

## Unit 5

# Systems of Linear Equations and Inequalities



Have you ever been in a situation where you had to accomplish two goals at once? Systems of equations and inequalities are helpful for finding a set of values that meets both constraints at the same time. In this unit, you'll explore how to solve systems of equations and inequalities using different strategies. You will also analyze the structure of the equations in a system to strategize about which solving method you choose.

## Essential Questions

- How can you solve systems of equations and inequalities symbolically and graphically?
- How can you use the structures of the equations, available tools, and knowledge of your personal mathematical preferences to select a solving method strategically?
- How can constraints be represented using systems of equations or inequalities?



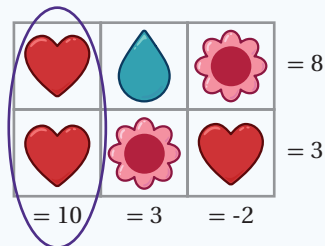
There are many different ways to solve problems and puzzles using math. Let's look at a few strategies for determining the values of shapes in a puzzle.

## Strategy 1: Look for single-shape rows/columns

If we know the value of two hearts, we can determine the value of one heart and substitute that value in the other parts of the puzzle.

$$2(\heartsuit) = 10$$

$$\heartsuit = 5$$



## Strategy 2: Substitute known shape values to solve for missing values

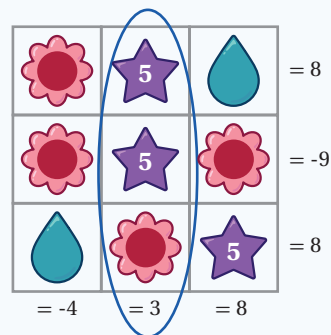
If we know the value of one star, then we can substitute that value in other parts of the puzzle.

$$2(\star) + \text{flower} = 3$$

$$2(5) + \text{flower} = 3$$

$$10 + \text{flower} = 3$$

$$\text{flower} = -7$$



## Strategy 3: Look for repeating shape patterns

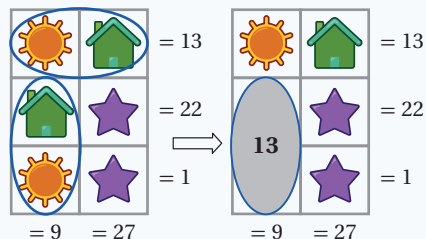
If we know the value of a shape combination and we see it repeat in the puzzle, then we can substitute the value for the shape combination.

$$\text{sun} + \text{house} = 13$$

$$\text{sun} + (\text{house} + \text{sun}) = 9$$

$$\text{sun} + 13 = 9$$

$$\text{sun} = -4$$



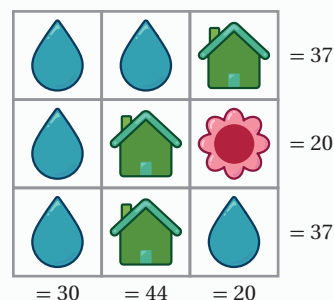
## Try This

Here is a shape puzzle. The sum of each row and column is shown.

- a** Determine the solution for this puzzle.

Shape	Value
House	
Flower	
Drop	

- b** Describe the strategy you used.



A **system of equations** is two or more equations that represent the same constraints using the same variables. The **solution to a system of equations** is the  $(x, y)$  ordered pair(s) that makes every equation in the system true. There are many solutions to a linear equation in two variables, but there might be only one (or no) solution to a system of linear equations in two variables.

There are many strategies for determining the ordered pair that makes both equations in a system true. One strategy is called **elimination**, where you add or subtract the equations to produce a new equation with one variable. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the  $y$ -variable.

$$\begin{array}{r} -2x + y = 9 \\ + (8x - y = 3) \\ \hline 6x + 0 = 12 \\ x = 2 \end{array}$$

$$\begin{array}{r} -2(2) + y = 9 \\ y = 13 \end{array}$$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the  $y$ -variable.

$$\begin{array}{r} x + 2y = 30 \\ - (x + y = 23) \\ \hline y = 7 \end{array}$$

$$\begin{array}{r} x + (7) = 23 \\ x = 16 \end{array}$$

## Try This

Here is a system of equations:

$$p + 3q = 14$$

$$p + 2q = 10$$

- a** Circle the action(s) that can be used to eliminate a variable in this system.
- Addition                      Subtraction                      Both                      Neither
- b** Determine the solution to this system of equations.

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Your goal is to end up with a system of equations where one variable has the same or *opposite* coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$\begin{aligned} 9x - 4y &= 2 \\ 3x + y &= 10 \end{aligned}$$

You can multiply the second equation by -3 to eliminate the  $x$ -variables.

$$\begin{aligned} 9x - 4y &= 2 \\ -3(3x + y) &= -30 \\ \hline 9x - 4y &= 2 \\ + -9x - 3y &= -30 \\ \hline 0 - 7y &= -28 \\ y &= 4 \\ 3x + (4) &= 10 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Or you can multiply the second equation by 4 to eliminate the  $y$ -variables.

$$\begin{aligned} 9x - 4y &= 2 \\ 4(3x + y) &= 40 \\ \hline 9x - 4y &= 2 \\ + 12x + 4y &= 40 \\ \hline 21x + 0 &= 42 \\ x &= 2 \\ 9(2) - 4y &= 2 \\ 18 - 4y &= 2 \\ -4y &= -16 \\ y &= 4 \end{aligned}$$

## Try This

Solve this system of equations in two different ways.

System of Equations	Strategy 1	Strategy 2
$\begin{aligned} 4x + 3y &= 3 \\ 8x + y &= 1 \end{aligned}$		

One strategy you can use to solve a system of equations is **substitution**, where you replace a variable with an equivalent expression. Substitution is a useful strategy when one variable is already isolated in an equation.

Here are two examples of systems of equations where substitution may be a useful strategy.

In this system, both  $y$ -variables are already isolated. We can substitute the expression  $-4x + 6$  in for  $y$  in the second equation.

$$\begin{aligned} y &= -4x + 6 \\ y &= 3x - 15 \end{aligned}$$

$y = -4x + 6$  →  $y = 3x - 15$

$$\begin{aligned} -4x + 6 &= 3x - 15 \\ -7x &= -21 \\ x &= 3 \\ y &= 3(3) - 15 \\ y &= -6 \end{aligned}$$

In this system of equations,  $y$  is already isolated, so we can substitute the expression  $2x - 5$  in for  $y$  in the first equation.

$$\begin{aligned} -3x - 2y &= 3 \\ y &= 2x - 5 \end{aligned}$$

$y = 2x - 5$  →  $-3x - 2y = 3$

$$\begin{aligned} -3x - 2(2x - 5) &= 3 \\ -3x - 4x + 10 &= 3 \\ -7x + 10 &= 3 \\ -7x &= -7 \\ x &= 1 \\ y &= 2(1) - 5 \\ y &= -3 \end{aligned}$$

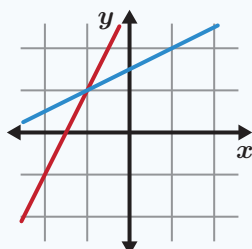
## Try This

- a** Determine the solution to this system of equations.
 
$$\begin{aligned} b &= 3a - 8 \\ 2a + b &= 2 \end{aligned}$$
  
- b** How is solving systems of equations by substitution like solving by elimination? How is it different?

You can solve systems of equations using strategies like elimination, substitution, or graphing. On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

## One Solution

The lines intersect at  $(-2, 2)$ .



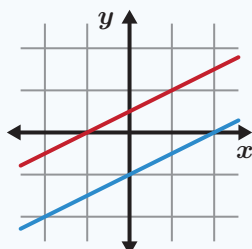
$$y = 2x + 6$$

$$y = \frac{1}{2}x + 3$$

The equations have different *slopes* and *y-intercepts*.

## No Solutions

The lines are *parallel*.



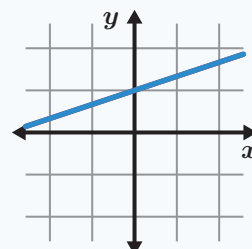
$$y = \frac{1}{2}x + 1$$

$$y = \frac{1}{2}x - 2$$

The equations have the same *slope* and different *y-intercepts*.

## Infinitely Many Solutions

The lines are the same.



$$y = \frac{1}{3}x + 2$$

$$3y - x = 6$$

The equations are *equivalent*.

## Try This

One equation in a system is  $y = 7x - 12$ .

Complete the table to create systems with one solution, no solution, and infinite solutions.

	One Solution	No Solution	Infinite Solutions
System	$y = 7x - 12$ .....	$y = 7x - 12$ .....	$y = 7x - 12$ .....

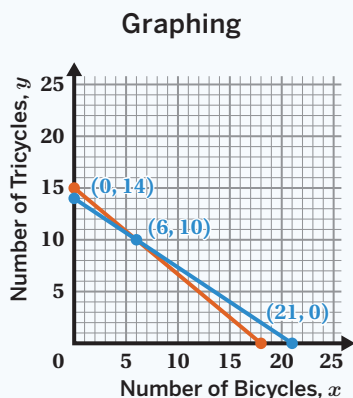
Systems of equations can represent *constraints* in a situation. There are different ways to solve systems of equations to determine the values that satisfy these constraints.

For example, a bike shop makes 2-wheel bicycles and 3-wheel tricycles. This week, they have 42 wheels and enough materials to make 16 bikes total.

Here is a system of equations about this situation:

- $x$  is the number of bicycles
- $y$  is the number of tricycles

$$\begin{aligned}x + y &= 16 \\2x + 3y &= 42\end{aligned}$$



The point of intersection  
(6, 10) is the solution.

**Substitution**

$$\begin{aligned}x + y &= 16 \\-y & -y \\x &= 16 - y\end{aligned}$$

$$\begin{aligned}2x + 3y &= 42 \\2(16 - y) + 3y &= 42 \\32 - 2y + 3y &= 42 \\y &= 10 \\x &= 16 - 10 \\x &= 6\end{aligned}$$

**Elimination**

$$\begin{aligned}-2 \cdot (x + y &= 16) \\2x + 3y &= 42 \\-2x - 2y &= -32 \\+ 2x + 3y &= 42 \\0 + y &= 10 \\y &= 10 \\x + (10) &= 16 \\x &= 6\end{aligned}$$

In this situation, the solution  $x = 6$  and  $y = 10$  means that the bike shop will use all of their wheels and materials if they make 6 bicycles and 10 tricycles.

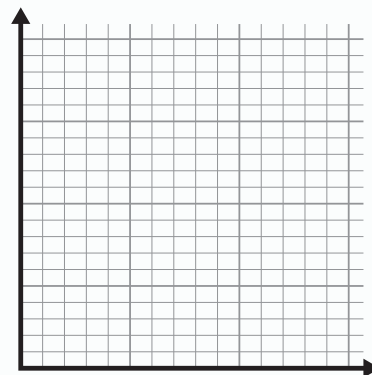
## Try This

A new 50-unit apartment building has space for 80 parking spaces. The city requires 2 parking spaces for every multi-bedroom apartment, and 1 parking space for every one-bedroom apartment.

This situation can be modeled by this system of equations, where  $x$  represents the number of multi-bedroom apartments and  $y$  represents the number of one-bedroom apartments.

$$\begin{aligned}x + y &= 50 \\2x + y &= 80\end{aligned}$$

- Graph the system of equations.  
Include a scale for the  $x$  and  $y$  axes.
- What does the solution represent in this situation?



You can write systems of equations to help represent constraints in real-world situations.

Here are some things to consider when you write a system of equations:

- Identify the constraints of the situation.
- Identify what each variable will represent.
- Determine whether you want to use *standard form* ( $Ax + By = C$ ) or *slope-intercept form* ( $y = mx + b$ ) to represent each constraint.

Let's look at an example. The architect of an apartment building has enough space to fit 50 apartments and 80 parking spaces. The city requires 2 parking spaces for every big apartment, and 1 parking space for every small apartment.

Constraints	Variables	System of Equations
• There is space to fit 50 apartments and 80 parking spaces.	$x$ represents the number of big apartments.	$x + y = 50$
• Every big apartment has 2 parking spaces and every small apartment has 1 parking space.	$y$ represents the number of small apartments.	$2x + y = 80$

## Try This

The knitting club sold 40 scarves and hats at a winter festival and made \$700. Each scarf costs \$18 and each hat costs \$14.

- a** If  $s$  represents the number of scarves sold and  $h$  represents the number of hats sold, which system of equations represents the constraints in this situation?

**A.**  $40s + h = 700$   
 $18s + 14h = 700$

**B.**  $18s + 14h = 40$   
 $s + h = 700$

**C.**  $s + h = 40$   
 $18s + 14h = 700$

**D.**  $40(s + h) = 700$   
 $18s = 14h$

- b** Solve the system of equations you chose.
- c** Describe what the solution means in this situation.



You can solve systems of equations symbolically using either substitution or elimination. Looking for specific structures in the equations can help you decide which strategy to use.

- It may be helpful to use substitution when at least one of the equations has an isolated variable or at least one is in *slope-intercept form*.
- It may be helpful to use elimination when both equations are in the same form or if the equations have a pair of same or opposite terms.

When solving a system of equations symbolically, sometimes all of the variables are eliminated.

## No Solutions

When the result is a false statement there are *no solutions* to the system of equations. This means the lines are parallel and will never intersect.

$$\begin{aligned} y &= 3x + 6 & y &= 3x - 6 \\ 3x + 6 &= 3x - 6 \\ 3x + 12 &= 3x \\ 12 &= 0 \end{aligned}$$

## Infinitely Many Solutions

When the result is a true statement there are *infinitely many solutions* to the system of equations. The equations are equivalent and represent the same line.

$$\begin{aligned} 2 \cdot (2x + 4y &= 6) \\ -4x - 8y &= -12 \\ 4x + 8y &= 12 \\ + -4x - 8y &= -12 \\ \hline 0 + 0 &= 0 \\ 0 &= 0 \end{aligned}$$

## Try This

Here are three systems of equations.

- a** Circle the strategy you would use to solve each system of equations.

$y = 3x - 10$ $2x - 3y = 16$	$y = 4x - 10$ $y = x + 2$	$9x + y = 25$ $3x + 2y = 5$
Elimination	Elimination	Elimination
Substitution	Substitution	Substitution
Both	Both	Both
Neither	Neither	Neither

- b** Select one system and solve it using the strategy you chose.

A **system of inequalities** is a system of two or more inequalities that represent the *constraints* on a shared set of variables.

You can use different strategies to determine if a point is a solution to a system of inequalities.

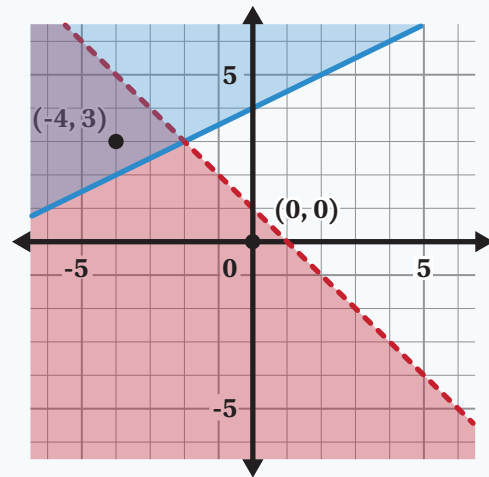
- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the  $x$ - and  $y$ -values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

Here is a graph for this system of inequalities.

$$\begin{aligned} x + y &< 1 \\ y &\geq \frac{1}{2}x + 4 \end{aligned}$$

You can see that the point  $(-4, 3)$  is a solution because it is in the shaded region for both inequalities.

You can also substitute points into both inequalities to determine if they are solutions.  $(0, 0)$  is not a solution and  $(-4, 3)$  is a solution.

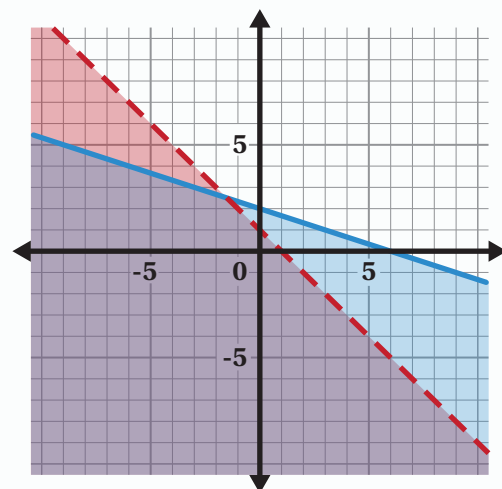


## Try This

This graph represents this system of inequalities:

$$\begin{aligned} x + 3y &< 6 \\ x + y &< 1 \end{aligned}$$

- Is the point  $(1, 0)$  a solution to the system?
- Select a point that does *not* represent a solution and show that it is not a solution.



The **solutions to a system of inequalities** are all the points that make both inequalities true. The solutions can be seen in the region where the graphs overlap, called the **solution region**.

One strategy for determining the location of the solution region is to test a point. Choose a point that is not on either *boundary line*, substitute the  $x$ - and  $y$ -values into each inequality to see if it makes the statement true, and shade based on the results of the test.

Let's look an example for this system of inequalities:

$$3x + y \geq 6$$

$$y > x + 2$$

You can test the point  $(3, 2)$  to help determine the solution region.

**Solid Line**

$$3x + y \geq 6$$

$$3(3) + 2 \geq 6$$

$$11 \geq 6$$

True ✓

Shade the side of the solid line  
that includes  $(3, 2)$

**Dashed Line**

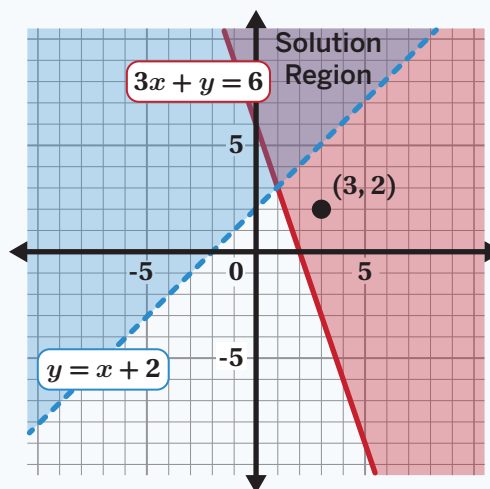
$$y > x + 2$$

$$0 > 3 + 2$$

$$0 > 5$$

False ✗

Shade the side of the dashed  
line that does not include  
 $(3, 2)$



## Try This

Here is a graph of this system of inequalities:

$$3x + y \geq 6$$

$$y > x + 2$$

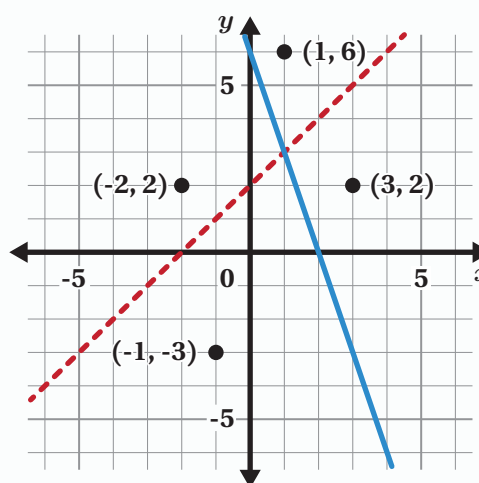
- a** Determine if each point is a solution to the system. Circle yes or no.

$(-2, 2)$       Yes      No

$(1, 6)$       Yes      No

$(3, 2)$       Yes      No

$(-1, -3)$       Yes      No



- b** Shade the solution region on the graph.

You can graph a system of linear inequalities by graphing the boundary line of each inequality and testing a point to determine which side of the boundary lines to shade.

You can use different strategies to help you graph boundary lines of inequalities.

- A strategy to graph boundary lines written in *slope-intercept form* ( $y = mx + b$ ) is to plot the  $y$ -intercept and use the slope to determine other points.
- A strategy to graph boundary lines written in *standard form* ( $Ax + By = C$ ) is to plot and connect the  $x$ - and  $y$ -intercepts.

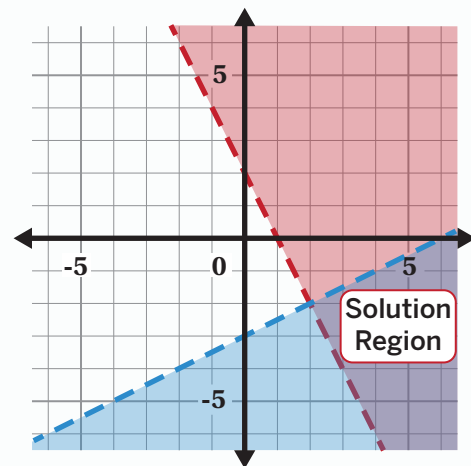
If an inequality uses a  $\leq$  or  $\geq$  symbol, then the boundary line is solid and included in the solution region. If an inequality uses a  $<$  or  $>$  symbol, then the boundary line is dashed and not part of the solution region.

You can also write a system of linear inequalities from a graph. Use the boundary lines and test points to help you determine the inequality symbols. You can also use test points to check the accuracy of your system of inequalities.

### Try This

Here is a graph of a system of inequalities.

- Write the system of inequalities that this graph could represent.
- Explain how you decided which inequality symbols to use.



Systems of inequalities can help you solve problems involving real-world *constraints*.

Here is an example about a juice shop.

You can write and graph a system of inequalities to represent these constraints.

Let  $x$  represent the number of 12-ounce jars and  $y$  can represent the number of 16-ounce jars.

$$12x + 16y \leq 144$$

$$2.50x + 4.50y > 33.50$$

You can graph each boundary line and use the test point  $(2, 7)$  to help us determine the solution region.

**Solid Line**

$$12x + 16y \leq 144$$

$$12(2) + 16(7) \leq 144$$

$$136 \leq 144$$

True ✓

**Shade the side of the solid line that includes  $(2, 7)$**

**Dashed Line**

$$2.50x + 4.50y > 33.50$$

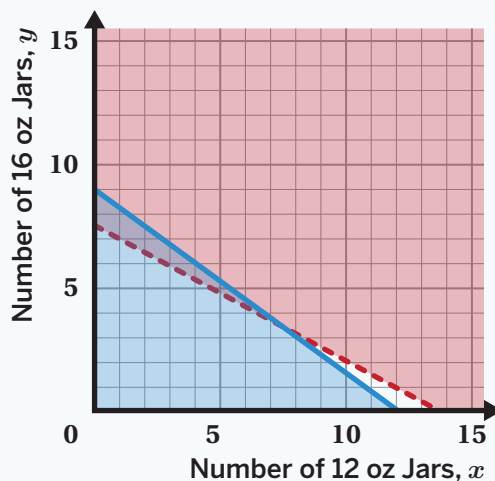
$$2.50(2) + 4.50(7) > 33.50$$

$$36.50 > 33.50$$

True ✓

**Shade the side of the dashed line that includes  $(2, 7)$**

A juice shop has 144 ounces of orange juice to fit in jars of 12 and 16 ounces. They earn \$2.50 for each 12-ounce jar and \$4.50 for each 16-ounce jar. They need to earn more than \$33.50 from the juice.



You can use the graph to help you determine some possible combinations of 12- and 16-ounce jars of juice that meet the constraints. Such as:

- Zero 12 oz jars of juice and eight 16 oz jars of juice
- Zero 12 oz jars of juice and nine 16 oz jars of juice
- One 12 oz jars of juice and eight 16 oz jars of juice

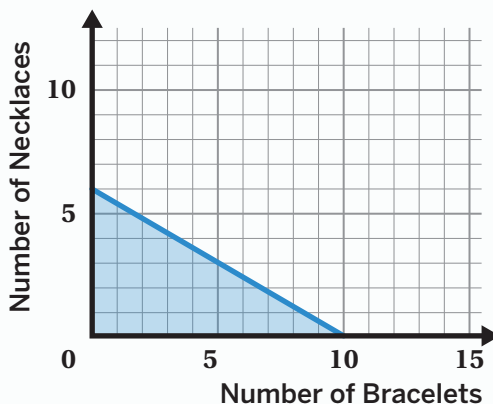
## Try This

Natalia wants to buy bracelets and necklaces as gifts for her friends. Each bracelet,  $b$ , costs \$3 and each necklace,  $n$ , costs \$5. She can spend no more than \$30 and needs at least 7 gifts.

This graph shows one of the inequalities that represents the constraints in this situation:

$$3b + 5n \leq 30$$

- Write the second inequality that represents the constraints in this situation.
- Graph the inequality you wrote.
- Use the graph to determine two possible combinations of bracelets and necklaces that meet the constraints.



## Lesson 1

**a** House = 17, Flower = -7 , Drop = 10

**b** Responses vary.

- Calculate the value of the drops first because the first column is only drops and has a total of 30. If 3 drops is 30, then one drop is 10.
- Substitute the value of the drop, 10, to find the value of the house in the first row.
- Substitute the value of the drop, 10, and the value of the house, 17, to find the value of the flower.

## Lesson 2

**a** Subtraction

**b**  $p = 2, q = 4$

*Explanation:*

$$\begin{array}{r} p + 3q = 14 \\ -(p + 2q = 10) \\ \hline q = 4 \end{array}$$

*Subtract the second equation from the first equation to eliminate  $p$ .*

$$\begin{array}{r} p + 2(4) = 10 \\ p + 8 = 10 \\ -8 \quad -8 \\ \hline p = 2 \end{array}$$

*After finding the value of  $q$ , substitute that value into the equation  $p + 2q = 10$  to solve for  $p$ .*

## Lesson 3

$x = 0, y = 1$

*Explanation:*

	Strategy 1		Strategy 2
$4x + 3y = 3 \Rightarrow -2(4x + 3y = 3)$	$-8x - 6y = -6$	$4x + 3y = 3$	$4x + 3y = 3$
$8x + y = 1$	$8x + y = 1$	$8x + y = 1 \Rightarrow -3(8x + y = 1)$	$-24x - 3y = -3$
<i>Multiply the first equation by -2 to eliminate <math>x</math> and solve for <math>y</math>.</i>	$-5y = -5$ $y = 1$	<i>Multiply the second equation by -3 to eliminate <math>y</math> and solve for <math>x</math>.</i>	$-20x = 0$ $x = 0$
<i>Substitute 1 for <math>y</math> to solve for <math>x</math>.</i>	$8x + (1) = 1$ $-1 \quad -1$ $8x = 0$ $x = 0$	<i>Substitute 0 for <math>x</math> to solve for <math>y</math>.</i>	$4(0) + y = 1$ $y = 1$

## Lesson 4

**a**  $a = 2, b = -2$

*Explanation:*

$$\begin{aligned} 2a + (3a - 8) &= 2 \\ 5a - 8 &= 2 \\ + 8 &+ 8 \\ 5a &= 10 \\ a &= 2 \end{aligned}$$

*Substitute the expression  $(3a - 8)$  for  $b$  into the equation  $2a + b = 2$  and then solve for  $a$ .*

$$b = 3(2) - 8$$

$$b = 6 - 8$$

$$b = -2$$

*Then substitute the value of 2 for  $a$ , and solve for  $b$ .*

**b** *Responses vary.*

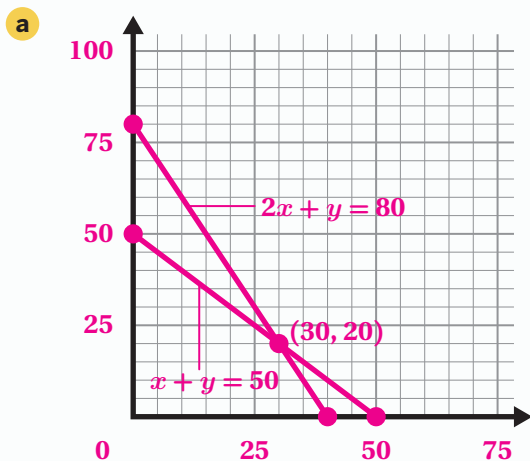
- Both strategies involve substituting the value of one variable to determine the value of the other.
- Both strategies lead to the creation of new equations.
- The elimination strategy involves adding or subtracting equations.
- The substitution strategy is useful when one variable is already isolated in one or both equations.

## Lesson 5

*Responses vary. Sample shown in table.*

	One Solution	No Solution	Infinite Solutions
Equation	$\begin{aligned} y &= 7x - 12 \\ \underline{y &= x - 4} \end{aligned}$ <p>[Any equation in <math>y = mx + b</math> form where the coefficient of <math>x</math> is not 7 is correct.]</p>	$\begin{aligned} y &= 7x - 12 \\ \underline{y &= 7x - 1} \end{aligned}$ <p>[Any equation in <math>y = mx + b</math> form where the coefficient of <math>x</math> is 7 and the constant is not -12 is correct.]</p>	$\begin{aligned} y &= 7x - 12 \\ \underline{2y &= 14x - 24} \end{aligned}$ <p>[Any equation equivalent to <math>y = 7x - 12</math> is correct.]</p>

## Lesson 6



- b (30, 20) represents the city building 30 multi-bedroom apartments and 20 one-bedroom apartments.

## Lesson 7

- a C.  $s + h = 40$   
 $18s + 14h = 700$

- b  $s = 35, h = 5$ .

*Explanation:*

$$s + h = 40 \Rightarrow -14(s + h = 40)$$

$$18s + 14h = 700$$

*Multiply the first equation by -14 to eliminate the  $h$  and solve for  $s$ .*

*Substitute 35 for  $s$  and solve for  $h$ .*

$$-14s - 14h = -560$$

$$18s + 14h = 700$$

$$4s = 140$$

$$s = 35$$

$$(35) + h = 40$$

$$-35 \quad -35$$

$$h = 5$$

- c The knitting club sold 35 scarves and 5 hats.



## Lesson 8

a

$y = 3x - 10$ $2x - 3y = 16$	$y = 4x - 10$ $y = x + 2$	$9x + y = 25$ $3x + 2y = 5$
Elimination	Elimination	Elimination
Substitution	Substitution	Substitution
Both	Both	Both
Neither	Neither	Neither

b

$y = 3x - 10$ $2x - 3y = 16$	$y = 4x - 10$ $y = x + 2$	$9x + y = 25$ $3x + 2y = 5$
(2, -4)	(4, 6)	(3, -2)

## Lesson 9

a

No.

*Explanation: The point (1, 0) is on a dashed boundary line, which means it is not included in the solution region.*

b

Responses vary.

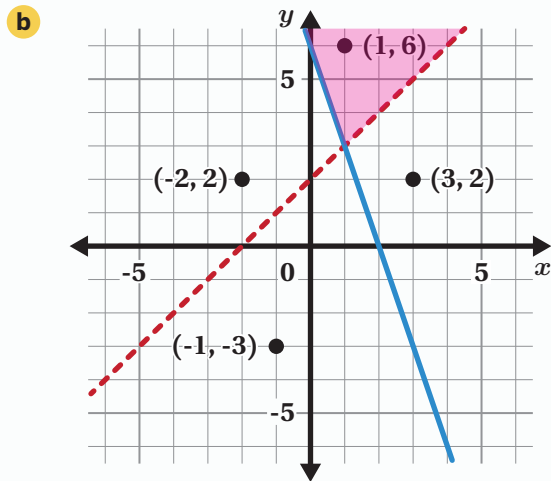
(-5, 5)

 $(-5) + 3(5) < 6$ 
 $-5 + 15 < 6$ 

*10 < 6 Explanation: This is not true, so (-5, 5) is not a solution.*

## Lesson 10

- a (-2, 2): No  
 (1, 6): Yes  
 (3, 2): No  
 (-1, -3): No

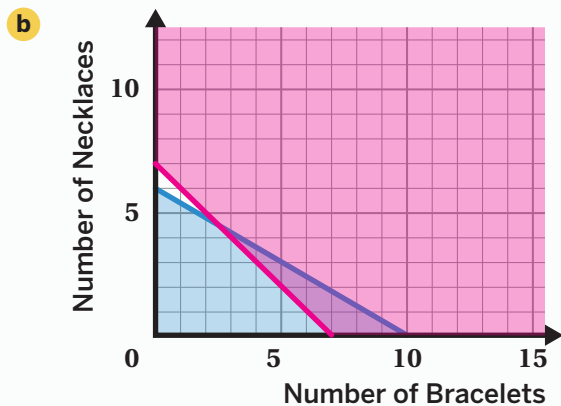


## Lesson 11

- a  $y > -2x + 2$  (or equivalent)  
 $y < \frac{1}{2}x - 3$  (or equivalent)
- b Responses vary. The boundary lines are dashed, which means that the inequality symbols can only be less than ( $<$ ) or greater than ( $>$ ).

## Lesson 12

- a  $b + n \geq 7$



- c Responses vary. 6 bracelets and 2 necklaces, 7 bracelets and 1 necklace.

# Algebra 1 Unit 5 Glossary/Álgebra 1 Unidad 5 Glosario

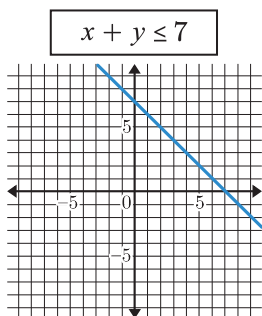
## English

### boundary line

The line that separates the solution region of a linear inequality from non-solutions.

A linear inequality (e.g.,  $y < 2x + 5$ ) has a boundary line that is represented symbolically by the corresponding equation (e.g.,  $y = 2x + 5$ ). A solid boundary line indicates that these points are included in the solution set (e.g.,  $y \leq x$ ). A dashed boundary line indicates that they are not (e.g.,  $y < x$ ).

The solution region to  $x + y \leq 7$  has a boundary line at  $x + y = 7$ . The line is solid because the points on the line  $x + y = 7$  are included in the solution region.



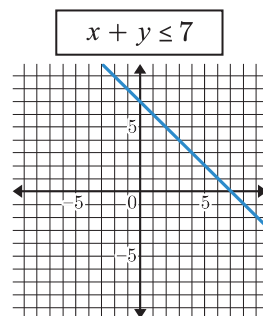
## B

## Español

### recta límite

La línea que separa la región solución de una desigualdad lineal de todos los valores que no son soluciones. Una desigualdad lineal (p. ej.,  $y < 2x + 5$ ) tiene una recta límite representada simbólicamente por la ecuación correspondiente (p. ej.,  $y = 2x + 5$ ). Una recta límite continua indica que esos puntos están incluidos en el conjunto de soluciones (p. ej.,  $y \leq x$ ). Una recta límite discontinua indica que no lo están (p. ej.,  $y < x$ ).

La región solución de  $x + y \leq 7$  tiene una recta límite en  $x + y = 7$ . La recta es continua porque los puntos en la recta  $x + y = 7$  se encuentran dentro de la región solución.



## C

**coefficient** A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression  $5x + 8$ , 5 is the coefficient of  $x$ .

**constraint** A limitation on the possible values of variables in a model. Equations and inequalities are often used to represent constraints.

The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality  $h \geq 36$ .

**coeficiente** Un número que se multiplica por una variable. Por lo general, no hay ningún signo entre el coeficiente y la variable.

En la expresión  $5x + 8$ , el coeficiente de  $x$  es 5.

**restricción** Una limitación de los posibles valores de las variables en un modelo. Suelen usarse ecuaciones o desigualdades para representar restricciones.

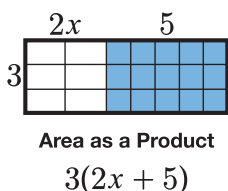
La restricción "debes medir 36 pulgadas o más para subirte a la rueda de la fortuna" puede representarse con la desigualdad  $h \geq 36$ .

## D

### distributive property

Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

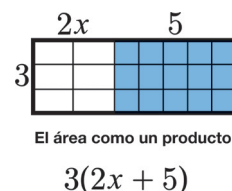
For example,  $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$ .



### propiedad distributiva

Multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente antes de sumarlos.

Ejemplo:  $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$ .



## English

## Español

## E

**elimination**

A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

In the example, subtraction is used to eliminate  $y$  and create an equation that can be solved for  $x$ .

**equivalent equations** Equations that have the exact same solution(s).

$3x + 4 = 10$  and  $9x + 12 = 30$  are equivalent equations because if you multiply the first equation by 3, you create the second. The solution to each equation is  $x = 2$ .

**equivalent systems** Two or more systems that have exactly the same solutions.

**eliminación**

Un método para resolver sistemas de ecuaciones en el que se suman o restan las ecuaciones para producir una nueva ecuación con menos variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

En el ejemplo, se usa la resta para eliminar  $y$  y producir una ecuación que puede resolverse para determinar el valor de  $x$ .

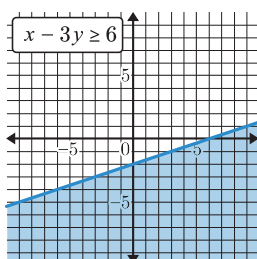
**ecuaciones equivalentes** Ecuaciones que tienen exactamente la misma o las mismas soluciones.

$3x + 4 = 10$  y  $9x + 12 = 30$  son ecuaciones equivalentes porque si se multiplica la primera por 3, se forma la segunda. La solución de cada ecuación es  $x = 2$ .

**sistemas equivalentes** Dos o más sistemas que tienen exactamente las mismas soluciones.

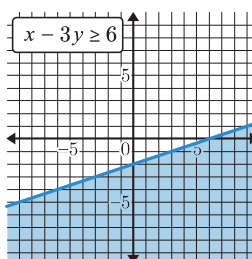
## H

**half-plane** A region that defines the solution set of a single two-variable linear inequality. The boundary line of the inequality splits the coordinate plane into two equal half-planes.



The graph shows the solution region to  $x - 3y \geq 6$ . The boundary line  $x - 3y = 6$  divides the coordinate plane into two equal half-planes.

**semiplano** Una región que define el conjunto de soluciones de una desigualdad lineal con dos variables. La recta límite de la desigualdad divide el plano de coordenadas en dos semiplanos iguales.



La gráfica muestra la región solución de  $x - 3y \geq 6$ . La recta límite  $x - 3y = 6$  divide el plano de coordenadas en dos semiplanos iguales.

# Algebra 1 Unit 5 Glossary/Álgebra 1 Unidad 5 Glosario

## English

## Español

### I

**infinitely many solutions** An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. In a system of equations with infinitely many solutions, every point on the graph is a solution to the system.

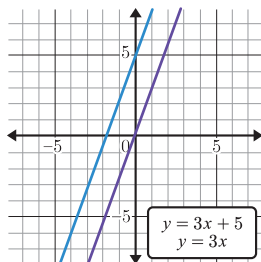
For example, the equation  $3x + 6 = 3(x + 2)$  has infinitely many solutions because the equation is true for any value of  $x$ .

**infinitas soluciones** Una ecuación tiene infinitas soluciones si es verdadera sea cual sea el valor de la variable. Un sistema de ecuaciones tiene infinitas soluciones si las ecuaciones del sistema son equivalentes. En un sistema de ecuaciones con infinitas soluciones, cada punto de la gráfica es una solución del sistema.

Por ejemplo, la ecuación  $3x + 6 = 3(x + 2)$  tiene infinitas soluciones porque la ecuación es verdadera con cualquier valor de  $x$ .

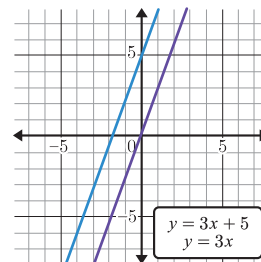
### N

**no solution** An equation has no solution if there is no value of the variable that will make the equation true. A system of equations has no solution if there is no set of values that makes all the equations in the system true. In a system of equations with no solution, there is no point that is on the graph of every equation in the system.



For example, the system of equations containing  $y = 3x + 5$  and  $y = 3x$  has no solution because the graphs are parallel and never intersect.

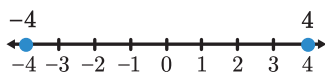
**sin solución** Una ecuación no tiene solución si no hay ningún valor de la variable que haga que la ecuación sea verdadera. Un sistema de ecuaciones no tiene solución si no hay ningún conjunto de valores que haga que todas las ecuaciones de ese sistema sean verdaderas. En un sistema de ecuaciones sin solución, no hay ningún punto que esté en la gráfica de cada una de las ecuaciones del sistema.



Por ejemplo, el sistema de ecuaciones que contiene  $y = 3x + 5$  y  $y = 3x$  no tiene solución porque las gráficas son paralelas y nunca se intersectan.

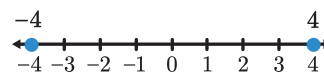
### O

**opposite** Two numbers that are the same distance from 0 and on different sides of the number line. Two terms that are opposites are also referred to as zero pairs and additive inverses.



For example, 4 and -4 are opposites.

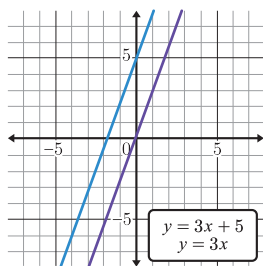
**opuestos** Dos números que están a la misma distancia del 0 y en diferentes lados de la recta numérica. Dos términos que son opuestos también se conocen como inversos aditivos.



Por ejemplo, 4 y -4 son opuestos.

## English

**parallel lines** Lines that never cross or intersect. On a graph, two lines with the same slope and different  $y$ -intercepts are parallel.

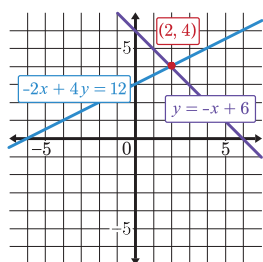


For example, the lines  $y = 3x + 5$  and  $y = 3x$  are parallel and never intersect.

**point of intersection**

A point where two lines or curves meet.

For example,  $(2, 4)$  is the point of intersection for the lines  $-2x + 4y = 12$  and  $y = -x + 6$ .



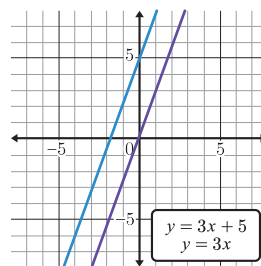
## P

## Español

**líneas (o rectas) paralelas**

Líneas que nunca se cruzan o intersecan.

En una gráfica, dos rectas con la misma pendiente e intersecciones diferentes con el eje  $y$  son paralelas.

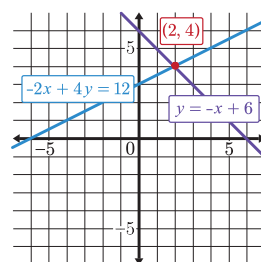


Por ejemplo, las rectas  $y = 3x + 5$  y  $y = 3x$  son paralelas y nunca se intersecan.

**punto de intersección**

Un punto donde se cruzan dos rectas o curvas.

Por ejemplo,  $(2, 4)$  es el punto de intersección de las rectas  $y = -x + 6$  y  $-2x + 4y = 12$ .

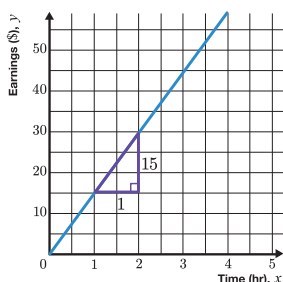


## S

**slope** A number that describes the direction and steepness of a line. Slope represents the amount that  $y$  changes when  $x$  increases by 1.

Since the slope between any two points on a line will be the same, we can say that a line has a constant rate of change. One way to calculate slope is to divide the vertical distance between any two points on the line by the horizontal distance between those points.

In this graph,  $y$  increases by 15 dollars when  $x$  increases by 1 hour. The slope of the line is 15, and the rate of change is 15 dollars per hour.

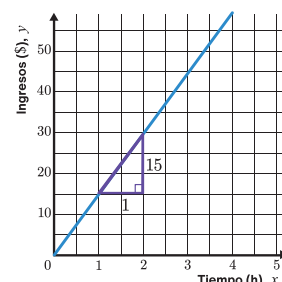


**pendiente**

Un número que describe la dirección e inclinación de una recta. La pendiente representa la cantidad en la que cambia  $y$  cuando

$x$  se incrementa en 1. Como la pendiente entre dos puntos cualesquiera de una recta es la misma, podemos decir que una recta tiene una tasa de cambio constante. Una manera de calcular la pendiente es dividir la distancia vertical entre dos puntos cualesquiera en la recta por la distancia horizontal entre dichos puntos.

En esta gráfica,  $y$  incrementa en 15 dólares cuando  $x$  incrementa en 1 hora. La pendiente de la recta es 15 y la tasa de cambio es de 15 dólares por hora.



# Algebra 1 Unit 5 Glossary/Álgebra 1 Unidad 5 Glosario

## English

**slope-intercept form** A way to write a linear equation that highlights the slope and the  $y$ -intercept of the line it represents. Slope-intercept form equations are written as  $y = mx + b$ , where  $m$  represents the slope,  $b$  represents the  $y$ -intercept of the line, and  $x$  and  $y$  are variables.

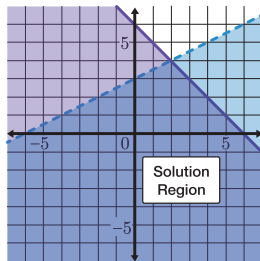
The equations  $y = 2x + 4$  and  $y = -5x - 10$  are in slope-intercept form. The equation  $2x + 5y = 20$  is not in slope-intercept form.

**solution** A value or set of values that makes an equation or inequality true.

For example,  $x = 2$  is a solution to the equation  $3x + 4 = 10$ .  $x > 2$  is the solution to the inequality  $3x + 4 > 10$ . The ordered pair  $(1, 2)$  is a solution to the equation  $3x + 4y = 11$ .

### solution region

The set of all ordered pairs that make an inequality or each inequality in a system true. For a two-variable linear inequality, the solution region is a half-plane. For a system of inequalities, the solution region is located where the graphs overlap.



## Español

**forma pendiente-intersección, forma pendiente-ordenada al origen** Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje  $y$  (o la ordenada al origen) de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben  $y = mx + b$ , donde  $m$  representa la pendiente,  $b$  representa la intersección con el eje  $y$  de la recta, y tanto  $x$  como  $y$  son variables.

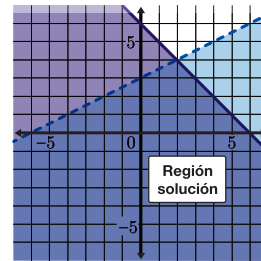
Las ecuaciones  $y = 2x + 4$  y  $y = -5x - 10$  están en la forma pendiente-intersección. La ecuación  $2x + 5y = 20$  no está en la forma pendiente-intersección.

**solución** Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

Por ejemplo,  $x = 2$  es una solución de la ecuación  $3x + 4 = 10$ .  $x > 2$  es la solución de la desigualdad  $3x + 4 > 10$ . El par ordenado  $(1, 2)$  es una solución de la ecuación  $3x + 4y = 11$ .

### región solución

El conjunto de todos los pares ordenados que hacen que una desigualdad, o cada desigualdad de un sistema, sean verdaderas. En una desigualdad lineal de dos variables, la región solución es un semiplano. En un sistema de desigualdades, la región solución se encuentra donde se superponen las gráficas.

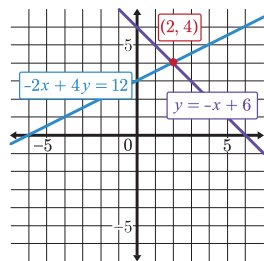


## English

### solution to a system of equations

A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the intersection point.

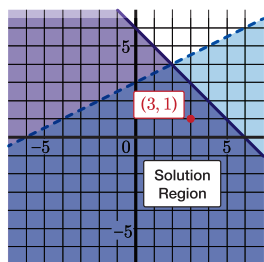
For the system  $y = -x + 6$  and  $-2x + 4y = 12$ ,  $(2, 4)$  is the solution to this system of equations and the intersection point on the graph.



### solution to a system of inequalities

An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the solution region where the graphs overlap.

For system  $y \leq -x + 6$  and  $-2x + 4y < 12$ ,  $(3, 1)$  is a solution to this system of inequalities because it makes both inequalities true and falls in the region where the inequalities overlap.



### standard form (of a linear equation)

Linear equations that are written in the form  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are constants and  $x$  and  $y$  are variables, are written in standard form.

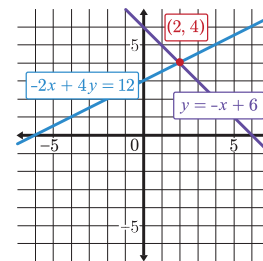
The equations  $2x + 5y = 20$  and  $3x - 4y = -10$  are in standard form. The equation  $y = 2x + 4$  is not in standard form.

## Español

### solución de un sistema de ecuaciones

Una solución de un sistema de ecuaciones es un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.

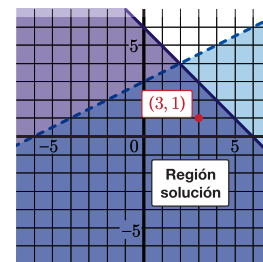
Por ejemplo, en el sistema de ecuaciones  $y = -x + 6$  y  $-2x + 4y = 12$ , la solución y el punto de intersección de la gráfica es  $(2, 4)$ .



### solución de un sistema de desigualdades

Un par ordenado que hace que cada desigualdad de un sistema sea verdadera. Cada par ordenado que sea una solución de un sistema se encuentra en la región solución donde se superponen las gráficas.

Por ejemplo, en el sistema de desigualdades  $y \leq -x + 6$  y  $-2x + 4y < 12$ , la solución es  $(3, 1)$  porque hace que ambas desigualdades sean verdaderas y se ubica en la región donde las desigualdades se superponen.



### forma estándar (de una ecuación lineal)

Las ecuaciones lineales que se escriben en la forma  $ax + by = c$ , donde  $a$ ,  $b$  y  $c$  son constantes, y tanto  $x$  como  $y$  son variables, se conocen como ecuaciones en forma estándar.

Las ecuaciones  $2x + 5y = 20$  y  $3x - 4y = -10$  están en forma estándar. La ecuación  $y = 2x + 4$  no está en forma estándar.



# Algebra 1 Unit 5 Glossary/Álgebra 1 Unidad 5 Glosario

## English

**substitute** Replace a variable or expression with a value or another expression.

In this example, 5 is substituted for  $x$  in the expression  $4x$ .

$$4x = 4(5)$$

### substitution

A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

$$\begin{array}{l} y = -4x + 6 \quad y = 3x - 15 \\ -4x + 6 = 3x - 15 \\ -7x = -21 \\ \boxed{x = 3} \\ y = 3(3) - 15 \\ \boxed{y = -6} \end{array}$$

For example, we can substitute  $-4x + 6$  in for  $y$  in  $y = 3x - 15$  because they are equivalent.

**sum** The value of two or more expressions when added together.

For example, the sum of 6 and 15 is 21, and the sum of  $5x$  and  $-2x + 4$  is  $3x + 4$ .

**system of equations** Two or more equations that represent the constraints on a shared set of variables form a system of equations.

These equations make a system:

$$\begin{array}{l} 3b + c = -2 \\ b - 5c = 12 \end{array}$$

**system of inequalities** Two or more inequalities that represent the constraints on a shared set of variables form a system of inequalities.

These inequalities make a system:

$$\begin{array}{l} 10m + 5n > -2 \\ m - 5n \leq 12 \end{array}$$

## Español

**sustituir** Reemplazar una variable o expresión por un valor u otra expresión.

$$4x = 4(5)$$

En este ejemplo, el 5 sustituye a la  $x$  en la expresión  $4x$ .

### sustitución

Un método para resolver sistemas de ecuaciones donde una variable se reemplaza con una expresión equivalente para producir una nueva ecuación con menos variables.

$$\begin{array}{l} y = -4x + 6 \quad y = 3x - 15 \\ -4x + 6 = 3x - 15 \\ -7x = -21 \\ \boxed{x = 3} \\ y = 3(3) - 15 \\ \boxed{y = -6} \end{array}$$

Por ejemplo, podemos introducir  $-4x + 6$  en lugar de  $y$  en  $y = 3x - 15$  porque son equivalentes.

**suma** El valor de dos o más expresiones al sumarse.

Por ejemplo, la suma de 6 y 15 es 21 y la suma de  $5x$  y  $-2x + 4$  es  $3x + 4$ .

**sistema de ecuaciones** Dos o más ecuaciones que representan las restricciones de un conjunto compartido de variables forman un sistema de ecuaciones.

Estas ecuaciones forman un sistema:

$$\begin{array}{l} 3b + c = -2 \\ b - 5c = 12 \end{array}$$

**sistema de desigualdades** Dos o más desigualdades que representan las restricciones de un conjunto compartido de variables forman un sistema de desigualdades.

Estas desigualdades forman un sistema:

$$\begin{array}{l} 10m + 5n > -2 \\ m - 5n \leq 12 \end{array}$$

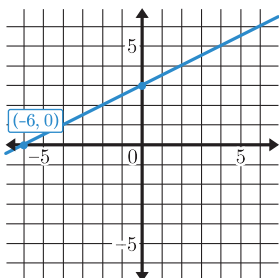
## English

## Español

## X

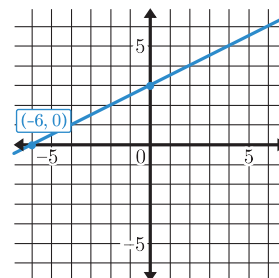
**$x$ -intercept** A point where the graph of an equation or function crosses the  $x$ -axis or when  $y = 0$ .

The  $x$ -intercept of the graph of  $-2x + 4y = 12$  is  $(-6, 0)$ , or just  $-6$ .



**intersección con el eje  $x$ , abscisa al origen** Un punto donde la gráfica de una ecuación o función cruza el eje  $x$ , o cuando  $y = 0$ .

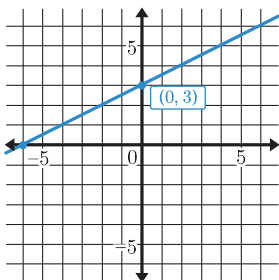
La intersección con el eje  $x$  de la gráfica de  $-2x + 4y = 12$  es  $(-6, 0)$ , o simplemente  $-6$ .



## Y

**$y$ -intercept** A point where the graph of an equation or function crosses the  $y$ -axis or when  $x = 0$ .

The  $y$ -intercept of the graph  $-2x + 4y = 12$  is  $(0, 3)$ , or just  $3$ .



**intersección con el eje  $y$ , ordenada al origen** Un punto donde la gráfica de una ecuación o función cruza el eje  $y$ , o cuando  $x = 0$ .

La intersección con el eje  $y$  de la gráfica de  $-2x + 4y = 12$  es  $(0, 3)$ , o simplemente  $3$ .

