height represented by h.

Complete this table with the volumes of other cylinders that have the same radius but different heights.

Height (cm)	Volume (cu. cm)
h	48π
$2h$	96π
$5h$	240π
$\frac{h}{2}$	24π
$\frac{h}{5}$	$\frac{48\pi}{5}$

FAST Practice

8. A bakery offers two types of celebration cakes, as shown. Select *all* the true statements.



- ☐ A. Cake B has more volume than cake A.
- ☑ B. Cake B has a volume of about 314 cubic inches.
- ☑ C. If cake A costs $130, that is about $0.26 per cubic inch.
- ☐ D. Cake A has a volume of 438 cubic inches.
- ☑ E. If cake B costs $150, that is about $0.48 per cubic inch.
- ☐ F. The difference in the cakes' volumes is 63 cubic inches.

Spiral Review

9. The area of a circle is approximately 201.06 square inches. What is its radius in inches?

8 inches

10. Match each circle with its area.

- Circle A has a radius of 4 units.
- Circle B has a radius of 10 units.
- Circle C has a radius of 8 units.

Area of the Circle	Circle
About 314 square units	B
64π square units	C
16π square units	A

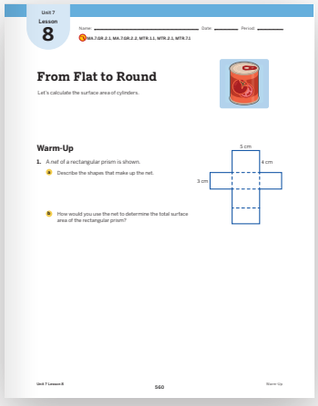
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3, 6–7	1	MA.7.GR.2.3
	2, 4–5	1	MA.7.GR.2.3
FAST Practice	8	2	MA.7.GR.2.3
Spiral Review			
	9	2	MA.7.GR.1.3
	10	2	MA.7.GR.1.4

Unit 7

Lesson

8



From Flat to Round

Surface Area of Cylinders

Let's calculate the surface area of cylinders.

Learning Outcomes

Today's Goals

1. **Goal:** Identify parts of the net of a cylinder.
2. **Goal:** Solve problems by calculating the *surface area* of a cylinder.
3. **Language Goal:** Describe the steps to take to determine the surface area of a cylinder using its net. **(Reading, Writing, Speaking, and Listening)**

Students develop strategies for calculating the **surface area** of a right circular cylinder. By understanding what the *net* of the cylinder looks like, they calculate surface area without having to rely on formulas. Students then solve real-world problems involving the surface area of a variety of different cylinders.

Coherence

Prior Learning

In Unit 3, students calculated the area of circles. In Lesson 6, students were introduced to the volume formula for a cylinder.

Future Learning

In Geometry, students will solve problems involving the surface area of cylinders, pyramids, prisms, cones, and spheres

Connections

- Students explore and calculate surface area using visuals of nets and applying it to real world situtations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.2.1

Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure's net.

MA.7.GR.2.2

Solve real-world problems involving surface area of right circular cylinders.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.4	**MA.912.GR.4.6**
MA.7.GR.1.4	

Vocabulary

New Vocabulary

surface area

Review Vocabulary

net

Lesson at a Glance

~ 45 min

MA.7.GR.2.1, MA.7.GR.2.2

Warm-Up

Independent | 5 min

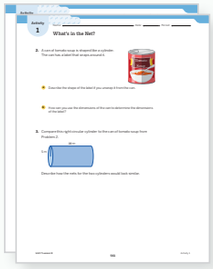
Students surface prior knowledge about *nets* and **surface area** of prisms.
Routine: Think-Pair-Share



Activity 1

Pairs | 15 min

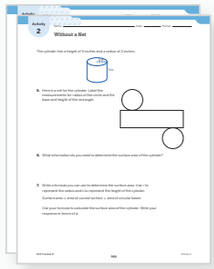
Students explore the nets of cylinders to support calculating surface area. **(MTR.2.1)**



Activity 2

Pairs | 15 min

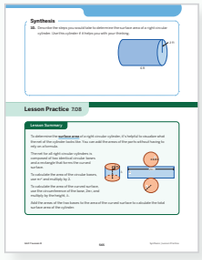
Students generalize a formula for calculating the surface area of a cylinder and solve mathematical and real-real-world problems involving the surface area of cylinders.
Routine: Decide and Defend



Synthesis

Whole Class | 5 min

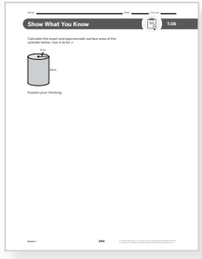
Students synthesize their understanding of how to calculate the surface area of a cylinder.



Show What You Know

Independent | 5 min

Students demonstrate their understanding of the surface area of a cylinder by using nets and applying formulas.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators (as needed)

Independent | 5 min

Warm-Up

Purpose: Students surface prior knowledge about *nets* and **surface area** of prisms.

1 Launch

To surface prior knowledge, consider asking, "What is the difference between a two-dimensional figure and a three-dimensional figure?"

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

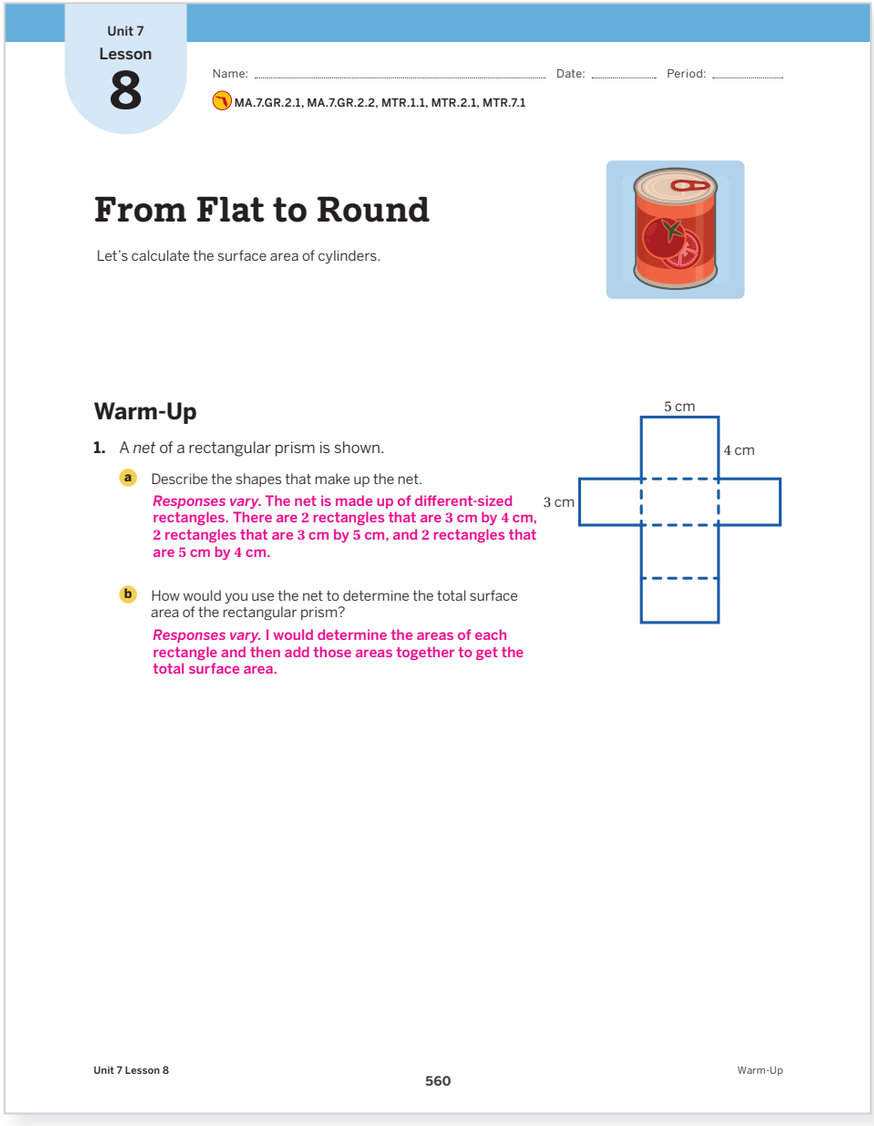
Consider asking, "How is a *net* similar to a composite figure?"

ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and inviting them to share in that language. **(Writing, Speaking, and Listening)**

2 Connect

Invite students to share their thinking on how they would determine the surface area of the the figure, such as:

- Using a generalized formula.
- Calculating the areas of each rectangle in the net and adding them together.



Unit 7
Lesson
8

Name: ..... Date: ..... Period: .....

MA.7.GR.2.1, MA.7.GR.2.2, MTR.1.1, MTR.2.1, MTR.7.1

From Flat to Round

Let's calculate the surface area of cylinders.

Warm-Up

1. A *net* of a rectangular prism is shown.

   a. Describe the shapes that make up the net.

   Responses vary. The net is made up of different-sized rectangles. There are 2 rectangles that are 3 cm by 4 cm, 2 rectangles that are 3 cm by 5 cm, and 2 rectangles that are 5 cm by 4 cm.

   b. How would you use the net to determine the total surface area of the rectangular prism?

   Responses vary. I would determine the areas of each rectangle and then add those areas together to get the total surface area.

Unit 7 Lesson 8 560 Warm-Up

Pairs | 15 min

Activity 1 What's in the Net?

Purpose: Students explore the nets of cylinders to support calculating surface area. **(MTR.2.1)**

Short on time? Consider completing Problem 4 as a class.

1 Launch

To surface connections, consider showing students a can of soup and unwrapping the label in real time. Encourage students to make a prediction about the shape of the label. **(MTR.7.1)**

Consider sharing that the cylinders in this lesson are called right circular cylinders.

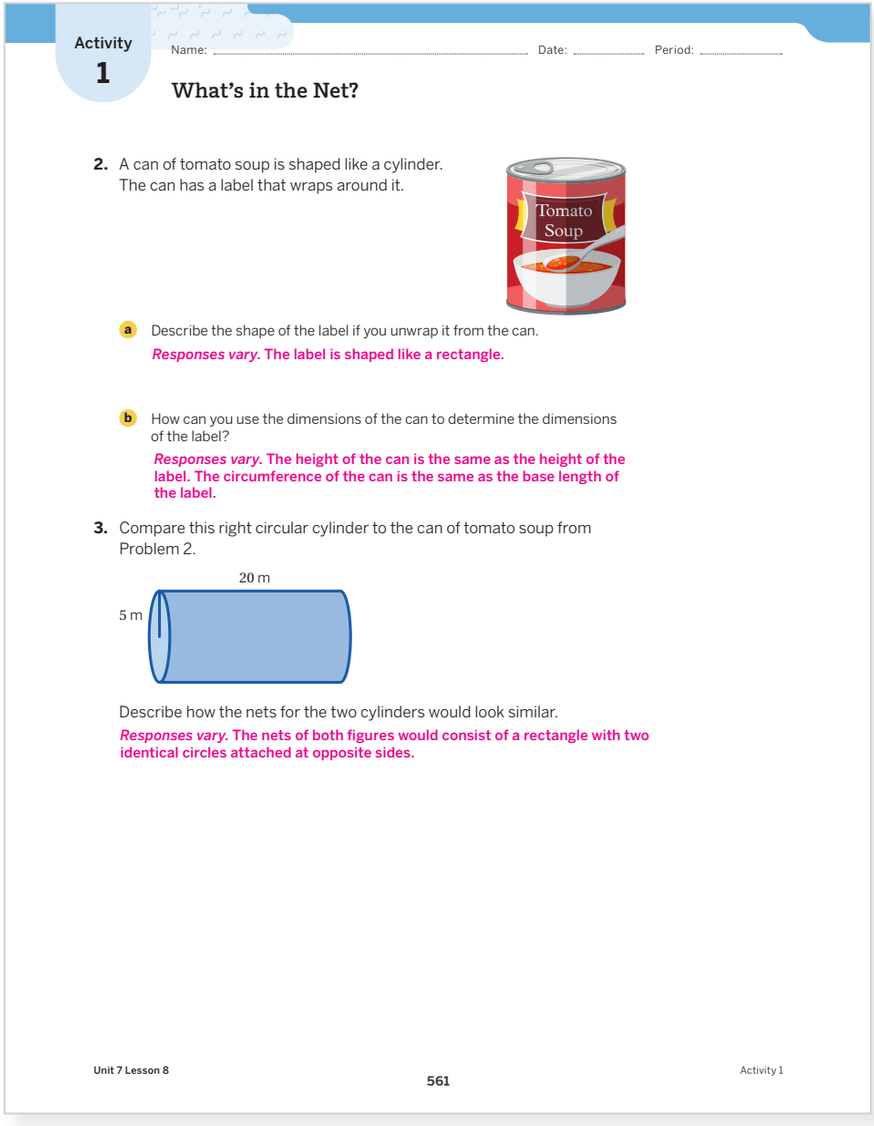
2 Monitor

Look for students who make the connection between the base length of the rectangle and the circumference of the can. **(MTR.2.1)**

Listen for student ideas to highlight during the Connect, such as:

- Understanding that the nets for both cylinders would be composed of two circles and a rectangle, regardless of different orientation or size.
- Describing the two circular bases of a cylinder as identical.
- Recognizing that the base of a cylinder does not always lie flat on a surface

Activity 1 continued >



Activity 1

Name: Date: Period:

What's in the Net?

2. A can of tomato soup is shaped like a cylinder. The can has a label that wraps around it.

a. Describe the shape of the label if you unwrap it from the can.

Responses vary. The label is shaped like a rectangle.

b. How can you use the dimensions of the can to determine the dimensions of the label?

Responses vary. The height of the can is the same as the height of the label. The circumference of the can is the same as the base length of the label.

3. Compare this right circular cylinder to the can of tomato soup from Problem 2.

Describe how the nets for the two cylinders would look similar.

Responses vary. The nets of both figures would consist of a rectangle with two identical circles attached at opposite sides.

Unit 7 Lesson 8 561 Activity 1

Pairs | 15 min

Activity 1 What's in the Net? (continued)

Purpose: Students explore the nets of cylinders to support calculating surface area. **(MTR.2.1)**

Short on time? Consider completing Problem 4 as a class.

2 Monitor

Consider sharing what it means when you determine an exact surface area.

Consider asking:

- "What does it mean when you write your answer in terms of π?"
- "When you are determining an approximate answer, what value do you use for π?"

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to identify the measurements that are the same in the three-dimensional figure and the net.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to analyze the first row of the table that is already completed. Consider asking, "Are the two circles from the net identical? What does this tell you about their areas?"
Enter answers without any units.	**Support** Consider asking, "What units do you use when you calculate an area?"
Correctly determine the total surface area.	**Stretch** Invite students to think about a formula that would generalize calculating the surface area for all right circular cylinders.

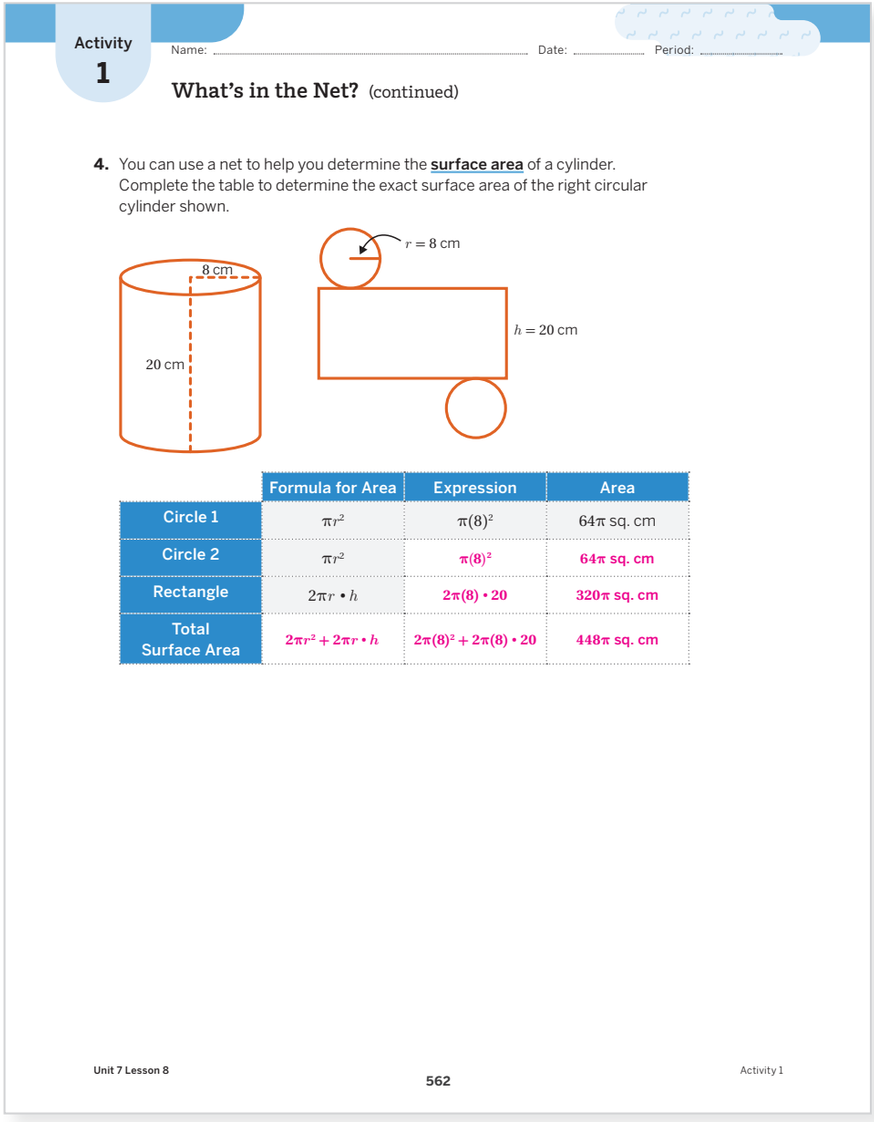
3 Connect

Invite students to share the connections they see between the three-dimensional drawing and the net of the cylinder.

Create a class definition of **surface area** based on student language. Record this definition for students to refer to throughout the lesson.

Create an anchor chart that displays the formulas for the area of a circle and the circumference of a circle so that students can revisit this information later in the lesson.

To surface the Key Takeaway, consider asking, "What does the net of a cylinder look like?"



Activity 1

Name: Date: Period:

What's in the Net? (continued)

4. You can use a net to help you determine the **surface area** of a cylinder. Complete the table to determine the exact surface area of the right circular cylinder shown.

	Formula for Area	Expression	Area
Circle 1	πr^2	$\pi(8)^2$	64π sq. cm
Circle 2	πr^2	$\pi(8)^2$	64π sq. cm
Rectangle	$2\pi r \cdot h$	$2\pi(8) \cdot 20$	320π sq. cm
Total Surface Area	$2\pi r^2 + 2\pi r \cdot h$	$2\pi(8)^2 + 2\pi(8) \cdot 20$	448π sq. cm

Unit 7 Lesson 8 562 Activity 1

Key Takeaway: The net of a cylinder consists of two identical circles and a rectangle.

Pairs | 15 min

Activity 2 Without a Net

Purpose: Students generalize a formula for calculating the surface area of a cylinder and solve mathematical and real-world problems involving the surface area of cylinders.

1 Launch

Encourage students to work independently first before discussing their thinking with a partner.

Consider asking, "What dimension do you need to calculate to label the net?"

2 Monitor

Look for students who:

- Identify the radius and the height correctly from the net.
- Multiply the area of one circle by 2 since the circles are identical.
- Use 3.14 when they are approximating their answers.
- Remember to include the correct units, sq in.

A **Accessibility: Executive Functioning** Invite students to review the anchor chart for the formulas for area and circumference of a circle.

Select and sequence a variety of correct and incorrect formulas to share during the Connect.

Encourage students to check their formula by adding the individual areas from the net and comparing the result to the surface area they calculated using their formula.

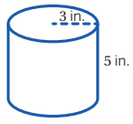
Activity 2 continued >

Activity 2

Name: Date: Period:

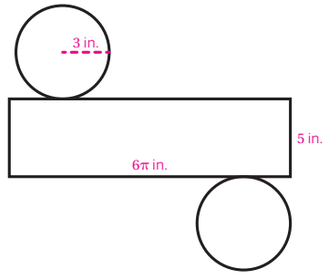
Without a Net

The cylinder has a height of 5 inches and a radius of 3 inches.



5. Here is a net for the cylinder. Label the measurements for radius of the circle and the base and height of the rectangle.

Responses shown on net.



6. What information do you need to determine the surface area of the cylinder?

Responses vary. I need to know the radius so I can calculate the areas of the circular bases.

I need to know the circumference of the circular base and height of the cylinder to calculate the area of the rectangle.

7. Write a formula you can use to determine the surface area. Use r to represent the radius and h to represent the height of the cylinder.

Surface area = area of curved surface + area of circular bases

$SA = 2\pi rh + 2\pi r^2$

Use your formula to calculate the surface area of the cylinder. Write your response in terms of π.

$SA = 2\pi(3)(5) + 2\pi(3)^2$

$SA = 30\pi + 18\pi$

$SA = 48\pi$ square inches

Unit 7 Lesson 8 563 Activity 2

Pairs | 15 min

Activity 2 Without a Net (continued)

Purpose: Students generalize a formula for calculating the surface area of a cylinder and solve mathematical and real-world problems involving the surface area of cylinders.

2 Monitor

Invite students to discuss their thinking with a partner.

Listen for student strategies and partner discussion, offering help or encouragement where needed. To support students getting started, consider asking:

- "What would the net of this cylinder look like?"
- "What do you need to calculate to solve the problem?" **(MTR.1.1)**

3 Connect

Invite students to share their formulas, calling attention to any conflict or consensus.

Consider asking, "What calculations produce square units in the formula for surface area of a cylinder?"

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:

- "What part of the expressions for surface area is the same for the two cylinders?"
- "What part of the expressions is different?"
- "Can you determine which cylinder has a greater surface area without doing calculations?" **(MTR.1.1)**

Invite students to share their strategies for solving the problem with the class.

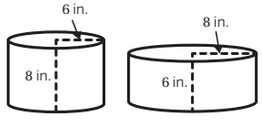
To surface the Key Takeaway, consider asking, "Can you use the net for all right circular cylinders to determine its surface area?"

Activity 2

Name: Date: Period:

Without a Net (continued)

8. Arjun says that the two cylinders have the same surface area.



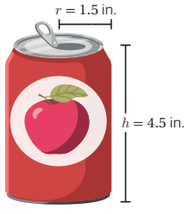
Is Arjun correct? Explain your thinking.

No. *Explanations vary.* The nets of the cylinders are similar. Each net would be composed of two identical circles and one rectangle. However, Arjun is not correct because you cannot interchange the values of the radius and height.

The surface area of the cylinder on the left is $2\pi(6)(8) + 2\pi(6)^2$, which gives 168π square inches.

The surface area of the cylinder on the right is $2\pi(8)(6) + 2\pi(8)^2$, which gives 224π square inches.

9. A can of apple juice like the one shown is cut from a large sheet of metal. Determine the amount of metal needed to make a similar can. Show your thinking. Use. 3.14 for π and round to the nearest tenth.



I need to calculate the total surface area to determine the amount of metal that is needed.

First, I can determine the combined area of the circles on the top and bottom of the can: $(3.14)(1.5)^2 \cdot 2 = 14.1$ square inches.

Then, I can determine the area of the of rectangle: $2(3.14)(1.5) \cdot 4.5 = 42.4$ square inches.

The amount of metal needed for another can is about 56.5 square inches.

Unit 7 Lesson 8 564 Activity 2

Key Takeaway: To determine the total surface area of a cylinder, add the areas of the circular bases, $2\pi r^2$, to the area of the curved surface, $2\pi r \cdot h$.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to calculate the surface area of a cylinder.

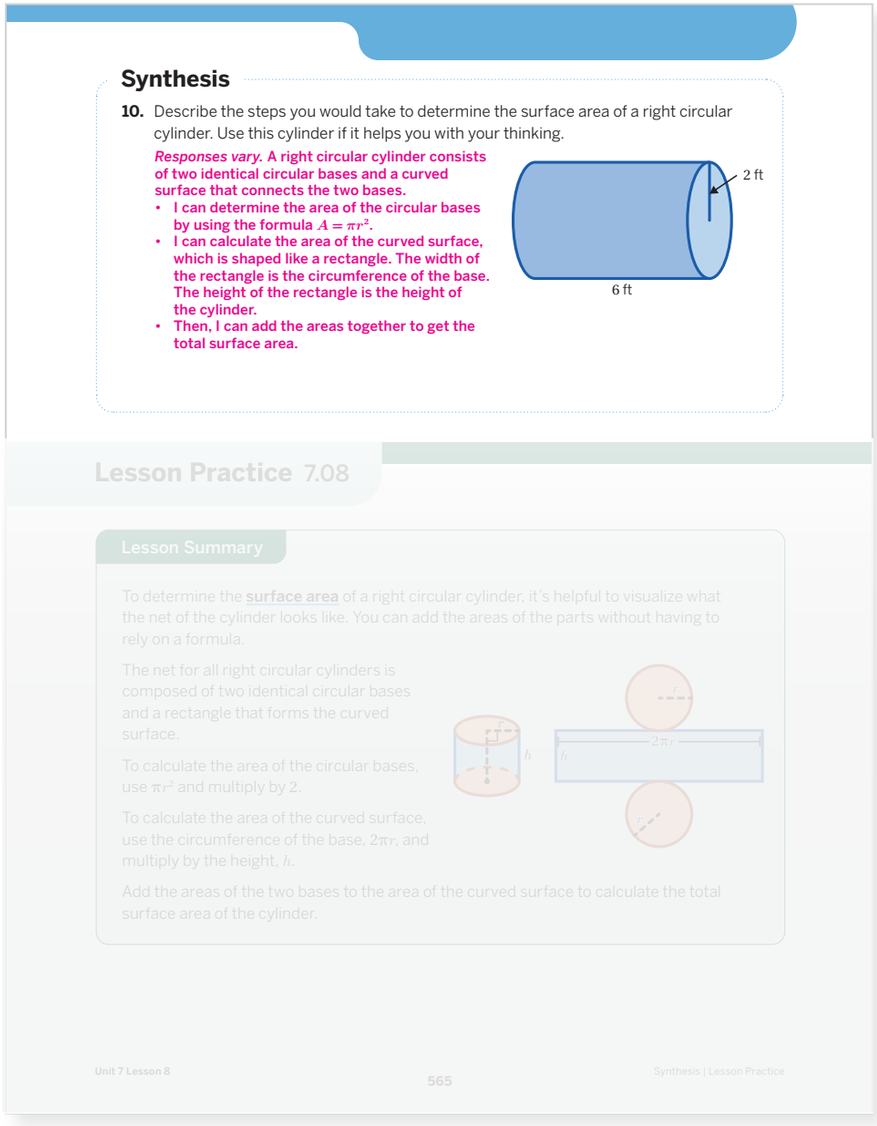
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- The net consists of two identical circular bases and a rectangle.
- Use the formulas for the area of a circle and circumference to help calculate the surface area.
- Add the areas of each part of the net to get the total surface area.

Consider adding information during the Synthesis to the class anchor chart.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe the steps you would take to determine the surface area of a right circular cylinder. Use this cylinder if it helps you with your thinking.

Responses vary. A right circular cylinder consists of two identical circular bases and a curved surface that connects the two bases.

- I can determine the area of the circular bases by using the formula $A = \pi r^2$.
- I can calculate the area of the curved surface, which is shaped like a rectangle. The width of the rectangle is the circumference of the base. The height of the rectangle is the height of the cylinder.
- Then, I can add the areas together to get the total surface area.

Lesson Practice 7.08

Lesson Summary

To determine the surface area of a right circular cylinder, it's helpful to visualize what the net of the cylinder looks like. You can add the areas of the parts without having to rely on a formula.

The net for all right circular cylinders is composed of two identical circular bases and a rectangle that forms the curved surface.

To calculate the area of the circular bases, use πr^2 and multiply by 2.

To calculate the area of the curved surface, use the circumference of the base, $2\pi r$, and multiply by the height, h.

Add the areas of the two bases to the area of the curved surface to calculate the total surface area of the cylinder.

Unit 7 Lesson 8 565 Synthesis | Lesson Practice

Lesson Takeaway: The surface area of a cylinder is the area of the curved surface plus the area of the two circular bases. This is represented by the formula $SA = 2\pi rh + 2\pi r^2$.

Independent | 5 min

Show What You Know

Purpose: Students their understanding of the surface area of a cylinder by using nets and applying formulas.

Today's Goals

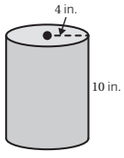
1. **Goal:** Identify parts of the net of a cylinder.
2. **Goal:** Solve problems by calculating the *surface area* of a cylinder.
3. **Language Goal:** Describe the steps to take to determine the surface area of a cylinder using its net. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.08

Calculate the exact and approximate surface area of the cylinder below. Use 3.14 for π.



Explain your thinking.

Exactly 160π square inches or approximately 502.4 square inches.
I used the formula $SA = 2\pi rh + 2\pi r^2 = 2\pi(4)(10) + 2\pi(4)^2 = 112\pi \approx 351.9$ sq.in.

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the surface area of the cylinder:

» **Respond to Student Thinking:**

- Revisit the table in Activity 1 and invite students to discuss the steps to fill in each column of the table.
- Select a few related Practice Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on your facilitation of the Think-Pair-Share routine in this lesson. How did this routine promote student discourse and build a conceptual understanding of surface area?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe the steps you would take to determine the surface area of a right circular cylinder. Use this cylinder if it helps you with your thinking.

Responses vary. A right circular cylinder consists of two identical circular bases and a curved surface that connects the two bases.

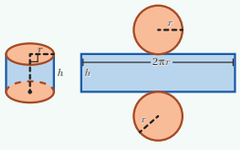
- I can determine the area of the circular bases by using the formula $A = \pi r^2$.
- I can calculate the area of the curved surface, which is shaped like a rectangle. The width of the rectangle is the circumference of the base. The height of the rectangle is the height of the cylinder.
- Then, I can add the areas together to get the total surface area.

Lesson Practice 7.08

Lesson Summary

To determine the **surface area** of a right circular cylinder, it's helpful to visualize what the net of the cylinder looks like. You can add the areas of the parts without having to rely on a formula.

The net for all right circular cylinders is composed of two identical circular bases and a rectangle that forms the curved surface.



To calculate the area of the circular bases, use πr^2 and multiply by 2.

To calculate the area of the curved surface, use the circumference of the base, $2\pi r$, and multiply by the height, h.

Add the areas of the two bases to the area of the curved surface to calculate the total surface area of the cylinder.

Lesson Practice 7.08

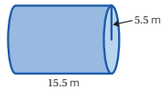
Name: _____ Date: _____ Period: _____

Problems 1–3: Determine if each statement is true or false.

1. The net of a right circular cylinder is composed of two circular bases and one rectangle.
   True
2. The two circular bases in the net of a cylinder have different areas.
   False
3. The area of the curved surface of a cylinder can be calculated by multiplying the circumference of the base by the height of the cylinder.
   True

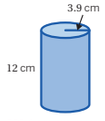
Problems 4–5: Calculate the surface area of each cylinder. Use 3.14 for π. Round to the nearest tenth.

4.



725.3 square meters

5.



389.4 square meters

6. A city used a large cylindrical water tank to store water. The tank has a radius of 10 meters and a height of 20 meters. The city painted the outside of the tank, including the top, bottom, and sides. Determine the exact area that was painted.
600π square meters

Lesson Practice 7.08

Name: _____ Date: _____ Period: _____

7. A baker made a round cake in the shape of a cylinder with a radius of 6 inches and a height of 3 inches. The baker covered the entire outside of the cake and the top of the cake with frosting. How much frosting did the baker use to cover these areas? Use 3.14 for π.
226.08 square inches

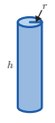
FAST Practice

8. The surface area of a cylinder is 154π sq. cm. The radius of the cylinder is 7 cm. What is the height of the cylinder?

A. 2 cm
B. 4 cm
C. 6 cm
D. 7 cm

Spiral Review

9. The cylinder shown has a height of $10\frac{1}{2}$ inches and a radius measuring $1\frac{1}{2}$ inches. What is the volume of the cylinder in cubic inches? Use 3.14 for π. Round to the nearest tenth.
74.2 cubic inches



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.GR.2.1
	4–5	2	MA.7.GR.2.1
	6–7	2	MA.7.GR.2.2
FAST Practice	8	2	MA.7.GR.2.1
Spiral Review			
	9	2	MA.7.GR.2.3

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining the area of parallelogram, the length of a base, or a height give the other two measurements	Lesson 1	1	MA.7.GR.1.1
2a	Determining the area of composite figures by decomposing quadrilaterals	Lesson 3	3	MA.7.GR.1.2
2b	Determining the area of complex shapes	Lesson 4	2	MA.7.GR.1.4
3a	Calculating the circumference of a circle given the diameter	Lesson 5	1	MA.7.GR.1.3
3b	Calculating the area of a circle given the diameter	Lesson 5	1	MA.7.GR.1.4
4a	Calculating cylindrical volume given the diameter and the height	Lesson 6	2	MA.7.GR.2.3
4b	Solving real-world problems involving volume of cylinders	Lesson 6	2	MA.7.GR.2.3
5	Determining the height of a cylinder given the radius and volume	Lesson 6	2	MA.7.GR.2.3
6	Calculating surface area of cylinders	Lesson 8	2	MA.7.GR.2.2

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

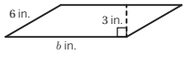
Name: Date: Period:

End-of-Unit Assessment

Unit 7

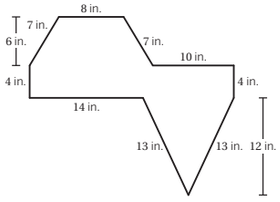
1. What is the length of the base for the given height Area: 24 square inches of this parallelogram?

A. 4 inches **C.** 8 inches **B.** 6 inches **D.** 12 inches



2. Calculate the area of the composite figures below.

a



$A_{trap} = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(8 + 14)6 = \frac{1}{2}(22)(6) = 66 \text{ in}^2$

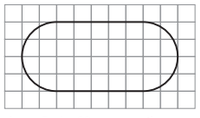
$A_{rect1} = bh = 14 \cdot 4 = 56 \text{ in}^2$

$A_{rect2} = bh = 10 \cdot 4 = 40 \text{ in}^2$

$A_{tri} = \frac{1}{2}(bh) = \frac{1}{2}(10)(12) = 60 \text{ in}^2$

$A_{total} = 66 + 56 + 40 + 60 = 222 \text{ in}^2$

b Each square has an area of 1 unit. (Use $\pi = 3.14$)



$A_{rect} = 5 \cdot 4 = 20$ square units

$A_{circle} = \pi r^2 = (3.14)(2^2) = (3.14)(4) = 12.56$ square units*

$A_{total} = 20 + 12.56 = 32.56$ square units

**2 half circles = 1 full circle*

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 7

3. A circular fountain in a park has a diameter of 10 meters. The park manager wants to install a decorative tile border around the fountain's edge.

a What is the circumference of the fountain? (Use $\pi = 3.14$.)

$C = \pi d = (3.14)(10) = 31.4$ meters

b If the manager also wants to cover the entire base of the fountain with blue tiles, what is the area of the base of the fountain?

$A = \pi r^2 = (3.14)(5)^2 = (3.14)(25) = 78.5$ square meters

4. A juice company packages its drinks in cylindrical cans. Each can has a diameter of 8 cm and a height of 12 cm.

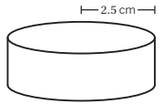
a Find the volume of one juice can. (Use $\pi = 3.14$.)

$V = \pi r^2 h = (3.14)(4)^2(12) = (3.14)(16)(12) = 602.88 \text{ cm}^3$

b If the factory produces 500 cans in one day, what is the total volume of juice produced?

$V_{total} = (500)(602.88) = 301{,}440 \text{ cm}^3$

5. A cylinder has a radius of 2.5 centimeters. Its volume is 37.5π cubic centimeters. What is the height of the cylinder?



$V = \pi r^2 h$

$37.5\pi = \pi(2.5)^2 h$

$\frac{37.5\pi}{6.25\pi} = \frac{\pi(6.25)h}{6.25\pi}$

$h = 6$ cm

$V = 37.5\pi \text{ cm}^3$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 7

6. Your neighbor owns a rain barrel with a radius of 14 inches and a height of 39 inches. He paints the entire outside and top of the barrel a different color. How much paint did your neighbor use to cover these areas? (Use $\pi = 3.14$.)

$SA = 2\pi rh + \pi r^{2*}$

$SA = 2(3.14)(14)(39) + (3.14)(14^2)$

$SA = 3{,}428.88 + 615.44$

$SA = 4{,}044.32 \text{ in}^3$

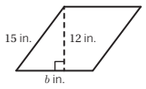
**only πr^2 instead of $2\pi r^2$ since the bottom is not painted*

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 7

1. What is the length of the base for the given height Area: 108 square inches of this parallelogram?



A. 7.2 inches

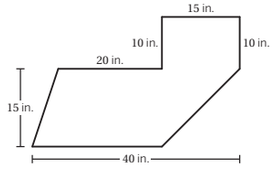
C. 9 inches

B. 8 inches

D. 15 inches

2. Calculate the area of the composite figures below.

a



$A_{tri1} = \frac{1}{2}bh = \frac{1}{2}(5)(15) = 37.5 \text{ in}^2$

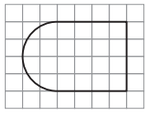
$A_{tri2} = \frac{1}{2}bh = \frac{1}{2}(15)(15) = 112.5 \text{ in}^2$

$A_{rect1} = bh = 20 \cdot 15 = 300 \text{ in}^2$

$A_{rect2} = bh = 10 \cdot 15 = 150 \text{ in}^2$

$A_{total} = 37.5 + 112.5 + 300 + 150 = 600 \text{ in}^2$

b Each square has an area of 1 unit. (Use $\pi = 3.14$.)



$A_{square} = 4 \cdot 4 = 16$ square units

$A_{circle} = \frac{1}{2}\pi r^2 = \frac{1}{2}(3.14)(2^2) = \frac{1}{2}(3.14)(4) = 6.28$ square units*

$A_{total} = 16 + 6.28 = 22.28$ square units

**image only has 1 half of a circle*

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Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 7

3. A gardener plans to create a circular flower bed with a diameter of 12 meters in a public park. She will place a stone border around the edge of the flower bed.

a What is the circumference of the flower bed? (Use $\pi = 3.14$.)

$C = \pi d = (3.14)(12) = 37.68$ meters

b If the gardener also wants to cover the entire base of the flower bed with mulch, what is the area of the base of the flower bed?

$A = \pi r^2 = (3.14)(6)^2 = (3.14)(36) = 113.04$ square meters

4. A candlemaker produces candles using cylindrical molds. Each mold has a diameter of 10 cm and a height of 15 cm.

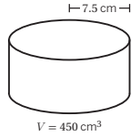
a Find the volume of one candle. (Use $\pi = 3.14$.)

$V = \pi r^2 h = (3.14)(5)^2(15) = (3.14)(25)(15) = 1{,}177.5 \text{ cm}^3$

b If the candlemaker produces 400 candles in one day, what is the total volume of wax used?

$V_{total} = (400)(1177.5) = 471{,}000 \text{ cm}^3$

5. A cylinder has a radius of 7.5 centimeters. Its volume is 450π cubic centimeters. What is the height of the cylinder?



$V = \pi r^2 h$

$450\pi = \pi(7.5)^2 h$

$\frac{450\pi}{56.25\pi} = \frac{\pi(56.25)h}{56.25\pi}$

$h = 8$ cm

$V = 450\pi \text{ cm}^3$

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 7

6. Your hometown is repainting a cylindrical water tower to match the high school colors. The water tower has a radius of 12 feet and a height of 42 feet. How much paint is needed to cover the top and the entire sides? (Use $\pi = 3.14$.)

$SA = 2\pi rh + \pi r^{2*}$

$SA = 2(3.14)(12)(42) + (3.14)(12)^2$

$SA = 3165.12 + 452.16$

$SA = 3{,}617.28 \text{ in}^3$

**only πr^2 instead of $2\pi r^2$ since the bottom is not painted*

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Calculate the areas of parallelograms, rhombi, trapezoids, polygons, and composite shapes. **(Lessons 1–5)**	1	**Support**	• **Mini Lesson:** Calculating the Area of Parallelograms (ML 7.01)
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 1, Activity 1 (Areas of Parallelograms and Rhombi)
	2a, 2b	**Support**	• **Mini-Lesson:** Calculating Areas of Complex Shapes (ML 7.04)
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 3 (Shapes Inside Shapes) Synthesis
		Stretch	• Invite students to create a poster using multiple representations to display their mathematical thinking.
Sub-Unit 2: • Calculate and compare the volumes of cylinders. **(Lessons 6–8)**	3a, 3b	**Support**	• **Teacher Move:** Consider revisiting Lesson 5 (Circle vs. Square)
	4a, 4b, 5	**Support**	• **Mini Lesson:** Calculating Volumes of Cylinders (ML 7.06)

		Strengthen	• **Teacher Move:** Consider revisiting these Lessons.
			  • Lesson 6 (Cylinder Volumes)
			  • Lesson 7 (Scaling Cylinders)

		Stretch	• **You're invited to explore more:** Lesson 6 (Cylinders)
	6	**Support**	• **Teacher Move:** Consider revisiting Lesson 8 (From Flat to Round) Synthesis

UNIT 8

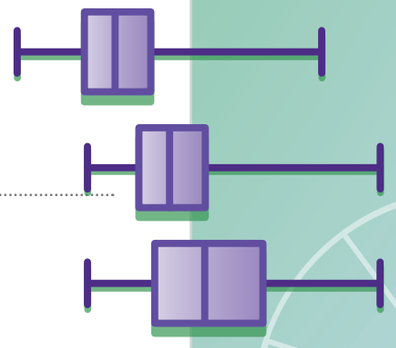
Probability and Sampling



Students learn about probability as a way to describe the likelihood of unknown events and estimate the probability of real-world situations. They use samples to draw conclusions about and compare populations.

Essential Questions

- How can we determine how likely an event is to happen?
- How do we choose the most appropriate display to represent data?
- How do samples help us make predictions about a larger population?



Sub-Unit Goals

Sub-Unit 1

- Compare probabilities to the results of repeated experiments.
- Determine the probability of events by identifying the sample space.

Sub-Unit 2

- Use measures of center and measures of variability from random samples to draw conclusions about and compare populations.
- Choose and create appropriate graphical representations for numerical and categorical data sets.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Rewrite numbers in equivalent forms. **(MA.7.NSO.1)**

Add, subtract, multiply and divide rational numbers. **(MA.7.NSO.2)**

Use percentages and proportional reasoning to solve problems. **(MA.7.AR.3)**

Analyze and represent two-variable proportional relationships. **(MA.7.AR.4)**

Represent and interpret numerical and categorical data. **(MA.7.DP.1)**

Develop an understanding of probability. Find and compare experimental and theoretical probabilities. **(MA.7.DP.2)**

Benchmarks Addressed in This Unit: MA.7.NSO.1.2 MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.AR.3.2 MA.7.AR.4.2 MA.7.AR.4.5 MA.7.DP.1.1 MA.7.DP.1.2 MA.7.DP.1.3 MA.7.DP.1.4 MA.7.DP.1.5 MA.7.DP.2.1 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Assess the reasonableness of solutions. **(MTR.6.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.3.1**

Coherence Telling a story within and across grades

In this Unit

Students describe the likelihood of unknown events and estimate the probabilities of real-world situations. They use samples to draw conclusions about and compare populations.

Prior Learning

- **Grade 6,** Unit 2: Use ratio reasoning **(MA.6.AR.3.1, MA.6.AR.3.5)**
- **Grade 6,** Unit 8: Create box plots and histograms **(MA.6.DP.1.5)**
- **Grade 6,** Unit 8: Calculate quantitative measures of center and variability **(MA.6.DP.1.2, MA.6.DP.1.3)**

Future Learning

- **Grade 8,** Unit 6: Scatter plots **(MA.8.DP.1.1, MA.8.DP.1.2)**
- **Grade 8,** Unit 6: Lines of best fit **(MA.8.DP.1.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Connections Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–4**, students describe the likelihood and probability of events and connect the results of repeated experiments to the accuracy of their own predictions.

Strategic Competence

In **Lessons 7 and 8**, students choose and create appropriate representations for data.

Procedural Fluency

In **Lesson 9**, students develop procedural fluency as they calculate mean and mean absolute deviation.

Adaptive Reasoning

In **Lessons 4 and 5**, students develop adaptive reasoning as they analyze results from experiments to approximate the probability of events.

Application

In **Lessons 9–12**, students apply their understanding of measures of center and variability to compare data and justify claims.

Productive Disposition

In **Lesson 13**, students have opportunities to develop a productive disposition towards mathematics as they explore a context that is meaningful to them through mathematics.

Unit at a Glance

Assess and Respond



A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.DP.1.2 MA.6.DP.1.3 MA.6.DP.1.4 MA.6.DP.1.5 MA.6.DP.1.6 MA.6.AR.3.4 MA.6.NSO.1.1 MA.6.NSO.3.5 MA.7.AR.3.1
MTR.4.1 MTR.5.1

Sub-Unit 1



1 How Likely?

Chance Experiments

Use phrases such as "likely" and "equally likely as not" to describe the likelihood of different events.

MA.7.DP.2.2
MTR.2.1 MTR.3.1



2 Prob-bear-bilities

Intro to Probability

Connect likelihood with probability written in different forms and use sample spaces to help determine probabilities.

MA.7.DP.2.1 MA.7.DP.2.2 MA.7.DP.2.3
MTR.2.1 MTR.3.1



3 Mystery Bag

Predicting Sample Spaces

Use repeated experiments and proportionality to make predictions and recognize that the number of repeated experiments affects the accuracy of a prediction.

MA.7.NSO.1.2 MA.7.AR.4.2 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.3.1 MTR.6.1



7 Choices, Choices

Choosing an Appropriate Representation for Data

Choose an appropriate graphical display for a given set of data.

MA.7.DP.1.4 MA.7.DP.1.5
MTR.2.1 MTR.4.1 MTR.7.1

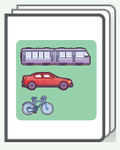


8 On Display

Creating an Appropriate Representation for Data

Create an appropriate graphical display for a given set of data.

MA.7.DP.1.4 MA.7.DP.1.5
MTR.2.1 MTR.7.1



9 Car, Bike, or Train?

Using Mean, Median, and Range to Compare Groups

Analyze and compare sets of data by calculating the mean, median, and range to make a recommendation for a real-world situation.

MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.DP.1.1 MA.7.DP.1.2
MTR.4.1 MTR.7.1



10 Crab Island

Sampling From Large Populations

Explore the advantages and disadvantages of using samples to answer a question about a population.

MA.7.DP.1.3
MTR.1.1 MTR.4.1 MTR.7.1

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.DP.1.1 MA.7.DP.1.2 MA.7.DP.1.3 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.4.1 MTR.6.1

Pacing: 17 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 13 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 1



4 Spin Class

Repeated Experiments

Recognize that the results of a repeated experiment will approach, but may not exactly match, the probability of the event.

MA.NSO.1.2 MA.7.AR.4.2 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.1.1 MTR.4.1



5 Is It Fair?

Comparing Probabilities and Experiments

Decide whether an object is fair by comparing probabilities from a model to the results of a repeated experiment.

MA.NSO.1.2 MA.7.AR.4.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.5.1 MTR.6.1

Assess and Respond



A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.DP.2.1 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4 MA.7.NSO.2.2
MTR.6.1 MTR.7.1

Sub-Unit 2



6 The County Fair

Creating Circle Graphs

Create and interpret data in circle graphs.

MA.7.DP.1.4
MTR.4.1 MTR.5.1

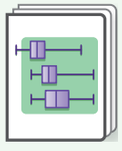


11 Flower Power

Using Percentages to Predict Populations

Use data from a sample to estimate the number of marigolds in a large flower garden using proportional relationships and percentages.

MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.AR.3.2 MA.7.AR.4.5 MA.7.DP.1.3
MTR.41 MTR.5.1 MTR.6.1



12 Plots and Samples

Using Statistical Measures to Make Predictions

Draw conclusions about a population using quantitative data and estimate a measure of center of a population based on one or more samples of the population.

MA.7.DP.1.1 MA.7.DP.1.2
MTR.5.1 MTR.6.1



13 School Newspaper

Comparing Populations

Students compare two populations using the difference between the centers of the samples, then compare the difference to the measures of variability.

MA.7.DP.1.2
MTR.4.1 MTR.7.1

M Math in Action

Connecting Area and Volume to Space Travel

Explore the connection between area, volume, probability and space travel, as well as the contributions of Franklin. R. Chang-Diaz to space vehicle construction.

MA.7.DP.2.1 MA.7.GR.1.4 MA.7.GR.2.3
MTR.3.1

Pacing Considerations

Lesson 1: This lesson surfaces what students know about likelihood, which will be addressed further in upcoming lessons. If omitted, the vocabulary from this lesson should be defined during Lesson 2.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know
- **Cut:** Activity 1 Sheet *(one per two groups)*
- **Materials:** coins, two different types *(e.g., a penny and a nickel)*, containers for paper letters *(e.g., bag, bowl, or cup)*, number cubes, paper clips for spinners

Lesson 2

- **Copy:** Show What You Know, Activity 2 and 3 Sheets *(one per pair)*, You're Invited to Explore More Sheet *(optional)*
- **Materials:** coloring tools *(optional)*, paper clips for spinners, coins, number cubes *(as needed)*

Lesson 3

- **Copy:** Show What You Know
- **Materials:** coloring tools (green and purple) *(optional)*

Lesson 4

- **Copy:** Show What You Know, Activity 3 Sheet *(one per student)*
- **Materials:** coloring tools (blue and red), paper clips *(optional)*

Lesson 5

- **Copy:** Show What You Know
- **Materials:** coins *(one per pair)*, number cubes *(one per pair)*

Lesson 6

- **Copy:** Show What You Know
- **Materials:** protractors

Lesson 7

- **Copy:** Show What You Know

Lesson 8

- **Copy:** Show What You Know
- **Materials:** protractors

Lesson 9

- **Copy:** Show What You Know

Lesson 10

- **Copy:** Show What You Know

Lesson 11

- **Copy:** Show What You Know

Lesson 12

- **Copy:** Show What You Know

Lesson 13

- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards



Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions



Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

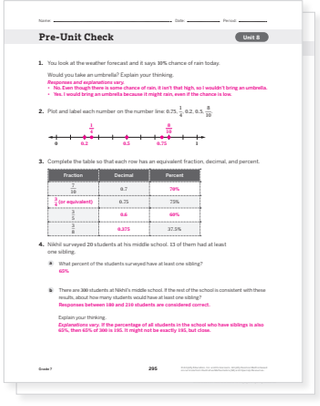
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.DP.1.2, MA.6.DP.1.3, MA.6.DP.1.4, MA.6.DP.1.5, MA.6.DP.1.6, MA.6.AR.3.4, MA.6.NSO.1.1, MA.6.NSO.3.5, MA.7.AR.3.1



Pre-Unit Check

Unit 8

1. You look at the weather forecast and it says 10% chance of rain today.

Would you take an umbrella? Explain your thinking.

Responses and explanations vary.
- No. Even though there is some chance of rain, it isn't that high, so I wouldn't bring an umbrella.
- Yes. I would bring an umbrella because it might rain, even if the chance is low.

2. Plot and label each number on the number line: 0.75, $\frac{1}{4}$, 0.2, 0.5, $\frac{45}{100}$

0 | 0.2 | 0.5 | 0.75 | 1

3. Complete the table so that each row has an equivalent fraction, decimal, and percent.

Fraction	Decimal	Percent
$\frac{2}{10}$	0.2	20%
$\frac{3}{4}$ (or equivalent)	0.75	75%
$\frac{5}{8}$	0.6	60%
$\frac{3}{8}$	0.375	37.5%

4. Nikhil surveyed 20 students at his middle school. 13 of them had at least one sibling.

a. What percent of the students surveyed have at least one sibling?
65%

b. There are 300 students at Nikhil's middle school. If the rest of the school is consistent with these results, about how many students would have at least one sibling?
Responses between 180 and 210 students are considered correct.

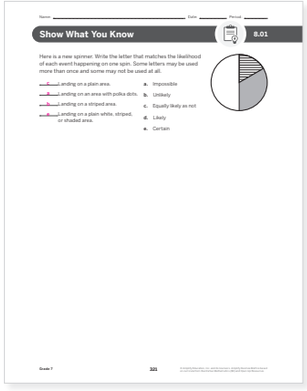
Explain your thinking.
Explanations vary. If the percentage of all students in the school who have siblings is also 65%, then 65% of 300 is 195. It might not be exactly 195, but close.

Grade 7 295

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 1 Show What You Know: MA.7.DP.2.2



Show What You Know 8.01

Here is a new spinner. Write the letter that matches the likelihood of each event happening on one spin. Some letters may be used more than once and some may not be used at all.

_____ landing on a plain area.
_____ landing on an area with polka dots.
_____ landing on a striped area.
_____ landing on a plain white, striped, or shaded area.

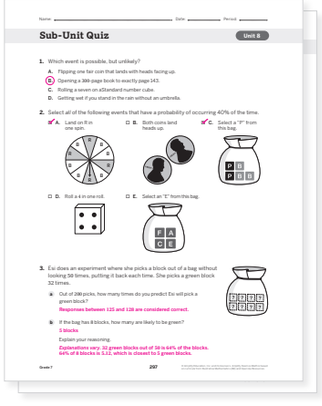
a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Grade 7 321

Sub-Unit Quizzes

Assign **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MA.7.NSO.2.2



Sub-Unit Quiz

Unit 8

1. Which event is possible, but unlikely?
A. Flipping one fair coin that lands with heads facing up.
B. Opening a 386-page book to exactly page 143.
C. Rolling a seven on a Standard number cube.
D. Getting wet if you stand in the rain without an umbrella.

2. Select all of the following events that have a probability of occurring 40% of the time.
☑ A. Land on R in one spin.
☐ B. Both coins land heads up.
☑ C. Select a "P" from this bag.
☐ D. Roll a 4 in one roll.
☐ E. Select an "E" from this bag.

3. Esi does an experiment where she picks a block out of a bag without looking 50 times, putting it back each time. She picks a green block 32 times.

a. Out of 200 picks, how many times do you predict Esi will pick a green block?
Responses between 125 and 128 are considered correct.

b. If the bag has 8 blocks, how many are likely to be green?
5 blocks

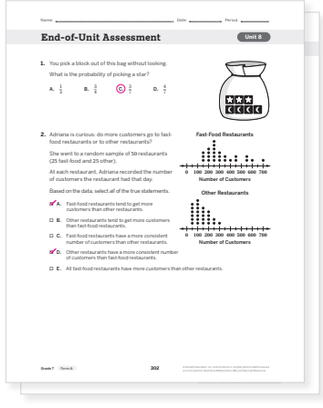
Explain your reasoning.
Explanations vary. 32 green blocks out of 50 is 64%, of the blocks. 64% of 8 blocks is 5.12, which is closest to 5 green blocks.

Grade 7 297

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.DP.1.1, MA.7.DP.1.2, MA.7.DP.1.3, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4



End-of-Unit Assessment

Unit 8

1. You pick a block out of this bag without looking.
What is the probability of picking a star?
A. $\frac{1}{3}$ B. $\frac{1}{4}$ C. $\frac{2}{5}$ D. $\frac{1}{2}$

2. Adriana is curious: do more customers go to fast-food restaurants or to other restaurants?

She went to a random sample of 50 restaurants (25 fast-food and 25 other).

At each restaurant, Adriana recorded the number of customers the restaurant had that day.

Based on the data, select all of the true statements.
☑ A. Fast-food restaurants tend to get more customers than other restaurants.
☐ B. Other restaurants tend to get more customers than fast-food restaurants.
☐ C. Fast-food restaurants have a more consistent number of customers than other restaurants.
☑ D. Other restaurants have a more consistent number of customers than fast-food restaurants.
☐ E. All fast-food restaurants have more customers than other restaurants.

Fast-Food Restaurants
0 100 200 300 400 500 600 700
Number of Customers

Other Restaurants
0 100 200 300 400 500 600 700
Number of Customers

Grade 7 Form A 302

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

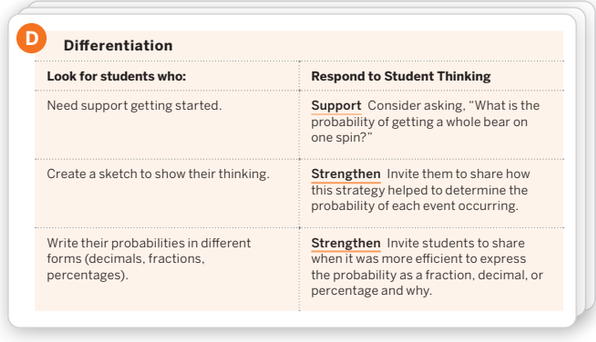
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Compare probabilities to the results of repeated experiments. • Determine the probability of events by identifying the sample space.	**Mini-Lesson** • *Predicting Sample Spaces Using Proportional Relationships*	**Repeated Challenges** • Lesson 2 (*Prob-bear-bilities*) **Challenge Creator** • Lesson 4 (*Spin Class*) **Practice** • Lessons 1–5	**You're invited to explore more.** • Lesson 2 (*Prob-bear-bilities*) • Lesson 5 (*Is It Fair?*)
Sub-Unit 2 (Lessons 6–13) • Use measures of center and measures of variability from random samples to draw conclusions about and compare populations. • Choose and create appropriate graphical representations for numerical and categorical data sets.	**Mini-Lesson** • *Calculating Measures of Center and Spread* • *Predicting Population Using Sample Data*	**Practice** • Lessons 6–13	**You're invited to explore more.** • Lesson 10 (*Crab Island*)

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 8**, where students determine the probability of an event.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is the probability of getting a whole bear on one spin?"
Create a sketch to show their thinking.	**Strengthen** Invite them to share how this strategy helped to determine the probability of each event occurring.
Write their probabilities in different forms (decimals, fractions, percentages).	**Strengthen** Invite students to share when it was more efficient to express the probability as a fraction, decimal, or percentage and why.

Accessibility

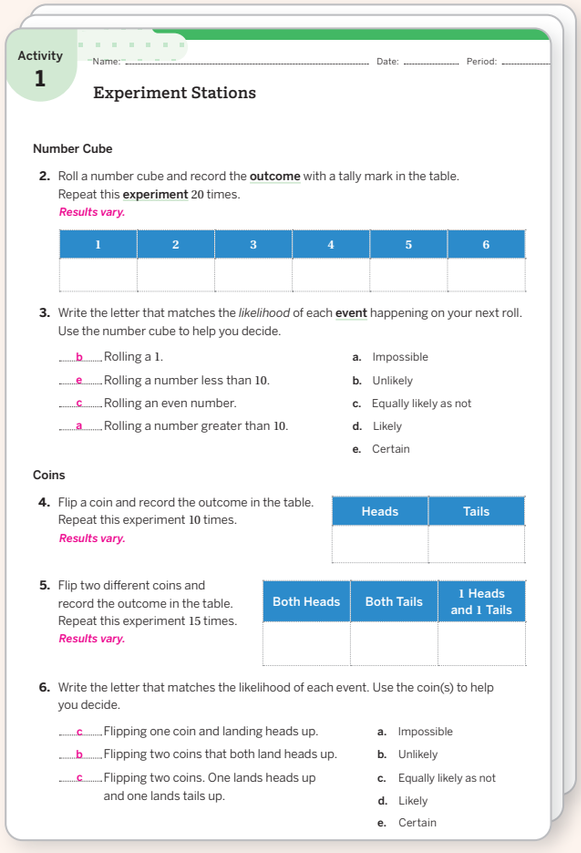
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action and Expression.

Spotlight on Interaction

In this lesson, students physically interact with manipulatives to perform experiments and move from station to station.



Activity 1

Name: Date: Period:

Experiment Stations

Number Cube

2. Roll a number cube and record the **outcome** with a tally mark in the table. Repeat this **experiment** 20 times.
Results vary.

1	2	3	4	5	6

3. Write the letter that matches the *likelihood* of each **event** happening on your next roll. Use the number cube to help you decide.

b Rolling a 1.
e Rolling a number less than 10.
c Rolling an even number.
a Rolling a number greater than 10.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Coins

4. Flip a coin and record the outcome in the table. Repeat this experiment 10 times.
Results vary.

Heads	Tails

5. Flip two different coins and record the outcome in the table. Repeat this experiment 15 times.
Results vary.

Both Heads	Both Tails	1 Heads and 1 Tails

6. Write the letter that matches the likelihood of each event. Use the coin(s) to help you decide.

c Flipping one coin and landing heads up.
b Flipping two coins that both land heads up.
c Flipping two coins. One lands heads up and one lands tails up.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Memory and Attention

Here are two suggestions in this unit for supporting students with activating prior knowledge to support making connections.

Lesson 11, Activity 1

Accessibility: Memory and Attention To activate prior knowledge, consider asking, "What do we know about percentages and ratios that can help us to analyze Eliza's and Javier's strategies?"

Lesson 10, Activity 2

Accessibility: Memory and Attention Invite students to refer to the anchor chart created previously on samples and population to activate prior knowledge. Add to the anchor chart as students share.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson
event **experiment** **outcome** **random**	1
probability **sample space**	2
relative frequency	4
circle graph	6
population **sample**	10

Review Vocabulary
bar chart
box plot
categorical data
center
constant of proportionality
dot plot
histogram
interquartile range
mean
median
outlier
percentage
proportional
range
shape
skewed
spread
stem-and-leaf plot
symmetrical

Additional Vocabulary
measure of center
measure of variability

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	6, 13
MLR2: Collect and Display	1–2, 7–10, 12
MLR7: Compare and Connect	11

Sub-Unit Language Goals

- Interpret the results from repeated experiments to determine fairness, probably, and the likelihood of events.
- Explain the purpose of sampling and which methods of obtaining a sample tend to produce representative samples.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 9**, where students are evaluating the measures of center and variability in a data set.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Scatter Plots

In this unit, students analyze numerical data in one variable using dot plots, box plots, and histograms. **In Grade 8, Unit 6**, students will analyze numerical data in two variables using scatter plots.

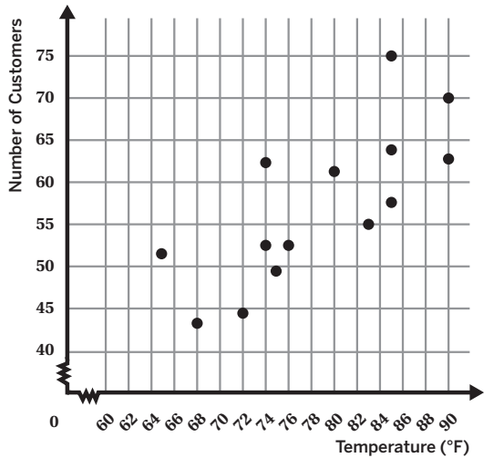
FL B.E.S.T. Math Benchmarks:
MA.8.DP.1.1, MA.8.DP.1.2

Example:

Scatter plots show us how two variables are related. This data shows temperature and the number of customers over time at an ice cream stand.

Some questions scatter plots can help answer:

- Is there an association between the outside temperature and the number of customers at an ice cream stand?
- What is the predicted number of customers if the temperature is 78°F?



Lines of Fit

In this unit, students analyze samples using measures of center and spread. **In Grade 8, Unit 6**, students will analyze numerical data in two variables using lines of fit.

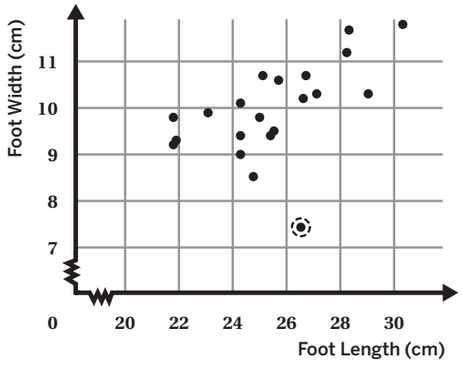
FL B.E.S.T. Math Benchmark:
MA.8.DP.1.3

Example:

We say that there is a positive association between foot length and foot width because in general, longer feet are wider than shorter feet.

The line on the graph shows the overall trend and can help us find the predicted foot width for a given foot length. This is called a line of fit.

Points that are not close to the line and to most of the data are called outliers.

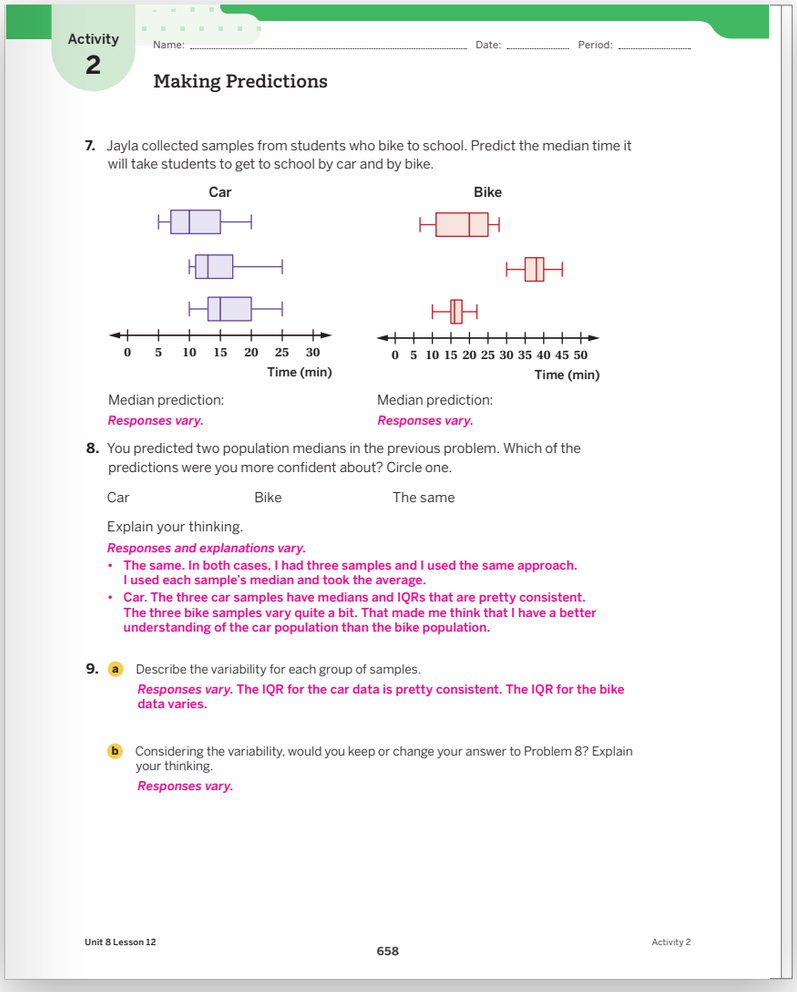


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Adaptive Reasoning

Lesson 12, Problems 7–9



Activity 2

Name: _____ Date: _____ Period: _____

Making Predictions

7. Jayla collected samples from students who bike to school. Predict the median time it will take students to get to school by car and by bike.

Car

Bike

Median prediction:
Responses vary.

Median prediction:
Responses vary.

8. You predicted two population medians in the previous problem. Which of the predictions were you more confident about? Circle one.

Car Bike The same

Explain your thinking.

Responses and explanations vary.

- The same. In both cases, I had three samples and I used the same approach. I used each sample's median and took the average.
- Car. The three car samples have medians and IQRs that are pretty consistent. The three bike samples vary quite a bit. That made me think that I have a better understanding of the car population than the bike population.

9. a. Describe the variability for each group of samples.

Responses vary. The IQR for the car data is pretty consistent. The IQR for the bike data varies.

b. Considering the variability, would you keep or change your answer to Problem 8? Explain your thinking.

Responses vary.

Unit 8 Lesson 12 658 Activity 2

Context

In this activity, students are developing adaptive reasoning as they move fluently between multiple samples using box plots to make sense of a problem.

Try This

Put on your student hat and complete Problems 7–9.

Points to Ponder

- Some students may refer to different box plots as separate populations. How might you support students to interpret each box plot as different samples of the same population?
- How does interpreting multiple box plots build from their understanding of representative sampling, measures of center, and variability?
- How will moving fluently between multiple samples using box plots support their understanding of determining whether two populations are very different?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Justifying an argument based on interpreting the probability of real world events	Lesson 1	2	MA.6.AR.3.4
2	Using the structure of the number line to locate fractions and decimals	Lesson 2	2	MA.6.NSO.1.1 MTR.5.1
3	Using the structure of place value and its relationship to fractions to determine equivalent fractions, decimals and percentages	Lesson 2	1	MA.6.NSO.3.5 MTR.5.1
4	Solving real-world problems with percentages and proportional relationships and justifying a claim about the results	Lesson 3	2	MA.6.AR.3.4, MA.7.AR.3.1
5	Communicating understanding and wonderings about measures of center and variability	Lesson 9	2	MA.6.DP.1.2, MA.6.DP.1.3, MA.6.DP.1.6
6	Interpreting a dot plot representation to solve problems and justify a claim	Lesson 9	2	MA.6.DP.1.2, MA.6.DP.1.4, MTR 4.1
7	Interpreting the structure of a box plot as it relates to a real-world context	Lesson 12	1	MA.6.DP.1.3, MA.6.DP.1.5, MTR.5.1

Assessment and Lesson Resources



- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 8

1. You look at the weather forecast and it says 10% chance of rain today.

Would you take an umbrella? Explain your thinking.

Responses and explanations vary.
- No. Even though there is some chance of rain, it isn't that high, so I wouldn't bring an umbrella.
- Yes. I would bring an umbrella because it might rain, even if the chance is low.

2. Plot and label each number on the number line: $0.75, \frac{1}{4}, 0.2, 0.5, \frac{8}{10}$.



3. Complete the table so that each row has an equivalent fraction, decimal, and percent.

Fraction	Decimal	Percent
$\frac{7}{10}$	0.7	70%
$\frac{3}{4}$ (or equivalent)	0.75	75%
$\frac{3}{5}$	0.6	60%
$\frac{3}{8}$	0.375	37.5%

4. Nikhil surveyed 20 students at his middle school. 13 of them had at least one sibling.

a. What percent of the students surveyed have at least one sibling?
65%

b. There are 300 students at Nikhil's middle school. If the rest of the school is consistent with these results, about how many students would have at least one sibling?
Responses between 180 and 210 students are considered correct.

Explain your thinking.
Explanations vary. If the percentage of all students in the school who have siblings is also 65%, then 65% of 300 is 195. It might not be exactly 195, but close.

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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

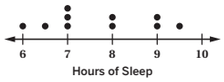
Unit 8

5. a. What are some things you know about mean and median?
Responses vary.
- To find the mean, add up all the numbers and divide by how many numbers you have.
- Median is the middle number.
- When figuring out the median, you need to put all the numbers in order.

b. What are some things you know about IQR (interquartile range)?
Responses vary.
- You can figure out IQR using a box plot.
- IQR is related to the median.

c. What are some things you still wonder about mean, median, or IQR?
Responses vary.
- What is IQR?
- When should I use median versus mean?

6. Eva is curious how much students at her school sleep. She asked 10 students how many hours they slept last night and recorded their answers in a dot plot.



a. How many students slept for 8 or more hours last night? 5 students

b. Calculate the mean number of hours that all 10 students slept. 7.7 hours

c. Do you think the mean you calculated is similar to the mean of all the students in Eva's school? Explain your reasoning.
Responses and explanations vary.
- Yes. I think so because the small group probably covers all different types of students: students who sleep a lot and students who don't sleep a lot.
- No. I don't think so because we don't know all the students in the school and how many hours they sleep for.

7. There are 11 dancers in a performance. Their ages (in years) are:

5.5, 6, 6, 6.5, 7, 7.5, 8, 8, 8.5, 9, 9

a. What is the median age of these dancers? 7.5 years

b. Determine the first quartile, median, and third quartile of the dancers' ages. Label them on the box plot.



c. What is the interquartile range (IQR) of the dancers' ages? 2.5 years

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Compare probabilities to the results of repeated experiments. • Determine the probability of events by identifying the sample space. **(Lessons 1–5)**	1	**Support** • **Teacher Move:** Before Lesson 1, consider surfacing students' arguments in order to come to a shared understanding of what chance and likelihood mean.
	2, 3	**Support** • **Mini-Lesson:** *Converting Between Fractions, Percents, and Decimals* • **Teacher Move:** Before Lesson 2, consider reviewing this problem and asking questions like, "Which value is closest to 1? 0? How do you know?" • **Teacher Move:** During Lesson 2, consider asking, "What would the probability look like written as a decimal? As a fraction? As a percentage? What form would be most helpful for comparing these probabilities?"
	4	**Support** • **Teacher Move:** During Lesson 3's Warm-Up, consider spending extra time discussing several different examples of bags that would lead to a 40% chance of picking a green block. • **Teacher Move:** Consider revisiting Grade 7, Unit 4, Lesson 4 *(Percent Machines)*

	5, 6	**Support**
		• **Teacher Move:** Consider revisiting the following Lessons from Grade 6, Unit 8:
		  • Lesson 6 *(Snack Time)*
		  • Lesson 8 *(Toy Cars)*
		  • Lesson 9 *(In the News)*

	7	**Support** • **Mini-Lesson:** *Interpreting Box Plots* • **Teacher Move:** Before Lesson 12, consider spending extra time reviewing what median and IQR represent about data and how to calculate them. • **Teacher Move:** Consider revisiting Grade 6, Unit 8, Lesson 12 *(Car, Plane, Bus, or Plane?)*

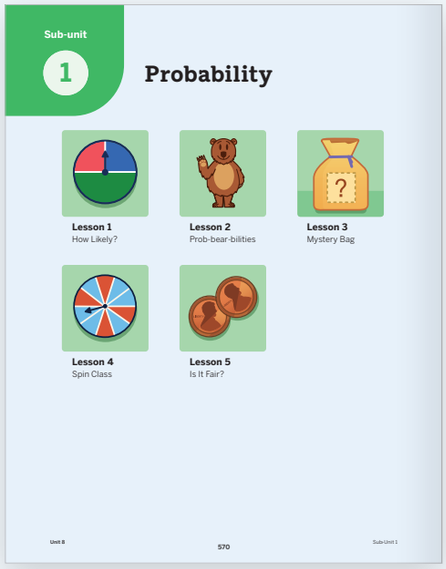
Sub-Unit 1

Probability

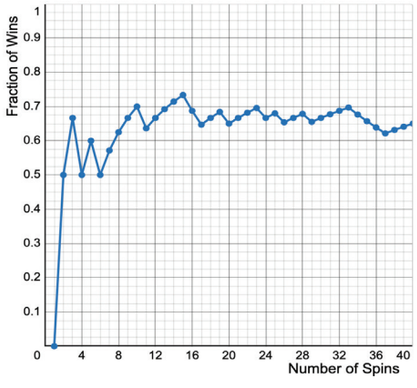
Sub-Unit 1 Goals

- **Goal:** Compare probabilities to the results of repeated experiments.
- **Goal:** Determine the probability of events by identifying the sample space.
- **Language Goal:** Interpret the results from repeated experiments to determine fairness, probability, and likelihood of events.

In this sub-unit, students determine the **probability** of **events** and compare the results of repeated **experiments** to the probability.



Math That Matters Most

Concept or Skill	For example . . .
Express the probability of events as a fraction, decimal, or percentage.	Calculate the probability that a spinner with equal parts red, yellow, and green lands on green on the next spin. $\frac{1}{3}$
Use data from a repeated experiment to approximate the probability of an event.	A spinner is spun 40 times for a game. This graph shows the fraction of games that are wins. Based on the graph, what is the probability of a spin winning this game?  *The probability is around 65% because the line on the graph approaches 0.65 as the number of spins increases.*

Unit 8

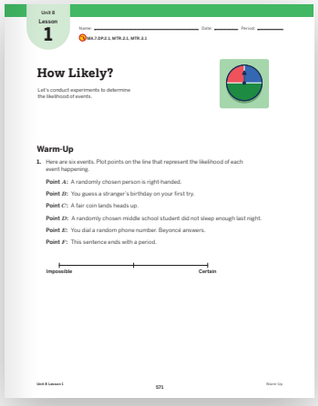
Lesson

1

How Likely?

Chance Experiments

Let's conduct experiments to determine the likelihood of events.


Unit 8
Lesson
1
Name _____ Date _____ Period _____
MA.7.DP.2.1, MTR.2.1, MTR.3.1
How Likely?
Let's conduct experiments to determine the likelihood of events.
Warm-Up
1. Here are six events. Plot points on the line that represent the likelihood of each event happening.
Point A: A randomly chosen person is right-handed.
Point B: You guess a stranger's birthday on your first try.
Point C: A fair coin lands heads up.
Point D: A randomly chosen middle school student did not sleep enough last night.
Point E: You dial a random phone number. Beyoncé answers.
Point F: This sentence ends with a period.
Impossible
Certain
Unit 8 Lesson 1
571
Warm-Up

Learning Outcomes

Today's Goals

1. **Language Goal:** Describe the likelihood of *events* using the words *impossible, unlikely, equally likely as not, likely,* or *certain*. **(Reading and Writing)**
2. **Language Goal:** Explain how *experiments* can help determine the likelihood of future *events*. **(Writing)**

Students experience chance as they conduct their own **experiments** and use phrases such as "likely" and "equally likely as not" to describe the likelihood of different **events**. They also encounter many of the tools they will use to simulate events later in the unit: coins, number cubes, spinners, and items in bags.

Coherence

Prior Learning

In Grade 6, students visualized data and calculated measures of center to describe and compare data sets.

Future Learning

In Sub-Unit 1, students will connect likelihoods to the word probability and to methods for calculating probability. They will use repeated experiments and proportionality to make predictions.

Connections

- Students explore experimental probability of flipping a coin or rolling a number cube and recording their outcomes on a table. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.1

Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	**MA.7.DP.2.3** **MA.7.DP.2.4**

Vocabulary

New Vocabulary

event	outcome
experiment	random

Lesson at a Glance

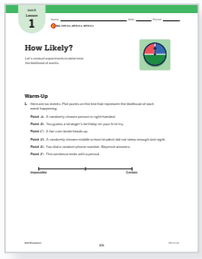
~ 45 min

MA.7.DP.2.2

Warm-Up

Pairs | 5 min

Students use their knowledge and intuition to rank the likelihoods of different **events**.



Activity 1

Small Groups | 30 min

Students describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.

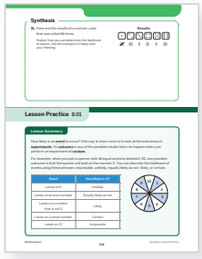


Synthesis

Whole Class | 5 min

Students synthesize their understanding of likelihoods by explaining how **experiments** can help determine the likelihood of events.

MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the likelihood of spinning a named section on a spinner.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

- Coins, two different types (e.g., a penny and a nickel), containers for paper letters (e.g., bag, bowl, or cup), number cubes, paper clips for spinners

Pairs | 5 min

Warm-Up

Purpose: Students use their knowledge and intuition to rank the likelihoods of different **events**.

1 Launch

Direct students' attention to the line and six events to read aloud as a class.

Consider asking, "What do you think *random* means in this context?"

Create a class definition for the terms **events** and **random** using students' language and the glossary.

To surface prior knowledge, consider asking:

- "What percentage is associated with impossible? Certain? The marker in between impossible and certain?"
- "Where on the line would you label as being unlikely? Likely?"

2 Connect

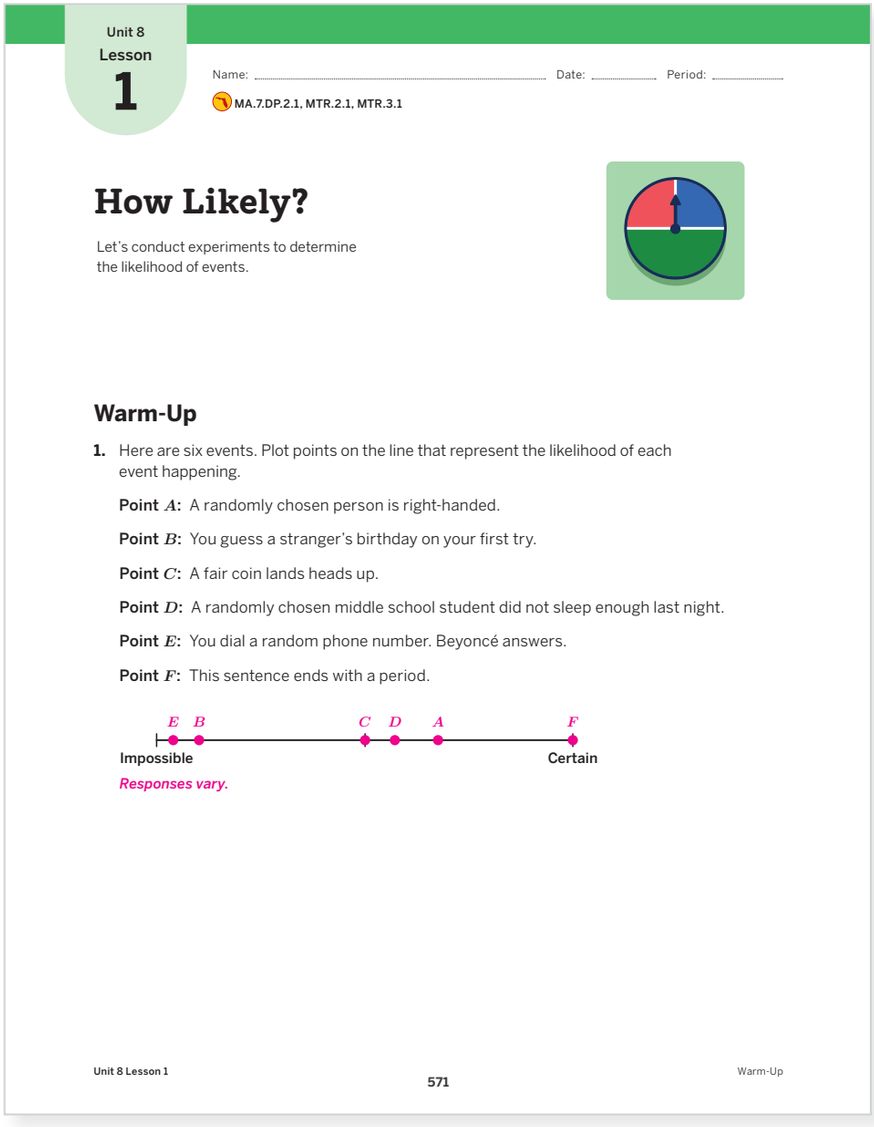
Invite students to share their responses and justify the placement of each point on the line. **(MTR.3.1)**

Encourage students to critique each other's reasoning.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning:

- I would place the statement _____ because _____.
- I think it should be placed farther left because _____.
- I think it should be placed farther right because _____.
- I think it should be placed right in the middle because _____.

Consider asking, "Which events did you think were likely to occur? Did you place them in the same position on the line? Why?"



Unit 8
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.DP.2.1, MTR.2.1, MTR.3.1

How Likely?

Let's conduct experiments to determine the likelihood of events.

Warm-Up

1. Here are six events. Plot points on the line that represent the likelihood of each event happening.

   Point A: A randomly chosen person is right-handed.

   Point B: You guess a stranger's birthday on your first try.

   Point C: A fair coin lands heads up.

   Point D: A randomly chosen middle school student did not sleep enough last night.

   Point E: You dial a random phone number. Beyoncé answers.

   Point F: This sentence ends with a period.

Responses vary.

Unit 8 Lesson 1 571 Warm-Up

Small Groups | 30 min

Activity 1 Experiment Stations

Purpose: Students describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.

Short on time? Consider allowing students to choose two of the four stations to complete.

1 Launch

To generate excitement, consider sharing that in this activity, students will be conducting experiments at different stations.

Note: You may set up stations with physical manipulatives including a number cube, coins, deck of letter cards, and a spinner.

Demonstrate how to use the physical tools for each experiment and discuss how to record the outcomes.

Create a class definition of the terms **outcome** and **experiment** using students' language and the glossary.

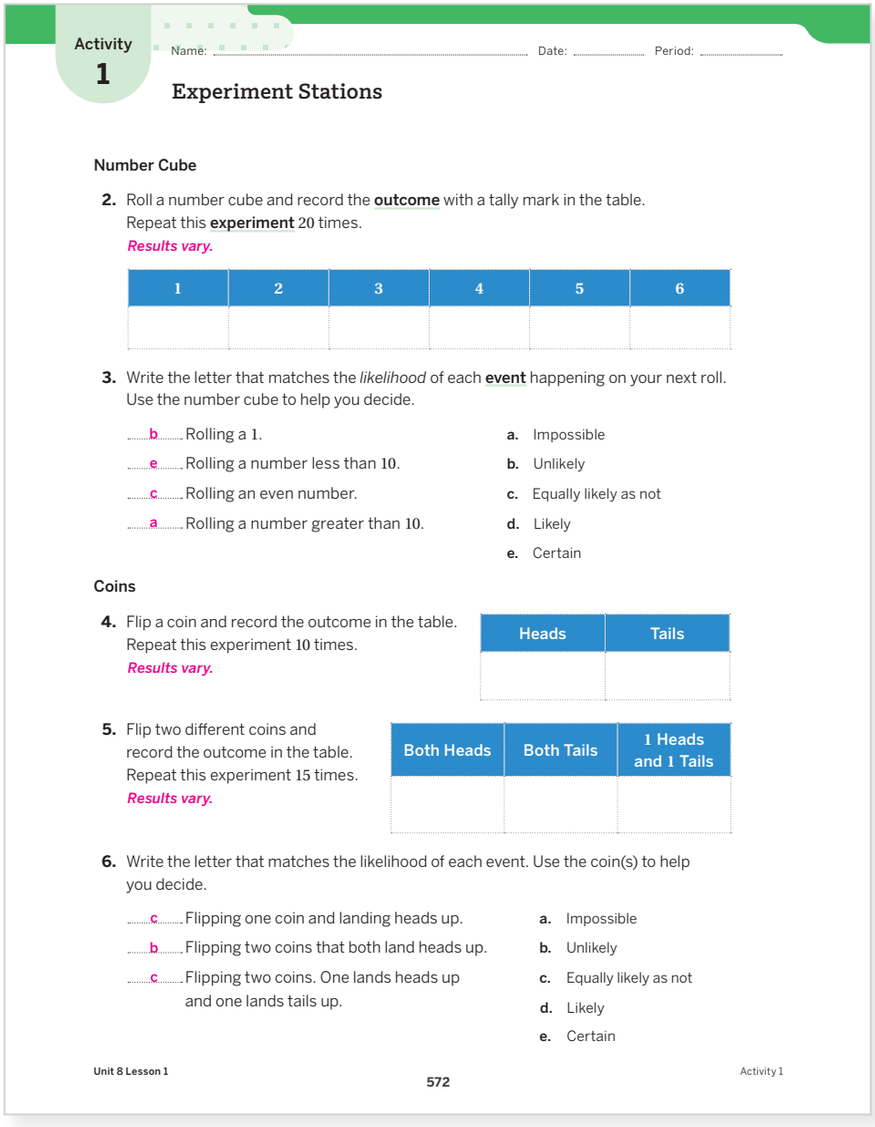
Note: Consider these options for student movement:

- As students finish a station, instruct them to move from station to station.
- After a set amount of time, instruct students to move as a group from station to station.
- Invite students to complete each station individually or in pairs.

2 Monitor

Note: Refer to the guidance on the next page to support students as they work through the experiments.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Experiment Stations

Number Cube

2. Roll a number cube and record the **outcome** with a tally mark in the table. Repeat this **experiment** 20 times.
Results vary.

1	2	3	4	5	6

3. Write the letter that matches the *likelihood* of each **event** happening on your next roll. Use the number cube to help you decide.

b Rolling a 1.
e Rolling a number less than 10.
c Rolling an even number.
a Rolling a number greater than 10.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Coins

4. Flip a coin and record the outcome in the table. Repeat this experiment 10 times.
Results vary.

Heads	Tails

5. Flip two different coins and record the outcome in the table. Repeat this experiment 15 times.
Results vary.

Both Heads	Both Tails	1 Heads and 1 Tails

6. Write the letter that matches the likelihood of each event. Use the coin(s) to help you decide.

c Flipping one coin and landing heads up.
b Flipping two coins that both land heads up.
c Flipping two coins. One lands heads up and one lands tails up.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Unit 8 Lesson 1 572 Activity 1

Activity 1 Experiment Stations (continued)

Purpose: Students describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.

Short on time? Consider allowing students to choose to complete two of the four stations.

2 Monitor

A Accessibility: Executive Functioning Invite students to only consider one event at a time. (*Problems 3, 6, 8, and 10*)

D Differentiation

Look for students who:	Respond to Student Thinking
Reason that an event is impossible because it did not happen in their experiment.	**Support** Consider asking, "Is the outcome possible based on the actual tool? Do you think that event happened in any group's experiment?"
Reason that a letter in TENNESSEE cannot have an equally likely as not chance of occurring. *(Problem 8)*	**Strengthen** Consider asking, "Which letter can you remove from TENNESSEE so that one of the letters has an equally likely as not chance of happening on your next pick?"
Would like to further explore the likelihood of events.	**Stretch** Invite students to use the tools at a station to create their own experiment where the outcome is unlikely but not impossible. Consider challenging students to create an experiment with the most unlikely outcome of anyone in their group.

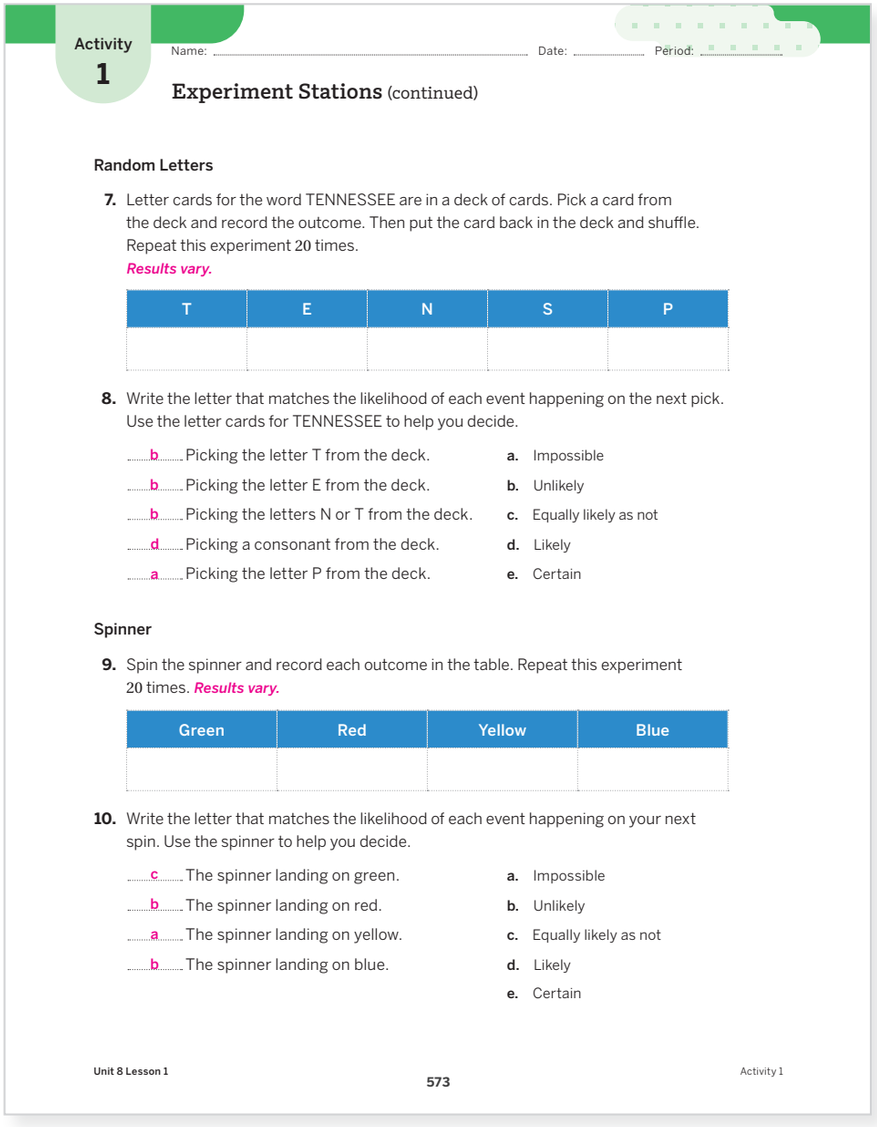
Listen for strategies students use at each station to determine the likelihood of each event to share during the Connect. **(MTR.2.1)**

3 Connect

Display a line from impossible to certain like the one from the Warm-Up on the board.

Invite students to revisit their responses from Problems 3, 6, 8, and 10, and then share where they would place the likelihood of each event on the line, one at a time.

To surface the Key Takeaway, consider asking, "Can you give an example of an event that is unlikely to occur? Where would you place its likelihood on the line?"



Activity 1

Name: _____ Date: _____ Period: _____

Experiment Stations (continued)

Random Letters

7. Letter cards for the word TENNESSEE are in a deck of cards. Pick a card from the deck and record the outcome. Then put the card back in the deck and shuffle. Repeat this experiment 20 times.

Results vary.

T	E	N	S	P

8. Write the letter that matches the likelihood of each event happening on the next pick. Use the letter cards for TENNESSEE to help you decide.

b Picking the letter T from the deck.
b Picking the letter E from the deck.
b Picking the letters N or T from the deck.
d Picking a consonant from the deck.
a Picking the letter P from the deck.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Spinner

9. Spin the spinner and record each outcome in the table. Repeat this experiment 20 times. *Results vary.*

Green	Red	Yellow	Blue

10. Write the letter that matches the likelihood of each event happening on your next spin. Use the spinner to help you decide.

c The spinner landing on green.
b The spinner landing on red.
a The spinner landing on yellow.
b The spinner landing on blue.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Unit 8 Lesson 1 573 Activity 1

Key Takeaway: **Events** can be described on a scale from impossible (0%) to certain (100%). While some events might have an unlikely chance of occurring, how unlikely they are can differ according to the specific event.

Whole Class | 5 min

Synthesis

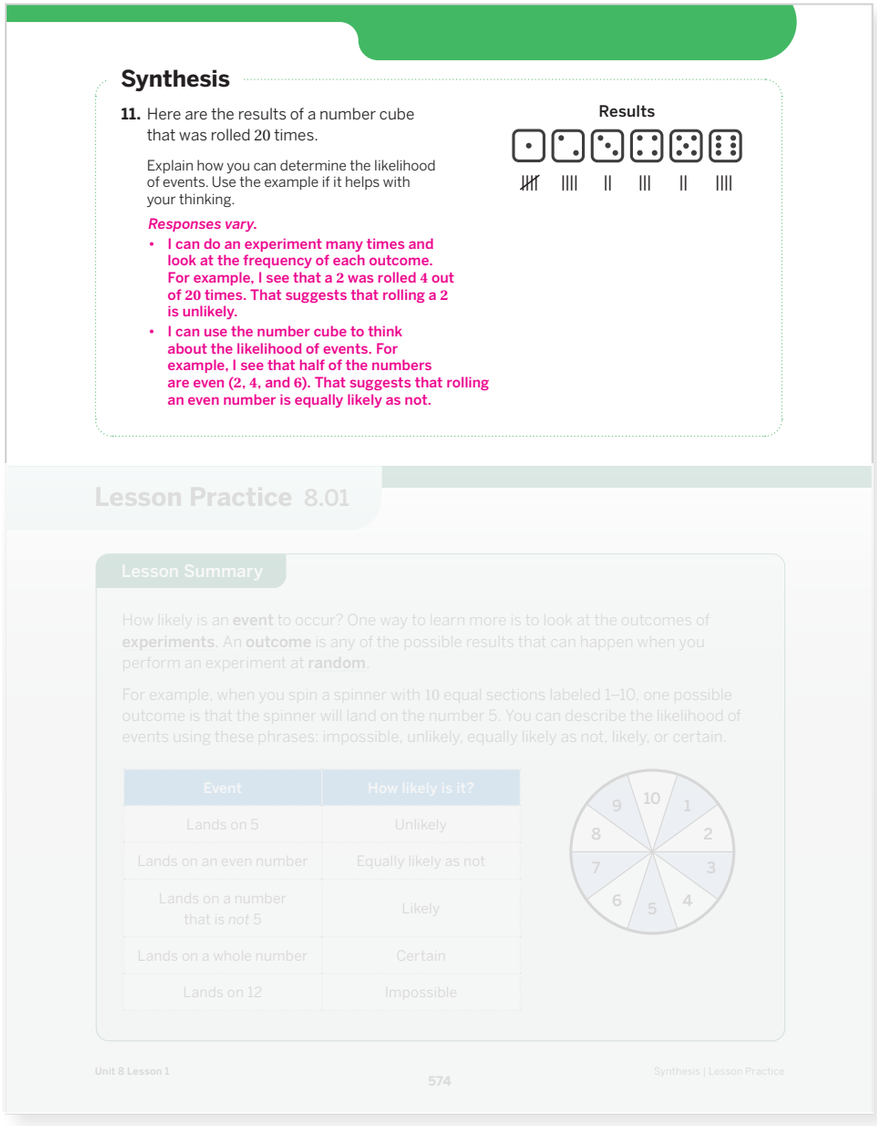
Purpose: Students synthesize their understanding of likelihoods by explaining how **experiments** can help determine the likelihood of events.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Conducting an experiment numerous times to gather data.
- Describing the likelihood using a range of impossible to certain.
- Analyzing the object used in an experiment.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *event, experiment, outcome,* and *random* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to make their response stronger and clearer based on the conversation.



Synthesis

11. Here are the results of a number cube that was rolled 20 times.

Explain how you can determine the likelihood of events. Use the example if it helps with your thinking.

Results

Responses vary.
- I can do an experiment many times and look at the frequency of each outcome. For example, I see that a 2 was rolled 4 out of 20 times. That suggests that rolling a 2 is unlikely.
- I can use the number cube to think about the likelihood of events. For example, I see that half of the numbers are even (2, 4, and 6). That suggests that rolling an even number is equally likely as not.

Lesson Practice 8.01

Lesson Summary

How likely is an **event** to occur? One way to learn more is to look at the outcomes of **experiments**. An **outcome** is any of the possible results that can happen when you perform an experiment at **random**.

For example, when you spin a spinner with 10 equal sections labeled 1–10, one possible outcome is that the spinner will land on the number 5. You can describe the likelihood of events using these phrases: impossible, unlikely, equally likely as not, likely, or certain.

Event	How likely is it?
Lands on 5	Unlikely
Lands on an even number	Equally likely as not
Lands on a number that is *not* 5	Likely
Lands on a whole number	Certain
Lands on 12	Impossible

Unit 8 Lesson 1 574 Synthesis | Lesson Practice

Lesson Takeaway: The likelihood of **events** can be described using the words impossible, unlikely, equally likely as not, likely, or certain. The results of an **experiment** can be used to help determine the likelihood of events. Analyzing the tools used to conduct an experiment can also be helpful to more precisely describe the likelihood.

Independent | 5 min

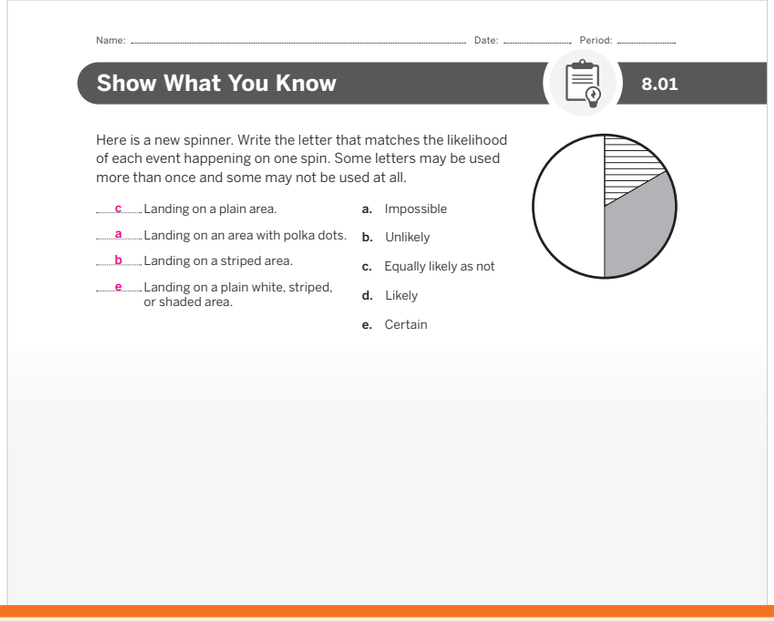
Show What You Know

Purpose: Students demonstrate their understanding by determining the likelihood of spinning a named section on a spinner.

Today's Goals

1. **Language Goal:** Describe the likelihood of *events* using the words *impossible, unlikely, equally likely as not, likely,* or *certain*. **(Reading and Writing)**
2. **Language Goal:** Explain how *experiments* can help determine the likelihood of future *events*. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 8.01

Here is a new spinner. Write the letter that matches the likelihood of each event happening on one spin. Some letters may be used more than once and some may not be used at all.

_c_ Landing on a plain area.
_a_ Landing on an area with polka dots.
_b_ Landing on a striped area.
_e_ Landing on a plain white, striped, or shaded area.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the likelihood of an event:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop fluency with these concepts in Lesson 2. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students described the likelihood of **events** occurring. How did students' work in the stations surprise you today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are the results of a number cube that was rolled 20 times.

Results

Explain how you can determine the likelihood of events. Use the example if it helps with your thinking.

Responses vary.

- I can do an experiment many times and look at the frequency of each outcome. For example, I see that a 2 was rolled 4 out of 20 times. That suggests that rolling a 2 is unlikely.
- I can use the number cube to think about the likelihood of events. For example, I see that half of the numbers are even (2, 4, and 6). That suggests that rolling an even number is equally likely as not.

Lesson Practice 8.01

Lesson Summary

How likely is an **event** to occur? One way to learn more is to look at the outcomes of **experiments**. An **outcome** is any of the possible results that can happen when you perform an experiment at **random**.

For example, when you spin a spinner with 10 equal sections labeled 1–10, one possible outcome is that the spinner will land on the number 5. You can describe the likelihood of events using these phrases: impossible, unlikely, equally likely as not, likely, or certain.

Event	How likely is it?
Lands on 5	Unlikely
Lands on an even number	Equally likely as not
Lands on a number that is *not* 5	Likely
Lands on a whole number	Certain
Lands on 12	Impossible



Lesson Practice 8.01

Name: _____ Date: _____ Period: _____

Problems 1–5: Write the letter that matches the likelihood of each event.



Use the image if it helps with your thinking.

1. a Rolling a 7.

2. d Rolling a number less than 5.

3. b Rolling a 3.

4. c Rolling an even number.

5. e Rolling a number greater than 0.

a. Impossible

b. Unlikely

c. Equally likely as not

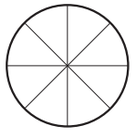
d. Likely

e. Certain

6. Label the sections on the spinner so that it has all of these likelihoods in one spin.

- Equally likely as not to spin green (G).
- Impossible to spin blue (B).
- Very unlikely to spin yellow (Y).
- Unlikely to spin red (R).

The spinner should contain four green sections, three red sections, one yellow section, and zero blue sections.



7. Sahana is curious how many of her classmates watch her favorite T.V. show, so she starts asking around at lunch. She gets the following responses:

yes yes yes no no no no no no yes no no no yes

If she asks one more person randomly in the cafeteria, do you think they will say "yes" or "no"? Explain your thinking.

***Responses vary.* I think they will say "no" because most people so far have said "no."**

Lesson Practice 8.01

Name: _____ Date: _____ Period: _____

FAST Practice

8. A bag contains 14 marbles. There are 6 blue, 7 yellow, and 1 red marble. One marble is selected at random. Determine whether each statement is true or false.

Statement	True	False
It's impossible that a green marble will be selected.	☑	☐
It's unlikely that a yellow marble will be selected.	☐	☑
It's certain that a blue marble will be selected.	☐	☑
It's unlikely that a red marble will be selected.	☑	☐

Spiral Review

Problems 9–12: Evaluate each expression.

9. $\left(\frac{1}{2}\right)^2 \cdot 8$

2

10. $3^2 + 2^3$

17

11. $(2 \cdot 3)^2$

36

12. $\left(\frac{1}{3}\right)^2 \cdot 3^2$

1

Lesson Practice Item Analysis

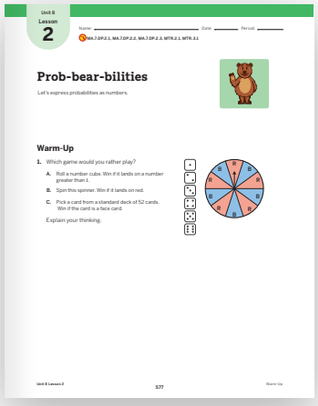
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	1	MA.7.DP.2.2
	6	3	MA.7.DP.2.2
	7	2	MA.7.DP.2.2
FAST Practice	8	1	MA.7.DP.2.2
Spiral Review			
	9–12	1	MA.7.NSO.2.1

Unit 8
Lesson
2

Prob-bear-bilities

Intro to Probability

Let's express probabilities as numbers.


Unit 8
Lesson
2
Name: _____ Date: _____ Period: _____
MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1
Prob-bear-bilities
Let's express probabilities as numbers.
Warm-Up
1. Which game would you rather play?
A. Roll a number cube. Win if it lands on a number greater than 1.
B. Spin this spinner. Win if it lands on red.
C. Pick a card from a standard deck of 52 cards. Win if the card is a face card.
Explain your thinking.
Unit 8 Lesson 2
577
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Understand *probability* as a number between 0 and 1 that represents the likelihood of an event occurring.
2. **Goal:** Compare the *probability* of events using fractions, decimals, and percentages.
3. **Language Goal:** Explain how to use a *sample space* to determine the *probability* of an event. **(Reading and Writing)**

Students are introduced to **probability** as a measure of the likelihood of an event. They make sense of probabilities using decimals, fractions, and *percentages*, and use **sample spaces** to help determine these probabilities.

Coherence

Prior Learning

In Lesson 1, students described the likelihood of events in words.

Future Learning

In Lesson 3, students will use repeated experiments and proportionality to make predictions and recognize that the number of repeated experiments affects the accuracy of their predictions.

Connections

- Students explore the likelihood of events happening, and understanding sample spaces to determine those probabilities such as spinning a spinner with five equal sections. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.1
Determine the sample space for a simple experiment.

MA.7.DP.2.2
Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.5	MA.8.DP.2.1 MA.8.DP.2.2 MA.8.DP.2.3

Vocabulary

New Vocabulary

probability
sample space

Review Vocabulary

percentage

Lesson at a Glance

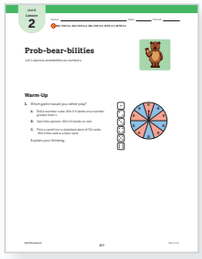
~ 45 min

MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3

Warm-Up

Independent | 5 min

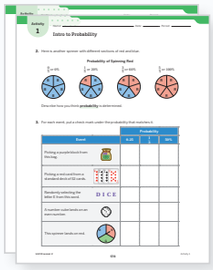
Students surface ideas about likelihood to decide which game they have a better chance of winning.



Activity 1

Pairs | 10 min

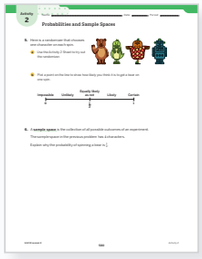
Students calculate the **probability** of an event and compare the probability of events expressed as fractions, decimals, and *percentages*. **(MTR.2.1)**



Activity 2

Pairs | 5 min

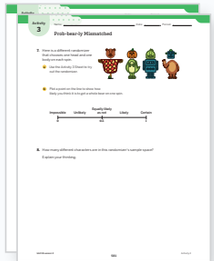
Students are introduced to the term **sample space** and use a sample space to determine the probability of an event.



Activity 3

Pairs | 15 min

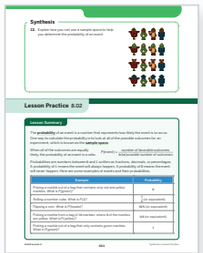
Students use a sample space to determine the probability of an event.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using a sample space to determine the probability of an event. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the probability of choosing a particular item when choosing at random from a given menu.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Coloring tools (optional)
  Paper clips for spinners, coins, number cubes (as needed)

Independent | 5 min

Warm-Up

Purpose: Students surface ideas about likelihood to decide which game they have a better chance of winning.

1 Launch

To surface connections, consider asking if students have ever played a game that involved number cubes, spinners, or cards.

Consider sharing that a standard deck of 52 playing cards has the following characteristics:

- There are 4 suits: hearts, diamonds, clubs, and spades
- Hearts and diamonds are red, clubs and spades are black.
- There are 13 cards of each suit: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king.
- The jack, queen, and king are called face cards.

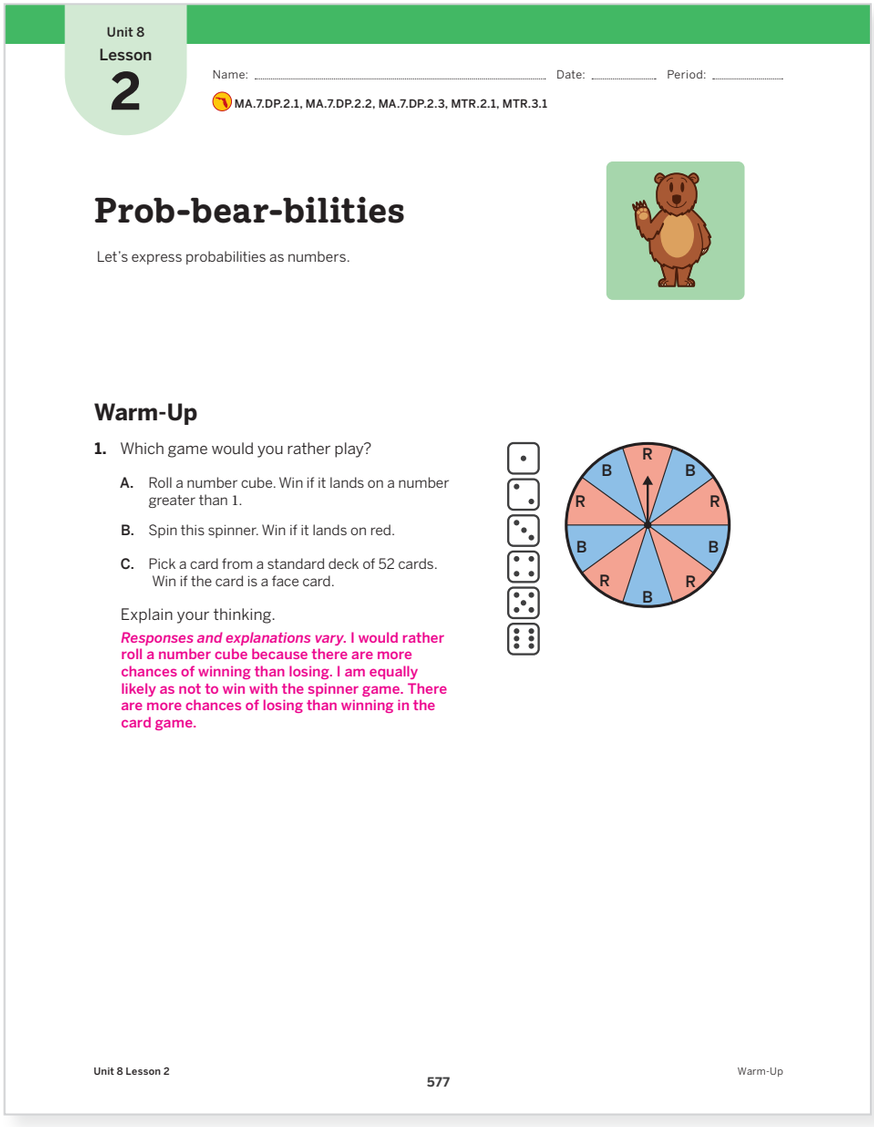
To generate engagement, invite students to imagine themselves at a fair where they get to choose which game in the Warm-Up to play to win something they've wanted.

2 Connect

Invite students to share their reasoning for which game they chose.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I would rather play _____ because _____.).

Consider asking, "What is the likelihood you would win the number cube game? The spinner game? The card game?"



Unit 8
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1

Prob-bear-bilities

Let's express probabilities as numbers.

Warm-Up

1. Which game would you rather play?

   A. Roll a number cube. Win if it lands on a number greater than 1.

   B. Spin this spinner. Win if it lands on red.

   C. Pick a card from a standard deck of 52 cards. Win if the card is a face card.

Explain your thinking.

Responses and explanations vary. I would rather roll a number cube because there are more chances of winning than losing. I am equally likely as not to win with the spinner game. There are more chances of losing than winning in the card game.

Unit 8 Lesson 2 577 Warm-Up

Pairs | 10 min

Activity 1 Intro to Probability

Purpose: Students calculate the **probability** of an event and compare the probability of events expressed as fractions, decimals, and *percentages*. **(MTR.2.1)**

Short on time? Consider having students complete only 1–2 challenges in Problem 4.

1 Launch

Invite students to study each spinner and ask what they notice about how the fraction and *percentage* were determined. **(MTR.2.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think probability is determined by _____.).

Create a class definition for the term **probability** using students' language and glossary. Consider sharing that probability is another way to describe the chance of an event happening.

2 Monitor

To support students getting started, invite them to write a list of the possible outcomes for each experiment.

Look for students who convert the probabilities to one form (decimal, fraction, or percentage).

A **Accessibility: Conceptual Processing** To support abstract thinking, provide probability tools such as coins or number cubes.

Activity 1 continued >

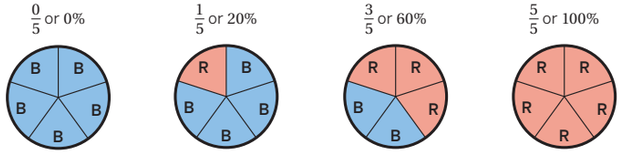
Activity 1

Name: _____ Date: _____ Period: _____

Intro to Probability

2. Here is another spinner with different sections of red and blue.

Probability of Spinning Red



Describe how you think **probability** is determined.

Responses vary. Probability is the number of red sections divided by the total number of sections, which is always 5 for this spinner.

3. For each event, put a check mark under the probability that matches it.

Event	Probability: 0.25	Probability: 1/3	Probability: 50%
Picking a purple block from this bag. 	✓		
Picking a red card from a standard deck of 52 cards. 			✓
Randomly selecting the letter E from this word. DICE	✓		
A number cube lands on an even number. 			✓
This spinner lands on red. 		✓	

Pairs | 10 min

Activity 1 Intro to Probability (continued)

Purpose: Students calculate the **probability** of an *event* and compare the probability of events expressed as fractions, decimals, and *percentages*. **(MTR.2.1)**

Short on time? Consider having students complete only 1–2 challenges in Problem 4.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation

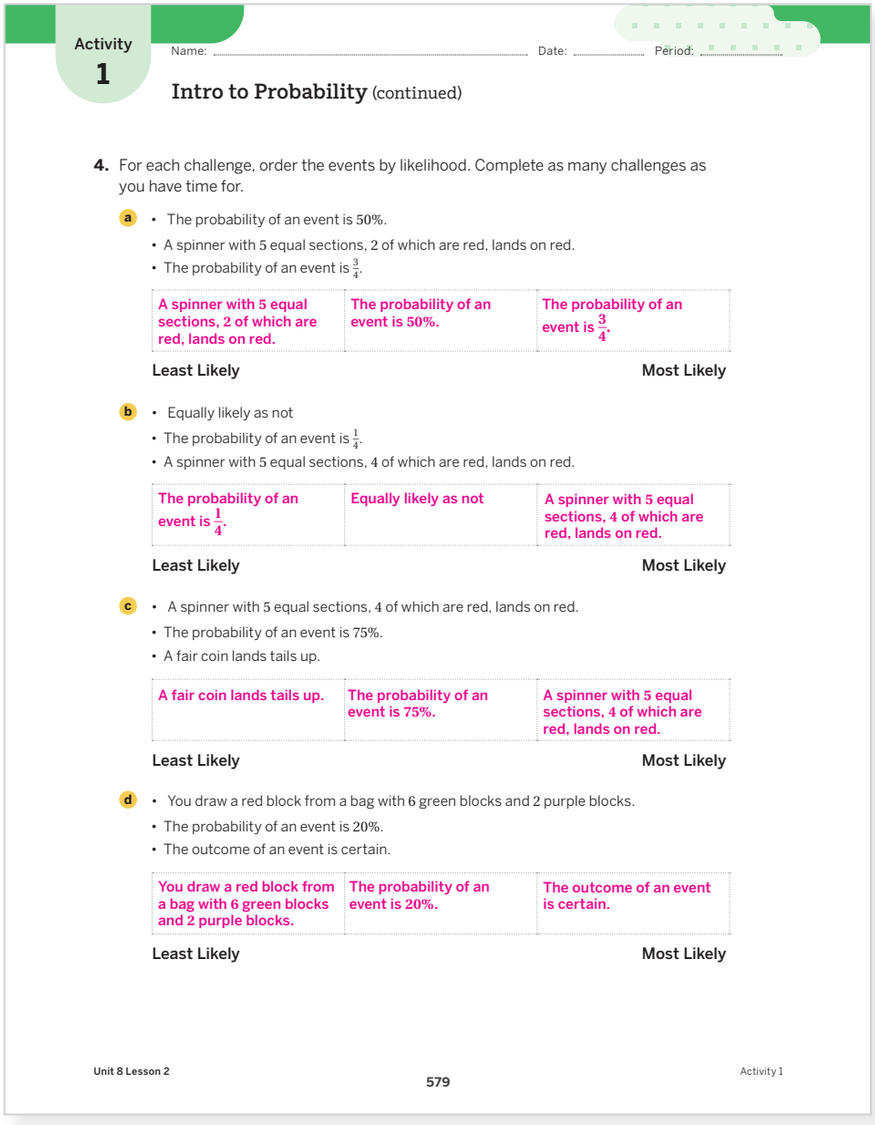
Look for students who:	Respond to Student Thinking
Use estimation or use words such as *unlikely*, *equally likely as not*, or *likely* to describe events.	**Strengthen** Consider asking, "How did your strategy help you compare the probabilities?"
Convert between forms (fractions, decimals, percentages). **(MTR.2.1)**	
Would like to explore ordering events by likelihood further.	**Stretch** Invite students to create their own challenge similar to the ones in Problem 4, and trade with a partner to complete each other's challenge.

3 Connect

Invite students to share their responses and the strategies they used to justify their placements and make revisions based on the discussion. **(MTR.3.1)**

Consider asking:

- "When is it helpful to convert between fractions, decimals, and percentages?"
- "Do you prefer using probabilities expressed as fractions, decimals, or percents to compare the likelihood of events? Why?"
- "How can you compare the likelihoods of events?"



Activity 1

Name: Date: Period:

Intro to Probability (continued)

4. For each challenge, order the events by likelihood. Complete as many challenges as you have time for.

a
- The probability of an event is 50%.
- A spinner with 5 equal sections, 2 of which are red, lands on red.
- The probability of an event is $\frac{3}{4}$.

A spinner with 5 equal sections, 2 of which are red, lands on red.	The probability of an event is 50%.	The probability of an event is $\frac{3}{4}$.

Least Likely — Most Likely

b
- Equally likely as not
- The probability of an event is $\frac{1}{4}$.
- A spinner with 5 equal sections, 4 of which are red, lands on red.

The probability of an event is $\frac{1}{4}$.	Equally likely as not	A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely — Most Likely

c
- A spinner with 5 equal sections, 4 of which are red, lands on red.
- The probability of an event is 75%.
- A fair coin lands tails up.

A fair coin lands tails up.	The probability of an event is 75%.	A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely — Most Likely

d
- You draw a red block from a bag with 6 green blocks and 2 purple blocks.
- The probability of an event is 20%.
- The outcome of an event is certain.

You draw a red block from a bag with 6 green blocks and 2 purple blocks.	The probability of an event is 20%.	The outcome of an event is certain.

Least Likely — Most Likely

Unit 8 Lesson 2 579 Activity 1

Pairs | 5 min

Activity 2 Probabilities and Sample Spaces

Purpose: Students are introduced to the term **sample space** and use a sample space to determine the probability of an event.

1 Launch

Invite students to make a prediction about how likely it is to get a bear on one spin before trying the randomizer.

Distribute the Activity 2 sheet and a paper clip to each pair of students.

2 Monitor

Listen for detailed or creative explanations.

D **Differentiation: (Support)** Students may think the probability will change if there is more than one spin. Consider asking, "If you flip a coin with heads and tails more than once, does the probability change?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., The probability of spinning a bear is $\frac{1}{4}$ or 25% because _____.).

3 Connect

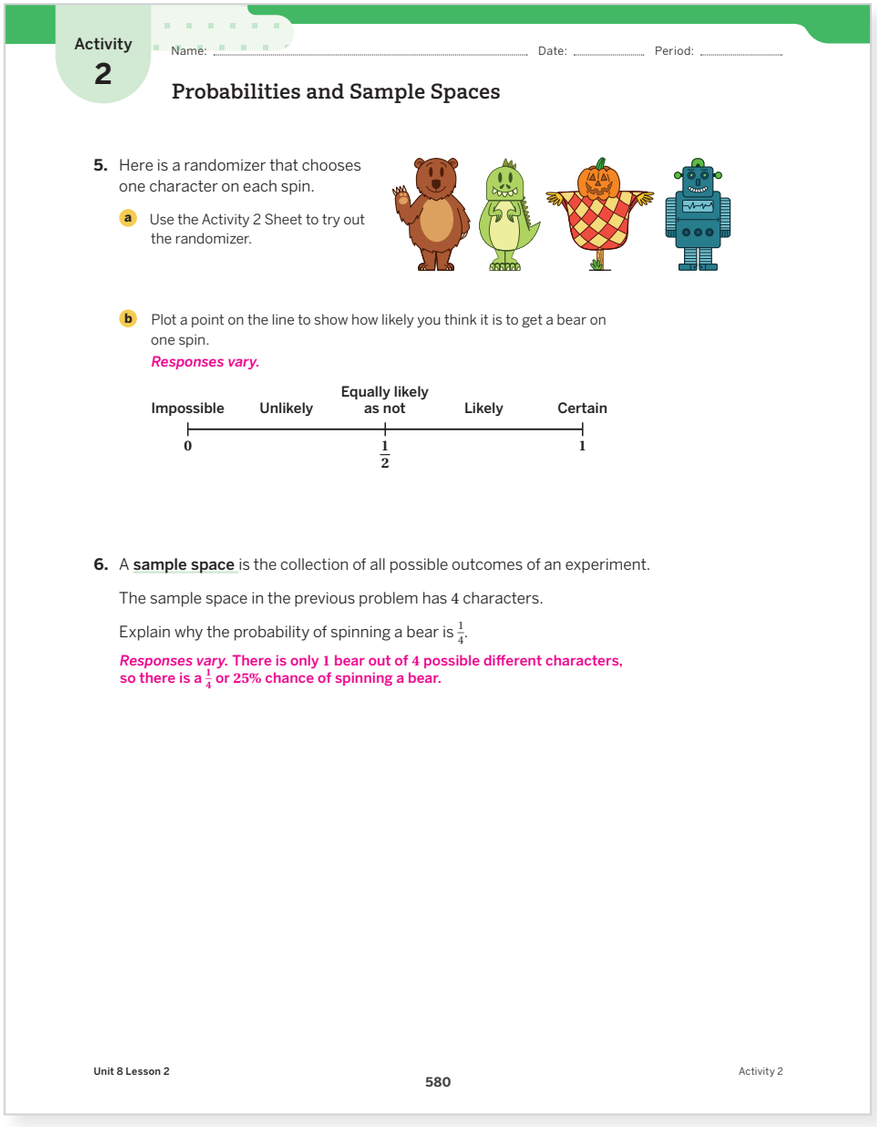
Invite students to share their explanations.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's explanations, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Consider asking:

- "What is the probability of spinning a character that is not a bear?"
- "What is the probability of spinning a character that has two legs? Two arms?"
- "How would you express these probabilities as a decimal? As a percentage?"

To surface the Key Takeaway, consider asking, "How does the sample space relate to the probability?"



Activity 2

Name: _____ Date: _____ Period: _____

Probabilities and Sample Spaces

5. Here is a randomizer that chooses one character on each spin.

a. Use the Activity 2 Sheet to try out the randomizer.

b. Plot a point on the line to show how likely you think it is to get a bear on one spin.

Responses vary.

Impossible — Unlikely — Equally likely as not — Likely — Certain

0 — $\frac{1}{2}$ — 1

6. A **sample space** is the collection of all possible outcomes of an experiment.

The sample space in the previous problem has 4 characters.

Explain why the probability of spinning a bear is $\frac{1}{4}$.

Responses vary. There is only 1 bear out of 4 possible different characters, so there is a $\frac{1}{4}$ or 25% chance of spinning a bear.

Unit 8 Lesson 2 580 Activity 2

Key Takeaway: The **sample space** of an experiment is the collection of all possible outcomes. The **probability** can be determined using the sample space and can be represented by a number between 0 and 1.

Pairs | 15 min

Activity 3 Prob-bear-ly Mismatched

Purpose: Students use the sample space to determine the probability of an event.

1 Launch

To surface connections, consider discussing how this problem and the one from Problem 5 are alike.

Distribute the Activity 3 sheet, a paper clip, and optional coloring tools to each pair of students.

Consider asking, "Will there be more or less than four characters in the sample space? Why?"

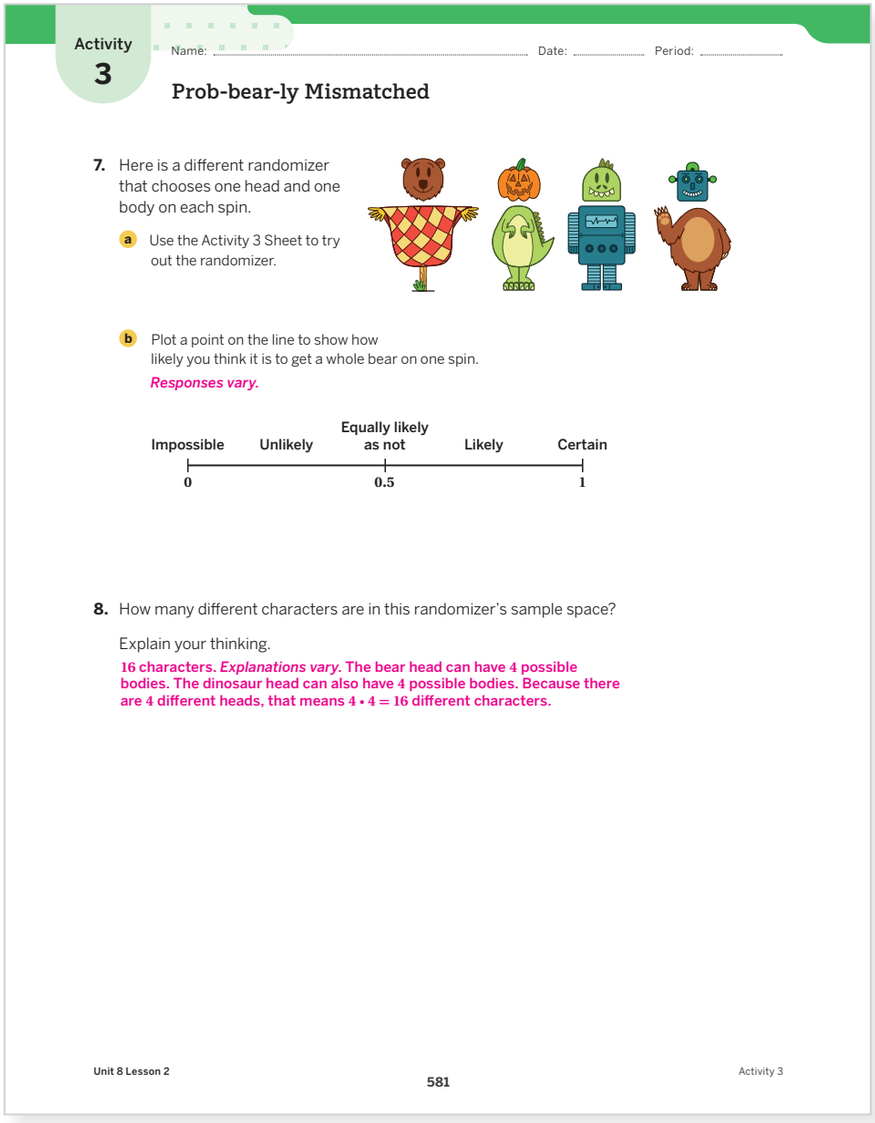
2 Monitor

Look for students who use drawings, organized lists, or other tools to help them determine the size of the sample space. **(MTR.2.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., I counted the different creatures by _____.).

Pause briefly to allow students to share their strategies and how they are alike or different from other students' strategies.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Prob-bear-ly Mismatched

7. Here is a different randomizer that chooses one head and one body on each spin.

a. Use the Activity 3 Sheet to try out the randomizer.

b. Plot a point on the line to show how likely you think it is to get a whole bear on one spin.

Responses vary.

Impossible — Unlikely — Equally likely as not — Likely — Certain

0 — 0.5 — 1

8. How many different characters are in this randomizer's sample space?

Explain your thinking.

16 characters. *Explanations vary.* The bear head can have 4 possible bodies. The dinosaur head can also have 4 possible bodies. Because there are 4 different heads, that means $4 \cdot 4 = 16$ different characters.

Unit 8 Lesson 2 581 Activity 3

Pairs | 15 min

Activity 3 Prob-bear-ly Mismatched (continued)

Purpose: Students use the sample space to determine the probability of an event.

2 Monitor

Listen for students making use of the structure of the sample space to organize their thinking about the missing character.

Select and sequence students' ideas to share during the Connect.

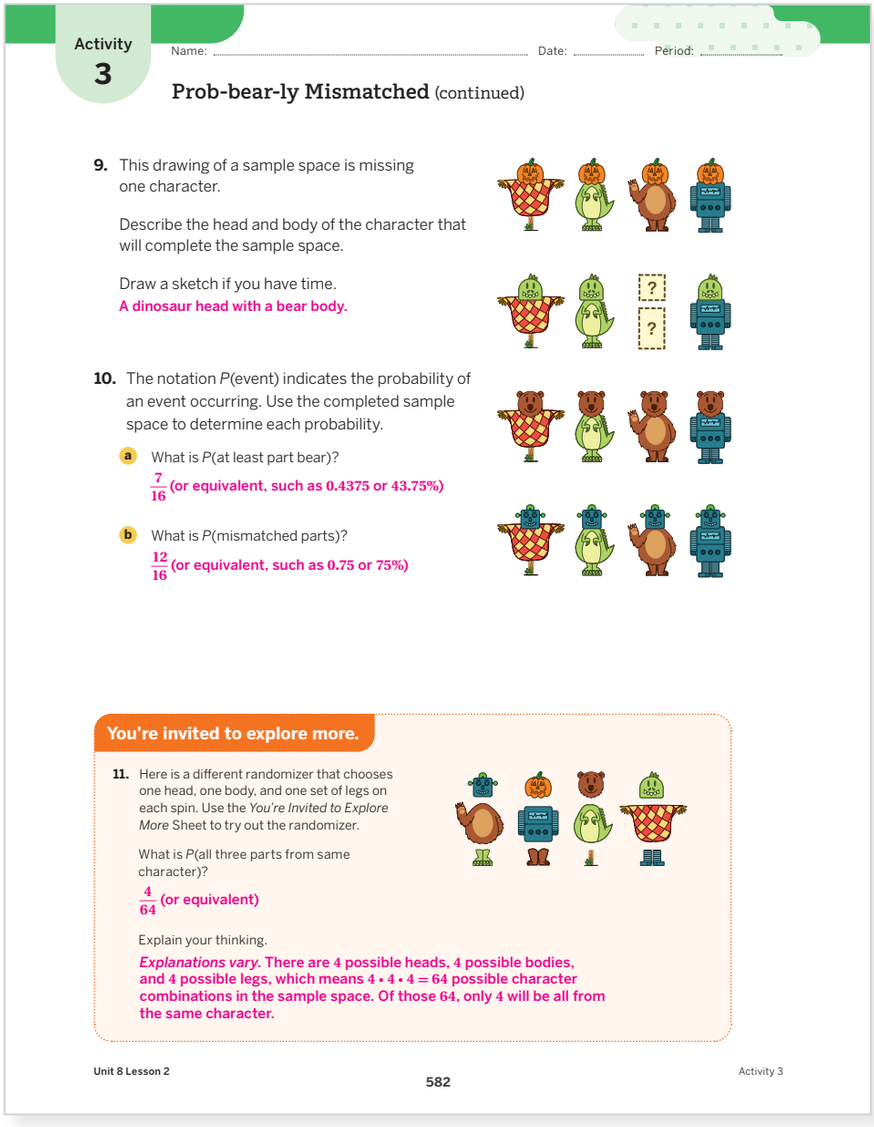
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is the probability of getting a whole bear on one spin?"
Need support reading probability notation.	**Support** Encourage students to write out the event in words. For example, *P*(at least part bear) means "the probability of spinning a character that is at least part bear."
Create a sketch to show their thinking.	**Strengthen** Invite them to share how this strategy helped to determine the probability of each event occurring.
Write their probabilities in different forms (decimals, fractions, percentages). **(MTR.2.1)**	**Strengthen** Invite students to share when it was more efficient to express the probability as a fraction, decimal, or percentage and why.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore probability using sample space further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their sketches from Problem 9, and discuss strategies they used to determine the probability. Invite students to share their responses and reasoning.



Activity 3

Name: Date: Period:

Prob-bear-ly Mismatched (continued)

9. This drawing of a sample space is missing one character.

Describe the head and body of the character that will complete the sample space.

Draw a sketch if you have time.

A dinosaur head with a bear body.

10. The notation *P*(event) indicates the probability of an event occurring. Use the completed sample space to determine each probability.

a. What is *P*(at least part bear)?

$\frac{7}{16}$ **(or equivalent, such as 0.4375 or 43.75%)**

b. What is *P*(mismatched parts)?

$\frac{12}{16}$ **(or equivalent, such as 0.75 or 75%)**

You're invited to explore more.

11. Here is a different randomizer that chooses one head, one body, and one set of legs on each spin. Use the *You're Invited to Explore More* Sheet to try out the randomizer.

What is *P*(all three parts from same character)?

$\frac{4}{64}$ **(or equivalent)**

Explain your thinking.

Explanations vary. **There are 4 possible heads, 4 possible bodies, and 4 possible legs, which means $4 \cdot 4 \cdot 4 = 64$ possible character combinations in the sample space. Of those 64, only 4 will be all from the same character.**

Unit 8 Lesson 2 582 Activity 3

Whole Class | 5 min

Synthesis

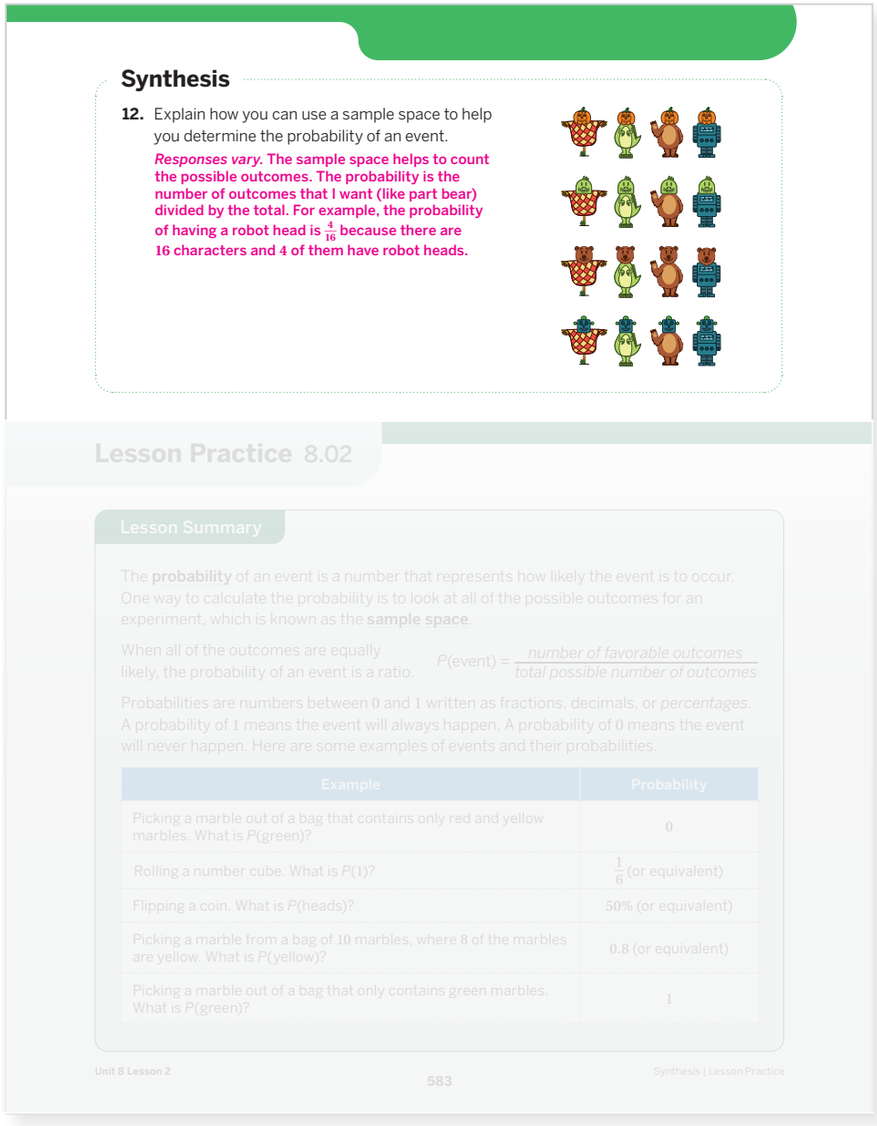
Purpose: Students synthesize their understanding of using a sample space to determine the probability of an event.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Counting the total possible outcomes.
- Using a ratio of outcomes desired over the total possible outcomes.
- Using benchmark percentages.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *probability* and *sample space* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to make their response stronger and clearer based on the conversation.



Synthesis

12. Explain how you can use a sample space to help you determine the probability of an event.

Responses vary. The sample space helps to count the possible outcomes. The probability is the number of outcomes that I want (like part bear) divided by the total. For example, the probability of having a robot head is $\frac{4}{16}$ because there are 16 characters and 4 of them have robot heads.

Lesson Practice 8.02

Lesson Summary

The **probability** of an event is a number that represents how likely the event is to occur. One way to calculate the probability is to look at all of the possible outcomes for an experiment, which is known as the **sample space**.

When all of the outcomes are equally likely, the probability of an event is a ratio. $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible number of outcomes}}$

Probabilities are numbers between 0 and 1 written as fractions, decimals, or *percentages*. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen. Here are some examples of events and their probabilities.

Example	Probability
Picking a marble out of a bag that contains only red and yellow marbles. What is $P(\text{green})$?	0
Rolling a number cube. What is $P(1)$?	$\frac{1}{6}$ (or equivalent)
Flipping a coin. What is $P(\text{heads})$?	50% (or equivalent)
Picking a marble from a bag of 10 marbles, where 8 of the marbles are yellow. What is $P(\text{yellow})$?	0.8 (or equivalent)
Picking a marble out of a bag that only contains green marbles. What is $P(\text{green})$?	1

Unit 8 Lesson 2 583 Synthesis | Lesson Practice

Lesson Takeaway: The **sample space** can be used to determine the **probability** of an event. The probability is the number of options desired divided by the total outcomes in the sample space and can be written as a fraction, decimal, or *percentage*.

Independent | 5 min

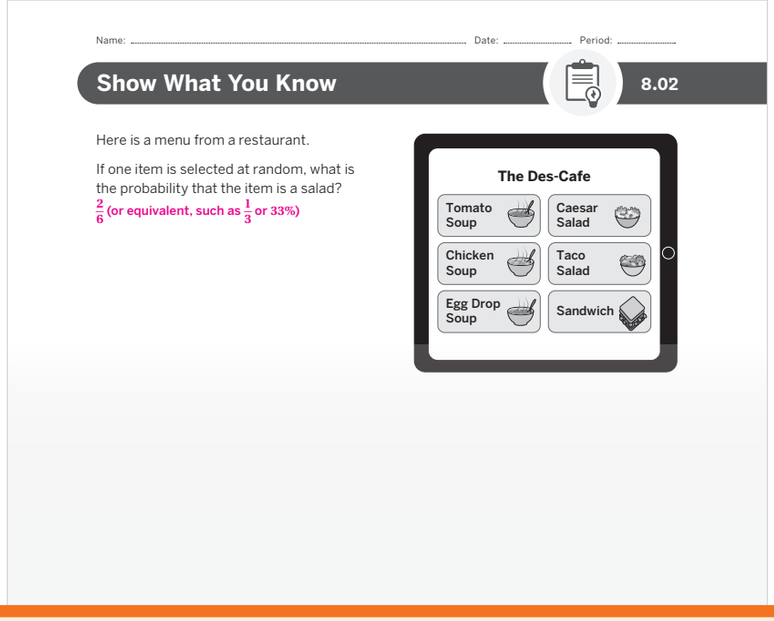
Show What You Know

Purpose: Students demonstrate their understanding by determining the probability of choosing a particular item when choosing at random from a given menu.

Today's Goals

1. **Goal:** Understand *probability* as a number between 0 and 1 that represents the likelihood of an event occurring.
2. **Goal:** Compare the *probability* of events using fractions, decimals, and percentages.
3. **Language Goal:** Explain how to use a *sample space* to determine the *probability* of an event. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 8.02

Here is a menu from a restaurant.

If one item is selected at random, what is the probability that the item is a salad?

$\frac{2}{6}$ (or equivalent, such as $\frac{1}{3}$ or 33%)

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine probabilities using sample space:

» **Respond to Student Thinking:**

- Revisit the definition of sample space in **Activity 2**. Invite students to share strategies for determining the sample space of an event and probability of an outcome.
- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **You're invited to explore more** task.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson.

- What opportunities were there to celebrate specific students' strategies for using **sample space** to determine the **probability**?
- What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Explain how you can use a sample space to help you determine the probability of an event.

Responses vary. The sample space helps to count the possible outcomes. The probability is the number of outcomes that I want (like part bear) divided by the total. For example, the probability of having a robot head is $\frac{4}{16}$ because there are 16 characters and 4 of them have robot heads.

Lesson Practice 8.02

Lesson Summary

The **probability** of an event is a number that represents how likely the event is to occur. One way to calculate the probability is to look at all of the possible outcomes for an experiment, which is known as the **sample space**.

When all of the outcomes are equally likely, the probability of an event is a ratio. $P(\text{event}) = \frac{\textit{number of favorable outcomes}}{\textit{total possible number of outcomes}}$

Probabilities are numbers between 0 and 1 written as fractions, decimals, or *percentages*. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen. Here are some examples of events and their probabilities.

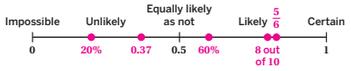
Example	Probability
Picking a marble out of a bag that contains only red and yellow marbles. What is P(green)?	0
Rolling a number cube. What is $P(1)$?	$\frac{1}{6}$ (or equivalent)
Flipping a coin. What is P(heads)?	50% (or equivalent)
Picking a marble from a bag of 10 marbles, where 8 of the marbles are yellow. What is P(yellow)?	0.8 (or equivalent)
Picking a marble out of a bag that only contains green marbles. What is P(green)?	1

Lesson Practice 8.02

Name: _____ Date: _____ Period: _____

1. Place a point on the line to show each probability's likelihood from impossible to certain.

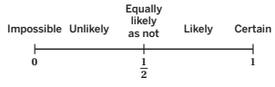
60% 8 out of 10 0.37 20% $\frac{5}{6}$



Problems 2–4: List the sample space for each experiment. For example, the sample space of flipping one fair coin is heads or tails.

2. Selecting a random season of the year.
   winter, spring, summer, fall
3. Selecting a random day of the week.
   Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday
4. Rolling a number cube.
   one, two, three, four, five, six

Problems 5–10: Determine the probability of each event. Use the line if it helps with your thinking.



5. You read this sentence.
   1
6. A fair coin lands heads up when flipped.
   $\frac{1}{2}$ (or equivalent)
7. A weekend day is selected randomly from the days of the week.
   $\frac{2}{7}$ (or equivalent)
8. You see a live unicorn outside today.
   0
9. A spinner with equal parts red, yellow, and green lands on green on the next spin.
   $\frac{1}{3}$ (or equivalent)
10. You pick a green block when you randomly pick one block from a bag with 7 green blocks and 3 blue blocks.
    $\frac{7}{10}$ (or equivalent)

Lesson Practice 8.02

Name: _____ Date: _____ Period: _____

Problems 11–13: A computer randomly selects a letter from the English alphabet.

11. How many different outcomes are possible?
    26 outcomes
12. What is P(A, E, I, O, or U)?
    $\frac{5}{26}$
13. What is P(first letter of your first name)?
    $\frac{1}{26}$

FAST Practice

14. This table shows the number of bags of different flavors of crackers in a large box. A bag will be selected at random from the box.

Flavor of Cracker	Number of Bags
Plain	12
Barbeque	16
Ranch	8
Cheddar Cheese	20

Which statement about the flavor of the crackers chosen is best supported by the information in the table?

A. Plain is the least likely flavor.
B. Choosing barbecue is twice as likely as choosing ranch.
C. There is an equal chance of choosing plain, barbeque, ranch, or cheddar cheese.
D. Choosing cheddar cheese is twice as likely as choosing ranch.

Spiral Review

Problems 15–16: The equation $m = \frac{1}{6}e$ represents the relationship between an object's weight on Earth, e, and the same object's weight on the Moon, m.

15. What does $\frac{1}{6}$ represent in this situation?
    Responses vary. $\frac{1}{6}$ is the constant of proportionality from an object's weight on Earth to its weight on the Moon. Something that weighs 1 pound on Earth weighs $\frac{1}{6}$ pounds on the Moon.
16. If a person weighs 24 pounds on the Moon, how many pounds would they weigh on Earth?
    144 pounds

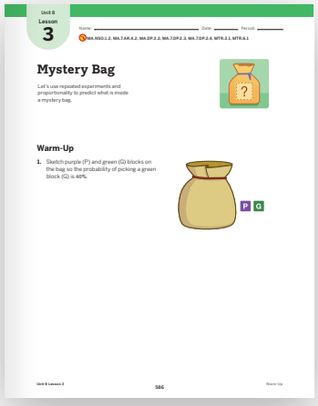
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.2.2
	2–4, 11	2	MA.7.DP.2.1
	5–10, 12–13	2	MA.7.DP.2.2
FAST Practice	14	2	MA.7.DP.2.2
Spiral Review			
	15	2	MA.7.AR.4.4
	16	2	MA.7.AR.4.5

Unit 8

Lesson

3



Mystery Bag

Predicting Sample Spaces

Let's use repeated experiments and proportionality to predict what is inside a mystery bag.

Learning Outcomes

Today's Goals

1. **Goal:** Use proportional reasoning and data from a repeated experiment to make predictions about the sample space.
2. **Language Goal:** Describe how to use the results from a repeated experiment to make predictions. **(Reading, Writing, Speaking, and Listening)**

Students use repeated experiments and proportionality to predict the contents of a mystery bag, recognizing that the number of repeated experiments affects the accuracy of their prediction. Students encounter situations in which each outcome is not equally likely. This lesson also surfaces important ideas about the difference between probabilities and the results of repeated experiments.

Coherence

Prior Learning

In Lessons 1 and 2, students described the likelihood of an event occurring and used the sample space to determine the probability of an event.

Future Learning

In Lesson 4, students will recognize that the results from repeated experiments approach the probability of the event.

Connections

- Using graphs students predict the contents of a mystery bag using repeated experiments and proportionality. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

MA.7.DP.2.4
Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.

Also Addressing: **MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.DP.2.1**
MA.6.AR.3.5	**MA.8.DP.2.2**
	MA.8.DP.2.3

Vocabulary

Review Vocabulary

constant of proportionality

Lesson at a Glance

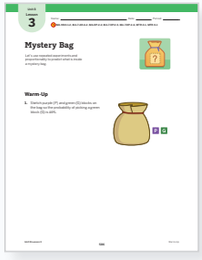
~ 45 min

MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4

Warm-Up

Pairs | 5 min

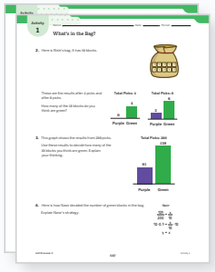
Students connect probability and the number of different colored blocks in a bag.



Activity 1

Pairs | 15 min

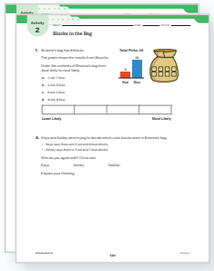
Students use probability and proportional reasoning to predict the sample space of a mystery bag. They recognize that repeated experiments lead to better predictions using the Notice and Wonder routine. **(MTR.6.1)**



Activity 2

Pairs | 15 min

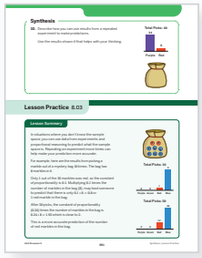
Students use proportional reasoning and the results from repeated experiments to make predictions about the sample space of a mystery bag.



Synthesis

Whole Class | 5 min

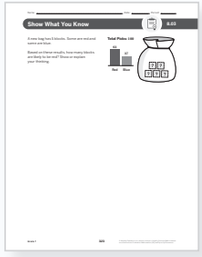
Students synthesize their understanding of using repeated experiments to make predictions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by predicting how many colored blocks are in a bag using the results from a repeated experiment.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools, green and purple (optional)

Pairs | 5 min

Warm-Up

Purpose: Students connect probability and the number of different colored blocks in a bag.

1 Launch

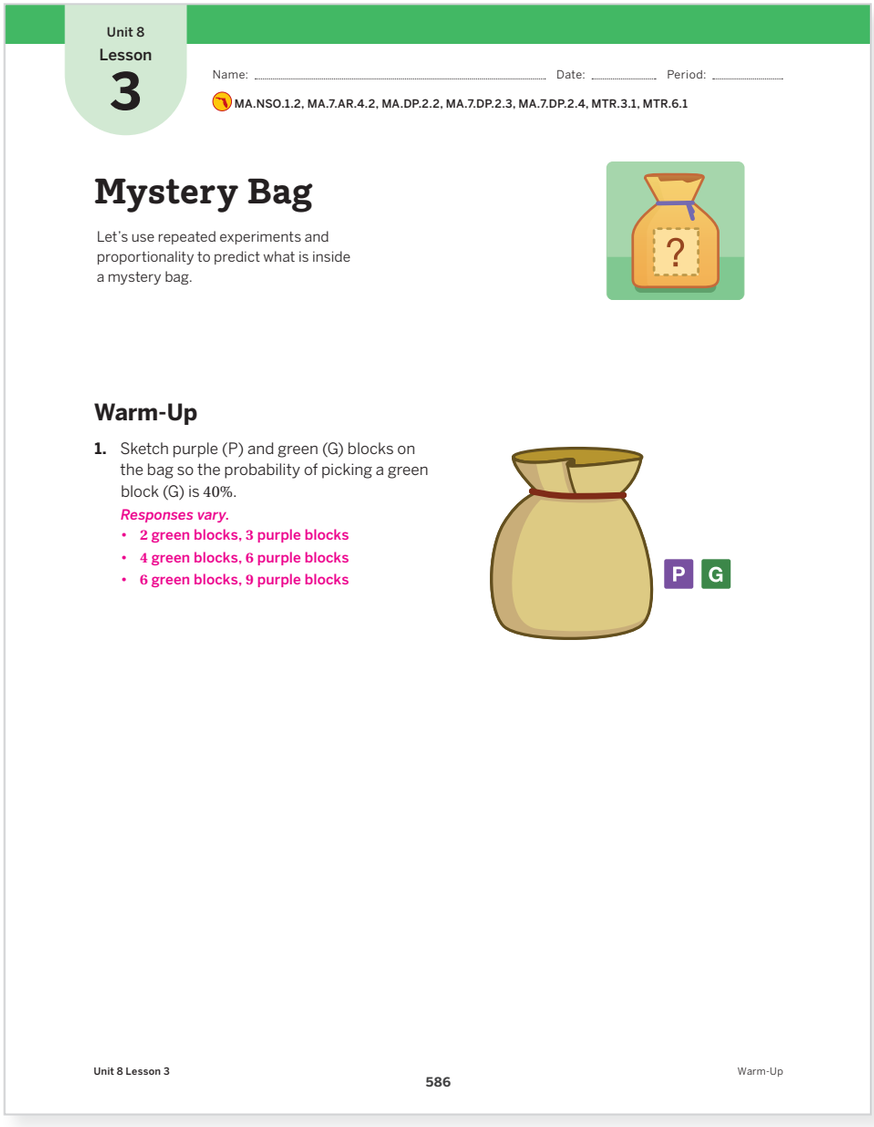
Invite students to use coloring tools, if desired.

Consider asking, "If one purple block and one green block are added to the bag, what would be the probability of picking a green block?"

2 Connect

Collect and share student responses of correct and incorrect bags. For each bag, consider asking, "What is the probability of picking a green block? How do you know?"

Invite students to share their strategies for determining how many purple and green blocks to add to the bag.



Unit 8
Lesson
3

Name: _____ Date: _____ Period: _____

MA.NSO.1.2, MA.7.AR.4.2, MA.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.3.1, MTR.6.1

Mystery Bag

Let's use repeated experiments and proportionality to predict what is inside a mystery bag.

Warm-Up

1. Sketch purple (P) and green (G) blocks on the bag so the probability of picking a green block (G) is 40%.

Responses vary.

- 2 green blocks, 3 purple blocks
- 4 green blocks, 6 purple blocks
- 6 green blocks, 9 purple blocks

Unit 8 Lesson 3 586 Warm-Up

Pairs | 15 min

Activity 1 What's in the Bag?

Purpose: Students use probability and proportional reasoning to predict the sample space of a mystery bag. They recognize that repeated experiments lead to better predictions using the Notice and Wonder routine. **(MTR.6.1)**

1 Launch

Invite students to make a prediction about how many blocks are green.

Note: It is okay if students are unsure at this stage. Students will be shown a strategy for approaching problems like this in Problem 4.

2 Monitor

To support students getting started, consider asking, "How many of the 200 blocks picked were green? How does this relate to how many of the 10 blocks might be green?"

A **Accessibility: Conceptual Processing** Invite students to solve a similar question with friendlier numbers to support recognizing connections between new problems and prior work with proportional reasoning.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think ___ of the 10 blocks are green because _____.).

Pause to discuss Nasir's strategy. Identify any students who used a strategy similar to Nasir's. **(MTR.3.1)**

To surface prior knowledge, consider asking, "What does 0.7 represent in Nasir's strategy?"

Note: If it does not come up naturally, revisit the definition of *constant of proportionality* and its connection to Nasir's strategy.

Activity 1 continued >

Activity 1

Name: Date: Period:

What's in the Bag?

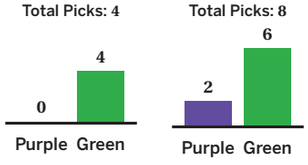
2. Here is Rishi's bag. It has 10 blocks.



These are the results after 4 picks and after 8 picks.

How many of the 10 blocks do you think are green?

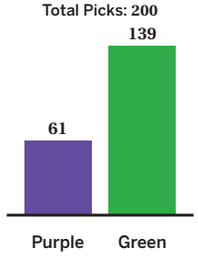
Responses vary.



3. This graph shows the results from 200 picks.

Use these results to decide how many of the 10 blocks you think are green. Explain your thinking.

Responses and explanations vary. 7 blocks. There are 139 green blocks out of 200 total, which is about 70% and 70% of 10 is 7.



4. Here is how Nasir decided the number of green blocks in the bag.

Explain Nasir's strategy.

Responses vary.

- Nasir set up a proportion. He divided 139 by 200 and got 0.7 after rounding. Then he multiplied the number of blocks in the bag (10) by 0.7 to determine the number of green blocks.
- Nasir calculated a constant of proportionality of about 0.7. To determine the number of green blocks, he multiplied the number of blocks in the bag (10) by the constant of proportionality.

Nasir

$$\frac{139}{200} = \frac{x}{10}$$

$$10 \cdot 0.7 = \frac{x}{10} \cdot 10$$

$$7 = x$$

Pairs | 15 min

Activity 1 What's in the Bag? (continued)

Purpose: Students use probability and proportional reasoning to predict the sample space of a mystery bag. They recognize that repeated experiments lead to better predictions using the Notice and Wonder routine. **(MTR.6.1)**

2 Monitor

Use the Notice and Wonder routine to promote curiosity and help students make sense of the two dot plots.

Pause to create a record of the things students noticed and wondered, along with the name of the student who shared each one.

Consider asking, "Did anyone notice a more similar response to Nasir's answer after 200 picks? Does this make you wonder about what happens to the data after more picks?" **(MTR.6.1)**

Direct students' attention to the contents of the bag.

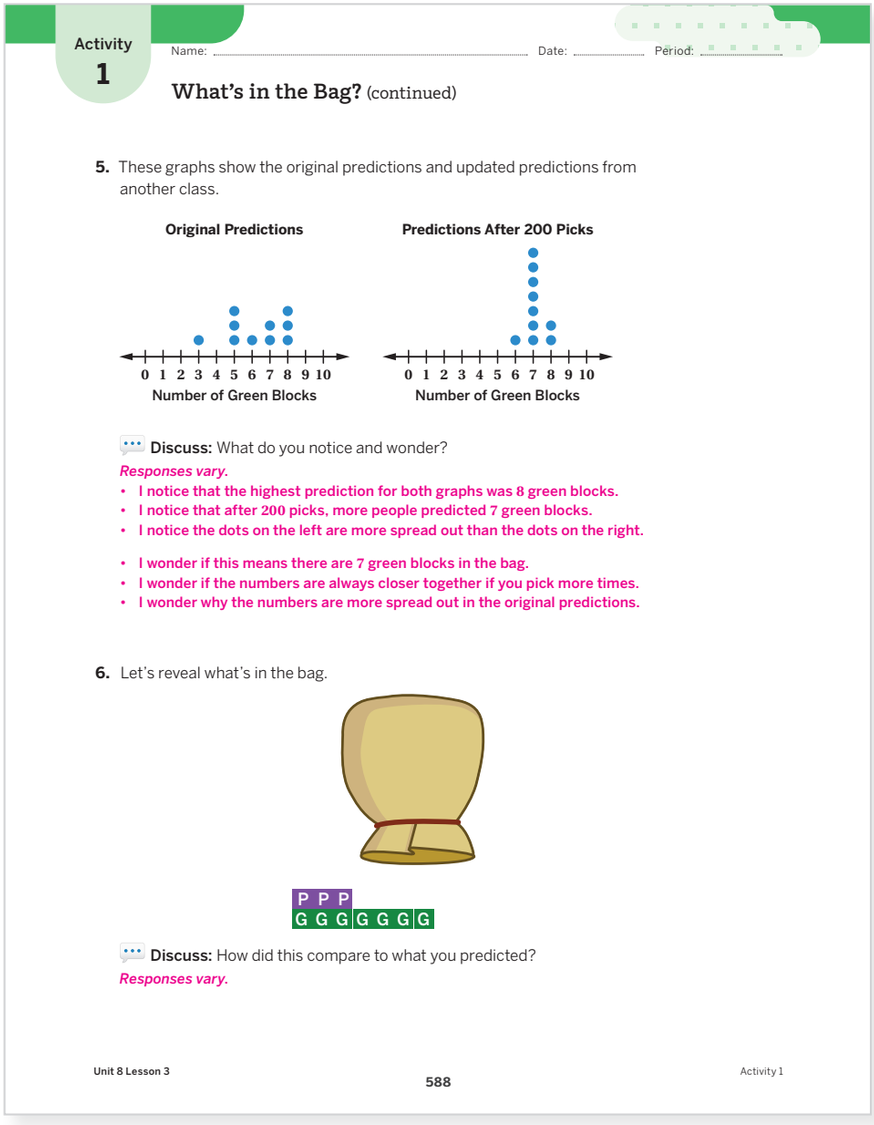
Encourage students to record their thinking.

3 Connect

Consider asking:

- "How did the predictions change after 200 picks? Why?"
- "What do these graphs tell you about the number of green blocks in the bag?"
- "Which graph would you rather use to decide? Why?"

Invite students to share if the number of green blocks matches their predictions. **(MTR.6.1)**



Activity 1

Name: Date: Period:

What's in the Bag? (continued)

5. These graphs show the original predictions and updated predictions from another class.

Original Predictions

Number of Green Blocks

Predictions After 200 Picks

Number of Green Blocks

Discuss: What do you notice and wonder?

Responses vary.

- I notice that the highest prediction for both graphs was 8 green blocks.
- I notice that after 200 picks, more people predicted 7 green blocks.
- I notice the dots on the left are more spread out than the dots on the right.

- I wonder if this means there are 7 green blocks in the bag.
- I wonder if the numbers are always closer together if you pick more times.
- I wonder why the numbers are more spread out in the original predictions.

6. Let's reveal what's in the bag.

P P P
G G G G G G

Discuss: How did this compare to what you predicted?

Responses vary.

Unit 8 Lesson 3 588 Activity 1

Pairs | 15 min

Activity 2 Blocks in the Bag

Purpose: Students use proportional reasoning and the results from repeated experiments to make predictions about the sample space of a mystery bag.

Short on time? Consider completing only 1–2 challenges in Problem 9.

1 Launch

To surface prior knowledge, consider asking, "When making comparisons, what does it mean for something to be *more likely* to occur?"

2 Monitor

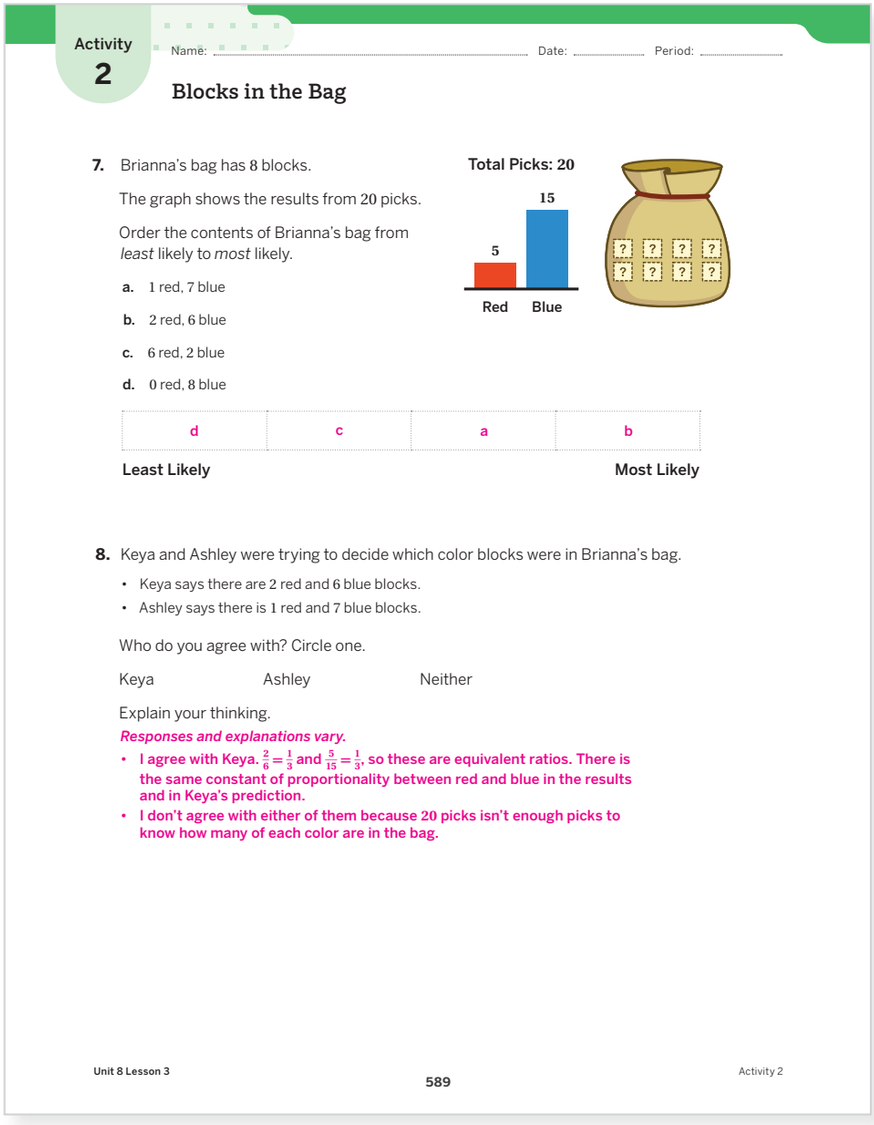
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to use proportional reasoning to order the contents of the bag. Consider asking, "If 5 of the 20 picks are red, what percentage is that?"
Calculate a constant of proportionality.	**Strengthen** Invite them to share how this helped them decide which options are more likely than others during the Connect.
Compare ratios.	

Collect a variety of student responses to share during the Connect.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because _____.).

Activity 2 continued >



Activity 2

Name: Date: Period:

Blocks in the Bag

7. Brianna's bag has 8 blocks.

The graph shows the results from 20 picks.

Order the contents of Brianna's bag from *least* likely to *most* likely.

a. 1 red, 7 blue

b. 2 red, 6 blue

c. 6 red, 2 blue

d. 0 red, 8 blue

d	c	a	b
Least Likely			Most Likely

8. Keya and Ashley were trying to decide which color blocks were in Brianna's bag.

- Keya says there are 2 red and 6 blue blocks.
- Ashley says there is 1 red and 7 blue blocks.

Who do you agree with? Circle one.

Keya Ashley Neither

Explain your thinking.

Responses and explanations vary.

- I agree with Keya. $\frac{2}{6} = \frac{1}{3}$ and $\frac{5}{15} = \frac{1}{3}$, so these are equivalent ratios. There is the same constant of proportionality between red and blue in the results and in Keya's prediction.
- I don't agree with either of them because 20 picks isn't enough picks to know how many of each color are in the bag.

Unit 8 Lesson 3 589 Activity 2

Pairs | 15 min

Activity 2 Blocks in the Bag (continued)

Purpose: Students use proportional reasoning and the results from repeated experiments to make predictions about the sample space of a mystery bag.

Short on time? Consider completing only 1–2 challenges in Problem 9.

2 Monitor

Listen for student strategies and partner discussions, offering help or encouragement where needed.

3 Connect

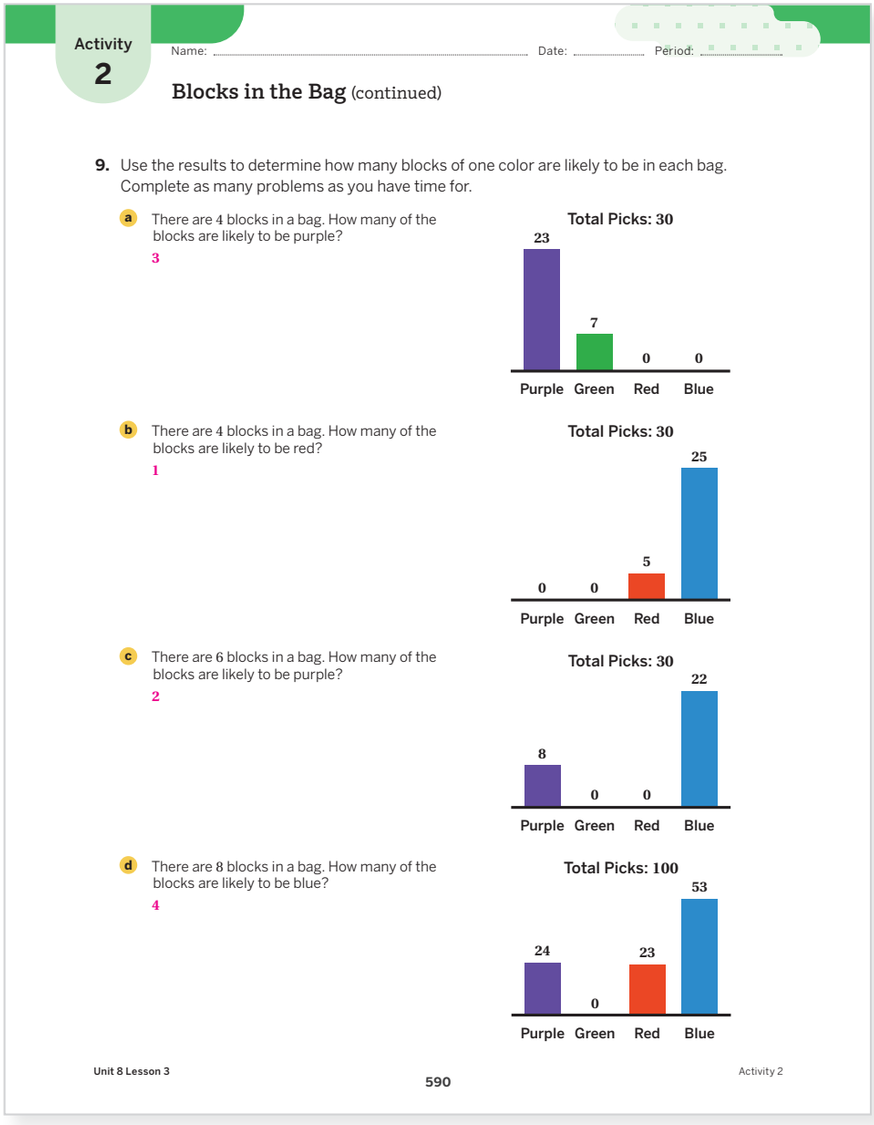
Invite students to compare their ordering with another pair of students and justify their reasoning for which descriptions were most and least likely. **(MTR.3.1)**

Consider asking, "Are this many picks enough to make you confident in your ordering?"

Share student responses, calling attention to any conflict or consensus. Invite students to share their reasoning for their choice.

Note: If it does not come up naturally, consider asking, "Are this many picks enough to know who is correct? Why or why not?"

To surface the Key Takeaway, consider asking, "How does the constant of proportionality help you to make a prediction about the contents of the bag?"



Activity 2

Name: Date: Period:

Blocks in the Bag (continued)

9. Use the results to determine how many blocks of one color are likely to be in each bag. Complete as many problems as you have time for.

a. There are 4 blocks in a bag. How many of the blocks are likely to be purple?

3

b. There are 4 blocks in a bag. How many of the blocks are likely to be red?

1

c. There are 6 blocks in a bag. How many of the blocks are likely to be purple?

2

d. There are 8 blocks in a bag. How many of the blocks are likely to be blue?

4

Unit 8 Lesson 3 590 Activity 2

Key Takeaway: You can use proportional reasoning to make predictions from an experiment. These predictions are likely to be more accurate when the experiment is repeated many times.

Whole Class | 5 min

Synthesis

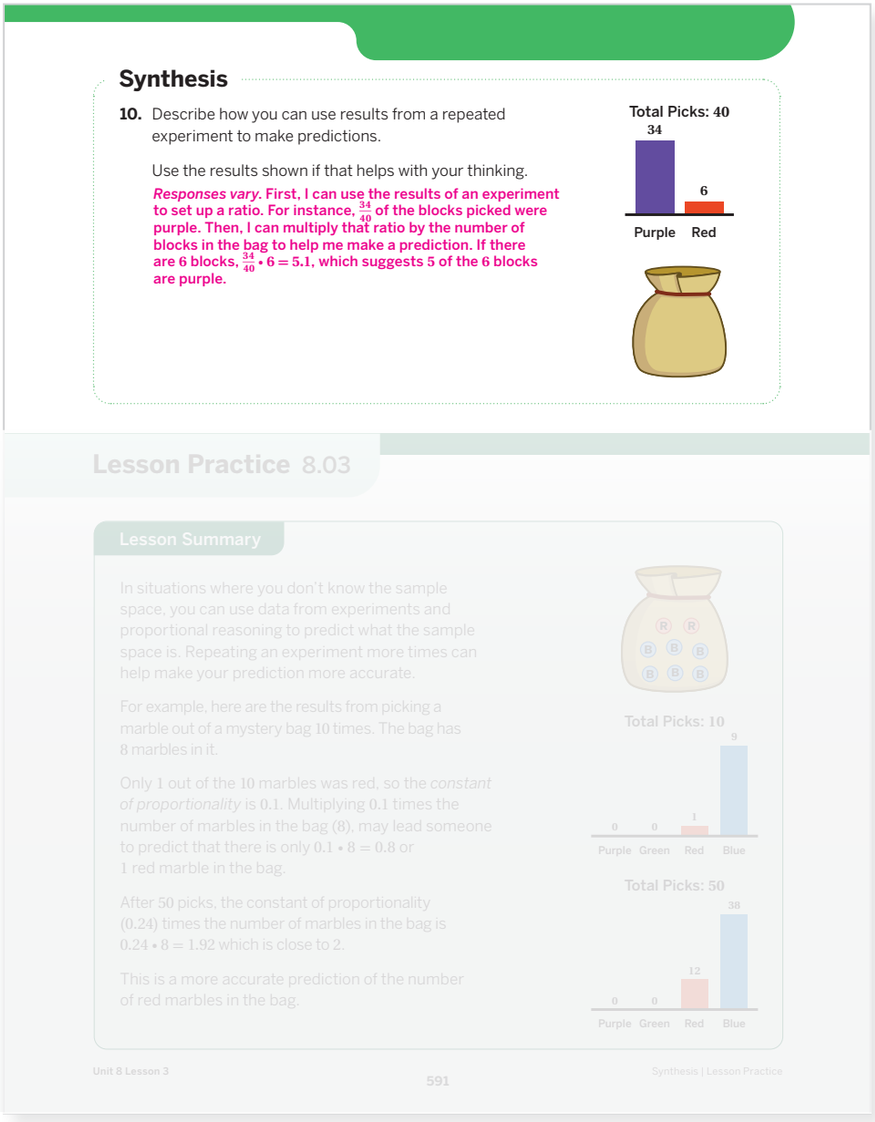
Purpose: Students synthesize their understanding of using repeated experiments to make predictions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Determining a constant of proportionality and multiplying or dividing to make a prediction.
- Rounding to the nearest whole number value if the prediction does not make sense.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe how you can use results from a repeated experiment to make predictions.

Use the results shown if that helps with your thinking.

Responses vary. **First, I can use the results of an experiment to set up a ratio. For instance, $\frac{34}{40}$ of the blocks picked were purple. Then, I can multiply that ratio by the number of blocks in the bag to help me make a prediction. If there are 6 blocks, $\frac{34}{40} \cdot 6 = 5.1$, which suggests 5 of the 6 blocks are purple.**

Lesson Practice 8.03

Lesson Summary

In situations where you don't know the sample space, you can use data from experiments and proportional reasoning to predict what the sample space is. Repeating an experiment more times can help make your prediction more accurate.

For example, here are the results from picking a marble out of a mystery bag 10 times. The bag has 8 marbles in it.

Only 1 out of the 10 marbles was red, so the *constant of proportionality* is 0.1. Multiplying 0.1 times the number of marbles in the bag (8), may lead someone to predict that there is only $0.1 \cdot 8 = 0.8$ or 1 red marble in the bag.

After 50 picks, the constant of proportionality (0.24) times the number of marbles in the bag is $0.24 \cdot 8 = 1.92$ which is close to 2.

This is a more accurate prediction of the number of red marbles in the bag.

Unit 8 Lesson 3 591 Synthesis | Lesson Practice

Lesson Takeaway: Results from repeated experiments can be helpful when making predictions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by predicting how many colored blocks are in a bag using the results from a repeated experiment.

Today's Goals

1. **Goal:** Use proportional reasoning and data from a repeated experiment to make predictions about the sample space.
2. **Language Goal:** Describe how to use the results from a repeated experiment to make predictions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

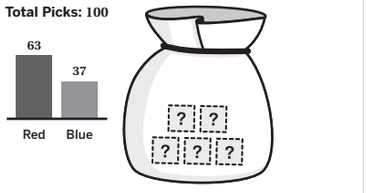
Name: _____ Date: _____ Period: _____

Show What You Know 8.03

A new bag has 5 blocks. Some are red and some are blue.

Based on these results, how many blocks are likely to be red? Show or explain your thinking.

3 blocks. *Responses vary.* I multiplied the ratio of $\frac{63}{100}$ and 5 to determine the the number of blocks out of 5 that would be red. I got 3.15 and, since there are only whole blocks, I rounded down to 3 blocks.



Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to make predictions about the sample space using results from a repeated experiment:

» **Respond to Student Thinking:**

- Use the ***Predicting Sample Spaces Using Proportional Relationships*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to set up their own mystery bag or other random experiment, then trade with a partner and conduct experiments to predict the sample space.

Professional Learning

Consider the next several lessons you will teach. How will students' work today in "What's in the Bag?" prepare them for understanding the relationship between the probability of an outcome and its relative frequency in repeated experiments?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you can use results from a repeated experiment to make predictions.

Use the results shown if that helps with your thinking.

Responses vary. First, I can use the results of an experiment to set up a ratio. For instance, $\frac{34}{40}$ of the blocks picked were purple. Then, I can multiply that ratio by the number of blocks in the bag to help me make a prediction. If there are 6 blocks, $\frac{34}{40} \cdot 6 = 5.1$, which suggests 5 of the 6 blocks are purple.

Lesson Practice 8.03

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In situations where you don't know the sample space, you can use data from experiments and proportional reasoning to predict what the sample space is. Repeating an experiment more times can help make your prediction more accurate.

For example, here are the results from picking a marble out of a mystery bag 10 times. The bag has 8 marbles in it.

Only 1 out of the 10 marbles was red, so the *constant of proportionality* is 0.1. Multiplying 0.1 times the number of marbles in the bag (8), may lead someone to predict that there is only $0.1 \cdot 8 = 0.8$ or 1 red marble in the bag.

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This is a more accurate prediction of the number of red marbles in the bag.



Lesson Practice 8.03

Name: _____ Date: _____ Period: _____

Problems 1–2: A month is chosen at random from the months of the year.

1. What is the probability of getting a month that starts with the letter "J"? Consider listing the sample space to show your thinking.
$\frac{1}{4}$ (or equivalent)

2. If you repeat this experiment 600 times, how many times do you expect to get a month that starts with "J"?
About 150 times

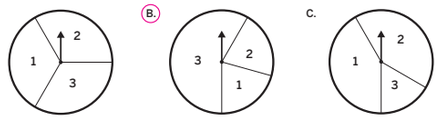
3. A bag has 6 blocks in it. Joel picks a block out of the bag 60 times. He gets a green block 43 times. Based on these results, how many blocks do you expect to be green? Explain your thinking.



4 blocks. *Explanations vary.* Joel picked a green block about 72% of the time because $\frac{43}{60} \approx 0.7167$. If the proportion of green blocks in the bag matched this, there would be 4.3 green blocks. Because 4.3 is closer to 4 than 5, it's likely that there are 4 green blocks in the bag.

4. Miko spun a spinner with numbered sections 15 times. Here are the results. Which spinner is most likely the one Miko used?

Miko's results:
3, 3, 3, 1, 3, 2, 2, 3, 3, 1, 3, 1, 3, 3, 2



Explain your thinking.

Explanations vary. Miko spun a 3 nine times, which is three times as many times that he spun a 1 or a 2. Because Spinner B has a large section for 3 and smaller equal sections for 1 and 2, this spinner seems to match Miko's results more than the other two spinners.

Problems 5–7: A textbook has 428 numbered pages, starting with 1. You are equally likely to stop on any of the pages if you flip through the book randomly.

5. What is the probability that you turn to page 45?
$\frac{1}{428}$ (or equivalent)

6. What is the probability that you turn to an even-numbered page?
$\frac{1}{2}$ (or equivalent)

7. If you repeat this experiment 50 times, about how many times do you expect to turn to an even-numbered page?
About 25 times

Lesson Practice 8.03

Name: _____ Date: _____ Period: _____

FAST Practice

8. The table shows the number of different colors of crayons in a case. A 7th grader will randomly choose one crayon from the case.

Color	Number of Crayons
Blue	2
Red	8
Orange	4
Yellow	5

Based on the information in the table, which statement is true?

A. The crayon is least likely to be orange.
B. The crayon is 4 times as likely to be red as it is to be blue.
C. The crayon is equally likely to be orange or yellow.
D. The crayon is more likely to be red than all other colors combined.

Spiral Review

Problems 9–11: For each word, determine the probability of selecting the letter "A" at random.

9. LAMB
$\frac{1}{4}$ (or equivalent)

10. SAFETY
$\frac{1}{6}$ (or equivalent)

11. ALABAMA
$\frac{4}{7}$ (or equivalent)

12. Select *all* the situations in which the surface area would be useful to know.

- [x] A. Ordering tiles to replace the roof of a house.
- [x] B. Estimating how long it will take to clean the windows of a greenhouse.
- [ ] C. Deciding whether leftover soup will fit in a container.
- [ ] D. Estimating how long it will take to fill a swimming pool with a garden hose.
- [x] E. Buying fabric to sew a couch cover.

Lesson Practice Item Analysis

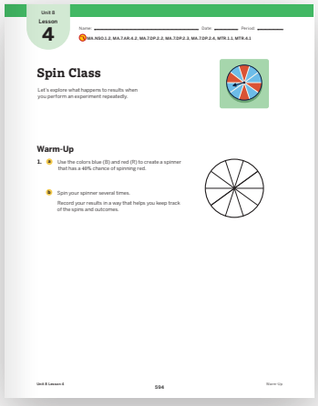
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 5–6	1	MA.7.DP.2.3
	2–4, 7	2	MA.7.DP.2.4
FAST Practice	8	2	MA.7.DP.2.2 MA.7.DP.2.3
Spiral Review			
	9–11	1	MA.7.DP.2.3
	12	2	MA.6.GR.2.4, MA.7.GR.2.1

Unit 8
Lesson
4

Spin Class

Repeated Experiments

Let's explore what happens to results when you perform an experiment repeatedly.


Unit 8
Lesson
4

Name _____ Date _____ Period _____

MA.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.1.1, MTR.4.1

Spin Class

Let's explore what happens to results when you perform an experiment repeatedly.

Warm-Up

1. Use the colors blue (B) and red (R) to create a spinner that has a 40% chance of spinning red.

   Spin your spinner several times.

   Record your results in a way that helps you keep track of the spins and outcomes.

Unit 8 Lesson 4 594 Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that the results of a repeated experiment approach the probability of the event.
2. **Language Goal:** Make connections between the probability of an event and a graph representing data from a repeated experiment. **(Writing, Speaking, and Listening)**

Students recognize that if an experiment is performed repeatedly, the relative frequency will approach, but may not exactly match, the probability of each outcome. Students analyze the results of repeated experiments and examine the structure of the results. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 3, students used a repeated experiment to predict the sample space.

Future Learning

In Lesson 5, students will decide whether an object is fair by comparing the probabilities of a model to the results of a repeated experiment.

Connections

- Students analyze visuals such as line graphs, scatter plots, or histograms of the results of repeating an experiment multiple times. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

MA.7.DP.2.4
Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.

Also Addressing: **MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	MA.8.DP.2.1
MA.6.AR.3.5	MA.8.DP.2.2
	MA.8.DP.2.3

Vocabulary

New Vocabulary

relative frequency

Lesson at a Glance

~ 45 min

MA.7.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4

Warm-Up

Independent | 5 min

Students connect spinners with probabilities and share different ways to keep track of spins. They create a spinner similar to the one they will analyze in Activity 1.

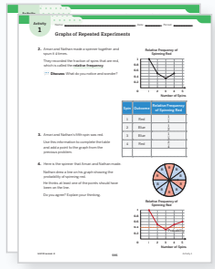


Activity 1

Pairs | 15 min

Students interpret a graph of a repeated experiment's results to surface the idea that the **relative frequency** approaches, but may not exactly match, the probability of an outcome.

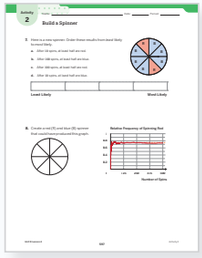
Routine: Notice and Wonder



Activity 2

Pairs | 7 min

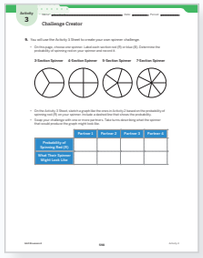
Students order the results of a repeated experiment and create a spinner to match the results of a repeated experiment on a graph.



Activity 3

Independent | 8 min

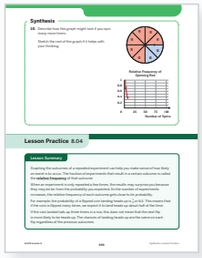
Students express their creativity by creating a spinner that could produce the results of a repeated experiment.



Synthesis

Whole Class | 5 min

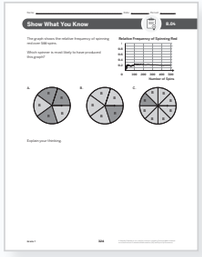
Students synthesize their understanding of how the results of a repeated experiment are related to the probability of an event.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by connecting the results of a graph to a probability spinner.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- coloring tools, blue and red
- paper clips (optional)

Independent | 5 min

Warm-Up

Purpose: Students connect spinners with probabilities and share different ways to keep track of spins. They create a spinner similar to the one they will analyze in Activity 1.

1 Launch

Demonstrate how to create the spinner.

Distribute coloring tools and paper clips to students.

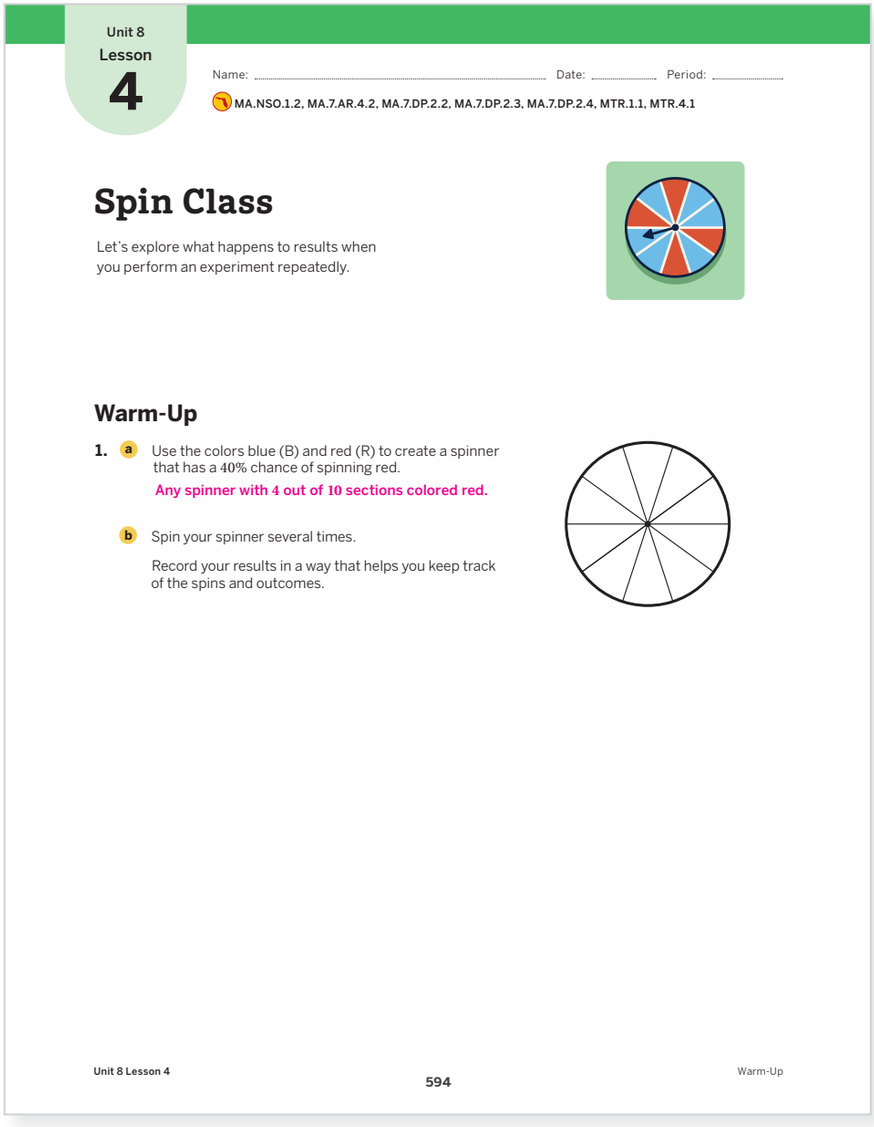
To surface prior knowledge, consider asking, "How can we change a percentage to a fraction?"

A **Accessibility: Memory and Attention** Create or review an anchor chart that displays the definition and examples of probability to aid in explanations and reasoning.

Invite students to use a paper clip as a pointer on their own spinner.

2 Connect

Invite students to share their spinners and their strategies for recording their results.



Unit 8
Lesson
4

Name: Date: Period:

MA.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.1.1, MTR.4.1

Spin Class

Let's explore what happens to results when you perform an experiment repeatedly.

Warm-Up

1. a Use the colors blue (B) and red (R) to create a spinner that has a 40% chance of spinning red.

   Any spinner with 4 out of 10 sections colored red.

   b Spin your spinner several times.

   Record your results in a way that helps you keep track of the spins and outcomes.

Unit 8 Lesson 4 594 Warm-Up

Pairs | 15 min

Activity 1 Graphs of Repeated Experiments

Purpose: Students interpret a graph of a repeated experiment's results to surface the idea that the **relative frequency** approaches, but may not exactly match, the probability of an outcome.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the table and graph representations. **(MTR.1.1)**

Create a class definition of the term **relative frequency** using students' language and the glossary. To help generate student language, consider asking:

- "After 4 spins, how many were red? Where do you see that information on the graph and in the table?"
- "How is the relative frequency of red related to the number of red spins?"
- "Does the relative frequency stay the same as Amari and Nathan spin more and more times?"

2 Monitor

To support students getting started, encourage them to analyze the table. Consider asking, "After 5 spins, how many were red? How does that help you determine the relative frequency?"

Record the distribution of responses. Consider selecting responses for each of the most popular choices to discuss in the Connect.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I agree or disagree because _____.).

Activity 1 continued >

Activity 1

Name: Date: Period:

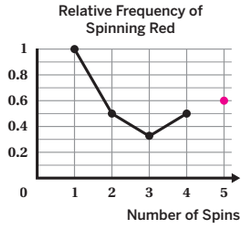
Graphs of Repeated Experiments

2. Amari and Nathan made a spinner together and spun it 4 times.

They recorded the fraction of spins that are red, which is called the **relative frequency**.

Discuss: What do you notice and wonder?

Responses vary.
- I notice that there are 4 spins in both the table and the graph.
- I notice that when the relative frequency is $\frac{1}{2}$, the graph is at 0.5.
- I notice that when they spun red on the fourth spin, the graph went back up.
- I wonder how the graph and the table are connected.
- I wonder where the fractions in the table came from.
- I wonder what the table and graph would look like if the spinner kept landing on red.



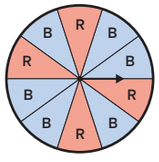
Spin	Outcome	Relative Frequency of Spinning Red
1	Red	$\frac{1}{1}$
2	Blue	$\frac{1}{2}$
3	Blue	$\frac{1}{3}$
4	Red	$\frac{2}{4}$
5	Red	$\frac{3}{5}$

3. Amari and Nathan's fifth spin was red.

Use this information to complete the table and add a point to the graph from the previous problem.

Responses shown in the table and on the graph.

4. Here is the spinner that Amari and Nathan made.

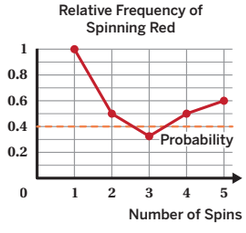


Nathan drew a line on his graph showing the probability of spinning red.

He thinks at least one of the points should have been on the line.

Do you agree? Explain your thinking.

Responses and explanations vary.
- Yes. If I perform an experiment many times, the relative frequency should match the probability at least some of the time.
- No. They only spun the spinner 5 times. That isn't enough to get close to the line. Also, because the spinner is random, we can't predict the exact fraction of times it will spin red.
- I'm not sure. The spinner is random, so it seems like the points could go anywhere on the graph.



Unit 8 Lesson 4 595 Activity 1

Pairs | 15 min

Activity 1 Graphs of Repeated Experiments (continued)

Purpose: Students interpret a graph of a repeated experiment's results to surface the idea that the **relative frequency** approaches, but may not exactly match, the probability of an outcome.

2 Monitor

Listen for observations that may surface connections to the discussion in the next problem.

Select and sequence student observations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Notice that the results of a repeated experiment do not exactly match the probability of the event.	**Strengthen** Consider asking, "Why do you think some graphs are closer to the probability line than others?"
Notice that the graph gets smoother toward 0.4, which is the probability.	**Strengthen** Invite them to share the relationship between the relative frequency and the probability of the outcome.
Notice that the graph varies during each of the 200 spins but gets closer as the number of spins increases.	

3 Connect

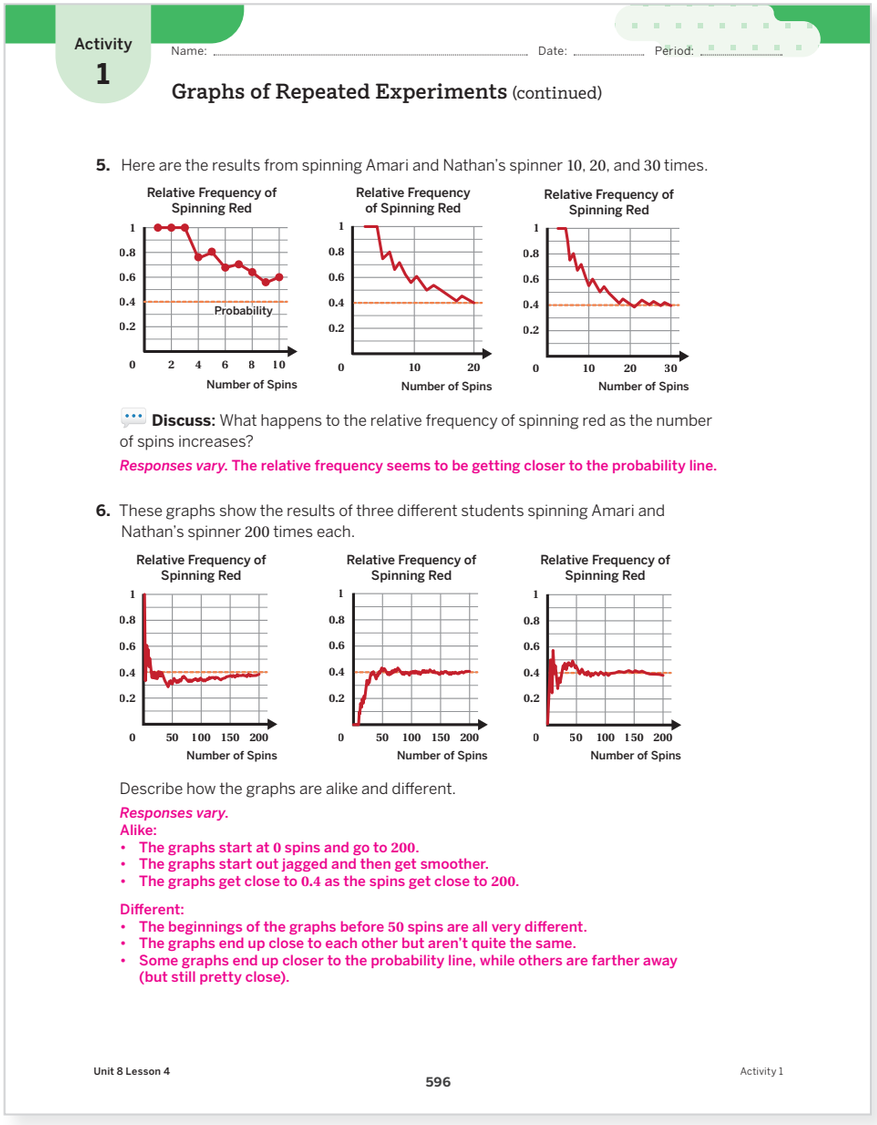
Invite students to share their responses, calling attention to any conflict or consensus.

Invite students to share why they believe the points should or should not be exactly on the line to surface that the relative frequency may not exactly match the probability, especially after only 5 spins. **(MTR.4.1)**

Invite students to share their descriptions.

Consider asking, "Why do you think parts of the graphs are more alike while other parts are different?"

To surface the Key Takeaway, consider asking, "What happens to the graph as the number of spins gets closer to 200?"



Activity 1

Name: Date: Period:

Graphs of Repeated Experiments (continued)

5. Here are the results from spinning Amari and Nathan's spinner 10, 20, and 30 times.

Discuss: What happens to the relative frequency of spinning red as the number of spins increases?

Responses vary. The relative frequency seems to be getting closer to the probability line.

6. These graphs show the results of three different students spinning Amari and Nathan's spinner 200 times each.

Describe how the graphs are alike and different.

Responses vary.
Alike:
- The graphs start at 0 spins and go to 200.
- The graphs start out jagged and then get smoother.
- The graphs get close to 0.4 as the spins get close to 200.

Different:
- The beginnings of the graphs before 50 spins are all very different.
- The graphs end up close to each other but aren't quite the same.
- Some graphs end up closer to the probability line, while others are farther away (but still pretty close).

Unit 8 Lesson 4 596 Activity 1

Key Takeaway: If you divide the number of times an outcome happened by the total number of experiments, you get the **relative frequency** of that outcome. As experiments are repeated (hundreds or thousands of times), the relative frequencies get closer to the probabilities of each outcome.

Pairs | 7 min

Activity 2 Build a Spinner

Purpose: Students order the results of a repeated experiment and create a spinner to match the results of a repeated experiment on a graph.

1 Launch

To surface connections, consider asking, "What is the probability of spinning red on this spinner? Blue?"

2 Monitor

To support students getting started, consider asking:

- "After 10 spins, what results would you expect to see? What would surprise you?"
- "Would more spins make it more or less likely to have mostly blue?"

ELL **English Language Learners** Invite one partner to read each prompt out loud before the pair discusses it. **(Reading and Listening)**

Look for students who have six colored sections of red and invite them to share their strategies during the Connect.

3 Connect

Invite students to share their strategies for how they ordered the results. Work together to reach a consensus about which results are least and most likely. **(MTR.4.1)**

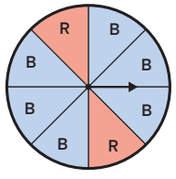
Consider asking, "How many sections of the spinner did you color red? How did you choose that number of sections?"

Activity 2

Name: Date: Period:

Build a Spinner

7. Here is a new spinner. Order these results from *least* likely to *most* likely.



a. After 10 spins, at least half are red.

b. After 100 spins, at least half are blue.

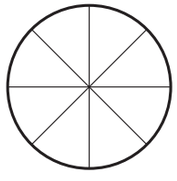
c. After 100 spins, at least half are red.

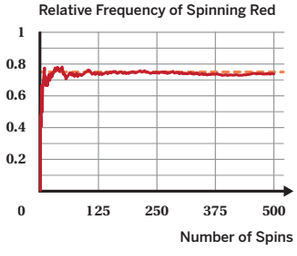
d. After 10 spins, at least half are blue.

c	a	d	b

Least Likely — Most Likely

8. Create a red (R) and blue (B) spinner that could have produced this graph.





Responses vary. Any spinner with 6 out of 8 sections colored red.

Unit 8 Lesson 4 597 Activity 2

Independent | 8 min

Activity 3 Challenge Creator

Purpose: Students express their creativity by creating a spinner that could produce the results of a repeated experiment.

Short on time? Consider omitting this Activity.

1 Launch

To generate excitement, share with students that they will create their own "secret" spinner today.

Distribute the Activity 3 Sheet and coloring tools.

2 Monitor

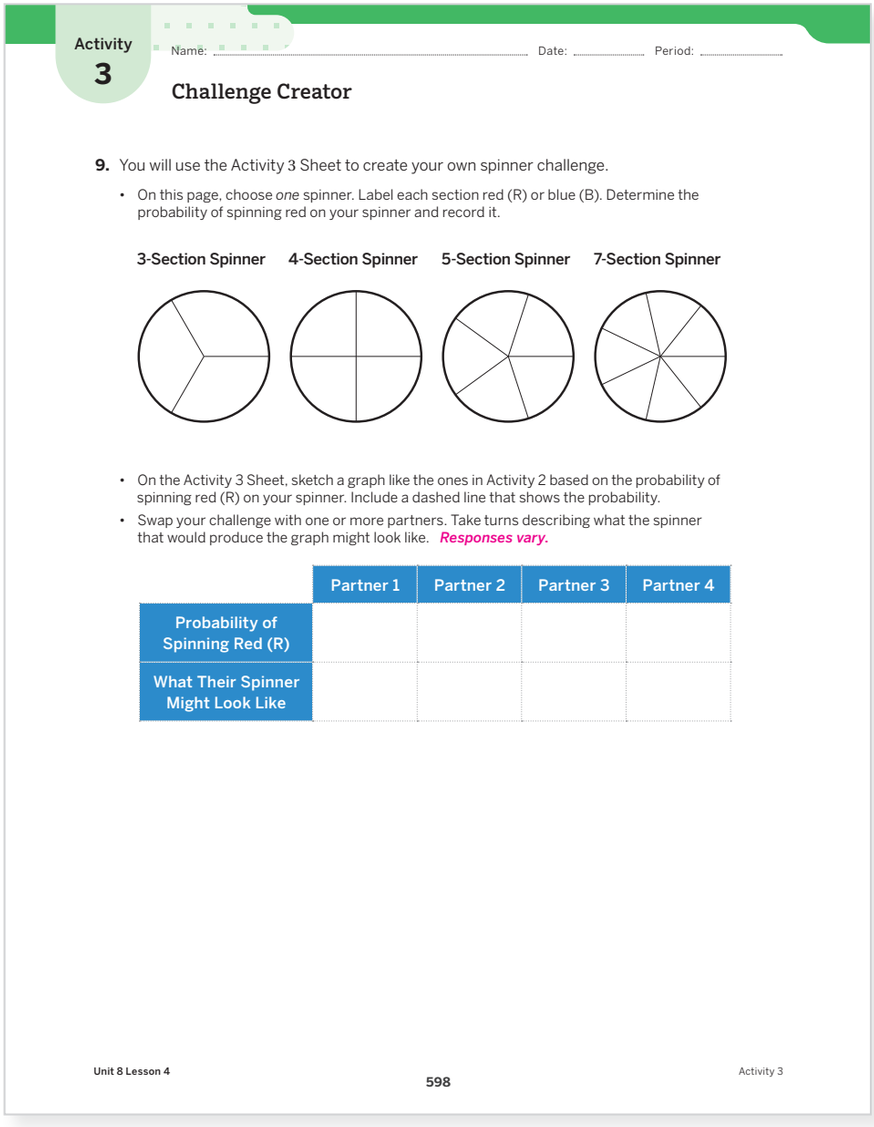
Look for creative challenges.

Note: Spinners in an equivalent ratio of red sections to total sections should be accepted.

3 Connect

Invite students to share how they determined what their classmate's spinner(s) look like.

Discuss how ratios are helpful when analyzing a graph to determine the sections of a spinner.



Activity 3

Name: Date: Period:

Challenge Creator

9. You will use the Activity 3 Sheet to create your own spinner challenge.

- On this page, choose *one* spinner. Label each section red (R) or blue (B). Determine the probability of spinning red on your spinner and record it.

3-Section Spinner 4-Section Spinner 5-Section Spinner 7-Section Spinner

- On the Activity 3 Sheet, sketch a graph like the ones in Activity 2 based on the probability of spinning red (R) on your spinner. Include a dashed line that shows the probability.
- Swap your challenge with one or more partners. Take turns describing what the spinner that would produce the graph might look like. *Responses vary.*

	Partner 1	Partner 2	Partner 3	Partner 4
Probability of Spinning Red (R)				
What Their Spinner Might Look Like				

Unit 8 Lesson 4 598 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how the results of a repeated experiment are related to the probability of an event.

Invite students to respond independently, and then share their thinking with a partner.

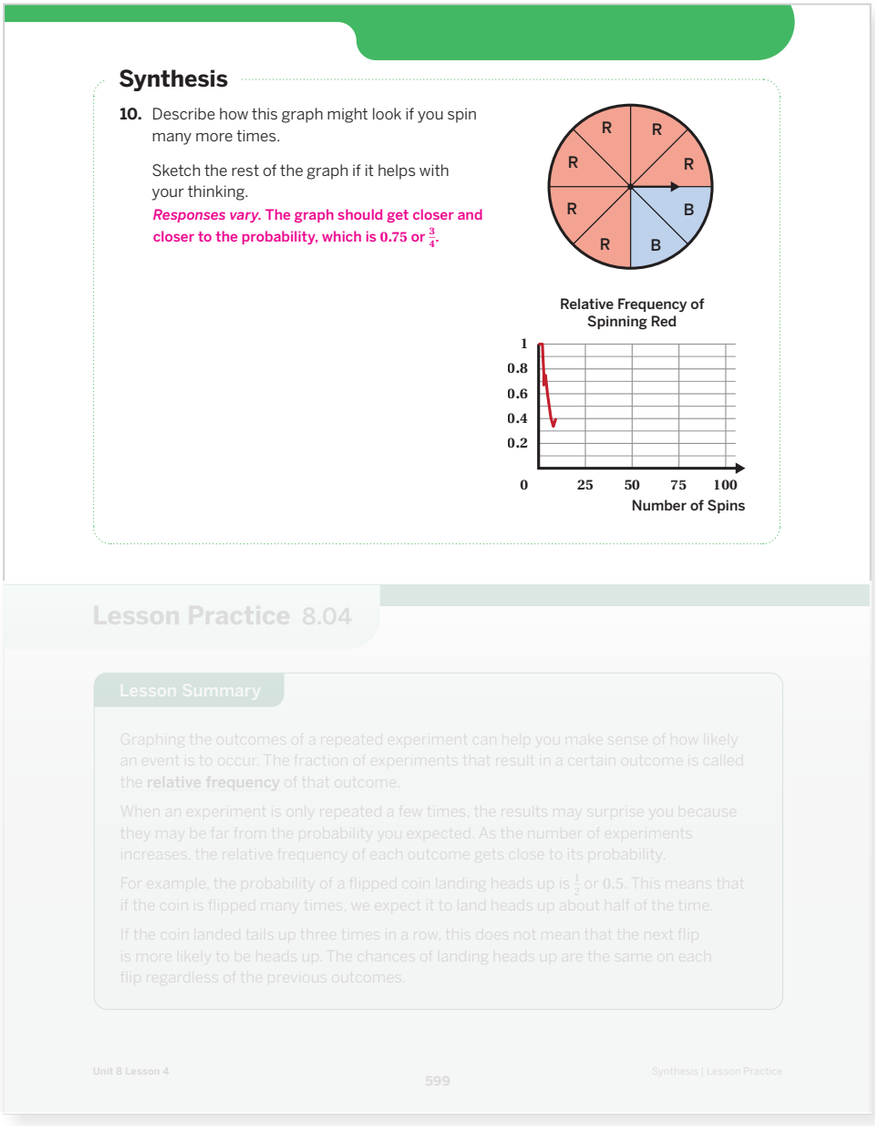
ELL **English Language Learners**

- Provide sentence frames to help students explain their reasoning (e.g., If you spin many more times, the graph will look like _____.).
- Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

Collect and share a variety of ideas, such as:

- Approaching the probability.
- The probability being 0.75 (or equivalent).

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe how this graph might look if you spin many more times.

Sketch the rest of the graph if it helps with your thinking.

Responses vary. The graph should get closer and closer to the probability, which is 0.75 or $\frac{3}{4}$.

Lesson Practice 8.04

Lesson Summary

Graphing the outcomes of a repeated experiment can help you make sense of how likely an event is to occur. The fraction of experiments that result in a certain outcome is called the **relative frequency** of that outcome.

When an experiment is only repeated a few times, the results may surprise you because they may be far from the probability you expected. As the number of experiments increases, the relative frequency of each outcome gets close to its probability.

For example, the probability of a flipped coin landing heads up is $\frac{1}{2}$ or 0.5. This means that if the coin is flipped many times, we expect it to land heads up about half of the time.

If the coin landed tails up three times in a row, this does not mean that the next flip is more likely to be heads up. The chances of landing heads up are the same on each flip regardless of the previous outcomes.

Unit 8 Lesson 4 599 Synthesis | Lesson Practice

Lesson Takeaway: When you repeat an experiment many times, the relative **frequency** of each outcome approaches its probability.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by connecting the results of a graph to a probability spinner.

Today's Goals

1. **Goal:** Recognize that the results of a repeated experiment approach the probability of the event.
2. **Language Goal:** Make connections between the probability of an event and a graph representing data from a repeated experiment. **(Writing, Speaking, and Listening)**

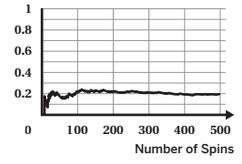
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

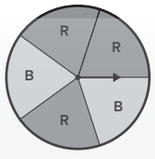
Show What You Know 8.04

The graph shows the relative frequency of spinning red over 500 spins.

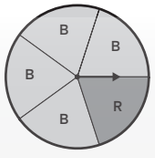
Which spinner is most likely to have produced this graph?



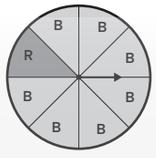
A.



B.



C.



Explain your thinking.

Explanations vary. The graph shows that after many spins, the relative frequency gets close to 0.2. This means the probability of red is likely $\frac{1}{5}$. A spinner with 1 red section out of 5 has a $\frac{1}{5}$ probability of landing on red.

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to recognize which spinner would have produced the results shown in the graph:

» **Respond to Student Thinking:**

- Revisit the spinners students built in Activity 2. Invite students to make connections between the observed relative frequencies and the characteristics of the spinner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

How did the Notice and Wonder routine support students in recognizing the idea that in repeated experiments, the relative frequency of each outcome approaches, but may not exactly match, its probability?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how this graph might look if you spin many more times.

Sketch the rest of the graph if it helps with your thinking.

Responses vary. The graph should get closer and closer to the probability, which is 0.75 or $\frac{3}{4}$.

Lesson Practice 8.04

Lesson Summary

Graphing the outcomes of a repeated experiment can help you make sense of how likely an event is to occur. The fraction of experiments that result in a certain outcome is called the **relative frequency** of that outcome.

When an experiment is only repeated a few times, the results may surprise you because they may be far from the probability you expected. As the number of experiments increases, the relative frequency of each outcome gets close to its probability.

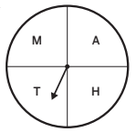
For example, the probability of a flipped coin landing heads up is $\frac{1}{2}$ or 0.5. This means that if the coin is flipped many times, we expect it to land heads up about half of the time.

If the coin landed tails up three times in a row, this does not mean that the next flip is more likely to be heads up. The chances of landing heads up are the same on each flip regardless of the previous outcomes.

Lesson Practice 8.04

Name: _____ Date: _____ Period: _____

Problems 1–2: A spinner has four equal sections, with one letter from the word MATH in each section.



1. You spin the spinner 20 times. About how many times do you expect it will land on "A"?

About 5 times

Explain your thinking.

Explanations vary. Because the spinner is split into four equal sections, there is a 25% chance of it landing on "A." 25% of 20 spins is a total of 5 times.

2. You spin the spinner 80 times. About how many times do you expect it will land on something other than "A"?

About 60 times

Explain your thinking.

Explanations vary. Because the spinner is split into four equal sections, there is a 75% chance of it landing on something other than "A." 75% of 80 is a total of 60 times.

Problems 3–5: A fair coin is flipped 100 times.

3. About how many times would you expect the coin to land heads up?

About 50 times.

4. What is another number of times that would be likely for it to land heads up?

Whole numbers between 40 and 60 times are considered correct.

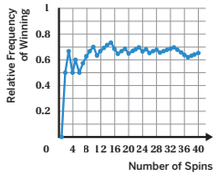
5. Adah says it's impossible for the coin to land heads up all 100 times. Basheera says it's possible, but unlikely. Who is correct?

Basheera

Explain your thinking.

Explanations vary. Because the probability of the coin landing heads up is 50%, it's very unlikely that every flip out of 100 lands heads up, but it's not impossible.

6. A spinner is spun 40 times for a game. This graph shows the relative frequency of winning the game.



Based on the graph, estimate the probability of winning this game.

Responses between 0.6 and 0.7 (or equivalent) are considered correct.

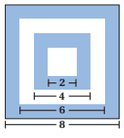
Explain your thinking.

Explanations vary. The relative frequency of winning approaches 0.65 the more times the spinner is spun.

Lesson Practice 8.04

Name: _____ Date: _____ Period: _____

Problems 7–8: Here is a square dartboard.



7. Is a dart thrown at random more likely to land in the shaded or the unshaded area? Show or explain your thinking.

Shaded. *Explanations vary.* The unshaded area is 24 square units because $2^2 + (6^2 - 4^2) = 24$. The shaded area is 40 square units because $(4^2 - 2^2) + (8^2 - 6^2) = 40$. The total area of the board is 64 square units. There is a 62.5% chance that a randomly thrown dart lands in the shaded area.

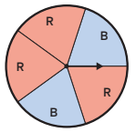
8. A dart is thrown at the board 100 times at random. About how many times is the dart likely to land in the shaded area? Explain your thinking.

Whole numbers between 60 and 65 times are considered correct. *Explanations vary.* There is a 62.5% chance that a randomly thrown dart lands in the shaded area, and 62.5% of 100 is 62.5 which rounds to 63.

FAST Practice

9. Janna spun the spinner shown 4 times. If it landed on R twice, what is the relative frequency of landing on R?

Write your answer as a decimal. 0.5



Spiral Review

10. Write each decimal as an equivalent fraction.

0.5 $\frac{1}{2}$ 0.4 $\frac{2}{5}$ 0.75 $\frac{3}{4}$ 0.25 $\frac{1}{4}$ $0.3\overline{3}$ $\frac{1}{3}$

11. Solve the inequality $12 < -3x$.

$x < -4$

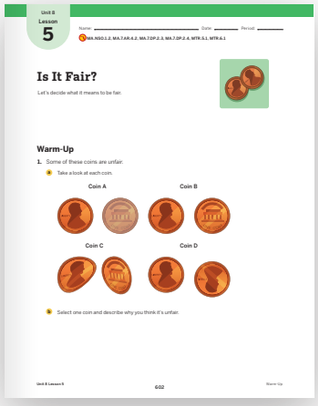
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.7.DP.2.4
	3	1	MA.7.DP.2.4
	4–6, 8	2	MA.7.DP.2.4
	7	3	MA.7.DP.2.4
FAST Practice	9	2	MA.7.DP.2.4
Spiral Review			
	10	1	MA.7.NSO.1.2
	11	1	MA.7.AR.2.1

Unit 8

Lesson

5


Unit 8
Lesson
5
Name _____ Date _____ Period _____
MA.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.5.1, MTR.6.1
Is It Fair?
Let's decide what it means to be fair.
Warm-Up
1. Some of these coins are unfair.
Take a look at each coin.
Coin A
Coin B
Coin C
Coin D
Select one coin and describe why you think it's unfair.
602
Warm-Up

Is It Fair?

Comparing Probabilities and Experiments

Let's decide what it means to be fair.

Learning Outcomes

Today's Goals

1. **Goal:** Use the results from a repeated experiment to approximate the probability of an event.
2. **Language Goal:** Explain whether an object is fair based on the results of a repeated experiment. **(Reading and Writing)**

Students are introduced to unfair objects. They decide whether an object is fair by comparing the probabilities of a model to the results of a repeated experiment. They also examine unfair coins and number cubes and explain possible sources of discrepancy between what they expect and the relative frequencies they observe.

Coherence

Prior Learning

In Lessons 3 and 4, students used the results of a repeated experiment to make predictions.

Future Learning

In Grade 8 Unit 9, students will solve real-world problems involving probabilities and will make predictions based on theoretical probability.

Connections

- Students investigate how the makeup of objects suchs a coin or die can affect the likelihood of certian outcomes. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

MA.7.DP.2.4
Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.

Also Addressing: **MA.7.NSO.1.2, MA.7.AR.4.2**

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.DP.2.1**
MA.6.AR.3.5	**MA.8.DP.2.2**
	MA.8.DP.2.3

Lesson at a Glance

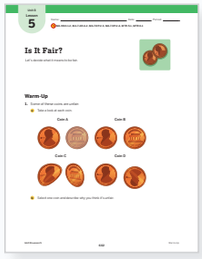
~ 45 min

MA.7.DP.2.3, MA.7.DP.2.4, MA.NSO.1.2, MA.7.AR.4.2

Warm-Up

Pairs | 5 min

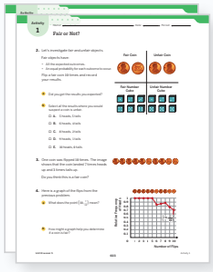
Students are introduced to fair and unfair objects.



Activity 1

Pairs | 15 min

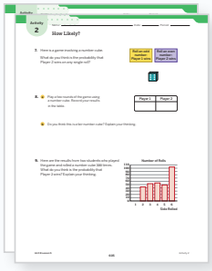
Students use the expected probabilities of fair objects and the results of a repeated experiment to decide if a coin is fair. They also approximate the probability of an unfair coin landing heads up. **(MTR.6.1)**



Activity 2

Pairs | 15 min

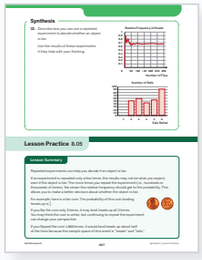
Students use the results of a repeated experiment (rolling an unfair number cube) to determine probabilities of events. Students also use these probabilities to design a game that is fair. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

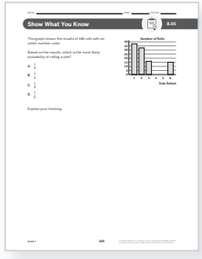
Students synthesize their understanding of using repeated experiments to decide whether an object is fair.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the probability of an event occurring based on the results of a repeated experiment.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Coins, one per pair
- Number cubes, one per pair

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to fair and unfair objects.

1 Launch

Consider asking, "Have you ever played a game you thought was unfair or set up as a trick? What made it unfair?"

To generate engagement, distribute a fair coin to each pair of students and invite students to examine it.

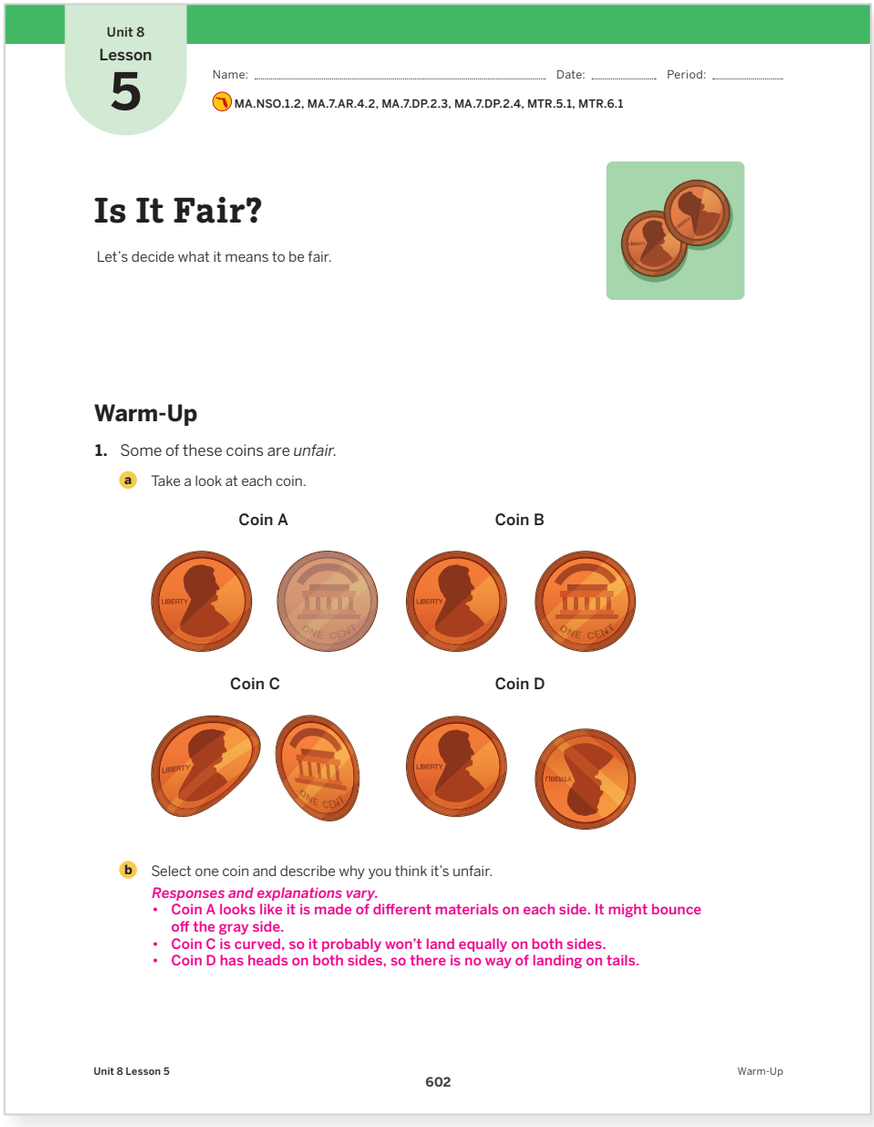
2 Connect

Invite students to share their responses.

Select and sequence 1–2 responses for each of the most popular choices to share and invite those students to share what they think makes the coin unfair.

Note: If it does not come up naturally when examining the coin, share with students that on a standard coin, the side with tails faces a different direction than the side with heads. Coin B is an example of a fair coin.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think Coin ___ is unfair because _____.).



Unit 8
Lesson
5

Name: _____ Date: _____ Period: _____

MA.NSO.1.2, MA.7.AR.4.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.5.1, MTR.6.1

Is It Fair?

Let's decide what it means to be fair.

Warm-Up

1. Some of these coins are *unfair*.

a. Take a look at each coin.

Coin A

Coin B

Coin C

Coin D

b. Select one coin and describe why you think it's unfair.

Responses and explanations vary.

- Coin A looks like it is made of different materials on each side. It might bounce off the gray side.
- Coin C is curved, so it probably won't land equally on both sides.
- Coin D has heads on both sides, so there is no way of landing on tails.

Unit 8 Lesson 5 602 Warm-Up

Pairs | 15 min

Activity 1 Fair or Not?

Purpose: Students use the expected probabilities of fair objects and the results of a repeated experiment to decide if a coin is fair. They also approximate the probability of an unfair coin landing heads up. **(MTR.6.1)**

1 Launch

Consider asking, "If you flip a fair coin 100 times, how many times do you expect it to land heads up? Why?" **(MTR.6.1)**

To encourage student engagement, distribute a coin and invite students to take turns flipping it 10 or more times.

A **Accessibility: Conceptual Processing** Invite students to read all statements or problems aloud to support extra processing time.

2 Monitor

Listen for a variety of student responses, including students who are not sure because the coin is not flipped enough times to make a prediction.

Pause to discuss student explanations, such as:

- You should compare the relative frequencies you see on the graph to 0.5, because that indicates the coin is fair.
- As the number of flips increases, you can look for the line to stay close to 0.5.
- You can pick a point on the graph with a high number of flips and see if the relative frequency is close to 0.5.

Consider asking, "How does that relate to the probability of a fair coin?" **(MTR.6.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., A graph might help you determine if a coin is fair by _____.).

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Fair or Not?

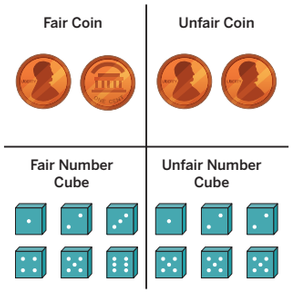
2. Let's investigate *fair* and *unfair* objects.

Fair objects have:

- All the expected outcomes.
- An equal probability for each outcome to occur.

Flip a fair coin 10 times and record your results.

Responses vary.



a. Did you get the results you expected?

Responses vary.

b. Select *all* the results where you would suspect a coin is unfair.

- ☐ A. 5 heads, 5 tails
- ☐ B. 6 heads, 4 tails
- ☐ C. 8 heads, 2 tails
- ☐ D. 9 heads, 1 tails
- ☐ E. 10 heads, 0 tails

Responses vary.

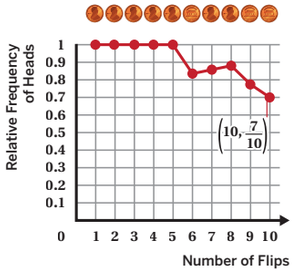
3. One coin was flipped 10 times. The image shows that the coin landed 7 times heads up and 3 times tails up.



Do you think this is a fair coin?

Responses vary.

4. Here is a graph of the flips from the previous problem.



a. What does the point $\left(10, \frac{7}{10}\right)$ mean?

Responses vary. The point $\left(10, \frac{7}{10}\right)$ means that after 10 flips, the relative frequency of heads was $\frac{7}{10}$.

b. How might a graph help you determine if a coin is fair?

Responses vary. If the graph gets close to 0.5, then the coin is fair. If not, then it isn't fair.

Unit 8 Lesson 5 603 Activity 1

Pairs | 15 min

Activity 1 Fair or Not? (continued)

Purpose: Students use the expected probabilities of fair objects and the results of a repeated experiment to decide if a coin is fair. They also approximate the probability of an unfair coin landing heads up. **(MTR.6.1)**

2 Monitor

Select and sequence a variety of explanations to share during the Connect.

D Differentiation

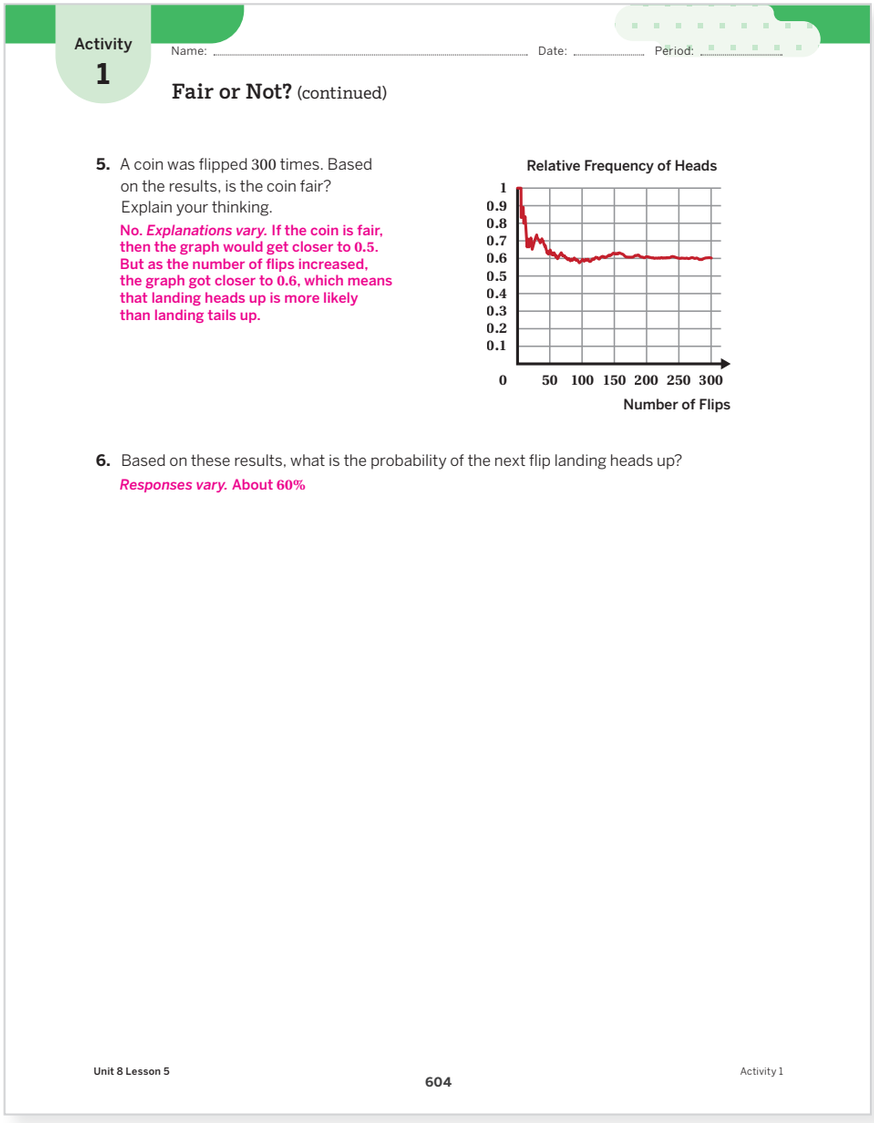
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What does the graph represent? The change in the graph over time?"
Think the coin is fair because 0.6 is close to 0.5.	**Strengthen** Encourage students to think about an example in a different context such as comparing \$0.50 per pound versus \$0.60 per pound when making a purchase at the grocery store.
Reason that heads facing up is more likely than tails.	**Strengthen** Invite them to share how they determined this during the Connect.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, share student explanations. Discuss what the graph of a fair coin might look like and how that differs from the graph of this coin.

MLR1: Stronger and Clearer Each Time (optional) Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Activity 1

Name: Date: Period:

Fair or Not? (continued)

5. A coin was flipped 300 times. Based on the results, is the coin fair? Explain your thinking.

No. *Explanations vary.* If the coin is fair, then the graph would get closer to 0.5. But as the number of flips increased, the graph got closer to 0.6, which means that landing heads up is more likely than landing tails up.

6. Based on these results, what is the probability of the next flip landing heads up?

Responses vary. About 60%

Unit 8 Lesson 5 604 Activity 1

Key Takeaway: Fair objects have an equal probability for each outcome to occur. When an experiment is repeated hundreds of times, the relative frequency of each outcome should be very close to the probability you expect. If it is not, the object may not be fair.

Pairs | 15 min

Activity 2 How Likely?

Purpose: Students use the results of a repeated experiment (rolling an unfair number cube) to determine probabilities of events. Students also use these probabilities to design a game that is fair. **(MTR.5.1)**

Short on time? Consider omitting Problem 8.

1 Launch

To generate excitement, invite students to choose who is Player 1 and Player 2.

Distribute a number cube to each pair of students.

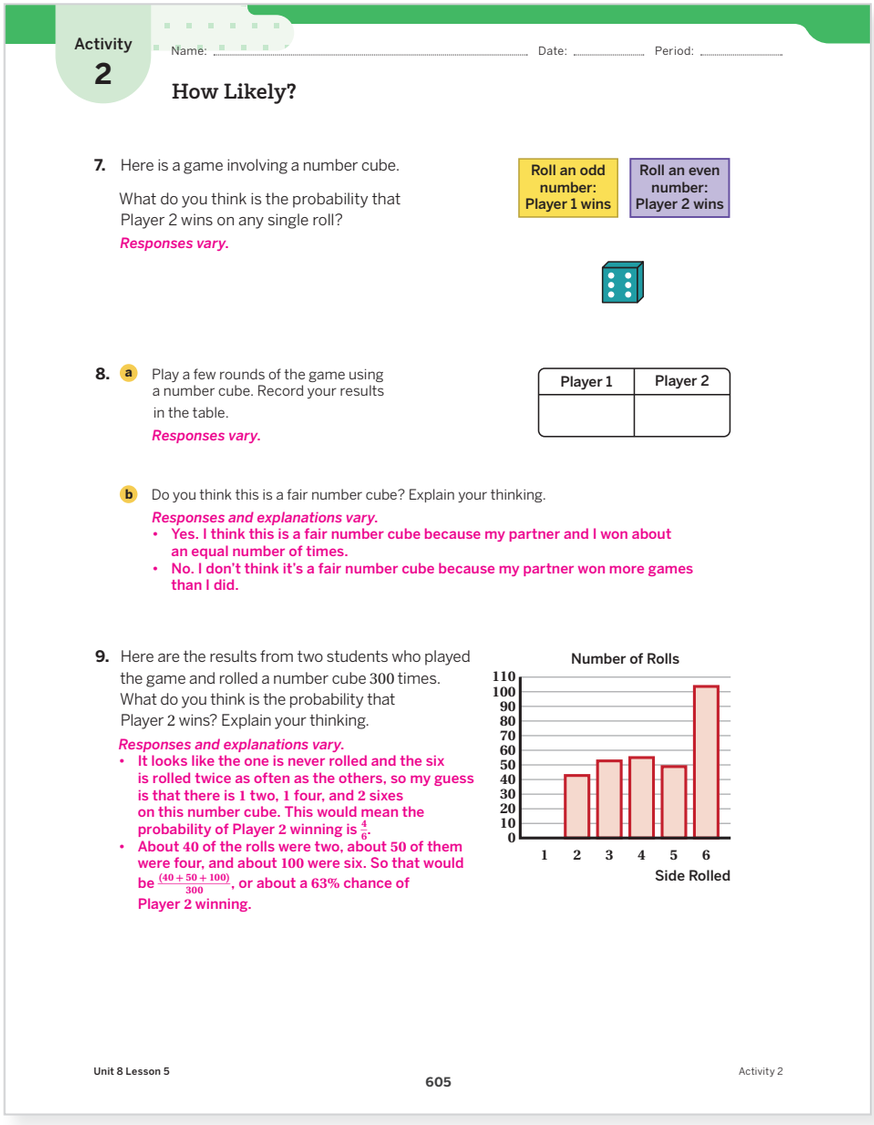
2 Monitor

Look for a variety of explanations on whether the game is fair.

To support students getting started, consider asking, "What numbers result in Player 2 winning?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think the probability that Player 2 wins is ___ because _____.).

Activity 2 continued >



Activity 2

Name: Date: Period:

How Likely?

7. Here is a game involving a number cube.

What do you think is the probability that Player 2 wins on any single roll?

Responses vary.

Roll an odd number: Player 1 wins	Roll an even number: Player 2 wins

8. a Play a few rounds of the game using a number cube. Record your results in the table.

Responses vary.

Player 1	Player 2

b Do you think this is a fair number cube? Explain your thinking.

Responses and explanations vary.

- Yes. I think this is a fair number cube because my partner and I won about an equal number of times.
- No. I don't think it's a fair number cube because my partner won more games than I did.

9. Here are the results from two students who played the game and rolled a number cube 300 times. What do you think is the probability that Player 2 wins? Explain your thinking.

Responses and explanations vary.

- It looks like the one is never rolled and the six is rolled twice as often as the others, so my guess is that there is 1 two, 1 four, and 2 sixes on this number cube. This would mean the probability of Player 2 winning is $\frac{4}{6}$.
- About 40 of the rolls were two, about 50 of them were four, and about 100 were six. So that would be $\frac{(40+50+100)}{300}$, or about a 63% chance of Player 2 winning.

Unit 8 Lesson 5 605 Activity 2

Pairs | 15 min

Activity 2 How Likely? (continued)

Purpose: Students use the results of a repeated experiment (rolling an unfair number cube) to determine probabilities of events. Students also use these probabilities to design a game that is fair. **(MTR.5.1)**

2 Monitor

Select and sequence games that are fair and unfair to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Explain fairness in terms of probability.	Strengthen Invite students to use their game as an example and justify why it is fair.
Explain fairness in terms of the number of winning outcomes.	

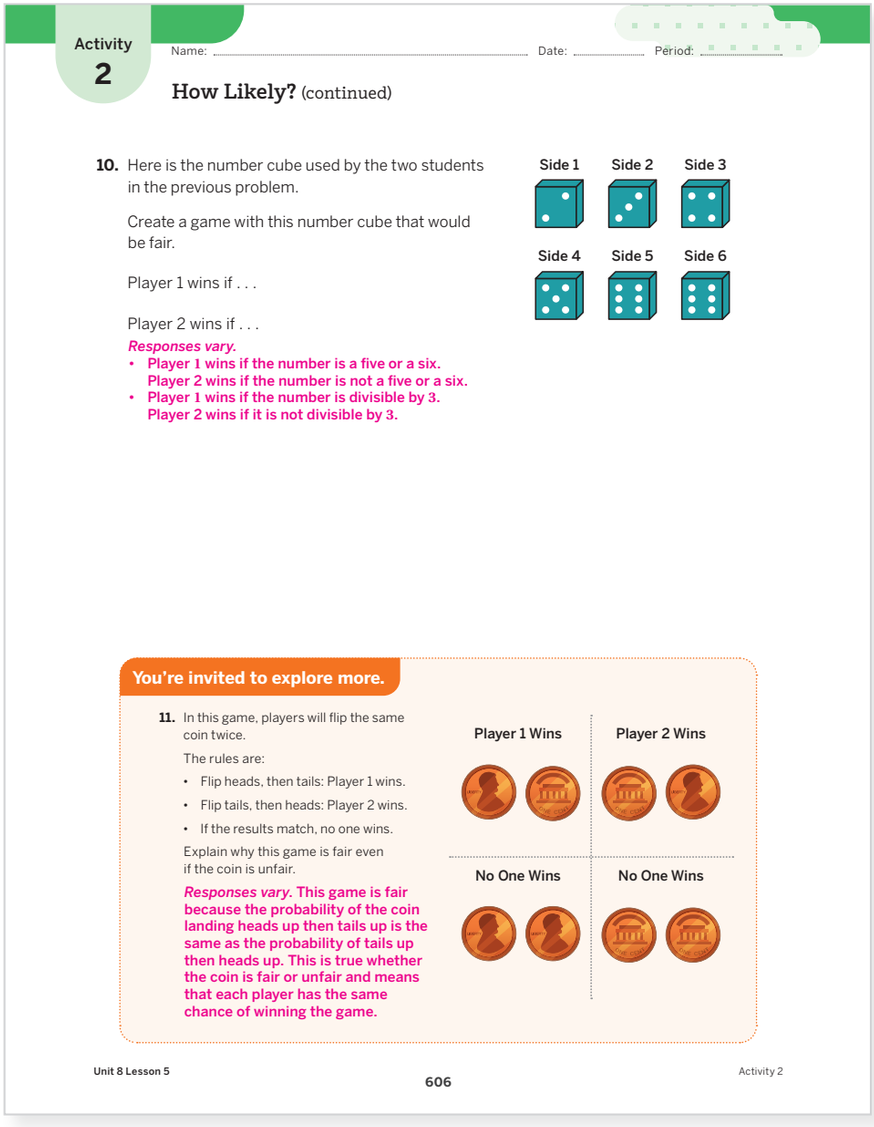
D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore fair games further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share the rules of their game and justify why the game is fair.

Consider asking:

- "If you roll a number cube 200 times and get a 6 every time, what can you reason about the number cube?" **(MTR.5.1)**
- "Would it be fair for two players to play your game with this number cube? Why? What is the probability each player would win?"



Activity 2

Name: Date: Period:

How Likely? (continued)

10. Here is the number cube used by the two students in the previous problem.

Side 1 Side 2 Side 3

Side 4 Side 5 Side 6

Create a game with this number cube that would be fair.

Player 1 wins if . . .

Player 2 wins if . . .

Responses vary.

- Player 1 wins if the number is a five or a six. Player 2 wins if the number is not a five or a six.
- Player 1 wins if the number is divisible by 3. Player 2 wins if it is not divisible by 3.

You're invited to explore more.

11. In this game, players will flip the same coin twice.

The rules are:

- Flip heads, then tails: Player 1 wins.
- Flip tails, then heads: Player 2 wins.
- If the results match, no one wins.

Explain why this game is fair even if the coin is unfair.

Responses vary. This game is fair because the probability of the coin landing heads up then tails up is the same as the probability of tails up then heads up. This is true whether the coin is fair or unfair and means that each player has the same chance of winning the game.

Player 1 Wins | Player 2 Wins

No One Wins | No One Wins

Unit 8 Lesson 5 606 Activity 2

Whole Class | 5 min

Synthesis

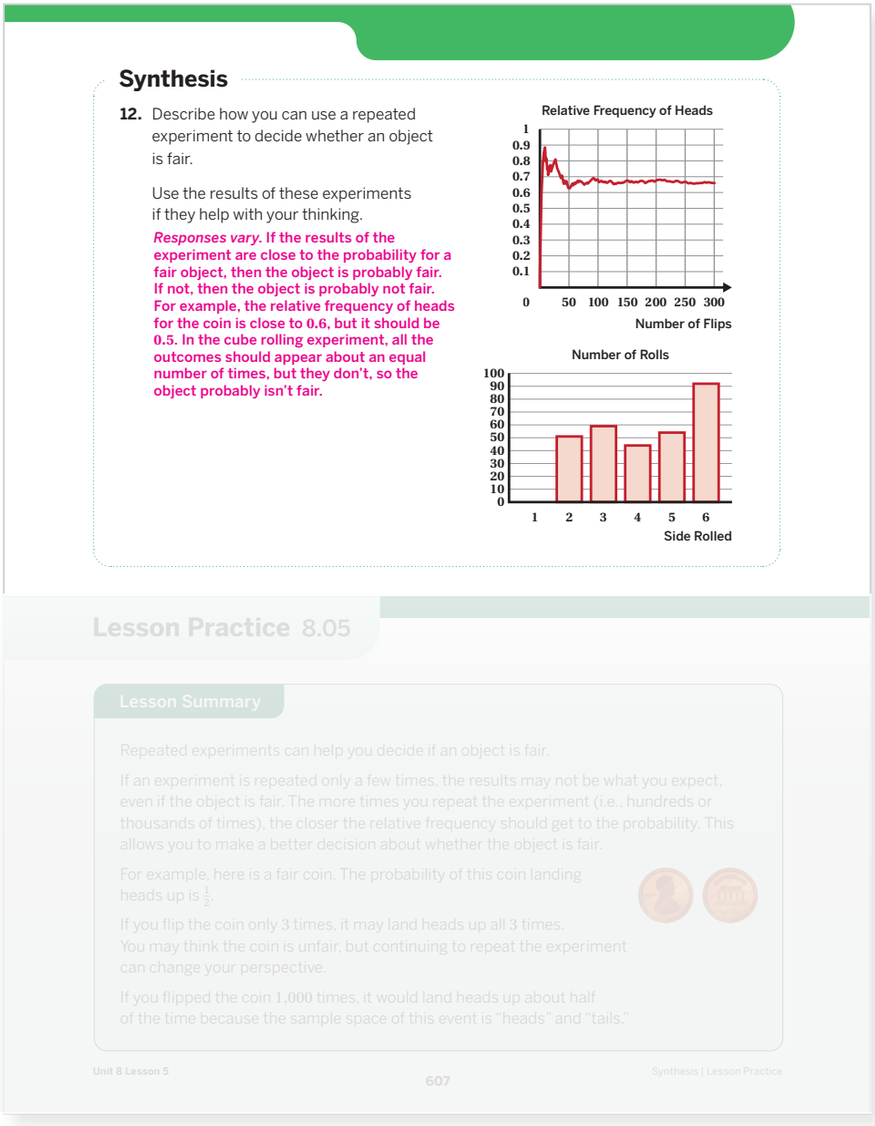
Purpose: Students synthesize their understanding of using repeated experiments to decide whether an object is fair.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Comparing the relative frequency from the experiment to the probability you expect from a fair object.
- The importance of repeating an experiment hundreds of times.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Describe how you can use a repeated experiment to decide whether an object is fair.

Use the results of these experiments if they help with your thinking.

Responses vary. **If the results of the experiment are close to the probability for a fair object, then the object is probably fair. If not, then the object is probably not fair. For example, the relative frequency of heads for the coin is close to 0.6, but it should be 0.5. In the cube rolling experiment, all the outcomes should appear about an equal number of times, but they don't, so the object probably isn't fair.**

Lesson Practice 8.05

Lesson Summary

Repeated experiments can help you decide if an object is fair.

If an experiment is repeated only a few times, the results may not be what you expect, even if the object is fair. The more times you repeat the experiment (i.e., hundreds or thousands of times), the closer the relative frequency should get to the probability. This allows you to make a better decision about whether the object is fair.

For example, here is a fair coin. The probability of this coin landing heads up is $\frac{1}{2}$.

If you flip the coin only 3 times, it may land heads up all 3 times. You may think the coin is unfair, but continuing to repeat the experiment can change your perspective.

If you flipped the coin 1,000 times, it would land heads up about half of the time because the sample space of this event is "heads" and "tails."

Unit 8 Lesson 5 607 Synthesis | Lesson Practice

Lesson Takeaway: The results from a repeated experiment can be used to approximate the probability of an event and help decide if an object is fair.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the probability of an event occurring based on the results of a repeated experiment.

Today's Goals

1. **Goal:** Use the results from a repeated experiment to approximate the probability of an event.
2. **Language Goal:** Explain whether an object is fair based on the results of a repeated experiment. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.05

This graph shows the results of 100 rolls with an unfair number cube.

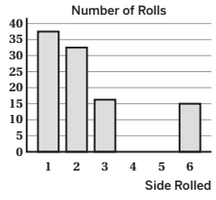
Based on the results, which is the most likely probability of rolling a one?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$



Explain your thinking.

Explanations vary. The results suggest that the number cube has one dot on 2 of its 6 sides. That means the probability of rolling a one is $\frac{2}{6}$ or $\frac{1}{3}$.

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use the results from a repeated experiment to approximate the probability of an event:

» **Respond to Student Thinking:**

- Review the **Lesson 4 Summary**.
- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task.

Professional Learning

Which students' ideas were you able to highlight when deciding if the coin is fair? Did learning on a device or learning on paper affect how often their work was shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe how you can use a repeated experiment to decide whether an object is fair.

Use the results of these experiments if they help with your thinking.

Responses vary. If the results of the experiment are close to the probability for a fair object, then the object is probably fair. If not, then the object is probably not fair. For example, the relative frequency of heads for the coin is close to 0.6, but it should be 0.5. In the cube rolling experiment, all the outcomes should appear about an equal number of times, but they don't, so the object probably isn't fair.

Lesson Practice 8.05

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For example, here is a fair coin. The probability of this coin landing heads up is $\frac{1}{2}$.



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If you flipped the coin 1,000 times, it would land heads up about half of the time because the sample space of this event is "heads" and "tails."

Lesson Practice 8.05

Name: _____ Date: _____ Period: _____

Problems 1–3: Deja has a six-sided number cube.

1. If this were a standard number cube, what would be the probability that the cube lands on a five?

$\frac{1}{6}$ (or equivalent)

2. Deja suspects the six-sided number cube is not standard.

- Deja rolled a five 40 times out of 100.
- Manuel rolled a five 21 times out 50.
- Santino rolled a five 11 times out of 30.

Based on these results, estimate the probability of rolling a five.

About 40% (or equivalent)

3. Is it likely this is a standard number cube? Explain your thinking.

No. *Explanations vary.* A standard number cube should land on five $\frac{1}{6}$ of the time. After 100 rolls, it should land on five about 16 or 17 times. All three people had it land on five more than twice as often. With this many rolls, there is strong evidence that this cube is not standard.

Problems 4–5: Santino wants to know if his coin is fair, so he flips it 10 times. It lands heads up 3 times and tails up 7 times.

4. Are these results enough to determine if the coin is fair? Explain your thinking.

No. *Explanations vary.* The relative frequency from experiments is usually a good approximation for the probability of an event, but only when the experiment is done many, many times. 10 flips may not be enough for the relative frequency to be close to the $\frac{1}{2}$ probability.

5. What could Santino do to be more sure of his results?

Santino should flip the coin many more times.

Problems 6–9: A game is played with two tetrahedral (4-sided) dice. The dice are in the shape of a pyramid with the numbers 1, 2, 3, and 4 written on each triangular side. After the dice are rolled, the numbers on the sides facing down are added together.

6. What are all of the possible sums for rolling two tetrahedral dice?

2, 3, 4, 5, 6, 7, 8

7. What sum do you think you will be most likely to get when rolling these dice? Least likely? Explain your thinking.

A sum of 5 is most likely and a sum of 2 or 8 are least likely. *Explanations vary.* I think rolling a sum of 5 will be the most likely because there are lot of ways to get 5: 1 and 4, 2 and 3, 3 and 2, and 4 and 1. I think rolling a sum of 2 or 8 will be the least likely because there is only one way to roll each of these: 1 and 1 and 4 and 4.

8. Create rules for a fair two-player game that uses these dice. *Responses vary.*

Player A wins if... the sum of the dice is 4. Player B wins if... the sum of the dice is 6.

Lesson Practice 8.05

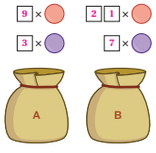
Name: _____ Date: _____ Period: _____

9. Explain why you think your game in Problem 8 is fair.

Responses vary. I think my game is fair because rolling a sum of 4 or a sum of 6 have the same likelihood of occurring. There are three ways to roll a sum of 4: $1+3$, $2+2$, and $3+1$. There are three ways to roll a sum of 6: $2+4$, $3+3$, and $4+2$. So, I assigned them the same number of points.

10. Fill in each blank using the digits 0 to 9 only once so that the probability of drawing a purple marble is the same for each bag.

Responses vary.



FAST Practice

11. A random number generator selects a digit from 1 to 5. Troy uses the generator 1,500 times. Which statement best predicts how many times the digit 3 will appear?

A. It will appear exactly 300 times.

B. It will appear exactly 340 times.

C. It will appear close to 300 times but probably not exactly 300 times.

D. It will appear close to 340 times but probably not exactly 340 times.

Spiral Review

Problems 12–14: There is a proportional relationship between sugar measured in cups and sugar measured in tablespoons. For example, 48 tablespoons of sugar is equivalent to 3 cups of sugar.

12. Plot and list the coordinates of two more points that represent the relationship.

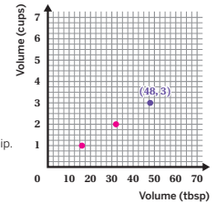
Points where the y-coordinate is $\frac{1}{16}$ of the x-coordinate are considered correct.

13. What is the constant of proportionality for this relationship?

$\frac{1}{16}$

14. Write an equation representing this relationship. Use c for cups and t for tablespoons.

$c = \frac{1}{16}t$ or $t = 16c$



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.2.3
	2–7	2	MA.7.DP.2.4
	8–10	3	MA.7.DP.2.4
FAST Practice	11	2	MA.7.DP.2.4
Spiral Review			
	12	1	MA.7.AR.4.3
	13	1	MA.7.AR.4.2
	14	1	MA.7.AR.4.4

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Describing the likelihood of events	Lesson 1	1	MA.7.DP.2.1
2	Identifying probability tools that can model real-world situations	Lesson 2	2	MA.7.DP.2.2, MTR.6.1
3a	Making predictions about future events of a repeated experiment	Lesson 3	2	MA.7.DP.2.4
3b	Making and justifying conjectures about the sample space of a repeated experiment	Lesson 3	2	MA.7.DP.2.4
4a	Using the structure of spinner diagrams to calculate the possible chance of events	Lesson 2	1	MA.7.DP.2.1
4b	Using the structure of spinner diagrams to calculate the probability of an event	Lesson 2	1	MA.7.DP.2.3
5a	Interpreting the fairness of a probability tool using the results of an experiment	Lesson 5	2	MA.7.DP.2.4, MTR.7.1
5b	Designing probability tools that can model real-world situations	Lesson 4	3	MA.7.DP.2.4, MA.7.NSO.2.2, MTR.7.1

Assessment and Lesson Resources



- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

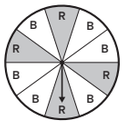
Unit 8

1. Which event is possible, but unlikely?

- **A.** Flipping one fair coin that lands with heads facing up.
- **B.** Opening a 300-page book to exactly page 143.
- **C.** Rolling a seven on aStandard number cube.
- **D.** Getting wet if you stand in the rain without an umbrella.

2. Select *all* of the following events that have a probability of occurring 40% of the time.

- ☑ **A.** Land on R in one spin.



- ☐ **B.** Both coins land heads up.



- ☑ **C.** Select a "P" from this bag.



- ☐ **D.** Roll a 4 in one roll.



- ☐ **E.** Select an "E" from this bag.



3. Esi does an experiment where she picks a block out of a bag without looking 50 times, putting it back each time. She picks a green block 32 times.



a Out of 200 picks, how many times do you predict Esi will pick a green block?

Responses between 125 and 128 are considered correct.

b If the bag has 8 blocks, how many are likely to be green?

5 blocks

Explain your reasoning.

***Explanations vary*. 32 green blocks out of 50 is 64% of the blocks. 64% of 8 blocks is 5.12, which is closest to 5 green blocks.**

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Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 8

4. The Spin N' Dine Restaurant has a special deal. You can either spin to select one appetizer at random for $5 or one entree for $10.

Appetizers

A: Chicken wings
B: Onion dip
C: Cheese sticks
D: Crab cakes
E: Fried pickles



Entrees

1: Cheese pizza
2: Barbecue ribs
3: Meatloaf



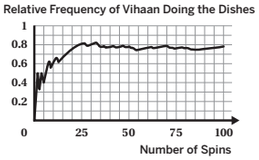
a Terrence's favorite appetizer is cheese sticks. What is the probability he will get this appetizer on his spin?

$\frac{1}{5}$

b Jenny likes cheese pizza and barbeque ribs. What is the probability she will get one of these appetizers on her spin?

$\frac{2}{3}$

5. Vihaan and Neena use a spinner to decide who has to do the dishes each night. They make a graph of the relative frequency of Vihaan doing the dishes.



a Is this spinner fair?

No

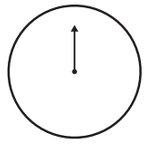
Explain your thinking. Use evidence to support your claim.

***Explanations vary*. If the spinner were fair, then the relative frequency should get close to 0.5 as you spin it more and more times. Looking at the graph, the relative frequency gets closer to 0.75 or 0.8.**

b Describe or sketch what you think their spinner could looks like.

Responses vary.

- **The spinner would have 4 sections with 3 of them labeled for Vihaan, so the probability of him doing the dishes would be $\frac{3}{4}$.**
- **The spinner would have 10 sections with 8 of them labeled for Vihaan, so the probability of him doing the dishes would be 0.8.**



D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Compare probabilities to the results of repeated experiments. • Determine the probability of events by identifying the sample space. **(Lessons 1–5)**	1, 2, 4	**Support**	• **Teacher Move:** Consider revisiting Unit 8, Lesson 1 (How Likely?) and Unit 8 Lesson 2 (Prob-bear-bilities)
		Stretch	• **You're invited to explore more:** Lesson 2 (Prob-bear-bilities)
	3	**Support**	• **Mini Lesson:** Calculating Probability Using Proportional Relationships (ML 8.03)
		Strengthen	• **Teacher Move:** Consider Revisiting Unit 8, Lesson 3 (Mystery Bag)
	5	**Support**	• **Teacher Move:** Consider revisiting Unit 8, Lesson 4 (Spin Class) and Unit 8, Lesson 5 (Is It Fair?)
		Strengthen	• **Challenge Creator:** Lesson 4 (Spin Class)
		Stretch	• **You're invited to explore more:** Lesson 5 (Is It Fair?)

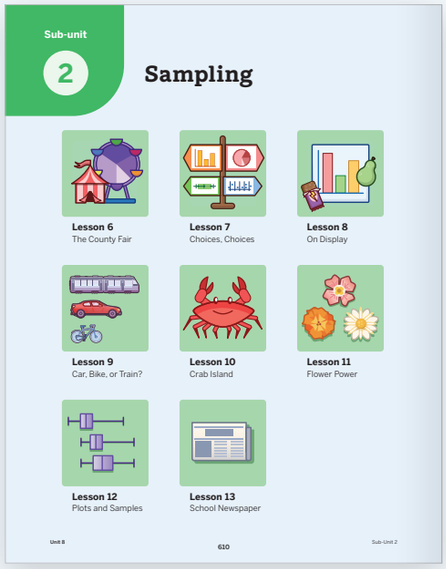
Sub-Unit 2

Sampling

Sub-Unit 2 Goals

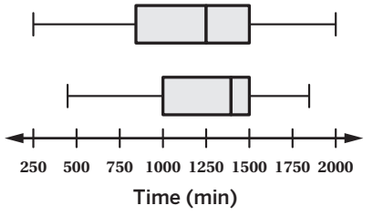
- **Goal:** Use measures of center and measures of variability from random samples to draw conclusions about and compare populations.
- **Goal:** Choose and create appropriate graphical representations for numerical and categorical data sets.
- **Language Goal:** Explain the purpose of sampling and which methods of obtaining a sample tend to produce representative samples.

In this sub-unit, students calculate mean and median to compare and contrast the measures of center for sets of data. They conceptualize the difference between a **sample** and a **population**, analyze whether sampling strategies would likely produce a **representative** sample, and explain how that affects their understanding of a population. Students create bar charts, circle graphs, histograms, box plots, line plots, and stem-and-leaf plots to display data.



Math That Matters Most

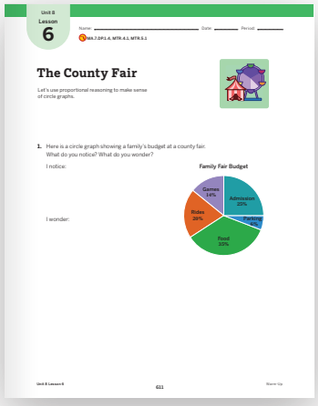
Concept or Skill	For example . . .
Calculate the mean and median for a data set.	Here is a data set: $4, 5, 5, 6, 8, 8$. **a.** Calculate the mean. *6* **b.** Determine the median. *5.5*
Use proportional reasoning to make predictions about a population based on a sample.	In a school of 580 students, one class was asked which hand they write with. "L" means they use their left hand, and "R" means they use their right hand. Here are the results: L, R, R, R, R, R, R, R, R, L, R, R, R, R, R Estimate the number of students at the school who write with their left hand. *About 77 students. In the sample, 2 out of 15 students write with their left hand, which is about 13.3%. 13.3% of 580 is 77 students.*

Concept or Skill	For example . . .
Understand why a sample is often used to study a population and determine whether a sampling method will generate a representative sample.	Jada wants to learn about the percentage of students who like the food in the cafeteria. Jada asks the first 25 students who purchase lunch at the cafeteria if they like the food. Is Jada's method likely to produce a representative sample? *No, because people who like school lunch might be more likely to line up first. Jada is also not asking any students who didn't want to buy lunch from the cafeteria.*
Use statistics from random samples to understand a population or compare two populations.	A video game developer wants to know how long it takes people to finish playing a new game. She surveyed two random samples of 20 players and asked how long it took each of them. Estimate the median time it takes a player to finish the game.  *It takes about 1300 minutes.*

Unit 8

Lesson

6



The County Fair

Creating Circle Graphs

Let's use proportional reasoning to make sense of circle graphs.

Learning Outcomes

Today's Goals

1. **Goal:** Create a *circle graph* to display data.
2. **Goal:** Use a *circle graph* to solve problems.
3. **Language Goal:** Explain how to use proportional reasoning to create a *circle graph*. **(Writing)**

Students are introduced to **circle graphs** as a way to visualize data. They use given data to create circle graphs and use circle graphs to solve problems.

Coherence

Prior Learning

In Unit 2, students learned to use proportional reasoning to solve problems. In Unit 4, students used percentages to describe mathematical situations and to solve problems.

Future Learning

In Lessons 7 and 8, students will choose and create an appropriate data display for a given set of data.

Connections

- Students use circle graphs visuals to represent data as parts of a whole and determine each slice's percentage directly related to the angle measure. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.4

Use proportional reasoning to construct, display and interpret data in circle graphs.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.3.1	**MA.7.DP.1.5**
MA.7.AR.3.2	
MA.7.AR.4.5	

Vocabulary

New Vocabulary

circle graph

Lesson at a Glance

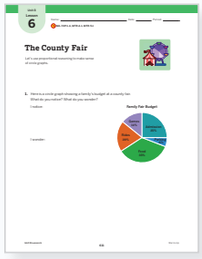
~ 45 min

MA.7.DP.1.4

Warm-Up

Whole Class | 5 min

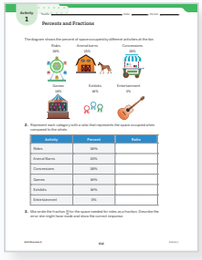
Students use the **Notice and Wonder** routine to discuss observations about a **circle graph**.



Activity 1

Pairs | 5 min

Students write percents as ratios to represent parts of a whole.



Activity 2

Pairs | 15 min

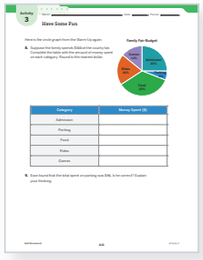
Students connect proportional reasoning to determining the angle measures in a circle graph from percents. **MLR1: Stronger and Clearer Each Time** **(MTR.5.1)**



Activity 3

Pairs | 10 min

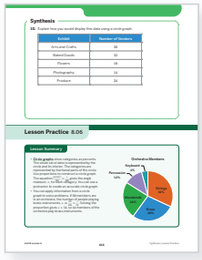
Students solve problems using a circle graph for a real-world context.



Synthesis

Whole Class | 5 min

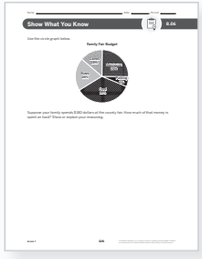
Students synthesize their understanding of circle graphs.



Show What You Know

Independent | 5 min

Students demonstrate their understanding using a circle graph to solve a problem involving a family's fair budget.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Protractors

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to discuss observations about a circle graph.

1 Launch

Direct students' attention to the *circle graph*.

Consider sharing that students will examine circle graphs as a way to represent data that are in categories.

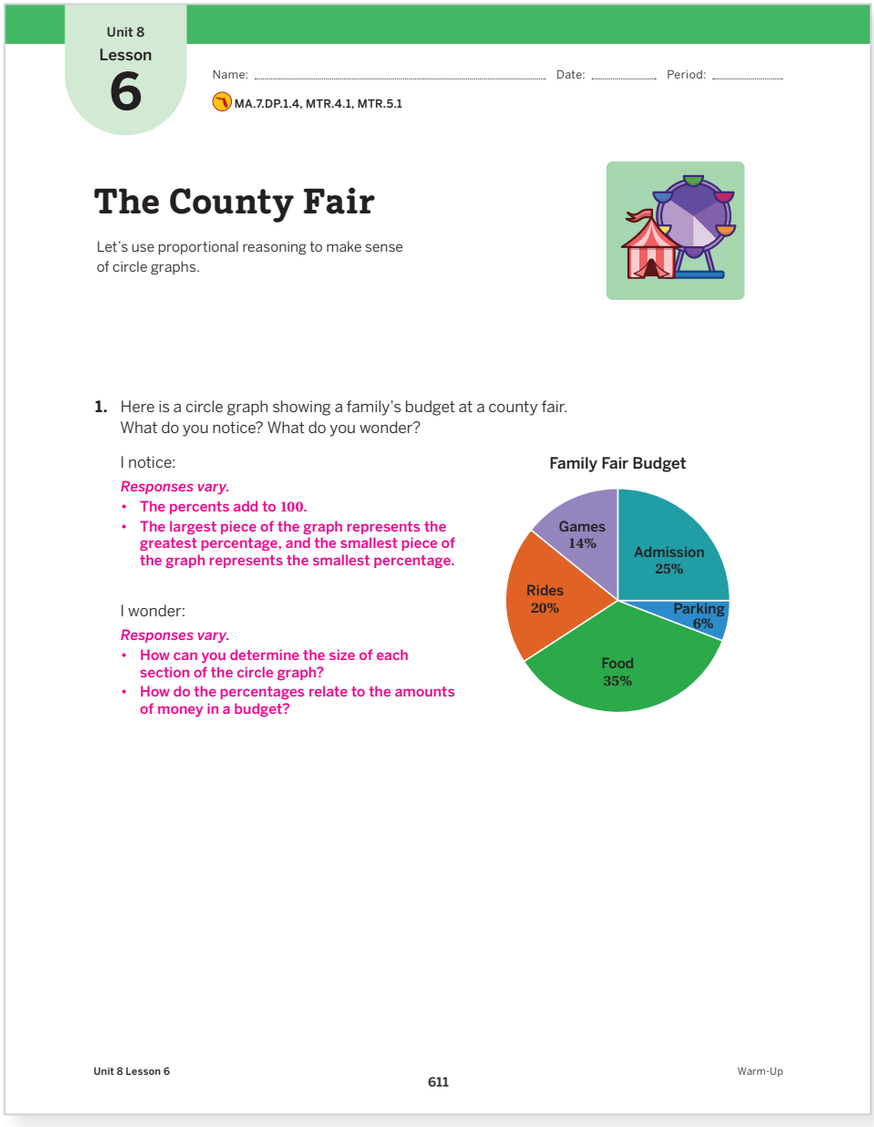
Use the Notice and Wonder routine to promote curiosity and help students make sense of the circle graph.

2 Connect

Invite students to share their observations, first in small groups and then with the whole class.

Consider asking:

- "How does the size of each piece of the graph compare with the percentage that each piece represents?"
- "What is the sum of the percentages in the graph?"
- "How are the percentages related to each other?"
- "What can you determine about the family's budget from the circle graph?"



Unit 8
Lesson
6

Name: ..... Date: ..... Period: .....

MA.7.DP.1.4, MTR.4.1, MTR.5.1

The County Fair

Let's use proportional reasoning to make sense of circle graphs.

1. Here is a circle graph showing a family's budget at a county fair. What do you notice? What do you wonder?

I notice:

Responses vary.

- The percents add to 100.
- The largest piece of the graph represents the greatest percentage, and the smallest piece of the graph represents the smallest percentage.

I wonder:

Responses vary.

- How can you determine the size of each section of the circle graph?
- How do the percentages relate to the amounts of money in a budget?

Family Fair Budget

Unit 8 Lesson 6 611 Warm-Up

Pairs | 5 min

Activity 1 Percents and Fractions

Purpose: Students write percents as ratios to represent parts of a whole.

1 Launch

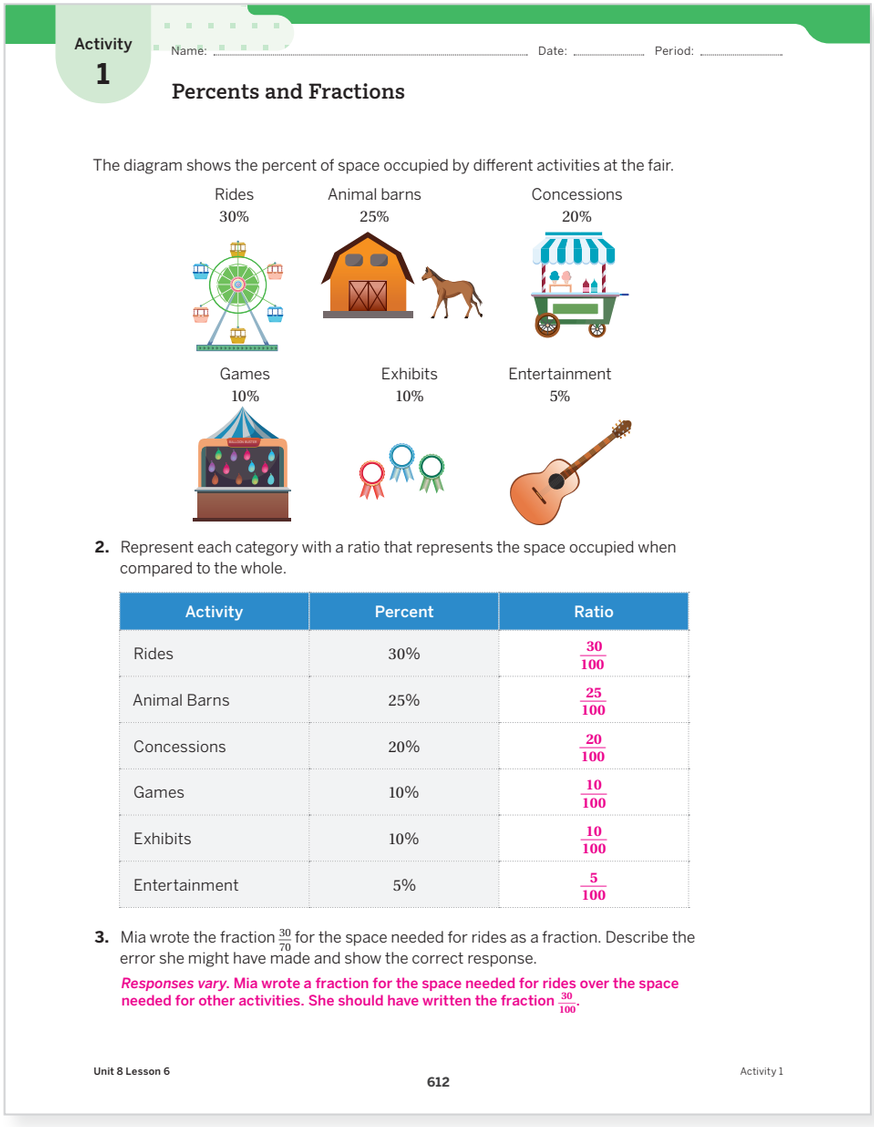
To increase engagement, ask students about what they know about fairs and the different fair activities shown in the diagram.

2 Monitor

Look for students who simplify the fractions. Note that it can be useful to use fractions with the same denominator to compare quantities.

3 Connect

Invite students to share their explanations of Mia's error and how to correct it. **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Percents and Fractions

The diagram shows the percent of space occupied by different activities at the fair.

Rides 30%

Animal barns 25%

Concessions 20%

Games 10%

Exhibits 10%

Entertainment 5%

2. Represent each category with a ratio that represents the space occupied when compared to the whole.

Activity	Percent	Ratio
Rides	30%	$\frac{30}{100}$
Animal Barns	25%	$\frac{25}{100}$
Concessions	20%	$\frac{20}{100}$
Games	10%	$\frac{10}{100}$
Exhibits	10%	$\frac{10}{100}$
Entertainment	5%	$\frac{5}{100}$

3. Mia wrote the fraction $\frac{30}{70}$ for the space needed for rides as a fraction. Describe the error she might have made and show the correct response.

Responses vary. Mia wrote a fraction for the space needed for rides over the space needed for other activities. She should have written the fraction $\frac{30}{100}$.

Unit 8 Lesson 6 612 Activity 1

Pairs | 15 min

Activity 2 Setting Up

Purpose: Students connect proportional reasoning to determining the angle measures in a circle graph from percents. **(MTR.5.1)**

Short on time? Consider assigning a row to a pair of students and having them work on their row to complete a class table.

1 Launch

Consider asking, "How do you know that you can use a circle graph to represent this data?"

Invite students to return to the Warm-Up so that they have an example of a completed circle graph, and discuss how they might create one for the data from Activity 1.

2 Monitor

Encourage students to think about proportional relationships between quantities throughout the activity.

Consider asking, "How would you solve a problem to determine a part when you know the percent of a whole?" **(MTR.5.1)**

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Discuss with students how to set up and solve a proportion between two quantities.
Use the percent for each category as the measure of the central angle.	**Support** Invite them to divide a circle into fourths. Consider asking, "What percent of the circle is one section? How many degrees is the angle measure?"
Determine the number of degrees by multiplying 360° by the percent.	**Stretch** Consider asking, "How can you use the percent equation to determine the measure of the central angle?"

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Setting Up

Circle graphs visually display data as parts of a whole. In a circle graph, the parts add up to 100 percent. There are 360° in a circle.

4. The data in this table matches the diagram from Activity 1. Use a proportional relationship to determine what part of a circle will represent each percent in the table.

Activity	Percent	Angle Measure
Rides	30%	108°
Animal Barns	25%	90°
Concessions	20%	72°
Games	10%	36°
Exhibits	10%	36°
Entertainment	5%	18°

5. Select one activity from the table and explain how you calculated the angle measure.

Responses vary. I calculated the angle measure for entertainment by writing the proportion using 5%: $\frac{x}{360} = \frac{5}{100}$. Then, I solved for x by multiplying both sides by 360. Finally, I simplified the right side of the equation.

Unit 8 Lesson 6 613 Activity 2

Pairs | 15 min

Activity 2 Setting Up (continued)

Purpose: Students connect proportional reasoning to determining the angle measures in a circle graph from percents. **(MTR.5.1)**

Short on time? Consider assigning a row to a pair of students and having them work on their row to complete a class table.

2 Monitor

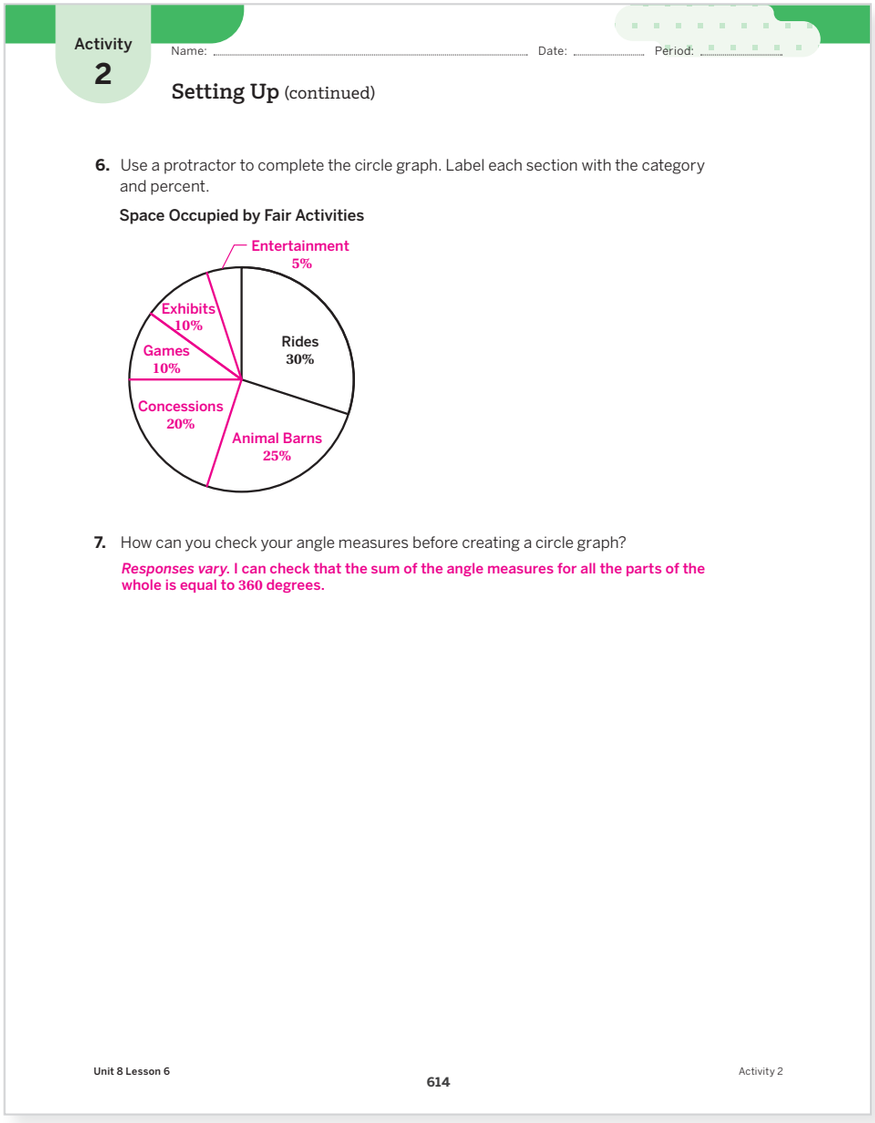
To support students getting started, show how to use a protractor to make one section of a circle graph when given a section of the circle graph by aligning the protractor along the edge of the given section.

3 Connect

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in creating step-by-step instructions for creating a circle graph. Invite students to meet with a partner, give feedback on each other's instructions, then use the feedback to write a second, clearer set of instructions. **(MTR.4.1)**

Create a class definition for the term **circle graph** using students' language and the glossary.

To surface the Key Takeaway, consider asking, "When would a circle graph be a good choice to display data?"



Activity 2

Name: Date: Period:

Setting Up (continued)

6. Use a protractor to complete the circle graph. Label each section with the category and percent.

Space Occupied by Fair Activities

7. How can you check your angle measures before creating a circle graph?

Responses vary. I can check that the sum of the angle measures for all the parts of the whole is equal to 360 degrees.

Unit 8 Lesson 6 614 Activity 2

Key Takeaway: **Circle graphs** are a good choice to display data that compares parts of data to a whole or parts to each other.

Pairs | 10 min

Activity 3 Have Some Fun

Purpose: Students solve problems using a circle graph for a real-world context.

1 Launch

Invite students to make a prediction about which category the family spent the most money on.

2 Monitor

Look for students to reason quantitatively to interpret the circle graph.

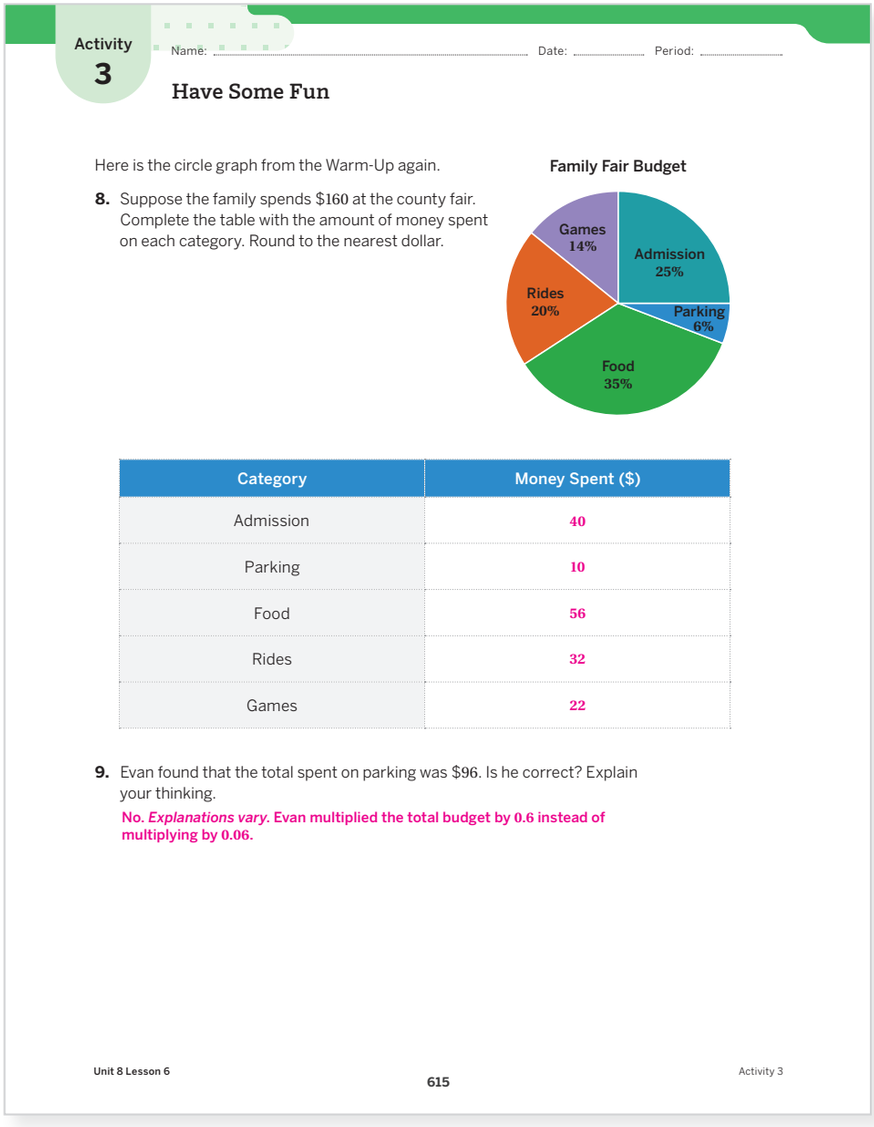
D Differentiation

Look for students who:	Respond to Student Thinking
Assume the percents translate directly into each amount spent in dollars.	**Support** Encourage students to consider the total for percentages in the circle graph and the total amount spent. Consider asking, "Are the total amount spent and the total percentage the same? How can you compare ratios when the denominators are not the same?"
Promptly explain their thinking and fluidly complete the table.	**Strengthen** Consider asking, "How would increasing the total money spent affect your answers? How would keeping the total money spent the same but increasing the money spent on games affect your answers?"

ELL **English Language Learners** Encourage students to visualize their thinking to help them explain how to use proportions to solve the problem. Demonstrate how to visualize thinking using a teacher think-aloud. **(Writing)**

3 Connect

Invite students to share the process they used to determine whether Evan was correct and how they identified the error that Evan made.



Activity 3

Name: Date: Period:

Have Some Fun

Here is the circle graph from the Warm-Up again.

Family Fair Budget

8. Suppose the family spends $160 at the county fair. Complete the table with the amount of money spent on each category. Round to the nearest dollar.

Category	Money Spent ($)
Admission	40
Parking	10
Food	56
Rides	32
Games	22

9. Evan found that the total spent on parking was $96. Is he correct? Explain your thinking.

No. *Explanations vary.* Evan multiplied the total budget by 0.6 instead of multiplying by 0.06.

Unit 8 Lesson 6 615 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of circle graphs.

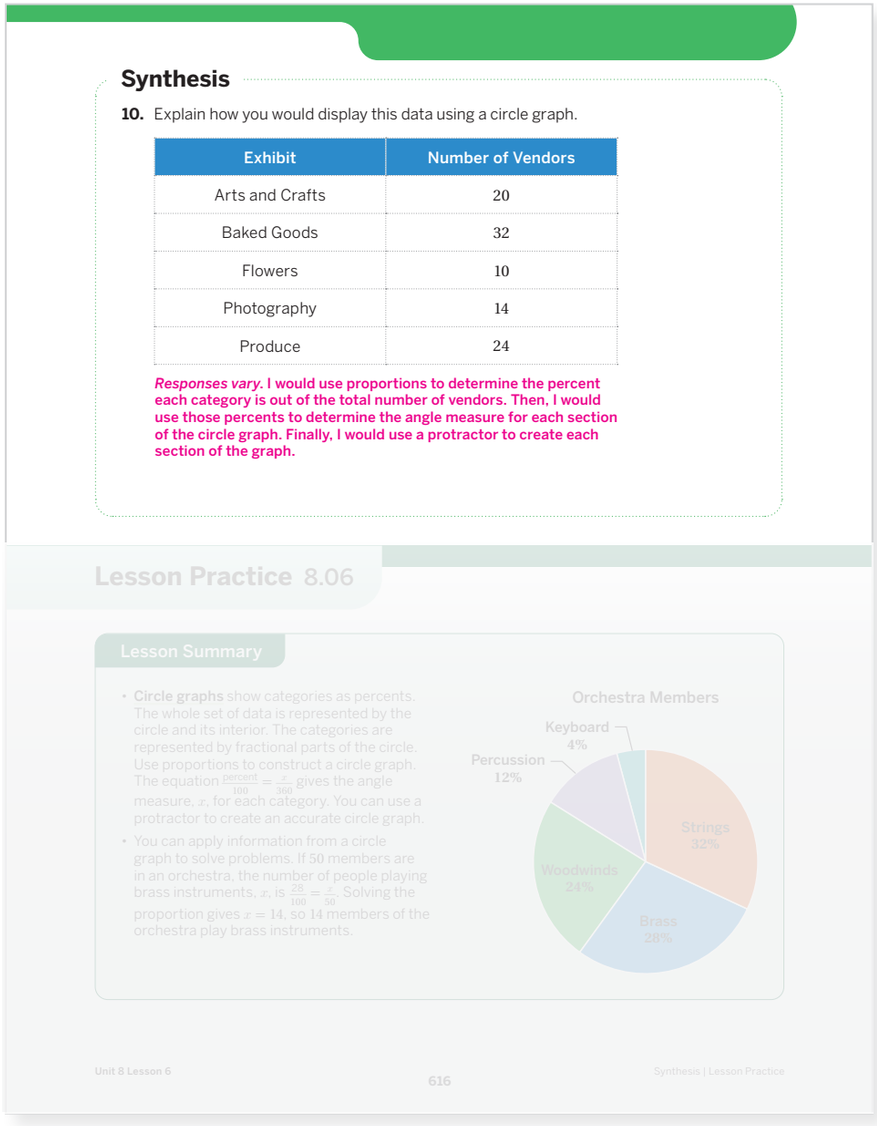
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas about understanding circle graphs, such as:

- Circle graphs display data as parts out of 100%.
- You can use proportional reasoning to determine the angle measure of each part of the circle graph.
- You can use proportional reasoning to interpret circle graphs, relating the percentages to other part-to-whole relationships.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for creating a circle graph from a set of data using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Explain how you would display this data using a circle graph.

Exhibit	Number of Vendors
Arts and Crafts	20
Baked Goods	32
Flowers	10
Photography	14
Produce	24

Responses vary. I would use proportions to determine the percent each category is out of the total number of vendors. Then, I would use those percents to determine the angle measure for each section of the circle graph. Finally, I would use a protractor to create each section of the graph.

Lesson Practice 8.06

Lesson Summary

- **Circle graphs** show categories as percents. The whole set of data is represented by the circle and its interior. The categories are represented by fractional parts of the circle. Use proportions to construct a circle graph. The equation $\frac{\text{percent}}{100} = \frac{x}{360}$ gives the angle measure, x, for each category. You can use a protractor to create an accurate circle graph.
- You can apply information from a circle graph to solve problems. If 50 members are in an orchestra, the number of people playing brass instruments, x, is $\frac{28}{100} = \frac{x}{50}$. Solving the proportion gives $x = 14$, so 14 members of the orchestra play brass instruments.

Unit 8 Lesson 6 616 Synthesis | Lesson Practice

Lesson Takeaway: You can use proportional reasoning and part-to-whole relationships to create and interpret **circle graphs**.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using a circle graph to solve a problem involving a family's fair budget.

Today's Goals

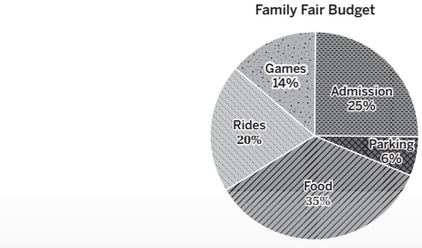
1. **Goal:** Create a *circle graph* to display data.
2. **Goal:** Use a *circle graph* to solve problems.
3. **Language Goal:** Explain how to use proportional reasoning to create a *circle graph*. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.06

Use the circle graph below.



Suppose your family spends $180 dollars at the county fair. How much of that money is spent on food? Show or explain your reasoning.

$63 was spent on food. .35(180) = $63

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use proportional reasoning to make sense of circle graphs:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on students' use of protractors in Activity 2. What support might students need to use this tool effectively in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Explain how you would display this data using a circle graph.

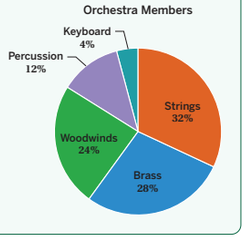
Exhibit	Number of Vendors
Arts and Crafts	20
Baked Goods	32
Flowers	10
Photography	14
Produce	24

Responses vary. I would use proportions to determine the percent each category is out of the total number of vendors. Then, I would use those percents to determine the angle measure for each section of the circle graph. Finally, I would use a protractor to create each section of the graph.

Lesson Practice 8.06

Lesson Summary

- **Circle graphs** show categories as percents. The whole set of data is represented by the circle and its interior. The categories are represented by fractional parts of the circle. Use proportions to construct a circle graph. The equation $\frac{\text{percent}}{100} = \frac{x}{360}$ gives the angle measure, x, for each category. You can use a protractor to create an accurate circle graph.
- You can apply information from a circle graph to solve problems. If 50 members are in an orchestra, the number of people playing brass instruments, x, is $\frac{28}{100} = \frac{x}{50}$. Solving the proportion gives $x = 14$, so 14 members of the orchestra play brass instruments.



Lesson Practice 8.06

Name: _____ Date: _____ Period: _____

Problems 1–3: The table shows the number of pigs of different breeds shown at the county fair. Use the table to answer the problems.

Breed	Number	Ratio	Percent	Number of Degrees
Berkshire	45	$\frac{45}{150}$	30%	108°
Landrace	11	$\frac{11}{150}$	7%	25°
Duroc	33	$\frac{33}{150}$	22%	79°
Chester White	42	$\frac{42}{150}$	28%	101°
Hampshire	19	$\frac{19}{150}$	13%	47°

1. What is the total number of pigs shown at the fair? Why do you need this total to complete the table?

150; *Responses vary.* I need the total number of pigs so that I can calculate what ratio each breed represents compared to the whole. Then I can use the ratio to determine the percent and the number of degrees that represents each breed in a circle graph.

2. For each breed, complete the table to show how it compares to the whole. For the Ratio column, do not simplify the fraction. For the Percent column and Number of Degrees column, round to the nearest whole.

Responses shown in table.

3. Explain how you determined the number of degrees for Duroc pigs.

Responses vary. First, I wrote a ratio of the number of Duroc pigs to the total number of pigs, $\frac{33}{150}$, to determine that Duroc pigs make up 22% of the whole. Then, I determined that 22% of 360° is 79.2°, which rounds to 79°.

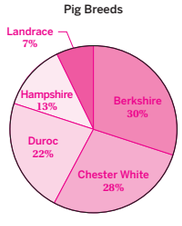
Lesson Practice 8.06

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 4–5: Use the completed table from the previous page.

4. Draw the circle graph that shows the number of pigs of different breeds shown at the county fair. Label each section with the breed and percent.



5. Which is an accurate comparison of the breeds of pigs at the county fair?

A. The angle for each breed of pig is $\frac{12}{5}$ times the number of pigs of each breed.

B. The difference in the angle measure for the Duroc and Landrace pigs is exactly 53°.

C. The ratio of Landrace pigs to Hampshire pigs is greater than the ratio of Duroc pigs to Berkshire pigs.

D. The percent of pigs that are Berkshire is greater than the sum of the percents for any two other breeds.

Spiral Review

6. Calculate the sum of $-\frac{3}{8} + \frac{4}{5}$. $\frac{17}{40}$

7. Calculate the product of $-\frac{3}{8} \times \frac{4}{5}$. $-\frac{3}{10}$

8. Calculate the difference of $-\frac{3}{8} - \frac{4}{5}$. $-1\frac{7}{40}$

9. Calculate the quotient of $-\frac{3}{8} \div \frac{4}{5}$. $-\frac{15}{32}$

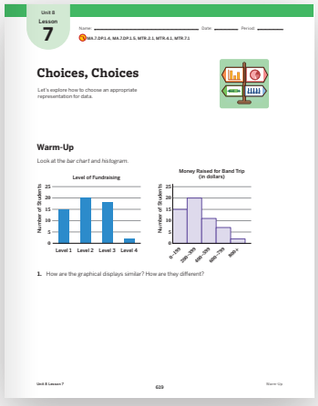
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.1.4
	2	2	MA.7.DP.1.4
	3	3	MA.7.DP.1.4
FAST Practice	4–5	2	MA.7.DP.1.4
Spiral Review			
	6–9	1	MA.7.NSO.2.2

Unit 8

Lesson

7



Choices, Choices

Choosing an Appropriate Representation for Data

Let's explore how to choose an appropriate representation for data.

Learning Outcomes

Today's Goals

1. **Goal:** Analyze different graphical displays that show the same data and choose the display that is most appropriate for answering a question or conveying certain information about the data.
2. **Goal:** Choose the most appropriate display for a given data set.
3. **Language Goal:** Describe why a graphical display is the best one for answering a question or representing the data. **(Reading, Writing, and Speaking)**

Students examine graphical displays that show the same data set and choose the ones that are most appropriate for conveying certain information about the data. They consider real-world numerical and *categorical data* and choose the graphical displays that are most appropriate for representing the data. **(MTR.2.1, MTR.7.1)**

Coherence

Prior Learning

In Lesson 6, students learned how to use proportional reasoning to construct and interpret data in circle graphs.

Future Learning

In Lesson 8, students will create appropriate representations for data.

Connections

- Students explore different ways to display data for a class reading contest. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.5

Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.

Also Addressing: **MA.7.DP.1.4**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.5	**MA.8.DP.1.1** **MA.912.DP.1.1**

Vocabulary

Review Vocabulary

bar chart	*box plot*
categorical data	*dot plot*
histogram	*stem-and-leaf plot*

Lesson at a Glance

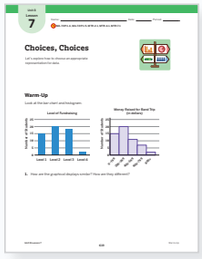
~ 45 min

MA.7.DP.1.4, MA.7.DP.1.5

Warm-Up

Pairs | 5 min

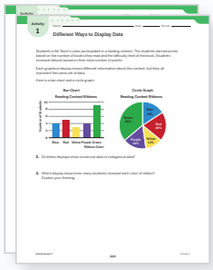
Students describe the similarities and differences between how a *bar chart* and a *histogram* display data. **Routine:** Think-Pair-Share



Activity 1

Pairs | 15 min

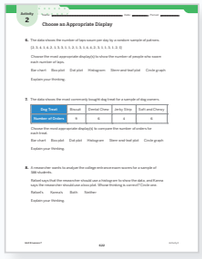
Students examine six graphical displays that represent the same data set and choose the ones that are most appropriate for conveying certain aspects of the data. **(MTR.2.1)**



Activity 2

Pairs | 15 min

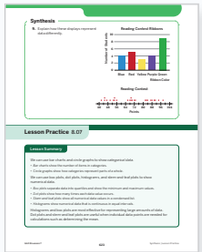
Students choose the most appropriate graphical displays for displaying real-world numerical and *categorical data*. **Routine:** Decide and Defend **(MTR.7.1)**



Synthesis

Whole Class | 5 min

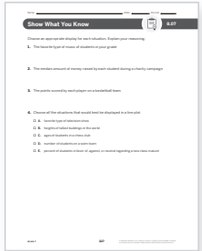
Students synthesize their understanding about how to choose an appropriate graphical display to represent data. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by choosing the most appropriate display for a given data set.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students describe the similarities and differences between how a *bar chart* and a *histogram* display data.

1 Launch

To surface prior knowledge, consider asking:

- "What is a *bar chart*? What does it show?"
- "What is a *histogram*? What does it show?"

Use the Think-Pair-Share routine to facilitate a discussion about how the amount of money raised is shown differently on the two displays and what the heights of the bars show. Invite students to think independently and then take turns sharing with a partner.

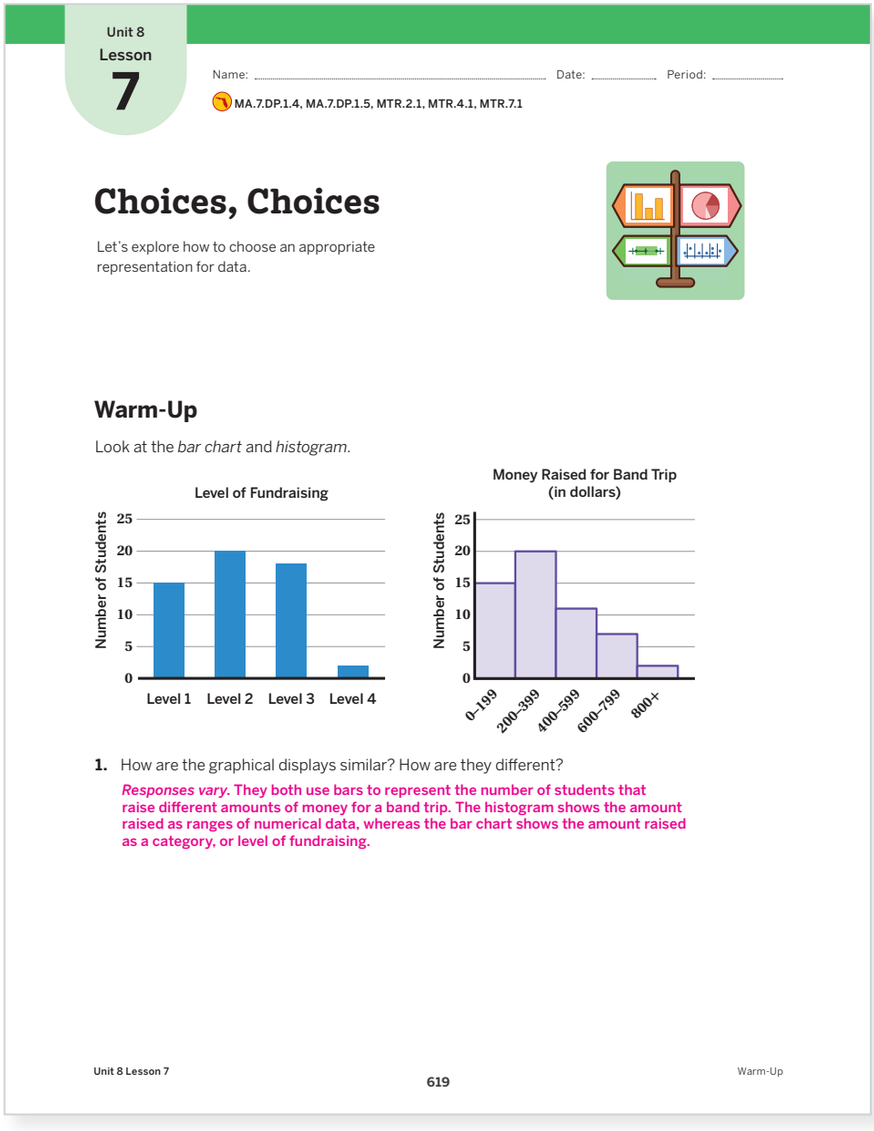
2 Connect

Collect and share students' observations, inviting each pair to share their ideas.

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting the visual representations. Consider asking, "What does the vertical axis show in each display? What does the horizontal axis show in each display?"

Consider asking:

- "Do the displays share a common characteristic?"
- "How do the vertical axes compare?"
- "How do the horizontal axes compare?"
- "What is a notable difference between the displays?"



Unit 8
Lesson
7

Name: _____ Date: _____ Period: _____

MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.4.1, MTR.7.1

Choices, Choices

Let's explore how to choose an appropriate representation for data.

Warm-Up

Look at the *bar chart* and *histogram*.

1. How are the graphical displays similar? How are they different?

Responses vary. They both use bars to represent the number of students that raise different amounts of money for a band trip. The histogram shows the amount raised as ranges of numerical data, whereas the bar chart shows the amount raised as a category, or level of fundraising.

Unit 8 Lesson 7 619 Warm-Up

Pairs | 15 min

Activity 1 Different Ways to Display Data

Purpose: Students examine six graphical displays that represent the same data set and choose the ones that are most appropriate for conveying certain aspects of the data. **(MTR.2.1)**

1 Launch

Direct students' attention to the situation and read the information aloud.

To generate interest, consider asking, "Have you ever participated in a contest?"

To support students getting started, consider asking them to describe how each display shows information about the reading contest. **(MTR.2.1)**

2 Monitor

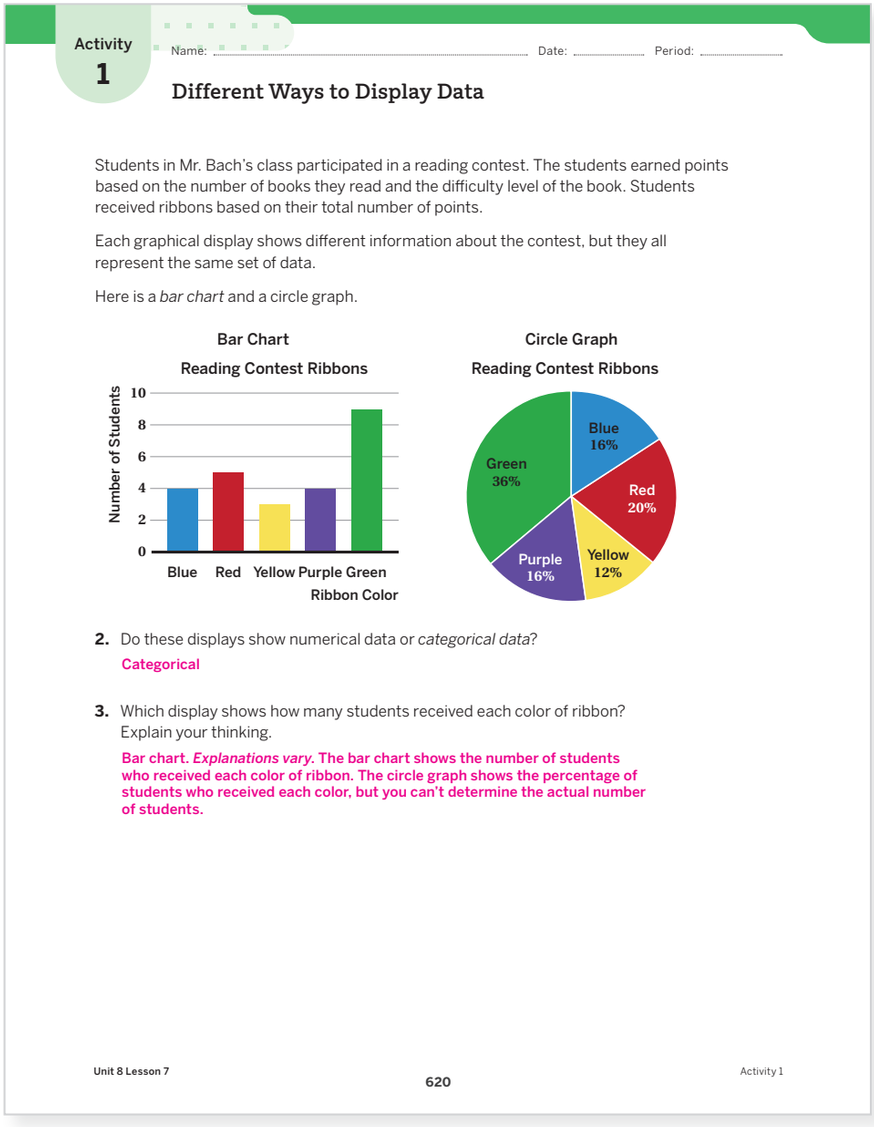
Look for students who do not understand the difference between numerical and *categorical data*.

D Differentiation *(Problems 2–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to write a brief description beside each display, explaining what it shows.
Need support getting started. *(Problem 5)*	**Support** Consider asking, "How does each display represent the individual data values?"
Show a clear understanding of how the displays are alike and different.	**Strengthen** For each pair of displays, consider asking, "Can you use the information in one display to create the other display?"
Finish early.	**Stretch** Encourage students to write questions that can be answered using one or more of the displays and trade with a partner to answer each other's questions.

To surface connections, consider asking, "How does each display use a number line to display information?" **(MTR.2.1)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Different Ways to Display Data

Students in Mr. Bach's class participated in a reading contest. The students earned points based on the number of books they read and the difficulty level of the book. Students received ribbons based on their total number of points.

Each graphical display shows different information about the contest, but they all represent the same set of data.

Here is a *bar chart* and a circle graph.

Bar Chart

Reading Contest Ribbons

Circle Graph

Reading Contest Ribbons

2. Do these displays show numerical data or *categorical data*?

Categorical

3. Which display shows how many students received each color of ribbon? Explain your thinking.

Bar chart. *Explanations vary.* The bar chart shows the number of students who received each color of ribbon. The circle graph shows the percentage of students who received each color, but you can't determine the actual number of students.

Unit 8 Lesson 7 620 Activity 1

Pairs | 15 min

Activity 1 Different Ways to Display Data (continued)

Purpose: Students examine six graphical displays that represent the same data set and choose the ones that are most appropriate for conveying certain aspects of the data. **(MTR.2.1)**

3 Connect

Consider asking:

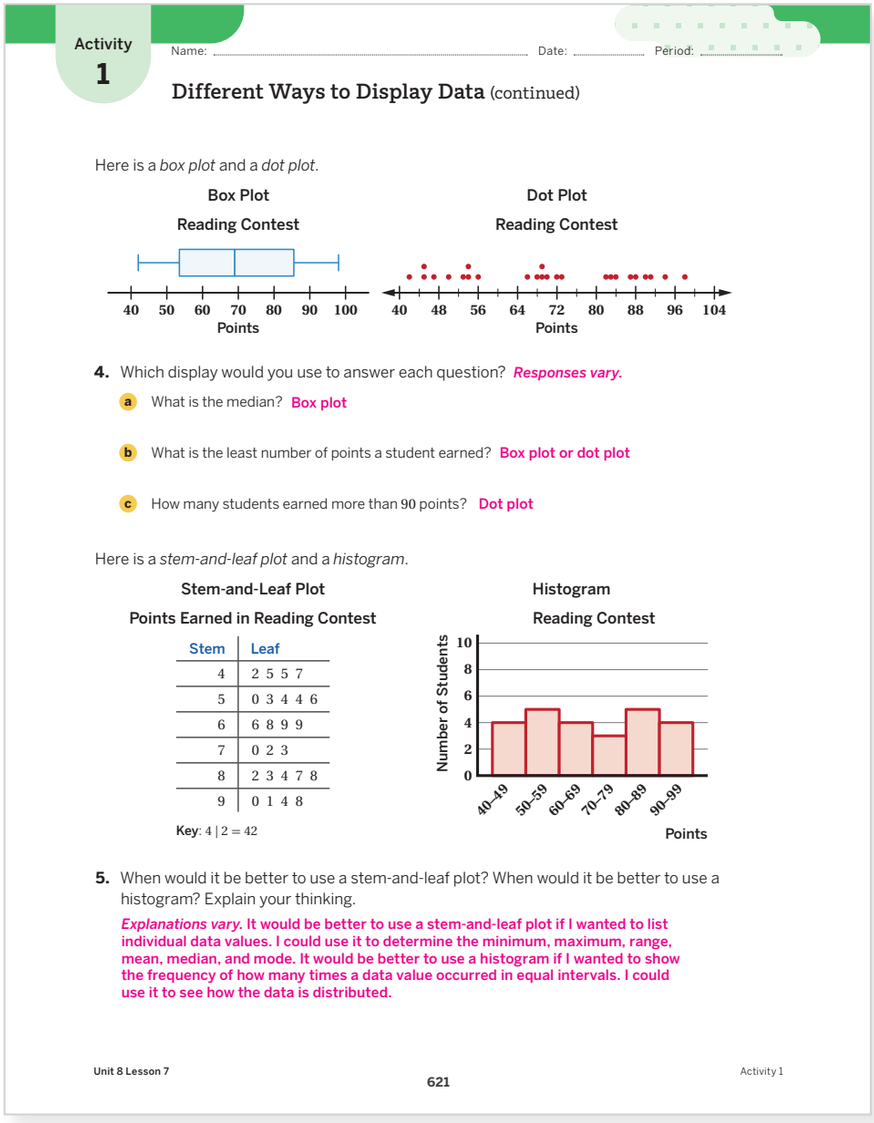
- "What statistics does the *box plot* show?"
- "What statistics can you calculate using the *dot plot*?"

Invite students to share their responses and justify their reasoning for each choice.

Create a class anchor chart describing what information each type of display shows. Display for students to reference later on.

To surface the Key Takeaway, consider asking:

- "What can you determine from graphical displays that represent categorical data?"
- "What can you determine from graphical displays that represent numerical data?"
- "Which displays show individual data values?"
- "Which displays would not be a good choice to represent large amounts of data?"**(MTR.2.1)**



Activity 1

Name: Date: Period:

Different Ways to Display Data (continued)

Here is a *box plot* and a *dot plot*.

Box Plot
Reading Contest

Dot Plot
Reading Contest

4. Which display would you use to answer each question? *Responses vary.*

a. What is the median? Box plot

b. What is the least number of points a student earned? Box plot or dot plot

c. How many students earned more than 90 points? Dot plot

Here is a *stem-and-leaf plot* and a *histogram*.

Stem-and-Leaf Plot
Points Earned in Reading Contest

Stem	Leaf
4	2 5 5 7
5	0 3 4 4 6
6	6 8 9 9
7	0 2 3
8	2 3 4 7 8
9	0 1 4 8

Key: 4 | 2 = 42

Histogram
Reading Contest

5. When would it be better to use a stem-and-leaf plot? When would it be better to use a histogram? Explain your thinking.

Explanations vary. It would be better to use a stem-and-leaf plot if I wanted to list individual data values. I could use it to determine the minimum, maximum, range, mean, median, and mode. It would be better to use a histogram if I wanted to show the frequency of how many times a data value occurred in equal intervals. I could use it to see how the data is distributed.

Unit 8 Lesson 7 621 Activity 1

Key Takeaway: Data can be displayed using several different graphical displays. Some displays make it easier to identify certain aspects of the data than others.

Pairs | 15 min

Activity 2 Choose an Appropriate Display

Purpose: Students choose the most appropriate graphical displays for displaying real-world numerical and *categorical data*. **(MTR.7.1)**

Short on time? Consider completing Problem 8 as a class.

1 Launch

To support students getting started, consider asking them to discuss what they notice about the real-world data.

Consider sharing that real-world situations can have more than one type of appropriate display. **(MTR.7.1)**

2 Monitor

Look for student responses to share during the Connect. Listen for student explanations about why they chose a particular display.

Encourage students to explain why they didn't choose one of the other displays.

3 Connect

Display the distribution of responses by polling the class, calling attention to any conflict or consensus. Invite students to justify their choices.

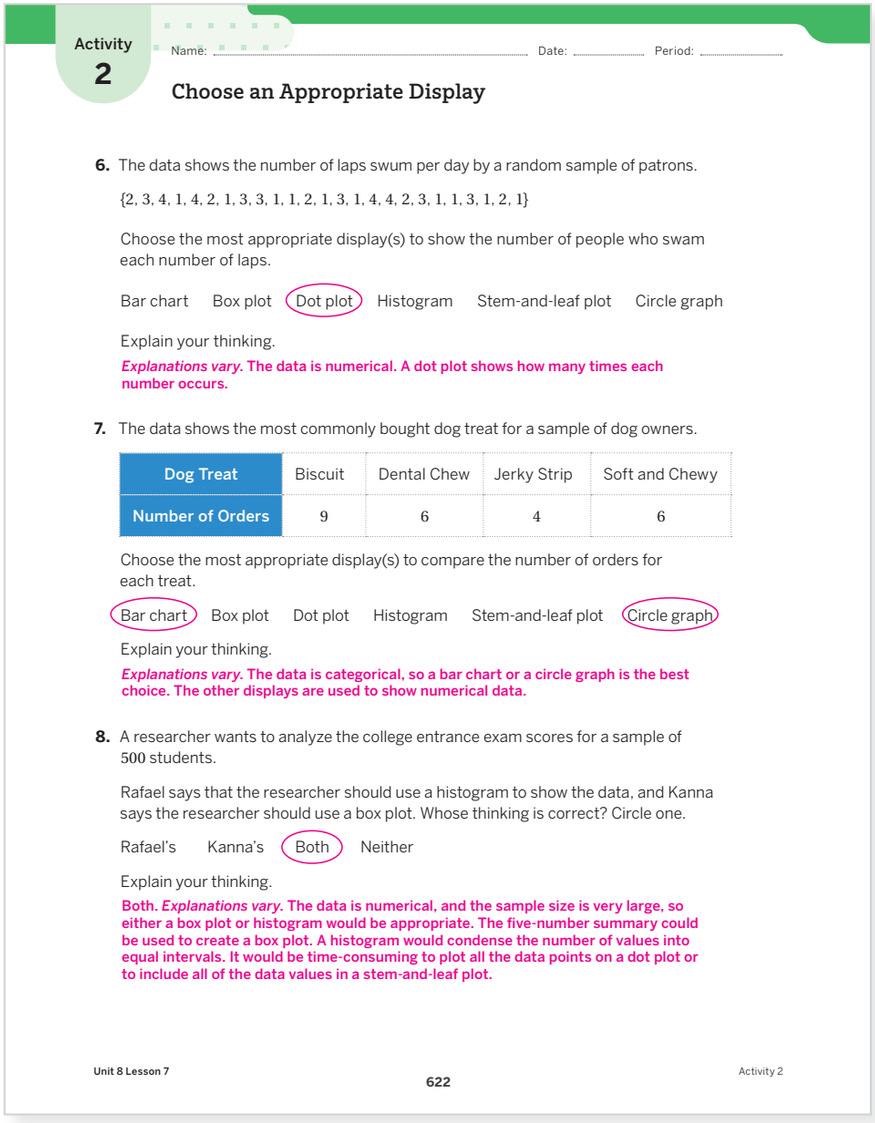
ELL **English Language Learners** Provide sentence frames to support students as they justify their choices. For example, "I think the ____ is the best display because it shows ____."

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Consider asking, "Would another type of display be appropriate to display 500 data values?"

To surface the Key Takeaway, consider asking:

- "What factors should you consider when choosing an appropriate display?"
- "What information is given?"
- "What information do you want the data to show?"



Activity 2

Name: Date: Period:

Choose an Appropriate Display

6. The data shows the number of laps swum per day by a random sample of patrons.

{2, 3, 4, 1, 4, 2, 1, 3, 3, 1, 1, 2, 1, 3, 1, 4, 4, 2, 3, 1, 1, 3, 1, 2, 1}

Choose the most appropriate display(s) to show the number of people who swam each number of laps.

Bar chart Box plot (Dot plot) Histogram Stem-and-leaf plot Circle graph

Explain your thinking.

Explanations vary. The data is numerical. A dot plot shows how many times each number occurs.

7. The data shows the most commonly bought dog treat for a sample of dog owners.

Dog Treat	Biscuit	Dental Chew	Jerky Strip	Soft and Chewy
Number of Orders	9	6	4	6

Choose the most appropriate display(s) to compare the number of orders for each treat.

(Bar chart) Box plot Dot plot Histogram Stem-and-leaf plot (Circle graph)

Explain your thinking.

Explanations vary. The data is categorical, so a bar chart or a circle graph is the best choice. The other displays are used to show numerical data.

8. A researcher wants to analyze the college entrance exam scores for a sample of 500 students.

Rafael says that the researcher should use a histogram to show the data, and Kanna says the researcher should use a box plot. Whose thinking is correct? Circle one.

Rafael's Kanna's (Both) Neither

Explain your thinking.

Both. *Explanations vary.* The data is numerical, and the sample size is very large, so either a box plot or histogram would be appropriate. The five-number summary could be used to create a box plot. A histogram would condense the number of values into equal intervals. It would be time-consuming to plot all the data points on a dot plot or to include all of the data values in a stem-and-leaf plot.

Unit 8 Lesson 7 622 Activity 2

Key Takeaway: When choosing the most appropriate display for a data set, consider the type of data given and what information about the data you want to communicate.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding about how to choose an appropriate graphical display to represent data.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The bar chart displays categorical data.
- The dot plot displays numerical data.
- The bar chart shows the number of students who earned each color of ribbon.
- The dot plot shows the number of students who earned each number of points.
- The number of points earned corresponds to the color of ribbon a student received.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about what to consider when choosing an appropriate graphical display. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to provide another real-world context and then discuss how they could use different displays to show different aspects of the data and reveal answers to different questions.



Synthesis

9. Explain how these displays represent data differently.

Responses vary. The bar chart displays categorical data.The bar chart shows the number of students who earned each color of ribbon.

The dot plot displays numerical data. The dot plot shows the number of students who earned each number of points.

Lesson Practice 8.07

Lesson Summary

We can use bar charts and circle graphs to show *categorical data*.

- *Bar charts* show the number of items in categories.
- Circle graphs show how categories represent parts of a whole.

We can use box plots, dot plots, histograms, and stem-and-leaf plots to show numerical data.

- *Box plots* separate data into quartiles and show the minimum and maximum values.
- *Dot plots* show how many times each data value occurs.
- *Stem-and-leaf plots* show all numerical data values in a condensed list.
- *Histograms* show numerical data that is continuous in equal intervals.

Histograms and box plots are most effective for representing large amounts of data. Dot plots and stem-and leaf plots are useful when individual data points are needed for calculations such as determining the mean.

Unit 8 Lesson 7 623 Synthesis | Lesson Practice

Lesson Takeaway: There are different ways to display data depending on whether the data is numerical or categorical. To choose the most appropriate graphical display, consider the number of data values, the information you want to present, and what questions could be answered from the data.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by choosing the most appropriate display for a given data set.

Today's Goals

1. **Goal:** Analyze different graphical displays that show the same data, and choose the display that is most appropriate for answering a question or conveying certain information about the data.
2. **Goal:** Choose the most appropriate display for a given data set.
3. **Language Goal:** Describe why a graphical display is the best one for answering a question or representing the data. **(Reading, Writing, and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.07

Choose an appropriate display for each situation. Explain your reasoning.

1. The favorite type of music of students in your grade

   Bar chart or circle graph. *Explanations vary.* Both displays show data in categories either as a number or as a percent.

2. The median amount of money raised by each student during a charity campaign

   Box plot. *Explanations vary.* A box plot shows five statistics, which includes the median.

3. The points scored by each player on a basketball team

   Line plot or stem-and-leaf plot. *Explanations vary.* Both displays show individual data values.

4. Choose *all* the situations that would best be displayed in a line plot.

   - ☐ A. favorite type of television show
   - ☑ B. heights of tallest buildings in the world
   - ☑ C. ages of students in a chess club
   - ☑ D. number of students on a swim team
   - ☐ E. percent of students in favor of, against, or neutral regarding a new class mascot

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to how to choose an appropriate representation for data:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

In this lesson, students connected their previous understanding of creating and interpreting different data displays to choosing appropriate representations for data. How will that work support their understanding of creating appropriate representations for data in Lesson 8?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Explain how these displays represent data differently.

Responses vary. The bar chart displays categorical data.The bar chart shows the number of students who earned each color of ribbon.

The dot plot displays numerical data. The dot plot shows the number of students who earned each number of points.

Lesson Practice 8.07

Lesson Summary

We can use bar charts and circle graphs to show *categorical data*.

- *Bar charts* show the number of items in categories.
- Circle graphs show how categories represent parts of a whole.

We can use box plots, dot plots, histograms, and stem-and-leaf plots to show numerical data.

- *Box plots* separate data into quartiles and show the minimum and maximum values.
- *Dot plots* show how many times each data value occurs.
- *Stem-and-leaf plots* show all numerical data values in a condensed list.
- *Histograms* show numerical data that is continuous in equal intervals.

Histograms and box plots are most effective for representing large amounts of data. Dot plots and stem-and-leaf plots are useful when individual data points are needed for calculations such as determining the mean.

Lesson Practice 8.07

Name: _____ Date: _____ Period: _____

Problems 1–4: Choose the appropriate graphical display.

1. The data shows the systolic blood pressure number for a random sample of patients at a hospital.

Systolic Blood Pressure									
149	147	109	117	147	93	125	115	123	129
123	137	115	120	133	137	123	96	126	114

Which type of display makes it easiest to see the mode?

A. Bar chart

B. Dot plot

C. Stem-and-leaf plot

D. Histogram

2. The data shows the field trip preferences for a random sample of students.

Destination	Number of Students
Anaheim	3
Palm Springs	5
San Diego	13
San Francisco	4

Which type of display would you use to compare the percentages of students who selected each field trip location?

A. Bar chart

B. Box plot

C. Histogram

D. Circle graph

3. The data shows the weights, in pounds, of a random sample of Chihuahuas.

{3.7, 4.7, 5.2, 3.9, 6.1, 5.8, 5.7, 4.5, 4.3, 5.6, 3.4, 6.9, 5.2, 3.3, 5.1, 4.2, 3.2, 4.4, 3.6, 6.1}

Which type of display should be used to represent the continuous data?

A. Bar chart

B. Dot plot

C. Stem-and-leaf plot

D. Histogram

Lesson Practice 8.07

Name: _____ Date: _____ Period: _____

Problems 4–6: Choose an appropriate display for each situation. Explain your reasoning.

4. The favorite scent of customers for a new line of body lotions

Bar chart or circle graph. *Explanations vary.* Both displays show data in categories either as a number or as a percent.

5. The median salary of teachers in a school district

Box plot. *Explanations vary.* A box plot shows five statistics, which includes the median.

6. The test scores for each student on a quiz

Dot plot or stem-and-leaf plot. *Explanations vary.* Both displays show individual data values.

FAST Practice

7. Choose *all* the situations that a dot plot would be a good choice to display the data.

- ☐ **A.** Movie genre preferences
- ☐ **B.** Heights of cactus plants in a nursery
- ☑ **C.** Ages of students in a chess club
- ☑ **D.** Number of students in each class
- ☐ **E.** Percent of students in favor of, against, or neutral regarding a new lunch item

Spiral Review

8. Tariq buys a notebook for $3.78 and 2 pens that cost $1.15 each. He pays with a $20 bill. How much change will he get back?

$13.92

9. The walking path around a pond is $\frac{5}{8}$ of a mile. Kayleen walks around the pond 3 times. How many miles does Kayleen walk in all?

$2\frac{1}{2}$ miles

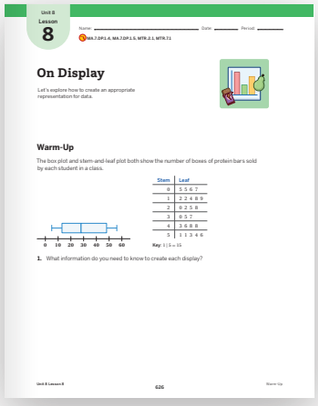
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.DP.1.5
	4–6	2	MA.7.DP.1.5
FAST Practice	**7**	2	MA.7.DP.1.5
Spiral Review			
	8–9	2	MA.7.NSO.2.3

Unit 8

Lesson

8


Unit 8
Lesson
8
Name _____ Date _____ Period _____
MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.7.1
On Display
Let's explore how to create an appropriate representation for data.
Warm-Up
The box plot and stem-and-leaf plot both show the number of boxes of protein bars sold by each student in a class.
0 10 20 30 40 50 60
Stem | Leaf
0 | 5 5 6 7
1 | 2 2 4 8 9
2 | 0 2 5 8
3 | 0 5 7
4 | 3 6 8 8
5 | 1 1 3 4 6
Key: 1 | 5 = 15
1. What information do you need to know to create each display?
626

On Display

Creating an Appropriate Representation for Data

Let's explore how to create an appropriate representation for data.

Learning Outcomes

Today's Goals

1. **Goal:** Create an appropriate representation for data and use the display to ask and answer questions.
2. **Goal:** Given a graphical display, create a different display to represent the same data.
3. **Language Goal:** Explain why the choice of a graphical display is appropriate. **(Reading, Writing, and Speaking)**

Students decide on the best display to represent real-world situations, create the display, and justify their choice. They use one display to create a different display that shows the same data. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 7, students chose appropriate displays to represent data.

Future Learning

In Lesson 9, students will compare data sets using the mean.

Connections

- Students efffectively use representations of data such as tables, bar graphs, pie charts, histograms, line graphs, and dot plots to represent flavor preference in a random sample. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.5
Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.

Also Addressing: **MA.7.DP.1.4**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.5	**MA.8.DP.1.1** **MA.912.DP.1.1**

Lesson at a Glance

~ 45 min

MA.7.DP.1.4, MA.7. DP.1.5

Warm-Up

Pairs | 5 min

Students look at two displays that show the same data and describe what information is needed to create each display.
Routine: Think-Pair-Share



Activity 1

Pairs | 15 min

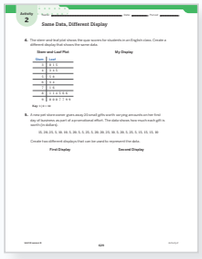
Students select an appropriate display for data, justify their choice, and then create the display. **(MTR.2.1)**



Activity 2

Pairs | 15 min

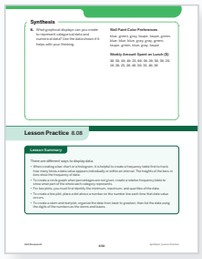
Students use a given display to create a different display that represents the same data and create two different displays to represent real-world data. **(MTR.2.1, MTR.7.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding about creating an appropriate graphical display to represent data. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating an appropriate representation for data and using the display to answer questions.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students look at two displays that show the same data and describe what information is needed to create each display.

Short on time? Consider omitting the Warm-Up.

1 Launch

To surface prior knowledge, consider asking:

- "What information can you gather from each display?"
- "Does one display make it easier to see certain aspects of the data than the other display?"
- "Why might you choose to use one display over the other?"

Use the Think-Pair-Share routine to facilitate a discussion about what information each display shows. Invite students to think independently and then share their descriptions of the displays.

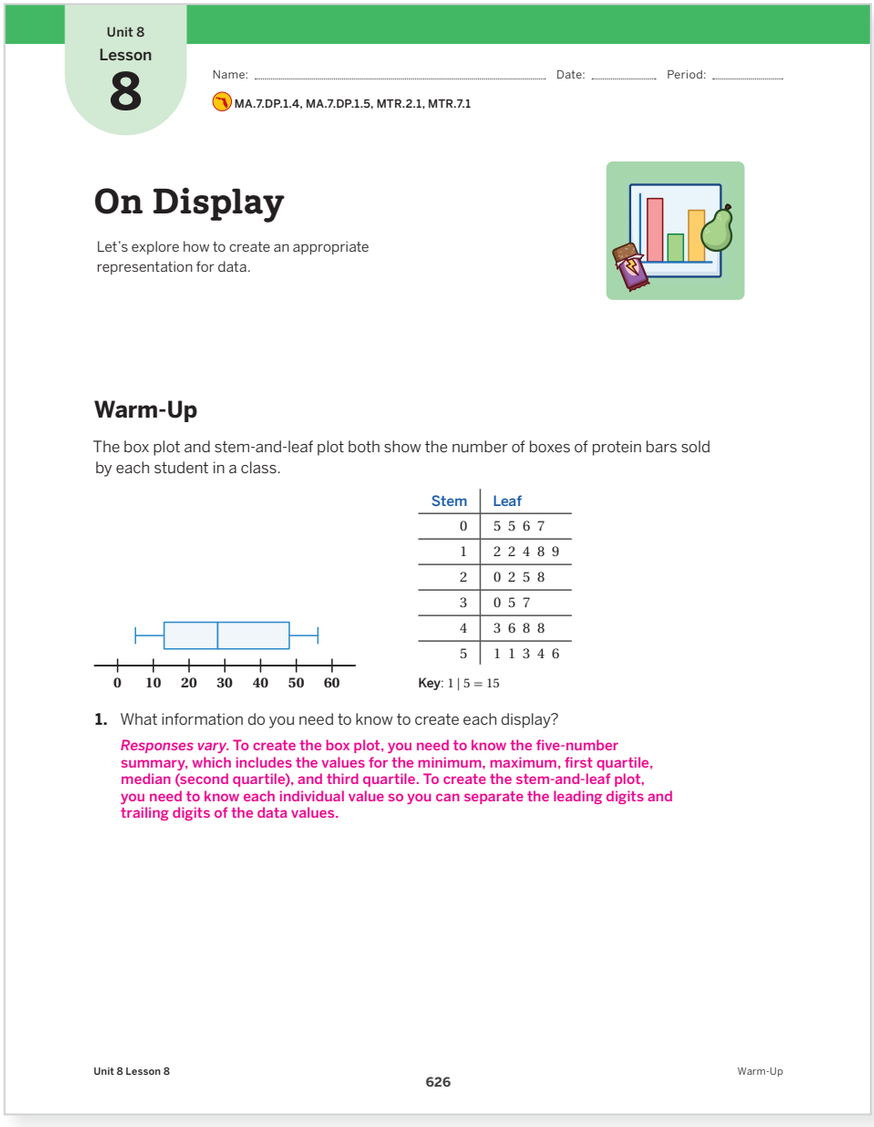
A **Accessibility: Visual-Spatial Processing**
To support processing of complex visual information, invite students to circle each part of each display to focus on what is included in each part.

2 Connect

Listen for and share students' answers, inviting each pair to share their ideas.

Consider asking:

- "Does one display require calculations in order to create it?"
- "Which display shows the individual data values?"
- "What do the vertical lines on the box plot show?"



Unit 8
Lesson
8

Name: _____ Date: _____ Period: _____

MA.7.DP.1.4, MA.7.DP.1.5, MTR.2.1, MTR.7.1

On Display

Let's explore how to create an appropriate representation for data.

Warm-Up

The box plot and stem-and-leaf plot both show the number of boxes of protein bars sold by each student in a class.

Stem	Leaf
0	5 5 6 7
1	2 2 4 8 9
2	0 2 5 8
3	0 5 7
4	3 6 8 8
5	1 1 3 4 6

Key: 1 | 5 = 15

1. What information do you need to know to create each display?

Responses vary. To create the box plot, you need to know the five-number summary, which includes the values for the minimum, maximum, first quartile, median (second quartile), and third quartile. To create the stem-and-leaf plot, you need to know each individual value so you can separate the leading digits and trailing digits of the data values.

Unit 8 Lesson 8 626 Warm-Up

Pairs | 15 min

Activity 1 Choose a Display

Purpose: Students select an appropriate display for data, justify their choice, and then create the display. **(MTR.2.1)**

1 Launch

Direct students' attention to the data of cracker flavor preferences.

To surface connections, consider asking:

- "What type of data is this?"
- "What are some displays that show this type of data?"

2 Monitor

Look for students who create different types of displays, such as using frequencies to create a bar chart or calculating percentages to create a circle graph. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support completing the frequency table.	**Support** Encourage students to use tally marks in the frequency table and to cross off each occurrence in the data when making the tally mark.
Need support creating a display.	**Support** Encourage students to use the frequency table to create a bar chart. Consider asking, "How does the frequency table help you to decide what scale to use on the bar chart?"
Create an appropriate display.	**Strengthen** Consider asking, "How can you check that your display includes all of the data?"
Finish early.	**Stretch** Invite students to create another type of display,

ELL **English Language Learners** Provide a word bank to support students in justifying their choice of display with words such as *categorical data*, *numerical data*, *frequency*, and *five-number summary*. **(Speaking)**

Collect the questions students create to share and answer during the Connect.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Choose a Display

2. The data shows the cracker flavor preferences for a random sample of students.

garlic	avocado	sour cream	avocado	avocado
avocado	sour cream	avocado	avocado	sour cream
garlic	avocado	avocado	original	garlic
avocado	garlic	avocado	sour cream	sour cream
original	garlic	garlic	garlic	avocado

a. Complete the table to show the frequency of each flavor preference.

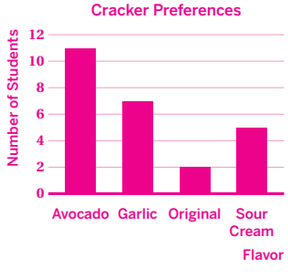
Flavor	Number of Students
Avacado	11
Garlic	7
Original	2
Sour Cream	5

b. Select an appropriate display to represent the data and justify your choice.

Responses vary. Bar chart; A bar chart is appropriate because it shows frequencies for categorical data.

c. Create the display.

Responses vary.



d. Create a question that can be answered by using the display. Then provide the answer.

Responses vary. Sample question: Which cracker flavor is preferred by most students? Answer: Avocado

Unit 8 Lesson 8 627 Activity 1

Pairs | 15 min

Activity 1 Choose a Display (continued)

Purpose: Students select an appropriate display for data, justify their choice, and then create the display. **(MTR.2.1)**

2 Monitor

To support students getting started, consider asking, "How can you create a frequency table for this data?"

Listen for students' strategies for creating the display to show the continuous data.

Look for students who create different types of displays, such as using equal intervals to create a histogram or calculating the five-number summary to create a box plot. **(MTR.2.1)**

Encourage students to share their reasoning with a partner about the most appropriate display to use.

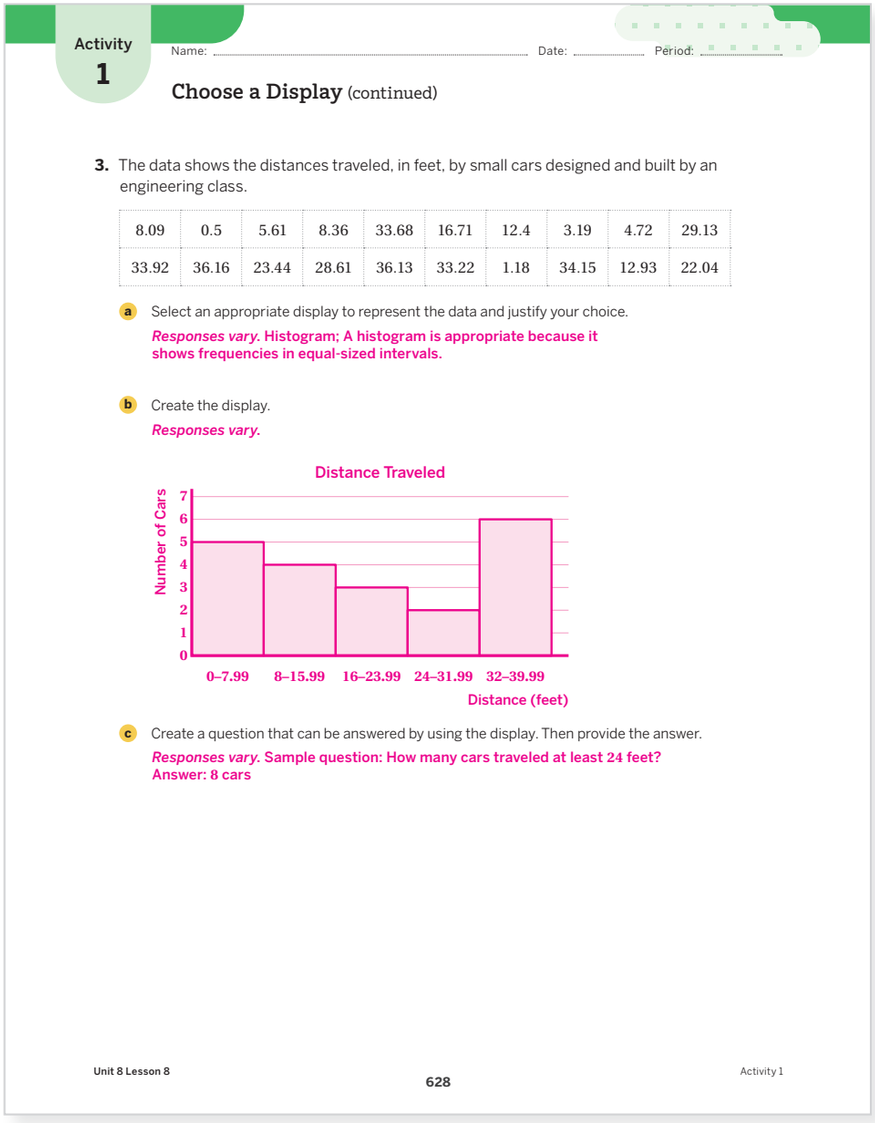
ELL **English Language Learners** Invite students to write their questions and answers in their primary language. **(Writing)**

3 Connect

Invite students to share their displays and the questions they posed. Invite the class to answer the questions using the display.

To surface the Key Takeaway, consider asking:

- "What details did you have to pay close attention to when creating your display?"
- "What upfront planning was required in order to create your display?"
- "What did you learn from creating your display that you can apply if you are asked to create this display again in the future?"



Activity 1

Name: Date: Period:

Choose a Display (continued)

3. The data shows the distances traveled, in feet, by small cars designed and built by an engineering class.

8.09	0.5	5.61	8.36	33.68	16.71	12.4	3.19	4.72	29.13
33.92	36.16	23.44	28.61	36.13	33.22	1.18	34.15	12.93	22.04

a. Select an appropriate display to represent the data and justify your choice.

Responses vary. Histogram; A histogram is appropriate because it shows frequencies in equal-sized intervals.

b. Create the display.

Responses vary.

c. Create a question that can be answered by using the display. Then provide the answer.

Responses vary. Sample question: How many cars traveled at least 24 feet?
Answer: 8 cars

Unit 8 Lesson 8 628 Activity 1

Key Takeaway: Some graphical displays are more appropriate than others when representing data. Creating a graphical display requires planning and attention to detail.

Pairs | 15 min

Activity 2 Same Data, Different Display

Purpose: Students use a given display to create a different display that represents the same data and create two different displays to represent real-world data. **(MTR.2.1, MTR.7.1)**

Short on time? Consider omitting Problem 5.

1 Launch

To increase participation, consider asking, "What is your favorite type of display to create?"

Direct students' attention to the stem-and-leaf plot. Invite students to read some of the values listed in the display.

Consider asking:

- "Do you know what the individual values are in the data set?"
- "What types of displays can you create from individual values?"
- "What statistics are shown in the stem-and-leaf plot?"

Encourage students to think for 1–2 minutes about how different types of displays communicate different information about the real-world data. **(MTR.7.1)**

2 Monitor

Select and sequence a variety of displays to share during the Connect.

Encourage students to create a different display than their partner.

Pause briefly to discuss some of the possible ways to display each data set. Consider asking, "How does your display represent the data? What aspect of the data does your display highlight?" **(MTR.2.1)**

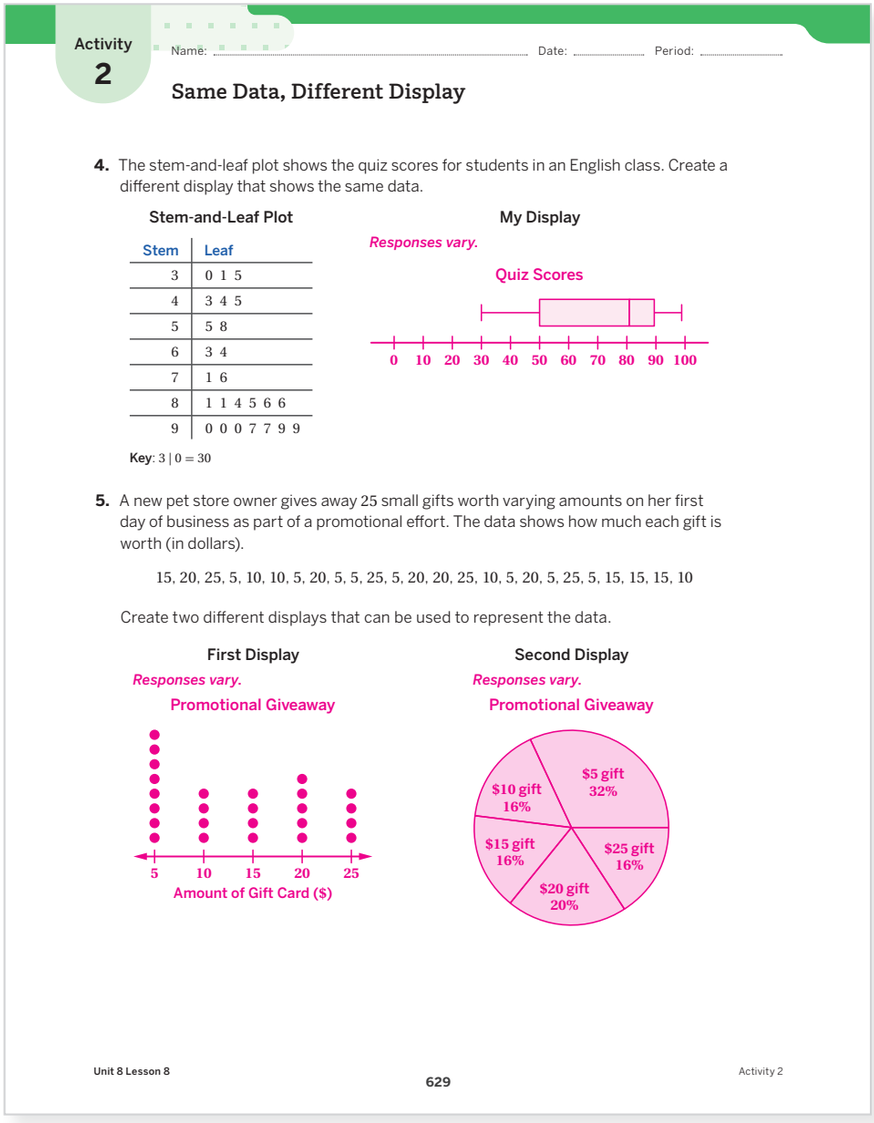
3 Connect

Invite students to share their displays and their reasoning for choosing a particular display.

Create a class anchor chart showing the different types of graphical displays and the steps used to create each one.

To surface the Key Takeaway, consider asking:

- "What type of displays can you use to create a circle graph?"
- "What type of display can you create from a box plot?"



Activity 2

Name: Date: Period:

Same Data, Different Display

4. The stem-and-leaf plot shows the quiz scores for students in an English class. Create a different display that shows the same data.

Stem-and-Leaf Plot

Stem	Leaf
3	0 1 5
4	3 4 5
5	5 8
6	3 4
7	1 6
8	1 1 4 5 6 6
9	0 0 0 7 7 9 9

Key: 3 | 0 = 30

My Display

Responses vary.

Quiz Scores

0 10 20 30 40 50 60 70 80 90 100

5. A new pet store owner gives away 25 small gifts worth varying amounts on her first day of business as part of a promotional effort. The data shows how much each gift is worth (in dollars).

15, 20, 25, 5, 10, 10, 5, 20, 5, 5, 25, 5, 20, 20, 25, 10, 5, 20, 5, 25, 5, 15, 15, 15, 10

Create two different displays that can be used to represent the data.

First Display

Responses vary.

Promotional Giveaway

5 10 15 20 25

Amount of Gift Card ($)

Second Display

Responses vary.

Promotional Giveaway

$5 gift 32%

$25 gift 16%

$20 gift 20%

$15 gift 16%

$10 gift 16%

Unit 8 Lesson 8 629 Activity 2

Key Takeaway: You can represent data with more than one type of display.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding about creating an appropriate graphical display to represent data.

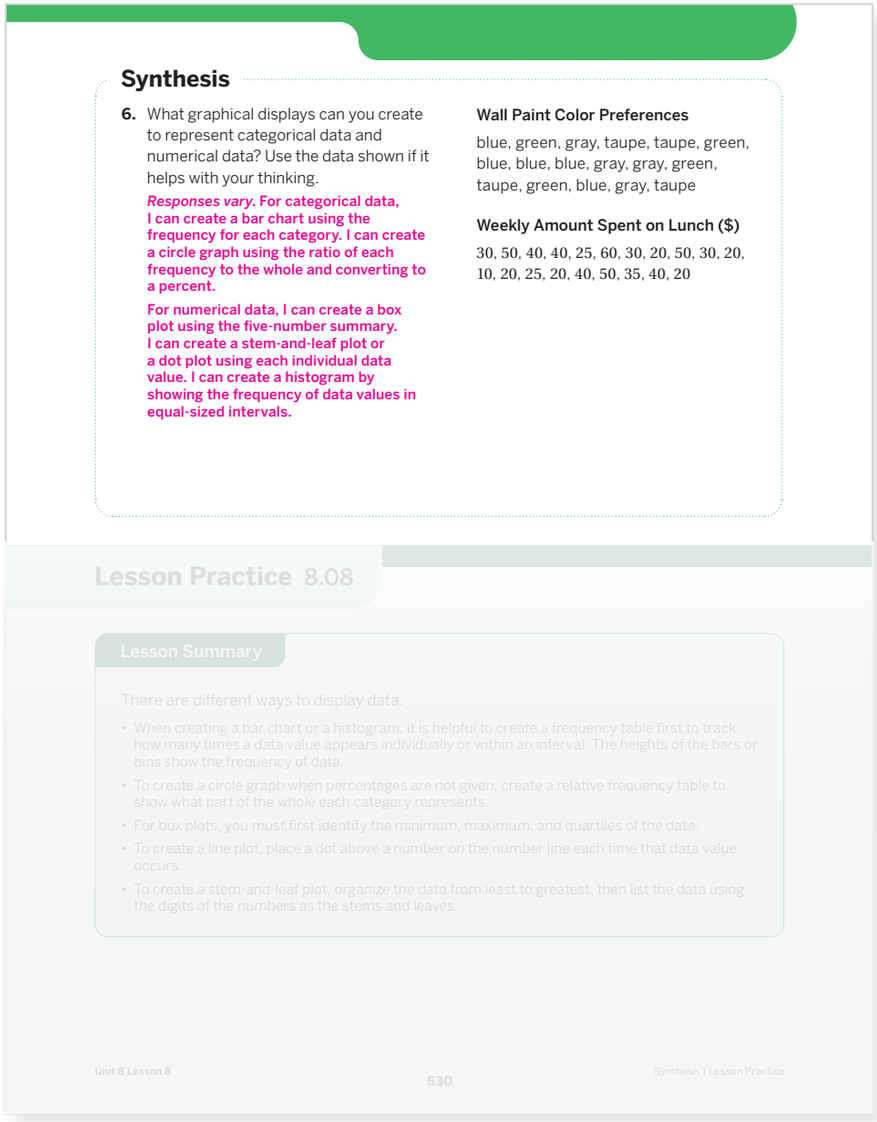
Invite students to respond independently, and then share their thinking with a partner.

Collect and share students' ideas about creating an appropriate graphical display, such as:

- Representing categorical data with bar charts or circle graphs.
- Representing numerical data with box plots, dot plots, histograms, or stem-and-leaf plots.
- Calculating the five-number summary to create box plots.
- Using frequency charts to organize data and record the number of times a data value occurs.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about the steps to use when creating each type of graphical display. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to provide real-world situations that could be represented by a particular display and then create the display.



Synthesis

6. What graphical displays can you create to represent categorical data and numerical data? Use the data shown if it helps with your thinking.

Responses vary. For categorical data, I can create a bar chart using the frequency for each category. I can create a circle graph using the ratio of each frequency to the whole and converting to a percent.

For numerical data, I can create a box plot using the five-number summary. I can create a stem-and-leaf plot or a dot plot using each individual data value. I can create a histogram by showing the frequency of data values in equal-sized intervals.

Wall Paint Color Preferences

blue, green, gray, taupe, taupe, green, blue, blue, blue, gray, gray, green, taupe, green, blue, gray, taupe

Weekly Amount Spent on Lunch ($)

30, 50, 40, 40, 25, 60, 30, 20, 50, 30, 20, 10, 20, 25, 20, 40, 50, 35, 40, 20

Lesson Practice 8.08

Lesson Summary

There are different ways to display data.

- When creating a bar chart or a histogram, it is helpful to create a frequency table first to track how many times a data value appears individually or within an interval. The heights of the bars or bins show the frequency of data.
- To create a circle graph when percentages are not given, create a relative frequency table to show what part of the whole each category represents.
- For box plots, you must first identify the minimum, maximum, and quartiles of the data.
- To create a line plot, place a dot above a number on the number line each time that data value occurs.
- To create a stem-and-leaf plot, organize the data from least to greatest, then list the data using the digits of the numbers as the stems and leaves.

Unit 8 Lesson 8 630 Synthesis | Lesson Practice

Lesson Takeaway: Often there is more than one way to display real-world data. Some graphical displays are more appropriate than others. Different displays require different information and steps to create the display.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by creating an appropriate representation for data and using the display to answer questions.

Today's Goals

1. **Goal:** Create an appropriate representation for data and use the display to ask and answer questions.
2. **Goal:** Given a graphical display, create a different display to represent the same data.
3. **Language Goal:** Explain why the choice of a graphical display is appropriate. **(Reading, Writing, and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.08

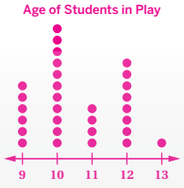
The data shows the ages of 30 students who are participating in a school play.

9, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 13

a. Create an appropriate display for the data.

b. Explain how your display might be used to answer questions.

c. Calculate the mean and median age of students who are participating in the school play.



Responses vary. A line plot is an appropriate display because the data values can be shown with dots that are placed vertically above the numbers. Each dot represents a data value. You can use the display to see how many students of each age participated in the play and make comparisons between ages.

The mean age of students participating in the school play is 10.6 and the median age is 10.

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support creating an appropriate representation for data and using the display to ask and answer questions:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on students' progress with statistical displays. In what ways have students' understanding and skills grown? How might you support students in making decisions about the most appropriate way to display data?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. What graphical displays can you create to represent categorical data and numerical data? Use the data shown if it helps with your thinking.

Responses vary. For categorical data, I can create a bar chart using the frequency for each category. I can create a circle graph using the ratio of each frequency to the whole and converting to a percent.

For numerical data, I can create a box plot using the five-number summary. I can create a stem-and-leaf plot or a dot plot using each individual data value. I can create a histogram by showing the frequency of data values in equal-sized intervals.

Wall Paint Color Preferences

blue, green, gray, taupe, taupe, green, blue, blue, blue, gray, gray, green, taupe, green, blue, gray, taupe

Weekly Amount Spent on Lunch ($)

30, 50, 40, 40, 25, 60, 30, 20, 50, 30, 20, 10, 20, 25, 20, 40, 50, 35, 40, 20

Lesson Practice 8.08

Lesson Summary

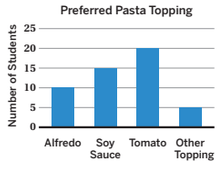
There are different ways to display data.

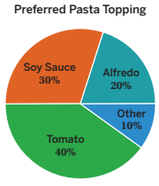
- When creating a bar chart or a histogram, it is helpful to create a frequency table first to track how many times a data value appears individually or within an interval. The heights of the bars or bins show the frequency of data.
- To create a circle graph when percentages are not given, create a relative frequency table to show what part of the whole each category represents.
- For box plots, you must first identify the minimum, maximum, and quartiles of the data.
- To create a line plot, place a dot above a number on the number line each time that data value occurs.
- To create a stem-and-leaf plot, organize the data from least to greatest, then list the data using the digits of the numbers as the stems and leaves.

Lesson Practice 8.08

Name: _____ Date: _____ Period: _____

1. Look at the bar chart and circle graph.





Do the displays show the same data? Explain your thinking.

Responses vary. **Yes, the bar chart and circle graph both show pasta topping preferences. The bar chart shows the frequencies for each preference, whereas the circle graph shows percentages for each preference.**

2. The data shows the number of hours students in a random sample volunteer per month at different organizations.

4, 28, 6, 8, 34, 21, 14, 34, 17, 2, 6, 35, 37, 40, 6, 31, 14, 24, 25, 13, 25, 37, 14, 28, 32

LaShawn represents the data using the histogram shown.



Is LaShawn's histogram correct? Explain your thinking.

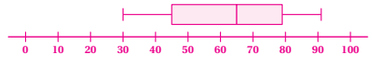
No. ***Explanations vary.*** **The histogram shows incorrect frequencies for two of the intervals. The frequency for the interval 10–19 should be 4, and the frequency for the interval 30–39 should be 8.**

3. The data shows the scores on a band playing test.

30, 34, 35, 42, 43, 43, 47, 48, 52, 53, 55, 65, 65, 67, 68, 71, 73, 74, 78, 80, 82, 82, 83, 87, 91

Create a display that you could use to show the interquartile range of the scores.

Responses vary.



Lesson Practice 8.08

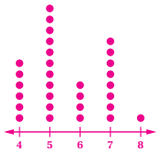
Name: _____ Date: _____ Period: _____

4. The data shows the grade levels of students who are participating in a community cleanup effort.

4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 8

Create an appropriate display for the data. Explain how your display might be used to answer questions.

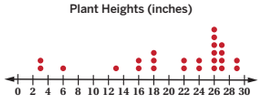
Grade Levels of Participants

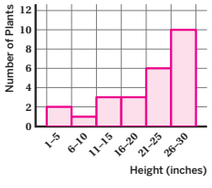


Responses vary. **A line plot is an appropriate display because the data values can be shown with dots that are placed vertically above the numbers. Each dot represents a data value. You can use the display to see how many students participated in each grade and make comparisons between grade levels.**

FAST Practice

5. The line plot shows the plant heights in a science experiment. Create a histogram to represent the same data.





Spiral Review

Problems 6–8: Determine the value of the expression.

6. $\frac{1}{2}\left|3-\frac{1}{4}\right|+3^3$

$28\frac{3}{8}$

7. $12 \div (6-4)^2 - 8$

-5

8. $(14.85 \div 3 + 6) \times 2$

21.9

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.7.DP.1.5
	4	3	MA.7.DP.1.5
FAST Practice	5	2	MA.7.DP.1.5
Spiral Review			
	6–8	2	MA.7.NSO.2.1

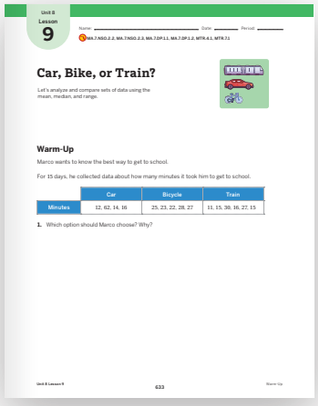
Unit 8

Lesson 9

Car, Bike, or Train?

Using Mean, Median, and Range to Compare Groups

Let's analyze and compare sets of data using the mean, median, and range.


Unit 8
Lesson 9

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1

Car, Bike, or Train?

Let's analyze and compare sets of data using the mean, median, and range.

Warm-Up

Marco wants to know the best way to get to school.

For 15 days, he collected data about how many minutes it took him to get to school.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15

1. Which option should Marco choose? Why?

633

Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the mean, median, and range for a data set.
2. **Goal:** Use measures of center and variability to compare data sets.
3. **Language Goal:** Justify a choice using evidence from the data set. **(Reading, Writing, Speaking, and Listening)**

Students revisit the tools they used in Grade 6 to analyze and compare sets of data. They calculate the *mean*, *median*, and *range*, and use these measures to compare data and make a recommendation. They also visually determine if a data point is an *outlier* and determine the effect the outlier has on the mean and median.

Coherence

Prior Learning

In Grade 6, students calculated the mean and mean, median, and mode of a data set and described the spread and distribution of data including outliers and range.

Future Learning

In Lesson 13, students will use measures of center and variability to determine if two populations are very different.

Connections

- Students effectively analyze and compare data sets for the best way to get to school by determining the mean, median, and range for a car, bicycle and train. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.1

Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.

MA.7.DP.1.2

Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.

Also Addressing: **MA.7.NSO.2.2, MA.7.NSO.2.3**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	**MA.912.DP.2.1**
MA.6.DP.1.6	

Vocabulary

Review Vocabulary

center	*mean*
median	*outlier*
range	*spread*

Lesson at a Glance

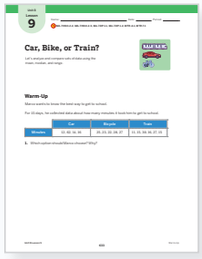
~ 45 min

MA.NSO.2.2, MA.NSO.2.3, MA.7.DP.1.1, MA..DP.1.2

Warm-Up

Pairs | 5 min

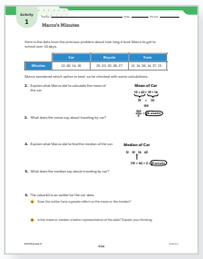
Students surface features of data that might be useful for making a decision about which transportation option to choose.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

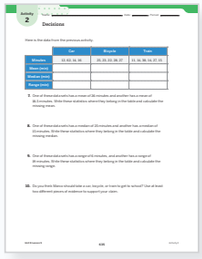
Students revisit how to measure the *center* of a data set using *mean* and *median*.



Activity 2

Pairs | 15 min

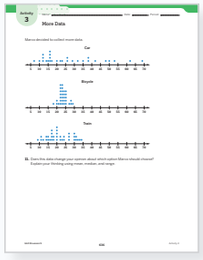
Students calculate the mean, median, and *range* and use these measures to make a recommendation. **Routine:** Think-Pair-Share **(MTR.4.1)**



Activity 3

Pairs | 5 min

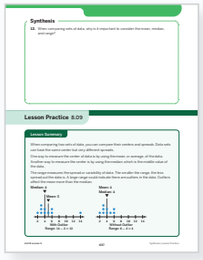
Students use their understanding of center and variability to compare and contrast data represented in dot plots.



Synthesis

Whole Class | 5 min

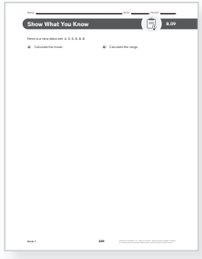
Students synthesize their understanding of using the mean, median, and range when analyzing a data set. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the mean and range of a data set.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students surface features of data that might be useful for making a decision about which transportation option to choose.

1 Launch

Direct students' attention to the data.

Invite students to discuss the three options with their groups before sharing as a whole class.

Use the Think-Pair-Share routine to support student-to-student discourse about data. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

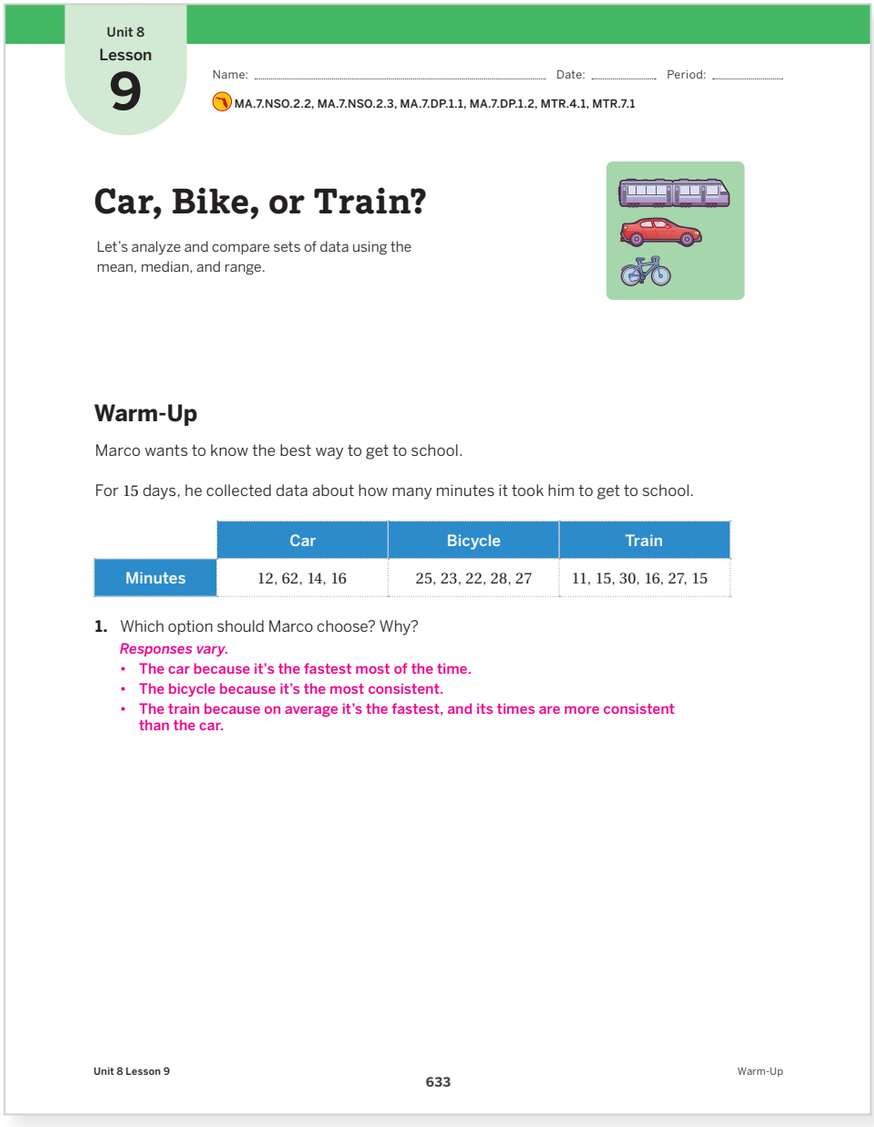
2 Connect

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Discuss the features of the data that students paid attention to when making their choices.

A **Accessibility: Conceptual Processing** Create an anchor chart that describes features of a data set to support learning new concepts.

Consider asking, "What would make you more confident in your choice?"



Unit 8
Lesson
9

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1

Car, Bike, or Train?

Let's analyze and compare sets of data using the mean, median, and range.

Warm-Up

Marco wants to know the best way to get to school.

For 15 days, he collected data about how many minutes it took him to get to school.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15

1. Which option should Marco choose? Why?

Responses vary.

- The car because it's the fastest most of the time.
- The bicycle because it's the most consistent.
- The train because on average it's the fastest, and its times are more consistent than the car.

Unit 8 Lesson 9 633 Warm-Up

Pairs | 10 min

Activity 1 Marco's Minutes

Purpose: Students revisit how to measure the *center* of a data set using *mean* and *median*.

1 Launch

Consider asking, "What makes a transportation option the best?"

Note: If it doesn't come up naturally, consider asking, "How can you measure which transportation option is best?

To surface prior knowledge, invite students to turn to their partner and discuss what they remember about mean and median.

Create a class anchor chart for *mean* and *median* including definitions, how to calculate the measures with examples, when they are useful, and the effect that *outliers* have on them.

Consider asking, "Why might Marco be interested in calculating the mean? How will this help him decide?" **(MTR.7.1)**

2 Monitor

To support students getting started, consider asking, "What operations did Marco use?"

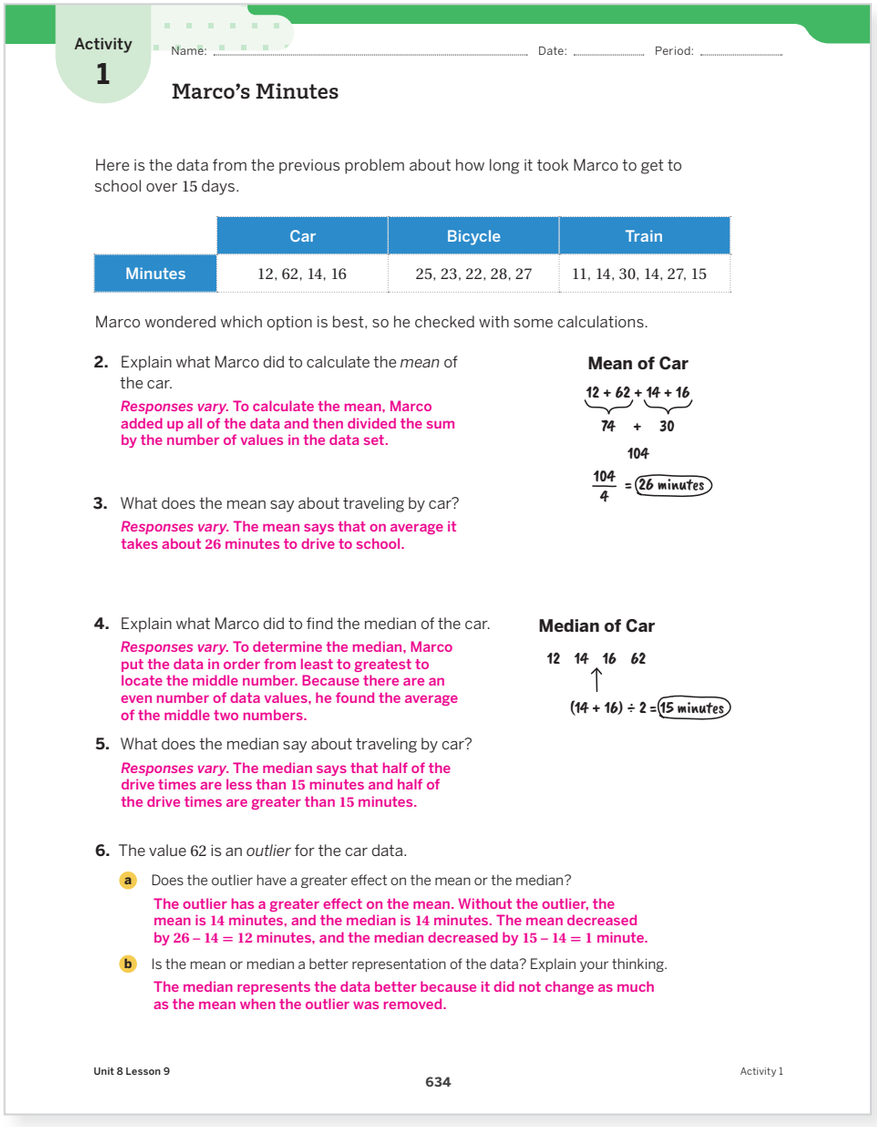
D Differentiation *(Problems 2–5)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 2)*	**Support** Refer students to Marco's car calculations and consider asking, "What do each of these numbers being added represent? What does the 4 in the denominator of the fraction represent?"
Would benefit from reviewing how to determine the median. *(Problem 4)*.	**Support** Consider asking, "Why is it important to order the data before identifying the median? How is determining the median different for an odd number of data values versus an even number of data values?"
Would like to practice calculating the mean and median.	**Stretch** Encourage students to calculate the mean and median of the other two data sets.

3 Connect

To surface the Key Takeaway, consider asking:

- "Does an outlier have a greater effect on the mean or the median? Why?"
- "How do you think an outlier that was much less than the other data would affect the mean and median?"



Activity 1

Name: Date: Period:

Marco's Minutes

Here is the data from the previous problem about how long it took Marco to get to school over 15 days.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 14, 30, 14, 27, 15

Marco wondered which option is best, so he checked with some calculations.

2. Explain what Marco did to calculate the *mean* of the car.

Responses vary. To calculate the mean, Marco added up all of the data and then divided the sum by the number of values in the data set.

Mean of Car

$12 + 62 + 14 + 16$

$74 + 30$

104

$\frac{104}{4} = 26$ minutes

3. What does the mean say about traveling by car?

Responses vary. The mean says that on average it takes about 26 minutes to drive to school.

4. Explain what Marco did to find the median of the car.

Responses vary. To determine the median, Marco put the data in order from least to greatest to locate the middle number. Because there are an even number of data values, he found the average of the middle two numbers.

Median of Car

12 14 16 62

$(14 + 16) \div 2 = 15$ minutes

5. What does the median say about traveling by car?

Responses vary. The median says that half of the drive times are less than 15 minutes and half of the drive times are greater than 15 minutes.

6. The value 62 is an *outlier* for the car data.

a. Does the outlier have a greater effect on the mean or the median?

The outlier has a greater effect on the mean. Without the outlier, the mean is 14 minutes, and the median is 14 minutes. The mean decreased by $26 - 14 = 12$ minutes, and the median decreased by $15 - 14 = 1$ minute.

b. Is the mean or median a better representation of the data? Explain your thinking.

The median represents the data better because it did not change as much as the mean when the outlier was removed.

Unit 8 Lesson 9 634 Activity 1

Key Takeaway: When analyzing a data set, it is important to look at the *center* (*mean* and *median*) and consider whether there are *outliers* that could have an effect on the center.

Pairs | 15 min

Activity 2 Decisions

Purpose: Students calculate the mean, median, and *range* and use these measures to make a recommendation. **(MTR.4.1)**

Short on time? Consider having students discuss Problem 8 instead of writing their response.

1 Launch

To surface connections, consider asking:

- "How could you use the mean and median to compare the three options?"
- "What does the *range* tell you about the data?"

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

2 Monitor

To support students getting started, invite them to review Marco's work from the previous activity.

Look for student strategies for estimating and calculating the missing values.

A **Accessibility: Memory and Attention** Invite students to review the anchor chart created previously on calculating the mean to activate prior knowledge.

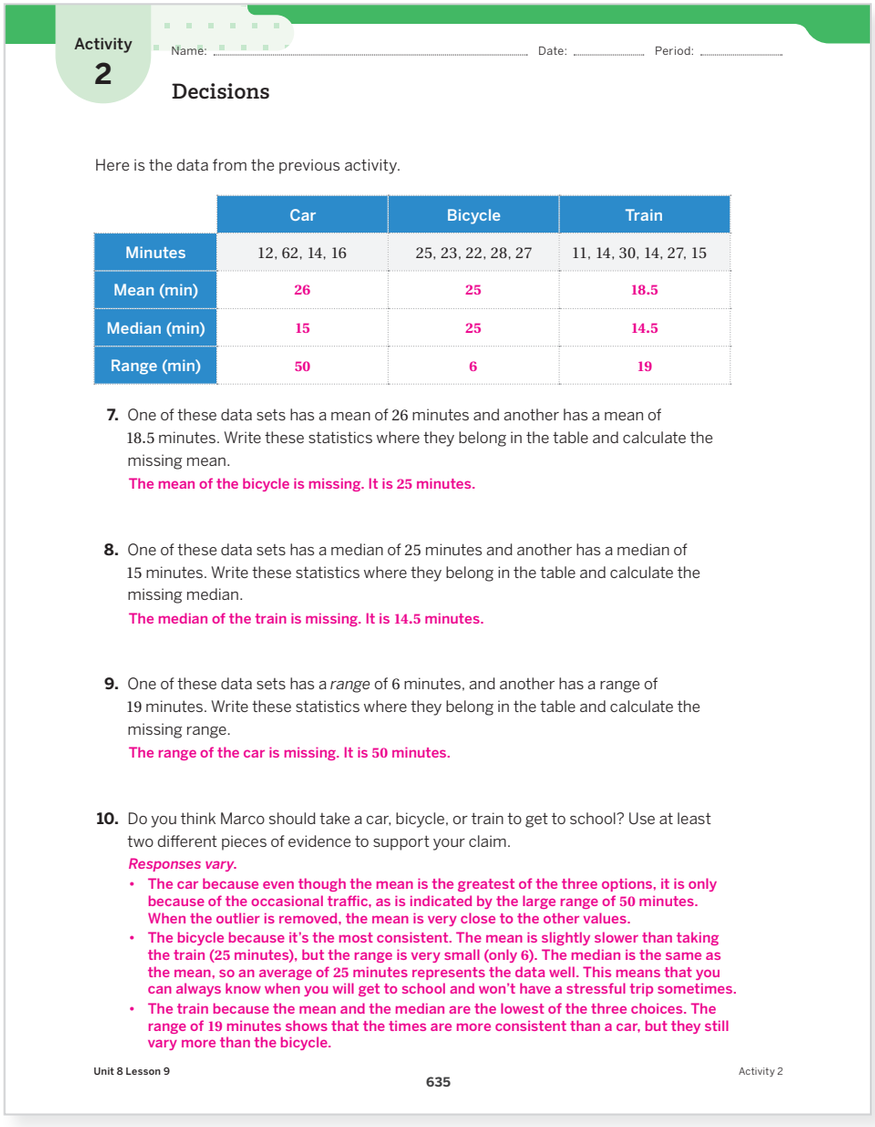
ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down.

3 Connect

Direct students' attention to the table. Poll the class to determine which mode of transportation Marco should use. Record the results to revisit later in the lesson.

Invite students to explain their reasoning and encourage them to use evidence from their calculations. **(MTR.4.1)**

Note: It is okay — even desirable — to lack consensus at this stage. The next activity will build toward consensus when students use more data to support their claim.



Activity 2

Name: Date: Period:

Decisions

Here is the data from the previous activity.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 14, 30, 14, 27, 15
Mean (min)	26	25	18.5
Median (min)	15	25	14.5
Range (min)	50	6	19

7. One of these data sets has a mean of 26 minutes and another has a mean of 18.5 minutes. Write these statistics where they belong in the table and calculate the missing mean.

The mean of the bicycle is missing. It is 25 minutes.

8. One of these data sets has a median of 25 minutes and another has a median of 15 minutes. Write these statistics where they belong in the table and calculate the missing median.

The median of the train is missing. It is 14.5 minutes.

9. One of these data sets has a *range* of 6 minutes, and another has a range of 19 minutes. Write these statistics where they belong in the table and calculate the missing range.

The range of the car is missing. It is 50 minutes.

10. Do you think Marco should take a car, bicycle, or train to get to school? Use at least two different pieces of evidence to support your claim.

Responses vary.

- The car because even though the mean is the greatest of the three options, it is only because of the occasional traffic, as is indicated by the large range of 50 minutes. When the outlier is removed, the mean is very close to the other values.
- The bicycle because it's the most consistent. The mean is slightly slower than taking the train (25 minutes), but the range is very small (only 6). The median is the same as the mean, so an average of 25 minutes represents the data well. This means that you can always know when you will get to school and won't have a stressful trip sometimes.
- The train because the mean and the median are the lowest of the three choices. The range of 19 minutes shows that the times are more consistent than a car, but they still vary more than the bicycle.

Unit 8 Lesson 9 635 Activity 2

Pairs | 5 min

Activity 3 More Data

Purpose: Students use their understanding of center and variability to compare and contrast data represented in dot plots.

1 Launch

Direct students' attention to the dot plots and invite students to share what they notice.

2 Monitor

To support students getting started, consider asking, "Looking at these dot plots, which mode of transportation do you predict to have the greatest and least range?"

Look for students who change their opinion or use mathematically precise language such as mean, median, range, *spread*, or variability and invite them to share during the Connect.

A **Accessibility: Affective Functioning** To support students in sharing their thinking, ask students in advance if they, you, or a classmate can share their thinking during the whole class discussion.

3 Connect

Invite students to share their thinking. Poll the class again to determine which mode of transportation Marco should use and compare the results to the previous activity.

Consider asking, "Did anyone's recommendation change? Why?" **(MTR.4.1)**

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

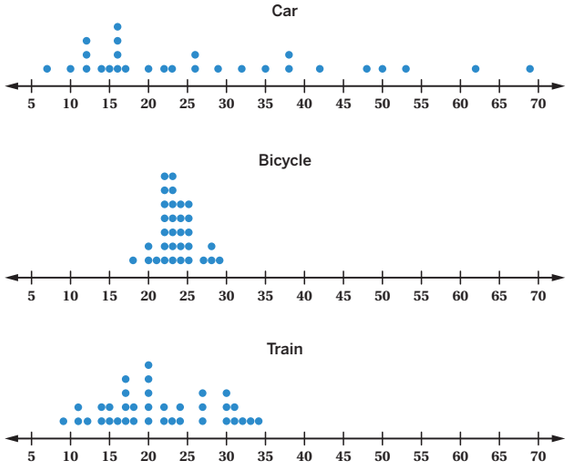
- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How do you know?

Activity 3

Name: Date: Period:

More Data

Marco decided to collect more data.



11. Does this data change your opinion about which option Marco should choose? Explain your thinking using mean, median, and range.

Responses and explanations vary.

- I would not take the car because it looks like there are outliers and the data has a large range.
- I think a bicycle is the best option because even the slowest trips are faster than some of the train and car trips. The mean and median are close in value, and the range is small, so you can consistently estimate the time it takes to get to school.
- I think the train is the best option because most of the time it is the fastest, and even the longest trips only take a few more minutes then riding a bicycle. The mean and median are both good approximations of the center of the data, and the range shows the data is not spread out too far.

Unit 8 Lesson 9 636 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using the mean, median, and range when analyzing a data set.

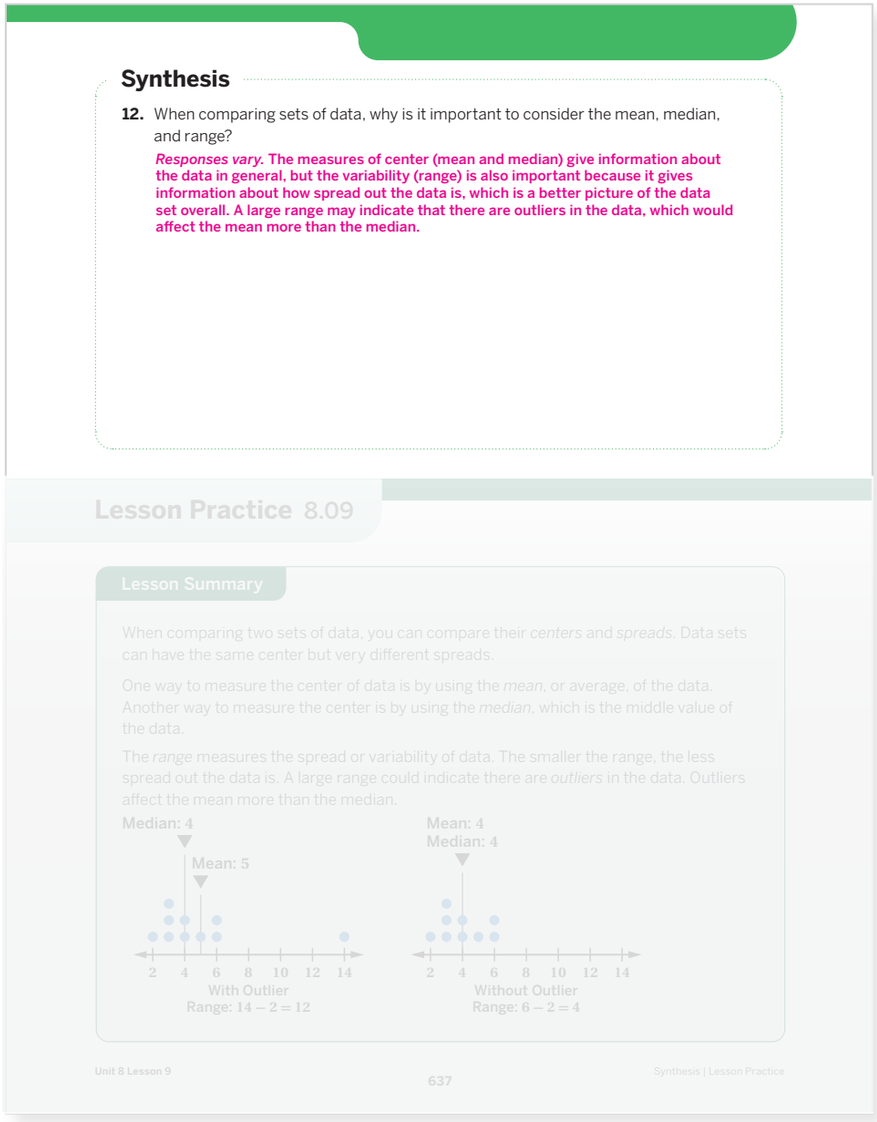
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The mean and median give information about the center of the data.
- The mean can be skewed one way or another by extremely high or low values (outliers) in the data set.
- The range gives information about the spread (variability) of the data.

Note: If time allows, invite students to revise their explanation based on the discussion.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *mean*, *median*, *range*, and *outlier* using students' language. Display any new language, examples, or diagrams for students to reference later.



Synthesis

12. When comparing sets of data, why is it important to consider the mean, median, and range?

Responses vary. The measures of center (mean and median) give information about the data in general, but the variability (range) is also important because it gives information about how spread out the data is, which is a better picture of the data set overall. A large range may indicate that there are outliers in the data, which would affect the mean more than the median.

Lesson Practice 8.09

Lesson Summary

When comparing two sets of data, you can compare their *centers* and *spreads*. Data sets can have the same center but very different spreads.

One way to measure the center of data is by using the *mean*, or average, of the data. Another way to measure the center is by using the *median*, which is the middle value of the data.

The *range* measures the spread or variability of data. The smaller the range, the less spread out the data is. A large range could indicate there are *outliers* in the data. Outliers affect the mean more than the median.

Median: 4
Mean: 5
2 4 6 8 10 12 14
With Outlier
Range: $14 - 2 = 12$

Mean: 4
Median: 4
2 4 6 8 10 12 14
Without Outlier
Range: $6 - 2 = 4$

Unit 8 Lesson 9 637 Synthesis | Lesson Practice

Lesson Takeaway: When comparing two sets of data, it is important to compare their *centers* and *spread*. Data sets can have a similar center but very different distributions. A larger *range* represents greater variability in the data set. A smaller range represents less variability. *Outliers* have a greater effect on the *mean* than on the *median*.

Independent | 5 min

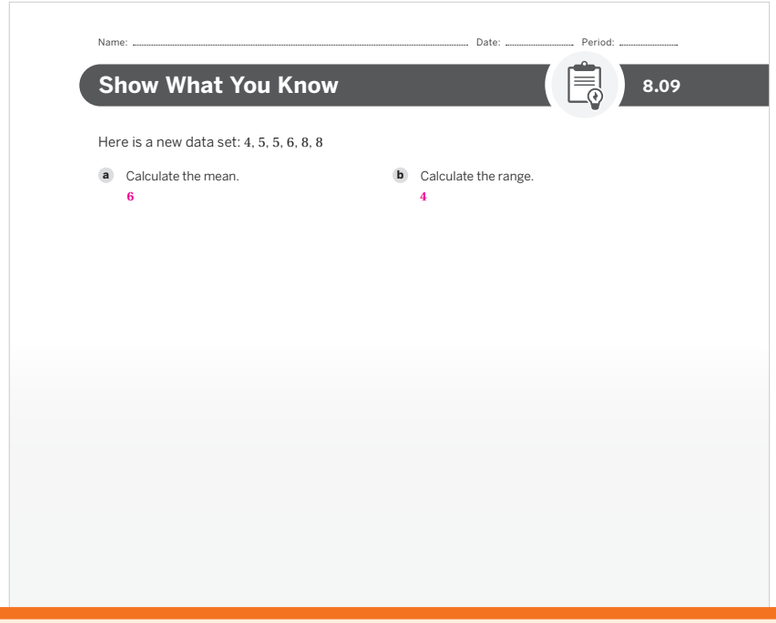
Show What You Know

Purpose: Students demonstrate their understanding by calculating the mean and range of a data set.

Today's Goals

1. **Goal:** Calculate the mean, median, and range for a data set.
2. **Goal:** Use measures of center and variability to compare data sets.
3. **Language Goal:** Justify a choice using evidence from the data set. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 8.09

Here is a new data set: 4, 5, 5, 6, 8, 8

a. Calculate the mean.
6

b. Calculate the range.
4

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the mean and median from a data set:

» **Respond to Student Thinking:**

- Use the ***Calculating Measures of Center and Spread*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

What strengths did your students exhibit today? How can you build from those strengths as students continue learning about *mean*, *median*, *range*, and *outliers?*

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. When comparing sets of data, why is it important to consider the mean, median, and range?

Responses vary. The measures of center (mean and median) give information about the data in general, but the variability (range) is also important because it gives information about how spread out the data is, which is a better picture of the data set overall. A large range may indicate that there are outliers in the data, which would affect the mean more than the median.

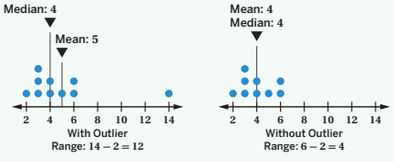
Lesson Practice 8.09

Lesson Summary

When comparing two sets of data, you can compare their *centers* and *spreads*. Data sets can have the same center but very different spreads.

One way to measure the center of data is by using the *mean*, or average, of the data. Another way to measure the center is by using the *median*, which is the middle value of the data.

The *range* measures the spread or variability of data. The smaller the range, the less spread out the data is. A large range could indicate there are *outliers* in the data. Outliers affect the mean more than the median.



Lesson Practice 8.09

Name: _____ Date: _____ Period: _____

Problems 1–3: A school's art club holds a bake sale on Fridays to raise money for art supplies. This table shows the number of cookies they sold each week in the fall and in the spring.

Fall		Spring
8 9	1	9
0 2 6 6 7	2	0 0 2 7 7
0 1 1	3	0 0 2 3
	4	

Key: 1 | 8 = 18

1. Calculate the mean and median number of cookies sold in the fall.

Mean: 25 cookies; median: 26 cookies

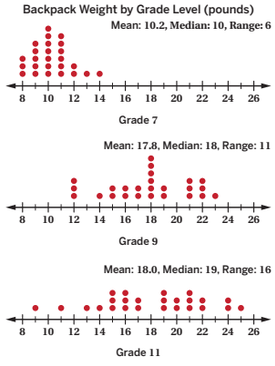
2. Calculate the mean and median number of cookies sold in the spring.

Mean: 26 cookies; median: 27 cookies

3. Based on this data, were sales generally higher in the spring than in the fall? Explain your thinking.

Yes. *Explanations vary.* Sales were slightly higher in the spring, but both the mean and median were only 1 more than in the fall, so they are not very different.

4. Compare the weights of the backpacks for students in these three grades. Include the mean and median weights and the range of weights in your response.



Responses vary.

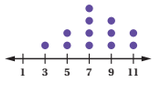
- **The backpacks tend to weigh less in grade 7 than in grades 9 and 11. The mean weight is 10.2 lb in grade 7 compared to 17.8 lb in grade 9 and 18.0 lb in grade 11. The median weight is 10 lb in grade 7 compared to 18 lb in grade 9 and 19 lb in grade 11. than for the 9th graders.**
- **The distribution of weights in grade 7 is less spread out than in grades 9 and 11. The range in grade 7 was 6 lb compared to a range of 11 lb in grade 9 and a range of 16 lb in grade 11. The backpack weights vary the most in grade 11.**

Lesson Practice 8.09

Name: _____ Date: _____ Period: _____

FAST Practice

5. Select *all* the true statements about the data shown in the dot plot.



- ☐ **A.** The mean is 7.
- ☑ **B.** The median is 7.
- ☐ **C.** The mean and median are the same.
- ☑ **D.** The mean is greater than the median.
- ☐ **E.** The range is 10.

Spiral Review

Problems 6–7: Evaluate each expression.

6. $\frac{(6-3)+(6-6)+(9-6)}{3}$ **= 2**

7. $\frac{(5-1)+(5-3)+(7-5)+(9-5)}{4}$ **= 3**

Problems 8–11: Rewrite each expression with fewer terms.

8. $4(5x-3y)-10x+6y$ **$= 10x-6y$ (or equivalent)**

9. $2.5(2x+4y)-5(4y-x)$ **$= 10(x-y)$ (or equivalent)**

10. $7.9(5x+3y)-4.2(5x+3y)-1.7(5x+3y)$ **$= 10x+6y$ (or equivalent)**

11. $6(x+2y)-2(y-2x)$ **$= 10(x+y)$ (or equivalent)**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.DP.1.2
FAST Practice	**5**	2	MA.7.DP.1.2
Spiral Review			
	6–7	1	MA.7.NSO.2.1
	8–11	2	MA.7.AR.1.1

Unit 8

Lesson 10

Crab Island

Sampling from Large Populations

Let's compare large populations of data.


Unit 8
Lesson 10
Name _____ Date _____ Period _____
MA.7.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1
Crab Island
Let's compare large populations of data.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
A. $\frac{1+3+9}{3}$
B. $\frac{1+2+3+4+5}{5}$
C. $\frac{2+4+5+6+8}{5}$
D. $\frac{3+5+13}{3}$
640
Warm-Up

Learning Outcomes

Today's Goals

1. **Goal:** Compare the mean of multiple *samples* from the same *population* to the mean of the population.
2. **Language Goal:** Describe how a *sample* may be used when it is unreasonable to gather data about a *population*. **(Reading, Writing, Speaking, and Listening)**

In this lesson, students use **samples** to draw conclusions about a **population**. Students compare sample means to population means and explore the advantages and disadvantages of using samples to answer a question about a population.

Coherence

Prior Learning

In Lesson 9, students calculated and interpreted the mean, median, and range of data sets in order to compare and contrast the center and spread of data.

Future Learning

In Lesson 11, students will use data from smaller samples to make estimates about a larger population.

Connections

- Students compare large populations of data for particular species of crabs using graphs. (mathematical, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.3
Given categorical data from a random sample, use proportional relationships to make predictions about a population.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.1	**MA.912.DP.1.4**
MA.6.DP.1.2	
MA.6.DP.1.6	

Vocabulary

New Vocabulary

population	**sample**

Lesson at a Glance

~ 45 min

MA.7.DP.1.3

Warm-Up

Pairs | 5 min

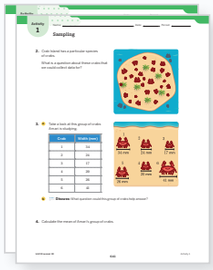
Students use the Which One Doesn't Belong? routine to analyze expressions.



Activity 1

Pairs | 10 min

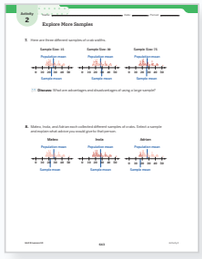
Students investigate the impact of **sample** size and randomness on how close a sample mean is to the **population** mean.
Routine: Notice and Wonder



Activity 2

Pairs | 10 min

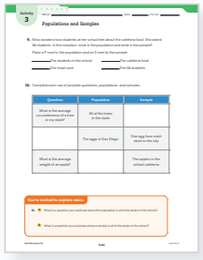
Students analyze how sample size can affect the quality of the data we collect. **(MTR.7.1)**



Activity 3

Pairs | 10 min

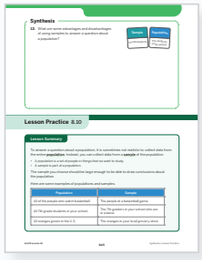
Students apply the terms population and sample to new contexts. **Routine:** Think-Pair-Share



Synthesis

Whole Class | 5 min

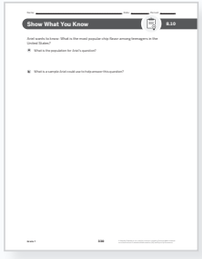
Students synthesize their understanding of the advantages and disadvantages of using samples to answer a question about a population. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the population and sample in a situation.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to analyze expressions.

1 Launch

Use the Which One Doesn't Belong? routine to support students in looking for structure in mean calculations and justifying their thinking precisely.

Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

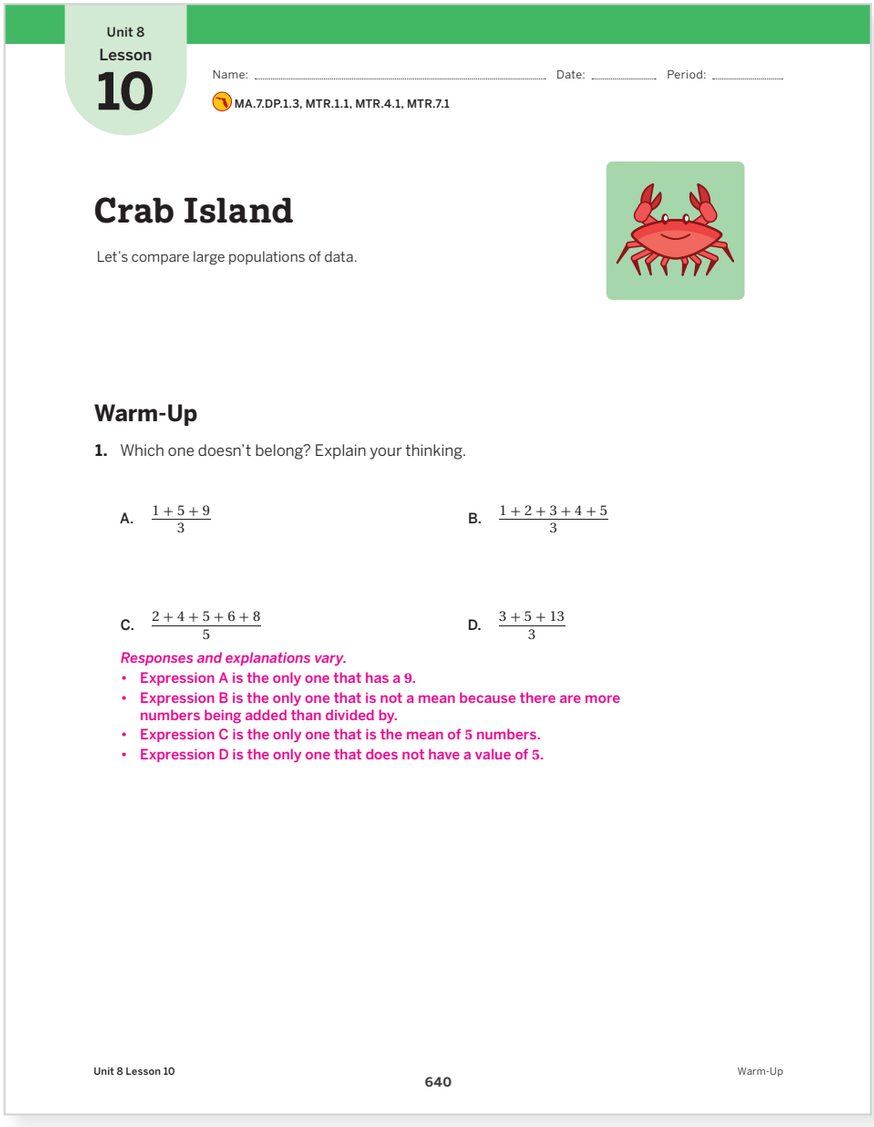
2 Connect

Discuss why each expression doesn't belong, asking students to justify their choice.

Collect and share responses and encourage students to share their thinking.

To surface connections, focus on expressions A, C, and D, and consider asking:

- "Is there a relationship between the denominator and the numerator? When have you seen this before?"
- "If you evaluate these expressions, what will the calculation represent?"



Unit 8
Lesson
10

Name: Date: Period:

MA.7.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1

Crab Island

Let's compare large populations of data.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. $\frac{1+5+9}{3}$

B. $\frac{1+2+3+4+5}{3}$

C. $\frac{2+4+5+6+8}{5}$

D. $\frac{3+5+13}{3}$

Responses and explanations vary.

- Expression A is the only one that has a 9.
- Expression B is the only one that is not a mean because there are more numbers being added than divided by.
- Expression C is the only one that is the mean of 5 numbers.
- Expression D is the only one that does not have a value of 5.

Unit 8 Lesson 10 640 Warm-Up

Pairs | 10 min

Activity 1 Sampling

Purpose: Students investigate the impact of **sample** size and randomness on how close a sample mean is to the **population** mean.

1 Launch

To generate excitement, direct students' attention to crab island and invite students to think about the prompt.

Invite students to share their questions.

2 Monitor

Direct students' attention to the group of crabs Amari is studying.

To support students getting started, invite them to review the expressions from the Warm-Up and describe a strategy for calculating the mean of a data set.

A **Accessibility: Memory and Attention** Invite students to use additional paper or a calculator to support them with performing calculations.

Activity 1 continued >

Activity 1

Name: Date: Period:

Sampling

2. Crab Island has a particular species of crabs.

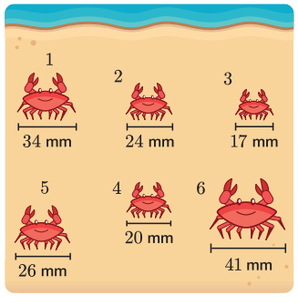
What is a question about these crabs that we could collect data for?

Responses vary.
- How big are the crabs' claws?
- What proportion of the crabs are babies?
- Are there more babies than adults?



3. a Take a look at this group of crabs Amari is studying.

Crab	Width (mm)
1	34
2	24
3	17
4	20
5	26
6	41



b **Discuss:** What question could this group of crabs help answer?

Responses vary. How wide are the crabs on this island?

4. Calculate the mean of Amari's group of crabs.

27 millimeters

Unit 8 Lesson 10 641 Activity 1

Pairs | 10 min

Activity 1 Sampling (continued)

Purpose: Students investigate the impact of **sample** size and randomness on how close a sample mean is to the **population** mean.

2 Monitor

Pause and use the Notice and Wonder routine to promote curiosity and help students make sense of using samples that reflect the population. **(MTR.1.1)**

Create a class definition for the terms **sample** and **population** using students' language and the glossary.

Consider asking:

- "What is the population that you are collecting data from?"
- "What is the sample that you are collecting data from?"
- "Why might we use samples when conducting experiments?" **(MTR.7.1)**

ELL **English Language Learners** Consider creating an anchor chart of examples of populations and samples that students can refer to throughout the rest of the unit.

Listen for student ideas to highlight during the Connect.

D Differentiation

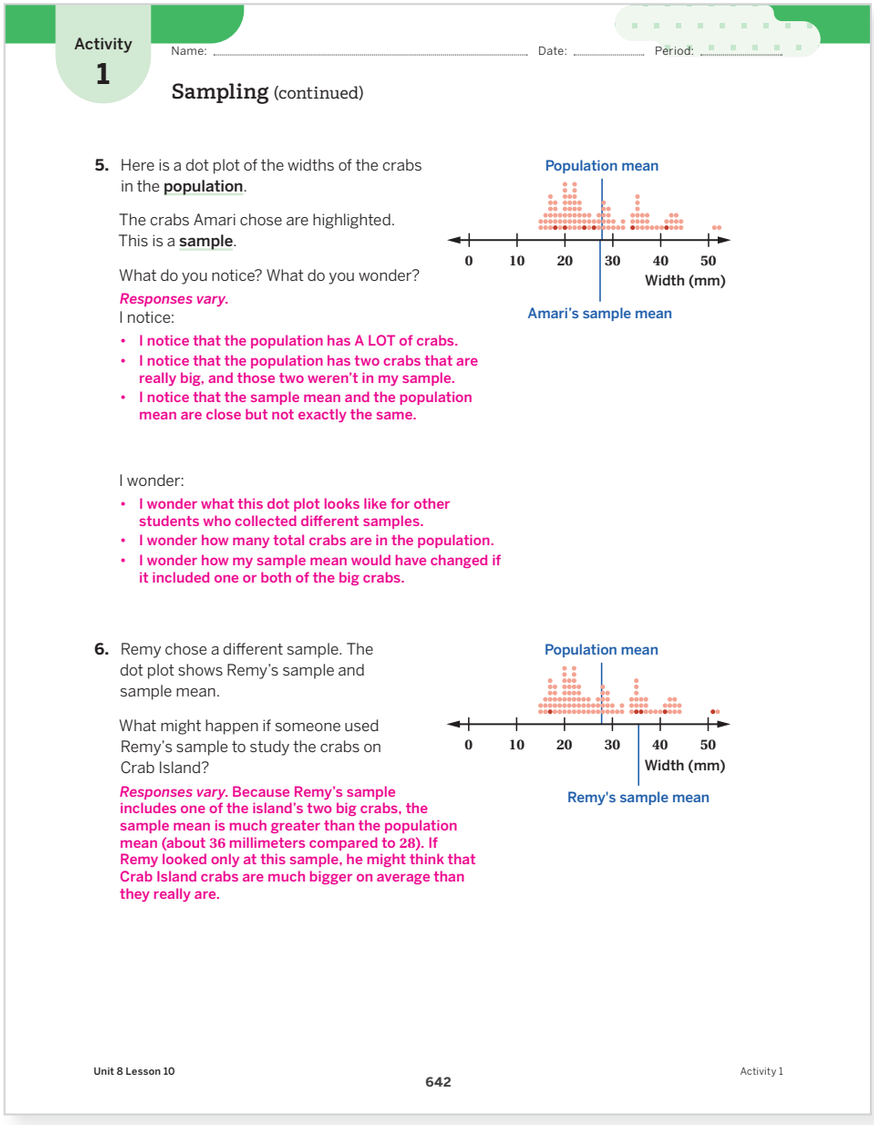
Look for students who:	Respond to Student Thinking
Only make observations about the population.	**Support** Consider asking, "What do you notice about the spread of Remy's sample?"
Would like to explore sample data more.	**Support** Encourage students to return to Problem 3 and try to select a sample that has a mean as close to the population mean as possible.
Notice that Remy's sample has more crabs that are larger.	**Stretch** Invite students to share why this may or may not reflect the population.

3 Connect

Share student responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, consider asking:

- "Why might we care how close a sample mean is to the population mean?"
- "Why do you think Amari's sample mean and Remy's sample mean are different?"
- "What could someone do to be more likely to get a sample mean close to the population mean?"



Activity 1

Name: Date: Period:

Sampling (continued)

5. Here is a dot plot of the widths of the crabs in the **population**.

The crabs Amari chose are highlighted. This is a **sample**.

What do you notice? What do you wonder?

Responses vary.

I notice:

- I notice that the population has A LOT of crabs.
- I notice that the population has two crabs that are really big, and those two weren't in my sample.
- I notice that the sample mean and the population mean are close but not exactly the same.

I wonder:

- I wonder what this dot plot looks like for other students who collected different samples.
- I wonder how many total crabs are in the population.
- I wonder how my sample mean would have changed if it included one or both of the big crabs.

6. Remy chose a different sample. The dot plot shows Remy's sample and sample mean.

What might happen if someone used Remy's sample to study the crabs on Crab Island?

Responses vary. Because Remy's sample includes one of the island's two big crabs, the sample mean is much greater than the population mean (about 36 millimeters compared to 28). If Remy looked only at this sample, he might think that Crab Island crabs are much bigger on average than they really are.

Unit 8 Lesson 10 642 Activity 1

Key Takeaway: When selecting a **sample** to gather data about a **population**, we should make sure that our sample has a variety of characteristics so that it reflects the larger population.

Pairs | 10 min

Activity 2 Explore More Samples

Purpose: Students analyze how sample size can affect the quality of the data we collect. **(MTR.7.1)**

1 Launch

Direct students' attention to the three different samples.

Share that the purpose of this activity is to explore how different samples and sample sizes affect the sample mean when compared to the population mean.

2 Monitor

Invite students to share their advice with another pair of students. Encourage them to support each other to make their advice stronger and clearer. **(MTR.4.1)**

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "How does each student's sample differ?"

D **Differentiation: (Stretch)** Invite students to return to Problem 3 and try to create a small sample with a mean as close to the population mean as possible.

3 Connect

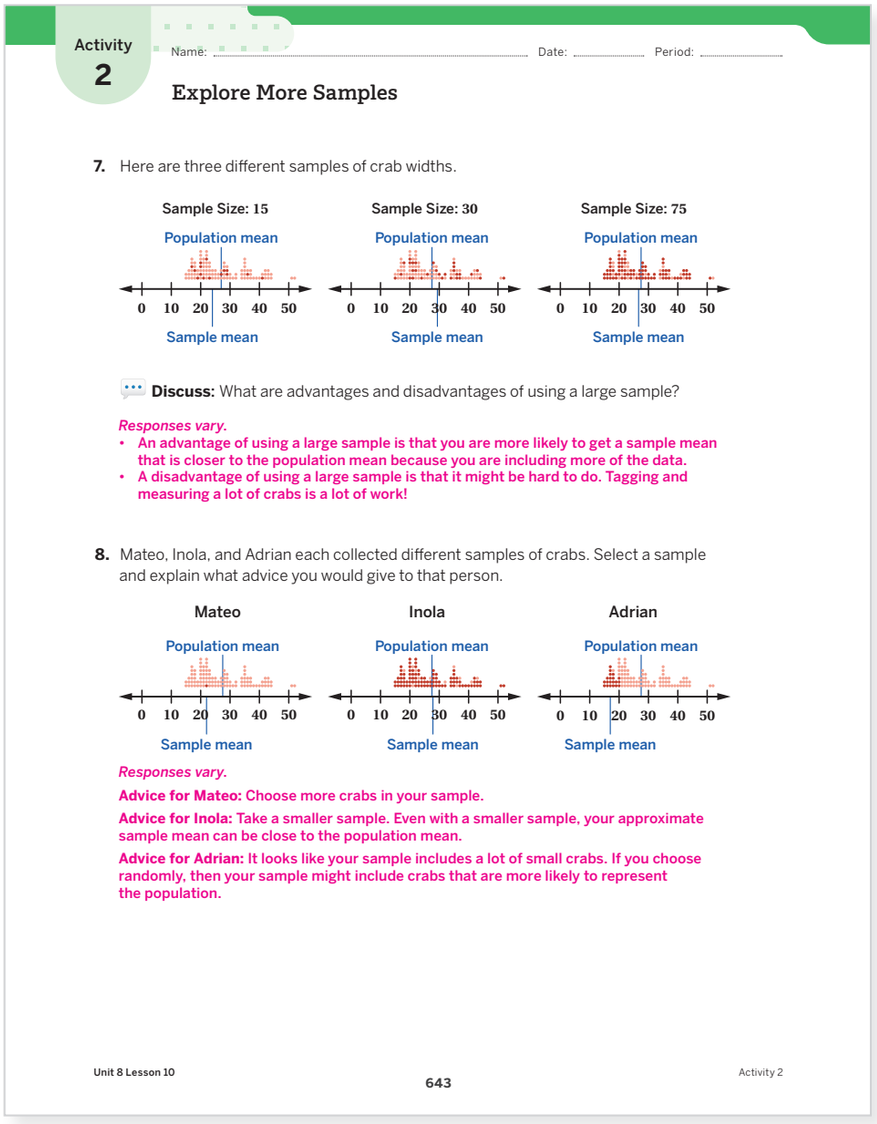
Consider asking:

- "What happened to the sample mean as the sample size increased?"
- "Why might it be challenging in real life to use a large sample?"
- "What other questions might you be interested in asking where surveying the whole population would be challenging?"

A **Accessibility: Memory and Attention** Invite students to refer to the anchor chart created previously on samples and population to activate prior knowledge. Add to the anchor chart as students share.

Invite students to share their advice for one of the characters, or share the advice of another student that was helpful.

To surface the Key Takeaway, consider asking, "Whose sample mean was closest to the actual population mean? Why do you think this was the case?" **(MTR.7.1)**



Activity 2

Name: Date: Period:

Explore More Samples

7. Here are three different samples of crab widths.

Discuss: What are advantages and disadvantages of using a large sample?

Responses vary.

- An advantage of using a large sample is that you are more likely to get a sample mean that is closer to the population mean because you are including more of the data.
- A disadvantage of using a large sample is that it might be hard to do. Tagging and measuring a lot of crabs is a lot of work!

8. Mateo, Inola, and Adrian each collected different samples of crabs. Select a sample and explain what advice you would give to that person.

Responses vary.

Advice for Mateo: Choose more crabs in your sample.

Advice for Inola: Take a smaller sample. Even with a smaller sample, your approximate sample mean can be close to the population mean.

Advice for Adrian: It looks like your sample includes a lot of small crabs. If you choose randomly, then your sample might include crabs that are more likely to represent the population.

Unit 8 Lesson 10 643 Activity 2

Key Takeaway: There is a relationship between the **sample** mean and the **population** mean. Larger samples are more likely to have means that are closer to the population mean. It may be difficult to collect a large sample, and when it is, selecting a random sample is the most efficient way to collect data that represents the entire population.

Pairs | 10 min

Activity 3 Populations and Samples

Purpose: Students apply the terms population and sample to new contexts.

Short on time? Consider completing Problems 9 and 10 as a whole class.

1 Launch

To surface connections, consider asking students, "Who would be the best group of people to ask if you wanted to know how students feel about something?"

Note: Students may think they need to place either P or S in all blanks. Consider sharing that there will be two unused options.

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Consider grouping students with different English language proficiency levels, so students can listen to and learn from their peers. **(Speaking and Listening)**

2 Monitor

To support students getting started, encourage them to think of any sample of trees that could be taken from the population.

Look for several responses to highlight the variety of possible answers.

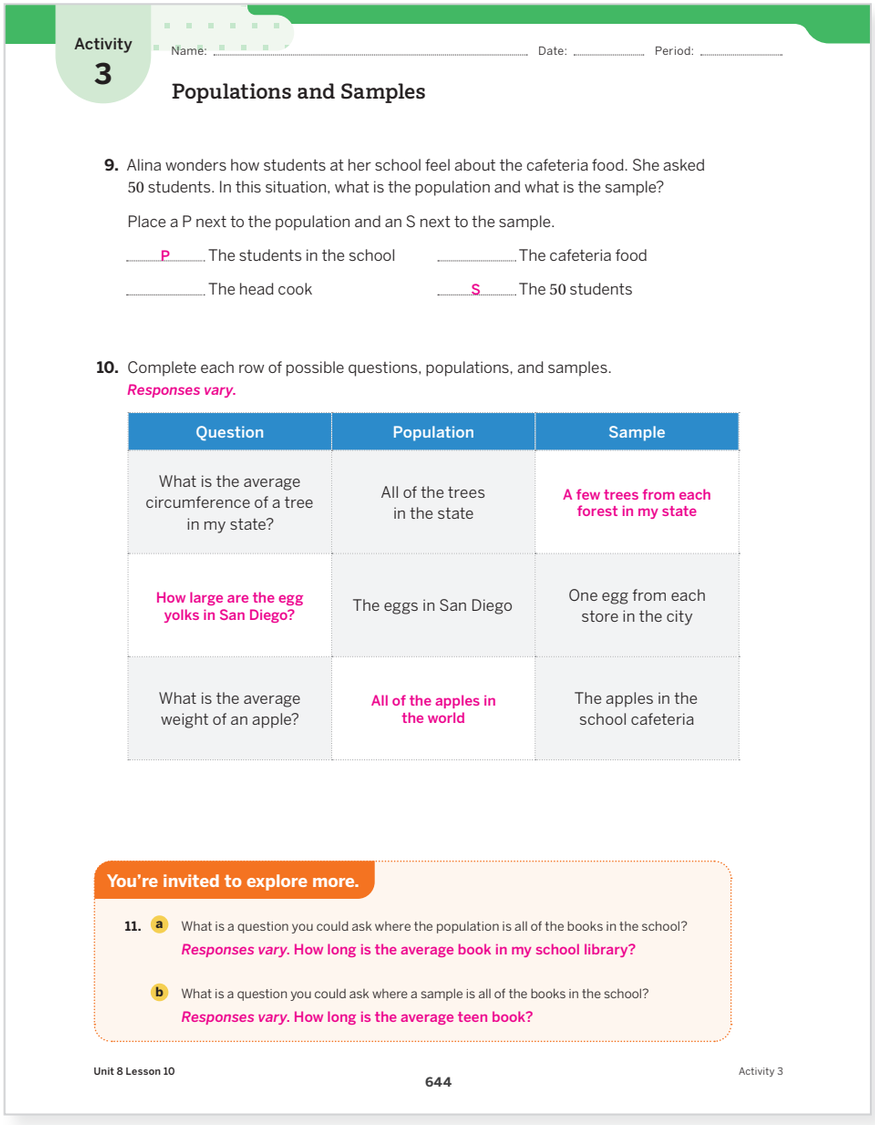
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore samples and populations further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their responses and discuss whether the responses coincide with the rest of the question, population, or sample.

Consider asking:

- "How do you know that is a sample?"
- "How do you know that is a population?"
- "What other samples could we use to answer this question?" **(MTR.7.1)**



Activity 3

Name: _____ Date: _____ Period: _____

Populations and Samples

9. Alina wonders how students at her school feel about the cafeteria food. She asked 50 students. In this situation, what is the population and what is the sample?

Place a P next to the population and an S next to the sample.

P The students in the school _____ The cafeteria food

_____ The head cook S The 50 students

10. Complete each row of possible questions, populations, and samples.

Responses vary.

Question	Population	Sample
What is the average circumference of a tree in my state?	All of the trees in the state	A few trees from each forest in my state
How large are the egg yolks in San Diego?	The eggs in San Diego	One egg from each store in the city
What is the average weight of an apple?	All of the apples in the world	The apples in the school cafeteria

You're invited to explore more.

11. a. What is a question you could ask where the population is all of the books in the school?

Responses vary. How long is the average book in my school library?

b. What is a question you could ask where a sample is all of the books in the school?

Responses vary. How long is the average teen book?

Unit 8 Lesson 10 644 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the advantages and disadvantages of using samples to answer a question about a population.

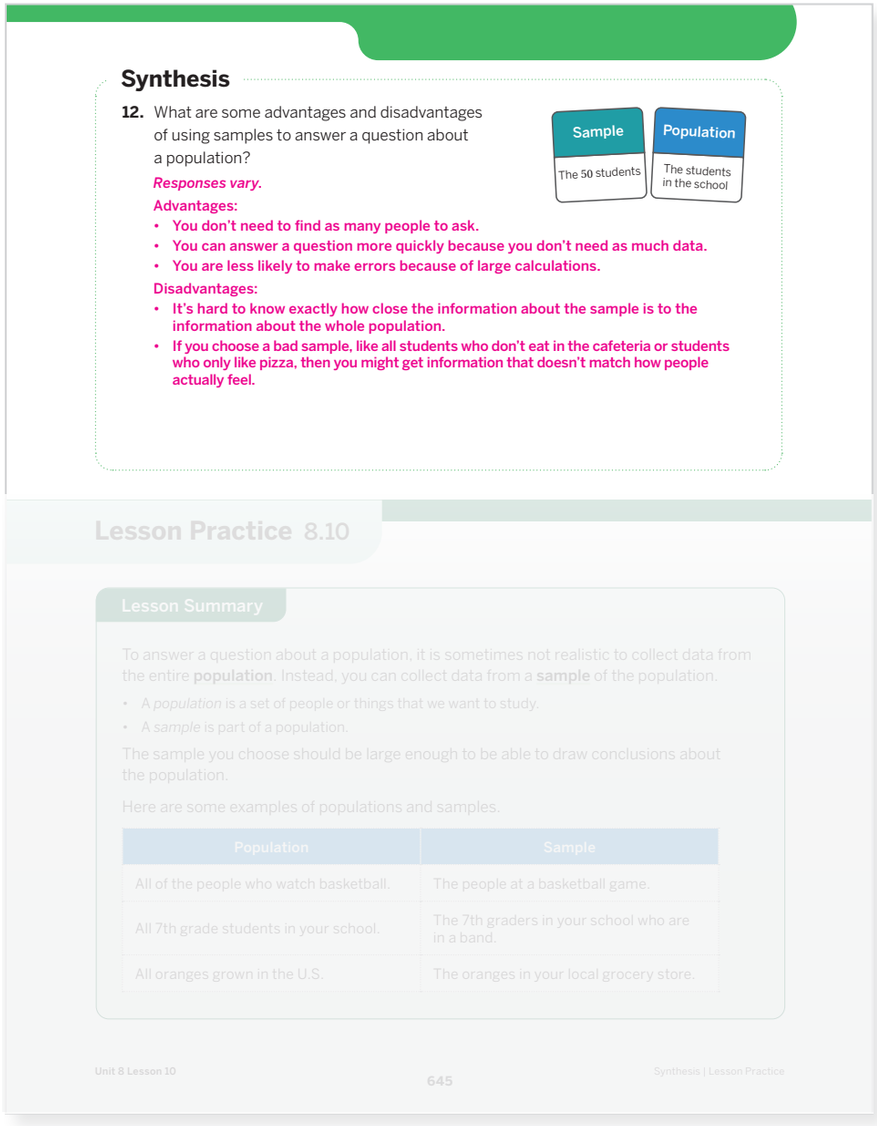
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Samples are smaller and it may be very difficult to ask an entire population a question.
- Samples may not always reflect an entire population.
- No two samples will produce the same results; depending on the sample the spread of data may be large and/or inaccurate.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *population* and *sample* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. What are some advantages and disadvantages of using samples to answer a question about a population?

Responses vary.

Advantages:

- You don't need to find as many people to ask.
- You can answer a question more quickly because you don't need as much data.
- You are less likely to make errors because of large calculations.

Disadvantages:

- It's hard to know exactly how close the information about the sample is to the information about the whole population.
- If you choose a bad sample, like all students who don't eat in the cafeteria or students who only like pizza, then you might get information that doesn't match how people actually feel.

Lesson Practice 8.10

Lesson Summary

To answer a question about a population, it is sometimes not realistic to collect data from the entire **population**. Instead, you can collect data from a **sample** of the population.

- A *population* is a set of people or things that we want to study.
- A *sample* is part of a population.

The sample you choose should be large enough to be able to draw conclusions about the population.

Here are some examples of populations and samples.

Population	Sample
All of the people who watch basketball.	The people at a basketball game.
All 7th grade students in your school.	The 7th graders in your school who are in a band.
All oranges grown in the U.S.	The oranges in your local grocery store.

Unit 8 Lesson 10 645 Synthesis | Lesson Practice

Lesson Takeaway: It is not always possible or reasonable to collect data from an entire **population**. In this case, data is collected from a **sample** of the population and used to answer any necessary questions. Samples can vary in size, but larger samples tend to better represent the population.

Independent | 5 min

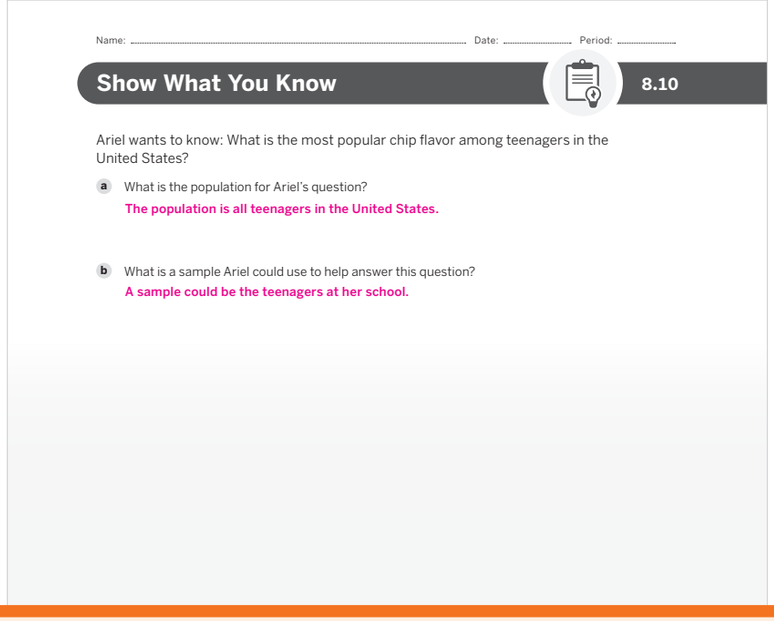
Show What You Know

Purpose: Students demonstrate their understanding by determining the population and sample in a situation.

Today's Goals

1. **Goal:** Compare the mean of multiple *samples* from the same *population* to the mean of the population.
2. **Language Goal:** Describe how a *sample* may be used when it is unreasonable to gather data about a *population*. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 8.10

Ariel wants to know: What is the most popular chip flavor among teenagers in the United States?

a. What is the population for Ariel's question?
The population is all teenagers in the United States.

b. What is a sample Ariel could use to help answer this question?
A sample could be the teenagers at her school.

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify the population and sample:

» **Respond to Student Thinking:**

- Revisit **Activity 3** as a class and invite students to discuss which of the phrases describe the sample and which describes the population.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task.

Professional Learning

Reflect on the facilitation of the **Which One Doesn't Belong?** routine in this lesson. Students will use this routine again in Lesson 13.

- What about the routine felt successful?
- What are some areas for growth for the next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What are some advantages and disadvantages of using samples to answer a question about a population?

Responses vary.

Advantages:

- You don't need to find as many people to ask.
- You can answer a question more quickly because you don't need as much data.
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Disadvantages:

- It's hard to know exactly how close the information about the sample is to the information about the whole population.
- If you choose a bad sample, like all students who don't eat in the cafeteria or students who only like pizza, then you might get information that doesn't match how people actually feel.

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- A *sample* is part of a population.

The sample you choose should be large enough to be able to draw conclusions about the population.

Here are some examples of populations and samples.

Population	Sample
All of the people who watch basketball.	The people at a basketball game.
All 7th grade students in your school.	The 7th graders in your school who are in a band.
All oranges grown in the U.S.	The oranges in your local grocery store.

Lesson Practice 8.10

Name: _____ Date: _____ Period: _____

Problems 1–3: Aniyah wonders: How much time do 7th graders at my school spend outdoors on a typical day?

1. What is the population for Aniyah's question?

7th graders at her school.

2. Select *all* possible samples for Aniyah's question.

- ☑ **A.** The 20 students in a 7th grade math class.
- ☐ **B.** The first 20 people to arrive at her middle school on a particular day.
- ☐ **C.** The 7th graders participating in a science fair with students from four middle schools.
- ☑ **D.** The 10 7th graders on the school soccer team.
- ☐ **E.** The students on the high school debate team.

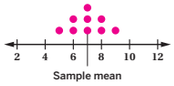
3. Select two samples and list a different possible population each sample could belong to. *Responses vary.*

Sample	Possible Population
The 20 students in a 7th grade math class.	All of the students in all math classes.
The first 20 people to arrive at her middle school on a particular day.	All of the people who visit the middle school.
The 7th graders participating in a science fair with students from four middle schools.	All of the students participating in the science fair.
The 10 7th graders on the school soccer team.	All of the students on the school soccer team.
The students on the high school debate team.	All of the students at the high school.

4. Here is a dot plot of a population. Create a dot plot of a sample with a mean that's the same as the population mean. Your sample should have more than 6, but fewer than 20 data points.



Population mean: 7



Sample mean

Lesson Practice 8.10

Name: _____ Date: _____ Period: _____

FAST Practice

5. The average math test score for the students in Grade 7 is 82. A teacher randomly selects 3 groups of 15 students each and calculates their average scores. The average score of Group A is 78, the average score of Group B is 84, and the average score of Group C is 82. What conclusions can you draw from these sample means?

- (A.) The sample means vary but are close to the population mean of 82.
- **B.** The sample means show that the population mean is incorrect.
- **C.** The samples cannot be used to estimate the population mean, as they differ.
- **D.** A new population mean should be calculated by averaging the sample means.

Spiral Review

6. There are 50 marbles in a bag. Students picked a marble, recorded its color, and put it back in. Each student carried out a different number of experiments. The table shows their results. Estimate the probability of getting a green marble from this bag. Explain your thinking.

	Number of Experiments	Green Marbles
Student 1	4	1
Student 2	12	5
Student 3	9	3

$\frac{9}{25}$ (or equivalent). *Explanations vary.* In a total of 25 experiments, the students picked a green marble 9 times.

Problems 7–9: Calculate the mean of each data set.

7. 8, 9, 9, 9, 10
9

8. 2, 6, 12, 16
9

9. 5, 6, 12, 13
9

Problems 10–11: A bookstore has a 15% discount on all books.

10. How much money is the 15% discount worth on a book that normally costs $18?
$2.70

11. After the discount, how much would the book cost?
$15.30

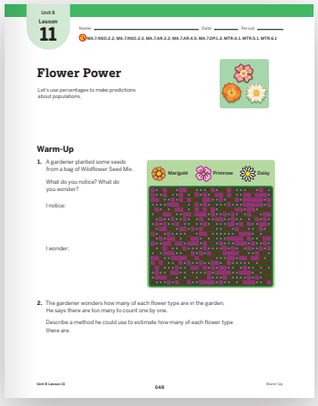
Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.1.3
	2–3	2	MA.7.DP.1.3
	4	3	MA.7.DP.1.3
FAST Practice	5	2	MA.7.DP.1.3
Spiral Review			
	6	2	MA.7.DP.2.4
Fluency	7–9	1	MA.6.DP.1.2
	10–11	1	MA.6.AR.3.4

Unit 8

Lesson

11


Unit 8
Lesson
11
Name _____ Date _____ Period _____
MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3, MTR.4.1, MTR.5.1, MTR.6.1
Flower Power
Let's use percentages to make predictions about populations.
Warm-Up
1. A gardener planted some seeds from a bag of Wildflower Seed Mix.
What do you notice? What do you wonder?
I notice:
I wonder:
Marigold
Primrose
Daisy
2. The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.
Describe a method he could use to estimate how many of each flower type there are.
Unit 8 Lesson 11
648
Warm-Up

Flower Power

Using Percentages to Predict Populations

Let's use percentages to make predictions about populations.

Learning Outcomes

Today's Goals

1. **Goal:** Estimate unknown information about a population using proportional reasoning.
2. **Language Goal:** Describe how a sample and proportional reasoning can be used to estimate information about a population. **(Reading, Writing, Speaking, and Listening)**

Students build on their knowledge of *proportional* relationships and percentages by using data from smaller samples to make estimates about a larger population. They test claims made by others by creating a new sample and comparing the results to form conclusions. **(MTR.6.1)**

Coherence

Prior Learning

In Lesson 10, students used samples to draw conclusions about a population. They explored advantages and disadvantages of using samples to answer questions about a population.

Future Learning

In Lessons 12 and 13, students will analyze how measures of center and variability can be used to compare sets of data. They will draw inferences by examining the spread of the data.

Connections

- Students understand the proportion or distribution of characteristics within a population to make predictions about wildflower seed mix. (cross curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.AR.4.5
Solve real-world problems involving proportional relationships.

MA.7.DP.1.3
Given categorical data from a random sample, use proportional relationships to make predictions about a population.

Also Addressing: **MA.7.NSO.2.2, MA.7.NSO.2.3**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	**MA.912.DP.1.4**
MA.6.AR.3.5	

Vocabulary

Review Vocabulary

proportional

Lesson at a Glance

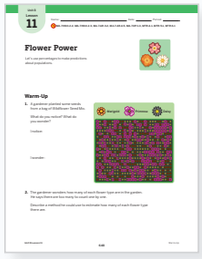
~ 45 min

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3

Warm-Up

Pairs | 5 min

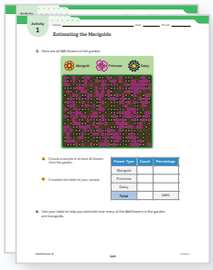
Students use the **Notice and Wonder** routine to explore flowers in a garden and consider methods for estimating how many flowers of each type there are.



Activity 1

Pairs | 15 min

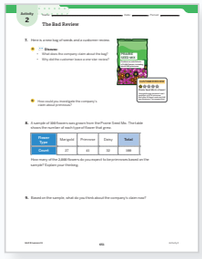
Students choose a sample of flowers from the garden to make an estimate about the total number of marigolds then analyze the estimation strategies of two other students.
MLR7: Compare and Connect



Activity 2

Pairs | 15 min

Students determine if a company's claim is true by first analyzing a customer review based on a small sample size, then using a second sample to test the claim.



Synthesis

Whole Class | 5 min

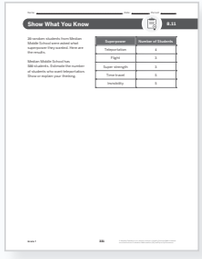
Students synthesize their understanding of using a sample and *proportional* reasoning to estimate information about a population.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using data from a sample to make an estimate about a larger population.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the **Notice and Wonder** routine to explore flowers in a garden and consider methods for estimating how many flowers of each type there are.

1 Launch

Direct students' attention to the illustration of the garden.

Use the Notice and Wonder routine to promote curiosity about the mixture of flowers in the garden and help students make sense of ways to analyze a population.

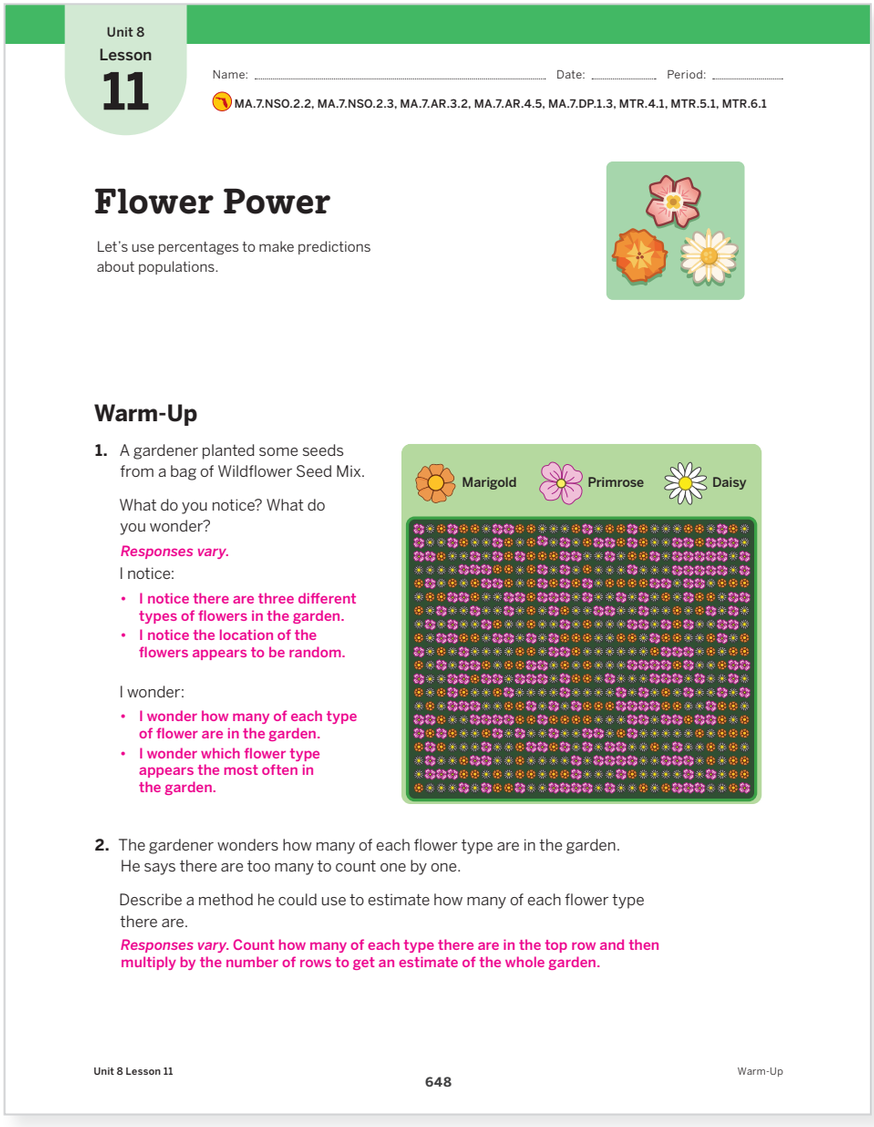
Invite students to think independently before discussing their ideas with a partner.

2 Connect

Invite students to share their methods with the class.

Consider asking:

- "What do you like about each method?"
- "Do you think this method would get a representative sample? Why or why not?"
- "What would you do if you took a random sample and one of the flower types wasn't included in your sample?"



Unit 8
Lesson
11

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3, MTR.4.1, MTR.5.1, MTR.6.1

Flower Power

Let's use percentages to make predictions about populations.

Warm-Up

1. A gardener planted some seeds from a bag of Wildflower Seed Mix.

What do you notice? What do you wonder?

Responses vary.

I notice:

- I notice there are three different types of flowers in the garden.
- I notice the location of the flowers appears to be random.

I wonder:

- I wonder how many of each type of flower are in the garden.
- I wonder which flower type appears the most often in the garden.

Marigold Primrose Daisy

2. The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.

Describe a method he could use to estimate how many of each flower type there are.

Responses vary. Count how many of each type there are in the top row and then multiply by the number of rows to get an estimate of the whole garden.

Unit 8 Lesson 11 648 Warm-Up

Pairs | 15 min

Activity 1 Estimating the Marigolds

Purpose: Students choose a sample of flowers from the garden to make an estimate about the total number of marigolds then analyze the estimation strategies of two other students.

Short on time? Consider completing the discussion for Problem 6 as a whole class.

1 Launch

To surface prior knowledge, consider asking, "How can you calculate what percentage of a whole is made up of a certain item?"

2 Monitor

Support students getting started by inviting them to select a different sample size than their partner and discuss their results.

Encourage students to use a calculator as needed.

Listen for student strategies for calculating the estimate to share during the Connect.

D **Differentiation: (Support)** Students may have difficulty using their sample to create an estimate. Consider asking:

- "What percent of the whole garden do you think is marigolds? Why?"
- "How can you use equivalent ratios to estimate the number of marigolds in the garden?" **(MTR.5.1)**

Invite students that finish early to choose a different sample and refine their estimate. **(MTR.6.1)**

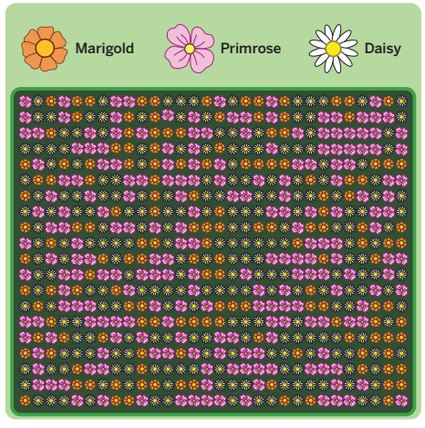
Activity 1 continued >

Activity 1

Name: Date: Period:

Estimating the Marigolds

3. Here are all 600 flowers in the garden.



a. Choose a sample of at least 25 flowers from the garden.

Samples vary.

b. Complete the table for your sample.

Responses vary based on students' sample.

Flower Type	Count	Percentage
Marigold	9	14%
Primrose	25	39%
Daisy	30	47%
Total	64	100%

4. Use your table to help you estimate how many of the 600 flowers in the garden are marigolds.

Responses vary based on students' sample.

Unit 8 Lesson 11 649 Activity 1

Pairs | 15 min

Activity 1 Estimating the Marigolds (continued)

Purpose: Students choose a sample of flowers from the garden to make an estimate about the total number of marigolds then analyze the estimation strategies of two other students.

Short on time? Consider completing the discussion for Problem 6 as a whole class.

2 Monitor

Listen to partner discussions for student ideas about *proportional* reasoning to highlight during the Connect.

A **Accessibility: Memory and Attention** To activate prior knowledge, consider asking, "What do we know about percentages and ratios that can help us to analyze Eliza's and Javier's strategies?"

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each student's strategy, and then connect to their strategy for estimating the number of marigolds in the garden. Consider asking:

- "How are Eliza's and Javier's strategies alike? How are they different?"
- "What do you think was helpful about each strategy?" **(MTR.4.1)**

Listen for a variety of student responses to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to think about how the sizes of samples could impact an estimate of the population.
Made an estimate that was much less or much greater than the actual number of marigolds in the garden.	**Support** Consider asking "How could you make adjustments to your sample size to improve your estimate?"
Made an estimate that was close to the actual number of marigolds in the garden.	**Strengthen** Encourage them to practice with different sample sizes and create new strategies for making estimates.

3 Connect

Invite students to share ideas about how Eliza and Javier used proportional reasoning in their estimation strategies. **(MTR.5.1)**

Invite students to share what they think they could do to improve their estimates. **(MTR.6.1)**

To surface the Key Takeaway, consider asking, "Why do you think your estimate might have been different from your classmates' estimates?"

Activity 1

Name: Date: Period:

Estimating the Marigolds (continued)

5. Eliza and Javier chose a 5-by-5 sample by Eliza closing her eyes and placing her finger on the image.



Here are their strategies for estimating the total number of marigolds.

Eliza

Marigolds are 28% of the sample.

Marigolds are probably 28% of the garden.

28% of 600 is 168 marigolds.

Javier

In the sample, 7 of 25 are marigolds.

Marigolds	Total
7	25
168	600

·24 ·24

Explain each student's strategy.

Explanations vary.

- Eliza's strategy: Figure out the percent of marigolds in the sample. Then calculate that same percent of flowers in the whole garden.
- Javier's strategy: Figure out how many times larger the population is than the sample. Multiply the number of marigolds in the sample by the same scale factor.

6. The actual number of marigolds in the entire garden is 151.

Discuss:

- How did the actual number of marigolds in the garden compare to the earlier estimates?
- What do you think could have improved the estimates?

Responses vary.

- There are 151 marigolds in the garden, which is less than my estimate of 172 marigolds. This is also 17 marigolds less than Eliza and Javier's estimate.
- Using a larger sample size or looking at multiple samples might have provided a more accurate representation of the population.

Unit 8 Lesson 11 650 Activity 1

Key Takeaway: Using a larger sample or multiple samples can provide a more accurate representation when making estimates about a population.

Pairs | 15 min

Activity 2 The Bad Review

Purpose: Students determine if a company's claim is true by first analyzing a customer review based on a small sample size, then using a second sample to test the claim.

1 Launch

To surface connections, consider asking, "What do you know about product reviews?"

Invite students to read the company's claim and the customer's review aloud then restate them in their own words.

2 Monitor

To support students getting started, encourage them to first compare the customer's review to the company's claim.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., "The customer left a one-star review because _____." or "To investigate the company's claim, I would _____."). **(Writing and Speaking)**

Direct students' attention to the table with the results of the 100-flower sample.

Listen for student strategies using proportional reasoning to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Make predictions that include a really high or low number of primroses.	**Support** Consider asking: • "Was there a big difference in the number of primroses in the sample compared to other flower types?" • "How did you determine how many of the 2,000 flowers were primroses?"
Make a prediction by applying proportional reasoning to the number of primroses from the sample.	**Strengthen** Consider asking, "How does your prediction compare to the customer's review about primroses?"

Collect a variety of student responses to share during the Connect.

3 Connect

Share strategies that used proportional reasoning and invite students to share how they used calculations to test the company's claim. **(MTR.6.1)**

To surface the Key Takeaway, consider asking:
- "How did using a random sample help you test the claim made by the company?"
- "How can proportional reasoning be used to make predictions about a larger population?"

Activity 2

Name: _____ Date: _____ Period: _____

The Bad Review

7. Here is a new bag of seeds and a customer review.



a. **Discuss:**
- What does the company claim about the bag?
- Why did the customer leave a one-star review?

Responses vary.
- The company claims that the prairie seed mix would produce a mix of 2,000 flowers, which would include about 800 primroses.
- The customer grew 10 flowers and only 1 was a primrose, which is 10%. That's a lower percentage than is implied from the description on the bag.

b. How could you investigate the company's claim about primroses? *Responses vary.* I could plant more flowers and see if about 40% are primroses, as the bag implies.

8. A sample of 100 flowers was grown from the Prairie Seed Mix. The table shows the number of each type of flower that grew.

Flower Type	Marigold	Primrose	Daisy	Total
Count	27	41	32	100

How many of the 2,000 flowers do you expect to be primroses based on the sample? Explain your thinking.

Explanations vary.
- I predict 820 of the flowers will be primroses since 41% of the flowers in the sample are primroses, and 41% of 2,000 is 820.
- I think there will be close to 820 primroses because in the sample there are 100 flowers and 41 are primroses. That's a ratio of $\frac{41}{100}$, which is equivalent to $\frac{820}{2,000}$.

9. Based on the sample, what do you think about the company's claim now?

Responses vary.
- I think the company's claim is true. In the sample, nearly half of the flowers are primroses, which would be more than 800 for a bag of 2,000.

Unit 8 Lesson 11 651 Activity 2

Key Takeaway: A random sample can be used to test a claim by applying *proportional* reasoning to the sample data to make predictions about the larger population.

Whole Class | 5 min

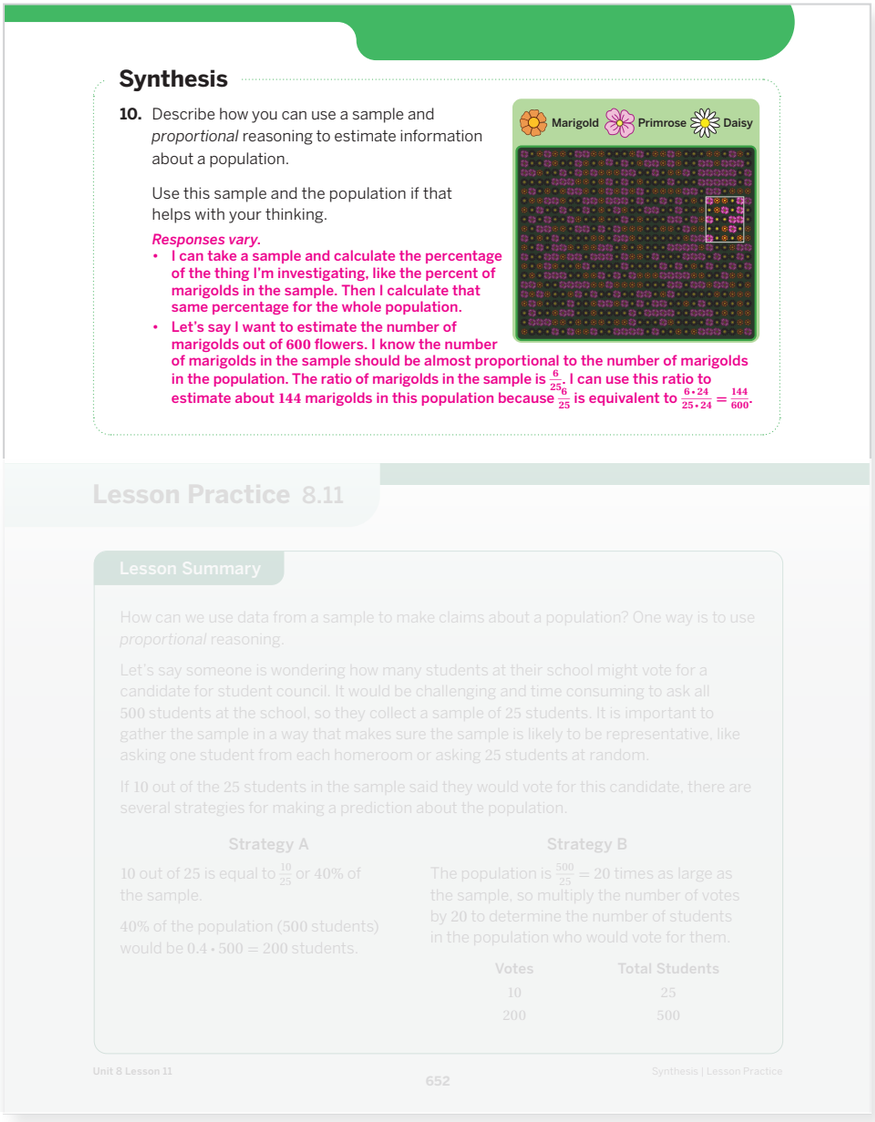
Synthesis

Purpose: Students synthesize their understanding of using a sample and *proportional* reasoning to estimate information about a population.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using proportional reasoning to calculate a percentage in a sample then calculate that percentage for the large population.
- Estimating a value for the larger population based on the number being proportional for that object in the sample.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe how you can use a sample and *proportional* reasoning to estimate information about a population.

Use this sample and the population if that helps with your thinking.

Responses vary.
- I can take a sample and calculate the percentage of the thing I'm investigating, like the percent of marigolds in the sample. Then I calculate that same percentage for the whole population.
- Let's say I want to estimate the number of marigolds out of 600 flowers. I know the number of marigolds in the sample should be almost proportional to the number of marigolds in the population. The ratio of marigolds in the sample is $\frac{6}{25}$. I can use this ratio to estimate about 144 marigolds in this population because $\frac{6}{25}$ is equivalent to $\frac{6 \cdot 24}{25 \cdot 24} = \frac{144}{600}$.

Lesson Practice 8.11

Lesson Summary

How can we use data from a sample to make claims about a population? One way is to use *proportional* reasoning.

Let's say someone is wondering how many students at their school might vote for a candidate for student council. It would be challenging and time consuming to ask all 500 students at the school, so they collect a sample of 25 students. It is important to gather the sample in a way that makes sure the sample is likely to be representative, like asking one student from each homeroom or asking 25 students at random.

If 10 out of the 25 students in the sample said they would vote for this candidate, there are several strategies for making a prediction about the population.

Strategy A

10 out of 25 is equal to $\frac{10}{25}$ or 40% of the sample.

40% of the population (500 students) would be $0.4 \cdot 500 = 200$ students.

Strategy B

The population is $\frac{500}{25} = 20$ times as large as the sample, so multiply the number of votes by 20 to determine the number of students in the population who would vote for them.

Votes	Total Students
10	25
200	500

Unit 8 Lesson 11 652 Synthesis | Lesson Practice

Lesson Takeaway: *Proportional* reasoning can be used to make connections between the sample data and the population data. Using larger or multiple samples is more likely to provide a more accurate representation of the population.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using data from a sample to make an estimate about a larger population.

Today's Goals

1. **Goal:** Estimate unknown information about a population using proportional reasoning.
2. **Language Goal:** Describe how a sample and proportional reasoning can be used to estimate information about a population. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.11

20 random students from Median Middle School were asked what superpower they wanted. Here are the results.

Superpower	Number of Students
Teleportation	4
Flight	3
Super strength	3
Time travel	5
Invisibility	5

Median Middle School has 500 students. Estimate the number of students who want teleportation. Show or explain your thinking.

100 students. Responses vary. First, I found the percentage of students from the random group who want teleportation as their superpower, which was $\frac{4}{20}$ or 0.20. Then, I multiplied 0.20 and 500 to estimate the number of students at the school who want the same superpower, which was $0.20 \times 500 = 100$ student.

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of make an estimate about population using proportional reasoning:

» **Respond to Student Thinking:**

- Use the ***Predicting Population Using Sample Data*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to pose a question about their school's student population, then gather a sample from students in their class and use it to draw a conclusion using proportional reasoning.

Professional Learning

How did the **MLR7: Compare and Connect** routine support students in using a sample to make an estimate about a population?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you can use a sample and *proportional* reasoning to estimate information about a population.

Use this sample and the population if that helps with your thinking.

Responses vary.

- I can take a sample and calculate the percentage of the thing I'm investigating, like the percent of marigolds in the sample. Then I calculate that same percentage for the whole population.
- Let's say I want to estimate the number of marigolds out of 600 flowers. I know the number of marigolds in the sample should be almost proportional to the number of marigolds in the population. The ratio of marigolds in the sample is $\frac{6}{25}$. I can use this ratio to estimate about 144 marigolds in this population because $\frac{6}{25}$ is equivalent to $\frac{6 \cdot 24}{25 \cdot 24} = \frac{144}{600}$.

Lesson Practice 8.11

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Votes	Total Students
10	25
200	500

Lesson Practice 8.11

Name: _____ Date: _____ Period: _____

Problems 1–3: Faaria and Ariel wondered how many students at school would dye their hair blue. They each surveyed a different random sample of the students.

1. Faaria asked 10 students, and 2 of them said they would. Based on Faaria's sample, what percent of the students would dye their hair blue?

20%

2. Ariel asked 100 students, and 17 of them said they would. Based on Ariel's sample, what percent of the students would dye their hair blue?

17%

3. Whose percentage is likely to be closest to the percentage of all the students? Explain how you know.

Ariel. *Explanations vary.* Ariel's sample is much larger than Faaria's. The larger the random sample, the closer it usually is to representing the actual population.

Problems 4–6: In a school of 580 students, one class was asked which hand they write with. "L" means they use their left hand, and "R" means they use their right hand.

Here are the results: L, R, R, R, R, R, R, R, L, R, R, R, R, R

4. Based on this sample, estimate the percentage of students at the school who write with their left hand.

About 13.3%

5. Estimate the number of students at the school who write with their left hand.

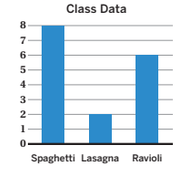
77 students

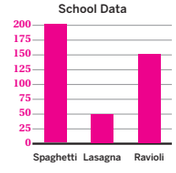
6. A different class of 18 students is surveyed. Estimate how many write with their left hand. Explain your thinking.

Responses and explanations vary. I estimate that 2 students write with their left hand because 13.3% of 18 is about 2.4 students and that number rounds down to 2.

7. 16 students at a school were asked about their favorite pasta dish. The results are shown in the bar graph. Create a new bar graph showing the possible results for all 400 students in the school. Be sure to scale and label the vertical axis.

Sample shown on graph.





Lesson Practice 8.11

Name: _____ Date: _____ Period: _____

FAST Practice

8. A gardener randomly selected 25 flowers from a garden with a total of 600 flowers. In the sample, 7 flowers were marigolds. Based on this sample, how many marigolds are likely in the entire garden?

A. 150
B. 168
C. 175
D. 200

Spiral Review

9. Match each expression in the first list with an equivalent expression from the second list.

Expression	Equivalent Expression
a. $(8x + 6y) - (2x + 4y)$	c. $10x - 10y$
b. $(8x + 6y) - (2x - 4y)$	d. $10x - 2y$
c. $8x - 6y - (-2x + 4y)$	a. $6x + 2y$
d. $8x - 6y - (-2x - 4y)$	b. $6x + 10y$

10. Complete the table so that each column has an equivalent fraction, decimal, and percent.

Fraction	$\frac{2}{5}$	$\frac{3}{4}$ (or equivalent)	$\frac{3}{10}$	$\frac{1}{8}$ (or equivalent)
Decimal	0.4	0.75	0.3	0.125
Percent	40%	75%	30%	12.5%

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.7.DP.1.3
	7	3	MA.7.DP.1.3
FAST Practice	8	2	MA.7.DP.1.3
Spiral Review			
	9	1	MA.7.AR.1.1
	10	2	MA.7.NSO.1.2

Unit 8

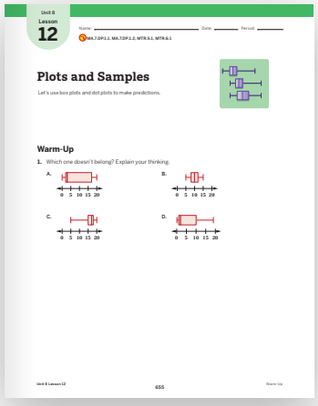
Lesson

12

Plots and Samples

Using Statistical Measures to Make Predictions

Let's use box plots and dot plots to make predictions.


Unit 8
Lesson
12
Plots and Samples
Let's use box plots and dot plots to make predictions.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
655

Learning Outcomes

Today's Goals

1. **Goal:** Estimate a measure of center of a population based on one or more samples.
2. **Language Goal:** Interpret box plots, dot plots, and lists of data that represent real-world situations. **(Reading, Writing, Speaking, and Listening)**

Students consider the variability between samples when drawing inferences about a population. They explore medians and *interquartile ranges (IQRs)* of box plots to make predictions about a population. Students decide whether multiple samples have measures of center that are alike or different and use this information to form claims about a population. They analyze data sets to determine the most appropriate measures of center and variability. **(MTR.5.1, MTR.6.1)**

Coherence

Prior Learning

In Lesson 11, students revisited proportional relationships and percentages to analyze samples and make predictions about a larger population.

Future Learning

In Lesson 13, students will compare two populations based on samples to determine whether there is a difference between the data sets.

Connections

- Students analyze box plots and dot plots to make perdictions on how long it takes students to get to school by car. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.1

Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.

MA.7.DP.1.2

Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	**MA.912.DP.2.1**
MA.6.DP.1.3	

Vocabulary

Review Vocabulary

interquartile range (IQR)	*shape*
skewed	*symmetrical (data)*

Additional Vocabulary

measure of center	*measure of variability*

Lesson at a Glance

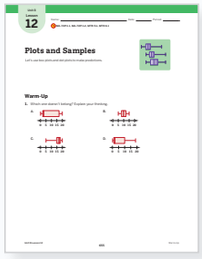
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MA.7.DP.1.1, MA.7.DP.1.2

Warm-Up

Pairs | 5 min

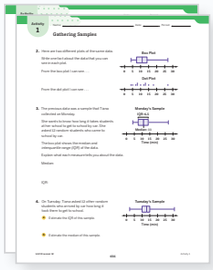
Students use the **Which One Doesn't Belong?** routine to compare the properties of four box plots and justify their thinking using precise language. **(MTR.5.1)**



Activity 1

Pairs | 14 min

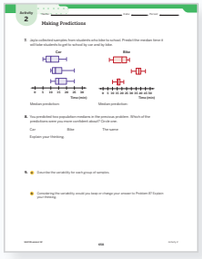
Students expand their prior knowledge of medians and *interquartile ranges (IQR)* by analyzing different samples represented by box plots and dot plots.



Activity 2

Pairs | 8 min

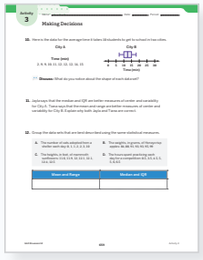
Students use data from multiple samples to draw inferences about a population then use the **Decide and Defend** routine to justify their confidence about the predictions. **(MTR.6.1)**



Activity 3

Pairs | 8 min

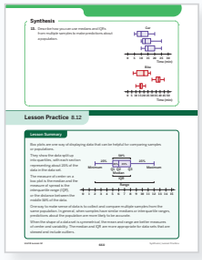
Students analyze data sets to determine the most appropriate measure of center and measure of variability. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

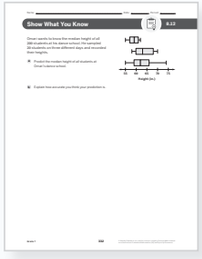
Students synthesize their understanding of using medians and interquartile ranges from multiple samples to make predictions about a population. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by predicting the median from multiple samples and determining the accuracy of their prediction.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to compare the properties of four box plots and justify their thinking using precise language. **(MTR.5.1)**

1 Launch

Use the Which One Doesn't Belong? routine to support students in analyzing properties of box plots and justifying their thinking precisely. **(MTR.5.1)**

Encourage students to look for more than one possibility.

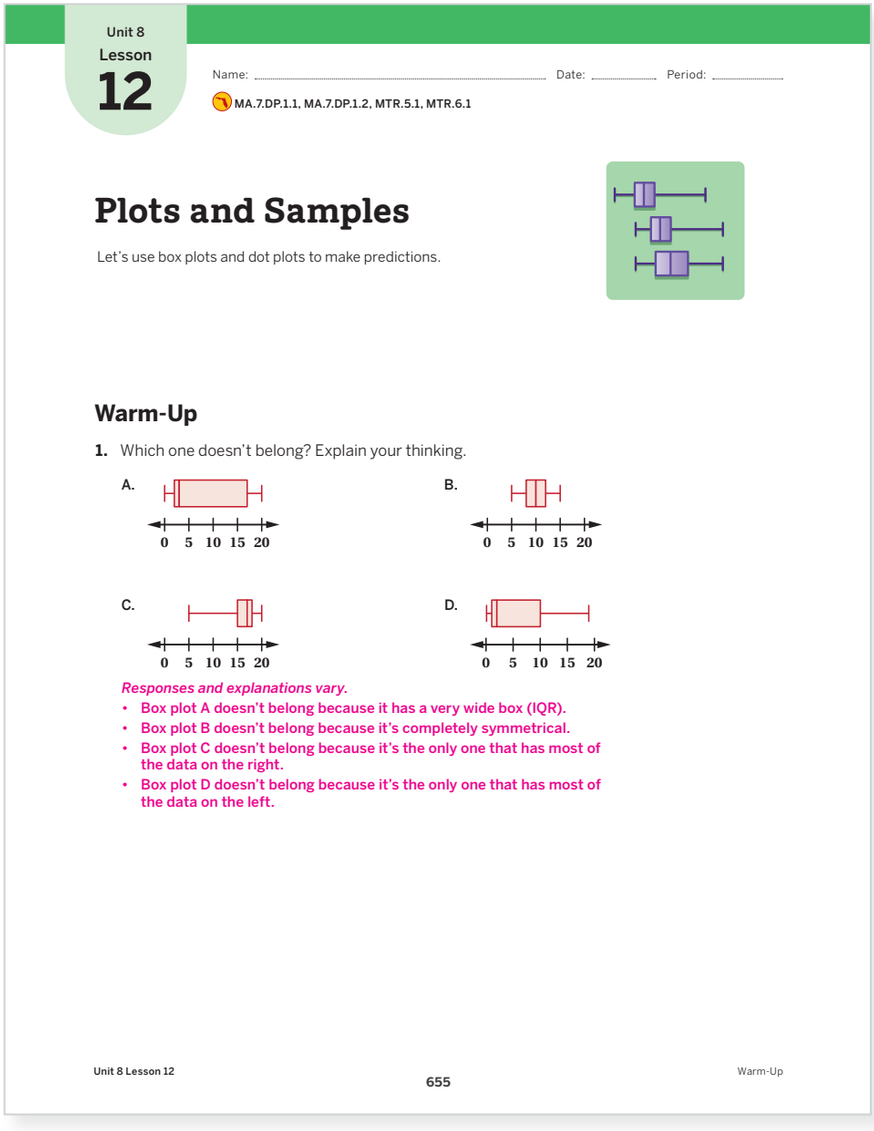
ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____ is the only one that has/doesn't have _____."). **(Writing and Speaking)**

2 Connect

Discuss why each representation doesn't belong, asking students to justify their choice.

Consider asking:

- "How is box plot A different from the other box plots?"
- "What is another way to say why box plot D doesn't belong?"



Unit 8
Lesson
12

Name: _____ Date: _____ Period: _____

MA.7.DP.1.1, MA.7.DP.1.2, MTR.5.1, MTR.6.1

Plots and Samples

Let's use box plots and dot plots to make predictions.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. 0 5 10 15 20

B. 0 5 10 15 20

C. 0 5 10 15 20

D. 0 5 10 15 20

Responses and explanations vary.

- Box plot A doesn't belong because it has a very wide box (IQR).
- Box plot B doesn't belong because it's completely symmetrical.
- Box plot C doesn't belong because it's the only one that has most of the data on the right.
- Box plot D doesn't belong because it's the only one that has most of the data on the left.

Unit 8 Lesson 12 655 Warm-Up

Pairs | 18 min

Activity 1 Gathering Samples

Purpose: Students expand their prior knowledge of medians and *interquartile ranges (IQR)* by analyzing different samples represented by box plots and dot plots.

Short on time? Consider completing the questions from Problem 2 as a whole class.

1 Launch

To surface prior knowledge, consider asking, "How can you compare samples when the data is displayed using two different types of representations?"

Direct students' attention to the box plot and dot plot to promote curiosity.

2 Monitor

To support students getting started, consider asking, "How is data on a box plot represented differently than data on a dot plot?"

Invite students to think independently about each measure before discussing their ideas with a partner.

Look for student responses that discuss the median of the box plot.

Consider asking, "What do you recall about medians and interquartile ranges (IQRs)?"

Create a class definition for the term *interquartile range (IQR)* using students' language and the glossary.

A **Accessibility: Visual-Spatial Processing** Add labels to the quartiles of the box plot to support processing visual information.

Encourage students to sketch to show their thinking.

Listen for opportunities to recognize the brilliance from students who have not been heard from recently.

D **Differentiation: (Support)** Students may think the measure of the IQR is determined by both the width of the box and the brackets. Consider inviting them to review the image in the previous problem for an example of the IQR.

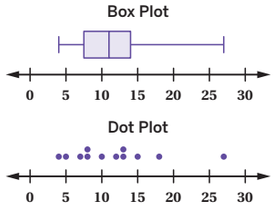
Activity 1 continued >

Activity 1

Name: Date: Period:

Gathering Samples

2. Here are two different plots of the same data.



Write one fact about the data that you can see in each plot.

From the box plot I can see . . .

Responses vary. The median appears to be about 11.

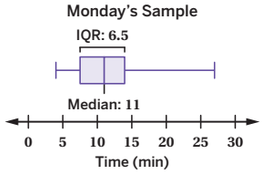
From the dot plot I can see . . .

Responses vary. The number of values in the data set.

3. The previous data was a sample that Tiana collected on Monday.

She wants to know how long it takes students at her school to get to school by car. She asked 12 random students who came to school by car.

The box plot shows the median and *interquartile range (IQR)* of the data.



Explain what each measure tells you about the data.

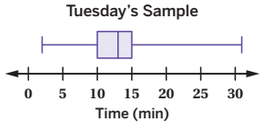
Median:

Responses vary. The median shows that about half of the students take 11 minutes or less to get to school by car and about half take 11 minutes or more.

IQR:

Responses vary. The IQR shows that about half of the students get to school within a 6.5-minute range between approximately 7.5 minutes and 14 minutes.

4. On Tuesday, Tiana asked 12 other random students who arrived by car how long it took them to get to school.



a. Estimate the IQR of this sample.
5 minutes

b. Estimate the median of this sample.
13 minutes

Unit 8 Lesson 12 656 Activity 1

Pairs | 18 min

Activity 1 Gathering Samples (continued)

Purpose: Students expand their prior knowledge of medians and *interquartile ranges (IQR)* by analyzing different samples represented by box plots and dot plots.

Short on time? Consider completing the questions from Problem 2 as a whole class.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the samples.

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

Listen for students who share strategies on how to match box plots and dot plots and how to determine the median and IQR from either representation. **(MTR.5.1)**

Select and sequence unique and creative student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to analyze the median and IQR of each box plot before comparing all three visually.
Identify the IQRs of the three box plots as the same size.	**Support** Encourage students to draw lines to connect the edges of the IQR boxes to the axes.
Notice that the IQRs of the three box plots are similar, but that each box plot has a different median.	**Strengthen** Invite students to compare their responses with a classmate.

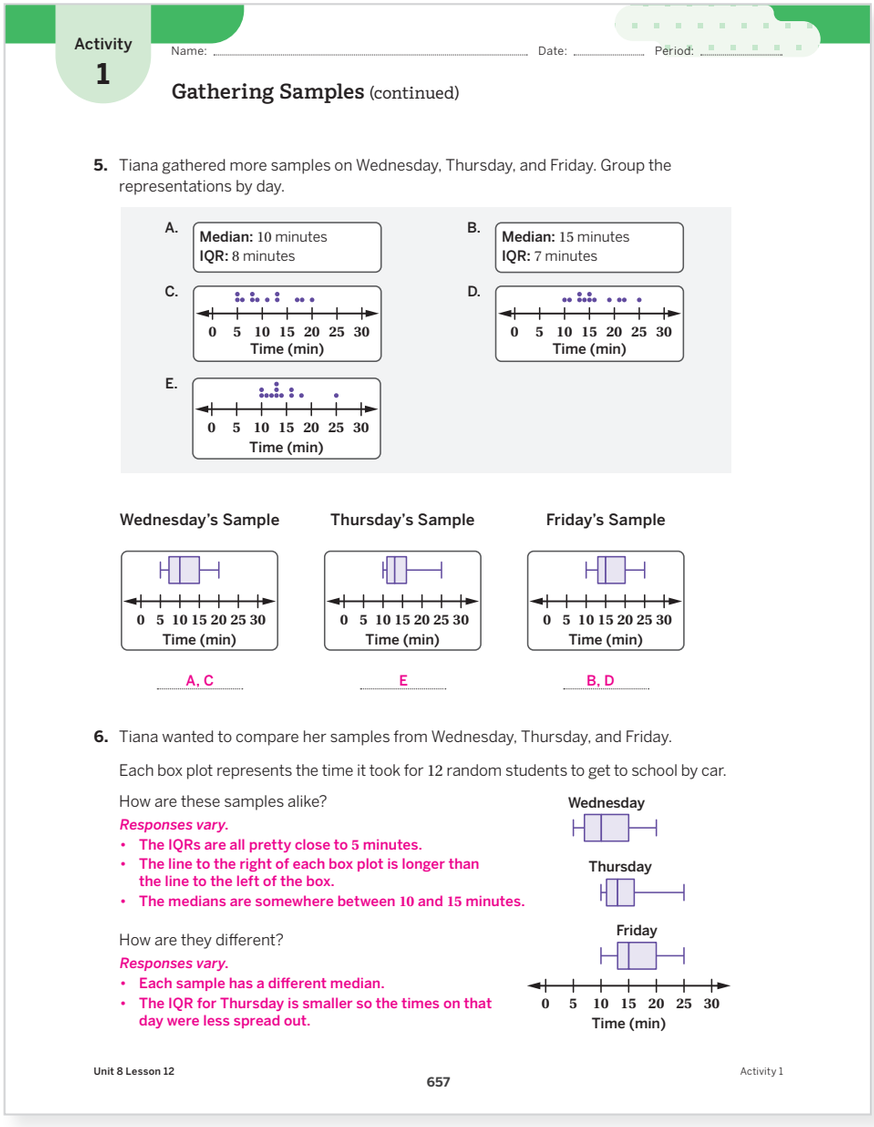
3 Connect

Invite students to share their strategies for grouping the different representations of Tiana's samples.

Consider asking, "How did you determine which box plots and dot plots represented data for the same day?"

Share student responses that show unique or creative ways of thinking.

To surface the Key Takeaway, consider asking, "How can you assess the accuracy of predictions made using the measures of center from multiple samples of a population?"



Activity 1

Name: Date: Period:

Gathering Samples (continued)

5. Tiana gathered more samples on Wednesday, Thursday, and Friday. Group the representations by day.

A. **Median:** 10 minutes **IQR:** 8 minutes

B. **Median:** 15 minutes **IQR:** 7 minutes

C.

D.

E.

Wednesday's Sample	Thursday's Sample	Friday's Sample
A, C	E	B, D

6. Tiana wanted to compare her samples from Wednesday, Thursday, and Friday.

Each box plot represents the time it took for 12 random students to get to school by car.

How are these samples alike?

Responses vary.

- The IQRs are all pretty close to 5 minutes.
- The line to the right of each box plot is longer than the line to the left of the box.
- The medians are somewhere between 10 and 15 minutes.

How are they different?

Responses vary.

- Each sample has a different median.
- The IQR for Thursday is smaller so the times on that day were less spread out.

Unit 8 Lesson 12 657 Activity 1

Key Takeaway: Predictions can be made about a population by comparing the visual overlap between the measures of center and variability of multiple samples. When medians and *IQRs* from multiple samples are closely aligned, similarities and differences in the data are easier to identify.

Pairs | 12 min

Activity 2 Making Predictions

Purpose: Students use data from multiple samples to draw inferences about a population then use the **Decide and Defend** routine to justify their confidence about the predictions. **(MTR.6.1)**

1 Launch

To surface connections, consider asking, "How can you use samples to make predictions about a population?"

Consider sharing an inaccurate location for the population median and invite students to explain why they believe the location would not be an accurate prediction.

2 Monitor

Invite students to continue to the next problem after they've discussed their median predictions with a partner.

Look for student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "In general, how can you tell when your predictions are more likely to be accurate?"
Are unsure about their confidence in their predictions.	**Support** Consider asking, "Which median prediction do you feel is a more accurate representation of the population?"

ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share them with others. **(Writing and Speaking)**

Consider asking, "How consistent is the IQR in each set of samples?"

3 Connect

Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.6.1)**

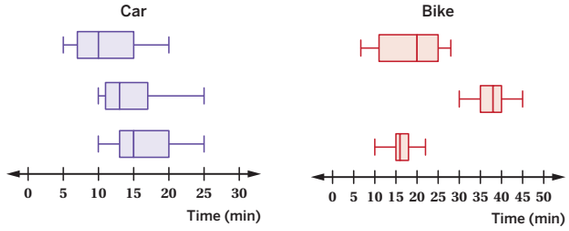
Consider asking, "How can the medians and IQRs from multiple samples help you to feel more confident that your prediction is accurate?"

Activity 2

Name: _____ Date: _____ Period: _____

Making Predictions

7. Jayla collected samples from students who bike to school. Predict the median time it will take students to get to school by car and by bike.



Median prediction:
Responses vary.

Median prediction:
Responses vary.

8. You predicted two population medians in the previous problem. Which of the predictions were you more confident about? Circle one.

Car Bike The same

Explain your thinking.

Responses and explanations vary.

- **The same. In both cases, I had three samples and I used the same approach. I used each sample's median and took the average.**
- **Car. The three car samples have medians and IQRs that are pretty consistent. The three bike samples vary quite a bit. That made me think that I have a better understanding of the car population than the bike population.**

9. a. Describe the variability for each group of samples.

***Responses vary.* The IQR for the car data is pretty consistent. The IQR for the bike data varies.**

b. Considering the variability, would you keep or change your answer to Problem 8? Explain your thinking.

Responses vary.

Unit 8 Lesson 12 658 Activity 2

Pairs | 8 min

Activity 3 Making Decisions

Purpose: Students analyze data sets to determine the most appropriate measure of center and measure of variability. **(MTR.5.1)**

1 Launch

Direct students' attention to the data sets.

To generate excitement, consider asking students how long it takes them to get to school each day, on average.

2 Monitor

Invite students to explore what is alike and different about the average times.

Listen for students using the terms *skewed*, *symmetrical*, and *outlier* in their responses to share during the Connect.

A **Accessibility: Visual-Spatial Processing** Invite students to create a data display for City A to help process the shape of the data.

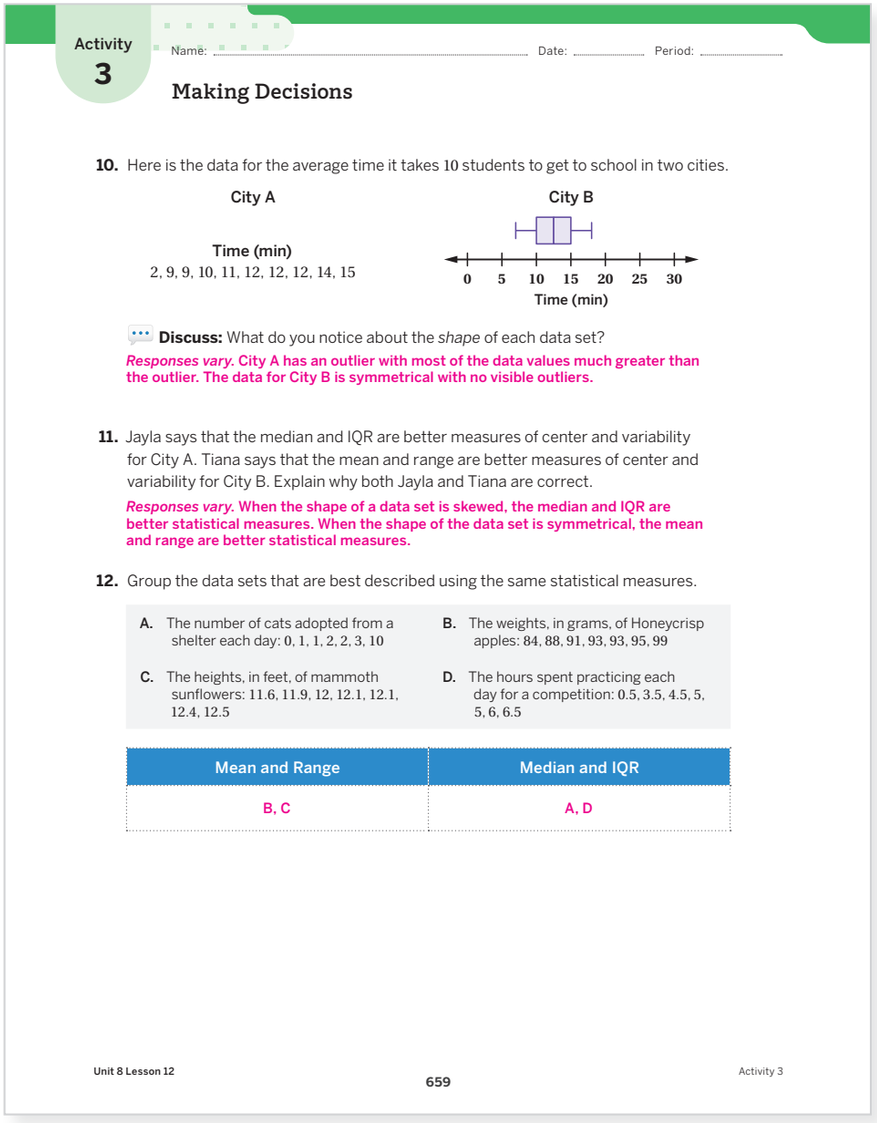
Consider asking, "How can the shape of the data help determine the most appropriate statistical measures to describe the data?" **(MTR.5.1)**

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the data sets.

3 Connect

Invite students to share their thinking about choosing the most appropriate measures of center and variability.

To surface the Key Takeaway, consider asking, "When is it appropriate to use the mean and range to describe a set of data? When is it more appropriate to use the median and IQR?"



Activity 3

Name: Date: Period:

Making Decisions

10. Here is the data for the average time it takes 10 students to get to school in two cities.

City A

Time (min)

2, 9, 9, 10, 11, 12, 12, 12, 14, 15

City B

Discuss: What do you notice about the *shape* of each data set?

Responses vary. City A has an outlier with most of the data values much greater than the outlier. The data for City B is symmetrical with no visible outliers.

11. Jayla says that the median and IQR are better measures of center and variability for City A. Tiana says that the mean and range are better measures of center and variability for City B. Explain why both Jayla and Tiana are correct.

Responses vary. When the shape of a data set is skewed, the median and IQR are better statistical measures. When the shape of the data set is symmetrical, the mean and range are better statistical measures.

12. Group the data sets that are best described using the same statistical measures.

A. The number of cats adopted from a shelter each day: 0, 1, 1, 2, 2, 3, 10

B. The weights, in grams, of Honeycrisp apples: 84, 88, 91, 93, 93, 95, 99

C. The heights, in feet, of mammoth sunflowers: 11.6, 11.9, 12, 12.1, 12.1, 12.4, 12.5

D. The hours spent practicing each day for a competition: 0.5, 3.5, 4.5, 5, 5, 6, 6.5

Mean and Range	Median and IQR
B, C	A, D

Unit 8 Lesson 12 659 Activity 3

Key Takeaway: The *shape* of a data set, along with the context of a problem, can help determine the most appropriate measure of center and measure of variability for the data.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using medians and interquartile ranges from multiple samples to make predictions about a population.

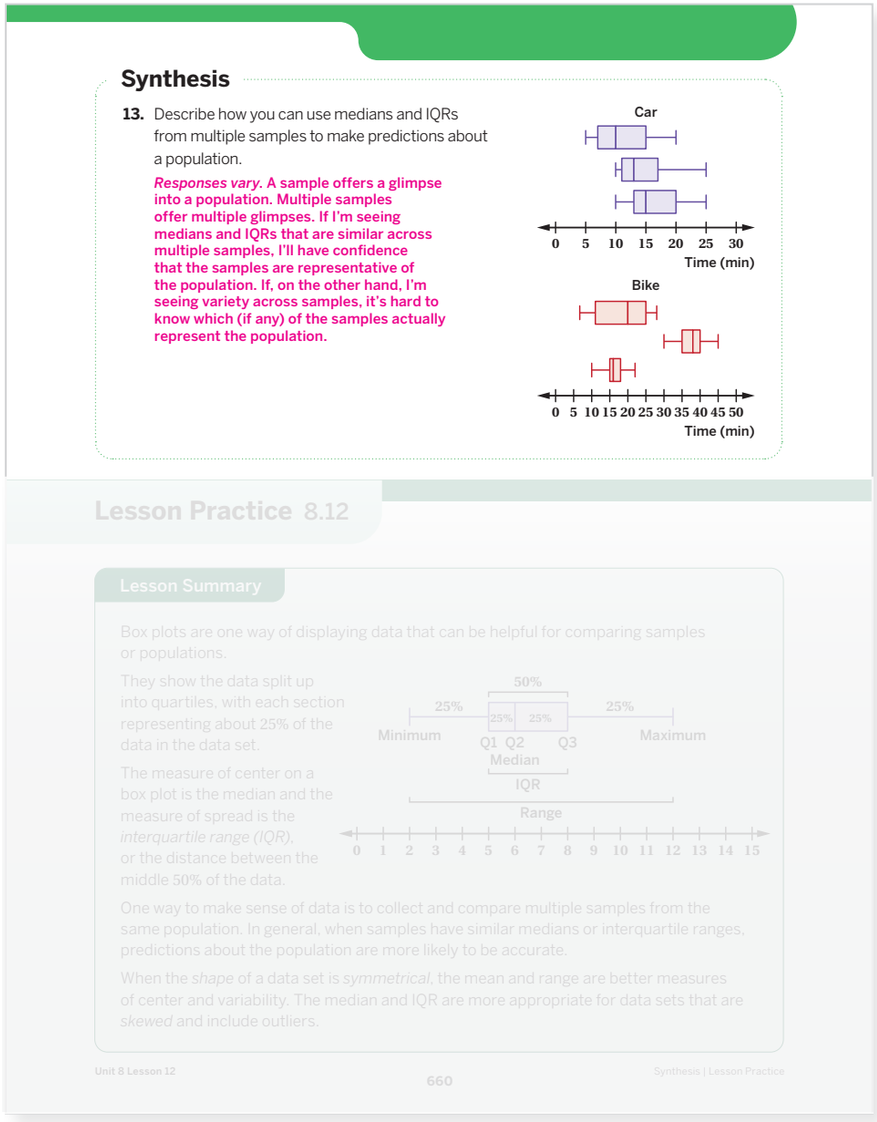
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using multiple samples with similar medians and IQRs to make predictions provides a higher level of confidence in the prediction.
- Predictions made about a population when the medians and IQRs of multiple samples are very different are less likely to be accurate.
- The shape of a data set can help determine whether the median and IQR are appropriate measures of center and variability.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *interquartile range* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. Describe how you can use medians and IQRs from multiple samples to make predictions about a population.

Responses vary. A sample offers a glimpse into a population. Multiple samples offer multiple glimpses. If I'm seeing medians and IQRs that are similar across multiple samples, I'll have confidence that the samples are representative of the population. If, on the other hand, I'm seeing variety across samples, it's hard to know which (if any) of the samples actually represent the population.

Lesson Practice 8.12

Lesson Summary

Box plots are one way of displaying data that can be helpful for comparing samples or populations.

They show the data split up into quartiles, with each section representing about 25% of the data in the data set.

The measure of center on a box plot is the median and the measure of spread is the *interquartile range (IQR)*, or the distance between the middle 50% of the data.

One way to make sense of data is to collect and compare multiple samples from the same population. In general, when samples have similar medians or interquartile ranges, predictions about the population are more likely to be accurate.

When the *shape* of a data set is *symmetrical*, the mean and range are better measures of center and variability. The median and IQR are more appropriate for data sets that are *skewed* and include outliers.

Unit 8 Lesson 12 660 Synthesis | Lesson Practice

Lesson Takeaway: Comparing the medians and *interquartile ranges* from multiple samples is useful for making estimates about a population. When the *shape* of a data set is *skewed* or includes an outlier, the median and IQR may be the best measures of center and variability. The accuracy of a prediction made about a population often depends on whether the medians and IQRs of the samples are alike or different.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by predicting the median from multiple samples and determining the accuracy of their prediction.

Today's Goals

1. **Goal:** Estimate a measure of center of a population based on one or more samples.
2. **Language Goal:** Interpret box plots, dot plots, and lists of data that represent real-world situations. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

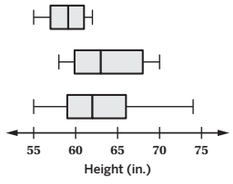
Name: _____ Date: _____ Period: _____

Show What You Know 8.12

Omari wants to know the median height of all 200 students at his dance school. He sampled 20 students on three different days and recorded their heights.

a. Predict the median height of *all* students at Omari's dance school.

Responses between 58 and 63 are considered correct.



b. Explain how accuracy you think your prediction is.

***Explanations vary.* Because the sample medians and IQRs are all similar, I think my prediction is pretty accurate.**

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of estimate the median from a box plot:

» **Respond to Student Thinking:**

- Revisit the launch of **Activity 2** as a class and invite students to share strategies for estimating a population's measure of center based on samples.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Consider the next lesson you will teach. How will students' work today in "Gathering Samples" prepare them for Lesson 13, where they will make comparisons between samples based on visual and quantitative differences in the measures of center?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. Describe how you can use medians and IQRs from multiple samples to make predictions about a population.

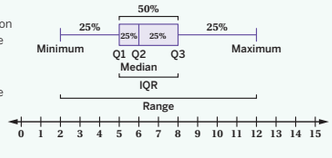
Responses vary. A sample offers a glimpse into a population. Multiple samples offer multiple glimpses. If I'm seeing medians and IQRs that are similar across multiple samples, I'll have confidence that the samples are representative of the population. If, on the other hand, I'm seeing variety across samples, it's hard to know which (if any) of the samples actually represent the population.

Lesson Practice 8.12

Lesson Summary

Box plots are one way of displaying data that can be helpful for comparing samples or populations.

They show the data split up into quartiles, with each section representing about 25% of the data in the data set.



The measure of center on a box plot is the median and the measure of spread is the *interquartile range (IQR)*, or the distance between the middle 50% of the data.

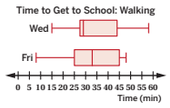
One way to make sense of data is to collect and compare multiple samples from the same population. In general, when samples have similar medians or interquartile ranges, predictions about the population are more likely to be accurate.

When the *shape* of a data set is *symmetrical*, the mean and range are better measures of center and variability. The median and IQR are more appropriate for data sets that are *skewed* and include outliers.

Lesson Practice 8.12

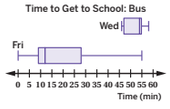
Name: _____ Date: _____ Period: _____

Problems 1–2: Kai wants to understand how long it takes for students to get to school using different methods. On Wednesday, Kai surveyed random samples of students who walked and students who took the bus. He repeated this on Friday.



1. Estimate the median time it takes to get to school by each method.

Walking: **Responses between 30 and 35 minutes are considered correct.**



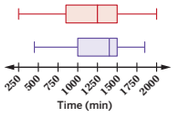
Bus: **Responses between 20 and 50 minutes are considered correct.**

2. Which estimate is more likely to be accurate? **Walking**

Explain your thinking.

***Explanations vary.* There is more variability between the samples of the time it took students to get to school by bus.**

Problems 3–5: A video game developer wants to know how long it takes people to finish playing a new game. She surveyed two random samples of 20 players and asked how long it took each of them.



3. Estimate the median time it takes a player to finish the game.

Responses between 1,250 and 1,400 minutes are considered correct.

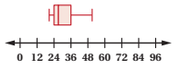
4. How close do you think your estimate is to the actual median time?

***Responses vary.* I expect my estimate to be close to the actual median time because the median and IQRs for each sample are similar.**

5. How could the developer change her sampling strategy to make you more confident that the data is representative of the population?

***Responses vary.* To get more representative samples, the developer could increase the size of each sample.**

6. Determine the best measure of center and measure of variability for the data set. Explain your thinking.



***Responses vary.* The box plot is skewed and appears to include an outlier, so the best measure of center is the median and the best measure of variability is the IQR.**

Lesson Practice 8.12

Name: _____ Date: _____ Period: _____

FAST Practice

7. The data shows the heights, in inches, of a team of volleyball players.

Heights of Volleyball Teammates (in.)

70, 70.5, 71, 71.5, 72, 72, 73

If the team adds a new player with a height of 71 inches, will the most appropriate statistical measures change? Explain your thinking.

The original data set is (A.) skewed B. fairly symmetrical, so the best measure of center is the (A.) mean B. median and the best measure of variability is the (A.) range B. IQR. The new player's height A. is (B.) is not an outlier, so the most appropriate statistical measures A. will (B.) will not change.

Spiral Review

8. Complete the table so that each column has an equivalent fraction, decimal, and percent.

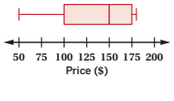
Fraction	$\frac{4}{5}$	$\frac{2}{3}$ (or equivalent)	$\frac{3}{8}$	$\frac{1}{100}$ (or equivalent)
Decimal	0.8	$0.\overline{6}$	0.375	0.01
Percent	80%	$66.\overline{6}\%$	37.5%	1%

9. The recipe for a signature pizza uses 3 cups of cheese, $\frac{3}{4}$ cup of olives, and $1\frac{1}{2}$ cups of onions. How many cups of onions are needed to make 10 signature pizzas?

A. 10 cups B. 30 cups C. $7\frac{1}{2}$ cups (D.) 15 cups

Problems 10–11: Determine the median and interquartile range (IQR) for each box plot.

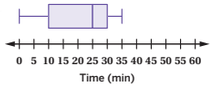
10.



Median: **150 dollars**

IQR: **75 dollars**

11.



Median: **25 minutes**

IQR: **20 minutes**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.DP.1.2
	6	2	MA.7.DP.1.1
FAST Practice	7	3	MA.7.DP.1.1
Spiral Review			
	8	1	MA.7.NSO.1.2
	9	2	MA.7.NSO.2.2
	10–11	2	MA.6.DP.1.3

Unit 8

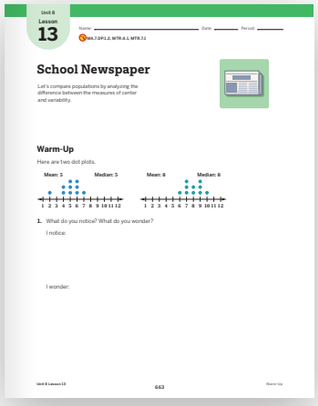
Lesson

13

School Newspaper

Comparing Populations

Let's compare populations by analyzing the difference between the measures of center and variability.



Learning Outcomes

Today's Goals

1. **Goal:** Decide whether two populations are very different using measures of center and variability.
2. **Language Goal:** Justify whether a claim is supported based on visual and quantitative differences in the measures of center and variability. **(Reading, Writing, Speaking, and Listening)**

Students compare two populations based on samples. They are introduced to the idea of deciding if there is a big difference between two populations by looking at the difference between the centers of the samples and comparing the difference to the measures of variability.

Coherence

Prior Learning

In Lesson 12, students used data from samples to draw conclusions about a population by estimating a measure of center.

Future Learning

In Grade 8 Unit 6, students will analyze numerical data in scatter plots and describe patterns of associations.

Connections

- Students use a school survey to compare populations, analyze the difference between measures of center and variability. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.2

Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	MA.8.DP.1.2 MA.912.DP.2.1

Lesson at a Glance

~ 45 min

MA.7.DP.1.2

Warm-Up

Small Groups | 5 min

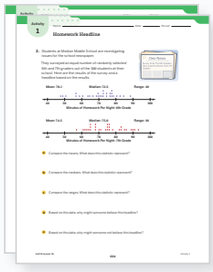
Students use the **Notice and Wonder** routine to make observations of two data sets represented on dot plots.



Activity 1

Small Groups | 20 min

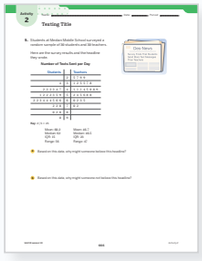
Students analyze data to determine the difference between two data sets by comparing their measures of center and variability.



Activity 2

Small Groups | 10 min

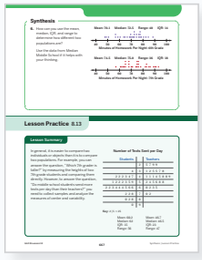
Students decide whether a headline is believable by comparing the difference between the means, medians, IQRs, and ranges of two data sets.
MLR1: Stronger and Clearer Each Time (MTR.7.1)



Synthesis

Whole Class | 5 min

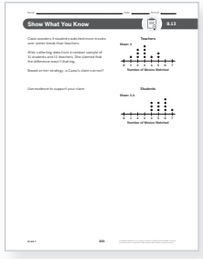
Students synthesize their understanding of using statistical measures to decide how different two data sets are.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing two different data sets from samples to decide whether they support a claim.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Small Groups | 5 min

Warm-Up

Purpose: Students use the **Notice and Wonder** routine to make observations of two data sets represented on dot plots.

1 Launch

Display the dot plots.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the data.

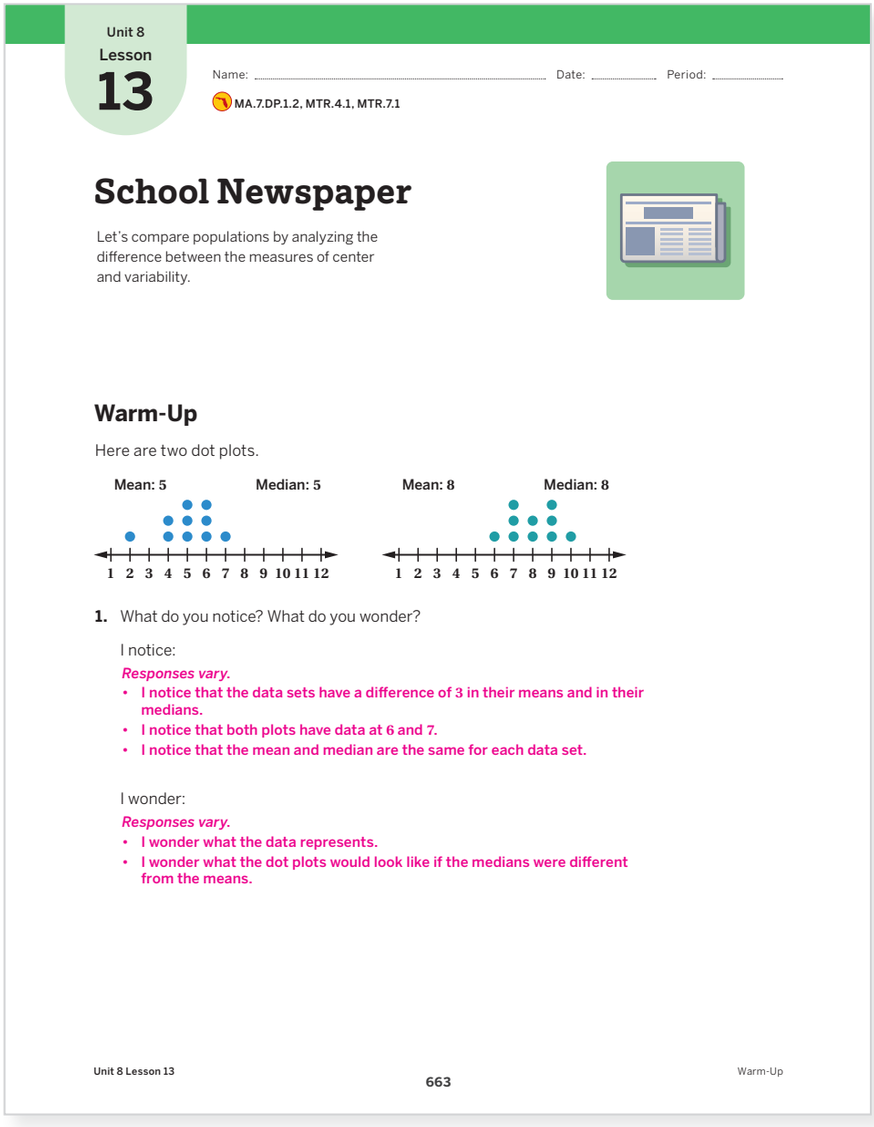
2 Connect

Invite students to share their observations.

Create a record of the things students noticed and wondered, along with the name of the students who shared them.

Note: If it does not come up naturally during the discussion, consider asking:

- "What could these dot plots represent?"
- "What do you notice about the difference in the means of both data sets? The medians?"
- "How much overlap is there between the points in the data sets?"
- "What do you notice about the spread of the data?"



Unit 8
Lesson
13

Name: ..... Date: ..... Period: .....

MA.7.DP.1.2, MTR.4.1, MTR.7.1

School Newspaper

Let's compare populations by analyzing the difference between the measures of center and variability.

Warm-Up

Here are two dot plots.

Mean: 5 Median: 5

Mean: 8 Median: 8

1. What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice that the data sets have a difference of 3 in their means and in their medians.
- I notice that both plots have data at 6 and 7.
- I notice that the mean and median are the same for each data set.

I wonder:

Responses vary.

- I wonder what the data represents.
- I wonder what the dot plots would look like if the medians were different from the means.

Unit 8 Lesson 13 663 Warm-Up

Small Groups | 20 min

Activity 1 Homework Headline

Purpose: Students analyze data to determine the difference between two data sets by comparing their measures of center and variability.

Short on time? Consider completing Problems 2a and 2b as a whole class.

1 Launch

Direct students' attention to the headline and the dot plots. Consider asking, "Do you think 6th graders or 7th graders have more homework?"

Invite students to share their thinking.

2 Monitor

Select and sequence student ideas to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problems 2a–2c)*	**Support** Consider asking, "How do you calculate the mean, median, and range?"
Describe the spread of data to justify their reasoning. *(Problems 2d–2e)*	**Strengthen** Invite them to share how the graphs can help someone decide whether they should believe the headline during the Connect.
Describe the overlap between data to justify their reasoning. *(Problems 2d–2e)*	

A Accessibility: Memory and Attention Provide students with an anchor chart that describes the meaning of mean, median, IQR, and range, and how to calculate them for students to refer to throughout the lesson.

Listen for explanations about why someone might and might not believe the headline in Problem 2 to share during the Connect.

Activity 1 continued >

Activity 1

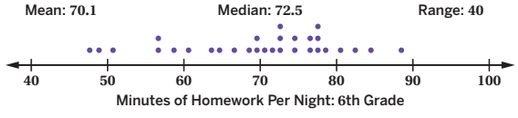
Name: Date: Period:

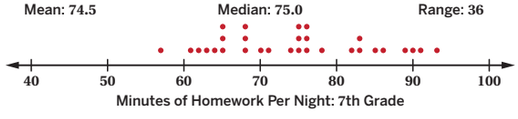
Homework Headline

2. Students at Median Middle School are investigating issues for the school newspaper.

They surveyed an equal number of randomly selected 6th and 7th graders out of the 500 students at their school. Here are the results of the survey and a headline based on the results.







a. Compare the means. What does this statistic represent?

Responses vary. A 6th grader has 70.1 minutes of homework and a 7th grader has 74.5 minutes of homeworkper night on average.

b. Compare the medians. What does this statistic represent?

About half of 6th graders spend 72.5 minutes or less on homework. About half of 7th graders spend 75.0 minutes or more on homework.

c. Compare the ranges. What does this statistic represent?

Responses vary. The range indicates how spread out the data is. The ranges are similar, but the 6th grade data set has slightly more variability in the data than the 7th grade data set.

d. Based on this data, why might someone believe this headline?

Responses vary. Someone might believe this headline because 6th graders have more dots on the left side of the dot plot, which shows less homework.

e. Based on this data, why might someone not believe this headline?

Responses vary. Someone might not believe this headline because there is a lot of overlap between the dots in each data set.

Unit 8 Lesson 13 664 Activity 1

Activity 1 Homework Headline (continued)

Purpose: Students analyze data to determine the difference between two data sets by comparing their measures of center and variability.

2 Monitor

Select and sequence explanations to share during the Connect.

D Differentiation *(Problems 3–4)*

Look for students who:	Respond to Student Thinking
Use the ranges to support their claim. (*Problem 3*)	**Strengthen** Invite them to share how the ranges helped them make their decision.
Reason that there is a big difference between the data sets. (*Problem 4*)	
Would like to further explore the difference in populations.	**Stretch** Consider asking, "Do you think people who write news articles use strategies similar to Ama's before writing headlines?"

3 Connect

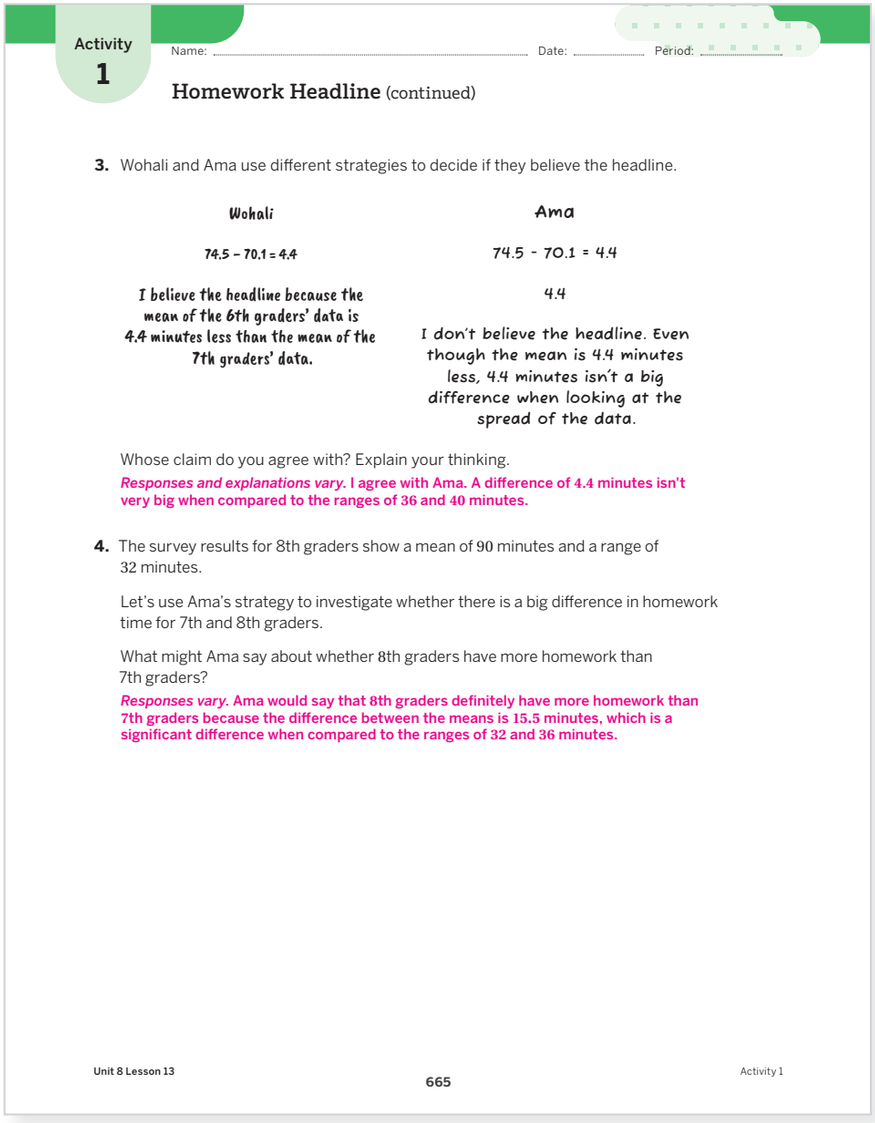
Discuss Ama's and Wohali's strategies.

Consider asking:
- "Whose strategy do you think makes more sense for this situation? Why?"
- "After analyzing each strategy, do you believe the headline in Problem 2? Why?"

Direct students' attention to the headline and data in Problem 2. Revisit their answers from the Launch to surface a connection between Ama's and Wohali's strategies and students' original answer choice. **(MTR.4.1)**

Invite students to share their responses for Problem 4 and their thinking behind it.

To surface the Key Takeaway, consider asking:
- "How does the range help you determine whether the difference in means is significant?"
- "Based on your calculations, what conclusions can you draw? How does this support Ama's claim?"



Activity 1

Name: Date: Period:

Homework Headline (continued)

3. Wohali and Ama use different strategies to decide if they believe the headline.

Wohali

74.5 − 70.1 = 4.4

I believe the headline because the mean of the 6th graders' data is 4.4 minutes less than the mean of the 7th graders' data.

Ama

74.5 - 70.1 = 4.4

4.4

I don't believe the headline. Even though the mean is 4.4 minutes less, 4.4 minutes isn't a big difference when looking at the spread of the data.

Whose claim do you agree with? Explain your thinking.

Responses and explanations vary. I agree with Ama. A difference of 4.4 minutes isn't very big when compared to the ranges of 36 and 40 minutes.

4. The survey results for 8th graders show a mean of 90 minutes and a range of 32 minutes.

Let's use Ama's strategy to investigate whether there is a big difference in homework time for 7th and 8th graders.

What might Ama say about whether 8th graders have more homework than 7th graders?

Responses vary. Ama would say that 8th graders definitely have more homework than 7th graders because the difference between the means is 15.5 minutes, which is a significant difference when compared to the ranges of 32 and 36 minutes.

Unit 8 Lesson 13 665 Activity 1

Key Takeaway: One way to compare two samples is by subtracting the means of the populations and comparing the result to the ranges of the samples. A difference in means of 15.5 might be much more significant than a difference in means of 4.4 depending on the range of data values.

Small Groups | 10 min

Activity 2 Texting Title

Purpose: Students decide whether a headline is believable by comparing the difference between the means, medians, IQRs, and ranges of two data sets. **(MTR.7.1)**

1 Launch

To generate excitement, share with students that they will use calculations to compare the number of texts students and teachers send each day. **(MTR.7.1)**

Direct students' attention to the headline and the stem-and-leaf plot. Consider asking, "Who do you think sends more text messages?"

2 Monitor

Select and sequence student reasoning to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Use a variety of strategies, such as: • Compare the means and medians. • Compare the spread of the data. • Mention sample size.	**Strengthen** Invite them to share how these strategies and ideas help in deciding whether this headline might or might not be believable.
Would like to further explore the difference in populations.	**Stretch** Consider asking, "When might you use median and IQR instead of mean and range?"

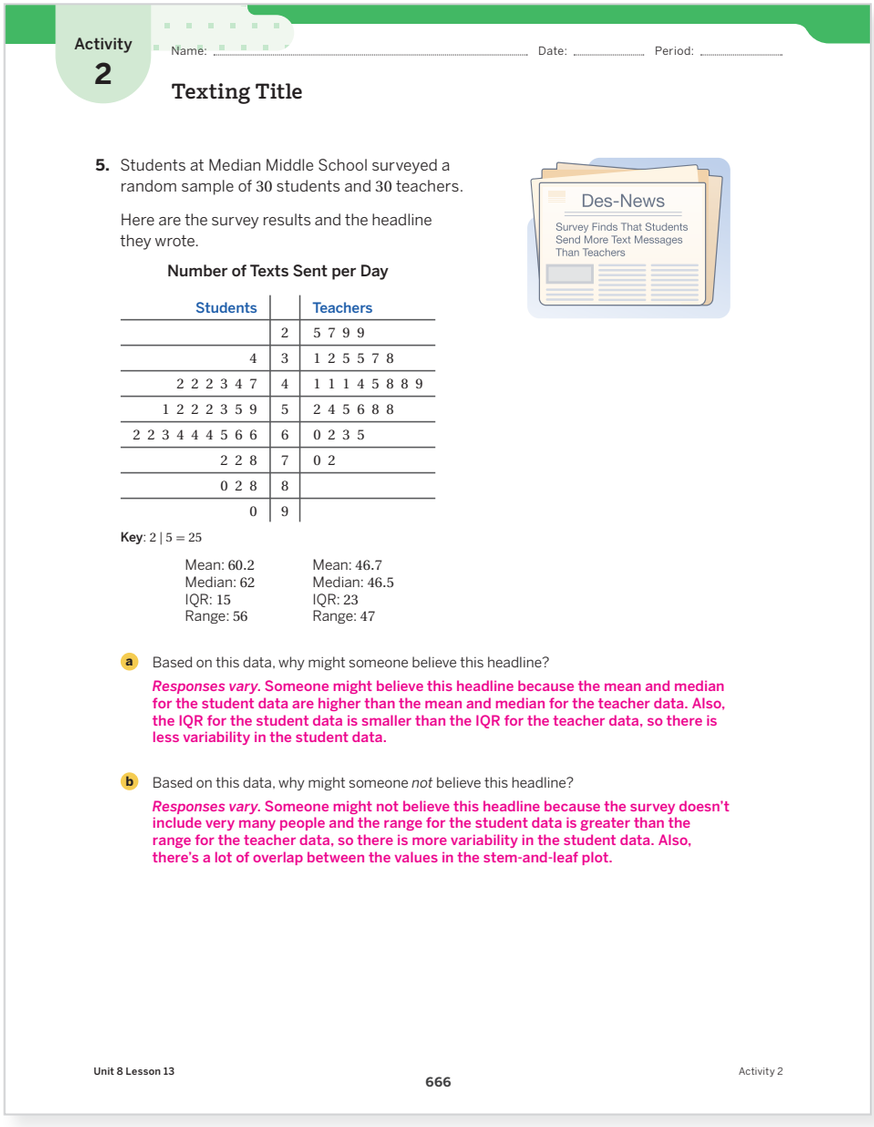
3 Connect

Invite students to share their responses from Problem 5 and if their original prediction about who sends more text messages was accurate.

MLR MLR1: Stronger and Clearer Each Time

Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Consider asking, "How might sample size affect the results of a survey?"



Activity 2

Name: Date: Period:

Texting Title

5. Students at Median Middle School surveyed a random sample of 30 students and 30 teachers.

Here are the survey results and the headline they wrote.

Number of Texts Sent per Day

Students		Teachers
	2	5 7 9 9
4	3	1 2 5 5 7 8
2 2 2 3 4 7	4	1 1 1 4 5 8 8 9
1 2 2 2 3 5 9	5	2 4 5 6 8 8
2 2 3 4 4 4 5 6 6	6	0 2 3 5
2 2 8	7	0 2
0 2 8	8	
0	9	

Key: 2 \| 5 = 25

Mean: 60.2	Mean: 46.7
Median: 62	Median: 46.5
IQR: 15	IQR: 23
Range: 56	Range: 47

a. Based on this data, why might someone believe this headline?

Responses vary. Someone might believe this headline because the mean and median for the student data are higher than the mean and median for the teacher data. Also, the IQR for the student data is smaller than the IQR for the teacher data, so there is less variability in the student data.

b. Based on this data, why might someone *not* believe this headline?

Responses vary. Someone might not believe this headline because the survey doesn't include very many people and the range for the student data is greater than the range for the teacher data, so there is more variability in the student data. Also, there's a lot of overlap between the values in the stem-and-leaf plot.

Unit 8 Lesson 13 666 Activity 2

Whole Class | 5 min

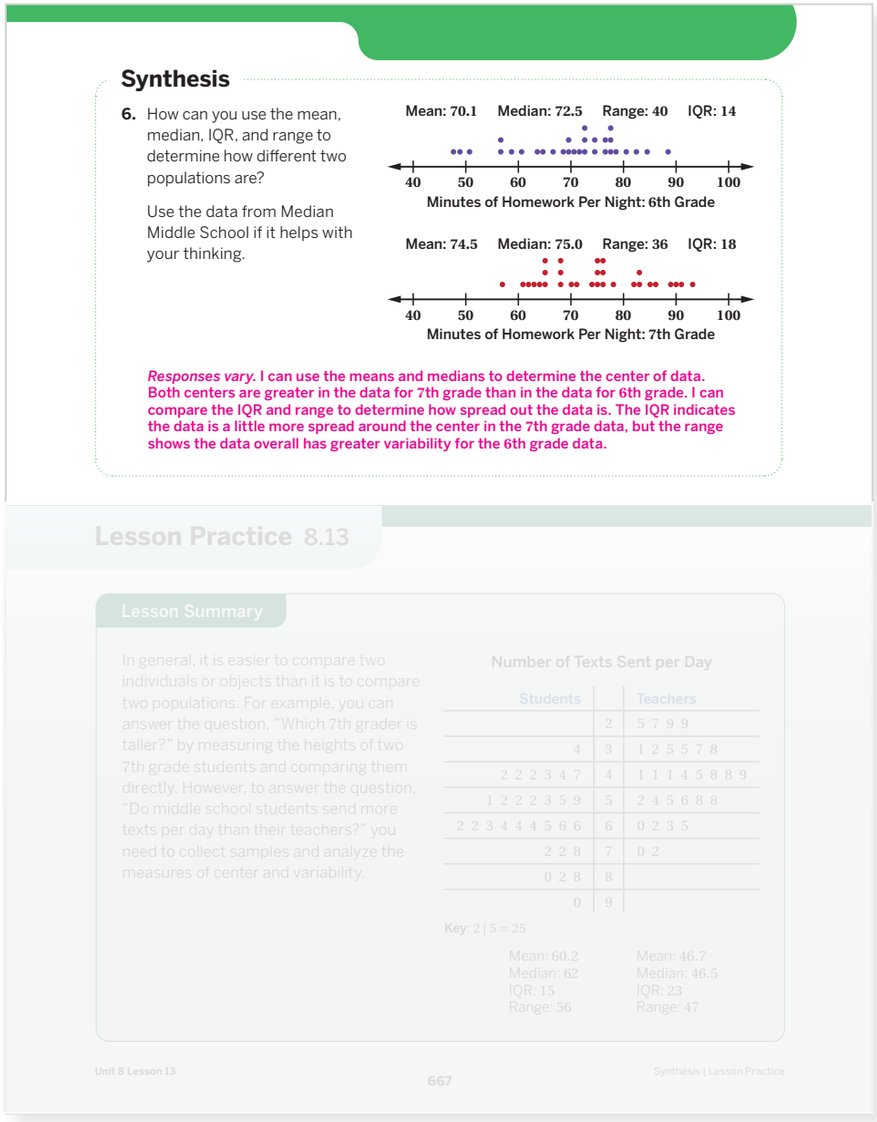
Synthesis

Purpose: Students synthesize their understanding of using statistical measures to decide how different two data sets are.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Comparing the means and medians.
- Analyzing the spread of the data.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

6. How can you use the mean, median, IQR, and range to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.

Responses vary. I can use the means and medians to determine the center of data. Both centers are greater in the data for 7th grade than in the data for 6th grade. I can compare the IQR and range to determine how spread out the data is. The IQR indicates the data is a little more spread around the center in the 7th grade data, but the range shows the data overall has greater variability for the 6th grade data.

Lesson Practice 8.13

Lesson Summary

In general, it is easier to compare two individuals or objects than it is to compare two populations. For example, you can answer the question, "Which 7th grader is taller?" by measuring the heights of two 7th grade students and comparing them directly. However, to answer the question, "Do middle school students send more texts per day than their teachers?" you need to collect samples and analyze the measures of center and variability.

Number of Texts Sent per Day

Students		Teachers
	2	5 7 9 9
4	3	1 2 5 5 7 8
2 2 2 3 4 7	4	1 1 1 4 5 8 8 9
1 2 2 2 3 5 9	5	2 4 5 6 8 8
2 2 3 4 4 4 5 6 6	6	0 2 3 5
2 2 8	7	0 2
0 2 8	8	
0	9	

Key: 2 | 5 = 25

Mean: 60.2
Median: 62
IQR: 15
Range: 56

Mean: 46.7
Median: 46.5
IQR: 23
Range: 47

Unit 8 Lesson 13 667 Synthesis | Lesson Practice

Lesson Takeaway: To calculate the differences between two data sets, first calculate the differences in the measures of center. Then compare the variability and consider the sample size of the data sets.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing two different data sets from samples to decide whether they support a claim.

Today's Goals

1. **Goal:** Decide whether two populations are very different using measures of center and variability.
2. **Language Goal:** Justify whether a claim is supported based on visual and quantitative differences in the measures of center and variability. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.13

Caasi wonders if students watched more movies over winter break than teachers.

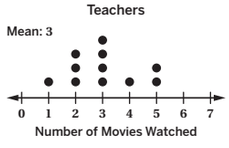
After collecting data from a random sample of 11 students and 11 teachers, She claimed that the difference wasn't that big.

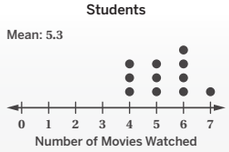
Based on her strategy, is Caasi's claim correct?

No

Use evidence to support your claim.

Explanations vary. Caasi's claim is incorrect because the difference between the means is $5.3 - 3 = 2.3$. There is a small difference between the number of movies watched by each population.





D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to compare populations using mean and median:

» **Respond to Student Thinking:**

- Revisit the launch of **Activity 2** as a class and invite students to share strategies for estimating a population's measure of center based on samples.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

How did you create learning opportunities when students needed support interpreting the measures of center and variability to compare two data sets and draw conclusions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. How can you use the mean, median, IQR, and range to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.

Responses vary. I can use the means and medians to determine the center of data. Both centers are greater in the data for 7th grade than in the data for 6th grade. I can compare the IQR and range to determine how spread out the data is. The IQR indicates the data is a little more spread around the center in the 7th grade data, but the range shows the data overall has greater variability for the 6th grade data.

Lesson Practice 8.13

Lesson Summary

In general, it is easier to compare two individuals or objects than it is to compare two populations. For example, you can answer the question, "Which 7th grader is taller?" by measuring the heights of two 7th grade students and comparing them directly. However, to answer the question, "Do middle school students send more texts per day than their teachers?" you need to collect samples and analyze the measures of center and variability.

Number of Texts Sent per Day

Students		Teachers
	2	5 7 9 9
4	3	1 2 5 5 7 8
2 2 2 3 4 7	4	1 1 1 4 5 8 8 9
1 2 2 2 3 5 9	5	2 4 5 6 8 8
2 2 3 4 4 4 5 6 6	6	0 2 3 5
2 2 8	7	0 2
0 2 8	8	
0	9	

Key: 2 | 5 = 25

Mean: 60.2
Median: 62
IQR: 15
Range: 56

Mean: 46.7
Median: 46.5
IQR: 23
Range: 47

Lesson Practice 8.13

Name: _____ Date: _____ Period: _____

Problems 1–3: A school's art club held a fundraiser to raise money for art supplies. This data shows the number of T-shirts sold each week during each season.

Fall

Stem	Leaf
1	9 9
2	0 0 4 4 4 5 6 9

Key: 1 | 9 = 19

Spring

Stem	Leaf
1	9
2	1 1 2 5 5 5 6 7 9

Key: 1 | 9 = 19

1. Calculate the mean number of T-shirts sold in the fall and in the spring.

Fall: 23 T-shirts; Spring: 24 T-shirts

2. Calculate the range of the number of T-shirts sold in the fall and in the spring.

Fall: 10 T-shirts; Spring: 10 T-shirts

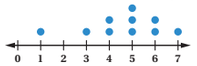
3. Based on this data, are sales generally higher in the spring than in the fall? Explain your thinking.

No. *Explanations vary.* There isn't much of a difference between the spring and fall sales because the ranges are the same and the means are very close together.

Problems 4–5: Abdullah compared the time that students and teachers spent watching television over the weekend. He took a random sample of 10 students and 10 teachers and made a dot plot of their responses.

Students

Mean: 4.6 Median: 5 Range: 6 IQR: 2



Teachers

Mean: 3.8 Median: 3.5 Range: 5 IQR: 1



4. Is there a big difference between the students' data and the teachers' data?

No

Explain your thinking.

***Explanations vary.* Teachers do watch less television than students, but the difference in the means (0.8 hours) and the difference in the medians (1.5 hours) are not very large. The IQRs and ranges are small, so there the data is pretty consistent for both groups.**

5. Abdullah then took a random sample of 10 parents. The mean was 2.6 hours, the median was 2.5 hours, the IQR was 1 hour, and the range was 5 hours.

Is there a big difference between how much television parents and students watch? Explain your thinking.

Yes. *Explanations vary.* The difference in the means is 2 hours, so parents watch 2 fewer hours of television each night on average. The median time for parents is half the median time for students. The IQR (1) and range (5) are small, so the data is consistent.

Lesson Practice 8.13

Name: _____ Date: _____ Period: _____

FAST Practice

6. These dot plots represent the time that it took new runners to run a mile.

Which statement is *best* supported by the information in the dot plots?

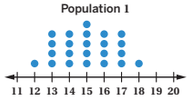
A. The two populations have an equal number of runners.

B. The two populations have the same range.

C. The two populations have different medians.

D. The two populations have the same mean.

Population 1



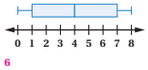
Population 2



Spiral Review

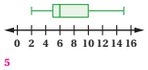
Problems 7–9: Determine the interquartile range (IQR) for each box plot.

7.



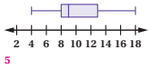
6

8.



5

9.



5

10. Sora says that 0.77 is a *repeating decimal* because both digits are the same. Is Sora's statement correct? Explain your thinking.

No. *Explanations vary.* Even though both digits in 0.77 are the same, it's not a repeating decimal because it stops. Repeating decimals have a pattern, but they're called repeating because they go on forever.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.DP.1.2
FAST Practice	6	2	MA.7.DP.1.2
Spiral Review			
Fluency	7–9	1	MA.6.DP.1.3
	10	2	MA.7.NSO.1.2

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students explored probability as a way to describe the likelihood of unknown events and used simulations to estimate the probability of events.

Career Connection | Meet Franklin R. Chang-Díaz

Invite students to read about engineer and former astronaut Franklin R. Chang-Díaz and his interest in space vehicle construction.

Consider asking:

- "What do you think makes up the payload for an airplane?" Sample response: The flight crew, passengers, luggage.

B.E.S.T. Benchmark Connections

Consider asking:

- "What other math concepts have you studied this year that could apply to work in the space industry?" Sample response: Engineers can apply scaled copies and drawing when they design equipment and space craft (MA.7.GR.1.5)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "NASA has facilities in several different U.S. states. How do you think the employees at NASA apply Mathematical Thinking and Reasoning MTR.1.1 and MTR.7.1 in their work?" Sample response: Since people work at different locations, they should work individually and with others to solve challenging problems and help each other when trying new methods or using new equipment (MTR.1.1). Work involving space travel uses lots of formulas and very large and small numbers, so they apply math concepts like solving equations and writing large numbers using exponents (MTR.7.1).



Unit 8 | Math in Action

Career Connection

There are over 5,000 people working in this science lab in California to support space missions.

Engineers, mechanics, designers, and researchers collaborate to apply astronomy, physics, robotics, and telecommunications ideas for space exploration.

Engineers are among the many careers that involve the space industry. They conduct experiments and collect data to determine safe equipment for space travel and to track the path of spacecraft before and during flights.

Jet Propulsion Laboratory" by NASA/JPL. Public Domain via Wikimedia Commons.

B.E.S.T. Mathematics Benchmark Connection

Engineers and others involved in space travel use many math concepts in their work. For example, they apply probability concepts (MA.7.DP.2.1) to test durability of materials so they are safe for space flight. They also apply geometry concepts involving area and volume (MA.7.GR.1.4, MA.7.GR.2.3) as they design and test the parts that are assembled for the spacecraft and other equipment that will be on board for the space missions.

Mathematical Thinking and Reasoning Connection

Engineers use thinking and reasoning skills like the ones you use for your math work! For example, they need to complete complex computations that involve the flight path of a spacecraft with fluency so that the mission is safe (MTR.3.1).

Meet Franklin R. Chang-Díaz

Franklin R. Chang-Díaz is an engineer who was also a NASA astronaut. He participated in seven spaceflights between 1986 and 2002, including three space walks! He applied his interests in engineering to projects such as vehicle construction and payload, which is the load that includes the people or instruments carried by a vehicle for a mission (and not the parts that make up the spacecraft).

JSC/NASA

Grade 7 Unit 8 670

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining the probability of events	Lesson 2	1	MA.7.DP.2.2
2	Comparing and contrasting data of boxplots in context	Lesson 9	2	MA.7.DP.1.2, MTR.6.1
3	Estimating and explaining predicted outcomes of probability events	Lesson 4	2	MA.7.DP.2.4, MTR.4.1
4a, 4b	Calculating the probability of events	Lesson 2	1	MA.7.DP.2.3
5a	Identifying the population of a random sample	Lesson 11	2	MA.7.DP.1.3, MTR.6.1
5b	Making predictions about a population	Lesson 11	2	MA.7.DP.1.3, MTR.6.1
5c	Explaining how results from multiple samples affect their confidence in predictions	Lesson 11	2	MA.7.DP.1.3, MTR.6.1
6a	Inferring about populations based on samples represented in dot plot diagrams	Lesson 13	2	MA.7.DP.1.1
6b	Comparing two populations using measures of center and variability for numerical data from random samples	Lesson 13	3	MA.7.DP.1.2

Assessment Resources



- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources



- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 8

1. You pick a block out of this bag without looking.



What is the probability of picking a star?

A. $\frac{1}{3}$ B. $\frac{3}{4}$ C. $\frac{3}{7}$ D. $\frac{4}{7}$

2. Adriana is curious: do more customers go to fast-food restaurants or to other restaurants?

She went to a random sample of 50 restaurants (25 fast-food and 25 other).

At each restaurant, Adriana recorded the number of customers the restaurant had that day.





Based on the data, select *all* of the true statements.

- ☑ A. Fast-food restaurants tend to get more customers than other restaurants.
- ☐ B. Other restaurants tend to get more customers than fast-food restaurants.
- ☐ C. Fast-food restaurants have a more consistent number of customers than other restaurants.
- ☑ D. Other restaurants have a more consistent number of customers than fast-food restaurants.
- ☐ E. All fast-food restaurants have more customers than other restaurants.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 8

3. Five friends share an apartment. Each week, they use this spinner with 5 equal sections to determine who will mop the floor.



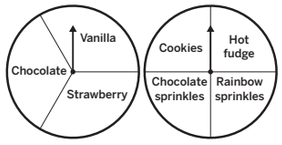
Estimate the number of times Abdel will be chosen to mop in a year (about 52 weeks).

10 or 11 are considered correct

Explain your thinking.

Explanations vary. The probability of the spinner landing on Abdel is $\frac{1}{5}$. That means the best estimate for how many spins out of 52 will land on Abdel is $\frac{1}{5}$ of 52, or 10.4, which rounds to 10.

4. The Spin N' Dine Restaurant sells ice cream for dessert. You can spin to either select a flavor of ice cream at random OR one topping at random.



a. What is the probability of getting chocolate ice cream on your spin?

$\frac{1}{3}$

b. What is the probability of NOT getting Hot fudge as a topping on your spin?

$\frac{3}{4}$

5. Rudra thinks school should have a longer lunch period and wants to know if others agree. They survey a random sample of 20 students and find that 12 of them agree.

a. What is the population for Rudra question?

All the students at Rudra school

b. If the school has 250 students, about how many do you predict would agree?

150 students

c. The next day, Rudra surveys another 20 random students and finds that 8 of them are in favor. Does this make you more or less confident in your prediction?

Less confident

Explain your thinking.

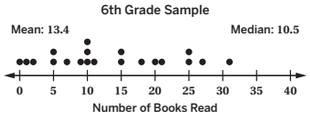
Explanations vary. In Rudra first sample, over 50% of students agreed. In their second sample, less than 50% agreed. This means that the variation is large, which makes me less confident in my prediction.

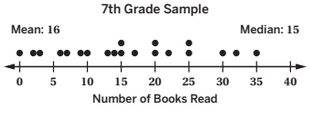
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 8

6. Maneli is wondering who reads more books at her school: the 6th graders or the 7th graders. She selects a random sample of 20 students from 6th grade and 20 students from 7th grade. Then she asks the students how many books they read in the past year.





a. Dylan looks at the dot plots and says that the school only has 20 6th graders and 20 7th graders.

Do you agree?

No

Explain your thinking.

Explanations vary. These are only samples of the 6th graders and 7th graders at Maneli's school, not the full populations.

b. Explain whether the data shows a big difference in the number of books read by 6th graders and 7th graders. Use evidence to justify your claim.

Responses vary. The median is a measure of center for the data. The measure of center changes from 10.5 to 15, which is a large change. This shows that there is a big difference in the number of books read by the two grades.

Name: Date: Period:

End-of-Unit Assessment

Unit 8

1. You pick a block out of this bag without looking.

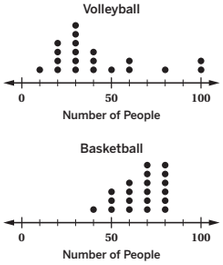


What is the probability of picking a block that is *not* a star?

A. $\frac{1}{4}$ B. $\frac{4}{5}$ C. $\frac{4}{9}$ D. $\frac{5}{9}$

2. Ebony is curious: do more people attend volleyball or basketball games?

She went to a random sample of 40 games (20 volleyball and 20 basketball) At each game, Ebony recorded the number of people.



Based on the data in the dot plots, select *all* of the true statements.

- [x] A. Basketball games tend to get more people to attend than volleyball games.
- [ ] B. Volleyball games tend to get more people to attend than basketball games.
- [x] C. Basketball games have a more consistent number of people than volleyball games.
- [ ] D. Volleyball games have a more consistent number of people than basketball games.
- [ ] E. All basketball games have more people attending than volleyball games.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 8

3. Six friends share an apartment. Each day, they use this spinner with 6 equal sections to determine who will wash the dishes.



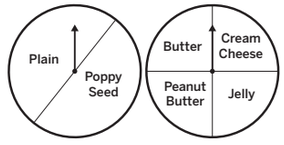
Estimate the number of times Binta will be chosen to do the dishes in a month (about 30 days).

5 times

Explain your thinking.

Explanations vary. A month has about 30 days. The probability of the spinner landing on Binta is $\frac{1}{6}$. That means the best estimate for how many spins in a month will land on Binta is about $\frac{1}{6}$ of 30, or 5.

4. The Spin N' Dine Restaurant sells bagels. You can spin to either select one random type of bagel OR one random spread.



a. What is the probability of getting a plain bagel on your spin?

$\frac{1}{2}$

b. What is the probability of NOT getting cream cheese as a topping on your spin?

$\frac{3}{4}$

5. Darius thinks school should start later and wants to know if others agree. He surveys a random sample of 40 students and finds that 26 of them agree.

a. What is the population for Darius's question?

All the students at Darius's school

b. If the school has 240 students, about how many do you predict would agree?

156 students

c. The next day, Darius surveys another 10 random students and finds that 6 of them agree. Does this make you more or less confident in your prediction?

More confident

Explain your thinking.

Explanations vary. In Darius's first sample, over 60% of the students agreed. In his second sample, 60% also agreed. This means that the variation is small, which makes me more confident in my prediction.

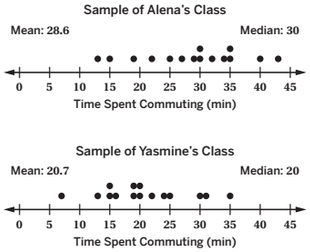
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 8

6. Alena is wondering who spends more time commuting: her class or her friend Yasmine's class. Alena selects a random sample of 15 students from her class. Yasmine does the same from her class.

They ask those students how long they spent commuting to school yesterday.



a. Dylan looks at the data and says that each class has 15 students. Do you agree?

No

Explain your thinking.

Explanations vary. They sampled 15 students from each class. This is not the whole population.

b. Explain whether the data shows a big difference in the time Alena's classmates and Yasmine's classmates spend commuting. Use evidence to justify your claim.

Responses vary. The median is a measure of center for the data. The measures of center is 30 for one class but 20 for another class, which is a large difference. This shows that there is a big difference in the the time that Alena's classmates and Yasmine's classmates spend commuting.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	

Sub-Unit 1:
• Compare probabilities to the results of repeated experiments.
• Determine the probability of events by identifying the sample space. **(Lessons 1–5)**
	1, 4a, 4b	Support	• **Teacher Move:** Consider revisiting:
			  • Unit 8, Lesson 1 (*How Likely?*)
			  • Unit 8, Lesson 2 (*Prob-bear-bilities*)

		Stretch	• **You're invited to explore more:** Lesson 2 (*Prob-bear-bilities*)

	3	Support	• **Teacher Move:** Consider revisiting:
			  • Unit 8, Lesson 4 (*Spin Class*)
			  • Unit 8, Lesson 5 (*Is It Fair?*)

		Strengthen	• **Challenge Creator:** Lesson 4 (*Spin Class*)
		Stretch	• **You're invited to explore more:** Lesson 5 (*Is It Fair?*)
Sub-Unit 2: • Use measures of center and measures of variability from random samples to draw conclusions about and compare populations. • Choose and create appropriate graphical representations for numerical and categorical data sets. **(Lessons 6–13)**	2	Support	• **Mini Lesson: Calculating Measures of Center and Spread** • **Teacher Move:** Consider revisiting Unit 8, Lesson 9 (*Car, Bike, or Train?*)
		Strengthen	• **Teacher Move:** Invite students to share with a partner why is it important to consider the mean, median, and range when comparing sets of data.
	5	Support	• **Mini Lesson:** Predicting Population Using Sample Data
		Strengthen	• **Teacher Move:** Consider revisiting Unit 8, Lesson 11 (*Flower Power*)
	6	Support	• **Teacher Move:** Consider revisiting Unit 8, Lesson 13 (*School Newspaper*)

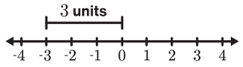
Glossary/Glosario

English

Español

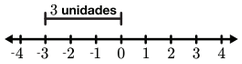
A

absolute value The distance from 0 to a number on a number line is its absolute value.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

valor absoluto La distancia del 0 a un número en una recta numérica es su valor absoluto.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

approximation A rounded value that you can use to represent a number that may be difficult to work with, such as a never-ending decimal.

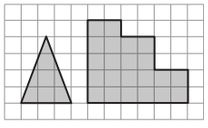
For example, the exact value of pi (π) is a never-ending decimal, so we often use the approximate value of 3.14 in calculations involving pi.

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un decimal infinito

Por ejemplo, el valor exacto de pi (π) es un decimal infinito, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

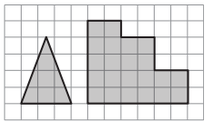
area The space inside a two-dimensional figure. It is expressed in square units.

The area of the triangle is 6 square units. The area of the other shape is 22 square units.



área El espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.

El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.



B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.

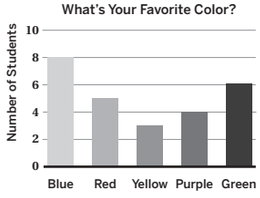
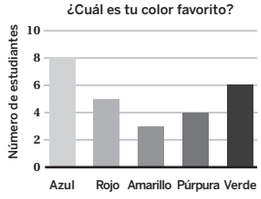


gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.



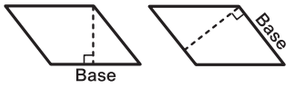
bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

For example, the repeating decimal $0.090909...$ can be written in bar notation as $0.\overline{09}$.

raya indicadora de decimales periódicos (vinculum) Una forma de representar los dígitos que se repiten en un número decimal. Sobre los dígitos que se repiten se traza una pequeña línea.

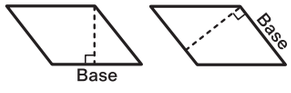
Por ejemplo, el decimal periódico $0.090909...$ puede escribirse con la raya indicadora de decimales periódicos (vinculum) como $0.\overline{09}$.

base (of a parallelogram) One side of a parallelogram. We can choose any side to be the base.



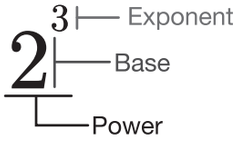
The base can also refer to the length of this side. The height of a shape is perpendicular to the base.

base (de un paralelogramo) Un lado de un paralelogramo. Podemos elegir cualquier lado como base.



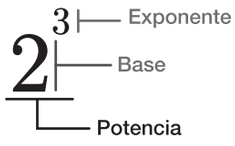
La base también puede referirse a la longitud de este lado. La altura de una figura es perpendicular a la base.

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.



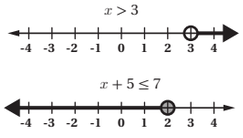
In the expression 2^3, 2 is the base.

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.



En la expresión 2^3, 2 es la base.

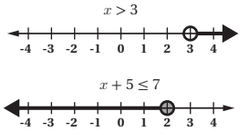
boundary point The point on a number line or graph that separates solutions of an inequality from non-solutions.



If the boundary point is a solution to the inequality (i.e., $\geq$ or $\leq$), it is represented with a closed circle on the graph. If it's not a solution (i.e., $>$ or $<$), it is represented with an open circle on the graph.

The boundary point of $x > 3$ is 3. The boundary point of $x + 5 \leq 7$ is 2.

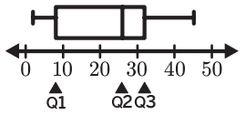
punto límite El punto en una recta numérica o una gráfica que separa las soluciones de una desigualdad de los valores que no son soluciones.
Si el punto límite es una solución de la desigualdad (es decir, $\geq$ o $\leq$), se representa con un círculo cerrado en la gráfica. Si no es una solución (es decir, $>$ o $<$), se representa con un círculo abierto en la gráfica.



El punto límite de $x > 3$ es 3. El punto límite de $x + 5 \leq 7$ es 2.

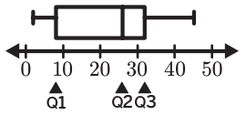
English

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



Español

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.

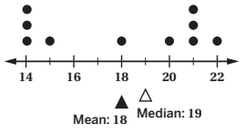


C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

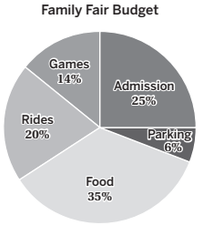
"What kind of pet do you have?" is a question that would result in categorical data.

center A single value that can be used to summarize the typical value in a data set. Mean and median are measures of center.



circle A shape made out of all the points that are the same distance from a center point.

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a pie chart.



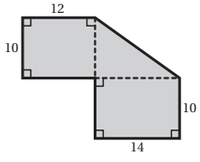
circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string. The circumference of a circle, C, can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression $5x + 8$, 5 is the coefficient of x.

composite figure A two- or three-dimensional figure that can be decomposed into smaller figures.

The composite figure can be decomposed into two rectangles and a triangle.



constant A value that does not change. In an algebraic expression, the constant term does not contain a variable.

In the expression $7x + 9$, 9 is the constant term.

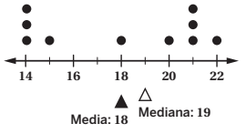
In the expression $5x - 8$, -8 is the constant term.

In the expression $12 - 4x$, 12 is the constant term.

datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

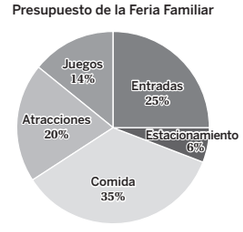
"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

centro Un solo valor que puede utilizarse para resumir el valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central, o sea, medidas de centro.



círculo Una figura formada por todos los puntos que están a la misma distancia de un punto central.

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado gráfico de torta.



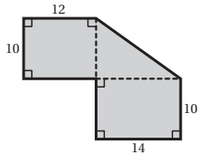
circunferencia La distancia alrededor de un círculo. Si imaginamos el círculo como un trozo de cuerda, es la longitud de la cuerda. La circunferencia de un círculo, C, puede calcularse mediante la fórmula $C = \pi d$, donde d es el diámetro del círculo, o $C = 2\pi r$, donde r es el radio.

coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

En la expresión $5x + 8$, el coeficiente de x es 5.

figura compuesta Figura bidimensional o tridimensional que puede descomponerse en figuras más pequeñas.

La figura compuesta se puede descomponer en dos rectángulos y un triángulo.



constante Un valor que no cambia. En una expresión algebraica, el término constante no contiene una variable.

En la expresión $7x + 9$, 9 es el término constante.

En la expresión $5x - 8$, -8 es el término constante.

En la expresión $12 - 4x$, 12 es el término constante.

Glossary/Glosario

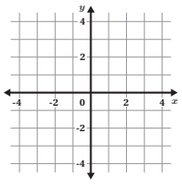
English

constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

Carpet (sq. ft.)		Cost (dollars)
10	$\times\frac{2}{3}$	15.00
20	$\times\frac{2}{3}$	30.00
50	$\times\frac{2}{3}$	75.00

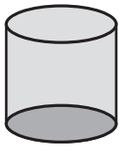
In this table, one constant of proportionality is $\frac{2}{3}$.

coordinate plane The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



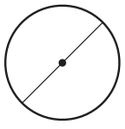
correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

cylinder A 3-D figure that has two parallel circular bases connected by a curved surface.

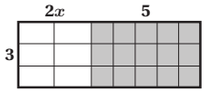


D

diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.



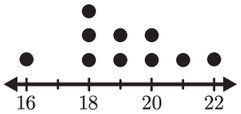
distributive property The property that says $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.



Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dot plot A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other. A dot plot is sometimes called a line plot.



Number of Jelly Beans

For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

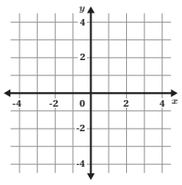
Español

constante de proporcionalidad En una relación proporcional, una constante de proporcionalidad es el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)		Costo (dólares)
10	$\times\frac{2}{3}$	15.00
20	$\times\frac{2}{3}$	30.00
50	$\times\frac{2}{3}$	75.00

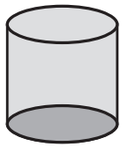
En esta tabla, una constante de proporcionalidad es $\frac{2}{3}$.

plano de coordenadas, plano cartesiano El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).

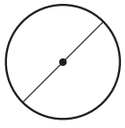


corresponder (partes correspondientes) Corresponder es coincidir. Cuando una parte de una figura original corresponde con una parte de una copia, se llaman partes correspondientes. Pueden ser puntos, segmentos, ángulos o distancias.

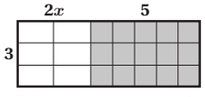
cilindro Una figura tridimensional que tiene dos bases circulares paralelas conectadas por una superficie curva.



diámetro La distancia entre un punto y otro en un círculo, pasando por el centro. También es la distancia mayor entre un punto y otro en un círculo.



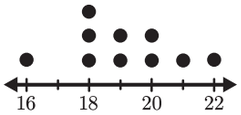
propiedad distributiva La propiedad que indica que $a(b + c) = ab + ac$. Significa que multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente y luego sumar los productos.



El área como un producto
$3(2x + 5)$

Por ejemplo, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

diagrama de puntos Una forma de visualizar conjuntos de datos numéricos, en la que cada punto de datos se representa mediante un punto en una recta numérica. Los puntos de datos con el mismo valor se apilan unos sobre otros. Un diagrama de puntos a veces se conoce como gráfica de puntos.



Cantidad de frijolitos

Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

English

Español

E

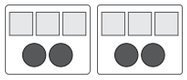
equivalent expressions Expressions that are equal for every value of a variable.

For example, $6x + 2x$ is equivalent to $5x + 3x$. No matter what the value of x is, the two expressions will be equal.

expresiones equivalentes Expresiones que son iguales para cualquier valor de una variable.

Por ejemplo, $6x + 2x$ es equivalente a $5x + 3x$. Independientemente del valor de x, las dos expresiones serán iguales.

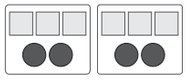
equivalent ratios Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



$3 : 2$ is equivalent to $6 : 4$ because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. Both recipes will taste the same.

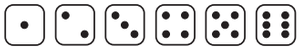
razónes equivalentes Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



$3 : 2$ es equivalente a $6 : 4$ porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

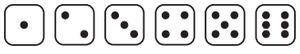
Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. Ambas recetas tendrán el mismo sabor.

event A set of one or more outcomes in a chance experiment.



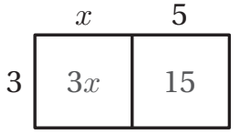
For example, if we roll a number cube, there are six possible outcomes.

evento Un conjunto de uno o más resultados en un experimento de azar.



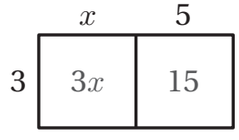
Por ejemplo, si tiramos un dado, hay seis resultados posibles.

expand To expand is to use the distributive property to multiply the factors in an expression and rewrite the expression as a sum. The new expression is equivalent to the original expression.



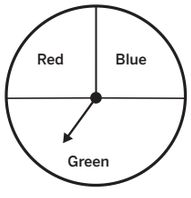
For example, we can expand $3(x + 5)$ to get the equivalent expression $3x + 15$.
The sum of the two boxes in the diagram show the expression in expanded form.

expandir Expandir significa usar la propiedad distributiva para multiplicar los factores en una expresión y reescribir la expresión como una suma. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos expandir $3(x + 5)$ para obtener la expresión equivalente $3x + 15$.
La suma de las dos casillas en el diagrama muestra la expresión en forma expandida.

experiment An action or activity that you can do over and over again without knowing what the outcome will be each time.



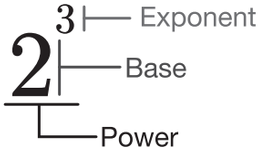
For example, each time you spin the spinner, it could land on red, blue, or green.

experimento Una acción o actividad que se puede llevar a cabo una y otra vez sin saber cuál será el resultado en cada ocasión.



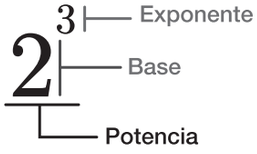
Por ejemplo, cada vez que se gira la ruleta, podría caer en rojo, azul o verde.

exponent A number used to describe repeated multiplication.



In this example, 3 is the exponent, which means $2^3 = 2 \cdot 2 \cdot 2$.

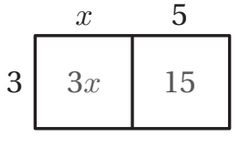
exponente Un número que se usa para describir multiplicaciones repetidas.



En este ejemplo, 3 es el exponente, lo cual significa que $2^3 = 2 \cdot 2 \cdot 2$.

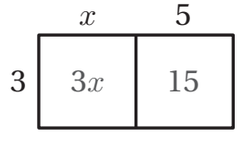
F

factor To factor is to use the distributive property to rewrite an expression as the product of two or more factors. The new expression is equivalent to the original expression.



For example, we can factor $3x + 15$ to get the equivalent expression $3(x + 5)$.
The product of the two sides in the diagram show the expression in factored form.

factorizar Factorizar significa usar la propiedad distributiva para reescribir una expresión como el producto de dos o más factores. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos factorizar $3x + 15$ para obtener la expresión equivalente $3(x + 5)$.
El producto de los dos lados del diagrama muestra la expresión en forma factorizada.

Glossary/Glosario

English

Español

H

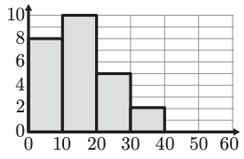
height (of a parallelogram) The shortest distance between a base and its opposite side. The height is always perpendicular to the base.



altura (de un paralelogramo) La distancia más corta entre una base y su lado opuesto. La altura siempre es perpendicular a la base.

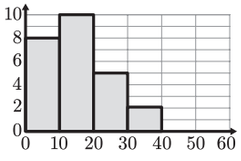


histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.



For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.

histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.



Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

I

integer All whole numbers and their opposites.

For example, 35, -15, and 1 are integers. 0.3 and $\frac{1}{3}$ are not.

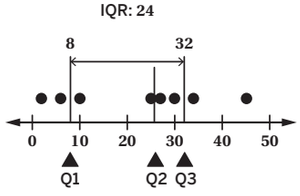
enteros Todos los números enteros y sus opuestos.

Por ejemplo, 35, -15 y 1 son números enteros. 0.3 y $\frac{1}{3}$ no lo son.

interest The extra money that you pay on a loan or the money that you earn when you save or invest.

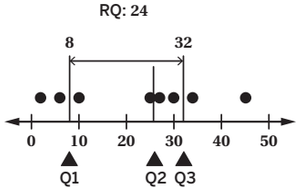
interés El dinero extra que pagas por un préstamo o el dinero que ganas cuando ahorras o inviertes.

interquartile range (IQR) One wayto measure how spread out a data set is. We also call this the IQR. The IQR is calculated as the distance from the first quartile (Q1) to the third quartile (Q3).



For example, the IQR of this data set is 24 because $32 - 8 = 24$.

rango intercuartílico (RQ) Una formade medir cuán disperso está un conjunto de datos. También lo denominamos RQ. El RQ se calcula como la distancia entre el primer cuartil (Q1) y el tercer cuartil (Q3).



Por ejemplo, el RQ de este conjunto de datos es 24 porque $32 - 8 = 24$.

inverse operations Two operations that undo each other are called inverse operations.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

L

like terms Terms with variables and exponents that are the same.

For example, $8x$ and $12x$ are like terms because both terms have a variable of x. $3x$ and $3x^2$ are not like terms because they have different exponents.

términos semejantes Términos con variables y exponentes iguales.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable que incluye x. $3x$ y $3x^2$ no son términos semejantes porque tienen exponentes diferentes.

M

markdown The amount of decrease between the original price of an item and its sale price.

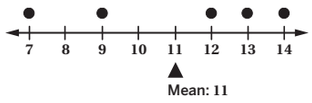
markup The amount of increase between the store's cost of an item and the item's selling price.

rebaja El monto de descuento entre el precio original de un artículo y su precio de venta.

sobreprecio La cantidad de aumento entre el costo de un artículo en la tienda y el precio de venta del artículo.

English

mean A measure of center. If you equally distribute a set of items into different groups, the mean is the number of items in each group.



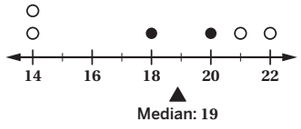
It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.

For example, for the data set $7, 9, 12, 13, 14$, the mean is 11 because $7 + 9 + 12 + 13 + 14 = 55$ and $\frac{55}{5} = 11$.

measure of center A single number that summarizes all of the values in a data set. It is usually a typical value for a data set. Mean and median are measures of center.

measure of variability A single number that describes the spread, or distribution, of a data set. Range and interquartile range are measures of variability. Also called a measure of variation.

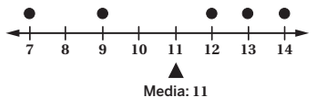
median A measure of center. The median is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values.



The median of this data set is 19 because it is the average of the two middle values: 18 and 20.

Español

media Una medida de tendencia central. Si distribuyes equitativamente un conjunto de elementos en diferentes grupos, la media es el número de artículos en cada grupo.



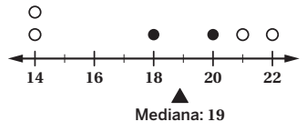
Para calcular la media, se pueden sumar los valores de todos los puntos de datos y luego dividir por la cantidad de puntos de datos.

Por ejemplo, en el conjunto de datos $7, 9, 12, 13, 14$, la media es 11 porque $7 + 9 + 12 + 13 + 14 = 55$ y $\frac{55}{5} = 11$.

medida de tendencia central, medida de centro Un solo número que resume todos los valores de un conjunto de datos. Suele ser un valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central.

medida de variabilidad Un solo número que describe la dispersión, o la distribución, de un conjunto de datos. El rango y el rango intercuartil son medidas de variabilidad. También llamada medida de variación.

mediana Una medida de tendenciacentral. La mediana es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay un número par de puntos de datos, la mediana es el promedio de los dos números centrales.

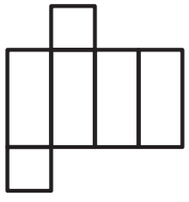


La mediana de este conjunto de datos es 19 porque es el promedio de los dos valores centrales: 18 y 20.

N

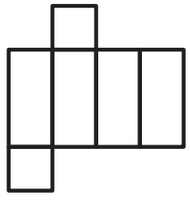
net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

Here is a net for a rectangular prism.



red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.

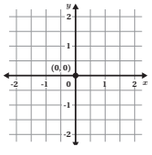
Esta es una red de un prisma rectangular.



O

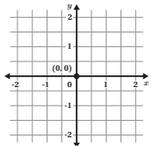
order of operations A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone. The standard order of operations is parentheses/grouping symbols, exponents/roots, multiplication/division, and then addition/subtraction.

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



orden de las operaciones Un orden coherente aplicado a una expresión con múltiples operaciones para que cualquiera pueda evaluar la expresión de la misma manera. El orden estándar de las operaciones es paréntesis/símbolos de agrupación, exponentes/raíces, multiplicación/división y, luego, suma/resta.

origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan el eje x y el eje y.



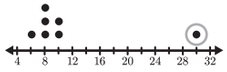
Glossary/Glosario

English

outcome One of the possible results of an experiment.

For example, the possible outcomes of tossing a coin are heads and tails.

outlier A value that is much higher or much lower than the other values in a set of data.



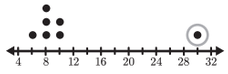
The circled point is an outlier.

Español

resultado Uno de los posibles resultados de un experimento.

Por ejemplo, los posibles resultados de lanzar una moneda son cara y sello.

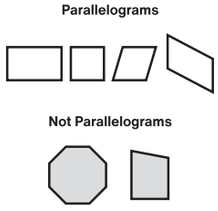
valor atípico Un valor que es mucho mayor o mucho menor que los demás valores de un conjunto de datos.



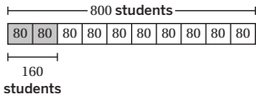
El punto encerrado en un círculo es un valor atípico.

P

parallelogram A quadrilateral that has two pairs of parallel sides. Opposite, or parallel, sides of a parallelogram are the same length.

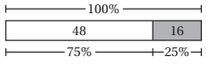


percent (percentage) Percent means for every 100. It is represented by the percent symbol: %. We use percentages to represent ratios and fractions.



For example, 20% means $20 : 100$. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number.
Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

percent decrease How much a quantity goes down, expressed as a percentage of the starting amount.

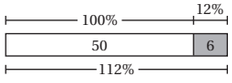


For example, a store had 64 hats in stock on Friday.
They had 48 hats left on Saturday. The amount of hats went down by 16.
This was a 25% decrease because 16 is 25% of 64.

percent error A way to describe the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a box is supposed to have 150 folders in it. Clare counts only 147 folders in the box. This is an error of 3 folders. The percent error is 2% because 3 is 2% of 150. $\frac{3}{150} = 0.02 = 2\%$

percent increase How much a quantity goes up, expressed as a percentage of the starting amount.



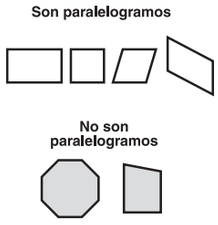
For example, Elena had $\$50$ on Monday. She helped a neighbor, so she had $\$56$ on Tuesday. The amount went up by $\$6$.
This was a 12% increase because 6 is 12% of 50. $\frac{6}{50} = 0.12 = 12\%$

perpendicular Describes a line that crosses or meets another line at a $90°$ angle.

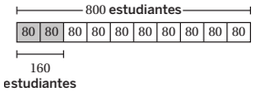
pi A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π.

Some common approximations for π are 3.14 and $\frac{22}{7}$.

paralelogramo Un cuadrilátero que tiene dos pares de lados paralelos. Los lados opuestos, o paralelos, de un paralelogramo tienen la misma longitud.

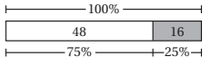


por ciento (porcentaje) Por ciento significa por cada 100. Se representa con el símbolo de porcentaje: %. Usamos porcentajes para representar razones y fracciones.



Por ejemplo, 20% significa $20 : 100$. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número.
Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos están en una excursión, entonces eso es 160 estudiantes porque 20 están de viaje por cada 100 estudiantes en total.

disminución porcentual Describe cuánto disminuye una cantidad, expresada como un porcentaje de la cantidad inicial.

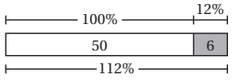


Por ejemplo, una tienda tenía 64 sombreros en inventario el viernes. El sábado le quedaban 48 sombreros.
La cantidad de sombreros se redujo en 16.
Fue una disminución del 25% porque 16 es el 25% de 64.

porcentaje de error, error porcentual Una forma de describir la diferencia entre un valor deseado y el valor real, expresada como un porcentaje del valor deseado.

Por ejemplo, se supone que una caja contiene 150 carpetas. Clare cuenta solo 147 carpetas en la caja. Ese es un error de 3 carpetas. El porcentaje de error es del 2% porque 3 es el 2% de 150. $\frac{3}{150} = 0.02 = 2\%$

aumento porcentual Cuánto aumenta una cantidad, expresada como un porcentaje de la cantidad inicial.



Por ejemplo, Elena tenía $\$50$ el lunes. Ayudó a un vecino, así que el martes tenía $\$56$. La cantidad aumentó en $\$6$.
Fue un aumento del 12% porque 6 es el 12% de 50. $\frac{6}{50} = 0.12 = 12\%$

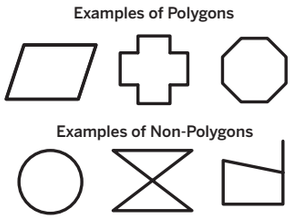
perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de $90°$.

pi Un número que representa la constante de proporcionalidad entre el diámetro y la circunferencia de cualquier círculo. El símbolo de pi es π.

Algunas aproximaciones comunes de π son 3.14 y $\frac{22}{7}$.

English

polygon A closed 2-D shape with straight sides that do not cross each other.



population A set of people or things that are being studied.

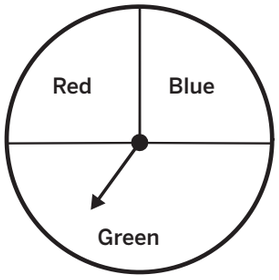
For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

power of ten A number written in the form 10^n, where n represents the number of times 10 is multiplied.

principal The total amount (initial value) of money borrowed or invested, not including any interest.

probability A number that tells us howlikely an event is to happen. A probability of 1 means the event is certain. A probability of 0 means the event is impossible.

For example, the probability of spinning red is $\frac{1}{4}$.



proportional Quantities are proportional if they form equivalent ratios.

For example, the heights and bases of these triangles are proportional because the ratio $6:12$ is equivalent to $4:8$.

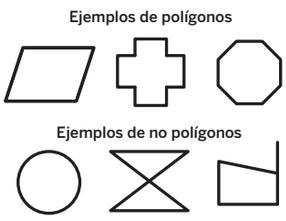
proportional relationship A collection of pairs of numbers that are in equivalent ratios. Two variables are in a proportional relationship if you can multiply the values for one variable by the same number to get the values for the other variable. The graph of a proportional relationship is a straight line passing through the origin, $(0, 0)$.

Carpet (sq. ft)	Cost (dollars)
10	15.00
20	30.00
50	75.00

For example, if $y = kx$, then y is said to be directly proportional to x and the constant of proportionality is k. Every cost in the table is equal to 1.5 times the number of square feet of carpet.

Español

polígono Una figura bidimensionalcerrada con lados rectos que no se cruzan.



población Conjunto de personas o cosas que se estudian.

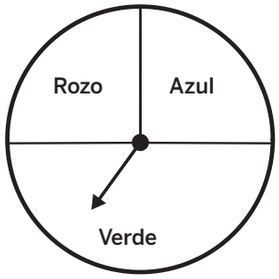
Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

potencia de diez Un número escrito de la forma 10^n, donde n representa el número de veces que se multiplica el 10.

principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, excluyendo los intereses.

probabilidad Número que indica qué tan probable es que ocurra un evento.Una probabilidad de 1 significa que el evento es seguro. Una probabilidad de 0 significa que el evento es imposible.

Por ejemplo, la probabilidad de girar y que caiga rojo es $\frac{1}{4}$.



proporcional Diferentes cantidades son proporcionales si forman razones equivalentes.

Por ejemplo, las alturas y las bases de estos triángulos son proporcionales porque la relación $6:12$ es equivalente a $4:8$.

relación proporcional Un conjunto de pares de números que están en proporciones equivalentes. Dos variables están en una relación proporcional si puedes multiplicar los valores de una variable por el mismo número para obtener los valores para la otra variable. La gráfica de una relación proporcional es una línea recta que pasa por el origen, $(0, 0)$.

Alfombra (pies cuadrados)	Costo (dólares)
10	15.00
20	30.00
50	75.00

Por ejemplo, si $y = kx$, entonces se dice que y es directamente proporcional a x y la constante de proporcionalidad es k. Cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

Q

quadrilateral A polygon that has four sides.

Squares, rectangles, and parallelograms are examples of quadrilaterals.

cuadrilátero Un polígono que tiene cuatro lados.

Los cuadrados, rectángulos y paralelogramos son ejemplos de cuadriláteros.

Glossary/Glosario

English

Español

R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.



radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.

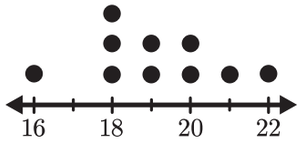


random Outcomes of a chance experiment are random if they are all equally likely to happen.

aleatorio Los resultados de un experimento de azar son aleatorios si todos tienen la misma probabilidad de suceder.

range A measure of spread. It is the difference between the maximum and minimum values in a data set.

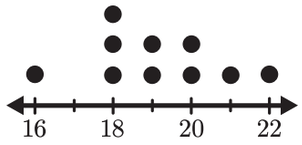
For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.



Number of Jelly Beans

rango Una medida de dispersión. Es la diferencia entre los valores máximo y mínimo de un conjunto de datos.

Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.



Cantidad de frijolitos

rational number Any number that can be written as a fraction, including whole numbers and negative numbers.

For example, -10, 2.5, $\frac{3}{7}$, and 82 are all rational numbers.

número racional Cualquier número que pueda escribirse como una fracción, incluidos los números enteros y los números negativos.

Por ejemplo, -10, 2.5, $\frac{3}{7}$ y 82 son números racionales.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \times \frac{2}{3} = 1$.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \times \frac{2}{3} = 1$.

relative frequency The number of times an outcome happens divided by the number of repeated experiments. After several experiments, the relative frequency of an outcome gets close to its probability.

For example, if you spin a spinner 12 times and it lands on red 5 times, that means the relative frequency of landing on red is $\frac{5}{12}$.

frecuencia relativa El número de veces que se produce un resultado dividido por el número de experimentos repetidos. Al cabo de varios experimentos, la frecuencia relativa de un resultado se aproxima a su probabilidad.

Por ejemplo, si se hace girar una ruleta 12 veces y cae en rojo 5 veces, la frecuencia relativa de que caiga en rojo es $\frac{5}{12}$.

repeating decimal A decimal with one or more digits (not all zeros) that repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (...) at the end.

If the repeating digits are all zeros, it would be called a terminating decimal.

For example, the decimal representation of $\frac{1}{3}$ is $0.\overline{3}$, which means $0.33333...$

The decimal representation of $\frac{25}{22}$ is $1.1\overline{36}$, which means $1.1363636...$

decimal periódico Un decimal con uno o más dígitos (no todos son ceros) que se repiten infinitamente. Un decimal periódico puede escribirse usando la raya indicadora de decimales periódicos (vinculum) encima de los dígitos que se repiten o con puntos suspensivos (...) al final.

Si todos los dígitos periódicos son ceros, entonces se denomina decimal exacto.

Por ejemplo, la representación decimal de $\frac{1}{3}$ es $0.\overline{3}$, lo que significa $0.33333...$

La representación decimal de $\frac{25}{22}$ es $1.1\overline{36}$, o sea $1.1363636...$

rhombus A quadrilateral containing four equal-length sides.



rombo Un cuadrilátero que contiene cuatro lados de igual longitud.



S

sample A part of a population.

For example, a population could be all the seventh grade students at one school. One sample of that population is all the seventh grade students who are in band.

muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

English

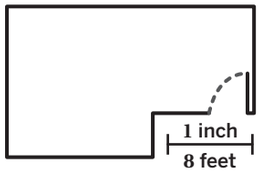
sample space The list of every possible outcome for a chance experiment.

For example, the sample space for tossing two coins is: heads-heads, tails-heads, heads-tails, tails-tails.



scale A scale tells us how the actual measurements of an object are represented in a drawing.

The scale of this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room.



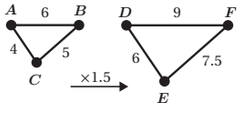
scale drawing A two-dimensional representation of an actual object or place. All the measurements in the drawing correspond to the measurements of the actual object by the same scale. Floor plans and maps are examples of scale drawings.



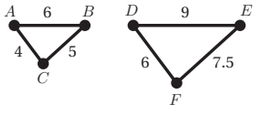
A scale may show that 1 centimeter on a map represents 30 miles on land. The distance marked on this map is 8 centimeters, or 240 miles.

scale factor The number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from ABC to DEF is 1.5.



scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a shape looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.



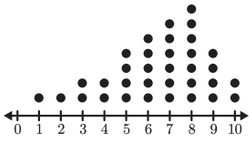
For example, triangle DEF is a scaled copy of triangle ABC.

shape A description of the distribution (or pattern) of the data within a data set. Descriptions of the shape of a distribution can include: symmetry, skew, clusters, and outliers.

simple interest Interest that is calculated solely based on the initial amount.

$I = prt$, where I = interest, p = principal, r = rate, and t = time

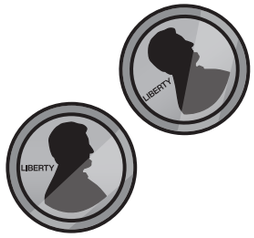
skewed A data distribution where there are more values concentrated on one end of the data, and few values on the other end.



Español

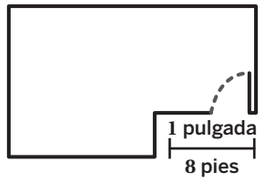
espacio muestral La lista de todos los posibles resultados de un experimento de azar.

Por ejemplo, el espacio muestral del lanzamiento de dos monedas es: cara-cara, sello-cara, cara-sello, sello-sello.



escala Una escala nos indica cómo están representadas en un dibujo o diagrama las medidas reales de un objeto.

La escala de este plano nos indica que 1 pulgada en el dibujo representa 8 pies en la habitación real.



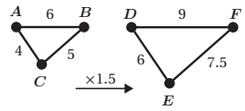
dibujo a escala Una representación bidimensional de un objeto o lugar reales. Todas las medidas del dibujo corresponden a las medidas del objeto real usando la misma escala. Los planos y los mapas son ejemplos de dibujos a escala.



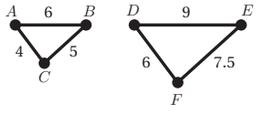
Una escala puede mostrar que 1 centímetro en un mapa representa 30 millas en tierra. La distancia mostrada en este mapa es 8 centímetros o 240 millas.

factor de escala El valor por el que se multiplica cada longitud de la figura original para producir la copia a escala.

Por ejemplo, el factor de escala de ABC a DEF es 1.5.



copia a escala Una copia de una imagen que puede cambiar de tamaño, pero que siempre conserva la forma y las medidas de los ángulos de la imagen original. Si una figura luce aplastada o estirada en comparación con la original, no es una copia a escala. Para crear una copia a escala, multiplicamos todas las longitudes de la figura original por el mismo número.



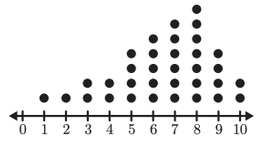
Por ejemplo, el triángulo DEF es una copia a escala del triángulo ABC.

figura Una descripción de la distribución (o patrón) de los datos en un conjunto de datos. Las descripciones de la forma de una distribución pueden incluir: simetría, asimetría, agrupaciones y valores atípicos.

interés simple Interés que se calcula solo según la cantidad inicial.

$I = prt$, donde I = interés, p = principal, r = tasa y t = tiempo

asimétrico Una distribución de datos en la que hay valores concentrados en un extremo de los datos y pocos valores en el otro extremo.



Glossary/Glosario

English

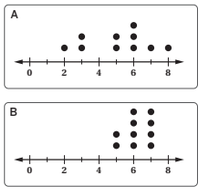
solution to an equation A value of a variable that makes the equation true.

For example, 5 is a solution to the equation $3x = 15$ because $3(5) = 15$. 6 is not a solution to the equation $3x = 15$ because $3(6) = 15$ is not true.

solutions to an inequality All of the values of a variable that make that inequality true.

For example, $x < 5$ represents the solutions to the inequality $3x < 15$ because any value that is less than 5 will make the inequality true.

spread A measure of how close together or far apart the values in a data set are from each other and from the center. The spread also describes the variability of a data set. Range and interquartile range measure the spread of data.



For example, Dot Plot A has a larger spread than Dot Plot B.

stem-and-leaf plot A table that organizes data by place value to compare data frequencies.

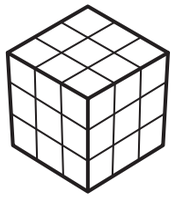
The stem-and-leaf plot shows the data set (5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64).

Stem	Leaf
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

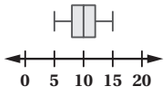
Key: 1 | 0 = 10

surface area The sum of the areas of the surfaces of a solid.

For example, the six faces of this cube each have an area of 9 square centimeters, so the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



symmetrical (data) A data display where the left side of the distribution is a mirror image of the right side.



Español

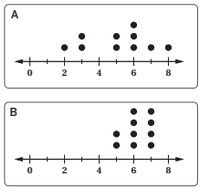
solución de una ecuación Un valor de una variable que hace que la ecuación sea verdadera.

Por ejemplo, 5 es una solución de la ecuación $3x = 15$ porque $3(5) = 15$. 6 no es una solución de la ecuación $3x = 15$ porque $3(6) = 15$ no es verdadero.

soluciones de una desigualdad Todos los valores de una variable que hacen que la desigualdad sea verdadera.

Por ejemplo, $x < 5$ representa las soluciones de la desigualdad $3x < 15$ porque cualquier valor que sea menor que 5 hará que la desigualdad sea verdadera.

dispersión Una medida que indica cuán agrupados o dispersos están los valores de un conjunto de datos entre sí y con respecto al centro. La dispersión también describe la variabilidad de un conjunto de datos. El rango y el rango intercuartil miden la dispersión de datos.



Por ejemplo, el diagrama de puntos A tiene una dispersión mayor que el diagrama de puntos B.

diagrama de tallo y hojas Una tabla que organiza los datos por valor posicional para comparar frecuencias de datos.

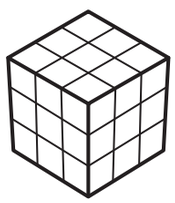
El diagrama de tallo y hojas muestra el conjunto de datos (5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64).

Tallo	Hoja
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

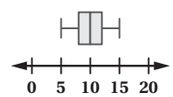
Leyenda: 1 | 0 = 10

área de superficie La suma de las áreas de las superficies de un sólido.

Por ejemplo, cada una de las seis caras de este cubo tiene un área de 9 centímetros cuadrados, por lo tanto, el área de superficie del cubo mide $6 \cdot 9$ o 54 centímetros cuadrados.

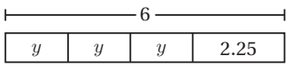


simétricos (datos) Una representación de datos donde el lado izquierdo de la distribución es una imagen reflejada del lado derecho.



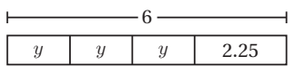
T

tape diagram A way to represent relationships between quantities (like ratios) as lengths of a tape. The entire tape diagram represents the whole and is divided to represent the parts.



This tape diagram shows a relationship that can be represented with the equation $3y + 2.25 = 6$.

diagrama de cinta Una forma de representar las relaciones entre cantidades (como razones) en forma de longitudes de una cinta. El diagrama de cinta completo representa el todo y se divide para representar las partes.

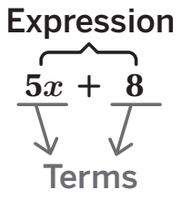


Este diagrama de cinta muestra una relación que puede representarse con la ecuación $3y + 2.25 = 6$.

English

term A part of an expression. A term can be a single number, a variable, or a number and a variable multiplied together.

For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.



terminating decimal A decimal with a finite number of non-zero digits after the decimal point.

For example, 0.08, 1.5, and 0.2563 are all terminating decimals.

trapezoid A quadrilateral with *at least* one pair of parallel sides.



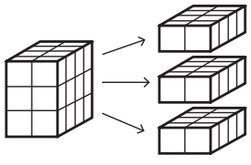
V

variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

volume The number of cubic units that fill a 3-D region without any gaps or overlaps.

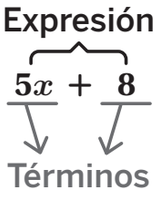
For example, the volume of this rectangular prism is 24 cubic units because it is made of 3 layers that are each 8 cubic units.



Español

término Una parte de una expresión. Un término puede ser un número individual, una variable, o una variable y un número multiplicados.

Por ejemplo, la expresión $5x + 8$ tiene dos términos. El primer término es $5x$ y el segundo término es 8.



decimal exacto Un decimal con un número finito de dígitos distintos de cero después del punto decimal.

Por ejemplo: 0.08, 1.5 y 0.2563 son decimales exactos.

trapezoide Un cuadrilátero con al menos un par de lados paralelos.

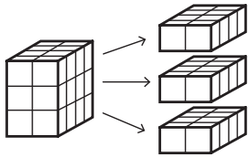


variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

volumen La cantidad de unidades cúbicas que llenan un espacio tridimensional, sin vacíos ni superposiciones.

Por ejemplo, el volumen de este prisma rectangular mide 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.



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