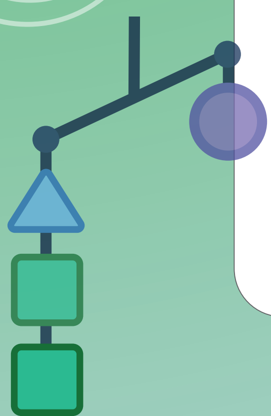
Unit 4

Linear Equations and Linear Systems

Equations can help you understand and solve problems. So far, you've solved single variable equations where the variable is on one side. In this unit, you'll solve equations with variables on both sides. You'll also solve two linear equations in a system and determine how many solutions there are.

Essential Questions

- How can you solve an equation with variables on both sides of the equal sign?
- How can systems of equations be used to represent situations and solve problems?
- What does it mean for an equation or system of equations to have *no*, *one*, or *infinitely many* solutions?



Summary | Lesson 1

You can use a number machine to perform a series of operations on a number.

- If only the *input* (the number put into the machine) is provided, the *output* (the number that comes out of the machine) can be determined by performing the series of operations in order.
- If only the output is provided, the input can be determined by guessing and checking, by working backward, or by writing and solving an equation.

For example, a number machine performs this series of operations in order:

- Pick an input
- Multiply by 3
- Add 8
- Subtract 1

The output is 19. Here are two ways you can determine the input:

Work backward from the output

Start with 19, add 1, subtract 8, and then divide by 3. $(19 + 1 - 8) \div 3 = 4$.

These steps show that the input is 4.

Write and solve an equation

The equation $(3x + 8) - 1 = 19$ represents the number machine, where x is the input.

Solving this equation shows that $x = 4$.

Try This

A number machine takes an input and performs these operations in order:

- Multiply by 2.
- Add 4.

The output is 18. What was the input?

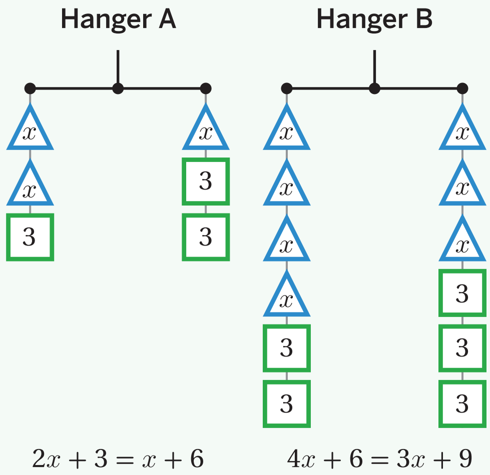
Summary | Lesson 2

A hanger diagram is balanced when its left and right sides are equal in weight. Adding or removing equally weighted items from each side of a hanger diagram keeps the hanger balanced. Balanced hanger diagrams can represent equations.

For example, Hangers A and B represent **equivalent equations**, which are equations that have the same solution.

These equations are equivalent because if you add $2x$ and 3 to each side of Hanger A, you create the also balanced Hanger B.

The solution to each equation is $x = 3$, which is the weight of each triangle on both hangers.



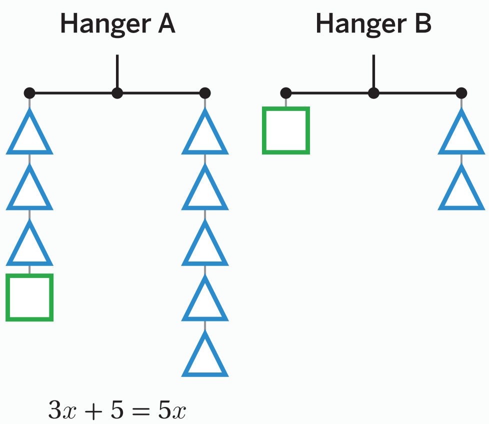
Try This

Hangers A and B are both balanced hangers. Each square weighs 5 pounds.

Hanger A can be represented by the equation $3x + 5 = 5x$.

a Write an equivalent equation to represent Hanger B.

b Determine the weight of the triangle.



Summary | Lesson 3

You can use hanger diagrams to visualize and represent equations. Hanger diagrams can help you see how moves that keep the hanger balanced create *equivalent equations*, such as:

- Adding or subtracting the same term on each side of an equation.
- Multiplying or dividing the expressions on each side of an equation by the same number.

You can use these moves to solve equations for an unknown variable. Here is an example of using balanced moves to solve an equation:

Solving Step	Balanced Equation Move
$4x + 9 = \text{-}2x - 3$ $-9 \qquad\quad -9$	Subtract 9 from both sides.
$4x = \text{-}2x - 12$ $+2x + 2x$	Add $2x$ to both sides.
$\frac{6x}{6} = \frac{\text{-}12}{6}$ $x = \text{-}2$	Divide by 6 on both sides.

You can check if a value is a *solution* to an equation by substituting it into the original equation. If it makes the equation true then the solution is correct.

$$4(\text{-}2) + 9 = \text{-}2(\text{-}2) - 3$$
$$\text{-}8 + 9 = 4 - 3$$
$$1 = 1$$

Try This

a Here are the steps to solve an equation. Match each step with a description of how it balances the equation.

Solving Steps

Step 1:

$4x + 9 = \text{-}2x - 3$

$4x = \text{-}2x - 12$

Step 2:

$4x = \text{-}2x - 12$

$6x = \text{-}12$

Step 3:

$6x = \text{-}12$

$x = \text{-}2$

Descriptions

Step _____: Add $2x$ to both sides.

Step _____: Subtract 9 from both sides.

Step _____: Divide both sides by 6.

b How can you check if -2 is a solution to the equation?

Summary | Lesson 4

There are many ways to solve an equation using balanced equation moves. Generally, you want to perform steps that will move you closer to an equivalent equation where the variable is isolated, such as an equation of the form $\text{variable} = \text{number}$. Here are some examples of moves that can be helpful in solving an equation.

Applying the *distributive property*	Combining *like* terms on one side of an equation	Adding or subtracting the same term from both sides	Multiplying or dividing both sides of the equation by the same value
$2x + 4 = 3(2x + 4)$ $2x + 4 = 6x + 12$	$-10x + 3x = 15$ $-7x = 15$	$-x - 8 = 4x + 7$ $-8 = 5x + 7$	$-15x = 5(2x + 4)$ $-3x = 2x + 4$

Try This

Solve the equation $2x = -3(x + 5)$. Show your thinking.

Summary | Lesson 5

You can use different steps to solve the same equation. For example, here are two ways to solve the equation $\frac{1}{3}(3x + 9) = 6x + 18$:

Start by distributing $\frac{1}{3}$ to $(3x + 9)$	**Start by multiplying both sides of the equation by 3**
$\frac{1}{3}(3x + 9) = 6x + 18$ $x + 3 = 6x + 18$ $-5x + 3 = 18$ $-5x = 15$ $x = -3$	$\frac{1}{3}(3x + 9) = 6x + 18$ $3x + 9 = 18x + 54$ $3x - 45 = 18x$ $-45 = 15x$ $-3 = x$

Sometimes you might unintentionally make a move that unbalances an equation. Here are some examples:

Description of Unbalanced Move	**Example**	**Alternative Balanced Move**
Distributing a factor to some terms in parentheses but not all of them	$\frac{1}{3}(3x + 9) = 5x$ $x + 9 = 5x$	$\frac{1}{3}(3x + 9) = 5x$ $x + 3 = 5x$
Multiplying some terms in an equation by a factor but not all of them	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 6$	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 30$
Adding or subtracting a term instead of distributing	$7 - 4(x + 1) = 2x + 5$ $3(x + 1) = 2x + 5$	$7 - 4(x + 1) = 2x + 5$ $7 - 4x + 4 = 2x + 5$

Try This

a What is one possible first step you could take to solve this equation?
$9 - 2b + 6 = -3(b + 5) + 4b$

b What is a different possible first step you could take to solve this equation?
$9 - 2b + 6 = -3(b + 5) + 4b$

c Does this move keep the equation balanced? Explain your thinking.
$9 - 2b + 6 = -3(b + 5) + 4b$
$9 + 6 = -3(b + 5) + 6b$

Summary | Lesson 6

Equations can have one solution, no solution, or **infinitely many solutions**.

Here are some examples.

One Solution

$$3x + 8 = 6 + 2 - 3x$$

This equation is only true when $x = 0$.

A linear equation has *one solution* when the expressions on either side of the equation have one value for the variable that makes them equal.

No Solution

$$3(x + 4) = 3x + 7$$

This equation is never true for any value of x.

A linear equation has *no solution* when the expressions on either side of the equation have no value for the variable that make them equal.

Infinitely Many Solutions

$$10 - 3x = 8 - 3x + 2$$

This equation is always true for any value of x.

A linear equation has *infinitely many solutions* when the expressions on either side of the equation are *equivalent*: always equal no matter the value of the variable.

Try This

Determine if each equation has one solution, no solution, or infinitely many solutions.

$2x = x + x$	$3x = x + 2$	$7 + x = x + 7$
$x = x + 1$	$-x = 1 - x$	$9x = 10$

One Solution	No Solution	Infinitely Many Solutions

Summary | Lesson 7

Equations can have many different features, including fractions, decimals, negative values, grouping symbols, and multiple terms. Based on the features, it can be helpful to think about what steps might be most useful in solving the equation.

When solving an equation with one solution, the goal is to end up with the variable isolated on one side of the equation and its value on the other. But this doesn't happen when there is no solution or infinitely many solutions.

One Solution

$$3x + 8 = 6 + 2 - 3x$$
$$3x + 8 = 8 - 3x$$
$$6x + 8 = 8$$
$$6x = 0$$
$$x = 0$$

This equation is only true when $x = 0$.

No Solution

$$3(x + 4) = 3x + 7$$
$$3x + 12 = 3x + 7$$
$$12 = 7$$

This equation is never true for any value of x.

Infinitely Many Solutions

$$10 - 3x = 8 - 3x + 2$$
$$10 - 3x = 10 - 3x$$
$$10 = 10$$

This equation is always true for any value of x.

Try This

Solve the equation. Show your thinking.

$3x - 7x + 5 = 3(x - 10)$

Summary | Lesson 8

We can write two expressions in one variable and set them equal to each other to represent a scenario in which two conditions are equal. We can solve this equation to determine the unknown quantity.

For example, imagine two hikers walking in the same direction on a flat trail. The hikers will meet each other when they are at the same location on the trail at the same time.

To determine when this occurs, an expression can be used to represent the location and walking speed of each hiker.

	Location (ft)	Walking Speed (ft/s)	Expression
Hiker 1	30	4	$30 + 4t$
Hiker 2	10	7	$10 + 7t$

You can set these two expressions equal to each other to form one equation that can be solved.

$30 + 4t = 10 + 7t$

$20 = 3t$

$t = \frac{20}{3}$ or about 6.7 seconds

Try This

A car is traveling at a constant speed of 16 meters per second. A scooter is traveling at 9 meters per second and is 42 meters ahead of the car.

This situation can be represented by the equation $16t = 9t + 42$, where t represents the number of seconds that the vehicles have been traveling.

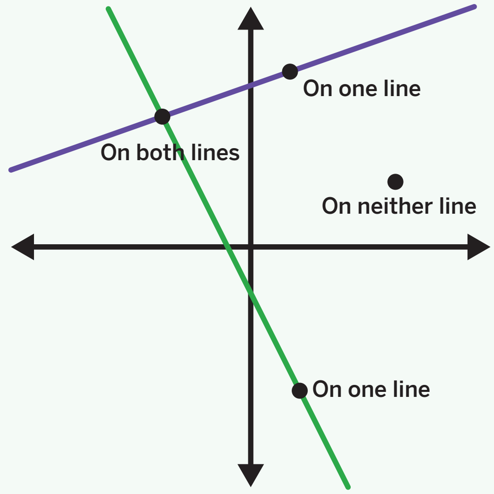
a. Solve this equation. Show your thinking.

b. What does the solution represent in this situation?

Summary | Lesson 9

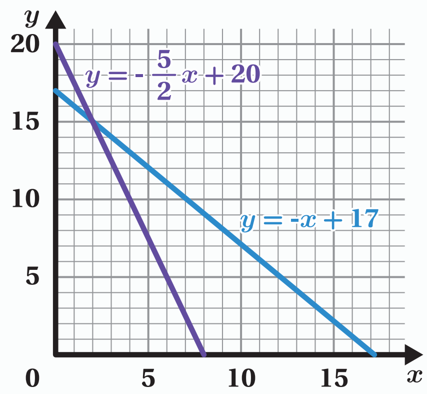
Linear relationships can represent many situations. Lines graphed on the same coordinate plane can simultaneously represent multiple conditions or relationships involving the same variables.

- The coordinates of a point that is on both lines make both relationships true.
- The coordinates of a point on only one line make only one relationship true.
- The coordinates of a point on neither line make neither relationship true.



Try This

This graph represents two relationships.



a What is a combination of values that makes both relationships true?

$x =$ ________ $y =$ ________

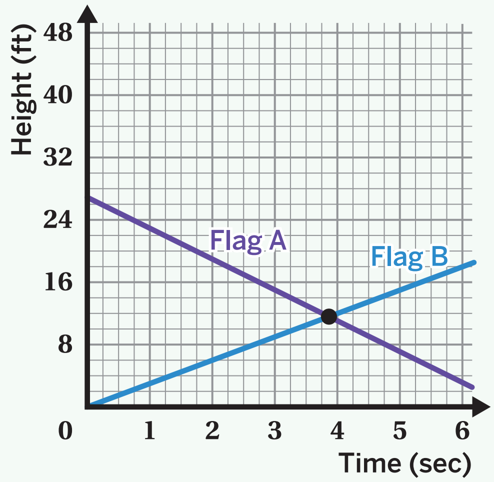
b What is a combination of values that makes *one* relationship true but not the other?

$x =$ ________ $y =$ ________

If there are two equations that share the same variables, you can find the solution that makes both equations true by locating the **point of intersection**, where the two lines meet on a graph.

For example, consider this graph.

Although you can't see the exact values of the point of intersection, you can tell that the flags are the same height, at about 11.5 feet, just before 4 seconds.



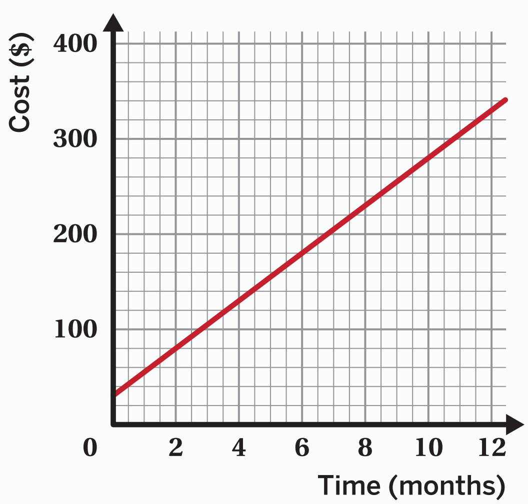
Try This

Gym A costs \$30 to join and \$25 per month. Gym B costs \$100 to join and \$15 per month.

The graph shows the cost of Gym A over time.

a Graph a line to represent the cost of Gym B over time.

b Explain what the point of intersection represents in this situation.



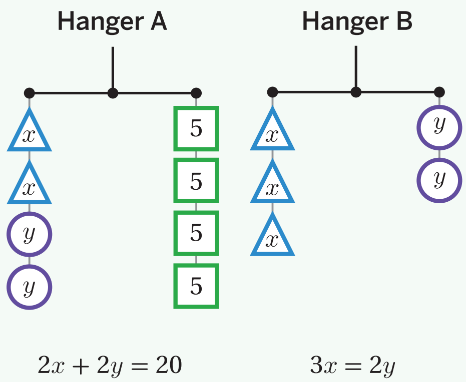
Summary | Lesson 11

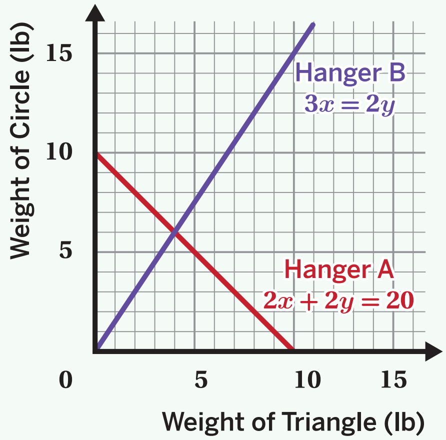
A **system of equations** is a set of two or more equations with the same variables and are meant to be solved together.

A **solution to a system** of equations is a set of values that makes all equations in that system true.

For example, here is a system of equations:

$$\begin{cases} 2x + 2y = 20 \\ 3x = 2y \end{cases}$$





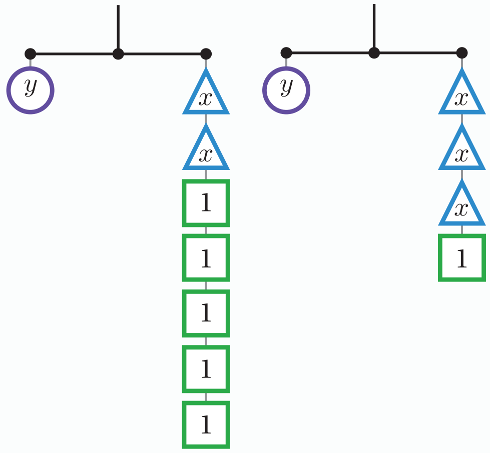
The ordered pair $(4, 6)$ is the point of intersection, which means that it will make both equations true. Both hangers will balance when the triangles weigh 4 pounds and the circles weigh 6 pounds.

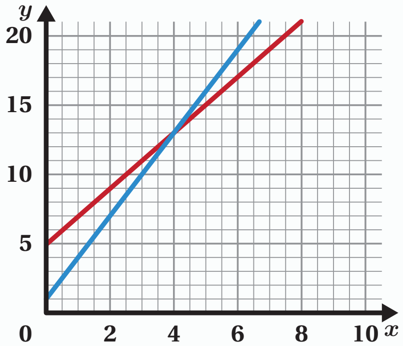
Try This

Determine the solution to this system of equations:

$y = 2x + 5$

$y = 3x + 1$





Summary | Lesson 12

For a point to be a solution to a system of equations, the x- and y-coordinates must make both of the equations true. This ordered pair is the *point of intersection* when the system is graphed.

For example, here is a system of equations:

$$\begin{cases} y = 4x - 5 \\ y = \text{-}2x + 7 \end{cases}$$

To determine the solution to the system, you can write a single equation by taking the two expressions that are equal to y and setting them equal to each other.

$4x - 5 = \text{-}2x + 7$
$6x - 5 = 7$
$6x = 12$
$x = 2$

You can then substitute the solution for x into either of the original equations to determine the value of y.

$y = 4x - 5$
$y = 4(2) - 5$
$y = 8 - 5$
$y = 3$

The solution to this system of equations is the point $(2, 3)$.

Try This

What is the solution to this system of equations?

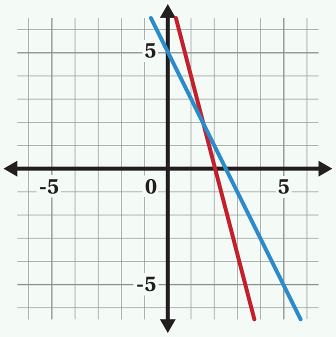
$y = 2x + 6$

$y = \text{-}3x - 4$

Summary | Lesson 13

Systems of two linear equations can have one solution, no solution, or infinitely many solutions. You can determine the number of solutions to a system of equations by graphing, comparing the slopes and y-intercepts, or solving the system algebraically.

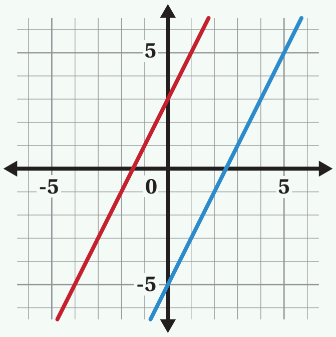
One solution:



$$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$$

- Different slopes

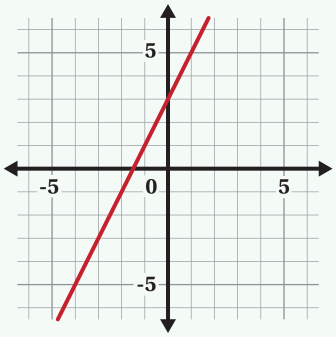
No solution:



$$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$$

- Same slopes
- Different y-intercepts

Infinitely many solutions:



$$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$$

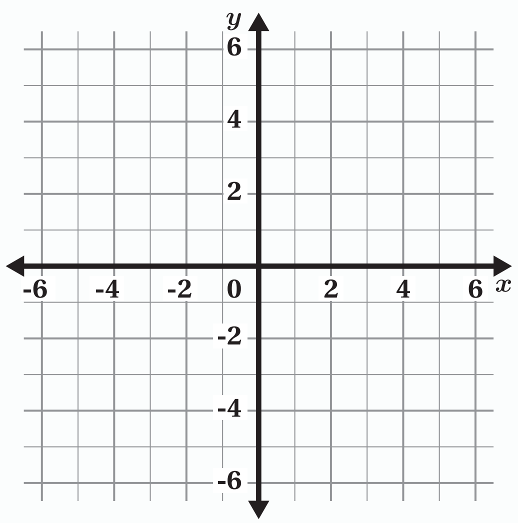
- Same slopes
- Same y-intercepts

Try This

Determine whether each system of equations will have one solution, no solution, or infinitely many solutions. Use the graph if it helps with your thinking. Show or explain your thinking.

a $\begin{cases} y = \frac{2}{3}x - 2 \\ y = \frac{2}{3}x + 3 \end{cases}$

b $\begin{cases} y = 1.5x -2 \\ y = 1.5x - 2 \end{cases}$



Summary | Lesson 14

An ordered pair (x, y) is a solution to a system of equations if it makes both equations true. If you know the value of one variable in one of the equations, you can substitute it into the other equation to solve for the second variable.

Here is a system of equations:

$$\begin{cases} y = 5x \\ 2x - y = 9 \end{cases}$$

Since $y = 5x$, you can substitute $5x$ for y in $2x - y = 9$, and then solve for x.

$$\begin{aligned} 2x - (5x) &= 9 \\ \text{-}3x &= 9 \\ x &= \text{-}3 \end{aligned}$$

You can then substitute the solution for x into either of the original equations to determine the value of y.

$$\begin{aligned} y &= 5x \\ y &= 5(\text{-}3) \\ y &= \text{-}15 \end{aligned}$$

The ordered pair (-3, -15) is the solution to the system of equations.

Try This

Solve this system of equations. Show your thinking.

$$\begin{cases} x = 7 \\ y = \text{-}3x + 42 \end{cases}$$

Lesson 1

7

[Here are two strategies for calculating the answer:

Work backward from the output:
Start with 18. Subtract 4, then divide by 2. $(18 - 4) \div 2 = 7$

Write and solve an equation:
The equation $2x + 4 = 18$ represents the number machine, where x is the input. Solving this equation shows that $x = 7$.]

Lesson 2

a. $5 = 2x$

b. 2.5 pounds

Lesson 3

a. Step __2__: Add $2x$ to both sides.

Step __1__: Subtract 9 from both sides.

Step __3__: Divide both sides by 6.

b. You can check if a value is a solution to an equation by substituting it into the original equation.
$4x + 9 = -2x - 3$
$4(-2) + 9 = -2(-2) - 3$
$(-8) + 9 = (4) - 3$
$1 = 1$

Lesson 4

$x = -3$. *Work varies.*

$2x = -3(x + 5)$

$2x = -3x - 15$

$5x = -15$

$x = -3$

Lesson 5

a *Responses vary*. [One possible first step is to distribute -3 to $(b + 5)$:
$9 - 2b + 6 = -3(b + 5) + 4b$
$9 - 2b + 6 = -3b - 15 + 4b$]

b *Responses vary*. [Another possible first step is to combine like terms 9 and 6:
$9 - 2b + 6 = -3(b + 5) + 4b$
$15 - 2b = -3(b + 5) + 4b$]

c Yes. *Explanations vary*. $2b$ was added to both sides of the equation, so the equation remains balanced.

Lesson 6

One Solution	No Solution	Infinitely Many Solutions
$3x = x + 2$ $9x = 10$	$x = x + 1$ $-x = 1 - x$	$7 + x = x + 7$ $2x = x + x$

Lesson 7

$x = 5$. *Work varies.*

$3x - 7x + 5 = 3(x - 10)$

$-4x + 5 = 3(x - 10)$

$-4x + 5 = 3x - 30$

$5 = 7x - 30$

$35 = 7x$

$5 = x$

Lesson 8

a $t = 6$. *Work varies.*

$16t = 9t + 42$

$7t = 42$

$t = 6$

b *Responses vary*. The car will catch up to the scooter after 6 seconds.

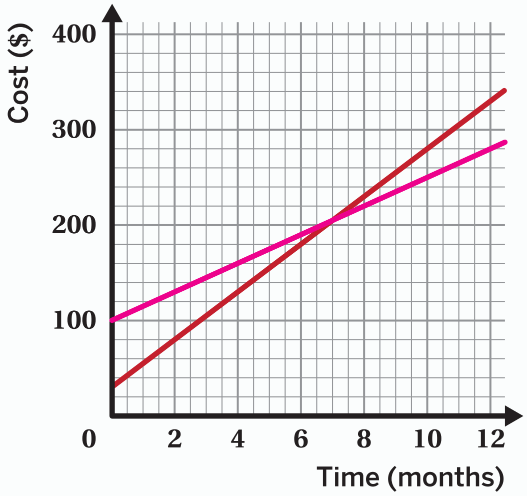
Lesson 9

a $x = 2$ and $y = 15$. [One strategy is to look for a point that is on both lines. $(2, 15)$ is that point.]

b *Responses vary.* $x = 6$ and $y = 5$. [One strategy is to look for a point that is on only one line, such as $(6, 5)$.]

Lesson 10

a



b The point of intersection represents the month during which both gyms will cost an equal amount. After 7 months both gyms will cost a bit more than $200.

Lesson 11

$(4, 13)$. [One strategy is to look at the point of intersection for the lines, which represents the x- and y-values that make both equations true.]

Lesson 12

(-2, 2). [One strategy is to write a single equation by taking the two expressions that are equal to y and setting them equal to each other.

$2x + 6 = -3x - 4$

$5x + 6 = -4$

$5x = -10$

$x = -2$

Then substitute the solution for x into one of the original equations:

$y = 2x + 6$

$y = 2(-2) + 6$

$y = -4 + 6$

$y = 2$

So the solution to this system of equations is (-2, 2).]

Lesson 13

a. No solution. Since these lines have the same slope and different y-intercepts, they will never intersect.

b. Infinitely many solutions. Since these are the same line, any point on one line will be on both lines.

Lesson 14

(7, 21). [Here is one strategy:

Since $x = 7$, you can substitute 7 for x in $y = -3x + 42$:

$y = -3x + 42$

$y = -3(7) + 42$

$y = -21 + 42$

$y = 21$

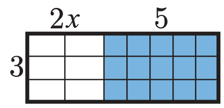
The ordered pair (7, 21) is the solution to the system of equations.]

Grade 8 Unit 4 Glossary/8.º grado Unidad 4 Glosario

English | **Español**

D

distributive property The distributive property says that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

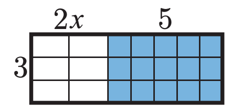


Area as a Product

$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

propiedad distributiva La propiedad distributiva indica que la multiplicación de un número por la suma de dos o más términos es igual a la suma de las multiplicaciones de dicho número por cada término individualmente.



El área como un producto

$3(2x + 5)$

Por ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

E

equivalent equations Equations that have the exact same solution(s).

$3x + 4 = 10$ and $9x + 12 = 30$ are equivalent equations because if you multiply the first equation by 3, you create the second one. The solution to each equation is $x = 2$.

ecuaciones equivalentes Ecuaciones que tienen la misma o las mismas soluciones.

$3x + 4 = 10$ y $9x + 12 = 30$ son equivalentes porque si se multiplica la primera ecuación por 3, se forma la segunda ecuación. La solución de cada ecuación es $x = 2$.

equivalent expressions Expressions that are equal for every value of a variable.

For example, $6x + 2x$ is equivalent to $5x + 3x$. No matter what value x is, the two expressions are always equal.

expresiones equivalentes Expresiones que son iguales para todos los valores de una variable.

Por ejemplo, $6x + 2x$ es equivalente a $5x + 3x$. Independientemente del valor de x, las dos expresiones son siempre iguales.

expression A set of numbers, symbols, and operators (such as + and •) grouped together to represent a value. Unlike equations, expressions do not include an equals sign (=).

For example, $5x$, $m - 4$, $3y + 0.5$, and $2(n + 8)$ are all expressions.

expresión Un conjunto de números, símbolos y operadores (como + y •) agrupados para representar un valor. A diferencia de las ecuaciones, las expresiones no incluyen un signo igual (=).

Por ejemplo, $5x$, $m - 4$, $3y + 0.5$, y $2(n + 8)$ son expresiones.

English	Español

I

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. Equivalent equations will create the same line on a graph, so every point on the line is a solution to each equation in the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

infinitas soluciones Una ecuación tiene un número infinito de soluciones si es verdadera independientemente del valor de la variable. Un sistema de ecuaciones tiene un número infinito de soluciones si las ecuaciones en el sistema son equivalentes. Las ecuaciones equivalentes producen la misma recta en una gráfica, por lo que cada punto en la recta es una solución de cada ecuación en el sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene un número infinito de soluciones porque la ecuación es verdadera para cualquier valor de x.

L

like terms Two or more terms that have the same variables and exponent values. Numbers, decimals, and fractions are all like terms.

For example, $8x$ and $12x$ are like terms because they both have a variable of x. $8x$ and 12 are not like terms. $8x$ and $12x^2$ are also not like terms because they have different exponents.

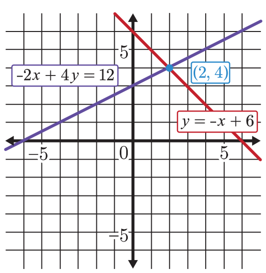
términos semejantes Dos o más términos que tienen variables y valores de exponentes iguales. Los números enteros, los decimales y las fracciones son términos semejantes.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable x. $8x$ y 12 no son términos semejantes. $8x$ y $12x^2$ tampoco son términos semejantes porque tienen exponentes diferentes.

P

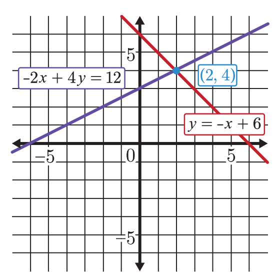
point of intersection A point where two lines meet.

For example, $(2, 4)$ is the point of intersection for the lines $y = -x + 6$ and $-2x + 4y = 12$.



punto de intersección Un punto donde se cruzan dos rectas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



R

rate A comparison, or ratio, that describes how two quantities change together.

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

Grade 8 Unit 4 Glossary/8.º grado Unidad 4 Glosario

English

S

solution The value or set of values that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y).

The solution to the equation $x + 15 = 8$ is $x =$ -7 because $(-7) + 15 = 8$.
One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.

solution to a system of equations A set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the point of intersection.

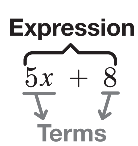
For example, $(2, 4)$ is the solution to this system of equations, and the point of intersection on the graph.
$y = -x + 6$
$-2x + 4y = 12$

system of equations Two or more equations that represent the constraints on a shared set of variables.

For example, these equations make up a system of equations:
$x + y = -2$
$x - y = 12$

T

term A part of an expression. A term can be a single number, a variable, or a number and variable multiplied together.



For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.

Español

S

solución El valor o conjunto de valores que hacen que la ecuación sea verdadera. Una solución de una ecuación con dos variables es un par de valores que hacen que la ecuación sea verdadera, y a menudo se escribe como un par ordenado, (x, y).

La solución de la ecuación $x + 15 = 8$ es $x =$ -7 porque $(-7) + 15 = 8$.
Una solución de la ecuación $4x + 3y = 24$ es $(6, 0)$ porque $4(6) + 3(0) = 24$.

solución de un sistema de ecuaciones Un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.

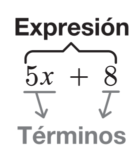
Por ejemplo, $(2, 4)$ es la solución de este sistema de ecuaciones y el punto de intersección en la gráfica.
$y = -x + 6$
$-2x + 4y = 12$

sistema de ecuaciones Dos o más ecuaciones que representan las restricciones en un conjunto compartido de variables.

Por ejemplo, estas ecuaciones forman un sistema de ecuaciones:
$x + y = -2$
$x - y = 12$

T

término Una parte de una expresión. Un término puede ser un número individual, una variable, o una variable y un número multiplicados.



Por ejemplo, la expresión $5x + 8$ tiene dos términos. El primer término es $5x$ y el segundo término es 8.