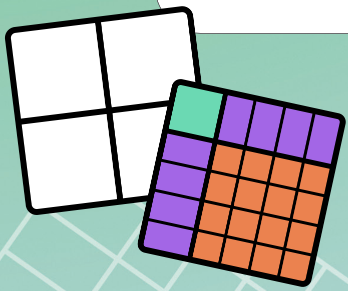
Unit 8

Quadratic Equations

Quadratic equations have fascinated mathematicians for thousands of years. In this unit, you'll learn different methods for solving quadratic equations and develop strategies for choosing your solving method strategically. You'll also connect your solving process to the graphs and key features of quadratic functions.

Essential Questions

- How can you solve quadratic equations symbolically and graphically?
- How can you use the structure of the equation, available tools, and your personal mathematical preferences to select a solving method strategically?
- How can rewriting expressions in different forms support both solving equations and identifying key features of functions?



Summary | Lesson 1

Quadratic and linear expressions are types of *polynomials*. When adding or subtracting quadratic and linear functions, you can predict what types of functions will be created.

The sum or difference of polynomials will always be a polynomial. For example, when adding or subtracting two quadratic expressions, the result can be either a quadratic, linear, or constant expression. The sum or difference of two quadratic expressions cannot become an exponential expression.

Often the goal of adding and subtracting quadratic and linear expressions is to write the expression with the fewest number of terms possible.

Here are some examples of sums and differences of quadratic and linear expressions written with the fewest number of terms possible.

Sums	Differences
$(2x - 1) + (x + 7) = 3x + 6$	$(4x^2 + 3x) - (3x - 5) = 4x^2 + 5$
$(-5x^2 + 9x - 4) + (5x^2 - 9x + 2) = -2$	$(x^2 + 4x + 1) - (x^2 + 2x - 5) = 2x + 6$

Try This

a Select *all* the true statements about the function $s(x) = (2x^2 - 6x) + (6x + 5)$.

- ☐ **A.** $s(x)$ is a concave up parabola.
- ☐ **B.** $s(x)$ is a concave down parabola.
- ☐ **C.** $s(x)$ is a line.
- ☐ **D.** $s(x)$ can be rewritten using two terms.
- ☐ **E.** $s(x)$ can be rewritten using three terms.

b Write the equation $d(x) = (3x^2 - 4x + 7) - (-2x^2 + 2x - 9)$ using the fewest number of terms.

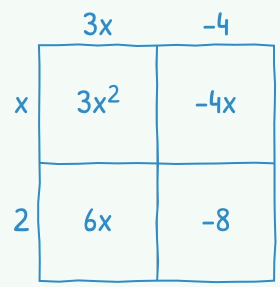
Summary | Lesson 2

Quadratic expressions can be written in *factored form* or *standard form*.

You can use an area model to help you rewrite a factored-form quadratic expression into an equivalent expression in standard form.

Here are two examples.

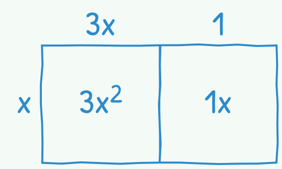
Factored form: $(3x - 4)(x + 2)$



$3x^2 + 6x - 4x - 8$

Standard form: $3x^2 + 2x - 8$

Factored Form: $x(3x + 1)$

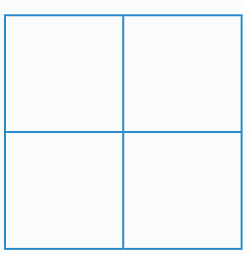


Standard form: $3x^2 + x$

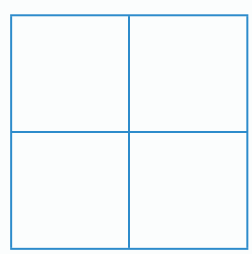
Try This

Rewrite each expression in standard form. Use the diagrams if they help with your thinking.

a $(2x - 5)(x + 3)$



b $(3x - 4)(2x + 1)$



Summary | Lesson 3

You can analyze the structure of a quadratic expression written in *factored form* to make predictions about what the equivalent expression in *standard form*, $ax^2 + bx + c$, will look like.

Here are two strategies for multiplying a factored-form expression to rewrite quadratic expressions in standard form.

Strategy 1

$(2x - 5)(x + 3)$

	2x	-5
x	$2x^2$	-5x
3	6x	-15

Standard form: $2x^2 + x - 15$

a = 2 b = 1 c = -15

Strategy 2

$(3x - 4)(3x + 4)$

$(3x - 4)(3x + 4)$

$9x^2 + 12x - 12x - 16$

Standard form: $9x^2 - 16$

a = 9 b = 0 c = -16

Here are some patterns demonstrated in these two examples:

- If the constants in factored form have opposite signs, the c-value in standard form will be negative.
- If the factors have the same *coefficients* but opposite constants, then $b = 0$ in standard form.

Try This

a Select *all* the expressions that have a negative c-value when written in standard form.

- ☐ **A.** $(x - 2)(2x + 2)$
- ☐ **B.** $(x - 5)(x - 4)$
- ☐ **C.** $-2x(x + 3)$
- ☐ **D.** $(x + 8)(3x - 7)$

b Select *all* the expressions that have a b-value of 0 when written in standard form.

- ☐ **A.** $(3x - 2)(x + 2)$
- ☐ **B.** $(x - 7)(x + 7)$
- ☐ **C.** $(4x - 2)(2x + 4)$
- ☐ **D.** $(6x - 3)(6x + 3)$

Summary | Lesson 4

A diagram can be a helpful tool for rewriting *standard-form* quadratic expressions in *factored form*. Here is an example: Rewrite the expression $3x^2 + 14x + 15$ in factored form.

Work

$3x^2$	
	15

Explanation

Place the ax^2 term in top-left corner of the diagram and the c-term in the bottom right.

	x	15
3x	$3x^2$	45x
1	x	15

Try different *factors* on the outside of the diagram that multiply to get ax^2 and c until the inside of the diagram matches standard form.

This attempt didn't work because $x + 45x \neq 14x$.
The first attempt might not work and that's okay! Try different factors or switching the positions of the current factors.

	3x	5
x	$3x^2$	5x
3	9x	15

This attempt works because the linear terms in the diagram combine to match bx in standard form: $5x + 9x = 14x$.

The factored form is $(3x + 5)(x + 3)$.

Try This

Rewrite each expression in factored form. Use the diagrams if they help with your thinking.

a $x^2 - 81$

b $2x^2 + 3x - 27$

Summary | Lesson 5

You can use the structure of a quadratic expression written in *standard form* to predict what the *factored form* will look like. Here are some strategies:

- Try to factor out a common factor first.
- If the standard form expression only has two terms, write the missing term with a coefficient of 0.
- If the c-value is negative, the signs of the constants in factored form will be different.

Here are some examples.

$3x^2 - 9x - 30$

3 is a common factor.
$3(x^2 - 3x - 10)$

Then factor $x^2 - 3x - 10$.

	x	-5
x	x^2	$-5x$
2	$2x$	-10

When $a = 1$, the constants in factored form multiply to c and add to b.

Factored form:
$3(x - 5)(x + 2)$

$x^2 - 81$

Expressions with this structure are called a *difference of squares*. Rewriting the expression with three terms might be helpful.

Factor $x^2 + 0x - 81$.

	x	9
x	x^2	$9x$
-9	$-9x$	-81

Factored form:
$(x - 9)(x + 9)$

$2x^2 + 3x - 27$

There is not a common factor in this quadratic expression.

Test pairs of expressions that multiply to $2x^2$ and -27.

The c-value is negative so the signs of the constants in factored form will be different.

	$2x$	9
x	$2x^2$	$9x$
-3	$-6x$	-27

Factored form:
$(2x + 9)(x - 3)$

Try This

Here are two expressions written in standard form.

Expression A	Expression B
$2x^2 + 22x + 60$	$3x^2 - 20x - 63$

a Factor both expressions. Create your own diagrams if they help with your thinking.

Expression A: ______________ Expression B: ______________

b How were your processes for factoring each expression alike? How were they different?

Summary | Lesson 6

An x-intercept is the coordinate point where the graph crosses the x-axis, and a **zero** is the x-value of the x-intercept.

To determine the zeros or x-intercepts, you determine the x-values that make the function equal to 0. Rewriting the function in factored form is a helpful step for determining those values.

Here are two examples of determining the x-intercepts of functions in different forms.

Factored Form: $h(x) = (x - 1)(x + 6)$

When $x = 1$, the factor $x - 1 = 0$, so $h(1) = 0$.

When $x = \text{-}6$, the factor $x + 6 = 0$, so $h(\text{-}6) = 0$.

The zeros are $x = 1$ and $x = \text{-}6$.

The x-intercepts are $(1, 0)$ and $(\text{-}6, 0)$.

Standard Form: $g(x) = x^2 - 6x - 40$

First, I can factor the function.

	x	-10
x	x^2	$\text{-}10x$
4	$4x$	-40

$g(x) = (x - 10)(x + 4)$

When $x = 10$, the factor $x - 10 = 0$, so $g(10) = 0$.

When $x = \text{-}4$, the factor $x + 4 = 0$, so $g(\text{-}4) = 0$.

The zeros are $x = 10$ and $x = \text{-}4$.

The x-intercepts are $(10, 0)$ and $(\text{-}4, 0)$.

Try This

Determine the zeros of each function. Use the diagram if it helps with your thinking.

a $h(x) = 3(x - 1)(x + 6)$

b $k(x) = 2x^2 - 14x + 24$

The **zero-product property** states that if the product of two or more *factors* is 0, then at least one of the factors is 0. You can use this property to determine the x-intercepts of a function or the *solutions* to quadratic equations using the following steps.

- Set the quadratic equation equal to 0.
- Factor the equation.
- Set each factor equal to 0.
- Solve for x.

Here are two examples of solving quadratic equations.

$(5x - 3)(2x + 3) = 0$

Set each factor equal to 0 and solve for x.

$$(5x - 3) = 0 \qquad (2x + 3) = 0$$
$$5x = 3 \qquad 2x = -3$$
$$x = \frac{3}{5} \text{ and } x = -\frac{3}{2}$$

$2x^2 - x = 21$

First, rewrite the equation so that it is equal to 0.

$2x^2 - x - 21 = 0$

Then factor the equation.
$(2x - 7)(x + 3) = 0$

Set each factor equal to 0 and solve for x.

$$(2x - 7) = 0 \qquad (x + 3) = 0$$
$$2x = 7 \qquad x = -3$$
$$x = \frac{7}{2} \text{ and } x = -3$$

Try This

Solve each equation.Show your work.

a $(10x - 5)(2x + 8) = 0$

b $40 = x^2 - 6x$

Summary | Lesson 8

To determine the number of solutions to a quadratic equation, you can use reasoning or use the structure of the equation. Here are some examples:

No Solutions	One Solution	Two Solutions
$(x + \underline{\quad})^2 =$ a negative number No value squared will result in a negative number.	$(x + \underline{\quad})^2 = 0$ Only one value squared will equal 0.	$(x + \underline{\quad})^2 =$ a positive number There are two values that when squared will equal a positive number.
$(x + 10)^2 = -25$ $(x - 3)^2 + 1 = 0$ $x^2 + 4 = 0$	$(x + 4)^2 = 0$ $x^2 + 9 = 9$ $(x - 3)(x - 3) = 0$	$(x + 4)^2 = 1$ $x^2 - 12 = -3$ $(x - 3)(x - 3) = 1$

Try This

For each equation, put a check for the number of solutions.

Determine the solution(s) for any equation with one or more solutions.

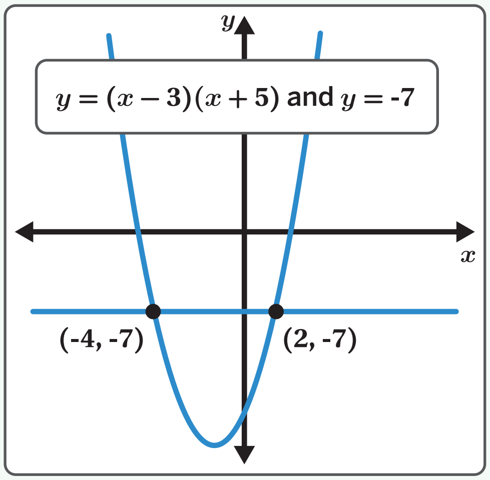
	No Solution	One Solution	Two Solutions	Solutions
$x^2 + 10 = 110$				
$(x - 8)^2 = 0$				
$x(x + 1) = 6$				
$(x - 3)^2 = -9$				

Graphs can be used to determine the solutions to a quadratic equation.

Here are two strategies using graphs to solve $(x - 3)(x + 5) = -7$.

Strategy 1

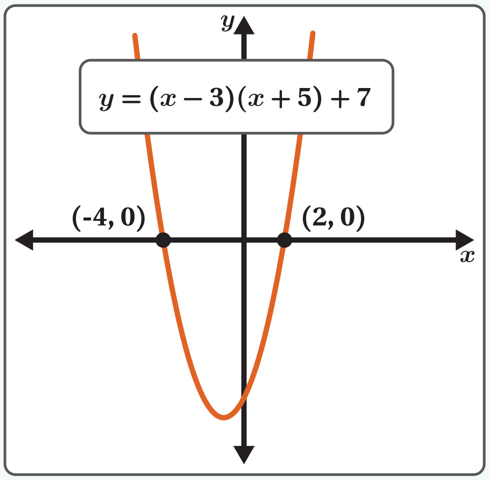
- Graph both sides of the equation as two separate graphs.
- Determine the x-coordinates where the graphs intersect.



Solutions: $x = -4$ and $x = 2$

Strategy 2

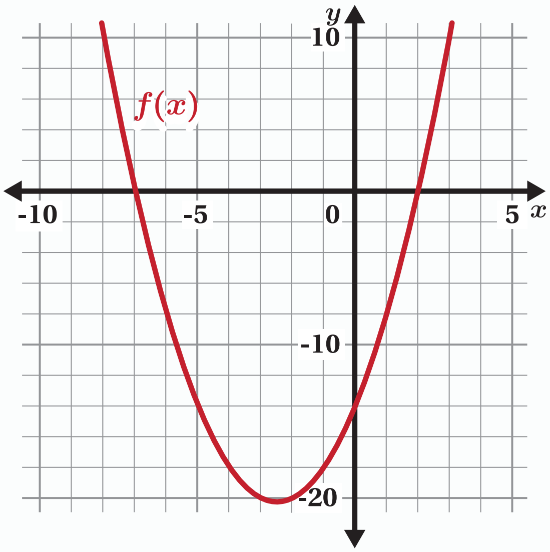
- Rewrite the equation so that it equals 0.
- Graph the equation. The solutions will be at the x-intercepts.



Solutions: $x = -4$ and $x = 2$

Try This

Use the graph of $f(x) = (x + 7)(x - 2)$ to determine the solutions to each equation.



a $0 = (x + 7)(x - 2)$

b $(x + 7)(x - 2) = 10$

Summary | Lesson 10

One strategy for solving equations in the form $(x + \_\_\_)^2 = \_\_\_$ is to take the *square root* ($\sqrt{\ }$) of both sides of the equation. When you take the square root of the equation while solving, you can use the ± **(plus/minus symbol)** to represent that there are two solutions that come from this process. Here are two examples:

$(x - 1)^2 = 36$

$$\begin{aligned} (x - 1)^2 &= 36 \\ \sqrt{(x - 1)^2} &= \sqrt{36} \\ x - 1 &= \pm\sqrt{36} \\ x - 1 &= \pm 6 \\ x &= 1 \pm 6 \\ x &= 7 \text{ and } x = -5 \end{aligned}$$

$2(x + 4)^2 = 12$

$$\begin{aligned} 2(x + 4)^2 &= 12 \\ (x + 4)^2 &= 6 \\ \sqrt{(x + 4)^2} &= \sqrt{6} \\ x + 4 &= \pm\sqrt{6} \end{aligned}$$

Exact solutions: $x = -4 \pm \sqrt{6}$

Approximate solutions: $x \approx -1.55$ and $x \approx -6.45$

If the solutions to a quadratic equation are *irrational numbers*, you can leave the square root in the expression to represent an exact solution or convert the square root to a decimal to express approximate solutions.

Try This

Determine the exact solution(s) to each equation. Show your work.

a $(x + 3)^2 = 7$

b $10(x - 5)^2 = 10$

c $(x + 4)^2 - 8 = 5$

Summary | Lesson 11

A quadratic expression is a **perfect square** if it can be represented as something multiplied by itself, like $(x +$ ______ $)^2$.

You can determine the missing constant value to add to make a perfect square by dividing the linear coefficient in half and then squaring that number.

Here are two examples of filling in the blanks to make each expression a perfect square.

$x^2 + 14x +$ ______

In perfect square expressions, the b-value, 14, is always double the constant in the factored form $(x + 7)^2$.

So the missing number must be the constant 7 squared, which is 49.

$x^2 + 14x + 49$

	x	7
x	x^2	$7x$
7	$7x$	49

$x^2 -$ ______ $+ 81$

The c-value, 81, is a perfect square so the factored form expression must be $(x - 9)^2$.

Rewrite the factored form to standard by using an area model or distributive property. The missing number must be $-18x$.

$x^2 - 18x + 81$

	x	-9
x	x^2	$-9x$
-9	$-9x$	81

Try This

Match each expression to the missing value that makes the expression a perfect square.

Expression	**Missing Value**
a. $x^2 +$ ______ $x + 9$	____ 16
b. $x^2 + 8x +$ ______	____ -16
c. $x^2 - 12x +$ ______	____ 36
d. $x^2 +$ ______ $x + 64$	____ -6

Summary | Lesson 12

You can solve quadratic equations by graphing, factoring, or **completing the square**, which is the process of rewriting a quadratic expression or equation to include a perfect square. You can analyze the structure of the equation to help you decide which strategy to use.

Here is an example: $x^2 + 10x = 2$. Solving by graphing will not produce exact solutions and factoring is not possible for this equation, so we can solve by completing the square:

Work	Explanation
$x^2 + 10x + 25 = 2 + 25$	$x^2 + 10x + 25$ is a perfect square, so add the constant value 25 to both sides of the equation.
$(x + 5)^2 = 27$	Rewrite the perfect square $x^2 + 10x + 25$ in factored form.
$x + 5 = \pm\sqrt{27}$	Take the square root and include both possibilities by writing $\pm$.
$x = -5 \pm \sqrt{27}$	Solve for x.

Try This

Determine the exact solutions to each equation. Show your work.

a $x^2 - 6x = 14$

b $x^2 + 16x - 9 = 0$

Summary | Lesson 13

Different forms of a quadratic function reveal different key features of its graph.

- Factored form reveals the x-intercepts or zeros.
- Standard form reveals the y-intercept.
- *Vertex form* reveals the maximum or minimum value of the quadratic function.

You can complete the square to rewrite a standard-form quadratic function into vertex form.

Here are two different strategies to rewrite $f(x) = x^2 - 6x + 17$ in vertex form.

Strategy 1	Strategy 2
$f(x) = x^2 - 6x + 17$	$f(x) = x^2 - 6x + 17$
$(x^2 - 6x + 9) - 9 + 17$	$(x^2 - 6x + 9) + 8$
$(x - 3)^2 - 9 + 17$	$(x - 3)^2 + 8$
$(x - 3)^2 + 8$	

The vertex of $f(x)$ is $(3, 8)$ and the graph of $f(x)$ is a concave up parabola, so 8 will be the minimum value.

Try This

Consider the function $f(x) = x^2 + 20x - 25$.

a Rewrite the function in vertex form. Show your work.

b Identify the vertex of $f(x)$.

Summary | Lesson 14

You can use the **quadratic formula** to solve quadratic equations written in standard form. The quadratic formula states that the solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The quadratic formula comes from completing the square.

Here is an example of completing the square and using the quadratic formula to solve $x^2 + 8x + 13 = 0$. Even though the answers look different, they are equivalent.

Completing the Square

Solve $x^2 + 8x + 13 = 0$

$$x^2 + 8x = -13$$

$$x^2 + 8x + 16 = -13 + 16$$

$$x^2 + 8x + 16 = 3$$

$$(x + 4)^2 = 3$$

$$x + 4 = \pm\sqrt{3}$$

$$x = -4 \pm \sqrt{3}$$

Quadratic Formula

Solve $x^2 + 8x + 13 = 0$

$a = 1,\ b = 8,\ c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{12}}{2}$$

Try This

For each quadratic equation, identify a, b, and c. Substitute those values into the quadratic formula.

	a	b	c	Quadratic Formula
$x^2 + 6x + 3 = 0$				$x = \frac{\quad \pm \sqrt{\quad}}{\quad}$
$3x^2 - 5x + 2 = 0$				$x = \frac{\quad \pm \sqrt{\quad}}{\quad}$
$-2.5x^2 + 6x - 8 = 0$				$x = \frac{\quad \pm \sqrt{\quad}}{\quad}$

Summary | Lesson 15

You can solve *any* quadratic equation written in standard form, $ax^2 + bx + c = 0$, using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here is an example of using the quadratic formula to solve the equation $2x^2 - 9x + 6 = 0$.

Work	Explanation
$a = 2$, $b = -9$, $c = 6$	Identify the a-, b-, and c-values from the standard form quadratic equation.
$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$	Substitute the a-, b-, and c-values into the quadratic formula.
$x = \frac{9 \pm \sqrt{33}}{4}$	Use order of operations to simplify the expression.
$x = \frac{9 + \sqrt{33}}{4}$ and $x = \frac{9 - \sqrt{33}}{4}$	Write the solutions.

Try This

Use the quadratic formula to determine the solutions to each equation.

a $x^2 + 6x + 3 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)\,(3)}}{2(1)}$$

b $3x^2 - 5x + 2 = 0$

$$x = \frac{\quad \pm \sqrt{\quad}}{\quad}$$

You can use quadratic functions to represent the paths of objects that are launched in the air, such as a stomp rocket. You can use functions to answer questions about the launch by:

1. Substituting the given information into the equation.
2. Solving for the missing variable using any strategy (e.g., the quadratic formula or any other available strategy).
3. Interpreting if the solution(s) make sense in the situation.

The function $h(t) = -2.5t^2 + 6t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

When will the rocket touch the ground?	When will the rocket be at a height of 10 meters?	Will the rocket reach a height of 15 meters?
Given information: $h(t) = 0$ $0 = -2.5t^2 + 6t + 8$ Using the quadratic formula, I get the following solutions: $t = -0.954$ and $t = 3.354$ Going back in time does not make sense in this situation, so the only answer is 3.354 seconds.	Given information: $h(t) = 10$ $10 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 2$ Using the quadratic formula, I get the following solutions: $t = 0.4$ and $t = 2$ The rocket will be at a height of 10 meters at both .4 seconds and 2 seconds.	Given information: $h(t) = 15$ $15 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 7$ Using the quadratic formula, I get no solution: $t = \frac{-6 \pm \sqrt{-34}}{-5}$ This means the rocket will never reach a height of 15 feet.

Try This

The function $h(t) = -2t^2 + 8t - 6$ represents the height of a stomp rocket, in meters, t seconds after it has been launched.

a Write an equation that can determine when the rocket will return to its original height of -6 meters.

b How long will it take for the rocket to return to its original height?

Rational numbers are numbers that you can write as a fraction with an integer numerator and denominator. *Irrational numbers* are numbers that are not rational, which means you cannot write them as a fraction with an integer numerator and denominator.

Here are some properties of sums and products of rational and irrational numbers.

	Sums		Products	
Properties	The sum of two rational numbers is always rational.	The sum of a rational number and an irrational number is always irrational.	The product of two rational numbers is always rational.	The product of a nonzero rational number and an irrational number is always irrational.
Examples	$\frac{1}{2} + 3$ $8 + \sqrt{25}$	$\sqrt{7} + 2$ $\frac{1}{2} + \sqrt{44}$	$\frac{2}{3} \cdot 0.\overline{1}$ $\frac{4}{5} \cdot \left(-\frac{7}{11}\right)$	$\pi \cdot \frac{1}{3}$ $2 \cdot \sqrt{5}$

Try This

Select *all* the expressions that are rational.

- ☐ **A.** $\frac{1}{2} + \frac{3}{4}$
- ☐ **B.** $\frac{2}{5} \cdot \sqrt{2}$
- ☐ **C.** $\sqrt{2} \cdot \sqrt{8}$
- ☐ **D.** $\frac{2}{3} \cdot \frac{13}{9}$
- ☐ **E.** $\sqrt{5} + \sqrt{7}$
- ☐ **F.** $\sqrt{3} - \sqrt{3}$

Summary | Lesson 18

To start solving a system of linear and quadratic equations, you can use *substitution* or *elimination*. After you create a new quadratic equation with only one variable, you pick any quadratic solving strategy to determine the values of x, such as:

- Factoring and using the zero-product property.
- Taking the square root.
- Completing the square.
- Using the quadratic formula.

Here's an example of using substitution, factoring, and the zero-product property to solve this *system of equations*:

$$y = 2x - 7$$

$$y = x^2 - 5x + 3$$

$$y = \boxed{2x - 7} \rightarrow y = x^2 - 5x + 3$$

$$2x - 7 = x^2 - 5x + 3$$

$$0 = x^2 - 7x + 10$$

$$0 = (x - 5)(x - 2)$$

$$\boxed{x = 5} \qquad \boxed{x = 2}$$

$$y = 2(5) - 7 \qquad y = 2(2) - 7$$

$$\boxed{y = 3} \qquad \boxed{y = -3}$$

Points of intersection:

$$\boxed{(5, 3)} \qquad \boxed{(2, -3)}$$

Try This

Solve each system of equations. Show your work.

a $y = 9x^2 - 7$
$y = -18x$

b $y = x^2 - 3x$
$y = 2x - 6$

Lesson 1

a A. $s(x)$ is a concave up parabola.

D. $s(x)$ can be rewritten using two terms.

E. $s(x)$ can be rewritten using three terms.

b $d(x) = 5x^2 - 6x + 16$

Lesson 2

a $2x^2 + x - 15$

b $6x^2 - 5x - 4$

Lesson 3

a A. $(x - 2)(2x + 2)$

D. $(x + 8)(3x - 7)$

b B. $(x - 7)(x + 7)$

D. $(6x - 3)(6x + 3)$

Lesson 4

a $(x - 9)(x + 9)$

b $(2x + 9)(x - 3)$

Lesson 5

a Expression A: $2(x + 5)(x + 6)$

Expression B: $(3x + 7)(x - 9)$

b *Responses vary.*

- In Expression A, I was able to factor out a 2 from every term, leaving the expression $2(x^2 + 11x + 30)$. However, in Expression B, there isn't a number that can be factored from every term.
- When factoring Expression B, I had to consider both the factors of 3 and -63. After factoring the 2 out of Expression A, I only had to focus on factors of 30.
- After listing the factors of both expressions, I had to find the right combination of factors that would add up to the b-value in both expressions.

Lesson 6

a $x = 1$ and $x = -6$

b $x = 4$ and $x = 3$

Lesson 7

a $x = 0.5$ and $x = -4$. *Work varies.*

$(10x - 5)(2x + 8) = 0$

$$\begin{array}{rcl} 10x - 5 &=& 0 \\ +5 &+& 5 \\ \hline 10x &=& 5 \\ x &=& \frac{5}{10} = \frac{1}{2} = 0.5 \end{array}$$

$$\begin{array}{rcl} 2x + 8 &=& 0 \\ -8 &-& 8 \\ \hline 2x &=& -8 \\ x &=& -4 \end{array}$$

b $x = 10$ and $x = -4$. *Work varies.*

$40 = x^2 - 6x$

$x^2 - 6x - 40 = 0$

$(x - 10)(x + 4) = 0$

$$\begin{array}{rcl} x - 10 &=& 0 \\ +10 &+& 10 \\ \hline x &=& 10 \end{array}$$

$$\begin{array}{rcl} x + 4 &=& 0 \\ -4 &-& 4 \\ \hline x &=& -4 \end{array}$$

Lesson 8

	No Solution	One Solution	Two Solutions	Solutions
$x^2 + 10 = 110$			✓	$x = 10$ $x = -10$
$(x - 8)^2 = 0$		✓		$x = 8$
$x(x + 1) = 6$			✓	$x = -3$ $x = 2$
$(x - 3)^2 = -9$	✓			

Lesson 9

a $x = -7$ and $x = 2$

b $x = -8$ and $x = 3$

Lesson 10

a $x = \text{-}3 \pm \sqrt{7}$. *Work varies.*

$(x+3)^2 = 7$

$\sqrt{(x+3)^2} = \sqrt{7}$

$x + 3 = \pm\sqrt{7}$

$-3 \quad -3$

$x = \text{-}3 \pm \sqrt{7}$

b $x = 5 \pm 1$ or $x = 6$ and $x = 4$. *Work varies.*

$10(x-5)^2 = 10$

$(x-5)^2 = 1$

$\sqrt{(x-5)^2} = \sqrt{1}$

$x - 5 = \pm 1$

$+5 \quad +5$

$x = 5 \pm 1$

c $x = \text{-}4 \pm \sqrt{13}$. *Work varies.*

$(x+4)^2 - 8 = 5$

$+8 \; +8$

$(x+4)^2 = 13$

$\sqrt{(x+4)^2} = \sqrt{13}$

$x + 4 = \pm\sqrt{13}$

$-4 \quad -4$

$x = \text{-}4 \pm \sqrt{13}$

Lesson 11

__b__ 16

__d__ -16

__c__ 36

__a__ -6

Lesson 12

a $x = 3 \pm \sqrt{22}$. *Work varies.*

$$x^2 - 6x = 14$$
$$x^2 - 6x + 9 = 14 + 9$$
$$(x - 3)^2 = 22$$
$$x - 3 = \pm\sqrt{22}$$
$$x = 3 \pm \sqrt{22}$$

b $x = -8 \pm \sqrt{73}$. *Work varies.*

$$x^2 + 16x - 9 = 0$$
$$x^2 + 16x = 9$$
$$x^2 + 16x + 64 = 9 + 64$$
$$(x + 8)^2 = 73$$
$$x + 8 = \pm\sqrt{73}$$
$$x = -8 \pm \sqrt{73}$$

Lesson 13

a $f(x) = (x + 10)^2 - 125$. *Work varies.*

$$f(x) = x^2 + 20x - 25$$
$$f(x) = (x^2 + 20x + 100) - 100 - 25$$
$$f(x) = (x + 10)^2 - 100 - 25$$
$$f(x) = (x + 10)^2 - 125$$

b (-10, -125)

Lesson 14

	a	b	c	Quadratic Formula
$x^2 + 6x + 3 = 0$	1	6	3	$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)}$
$3x^2 - 5x + 2 = 0$	3	-5	2	$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$
$-2.5x^2 + 6x - 8 = 0$	-2.5	6	-8	$x = \frac{-6 \pm \sqrt{6^2 - 4(-2.5)(-8)}}{2(-2.5)}$

Lesson 15

a. $x = \frac{-6 \pm \sqrt{24}}{2}$ (or equivalent)

b. $x = \frac{2}{3}$ and $x = 1$

Lesson 16

a. $-6 = -2t^2 + 8t - 6$

b. 4 seconds

Lesson 17

A. $\frac{1}{2} + \frac{3}{4}$

C. $\sqrt{2} \cdot \sqrt{8}$

D. $\frac{2}{3} \cdot \frac{13}{9}$

F. $\sqrt{3} - \sqrt{3}$

Lesson 18

a $(\frac{1}{3}, -6)$ and $(-\frac{7}{3}, 42)$. *Work varies.*

$y = 9x^2 - 7$

$y = -18x$

$-18x = 9x^2 - 7$

$0 = 9x^2 + 18x - 7$

$$x = \frac{-18 \pm \sqrt{18^2 - 4(9)(-7)}}{2(9)} = \frac{-18 \pm \sqrt{576}}{18} = \frac{-18 \pm 24}{18} = \frac{-3 \pm 4}{3}$$

$x = \frac{-7}{3}$ and $x = \frac{1}{3}$

$-18\left(\frac{-7}{3}\right) = 42$ and $-18\left(\frac{1}{3}\right) = -6$

b $(2, -2)$ and $(3, 0)$. *Work varies.*

$y = x^2 - 3x$

$y = 2x - 6$

$2x - 6 = x^2 - 3x$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2$ and $x = 3$

$2(2) - 6 = -2$ and $2(3) - 6 = 0$

Algebra 1 Unit 8 Glossary/Álgebra 1 Unidad 8 Glosario

English

C

completing the square The process of rewriting a quadratic expression or equation to include a perfect square.

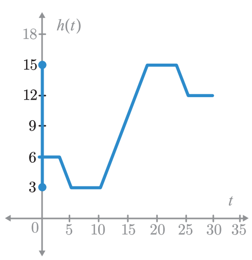
$$f(x) = x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

D

difference of squares An expression that can be written as two perfect squares subtracted from one another. It has the structure $r^2 - s^2$ in standard form and $(r - s)(r + s)$ in factored form.

These expressions are differences of squares: $x^2 - 25$, $9x^2 - 100$, and $16x^2 - 1$.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as: All numbers from 0 to 30 or $0 \le t \le 30$.

E

elimination A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

$$\begin{aligned} 9x + y &= 2 \\ -(3x + y &= 10) \\ \hline 6x + 0 &= -8 \end{aligned}$$

In the example, subtraction is used to eliminate y and create an equation that can be solved for x.

Español

C

completación del cuadrado El proceso de reescribir una expresión o ecuación cuadrática para incluir un cuadrado perfecto.

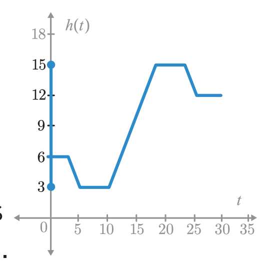
$$f(x) = x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

D

diferencia de cuadrados Una expresión que puede escribirse como dos cuadrados perfectos restados entre sí. Tiene la estructura $r^2 - s^2$ en forma estándar y $(r - s)(r + s)$ en forma factorizada.

Estas expresiones son diferencias de cuadrados: $x^2 - 25$, $9x^2 - 100$ y $16x^2 - 1$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera:
Todos los números del 0 al 30 o $0 \le t \le 30$.

E

eliminación Un método para resolver sistemas de ecuaciones en el que se suman o restan las ecuaciones para producir una nueva ecuación con menos variables.

$$\begin{aligned} 9x + y &= 2 \\ -(3x + y &= 10) \\ \hline 6x + 0 &= -8 \end{aligned}$$

En el ejemplo, se usa la resta para eliminar y y producir una ecuación que puede resolverse para determinar el valor de x.

English | Español

F

factor (of a number or expression) A number or expression multiplied with other numbers or expressions to make a product.

For example, 1, 2, 4, and 8 are all factors of the number 8 because $1 \cdot 8 = 8$ and $2 \cdot 4 = 8$. Also, $(x + 3)$ and $(x - 5)$ are factors of $x^2 - 2x - 15$ because $(x + 3)(x - 5) = x^2 - 2x - 15$.

factor (de un número o una expresión) Un número o una expresión que se multiplica por otros números o expresiones para dar como resultado un producto.

Por ejemplo, 1, 2, 4 y 8 son factores del número 8 porque $1 \cdot 8 = 8$ y $2 \cdot 4 = 8$. Además, $(x + 3)$ y $(x - 5)$ son factores de $x^2 - 2x - 15$ porque $(x + 3)(x - 5) = x^2 - 2x - 15$.

factored form One of three common forms of a quadratic equation. A quadratic equation in factored form looks like: $f(x) = a(x - m)(x - n)$.

These equations are in factored form:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

forma factorizada Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma factorizada tiene el siguiente orden: $f(x) = a(x - m)(x - n)$.

Estas ecuaciones están en forma factorizada:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

I

irrational number A number that cannot be written as a fraction with integers as the numerator and denominator.

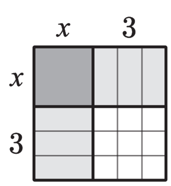
2 is a rational number because it can be written as $\frac{2}{1}$, whereas $\sqrt{3}$ is irrational because it cannot be written as a fraction made up of two integers.

número irracional Números que no pueden escribirse como una fracción con números enteros en el numerador y el denominador.

2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que $\sqrt{3}$ es irracional porque no se puede escribir como una fracción de dos números enteros.

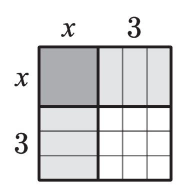
P

perfect square An expression that can be written as something multiplied by itself.



These expressions are perfect squares: 3^2, 9, $(x + 3)^2$, and $x^2 + 6x + 9$.

cuadrado perfecto Una expresión que puede escribirse como algo multiplicado por sí mismo.



Estas expresiones son cuadrados perfectos: 3^2, 9, $(x + 3)^2$ y $x^2 + 6x + 9$.

Algebra 1 Unit 8 Glossary/Álgebra 1 Unidad 8 Glosario

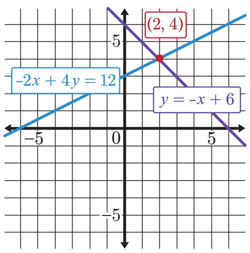
English

$\pm$ (plus/minus symbol) A symbol used to represent both the positive and negative values of a number. It also can be used to represent two expressions.

± 9 represents -9 and +9.
2 ± 3 represents $2 - 3$ and $2 + 3$.

point of intersection A point where two lines or curves meet.

For example, $(2, 4)$ is the point of intersection for the lines $-2x + 4y = 12$ and $y = -x + 6$.



polynomial An expression that can be written as the sum of terms, each of which is a number multiplied by a power of a variable. The word polynomial is used to refer both to the expression and to the function it defines. Linear and quadratic functions are both examples of polynomial functions.

$2x - 7$ and $3x^2 - x + 4$ are polynomial expressions.
$f(x) = 2x - 7$ and $g(x) = 3x^2 - x + 4$ are polynomial functions.

Q

quadratic formula A formula that can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

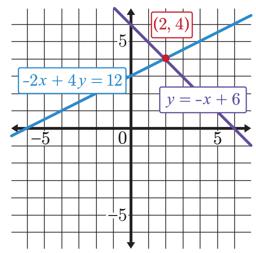
Español

$\pm$ (signo más menos) Un símbolo que se usa para representar los valores positivos y negativos de un número. También puede usarse para representar dos expresiones.

± 9 representa -9 and +9.
2 ± 3 representa $2 - 3$ y $2 + 3$.

punto de intersección Un punto donde se cruzan dos rectas o curvas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



polinomio Una expresión que puede escribirse como la suma de términos, cada uno de los cuales es un número multiplicado por una potencia de una variable. La palabra polinomio se emplea para referirse tanto a la expresión como a la función que define. Las funciones lineales y cuadráticas son ejemplos de funciones polinómicas.

$2x - 7$ y $3x^2 - x + 4$ son expresiones polinómicas.
$f(x) = 2x - 7$ y $g(x) = 3x^2 - x + 4$ son funciones polinómicas.

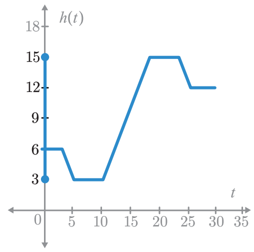
fórmula cuadrática Una fórmula que puede usarse para determinar las soluciones de una ecuación cuadrática $ax^2 + bx + c = 0$, donde $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

English	Español

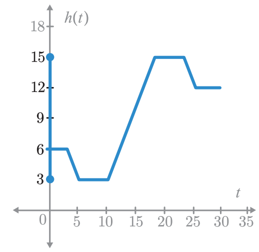
R

range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.



The range of this graph can be described as:
All numbers from 3 to 15 or $3 \leq h(t) \leq 15$.

rango (de una función) El conjunto de todos los posibles valores de salida de una función o relación. El rango puede describirse con palabras o como una desigualdad.



El rango de esta gráfica puede describirse de la siguiente manera:
Todos los números del 3 al 15 o $3 \leq h(t) \leq 15$.

rational number A number that can be written as a fraction with a non-zero denominator.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5, and $\sqrt{9}$ are rational numbers.

número racional Números que pueden escribirse como una fracción con números enteros en el numerador y el denominador.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5 y $\sqrt{9}$ son números racionales.

S

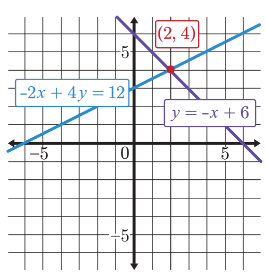
solution A value or set of values that makes an equation or inequality true.

For example:
$x = 2$ is a solution to the equation $3x + 4 = 10$.
$x > 2$ is the solution to the inequality $3x + 4 > 10$.
The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.

solución Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

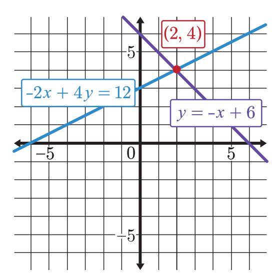
Por ejemplo:
$x = 2$ es una solución de la ecuación $3x + 4 = 10$.
$x > 2$ es la solución de la desigualdad $3x + 4 > 10$.
El par ordenado $(1, 2)$ es una solución de la ecuación $3x + 4y = 11$.

solution to a system of equations A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the intersection point.



For the system $y = -x + 6$ and $-2x + 4y = 12$, $(2, 4)$ is the solution to this system of equations and the intersection point on the graph.

solución de un sistema de ecuaciones Una solución de un sistema de ecuaciones es un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.



Por ejemplo, en el sistema de ecuaciones $y = -x + 6$ y $-2x + 4y = 12$, la solución y el punto de intersección de la gráfica es $(2, 4)$.

Algebra 1 Unit 8 Glossary/Álgebra 1 Unidad 8 Glosario

English

square root The square root of a number n (written as $\sqrt{n}$) is the positive number that can be squared to get n. The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

standard form (of a quadratic equation) One of three common forms of a quadratic equation. A quadratic equation in standard form looks like: $f(x) = ax^2 + bx + c$.

These equations are in standard form:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

substitution A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

$y = -4x + 6$ → $y = 3x - 15$
$-4x + 6 = 3x - 15$
$-7x = -21$
$x = 3$
$y = 3(3) - 15$
$y = -6$

For example, we can substitute $-4x + 6$ in for y in $y = 3x - 15$ because they are equivalent.

system of equations Two or more equations that represent the constraints on a shared set of variables form a system of equations.

These equations make a system:
$3b + c = -2$
$b - 5c = 12$

Español

raíz cuadrada La raíz cuadrada de un número n (se escribe $\sqrt{n}$) es el número positivo que puede elevarse al cuadrado para obtener n. La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

forma estándar (de una ecuación cuadrática) Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma estándar tiene el siguiente orden: $f(x) = ax^2 + bx + c$.

Estas ecuaciones están en forma estándar:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

sustitución Un método para resolver sistemas de ecuaciones donde una variable se reemplaza con una expresión equivalente para producir una nueva ecuación con menos variables.

$y = -4x + 6$ → $y = 3x - 15$
$-4x + 6 = 3x - 15$
$-7x = -21$
$x = 3$
$y = 3(3) - 15$
$y = -6$

Por ejemplo, podemos introducir $-4x + 6$ en lugar de y en $y = 3x - 15$ porque son equivalentes.

sistema de ecuaciones Dos o más ecuaciones que representan las restricciones de un conjunto compartido de variables forman un sistema de ecuaciones.

Estas ecuaciones forman un sistema:
$3b + c = -2$
$b - 5c = 12$

English	Español

V

vertex form One of three common forms of a quadratic equation. A quadratic equation in vertex form looks like: $f(x) = a(x - h)^2 + k$.

These equations are in vertex form:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

forma de vértice Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma de vértice tiene el siguiente orden: $f(x) = a(x - h)^2 + k$.

Estas ecuaciones están en forma de vértice:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

Z

zero-product property A property which states that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to help solve equations.

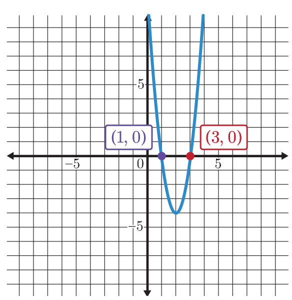
If $(2x - 3)(x + 1) = 0$, then either $2x - 3 = 0$ or $x + 1 = 0$.

propiedad del producto cero Una propiedad que establece que si el producto de dos o más factores es 0, entonces al menos uno de los factores es 0. Esta propiedad puede usarse como ayuda para resolver ecuaciones.

Si $(2x - 3)(x + 1) = 0$, entonces $2x - 3 = 0$ o $x + 1 = 0$.

zeros The x-values that make a function equal zero, or $f(x) = 0$.

The zeros of $f(x) = 4(x - 1)(x - 3)$ are 1 and 3.



ceros Los valores x que hacen que una función sea igual a cero, o $f(x) = 0$.

Los ceros de $f(x) = 4(x - 1)(x - 3)$ son 1 y 3.