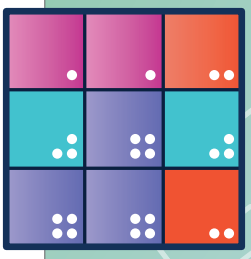


Unit 6

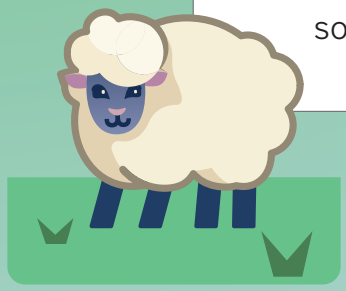
Expressions, Equations, and Inequalities

Sometimes values are unknown. A fair charges $7 per ticket and $2.50 for each ride. You have $20 to spend. How many rides can you take? Is there only one value that keeps you within budget? In this unit, you will explore mathematical problems and make sense of them using variables, symbols, and drawings.



Essential Questions

- Which representations help you make sense of different mathematical situations?
- Which strategies for solving simple equations or inequalities can help you solve more complex ones?
- What are ways to increase efficiency when solving mathematical problems?



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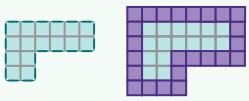
Summary | Lesson 1

Analyzing shape patterns can help you understand number patterns. Here is a design made with border toothpicks and border tiles.

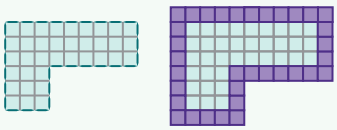
Stage 1



Stage 2



Stage 3



In Stage 1, there are 10 toothpicks. In Stage 2, there are 20 while in Stage 3, there are 30. The number of toothpicks increases by 10 each time. The table shows that the number of tiles is always 4 more than the number of toothpicks.

Stage	Border Toothpicks	Border Tiles
1	10	14
2	20	24
3	30	34

We can extend these rules to make predictions about any stage of the pattern. For example, in Stage 5 there will be 50 toothpicks and the number of border tiles will be 4 greater, 54.

Try This

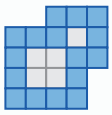
Here is a pattern. The tiles around the edge are called border tiles.

Predict how many border tiles will be used in Stage 4.

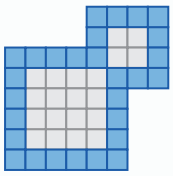
Enter the missing information in the table.

Stage	Border Tiles
1	16
2	28
3	
4	

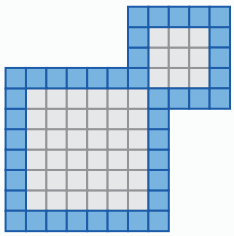
Stage 1



Stage 2



Stage 3



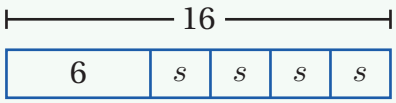
Summary | Lesson 2

You can use *tape diagrams* and expressions to represent situations.

Here are two situations and the tape diagrams that represent them.

Situation A

The local farmer's market is selling fresh fruit and jam. Alma buys 4 pounds of strawberries for s dollars each and 1 jar of raspberry jam for \$6. The total bill is \$16.

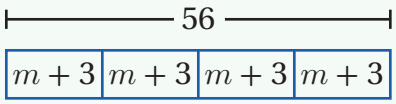


$s = 2.5$

Each pound of strawberries costs \$2.50.

Situation B

4 friends buy lunch from a food truck. The food truck charges a set price of m dollars for a meal and \$3 for a beverage. The total bill is \$56.



$m = 11$

Each meal costs \$11.00.

Try This

Aba bought a loaf of bread for \$2.25, as well as 3 apples. She spent a total of \$6.

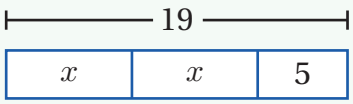
a Draw a tape diagram to represent this situation.

b What is the price of one apple?

Summary | Lesson 3

We can use tape diagrams and equations to make sense of stories and determine unknown amounts.

Here is a story with the equation and tape diagram that represent it.

Story	Equation	Tape Diagram
Two students go to the movie theater. They purchase two tickets and a $5 popcorn to share. In total, they spend $19.	$2x + 5 = 19$	

In the equation and tape diagram, x represents the unknown price of a movie ticket, 2 represents the number of tickets that were purchased, 5 represents the $5 spent on popcorn, and 19 represents the total amount spent.

Try This

A drive-in movie theater charges $6.00 per car, plus a fee for each person in the car. A family of 3 came in one car and paid $22.50 total.

a Circle the tape diagram that best matches this situation.

Diagram A

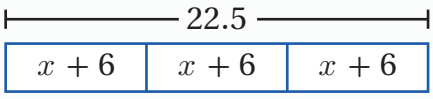
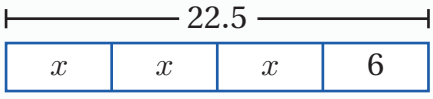


Diagram B



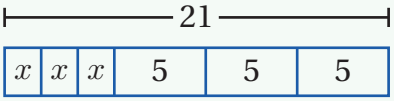
b Write an equation to represent this situation.

c How much was the fee for each family member?

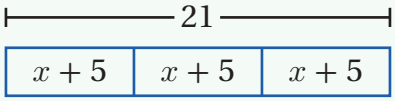
We can use a tape diagram to help us make sense of a situation. Two different tape diagrams and equations can often represent the same situation.

For example, Elena is training for a race. She trained 3 days this week for a total of 21 miles. On each training day, she ran several miles and biked 5 miles. If x represents the number of miles Elena ran, then we can model this situation using two equations and tape diagrams.

$3x + 15 = 21$



$3(x + 5) = 21$



We can use either of the tape diagrams and equations to determine that Elena ran 2 miles on each training day.

Try This

Deiondre bought a juice for $\$3$ and 2 sandwiches that cost x dollars each. Together, the items cost $\$11.50$. Complete each of these sections to represent this situation.

Tape Diagram	Equation
Solution	**Meaning of Solution**

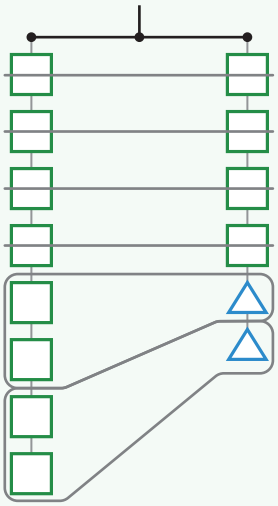
Hangers are balanced when the weight on both sides is the same. When you remove the same weight from both sides, the hanger will remain balanced.

Here are two examples of strategies you can use to determine an unknown value.

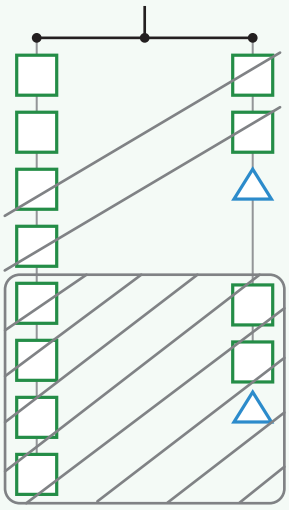
In Hanger A, you can cross off 4 squares on each side while still keeping the sides balanced. Then, 4 squares to the left are balanced with 2 triangles to the right. This means they have the same weight, so 1 triangle has the same weight as 2 squares.

In Hanger B, we can divide each side into two equal groups and remove one group on each side. This leaves 4 squares on the left and a cluster of 1 triangle and 2 squares on the right. Removing 2 more squares from each side leaves the weight of 1 triangle equal to the weight of 2 squares.

Hanger A



Hanger B



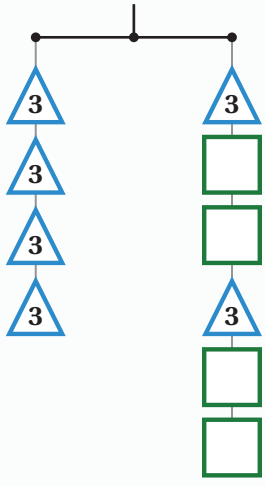
Try This

a Complete the table so that Hanger A stays balanced.

Weight of One Triangle (lb)	Weight of One Square (lb)
3	

b Draw another balanced hanger that uses the same shapes and weights.

Hanger A



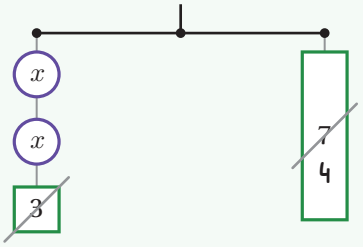
We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$\begin{aligned} 2x + 3 &= 7 \\ 2x + 3 - 3 &= 7 - 3 \\ 2x &= 4 \end{aligned}$$

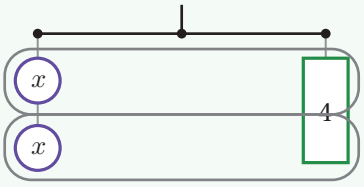
Remove 3 from both sides.



Divide both sides by 2.

$$\begin{aligned} 2x &= 4 \\ 2x \div 2 &= 4 \div 2 \\ x &= 2 \end{aligned}$$

Divide into two equal groups.

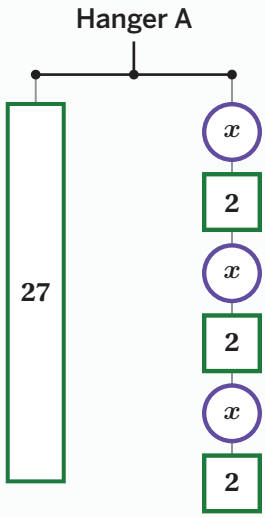


Try This

a Write an equation to represent this hanger.

b What is the value of x?

Write out the steps if it helps with your thinking.



When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1

$$\begin{aligned} 3x - 6 &= 9 \\ 3x - 6 + 6 &= 9 + 6 \\ 3x &= 15 \\ 3x \div 3 &= 15 \div 3 \\ x &= 5 \end{aligned}$$

Equation 2

$$\begin{aligned} 3(x - 6) &= 9 \\ 3(x - 6) \div 3 &= 9 \div 3 \\ x - 6 &= 3 \\ x - 6 + 6 &= 3 + 6 \\ x &= 9 \end{aligned}$$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

Equation 1

$$\begin{aligned} 3(5) - 6 &= 9 \\ 15 - 6 &= 9 \\ 9 &= 9 \text{ ✅} \end{aligned}$$

Equation 2

$$\begin{aligned} 3(9 - 6) &= 9 \\ 3(3) &= 9 \\ 9 &= 9 \text{ ✅} \end{aligned}$$

Try This

a Solve the equation $3(x - 7) = 33$. Show your thinking.

b Check your solution.

Summary | Lesson 8

Some equations have the form $p(x + q) = r$. We call this the **factored** form of the equation because it shows the product of two factors, p and $(x + q)$.

You can solve these equations by **expanding** or dividing as a first step. When expanding first, you can use the distributive property to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the *solution to the equation*, is the same.

Expanding First
(Using the distributive property)

$$\begin{aligned} 3(x + 1) &= 9 \\ 3x + 3 &= 9 \\ 3x + 3 - 3 &= 9 - 3 \\ 3x &= 6 \\ 3x \div 3 &= 6 \div 3 \\ x &= 2 \end{aligned}$$

Dividing First

$$\begin{aligned} 3(x + 1) &= 9 \\ 3(x + 1) \div 3 &= 9 \div 3 \\ x + 1 &= 3 \\ x + 1 - 1 &= 3 - 1 \\ x &= 2 \end{aligned}$$

Try This

Solve the equation $4(x - 1) = 30$.

Show your thinking.

Expressions that give the same output for every input are called **equivalent expressions**. To determine whether expressions are equivalent, you can test several inputs to see if they produce the same output.

In this example, $3x + 6$ and $3(x + 2)$ are equivalent because they give the same output for every input. You can test other values and they will always give matching outputs. $6(x + 3)$ is not equivalent because it doesn't give the same output for every input (it is only the same when $x = -4$).

x	$3x + 6$	$3(x + 2)$	$6(x + 3)$
10	36	36	78
7	27	27	60
-4	-6	-6	-6

Try This

Write an equivalent expression for each of the following expressions.

a $10 - 25x$

b $-4x + 14$

Summary | Lesson 10

You can expand, add, and subtract terms to write equivalent expressions. This helps us write expressions with the same value that have fewer terms. Terms with variables and exponents that are the same are called *like terms* and can be combined. For example, let's find the sum of $-\frac{1}{3}(3x - 6)$ and $2x - 4x + 5$.

We can write the sum as $-\frac{1}{3}(3x - 6) + 2x - 4x + 5$.

Let's write an equivalent expression with fewer terms.

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$	Expand using the distributive property.
$-x + 2 + 2x - 4x + 5$	Add terms that are alike ($-x$, $2x$, and $-4x$).
$-3x + 2 + 5$	Add terms that are alike (2 and 5).
$= -3x + 7$	

Try This

Write the sum of $5(x - 2)$ and $3x + 6$ using the fewest number of terms.

Show your thinking.

Summary

You can solve an equation in many different ways.

$-6(3x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(3x + (-5)) = 75$	Distribute the -6 to the $3x$ and -5.
$-18x + 30 = 75$	Subtract 30 from both sides.
$-18x = 45$	Divide each side by -18.
$x = -2.5$	

Here is a different strategy for solving the same equation.

$-6(3x - 5) = 75$	Divide both sides of the equation by -6.
$3x - 5 = -12.5$	Add 5 to both sides.
$3x = -7.5$	Divide each side by 3.
$x = -2.5$	

Try This

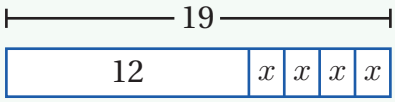
Solve the equation $3 + 5(x - 1) = 48$.

Show your thinking.

Summary | Lesson 12

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for $12. She pays a total of $19. This tape diagram represents Zahra's situation.



Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$12 + 4x = 19$$

$$4x = 7$$

$$x = 1.75$$

Zahra pays $1.75 for each pen.

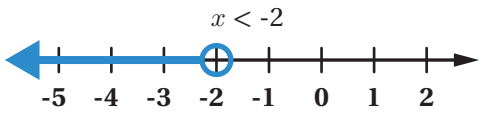
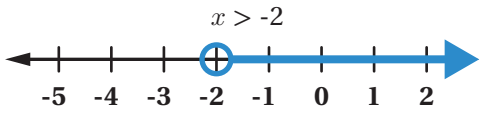
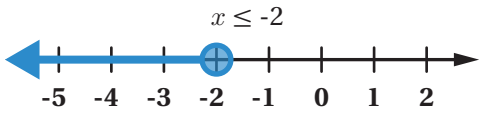
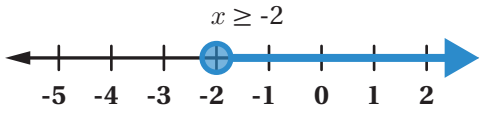
Try This

Valeria is planning a fundraiser for her school's running club. The decorations for the fundraiser cost $10. If 20 people attend, how much will Valeria need to charge each person to raise a final total of $300?

Use a visual representation or equation to help show your thinking.

You can use an inequality to describe a range of values. For example, the inequality $x > 2$ describes all the values that are greater than 2. The value 2 is not included. The inequality $x \geq 2$ includes the value 2. When graphing, we fill in the circle to represent including the boundary point. An open circle means the boundary point is not included.

Here are the symbols used to write inequalities.

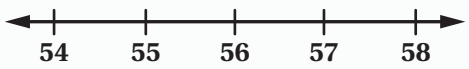
Symbol	Name	Meaning	Example
$<$	Less than	Lower, smaller	
$>$	Greater than	More than, higher, larger	
$\leq$	**Less-than-or-equal-to**	No more than, the maximum amount	
$\geq$	**Greater-than-or-equal-to**	At least, the minimum amount	

Try This

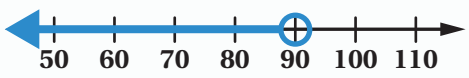
Riders must be less than 56 inches tall to ride a carousel.

a Darryl is 56 inches tall. Can he go on the carousel? Explain your thinking.

b Graph the possible heights of the carousel riders on this number line.



c Write an inequality that represents this new graph.



You can use many of the same strategies you use for solving equations to solve inequalities. The values of x that make the inequality true are known as the **solutions to an inequality**. You can test values by substituting them into the inequality.

For example, consider the inequality $4x + 2 < 22$.

- To determine the value of x that balances the hanger, solve the equation $4x + 2 = 22$.
- When $x = 5$, the hanger is balanced. All values less than 5 will make the inequality true because $4x + 2$ needs to be less than 22.

The solution shown on the graph means that all values of x *less than* 5 will make the inequality true.



To check the solution, substitute any value less than 5 into the original inequality.

$$4(4) + 2 < 22$$

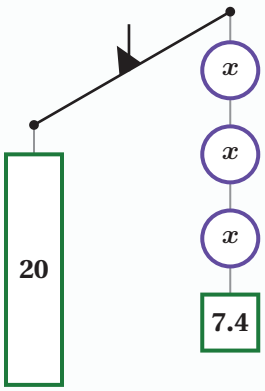
$$16 + 2 < 22$$

$$18 < 22 \checkmark$$

Try This

Solve the inequality $20 > 3x + 7.4$.

Use the hanger if it helps with your thinking.



Inequalities can be used in real-world situations, like budgeting for a project.

For example, Aditi has $5 and sells homemade greeting cards for $1.50 each. Her goal is to have $20 total.

The solution to the equation $1.50x + 5 = 20$ represents the number of greeting cards, x, that she needs to sell in order to have exactly $20. If she sells 10 greeting cards, she will have $20 because $1.50(10) + 5 = 20$.

What if Aditi wants to have more than $20? The inequality $1.50x + 5 > 20$ represents this situation.

While the solution to the equation was $x = 10$, the solution to the inequality is $x > 10$. Aditi will need to sell *more than* 10 cards to have *more than* $20 total.

Aditi

$1.50x + 5 = 20$

$1.50x + 5 - 5 = 20 - 5$

$1.50x = 15$

$x = 10$

10 cards makes $20

$x > 10$ represents making more than $20

Try This

Koharu is making candies for a party. She plans to give 10 candies to her sister and then include 5 candies in each gift bag. She has enough ingredients to make 100 candies.

a Solve the inequality $10 + 5x \leq 100$.

b Explain what the solutions to this inequality mean.

When solving an inequality, it can help to start by solving a related equation. The solution to the equation tells you the boundary point of the graph of the solutions to the inequality. Once you determine the boundary point, you still need to decide whether the solutions include values greater or less than the boundary point. This can be done by testing a value to the right or left of the boundary point on the number line.

Let's solve the inequality $-3x + 6 < 18$ by starting with the related equation.

Equation	Explanation
$-3x + 6 = 18$	Write the inequality as an equation.
$-3x + 6 - 6 = 18 - 6$	Subtract 6 on both sides.
$-3x \div (-3) = 12 \div (-3)$	Divide by -3 on both sides.
$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle at -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) + 6 < 18$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

Try This

a Solve the inequality $3(10 - 2x) < 18$.

b Graph the solutions of this inequality.



Summary | Lesson 17

You can use inequalities to represent and solve real-world problems. When you write an inequality, it can be helpful to first decide what quantity the variable represents. Then, write an inequality based on the relationships between the quantities in the situation.

Once you have solved your inequality, you will need to interpret the solution to ensure it makes sense for the situation. Some quantities only make sense with whole number values (e.g., number of people, number of buses, items that can be purchased, etc.), while other solutions can include decimal or fractional values (e.g., height of a roller-coaster rider, weight of a package, etc.).

Try This

Sol and their three siblings plan to order lunch from a restaurant. They each order juice, which is $\$2.50$ per person. If Sol has $\$52$ to pay for lunch, how much can each person spend on the rest of their meal?

a Write an inequality that describes Sol's situation.

b Solve the inequality you wrote. Show your thinking.

c What is the meaning of the solutions to your inequality?

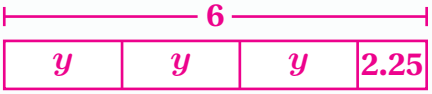
Lesson 1

Stage	Border Tiles
1	16
2	28
3	**40**
4	**52**

Caregiver Note: One strategy for predicting the number of border tiles in Stage 4 is to think of the patterns as a large square and a small square. The large square will have $4(8) + 3 = 35$ border tiles and the small square will have $4(4) + 3 = 19$. Two border tiles overlap, so there are $35 + 19 - 2 = 52$ border tiles in total.

Lesson 2

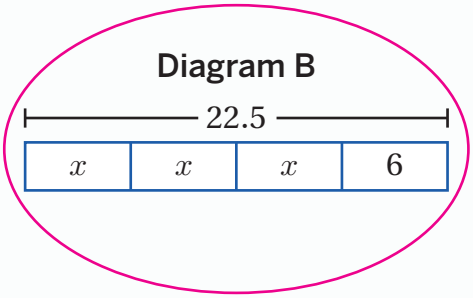
a



b **$1.25**

Lesson 3

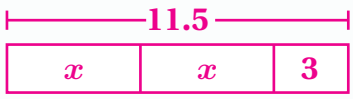
a



b $6 + 3x = 22.50$ (or equivalent)

c **$5.50**

Lesson 4

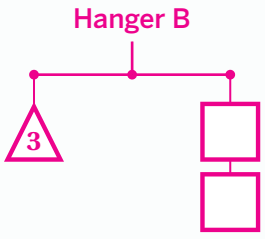
Tape Diagram	Equation
	$3 + 2x = 11.5$
Solution	**Meaning of Solution**
$x = 4.25$	\$4.25 is the cost of one sandwich.

Lesson 5

a

Weight of One Triangle (lb)	Weight of One Square (lb)
3	1.5

b *Drawings vary.*



Lesson 6

a $27 = 3x + 6$ (or equivalent)

b $x = 7$

Caregiver Note: One strategy is to subtract 6 from both sides, then divide both sides by 3.

$27 - 6 = 3x + 6 - 6$

$21 = 3x$

$21 \div 3 = 3x \div 3$

$7 = x$

Lesson 7

a $x = 18$. *Work varies.*

$$3(x - 7) \div 3 = 33 \div 3$$
$$x - 7 = 11$$
$$x - 7 + 7 = 11 + 7$$
$$x = 18$$

b $3((18) - 7) = 33$
$3(11) = 33$
$33 = 33$

Caregiver Note: One strategy to check a solution is to substitute the value of x into the equation and see if it remains true.

Lesson 8

$x = 8.5$. *Work varies.*

Expanding First		Dividing First	
$4(x - 1) = 30$	Step One: Multiply 4 by each term inside the parentheses.	$4(x - 1) = 30$	Step One: Divide both sides by 4.
$4x - 4 = 30$		$4(x - 1) \div 4 = 30 \div 4$	
$4x - 4 + 4 = 30 + 4$		$x - 1 = 7.5$	
$4x = 34$		$x - 1 + 1 = 7.5 + 1$	
$4x \div 4 = 34 \div 4$		$x = 8.5$	
$x = 8.5$			

Lesson 9

a $5(2 - 5x)$ (or equivalent)

Caregiver Note: 10 and $25x$ both have a factor of 5.

b -2 $(2x - 7)$ (or equivalent)

Caregiver Note: -4 and 14 both have a factor of -2.

Lesson 10

$8x - 4$ (or equivalent). *Work varies.*

$5(x - 2) + 3x + 6$	Expand using the distributive property.
$5x - 10 + 3x + 6$	Add terms that are alike ($5x$ and $3x$).
$8x - 10 + 6$	Add terms that are alike (-10 and 6).
$8x - 4$	

Lesson 11

$x = 10$. *Work varies.*

$3 + 5(x - 1) = 48$	Rewrite subtracting 1 as adding (-1).
$3 + 5(x + (-1)) = 48$	Distribute the 5 to the x and -1.
$3 + 5x - 5 = 48$	Add like terms (3 and -5).
$5x - 2 = 48$	Add 2 to both sides.
$5x = 50$	Divide both sides by 5.
$x = 10$	

Lesson 12

$\$15.50$. *Visual representations* and *equations vary*. $20x - 10 = 300$

Lesson 13

a No. *Explanations vary*. Riders must be *less* than 56 inches tall, and Darryl is exactly 56 inches tall.

b



c $x < 90$ (or equivalent)

Lesson 14

$4.2 > x$

Caregiver Note: One strategy is to solve a related equation. $20 > 3x + 7.4$ *is similar to* $20 = 3x + 7.4$. *Because the solution to this related equation is* $4.2 = x$, *the solution to the inequality is* $4.2 > x$.

Lesson 15

a $x \leq 18$

b *Responses vary*. Koharu can make gift bags for no more than 18 party guests.

Lesson 16

a $x > 2$

Caregiver Note: One strategy is to solve a related equation to find the boundary point. Then you can test a value to the right or left of the boundary point on the number line. The solution to $3(10 - 2x) = 18$ is $x = 2$, so the boundary point for the inequality is 2. When you substitute the value 0 into the inequality, the inequality is false. Because 0 is to the left of the boundary point and causes the inequality to be false, the solutions must be to the right of 2.

b



Lesson 17

a $4(x + 2.50) \leq 52$

b $x \leq 10.50$

A strategy for solving this inequality is to use the same steps you would use to solve a related equation, like $4(x + 2.50) = 52$.

$4(x + 2.50) = 52$

$4x + 10 = 52$

$4x = 42$

$x = 10.50$

c *Responses vary.* Sol and their siblings can spend up to $\$10.50$ each on their meals.