Lesson Summary

When you're trying to visualize how a pattern will grow, it helps to have a toolbox of different strategies.

Here are the first three figures in a pattern and a table showing the number of tiles at each stage. You could determine how many tiles there will be in Figure 7 by:

- Drawing the next four figures and adding a row of 3 each time.
- Continuing the table, increasing the number of tiles by 3 in each new row.
- Noticing that the number of tiles is 3 times the figure number, plus 2, and then calculating the tiles for Figure 7.

Figure 1 **Figure 2** **Figure 3**



Figure	Number of Tiles
1	5
2	8
3	11

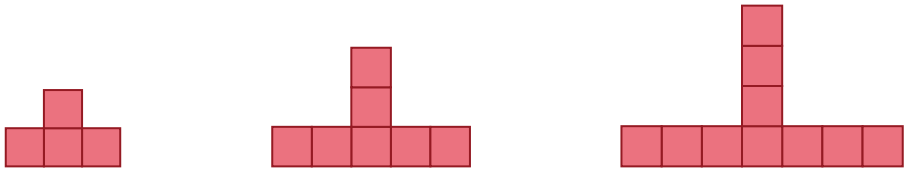
Things to Remember:

Lesson Practice
A1.1.01

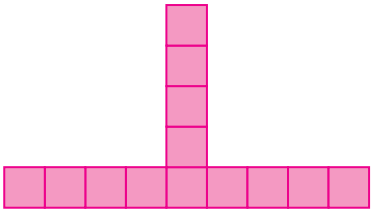
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Problems 1–3: Here is a visual pattern.

Figure 1 **Figure 2** **Figure 3**



1. Draw Figure 4.



2. Complete the table with the number of tiles in each figure.

Figure	1	2	3	4	…	10
Number of Tiles	4	7	10	13	…	31

3. Which expression represents the number of tiles in Figure 11?

A. $1 + 3(11)$

B. $1 + 4(11)$

C. $4 + 3(11)$

D. $4 + 1(11)$

For Problems 4–5: Create your own pattern.

4. Draw Figures 1 and 3. Then complete the table. ***Patterns vary.***

Figure 1 **Figure 2** **Figure 3**

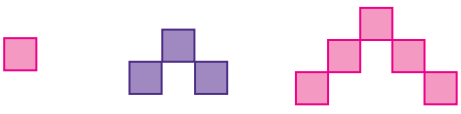


Figure	Number of Tiles
1	1
2	3
3	5

5. How many tiles will Figure 6 of your pattern have?

Responses vary depending on the patterns students create.

Lesson Practice
A1.1.01

Name: _____ Date: _____ Period: _____

Spiral Review

6. Create two different patterns that begin with the numbers 5 and then 10.

Patterns vary.

Pattern A: 5, 10, 15, 20, 25

Pattern B: 5, 10, 20, 40, 80

Describe how each pattern is changing.

Responses vary depending on the patterns students create.

Pattern A: Add 5 each time. **Pattern B:** Double the previous number.

7. Create two different patterns that each include the numbers 2 and 12.

Patterns vary.

Pattern C: 2, 4, 6, 8, 10, 12

Pattern D: 2, 12, 22, 32, 42

8. Determine the value of each expression.

Expression	Value
$4 + 4 + 4 + 4$	16
$4 \cdot 4 \cdot 4$	64
$4(3)$	12
3^4	81

9. Select *all* the expressions that are equal to 16.

☐ **A.** 8^2 ☑ **B.** 2^4 ☐ **C.** 2^8 ☑ **D.** 4^2 ☑ **E.** 16^1

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A **sequence** is a list of numbers in a particular order.

Sequences can change in predictable ways. Two examples of predictable change are constant differences and constant ratios.

A sequence has a **constant difference** when the difference between any two consecutive terms is the same.

$+6 \quad +6 \quad +6$

$5, \ 11, \ 17, \ 23$

A sequence has a **constant ratio** when the ratio between any two consecutive terms is the same.

$\times 4 \quad \times 4 \quad \times 4$

$2, \ 8, \ 32, \ 128$

Sequences can also change in predictable ways that are neither a constant difference or ratio.

$-1 \quad -2 \quad -3$

$13, \ 12, \ 10, \ 7$

Once you know how a sequence changes, you can use that change to determine unknown terms.

Here is an example:

- The known terms in this sequence have a constant ratio of 0.5.
- The value of the unknown term should be 0.5 times the term before, 40, or 2 times the term after, 10.
- That means the unknown term is 20.

$\times 0.5 \qquad \times 0.5$

$80, \ 40, \ ?, \ 10, \ 5$

Things to Remember:

Lesson Practice
A1.1.02

Name: _____ Date: _____ Period: _____

1. Determine the *constant ratio* for each sequence.

Sequence	Constant Ratio
256, 128, 64	0.5 or $\frac{1}{2}$
18, 54, 162	3
0.8, 0.08, 0.008	0.1 or $\frac{1}{10}$

2. Determine the *constant difference* for each sequence.

Sequence	Constant Difference
12, 17, 22	5
102, 85, 68	-17
$\frac{1}{4}$, 1, $\frac{7}{4}$	0.75 or $\frac{3}{4}$

3. Here's the start of a sequence: 1, -1, . . .

a Write two different rules the sequence could follow.

Rule 1: Constant ratio of -1 **Rule 2:** Constant difference of -2

b Determine the next three terms for each rule.

Rule 1: 1, -1, 1, -1, 1 **Rule 2:** 1, -1, -3, -5, -7

Problems 4–7: Fill in the blanks to complete each sequence. Each sequence has a constant difference.

4. -3, -2, -1, 0, 1

5. 1, 13, 25, 37, 49

6. 1, 0.25, -0.5, -1.25, -2

7. 92, 89, 86, 83, 80

Problems 8–9: Here are Sequence A and Sequence B. One sequence has a constant difference and one has a constant ratio.

8. Complete the table.

Sequence A	70, 90, 110, . . .	Constant ratio or (Constant difference)	Fourth term: 130
Sequence B	10, 20, 40, . . .	(Constant ratio) or Constant difference	Fourth term: 80

9. Which sequence will have the greater 10th term? Show or explain your thinking.

Sequence B. *Explanations vary.* Sequence B will have a greater 10th term. Even though the beginning terms in Sequence B are less than the terms in Sequence A, they increase at a faster rate.

Name: ..... Date: ..... Period: .....

10. A sequence has a first term of 6 and a constant ratio of -2. What are the first four terms of the sequence?

A. 6, 4, 2, 0 **B.** -2, 4, 10, 16 **C.** 6, 8, 10, 12 **D.** 6, -12, 24, -48

Spiral Review

11. Square A has an area of 64 square feet. Select *all* the expressions that are equal to the side length of this square in feet.

- ☐ **A.** $\sqrt{8}$
- ☑ **B.** $\sqrt{64}$
- ☐ **C.** 4
- ☑ **D.** 8
- ☐ **E.** $\frac{64}{2}$

12. The points (7, 21) and (-5, 17) lie on a line. What is the slope of the line?

$\frac{4}{12}$ (or equivalent)

13. Determine the value of each expression when $n = 4$.

Expression	$n^2 - 5$	$n(n + 6)$	$3n^2$
Value When $n = 4$	11	40	48

Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

There are several ways to define, or describe, a sequence. When you define a sequence recursively, you're determining each term using the previous term.

A **recursive definition** of a sequence includes a first term and a rule for finding every term that follows.

Here are some examples of recursive definitions for this sequence: $32, 16, 8, 4, 2, 1, 0.5$.

First term: 32

Rule: Half of the previous term

First term: 32

Rule: Constant ratio of $\frac{1}{2}$

First term: 32

Rule: Multiply the previous term by 0.5

Things to Remember:

Lesson Practice
A1.1.03

Name: _____ Date: _____ Period: _____

Problems 1–2: Use the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

1. Write a recursive definition for this sequence.

First term: $\frac{1}{2}$

Rule: Constant ratio of $\frac{1}{2}$ (or equivalent)

2. Write the next three terms of the sequence.

$\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$

3. Complete the recursive definition for each sequence.

	Sequence A	Sequence B	Sequence C	Sequence D
Sequence	2, 4, 6, 8, . . .	5, 7, 9, 11, . . .	50, 25, 0, -25, . . .	$\frac{1}{3}$, 1, 3, 9, . . .
First Term	2	5	50	$\frac{1}{3}$
Constant Difference or Constant Ratio?	Constant difference	Constant difference	Constant difference	Constant ratio
Rule	2	2	-25	3

4. This sequence has a constant difference of 5. Fill in the missing terms.

-3, 2, 7, 12, 17

Problems 5–6: Here is the start of a sequence: 1, 5, . . .

5. Write a rule and the next three terms the sequence could follow.

Rule: Constant difference of 4 Terms: 9, 13, 17

6. Write a *different* rule and the next three terms the sequence could follow.

Rule: Constant ratio of 5 Terms: 25, 125, 625

7. Remy is studying this sequence: 20, ?, 80. Remy thinks the missing term could be less than the first and third terms. Is Remy correct? Explain your thinking.

Yes. *Explanations vary.* The sequence could have a constant ratio of -2, which would make the second term -40.

Name: _____ Date: _____ Period: _____

8. A swimming pool is filling with water at a rate of 17 gallons per minute. The pool started with 100 gallons of water. Write a recursive definition to model the situation.

First term: **100** Rule: **Constant difference of 17**

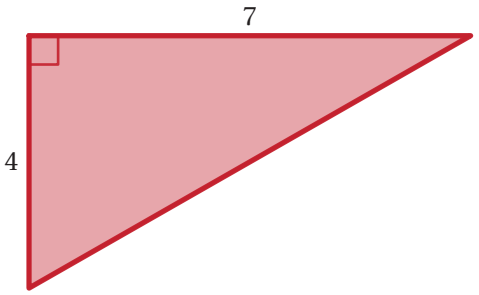
Spiral Review

9. Explain how you know that $\sqrt{42}$ is between 6 and 7.

Responses vary. **$\sqrt{36} = 6$ and $\sqrt{49} = 7$. $\sqrt{42}$ is between 6 and 7 because 42 is between 36 and 49.**

10. Determine the exact length of the unlabeled side.

$\sqrt{65}$



11. Write each expression as a single power of 10.

Expression	$\frac{10^7}{10^3}$	$10^2 \cdot 10^5$	$10^0 \cdot 10^9$	$\frac{10^5}{10^0}$
Single Power of 10	**10^4**	**10^7**	**10^9**	**10^5**

Reflection

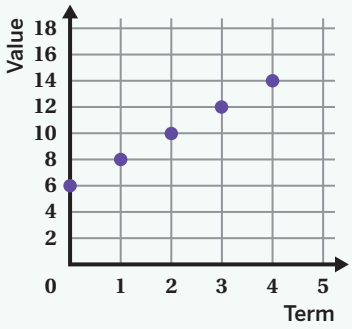
1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

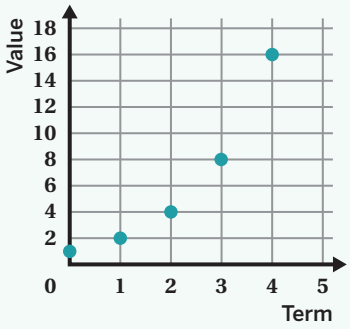
Sequences that change by a constant difference are called **arithmetic sequences**, while sequences that change by a constant ratio are called **geometric sequences**.

Sequences can be represented in multiple ways: as a list of numbers, in a table, on a graph, or with a recursive definition. In each representation, there are ways to identify if the sequence is arithmetic, geometric, or neither. Here are some examples of graphs of different kinds of sequences.

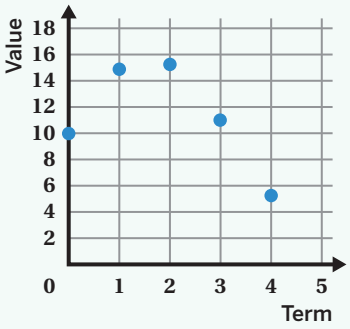
Arithmetic



Geometric



Neither



Things to Remember:

Lesson Practice
A1.1.04

Name: _____ Date: _____ Period: _____

1. Determine whether each sequence is arithmetic, geometric, or neither.

Sequence	1000, 200, 40, 8	2, 4, 16, 256	10, 20, 30, 40	500, 100, 20, 4
Arithmetic, Geometric, or Neither	Geometric	Neither	Arithmetic	Geometric

2. Complete each arithmetic sequence with its missing terms.

-2, 4, **10**, 16, **22**

11, 111, **211**, **311**, 411

5, 7.5, 10, **12.5**, **15**

5, **-4**, -13, -22, **-31**

3. Complete each geometric sequence with its missing terms.

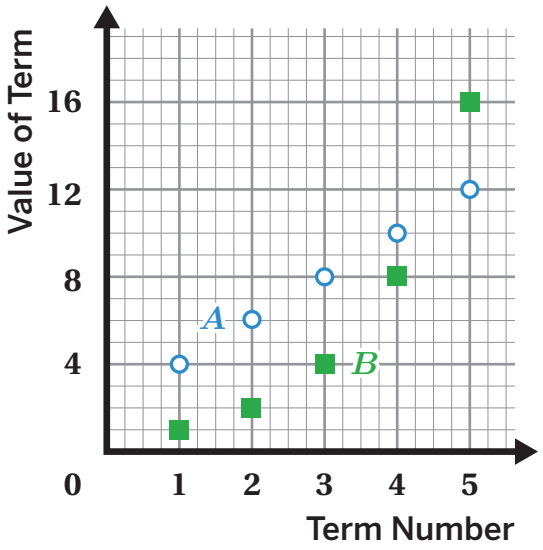
1, 5, 25, **125**, 625

-1, **6**, -36, 216, **-1296**

10, 5, **2.5**, **1.25**, 0.625

-4, **12**, -36, 108, **-324**

Problems 4–6: Here are the graphs of two sequences.



4. For Sequence A, describe a way to produce a new term from the previous term.

Responses vary. **Add 2 to the previous term.**

5. For Sequence B, describe a way to produce a new term from the previous term.

Responses vary. **Multiply the previous term by 2.**

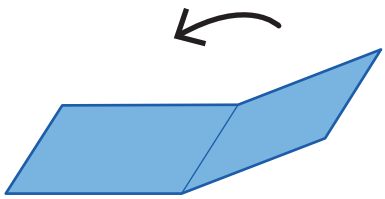
6. Which of these is a geometric sequence? Explain your thinking.

Sequence B. *Explanations vary.* Sequence B is a geometric sequence because it has the same multiplier from one term to the next: 2.

Name: _____ Date: _____ Period: _____

7. Ahmed is trying to see how many times he can fold a piece of paper in half. The thickness of the piece of paper he starts with is 0.001 centimeters. How thick will the paper be after 10 folds?

1.024 centimeters



Spiral Review

8. Select *all* the sets of side lengths that form a right triangle.

- ☐ **A.** $2, 3, 5$
- ☑ **B.** $\sqrt{7}, 9, \sqrt{88}$
- ☑ **C.** $\sqrt{12}, 6, \sqrt{48}$
- ☑ **D.** $4, 5, \sqrt{41}$
- ☐ **E.** $4, 5, 9$

9. If a line has a negative slope and contains the point $(4, 6)$, which of these points could it also contain?

A. $(7, 6)$ **B.** $(3, 1)$ **C.** $(6, 5)$ **D.** $(5, 8)$

10. Evaluate each expression.

Expression	$\left(\frac{1}{3}\right)^2$	$\left(\frac{1}{3}\right)^{-2}$
Value	$\frac{1}{9}$	9

Reflection

1. Circle a problem you're still curious about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

An **explicit definition** is a formula that determines the value of any term in a sequence using the term number. An explicit definition of a sequence can be written as an *expression* or in words.

Here are two situations and the explicit expressions that define them.

Situation: Julian's family is 250 miles away from home. They drive 50 miles toward their home for every n hours.

Expression: $250 - 50n$

The expression represents the number of miles Julian's family is from home after n hours.

- 250 represents the starting distance.
- -50 represents the distance traveled each hour.
- $-50n$ represents the total distance traveled.
- n represents the number of hours driven.

Situation: A scientist tracks a group of 250 bugs. Each month, the number of bugs increases by 1.5 times, for n months.

Expression: $250 \cdot 1.5^n$

The expression represents the number of bugs after n months.

- 250 represents the initial number of bugs.
- 1.5 represents the ratio that the bugs grow by each month.
- n represents the number of months.

Things to Remember:

Lesson Practice
A1.1.05

Name: _____ Date: _____ Period: _____

1. Jamar had 80 followers on social media. His number of followers *tripled* every month for 4 months. Select *all* the expressions that represent Jamar's followers after 4 months.

- [x] **A.** $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
- [ ] **B.** $80 + 4^3$
- [ ] **C.** $80 \cdot 4 \cdot 4 \cdot 4$
- [ ] **D.** $80 + 3 + 3 + 3 + 3$
- [x] **E.** $80 \cdot 3^4$

2. The weather forecast predicts a snowfall rate of 0.75 inches of snow per hour overnight. There are already 9 inches of snow on the ground in Anushka's neighborhood.

Complete the table to show the amount of snow on the ground, in inches, after n hours.

Hours, n	0	1	2	3	…	8
Amount of Snow (in.)	**9**	**9.75**	**10.5**	**11.25**	…	**15**

3. The population of a city was 100,000 in 1970 and has doubled 3 times since then. Select *all* the expressions that represent the population today.

- [ ] **A.** 300000
- [x] **B.** 800000
- [x] **C.** $100000 \cdot 2 \cdot 2 \cdot 2$
- [ ] **D.** $100000 \cdot 3^2$
- [x] **E.** $100000 \cdot 2^3$

Problems 4–5: Here is a table representing a pattern.

x	y
0	60
1	30
2	15
3	7.5
4	3.75

4. Circle the equation that represents the table.

A. $y = 60 + \frac{1}{2}x$

B. $y = 60 \cdot \left(\frac{1}{2}\right)^x$

C. $y = 60 \cdot 2^x$

D. $y = 60 - \frac{1}{2}x$

5. Explain your thinking.

Explanations vary. **The starting value is 60 and the common ratio is $\frac{1}{2}$.**

Name: ..... Date: ..... Period: .....

Problems 6–7: Maia earns money by shoveling snow in her neighborhood. She starts the winter with $52 in her bank account and will deposit $10 for every driveway she shovels.

6. Complete the table with Maia's account balance, in dollars, after she shovels n driveways.

Number of Driveways, n	0	1	2	3
Account Balance ($)	52	**62**	**72**	**82**

7. Write an explicit expression for this situation.

$52 + 10n$

Problems 8–9: A group of students tracked the number of likes on a social media post. They represented this number using the equation $n = 40 \cdot 1.5^t$, where n is the total number of likes and t is the number of days since the students started counting.

8. Explain what the 40 and 1.5 mean in this situation.

40 means that there were 40 likes on the social media post when the students started counting. 1.5 means that the number of likes on the post grew by 1.5 times every day.

9. How many likes do the students predict there will be 2 days after they started counting?

90 likes

Spiral Review

Problems 10–11: Write each expression using an exponent.

10. $\left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

$\left(\frac{4}{5}\right)^3$

11. $9.3 \cdot 9.3 \cdot 9.3 \cdot 9.3 \cdot 9.3$

9.3^5

12. An arithmetic sequence starts with $10, 5, \ldots$
Explain how you would calculate the value of the 100th term.

Responses vary. **Starting with the first term, I can add the constant difference multiplied by 99 to get the 100th term. In this example, the first term is 10 and the constant difference is -5, so the 100th term would be $10 + (-5)(99)$ or -485.**

Reflection

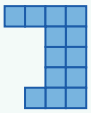
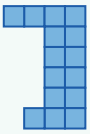
1. Put a star next to a problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can write an explicit expression for a pattern or sequence in multiple equivalent ways, just by referencing different term numbers.

Here's an example of a pattern and its matching sequence.

Figure 1	Figure 2	Figure 3	Figure 4	Sequence
				$9, 11, 13, 15$ Term 1
$7 + 2(1)$	$7 + 2(2)$	$7 + 2(3)$	$7 + 2(4)$	$9 + 2(n - 1)$

One expression that represents this sequence is $9 + 2(n - 1)$; this expression uses Term 1 as the starting value and $n - 1$ to calculate the change.

The expression $7 + 2n$ also represents this sequence, using Term 0 as the starting value, and n to calculate the change.

Things to Remember:

Lesson Practice
A1.1.06

Name: _____ Date: _____ Period: _____

1. Match each sequence to its explicit expression.

Sequence	Explicit Expression
a. 4, 10, 16, 22	c. $320 \cdot \left(\frac{1}{2}\right)^n$
b. 4, 12, 36, 108	a. $4 + 6(n - 1)$
c. 160, 80, 40, 20	d. $320 + \frac{1}{2}(n - 1)$
d. 320, 320.5, 321, 321.5	b. $4 \cdot 3^{(n-1)}$

2. Select *all* the expressions that could represent the number of tiles in Figure n of this pattern.

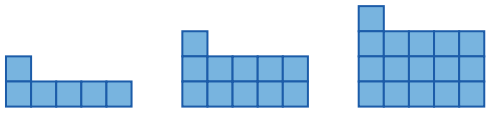


Figure 1 Figure 2 Figure 3

- ☐ A. $5 + n$
- ☑ B. $5n + 1$
- ☐ C. $6n - 1$
- ☑ D. $6 + 5(n - 1)$
- ☐ E. $10n - 4$

Problems 3–4: Here is a visual pattern.

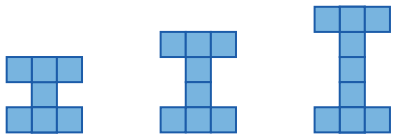


Figure 1 Figure 2 Figure 3

3. Complete the table with the number of tiles in each figure.

Figure	# of Tiles
1	7
2	8
3	9
4	10
…	…
15	21

4. Write an explicit expression for the number of tiles in Figure n.

- $6 + n$ (or equivalent)
- $7 + (n - 1)$ (or equivalent)

5. The first four terms of a geometric sequence are: 6, 18, 54, 162. Which explicit expression can be used to describe this sequence?

A. $2 + 3n$ B. 3^n C. $2 \cdot 3^n$ D. $6 \cdot 3^n$

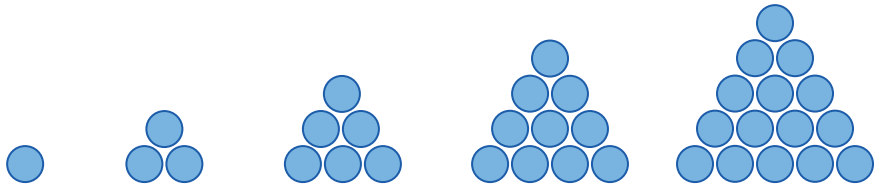
Name: _____ Date: _____ Period: _____

6. The first four terms in a sequence are 15, 22, 29, 36. Write two explicit expressions for Term n of this sequence.

Expression 1: $15 + 7(n - 1)$ (or equivalent)

Expression 2: $8 + 7n$ (or equivalent)

7. A *triangular number* can be represented as the number of dots in an equilateral triangle.



What is the 8th triangular number?
36

Spiral Review

Problems 8–10: Determine the value of each expression when $x = 3$.

8. $7x - 8$
13

9. $5(x + 9)$
60

10. $\frac{x - 6}{4x}$
$-\frac{1}{4}$ (or equivalent)

Problems 11–13: Decide whether each sequence is arithmetic, geometric, or neither. Circle one.

11. 25, 5, 1, . . . Arithmetic (Geometric) Neither

12. 25, 19, 13, . . . (Arithmetic) Geometric Neither

13. 25, 52, 25, 52, . . . Arithmetic Geometric (Neither)

Reflection

1. Put a heart next to the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A **model** is a mathematical representation (such as a graph, equation, or relationship) of a situation. You can use a model to make predictions or decisions.

For example, let's say a group of scientists track the amount of algae on the surface of a lake, in square feet, over n weeks.

Weeks (n)	0	1	2	3	4
Amount of Algae (sq. ft)	2	2.8	3.92	5.49	7.68

The scientists model this information with the expression $2 \cdot 1.4^n$, which can be used to predict how much algae there will be in the upcoming weeks. Predictions might not be exactly accurate because factors like weather or human action can affect algae growth. The model might also be limited by the size of the lake.

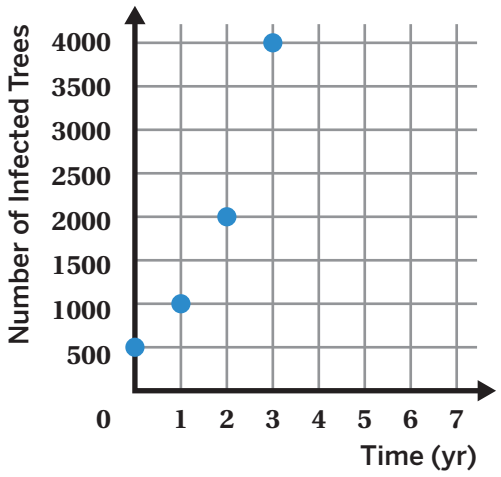
No mathematical model applies perfectly to real life, but a model can still be a powerful and useful tool. We can even improve models when we take into account more information.

Things to Remember:

Lesson Practice
A1.1.07

Name: ..... Date: ..... Period: .....

Problems 1–3: A group of scientists are tracking the number of trees infected with a disease. They count the number of infected trees each year.



1. How many infected trees were there when the scientists began counting?

   500 infected trees

2. After how many years will the number of infected trees reach 4,000?

   3 years

3. Write an explicit expression to represent the number of infected trees after n years.

   $500 \cdot 2^n$

Problems 4–5: A skateboarder scrapes their arm. 1 day after they apply an antibiotic, the population of bacteria is 1,000,000. After every day, the population decreases by half.

Number of Days, n	Population of Bacteria
1	1000000
2	500000
3	**250000**
4	**125000**
5	**62500**

4. Complete the table with the population of bacteria that is present after n days.

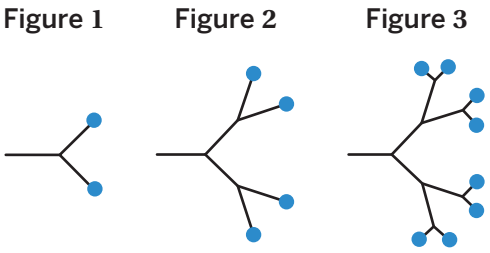
5. Is the population of bacteria arithmetic, geometric, or neither? Circle one.

   Arithmetic (Geometric) Neither

   Explain how you know.

   Explanations vary. **The population of bacteria changes by a constant ratio of $\frac{1}{2}$ each day, so it's geometric.**

Problems 6–7: Here are the first three figures in a pattern.



6. Is the number of dots arithmetic, geometric, or neither? Circle one.

   Arithmetic (Geometric) Neither

   Explain how you know.

   Explanations vary. **The pattern is a geometric sequence because it grows by a constant ratio of 2.**

7. Write an explicit expression to determine the number of dots in Figure n.
   - **$2 \cdot 2^{(n-1)}$ (or equivalent)**
   - **$1 \cdot 2^n$ (or equivalent)**

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Problems 8–10: A paper clip weighs 0.5 grams and an empty envelope weighs 6.75 grams. Sai adds paper clips one at a time to an empty envelope.

8. Complete the table with the weight of the envelope, in grams, after Sai adds n paper clips.

Number of Paper Clips, n	Weight (g)
0	6.75
1	**7.25**
2	**7.75**
3	**8.25**

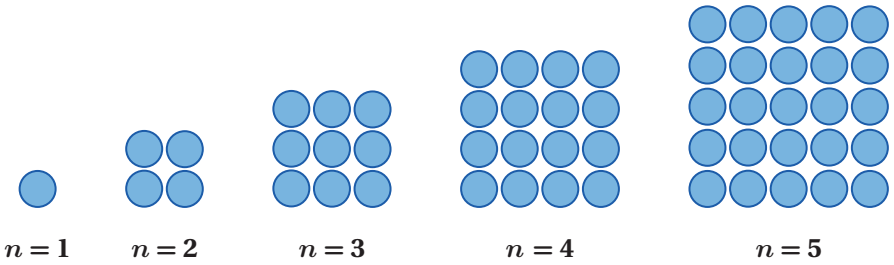
9. Does it make sense to have a row in the table where $n = 10.25$? Explain your thinking.

No. *Explanations vary.* Because n represents the number of paper clips, the value should always be a whole number.

10. How many paper clips will be in the envelope when the weight is 9.75 grams?

6 paper clips

Problems 11–12: A *square number* can be represented as the number of dots in a square.



11. Complete the table for square numbers 1 to 5.

n	1	2	3	4	5
Number of Dots	**1**	**4**	**9**	**16**	**25**

12. Write an expression for the number of dots in square number n.

n^2 (or equivalent)

Spiral Review

Problems 13–14: Determine the value of each expression.

13. $2(3^3) =$ **54**

14. $2 + 3^3 =$ **29**

Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or to share something you're proud of.