Solving an equation means taking steps to determine a **solution**. A solution to an equation is a value that makes the equation true. There are many ways to solve an equation, including working backwards, inverse operations, and moves that keep the equation balanced.

Here are two strategies for solving the equation -4(x + 2) = 20:

$$x$$
 $x+2$ $-4(x+2)$ -7 -5 20

$$\frac{-4(x+2)}{-4} = \frac{20}{-4}$$

$$x + 2 = -5$$

$$-2 - 2$$

$$x = -7$$

You can check that the value you determined is a solution to an equation by substituting the value back into the equation to see if it makes the equation true. The solution to the equation -4(x + 2) = 20 is x = -7 because -4(-7 + 2) = 20 is a true statement.

Try This

Here is an equation: -30 = 5(x - 2).

a Solve the equation. Use the table if it helps with your thinking.

x	x-2	5(x-2)

b Check your solution.

You can solve one-variable equations by creating *equivalent equations*. To create equivalent equations, use moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of solving moves that keep an equation balanced:

$$-3m+5+m=2(6m+3) \qquad \text{This is the original equation.}$$

$$-2m+5=12m+6 \qquad \text{Combine like terms on the left and distribute on the right.}$$

$$5=14m+6 \qquad \text{Add } 2m \text{ to each side of the equation.}$$

$$-1=14m \qquad \text{Subtract 6 from each side of the equation.}$$

$$-\frac{1}{14}=m \qquad \text{Divide each side of the equation by } 14.$$

All of the equations created at each step of this solution process are equivalent equations because they have the same solution: $m = -\frac{1}{14}$.

Try This

Emma made an error while solving the equation

$$\frac{1}{2}(x+4) = -10 + 3x.$$

- a Describe one thing that Emma did well.
- **b** Circle the error and explain what the error was.
- Solve $\frac{1}{2}(x+4) = -10 + 3x$.

$$\frac{1}{2}(x+4) = -10 + 3x$$

$$x + 4 = -20 + 3x$$

$$x + 24 = 3x$$

$$24 = 2x$$

$$12 = x$$

Not all one-variable linear equations have a single solution. Some linear equations have *infinitely many solutions*, and some have *no solution*.

In the process of solving, there is a difference between equations with one solution, no solution, or infinitely many solutions:

- In an equation with one solution, a single value of x will make the equation true.
- In an equation with no solution, *no value* of *x* will make the equation true.
- In an equation with infinitely many solutions, any value of x will make the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
3x + 4 = 2x + 10	2x + 4 = 2x + 10	2(x+5) = 2x + 10
3x = 2x + 6	4 = 10	2x + 10 = 2x + 10
x = 6	This is <i>never</i> true!	10 = 10
		This is <i>alway</i> s true!

If the variable in an equation is eliminated during the solving process, the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

Try This

Group the equations based on their number of solutions.

$$4t + 7 = 2(2t + 2)$$
 $3x = 10 - 3x$ $10 - 2(t + 5) = -2t$ $4t = 6t$ $5t + 7 = -3 + 5t$

One Solution	No Solution	Infinitely Many Solutions

Two-variable linear equations can be written in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations that represent the number of seats and handholds possible in a subway car that has 300 square feet of floor space. They are each written in different forms, and they each reveal different information about the situation. In each equation, t is the seating capacity and d is the standing capacity.

$$4t + 2d = 300$$

- Each seat takes up 4 square feet of floor space.
- · Each standing spot requires 2 square feet of floor space.
- The total amount of floor space is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats (t = 0), 150 passengers can stand in the car.
- · For every seat that is added, the number of spots for standing decreases by 2.



Try This

Tiara is trying to save \$240 for a new gaming console. To earn the money she needs, she works at the pool for \$8 an hour and tutors Spanish for \$12 an hour.

Tiara wrote the equation 8p + 12t = 240 to represent her situation.

- **a** What do the variables p and t represent in Tiara's situation?
- **b** Which equation is equivalent to Tiara's equation?

A.
$$t = 240 - 8p$$

B.
$$t = 20 - \frac{2}{3}t$$

C.
$$t = 30 - \frac{3}{2}t$$

A.
$$t = 240 - 8p$$
 B. $t = 20 - \frac{2}{3}p$ **C.** $t = 30 - \frac{3}{2}p$ **D.** $t = -\frac{2}{3}p + 30$

You can solve equations that contain multiple variables using some of the same strategies for solving equations with one variable. These strategies include working backwards, using inverse operations, and keeping the equation balanced.

When solving problems that model real-world situations, it can be helpful to rearrange an equation to highlight a variable of interest or make calculations easier. Rearranging equations can also reveal different relationships between the variables and the quantities that they represent.

Here is how you could rearrange the equation y = mx + b to solve for m or b.

Solving for m y = mx + b y - b = mx + b - b $\frac{y - b}{x} = \frac{mx}{x}$ $\frac{y - b}{x} = m$ y = mx + b y - mx = mx + b - mx y - mx = b

Rearranging equations for a specific variable can help make some calculations easier. For example, if you know the values of r and s for 6r + 4s = 240, solving the equation for s can make it simpler to test different values of r and s to see how they affect the value of s.

Try This

Solve each equation on the left. Then use the same strategy to solve the equation on the right.

a Solve for
$$t$$
.

$$10 = 4 - 3t 10 = v - at$$

b Solve for
$$m$$
.

$$\frac{m}{3} + 7 = -4$$

$$\frac{m}{a} + t = h$$

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation (make the equation true).

Linear equations can be written in different but equivalent forms. Rearranging equations helps reveal new information, such as the $\underline{x\text{-intercept}}$ and $\underline{y\text{-intercept}}$, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for \$3 per cup and cookies for \$2 each. The stand made \$12. ℓ represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in many different ways:

Equation

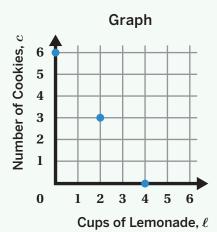
$$3\ell + 2c = 12$$

Equation Solved for \boldsymbol{c}

$$c = 6 - \frac{3}{2}\ell$$

Table

ℓ	0	2	4
c	6	3	0



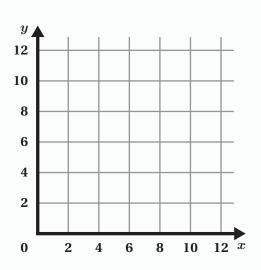
Try This

Here is an equation: 6x + 2y = 24.

- **a** Solve the equation for y.
- **b** Graph the equation.

Use the table if it helps with your thinking.

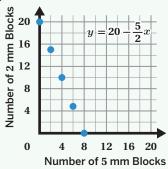
$oldsymbol{x}$	y



There are many ways to represent a situation that involves two variables. Each of these representations is connected and reveals information about how the quantities in the situation are related.

A description explains how the quantities are related verbally or in writing. For example: A snail needs to cross a gap that is 40 mm tall using 5 mm and 2 mm blocks.

A graph reveals the slope and y-intercept. This graph shows a slope of $-\frac{5}{2}$ and a y-intercept of 20.



A table reveals several combinations of 5 mm and 2 mm blocks that let the snail cross the gap.

x	0	2	4	6
y	20	15	10	5

The ordered pairs can be used to make a graph or find the rate of change (slope).

Equations reveal how the variables are related. Standard form mirrors the language of the situation. Slope-intercept form shows how the number of 2-mm blocks change as you change the number of 5-mm blocks.

Standard form

Slope-intercept form (solved for one variable)

$$5x + 2y = 40$$

$$y = 20 - \frac{5}{2}x$$

Try This

Neel won 20 tickets at the fair to trade in for prizes. He can get a sticker for 2 tickets and a pencil for 3 tickets. Let s represent the number of stickers Neel gets and p represent the number of pencils.

- a Write an equation to represent how many of each kind of prize Neel can get.
- **b** Solve your equation for s.
- f c What can you determine about the situation by solving the equation for s?

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs \$12 and there is a delivery fee of \$8.	Tasia can spend <i>up to</i> \$140.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols >, \geq , <, and \leq to represent the nature of the constraint. Terms like *greater than*, *less than*, *at most*, *at least*, or *up to* can help you determine which inequality symbol to use.

Constraints modeled by inequalities produce a set of solutions. When using inequalities to model constraints, it is important to note that not all solutions to inequalities are viable solutions or make sense for the situation.

Try This

Valeria wants to donate at least \$120 to her local food bank. She has already saved \$64 and is planning to save \$8 each week.

a Why is Valeria's situation an example of a constraint?

b Write an inequality to match Valeria's situation.

Use w to represent the number of weeks that Valeria will save \$8.

The *solution* set of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution region* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for $2x - 4 \ge 8$:

Determine the boundary point:

$$2x - 4 = 8$$

$$2x = 12$$

$$x = 6$$

- 6 is the boundary point and the ≥ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.

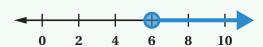
When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution -0.5 > x graphed on a number line.

Determine all the solutions:

$$2x - 4 \ge 8$$

$$2(0) - 4 \ge 8$$

$$4 \ge 8$$
 False!

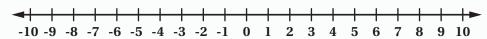




Try This

Here is an inequality: 5 + 3x < 32.

- a Determine the boundary point.
- **b** Graph all the solutions to the inequality. Show or explain your thinking.



You can solve any one-variable inequality by solving its corresponding equation to determine the boundary point, then testing values to determine the rest of the solutions.

Here's an example of how you could solve the inequality 10 > -3x - 2.

$$10 = -3x - 2$$
.

$$10 = -3x - 2$$
$$12 = -3x$$

The solution to this equation is -4, so the boundary point is x = -4.

$$-4 = x$$

Step 2: Test values to determine the rest of the solutions.

$$x = -5$$

 $10 > -3(-5) -2$

$$x = 0$$

In this example, we tested
$$x = -5$$
 and $x = 0$.

$$10 > -3(0) - 2$$

 $10 > -2$

When x= -5, the inequality is false, so x= -5 is not a solution. This means that the solutions

are greater than -4.

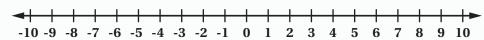
False! True!

The solutions to an inequality do not always have the same inequality symbol as the original inequality. Since the solutions are *greater than -4*, the solution set can be written as -4 < x or x > -4.

Try This

Here is an inequality: $11 - 2x \ge 5$.

- a Solve this inequality.
- **b** Graph all the solutions to this inequality.



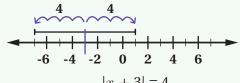
The absolute value of a number is its distance from 0 on a number line. Equations with absolute values often have two solutions because there are often two numbers that are the same distance from a number.

For example, the equation |x + 3| = 4 has two solutions, x = 1 and x = -7, because 1 and -7 are both 4 units away from -3.

You can determine the solutions using a number line or by solving two equations.

To determine the solutions to an inequality with an absolute value, you can:

- Step 1: Solve for the boundary points.
- Step 2: Determine whether the boundary points are included in the solution set. 1 and -7 are *not* included.
- Step 3: Test a value between the two boundary points to decide which values make the inequality true.
- Step 4: Graph all the solutions.



$$|x+3|=4$$

$$x + 3 = 4$$
 $-(x + 3) = 4$
 $x = 1$ $x = -7$

$$|x + 3| > 4$$

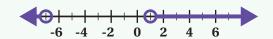
Boundary points:

$$x = 1$$
 $x = -7$

Test
$$x = 0$$
:

$$|(0) + 3| > 4$$

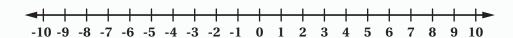
False!



Try This

Here is an inequality: $|x - 3| \le 2$.

- a List four solutions to this inequality.
- **b** Graph all the solutions to this inequality. Show or explain your thinking.

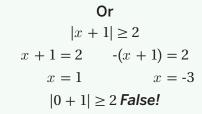


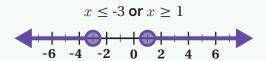
We can write the solution to an absolute value inequality using a **compound inequality** that joins two or more inequalities with the words *and* or *or*. Using the word "and" communicates that the solutions are located between the two boundary points. Using the word "or" communicates that the solutions are located to the left of one boundary point and to the right of the other.

Here are two examples of absolute value inequalities and their solutions written as compound inequalities.

And
$$|x+1| \ge -2$$
 $x+1=-2$ $-(x+1)=-2$ $x=-3$ $x=1$ $|0+1| \ge -2$ *True!*

$$x \ge -3$$
 and $x \le 1$
 -6 -4 -2 0 2 4 6





Try This

Match each absolute value inequality with its solutions. One set of solutions will have no match.

$$x < 4$$
 and $x > -5$ $x > 4$ or $x < -5$ $x < -4$ or $x > 5$ $x > -4$ and $x < 5$

2x + 1 > 9	2x + 1 < 9	2x - 1 < 9

The solutions to a two-variable inequality are all of the ordered pairs that make the inequality true.

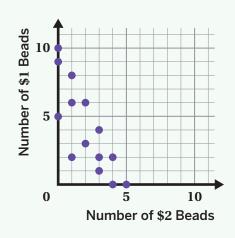
Here is an example of how you can determine if an ordered pair is a solution to an inequality: Marco is making bracelets. Each bracelet needs to cost no more than \$10. Planet beads cost \$1 and oval beads cost \$2. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

Marco is wondering if (4,3) is a solution to the inequality $2x + y \le 10$, where x represents the number of \$2 beads and y represents the number of \$1 beads. To check, Marco substitutes x = 4 and y = 3 into the inequality:

$$2(4) + (3) \le 10$$

$$8 + (3) \le 10$$

$$11 \le 10$$
 False!



That means that (4, 3) is *not* a solution and Marco *cannot* make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

Try This

The Theater Club makes \$5 for every student ticket that they sell, x, and \$7 for every adult ticket, y. They want to make at least \$180 to buy costumes for their next show.

a Which inequality or equation represents this situation?

A.
$$5x + 7y \le 180$$

B.
$$5x + 7y = 180$$

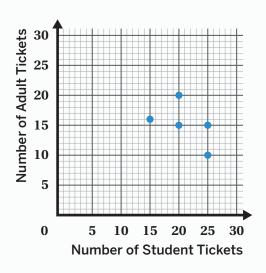
C.
$$5x + 7y \ge 180$$

D.
$$7y = 5x + 180$$

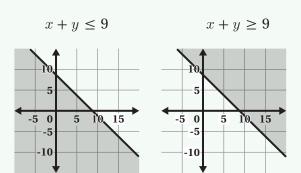
b This graph shows some solutions to the Theater Club's situation.

Choose one solution.

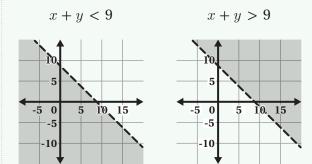
Show that this point is a solution to the inequality or equation that you chose in the previous problem.



The solutions to a two-variable linear inequality can be represented on a graph as a half-plane. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.



A solid line means that the points on the boundary line are included in the solutions. This is represented by the \leq and \geq symbols.



A dashed line means the points on the boundary line are not included in the solutions.

This is represented by the < and > symbols.

To determine which of the half-planes is the **solution region**, you can test points on either side of the boundary line to see whether they make the inequality true or false.

Try This

Here is the graph of an inequality.

Which inequality does this graph represent?

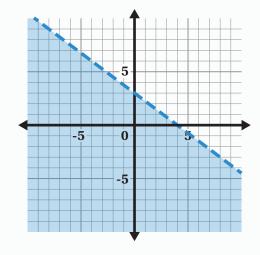
A.
$$3x + 4y < 12$$

B.
$$3x + 4y \le 12$$

C.
$$3x + 4y > 12$$

D.
$$3x + 4y \ge 12$$

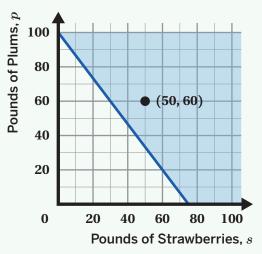
How would the inequality change if the boundary line were solid instead of dashed? Explain your thinking.



Looking at solutions to two-variable inequalities on a graph can help us make sense of different situations.

Here is an example: Angel makes \$4 per pound of strawberries and \$3 per pound of plums that they sell. The inequality $4s + 3p \ge 300$ represents the pounds of strawberries, s, and pounds of plums, p, that Angel needs to sell to meet their goal of making at least \$300.

- To determine the solutions to the inequality, graph the corresponding equation 4s + 3p = 300.
- Decide whether the points on the line will reach the goal by looking at the original inequality symbol.
- Then test a value, such as (50, 60), to identify the solution region. $4(50) + 3(60) \ge 300$ *True!*



Because the point (50, 60) makes the inequality true, the *half-plane* that includes (50, 60) is the solution region. So any combination of strawberries and plums in the shaded region, including those on the line, would meet Angel's goal.

But not all solutions to the inequality will make sense for the situation. For example, the point (90, -20) makes the inequality true, but it doesn't make sense for Angel to sell -20 pounds of plums.

Try This

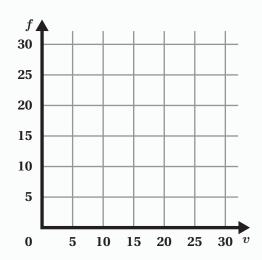
A group of students is installing a garden at their school. A vegetable bed will cost \$15 per square foot and a flower bed will cost \$12 per square foot. Their budget for the project is \$300.

a If $15v + 12f \le 300$ represents this situation, define v and f.

v represents . . .

f represents . . .

- **b** Graph the corresponding equation: 15v + 12f = 300.
- Shade in the region that represents the solutions to the inequality $15v + 12f \le 300$.

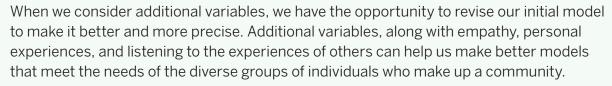


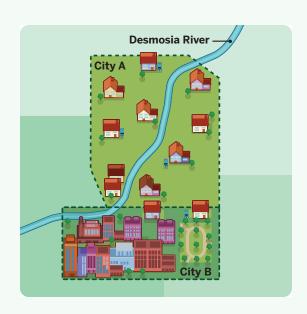
We can use two-variable linear inequalities to model real-world constraints and make sense of issues in society, such as fair water distribution.

When modeling, we often make an initial decision based on the information and variables we're given. But we might find additional variables to consider when we look deeper.

In the example of fair water distribution, some additional variables to consider are:

- The population of each city.
- The amount of land in each city.
- · The amount of water each city uses.
- Any future predictions for change in the river's flow.





Try This

A theater wants to pay its actors and other workers more. To do this, it needs to make \$1,800 in ticket sales for each performance. Each adult ticket costs \$12 and each child ticket costs \$8.

- **a** Why might someone want to create a mathematical model for this situation?
- **b** Write an inequality to represent this situation.
 - Use *x* to represent the number of adult tickets sold.
 - Use y to represent the number of child tickets sold.

a x = -4.

Caregiver Note: One strategy for solving the equation is to determine the order of operations to evaluate the expression and then do the inverse operations backwards. For example, since the last step in evaluating 5(x-2) is to multiply by 5, the first step in solving is to divide both sides by 5.

b
$$-30 = 5([-4] - 2)$$

 $-30 = 5(-6)$
 $-30 = -30$

Lesson 2

a Responses vary.

• Emma tried to multiply by 2 on both sides because $2 \cdot \frac{1}{2} = 1$.

• She added 20 to both sides because -20 + 20 = 0.

• She subtracted x from both sides because x - x = 0.

b Emma made an error in the first step when she multiplied -10 by 2 but not 3x.

x = 4.8 (or equivalent)

Caregiver Note: Here is one way to solve the equation that builds on Emma's strategy.

$$\frac{1}{2}(x+4) = -10 + 3x$$

$$x+4 = -20 + 6x$$

$$4 = -20 + 5x$$

$$24 = 5x$$

$$4.8 = x$$

One Solution	No Solution	Infinitely Many Solutions
4t = 6t $3x = 10 - 3x$	5t + 7 = -3 + 5t $4t + 7 = 2(2t + 2)$	10 - 2(t+5) = -2t

a The variable p represents the number of hours Tiara works at the pool. The variable t represents the number of hours she works tutoring Spanish.

b B.
$$t = 20 - \frac{2}{3}p$$

Caregiver Note: One strategy for determining which equation is equivalent is to rewrite the equation in the form t=.

$$8p + 12t = 240$$

$$12t = 240 - 8p$$

$$\frac{12t}{12} = \frac{240 - 8p}{12}$$

$$t = 20 - \frac{2}{3}p$$

Lesson 5

a Left:
$$t = -2$$

Right:
$$t = \frac{10 - v}{-a}$$
 (or equivalent)

Caregiver Note: Here is one strategy for solving the equation on the right.

$$10 = v - at$$

$$10 - v = -at$$

$$\frac{10 - v}{-a} = \frac{-at}{-a}$$

$$\frac{10 - v}{-a} = t$$

b Left:
$$m = -33$$

Right:
$$m = a(h - t)$$
 (or equivalent)

Caregiver Note: Here is one strategy for solving the equation on the right.

$$\frac{m}{a} + t = h$$

$$\frac{m}{a} = h - t$$

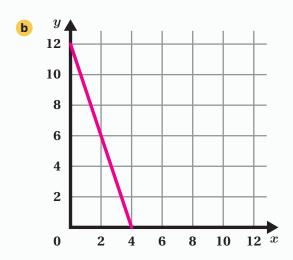
$$a\left(\frac{m}{a}\right) = a(h - t)$$

$$m = a(h - t)$$

a y = 12 - 3x (or equivalent)

Caregiver Note: Here is one way to solve the equation for y.

$$6x + 2y = 24$$
$$2y = 24 - 6x$$
$$\frac{2y}{2} = \frac{24 - 6x}{2}$$
$$y = 12 - 3x$$



Caregiver Note: Here are two strategies for graphing this relationship.

- Use the equation 6x + 2y = 24 to identify the x-intercept and y-intercept. The x-intercept is the value of x when y = 0. The x-intercept is (4,0) because 6x + 2(0) = 24 and 6x = 24. The y-intercept is the value of y when x = 0. Using similar reasoning, the y-intercept is (0,12).
- Use the equation y = 12 3x to identify the y-intercept and slope. The y-intercept is (0, 12) and the slope is -3.

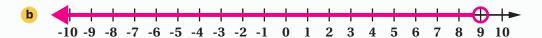
Lesson 7

- a 20 = 2s + 3p (or equivalent)
- **b** $10 \frac{3}{2}p = s$ (or equivalent)
- **c** Responses vary. Solving the equation for s tells you how many stickers Neel can get if he gets no pencils. It also shows the slope of the graph because the equation is in slope-intercept form.

- a Responses vary. Valeria wants to donate at least \$120. This is a constraint because she can save more than that amount but not less.
- **b** $120 \le 64 + 8w$ (or equivalent)

a x = 9.

Caregiver Note: One strategy for calculating the boundary point is to solve the corresponding equation, 5 + 3x = 32.

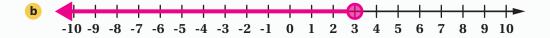


Explanations vary. There is an open circle at x=9 because that is the boundary point but the inequality has a < symbol, which means that the inequality is not true when x=9. If you test x=0, 5+3(0)<32 is true, so the solutions include x=0 (and all of the numbers in the same direction).

Lesson 10

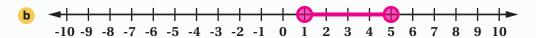
a $x \leq 3$.

Caregiver Note: One strategy for solving this inequality is to determine the boundary point, then test values. To determine the boundary point, solve the corresponding equation 11 - 2x = 5. If you test x = 0, $11 - 2(0) \ge 5$ is true, so the solutions include x = 0 (and all of the numbers in the same direction).



Lesson 11

a Responses between 1 and 5 are considered correct.



Explanations vary. The equation $|x-3| \le 2$ is saying that the distance between x and 3 has to be less than or equal to 2. The numbers that are exactly 2 units away from 3 are 1 and 5. Since the distance has to be less than or equal to 2, any number between 1 and 5 is a solution.

2x + 1 > 9	2x + 1 < 9	2x - 1 < 9
x>4 or $x<-5$	x < 4 and x > -5	x > -4 and $x < 5$

- a $C.5x + 7y \ge 180$
- **b** Responses vary. One solution on the graph is (25, 10).

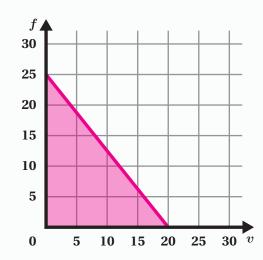
$$5(25) + 7(10) \ge 180$$
$$125 + 70 \ge 180$$
$$195 \ge 180$$
True!

Lesson 14

- a A. 3x + 4y < 12
- **b** The equation would become $3x + 4y \le 12$. Explanations vary. The \le symbol means less than or equal to, which means that the boundary line is included in the solutions to the inequality. The way to show that on a graph is by using a solid line.

Lesson 15

- a v represents the number of square feet of vegetable bed the students want to install. f represents the number of square feet of flower bed they want to install.
- **b** Response shown on graph.
- c Response shown on graph.



- a Responses vary. The theater needs to make at least a certain amount of money (or more), so it might be helpful to have an inequality or graph to see all the combinations of tickets they could sell and still earn enough.
- **b** $12x + 8y \ge 1800$ (or equivalent)